

# COMMODITIES AS AN ASSET CLASS THROUGHOUT THE FINANCIAL CRISIS

by

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## **Abstract**

This project investigates the performance of commodities as an asset class from September 24, 2003 to June 30, 2011, in the context of its inclusion within a broader portfolio of equities and bonds. Specifically, we examined whether the Goldman Sachs Commodity Index (GSCI), a fully-collateralized index of commodity futures, performed better or worse than the equity and bond marketplaces leading up to, during, and following the financial crisis of the late-2000s, and whether or not it provided any diversification benefits to a traditional portfolio.

Our findings were that the GSCI outperformed U.S. bonds but generally not U.S. stocks during the study period, that it was more volatile than both traditional asset classes, offered modest diversification benefits, especially after the crisis began, and that it fared worse than equities in a review of higher moments. Canadian equity investors would have found the GSCI more appealing in a portfolio context than U.S. equity investors would have during the study period, due to a more favourable return weak performance of the U.S. Dollar.

These results are in marked contrast to studies of commodity futures prior to the financial crisis, and provide a cautionary note for investors with respect to incorporating a basket of commodities that is heavily weighted in a particular commodity type, such as the GSCI, into their traditional portfolios. Nevertheless commodities clearly have maintained certain diversification benefits, especially during the worst of crisis where they have tended to outperform equities.

On the other hand, an extension of the study period to include the Dotcom crisis revealed that commodities offered substantial diversification benefits to a traditional portfolio during that

time. In addition, adding commodity futures to a portfolio of stocks and bonds significantly reduces downside risk, as measured by Value-at-Risk (VaR).

On balance, we recommend that a basket of commodity futures be considered for inclusion into a traditional portfolio with a long-term investment horizon.

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## Introduction

Investors are generally familiar with traditional asset classes such as equities, bonds and real estate. Commodities, however, are less widely understood. The question as to whether commodities can even be considered a separate asset class is an intriguing one. A primary argument against the idea of commodities as an asset class is that the constituents within the commodities category are not necessarily economically related (Dennis Gartman 2010). For example, the factors that drive the price of natural gas, cocoa, and lean hogs are appreciably different from one another. On the other hand, it can be argued that the price dynamics of commodities share a commonality in that they are principally determined by global supply and demand forces. Furthermore, commodity price dynamics often differ with those that apply to the equity asset class, where prices are mainly driven by business fundamentals and future expected cashflows. In the case of bonds, a movement in interest rates affects all bond prices, regardless of type.

Our thesis will focus on the case for and against commodities as an asset class throughout the Late-2000s Financial Crisis (“the Financial Crisis”), based on empirical studies of correlation and risk diversification with other traditional asset classes in an asset allocation context. This research will also attempt to answer various questions relating to the investment qualities of commodities in the framework of portfolio management, and how these assets interact with the traditional asset classes of stocks and bonds. Specifically:

- 1) How risky are commodity futures relative to other traditional asset classes?
- 2) Do commodities continue to exhibit low or negative correlation with stocks and bonds during periods of crisis?
- 3) Have any new trends emerged? And if so, what is their meaning?



Answering these questions and other related concerns could provide a platform for increased direct institutional and retail investment into commodities, especially in the case where IPS restrictions currently prevent such exposure due to perceived riskiness. It may also provide a compelling case for private investors to diversify their own assets into commodities in order to protect their portfolio from market and inflation risk.

Previous studies have shown that a basket of commodity futures exhibit similar returns to U.S. equities, are negatively correlated with equities and bonds, and positively correlated with inflation (Erb & Harvey, 2006), (Gorton & Rouwenhorst, 2006). These studies concluded that a portfolio of commodity futures was indeed an asset class and that their inclusion into a broader portfolio with traditional assets could significantly reduce portfolio risk. On the other hand, these studies were completed prior to the Financial Crisis of the Late-2000s<sup>1</sup>. We will investigate whether the correlation and diversification effects have changed prior to, during, and after the financial crisis. Furthermore, if a change has occurred, we will investigate the possible causes and examine whether the benefits of including the commodity asset class into a traditional portfolio of assets still apply.

### *Commodity Futures*

A commodity futures contract is a standardized contract between a buyer and seller that stimulates the exchange of a specified commodity at a set price and quantity. The price is agreed upon as at the date of inception but not paid until the settlement date. As such, the price that is initially set will include an expectation of future spot prices. The future spot price is never known with certainty and as such, the investor who purchased the future risks gains or losses

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<sup>1</sup> Erb and Harvey (2006) examine historical returns up to and including May, 2004, and Gary and Rouwenhorst study data from July, 1959 to December, 2004 inclusive.

from unexpected movement in the spot price. As the futures contract reaches expiration, its price will converge to the spot price at maturity. Thus, an investor who has purchased a futures contract will profit if the spot price at maturity is greater than the futures price. As described by Gorton and Rouwenhorst (2006), Keynes (1930) and Hicks (1939) hypothesized the theory of normal backwardation, where a risk premium accrues, on average, to the buyers of future contracts in order to compensate them for this market risk.

For the purpose of analyzing commodities as an asset class, we will use commodity futures as opposed to spot prices. Gorton and Rouwenhorst (2006) showed that the historical inflation-adjusted performance of an index of commodity futures significantly exceeded that of spot commodity prices, despite strong, positive correlation. The authors explain that this difference is due to the risk premium: that commodity futures, unlike spot prices, rise with any risk-premium earned. Commodities also exhibit seasonality patterns that affect spot prices. However, as seasonality is foreseeable, an index of futures is unaffected. In addition, investing in commodity futures requires collateral, invested in treasury bills, which contributes a risk-free rate of return to the total return of the futures. Lastly, investment in commodities is generally in the form of futures contracts. Taking the above factors into account, analyzing commodity futures as an asset class as opposed to spot prices is to more truly reflective of the nature of commodities investing, and it provides a more provide a more meaningful comparison between commodities and traditional asset classes that earn a risk-premium.

## Literature Review

*Robert J. Greer – “What is an Asset Class, Anyway?”*

In order to properly classify commodities as an asset class, we began with a review of what an asset class actually is. According to Greer (1997), “an asset class is a set of assets that bear some fundamental economic similarities to each other, and that have characteristics that make them distinct from other assets that are not part of the class”. Consequently, it is easy to see the argument for commodities not being an asset class as the fundamental drivers that affect one commodity can be quite different from that of another.

Greer defines three broad classes of assets: capital assets, consumable/transformable assets, and store of value assets. Capital assets provide an ongoing source of value and include such assets as equities, bonds, and real estate. Equities offer the expectation of dividends, bonds give the expectation of interest payments and principal, and real estate offers an ongoing stream of operating income in addition to residual value. Moreover, each class has characteristics that distinguish it economically from each other. For instance, equities return the residual value of the asset while bonds have a prior claim to equity in a fixed amount.

The second broad category of assets is consumable and transformable assets. These assets can be transformed into another form, consumed, but does not yield an ongoing stream of value. Included in this category are physical commodities. Compared to capital assets, consumable and transformable assets are cannot be valued using net-present-value analysis. Global supply and demand are the main determinants of price for consumable and transformable assets. Another economically distinguishing characteristic is that consumable and transformable assets cannot be explained by the CAPM. Black (1976) mentions that commodity futures are not included in the market portfolio. As such, it is only logical that the model cannot explain this category of non-

capital assets. In this context, it is clear that commodity futures should be considered as an asset class.

The last broad category described by Greer is store of value assets. These assets cannot be consumed and does not generate income. Examples are fine art and currency. Investors will hold one currency over another if they feel that it will appreciate and represent a better store of value. Likewise, investors may hold currency or cash if they are uncertain about other asset classes.

*Gary Gorton & K. Geert Rouwenhorst – “Facts and Fantasies about Commodity Futures”*

Gorton and Rouwenhorst (2006) also studied the simple properties of commodity futures as an asset class. They sought to answer commonly raised questions about investment properties of commodity futures as an asset class by analyzing a long-term, monthly time series of an equally-weighted index of futures. Noting that futures positions are typically levered due to posted collateral being only a fraction of the notional value of the contract, they controlled for leverage by assuming full collateralization of the index, prior to calculating returns. This allowed them to offer a more meaningful comparison between the performance of commodity futures and the performance of traditional asset classes.

As a first task, the authors compared the real returns of the fully-collateralized futures asset class to the real returns of an equally-weighted portfolio of commodity spot prices from 1959 to 2004. They concluded that the fund of collateralized futures dramatically outperformed the spot commodities, and that both indices outpaced CPI. They also noted the high positive correlation and strong divergence of commodity futures and spot values. Since commodity futures rise with the risk-free rate plus any risk premium earned, and returns do not include

expected future movements in the spot price, the accompanying returns are subject to a different trend than spot returns are. They further recognize that by using equally-weighted indices, they are applying an embedded trading strategy which would outperform a buy-and-hold strategy if returns aren't independently distributed over time (as shown by Blume and Stambaugh (1983) and Roll (1983)). Their research demonstrated that a buy-and-hold strategy for spot commodities radically lowers the spot index returns versus a strategy of frequent rebalancing, and that spot commodity returns did not keep pace with CPI, over the 45-year study period.

The authors also found that commodity futures had a similar cumulative real return to the S&P 500, albeit with long interim periods (1970s and 1990s) of wildly dissimilar performance, and that both asset classes outperformed bonds by a wide margin. Furthermore, the calculated risk premium was similar for stocks and commodities and was more than double that of bonds. Intriguingly, collateralized commodity futures had positive skewness and a large excess kurtosis, implying that the return distribution of such futures deviate from the normal distribution in that it has fatter tails, with especially more weight on the right tail. Stocks also had positive excess kurtosis, but had negative skewness and thus more weight on the left tail. Thus, stocks appear to have more downside risk than commodity futures. In addition, stocks were shown to have had a higher volatility than commodity futures.

A look at correlations between the traditional asset classes and commodity futures revealed that, over quarterly, annual and 5-year holding periods, collateralized commodity future returns were negatively correlated with those of stocks and bonds, with a small (near-zero) positive correlation for monthly holding periods. The hypothesis that correlations of futures with stocks and bonds is zero could not be rejected, however the results indicated that commodity futures are effective in diversifying portfolios composed of traditional asset classes, with

increasing effectiveness over longer holding periods. Gorton and Rouwenhorst also discovered that commodity futures have a positive correlation with inflation (CPI), and that the correlation increased over longer holding periods. This suggests that over the long-term, commodities have inflationary properties.

Commodity futures also offer diversification benefits due to their return behaviour throughout the business cycle. Whereas stocks and bonds outperform commodity futures in the late stages of recession and early stages of expansion, the reverse is true during the late stages of expansion and early stages of recession. Furthermore, whenever stock and bond returns are below their average for a business cycle, it is precisely the time that commodity future returns are positive and outperform the traditional asset classes.

Noting that in an efficient market, a profitable trading strategy of selecting commodity futures according to the size of their basis (difference between futures price and current spot price) must be due to variation of required risk premiums or changing risk over time, the authors established such a trading strategy. They found that a strategy of rebalancing monthly a portfolio towards a low basis outperformed the equally weighted index by about the same amount that the strategy of rebalancing a portfolio towards a high basis underperformed. Furthermore, the low basis strategy outperformed the high basis strategy by 10% per annum on average. They concluded that “the futures basis seems to hold important information about the risk premium of individual commodities”.

Finally, Gorton and Rouwenhorst studied both the real return history of commodity futures from the perspective of foreign investors, and investigated whether an investment in companies that produce commodities is a substitute for an investment in commodity futures.

They discovered that UK and Japanese investors in commodity futures would have fared similarly over time to U.S. investors, and that commodity equities are not an appropriate substitute for collateralized futures of the same commodity. They also discovered that commodity equities had a stronger positive correlation with the S&P 500 than they did with the commodity futures.

Through their study, Gorton and Rouwenhorst demonstrated that an equally weighted index of commodity futures would have greatly outperformed spot commodities from July, 1959 to December, 2004. They also showed that a long position in such an index had a positive risk premium that exceeded that of bonds and matched that of equities, that the historical risk of an investment in commodity futures had been relatively low in terms of variation and downside risk, and that commodity futures provided substantial diversification benefits to a portfolio of stocks and bonds due to its negative correlation to the traditional asset classes over the long-term.

## **Data and Methodology**

### *The Goldman Sachs Commodity Index (GSCI)*

In order to measure the performance of commodities, we used the S&P GSCI Total Return Index as a proxy. This index is unlevered and takes long-only positions in commodity futures contracts. Although leveraged commodity indices exist, we assume that an unlevered index would be more representative of a typical commodities investment style by an average investor or by institutional investors constrained by IPS restrictions on leverage. It is fully collateralized meaning that for every \$1 invested in a futures contract, there is \$1 invested in T-bills. This is consistent with the actual dynamics of futures contract trading as some collateral is always posted with the exchange when an investor takes a position; the difference in this case is that leverage is removed by matching full collateral against the notional value, instead of just a fraction thereof. Each month, the collateralized commodity futures investment is rolled forward to the next month.

The GSCI is composed of 24 commodities that are included based on liquidity and weighted according to their corresponding world production quantities. The index includes six energy products, five industrial metals, eight agricultural products, three livestock products, and two precious metals. This process therefore reflects the liquidity of the market and the futures contracts that would be available to an investor. In contrast to an equal-weighted index, a production-weighted index avoids issues relating to a lack of futures contracts for thinly traded commodities.



**Table 1 – S&P GSCI Index Components (as of 12/31/2010)**

<b>Energy</b>		<b>Industrial Metals</b>		<b>Agriculture</b>	
Crude Oil	34.6%	Aluminum	2.4%	Wheat	3.8%
Brent Crude	14.3%	Copper	4.0%	Kansas Wheat	0.8%
Unleaded Gasoline	4.3%	Lead	0.5%	Corn	4.3%
Heating Oil	4.5%	Nickel	0.8%	Soybeans	2.7%
Gas Oil	5.5%	Zinc	0.6%	Cotton	1.8%
Natural Gas	3.2%		8.3%	Sugar	2.8%
	66.5%			Coffee	1.0%
				Cocoa	0.3%
					17.4%
<b>Precious Metals</b>		<b>Livestock</b>			
Gold	2.9%	Feeder Cattle	0.4%		
Silver	0.5%	Live Cattle	2.5%		
	3.4%	Lean Hogs	1.4%		
			4.3%		

### *Data*

To compare the performance of the GSCI, the S&P 500 was used as a proxy for equities and the iShares Barclays Aggregate Bond Fund was used as a proxy for bonds. In addition, the National Council of Real Estate Investment Fiduciaries Index (NCREIF Index) was used as a proxy for real estate. As this index is reported only quarterly, the GSCI was compared based on a quarterly interval as well. Our analysis included a look at inflation and whether the positive correlation with commodities still existed after the financial crisis.

The time period chosen for our analysis was September 24, 2003 to June 30, 2011. Notably, there is a data limitation for the Barclays Aggregate Bond Fund prior to this date. Nevertheless, this period includes the effects of the 2007-2009 financial crisis with roughly the same length of time before and after the event. For the analysis, this period was separated into three sub-intervals: a pre-crisis period (09/24/2003 to 06/29/2007), a crisis period (06/29/2007 to 03/31/2009), and a post-crisis period (03/31/2009 to 06/30/2011). These intervals were chosen so

that we are able to analyze the performance of commodities before, during, and after the financial crisis.

For the NCREIF Index and CPI, the period chosen for analysis was June 30, 2003 to June 30, 2011. Similar to the analysis with respect to equities and bonds, this period was split into three sub-intervals: a pre-crisis period (06/30/2003 to 06/30/2007), a crisis period (06/30/2007 to 03/31/2009), and a post-crisis period (03/31/2009 to 06/30/2011). As reporting for the NCREIF and CPI was less frequent, we looked at a slightly longer period than that used for the S&P 500 and Barclays Aggregate Bond Fund.

## Analysis

### *Comparison with Equities and Bonds*

For the full and sub-periods, returns and volatility for the GSCI Index, S&P 500 Index, and Barclays Aggregate Bond Fund were calculated:

**Table 2 – Full Period, 09/24/2003 – 06/30/2011**

	S&P GSCI Total Return Index	S&P 500 Index	Barclays Aggregate Bond Index
Annualized Return	3.24%	3.52%	0.59%
Annualized Standard Deviation	26.98%	21.10%	5.97%
Sharpe Ratio	-0.03	-0.02	-0.57
<i>t</i> -Statistic	5.31	7.38	4.39

A look at the full period from 09/24/2003 to 06/30/2011 indicates that the return of the GSCI was roughly similar to that of the S&P 500 but with a higher amount of volatility. Both of these indices outperformed the Barclays Aggregate Bond Index. However, all asset classes

underperformed the risk-free rate of 3.97%, as measured as the average 10 year U.S. Government treasury yield for that period.

**Table 3 – Pre-Crisis Period, 09/24/2003 – 06/29/2007**

	S&P GSCI Total Return Index	S&P 500 Index	Barclays Aggregate Bond Index
Annualized Return	11.64%	11.16%	-0.93%
Annualized Standard Deviation	23.12%	10.67%	3.89%
Sharpe Ratio	0.31	0.63	-1.39
<i>t</i> -Statistic	15.51	32.20	-7.38

During the pre-crisis period, the volatility of the S&P 500 was less than half of the volatility of the GSCI. With much higher volatility and incrementally higher return, the idea of holding commodities within a broader portfolio ostensibly loses appeal. However, before ruling out the feasibility of commodities as an asset class, we must examine the correlations which could reveal potential diversification benefits.

**Table 4 – Crisis Period, 06/29/2007 – 03/31/2009**

	S&P GSCI Total Return Index	S&P 500 Index	Barclays Aggregate Bond Index
Annualized Return	-25.62%	-30.31%	1.79%
Annualized Standard Deviation	36.62%	36.20%	10.02%
Sharpe Ratio	-0.80	-0.94	-0.20
<i>t</i> -Statistic	-14.71	-17.61	3.76

Interestingly, during the crisis period, the volatility of the GSCI was almost identical to that of the S&P 500. However, the S&P 500's return was lower than that of the GSCI. Both the S&P 500 and GSCI underperformed the Barclays Aggregate Bond Index which returned 1.79%. During this period, the volatility of the Barclays Index increased substantially as well, demonstrating the ferocity of the crisis and its impact on all asset classes.

**Table 5 – Post-Crisis Period, 03/31/2009 – 06/30/2011**

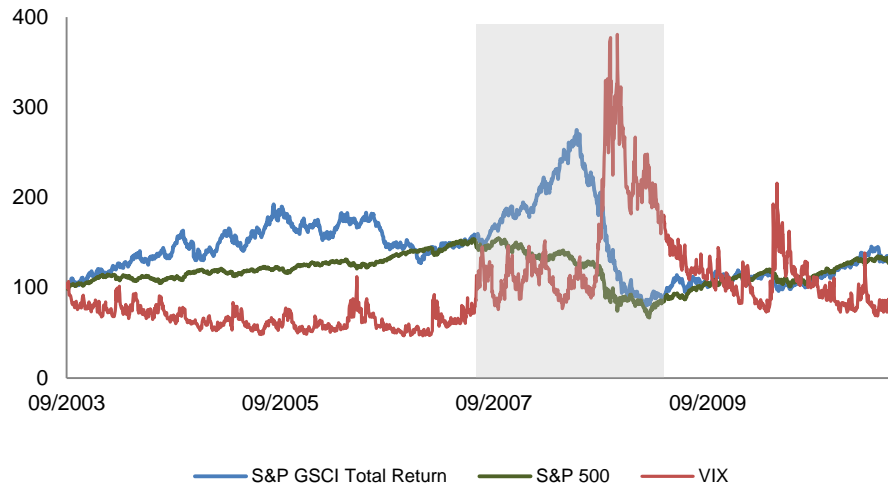
	S&P GSCI Total Return Index	S&P 500 Index	Barclays Aggregate Bond Index
Annualized Return	16.88%	25.00%	2.24%
Annualized Standard Deviation	23.95%	17.92%	4.41%
Sharpe Ratio	0.57	1.21	-0.24
<i>t</i> -Statistic	16.81	33.28	12.11

Finally, in the post-crisis period, we see that the GSCI, the S&P 500, and the Barclays Aggregate Index all garnered higher returns relative to the full and sub-periods. The volatility of the GSCI and Barclays indices returned to pre-crisis levels. In the case of equities, volatility also decreased, but was still significantly higher than the levels seen prior to the crisis. This could be due to the ongoing fragility of the world economy generally, and the U.S. economy specifically. In particular, U.S. unemployment has risen dramatically post-crisis and the U.S. housing market has yet to gain traction, likely contributing to a continued lack of overall demand. Against a backdrop of trillions of dollars of global wealth having vanished during the financial crisis, and a widespread sovereign debt crisis in Europe, the U.S. stock market has found itself under continued downward pressure.

More to the point, a look at the Volatility S&P 500 Index (VIX) in the chart below reveals that our post-crisis study period begins at a time where Implied Volatility of the S&P 500 was still extremely high, though decreasing rapidly, as the stock market finally rose from its crisis-induced low. Then, thirteen months later, in April, 2010, Greece bonds were downgraded to junk status, Portuguese and Spanish credit was also downgraded, and the stock market reacted with high volatility once again. Thus, the early stages of the post-crisis period and the later aftershock of the European credit downgrades contributed to massive volatility for stocks and have kept S&P returns from reverting to their pre-crisis, relative stability.

**Figure 1 – Relative Performance of the GSCI Total Return, S&P 500 & VIX Indices**

Sep 24 2003 = 100



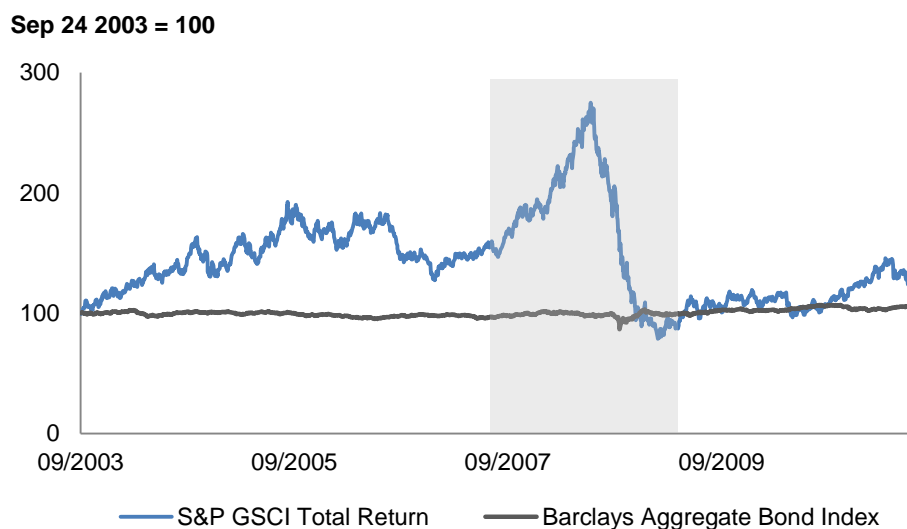
We also calculated the correlations between the GSCI and VIX, and the S&P 500 and VIX.

**Table 6 – VIX Index Correlation with S&P GSCI Total Return and S&P 500 Indices**

	GSCI	S&P 500 Index
Pre-crisis 09/24/2003 – 06/29/2007	-0.0212	-0.7864
Crisis 06/29/2007 – 03/31/2009	-0.2431	-0.7991
Post-Crisis 03/31/2009 – 06/30/2011	-0.4134	-0.7363
Full Period 09/24/2003 – 06/30/2011	-0.2056	-0.7364

Prior to the crisis, swings in volatility as measured by the VIX index were uncorrelated with the returns of the GSCI and bore a strong negative correlation with the S&P. Once the crisis began, however, high volatility became increasingly correlated with negative returns for not only the S&P 500, but for the GSCI as well. Thus, from the crisis onwards, the GSCI has provided less function as a risk-diversifying investment.

**Figure 2– Relative Performance of the GSCI Total Return Index and Barclays Aggregate Bond Index**



*Correlations*

Given that the returns of the S&P GSCI Total Return Index are comparable to those of the S&P 500 but with greater volatility, it begs the question as to why anyone would want to invest in commodities. To begin to answer this query, we examine the correlation and diversification effects of adding the GSCI Commodity Index to a portfolio of traditional assets.

**Table 7 – S&P GSCI Total Return Index Correlation with Stocks and Bonds**

	S&P 500 Index	Barclays Aggregate Bond Index	Stocks and Bonds Correlation
Pre-crisis 09/24/2003 – 06/29/2007	-0.0185	0.0469	-0.0704
Crisis 06/29/2007 – 03/31/2009	0.3216	0.0982	-0.0474
Post-Crisis 03/31/2009 – 06/30/2011	0.5835	-0.2316	-0.3071
Full Period 09/24/2003 – 06/30/2011	0.2943	0.0179	-0.0989

**Table 8 – S&P GSCI Total Return Index Correlation with Inflation and the NCREIF Index**

	Inflation (CPI)	Inflation (Core)	NCREIF Index (Quarterly)
Pre-crisis 06/30/2003 – 06/30/2007	0.2556	-0.1746	-0.3237
Crisis 06/30/2007 – 03/31/2009	0.5647	0.1966	0.6864
Post-Crisis 03/31/2009 – 06/30/2011	0.2786	0.3717	0.0641
Full Period 06/30/2003 – 06/30/2011	0.4285	0.3503	0.3222

Prior to the financial crisis, the small negative correlation between the GSCI and equities and the near-zero correlation with bonds meant that the GSCI could help reduce overall portfolio volatility. In addition, the positive correlation of 0.2556 with inflation meant that the GSCI Index was helping, modestly, to preserve purchasing power. However we see that the GSCI is negatively correlated with core inflation, a measure that excludes food and energy. Because real estate was increasing steadily while commodities exhibited volatility, the GSCI had a negative correlation with the NCREIF Index.

During the crisis, however, some of these relationships have changed substantially. Most notably, the correlation of the GSCI Index with the S&P 500 surged from -0.0185 to 0.3216 while the correlation with the Barclays Aggregate was still relatively low. The correlation between the S&P 500 and Barclays Aggregate also remained relatively constant and negatively correlated. Furthermore, the economy contracted while asset prices fell. Thus, the correlation with inflation increased during the crisis, and the correlation with real estate went from negative to a strong positive correlation.

In the post-crisis period, the performance of the GSCI became even more strongly correlated with the S&P 500 and more negatively correlated with the Barclays Aggregate. Similarly, the correlation between the S&P 500 and Barclays Aggregate became more negatively

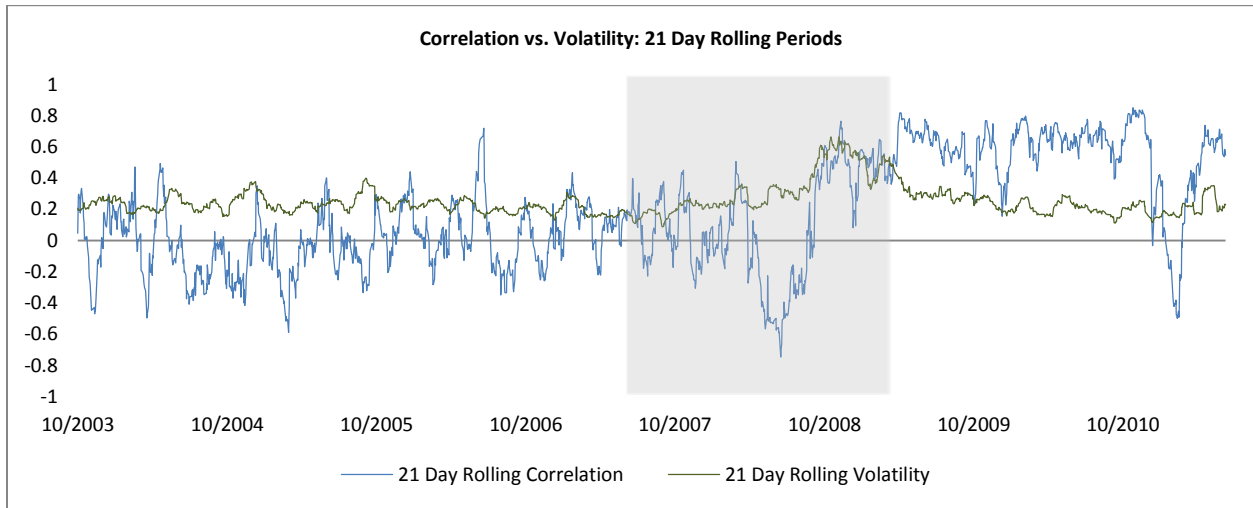
correlated as well. Taking into account the fact that the GSCI had twice the amount of volatility and roughly the same return as the S&P 500 in the post-crisis period, this high correlation inevitably dampens the diversification benefits of the GSCI Index. During this time the GSCI reverted to its pre-crisis correlation level with CPI but did not return to a negative correlation with core inflation. Instead of returning to being negatively correlated with core inflation, the GSCI increased its correlation instead. This time, we find a small, positive correlation with the NCREIF suggesting that the GSCI was moving independently of real estate.

#### *Rolling Period Volatility and Correlations*

In an attempt to gain insight into the behaviour of correlation and volatility, we calculated 21 day rolling standard deviations and correlations between the GSCI and the S&P 500 for the full period. This frequency was selected based on there being 252 trading days and 12 months in a year (252 divided by 12 yields 21). However, there were no conclusions that could be drawn with respect to the relationship between the rolling correlation and standard deviations. Nevertheless, it is a striking image of how the average correlation between stocks and commodities has increased dramatically from prior to the crisis to present day.



Figure 3 – 21 Day Rolling Correlations and Volatilities



Following the collapse of Lehman Brothers in September, 2008, and then again at the very bottom of the S&P 500 market in March, 2009, we see the dramatic increase in correlation between the GSCI and S&P 500. To verify this change in correlation levels, we performed a Paired Two-Sample  $t$ -Test for Equal Means (Snedecor & Cochran, 1989). The trend in correlation seems to change most noticeably at September 9, 2008, coinciding with the onset of the bankruptcy of Lehmann Brothers<sup>2</sup>, and so we chose that date as the break-point between the two samples of equal size.

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<sup>2</sup> Lehman Brothers shares fell 45% after state-owned Korean Development Bank pulled out of talks to invest \$6 billion into the company. As reported by The Sunday Times

**Table 9 – Two-Sample t-Test for Equal Means: Correlation Between GSCI & S&P 500 Pre- and Post-Lehman**

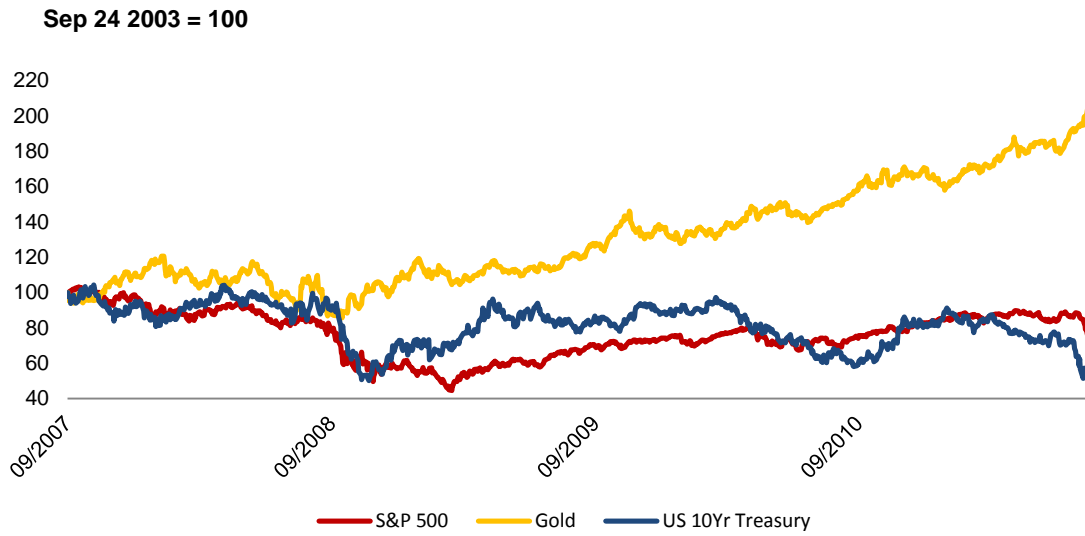
	Pre-Lehman Brothers Bankruptcy 11/14/2005 – 9/8/2008	Post-Lehman Brothers Bankruptcy 9/9/2008 – 6/30/2011
Mean correlation	0.0091	0.5297
Observations	709	709
Pearson correlation	0.1243	
Hypothesized Mean Difference	0	
Degrees of Freedom	708	
<i>t</i> Stat (T)	-44.073	
<i>t</i> Critical two-tail ( $t_{0.025,708}$ )	1.6470	

We see that the mean correlation of the GSCI and S&P 500 in the 709 days prior to the collapse of Lehman Brothers is close to zero, at 0.0091. Afterwards, the sample mean correlation is much higher, at 0.5297. Since  $T < -t_{0.025,708}$  we reject the null hypothesis that the two means of the distribution are equal. The test confirms that a material change in correlation between the GSCI and S&P 500 occurred at or around the time of the collapse of Lehman Brothers.

As mentioned earlier, the GSCI is heavily weighted in energy commodities (66.5% of total index) in general and Crude Oil (34.6%) in particular, the prices of which are dictated largely by global expected supply and demand. When many trillions of dollars of global wealth were wiped out during the depths of the Financial Crisis and global economic outlook worsened immediately, the price of oil entered into a steep decline alongside (and surpassing that of) equity values. With each bit of positive and negative economic news since that time, it is likely that investor expectations of both equity and oil price performance have hinged on the same concept: that economic growth expectations drive value above all else.

An additional, related reason for the increased correlation between commodities and equities post-crisis could be a higher level of risk aversion among investors. The idea that investor behaviour has followed an indiscriminate, “risk-on, risk-off” pattern of buying and selling has often appeared in the financial press (where “risk-on” is synonymous with a bull market and “risk-off” is synonymous with a bear market). The idea is that, as the Financial Crisis worsened and eventually segued into repeated aftershock crises, it is plausible that frequent, renewed economic pessimism has led to investors entering and exiting the financial markets en masse. The above *t*-test provides support for this conclusion as correlations increased dramatically following the Lehman Brothers bankruptcy, and have stayed high ever since. A companion idea to “risk-on, risk-off” is that of “flight to safety,” where investors in risky assets like stocks and commodities flee into “safe” assets like U.S. Treasuries and gold. We can see below that the stock market and U.S. 10-year Treasury yields have both stayed at or below pre-crisis levels, and have even begun to move more often in the same direction. This suggests that as value has been removed from the equity market, demand for relatively safe U.S. bonds has increased, leading to high prices and low yields. In the same fashion, demand for gold, long-considered a “safe” asset during turbulent economic periods, has also led to a rise in prices.

Figure 4– Relative Performance of the S&P 500, Gold Spot Price and 10-Year U.S. Treasuries



### *ETFs and High Frequency Trading*

We believe that this increase in correlation is due to indexation, ETFs, and high frequency trading that grew in popularity in the late-2000s. As discussed by Sakoui and Kaminska (2010), when so-called indexed ETFs are purchased or sold, the underlying constituents get bought and sold as well. As such, during times of market distress, investors may sell their indexed ETFs regardless of the asset classes tracked by that ETF. Thus, even if commodities futures were not directly impacted by market distress, the sale of commodity-linked ETFs would cause the underlying constituents (commodities futures in this case) to be liquidated and prices to decrease. High frequency trading, which allows investors to buy and sale securities in mere seconds helps facilitate the effects mentioned above.

## Higher Moments

In order to gain a further insight into the distribution of returns, we looked at the skewness and kurtosis of the three asset classes:

**Table 10 – Comparison of Skewness**

	S&P GSCI Total Return Index	S&P 500 Index	Barclays Aggregate Bond Index
Pre-crisis 09/24/2003 – 06/29/2007	0.1581	-0.1735	-0.0787
Crisis 06/29/2007 – 03/31/2009	-0.1476	0.1845	-2.2530
Post-Crisis 03/31/2009 – 06/30/2011	-0.2104	-0.1910	-0.4890

**Table 11 – Comparison of Kurtosis**

	S&P GSCI Total Return Index	S&P 500 Index	Barclays Aggregate Bond Index
Pre-crisis 09/24/2003 – 06/29/2007	3.3962	3.9658	3.6931
Crisis 06/29/2007 – 03/31/2009	4.3725	7.2861	37.7039
Post-Crisis 03/31/2009 – 06/30/2011	4.0313	4.5479	5.1299

In general, positive skewness is desirable in financial assets it describes an asymmetry with greater weight in the right tail; this translates into more frequent, higher realizable returns than would otherwise be seen in a symmetrical return distribution. While the Barclays Aggregate Bond Index maintained its negative skewness in all periods, the GSCI and S&P 500 reversed their skewness during the financial crisis. In other words, during the financial crisis, the GSCI Index had disproportionate, negative returns compared with the period prior, whereas the S&P 500 encountered larger positive returns than before. Thus, during times of distress to its component assets, the GSCI is more likely than the S&P 500 to experience outlying negative returns that are generally not encountered in normal periods. This is a finding that reduces the creditability of commodities as a diversifying asset class. Consistent with previous results, it

appears that the dynamics of commodities as an asset class may have changed subsequent to the financial crisis.

Kurtosis is a measure of “peakedness” and a higher value represents that the distribution is likely to have more extreme values (both negative and positive) than a normal distribution. In general, a lower kurtosis is desired as it translates into a lower likelihood of extreme outcomes. It is expected that the shock of the financial crisis would cause all asset classes to realize a higher kurtosis. The kurtosis of the S&P 500 increased more substantially than that of the GSCI Index while the Barclays Aggregate jumped the most. This is consistent with the previous analysis of standard deviation where the volatility of the S&P 500 more than doubled whereas the GSCI’s increased by roughly 30%.

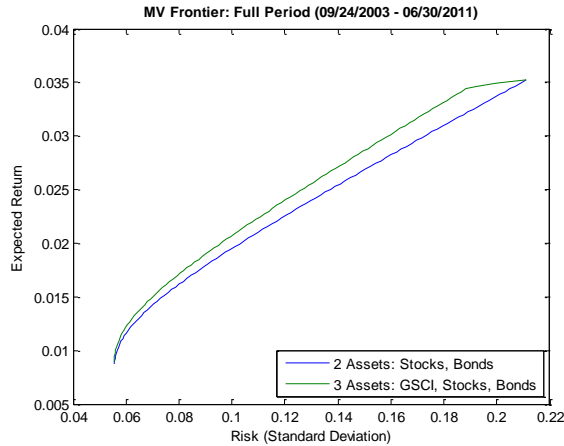
Following the crisis, the S&P 500 and the GSCI Index both exhibit similar levels of negative skewness and positive kurtosis. Prior to the crisis, the GSCI Index displayed a favourably positive skewness but it appears this advantage has now disappeared. The combination of the S&P 500’s comparable return at a lower risk and higher kurtosis signal that the equities asset class is making a more consistent recovery than the GSCI Index. From a diversification perspective, inclusion of a commodities index can still provide lowered volatility but at much less pronounced of an effect than prior to the financial crisis.

### *Mean Variance Efficient Frontiers*

To gain further insight into the portfolio diversification effects of commodities, we looked at the mean-variance efficient frontier with only stocks (represented by the S&P 500) and bonds (represented by the Barclays Aggregate Bond Index) and then again with the S&P GSCI

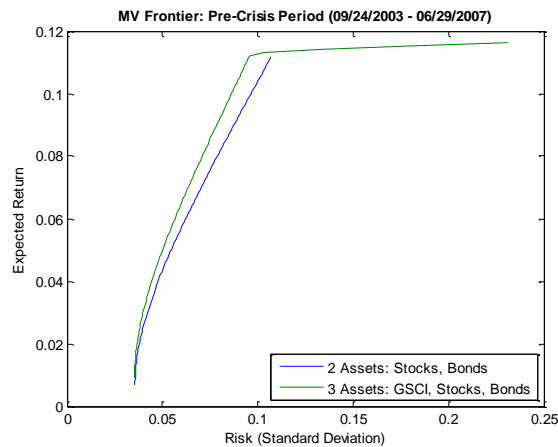
Total Return Index included. Once again, the analysis was conducted based on the full period and also the sub-periods.

**Figure 4 – Full Period Mean-Variance Efficient Frontier**



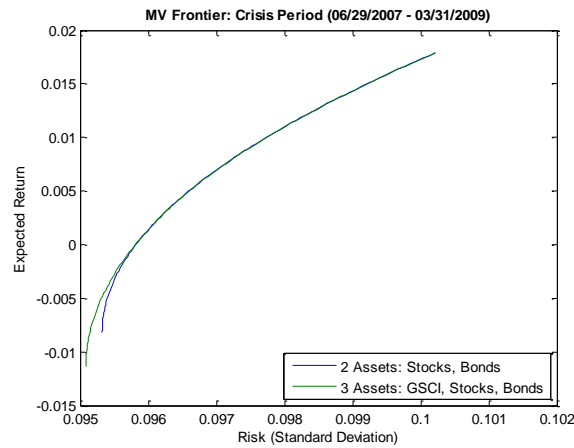
Taking the full period into perspective, the three asset frontier is higher than the two asset frontier at all points. At every level of risk, inclusion of the GSCI Index can increase expected returns. Put another way, the addition of the GSCI Index can help generate the same expected return with less risk. This is consistent with the findings of Jensen et al. (2002), who showed that commodity futures provided a higher rate of return when added to a traditional portfolio.

**Figure 5 – Pre-Crisis Mean-Variance Efficient Frontier**



As shown in Figure 5, in the pre-crisis period, the inclusion of the GSCI Index shifts the efficient frontier to the left representing a decrease in risk for certain levels of returns. The addition of the GSCI also allows for a higher expected return overall but this increment comes with a greater amount of risk as well. Because the GSIC had a slightly higher return than the S&P 500, there are additional points available on the frontiers that are not obtainable with just the two assets of stocks and bonds. However, this marginally higher point of expected return comes with substantially higher risk and as such may not be a feasible choice for most investors.

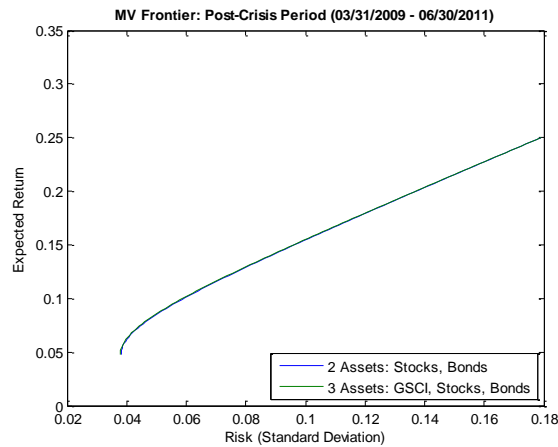
**Figure 6– Crisis-Period Mean-Variance Efficient Frontier**



During the crisis, the efficient frontier with two assets is nearly identical to that of the three asset frontier except at certain lower risk points. Taking into account the scale of the axis, these points represent an insignificantly minor range. Thus, during the crisis, we can say that the inclusion of the GSCI Index into a portfolio of equities and bonds no longer yields the same magnitude of diversification benefits as before.



**Figure 7 – Post-Crisis Mean-Variance Efficient Frontier**



Finally, subsequent to the financial crisis, it is apparent that the efficient frontier with three assets is entirely identical to that with two assets. Even the previously more efficient points between the 0.095 and 0.096 standard deviation intervals (as shown in Figure 6 – Crisis-Period Mean-Variance Efficient Frontier) have disappeared. As such, despite the less than one correlation with the S&P 500, the lower expected return and higher standard deviation of the GSCI Index has made it an obsolete asset class in terms of portfolio allocation.

### *Commodity Index Decomposition*

The GSCI index is heavily weighted according to energy commodities, holding 66.5% in various oil and gasoline products as at December 31, 2010. Thus, GSCI returns are disproportionately correlated with the returns of energy commodities. Crude oil, for instance, accounts for nearly half of the entire index, so it is no surprise to discover that the correlation between the index itself and the All Crude Oil sub-index is 96.3% for the study period. Conversely, the correlation between the GSCI and its Non-Energy sub-index, which accounts for 33.5% of the index, is merely 63.9% for the same period.

### *S&P GSCI Index vs. S&P TSX Composite*

With the performance of the GSCI throughout the Financial Crisis so heavily influenced by energy commodities, we extended our analysis to the Canadian equity market, which is very heavily weighted by energy stocks. We thus compared the S&P GSCI Total Return Index (adjusted for currency) to the S&P TSX Composite.

**Table 12 – Full Period, Comparison with the S&P TSX Composite**

	S&P GSCI Total Return Index (Adj.)	S&P 500 Index	S&P TSX Composite
Annualized Return	8.25%	3.52%	7.39%
Annualized Standard Deviation	33.32%	21.10%	19.84%

**Table 13 – Pre-Crisis Period, Comparison with the S&P TSX Composite**

	S&P GSCI Total Return Index (Adj.)	S&P 500 Index	S&P TSX Composite
Annualized Return	19.86%	11.16%	17.22%
Annualized Standard Deviation	26.37%	10.67%	11.49%

**Table 14 – Crisis Period, Comparison with the S&P TSX Composite**

	S&P GSCI Total Return Index (Adj.)	S&P 500 Index	S&P TSX Composite
Annualized Return	-32.41%	-30.31%	-23.36%
Annualized Standard Deviation	45.43%	36.20%	33.39%

**Table 15 – Post-Crisis Period, Comparison with the S&P TSX Composite**

	S&P GSCI Total Return Index (Adj.)	S&P 500 Index	S&P TSX Composite
Annualized Return	31.64%	25.00%	20.56%
Annualized Standard Deviation	32.50%	17.92%	16.40%

The S&P TSX Composite adjusted for currency generally underperformed the GSCI and S&P 500, with the exception of during the crisis period. This was due to the fact that the CAD was strengthening against the USD during the crisis. As such, the returns of the GSCI were amplified by the currency effects.

**Figure 8 – USD/CAD Exchange Rate**



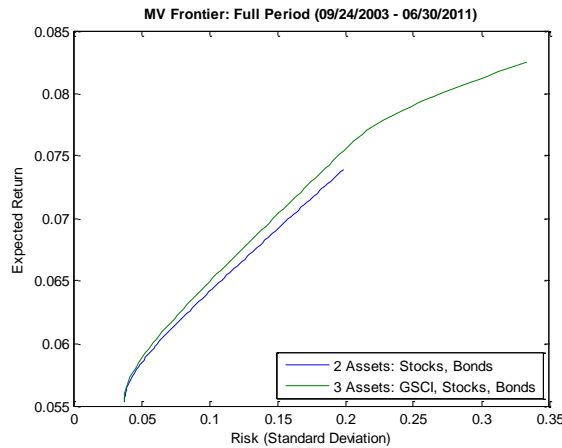
**Table 16 – S&P GSCI Total Return Index Correlation with S&P 500 and TSX**

	S&P 500 Index	S&P TSX Composite
Pre-crisis 09/24/2003 – 06/29/2007	-0.0185	0.3922
Crisis 06/29/2007 – 03/31/2009	0.3216	0.5589
Post-Crisis 03/31/2009 – 06/30/2011	0.5835	0.6857
Full Period 09/24/2003 – 06/30/2011	0.2943	0.5350

As shown in Table 14, the GSCI Index is far more positively correlated with the S&P TSX than it is with the S&P 500. This was expected, given that the TSX is heavily weighted

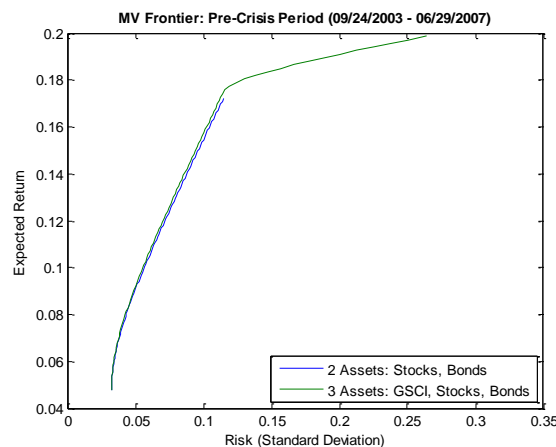
towards energy and materials (these sectors comprise approximately half of the TSX), however the magnitude of this positive correlation was eye-catching.

**Figure 9 – Full Period MV Efficient Frontier with TSX**

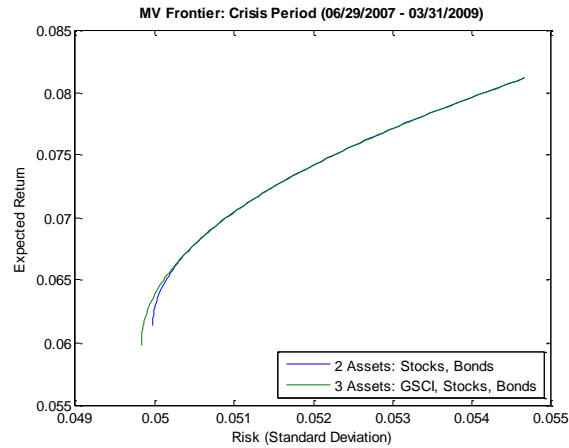


The addition of the GSCI, adjusted for currency into a two-asset context significantly improves the mean variance efficient frontier for a Canadian investor. This is not surprising given that the return of the GSCI (Adj) outperformed that of the TSX Index. Risk-tolerant investors stand to gain the most benefits due to the relatively high volatility of the GSCI (Adj), which accompanies its high return.

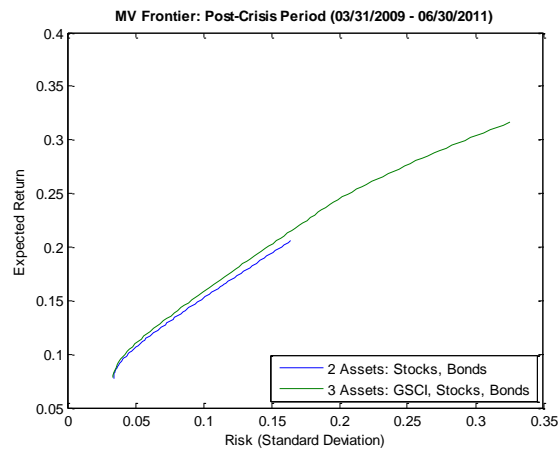
**Figure 10 – Pre-Crisis Period MV Efficient Frontier with TSX**



**Figure 11 – Crisis Period MV Efficient Frontier with TSX**



**Figure 12 – Post-Crisis Period MV Efficient Frontier with TSX**



The diversification benefits of the GSCI Index with the TSX are more pronounced than they were with the S&P 500, and conservative investors would have especially had reason to hold the GSCI during the crisis due to the better correlations. In the pre- and post-crisis periods, the superior return of the GSCI (Adj) to the TSX results in an improved frontier, especially for the more risk-tolerant individuals. This is attributable to the higher return and lower volatility of the GSCI compared to the TSX. Overall, however, the analysis indicates that commodities, purchased in U.S. dollars, as an asset class was more meaningful in a Canadian portfolio asset

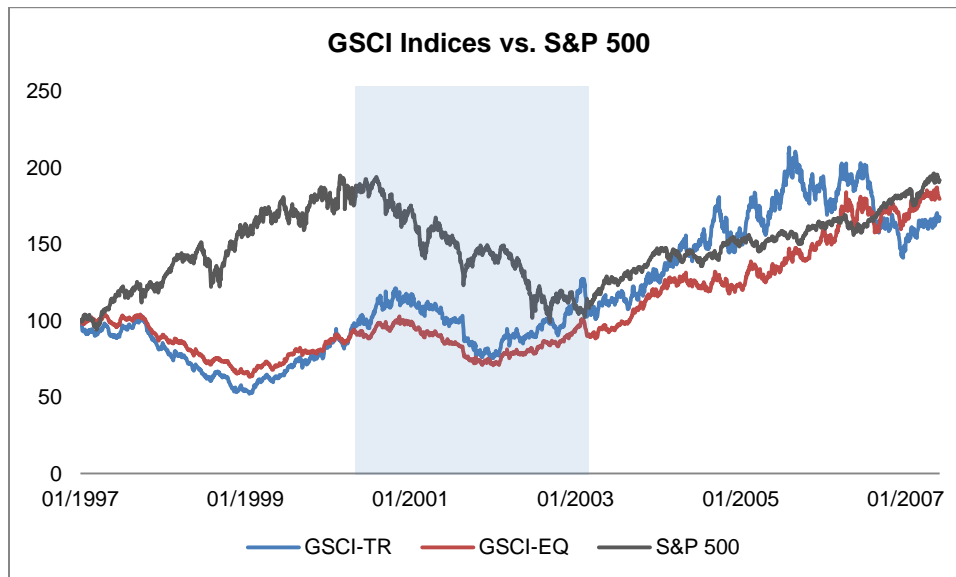
allocation context during the study period than it was in the U.S. portfolio asset allocation context.

## Extension of the Study Period

### *Inclusion of the Dotcom Crisis*

In order to confirm that the results observed from the “financial crisis” periods were not a one-time phenomenon, we extended our analysis backwards to another familiar event, the “Dotcom Crash”. In this scenario, we looked at the period from January 31, 1997 to June 29, 2007 (which excludes the financial crisis period). It should be noted that this scenario also includes the effects of the 2002 stock market downturn that was initiated by September 11 in addition to the crash of technology stocks after the Dotcom bubble burst in March 2000. The sub-intervals were as follows: a pre-Dotcom crisis period (01/31/1997 to 03/31/2000), a Dotcom crisis period (03/31/2000 to 01/31/2003), and a post-Dotcom crisis period (01/31/2003 to 06/29/2007).

**Figure 13 – Performance of the GSCI Total Return Index, GSCI Equal-Weighted Index, and S&P 500**



**Table 17 – Pre-Dotcom Crisis Period, 01/31/1997 – 03/31/2000**

	S&P GSCI Total Return Index	S&P GSCI Equal-Weighted Index	S&P 500 Index	JPM Bond Index
Annualized Return	-3.64%	-3.76%	22.56%	6.04%
Annualized Standard Deviation	16.93%	11.39%	19.47%	4.10%
Sharpe Ratio	-0.56	-0.84	0.86	0.07
<i>t</i> -Statistic	-6.07	-9.34	32.76	41.65

In the period leading up to the Dotcom crash and subsequent market downturn, we find that the GSCI Indices (both the production-weighted and equal-weighted) have significantly underperformed the S&P 500 and the JP Morgan Bond Index. This is primarily due to the “Great Commodities Depression” where commodities had been in a bear market for over two decades during the 1980’s and 1990’s. The S&P 500 on the other hand, was immune to this trend and posted an annual return of 22.6% during this period.

**Table 18 – Dotcom Crisis Period, 03/31/2000 – 01/31/2003**

	S&P GSCI Total Return Index	S&P GSCI Equal-Weighted Index	S&P 500 Index	JPM Bond Index
Annualized Return	9.83%	2.59%	-18.01%	10.15%
Annualized Standard Deviation	22.47%	12.61%	23.27%	4.03%
Sharpe Ratio	0.22	-0.19	-0.99	1.28
<i>t</i> -Statistic	11.67	5.48	-20.64	67.14

We begin to see some interesting results during the Dotcom crisis period where the returns of the GSCI Indices significantly outperformed the S&P 500 with similar or less levels of risk. It should be noted that the commodities bear market that was present during the pre-Dotcom period ended in the late-1990s. However, equity and commodity asset classes alike both underperformed the JPM Bond Index which also demonstrated minimal volatility. During this period, the GSCI production-weighted index

returned 9.8% compared to the equal-weighted index which returned 2.6%. This was due to further decline in agriculture and metals prices, while energy prices increased substantially.

**Table 19 – Post-Dotcom Crisis Period, 01/31/2003 – 06/29/2007**

	S&P GSCI Total Return Index	S&P GSCI Equal-Weighted Index	S&P 500 Index	JPM Bond Index
Annualized Return	9.24%	15.57%	13.64%	3.72%
Annualized Standard Deviation	23.23%	15.42%	11.97%	3.57%
Sharpe Ratio	0.21	0.73	0.77	-0.19
<i>t</i> -Statistic	13.26	33.67	37.97	34.80

In the post-Dotcom crisis period, the equal-weighted GSCI Index has now overtaken its production-weighted counterpart. Both GSCI Indices and the S&P 500 outperformed the JPM Bond Index with greater Sharpe ratios. Once again, we find that the decision on how a commodities index is weighted has a substantial impact on its performance. During this period, the production-weighted GSCI Index was heavily weighted in energy, a sub-index that, despite almost doubling in prices over the period, underperformed relative to metals. Metals tripled in value over the same time period, and agricultural commodities also increased in value, giving the equally-weighted GSCI a better return.

### *Correlations – Dotcom Crisis*

**Table 20 – Pre-Dotcom crisis, 01/31/1997 – 03/31/2000**

Correlation	GSCI-TR	GSCI-EQ	Equities	Bond
GSCI-TR	1.0000	-	-	-
GSCI-EQ	0.8657	1.0000	-	-
Equity	-0.0255	0.0264	1.0000	-
Bonds	-0.0652	-0.0892	0.0453	1.0000



**Table 21 – Dotcom Crisis, 03/31/2000 – 01/31/2003**

Correlation	GSCI-TR	GSCI-EQ	Equities	Bonds
GSCI-TR	1.0000	-	-	-
GSCI-EQ	0.8639	1.0000	-	-
Equities	0.0363	0.0822	1.0000	-
Bonds	-0.1123	-0.1177	-0.3210	1.0000

**Table 22 – Post-Dotcom Crisis, 01/31/2003 – 06/29/2007**

Correlation	GSCI-TR	GSCI-EQ	Equities	Bonds
GSCI-TR	1.0000	-	-	-
GSCI-EQ	0.8338	1.0000	-	-
Equities	-0.0910	0.0111	1.0000	-
Bonds	0.0789	0.0507	-0.0620	1.0000

Interestingly enough, we find that the correlations during the Dotcom crisis are much more resilient than that during the financial crisis. Specifically, both GSCI Indices retained their high correlation (over 0.80) throughout the full period. Furthermore, the GSCI Indices also exhibited a negative to near-zero correlation with the S&P 500 before, during, and after the Dotcom crisis. This finding is contrary to what was observed during the Financial Crisis periods. We believe that this difference can be explained by the growing popularity of indexation and high frequency trading that became more prominent in the late-2000s (Kaminska and Sakoui 2010). The result of which, was a rise in correlation across asset classes. As such, if we viewed only this event in isolation, it would appear that commodities as an asset class did in fact add significant diversification benefits and improved risk-reward in relation to other traditional asset classes.

### *Higher Moments – Dotcom Crisis*

As discussed previously, returns of assets are rarely normally distributed. In reality, we find that skewness and kurtosis exist. A positive skewness is preferred as it signifies that there is more weight in the right tail of the distribution. A lower kurtosis value is also desired as it translates into a lower likelihood of extreme outcomes (a more consistent return).

**Table 23 – Comparison of Skewness (Dotcom Crisis)**

Skewness	S&P GSCI Total Return Index	S&P GSCI Equal-Weighted Index	S&P 500 Index	JPM Bond Index
Pre-crisis 01/31/1997 – 03/31/2000	0.0063	0.2247	-0.2767	-0.1401
Crisis 03/31/2000 – 01/31/2003	-0.4469	-0.2994	0.2767	-0.3980
Post-Crisis 01/31/2003 – 06/29/2007	0.1080	-0.0646	-0.0034	-0.1023

Prior to the Dotcom crisis, we find that both GSCI Indices are positively skewed while the S&P 500 is negatively skewed. During the Dotcom crisis however, these indices and the S&P 500 all reversed their skewness. Going back to the analysis of returns and risk, we found that during this period, returns of the GSCI Indices were positive while the S&P 500 was negative. This can be interpreted as, during the crisis period, returns of the GSCI Indices were positive on the whole, but at times saw large negative returns. The reverse can be said for the S&P 500.

**Table 24 – Comparison of Kurtosis (Dotcom Crisis)**

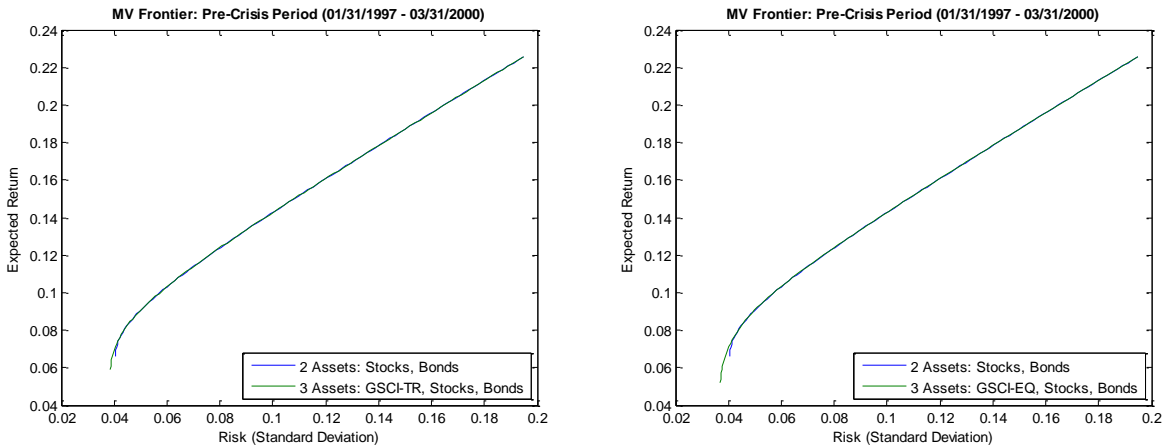
Kurtosis	S&P GSCI Total Return Index	S&P GSCI Equal-Weighted Index	S&P 500 Index	JPM Bond Index
Pre-crisis 01/31/1997 – 03/31/2000	1.6212	1.0408	3.2424	1.0669
Crisis 03/31/2000 – 01/31/2003	2.0766	0.9320	1.1840	0.9388
Post-Crisis 01/31/2003 – 06/29/2007	0.4244	0.4402	1.6816	1.7990

We found that before and after the Dotcom crisis, the GSCI Indices had lower kurtosis than the S&P 500. As mentioned previously, this is a desirable trait for asset classes as it means that returns are

less dispersed. It should be noted that the S&P 500's kurtosis went down significantly during the crisis period. However, the S&P 500 returned -18.0% during this period and the low kurtosis translates into the fact that most of the return observations were negative and of a similar magnitude.

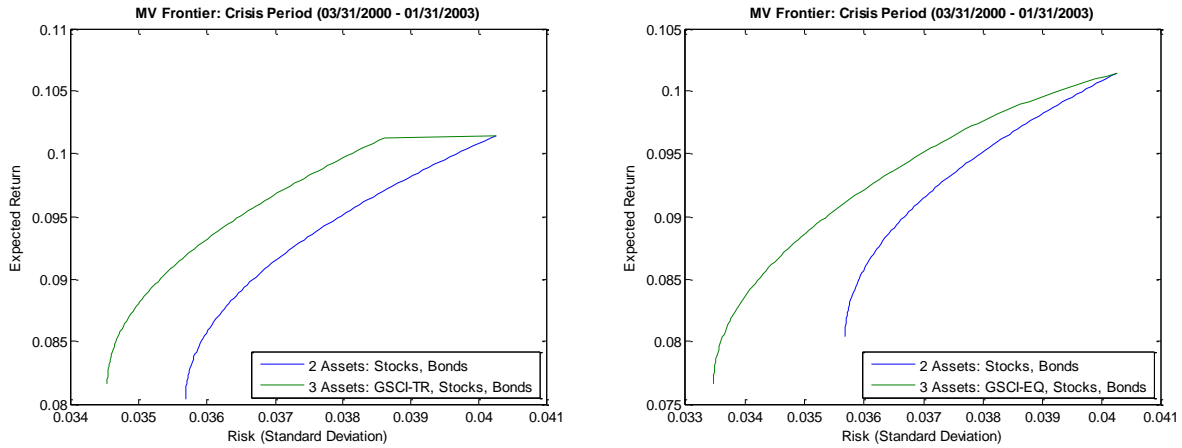
*Mean Variance Efficient Frontiers - Dotcom Crisis (Production-Weighted GSCI)*

**Figure 14 – Pre-Dotcom Crisis Period MV Efficient Frontiers**



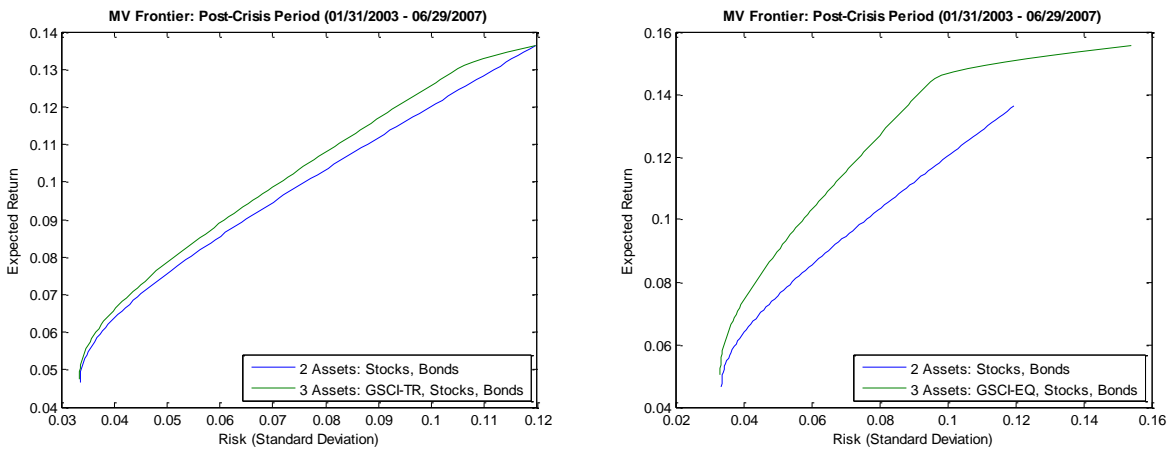
Consistent with the results from the analysis of return and volatility, we find that the efficient frontiers with two traditional asset classes (stocks and bonds) are not significantly improved by the introduction of the GSCI Index (regardless of it being production-weighted or equal-weighted) during the pre-crisis period. As mentioned earlier, this was due to the poor performance of commodities as a result of the commodities bear market.

**Figure 15 – Dotcom Crisis Period MV Efficient Frontiers**



In the Dotcom crisis period, the efficient frontiers are significantly improved with risk being reduced along most points of the frontier. This is consistent with past studies indicating that commodities offer diversification benefits when most desired (during the crisis period for example).

**Figure 16 – Post-Dotcom Crisis Period MV Efficient Frontiers**



Lastly, in the post-Dotcom crisis period, we still find the presence of diversification benefits associated with commodities. In this situation, not only is risk reduced on the frontier, but an investor can also achieve higher expected returns given the equal-weighted GSCI Index. In short, the benefits of adding commodities to a traditional portfolio are clearly significant in the context of a Dotcom crisis scenario.

These results are in sharp contrast to those observed in the Financial Crisis scenario and indicates that commodities are indeed a viable asset class under most conditions. However disappointing the asset allocation implications are during the Financial Crisis scenario, it is possible that this was a one-time phenomenon due to the magnitude of the shock. Although we do not hope for another stock market crisis, such an event could clarify whether the diversification benefits of commodities as an asset class still exist. In the absence of such an event, we can only base our study on past empirical studies and observations and conclude that commodities should be deemed a viable asset class that offers potential diversification benefits when properly integrated with a traditional portfolio.

## **Portfolio Value-at-Risk**

### *Measuring Value-at-Risk*

Value-at-Risk (VaR) is a measure of downside risk. The VaR of a portfolio at the 95% confidence level, for example, is the smallest amount of loss such that the probability of exceeding that amount is not greater than 5%.

First we constructed a hypothetical, “base” U.S. portfolio using the full extended study period of January 31, 1997 to June 30, 2011. We chose a 60%/40% weighting of equities (S&P 500 Index) and bonds (JPMorgan Funds – U.S. Aggregate Bond Fund), respectively. The skewness of this base portfolio was close to zero, at 0.0324, however the kurtosis of the historical sample of returns of this portfolio was 7.2369, indicating a non-normal distribution. We thus chose to use historical returns to estimate VaR, rather than use parametric or simulation methods.

### *U.S. Portfolio*

From our starting point of a balanced portfolio of 60% equities and 40% bonds, we replaced part of the equity holdings with the GSCI. The resulting allocation was 50% S&P 500, 40% JPMorgan Funds Aggregate Bond Fund, and 10% GSCI. We then repeated this exercise with the GSCI Equal-Weighted

Index. The results are below for the extended study period, for the DotCom Crisis study period (from pre- to post-, inclusive), and for the Financial Crisis study period (from pre- to post-, inclusive).

**Table 25 – Comparison of VaR (Full Study Period)**

	US 60%/40%	US 50%/40%/10% - GSCI	US 50%/40%/10% - GSCI-EQ
99% VaR	-2.16%	-1.94%	-1.87%
95% VaR	-1.20%	-1.03%	-1.01%

From the above, we see that the base portfolio can be expected to lose in one day at least 2.16% of its value, with probability 1% or less. As we add commodities, however, we see that the daily VaR is reduced at both the 99% and 95% confidence levels. This indicates that the introduction of commodities into a balanced, traditional portfolio, reduces downside risk.

We repeated this exercise for the Dotcom Crisis study period, and the Financial Crisis study period. The results are similar.

**Table 26 – Comparison of VaR (Dotcom Crisis Study Period)**

	US 60%/40%	US 50%/40%/10% - GSCI	US 50%/40%/10% - GSCI-EQ
99% VaR	-1.74%	-1.48%	-1.49%
95% VaR	-1.10%	-0.94%	-0.94%

**Table 27 – Comparison of VaR (Financial Crisis Study Period)**

	US 60%/40%	US 50%/40%/10% - GSCI	US 50%/40%/10% - GSCI-EQ
99% VaR	-3.04%	-1.70%	-1.65%
95% VaR	-1.57%	-0.59%	-0.57%

We can see that in terms of downside risk, the Financial Crisis (including the pre- and post- time periods) was much more severe than the Dotcom Crisis (including both pre- and post- time periods). In both eras, the inclusion of commodities significantly reduced Value-at-Risk. The equally-weighted GSCI is even more effective at this than the GSCI.

### *Canadian Portfolio*

For Canadian Investors, we created a 60%/40% equities/bonds “base” portfolio using the S&P/TSX Composite and the Government of Canada, 10-year, 3.25% fixed bond index. We then adjusted the GSCI indices by the USD-CAD exchange rate.

**Table 28 – Comparison of VaR for a Canadian Investor (Full Study Period)**

	Cdn 60%/40%	Cdn 50%/40%/10% - GSCI	Cdn 50%/40%/10% - GSCI-EQ
99% VaR	-2.10%	-2.05%	-1.77%
95% VaR	-1.20%	-1.15%	-1.02%

While the above results are similar to the earlier ones for a U.S. investor, it is interesting to note that introducing the GSCI has only a small effect in reducing downside risk for the Canadian portfolio, but introducing the equally-weighted GSCI has a significant such effect. This is in contrast to the U.S. portfolio, where the introduction of the GSCI provided significant risk-reduction benefits while the equally-weighted GSCI enhanced those benefits only modestly. This, again, is due to the fact that the TSX Composite and GSCI are both heavily weighted towards energy.

**Table 29 – Comparison of VaR for a Canadian Investor (DotCom Crisis Study Period)**

	Cdn 60%/40%	Cdn 50%/40%/10% - GSCI	Cdn 50%/40%/10% - GSCI-EQ
99% VaR	-1.85%	-1.77%	-1.49%
95% VaR	-1.12%	-1.04%	-0.97%

**Table 30 – Comparison of VaR for a Canadian Investor (Financial Crisis Study Period)**

	Cdn 60%/40%	Cdn 50%/40%/10% - GSCI	Cdn 50%/40%/10% - GSCI-EQ
99% VaR	-1.74%	-1.72%	-1.40%
95% VaR	-0.63%	-0.64%	-0.56%

Generally speaking, the Dotcom crisis was more severe with respect to portfolio downside risk to a Canadian portfolio than it was for a U.S. portfolio, and the reverse was true for the Financial Crisis study period. Nevertheless, for all investors adding commodities appears to reduce Value-at-Risk at both the 95% and 99% levels.

## **Conclusion**

Past studies have shown that commodity values “are driven by economic factors distinct from the factors determining [equity, bond and real estate] values,” (Greer, 1997) and as such, have natural, diversifying qualities versus traditional asset classes. Furthermore, it has been shown (Greer, 1997), (Gorton & Rouwenhorst, 2006), (Erb & Harvey, 2006) that various baskets of commodities demonstrate negative correlation with equities, bonds and real estate, especially over long-term holding periods (Gorton & Rouwenhorst, 2006), thus offering diversifying benefits to a traditional portfolio. Gorton and Rouwenhorst also determined that an equally-weighted basket of commodities had slightly lower volatility versus stocks, and that the distribution of commodity returns with the equally-weighted index had positive skewness relative to stock performance.



These studies, however, examined asset returns prior to the Financial Crisis of the Late-2000s, the worst financial crisis the world had seen since the Great Depression. Our analysis suggests that, prior to, during and after the Financial Crisis, commodities as an asset class (as represented by the GSCI) did not offer significant diversification benefits when added to a traditional portfolio of stocks and bonds. While commodities did have a small negative correlation with stocks and near-zero correlation with bonds prior to the crisis, and a relatively low positive correlation with stocks during the crisis, the correlation with equities increased dramatically during the post-crisis period (commodities have become strongly, negatively correlated with bonds during that time). A two-sample t-Test demonstrated that the average correlation between the GSCI and S&P 500 increased sharply after the onset of the bankruptcy of Lehman Brothers. We believe that some combination of higher investor risk-aversion, and the rise of ETFs and Indexation has contributed to the newfound, high correlation levels. Furthermore, commodities have largely underperformed stocks according to returns, the distribution of returns (skewness and kurtosis), risk-adjusted return (Sharpe Ratio) and volatility since the crisis period began. The GSCI does have a strong correlation with inflation, however, though not with core inflation. Thus, commodities as an asset class, as represented by the Goldman Sachs Commodity Index (GSCI), have provided only modest diversification benefits to a traditional portfolio throughout the Financial Crisis.

As noted earlier, the GSCI is highly correlated with the performance of energy commodities, especially Crude Oil. Following a price boom before the Financial Crisis, energy commodities performed particularly poorly as liquidity dried up leading up to, and sovereign debt troubles took hold in Europe following, the crisis. As such, the GSCI, 66.5% of which is

comprised of energy commodities, experienced particularly high volatility and low returns during lengthy periods throughout the entire study.

For a Canadian equity investor, an investment in commodities offered more appealing of an opportunity in the context of a portfolio, as the GSCI, adjusted for currency, had a far higher return than the S&P/TSX Composite (albeit with much higher volatility). The return was greatly influenced by the U.S. Dollar, which performed poorly versus the Canadian Dollar during and after the crisis. The correlations between the GSCI and S&P/TSX Composite (adjusted) are relatively high due to the heavy weighting of the Canadian marketplace to energy and materials equities.

By extending the study period to include the Dotcom Crisis, we determined that commodities performed well as an asset class, and offered proper diversification benefits to a portfolio. We also examined Value-at-Risk (VaR), and showed that for the extended study period as a whole, for the Dotcom Crisis study period and for the Financial Crisis study period, adding commodity futures to a balanced, traditional portfolio reduces overall downside risk. This is also true for Canadian investors, who from a VaR perspective stand to benefit even more greatly from the introduction of the equally-weighted GSCI.

In summary, whereas prior studies indicated that a basket of commodities could serve as a viable asset class in the portfolio context due to favourable risk-return characteristics and substantial diversification benefits, we found that from 9/30/2003 to 6/30/2011, the GSCI contributed only modestly toward the optimization of a traditional portfolio of assets, in large part due to its heavy weighting in energy commodities. An investor in the Canadian equity marketplace, however, would have found more reason than an investor in the American equity

marketplace to add the GSCI to a portfolio of stocks and bonds. This latter finding was due to substantially higher return of the GSCI (Adj.) over the S&P/TSX Composite and the weak performance of the U.S. Dollar during the study period. The Financial Crisis appears to be a new kind of crisis in terms of how it has impacted the benefits of including commodity futures into a portfolio. Commodity futures as an asset class appear to be a suitable addition to a prudently diversified portfolio. While some of our analysis strikes a cautionary note for investors with respect to incorporating a basket of commodities that is heavily weighted in a particular commodity type, such as the GSCI, investors can expect at least some diversification benefits and a reduction in downside risk, as measured by VaR. And of course if correlations between the GSCI and equities revert to lower levels as in the past, commodities would offer even greater benefits to a portfolio.

It is our recommendation that investment funds consider exposure to a basket of commodity futures, as, over the long-term and through crises, they have enhanced the performance of traditional portfolios of stocks and bonds. It will be important to identify whether the high correlations between the GSCI and equity indices in the post Financial Crisis time period represent a new reality, or if they are merely of a temporary pattern.

## Appendix A – S&P GSCI Commodities List

<b>Industrial Metals</b>	<b>Energy Products</b>
Aluminum	Crude Oil
Copper	Brent Crude Oil
Nickel	Heating Oil
Zinc	Gas Oil
Lead	RBOB Gasoline
	Natural Gas
<b>Agricultural Products</b>	
Wheat	<b>Livestock Products</b>
Kansas Wheat	Lean Hogs
Corn	Live Cattle
Soybeans	Feeder Cattle
Coffee	
Sugar	<b>Precious Metals</b>
Cocoa	Gold
Cotton	Silver

## Appendix B – S&P GSCI Contract Months (as of 2007)

1	Crude Oil	All	13	Cocoa	Mar, May, Jul, Sep, Dec
2	Brent Brude Oil	All	14	Cotton	Mar, May, Jul, Dec
3	Heating Oil	All	15	Aluminum	All
4	Gas Oil	All	16	Copper	All
5	RBOB Gasoline	All	17	Nickel	All
6	Natural Gas	All	18	Zinc	All
7	Wheat	Mar, May, Jul, Sep, Dec	19	Lead	All
8	Kansas Wheat	Mar, May, Jul, Sep, Dec	20	Lean Hogs	Feb, Apr, Jun, Jul, Aug, Oct, Dec
9	Corn	Mar, May, Jul, Sep, Dec	21	Live Cattle	Feb, Apr, Jun, Aug, Oct, Dec
10	Soybeans	Jan, Mar, May, Jul, Nov	22	Feeder Cattle	Jan, Mar, Apr, May, Aug, Sep, Oct, Nov
11	Coffee	Mar, May, Jul, Sep, Dec	23	Gold	Feb, Apr, Jun, Aug, Dec
12	Sugar	Mar, May, Jul, Oct	24	Silver	Mar, May, Jul, Sep, Dec

## Appendix C – MatLab Code for the Analysis of Correlation, Skewness, and Kurtosis

```
clc
close all
clear all
format compact

[Returns,Titles1]=xlsread('C:\Users\DW\Dropbox\Commodities Thesis\Thesis (Data and
Outline)\MatLab\Data.xlsx');
% [INF,Titles2]=xlsread('C:\Users\DW\Dropbox\Commodities Thesis\Thesis (Data and
Outline)\MatLab\Inflation.xlsx');
% [RE,Titles3]=xlsread('C:\Users\DW\Dropbox\Commodities Thesis\Thesis (Data and
Outline)\MatLab\NCREIF.xlsx');
[USTreas,Titles4]=xlsread('C:\Users\DW\Dropbox\Commodities Thesis\Thesis (Data and
Outline)\MatLab\USTreasChange.xlsx');

%%%%%%%%%% STOCK & BOND ANALYSIS %%%%%%%%%%%
% 09/24/2003 - 06/29/2007 (PRE-CRISIS)
GSCIRetB = Returns(1:949,1);
StockRetB = Returns(1:949,2);
BondRetB = Returns(1:949,3);
USGovB = USTreas(1:949,1);

% 06/29/2007 - 03/31/2009 (CRISIS)
GSCIRetD = Returns(949:1390,1);
StockRetD = Returns(949:1390,2);
BondRetD = Returns(949:1390,3);
USGovD = USTreas(949:1390,1);

% 06/29/2007 - 06/30/2011 (POST-CRISIS)
GSCIRetC = Returns(1390:end,1);
StockRetC = Returns(1390:end,2);
BondRetC = Returns(1390:end,3);
USGovC = USTreas(1390:end,1);

% 09/24/2003 - 06/30/2011 (ALL DATA)
GSCIRet = Returns(1:end,1);
StockRet = Returns(1:end,2);
BondRet = Returns(1:end,3);
USGov = USTreas(1:end,1);

disp('-----')
disp('09/24/2003 - 06/29/2007 (PRE-CRISIS)')
GSCI_skewB = skewness(GSCIRetB)
Stock_skewB = skewness(StockRetB)
Bond_skewB = skewness(BondRetB)
GSIK_kurtB = kurtosis(GSCIRetB)
Stock_kurtB = kurtosis(StockRetB)
Bond_kurtB = kurtosis(BondRetB)
Rho_GSCI_StockB = corr(GSCIRetB, StockRetB)
Rho_GSCI_BondB = corr(GSCIRetB, BondRetB)
```

```
Rho_Stock_BondB = corr(StockRetB, BondRetB)
Rho_GSCI_USGovB = corr(GSCIRetB, USGovB)
```

```
disp('-----')
disp('06/29/2007 - 03/31/2009 (CRISIS)')
GSCI_skewD = skewness(GSCIRetD)
Stock_skewD = skewness(StockRetD)
Bond_skewD = skewness(BondRetD)
GSIC_kurtD = kurtosis(GSCIRetD)
Stock_kurtD = kurtosis(StockRetD)
Bond_kurtD = kurtosis(BondRetD)
Rho_GSCI_StockD = corr(GSCIRetD, StockRetD)
Rho_GSCI_BondD = corr(GSCIRetD, BondRetD)
Rho_Stock_BondD = corr(StockRetD, BondRetD)
Rho_GSCI_USGovD = corr(GSCIRetD, USGovD)
```

```
disp('-----')
disp('03/31/2009 - 06/30/2011 (POST-CRISIS)')
GSCI_skewC = skewness(GSCIRetC)
Stock_skewC = skewness(StockRetC)
Bond_skewC = skewness(BondRetC)
GSIC_kurtC = kurtosis(GSCIRetC)
Stock_kurtC = kurtosis(StockRetC)
Bond_kurtC = kurtosis(BondRetC)
Rho_GSCI_StockC = corr(GSCIRetC, StockRetC)
Rho_GSCI_BondC = corr(GSCIRetC, BondRetC)
Rho_Stock_BondC = corr(StockRetC, BondRetC)
Rho_GSCI_USGovC = corr(GSCIRetC, USGovC)
```

```
disp('-----')
disp('09/24/2003 - 06/30/2011 (ALL DATA)')
GSCI_skew = skewness(GSCIRet)
Stock_skew = skewness(StockRet)
Bond_skew = skewness(BondRet)
GSIC_kurt = kurtosis(GSCIRet)
Stock_kurt = kurtosis(StockRet)
Bond_kurt = kurtosis(BondRet)
Rho_GSCI_Stock = corr(GSCIRet, StockRet)
Rho_GSCI_Bond = corr(GSCIRet, BondRet)
Rho_Stock_Bond = corr(StockRet, BondRet)
Rho_GSCI_USGov = corr(GSCIRet, USGov)
```

## Appendix D – MatLab Output for the Analysis Code

-----  
09/24/2003 - 06/29/2007 (PRE-CRISIS)

GSCI\_skewB =  
0.1581  
Stock\_skewB =  
-0.1735  
Bond\_skewB =  
-0.0787  
GSIC\_kurtB =  
3.3962  
Stock\_kurtB =  
3.9658  
Bond\_kurtB =  
3.6931  
Rho\_GSCI\_StockB =  
-0.0185  
Rho\_GSCI\_BondB =  
0.0469  
Rho\_Stock\_BondB =  
-0.0704  
Rho\_GSCI\_USGovB =  
0.0409

-----  
06/29/2007 - 03/31/2009 (CRISIS)

GSCI\_skewD =  
-0.1476  
Stock\_skewD =  
0.1845  
Bond\_skewD =  
-2.2530  
GSIC\_kurtD =  
4.3725  
Stock\_kurtD =  
7.2861  
Bond\_kurtD =  
37.7039  
Rho\_GSCI\_StockD =  
0.3216  
Rho\_GSCI\_BondD =  
0.0982  
Rho\_Stock\_BondD =  
-0.0474  
Rho\_GSCI\_USGovD =  
-0.2717

-----  
03/31/2009 - 06/30/2011 (POST-CRISIS)

GSCI\_skewC =  
-0.2104  
Stock\_skewC =  
-0.1910  
Bond\_skewC =  
-0.4890  
GSIC\_kurtC =  
4.0313  
Stock\_kurtC =  
4.5479  
Bond\_kurtC =



5.1299  
 Rho\_GSCI\_StockC =  
 0.5835  
 Rho\_GSCI\_BondC =  
 -0.2316  
 Rho\_Stock\_BondC =  
 -0.3071  
 Rho\_GSCI\_USGovC =  
 -0.3108  
 -----  
 09/24/2003 - 06/30/2011 (ALL DATA)  
 GSCI\_skew =  
 -0.1271  
 Stock\_skew =  
 -0.0027  
 Bond\_skew =  
 -2.4449  
 GSIC\_kurt =  
 4.9832  
 Stock\_kurt =  
 14.9696  
 Bond\_kurt =  
 68.1904  
 Rho\_GSCI\_Stock =  
 0.2943  
 Rho\_GSCI\_Bond =  
 0.0179  
 Rho\_Stock\_Bond =  
 -0.0989  
 Rho\_GSCI\_USGov =  
 -0.2003  
 -----  
 NCREIF PRE-CRISIS: JUN 30 2003 - JUN 30 2007  
 Rho\_GSCI\_NCREIFA =  
 0.7071  
 NCREIF DURING-CRISIS: JUN 30 2007 - MAR 31 2009  
 Rho\_GSCI\_NCREIFC =  
 0.6298  
 NCREIF POST-CRISIS: MAR 31 2009 - JUN 30 2011  
 Rho\_GSCI\_NCREIFB =  
 0.6684  
 NCREIF FULL PERIOD: JUN 30 2003 - JUN 30 2011  
 Rho\_GSCI\_NCREIF =  
 0.3453  
 -----

## Appendix E – MatLab Code for Efficient Frontiers

```
clc
clear all
close all
format compact

%%%%% FULL PERIOD %%%%%
% 2 Assets - Stocks and Bonds
Ret1 = [0.03519804 0.005917969]
VCV1 = [0.044501127 -0.001244403; -0.001244403 0.003558143]
NumPorts = 100;
Bounds1 = [0 0; 1 1];

% 3 Assets - GSCI, Stocks, and Bonds
Ret2 = [0.032399159 0.03519804 0.005917969]
VCV2 = [0.072808555 0.016750254 0.000287646; 0.016750254 0.044501127 -0.001244403; 0.000287646 -
0.001244403 0.003558143]
Bounds2 = [0 0 0; 1 1 1];

[PortRisk1, PortRet1, PortWts1] = frontcon(Ret1, VCV1, NumPorts, [], Bounds1);
[PortRisk2, PortRet2, PortWts2] = frontcon(Ret2, VCV2, NumPorts, [], Bounds2);

hold on
figure(1);
plot(PortRisk1, PortRet1, PortRisk2, PortRet2)
title('MV Frontier: Full Period (09/24/2003 - 06/30/2011)','fontweight','b')
xlabel('Risk (Standard Deviation)')
ylabel('Expected Return')
legend('2 Assets: Stocks, Bonds','3 Assets: GSCI, Stocks, Bonds')
legend('Location','SouthEast')
hold off

%%%%% PRE-CRISIS PERIOD %%%%%
% 2 Assets - Stocks and Bonds
Ret1A = [0.111578962 -0.009319228]
VCV1A = [0.011393161 -0.000292452; -0.000292452 0.001514398]

% 3 Assets - GSCI, Stocks, and Bonds
Ret2A = [0.116414747 0.111578962 -0.009319228]
VCV2A = [0.053452704 -0.000457406 0.000422114; -0.000457406 0.011393161 -0.000292452; 0.000422114 -
0.000292452 0.001514398]

[PortRisk1A, PortRet1A, PortWts1A] = frontcon(Ret1A, VCV1A, NumPorts, [], Bounds1);
[PortRisk2A, PortRet2A, PortWts2A] = frontcon(Ret2A, VCV2A, NumPorts, [], Bounds2);

hold on
figure(2);
plot(PortRisk1A, PortRet1A, PortRisk2A, PortRet2A)
title('MV Frontier: Pre-Crisis Period (09/24/2003 - 06/29/2007)','fontweight','b')
xlabel('Risk (Standard Deviation)')
ylabel('Expected Return')
legend('2 Assets: Stocks, Bonds','3 Assets: GSCI, Stocks, Bonds')
legend('Location','SouthEast')
hold off
```

```

%%%%% CRISIS PERIOD %%%%%
% 2 Assets - Stocks and Bonds
Ret1B = [-0.303149347 0.017906166]
VCV1B = [0.131011289 -0.001720922; -0.001720922 0.010044268]

% 3 Assets - GSCI, Stocks, and Bonds
Ret2B = [-0.256181424 -0.303149347 0.017906166]
VCV2B = [0.13412341 0.042633766 0.003604121; 0.042633766 0.131011289 -0.001720922; 0.003604121 -
0.001720922 0.010044268]

[PortRisk1B, PortRet1B, PortWts1B] = frontcon(Ret1B, VCV1B, NumPorts, [], Bounds1);
[PortRisk2B, PortRet2B, PortWts2B] = frontcon(Ret2B, VCV2B, NumPorts, [], Bounds2);

hold on
figure(3);
plot(PortRisk1B, PortRet1B, PortRisk2B, PortRet2B)
title('MV Frontier: Crisis Period (06/29/2007 - 03/31/2009)', 'fontweight', 'b')
legend('2 Assets: Stocks, Bonds', '3 Assets: GSCI, Stocks, Bonds')
legend('Location', 'SouthEast')
xlabel('Risk (Standard Deviation)')
ylabel('Expected Return')
hold off

%%%%% POST-CRISIS PERIOD %%%%%
% 2 Assets - Stocks and Bonds
Ret1C = [0.250045451 0.022380594]
VCV1C = [0.032124357 -0.002425964; -0.002425964 0.001942924]

% 3 Assets - GSCI, Stocks, and Bonds
Ret2C = [0.168774572 0.250045451 0.022380594]
VCV2C = [0.057361505 0.025049609 -0.002444537; 0.025049609 0.032124357 -0.002425964; -0.002444537 -
0.002425964 0.001942924]

[PortRisk1C, PortRet1C, PortWts1C] = frontcon(Ret1C, VCV1C, NumPorts, [], Bounds1);
[PortRisk2C, PortRet2C, PortWts2C] = frontcon(Ret2C, VCV2C, NumPorts, [], Bounds2);

hold on
figure(4);
plot(PortRisk1C, PortRet1C, PortRisk2C, PortRet2C)
title('MV Frontier: Post-Crisis Period (03/31/2009 - 06/30/2011)', 'fontweight', 'b')
xlabel('Risk (Standard Deviation)')
ylabel('Expected Return')
legend('2 Assets: Stocks, Bonds', '3 Assets: GSCI, Stocks, Bonds')
legend('Location', 'SouthEast')
hold off

```

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