Modeling the Dynamics of Implied Volatility Surface

Of

S&P CNX NIFTY

by

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Abstract

Of all the inputs that go into the Black, Scholes & Merton option pricing formula, all but one – volatility - can be directly measured. There have been several attempts in the past to model the volatility of an underlying asset. Models developed till date has not been able to provide an accurate recipe for estimating future volatility; nonetheless these models are partially successful in capturing volatility behaviour.

This study is intended to apply and extend the accepted implied volatility modelling principles to the S&P CNX NIFTY (Index from National Stock Exchange of India - NSE) index options and account for the deviations in the volatility surface and the corresponding risk factors. The methodology followed for modelling implied volatility is similar to Dumas, Fleming and Whaley (DFW 1998) and Ishan Ullah Badshah (IUB – working paper 2008) and the methodology used for Principal Component Analysis is similar to the one applied by Skiadopolous, Hodges and Clewlow (SHC 1999). We compare the implied volatility surface generated using one linear model (constant volatility) and three nonlinear models that take into consideration varying levels of skew or smile and maturities. We find that the fourth model best captures all the characteristics of implied volatility. Secondly, we apply Principal Component Analysis (PCA) to the implied volatility surface and extract the most relevant principal components that explain most of the dynamics of the volatility surface. We determined that 80.66% to 94.47% of the variation in the IV surface is explained by the first three principal components. Lastly, we study the behaviour of the implied volatility surface of the S&P CNX NIFTY for two distinct periods – pre crisis (2006) and post crisis (2009). Specific applications of the model include pricing and hedging of derivatives and risk management.

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At the very outset, I bow my head with reverence to the Supreme God, by whose blessings I have been able to reach another milestone of my life.

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Deepak Prashanth Reddy. V

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1. Introduction

There have been numerous studies by both practitioners and academicians trying to explore and model asset volatilities, which are crucial to pricing and valuation of options. Though most of the studies tried to model historical volatilities using time series applications, advances in academia and breakthrough studies in the past 15 years have shifted the focus from historical volatility to implied volatility as a better measure. Famous studies such as the ones by DFW, SHC and Amadeo Alentorn (2004) have asserted the superiority of implied volatility over historical volatility. They showed that the implied volatility, which is the collective average market's expectation of future volatility, gives a precise notion of the market's judgement. Furthermore, implied volatility is forward looking whereas historical volatility is backward looking. The collective market's expected volatility is recovered from the observed option prices. The observed option price is inverted by using the Black, Scholes & Merton (BSM 1973) option pricing formula to deduce the implied volatility. However, it is well established that the implied volatilities computed from the option prices on stocks/indices are different across terms and strikes if examined at the same point in time. For example, the implied volatilities if calculated from the option prices of 3 month expiry will be different from the implied volatilities of options with 12 month expiry, everything else remaining the same. Similarly, for a given expiry, the implied volatility calculated from in-the-money options will be different from the implied volatility calculated from out-of-the-money options. These two dimensions (term-to-maturity and strike) provide a frame of reference for modelling the implied volatility as a two dimensional surface.

The BSM option pricing formula is the most widely used model for pricing options. It assumes that the underlying asset follows a geometric Brownian motion and has constant volatility. The constant volatility assumption implies that the options with different strikes and maturities on a same underlying asset should have the same volatility. Contrary to this, the volatility observed in practice varies with maturities and strikes. This deviation from the theoretical BSM model can be attributed to various risk factors such as market shocks, transaction costs, information asymmetry and irrational investor behaviour. The primary motivation to model implied volatility is to account for this variation in volatility that is not captured by the BSM model.

Another factor worth considering is the degree of correlation that exists between the various market risk factors. There may be several risk factors that affect the volatility of options. Identifying and modelling each of these factors individually is tedious, time consuming and computationally demanding. Since these factors are correlated, an alternate approach would be to identify only the major factors that account for most of the variation and model those factors only. We use Principal Component Analysis (PCA) to identify and extract the most important factors that can help us explain most of the dynamics of implied volatility and reduce the dimensionality of the correlated factors.

The objectives of our study are to

1. Estimate the implied volatility from the observed S&P CNX NIFTY option prices.

2. Apply non-linear parametric models to generate a smooth implied volatility surface that characterizes the volatility of NIFTY index.

3. Study the dynamics of the implied volatility surface by applying PCA.

4. Comparative study of the implied volatility behaviour of NIFTY during, before and after the sub-prime mortgage crisis.

Our study is very closely related to the papers by Dumas, Fleming and Whaley (DFW 1998) (for modelling implied volatility), Skiadopolous, Hodges and Clewlow (SHC 1999) (for PCA). A similar study was done by Ihsan Ullah Badshah using the combined methodology of DFW and SHC for his study of FTSE 100 Index. The methodology that we have applied is similar to DFW, SHC and IUB. However, there are certain aspects of our study that differ from our precedent papers in some ways.

1. DFW modelled the implied volatilities for the S&P 500 for the period June 1988 – December 1993 and IUB studied options on FTSE100 for the period January 2004 to October 2006. Our study models the options on the S&P CNX NIFTY from January 2006 to December 2010.

2. DFW estimated the implied volatilities by excluding out-of-money puts for low strikes and out-of-the-money calls for high strikes. IUB estimated IV`s by considering only the data for out-of-the-money put and call options. In comparison, our study considers the entire spectrum of available moneyness of options in order to give us a holistic view and to efficiently utilize all available market data to make any inference.

3. DFW model incorporates the strike price when estimating implied volatility surface. However, we incorporate moneyness instead of strike, similar to the approach of IUB. This is because of the observation in Gross and Waltner's (1995), who concluded that variations in IV surface vary less in time when expressed in terms of moneyness rather than strike.

4. We compare the implied volatilities before, during and after the financial crisis of 2008. For the purpose of comparison it is assumed that major after effects of the crisis were witnessed during 2008-2009 after the market crash of January 2008 and October 2008 when NIFTY plunged more than 50% from its peak, whereas no such comparison is made in the precedent papers.

5. In PCA, we try to extract the risk factors for *index* options on the NIFTY, whereas our precedent paper SHC extracts the risk factors for the *futures* options on S&P 500.

Our main findings from this study are that the constant volatility model (Model 1) fails to capture the variations in the implied volatility surfaces and, consequently, generates flat volatility surfaces. Model 2 which captures the smile (skew) effects of moneyness is good enough; however, there are some variations that can be attributed to the time-to-maturity dimension in the implied volatilities. Therefore, models 3 and 4, which account for variations in both (time to maturity and moneyness) dimensions, fit well to the surfaces. They produce smoother and tighter volatility surfaces to the observed data. Finally, we find that the first three principal components can explain about 80.66% to 94.47% of the variances in the implied volatility surfaces.

Even though the results we obtained varied quantitatively from our precedent papers, this variation was not unexpected given the following reasons. Firstly the fundamental nature of the underlying market that we studied was different from ones studied in our precedent papers. While NIFTY represents an emerging economy, FTSE 100 and S&P 500 both indices belong to developed economies. Secondly the time period and time frame of study varied from our

precedent papers. DFW studied data from 1988 to 1993 and IUB investigated FTSE 100 data from 2004 to 2006.

This paper is organized as follows. Section 2 reviews the existing literature on the subject. Section 3 describes the sources and screening of data. Section 4 provides an outline of the methodology used and finally, we present our empirical results in section 5 and conclusion in sections 6.

2. Literature Review

In keeping with the assumptions of the BSM, early attempts to model volatility as a deterministic function were undertaken by many academicians including Derman and Kani (1994), Rubinstein (1994) and Dupire (1994). These authors tried to model volatility by using implied binomial and trinomial trees. The observed implied tree generated at time 't' should include the implied tree at time 't+1'. Failure to explain this phenomenon is the primary shortcoming of these models. Gross and Waltner's (1995) studied volatility on S&P 500 Index and concluded that moneyness rather than strike is better measure to capture volatility smile/skew, because implied volatility is less sensitive to moneyness than strike.

Several papers were studied and reviewed to serve as a foundation for non-linear parametric modelling of implied volatility surface. The research by Buraschi and Jackwerth (1998) and Dumas, Fleming and Whaley (1998) provide conclusive evidence that constant volatility models are not capable of representing volatility surface in two dimensions. Kamal and Derman (1997) analyzed the dynamics of implied volatilities of over the counter (OTC) S&P 500 and Nikkei 225 Index options and observed that first three principal components explain about 95% of the variance of the volatility surface. Their interpretation of the first three principal components is as follows: the first principal component represents a degree of parallel shift, i.e. any changes in implied volatility will lead to a constant shift in the IV surface across both parameters. The second principal component represents the tilt of the implied volatility i.e. it captures the effect of the changes in term-to-maturity on the implied volatility surface. The third principal component represents the skew or curvature of the IV surface. In their paper, Fung and Hsieh (1991) show that the smile structure of implied volatilities is influenced not only by the term and

strike, but also by the underlying asset. This paper implied that the dynamics of the two-dimensional implied volatility surface will vary with the selection of the asset.

Skiadopolous, Hodges and Clewlow (SHC 1999) conducted PCA by segregating the data into two categories – (1) Smile, considering both the term-to-maturity and the moneyness for performing PCA and (2) Surface, consisting of all maturities, but in different moneyness buckets. They concluded that both smile analysis and surface analysis show similar principal components. On average, the first two components explained about 78% of variation in volatility smiles and 60% of the variation in volatility surface. Another important study was done by Cont and Fonseca (2002), who examined the dynamics of the implied volatility for options on both FTSE100 and S&P 500. The findings of the aforementioned studies were threefold.

1. The implied volatility that reflects both strike and term to maturity has a non-flat structure.
2. The implied volatility surface changes with time.
3. The movements in implied volatility are independent of the movements in the underlying asset.

This paper primarily draws from DFW (1998) and IUB (2008) in modelling implied volatility. DFW and IUB used the non-linear parametric form of modelling implied volatility to estimate the implied volatility surface. For Principal Component Analysis, this paper relies heavily on the model developed by Skiadopolous, Hodges and Clewlow (1999).

3. Data

3.1 Source Data

We used daily closing price data of options with underlying as S&P CNX NIFTY index of National Stock Exchange of India (NSE). S&P CNX NIFTY is a diversified 50 stock index accounting for 24 sectors of the economy and is used for multiple purposes, such as benchmarking fund portfolios, index based derivatives and index funds. S&P CNX NIFTY is owned and managed by India Index Services and Products Ltd. (IISL). Historical data provided by NSE contains the following daily information for each option traded on the exchange: trading date, expiration date, the opening, high, low, close, last traded price (LTP), settlement price, number of contracts, turnover, open interest and the change in open interest. The results reported in the study are based on the daily closing price of the option contracts under study. The options are european in style and traded between January 1, 2006 and December 31, 2010. We used 91 day T-Bill (Issued by Government of India) yield as a risk free rate for use in the Black, Scholes & Merton model.

For Principal Component Analysis (PCA), we use the same maturities and strikes that were used in surface analysis. Series of implied volatility observations segregated on the basis of different levels of moneyness is the only input data in PCA. Implied volatilities are calculated by inverting option prices available in the market. We filtered observations of implied volatilities on the basis of moneyness and then applied differencing technique to make the data stationary, because PCA is usually misleading when applied to non-stationary data series. Input data must be a stationary series in order to avoid any effect of correlation between the observations. Results thus obtained

are more accurate measure of variance explained by principal components, and each principal component being independent of the other.

3.2 Screening of Data

We filtered the options on the basis of the number of contracts traded, time to expiry and arbitrage violation. We have observed that a majority of abnormal option prices (indicating zero implied volatility) occurred where the number of daily traded contracts was less than 100. This can probably be attributed to irrational trading practices by individual or retail investors. Illiquid options, with traded number of contacts less than 100 on a single trading day were ignored to avoid any extreme observations resulting in distortion of implied volatility surface or data fitting. This is one more area of departure from the data used in precedent paper.

NIFTY options with maturity more than ninety days were not traded in NSE until 2008. Since our study includes 2006 and 2007, we have incorporated the additional maturity options that were introduced in 2008 onwards into our analysis. This is somewhat different from the approach used in DFW and IUB, who used options with all maturities in their study. We also ignored the options with time to expiry equal to zero for fitting of data, estimation of implied volatility surface and for performing Principal Component Analysis.

4. Methodology

There are four primary objectives of the study. We calculated the implied volatility from the option prices available in the market; we then estimated the parameters for fitting and modeling the IV surface. Principal Component Analysis is applied to study the dynamics of implied volatility surface and lastly, we compared the behaviour of the IV surface through the crisis years. The methodology for modelling implied volatility is similar to DFW, with the exception of using moneyness instead of strike price. The methodology for PCA is similar to the one applied by SHC (1999).

4.1 Calculating Implied Volatility

Although there is varying degree of agreement on the relevance of the assumptions of the Black, Scholes and Merton (BSM) option pricing model, this model is generally accepted in the industry as sufficiently accurate to derive the implied volatilities of the underlying. The price of call or put options as given by the BSM model is:

$$C(S_t,t) = S_t N(d_1) - k \exp(-r(T-t)) N(d_2)$$

where

$$d_1 = \frac{Ln\left(S_t/k\right) + \left(r + \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

All the variables except volatility are directly observable in the market. Generally, historical volatility is used as a good estimator for volatility and is used as an input to price options.

However, as widely acknowledged, option prices calculated this way are different from the market observed prices of options. The only discernable reason for this variation in theoretical vs. observed options prices is the variation in the volatility that is used by the market to price different time-to-maturities and strikes. Since we observe the market prices of options but not their volatilities, we invert the BSM option pricing formula to derive the volatilities implied by the market. In other words, the implied volatility of a particular underlying asset is the volatility at which the market price of the options equals the BSM model price. The BSM implied volatility is a unique solution because, in the volatility parameter, the BSM formula is monotonic.

$$\frac{\partial B}{\partial \sigma} > 0$$

Initial data was screened and options with either time to maturity equal to zero or number of contract less than 100 were ignored. Further we used option price, time to maturity, risk free rate, underlying price, strike price and option type (call or put) as input data to calculate implied volatility. Mentioned below are the descriptive statistics of implied volatilities obtained from observed market prices.

Year	Mean	Median	Std. Dev.	Kurtosis	Skewness
2006	0.3219	0.2788	0.1940	42.7872	4.7581
2007	0.3353	0.3074	0.1776	41.4806	4.5720
2008	0.4877	0.4188	0.2878	42.2853	4.8212
2009	0.4414	0.3847	0.3029	83.8514	7.0221
2010	0.2714	0.2346	0.2009	188.8230	9.4288

Table 1: Descriptive statistics for Implied Volatilities

The above table mentions the descriptive statistics of the implied volatilities of all types of options, i.e. in-the-money (ITM), out-of-the-money (OTM) and at-the-money (ATM) in different years of study. There is an evident non–normality in the data as indicated from high kurtosis and skewness in the implied volatilities of the options. This non normality of data further reiterates the empirical observation of varying volatility in the underlying asset and refutes the BSM assumption of constant volatility. Extreme measures of skewness and kurtosis can be attributed to three primary characteristics of emerging markets,

1. Relative shallowness of emerging markets in comparison to developed markets as evident by emerging market's lower average trading volumes and market capitalization relative to developed markets.

2. Lower market efficiency due to lenient regulations, in addition to other detrimental factors like political interference, corporate and trader lobbies etc.

3. Irrational trading practices driven by irrational behaviour, lower liquidity and other influencing factors leading to mispricing of financial assets in either direction.

The implied volatility is calculated using 'blsimpv' function in the MATLAB finance toolbox. This method uses the interpolation (bisection) method. Two initial seed values of volatilities corresponding to a lower and higher option prices are used. A linear interpolation of these two volatilities is used as an input to calculate the BSM option price. Depending on the outcome of the trial, the seed values are altered in either direction and the process repeats. For a desired level of accuracy, the iteration is repeated until the accurate implied volatility is obtained.

4.2 Modelling IV Surface

Fitting parametric volatility models to observed implied volatilities has been studied and implemented extensively, but none have been applied to S&P CNX NIFTY. We implement the approach developed by Dumas, Fleming and Whaley (1998) in parametric modelling across two dimensions – time-to-maturity and strike. We however use moneyness instead of strike. Although a ratio of strike to underlying may be a good measure to strike, we use the industry accepted measure of 'moneyness', denoted by `m`. The reason for this is that the implied volatility varies less in time when moneyness is used instead of strikes.

$$m = \frac{\log(\frac{S\,e^{r\tau}}{k})}{\sqrt{\tau}}$$

Where, S is market price of underlying asset, r is risk free rate, k is the strike price of an option and τ represents term to maturity. In order to obtain a smooth implied volatility surface, we can use the four structural models used by DFW to characterize the implied volatility in two dimensions:

Model 1: $\quad I(m,\tau) = \beta_0 + \varepsilon$

Model 2: $\quad I(m, \tau) = \beta_0 + \beta_1\,m + \beta_2\,m^2 + \varepsilon$

Model 3: $\quad I(m, \tau) = \beta_0 + \beta_1\,m + \beta_2\,m^2 + \beta_3\tau + \beta_4\tau m + \varepsilon$

Model 4: $\quad I(m, \tau) = \beta_0 + \beta_1\,m + \beta_2\,m^2 + \beta_3\tau + \beta_4\tau m + \beta_5\tau^2 + \varepsilon$

Model 1 represents the constant volatility model of BSM that yields a constant volatility across moneyness and strikes. The β_0 in the model is the constant volatility of the BSM model. The

second model tries to capture the quadratic volatility smile and skew. β_1 and β_2 in the model represent the slope and curvature of moneyness. The third model captures the time effect by including the time-to-maturity (τ) and the combined effect of moneyness and the time-to-maturity. β_3 and β_4 in the model represent the effect of time-to-maturity and the combined effect of time-to-maturity and moneyness. The fourth model captures the curvature effect of the time-to-maturity β_5.

In order to find the appropriate model parameters, we use the optimization technique – specifically the non-linear least square function 'lsqnonlin' from the MATLAB finance toolbox. In order to test the accuracy of the model and the goodness of fit, we use two statistical measures of root mean square error. The first loss function we use to measure the effectiveness of the fit is the implied volatility root mean square error (IVRMSE). The IVRMSE is the root mean square difference between the BSM model implied volatilities and model volatility.

$$\text{IVRMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\sigma_i - \sigma_i(\theta))^2}$$

The second method is the percentage root mean squared error (%RMSE). This loss function minimizes the difference between the BSM implied volatility and model implied volatility. This loss function assigns more weight to deep out-of-the-money options by dividing the difference (between BSM implied volatility and model implied volatility) by the option price.

$$\%\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{e_i(\theta)}{Option\,Price_i}\right)^2}$$

In addition to the root mean square measures mentioned above, we also test our models by checking the adjusted R^2 and the Akaike Information Criterion (AIC).

Adjusted R^2 is used to check for any statistically significant improvement in the model due to the addition of new variables and is defined as:

$$\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-p-1}$$

Where n is the number of observations and p is the number of parameters in the model.

The Akaike information criterion is a measure of the relative goodness of fit of a statistical model. It is used to check if the additional complexity of the model provides any statistically significant improvement in the model results. Like adjusted R^2, AIC also penalizes the model for over-fitting. AIC can be calculated using residual sums of squares from regression:

$$AIC = n * \ln (RSS/n) + 2 * p$$

where n is the number of data points (observations), RSS is the residual sums of squares and p is the number of parameters in the model.

4.3 Principal Component Analysis

It has been observed that there are multiple sources of risk and uncertainty associated with an individual asset in the market. Also there are various common sources of risk which affect numerous assets at the same time, thus indicating certain degree of correlation within the source of risk itself and within assets too. Nonetheless the degree of correlation can differ with varying nature of sources of risk and assets under investigation. Considering the large number of sources of risk affecting the prices of a single asset and the degree of correlation between the sources of price variance, measuring the effect of each source separately becomes a tedious task. Further it tends to be less accurate owing to the correlation among the sources of uncertainty. Nonetheless it is more efficient to explain the variance of an asset in a limited number of independent variables, rather than large number of correlated variables. This can be achieved by using Principal Component Analysis (PCA) as suggested by Skiadopoulos et al (1999): "*Principal Component Analysis explains the systematic behaviour of multiple observed variables, in terms of a smaller set of unobserved variables known as principal components. Its objective is to transform correlated variables into an orthogonal set to reproduce the original variance-covariance structure*". The process of PCA comprises of decomposition of an input matrix into a matrices of Eigenvectors and Eigenvalues. We follow the approach used by SHC in implementing PCA for calculated IV's.

Consider a grouping of all options within the observation time-frame into N moneyness levels. These N moneyness levels represent dependent variables affecting the combined implied volatility surface. If we have M implied volatility observations, under each moneyness level

then we have a matrix [V = MxN] representing the implied volatility surface. Since we have N variables, the variance-covariance matrix will be of the dimension [NxN]. Our primary purpose is to reduce the number of variables that affect implied volatility by isolating the independent components that explain most of the variation. This can be done in PCA by decomposing the N x N variance - covariance matrix (VCV hereafter) of an input matrix V into Eigen vectors and Eigen values matrices. PCA determines a linear combination of volatilities that explains maximum variation in the implied volatility surface. The factor loadings, F ($f_1, f_2 \ldots f_N$) in the linear combination of volatilities, (also known as coefficients of principal component) are derived from set of Eigen vectors of VCV Matrix. This is done in MATLAB using the 'princomp' function. The resulting co-efficient weights are the factor loadings F mentioned above. Hence the m_{th} principal component can be given as

$$P_m = f_{m1}V_{1m} + f_{m2}V_{2m} + f_{m3}V_{3m} + \ldots\ldots + f_{mp}V_{pm}$$

The full M x N matrix of principal components is P = V x F. We performed Principal Component Analysis on the implied volatility surface determined for S&P CNX NIFTY. The input data for Principal Component Analysis of the implied volatility surface comprises of calculated implied volatilities for the selected time period under study. For each time period, implied volatilities obtained from option prices are filtered on the basis of different levels of moneyness, which is then presented in a form of a matrix. Each matrix [M x N] so formed comprises of 100 -150 rows (M) of implied volatilities observations and 7-10 columns of moneyness (N). The input data series was made stationary using differencing approach to avoid any directional bias of high implied volatility input on the end results. We performed PCA on implied volatility surface for each year and compared the dynamics of the same.

5. Empirical Results

5.1 Parametric Modelling of Implied Volatility

We have applied each of the four parametric models described in section 4 to generate the model betas for comparison. Table 2 shows the constant volatility model (model 1) of the Black, Scholes & Merton. As expected, the model generates a flat volatility surface (figure 1) with a constant β_0 indicating level of constant volatility generated by the model. The flat volatility surface in figure 1 indicates constant volatility assumption of Black, Scholes & Merton, and it shows considerable deviations from observed implied volatilities. This, however, is not an accurate model as empirical data clearly indicates volatility is not constant. The high values of %RMSE and IVRMSE indicate a very large variation in model results as compared to the implied volatilities. The low value of average adjusted R^2 also supports the inappropriateness of the model. Further, on average, this model is having the lowest AIC in 0.90% of our sample data.

Model 2 results for the same time periods are shown in table 3 below. This model attempts to capture the moneyness effects of the implied volatility. Judging the goodness of fit by the %RMSE and the IVRMSE values, we see a great improvement from model 1. The average %RMSE has decreased from 0.5017 to 0.1940 and the average IVRMSE has decreased from 0.2128 to 0.0939. The average adjusted R^2 for this model is 0.7214. Further, on average, this model is having the lowest AIC in 10.60% of our sample data, indicating a better fit relative to model 1, but not as good as next two models. Since this model does not include the term-to-maturity variable, this figure (Figure 3) shows a constant volatility term structure.

Model 3 results for the same time periods are shown in table 4 below. In addition to moneyness, this model attempts to capture the term-to-maturity tilt effects on the implied volatility surface by including a τ component in the model. As a result we see that the goodness of fit factors for every period of observation is lower than that for model 2, indicating a much better fit. The average %RMSE has decreased from 0.1940 to 0.1919 and the average IVRMSE has decreased from 0.0939 to 0.0929. The average adjusted R^2 for this model has improved marginally from 0.7214 to 0.7288. Further, on average, this model is having the lowest AIC in 17.59% of our sample data as compared to 10.60% in model 2. Also, in comparison to model 2, we can see that model 3 exhibits the tilt effect of term-to-maturity of implied volatility.

Model 4 results for the same time periods are shown in table 5 below. This model attempts to capture all the factors effecting implied volatility including moneyness, term-to-maturity and the combined effect of both. As a result we see a compact fit of the data points to the model surface. The goodness of fit factors for every period of observation is lower than that for model 3, indicating better fit. The average %RMSE has decreased from 0.1919 to 0.1894 and the average IVRMSE has decreased from 0.0929 to 0.0922. The average adjusted R^2 for this model has improved marginally from 0.7288 to 0.7316. However, the lowest average AIC has increased considerably from 17.59% to 73.94% of our sample data, which lends further support to conclude that model 4 best captures the implied volatility surface. We can see that model 4 captures the parallel shift, tilt and curvature effect of implied volatility.

Model 1					
Month	**β_0**	**AIC**	**Adj R^2**	**IVRMSE**	**%RMSE**
Mar-06	0.27	0.0000	0.0000	0.1979	0.0735
Sep-06	0.27	0.0000	0.0000	0.0802	0.0526
Mar-07	0.33	0.0000	0.0000	0.1366	0.0557
Sep-07	0.35	0.0000	0.0000	0.2591	0.0759
Mar-08	0.43	0.0000	0.0000	0.1472	0.1813
Sep-08	0.40	0.0000	0.0000	0.1405	0.2343
Mar-09	0.46	0.0000	0.0000	0.2984	0.5044
Sep-09	0.40	0.0000	0.0000	0.3605	1.7764
Mar-10	0.27	0.0000	0.0000	0.2325	1.4169
Sep-10	0.29	0.0895	0.0000	0.2750	0.6460
Average	**0.35**	**0.0090**	**0.0000**	**0.2128**	**0.5017**

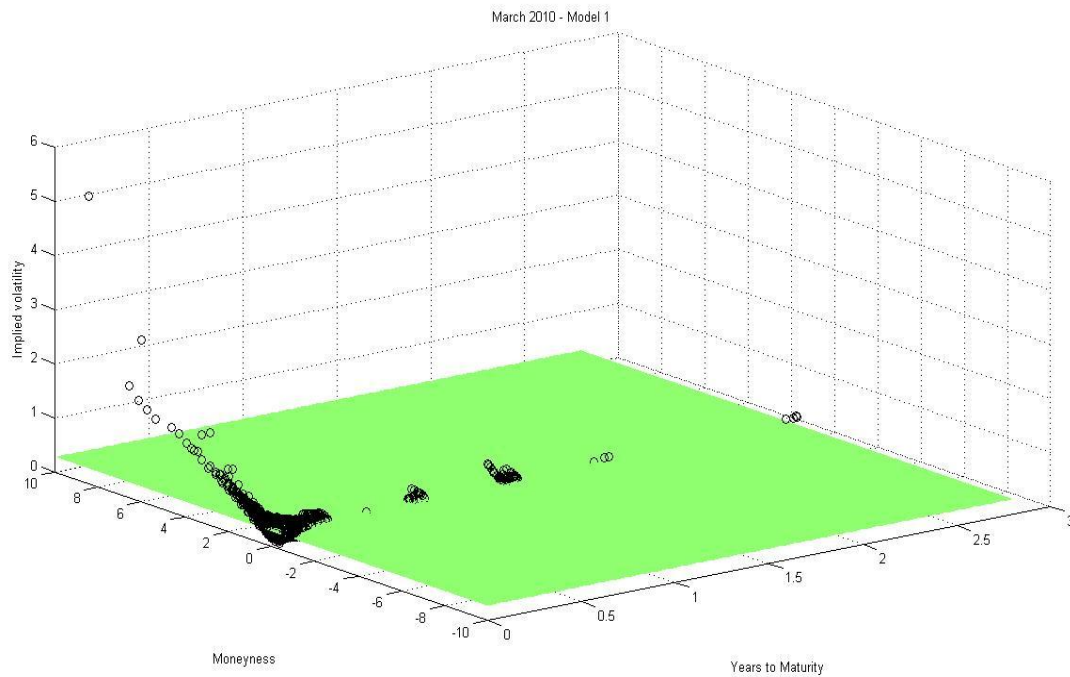**Table 2: Estimated parameters for model 1 with corresponding adjusted R^2, AIC, IVRMSE and %RMSE**

Figure 1: Estimated volatility surface for model 1 for March 2010. The black circles are the observed implied volatilities.

Model 2							
Month	β_0	β_1	β_2	AIC	Adj R^2	IVRMSE	%RMSE
Mar-06	0.19	0.27	0.09	0.0002	0.8676	0.0719	0.0225
Sep-06	0.24	0.09	0.15	0.0000	0.6928	0.0444	0.0123
Mar-07	0.30	0.00	0.28	0.0000	0.2658	0.1169	0.0690
Sep-07	0.24	0.26	0.07	0.0000	0.8578	0.0976	0.0319
Mar-08	0.40	0.08	0.14	0.0059	0.4006	0.1138	0.0794
Sep-08	0.37	0.06	0.09	0.0000	0.6512	0.0829	0.0626
Mar-09	0.41	0.02	0.06	0.2839	0.8135	0.1288	0.1540
Sep-09	0.30	0.15	0.03	0.7698	0.9178	0.1033	0.5382
Mar-10	0.21	0.14	0.04	0.0000	0.8583	0.0874	0.7076
Sep-10	0.18	0.27	0.02	0.0000	0.8890	0.0916	0.2620
Average	0.28	0.13	0.10	0.1060	0.7214	0.0939	0.1940

Table 3: Estimated parameters for model 2 with corresponding adjusted R^2, AIC, IVRMSE and %RMSE
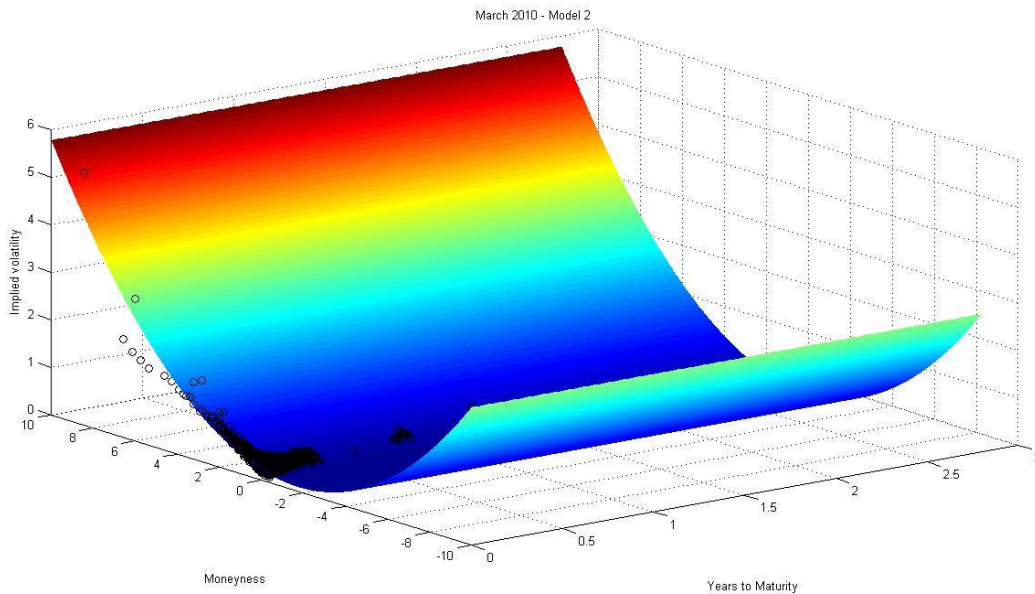
Figure 2: Estimated volatility surface for model 2 for March 2010. The black circles are the observed implied volatilities.

Model 3									
Month	**β₀**	**β₁**	**β₂**	**β₃**	**β₄**	**AIC**	**Adj R²**	**IVRMSE**	**%RMSE**
Mar-06	0.18	0.33	0.08	0.22	-1.74	0.2214	0.8717	0.0706	0.0257
Sep-06	0.22	0.11	0.15	0.26	-0.26	0.6063	0.7106	0.0430	0.0126
Mar-07	0.33	0.04	0.26	-0.43	-1.31	0.4563	0.2872	0.1152	0.0547
Sep-07	0.25	0.28	0.07	-0.06	-0.57	0.0000	0.8591	0.0970	0.0313
Mar-08	0.41	0.07	0.14	-0.06	0.38	0.0333	0.4047	0.1133	0.0812
Sep-08	0.38	0.05	0.09	-0.05	0.22	0.0000	0.6698	0.0806	0.0595
Mar-09	0.41	0.02	0.06	-0.01	0.03	0.3013	0.8137	0.1286	0.1524
Sep-09	0.30	0.14	0.03	-0.01	0.03	0.1406	0.9177	0.1033	0.5369
Mar-10	0.22	0.13	0.04	-0.04	0.25	0.0000	0.8639	0.0856	0.7039
Sep-10	0.19	0.26	0.02	-0.06	0.09	0.0000	0.8896	0.0913	0.2605
Average	**0.29**	**0.14**	**0.09**	**-0.02**	**-0.29**	**0.1759**	**0.7288**	**0.0929**	**0.1919**

**Table 4: Estimated parameters for model 3 with corresponding adjusted R², AIC, IVRMSE and %RMSE**
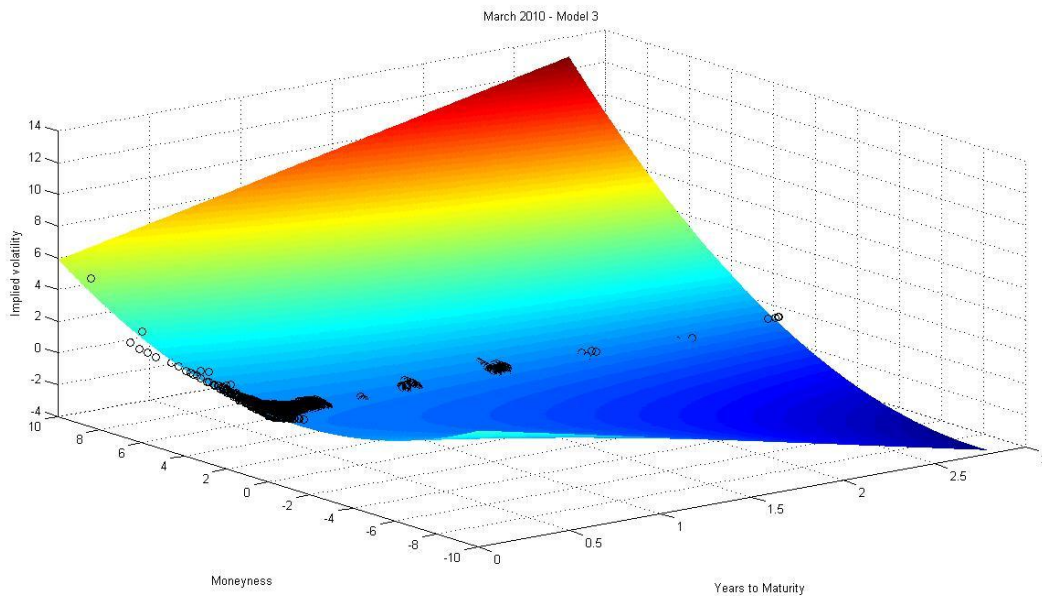
Figure 3: Estimated volatility surface for model 3 for March 2010. The black circles are the observed implied volatilities.

Model 4										
Month	**β₀**	**β₁**	**β₂**	**β₃**	**β₄**	**β₅**	**AIC**	**Adj. R²**	**IVRMSE**	**%RMSE**
Mar-06	0.19	0.31	0.08	-0.20	-1.45	1.89	0.7783	0.8726	0.0703	0.0256
Sep-06	0.22	0.11	0.15	0.39	-0.33	-0.60	0.6063	0.7107	0.0430	0.0126
Mar-07	0.34	0.04	0.25	-0.91	-1.33	2.79	0.5437	0.2882	0.1150	0.0534
Sep-07	0.29	0.25	0.07	-1.09	-0.27	4.72	1.0000	0.8631	0.0956	0.0309
Mar-08	0.41	0.08	0.14	-0.12	0.16	0.04	0.9608	0.4105	0.1127	0.0762
Sep-08	0.40	0.05	0.09	-0.19	0.17	0.05	1.0000	0.6793	0.0794	0.0568
Mar-09	0.41	0.02	0.06	-0.08	0.03	0.03	0.4149	0.8140	0.1285	0.1507
Sep-09	0.30	0.14	0.03	-0.05	0.03	0.02	0.0895	0.9177	0.1032	0.5357
Mar-10	0.23	0.12	0.04	-0.15	0.29	0.06	1.0000	0.8673	0.0845	0.6979
Sep-10	0.22	0.26	0.02	-0.27	0.07	0.29	1.0000	0.8928	0.0899	0.2541
Average	**0.30**	**0.14**	**0.09**	**-0.27**	**-0.26**	**0.93**	**0.7394**	**0.7316**	**0.0922**	**0.1894**

Table 5: Estimated parameters for model 4 with corresponding adjusted R², AIC, IVRMSE and %RMSE
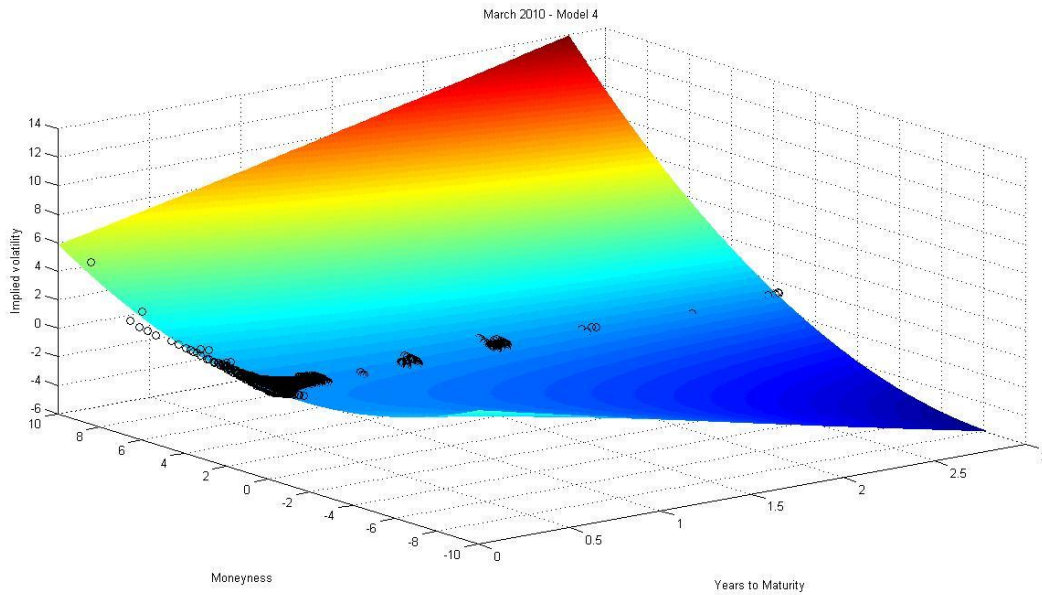
Figure 4: Estimated volatility surface for model 4 for March 2010. The black circles are the observed implied volatilities.

5.2 Results for Principal Component Analysis

We performed PCA on implied volatility surface for year 2006-2010 and used the 'proportion of variance accounted for' method to retain components. We found that first three components accounts for majority of variance explained in all the years of observation. Therefore, only the first three components were retained. The following table shows the variance explained by retained principal components and factor loadings for the duration of 2006 – 2010.

Year	Component	(%) Variance Explained	Factor loadings		
			f1	f2	f3
2006	1	55.55	0.041	-0.016	0.022
	2	19.98	0.128	-0.144	0.057
	3	17.85	-0.203	-0.972	0.008
	Total	**93.38**			
2007	1	55.49	0.006	-0.002	0.003
	2	28.02	-0.038	0.022	-0.021
	3	10.96	0.009	-0.001	0.014
	Total	**94.47**			
2008	1	45.49	0.904	-0.005	0.033
	2	31.76	-0.419	-0.086	0.125
	3	12.19	-0.044	-0.359	0.127
	Total	**89.43**			
2009	1	32.64	0.052	-0.023	0.005
	2	26.5	-0.039	-0.005	-0.005
	3	21.52	0.032	0.015	-0.005
	Total	**80.66**			
2010	1	41.88	0.049	-0.023	0.004
	2	27.8	-0.039	-0.015	0.003
	3	19.13	0.04	0.017	0.014
	Total	**88.81**			

Table 6: Principal Components and Factor Loadings

First three principal components of the implied volatility surface commonly represent parallel shift, tilt and curvature. The first principal component is considered as the level factor and is the primary component representing uniform shift in the IVS in similar direction and across all dimensions i.e. parallel shift. Proportion of variance explained by first principal component for the year 2006 to 2010 ranges from 32.64% to 55.55%. On average the first PC explained 46.21% of variance in IVS during 2006-2010.

The second PC explains the tilt effect i.e. the effect of the changes in term-to-maturity on the implied volatility surface. It has a uniform bearing on IVS across its moneyness axis. The second PC explained 19.98% to 31.76% of variance in the IVS during the period. The average variance explained in the corresponding period was 26.81%. The effect of tilt factor on IVS varies with maturities and can be attributed to long term risk factors.

The third major principal component, called as curvature, is also known as jump factor. This gives a representation of the steepness in volatility skews and has a varying effect on both type OTM calls and OTM puts. The proportionate variance explained by jump factor varied from 10.96% to 21.52% during year 2006 to 2010, with an average of 16.33%.

When compared on year to year basis, first three PC's explained 80.66% to 94.47% of total variance in the year 2006 to 2010. The results of the PCA are in agreement with the IVS surface generated for the corresponding years. We notice that the shape of the IVS changed from almost a flat surface in 2006 to a tilted and curved (skew) surface in 2008. As can be expected, the percentage of variation explained by the first PC has reduced from 55.55% in 2006 to 45.49% in

2008 while the corresponding second PC has increased from 19.98% in 2006 to 31.76% in 2008. The reduction in variance explained by 1st PC was redistributed amongst 2nd and 3rd PC in 2008. The variation explained by the first three PC's change (redistributed) in time in accordance with the changes in the observed IVS.

Even though our results of non-linear parameterization and PCA varied quantitatively when compared to our precedent papers, the inference and the logical conclusion of our study is in line with that of the precedent papers. For example, similar to empirical results of DFW & IUB, our study also concludes that the goodness of fit of the parametric models improved non-linearly from Model 1 to Model 4. Furthermore, depending on the period of observation, the first three PC`s of study by SHC explained between 66.9% to 72% of the variance in IV surface and the first three PC`s of study by IUB explained between 69.7% to 82.1% of the variance in IV surface. Comparatively our study estimates the variance explained by the first three PC's in the range of 80.66% to 94.47%. This increase in percentage of variation explained by the first three PC's indicates that a fewer number of principal variables explain most of the variation in NIFTY. This reasoning is supported by the high correlation of NIFTY with the stock markets of the developed economies, especially the USA, especially in times of distress. Also, a sizeable portion of investments in India is through FDI's (Foreign Direct Investments), which vary directly (high correlation) with state of the western economies.

5.3 Comparative Study of IV behaviour during financial turbulence

In order to compare the behaviour of implied volatility, we have generated the IV surfaces for each year from 2006 through 2010. Visual inspection of the IV surfaces through these years gives us a fair understanding of the financial market expectation and behaviour during these years.

If we look at the IV surface that was generated for March 2006 (figure 5(a)), we notice that most of the IV observations had positive moneyness. Additionally it was observed that implied volatility has more or less a linear relationship with term to maturity of an option. When we notice the dynamics of the IVS from 2006 to 2010, we can observe considerable changes in the shift, tilt and curvature over these years. These changes were most significant with respect to tilt and curvature i.e. the second and third principal components, while the effect was most prominent with respect to the tilt (second principal component). Further, the IVS seems to suggest that the wider dispersion of the observed data on both sides of zero moneyness reflect an increase in market volatility.

As the financial crisis started to unfold in 2007, we observe that the IVS had more tilt and curvature exhibiting a higher volatility compared to 2006. These observations are supported by the average IV figures described in the table 1. We observe that the average IV's for the years 2006 and 2010 (when the IVS was flat) were 32.19% and 27.18% respectively. Comparatively, the average IV's for the years 2007, 2008 and 2009 should be higher because of the increase in the curvature of the IVS. This too is supported by the average IV's for the years 2007, 2008 and 2009 which were 33.53%, 48.77% and 44.14% respectively.
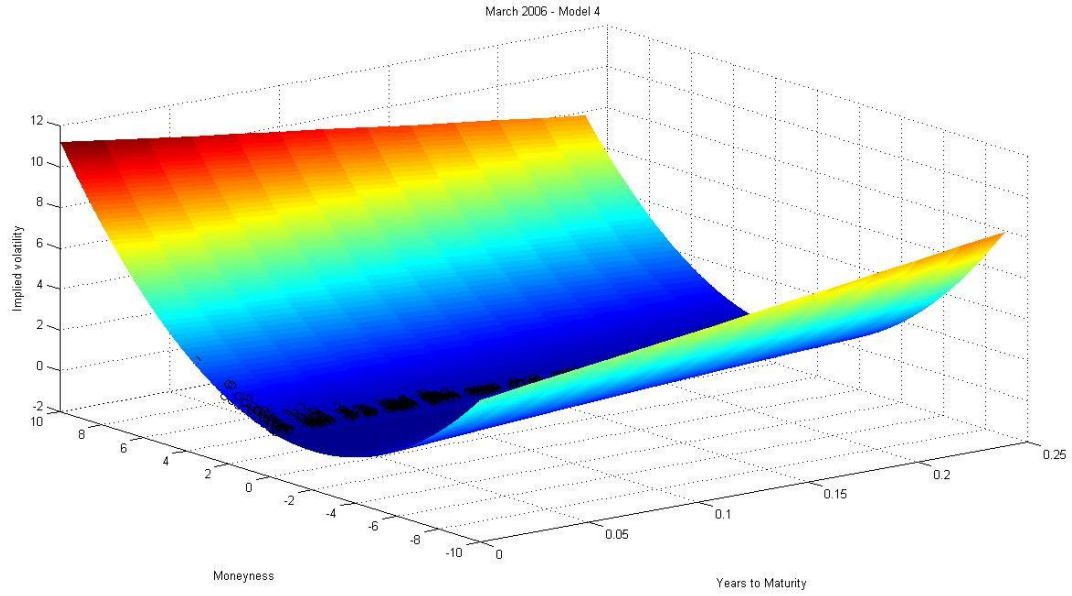
Figure 5(a): Estimated volatility surface for model 4 for March 2006. The black circles are the observed implied volatilities.
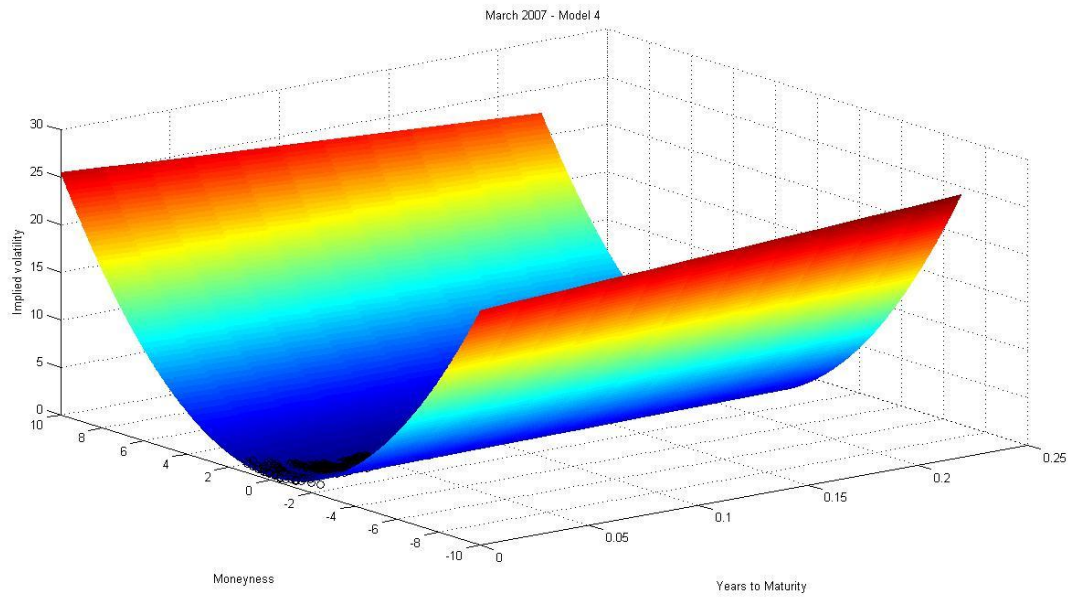
Figure 5(b): Estimated volatility surface for model 4 for March 2007. The black circles are the observed implied volatilities.
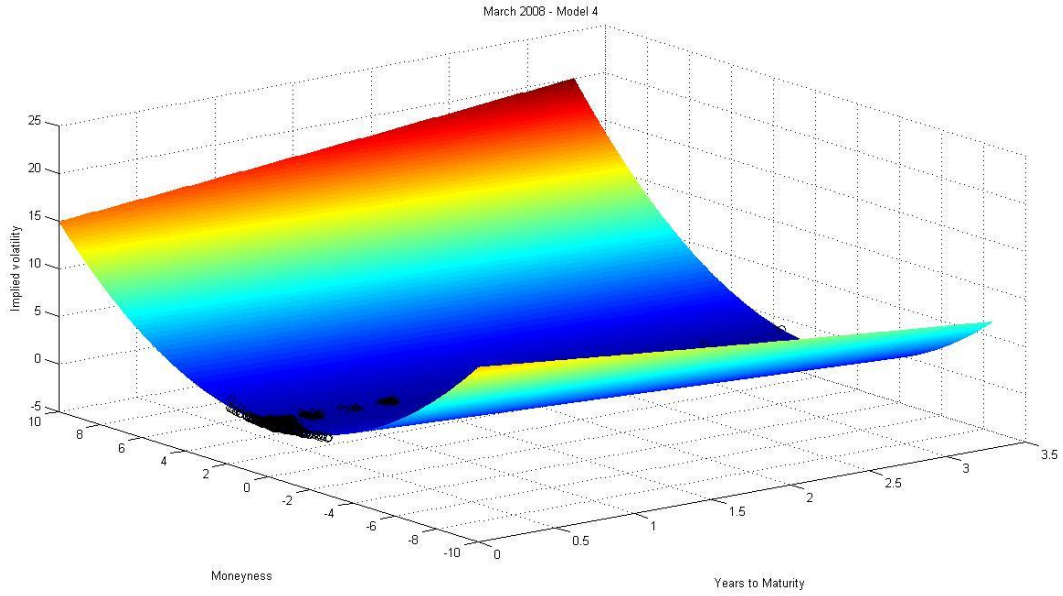
Figure 5(c): Estimated volatility surface for model 4 for March 2008. The black circles are the observed implied volatilities.
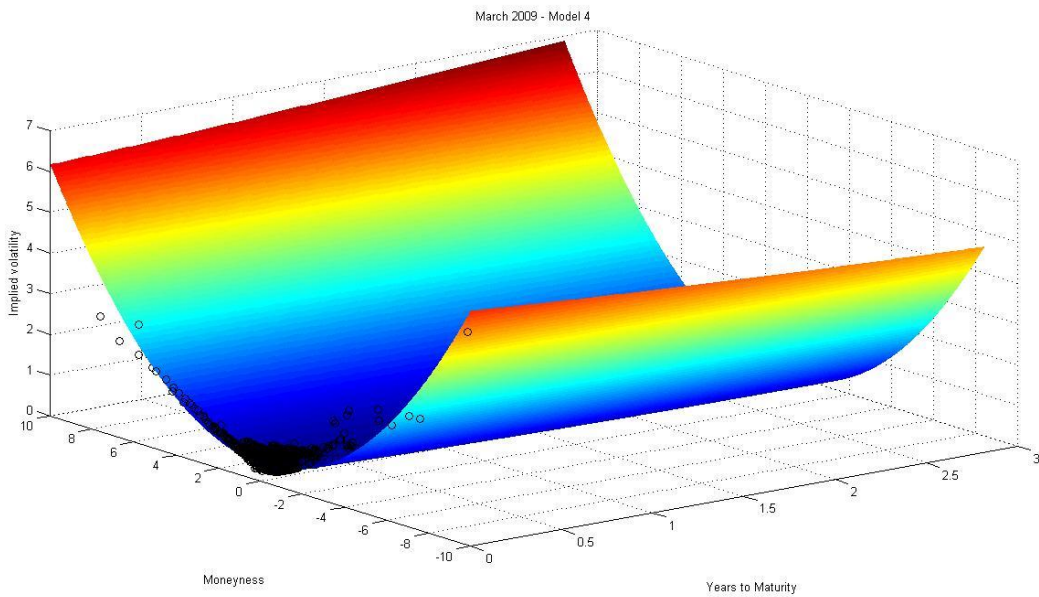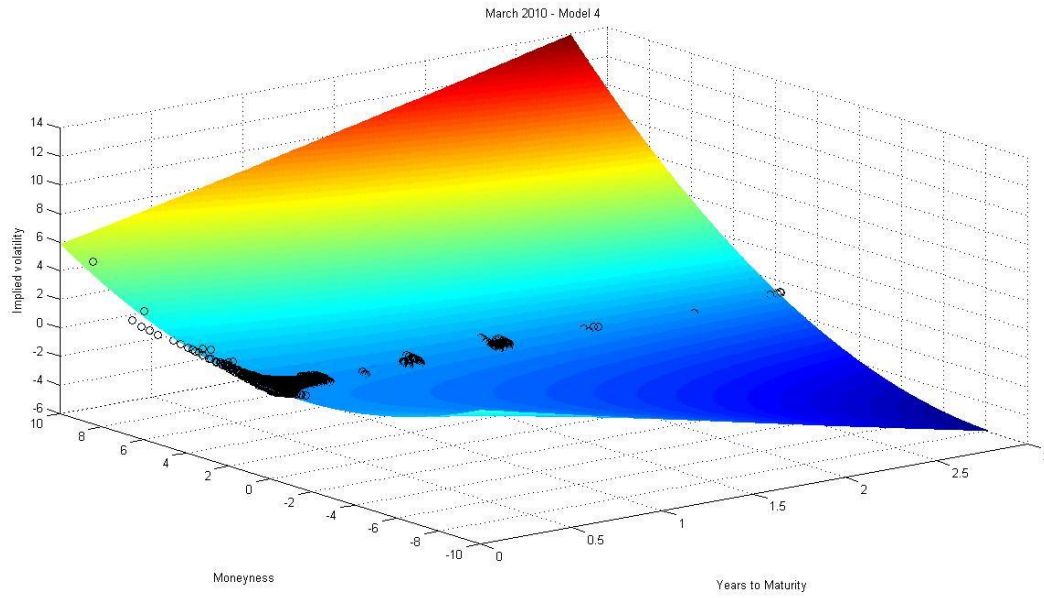

Figure 5(d): Estimated volatility surface for model 4 for March 2009. The black circles are the observed implied volatilities.

Figure 5(e): Estimated volatility surface for model 4 for March 2010. The black circles are the observed implied volatilities.

6. Conclusion

We have determined the implied volatility and modelled the implied volatility surface for the S&P CNX NIFTY index. We use three variations of non-linear parametric optimization techniques to model implied volatilities. Model 1 is constant volatility assumption of BSM, thus is inappropriate. Model 2 uses only the effects of moneyness to estimate the IV surface. We observed that though this model explains certain degree of volatility smile, it fails to account for the tilt and the term structure of volatility. We include the effect of moneyness and the combined effect of moneyness and term to maturity in the model 3 and 4. These two models effectively explain and account for both the term structure and smile effects of volatility. These two models provide a very close fit to the observed data as seen in the volatility surface and established by the goodness of fit factors of IVRMSE, %RMSE, adjusted R^2 and AIC. Model 4 can therefore be used in practise for pricing exotic options that are consistent with the observed volatility surface. Most of the variation in IV surface can be best captured by top three principal components that explain shift, tilt and curvature respectively. Lastly, variations in market sentiment are best expressed by the shape and movements in the implied volatility surface.

Applications:

1. We can use the non-linear parametric model 4 for pricing exotic options with volatilities consistent with those observed in the market.

2. Since the parametric approach to model 4 in an unbiased estimation of future volatility, this model can be used for risk management applications. For options that are dependent on the S&P CNX Nifty, we can use model 4 to estimate the Value at Risk (VaR) for a portfolio of options.

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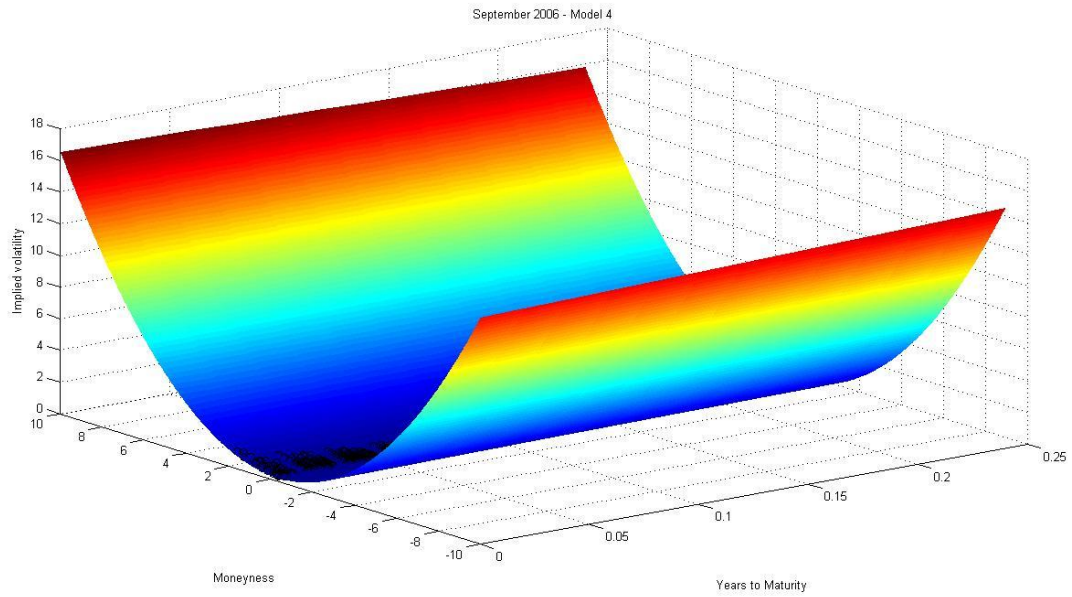# Appendix A: Implied volatility surfaces for September 2006 through September 2010

Figure 6(a): Estimated volatility surface for model 4 for September 2006. The black circles are the observed implied volatilities.
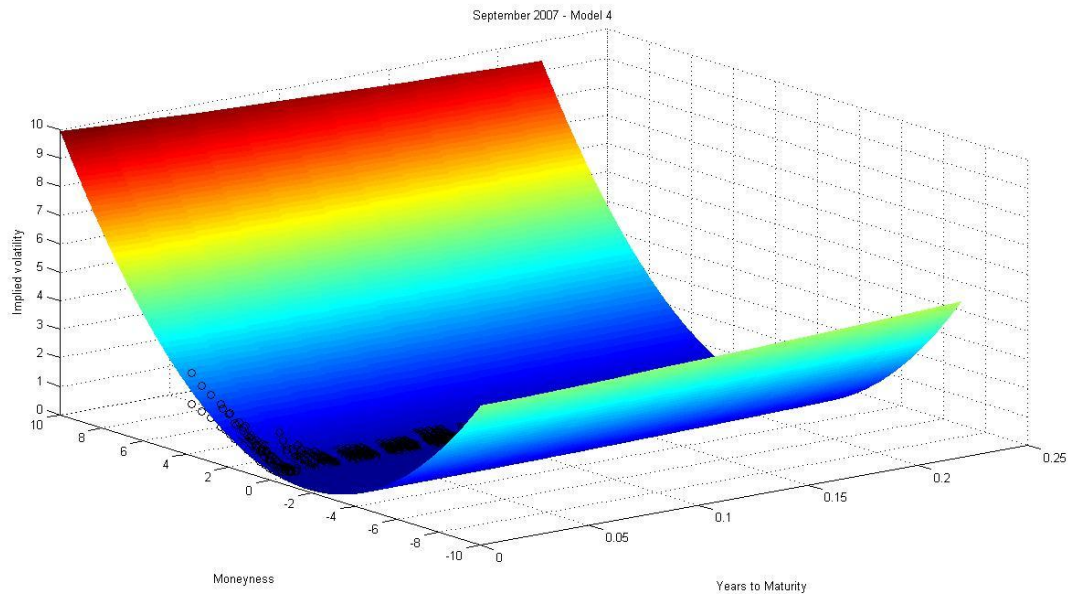

Figure 6(b): Estimated volatility surface for Model 4 for September 2007. The black circles are the observed implied volatilities.
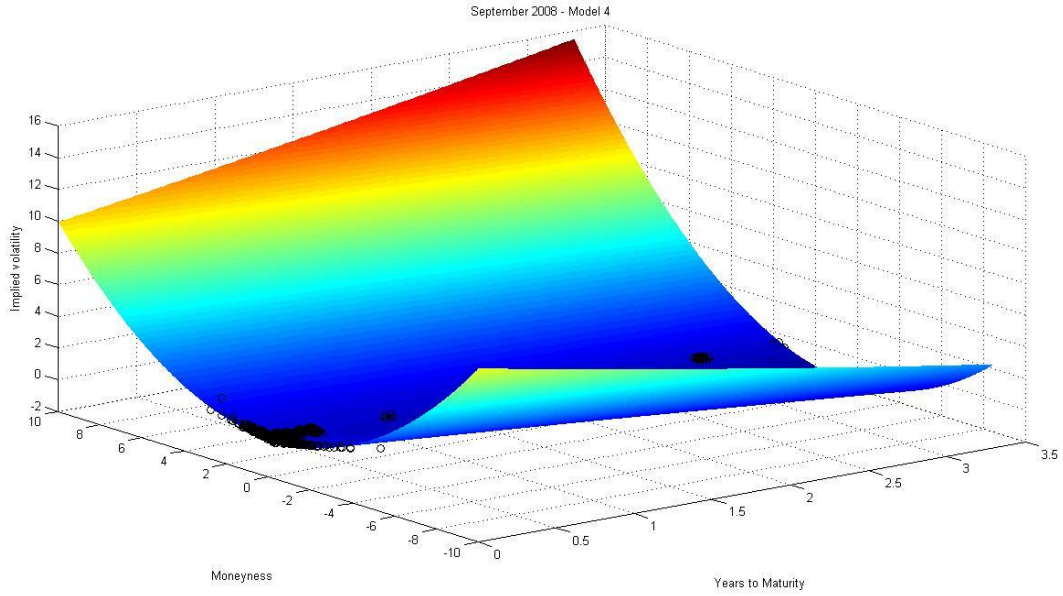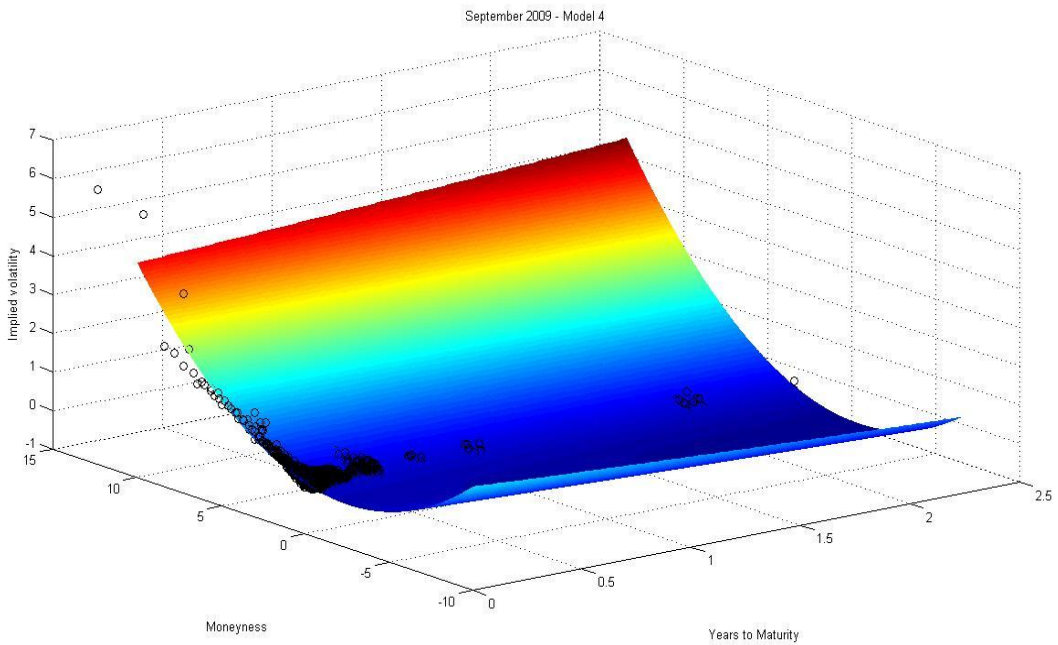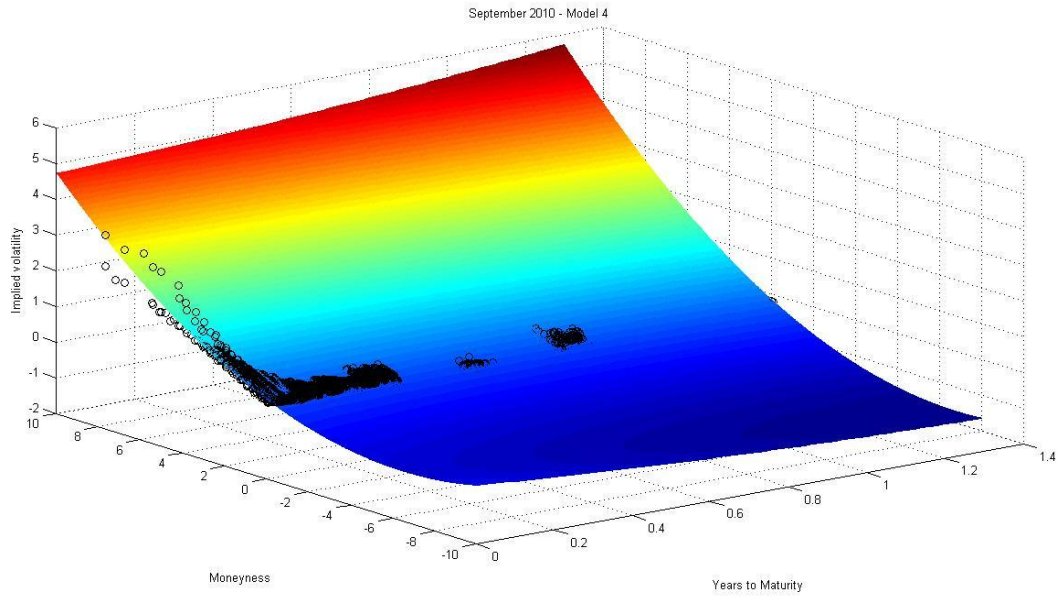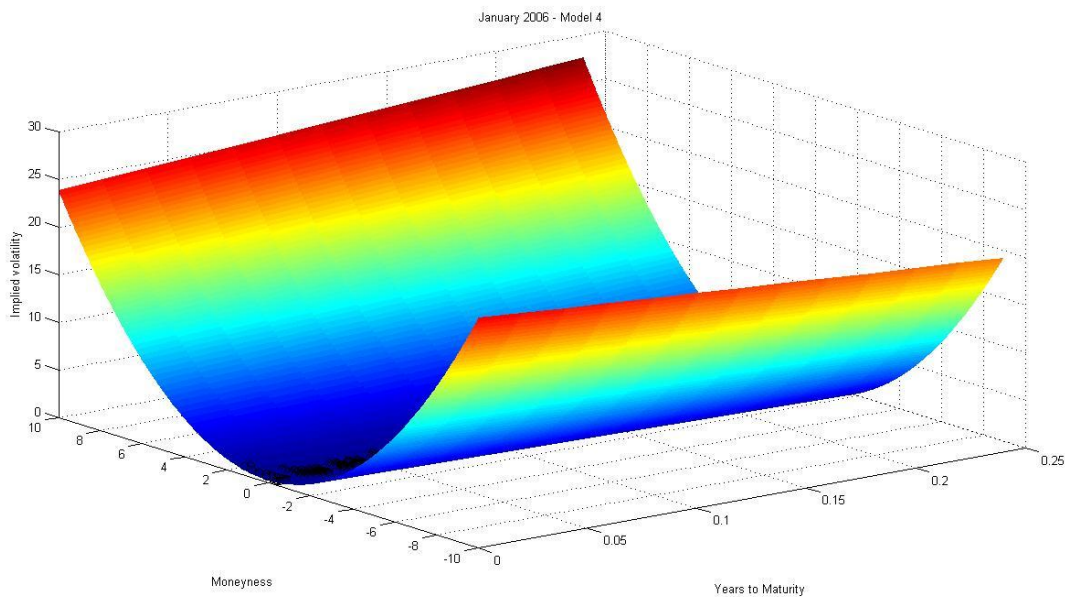
Figure 6(c): Estimated volatility surface for model 4 for September 2008. The black circles are the observed implied volatilities.

Figure 6(d): Estimated volatility surface for model 4 for September 2009. The black circles are the observed implied volatilities.

Figure 6(e): Estimated volatility surface for model 4 for September 2010. The black circles are the observed implied volatilities.

Appendix B: Implied volatility surfaces for January 2006 through January 2010

Figure 7(a): Estimated volatility surface for model 4 for January 2006. The black circles are the observed implied volatilities.
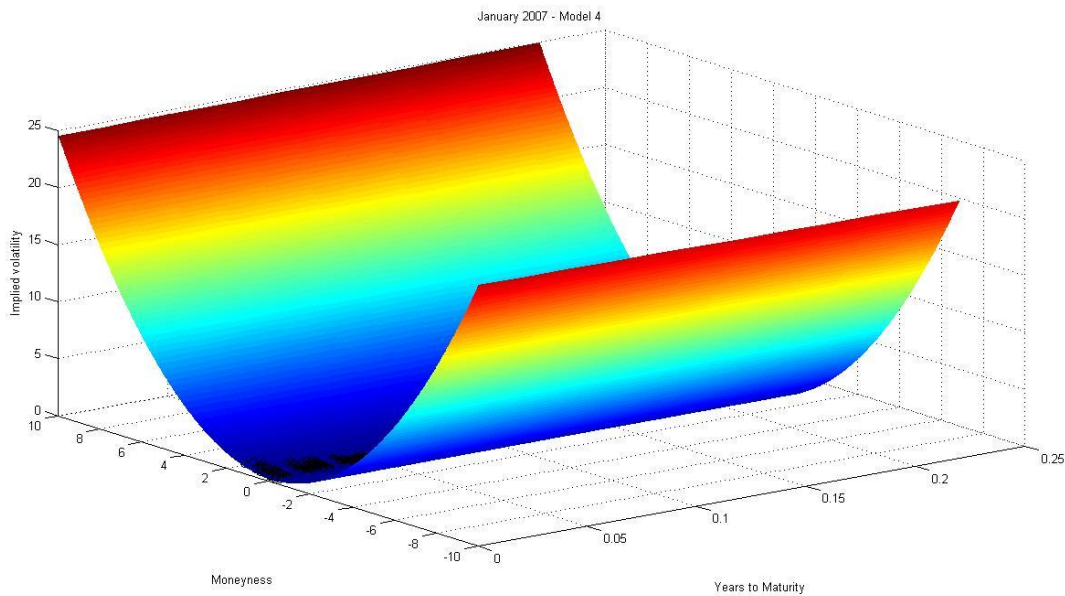

Figure 7(b): Estimated volatility surface for model 4 for January 2007. The black circles are the observed implied volatilities.
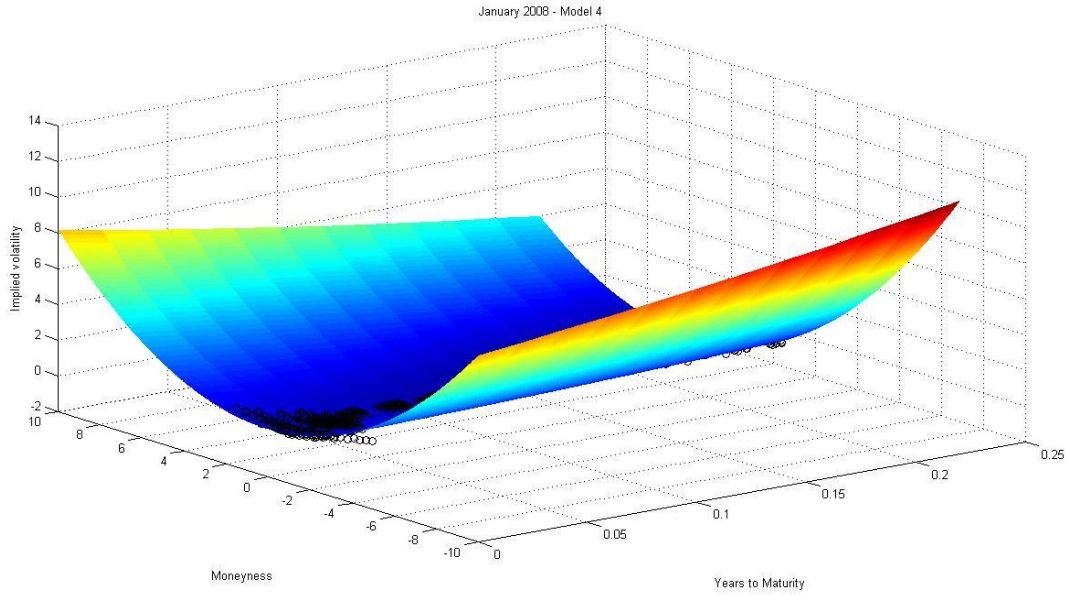
Figure 7(c): Estimated volatility surface for model 4 for January 2008. The black circles are the observed implied volatilities.
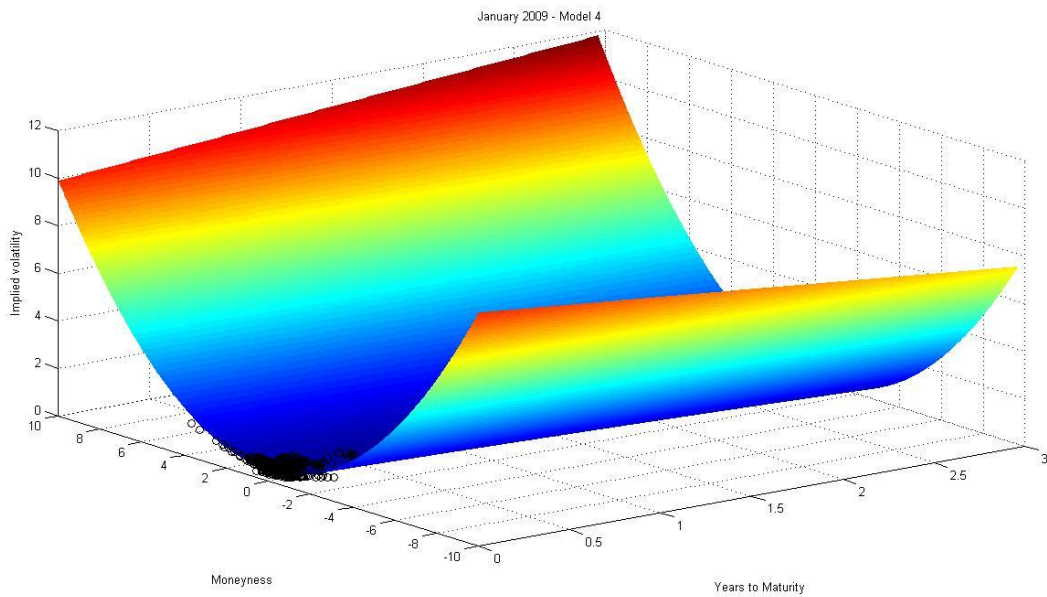

Figure 7(d): Estimated volatility surface for model 4 for January 2009. The black circles are the observed implied volatilities.
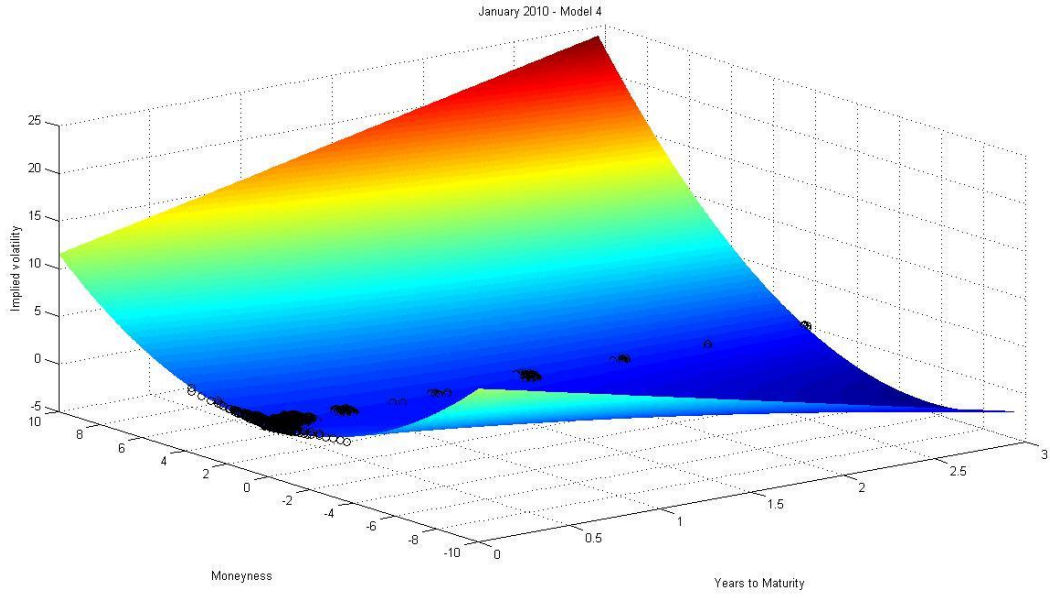
Figure 7(e): Estimated volatility surface for model 4 for January 2010. The black circles are the observed implied volatilities.