

Forecasting Canadian Equity Volatility: the information content of the MVX Index

by

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RESEARCH PROJECT SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE IN FINANCE

BEEDIE SCHOOL OF BUSINESS

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SIMON FRASER UNIVERSITY

Summer 2012

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Abstract

The information content of the option implied equity volatility index (MVX) in Canada is examined. We compare the in-sample and out-of-sample forecasting performance of the GJR model and the combination of GJR and implied volatility index. Forecasts of two measures of volatility are obtained by estimation using an ARCH model based on daily index stock returns and the daily MVX index. The in-sample estimates show that nearly all relevant information is provided by the index return. For out-of-sample forecasting, the MVX index provides the most accurate forecast for all forecast horizons and performance measures considered.

Keywords: GJR model; Forecasting; Stock index return; Implied volatility

Acknowledgements

We express our deepest gratitude to Dr. Christina Atanasova for her valuable feedback and guidance throughout the development and completion of this project, and for her generous support for us during the entirety of the Master of Science in Finance program.

We also extend our thanks to our second reader, Dr. Evan Gatev for reviewing and providing feedback regarding this thesis paper. Last but not least, we thank the SFU MSc Finance faculty and our classmates for making this year so enjoyable, as well as our families for their unconditional support and encouragement.

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1 Introduction

Volatility is of great importance in option trading, firm risk management, portfolio management, and evaluating investor sentiment, among other things. For example, the determination of option value, volatility has the single biggest effect. The recent financial crisis has highlighted the importance of prudent financial risk management, including volatility forecasting.

There are different ways to forecast volatility, such as using intra-day or daily index returns, option implied volatility or the combination of various factors. Some researchers have concluded that in US, implied volatility provides the best method to forecast performance. In this thesis paper, we summarize our attempt to determine which information content is provided by the implied volatility index in Canada, the MVX.

We examined the in-sample estimate and out-of-sample forecasting ability of implied volatilities. Three models were tested:

1. ARCH model based on daily index returns
2. Implied volatility model, and
3. a combination of both 1 and 2 above.

We used a maximum log-likelihood function to estimate the parameters of each model, and examined the models' performance by calculating P, R^2 , RMSE and MAE.

2 Literature Review

ARCH model and Implied Volatility model have been researched in various studies to determine their ability to accurately predict volatility. Some studies (Mayhew and Stivers, 2003 and Szakmarya et al, 2003) conclude that the Implied Volatility model outperforms the other models, including GARCH. Mayhew and Stivers (2003) examine 50 firms with the highest option volume on the Chicago Board Options Exchange between 1988 and 1995 and determine that the ability of implied volatility to subsume all relevant information about conditional variance depends on option trading volume. For most of the active options in the sample, implied volatility outperforms GARCH and subsumes all information in return shocks beyond the first lag. Significantly, for

lower option-volume firms, the performance of implied volatility deteriorates relative to time-series volatility models. Finally, compared to a time-series approach, the implied volatility of equity index options provides reliable incremental information about future firm-level volatility.

A similar study by Szakmarya et al. (2003) uses data from 35 futures options markets from eight separate exchanges to test how well the Implied Volatilities (IVs) embedded in option prices predict subsequently Realized Volatility (RV) in the underlying futures. Their results show that IV outperforms Historical Volatility (HV) as a predictor of the subsequently RV in the underlying futures prices. In most markets examined, they find that HV contains no economically significant predictive information beyond what is already incorporated in IV. These results are consistent with the hypothesis that futures options markets in general, with their minimal trading frictions, are efficient.

The use of high frequency returns (intraday returns) proves to outperform even the Implied Volatility model. Koopmana et al. (2004) compare the forecasting value of historical volatility (extracted from daily return series), implied volatility (extracted from option pricing data) and realized volatility (computed as the sum of squared high frequency returns within a day). They take unobserved component (UC-RV) and long memory models into consideration for realized volatility ¹. Their empirical results show that realized volatility models produce far more accurate volatility forecasts, when compared to models based on daily returns and that long memory models seem to provide the most accurate forecasts.

The volatility forecast model is not only tested on equity return, but is also tested on foreign exchange rates (Ponga et al. 2004), commodities (Martens and Zein, 2004) and futures (Noha and Kimbc, 2006). Ponga et al. (2004) compare forecasts of foreign exchange realized volatility from a short memory ARMA model, long memory ARFIMA model, GARCH model and option implied volatilities. They find that intraday rates provide the most accurate forecasts for the one-day and one-week forecast horizons which implied volatilities are at least as accurate as the historical forecasts for the one-month and three-month horizons. The superior accuracy of the historical forecasts, relative to implied volatilities, comes from the use of high frequency returns, and not from a long memory specification. They also find significant incremental information in historical

¹The true volatility is not observable. Both the realized and the implied volatilities are measures of the true volatility. Realized is backward looking (historical) measure based on standard deviation of realized returns. Implied is forward looking, calculated from option prices using Black-Scholes model.

forecasts, beyond the implied volatility information, for forecast horizons up to one week.

Martens and Zein (2004) suggest that both the measurement and the forecasting of financial volatility is improved using high-frequency data and long memory modeling based on three separate asset classes, equity, foreign exchange, and commodities. Their results for S&P 500, YEN/USD, and Light, Sweet Crude Oil indicate that volatility forecasts based on historical intraday returns do provide good volatility forecasts that can compete with and even outperform implied volatility.

Noha and Kimbc (2006) forecast the volatility of futures market of S&P 500 and FTSE 100 futures using high frequency returns and implied volatility. They find that, for the FTSE 100 futures, historical volatility using high frequency returns outperform implied volatility, while for S&P 500 futures, implied volatility outperform historical volatility. Their results also indicate that historical volatility using high frequency returns could be an unbiased forecast for the FTSE 100 futures.

This thesis paper will concentrate on Canadian equity returns, and the forecast accuracy between ARCH and Implied Volatility models. The reference period used in this research includes the recent financial crisis period, but the data will not be separated into two separate periods, due to the lack of implied volatility index data. Only daily index returns and implied volatility data will be used in this paper.

3 Data

There are two main types of data: daily index returns (TSX 60) and daily implied volatility (MVX). The data are from 2 December 2002 to 15 October 2010 inclusive, with dates adjusted according to the availability of returns and implied volatility data. The in-sample period is from 2 December 2002 to 13 March 2007 providing 1078 daily observations, followed by the out-of-sample period from 14 March 2007 to 15 October 2010 providing 900 daily observations. The daily index returns are obtained from Google Finance website, whereas the daily implied volatilities are downloaded from Montreal Exchange website. The period chosen above is based on the availability of MVX data.

3.1 Daily Index Returns

Daily returns for the TSX 60 index are defined as the natural logarithm of the ratio of consecutive daily closing levels.

3.2 Implied Volatilities

Implied volatilities are considered to be the market expectation of the volatility of the underlying asset of an option, which is reflected in option prices. We can calculate implied volatility from the Black Scholes model, given index level, risk free rate, dividends and contractual provisions. However, the calculated volatilities are subject to biases due to measurement error in those variables. Therefore, we use MVX, introduced by Montreal Exchange, as a substitute.

MVX is calculated from current prices of nearby at-the-money options on the iShares of the CND S&P/TSX 60 Fund (XIU) that are traded on the Montreal Exchange. MVX is an implied volatility index that is updated every minute of a trading day, and is a good proxy of investor sentiment for the Canadian equity market; the higher the Index, the higher the risk of market turmoil.

4 Methodology

4.1 In Sample Model

The model estimations performed for in-sample data is primarily based on a Generalized Autoregressive Heteroskedasticity (GARCH) model. To account for the effect of both good news and bad news, an asymmetric volatility model (GJR-GARCH) developed by Glosten, Jagannathan, and Runkle (1993) is used instead. The implied volatilities are then added to the model(GJR-GARCH) to verify the significance of its informational content. The following three models are estimated (r_t and ε_t apply for all models):

1. GJR-GARCH(1,1) model that utilizes only index returns.

$$r_t = \mu + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_t z_t, \quad z_t \sim \Phi(0, 1) \tag{2}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3}$$

2. MVX Volatility model - based on Peng He (2007)

$$\sigma_t^2 = \alpha_0 + \delta \text{MVX}_{t-1}^2 \tag{4}$$

3. Model that uses both MVX and index returns.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \delta \text{MVX}_{t-1}^2 \tag{5}$$

Here σ_t^2 is the conditional variance of return in period t , s_{t-1} is 1 when $\varepsilon_{t-1} < 0$ and otherwise it is zero, MVX_{t-1} is the daily implied index volatility computed from monthly volatility as $\text{MVX}/\sqrt{22}$. Model 2 is estimated to test whether using MVX implied volatility only will provide result similar to GJR-GARCH(1,1), whereas model 3 is estimated to test whether MVX volatility offers additional information content not available in index returns.

The parameters are estimated by quasi-likelihood methodology by assuming that the standardized returns, z_t , have normal distributions. The log-likelihood function is defined as:

$$\text{LLF} = -\frac{N}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^N \log \sigma_t^2 - \frac{1}{2} \sum_{t=1}^N \frac{\varepsilon_t^2}{\sigma_t^2} \quad (6)$$

The general parameters include: $(\mu, \alpha_0, \alpha_1, \alpha_2, \beta, \delta)$. The log-likelihood function is maximized with constraints: $\alpha_1, \alpha_1 + \alpha_2 \geq 0$.

To assess the predictive power of the model, R^2 value is estimated based on in-sample fitted data (conditional variance σ_t^2) and realized volatility. The realized volatility will be described in the later section of this paper. Higher values of R^2 indicates more accurate in-sample forecasts of volatilities. In addition to that, LLF values are also evaluated for each model, with higher LLF values indicating a better model.

4.2 Calculating t-statistic

Before calculating the t-statistic for each coefficient, the standard errors for each coefficient need to be determined. The standard errors of estimators or coefficients, $\hat{\theta}$, are the square roots of the diagonal terms in the variance-covariance matrix.

$$\text{var}(\theta) = [I(\theta)]^{-1} \quad (7)$$

$$= \left(-E \left[\frac{\partial^2 L}{\partial \theta \partial \theta'} \right] \right)^{-1} \quad (8)$$

The variance covariance matrix is simply the negative of inverse of the information matrix. The score is the gradient of the likelihood ($\frac{\partial L}{\partial \theta}$). If the model is correctly specified, the expectation of the outer product of the scores is equal to the information matrix. The steps to calculate the t-statistics are defined as follow:

1. Given the set of coefficients c , the LLF values are calculated for each data.
2. To calculate the gradient of likelihood, define delta δ for coefficients, for example $1e - 10$. c_delta is then defined as $c \times (1 + \delta)$, dp is defined as $\delta \times c$.
3. Another set of LLF values (LLF_delta) are calculated at the coefficients c_delta .

4. Score value for the coefficient can then be calculated as $\frac{(\text{LLF}-\text{LLF}_{\text{delta}})}{dp}$. Score is a matrix of size $(\text{data_length} \times \text{no_of_coefficients})$.
5. The variance covariance matrix can then be determined by taking the inverse of outer product of score matrix, i.e. $(\text{score}' \text{ score})^{-1}$.
6. The standard errors are determined by taking the square roots of the diagonal elements.
7. Finally, the t-statistic for each coefficients can be calculated by $\frac{\text{coefficient}}{\text{std_error}}$

4.3 Forecasting Methods

The time series of forecasts are estimated based on rolling ARCH models. The in sample size is 1078 trading days, while the out of sample size is 900 trading days. Each model is estimated based on the final 1000 trading days before the forecasted day. The model parameters are then used to forecast the volatility for the next day ($T + 1$). The model and data are then rolled forward one day, deleting the observation at time $(T - 999)$, and adding the observation at time $(T + 1)$. Next, the same estimation is performed again, and the parameters are used to forecast the volatility at time $(T + 2)$. This rolling method is applied until the end of the out of sample period. The method described here is shown on the following figure.

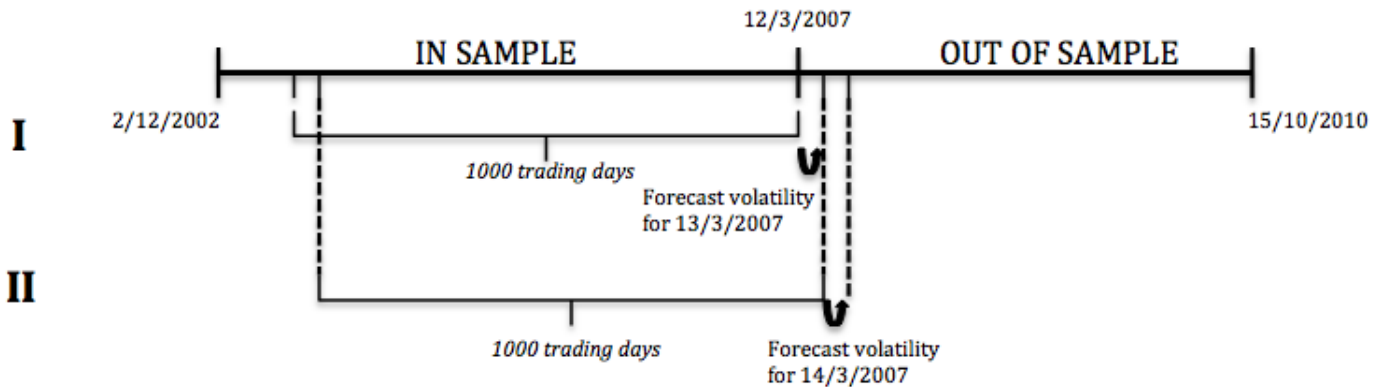


Figure 1: Rolling ARCH Forecast

On each day, forecasts are also made for 5, 10 and 20 day volatility. Realized volatility which is used to check on the forecast accuracy is calculated as squared excess returns. The forecasts made at time T are:

$$(r_{T+1} - \mu)^2 \text{ and } \sum_{j=1}^N (r_{T+j} - \mu)^2, N = 5, 10, 20$$

When predicting these values, we assume that the conditional expected return μ is constant, such that the results are not sensitive to the choice of μ ; annual expected return of 10% is used. The daily μ is then calculated as:

$$\mu = (1 + 0.1)^{1/252} - 1 = 0.000378$$

The forecasts are calculated based on the four models specified on the following subsections.

4.3.1 Historic Volatility

Historic volatility is based on a simple method, and is used as a comparison against more sophisticated models. The one step ahead forecast is calculated as sample variance of daily returns over recent 100 trading days (from time $T - 99$ to T inclusive).

$$\sigma_{T+1}^2 = \frac{1}{100} \sum_{j=0}^{99} (r_{T-j} - \bar{r}_T)^2, \quad \bar{r}_T = \frac{1}{100} \sum_{j=0}^{99} r_{T-j} \quad (9)$$

To calculate 5, 10 and 20 day volatility, the one-step ahead forecast is multiplied by 5, 10 and 20 respectively.

4.3.2 GJR(1,1) Forecast

The one-step ahead forecast, σ_{T+1}^2 is defined by the following recursive formula,

$$\sigma_{T+1}^2 = \alpha_0 + \alpha_1 \varepsilon_T^2 + \alpha_2 s_T \varepsilon_T^2 + \beta \sigma_T^2 \quad (10)$$

where s_T equals 1 when $\varepsilon_T < 0$ and otherwise equals 0. The parameters above are estimated based on 1000 trading days immediately preceding the forecasted day. Forecasts for 5, 10, and 20 day

volatility are calculated by aggregating expectations.

$$E(\sigma_{T+j}^2 | I_T^{(1)}) = \alpha_0 + p_{gjr} E(\sigma_{T+j-1}^2 | I_T^{(1)}), j > 1 \quad (11)$$

where $I_T^{(1)} = \{r_{T-i}, 0 \leq i \leq 999\}$ and persistence $p_{gjr} = \alpha_0 + \frac{1}{2}\alpha_2 + \beta$ for GJR(1,1) model, assuming that returns have symmetric distribution. Then the volatility forecast for N days becomes $\sum_{j=1}^N E(\sigma_{T+j}^2 | I_T^{(1)})$.

4.3.3 Volatility Forecast using MVX

This forecast is based on solely MVX data. The model is based on a simple regression on MVX volatility. The one step ahead forecast is defined by:

$$\sigma_{T+1}^2 = \alpha_0 + \delta MVX_T^2 \quad (12)$$

Parameters α_0 and δ are estimated from $I_T^{(2)} = \{r_{T-i}, MVX_{T-i}, 0 \leq i \leq 999\}$. The 5, 10 and 20 day volatility forecast is produced by multiplying the one-step ahead forecast by 5, 10 and 20 respectively.

4.3.4 Volatility Forecast using Index Return and MVX

The last forecast is based on the fourth model, combining both index returns and MVX volatility data. The one-step ahead forecast is given by:

$$\sigma_{T+1}^2 = \alpha_0 + \alpha_1 \varepsilon_T^2 + \alpha_2 s_T \varepsilon_T^2 + \beta \sigma_T^2 + \delta MVX_T^2 \quad (13)$$

where s_T equals 1 when $\varepsilon_T < 0$ and otherwise equals 0. Parameters $\alpha_0, \alpha_1, \alpha_2, \beta$ and δ are estimated from $I_T^{(2)} = \{r_{T-i}, MVX_{T-i}, 0 \leq i \leq 999\}$. To produce 5, 10 and 20 day volatility forecasts, the simple multiplicative method is used, i.e. multiplying one-step ahead forecast by 5, 10 and 20 respectively.

4.4 Forecast Evaluation

There are several evaluation criteria to assess the relative predictive accuracy of the four forecasting methods. Given forecasts $x_{T,N}$ made at times $T = s, \dots, n - N$ and realized volatilities $y_{T,N}$ at the same time range, the following values are calculated:

1. Proportion of variance explained by forecasts, suggested by Blair et al. (2001)

$$P = 1 - \frac{\sum_{T=s}^{n-N} (y_{T,N} - x_{T,N})^2}{\sum_{T=s}^{n-N} (y_{T,N} - \bar{y})^2} \quad (14)$$

Higher value of P-statistic indicates better accuracy.

2. Root Mean Square Error (RMSE)

$$\text{RMSE} = \frac{1}{n - N} \sum_{T=s}^{n-N} (y_{T,N} - x_{T,N})^2 \quad (15)$$

Lower value of RMSE indicates better accuracy.

3. Mean Absolute Error (MAE)

$$\text{MAE} = \frac{1}{n - N} \sum_{T=s}^{n-N} |y_{T,N} - x_{T,N}| \quad (16)$$

Lower value of MAE indicates better accuracy.

4. Squared correlation, R^2 from regression

$$y_{t,N} = \alpha + \beta x_{t,N} + \varepsilon_t \quad (17)$$

Higher value of R^2 indicates better fit and accuracy.

P statistic, which measures the forecast accuracy, is at most equal to R^2 that is often interpreted as a measure of information content.

5 Results

5.1 In Sample ARCH Results

The parameter estimates, along with the log-likelihoods and squared correlations R^2 are presented in Table 1.

Models for S&P TSX 60 index daily returns from 2 Dec 2002 to 15 Oct 2010

$$r_t = \mu + \varepsilon_t$$

$$\varepsilon_t = \sigma_t z_t, \quad z_t \sim \Phi(0, 1)$$

Model 1: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$

Model 2: $\sigma_t^2 = \alpha_0 + \delta \text{MVX}_{t-1}^2$

Model 3: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 s_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \delta \text{MVX}_{t-1}^2$

s_t is 1 if ε_t is negative otherwise s_t is zero

MVX is a measure of implied volatility.

t-ratios are shown in parentheses.

Parameter	Model		
	(1)	(2)	(3)
$\alpha_0 \times 10^{-6}$	3.1690 (2.79)	15.9616 (2.66)	5.6279 (2.64)
α_1	0.0014 (0.08)		0 (0)
α_2	0.0845 (2.97)		0.1349 (2.67)
β	0.8986 (30.61)		0.7044 (7.58)
δ		0.0437 (6.18)	0.0077 (2.06)
$\mu \times 10^{-4}$	6.284 (2.76)	6.9542 (3.06)	6.5796 (2.88)
log-L	3749.69	3746.53	3753.43
Excess log-L		-3.16	3.74
R^2	0.0314	0.0234	0.0273

Table 1: In-Sample ARCH Result

The results are obtained from 2 December 2002 to 12 March 2007 inclusive. Combination of GJR and MVX has the highest log-likelihood value, while MVX model has the lowest log-likelihood. The excess log-likelihood value is calculated by using GJR model (model 1) as the base. The excess log-likelihood value for MVX model is negative, indicating that MVX data does not have incremental information, i.e. the data from index return is sufficient to model the volatility. The third model has positive excess log likelihood, but the value is not significant. This indicates that MVX data has little incremental value to the model.

The predictive power indicated by R^2 shows that GJR model has the best fit among the models. The comparison between realized volatilities and estimated in-sample volatilities for each model is shown on the three plots below.

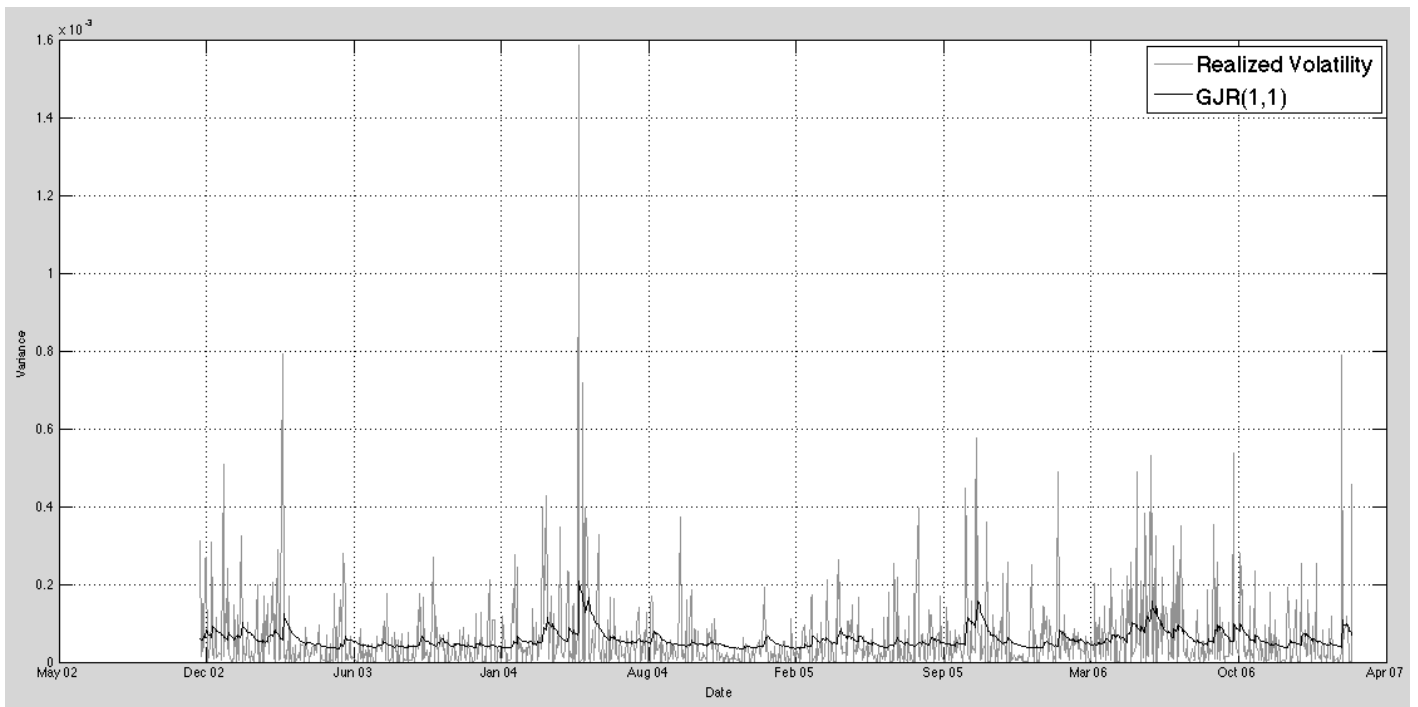


Figure 2: GJR(1,1) Model - In Sample Estimates

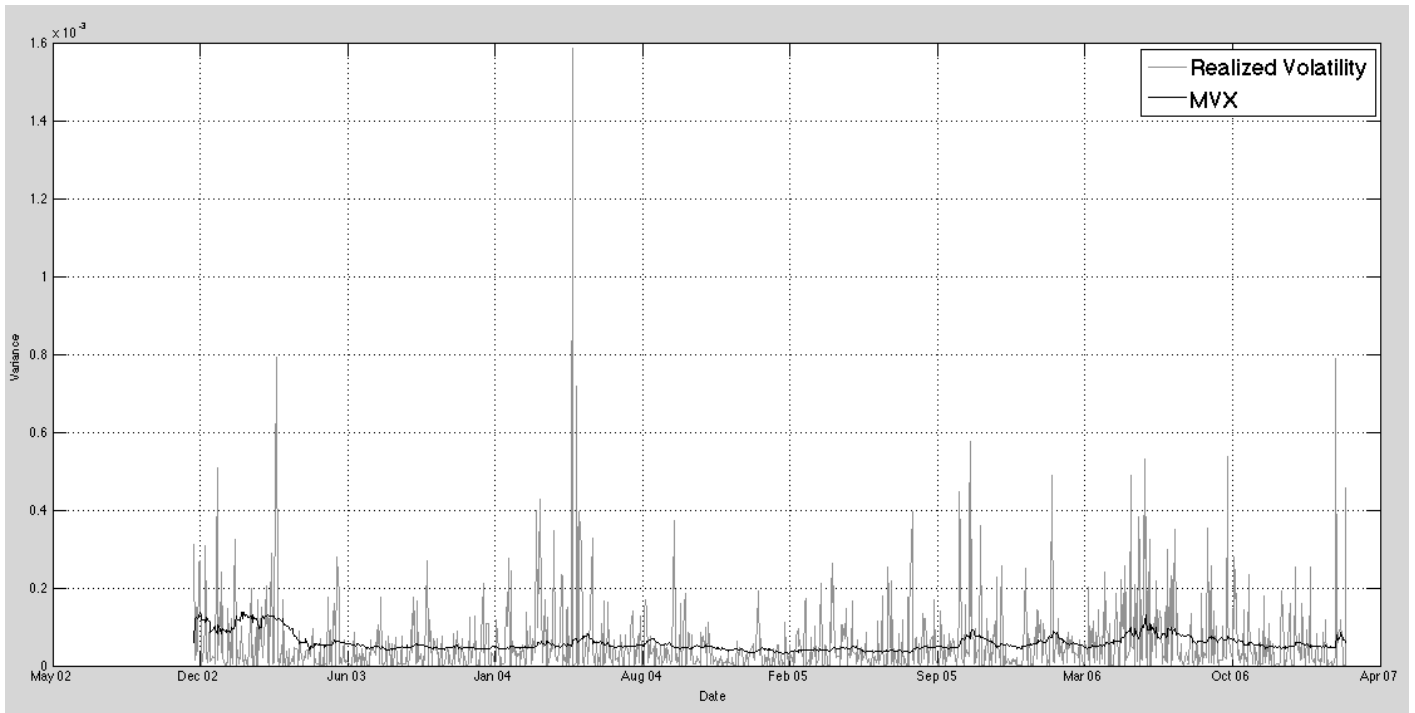


Figure 3: MVX Model - In Sample Estimates

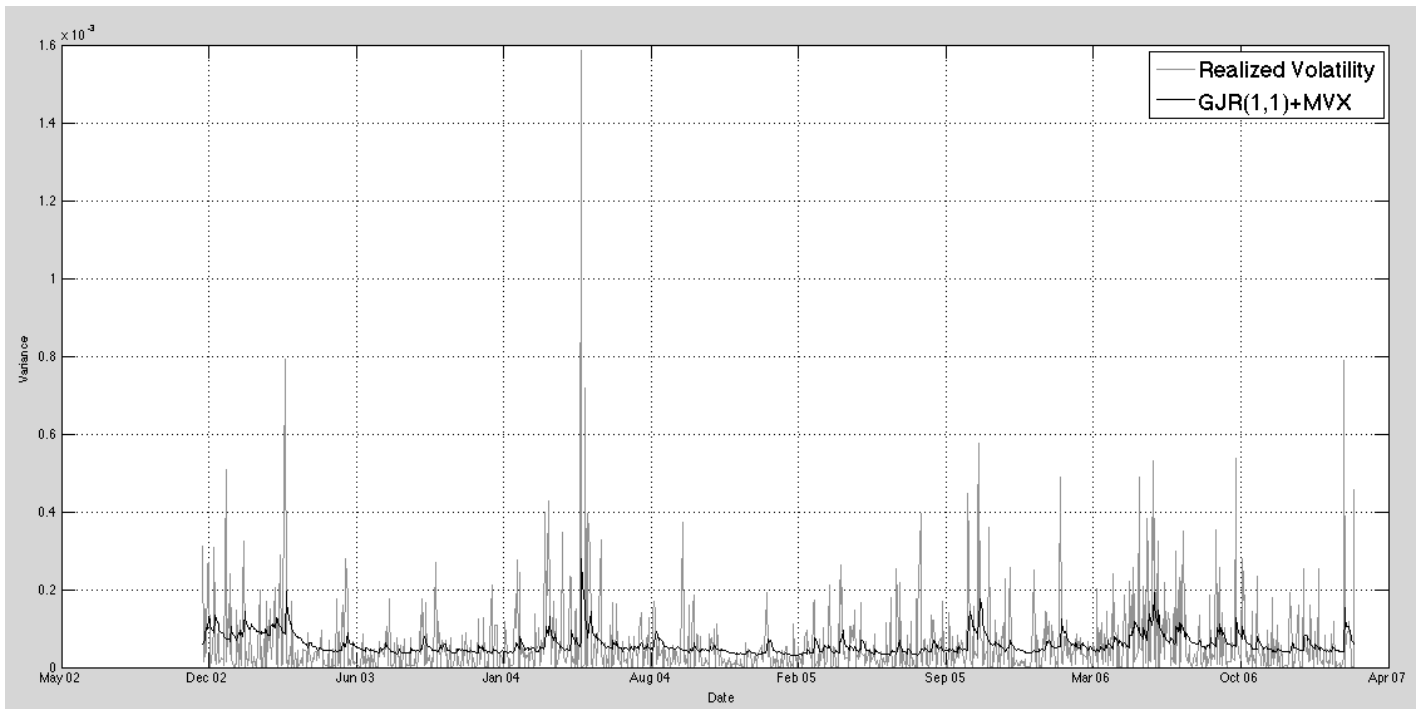


Figure 4: GJR(1,1) + MVX Model - In Sample Estimates

The first model is the standard GJR(1,1) model, that uses only index returns to characterize the conditional variance. The ratio between multiplier for squared negative returns $\alpha_1 + \alpha_2$ and squared positive returns α_1 is fairly large, indicating a substantial asymmetric effect. The persistence estimate is $\alpha_1 + \frac{1}{2}\alpha_2 + \beta = 0.94425$.

The second model is the MVX model which makes use of MVX implied volatility data to estimate the conditional variance. When compared to the other two models, the MVX model performs worse in data fitting. However, the high t-statistic value of δ (6.18) shows the significance of the MVX implied volatility index data on this model.

The third model is combination of GJR(1,1) and MVX model. Comparing Figure 2 and Figure 4, the in sample estimates' difference is fairly minimal, confirming that MVX volatility data has little incremental information. Another observation is that the t-statistic value of δ (2.06) is no longer significant, assuming 99% confidence interval (critical t-statistic value = 2.58). It can be seen that the realized volatilities are very noisy on the three plots, but the estimates are not so volatile.

5.2 Out of Sample Forecasting

The out-of-sample forecast accuracy is compared from 14 March 2007 to 15 October 2010. Table 2 summarizes the four accuracy measures (P-statistic, RMSE, MAE and R^2) for each model, including additional Historical Volatility model. The results obtained are different with those from in-sample estimates.

The relative accuracy of volatility forecasts from March 2007 to October 2010

The accuracy of forecasts are measured by:

$$P = 1 - \frac{\sum_{T=s}^{n-N} (y_{T,N} - x_{T,N})^2}{\sum_{T=s}^{n-N} (y_{T,N} - \bar{y})^2}$$

$$RMSE = \frac{1}{n-N} \sum_{T=s}^{n-N} (y_{T,N} - x_{T,N})^2$$

$$MAE = \frac{1}{n-N} \sum_{T=s}^{n-N} |y_{T,N} - x_{T,N}|$$

A. Values of P for forecasts of sums of squared excess returns

Forecast	N=1	N=5	N=10	N=20
GJR	0.2542	0.5644	0.5646	0.4945
MVX	0.2773	0.5678	0.5827	0.5036
GJR + MVX	0.2033	0.4060	0.4383	0.3913
HV	0.0696	0.1384	0.1196	0.058

B. Values of R^2 for forecasts of sums of squared excess returns				
Forecast	N=1	N=5	N=10	N=20
GJR	0.2564	0.5736	0.5796	0.5229
MVX	0.3147	0.6223	0.6239	0.5213
GJR + MVX	0.2702	0.5224	0.5590	0.4889
HV	0.0880	0.1887	0.1909	0.1715
C. Values of RMSE for forecasts of sums of squared excess returns				
Forecast	N=1	N=5	N=10	N=20
GJR	0.00078	0.00195	0.00365	0.00741
MVX	0.00076	0.00194	0.00358	0.00734
GJR + MVX	0.00080	0.00228	0.00415	0.00813
HV	0.00087	0.00274	0.00519	0.01011
D. Values of MAE for forecasts of sums of squared excess returns				
Forecast	N=1	N=5	N=10	N=20
GJR	0.00029	0.00087	0.00163	0.00315
MVX	0.00029	0.00088	0.00162	0.00305
GJR + MVX	0.00027	0.00093	0.00180	0.00355
HV	0.00034	0.00126	0.00246	0.00496

Table 2: Out of Sample Forecast Accuracy

Based on the table, following are the observations:

1. Comparing the different N-day ahead forecasts, the accuracy of the forecast is increasing up to 10-day ahead generally, as shown by the P-statistic and R^2 value. The value of these statistics for 20-day ahead forecast are decreasing for every model.
2. MVX Volatility model has the best accuracy for out of sample forecasting. Generally, it has the highest P-statistic and R^2 , and the lowest RMSE and MAE.
3. GJR(1,1) ranks second to MVX. Combining both models results in decrease of accuracy.
4. Simple forecasting model, like Historical Volatility, are not better than the more sophisticated models.
5. The values of RMSE and MAE are increasing for all models when N is increased.
6. Combining GJR and MVX causes a decline in forecasting accuracy, as shown by the lower P and higher error values. The stand-alone models work better in forecasting.

7. The increase in value of R^2 after 5-day ahead forecast is minimal, indicating the lack of incremental information for 10-day and 20-day ahead forecasts.
8. 5-day ahead forecast is typically more accurate than 1-day ahead forecast, shown by the higher values of P and R^2 .
9. The values of P and R^2 are quite similar, the difference $R^2 - P$ is pretty small.

The following four figures show the plot of squared excess returns and forecasts for each model. Since the accuracy of forecasts between GJR and MVX is difficult to observe based on the plots, the accuracy needs to be determined based on the measures described above.

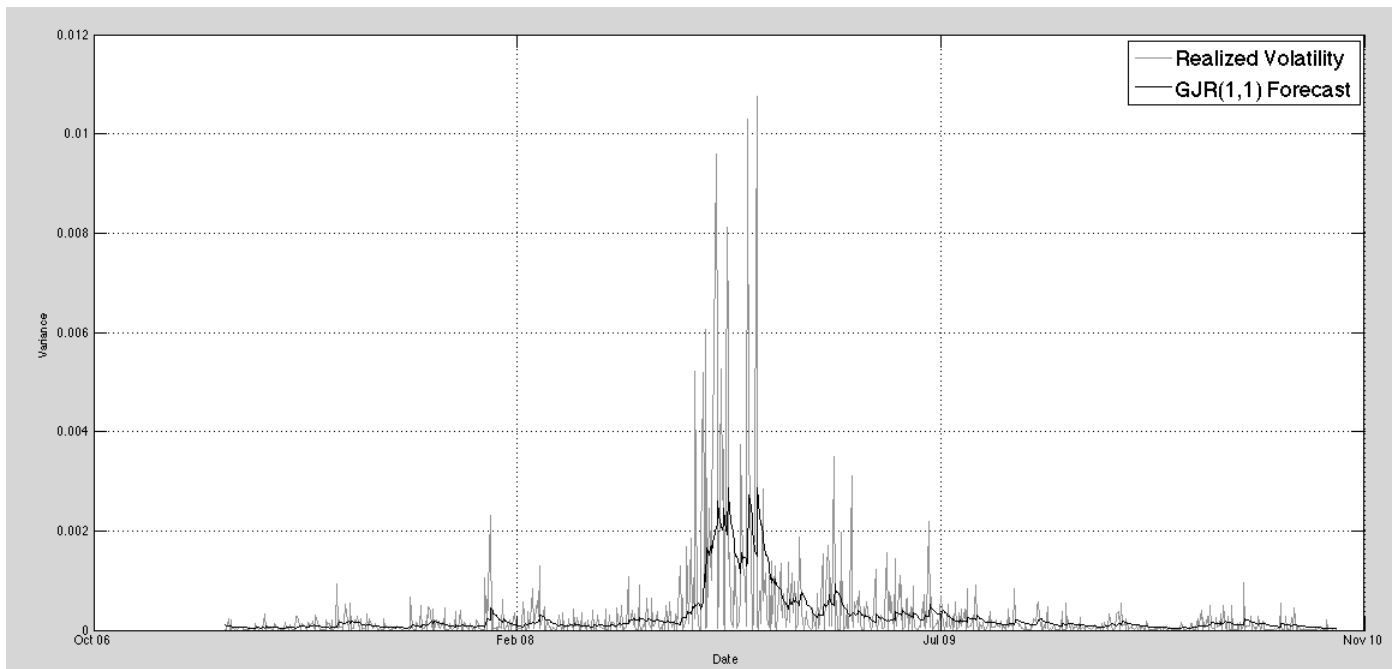


Figure 5: GJR(1,1) Model - Out of Sample Forecast

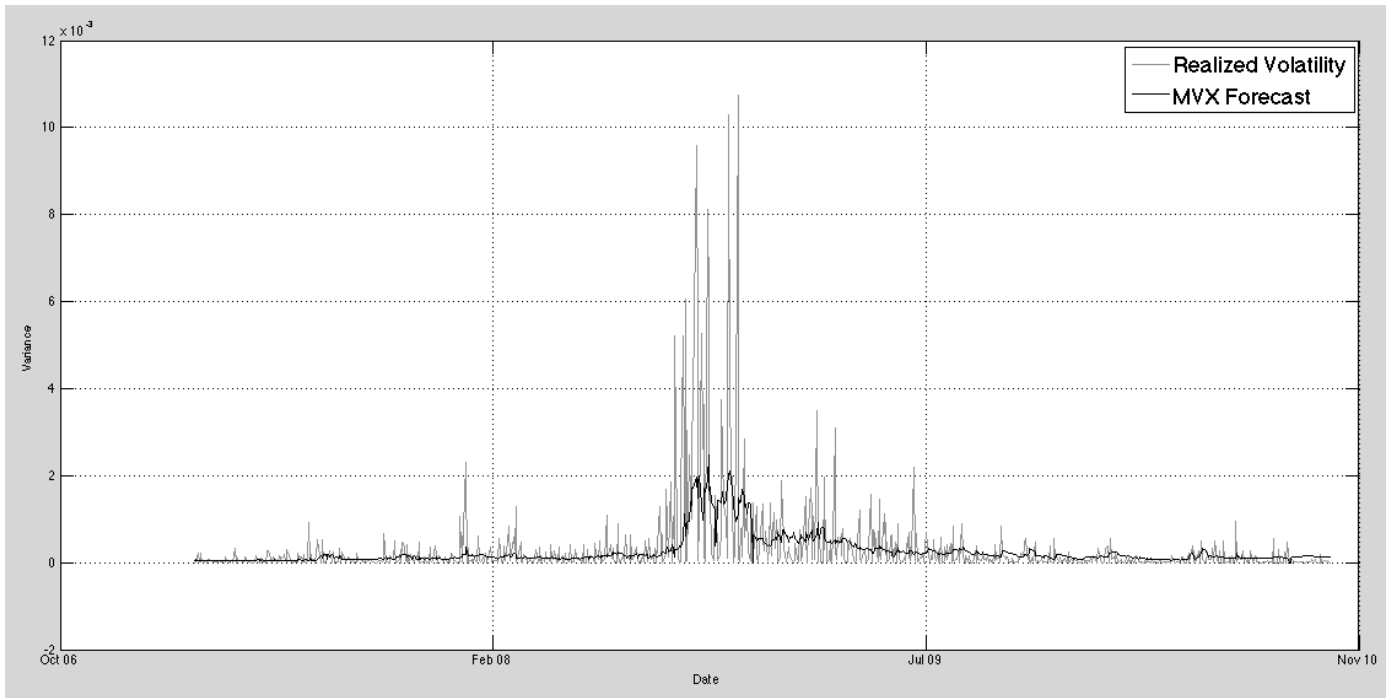


Figure 6: MVX Model - Out of Sample Forecast

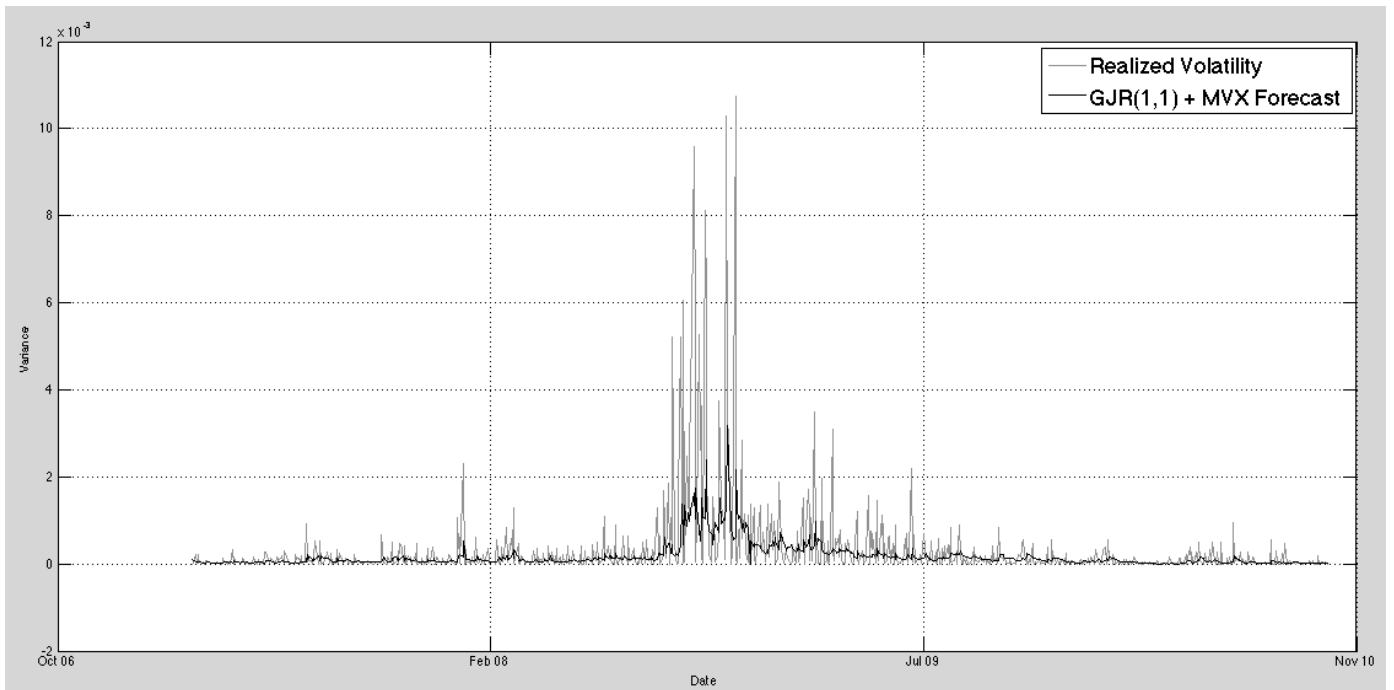


Figure 7: GJR(1,1) + MVX Model - Out of Sample Forecast

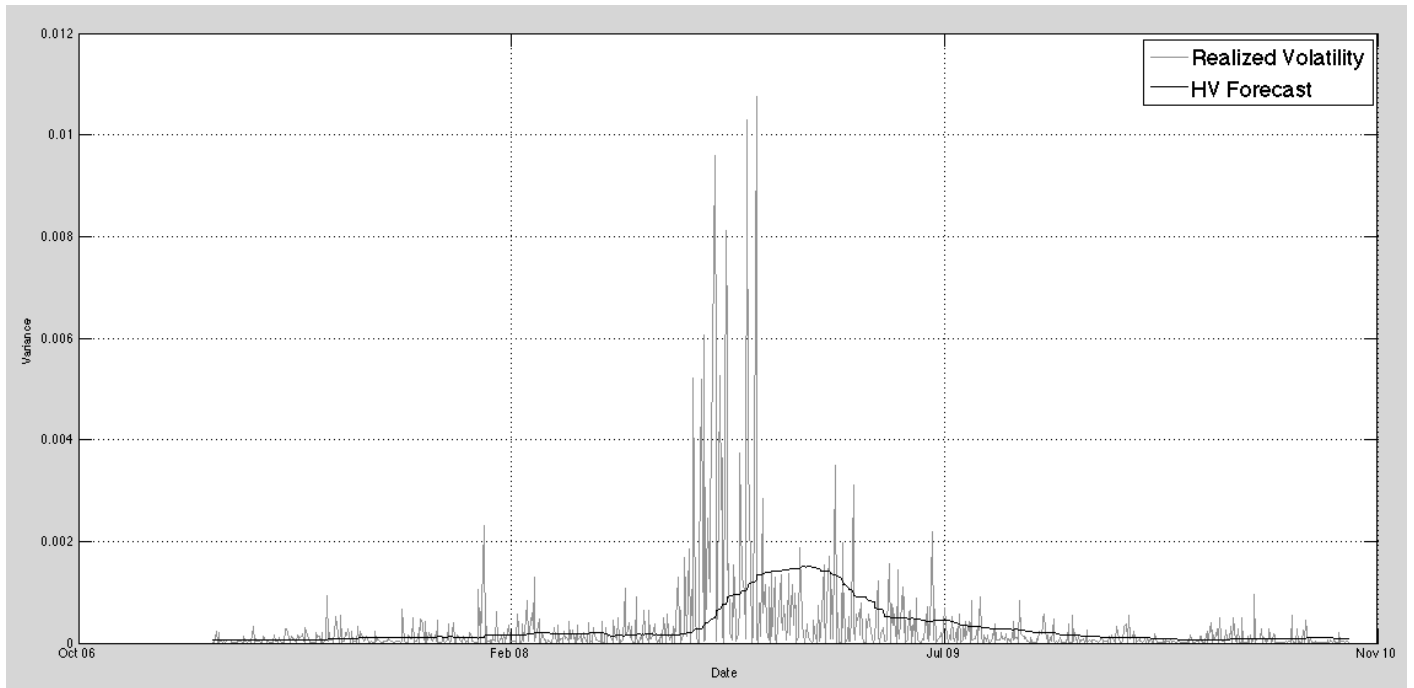


Figure 8: HV Model - Out of Sample Forecast

6 Conclusions

Previous studies of index returns and implied volatilities have produced several differing outcomes. The most often cited result states that the implied volatility model outperforms the other models, but it does not outperform the high frequency return (intraday return) model. Our in-sample analysis shows that the Implied Volatility model performs slightly worse than GJR-GARCH model (using daily index return), but the MVX volatility data is comparable to the daily index return as shown by the high significance in MVX volatility model. Combining the two models shows that MVX implied volatility does not offer incremental information. This result is in agreement with the conclusion of Blair et al (2001) for VIX on S&P 100.

Out-of-sample volatility forecasts show that MVX volatility model performs the best. A combination of daily index returns and MVX volatility data results in a decline in forecasting accuracy, showing that MVX has little incremental information when combined with daily index returns. Additionally, the 5-day ahead forecast is better than the 1-day ahead forecast, but the forecast accuracy does not increase for 10-day and 20-day ahead forecasts. This result confirms the conclusion produced by Mayhew and Stivers (2003) and Szakmarya et al (2003).

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