

COMPUTER TESTBED FOR EXPERIMENTS
ON COORDINATION

by

Sultan Orazbayev

B.A, Simon Fraser University, 2004

A PROJECT SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS
in the Department
of
Economics

© Sultan Orazbayev 2005
SIMON FRASER UNIVERSITY
Fall 2005

All rights reserved. This work may not be
reproduced in whole or in part, by photocopy
or other means, without the permission of the author.

APPROVAL

Name: **Sultan Orazbayev**
Degree: **M. A. (Economics)**
Title of Project: **Computer Testbed For Experiments On Coordination**

Examining Committee:

Chair: Ken Kasa

Jasmina Arifovic
Senior Supervisor

Alexander Karaivanov
Supervisor

Steve Mongrain
Internal Examiner

Date Approved: Wednesday, November 30, 2005



**SIMON FRASER
UNIVERSITY library**

DECLARATION OF PARTIAL COPYRIGHT LICENCE

The author, whose copyright is declared on the title page of this work, has granted to Simon Fraser University the right to lend this thesis, project or extended essay to users of the Simon Fraser University Library, and to make partial or single copies only for such users or in response to a request from the library of any other university, or other educational institution, on its own behalf or for one of its users.

The author has further granted permission to Simon Fraser University to keep or make a digital copy for use in its circulating collection, and, without changing the content, to translate the thesis/project or extended essays, if technically possible, to any medium or format for the purpose of preservation of the digital work.

The author has further agreed that permission for multiple copying of this work for scholarly purposes may be granted by either the author or the Dean of Graduate Studies.

It is understood that copying or publication of this work for financial gain shall not be allowed without the author's written permission.

Permission for public performance, or limited permission for private scholarly use, of any multimedia materials forming part of this work, may have been granted by the author. This information may be found on the separately catalogued multimedia material and in the signed Partial Copyright Licence.

The original Partial Copyright Licence attesting to these terms, and signed by this author, may be found in the original bound copy of this work, retained in the Simon Fraser University Archive.

Simon Fraser University Library
Burnaby, BC, Canada

Abstract

Experimental studies of coordination games consistently show that large groups are unable to escape the inefficient equilibrium. Weber (2005) modifies experimental design and obtains large groups that coordinate on the efficient equilibrium. This feature is incorporated into a computer testbed. After examining both individual and social learning, it is found that experimental results cannot be described with a simple learning process. A discussion on possible explanations concludes the project.

Acknowledgments

I sincerely appreciate help from Prof. Arifovic and Prof. Karaivanov. Their advice and patience have kept me on the track throughout the difficult times.

I would also like to thank faculty and staff at the Department of Economics for guiding me throughout the graduate program at SFU.

Contents

Approval	ii
Abstract	iii
Acknowledgments	iv
Contents	v
Introduction	1
Experimental Data	4
Computer Testbed	8
Conclusion	21
Appendix	22
Bibliography	23

Introduction

Coordination games are an important tool in macro and development economics. The team production model is used in macroeconomics to model fluctuations in macroeconomic activity that result from changes in agents' expectations and not economic fundamentals, Cooper and John (1988). In development economics coordination games describe situations where an economy can get stuck in a poverty trap as a result of poor coordination.

Experiments with coordination games have shown that large groups are unable to coordinate on the payoff dominant equilibrium (e.g. Van Huyck et al (1990)). This result seems to contradict the real world observations of efficiently-operating large firms and suggests that a particular feature is missing in the experiments. An example from airline business is preparation of an airplane for departure. The plane needs to be fueled, catered, checked for safety and prepared in other ways for the flight. A delay in one of these procedures will lead to a delay in the departure, Camerer (2003).

Weber (2005) modifies the experimental design by allowing groups that are increasing in size at an exogenously determined rate. In his experiments the groups that grow sufficiently slow are able to coordinate on the efficient equilibrium. In a related set of experiments, Weber (2005) allows for endogenous growth by selecting a "manager", who decides whether to increase group size or not at each time period.

The motivation for this project is to see whether it is possible to construct a computer model that would generate results similar to the experimental data. While such a 'testbed' cannot substitute an actual experiment it would allow forming a fast and cheap prediction of the likely experimental outcome. For other examples of

testbeds see Arifovic and Ledyard (2005) and Haruvy et al (2003).

The working assumption of this project is that human subjects' behavior can be modeled by an adaptive process. A good tool for modeling adaptive processes is genetic algorithm (GA). The paper begins with presentation of experimental results from Weber (2005). In the following section I present several attempts to replicate these experimental results with GA models. Starting from simple models and increasing their sophistication does not generate results that strongly resemble experimental data.

Both social and individual learning are considered. Social learning corresponds to the environment where players are able to learn from each other, either by means of genetic inheritance or by imitation. This implicitly assumes either some communication or large populations and biological inheritance. In individual learning, the player learns from her own potential choices. While in general the player can be allowed to communicate with others and modify her strategies to accommodate for more optimal strategies, here no such communication is assumed to mimic the experimental setup.

The simplest model forces players to pick strategies that pick an action, unconditional of group size or past minima. Sophistication of players is improved when they base their action on past period's choice as in Arifovic (2001) specification. In that paper each agent is represented by a string that encodes initial choice and actions to be taken conditional on past observation. Arifovic (2001) uses social learning and it results in transitions across Nash equilibria, with negative correlation between group size and average minimum.

Individual learning with similar specification results in convergence to the low action equilibrium. In individual learning each agent is represented by a population of strings, with each string encoding a strategy as in social learning model. Only one string from that population is chosen to represent the player during a game. Other strings do not participate and their fitness needs to be estimated. The conventional potential payoff evaluation results in convergence of average minimum towards the lowest action level.

Social learning version of Arifovic (2001) generates results that are better than individual learning considered here, despite being inappropriate for experiments on

coordination games by Weber (2005). Social learning requires agents to be able to observe others' strategies, which is not a feature that is available in Van Huyck et al (1990) or Weber (2005). This suggests that the potential payoff evaluation principles used in individual learning need to be examined further. I conclude that a revision to individual learning procedure will help in construction of a useful tesbed for coordination games.

Experimental results

Experimental design

Overall design of Weber (2005) is similar to previous work (e.g. Van Huyck et al (1990)). The main difference is that in Weber (2005) the group size is not fixed and weakly increases over time. There are two sets of experiments: in the first set the rate of growth is exogenous (pre-determined by the experimenter) and in the second one it is endogenous and is determined by a player ("manager") whose payoffs are tied to equilibrium of the group that is playing the coordination game. Payoffs for subjects playing the coordination game are given in Table 1.

Player's choice	Minimum choice of all players						
	7	6	5	4	3	2	1
7	.9	.7	.5	.3	.1	-.1	-.3
6		.8	.6	.4	.2	.0	-.2
5			.7	.5	.3	.1	-.1
4				.6	.4	.2	.0
3					.5	.3	.1
2						.4	.2
1							.3

Table 1: Payoffs for Weber(2005)

Exogenous growth: design

Groups of 12 students were presented the game (framed in the context of report completion) and anonymously assigned participant numbers. The subjects were given the pre-determined exogenous growth path that specified the period at which a participant would join the game. For example, the path might specify that player # 7 will join the game in 15th period. All games started with participants #1 and #2 playing the game and other players observing the game quietly (participants were told that they would receive a 'fair' compensation for the periods not spent in the game). In all growth paths, all 12 subjects were participating by the last few periods and games after 22 periods.

In each period, the participants recorded a number from 1 to 7 (action) on a piece of paper and handed it to the experimenter. At the end of each period, the minimum was announced to all the subjects, including those who weren't actively participating yet. This was the only feedback provided.

Exogenous growth: results

Nine sessions were conducted with three growth paths that were chosen to give subjects opportunity for a successful experience with initial growth. The main results are:

- in the initial stages when the group size is small they coordinated on a high-level of efficiency for at least two consecutive periods (8 out of 9 groups coordinated on the highest level)
- growth does not always work, in 4 out 9 sessions the minimum by period 20 is 1
- however, in 3 out of 9 sessions the groups coordinate on the highest level and in the remaining 2 sessions the minimum does not fall below 3
- in several sessions a pattern emerged whereby the minimum would drop by 1 whenever a new player joined

Weber (2005) conducts a series of formal tests to compare choices in the growth sessions with control session (where group size was fixed at 12). The statistical tests mildly support that the distribution of choices was different, with growth sessions giving higher number of players playing the efficient strategy.

Endogenous growth: design

The only modification from previous design is addition of a "manager" who determines the growth path during the experiment (note that manager was allowed to decrease the group size). The manager was placed in a separate room from the group and an experimenter carried information between the two rooms. The coordination game played by the group was described to the manager. Manager's payoffs rewarded large groups that are efficiently coordinated and punished large groups that are inefficiently coordinated.

In the first period the group size was restricted to two, but at the beginning of each subsequent period the manager wrote down a number between 2 and 12, and then the participants from 1 to that number played the game in that period (note that the manager had the option to reassign participant numbers). The subjects were informed that the number of active participants would be determined at the beginning of each period by the manager.

Endogenous growth: results

Four sessions were conducted, each lasting for 35 periods. The four managers grew their groups very quickly initially, but later reduce the group size and attempted slower growth paths. Results:

- the initial rapid growth resulted in coordination failure (medium to low levels of efficiency)
- two managers reduced group size following coordination failure, but did not recognize the need for slow growth and continued to add players at a fast rate

- two managers did realize the need for slower growth and managed to coordinate on high levels of efficiency (6 and 7)
- in one of the sessions, the minimum increased by 1 for each period that the group stayed the same size and dropped every time a new player was added. This is similar to the pattern from exogenous growth experiments and suggests a strong effect of previous experience with growth.

Computer testbed

Specifics of the game

There are two approaches to modeling strategy in the coordination games¹:

- directly allow the agent to pick an action (this action need not be the optimal one, perhaps due to bounded rationality of the agent, e.g. the past minimum choices were 1 during the last several games yet an agent chooses to play 5);
- allow the agent to form an expectation and, imposing rationality, the agent's optimal strategy is given by her expectation of the minimum choice, e.g. if the agent expects the minimum to be 3 she is better off playing 3.

Each approach has its advantages and problems. The first approach is easy to model, but it takes a simplistic view of human behavior. The second approach is more sophisticated but requires a mechanism through which expectations are formed.

This project attempts to use adaptive learning to explain the experimental results. In coordination games explicit and implicit (beliefs) models are equivalent since a belief and an optimal choice given that belief coincide. For example, if an agent's string encodes 4 then in an explicit model she would choose 4 (because she takes the action that is prescribed by the string) and in the implicit model she would believe that the minimum will be 4. By construction, the optimal response to that belief is 4.

Strategies (or expectations) could be made more sophisticated by allowing conditioning on some information, for example group size, past action. The first model will

¹This project assumes basic familiarity with genetic algorithms, for a good introduction to this subject see Arifovic (1994).

use simple (unconditional) strategies and the next step is to use strategies conditional on group size (for model with strategies conditional upon past choice, see Arifovic (2001)). Due to inability of even sophisticated models to explain successful coordination in small groups, we had to resort to the specification that achieved it: Arifovic (2001). Following this demonstration we extend the model to allow for conditioning the strategy on the group size as well.

Finally, learning could occur at an individual level or at a social level. Weber's experimental design did not allow for communication between players and the only information obtained after each period was the game minimum. This precludes possibility of social learning as subjects are not made aware of others' choices (e.g. through mean statistic). Generally GA models yield different results for the social and individual learning. In this game the two types of learning give similar results in simple specification (unconditional strategies) and different results in Arifovic (2001) specification.

For simple specifications, the results are similar because in individual learning with potential payoff evaluation all strategies aside from the observed one will give strictly less payoff. For example, if a minimum during previous period was 4, then social learning would generate a new population that would have 4 as the fittest strategy. If an individual consists of a population of strings that decode in 1-7 range, individual selection would generate a new population with 4 as the fittest string. (Note: an assumption is needed to assign potential payoffs if an individual's potential choice is below the actual minimum in the game. For example, if minimum was 4 and one of potential choices is 1, then the payoff is not defined from Table 1. I assume that in this situation agent would imagine his potential choice would also be the lowest and hence pick the corresponding payoff.)

Bearing the above in mind, the models below use explicit, social and individual learning models with conditional and unconditional strategies.

Social learning with unconditional strategies

Fixed group size

Each agent is modeled as a binary string of length k . This string can be decoded to an integer between zero and $(2^k - 1)$. This integer represents the strategy that the player will choose, independent of group size or past choice (hence, unconditional). The strategy space is $(1, 7)$ and so the string values are normalized with each agent's choice given by $(6 \times \frac{\text{decodedvalue}}{(2^k - 1)} + 1)$.

A population of N such agents is generated randomly and the game is started. Each period the strings are decoded to represent the agents' choices. The minimum choice is found and the payoffs are assigned using table 1. Following this the genetic operators of reproduction, crossover and mutation are applied.

The results for $N = 2$ show that high effort levels were not always achieved, and when they were achieved they were not stable. For example, even if initialized at the high effort level the coordination does not persist because the first mutant will bring the minimum down and make 7 a less fit strategy. Note, while this could in principle be corrected by addition of the election operator it would require several further assumptions and would only improve stability. At the same time it would hinder the dynamics, since by construction of the game any choices apart from the minimum are less fit. With election this would imply history dependence, in the sense that the minimum choice in the first period would persist.

The sample simulation in Figure 1 shows the case when population starts off at a high effort level. The first mutant at period 4, brings the minimum down.

If the initial population is generated randomly then high levels of efficiency are not always attained. For example, in the following simulation the minimum stays below 4 (figure 2).

With larger group size coordination on high effort becomes impossible, because it requires all agents to suddenly switch to high effort (figure 3).

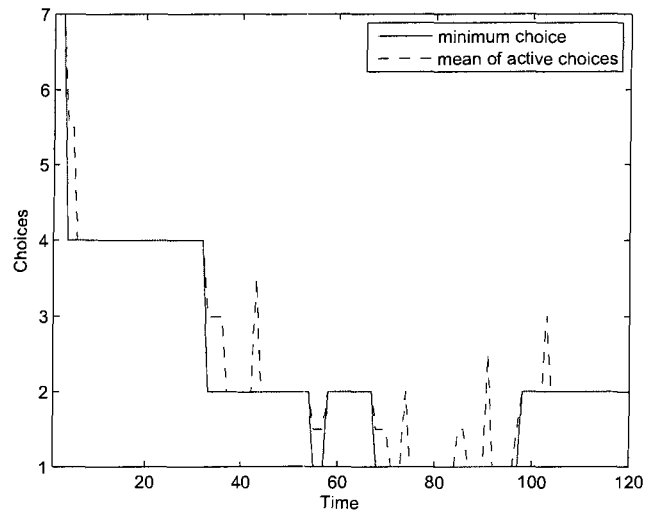


Figure 1: Parameter set 1, $N = 2$, high effort initialization

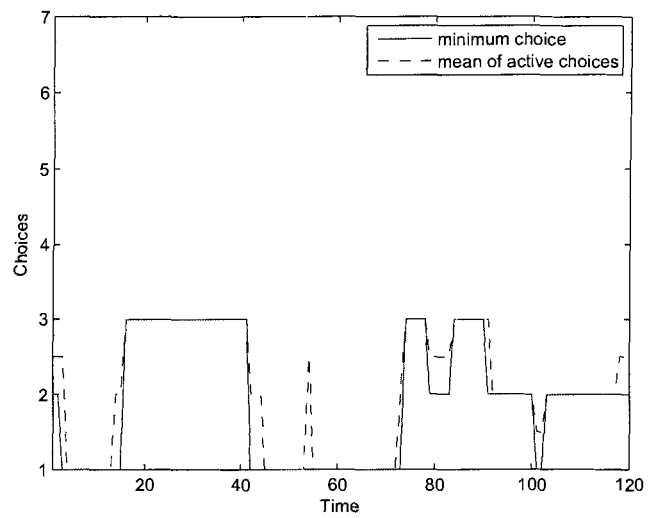


Figure 2: Parameter set 1, $N = 2$, random initialization

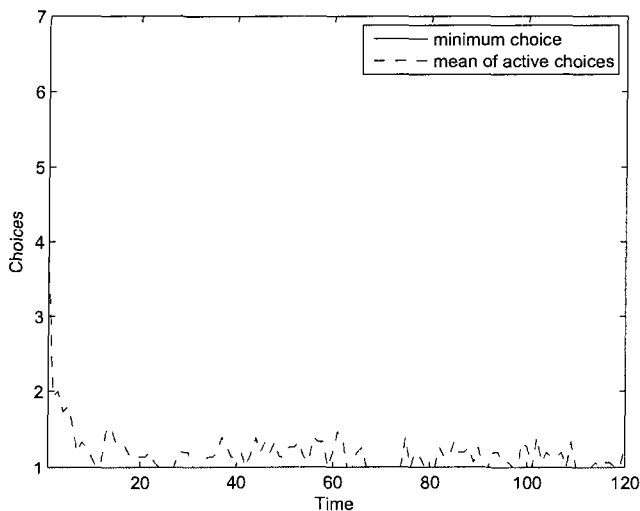


Figure 3: Parameter set 1, $N = 15$, random initialization

Growing groups

This is a continuation of the previous model with group size weakly increasing over time. At the beginning of simulation a growth path is specified. Growth path contains information on the periods of the game at which the number of players changes. Note that since the strategies are simple (unconditional) this does not require a change in the strategies of agents. The major change that is required now is specifying the learning for agents that are not actively playing the game.

These players' choices are decoded and their potential payoff is computed from Table 1, if possible. If an agent's choice is below the actual minimum in the game then she assumes that her choice would be the minimum one and computes the corresponding payoff.

The growth path used for all simulations was: start with 2 players and add 1 player every 30 periods until the group has 9 agents (faster growth paths did not allow to distinguish between random effects and growth effects). Simulation results show that increasing group size 'dooms' the group to low level of effort (figure 4).

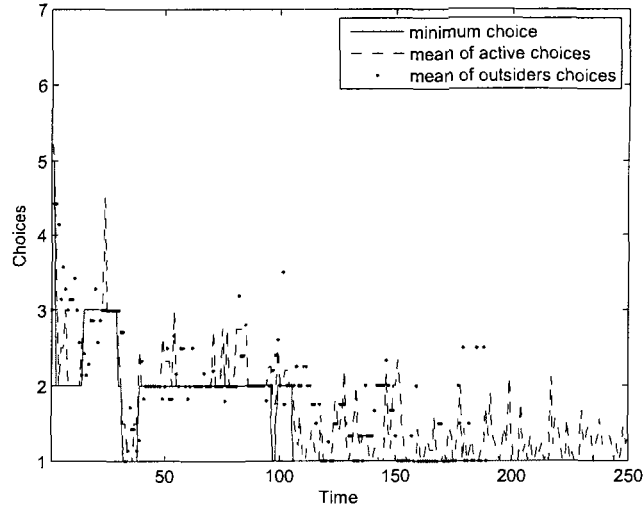


Figure 4: Parameter set 2, $N_0 = 2$, random initialization

Strategies conditional on size

This model extends the previous one by allowing agents to condition their strategies upon group size, since growth path was announced at the beginning of the game. It was an experimentally observed result that adding a player lead to a drop in the minimum choice. To add possibility for this feature the model was modified as follows:

- each agent's strategy prescribes an action to be taken given a group size
- if an agent's action differs from the minimum that occurs when group of that size actually plays then the agent adjusts her future actions; for example, if agent's strategy prescribed to play 5 when group size was 4 people but the actual minimum effort in that game was 3 then the agent decreases her choices for larger group sizes by 1.

Note, that for agents that are actively playing the game it is not possible to adjust their future actions upward since they do not have information other than the minimum outcome of a game. For agents that are not actively in the game it is possible

to adjust their future actions both upward and downward. A sample simulation is shown on Figure 5.

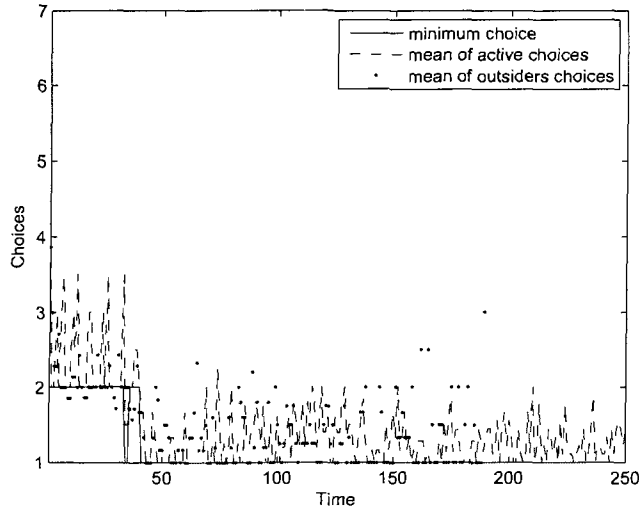


Figure 5: Parameter set 2, $N_0 = 2$, random initialization

This model points out the difficulties in estimating potential payoffs (and foreshadows the results for individual learning). The conventional procedure results in estimation that is negatively biased and hence results in a very fast convergence to the inefficient action. At the same time there is no clear alternative.

Arifovic (2001) specification

In Arifovic (2001) the strategies are conditional on the previous period's minimum effort. This specification allows the groups of any (fixed) size to achieve any equilibrium, but the time spent in high effort equilibria is negatively correlated with group size. First, we attempt to reconstruct this model and then extend it to allow for increasing group size.

Since social learning model is used, each agent is represented by a string that encodes actions to be taken in the first and subsequent periods. The first part of the

string tells what the player should choose in the first round, the latter parts of the string tell what the player should choose given previous period's minimum.

This specification is interesting because it allows multiple equilibria in the repeated game, including cyclical equilibria for 'monomorphic' (i.e. equilibrium strategies are homogenous) populations. The simulation results from Arifovic (2001) show that the evolved strategies appear to have 'hedging' against mutant invasions. This hedging takes form of encoding strings so that (almost) no matter what the previous outcome was the player picks the minimum that is the prevalent equilibrium at the moment. For example, if the minimum in the game was 5, then the strings will encode 'play 5, if past outcome was 1', 'play 5, if past outcome was 2', and so on.

The whole population is divided into groups of fixed size and each group plays for T periods. After T periods the fitnesses are assigned to each player and genetic operators are applied. Arifovic (2001) uses tournament selection: pick two random strings (with replacement) from the population and compare their fitness, then transfer a copy of the more fit one into new population. Tournament selection is repeated until a new population of the same size is generated.

Fixed group size

The results from Arifovic (2001) were replicated. A sample simulation in Figure 6 (30 players; 2 players per group) shows that there are transitions across Nash equilibria. Such transitions take place even in larger groups but with less time spent in high effort equilibrium, see Figure 7 (30 players; 5 players per group).

Growing groups within or between epochs

There were two possibilities for growing groups. One was to allow groups play with each other and then increase group every tenth epoch. The other possibility was to allow groups play with increasing group size each epoch.

In the second approach, an assumption had to be made about how updating took place for players that weren't playing. The same potential payoff evaluation was used as in the case of individual learning with simple strategies.

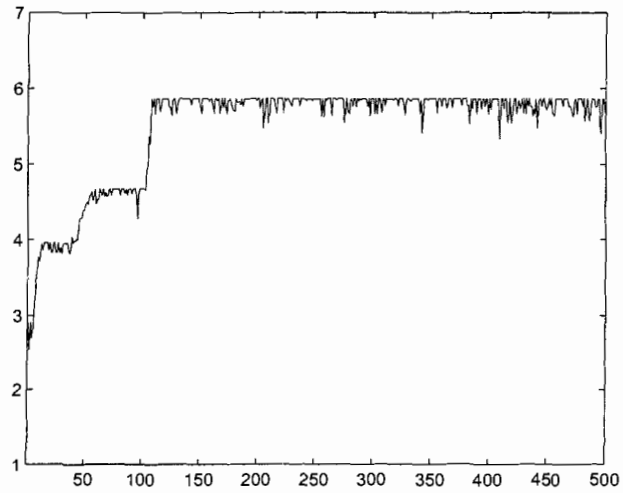


Figure 6: Parameter set 3, 2 player per group, random initialization, fixed group size

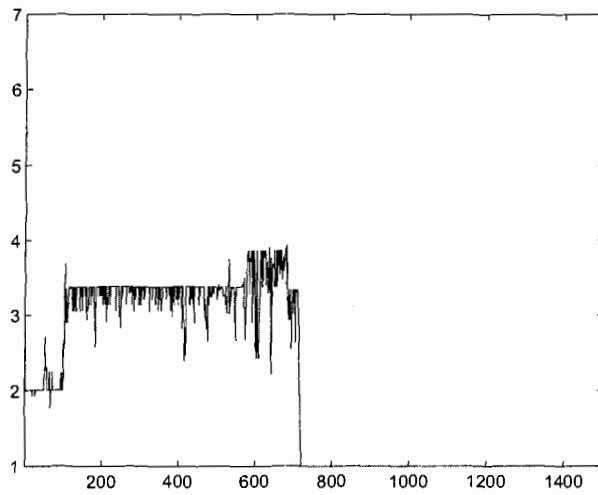


Figure 7: Parameter set 3, 5 player per group, random initialization, fixed group size

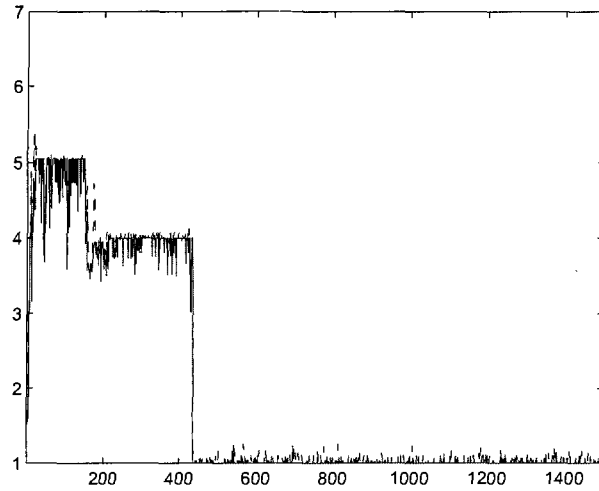


Figure 8: Parameter set 4, $N_0 = 2$, random initialization, group grows between epochs

For simulations presented, the group started with 2 agents and was grown to 16 agents. Total number of players was 90 agents, so that in the end there were 5 groups. The growth path was add 1 player every 10 epochs (between epochs) or add 1 player every 5 periods (within epochs).

Overall, results show that growing groups between epochs proved to be somewhat better (Figure 8). As group size increased the effort levels gone down to the lowest level (longer simulations do not show transitions across Nash equilibria). This fall was more drastic for growing groups within an epoch (Figure 9).

The problem here is with evaluation of potential payoffs in growing-within-epoch specification, since they are biased downwards. While growing groups between epochs is essentially smooth transitions between groups as in Arifovic (2001), adding players within epochs is similar to individual learning treatments.

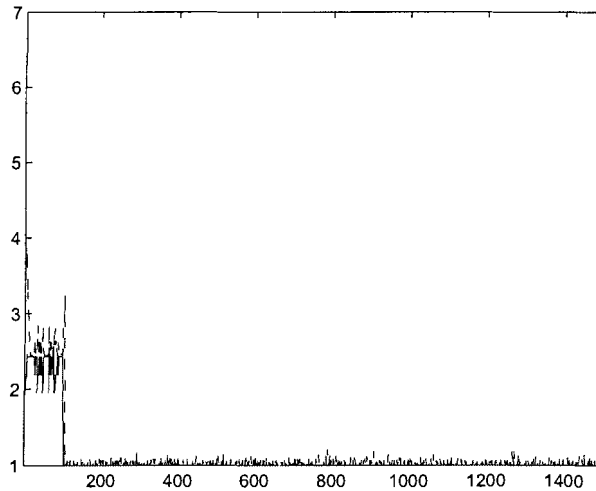


Figure 9: Parameter set 4, $N_0 = 2$, random initialization, group grows within epochs

Individual learning

Individual learning was implemented in a standard way (e.g. Arifovic and Ledyard (2005)). Each agent is represented by a population of strings. One string is selected randomly from population each epoch and it represents the agent's strategy for that epoch. The strings that are not playing are evaluated using the same criteria as mentioned in the simple GA with growing groups.

In particular, if a strategy prescribed an action that is higher than the current period's minimum then this strategy would receive payoff according to Table 1. If a strategy prescribed action that is lower than minimum effort, then the agent would assume that this action is the lowest during period. The payoff would be obtained from the main diagonal of Table 1.

Fixed group size and growing groups

The two treatments are put together in this section because they did not yield qualitatively different predictions. What seemed to matter is the size of each agent's

population. When size of the group was 5 (total 80 players) and each agent had 52 strings, the high effort equilibrium was not sustainable, even when initialized at it (Figure 10). From Figure 11, the qualitative results did not change in small groups (2 people per group, everything else the same).

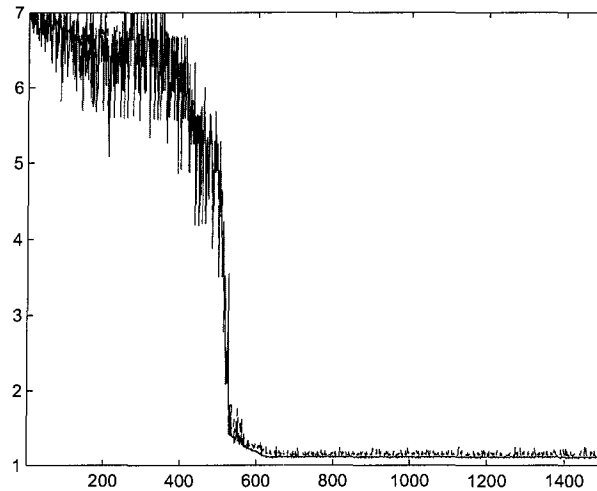


Figure 10: Parameter set 3, 5 players per group, random initialization

The description of individual learning seems to match the experimental conditions. In particular, without information about other players' strategies and no communication, the subjects had to learn only from their own choices and experience. Yet this type of learning fails to describe experimental data. This confirms the earlier findings that suggest that, at least in experiments on coordination, an alternative payoff evaluation scheme must be used.

Intuition for why the conventional payoff evaluation fails can be seen from the following example: suppose a player has a population of strings and the string she is chosen to play for next T periods encodes to play 7 no matter what. The potential payoff evaluation then would take the realized minima and use them to estimate fitness for other rules. If another rule prescribes actions equal to or above the realized minima then the payoffs are obtained from Table 1, otherwise the actions are assumed

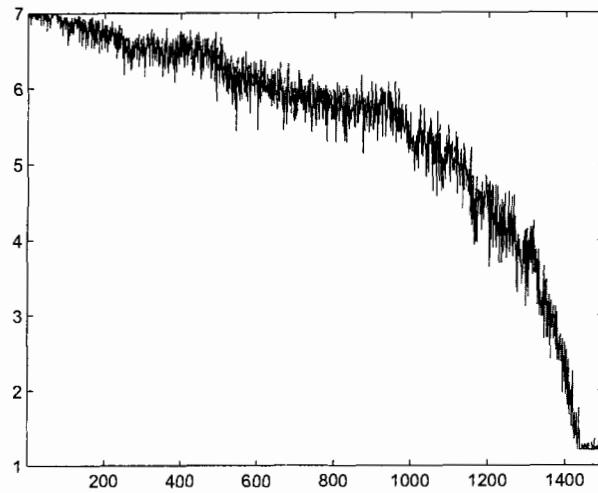


Figure 11: Parameter set 3, 2 players per group, random initialization

to be the minima and payoffs are obtained from the diagonal of Table 1. With this evaluation scheme, a more conservative strategy would have a higher fitness (a mutant strategy could invade).

This invasion is even faster when the updating is done within an epoch, because early within any epoch there are players that are not actively playing the game. None of their strategies are used and hence the potential payoff evaluation applies even to the rule that would be selected otherwise (the one that would get actual payoff, if played). Due to negative bias in evaluation, there is a fast convergence to the low effort equilibrium.

Conclusion

The aim of the paper was to test if a computer testbed could be built with the basic assumptions about individual learning. Such a testbed would allow for a fast and cheap prediction of aggregate experimental results (e.g. Arifovic and Ledyard (2005)). The simple models constructed in this project fail to replicate the experimental data.

In many cases genetic algorithm have been shown to replicate human-generated data, and the reason it was not successful here is due to a very limited information transfer and potential payoff evaluation procedure. By observing the minimum effort only, agents are not able to figure out if there's only one person that is 'shirking' or more. If agents could observe more information, e.g. mean action, then genetic algorithm might have performed better.

Finally, improving the potential payoff evaluation might lead to better predictions of experimental results when the experiment subjects have to rely only on individual learning. Given the particular design of coordination games, a model that combines elements of forecasting and adaptation might be the most appropriate candidate for a testbed.

Appendix

Parameter	Parameter set number				
	1	2	3	4	5
Prob(crossover)	.6	.6	.6	.6	.6
Prob(mutation)	.0033	.0033	.0033	.0033	.0033
updating period	1	10	50	.20	50
string length	5	10	5	28	35

Table 2: The parameter values used.

Treatment	Action						
	1	2	3	4	5	6	7
Soc, uncond, fixed, N=2	18	26	23	10	9	9	5
Soc, uncond, fixed, N=5	99	1	0	0	0	0	0
Soc, uncond, growing	100	0	0	0	0	0	0
Soc, cond. on group size	99	1	0	0	0	0	0
Soc, cond. on past min, fixed, N=2	6	6	3	6	14	25	40
Soc, cond. on past min, fixed, N=5	27	20	15	6	9	12	11
Soc, cond. on past min, gr. within	100	0	0	0	0	0	0
Soc, cond. on past min, gr. between	100	0	0	0	0	0	0
Ind, cond. on past min, fixed	100	0	0	0	0	0	0
Ind, cond. on past min, growing	100	0	0	0	0	0	0

Table 3: Rounded frequency of average minimum choice in the past 100 periods in %, 1000 simulations

Bibliography

- [1] Jasmina Arifovic. Genetic algorithm and the cobweb model. *Journal of Economic Dynamics and Control*, 18:3–28, 1994.
- [2] Jasmina Arifovic. Evolutionary dynamics of the transitions across the nash equilibria of a tacit coordination game. *European Journal of Economic and Social Systems*, 15:109–29, 2001.
- [3] Jasmina Arifovic and John Ledyard. Computer testbeds for mechanism design. *manuscript*, April 2005.
- [4] Colin Camerer. *Behavioral Game Theory*. New Jersey : Princeton University Press, 2003.
- [5] Russell Cooper and Andrew John. Coordinating coordination failures in keynesian models. *Quarterly Journal of Economics*, pages 441–63, 1988.
- [6] Herbert Dawid. *Adaptive learning by genetic algorithms : analytical results and applications to economic models*. Berlin ; New York : Springer, 1999.
- [7] Ernan Haruvy, Alvin Roth, and Unver Utku. The dynamics of law clerk matching: An experimental and computational investigation of proposals for reform of the market. *Journal of Economic Dynamics and Control*, 2005, forthcoming.
- [8] Thomas Riechmann. *Learning in economics : analysis and application of genetic algorithms*. Heidelberg ; New York : Physica-Verlag, 2001.
- [9] JB Van Huyck, RC Battalio, and RO Beil. Tacit coordination games, strategic uncertainty, and coordination failure. *American Economic Review*, 80:234–48, 1990.
- [10] Roberto Weber. Managing growth to achieve efficient coordination in large groups. *American Economic Review*, 2005, forthcoming.