LEARNING AND MONETARY POLICY

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY
in the Department
of
Economics

SIMON FRASER UNIVERSITY
Spring 2008

APPROVAL

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Degree: **Doctor of Philosophy**

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Abstract

This thesis studies implications of different learning mechanisms in various monetary environments.

In Chapter 2, adaptive step-size algorithm (Kushner, Yin 2003) is used to model time-varying learning and is studied in the environment of Marcet, Nicolini (2003). The resulting model gives qualitatively similar results to MN and performs quantitatively somewhat better based on the criterion of mean squared error. This model generates increasing gain during hyperinflations that matches findings in Cagan (1956), Khan (1977). An agent behaves cautiously when faced with sudden changes in policy, and is able to recognize a change in regime after acquiring sufficient information.

Chapter 3 analyzes the effects of social learning in New Keynesian model described in Woodford (2003). The question is whether the economy will converge to a rational expectations equilibrium under this more realistic learning dynamics. A key result from the literature in this version of the model is that the Taylor Principle governs both the uniqueness and the expectational stability of the rational expectations equilibrium when all agents learn homogeneously using recursive algorithms. The finding is that the Taylor Principle is not necessary for convergence in a social learning context. This paper also contributes to the use of genetic algorithm learning in stochastic environments.

Chapter 4 studies cheap talk announcement in an agent-based dynamic extension of Kydland-Prescott model. The government choose inflation announcement and actual inflation and updates its decisions using a model of individual, evolutionary learning (Arifovic, Ledyard 2004). Private agents use nave and more sophisticated inflation forecasts and switch between them based on their payoffs. Agents and government can coordinate on Pareto-superior outcomes with positive fraction of naive agents. However, the economy does not stay there. It exhibits recurrent fluctuations in announced and actual inflation as government repeatedly builds up and exploits the proportion of believers. Outcomes with higher fraction of naive forecasters have higher average welfare of agents and government. When cost of sophisticated forecast goes up, the proportion of naive believers goes up. When nonbelievers update slower, naive believers are more likely to disappear. Therefore, quick and accurate sophisticated forecasters ensure positive number of naive agents.

Keywords: time-varying gain, adaptive expectations, hyperinflation, learning in macroeconomics, New Keynesian macroeconomics, genetic algorithm learning

Subject terms: learning, macroeconomics, adaptive expectations

Acknowledgments

I would like to thank my supervisors Jasmina Arifovic, Ken Kasa and James Bullard for excellent teaching and mentoring. I thank all the faculty at the Department of Economics, and especially Geoffrey Dunbar, Stephen Easton, Alex Karaivanov, and Fernando Martin. I also thank Laura Nielson and Gwen Wild.

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Chapter 1

Introduction.

This thesis studies models where the assumption of rational expectations is relaxed, and agents are assumed to be boundedly rational (learning). Learning is used to provide an explanation of how agents arrive/converge to a rational expectations equilibrium. Learning can serve as a selection tool if a model with rational expectations has multiple equilibria (Arifovic 1995, 1996 , Marcet and Sargent 1989a, 1989b). Models with bounded rationality are used to explain the puzzles that rational expectations models have difficulty to explain. The examples of this research include the following. Evans, Chakraborty (2007) address forward premium puzzle. Arifovic (1996), Lux and Marchesi (2000), Lux and Schornstein (2002) build model with evolutionary learning agents to explain facts about foreign exchange rates - unit roots, fat tails and volatility clustering. Timmermann (1993) explains excess volatility and predictability of excess returns for stocks; Orphanides and Williams (2004) introduce learning to address the persistence of inflation; Marcet and Nicolini (2005) study the short-run correlation between money supply and inflation; Sargent (1999) explains the change in post World War II US inflation.

The use of bounded rationality in macroeconomics is motivated by many reasons. Rational expectations impose high computational, information and cognitive requirements. It is not likely that people maximize all the time: there can be inertia due to transaction costs to change decisions, people can use rules of thumb in their decision making. In econometrics, often the number of variables and lags is limited, these misspecified models are hardly rational. McCallum (2005) points out that the implication of rational expectations is that agents do not know about regime change ex ante, and it is completely credible ex post. Linear approximations are used for solving nonlinear macro models. Hommes and Sorger (1998) define consistent expectations equilibrium (CEE) in which agents using linear model cannot discover their misspecification of nonlinear environment. Other equilibrium concepts involving bounded rationality are self-confirming equilibrium (SCE) by Sargent (1999), restricted perception equilibrium (RPE) and misspecification equilibrium (ME) by Branch and Evans(2006 a,b, 2007), Branch (2004b).

Many empirical studies reject the rational expectations hypothesis in survey data on inflationary expectations (surveyed by Branch 2004a). Several studies find that certain models of learning provide good fit for survey data and address the persistent heterogeneity of expectations in the survey data. Branch and Evans (2004) find that constant gain learning provides the best fit for data on inflation expectation in the Survey of Professional Forecasters. Carrol (2003) finds evidence in favor of the epidemiological model of expectation formation. Branch (2004a) builds a model with rationally heterogenous expectations and fits it to the data of Michigan Survey of Households.

It is important to take into account learning behavior. Gaspar, Smets and Vestin (2006) show that when the private sector is adaptive with backward looking inflation expectations, if the central bank acts as if the private sector had rational expectations, this leads to higher volatility of inflation and output. Akerlof, Yellen (1985) show that small deviations from rationality can have first order effects on the equilibrium outcome in microeconomic models.

One frequently used type of learning describes agents as econometricians. Agents estimate model's parameters recursively as new data arrives and make decisions based on the estimated model. The important parameter in recursive estimation is the gain. Gain determines how much weight is placed on the recent data relative to the past data. In least squares, gain is decreasing so that all data observations receive equal weight. Decreasing gain is used when the environment is stationary. Work with least squares learning includes Evans and Honkapohja (2001), Marcet and Sargent (1989a, 1989b), Bullard and Mitra (2002). In general, least squares learning can converge to a rational expectations equilibrium under certain conditions (Evans and Honkapohja (2001), Bullard and Mitra (2002), Bullard (1991), (2006)). Agents can also use constant gain that means that they place more weight on the recent observations and discard past data. This is useful in nonstationary environments for tracking changes. The examples of research using constant gain learning are Evans and Honkapohja (1993, 2001), Sargent (1999), Orphanides and Williams (2005), Chakraborty and Evans (2006), Branch and Evans (2006 a,b, 2007), Kasa (2001), Cho and Kasa (forthcoming). In a changing environment, constant gain learning does not converge to a rational expectations equilibrium but to an ergodic distribution around it (Sargent (1999), Evans and Honkapohja (2001)).

The important decision in using constant gain is the choice of value of gain. The optimal value depends on how the parameters to be estimated, the probability distribution of data vary relative to the observation noise. If parameters to be estimated and probability distribution of data vary more relatively to noise, then higher value of gain is optimal. If noise varies more, then lower gain is optimal. Sargent, Williams, Zha (2006), Milani (2005 a,b), Orphanides, Williams (2004), Branch, Evans (2006a) estimate the value of gain from the data and Survey of Professional Forecasters. Another important aspect in choosing gain is that agents can update at different rates depending on the economic environment. Cagan (1956), Khan (1977), Silveira (1973) present empirical evidence that if agents use adaptive expectations, their gain (coefficient of expectation) increases during hyperinflation.

In Chapter 2, "Application of adaptive step-size algorithm in a model of hyperinflation", I introduce an algorithm that allows to model time varying speed of update. It is an adaptive step-size algorithm described in Benveniste et al. (1990), Kushner and Yang (1995), Kushner and Yin (2003). It allows agents to estimate the parameters of the model and gain at the same time based on their experience in the economy. The advantages of using this algorithm are that it is adaptive to the environment, and agents' performance does not depend on the choice of one value of gain at the beginning of estimation. For an application of this algorithm, I use the Marcet and Nicolini (2003) model that introduces learning into Sargent, Wallace (1987). Using bounded rationality in the model of hyperinflations can be justified as: most agents are not likely to understand what happens during hyperinflation as the prices increase tremendously in short period of time. Using an adaptive step-size algorithm provides a unified specification of how gain behaves and avoids arbitrary/ad hoc mechanism of Marcet and Nicolini (2003) combining constant and decreasing gains.

The results are qualitatively the same as in Marcet and Nicolini (2003) (MN) and quantitatively adaptive step-size algorithm performs better based on the criterion of Mean Forecast Squared Error. The model economy addresses the same stylized facts about hyperinflations as MN: recurrent hyperinflations, low correlation of seignorage and inflation in hyperinflationary countries, high correlation between average seignorage and inflation across countries, Exchange rate rules stop hyperinflation temporarily, and hyperinflations can be eliminated by permanently lower average seignorage. In addition, the introduction of an adaptive step-size algorithm produces increasing gain during hyperinflation that addresses findings in Cagan (1956), Khan (1977), Silveira (1973).

The behavior of gain is similar to MN during low inflation, but it is different from MN during and after the end of hyperinflations. When inflation is low and stable, the speed of update is low as agents do not make big forecast errors and so do not need to update quickly. During hyperinflations the speed of update is increasing as agents repeatedly underpredict inflation and need to put increasingly more weight on the recent observations to catch up with inflationary process. In comparison, the agents in MN use high but constant speed of update during hyperinflations. After hyperinflations end with fixed Exchange Rate Rule, Marcet and Nicolini's agents keep updating at high speed. Agents using my algorithm reduce the speed of update. I interpret this behavior such that agents are cautious and do not rush to revise their expectations down as they do not trust that government's new low inflation policy will prevail successfully. This is similar to the learning specification in Cho and Sargent: agents do not believe that government can use good economic policy. During hyperinflation agents repeatedly underpredict inflation, and when hyperinflations end, they do not hurry to reduce their inflation forecasts as they do not trust government.

I also estimate the model using a simulated method of moments approach. I compare the learning mechanism based on an adaptive step-size algorithm to Bayesian learning. I find that based on the criterion of Mean Squared Error, the loss of forecasting accuracy is not big when using an adaptive step-size algorithm in comparison to Bayesian learning. This is evidence in favour of an adaptive step-size algorithm because it can be used by an agent with little knowledge about the data generating process whereas a Bayesian agent knows much more about DGP.

In two other chapters, I use a different learning mechanism - evolutionary learning based on genetic algorithm. Genetic algorithm is a numerical optimization technique first introduced by Holland (1975) and described in Goldberg (1987), Michalewicz (1996), Back et. al. (2000). Among the advantages of using genetic algorithm for optimization are that it starts with a set of random solutions and so does not rely on the starting point, and that it is applicable for discontinuous, nondifferentiable, noisy, multimodal and other unconventional surfaces (Schwefel 2000). For example, Bullard and Duffy (2004) use simulated method of moments with a genetic algorithm to estimate growth model with structural breaks. Evolutionary learning is convenient to study economies with heterogeneous agents, it does not impose high information and computational requirements on agents and is able to explain actual and experimental data better than models with rational expectations. Evolutionary learning has been used in different economic environments. Arifovic (1994, 1995, 1996), LeBaron (2000), Arifovic, Bullard and Duffy (1997, 1998 a,b,c), Dawid (2006), Lux and Marchesi (2000), Lux and Schornstein (2002) are some examples of research using evolutionary learning agents.

Chapter 3, "Social Learning and Monetary Policy", analyzes the effects of social learning in a widely-studied monetary policy context - New Keynesian model described in Woodford (2003). Social learning might be viewed as more descriptive of actual learning behavior in complex market economies. Ideas about how best to forecast the economy's state vector are initially heterogeneous. Agents can copy better forecasting techniques and discard those techniques which are less successful. The question is whether the economy will converge to a rational expectations equilibrium under this more realistic learning dynamic. A key result from the literature in the version of the model we study is that the Taylor Principle governs both the uniqueness and the expectational stability of the rational expectations equilibrium when all agents learn homogeneously using recursive algorithms. The finding is that the Taylor Principle is not necessary for convergence in a social learning context. This paper also contributes to the use of genetic algorithm learning in stochastic environments.

Chapter 4, "Learning Benevolent Leadership in a Heterogeneous Agents Economy", studies cheap talk announcement in an agent-based dynamic extension of Kydland-Prescott model. The government decides on inflation announcement and actual inflation and updates its decisions using a model of individual, evolutionary learning (Arifovic, Ledyard

2004). Private agents use naive and more sophisticated inflation forecasts and switch between forecast mechanisms based on their payoffs. Agents and government can coordinate on Pareto superior outcomes with a positive fraction of naive agents. The economy exhibits recurrent fluctuations in the inflation announcement and actual inflation as a result of the government's actions. The fluctuations happen as the government builds up and exploits the population of believers. In a typical sequence of events, the government first builds up a positive proportion of believers by keeping the difference between announced and actual inflation relatively small. Having achieved this, the government starts exploiting the existing believers by increasing the discrepancy between actual inflation and inflation announcement in a short-sighted attempt to lower unemployment and increase its payoff. This has a negative impact on the believers' payoffs and their proportion decreases. Recurrent fluctuations in the proportion of believers represent changes in the level of trust to the government. Our model generates endogenous credibility that changes over time as a result of the government actions and its impact on private sector's payoffs. When it is costlier for nonbelievers to make their own forecast, the proportion of believers goes up. When the speed of update of nonbelievers is lower, the proportion of believers fluctuates faster and naive believers are more likely to disappear. Therefore, quick and accurate sophisticated forecasters ensure positive number of naive agents.

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Chapter 2

Application of an Adaptive Step-Size Algorithm in Models of Hyperinflation.

2.1 Introduction

Models departing from rational expectations are used to explain how agents converge to a rational expectations equilibrium, to select among multiple equilibria, and to explain puzzles that cannot readily be explained by models with rational expectations. The explanations of hyperinflations using rational expectations rely on bubble equilibria (Sargent and Wallace (1987)), and recurrence of hyperinflations is explained by sunspots (Funke et al. (1994)). Other explanations employ departures from rational expectations and introduce learning (Marcet and Nicolini (2003), Sargent, Williams and Zha (2006), Adam et al. (2006)). The latter use learning mechanisms that differ in the specification of the speed of adjustment to new information.

Two common assumptions about the gain used in the literature are least squares learning and constant gain (perpetual) learning. In least squares learning, the gain decreases with time and gives equal weight to all past observations. If an agent knows that he is in a stationary environment, he will use a decreasing gain. Examples of research using least squares learning include Evans and Honkapohja (2001), and Marcet and Sargent (1989a, 1989b). In general, least squares learning can converge to a rational expectations equilibrium under certain conditions (Evans and Honkapohja (2001), Bullard and Mitra (2002), Bullard (1991), (2006)).

In contrast, a constant gain algorithm discounts past data and gives more weight to recent observations. A constant gain is thus better at tracking structural changes. If an agent believes that the environment is not stationary, and that regime changes are possible, constant gain learning is used. Constant gain learning is normally interpreted as agents do not trust the government and are alert to possible changes in the government's policy. Once change happens, the agent using constant gain is ready to view it as a regime switch and updates his estimates accordingly. Evans and Honkapohja (1993, 2001), Sargent (1999), Orphanides and Williams (2005), Chakraborty and Evans (2006), and Cho and Kasa (forthcoming) are several examples of research using constant gain learning. Sargent (1999) finds that least squares learning converges to a Nash equilibrium in the Kydland-Prescott model (Kydland and Prescott (1977)). Sargent (1999) suggests using constant gain learning - it can allow a government using a misspecified model to achieve a superior Ramsey outcome and escape from Nash inflation. In a changing environment, constant gain learning does not converge to a rational expectations equilibrium, but to an ergodic distribution around it (Sargent 1999, Evans and Honkapohja (2001).

Several questions about constant gain learning remain be addressed. First is the choice of the gain parameter. In most of the literature, this parameter is chosen in an ad hoc manner to produce desired properties in the model at hand. The value of the gain is generally quite important for the results. Recent literature either estimates the constant gain from the data or from surveys of professional forecasters (Sargent, Williams and Zha (2006), Milani (2005 a,b), Orphanides and Williams (2004), Branch and Evans (2005)).

A second question is whether constant gain beliefs can be self-confirming. The stated motivation for using a constant gain algorithm is a nonstationary environment, but constant gain learning is often used in models that are stationary. In these models, if agents were to use decreasing gain, their expectations would be validated just as well as they are validated when they use constant gain expectations (Chakraborty and Evans (2007)).

A third issue is that one would expect agents to learn at different speeds depending on the economic environment. Cagan (1956) uses adaptive expectations to study hyperinflation and points out the limitations of this scheme: adaptive mechanisms with constant gain [1] constrain agents to adjust their forecasts by a constant proportion of their forecast error. Cagan (1956) estimates gains during hyperinflation and finds that agents update slower in the earlier periods of hyperinflation, and quicker in the later periods of hyperinflation. Khan (1977) finds support for a variable gain in data as well - the update speed increases with the variability of inflation. Khan (1977) specifies the gain as a function of the absolute level of inflation (using Cagan's finding that the gain increases as hyperinflation unfolds) and the variability of inflation measured by the absolute change of inflation. A similar specification for a variable gain is derived in a model with rational expectations by Mussa (1981). Silveira (1973) finds support for an increasing gain in hyperinflation data from Brazil. These papers explain that agents need to revise their expectations quicker and so use a higher gain because slow adjustment to forecast errors can be costly in hyperinflationary environments.

There are more recent examples using time-varying gain parameters. Marcet and Nicolini (2003) introduce a learning mechanism that combines a constant gain and a decreasing gain to model recurrent hyperinflation in Latin America. Their agent switches from a decreasing gain to a constant gain when forecast errors rise above a critical level, and then returns to a decreasing gain when forecast errors fall below the critical level. The value of the constant gain is chosen so that the learning algorithm satisfies lower bound on rationality. Timmermann (1993) models learning in an environment with infrequent structural changes, where the timing of the regime shifts are known to the agent. The agent uses a decreasing gain when the structure is unchanged and a constant gain when a structural break occurs. Milani (2005a) allows for a structural break that changes the value of constant gain. Evans and Ramey (2005) derive Nash equilibrium gain and show that agents using Recursive Prediction Error (RPE) algorithm (Ljung and Soderstrom 1983) are able to adjust their gain to an equilibrium value that varies with policy changes.

In this paper, I introduce a new time-varying gain algorithm. I use an adaptive step size algorithm as described in Benveniste et al. (1990), Kushner and Yang (1995), Kushner

[1] Literature discussed in this paragraph uses term 'coefficient of expectations' for gain/speed of update.

and Yin (2003) [pp.69-73]. The successful use of a constant gain algorithm depends on the choice of the gain. The optimal choice of the gain depends on how fast the time-varying parameters to be estimated, the probability distributions of data and the observation noise vary relative to each other. If the probability distributions of data and parameters vary a lot, then a higher gain is optimal. However, if observation noise is high, then a smaller gain is better. The adaptive step-size algorithm allows the gain to evolve in response to changes in the environment. Estimation consists of two parts: (i) estimation of the model parameters, and (ii) estimation of the gain (the details are provided in the section "Specification of expectations"). This algorithm is commonly used in engineering applications, and should be useful in economic learning models. Its economic interpretation is that agents adjust the speed of their learning depending on their recent experience. This procedure thus avoids the problems of committing to a single value for the gain and specifying an ad hoc learning mechanism.

I apply the adaptive step size algorithm to the environment from Marcet, Nicolini (2003), in which an agent learns about forming inflation expectations. He does not know whether the model has regime changes or not, and adapts his forecasts and learning speed based on his observations using the adaptive step-size algorithm.

2.1.1 Main findings

The adaptive step size algorithm specifies a learning scheme endogenous to the model and changes in policy. This addresses the criticism that boundedly rational mechanisms are exogenous to the model. This algorithm provides a universal approach to how gain parameters change in response to changes in the model environment, and avoids the arbitrariness of gain parameter specifications in previous literature.

Interestingly, the simulations show that the model in this paper produces qualitatively similar behavior of inflation and inflation expectations as in Marcet and Nicolini's (2003) model (MN below). Mean Squared Errors of the learning specification in this paper are similar to or smaller than those of MN, which implies that an adaptive step-size algorithm performs somewhat better.

The value of gain increases as hyperinflation develops. The increasing gain during hyperinflations matches the empirical findings in Cagan (1956), Khan (1977) and Silveira (1973). The adaptive step-size learning specification matches findings of increasing speed of update during hyperinflation better than the mechanism in MN. In the MN model, the agent switches from a decreasing gain to a constant gain at the beginning of hyperinflations and continues to use a constant gain during hyperinflations. In my model, the agent revises his update speed optimally based on the adaptive step-size algorithm.

In my model, the gain behaves differently than in MN after a hyperinflation ends with the implementation of an exchange rate rule (ERR). In MN, the agent continues to update using constant gain. In my model, the agent switches to *lower* gain right after an ERR is implemented. The decrease in gain has the following behavioral interpretation. The agent does not believe that a regime change has happened and inflation will be low, and/or he does not believe that new policy regime is credible. The agent has repeatedly underpredicted inflation during hyperinflation. When inflation drops, the agent does not discard his past experience quickly, and does not rush to revise down his inflation forecasts because he does not believe that a new regime has started or will prevail successfully.

This behavior of the gain (increases during hyperinflation, decreases after implementation of the exchange rate rule) is similar to the mechanism modeled in Cho and Sargent (1997). They specify a learning algorithm in which the agents are skeptical that the government can stick to good economic policy [p.10]. When applied to hyperinflationary environments, this mechanism implies the following. During a hyperinflation, agents update their beliefs by putting more weight on recent data because they understand that the government does not use good economic policy. When agents observe a sudden drop in inflation, they update by placing less weight on recent data because they do not expect the government to maintain good policy.

I estimate the model for Argentina, Bolivia, Brazil and Peru using simulated method of moments with a genetic algorithm. Simulations for the estimated parameters exhibit hyperinflations for all countries except Brazil. This suggests that a more sophisticated estimation (for example, based on transition probabilities for inflation) may be needed.

2.1.2 Related literature

Sargent and Wallace (1987) show that a high inflation steady state (with perverse comparative statics) is stable under perfect foresight and explain hyperinflations as rational bubble equilibria. Marcet and Sargent (1989) study the model in Sargent and Wallace (1987) with least squares learning and find that low inflation steady state is stable under least squares learning, whereas high inflation steady state is not. Adam et al. (2006) show that hyperinflationary paths near the high inflation steady state are stable if agents use contemporaneous data.

Marcet and Nicolini (2003) study the Sargent and Wallace (1987) model (with stochastic iid seignorage) by introducing an endogenous learning scheme that combines least squares learning and constant gain learning. One of the mechanisms is used depending on the size of forecast error. Marcet and Nicolini (2003) are able to address stylized facts of recurrent hyperinflations in Latin American countries.

Sargent, Williams and Zha (2006) estimate the same model with constant gain learning and some modifications (e.g. specification of seignorage as a Markov switching process) for Latin American countries that experienced hyperinflation. They conclude that change in inflation can be attributed to the learning dynamics (switches of perceived inflation between low and high Self Confirming Equilibria) and/or to change in fundamentals (change in seignorage).

2.1.3 Organization

I describe the environment and expectation formation mechanism in section 2. The results of the simulations and analysis are presented in section 3, and the model is estimated in section 4. Section 5 compares the adaptive step size algorithm to Bayesian learning. Section 6 compares related models and is followed by the conclusion.

2.2 Model

I study a model that consists of a money demand equation, a government budget constraint, an exogenous process for seignorage, and a specification of expectation formation that is different from rational expectations. Marcet and Nicolini (2003) and Sargent, Williams and Zha (2006) use the same model with different expectation mechanisms.

The demand for money is given by a Cagan style specification:

$$\frac{M_t}{P_t} = \phi - \gamma\phi\frac{P^e_{t+1}}{P_t} \tag{2.1}$$

where ϕ, $\gamma > 0$ are parameters, M_t is nominal balances as a percent of output at time t, P_t is the price level at time t, P^e_{t+1} is the expected price level for time $t+1$.

The government supplies money to finance seignorage. If inflation is above a certain critical level, the government implements an exchange rate rule (ERR). If there is no need for the ERR, then the government budget constraint is:

$$M_t = M_{t-1} + d_t P_t \tag{2.2}$$

where d_t is seignorage, it is an *iid* stochastic process. I will follow the assumption in Marcet and Nicolini (2003) that seignorage is normally distributed, $N(E(d), \sigma_d^2)$, $\sigma_d = 0.01$, and truncated to have positive values. The equilibrium values of nominal balances and prices $\{M_t, P_t\}_{t=0}^{\infty}$ are determined from (2.1), (2.2) and an expectation formation equation. I denote the inflation rate as $\pi_t = \frac{P_t}{P_{t-1}}$. Under rational expectations, there are 2 deterministic steady state inflation rates for $d = E(d_t)$:

$$\pi_{1,2} = \frac{1 + \gamma - \frac{d}{\phi} \pm \sqrt{(1 + \gamma - \frac{d}{\phi})^2 - 4\gamma}}{2\gamma} \tag{2.3}$$

The government buys or sells foreign reserves R_t at exchange rate e_t to peg the exchange rate satisfying:

$$\frac{P^f_t}{P^f_{t-1}}\frac{e_t}{e_{t-1}} = \overline{\pi} \tag{2.4}$$

where $\overline{\pi}$ is the targeted inflation rate, and P^f_t is the foreign price level. The targeted inflation rate is assumed to be equal to the low steady state inflation from (2.3). Perfect goods mobility and purchasing power parity imply that the targeted inflation rate is achieved:

$$\frac{P_t}{P_{t-1}} = \overline{\pi} \tag{2.5}$$

Under the ERR, the equilibrium price level is determined from (2.5). Money demand is solved from (2.1) together with the expectation formation specification. If the money demand determined from (2.1) is not equal to the money supply from (2.2), then the government adjusts international reserves such that the money supply is determined by:

$$M_t = M_{t-1} + d_t P_t + e_t(R_t - R_{t-1}) \tag{2.6}$$

The exchange rate rule (ERR) is imposed when inflation is above the critical level π^U:

$$\frac{P_t}{P_{t-1}} > \pi^U \tag{2.7}$$

or when there is no positive price level that clears the market if $R_t = R_{t-1}$. The implicit assumption is that the ERR can always be enforced, i.e. there are always enough international reserves to peg the exchange rate.

2.2.1 Expectation formation

Agents form their expectations of inflation, $\beta_{t+1} = \frac{P^e_{t+1}}{P_t}$, adaptively as:

$$\beta_t = \beta_{t-1} + a_t(\pi_{t-1} - \beta_{t-1}) \tag{2.8}$$

for some initial value β_0. Thus inflation forecasts are revised by the last forecast error weighted by the gain a_t.

In this paper, the gain a_t evolves based on the adaptive step-size algorithm suggested in Benveniste, Metivier and Priouret(1990), proved in Kushner and Yang (1995) and described in Kushner and Yin (2003) [p.71]. This algorithm specifies endogenous behavior of the gain in response to developments in the economy. The algorithm can therefore be thought of as providing a unified specification of how gains evolve over time, and so avoids an ad hoc mechanism of discrete changes in gain in response to big forecast errors as in Marcet and Nicolini (2003). In my model, the gain follows:

$$a_t = \prod_{[a_-,a_+]} [a_{t-1} + \mu * (\pi_{t-1} - \beta_{t-1}) * V_{t-1}] \tag{2.9}$$

$$V_t = V_{t-1} - a_{t-1} * V_{t-1} + (\pi_{t-1} - \beta_{t-1}), V_0 = 0 \tag{2.10}$$

In (2.9), μ is the step size in the stochastic approximation of the a_t process (loosely speaking, μ is 'gain on the gain'). $\prod_{[a_-,a_+]}$ is a projection operator that sets the gain a_t equal to a_- when it falls below this value, and sets the gain equal a_+ when it rises above this value. Kushner and Yin (2003) show that the performance of a nonadaptive algorithm (with constant gain) is much more sensitive to the choice of constant gain a than the adaptive step size algorithm is to the choice of the step size μ. While it is necessary that $0 < \mu \ll a_-$ for the proofs, the lower bound a_- is not so important in applications. However, the upper bound a_+ is very important for the performance, and is often chosen close to the point where the algorithm becomes unstable.

In (2.10), V_t denotes the "derivative" of the estimated parameter (β in this case) with respect to the gain a for the stationary process. The process β_t is not a classical function of a, but its distribution depends on a. Kushner and Yang (1995) interpret V_t as the desired derivative (also see Kushner and Yin 2003).

The intuition behind this learning mechanism is as follows. The change in gain (2.9) is driven by the discounted past errors, V, and the last period forecast error, $\pi_t - \beta_t$. When the last period forecast error is in the same direction as the discounted past errors, the agent increases the gain. This means that if the agent keeps making the same error, he wants to increase his response to the last period forecast error when updating forecasts. If the last period's forecast error is in a different direction from the discounted past errors, the agent decreases the gain. This means that when the agent encounters something contradictory to his past experience, the agent reduces his response to forecast errors until he learns more. The value of V (2.10) depends on the size of the last period forecast error $(\pi_{t-1} - \beta_{t-1})$ relative to the size of the past discounted errors $(V_{t-1} - a_{t-1} * V_{t-1})$. For example, if the latest forecast error is small relative to the past discounted errors, then V changes slowly.

2.3 Simulation results

I simulate the economy for the same parameter values as in Marcet and Nicolini (2003). [2] The parameters in the money demand equation are $\gamma = 0.4$, $\phi = 0.37$, and mean seignorage

[2] Model in Sargent, Williams and Zha (2006) is estimated with seignorage as a Markov switching process, and so their values of the model parameters are different from Marcet and Nicolini (2003).

is $E(d) = 0.049$. The model has 2 steady states that become closer for higher values of average seignorage. For the above money demand parameters, the maximum value of average seignorage for which rational expectations equilibrium exists is $E(d) = 0.05$. When mean seignorage is closer to its maximum value, it is easier for the system to move above the high inflation steady state and to explode into a hyperinflation.

The parameters of the step size algorithm are $\mu = 0.001$, $a_- = 0.01$, $a_+ = 0.6$. The initial value of the gain is 0.2.[3] It is important to note that this upper boundary for the gain is not binding as in the simulations gain rarely reached values equal or above it. I choose the initial expectation as $\beta_0 = \pi_0 = \pi_{low}$ equal to the initial inflation started at the low inflation steady state.

I will compare the simulations based on Marcet and Nicolini (2003) and the simulations based on the gain specification in (2.9). Figure 2.1 presents the replication of Marcet and Nicolini's (2003) model. Figures 2.2 and 4.3 present the results of the simulations with my adaptive step-size algorithm.

Figures 2.1 and 2.2 are based on the same seed, the only difference is the specification of the gain. These two figures look very similar in terms of the behavior of actual inflation (thick blue line) and inflation expectations (thin black line). For example, the timing and magnitudes of inflation are similar. Expectations of inflation adapt slowly after the end of hyperinflation in both models. The key difference between the two models is in how agents revise their inflation expectations, i.e. the behavior of the gain is different.

As hyperinflation develops, the agent using an adaptive step-size algorithm increases his gain. The agent using an adaptive step-size algorithm can adjust his gain flexibly as the economic environment changes. As hyperinflation unfolds, the agent's inflation forecasts are repeatedly below actual inflation. Therefore, the agent begins to increase gain so that his inflation expectations catch up to actual inflation. A higher gain means that the agent updates his inflation expectations with higher responses to forecast errors. If the forecast errors are positive (as is the case during hyperinflation), increasing the gain means that the

[3] This is value of constant gain Marcet, Nicolini (2003) use.

agent increases his inflation expectation by a higher proportion of forecast error.

Cagan (1956) estimates a model with adaptive expectations for subperiods of hyperinflation and finds that gain increases as hyperinflation evolves. Khan (1977) specifies the gain as a function of inflation and inflation variability and finds that the gain is positively related to inflation variance. Khan's specification is related to Mussa (1981) who finds that update speed is a function of inflation variance in a model with rational agents. In this paper, the agent revises his speed of update optimally based on the adaptive step-size algorithm (2.9), and the resulting behavior of the gain matches the above empirical findings of increasing gain.

The adaptive step-size algorithm captures an increasing speed of update during hyperinflation better than the mechanism in MN. In the MN model, as hyperinflation progresses the agent makes forecast errors that are higher than the critical level, and therefore, the agent switches from a decreasing gain to the constant gain of 0.2. The gain then stays constant during hyperinflation and after hyperinflation is terminated by the ERR. Behavioral interpretation is that the agent realizes that he makes large forecast errors and interprets that as a change in regime, not as an exceptionally large shock in a stationary environment. The agent wants to learn about this new regime as quickly as possible in order not to make large forecast errors, and so he switches to high constant gain. As soon as forecast errors are below the critical level, the agent resumes the use of a decreasing gain.

To summarize, the model in this paper produces hyperinflation as in Marcet and Nicolini (2003) and, in addition, matches the empirical finding that gains increase during hyperinflation.

Another difference in the behavior of the gain occurs after hyperinflations end. The second panel of Figure 1 illustrates behavior in the MN model, and the second panel of Figure 2 illustrates behavior of adaptive step-size gain on the left y-axis, and the derivative V on the right y-axis. The agent in this paper *decreases* his gain whereas the MN agent continues to update at a high constant gain after hyperinflation is ended. The MN agent uses constant gain because he makes forecast errors above the critical level during hyperinflation and after it. The behavior of the agent using the adaptive step-size algorithm can be described as follows. During hyperinflation, the agent increases his gain to speed up his learning as explained above. By the time hyperinflation reaches its peak, the agent has experienced a long history of underpredicting actual inflation. When hyperinflation is terminated by imposing ERR, the agent makes negative forecast error ($\pi_t - \beta_t < 0$), i.e. he overpredicts actual inflation. When the forecast error is negative in (3.10), the agent will reduce his inflation forecast. Decrease in gain means that the forecast will be reduced by a smaller fraction of the last forecast error and achieves smaller revision of expectations. This can be interpreted as the agent not wanting to rush to lower his forecast.

There can be several reasons for an agent's unwillingness to update quickly. If the ERR is unknown to the agent, or is perhaps not credible, then the agent will be cautious, and not revise his inflation forecasts too quickly. Therefore, he reduces gain to decrease response to the negative forecast error. The agent needs to confirm that low inflation is not due to a temporary shock; and if low inflation is a new government's policy, the agent needs to observe that government is successful in implementing it. After the agent observes low inflation for some time, he will believe it is caused by a change in regime, and thus he increases his gain to learn it more quickly. Figure 4.3 shows that this behavior is typical after the end of hyperinflations.

I would like to provide a technical explanation of why the gain decreases after hyperinflation is stopped. Technical interpretation of the 'long history of underpredicting the inflation' is a high and positive value of V in (2.9, 2.10). V stores discounted past forecast errors. By the peak of hyperinflation, V is high and positive. When hyperinflation ends, negative forecast errors $\pi_t - \beta_t < 0$ lower V according to (2.10), but not by much and so V remains positive. From (2.9), the gain decreases. The key aspect of this behavior is the long history of past mistakes (high positive V) relative to negative forecast error. For the gain to increase, it is necessary to repeatedly experience negative forecast errors such that V decreases and becomes negative. When V and $\pi_t - \beta_t$ are both negative, the gain can increase. This means the agent's response to the latest forecast error increases, i.e. he starts to revise down his inflation expectations by larger increments.

This adaptive step-size algorithm produces behavior of the gain that is directly related to the learning mechanism specified in Cho and Sargent (1997). Their specification is set up so that the agent is suspicious about the government's implementation of good policy. When applied to a hyperinflationary environment, Cho and Sargent (1997) implies the following behavior. During hyperinflation the agent realizes that the government's policy is bad, and so he updates his beliefs with a high gain so as to be alert to possible repercussions. When hyperinflation is ended with the exchange rate rule, the agent no longer trusts the government's intentions, or possibly does not believe the new government's ability to implement the new policy successfully, and so the agent slows down the update of his beliefs.

Next, I evaluate the performance of the forecasting mechanism based on the adaptive step-size algorithm and the mechanism in MN by comparing mean squared errors (MSE) in the simulations. I run 100 simulations of each type, compute MSE for each run, and then average over 100 simulations. The values of model parameters are as described above. I compute MSE for the simulations of different lengths. The results are summarized in Table 2.1. For a simulation length of 100 periods, the values of MSE are comparable for the two mechanisms. The adaptive step-size gain performs better for simulation lengths greater or equal 200. This means that it takes some time for this specification to be put to its best use.

2.4 Estimation of the model parameters

This model is stylized, and the data on inflation is nonstationary. These factors can make econometric estimation of the model problematic. The estimation approach in this paper is based on indirect inference (Gourieroux et al. (1993)). The indirect inference method is useful for complex models with intractable likelihood function, and the only requirement for estimation is that the model can be simulated. The procedure can be summarized as follows. First, I compute the auxiliary parameter from the real data on inflation. I use skewness as an auxiliary parameter. This may be a quick fix but may be a useful start. Second, I simulate the model for different sets of model parameters, collect data from these simulations and compute the skewness of simulated inflation. The objective is to find the set of model parameters for which the distance between the auxiliary parameter from the real and simulated data is the closest. In other words, I aim to match the moment (skewness) in real data and in simulated data.

To match the moments in the data and the model, I use a genetic algorithm - a numerical optimization technique first introduced by Holland (1975) and described in Goldberg (1987), Michalewicz (1996), and Back et. al. (2000). Among the advantages of using a genetic algorithm are that it starts with a set of random solutions and so does not rely on the starting point, and that it is applicable for discontinuous, nondifferentiable, noisy, multimodal and other unconventional surfaces. (Schwefel 2000). Bullard and Duffy (2004) use a simulated method of moments with a genetic algorithm to estimate a growth model with structural breaks. The optimization problem here is to minimize the distance between the auxiliary parameters computed from real data and the auxiliary parameters from simulated data with respect to the values of model parameters. The nature of landscape is not known in advance, and so the application of a genetic algorithm is appropriate.

The parameters to be estimated are money demand parameters, γ and ϕ, mean $E(d)$ and standard deviation σ_d of seignorage, and the initial value of the gain a_1. Parameter $\mu = 0.001$ is fixed because the performance of the adaptive step-size algorithm is not sensitive to this parameter (as shown in Kushner and Yin (2003)).

2.4.1 Description of the genetic algorithm

The algorithm starts with N rules. Each rule consists of the model parameters to be estimated: γ, ϕ, $E(d)$, σ_d), (a_1).

The *initial pool of rules* is generated randomly from uniform distributions with supports that are different for each parameter and are given in Table 2.2 along with the standard deviations for mutation. Other genetic algorithm parameters are given in Table 2.3. The ranges for the parameters are chosen taking into consideration the values of the corresponding parameters in the related literature (Sargent, Wallace (1987), Marcet, Sargent (1989 b), Sargent, Williams, Zha (2006)). Attention must be paid to the value of mean seignorage because it has the maximum value for which a rational expectation equilibrium exists, and this maximum value is determined by the values of the money demand parameters. The value of mean seignorage is restricted to be close to the maximum value because hyperinflations can arise when mean seignorage is sufficiently close to its maximum value. I deal with these aspects by initializing and performing mutation on the mean seignorage after initialization and mutation of the parameters of the money demand in the following way. For each rule's values of money demand parameters γ, ϕ, I compute the maximum value of mean seignorage as:

$$max(d) = (1 + \gamma - 2\sqrt{\gamma})\phi$$

Then I restrict the range for mean seignorage $[min_d, max_d]$ within some distance from the maximum value $max(d)$ as $min_d = 0.9max(d)$, $max_d = 0.99max(d)$.

To evaluate the performance of each rule, the simulation is run for the parameter values of this rule, simulated inflation data is collected, and skewness of simulated inflation is computed. The length of the simulation is equal to the number of observations in the data on inflation plus 20 periods. The initial 20 periods of simulated data are discarded to avoid dependence on the initial values.

Fitness criterion is very important for the performance of the algorithm. Mean squared error is often used as a performance measure. In this estimation, fitness is the squared distance between skewness in the real data and skewness in the simulated data computed and added for 100 realizations.

Once fitness is computed, the pool of rules is updated by replication, crossover, and mutation.

Replication is done by tournament selection. Two rules are randomly selected with replacement from an old pool of rules. The fitness measures of these two rules are compared, and the rule with higher fitness is chosen into the new pool of rules. This procedure is repeated N times to form a new pool of rules. Replication provides all the selection pressure in this genetic algorithm.

Crossover is done with probability $pcross = 0.5$ for a pair of randomly selected (without replacement) rules. Once a pair of rules is chosen for crossover, the values of each parameter are exchanged between the rules with probability 0.5. The role of the crossover can be very important for the performance of the algorithm when searching for multiple parameters. [4]

[4] See Arifovic, Bullard, Kostyshyna (2007)

Mutation is done for each parameter in the rule with probability $pmut = 0.1$. The new value of the parameter is computed as:

$$new \quad rule = old \quad rule + randn * std$$

where $randn$ is the random number from the standard normal distribution (N(0,1)), std is the standard deviation for a specific parameter. The values of std for each parameter are presented in Table 2.2. If the value of a new rule is below (above) the low (high) boundary value, the new rule is set equal to low (high) boundary value.

The genetic algorithm is repeated for T iterations. The purpose is for the parameters to converge to their globally optimal values by the end of the genetic algorithm simulation.

2.4.2 Results of the computations

I now present the estimated parameters from the genetic algorithm optimization with skewness as an auxiliary parameter. For each parameter, I report the average and standard deviation (in parenthesis) of 30 rules after 40,000 iterations of the genetic algorithm.

The estimated parameter values for Argentina, Bolivia, Brazil and Peru are presented in Table 2.4. The data used for these countries are monthly inflation rates for the samples indicated in Table 2.4 and computed from International Financial Statistics Consumer Price Indexes. This data is plotted in Figure 2.4. The skewness in data for each country is given in column 2 in parenthesis. The skewness for simulated data for the estimated parameters is given in column 2.

Using these estimated parameter values, I simulate the model to obtain the path of the time-varying gain and present the simulations in Figure 2.6 for Argentina, in Figure 2.7 for Bolivia, in Figure 2.8 for Brazil, and in Figure 2.9 for Peru. For all countries except Brazil, the model with estimated parameters exhibits hyperinflation. The figures for the simulated data show that the model captures the general features in the actual data when compared to the figures with the actual data shown in Figures 2.5 and 2.4. Matching model skewness with skewness in the data can be viewed as a first step, which in the case of Brazil, does not produce the desired outcome. Therefore, a more sophisticated approach may be warranted, for example, matching transition probabilities in inflation data and simulated inflation for an autoregressive Markov-switching model.

2.5 Comparison of learning mechanisms

In this section, I compare Bayesian learning and learning based on the adaptive step-size algorithm to evaluate the loss of statistical efficiency from using the latter mechanism. I evaluate these learning mechanisms based on the criterion of Mean Squared Error in a simple case where agents need to make forecasts and learn about a Markow Switching inflation process.

I consider a process with first order 2-state Markov Switching constant and variance:

$$y_t = c_{S_t} + e_t, \quad e_t \sim N(0, \sigma^2_{S_t})$$

$$Pr[S_t = j/S_{t-1} = i] = p_{ji}, \quad i, j = 1, 2$$

$$\sum_{i=1}^{2} p_{ji} = 1$$

$$c_{S_t} = c_1 S_{1t} + c_2 S_{2t}$$

$$\sigma^2_{S_t} = \sigma^2_1 S_{1t} + \sigma^2_2 S_{2t},$$

where $S_{mt} = 1$ if $S_t = m$, $S_{mt} = 0$, otherwise, $m = 1, 2$

I use the following parameter values: $c_1 = 0.9$, $c_2 = 0.1$, $\sigma^2_1 = 0.1$, $\sigma^2_2 = 0.01$, $p_{11} = 0.9$, $p_{22} = 0.8$.

These parameters are used to simulate time paths, and allow agents to forecast inflation using two approaches - (i) Bayesian learning, and (ii) adaptive step-size algorithm. Bayesian agents have the following information about the data generating process - transition probabilities, means and variance in each state. Bayesian agents update probabilities of being in each state using Bayesian updating and form forecasts using this information. I compute an implicit gain from the forecasts of Bayesian agents as follows:

$$\beta_t = \beta_{t-1} + gain_t(\pi_t - \beta_{t-1}) \tag{2.11}$$

where β_t is the forecast in period t, π_t is actual data in period t. The implicit gain for Bayesian updating is:

$$gain_t = \frac{\beta_t - \beta_{t-1}}{\pi_t - \beta_{t-1}} \tag{2.12}$$

Agents using the adaptive step-size algorithm update their forecasts and gain at the same time. Their model is follows:

$$\beta_t = \beta_{t-1} + a_t(\pi_t - \beta_{t-1}) \tag{2.13}$$

$$a_t = \prod_{[a_-, a_+]} [a_{t-1} + \mu * (\pi_{t-1} - \beta_{t-1}) * V_{t-1}] \tag{2.14}$$

$$V_t = V_{t-1} - a_{t-1} * V_{t-1} + (\pi_{t-1} - \beta_{t-1}), V_0 = 0 \tag{2.15}$$

where $\beta_1 = y_1$, $a_1 = 0.2$, $\mu = 0.001$, $V_1 = 0$. I choose the same values of a_1 and μ as in Section 3.

A typical realization of the simulation is illustrated on Figure 2.10. This figure shows that each learning algorithm forecasts better or worse than the other during certain periods of time. For the simulation illustrated on Figure 2.10, forecasting based on Bayesian learning performs somewhat better than forecasting based on the adaptive step-size mechanism as measured by Mean Squared Error.

I perform 100 simulations and report Mean Squared Errors averaged over 100 simulations. These results are reported in Table 2.5. Based on the data in this Table, the Bayesian forecasting model performs somewhat better than the adaptive step size algorithm, but the mean squared forecast error from using the adaptive step size algorithm is not much bigger. The order of magnitudes of MSE are similar for the two mechanisms, and so the loss of forecasting accuracy is not high. This finding actually favors the adaptive algorithm, as it can perform almost the same without the substantial information requirements of Bayesian learning.

2.6 Comparison of related models

In Sargent, Williams and Zha (2006), hyperinflations occur when a sequence of seignorage shocks push inflation expectations above the high unstable self-confirming equilibrium (SCE), which means that inflation dynamics escape the domain of attraction of the low SCE. (SCE are good approximations of Rational Expectations Equilibria for very persistent average deficit states). In this escape region, actual inflation is higher than expected inflation, and so both actual and perceived inflation increase, and thus hyperinflations occur. The end of hyperinflations is explained by learning dynamics or changes in fundamentals.

A similar mechanism is in place in Marcet and Nicolini (2003). If inflation starts below the high inflation steady state, actual inflation is on average closer to the low inflation steady state than perceived inflation, and so learning moves perceived inflation towards the low inflation steady state. If perceived inflation is above the high inflation steady state, then actual inflation is on average higher than perceived inflation, and so perceived inflation increases. There is also an additional amplifying impact of the increased gain during the periods leading to hyperinflation and during hyperinflation: as the agent makes large forecast errors, the gain increases, and so the agent updates perceived inflation quicker which feeds back into the actual inflation and further increases it. When inflation reaches the critical level, the ERR is implemented and actual inflation is set to the low inflation steady state.

The hyperinflations happen in the adaptive step size algorithm application in this paper and in the model of Marcet and Nicolini (2003) in the same way. The difference between Marcet and Nicolini (2003) and the specification in this paper is the behavior of gain during hyperinflations and right after hyperinflation is ended with implementation of the ERR. In MN, the agent updates his forecasts using high gain (0.2) during and after hyperinflations until his forecast error is lower than the critical level. In my model, the agent increases speed of update as hyperinflation develops. Agent meets the ERR reform with suspicion, and lowers the speed of update after he observes a sudden large drop of inflation. After a sufficiently long period of low inflation, the agent is convinced that the reform works and proceeds to update his forecasts quickly to learn the new regime.

2.7 Conclusion

An adaptive step-size algorithm is introduced to model time-varying learning in the environment of Marcet and Nicolini (2003). This model behaves qualitatively similar to Marcet and Nicolini (2003). The model in this paper performs quantitatively somewhat better based on the criterion of mean squared error. Agent increases the speed of update during hyperinflations that matches empirical findings in Cagan (1956), Khan (1977), and Silveira (1973). The agent using this model shows caution when faced with sudden changes in policy, and is able to recognize the change in regime after acquiring sufficient information.

Model parameters are estimated using simulated method of moments with a genetic algorithm to minimize the distance between skewness in the real and the simulated data. The simulations for the estimated parameters exhibit hyperinflations for all countries except Brazil.

The loss of statistical efficiency of using adaptive step-size algorithm is not large in comparison to Bayesian learning based on the criterion of mean squared error.

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2.9 Appendix

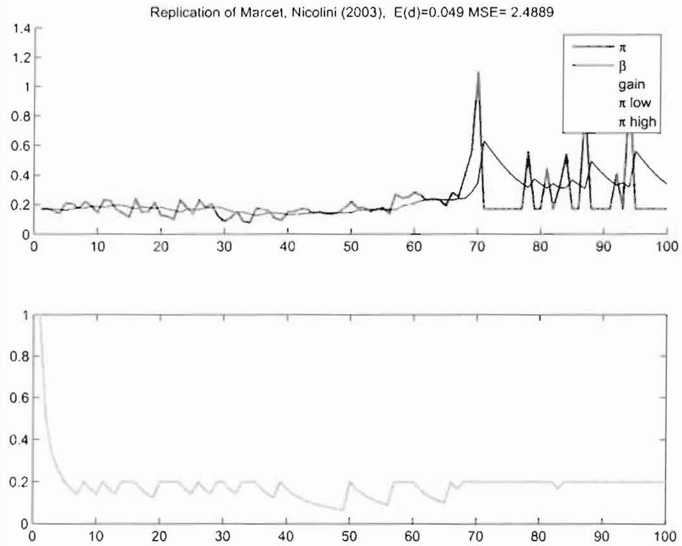

Figure 2.1: The replication of the model by Marcet and Nicolini (2003)

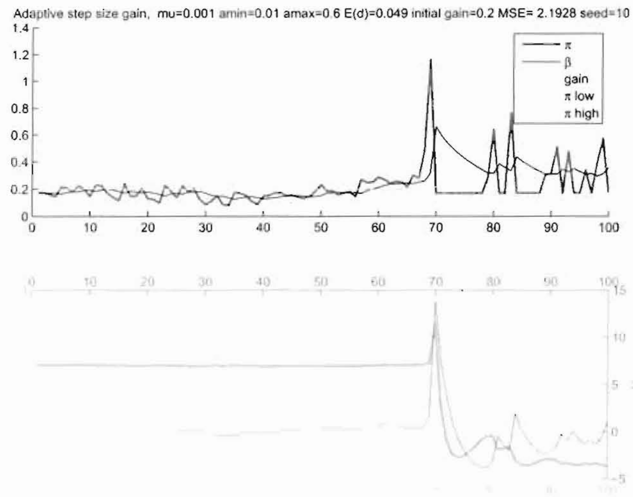

Figure 2.2: Typical simulation with the adaptive step-size algorithm. The first panel shows actual inflation (pai) and expectations of inflation (beta). The second panel shows adaptive step-size gain on the left y-axis (thick line), and the derivative V on the right y-axis (thin line).

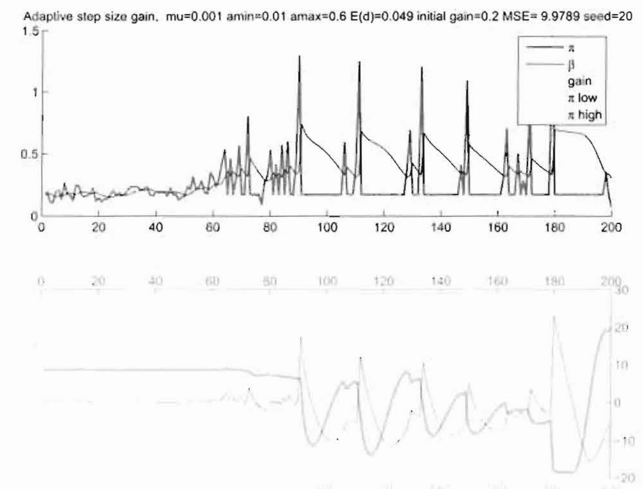

Figure 2.3: Typical simulation with the adaptive step size algorithm. The first panel shows actual inflation (pai) and expectations of inflation (beta). The second panel shows adaptive step-size gain on the left y-axis (thick line), and the derivative V on the right y-axis (thin line).

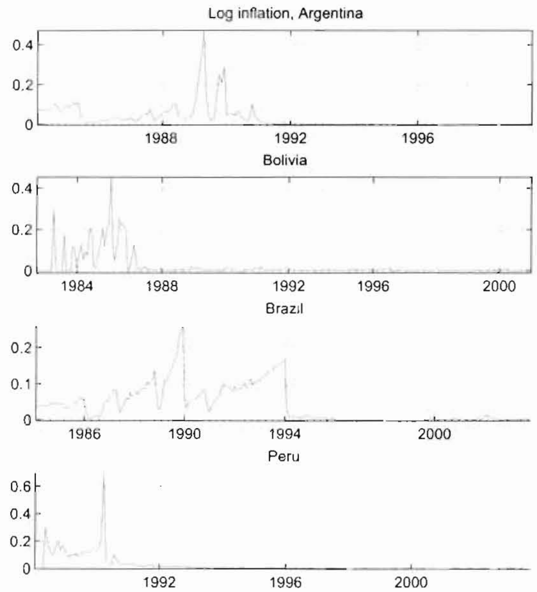

Figure 2.4: Monthly inflation rates (in logs).

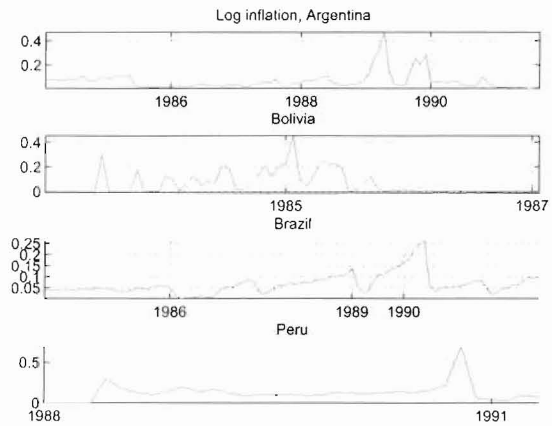

Figure 2.5: Monthly inflation rates (in logs) during periods of hyperinflation, closer look at the data from Figure 2.5.

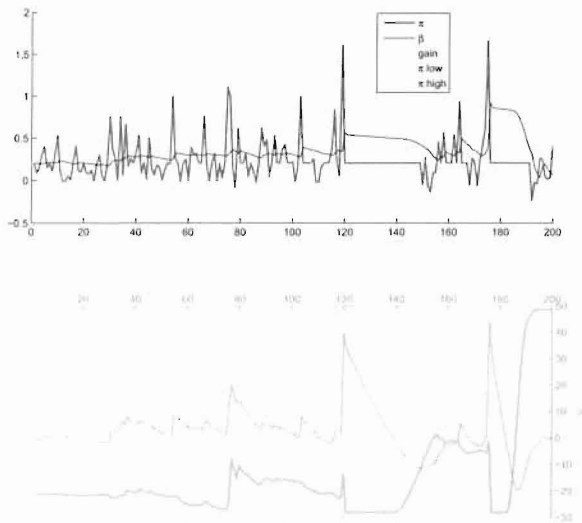

Figure 2.6: Typical simulation with estimated parameters based on skewness for Argentina. The first panel shows actual inflation (pai) and expectations of inflation (beta). The second panel shows adaptive step-size gain on the left y-axis (thick line), and the derivative V on the right y-axis (thin line).

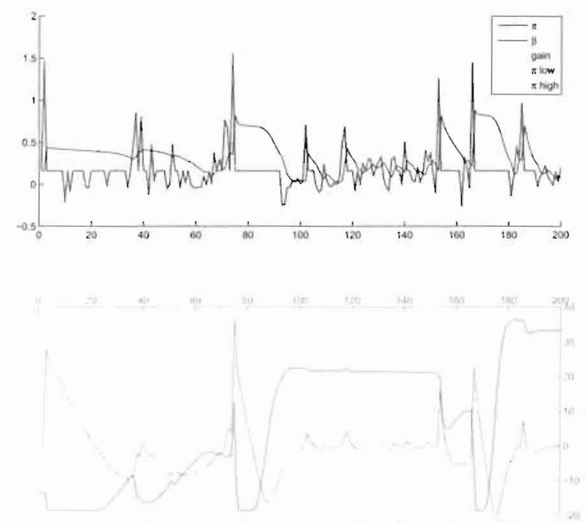

Figure 2.7: Typical simulation with estimated parameters based on skewness for Bolivia. The first panel shows actual inflation (pai) and expectations of inflation (beta). The second panel shows adaptive step-size gain on the left y-axis (thick line), and the derivative V on the right y-axis (thin line).

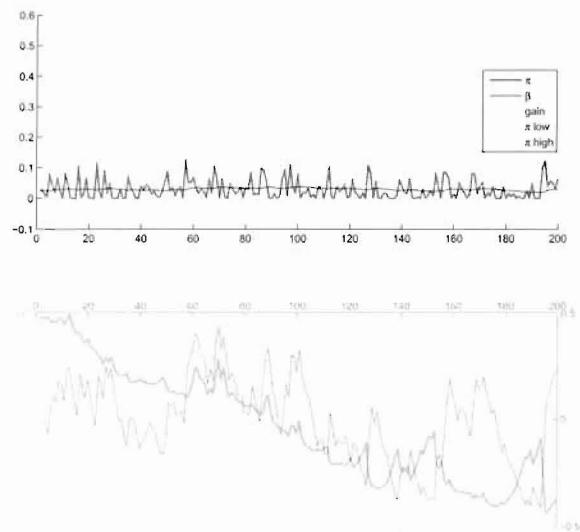

Figure 2.8: Typical simulation for estimated parameters based on skewness for Brazil. The first panel shows actual inflation (pai) and expectations of inflation (beta). The second panel shows adaptive step-size gain on the left y-axis (thick line), and the derivative V on the right y-axis (thin line).

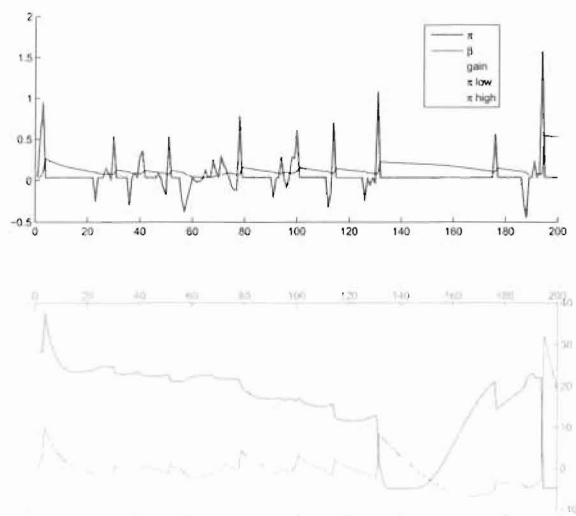

Figure 2.9: Typical simulation for estimated parameters based on skewness for Peru. The first panel shows actual inflation (pai) and expectations of inflation (beta). The second panel shows adaptive step-size gain on the left y-axis (thick line), and the derivative V on the right y-axis (thin line).

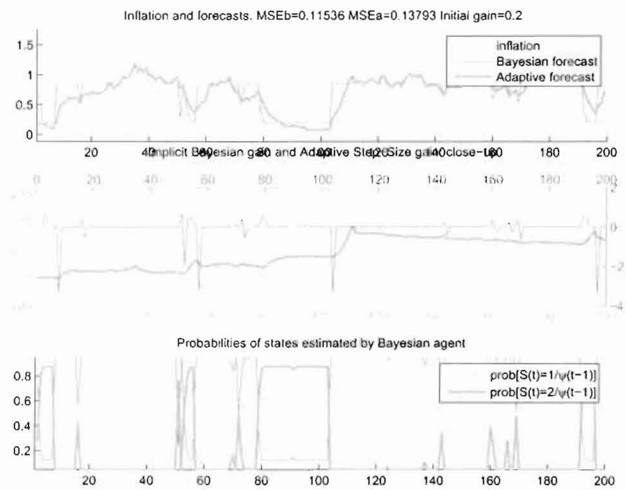

Figure 2.10: Typical simulation with Bayesian learning and adaptive step-size algorithm learning about Markow Switching process. The second panel shows adaptive step-size gain on the left y-axis (thick line), and the implicit Bayesian gain on the right y-axis (thin line)

TABLE 1.

Periods	Marcet, Nicolini (2003)	model with adaptive step size
100	7.00860 (8.4092)	7.30467 (8.68631)
200	9.21476 (5.46953)	8.71826 (5.48384)
300	11.21859 (5.52852)	8.00149 (4.22205)
400	11.57840 (4.62655)	7.35961 (3.07069)
500	11.74324 (4.29320)	7.10317 (2.80545)

Table 2.1: Mean squared errors in two models for simulations with different number of periods. Standard deviations are given in the parenthesis.

TABLE 2.

Parameters	Ranges	Standard deviations (std)
γ	[0.3, 0.9]	0.1
ϕ	[0.3, 1.9]	0.1
$E(d)$	range depends on γ, ϕ	0.05
σ_d	[0.01, 0.2]	0.05
a_1	[0.01, 0.4]	0.05

Table 2.2: Ranges and standard deviations for mutation for different parameters.

TABLE 3.

Parameters	Values
N	30
T	500
pcross	0.5
pmut	0.1

Table 2.3: Genetic algorithm parameters.

TABLE 4.

Country Sample	Skewness [data]	γ	ϕ	$E(d)$	σ_d	a_1
Argentina 1984:01-2000:04	6.2538 [5.6072]	0.35 (0.11)	1.17 (0.11)	0.194 (0.06)	0.18 (0.01)	0.05 (0.045)
Bolivia 1982:02-2001:09	6.2021 [5.2451]	0.38 (0.10)	0.71 (0.13)	0.099 (0.03)	0.16 (0.02)	0.05 (0.038)
Brazil 1984:01-2004:09	1.1931 [1.9263]	0.34 (0.11)	1.82 (0.31)	0.068 (0.13)	0.10 (0.04)	0.04 (0.10)
Peru 1988:04-2004:09	8.2743 [10.5472]	0.77 (0.09)	1.62 (0.23)	0.019 (0.002)	0.17 (0.02)	0.07 (0.07)

Table 2.4: Estimated values of the model parameters. The standard deviations of the estimated parameters are given in the parenthesis. The first row of column 2 gives skewness of the simulated data based on the estimated parameters, the second row of column 2 gives skewness in the data.

TABLE 5.

Number of periods	Bayesian learning	Adaptive step-size
100	0.1497	0.1862
200	0.1514	0.1862
300	0.1508	0.1857
400	0.1512	0.1856
500	0.1519	0.1859

Table 2.5: Mean Squared Errors averaged over 100 simulations.

Chapter 3

Social Learning and Monetary Policy Rules [1]

3.1 Introduction

3.1.1 Overview

Recent research has emphasized how policy choices may influence the stability properties of rational expectations equilibrium. In a typical analysis, a policymaker may commit to a particular policy rule, stating how adjustments to a control variable will be made in response to disturbances to the economy. The policy rule, together with optimizing private sector behavior, may imply that there is a unique rational expectations equilibrium associated with the policy rule, and that the equilibrium has desirable welfare properties. However, the equilibrium may or may not be robust to small expectational errors. If the expectations of the players in the economy are initially not rational, but deviate from rational expectations by a small amount, behavior of the players in the economy will be changed. This will then have effects on the price and quantity outcomes in the economy, feeding back into the learning process. Such a dynamic may or may not converge to the rational expectations equilibrium which is the policymaker's target. When the process does converge, it is called an *expectationally stable*, or *learnable* equilibrium.

We study learnability in a standard context, the model of monetary policy of Woodford

[1] **This chapter is based on a work cowritten with Jasmina Arifovic and James Bullard.**

(2003). A standard result, discussed in Woodford (2003) and Bullard and Mitra (2002), is that in a simple version of the model, the rational expectations equilibrium will be learnable provided the policymaker follows the *Taylor Principle*.[2] This means that the policymaker must react sufficiently aggressively to economic developments, such as deviations of inflation from target or the deviation of output from the flexible price, or potential, level of output. Failure to do so will create a rational expectations equilibrium which is unstable in the recursive learning dynamic. Such an equilibrium is unlikely to be successfully implemented in actual policymaking. Even small expectational errors would drive the economy away from the intended equilibrium.

The standard results are derived under the assumption of homogeneous expectations which are updated via recursive algorithms. This is the approach discussed extensively in Evans and Honkapohja (2001). By assuming homogeneous expectations and recursive algorithms, analytical results can be obtained concerning the expectational stability properties of equilibria across a wide variety of models. In this paper we study an alternative approach to learning, one that can be viewed as more realistic in terms of actual learning in complicated market economies. In it, agents are initially heterogeneous with respect to the models they use to forecast the future. Forecast rules are updated via genetic operators, meant to simulate the process of learning from neighbors and others in the economy. Results are not analytic but are based on computational experiments. We will call this alternative approach *social learning*.

Social learning has been studied in a wide variety of contexts in economics, but not in the standard New Keynesian model where many of the other findings concerning learnability have been presented. One reason is that the New Keynesian model is inherently stochastic, and the genetic algorithm applications which are drawn from the artificial intelligence literature are deterministic.[3] The genetic algorithm is meant to find "good" solutions to complicated problems with no known best solution. One purpose of this paper is to understand how insights from the genetic algorithm learning literature may be applied in a stochastic context.

[2]For a discussion of the Taylor Principle, see Woodford (2001).

[3]That is, the set of problems which have been considered are deterministic, although the algorithm itself is necessarily stochastic.

3.1.2 Main findings

We conduct a series of computational experiments with social learning in the setting studied by Bullard and Mitra (2002). Our main finding is that the Taylor Principle does not have to be met in order for agents to coordinate on a rational expectations equilibrium of the model via the social learning dynamic. This stands in marked contrast to the findings in the recent learning literature.

3.1.3 Recent related literature

Woodford (2003) contains the definitive statement of the nature of the New Keynesian model of monetary policy. Bullard (2006) surveys some of the literature on monetary policy and expectational stability, along with related issues. Genetic algorithm learning in economic contexts has been surveyed by Arifovic (2000).

Our paper is related to the recent literature on heterogenous learning. For example, Giannitsarou (2003) as well as Honkapohja and Mitra (2005, 2006) distinguish the following forms of heterogeneity in learning: different initial perceptions, different learning rules, and different degrees of inertia in updating in the same learning rule. Giannitsarou (2003) finds that when agents use least squares learning, E-stability implies learnability in the case of different initial perceptions. But for the other types of heterogeneity, the stability under homogenous learning does not necessarily imply stability under heterogenous learning. Our social learning approach encompasses a greater degree of heterogeneity than previous studies in this area, as a finite number of agents each have a different model within a given class of models.

Honkapohja and Mitra (2006) add structural heterogeneity to their analysis (agents respond differently to their forecasts), and study how transient and persistent heterogeneity in learning affects the learnability of the fundamental (MSV) solution. They find that transient heterogeneity in learning does not change the convergence conditions even in the presence of structural heterogeneity. But in case of persistent heterogeneity in learning, E-stability conditions do not in general imply learnability in structurally homogeneous and heterogeneous economies. Honkapohja and Mitra (2005) study the performance of interest rate rules in the presence of heterogeneous forecasts by the private sector and the central bank in New Keynesian model. They find that E-stability conditions are necessary but not sufficient for learnability with heterogeneity in learning.

Negroni (2003) studies heterogeneity in adaptive expectations. He considers two sources: heterogeneity of expectations (different gains) and heterogeneity of fundamentals. He finds that in the presence of heterogeneity, the conditions for convergence of heterogeneous adaptive beliefs to the stationary REE are not the same as for homogeneous beliefs.

Our agents have the same form of the learning rule but different initial beliefs about the values of the coefficients in the perceived law of motion. They update their beliefs at the same rate, so the economy is structurally homogeneous. Our agents are able to learn from the other agents (social learning), whereas in all the models with heterogenous learning mentioned above agents proceed to update their beliefs without knowing what and how well the rest of the agents are doing. The social aspect of the learning seems to be important for learning of the rational expectations equilibrium.

Branch and Evans (2004) show that heterogeneity can arise under certain conditions as an endogenous outcome when agents choose between misspecified models. In our study, agents have the correct specification of the REE model, although they start with different beliefs about the coefficients in the correct specification. Our question is whether agents are able to learn the fundamental (MSV) values of the coefficients.

3.1.4 Organization

In the next section we discuss the New Keynesian model that we wish to study in this paper. Much has been written about this model, but here we only provide the reader with a minimal outline of the key equations, as the model itself is not the focus of this analysis. We then turn to a discussion of the social learning dynamic as we have implemented it in the New Keynesian model. Our main findings are the results of computational experiments, which we compare to standard results from the literature. The concluding section summarizes our findings and suggests a few directions for future research.

3.2 Environment

3.2.1 Overview

We study the simple version of the New Keynesian model employed by Bullard and Mitra (2002) and Woodford (2003). The economy is populated by a continuum of infinitely-lived household-firms that maximize utility and profits. Household-firms consume all goods but produce only one good on the continuum. Firms are monopolistically competitive and face a Calvo-style sticky price friction when determining their price. The model consists of three equations along with an exogenously specified stochastic process. The first equation is the linearized version of the first order condition for household utility maximization. The second equation is the linearized version of the first order condition for firm maximization of profits. The third equation is a Taylor-type interest rate feedback rule that describes the behavior of the monetary authority.[4] The system is given by

$$z_t = z^e_{t+1} - \sigma^{-1}[r_t - \pi^e_{t+1}] + \sigma^{-1} r^n_t \quad (3.1)$$

$$\pi_t = \kappa z_t + \beta \pi^e_{t+1} \quad (3.2)$$

where z_t is the output gap, π_t is the deviation of the inflation rate from a prespecified target, r_t is the deviation of the short-term nominal interest rate from the value that would hold in a steady state with the level of inflation at target and output at the level consistent with fully flexible prices. A superscript e denotes a subjective expectation that can initially be different from a rational expectation. All variables are expressed in percentage point terms and the steady state is represented by the point $(z_t, \pi_t, r_t) = (0,0,0)$. The parameter $\beta \in (0,1)$ is the discount factor of the representative household, $\sigma > 0$ controls the intertemporal elasticity of substitution of the household, and $\kappa > 0$ relates to the degree of price stickiness in the economy. A standard calibration suggested by Woodford (2003) and widely used in the literature sets $(\beta, \sigma, \kappa) = (0.99, 0.157, 0.024)$. The natural rate of interest, r^n_t, is a stochastic term which follows the process

$$r^n_t = \rho r^n_{t-1} + \epsilon_t, \quad (3.3)$$

where ϵ_t is *i.i.d.* noise with variance σ^2_ϵ, and $0 \leq \rho < 1$ is a serial correlation parameter. The interest rate feedback rule of the monetary authority is given by

$$r_t = \varphi_\pi \pi_t + \varphi_z z_t, \quad (3.4)$$

where φ_π and φ_z are policy parameters taken to be strictly positive. The policymaker is committed to this rule and does not deviate from it. Substituting (3.4) into (3.1), we obtain

$$z_t = z^e_{t+1} - \sigma^{-1}[\varphi_\pi \pi_t + \varphi_z z_t - \pi^e_{t+1}] + \sigma^{-1} r^n_t. \quad (3.5)$$

[4]Optimal policy and learnability can also be studied—see Evans and Honkapohja (2003).

3.2.2 Determinacy and learnability

Equations (3.2),(3.3), and (3.5) can be written as:

$$y_t = \alpha + By_{t+1}^e + \chi r_t^n \tag{3.6}$$

where $\alpha = 0$, $y_t = [z_t, \pi_t]'$,

$$B = \frac{1}{\sigma + \varphi_z + \kappa\varphi_\pi} \begin{bmatrix} \sigma & 1 - \beta\varphi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \varphi_z) \end{bmatrix}, \tag{3.7}$$

and

$$\chi = \frac{1}{\sigma + \varphi_z + \kappa\varphi_\pi} \begin{bmatrix} 1 \\ \kappa \end{bmatrix}. \tag{3.8}$$

In order to analyze the effects of homogeneous recursive learning in this environment, Bullard and Mitra (2002) proceeded as follows. Assume that all agents have the following perceived law of motion (PLM)[5]

$$y_t = a + cr_t^n, \tag{3.9}$$

which describes their belief concerning the equilibrium law of motion of the economy. With this perceived law of motion, they form expectations as

$$E_t y_{t+1} = a + c\rho r_t^n = \begin{bmatrix} z_{t+1}^e \\ \pi_{t+1}^e \end{bmatrix}. \tag{3.10}$$

The actual law of motion (ALM) is then found by substituting (3.10) into (3.6)

$$y_t = Ba + (Bc\rho + \chi)r_t^n. \tag{3.11}$$

The minimal state variable (MSV) solution is

$$y_t = \bar{a} + \bar{c}r_t^n \tag{3.12}$$

where $\bar{a} = 0$ and $\bar{c} = [I - \rho B]^{-1}\chi$. At $(\bar{a}, \bar{c})$, the actual law of motion coincides with the perceived law of motion and rational expectations equilibrium has been attained. If the actual law of motion has dynamics which tend to this fixed point, we say that the equilibrium is learnable.

[5]The assignment of the PLM is not arbitrary but corresponds to the equilibrium law of motion of the economy.

Bullard and Mitra (2002) determine the necessary and sufficient condition for a rational expectations equilibrium to be determinate in the sense of Blanchard and Kahn (1980) as

$$\kappa(\varphi_\pi - 1) + (1 - \beta)\varphi_z > 0 \tag{3.13}$$

Bullard and Mitra (2002) also show that this same condition is necessary and sufficient for the expectational stability of rational expectations equilibrium. Inequality (3.13) is a statement of the Taylor Principle. In particular, consider the simplified condition $\varphi_z = 0$, so that the central bank does not respond to deviations of output from potential when setting its nominal interest rate target. Since $\kappa > 0$, the condition requires $\varphi_\pi > 1$, which is to say that the nominal interest rate must be adjusted more than one-for-one in response to deviations of inflation from target.

Bullard and Mitra (2002) concluded that condition (3.13) governs both uniqueness of rational expectations equilibrium as well as expectational stability of that equilibrium in this simple model.[6] Expectational stability is a notional time concept, but Evans and Honkapohja (2001) show that it governs the stability of the real time system formed when agents estimate the coefficients in (3.9) using recursive algorithms such as least squares. We now turn to examine the robustness of this finding when homogenous recursive learning is replaced with social learning.

3.3 Social learning

3.3.1 Overview

We study the behavior of evolutionary learning agents. Agents are initially heterogeneous with respect to their perceived law of motion (3.9), in the sense that each agent has a separate and possibly different set of coefficients. Thus each agent initially has a different forecasting model. The coefficients are updated using social evolutionary learning instead of least squares learning. Our objective is to see whether MSV solutions are learnable by evolutionary learning agents.

[6]The relationship between determinacy and learnability is less clear in more complicated settings.

3.3.2 Initialization

We follow the artificial intelligence literature and take equations (3.1) and (3.2) as fixed and immutable in the analysis under evolutionary learning. In this interpretation, we are viewing the formal model with homogeneous expectations the only way it can be viewed, namely, as an approximation to a more realistic model with heterogeneous expectations. The core model results of the previous section can only be useful to the economics community if they are intended to be reasonably robust to the introduction of some heterogeneity among agents, especially with respect to agent expectations. We now introduce that heterogeneity.

There are N agents in the private sector. Each agent, $i = 1, ..N$ has a perceived law of motion (PLM)

$$z_t = a_{1,i,t} + c_{1,i,t} r_t^n \tag{3.14}$$

$$\pi_t = a_{2,i,t} + c_{2,i,t} r_t^n \tag{3.15}$$

We stress that r^n is a stochastic term, and that finding equilibrium values of a and c will depend on evaluating how well each forecast rule works even though there is noise in the system. This is not a typical feature of evolutionary learning environments. It is true that the genetic operators we discuss below are inherently stochastic, but the fitness calculation does not normally have to contend with exogenous stochastic terms.

The initial values for the coefficients are each randomly generated from a normal distribution with mean equal to the respective MSV value. The standard deviation for coefficients c_1 and c_2 is equal the largest of the absolute values of the MSV values of these coefficients. We used a smaller initial standard deviation for the coefficients a_1 and a_2. The MSV values for these coefficients are 0 and are smaller than MSV values for the coefficients c_1 and c_2. Therefore, we used initial standard deviation for coefficients a half as large as for the coefficients c. When setting the values of initial standard deviations we have pursued several objectives - starting with diverse population of rules, injecting diverse new rules through mutation and keeping the diversity of new rules commensurate with the MSV values.

3.3.3 Expectations and the actual law of motion

Agents form their expectations of the output gap and deviation of inflation from target using (3.3), (3.14), (3.15) as

$$z^e_{i,t+1} = a_{1,i,t} + c_{1,i,t}\rho r^n_t, \tag{3.16}$$

$$\pi^e_{i,t+1} = a_{2,i,t} + c_{2,i,t}\rho r^n_t. \tag{3.17}$$

The average expectations of the output gap and the deviation of inflation from target are computed as

$$z^e_{t+1} = \frac{1}{N}\sum_{i=1}^{N} z^e_{i,t+1}, \tag{3.18}$$

$$\pi^e_{t+1} = \frac{1}{N}\sum_{i=1}^{N} \pi^e_{i,t+1}. \tag{3.19}$$

The actual values of the output gap and deviation of inflation from target are obtained from:

$$y_t = \alpha + B\begin{bmatrix} z^e_{t+1} \\ \pi^e_{t+1} \end{bmatrix} + \chi r^n_t. \tag{3.20}$$

3.3.4 Forecast rule performance

Agents assess the performance, or *fitness*, of their forecasting model using mean squared forecast error as a criterion. Agents compute the mean squared forecast error for the output gap and the deviation of inflation over all periods following an initial history. We stress that it is important not to base the performance on only the most recent forecast error because the environment is stochastic.[7]

The fitness is computed as

$$F_{i,t} = -\frac{1}{t}\sum_{k=1}^{t}(z_k - z^f_{i,k})^2 - w\frac{1}{t}\sum_{k=1}^{t}(\pi_k - \pi^f_{i,k})^2 \tag{3.21}$$

where z^f_k is the forecast value of z for period k, and π^f_k is the forecast value of π for period k, and w is the relative weight on the MSE for inflation. An agent is characterized by a set of

[7]Branch and Evans (2004, p. 3) assume that "... agents to make their on unconditional mean payoffs rather than on the most recent period's realized payoff. This is more appropriate in our stochastic environment since otherwise agents would frequently be misled by single period anomalies."

coefficients ($a_{1,i,t}$, $a_{2,i,t}$, $c_{1,i,t}$, $c_{2,i,t}$) at each date t. The terms z^f_k and π^f_k are the forecasts of the output gap and the deviation of inflation from target that agent i could have computed in period k, if he had used the current, date t, set of coefficients ($a_{1,i,t}$, $a_{2,i,t}$, $c_{1,i,t}$, $c_{2,i,t}$). The forecasts z^f_k, π^f_k are computed by agent i as

$$z^f_{i,k} = a_{1,i,t} + c_{1,i,t}\rho r^n_{k-1} \tag{3.22}$$

$$\pi^f_{i,k} = a_{2,i,t} + c_{2,i,t}\rho r^n_{k-1} \tag{3.23}$$

The weight w is used to give equal importance to the prediction error for the output gap and the deviation of inflation from target as the values of the MSE for these two variables can differ in order of magnitude. Without reasonable weighting, the fitness measure puts insufficient emphasis on the first or the second term in (3.21), leading to drift in coefficients away from MSV values.

First, we considered simulations with weight $w = 1$, implying output forecast error volatility and inflation forecast error volatility have the same weight in the assessment of forecast rules.[8] From these simulations, we collected the data on fitness and its composition: the first and the second summation terms in (3.21). This data indicated that the MSE for z was several orders of magnitude larger than the MSE for π, and therefore, agents effectively did not care very much about the accuracy of their prediction for π when assessing their forecast rule. As a result, the coefficients diverged away from MSV values (see quantitative details in section (3.5.3)).

The difference in magnitudes of MSE for output gap and MSE for inflation deviation can be explained by the difference in values of output gap and inflation deviations. From time series of z and π, we observed that output gap assumes larger values than inflation deviations. This comes from the values of coefficients in equation (3.20) for the computation of the actual output gap, z, and inflation deviation, π. At the standard calibration we use, the coefficients for the computation of z are several times larger than the coefficients for the computation of π. This makes values of z larger than values of π, and so the squared prediction error for z larger than for π. In turn, this implies that in the fitness calculation, the first summation term in (3.21) is considerably larger than the second summation term (most frequently by a factor of 100). We used the weight w to adjust for this asymmetry. In particular, we set w such that the first and second summation terms in (3.21) are of

[8]The genetic operators used in these simulations are described below. Here, we simply wanted to discuss the fitness criterion.

the same order of magnitude. We use weight equal 100 for the simulations reported in this paper.

The criterion (3.21) with z_k^f set to zero is a version of the objective function for the central bank that is often employed in models of optimal monetary policy. In studies of this type, w would represent the central bank's relative preference for inflation versus output volatility. This objective is also often rationalized as an approximation to the utility of the representative household in this economy, as suggested by Woodford (2003). In the optimal policy literature, w takes on a relatively large value. There the weight on inflation stabilization is typically set to one, and the weight on output stabilization is close to zero, so that the relative weight on inflation stabilization is quite large. In the present paper, the agents are concerned with the forecasting performance of their forecasting model, and so forecast performance matters and z_k^f as well as π_k^f are non-zero. However, the relatively large value of w that delivers the best performance of the learning model is similar.

3.3.5 Genetic operators

A hallmark of the evolutionary learning literature is that agents update their current state using genetic operators. These operators are meant to simulate the exchange of information in a large, complex economy, and are based on the principles of population genetics. Agents can meet other agents, exchange information concerning their current forecast rule, and possibly copy the partner's forecast rule, either in whole or in part. This process is implemented as described below.

We follow the literature in this area and use three genetic operators, namely crossover, mutation, and tournament selection. Our genetic system is real-valued. Crossover is implemented first. Two agents in the set of N agents are randomly matched without replacement. With probability of crossover *mcross*, their sets of coefficients can be subjected to crossover: If a random draw from a uniform distribution is less than or equal *mcross*, the agents exchange each type of coefficient with probability 0.5.

Mutation is implemented following crossover. An agent changes each coefficient with probability of mutation *mprob* in the following way

$$new = old + random * mutdeviation, \tag{3.24}$$

where *random* is a random number drawn from a standard normal distribution, *old* is the current value of the coefficient, and *mutdeviation* is the standard deviation used for mutation. We set *mutdeviation* to be decreasing over time according to

$$mutdeviation = deviation * (1 - decrease * t/T) \tag{3.25}$$

where *deviation* is the standard deviation used to generate initial set of rules, t is current date, T is the total number of periods in the simulation, and *decrease* is a coefficient. We set *decrease* equal 0.95, it is intended to allow non-zero mutation standard deviation even in the last period of the simulation. Mutation can be very destructive late in a simulation when the N forecast rules may be very close to optimal, REE forecast rules, because a random choice of a new coefficient will cause a new round of genetic variation. The term (3.25) is meant to control this effect.

After mutation, agents compute the fitness of their coefficients according to (3.21).

The final genetic operator is tournament selection. Agents are randomly selected in pairs with replacement N times. For each pair of agents, the fitness values of the forecast rules are compared. The agent with the higher fitness value is copied into the next generation of agents. This creates a new generation of N agents. After this update is finished, agents go to the next period of the simulation. Tournament selection will provide most of the selection pressure in this evolutionary learning environment, as weaker forecasting rules are systematically discarded during this process.

3.4 Computational experiments

3.4.1 Overview

We conduct a set of computational experiments in order to understand the behavior of the economy under social learning. We begin our simulations by generating an initial history for the system at the rational expectations equilibrium, that is, using the MSV values for the coefficients a and c. We then conduct simulations that last for 3000 periods, and we set the length of the initial history to 100 periods. We use the parameter values from Woodford (2003), namely, $\sigma = 0.157$, $\kappa = 0.024$, $\beta = 0.99$, and $\rho = 0.35$. The standard deviation of r^n is 3.72. We consider a range of values for the parameters in the Taylor-type monetary policy rule. Generally, these were for values of the coefficient on the output gap, $\varphi_z \in [0.2, 1.1]$. For the coefficient on inflation, we considered $\varphi_\pi \in [0.5, 2]$. At these parameter values, condition (3.13) is met for some policy parameter pairs but not for others, and is governed primarily by the value of φ_π.

We use the following parameter values in the genetic algorithm. The probability of mutation is 0.1 and the probability of crossover is 0.5. The number of agents is 30. The value of w in the fitness criterion is 100.

3.4.2 Main findings

We found that agents are able to learn MSV values of coefficients for most of the policy parameter pairs (φ_z, φ_π), both in determinate and E-stable region as well as in the indeterminate and E-unstable region. A series of four figures shows our main results.

A typical simulation result for the policy rule characterized by $\varphi_\pi = 2.0$ and $\varphi_z = 0.2$ is given in Figure 1, and for the policy rule $\varphi_\pi = 1.5$ and $\varphi_z = 0.5$ in Figure 2. These policy rules are associated with a determinate rational expectations equilibrium and E-stability. The figures show the time series of the deviation of each of the four coefficients from their MSV values averaged across all agents. The figure also shows ± 1 standard deviation for each coefficient's deviation from MSV values, showing the extent of the dispersion in coefficients in use at date t in the population of agents. Figures 1 and 2, along with other simulations using policy rules consistent with determinacy and learnability, suggest that long-run predictions from analyses using recursive learning and analyses using evolutionary learning are similar. In particular, both approaches predict convergence to the rational expectations equilibrium. This result breaks down when we consider other policy rules, however.

Figures 3 and 4 show typical simulation results for $\varphi_\pi = 0.5$ and $\varphi_z = 0.5$ or $\varphi_z = 0.3$, respectively. These policy rules are associated with indeterminacy and expectational instability. The figures again show the time series of the deviation of each of the four coefficients from their MSV values averaged across all agents, and ± 1 standard deviation. Here, the evolutionary learning dynamic converges to the MSV solution once again, even though the prediction from an analysis based on least squares learning would predict instability in the learning dynamics. These findings suggest that, provided one is willing to take an evolutionary learning perspective, the less aggressive policy rules are not as disturbing as they may have appeared to be.

In order to provide more details concerning these results, we performed 1000 simulations for different policy rules (φ_π, φ_z) and collected data for the deviations of coefficients from their MSV values for each simulation. During each simulation, for each coefficient, we computed the average value of deviation from the MSV value for each period. Then we computed average value of the average deviations during the last 100 periods of simulation.[9] We also compute average of absolute values of deviations from MSV values during last 100 periods. In addition we collected data on percentage deviations from MSV values for coefficients $c1$ and $c2$ and computed average of (absolute) percentage deviations during last 100 periods of the simulations. (We cannot compute percentage deviations for coefficients $a1$ and $a2$ as their MSV values are zero). For each policy rule (φ_π, φ_z), we perform 1000 simulations, collect the above described statistics for each simulation, and report means and standard deviations for each statistic over 1000 simulations.

Table 3.1 reports means and standard deviations for average deviations from MSV values for a variety of fixed policy rules. The policy rules presented in this table include some that induce a determinate and E-stable rational expectations equilibrium, as well as others that induce indeterminacy and expectational instability. The policy rules that induce determinacy and learnability according to condition (3.13) will have larger values of φ_π and φ_z, which tend to be located toward the northeast part of the table. Relatively small values for φ_π and φ_z are associated with indeterminacy and expectational instability, and tend to be located in the southwest portion of the table.

We can make the following observations from Table 3.1. Perhaps most importantly, for the policy rules considered, regardless of whether they are consistent with determinacy and learnability or not, the population coefficients are quite close to their MSV values. The genetic algorithm we have implemented allows mutation up to date T in the simulation and so does not attempt to eliminate variation entirely, yet the table indicates that the population is quite close to the one that would use MSV values exclusively (all values in the table are very close to zero). To the extent there are differences from MSV values, the deviations for the constant coefficients a_1 and a_2 can be somewhat higher than those for the slope coefficients c_1 and c_2. Standard deviations indicate that there is some variety in the population even during the last 100 periods of the simulation, but the extent of the variety is not very large.

Table 3.2 presents means and standard deviations for absolute values of the deviations from the MSV values. This table also presents the percentage absolute deviation for the slope coefficients c_1 and c_2. These percentages for the absolute deviations range from about 3.0 to 11.0, and do not seem to vary systematically with the policy rule.

[9]The results are not qualitatively different for data computed for last 100-period, last 10-period and last 1-period; therefore, we only report results for last 100-period data.

The previous tables illustrate convergence of each individual coefficient. We would also like to present a measure of convergence for a complete set of coefficients - how close all coefficients are to MSV values at the same time. Table 3.3 reports the number of simulations out of 1000 that satisfy specific convergence criteria based on averages of absolute deviations over last 100 periods of simulation. As different coefficients deviate from MSV values by different values, we present results of application of two criteria for convergence. Criterion 1 requires that absolute deviations from MSV values for all coefficients are less than or equal 0.2. Criterion 2 requires that the absolute deviation from the MSV value for $a1$ is less than or equal 0.5, and that the absolute deviations from the MSV values for $a2$, $c1$, and $c2$ are less than or equal to 0.3.[10]

Table 3.3 perhaps indicates a result more in conformity with previous findings in the learning literature: The number of simulations out of $1,000$ satisfying either convergence criterion clearly tends to decline as one moves toward the southwest in Table 3.3, that is, as one moves toward the region of the parameter space that is associated with indeterminacy and expectational instability. This is perhaps clearest when comparing the most northeasterly cell in the table with the cell in the southwest corner. The former is associated with determinacy and expectational stability, while the latter is not. In the northeast corner we observe values of 963 and 995, respectively, for the two convergence criteria, while in the southwest corner we observe values of 145 and 403. This would seem to be a clear indication that it is somehow "more difficult" for the social learning system to converge upon the MSV solution when expectational stability and determinacy conditions fail. However, we do not wish to press this point too hard. The cell associated with $\varphi_\pi = 1.0$ and $\varphi_z = 0.2$ has values of 208 and 578 for the two convergence criteria, respectively, not very different from the results for the cell in the southwest corner. Yet these parameter values satisfy condition (3.13); rational expectations equilibrium here is unique and expectationally stable. One other point is that Tables 3.1 and 3.2 indicated that whatever failure to converge may exist, actual values are not very different from MSV values, and would probably not be meaningful in economic terms.

In some simulations, we can observe deviations of average values of coefficients a_1 and a_2 from their MSV counterparts, even though agents are always able to learn MSV values of

[10]The number of simulations satisfying criterion 2 is very close to the number of simulations satisfying a criterion which requires that the absolute value of the deviation of coefficient c2 from its MSV value is less than 0.03, and the rest of the coefficients satisfy criterion 2.

c_1 and c_2 quite closely. Again considering Table 2, to the extent that agents are inaccurate in learning MSV values, it is due to the coefficients a_1 and a_2, as the deviation of these coefficients from MSV is the largest among all coefficients. In the least squares learning model of Bullard and Mitra (2002), as pointed by Woodford (2003, pp. 271-272), "... it is in fact the possible instability of the dynamics of estimates of the constant terms Γ_0 in the forecasting model that is the relevant threat; and whether this occurs or not is determined by whether or not the Taylor principle is adhered to ...." In our notation, Γ_0 corresponds to the coefficients a_1 and a_2. Similarly, Honkapohja and Mitra (2004) point out that "In Bullard and Mitra (2002, p.1757), the constant term was the key to E-stability of the MSV solution .... " However, our simulations show that the system under evolutionary learning behaves somewhat differently. While the values of a_1 and a_2 may not be as close to their MSV values as the values of c_1 and c_2, this effect occurs whether or not the Taylor principle holds.

3.5 Modifications and robustness

We performed several modifications of the simulations described above. These included using different fitness criterion and not using crossover. We now turn to a description of these modifications and their effects on the results.

3.5.1 Different performance evaluation

As we stressed earlier, the weighting of the two dimensions in the fitness criterion is essential to convergence of the social learning systems we study. Without reasonable weighting, the fitness measure puts insufficient emphasis on one dimension or the other, leading to drift in coefficients away from MSV values. The modification considered in this section has each agent compute the mean squared error for forecasting deviation of inflation from target and the output gap separately, and simply consider them separately without combining them into one fitness measure. In particular, agent i computes mean squared errors for the output gap and inflation as

$$F^z_{i,t} = -\frac{1}{t}\sum_{k=1}^{t}(z_k - z^f_{i,k})^2, \quad (3.26)$$

$$F^\pi_{i,t} = -\frac{1}{t}\sum_{k=1}^{t}(\pi_k - \pi^f_{i,k})^2, \quad (3.27)$$

where $z^f_{i,k}$, $\pi^f_{i,k}$ are computed as in (3.22) and (3.23).

The change in performance criterion also has affects on the tournament selection operator. We modified the operator as follows. Again, N pairs of agents are randomly selected from the current generation with replacement, and fitness is compared for each pair. A new member of the next generation adopts the coefficients for forecasting output gap from the agent with higher $F^z_{i,t}$ (lower mean squared error for forecasting the output gap) and the coefficients for forecasting the deviation of inflation from the target from the agent with higher $F^\pi_{i,t}$ (lower mean squared error for forecasting inflation). In this way the next generation of agents is created, and more fit forecasting rules are systematically selected while weaker rules are systematically discarded.[11]

The results of these simulations are reported in Table 3.4. This table reports the same data as Table 3.2 for the baseline simulations. The results are qualitatively the same as for the baseline simulations. Table 3.5 reports the number of simulations that satisfy convergence criteria. We find similar effects when moving from northeast to southwest in this table as we did in Table 3.3.

3.5.2 Simulations without crossover

Crossover is considered a powerful operator in the genetic algorithm literature. One is taking "building blocks of good solutions" and combining them to create new possible solutions. This is thought to be a much faster way to find a good solution to a difficult problem than to merely rely on a mutation process. Especially for our real-valued, multidimensional problem, it can take a long time for mutation alone to find the best solution. In this subsection, we show that crossover is essential to our findings. To do this, we consider systems in which crossover has been discarded completely from the genetic algorithm. These simulations are done in the same way as the baseline simulations described above, with

[11] One of the principles of the GA literature is to accomplish this task without losing genetic diversity too rapidly.

the only modification of no crossover. Table 3.6 reports the some of the same data for simulations without crossover as Table 3.2 for the baseline simulations. The simulations without crossover have the following results. The constant coefficients, a_1 and a_2, approach the MSV values of zero. However, the slope coefficients, c_1 and c_2, deviate from the MSV values by $96 - 99$ percent. We conclude that crossover is an important GA operator for learning the MSV solution in this model.

3.5.3 Weight equal 1.

For completeness, we also report results of the simulations in which the weight w was simply set equal to one. As we have indicated, the convergence properties are not as good for this parameterization. The results are shown in Table 3.7 where the coefficients tend to be farther from MSV values at the end of the simulation as compared to the baseline simulation. Table 3.8 shows that the convergence criteria are met less often as well.

3.6 Conclusion

A key finding in the literature on learning in New Keynesian models of monetary policy is that nominal interest rate feedback policies which are too close to an interest rate peg tend to be associated with indeterminacy and instability in the recursive learning dynamics. The policymaker must react sufficiently aggressively to economic developments in order to assure determinacy of rational expectations equilibrium and expectational stability of that equilibrium. This has been promoted as an important reason to discard policy rules which are insufficiently aggressive,[12] and this idea has gained widespread acceptance in monetary policy discussions.

We have investigated whether this result is robust to the substitution of an evolutionary learning dynamic for the recursive learning dynamic. Our main finding is that the evolutionary learning dynamic does not put a premium on policy rules which obey the Taylor Principle. Instead, evolutionary learning converges to a small neighborhood of the MSV solution whether or not the policymaker obeys that principle.

When the Taylor Principle is violated, equilibrium is indeterminate. It is well-known that sunspot equilibria exist in a neighborhood of an indeterminate rational expectations

[12] See, for instance, Woodford (2003).

equilibrium. This is another important reason why insufficiently aggressive policy rules may be considered poor policy. In the recursive learning literature, it has generally been difficult to obtain expectational stability of sunspot equilibria.[13] An interesting extension of our analysis would be to analyze the stability of sunspot equilibria under the evolutionary learning dynamic.

[13]See, for instance, Honkapohja and Mitra (2004).

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3.8 Appendix

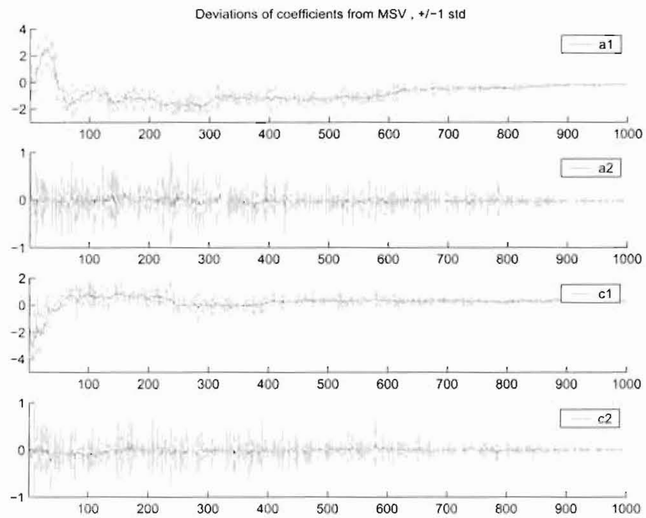

Figure 1. Simulation for determinate and E-stable region: $\phi_\pi = 2$, $\phi_z = 0.2$.

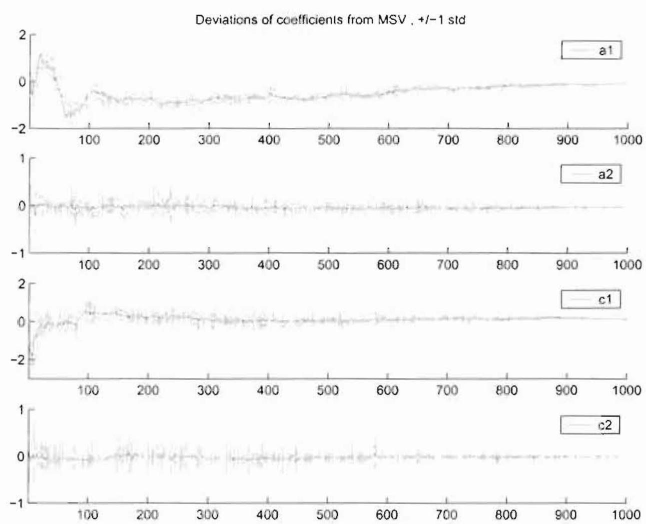

Figure 2. Simulation for determinate and E-stable region: $\phi_\pi = 1.5$, $\phi_z = 0.5$.

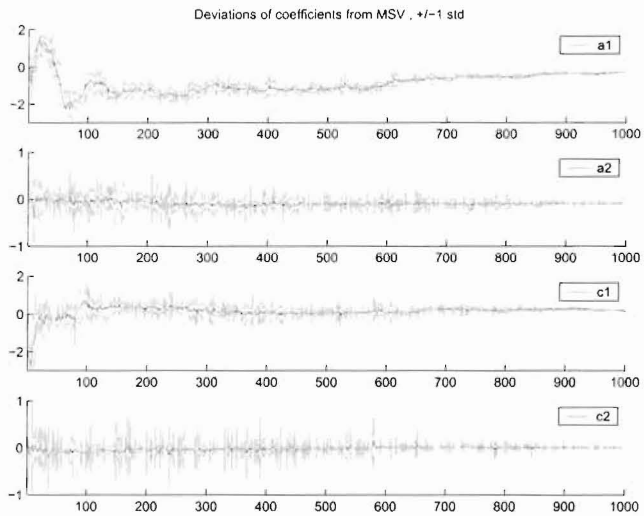

Figure 4. **Simulation for indeterminate and E-unstable region:** $\phi_\pi = 0.5$, $\phi_z = 0.5$.

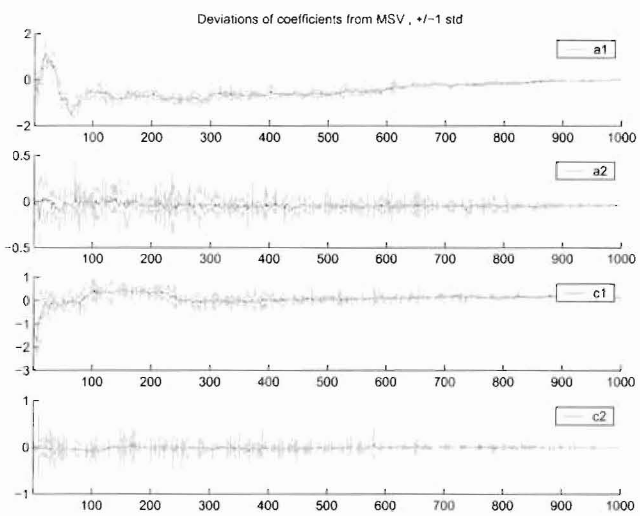

Figure 3. **Simulation for indeterminate and E-unstable region:** $\phi_\pi = 0.5$, $\phi_z = 0.3$.

TABLE 1.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	-0.001	0.078	0.000	0.063	0.001	0.066	0.001	0.078
	a2	-0.002	0.081	-0.001	0.076	-0.002	0.071	-0.002	0.066
	c1	-0.002	0.044	-0.001	0.043	0.000	0.042	0.001	0.042
	c2	-0.002	0.003	-0.002	0.003	-0.002	0.003	-0.002	0.003
1.0	a1	-0.001	0.089	0.000	0.070	0.001	0.073	0.002	0.089
	a2	-0.002	0.089	-0.001	0.082	-0.002	0.077	-0.002	0.071
	c1	-0.002	0.048	-0.001	0.047	0.000	0.046	0.001	0.045
	c2	-0.002	0.003	-0.002	0.003	-0.002	0.003	-0.002	0.003
0.9	a1	-0.001	0.102	0.000	0.078	0.001	0.082	0.002	0.103
	a2	-0.002	0.098	-0.002	0.090	-0.002	0.084	-0.002	0.078
	c1	-0.002	0.053	-0.001	0.052	0.000	0.051	0.001	0.050
	c2	-0.002	0.004	-0.002	0.004	-0.002	0.004	-0.002	0.003
0.8	a1	-0.001	0.120	0.000	0.088	0.001	0.094	0.003	0.121
	a2	-0.002	0.110	-0.002	0.100	-0.003	0.092	-0.002	0.084
	c1	-0.003	0.060	-0.001	0.058	0.000	0.056	0.001	0.055
	c2	-0.003	0.004	-0.003	0.004	-0.003	0.004	-0.002	0.004
0.7	a1	-0.001	0.146	0.000	0.101	0.002	0.111	0.003	0.145
	a2	-0.002	0.125	-0.002	0.113	-0.003	0.102	-0.002	0.092
	c1	-0.004	0.068	-0.002	0.066	0.000	0.064	0.002	0.062
	c2	-0.003	0.005	-0.003	0.005	-0.003	0.004	-0.003	0.004

Table 3.1: Means and standard deviations of coefficients for last 100 periods for a variety of policy rules.

TABLE 1 CONTINUED.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
0.6	a1	-0.002	0.183	0.000	0.120	0.003	0.134	0.003	0.182
	a2	-0.002	0.145	-0.003	0.129	-0.003	0.115	-0.002	0.103
	c1	-0.005	0.078	-0.002	0.075	0.000	0.073	0.002	0.070
	c2	-0.004	0.005	-0.003	0.005	-0.003	0.005	-0.003	0.005
0.5	a1	-0.002	0.240	0.000	0.146	0.003	0.168	0.004	0.234
	a2	-0.003	0.170	-0.003	0.150	-0.003	0.131	-0.002	0.115
	c1	-0.007	0.092	-0.003	0.088	0.000	0.085	0.003	0.082
	c2	-0.004	0.006	-0.004	0.006	-0.004	0.006	-0.003	0.005
0.4	a1	-0.003	0.343	0.001	0.186	0.005	0.222	0.005	0.318
	a2	-0.003	0.212	-0.004	0.178	-0.003	0.152	-0.002	0.131
	c1	-0.010	0.112	-0.004	0.107	0.000	0.102	0.004	0.097
	c2	-0.005	0.008	-0.005	0.007	-0.004	0.007	-0.004	0.007
0.3	a1	-0.002	0.548	0.003	0.256	0.008	0.320	0.010	0.463
	a2	-0.003	0.280	-0.004	0.221	-0.004	0.182	-0.002	0.151
	c1	-0.016	0.144	-0.007	0.135	0.001	0.127	0.007	0.121
	c2	-0.007	0.010	-0.006	0.009	-0.006	0.008	-0.005	0.008
0.2	a1	-0.004	1.051	0.007	0.405	0.017	0.523	0.016	0.735
	a2	-0.005	0.399	-0.004	0.291	-0.005	0.226	-0.002	0.173
	c1	-0.030	0.200	-0.013	0.183	0.001	0.170	0.011	0.158
	c2	-0.010	0.014	-0.008	0.012	-0.007	0.011	-0.006	0.010

TABLE 2.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	0.061	0.049	0.050	0.039	0.052	0.040	0.059	0.051
	a2	0.046	0.067	0.043	0.062	0.041	0.059	0.038	0.054
	c1	0.035	0.027	0.035	0.026	0.034	0.025	0.033	0.025
	c2	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
	perc c1	4.265	3.226	4.236	3.204	4.205	3.172	4.178	3.165
	perc c2	10.129	6.779	10.015	6.833	9.992	6.717	9.819	6.478
1.0	a1	0.069	0.056	0.055	0.043	0.058	0.045	0.066	0.059
	a2	0.050	0.073	0.047	0.067	0.044	0.063	0.041	0.058
	c1	0.039	0.029	0.038	0.028	0.037	0.028	0.036	0.027
	c2	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
	perc c1	4.280	3.238	4.253	3.206	4.213	3.196	4.183	3.177
	perc c2	10.145	6.843	9.937	6.763	9.967	6.699	9.873	6.533
0.9	a1	0.079	0.065	0.062	0.047	0.065	0.051	0.076	0.070
	a2	0.056	0.081	0.052	0.074	0.048	0.069	0.045	0.063
	c1	0.043	0.032	0.042	0.031	0.040	0.031	0.040	0.030
	c2	0.004	0.002	0.004	0.002	0.004	0.002	0.003	0.002
	perc c1	4.294	3.254	4.272	3.227	4.224	3.208	4.210	3.190
	perc c2	10.231	6.818	10.059	6.785	10.048	6.703	9.814	6.532
0.8	a1	0.093	0.077	0.070	0.054	0.074	0.059	0.088	0.083
	a2	0.063	0.090	0.058	0.082	0.053	0.076	0.048	0.068
	c1	0.048	0.036	0.046	0.035	0.045	0.034	0.044	0.033
	c2	0.004	0.003	0.004	0.003	0.004	0.003	0.004	0.002
	perc c1	4.339	3.275	4.285	3.235	4.247	3.222	4.226	3.198
	perc c2	10.284	6.834	10.048	6.883	9.981	6.842	9.700	6.530
0.7	a1	0.111	0.094	0.080	0.062	0.086	0.070	0.104	0.101
	a2	0.072	0.103	0.065	0.092	0.059	0.084	0.053	0.075
	c1	0.054	0.041	0.052	0.040	0.051	0.038	0.049	0.037
	c2	0.005	0.003	0.005	0.003	0.004	0.003	0.004	0.003
	perc c1	4.371	3.307	4.320	3.269	4.278	3.235	4.250	3.217
	perc c2	10.408	7.025	10.180	6.934	10.085	6.770	9.690	6.552

Table 3.2: Means and standard deviations for absolute values of 100-period data for a variety of policy rules. The values of the a1 and a2 coefficients are sometimes farther from the MSV solution, which is zero.

TABLE 2 CONTINUED.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
0.6	a1	0.138	0.120	0.095	0.073	0.103	0.085	0.128	0.130
	a2	0.083	0.119	0.074	0.105	0.067	0.094	0.059	0.084
	c1	0.062	0.047	0.060	0.046	0.058	0.044	0.056	0.042
	c2	0.005	0.004	0.005	0.003	0.005	0.003	0.005	0.003
	perc c1	4.411	3.336	4.362	3.308	4.307	3.254	4.284	3.231
	perc c2	10.443	7.044	10.286	6.908	10.093	6.750	9.645	6.479
0.5	a1	0.180	0.159	0.115	0.089	0.128	0.109	0.160	0.171
	a2	0.098	0.139	0.087	0.122	0.076	0.107	0.067	0.094
	c1	0.074	0.056	0.071	0.053	0.068	0.051	0.065	0.049
	c2	0.006	0.004	0.006	0.004	0.006	0.004	0.005	0.004
	perc c1	4.484	3.375	4.418	3.342	4.356	3.303	4.316	3.278
	perc c2	10.522	7.044	10.312	6.918	10.172	6.816	9.701	6.569
0.4	a1	0.254	0.230	0.147	0.113	0.166	0.147	0.212	0.237
	a2	0.123	0.173	0.104	0.145	0.090	0.123	0.077	0.106
	c1	0.090	0.068	0.085	0.065	0.081	0.062	0.077	0.059
	c2	0.008	0.005	0.007	0.005	0.007	0.005	0.006	0.004
	perc c1	4.567	3.453	4.487	3.398	4.409	3.350	4.353	3.309
	perc c2	10.941	7.189	10.346	6.994	10.188	6.845	9.816	6.710
0.3	a1	0.395	0.379	0.204	0.154	0.233	0.219	0.300	0.352
	a2	0.162	0.228	0.130	0.179	0.107	0.147	0.090	0.121
	c1	0.116	0.087	0.108	0.082	0.101	0.077	0.096	0.073
	c2	0.010	0.007	0.009	0.006	0.008	0.006	0.008	0.005
	perc c1	4.716	3.552	4.584	3.473	4.504	3.410	4.463	3.360
	perc c2	11.160	7.344	10.716	7.166	10.165	6.796	9.596	6.470
0.2	a1	0.739	0.747	0.324	0.244	0.369	0.371	0.462	0.571
	a2	0.234	0.323	0.174	0.233	0.137	0.180	0.107	0.136
	c1	0.162	0.122	0.146	0.111	0.135	0.103	0.126	0.095
	c2	0.014	0.009	0.012	0.008	0.011	0.007	0.010	0.007
	perc c1	4.972	3.752	4.774	3.609	4.649	3.544	4.570	3.462
	perc c2	11.647	7.625	10.918	7.340	10.204	6.906	9.753	6.623

TABLE 3.

Parameter	φ_π	0.5	1.0	1.5	2.0
φ_z	Criterion				
1.1	1	950	960	964	963
	2	990	992	994	995
1.0	1	933	947	955	957
	2	982	990	989	991
0.9	1	901	931	945	935
	2	973	981	989	991
0.8	1	866	911	921	902
	2	967	972	979	987
0.7	1	810	863	880	860
	2	952	962	971	974
0.6	1	725	801	822	807
	2	932	945	961	966
0.5	1	604	694	739	717
	2	901	925	944	938
0.4	1	453	554	607	619
	2	830	888	919	889
0.3	1	280	379	467	490
	2	668	800	847	815
0.2	1	145	208	275	306
	2	403	578	677	690

Table 3.3: The number of simulations for which each of the criteria is satisfied, the total number of simulations is 1000. Criterion 1 means that absolute deviations from MSV values for all coefficients are less than or equal 0.2. Criterion 2 means that absolute deviations from MSV value for $a1$ is less than or equal 0.5 and that absolute deviations from MSV values for $a2$, $c1$, and $c2$ are less than or equal to 0.3.

TABLE 4.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	0.064	0.053	0.050	0.039	0.054	0.044	0.066	0.062
	a2	0.061	0.083	0.057	0.078	0.053	0.072	0.050	0.066
	c1	0.034	0.027	0.033	0.026	0.033	0.026	0.032	0.025
	c2	0.003	0.002	0.003	0.002	0.003	0.002	0.003	0.002
	perc c1	4.117	3.230	4.091	3.199	4.059	3.187	4.036	3.168
	perc c2	9.102	6.030	8.946	6.035	8.812	5.899	8.687	5.867
0.8	a1	0.099	0.086	0.069	0.055	0.079	0.068	0.100	0.102
	a2	0.084	0.114	0.077	0.103	0.070	0.093	0.063	0.084
	c1	0.046	0.036	0.045	0.035	0.044	0.034	0.042	0.033
	c2	0.004	0.002	0.004	0.002	0.003	0.002	0.003	0.002
	perc c1	4.189	3.280	4.149	3.250	4.112	3.224	4.081	3.204
	perc c2	9.208	6.131	9.037	6.042	8.893	5.929	8.678	5.868
0.5	a1	0.202	0.190	0.115	0.091	0.141	0.130	0.188	0.211
	a2	0.133	0.179	0.116	0.154	0.100	0.130	0.087	0.113
	c1	0.071	0.056	0.068	0.054	0.065	0.051	0.063	0.049
	c2	0.006	0.004	0.005	0.004	0.005	0.003	0.005	0.003
	perc c1	4.330	3.400	4.262	3.349	4.208	3.303	4.151	3.263
	perc c2	9.469	6.365	9.200	6.211	8.946	5.981	8.759	5.936
0.2	a1	0.886	0.938	0.325	0.258	0.417	0.460	0.567	0.715
	a2	0.314	0.410	0.236	0.301	0.178	0.218	0.138	0.164
	c1	0.156	0.122	0.141	0.111	0.130	0.102	0.121	0.095
	c2	0.012	0.008	0.011	0.007	0.010	0.007	0.009	0.006
	perc c1	4.794	3.756	4.608	3.627	4.477	3.520	4.380	3.442
	perc c2	10.346	6.848	9.801	6.478	9.243	6.309	8.841	6.074

Table 3.4: Means and standard deviations for absolute values of 100-period data for a variety of policy rules for simulations with separate mean squared error for output gap and deviation of inflation from target.

TABLE 5.

Parameter	φ_π	0.5	1.0	1.5	2.0
φ_z	Criterion				
1.1	1	921	941	952	949
	2	975	980	984	987
0.8	1	828	874	893	869
	2	946	959	968	974
0.5	1	553	669	710	682
	2	861	898	924	920
0.2	1	127	193	267	275
	2	371	549	620	619

Table 3.5: The number of simulations for which each of the criteria is satisfied for simulations with separate fitness, the total number of simulations is 1000. Criterion 1 means that absolute deviations from MSV values for all coefficients are less than or equal 0.2. Criterion 2 means that absolute deviations from MSV value for $a1$ is less than or equal 0.5 and that absolute deviations from MSV values for $a2$, $c1$, and $c2$ are less than or equal to 0.3.

TABLE 6.

Parameter φ_z	φ_π	0.5 mean	0.5 std deviation	1.0 mean	1.0 std deviation	1.5 mean	1.5 std deviation	2.0 mean	2.0 std deviation
1.1	a1	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000
	a2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	c1	0.825	0.001	0.812	0.001	0.800	0.001	0.789	0.001
	c2	0.030	0.000	0.029	0.000	0.029	0.000	0.028	0.000
	perc c1	99.570	0.063	99.570	0.063	99.569	0.063	99.569	0.063
	perc c2	96.999	0.632	96.972	0.637	96.947	0.639	96.919	0.641
0.8	a1	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	a2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	c1	1.097	0.001	1.075	0.001	1.054	0.001	1.034	0.001
	c2	0.039	0.000	0.038	0.000	0.038	0.000	0.037	0.000
	perc c1	99.570	0.063	99.570	0.063	99.570	0.063	99.569	0.063
	perc c2	97.075	0.620	97.037	0.623	97.001	0.631	96.967	0.637
0.5	a1	0.002	0.001	0.002	0.001	0.002	0.001	0.002	0.001
	a2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	c1	1.639	0.001	1.591	0.001	1.545	0.001	1.503	0.001
	c2	0.059	0.000	0.057	0.000	0.055	0.000	0.054	0.000
	perc c1	99.571	0.063	99.571	0.063	99.570	0.063	99.570	0.063
	perc c2	97.214	0.597	97.160	0.603	97.108	0.614	97.055	0.623
0.2	a1	0.004	0.002	0.003	0.002	0.003	0.001	0.003	0.001
	a2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	c1	3.238	0.002	3.055	0.002	2.892	0.002	2.746	0.002
	c2	0.117	0.001	0.110	0.001	0.104	0.001	0.099	0.001
	perc c1	99.574	0.063	99.573	0.063	99.572	0.063	99.572	0.063
	perc c2	97.566	0.553	97.460	0.569	97.366	0.580	97.271	0.589

Table 3.6: Means and standard deviations for absolute values of 100-period data for a variety of policy rules for simulations with no crossover.

TABLE 7.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
1.1	a1	0.072	0.060	0.050	0.039	0.062	0.050	0.087	0.082
	a2	0.093	0.107	0.088	0.102	0.084	0.096	0.081	0.092
	c1	0.035	0.027	0.035	0.026	0.034	0.026	0.033	0.025
	c2	0.004	0.003	0.004	0.003	0.004	0.003	0.004	0.003
	perc c1	4.266	3.229	4.236	3.189	4.212	3.181	4.177	3.165
	perc c2	13.579	10.732	13.243	10.246	13.147	10.178	13.113	10.429
1.0	a1	0.084	0.070	0.055	0.043	0.071	0.058	0.102	0.095
	a2	0.103	0.118	0.095	0.111	0.092	0.105	0.088	0.099
	c1	0.039	0.029	0.038	0.028	0.037	0.028	0.036	0.027
	c2	0.005	0.004	0.004	0.003	0.004	0.003	0.004	0.003
	perc c1	4.278	3.238	4.254	3.199	4.221	3.188	4.190	3.173
	perc c2	13.574	10.894	13.488	10.378	13.030	9.919	13.064	9.981
0.9	a1	0.098	0.082	0.062	0.048	0.081	0.068	0.119	0.112
	a2	0.113	0.127	0.106	0.123	0.099	0.114	0.094	0.105
	c1	0.043	0.032	0.042	0.031	0.041	0.031	0.040	0.030
	c2	0.005	0.004	0.005	0.004	0.005	0.004	0.004	0.003
	perc c1	4.320	3.255	4.275	3.219	4.234	3.205	4.199	3.190
	perc c2	13.529	10.498	13.678	10.678	13.196	10.409	12.900	10.094
0.8	a1	0.117	0.101	0.070	0.054	0.094	0.080	0.142	0.137
	a2	0.126	0.144	0.116	0.131	0.109	0.125	0.103	0.116
	c1	0.048	0.036	0.046	0.035	0.045	0.034	0.044	0.033
	c2	0.006	0.004	0.005	0.004	0.005	0.004	0.005	0.004
	perc c1	4.345	3.288	4.292	3.239	4.270	3.210	4.231	3.191
	perc c2	13.735	11.008	13.519	10.442	13.160	10.672	12.865	9.970
0.7	a1	0.145	0.126	0.081	0.062	0.114	0.100	0.175	0.168
	a2	0.146	0.161	0.131	0.149	0.122	0.141	0.114	0.125
	c1	0.054	0.041	0.052	0.039	0.051	0.038	0.049	0.037
	c2	0.006	0.005	0.006	0.005	0.006	0.005	0.005	0.004
	perc c1	4.375	3.317	4.329	3.259	4.296	3.241	4.253	3.217
	perc c2	13.886	11.064	13.414	10.421	13.236	10.616	12.421	9.706

Table 3.7: Means and standard deviations for absolute values of 100-period data for a variety of policy rules for weight=1. The values of the a1 and a2 coefficients are sometimes farther from the MSV solution, which is zero.

TABLE 7 CONTINUED.

Parameter	φ_π	0.5		1.0		1.5		2.0	
φ_z		mean	std deviation	mean	std deviation	mean	std deviation	mean	std deviation
0.6	a1	0.188	0.166	0.095	0.073	0.143	0.128	0.219	0.220
	a2	0.170	0.188	0.148	0.168	0.139	0.159	0.125	0.139
	c1	0.062	0.047	0.060	0.045	0.058	0.044	0.056	0.043
	c2	0.007	0.006	0.007	0.005	0.006	0.005	0.006	0.005
	perc c1	4.420	3.348	4.361	3.294	4.315	3.263	4.288	3.244
	perc c2	13.889	10.786	13.384	10.591	13.003	10.128	12.418	9.586
0.5	a1	0.250	0.224	0.116	0.089	0.184	0.166	0.278	0.286
	a2	0.201	0.216	0.177	0.198	0.158	0.176	0.137	0.152
	c1	0.074	0.056	0.070	0.053	0.068	0.051	0.066	0.049
	c2	0.008	0.007	0.008	0.006	0.007	0.006	0.007	0.005
	perc c1	4.496	3.398	4.409	3.338	4.354	3.299	4.341	3.278
	perc c2	13.886	11.099	13.256	10.314	12.960	10.521	12.636	9.875
0.4	a1	0.362	0.333	0.148	0.114	0.255	0.234	0.378	0.387
	a2	0.249	0.268	0.214	0.243	0.184	0.206	0.153	0.169
	c1	0.090	0.068	0.085	0.065	0.081	0.061	0.078	0.060
	c2	0.010	0.008	0.009	0.007	0.009	0.007	0.008	0.006
	perc c1	4.593	3.436	4.491	3.392	4.422	3.341	4.365	3.355
	perc c2	13.874	10.723	13.386	10.320	12.975	9.905	12.438	9.436
0.3	a1	0.577	0.548	0.207	0.157	0.368	0.352	0.568	0.602
	a2	0.315	0.345	0.265	0.292	0.212	0.239	0.182	0.198
	c1	0.116	0.087	0.108	0.081	0.101	0.077	0.097	0.075
	c2	0.013	0.010	0.012	0.009	0.011	0.008	0.010	0.007
	perc c1	4.740	3.564	4.590	3.465	4.505	3.437	4.503	3.450
	perc c2	14.270	10.902	13.578	10.776	13.438	9.905	12.359	9.337
0.4	a1	1.130	1.111	0.333	0.248	0.621	0.627	0.931	1.002
	a2	0.453	0.498	0.353	0.396	0.268	0.291	0.210	0.225
	c1	0.162	0.124	0.147	0.111	0.135	0.103	0.127	0.097
	c2	0.018	0.014	0.016	0.012	0.014	0.010	0.013	0.009
	perc c1	4.996	3.801	4.779	3.609	4.641	3.536	4.597	3.524
	perc c2	15.276	11.323	13.768	10.315	13.254	9.774	12.555	9.319

TABLE 8.

Parameter	φ_π	0.5	1.0	1.5	2.0
φ_z	Criterion				
1.1	1	850	881	881	850
	2	934	953	953	955
1.0	1	819	858	861	845
	2	921	941	940	946
0.9	1	781	826	840	802
	2	908	921	929	936
0.8	1	741	793	802	739
	2	878	907	915	928
0.7	1	662	734	749	682
	2	853	874	889	905
0.6	1	569	676	674	596
	2	806	847	867	876
0.5	1	447	556	579	514
	2	759	805	838	822
0.4	1	299	418	422	399
	2	663	751	791	733
0.3	1	201	261	308	269
	2	507	633	677	608
0.2	1	78	137	157	163
	2	276	418	484	452

Table 3.8: The number of simulations for which each of the criteria is satisfied, the total number of simulations is 1000, weight=1. Criterion 1 means that absolute deviations from MSV values for all coefficients are less than or equal 0.2. Criterion 2 means that absolute deviations from MSV value for $a1$ is less than or equal 0.5 and that absolute deviations from MSV values for $a2$, $c1$, and $c2$ are less than or equal to 0.3.

Chapter 4

Learning Benevolent Leadership in a Heterogenous Agents Economy [1]

4.1 Introduction

The problems of time consistency, credibility and commitment have been studied since Kydland and Prescott (1977). Literature offers many extensions and solutions to the inflation bias problem, for example, models with reputation and trigger strategies (Barro and Gordon (1983), Stokey (1989)).

We build a model with cheap talk inflation announcement to study the following question. Can non-binding policy announcements be a useful instrument to improve outcomes in situations where time consistency of policy is a problem and where private agents are boundedly rational? The answer to this question depends on the dynamics of the response of private agents to the changes of government's policy.

We study a dynamic agent-based extension of the Kydland-Prescott model with cheap talk announcement. The government makes inflation announcements and decides on the actual level of inflation. There are two types of private agents who form inflation forecasts, believers and nonbelievers. Government's payoff depends on the weighted average of unemployment of both types of agents and on the level of inflation. Private agents' payoffs

[1] **This chapter is based on a work cowritten with Jasmina Arifovic, Herbert Dawid and Christophe Deissenberg.**

depend on their forecast errors and the level of inflation.

The first type of private agents, the *believers*, naively set their forecast equal to the government's inflation announcement. The second type, the *nonbelievers*, do not believe that actual inflation will be equal to the announcement. They pay a small calculation cost for forming their own inflation forecast. Nonbelievers know what the best response would be in a static environment when government is rational. However, they are also aware that they live in a dynamic environment and that government is learning. Thus, they use an error correction mechanism to correct for their forecasting mistakes.

We study two variants of the error correction mechanism. In the first one, all nonbelievers share the same error correction term and thus all nonbelievers make *homogenous* forecasts. According to the second one, each nonbeliever has her own, individual value of the error correction term. This results in formation of *heterogenous* forecasts. These two cases should capture different scenarios with respect to information flow between individual agents.

The fraction of believers changes over time through its response to the differences in the payoffs of the two types of agents. [2] Changes of the fraction of believers can serve as an indicator of government's evolving credibility.

The government knows the structure of the economy: expectational Phillips curve. Government knows how agents form forecasts and learn, i.e. it knows how the expected change in the proportion of believers is determined based on the payoff differences of two types of agents. The government needs to make policy decisions in this rich dynamic environment. We assume that government does not know optimal response function and needs to learn about which policy to use. The government adjusts its announcements of inflation and actual inflation using individual evolutionary learning (Arifovic, Ledyard 2004). [3]

[2] In this respect, the framework is similar to the models of rationally heterogenous expectations Evans and Ramey, (1992, 1998), Brock and Hommes (1997, 1998), Branch (2004), Branch and Evans (2006).

[3] There is a number of different models of individual learning discussed in the literature (see Erev and Haruvi (2008) for a thorough overview). However, most of them have been used in the game-theoretic settings with small strategy space. Arifovic and Ledyard (2004) have shown that IEL handles environments with large strategy spaces well, and that its behavior captures, in real time, behavior observed in the experiments

Individual evolutionary learning is based on updating of the entire collection of decision rules, in case of our government, rules that consist of the announcement and the actual level of inflation. The frequency of well performing strategies increases over time and the choice of a particular decision rule to be used in a given period is probabilistic. The payoffs of decision rules in any given period is based on calculation of 'foregone' payoffs. Initial set of decision rules is randomly generated. Occasional experimentation ensures that new decision rules, not represented in the initial set, enter into the collection.

In a related paper, Dawid and Deissenberg (2005) study a continuous-time version of the model, with heterogenous agents (naive believers and rational nonbelievers) and with a government that optimizes over an infinite horizon. Their analytical results indicate the existence of two stable equilibria, one without believers, and the other with a positive fraction of believers. The second equilibrium Pareto dominates the first.

Our agent based environment allows to concentrate on issues related to heterogeneity of expectations. Thus, we investigate whether heterogeneity matters in a qualitative way. Is the dynamics with heterogenous agents equivalent to the one with homogeneous agents with the same parameter constellation, can it be 'proxied' by homogeneous agent simulations with changed parameter constellation or do phenomena occur that cannot be observed in any homogeneous simulation? Related to this is the question of whether the coordination of the expectation formation of non-believers is beneficial for the government and whether this can have policy implications regarding the information flows.

4.1.1 Main findings

We find that government is able to use appropriate policy announcements to steer the economy to the Pareto superior outcomes. Government does not commit to set actual inflation at the announced level, but even so, the appropriate inflation announcement and actual inflation can result in higher payoffs for both the government and the private sector. Cheap talk in our environment can affect actual inflation and output. It can be beneficial to act

with human subjects.

as if believing the announcement under the condition that government uses appropriate monetary policy values.

This economy exhibits recurrent fluctuations in the inflation announcement and actual inflation that result from government's actions. These fluctuations happen as government builds up and exploits the population of believers. In a typical sequence of events, the government first builds up a positive proportion of believers by keeping the difference between announced and actual inflation relatively small, and by maintaining a relatively small level of actual inflation. (During this period, payoffs of believers are, on average, higher than the payoffs of nonbelievers.) Having achieved this, government starts exploiting the existing believers by increasing the discrepancy between actual inflation and inflation announcement in a short-sighted attempt to lower unemployment and increase its payoff. This has a negative impact on the believers' payoffs and their proportion decreases. In response to a shrinking fraction of believers, the government tries to restore economy's trust in its actions by reverting to the more 'credible' policy with lower difference between actual and announced inflation as well as low inflation levels. As a result, a proportion of believers increases again.

Recurrent fluctuations in the proportion of believers represent changes in the level of trust to the government. Our model generates endogenous credibility that changes over time as a result of government actions and its impact on private sector's payoffs. [4] During periods of relatively low inflation and small discrepancy between announced and actual inflation, the level of credibility is high. This credibility deteriorates as the government starts, myopically, to exploit the trust. The resulting outcome is higher rates of inflation and loss of credibility. Both the government and the private agents are better off during times when government enjoys high credibility than during the times when there is little trust.

We examine the impact of changes in how nonbelievers form their forecasts. First, we increase the cost associated with computation of one's own forecasts. Second, we lower the speed of adjustment of the error correction (EC) term. An increase in cost induces the government to move towards a policy that primarily aims at low inflation, a decrease in adjustment speed triggers a shift of attention towards a low unemployment policy. Higher

[4]The changes in the proportion of believers, and thus the level of trust in government's announcements, happen gradually in both directions (increasing and decreasing. This is in contrast to Phelan (2005).

cost results in higher payoffs for everyone, whereas for lower adjustment speed, only discounted government payoff is higher, average government payoff and private agents' payoffs are lower. Both higher cost and lower speed of update lead to initially higher proportion of believers that later decreases. Changes happen slower in response to higher cost than in response to lower speed of update. Increase in cost results in higher proportion of believers that can be sustained longer than in the case of the lower adjustment speed. For lower adjustment speed, the proportion of believers increases faster but also goes down faster than in the case of higher cost. Eventually believers can disappear. This leads us to a policy implication - well updated/informed nonbelievers are necessary to sustain high proportion of believers and hence to maintain higher level of payoffs. Therefore, it is important that public is able to adjust its forecasts quickly.

The response to introduction of heterogeneous nonbelievers is similar to the response to the lower speed of adjustment. In heterogeneous case, when believers switch to 'not believing', they start from scratch. In contrast, in homogeneous case after the switch, new nonbelievers starts with the population value of beliefs that includes the past experience. As a result, the speed of update of population of nonbelievers is slowed down in heterogeneous case. Therefore, introduction of heterogeneous nonbelievers' expectations is similar to lowering the speed of update.

Our results show that the response to lower speed of update of error correction term are qualitatively similar for homogeneous and heterogeneous cases, but they differ in the speed of adjustment.

4.1.2 Related literature

Cho and Sargent (1997) use stochastic gradient learning in infinitely repeated Kydland-Prescott model and show that the choice of equilibria narrows down to Nash and Ramsey outcome. However, in the simulations Nash equilibrium occurs most frequently as the eventual outcome. Based on this, they conclude that commitment is important for achieving low inflation and welfare improvement. In our paper, government does not commit to set actual inflation at the announced level, but even so, the appropriate inflation announcement and actual inflation can result in higher payoffs for government and private sector.

Arifovic and Sargent (2003) provide evidence from experiments in Kydland-Prescott model. They find that experimental economies stay in Ramsey outcome most of the time, and slide to Nash equilibrium occasionally. The inflation in our model fluctuates as government changes its monetary policy to build up and exploit naive forecasters.

The behavior of inflation in our simulations is similar to that in Sargent (1999): both setups exhibit recurrent fluctuations in inflation. Two environments have some similarities and some differences. In Sargent's model, government learns from misspecified Phillips curve that occasionally approximates the truth, and agents are rational. The dynamics consists of mean dynamics that pushes the economy towards self-confirming equilibrium (with Nash inflation) and escape dynamics that sets the economy on the path away from self-confirming equilibrium toward Ramsey outcome. Thus system escapes from self-confirming equilibrium toward outcome that is not an equilibrium. This dynamics is characterized analytically in Cho, Williams, Sargent (2002). The escapes are possible as government discounts past data in its estimation of Phillips curve by using constant gain recursive least squares.

In contrast, in our setup both government and agents are nonrational. Government learns about its decisions and economy based on individual evolutionary learning. Our government does not estimate an econometric model, it tries different combinations of inflation announcement and actual inflation, observes their performance and learns from that. The reason for recurrent fluctuations in our environment is the myopic attempts of the government to reduce unemployment by exploiting the naive agents through increasing the gap between inflation announcement and actual inflation. If government is quick to realize that this depletes the proportion of naive believers, it reverses its policy. When government repeats the sequence of building up the proportion of believers and exploiting them, we observe recurrent inflation fluctuations. These two models also have a common feature. In Sargent's model, government sets low inflation when its model approximates the truth correctly and it stops trying to exploit short run inflation-unemployment tradeoff. In our setup, government sets low inflation when it stops exploiting the same tradeoff to restore the positive proportion of believers.

Phelan (2005) builds on the reputation literature (Barro, Gordon (1983)) and studies a model in which government's type is hidden information and changes over time according to Markow switching process, agents are Bayesian learning. He finds that betrayals of public trust happen abruptly, and trust is rebuilt gradually. Trust is rebuilt because there is a positive probability that government is of trustworthy type (Markov switching assumption). In our paper, private agents do not know whether government is going to respect its announcements, and the level of trust in government's announcements is shown by the proportion of believers. In our setup, it can be beneficial to act as if believing the announcement under the condition that government uses appropriate monetary policy values. We can observe recurrent fluctuations in government's decisions and consequently in the proportion of believers as a manifestation of trust. These fluctuations happen gradually as government builds up and exploits the population of believers.

Stein (1989) studies cheap talk announcements in a version of the Crawford, Sobel's (1982) environment. As the government has an incentive to manipulate expectations, it cannot use precise announcement credibly. Stein's results indicate that the government should thus use imprecise announcements only. In contrast, in our model, government uses inflation announcements to try to influence agents' expectation, and agents may be willing to trust the announcement to avoid the costs of making their own forecasts. Inflation announcement is the policy instrument that can steer the economy towards the payoff superior outcome.

4.1.3 Organization

In section 2, we present summary of analytical results of the model by Dawid and Deissenberg (2005). We describe the learning mechanism in section 3. Section 4 presents the results of the simulations. And the last section concludes.

4.2 Description of the Model

In this section, we summarize the model from Dawid and Deissenberg (2005) that is the basis of our agent-based analysis. The economy consists of government G and a large number [5]

[5]In the analytical model which is the basis for our agent-based analysis we formally have a continuum of private agents, in the actual agent-based model obviously there is a finite but large number of agents.

of private agents i. At any point of time t, government and agents play the following game.

1. Government makes an announcement of inflation, y^a, that is its anticipation of actual inflation, y, in period t.
2. Agents form their forecasts of inflation, x^i, for period t.
3. Government sets actual inflation in period t.

The agent's unemployment rate u^i is given according to an expectation augmented Phillips curve:

$$u^i = U^* - \theta(y - x^i) \tag{4.1}$$

and agent i receives a payoff of:

$$J^i(x^i, y) = -\frac{1}{2}((y - x^i)^2 + y^2) \tag{4.2}$$

There are two types of agents in this economy: believers and nonbelievers. They differ by their attitude to the government's announcement of inflation. Believers trust the announcement of inflation, and they do not observe any other information about the economy. Thus, they set their forecast of inflation, x^B, equal to the announcement of inflation. Nonbelievers do not trust the government and use all the information available in the economy to set their forecast of inflation, x^{NB}. The fraction of believers in the economy, denoted by π, is common knowledge.

The government's payoff depends on the weighted average of the squared rate of unemployment of believers and nonbelievers, and on the squared rate of inflation, i.e.:

$$J^G = -\frac{1}{2}[\pi(u^B)^2 + (1 - \pi)(u^{NB})^2 + y^2] \tag{4.3}$$

Static solution. After believers and nonbelievers have chosen their forecasts of inflation, government sets its actual inflation be minimizing (4.3) with respect to y, and so its reaction function is given by:

$$y = R^G(x^B, x^{NB}; \pi) = \frac{\theta}{1 + \theta^2}(U^* + \theta\pi x^B + \theta(1 - \pi)x^{NB}) \tag{4.4}$$

The believers minimize (4.2) with respect to x^i taking into account that $y = y^a$. And so the believers' reaction function is:

$$x^B = R^B(y^a) = y^a \tag{4.5}$$

The nonbelievers know that there is fraction π of believers in the economy who set their expectations according to (4.5), and that the government's chooses inflation from (4.4). Nonbelievers substitute reaction functions (4.5, 4.4) into their payoff function (4.2) and maximize it with respect to x^i taking $(1-\pi)x^{NB}$ as given (the representative non-believer knows that s/he is too small to influence the average nonbelievers' forecast $(1-\pi)x^{NB}$). Then using the equilibrium condition $x^i = x^{NB}$ we obtain for the non-believers' forecast of inflation:

$$x^{NB} = R^{NB}(y^a, \pi) = \frac{\theta^2 \pi y^a + \theta U^*}{1+\theta^2\pi} \tag{4.6}$$

Given the reaction functions of believers and nonbelievers (4.5, 4.6), government can solve for optimal announcement of inflation, y^a, and actual inflation, y:

$$y^{a*} = -\frac{U^*}{\theta} \tag{4.7}$$

$$y^* = \frac{\theta(1-\pi)}{1+\theta^2\pi}U^* \tag{4.8}$$

Actual inflation, y, and the discrepancy $y^* - y^{a*}$ decrease with π. The optimal choices of believers and nonbelievers are $x^{B*} = y^{a*}, x^{NB*} = y^*$, i.e. nonbelievers can accurately forecast actual inflation. The difference between forecasts of nonbelievers and believers is equal the difference between actual inflation and announcement of inflation, and so it decreasing in π.

Government's payoff in the equilibrium is:

$$J^{G*} = -\frac{1+\theta^2}{1+\theta^2\pi}(1-\pi)U^* \tag{4.9}$$

Believers' payoff in the equilibrium is:

$$J^{B*} = -\frac{1}{2}\frac{(1+2\theta^2+2\theta^4-2\pi\theta^4+\pi^2+2\theta^4)}{\theta^2(1+\pi+2\theta^2)^2}U^* \tag{4.10}$$

Nonbelievers' payoff in the equilibrium is:

$$J^{NB*} = -\frac{1}{2}y^{*2} = -\frac{1}{2}[\frac{\theta(1-\pi)}{1+\theta^2\pi}U^*] \tag{4.11}$$

The average payoff of believers and nonbelievers is:

$$J^{P*} = \pi J^{B*} + (1-\pi)J^{NB*} \tag{4.12}$$

For all $\pi \in (0,1)$, the payoff of believers is always lower than payoff of nonbelievers. However, all payoffs decrease in π. This means that government and private agents are better off if all private agents act as believers. For $\pi = 0$, the announcement of the government becomes irrelevant and since private agents are atomistic the fact that they build observable expectations before the government decides on y is of no significance and the equilibrium outcome is given by the Nash equilibrium of the game where y and x^{NB} are chosen simultaneously. For $\pi = 1$, government solves an optimization problem where the believers reaction to the announcements are internalized. In this case, government gets the highest possible payoff of zero while agents still have non-zero, negative, payoffs. The possibility to influence beliefs by unbinding announcements of inflation increase payoffs as long as the fraction of believers, π is positive. As π increases, actual inflation decreases which is beneficial for both the government and the agents. Nonbelievers' unemployment stays at U^*, believers' converges towards the government's preferred unemployment level of 0. Government benefits from decrease of actual inflation and from convergence of believers' unemployment rate to 0.

Dynamic model. In a dynamic version of the model, the fraction of believers changes over time and government maximizes cumulative discounted payoff over an infinite horizon. Private agents switch between believing and non-believing based on information about the relative profitability of the two options. The government is aware of the mechanism through which the fraction of believers changes over time and takes the resulting intertemporal effects into account when determining the time path of its two decision variables. Accordingly, the government faces an intertemporal optimization problem with infinite time horizon.

The main results are that depending on the parameter constellation the model can have either two stable equilibria separated by an threshold point or a unique stable equilibrium. In the dynamic model (just as in the static model), the payoff of the government is increasing in π, i.e. government is always better off with having more believers in the economy. However, as long as the government sticks to the action that maximizes its current payoff – this action would correspond to the government's equilibrium action in the static game described above – nonbelievers always have a larger payoff than believers and accordingly the fraction of believers would decrease over time. Accordingly, in the dynamic setting the government, due to inter-temporal considerations, has incentives to deviate with the actual value of inflation y from the static equilibrium action and move it towards the announced inflation y^a. The interplay between the strategic effect captured in the static model and this inter-temporal effect drives the dynamics of government actions both in the analytical and the agent-based model we consider here.

4.3 Learning

In our agent-based model, both the government and the private sector learn and adjust their decisions and forecasts over time. The government learns about its decision variables with individual evolutionary learning (IEL). The private agents adapt their strategies through the process of the 'word of mouth' learning. The timing of the decision making in a given time period is as follows. First, the government makes announcement of inflation. Second, private agents set their forecasts of inflation. Third, the government sets actual inflation. At the end of the period both the government and private agents compute their payoffs and update their rules before making decisions in the next period. We provide the flow chart of the events that take place during a course of a simulation in Appendix A.

4.3.1 Decisions of the government and private agents

The government has two decision variables - announcement of inflation, y^a, and actual inflation, y. At each time t, the government has a collection of J rules. Each rule j, $j \in \{1, J\}$ consists of 2 elements which are the two decision variables, y^a and y. In each period, the government chooses one of these rules as its *actual* decision rule. The choice of the actual rule is probabilistic. Selection probabilities are based on the rules' hypothetical (foregone) payoffs. Over time, as a result of accumulated information about the performance of individual rules, the collection is updated in way that increases frequency of representation of relatively well performing rules. In addition, rules are subjected to occasional mutation that brings in diversity.

Initial collection of rules is randomly generated from uniform distribution with support [-10,15]. In addition, a rule (y_j^a, y_j) that government uses for its decisions at $t = 1$, y_1^a and y_1, is randomly selected.

Once the government makes an announcement of inflation y_t^a at the beginning of period t, private agents make their forecasts of inflation for that period. Each private agent is initialized with probability $\pi_{initial} = 0.5$ to be a believer and with probability $1-\pi_{initial}$ to be a nonbeliever. Believers set their inflation forecast equal to the government's announcement of inflation:

$$x_t^B = y_t^a \tag{4.13}$$

Nonbelievers set their forecast of inflation based on the reaction function from the static problem (given in 4.6) plus an error correction term d.[6] Thus, in period t, nonbeliever i's forecast is:

$$x_t^{NB,i} = \frac{\theta^2 \pi_t y^a + \theta U^*}{1 + \theta^2 \pi_t} + d_t^i \tag{4.14}$$

where d_t^i is the EC term of nonbeliever i in period t.

The initial value of EC term is zero. At the end of each period, each nonbeliever i updates her error term d_t^i using an error correction method:

$$d_{t+1}^i = d_t^i + \gamma(y_t - x_t^{NB,i}) \tag{4.15}$$

where y_t is actual inflation in period t, $\gamma = 0.1$ is a parameter controlling the speed of update.

In our simulations we will consider the case of heterogeneous beliefs of non-believers as well as the case where all non-believers share identical expectations. In the first case, each nonbeliever has her own value of d and proceeds to update it on her own as in (4.15). In the second case, all nonbelievers can have the same value of d (and hence all nonbelievers have the same inflation forecast). In this case, the error term is updated as:

$$d_{t+1} = d_t + \gamma(y_t - x_t^{NB}),$$

where y_t^{NB} is the (homogeneous) expectation of all nonbelievers. Once agents form their inflation forecasts, government chooses actual inflation y_t for period t. At the end of the period, agents and the government compute their payoffs.

[6] As noted in the introduction, if the government made rational decision, nonbelievers would set their forecasts based on reaction function from the rational expectations static solution. However, in our setup, they know that the government is learning, and thus they add, to their forecast, an error correction term that they update over time.

Payoff of believers is computed as:

$$J_t^B = -\frac{1}{2}[(y_t - x_t^i)^2 + y^2] \tag{4.16}$$

Payoff of nonbelievers is computed as:

$$J_t^{NB} = -\frac{1}{2}[(y_t - x_t^i)^2 + y^2] - c \tag{4.17}$$

where c is the cost of forming own forecast by nonbeliever. This cost can include computational, informational and other efforts nonbeliever has to apply to form own forecast in contrast to believing the announced inflation.

Government's payoff is calculated as:

$$J^G t = -\frac{1}{2}[\pi_t(u_t^B)^2 + (1-\pi_t)(u_t^{NB})^2 + y^2] \tag{4.18}$$

where u^B, u^{NB} are unemployment of believers and nonbelievers defined as:

$$\begin{aligned} u_t^B &= U^* - \theta(y_t - x_t^B) \\ u_t^{NB} &= U^* - \theta(y_t - \overline{x}_t^{NB}) \end{aligned} \tag{4.19}$$

where u_t^{NB} is the average unemployment of nonbelievers computed using average forecast of nonbelievers $\overline{x}_t^{NB} = \frac{1}{N_{NB}} \sum_{i=1}^{N_{NB}} x_t^{NB,i}$, N_{NB} is the number of nonbelievers. We use parameter values $\theta = 1$ and natural rate of unemployment $U^* = 5.5$ that are accepted in macro literature, e.g. Sargent (1999).

4.3.2 Updating

Agents' updating. In each period, agents learn by 'word of mouth'. A fraction of agents β is chosen randomly without replacement to undergo the following learning process (each of these 'chosen' agents is called agent i). Agent i meets agent j chosen randomly with replacement from the population of agents. Agent i can observe agent j's action (believe/ not believe). However, she cannot observe payoff of agent j, J^j, perfectly. Instead, agent i observes the value of the payoff that is disturbed by a random shock:

$$J^j_{observed} = J^j + shock$$

where

$$shock = 2 * tan(pi * (rand - 0.5))/(pi)$$

where $rand$ is drawn from uniform distribution [0,1], $pi = 3.14$[7].

If the observed payoff of agent j is higher than agent i's own payoff, $J^j_{observed} > J^i$, agent i adopts the action of agent j.

As a result of this learning mechanism, the expected change in proportion of believers can be computed as:

$$\Delta\pi_t = \pi_{t+1} - \pi_t = \beta\pi_t(1 - \pi_t)arctan(J_t^B - J_t^{NB}), \tag{4.20}$$

Under our *common error correction term* scenario (CEC), when an agent switches from believe to not believe, she observes the common value of the term d_t and adopts it in forming her own forecast. Thus, in this case, the knowledge of EC term is public and shared by nonbelievers.

Under our *individual error correction term* scenario (IEC), individual error terms, d^i's, are not observable by other agents. When an agent switches from being a believer to being a nonbeliever, his d^i is set equal to zero as she cannot observe the value of the other agent's EC term. Thus, every time agents become nonbelievers, they have to evolve and adapt their own error correction terms as these values are not shared across the agents.

Government's updating Government updates its collection of rules using experimentation and replication.

Experimentation. Each element of a rule j, $j \in \{1, J\}$, inflation announcement y_j^a and actual inflation y_j, is changed independently with the probability of experimentation equal to *mprob*=0.2.[8] The new value after experimentation is computed as:

$$new \quad value = old \quad value + randn * deviation,$$

[7] The distribution of shocks is unimodal with mean zero and qualitatively similar to a Gaussian distribution. We choose this formulation to obtain a simple analytical expression for the expected change in the fraction of believers.

[8] Compared to other studies that use evolutionary learning, this is a fairly large rate of experimentation. However, this higher rate is necessary to build positive fractions of believers.

where *randn* is random number drawn from standard normal distribution, $N \sim (0,1)$, deviation is the standard deviation of the distribution for the rule that is mutated. We use standard deviation of 1 for both inflation announcement and actual inflation.

The next step in government's updating is computation of hypothetical payoffs for each rule (y_j^a, y_j), $j = 1...100$. Hypothetical payoff shows how each pair of rules would have performed if it had been chosen in the previous period. It is calculated with government realizing that its announcement y^a influences forecasts of agents of both types and, therefore, influences their unemployment rates. And so for each rule (y_j^a, y_j), government computes the hypothetical values of inflation forecasts and hypothetical unemployment rates ($u_{hyp,j}^B$ and $u_{hyp,j}^{NB}$) that would have resulted if this pair had been chosen in the previous period. The hypothetical payoff also takes into account the impact on the change of the proportion of believers $(\Delta\pi_j^{exp})$.

The hypothetical payoff for each pair of rules j is calculated as:

$$J_j^G = -\frac{1}{2}[\pi_t(u_{hyp,j}^B)^2 + (1-\pi_t)(u_{hyp,j}^{NB})^2 + y_j^2] + \Omega \ \ \Delta\pi_j^{exp} \tag{4.21}$$

This payoff includes expected change in the proportion of believers $\Delta\pi_j^{exp}$ weighted by Ω. This formulation of payoff is equivalent to the sum of expected future payoffs. Parameter Ω evaluates how much government cares about the future: higher Ω means that government cares more about future. The value of this parameter is important for the results, and the sensitivity analysis will be presented later. The parameter Ω can be seen as a proxy of the shadow value of π in a dynamic optimization problem solved by the government. The details of payoff computations are presented in Appendix B.

Replication. Replication reinforces rules that would have been good choices in previous periods. It allows potentially better paying alternatives to replace those that might pay less. We implement *tournament selection* variant of replication. For $j = 1$ to J, 2 rules are drawn randomly with replacement from the existing pool of rule. Rule j in the new pool of rules is set equal to the rule with a higher hypothetical payoff.

Selection of rule. Having updated its pool of rules with experimentation and replication, government chooses announcement of inflation and actual inflation in the next period based on the rules' hypothetical payoffs $J_j^G, j = 1...100$. The probability for a pair of rules j to be selected is higher for a rule with higher payoff and is computed as:

$$P(rule_j) = \frac{J_j^G}{\sum_{i=1}^{100} J_i^G}$$

It is worth pointing out that experimentation is not a *trembling hand mistake*, mutation that is traditionally discussed in the literature on learning or evolutionary game theory. It is rather *purposeful experimentation* intended to improve government's payoff as a choice generated through experimentation is implemented only if it demonstrates a potential for bringing a higher payoff. In addition to having a high hypothetical payoff has to have a high hypothetical payoff, it also has to increase in frequency in order to increase its selection probability.

Directed experimentation: As pointed out earlier, directed experimentation is not as random as it may look. While it is true that an alternative is selected at random from the set the alternative selected must have a reasonably high hypothetical payoff relative to the last period or future periods to have any chance of ever being used. A newly generated alternative has to increase in frequency in order to increase its selection probability. This can happen only if it proves successful over several periods.

4.4 Simulation Results

In the discussion of the simulation results we use the case of error correction learning of private agents with common EC term as our default case. The standard parameter setting for our simulations is $\beta = 0.05$, $U^* = 5.5$, $\theta = 1$, $c = 0.1$, and $\gamma = 0.1$. Simulations are run for 300 periods. All data presented in the tables are averages over 100 runs.

4.4.1 Emergence of policy announcements and evolution of credibility

The first major question is whether in our environment policy announcements emerge as a useful tool to improve economic performance relative to the Nash equilibrium in which there are no believers. As discussed above, policy announcements can only emerge as a useful tool if the government chooses announcements in such a way that, on the one hand, a positive stock of believers evolves and sustains over time and, on the other hand, announcements influence private beliefs such that positive effects on both the government's and agents payoffs result.

We illustrate the typical behavior exhibited in one of our runs in Figure 4.1 where we present the dynamics of the behavior of the stock of believers, government payoffs, actual inflation, the announced inflation and forecast errors of believers and nonbelievers.

After a rather brief initial time span, a clear pattern arises between periods 50 and 200. The actual inflation oscillates around zero and announced inflation mirrors these oscillations with some downward shift. Due to these oscillations nonbelievers are not able to effectively adjust their learning parameter d_t and, therefore, their expectation errors stay relatively large compared to that of the believers. Accordingly, the stock of believers keeps increasing. Once the government has built up a high proportion of believers, approximately in period 150, it starts to 'exploit' their trust by increasing the difference between inflation announcement and actual inflation. Payoffs of nonbelievers rise above payoffs of believers, and π_t starts to decrease. As this happens, the payoff of the government increases due to lower unemployment of believers. But this temporary increase is bought at the price of a decrease in the proportion of believers and the associated decrease in future payoffs. Now, the government tries to stop this downward trend by reducing the discrepancy between inflation announcements and the actual inflation around period 220. The payoffs of believers increase and as a result π goes up reaching the value of 0.7 in period 262. Now, the government goes back to try to 'exploit' this build-up in the fraction of believers. It sets inflation announcement at about -5.5 and actual inflation close to 0, as these are the optimal values for the government in the static game that give it the highest payoff of 0. This leads to the reduction of the proportion of believers. In this simulation, π stays at the values higher than 0.5 until the end of the simulation. During the entire simulation, the government manages to build-up trust after each of the periods during which the gap between announced and actual inflation increases, and thus the economy maintains a fairly large fraction of believers. However, there can be instances that we discuss later in the text, where the gap might get high enough that it is not possible for the government to reverse its policy before believers disappear from the economy.

During the fluctuations described here the average payoff of the private agents and the payoff of the government are higher during the periods with high π. Given our parameter values, the government payoff in Nash equilibrium is $J^{G*} = -30.25$. It can be seen that apart from the first few periods, where due to the random initialization of private agents' inflation expectations large expectation errors and unemployment rates might occur, the government is typically able to obtain payoffs that are substantially higher than J^{G*}.

Looking at table 1, row one which provides average values over 100 runs for our baseline case, we can see that the government's average payoffs are higher (-16) than the Nash equilibrium ones (-30.25). In the Nash equilibrium, all agents are nonbelievers, their forecasts are correct, and the payoff is equal to the negative of the squared inflation rate, y^N which is equal to $U^* = 5.5$. Thus, $J^P = -30.25$. The data in row 1, table 1 show that average payoffs of both types of agents exceed the Nash equilibrium values (believers receive -11.07, and nonbelievers -4.40).

4.4.2 Tradeoff in government's decision

As mentioned above, the government tries to stop the downward trend in the proportion of believers sooner or later by reducing the discrepancy between inflation announcements and the actual inflation. The lower the values of parameter Ω, the later the government returns to a 'believers buildup', and the larger is the probability the stock of believers becomes zero before the government reduces the discrepancy sufficiently. Thus, the parameter Ω determines how strongly the government takes into account the effect of its current action on the stock of believers in the next period. It is very plausible that increasing the value of the parameter Ω makes government more biased towards actions that increase the stock of believers. Figure 4.2 shows the impact of Ω on the stock of believers. We can see that an increase of Ω leads to an increase in the stock of believers. Since the government's objective function increases with the stock of believers this also implies higher government payoffs for higher values of Ω. However, the stronger bias towards actions that increase the stock of believers also means that lower current payoffs are accepted. Therefore, it is not clear that increasing Ω leads to higher accumulated payoffs of the government.

In order to further investigate this issue, we define discounted payoff of the government as:

$$J^{G,disc} = \sum_{t=0}^{T} \rho^t J_t^G$$

where $\{J_t^G\}_{t=1}^T$ is the time series of government payoffs and $\rho \in (0,1)$ is the discount factor. We examine the dependence of $J^{G,disc}$ on the value of the parameter Ω. Figure 4.3 presents discounted payoffs for different values of $\Omega \in [800, 2000]$, $\rho = 0.98$ [9] and $T = 300$.

We can see that the discounted payoff of the government is maximized at an interior value of approximately $\Omega^{opt} = 1000$. To understand this result it is important to realize that a government with a high value of Ω is strongly concerned with the stock of believers and, therefore, with the believers' payoff that depends on the deviation between believers' inflation expectations and the actual inflation. Thus, the higher Ω is, the lower is,ceteris paribus, the difference between actual inflation and the inflation announcement. This increases current unemployment and has negative effects on short-term government profits. If the value of Ω exceeds the optimal value, the short term losses become too large compared to the long run gains from a high stock of believers, and discounted government payoffs decrease for increasing Ω.[10].

4.4.3 Costlier and slower adaptations of nonbelievers' forecasts

Our discussion so far has reinforced the point that it is desirable for the government if the success of nonbelievers is relatively small compared to that of believers. Basically, the attractiveness of building and adapting beliefs that are different from the policy announcements

[9] In how far such a value of the discount factor can be considered as plausible depends on the interpretation of the time unit. If we interpret one unit of time as one month, which seems reasonable given the value of parameter $\beta = 0.05$ and the speed of the dynamics of π_t, this corresponds to a yearly discounting of about 20%. This rate might look very high but it has to be taken into account that governments act under uncertainty of how long they will be in the office and therefore one should assume rather high discounting of future payoffs.

[10] It should be noted that choosing the value of Ω that maximizes discounted payoffs typically leads to the elimination of the stock of believers in the long-run. This means that even if a government is interested in the maximization of a discounted payoff stream with infinite time horizon, the corresponding optimal policy rule will in the long run eliminate any trust of the private agents in policy announcements and make them irrelevant. The mechanism that leads to the eventual extinction of believers is closely related to the phenomenon of persistent fluctuations in the values of the policy, its announcements and the stock of believers that can be observed in the simulations. We discussed this phenomenon in more detail in the previous subsection

may be decreased for two reasons. First, the cost of building such beliefs might increase and, second, the speed of adjustment of the EC term might decrease. As we will show in this subsection both of these changes indeed result in an increase in government's payoffs (both J^G and $J^{G,disc}$), but otherwise have quite different effects on the key economic variables. To explore this issue we compare the qualitative effect of increasing costs of nonbelievers from $c = 0.1$ to $c = 1$ with that of decreasing the speed of adjustment of the EC term, d, from $\gamma = 0.1$ to $\gamma = 0.01$. [11]

In table 4.1 we show the discounted government payoff ($J^{G,disc}$) as well as time averages over periods 21 to 300 of the main variables of interest for the two values of c that we consider. All reported values are averages of 100 runs. In row 3 of the table we provide the significance levels of Wilcoxon test that show the observed signs in almost all the differences of means are statistically significant at a 95% level[12]

As mentioned above, both parameter changes lead to an increase in the discounted payoff of the government. However, the mechanisms leading to this increase are quite different in two cases. If the costs of forming individual forecasts increase, ceteris paribus, the stock of believers goes up and the government policy changes such that actual and announced inflation decreases (the change of policy announcements is less significant). Interestingly enough, the government does not exploit the increased costs of nonbelievers but rather reduces the gap between its announced and actual inflation policy. The level of unemployment among believers therefore goes up, unemployment of nonbelievers increases [13], but since unemployment among believers is still smaller than unemployment among nonbelievers this effect is at least partly offset by the increase in the fraction of believers. In combination with the decrease in actual inflation this leads to higher government payoffs. Furthermore,

[11] The weight parameter is set to *weight* = 1000 for this comparison, which is the optimal value of this parameter identified above.

[12] The data for Wilcoxon test is generated as follows. We generate 2 samples of 100 observations each: one for cost $c = 0.1$, the other for cost $c = 1$. For each sample, parameters $\beta = 0.05$, $U^* = 5.5$, $\lambda = 2$, $\theta = 1$ are fixed at their baseline values for all observations. We draw randomly values of γ from uniform distribution $[0.01, 0.1]$ and values of Ω from uniform distribution $[800, 1300]$ for each of 100 observations, the sets of these values are the same for samples with $c = 0.1$ and $c = 1$. We collect data on averages and discounted payoffs for each simulation. Then we compute differences between observations for cost $c = 0.1$ and cost $c = 1$ for the same values of γ and Ω. We perform one-tailed Wilcoxon test for these differences.

[13] Unemployment of nonbelievers becomes closer to the natural rate $U^* = 5.5$ which implies that nonbelievers' forecasts are more accurate.

due to the stronger alignment between announced and actual policy and to the decrease in inflation both groups of individual agents are better off after an increase in c and it is easy to see that also the average payoff in the population increases. This is directly the result of higher average fraction of believers.

Figure 4.4 illustrates behavior for high cost $c = 1$. After the initial adjustment, the government implements the policy where it keeps actual inflation very close to the announced values. In addition the value of y_t fluctuates around zero. The fraction of believers grows over time, and remains at high levels of around 0.8. However, at period 223, the government suddenly decreases y_t^a to -4.04, while keeping y_t at first at level close to zero. In subsequent periods, it maintains low values of y_t^a, but starts increasing the level of y_t. This results in large decrease in believers' unemployment. Believers' payoffs suffer from huge forecasting errors they make, and π decreases to 0.33 (period 268). The government is able to recover from this episode and brings the values of y_t^a and y_t closer together. Inflation stays high thus stabilizing the value of π (which at the end of the simulation is remains at its lower level of 0.33).

Quite a different picture emerges if the speed of adaptation of nonbelievers error correction term goes down. Table 2 shows the effects of the decrease in γ[14]. Here the government reacts to the parameter change by strongly widening the gap between its announcement and its actual policy. The announcement is decreased whereas at the same time inflation goes up. Because of this shift of the government policy a decrease of γ actually leads to an *decrease* in the long run stock of believers. The government is able to quickly reduce unemployment among believers to zero and, therefore, gains in the short run which results in an increase of its discounted payoff. Note that the decrease of the learning speed of nonbelievers leads to a decrease in the payoff of the government in the long run. Also, both believers and nonbelievers have lower payoffs after a decrease of γ. [15] Put differently, whereas an increase in c induces the government to move towards a policy that primarily aims at low inflation, a decrease in γ triggers a shift of attention towards a low unemployment policy.

[14] In this case, for Wilcoxon test we generate 2 samples: one for $\gamma = 0.01$, the other for $\gamma = 0.1$. Cost is drawn randomly from uniform distribution $[0.1, 1]$, weight - from uniform distribution $[800, 1300]$, the rest of the parameters are at the baseline values.

[15] It should be noted here that the rate of unemployment does not enter the individuals objective function in this formulation and, therefore, positive employment effects do not influence the individual payoffs.

Figure 4.5 illustrates the behavior observed in one of the simulations with $\gamma = 0.01$. At the beginning of the simulation the stock of believers gradually grows, from 0.5 (which is a result of random initialization) to the values close or equal to 1. Around period 50, virtually everyone in the economy is a believer. But, a few nonbelievers that stay in there continue updating their d_ts. In the process, their forecast errors become smaller and their payoffs higher. At period 55, their payoffs surpass the believers' payoffs and being a non-believer becomes a lot more attractive. As the figure shows, at this point, the government set of rules consists mostly of those that prescribe large gaps between actual and announced inflation. As government continues to use policy with high gap between actual and announced inflation, believers make worse forecasts than nonbelievers. As a result, a fraction of believers starts declining. Government follows up with policy that restores the fraction of believers to 0.72 in period 147, but later repeats policy with high gap between announced and actual inflation starting in period 158. The end result is the total extinction of believers from the economy.

Overall, while decrease in the speed of adjustment results in worse economic outcomes, increase in the cost results in greater trust in government policies, which in turn is supported by moderate gaps between announced and actual inflation, lower inflation rates, and higher government's and private sector's payoffs. An interesting question is then whether further increases in the cost c result in further increases in the payoffs? Is there an optimal level of c that maximizes private agents' and government's payoffs given other parameters of the model? We present the results of our investigation in Figure 4.6 in which we vary c between 0 and 4, in the increments of 0.1 and simulate 100 runs for each value of c. It turns out that, for our set of parameter values, the payoff of believers is the highest for cost $c = 1.3$, the payoff of nonbelievers is the highest for cost $c = 0.8$. The average payoff (weighted by fractions of believers and nonbelievers) of agents achieves the highest value for cost $c = 0.8$. Government's discounted payoff and average payoff increases monotonically with higher cost.

Ones the cost starts increasing over the 'optimal' value, we observe a shift in the governments policy. It reverses its policy of low inflation and starts increasing the gap between the actual and announced inflation which result in the decrease of believers' unemployment. This is similar to the policy government uses in the case of lower γ.

4.5 Heterogeneity of expectations of nonbelievers

So far we have assumed that all nonbelievers share the same beliefs. This in particular requires that any private agent who shifts from believing to non-believing is able to predict the actual inflation rate as well as other nonbelievers who have already acquired experience in the past. Formally, this is expressed by the fact that all nonbelievers share the same value of the EC term d_t. Whereas such an assumption is plausible if inflation forecasts are available at institutions like economic research institutes, we will now assume that individuals indeed profit from past experience with expectation formation. Nonbelievers differ in the d-values they are using and privately update these values based on information they obtain. Note that such heterogeneity of expectations introduces additional costs of switching to non-believing in a way that is quite different from change in cost and speed of adjustment we have considered in this subsection.

As described previously, once an agent switches from being a believer to being a non-believer, she starts with an EC term of the forecast equal to zero. She keeps adjusting it based on her own experience.

In order to estimate the effect of its policy, government must have an estimate of the non-believers expectations, which is harder when nonbelievers' expectations are heterogeneous. We assume that the government knows the average value of the expectation parameters of all nonbelievers and, based on this, computes the hypothetical value of nonbelievers' forecast as described in Appendix B.

4.5.1 Comparison of homogeneous and heterogenous baseline cases

Table 4.3 reports data for baseline parameter values $\gamma = 0.1$ and $cost = 0.1$ for homogenous and heterogenous nonbelievers' expectations. It also presents the significance levels for Wilcoxon test that show that the observed signs for most of the differences in the means for homogeneous and heterogeneous cases are significant at 95%[16].

We can see from Table 4.3 that the proportion of believers, government's discounted payoffs, government's average payoffs, actual inflation, announced inflation and unemployment of believers are lower and unemployment of nonbelievers is higher in heterogeneous case than in homogeneous case.

The proportion of believers is lower for heterogeneous case. Simulation with heterogeneous nonbelievers is illustrated in Figure 4.12 which shows that, compared to the homogenous case (Figure 4.1), proportion of believers increases somewhat faster and starts to decrease earlier. This explains lower proportion of believers averaged over the length of simulation (as shown in Table 4.3). The dynamics can be explained in the following way. Initially, it is easier for the government to build up the proportion of believers because the alternative action 'not believe' is less attractive in the heterogeneous case due to the zero EC term after switching to the action 'not believe'. Once government builds up the stock of believers, it starts to 'exploit' them by increasing the gap between actual and announced inflation, and this leads to decreasing proportion of believers. We can see higher value of this gap and lower unemployment of believers in the heterogeneous case in Table 4.3.

In the heterogeneous case there is a positive variance of nonbelievers' expectations that makes it more difficult for the government to adapt its actual inflation rate to the distribution of beliefs in the population compared to the homogenous case where only two levels of expectations exist in the entire population. As a result, we observe that even in the long run the average expectation of heterogeneous non-believers is above the actual inflation rate. Such a phenomenon has negative implications for both sides as unemployment of nonbelievers is above the natural rate of $U^* = 5.5$ (see table 4.3). The fact that such a negative expectation gap exists nevertheless shows the coordination problems arising in a system with heterogeneous agents.

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[16]For this test, we generate two samples of data: one - for homogeneous, the other - for the heterogeneous nonbelievers. Each sample is generated for *cost* drawn from uniform distribution $[0.1, 1]$, γ drawn from uniform distribution $[0.01, 0.1]$, and weight from uniform $[800, 1300]$, the other parameters are at the baseline values described above.

Further, the differences in discounted and average government's payoffs can be explained by the changes in actual inflation and unemployment rates. Although unemployment rate of nonbelievers is higher for heterogeneous case, unemployment of believers and actual inflation are lower, and so for lower proportion of believers, these changes lead to lower government's payoffs in heterogeneous case.

Finally, the same differences in the behavior of the homogenous and heterogenous case are observed for a range of values of the parameter Ω. These are illustrated in figures 4.7, 4.8, 4.9 and 4.10 in which the behavior of government payoffs, unemployment, proportion of believers, actual inflation and announced inflation for common and individual nonbelievers' expectations presented for range of values of Ω between 800 and 1,300.

4.5.2 Costlier and slower adaptations of nonbelievers' forecasts - heterogenous agents

As noted earlier, in case of homogenous nonbelievers, the government responds in qualitatively different ways to changes in the parameters (c and γ). An increase in the cost of forming individual expectations induces a change towards a more inflation oriented policy of the government, whereas a decrease in the speed of update of nonbelievers leads to an orientation towards unemployment reduction. Costlier forecasts (up to the point) also make, on average, both the government and agents better off, while slower speed of adjustment increases government's discounted payoff, but lowers its average payoff, as well as the average payoff of both types of agents.

In table 4.4 we show the effect of an increase of cost c on the key variables of the model where nonbelievers have heterogenous expectations; row 3 provides significance levels of Wilcoxon test performed analogously to the case of homogeneous nonbelievers' expectations. Table 4.5 reports the effect of a decrease in the speed of adjustment and significance levels for Wilcoxon test.

In this environment, an increase in c no longer induces an inflation oriented policy change of the government, but rather an unemployment oriented one. Inflation goes up, announced inflation goes down, and the gap between actual and announced inflation widens. This induces very low unemployment among believers, and the government is better off. The proportion of believers goes down for higher cost in the heterogeneous case in contrast to an increase of the proportion of believers in the homogeneous case. Higher cost, c, results in higher government's discounted payoff and lower average payoff. The lower average payoff is due to lower proportion of believers. Agents' average payoff goes down in heterogeneous case whereas it goes up in homogeneous case.

Thus, with heterogeneity of expectations, higher cost of forecast no longer provides beneficial effects for the entire economy. However, in the homogenous expectations case, there is a maximum level of cost that results in increased payoffs. Beyond that point, further increases in cost have negative impact on the payoffs. In the economy with heterogenous agents, the effect of having to learn the value of d_t from scratch, each time there is a switch, might act as an addition to the total cost of forecasting, pushing it beyond the point where increases in costs cease to have positive effects. We present the results of our investigation in Figure 4.11. We varied c between 0 and 0.4, in the increments of 0.01 and simulated 100 runs for each value of c. For our set of parameter values, the payoff of believers is the highest for cost $c = 0.02$, the payoff of nonbelievers is the highest for cost $c = 0.01$. The average payoff of agents achieves the highest value at cost $c = 0.01$. Government's discounted payoff and average payoff keeps increasing with higher cost although average payoff exhibits some volatility.

When the speed of adjustment, γ, goes down in the environment with heterogenous expectations, actual inflation and announced inflation both decrease, but the gap between them increases. The higher gap leads to the reduction of the unemployment of believers. The unemployment of nonbelievers increases as their forecast error is higher due to lower speed of adjustment. The proportion of believers increases in contrast to the reduction in case of homogeneous nonbelievers (see Table 4.2). As both actual inflation and unemployment of believers go down together with an increase of the proportion of believers, government's payoff J^G increases, and so does the discounted payoff. Agents' payoffs go down as in the case of homogeneous nonbelievers. The unemployment policy is the same and inflation policy is opposite in homogeneous and heterogeneous cases in response to lower γ. Responses of inflation and unemployment of nonbelievers to changes in cost and γ are opposite in the heterogeneous compared to the homogenous case. Unemployment of believers on the other hand moves in the same direction in both cases.

Different responses to costlier and slower adjustment in case of homogeneous and heterogeneous expectations of nonbelievers (Table 4.1 and Table 4.4, Table 4.2 and Table 4.5) illustrate the importance of explicit consideration of the impact of heterogeneity among individuals when analyzing policy measures. The only difference between the scenarios considered in these tables is homogeneity and heterogeneity of nonbelievers' expectations. Nevertheless two qualitatively very different policy reactions to a change of c or γ are observed. These observations reinforce the point that heterogenous agent models are needed to fully examine the qualitative feature of the problem at hand.

It is in the interest of the government to facilitate the information flow between nonbelievers such that agent that just switched to non-believing can build on the experience of the other nonbelievers. *Making too much data publicly available might however reduces the costs c of being a non-believer, which, as we have seen above, is not desirable for the government.* So, if the government intends to interfere with the way nonbelievers build their expectations it faces the non-trivial problem to keep the incentives for taking government announcements at face value as high as possible, and at the same time to avoid too much heterogeneity and large expectation errors among those agents who do not believe the announcements.

4.5.3 Understanding the difference in responses to parameter changes in homogeneous and heterogeneous cases

We have seen that economies with homogeneous and heterogeneous nonbelievers respond differently in the experiments conducted in the previous subsections. We would like to better understand these differences, and so we perform simulations and take 2 snapshots at different points of time. We report averages over the following periods of time: [21, 100], [21,300]. Reporting the results at different points of time allows to understand how the economy evolves over time. We summarize the direction of changes in response to parameter changes in table 4.10. We will analyze the impact of lower γ first and then the impact of higher c.

The results of the simulations for two values of γ are reported in Tables 4.6 and 4.7

for homogeneous and heterogeneous cases. To make comparison easier, we present both averages over periods [21,100] and averages over periods [21,300], the latter results are also in tables 4.2 and 4.5. We can summarize these tables as follows. For lower γ, the proportion of believers increases during the initial 100 periods for homogenous and heterogenous cases (as expected). During 300 periods, proportion of believers decreases with γ for homogenous case, but it is higher for lower γ in heterogenous case. For each value of γ, proportion of believers is lower over 300 periods than over 100 periods that means that proportion of believers declines over time. Together these facts mean that the proportion of believers decline slower in heterogeneous case than in homogeneous case.

Now we will compare inflation and unemployment approaches of the government in response to lower γ in homogeneous and heterogeneous cases. The level of inflation is lower in heterogeneous case during both time intervals. In homogeneous case, inflation decreases for lower γ during the initial 100 periods, and then increases over 300 periods. We can see that the gap between actual and announced inflation increases as γ decreases both during the initial 100 periods and during 300 periods for homogeneous and heterogeneous cases. Increasing gap leads to lower unemployment of believers for lower γ, whereas unemployment of nonbelievers is higher for lower γ due to higher forecast error. Higher gap between announced and actual inflation also contributes to the decreasing proportion of believers over 300 periods. The combination of lower proportion of believers and higher unemployment of nonbelievers cause lower government's payoff J^G, whereas government's discounted payoff increases due to short-term lower inflation, higher proportion of believers and lower unemployment of believers.

Based on the observations in Tables 4.6 and 4.7, we can summarize the dynamics in the economy. The government is able to build up a high proportion of believers quicker for lower γ as the option 'not believe' does not give good forecasts due to the slower update of nonbelievers. Then the government starts to exploit existing believers substantially. As government continues its exploitative policy, the proportion of believers declines. If the government does not reverse its policy, believers disappear. For lower γ, proportion of believers starts to decrease earlier. This is illustrated in Figures 4.1 and 4.5 for homogeneous case, and in Figures 4.12 and 4.13 for heterogeneous case. We can make an *important* conclusion from this exercise: to maintain positive proportion of believers, it is necessary that nonbelievers are quick and accurate in their forecasts. This contains government from exploiting believers too much to the extent of extinction and protects believers.

The response of the economy to the introduction of heterogeneous nonbelievers is similar to the response to the lower speed of adjustment of nonbelievers (see Table 4.10). We can see that proportion of believers starts to decline earlier in heterogeneous case (Figure 4.12) than in homogeneous case (4.1). This is similar to the impact of lower adjustment speed: proportion of believers declines quicker for lower γ in Figures 4.5 compared to baseline γ in Figure 4.1 for homogeneous case. Similarly, in Figure 4.13 the proportion of believers declines quicker for lower γ compared to Figure 4.12 in heterogeneous case. When believers switch to 'not believing', they start from scratch at $d = 0$ in heterogeneous case. As a result, the speed of update of population of nonbelievers is slowed down in comparison to homogeneous case where, after the switch, new nonbelievers start with the population value of d that includes the past experience. Therefore, introduction of heterogeneous nonbelievers' is similar to lowering the speed of update, γ.

Now we analyze the effects of higher cost. The results of simulations are reported in Tables 4.8 and 4.9. We can summarize these tables as follows. The proportion of believers increases during the initial 100 periods for homogenous and heterogenous cases. During 300 periods, proportion of believers is higher for higher cost for homogeneous case, but it is lower for heterogeneous case. For the same cost, proportion of believers is lower over 300 periods than over initial 100 periods in heterogeneous case, but it is higher in homogeneous case. This means that initially proportion of believers increases in homogeneous and heterogeneous cases, it keeps increasing within 300 periods in homogeneous case , but starts declining in heterogeneous case. The initial response to higher cost bears resemblance to the response to lower γ: proportion of believers increases as cost of forming own forecasts for nonbelievers is higher. As proportion of believers is high, government starts exploiting them that decreases proportion of believers in heterogeneous case. The dynamics for economy with higher cost is illustrated in Figures 4.4 for homogeneous case and 4.14 for heterogeneous case.

The level of inflation decreases with cost for homogeneous case during both time intervals, whereas inflation goes down initially and then increases in homogeneous case. Gap between announced and actual inflation goes down for higher cost that brings higher unemployment of nonbelievers in homogeneous case. Gap between announced and actual inflation goes up causing lower unemployment of believers in heterogeneous case. Unemployment of nonbelievers goes up in homogeneous and heterogeneous case: in homogeneous case unemployment of nonbelievers becomes closer to natural rate, whereas in heterogeneous case, it goes up above natural rate. Both believers and nonbelievers' payoffs are higher for higher cost in homogeneous case, but they are lower for higher cost in heterogeneous case. Government's discounted payoff goes up for higher cost in both cases. Government's payoff J^G goes up in homogeneous case, and it goes up during initial 100 periods and goes down during 300 periods in heterogeneous case. We can see that government's response to higher cost is different in homogeneous and heterogeneous case. In homogeneous case, government concentrates on lower inflation; in heterogeneous case, government aims at lower unemployment of believers. This difference of government's response can be explained by our finding in Figures 4.6 and 4.11 that 'optimal' level of cost is different for homogeneous and heterogeneous cases: 0.8 for the former case, 0.01 for the latter case. Thus, cost increases from 0.1 to 1, this means movement closer to the optimal level for homogeneous case, but it is movement away from the optimal level for heterogeneous case.

The response to higher cost and lower speed of update is qualitatively similar for homogenous and heterogeneous cases: proportion of believers initially increases due to costlier/slower update of nonbelievers, and then decreases due to government's exploitation of believers to reduce unemployment. The difference in responses is in the speed of these adjustments: proportion of believers goes up and stays high longer for higher cost than for lower γ. This explains the differences observed in data averaged over period [21, 300].

4.6 Conclusion

In this paper, we study a dynamic version of Kydland-Prescott model in which the government uses announcements to improve the economic outcome. The simulations show that government is able to learn a Pareto-superior outcome by making inflation announcements and setting actual inflation that maintain the number of believers at the level necessary for an increase in government's and average private sector's payoffs. The Pareto improving outcome is achieved despite the fact that the announcements are not respected. The economy exhibits recurrent fluctuations in announced and actual inflation as government repeatedly builds up and exploits the proportion of believers.

The size of the fraction of believers reflects the level of credibility that the government enjoys. This credibility evolves endogenously and changes with the changes in the fraction of believers. By setting their beliefs equal to the announced inflation, the believers might not necessarily believe that the government is going to set the actual inflation at that level. Rather they might believe that the government is not going to make the gap between the announced and actual inflation too large. As long as the gap is small enough to keep their forecast errors small enough and thus switching to not believing not worthwhile, believers will 'trust' the government. Thus, credibility in this economy is not about the announced versus actual inflation but about the size of the gap. Credibility can be sustained only with a healthy fraction of nonbelievers whose presence keeps the government in check.

Changes in different parameter values lead to different government policy responses. While the increase in the cost of acquiring individual forecast induces inflation oriented government policy, a decrease in a speed of adjustment of nonbelievers EC term induces policy oriented towards reduction of unemployment. Further, increase in the cost results in the higher average payoffs for both the government and the agents. We also find that there is, for a given set of other parameter values, an optimal size of the cost that brings about the highest payoffs. Further increases in the cost beyond that value result in deterioration of everyone's payoffs. On the other hand, decreases in the speed of adjustment result in lower average payoffs for the government and both types of agents (the government discounted payoff goes up though).

In the heterogeneous case there is a positive variance of nonbelievers' expectations that makes it more difficult for the government to adapt its actual inflation rate to the distribution of beliefs in the population compared to the homogenous case where only two levels of expectations exist in the entire population. As a result, we observe that even in the long run the average expectation of heterogeneous non-believers is above the actual inflation rate. Such a phenomenon has negative implications for both sides as unemployment of nonbelievers is above the natural rate of $U^* = 5.5$. The fact that such a negative expectation gap exists nevertheless shows the coordination problems arising in a system with heterogeneous agents.

It is in the interest of the government to facilitate the information flow between nonbelievers such that agent that just switched to non-believing can build on the experience of the other nonbelievers. *Making too much data publicly available might however reduce the costs c of being a non-believer, which, as we have seen above, is not desirable for the government.* So, if the government intends to interfere with the way nonbelievers build their expectations it faces the non-trivial problem to keep the incentives for taking government announcements at face value as high as possible, and at the same time to avoid too much heterogeneity and large expectation errors among those agents who do not believe the announcements.

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4.8 Appendix

APPENDIX A. Computation of hypothetical payoffs.

The hypothetical payoff for each pair of rules j is calculated as:

$$J_j^G = -\frac{1}{2}[\pi_t(u_{hyp,j}^B)^2 + (1-\pi_t)(u_{hyp,j}^{NB})^2 + y_j^2] + weight\ \ \Delta\pi_j^{exp} \tag{4.22}$$

The hypothetical values of inflation forecasts for believers are calculated for each pair j's announcement of inflation, y_j^a, as:

$$x_{hyp,j}^B = y_j^a \tag{4.23}$$

When government computes hypothetical forecasts by nonbelievers, it needs to take into consideration that nonbelievers have an error term d in their forecasts. In the case of individual values of d, government knows the average value of error terms in the last period and computes the hypothetical value of nonbelievers' forecast as:

$$x_{hyp,j}^{NB} = \frac{\theta^2\pi_t y_j^a + \theta U^*}{1+\theta^2\pi_t} + d_t^{aver} \tag{4.24}$$

where $d_t^{aver} = \frac{1}{N_{NB}}\sum_{i=1}^{N_{NB}} d_t^i$, N_{NB} is the number of nonbelievers.

In the case of common value of d for all nonbelievers, the government knows last period value of d_t and computes hypothetical forecast of nonbelievers as:

$$x_{hyp,j}^{NB} = \frac{\theta^2\pi_t y_j^a + \theta U^*}{1+\theta^2\pi_t} + d_t \tag{4.25}$$

In the case of individual values of d, the government is assumed to know the average value of the expectation parameters of all nonbelievers and computes the hypothetical value of nonbelievers' forecast as:

$$x_{hyp,j}^{NB} = \frac{\theta^2\pi_t y_j^a + \theta U^*}{1+\theta^2\pi_t} + d_t^{aver} \tag{4.26}$$

where $d_t^{aver} = \frac{1}{N_{NB}}\sum_{i\in A_{NB}} d_t^i$ and A_{NB} is the set of nonbelievers.

The hypothetical unemployment rates are computed as

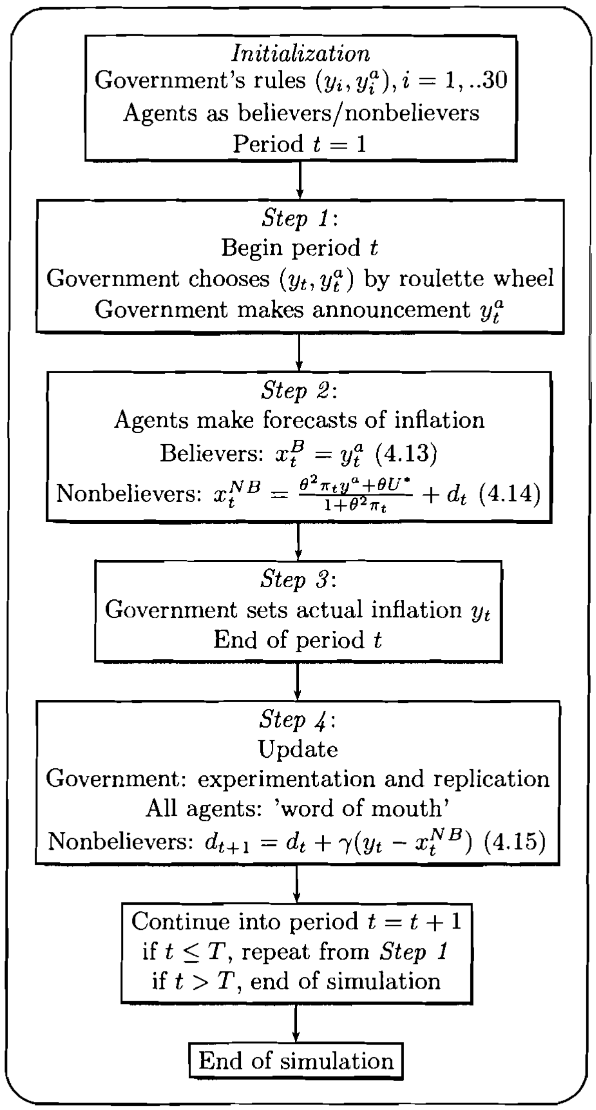
$$u_{hyp,j}^i = U^* - \theta(y_j - x_{hyp,j}^i), i = B, NB \tag{4.27}$$

Government computes the expected change in the proportion of believers, $\dot{\pi}_j^{exp}$, knowing that it is determined by (4.20). The necessary computations include the following:

$$\begin{aligned} J_{hyp,j}^{B}(x_{hyp,j}^{B}, y_j) &= -\frac{1}{2}((y_j - x_{hyp,j}^{B})^2 + y_j^2) \\ J_{hyp,j}^{NB}(x_{hyp,j}^{NB}, y_j) &= -\frac{1}{2}((y_j - x_{hyp,j}^{NB})^2 + y_j^2) \\ \Delta\pi_j^{exp} &= \beta\pi_t(1-\pi_t)arctan(J_{hyp,j}^{B} - J_{hyp,j}^{NB}) \\ \pi_{t+1}^{exp} &= \pi_t + \Delta\pi_j^{exp} \end{aligned}$$

If $\pi_{t+1}^{exp} > 1$, $\pi_{t+1}^{exp} = 1$, and so $\Delta\dot{\pi}_j^{exp} = 1 - \pi_t$. If $\pi_{t+1}^{exp} < 0$, $\pi_{t+1}^{exp} = 0$, and then $\dot{\pi}_j^{exp} = 0 - \pi_t$. This keeps π in interval $[0, 1]$.

APPENDIX B. The algorithm of the simulation.



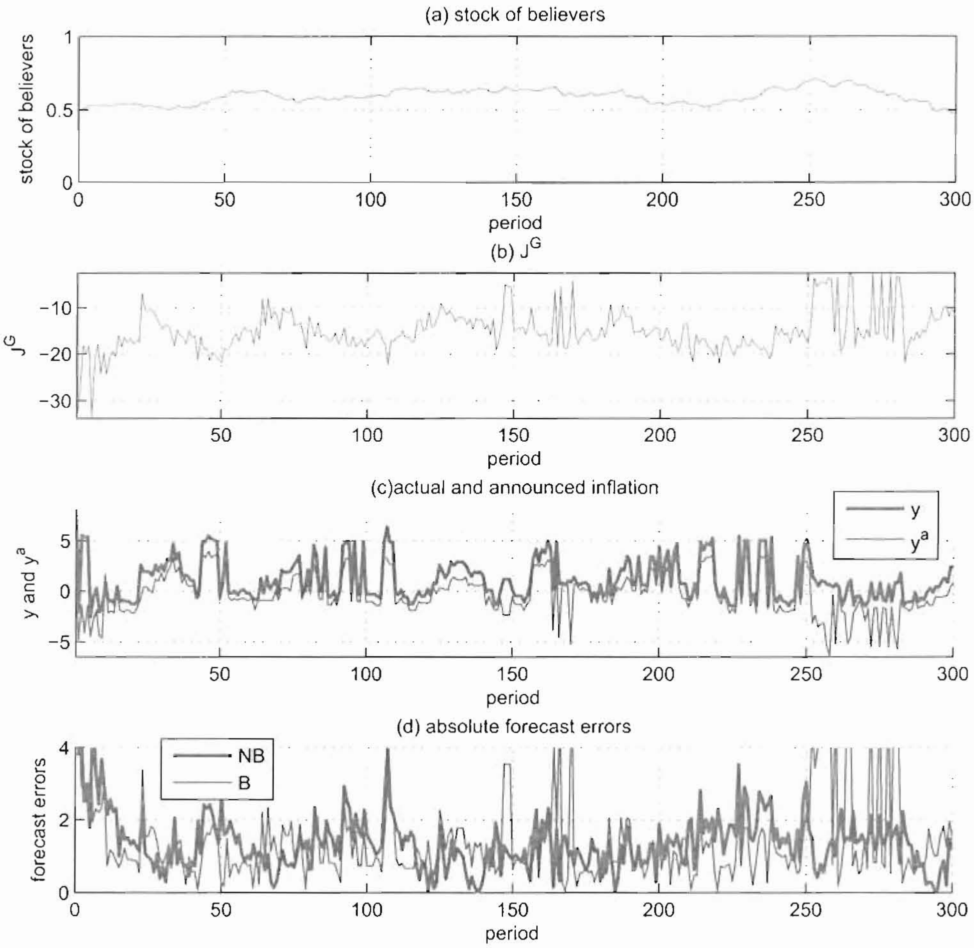


Figure 4.1: Evolution of the stock of believers (a), government payoffs (b), inflation rate and announced inflation rate (c), absolute forecast errors (d) for $cost = 0.1, \gamma = 0.1$ in the economy with homogeneous nonbelievers.

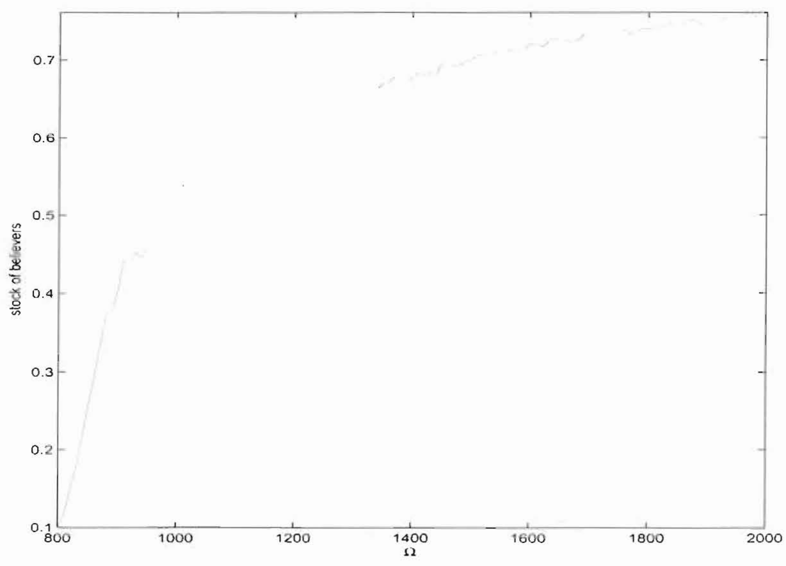


Figure 4.2: Long-run stock of believers for different values Ω.

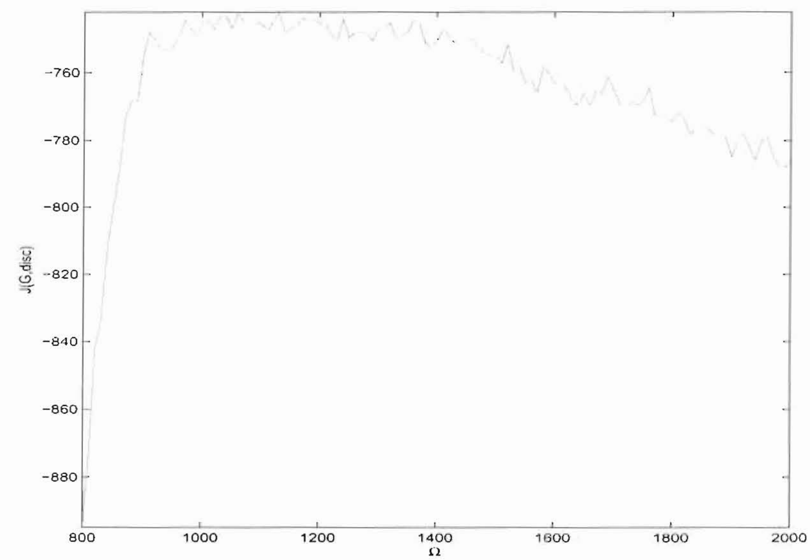


Figure 4.3: Discounted government payoff for different values Ω.

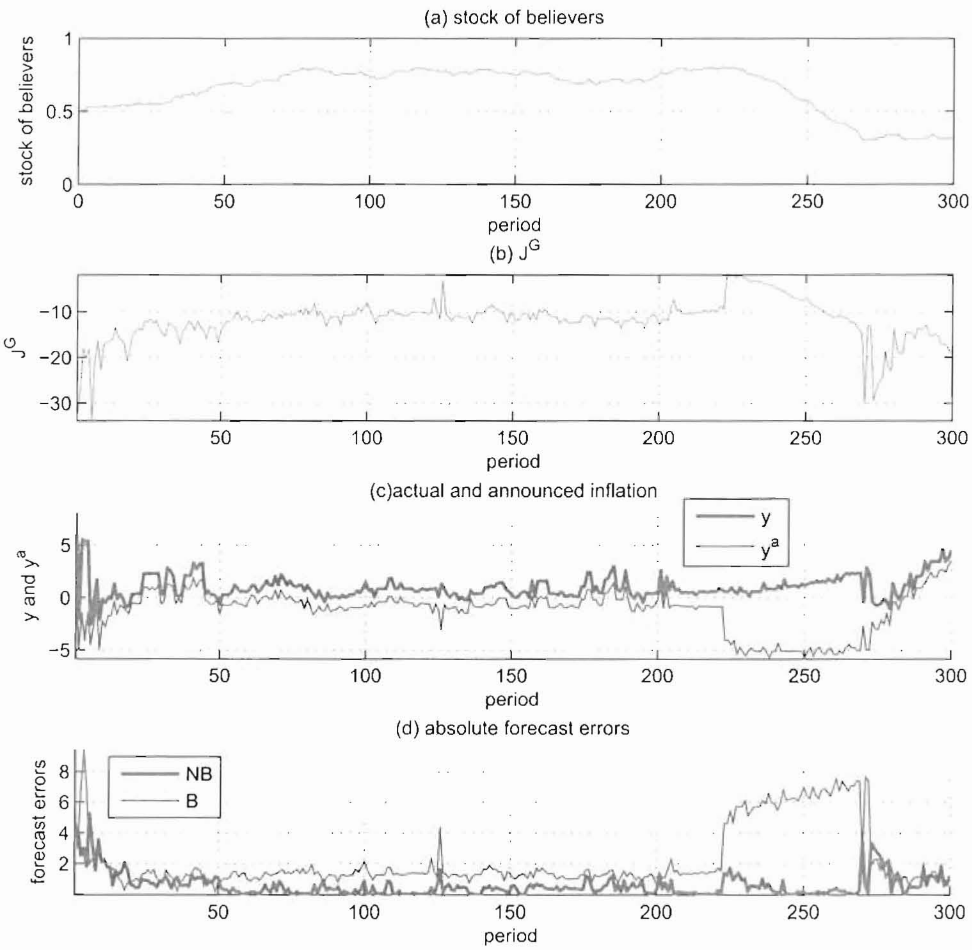


Figure 4.4: Evolution of the stock of believers (a), government payoffs (b), inflation rate and announced inflation rate (c), absolute forecast errors (d) for $cost = 1, \gamma = 0.1$ in the economy with homogeneous nonbelievers.

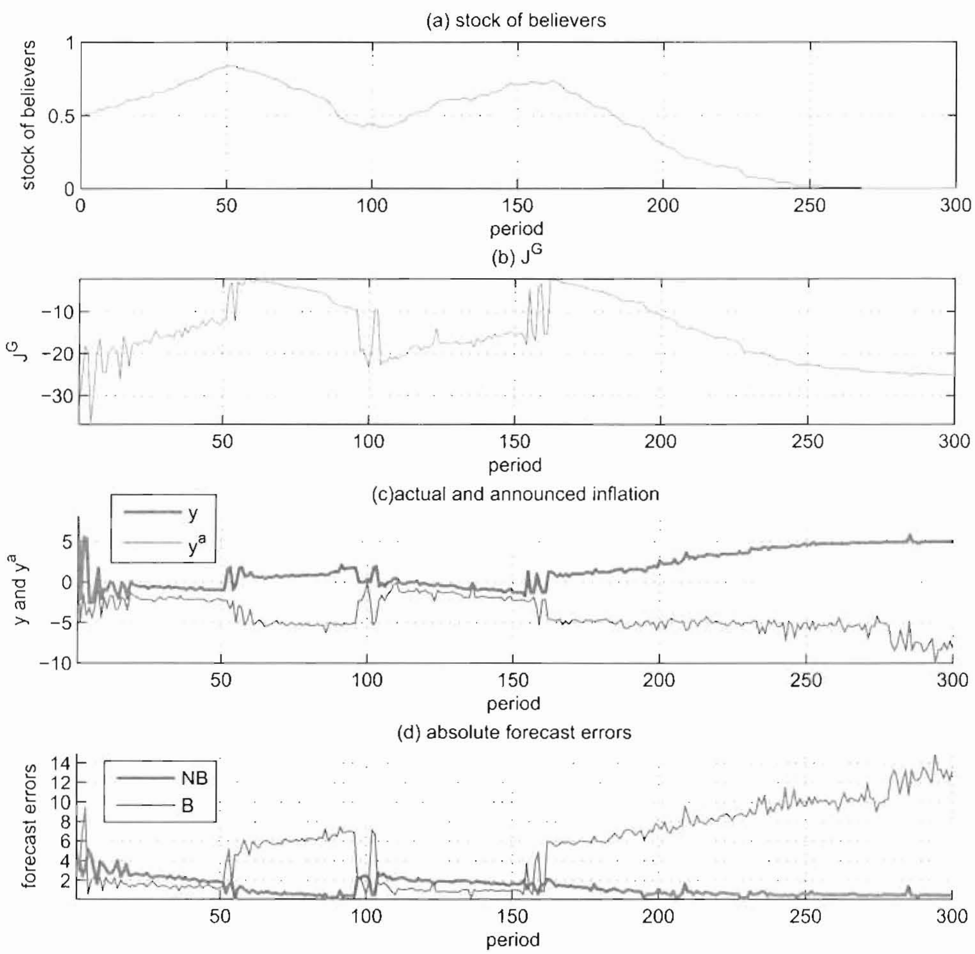


Figure 4.5: Evolution of the stock of believers (a), government payoffs (b), inflation rate and announced inflation rate (c), absolute forecast errors (d) for $cost = 0.1, \gamma = 0.01$ in the economy with homogeneous nonbelievers.

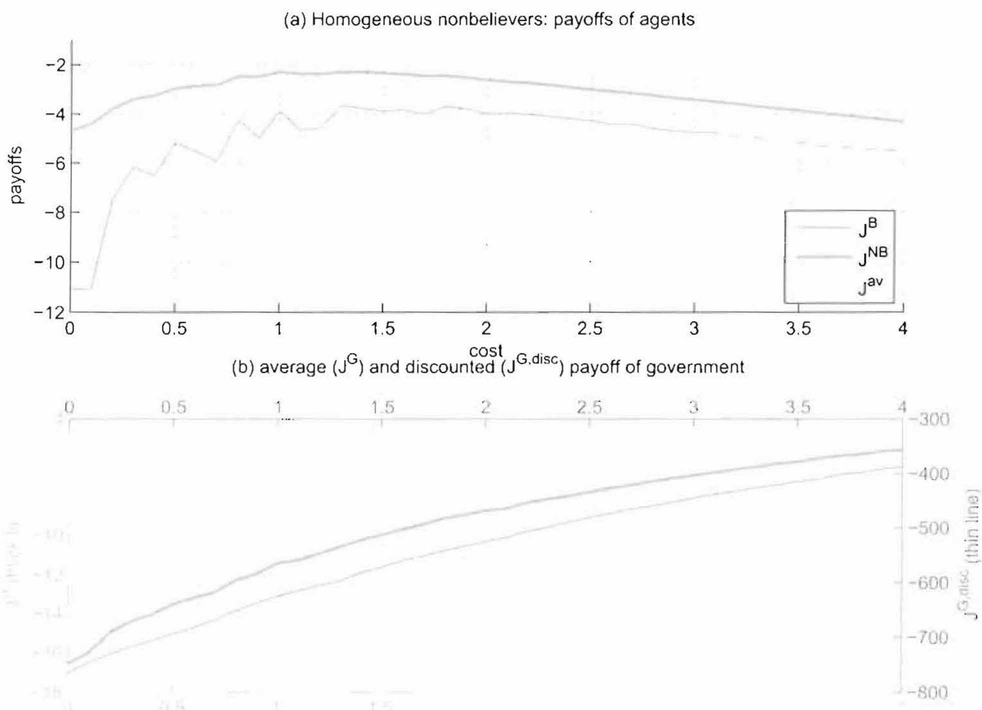


Figure 4.6: Payoffs of believers and nonbelievers and average agents' payoffs (a) and discounted and average payoff of government (b) for different cost c for the case of homogeneous nonbelievers.

Figure 4.7: Discounted payoff of the government with homogeneous and heterogeneous expectations of nonbelievers.

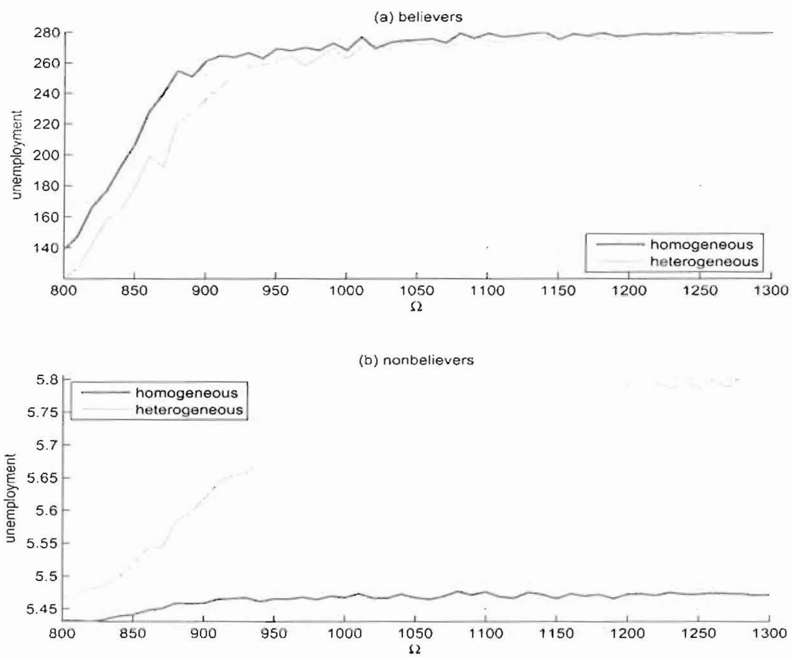


Figure 4.8: Unemployment among believers (a) and nonbelievers (b) with homogeneous and heterogeneous expectations of nonbelievers.

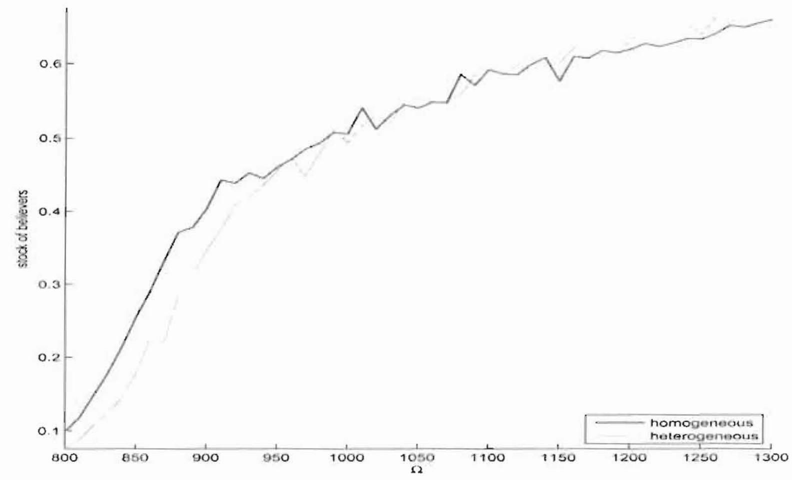


Figure 4.9: The stock of believers with homogeneous and heterogeneous expectations of nonbelievers.

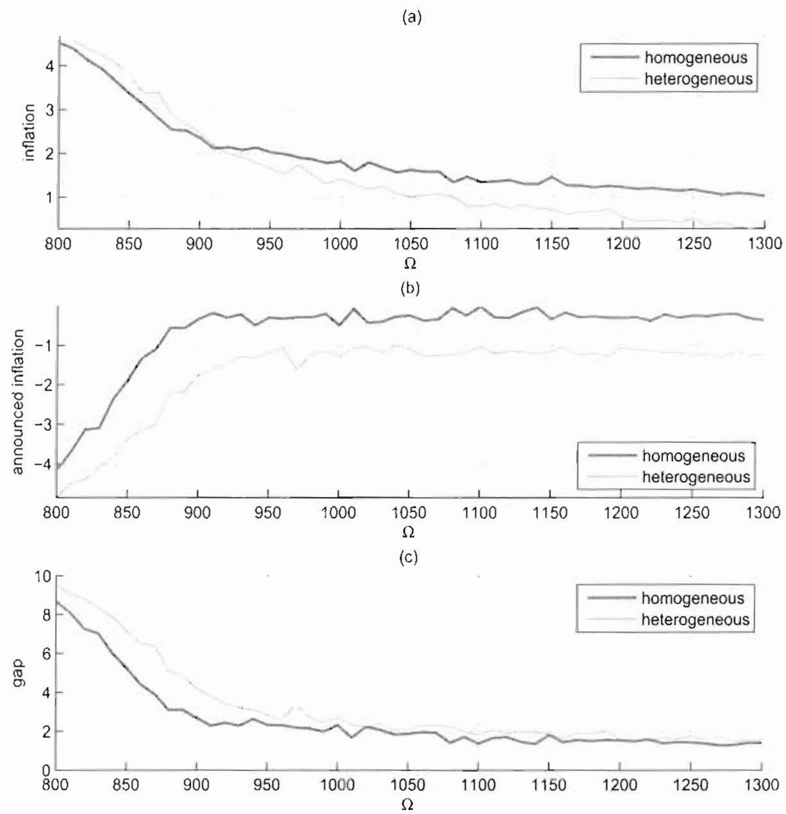


Figure 4.10: Inflation (a) and announced inflation (b) with homogeneous and heterogeneous expectations of nonbelievers.

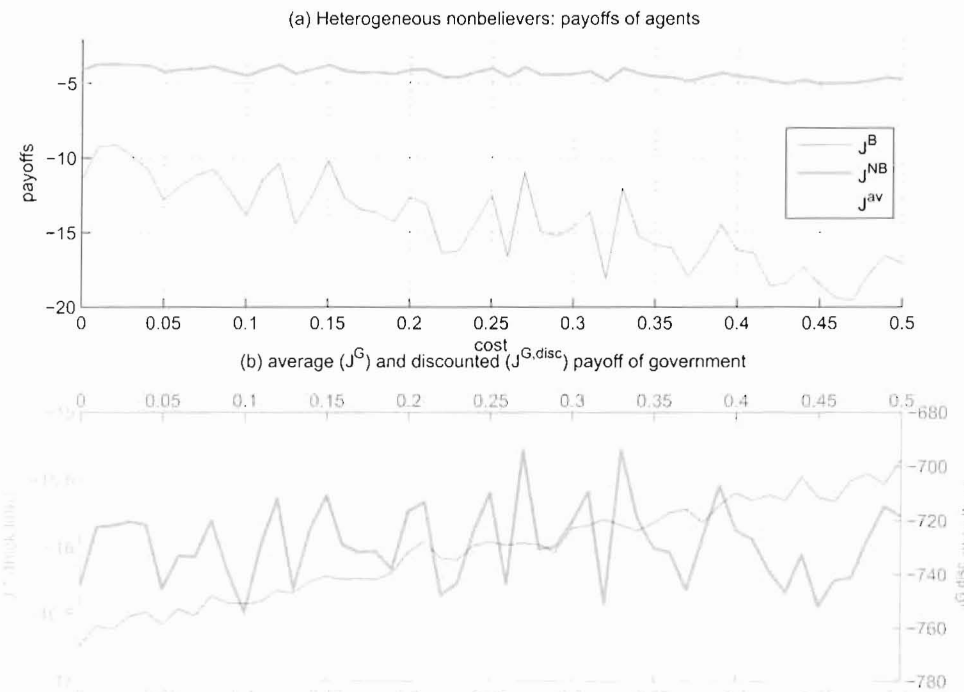


Figure 4.11: Payoffs of believers and nonbelievers and average agents' payoffs (a) and discounted and average payoff of government (b) for different cost c for the case of heterogeneous nonbelievers.

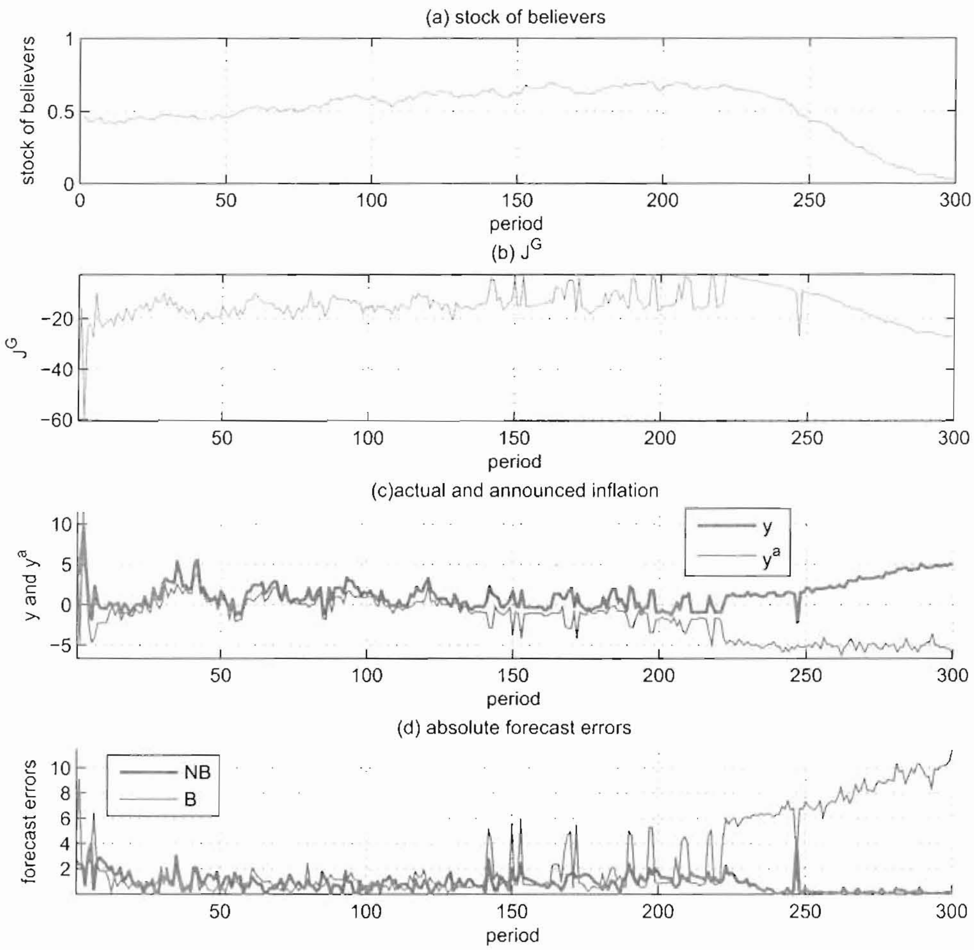


Figure 4.12: Evolution of the stock of believers (a), government payoffs (b), inflation rate and announced inflation rate (c), absolute forecast errors (d) for $cost = 0.1, \gamma = 0.1$ in the economy with heterogeneous nonbelievers.

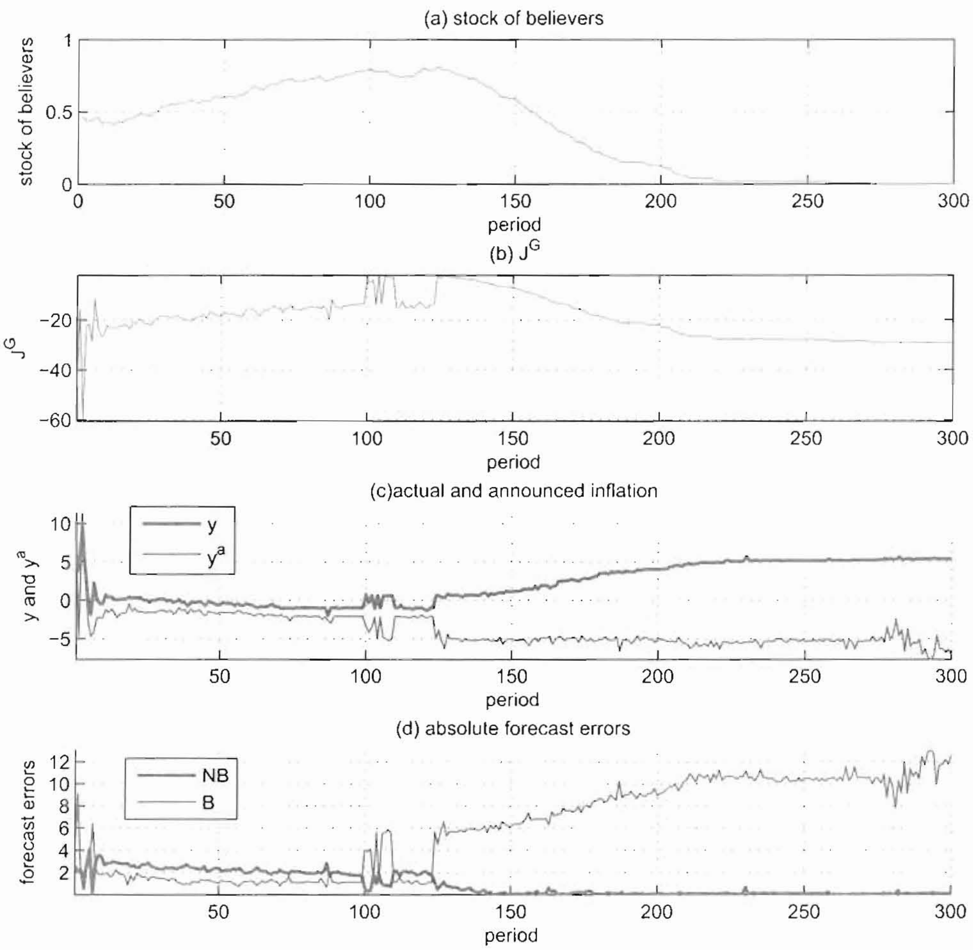


Figure 4.13: Evolution of the stock of believers (a), government payoffs (b), inflation rate and announced inflation rate (c), absolute forecast errors (d) for $cost = 0.1, \gamma = 0.01$ in the economy with heterogeneous nonbelievers..

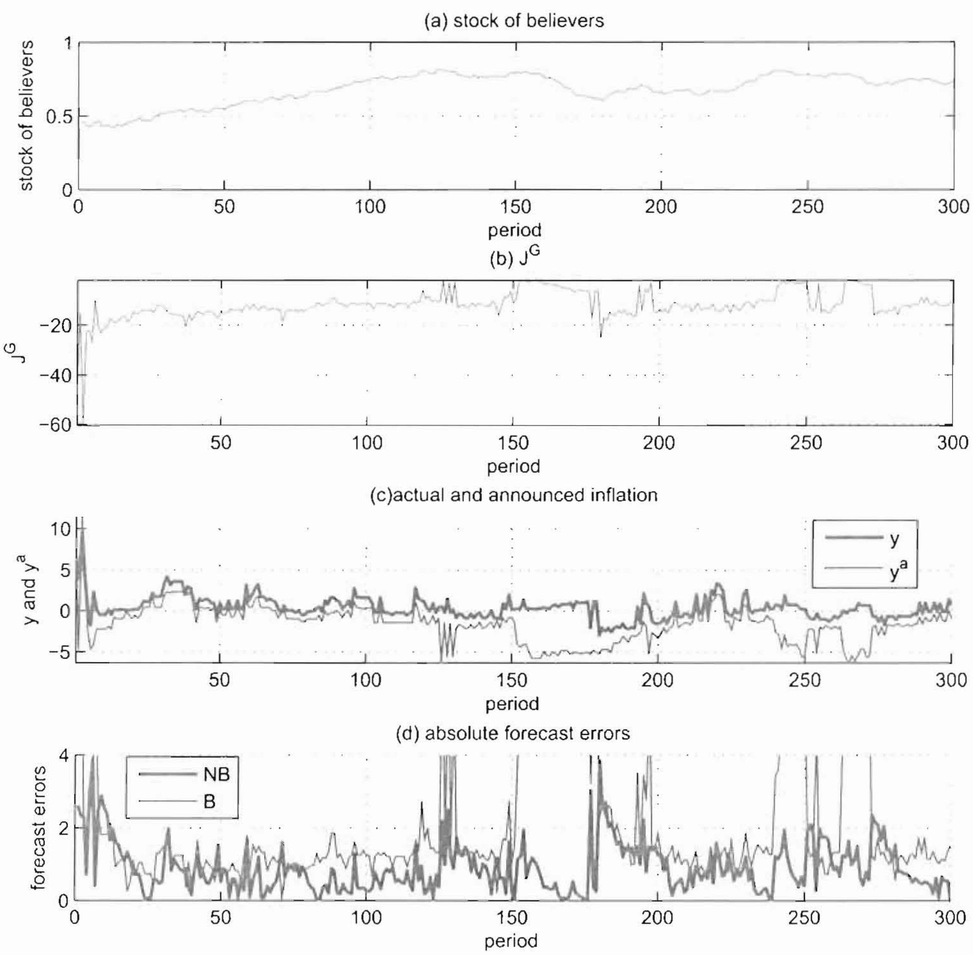


Figure 4.14: Evolution of the stock of believers (a), government payoffs (b), inflation rate and announced inflation rate (c), absolute forecast errors (d) for $cost = 1, \gamma = 0.1$ in the economy with heterogeneous nonbelievers..

cost	$J^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$
$c = 0.1$	-745.84	-16.00	0.51	1.82	-0.50	3.18	5.47	-11.07	-4.40
$c = 1$	-625.99	-11.42	0.69	0.94	-0.83	3.73	5.48	-3.89	-2.30
α	0.000005	0.3859	0.0129	0.00005	0.0885	0.4602	0.0007	0.000005	0.000005

Table 4.1: Effect of an increase in *cost* with homogeneous expectations of believers.

γ	$J^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$
$\gamma = 0.1$	-745.84	-16.00	0.51	1.82	-0.50	3.18	5.47	-11.07	-4.40
$\gamma = 0.01$	-689.80	-17.42	0.35	2.33	-4.04	-0.87	5.63	-33.87	-6.55
α	0.000005	0.3859	0.1492	0.0005	0.000005	0.000005	0.000005	0.000005	0.000005

Table 4.2: Effect of a decrease of γ with homogeneous expectations of believers.

type	$J^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$
1	-745.84	-16.00	0.51	1.82	-0.50	3.18	5.47	-11.07	-4.40
2	-750.71	-16.51	0.48	1.54	-1.38	2.58	5.70	-14.00	-4.51
α	0.0005	0.0007	0.0009	0.242	0.000005	0.000005	0.000005	0.000005	0.000005

Table 4.3: The difference between simulations with homogeneous and heterogeneous nonbelievers. Row 1 is for homogeneous case, row 2 is for heterogeneous case.

cost	$J^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$
$c = 0.1$	-750.71	-16.51	0.48	1.54	-1.38	2.58	5.70	-14.00	-4.51
$c = 1$	-659.30	-16.59	0.44	1.85	-3.04	0.61	5.63	-25.01	-6.15
α	0.000005	0.00005	0.0015	0.0019	0.000005	0.000005	0.000005	0.000005	0.000005

Table 4.4: Effect of an increase of *cost* with heterogenous expectations of nonbelievers.

γ	$J^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$
$\gamma = 0.1$	-750.71	-16.51	0.48	1.54	-1.38	2.58	5.70	-14.00	-4.51
$\gamma = 0.01$	-697.85	-15.59	0.54	1.03	-3.78	0.69	6.46	-22.34	-4.98
α	0.00005	0.00005	0.4286	0.1562	0.000005	0.0005	0.000005	0.00005	0.00005

Table 4.5: Effect of a decrease of γ with heterogenous expectations of nonbelievers.

periods		$J^{G,disc}$	J^G	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	U^B	U^{NB}	$J^{P,B}$	$\bar{J}^{P,NB}$	$y - x^{NB}$
[21, 100]	$\gamma = 0.1$	-597.41	-15.41	0.53	1.69	0.38	4.19	5.43	-4.86	-3.79	0.07
	$\gamma = 0.01$	-552.65	-12.40	0.69	-0.24	-2.77	2.97	6.72	-5.82	-2.22	-1.22
[21, 300]	$\gamma = 0.1$	-745.84	-16.00	0.51	1.82	-0.50	3.18	5.47	-11.07	-4.40	0.03
	$\gamma = 0.01$	-689.80	-17.42	0.35	2.33	-4.04	-0.87	5.63	-33.87	-6.55	-0.13

Table 4.6: Effect of a decrease in γ with homogeneous expectations of nonbelievers.

periods		$J^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$	$y - x^{NB}$
[21, 100]	$\gamma = 0.1$	-604.71	-15.37	0.53	1.18	-0.17	4.15	5.69	-4.38	-3.36	-0.19
	$\gamma = 0.01$	-577.28	-13.28	0.70	-0.39	-2.62	3.27	7.02	-4.84	-2.59	-1.52
[21, 300]	$\gamma = 0.1$	-750.71	-16.51	0.48	1.54	-1.38	2.58	5.70	-14.00	-4.51	-0.20
	$\gamma = 0.01$	-697.85	-15.59	0.54	1.03	-3.78	0.69	6.46	-22.34	-4.98	-0.96

Table 4.7: Effect of a decrease in γ with heterogenous expectations of nonbelievers.

periods		$J^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$	$y - x^{NB}$
[21, 100]	$c = 0.1$	-597.41	-15.41	0.53	1.69	0.38	4.19	5.43	-4.86	-3.79	0.07
	$c = 1$	-521.47	-13.00	0.63	1.29	0.10	4.31	5.46	-2.35	-2.71	0.04
[21, 300]	$c = 0.1$	-745.84	-16.00	0.51	1.82	-0.50	3.18	5.47	-11.07	-4.40	0.03
	$c = 1$	-625.99	-11.42	0.69	0.94	-0.83	3.73	5.48	-3.89	-2.30	0.02

Table 4.8: Effect of an increase of *cost* with homogeneous expectations of nonbelievers.

periods		$J^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$	$y - x^{NB}$
[21, 100]	$c = 0.1$	-604.71	-15.37	0.53	1.18	-0.17	4.15	5.69	-4.38	-3.36	-0.19
	$c = 1$	-519.34	-12.94	0.59	0.81	-1.55	3.14	5.63	-7.22	-3.47	-0.13
[21, 300]	$c = 0.1$	-750.71	-16.51	0.48	1.54	-1.38	2.58	5.70	-14.00	-4.51	-0.20
	$c = 1$	-659.30	-16.59	0.44	1.85	-3.04	0.61	5.63	-25.01	-6.15	-0.13

Table 4.9: Effect of an increase of *cost* with heterogenous expectations of nonbelievers.

Cases		$\bar{J}^{G,disc}$	$\bar{J}^G$	$\bar{\pi}$	$\bar{y}$	$\bar{y}^a$	$\bar{y}-\bar{y}^a$	$\bar{U}^B$	$\bar{U}^{NB}$	$\bar{J}^{P,B}$	$\bar{J}^{P,NB}$	$y-x^{NB}$	$\lvert y-x^{NB}\rvert$
Homogeneous nonbelievers													
[21, 100]	$\gamma\downarrow$	↑	↑	↑	↓	↓	↑	↓	↑	↓	↑	↓	↑
[21, 300]	$\gamma\downarrow$	↑	↓	↓	↑	↓	↑	↓	↑	↓	↓	↓	↑
Heterogeneous nonbelievers													
[21, 100]	$\gamma\downarrow$	↑	↑	↑	↓	↓	↑	↓	↑	↓	↑	↓	↑
[21, 300]	$\gamma\downarrow$	↑	↑	↑	↓	↓	↑	↓	↑	↓	↓	↓	↑
Homogeneous nonbelievers													
[21, 100]	$c\uparrow$	↑	↑	↑	↓	↓	↓	↑	↑	↑	↑	↓	↓
[21, 300]	$c\uparrow$	↑	↑	↑	↓	↓	↓	↑	↑	↑	↑	↓	↓
Heterogeneous nonbelievers													
[21, 100]	$c\uparrow$	↑	↑	↑	↓	↓	↑	↓	↓	↓	↓	↓	↓
[21, 300]	$c\uparrow$	↑	↓	↓	↑	↓	↑	↓	↓	↓	↓	↓	↓
Summary of change from homogeneous (1) nonbelievers to heterogeneous (2) nonbelievers													
[21, 100]	$1\rightarrow 2$	↓	↑	no Δ	↓	↓	↓	↓	↑	↑	↑	↓	↑
[21, 300]	$1\rightarrow 2$	↓	↓	↓	↓	↓	↑	↓	↑	↓	↓	↓	↑

Table 4.10: Summary of the directions of change.

Chapter 5

Conclusion

This thesis studies a variety of monetary models with boundedly rational agents. For future work, I consider the following extensions of the presented articles. The adaptive step-size algorithm can be used in other environments to study its implications. In the presented article, agent evaluates his performance based on forecast squared error. As the money demand function comes from overlapping generations models, an interesting extension would be to translate agent's inflation forecasts into choices of consumption and evaluate his utility. Then it would be possible to estimate how much consumption an agent is willing to forego to pay for better performing forecasting mechanism. Another extension is to evaluate the performance of adaptive step-size algorithm in different types of data generating processes to better understand its advantages.

The evaluation of decisions based on the utility can be possible in the New Keynesian environment of the second article. The agent-based environment has the advantage of studying the heterogeneity of agents, and in this paper agents are able to learn from other agents through imitation. It could be interesting to see the implications if agents need to trade their forecasting models and pay for better performing forecasts.