# DO BANKS UNDERESTIMATE VAR DIVERSIFICATION? 

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#### Abstract

In this paper we study empirically the diversification effect in banks' aggregated Value-at-Risk (VaR). Using actual data from the six largest Canadian commercial banks and five leading US commercial banks, we estimate the benchmark VaR based on individual VaRs for each risk factor and an historical correlation matrix, and then compare the benchmark with the aggregated VaR disclosed by the bank. Our main result is that the diversification effect reported by Canadian banks tends to be smaller than the one estimated by our correlation model over the period from 1999 to 2006. For the US banks, there is no supportive evidence for the underestimation of VaR diversification; however, there are very interesting results among different banks.


Keywords: Bank, Value-at-Risk (VaR), Diversification, Correlation

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## 1 INTRODUCTION

In recent years, the trading portfolios of large commercial banks have grown rapidly both in size and in complexity, as banks have taken massive positions in the derivatives market. For example, the trading portfolio of Royal Bank of Canada increased from CAD18,740 million in 1996 to CAD147,237 million in 2006, a 680\% increase, whereas the total assets increased by only $120 \%$ over the same period. This fact shows that large commercial banks are more exposed to market risk than they used to. As a result, regulators, banks, and academics have been intensively debating about how to measure and manage market risk.

Commercial banks have been computing their Value-at-Risk (VaR) since the mid 90 's. VaR is a measure of the maximum expected loss (in dollars) on a given position at a given confidence level (e.g. 99\%) over a given forecast period (e.g. 1 day). Besides the aggregated VaR, some banks disclose their individual VaRs associated with different risk categories, as well as the effect of diversification across different risk categories. The five main risk categories are interest rate risk, equity risk, foreign exchange risk, commodity risk and credit spread risk.

Previous empirical studies have shown that banks tend to overstate their VaR. This finding is surprising since under the 1996 Amendment of the Basel Accord, higher VaR induces higher regulatory capital for banks. Pérignon, Deng and Wang (2007) uncover that the six largest Canadian commercial banks exhibit a systematic excess of conservatism in their VaR estimates. Out of the

7,354 trading days analyzed in their study, there are only two exceptions (days when the trading loss exceeds the VaR ), whereas the expected number of exceptions with a $99 \%$ VaR is 74. Berkowitz and O'Brien (2002) also find that for a sample of US commercial banks, the $99 \%$ VaRs are too high, and sometimes inaccurate. Furthermore, Pérignon and Smith (2007a) report consistent results for a sample of international banks.

Academics have put forward several reasons to explain the 'inflated VaR puzzle', including: (1) extreme cautiousness due to the penalty imposed by regulators; (2) reputation risk of the banks if their actual loss exceeds the VaR too frequently; (3) the obligation of risk managers to explain the reason of each exception; and (4) underestimation of the diversification effect of the individual risk factors. To the best of our knowledge, there has been no direct empirical test of the last assertion yet.

Our study moves one step forward to test whether banks underestimate the diversification effects when computing their VaR. By doing so, we try to understand why banks overstate their VaR. Our empirical analysis is based on the VaR disclosed by the six largest Canadian commercial banks and five leading US commercial banks. For each bank, we collect from the bank's financial reports the aggregated VaR, the individual VaR of each risk category, and the diversification effect. We also collect the Canadian and US market indices to calculate the correlation matrix among the five risk categories, i.e. interest rate, equity, foreign exchange rate, commodity price, and credit spread. These individual VaRs and correlation matrix are used to compute our own
estimated VaR that we call the benchmark VaR. Our central test is to compare the total VaR disclosed by the banks with our estimated VaR.

Our main result is that the diversification effect of Canadian banks tends to be smaller than the one estimated by our correlation model over the period 1999 - 2006. This finding suggests that Canadian banks underestimate the VaR diversification effect. Bank of Montreal (BMO) is the most extreme case in our Canadian sample. BMO's aggregated VaRs are larger than our estimates throughout the sample period, implying that BMO's diversification effect is smaller than the one calculated by the correlation matrix. On average, BMO's estimated VaR is $30 \%$ less than the disclosed VaR. We also find that there is a difference between Canadian and US banks. In our US sample, we do not find any supportive evidence for the claim that banks underestimate the VaR diversification effect. Indeed, there is no obvious excess estimation of the aggregated VaR disclosed by banks over our benchmark. For instance, between 2000 and 2004 the aggregated VaRs of Bank of America (BoA) were systematically smaller than our estimated VaRs, which suggests that BoA was overestimating its VaR diversification.

The paper proceeds as follows. Section 2 presents the data used in our empirical test. In Section 3, we describe our methodology for estimating the VaR after diversification. Section 4 analyzes the empirical results and Section 5 concludes our study.

## 2 DATA

Our study covers the six largest Canadian banks and five leading US banks. Market indices are also collected for both Canadian and US markets. We describe the details regarding to the data collection for Canadian and US banks separately in this section.

### 2.1 Canadian Banks' VaR

We study the year end disclosed VaR of six Canadian commercial banks over the period from 1996 to 2006. The six banks are Bank of Montreal (BMO), Bank of Nova Scotia (BNS), Canadian Imperial Bank of Commerce (CIBC), National Bank of Canada (NBC), Royal Bank of Canada (RBC), and TorontoDominion Bank (TD). These six banks are the largest commercial banks in Canada and have important trading positions. The data are collected from their annual reports (fiscal year ended October 31).

In Canada, the VaR calculation has been made compulsory since 1997 by the Office of the Superintendent of Financial Institutions Canada (OSFI), but the public disclosure is optional, and the format of disclosure is not restricted. Canadian banks started to disclose their VaR at different time. RBC was the first to disclose its VaR, starting from1996. CIBC started from 1999, followed by BMO, BNS, and NBC in 2001. TD only disclosed the year end VaR in risk categories after 2005. As a result, we have VaR data from six Canadian commercial banks with different sample periods from 2 years to 11 years.

All the VaR data are 1-day VaR with $99 \%$ confidence level, except for BNS that used a 10-day VaR with 99\% confidence level in 2001 and 2002. To be consistent with other banks, we convert BNS's 10-day VaR to 1-day VaR. ${ }^{1}$ BMO and RBC started to disclose the credit spread VaR separately from the interest rate VaR from 2004, whereas CIBC has separated the credit spread VaR from the very beginning. BNS and NBC do not disclose the credit spread VaR but instead combine it with the interest rate VaR. RBC combined the foreign exchange VaR with the commodity VaR before 2005 and started to separate them in 2005.

We present the disclosed VaR for the sample Canadian banks in Table 1.

### 2.2 Canadian Market Indices

We use the five market indices as proxies to estimate the correlation among the five risk factors. The Canadian market indices include yield for oneyear zero coupon government bonds, the S\&P/TSX Composite index, the Dow Jones-AIG Spot Commodity Index, the exchange rate between Canadian and US dollars, and the difference between the yield of Canadian corporate bond and government bond, with each index representing one risk category. We retrieve the time-series indices for the period from November 11995 to October 31 2006, and details about the indices can be found in Table 2.

As Canadian corporate bond yield data are only available with a weekly frequency measured on Wednesdays, we use weekly data for all the Canadian market indices (also measured on Wednesdays) to estimate the correlation

[^0]matrix. Out of 581 weekly data points, there are 20 trading days when there is one or more than one index missing data. In each case we interpolate linearly the missing data points.

The five Canadian market indices are plotted in Figure 1. From the figures, we see that the market indices display an interesting correlation structure. Table 3 shows the in-sample correlation matrix among the five index returns which equal to the difference between index ( t ) and index ( $\mathrm{t}-1$ ) divided by index $(\mathrm{t}-1$ ). We find that the correlation coefficients are small with the highest one being 0.28 , occurring between foreign exchange rate and the commodity price. The average of the correlation coefficients is only 0.037 . In addition, 5 out of 10 correlation coefficients in the matrix are negative, and the credit spread index has negative correlation with all the other four indices. This implies that the diversification effect is expected to be strong in the sample.

### 2.3 US Banks' VaR

The five sample US commercial banks in our study are JPMorgan Chase and Co. (JPMorgan), Citigroup Inc. (Citibank), HSBC USA Inc. (HSBC), Bank of America (BoA), and Wachovia Bank (Wachovia), We examine these five banks because they are among the ten largest US commercial banks in terms of consolidated assets. ${ }^{2}$ More importantly, these five banks have disclosed their aggregated VaRs, the individual VaRs in different risk groups, and the diversification effect in their $10-\mathrm{K}$ and $10-\mathrm{Q}$ forms. The sample period covers from the last quarter of 1998 to the first quarter of 2007 with different banks

[^1]having different starting points. In the U.S., market risk disclosure is mandatory in financial reports required by the SEC, however banks can choose between three different formats of the disclosure, and the disclosure of individual VaRs is voluntary.

Among the five US banks, JPMorgan and Citibank disclose their 1-day VaR at 99\% confidence level at each quarter end. HSBC reported its 10-day VaR at the $99 \%$ confidence level until June 30, 2006 and changed to 1-day VaR at the 99\% confidence level afterwards. To obtain consistent measurement, we convert the 10 -day VaRs to 1-day VaRs for HSBC. Unlike the first three banks, BoA and Wachovia disclose their average VaR, instead of the period end VaR. BoA discloses its average VaR for the past twelve months at each quarter end, while Wachovia only discloses its yearly average VaR at year end. Both report 1-day VaR at 99\% confidence level.

In case of restated VaR because of changed assumption and estimation approach used in the VaR models, we collect from the most recent reports and systematically use the most recent VaRs. We recalculate the VaRs for Citigroup before 2000 by adding the VaRs of Salomon Smith Barney, a division of Citigroup Global markets Inc. because (1) VaRs for Salomon Smith Barney were disclosed in Citibank's 10-K and 10-Q before 2000, (2) we think it is a reasonable conversion suggested by the VaRs released by Citibank after 2000, and (3) our approach is consistent with Jorion (2007) who adds up data from separate entities.

We present the disclosed VaR for the sample US banks in Table 4.

### 2.4 US Market Indices

The US market indices are the market yield on one-year US Treasury securities, S\&P500 Composite index, the major currencies index, and the difference between Moody's corporate BAA yield and one-year Treasury yield. We retrieve the time-series US indices for the years from 1998 to the first quarter of 2007 with details explained in Table 5. Out of 2,313 data points, there are 47 trading days when there is one or more than one index missing data. We interpolate the missing data linearly. The five US market indices are plotted in Figure 2. Table 6 shows the correlation matrix between the five index returns. We find that the positive correlation coefficients are smaller than the Canadian ones, with the highest one being 0.20 , and an average of 0.01 .

## 3 METHODOLOGY

The diversified $\operatorname{VaR}$ (or aggregated VaR ) is determined by two factors: (1) the level of individual VaRs, and (2) the diversification effect in aggregating the VaR of each risk factor.

If the risk factors in the portfolio are perfectly correlated with $\rho_{i j}=1$, the diversified $\operatorname{VaR}$ of the portfolio is equal to the sum of the individual VaRs. However, in general, the correlations for the market indices differ from one, resulting in diversification effect. The diversification can be measured by the difference between the diversified VaR and the sum of the individual VaRs .

There are two potential causes for banks' inflated VaR. First, bank's VaR models can overstate the individual VaR of each risk factor. Second, bank's VaR models can underestimate the diversification effect. Our paper focuses on testing the diversification effect story.

We estimate the diversified VaR based on the historical correlation among the market indices and the disclosed individual VaRs. By comparing our estimated VaR with the banks' disclosed VaR after diversification, we investigate whether there is any evidence for the claim that banks underestimate the diversification among risk factors.

The formula for the diversified VaR is derived from Jorion (2006) and Pérignon and Smith (2007b).

The $\operatorname{VaR}$ of the investment in asset i is given by:

$$
\begin{equation*}
V a R_{i}=\alpha \sigma_{i} x_{i} \tag{1}
\end{equation*}
$$

where $\alpha$ is the standard normal deviate, $\sigma_{i}$ is the standard deviation of the rate of return of asset i , and $x_{i}$ is the dollar exposure of asset i .

As $\sigma_{p}^{2} W^{2}=x^{\prime} \sum x$, the portfolio $\operatorname{VaR}$ is given by:

$$
\begin{equation*}
V_{a} R_{p}=\alpha \sigma_{p} W=\alpha \sqrt{x^{\prime} \sum x} \tag{2}
\end{equation*}
$$

where $\sigma_{P}$ is the standard deviation of the portfolio, $W$ is the initial portfolio value, $x$ is the vector of the dollar exposure of the portfolio, and $\sum$ is the covariance matrix of the asset returns.

We know that the covariance matrix can be decomposed as follows:

$$
\begin{equation*}
\Sigma=D R D \tag{3}
\end{equation*}
$$

where $D$ is the diagonal matrix containing the standard deviations of return of asset i , and R is the correlation matrix of the returns of assets in the portfolio.

The diversified VaR (DVaR), which aggregates all the risk categories' VaRs, follows the same formula of $\mathrm{VaR}_{\rho}$, so by substituting formula (2) and (3):

$$
\begin{equation*}
D V a R=V_{a} R_{p}=\alpha \sqrt{x^{\prime} \sum x}=\sqrt{\alpha^{2} x^{\prime} D R D x} \tag{4}
\end{equation*}
$$

In the case of calculating the $D V a R, D$ represents the diagonal matrix with the standard deviation of each risk category's VaR , and $R$ stands for the correlation matrix among the risk factors.

By rearranging formula (4), we get:

$$
\begin{equation*}
D V a R=\sqrt{\left(\alpha x^{\prime} D\right) R(\alpha D x)} . \tag{5}
\end{equation*}
$$

As $V a R_{i}=\alpha \sigma_{i} x_{i}, \alpha D x$ forms a vector comprising of each risk category's VaR, therefore:

$$
\begin{equation*}
D V a R=\sqrt{V^{\prime} R V} \tag{6}
\end{equation*}
$$

where $D V a R$ is the diversified $\operatorname{VaR}, V$ is the column vector containing the individual VaRs, and $R$ is the correlation matrix of the market indices. The formula (6) is used to calculate the diversified VaR in our study.

We use the following example to illustrate the importance of the diversification effect in determining the diversified VaR for a bank. Assume that Bank $A$ has calculated its individual VaRs: $\$ 10 \mathrm{~m}$ for interest risk, $\$ 10 \mathrm{~m}$ for equity risk, $\$ 10 \mathrm{~m}$ for foreign exchange risk, $\$ 0$ for commodity risk and $\$ 10 \mathrm{~m}$ for credit spread risk, and the correlation coefficients among the five risk factor indices are all 0.1. The DVaR is:

$$
D V a R=\sqrt{\left(\begin{array}{lllll}
10 & 10 & 10 & 0 & 10
\end{array}\right)\left(\begin{array}{ccccc}
1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0.1 & 1 & 0.1 & 0.1 & 0.1 \\
0.1 & 0.1 & 1 & 0.1 & 0.1 \\
0.1 & 0.1 & 0.1 & 1 & 0.1 \\
0.1 & 0.1 & 0.1 & 0.1 & 1
\end{array}\right)\left(\begin{array}{c}
10 \\
10 \\
10 \\
0 \\
10
\end{array}\right)}=22.8
$$

whereas the sum of the individual VaRs is $\$ 40 \mathrm{~m}$. As a result the diversification effect is $\$ 17.2 \mathrm{~m}$ ( $\$ 40 \mathrm{~m}-\$ 22.8 \mathrm{~m}$ ), or $43 \%$ of the undiversified VaR.

If we decrease the correlation coefficients to 0 , we get the new DVaR equal to $\$ 20 \mathrm{~m}$, and the diversification effect increases to $\$ 20 \mathrm{~m}$. If the correlation coefficients are all -0.1 , the DVaR changes to $\$ 16.73$ and the diversification effect increases by $\$ 2.27 \mathrm{~m}$.

This example shows that the lower the correlation coefficients among risk factors the higher the diversification effect. The bottom line of the methodology section is that the diversification is a key factor in determining the diversified VaR.

## 4 ANALYSIS AND EMPIRICAL RESULTS

### 4.1 Disclosed VaR Diversification

The actual diversification effect is the difference between the undiversified VaR and diversified VaR scaled by the undiversified VaR. We summarize the disclosed diversification effects of the Canadian banks and US banks in Table 7 and Table 8, respectively.

We cannot find a similar pattern for the diversification effect of the sample Canadian banks (Figure 3). Thus, we calculate the time average and the crossbank average to investigate whether there is any pattern for the Canadian banking industry. For the cross-bank average diversification, we omit the first three years (1996-1998) since RBC is the only bank that disclosed its individual VaRs during that period and one bank does not represent the industry properly. The cross-bank average diversification effect (Figure 4) is stable over the period from 1999 to 2006, with some movement from $28 \%$ to $42 \%$. CIBC has the largest diversification effect of $46.8 \%$, followed by RBC (40.9\%), BNS (40.25\%), TD (35.99\%), NBC (31.79\%), and BMO (20.46\%). BMO's has the least diversification, less than half of CIBC's.

Compared to the Canadian banks, our sample US banks show more characteristics. The disclosed diversification effect of the five US banks is summarized in Table 8 and plotted in Figure 5. We notice that there is an upward trend for the diversification effect curve for JPMorgan and Wachovia. JPMorgan had a diversification effect of $17.72 \%$ in the first quarter of 2001 and reached its highest level of $52.82 \%$ in the second quarter of 2006. Wachovia has an even
stronger increase. It started from $10.77 \%$ in 1999, increased to $31.10 \%$ in 2001, fell back to $23.67 \%$ in 2004 , and jumped to $51.78 \%$ in 2006 . The curves for Citibank and BoA are smoother with several exceptions. For Citibank, we see an upward trend after the second quarter of 2004. BoA's diversification effects are around $52 \%$, but with a bump in 2003. HSBC's diversification effect is the most volatile among the five US banks. It had the least diversification, a ratio as low as $4.54 \%$ in the third quarter of 2004, and also a peak of $57.14 \%$ in the fourth quarter of 2006. However, when we average the diversification effect across banks as shown in the first graph of Figure 6, we can not tell there is any trend for all the banks over the period from 1999 to 2006. The rank of the average diversification effect (the second graph of Figure 6) is BoA (53.23\%), Citibank (37.6\%), JPMorgan (36.70\%), HSBC (29.12\%), and Wachovia (29.11\%), and Wachovia's diversification effect is the lowest, almost half of BoA's.

Overall, US banks report an average of diversification of $39 \%, 2 \%$ more than Canadian banks. This result agrees with our findings from the correlation matrix. As the average of US correlation coefficients is smaller than the Canadian one, the diversification effects of US banks are supposed to be larger.

### 4.2 Determinants of Diversification Effects

As a preliminary analysis, we run some regressions to investigate what factors have impact on the diversification effect of VaR. The most intuitive independent variables are the individual VaRs disclosed by banks. In addition, the standard deviation of the individual VaRs is included in our regression, since
the balance among individual VaRs might affect the diversification as well. Thus, the first regression we run is:
$\frac{D V a R}{\sum V a R_{i}}=\alpha+\beta_{I R} \times V_{I R}+\beta_{\text {equitr }} \times V_{\text {equity }}+\beta_{F X} \times V_{F X}+\beta_{C} \times V_{c}+\beta_{\text {crodi }} \times V_{\text {cradn }}+\beta_{\sigma} \times V_{o}+\varepsilon$
where $D V a R$ is the diversified VaR disclosed by bank, $\Sigma V_{a R}$ is the sum of the individual VaRs for each risk factor, $V_{\text {credit }}$ is the interest rate $\mathrm{VaR}, V_{\text {equity }}$ is the equity $\operatorname{VaR}, V_{F x}$ is the foreign exchange $V a R, V c$ is the commodity $V a R$, and $V_{\text {crodit }}$ is the credit risk VaR.

The regression results for both Canadian and US banks are summarized in Table 9. In the case of Canadian banks, the t-statistics suggest that the coefficients of $V_{F X}$ and $V_{\text {credit }}$ are significant at $95 \%$ confidence level. The two coefficients are all negative, suggesting that higher $V_{F X}$ and $V_{\text {credit }}$ lead to more diversification. For US banks, only the coefficient of $V_{\text {credit }}$ is significant and it is negative. We notice that the standard deviation of the individual VaRs is insignificant in either case. Thus, we run our regression B without the standard deviation of the VaRs.

When we regress $\frac{D V a R}{\sum V a R_{i}}$ against the five individual VaRs only, we find that $V_{I R}, V_{F X}$, and $V_{\text {credit }}$ are significant for Canadian banks and only $V_{\text {credit }}$ is significant for US banks. Unlike other significant coefficients, the coefficient of $V_{I R}$ for Canadian banks is positive, implying that larger $V_{\mathbb{R}}$ is associated to lower diversification.

We notice that the standard deviation of $V a R i$ is highly correlated with $V_{I R}$. The correlation coefficients are 0.80 and 0.99 for Canadian banks and US banks
respectively. This is because that $V_{\mathbb{R}}$ is the largest component among individual VaRs as interest rate risk is the biggest risk that banks face. We suspect the high correlation is the reason for why $V_{\mathbb{R}}$ becomes significant when we exclude the standard deviation in our regression B.

Our preliminary analysis shows that interest rate VaR is negatively correlated with the diversification effect while foreign exchange VaR and credit spread VaR are positively correlated with the diversification. The level of balance among the individual VaRs does not affect the diversification significantly.

### 4.3 Comparison between Bank VaR and Estimated VaR

As we discussed in Section 3, our benchmark VaR is computed by the estimated correlation matrix and the individual VaR disclosed by the banks. In our DVaR estimation model, we use a one-year moving window to estimate the correlation matrix at each year/quarter end. In the case that the banks do not disclose the VaR for particular risk categories, we put zero VaR in the vector of individual VaRs.

The comparison between the disclosed DVaR and estimated DVaR is summarized in Table 10 and Table 11 for Canadian banks and US banks, respectively. We calculate the VaR excess ratio as the difference between the disclosed DVaR and the estimated DVaR over the disclosed one.

For the Canadian sample as shown in Figure 7, we notice that BMO's disclosed DVaRs are always greater than our estimation from 2001 to 2006, by an average of $30 \%$. CIBC takes the second place, and seven out of eight disclosed DVaRs are greater than the estimated ones, with an average of $18 \%$
exceeding the estimated DVaR. TD's disclosed DVaR in 2005 is very close to our estimated DVaR, however the 2006 disclosed DVaR is in excess of $20 \%$ of our estimated DVaR. We notice that the difference for RBC in 1996 and 1997 is $102 \%$ and $-54 \%$, respectively, the largest difference among all data. A small difference suggests that the banks reasonably analyze the diversification effect, while a large difference implies an inaccurate estimation for the diversification effect. One possible explanation for RBC's large difference is that RBC is the first bank to disclose quantitative market risk, and it might not be experienced enough to estimate the VaR in the first couple of years. Disregarding the first two years' impact, the results for NBC and RBC do not show much tendency toward either overestimation or underestimation, with half of the disclosed VaRs greater than the estimated ones and the other half smaller. Unlike the case of BMO, BNS has only one disclosed DVaR exceeding the estimated DVaR from 2001 to 2006.

We perform the same analysis for the five US banks as well. The results are presented in Table 11, and Figure 8 is the plot of the VaR excess ratio of each bank. The graph of BoA is interesting. We find that the estimated DVaRs are very close to the disclosed ones since the second quarter of 2005. However, the difference is big from the second quarter of 2001 to the second quarter of 2004, underestimating by more than $20 \%$, with an extreme value over $60 \%$ in the third quarter of 2003. The overestimation of VaR diversification might be the reason why BoA experienced three exceptions of actual trading loss exceeding VaR in 2003. On the contrary, HSBC overestimates its DVaR 73\% of the time, and the magnitude of the excess ratio is as high as $40 \%$ on average from the second quarter of 2002 to the third quarter of 2005. However the DVaRs of

HSBC become smaller than our estimation from the fourth quarter of 2005, with a peak value of $-71 \%$ in the second quarter of 2006. The disclosed DVaRs of JPMorgan and Wachovia are close to the estimated DVAR with a maximum difference of $29 \%$ and $21 \%$ respectively. Citibank, the first one to disclose VaR among the five, has the smallest variations for the sample period.

If we analyze each bank individually, we cannot find a typical trend of the difference between the disclosed DVaR and the estimated one across the Canadian banks or the US banks. However, when we average the excess percentage of disclosed DVaR over estimated DVaR among Canadian banks for the period from 1999 to 2006, Figure 9 shows that in all the seven years Canadian banks overestimate the DVaR by a level around $1 \%$ to $20 \%$, with an average of $11 \%$. The years from 1996 to 1998 are not included in Figure 9, because no other banks disclosed VaR during the three years except RBC. The analysis from the perspective of average effect across Canadian banks supports our claim that Canadian banks underestimate the VaR diversification effect which leads to the VaR overestimation. The same analysis for US banks does not reach the conclusion of a general underestimation of VaR diversification.

In addition, we calculate the estimated DVaR based on the correlation matrix derived from the five-year moving window market index data, and find there is no material difference compared to the previous estimated DVaR. Besides that, we also form a market index matrix by deleting those trading days with missing data instead of interpolating them, and the estimated DVaR based on this approach is very close to the result derived from the interpolated indices.

## 5 CONCLUSION

Our empirical analysis reveals that Canadian banks underestimate the diversification effect of the VaR, thus overstating their aggregated VaR. However, there is no clear evidence indicating that the US banks are also underestimating the diversification effect.

We also find some very different cases among the sample banks. BMO always underestimates the VaR diversification effect and has the highest percentage in terms of the VaR overestimation, so BMO is likely to underestimate the VaR diversification. This finding is further verified by the fact that BMO reports the least average diversification effect during our sample period. BoA, on the contrary, constantly underestimated the diversified VaR with a significant magnitude from 2000 to 2004. In cases when the VaR diversification is overestimated, we suspect that the overestimation of the disclosed individual VaRs contributes a partial effect to the overestimation of the diversified VaR, More empirical tests need to be done to validate the individual VaRs. However, the portfolio data of each risk category are not publicly available, which makes it hard to investigate the individual VaRs.

We use the historical correlation model to estimate the correlation matrix for the individual VaRs. Instead of the historical data as a proxy, more sophisticated correlation models, such as the conditional correlation model proposed by Robert Engle (2002), can be applied to obtain a better estimation of the time-varying correlation matrix, thus a closer measurement of the diversification effect.

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## TABLES

Table 1 Disclosed VaR for Canadian Banks

| Bank |  | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BMO | Interest rate | - | - | - | - | - | 15.8 | 16.9 | 21.4 | 10.1 | 11.8 | 19.1 |
|  | Equity | - | - | - | - | - | 4.8 | 6.6 | 5.1 | 3.9 | 3.8 | 9.8 |
|  | Foreign exchange | - | - | - | - | - | 7.0 | 3.8 | 6.3 | 0.5 | 0.4 | 3.3 |
|  | Commodity | - | - | - | - | - | 2.0 | 1.1 | 0.8 | 1.1 | 3.2 | 8.4 |
|  | Credit spread | - | - | - | - | - | n/a | n/a | n/a | 4.0 | 4.1 | 5.8 |
|  | Undiversified VaR | - | - | - | - | - | 29.6 | 28.4 | 33.6 | 19.6 | 23.3 | 46.4 |
|  | Diversification | . | - | - | - | - | (5.6) | (5.2) | (5.4) | (4.6) | (5.5) | (10.4) |
|  | Diversified VaR | - | - | - | - | - | 24.0 | 23.2 | 28.1 | 15.0 | 17.8 | 36.0 |
| BNS | Interest rate | - | - | - | - | - | 4.2 | 7.4 | 8.5 | 3.6 | 4.6 | 9.5 |
|  | Equity | - | - | - | - | - | 4.3 | 3.6 | 6.4 | 4.0 | 4.3 | 2.8 |
|  | Foreign exchange | - | - | - | - | - | 1.2 | 1.7 | 1.0 | 1.5 | 1.0 | 0.6 |
|  | Commodity | - | - | - | - | - | 0.7 | 0.8 | 1.2 | 0.7 | 1.7 | 0.5 |
|  | Credit spread | - | - | - | - | - | n/a | n/a | n/a | n/a | n/a | n/a |
|  | Undiversified VaR | - | - | - | - | - | 10.5 | 13.6 | 17.1 | 9.8 | 11.6 | 13.4 |
|  | Diversification | - | . | - | - | - | (3.9) | (5.7) | (6.5) | (4.5) | (5.0) | (4.8) |
|  | Diversified VaR | - | - | - | - | - | 6.6 | 7.9 | 10.6 | 5.3 | 6.6 | 8.6 |
| CIBC | Interest rate | - | - | - | 12.2 | 6.4 | 6.1 | 7.3 | 2.5 | 6.0 | 3.4 | 6.1 |
|  | Equity | - | - | - | 15.6 | 14.1 | 8.3 | 9.3 | 5.4 | 4.7 | 5.1 | 6.1 |
|  | Foreign exchange | - | - | - | 1.0 | 0.9 | 0.9 | 0.5 | 1.0 | 0.2 | 0.1 | 0.4 |
|  | Commodity | - | - | - | 2.1 | 1.0 | 1.1 | 2.6 | 0.8 | 2.0 | 1.1 | 1.2 |
|  | Credit spread | - | - | - | 14.4 | 13.3 | 6.7 | 5.7 | 2.6 | 2.9 | 2.6 | 5.7 |
|  | Undiversified VaR | - | - | - | 45.3 | 35.5 | 23.1 | 25.4 | 12.3 | 15.8 | 12.3 | 19.5 |
|  | Diversification | - | - | - | (20.6) | (15.0) | (12.0) | (10.0) | (6.1) | (7.0) | (6.0) | (10.3) |
|  | Diversified VaR | - | - | - | 24.7 | 20.5 | 11.2 | 15.4 | 6.2 | 8.8 | 6.3 | 9.2 |
| NBC | Interest rate | - | - | - | - | - | 2.0 | 3.0 | 2.0 | 2.1 | 3.5 | 4.1 |
|  | Equity | - | - | - | - | - | 1.0 | 1.0 | 1.2 | 1.1 | 5.1 | 4.1 |
|  | Foreign exchange | - | - | - | - | - | 1.0 | 0.0 | 0.2 | 0.2 | 0.9 | 1.2 |
|  | Commodity | - | - | - | - | - | n/a | n/a | 0.2 | 0.2 | 0.6 | 1.5 |
|  | Credit spread | - | - | - | - | - | n/a | n/a | n/a | n/a | n/a | n/a |
|  | Undiversified VaR | - | * | - | - | - | 4.0 | 4.0 | 3.6 | 3.6 | 10.1 | 10.9 |
|  | Diversification | - | - | - | - | - | (1.0) | (1.0) | (0.7) | (0.9) | (5.0) | (5.1) |
|  | Diversified VaR | - | - | - | - | - | 3.0 | 3.0 | 2.9 | 2.7 | 5.1 | 5.8 |
| RBC | Interest rate | 15.0 | 12.0 | 5.0 | 6.0 | 7.0 | 3.0 | 11.0 | 8.0 | 8.0 | 12.0 | 13.0 |
|  | Equity | 2.0 | 10.0 | 17.0 | 9.0 | 14.0 | 8.0 | 7.0 | 4.0 | 4.0 | 7.0 | 7.0 |
|  | Foreign exchange | 10.0 | 20.0 | 5.0 | 6.0 | 4.0 | 2.0 | 2.0 | 2.0 | 2.0 | 1.0 | 2.0 |
|  | Commodity | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a | 1.0 | 1.0 |
|  | Credit spread | n/a | n/a | n/a | n/a | n/a | n/a | n/a | n/a | 2.0 | 2.0 | 3.0 |
|  | Undiversified VaR | 27.0 | 42.0 | 27.0 | 21.0 | 25.0 | 13.0 | 20.0 | 14.0 | 16.0 | 23.0 | 26.0 |
|  | Diversification | (17.0) | (27.0) | (9.0) | (8.0) | (7.0) | (5.0) | (7.0) | (6.0) | (6.0) | (8.0) | (9.0) |
|  | Diversified VaR | 10.0 | 15.0 | 18.0 | 13.0 | 18.0 | 8.0 | 13.0 | 8.0 | 10.0 | 15.0 | 17.0 |
| TD | Interest rate | - | - | - | - | - | - | - | - | - | 4.0 | 7.3 |
|  | Equity | - | - | - | - | - | - | - | - | - | 6.0 | 5.5 |
|  | Foreign exchange | - | - | - | - | - | - | - | - | - | 1.2 | 1.9 |
|  | Commodity | - | - | - | - | - | - | - | - | - | 1.0 | 0.8 |
|  | Credit spread | - | - | - | - | - | - | - | - | - | n/a | n/a |
|  | Undiversified VaR | - | - | - | - | - | - | - | - | - | 12.2 | 15.5 |
|  | Diversification | - | - | - | - | - | - | - | - | - | (5.0) | (4.8) |
|  | Diversified VaR | - | - | - | - | - | - | - | - | - | 7.2 | 10.7 |

Notes: This table summarizes the year end individual VaRs and diversified VaRs (1-day, 99\%) disclosed by the sample Canadian banks in their annual reports for the period from 1996 to 2006. The blank cells mean that the data are not available in the annual reports. BNS only disclosed $10-$ day VaRs in 2001 and 2002. To be consistent with other reported VaRs, we convert the 10-day VaRs to 1-day VaRs for BNS in 2001 and 2002.
Table 2 Canadian Market Indices

| Risk Category | Market Index | Frequency | Source | Notes |
| :--- | :--- | :---: | :--- | :--- |
| Interest Rate | Yield for 1 year zero-coupon <br> Canadian government bonds | Daily | Bank of Canada |  |
| Equity | S\&P/TSX Composite index | Daily | CFMRC | The Canadian Financial Markets Research Centre <br> (CFMRC) Summary Information Database includes daily <br> and monthly Toronto Stock Exchange trading ingormation. <br> The subscription to CFMRC is accessed through CHASS <br> Data Centre via the Simon Fraser University Library. |
| Foreign <br> Exchange | Canada/United States dollar, <br> closing spot rate | Daily | CANSIM | CANSIM (Canadian Socioeconomic Information <br> Management System), , is Statistics Canada's computerized <br> data base and information retrieval service. The <br> subscription to CANSIM is accessed through CHASS Data <br> Centre via the Simon Fraser University Library. |
| Commodity | The multiplication of Dow Jones- <br> AIG Spot Commodity Index and the <br> spot exchange rate of CAD/USD | Daily | AIG Financial Products <br>  <br> Company Inc. | The Dow Jones - AIG Spor Commodity Index provides a <br> general estimate of the trend in commodity prices, and <br> represents a diversified group of commodities. Data are <br> retrieved from http://www.djindexes.com. |
| Credit Spread | Difference between Canadian <br> corporate and government bond <br> yield | Weekly | The Economist | The subscription is accessed through <br> http://www.globalfindata.com. |

Notes: This table summarizes the Canadian market indices we use for the proxies of risk factors and the sources we retrieve data from.
Table 3 Correlation Matrix of Canadian indices

| CA Correlation Matrix | Interest <br> Rate | Equity | FX | Commodity | Credit <br> Spread |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Interest Rate | 1.0000 |  |  |  |  |
| Equity | 0.0160 | 1.0000 |  |  |  |
| FX | 0.0811 | $(0.0859)$ | 1.0000 |  |  |
| Commodity | 0.0447 | 0.1222 | 0.2766 | 1.0000 |  |
| Credit Spread | $(0.0718)$ | $(0.0030)$ | $(0.0007)$ | $(0.0062)$ | 1.0000 |

Notes:This historical correlation matrix is based on 581 weekly (Wednesdays) samples in total from 11/1/1995 to 10/31/2006.
Table 4 Disclosed VaR for US Banks

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Notes：This table summarizes the individual VaRs and diversified VaRs（1－day， $99 \%$ ）disclosed by the sample US banks in their 10－Ks and 10－Qs．for the period from 1998 to 2007．The blank cells mean that the data are not available in the reports．The VaRs of JPMorgan，Citibank，and HSBC are quarter end VaRs．We are only able to get the yearly average VaRs at quarter end for BoA and the yearly average at year end for Wachovia，respectively．The 1－day VaRs for HSBC before the third quarter of 2006 are converted from the 10 －day VaRs．
Table 5 US Market Indices

| Risk Category | Market Index | Frequency | Source | Notes |
| :---: | :---: | :---: | :---: | :---: |
| Interest Rate | Market yield on U.S. Treasury securities at 1-year constant maturity | daily | Federal Reserve |  |
| Equity | S\&P500 Composite index | daily | CRSP | Center for Research in Security Prices (CRSP) provides access to NYSE/AMEX/Nasdaq Daily and Monthly Security Prices. The subscription to CRSP is accessed through CHASS Data Centre via Simon Fraser University Library. |
| Foreign Exchange | The invert of the major currencies index | daily | Federal Reserve | The major currencies index is a weighted average of the foreign exchange values of the U.S. dollar against a subset of currencies in the broad index that circulate widely outside the country of issue. The weights are derived from those in the broad index. |
| Commodity | Dow Jones AIG Spot Commodity Index | daily | AIG Financial Products Corp. and Dow Jones \& Company Inc. | Data are retrieved from http://www.djindexes.com. |
| Credit Spread | Difference between Moody's corporate BAA yield and 1 year Treasury yield | daily | Bank of Canada | The subscription is accessed through http://www.globalfindata.com. |

Notes: This table summarizes the US market indices we use for the proxies of risk factors and the sources we retrieve data from.
Table 6 Correlation Matrix of US indices

| US Correlation <br> Matrix | Interest <br> Rate | Equity | FX | Commodity | Credit <br> Spread |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Interest Rate | 1.0000 |  |  |  |  |
| Equity | 0.2049 | 1.0000 |  |  |  |
| FX | $(0.1925)$ | $(0.0907)$ | 1.0000 |  |  |
| Commodity | 0.0234 | 0.0232 | 0.1966 | 1.0000 |  |
| Credit Spread | $(0.2758)$ | $(0.0929)$ | 0.0682 | 0.0284 | 1.0000 |

Notes:This correlation matrix is based on 2,313 daily samples in total from 1/1/1998 to 3/31/2007.
Table 7 Disclosed Diversification Effect for Canadian Banks

| Bank | 1996 | 1997 | 1998 | 1999 | $\mathbf{2 0 0 0}$ | $\mathbf{2 0 0 1}$ | $\mathbf{2 0 0 2}$ | $\mathbf{2 0 0 3}$ | $\mathbf{2 0 0 4}$ | $\mathbf{2 0 0 5}$ | $\mathbf{2 0 0 6}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BMO | - | - | - | - | - | $19 \%$ | $18 \%$ | $16 \%$ | $23 \%$ | $24 \%$ | $22 \%$ | $20 \%$ |
| BNS | - | - | - | - | - | $37 \%$ | $42 \%$ | $38 \%$ | $46 \%$ | $43 \%$ | $36 \%$ | $40 \%$ |
| CIBC | - | - | - | $45 \%$ | $42 \%$ | $52 \%$ | $39 \%$ | $50 \%$ | $44 \%$ | $49 \%$ | $53 \%$ | $47 \%$ |
| NBC | - | - | - | - | - | $25 \%$ | $25 \%$ | $19 \%$ | $25 \%$ | $50 \%$ | $47 \%$ | $32 \%$ |
| RBC | $63 \%$ | $64 \%$ | $33 \%$ | $38 \%$ | $28 \%$ | $38 \%$ | $35 \%$ | $43 \%$ | $38 \%$ | $35 \%$ | $35 \%$ | $41 \%$ |
| TD | - | - | - | - | - | - | - | - | - | $41 \%$ | $31 \%$ | $36 \%$ |
| Average | $63 \%$ | $64 \%$ | $33 \%$ | $42 \%$ | $35 \%$ | $34 \%$ | $32 \%$ | $33 \%$ | $35 \%$ | $40 \%$ | $37 \%$ | - |

Notes: This table presents the disclosed diversification effect of the sample Canadian banks for the period from 1996 to 2006. The diversification effect is the difference between the undiversified VaR and diversified VaR scaled by the undiversified VaR, and the undiversified VaR is the sum of individual VaRs in each risk category. The blank cells mean that the data are not available in the annual reports.
Disclosed Diversification Effect for US Banks

| Bank | 98Q4 | $99 \mathrm{Q1}$ | 99Q2 | 99Q3 | 99Q4 | 00Q1 | 00Q2 | 00Q3 | 00Q4 | 01Q1 | 01Q2 | 01Q3 | 0104 | 02Q1 | 02Q2 | 02Q3 | 02Q4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| JPMorgan | - | - | - | - | - |  | - | - | 35\% | 18\% | 19\% | 28\% | 23\% | 30\% | 38\% | 27\% | 29\% |  |
| Citibank | 34\% | 34\% | 39\% | 43\% | 41\% | 41\% | 40\% | 42\% | 38\% | 33\% | 36\% | 31\% | 36\% | 38\% | 30\% | 37\% | 29\% |  |
| HSBC | - |  | - | - | - |  | - | - |  | - |  | - | 32\% | 37\% | 23\% | 30\% | 27\% |  |
| BoA | - | - | - | - | 39\% | 56\% | 54\% | 51\% | 50\% | 48\% | 49\% | 49\% | 50\% | 50\% | 51\% | 51\% | 52\% |  |
| Wachovia | - | - | - | - | 11\% |  | - |  | 28\% | - | - | - | 31\% | - | - |  | 30\% |  |
| Average | 34\% | 34\% | 39\% | 43\% | 30\% | 48\% | 47\% | 47\% | 38\% | 33\% | 35\% | 36\% | 35\% | 39\% | 35\% | 36\% | 34\% |  |
|  | 03Q1 | 03Q2 | 03Q3 | 03Q4 | 04Q1 | 04Q2 | 04Q3 | 04Q4 | 05Q1 | 05Q2 | 05Q3 | 05Q4 | 06Q1 | 06Q2 | 06Q3 | 06Q4 | 07Q1 | Average |
| JPMorgan | 40\% | 34\% | 41\% | 41\% | 39\% | 35\% | 36\% | 40\% | 41\% | 43\% | 44\% | 40\% | 46\% | 53\% | 47\% | 41\% | 45\% | 37\% |
| Citibank | 34\% | 35\% | 35\% | 35\% | 38\% | 35\% | 30\% | 33\% | 35\% | 36\% | 38\% | 41\% | 42\% | 51\% | 48\% | 44\% | 47\% | 38\% |
| HSBC | 35\% | 24\% | 37\% | 23\% | 28\% | 12\% | 5\% | 12\% | 20\% | 39\% | 33\% | 41\% | 25\% | 26\% | 48\% | 57\% | 27\% | 29\% |
| BoA | 59\% | 64\% | 65\% | 64\% | 59\% | 56\% | 55\% | 54\% | 51\% | 51\% | 52\% | 53\% | 54\% | 52\% | 52\% | 52\% | 52\% | 53\% |
| Wachovia | - | - | - | 29\% | - | - | - | 24\% | - | - | - | 28\% | - | - | - | 52\% | - | 29\% |
| Average | 42\% | 39\% | 45\% | 38\% | 41\% | 35\% | 31\% | 32\% | 37\% | 42\% | 42\% | 41\% | 42\% | 46\% | 49\% | 49\% | 43\% | - |

Notes: This table presents the diversification effect of the sample US banks for the period from 1998 to 2007. The blank cells mean that the data in not available in the financial reports.
Regression Results for Diversification Effect

| Regression | obs | $\alpha$ | $\beta_{I R}$ | $\beta_{\text {equity }}$ | $\beta_{F X}$ | $\beta_{c}$ | $\beta_{\text {credit }}$ | $\beta_{o}$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Canadian Banks (A) t -statistics | 39 | $\begin{array}{r} 0.5768 \\ (14.7760) \\ \hline \end{array}$ | $\begin{array}{r} -0.0010 \\ (-0.0706) \\ \hline \end{array}$ | $\begin{array}{r} -0.0071 \\ (-0.5898) \\ \hline \end{array}$ | $\begin{array}{r} -0.0283 \\ (-3.2807) \\ \hline \end{array}$ | $\begin{array}{r} 0.0191 \\ (0.9656) \\ \hline \end{array}$ | $\begin{array}{r} -0.0188 \\ (-2.8503) \\ \hline \end{array}$ | $\begin{array}{r} 0.0488 \\ (1.0836) \\ \hline \end{array}$ | 0.4135 |
| Canadian Banks (B) t-statistics | 39 | $\begin{array}{r} 0.5712 \\ (14.7236) \end{array}$ | $\begin{array}{r} 0.0137 \\ (3.1662) \end{array}$ | $\begin{array}{r} 0.0046 \\ (0.8625) \end{array}$ | $\begin{array}{r} -0.0215 \\ (-3.6011) \\ \hline \end{array}$ | $\begin{array}{r} 0.0039 \\ (0.2795) \end{array}$ | $\begin{array}{r} -0.0170 \\ (-2.6585) \\ \hline \end{array}$ |  | 0.3919 |
| US Banks (A) <br> t-statistics | 120 | $\begin{array}{r} 0.8125 \\ (26.3125) \\ \hline \end{array}$ | $\begin{array}{r} 0.0004 \\ (0.0768) \end{array}$ | $\begin{array}{r} -0.0027 \\ (-1.7788) \end{array}$ | $\begin{array}{r} 0.0009 \\ (0.2954) \end{array}$ | $\begin{array}{r} -0.0012 \\ (-0.6506) \end{array}$ | $\begin{array}{r} -0.0100 \\ (-5.3176) \\ \hline \end{array}$ | $\begin{array}{r} -0.0040 \\ (-0.3409) \\ \hline \end{array}$ | 0.3588 |
| US Banks (B) t-statistics | 120 | $\begin{array}{r} 0.8099 \\ (27.1810) \\ \hline \end{array}$ | $\begin{array}{r} -0.0012 \\ (-1.6803) \end{array}$ | $\begin{array}{r} -0.0028 \\ (-1.8906) \end{array}$ | $\begin{array}{r} 0.0012 \\ (0.4308) \\ \hline \end{array}$ | $\begin{array}{r} -0.0009 \\ (-0.5607) \\ \hline \end{array}$ | $\begin{array}{r} -0.0097 \\ (-5.8326) \\ \hline \end{array}$ |  | 0.3582 |

Notes: This table presents the results for the two regressions as preliminary analysis to find out which factors have impact on the level of diversification of VaR. Regression A regresses the ratio of DVaR over the undiversified VaR against the five individual VaRs and the standard deviation of these individual VaRs. Regression B only has the five individual VaRs as the independent variables. Both regressions are run for the Canadian banks and US banks.
Table 10 Comparison between Disclosed VaR and Estimated VaR for Canadian Banks

| Bank | Diversified VaR | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BMO | Disclosed | - | - | - | - | - | 24.00 | 23.20 | 28.10 | 15.00 | 17.80 | 36.00 |  |
|  | Estimated | - | - | - | - | - | 18.21 | 19.58 | 22.73 | 8.58 | 10.44 | 22.67 |  |
|  | Excess Ratio | - | - | - | - | - | 24\% | 16\% | 19\% | 43\% | 41\% | 37\% | 30\% |
| BNS | Disclosed | - | - | - | - | - | 6.64 | 7.91 | 10.60 | 5.30 | 6.60 | 8.60 |  |
|  | Estimated | - | - | - | - | - | 6.68 | 9.08 | 11.11 | 5.19 | 6.62 | 9.53 |  |
|  | Excess Ratio | - | - | - | - | - | -1\% | -15\% | -5\% | 2\% | 0\% | -11\% | -5\% |
| CIBC | Disclosed | - | - | - | 24.68 | 20.49 | 11.16 | 15.35 | 6.20 | 8.80 | 6.30 | 9.20 |  |
|  | Estimated | - | - | - | 18.99 | 16.59 | 10.13 | 11.58 | 6.20 | 6.45 | 5.50 | 9.66 |  |
|  | Excess Ratio | - | - | - | 23\% | 19\% | 9\% | 25\% | 0\% | 27\% | 13\% | -5\% | 14\% |
| NBC | Disclosed | - | - | - | - | - | 3.00 | 3.00 | 2.90 | 2.70 | 5.10 | 5.80 |  |
|  | Estimated | - | - | - | - | - | 2.44 | 3.35 | 2.41 | 2.22 | 6.10 | 5.85 |  |
|  | Excess Ratio | . | - | - | - | - | 19\% | -12\% | 17\% | 18\% | -20\% | -1\% | 3\% |
| RBC | Disclosed | 10.00 | 15.00 | 18.00 | 13.00 | 18.00 | 8.00 | 13.00 | 8.00 | 10.00 | 15.00 | 17.00 |  |
|  | Estimated | 20.18 | 23.15 | 17.42 | 12.12 | 14.21 | 8.97 | 13.93 | 9.13 | 7.80 | 12.23 | 13.36 |  |
|  | Excess Ratio | -102\% | -54\% | 3\% | 7\% | 21\% | -12\% | -7\% | -14\% | 22\% | 19\% | 21\% | -9\% |
| TD | Disclosed | - | - | - | - | - | - | - | - | - | 7.20 | 10.70 |  |
|  | Estimated | - | - | - | - | - | - | - | - | - | 7.25 | 8.51 |  |
|  | Excess rate | - | - | - | - | - | - | - | - | - | -1\% | 20\% | 10\% |
| Average | Excess Ratio | -102\% | -54\% | 3\% | 15\% | 20\% | 8\% | 1\% | 3\% | 22\% | 9\% | 10\% |  |

Notes: This table presents the comparison between the DVaRs disclosed by the sample Canadian banks and the DVaRs estimated by our correlation matrix. The excess ratio represents the difference between the disclosed DVaR and our estimation as a percentage of the disclosed DVaR.
Table 11 Comparison between Disclosed VaR and Estimated VaR for US Banks

Notes: This table presents the comparison between the DVaRs disclosed by the sample US banks and the DVaRs estimated by our correlation

## FIGURES

Figure 1 Canadian Market Indices







Notes: These are the plots of the weekly (Wednesdays) Canadian market indices for the period from 11/1/1995 to 10/31/2006.

Figure 2 US Market Indices






Notes: These are the plots of the daily US market indices for the period from 1/1/1998 to 3/31/2007.

Figure 3 Disclosed Diversification Effect for Canadian Banks


Notes: These figures display the disclosed diversification effect for each Canadian bank, which is the disclosed diversification effect divided by the sum of individual VaRs in each risk category.

Figure 4 Average Diversification Effect for Canadian Banks


Ranked Average Diversification For Canadian Banks


Notes: The first figure plots the cross-bank average of the diversification effect of the selected Canadian banks for the period from 1999 to 2006, which is the sum of the diversification percentage of all the Canadian banks divided by the number of Canadian banks in each particular year. The bottom figure plots the time average of the diversification effect for each bank, which is the sum of the diversification percentage throughout the available years for each bank divided by the number of years.

Figure 5 Disclosed Diversification Effect for US Banks


Figure 6 Average Diversification Effect for US Banks



Notes: The first figure plots the cross-bank average of the diversification effect of the selected US banks for the period from 1999 Q4 to 2007 Q1. The cross-bank average is calculated as the sum of the diversification percentage of all the US banks divided by the number of US banks in each particular period. The bottom figure plots the time average of the diversification effect for each bank, which is the sum of the diversification percentage throughout the available years for each bank divided by the number of periods.

Figure 7 Excess Ratio of Diversified VaR for Canadian Banks


Notes: These figures display the ratio of VaR overestimation or underestimation for Canadian banks, which is the excess of the disclosed DVaR over our estimated DVaR divided by the disclosed DVaR. Excess ratio is positive when the disclosed DVaR is greater than the estimated one. To be consistent with all the sample banks, we set the $y$-axis from $-25 \%$ to $45 \%$. The excess ratio for RBC is $-102 \%$ and $-54 \%$ in 1996 and 1997, respectively.

Figure 8 Excess Ratio of Diversified VaR for US Banks




Notes: These figures display the ratio of VaR overestimation or underestimation for US banks, which is the excess of the disclosed DVaR over our estimated DVaR divided by the disclosed DVaR. The excess ratio is positive when the disclosed DVaR is greater than the estimated one. To be consistent with all the sample banks, we set the $y$-axis from $-80 \%$ to $80 \%$.

Figure 9 Average of Excess Ratio of Diversified VaR


US Banks


Notes: The figure is the plot of average percentage of overestimation/underestimation of VaR , which is the sum of the overestimation/underestimation percentage of all the Canadian banks or US banks divided by the number of banks in each particular year.


[^0]:    ${ }^{1}$ 1-day VaR equals to 10-day VaR divided by the square root of 10 . See Jorion (2006), p122 for details.

[^1]:    ${ }^{2}$ Source: The Federal Reserve Board, 2007.

