A Non-linear Model on Canadian Output and Unemployment

by

Lingyi Kong
Bachelor of Arts, Renmin University of China, 2002

PROJECT SUBMITTED IN PARTIAL FULFULLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

In the Department
of
Economics

SIMON FRASER UNIVERSITY

August 2004

APPROVAL

Name: **Lingyi Kong**

Degree: Master of Arts (Economics)

Title of Project: A Non-linear Model on Canadian Output and Unemployment

Examining Committee:

Chair: **Professor Gorden Mayers**

Ken Kasa
Senior Supervisor
Associate Professor

David Andolfatto
Supervisor
Associate Professor

Simon Woodcock
Internal Examiner
Assistant Professor

Date Approved: July 23, 2004

ABSTRACT

This project employs a non-linear VAR (threshold VAR) to study the joint dynamics of Canadian output and unemployment rate. A lot of recent studies showed that the exogenous shocks have asymmetric effect on economy, but the traditional linear analysis fails in this circumstance. In order to capture this asymmetry, I use a feedback variable that endogenously augments the real GDP lags of the VAR in recession phases. To implement the threshold VAR, a so-called qausi-maximum likelihood estimator (QMLE) is employed to estimate this threshold. Based on the estimate of the threshold VAR, I will analyze this feedback effect by the Generalized Impulse Response function.

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank everyone who gave their value time, skills and enthusiasm to make this project work. I would like to thank my senior supervisor, Kenneth Kasa, my junior supervisor, David Andolfatto and my internal examiner, Simon Woodcock, for their assistant, and patience. I also need to thank professor Peter Kennedy for his help.

I thank my best friends, Cherry, Crystal, May, Tommy and Yi for their encouragement and sharing their wisdom. Special thanks to my parents for their encouragement and support. Finally, thank you Lei for your unconditional love, patience and encouragement.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

It has been a long time for economists to employ the reduced form VAR and impulse a response function to study the propagation and persistence of shocks in economy as well as the transmission mechanism between output and labor market. The standard reduced form models are linear, which implicitly impose a number of symmetry restrictions on economic fluctuation. For instance, multiplying interested magnitude on basic shock will have corresponding impulse response function. Furthermore, the effects of shocks do not change over business cycle. However, the mounting empirical evidences on asymmetries in the effect of shocks make these restrictions questionable.

To capture these asymmetries on economic fluctuation, growing literatures employ the nonlinear models to analyze the dynamics of real economy. Beaudy and Koop (1993) (hereafter BK) introduced the nonlinear specification in a univariate model of US output where the asymmetry is captured by allowing the feedback variable defined as the current depth of recession to enter the model with multiple lags. The threshold parameter is assumed to be zero in BK. Pesaran and Potter (1994) (hereafter PP) extended BK's specification into three regimes (recession, corridor, overheating) with the threshold parameter endogenously estimated. Koop, Pesaran and Potter (1996) (hereafter KPP) developed the Generalized Impulse Response function to analyze the impulse dynamics under the context of nonlinear time series. Altissimo and Violante (2001) (hereafter AV) use the specification suggested by BK and Generalized impulse response function on the US output and unemployment bivariate system with threshold parameter endogenously estimated.

Since most of the literatures are concerning about the dynamics of the US economy, it will be interesting to see how these non-linear framework works on Canadian economy. This project uses Matlab to implement the bivariate system of output and unemployment rate on Canadian data (1976Q1 to 2004Q1). The organization of this project is as follows. Section 2 presents the nonlinear time series model. Section 3 describes the method used in estimating the threshold parameter and provides the estimate of the Threshold VAR (TVAR). In section 4, the procedure of the GIRF based the estimates in section 3 is described first and then the results of the GIRF is presented.

2. A THRESHOLD VAR ON REAL GDP AND UNEMPLOYMENT RATE

2.1 Model Specification

Following BK, PP, KPP and AV, I constructed a feedback variable measuring the current depth of the recession (CDR_t), which is defined as the gap between current level of log-GDP and historical maximum level augmented by a threshold parameter r. Formally:

$$CDR_t(r,\tau) = y_t - \max\{y_t, y_{t-1} + r, \ldots, y_{t-\tau} + r\} \quad (1)$$

where r is a finite integer. When the growth of log-GDP is lower than the threshold parameter r, CDR_t is activated. As long as the growth of log-GDP below r, which means that when the economy does not fully recovered from recession, CDR_t will remain as a negative value. Let F_t=1($CDR_t < 0$) be the indicator function of whether current economy in recession (F_t=1) or in expansion (F_t=0). And the recession regime is defined as at least one of lags of CDR_t nonzero $(\sum_{i=1}^{q} CDR_{t-i} < 0)$, while expansion regime is defined as $\sum_{i=1}^{q} CDR_{t-i} = 0$.

I construct a bivariate nonlinear model of the change in log-GDP times 100, (ΔY_t), and the unemployment rate (U_t). X_t denotes $(\Delta Y_t, U_t)$, and $\Phi(L)$ and $\Theta(L)$ are matrix polynomial in the lag operator with order p and q respectively. So the model specification is:

$$X_t = a + \Phi(L)X_{t-1} + \Theta(L)CDR_{t-1}(r,\tau) + \eta_t \qquad (2)$$

To allow for the regime dependent heteroskedasticity, I assume $\eta_t = V_t^{1/2}\varepsilon_t$, where $\varepsilon_t \sim \mathrm{II}D(0, I_2)$. Thus, the conditional variance covariance matrix takes the form of

$$V_t = 1(\sum_{i=1}^{q} CDR_{t-i}) \cdot (\Sigma_r - \Sigma_e) + \Sigma_e \qquad (3)$$

where Σ_r (or Σ_e) is the covariance matrix in recession (or expansion) regime. Since CDR_t depends on the value of the threshold parameter r, r enters both the conditional mean and conditional variance.

2.2 An interpretation of the model specification

Hamilton (1989) employed the Markov switching regime model to capture the non-linearity, positing the regime switching as exogenous and determined by Markov chain. BK first introduced the specification with the feedback variable of current depth of recession to study the dynamics of the US output. As PP, this specification can be interpreted as a member of the Threshold Autoregression (TAR) class[1]. TAR model assumes an observed variable relative to a threshold value to determine the regime switching. Tong (1990) suggested a Self-Exciting Threshold Autoregression (SETAR) model, which assumes that the threshold variable is chosen to be some delayed value of the time series itself. The SETAR model is linear within regime, but with different

[1] I provide a detailed description of this TAR style specification in appendix.

coefficients across regimes. For example, a typical univariate SETAR model can be specified as $y_t = \phi_1 x_t 1(y_{t-d} > r) + \phi_2 x_t 1(y_{t-d} < r) + \varepsilon_t$. Where $\phi_i = (\phi_{0,i}, \cdots, \phi_{p,i})$, i=1,2, $x_t = (1, y_{t-1}, \cdots, y_{t-p})'$, and y_{t-d} as the threshold variable. Potter (1995) used second lag of output growth (Δy_{t-2}) as the delay variable with threshold $r = 0$. Though the model performs much better than linear model, it is hard to answer the question "why only Δy_{t-2} matters". Tiao and Tsay (1994) introduced two "subregimes" conditional on Δy_{t-2}<0, one as worsening regime ($\Delta y_{t-1} < \Delta y_{t-2}$) and another as improving regime (Δy_{t-1}>Δy_{t-2}). It is still possible to go further. For instance, one can check whether Δy_{t-3}<0 conditional on recession regime. However, unless imposing restrictions across some regimes, it does not seem to be a very attractive way since this specification may exhaust the degree of freedom quickly.

The bivariate system of equation (2) can be interpreted as a SETAR model with many regimes. If the economy is in recession regime and $CDR_t = y_t - \max\{y_t, y_{t-1} + r, \ldots, y_{t-\tau} + r\}$ is activated, assuming historical maximum is Δy_{t-s}, $1 \le s \le \tau$, then we have:

$CDR_t = y_t - y_{t-s} - r = (y_{t-1} - y_{t-s} - r) + (y_t - y_{t-1}) = CDR_{t-1} + \Delta y_t$, so that

$$CDR_t = \begin{cases} 0 & if \quad F_t = 0 \\ \Delta y_t - r & if \quad F_t = 1 \quad and \quad F_{t-1} = 0 \\ CDR_{t-1} + \Delta y_t & if \quad F_t = 1 \quad and \quad F_{t-1} = 1 \end{cases}$$

Since $F_t = 1(CDR_t < 0)$ is the indicator function of whether CDR_t<0, it is appropriate to write $CDR_t = CDR_t \cdot F_t$. By substituting CDR_t by CDR_{t-1} on the right hand side, we have $CDR_t = (CDR_{t-1} + \Delta y_t) \cdot F_t$. Continuing this substitution until some $F_{t-k} = 0$ and $CDR_t = CDR_{t-k-1} \prod_{i=0}^{k} F_{t-i} + \sum_{i=0}^{k} \Delta y_{t-i} \prod_{l=0}^{i} F_{t-l}$. So, there are τ possible subregimes conditional on recession regime.

As mentioned in section 2.1, we have $X_t = (\Delta Y_t, U_t)$, $\Phi(L)$ and $\Theta(L)$ are in the lag operator with order p and q respectively. Let $Z_t = (X_t, X_{t-1}, \ldots, X_{t-q-\tau+2})'$[2], which is a $1 \times 2(t-q-\tau+2)$ vector. Because the model includes q order lags of CDR_t, equation (2) can be written as a SETAR model with $(1+\tau)^q$ regimes. Define J_t as an indicator random variable with values in the set {0,1, 2, …, $(1+\tau)^q$-1}, and J_t=j indicates state j happening. The equation (2) can be written as:

$$Z_t = \begin{pmatrix} a \\ 0 \end{pmatrix} + r \sum_{i=1}^{q} \Theta_i F_{t-i} + \begin{pmatrix} \beta_1^{\{j\}}, \beta_2^{\{j\}}, \ldots, \beta_{t-q-\tau+1}^{\{j\}} & \beta_{t-q-\tau+2}^{\{j\}} \\ \hline I_{t-q-\tau+1} & 0 \end{pmatrix} Z_{t-1} + \begin{pmatrix} \eta_t \\ 0 \end{pmatrix} \qquad (4)$$

[2] Since CDR_{t-1} enters the model, $q-1+\tau$ lags of output growth may have influence on the dependent variable. Furthermore, this implicitly assumes p<$q-1+\tau$, which is not a problem in this project (τ is assumed to be greater than the sample size).

where, a is a 2×1 vector of constant, and $\beta_i^{\{j\}}$ is the coefficient matrix on Z_{t-i} for state j, $i=1,\ldots,(t-q-\tau+2)$[3]. Equation (4) is the standard SETAR model suggested by Tong (1990).[4]

---

[3] To gain more insight of this SETAR specification, I provide an examples in appendix.
[4] A more detailed discussion can be found in AV.

3. MODEL ESTIMATION

3.1 The estimate of the threshold parameter *r*

The difficulty of estimating the threshold parameter r is that the log likelihood function is not differentiable in r, i.e. the conditional covariance matrix is discrete between regimes. Following PP and AV, I implement grid search for the estimation of r.

Chan (1993) proved that under certain conditions, the estimate of grid search for threshold autoregression is consistent. AV showed that the model in 2.1 can be written as SETAR which satisfies those conditions.[5] Equation (2) can be written as:

$$X_t = C + X_{t-1}B(L) + CDR_{t-1}\theta(L) + V_t\varepsilon_t \qquad (5)$$

$$= W_t(r)\beta + V_t\varepsilon_t$$

Let $e = X_t - W_t(r)\beta$ denotes the error term of the regression. For simplicity, I assume τ is greater than the sample size, which means that I search all past growth of output when calculating certain CDR. Furthermore, I assume that the value before the sample and the first q observations of CDR_t and F_t are zero.

The grid search steps are as follows:

1. Generate a grid with 400 points in the interval [-1.5, 1.5].

---

[5] In finite sample, since the threshold r enters the definition of the current depth of recession (CDR_t), there is some information between grid points. As the sample size goes to infinity, the gaps between the grid points go to arbitrarily small.

2. Estimate the log-likelihood function conditional on each point of r. The log-likelihood function takes the forms of

$$\ell \propto -\frac{T_r}{2}\log(|\Sigma_r|) - \frac{T_e}{2}\log(|\Sigma_e|) - \frac{1}{2}\sum_{i=p}^{T} F_{i-1}(e\Sigma_r^{-1}e') - \frac{1}{2}\sum_{j=p}^{T}(1-F_{t-1})(e\Sigma_e^{-1}e)$$

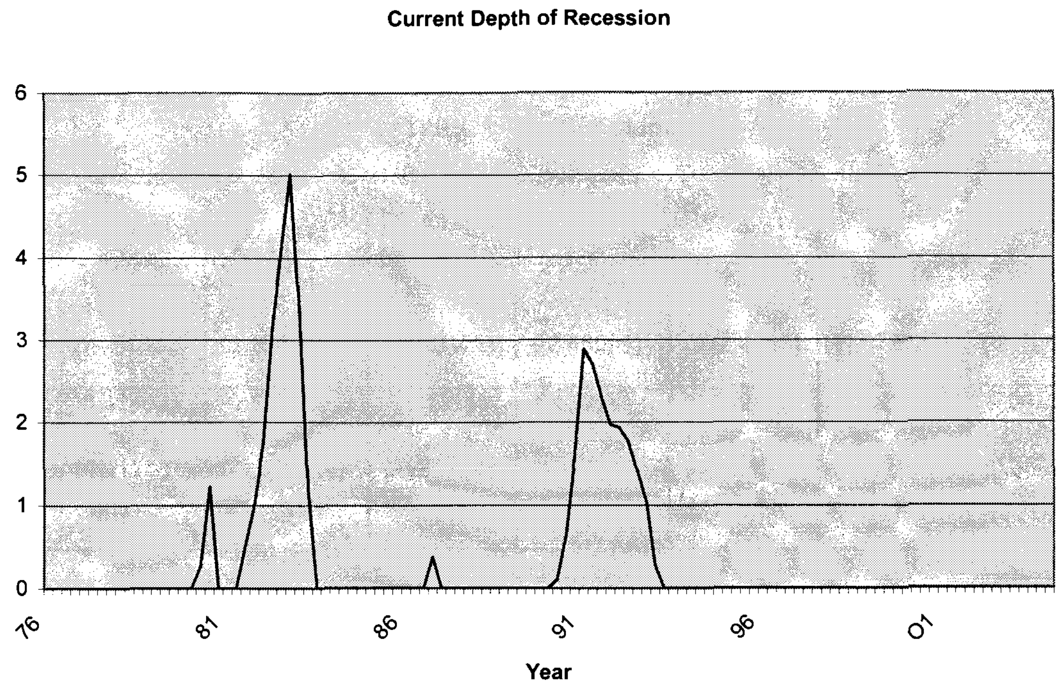
where T_r (or T_e) is the number of observations in recession (or expansion) regime. Since the elements in Σ_r and Σ_e are unknown, it is very painful to try to figure out the analytical Hessian matrix. I implement the Newton-Raphson algorithm to estimate the remaining parameters (other than r) through the numerical derivative with LS estimates as starting values. On each point of grid, I allow 100 iterations over the likelihood function and with convergence criterion of 10^{-4} on each parameter.

3. The point in the grid, which maximizes the log likelihood function, is chosen as the estimate of r.

The estimate of the threshold parameter $\hat{r}$[6] is –0.1917, which is far from AV's estimate of $\hat{r}$ -0.140 for the US case. $\hat{r}$ takes a small negative value since it is reasonable to expect that the small drops of growth of log GDP do not trigger the threshold effect, especially when the drop followed by a strong growth. Figure1 give the plot of |CDR| conditional $\hat{r}$.

[6] The estimates of the covariance matrices are: $\hat{\Sigma}_r = \begin{pmatrix} 0.5381 & -0.1936 \\ -0.1936 & 0.2309 \end{pmatrix}$ and $\hat{\Sigma}_e = \begin{pmatrix} 0.3174 & -0.0166 \\ -0.0166 & 0.0408 \end{pmatrix}$.

Figure 1: CDR conditional on $\hat{r}$



3.2 The Estimate of VAR conditional on *r*

The result of the estimation of Threshold VAR[7] is reported in table 1 while the t-statistics is in the parentheses. Since there exists obviously high muticollinearity between independent variables, the t-statistics is not trustable. I implement F test on the joint zero null on the CDRs. The p-value of the F-test on the second equation (u_t) is 0.0000[8]. For first equation (Δy_t), the F-test with p-value 0.9301 implies that the CDRs are jointly insignificant in the equation of output, which is a very surprising result. Since BK and

[7] I tried several specifications with different number of lags, and the estimate is not sensitive.

[8] I tried to include either of the two CDRs in the equation of u, but none of them gives significant t-statistics. This may happen since the CDR is calculated from y_t and corresponding lags conditional on the estimate of threshold parameter. And AV has the same results on the US case (their t-statistics based asymptotic standard errors conditional on the estimate of r).

KPP's univariate models of Δy_t have significant effects on CDRs. The omission of the unemployment rate in univariate model may be the reason for this different finding.

Table 1: TVAR Estimate

	Cons.	Δy_{t-1}	Δy_{t-2}	u_{t-1}	u_{t-2}	CDR_{t-1}	CDR_{t-2}
	-0.085063	0.349012	-0.126009	-0.63604	0.709289	0.018761	-0.022734
Δy	(-0.21800)	(3.158299)	(-1.20421)	(-2.65296)	(3.054683)	(0.078737)	(-0.10839)
	0.475028	-0.142389	-0.013149	1.235610	-0.279882	-0.087070	0.042074
u	(3.212503)	(-3.371224)	(-0.31962)	(12.35352)	(-2.91191)	(-0.67821)	(0.350911)

Since the VAR is in the reduced form with over-identification, it is hard to give the exact interpretation of coefficients. Loosely speaking, in the equation of the unemployment rate, the combination of coefficients on the lags of CDR implies that, when the recession begins (at t-1, say), the feedback variable CDR_{t-1} (negative) makes the unemployment rate (at t) increased. As long as the economy worsens ($|CDR_t|>|CDR_{t-1}|$), the unemployment rate will sharply decrease. This is because that the positive effect of CDR_t (-0.087) is greater than the negative effect of CDR_{t-1} (0.042). On the contrary, if the economy is in weak recovering[9], the combined effect of CDRs will still be positive but with a downward pressure on the unemployment rate. Until the effect of CDR_{t-1} dominates the effect of CDR_t, the decreasing of unemployment rate will be accelerated

[9] $|CDR_t|<|CDR_{t-1}|$, but $|-0.08\,CDR_t|>|0.04\,CDR_{t-1}|$,

by the combination of CDRs. CDRs will also influent the dynamics of the output growth through the unemployment.

4 THE GENERALIZED IMPULSE RESPONSE FUNCTION

4.1 An introduction to Generalized Impulse Response Function (GIRF)

Based on Pesaran and Shin (1999), the traditional impulse response function (IR) is aiming on answering the question "What is the response to a unit impulse today when all future shocks are sent to zero". Particularly, the IR is defined as

$$IR_X(n,\delta,\omega_{t-1}) = E[X_{t+n} \mid \varepsilon_t = \delta, \varepsilon_{t+1} = 0, \ldots \varepsilon_{t+n} = 0, \omega_{t-1}] - E[X_{t+n} \mid \varepsilon_t = 0, \ldots, \varepsilon_{t+n} = 0, \omega_{t-1}]$$

where n is the maximum time horizon taken into account, and δ is a shock at time t. Let's denote Ω_{t-1} as the information set to forecast X_t, and ω_{t-1} is a particular realization of Ω_{t-1}. As explained by KPP, this IR specification is history, shock, and composition dependence. The IR with zero future shocks may lead to substantial bias. Since setting all future shocks as zeros implies that whether threshold effect activated depends only on present shock and will be independent with future shocks, even though the present economy is at the boundary between regimes.

To deal with the problems arises with using IR on nonlinear time series, KPP and Potter (1999) constructed the GIRF as a random variable on the same probability space as the time series. That is, fixing the ith shock from the vector of all shocks ξ_t, and then integrating out the effects of all other future shocks. The GIRF takes the form of:

$$GIRF_X(n,\delta,\omega_{t-1}) = E[X_{t+n} \mid \varepsilon_t = \delta, \Omega_{t-1}] - E[X_{t+n} \mid \Omega_{t-1}] \quad (6)$$

$E[X_{t+n} \mid \varepsilon_t = \delta, \Omega_{t-1}]$ stands for the expectation conditional on the information set Ω_{t-1} and the ith shock at time t fixed at the value of δ. The time profile of shocks can be constructed conditional on a specific history and/or type of present shocks. The GIRF is a random variable which is a difference between two random variables. The first term on the right hand side of equation (6) is the expectation of X_{t+n} conditional on history and the chosen shock δ. The second term is the expectation of X_{t+n} conditional on history only. Both terms are themselves random variables. The GIRF assumes the "regular shocks" (i.e., the mean of past shocks) keeping hitting the system over future horizons and the conditional expectation operator averages out the future shocks. And all the contemporaneous and future shocks are integrated out except the shock δ in the first term. Thus, comparing with the traditional impulse response function, the GIRF here is more likely to answer the question "What is the response to a shock δ today when all future shocks are averaged out?" Since generally there is no analytical expression for the conditional expectation on nonlinear models, Monte Carlo study is employed to numerically integrate the expectation.

Since the main focus of this project is on the asymmetries of the GIRF across regimes, following AV, I define the GIRF as:

$$GIRF_X(n, \delta, H_{t-1}) = E[E[X_{t+n} \mid \varepsilon_t = \delta, H_{t-1}] - E[X_{t+n} \mid H_{t-1}] \mid H_{t-1} \in R] \qquad (7)$$

where $R \equiv \{recession, expansion\}$, and H_{t-1} is the history until time (t-1) which is given by $\{X_{t-i}, CDR_{t-i}\}$ with i=1,2. The estimated GIRF conditional on each regime is computed by:

$$\hat{GIRF}_X(n,\delta,H_{t-1}) = \frac{1}{T_R}\sum_{i=1}^{T_R}\{\frac{1}{M}\sum_{j=1}^{M}[X_{t+n}^j(\delta,H_i) - X_{t+n}^j(\varepsilon_t^i,H_i)]\} \quad (8)$$

T_R is number of observations for regime R, H_i is the ith history[10] for regime R, and M is number of bootstrapping replication.

4.2 The Computation of the GIRF

The procedures of computing the GIRF are as follows:

1. Separating the data into two regimes based on the value of the recession indicator F_{t-1}. There are 27 observations fell into the recession regime and 83 observations leading to expansion regime. Following the graphic method suggested by Gallant, Rossi and Tauchen (1993), I choose the typical shock by inspecting the scatter plot of the residues[11] from the VAR estimation. I pick up δ =(1, -1) for $(\Delta y_t, u_t)$ as the representative positive shock (denoted as P-shock). By changing the sign, I get the representative negative shock (N-shock) and by doubling the magnitude, I obtain the PP-shock and NN-shock by double the representative shock to analyze the size of asymmetry of GIRF. The maximum time horizon is set to 60 quarters.

2. Since equation (2) assumes that the innovations are independent with the history of time series, all shocks within regime take the equal weight $1/T_R$. Given regime, for each

[10] $\{X_{t-i}, CDR_{t-i}\}$ with i=1,2.

[11] Please refer to Appendix B for the scatter plot of the residues.

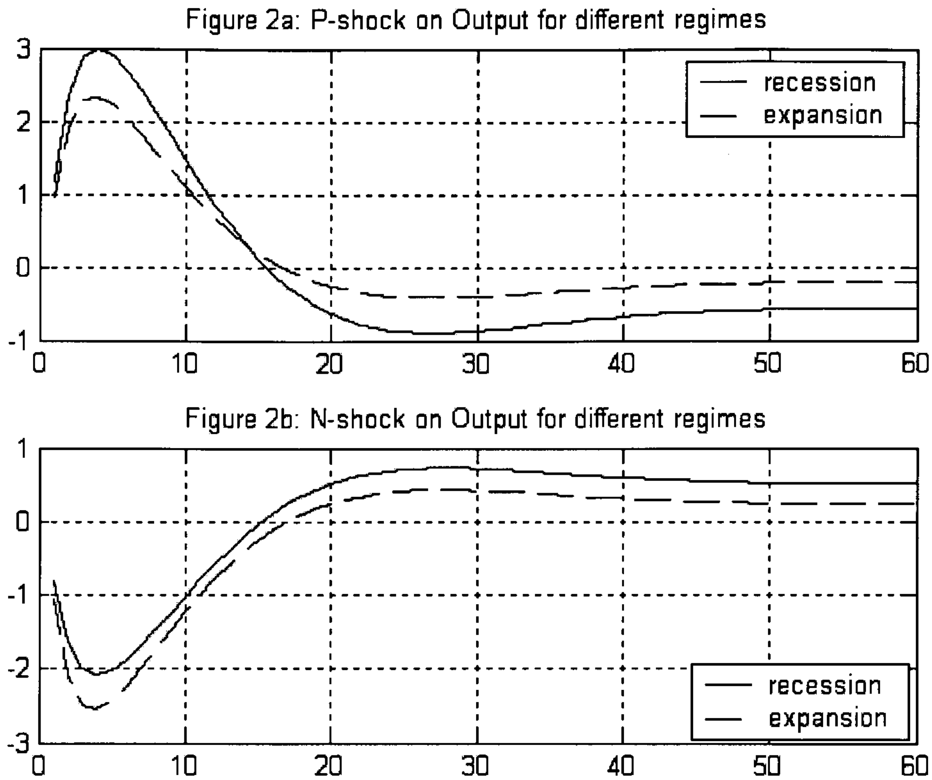
history, I bootstrap 1000×59 realizations from the within regime residuals for the shocked economy and 1000×60 realizations for the baseline economy. The future of 60 quarters of $(\Delta y_t, u_t)$ and CDR is calculated recursively for both economies. Then calculating $\bar{X}_{t+n}(\delta, H_i) = \frac{1}{M}\sum_{j=1}^{M} X_{t+n}^{j}(\delta, H_i)$ for the shocked economy and $\bar{X}_{t+n}(H_i) = \frac{1}{M}\sum_{j=1}^{M} X_{t+n}^{j}(H_i)$ for the baseline economy. For each history, as $M \to \infty$, the Law of Large Numbers ensures the convergence of the average across bootstrapping to the conditional expectation.

3. For each regime, by taking the average across all histories, the $\hat{GIRF}$ for given regime is obtained.

4.3 Results of GIRF

4.3.1 The Impulse Response on Output

Figure 2. Aggregate Shocks on Output[12]



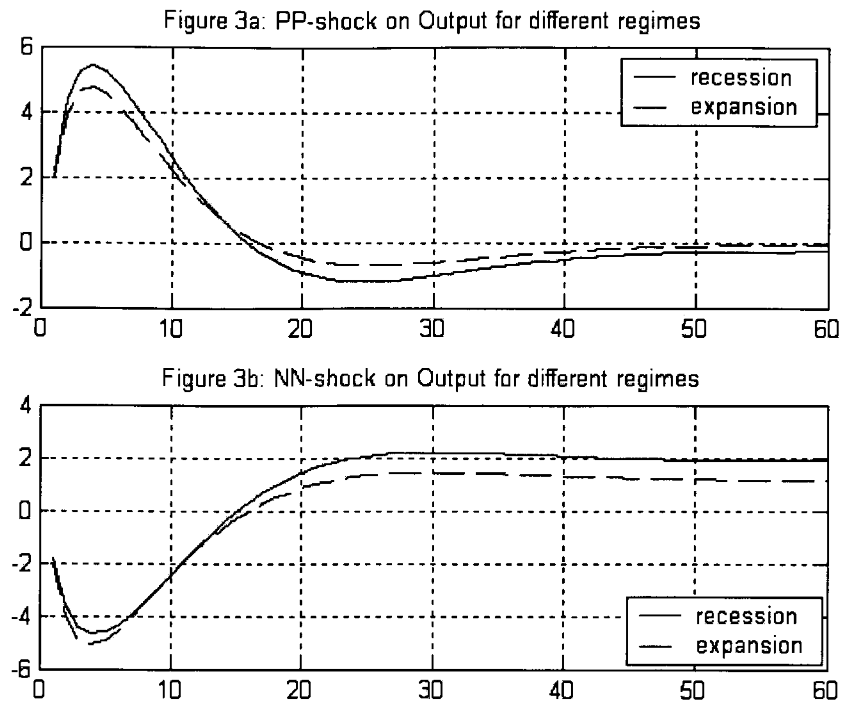
It is obvious that the long-run persistence is quite different between regimes. For example, when there is a negative shock hit the economy at time t (figure 2b), the GIRF[13] will perform differently across regimes. If the economy is in recession regime, output will decrease at time t to response to the negative shock so that CDR_t is activated. But since it is in the recessionary regime ($CDR_{t-1} < 0$), this CDR_{t-1} will have

[12] AV has very similar shape of GIRF on the US data with level value of unemployment rate.
[13] I tried some values on the neighbour of the estimate of *r*, and the GIRF is not sensitive to these different threshold values. That's because the estimates of coefficients of CDRs are not sensitive to small changes of *r*.

a positive effect on economy at time (t+1)[14]. Furthermore the CDR_t will also have a positive effect on economy only two quarters later. But in expansion regime ($\sum_{i=1}^{q} CDR_{t-i} = 0$), the negative effect will be weaker because of lacking the offsetting component. Similar mechanism makes the output in recession regime to have a greater response to a positive shock. Since the economy experienced a positive shock at time t, CDR_t will be zero. If the economy is in recession regime at time t, CDR_{t-1} <0, which will have positive effect on the output at time t+1. Thus, the output of recessionary regime has greater responses to positive shock than that of expansionary regime.

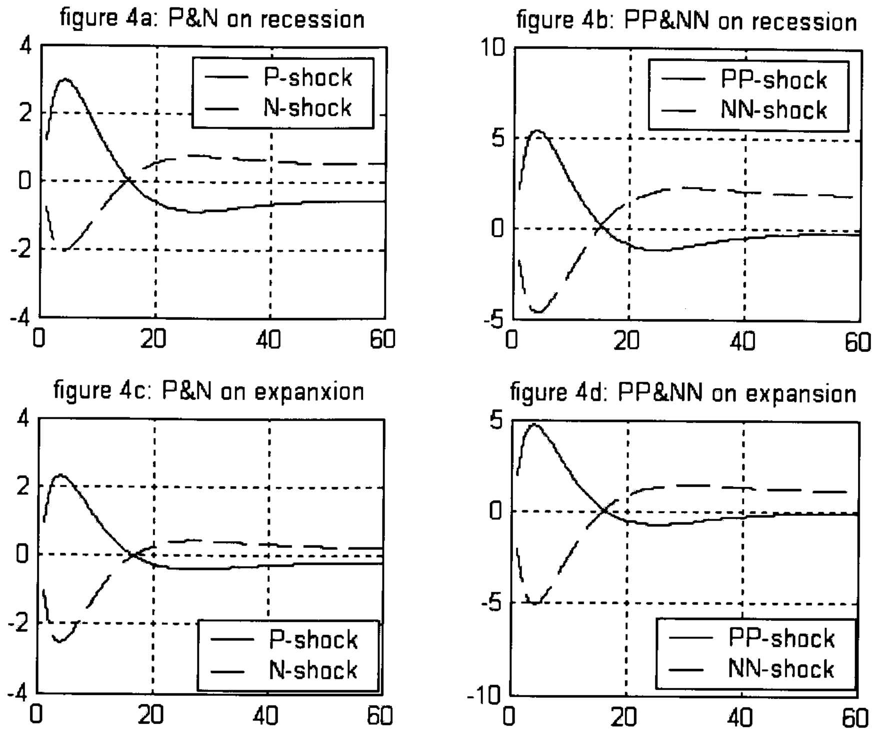
[14] Even if CDR_{t-1} =0 at time t, as long as $CDR_{t-q} < 0$ which will have a positive effect on output at time t, the effect of the negative shock at t+1 will still be weaken.

Figure 3. Double magnitude Shocks on Output



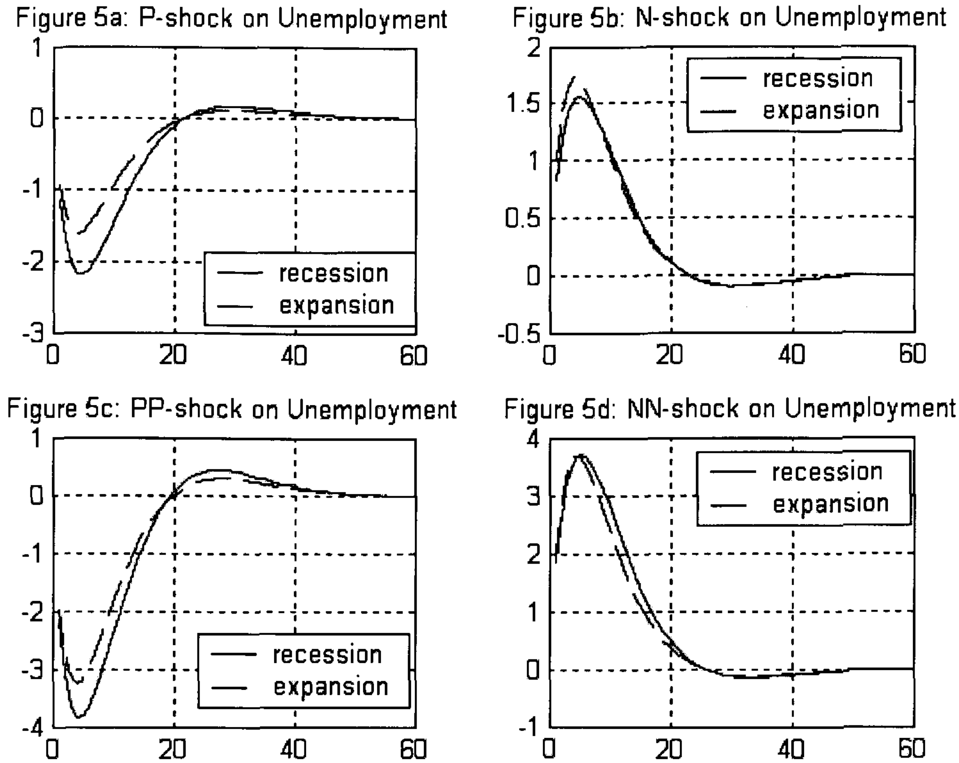
Comparing figure 2 and figure 3, the dynamics looks pretty similar. But we can notice that except the magnitude asymmetry, the PP-shock is more persistence than the P-shock, while NN-shock and N-shock share a similar length of memory. Figure 4 shows the comparison of shocks with different signs on a certain regime. By looking at figure 4, it is obvious that the recessionary regime exposes more asymmetry than the expansionary regime. And it is due to the feedback variables (CDR), which is the main source of the asymmetry.

Figure 4. Shocks with different signs on output conditional on regime



4.3.2 The Impulse Response on Unemployment

Figure 5. Shocks on Unemployment



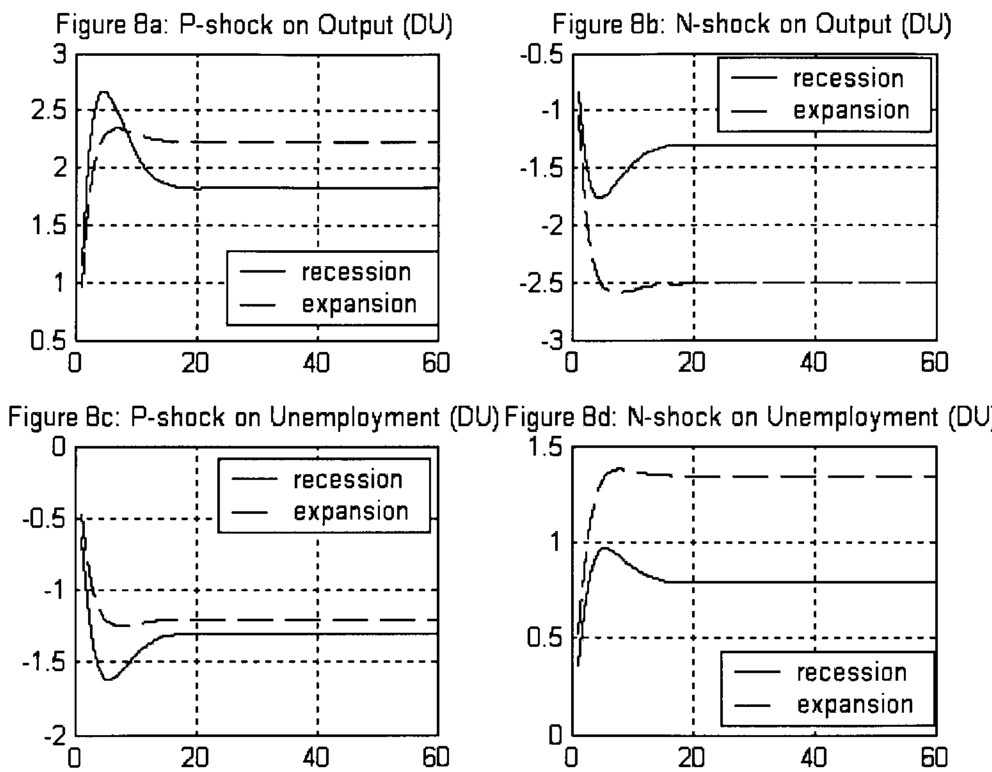
The Graphs looks pretty similar as those in figure 4, but the expansion seems less persistent for the case of unemployment. Imagine the economy experienced a negative shock (figure 5b). If initially in recession regime, since the coefficient of the second lag of CDR is positive, the upward pressure of the unemployment decreases. As we explained in section 3.2, in recession, the unemployment rate must be relatively high. The unemployment rate may still continue to increase if the economy worsens, but not much. Even though the GIRF captures the asymmetries of the bivariate dynamics pretty well, it seems the adjustment may slow (about 5 years).

4.3.3 Sensitivity of Modelling Unemployment I(1)

AV argues that the standard unit root test may fail under the context of the nonlinear model specification. They implement a unit root test robust to this nonlinear model suggested by Caner and Hansen (1997). It seems that there may be some evidence of unit root existing in unemployment rate in their context (including unit root test and some extra evidence), and AV choose the differenced unemployment as the benchmark model. But I still cannot persuade myself that the unemployment rate is a nonstationary time series. Following exactly the same procedure described above, I find that though the threshold ($\hat{r}$ =-0.2068) is not sensitive to I(1) specification, the GIRF is very sensitive the differencing of unemployment rate.

The plot of the GIRF under this context is very similar as AV reported in their paper. The regime dependence of the long run persistence is much greater than that of the I(0) specification model and the adjustment is also much faster (about 12 quarters). The feed back variable CDR plays very similar roles as that in the I(0) unemployment rate specification. The sensitivity is reasonable, since the GIRF is based on the estimation of the Threshold VAR. Both the problems of spurious regression and over-differencing can have serious consequence. But the discussion of unit root test is not the focus of this project. This dilemma can be open to future research.

Figure 6. Shocks on Output and Unemployment I(1)



5 CONCLUSION

By building on the measurement of current depth of recession into the VAR model, the effect of shock is allowed to vary across business cycle. The model specification in section 2 can be written a model with fixed lags in expansion regime, and with constrained time varying lags in the recession regime. Through the estimates of TVAR and the analysis of the GIRF, it is obvious that the specification with feedback variables (i.e. CDR) captures the important asymmetric behavior of the bivariate dynamics of Canadian output and unemployment rate.

APPENDICES

Appendix A: An Example of SETAR

To see how equation (2) can be expressed as a SETAR model easier, let's assume $\tau=1$, q=2, p=1. $CDR_t = y_t - \max\{y_t, y_{t-1}+r\} = \min\{0, \Delta y_{t-1} - r\}$. The model specified in section 2 can be written as:

$$\begin{pmatrix}\Delta y_t \\ u_t\end{pmatrix} = a + \begin{pmatrix}\varphi_1 & \delta_1 \\ \phi_2 & \delta_2\end{pmatrix}\begin{pmatrix}\Delta y_{t-1} \\ u_t\end{pmatrix} + \begin{pmatrix}\theta_1^y \\ \theta_1^u\end{pmatrix} CDR_{t-1} + \begin{pmatrix}\theta_1^y \\ \theta_2^u\end{pmatrix} CDR_{t-2} + \eta_t$$

There will be four regimes,

$$\begin{pmatrix}\Delta y_t \\ u_t\end{pmatrix} = \begin{cases} a + \begin{pmatrix}\varphi_1 & \delta_1 \\ \phi_2 & \delta_2\end{pmatrix}\begin{pmatrix}\Delta y_{t-1} \\ u_{t-1}\end{pmatrix} + \eta_t & \text{if } F_{t-1}=0 \,\&\, F_{t-2}=0 \\ a + \begin{pmatrix}\varphi_1+\theta_1^y & \delta_1 \\ \phi_2+\theta_1^u & \delta_2\end{pmatrix}\begin{pmatrix}\Delta y_{t-1} \\ u_{t-1}\end{pmatrix} - r\begin{pmatrix}\theta_1^y \\ \theta_1^u\end{pmatrix} + \eta_t & \text{if } F_{t-1}=1 \,\&\, F_{t-2}=0 \\ a + \begin{pmatrix}\varphi_1 & \delta_1 \\ \phi_2 & \delta_2\end{pmatrix}\begin{pmatrix}\Delta y_{t-1} \\ u_{t-1}\end{pmatrix} + \begin{pmatrix}\theta_2^y \\ \theta_2^u\end{pmatrix}\cdot(\Delta y_{t-2} - r) + \eta_t & \text{if } F_{t-1}=0 \,\&\, F_{t-2}=1 \\ a + \begin{pmatrix}\varphi_1+\theta_1^y & \delta_1 \\ \phi_2+\theta_1^u & \delta_2\end{pmatrix}\begin{pmatrix}\Delta y_{t-1} \\ u_{t-1}\end{pmatrix} - r\begin{pmatrix}\theta_1^y+\theta_2^y \\ \theta_1^u+\theta_2^u\end{pmatrix} + \Delta y_{t-2}\begin{pmatrix}\theta_2^y \\ \theta_2^u\end{pmatrix} + \eta_t & \text{if } F_{t-1}=1 \,\&\, F_{t-2}=1 \end{cases}$$

which can be written as a SETAR, which takes the form of

$$\begin{pmatrix}X'_t \\ X'_{t-1}\end{pmatrix} = \begin{pmatrix}a \\ 0\end{pmatrix} + r\begin{pmatrix}\sum_{i=1}^{q}\Theta_i F_{t-i} \\ 0\end{pmatrix} + \begin{pmatrix}\beta_1^{\{j\}} & \beta_2^{\{j\}} \\ I & 0\end{pmatrix}\begin{pmatrix}X'_{t-1} \\ X'_{t-2}\end{pmatrix} + \begin{pmatrix}\eta_t \\ 0\end{pmatrix}$$

where $X_t = (\Delta Y_t, U_t)$, $\Theta_i = \begin{pmatrix} \theta_i^y \\ \theta_i^u \end{pmatrix}$. $\beta_i^{\{j\}}$ is a 2×2 matrices associated to the four regimes, i=1,2. Let j=0 stands for $F_{t-1} = 0 \,\&\, F_{t-2} = 0$, j=1 for $F_{t-1} = 1 \,\&\, F_{t-2} = 0$, j=2 for $F_{t-1} = 0 \,\&\, F_{t-2} = 1$ and j=3 for $F_{t-1} = 1 \,\&\, F_{t-2} = 1$. Then we have:

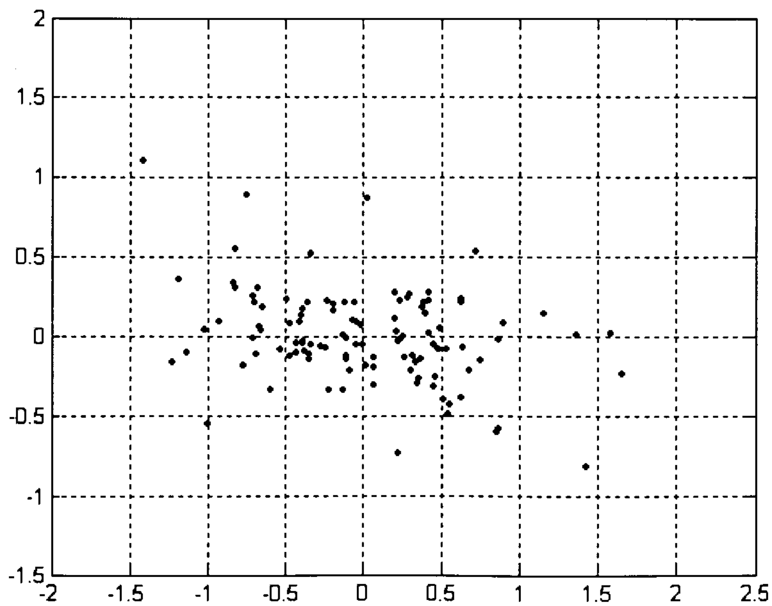
$$\beta_1^{\{0\}} = \begin{pmatrix} \varphi_1 & \delta_1 \\ \phi_2 & \delta_2 \end{pmatrix}, \quad \beta_2^{\{0\}} = 0; \quad \beta_1^{\{1\}} = \begin{pmatrix} \varphi_1 + \theta_1^y & \delta_1 \\ \phi_2 + \theta_1^u & \delta_2 \end{pmatrix}, \quad \beta_2^{\{1\}} = 0; \quad \beta_1^{\{2\}} = \begin{pmatrix} \varphi_1 & \delta_1 \\ \phi_2 & \delta_2 \end{pmatrix},$$

$$\beta_2^{\{2\}} = \begin{pmatrix} \theta_2^y & 0 \\ \theta_2^u & 0 \end{pmatrix}; \text{ and } \beta_1^{\{3\}} = \begin{pmatrix} \varphi_1 + \theta_1^y & \delta_1 \\ \phi_2 + \theta_1^u & \delta_2 \end{pmatrix}, \beta_2^{\{3\}} = \begin{pmatrix} \theta_2^y & 0 \\ \theta_2^u & 0 \end{pmatrix}.$$

exchange rate volatility.

Appendix B: Scatter plot of the residues.

Figure 7. Scatter plots graph of the residues



REFERENCE LIST

Altissimo, F. and G. Violante, 2001, The non-linear dynamics of output and unemployment in the US, Journal of Applied Econometrics

Beaudry, P. and G. Koop, 1993, Do recessions permanently change output?, Journal of Monetary Economics 31, 149-163

Fan, J. and Q. Yao, 2003, Nonlinear Time series: Nonparametric and Parametric Methods, Springer, New York

Gallant, AR., Rossi, PE. and G. Tauchen, 1993, Nonlinear Dynamic Structures, Econometrica 61,871-908

Hanselman, D and B. Littlefield, 2001, Matering Matlab 6, Prentice

Hansen,BE., 2000, Sample spliting and threshold estimation, Econometrica 68, 573-603

Koop G, Pesaran, MH and S. Potter,1996, Impulse response analysis in nonlinear multivariate models, Journal of Econometrics 74,119-147

Koop, G. and S. Potter, 1999, Asymmetries in US unemployment, Journal of Business and Economic Statistics 17, 298-312

Pesaran, HH and Y. Shin, 1998, Generalized impulse response analysis in linear multivariate models, Economics Letters 58,17-29

Pesaran, MH. and S. Potter, 1997, A floor and ceiling model of US output, Journal of economic Dynamics & control 21, 661-695

Potter, S., 1995, A nonlinear approach to US GDP, Jornal of Applied Econometrics 10,109-125

Potter, S., 1999, Nonlinear time series modelling: an introduction, Journal of Economic Surveys 13, 505-524

Tiao, G. and R. Tsay, 1994, Some advance in nonlinear and adaptive modelling in time series analysis, Journal of Forecasting 13,109-131

Tong, H., 1990, Non-linear Time series: A dynamical system approach, Oxford: Clarendon Press