ESTIMATING IMPLIED DEFAULT PROBABILITIES AND RISK MEASURES FOR CREDIT BONDS

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Abstract

This paper implements the reduced form approach to model the credit risk term structure of the 16 SIAS fixed income portfolio's debt issuers. The major advantage of reduced form model risk measures is that they explicitly take the default risk and recovery rate into consideration. The default-risk-adjusted duration and convexity will be smaller than the traditional measures because of the possibility of receiving the recovery value. By analyzing the credit risk term structure, we can observe the time-varying pattern in market's expectation on the issuer's ability to fulfill its debt obligation. Discrepancy between bonds' rating and their implied default probability is also observed.

Keywords: Reduced Form Model; Implied Default Intensity; Credit Risk Term Structure; Default-Risk-Adjusted Duration; Default-Risk-Adjusted Convexity

Dedication

This paper is dedicated to our families and FRM colleagues, who always show their supports and encouragements during this past year. Without their support, we would never have made it this far and be the person we want to be.

Wei

I would like to dedicate this paper to my mom Alice, my husband Powell, and my dearest son Jovi. They are the motivation for me to complete this degree and the final project. I would also like to dedicate this paper to my FRM colleagues, especially Ted, Tanya, Brandon, and Jerry. Their advices and support made this paper possible!

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1. Introduction

Effective risk management relies on a comprehensive integration of market, credit, and liquidity risk. Therefore, the parameterization of a particular general model and the estimation of its risk factors are critical to the success of the implementation of risk management procedures. For credit-risky bonds, risk parameters can be categorized into three aspects: the term structure of Treasury interest rates, credit risk, and issue-specific features, such as liquidity, degree of subordination, and call structure. Credit risk has been proven to be the main contributor for spreads of risky bonds. Longstaff, Mithal, and Neis (2006) found that the default risk in the highest-rated firm accounts for more than 50% of the total corporate spread.

For modeling the credit risk, the available tools have two main types – structural and reduced form. In the structural approach, equity prices and balance sheet data are used to estimate the possibility of bankruptcy and the possible residual value of the debt issuers. The reduced form model, on the other hand, does not look into the volatility of the issuers' asset, but rather treats default as an exogenous event, and the dynamics of the default intensity can be calibrated from market prices. The purpose of this paper is to construct the credit risk term structures for issuers of bonds in the fixed income portfolio of Simon Fraser University Student Investment Advisory Service (SIAS) endowment fund by applying the reduced form model and compute default-risk-adjusted risk measures. One major assumption we apply to estimate bondholders' residual claim given default is the Recovery of Face Value assumption. We analyze and compare the credit risk term structures of issuers in the same sector (Government, Provincial, Municipal, and Corporate). We also conduct a comparison between the implied default probability we derived and the objective default probability measured by the credit rating agency. Default-risk-adjusted risk measures such as the reduced form duration and convexity are calculated for each of the 19 bond in the portfolio. Since the default-risk adjustment is more prominent for risk measures of fixed income securities with higher credit risk, we also include two lower graded bonds to demonstrate the larger effect. We further conduct relative pricing test using credit term structures we constructed on bonds not included in the portfolio.

We start by presenting findings of credit risk term structure analysis. Section 3 outlines the basic concept of reduced form model. Section 4 gives an overview on SIAS fund fixed income investment philosophy and our data source. Section 5 explains our methodology. Section 6 discusses our results. The final section concludes and suggests direction for further investigation.

2. Credit Risk Investigation

2.1 Patterns in Default Rates

When the maturity of corporate bond increases, the bond's credit spread may widen or narrow based on the bond's credit risk. By looking at rating category (Fons, 1994), lower-rated (smaller or younger) issuers normally have wider credit spreads that narrow with the time to maturity (TTM). In contrast, higher-rated (mature or stable) issuers have narrower credit spreads that widen with the maturity time. The pattern reflects a typical company's life cycle, and assumes a highly leveraged firm may run into refinancing difficulty when their short-term debt matures. This higher default risk is normally reflected in a higher spreads at shorter maturities. Bernt et al (2004), Hull, Predescu, and White (2004) also pointed out that the risk premiums have varied through time.

2.2 Credit Risk vs. Issue-Specific Factors

Liquidity plays a role in the determination of spreads (Covitz and Downing, 2007); however, the effect of liquidity is more prominent in the short-term period, even though credit risk still plays a more significant role than liquidity. As shown by Longstaff, Mithal, and Neis (2006), there is a strong relationship between non-credit component of spread and bond-specific characteristics. In addition, the measures of Treasury richness such as the on/off- the-run spread and the overall liquidity of fixed income markets are all relevant factors of change in the non-credit component.

2.3 Risk-Neutral Default Probabilities vs. Objective Probabilities

The default probability calculated from historical data is an objective measure, which is usually much smaller than the risk-neutral default probability implied from bond prices. Altman (1989) initiated the investigation on the huge difference between the objective default probabilities and risk-neutral default probabilities. Possible explanations to this puzzle include the market's recognition of contagion risk (Collin-Dufresne et al, 2003), underestimation of liquidity risk premium, agency costs, supply/demand effects and/or other institutional factors (Hull, Predescu, and White, 2004). Courtois and Quittard-Pinon (2007) further examined the relations between the actual and risk-neutral world with a structural approach that excluded liquidity issue.

2.4 Fusion of Reduced Form and Structural Model

Although the two approaches require two different intakes: structural models use equity information, and the reduced form models use debt prices, ways of combining the two's advantages and trimming the weaknesses have been investigated. Portfolio theories often incorporate equity in the reduced form model (Duffie & Singleton, 1999). In his 2001 paper, Jarrow argued that the partition of debt and equity market is unnecessary since both markets present useful information that can lead to parameterization of defaulting process (Jarrow 2001). He presented a methodology that allows default probabilities and recovery rates to be correlated and to be dependent on the macroeconomic status. Darrell & Lando (2001), Giesecke (2001), and Cetin, Jarrow, Protter & Yildirim (2002) introduced the incomplete information credit models that contained new structural/reduced form hybrids.

3. The Reduced Form Model

The reduced form model was initiated by Philippe & Delbaen (1995), Jarrow and Jurnbull(1995), and Duffie & Singleton (1999). Different from the other school of thought, where the endogenized default probability is explicitly modeled using fundamental information such as the asset and liability on the company's balance sheet (Merton 1974), the reduced from models treat bankruptcy event as an exogenous event and aim to explain the occurrence of default in an actuarial way. This stream leads to a pricing methodology that shares similar concepts to the term structure models.

3.1 Default Intensity

In a reduced form model, we model the default count, *N*, as a stochastic process which only takes on integer value. *N(0,t)* represents the number of credit events that have happened from time *0* to time *t*. If we assume that the economic life of a company ends with the first default event, we are only interested in the time when the first default arrives, which is denoted by τ:

$$\tau = \min\{t \geq 0 \mid N(0,t) \geq 1\} \quad (1)$$

Poisson process is the simplest way to express the counting process of credit event. The probability of having *N* defaults in the time interval *0* to *t* therefore is:

$$P(N) = (\frac{\lambda t^N}{N!})e^{-\lambda t} \quad (2)$$

Here we use the default intensity, or hazard rate, λ, as a determinant of the dynamic of the process:

$$P[\tau \leq t + dt \,|\tau \geq t\,] = \lambda\,(t)dt \quad (3)$$

Equation (3) shows that, given that the company has survived to time *t*, the probability of defaulting in the time interval *dt* is proportional to $\lambda(t)$ and the length of *dt*. The survival probability, *Q(0,t)*, which is the probability that τ does not occur between the time interval *0* to *t* (*N* equal *0*), is

$$P\,(\tau > t) = Q(0,t) = e^{-\lambda t} \quad (4)$$

And the probability that default happens in the time interval is:

$$P\,(\tau \leq t) = 1 - Q\,(0,t) = 1 - e^{-\lambda t} \quad (5)$$

Literally, λ is the conditional default probability per unit time, and can be constant, time – deterministic, or time – stochastic. Specifying the intensity function λ therefore determines the risk-neutral default probability measure, which is different from the objective default frequencies, and parameterizes the default factor in the no-arbitrage valuation.

3.2 Implied Default Probabilities from Security Prices

Under the risk-neutral assumption, the present value of a credit-risky bond should be the risk neutral expectation of its cash flows. The simplest scenario: for a risky zero bond with no recovery, its value at time *0*, *B(0,t)*, has a relationship with the risk-free zero bond, *b(0,t)*, such that:

$$\mathrm{B}(0,\mathrm{t}) = \mathrm{b}(0,\mathrm{t}) * \mathrm{P}(\tau > t) = \mathrm{b}(0,\mathrm{t}) * \mathrm{Q}(0,\mathrm{t}) \qquad (6)$$

Since the bond prices can be observed from the market, we can derive the implied default probabilities from the readily available prices. However, corporate zeroes are rare. The variability in recovery values plus the issue-specific features of the securities further complicate the application of the model. Therefore, some methodologies have been developed to resolve the problems.

3.2.1 Piece-wise Constant Default Intensity

One assumption that we consider is the piece-wise constant default intensity. The function λ takes on the form of:

$$\lambda(\mathrm{t}) = \lambda_0 + \lambda_1 1_{\{t \geq t1\}} + \lambda_2 1_{\{t \geq t2\}} \cdots \qquad (7)$$

This means that λ is constant between each time interval.

Consider the simplest scenario again: a zero recovery, zero coupon bond. By rearranging equation (6) we can determine the spot λ :

$$\lambda(0,\mathrm{t}) = \frac{1}{\mathrm{t}} \ln\left[\frac{\mathrm{b}(0,\mathrm{t})}{\mathrm{B}(0,\mathrm{t})}\right] \qquad (8)$$

To avoid arbitrage opportunity, the bond price of a two-period zero coupon bond must equal to:

$$B(0,t_2) = B(0,t_1)B(t_1, t_2) \quad (9)$$

By replacing the bond prices with equation (6), the forward λ can be calculated:

$$\lambda(t_1,t_2) = \frac{1}{t_2 - t_1}\ln\left[\left(\frac{b(0,t_2)}{b(0,t_1)}\right)\left(\frac{B(0,t_1)}{B(0,t_2)}\right)\right] \quad (10)$$

Adding coupon to our simple scenario, the value of a zero-recovery risky bond, *V(0,T)*, equal to:

$$V(0,T) = \sum_{i=1}^{N} C_i \cdot b(0,t_i) \cdot Q(0,t_i) + FV \cdot b(0,T) \cdot Q(0,T) \quad (11)$$

Where *N* is the number of coupon payments.

3.2.2 Recovery Value

One advantage of structural models over the reduced form models is their accessibility to the recovery value – it is the by-product of the asset – liability simulation. The reduced form approach requires an explicit method of parameterizing the recovery value, *R*. Several conventional methods are described below.

Equivalent Recovery: Introduced by Jarrow & Turnbull (1995), this assumption replaces the defaulting security by *R* of non-defaultable securities. The value of a zero bond with *R>0* thus becomes:

$$B(0,T) = R \cdot b(0,T) + (1-R)b(0,T) \cdot Q(0,T) \quad (12)$$

Fractional Recovery: This assumption was made by Duffie & Singleton (1999) and further developed to multiple default by Schonbucher(1998). The idea is to allow the bond to continue to trade after losing a fraction *q* of its face value at each credit event. Therefore, we deem the bond to be default free, where the value of a zero risky bond is the sum of the expected cash flow discounted using an adjusted interest rate:

$$B(0,T) = b(0,T)e^{-(q+\lambda)t} \quad (13)$$

Recovery of Face Value: Under this assumption, bondholders receive a fraction *R* of the bond's principal value but not the outstanding coupon payments. This assumption is in line with conventional bankruptcy practices, where bondholders entitle to receive fraction of the company's residual value weighted by the contractual promised face value of their debt.

For a risky coupon bond, under the piece-wise constant default intensity and recovery of face value assumption, its value, *V(0,T)*, can be expressed as:

$$V(0,T) = \sum_{i=1}^{N} C_i \cdot b(0,t_i) \cdot Q(0,t_i) + FV \cdot b(0,T) \cdot Q(0,T) + \sum_{i=1}^{N} R \cdot b(0,t_i) \cdot [Q(0,t_{i-1}) - Q(0,t_{i-1})] \quad (14)$$

3.3 Default-Risk-Adjusted Risk Measures

One of the first steps to monitor the risk of a fixed income portfolio is to accurately measure the sensitivity parameters. Berd, Mashal & Wang (2004) adopted the reduced from approach for their duration and convexity calculation. By explicitly incorporating the default risk and the possibility of receiving recovery values in the traditional duration and convexity calculation, we can obtain the interest rate sensitivity and credit risk sensitivity measure of a particular credit risky bond.

Reduced-form Macaulay duration is:

$$D = \frac{1}{P}\left[\sum_{i=1}^{N} t_i \cdot C_i \cdot b(0,t_i) \cdot Q(0,t_i) + T \cdot FV \cdot b(0,T) \cdot Q(0,T) + \sum_{i=1}^{N} t_i \cdot R \cdot b(0,t_i) \cdot [Q(0,t_{i-1}) - Q(0,t_{i-1})]\right] \quad (15)$$

Where *P* is the market price of the bond.

In this equation, the cash flow is both weighted by the risk-free discount factor and the probability of realization of the cash flow. The reduced-form Macaulay duration is always less than the traditional Macaulay duration since we take into account the possibility of receiving the recovery value if the company defaults. If the default risk is high, the difference between the reduced-form duration and traditional duration will be large[1]. Thus, the interest rate sensitivity will be overestimated with the traditional duration measures.

We can modify the traditional convexity with the same approach to arrive at a reduced-form convexity:

$$\Gamma = \frac{1}{P}\left[\sum_{i=1}^{N} t_i^2 \cdot C_i \cdot b(0,t_i) \cdot Q(0,t_i) + T^2 \cdot FV \cdot b(0,T) \cdot Q(0,T) + \sum_{i=1}^{N} t_i^2 \cdot R \cdot b(0,t_i) \cdot [Q(0,t_{i-1}) - Q(0,t_{i-1})]\right] \quad (16)$$

The reduced-form convexity is expected to exhibit the same deviation from its traditional measure as the reduced-form duration.

[1] Berd, Mashal, and Wang have further demonstrated the closer relationship between a company's financial stands and the reduced-form duration with a particular Calpine bond. Beside interest rate

4. Fixed Income Portfolio and Data

4.1 SFU SIAS Fixed Income Portfolio

Our paper applies the reduced form approach to the fixed income portfolio of Simon Fraser University SIAS fund, which consists of 19 Canadian bonds with 16 different issuers. The fund adopts a value investment philosophy, where the major objective is the preservation of the fund value. Based on the fund's Investment Policy Standard, 50% to 100% the fixed income portfolio has to consist of bonds with A rating or above. Purchasing any bond which has a rating below BBB is restricted. Therefore, bonds in SIAS fund portfolio generally have a relatively low default risk

4.2 Data

To construct credit term structures for each of the 19 bond in SIAS fixed income portfolio, we use the bullet bond portfolio of each bond's issuer as our starting point. Bond information, including price, coupon, payment frequency, rating, and maturity time, is retrieved from Bloomberg on July 12, 2010. For bond issuers with insufficient number of bonds outstanding to extrapolate a legitimate credit term structure, we select bullet bonds with equivalent rating from issuers' parent companies or peer companies (same sector with similar capitalization size) to create peer bond portfolio that serves as the basis of our credit curve construction.

For the two U.S. corporate bond issuers, Ford Motor Company and Ally Financial Inc, their bond information is retrieved from Bloomberg on August 12, 2010.

We use LIBOR Swap rates as our risk-free discount rates. Canadian dollar LIBOR Swap rates on July 12, 2010 and U.S. dollar LIBOR Swap rates on August 12, 2010 are collected from Bloomberg for the following maturity: 0.5 year, 1 to 10 years, 12, 15, 20, 25, and 30 years.

5. Methodology

There are three assumptions we employed:

Fairness of Market Bond Price: We assume that market is efficient and the observable bond prices on the market fairly represent the intrinsic value of the securities.

Correlation between Risk free Rate and default probability: We assume independence between interest rate and the default intensity.

Recovery Rate: We use the recovery of face value assumption for our credit risk modeling. For the recovery rate for Sovereign bonds, we take the issuer-weighted average recovery rate for the period from 1983 to 2008 of 50% (Moody's 2009), The recovery rate for corporate bonds is set to be 41.44% for constructing credit risk term structure, which is the average world-wide corporate bond recovery rate from 1920 to 2008 (Moody's 2009). However, for calculating individual bonds' default-risk-adjusted duration and convexity, we assign recovery rate to each bond according to its seniority – here we use the average world-wide corporate bond recovery rate based on seniority from 1982 to 2008 (Moody's 2009).

5.1 Extracting Implied Piece-wise Constant Default Intensity

Our bootstrapping procedure assumes a semiannual piece-wise constant default intensity. A semiannual time interval is chosen to better accommodate the traditional semiannual coupon payments of bonds. The process starts with arranging the bonds in a particular issuer's portfolio from the shortest TTM to the longest. The value of a risky coupon bond with a TTM = T_1, where $0 < T_1 \leq 0.5$, can be expressed as:

$$V(0, T_1) = (C + FV) \cdot b(0, T_1) \cdot Q(0, T_1) + R \cdot b(0, T_1) \cdot [Q(0,0) - Q(0, T_1)] \quad (17)$$

Where $Q(0, 0) = 1$ because the survival probability at $t = 0$ must equal to 1.

Since $Q(0,T_1)$ is the only unknown in the equation, we can rearrange equation (17) to calculate the $Q(0,T_1)$:

$$Q(0, T_1) = \frac{V(0,T_1) - R \cdot b(0,T_1)}{b(0,T_1)[(C+FV) - R]} \quad (18)$$

And, from equation (4), we can compute the piece-wise constant default intensity:

$$\lambda(0,0.5) = \frac{-\ln Q(0,T_1)}{T_1} \quad (19)$$

Once we obtain the first 0.5-year's implied default intensity from the bond with the shortest TTM, we use that as an input to calculate the 1-year implied default intensity from the second bond. The implied default intensities of longer period are extracted in the same fashion.

During the process, if the difference between the TTM of a particular bond and the consecutive bond is longer than 0.5 year, we will assume that the default intensity in the period between the two maturity dates is constant. Also, the survival probability must satisfy the constraint:

$$Q(t) \leq 1$$

5.2 Fitted Implied Survival, Default, and Default Intensity Term Structures

After the semiannual piece-wise constant default intensities and the survival probabilities are extracted from each issuer's bond portfolio, we extrapolate the credit risk term structures using cubic smoothing spline[2]. We also derive the fitted implied forward default intensity term structure from the credit information recovered. Now we can analyze bonds in SIAS fixed income portfolio using the issuer-specific fitted implied credit term structures and calibrate their calculated price to the market price with a constant, issue-specific OAS-to-Fit rate (OASF).

The value of individual bond, *V(0,T)*, is:

$$V(0,T) = \sum_{i=1}^{N} C_i \cdot b(0,t_i) \cdot Q(0,t_i) \cdot e^{-OASF \cdot t_i} + FV \cdot b(0,T) \cdot Q(0,T) \cdot e^{-OASF \cdot T} + \sum_{i=1}^{N} R \cdot b(0,t_i) \cdot [Q(0,t_{i-1}) - Q(0,t_i)] \cdot e^{-OASF \cdot t_i} \quad (20)$$

Where $Q(0,0) = 1$.

The survival probability $Q(0,t_i)$ can be obtained from the fitted implied credit term structure. Therefore, we solve for the constant OASF rate and calibrate the model to the bond's market price.

5.3 Reduced-Form Duration and Convexity

We adopted Berd, Mashal & Wang's (2004) approach to calculate the reduced-form duration and convexity.

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[2] The cubic smoothing spline *f* for a given data *x* and *y* – in our case, TTM and default risk measures – approximates the data value *y* at each smaller, intermediate *x* value. This smoothing spline *f* minimizes the value:

$$P \sum_{j=1}^{n} w(j)\, |y(j) - f(x(j))|^2 + (1-p) \int \lambda(t) |D^2 f(t)|^2 dt$$

where *j* is the smaller intermediate *x* value, *p* is the smoothing parameter, λ is the weight function, and $D^2 f$ is the second derivative of the function *f*.

Reduced-form Macaulay duration is:

$$D = \frac{1}{P}\left[\sum_{i=1}^{N} t_i \cdot C_i \cdot b(0,t_i) \cdot Q(0,t_i) \cdot e^{-OASF \cdot t_i} + T \cdot FV \cdot b(0,T) \cdot Q(0,T) \cdot e^{-OASF \cdot T} + \sum_{i=1}^{N} t_i \cdot R \cdot b(0,t_i) \cdot [Q(0,t_{i-1}) - Q(0,t_{i-1})] \cdot e^{-OASF \cdot t_i}\right] \tag{21}$$

The reduced-form convexity:

$$\Gamma = \frac{1}{P}\left[\sum_{i=1}^{N} t_i^2 \cdot C_i \cdot b(0,t_i) \cdot Q(0,t_i) \cdot e^{-OASF \cdot t_i} + T^2 \cdot FV \cdot b(0,T) \cdot Q(0,T) \cdot e^{-OASF \cdot T} + \sum_{i=1}^{N} t_i^2 \cdot R \cdot b(0,t_i) \cdot [Q(0,t_{i-1}) - Q(0,t_{i-1})] \cdot e^{-OASF \cdot t_i}\right] \tag{22}$$

6. Results

6.1 Implied Credit Risk Term Structure

Looking at the implied credit risk term structures for the 16 debt issuers of bonds in the SIAS portfolio (**Figure 1 – 47**), overall, the market appears to anticipate an increase in default intensity with TTM between 0 to 20 years. After that, the expectation of default intensity gradually decreases with TTM. Each issuer's cumulative default probabilities for the next 10 years are summarized in **Table A**. The following sections give a more detailed analysis on the credit term structure based on the issuers' sector.

6.1.1 Government Bonds

The three Canadian Government issuers in our portfolio exhibit a relatively low default risk as predicted. Constructing the credit term structure for Government of Canada bonds mainly serves as a check on our model, since the yield is close to, or sometimes even lower, than the Canadian LIBOR swap rates. That means a flat fitted implied survival probability term structure of 1. However, some irregularities appear for bonds with time-to-maturity less than 2 years (**Figure 1 & 2**). A closer look at the credit risk term structure reveals a small discount of 0.03% for a bond due immediately, and this discount decreases steadily and disappears when the TTM reaches 2 years. We suspect the existence of this deviation is due to other issues, rather than credit risk. For Canadian Mortgage & Housing Corporation (CANMOR) and Export Development Canada (EDC), their default probabilities are low, with an implied cumulative default probability of 2.73% for TTM of 7.5 years and 3.48% for TTM of 6.5 years, respectively (**Figure 3 & 6**). However, CANMOR's fitted implied hazard rate and forward hazard rate show a decreasing trend after TTM of 5 years, indicating the market's stronger confidence in its ability to meet the debt obligation in the longer term (**Figure 4 & 5**).

6.1.2 Provincial and Municipal Bond

We analyze three Canadian Provinces – British Columbia (BC), Ontario, and Quebec, with S&P rating of AAA, AA-, and A+, respectively. Both BC and Ontario bonds show an inclining implied hazard rate for TTM up to 15 years and a declining rate after (**Figure 10 & Figure 13**). For Ontario bonds, the hazard rate even slightly raises after TTM reaches 30 years. This might imply a different view on Ontario bonds' credit risk with respect to their TTM.

Comparing the three provinces' default probability, some interesting facts emerge after taking their rating into consideration. Although Ontario's rating is between BC's and Quebec's, its annual cumulative default probabilities for the next 8 years is lower than those of the other two provinces. Only if we extend TTM after 9 years, we would see default probabilities for Ontario higher than BC's. Moreover, BC actually has the highest default probability for the next 6 years, despite its highest credit rating. This may indicates that the market is more vigilant about BC's short-term financial stands, and this concern over the short run has not been accounted for in the rating system. The rating is relatively accurate for Quebec in the longer time period, as its default probability becomes the highest among the three after TTM of 7 years.

For our two municipal bond issuers, Municipal Finance Authority of British Columbia (BCMFA) and London Ontario (London), their hazard rates show an upward trend up to TTM of the longest bonds in their debt portfolios (**Figure 22 & 19**).

6.1.3 Corporate Bond

SIAS portfolio contains 8 corporate bonds, with 3 of them issued by Canadian Banks – Bank of Montreal (BMO), Toronto-Dominion Bank (TD), and Canadian Imperial Bank of Commerce (CIBC). In the 1-to-15-year time period, the three banks' implied hazard rates increase steadily (**Figure 24-26, 30-32, 42-44**). However, when TTM reaches 15 years, there is a sharp elevation in hazard rate for CIBC and BMO, and the rate curve quickly flattens out after TTM of 20 years. Therefore, the market seems to be suspicious about both banks' creditworthiness for fulfilling their long-term debt. In terms of credit rating, BMO's implied default probability is always lower than CIBC's for the time period analyzed, although BMO has a lower rating than CIBC.

The other 5 corporate bonds scatter over different industrial sectors. Among them, General Electric Capital Canadian Funding has the highest S&P rating of AA+. However, its cumulative implied default probability for the next 10 years is not the lowest – only higher than the two lowest grading corporate bonds (S&P A- for Industrial Alliance Capital Trust, S&P BBB- for Shaw Communications Inc). There is also a discrepancy between the default probabilities of the two A-rating bonds: 407 International Inc. (ETHWAY) and Greater Toronto Air Authority (GTAA). According to our result, ETHWAY has cumulative default probabilities that are more than double, sometimes triple, than GTAA's numbers. In fact, GTAA has the lowest cumulative default probability for the next 10-year period out of all 8 corporate bonds.

6.2 Comparison of Implied Default & Objective Default Probability

As the aforementioned, a large difference between the implied default probability and objective default probability exists because of the difference between the fundamental assumptions of the two. Now we would like to do a comparison between the implied default probability we derived and the objective default probability measured by credit rating agency.

Table B summarizes the comparison of our corporate bonds' cumulative implied default probability and the average real-world default probability of every Moody's rating category for 1970-2008 (Moody's 2009). **Figure 48 - 51** are the graphic presentations of the comparison. It is clear that the implied default probability is much higher than the objective default probability. However, the magnitude of the difference appears to decrease with the rating.

6.3 Survival-based Duration and Convexity

Table C presents the OASF, duration, and convexity of the 19 bonds in SIAS portfolio. Most bullet bonds have low OASF, where three of the four callable bonds have relatively higher OASFs due to their call feature. Although a more detailed investigation should be done in order to prove the assumption, the credit risk appears to be able to explain the majority portion of the yield spread because of the small OASF values.

We expect the reduced form duration and convexity to be smaller than their traditional numbers because of the possibility of receiving the recovery value if the issuer defaults. The deviation would not be significant since the bonds in SIAS portfolio generally have a relatively low default risk. Our model confirms the low deviation. Moreover, the magnitude of the difference increases as the bond's TTM becomes longer, since the default probability grows with time. However, the BMO callable bond has a significantly higher reduced form duration than their traditional duration (34.75% higher). This means the sum of time-weighted cash flow from recovery is smaller than the time-weighted difference between the spreads and the default probability of the bonds. Except for the BMO callable bond, the reduced form convexities are smaller than the traditional numbers.

For our two U.S. corporate bonds, Ford Motor Company and Ally Financial Inc., their duration and convexity values do show a more significant difference between the reduced-form and the traditional than bonds with similar TTM in our SIAS fixed income portfolio. Their higher default probability leads to a higher chance of receiving the recovery value when default occurs; therefore, this larger deviation is expected.

6.4 Relative Pricing Using Fitted Credit Term Structure

In this section, we test our credit term structures by pricing bonds that are not included in SIAS portfolio. For provincial bond, we choose two Province of Alberta bonds (ALTA), with S&P rating of AAA. For banking sector, two bonds from Bank of Nova Scotia (BNS) with S&P rating of AA- and three bonds from Royal Bank of Canada (RBC) with S&P rating of AA- are selected.

6.4.1 Provincial Bond

The ALTA bond with a shorter TTM (Maturity Date: June 1st, 2012) has a market price that is closer to the calculated prices using BC's and Quebec's credit term structure, where BC has the same rating and Quebec has a lower rating (**Table D**). The calculated price using ONT's credit term structure is higher than the market price. The result indicates that the implied default probability of ALTA is higher than Ontario's, and similar to that of BC or Quebec, for the bond duration. For the ALTA bond with a longer TTM (Maturity Date: December 1st, 2019), the calculated price using BC's and ONT's term structure is very close to its market price. Using the credit term structure of QBC undervalued the bond.

6.4.2 Banking Sector Bond

For the BNS bond with shorter TTM (Maturity Date: June 4th, 2012), the calculated price using CIBC's credit term structure is close to its market price (**Table E**). The value of the bond is overpriced with the credit term structure of TD or BMO. Therefore, the short-run default probability of BNS is similar to CIBC's and higher than TD's or BMO's, although BNS has a rating (AA-) same as TD's and higher than CIBC's (A+) and BMO's (A). This result is not surprising since we have observed the discrepancy between the rating and the implied default probability. For BNS bond with longer TTM (Maturity Date: June 8th, 2017), using CIBC's or BMO's credit term structure would overvalue the bond. The default probability of BNS in the period corresponding to the bond's TTM is actually higher than the two bonds with lower rating.

Pricing the three RBC bonds with the three banks' credit term structures posts similar result (**Table F**). The RBC bond with the shortest TTM (Maturity Date: July 6th, 2011) has a market price that is similar to value calculated with CIBC's credit term structure. The mid-term bond (Maturity Date: January 25th, 2017) is relatively overpriced by less than 1% using BMO or CIBC credit term structure. The long-term default risk for RBC is higher than that of BMO or CIBC since the market price of its long-term bond (Maturity Date: June 8th, 2023) is lower than the price computed using BMO's or CIBC's credit term structure, despite RBC's higher rating.

7. Conclusion and Discussion

In this paper, we employ the reduced form approach to model the credit term structure of 16 debt issuers and compute the credit-risk-adjusted risk measures. Different from the traditional duration and convexity value that only measure interest rate risk, the reduced from duration and convexity explicitly take the default risk and recovery value into consideration, therefore give a more comprehensive and detailed view on the risk of the portfolio. Moreover, incorporating the default risk parameters into the estimation of duration and convexity usually results in risk measures lower than the traditional forms because of the possibility of receiving the recovery value, and that means the traditional numbers may overestimate the interest rate sensitivity of the security. The overstatement is higher for bonds with higher default risk and/or longer TTM.

By analyzing the credit term structure of 16 bond issuers, we find that, although the default probability increases with TTM, the default intensity exhibits patterns that might correspond to the market's expectation in the issuer's ability to fulfill its debt obligation in different time period. Even for high-graded bond, such as the provincial bonds we examined, the long-term default intensity shows a decreasing trend. Further investigation can be done to the issuers' financial stands or economic outlook in order to analyze the pattern. This approach thus leads to the incorporation of structural model and the reduced form model.

The risk-neutral default probability we derived is much higher than the objective default probability calculated by Moody's. However, the discrepancy between credit rating and risk-neutral implied default probability is a puzzling result. Some issuers have a higher implied default probability, although their ratings show a higher creditworthiness than issuers with a lower implied default probability. Moreover, our credit term structure analysis and relative pricing test shows that bonds with different TTM, even though issued by the same institution, may exhibit different credit risk pattern and thus fall into different credit rating category. Considering this and the fluctuation in default intensity, the credit risk modeling process should take the pattern in credit risk term structure into account for better implementation of risk management.

8. Tables

Table A : 10 years Implied Cumulative Default Probability

Issue	S&P	Moody's	Cumulative Default Probability (%)									
Name	**Rating**	**Rating**	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**	**Year 7**	**Year 8**	**Year 9**	**Year 10**
Government												
Canada	AAA	Aaa	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CMHC	AAA	Aaa	0.20%	0.75%	1.28%	1.79%	2.18%	2.43%	2.64%			
Export Development Corp.	AAA	Aaa	0.10%	0.53%	1.08%	1.68%	2.33%	3.08%				
Provincial												
British Columbia	AAA	Aaa	0.00%	1.11%	2.52%	3.93%	5.36%	6.67%	8.09%	9.50%	10.90%	12.27%
Ontario	AA-	Aa1	0.00%	0.00%	1.09%	2.37%	3.73%	5.37%	7.20%	9.07%	11.15%	13.23%
Quebec	A+	Aa2	0.08%	0.81%	1.76%	2.95%	4.53%	6.32%	8.29%	10.25%	12.22%	14.11%
Municipal												
London Ontario	AAA	Aaa	0.55%	1.79%	3.33%	5.05%	6.98%	9.29%	11.76%	14.33%	16.82%	
B.C. MFA	AAA	Aaa	0.00%	1.00%	2.24%	3.72%	5.33%	7.17%	9.10%	11.19%	13.31%	15.34%
Corporate												
C.I.B.C.	A+	Aa2	0.36%	1.45%	2.61%	3.88%	5.32%	6.92%	8.63%	10.35%	11.93%	13.31%
GE Capital Cda Funding	AA+	Aa2	0.94%	2.81%	4.88%	7.11%	9.44%	11.80%	14.27%	16.97%	19.84%	22.81%
Toronto-Dominion Bk	AA-	Aaa	0.00%	0.33%	2.39%	7.60%	16.48%					
407 International Inc.	A	NA	0.08%	1.70%	3.28%	5.03%	6.89%	8.77%	10.86%	12.96%	15.29%	17.69%
Grtr Tor Air Authority	A	A2	0.37%	0.67%	0.98%	1.31%	1.75%	2.39%	3.39%	4.92%	7.07%	9.66%
Ind Alliance Cap Trust	A-	NA	0.00%	0.77%	3.69%	6.86%	10.35%	14.15%	18.14%	22.01%	25.34%	27.98%
BMO Capital Trust	A-	NA	0.00%	0.04%	1.17%	2.49%	3.87%	5.42%	7.13%	8.98%	11.00%	13.21%
Shaw Communications Inc.	BBB-	Baa3	1.06%	3.18%	5.50%	8.16%	11.02%	13.96%	17.05%	20.26%	23.60%	

Table B: Comparison of Risk-Neutral and Objective Default Probability

Issuer	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**	**Year 7**	**Year 8**	**Year 9**	**Year 10**
Aaa Category										
Toronto-Dominion Bk	0.00%	0.33%	2.39%	7.60%	16.48%					
Objective Default Rate	0.00%	0.01%	0.01%	0.04%	0.11%					
Aa Category										
GE Capital Cda Funding	0.94%	2.81%	4.88%	7.11%	9.44%	11.80%	14.27%	16.97%	19.84%	22.81%
C.I.B.C.	0.36%	1.45%	2.61%	3.88%	5.32%	6.92%	8.63%	10.35%	11.93%	13.31%
Objective Default Rate	0.02%	0.05%	0.09%	0.16%	0.23%	0.31%	0.39%	0.46%	0.50%	0.55%
A Category										
Grtr Tor Air Authority	0.37%	0.67%	0.98%	1.31%	1.75%	2.39%	3.39%	4.92%	7.07%	9.66%
Objective Default Rate	0.03%	0.12%	0.27%	0.43%	0.61%	0.81%	1.03%	1.27%	1.52%	1.75%
BBB Category										
Shaw Communications Inc.	1.06%	3.18%	5.50%	8.16%	11.02%	13.96%	17.05%	20.26%	23.60%	
Objective Default Rate	3.44%	9.75%	15.11%	19.86%	24.18%	28.26%	32.16%	35.43%	38.44%	

Table C: OASF, Reduced Form and Traditional Duration and Convexity

Gonvernment, Provincial, and Municipal

Issue Name	Coupon	Price	S&P Rating	Moody's Rating	Maturity Date	OASF	Duration			Convexity		
							Reduced Form	Maculay	% Difference	Reduced Form	Traditional	% Difference
Government												
Canada	3.75%	102.92	AAA	Aaa	Sep 1, 2011	0.31%	1.12	1.11	0.45%	1.26	1.71	-26.42%
CMHC	5.50%	106.87	AAA	Aaa	Jun 1, 2012	0.12%	1.81	1.80	0.58%	3.36	4.08	-17.66%
Export Development Corp.	5.10%	109.00	AAA	Aaa	Jun 2, 2014	-0.93%	3.68	3.57	3.22%	14.04	14.62	-3.95%
Provincial												
British Columbia	7.50%	117.67	AAA	Aaa	Jun 9, 2014	0.30%	3.59	3.47	3.32%	14.12	14.13	-0.13%
British Columbia	5.40%	111.43	AAA	Aaa	Jun 18, 2035	0.02%	13.30	14.52	-8.42%	252.39	282.79	-10.75%
Ontario	4.50%	106.54	AA-	Aa1	Mar 8, 2015	0.10%	4.19	4.2	-0.24%	18.80	20.17	-6.79%
Ontario	5.35%	110.34	AA-	Aa1	Jun 2, 2019	0.12%	7.07	7.24	-2.41%	57.50	60.36	-4.74%
Ontario	5.85%	116.06	AA-	Aa1	Mar 8, 2033	0.12%	12.11	13.32	-9.09%	211.07	238.48	-11.49%
Quebec	5.25%	108.51	A+	Aa2	Oct 1, 2013	0.23%	2.97	2.97	-0.11%	9.28	10.34	-10.28%
Municipal												
London Ontario	5.88%	112.69	AAA	Aaa	Aug 6, 2017	0.22%	5.66	5.83	-2.92%	36.91	39.51	-6.59%
B.C. MFA	4.90%	107.39	AAA	Aaa	Dec 3, 2013	0.31%	3.14	3.15	-0.44%	10.30	11.49	-10.41%

Corporate

Issue Name	Coupon	Price	Type	Recovery Rate	S&P Rating	Moody's Rating	Maturity Date	OASF	Duration			Convexity		
									Reduced Form	Maculay	% Difference	Reduced Form	Traditional	% Difference
Corporate														
C.I.B.C.	3.05%	101.27	Deposit Notes	71.38%	A+	Aa2	Jun 3, 2013	0.10%	2.75	2.78	-1.07%	7.83	8.88	-11.88%
GE Capital Cda Funding	5.53%	105.81	Company Guarnt	58.56%	AA+	Aa2	Aug 17, 2017	0.29%	5.56	5.87	-5.31%	36.89	39.95	-7.66%
Toronto-Dominion Bk	5.14%	106.55	Sr. Unsecured	45.49%	AA-	Aaa	Nov 19, 2012	0.53%	2.22	2.23	-0.35%	5.13	5.95	-13.82%
407 International Inc.	5.96%	111.05	Sr. Secured	58.56%	A	n/a	Dec 3, 2035	-0.03%	11.56	13.91	-16.89%	208.19	266.43	-21.86%
Corporate - Callable														
Grtr Tor Air Authority	6.25%	106.45	Sr. Secured	58.56%	A	A2	Jan 30, 2012	0.98%	1.46	1.46	0.10%	2.23	2.78	-19.83%
Ind Alliance Cap Trust	5.71%	105.44	Subordinated	35.82%	A-	n/a	Dec 31, 2013	0.54%	3.21	3.19	0.58%	10.72	11.62	-7.73%
BMO Capital Trust	6.69%	106.12	Jr. Subordinated	28.89%	A-	n/a	Dec 31, 2011	0.91%	1.90	1.41	34.75%	3.64	2.58	41.13%
Shaw Communications Inc.	6.15%	109.17	Sr. Unsecured	45.49%	BBB-	Baa3	May 9, 2016	0.16%	4.84	4.97	-2.70%	26.28	28.35	-7.30%
Non - SIAS Corporate														
Ford Motor Company	6.50%	101.00	Sr. Unsecured	45.49%	B	B2	Aug 1, 2018	-0.19%	5.61	6.02	-6.77%	38.71	42.59	-9.11%
Ally Financial Inc.	8.00%	99.87	Subordinated	35.82%	CCC+	B3	Dec 31, 2018	0.05%	5.55	5.85	-5.13%	38.59	41.45	-6.90%

Table D: Alberta Price Comparison

Issue Name	Coupon	Maturity Date	S&P Rating	Market Price	Calculated Price		
					BC (AAA)	ONT (AA-)	QBC (A+)
Alberta	4.25%	2012/6/1	AAA	104.583	104.728	105.320	104.757
Alberta	4.00%	2019/12/1	AAA	101.996	101.006	100.885	100.050

Table E: Bank of Scotia Bank (BNS) Price Comparison

Issue Name	Coupon	Maturity Date	S&P Rating	Market Price	Calculated Price		
					BMO (A)	TD(AA-)	CIBC (A+)
BNS	3.03%	2012/6/4	AA-	101.996	103.893	102.630	101.968
BNS	4.10%	2017/6/8	AA-	101.344	102.447	n/a	102.396

Table F: Royal Bank Price Comparison

Issue Name	Coupon	Maturity Date	S&P Rating	Market Price	Calculated Price		
					BMO (A)	TD(AA-)	CIBC (A+)
RBC	4.92%	2011/7/16	AA-	103.315	105.858	108.080	103.430
RBC	3.66%	2017/1/25	AA-	100.312	101.277	n/a	101.126
RBC	9.30%	2023/6/8	AA-	143.012	145.257	n/a	148.208

9. List of Figures

Government of Canada

Figure 1. Fitted Implied Survival Probability of Government of Canada Bond

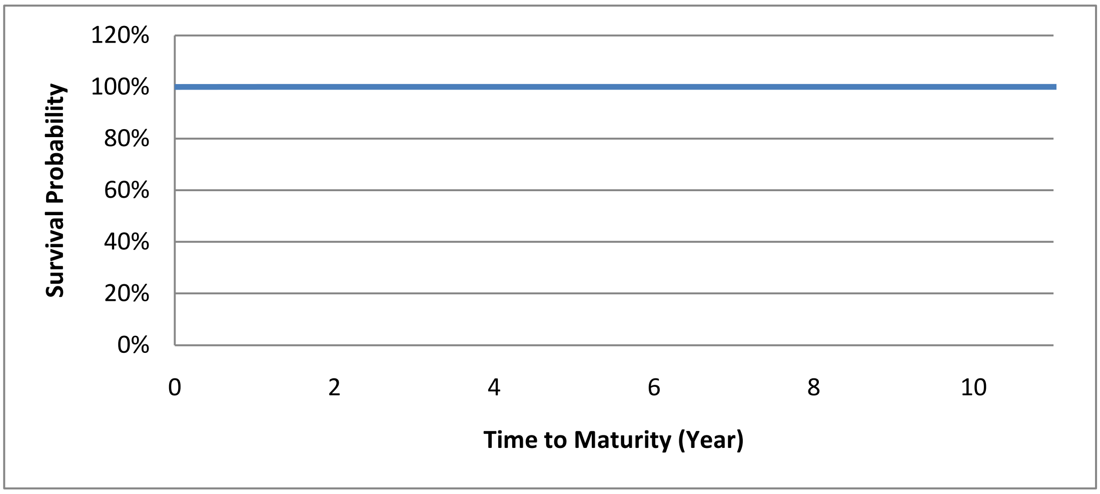
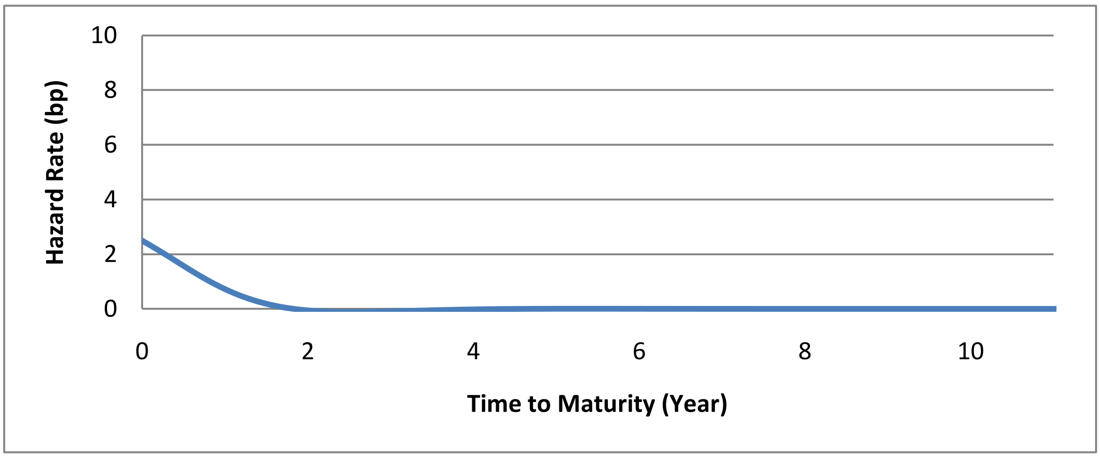


Figure 2. Fitted Implied Hazard Rate Government of Canada Bond



Canadian Mortgage Housing Corporation (CANMOR)

Figure 3. Fitted Implied Survival Probability of CANMOR Bond

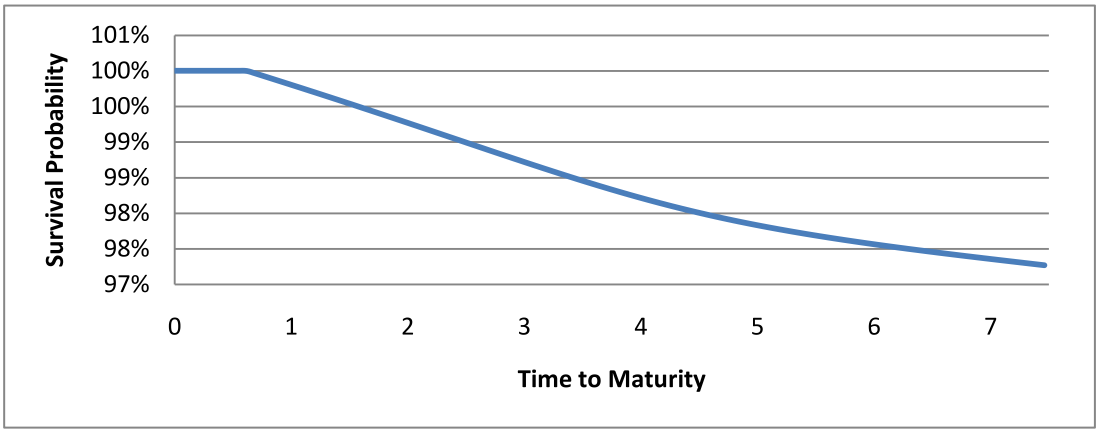


Figure 4. Fitted Implied Hazard Rate Term Structure of CANMOR Bond

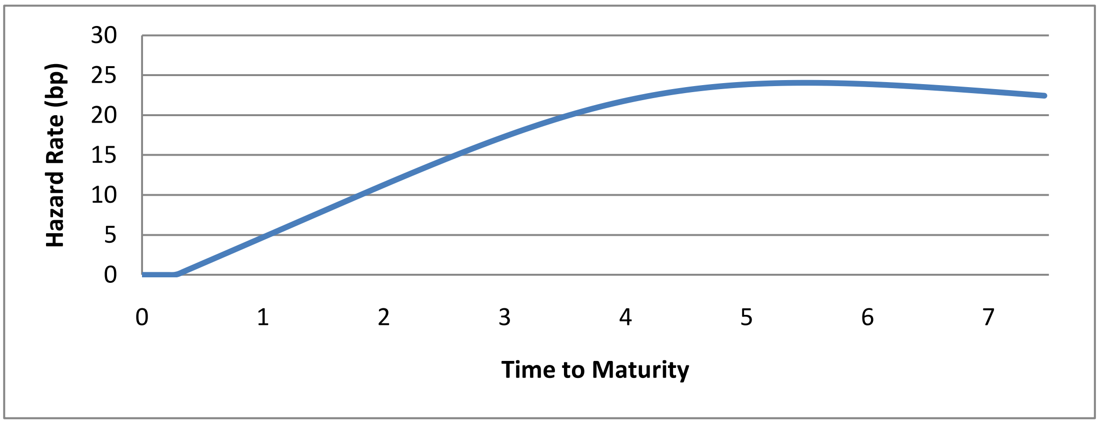


Figure 5. Fitted Implied Forward Hazard Rate Term Structure CANMOR Bond

Export Development Canada (EDC)

Figure 6. Fitted Implied Survival Probability of EDC Bond

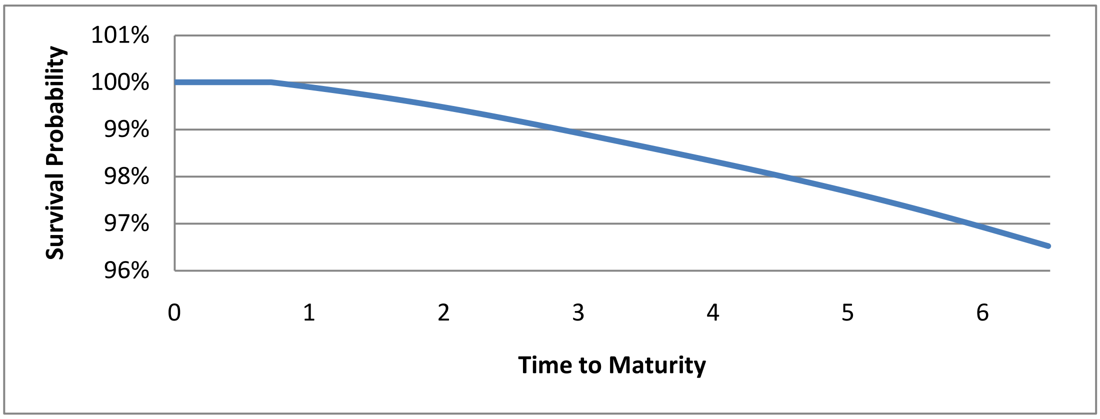


Figure7. Fitted Implied Hazard Rate Term Structure of EDC Bond

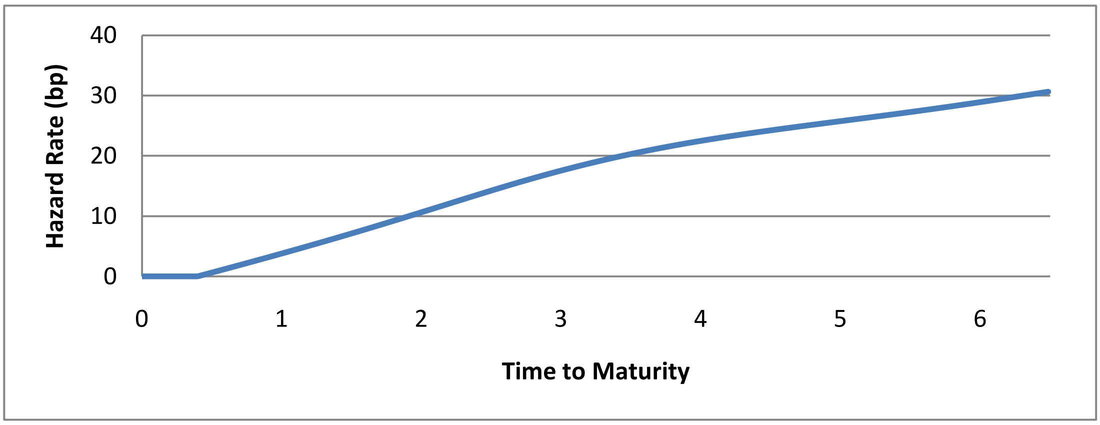


Figure 8. Fitted Implied Forward Hazard Rate Term Structure of EDC Bond

Province of British Columbia

Figure 9. Fitted Implied Survival Probability of British Columbia Bond

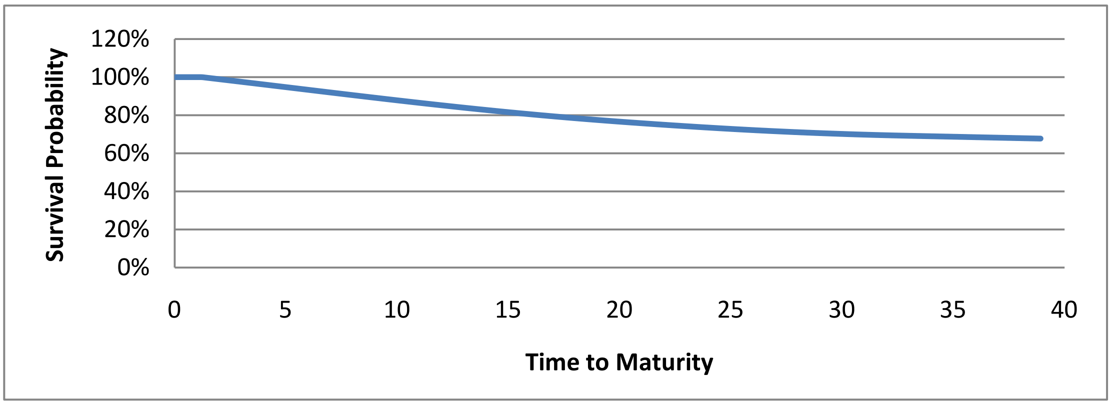


Figure 10. Fitted Implied Hazard Rate of British Columbia Bond

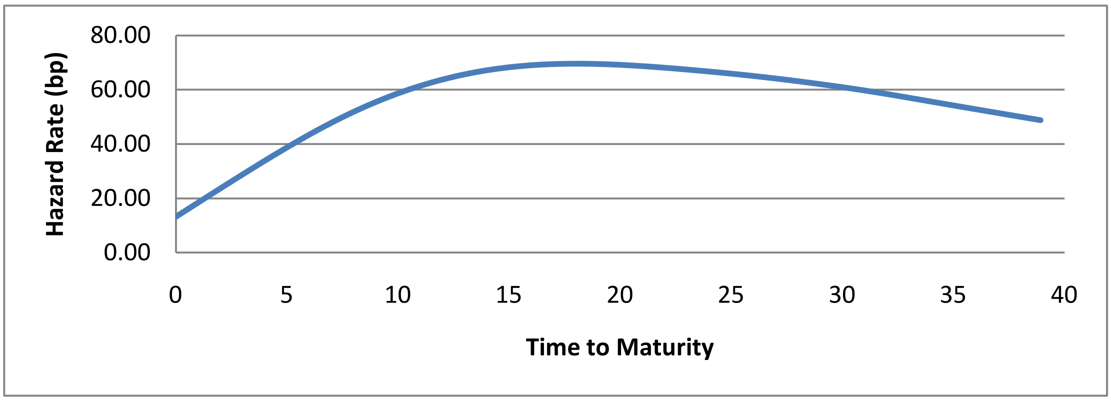
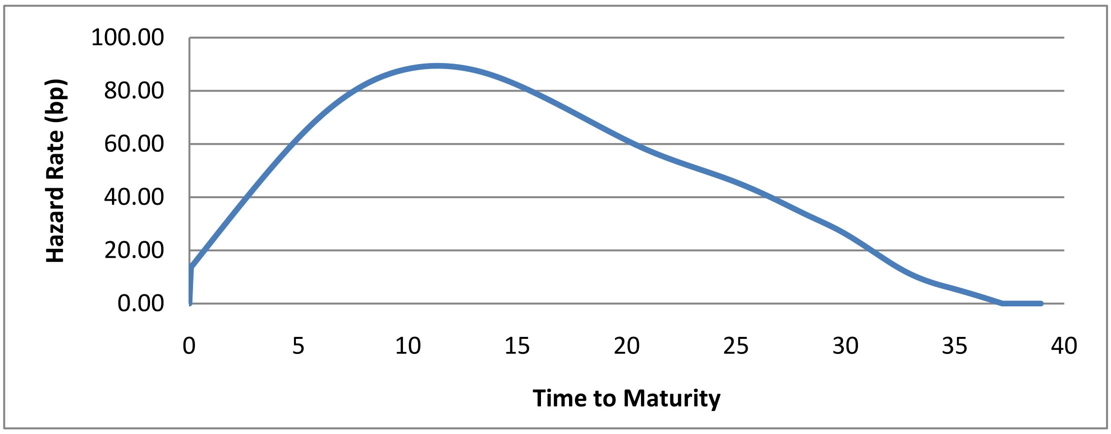


Figure 11. Fitted Forward Hazard Rate of British Columbia Bond



Province of Ontario

Figure 12. Fitted Implied Survival Probability of Ontario Bond

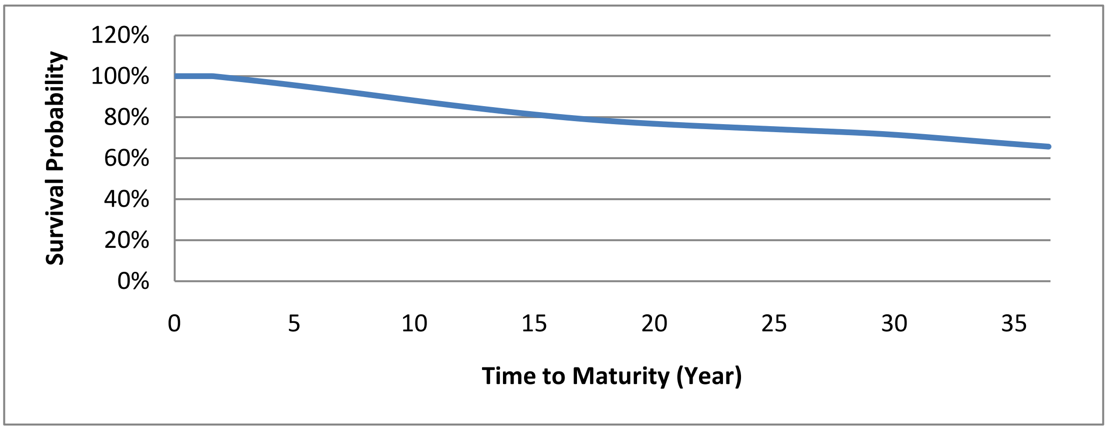


Figure 13. Fitted Implied Hazard Rate Term Structure of Ontario Bond

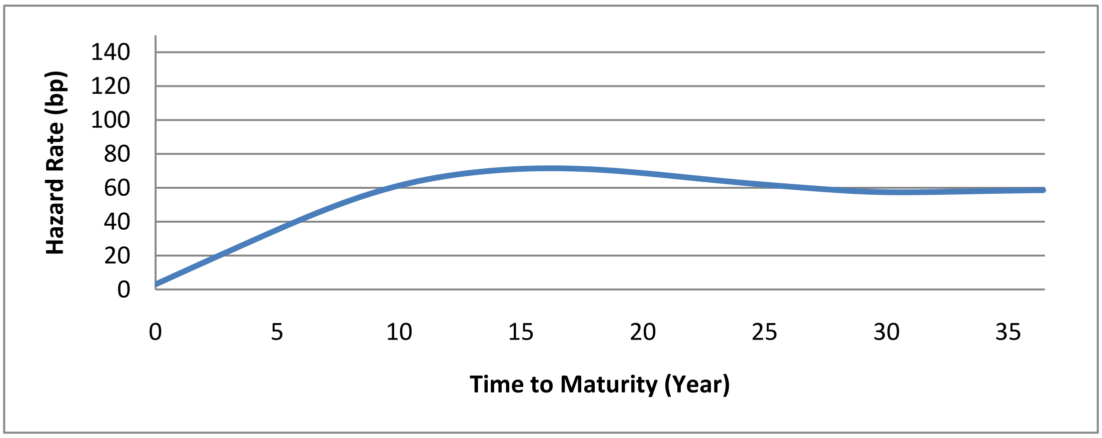
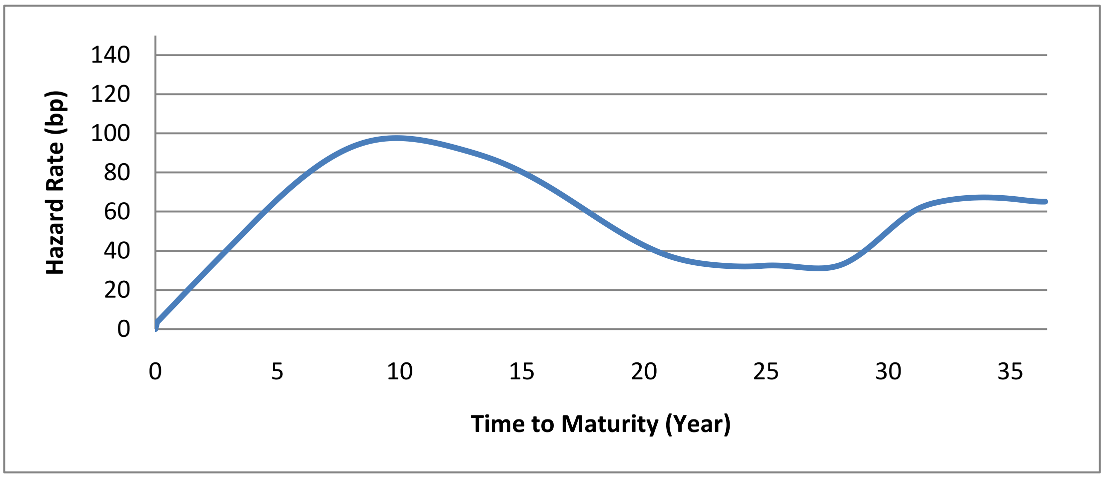


Figure 14. Fitted Implied Forward Hazard Rate of Ontario Bond



Province of Quebec

Figure 15. Fitted Implied Survival Probability of Quebec Bond

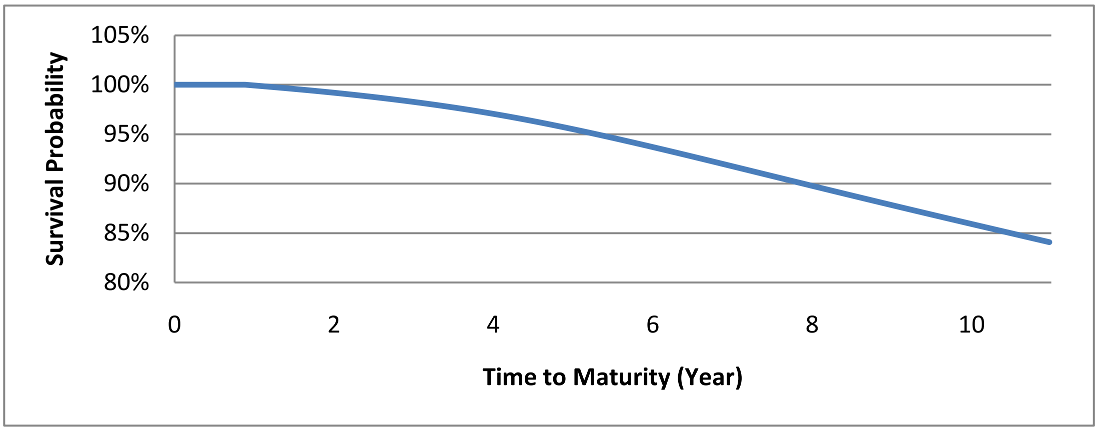


Figure 16. Fitted Implied Hazard Rate Term Structure of Quebec Bond

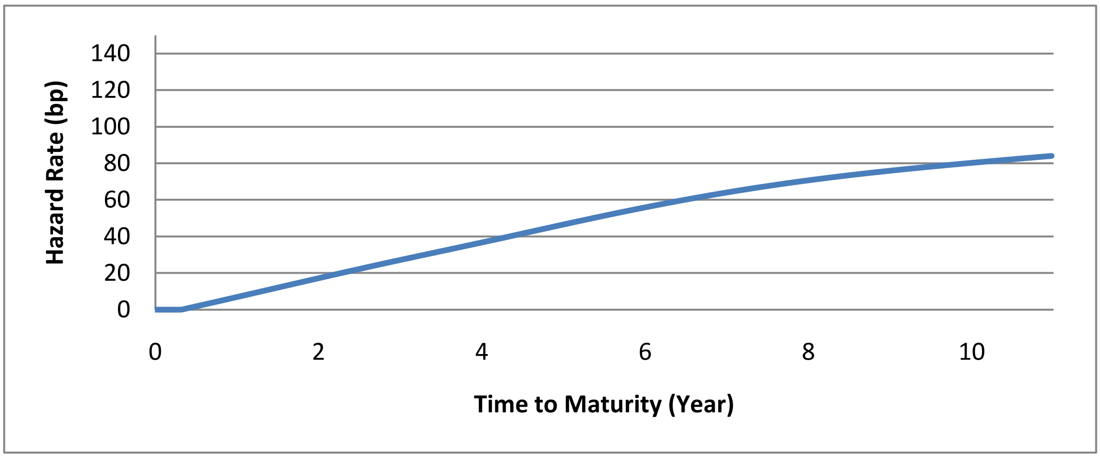
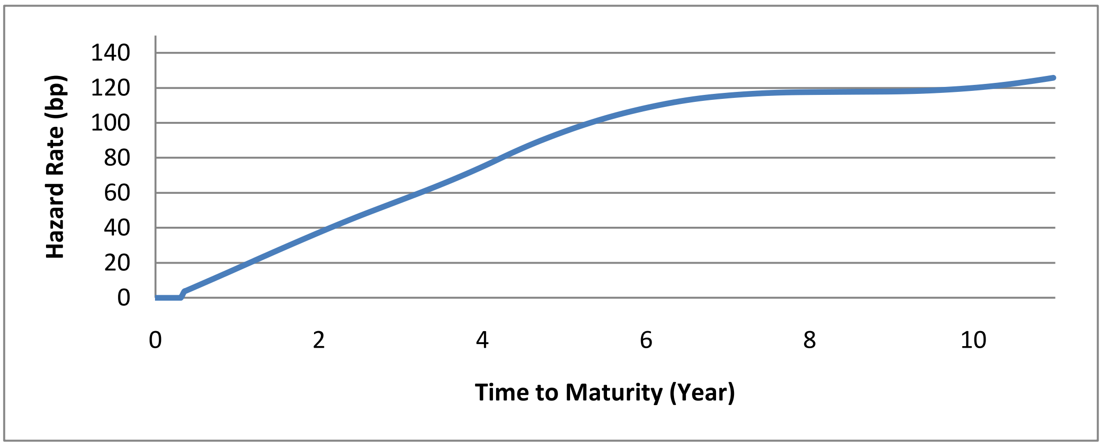


Figure 17. Fitted Implied Forward Hazard Rate of Quebec Bond



London Ontario

Figure 18. Fitted Implied Survival Probability of London Bond

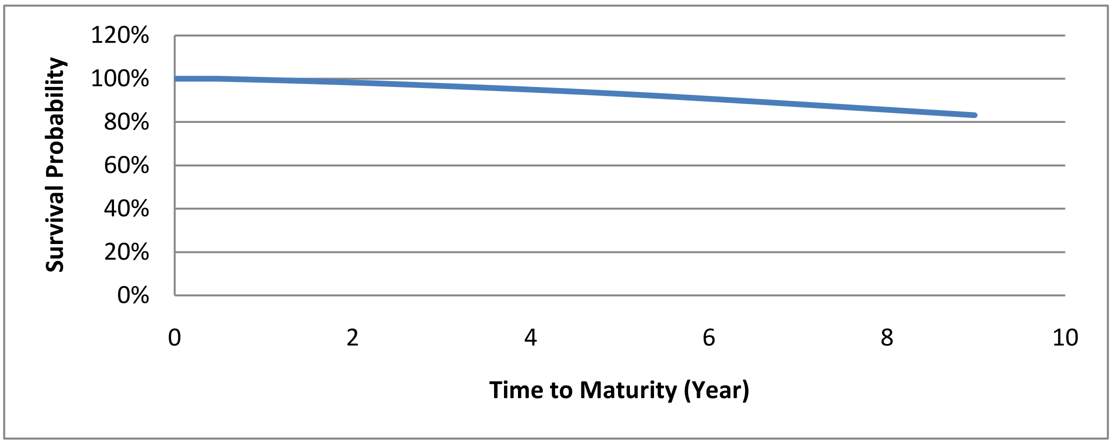


Figure 19. Fitted Implied Hazard Rate Term Structure of London Bond

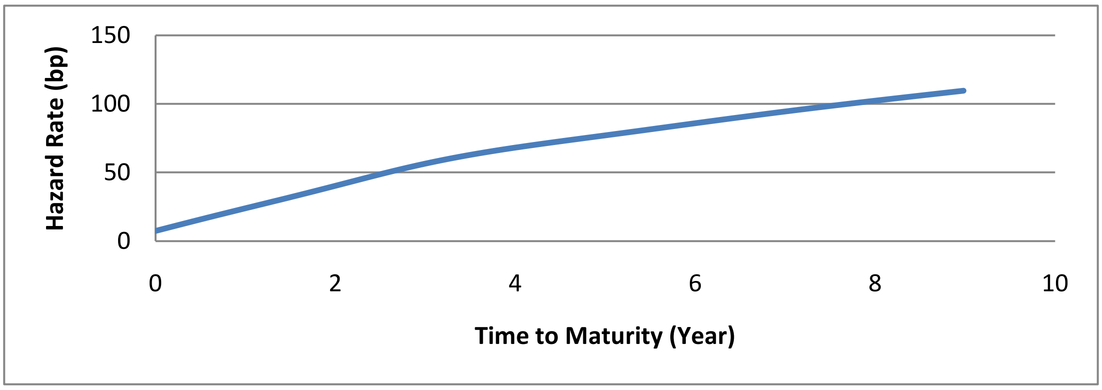
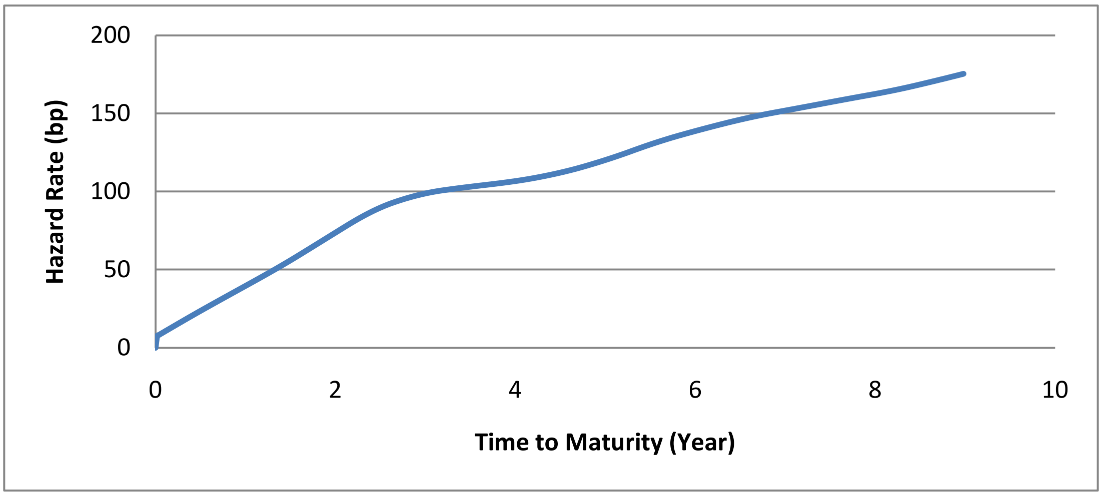


Figure 20. Fitted Implied Forward Hazard Rate of London Bond



Municipal Finance Authority of BC

Figure 21. Fitted Implied Survival Probability of BCMFA Bond

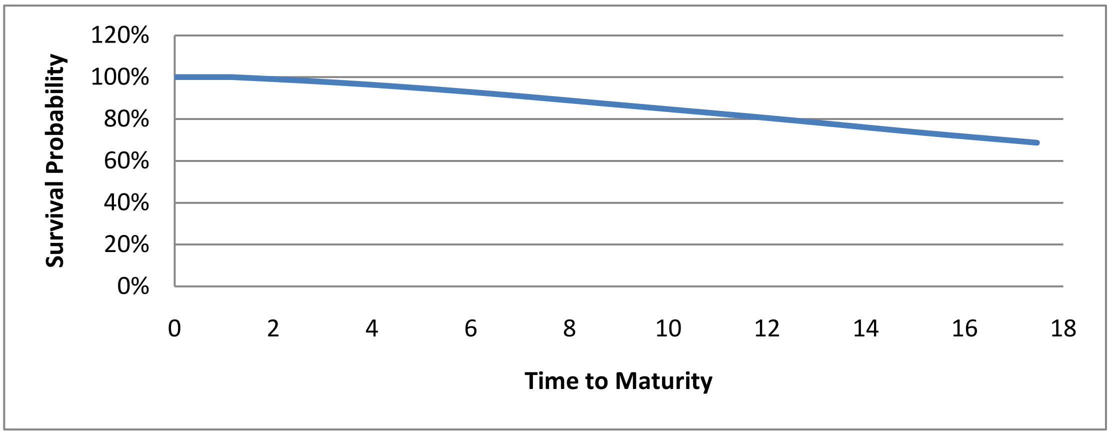


Figure 22. Fitted Implied Hazard Rate Term Structure of BCMFA Bond

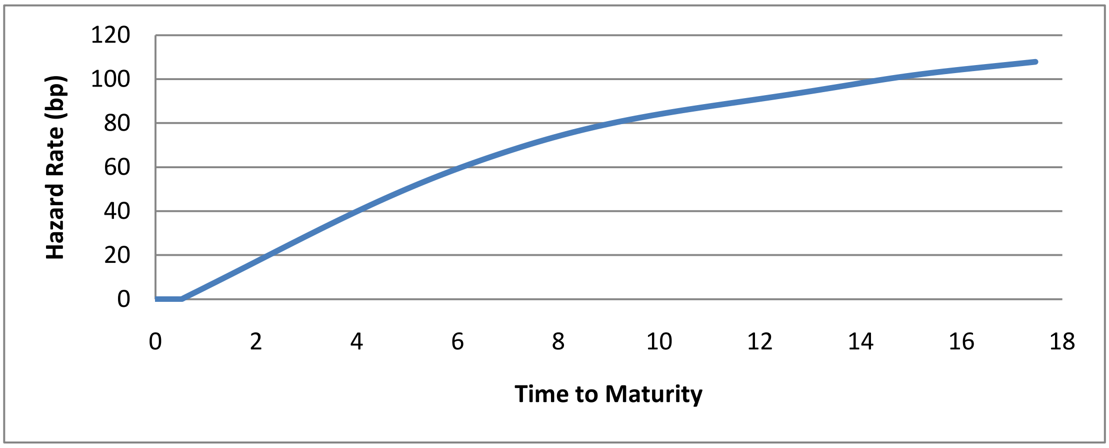
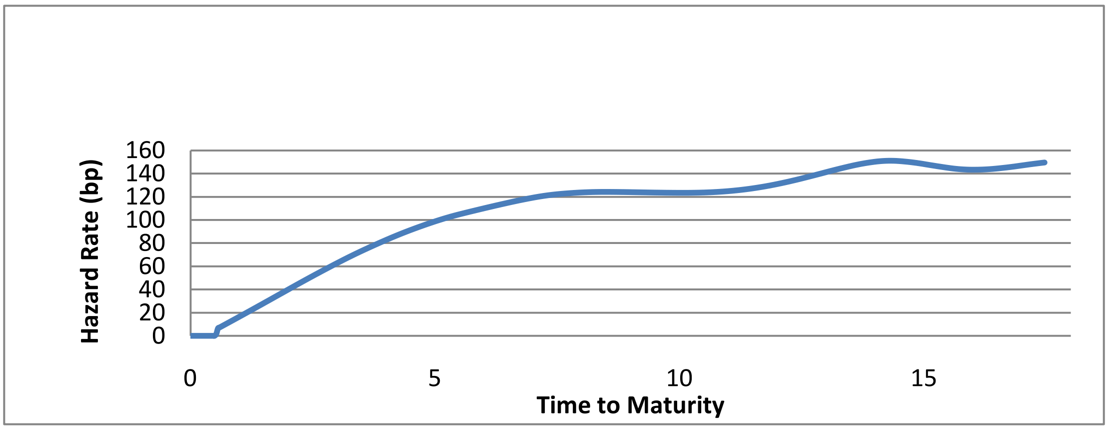


Figure 23. Fitted Implied Forward Hazard Rate Term Structure of BCMFA Bond



Canadian Imperial Bank of Commerce

Figure 24. Fitted Implied Survival Probability of CIBC Bond

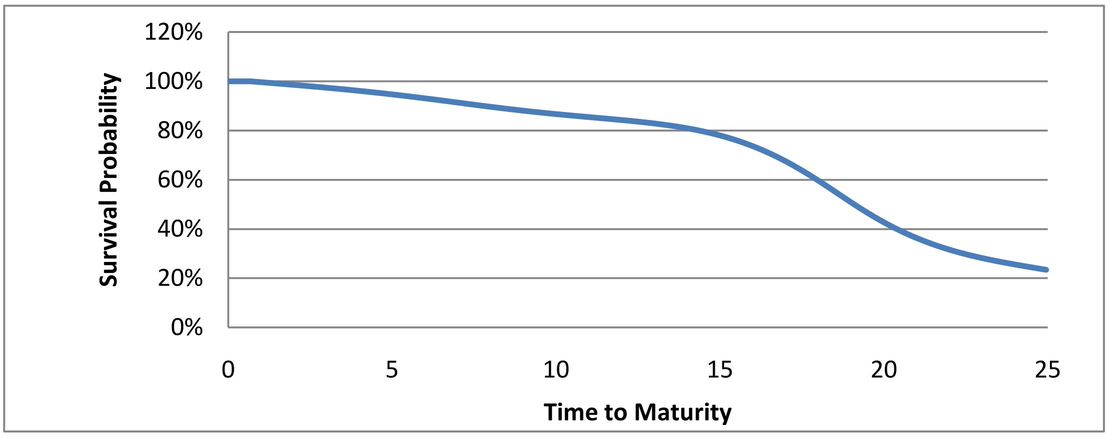


Figure 25. Fitted Implied Hazard Rate Term Structure of CIBC Bond

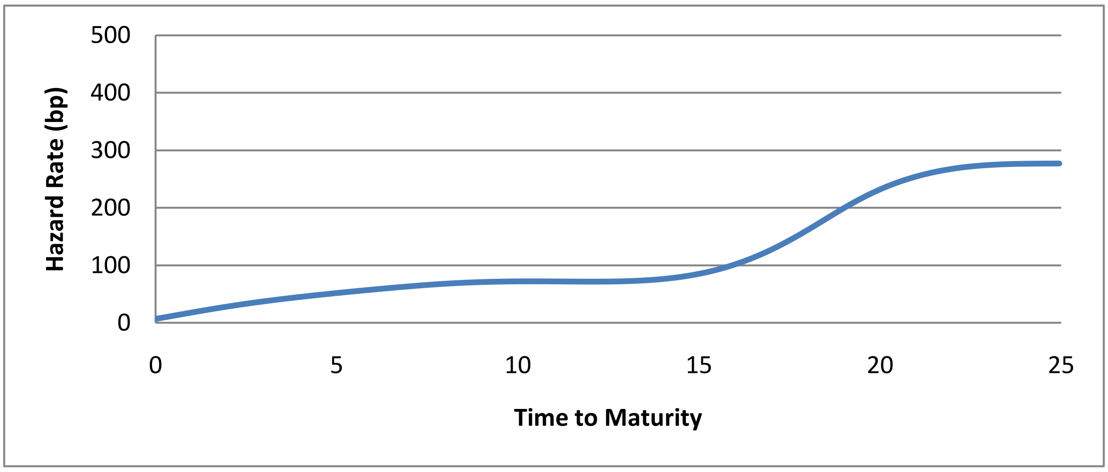
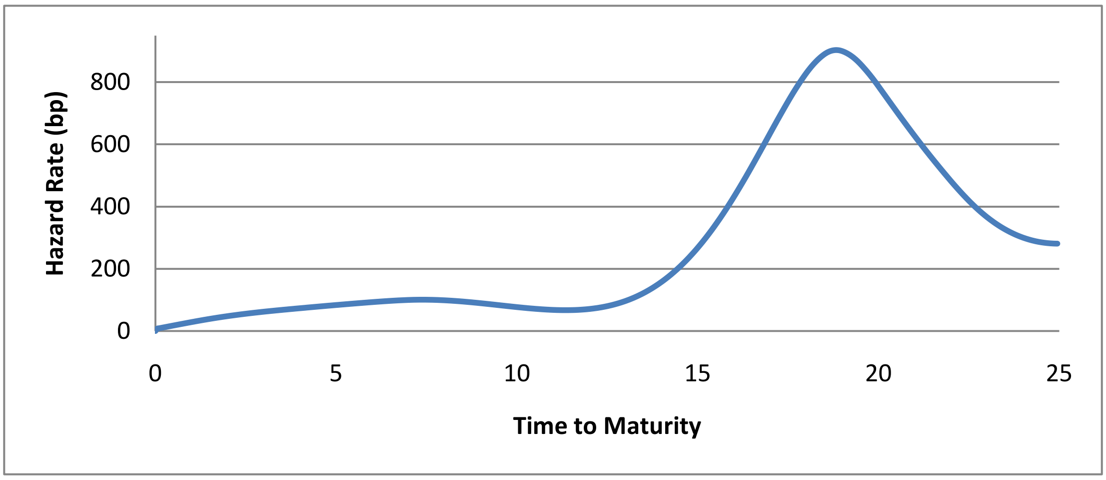


Figure 26. Fitted Implied Forward Hazard Rate Term Structure of CIBC Bond



General Electric Corporation

Figure 27. Fitted Implied Survival Probability of GE Bond

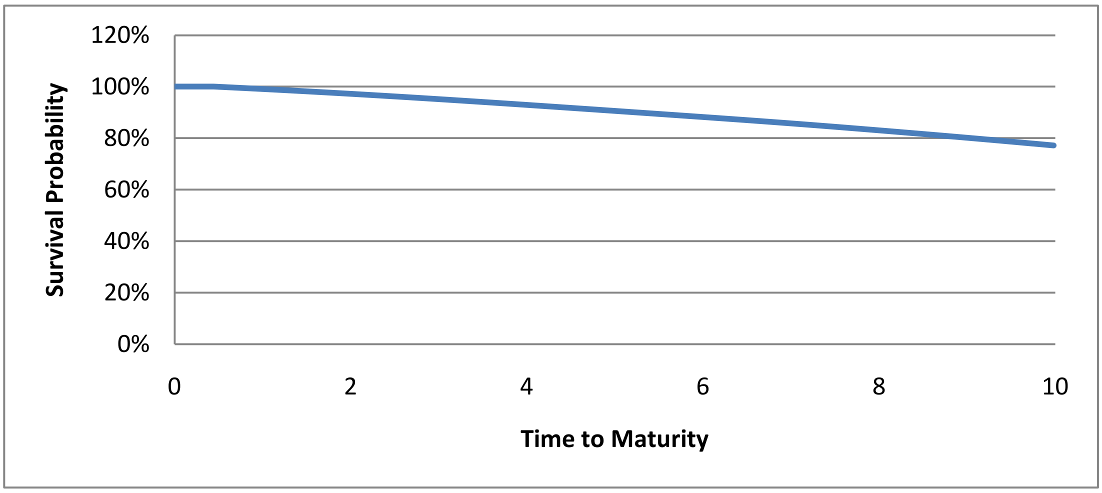


Figure 28. Fitted Implied Hazard Rate Term Structure of GE Bond

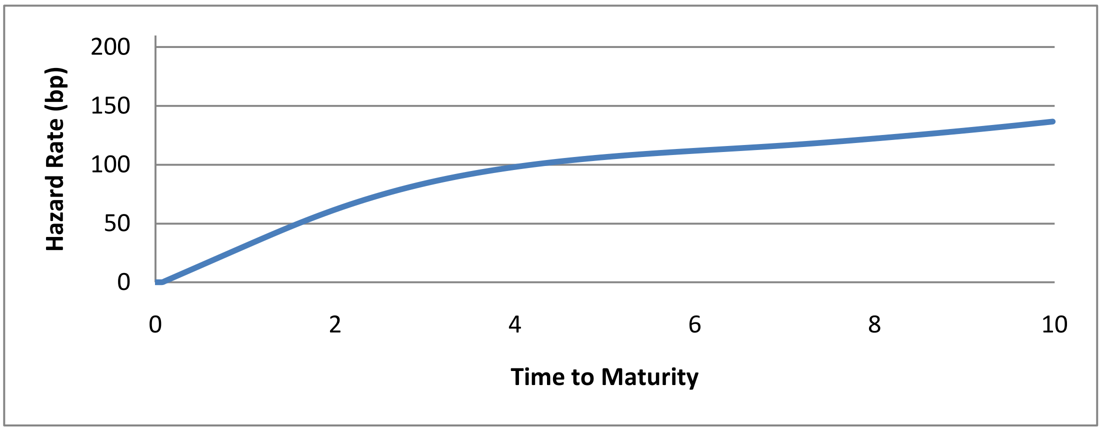
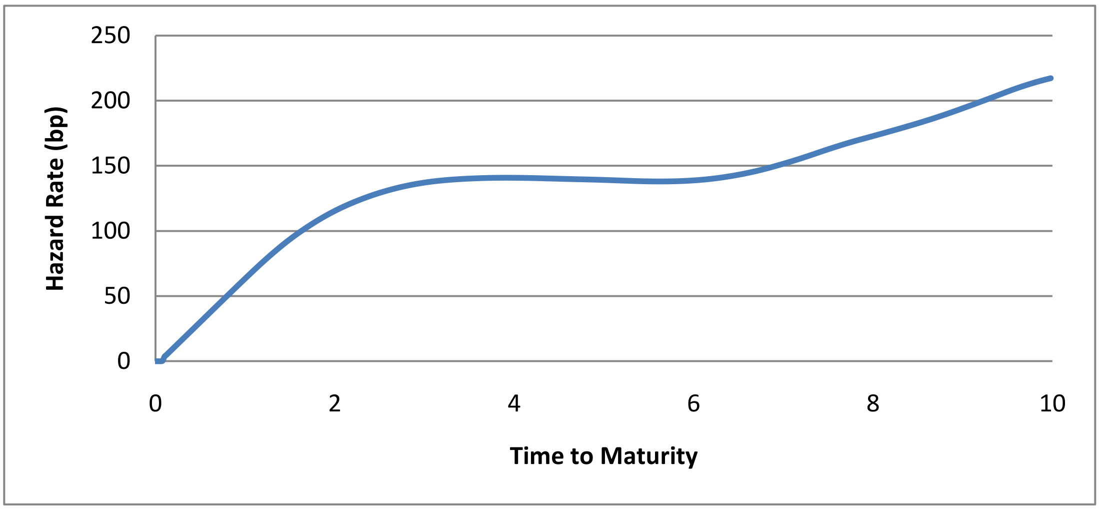


Figure 29. Fitted Implied Forward Hazard Rate Term Structure of GE Bond



Toronto Dominion

Figure 30. Fitted Implied Survival Probability of TD-Peer Bond

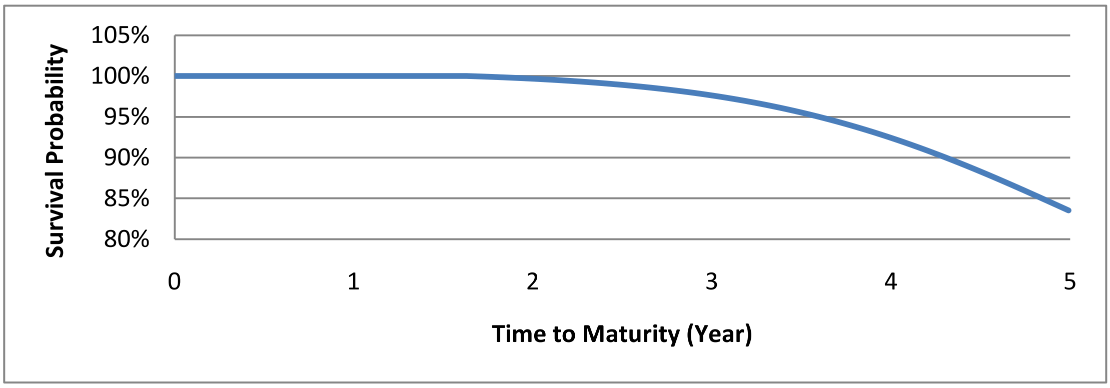


Figure 31. Fitted Implied Hazard Rate Term Structure of TD-Peer Bond

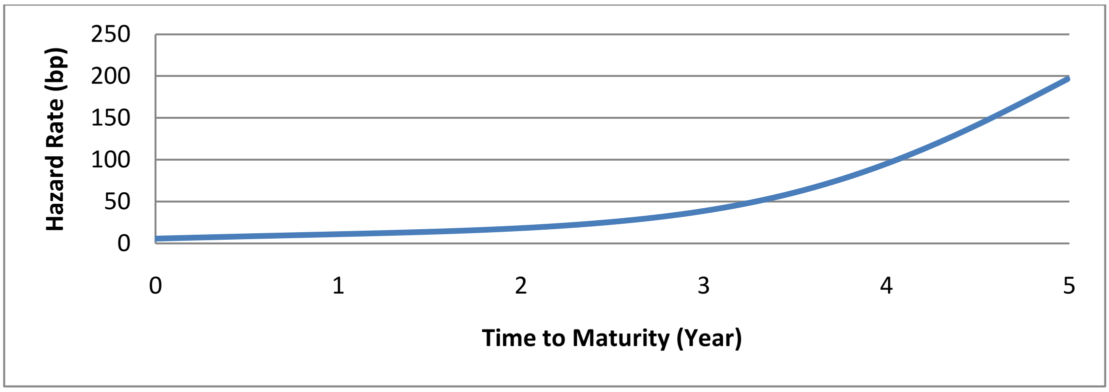
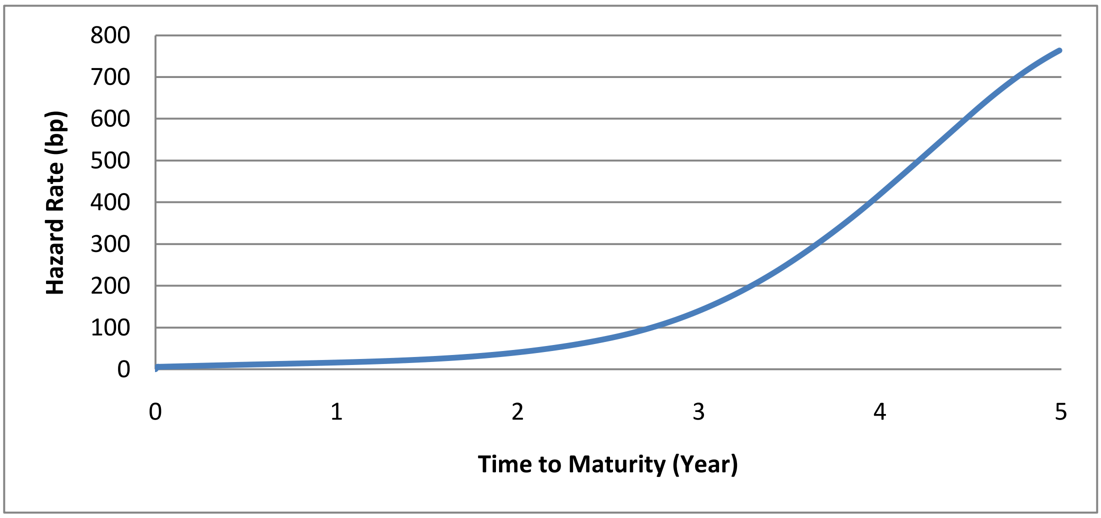


Figure 32. Fitted Implied Forward Hazard Rate of TD-Peer Bond



407 International Inc. Corporations

Figure 33. Fitted Implied Survival Probability of ETHWAY Bond

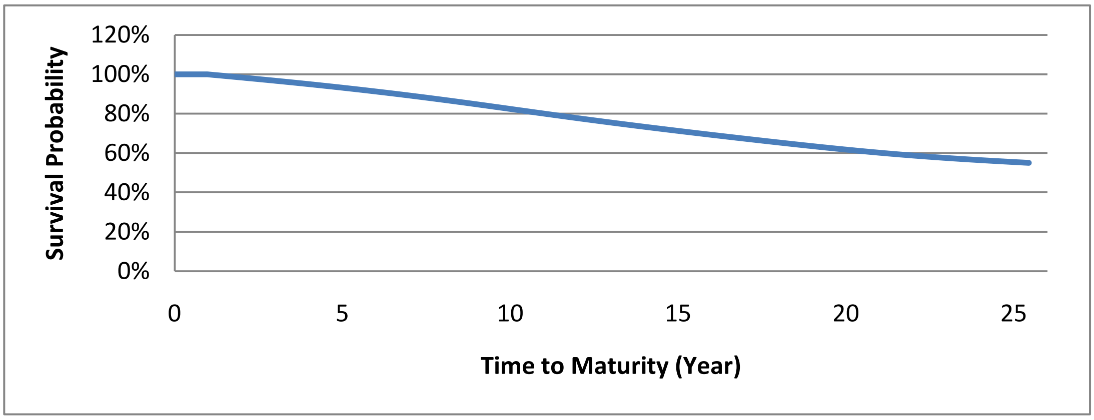


Figure 34. Fitted Implied Hazard Rate Term Structure of ETHWAY Bond

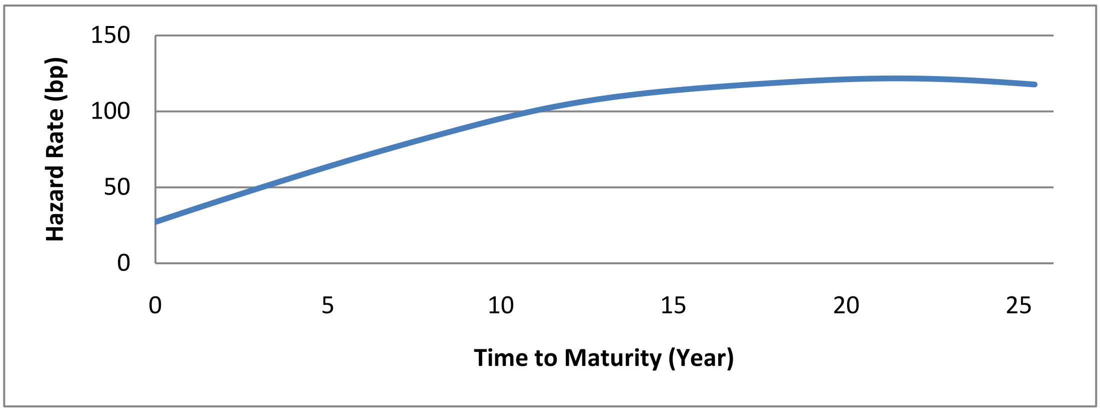
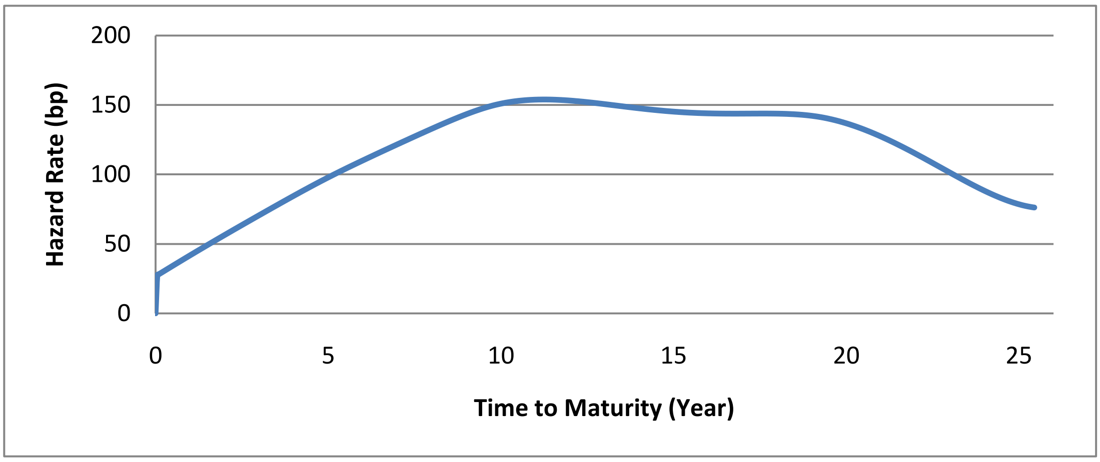


Figure 35. Fitted Implied Forward Hazard Rate of ETHWAY Bond



Grtr Tor Air Authority

Figure 36. Fitted Implied Survival Probability of GTAA Bond

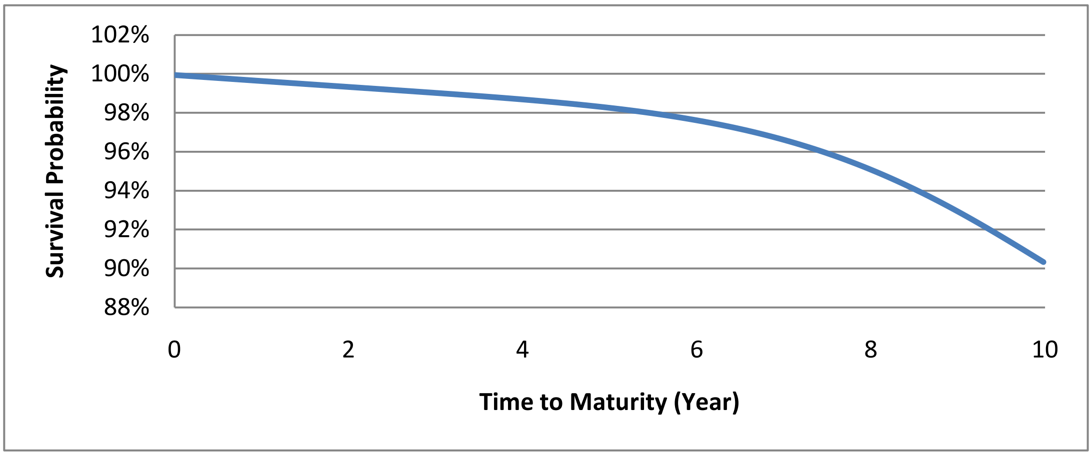


Figure 37. Fitted Implied Hazard Rate Term Structure of GTAA Bond

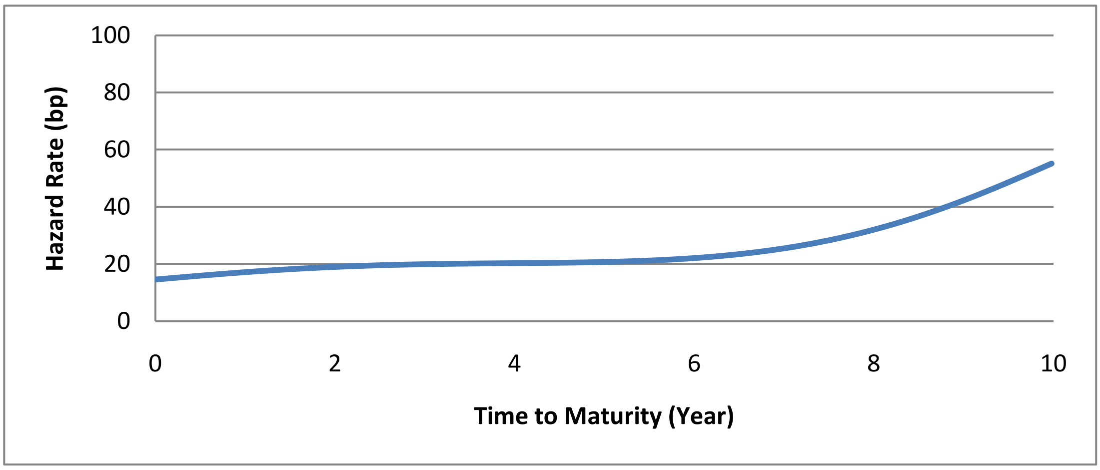
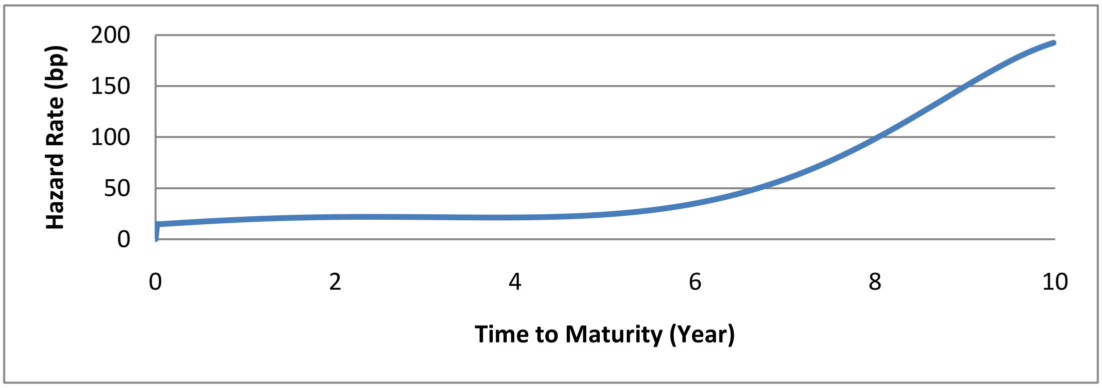


Figure 38. Fitted Implied Forward Hazard Rate Term Structure of GTAA Bond



Industrial Alliance Capital Trust

Figure 39. Fitted Implied Survival Probability of IDAL Bond

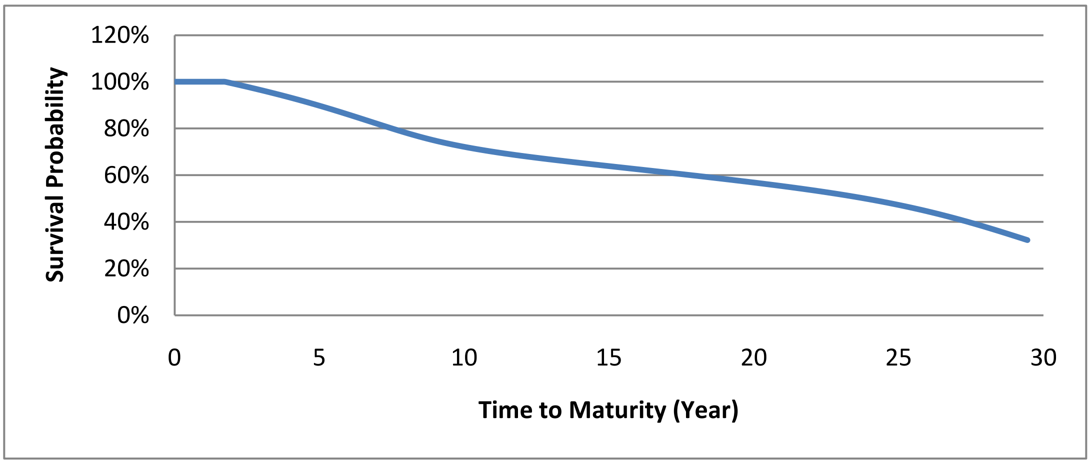


Figure 40. Fitted Implied Hazard Rate Term Structure of IDAL Bond

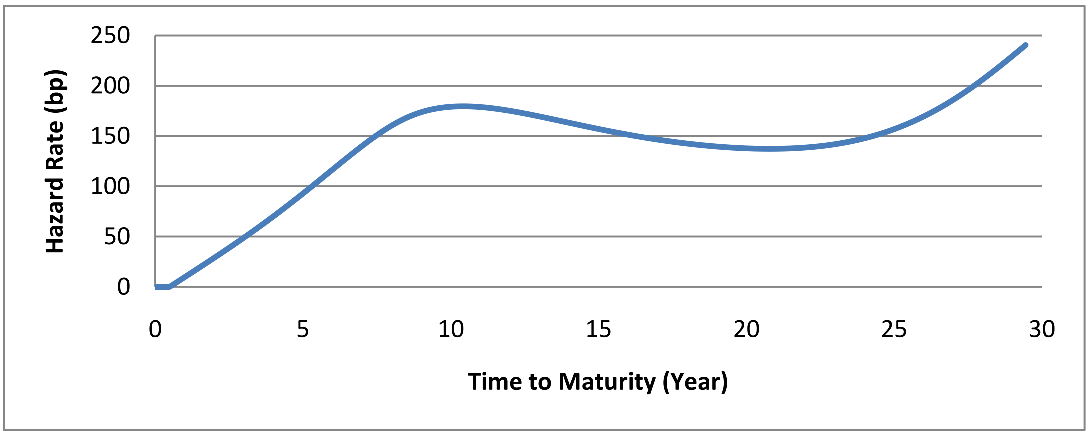
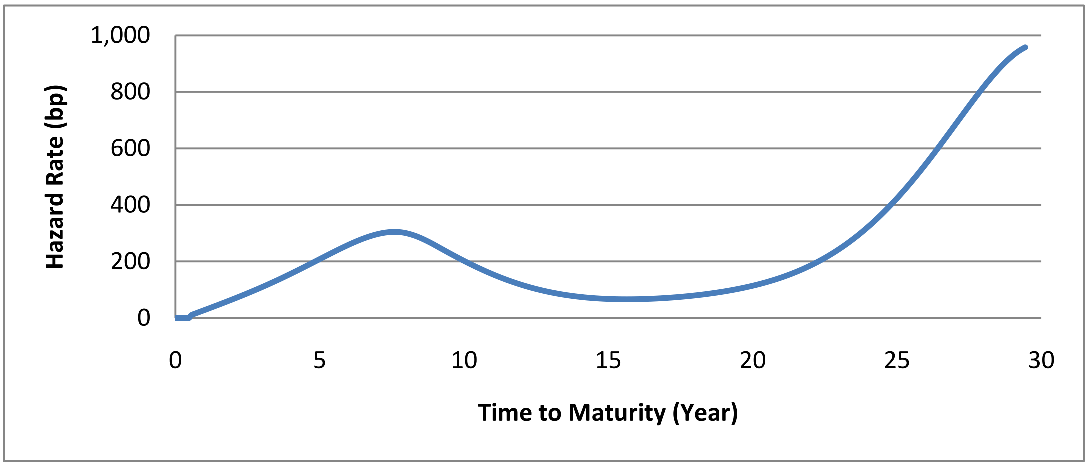


Figure 41. Fitted Implied Forward Hazard Rate Term Structure of IDAL Bond



BMO Capital Trust

Figure 42. Fitted Implied Survival Probability of Bank of Montreal Bond

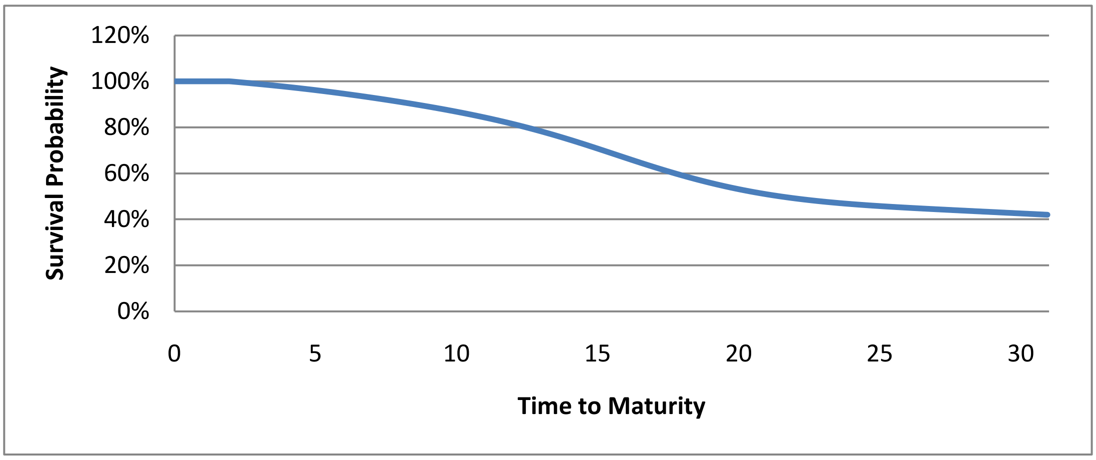


Figure 43. Fitted Implied Hazard Rate of Bank of Montreal Bond

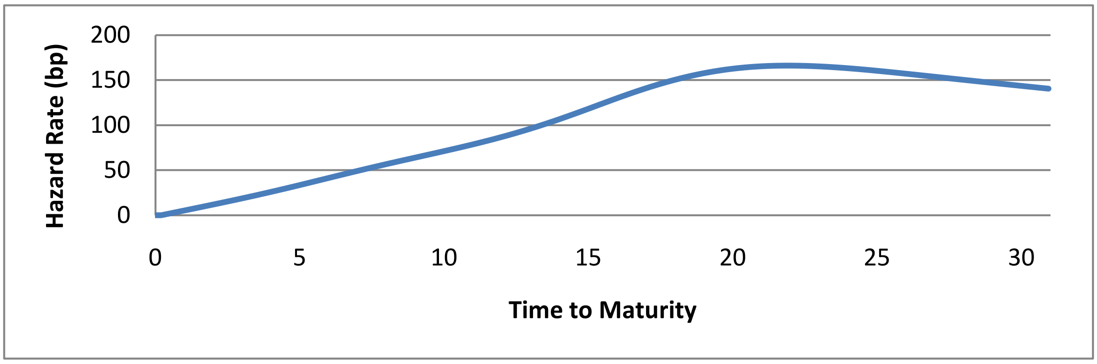
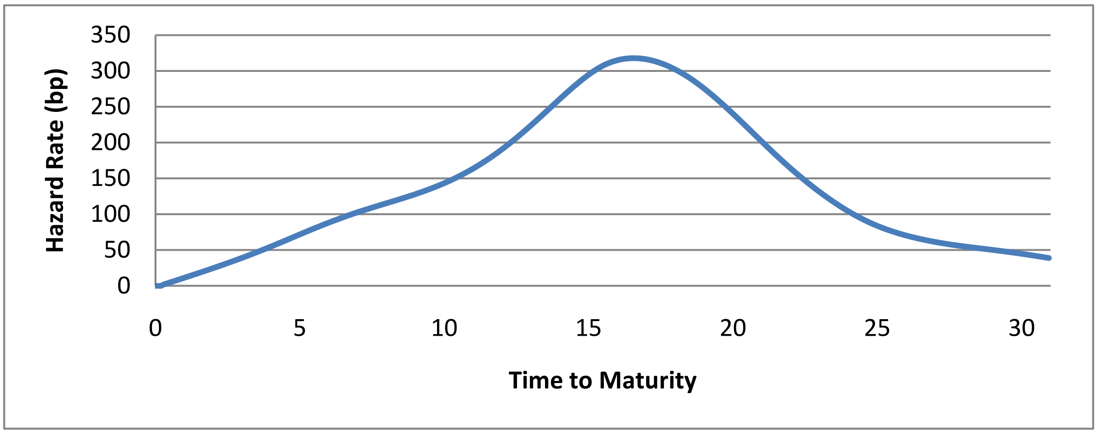


Figure 44. Fitted Implied Forward Hazard Rate Bank of Montreal Bond



Shaw Communications Inc.

Figure 45. Fitted Implied Survival Probability of Shaw Bond

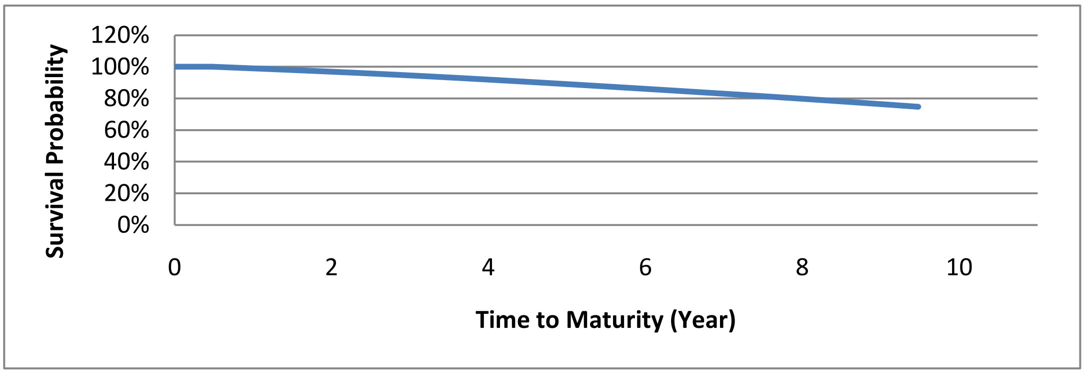


Figure 46. Fitted Implied Hazard Rate Term Structure of Shaw Bond

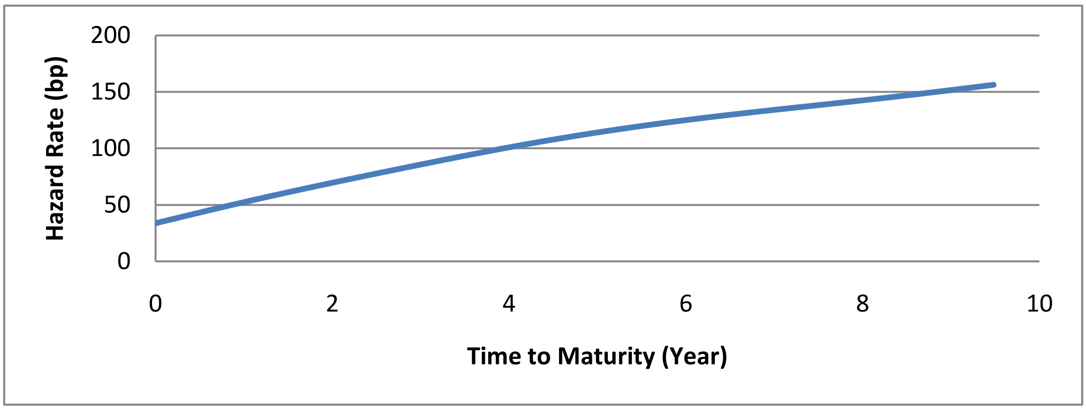


Figure 47. Fitted Implied Forward Hazard Rate Term Structure of Shaw Bond

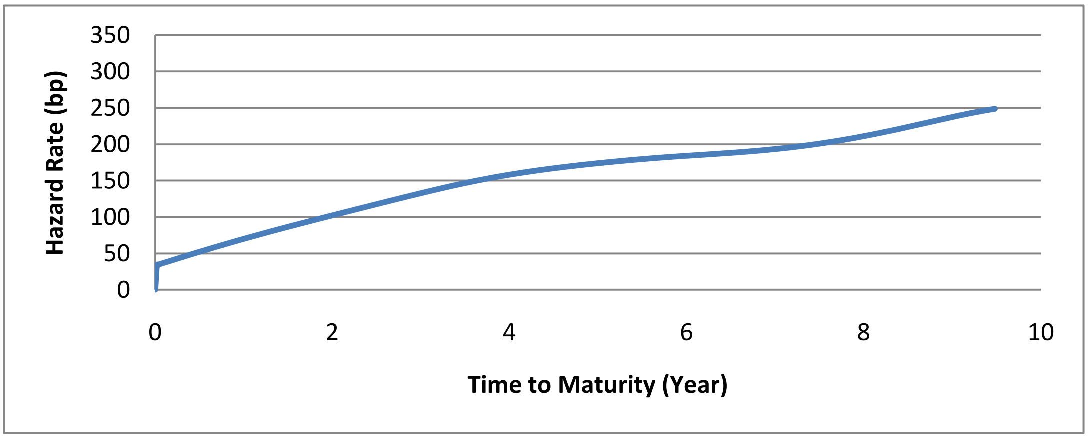


Figure 48. Objective and Risk-Neutral Default Probability – Aaa Rating

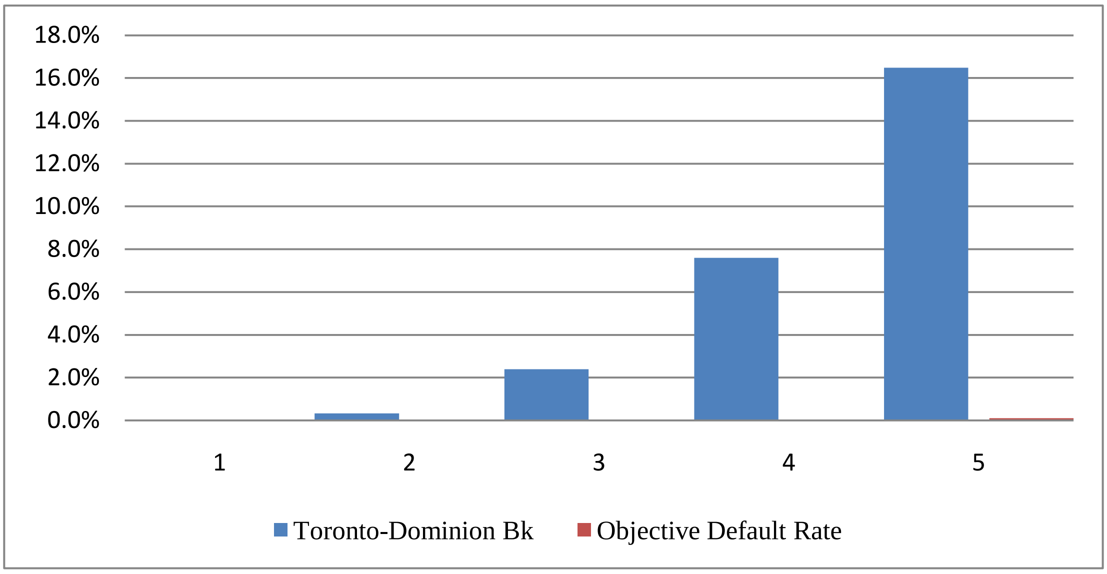


Figure 49. Objective and Risk-Neutral Default Probability – Aa Rating

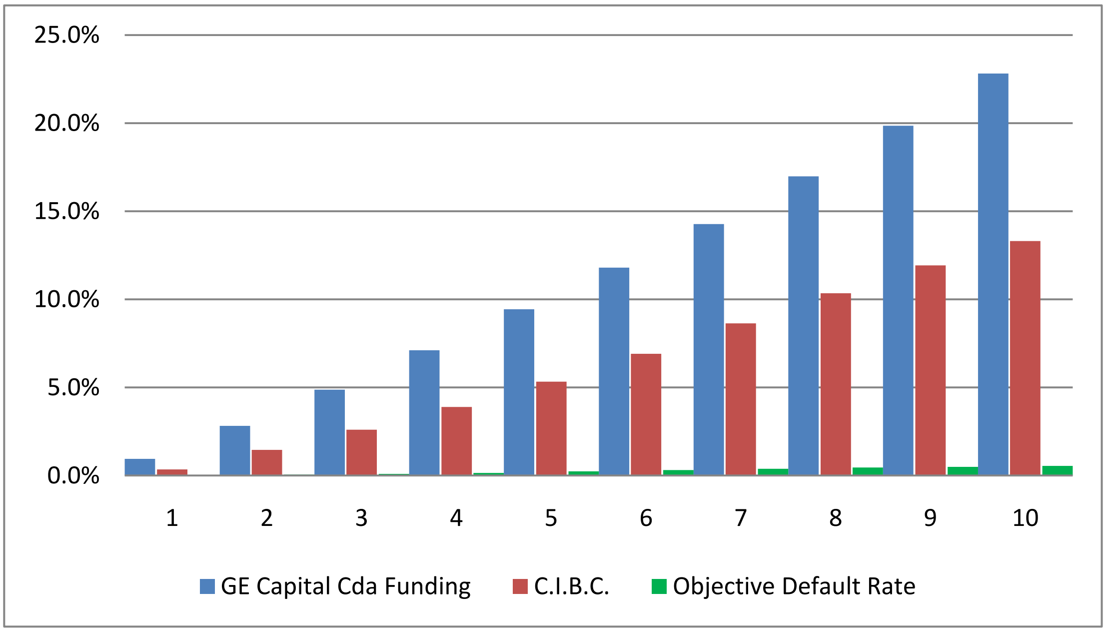


Figure 50. Objective and Risk-Neutral Default Probability - A Rating

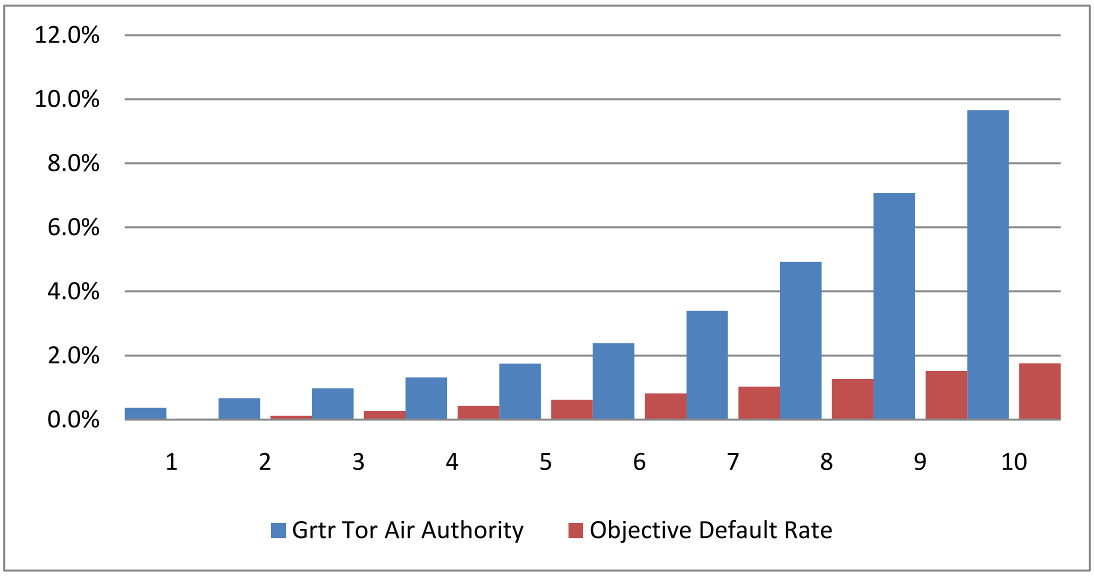
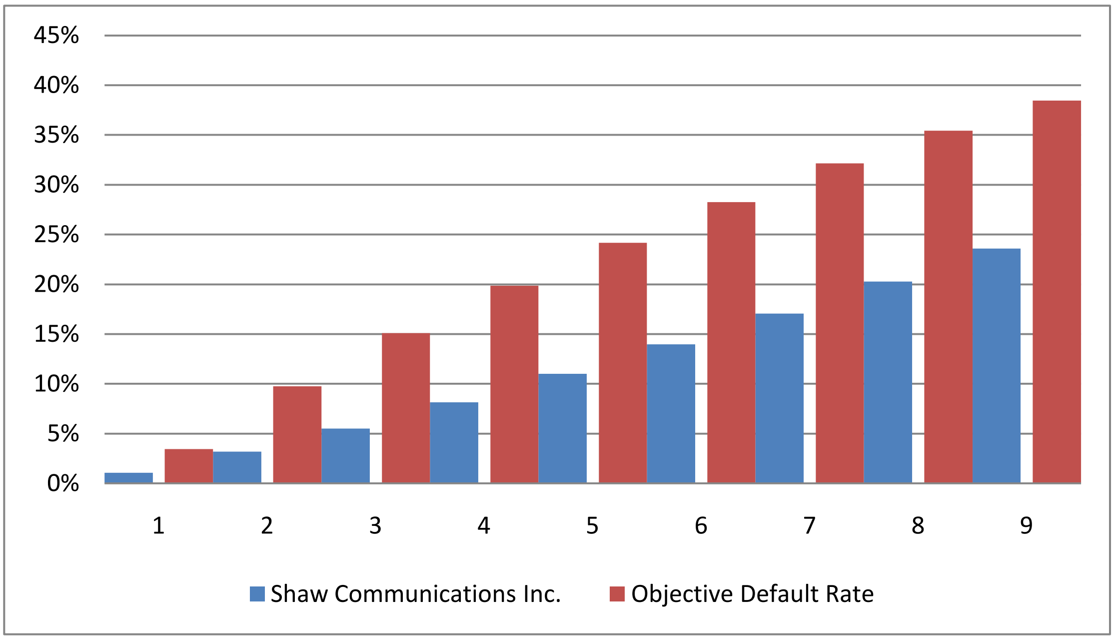


Figure 51. Objective and Risk-Neutral Default Probability - BBB Rating



10. Sample of Matlab Code

Here we show the set of code for constructing the credit risk term structure and estimating the default-risk-adjusted duration and convexity for Province of Quebec bonds. There are a few limitations to our model; for example the model cannot handle a portfolio with two consecutive bonds where the difference between their TTM is shorter than six months. That means we have to cherry-pick the bonds and may produce inaccurate result. Therefore, we will continue to improve our codes in terms of accuracy and efficiency. One modification we will be working on is to use exponential cubic spline to calibrate the term structure, instead of extracting piece-wise constant default intensity and forming the curve with cubic smoothing spline. This will solve the problem we mentioned and ensure a more reliable result.

```
% Codes for constructing credit risk term structure and estimating
% default-risk-adjusted duration and convexity.
% This set is for Pronvince of Quebec bonds
% June 24, 2010
% By Belinda Liao

clc
clear all
close all
load qbc
addpath(genpath('C:\Users\EPC\Documents\matlab\finfixed'))
% Converting Excel date format to Matlab date format
MatTime(:,1) = x2mdate(qbcdata(:,1));
MatTime(:,2) = x2mdate(qbcdata(:,2));
MatTime(:,3) = qbcdata(:,3);
C=qbcdata(:,4);
P0=qbcdata(:,5);
% Calculating time factor for each cashflow of bonds
%TF = cftimes(MatTime(:,1),MatTime(:,2),MatTime(:,3));
%Calculating discounted cashflow for each bonds
C = C/100;
L = length(C);
C = C';
[CFlowAmounts, CFlowDates, TFactors, CFlowFlags] = cfamounts(C,
```

```
MatTime(:,1), MatTime(:,2));
%Calculating discount factor Z(t)
U = length(R);
R = R';
A = nan(L,R);
% for v = 1:L
% A(v,:) = exp(-(TFactors(v,:).*R));
% end
for v = 1:L
    for w = 1:U
A(v,w) = exp(-(TFactors(v,w)*R(w)));
    end
end
Z = A(:,2:end);
CF = CFlowAmounts(:,2:end);
TF = TFactors(:,2:end);
Z = Z';
W = length(Z);
Z = Z';
b=nan(L,W);
for i=1:L
    for j= 1:W
        if isnan(Z(i,j)) == 1
    b(i,j) = 0;
        else
    b(i,j) = 1;
        end
    end
end
b = b';
c = sum(b);
c = c';
RecRate = 0.5;
FV = 100;
Rec = RecRate*FV;
Coef = nan(L,W);
for i = 1:L
      n = c(i,1);
```

```
        if n == 1
        Coef(i,n) = (CF(i,n)-Rec)*Z(i,n);
        elseif n > 1
                for k = 1:n-1
                Coef(i,k) = (CF(i,k) - Rec)*Z(i,k)+Rec*Z(i,k+1);
                end
                Coef(i,n) = (CF(i, n) - Rec)*Z(i,n);
                for m = 1+n:W
                    Coef(i,m) = 0;
                end
        end
end
%calculating cash price for bonds
AITime = -(TF(:,1)-1);
C = C';
AI = (AITime.*C/2)*100;
P0 = P0+AI;
LHS = P0 - Rec*Z(:,1);

Q(1,1) = LHS(1)/Coef(1,1);
if Q(1,1)>1
    Q(1,1) = 1;
end
Lambda(1,1) = -log(Q(1,1))/TF(1,1);
for p = 2:L
    n = c(p,1);
    d = c(p-1,1);
    u = n - d;
    if u == 1
    for q = 1:n-1
        Q(p,q) = exp(-Lambda(p-1,q)*TF(p,q));
        if Q(p,q)>1
            Q(p,q) = 1;
        end
        Lambda(p,q) = -log(Q(p,q))/TF(p,q);
        J(p,q) = Q(p,q)*Coef(p,q);
    end
        RTerm(p) = sum(J(p,1:n-1));
```

```
        Q(p,n) = (LHS(p)-RTerm(p))/Coef(p,n);
        Lambda(p,n) = -log(Q(p,n))/TF(p,n);

    elseif u > 1
        Lambda(p,1:d) = Lambda(p-1,1:d);
        Lambda(p,d+1:n) = Lambda(p,d);
        for q = 1:n-1
            Q(p,q) = exp(-Lambda(p,q)*TF(p,q));
            J(p,q) = Q(p,q)*Coef(p,q);
        end
        RTerm(p) = sum(J(p,1:n-1));
        Q(p,n) = (LHS(p)-RTerm(p))/Coef(p,n);
        if Q(p,q)>1
            Q(p,q) = 1;
        end
        Lambda(p,n) = -log(Q(p,n))/TF(p,n);
    end
end

%% Piece-wise Duration calculation
%Weighting bond cashflow
for i = 1:L
    for j = 1:W
SumBCF(i,j) = TF(i,j)*CF(i,j)*Z(i,j)*Q(i,j);
    end
end
for i=1:L
    for j= 1:W
        if isnan(SumBCF(i,j)) == 1
        DurBCashflow(i,j) = 0;
        else
            DurBCashflow(i,j) = SumBCF(i,j);
        end
    end
end
DurBCashflow = DurBCashflow';
DurWeightBCF = sum(DurBCashflow);
%Weighting recovery
```

```
for y = 1:L
    x(1) = 1;
    for x = 2:W
        SumRec(y,x) = Rec*TF(y,x)*Z(y,x)*(Q(y,x-1) - Q(y,x));
    end
end
for i=1:L
    for j= 1:W
        if isnan(SumRec(i,j)) == 1
        DurRec(i,j) = 0;
        else
            DurRec(i,j) = SumRec(i,j);
        end
    end
end
DurRec = DurRec';
DurWeightRec = sum(DurRec);
% Summing the Weighted Cashflows
DurWeightCF = DurWeightBCF+DurWeightRec;
%Calculating Duration
DurWeightCF = DurWeightCF';
Duration = DurWeightCF./P0;

%% Piece-wise Convexity Calculation

%Weighting bond cashflow
for i = 1:L
    for j = 1:W
ConvSumBCF(i,j) = (TF(i,j)^2)*CF(i,j)*Z(i,j)*Q(i,j);
    end
end
for i=1:L
    for j= 1:W
        if isnan(ConvSumBCF(i,j)) == 1
        ConvBCashflow(i,j) = 0;
        else
            ConvBCashflow(i,j) = ConvSumBCF(i,j);
        end
```

```
    end
end
ConvBCashflow = ConvBCashflow';
ConvWeightBCF = sum(ConvBCashflow);
%Weighting recovery
for y = 1:L
    x(1) = 1;
    for x = 2:W
        ConvSumRec(y,x) = Rec*(TF(y,x)^2)*Z(y,x)*(Q(y,x-1) - Q(y,x));
    end
end
for i=1:L
    for j= 1:W
        if isnan(ConvSumRec(i,j)) == 1
        ConvRec(i,j) = 0;
        else
            ConvRec(i,j) = ConvSumRec(i,j);
        end
    end
end
ConvRec = ConvRec';
ConvWeightRec = sum(ConvRec);
% Summing the Weighted Cashflows
ConvWeightCF = ConvWeightBCF+ConvWeightRec;
%Calculating Duration
ConvWeightCF = ConvWeightCF';
Convexity = ConvWeightCF./P0;

%% Standardizing Survival Probability
Time = [0:W];
SAQ(1) = 1;
SAQ(1,2:W+1) = exp(-Lambda(L,:).*Time(:,2:W+1));
%SAQ = exp(-Lambda(L,:).*Time);
plot(Time,SAQ,'.')
title('Survival Probability of Qebec Bond')
figure
p = 0.02;
lx = 500;
```

```
xx = linspace(0,W,lx);
CSQ = csaps(Time,SAQ,p,xx);
for i = 1:lx
    if CSQ(1,i)>1
        CSQ(1,i) = 1;
    end
end

plot(Time, SAQ,'o',xx,CSQ,'-');
title('Fitted Survival Probability of Qebec Bond')
figure
%% plot Lambda
PlotLambda(1) = 0;
PlotLambda(2:W+1) = Lambda(L,:);

plot(Time, PlotLambda, '.')
title('Lambda of Qebec Bond')
figure
p = 0.02;
lx = 500;
xx = linspace(0,W,lx);
CSLambda = csaps(Time,PlotLambda,p,xx);

plot(Time, PlotLambda, 'o',xx,CSLambda,'-');
title('Fitted Lambda of Qebec Bond')

%% Option Value - QBC
%[OptionValue] =
OptionValueCal(0.0625,40938,40371,Rec,R,Time,SAQ,p,106.278)
Coupon = 0.0525;
MTD = x2mdate(41548);
SET = x2mdate(40371);
[CallCFlow, CallCFlowDates, CallTFactors, CallCFlowFlags] =
cfamounts(Coupon, SET, MTD);
CallCFlow = CallCFlow';
s = length(CallCFlow);
CallCFlow = CallCFlow';
CallCFlow = CallCFlow(1,2:s);
```

```
CallFlowDates = CallCFlowDates(1,2:s);
CallTFactors = CallTFactors(1,2:s);

 for w = 1:s-1
   Z2(1,w) = exp(-(CallTFactors(1,w)*R(w+1)));
   Q2(1,w) = csaps(Time,SAQ,p,CallTFactors(1,w));
 end
 DCallCF = CallCFlow.*Z2;
 DCallCF = DCallCF.*Q2;

 DCallRec(1,1) = Z2(1,1)*(1-Q2(1,1))*Rec
 for x = 2:s-1
    DCallRec(1,x) = Z2(1,x)*(Q2(1,x-1)-Q2(1,x))*Rec;
 end
 BulletPrice = sum(DCallCF)+sum(DCallRec)
 CallPrice = 108.512;
 OptionValue = BulletPrice - CallPrice


%% OAS-adjusted Duration and Convexity
%Sloved using Excel Solver
OAS = 0.002277539;
% Weighting Cashflow
   for j = 1:s-1
CallBCF(1,j) =
CallTFactors(1,j)*CallCFlow(1,j)*Z2(1,j)*Q2(1,j)*exp(-OAS*(CallTFact
ors(1,j)));
   end
CallConvBCF = CallBCF.*CallTFactors;
CallBCFsum = sum(CallBCF);
CallConvsum = sum(CallConvBCF);
% Weighting Recovery Value
CallRec(1,1) = Rec*CallTFactors(1,1)*Z2(1,1)*(1 -
Q2(1,1))*exp(-OAS*(CallTFactors(1,1)))
   for x = 2:s-1
       CallRec(1,x) = Rec*CallTFactors(1,x)*Z2(1,x)*(Q2(1,x-1) -
Q2(1,x))*exp(-OAS*(CallTFactors(1,x)));
   end
```

```
CallConvRec = CallRec.*CallTFactors;
CallRecsum = sum(CallRec);
CallConvRecsum = sum(CallConvRec);

CallAITime = -(TF(1,1)-1);
CallAI = (CallAITime*Coupon)*100;
CallDirPrice = CallPrice+CallAI;
CallDur = (CallBCFsum+CallRecsum)/CallPrice/2
CallConv = (CallConvsum+CallConvRecsum)/CallPrice/4
```

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