EMPIRICAL ANALYSIS OF VALUE AT RISK MODELS: SIAS EQUITY PORTFOLIO RISK MODEL SELECTION AND FORMULATION

by

Shao Tai (Ted) Hsieh
BBA of Simon Fraser University, 2009

and

Issam El-Hourani
BACS of University of Western Ontario, 2003

PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF FINANCIAL RISK MANAGEMENT

In Faculty of Business Administration

SIMON FRASER UNIVERSITY
Summer Year

Approval

Name:	**Shao Tai (Ted) Hsieh & Issam El-Hourani**
Degree:	**Master of Financial Risk Management**
Title of Project:	**Empirical Analysis of Value at Risk Models: SIAS Equity Portfolio Risk Model Selection and Formulation**

Supervisory Committee:

Daniel Smith
Senior Supervisor
Associate Professor of Finance

Robert Jones
Second Reader
Professor of Economics

Date Approved:

Abstract

This paper is to determine an appropriate Value-at-Risk model that can improve the overall management of the SIAS fund, particular for the two equity portfolios. We consider the four candidate models: Historical Simulation, Dynamic Conditional Correlation Generalized AutoRegressive Conditional Heteroskedastic , Filtered Historical Simulation, and Hybrid Approach. Using historical information from 2003, all the models are implemented, and their specifications and performances are discussed in detail and examined with four backtesting procedures, including Unconditional Coverage, Independence, Conditional Coverage, and Quantile Regression tests.

Our findings confirm that the Historical Simulation model performs poorly in capturing the volatility dynamics, and we also have a comprehensive discussion about the factors that are used in the Hybrid Approach model. Those two are highly rejected from all the test procedures. On the other hand, Filtered Historical Simulation is the only model that passes the likelihood ratio tests. However, the likelihood ratio test may be flawed and biased; therefore, we employ Quantile Regression test that is believed to be a more powerful backtesting procedure. The results turn out that DCC GACRH is the best model among others. In addition, its other properties allow the risk management process to be more in depth. Therefore, DCC GACRH is strongly recommended.

Keywords: Value-at-Risk; Historical Simulation; DCC GARCH; Filtered Historical Simulation; Hybrid Approach; Backtesting; Quantile Regression; Diversification

Dedication

We dedicate this project to the SIAS fund and to incoming cohorts. We hope that a more comprehensive risk model including fixed income will emerge in the future enabling students to make better-informed decision.

Acknowledgements

We would like to thank our supervisor Dr. Daniel Smith for his support, feedback and encouragement in writing this thesis paper. Special thanks to Dr. Robert Jones and Dr. Anton Theunissen for their support and feedback.

We are grateful to Segal Graduate School of Business and its faculty for giving us the opportunity to manage the SIAS fund and apply new concepts learned throughout this year.

Table of Contents

Approval ii
Abstract iii
Dedication iv
Acknowledgements v
Table of Contents vi
List of Figures vii
List of Tables x

1: Introduction 1

2: Overview of SIAS Fund 4

3: Methodology 7
3.1 Historical Simulation 7
3.2 Dynamic Conditional Correlation GACRH 8
3.3 Filtered Historical Simulation 11
3.4 Hybrid Approach 12
3.5 Diversification 14

4: Data 16

5: Backtesting 17
5.1 Unconditional Coverage, Independence, and Conditional Coverage Test 17
5.2 Quantile Regression Test 18

6: Empirical Results and Analysis 20
6.1 Normality Analysis 20
6.2 Implementation and discussion of the models 20
6.3 Number of Exception 23
6.4 Backtesting 25
6.5 Speed and Accuracy 28

7: Diversification 29

8: Conclusion 31

References 34

Figures and Tables 37

List of Figures

Figure 1: The Average Holdings of Decomposed SIAS Portfolio from 08/04/2003 to 31/05/2010 37

Figure 2: Summary of Investment Policy Statement of SIAS Fund....37

Figure 3: 1-Year Exponential Smoothing Weights with Different Decay Factors....38

Figure 4: 2-Year Exponential Smoothing Weights with Different Decay Factors....38

Figure 5: Histogram of SIAS Canadian Equity Portfolio Returns from 04/30/2003 to 05/31/2010 (1782 samples)....39

Figure 6: Histogram of SIAS Global Equity Portfolio Returns from 04/08/2004 to 05/28/2010 (1546 samples)....39

Figure 7: Canadian Equity P&L and 1% Historical Simulation VaR from 04/30/2003 to 05/31/2010 40

Figure 8: Canadian Equity P&L and 1% DCC GARCH VaR from 04/30/2003 to 05/31/2010 40

Figure 9: Canadian Equity P&L and 1% Filtered HS VaR from 04/30/2003 to 05/31/2010 40

Figure 10: Canadian Equity P&L and 1% Hybrid Approach VaR from 04/30/2003 to 05/31/2010 41

Figure 11: Canadian Equity P&L and 5% Historical Simulation VaR from 04/30/2003 to 05/31/2010 41

Figure 12: Canadian Equity P&L and 5% DCC GARCH VaR from 04/30/2003 to 05/31/2010 41

Figure 13: Canadian Equity P&L and 5% Filtered HS VaR from 04/30/2003 to 05/31/2010 42

Figure 14: Canadian Equity P&L and 5% Hybrid Approach VaR from 04/30/2003 to 05/31/2010 42

Figure 15: Global Equity P&L and 1% Historical Simulation VaR from 04/08/2004 to 05/28/2010 42

Figure 16: Global Equity P&L and 1% DCC GARCH VaR from 04/08/2004 to 05/28/2010 43

Figure 17: Global Equity P&L and 1% Filtered HS VaR from 04/08/2004 to 05/28/2010....43

Figure 18: Global Equity P&L and 1% Hybrid Approach VaR from 04/08/2003 to 05/28/2010 43

Figure 19: Global Equity P&L and 5% Historical Simulation VaR from 04/08/2004 to 05/28/2010 44

Figure 20: Global Equity P&L and 5% DCC GARCH VaR from 04/08/2004 to 05/28/2010 44

Figure 21: Global Equity P&L and 5% Filtered HS VaR from 04/08/2004 to 05/28/2010 44

Figure 22: Global Equity P&L and 5% Hybrid Approach VaR from 04/08/2003 to 05/28/2010 45

Figure 23: Canadian Equity P&L and 1-Year Hybrid Approach VaR from 01/01/2007 to 05/31/2010 45

Figure 24: Canadian Equity P&L and 2-Year Hybrid Approach VaR from 01/01/2007 to 05/31/2010 45

Figure 25: Global Equity P&L and 1-Year Hybrid Approach VaR from 01/01/2007 to 05/28/2010 46

Figure 26: Global Equity P&L and 2-Year Hybrid Approach VaR from 01/01/2007 to 05/28/2010 46

Figure 27: Canadian Equity P&L and 1-Year Hybrid Approach VaR with Interpolation from 01/01/2007 to 05/31/2010 47

Figure 28: Canadian Equity P&L and 2-Year Hybrid Approach VaR with Interpolation from 01/01/2007 to 05/31/2010 47

Figure 29: Global Equity P&L and 1-Year Hybrid Approach VaR with Interpolation from 01/01/2007 to 05/28/2010 48

Figure 30: Global Equity P&L and 2-Year Hybrid Approach VaR with Interpolation from 01/01/2007 to 05/28/2010 48

Figure 31: Canadian Equity P&L and 1% / 1-Year HB VaR with Different λ from 01/01/2007 to 05/31/2010 49

Figure 32: Canadian Equity P&L and 1% / 2-Year HB VaR with Different λ from 01/01/2007 to 05/31/2010 49

Figure 33: Canadian Equity P&L and 5% / 1-Year HB VaR with Different λ from 01/01/2007 to 05/31/2010 50

Figure 34: Canadian Equity P&L and 5% / 2-Year HB VaR with Different λ from 01/01/2007 to 05/31/2010 50

Figure 35: Global Equity P&L and 1% / 1-Year Hybrid Approach VaR with Different λ from 01/01/2007 to 05/31/2010 51

Figure 36: Global Equity P&L and 1% / 2-Year Hybrid Approach VaR with Different λ from 01/01/2007 to 05/31/2010 51

Figure 37: Global Equity P&L and 5% / 1-Year Hybrid Approach VaR with Different λ from 01/01/2007 to 05/31/2010 52

Figure 38: Global Equity P&L and 5% / 2-Year Hybrid Approach VaR with Different λ from 01/01/2007 to 05/31/2010 52

Figure 39: Total Equity Portfolio P&L and 1% Diversified Historical Simulation VaR from 01/03/2006 to 05/28/2010 53

Figure 40: Total Equity Portfolio P&L and 1% Diversified DCC GARCH VaR from 01/03/2006 to 05/28/2010 53

Figure 41: Total Equity Portfolio P&L and 1% Diversified Filtered HS VaR from 01/03/2006 to 05/28/2010 53

Figure 42: Total Equity Portfolio P&L and 1% Diversified Hybrid Approach VaR from 01/03/2006 to 05/28/2010 54

Figure 43: Total Equity Portfolio P&L and 5% Diversified Historical Simulation VaR from 01/03/2006 to 05/28/2010 54

Figure 44: Total Equity Portfolio P&L and 5% Diversified DCC GARCH VaR from 01/03/2006 to 05/28/2010 54

Figure 45: Total Equity Portfolio P&L and 5% Diversified Filtered HS VaR from 01/03/2006 to 05/28/2010 55

Figure 46: Total Equity Portfolio P&L and 5% Diversified Hybrid Approach VaR from 01/03/2006 to 05/28/2010 55

List of Tables

Table 1: Summary of Assigned Weights of Hybrid Approach..13

Table 2: Summary of Conditional Probability of Exceptions 18

Table 3: Selected Periods of Hybrid Approach VaR without Interpolation 56

Table 4: Selected Periods of Hybrid Approach VaR with Interpolation 57

Table 5: Statistics Summary of 1% VaR of Canadian Equity Portfolio.......................................58

Table 6: Statistics Summary of 5% VaR of Canadian Equity Portfolio.......................................59

Table 7: Statistics Summary of 1% VaR of Global Equity Portfolio 60

Table 8: Statistics Summary of 5% VaR of Global Equity Portfolio 61

Table 9: Summary of Quantile Regression Test Results for 1% VaR of Canadian Equity Portfolio 62

Table 10: Summary of Quantile Regression Test Results for 5% VaR of Canadian Equity Portfolio 63

Table 11: Summary of Quantile Regression Test Results for 1% VaR of Global Equity Portfolio 64

Table 12: Summary of Quantile Regression Test Results for 5% VaR of Global Equity Portfolio 65

Table 13: Summary of Speed and Accuracy of VaR Models in Canadian Equity Portfolio.........66

Table 14: Summary of Speed and Accuracy of VaR Models in Global Equity Portfolio.............66

Table 15: Summary of Diversification of the Equity Portfolio 67

1: Introduction

Value-at-Risk (VaR) plays a big part in the risk management of financial and non-financial institutions in today's world. The popularity of VaR is due to its simple interpretation as it quantifies the size of future losses in any currency at a predetermined probability. This model gained wide acceptance in the early 90's by banks, and later in 1996 was amended to meet the requirements of the Basel Accord. VaR is one of three ways in which banks report market risk capital requirements. In 2006, an enhanced version of regulation, Basel II, has been introduced to improve the risk disclosure and supervision of financial firms with the three pillars and further taking operational risk factor into account. Unlike banks that have a mandate to adhere to the Basel II Accord, an increasing number of asset management firms such as Canaccord Wealth Management quickly adopted the VaR model to manage their market risk exposures. However, VaR models impose strong assumptions about the underlying data as it assumes that the density function of daily return follow a normal distribution with a constant mean and variance (Barone-Adesi et al., 2002). This assumption is flawed as empirical evidence suggests that stock returns do not follow a normal distribution. VaR has been criticized that it assumes that extreme events are rare in nature but as we saw in 2008, this was not the case. Another criticism is that VaR gives firms false confidence about their financial health, and as a result, firms might take on excessive risks or leverage assuming they are safe from any undesirable consequences. In spite of these criticisms, the VaR model is the first and most developed model to date which can be improved to capture extreme events in the future. In 1997, Philippe Jorion wrote:

> *"The greatest benefit of VaR lies in the imposition of a structured methodology for critically thinking about risk. Institutions that go through the process of computing their VaR are forced to confront their exposure to financial risks and to set up a proper risk management function. Thus the process of getting to VaR may be as important as the number itself." (Philippe Jorion – Value at Risk 2007, p. xi)*

In addition, there has been an increasing trend of reporting each individual line item of market risk factors, including equity, interest rate, commodity, credit, and foreign exchange risk, thus promoting higher transparency to the public (Pérignon and Smith 2010).

For these reasons, we believe that the VaR model is adequate, in terms of aggregating risk factors in a portfolio composed of different asset classes to closely manage and report the overall risk exposure.

Even though VaR has become the standard for risk assessment, the majority of financial institutions are incapable of estimating VaR precisely and tend to overestimate reported VaR. Berkowitz and O'Brien (2002) investigated large banks adhering to Basel regulation, and indicated that their VaR model specification was inadequate. Pérignon, C., Deng, Z.Y., and Wang, Z.Y., (2008) showed that Canadian banks are too conservative in estimating VaR, and this may be due to imposed penalties by regulators. The other explanation is that these banks fail to consider the diversification effect among different categories of risk factors associated with their assets or investments (Berkowitzand O'Brien, 2002; Pérignon et al., 2008; Pérignon and Smith, 2010), which also somehow violates the sub-additive rule by aggregating the risk exposure with the basic sum of each individual VaR without considering their correlation.

Similar to the other investment funds, the Student Investment Advisory Services (SIAS) fund of Simon Fraser University (SFU) was initiated in the year of 2003 as a part of the Global Asset and Wealth Management (GAWM) MBA program. Originally, the fund consisted of Canadian Equity and Canadian Fixed Income. After several revisions of the Investment Policy Statement (IPS), the dynamic of the funds expanded with an additional asset class, including Global Equity, mainly U.S stocks and ETF's. In 2003, few students from the GAWM MBA program were involved in managing the fund. The fund originally started with a $5.6 million contribution and has grown to around $9.5 million in 2010, in spite of going through a tough period during the economic crisis of 2008. Up until 2009, the main focus of the students managing the fund was on equity and bond research, portfolio allocations, performance measurement, updating policies and procedures and formulating the IPS. In 2005, the Master of Financial Risk Management (MFRM) was established and the MFRM students have participated in the management of the SIAS fund side by side with the GAWM MBA students. In 2009-2010, the GAWM MBA and MFRM merged and the MFRM class was fully in charge of the fund.

Up until now, there has been a minimal effort for formulating a RiskMetrics procedure for the SIAS portfolio. In the past, risk measures were focused on individual stock volatility based on the traditional formulas, industry risk and credit ratings of bonds but an overall look at the risk modelling of the SIAS portfolio was lacking. In this paper, we present the procedures to select and backtest VaR models for measuring the risk of the SIAS equity portfolios, Canadian and Global, in which we also taking into account the pros and cons of various VaR methodologies. We believe that this paper will be beneficial, in terms of quantifying risk that the SIAS portfolio is exposed to at different time intervals and will enable future students to make better-informed decisions.

This paper proceeds as following. In Section 2, we give a brief overview about the operation and management of the SIAS fund. In Section 3, we discuss the different selected VaR models and the technique used taking into account diversification. In Section 4, we discuss the data we used. In Section 5, the backtesting procedures are discussed. In Section 6, we implement and backtest the models, and a quick analysis of the speed and accuracy of the selected models is presented. In Section 7, we implement the technique to aggregate the VaRs of different portfolios. Finally, we conclude in Section 8.

2: Overview of SIAS Fund

As mentioned, the SIAS fund was established from 2003, giving the GAWM MBA graduates, and a few selected elite undergraduates' hands on experiences managing a fund according to the pre-specified rules in IPS (see Figure 1). The management of the fund is divided into 7 teams: Canadian Equity, Global Equity, Fixed Income, Compliance, Trading, Economics, and RiskMetrics. The heads and fund supervisor (i.e. faculty members) have regular meetings to make top-level strategic decisions of the asset mix of the portfolio based on the collaborative efforts from different functional teams. For example, the economists provide the overall economic outlooks (e.g. interest rates), while heads of equity and fixed income provide the overall research results from their analysts. The variation of the asset mix is strictly defined in the IPS, such that Canadian Equity is 35%, Global Equity is 35%, Fixed Income is 28% and the remaining Cash is 2%. The IPS allows ± 10% deviation from each asset class, depending on the strategic decisions.

At the functional level, the heads are responsible to determine the strategic weights (i.e. underweigh/overweigh) of each sector and to co-ordinate and communicate with their analysts research assignments. As the SIAS fund follows a value investment philosophy, mostly, the research is done with a thoughtful top-down approach analyzing the industry, company's management, business fundamentals, and the valuation based on value criteria, and then it is summarized in few-page reports along with recommendations. Those potential buys or sells have to be approved by Compliance according to the IPS that clearly states the constraints regarding to the criteria for selecting investment opportunities, the limits of initial holdings, and the minimum required allocation to each sector. Then, all the participations of the SIAS fund would vote on the approved transactions, and the trader would execute the orders. Due to these lengthy processes, the trading frequency is more likely to occur once per quarter, and the turnover of holdings tend to be rigid, which is typical for a value fund.

Besides all these rules, IPS also establishes the specific goals that the fund should fulfil going forward. For instance, Canadian Equity portfolio has a mandate of 1.5% above the S&P/TSX as the benchmark. The Global Equities portfolio has a mandate of 2% the MSCI/Barra Ex Canada as the benchmark. The Fixed Income portfolio is compared to the DEX universe as a benchmark plus 0.4%. Other restrictions imposed on the SIAS portfolio include no margins, short positions, or derivatives related investment.

Every quarter, the review of the SIAS' performance is formally held. The SFU treasurers, sponsors, custodians, and industry members, including HSBC, CIBC, BCIMC, and more, are invited. In this gathering, the presenters would review how effectively the SIAS team managed the fund in the previous quarter. The compliance presenter will evaluate the past strategies based on the performance attribution and some simple risk-adjusted performance measurement such as Sharpe ratio or Treynor ratio. With the support of the Economists, the presenters of the three asset classes would present strategies going forward.

The SIAS fund have steadily grow from 5.6 million to 9.5 million from 2003 to 2010, and according to the treasurer, the SIAS fund has outperformed other student endowment funds in North American based on a 5-year performance measure. As the fund grew and revisions of IPS were implemented, the portfolio holdings have become more diversified and enriched. As shown in Figure 2, the average total holdings from 2003 to 2004 increased from 45 holdings, majority from Canadian Equities, to approximately 90 holdings after the revised IPS, which required exposure in the global markets. In addition, the Equity portfolios are more likely to be active managed, whereas the Fixed Income portfolio is passively managed; a simple buy and hold to maturity strategy. Figure 2 reveals a change in strategy to underweigh fixed income but overweigh equity after the economic crisis of 2008. During the catastrophic sub-prime crisis, all markets almost collapsed. From August 2008 to February 2009, the S&P/ TSX Composite Index and S&P 500 index went down by 41% and 47% respectively, but the SIAS fund only decreased by 26% from 10 to 7.4 million. This demonstrates the effectiveness of the value investment philosophy, in which selected value brand name stocks, and large established firms could survive even during the crisis.

Up until now, we realize that the effort put into risk management for the entire portfolio is not sufficient even though the IPS has restrictions on position limits in each individual stock and sector, and the investment philosophy tends to be protective of the entire fund. In addition, those required risk adjusted performance measurements such as Sharpe ratio and Treynor ratio are flawed because the volatility cannot be estimated properly based on the generic formula. Nevertheless, a standardized procedure in risk management has been lacking. Some cohorts attempted to establish Mean-Variance analysis, but it is not sustainable due to problems in forecasting volatility. Thus, to make the entire management team more complete, a proper risk model and procedures should be launched, so that the precise statistics of risk factors can be provided and the fund can be monitored more effectively and not only based on returns. Therefore, we would like to consider the potential VaR models that have intuitive representations in the risk factors and other extendable properties that can be used in more detailed risk management, and perhaps form future strategies.

3: Methodology

VaR is defined as the statistical estimated worst loss, given a level of confidence interval($\boldsymbol{c}$), volatility of risk factor ($\boldsymbol{\sigma}$), and mark-to-market portfolio value ($\boldsymbol{W}$), over a horizon of time ($\sqrt{\boldsymbol{t}}$):

$$\boldsymbol{VaR = \alpha\sigma}\mathbf{W}\sqrt{\boldsymbol{t}} \tag{1}$$

where α is the targeted probability that the actual loss would exceed the VaR, which is equivalent to 1 – c (e.g. 99% confident interval is equal to 1% VaR). VaR is driven by two most essential components of the equation (1): the estimated risk factor volatility and the confidence level for a pre-specified distribution. Thus, different quantitative assessments of these two contribute to the two main classes of VaR – the parametric and non-parametric approaches. The former focuses on forecasting the volatility with various alternatives of econometrics and statistical tools, while the later focuses on the empirical distribution of the portfolio returns and determines VaR based on the quantile of the distribution. Beside these two classes, some other models tend to take the advantages from both, which is so-called semi-parametric. To select the potential VaR models, we consider ones from various classes as mentioned in the following.

3.1 Historical Simulation

Historical Simulation is characterized as the non-parametric approach, and it is by far the most commonly used method for the financial industry worldwide (Pritsker, 2006). Pérignon and Smith (2010) reported that 73% of banks employing this method to determine their risk measurements. The wide acceptance may be due to its application simplicity by taking the percentile of the formed distribution, in addition to other practical properties. One advantage this method is that it does not make assumptions about the distribution (e.g. normal distribution). Realistically, financial data rarely fit the parameters of a normal distribution and usually exhibit fat tails, skewness and unstable correlations which make the VAR estimates unreliable. This approach simply lets the data state the shape of the distribution, thus allowing the distribution to take into consideration all these concerns. However, HS approach suffers from two main problems: Extreme percentiles are hard to estimate with little data and it assumes that returns are independent and identically distributed (i.i.d.), but indeed, most returns are subject to the serial correlation and heteroskedasticity issues. Nevertheless, this method is criticized for its inability of capturing volatility and therefore, leads to biased VaR estimates. (Pérignon and Smith, 2010). Also, Van den Goorbergh and Vlaar (1999) and Vlaar (2000) claimed that VaR model using the HS approach is not capable to reflect the volatility dynamic appropriately because it assumes the weights assigned to the most recent and distant observations are flat.

The steps to implement the HS model as the following:

1. Simulate the portfolio return distribution with the fixed positions of assets at the date used in computing the VaR with past 1-year prices
2. Take the percentile of the simulated distribution (i.e. 1 % and 5% VaR)

3.2 Dynamic Conditional Correlation GACRH

Dynamic Conditional Correlation Generalized AutoRegressive Conditional Heteroskedastic Model (DCC GARCH) of Engle (2002) is categorized as the parametric approach, and belongs to multivariate model family endeavouring to estimate the correlations and form entire covariance matrix of a portfolio with multiple-assets. The two main problems that the multivariate models have been facing: (1) The number of the estimators significantly increases as the number of assets increases within the portfolio, which leads to computational burden. (2) The estimated covariance matrix must be positive semi-definite (psd.) to produce sensible results of the portfolio volatility.

The revolution of multivariate models is motivated by resolving those two problems, and also relaxing some unrealistic simplicity such as constant correlations. Initiating with the VECH model of Bollerslev, Engle, and Wooldridge (1988), Constant Correlation (CC) model of Bollerslev (1990), VEC model of Engle and Kroner (1995), those models seriously suffer from the both problems of proliferating estimated parameters and not guaranteeing the psd. covariance matrix. Soon after, BEKK model Engle and Kroner

(1995) and FlexMGARCH model of Ledoit, Santa-Clara and Wolfe (2004) were proposed to remedy the problem of psd. covariance matrix. Furthermore, DCC GARCH of Engle (2002) has been introduced, and contributed to a more realistic covariance forecasting by allowing the time-varying correlations and relatively fewer number of parameters to be estimated. Numerous literatures have been providing nice reviews and discussions of the multivariate models (Engle and Sheppard, (2001), Bauwens et al. (2003), Patton and Sheppard, (2007), and Smith (2010)).

DCC GARCH employs the techniques from CC model by decomposing the covariance matrix ($\boldsymbol{H_t}$) into the diagonal matrix($\boldsymbol{D_t}$), where the conditional standard deviations located diagonally, and the conditional correlation matrix ($\boldsymbol{R_t}$). Then, with respected to log-likelihood returns, the route is to apply Maximum Likelihood Estimation (MLE) to not only $\boldsymbol{D_t}$ but also $\boldsymbol{R_t}$ based upon $\boldsymbol{D_t}$ to allow time varying correlations. Thus, the returns is multivariate distributed relatively to $\boldsymbol{H_t}$.

$$\boldsymbol{r_t \sim MVN(0, H_t)} \tag{2}$$

$$\boldsymbol{H_t = D_t R_t D_t} \tag{3}$$

$\boldsymbol{D_t}$ can be estimated through the general multivariate GARCH process.

$$\boldsymbol{D_t^2 = diag(\omega_i) + diag(\alpha_i) \odot (r_{t-1} r'_{t-1}) + diag(\beta_i) D_{t-1}^2} \tag{4}$$

where $\odot$ is the element to element mortification operator, and R_t can be expressed as:

$$\boldsymbol{R_t = diag(Q_t)^{-1/2}\, Q_t diag(Q_t)^{-1/2}} \tag{5}$$

$\boldsymbol{Q_t}$, the conditional covariance matrix, is modelled based on the diagonal VECH model

$$\boldsymbol{Q_t = S \odot (\iota\, \iota' - A - B) + A \odot (e_{t-1} e'_{t-1}) + B \odot Q_{t-1}} \tag{6}$$

$$\mathbf{e_t = \varepsilon_t / \sqrt{h_t}} \tag{7}$$

where $\boldsymbol{S}$ is the unconditional covariance matrix, $\boldsymbol{e}$ is the standardized residuals, and $\boldsymbol{h}$ conditional variance. Based on above, the log-likelihood function can be stated and decomposed as two below. Then, systematically using MLE estimates the conditional variances first, and estimates the conditional correlations based upon.

$$\boldsymbol{L(\{r_t\};\ \theta_h, \theta_r) = -\frac{1}{2}\sum_{t=1}^{T}[n \log(2\pi) + \log|H_t| + r'_t H_t^{-1} r_t]}$$

$$\boldsymbol{= -\tfrac{1}{2}\textstyle\sum_{t=1}^{T}[n \log(2\pi) + \log|D_t| + r'_t D_t^{-2} r_t - e'_t e_t + \log|R_t| + e'_t R_t^{-1} e_t]}$$

$$= \boldsymbol{L_H(\{r_t\};\ \theta_h) + L_R(\{r_t\}; \theta_r)} \qquad \textbf{(8)}$$

where $\boldsymbol{\theta_h}$ and $\boldsymbol{\theta_r}$ are the describers of the parameters of conditional variances and correlations respectively. Then, maximizing the $\boldsymbol{L_H}$ first,

$$\boldsymbol{L_H(\{r_t\};\ \theta_h) = -\frac{1}{2}\sum_{t=1}^{T}\left[n\log(2\pi) + \log\left|D_t^2\right| + r_t'D_t^{-2}r_t\right]}$$

$$\boldsymbol{= \sum_{i=1}^{n}\left\{-\frac{1}{2}\sum_{t=1}^{T}\left(\log(2\pi) + \log(h_{it}) + \frac{r_{it}^2}{h_{it}}\right)\right\}} \qquad \textbf{(9)}$$

Then, maximizing the $\boldsymbol{L_R}$ based upon the conditional variances systematically

$$\boldsymbol{L_R(\{r_t\}; \theta_r) = -\frac{1}{2}\sum_{t=1}^{T}[\log|R_t| + e_t'R_t^{-1}e_t - e_t'e_t]} \qquad \textbf{(10)}$$

Smith (2010) summarized the derivations above, and Kevin Sheppard provides the exact algorithm as above to implement the DCC GARCH in the UCSD GARCH Toolbox[1] for Matlab. DCC GARCH (1, 1) is our specification of the model. Due to the fact that DCC GARCH is highly sensitive, we input the data that at least contains 1-year historical returns to compute VaRs. This is why some holdings are excluded since they cannot satisfy this criterion as mentioned in Section 2. In other words, DCC GARCH is flawed in handling the new name company or spin-off that do not have sufficient historical information for a pre-specified amount of data required.

Once the conditional covariance matrix is estimated, the portfolio volatility can be computed as:

$$\boldsymbol{\sigma_t = \sqrt{w_t'H_tw_t}} \qquad \textbf{(11)}$$

where $\boldsymbol{w}$ is the column vector for the individual holding weights within the portfolio. Thus, percentage VaR can be calculated by applying equation (1), while dollar VaR can be calculated as:

$$\boldsymbol{VaR(dollar) = \alpha\sqrt{x'Hx}} \qquad \textbf{(12)}$$

where $\boldsymbol{x}$ is the column vector of the value of individual asset.

[1] UCSD Toolbox by Kevin Sheppard can be download from
http://www.kevinsheppard.com/wiki/UCSD_GARCH

3.3 Filtered Historical Simulation

As mentioned previously, the traditional HS has some serious drawbacks as it makes very strong assumption that the asset returns are i.i.d., which is rarely the case. From empirical evidence, asset returns exhibit patterns of volatility clustering as we saw in the 2008 economic crisis. In addition, HS ignores the fact that asset risks are changing all the time. In order to overcome these deficiencies, Filtered Historical Simulation (FHS) has been proposed and extended in order to take into account the changes in past and present volatilities of historical returns and makes the least number of assumptions about the statistical properties of future return distribution (Barone-Adesi and Giannopoulos, 1996, Barone-Adesi, Bourgoin and Giannopoulos, 1998, Barone-Adesi, Giannopoulos, and Vosper, 1999, and Barone-Adesi, Giannopoulos, and Vosper, 2002). FHS is one of the semi-parametric approaches, in the way that it employs the parametric method (e.g. GARCH) to capture the volatility dynamics, and non-parametric method to reconstruct a cumulative distribution, which is similar to HS. The intuition of FHS is simple. It estimates the conditional volatilities associated to each time series of historical returns, and then standardizes those returns to strip the effects of time varying risk factors, serial correlations, and heteroskedasticity so that the historical standardized returns become close to i.i.d.. Then these standardised historical returns are scaled by forecasted volatility and the results are used to generate scenarios for computing portfolio VaR. FHS is proceed in the following steps and specifications.

a) Computing historical portfolio returns $(\boldsymbol{R_t})$

b) Estimating conditional variance $(\boldsymbol{h_t})$ based on GARCH(1,1) through the time series of portfolio returns

$$\boldsymbol{h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}} \tag{13}$$

c) Standardizing the historical residuals $(\boldsymbol{\varepsilon_t})$ corresponding to $(\boldsymbol{h_t})$. Since none of the specification for the return forecasting model is imposed. Thus,

$$\boldsymbol{R_t = \varepsilon_t} \tag{14}$$

$$\boldsymbol{\varepsilon_t \sim N(0, h_t)} \tag{15}$$

$$\boldsymbol{e_t = \varepsilon_t / \sqrt{h_t}} \tag{16}$$

d) Scaling the standardized residuals $(\boldsymbol{e})$ based on the forecasted conditional volatility one day forward so that we have a pool of the $\boldsymbol{z}$

$$\boldsymbol{z_{t+1} = e\sqrt{h_{t+1}}} \quad \textbf{(17)}$$

e) Constructing the cumulative distribution based on $\boldsymbol{z}$ and taking the percentile to determine the VaR (i.e. 1% and 5% VaR). Many alternatives to form the distribution could be considered, and here we simply use the 252 days moving windows to draw risk adjusted returns to construct the distribution

3.4 Hybrid Approach

The Hybrid Approach is an interesting semi-parametric VaR methodology that combines the methodologies of two popular approaches; Historical Simulation and exponential smoothing (Richardson, Boudoukh, & Whitelaw, 1997). Having given enough details regarding HS, the exponential smoothing approach, on the other hand, assigns predetermined declining weights to past returns and volatilities to forecast the conditional volatility. The most classic example is RiskMetrics (RM) approach introduced by JP Morgan (1994), in which the conditional variance is compute as,

$$\boldsymbol{h_t = \lambda h_{t-1} + (1-\lambda)R_{t-1}^2} \quad \textbf{(18)}$$

where $\boldsymbol{\lambda}$ is the predetermined decay factors, (i.e. 0.94 for daily data and 0.96 for monthly data), $\boldsymbol{h}$ is the conditional variance, and **R** is the returns of the portfolio. RM follows a recursive process to calculate the conditional volatility, so this implies that RM has memory for the past volatility dynamics but less significant in distant and depends on the decay factors.

However, HB, instead assigning declining weights to the past conditional volatilities, assigns weights to the historical returns so that it could capture the current market dynamics more, which may deviate from the parametric model, in the sense of mathematical computation to estimate volatility but the intuition is similar by weighting current market information more. As shown on Figures 3 and 4, the assigned weights relatively to the different $\boldsymbol{\lambda}$ reveal different diminishing rates. The lower the $\boldsymbol{\lambda}$ is, the faster the weights are declining. On the other hand, when considering different sample (i.e. 252 trading day vs. 504 trading days), the impact on the weights seems insignificant.

The weights tend to drop at the same magnitude at the first 50 days. This may suggest that the actual returns that occurred within this past 50-day period are influential, especially the negative returns. Moreover, combining this finding with the fact that $\boldsymbol{\lambda}$ also determine the first assigned weight as $(\mathbf{1-\lambda})$**,** it is conclusive that the interaction between selected $\boldsymbol{\lambda}$ and the target failure rates could be significant to the measurement of VaR. HB is similar to HS in the method of constructing the distribution based on the historical returns, but with their attached weights. HB is implemented in following three steps.

a) Simulate and portfolio returns based on the fixed positions at that date and with their historical prices
b) Assigning weights to the most recent $(\boldsymbol{R_t})$ to most distant return $(\boldsymbol{R_{t-K+1}})$ in a predetermined period and making sure weights add up to 1. In our specification, we select the 1-year moving window as the predetermined period, in total 252 observations $(\boldsymbol{K})$, and the decay factor $\boldsymbol{\lambda}$ is 0.94. The weights are assigned as shown in Table 1.

Table 1: *Summary of Assigned Weights of Hybrid Approach*

Days After	Realized Returns	Assigned Weights
0	R_t	$[(1-\lambda)/(1-\lambda^K)]\lambda^0$
1	R_{t-1}	$[(1-\lambda)/(1-\lambda^K)]\lambda^1$
2	R_{t-2}	$[(1-\lambda)/(1-\lambda^K)]\lambda^2$
⋮	⋮	⋮
⋮	⋮	⋮
K-1	R_{t-K+1}	$[(1-\lambda)/(1-\lambda^K)]\lambda^{K-1}$

c) Ascending the order based on the returns and attached weights align to the original corresponding returns.
d) Starting with the lowest returns and accumulating weights attached to them until approaching the target x% VaR of the portfolio. **Note that** our original specification is that the VaR is determined at the time, where the summed

probability exceeds the target percentile, and later linear interpolation is applied, which is discussed in the Section 4 in details.

3.5 Diversification

As mentioned, many studies presented empirical results that stress the importance of diversified VaR for different asset classes or lines of business into an aggregated level of VaR. On the other hand, Berkowitz, Christoffersen, and Pelletier (forthcoming) encouraged studying the interactions among different risk categories by decomposing the aggregated level of VaR. All these recommendations help to promote the higher level of quality of risk reporting and monitoring process. The diversification ($\boldsymbol{d}$) of a portfolio is basically the relative ratio of the undiversified VaR ($\boldsymbol{UDVaR}$) to the diversified ($\boldsymbol{DVaR}$) (Jorion, 2007, p. 162-165):

$$\boldsymbol{UDVaR = \sum_{i=1}^{n} VaR_i} \qquad \textbf{(19)}$$

$$\boldsymbol{VaR_i = \alpha\sigma_i w_i \mathrm{W} = \alpha\sigma_i x_i} \qquad \textbf{(20)}$$

where $\boldsymbol{w_i}$is the weight of each asset, $\boldsymbol{W}$ is the total portfolio value, and $\boldsymbol{x_i}$ is the value of the each asset in the portfolio. $\boldsymbol{UDVaR}$ is the sum of individual VaR in dollar term for **N** assets. Thus, the diversification effect of the portfolio is defined as (Pérignon and Smith (2010)):

$$\boldsymbol{d = \frac{UDVaR - DVaR}{UDVaR}} \qquad \textbf{(21)}$$

$\boldsymbol{DVaR}$ can be computed as following:

$$\boldsymbol{DVaR = \alpha\sqrt{x'Hx}} \qquad \textbf{(22)}$$

where $\boldsymbol{x}$ is the column vector of the value of individual asset, and $\boldsymbol{H}$ is the covariance matrix among assets, which can be decomposed as:

$$\boldsymbol{H = DRD} \qquad \textbf{(23)}$$

where $\boldsymbol{D}$ is the diagonal standard deviation matrix, and $\boldsymbol{R}$ again is the correlation matrix. Thus,

$$\boldsymbol{DVaR = \alpha\sqrt{x'DRDx}} \qquad \textbf{(24)}$$

$$\boldsymbol{DVaR = \sqrt{\alpha^2 x'DRDx}} \qquad \textbf{(25)}$$

$$\boldsymbol{DVaR = \sqrt{V'RV}} \qquad \textbf{(26)}$$

where $\boldsymbol{V}$ is the column vector of each individual VaR of assets. Therefore, the ultimate goal is to estimated the correlation matrix, and then determine the diversification of an overall portfolio. Pérignon and Smith (2010) suggested various alternatives, including the BEKK, DCC GARCH and copula models. Among those, DCC GARCH is one and only one of our selected models can perform the tasks, so the procedure is the following:

a) Estimating the conditional covariance matrices ($\boldsymbol{H}$) based on DCC GARCH (1,1) with the two time series of VaRs from Canadian and Global Equity portfolios with difference models
b) Converting the $\boldsymbol{H}$ into the correlation matrices ($\boldsymbol{R}$)
c) Determining the $\boldsymbol{DVaR}$, and computing diversification ($\boldsymbol{d}$) of the overall equity portfolio

4: Data

Historical holdings data is gathered from CIBC's Workbench monthly audited reports. For the Canadian Equity portfolio, the trading day data is from April 30, 2003 to May 31 2010 for a total of 1782 samples. As for the Global Equity portfolio, trading day data is from April 8, 2004 to May 28, 2010 for a total 1546 samples.

As the setup of the risk model uses in-sample data and requires at least 1-year of past data, some holdings that do not meet the criteria are excluded, and other holdings are ignored due to the fact that the firm had changed its ticker or merged with another company. Our data gathering focuses on multiple sources including Bloomberg, Yahoo Finance, and Google Finance. The total number of exclusions from our data adds up to eight, six[2] of which were from the Canadian Equity portfolio and two[3] from the Global equity portfolio. Thus, our data is based on 78 historical holdings for the Canadian Equity portfolio and 76 historical holdings for Global Equity portfolio.

All the closing prices, except for the price of purchase or sale, and cash flows such as dividends, interest payments and permanent realized long term or short capital gains attached to the holdings are extracted from Bloomberg. Daily stock returns are calculated based upon all the cash flow factors and are adjusted with stock dividends, stock split, and spin off, and the U.S currency denominated investments are converted into Canadian currency corresponding to the exchange rate at the date.

[2] Cenovus Energy Inc, Fraser Paper Inc, and Novelis Inc/GA are excluded due to spin-off and the holding period shorter than 1 year. MI Developments Inc is excluded due to incomplete data. Bell Aliant Regional Communications Income, and Grande Cache Coal Corp are excluded due to new issue.

[3] US Physical Therapy Inc and Valley Forge Corp are excluded because historical information is not available in Bloomberg and other website resources such as Yahoo!Finance.

5: Backtesting

5.1 Unconditional Coverage, Independence, and Conditional Coverage Test

In order to verify the specifications and accuracy of select VaR models, we apply the most widely used log-likelihood ratio backtesting procedures. Unconditional Coverage test ($\mathbf{LR_{uc}}$) is basically testing the specification of the confidence level $(\mathbf{c})$ to compute $\mathbf{p}$% VaR ($\mathbf{p} = \mathbf{1} - \mathbf{c}$, i.e. 1% and 5% VaR) whether it is unbiased relatively to the actual exceptions ($\mathbf{N}$), where realized returns over the estimated VaRs over the sampled period ($\mathbf{T}$) (Kupiec, 1995). In other word, this is to test how likely the null hypothesis of failure rate ($\frac{\mathbf{N}}{\mathbf{T}}$) aligned to the expected probability $(\mathbf{p})$ of exceptions is true, which could be performed as the following log-likelihood ratio equation (Jorion, 2006, p. 147-151), and the result is asymptotical to the χ^2 distribution with one degree of freedom:

$$\mathbf{LR_{uc}} = -\mathbf{2}\,\mathbf{ln}[(\mathbf{1}-\mathbf{p})^{\mathbf{T}-\mathbf{N}}\mathbf{p}^{\mathbf{N}}] + \mathbf{2}\,\mathbf{ln}\left\{\left[\mathbf{1}-\frac{\mathbf{N}}{\mathbf{T}}\right]^{\mathbf{T}-\mathbf{N}}\left[\frac{\mathbf{N}}{\mathbf{T}}\right]^{\mathbf{N}}\right\} \qquad \mathbf{(27)}$$

In fact, the unconditional test has its limitations. For instance, it cannot produce adequate results with insufficient samples. Also, it ignores the clustering of the exceptions. Especially testing the period involving several fluctuations or economic shocks, it may generate misleading results that are subjected to higher type I (rejecting the good model) and type II (accepting the bad model) errors. To solve the clustering problem, Christoffersen (1998) enhanced the unconditional coverage test with an additional independence test ($\mathbf{LR_{ind}}$) of the exception, and the Conditional Coverage test **($\mathbf{LR_{cc}}$)** has been proposed as the sum of the two:

$$\mathbf{LR_{ind}} = -\mathbf{2}\,\mathbf{ln}\left[(\mathbf{1}-\boldsymbol{\pi})^{(\mathbf{T_{00}}+\mathbf{T_{10}})}\boldsymbol{\pi}^{(\mathbf{T_{01}}+\mathbf{T_{11}})}\right] + \mathbf{2}\,\mathbf{ln}[(\mathbf{1}-\boldsymbol{\pi}_{\mathbf{0}})^{\mathbf{T_{00}}}\boldsymbol{\pi}_{\mathbf{0}}^{\mathbf{T_{01}}}(\mathbf{1}-\boldsymbol{\pi}_{\mathbf{1}})^{\mathbf{T_{00}}}\boldsymbol{\pi}_{\mathbf{1}}^{\mathbf{T_{11}}}] \qquad \mathbf{(28)}$$

$$\mathbf{LR_{cc}} = \mathbf{LR_{uc}} + \mathbf{LR_{ind}} \qquad \mathbf{(29)}$$

where $\mathbf{T}$ and $\boldsymbol{\pi}$ are the conditional counts of the number of exceptions and probabilities of the exception occurrences , which is summarized in Table 2 (Jorion, 2006, p. 152), in which the subscripts are represented the conditions of previous day and current day (e.g. $\mathbf{T_{01}}$ is the counts of the sample that has no exception yesterday and has exception today). The idea is that if the exceptions do not tend to herd and to spread out equally regardless what the happened before, the conditional probabilities of $\boldsymbol{\pi_0}, \boldsymbol{\pi_1}$, and $\boldsymbol{\pi}$ would be similar, which is the null of this test. The $\mathbf{LR_{ind}}$ itself is asymptotical to the χ^2 distribution with one degree of freedom, while the aggregated $\mathbf{LR_{cc}}$ is asymptotical to the χ^2 distribution with two degree of freedom.

Table 2: *Summary of Conditional Probability of Exceptions*

	Conditional Day Before		
	No Exception	**Exception**	**Unconditional**
Current Day			
No Exception	$T_{00} = T_0(1-\pi_0)$	$T_{10} = T_1(1-\pi_1)$	$T(1-\pi)$
Exception	$T_{01} = T_0\pi_0$	$T_{11} = T_1\pi_1$	$T\pi$
Total	T_0	T_1	T

5.2 Quantile Regression Test

Even through the above likelihood tests are most commonly used procedures to detect misspecification of VaR models, they suffer from some drawbacks. One is that they require sufficient amount of exceptions to study the model. If there are too few exceptions, these tests cannot even produce a numerical result. The other disadvantage is that they barely give a clue how the model can be improved even though they can somehow examine the effectiveness of the model whether the average failure aligned to the expected failure rate and the hits do not happen at a herd. Nevertheless, the unconditional coverage test only focus on the average exception rate. This may lead to inaccurate results as well as the conditional coverage test to reject a good model or accept a bad model. Other conditional backtesting such as Dynamic Quantile test of Engle and Manganelli (2004) uses a linear regression framework to backtest the VaR model, and the results would be asymptotically to the unconditional coverage test above, but this encounters the same problem that it cannot compute the numerical solution when there are too few exceptions.

To improve the power of the backtesting procedure, Gaglianone, Lima, Linton and Smith (2009) propose and advanced backtesting procedure - Quantile Regression (QR) test that solves the problems that the likelihood tests encounter. Most importantly, this test provides useful guides to improve the model. The idea to regress the actual returns and VaR and see if the $\boldsymbol{\alpha}$ quantile is true since knowing that the VaR measure is essentially the quantile of a distribution as:

$$\boldsymbol{F_t}(\boldsymbol{\pi})^{-1} = \boldsymbol{\alpha} + \boldsymbol{\beta V_t} \tag{30}$$

where $\boldsymbol{V_t}$ is the estimated VaR vector, and $\boldsymbol{F_t}(\boldsymbol{\pi})^{-1}$is the inverse function of the realized return distribution. Then, a simulated Bernoulli distribution with probability of $\boldsymbol{\pi}$ is used to solve the equation (30) to zero based on a bootstrap procedure. While each step, the parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are estimated based on the random draws from the simulated and actual return distribution, and the Wald joint test is computed based on the covariance matrix between the simulated and the realized return distributions. When specification of the VaR model is true, then the regressed results would show intercept of zero and slope of unity. These two parameters are the critical indicators that provide ways to improve the VaR model. The intercept shows the magnitude of over or underestimate (e.g. a positive intercept means underestimate), while the slope indicates the ability of the VaR model react to the volatility of the risk factor (e.g. a slope smaller than one means under reaction). Therefore, the flawed model could be calibrated and improved based upon, and the process of models' comparison could be facilitated.

6: Empirical Results and Analysis

6.1 Normality Analysis

Before analysing our statistical results for the four selected models, we examine the returns of the two portfolios to observe the actual distributions relative to the underlying assumptions used in each VaR model. The sample periods are divided into sub-periods and the entire period included is from April 30 2003 to May 31 2010, and from April 8 2004 to May 28 2010 for Canadian Equity and Global Equity portfolio respectively (see Figure 5 and 6 for the histograms). As the statistics show in Table 5 and 7 Panel I, the overall means of the two portfolios are close to zero and the skewness is slightly negative for Canadian Equity portfolio and slightly positive for Global Equity portfolio, which is also shown in the histograms. However, kurtosis showed significant differences for the majority of the period, especially in the sub-prime crisis in 2008. As expected, the kurtosis of the two portfolios is quite high and the skewness is negative, which indicates that the left fat tail exists. Jarque-Bera (JB) test is applied, and the statistics at the 95% confidence interval show that 75% and 57% of the time, the normalities are rejected for the Canadian and Global portfolio respectively over the entire time horizon. As expected in 2008, the normalities are highly rejected, and these rejections most likely lead to the significant high JB statistics for the full sample period. As a result, we can strongly confirm that the returns of the two portfolios are not normally distributed, and this is expected as financial data rarely exhibit a normal distribution.

6.2 Implementation and discussion of the models

As we implement the four models (see Figure 7 to Figure 14 for Canadian Equity Portfolio, and Figure 15 to 22 for Global Equity Portfolio), we can demonstrate HS's inability of capturing the volatility dynamics, and this would be largely due to its strong assumption that returns are i.i.d.. HS has been criticized over overestimating VaR if the coverage of empirical distribution used to compute VaR involves a fluctuating period (i.e.

SIAS fund encountered the sub-prime crisis in 2008). The 1% HS VaRs tend to be overestimated as shown in Figure 7 and Figure 15, with respect to both Canadian and Global Equity portfolio realized returns. The length of overestimation depends on the coverage of samples. Ideally, under the assumption that the asset mix of the portfolio does not change frequently, the wider the coverage, the longer period VaR would be overestimated. This is the case for the SIAS fund as purchased stocks are more likely to be held for a longer period. However, if the holdings of the portfolio tend to change frequently, then this would be a different result that shows more fluctuating rather than flat VaR due to the fact that the routes of HS is to perform a full valuation of the current holdings based upon the historical returns accordingly. In addition, the overestimation is due to the specification and assumption of the model. When using a 1-year moving window coupled with HS's assumption that weights each historical return equally regardless of the time horizon, the VaR given by targeted quantile would not vary if the replacement of return occurs in the right of the targeted quantile over time. Until the window reaches a time that returns are drawn from the left of the targeted quantile, left tail distribution would change as well as the estimated VaR. Definitely, instead of a moving fixed window, using a full period return distribution would lead to different outcomes because the relative cumulative probabilities of the distribution will change over time. However, how many historical scenarios should be considered to do the full valuation in order to measure VaRs accurately or efficiently is not definable. This is also related to speed and accuracy issue. As a result, after the sub-prime crisis of 2008, the overestimation period was prolonged almost one year after the shock.

At a first glance, DCC GARCH and FHS VaRs seem to capture the volatility dynamics properly, and the estimated VaRs vary correspondingly to the volatility of the portfolio (see Figure 8, 9, 12, 13, 16, 17, 20, 21). However, the 1 % FHS VaRs is perhaps slightly overestimated for both portfolios, especially during the period from the middle of 2007 to end of 2009. This is because FHS model assumes that the standardized returns or risk-adjusted returns behave accordingly. This also means that FHS assumes that trade-off between returns and risks is constant or identical over time even though FHS model tends to employ the process of GARCH to reduce the impact of serial correlation and heteroiskedasticity of returns and make returns sample close to i.i.d.. However, this would never be the case. When the sub-prime crisis exploded in 2008, the volatilities of the market and SIAS equity portfolios were amplified to an abnormal high level. Those volatilities are used to scale up the standardized residuals. Consequently, the FHS model takes the percentile of the distribution formed by those extremely scaled samples, and this could result in overestimated VaR. On the other hand, the DCC GARCH model is believed to produce more accurate and unbiased results because it allows time varying return correlations rather than assuming it is constant.

As shown in Figure 10,14,18,22, the HB model, interestingly, seems to perform well. Considering the 1% HB VaRs, the plotted line behaves in an interesting manner, just like a plateau area consisting of plains, cliffs, and terraces. On the other hand, the 5% HB VaRs tend to behave normally. When we see this significant difference, it is inspirable to discuss each factor that determines the HB VaR measurement. Firstly, the 1 year specification of past returns extends to 2-year past returns (see figure 23 to 26). Surprisingly, VaRs with the same targeted quantile but different past data are identical graphically and computationally (see Table 3). If it is due to our specification that the HB VaR is determined at the point, where the cumulative probability is equal or over the targeted percentile without using interpolation, therefore, the linear interpolation is added as the second consideration (see Figure 27 to 30). As expected, the plotted VaRs become a little smoother, which also remedy the potential underestimated VaRs by choosing the exact return where the summed up weights exceed the targeted percentile. Hereafter, to be precise, the HB model is improved with the linear interpolation and the results would be discussed and tested in the later section. Despite this finding, the figures still look very similar. Therefore, we checked the actual VaR numbers and found out that the differences are relatively small for both portfolios as shown in Table 4. One possible explanation is due to the chosen targeted failure rate and decay factor. As the weights based on the exponential smoothing method rapidly decline within the first 50 days, regardless of the size of the input historical returns as in Figure 3 and 4 before, and the data of the past 50 days is identical for both sample sizes, these 50 samples could have the same significant influence in computing HB VaR. Moreover, the HB model assigns the first weight to (1-λ) (e.g. λ is 0.90, the first assigned weight is 0.1), so it is conclusive that if one significant downside return happened yesterday and the targeted percentile is set to be smaller than

($1-\lambda$) (e.g. 1% VaR v.s. 0.94 decaying rate). This negative return is dominant in computing VaR at least 3 days by simple math, and the adjacent negative numbers would become significant. As long as the size of observation increases, most likely the more negative returns would be involved in a wider coverage of historical returns and they would fit into gaps even through the additional observations have extremely tiny weights (e.g. the weight of returns 201 day ago). When the recent negative returns largely contribute to the cumulative probability that hits the targeted quantile, the VaR is determined by the ones around the dominant negative return, but this may need a considerable amount of those distant negative returns to reach the targeted probability since they all have tiny weights. Moreover, the degree of those dominant weights could explain the flat parts of the HB VaRs, the higher degree would lead to the repeatedly flat HB VaR estimate in a prolonged period.

Knowing the size effect of data used to calculate the HB VaR seems to be minimal, the final necessary input of HB model, the decay factor λ is discussed (see Figure 31 to 38), with changing λ ranging from 0.90 to 0.98. Once again we cannot visualize any difference between two different sizes of data (i.e. 1year vs. 2 year). However, we can conclude that the interaction between selected λ and the targeted quantile is much more significant. In the 1% HB VaR, the weight dominance effect as just mentioned appears very strong since all $1-\lambda$ are smaller than the 1%. Thus, we can see the plot is smoother from high λ to low λ, and the low λ (i.e. λ is 0.9) reveals more kinks and flat areas. On the other hand, among 5% HB VaRs, the VaR behaves much better than the 1% HB VaR does. At minimum, before performing any backtests, the graphs illustrate HB's capability of capturing the volatility dynamics at a lower confidence interval VaR with less weight dominance. The proper selection of λ is still undetermined, but it seems that the middle range ones (i.e. 0.96 and 0.94) perform better.

6.3 Number of Exception

In this section, each model would be discussed, in terms of its effectiveness. The simplest way to verify the accuracy of the model is to record the exception rate or failure rate (i.e. number of exceptions over observations) to find the proportion of time the VaR is exceeded. When the VaR model is perfectly calibrated, the failure rate should be in line with the confidence level. If the failure rate is too high due to large number of exceptions, the model underestimates risk in a normal market condition. Definitely, if the market is so volatile, resulting in high fluctuating returns, the number of exception could not provide a clear conclusion of the effectiveness of the VaR model, but the large number of hits could be explained. On the other hand, too few exceptions also indicate misspecification of the VaR model. Indeed, the consequences of misestimating VaR for the regulated financial firms would be more serious. If the number of exception is too high, they would be penalized by allocating more funds to the regulatory capital, while if it is too low, they tend to set too much capital aside. In either case, they are subjected to inefficient use of their capitals.

The numbers of exceptions produced by the four models are summarized in Panel II of Table 5 to 8. As a simple rule, it is more favourable to see the actual failure rate close to the expected failure rate (e.g. 2.5 and 12.5 times of exception for 1% and 5% VaR at the one-year interval). Among all models, FHS produces the most reasonable results with the fewest hits in 1% and 5% VaR, while the HB produces 46 exceptions in 1% VaR for the entire period which is the highest number of hits of any of the four models followed by the HS approach. However, we can confirm that HB can perform better if we lower the confidence level (i.e. 5 % VaR) with moderate decay factor (λ = 0.94 in our case) to optimize the effect of assigned weights dominance, and the results turn out that the HS is the worst in the 5% VaR. Once again, the evidence shows HS is not capable of responding to fluctuations efficiently and results in the highest number of exceptions. After 2008, HS consistently overestimates the VaRs and produces relatively low exceptions. In general, the higher numbers of exception are expected in the volatile period such as in the economic crisis of 2008, where most models produce relatively large number of hits. However, FHS performs almost a perfect job in 1% VaR, and results in two and three hits in the Canadian Equity and Global Equity portfolio respectively, but this may be questionable. DCC GARCH is ranked the second among all the models in Canadian Equity VaR. However, a larger discrepancy is found between DCC GARCH and FHS when comparing portfolios at the 1% VaR. The 5% VaR producs consistent results with our previous findings with the highest number of exceptions during the 2007 and 2008. The DCC GARCH and the FHS results are quiet similar, unlike the results obtained with the 1% VaR.

6.4 Backtesting

As we examine the unconditional coverage, independence and conditional coverage for the Canadian Equity Portfolio and the Global Equity Portfolio at the 1% and 5% VaR, we assigned *, **, and *** to indicate the rejection of the likelihood ratio test at 90%, 95% and 99 % confidence level, respectively and for $\chi 2$ distribution with one or two degree of freedom (i.e. one for Unconditional Coverage and Independence test, two for Conditional Coverage test see Table 5 to 8, Panel III to V).

As shown in Panel III, the unconditional coverage test for the 1% VaR for both Equity portfolios are very consistent. The results for the full sample period highly reject all models at 99% confidence interval with the exception of the FHS model. For the HS and HB, these results are expected since the unconditional coverage tests show the high rejection not only for the full period but also for the individual sub-period. However, such high rejection rate may not be reasonable for DCC GARCH as we know that the significant large exceptions in 2008 may cause the overall results to be misleading. This is why this likelihood test is subjected to less explained power to the effectiveness of the model. When considering the unconditional coverage test for the 5% VaR, HS remains the worst again. However, the HB model significant improves with a lower target quantile to compute VaRs, and the unconditional coverage test provides favourable results once our findings are confirmed.

As mentioned in Section 5.1, the unconditional coverage ignores time variation in the data and the observations could cluster, which would lead to model invalidation. The more clustering also indicates the model is insufficient to respond to the current fluctuation. Thus, the independence coverage test is used to detect this weakness of the model. The overall independence coverage test results for both portfolios are very positive except for the HS that shows 99% significant rejection, and for the FHS that shows rejection at 90% confidence interval once (See Panel IV in Table 5 to 8). These results can demonstrate most selected models (except for HS chosen as the comparison) are well calibrated and capable of reacting to the return volatility accurately. However, it is worth mentioning that one of the disadvantages for the likelihood test is that if insufficient information is given (e.g. zero exception), then the test may not be applicable as shown the statistics of 2009.

The conditional coverage back test, which combines unconditional coverage and independence test would largely depend on the unconditional coverage test and produce the similar outcomes since the overall independence statistics are very low. At the 1% VaR, the models are highly rejects in both portfolios, the HS and HB at the 99% confidence level. DCC GARCH is rejected at the 95% confidence level and the FHS passes for Canadian Equity portfolio, while the DCC GARCH passes and the FHS is rejected at the 90% confidence level. Again, high rejection rates appear for all the models except for FHS even during 2008 economic recession. The results obtained for the 5% VaR are similar as the HS is highly rejected, while they are different as FHS is rejected at the 99% and 90% respectively. DCC GARCH and HB passes this time.

Due to concerns that likelihood ratio tests lack power of test, this may lead to inaccurate results that are subjected either type I or type II error. Quantile Regression test is applied, and this test is believed not only to have more power to detect the misspecification of a VaR model but also provides useful guides to improve or calibrate the models further. Keep in mind, the intercept of the regression provides the information regarding how an underlying model over or underestimates VaR (e.g. negative intercept means overestimate), while the slope of the regression provides the information regarding how well a model responds to fluctuations (e.g. slope greater than one means overreaction). Thus, a perfect model would produce a zero intercept and unity slope, which is the null hypothesis of this backtesting procedure. The statistic results are shown in Table 9 to 12. Overall, all intercepts are very close to zero, but it is worth mentioning that these intercept numbers reveal some information consistent with previous findings (see Panel I). One is that the intercepts of HS start from negative and positive after 2008. This exactly implies that HS VaRs tend to be overestimated after the shocks. The largest positive intercept happens at the HS 1% VaR in the Global Equity portfolio, and this reflects the highest number of the exceptions. Furthermore, FHS consistently has negative intercepts at the 1% VaR for both portfolios, and the largest negative value of intercept over the full period. This can confirm the graphical interpretation in the previous discussion even through FHS is considered as the best model based on the likelihood tests. However, some regression results may not match the actual situation. For instance, at 1% VaR, HB produces the highest number of the exception, and this would indicate that HB systematically produces underestimated results, but the resulting intercepts all appear in negative values.

When considering the slopes of regressions, mostly results in the sub-period show wide range of the slopes and no clear pattern exists. On the other hand, when examining the full period results in both portfolios, it could be conclusive that the DCC GARCH performs the best in capturing the volatility dynamics, in which most of the results are close to unity (see Panel II to IV).

When applying Wald test that examines jointly whether the intercept of zero and slope of unity, at 1% VaR in Canadian Equity portfolio (see Panel IV), all the models are highly rejected at 99% significance, except DCC GARCH which is rejected at 95% level. The rest are rejected at 99% level, while at 1% VaR in Global Equity portfolio, DCC GARCH passes, and HS, FHS, and HB are rejected at 90% confidence level. At the 5% VaR, HS is rejected at 95% significance in both portfolios. In addition, FHS and HB pass the test, while FHS and HB are rejected at 95% and 99% significance level respectively in Global Equity portfolio. Overall, the 1% and 5% VaR Quantile Regression results lead to very deviating conclusions of the effectiveness. If all the models are rejected in 1% VaR, this may indicate that choosing the 1% as the cutting point for the expected failure rate may be not reasonable, and this could be linked to the findings that highly reject the normality. As a result, this would be very distorting to the effectiveness of a model with normality as the underlying assumption, especially for DCC GARCH using MLE and taking percentile for normal distribution. The HS and HB are rejected, and this is consistent with the likelihood backtesting results previously. However, the results that are not consistent to the previous tests is that FHS that passes all the likelihood tests but could not survive based on Quantile Regression test that has more conclusive power than other backtesting procedures. This may largely due to the fact that FHS systematically overestimates VaRs. As a result, DCC GARCH is considered as the best among the four models.

6.5 Speed and Accuracy

The trade-off between the speed and accuracy would be an important concern that depends on various situations. For instance, in order to retrieve timely risk information some accuracy may be scarified, especially when dealing with a large size of data and a wide range of the risk factors. Thus, a quicker model providing moderate results may be chosen. The speed and accuracy of the models shows significant differences (see Table 13 and 14). The DCC GARCH shows a speed of 20.7 seconds compared to under a second for the remaining three models at both the 1% & 5% confidence level for the Canadian Equity Portfolio. The results for the Global Equity Portfolio are quiet similar, showing the DCC GARCH with a speed of 15.8 seconds compared to others with less than one second for both the 1% and 5% confidence level. The 5-second difference would be mainly caused by the number of holdings. As just mentioned that DCC GARCH is the model that estimates the entire covariance matrix for a portfolio, and the number of the estimated parameters grows exponentially as the number of the holding increases. Thus, a longer running time is expected. In addition, DCC GARCH also produces the highest accuracy relatively to others. Surprisingly, HS is superior to HB and FHS in all the aspects even through FHS is considered the best based on the backtesting results.

7: Diversification

As shown in Table 8, we apply equation (26) to determine the diversification between two equity portfolios in order to merge their VaR into one aggregated number. We select the past 5 year data to see how well diversified in the SIAS equity portfolios. From 2006 to 2010, we can see there were not significant changes in the relative weights between the two portfolios. This is largely due to the fact the SIAS fund has been operated with strict parameters according to the IPS. Of course, the asset allocation would be one of them. However, the average holdings has gradually increased and reached 66.3 holdings in average in 2010. Based on a rule of portfolio theory, the increase holdings should improve the diversification and lower the risk. However, we can surprisingly recognize that all the diversification effects in Panel IV and VII are very tiny, not even 1% reduce in the risk. Indeed, the Global Equity portfolio largely consisted of U.S equities and European ETFs approximately 50% and 25% respectively of the overall Global Equity portfolio. Since these stock markets are highly correlated, it is reasonable to think that even the individual systematic risks are diversified away, but the correlations among these investments matter. When checking the correlation between the two time series of VaRs in Canadian and Global Equity portfolios, they are almost perfectly positive correlated, approximately 0.98+. As considering the diversifications computed by different models, we can ignore the HS and HB model that are highly rejected because they produce the biased VaR estimates. One of the fundamental assumptions to determine the diversification among VaRs is that the model should be qualified and be able to estimated VaR properly Pérignon and Smith (2010). Thus, DCC GARCH and FHS models are the qualified ones. By comparing the diversification of 1% and 5% VaRs, we can note that the diversifications are identical for DCC GARCH. This is because DCC GARCH is a pure parametric approach that focuses on forecasting the volatilities. Then, VaR is computed by multiplying the forecasted volatility and the targeted percentile from the assumed distribution, and thus results in no difference. On the other hand, FHS still uses the empirical distributions formed by standardized returns. The shapes of these distributions vary; therefore, FHS results in different VaRs even at the same cut off point. The diversification based upon the VaR estimates would be different. In addition, the other two semi-parametric models can demonstrate this point. Comparing the two adequate models, the computed diversification is very similar except the ones of Global Equity portfolio in 2009, where the DCC GARCH is rejected. The aggregated portfolio returns and the diversified VaRs of four models are shown in Figure 39 to 46.

8: Conclusion

In this paper, our purpose is to select a risk model as a backbone used to develop a comprehensive and sustainable risk management system and procedures that could enhance the overall portfolio risk management of SIAS fund. This is the first attempt to create a risk model based on the concept of VaR. The desired properties of the candidate models should produce accurate measurements of risk factors and allow more thoughtful risk management not only at the individual holdings level, but also at the overall portfolio-level that is compatible for different asset classes. In order to examine the candidate risk models' performance, we first gather all the historical information of SIAS from various resources such as SFU treasurers, custodians, brokers, and clients, and the information is well filed and stored so that future cohorts can have easy access for later reference. Then, we have a detailed analysis, including not only standard backtesting procedures such as likelihood ratio tests but also a far more powerful test (i.e. Quantile Regression test).

We select four models that represent different approaches. Historical Simulation is pure non-parametric, Dynamic Conditional Correlation Generalized AutoRegressive Conditional Heteroskedastic is highly parametric and sensitive to estimating correlations, Filtered Historical Simulation is semi-parametric in a way that combines Historical Simulation and GARCH, and finally the Hybrid Approach is the interesting variation from the semi-parametric group. We present extensive discussions about each model. We confirm the general criticisms about Historical Simulation in its inability of capturing volatility dynamics, while we discuss the main inputs of Hybrid Approach. The size of the data used seems to have a minimum impact on VaR measurement, while the interaction between the selected target failure rate and decay factor has a more significant impact, affirming the case that the current weights assigned is dominant in computing VaR. We also find that if the targeted confidence interval used to determine VaR is set to a lower value, Hybrid Approach does a good job.

Based on the backtesting results, Historical Simulation in all cases and Hybrid Approach except the 5% VaR are highly rejected. Filtered Historical Simulation is the best among all the models. It survives all the likelihood ratio tests and has a good trade off between the speed and accuracy. DCC GARCH is rejected sometimes, largely due to extreme high rejection rate in 2008. This may indicate weakness of the backtesting procedures, including the unconditional coverage, independence, and conditional coverage tests. By applying Quantile Regression that is considered more powerful than the commonly used likelihood test, the results confirm that DCC GARCH is the best.

The beauty of the DCC GARCH is its ability to estimate correlations not only from two streams of VaR but also between different assets within a portfolio so that diversification in different levels can be determined. This allows SIAS to manage the risk of the portfolio more precisely by using the concepts of components and marginal VaRs. Thus, it is feasible for SIAS to further monitor the risk components in specific sectors in the Canadian Equity portfolio or in specific regions in the Global Equity portfolio or even going two steps further into individual holdings. Thus, the strategic decision, asset allocation, and management of SIAS fund can be evaluated from a different prospective. Therefore, we strongly recommend DCC GARCH model after thoroughly considering all of these remarkable benefits. The procedures of monthly risk report and model programming have been standardized and stored at the SIAS SharePoint website. Of course, we look forward to any extensive improvements from future cohorts.

Even though considerable efforts have been in place to form the Equity Portfolio risk model, some procedures are still essential such as stress tests of the model. In addition, the procedure to further calibrate the risk model is another consideration. For example, investigating the autocorrelation to determine the superior specification of DCC GACRH, or relaxing the normal distribution assumption to estimate the parameters by using other distribution such as Generalized Error Distribution in order to take the fat tail effect into consideration and promote more precise estimations, those would be excellent future projects. Most importantly, for the SIAS portfolio as whole, the fixed income risk model is lacking. In addition to the interest rate risks, the fixed income risk model should be carefully selected and equipped with procedures to measure credit and liquidity risks, the two main concerns of the real world when managing fixed income portfolios[4].

[4] The two FRM 2010 final projects are would be nice references for future cohorts or FRM students with the same interests in the credit risk of fixed income. "Estimating Implied Default Probability and Risk Measurement for Credit Bonds" by Belinda Liao and Wei Hung employed reduced form model to estimate credit risks and their impacts on bond duration and convexity measurements. "The Impact on Portfolio Credit with Different Correlation Assumptions" by Jesse Jia, and Dabria Guo discussed how to determine an aggregated level of VaR and expected tail losses to management credit risk of a fixed income portfolio based on reduced form model and copula model.

References

Barone-Adesi, G., Bourgoin, F., and Giannopoulos, K., 1998. Don't Look Back". *Risk 11*, 100-104.

Barone-Adesi, G., and Giannopoulos, K., 1996. A Simplified Approach to the Conditional Estimation of Value at Risk. *Future and Options World*, 68-72.

Barone-Adesi, G., Giannopoulos, K., and Vosper, L., 1999. VaR without Correlations for Non-linear Portfolios. *Journal of Future Markets 19*, 583-602.

Barone-Adesi, G., Giannopoulos, K., and Vosper, L., 2002. Backtesting Derivative Portfolios with Filter. *European Financial Management 8(1)*, 31-58.

Basel Committee on Banking Supervision, 1996. *Amendment to the capital accord to incorporate market risks*. Bank for International Settlements.

Basel Committee on Banking Supervision, 2006. *Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework*. Bank for International Settlements.

Bauwens, L., Sébastien, L., and Rombouts, J. V. K., 2003. Multivariate GARCH Models: A Survey. CORE Discussion Paper.

Berkowitz, J., Christoffersen, P.F., and Pelletier, D., forthcoming. Evaluating Value-at-Risk Models with Desk-Level Data. *Management Science*.

Berkowitz, J., and O'Brien, J., 2002. How Accurate are Value-at-Risk Models at Commercial Banks?. *Journal of Finance 57*, 1093–1111.

Bollerslev, T., 1990. Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized Arch Model. *Review of Economics and Statistics 72*, 498-505.

Bollerslev, T., Engle, R., and Wooldridge, J., 1988. A Capital Asset Pricing Model with Time Varying Covariances. *Journal of Political Economy 96*, 116-131.

Christoffersen, P.F., 1998. Evaluating interval forecasts. *International Economic Review 39*, 841–862.

Engle, R. F., 2002. Dynamic Conditional Correlation-A Simple Class of Multivariate GARCH Models. *Journal of Business and Economic Statistics 20*, 339-350.

Engle, R. F., and Kroner, K. F., 1995. Multivariate Simultaneous Generalized Arch. *Econometric Theory 11*, 122-150.

Engle, R., and Manganelli, S., 2004. CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles. *Journal of Business and Economic Statistics 22*, 367-381.

Engle, R. F., and Sheppard, K. K., 2001. Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH.Working Paper. (New York University)

Gaglianone, W. P., Lima, L. R., Linton, O., and Smith D. R., 2009. Evaluating Value-at-Risk Models via Quantile Regression. Working Paper.

Jorion, P., 2007. *Value-at-Risk: The New Benchmark for Managing Financial Risk* (3th Ed.). McGraw-Hill, New York, International Edition.

JP Morgan,1994. RiskMetrics. *Technical Document*. New York.

Kupiec, P.H., 1995. Techniques for Verifying the Accuracy of Risk Measurement Models. *Journal of Derivatives 3*, 73–84.

Ledoit, O., Santa-Clara P., and Wolf, M., 2003. Flexible Multivariate GARCH Modeling With an Application to International Stock Markets. *Review of Economics and Statistics 85*, 735-747.

Patton, A., and Sheppard, K., 2007. Evaluating Volatility and Correlation Forecasts. (T. G. Andersen, R. A. Davis, J.-P. Kreib, & T. Mikosch, Eds.). *Handbook of Financial Time Series* (Springer Verlag).

Pérignon, C., Deng, Z.Y., and Wang, Z.Y., 2008. Do Banks Overstate Their Value-at-Risk?. *Journal of Banking and Finance 32*, 783–794.

Pérignon, C., Smith, D.R., 2010. Diversification and Value-at-Risk. *Journal of Banking and Finance 34*, 55-66.

Pérignon. C., and Smith, D. R., 2010. The Level and Quality of Value-at-Risk Disclosure by *Commercial Banks. Journal of Banking and Finance 34*, 362–377.

Pritsker, M., 2006. The Hidden Dangers of Historical Simulation. *Journal of Banking and Finance 30*, 561–582.

Richardson, M. P., Boudoukh, J., and Whitelaw, R. F., 1997. The Best of Both Worlds: A Hybrid Approach to Calculating Value at Risk.

Smith, D. R., 2010. Multivariate GARCH Models. Manuscript. (Simon Fraser University University)

Van den Goorbergh, R. and Vlaar, P., 1999. Value at risk analysis of stock returns: historical simulation, variance techniques or tail index estimation?. Manuscript. (Tilburg University)

Vlaar, P., 2000.Value at risk models for Dutch bond portfolios. *Journal of Banking and Finance 24,* 1131–1154.

Figures and Tables

Figure 1: *The Average Holdings of Decomposed SIAS Portfolio from 08/04/2003 to 31/05/2010*

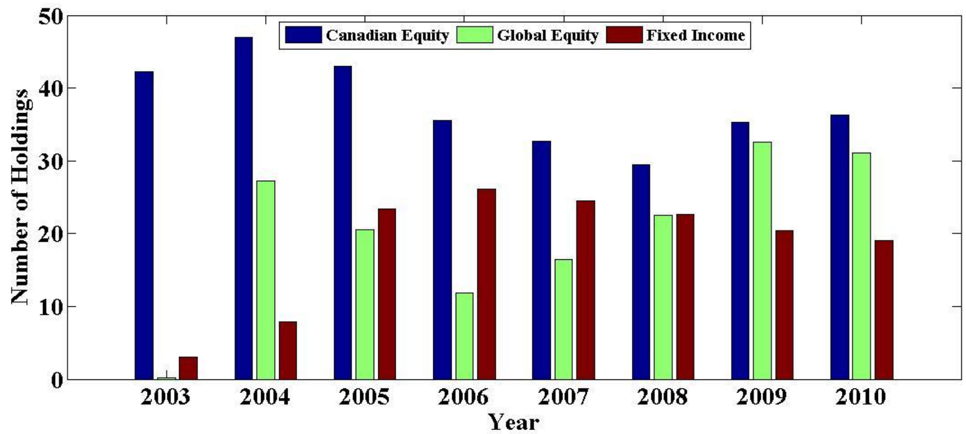

Figure 2: *Summary of Investment Policy Statement of SIAS Fund*

SIAS Proposed IPS Summary Table

	Equities			Fixed Income	Cash
Benchmark	70%			28%	2%
Range	60-80%			20-40%	0-10%
Risk	Sharpe Ratio>=90% of benchmark SR				
	Canadian Equity	Global Equity			
Bogie	S&P/TSX+1.5%	MSCI/Barra Ex Can +2%		DEX Universe +0.4%	DEX 91 day T-Bill
Benchmark	35%	35%			
Range	30-40%	30-40%			
		US	ROW		
Restrictions	7/10 GIC Groups >=25 stocks Advice: <=35 Stocks <=5% any issuer * 15 largest<=70% 100-900 mm Cap <=5% No small cap stock >2% No < 100 mm cap	>=10 Stocks + ETFs Market Cap>=$900 mm	>= 30% of Global >= 6 ETFs Must buy on US Exchanges <=5% in one position * <= 15% emerging markets No 'frontier' markets	Canada only <=5% any 1 security <=1% any security <A 50-100% in >A 0-50% in <= A 0-10% in BBB no BB or less Duration: DEX Univ +-1 yr Tracking error <= 80 bps Info Ratio >= 0.5 benchmark	min R-1/Low by DBRS or equivalent Can include GICs, Term Deposits

*for initial positions; can increase to 10%.

No margin, no short positions (including in funds/ETFs), no derivatives. No currency hedging.

__Figure 3:__ 1-Year Exponential Smoothing Weights with Different Decay Factors

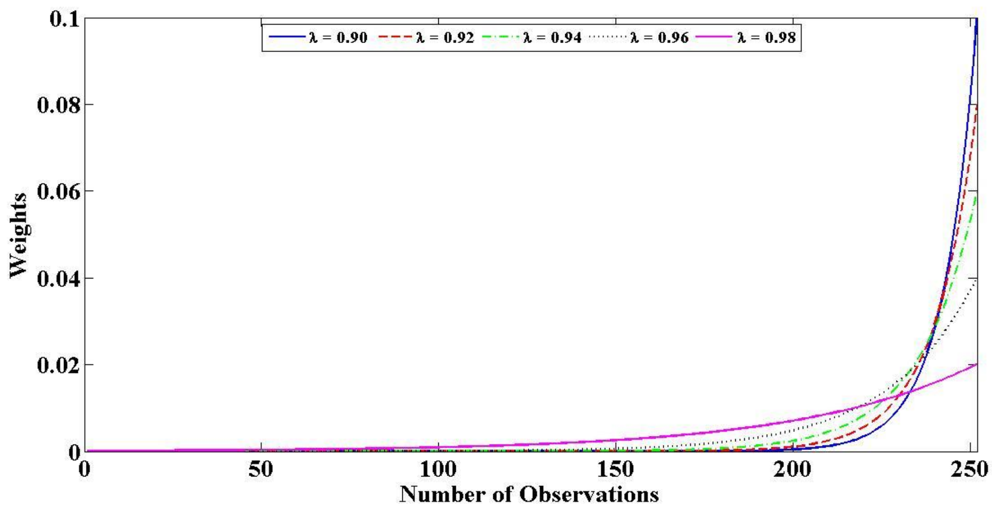

__Figure 4:__ 2-Year Exponential Smoothing Weights with Different Decay Factors

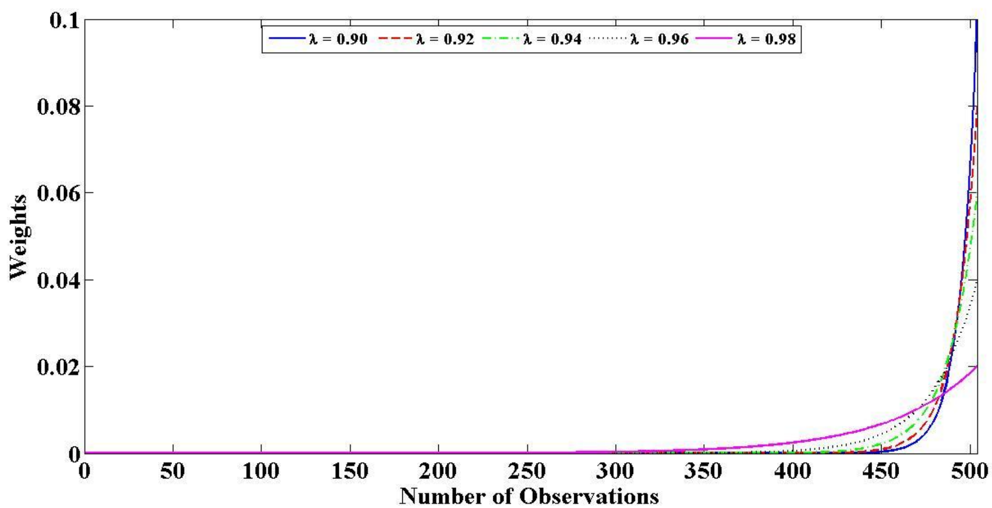

Figure 5: *Histogram of SIAS Canadian Equity Portfolio Returns from 04/30/2003 to 05/31/2010 (1782 samples)*

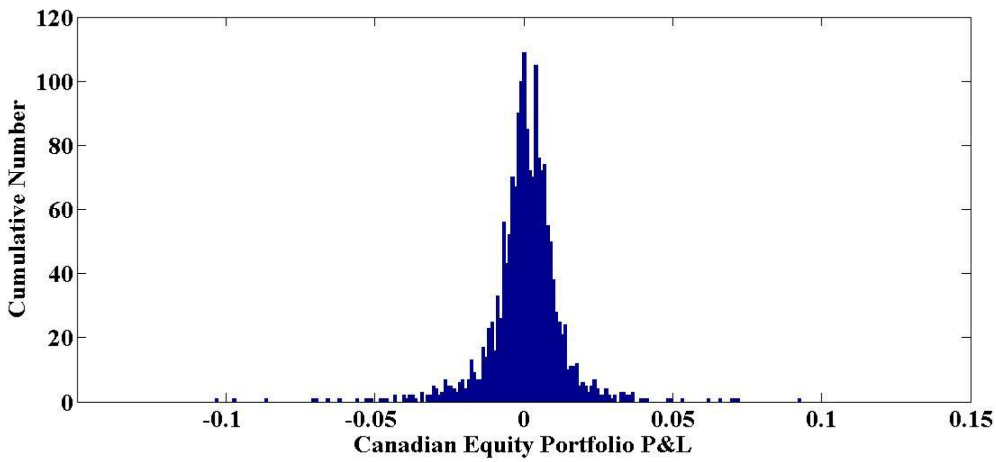

Figure 6: *Histogram of SIAS Global Equity Portfolio Returns from 04/08/2004 to 05/28/2010 (1546 samples)*

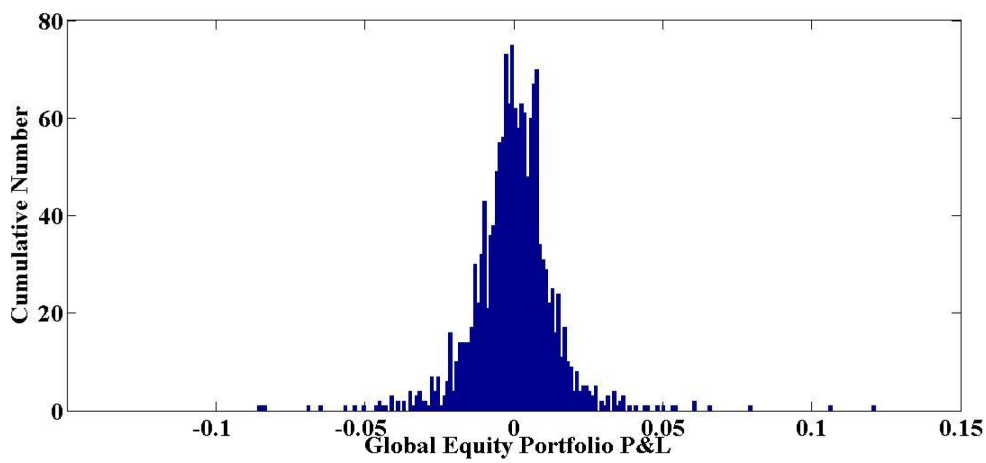

Figure 7: Canadian Equity P&L and 1% Historical Simulation VaR from 04/30/2003 to 05/31/2010

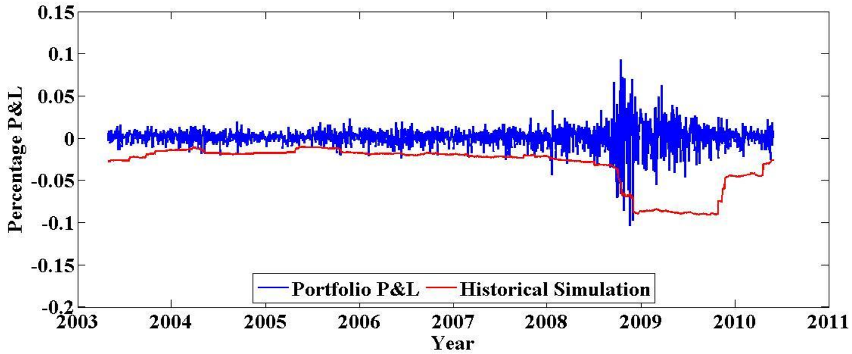

Figure 8: *Canadian Equity P&L and 1% DCC GARCH VaR from 04/30/2003 to 05/31/2010*

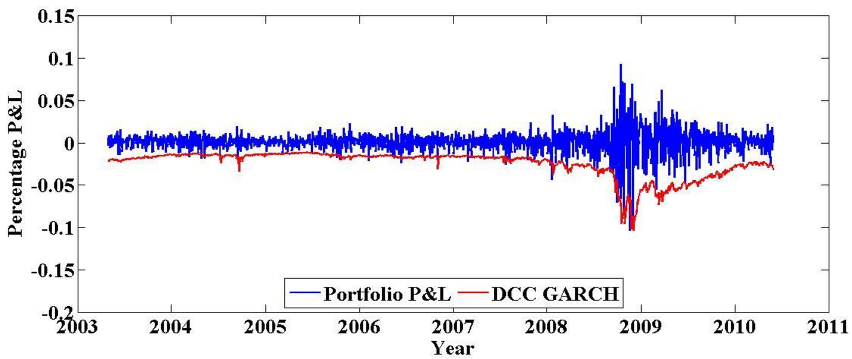

Figure 9: *Canadian Equity P&L and 1% Filtered HS VaR from 04/30/2003 to 05/31/2010*

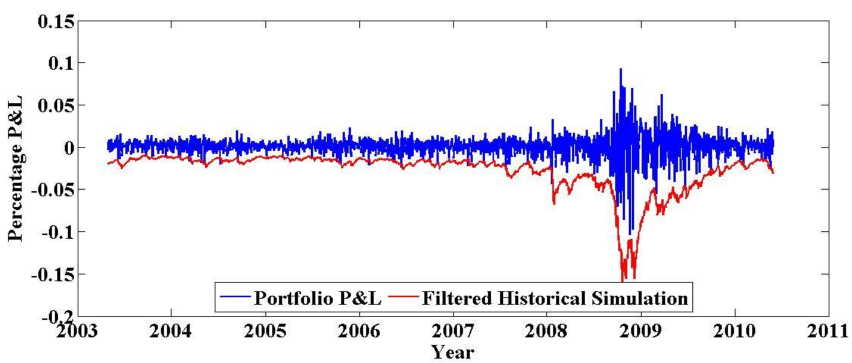

Figure 10: *Canadian Equity P&L and 1% Hybrid Approach VaR from 04/30/2003 to 05/31/2010*

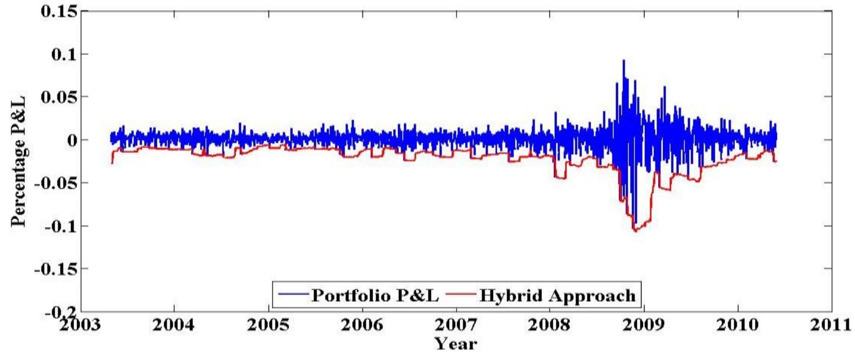

Figure 11: *Canadian Equity P&L and 5% Historical Simulation VaR from 04/30/2003 to 05/31/2010*

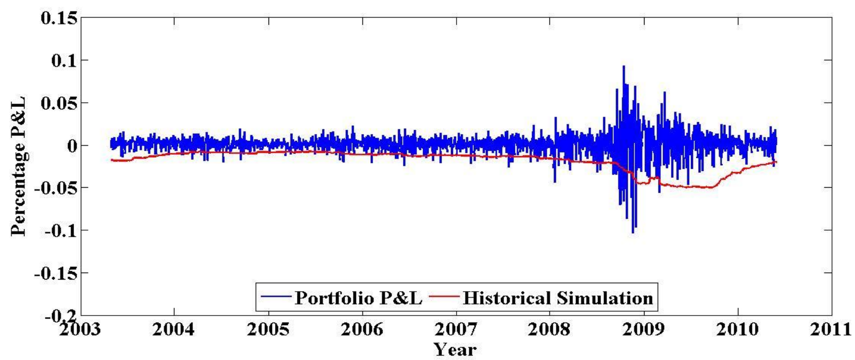

Figure 12: *Canadian Equity P&L and 5% DCC GARCH VaR from 04/30/2003 to 05/31/2010*

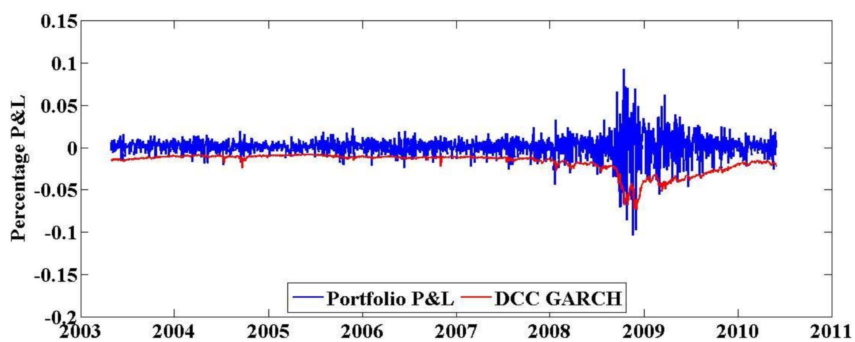

Figure 13: *Canadian Equity P&L and 5% Filtered HS VaR from 04/30/2003 to 05/31/2010*

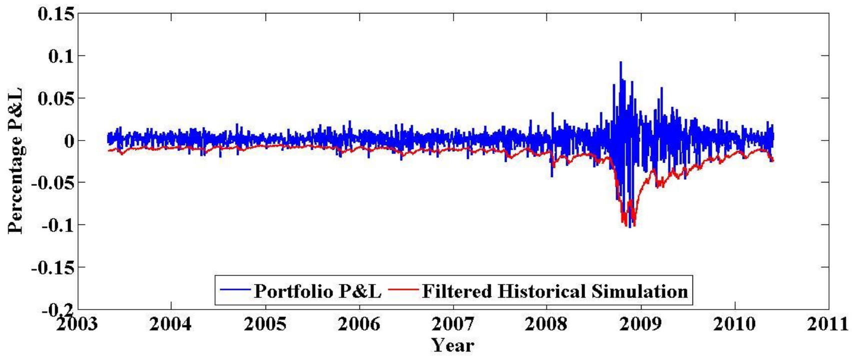

Figure 14: *Canadian Equity P&L and 5% Hybrid Approach VaR from 04/30/2003 to 05/31/2010*

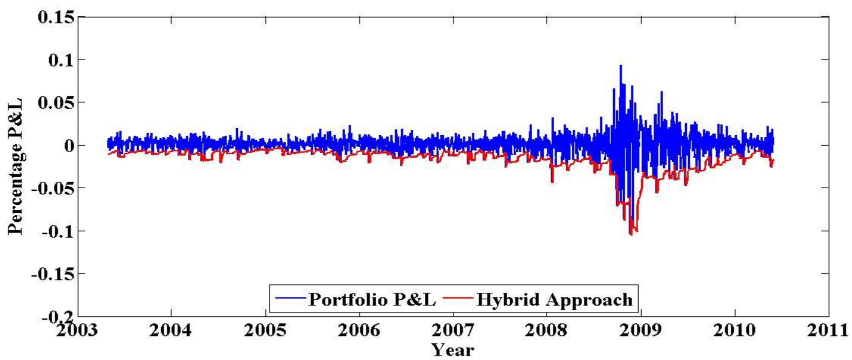

Figure 15: *Global Equity P&L and 1% Historical Simulation VaR from 04/08/2004 to 05/28/2010*

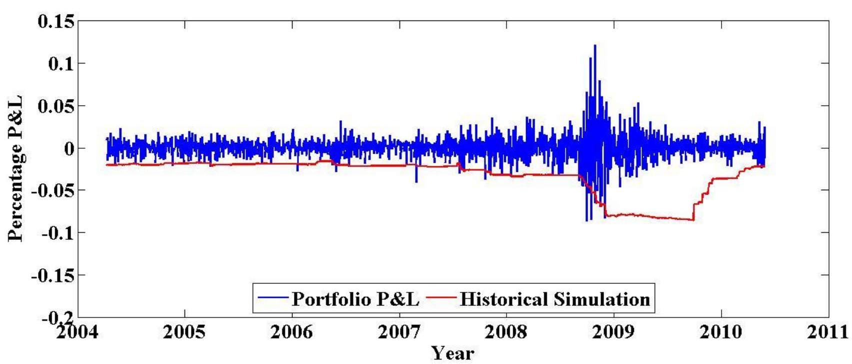

Figure 16: *Global Equity P&L and 1% DCC GARCH VaR from 04/08/2004 to 05/28/2010*

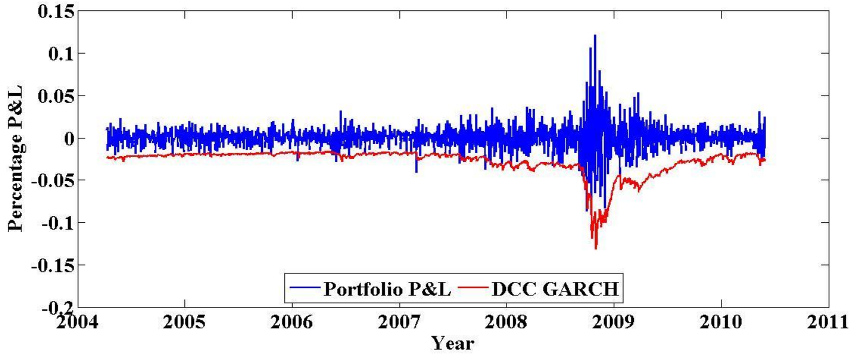

Figure 17: *Global Equity P&L and 1% Filtered HS VaR from 04/08/2004 to 05/28/2010*

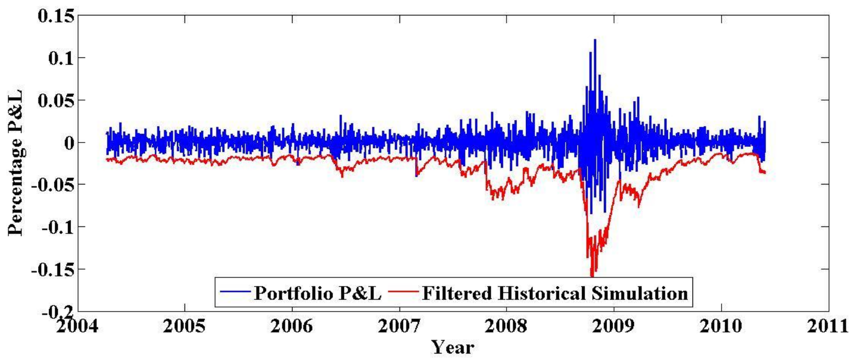

Figure 18: *Global Equity P&L and 1% Hybrid Approach VaR from 04/08/2003 to 05/28/2010*

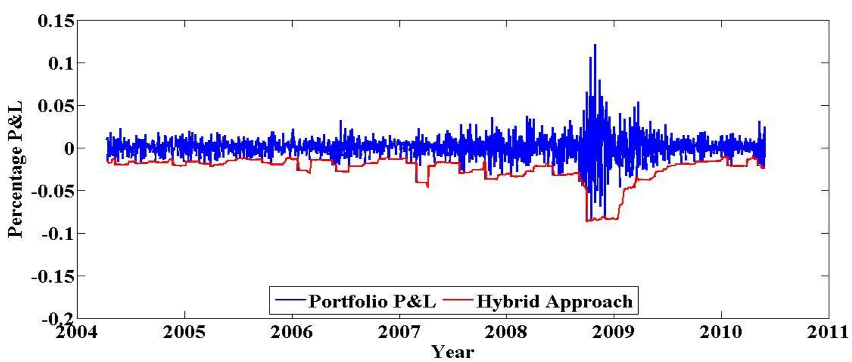

__Figure 19:__ Global Equity P&L and 5% Historical Simulation VaR from 04/08/2004 to 05/28/2010

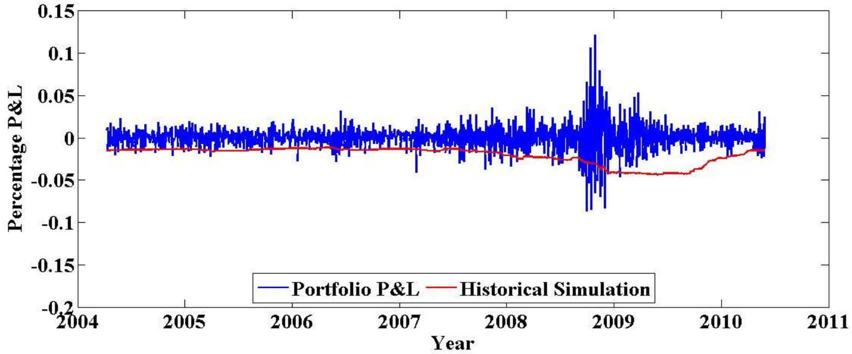

__Figure 20:__ Global Equity P&L and 5% DCC GARCH VaR from 04/08/2004 to 05/28/2010

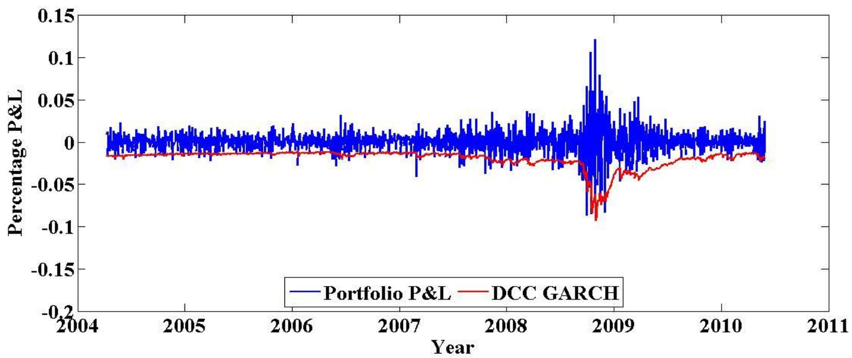

__Figure 21:__ Global Equity P&L and 5% Filtered HS VaR from 04/08/2004 to 05/28/2010

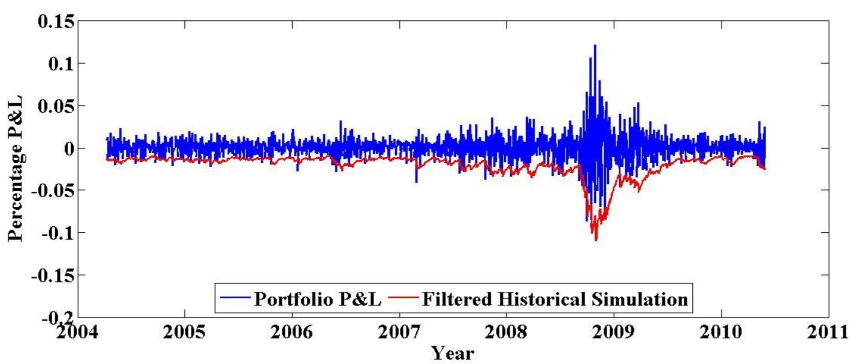

Figure 22: *Global Equity P&L and 5% Hybrid Approach VaR from 04/08/2003 to 05/28/2010*

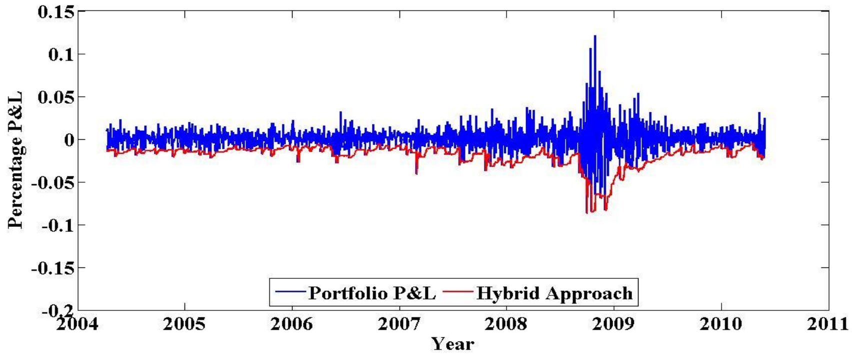

Figure 23: *Canadian Equity P&L and 1-Year Hybrid Approach VaR from 01/01/2007 to 05/31/2010*

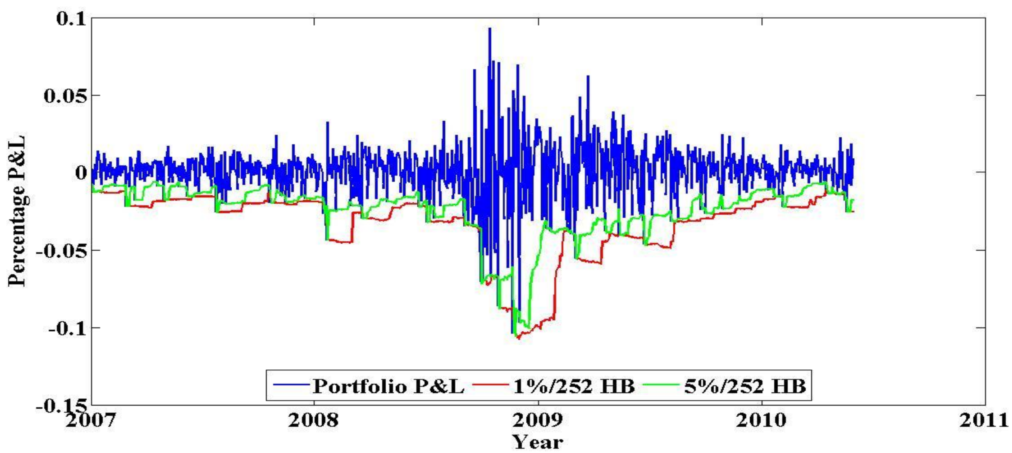

Figure 24: *Canadian Equity P&L and 2-Year Hybrid Approach VaR from 01/01/2007 to 05/31/2010*

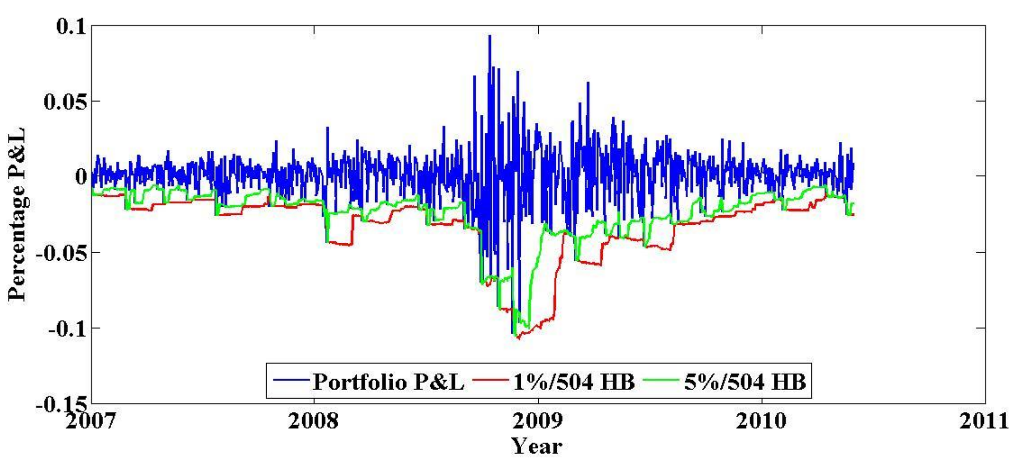

Figure 25: *Global Equity P&L and 1-Year Hybrid Approach VaR from 01/01/2007 to 05/28/2010*

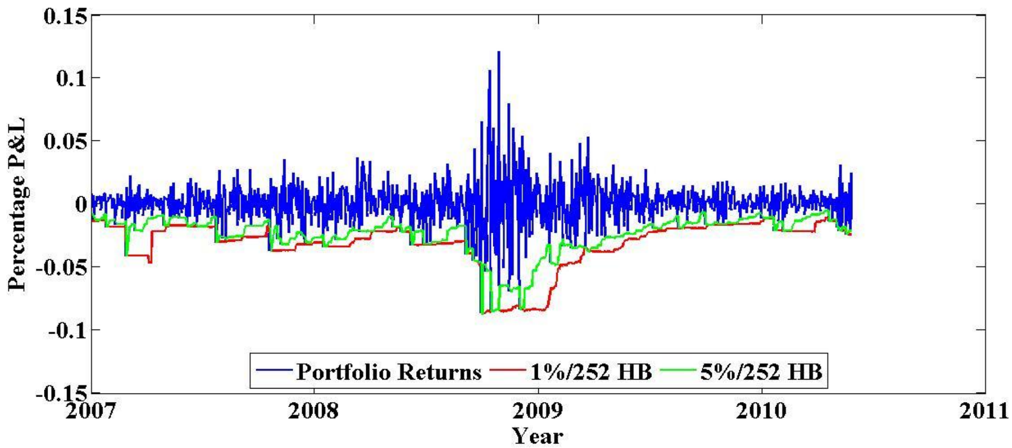

Figure 26: *Global Equity P&L and 2-Year Hybrid Approach VaR from 01/01/2007 to 05/28/2010*

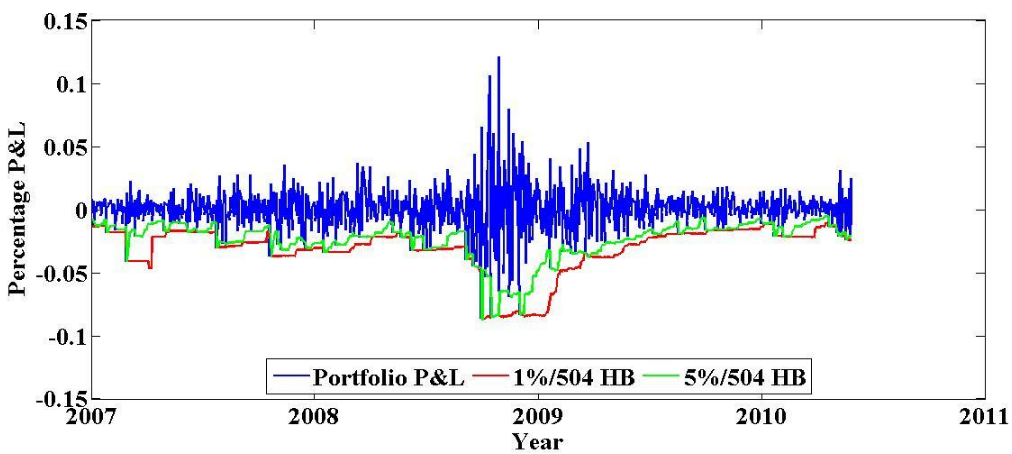

Figure 27: *Canadian Equity P&L and 1-Year Hybrid Approach VaR with Interpolation from 01/01/2007 to 05/31/2010*

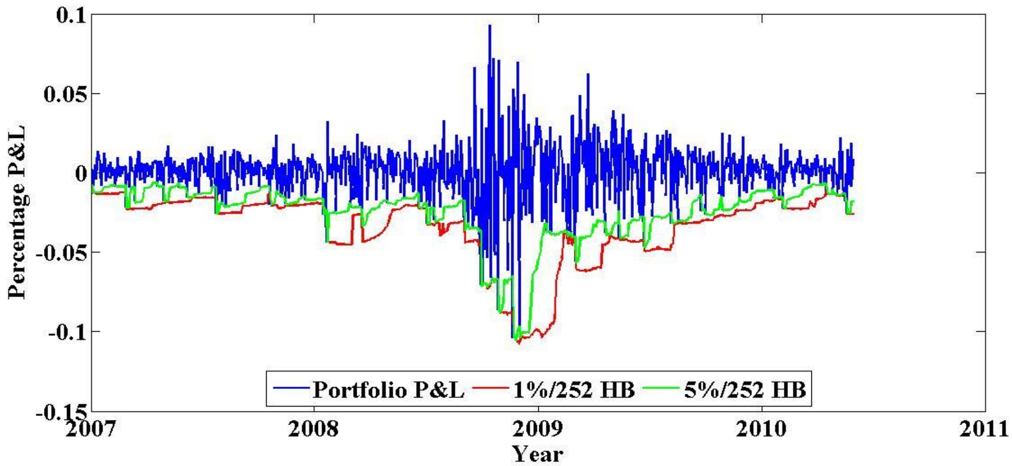

Figure 28: *Canadian Equity P&L and 2-Year Hybrid Approach VaR with Interpolation from 01/01/2007 to 05/31/2010*

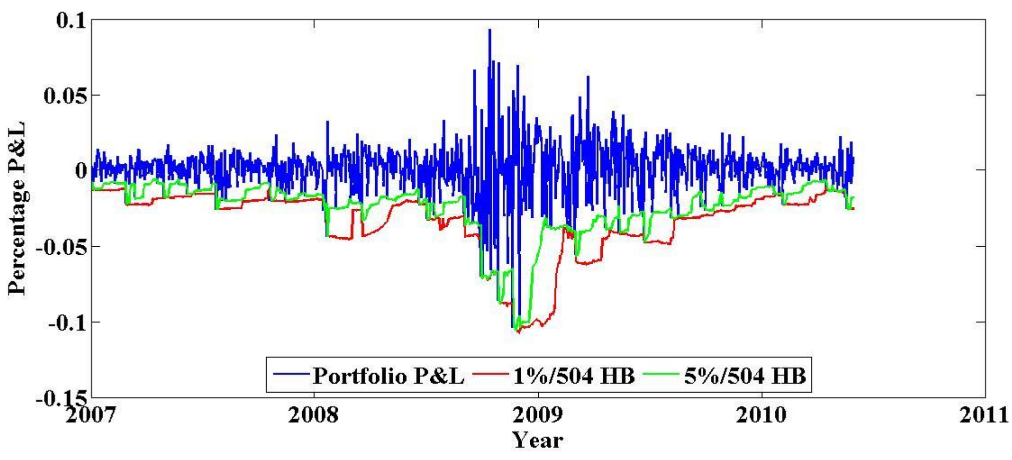

Figure 29: *Global Equity P&L and 1-Year Hybrid Approach VaR with Interpolation from 01/01/2007 to 05/28/2010*

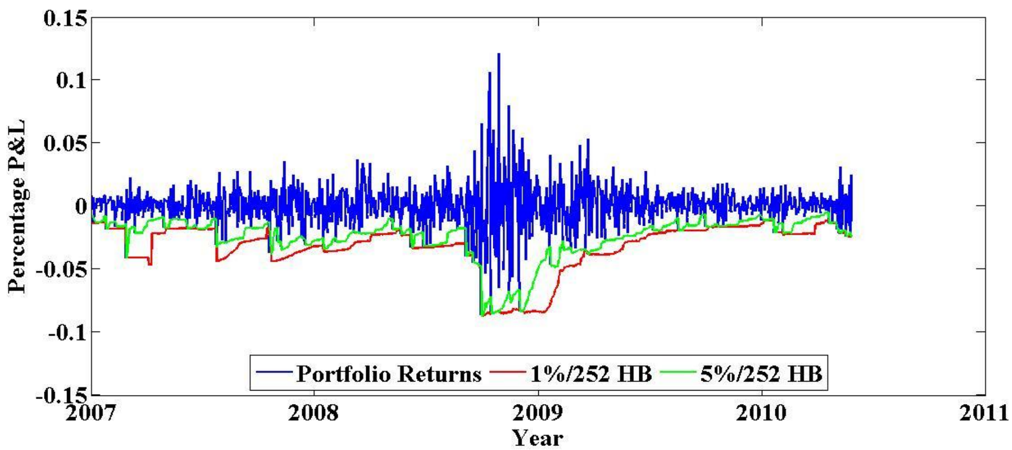

Figure 30: *Global Equity P&L and 2-Year Hybrid Approach VaR with Interpolation from 01/01/2007 to 05/28/2010*

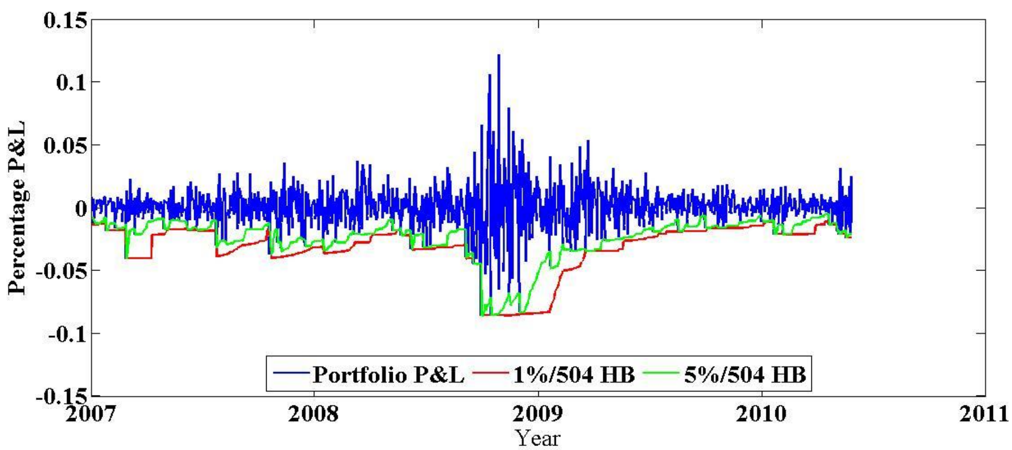

Figure 31: *Canadian Equity P&L and 1% / 1-Year HB VaR with Different λ from 01/01/2007 to 05/31/2010*

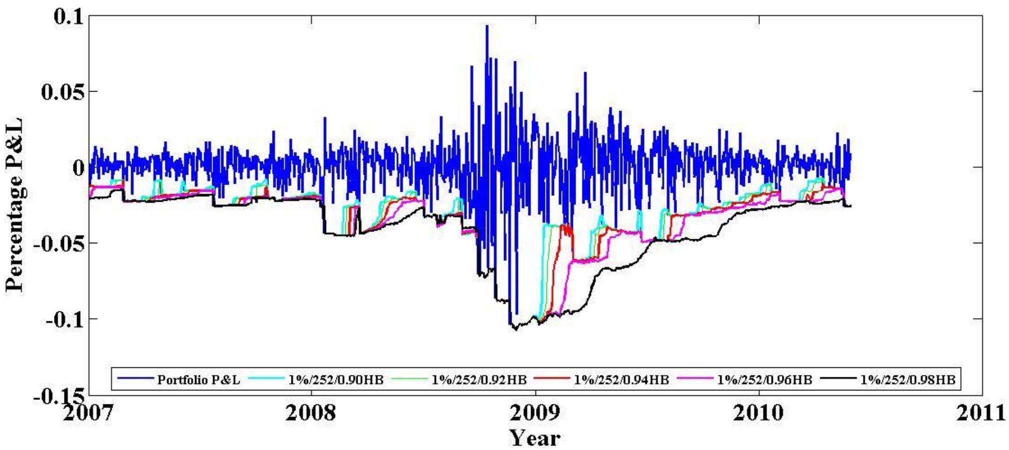

Figure 32: *Canadian Equity P&L and 1% / 2-Year HB VaR with Different λ from 01/01/2007 to 05/31/2010*

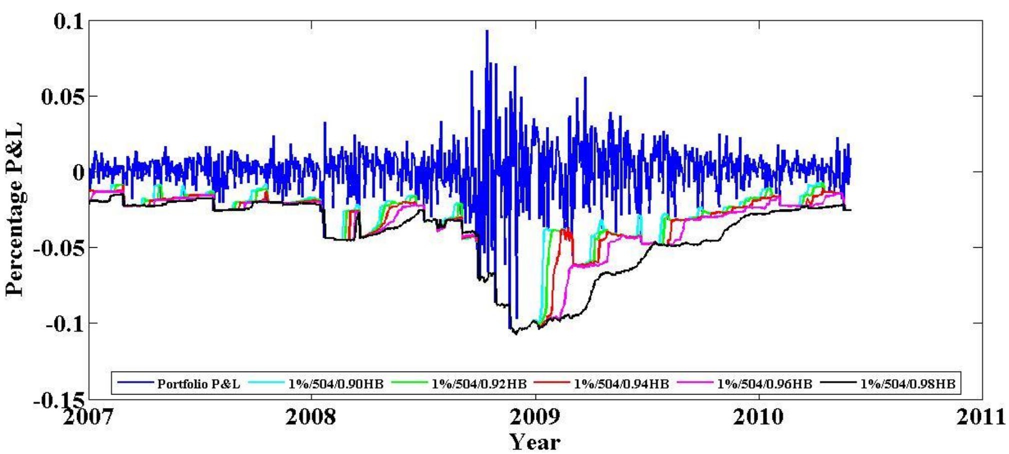

Figure 33: *Canadian Equity P&L and 5% / 1-Year HB VaR with Different λ from 01/01/2007 to 05/31/2010*

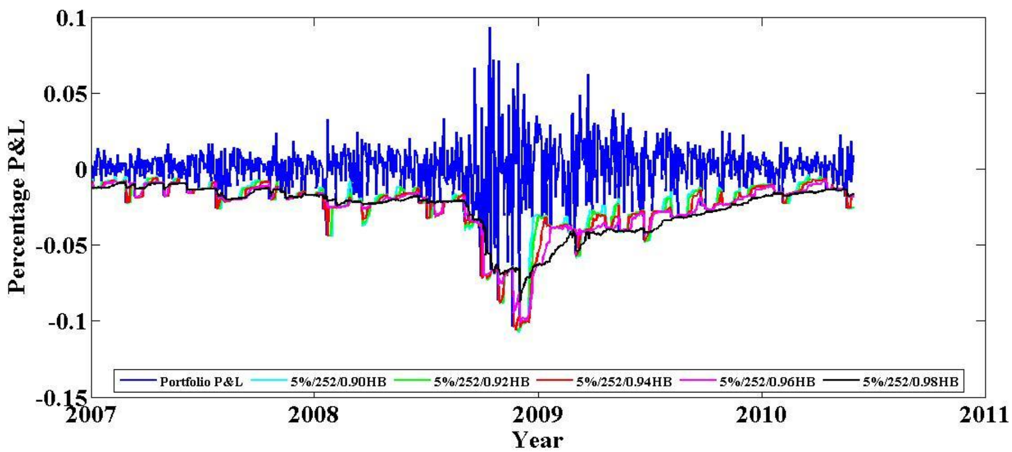

Figure 34: *Canadian Equity P&L and 5% / 2-Year HB VaR with Different λ from 01/01/2007 to 05/31/2010*

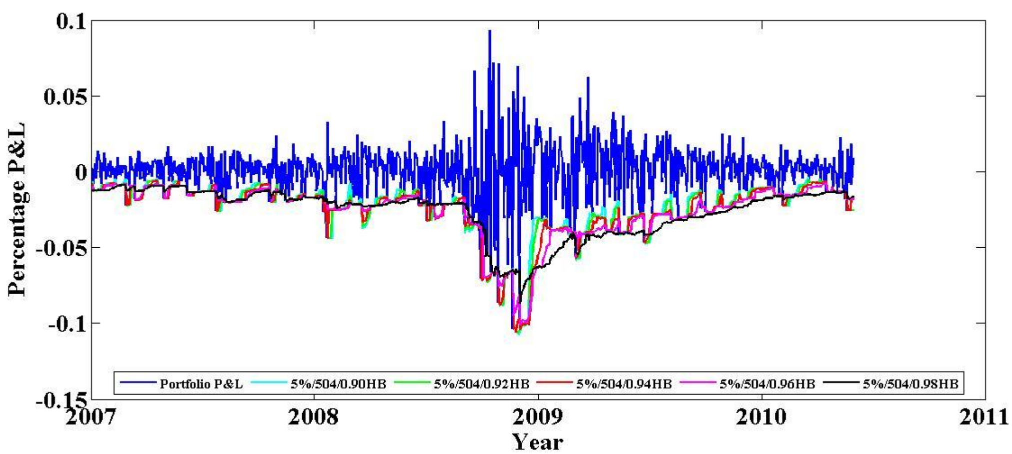

Figure 35: *Global Equity P&L and 1% / 1-Year Hybrid Approach VaR with Different λ from 01/01/2007 to 05/31/2010*

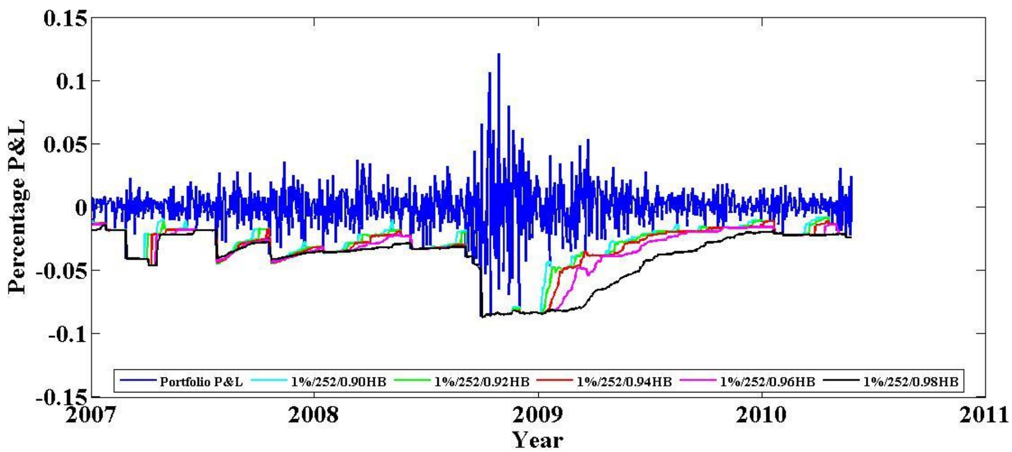

Figure 36: *Global Equity P&L and 1% / 2-Year Hybrid Approach VaR with Different λ from 01/01/2007 to 05/31/2010*

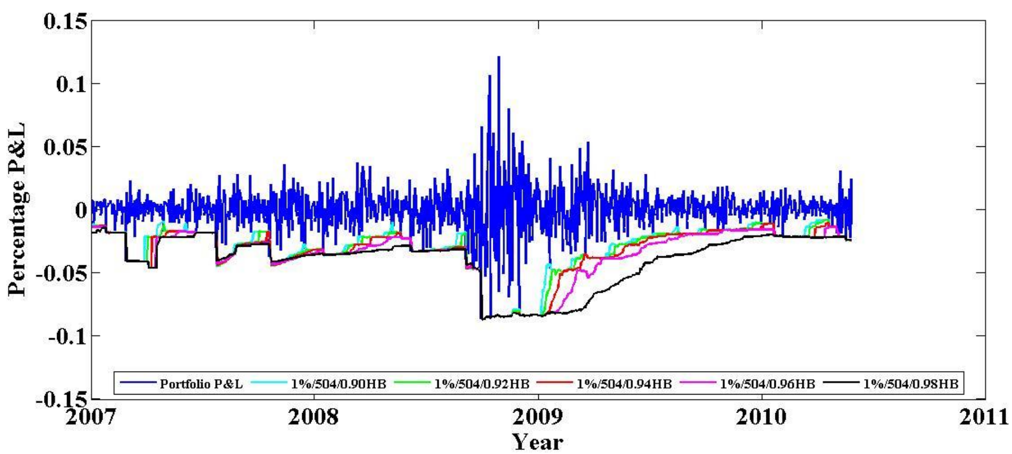

__Figure 37:__ Global Equity P&L and 5% / 1-Year Hybrid Approach VaR with Different λ from 01/01/2007 to 05/31/2010

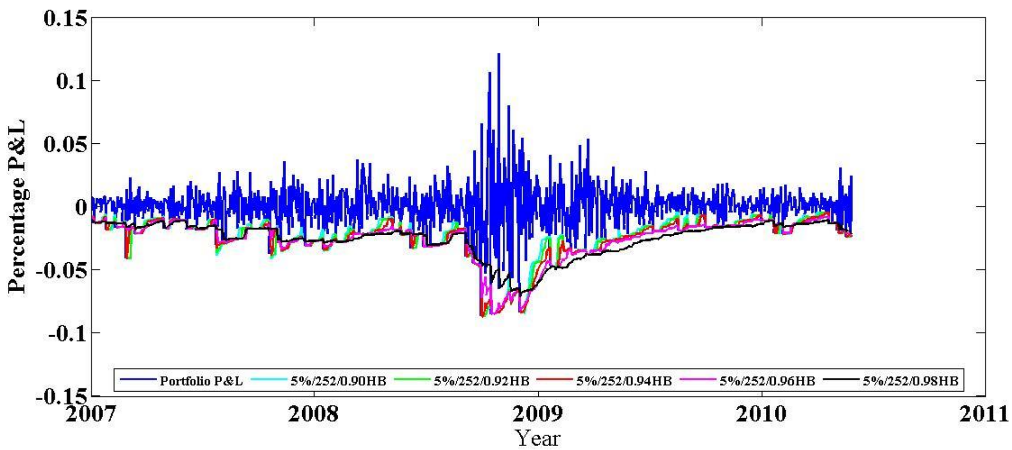

__Figure 38:__ Global Equity P&L and 5% / 2-Year Hybrid Approach VaR with Different λ from 01/01/2007 to 05/31/2010

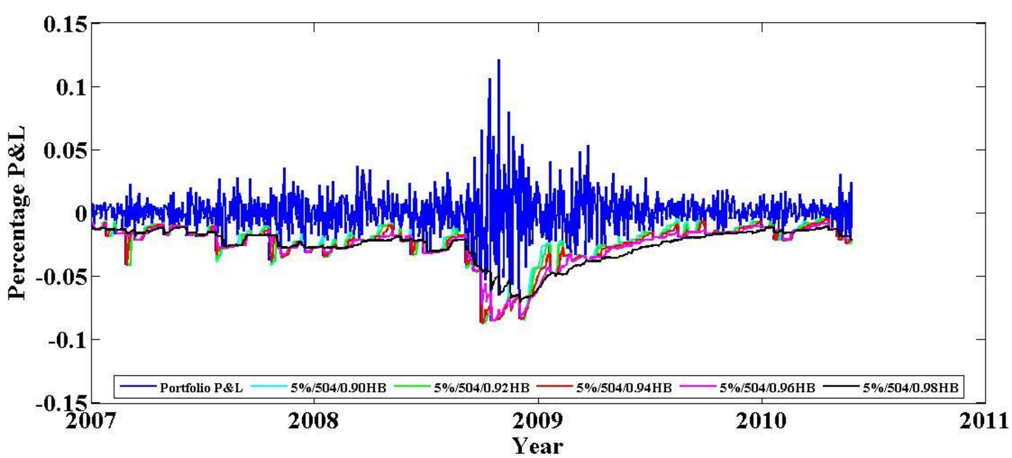

Figure 39: *Total Equity Portfolio P&L and 1% Diversified Historical Simulation VaR from 01/03/2006 to 05/28/2010*

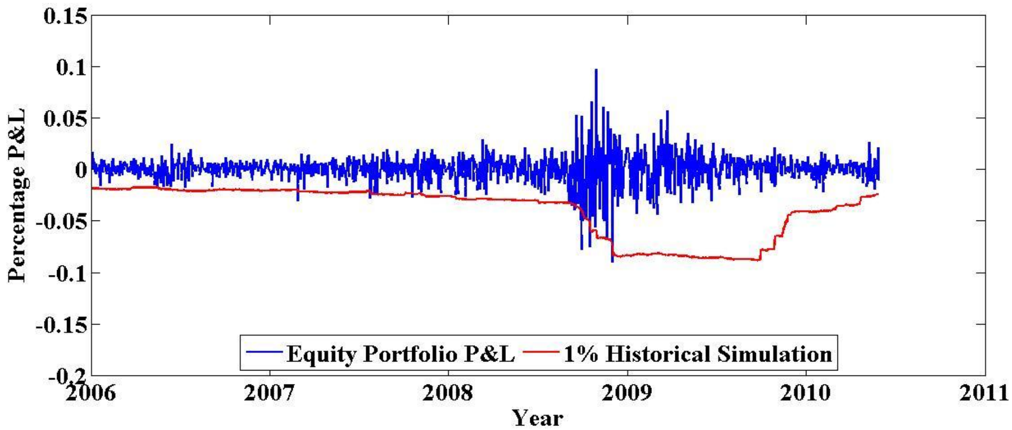

Figure 40: *Total Equity Portfolio P&L and 1% Diversified DCC GARCH VaR from 01/03/2006 to 05/28/2010*

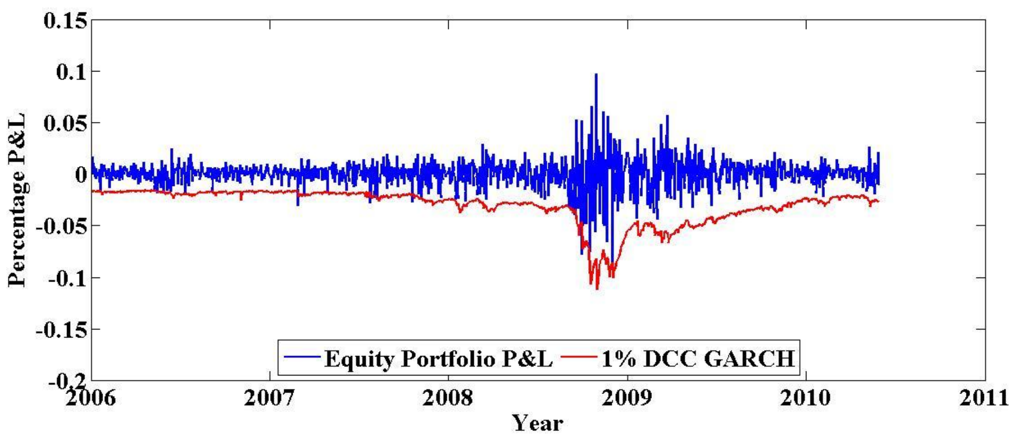

Figure 41*: Total Equity Portfolio P&L and 1% Diversified Filtered HS VaR from 01/03/2006 to 05/28/2010*

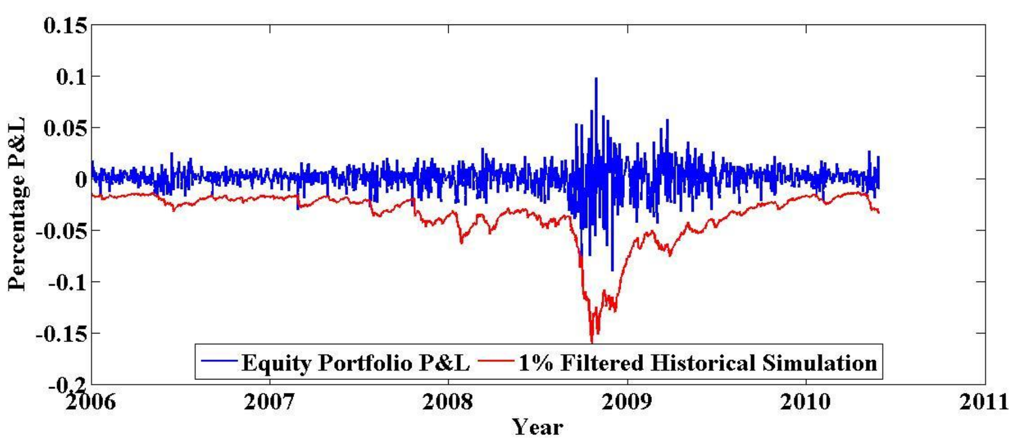

Figure 42: *Total Equity Portfolio P&L and 1% Diversified Hybrid Approach VaR from 01/03/2006 to 05/28/2010*

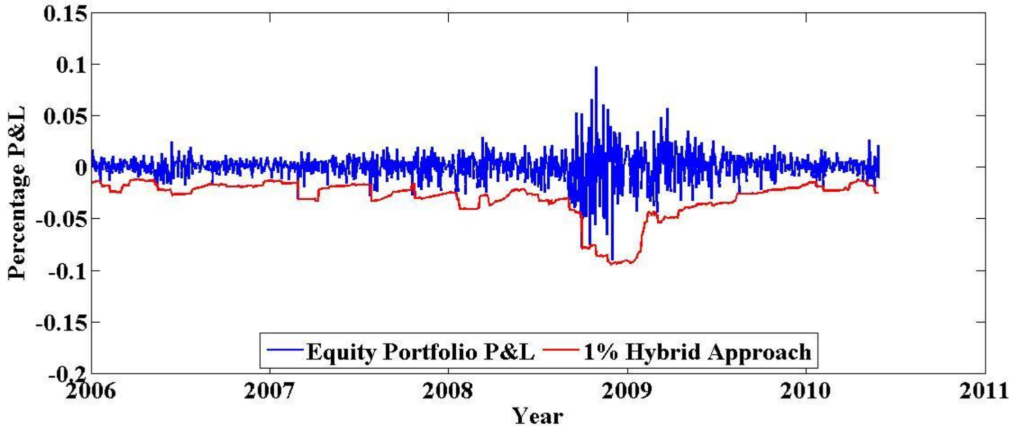

Figure 43: *Total Equity Portfolio P&L and 5% Diversified Historical Simulation VaR from 01/03/2006 to 05/28/2010*

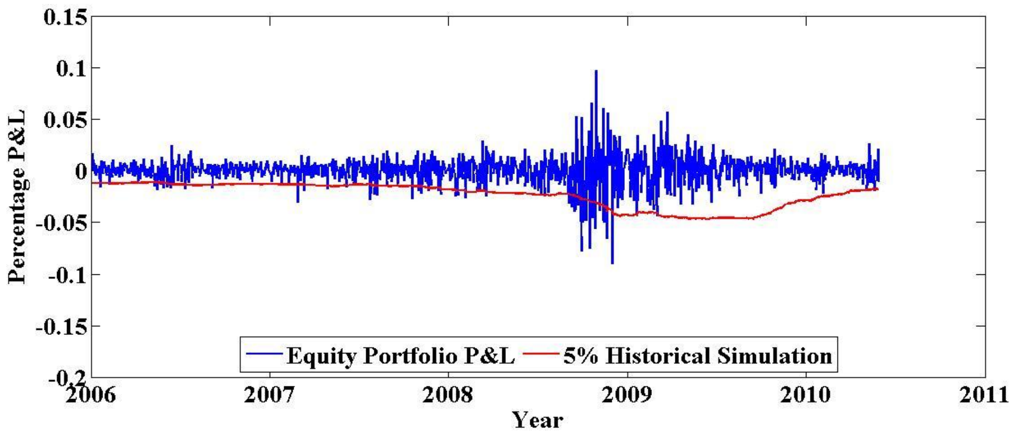

Figure 44: *Total Equity Portfolio P&L and 5% Diversified DCC GARCH VaR from 01/03/2006 to 05/28/2010*

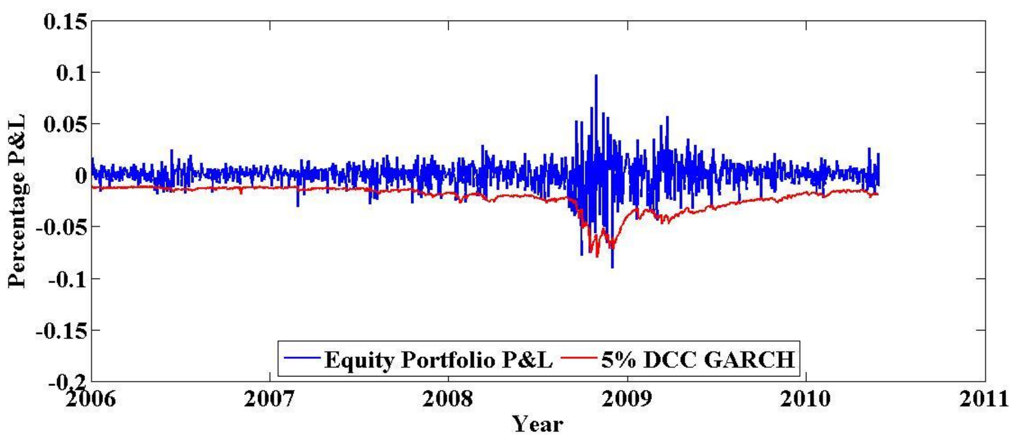

Figure 45: *Total Equity Portfolio P&L and 5% Diversified Filtered HS VaR from 01/03/2006 to 05/28/2010*

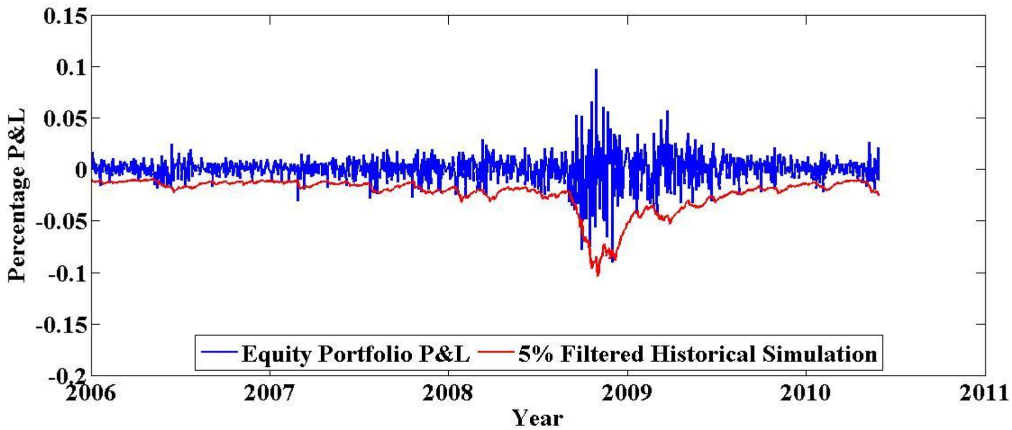

Figure 46: *Total Equity Portfolio P&L and 5% Diversified Hybrid Approach VaR from 01/03/2006 to 05/28/2010*

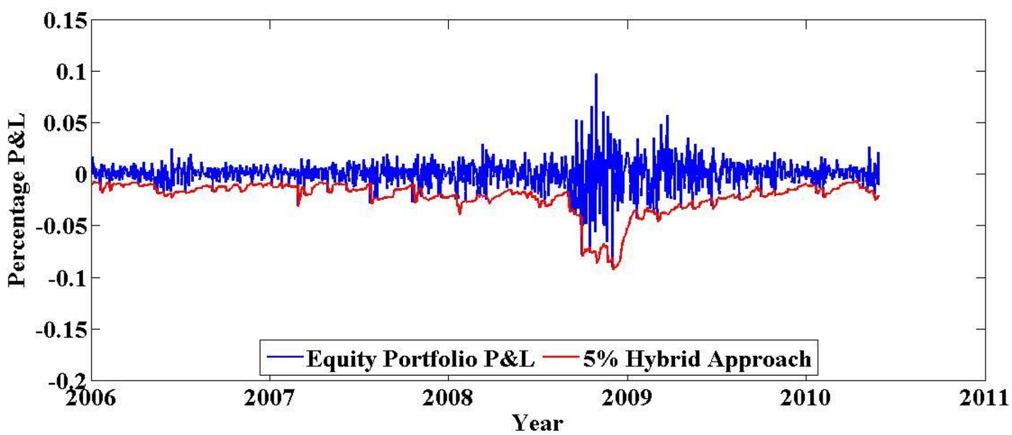

Table 3: *Selected Periods of Hybrid Approach VaR without Interpolation*

July 2007 to May 2010: 858 obervations									
Canadian Equity Portfolio					Global Equity Portfolio				
Size of Data	252	504	252	504	Size of Data	252	504	252	504
Date	1% VaR	1% VaR	5% VaR	5% VaR	Date	1% VaR	1% VaR	5% VaR	5% VaR
02/05/2007	-1.8519606%	-1.8519606%	-1.7533364%	-1.7533364%	07/05/2007	-1.8118404%	-1.8118404%	-1.7071791%	-1.7071791%
03/05/2007	-1.8576460%	-1.8576460%	-1.7565960%	-1.7565960%	08/05/2007	-1.7075069%	-1.7075069%	-1.7075069%	-1.7075069%
04/05/2007	-1.7565752%	-1.7565752%	-1.7565752%	-1.7565752%	09/05/2007	-1.7072750%	-1.7072750%	-1.7072750%	-1.7072750%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
28/05/2007	-1.7370593%	-1.7370593%	-0.9288063%	-0.9288063%	31/05/2007	-1.7075295%	-1.7075295%	-1.0792205%	-1.0792205%
29/05/2007	-1.7371282%	-1.7371282%	-0.9268601%	-0.9268601%	01/06/2007	-1.7091478%	-1.7091478%	-1.0812081%	-1.0812081%
30/05/2007	-1.7357779%	-1.7357779%	-0.9218270%	-0.9218270%	04/06/2007	-1.7098052%	-1.7098052%	-1.0817610%	-1.0817610%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
06/03/2008	-2.6186864%	-2.6186864%	-2.1389537%	-2.1389537%	10/03/2008	-2.7707466%	-2.7707466%	-2.0958416%	-2.0958416%
07/03/2008	-2.6233687%	-2.6233687%	-2.1441599%	-2.1441599%	11/03/2008	-2.7651175%	-2.7651175%	-2.0883308%	-2.0883308%
10/03/2008	-2.6161561%	-2.6161561%	-2.1456523%	-2.1456523%	12/03/2008	-2.7779569%	-2.7779569%	-2.1015898%	-2.1015898%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
03/07/2009	-4.6348215%	-4.6348215%	-3.1673768%	-3.1673768%	06/07/2009	-2.2161499%	-2.2161499%	-1.6146628%	-1.6146628%
06/07/2009	-4.6387306%	-4.6387306%	-2.9178927%	-2.9178927%	07/07/2009	-2.1995170%	-2.1995170%	-1.5488339%	-1.5488339%
07/07/2009	-4.5925377%	-4.5925377%	-2.8995566%	-2.8995566%	08/07/2009	-2.1888904%	-2.1888904%	-1.9075626%	-1.9075626%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
23/09/2009	-3.0086348%	-3.0086348%	-1.3803905%	-1.3803905%	23/09/2009	-1.9355984%	-1.9355984%	-0.9465576%	-0.9465576%
24/09/2009	-3.0045784%	-3.0045784%	-1.3708993%	-1.3708993%	24/09/2009	-1.9279534%	-1.9279534%	-0.7760759%	-0.7760759%
25/09/2009	-2.9975745%	-2.9975745%	-2.3856104%	-2.3856104%	25/09/2009	-1.9180912%	-1.9180912%	-0.7063658%	-0.7063658%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
16/11/2009	-2.6311177%	-2.6311177%	-2.0003679%	-2.0003679%	13/11/2009	-1.6474346%	-1.6474346%	-1.5001975%	-1.5001975%
17/11/2009	-2.6460187%	-2.6460187%	-1.9974926%	-1.9974926%	16/11/2009	-1.6445455%	-1.6445455%	-1.5003871%	-1.5003871%
18/11/2009	-2.6504614%	-2.6504614%	-1.9929158%	-1.9929158%	17/11/2009	-1.6007586%	-1.6007586%	-1.5074156%	-1.5074156%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
19/01/2010	-1.7259220%	-1.7259220%	-1.0846459%	-1.0846459%	19/01/2010	-1.1305867%	-1.1305867%	-1.0492167%	-1.0492167%
20/01/2010	-1.7235712%	-1.7235712%	-1.0854587%	-1.0854587%	20/01/2010	-1.1290043%	-1.1290043%	-1.0132371%	-1.0132371%
21/01/2010	-1.7205654%	-1.7205654%	-1.0711102%	-1.0711102%	21/01/2010	-1.1337949%	-1.1337949%	-1.0150093%	-1.0150093%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
25/03/2010	-2.0865130%	-2.0865130%	-0.7159432%	-0.7159432%	25/03/2010	-2.1817238%	-2.1817238%	-0.8327032%	-0.8327032%
26/03/2010	-2.0780674%	-2.0780674%	-0.7101997%	-0.7101997%	26/03/2010	-2.1779143%	-2.1779143%	-0.8336527%	-0.8336527%
29/03/2010	-2.0881797%	-2.0881797%	-0.7149763%	-0.7149763%	29/03/2010	-2.1800496%	-2.1800496%	-0.8336311%	-0.8336311%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
13/04/2010	-1.5211978%	-1.5211978%	-0.6413241%	-0.6413241%	13/04/2010	-1.3245781%	-1.3245781%	-0.6552584%	-0.6552584%
14/04/2010	-1.5205903%	-1.5205903%	-0.6442511%	-0.6442511%	14/04/2010	-1.3254859%	-1.3254859%	-0.5980787%	-0.5980787%
15/04/2010	-1.4419972%	-1.4419972%	-0.6387691%	-0.6387691%	15/04/2010	-1.2087065%	-1.2087065%	-0.5342876%	-0.5342876%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
27/05/2010	-2.5422227%	-2.5422227%	-2.2153236%	-2.2153236%	26/05/2010	-2.3980714%	-2.3980714%	-2.0883284%	-2.0883284%
28/05/2010	-2.5431968%	-2.5431968%	-1.7998411%	-1.7998411%	27/05/2010	-2.4025302%	-2.4025302%	-2.2092451%	-2.2092451%
31/05/2010	-2.5384503%	-2.5384503%	-1.7869457%	-1.7869457%	28/05/2010	-2.4027890%	-2.4027890%	-2.2101358%	-2.2101358%
Ave. Diff.	0.0000000%		0.0000000%		Ave. Diff.	0.0000000%		0.0000000%	

Table 4: *Selected Periods of Hybrid Approach VaR with Interpolation*

July 2007 to May 2010: 858 obervations									
Canadian Equity Portfolio					Global Equity Portfolio				
Size of Data	252	504	252	504		252	504	252	504
Date	1% VaR	1% VaR	5% VaR	5% VaR	Date	1% VaR	1% VaR	5% VaR	5% VaR
02/05/2007	-1.8895475%	-1.8595015%	-1.7842811%	-1.7637588%	07/05/2007	-1.8158772%	-1.8158772%	-1.7229108%	-1.7126713%
03/05/2007	-1.8773531%	-1.8621624%	-1.7825837%	-1.7658831%	08/05/2007	-1.8104934%	-1.7432312%	-1.7160511%	-1.7104707%
04/05/2007	-1.8569667%	-1.7977398%	-1.7763575%	-1.7646867%	09/05/2007	-1.8062108%	-1.7422653%	-1.7082586%	-1.7076228%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
28/05/2007	-1.8391429%	-1.7470936%	-0.9340219%	-0.9340219%	31/05/2007	-1.7714813%	-1.7267769%	-1.0957064%	-1.0792382%
29/05/2007	-1.8372753%	-1.7401710%	-0.9325860%	-0.9325860%	01/06/2007	-1.7682183%	-1.7258736%	-1.0941428%	-1.0930525%
30/05/2007	-1.8286708%	-1.7988713%	-0.9284584%	-0.9284584%	04/06/2007	-1.7635999%	-1.7248571%	-1.0922939%	-1.0913575%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
06/03/2008	-2.6711831%	-2.6283177%	-2.1821914%	-2.1821913%	10/03/2008	-2.8598967%	-2.7752499%	-2.1318314%	-2.1082410%
07/03/2008	-2.6728612%	-2.6233765%	-2.1806653%	-2.1806653%	11/03/2008	-2.8529874%	-2.7711032%	-2.0884797%	-2.0884797%
10/03/2008	-2.6606997%	-2.6606997%	-2.1770655%	-2.1770655%	12/03/2008	-2.8493604%	-2.7804876%	-2.1291519%	-2.1061280%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
03/07/2009	-4.8902743%	-4.7684346%	-3.2180015%	-3.2180003%	06/07/2009	-2.2401796%	-2.2260434%	-1.6198192%	-1.6198192%
06/07/2009	-4.8662919%	-4.7575064%	-2.9620352%	-2.9228325%	07/07/2009	-2.2188201%	-2.2151585%	-1.5533086%	-1.5533086%
07/07/2009	-4.8318263%	-4.7312648%	-2.9182968%	-2.9074780%	08/07/2009	-2.2051415%	-2.2033301%	-1.9304061%	-1.9169898%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
23/09/2009	-3.0972295%	-3.0499980%	-1.3858232%	-1.3858232%	23/09/2009	-1.9568002%	-1.9370106%	-0.9539301%	-0.9510573%
24/09/2009	-3.0660068%	-3.0405544%	-1.4014378%	-1.4014379%	24/09/2009	-1.9474842%	-1.9288917%	-0.7885148%	-0.7864646%
25/09/2009	-3.0314975%	-3.0314975%	-2.3998138%	-2.3998138%	25/09/2009	-1.9357952%	-1.9220247%	-0.7466188%	-0.7140545%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
16/11/2009	-2.6510013%	-2.6483004%	-2.2068580%	-2.0111490%	13/11/2009	-1.6484504%	-1.6484505%	-1.5008498%	-1.5008498%
17/11/2009	-2.6594586%	-2.6566368%	-2.1824653%	-2.0157527%	16/11/2009	-1.6451671%	-1.6451671%	-1.5009116%	-1.5009116%
18/11/2009	-2.6569866%	-2.6559136%	-1.9951747%	-1.9951747%	17/11/2009	-1.6500761%	-1.6007634%	-1.5236320%	-1.5108468%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
19/01/2010	-1.7751535%	-1.7488825%	-1.0981786%	-1.0981786%	19/01/2010	-1.1345716%	-1.1345716%	-1.0494817%	-1.0492204%
20/01/2010	-1.7604011%	-1.7393129%	-1.0934833%	-1.0883182%	20/01/2010	-1.1337748%	-1.1337748%	-1.0457075%	-1.0223776%
21/01/2010	-1.7428236%	-1.7323326%	-1.0878927%	-1.0760568%	21/01/2010	-1.1870138%	-1.1668534%	-1.0374068%	-1.0225224%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
25/03/2010	-2.1081830%	-2.0928059%	-0.7934385%	-0.7731985%	25/03/2010	-2.1899538%	-2.1843987%	-0.8697955%	-0.8337631%
26/03/2010	-2.1414647%	-2.0873634%	-0.7144950%	-0.7144950%	26/03/2010	-2.1834584%	-2.1782346%	-0.8676744%	-0.8637045%
29/03/2010	-2.1239716%	-2.0910822%	-0.7790013%	-0.7701485%	29/03/2010	-2.1827741%	-2.1801206%	-0.8628260%	-0.8584998%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
13/04/2010	-1.5373501%	-1.5327638%	-0.6809625%	-0.6692400%	13/04/2010	-1.3299863%	-1.3281451%	-0.6877023%	-0.6631261%
14/04/2010	-1.5255577%	-1.5231841%	-0.6731495%	-0.6467621%	14/04/2010	-1.3258964%	-1.3257124%	-0.6124758%	-0.6124761%
15/04/2010	-1.4555879%	-1.4523365%	-0.6743085%	-0.6427043%	15/04/2010	-1.2183417%	-1.2102057%	-0.5401056%	-0.5401061%
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
27/05/2010	-2.6006253%	-2.5480676%	-2.2566490%	-2.2566504%	26/05/2010	-2.3980714%	-2.4257930%	-2.1319139%	-2.1319139%
28/05/2010	-2.6153901%	-2.5468658%	-1.8250825%	-1.8086078%	27/05/2010	-2.4025302%	-2.4324645%	-2.3748008%	-2.2248769%
31/05/2010	-2.6098257%	-2.5450551%	-1.8155256%	-1.7933462%	28/05/2010	-2.4027890%	-2.4035566%	-2.3649028%	-2.2445598%
Ave. Diff.	-0.0112701%		-0.0081901%		Ave. Diff.	-0.0048493%		-0.0122776%	

Table 5: *Statistics Summary of 1% VaR of Canadian Equity Portfolio*

Statistics & Back Testing 1% Value at Risk of Canadian Equity Portfolio
April 30 2003 to May 31 2010: 1782 observations

Panel I: Normality Analysis

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Observations	169	253	251	251	252	252	251	103	1782
Mean	0.0012	0.0006	0.0008	0.0007	0.0005	-0.0013	0.0017	0.0000	0.0005
Variance	0.0000	0.0000	0.0000	0.0001	0.0001	0.0006	0.0003	0.0001	0.0002
Skewness	-0.1350	-0.2790	-0.0749	-0.4237	-0.4660	-0.4331	-0.2480	-0.1955	-0.7086
Kurtosis	3.0359	3.7156	4.1935	3.3426	3.3624	6.4921	3.6177	3.4705	14.7138
GED Factor	1.9460	1.5754	1.5074	1.8626	1.7937	1.6235	1.7100	1.2389	1.6432
Jarque-Bera	0.5224	8.6816	15.1329	8.7379	10.5009	135.9220	6.5623	1.6065	11184.2522
JB Crtitical	5.6303	5.7370	5.7352	5.7352	5.7361	5.7361	5.7352	5.4452	5.9613
Normality	Accept	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Reject

Panel II: Number of Exception

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0	5	7	4	2	16	0	0	34
DCC GARCH	0	5	3	4	6	11	0	1	30
Filtered Historical Simulation	1	5	2	4	4	2	1	2	21
Hybrid Approach	5	5	9	6	4	9	3	5	46

Panel III: Unconditional Coverage Back Testing

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	*3.3970	1.8966	**5.4604	0.7570	0.1166	***32.9284	**5.0453	2.0704	***11.7195
DCC GARCH	*3.3970	1.8966	0.0909	0.7570	*3.4988	***15.7516	**5.0453	0.0009	***6.9768
Filtered Historical Simulation	0.3334	1.8966	0.1125	0.7570	0.7451	0.1166	1.1886	0.7236	0.5422
Hybrid Approach	**4.2930	1.8966	***10.1760	*3.5270	0.7451	***10.1232	0.0909	***8.0154	***31.338

Panel IV: Independence Back Testing

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	N/A	*3.1764	***6.7514	0.1301	0.0321	0.0004	N/A	N/A	***8.5498
DCC GARCH	N/A	*3.1764	0.0729	0.1301	0.2939	1.0089	N/A	0.0198	0.3926
Filtered Historical Simulation	0.0120	0.2024	0.0323	0.1301	0.1296	0.0321	0.0080	0.0800	0.5011
Hybrid Approach	0.3068	0.2024	1.0122	0.2951	0.1296	0.6696	0.0729	0.5157	0.0331

Panel V: Conditional Coverage Back Testing

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	N/A	*5.0730	***12.2118	0.8871	0.1488	***32.9288	N/A	N/A	***20.2694
DCC GARCH	N/A	*5.0730	0.1638	0.8871	3.7927	***16.7603	N/A	0.0207	**7.3694
Filtered Historical Simulation	0.3454	2.0991	0.1448	0.8871	0.8746	0.1488	1.1966	0.8036	1.0433
Hybrid Approach	4.5998	2.0991	***11.1881	3.8221	0.8746	***10.7927	0.1638	**8.5311	***31.3711

Note that in the normality analysis, the Jarque-Bera test is based on 95% confidence level of $\chi2$ distribution with two degree of freedom. The Generalized Error Distribution (GED) factor is the shape describer (v). If the value of v is 2, the GED is equivalent to normal distribution. If v is less than 2, the fat tails appear. The *, **, and *** indicate the rejection of the likelihood ratio test for conifdence level of 90%, 95%, and 99% respectively, and for $\chi2$ distribution with one or two degree of freedom (i.e. one for Unconditional Coverage and Independenc test, two for Conditional Coverage tets)

__Table 6:__ Statistics Summary of 5% VaR of Canadian Equity Portfolio

Statistics & Back Testing 5% Value at Risk of Canadian Equity Portfolio

April 30 2003 to May 31 2010: 1782 observations

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Panel I: Normality Analysis									
Observations	169	253	251	251	252	252	251	103	1782
Mean	0.0012	0.0006	0.0008	0.0007	0.0005	-0.0013	0.0017	0.0000	0.0005
Variance	0.0000	0.0000	0.0000	0.0001	0.0001	0.0006	0.0003	0.0001	0.0002
Skewness	-0.1350	-0.2790	-0.0749	-0.4237	-0.4660	-0.4331	-0.2480	-0.1955	-0.7086
Kurtosis	3.0359	3.7156	4.1935	3.3426	3.3624	6.4921	3.6177	3.4705	14.7138
GED Factor	1.9460	1.5754	1.5074	1.8626	1.7937	1.6235	1.7100	1.2389	1.6432
Jarque-Bera	0.5224	8.6816	15.1329	8.7379	10.5009	135.9220	6.5623	1.6065	11184.2522
JB Crtitical	5.6303	5.7370	5.7352	5.7352	5.7361	5.7361	5.7352	5.4452	5.9613
Normality	Accept	Reject	Reject	Reject	Reject	Reject	Reject	Accept	Reject
Panel II: Number of Exception									
Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	1	17	19	15	15	34	2	1	104
DCC GARCH	1	12	14	16	16	27	8	2	96
Filtered Historical Simulation	6	12	16	12	14	20	8	7	95
Hybrid Approach	7	12	16	16	16	16	14	8	105
Panel III: Unconditional Coverage Back Testing									
Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	***10.9721	1.4281	*3.0353	0.4751	0.4547	***26.6738	***14.2137	**5.1956	2.4949
DCC GARCH	***10.9721	0.0357	0.1703	0.9219	0.8931	***13.2396	1.9818	2.6169	0.5492
Filtered Historical Simulation	0.8283	0.0357	0.9219	0.0257	0.1583	**3.9126	1.9818	0.6320	0.4029
Hybrid Approach	0.2775	0.0357	0.9219	0.9219	0.8931	0.8931	0.1703	1.4309	*2.8320
Panel IV: Independence Back Testing									
Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.0120	***9.1319	0.2280	1.9162	1.9081	*2.9230	0.0323	0.0198	***11.0017
DCC GARCH	0.0120	2.5358	1.5516	2.1897	0.0004	0.4765	0.5290	0.0800	2.6475
Filtered Historical Simulation	1.7372	2.5358	0.8601	1.2106	0.0639	0.1146	1.3873	0.5153	*4.2965
Hybrid Approach	1.2060	1.2005	1.0134	2.1897	0.0004	2.1804	0.0620	1.3634	0.2746
Panel V: Conditional Coverage Back Testing									
Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	***10.9841	***10.5560	3.2633	2.3913	2.3628	***29.5968	***14.2460	*5.2154	***13.4966
DCC GARCH	***10.9841	2.5716	1.7218	3.1117	0.8935	***13.7162	2.5108	2.6969	3.1967
Filtered Historical Simulation	2.5655	2.5716	1.7821	1.2363	0.2222	4.0272	3.3691	1.1472	*4.6994
Hybrid Approach	1.4835	1.2362	1.9354	3.1117	0.8935	3.0735	0.2322	2.7943	3.1066

Note that in the normality analysis, the Jarque-Bera test is based on 95% confidence level of $\chi2$ distribution with two degree of freedom. The Generalized Error Distribution (GED) factor is the shape describer (v). If the value of v is 2, the GED is equivalent to normal distribution. If v is less than 2, the fat tails appear. The *, **, and *** indicate the rejection of the likelihood ratio test for conifdence level of 90%, 95%, and 99% respectively, and for $\chi2$ distribution with one or two degree of freedom (i.e. one for Unconditional Coverage and Independenc test, two for Conditional Coverage tets)

Table 7: Statistics Summary of 1% VaR of Global Equity Portfolio

Statistics & Back Testing 1% Value at Risk of Global Equity Portfolio
April 8 2004 to May 28 2010: 1546 observations

Panel I: Normality Analysis

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Observations	185	252	251	251	253	252	102	1546
Mean	0.0000	0.0001	0.0007	-0.0003	-0.0013	0.0007	-0.0005	0.0000
Variance	0.0001	0.0001	0.0001	0.0001	0.0007	0.0002	0.0001	0.0002
Skewness	-0.1159	-0.1082	-0.2275	-0.3454	0.4362	0.1502	0.0640	0.2926
Kurtosis	2.7770	2.8710	4.2609	3.9696	6.9185	4.4843	3.4928	13.5379
GED Factor	2.0323	2.1040	1.3891	1.4817	1.6781	1.7303	1.7669	1.6183
Jarque-Bera	0.7977	0.6662	18.7912	14.8240	169.8849	24.0802	1.1019	7749.3956
JB Crtitical	5.6568	5.7361	5.7352	5.7352	5.7370	5.7361	5.4407	5.9613
Normality	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Reject

Panel II: Number of Exception

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	1	2	5	7	14	0	2	31
DCC GARCH	1	2	6	9	7	1	0	26
Filtered Historical Simulation	3	1	3	4	3	0	2	16
Hybrid Approach	4	7	5	5	8	0	6	35

Panel III: Unconditional Coverage Back Testing

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.4736	0.0870	1.9366	**5.4604	***22.0589	**5.0654	0.7429	***12.2137
DCC GARCH	0.4736	0.1166	*3.5270	***10.1760	**5.3879	1.2007	2.0503	**6.0245
Filtered Historical Simulation	0.6078	1.2007	0.0909	0.7570	0.0832	**5.0654	0.7429	0.0188
Hybrid Approach	1.8942	**5.4241	1.9366	1.9366	***7.5999	**5.0654	***11.5532	***18.3670

Panel IV: Independence Back Testing

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.0109	0.0726	0.2041	0.4033	1.4149	N/A	0.0808	1.2696
DCC GARCH	0.0109	0.0321	0.2951	0.6724	0.4001	0.0080	N/A	0.8901
Filtered Historical Simulation	0.0995	0.0080	0.0729	0.1301	0.0723	N/A	0.0808	0.3349
Hybrid Approach	0.1778	0.4017	0.2041	0.2041	0.5247	N/A	0.7584	1.6227

Panel V: Conditional Coverage Back Testing

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.4845	0.1596	2.1407	*5.8638	***23.4738	N/A	0.8237	***13.4832
DCC GARCH	0.4845	0.1488	3.8221	***10.8483	*5.7880	1.2087	N/A	**6.9146
Filtered Historical Simulation	0.7072	1.2087	0.1638	0.8871	0.1555	N/A	0.8237	0.3537
Hybrid Approach	2.0720	5.8257	2.1407	2.1407	**8.1246	N/A	***12.3116	***19.9896

Note that in the normality analysis, the Jarque-Bera test is based on 95% confidence level of $\chi 2$ distribution with two degree of freedom. The Generalized Error Distribution (GED) factor is the shape describer (ν). If the value of ν is 2, the GED is equivalent to normal distribution. If ν is less than 2, the fat tails appear. The *, **, and *** indicate the rejection of the likelihood ratio test for conifdence level of 90%, 95%, and 99% respectively, and for $\chi 2$ distribution with one or two degree of freedom (i.e. one for Unconditional Coverage and Independenc test, two for Conditional Coverage tets)

Table 8: *Statistics Summary of 5% VaR of Global Equity Portfolio*

Statistics & Back Testing 5% Value at Risk of Global Equity Portfolio
April 8 2004 to May 28 2010: 1546 observations

Panel I: Normality Analysis

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Observations	185	252	251	251	253	252	102	1546
Mean	0.0000	0.0001	0.0007	-0.0003	-0.0013	0.0007	-0.0005	0.0000
Variance	0.0001	0.0001	0.0001	0.0001	0.0007	0.0002	0.0001	0.0002
Skewness	-0.1159	-0.1082	-0.2275	-0.3454	0.4362	0.1502	0.0640	0.2926
Kurtosis	2.7770	2.8710	4.2609	3.9696	6.9185	4.4843	3.4928	13.5379
GED Factor	2.0323	2.1040	1.3891	1.4817	1.6781	1.7303	1.7669	1.6183
Jarque-Bera	0.7977	0.6662	18.7912	14.8240	169.8849	24.0802	1.1019	7749.3956
JB Crtitical	5.6568	5.7361	5.7352	5.7352	5.7370	5.7361	5.4407	5.9613
Normality	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Reject

Panel II: Number of Exception

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	10	11	14	21	25	1	7	89
DCC GARCH	7	14	14	20	19	2	7	83
Filtered Historical Simulation	10	13	14	17	14	7	9	84
Hybrid Approach	10	14	17	17	15	10	7	90

Panel III: Unconditional Coverage Back Testing

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.0624	0.2230	0.1703	**5.0247	***10.0067	***18.6858	0.6709	1.7812
DCC GARCH	0.6267	0.1583	0.1703	*3.9757	*2.9270	***14.3004	0.6709	0.4325
Filtered Historical Simulation	0.0624	0.0132	0.1703	1.5023	0.1468	*3.1010	2.5828	0.5952
Hybrid Approach	0.0624	0.1583	1.5023	1.5023	0.4348	0.6059	0.6709	2.0910

Panel IV: Independence Back Testing

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	1.1501	0.4765	0.0620	0.0363	1.3449	0.0080	1.0435	0.0036
DCC GARCH	0.5538	0.0639	0.0620	0.1108	*3.1021	0.0321	1.0435	0.0544
Filtered Historical Simulation	1.1501	2.0056	0.0620	0.0252	1.6480	2.1440	0.0554	0.4623
Hybrid Approach	1.1501	1.5624	0.5996	0.0252	1.9000	0.8627	1.0435	0.1185

Panel V: Conditional Coverage Back Testing

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	1.2125	0.6995	0.2323	*5.0611	***11.3515	***18.6938	1.7144	1.7848
DCC GARCH	1.1805	0.2222	0.2323	4.0865	*6.0291	***14.3325	1.7144	0.4869
Filtered Historical Simulation	1.2125	2.0189	0.2323	1.5276	1.7948	*5.2449	2.6382	1.0576
Hybrid Approach	1.2125	1.7207	2.1019	1.5276	2.3348	1.4686	1.7144	2.2094

Note that in the normality analysis, the Jarque-Bera test is based on 95% confidence level of $\chi 2$ distribution with two degree of freedom. The Generalized Error Distribution (GED) factor is the shape describer (ν). If the value of ν is 2, the GED is equivalent to normal distribution. If ν is less than 2, the fat tails appear. The *, **, and *** indicate the rejection of the likelihood ratio test for conifdence level of 90%, 95%, and 99% respectively, and for $\chi 2$ distribution with one or two degree of freedom (i.e. one for Unconditional Coverage and Independenc test, two for Conditional Coverage tets)

Table 9: *Summary of Quantile Regression Test Results for 1% VaR of Canadian Equity Portfolio*

Statistics of Quantile Regression Test of 1% Value at Risk of Canadian Equity Portfolio
April 30 2003 to May 31 2010: 1782 observations

Panel I: Intercept

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	-0.0078	-0.0191	-0.0283	-0.0978	-0.0329	-0.0094	0.0080	-0.0314	-0.0028
DCC GARCH	-0.0062	-0.0206	0.0189	-0.0446	-0.0265	-0.0033	0.0140	-0.0467	-0.0013
Filtered Historical Simulation	-0.0079	-0.0157	0.0031	-0.0094	-0.0230	-0.0064	-0.0075	0.0064	-0.0056
Hybrid Approach	-0.0192	-0.0159	-0.0088	-0.0262	-0.0226	-0.0173	-0.0421	-0.0627	-0.0031

Panel II: Standard Error - Intercept

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.0170	0.0228	0.0243	0.0454	0.0946	0.0363	0.1421	0.0817	0.0040
DCC GARCH	0.0112	0.0192	0.0153	0.0539	0.0175	0.0248	0.0316	0.0643	0.0025
Filtered Historical Simulation	0.0050	0.0114	0.0184	0.0231	0.0104	0.0146	0.0218	0.0231	0.0019
Hybrid Approach	0.0085	0.0126	0.0061	0.0141	0.0066	0.0211	0.0330	0.0249	0.0021

Panel III: Slope

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.2373	-0.0739	-0.9278	-4.1526	-0.5696	1.4414	0.5587	-0.2081	1.0000
DCC GARCH	0.3644	-0.2056	2.5141	-1.5567	-0.3240	1.5386	1.1002	-0.8607	1.1001
Filtered Historical Simulation	0.2976	0.1045	1.1978	0.5656	-0.0868	0.8292	0.6971	1.4130	0.7618
Hybrid Approach	-0.5963	0.1004	0.5984	-0.5214	-0.0970	1.0215	-0.0435	-2.0719	1.1504

Panel IV: Standard Error - Slope

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.7671	1.3774	1.7680	2.4570	4.3119	0.9814	1.6496	1.9522	0.1855
DCC GARCH	0.5600	1.3934	1.1141	3.4035	1.0055	0.6158	0.6731	2.4734	0.1293
Filtered Historical Simulation	0.2815	0.8175	1.3508	1.3707	0.4626	0.2724	0.4588	0.9597	0.0760
Hybrid Approach	0.8316	0.8364	0.4977	0.8053	0.3660	0.4967	0.8014	1.4436	0.1300

Panel IV: Wald Joint Test

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	*4.6835	0.8193	1.7396	*5.3697	0.3831	**6.8672	***35.5084	3.1080	***10.5045
DCC GARCH	3.1920	**7.9966	2.8479	3.0081	3.3583	*5.4142	3.9137	0.5989	**6.9499
Filtered Historical Simulation	**6.5452	3.8382	0.0683	0.5458	5.5207	0.4082	1.3803	0.5232	***10.0620
Hybrid Approach	**6.6274	2.2229	3.2667	3.5817	***11.6717	3.4663	1.6964	***8.8643	***27.4983

The *, **, and *** indicate the rejection of the Wald joint test for conifdence level of 90%, 95%, and 99% respectively, and for $\chi 2$ distribution with one or two degree of freedom

Table 10: *Summary of Quantile Regression Test Results for 5% VaR of Canadian Equity Portfolio*

Statistics of Quantile Regression Test of 5% Value at Risk of Canadian Equity Portfolio
April 30 2003 to May 31 2010: 1782 observations

Panel I: Intercept

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	-0.0078	-0.0191	-0.0283	-0.0978	-0.0329	-0.0094	0.0080	-0.0314	-0.0039
DCC GARCH	-0.0062	-0.0206	0.0189	-0.0446	-0.0265	-0.0033	0.0140	-0.0467	0.0011
Filtered Historical Simulation	-0.0079	-0.0157	0.0031	-0.0094	-0.0230	-0.0064	-0.0075	0.0064	-0.0018
Hybrid Approach	-0.0192	-0.0159	-0.0088	-0.0262	-0.0226	-0.0173	-0.0421	-0.0627	-0.0018

Panel II: Standard Error - Intercept

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.0170	0.0228	0.0243	0.0454	0.0946	0.0363	0.1421	0.0817	0.0016
DCC GARCH	0.0112	0.0192	0.0153	0.0539	0.0175	0.0248	0.0316	0.0643	0.0014
Filtered Historical Simulation	0.0050	0.0114	0.0184	0.0231	0.0104	0.0146	0.0218	0.0231	0.0015
Hybrid Approach	0.0085	0.0126	0.0061	0.0141	0.0066	0.0211	0.0330	0.0249	0.0012

Panel III: Slope

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.2373	-0.0739	-0.9278	-4.1526	-0.5696	1.4414	0.5587	-0.2081	0.7476
DCC GARCH	0.3644	-0.2056	2.5141	-1.5567	-0.3240	1.5386	1.1002	-0.8607	1.1075
Filtered Historical Simulation	0.2976	0.1045	1.1978	0.5656	-0.0868	0.8292	0.6971	1.4130	0.8944
Hybrid Approach	-0.5963	0.1004	0.5984	-0.5214	-0.0970	1.0215	-0.0435	-2.0719	0.9130

Panel IV: Standard Error - Slope

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.7671	1.3774	1.7680	2.4570	4.3119	0.9814	1.6496	1.9522	0.1201
DCC GARCH	0.5600	1.3934	1.1141	3.4035	1.0055	0.6158	0.6731	2.4734	0.1086
Filtered Historical Simulation	0.2815	0.8175	1.3508	1.3707	0.4626	0.2724	0.4588	0.9597	0.1020
Hybrid Approach	0.8316	0.8364	0.4977	0.8053	0.3660	0.4967	0.8014	1.4436	0.0763

Panel IV: Wald Joint Test

Periods	2003	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	***-36.9889	***-9.3594	***12.1570	1.2339	**6.5088	***55.7155	***-54.0469	***-19.3981	**6.1324
DCC GARCH	***-28.3121	-0.6240	4.5603	3.0599	***9.4135	***27.5080	***-13.1673	-4.4148	1.0977
Filtered Historical Simulation	**-7.7804	1.5043	2.5071	2.7405	-4.5733	*5.9018	***-15.7590	0.4498	1.4056
Hybrid Approach	**-7.8540	-3.1593	2.7326	***-9.2334	0.6816	*4.8494	-2.2277	2.2605	2.2298

The *, **, and *** indicate the rejection of the Wald joint test for conifdence level of 90%, 95%, and 99% respectively, and for $\chi 2$ distribution with one or two degree of freedom

***Table 11**: Summary of Quantile Regression Test Results for 1% VaR of Global Equity Portfolio*

Statistics of Quantile Regression Testing 1% Value at Risk of Global Equity Portfolio
April 8 2004 to May 28 2010: 1546 observations

Panel I: Intercept

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	-0.0527	-0.0845	-0.0831	-0.0444	0.0555	0.0078	-0.0273	-0.0034
DCC GARCH	-0.0103	-0.0649	-0.0442	-0.0520	-0.0164	0.0028	0.0071	-0.0053
Filtered Historical Simulation	-0.0388	-0.0253	-0.0184	-0.0409	-0.0171	-0.0056	-0.0176	-0.0075
Hybrid Approach	-0.0325	-0.0101	-0.0186	-0.0434	-0.0128	-0.0058	-0.0173	-0.0044

Panel II: Standard Error - Intercept

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.0755	0.0466	0.0686	0.0272	0.0785	0.0847	0.0361	0.0058
DCC GARCH	0.0318	0.0544	0.0290	0.0209	0.0383	0.0218	0.0569	0.0053
Filtered Historical Simulation	0.0211	0.0277	0.0210	0.0133	0.0227	0.0161	0.0213	0.0031
Hybrid Approach	0.0210	0.0249	0.0163	0.0130	0.0264	0.0097	0.0245	0.0023

Panel III: Slope

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	-1.7329	-3.5124	-2.9683	-0.4387	2.9409	0.5256	-0.1616	1.0185
DCC GARCH	0.3871	-2.3927	-1.0213	-0.6562	0.7423	0.7858	1.2429	0.8920
Filtered Historical Simulation	-0.9676	-0.3771	0.1087	-0.1683	0.6181	0.5670	0.1788	0.7075
Hybrid Approach	-0.7809	0.5225	0.1129	-0.3659	0.8567	0.6099	0.3019	0.9333

Panel IV: Standard Error - Slope

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	3.8402	2.4785	3.4251	1.0955	2.0824	1.0891	1.1797	0.2159
DCC GARCH	1.5132	2.9250	1.6216	0.9090	0.9900	0.5401	2.5265	0.2032
Filtered Historical Simulation	1.0516	1.3363	0.9966	0.3895	0.4151	0.4221	1.0827	0.1291
Hybrid Approach	1.1347	1.3771	0.8020	0.4748	0.7121	0.3646	1.5726	0.0930

Panel V: Wald Joint Test

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.5381	3.3290	2.5450	4.4995	3.0816	***64.9460	1.9370	*5.2988
DCC GARCH	0.5758	2.2994	**7.0464	***10.4283	0.4732	3.2195	0.0702	4.2980
Filtered Historical Simulation	3.5088	3.1065	0.8000	***9.7841	0.8558	3.6763	0.7165	*5.8284
Hybrid Approach	2.4708	0.4902	1.3002	***11.1538	0.5238	2.1602	1.1258	*5.4049

The *, **, and *** indicate the rejection of the Wald joint test for conifdence level of 90%, 95%, and 99% respectively, and for $\chi 2$ distribution with one or two degree of freedom

Table 12: *Summary of Quantile Regression Test Results for 5% VaR of Global Equity Portfolio*

Statistics of Quantile Regression Testing 5% Value at Risk of Global Equity Portfolio
April 8 2004 to May 28 2010: 1546 observations

Panel I: Intercept

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	-0.0527	-0.0845	-0.0831	-0.0444	0.0555	0.0078	-0.0273	-0.0069
DCC GARCH	-0.0103	-0.0649	-0.0442	-0.0520	-0.0164	0.0028	0.0071	-0.0008
Filtered Historical Simulation	-0.0388	-0.0253	-0.0184	-0.0409	-0.0171	-0.0056	-0.0176	-0.0043
Hybrid Approach	-0.0325	-0.0101	-0.0186	-0.0434	-0.0128	-0.0058	-0.0173	-0.0054

Panel II: Standard Error - Intercept

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.0755	0.0466	0.0686	0.0272	0.0785	0.0847	0.0361	0.0032
DCC GARCH	0.0318	0.0544	0.0290	0.0209	0.0383	0.0218	0.0569	0.0023
Filtered Historical Simulation	0.0211	0.0277	0.0210	0.0133	0.0227	0.0161	0.0213	0.0017
Hybrid Approach	0.0210	0.0249	0.0163	0.0130	0.0264	0.0097	0.0245	0.0017

Panel III: Slope

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	-1.7329	-3.5124	-2.9683	-0.4387	2.9409	0.5256	-0.1616	0.6721
DCC GARCH	0.3871	-2.3927	-1.0213	-0.6562	0.7423	0.7858	1.2429	0.9907
Filtered Historical Simulation	-0.9676	-0.3771	0.1087	-0.1683	0.6181	0.5670	0.1788	0.7555
Hybrid Approach	-0.7809	0.5225	0.1129	-0.3659	0.8567	0.6099	0.3019	0.7343

Panel IV: Standard Error - Slope

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	3.8402	2.4785	3.4251	1.0955	2.0824	1.0891	1.1797	0.1856
DCC GARCH	1.5132	2.9250	1.6216	0.9090	0.9900	0.5401	2.5265	0.1397
Filtered Historical Simulation	1.0516	1.3363	0.9966	0.3895	0.4151	0.4221	1.0827	0.1072
Hybrid Approach	1.1347	1.3771	0.8020	0.4748	0.7121	0.3646	1.5726	0.1028

Panel V: Wald Joint Test

Periods	2004	2005	2006	2007	2008	2009	2010	Full Period
Historical Simulation	-1.6557	*-5.9754	-3.3939	***13.6891	***24.3234	***-82.4782	-0.1045	**6.0365
DCC GARCH	***-12.6344	-0.0107	1.0283	***9.7002	***14.4102	***-32.1963	1.5105	0.6569
Filtered Historical Simulation	-3.5232	-0.8615	2.3184	3.3514	0.6429	***-39.9612	4.3077	**6.8563
Hybrid Approach	***-24.9934	-2.1144	**7.3411	3.2021	1.2615	***-13.7053	3.8165	***10.8013

The *, **, and *** indicate the rejection of the Wald joint test for conifdence level of 90%, 95%, and 99% respectively, and for $\chi 2$ distribution with one or two degree of freedom

Table 13: Summary of Speed and Accuracy of VaR Models in Canadian Equity Portfolio

Canadian Equity Porfolio May 31 2010: 33 Holdings			
	Speed per Estimate (Seconds)	1% VaR 1000*MSE	5% VaR
Historical Simulation	0.0049	1.7838	0.6923
DCC GARCH	20.7127	1.0442	0.6088
Filtered Historical Simulation	0.2537	1.5707	0.7506
Hybrid Approach	0.0056	1.1244	0.7235

Resources of Computing and Programming	
Manufacturer:	Dell
Model:	Optiplex 760
Processor:	Intel Core2 Due CPU E8400 @3.00 GHz 2.99 GHz
RAM:	4.00 GB (3.87 GB usable
System type:	64-bit Operating System
Programming software:	Matlab R2009b

Table 14: Summary of Speed and Accuracy of VaR Models in Global Equity Portfolio

Global Equity Portfolio May 28 2010: 29 Holdings			
	Speed per Estimate (Seconds)	1% VaR 1000*MSE	5% VaR
Historical Simulation	0.0003	1.7720	0.7188
DCC GARCH	15.8090	1.2241	0.7075
Filtered Historical Simulation	0.2594	1.6779	0.8275
Hybrid Approach	0.0010	1.1332	0.8035

Resources of Computing and Programming	
Manufacturer:	Dell
Model:	Optiplex 760
Processor:	Intel Core2 Due CPU E8400 @3.00 GHz 2.99 GHz
RAM:	4.00 GB (3.87 GB usable
System type:	64-bit Operating System
Programming software:	Matlab R2009b

Table 15: *Summary of Diversification of the Equity Portfolio*

Summary of Undiversified and Diversified Value at Risk of SIAS Equity Portfolio
January 4 2006 to May 31 2010: 1095 observations

Panel I: Holdings and Weights of the Portfolio

Period	2006	2007	2008	2009	2010	Full Period
Observations	248	246	251	249	101	1095
Avergae Holding of CE	34.75	31.88	29.49	35.27	35.28	33.12
Avergae Holding of GE	11.87	16.48	22.07	32.40	31.02	21.67
Avergae Weight of CE	55.69%	57.99%	54.60%	49.79%	51.46%	56.95%
Avergae Weight of GE	44.31%	42.01%	45.40%	50.21%	48.54%	43.05%

Panel II: 1% Undiversified VaR

Periods	2006	2007	2008	2009	2010	Full Period
Historical Simulation	96243.45	124411.59	196914.76	433949.56	233337.18	184998.87
DCC GARCH	87141.33	111041.40	219151.89	234718.91	161723.79	142476.90
Filtered Historical Simulation	98388.71	146144.73	306626.12	230749.71	130070.50	164927.15
Hybrid Approach	92382.00	125959.69	225970.21	202709.45	124518.07	137792.71

Panel III: 1% Diversified VaR

Periods	2006	2007	2008	2009	2010	Full Period
Historical Simulation	96197.00	124356.54	196821.08	433777.80	233239.59	184911.90
DCC GARCH	87032.58	110923.99	218887.40	234523.55	161559.06	142324.01
Filtered Historical Simulation	98201.15	145880.20	306088.09	230359.11	129834.60	164622.89
Hybrid Approach	92262.93	125803.33	225705.63	202471.69	124363.85	137620.06

Panel IV: Diversification Effect of 1% VaR

Periods	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.049%	0.044%	0.045%	0.040%	0.043%	0.059%
DCC GARCH	0.124%	0.108%	0.107%	0.084%	0.101%	0.112%
Filtered Historical Simulation	0.193%	0.183%	0.174%	0.170%	0.182%	0.205%
Hybrid Approach	0.130%	0.126%	0.118%	0.117%	0.125%	0.140%

Panel V: 5% Undiversified VaR

Periods	2006	2007	2008	2009	2010	Full Period
Historical Simulation	61417.75	77614.06	123866.08	234522.15	147863.61	109667.56
DCC GARCH	61613.63	78512.27	154952.23	165958.95	114347.46	100738.87
Filtered Historical Simulation	65267.09	87239.58	186748.34	160153.60	98314.32	106389.63
Hybrid Approach	63954.12	88013.55	186324.37	144793.82	91260.75	102948.35

Panel VI: 5% Diversified VaR

Periods	2006	2007	2008	2009	2010	Full Period
Historical Simulation	61404.19	77597.43	123837.56	234476.41	147833.39	109643.48
DCC GARCH	61536.73	78429.26	154765.22	165820.82	114230.99	100630.76
Filtered Historical Simulation	65163.84	87103.64	186459.84	159929.35	98166.91	106223.36
Hybrid Approach	63730.16	87730.76	185764.34	144370.60	90965.11	102618.31

Panel VII: Diversification Effect of 5% VaR

Periods	2006	2007	2008	2009	2010	Full Period
Historical Simulation	0.022%	0.021%	0.022%	0.020%	0.021%	0.025%
DCC GARCH	0.124%	0.108%	0.107%	0.084%	0.101%	0.112%
Filtered Historical Simulation	0.160%	0.156%	0.154%	0.139%	0.149%	0.172%
Hybrid Approach	0.359%	0.336%	0.307%	0.294%	0.330%	0.361%

Note that the undiversified and diversified VaRs are stated in the nominal Canadian dollars. The samples of each period are chosen to match the exact trading day of Canada and U.S in order to compute the diversification more precisely. Thus, the obvervations in each period may be different from the previous tables.