## **MODEL RISK AND MARKET RISK IN DERIVATIVE TRADING**

by

Grace Yi Zhao B. Commerce (Accounting & Finance) University of Manitoba, 2008

and

Shuoyan Wang B. Economy (Insurance & Risk Management) Guangdong University of Foreign Studies, 2008

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# <span id="page-1-0"></span>**Approval**



**Supervisory Committee:**

**Peter Klein** Senior Supervisor Professor of Finance

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**Evan Gatev** Second Reader Assistant Professor of Finance

Date Approved:

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# <span id="page-2-0"></span>**Abstract**

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Figlewski & Green (1999) develop a methodology to assess the model risk and market risk faced by a financial institution that follows two option-trading strategies: writing standard European options, pricing them by Black-Scholes model with volatilities forecasted from historical data, and carrying the position to expiration, either with or without delta hedging. Specifically, they try to examine the impact of volatility forecasting errors on returns and standard deviation of returns of above two trading strategies. The purpose of this paper is to test the robustness of this methodology. First, we replicated their methodology with the same S&P 500 data (Jan 1976-Dec 1991) used in the paper. Then we updated the results with recent S&P 500 data (Jan 1992- Dec 2003), and applied this methodology on NASDAQ with data from Jan 1992 to Dec 2003. Our robustness testing results indicate that Figlewski  $\&$  Green's methodology is quite robust for different period and different market, since we can draw similar conclusions from our testing results.

#### **Keywords: Model Risk; Market Risk; Derivative Trading and Valuation**

# <span id="page-3-0"></span>**Dedication**

We wish to dedicate this paper to our dear parents for encouraging and supporting us.

# <span id="page-4-0"></span>**Acknowledgements**

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# <span id="page-9-0"></span>**1: Introduction**

Derivative trading activities have been growing over recent years for many large financial institutions. Much of the growth is in derivatives with option features. Given the fact that only the option buyer has liability limited to the initial amount invested, and the option writer has great potential to lose money, which can significantly exceed the initial premium received, general public prefer to buy options rather than write options. In order to satisfy the public's demand, typical financial institutions entering the derivative market will primarily write option contracts. In doing so, financial institutions will expose to a variety of risks, such as market risk, credit risk and legal risk.(Figlewski & Green, 1999). In addition, derivative trading relies heavily on quantitative models for valuation, and risk management. These models are never perfect, and not all the model input parameters are directly observable. These introduce another type of risk: model risk.

In Figlewski & Green"s paper (1999), they develop a methodology to measure the model risk and market risk confronted by a financial institution that follows two optiontrading strategies: writing standard European options, pricing them by Black-Scholes model with volatilities estimated from historical data, and carrying the position to expiration, either with or without delta hedging. Specifically, they examine the impact of volatility estimation error on returns and standard deviations of returns of the above two trading strategies. To see the effect of volatility forecasting error on overall results, they also compute returns and standard deviation of returns with realized volatilities over the life of options, and compare them with the results calculated from using forecasted volatility. The purpose of this paper is to test the robustness of this methodology. First, we replicated their methodology with the same period of data used in the paper. Then we updated the results of this methodology with recent data and applied this methodology on different market (NASDAQ).

Section 2 discusses various sources of model risk and market risk. Section 3 describes the details of the methodology developed in Figlewski & Green (1999). Section 4 analyzes the robustness testing results, and Section 5 draws conclusions.

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# <span id="page-10-0"></span>**2: Literature Review**

Derivative instruments have been traded for a long time, but the recent growth in the number of traded contracts is remarkable. Concerns about risks in derivative trading have grown along with the growth of derivatives market. Since early 1990s, a series of significant losses related to derivative trading have been reported. For example, in 1995, Barings PLC, a 233-year-old British investment bank, went bankrupt due to a single derivative trader, Nicholas Leeson, who lost \$ 1.3 billion from unauthorized derivative speculation (Jorion, 2007). Bank Negara, Malaysia"s central bank, lost more than \$3 billion in 1992 and \$2 billion in 1993 from derivative trading (Jorion, 2007). These great losses caused by derivative trading have brought public's attention to derivative risks. There are two major derivative risks: market risk and model risk.

## <span id="page-10-1"></span>**2.1 Market Risk**

According to Jorion (2007), market risk is the risk of losses due to movement of the market price. In other words, market risk is the risk that market price changes will cause losses on a security position (Figlewski, 1998). If the security position is stock portfolio, the market risk exposure is relatively easy to understand. For example, if the stock market falls, a portfolio of stocks will expect to experience loss in size directly related to the portfolio"s beta (Jorion, 2007). Market risk exposure for derivative positions is the same thing. Because the value of derivatives depends on the value of the underlying assets, price change of underlying assets will cause the value of derivatives to fluctuate. For example, a financial institution who writes options based on stock market index will experience losses and gains as the priced stock market fluctuates (Jorion, 2007).

Option delta measures the sensitivity of option price to the change of the underlying asset price. In order to eliminate the market risk exposure of options, option traders usually delta hedge their positions by entering a position in the underlying assets, which has a delta equal in size and opposite in sign from the position to be hedged. The resulting hedged position is said to be delta neutral (Figlewski, 1998). This delta hedging strategy requires instant rebalance as underlying assets price changes. However, in real life, this continuous rebalance is impossible in terms of transaction cost. In practice, delta hedging is done approximately, and it produces approximation error. The

source of this error is the curvature of the option pricing function. The extent of curvature is measured by option gamma. To remove delta and gamma risk exposure, option traders usually purchase a combination of underlying assets and other options to offset both delta and gamma exposure. The net position will be delta neutral and gamma neutral.

The market risk exposure for derivative positions has several unique features. First of all, the moving direction of derivative value, caused by price movement of underlying assets, depends on the type of option. As the underlying asset price rises, the value of the call option will increase, but the price of the put option will decrease. Secondly, the dollar change in option value is generally smaller than the change in value of the underlying assets, but it is large in percentage, which is measured by lambda. For example, 1% change in stock price may produce 5% change in option price. Thirdly, According to Figlewski (1998), "market risk exposure relative to dollar value of a position is greater for derivative instrument than for a portfolio of stocks." For example, a long call position may end up out of the money with 100% loss in premium, even though price of the underlying asset may just have changed a little bit.

Some other factors, which also influence the value of the option, include Vega, Theta, Rho, and Phi.

Vega: measures the sensitivity of option price to the change of volatility (McDonald, 2006).

Theta: measures the change in the option price when there is a one-day decrease in the time to maturity (McDonald, 2006).

Rho: measures the sensitivity of option price to the change of interest rate (McDonald, 2006).

Phi: measure the sensitivity of option price to the change of dividends yield (McDonald, 2006)

## <span id="page-11-0"></span>**2.2: Model Risk**

One important feature of derivative trading is that derivative trading depends heavily on quantitative models for pricing and risk management. Virtually all derivative traders have access to computer programs of these models and use them to price options, assess risk exposures and implement risk management and hedging strategies. Even though these derivative models are derived from well-developed theoretical principles and mathematical

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models, they remain true only based upon assumptions made by the theoretical principles and mathematical models. Since real world is different from models, reliance on models will lead to a new type of risk that has not been paid much attention in investment before: model risk. Many papers have been published to discuss various sources of model risk.

According to Figlewski and Green (1999), " In order to derive a derivative valuation model, it is necessary to assume a stochastic process for the derivative's underlying asset." The original Black-Scholes (1973) option-pricing model assumes the stock price follows a random walk in continuous time, the distribution of possible stock price at the end of any finite interval is lognormal, the continuously compounded returns on stocks are normally distributed and the volatility of continuously compounded returns is known and constant. However, these ideal condition assumptions about the market and the return process are not supported by the empirical data. Figlewski (1998) states that actual distribution of return for all examined market has fat tail. In other words, for every market that has been examined, there are more returns realized in extreme tails. Also, it is well known that volatility varies over time. In practice, volatility never remains constant over the entire lifetime of the contracts, which creates additional model error.

A second source of model risk is that derivative models require users to input a number of parameters, including some that are not directly observable, such as the volatility of the underlying asset.(Figlewski, 1998). The standard Black-Scholes model(1973) requires a forward-looking volatility over the entire life of the options, however, in practice, nobody will know the realized volatility over the life of the option ahead of time. This creates a forecasting problem. Even the "best" estimation procedure never be able to accurately forecast the realized volatility.

Thirdly, option trader can eliminate the option"s market risk exposure by delta hedging in which the option trader computes the option delta and take an offsetting position in the underlying assets according to the calculated delta. The option delta is part of the option valuation formula. Perfect delta hedging requires an accurate option delta; accurate option delta requires the correct volatility input (Figlewski and Green, 1999), which goes back to the second source of model risk: not all input parameters are directly observable. In addition, in order to maintain the "delta neutrality" of the hedged position, the delta hedging strategy requires continuous rebalance for every change in the option delta. In real life, it is not

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feasible for option traders to rebalance at every minute when the option delta changes because instant rebalance will incur a huge amount of transaction cost. In addition, the market is not open all the time, option trader may not be able to rebalance the hedged position when it is needed. In practice, option trader will wait until the position becomes considerably far away from delta neutral, and then execute trades to re-establish the delta neutrality. In other words, practically, option trader does delta hedging approximately, as a result, the market exposure of the option is not fully eliminated (McDonald, 2006)

These various sources of market risk and model risk are generally known, but the quantitative impact is unknown. In this paper, we try to develop a quantitative measurement to assess the extent to which model risk and market risk can be expected to affect two basic option-trading strategies that might be followed by some financial institutions. Then we try to give some suggestion to limit the damage caused by imperfect option pricing model and market risk exposure.

# <span id="page-14-0"></span>**3: Methodology**

This section is intended to describe the methodology developed in Figlewski & Green (1999) in details, discuss some difficulties we had when we try to replicate the methodology with the same data used in Figlewski & Green" paper, and talks about some variations of the methodology we made when we update the results and apply this methodology on different market (NASDAQ).

## <span id="page-14-1"></span>**3.1 Forecasting Volatility**

According to Figlewski and Green (1998), the simplest method to forecast volatility is to calculate the volatility from a sample of historical data and assume this volatility will apply over the future life of the options. Variation of this method includes: different sample size, whether the volatility is calculated around the sample mean or around an imposed value, and whether to take account of the age of each data and weight each data in proportion to its age.

In their paper, they forecasted volatility of S& P 500 index over two forecast horizon (2-year and 5-year) using three different sizes of samples ( 2-years historical data, 5-year historical data, and all available historical data).

To demonstrate the details of the forecasting procedures, we give an example of how they forecast volatilities on S&P 500 over 5-year forecast horizon using 5 years of historical data. In their paper, they try to use a 5-year rolling window to forecast the volatility over a five-year horizon every month from January 1976 to December 1991, so the first forecast date is January 1976. In this case, first, they calculate the monthly continuous compound return of S&P 500 for the past 5 years (Jan 1971 to Dec 1975), then they calculate the monthly variance of these monthly returns. When they calculate the monthly variance, they compute the monthly variance around an imposed value: zero, instead of the true sample mean. So the monthly variance is just the average of the squared monthly continuous compound return. Then they annualize the monthly variance by multiplying the monthly variance by 12, then they take the square root of the annualized variance to get the annualized volatility. They use this volatility as the forecast volatility going forward from January 1976 to December 1980. The forecast date is then advanced one month , and they repeat the procedure using historical data from February 1971 to January 1976 to get the forecast volatility over a 5-year horizon for February 1976. They keep doing this until December 1991.

For the 5-year forecasting horizon, they also use another two different sizes of samples to forecast the volatilities. The forecasting procedure is the same for the two-year sample. They just use a two-year historical rolling window instead of a five-year rolling window to forecast the volatilities over a 5-year horizon. For example, for the first forecast date January 1976, they calculate historical volatility using historical data of past two years (January 1974 to December 1975), and take this volatility as the forecast volatility going forward over next 5 years (January 1976 to December 1980). Unlike the forecasting procedure for 2-year sample and 5-year sample, the forecasting procedures for "all available data" sample are slightly different. For each forecast date, it uses all past available data back to the beginning of the sample instead of a rolling window. For example, for the first forecast date January 1976, it uses past 5 years data from January 1971 to December 1975. For the second forecast date February 1992, it uses all past data from January 1971 to January 1976.

## <span id="page-15-0"></span>**3.2 Calculating Root Mean Squared Forecast Error**

The main purpose of figlewski  $\&$  Green's paper is to examine the impact of volatility forecasting error on returns of two basic option-trading strategies. They use the Root Mean Squared Forecast Error (RMSE) as a measure of the accuracy of forecasted volatilities.

To demonstrate the procedures of calculating RMSE, we give an example of how they calculate the RMSE for volatility on S&P 500 over 5-year forecast horizon using 5 years of historical data. As we described in previous section, they take the historical volatility from past 5 years (January 1971-December 1975) as the forecasted volatility for next 5 years for the first forecast date January 1976. Then, they calculate the actual realized volatility over the next 5 years from January 1976 to December 1980. The difference between the forecast and realized volatility is the volatility forecast error for January 1992, and they square this volatility forecast error. After that, they advance the forecast date one month and repeat this procedure until December 2003. Finally, they average the squared forecast errors and take squared root of it to get the Root Mean Squared Forecast Error (RMSE).

## <span id="page-15-1"></span>**3.1: Design of Simulation**

In Figlewski and Green"s paper (1999), they simulate the performance of two basic optiontrading strategies that might be followed by a financial institution. They consider a financial institution who writes standard European call and put options on S&P 500 index every month from January 1976 to December 1991. There are two different maturities ( 2 years and 5 years) and two degree of "moneyness" ( at the money and out of the money). For at the money option, the strike

price is equal to the current spot price of the underlying asset. For out of the money option, the strike price is equal to 0.4 standard deviations away from the current spot price. Therefore, for the out of money call options, the strike price is equal to the current spot price of the underlying asset plus 0.4 standard deviations. For the out of money put option, the strike price is equal to the current spot price minus 0.4 standard deviations.

## <span id="page-16-0"></span>**3.2: Pricing Formula**

They price the options basing on the following Black-Scholes formula ( McDonal, 2006):

For call option:

$$
C(S, K, \sigma, \delta, r, T) = S e^{-\delta T} N[d1] - K e^{-rT} N[d2]
$$

Where, S is the current spot price of the underlying stock index. K is the strike price of the option. σ is the annualized volatility of the underlying stock index. δ is the annualized continuously compounded dividend yield of the underlying stock index. r is the continuously compounded annual risk free rate. T is the time to maturity. N[.] is the cumulative normal distribution function, and d1 and d2 are given by the following equations:

$$
d1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}
$$

$$
d2 = d1 - \sigma\sqrt{T}
$$

For put option:

$$
P(S, K, \sigma, \delta, r, T) = Ke^{-rT}N[-d2] - Se^{-\delta T}N[-d1]
$$

The call and put deltas are given by:

call delta = 
$$
e^{-\delta T} N[d1]
$$
  
put delta =  $-e^{-\delta T} N[-d1]$ 

## <span id="page-16-1"></span>**3.3 Trading Strategies**

In Figlewski  $\&$  Green's paper, there are two option-trading strategies: writing options without hedging and writing options with hedging.

#### <span id="page-17-0"></span>**3.3.1 Strategy1: Writing Options without Delta Hedging**

This option trading strategy is to price option by Black-Scholes formula with lowest RMSE forecasted volatility, sell enough options to produce \$100 premium, and simply hold the short position of options until maturity. Alongside the option position, there is also a cash account. The initial \$100 premium is placed at the cash account and roll over every month. Therefore, the cash account earns interest every month at an annual rate equal to that month"s 90 days Euro-dollar interest rate. At maturity, any option ends up in the money will be paid off from the cash account. The return of this option trading strategy is just the balance of the cash account after the options had been paid off. Since this strategy writes options every month from January 1976 to December 1991, the return is on "per trade" basis. Then they take the average of the "per trade" returns to get the mean return. They also calculate the standard deviation of these "per trade" returns.

#### <span id="page-17-1"></span>**3.3.2 Strategy 2: Writing Options with Delta Hedging**

This option trading strategy is the same as the first option-trading strategy, except the short positions in options are delta hedged by the underlying assets  $(S\& P 500)$  over the life of the contracts. The hedge ratio is the option deltas. The hedged position is rebalanced very month, and all the subsequent cash flows from rebalance are assumed to come out of the cash account, which makes this option trading strategy self-financing. Therefore, the cash account goes up and down every month as rebalance cause the underlying asset to be bought and sold. At maturity, options expired in the money are also paid off from this cash account. Return of this trading strategy is also balance of the cash account after the in-the-money options had been paid.

The main purpose of Figlewski  $\&$  Green's paper is to examine the impact of volatility forecasting error on returns and return standard deviations of the above two option trading strategies. In order to see how important volatility estimation error on overall results, for both trading strategies and each kind of option, Figlewski  $\&$  Green also compute the returns and return standard deviations with realized volatility over the life of the options, and compare them with the results calculated from using forecasted volatility with lowest RMSE.

## <span id="page-17-2"></span>**3.4 Replicating Difficulties with Same Period of Data**

We had some difficulties when we were trying to replicate the second trading strategy with the same period of S&P 500 data (from Jan 1976-Dec 1991) used in Figlewski & Green"s paper. For this trading strategy: writing options with delta hedging, we need to recalculate the option deltas at each rebalance point. In Figlewski & Green's paper, they only state that they use up-to-date volatility to recalculate option delta. There is no information about what kind of dividend yield and interest

rate they used when they recalculate the option deltas. So we assume they use up-to-date information on everything, including interest rate, dividend yield, and volatility to recalculate the option deltas. However, based upon these assumptions, we could not get similar results. Therefore, we tried different ways to recalculate option deltas. We found that when we use up-to-date volatility but keep dividend yield and interest rate the same as we initially price the option to recalculate the option delta and rebalance accordingly, we could get results that are very close to the results found in Figlewski & Green"s paper.

Since we do not know exactly how Figlewski & Green recalculate the option deltas in their paper, based on our best assessment, we assume they use up-to-date volatility but the same dividend yield and interest rate as they initially price the options to recalculate option deltas. However, these assumptions are not very realistic in real life. Therefore, for the second trading strategy, when we robustness test Figlewski & Green"s methodology with recent S&P 500 data and different market data (NASDAQ), we applied both methods to recalculate option delta. We either use up-to-date volatility but the same dividend yield and interest rate as we initial price the option and to recalculate the option deltas, or we use up-to-date information on everything, including interest rate, dividend yield, and volatility to recalculate the option deltas. Then we rebalance the hedged position and calculate returns and standard deviation of returns respectively.

## <span id="page-18-0"></span>**3.5 Innovation of Figlewski & Green's Methodology**

In Figlewski & Green"s paper, for both trading strategy, they price each kind of option only with minimum RMSE forecast volatility and realized volatility, and calculate mean return and standard deviation of returns respectively. When we robustness test their methodology, we extend their methodology. For both trading strategies, we price each kind of option with all three forecasted volatilities estimated from three different historical samples and the realized volatility, and calculate the mean return and standard deviation respectively.

# <span id="page-19-0"></span>**4: Results and Analysis**

## <span id="page-19-1"></span>**4.1 Replicating Results with Same S&P 500 Data**

#### <span id="page-19-2"></span>**4.1.1 Replicating Results for RMSE**

As results shown in Table 1, we are able to exactly replicate the Root Mean Squared Forecast Error (RMSE) with the same period S&P 500 data used in Figlewski & Green's paper. For example, in their paper, the RMSE for forecast volatility over a 2-year horizon using 2 years historical data is 5.9%. Our replicating result is 5.8503%.

#### <span id="page-19-3"></span>**4.1.2 Replicating Results for Trading Strategy 1: Writing Options without Delta Hedging**

When we use the same S&P 500 data to replicate this trading strategy, we can get results that are very similar to the results found in Figlewski & Green"s paper. We present our results and results in Figlewski & Green"s paper in Table 2. As we can see from Table 2, our results are very close but generally smaller than results in Figlewski & Green"s paper. The slightly deviation may caused by the different way of constructing dividend yield when we replicate their paper.

Since options are priced by Black-Scholes formula, we need continuous dividend yield as an input for the Black-Scholes formula. In Figlewski & Green's paper, the continuous annual dividend yield is constructed as follows: first, they take the difference between two series of data obtained from CRSP: daily dividend inclusive return and daily dividend exclusive return, to get the daily dividend yield. Then they convert that daily dividend yield into continuous annual dividend yield. However, we construct the continuous annual dividend yield as follows: first, we divide the monthly dollar dividends obtained from Bloomberg by the end of month index price to get the monthly dividend yield, and then convert that monthly dividend yield into continuous annual dividend yield.

#### <span id="page-19-4"></span>**4.1.3 Replicating Results for Trading Strategy 2: Writing Options with Delta Hedging**

As we mentioned in previous section, we has some difficulties when we were trying to replicate the second trading strategy with same data. From results presented in Table 3, we can see when we use up-to-date volatility, dividend yield, and interest rate to recalculate option delta at each rebalance point and rebalance the hedged position accordingly, we could not get rational results. For example, for the two year at the money put option, the standard deviation of returns for forecasted

volatility with minimum RMSE is  $206.45$ , which is very different from Figlewski & Green's result (39.23), and it is even larger than the return standard deviation for same kind of option without delta hedging (50.13). However, when we use up-to-date volatility but the same dividend yield and interest rate as we initially price the options to recalculate option delta and rebalance the hedged position according, the standard deviation of returns decrease sharply to 47.8, which is very close to Figlewski & Green's result. Since we do not know exactly what the authors did in their paper, based on our best assessment, we think they use up-to-date volatility but the same dividend yield and interest rate to recalculate the deltas, however, if they went through their study in this way, it is not very realistic in real life. In practice, option traders usually use best available information on everything to recalculate option delta and rebalance the hedged position.

## <span id="page-20-0"></span>**4.2 Robustness Testing Results with up-date S&P 500 Data (Jan1992 to Dec 2003)**

In this section, we update the results of Figlewski & Green's methodology with recent  $S\&P$ 500 data (January 1992 to December 2003). Our intention is to test whether this methodology is robust for different period of data.

#### <span id="page-20-1"></span>**4.2.1 Analysis on Root Mean Squared Forecast Error (RMSE)**

Table 4 demonstrates that the forecasted volatility with the minimum Root Mean Squared Forecast Error (RMSE) is still calculated from utilizing "all available historical data" sample, for both 2-year and 5-year horizon. Comparing results in Table 4 to those in Table 1, we can discover that the average realized volatility of S&P 500 Index for the recent period (January 1992 to December 2003) is smaller than the one for the previous period, but forecasting errors during this period are generally slightly larger. This indicates that, even though the return on S&P 500 Index is less volatile than before, accurately forecasting volatility is more difficult for recent period.

#### <span id="page-20-2"></span>**4.2.2 Risk Analysis for Strategy 1: Writing Options without Hedging**

Options are considered to be a type of risky asset. Financial institutions who write options may simply invest the premium received from writing options at risk-free rate and hold the short position until maturity without delta hedging. This strategy remains significant amount of risks in financial institution"s asset portfolio.

Table 5 examines the impact of the volatility forecasting error on writing options without delta hedging. As we described in previous section, this option trading strategy is simply to price

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options at their model value, write options amount to \$100 in premium, rolling invest this premium at risk free rate each month, and hold this short position until options mature. (2 years or 5 years)

Results presented in Table 5 are mean return and standard deviation of returns of options written on S&P 500 Index over two horizons (2-year and 5-year). Each kind of option is priced with all four kinds of volatilities. The left side is the results from inputting three forecasted volatilities, and the right side applies realized volatility.

From results shown in Table 5, we can easily discover that mean returns for call options are usually negative, and the majority of mean returns for put options are positive. This tendency is similar to what we discovered from Figlewski  $\&$  Green's results. The reason for that tendency is that, even though there are up and down movement in the stock market, the overall trend for stock market for a relative long period is usually upward. For example, the average returns on S&P 500 Index are 6%-8% higher than the risk-free rate during 1992 to 2003. In this case, it is more likely for the call options to end up in the money at expiration. Therefore, mean returns are more likely to be positive for call options, but negative for put options, and financial institutions will be more likely to lose money from writing call options, but profit from writing put options.

From results of Table 5, we can draw similar conclusion to what discovered in Figlewski  $\&$ Green"s paper: writing options without hedging creates significant amount of risk no matter the volatility is known or forecasted. We can see from Table 5, for each kind of option, the standard deviation of returns is very large, usually several times larger than \$100 initial premium, and they are not very different for options priced either with forecasted volatilities or with realized volatility. Additionally, compared to results found in Figlewski  $\&$  Green's paper, we could easily discover that mean returns of this strategy are lower, and return standard deviations are much higher in the recent period, especially for put options. It suggests that the risk of writing options without hedging has been increasing through time.

As we mentioned earlier, for each kind of option, the financial institution writes contracts every month from January 1992 to December 2003. Figure 3 illustrates the distribution of the series of returns of each kind of option written on S&P 500 Index. Each graph under Figure 3 shows the distribution of the series returns for one kind of option priced with four different volatilities for both trading strategies. In the graph, each box covers the distribution range from  $25<sup>th</sup>$  to  $75<sup>th</sup>$  percentile. On each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, the whiskers extend to the range  $q_3 + 1.5(q_3 - q_1)$  and  $q_1 - 1.5(q_3 - q_1)$ , where  $q_3$  is the upper edge of box and  $q_1$  is the lower edge of box, and outliers are plotted individually by  $+$ . Studying on Figure 3, we observe that , for call options, the boxes are large no matter which volatility is used, indicating that

returns between 25th and 75th percentiles are scattering across a large range. However, for put options, even though boxes are extremely small, the amount of outliers beyond the whiskers is very large. Great amount of outliers increase the return standard deviation of the process.

A problem with this strategy is that forecasted volatilities are estimated from large amount of historical data. This leads to overlapping of a majority of data for consecutive forecast date, as well as serial correlation in forecasted volatilities. Since forecast volatilities do not change too much from month to month, it easily causes a grouping effect or a string of losing trading, in which writing options has negative return over a long period.

#### <span id="page-22-0"></span>**4.2.3 Risk Analysis for Strategy 2: Writing Options with Delta Hedging**

Since writing options without delta hedging exposes to a high level of risk, financial institutions may apply delta hedging strategy to reduce risk exposure in writing option. As we described in previous section, for the second strategy: writing options with delta hedging, we assume that the financial institution write each kind of option amount to \$100 in premium, and rolling invest this \$100 premium every month. Simultaneously, financial institutions reduce risk exposure by delta hedging, and rebalance the hedged position according to up-to-date stock price and volatility. Right here, we employ two methods to recalculate delta at each rebalance date:

Constant dividend and risk-free rate: dividend and risk-free rate are kept constant as the initial ones when the options are written;

Inconstant Dividend and risk-free rate: dividend and risk-free rate are up to date ones when doing rebalance every month.

Comparing results shown in Table 6 and Table7 (hedging strategy) with those shown in Table 5 (no hedging strategy), we can discover similar results as Figlewski & Green discovered in their paper: delta hedging (for both methodology of rebalance) greatly decreases risk exposure from writing options. However, the return standard deviations are still sizable. One factor that possibly plays a role in causing the large return standard deviations is the nonlinear relationships among the estimated variance, the volatility, and the model value for an option's price and delta (Figlewski  $\&$ Green, 1999). Specifically, the change of underlying asset price causes the change of option value, but the change of option value is not linearly correlated with the change of the underlying asset price. This is the reason why we need to rebalance the hedged position. However, since our rebalance frequency is once per month, it could not perfectly hedge all the risks caused by the movement of underlying asset price. Another factor causing the large standard deviation of returns is the estimation error of forecast volatility. The effect of this factor is significant when we use the forecast volatility

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as input to price options. Still, except model risk caused by utilizing forecast volatility which does not equal to the real volatility, other model risks, such as the true distribution of returns of the underlying asset is not normal, could also cause this large return standard deviation.

The results shown in Table 6 and Table7 demonstrate the influence of volatility estimation error on mean return and return standard deviations. Regarding the standard deviation of returns for options priced with three different forecasted volatilities, it is clear that using forecast volatility with the minimum RMSE leads to lowest return standard deviation. Additionally, comparing results with minimum RMSE forecasted volatility to results with realized volatility, we could discover that inputting realized volatility in Black-Sholes model and delta hedging provides the lowest return standard deviation. This indicates that knowing the true volatility would substantially reduce the risk.

When we compare the update results with results found in Figlewski  $\&$  Green's paper, we can see that the risk of writing options even with delta hedging has been increase for recent period. For example, from results shown in Table 3 (previous data results) and Table 6 ( up-date data results inputing constant dividend and risk-free rate for rebalance), it is apparent that, no matter whether forecasted volatility with minimum error or realized volatility is used, the recent period data produces larger return standard deviation in writing call options. Also, comparing Table 3(previous data results) & Table 7 (update results inputing up-to-date dividend and risk-free rate for rebalance), we observe that the recent period produces larger return standard deviation in writing both call and put options.

Table 8 compares the effectiveness of delta hedging for options written on S&P 500 Index using two methodologies for rebalance. The effectiveness of delta hedging is measured by the following formula:

## $\sigma$  with hedging  $-\sigma$  without hedging σ with hedging

In this table, we exam results obtained from using the following 4 methods to recalculate deltas in rebalance:

- 1. Using minimum RMSE forecasted volatility and constant dividend and risk-free rate as input in rebalance;
- 2. Using minimum RMSE forecasted volatility and up to date dividend and risk-free rate as input in rebalance;
- 3. Using realized volatility and constant dividend and risk-free rate as input in rebalance;

4. Using realized volatility and up to date dividend and risk-free rate as input in rebalance.

According to Table 8, delta hedging is most effective when we use the first method to recalculate deltas, which reduces return standard deviation by approximately 80% to 95%. In addition, from Figure 3, we can see that the distribution of the series returns with the first method is more likely to cluster around the mean. All these indicate that delta hedging is more effective when we use the first method to recalculate delta.

Based on the up-date results, we conclude that writing options without delta hedging creates a significant amount of risk no matter the forecasted volatility is known or forecasted; delta hedging significantly reduces risk exposure of writing options, but volatility estimation error still creates a great amount risk. Moreover, compared to results found in previous period, we discover that it is more difficulty to accurately forecast the volatility of S&P 500 in recent years even though S&P 500 market becomes less volatile. And due to this growing estimation error in forecast volatility, the risk of the two option trading strategies has been growing during the recent period (Jan 1992 to Dec 2003) since the return standard deviations for both strategies become larger.

## <span id="page-24-0"></span>**4.3 Robustness Testing Results with NASDAQ Data( Jan1992 to Dec 2003)**

In this section, we apply Figlewski & Green"s methodology into different market: NASDAQ Composite Index. Our intuition is to test whether this methodology still work in the different market. The study period in this section is also from January 1992 to December 2003

#### <span id="page-24-1"></span>**4.3.1 Analysis on Root Mean Squared Forecast Error (RMSE)**

Figure 1& 2 and the average realized volatilities shown in Table 4 depict that NASDAQ Composite Index is more volatile than S&P 500 Index, since NASDAQ Index is an indicator of the performance of stocks of technology companies and growth companies. Specifically, the technique bubble and its burst during 1998 and 2002 causes NASDAQ become a more volatile Index compared to S&P 500 Index. As Table 4 demonstrated, the average realized volatility of NASDAQ Index is approximately 25% while the average realized volatility of S&P 500 Index is around 14%. Considering estimation error in forecast volatility, we find that NASDAQ Index leads to higher RMSEs which are two to three times larger than RMSEs computed from S&P 500 Index during the same period. Furthermore, for 2-year horizon, NASDAQ Index forecast volatility calculated from the sample of 2-year historical data has the minimum RMSE; for 5-year horizon, the forecast volatility calculated from the sample of all available historical data has the minimum RMSE.

#### <span id="page-25-0"></span>**4.3.2 Risk Analysis for Strategy 1: Writing Options without Hedging**

We apply the same strategy and price the options with all four types of volatilities as we did in Section 4.2.2. Regarding the results for call and put options, they have the same tendency as what we discovered for S&P 500 Index: the return of writing call options tends to be negative and positive for writing put options; and return standard deviations are lower for put options than for call options. Simultaneously, similar to what we found from S&P 500 Index, the return standard deviations for each kind of option are always large (usually several times larger than \$100 initial premium) no matter which volatility is used. This fact reveals the same indications: different volatility inputs make a little difference in risk reduction.

#### <span id="page-25-1"></span>**4.3.3 Risk Analysis for Strategy 2: Writing Options with Delta Hedging**

In order to study the impact of delta hedging on options written on NASDAQ Index, we still employ the same methods for rebalance: input constant and inconstant dividend and risk-free rate to computing delta in rebalance. Comparing results using hedging strategy to those applying no hedging strategy in NASDAQ market, we can find similar conclusions as what we find in S&P 500 market: delta hedging significantly reduce risk exposure of writing options, but a great amount risk still remains due to volatility estimation error. In this section, we focus on analyzing the effectiveness of delta hedging for options written on the two different stock indices.

Table 9 & 12 compare the effectiveness of delta hedging for options written on the NASDAQ Index and S&P 500 Index using two methodologies for rebalance. If forecast volatility with the minimum RSME is used, delta hedging is more effective for options written on S&P 500 since the decrease in return standard deviations due to delta hedging is larger for S&P 500 than for NASDAQ Index. The possible reason is that volatility-forecasting error for NASDAQ is two to three times larger than that for S&P 500. This leads to the delta hedging is less effective for NASDAQ when we use the forecasted volatility. Therefore, hedging efficiency calculated from realized volatility is more reasonable for us to study. When we use realized volatility for pricing options and implementing delta hedging, for call options, delta hedging strategy is more effective for NASDAQ however, for put options, delta hedging strategy is more effective for S&P 500 Index. This suggests that if there"s no estimation error in forecasted volatility, or the estimation error is approximately same, implementing delta hedging strategy is usually more effective in more volatile market when writing call options, but less effective in more volatile market when writing put options.

After analyzing the results from two different option-trading strategies in different markets, we conclude that delta hedging does reduce risk exposure of writing options. However, implementing

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delta hedging strategy based on forecast volatility still leave a substantially high degree of risk exposure, and this risk exposure is even larger for the more volatile market, and it has been growing through the time. Therefore, in order to reduce the risk exposure of writing options due to volatility estimation error, financial institutions who write options usually sell options at a price higher than the model value.

## <span id="page-26-0"></span>**4.4 Reducing Loss by Volatility Markup**

From the above analysis, we discover that delta hedging still keeps a considerable risk exposure for financial institutions who write options. The most common procedure to reduce loss is to find the forecasted volatility with the minimum RMSE, and increase it by a reasonable amount to price options. Higher volatility produces a higher option price, which compensates risk exposed by financial institutions.

In the markup strategy, we increase volatility by 10%, 25%, 50%, 75% and 85% when we price options, and use the unadjusted volatility in delta hedging and rebalance. Table 13-16 shows the mean return, standard deviation of returns and percentage of trades that lose money for each kind of option written on both S&P 500 and NASDAQ index.

As results presented in Table 13-16, it is apparent that increasing volatility helps financial institutions increase the mean returns and reduce the fraction of trades that lose money. In addition, for more volatile underlying asset market, financial institutions should adjust the volatility into a higher level to compensate for the higher risk exposure of the more volatile market. For example, for options written on NASDAQ Index, boosting volatility by 85% reduces the percentage of losing trades to a satisfied level (within 5%), but we only need to increase volatility by 50% to get the same level of fraction of losing trades for options written on S&P 500 Index.

# <span id="page-27-0"></span>**5: Conclusion**

In this paper, we first try to replicate Figleski  $\&$  Green's Methodology using the same historical data on S&P 500. After that, we update the results of this methodology with recent S&P 500 data and apply this methodology on NASDAQ Index with data from January 1992 to December 2003. From the results we obtained, we learn how volatility estimation error can affect option writers' risk exposure, and we find that Figlewski  $\&$  Green's methodology is robust for different period data and market data since we can draw similar conclusions as Figlewski & Green presented in their paper.

Financial institutions who write options, invest the premium at risk-free rate, and simply hold the short positions to maturity, expose to a substantially large risk, since the standard deviation of returns of this strategy could reach several times above the initial premium. This large risk exposure encourages financial institutions to implement hedging strategy.

The second trading strategy: writing option with delta hedging implies that delta hedging provides a significant contribution in reducing risk exposure. However, imperfect models and volatility estimation error still creates a large risk exposure even using delta hedging. One way to eliminate the risk cause by the imperfect models and volatility forecasting error is to price the options with a higher volatility than its best estimates from its historical data. Our results show that on average, increase volatility by 50% could reduce the risk exposure of writing options to a satisfied level. Additionally, Black-Sholes-Merton model assumes that dividend and risk-free rate are constant during the option"s lifetime, which is not very realistic in real life. Also, through our replication process, we examine results under two different assumptions: constant dividend and risk-free rate, as well as up-to-date dividend and risk-free rate for recalculate option deltas at each rebalance point. And eventually we find that results applying constant dividend and risk-free rate is closer to the original results presented in Figlewski & Green (1999). However, when up-to-date dividend and riskfree rate are used, the return standard deviations of delta hedging strategy in both periods and both markets are larger. This indicates that, in the real world, financial institutions actually expose to much higher degree of risk exposure, since it is impossible for them to receive constant dividend and risk-free rate. This calls for a more practical model that considers the change of dividend yield and risk-free rate during the option"s lifetime.

# <span id="page-28-0"></span>**Appendices**

# <span id="page-28-1"></span>**Appendix A: Sources of Data**

Standard & Poors Stock Index: the monthly index level for 1/1971-12/2008 comes from CRSP. The monthly continuously compounded returns are constructed from the monthly index level.

NASDAQ Stock Index: the monthly index level for 1/1971-12/2008 comes from CRSP. The monthly continuously compounded returns are constructed from the monthly index level.

Dollar Dividends on Standard & Poors Stock Index: the dollar dividends for 1/1971-12/2008 comes from Bloomberg. The annualized continuous dividends yield is constructed from the dollar dividends. The method is discussed in the paper.

Dollar Dividends on NASDAQ Stock Index: the dollar dividends for 1/1971-12/2008 comes from Bloomberg. The annualized continuous dividends yield is constructed from the dollar dividends. The method is the same as constructing dividend yield of Standard & Poors dividend yield.

US Risk Free Interest Rate: the 90-days Eurodollar interest rate after conversion to the equivalent annualized continuous compounded rate is used as the risk free rate. The data for 1/1971-12/2008 comes from Federal Reserve Statistical Release

(http://www.federalreserve.gov/releases/h15/data.htm)

# <span id="page-29-0"></span> **Appendix B: Graphs**

<span id="page-29-1"></span>



<span id="page-29-2"></span>Figure 2: Price Comparison of NASDAQ Composite Index and Standard & Poor's 500 Index (1971-2008)



Figure 3: Return Distribution of Options Written on Standard & Poor's 500 Index

<span id="page-30-0"></span>The following graphs show returns of options on per trade basis for different volatility inputs &trading strategies













VF1NOH: no hedging strategy applies forecast volatility using 2-year historical data; VF2NOH: no hedging strategy applies forecast volatility using 5-year historical data; VF3NOH: no hedging strategy applies forecast volatility using all available historical data without hedging; VRNOH: no hedging strategy applies realized volatility; VF1HCON: delta hedging strategy applies forecast volatility using 2 year historical and constant dividend and risk-free rate in rebalance; VF2HCON: delta hedging strategy applies forecast volatility using 5 year historical data and constant dividend and risk-free rate in rebalance; VF3HCON: delta hedging strategy applies forecast volatility using all available historical data and constant dividend and risk-free rate in rebalance; VRHON: delta hedging strategy applies realized volatility and constant dividend and risk-free rate in rebalance; VF1HIN: delta hedging strategy applies forecast volatility using 2-year historical and up to date dividend and risk-free rate in rebalance; VF2HIN: delta hedging strategy applies forecast volatility using 5-year historical data and up to date dividend and risk-free rate in rebalance; VF3HIN: delta hedging strategy applies forecast volatility using all available historical data and up to date dividend and risk-free rate in rebalance; VRHIN: delta hedging strategy applies realized volatility and up to date dividend and risk-free rate in rebalance

## Figure 4: Return Distribution of Options Written on NASDAQ Composite Index

<span id="page-34-0"></span>The following graphs show returns of options on per trade basis for different volatility inputs &trading strategies













VF1NOH VF2NOH VF3NOH VRNOH VF1HCON VF2HCON VF3HCON VRHCON VF1HIN VF2HIN VF3HIN VRHIN

VF1NOH: no hedging strategy applies forecast volatility using 2-year historical data; VF2NOH: no hedging strategy applies forecast volatility using 5-year historical data; VF3NOH: no hedging strategy applies forecast volatility using all available historical data without hedging; VRNOH: no hedging strategy applies realized volatility; VF1HCON: delta hedging strategy applies forecast volatility using 2 year historical and constant dividend and risk-free rate in rebalance; VF2HCON: delta hedging strategy applies forecast volatility using 5 year historical data and constant dividend and risk-free rate in rebalance; VF3HCON: delta hedging strategy applies forecast volatility using all available historical data and constant dividend and risk-free rate in rebalance; VRHON: delta hedging strategy applies realized volatility and constant dividend and risk-free rate in rebalance; VF1HIN: delta hedging strategy applies forecast volatility using 2-year historical and up to date dividend and risk-free rate in rebalance; VF2HIN: delta hedging strategy applies forecast volatility using 5-year historical data and up to date dividend and risk-free rate in rebalance; VF3HIN: delta hedging strategy applies forecast volatility using all available historical data and up to date dividend and risk-free rate in rebalance; VRHIN: delta hedging strategy applies realized volatility and up to date dividend and risk-free rate in rebalance

## **Appendix C: Comparison of Paper Results and Our Results: S&P500 (January 1987 to December 1991)**

Table 1: RMSE for S&P 500 Index (January 1987 to December 1991)

The following table shows Root Mean Squared Forecast Errors for volatilities forecasted from three different sample sizes (2 years historical data, 5 years historical data, and all available historical data) for two forecast horizons (2 year horizon and 5 year horizon). The left one is the original results presented in Figlewski and Green (1998), and the right side one is our results. Shading indicates the minimum RMSE estimation method.

<span id="page-38-0"></span>

Table 2: Return and Risk of Writing Options without Hedging: S&P 500 Index (January 1987 to December 1991)

The table compare the results presented in Figlewski and Green (1998) and our results. The strategy applied is writing S&P 500 Index options each month without hedging and investing \$100 premium at risk-free rate. For out of the money call options, the strike prices are always set 0.4 standard deviation above the initial S&P 500 Index price. And for out of money put options, the strike prices are always set 0.4 standard deviation below the initial S&P 500 Index price. Option price are computed by inputting forecast volatility with the minimum RMSE (forecast volatility using all available historical date), and realized volatility.



## Table 3: Return and Risk of Writing Options with Hedging: S&P 500 Index (January 1987 to December 1991)

The table reports the results presented in Figlewski and Green (1998) and our results with the strategy of writing S&P 500 Index options each month with hedging, investment \$100 premium at risk-free rate, and reduce risk exposure using delta hedging. Our results are presented by using both constant and up-to-date dividend and risk-free rate for calculating delta when implementing rebalance. For out of the money call options, the strike prices are always set 0.4 standard deviation above the initial S&P 500 Index price. And for out of money put options, the strike prices are always set 0.4 standard deviation below the initial S&P 500 Index price. Option price are computed by inputting forecast volatility with the minimum RMSE (forecast volatility using all available historical date), and realized volatility



# **Appendix D: Root Mean Squared Forecast Error (RMSE)**

Table 4: RMSE for S&P 500 Index and NASDAQ Composite Index (Jan 1991-Dec 2003)

The following table shows Root Mean Squared Forecast Errors for volatilities forecasted from three different sample sizes (2 years historical data, 5 years historical data, and all available historical data) for two forecast horizons (2 year horizon and 5 year horizon) and two stock market indices (NASDAQ and S&P 500). Shading indicates the minimum RMSE estimation method.

<span id="page-41-1"></span><span id="page-41-0"></span>

## **Appendix E: Return and Standard Deviation of Different Option Trading Strategies: S&P 500 Index**

Table 5: Return and Risk of Writing Options without Hedging: S&P 500 (Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing S&P 500 Index options each month without hedging and investment \$100 premium at risk-free rate. For out of the money call options, the strike prices are always set 0.4 standard deviation above the initial S&P 500 Index price. And for out of money put options, the strike prices are always set 0.4 standard deviation below the initial S&P 500 Index price. Option price are computed by inputting forecast volatility using 2-year historical data, forecast volatility using 5-year historical data, forecast volatility using all available historical data, and realized volatility

<span id="page-42-1"></span><span id="page-42-0"></span>

### Table 6: Return and Risk of Writing Options with Hedging Using Constant Dividend Yield and Interest Rate: S&P 500

(Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing S&P 500 Index options each month with hedging, investment \$100 premium at riskfree rate, and reduce risk exposure using delta hedging. Constant dividend and risk-free rate are used here for calculating delta when implementing rebalance. For out of the money call options, the strike prices are always set 0.4 standard deviation above the initial S&P 500 Index price. And for out of money put options, the strike prices are always set 0.4 standard deviation below the initial S&P 500 Index price. Option price are computed by inputting forecast volatility using 2-year historical data, forecast volatility using 5-year historical data, forecast volatility using all available historical data, and realized volatility

<span id="page-43-0"></span>

## Table 7: Return and Risk of Writing Options with Hedging Using Up-to-Date Dividend Yield and Interest Rate: S&P500

(Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing S&P 500 Index options each month with hedging, investment \$100 premium at riskfree rate, and reduce risk exposure using delta hedging. Up to date dividend and risk-free rate are used here for calculating delta when implementing rebalance. For out of the money call options, the strike prices are always set 0.4 standard deviation above the initial S&P 500 Index price. And for out of money put options, the strike prices are always set 0.4 standard deviation below the initial S&P 500 Index price. Option price are computed by inputting forecast volatility using 2-year historical data, forecast volatility using 5-year historical data, forecast volatility using all available historical data, and realized volatility

<span id="page-44-0"></span>

## Table 8: Effectiveness of Hedging Strategy for S&P 500 Index (Jan 1992-Dec 2003)

The table reports the hedging efficiency in S&P 500 market. In the upper panel, constant dividend and interest rate are used to compute delta in rebalance, and in the lower panel, up-to-date dividend and interest rate are used. On the left side, the forecast volatility employed is the forecast volatility with the minimum RSME (forecast volatility using all available historical data for 2-year and 5-year S&P 500 Index options). On the right side, the realized volatility is applied. Return standard deviations both without and with delta hedging and decrease in standard deviation are all presented. The decrease in standard deviation is presented as percentile.

<span id="page-45-0"></span>

## Table 9: Return and Risk of Writing Options without Hedging: NASDAQ

(Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing NASDAQ Index options each month without hedging and investment \$100 premium at risk-free rate. For out of the money call options, the strike prices are always set 0.4 standard deviation above the initial NASDAQ Index price. And for out of money put options, the strike prices are always set 0.4 standard deviation below the initial NASDAQ Index price. Option price are computed by inputting forecast volatility using 2-year historical data, forecast volatility using 5-year historical data, forecast volatility using all available historical data, and realized volatility

<span id="page-46-0"></span>

## Table 10: Return of Risk of Writing Options with Hedging Using Constant Dividend Yield and Interest Rate: NASDAQ

(Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing NASDAQ Index options each month with hedging, investment \$100 premium at riskfree rate, and reduce risk exposure using delta hedging. Constant dividend and risk-free rate are used here for calculating delta when implementing rebalance. For out of the money call options, the strike prices are always set 0.4 standard deviation above the initial NASDAQ Index price. And for out of money put options, the strike prices are always set 0.4 standard deviation below the initial NASDAQ Index price. Option price are computed by inputting forecast volatility using 2-year historical data, forecast volatility using 5-year historical data, forecast volatility using all available historical data, and realized volatility

<span id="page-47-0"></span>

### Table 11: Return and Risk of Writing Options with Hedging Using Up-to-Date Dividend Yield and Interest Rate: NASDAQ

(Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing NASDAQ Index options each month with hedging, investment \$100 premium at riskfree rate, and reduce risk exposure using delta hedging. Up to date dividend and risk-free rate are used here for calculating delta when implementing rebalance. For out of the money call options, the strike prices are always set 0.4 standard deviation above the initial NASDAQ Index price. And for out of money put options, the strike prices are always set 0.4 standard deviation below the initial NASDAQ Index price. Option price are computed by inputting forecast volatility using 2-year historical data, forecast volatility using 5-year historical data, forecast volatility using all available historical data, and realized volatility

<span id="page-48-0"></span>

## Table 12: Effectiveness of Hedging Strategy for NASDAQ Composite Index (Jan 1992-Dec 2003)

The table reports the hedging efficiency in NASDAQ market. In the upper panel, constant dividend and interest rate are used to compute delta in rebalance, and in the lower panel, up-to-date dividend and interest rate are used. On the left side, the forecast volatility employed is the forecast volatility with the minimum RSME (forecast volatility using 2-year historical data for 2-year horizon, and forecast volatility using all available historical data for 5-year horizon). On the right side, the realized volatility is applied. Return standard deviations both without and with delta hedging and decrease in standard deviation are all presented. The decrease in standard deviation is presented as percentile.

<span id="page-49-0"></span>

## Table 13: Return & Risk of Writing and Hedging Options with Volatility Markup Using Constant Dividend & Interest Rate: S&P 500

## **(**Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing and delta hedging options each month. Options prices are set by valuing the option with the appropriate model but with volatility input that is 'marked up' by 10%, 25%, or 50%. Hedging and hedge rebalance are done with the unadjusted volatility. Dividend and risk-free rate for calculating delta when implementing rebalance are constant as the initial ones when the options are write. The table demonstrates the mean return per \$100 of option premium, the return standard deviation, the percentage of positions that lose money.

<span id="page-50-0"></span>

## Table 14:Return & Risk of Writing & Hedging Options with Volatility Markup Using Up-to-Date Dividend & Interest Rate: S&P 500

(Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing and delta hedging options each month. Options prices are set by valuing the option with the appropriate model but with a volatility input that is 'marked up' by 10%, 25%, or 50%. Hedging and hedge rebalance are done with the unadjusted volatility. Dividend and risk-free rate for calculating delta when implementing rebalance are up to date ones of each month. The table demonstrates the mean return per \$100 of option premium, the return standard deviation, the percentage of positions that lose money.

<span id="page-51-0"></span>

### Table 15:Return & Risk of Writing and Hedging Options with Volatility Markup Using Constant Dividend & Interest Rate: NASDAQ

(Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing and delta hedging options each month. Options prices are set by valuing the option with the appropriate model but with a volatility input that is 'marked up' by 10%, 25%, 50%, 75% or 85%.. Hedging and hedge rebalance are done with the unadjusted volatility. Dividend and risk-free rate for calculating delta when implementing rebalance are constant as the initial ones when the options are write. The table demonstrates the mean return per \$100 of option premium, the return standard deviation, the percentage of positions that lose money.

<span id="page-52-0"></span>

### Table 16:Return &Risk of Writing &Hedging Options with Volatility Markup using Up-to-Date Dividend and Interest rate: NASDAQ

(Jan 1992-Dec 2003)

The table reports the performance of the strategy of writing and delta hedging options each month. Options prices are set by valuing the option with the appropriate model but with a volatility input that is 'marked up' by 10%, 25%, 50%, 75% or 85%. Hedging and hedge rebalance are done with the unadjusted volatility. Dividend and risk-free rate for calculating delta when implementing rebalance are up to date ones of each month. The table demonstrates the mean return per \$100 of option premium, the return standard deviation, the percentage of positions that lose money.

<span id="page-53-0"></span>

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