CREDIT MIGRATION INDEX

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Contents

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1. Introduction

Credit migration matrices are inputs to many risk management applications, such as portfolio risk assessment, modeling the term structure of credit risk premia, pricing credit derivatives and assessment of regulatory capital. The simplest application of credit migration matrices probably is bond valuation. A bond will appreciate when upgraded or depreciate when downgraded. Gupton and Finger (2007) present an example for a BBB bond in table (1.1), showing strong economic implication of credit migration of rated assets. In the New Basel Accord (BIS, 2001), capital requirements are related to rating migrations. Credit migration matrices are also inputs in risky bond pricing methods by Jarrow and Turnbull (1995) and Jarrow et al. (1997), and in credit derivatives models from Kijima and Komoribayashi (1998) and Acharya et al. (2002). In risk management, credit migration matrices are required by credit portfolio models such as CreditMetrics too.

Acknowledging the importance of credit migrations, researchers have conducted many studies to estimate and evaluate credit migrations. Some studies compare and interpret differences among credit migration matrices in a meaningful way. Jafry and Schuerman (2004) provide a summary on the comparison of credit migration matrices. In the literature, mobility indexes which are applied to the transition matrices for general Markov chains measure the amount of migration (mobility). Shorrocks (1978) computes indexes for Markov matrices using eigenvalues and determinants, a line of inquiry extended in Geweke et al. (1986). They propose a set of criteria to judge the performance of a proposed metric (for arbitrary transition matrices). Jafry and Schuermann (2004) suggest an additional criterion, distribution discrimination. Distribution discrimination is particularly relevant for credit migration matrices because forward bond values in principle are sensitive to the distribution of off-diagonal probability mass since in real world far migrations have greater impacts than near migrations. For example, an AAA rated bond will depreciate about 50%[1] when it defaults (typically the last column of the migration matrix) but only several percents of price drop when it downgrades to AA grade(i.e. one off the diagonal). Jafry and Schuermann (2004) propose a formal scalar metric, average of the singular values of the mobility matrix, suitable for credit migration matrices.

2. Economic Implication of Credit Migration

Before discussing the existing techniques for comparing credit migration matrices, we will use a hypothetical example to illustrate the economic implication of credit migrations matrices to rated assets. Through the example, the key factors associated with credit migration and with forward values of rated assets are addressed. We make some assumptions for our hypothetical example.

[1]approximate according to Moody's report, see table (2.2)

Table 1.1. Value and Normalized Value Change of a BBB bond 1 year hence, Gupton and Finger(2007)

Rating	AAA	AA	A	BBB	BB	B	CCC	D
Forward Values	109.37	109.19	108.66	107.55	102.02	98.10	83.64	51.13
Normalized Forward Values	101.7	101.5	101.0	100.0	94.9	91.2	77.8	47.5

Table 2.1. Spot rate and 1-year forward rate one year from now (%)

	Spot Rate					Forward Rate			
Period	1	2	3	4	5	1	2	3	4
Grade A	3.73	4.33	4.92	5.31	5.50	4.93	6.10	6.48	6.26
Grade B	6.05	7.04	8.03	8.51	8.80	8.04	10.03	9.96	9.97
Grade C	15.01	15.02	15.40	15.80	16.00	15.03	16.16	17.00	16.80

Table 2.2. Average corporate debt recovery rates 1982-2007

Lien Position	Sr.Secured bonds	Subordinated bonds
Recovery Values $	51.89	31.19
Issuer-Weighted (Par is $100), measured by post-default trading prices		

Assumption. *We assume that there are only four credit ratings, A, B, C and D(Default). At $t = 0$, bond M is rated* B. *Bond M matures at $t = 5$ and pays a 6% coupon annually. Spot rates are given in table (2.1). 1-year forward rates one year from now derived from spot rates are shown in table (2.1) too.*

According to no arbitrage bond pricing model, we can determine the price for bond M on spot rates.

$$M_0^B = \sum_{i=1}^{5} \frac{CF_i}{(1+r)^i} = \frac{6}{(1+0.0605)^1} + \frac{6}{(1+0.0704)^2} + \frac{6}{(1+0.0803)^3} + \frac{6}{(1+0.0851)^4} + \frac{106}{(1+0.0880)^5} = 88.83 \tag{2.1}$$

At $t = 1$ just after coupon payments, if no credit event happens, bond M will keep its rating B. We assume the following migration probabilities for the bond[2]:

- *93.4% probability to stay in Grade B , or*
- *6.3% probability to upgrade to Grade A, or*
- *0.12% probability to downgrade to Grade C, or*
- *0.18% probability to default.*

The forward value of bond M at time $t = 1$ is uncertain due to the credit event. The expected forward value is a meaningful measure for us to understand the bond value at $t = 1$.

2.1. **Expected forward value.** To estimate the expected forward value of bond M at $t = 1$, we must know the probability and forward value of each possible outcome in case of the credit migrations. Probabilities are already known and we just need forward values in all possible states. We discount the cash flows of bond M by suitable forward rates to attain the forward values in state Grade A, B and C.

$$\begin{aligned} M_1^A &= \textstyle\sum_{i=1}^4 \frac{CF_i}{(1+r)^i} = \frac{6}{1.0493} + \frac{6}{1.0493\times1.061} + \frac{6}{1.0493\times1.061\times1.0648} + \frac{106}{1.0493\times1.0610\times1.0648\times1.0626} = 101.64 \\ M_1^B &= \textstyle\sum_{i=1}^4 \frac{CF_i}{(1+r)^i} = \frac{6}{1.0803} + \frac{6}{1.0803\times1.1002} + \frac{6}{1.0803\times1.1002\times1.0996} + \frac{106}{1.0803\times1.1002\times1.0996\times1.0997} = 90.51 \\ M_1^C &= \textstyle\sum_{i=1}^4 \frac{CF_i}{(1+r)^i} = \frac{6}{1.1503} + \frac{6}{1.1503\times1..1616} + \frac{6}{1.1503\times1.1616\times1.1700} + \frac{106}{1.1503\times1.1616\times1.1700\times1.1680} = 70.72 \end{aligned} \tag{2.2}$$

If bond M defaults, we assign a subjective residual value net of recoveries of \$51.89 per face value of \$100 based on a report by Moodys (see table (2.2))[3], denoted as $M_1^D = 51.89$.

We can normalize forward values of bond M in term of the value when bond M remains its rating, getting the following result $NM = [112.30\ 100\ 78.14\ 57.33]$.

$$NM_1^A = \frac{M_1^A}{M_1^B} = \frac{101.64}{90.51} = 112.30 \ NM_1^C = \frac{M_1^C}{M_1^B} = \frac{70.72}{90.51} = 78.14 \ NM_1^D = \frac{M_1^D}{M_1^B} = \frac{51.89}{90.51} = 57.33 \tag{2.3}$$

The expected forward value and normalized expected forward value of bond M is,

$$\begin{aligned} V &= \textstyle\sum M \times P = 101.64 \times 6.3\% + 90.51 \times 93.4\% + 70.72 \times 0.12\% + 51.89 \times 0.18\% = 91.12 \\ NV &= \textstyle\sum NM \times P = 112.30\% \times 6.3\% + 100 \times 93.4\% + 78.14\% \times 0.12\% + 57.33\% \times 0.18\% = 100.67\% \end{aligned} \tag{2.4}$$

We summarize information for bond M in table (2.3). In table (2.3), probabilities presents likelihood of each outcome of the credit migration. Forward values are calculated from bond valuation model using rating specific forward rates at $t = 1$. Normalized forward values are obtained by dividing the bond forward value by its

[2]We use a variant of data from Gupton and Finger(2007); we sum the probabilities of AAA, AA and A as A, BBB, BB and B as B.
[3]The residual value is just for example. In practice, it differs case by case.

forward value when the bond rating remains unchanged. Summing up all the expected forward value at all possible states, we can get the expected forward value and normalized expected forward value.

Without credit migration, the value of bond M is 90.51 at $t = 1$. However, due to possible credit migration, bond M has other three possible outcomes except its original rating. If the forward rates are known, we can find out the expected bond value. In our example, it is 91.12 for bond M, or 100.67% of the value of bond M without rating changed. The expected value of bond M is determined by two factors, the credit migration matrix and forward values in each state; values in each state are in turn depended on the forward rates which are not fixed in long term.

2.2. **A portfolio of bonds.** Our analysis applies to a single bond or a portfolio of bonds. The expected value of a portfolio of bonds is independent of the correlation accross bond values. It is just the sum of the expected value of each bond.

2.3. **Key Factors in credit migration.** From the above examples, we see that the expected value is determined by several factors.

- The direction of rating movements

A bond will appreciate when it upgrades or depreciate when it downgrades in term of the value of the bond without credit migration, as shown in table (2.3). We refer this characteristic as the direction of rating movements and upgrade is believed better than downgrade. For example, the bond M will appreciate 12.30%[4] if it upgrades to Grade A or depreciate 21.86%[5] if it downgrade to Grade C.

- The extent of rating movements

With same direction movement, the extent of the rating movements also affects the expected bond value. As we see in the table (2.3), we find that the further the upgrade or downgrade, the greater the bond forward value will change. For example, if bond M defaults, it will depreciate 44.37% [6], comparing to 17.64% for its downgrade to Grade C.

- The probability in each possible state

As we have shown in above examples, the probability in each possible state is one component used to evaluate the expected bond or portfolio value.

- The forward value in each possible state

As we have shown in above examples, the forward value in each possible states is one component used to evaluate the expected bond or portfolio value, too.

- The composition of the portfolio

For a portfolio, its expected value also depends on the weight of different bonds.

[4] 112.30%-1=12.30%, see table (2.3)
[5] 1-78.14%=21.86%, see table (2.3)
[6] 1-55.63%=44.37%, see table (2.3)

TABLE 2.3. Summary of bond M at $t = 1$

Grade	A	B	C	D
Probabilities %	6.3	93.4	0.12	0.18
Forward values	101.64	90.51	70.72	51.89
Normalized forward values	112.30	100	78.14	57.33
Expected Forward value				91.12
Normalized expected Forward value				100.67

While the credit migration matrix does not include any information on the forward value in each possible state and the composition of the portfolio, it captures all information about the direction of rating movements, the extent of rating movements and the probability in each possible state.

3. Matrix Comparison Techniques

Knowing that credit migration matrices contain important information related to the bond or portfolio value expectation, we need to evaluate and compare credit migration matrices. Comparing two scalars are simple because they are at the single dimension space but matrices have at least two dimensions, making the traditional methods fit for scalar useless. Several methods have been advised, including cell-by-cell distance metrics, eigenvalue-based metrics, eigenvector distance metric and singular values. We just review these methods drawing on Jafry and Schuerman (2004).

3.1. **Cell-by-cell distance metrics.** There are two common approaches, the L1 and L2 (Euclidean) distance metrics, for comparing two matrices (say A and B, each with dimension N · N).

The L1 distance metric[7] computes the average absolute difference corresponding elements of matrices. The formula is as follow,

(3.1)
$$L_1 \triangleq \frac{1}{N^2} \times \sum_{i=1}^{N} \sum_{j=1}^{N} \mid A_{ij} - B_{ij} \mid$$

The L2 distance metric[8] measures the average root-mean-square difference between corresponding elements of the matrices. The formula is given as follow,

(3.2)
$$L_2 \triangleq \frac{1}{N^2} \times \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} (A_{ij} - B_{ij})^2}$$

These two approaches are simple. While providing a relative comparison between two matrices, these methods do not provide absolute measure for an individual matrix[9]. Given a distance, we have difficulty to judge whether this distance is large or small. Meanwhile, it is not easy to interpret the economic meaning of the distance.

Although appealing in their simplicity, these methods offer no absolute measure for an individual matrix: they only provide a relative comparison between two matrices. For example, if the Euclidean distance between two matrices turns out to be, say, 0.1, it is not clear if this is a “large” or a “small” distance, nor is it possible to infer which matrix is the “larger” of the two.

3.2. **Eigenvalue-based metrics.** Based on the eigenvalues of P, Geweke et al. (1986) advised the mobility indexes for general transition matrices.Their formula are as follow.

(3.3)
$$\begin{cases} M_p(P) & = \dfrac{1}{N-1}(N - tr(P))) \\ M_D(P) & = 1 - \mid det(P) \mid \\ M_E(P) & = \dfrac{1}{N-1}\left(N - \sum_{i=1}^{N} \mid \lambda_i(P) \mid\right) \\ M_2(P) & = 1 - \mid \lambda_2(P) \mid \end{cases}$$

where $tr(P)$ refers to trace of the matrix, $det(P)$ is the determinant, and $\lambda_i(P)$ denotes the *ith* eigenvalue (arranged in the sequence from largest to smallest absolute value, with λ_2 denoting the largest less than unity).

[7]The L1 metric is used in Israel et al.(2001)

[8]The L2 metric is used in Bangia et al. (2002)

[9]It is possible to set up a reference matrix and to compare individual matrix with the reference matrix thus to work around the problem.

3.3. **Eigenvector distance metric.** Due to the an absorbing state within the credit migration matrices, they will have identical steady-state solutions. Therefore the steady-state solution, or, equivalently, the first eigenvector of (the transpose of) the migration matrix contain no useful information as a basis for comparing matrices. Yet, the remaining eigenvector include useful information. Using these eigenvector, AGL construct a relative metric, proposing to assess the similarity of all eigenvector between two matrices by computing a (scalar) ratio of matrix norms. The scalar is defined as follow

$$M(A,B) = \frac{\| AB - BA \|}{\| A \| \cdot \| B \|} \tag{3.4}$$

where for a vector X, $\| X \|$ refers the length of the vector.

This scalar is bounded between zero and two; it is equal to zero when A and B have exactly the same eigenvector (regardless of their eigenvalues), and it increases if the more dissimilar the eigenvector become. AGL conclude that a value of this metric not greater than 0.08 will be sufficiently judge that two vectors vary by only small and can thus assumed to be similar at an annual base. Yet, they do not explain the reason for choosing 0.08 as the threshold.

3.4. **Singular value metric.** Jafry and Schuermann (2004) introduce a new singular value metric, *the average of the singular values of the mobility matrices,* which is defined as follow,

$$\widetilde{P} \triangleq P - I,\ M_{SVD}(P) = \frac{\sum_{i=1}^{N} \sqrt{\lambda_i \left(\widetilde{P}^T \widetilde{P} \right)}}{N} \tag{3.5}$$

Giving the state equation $X(n+1) = X(n) \times P$, we get $X(n+1) = X(n) \times \left(\widetilde{P} + I \right)$ or $X(n+1) - X(n) = X(n) \times \widetilde{P}$. We then know $\| X(n+1) - X(n) \| = \| X(n) \times \widetilde{P} \| \leqq \| X(n) \| \| \widetilde{P} \|$, meaning that the degree of migration is partly determined and bound by the "magnitude" of $\widetilde{P}$. According to the norm of a given matrix, as described in Strang(1998), whereby the norm of a given matrix A is defined as

$$\| A \| = \max_{x \neq 0} \frac{\| Ax \|}{\| x \|}$$

The norm is the largest singular value of A while the norm of $A'A$ equal to the square of the norm of A. Hoping to capture more information of $\widetilde{P}$ on a feasible state vector, Jafry and Schuermann propose the average of the singular values of the mobility matrix average. Furthermore, they calculate the following hypothetical "average" migration matrix P_{avg} with diagonal elements equal to $(1-p)$ and all off-diagonal elements equal to $p/(N-1)$. The important characteristics of P_{avg} is that its $M_{SVD}(P) = p$.

$$P_{avg} = \begin{bmatrix} 1-p & p/(N-1) & \ddots & & \\ p/(N-1) & 1-p & p/(N-1) & \ddots & \\ \ddots & p/(N-1) & 1-p & p/(N-1) & \ddots \\ & \ddots & p/(N-1) & 1-p & \ddots \\ & & & & \ddots \end{bmatrix} \tag{3.6}$$

If a matrix has $M_{SVD}(P) = p$, Jafry and Schuermann (2004) suggest that the matrix has an effective average probability of migration of p.

3.5. **Problems with Existing Techniques for Comparing Matrices.** Consider the following simple transition metrics:

$$A = \begin{bmatrix} & A & B & C & D \\ A & 1 & & & \\ B & & 0.1 & 0.9 & \\ C & & & 1 & \\ D & & & & 1 \end{bmatrix} \quad B = \begin{bmatrix} & A & B & C & D \\ A & 1 & & & \\ B & & 1 & & \\ C & & & 0.1 & 0.9 \\ D & & & & 1 \end{bmatrix} \quad C = \begin{bmatrix} & A & B & C & D \\ A & 1 & & & \\ B & & 0.1 & & 0.9 \\ C & & & 1 & \\ D & & & & 1 \end{bmatrix} \tag{3.7}$$

We apply the existing matrices comparison techniques to above matrices and get following results.

Comparison results using Cell-by-Cell distance metric

$$L_1(A,B) \triangleq \frac{1}{4^2} \times \sum_{i=1}^{4}\sum_{j=1}^{4} \mid A_{ij} - B_{ij} \mid = 0.23 \qquad L_2(A,B) \triangleq \frac{1}{4^2} \times \sqrt{\sum_{i=1}^{4}\sum_{j=1}^{4} (A_{ij} - B_{ij})^2} = 0.11 \tag{3.8}$$

$$L_1(B,C) \triangleq \frac{1}{4^2} \times \sum_{i=1}^{4}\sum_{j=1}^{4} \mid B_{ij} - C_{ij} \mid = 0.23 \qquad L_2(B,C) \triangleq \frac{1}{4^2} \times \sqrt{\sum_{i=1}^{4}\sum_{j=1}^{4} (B_{ij} - C_{ij})^2} = 0.11 \tag{3.9}$$

$$L_1(A,C) \triangleq \frac{1}{4^2} \times \sum_{i=1}^{4}\sum_{j=1}^{4} \mid A_{ij} - C_{ij} \mid = 0.11 \qquad L_2(A,C) \triangleq \frac{1}{4^2} \times \sqrt{\sum_{i=1}^{4}\sum_{j=1}^{4} (A_{ij} - C_{ij})^2} = 0.08 \tag{3.10}$$

Comparison results using eigenvector distance metric

$$M(A,B) = \frac{\parallel AB - BA \parallel}{\parallel A \parallel \cdot \parallel B \parallel} = 0.63 \; M(A,C) = \frac{\parallel AC - CA \parallel}{\parallel A \parallel \cdot \parallel C \parallel} = 0.63 \; M(B,C) = \frac{\parallel BC - CB \parallel}{\parallel B \parallel \cdot \parallel C \parallel} = 0 \tag{3.11}$$

Comparison results using eigenvalue-based metrics

$$M_p(A) = \frac{1}{4-1}(4 - tr(A))) = 0.3 \quad M_D(A) = 1 - \mid det(A) \mid = 0.9 \tag{3.12}$$

$$M_p(B) = \frac{1}{4-1}(4 - tr(B))) = 0.3 \quad M_D(B) = 1 - \mid det(B) \mid = 0.9 \tag{3.13}$$

$$M_p(C) = \frac{1}{4-1}(4 - tr(C))) = 0.3 \quad M_D(C) = 1 - \mid det(C) \mid = 0.9 \tag{3.14}$$

$$M_E(A) = \frac{1}{4-1}\left(4 - \sum_{i=1}^{4} \mid \lambda_i(A) \mid\right) = 0.3 \qquad M_2(A) = 1 - \mid \lambda_2(A) \mid = 0.9 \tag{3.15}$$

$$M_E(B) = \frac{1}{4-1}\left(4 - \sum_{i=1}^{4} \mid \lambda_i(B) \mid\right) = 0.3 \qquad M_2(B) = 1 - \mid \lambda_2(B) \mid = 0.9 \tag{3.16}$$

$$M_E(C) = \frac{1}{4-1}\left(4 - \sum_{i=1}^{4} \mid \lambda_i(C) \mid\right) = 0.3 \qquad M_2(C) = 1 - \mid \lambda_2(C) \mid = 0.9 \tag{3.17}$$

Comparison results using average of the singular values of the mobility matrix

$$M_{SVD}(A) = \frac{\sum_{i=1}^{4} \sqrt{\lambda_i((A-I)'(A-I))}}{4} = 0.32 \tag{3.18}$$

TABLE 3.1. Summary of matrix comparison

	Metric				Metric				
Situation	L1	L2	M	Matrix	ME	M2	MP	MD	MSVD
A vs. B	0.23	0.11	0.63	A	0.3	0.9	0.3	0.9	0.32
B vs. C	0.23	0.11	0.63	B	0.3	0.9	0.3	0.9	0.32
A vs. C	0.11	0.08	0.00	C	0.3	0.9	0.3	0.9	0.32

$$M_{SVD}\left(B\right)=\frac{\sum_{i=1}^{4}\sqrt{\lambda_i((B-I)'\left(B-I\right))}}{4}=0.32 \tag{3.19}$$

$$M_{SVD}\left(C\right)=\frac{\sum_{i=1}^{4}\sqrt{\lambda_i((C-I)'\left(C-I\right))}}{4}=0.32 \tag{3.20}$$

CMI CMI supposed by us[10]

$$CMI\left(A\right)=94.34\ CMI\left(B\right)=94.61\ CMI\left(C\right)=88.96 \tag{3.21}$$

We summarize above results in table (3.1) and raise following concerns on these techniques.

- Cell-by-Cell distance metrics always can judge whether two matrices are different, but other techniques may give identical results even when two matrices are different.
- When giving identical results, these techniques do not state what it means. For example, if we know $M_{SVD}\left(C\right)=M_{SVD}\left(B\right)$, we are unable to determine whether $C=A$ or not. If $C\neq A$, we do not have clear idea what that means in terms of MSVD.
- No standard exists to judge the significance of the difference between two matrices.
- No clear economic implication for us to compare two results calculated from two matrices, for example, we do not have clear idea whether the value change of portfolio A is less than B or not given $M_{SVD}\left(A\right)<M_{SVD}\left(B\right)$.
- These techniques do not discriminate between credit upgrades and downgrades, as shown for the results for matrices A and B.
- These techniques can not give useful economic information of a rating movement, for example, B and C are seem to be similar, yet they have quite different economic consequence for a portfolio manager.

4. Credit Migration Index

All matrices comparison methodology mentioned above have their advantages. We suggest a supplement to the existing techniques, Credit Migration Index (CMI). Compared to the existing techniques, CMI emphasizes change in forward value from credit migrations and pay attention to evaluate the identical result from CMI.

4.1. Constructing the CMI.

The main economic impacts associated with credit migration is value change. Fortunately, the value change is a scalar rather than a matrix. We suggest *Credit Migration Index(CMI),* a new index as the proxy to capture the economic movement of credit migration.

Definition. The value of portfolio[11] without credit migration is V_0 and the expected forward values of the portfolio associated with a possible credit migration is V_T under forward value setting FV; Credit Migration

[10] The calculation formula are in later parts.

[11] a single bond is also seen as a portfolio of one bond.

Index(CMI) under forward rate setting FV is defined as follow[12],

$$CMI\,(P,\ FV) \triangleq \frac{V_T}{V_0} \times 100 \tag{4.1}$$

Supposedly, a portfolio has n different components $B_i\,(i = 1 \cdots n)$, and each components has its own credit migration matrix P_i. Without credit migration B_i has value V_i; and it has forward values FV_i in its all possible states while NFV_i denotes the normalized forward values. According to the definition of CMI, we can rewrite the CMI formula as follow,

$$CMI\,(P) \triangleq \frac{V_T}{V_0} \times 100 = \frac{E\left(\sum B_i\right)}{\sum V_i} \times 100 = \frac{\sum E\left(\frac{B_i}{V_i}\right) \cdot V_i}{\sum V_i} \times 100 = \sum CMI(B_i) \times w_i \quad w_i = \frac{V_i}{\sum V_i} \tag{4.2}$$

In this case, CMI of a portfolio of bonds is the sum of weighted CMI of each portfolio components. The decomposition can simplify the calculation of CMI. Supposedly, a portfolio has following data $CMI\,(M) = 98.08$, $CMI\,(N) = 97.04$, $V_M = 106.5$ and $V_N = 90.79$. We can get that

$$w_M = \frac{V_M}{V_M + V_N} = 0.54 \; w_N = \frac{V_N}{V_M + V_N} = 0.46 \; CMI\,(P) = w_M \cdot CMI\,(M) + w_N \cdot CMI\,(N) = 97.60 \tag{4.3}$$

When a portfolio is known without credit migration, its CMI is 100 since $V_T \equiv V_0$.

4.2. **Characteristics of CMI.**

- Scalar

CMI is scalar, which means we can apply a CMI to operations which accept a scalar.

- Positive

From the definition, we know that the value expectation can not be negative which means CMI is great than zero.

- CMI comparison

Since the CMI is scalar, two CMIs are comparable, meaning it must fall in and only fall in one of these three situation, $CMI1 > CMI2$, or $CMI1 = CMI2$, or $CMI1 < CMI2$, . The comparison is meaningful in economic terms, because a higher CMI means a greater normalized expected forward value. The comparison is transitive, which means if $CMI1 > CMI2$ and $CMI2 > CMI3$, then $CMI1 > CMI3$.

- Sensitive to the forward values setting

As from the example in CMI calculation, we know that CMI is sensitive to the forward values in all possible states. Two mathematically equal credit migration matrices can have different CMI under different forward value matrices.

- Distribution discrimination

Since the forward values setting have larger magnitude in the off-diagonal elements, the CMI is sensitive to the off-diagonal element of the credit migration matrix.

5. Applying CMI on S&P corporate bonds

We demonstrate the application of CMI to S&P corporation bond transitions. Estimates of transition matrices are taken from Theunissen (2008). Detail data is in table (8.1-8.5).

[12] Multiple of 100 in the formula is just for convenience.

Table 5.1. Forward zero curves and normalized forward value matrix

One-year forward zero curves (%)					Normalized Forward Value								
Category	Year 1	Year 2	Year 3	Year 4		AAA	AA	A	BBB	BB	B	CCC	D
AAA	3.60	4.17	4.73	5.12	AAA	100.0	99.8	99.3	98.2	92.6	88.6	74.0	52.1
AA	3.65	4.22	4.78	5.17	AA	100.2	100.0	99.5	98.3	92.8	88.8	74.1	52.2
A	3.72	4.32	4.93	5.32	A	100.7	100.5	100.0	98.9	93.3	89.3	74.5	52.5
BBB	4.10	4.67	5.25	5.63	BBB	101.9	101.7	101.1	100.0	94.3	90.3	75.3	48.6
BB	5.55	6.02	6.78	7.27	BB	108.0	107.8	107.2	106.0	100.0	95.7	79.9	33.8
B	6.05	7.02	8.03	8.52	B	112.8	112.6	112.0	110.8	104.5	100.0	83.5	35.4
CCC	15.05	15.02	14.03	13.52	CCC	135.2	135.0	134.2	132.7	125.2	119.8	100.0	42.4

The expansion of the forward value matrix. From the definition of CMI, CMI is sensitive to the forward value matrix used to calculate the CMI. To calculate CMIs for S&P corporate bonds, in principal, we need twenty-six actual forward value matrices for year from 1982 to 2007. Since in this paper, we just want to present an application of CMI, we take a shortcut to work around the problem. We will use only one forward value matrix for all years although it is not perfect. We attain the one-year forward zero curves from Gupton and Finger(2007), shown in table (5.1). We use forward zero curves to calculate the forward values. Assuming all category bonds mature in five years and pay 5% coupon annually[13], we can find out the normalized forward values matrix as shown in table (5.1).[14]

CMI for different category bonds. Having the required data, we can start the CMI calculation. As we mentioned before, we apply a real forward value matrix to calculate the CMI. We put the information of CMI using a real forward value matrix in table (5.2) and figure (5.1)

Table 5.2. CMI for S&P corporate bond from 1982-2007

Year	Mean	SD	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
AAA	100.0	0.0	100.0	100.0	100.0	99.9	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
AA	99.9	0.1	100.0	99.9	100.0	99.7	100.0	100.0	99.9	100.0	100.0	99.9	100.0	100.0
A	99.9	0.1	99.9	100.0	99.9	99.9	99.8	99.8	99.9	99.8	99.8	99.9	99.7	99.9
BBB	99.6	0.2	99.6	99.8	99.4	99.4	99.3	99.3	99.5	99.2	99.0	99.5	99.7	99.6
BB	99.2	1.0	99.8	99.7	100.2	99.6	99.1	99.6	99.4	99.1	97.2	96.9	99.7	99.2
B	96.3	3.2	97.7	98.4	99.4	96.7	95.4	98.3	97.7	97.5	91.8	89.2	96.5	98.1
CCC	81.6	11.9	100.0	96.1	68.1	83.9	86.8	98.3	85.1	72.2	67.7	74.1	82.2	98.9
Portfolio	97.2	1.8	99.6	99.3	96.2	97.5	97.6	99.4	97.8	96.2	94.7	95.0	97.4	99.4
Year	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
AAA	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
AA	100.0	99.9	100.0	100.0	100.0	100.0	100.0	99.9	99.9	99.9	100.0	100.0	100.0	100.0
A	99.9	99.9	100.0	99.9	99.9	99.9	99.9	99.7	99.8	99.9	100.0	100.0	100.0	99.9
BBB	99.7	99.7	99.9	99.6	99.4	99.6	99.7	99.3	99.3	99.6	99.9	99.9	99.9	99.8
BB	100.2	99.9	99.6	99.9	99.1	99.3	98.6	97.3	97.4	98.8	99.5	99.9	99.9	99.8
B	97.4	97.4	98.5	98.7	95.8	94.9	94.0	88.1	89.9	96.1	98.9	99.0	99.4	99.5
CCC	80.0	75.7	82.0	85.1	65.7	65.4	74.4	61.8	64.9	80.5	90.8	93.8	93.9	95.2
Portfolio	97.3	96.8	97.7	98.1	95.3	95.2	96.0	93.5	94.1	97.0	98.7	99.1	99.2	99.3

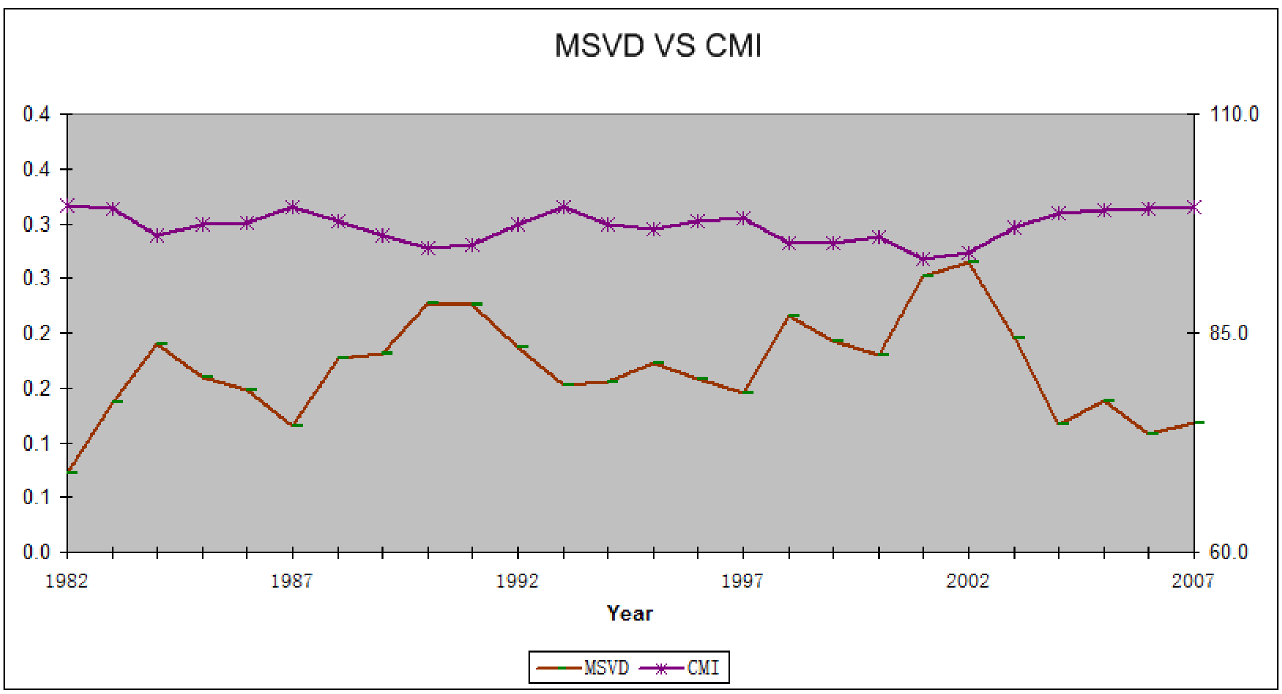
[13]This assumption is not perfect in real world. In real world, there are many coupon patterns.

[14]We use $51.89 for default value per par for bond rated AAA, AA, A, $47.5 for BBB rated and $31.19 for BB, B and CCC rated bonds

TABLE 5.3. Summary of the comparison of credit migration matrix by other techniques

Year	Mean	SD	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
MP	0.16	0.04	0.07	0.13	0.17	0.16	0.15	0.12	0.18	0.17	0.21	0.21	0.19	0.16
MD	0.74	0.11	0.39	0.70	0.79	0.74	0.69	0.61	0.78	0.78	0.85	0.85	0.81	0.74
ME	0.16	0.04	0.07	0.13	0.17	0.16	0.15	0.12	0.18	0.17	0.21	0.21	0.19	0.16
M2	0.01	0.01	0.02	0.01	0.00	0.02	0.02	0.01	0.01	0.01	0.05	0.03	0.01	0.01
M_{SVD}	0.17	0.05	0.07	0.14	0.19	0.16	0.15	0.11	0.18	0.18	0.23	0.23	0.19	0.15
L1	0.04	0.01	0.01	0.03	0.04	0.03	0.03	0.03	0.04	0.04	0.05	0.05	0.04	0.03
L2	0.01	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
M_{AGL}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Year	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
MP	0.15	0.17	0.16	0.15	0.20	0.18	0.17	0.23	0.24	0.19	0.12	0.14	0.11	0.12
MD	0.72	0.78	0.74	0.72	0.88	0.82	0.77	0.91	0.90	0.82	0.62	0.70	0.60	0.64
ME	0.15	0.17	0.16	0.15	0.20	0.18	0.17	0.23	0.24	0.19	0.12	0.14	0.11	0.12
M2	0.00	0.01	0.00	0.01	0.02	0.02	0.02	0.03	0.05	0.03	0.00	0.00	0.00	0.00
M_{SVD}	0.16	0.17	0.16	0.15	0.22	0.19	0.18	0.25	0.27	0.20	0.12	0.14	0.11	0.12
L1	0.03	0.04	0.03	0.03	0.04	0.04	0.04	0.05	0.05	0.04	0.03	0.03	0.02	0.03
L2	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01
M_{AGL}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

FIGURE 5.1. MSVD and CMI

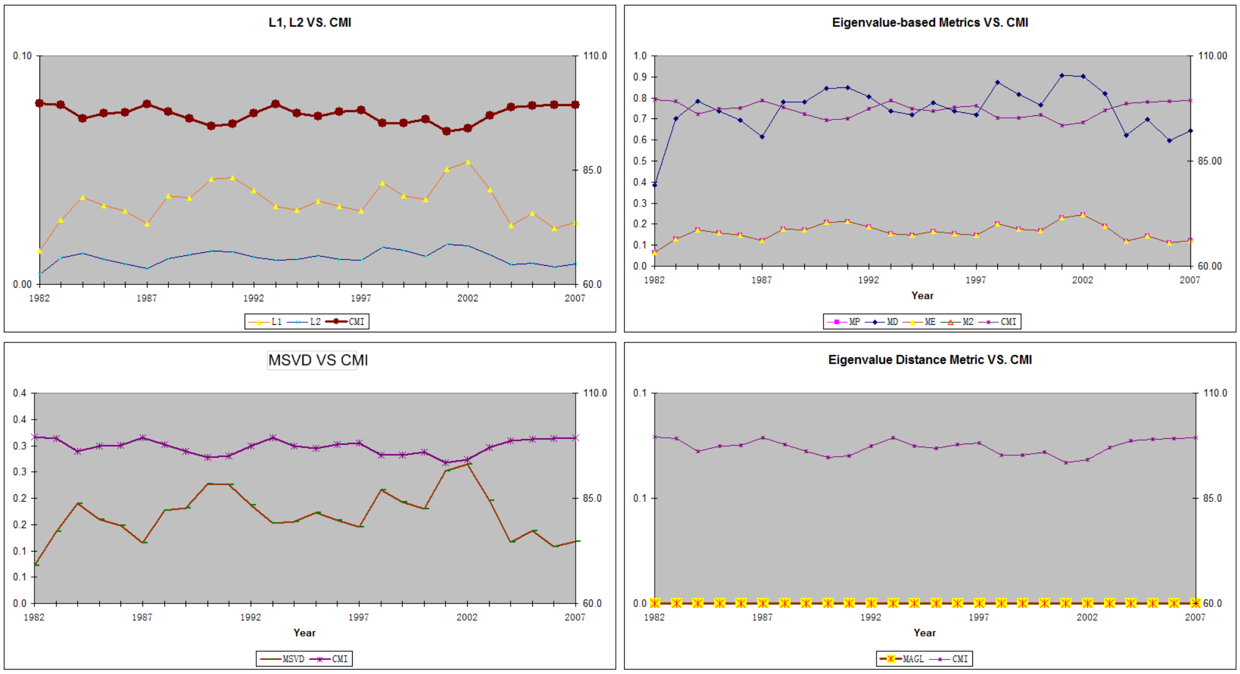


CMI for portfolios. When we calculate CMI for portfolios we need to know the weights of each bonds within the portfolio. Here, we make a simple assumption of our portfolio that all bonds will have just one units. We also calculate the portfolio CMI using a real forward value matrix. The result is in table (5.2). Please refer to figure (5.2) to compare the CMI with other techniques.

Results from other techniques. We apply other techniques on the S&P Bond's credit matrices too and the result is in table (5.3) and figure (5.2)[15].

[15]For metrics require two matrices input, we compare the migration matrix to unit matrix.

FIGURE 5.2. Results from other matrix comparison techniques



Comments on the CMI of S&P corporate bonds. From the result, we can find that except bonds rated grade CCC, all other bonds have CMI close to 100, which means the expected value change due to credit migration is very small. Meanwhile, the CMI of bonds rated grade CCC is quite below 100, meaning a great value change due to credit migration. As we can expected the portfolio CMI is quite stable because the weight of CCC bond is just about 1/7 of total values. From the result, it suggests we should focus on one single category bonds rather than portfolio of all bonds.

6. CMI and other matrix comparison techniques

It is useful to compare the CMI with other techniques for comparing credit migration matrices. We separate the comparisons into several aspects.

- **Scalar:** All the techniques including CMI, give scalar results. Generally, we can apply general scalar analytical techniques on the results from comparison of matrices.
- **Applicable to single matrix:** The Cell-by-cell distance metrics and eigenvector distance metric must have two inputs, making them unable to apply to single matrix[16]. Eigenvalue-based metrics, the average of the singular values of the mobility matrix and CMI can have single input.
- **Rule for transitive Comparison:** No rules on the transferable comparison are given on cell-by-cell distance metrics and eigenvalue-based metrics. For example, $L1\left(A,B\right)>0,\ L1\left(C,B\right)>0$, we do not know the relationship between A and C. It is not sure whether we can derive a transferable comparison rules for eigenvector distance metric and the average of the singular values of the mobility matrix. CMI has a very simple rule for transferable comparison and direct economic meaning for transferable comparison.
- **Duplicate value:** If two mathematical different matrices are given the same results under these techniques, they do not provide straightforward explanation. Yet, CMI clearly states that in that situation, those two CMIs are economically equal.

[16]Normally, we can workaround this issue by comparing a matrix with unit matrix.

Economic consideration: Those techniques come from mathematical requirement and have not been designed specially for economic application. CMI is designed for economic application.

Direction of rating movement: The direction of the rating movement is important and has strong economic implication. Downgrade of one grade will be terrible but upgrade with one grade has minor effect. Only CMI reflects this direction from its calculation while others ignore this feature of the credit migration.

Time Evolution: Supposedly, a portfolio with value V_0 now will has a series of credit migration $P_i(i = 1 \cdots n)$ and the expected values of the portfolio are $V_i(i = 1 \cdots n)$. We can at its expected value at $t = N$ as follow,

$$V_N = \frac{V_N}{V_{N-1}} \cdot \frac{V_{N-1}}{V_{N-2}} \cdots \frac{V_1}{V_0} \cdot V_0 = CMI_N \cdot CMI_{N-1} \cdots CMI_1 \cdot V_0 = \prod_{i=1 \cdots N} CMI_i \cdot V_0 \tag{6.1}$$

This formula provides a simple way to calculate the long-term expected value of a portfolio, or a way to calculate a CMI spanning several years. There are no evidence that other matrix comparison techniques also provide such convenience.

7. Conclusion

In this paper, we mainly focus on the techniques for comparing credit migration matrices. After reviewing the existing techniques, we believe that those techniques ignore two important issues. The first one is that when the calculation of two matrices gives identical results, these techniques do not present further explanation. Another issues is that these techniques view credit migration matrices as normal matrices, ignoring their economic implications. An example is given to present these two issues. To address these two issues, we distinguish economic difference from mathematical difference. Although these two concepts are quite simple, yet they provide a straightforward way for us understand those problems. Consequently, we propose another measure, credit migration index. This index reflects our consideration on the economic impacts raised by credit migration.We define and present how to construct CMI. We compare the CMI method with other techniques, finding that CMI is a good supplement to other techniques.. As we believe it is necessary and best to compare credit migration matrices in term of economic. CMI supports us to capture the economic information of a credit migration, the direction of migration, the magnitude of the migration and the impacts of the migrations. In a conclusion, it is good to adopt this new measure.

8. Appendix

8.1. S&P Credit Migration Data.

Table 8.1. S&P corporation bonds credit transition matrices 1982-1987

1982	AAA	AA	A	BBB	BB	B	CCC	D
AAA	92.19%	7.59%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.00%	94.37%	5.45%	0.17%	0.00%	0.00%	0.00%	0.00%
A	0.00%	0.00%	93.81%	5.96%	0.23%	0.01%	0.00%	0.00%
BBB	0.00%	0.20%	0.00%	92.48%	6.90%	0.41%	0.00%	0.00%
BB	0.00%	5.11%	0.15%	0.00%	84.21%	10.33%	0.00%	0.19%
B	0.00%	0.00%	0.00%	0.00%	0.00%	96.50%	0.00%	3.50%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1983	AAA	AA	A	BBB	BB	B	CCC	D
AAA	97.44%	2.42%	0.14%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.00%	89.33%	10.51%	0.16%	0.00%	0.00%	0.00%	0.00%
A	0.00%	1.42%	95.72%	2.83%	0.00%	0.03%	0.00%	0.00%
BBB	0.00%	2.23%	2.42%	93.01%	0.00%	2.30%	0.02%	0.02%
BB	0.00%	0.00%	0.00%	0.00%	94.75%	5.16%	0.04%	0.05%
B	0.00%	0.00%	0.00%	0.00%	0.00%	96.61%	1.27%	2.11%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	37.63%	42.71%	19.65%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1984	AAA	AA	A	BBB	BB	B	CCC	D
AAA	78.94%	19.97%	1.07%	0.02%	0.00%	0.00%	0.00%	0.00%
AA	3.61%	86.93%	9.25%	0.20%	0.01%	0.00%	0.00%	0.00%
A	0.02%	1.01%	94.69%	4.05%	0.19%	0.04%	0.00%	0.00%
BBB	0.00%	0.02%	3.45%	86.37%	8.30%	1.85%	0.00%	0.01%
BB	0.00%	0.01%	1.82%	3.39%	91.22%	3.55%	0.00%	0.02%
B	0.00%	0.00%	0.03%	0.06%	3.45%	95.25%	0.00%	1.20%
CCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	44.73%	55.27%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1985	AAA	AA	A	BBB	BB	B	CCC	D
AAA	91.02%	7.14%	0.27%	0.05%	1.41%	0.08%	0.03%	0.01%
AA	0.91%	90.50%	6.51%	1.06%	0.04%	0.10%	0.69%	0.18%
A	0.01%	1.25%	93.72%	4.79%	0.16%	0.07%	0.01%	0.00%
BBB	0.01%	1.18%	3.64%	86.71%	5.75%	2.61%	0.02%	0.07%
BB	0.00%	0.02%	1.90%	0.95%	87.63%	9.18%	0.06%	0.26%
B	0.00%	0.00%	0.04%	0.02%	3.96%	89.76%	1.02%	5.20%
CCC	0.00%	0.00%	0.00%	0.00%	0.38%	15.50%	50.71%	33.40%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1986	AAA	AA	A	BBB	BB	B	CCC	D
AAA	93.73%	6.04%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.00%	92.89%	6.85%	0.20%	0.03%	0.04%	0.00%	0.00%
A	0.00%	4.34%	88.79%	5.05%	0.75%	0.99%	0.05%	0.03%
BBB	0.00%	0.13%	5.17%	83.74%	8.74%	1.99%	0.17%	0.07%
BB	0.00%	0.00%	0.04%	1.29%	90.40%	5.87%	1.93%	0.47%
B	0.00%	0.00%	0.01%	0.41%	2.86%	83.95%	7.20%	5.57%
CCC	0.00%	0.00%	0.00%	0.02%	0.17%	9.48%	63.98%	26.34%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1987	AAA	AA	A	BBB	BB	B	CCC	D
AAA	95.90%	2.99%	0.09%	0.95%	0.05%	0.01%	0.00%	0.00%
AA	1.34%	93.88%	4.62%	0.12%	0.00%	0.02%	0.00%	0.00%
A	0.01%	1.32%	92.85%	4.62%	0.26%	0.91%	0.01%	0.01%
BBB	0.00%	0.02%	3.16%	85.71%	9.02%	2.02%	0.02%	0.05%
BB	0.00%	0.00%	0.57%	4.88%	85.15%	8.72%	0.08%	0.60%
B	0.00%	0.00%	0.34%	0.16%	4.34%	90.96%	1.58%	2.62%
CCC	0.00%	0.02%	2.64%	2.60%	2.84%	10.31%	70.78%	10.82%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Table 8.2. S&P corporation bonds credit transition matrices 1988-1993

1988	AAA	AA	A	BBB	BB	B	CCC	D
AAA	93.56%	4.29%	2.10%	0.04%	0.01%	0.00%	0.00%	0.00%
AA	2.31%	84.69%	12.66%	0.25%	0.05%	0.03%	0.00%	0.00%
A	0.03%	2.14%	92.99%	3.57%	0.78%	0.46%	0.02%	0.01%
BBB	0.01%	0.69%	7.04%	85.18%	4.17%	2.77%	0.10%	0.04%
BB	0.00%	0.04%	1.61%	5.76%	79.48%	11.34%	1.34%	0.42%
B	0.00%	0.00%	0.04%	0.45%	3.56%	88.58%	4.56%	2.80%
CCC	0.00%	0.01%	0.13%	2.14%	4.06%	10.36%	50.89%	32.41%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1989	AAA	AA	A	BBB	BB	B	CCC	D
AAA	94.59%	5.34%	0.07%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.00%	97.51%	2.38%	0.08%	0.02%	0.01%	0.00%	0.00%
A	0.00%	2.78%	88.96%	5.79%	2.00%	0.44%	0.01%	0.02%
BBB	0.00%	0.08%	5.12%	82.09%	10.99%	1.57%	0.04%	0.11%
BB	0.00%	0.03%	1.07%	6.36%	83.90%	6.95%	0.18%	1.51%
B	0.00%	0.27%	0.10%	0.47%	5.95%	85.78%	4.07%	3.36%
CCC	0.00%	0.03%	1.69%	0.13%	1.64%	0.08%	46.28%	50.15%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1990	AAA	AA	A	BBB	BB	B	CCC	D
AAA	93.10%	5.77%	1.07%	0.06%	0.00%	0.00%	0.00%	0.00%
AA	0.40%	90.81%	8.28%	0.47%	0.04%	0.00%	0.00%	0.00%
A	0.00%	0.65%	88.26%	9.94%	1.00%	0.11%	0.02%	0.01%
BBB	0.00%	0.02%	3.57%	82.57%	11.54%	1.82%	0.28%	0.22%
BB	0.00%	0.02%	0.08%	3.79%	78.41%	11.79%	3.04%	2.87%
B	0.00%	0.30%	0.03%	0.62%	2.51%	77.94%	7.58%	11.02%
CCC	0.00%	0.00%	0.00%	0.01%	0.03%	1.49%	41.88%	56.59%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1991	AAA	AA	A	BBB	BB	B	CCC	D
AAA	86.01%	13.29%	0.68%	0.02%	0.00%	0.00%	0.00%	0.00%
AA	0.36%	90.09%	9.15%	0.36%	0.04%	0.00%	0.00%	0.00%
A	0.28%	0.64%	91.03%	7.16%	0.83%	0.04%	0.01%	0.02%
BBB	0.01%	0.93%	3.28%	88.18%	7.00%	0.39%	0.06%	0.15%
BB	0.00%	0.04%	0.08%	4.07%	81.36%	8.88%	1.46%	4.11%
B	0.00%	0.36%	0.02%	0.24%	5.91%	70.63%	7.48%	15.36%
CCC	0.00%	0.02%	0.03%	1.47%	2.89%	3.86%	43.36%	48.37%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1992	AAA	AA	A	BBB	BB	B	CCC	D
AAA	90.51%	9.12%	0.35%	0.01%	0.00%	0.00%	0.00%	0.00%
AA	1.13%	91.51%	7.10%	0.23%	0.00%	0.00%	0.00%	0.02%
A	0.01%	1.00%	92.22%	6.05%	0.19%	0.02%	0.00%	0.52%
BBB	0.00%	0.41%	5.02%	88.47%	5.46%	0.54%	0.07%	0.03%
BB	0.00%	0.02%	0.29%	9.75%	82.13%	5.83%	1.48%	0.49%
B	0.00%	0.01%	0.48%	1.87%	10.27%	74.92%	7.86%	4.60%
CCC	0.00%	0.00%	0.04%	0.31%	3.59%	10.80%	48.86%	36.41%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1993	AAA	AA	A	BBB	BB	B	CCC	D
AAA	95.57%	4.28%	0.15%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.01%	93.46%	6.40%	0.12%	0.00%	0.00%	0.00%	0.01%
A	0.44%	1.30%	94.35%	3.58%	0.10%	0.01%	0.01%	0.22%
BBB	0.01%	0.05%	5.21%	88.96%	4.93%	0.51%	0.26%	0.07%
BB	0.00%	0.34%	0.88%	6.28%	84.52%	5.89%	0.84%	1.24%
B	0.00%	0.03%	0.41%	0.76%	11.34%	82.18%	1.85%	3.44%
CCC	0.00%	0.00%	0.08%	0.13%	2.35%	31.63%	51.91%	13.90%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Table 8.3. S&P corporation bonds credit transition matrices 1994-1999

1994	AAA	AA	A	BBB	BB	B	CCC	D
AAA	88.26%	10.54%	1.18%	0.02%	0.00%	0.00%	0.00%	0.00%
AA	0.49%	90.60%	8.72%	0.18%	0.00%	0.01%	0.00%	0.00%
A	0.00%	0.90%	95.01%	3.83%	0.06%	0.18%	0.00%	0.01%
BBB	0.00%	0.26%	2.60%	94.06%	2.76%	0.06%	0.00%	0.27%
BB	0.00%	0.01%	0.36%	6.27%	89.51%	3.69%	0.07%	0.08%
B	0.00%	0.00%	0.01%	0.41%	4.52%	88.18%	3.24%	3.64%
CCC	0.00%	0.00%	0.00%	0.02%	0.30%	10.71%	50.44%	38.52%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1995	AAA	AA	A	BBB	BB	B	CCC	D
AAA	91.47%	7.42%	1.09%	0.02%	0.00%	0.00%	0.00%	0.00%
AA	0.72%	89.32%	9.75%	0.19%	0.01%	0.01%	0.00%	0.00%
A	0.01%	2.42%	93.57%	3.62%	0.22%	0.14%	0.00%	0.01%
BBB	0.00%	0.43%	2.62%	92.79%	3.84%	0.11%	0.00%	0.21%
BB	0.00%	0.26%	1.30%	4.87%	88.04%	5.08%	0.09%	0.36%
B	0.00%	0.01%	0.29%	0.95%	6.98%	85.05%	2.72%	3.99%
CCC	0.00%	0.01%	0.05%	2.65%	0.40%	7.45%	43.01%	46.43%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1996	AAA	AA	A	BBB	BB	B	CCC	D
AAA	90.83%	8.12%	1.04%	0.01%	0.00%	0.00%	0.00%	0.00%
AA	1.14%	90.84%	7.95%	0.06%	0.01%	0.01%	0.00%	0.00%
A	0.02%	3.52%	94.78%	1.41%	0.14%	0.13%	0.00%	0.00%
BBB	0.16%	0.10%	4.88%	93.39%	1.10%	0.20%	0.13%	0.04%
BB	0.01%	0.01%	0.60%	7.10%	85.88%	5.04%	0.54%	0.82%
B	0.00%	0.00%	0.27%	1.39%	8.18%	84.93%	2.84%	2.38%
CCC	0.00%	0.00%	0.02%	0.20%	3.09%	10.24%	50.15%	36.30%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1997	AAA	AA	A	BBB	BB	B	CCC	D
AAA	93.73%	6.14%	0.11%	0.02%	0.00%	0.00%	0.00%	0.00%
AA	0.84%	95.06%	3.37%	0.71%	0.02%	0.01%	0.00%	0.00%
A	0.01%	2.15%	92.81%	4.47%	0.31%	0.24%	0.00%	0.01%
BBB	0.00%	0.16%	3.46%	91.90%	3.58%	0.61%	0.01%	0.27%
BB	0.00%	0.01%	0.32%	6.85%	85.96%	6.21%	0.34%	0.32%
B	0.00%	0.01%	0.54%	0.99%	5.91%	88.18%	2.32%	2.06%
CCC	0.00%	0.00%	0.10%	3.02%	3.20%	11.20%	49.64%	32.84%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1998	AAA	AA	A	BBB	BB	B	CCC	D
AAA	88.58%	10.41%	0.34%	0.64%	0.02%	0.00%	0.00%	0.00%
AA	0.56%	93.38%	5.86%	0.19%	0.01%	0.00%	0.00%	0.00%
A	0.11%	2.02%	91.49%	5.80%	0.52%	0.04%	0.00%	0.01%
BBB	0.01%	0.14%	2.36%	89.42%	6.80%	0.95%	0.05%	0.27%
BB	0.27%	0.16%	0.35%	3.90%	85.03%	8.56%	0.72%	1.01%
B	0.01%	0.12%	0.31%	0.77%	4.47%	84.02%	4.34%	5.96%
CCC	0.00%	0.03%	1.44%	0.13%	1.58%	6.86%	26.40%	63.56%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

1999	AAA	AA	A	BBB	BB	B	CCC	D
AAA	95.56%	4.27%	0.15%	0.02%	0.00%	0.00%	0.00%	0.00%
AA	0.55%	91.96%	6.70%	0.74%	0.03%	0.00%	0.00%	0.00%
A	0.01%	2.22%	91.75%	5.46%	0.42%	0.03%	0.00%	0.11%
BBB	0.00%	0.24%	2.71%	92.10%	4.20%	0.50%	0.02%	0.24%
BB	0.00%	0.12%	0.06%	3.16%	86.54%	8.99%	0.39%	0.74%
B	0.00%	0.00%	0.11%	0.56%	3.07%	84.19%	5.09%	6.96%
CCC	0.00%	0.00%	0.03%	1.87%	0.09%	1.78%	34.43%	61.80%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Table 8.4. S&P corporation bonds credit transition matrices 2000-2005

2000	AAA	AA	A	BBB	BB	B	CCC	D
AAA	90.38%	6.43%	3.06%	0.12%	0.00%	0.00%	0.00%	0.00%
AA	0.88%	91.47%	7.34%	0.30%	0.01%	0.00%	0.00%	0.00%
A	0.01%	2.62%	89.49%	7.50%	0.34%	0.02%	0.00%	0.01%
BBB	0.00%	0.32%	2.39%	92.95%	3.76%	0.42%	0.02%	0.14%
BB	0.00%	0.24%	0.18%	3.77%	85.96%	7.48%	0.54%	1.84%
B	0.00%	0.11%	0.24%	0.38%	3.24%	81.96%	5.95%	8.12%
CCC	0.00%	0.02%	0.81%	0.05%	0.08%	3.82%	48.97%	46.25%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

2001	AAA	AA	A	BBB	BB	B	CCC	D
AAA	91.85%	6.95%	1.15%	0.04%	0.00%	0.00%	0.00%	0.00%
AA	0.35%	89.42%	9.81%	0.38%	0.02%	0.00%	0.00%	0.02%
A	0.00%	2.21%	89.91%	7.03%	0.39%	0.04%	0.00%	0.42%
BBB	0.00%	0.12%	3.01%	90.01%	5.50%	0.67%	0.05%	0.64%
BB	0.00%	0.01%	0.26%	2.68%	79.63%	13.05%	1.14%	3.23%
B	0.00%	0.10%	0.01%	0.14%	2.64%	72.01%	8.65%	16.46%
CCC	0.00%	0.00%	0.00%	0.01%	0.12%	5.81%	25.78%	68.27%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

2002	AAA	AA	A	BBB	BB	B	CCC	D
AAA	88.83%	9.21%	1.02%	0.88%	0.04%	0.00%	0.00%	0.00%
AA	0.36%	79.57%	18.19%	1.74%	0.12%	0.02%	0.00%	0.00%
A	0.01%	0.58%	88.40%	9.83%	0.98%	0.16%	0.01%	0.03%
BBB	0.08%	0.08%	2.20%	87.84%	8.48%	0.93%	0.08%	0.31%
BB	0.10%	0.01%	0.34%	3.17%	79.71%	12.32%	1.26%	3.09%
B	0.00%	0.00%	0.01%	0.31%	4.58%	72.64%	8.63%	13.83%
CCC	0.00%	0.00%	0.00%	0.02%	0.58%	4.73%	31.94%	62.73%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

2003	AAA	AA	A	BBB	BB	B	CCC	D
AAA	87.49%	10.71%	1.72%	0.07%	0.02%	0.00%	0.00%	0.00%
AA	0.22%	87.04%	11.79%	0.70%	0.24%	0.01%	0.00%	0.01%
A	0.00%	1.03%	91.31%	7.24%	0.30%	0.03%	0.00%	0.10%
BBB	0.00%	0.01%	1.72%	92.19%	5.28%	0.66%	0.03%	0.12%
BB	0.00%	0.00%	0.03%	2.72%	85.38%	10.11%	0.60%	1.17%
B	0.00%	0.00%	0.00%	0.21%	6.21%	82.82%	5.78%	4.98%
CCC	0.00%	0.00%	0.00%	0.03%	1.20%	17.12%	41.43%	40.22%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

2004	AAA	AA	A	BBB	BB	B	CCC	D
AAA	95.39%	4.49%	0.11%	0.01%	0.00%	0.00%	0.00%	0.00%
AA	0.24%	94.58%	4.61%	0.56%	0.01%	0.00%	0.00%	0.00%
A	0.00%	1.20%	95.51%	3.08%	0.20%	0.01%	0.00%	0.00%
BBB	0.00%	0.09%	3.00%	94.74%	2.02%	0.14%	0.00%	0.01%
BB	0.00%	0.00%	0.06%	3.98%	88.82%	6.31%	0.18%	0.66%
B	0.00%	0.00%	0.00%	0.24%	6.19%	89.38%	2.63%	1.57%
CCC	0.00%	0.00%	0.00%	0.02%	0.65%	17.60%	59.39%	22.33%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

2005	AAA	AA	A	BBB	BB	B	CCC	D
AAA	92.98%	5.67%	1.31%	0.05%	0.00%	0.00%	0.00%	0.00%
AA	0.00%	93.50%	5.67%	0.80%	0.01%	0.00%	0.00%	0.00%
A	0.16%	1.54%	94.50%	3.72%	0.06%	0.01%	0.00%	0.00%
BBB	0.01%	0.19%	6.44%	89.98%	3.05%	0.32%	0.01%	0.00%
BB	0.00%	0.01%	0.30%	5.62%	86.44%	7.23%	0.25%	0.15%
B	0.00%	0.00%	0.10%	0.45%	8.04%	86.10%	4.09%	1.22%
CCC	0.00%	0.00%	0.04%	0.72%	1.12%	22.18%	56.65%	19.28%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

TABLE 8.5. S&P corporation bonds credit transition matrices 2006-2007

2006	AAA	AA	A	BBB	BB	B	CCC	D
AAA	97.55%	2.43%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.45%	97.77%	1.74%	0.04%	0.00%	0.00%	0.00%	0.00%
A	0.01%	4.21%	91.39%	4.08%	0.29%	0.01%	0.00%	0.00%
BBB	0.07%	0.18%	4.57%	91.88%	2.94%	0.36%	0.01%	0.00%
BB	0.00%	0.10%	0.11%	4.33%	89.39%	5.67%	0.18%	0.22%
B	0.00%	0.01%	0.16%	0.19%	7.52%	88.28%	3.10%	0.76%
CCC	0.00%	0.00%	0.02%	0.01%	0.77%	17.17%	65.20%	16.83%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%
2007	AAA	AA	A	BBB	BB	B	CCC	D
AAA	97.66%	2.30%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	0.60%	95.85%	3.49%	0.06%	0.01%	0.00%	0.00%	0.00%
A	0.01%	3.12%	93.11%	3.15%	0.50%	0.04%	0.06%	0.01%
BBB	0.00%	0.34%	3.78%	92.55%	2.79%	0.53%	0.01%	0.01%
BB	0.00%	0.01%	0.30%	5.78%	85.81%	7.67%	0.13%	0.29%
B	0.00%	0.00%	0.01%	0.28%	6.47%	89.86%	2.81%	0.56%
CCC	0.00%	0.00%	0.00%	0.03%	0.91%	23.33%	58.90%	16.83%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

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