DETERMINANTS ANALYSIS OF GROWTH RATE IN PRIVATE EQUITY MARKET

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Abstract

This paper focuses on the analysis of modeling the growth rate of private equity investments and determining its related underlying factors. We calculate the growth rate based on the formula presented by Takahashi and Alexander (2001) and then model the growth rate to be related to the cumulative contribution rate and growth rate of price and earnings(with some lags) in public equity market. First we test the reliability of the model constructed by Hoek using the data of US private equity market from 1990 to 2002. Then we update the dataset up to 2006 for both US and European markets and redo the ordinary linear regression to test the reliability of our newly constructed model. We find that Hoek's model is not quite reliable and persuasive in associating the growth rate of US private equity market to excess stock market return. In contrast, based on the derivation of change in net asset value (NAV) and results of statistical tests, the growth rate of US and European private equity markets seem to only depend on changes in the price of public equity markets but with the lag of one year for the US market and three years for European market.

Keywords: Private Equity, Growth Rate

Dedication

I wish to dedicate this paper to my dearest parents and grandparents for their endless love and support in my life. Also, I wish to dedicate this paper to my boyfriend Mr. Pan for his understanding and all my best friends, who care and support me all the time.

Sijing

I wish to dedicate this paper to my dearest parents and parents in law for their endless love, encouragement and fully support over my studies. I also wish to dedicate this paper to my husband BaoWang for his understanding and taking care of me throughout my study in this program.

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1. Introduction

Private Equity refers to any type of equity investment in which the equity is not freely tradable on a public stock market. It generally exists in three different forms: leveraged buyout, venture capital and growth capital. Over the past 15 years, the size of private equity market has grown dramatically. The funds in Europe and in the US have grown from 5 to 432 billion dollars in 2006 (Bespoke Data, 2008). While the growth of private equity is striking at present, the potential for future development is likely to be equally remarkable.

Private equity investment is by nature an illiquid asset class which often demand long holding periods to allow for the turnaround of a distressed company or a liquidity event such as an IPO or sale to a public company. In return for the additional risks and liquidity constraints associated with such investments, private equity investors always demand a significant premium. In order to effectively manage the investment portfolio, both investors and managers need to estimate the risks and the potential returns of private equity investments in a quantitative way. As a result, in recent years, we have seen a growing number of papers discussing the risks and return of either individual private equity investment or private equity funds.

The typical measure used to evaluate the performance of private equity funds is the internal rate of return (IRR). In reality, its unique characteristic of illiquid nature and lack of transparency make it difficult to judge their performance, as the true performance of a fund is only observable once the fund has been liquidated. However, it usually takes several years before investee companies can be profitably exited, and then a time lag between draw downs and distributions exists. This fact renders IRR performance measures inappropriate for not yet liquidated funds as IRR is entirely cash flow-based and only covers the realized rate of return. In fact, Blaydon and Horvath (2003) showed how cash flow-based measures severely underestimate the performance of young, still operating funds (Blaydon, C. and M. Horvath, 2003,March). In addition, in the context of portfolio management, IRR does not allow the estimation of a standard deviation of returns and a correlation of private equity returns to other asset classes, such as publicly traded stocks (Axel Buchner, Christoph Kaserer and Niklas Wagner, 2007). Some empirical studies on the performance of private equity funds try to avoid the drawbacks of IRR by calculating returns (G(t)) on the fund's disclosed net asset values (NAV), which includes the aggregation of all unrealized returns of an investment. The younger the fund, the more important the role of NAV is in assessing performance. Moreover, growth rate G(t) is important especially for institutional investors in annual valuations since IRR focuses on the rate of return of whole investment period instead of interim intervals. In 2001, Takahashi and Alexander presented a model that covered the illiquidity aspect of private equity in terms of contributions, distributions and net asset value. The growth rate G(t) of net asset value calculated from their model is a combination of both unrealized and realized returns. However, little research has been done to identify the determinants of G(t).

In 2007, Hoek modifies the growth rate G(t) to be stochastic and dependent on a series of underlying factors such as cumulative contribution rate CUM(t) and the excess return of the stock market (current and previous years) in the working paper "An ALM Analysis of Private Equity". We are not fully convinced by his analysis and experiment. We prefer to associate G(t) with change in price and change in earnings of public equity market (see derivation in section 3) instead of excess stock market return but we agree that there is a lag between G(t) and public market metrics. The reason why cumulative contribution rate CUM(t) should be relevant to G(t) is not explained well in Hoek's papaer, but we will nevertheless test the relationship and significance of CUM(t) with G(t) based on updated US and European private equity market data by running an ordinary linear regression. Moreover, the optimal value of p[1]which is assumed to be "1" in Hoek's paper will also be selected. All in all, if we can accurately construct a model of the growth rate G(t), then we can better estimate the future values of private equity investments.

This paper is organized as follows. In section 2 we will give a literature review of available researches on performance modeling of illiquid asset funds; section 3 describes methodologies which the paper is based on and the procedures of doing the significance tests of potential parameters; the fourth section introduces database used in the model; the dependence of the growth rate on the underlying factors and the best value of "p" are analyzed in section 5 and conclusions are drawn in section 6.

[1] "p" means the number of lags.

2. Literature Review

During the last decade, private equity industry grew tremendously and gained considerable awareness among the general public. At the beginning there was little research about the return of private equity investment. Most investors just take into account the Sharpe Ratio of these investments and its correlation with equity benchmark. In recent years, a growing number of academic papers on the risk and return of private equity have been published.

Peng (2001) used two innovative methods, a re-weighting procedure and moment repeat sales regression to solve three problems - missing data, censored data, and sample selection. He reported the capital flows, net asset value, and returns of the venture capital during the sample years and found that the venture capital industry experienced an impressive growth in the number of financing rounds and the net asset value of the index. The average return to venture capital was 55.18% per year in his paper, with the lowest annual return in 1990 (-5.94%) and the highest in 1999 (681.22%). He also found that the venture capital index has much higher volatility than both S&P 500 and NASDAQ and there is a significant correlation between the venture capital index and NASDAQ in returns and volatility.

Generally, we measure return of a company only when there is a liquid activity in private equity such like going public or a new finance round. Cochrane (2003) focused on the individual portfolio company level, rather than on the funds as a whole with data from Venture Economics. He measured 7,765 companies financing rounds and considered the estimate has selection bias. In his paper, he created a model to estimate the expected return, standard deviation, alpha and beta of venture capital investment by using maximum likelihood estimation. He found that the estimates are driven by stylized facts in the data, pattern of returns and project age. He showed that mean average returns decline from 698 percent to 59 percent if he controls for selection bias. Accordingly, the arithmetic alpha declined from 462 percent to 32 percent after correction. He also found that the smallest NASDAQ stocks have similar large means, volatilities, and arithmetic alphas in this time period, demonstrating that the puzzles are not only special to venture capital. Cochrane concluded that the venture returns are very volatile and deals of later stage have less volatility than early stage deals.

However, Kaplan and Schoar (2004) analyzed the risk and return characteristics of the private equity industry based on fund level data of a sample of 746 private equity funds, which is also obtained from Venture Economics. The funds included were either already liquidated or mature funds which were close to liquidation. Based on a cash flow approach, they measured the funds' IRR, PME and TVPI. Kaplan and Schoar find large heterogeneity through these fund's returns. They reported that the median IRR in their sample is 12 percent, whereas the average IRR is 17 percent equally weighted and 18 percent value weighted. In their PME calculation, Kaplan and Schoar discovered that the private equity funds underperformance relative to the S&P 500 by using an equally weighted average. Since their sample is net of fees they concluded that private equity generally performs better than public equity if fees are added back to the fund's returns. A key difference is that the determination of substantial persistence of the fund's returns is markedly different from that for mutual funds, where persistence is hard to detect. Furthermore, Kaplan and Schoar reported improving returns with increasing experience of the General Partners.

Despite private equity plays an important role in fostering innovative firms and reallocating capital to more productive sectors of the economy, relatively little information is known about the key characteristics of private equity: liquidity, risk, and return and investors have less control over how the capital is invested. Ljungqvist & Richardson (2003) provided the first analysis of private equity returns based on actual cash flows of venture capital and buyout funds in US market. They focused on mature (close to liquidation) and liquidated funds and their sample consists of 73 private equity funds from1981 to 1993. They used this unique dataset to document the degree of liquidity and in terms of risk, how much compensation investor should get. Specifically, they empirically measured how quickly capital is returned to investors and the overall performance of private equity by analyzing draw down (contribution) and capital return schedules for the typical private equity fund. Besides computing IRR and TVPI for the funds, they investigated the relative performance to a public market investment on the basis of excess IRR. They reported an average outperformance of the private equity investments of 6 to 8 percent relative to the S&P 500 and 3 to 6 percent relative to the Nasdaq Composite Index. Different to other studies, Ljungqvist and Richardson focused on realized cash flows only, leaving net asset values aside. They estimated the fund's betas. The betas average 1.08 for the buyout and 1.12 for the venture funds. On the basis of their estimated beta, they calculated a risk-adjusted profitability index. The index is computed by discounting capital calls at the risk free rate of interest and the distributions by discounting at the risk free rate plus the fund's estimated beta times the risk premium (Fama and French measure). Moreover, they found that IRR of the average fund does not turn positive until the eighth year of the fund's life. Thus, once adjust for the cost of capital, the excess returns only realized at the very end. This highlighted the illiquidity of private equity investments and also suggested that "interim" IRRs computed before a fund reaches maturity are not very reliable. Therefore, measuring a fund's performance requires precisely dated cash flows over a fund's life, rather than relying on arbitrary assumptions about the time of capital returns. i.e. realized return.

In order to guide investors of private equity to assess the impact of changing in fund commitment levels and varying assumptions including contributions, distributions and net asset values, Takahashi and Alexander (2001) build a model that enables institutional investors to estimate future asset values and cash flows in illiquid alternative asset classes, such as venture capital, leveraged buyouts, real estate and natural resources. They assumed future funds would have the same patterns of capital contributions, distributions and asset values as historical averages. First, they assume the NAV development would grow at a certain rate of the residual value. The cash flows are then modeled by assuming a rate of contribution (a percentage of the undrawn commitments) and rate of distribution (a percentage of the forecasted residual value). This approach is quite intuitive to easily be applied to single funds or large portfolios and the model is superior to simple rules of thumb, which is simply relating commitment levels to expected asset values of funds.

Furthermore, in order to better diversify and manage the asset and liability, Hoek (2007) extended this model to evaluate whether the dynamics generated by the model are sufficiently with the behavior of private equity in real life. For application purpose, he also analyzed the added value of private equity in strategic asset allocation. Hoek assumed investment in a private equity fund starts with committing capital CC to the fund gradually. Once the investment has started, NAV will change over time, due to new contributions, “unrealized” returns and realized returns. And he measured the rate of return G (t) on invested capital in year t which showed the whole process of change in NAV. Hoek set new private equity investment growth rate model by regression the underlying related factors from a stochastic perspective. He concluded that G (t) is related to cumulative contribution rate, public stock market return and one year lag return. The paper pointed that the institutional investors should pay more attention on asset growth rate which combined by realized and unrealized return to lower their risk and better allocate illiquid asset portfolio.

“The internal rate of return (IRR) is a usually used measure in assessing the performance of private equity investments. It is defined as the discount rate, making the present value of all investment cash flows equal to zero. However, performance evaluation on basis of IRR calculation has several drawbacks. First, the IRR may not be unique when future cash flows vary in sign. Second, the IRR is based on the implicit assumption that intermediate cash flows can be reinvested at the discount rate. Last, but maybe most important in the context of portfolio management is the fact that the IRR does not allow the estimation of a standard deviation of returns and a correlation of private equity returns to other asset classes, such as publicly traded stocks. Some empirical studies on the performance of private equity funds try to avoid these drawbacks of the IRR by calculating returns based on the funds disclosed net asset values (NAV).”(Axel Buchner, Christoph Kaserer and Niklas Wagner, 2007) In our paper, we will focus on G (t) the realized and unrealized growth rate based on NAV which was in the formula set by Takahashi and Alexander (2002). In order to check the model by Hoek (2007), we will demonstrate whether it is still reliable for further research from a market perspective and whether growth rate of private equity investment is correlated with those underling factors by using data of private equity market (buyout and venture capital combined) in both US and European markets. Also we will build a new model of regression which changes the factor of stock market return to price earnings ratio to forecast the relationship among these factors with G (t). What’s more, we will assume more time lags return to suggest whether $p = 1$is the best estimator.

3. Methodology

Previous methodologies used in characterizing the returns of private equity investment mainly focus on the cash flow-based internal rate of return (IRR). However, the use of IRR as a measurement of performance has several drawbacks. First, IRR is based on the implicit assumption that intermediate cash flows can be reinvested at the discount rate. Second, it can be only applied when all the liquidation has been achieved. Therefore, we believe that the growth rate of net asset value is a better measurement in assessing the performance of private equity investments. In 2001, Takahashi and Alexander first introduced the performance measurement of private equity from the perspective of net asset value. Later on, Hoek modeled the growth rate to be stochastic and dependent on the excess stock market return (of current and previous years) and cumulative contribution rate CUM(t), which we think the logic behind is not quite persuasive. Here we would like to propose a new method for modeling the growth rate G(t) by replacing the term of excess stock market return with change in prices and earnings of the public equity market, which we think are better proxies of the determinants of G(t) compared with excess return of stock market. In the following sections, the illiquid asset fund model (Takahashi and Alexander, 2001), Hoek's private equity growth rate model and our newly proposed model will be introduced separately.

3.1 Illiquid Alternative Asset fund Model (Takahashi and Alexander, 2001)

Takahashi and Alexander created the illiquid alternative asset fund model in 2001, in searching for better estimates of future values of private equity market and real asset capital commitments. The original model depicts the process of the private equity investment in terms of contributions (cash inflows), distributions (cash outflows) and the valuation of the committed capital (net asset value) (Hoek, 2007).

3.1.1 Contribution

Investment in a private equity fund starts with limited partners committing capital to the fund. The manager (general partner) of a limited partnership is responsible for the investment decisions and has full liability for the partnership's obligations. This committed capital (CC) is not invested immediately as a lump sum, but gradually. When the general partner identifies an investment opportunity, she/he typically issues a capital call (also known as a drawdown price) to each of the limited partners, who must then provide capital, i.e. contribution C(t) to the partnership in proportion to their commitments. For example, in year t the institutional investor obtains a capital call and has to contribute part of the remaining capital commitment:

$$\mathrm{C(t)} = \mathrm{CR(t)}\big(\mathrm{CC} - \mathrm{PIC(t)}\big) \tag{1}$$

Therefore, capital contributions are calculated by multiplying the rate of contribution by the remaining capital commitment, or the initial capital commitment minus the sum of the paid in capital (PIC), where

$$PIC(t) = \sum_{0}^{t-1} C(t) \quad (2)$$

In practice, the sum of the capital contributions rarely equals to the capital commitment. Typically, only 50% - 70% (rule of thumb) of an investor's commitment is required to meet its obligations to the fund, because some portion of the early distributions can be used to fund later obligations. Moreover, based on the analysis of historical data (Private Equity, 2004), Exhibit 2.1 illustrates the incremental and cumulative cash-flow patterns of a typical private equity fund and shows that private equity funds invest the majority of their commitment during the first five years, i.e. the investment period.

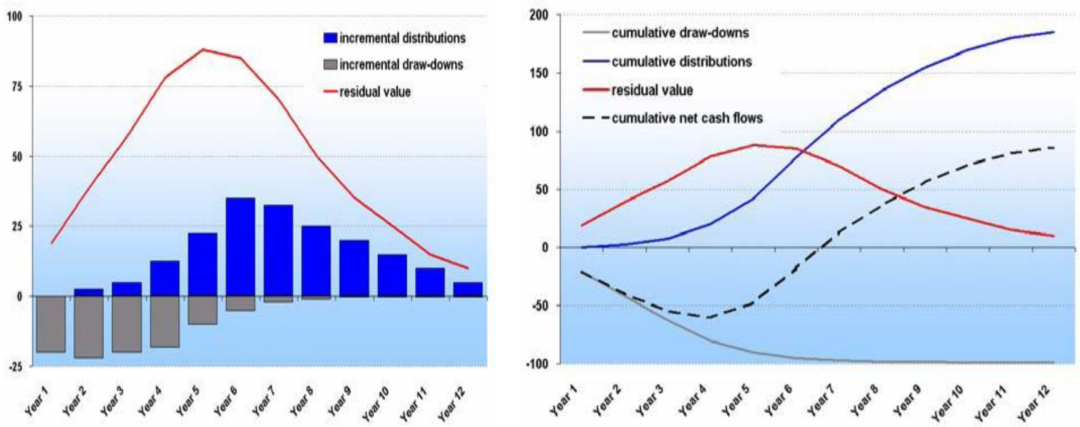


Figure 3.1: The above pictures illustrate incremental and cumulative private equity cash flows (draw-downs in light grey, distributions in dark grey) and the development of the residual value (blue).

3.1.2 Distribution

Distribution can be regarded as the realized return of the investment. As described by Takahashi and Alexander (2001), "it varies in different stages of a fund's life. In the early years of a fund, distributions tend to be scarce as investments have not had time to be harvested. The middle years of a fund tend to have the most affluent distributions as investments come to fruition. Later years are marked by a steady decline in distributions as fewer investments are left to be harvested."

3.1.3 Net Asset Value

The valuation of private equity investments (i.e. net asset value) is the sole responsibility of general partners. According to EVCA[2], the conservative and fair market value methodologies are most commonly used in valuing private equity investments. The conservative methodology applies to companies both without profits and in the initial year of investment. Under the conservative methodology, companies are valued at cost unless (1) a new price was determined in a transaction involving a new financing round or an investment by a third party at arm's length; and (2) the financial outlook of the company aggravated due to, inter alia, a breach of covenant, change in senior management or worsening market conditions. In this case, the companies should be written-down by multiples of 25%.

The fair market value methodology should be applied when twelve months have elapsed since the initial investment and the investee company becomes profit-generating or has positive cash flows. This methodology relies on factors such as price/earnings (P/E) or price/cash flow (P/CF) of quoted companies comparable to the investee company. As illustrated by EVCA, "the GPs identify the sample of comparable public companies, calculate the mean of the sample multiples and apply

[2] EVCA stands for "European Private Equity and Venture Capital Association".

them to their investee company. The resulting valuation is then discounted by at least 25% to account for illiquidity. If comparable companies are not identifiable, multiples at the time of the initial investment can be applied to the most recent trading figures". Since we are looking at the annual growth rate ranging from 1990 to 2006, valuation is fair market value, i.e. price/earnings (P/E) or price/cash flow (P/CF).

The development of net asset value captures the unrealized return of the private equity investment compared with cash flow-based rate of return. Once the investment started, its value, measured by the Net Asset Value, will change over time, and is affected by three variables: investment performance G(t), capital inflows (contributions) and capital outflows (distributions). In the model, as shown below, the net asset value at time t (NAV(t)) is calculated by multiplying the previous year's net asset value NAV(t-1) by one plus the growth rate (G(t)), adding contributions (C(t)), and subtracting distributions (D(t)):

$$\mathrm{NAV}(t) = [\mathrm{NAV}(t-1) \times (1 + G(t)] + C(t) - D\ (t) \qquad (3)$$

From this equation, we can see that $G(t)$ measures both unrealized and realized return and can be calculated given that net asset value, contribution and distribution are all known.

3.2 Growth Rate Models

3.2.1 Hoek's Model

In the paper “An ALM Analysis of Private Equity” (Hoek, 2007), the growth rate G(t) is modified to be stochastic and dependent on a series of underlying factors: cumulative contribution rate CUM(t) and excess stock market return (of current year and with some lags). Denoting returns on the stock market by R(t), the average stock return by AR, this gives the following model:

$$\mathrm{Ln}\big(1+\mathrm{G}(\mathrm{t})\big) = \mathrm{a}(0) + \mathrm{a}(1)\mathrm{CUM}(\mathrm{t}) + \mathrm{b}(0)(\mathrm{R}(\mathrm{t}) - \mathrm{AR}) + \mathrm{b}(1)(\mathrm{R}(\mathrm{t}-1) - \mathrm{AR}) + \cdots \mathrm{b}(\mathrm{p})(\mathrm{R}(\mathrm{t}-\mathrm{p}) - \mathrm{AR}) + \varepsilon(\mathrm{t}) \quad (4)$$

During the process of an ordinary linear regression analysis, Hoek assumed “p” to be “1”, and then the above equation is simplified to:

$$\mathrm{Ln}\big(1+\mathrm{G}(\mathrm{t})\big) = \mathrm{a}(0) + \mathrm{a}(1)\mathrm{CUM}(\mathrm{t}) + \mathrm{b}(0)(\mathrm{R}(\mathrm{t}) - \mathrm{AR}) + \mathrm{b}(1)(\mathrm{R}(\mathrm{t}-1) - \mathrm{AR}) + \varepsilon(\mathrm{t}) \quad (5)$$

Here, $\varepsilon(\mathrm{t})$ is an error term and normally distributed with mean 0 and variance σ^2. He expected the returns on private equity to be positive skewed (right skewed), therefore, chose to use a logarithmic transformation. The logarithmic transformation will reduce positive skewness, as it compresses the upper end (tail) of the distribution while stretching out the lower end. As described in Hoek’s paper, he included CUM(t) to indicate that “more and higher capital calls reflect increasing profit opportunities”. However, we are a little suspicious of that argument as need for capital may not only imply good growth but may also imply financial difficulty and can’t find any evidence in the literature why CUM(t) should be related to G(t). We will nevertheless test its significance and relationship with G(t) by employing the “F test”. More clarification will be provided in later sections.

3.2.2 New Constructed Model

When valuing the private equity each year, NAV is calculated as the product of P/E and E, i.e. the valuator calculates P=P/E*E. The P/E applied by the valuator is determined by comparison with public P/E multiples. The E used by the valuator should be the earnings of the private equity. Here we hypothesize E is related to earnings in public equity markets. Therefore, when we look at changes in NAV (which can be calculated by taking Ln(NAV_t/NAV_{t-1}), the product on the right hand side of the valuation equation will be the sum of change in P/E and change in E. Rearrange it to isolate E from P in the first term and the formula will end up with:

$$\mathrm{Ln}(\frac{\mathrm{NAV_t}}{\mathrm{NAV_{t-1}}}) = \gamma_1 \mathrm{Ln}\left(\frac{\frac{P_1}{E_1}}{\frac{P_0}{E_0}}\right) + \gamma_2 \mathrm{Ln}\left(\frac{E_1}{E_0}\right)$$

$$= \gamma_1(\mathrm{LnP_1} - \mathrm{LnE_1} - \mathrm{LnP_0} + \mathrm{LnE_0}) + \gamma_2(\mathrm{LnE_1} - \mathrm{LnE_0})$$

$$= \gamma_1(\mathrm{Ln}\ \frac{P_1}{P_0}) + (\gamma_2 - \gamma_1)(\mathrm{Ln}\frac{E_1}{E_0})$$

Based on the above derivation, it is quite reasonable to model G(t) to rely on the growth rates of price and earnings of public equity market. Second, since valuation reports of private equity market need to be compiled by someone such as a business valuator, there may be some lags between the valuations of public equity and private equity markets. Therefore, we agree with Hoek to the point of lag's existence and will confirm the best value of "p" through statistical "F test". In terms of the cumulative contribution rate, we see no justification in the literature why it should be relevant. We will include it and use "F test" to determine its significance. If it is significant, its relationship (positive or negative) with G(t) can be shown by the estimated coefficient, which helps us to verify that "need for capital may not only imply good growth but may also imply financial difficulty in a given project". Denoting the annual growth of price and earnings as R(t) and E(t), the new model is constructed as:

$$\mathrm{Ln}\big(1 + \mathrm{G(t)}\big) = \mathrm{a(0)} + \mathrm{a(1)CUM(t)} + \mathrm{b(0)}\big(\mathrm{R(t)}\big) + \mathrm{b(1)}\big(\mathrm{R(t-1)}\big) + \cdots$$
$$+ \mathrm{b(P)}\big(\mathrm{R(t-p)}\big) + \mathrm{c(0)}\big(\mathrm{E(t)}\big) + \mathrm{c(1)}(\mathrm{E(t-1)} + \cdots$$
$$+ \mathrm{c(p)}(\mathrm{E(t-p)} + \varepsilon(\mathrm{t}) \qquad (6)$$

3.3 Model Test

3.3.1 Intuition

Using data on C(t), D(t) and NAV(t), the return series G(t) can be computed. Given all factors on the left hand side and right hand side of formula (4) or (6) are known, by running an ordinary linear regression, we can obtain the estimated values for each of the coefficients and their correlated t-stats values and the model's R^2 value. Comparing their t-stats values with the critical value of 95% confidence interval $\pm$ 1.96 (sample size less than 20) and R^2 value with 1, which stands for perfect fitness, we can judge whether the underlying factors are significant in determining the value of G(t) and further the reliability of the model for G(t) based on our dataset. In addition, we use F test to select the best value of "p". The intuition of "F test" can be illustrated as follows. Considering two models, 1 and 2, where model 1 is nested within model 2, which means model 1 has p_1 parameters, and model 2 has p_2 parameters, where $p_2 > p_1$. (Any constant parameter in the model is included when counting the parameters.) If there are n data points to estimate parameters of both models, then F is calculated as

$$F = \frac{\frac{RSS_1 - RSS_2}{p_2 - p_1}}{\frac{RSS_2}{n - p_2}} \quad (7)$$

The null hypothesis for F test is none of the additional $p_2 - p_1$ parameters differ from zero. If the calculated F is greater than the F given by the critical value of F for some desired rejection probability, then we reject the null hypothesis and conclude that the additional parameters are necessary; otherwise accept it.

3.3.2 Procedure

In order to compare with Hoek's results, we will use the same model (formula (5)) and market data from the same time period (1990-2002) to run the ordinary linear regression to see whether the influences of the cumulative contribution rate and excess stock market return on G(t) from the perspective of private equity market is the same as those on venture capital and leveraged buyouts separately. Next we will construct a new model by substituting the term of excess stock market return with the annual growth of price and earnings which we think more accurately captures the changes in net asset value. Moreover, the necessity of term CUM(t) and the optimal value of "p" will be selected through F test. Based on the information we have got, we cannot see clearly the way the author used in calculating CC, i.e. the committed capital. We try to contact the author via email, but unfortunately we haven't got any reply until now. However, we have the total capitalization number showing up in our dataset. Therefore, we assume the total capitalization is the committed capital. The number of 74%, which is the percentage of the sum of all contributions to total capitalization, helps to confirm our assumption, as 74% is consistent with the rule of thumb: Total contributions are about 70% of committed capital.

First of all, we will test the significances of the parameters according to formula (5) using the data that consists of annual numbers on contributions, total distributions and net asset values of the US private equity market, ranging from 1990 to 2002, which corresponds to the same time period addressed by Hoek. As the number of total capitalization is for the whole period of the dataset we have got, ranging from 1990 to 2006, we calculate the committed capital for 1990 to 2002 by the total capitalization number minus the sum of all contributions from 2003 to 2006.

Second, we run the ordinary linear regression based on the updated data ranging from 1990 to 2006 for both the US and European markets and using the new model that includes parameters of growth of price and earnings instead of excess stock market return. Therefore, the influences of these factors on the growth rate G(t) for the US and European market can evidently be shown. Meanwhile, the best value of “p” for each market will be selected according to F test.

4. The Data

We obtained the data from venture economics by calling Investment Banking Helpdesk of Thomson Reuters. The data covers the whole private equity market and consists of quarterly numbers on contributions, total distributions and net asset values (net of management fees and carried interest). All data are at the aggregate level, and give, per vintage year, the sum of all contributions, distributions and net asset values of all private equity funds that are included in the database. We convert the quarterly data into yearly data by adding up the quarterly cash flows per year, but only consider the net asset value in the final quarter of each year as the yearly net asset value. To be consistent with Hoek's procedure, we assume that on average cash flows take place in the middle of the year (This assumption is only applied for comparison with Hoek's results), so the development of the net asset value becomes (Hoek, 2007):

$$\mathrm{NAV}(t) = \left(\mathrm{NAV}(t-1) + \frac{C(t) - D(t)}{2}\right) \times \left(1 + G(t)\right) + \frac{C(t) - D(t)}{2} \qquad (8)$$

Based on those yearly data, the growth rate G(t) and cumulative contribution rate CUM(t) can be calculated according to the following equations and their trends through the whole period can be shown in appendix 1and appendix 2.

$$G(t) = \frac{2\mathrm{NAV}(t) + D(t) - C(t)}{[2\mathrm{NAV}(t-1) + C(t) - D(t)]} - 1 \quad \text{(for comparison only)} \qquad (9)$$

$$G(t) = \frac{\mathrm{NAV}(t) + D(t) - C(t)}{\mathrm{NAV}(t-1)} - 1 \qquad (10)$$

$$CUM(t) = 1 - (1 - C(1)) \times (1 - C(2)) \times \ldots \times (1 - C(T)) \quad (11)$$

In our newly constructed model, the annual growth of price and earnings are two potential underlying factors in determining G(t). We use S&P 500 yearly stock market rate of return as the growth of price in US public equity market ranging from 1990 to 2006. We also get the price/earnings (P/E) of S&P 500 in order to calculate the value of earnings (P/(P/E)). The data of the yearly P/E of European market is not available. However, according to our US results, the price (ignoring E) of private equity investments can be well estimated by the return on the stock market. Therefore, for the European market, we assume the growth rate G(t) to be only dependent on the rate of return of public stock market (ignoring the influence of change in earnings) and use the data of MSCI (Europe) yearly stock market return in our study of the European market. Here are the plotted figures for G(t), R(t), CUM(t) and change in Earnings for both US and European markets.

Figure 4.1: Relationship among G(t), R(t) and CUM(t) for US market

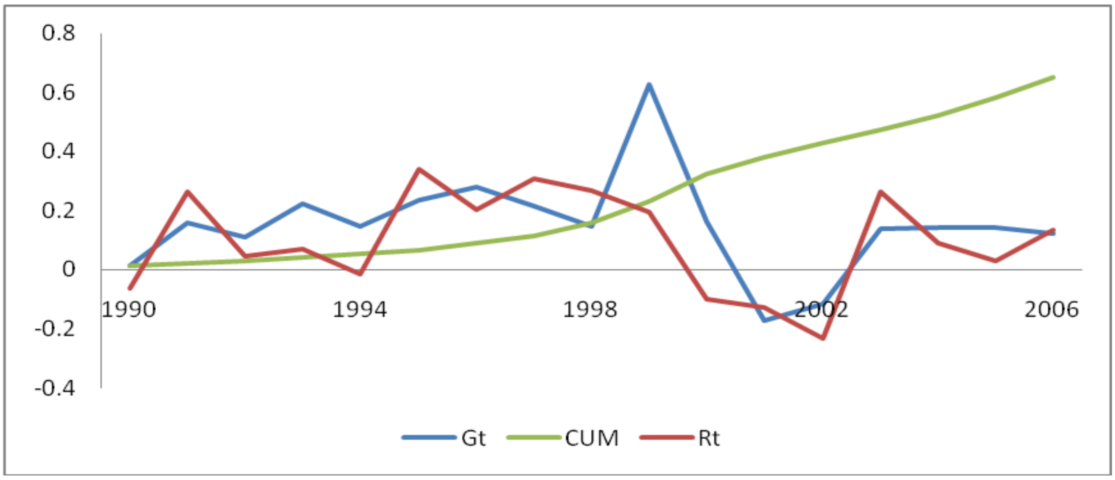


Figure 4.2: Relationship among G(t), CUM(t) and Change in Earning for US market

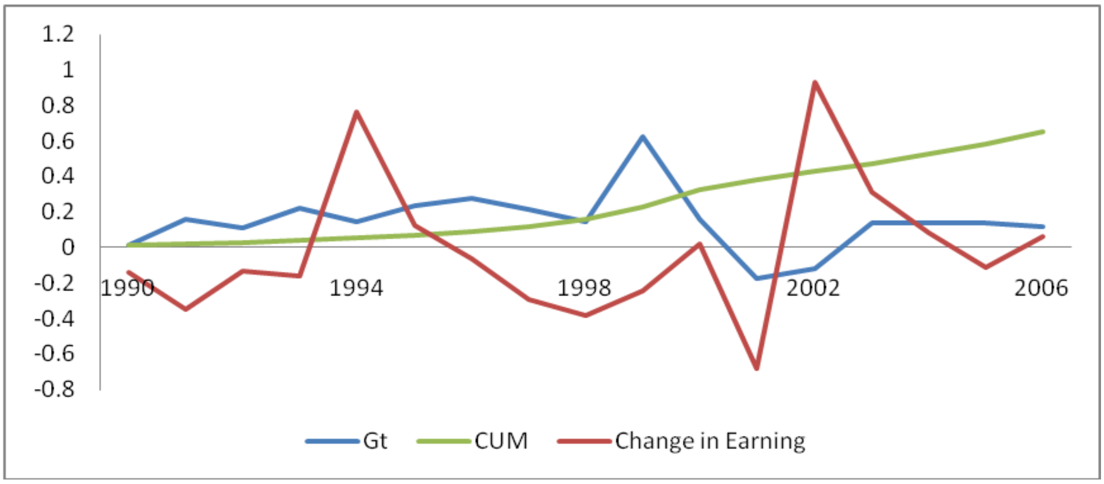
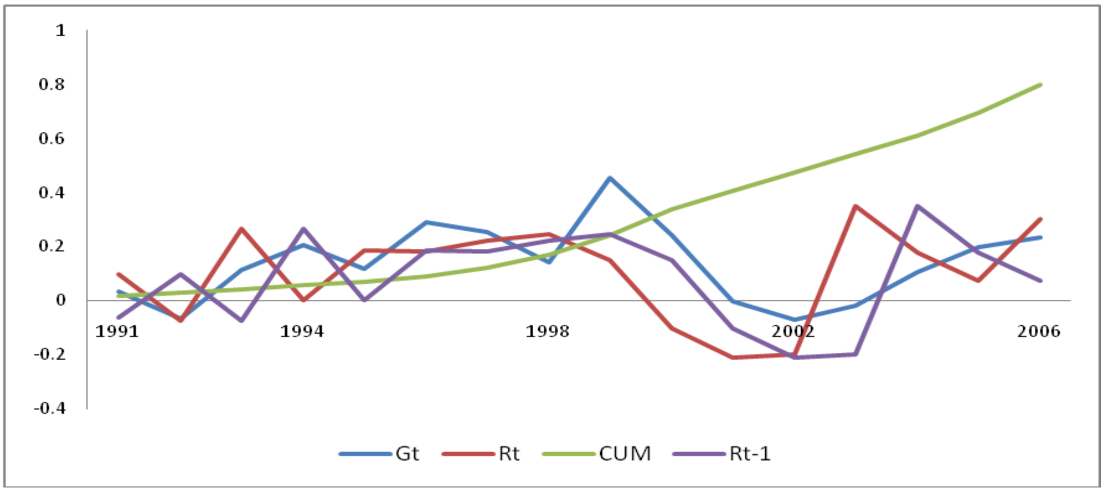


Figure 4.3: Relationship among G(t), R(t), R(t-1), CUM(t) for European market



As observed from the above tables, we expect R(t) (or with some lags) is the most important indicator of modeling G(t) and this will be confirmed in the next section by statistical tests.

5. Results and Analysis

5.1 Comparison with Hoek's Results

In order to compare with the results described in Hoek's paper, we assume that the growth rate G(t) only depends on the cumulative contribution rate, the excess stock market return (contemporarily and with a lag of one year) which is exactly the model (see formula (5)) used by the paper to run the ordinary linear regression and G(t) is calculated according to equation (9), which assumes cash flows take place in the middle of the year. Moreover, we need to use dummy variables (see table 5.3) to take account of 1999 peak for the US market as Hoek did. Here are the results from Hoek's paper for the US venture capital and leveraged buyouts separately and our results for the US private equity market as a whole, ranging from 1990 to 2002.

Table 5.1: Estimation results, G (t) US VC

R-squared:.0.412		
Variable	Estimate	t-value
CONSTANT	-0.077986	-0.666459
CUM(t)	0.400324	2.379946
R(t)-AR	0.609829	3.689711
R(t-1)-AR	1.077579	2.731174

Table 5.2: Estimation results, G (t) US BO

R-squared:.0.319		
Variable	Estimate	t-value
CONSTANT	-0.033113	-0.666176
CUM(t)	0.174899	2.853225
R(t)-AR	0.472003	5.767640

Table 5.3: Estimation results, G (t) of US private equity market from 1990 to 2002

R-squared: .8463		
Variable	Estimate	t-value
CONSTANT	.1730	4.059
CUM(t)	-.1920	-1.852
R(t)-AR	.1783	.9248
R(t-1)-AR	.2524	1.512
Dummy Variable	0.3166	3.295

As shown in table 5.1 and table 5.2 (Hoek's results), both the excess stock market return (contemporarily and with a lag of one year) and the cumulative contribution rate are all significant and have a positive effect on the value of the growth rate of both the US venture capital and leveraged buyouts. In contrast, based on our US private equity market data, as shown in table 5.3, the t-stats values for the coefficients of CUM(t), R(t)-AR and R(t-1)-AR are all in the range of critical value $\pm$ 1.96 of 95% confidence interval for sample size less than 20. Therefore, we accept the null hypotheses that their coefficients are equal to 0 and conclude that CUM(t), R(t)-AR and R(t-1)-AR are all insignificant in modeling the growth rate G(t) of US private equity market from1990 to 2002.

The distinct results can be caused by the following reasons. First of all, the data used in this project is different from the data used in Hoek's paper. The original paper splits the data into venture capital and leveraged buyouts from the perspective of asset and liability management. Here we are testing the reliance of growth rate on the underlying factors CUM(t), R(t)-AR, R(t-1)-AR from the whole private equity market point of view. Moreover, our data for net asset values are net of management fee and carried interest while data used by Hoek is net of carried interest only.

In addition, we assume that the committed capital during 1990 to 2002 is calculated by total capitalization minus the sum of all contributions from 2003 to 2006. Different methods used to calculate the amount of committed capital lead to different results of contribution rate and then consequently give different cumulative contribution rate. This is one possible reason causing the insignificance of CUM(t) to the growth rate G(t) of the whole private equity market.

Although there are many potential reasons resulting in the insignificance of all parameters, we are not quite convinced by the logics behind why G(t) should be related with excess stock market return. Thus we choose to derive the formula of G(t) from the perspective of fair market value valuation and then associate G(t) with the changes of price and earnings in public equity market. We will construct a new model with and without CUM(t) separately in the following part, and the significance of CUM(t) will be demonstrated by "F test".

5.2 Analysis for New Constructed Model

5.2.1 with CUM(t) Term

With the purpose of modeling growth rate G(t), we will presumably relate it to change in price (i.e. rate of return of public stock market) and change in earnings in public stock market. As explained in section 4, P/E data is not available for European public equity market and thus we assume rate of return of public stock market is the only determinant of G(t) except for CUM(t) for European private equity market. However, we are not certain of the influence of CUM(t) on G(t) and thus decide to include it in our model at first and then examine its effectiveness by statistical F test. To simplify the procedure, we will assume p=1 here and leave the selection of the optimal value of "p" to next step. Thus the new model will be constructed as:

$$\mathrm{Ln}\big(1+G(t)\big) = a(0) + a(1)\mathrm{CUM}(t) + b(0)\big(R(t)\big) + b(1)\big(R(t-1)\big) + c(0)\big(E(t)\big) + c(1)\big(E(t-1)\big) + \varepsilon(t) \qquad (12)$$

Based on this new model and updated data ranging from 1990 to 2006 for the US as well as the European markets, we run the ordinary linear regression and the estimated results are shown as follows:

Table 5.4: Estimation results for new model, G (t) of US private equity market from 1990 to 2006

R-squared: .6246		
Variable	Estimate	t-value
CONSTANT	.1111	2.191
CUM	-.1978	-1.489
R(t)	.4773	2.420
R(t-1)	.3439	1.964
E (t)	.01151	.1606
E (t-1)	-.02995	-.3556

Table 5.5: Estimation results for new model, G (t) of European private equity market 1990-2006

R-squared: .5345 F: 0		
Variable	Estimate	t-value
CONSTANT	.06507	1.628
CUM(t)	.001208	.01297
R(t)	.1866	1.327
R(t-1)	.4590	3.194

Based on the estimated values shown in table 5.4, we find that for US private equity market from 1990 to 2006, the stock market rate of return (contemporarily and with a lag of one year) is quite significant and has a positive effect on the growth rate G(t) while change in earnings and CUM(t) are insignificant in valuing G(t). The relationships among G(t), R(t), E(t) and CUM (t) can be evidently shown in figure 4.1 and figure 4.2. Therefore, we conclude that G(t) is mostly and positively dependent on the stock market rate of return. When stock market price goes up, net asset value of private equity market will go up as well.

For European private equity market during the period from 1990 to 2006, only the stock market rate of return (with one year lag) is significant in modeling G(t) and has a positive effect on it (see table 5.5). All the other terms are insignificant especially CUM(t)[3] with a relatively small t-stats value. The relationship can also be evidently shown in figure 4.3. In order to be more persuasive, we will prove the insignificance

[3] We also replace the term CUM(t) with the difference between CUM(t) and CUM(t-1) on the right hand side of our regression and find out the similar results, i.e. the insignificance of the difference of CUM(t) and CUM(t-1) for US and European markets(see Appendix 3).

of CUM(t) to G(t) of European private equity market by F test in the following part.

5.2.2 Without CUM(t) Term

Table 5.6: Estimation results for new model with cum(t) in US private equity market

R-squared: .6189 F: 1.784964		
Variable	Estimate	t-value
CONSTANT	.1158	2.598
CUM(t)	-.2014	-1.689
R(t)	.4362	2.878
R(t-1)	.3563	2.297

Table 5.7: Estimation results for new model without cum(t) in US private equity market

R-squared: .5621		
Variable	Estimate	t-value
CONSTANT	.05314	1.589
R(t)	.4725	2.911
R(t-1)	.3555	2.257

Table 5.8: Estimation results for new model without cum(t) European private equity market

R-squared: .5345		
Variable	Estimate	t-value
CONSTANT	.06543	2.335
R(t)	.1867	1.384
R(t-1)	.4589	3.330

Based on the estimation results (see table 5.7 and 5.8) of the model that relates G(t) with stock market rate of return only (concurrent and with a lag of one year), comparing table 5.6 with table 5.7, we find that R(t) and R(t-1) are still the two main indicators of G(t) in US private equity market either with or without CUM(t) term. Moreover, the calculated F value of 1.784964, which aims to test the necessity of adding CUM(t) term, is smaller than its critical value 2.554 of 95% confidence interval. Therefore, we accept the null hypothesis that the coefficient of CUM(t) term is 0 and conclude that CUM(t) is insignificant and unnecessary in modeling the growth rate of US private equity market, which is consistent with the information shown in figure 4.1 and 4.2.

In terms of European private equity market, comparing table 5.5 with table 5.8, we find that after taking CUM(t) out of the model , meaning G(t) only depends on the rate of return in stock market, the results doesn't show any difference in terms of parameters' significance. The rate of return of public stock market is still the only significant parameter in modeling G(t) for European private equity market. Furthermore, the calculated F value of 0 helps to verify the insignificance of CUM(t) in valuing G(t).

All in all, for US and European private equity markets, G(t) should be constructed as:

$$\mathrm{Ln}\big(1+\mathrm{G}(\mathrm{t})\big)=\mathrm{a}(0)+\mathrm{b}(0)\big(\mathrm{R}(\mathrm{t})\big)+\mathrm{b}(1)\big(\mathrm{R}(\mathrm{t}-1)\big)+\cdots+\mathrm{b}(\mathrm{P})\big(\mathrm{R}(\mathrm{t}-\mathrm{p})\big)+\varepsilon(\mathrm{t}) \quad (13)$$

The best value of "p" will be tested in the following part.

5.3 Selection of Optimal Value for P

5.3.1 Optimal Value for "p" in US Market

Based on the most appropriate model constructed for US market in last part (equation (13)), we select the optimal value for p in US market based on F test.

Table 5.9: p=1, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .5621		
Variable	Estimate	t-value
CONSTANT	.05314	1.589
R(t)	.4725	2.911
R(t-1)	.3555	2.257

Table 5.10: p=2, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .5901 F: 0.754286		
Variable	Estimate	t-value
CONSTANT	.04284	1.130
R(t)	.4674	2.603
R(t-1)	.3250	1.838
R(t-2)	.1446	.8664

Table 5.11: p=3, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .6049 F:0.373532		
Variable	Estimate	t-value
CONSTANT	.05619	1.205
R(t)	.4220	2.021
R(t-1)	.3643	1.766
R(t-2)	.1432	.7385
R(t-3)	-.09786	-.5104

Higher "p" value means there is longer lag between the public stock market rate of return and G(t). We stop our test of optimal value "p" at "3" for US market, as we find that the added parameters are all insignificant when p=2 and 3 and F value decreases as p increases as well. As observed from the above tables, when "p" is equal to 2, the value of F test 0.754286 is much smaller than its critical value 2.554 of 95% confidence interval. Therefore, we accept the null hypothesis that the coefficient b(2) of term R(t-2) is equal to 0, which means it is unnecessary to add an extra term R(t-2) to the model of G(t). In addition, if we extend "p" to 3, its lower value of F test 0.373532, which is also much smaller than its critical value 2.660 of 95% confidence interval, helps to show the uselessness of adding term R(t-3) as well. Moreover, the characteristics of insignificance of parameters R(t-2) and R(t-3) shown by their t-stats values help to further verify our assumption. Therefore, the optimal value of "p" for US private equity market is "1" and its optimal model of G(t) is:

$$\mathrm{Ln}\big(1 + \mathrm{G(t)}\big) = \mathrm{a}(0) + \mathrm{b}(0)\big(\mathrm{R(t)}\big) + \mathrm{b}(1)\big(\mathrm{R}(\mathrm{t}-1)\big) + \varepsilon(\mathrm{t}) \qquad (14)$$

5.3.2 Optimal Value for "p" in European Market

Table 5.12 p=1, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .5345		
Variable	Estimate	t-value
CONSTANT	.06543	2.335
R(t)	.1867	1.384
R(t-1)	.4589	3.330

Table 5.13 p=2, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .6878 F:5.273133		
Variable	Estimate	t-value
CONSTANT	0.4709	1.718
R(t)	.1913	1.586
R(t-1)	.4046	3.147
R(t-2)	.3074	2.490

Table 5.14: p=3, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .7865 F:7.12987		
Variable	Estimate	t-value
CONSTANT	.04860	1.838
R(t)	.1792	1.657
R(t-1)	.4317	4.043
R(t-2)	.2214	2.069
R(t-3)	.1870	1.739

Table 5.15 p=4, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .8119 F:1.008		
Variable	Estimate	t-value
CONSTANT	.03147	.9482
R(t)	.1894	1.533
R(t-1)	.5012	3.771
R(t-2)	.2097	1.832
R(t-3)	.1972	1.697
R(t-4)	.1154	.8091

Table 5.16: p=5, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .8627 F: 1.916996		
Variable	Estimate	t-value
CONSTANT	.05856	1.343
R(t)	.1453	1.026
R(t-1)	.5083	3.540
R(t-2)	.1007	.6218
R(t-3)	.2064	1.754
R(t-4)	.1916	1.246
R(t-5)	-.2527	-1.298

For European private equity market, we find that when p=2 and 3, the calculated values of F test are much bigger than their critical values of 95% confidence interval: 2.554 and 2.660 respectively, which means that it is necessary to add term R(t-2) and R(t-3) to model G(t). In addition, higher R^2 value of 0.7865 shows the model's good fitness when p=3 and it also helps to confirm our assumption of ignoring the influence of change in earnings due to unavailable data mentioned in section 4. Therefore, for European private equity market, the optimal value of p is "3" and G(t) can be best modeled as:

$$\mathrm{Ln}\big(1+\mathrm{G(t)}\big) \\ = a(0) + b(0)R(t) + b(1)\big(R(t-1)\big) + b(2)\big(R(t-2)\big) + b(3)\big(R(t-3)\big) \\ + \varepsilon(t) \qquad (15)$$

However, based on our understanding of the valuation process when NAVs are reported, we expect lags beyond one year are not significant. Therefore, finding significant lags greater than 1 (p=3) is not what we expected in advance.

5.4 Comparison between US and European Markets (1990-2006)

According to table 5.9 and 5.14 (estimation results for their optimal models), we find that the influences of the stock market rate of return on the growth rate G(t) of these two markets are quite similar except that European private equity market are more reliant on the lagged stock market rate of return than US market, which is shown by optimal value p=3 for European market while p=1 for US market. Moreover, it shows insignificant dependence of the growth rate G (t) on the cumulative contribution rate CUM (t) for both markets from a long run[4].

In short, we conclude that the effects of the underlying factors on the growth rate G (t) in both US and European private equity markets are quite similar. Therefore, we

[4] To be more convictive, we run our regression based on quarterly data as well. The results are quite consistent with the yearly data in terms of parameters' significance and optimal time lag for both US and European markets (see appendix 4).

expect similar trends of growth rate in these two markets and that has been confirmed by the plotted figures (see appendix 1).

6. Conclusion

In this paper, we first presented a model by Takahashi and Alexander (2001) describing the growth rate G(t) of private equity market in terms of contributions, distributions and net asset value. Later on, Hoek (2007) modified G(t) to be stochastic and dependent on the cumulative contribution rate CUM(t) and excess stock market return (contemporarily and with some lags). We agree with Hoek to the point that there are some valuation lags of private equity market compared with public equity market, but we think the growth rate of price and earnings should be more closely related to G(t) and our concern for the significance of term CUM(t) has been resolved through statistical F test.

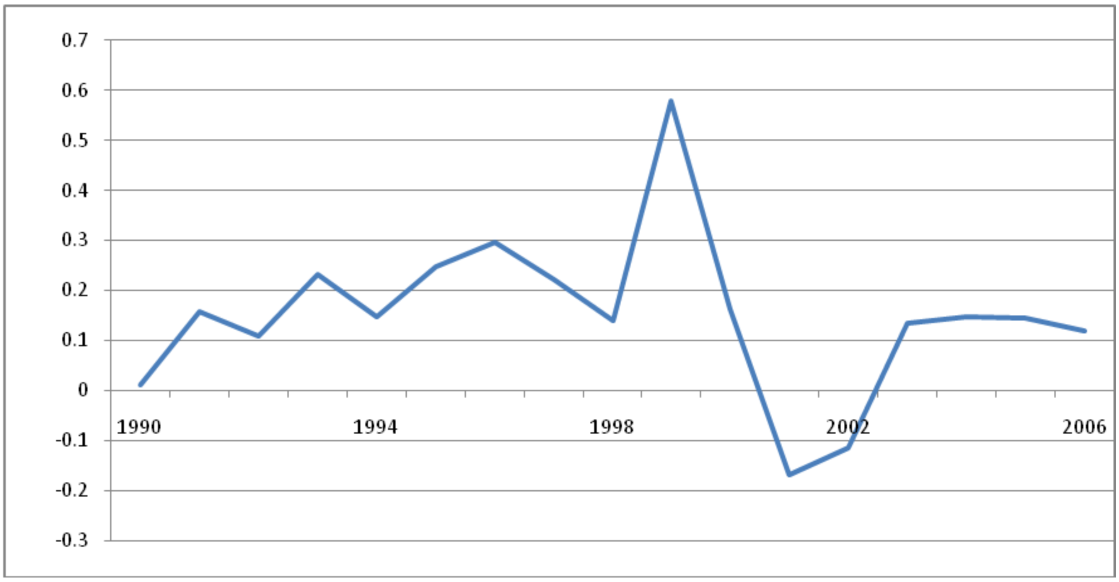
By running an ordinary linear regression and comparing t-stats values of the estimated coefficients with the critical value of 95% confidence interval, we conclude that public stock market rate of return is significant in valuing the growth rate of private equity market for both US and European markets; while the other terms (CUM(t), change in earnings) are insignificant in modeling the growth rate in both markets. Moreover, the optimal values of “p” for US and European markets are selected: p= “1” for US market and p= “3” for European market.

Appendices

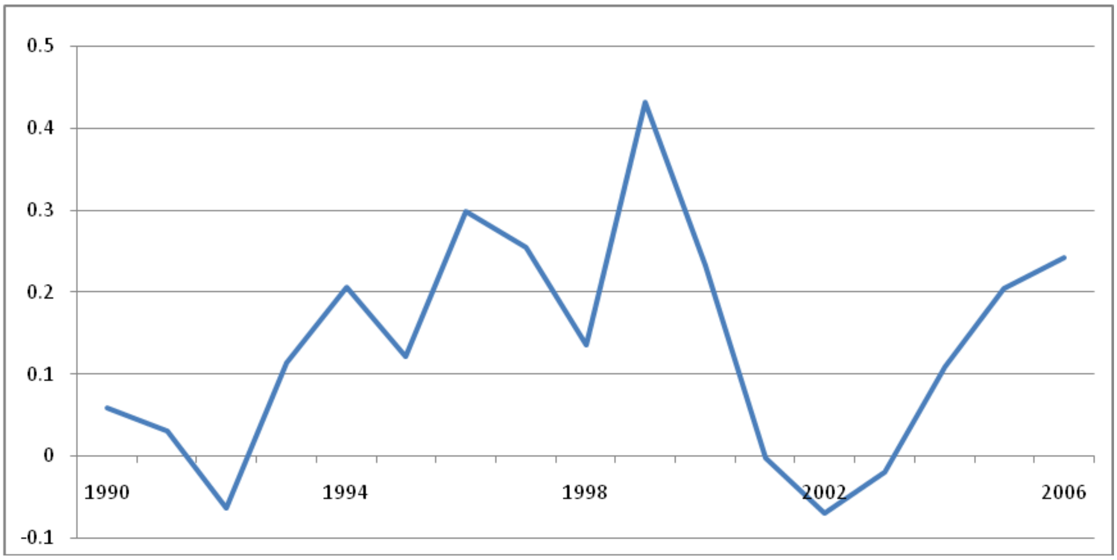
Appendix 1

G (t)

G (t) of US Private Equity Investment from 1990 to 2006



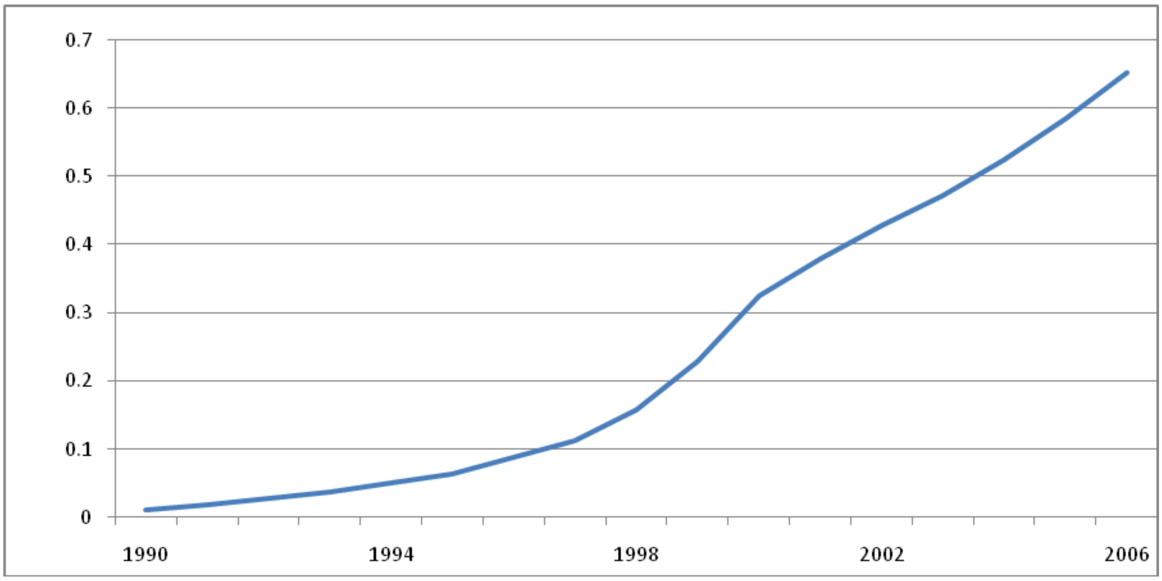
G (t) of European Private Equity Investment from 1990 to 2006



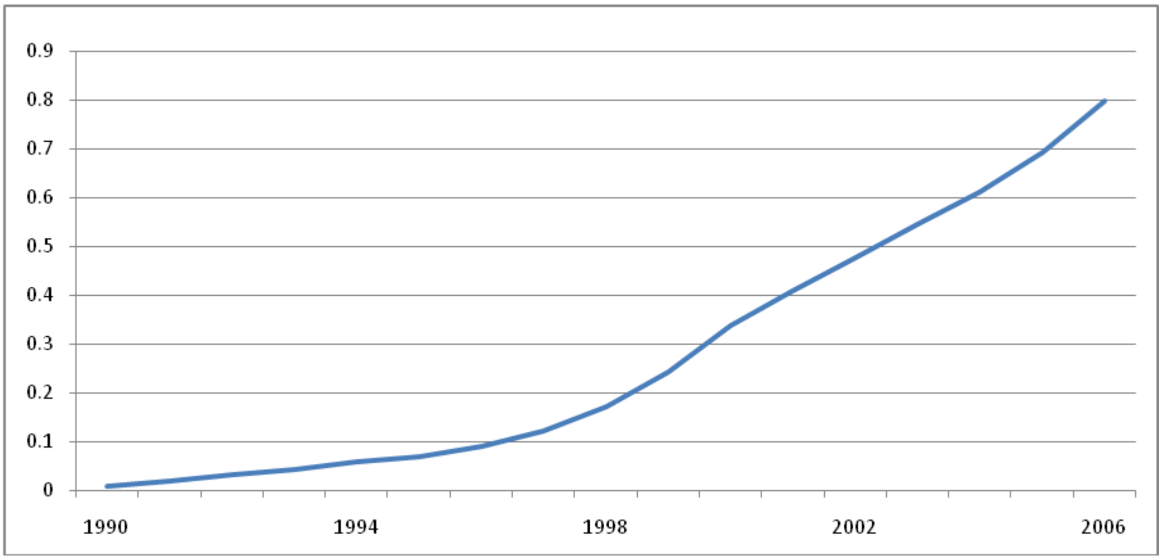
Appendix 2

Cumulative contribution rate

Cumulative contribution rate of US market from 1990 to 2006



Cumulative contribution rate of European market from 1990 to 2006



Appendix 3

Table 1: Estimation results with cum(t)-cum(t-1), G(t) of US market from 1990-2006

R-squared: .5289		
Variable	Estimate	t-value
CONSTANT	.06790	1.172
CUM(t)-CUM(t-1)	-.1321	-.1166
R(t)	.4277	2.486
R(t-1)	.3772	2.147

Table 2: Estimation results with cum(t)-cum(t-1), G(t) of European market from 1990-2006

R-squared: .1771		
Variable	Estimate	t-value
CONSTANT	.01239	1.490
CUM(t)-CUM(t-1)	.9160	1.723
R(t)	.1600	2.844
R(t-1)	.1201	2.120

Notes: The above two tables show the insignificance of term CUM(t)-CUM(t-1) for both US and European markets.

Appendix 4

Results for quarterly data

Table 3: Estimation results, G(t) of US private equity market from 1990 to 2006

R-squared: .5160		
Variable	Estimate	t-value
CONSTANT	.02147	2.898
CUM(t)	-.02450	-1.210
R(t)	.4054	6.744
R(t-1)	.1909	3.109
E(t)	.09284	1.658
E(t-1)	.05329	.9686

Notes: The above table shows the insignificance of CUM(t) and change in earnings based on quarterly data.

Table 4: Estimation results for new model without cum(t) in US private equity market

R-squared: .4515		
Variable	Estimate	t-value
CONSTANT	.01791	3.513
R(t)	.4097	6.658
R(t-1)	.2148	3.503

Table 5: Estimation results for new model with cum(t) in US private equity market

R-squared: .4567 F: 0.612676		
Variable	Estimate	t-value
CONSTANT	.02239	2.903
CUM(t)	-.01620	-.7761
R(t)	.4026	6.451
R(t-1)	.2075	3.334

Notes: The low F value 0.612676 helps to confirm the insignificance of CUM(t) in modeling G(t) for US market.

Table 6: Estimation results for new model without cum(t) in European private equity market

R-squared: .1384		
Variable	Estimate	t-value
CONSTANT	.02361	4.495
R(t)	.1604	2.808
R(t-1)	.1193	2.074

Table 7: Estimation results for new model with cum(t) in European private equity market

R-squared: .1389 F: 0.063809		
Variable	Estimate	t-value
CONSTANT	.02463	3.341
CUM(t)	-.003867	-.1984
R(t)	.1611	2.793
R(t-1)	.1197	2.065

Notes: The low F value 0.063809 helps to confirm the insignificance of CUM(t) in modeling G(t) for European market based on quarterly data.

Table 8: p=1, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .4515		
Variable	Estimate	t-value
CONSTANT	.01791	3.513
R(t)	.4097	6.658
R(t-1)	.2148	3.503

Table 9: p=2, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .4948 F:5.565217		
Variable	Estimate	t-value
CONSTANT	.01509	2.915
R(t)	.3934	6.508
R(t-1)	.2252	3.735
R(t-2)	.1390	2.306

Table 10: p=3, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .5388 F:6.134618		
Variable	Estimate	t-value
CONSTANT	.01143	2.181
R(t)	.4008	6.519
R(t-1)	.2082	3.527
R(t-2)	.1557	2.646
R(t-3)	.1173	1.980

Table 11: p=4, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .5715 F:6.820357		
Variable	Estimate	t-value
CONSTANT	.01121	2.148
R(t)	. .3976	6.655
R(t-1)	.1654	2.776
R(t-2)	.1424	2.490
R(t-3)	.1215	2.109
R(t-4)	.1092	1.904

Table 12: p=5, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .5862 F:2.072984		
Variable	Estimate	t-value
CONSTANT	.01066	1.984
R(t)	.4240	6.764
R(t-1)	.1663	2.767
R(t-2)	.1148	1.893
R(t-3)	.1104	1.894
R(t-4)	.1085	1.873
R(t-5)	.05969	1.024

Table 13: p=6, Estimation results, G (t) of US private equity market from 1990 to 2006

R-squared: .5915 F: 0.793177		
Variable	Estimate	t-value
CONSTANT	.01156	2.081
R(t)	.4344	6.716
R(t-1)	.1598	2.511
R(t-2)	.1183	1.916
R(t-3)	.1148	1.847
R(t-4)	.1150	1.945
R(t-5)	.05592	.9419
R(t-6)	-.05153	-.8571

Notes: According to table 8 to table 13, p=4 (four quarters lag, i.e. one year lag) is the optimal value for US market, which is consistent with the result (p=1, one year lag) of yearly data.

Table 14: p=1, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .1384		
Variable	Estimate	t-value
CONSTANT	.02361	4.495
R(t)	.1604	2.808
R(t-1)	.1193	2.074

Table 15: p=2, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: 1548 F:1.300813		
Variable	Estimate	t-value
CONSTANT	.02207	4.034
R(t)	.1507	2.589
R(t-1)	.1312	2.231
R(t-2)	.06533	1.116

Table 16: p=3, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .1784 F: 2.030227		
Variable	Estimate	t-value
CONSTANT	.02029	3.568
R(t)	.1459	2.403
R(t-1)	.1183	1.986
R(t-2)	.08067	1.354
R(t-3)	.08216	1.399

Table 17: p=4, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .2268 F:4.183227		
Variable	Estimate	t-value
CONSTANT	.01859	3.227
R(t)	.1335	2.223
R(t-1)	.1039	1.704
R(t-2)	.06586	1.114
R(t-3)	.1044	1.774
R(t-4)	.1159	1.985

Table 18: p=5, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .2621 F:4.010083		
Variable	Estimate	t-value
CONSTANT	.01773	3.032
R(t)	.1512	2.530
R(t-1)	.09756	1.619
R(t-2)	.03695	.6074
R(t-3)	.09292	1.587
R(t-4)	.1316	2.254
R(t-5)	.09117	1.572

Table 19: p=6, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .2851 F: 1.825483		
Variable	Estimate	t-value
CONSTANT	.01591	2.643
R(t)	.1484	2.406
R(t-1)	.1016	1.667
R(t-2)	.03145	.5147
R(t-3)	.1022	1.673
R(t-4)	.1224	2.074
R(t-5)	.1015	1.726
R(t-6)	.05803	.9778

Table 20: p=7, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .3176 F: 2.580779		
Variable	Estimate	t-value
CONSTANT	.1403	2.289
R(t)	.1499	2.402
R(t-1)	.08822	1.414
R(t-2)	.04033	.6588
R(t-3)	.09636	1.582
R(t-4)	.1216	1.977
R(t-5)	.08966	1.518
R(t-6)	.07168	1.200
R(t-7)	.08898	1.498

Table 21: p=8, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .3977 F:8.934409		
Variable	Estimate	t-value
CONSTANT	.01388	2.364
R(t)	.1079	1.772
R(t-1)	.1109	1.858
R(t-2)	.004232	.07174
R(t-3)	.1168	2.019
R(t-4)	.1223	2.108
R(t-5)	.04424	.7564
R(t-6)	.06797	1.202
R(t-7)	.09949	1.760
R(t-8)	.1410	2.429

Table 22: p=9, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .4021 F: 1.120274		
Variable	Estimate	t-value
CONSTANT	.01360	2.254
R(t)	.1109	1.799
R(t-1)	.09653	1.559
R(t-2)	.01088	.1799
R(t-3)	.1037	1.733
R(t-4)	.1310	2.213
R(t-5)	.04434	.7503
R(t-6)	.05428	.9078
R(t-7)	.09610	1.673
R(t-8)	.1449	2.458
R(t-9)	.05714	.9545

Table 23: p=10, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .4791 F: 7.09841		
Variable	Estimate	t-value
CONSTANT	.01089	1.850
R(t)	.1546	2.521
R(t-1)	.1086	1.833
R(t-2)	-.02831	-.4741
R(t-3)	.1053	1.820
R(t-4)	.1015	1.744
R(t-5)	.07084	1.235
R(t-6)	.04481	.7842
R(t-7)	.08121	1.417
R(t-8)	.1204	2.113
R(t-9)	.07451	1.296
R(t-10)	.1543	2.596

Table 24: p=11, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .5005 F: 6.799098		
Variable	Estimate	t-value
CONSTANT	.01376	2.379
R(t)	.1341	2.267
R(t-1)	.1075	1.819
R(t-2)	-.03214	-.5627
R(t-3)	.1081	1.887
R(t-4)	.1275	2.254
R(t-5)	.05569	.9869
R(t-6)	.04831	.8713
R(t-7)	.08741	1.594
R(t-8)	.09284	1.642
R(t-9)	0.8653	1.558
R(t-10)	.1379	2.400
R(t-11)	-.03311	-.5811

Table 25: p=12, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .5062 F: 2.028737		
Variable	Estimate	t-value
CONSTANT	.01361	2.273
R(t)	.1278	2.135
R(t-1)	.1046	1.747
R(t-2)	-.01035	-.1734
R(t-3)	.1125	1.957
R(t-4)	.1088	1.847
R(t-5)	.06548	1.143
R(t-6)	.02922	.5111
R(t-7)	.1019	1.822
R(t-8)	.09258	1.629
R(t-9)	.06354	1.091
R(t-10)	.1302	2.246
R(t-11)	-.03039	-.5260
R(t-12)	.06093	1.063

Table 26: p=13, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .5639 F: 5.651095		
Variable	Estimate	t-value
CONSTANT	.009993	1.674
R(t)	.1349	2.339
R(t-1)	.1248	2.141
R(t-2)	.009198	.1583
R(t-3)	.1291	2.243
R(t-4)	.1185	2.074
R(t-5)	.04270	.7413
R(t-6)	1757	.3143
R(t-7)	.09296	1.655
R(t-8)	.09825	1.770
R(t-9)	.05816	1.035
R(t-10)	.1412	2.420
R(t-11)	-.04232	-.7551
R(t-12)	.07963	1.429
R(t-13)	.09188	1.563

Table 27: p=14, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .5875 F: 2.535576		
Variable	Estimate	t-value
CONSTANT	.009938	1.595
R(t)	.1480	2.509
R(t-1)	.1279	2.195
R(t-2)	.001498	.02564
R(t-3)	.1228	2.121
R(t-4)	.1385	2.344
R(t-5)	.05116	.8841
R(t-6)	-.001699	-.02906
R(t-7)	.1052	1.857
R(t-8)	.07196	1.229
R(t-9)	.0750	1.308
R(t-10)	.1503	2.576
R(t-11)	-.06795	-1.168
R(t-12)	.07311	1.307
R(t-13)	.09384	1.576
R(t-14)	.05714	.9393

Table 28: p=15, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .5981 F:1.081958		
Variable	Estimate	t-value
CONSTANT	.007721	1.154
R(t)	.1517	2.515
R(t-1)	.1462	2.355
R(t-2)	.008000	.1344
R(t-3)	.1289	2.171
R(t-4)	.1504	2.455
R(t-5)	.06425	1.055
R(t-6)	.007343	.1225
R(t-7)	.08216	1.314
R(t-8)	.06789	1.134
R(t-9)	.06469	1.077
R(t-10)	.1610	2.643
R(t-11)	-.06573	-1.109
R(t-12)	.06724	1.129
R(t-13)	.09493	1.574
R(t-14)	.07387	1.157
R(t-15)	.06443	.9595

Table 29: p=16, Estimation results, G (t) of European private equity market from 1990 to 2006

R-squared: .6025 F:0.4756		
Variable	Estimate	t-value
CONSTANT	.008440	1.187
R(t)	.1531	2.468
R(t-1)	.1428	2.237
R(t-2)	-.003514	-.05491
R(t-3)	.1264	2.071
R(t-4)	.1532	2.414
R(t-5)	.06174	.9734
R(t-6)	-.003250	-.05126
R(t-7)	.07408	1.134
R(t-8)	.08294	1.260
R(t-9)	.06404	1.033
R(t-10)	.1721	2.670
R(t-11)	-.07603	-1.217
R(t-12)	.06229	1.016
R(t-13)	.1092	1.666
R(t-14)	.07519	1.152
R(t-15)	.05978	.8560
R(t-16)	-.04092	-.5856

Notes: Based on table 14 to table 29, we find that for p bigger than 14, the F values are decreasing steadily and are all smaller than their critical values of 95% confidence interval, therefore, we believe 14 is the best value for p (three and a half years lag), which is quite close to the result getting from yearly data, which is p=3 meaning three years lag.

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