INVESTIGATING TRANSITION MATRICES ON U.S. RESIDENTIAL BACKED MORTGAGE SECUTIRES

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Abstract

The purpose of our research is to expand on the work of Kavvathas (2001) that studies credit rating transition probabilities for corporate bonds. This paper, for the period of 1991-2007 will be focused on rating transition matrices for US residential mortgage-backed securities (RMBS). In particular, we extend their techniques to a different data set and more recent time period by estimating credit rating transition matrices through the cohort method and the time-homogeneous duration method. In addition, we apply an alternative approach to calculate the average transition matrices.

Keywords: Credit risk; rating transitions; transition matrices

Dedication

We wish to dedicate this project to our families for their supports.

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Table of Contents

List of Figures

List of Tables

1. Introduction

The haze of global economy has not completely cleared off since the current explosion of sub-prime mortgage in the United States. Back to two years ago, the U.S. housing market started to experience a significant downturn. According to the report from Standard & Poor's (2007), a large number of U.S. Residential Mortgage-Backed Securities (RMBS) were downgraded during the last half year of 2007. Many researchers have investigated rating transitions since they are the core of credit risk management in financial market. The purpose of our research is to expand on the work of Kavvathas (2000) that studies credit rating transition probabilities for corporate bonds. This paper will be focused on rating transition matrices for the U.S. RMBS. In particular, we extend their techniques to a different data set and more recent time period by illustrating two techniques for estimating credit rating transition matrices – the cohort method and the time-homogeneous duration method. This paper shows that the duration approach yields greater default probabilities for high credit ratings and low credit rating compared to the cohort approach. In addition, we find that the duration method generates smaller downgrade and upgrade probabilities.

The outline of our paper is as follows. In Section 2, we review some academic literature on credit transition matrices. Section 3 describes the data and methodology. The results are presented and discussed in Section 4. Finally, we will summarize our work and draw the conclusion in the Section 5.

2. Literature Review

There have been many researches discussing various techniques for estimating transition matrices.

Lando and Skodeberg (2002) compare transition matrices through the cohort method and the non-parametric duration method using the data from 1981 to 1997. They find higher default probabilities for CCC rating category in the duration case. They also confirm that the default probabilities are non-zero for the AAA rating category using either the maximum likelihood estimator in the time-homogeneous case or the time-inhomogenous case. Furthermore, they find significant downgrade momentum (except for the categories of BB, CCC+ and CCC) which increases the downgrade intensity by a factor of three.

Kavvathas (2000) conducts many extensive empirical investigations into credit rating transition probabilities. Kavvathas concentrates on US corporate bonds and uses S&P data from 1981 to 1998. They examine term structure and equity factors, reporting that an increase in nominal short and long and real interest rates, low equity returns and high equity return volatility are usually associated with high downgrade probabilities. In addition, Kavvathas presents many extensions to capture rating momentum and economic cycles. They discover that macroeconomic variables and industry effects are significant to rating intensities.

In a more recent study, Jobst and Gikes (2005) find that duration approach significantly increases default probabilities for investment grade. They also confirm that the default probabilities for non-investment grade ratings are generally higher compared to the cohort method.

Currently, most studies are investigated on the ratings for corporate bonds.

3. Data and Methodology

3.1 The Data

In the paper, we use the data set from Standard & Poor's CreditPro. The raw data covers 18 years of rating history from 1990 to 2007, totaling 51,258 U.S. RMBS and 28,838 rating transitions. However, we observe sample period from 1991 to 2007, omitting 401 observations of the first year data set. The reason is that the first year data ignores default securities by only recording the active securities. Such biased selection can substantially affect the results of default probabilities. The ratings are listed based on 23 categories including NR. The highest rating is AAA and the lowest is C. Each of the categories of AA, A, BBB, BB, B and CCC then contains two modifiers "+" or "-". Finally, there are a default category denoted D and a not rated category denoted NR. For our results, we use full categories for simplicity.

3.2 Methodology

Broadly there are the two approaches for modeling rating transitions, the cohort method and duration method.

3.2.1 The Cohort Method

The most common and straight-forward approach to estimate the transition matrices is the cohort method. It observes the starting states and the ending states for each security. The probabilities of rating transitions are calculated through dividing the number of transitions by the number of securities from the original ratings. We denote $n_i(t)$ the total number of securities in rating i at time t, and $n_{ij}(0,t)$ the number of transitions of the securities from rating i at time 0 to rating j at time t. The maximum likelihood estimator of q_{ij} is

$$\hat{q}_{ij} = \frac{n_{ij}(0,t)}{n_i(0)}$$

The intensity matrices $\hat{\Lambda}$ can be obtained through

$$\hat{Q} = e^{\hat{\Lambda}}$$

The cohort method conceals a significant drawback. It captures only the terminal information in each year, but ignores the intra-year information which can be important for transition matrices.

3.2.2 The Duration Method

To avoid the cohort method problems, Lando and Skodeberg (2002) propose duration method to estimate the intensity matrices $\hat{\Lambda}$, which require the knowledge of the precise transition time during the rating years. The duration method can be based on two assumptions: time-homogeneity and time-inhomogeneity. The assumption of time homogeneity implies that the intensity matrices are constant over time. The estimator of time-homogeneous transition intensities is

$$\hat{\lambda}_{ij} = \frac{m_{ij}(T)}{\int_0^T n_i(s)ds}$$

where $m_{ij}(T)$ is the total number of transitions from rating i to rating j, $i \neq j$ over the interval $[0, T]$. See Appendix A for the details. The $m_{ij}(T) \geq n_{ij}(T)$, because $m_{ij}(T)$ includes the transitions entering rating i during the interval. The denominator $\int_0^T n_i(s)ds$ is the total amount of time that securities stay in the rating i.

There is a much more complex estimation based on the assumption of time-inhomogeneous transition intensity. By using the Nelson-Aalen estimator, Lando and Skodeberg (2000) propose

$$\hat{A}_{ij}(t) = \sum_{T_{ij}(k) \leq t} \frac{1}{n_i(T_{ij}(k)-)}$$

where $T_{ij}(k)$ is the time of the k-th transition from rating i to rating j, and $n_i(t-)$ is the number of i rated securities, evaluated exactly before time t. And the formula $\hat{A}_{ij}(t) = \int_0^t \lambda_{ij}(s)ds$ applies to convert the Nelson-Aalen estimator to intensity estimators. The probability density of the transition time from rating i to rating j for each rating state:

$$f(t_{ij}) = \lambda_{ij} e^{-(t_{ij}-t_{ij}^{'})\sum_{k \neq i}^{J} \lambda_{ik}}$$

Lando and Skodeberg (2002) reveal that the time-inhomogeneous estimator is not much different to the time-homogeneous estimator. Therefore, we restrict our investigation to cohort method and time-homogeneous duration method.

4. Results and Discussion

4.1 Simple Average Calculation

We obtain the average transition matrix by simply calculating the mean of yearly transition matrices.

$$\hat{q}_{i,j} = \overline{q_{i,j,k}}$$

4.1.1 Simple Average (1991-2007) one-year transition matrices: NR-adjusted

To begin our study, we first calculate transition matrices on a NR-adjusted base. Table 1 reports the average one year transition matrix by the cohort method. Table 2 presents the one year transition matrix by the time-homogeneous duration method.

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D
AAA	99.536	0.436	0.013	0.012	0.000	0.000	0.000	0.000	0.000	0.004
AA	6.481	93.156	0.225	0.030	0.016	0.055	0.035	0.000	0.000	0.002
A	1.821	3.955	91.687	1.934	0.303	0.022	0.253	0.000	0.000	0.024
BBB	0.182	1.766	2.618	93.479	0.968	0.377	0.208	0.115	0.000	0.288
BB	0.034	0.042	1.447	3.754	90.909	1.090	1.253	0.439	0.000	1.033
B	0.130	0.042	0.029	1.102	3.207	91.701	1.691	0.421	0.000	1.678
CCC	0.000	0.000	0.000	0.000	0.000	0.000	41.539	11.985	1.861	44.615
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	57.814	0.000	42.186
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Table 1: Cohort method: Simple Average 1-year (NR adjusted) transition matrix (1991-2007)

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D
AAA	99.671	0.301	0.007	0.013	0.000	0.000	0.000	0.000	0.000	0.008
AA	0.119	99.738	0.049	0.048	0.006	0.006	0.025	0.000	0.000	0.010
A	0.364	0.137	99.134	0.147	0.107	0.034	0.053	0.000	0.000	0.024
BBB	0.061	0.087	0.139	99.310	0.123	0.065	0.088	0.022	0.000	0.106
BB	0.071	0.053	0.159	0.176	98.213	0.333	0.401	0.193	0.000	0.401
B	0.276	0.029	0.459	0.184	0.248	97.268	0.726	0.320	0.000	0.489
CCC	0.000	0.000	0.000	0.000	0.000	0.000	36.279	32.305	1.699	29.717
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	53.938	0.000	46.062
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Table 2: Duration method: Simple Average 1-year (NR adjusted) transition matrix (1991-2007)

By comparing the two methods, several major differences stand out:

First of all, the default probabilities of investment grade are non-zero under both cases. However, the duration method generates greater default probabilities for high credit ratings (AAA, AA and A) compared to the cohort approach.

Secondly, the default probabilities for low credit ratings are smaller for the duration method. For example, the CCC default probability of cohort method is1.5 times as big as that of the duration method. Note that other low credit rating such as CC is not indicative because there are only 54 CC-rating securities out of 51,258.

Moreover, both matrices are diagonally dominant – in other words, there are high probabilities that securities maintain their ratings. But the probabilities of no migration are mostly higher for the duration method. We also find that the duration approach yields smaller downgrade probabilities (the entries right to the diagonal) and upgrade probabilities (the entries left to the diagonal). Therefore, as the cohort method ignores the intra-year transitions it may overestimate both upgrade and downgrade probabilities.

4.1.2 Simple Average (1991-2007) one-year transition matrices: NR-included

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D	NR
AAA	86.764	0.428	0.012	0.011	0.000	0.000	0.000	0.000	0.000	0.003	12.782
AA	5.955	87.636	0.221	0.029	0.016	0.053	0.032	0.000	0.000	0.002	6.056
A	1.748	3.771	88.466	1.914	0.294	0.021	0.253	0.000	0.000	0.023	3.511
BBB	0.173	1.685	2.507	90.482	0.951	0.364	0.205	0.114	0.000	0.276	3.243
BB	0.030	0.038	1.371	3.592	88.766	1.076	1.237	0.436	0.000	1.000	2.455
B	0.127	0.036	0.026	1.055	3.094	89.387	1.653	0.416	0.000	1.629	2.576
CCC	0.000	0.000	0.000	0.000	0.000	0.000	39.796	11.897	1.772	38.230	8.305
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	43.076	0.000	29.940	26.984
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 3: Cohort method: Simple Average 1-year (NR included) transition matrix (1991-2007)

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D	NR
AAA	99.514	0.300	0.007	0.013	0.000	0.000	0.000	0.000	0.000	0.008	0.158
AA	0.119	99.606	0.049	0.048	0.006	0.006	0.025	0.000	0.000	0.010	0.132
A	0.363	0.137	98.987	0.147	0.106	0.034	0.052	0.000	0.000	0.024	0.148
BBB	0.061	0.087	0.138	99.174	0.123	0.065	0.088	0.022	0.000	0.106	0.137
BB	0.071	0.053	0.158	0.176	97.924	0.332	0.400	0.192	0.000	0.400	0.295
B	0.275	0.029	0.457	0.184	0.247	96.940	0.724	0.319	0.000	0.487	0.336
CCC	0.000	0.000	0.000	0.000	0.000	0.000	33.959	30.239	1.591	27.816	6.395
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	11.270	0.000	9.625	79.105
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 4: Duration method: Simple Average 1-year (NR included) transition matrix (1991-2007)

We now include the NR category in the cohort and duration methods. Table 3 and Table 4 report the results when the transitions from and to NR are observed. We find that including the NR transition has little influence on the results of the duration method. For the cohort case, the default probabilities and the probabilities of no migration become smaller. Our explanation is that the cohort method ignores the amount of time that securities spend in each rating state before they migrate to NR. Hence, the less efficient cohort method underestimates the default probabilities and probabilities of no migration.

4.2 Weighted Average Calculation

In previous results, we equally averaged the probability matrices. However, Table 5 shows that the number of data is unevenly distributed as most securities receive their ratings after the year 2000. Thus, we count the numbers of active securities in each year and calculate their weights by

$$w_k = \frac{N_k}{\sum_{i=1}^{k} N_i}$$

The weights of active securities are shown in the Figure 1.

Initial Rating Year	Numbers of securities	Weights in Total
1991	455	0.89%
1992	619	1.22%
1993	567	1.11%
1994	502	0.99%
1995	370	0.73%
1996	305	0.60%
1997	521	1.02%
1998	776	1.53%
1999	518	1.02%
2000	573	1.13%
2001	1466	2.88%
2002	3004	5.91%
2003	4927	9.69%
2004	7364	14.48%
2005	10464	20.58%
2006	11085	21.80%
2007	7341	14.43%
Total	50857	100.00%

Table 5: US RMBS 1991-2007: Numbers of Securities Entering Rating

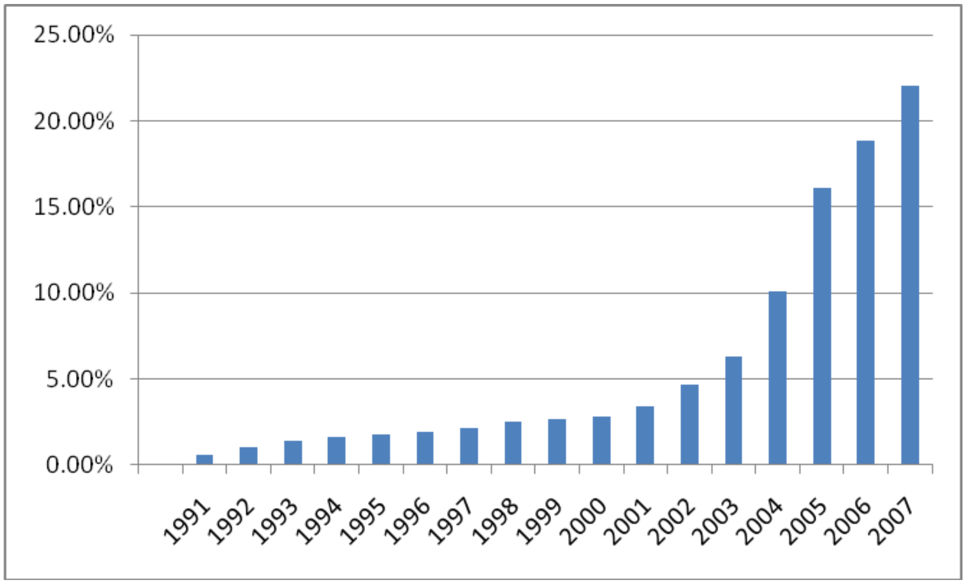

Figure 1: US RMBS 1991-2007: Weights of Active Securities

Furthermore, the comparison of transition matrices for 2002 and 2006 indicates that transition probabilities are not constant over the rating histories (Appendix B). Moreover, the variation by calculating average singular value metrics confirms that transition probabilities fluctuate over our sample period (Appendix C).

Therefore, we average the probability matrices based on the weights of data over 17 years. In particular, we use the numbers of rated securities as the input data for calculating the weights. The average transition probabilities matrix is modified as follows

$$\hat{q}_{i,j} = \sum_{k=1}^{l} q_{i,j,k} * w_k$$

4.2.1 Weighted Average (1991-2007) one-year transition matrices: NR-adjusted

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D
AAA	99.885	0.094	0.004	0.004	0.000	0.000	0.000	0.000	0.000	0.013
AA	4.976	94.901	0.068	0.016	0.004	0.015	0.014	0.000	0.000	0.006
A	1.254	3.597	94.355	0.460	0.152	0.039	0.080	0.000	0.000	0.062
BBB	0.144	1.464	2.134	94.655	0.591	0.331	0.173	0.040	0.000	0.468
BB	0.058	0.081	1.414	3.565	91.384	0.699	0.764	0.153	0.000	1.881
B	0.059	0.065	0.056	0.798	2.454	93.658	1.013	0.169	0.000	1.728
CCC	0.000	0.000	0.000	0.000	0.000	0.000	25.288	4.169	0.499	70.045
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	67.970	0.000	32.030
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Table 6: Cohort method: Weighted Average 1-year (NR adjusted) transition matrix (1991-2007)

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D
AAA	99.892	0.086	0.001	0.003	0.000	0.000	0.000	0.000	0.000	0.018
AA	0.053	99.891	0.020	0.014	0.001	0.001	0.004	0.000	0.000	0.017
A	0.065	0.055	99.657	0.040	0.075	0.038	0.047	0.000	0.000	0.023
BBB	0.022	0.048	0.053	99.637	0.048	0.044	0.071	0.005	0.000	0.072
BB	0.048	0.051	0.100	0.107	99.131	0.132	0.170	0.037	0.000	0.225
B	0.128	0.021	0.513	0.095	0.148	98.457	0.278	0.071	0.000	0.288
CCC	0.000	0.000	0.000	0.000	0.000	0.000	43.786	18.471	0.655	37.088
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	52.941	0.000	47.059
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Table 7: Duration method: Weighted Average 1-year (NR adjusted) transition matrix (1991-2007)

4.2.2 Weighted Average (1991-2007) one-year transition matrices: NR-included

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D	NR
AAA	80.069	0.086	0.004	0.004	0.000	0.000	0.000	0.000	0.000	0.011	19.827
AA	4.840	88.945	0.064	0.015	0.004	0.015	0.013	0.000	0.000	0.006	6.099
A	1.258	3.601	89.977	0.442	0.146	0.039	0.080	0.000	0.000	0.058	4.399
BBB	0.143	1.464	2.116	89.502	0.569	0.317	0.167	0.041	0.000	0.425	5.257
BB	0.053	0.074	1.411	3.617	87.453	0.689	0.741	0.155	0.000	1.727	4.082
B	0.055	0.057	0.051	0.793	2.457	90.024	0.998	0.165	0.000	1.660	3.741
CCC	0.000	0.000	0.000	0.000	0.000	0.000	24.051	4.167	0.468	59.486	11.827
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	31.831	0.000	16.614	51.555
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 8: Cohort method: Weighted Average 1-year (NR included) transition matrix (1991-2007)

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D	NR
AAA	99.791	0.086	0.001	0.003	0.000	0.000	0.000	0.000	0.000	0.017	0.102
AA	0.055	99.820	0.020	0.014	0.001	0.001	0.005	0.000	0.000	0.016	0.068
A	0.066	0.058	99.578	0.041	0.077	0.038	0.048	0.000	0.000	0.023	0.071
BBB	0.024	0.050	0.055	99.562	0.049	0.045	0.071	0.005	0.000	0.074	0.065
BB	0.050	0.052	0.104	0.111	98.939	0.137	0.174	0.039	0.000	0.229	0.165
B	0.134	0.022	0.516	0.100	0.153	98.209	0.283	0.074	0.000	0.292	0.218
CCC	0.000	0.000	0.000	0.000	0.000	0.000	37.812	15.686	0.542	31.318	14.641
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	8.538	0.000	7.576	83.886
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 9: Duration method: Weighted Average 1-year (NR included) transition matrix (1991-2007)

By calculating the transition matrices on a weighted average base, we find the default probabilities for investment grade ratings are slightly higher. Generally, the results of Table 5 to Table 8 show that the differences between the duration and cohort method on the weighted average base are similar to the ones on the simple average base.

According to the Central Limit Theorem in statistics, the larger the data observed, the more precise information will be extracted from the samples. Given that US RMBS securities are unevenly distributed, the securities before 2000 are over-weighted on the simple average base because of small sample size. Thus, we prefer to adopt weighted average calculation for more accuracy.

5. Conclusion

In this work we estimate rating transition matrices for US RMBS securities for the period of 1991 to 2007. Similar to the findings for corporate bonds, our results do not report significant change on the new data set and time period. Comparisons between the cohort method and the time-homogeneous duration method reveal that the latter yields greater default probabilities for high credit ratings and lower default probabilities for low credit ratings. In addition, we find that the duration method generates smaller downgrade and upgrade probabilities.

Furthermore, we conduct an alternative approach for calculating the average transition matrices. We suggest that, in the case of unevenly distributed data set, the weighted average calculation approach is more efficient and accurate.

Appendices

Appendix A: Derivation of the Estimator of Homogeneous Transition Intensities using Duration Method

Lando and Skodeberg (2000) propose the direct maximum-likelihood estimator for the generator matirces $\hat{\Lambda}$ in time-homogeneous case. This estimator process requires the precise information in time at which the ratings transitions take place.

We use the following notations:

m_{ij} = number of transitions to rating j from rating i

t_{ij} = time of the security transits from rating i to rating j

t'_{ij} = time of the security entered rating i and finally transits to rating j

m_{ii} = number of the securities stay in the same ratings

t'_{i} = time of security entered rating i but not transitioning

m = total number of the securities entered rating i

λ_{ij} = transition intensity from rating i to rating j

For time homogeneous case, the λ_{ij} is a fixed intensity estimator. The probability density of the transition time from rating i to rating j for each rating state:

$$f(t_{ij}) = \lambda_{ij} e^{-(t_{ij}-t'_{ij})\sum_{k\neq i}^{J}\lambda_{ik}}$$

For the time in $[t'_{ij}, t]$, otherwise 0.

The likelihood function for a portfolio of securities occupying rating i at some time during $[0, t]$ is the product off the above density functions:

$$L = \left(\prod_{i=1}^{I}\prod_{j\neq i}^{m_{ij}} \lambda_{ij} e^{-(t_{ij}-t'_{ij})\sum_{k\neq i}^{J}\lambda_{ik}}\right)\prod_{i=1}^{m_{ii}} e^{-(t_i-t'_i)\sum_{k\neq i}^{J}\lambda_{ik}}$$

Taking the log of L

$$\ln L = \sum_{i=1}^{I}\sum_{j\neq i}^{m_{ij}}\left(\ln\lambda_{ij} - (t_{ij}-t'_{ij})\sum_{k\neq i}^{J}\lambda_{ik}\right) - \sum_{i=1}^{I}\left((t_i - t'_i)\sum_{k\neq i}^{J}\lambda_{ik}\right)$$

Maximizing with respect to a particular λ_{ij} and take the first order conditions then set it to zero

$$\frac{\partial \ln L}{\partial \lambda_{ij}} = \frac{m_{ij}}{\lambda_{ij}} - \sum_{i=1}^{I}\sum_{j\neq i}^{m_{ij}}(t_{ij}-t'_{ij}) - \sum_{i=1}^{I}(t_i - t'_i) = 0$$

Therefore, by solving the above partial differentiation equations, we can get the estimator for at maximum likely:

$$\hat{\lambda}_{ij} = \frac{m_{ij}}{T}$$

$$T = \sum_{i=1}^{I}\sum_{j\neq i}^{m_{ij}}(t_{ij}-t'_{ij}) - \sum_{i=1}^{I}(t_i - t'_i)$$

Appendix B: Comparison of Transition Matrices

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D
AAA	99.922	0.078	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	0.385	99.494	0.081	0.040	0.000	0.000	0.000	0.000	0.000	0.000
A	0.158	0.454	99.034	0.197	0.099	0.059	0.000	0.000	0.000	0.000
BBB	0.000	0.070	0.334	97.942	0.862	0.317	0.317	0.000	0.000	0.158
BB	0.000	0.000	0.083	0.579	97.685	0.703	0.496	0.000	0.000	0.455
B	0.000	0.000	0.000	0.000	0.321	98.526	0.513	0.000	0.000	0.641
CCC	0.000	0.000	0.000	0.000	0.000	0.000	33.333	0.000	0.000	66.667
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Table 10: Cohort method (NR adjusted) transition matrix (2006)

	AAA	AA	A	BBB	BB	B	CCC	CC	C	D
AAA	100.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AA	16.990	83.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
A	3.075	11.567	85.066	0.293	0.000	0.000	0.000	0.000	0.000	0.000
BBB	0.451	3.759	5.113	90.075	0.301	0.301	0.000	0.000	0.000	0.000
BB	0.000	0.000	2.279	13.960	82.336	0.570	0.000	0.000	0.000	0.855
B	0.000	0.000	0.000	0.906	7.553	86.707	2.719	0.000	0.000	2.115
CCC	0.000	0.000	0.000	0.000	0.000	0.000	75.000	10.000	0.000	15.000
CC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	66.667	0.000	33.333
C	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Table 11: Cohort method (NR adjusted) transition matrix (2002)

The comparison indicates that probabilities of no migration and default probabilities are not constant for different years.

Appendix C: US RMBS 2000-2007: M_{SVD}

We comput the average of the sigular values of the mobility matrix M_{SVD} for each year by following Jafry and Schuermann (2004).

$$M = Q - I$$

$$s(M) = \sqrt{eig(M'M)}$$

$$M_{SVD} = \overline{s(M)} = \frac{\Sigma_{j=1}^{N}\left(\sqrt{e_j(M'M)}\right)}{N}$$

where M and Q represent the mobility matrix and transition matrix, respectively.

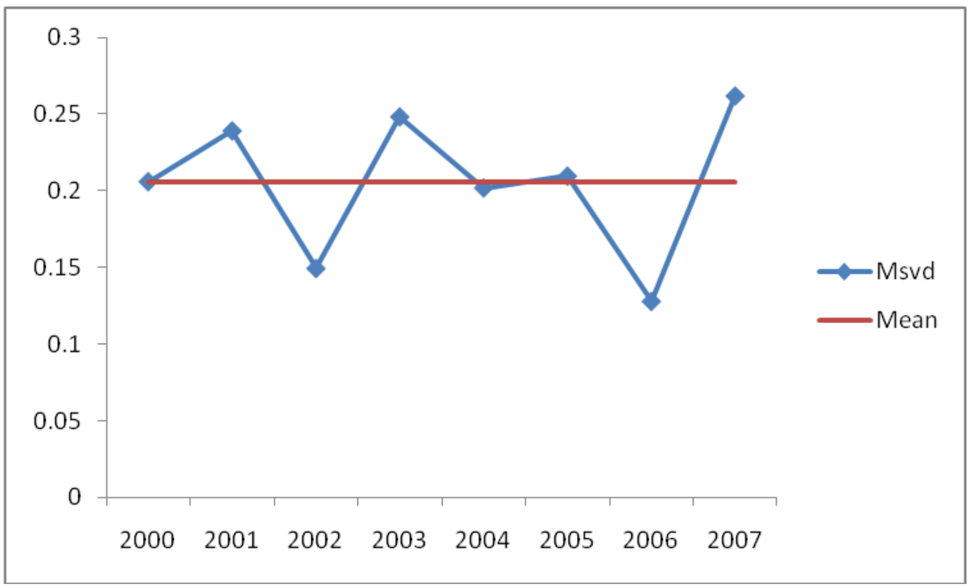

Figure 2: US RMBS 2000-2007 M_{SVD}

Figure 2 plots the singular value metric for the 8 annual migration matrices (2000-2007). The horizontal line represents the average of the 8 M_{SVD} values. This figure shows significant amount of variations over time.

Appendix D: Matlab Code - Cohort Method

```
% Cohort Method RMBS

% This code is designed to estimate the transition matrix of the credit ratings

clear all

close all

clc

load RMBS_Raw.mat



% preset an 3-D transition matrix by zeros formating: (Rating@t-1,Rating@t,28-1years)

Q=nan(23,23,18);

Q(:,:)=0; % set all cells to zero

% preset a annual rating matrix from 01-Jan-1991 to 01-Jan-2008

RMBS_Raw(:,18:19)=nan;

Ratings=nan(length(RMBS_Raw),19);

Ratings(:,1)=RMBS_Raw(:,1);

% preset a date table

NewYearDays=[727199,727564,727930,728295,728660,729025,729391,729756,730121,730486,730852,731217,7315
82,731947,732313,732678,733043,733408,733774];

Observations_initial=[401,455,619,567,502,370,305,521,776,518,578,1466,3004,4927,7364,10464,11085,7341];

%datedisp(NewYearDays)
```

```
% Set Parameters

% "No Ratings" NR=1 (NR adjusted) NR=0 (NR included)

NR=1;

% "Rating Modifiers" RM=1 (without RM) RM=0 (with RM)

RM=1;

% "Weighted Matrix" Weighted=1 (Weighted Average) Weighted=0 (Equal Average)

Weighted=0;



tic

for idx=1:length(RMBS_Raw(:,1))

  jdx=1;

  p=RMBS_Raw(idx,jdx*2+1);

  Position_p=find(p<NewYearDays);

  q=RMBS_Raw(idx,jdx*2+3);

  if isnan(q)==1 & isnan(p)~=1

    Ratings(idx,Position_p+1:end)=RMBS_Raw(idx,jdx*2);

  end

  while isnan(p)~=1 & isnan(q)~=1

    Position_p=find(p<=NewYearDays);

    Position_q=find(q<=NewYearDays);
```

```
        Ratings(idx,Position_p(1)+1:Position_q(1))=RMBS_Raw(idx,jdx*2);
        jdx=jdx+1;
        p=RMBS_Raw(idx,jdx*2+1);
        q=RMBS_Raw(idx,jdx*2+3);
        % Recheck with the "tail"
        if isnan(q)==1
            switch RMBS_Raw(idx,jdx*2)
                case 22 % Default as the last state
                    Ratings(idx,Position_q(1)+1:end)=22; %??? Default forever?
                case 23 % No Ratings as the last state
                    Ratings(idx,Position_q(1)+1)=23;
                otherwise % Lasting same ratings until present
                    Ratings(idx,Position_q(1)+1:end)=RMBS_Raw(idx,jdx*2+2);
            end
        end
    end
end
toc

% Adjusting the Rating Modifiers
```

```
if RM
    for idx=3:3:18
        Ratings(Ratings==idx-1)=idx;
        Ratings(Ratings==idx+1)=idx;
    end
end


% Calibrate the Migration Matrices
tic
Ratings_form(:,1)=1:23;
Ratings_2years=nan(length(Ratings),2);
for kdx=1:length(NewYearDays)-1
    % Import Rating data as two years rolling window
    Ratings_2years=Ratings(:,kdx+1:kdx+2);
    % Exclude the NaN points in the matrix
    Ratings_2years=Ratings_2years(~any(isnan(Ratings_2years),2),:);
    % Exclude NR ratings
    if NR
        Ratings_2years=Ratings_2years(any((Ratings_2years(:,1)~=23 & Ratings_2years(:,2)~=23),2),:);
    end
```

```
% Summary the moving year transition by filter stays and trans

Ratings_2years_stays=Ratings_2years(any(Ratings_2years(:,1)==Ratings_2years(:,2),2),:);

Ratings_2years_trans=Ratings_2years(any(Ratings_2years(:,1)~=Ratings_2years(:,2),2),:);

% Count all data stays in the same ratings during the moving window

Count_stays=histc(Ratings_2years_stays(:,1),1:23);

% Store the number of Active Rating Securities

Observations_Act_Raw=Ratings_2years(:,1);

Observations_Act(kdx)=length(Observations_Act_Raw(Observations_Act_Raw~=22 &
Observations_Act_Raw~=23));

% Fill the Q matrics

for idx=1:23

  for jdx=1:23

      % Fill Q with the observations of changed ratings

      Count_trans=length(Ratings_2years_trans(any((Ratings_2years_trans(:,1)==idx &
Ratings_2years_trans(:,2)==jdx),2),:));

      Q(idx,jdx,kdx)=Count_trans;

      % Fill Q with the observations of unchanged ratings

      if idx==jdx

          Q(idx,jdx,kdx)=Count_stays(idx);

      end

  end
```

```
      if sum(Q(idx,:,kdx))~=0
         % Convert Q into probabilities
         Q(idx,:,kdx)=Q(idx,:,kdx)/sum(Q(idx,:,kdx));
         % Check the Q matrix stability
         check_SMO(idx,kdx)=sum(Q(idx,:,kdx));
      end
   end
   Default(:,kdx)=Q(:,end-1,kdx);
end
toc


% Calculate the weights for each year
%%%Yearly_Weights=Observations_initial(2:end)/sum(Observations_initial(2:end));
Yearly_Weights=Observations_Act(2:end)/sum(Observations_Act(2:end));


% Average the Q matrix by annulizing migration matrix
tic
for idx=1:23
   for jdx=1:23
       if Weighted
```

```
            Q_weighted(1:17)=Q(idx,jdx,1:17);

            Q_weighted=Yearly_Weights.*Q_weighted;

            Q_hat(idx,jdx)=mean(Q_weighted);

        else

            Q_hat(idx,jdx)=mean(Q(idx,jdx,1:17));

        end

    end

    if sum(Q_hat(idx,:))~=0

        Q_hat(idx,:)=Q_hat(idx,:)/sum(Q_hat(idx,:));

    end

end

toc
```

Appendix E: Matlab Code - Duration Method

```
% Duration Method RMBS

% This code is designed to estimate the transition matrix of the credit ratings

clear all

close all

clc

load RMBS_Raw.mat



% preset an 3-D transition matrix by zeros formating: (Rating@t-1,Rating@t,28-1years)

L=nan(23,23,18);

L(:,:)=0; % set all cells to zero

% preset a annual rating matrix from 01-Jan-1991 to 01-Jan-2008

RMBS_Raw(:,18:19)=nan;

Ratings=nan(length(RMBS_Raw),29);

Ratings_2years=nan(length(RMBS_Raw),4);

Ratings(:,1)=RMBS_Raw(:,1);

% preset a date table

NewYearDays=[727199,727564,727930,728295,728660,729025,729391,729756,730121,730486,730852,731217,7315
82,731947,732313,732678,733043,733408,733774];

Observations=[401,455,619,567,502,370,305,521,776,518,578,1466,3004,4927,7364,10464,11085,7341];
```

```
%datedisp(NewYearDays)

% Set Parameters

% "No Ratings" NR=1 (NR adjusted) NR=0 (NR included)

NR=1;

% "Rating Modifiers" RM=1 (without RM) RM=0 (with RM)

RM=1;

% "Weighted Matrix" Weighted=1 (Weighted Average) Weighted=0 (Equal Average)

Weighted=1;



tic

for idx=1:length(RMBS_Raw(:,1))

  jdx=1;

  p=RMBS_Raw(idx,jdx*2+1);

  Position_p=find(p<NewYearDays);

  q=RMBS_Raw(idx,jdx*2+3);

  if isnan(q)==1 & isnan(p)~=1

    Ratings(idx,Position_p+1:end)=RMBS_Raw(idx,jdx*2);

  end

  while isnan(p)~=1 & isnan(q)~=1

    Position_p=find(p<=NewYearDays);
```

```
        Position_q=find(q<=NewYearDays);

        Ratings(idx,Position_p(1)+1:Position_q(1))=RMBS_Raw(idx,jdx*2);

        jdx=jdx+1;

        p=RMBS_Raw(idx,jdx*2+1);

        q=RMBS_Raw(idx,jdx*2+3);

        % Recheck with the "tail"

        if isnan(q)==1

            switch RMBS_Raw(idx,jdx*2)

                case 22 % Default as the last state

                    Ratings(idx,Position_q(1)+1:end)=22; %??? Default forever?

                case 23 % No Ratings as the last state

                    Ratings(idx,Position_q(1)+1)=23;

                otherwise % Lasting same ratings until present

                    Ratings(idx,Position_q(1)+1:end)=RMBS_Raw(idx,jdx*2+2);

            end

        end

    end

end

toc
```

```
% Adjusting the Rating Modifiers

if RM

    for idx=3:3:18

        Ratings(Ratings==idx-1)=idx;

        Ratings(Ratings==idx+1)=idx;

    end

end



% Loop the Duration Method yearly transition matrix

tic

for kdx=1:length(NewYearDays)-1

    % Filter the data to rolling two years with detail of transition records

    Ratings_2years=Ratings(:,kdx+1:kdx+2);

    % Transited records

    Position_Trans=find(isnan(Ratings_2years(:,1))==0 & isnan(Ratings_2years(:,2))==0 &
Ratings_2years(:,1)~=Ratings_2years(:,2));

    for idx=1:length(Position_Trans)

        o=find(RMBS_Raw(Position_Trans(idx),:)<=NewYearDays(kdx+1) &
RMBS_Raw(Position_Trans(idx),:)>=NewYearDays(kdx));

        Ratings_2years(Position_Trans(idx),3)=RMBS_Raw(Position_Trans(idx),o(1))-NewYearDays(kdx);

    end
```

```
    % Exclude the NaN points in the matrix %Ext.: Exclude NR ratings

    Ratings_2years=Ratings_2years(~any(isnan(Ratings_2years),2),:);

    % Exclude NR ratings

    if NR

        Ratings_2years=Ratings_2years(any((Ratings_2years(:,1)~=23 & Ratings_2years(:,2)~=23),2),:);

    end

    Ratings_2years_stays=Ratings_2years(any(Ratings_2years(:,1)==Ratings_2years(:,2),2),:);

    Ratings_2years_trans=Ratings_2years(any(Ratings_2years(:,1)~=Ratings_2years(:,2),2),:);

    Count_stays=histc(Ratings_2years_stays(:,1),1:23);

    %%%Ratings_2years(Ratings_2years==0)=nan;

    % Store the number of Active Rating Securities

    Observations_Act_Raw=Ratings_2years(:,1);

    Observations_Act(kdx)=length(Observations_Act_Raw(Observations_Act_Raw~=22 &
Observations_Act_Raw~=23));

    % Fill the L matrices

    for idx=1:23

        for jdx=1:23

            Ratings_2years_trans_ij=Ratings_2years_trans(any((Ratings_2years_trans(:,1)==idx &
Ratings_2years_trans(:,2)==jdx),2),:);

            Count_trans=size(Ratings_2years_trans_ij);

            if idx~=jdx & sum(Ratings_2years_trans_ij(:,3))~=0
```

```
        %t=sum(Ratings_2years_trans_ij(:,3)/(NewYearDays(kdx+1)-NewYearDays(kdx)));

        L(idx,jdx,kdx)=Count_trans(1)/sum(Ratings_2years_trans_ij(:,3)/(NewYearDays(kdx+1)-
NewYearDays(kdx)));

      elseif idx==jdx

        %L(idx,jdx,kdx)=0;

        L(idx,jdx,kdx)=Count_stays(idx);

      end

    end

    %if sum(L(idx,:,kdx))~=0

    %    L(idx,:,kdx)=L(idx,:,kdx)/sum(L(idx,:,kdx));

    %end

  end

  Default(:,kdx)=L(:,end-1,kdx);

end

toc



% Calculate the weights for each year

%%%Yearly_Weights=Observations_initial(2:end)/sum(Observations_initial(2:end));

Yearly_Weights=Observations_Act(2:end)/sum(Observations_Act(2:end));
```

```
% Average the L matrix by annulizing migration matrix
tic
for idx=1:23
    for jdx=1:23
        if Weighted
            L_weighted(1:17)=L(idx,jdx,1:17);
            L_weighted=Yearly_Weights.*L_weighted;
            L_hat(idx,jdx)=mean(L_weighted);
        else
            L_hat(idx,jdx)=mean(L(idx,jdx,1:17));
        end
    end
    if sum(L_hat(idx,:))~=0
        L_hat(idx,:)=L_hat(idx,:)/sum(L_hat(idx,:));
    end
end
toc
```

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