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Nonlinear theory of resonant slow MHD waves in twisted magnetic flux tubes

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Abstract. The nonlinear dynamics of resonant slow MHD waves in weakly dissipative plasmas is investigated in cylindrical geometry with a twisted equilibrium magnetic field. Linear theory has shown that the wave motion is governed by conservation laws and jump conditions across the resonant surface considered as a singularity – first derived in linear ideal MHD theory by Sakurai, Goossens and Hollweg [*Solar Phys.* **133**, 227 (1991)]. By means of the simplified method of matched asymptotic expansions, we obtain the generalized connection formulae for the variables across the dissipative layer, and we derive a non-homogeneous nonlinear partial differential equation for the wave dynamics in the dissipative layer.

1. Introduction

A continuous field distribution is a superposition of elemental flux tubes, each tube crowding against its neighbors with pressure $B^2/2\mu$ and striving to shorten its length with tension B^2/μ. In many circumstances in nature, such as the solar atmosphere, magnetic fields are observed to split into separated flux tubes (sunspots, magnetic knots, spiculae, coronal loops, etc.). Thus the dynamic properties of the individual tubes provide an understanding of large-scale continuous field distribution. The concentration of the magnetic field into flux tubes occurs spontaneously, in opposition to the considerable magnetic pressure of the concentrated field. This phenomenon challenges our understanding of the basic physics of flux tubes. Owing to the complexity of the problem, in general, the effect of the magnetic twist is neglected.

The study of the *twisted tubes* can be important in the context of controlled fusion physics, astrophysics and, in particular, solar physics. For example, many observational studies have revealed that the solar magnetic field is twisted. This is manifested in various kinds of observations, such as the morphology of H_α structures, the morphology of filaments and coronal loops, and the signs of current helicity derived from vector magnetograms. All these observations demonstrate a hemispheric preference of the sense of the twist, i.e. left-handedness in the northern hemisphere and right-handedness in southern hemisphere in the current solar cycle. It is most likely that the sense of the twist is associated with the solar dynamo and physical conditions in the convection zone.

Two mechanisms have been proposed to explain the formation of the twist, namely vertical motions of the magnetic plasma and the emergence of subsurface twisted flux. Many recent investigations of the emerging flux regions have showed that the magnetic flux may already be twisted below the photosphere.

The solar corona is a very hot tenuous plasma at a typical temperature of $(2\text{–}3) \times 10^6$ K, much higher than that of the underlying layers (the transition region, the chromosphere and the photosphere), so the nonthermal energy must be transported into the corona and dissipated there. The high-temperature coronal plasma mainly radiates in the soft X-ray range corresponding to typical wavelengths of the order of 10–100 Å. This radiation is not homogeneous either in space or in time, with a wide range of spatial and time scales. Large-scale structures of typical lengths $10^6\text{–}10^8$ m persist during a characteristic time much longer than the Alfvén time, while small bright points of dimension of the order of 10^4 m evolve on a few Alfvén times and in some cases even more rapidly. High-resolution observations have now given an image of the solar corona as a rapidly evolving dynamic plasma where energetic phenomena occur mainly on very 'small' scales.

The main problem related to the understanding of the coronal heating mechanism is to perform sufficiently high-resolution observations (i.e. observations at typical wavelengths of the order of the dissipative length scales), capable of shedding light on the physical dissipative process at work in the solar corona. As a consequence, no realistic models, even very simplified, have been developed so far, while a number of conceptual models starting from the available data try to roughly describe the small-scale dynamics and to derive all the possible consequences on the mean dynamics in order to fit the observational large-scale constraints.

Nevertheless, some basic ideas have now reached a large general consensus. First of all, photospheric and sub-photospheric random motions are considered as the energy source of the heat deposited into the corona, since the corresponding energy flux flowing outwards from the photosphere towards the outer layers of the Sun is large enough to compensate the radiative and conductive energy losses in the corona. The second important point that is now accepted is the key role of the magnetic field as the link between the energy source, the photosphere and the region where the energy is converted into heat, the corona, as well as the main agent in the energy-transfer process from the injection scales to the small dissipative scales. Many proposals have been made in the last few decades concerning the nature of the mechanism acting to dissipate the energy in the solar corona.

The possible heating mechanisms can be classified as a function of the characteristic time t_{ph} of the photospheric random perturbations with respect to the Alfvén time t_A defined as $t_A = l/v_A$, where l and v_A are a characteristic length of the system and the Alfvén velocity. In the limit of 'slow' perturbations, $t_{\text{ph}} \gg t_A$, the large-scale coronal structures can evolve through a series of magnetostatic force-free equilibria. The typical case is that of a coronal loop continuously stressed at its footpoints. Eventually, strong current sheets are generated near the separatrix and magnetic energy is released, for example via magnetic reconnection. However, further dynamical investigations of the evolution of such magnetostatic configurations are necessary in order to estimate the characteristic time needed to generate the current sheets and to dissipate the magnetic energy.

In the limit of 'fast' photospheric perturbations, $t_{\text{ph}} \ll t_A$, most of the energy is converted into MHD waves, which propagate outwards. It is still an open problem how MHD waves can reach the upper part of the solar atmosphere. It is believed that only the Alfvén waves, owing to their highly anisotropic dispersion relation, are able to reach the corona, the other MHD waves being dissipated or reflected at lower altitudes. Recently, however, it was realized that part of the coronal MHD waves may be generated *in the corona itself* by, for example, small and large magnetic reconnection events, solar tornados or wave transformation due to nonlinear effects. This may turn out to be important for coronal heating, since it removes the difficulty of energy transfer from the turbulent convection zone through the transition region to the corona. The problem is then how to dissipate the energy carried by the Alfvén waves due to their strong dissipative inefficiency in a perfectly homogeneous plasma, even when considering nonlinear interactions. For this reason, wave heating theories, faced with the problem of how to speed up the dissipative effects, have mostly considered the interaction of Alfvén waves with an inhomogeneous medium. In this context, the most promising mechanisms are the phase mixing and the resonant absorption that occur whenever the wave propagates along a magnetic field inhomogeneous in the transverse direction. In particular, phase mixing appears as a result of the spontaneous decay of the free oscillations of the system. If oscillations on different magnetic surfaces are initially excited coherently (i.e. in phase), then, in time, the oscillations become gradually out of phase among neighboring surfaces because each surface oscillates with a different eigenfrequency. As a consequence, large gradients develop across the magnetic surfaces. Owing to progressive creation of smaller length scales comparable to the scales where resistivity and viscosity operate, this process will lead to wave damping.

Resonant transfer of energy is a natural phenomenon of interacting dynamic systems. In the solar atmosphere, where the excited and propagating MHD waves interact with an inhomogeneous plasma, these waves can transfer their energy to each other. The energy transfer from the waves to the background plasma is related to the fact that in inhomogeneous plasmas externally driven waves can resonantly interact with the local oscillation eigenmodes. After all, resonant absorption can be considered as an effective process of generation of small length scales comparable to the dissipation length scales.

The local oscillation modes of an inhomogeneous plasma are represented by continuous spectra for slow MHD and Alfvén waves and a discrete spectrum for fast MHD waves. Resonant absorption occurs when the frequency of a laterally driven oscillation matches the local slow and/or Alfvén frequency and a resonant field line is created that transfers energy from the surface disturbance to its environment.

In ideal plasmas, the resonant waves are confined to an individual magnetic surface that cannot interact with its neighbors. Since we suppose that a driven external mode can exist for an infinitely long period, this energy accumulation results in an infinite wave amplitude at the resonant position. But a real plasma is far from being an ideal medium. To describe a realistic situation, dissipative effects have to be taken into account. Usually, the importance of dissipation is characterized by the viscous and magnetic Reynolds numbers (if viscosity and magnetic diffusion are considered as dissipative effects). Dissipative effects cause coupling of the resonant magnetic surface to neighboring magnetic surfaces, and the disturbance provoked at the resonant surface is transmitted to neighboring field lines. We can define a 'resonant layer', considering that region where the disturbances do not go out of phase relative to the driven oscillations. For large values of the viscous and magnetic Reynolds numbers (as in the solar atmosphere or in tokamak physics), this coupling is weak and the local resonant slow and Alfvén oscillations are characterized by steep gradients across the magnetic field. Now, the energy of the external oscillations is dissipated to the plasma and can be converted into heat.

Ionson (1978) pointed out first that resonant absorption can be a viable candidate for coronal heating. Since then, this process has become one of the most popular mechanisms to explain the anomalous behavior of the coronal temperature (see e.g. Kuperus et al. 1981; Davila 1987; Narain and Ulmschneider 1990, 1996; Hollweg 1991; Goossens 1991; Goossens and Ruderman 1995; and references therein). The same mechanism is used to explain the observed energy loss of p-modes in the vicinity of sunspots (see e.g. Hollweg 1988; Lou 1990; Goossens and Poedts 1992; Goossens and Hollweg 1993; Erdélyi and Goossens 1994).

A turning point in the study of resonant absorption was the development of so-called *connection formulae* by Sakurai et al. (1991) for ideal plasmas and by Goossens et al. (1995) and Erdélyi (1997) for dissipative plasmas. This approach is based on the very simple idea that a thin dissipative layer acts as a surface of discontinuity when solving the MHD equations. At both sides of this surface of discontinuity, the plasma motion is governed by the ideal MHD equations. The solution of dissipative MHD inside the dissipative layer is used to obtain the connection formulae that provide boundary conditions at the surface of discontinuity.

The linear theory of resonant absorption has shown that in the vicinity of a resonant position, perturbations have steep gradients and large amplitudes, and therefore linear theory can break down in this region and nonlinear theory has to be considered. Nonlinearity in the dissipative layer was first taken into account in the theory of resonant absorption by Ruderman et al. (1997a), who studied the nonlinear evolution of slow resonant MHD waves in a dissipative layer using Cartesian geometry and considering isotropic dissipative effects. Later, this theory was extended to anisotropic plasmas (e.g. the solar corona) by Ballai et al. (1998a), where anisotropic viscosity and field-aligned thermal conductivity played the role of dissipative effects. These theories were applied to study the resonant absorption of sound and fast magnetoacoustic waves in solar structures (see e.g. Ruderman et al. 1997b; Ballai et al. 1998b; Erdélyi and Ballai 1999; Erdélyi et al. 2001). One of the main results was that, in contrast to linear theory, the coefficient of wave energy absorption was dependent on the particular type of dissipation. It was also found that the general tendency of nonlinearity is to decrease the absolute value of the coefficient of wave energy absorption when the wavelength of the incoming wave is much larger than the characteristic scale of the inhomogeneity and nonlinearity is considered weak. This is no longer the case in the limit of strong nonlinearity, as pointed out by Ruderman (2000), at least for intermediate wavelengths. In the long-wavelength approximation, nonlinearity again decreases the net coefficient of energy absorption, and the difference relative to the result found by means of linear theory is about 20%.

The characteristic quantities used in the present paper are the thickness of the dissipative layer, $l_{\rm dis}$, the characteristic length scale of the inhomogeneity, $l_{\rm inh}$, and the dimensionless amplitude of the oscillations far away from the dissipative layer, ϵ. The total Reynolds number, which measures the magnitude of the dissipative effects, is defined by

$$\frac{1}{R} = \frac{1}{R_v} + \frac{1}{R_m}, \tag{1.1}$$

where R_v and R_m are the viscous and magnetic Reynolds numbers respectively.

Since solar observations reveal that R is very large in astrophysical plasmas (10^6 in the photosphere and up to 10^{12} in the corona), we can use the approximation of a weakly dissipative plasma.

The goal of the present paper is to investigate how nonlinearity affects the dynamics of resonant slow MHD waves in *twisted* magnetic flux tubes and to investigate the effect of magnetic twisting on the connection formulae. The paper is organized as follows. In the next section, we introduce the basic equations and the equilibrium state. In order to obtain an equation that contains the effects of the nonlinearity and dissipation, a scaling law is introduced in this section. In Sec. 3, we calculate the governing equations for the wave dynamics outside and inside the dissipative layer. Since the resonant surface acts as a singular surface, the connection formulae are calculated by means of the method of asymptotic expansions. Finally, we summarize and discuss our results.

2. Basic equations

In what follows, we adopt cylindrical coordinates (r, φ, z) and restrict our analysis to a static equilibrium state, i.e. $\mathbf{v}_0 = 0$. The effect of gravity is also neglected. The components of the background magnetic field are $(0, B_0\varphi(r), B_{0z}(r))$, and the other equilibrium quantities depend on the radial coordinate only.

We use the full set of nonlinear viscoresistive MHD equations

$$\frac{D\bar{\rho}}{Dt} + \bar{\rho}\boldsymbol{\nabla}\cdot\mathbf{v} = 0, \qquad \boldsymbol{\nabla}\cdot\bar{\mathbf{B}} = 0, \tag{2.1}$$

$$\bar{\rho}\frac{D\mathbf{v}}{Dt} = -\boldsymbol{\nabla}\bar{p} + \frac{1}{\mu}[(\boldsymbol{\nabla}\times\bar{\mathbf{B}})\times\bar{\mathbf{B}}] + \bar{\rho}\bar{\nu}[\nabla^2\mathbf{v} + \tfrac{1}{3}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{v})], \tag{2.2}$$

$$\frac{\partial\bar{\mathbf{B}}}{\partial t} = \boldsymbol{\nabla}\times(\mathbf{v}\times\bar{\mathbf{B}}) + \bar{\eta}\nabla^2\bar{\mathbf{B}}, \tag{2.3}$$

$$\frac{D}{Dt}\left(\frac{\bar{p}}{\bar{\rho}^\gamma}\right) = 0, \tag{2.4}$$

where $D/Dt = \partial/\partial t + \mathbf{v}\cdot\boldsymbol{\nabla}$ is the convective derivative. In the above equations, $\mathbf{v}$ and $\bar{\mathbf{B}}$ are the velocity and magnetic induction vectors, $\bar{p}$ and $\bar{\rho}$ are the pressure and density, and $\bar{\nu}$, $\bar{\eta}$ and γ are the coefficient of kinematic shear viscosity, the coefficient of magnetic diffusion and the adiabatic index respectively. The perturbations of the magnetic field and velocity are denoted by $\mathbf{b} = (b_r, b_\varphi, b_z)$ and $\mathbf{v} = (u, v, w)$. In spite of the presence of dissipation, we use the adiabatic equation as an approximation of the energy equation. Ballai et al. (2000) have shown that the dissipation due to viscosity and finite electrical conductivity present in the energy equation does not lead to a significant change in the behavior of nonlinear slow resonant MHD waves in a driven problem.

We consider finite perturbations of the form

$$\bar{f} = f_0(r) + f(r, \varphi, z, t), \tag{2.5}$$

where f_0 is the equilibrium value of a variable and f its Eulerian perturbation.

The equilibrium variables satisfy the radial force balance

$$\frac{d}{dr}\left(p_0 + \frac{B_0^2}{2\mu}\right) = -\frac{B_{0\varphi}^2}{\mu r}, \tag{2.6}$$

where $B_0 = (B_{0\varphi}^2 + B_{0z}^2)^{1/2}$. With these variables, we can define the total pressure perturbation as

$$P = p + \frac{\mathbf{B}_0 \cdot \mathbf{b}}{\mu} + \frac{\mathbf{b}^2}{2\mu}. \tag{2.7}$$

In linear theory, all physical variables oscillate with the same real frequency ω and they can be Fourier-analyzed. However, in nonlinear theory, this procedure cannot be applied, since the oscillations are no longer in phase. To be as close as possible to the linear theory, we suppose that the oscillations are plane-periodic propagating waves with permanent shape, and we take all variables to depend on the combination $\theta = m\varphi + kz - \omega t$ of the independent variables φ, z and t rather than on φ, z and t separately. Here m and k are the azimuthal and longitudinal wavenumbers. Convenient quantities for later use are

$$f_B = \frac{m}{r}B_{0\varphi} + kB_{0z}, \qquad g_B = \frac{m}{r}B_{0z} - kB_{0\varphi}. \tag{2.8}$$

In this notation, the squares of the Alfvén speed, the sound speed, the Alfvén and the cusp frequencies are defined as

$$v_A^2 = \frac{B_0^2}{\mu\rho_0}, \qquad c_S^2 = \frac{\gamma p_0}{\rho_0}, \qquad \omega_A^2 = \frac{f_B^2}{\mu\rho_0}, \qquad \omega_T^2 = \frac{\omega_A^2 c_S^2}{v_A^2 + c_S^2}.$$

Let us introduce the parallel and perpendicular components of the velocity and magnetic field relative to the equilibrium magnetic field as

$$(v_\parallel, b_\parallel) = (\mathbf{v}, \mathbf{b}) \cdot \frac{\mathbf{B}_0}{B_0}, \qquad (v_\perp, b_\perp) = \frac{1}{B_0}[(v, b_\varphi)B_{0z} - (w, b_z)B_{0\varphi}]. \tag{2.9}$$

With these considerations, the modified MHD equations are

$$\rho_0\left(\frac{1}{r}\frac{\partial(ur)}{\partial r} + \frac{1}{B_0}\frac{\partial}{\partial\theta}(f_B v_\parallel + g_B v_\perp)\right) + u\frac{d\rho_0}{dr} - \omega\frac{\partial\rho}{\partial\theta} = N_{1t}, \tag{2.10}$$

$$\frac{\partial P}{\partial r} - \rho_0\frac{\partial}{\partial\theta}\left(\omega u + \frac{f_B}{\mu\rho_0}b_r\right) + \frac{2B_{0\varphi}\gamma_\parallel^\perp}{B_0\mu r} = N_{2t} + D_{1t}, \tag{2.11}$$

$$\frac{\partial}{\partial\theta}\left(\frac{f_B}{B_0}P - \omega\rho_0 v_\parallel - \frac{f_B}{\mu}b_\parallel\right) - \frac{B_{0\varphi}^2}{B_0\mu r}b_r - \frac{1}{\mu}\frac{dB_0}{dr}b_r = N_{3t} + D_{2t}, \tag{2.12}$$

$$\frac{\partial}{\partial\theta}\left(\frac{g_B}{B_0}P - \omega\rho_0 v_\perp - \frac{f_B}{\mu}b_\perp\right) - \frac{B_{0\varphi}B_{0z}}{B_0\mu r}b_r - \frac{B_{0\varphi}^2}{\mu}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)b_r = N_{4t} + D_{3t}, \tag{2.13}$$

$$B_0 f_B\frac{\partial}{\partial\theta}\left(u + \frac{\omega}{f_B}b_r\right) = N_{5t} + D_{4t}, \tag{2.14}$$

$$g_B\frac{\partial}{\partial\theta}\left(v_\perp - \frac{\omega}{g_B}b_\parallel\right) + \frac{dB_0}{dr}u + \frac{B_0}{r}\frac{\partial(ur)}{\partial r} - \frac{B_{0\varphi}^2}{B_0 r}u = N_{6t} + D_{5t}, \tag{2.15}$$

$$f_B\frac{\partial}{\partial\theta}\left(v_\perp + \frac{\omega}{f_B}b_\perp\right) - B_{0\varphi}^2\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)u - \frac{B_{0\varphi}B_{0z}}{B_0 r}u = N_{7t} + D_{6t}, \tag{2.16}$$

$$\omega\left(\frac{\partial p}{\partial\theta} - c_S^2\frac{\partial\rho}{\partial\theta}\right) - u\left(\frac{dp_0}{dr} - c_S^2\frac{d\rho_0}{dr}\right) = N_{8t}, \tag{2.17}$$

$$P - p - \frac{B_0}{\mu}b_\parallel = N_{9t}, \tag{2.18}$$

where

$$\gamma_{\parallel}^{\perp} = b_{\parallel} B_{0\varphi} + b_{\perp} B_{0z}, \tag{2.19}$$

and we have collected all nonlinear terms ($N_{it}, i = 1, \ldots, 9$) on the right-hand side of the equations, and D_{jt}, ($j = 1, \ldots, 6$) denote the dissipative terms. The exact expressions for the nonlinear and dissipative terms are given in Appendix A.

This set of equations is used to derive the governing equation for slow MHD wave dynamics in the dissipative layer in the following sections.

3. Wave dynamics in the dissipative layer

The mathematical procedure used to derive the equation that describes nonlinear resonant slow-wave motion in the dissipative layer is based on the simplified method of matched asymptotic expansions developed by Ballai et al. (1998a) from a more general method due to Ruderman et al. (1997a). The method consists of finding the so-called inner and outer expansions and matching them in the overlap regions. Adapted to our problem, the inner and outer expansions will mean the expansions of the solution inside and outside the dissipative layer.

The simplified version of the method of matched asymptotic expansions is based on very simple ideas. Since we are dealing with a weakly dissipative plasma, the viscosity and the magnetic diffusivity are essential only in the dissipative layer. Far from the dissipative layer, the amplitudes of perturbations are small. These two facts enable us to consider that outside the dissipative layer the plasma motion is described by the ideal linear MHD equations. Another assumption is related to the behavior of the equilibrium quantities. We suppose that each of these variables changes only slightly across the dissipative layer and can be approximated by the first non-vanishing term in its Taylor series expansion with respect to the inhomogeneity coordinate r. Similar to the linear theory, we assume that these expansions provide suitable approximations for the equilibrium quantities in the region embracing the ideal resonant position, which is much wider than the dissipative layer. This implies that there are two overlap regions on both sides of the dissipative layer where both the outer (the solution of linear ideal MHD) and inner solutions (the solution of nonlinear dissipative MHD) are valid. The two solutions have to coincide in the overlap regions, which provides the matching condition.

Let us proceed to the derivation of the governing equation. In the first step, we obtain the solution outside the dissipative layer. As has been pointed out, in this region, the wave dynamics is described by ideal linear MHD, so we set $N_{it} = 0$ and $D_{jt} = 0$ in (2.10)–(2.18).

The system of linearized ideal MHD equations can be reduced to a system of two coupled first-order partial differential equations for the radial component of the velocity u, and the Eulerian perturbation of the total pressure P:

$$D\frac{\partial(ur)}{\partial r} = C_1 ur + \omega C_2 r \frac{\partial P}{\partial \theta}, \tag{3.1}$$

$$\omega r D \frac{\partial^2 P}{\partial r\,\partial \theta} = C_3 ur - \omega r C_1 \frac{\partial P}{\partial \theta}, \tag{3.2}$$

where

$$D = \rho_0 D_A D_C, \qquad D_A = \omega^2 - \omega_A^2, \qquad D_C = (c_S^2 + v_A^2)(\omega^2 - \omega_T^2), \tag{3.3}$$

and

$$C_1 = 2\omega^4 \frac{B_{0\varphi}^2}{\mu r} - 2\frac{m f_B B_{0\varphi}}{\mu r^2} D_C, \qquad C_2 = \omega^4 - \left(\frac{m^2}{r^2} + k^2\right) D_C,$$

$$C_3 = D\left[\rho D_A \frac{\partial^2}{\partial \theta^2} + \frac{2B_{0\varphi}}{\mu}\frac{d}{dr}\left(\frac{B_{0\varphi}}{r}\right)\right] - 4\omega^4\left(\frac{B_{0\varphi}^2}{\mu r}\right)^2 + \frac{4\rho_0 D_C \omega_A^2}{\mu r^2} B_{0\varphi}^2. \tag{3.4}$$

All the other variables can be calculated in terms of these two variables:

$$\frac{\partial v_\parallel}{\partial \theta} = \frac{\omega f_B c_S^2}{\rho_0 B_0 D_C}\frac{\partial P}{\partial \theta} + \frac{2B_0 B_{0\varphi}}{\mu r D}\left(\frac{\omega^2 B_{0z} g_B}{\mu \rho_0} + \frac{m}{r} c_S^2 D_A\right) u, \tag{3.5}$$

$$v_\perp = \frac{g_B \omega}{\rho_0 B_0 D_A} P, \qquad b_r = -\frac{f_B}{\omega} u, \tag{3.6}$$

$$\frac{\partial b_\parallel}{\partial \theta} = \frac{B_0(\omega^2 v_A^2 - \omega_A^2 c_S^2)}{\rho_0 v_A^2 D_C}\frac{\partial P}{\partial \theta}$$

$$+\frac{B_{0\varphi}\rho_0}{\omega f_B B_0 r D}[B_{0\varphi} f_B D_A(\omega^2 v_A^2 - c_S^2 D_A) - 2\omega^2 B_{0z} g_B D_C] u + \frac{u}{w}\frac{dB_0}{dr}, \tag{3.7}$$

$$\frac{\partial b_\perp}{\partial \theta} = -\frac{f_B g_B}{\rho_0 B_0 D_A}\frac{\partial P}{\partial \theta} + \frac{B_{0\varphi} B_{0z}}{r\omega B_0} u + \frac{B_{0\varphi}^2}{\omega}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right) u, \tag{3.8}$$

$$\frac{\partial p}{\partial \theta} = \frac{\omega^2 c_S^2}{D_C}\frac{\partial P}{\partial \theta} + \frac{B_{0\varphi}^2}{\omega^2 \mu r D_C}[\omega^2(c_S^2 - v_A^2) + \omega_A^2 c_S^2] - \frac{B_0}{\mu\omega} u \frac{dB_0}{dr}, \tag{3.9}$$

$$\frac{\partial \rho}{\partial \theta} = \frac{\omega^2}{D_C}\frac{\partial P}{\partial \theta} + \frac{2B_{0\varphi}^2 \omega}{\mu r D_C} u + \frac{u}{\omega}\frac{d\rho_0}{dr}. \tag{3.10}$$

Eliminating the pressure from (3.1) and (3.2), we obtain a second-order differential equation for the radial component of the velocity:

$$\frac{\partial}{\partial r}\left[f(r)\frac{\partial (ur)}{\partial r}\right] - g(r)\frac{\partial^2 (ur)}{\partial \theta^2} = 0, \tag{3.11}$$

where

$$f(r) = \frac{D}{rC_2}, \qquad g(r) = \frac{\partial}{\partial r}\left(\frac{C_1}{rC_2}\right) - \frac{1}{rD}\left(\frac{C_3}{\omega} - \frac{C_1^2}{C_2}\right). \tag{3.12}$$

If we eliminate the radial component of the velocity, we obtain a second order differential equation for the Eulerian perturbation of the total pressure

$$\frac{\partial}{\partial r}\left(\hat{f}(r)\frac{\partial P}{\partial r}\right) - \hat{g}(r)\frac{\partial^2 P}{\partial \theta^2} = 0, \tag{3.13}$$

where

$$\hat{f}(r) = \frac{rD}{C_3}, \qquad \hat{g}(r) = -\frac{\partial}{\partial r}\left(\frac{rC_1}{C_3}\right) - \frac{r}{D}\left(C_2 - \frac{C_1^2}{C_3}\right). \tag{3.14}$$

We are interested in the solutions of the system (3.1) and (3.2) in the vicinity of the slow-wave resonant point, i.e. at $r = r_c$ determined by the condition $\omega = \omega_T(r = r_c)$, where ω is the driver frequency. The waves that satisfy this condition are called *resonant waves*. We introduce a new radial variable s, defined by

$$s = r - r_c. \tag{3.15}$$

The point $s = 0$ (the condition for resonance) is a regular singular point of the system (3.1)–(3.2) and therefore we look for solutions in the form of a Fröbenius expansion around the resonant position $s = 0$. The solutions take the form

$$P = P_1(\theta) + P_2(\theta)s \ln|s| + P_3(\theta)s + \dots \tag{3.16}$$

and

$$u = u_1(\theta) \ln|s| + u_2(\theta) + u_3(\theta)s \ln|s| + \dots \quad . \tag{3.17}$$

Here dots $(\cdots)$ denote terms that are of higher order with respect to s. In general, the coefficient functions of θ in (3.16) and (3.17) are different for $s < 0$ and $s > 0$.

Using the relations (3.5)–(3.10), we eventually find that the perpendicular components of the velocity and magnetic field perturbation behave like the total pressure perturbation, so they are regular at $s = 0$. The other quantities are singular. The quantities u and b_r behave like $\ln|s|$, while the quantities $v_\parallel, b_\parallel, p$ and ρ have an s^{-1} singularity. These quantities are called large variables.

The outer solution is the so-called large-scale mode, because the transverse scale of this motion is of the order of the characteristic domain of interest (e.g. the diameter of a coronal loop). However, near the resonant position, in the *inner region*, the characteristics of the solutions are changed and the assumption of ideal MHD is no longer valid. In this region, we must include the dissipative, small-scale inner solution.

The magnitude of the dissipation is given by the total Reynolds number defined in Sec. 1, where the magnetic and viscous Reynolds numbers have the properties

$$R_m \sim \frac{1}{\bar{\eta}}, \qquad R_v \sim \frac{1}{\bar{\nu}}. \tag{3.18}$$

If f is a large variable with the dimension of velocity (e.g. the parallel component of the velocity relative to the equilibrium magnetic field lines), then a typical representation of the large nonlinear terms is of the form $f\partial f/\partial z$. The typical representation of large dissipative terms is of the form $\partial^2 f/\partial s^2$ multiplied by one of the dissipative coefficients. Since inside the dissipative layer the large variables are of the order of $f \sim \epsilon R^{1/3}$ and $\partial/\partial z \sim l_{\rm inh}^{-1}$, $\partial/\partial s \sim l_{\rm dis}^{-1}$, the ratio of the nonlinear terms to the dissipative terms is estimated to be

$$\frac{f\partial f/\partial z}{\bar{\nu}\partial^2 f/\partial s^2} \sim \epsilon R^{2/3}, \tag{3.19}$$

and $\epsilon R^{2/3}$ emerges as the nonlinearity parameter.

If the condition $\epsilon R^{2/3} \ll 1$ is satisfied, then linear theory gives an adequate description of the motions in the dissipative layer. However, for combinations of ϵ and R such that $\epsilon R^{2/3} \sim 1$, and definitely for $\epsilon R^{2/3} \gg 1$, nonlinearity has to be taken into account when studying resonant waves in the dissipative layer. Linear theory is a valid approximation for the description of the wave dynamics in the dissipative layer if the dissipative terms are much larger than the nonlinear terms, and so linear theory can be used if $\epsilon \ll R^{-2/3}$.

In linear theory, the terms describing dissipation in the MHD equations are retained inside the dissipative layer to remove the singularity. The nonlinear terms have to be taken into account in the dissipative layer if they are of the same order as or larger than the dissipative terms, so that $\epsilon R^{2/3} \sim 1$, i.e. $R \sim \epsilon^{-3/2}$. Linear studies of velocity scaling laws ($v \sim R^{1/3}$) indicate that the predicated velocities in the dissipative layer are several orders of magnitude larger than the observed nonthermal velocities if the linear results are scaled to match the observed heating rate. This leads to the suggestion that nonlinear effects, important in the dissipative layer, might enhance dissipation and alter the linear velocity scaling law. Therefore, according to (3.18), we can scale the dissipative coefficients as

$$\bar{\upsilon} = \epsilon^{3/2}\nu, \qquad \bar{\eta} = \epsilon^{3/2}\eta. \tag{3.20}$$

In order to obtain the solutions in the internal region, we introduce a new stretching variable in the system (2.10)–(2.18). The thickness of the dissipative layer is of the order of $l_{\text{inh}}R^{-1/3}$, and, since we assume that in the dissipative layer the nonlinear and dissipative terms are of the same order ($R \sim \epsilon^{-3/2}$), the new variable is $\tau = \epsilon^{-1/2}s$, i.e. $r = r' = r_c + \epsilon^{1/2}\tau$. The new form of the MHD equations is given in Appendix B.

The above equations contain $\epsilon^{1/2}$, so we use this quantity as an expansion parameter. To describe the expansion form for the variables in the dissipative layer, we have to analyze the form of the outer expansions given by (3.16) and (3.17). Since the quantities $v_\perp$, $b_\perp$ and P are regular in the vicinity of $s = 0$, their amplitudes in the dissipative layer have to be the same as the amplitudes outside this layer. Therefore, the expansion of these quantities is

$$f = \epsilon f^{(1)} + \epsilon^{3/2} f^{(2)} + \cdots \quad . \tag{3.21}$$

It is easy to verify that the amplitude of the large variables in the dissipative layer is of the order of $\epsilon^{1/2}$, so we can write the expansions for $v_\parallel$, $b_\parallel$, p and ρ in the form

$$h = \epsilon^{1/2} h^{(1)} + \epsilon h^{(2)} + \cdots \quad . \tag{3.22}$$

The quantities u and b_r have a $\ln|s|$ behavior near the resonant position $s = 0$, so they are of the order of $\epsilon \ln \epsilon$ in the dissipative layer, which means that we have to start the expansions of these quantities with this term. It was shown by Ruderman et al. (1997a) that the expansions (3.21) and (3.22) also contain terms proportional to $\epsilon^{3/2}\ln\epsilon$ and $\epsilon\ln\epsilon$. However, it was shown by Ballai et al. (1998a) that in the simplified version of the method of matched asymptotic expansions we can use $\ln|\epsilon| \ll \epsilon^{-\kappa}$ for any positive κ and $\epsilon \to +0$, and we consider $\ln\epsilon$ to be of the order of unity in the dissipative layer. This enable us to write the expansions for u and b_r in the form (3.21).

In the first-order approximation, we obtain a system of homogeneous linear equations for the variables with the superscript (1). The quantities that we need to express the variables of second order can be written with the aid of $P^{(1)}$, $u^{(1)}$ and $v_\parallel^{(1)}$ as

$$b_\parallel^{(1)} = -\frac{\omega f_B}{\omega_A^2} v_\parallel^{(1)}, \qquad b_r^{(1)} = -\frac{f_B}{\omega} u^{(1)}, \tag{3.23}$$

$$p^{(1)} = \frac{B_0 \omega \rho_0}{f_B} v_\parallel^{(1)}, \qquad \rho^{(1)} = \frac{B_0 \omega \rho_0}{c_S^2 f_B} v_\parallel^{(1)}. \tag{3.24}$$

The radial component of the momentum equation connects the derivative of the total pressure perturbation and the parallel component of the velocity,

$$\frac{\partial P^{(1)}}{\partial r} = \frac{2B_{0\varphi}^2 \omega \rho_0}{B_0 f_B r_c} v_\parallel^{(1)}, \tag{3.25}$$

and the equation that relates the normal and parallel components of velocity is

$$\frac{\partial u^{(1)}}{\partial \tau} + \frac{\omega^2}{\omega_A^2}\frac{f_B}{B_0}\frac{\partial v_\parallel^{(1)}}{\partial \theta} = 0, \tag{3.26}$$

where the equilibrium quantities are evaluated at the resonant point. We can see that for $B_{0\varphi} = 0$ we recover the results found for a straight equilibrium magnetic field by Ballai et al. (2000). Combining the last two equations, we obtain a conservation law similar to the linear theory, i.e.

$$\frac{dP^{(1)}}{d\theta} + \frac{2B_{0\varphi}^2}{\omega\mu r_c}u^{(1)} = \mathscr{C}^{(1)}(\theta), \tag{3.27}$$

where $\mathscr{C}^{(1)}(\theta)$ is the first term in a series expansion of the coefficient $\mathscr{C}(\theta)$, i.e. $\mathscr{C}(\theta) \approx \epsilon\mathscr{C}^{(1)}(\theta)$.

In the second-order approximation, we use only the equations obtained from (B 1), (B 3), (B 6), (B 8) and (B 9) and the relations (3.23)–(3.27), and we have

$$\begin{aligned}
&\rho_0\frac{\partial u^{(2)}}{\partial \tau} - \omega\frac{\partial \rho^{(2)}}{\partial \theta} + \frac{\rho_0 f_B}{B_0}\frac{\partial v_\parallel^{(2)}}{\partial \theta} \\
&\quad = \frac{\rho_0 m B_{0\varphi}}{B_0 r_c^2}\tau\frac{\partial v_\parallel^{(1)}}{\partial \theta} + \frac{f_B}{\mu v_A^2}\frac{dB_0}{ds}\tau\frac{\partial v_\parallel^{(1)}}{\partial \theta} + \frac{\omega^3\rho_0}{c_S^2\omega_A^2}v_\parallel^{(1)}\frac{\partial v_\parallel^{(1)}}{\partial \theta} - \frac{\rho_0}{r_c}u^{(1)} \\
&\quad - \frac{f_B v_A^2}{B_0(v_A^2 + c_S^2)}\frac{d\rho_0}{ds}\tau\frac{\partial v_\parallel^{(1)}}{\partial \theta} - \frac{\rho_0 g_B}{B_0}\frac{\partial v_\perp^{(1)}}{\partial \theta} - \frac{\rho_0}{B_0}\left(\frac{m}{r_c}\frac{dB_{0\varphi}}{ds} + k\frac{dB_{0z}}{ds}\right)\tau\frac{\partial v_\parallel^{(1)}}{\partial \theta} \\
&\quad - \frac{d\rho_0}{ds}u^{(1)} - \frac{B_0\omega\rho_0}{c_S^2 f_B}u^{(1)}\frac{\partial v_\parallel^{(1)}}{\partial \tau} - \frac{2\omega\rho_0}{c_S^2}v_\parallel^{(1)}\frac{\partial v_\parallel^{(1)}}{\partial \theta},
\end{aligned} \tag{3.28}$$

$$\begin{aligned}
\omega\rho_0\frac{\partial v_\parallel^{(2)}}{\partial \theta} + \frac{f_B}{\mu}\frac{\partial b_\parallel^{(2)}}{\partial \theta} &= -\frac{\omega\rho_0 m B_{0\varphi}}{r_c^2 f_B}\tau\frac{\partial v_\parallel^{(1)}}{\partial \theta} - \omega\frac{d\rho_0}{ds}\tau\frac{\partial v_\parallel^{(1)}}{\partial \theta} \\
&= \frac{f_B}{B_0}\mathscr{C}(\theta) + \frac{\omega\rho_0}{f_B}\left(\frac{m}{r_c}\frac{dB_{0\varphi}}{ds} + k\frac{dB_{0z}}{ds}\right) \\
&\quad - \frac{f_B B_{0\varphi}^2}{B_0\mu\omega r_c}u^{(1)} + \frac{f_B}{\mu\omega}\frac{dB_0}{ds}u^{(1)} - \frac{\rho_0}{B_0^2}\nu\frac{\partial^2 v_\parallel^{(1)}}{\partial \tau^2},
\end{aligned} \tag{3.29}$$

$$\begin{aligned}
\omega\frac{\partial b_\parallel^{(2)}}{\partial \theta} - B_0\frac{\partial u^{(2)}}{\partial \tau} &= gB\frac{\partial v_\perp^{(1)}}{\partial \theta} - \frac{\omega^2 f_B}{B_0\omega_A^2}\frac{dB_0}{ds}\tau\frac{\partial v_\parallel^{(1)}}{\partial \theta} + \frac{dB_0}{ds}u^{(1)} \\
&\quad + \frac{B_{0z}^2}{B_0 r_c}u^{(1)} + \frac{f_B v_A^2}{\omega(c_S^2 + v_A^2)}u^{(1)}\frac{\partial v_\parallel^{(1)}}{\partial \tau} \\
&\quad - \frac{\omega B_0}{c_S^2 + v_A^2}v_\parallel^{(1)}\frac{\partial v_\parallel^{(1)}}{\partial \theta} + \eta\frac{\omega}{f_B v_A^2}\frac{\partial^2 v_\parallel^{(1)}}{\partial \tau^2},
\end{aligned} \tag{3.30}$$

$$\omega\left(\frac{\partial p^{(2)}}{\partial\theta}-c_S^2\frac{\partial\rho^{(2)}}{\partial\theta}\right)=-\frac{B_0\omega^2}{f_B}\frac{d\rho_0}{ds}\tau\frac{\partial v_\parallel^{(1)}}{\partial\theta}-\frac{\gamma\omega^2B_0B_{0\varphi}^2}{\mu c_S^2f_Br_c}\tau\frac{\partial v_\parallel^{(1)}}{\partial\theta}$$
$$-\frac{B_0}{\mu}\frac{dB_0}{ds}u^{(1)}-c_S^2\frac{d\rho_0}{ds}u^{(1)}-\frac{\gamma\omega^2B_0^2}{c_S^2f_B\mu}\frac{dB_0}{ds}\tau\frac{\partial v_\parallel^{(1)}}{\partial\theta}-\frac{B_{0\varphi}^2}{\mu r_c}u^{(1)}$$
$$+\frac{\omega^3v_A^2\rho_0}{\omega_A^2c_S^2}(\gamma-1)v_\parallel^{(1)}\frac{\partial v_\parallel}{\partial\theta}, \tag{3.31}$$

$$\frac{\partial p^{(2)}}{\partial\theta}+\frac{B_0}{\mu}\frac{\partial b_\parallel^{(2)}}{\partial\theta}=\frac{\omega\rho_0}{f_B}\frac{dB_0}{ds}\tau\frac{\partial v_\parallel^{(1)}}{\partial\theta}-\frac{\omega^2\rho_0}{\omega_A^2}v_\parallel\frac{\partial v_\parallel^{(1)}}{\partial\theta}+\mathscr{C}(\theta)-\frac{2B_{0\varphi}^2}{\omega\mu r_c}u^{(1)}. \tag{3.32}$$

In the above equations, all equilibrium quantities and their derivatives are calculated at the resonant position $s=0$.

The left-hand sides of the set of equations (3.28)–(3.32) can be obtained from the left-hand sides of the corresponding equations of the first-order approximation by substituting variables with superscripts (2) for the corresponding variables with superscripts (1). Since the set of first-order approximation equations possess a nontrivial solution, (3.28)–(3.32) are compatible only if their right-hand sides satisfy a compatibility condition. In order to derive this condition, we first express $b_\parallel^{(2)}$ and $\rho^{(2)}$ in terms of $v_\parallel^{(2)}$ and variables of the first-order approximation using (3.29), (3.31) and (3.32). Subsequently, we introduce these expressions into (3.28) and (3.30). In this way, we obtain two equations with the same left-hand sides. Subtracting these two equations, we obtain an equation that connects $v_\parallel^{(1)}$ and $\mathscr{C}^{(1)}$, which constitutes the *compatibility equation* for the system obtained in the second-order approximation:

$$\Delta\tau\frac{\partial v_\parallel^{(1)}}{\partial\theta}-\frac{\omega^3B_0[(\gamma+1)v_A^2+3c_S^2]}{f_Bc_S^2(c_S^2+v_A^2)}v_\parallel^{(1)}\frac{\partial v_\parallel^{(1)}}{\partial\theta}+\omega\left(\nu+\frac{\omega_T^2}{\omega_A^2}\eta\right)\frac{\partial^2v_\parallel^{(1)}}{\partial\tau^2}$$
$$=\frac{\omega^3B_0}{\rho_0v_A^2f_B}\mathscr{C}^{(1)}(\theta), \tag{3.33}$$

where

$$\Delta=-\frac{d\omega_c^2}{ds}(s=0).$$

Similar to the $B_{0\varphi}=0$ case investigated by Ballai et al. (2000), the driving term is the quantity that does not change across the dissipative layer. Equation (3.33) is the nonlinear governing equation for the parallel velocity in the dissipative layer for slow resonant waves. The second term on the left-hand side is the nonlinear term and the third term is the dissipative term. The term on the right-hand side of (3.33) is determined by the solution outside the dissipative layer, and its form can be prescribed. Neglecting the driving term (necessary for resonance) in a homogeneous plasma, (3.33) becomes very close to another very important family of nonlinear wave equations describing solitons in dissipative media (e.g. the Korteweg–de Vries–Burgers and Benjamin–Ono–Burgers equations). The only difference is that in the present case, the dispersion is not present and the plasma is inhomogeneous.

Inspecting this formula and the result obtained by Ruderman et al. (1997a) and Ballai and Erdélyi (1998), we can conclude that the equation, first found by

Ruderman et al. (1997a), *has an universal character describing nonlinear resonant slow waves in the dissipative layer in static and steady-state isotropic plasmas*. It is independent of the actual structure of the geometry and magnetic field.

The resonant surface $s = 0$ can be considered as a surface of discontinuity when solving the system of equations that govern plasma motion outside the dissipative layer. Therefore, we have to calculate the jumps in the perturbation of the total pressure and radial component of the velocity.

The jump of a function $f(r)$ across the dissipative layer is defined by

$$[f] = \lim_{s\to+0}\{f(s) - f(-s)\}. \tag{3.34}$$

Let us introduce new dimensionless variables

$$\sigma = \delta_c^{-1}\epsilon^{1/2}\tau, \qquad q = \epsilon^{1/2}\frac{\omega\delta_c}{v_A^2}v_\parallel^{(1)}, \tag{3.35}$$

where δ_c, measures the thickness of the dissipative layer in isotropic plasmas and is defined by the condition that the first and third terms on the left-hand side of the governing equation (3.33) are of the same order. Its form is given by

$$\delta_c = \left\{\frac{\omega}{|\Delta|}\left(\bar{v} + \frac{\omega_T^2}{\omega_A^2}\bar{\eta}\right)\right\}^{1/3}. \tag{3.36}$$

Let r_0 be the characteristic width of the overlap regions to the left and right of the dissipative layer, where both the linear ideal and the nonlinear and dissipative MHD equations with coefficients approximated by the first nonzero terms of the Taylor expansions are valid. The main property of the variable σ introduced in (3.35) is that $\sigma = O(1)$ in the dissipative layer, while $|r| \to r_0$ corresponds to $|\sigma| \to \infty$. In agreement with the matching procedure, the inner and outer solutions have to be the same in the overlap regions. This condition provides us with another expression for the jump in the function $f(r)$ across the dissipative layer:

$$[f] = \lim_{\sigma\to\infty}\{f(\sigma) - f(-\sigma)\}. \tag{3.37}$$

In the new variables, the governing equation becomes

$$\sigma\frac{\partial q}{\partial\theta} + \Lambda q\frac{\partial q}{\partial\theta} - \frac{\partial^2 q}{\partial\sigma^2} = -\frac{\omega^4 B_0}{v_A^4 f_B\rho_0|\Delta|}\mathscr{C}(\theta), \tag{3.38}$$

where we have used the fact that outside the dissipative layer the approximations $u \approx \epsilon u^{(1)}$ and $\mathscr{C}(\theta) \approx \epsilon\mathscr{C}(\theta)^{(1)}$ are valid.

In order to derive the two connection formulae, we introduce the new variables in the relations (3.25) and (3.26), and, using (3.37), we finally obtain

$$[P] = \frac{2B_{0\varphi}^2 B_0}{\mu r_c f_B}\mathscr{P}\int_{-\infty}^{\infty} q\,d\sigma, \tag{3.39}$$

and

$$[u] = -\frac{\omega f_B v_A^2}{\omega_A^2 B_0}\mathscr{P}\int_{-\infty}^{\infty}\frac{\partial q}{\partial\theta}d\sigma, \tag{3.40}$$

where we have used the symbol for the Cauchy principal part $\mathscr{P}$ since the integrals are divergent at infinity. We can see that for a magnetic field with straight lines ($B_{0\varphi} = 0$), the jump in the total pressure becomes zero, i.e. this quantity is *conserved* across the singularity. These two equations are the nonlinear analogues of the connection formulae for the radial component of the velocity and the total pressure perturbation obtained in linear theory by Sakurai et al. (1991). However, in contrast to linear theory, where the jump conditions were given in terms of equilibrium quantities and the perturbation of the total pressure, the nonlinear connection formulae are given in term of integrals of an unknown function q. Therefore, we have to solve simultaneously the system (3.1), (3.2) describing wave motion *outside* the dissipative layer and the equation (3.38) describing wave dynamics *inside* the dissipative layer, with (3.39) and (3.40) providing boundary conditions for the problem.

It is straightforward to see from (3.38) that the nonlinearity parameter, which is the ratio of the second term to the third term in (3.38), is

$$\lambda = O(\epsilon^{1/2} R^{1/3}), \tag{3.41}$$

which means that the nonlinearity is important if $\lambda \geqslant 1$; otherwise, the nonlinear term in (3.38) can be neglected, and the system is described fully by linear theory. This result is in perfect agreement with the result obtained in Sec. 2 from qualitative considerations.

4. Conclusions

The aim of the present paper was to study analytically the nonlinear behavior of resonant slow MHD waves in *twisted* magnetic flux tubes. The present study is a natural extension of the work of Ballai et al. (2000).

The applied scaling method divides the domain of interest into two regions, where the wave behavior and dynamics are governed by different sets of equations.

In the outer domain, i.e. outside the dissipative layer, the wave motion is described by the ideal linear MHD equations, which can be reduced to a pair of coupled first-order partial differential equations derived, for example, for the radial component of the velocity and the total pressure perturbation.

In the inner domain, i.e. in the dissipative layer, the wave dynamics is governed by an inhomogeneous nonlinear partial differential equation derived for the parallel component of the velocity, where the inhomogeneous part originates from the driving term. Since the dissipative layer embracing the resonant surface is very narrow, it can be considered as a surface of discontinuity when solving the governing partial differential equations outside the dissipative layer. The *connection formulae* obtained for example for the total pressure perturbation and the radial component of the velocity give the jumps in these quantities across the dissipative layer, thereby providing boundary conditions at the resonant surface of discontinuity. In contrast to the case with a straight equilibrium magnetic field, the Eulerian perturbation of the total pressure is no longer a conserved quantity. Instead, we have found the conserved quantity to be a combination of the total pressure and the radial component of the velocity, similar to its counterpart in linear theory.

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Appendix A. The nonlinear and dissipative terms in the governing equations outside the dissipative layer

The nonlinear quantities $N_{it} (i = 1, \ldots, 9)$ in the case of a twisted equilibrium magnetic field are as follows:

$$N_{1t} = -u\frac{\partial \rho}{\partial r} - \frac{1}{B_0}\left(f_B\frac{\partial(v_\parallel \rho)}{\partial \theta} + g_B\frac{\partial(v_\perp \rho)}{\partial \theta}\right) - \frac{\rho}{r}\frac{\partial(ur)}{\partial r}, \tag{A 1}$$

$$\begin{aligned} N_{2t} = {} & \rho\omega\frac{\partial u}{\partial \theta} - \bar{\rho}\left(u\frac{\partial u}{\partial r} + (f_B v_\parallel + g_B v_\perp)\frac{\partial u}{\partial \theta} - \frac{v^2}{r}\right) \\ & + \frac{1}{\mu B_0}(f_B b_\parallel + g_B b_\perp)\frac{\partial b_r}{\partial \theta} - \frac{b_\varphi^2}{\mu r} + \frac{b_r}{\mu}\frac{\partial b_r}{\partial r}, \end{aligned} \tag{A 2}$$

$$\begin{aligned} N_{3t} = {} & \rho\omega\frac{\partial v_\parallel}{\partial \theta} - \bar{\rho}\left[\frac{B_{0\varphi}^2}{B_0^2}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)uv_\perp + u\frac{\partial v_\parallel}{\partial r}\right] + \frac{f_B v_\parallel + g_B v_\perp}{B_0}\frac{\partial v_\parallel}{\partial \theta} \\ & + \frac{uvB_{0\varphi}}{rB_0} + \frac{b_r b_\varphi B_{0\varphi}}{\mu r B_0} + \frac{b_r}{\mu B_0}\left[\frac{B_{0\varphi}^2}{B_0}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)b_\perp + B_0\frac{\partial b_\parallel}{\partial r}\right] \\ & + \frac{f_B b_\parallel + g_B b_\perp}{\mu B_0}\frac{\partial b_\parallel}{\partial \theta}, \end{aligned} \tag{A 3}$$

$$\begin{aligned} N_{4t} = {} & \rho\omega\frac{\partial \omega}{\partial \theta} - \bar{\rho}\left[\frac{uvB_{0z}}{rB_0} - \frac{B_{0\varphi}^2}{B_0^2}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)uv_\parallel + u\frac{\partial v_\perp}{\partial r} + \frac{f_B v_\parallel + g_B v_\perp}{B_0}\frac{\partial v_\perp}{\partial \theta}\right] \\ & + \frac{b_r b_\varphi B_{0z}}{\mu r B_0} - \frac{B_{0\varphi}^2}{\mu B_0^2}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)b_r b_\parallel + \frac{b_r}{\mu}\frac{\partial b_\perp}{\partial r} + \frac{f_B b_\parallel + g_B b_\perp}{\mu B_0}\frac{\partial b_\perp}{\partial \theta}, \end{aligned} \tag{A 4}$$

$$N_{5t} = \frac{m}{r}\frac{\partial}{\partial \theta}(vb_r - ub_\varphi) - k\frac{\partial}{\partial \theta}(ub_z - wb_r), \tag{A 5}$$

$$\begin{aligned} N_{6t} = {} & \frac{B_{0z}}{B_0}(wb_r - ub_z) + v_\parallel\frac{\partial b_r}{\partial r} + \frac{b_r}{B_0}\left[\frac{B_{0\varphi}^2}{B_0}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)v_\perp + B_0\frac{\partial v_\parallel}{\partial r}\right] \\ & - \frac{u}{B_0}\left[\frac{B_{0\varphi}^2}{B_0}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)b_\perp + B_0\frac{\partial b_\parallel}{\partial r}\right] - b_\parallel\frac{\partial u}{\partial r} - \frac{g_B}{B_0}\frac{\partial}{\partial \theta}(b_\parallel v_\perp - b_\perp v_\parallel), \end{aligned} \tag{A 6}$$

$$\begin{aligned} N_{7t} = {} & \frac{B_{0\varphi}}{B_0}(wb_r - ub_z) - v_\perp\frac{\partial b_r}{\partial r} + b_\perp\frac{\partial u}{\partial r} + \frac{b_r}{B_0}\left[\frac{B_{0\varphi}^2}{B_0}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)v_\parallel - B_0\frac{\partial v_\perp}{\partial r}\right] \\ & - \frac{u}{B_0}\left[\frac{B_{0\varphi}^2}{B_0}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)b_\parallel - B_0\frac{\partial b_\perp}{\partial r}\right] - \frac{f_B}{B_0}\frac{\partial}{\partial \theta}(v_\perp b_\parallel - v_\parallel v_\perp), \end{aligned} \tag{A 7}$$

$$N_{8t} = \frac{1}{\rho_0}\left[\omega\left(\gamma p\frac{\partial\rho}{\partial\theta} - \rho\frac{\partial p}{\partial\theta}\right) + u\left(\rho\frac{dp_0}{dr} - \gamma p\frac{d\rho_0}{dr} + \bar{\rho}\frac{\partial p}{\partial r} - \bar{p}\gamma\frac{\partial\rho}{\partial r}\right)\right.$$

$$\left. - \left(\bar{p}\gamma\frac{\partial\rho}{\partial\theta} - \bar{\rho}\frac{\partial p}{\partial\theta}\right)\frac{f_B v_\parallel + g_B v_\perp}{B_0}\right], \tag{A 8}$$

$$N_{9t} = \frac{1}{2\mu}\frac{\partial}{\partial\theta}(b_r^2 + b_\varphi^2 + b_z^2). \tag{A 9}$$

The dissipative terms $D_i (i = 1, \ldots, 6)$ in the MHD equations are given by

$$D_{1t} = \bar{\rho}\bar{\nu}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \left(\frac{m^2}{r^2} + k^2\right)\frac{\partial^2 u}{\partial\theta^2} - \frac{2m}{r^2}\frac{\partial v}{\partial\theta} - \frac{u}{r^2}\right.$$

$$\left. + \frac{1}{3}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ur)}{\partial r} + \frac{m}{r}\frac{\partial v}{\partial\theta} + k\frac{\partial w}{\partial\theta}\right)\right], \tag{A 10}$$

$$D_{2t} = \bar{\rho}\bar{\nu}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) + \left(\frac{m^2}{r^2} + k^2\right)\frac{\partial^2 v}{\partial\theta^2} + \frac{2m}{r^2}\frac{\partial u}{\partial\theta} - \frac{v}{r^2}\right.$$

$$\left. + \frac{1}{3}\frac{m}{r}\left(\frac{1}{r}\frac{\partial^2(ur)}{\partial r\partial\theta} + \frac{m}{r}\frac{\partial^2 v}{\partial\theta^2} + k\frac{\partial^2 w}{\partial\theta^2}\right)\right], \tag{A 11}$$

$$D_{3t} = \bar{\rho}\bar{\nu}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \left(\frac{m^2}{r^2} + k^2\right)\frac{\partial^2 w}{\partial\theta^2} + \frac{k}{3}\left(\frac{1}{r}\frac{\partial^2(ur)}{\partial r\,\partial\theta} + \frac{m}{r}\frac{\partial^2 v}{\partial\theta^2} + k\frac{\partial^2 w}{\partial\theta^2}\right)\right], \tag{A 12}$$

$$D_{4t} = \bar{\eta}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial b_r}{\partial r}\right) + \left(\frac{m^2}{r^2} + k^2\right)\frac{\partial^2 b_r}{\partial\theta^2} - \frac{2m}{r^2}\frac{\partial b_\varphi}{\partial\theta} - \frac{b_r}{r^2}\right], \tag{A 13}$$

$$D_{5t} = \bar{\eta}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial b_\varphi}{\partial r}\right) + \left(\frac{m^2}{r^2} + k^2\right)\frac{\partial^2 b_\varphi}{\partial\theta^2} + \frac{2m}{r^2}\frac{\partial b_r}{\partial\theta} - \frac{b_\varphi}{r^2}\right], \tag{A 14}$$

$$D_{6t} = \bar{\eta}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial b_z}{\partial r}\right) + \left(\frac{m^2}{r^2} + k^2\right)\frac{\partial^2 b_z}{\partial\theta^2}\right]. \tag{A 15}$$

Appendix B. The modified MHD equations in the dissipative layer

$$-\omega\tau'\epsilon^{1/2}\frac{\partial\rho}{\partial\theta} + \bar{\rho}\left(\frac{\partial(\tau' u)}{\partial\tau} + \frac{m}{B_0}\epsilon^{1/2}\frac{\partial\Gamma_\parallel^\perp}{\partial\theta} + \frac{k}{B_0}\tau'\epsilon^{1/2}\frac{\partial\tilde{\Gamma}_\parallel^\perp}{\partial\theta}\right)$$

$$+ u\tau'\epsilon^{1/2}\frac{d\rho_0}{dr} + u\tau'\frac{\partial\rho}{\partial\tau} + \frac{m}{B_0}\epsilon^{1/2}\Gamma_\parallel^\perp\frac{\partial\rho}{\partial\theta} + \frac{k}{B_0}\tau'\epsilon^{1/2}\tilde{\Gamma}_\parallel^\perp\frac{\partial\rho}{\partial\theta} = 0, \tag{B 1}$$

$$-\omega\bar{\rho}\tau'\epsilon^{1/2}\frac{\partial u}{\partial\theta} + \bar{\rho}\left(u\tau'\frac{\partial u}{\partial\tau} + \frac{m}{B_0}\epsilon^{1/2}\Gamma_\parallel^\perp\frac{\partial u}{\partial\theta} + \frac{k}{B_0}\epsilon^{1/2}\tau'\tilde{\Gamma}_\parallel^\perp\frac{\partial u}{\partial\theta} - \frac{\epsilon^{1/2}}{B_0^2}\Gamma_\parallel^{\perp 2}\right)$$

$$= -\tau'\frac{\partial P}{\partial\tau} + \frac{1}{\mu}\left(-2\epsilon^{1/2}\frac{B_{0\varphi}}{B_0}\gamma_\parallel^\perp + \epsilon^{1/2}(mB_{0\varphi} + kB_{0z}\tau')\frac{\partial b_r}{\partial\theta}\right.$$

$$\left. + \frac{m\epsilon^{1/2}}{B_0}\gamma_\parallel^\perp\frac{\partial b_r}{\partial\theta} + \frac{k}{B_0}\epsilon^{1/2}\tau'\tilde{\gamma}_\parallel^\perp\frac{\partial b_r}{\partial\theta} + \tau' b_r\frac{\partial b_r}{\partial\tau} - \frac{\epsilon^{1/2}}{B_0^2}\gamma_\parallel^{\perp 2}\right), \tag{B 2}$$

$$
-\omega\bar{\rho}\epsilon^{1/2}\tau'\frac{\partial v_\parallel}{\partial\theta} + \bar{\rho}\left[u\tau'\frac{\partial v_\parallel}{\partial\tau} + \epsilon^{1/2}\tau'\frac{B_{0\varphi}^2}{B_0^2}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)uv_\perp\right.
$$

$$
\left.+\frac{m}{B_0}\epsilon^{1/2}\Gamma_\parallel^\perp\frac{\partial v_\parallel}{\partial\theta} + \frac{k}{B_0}\epsilon^{1/2}\tau'\tilde{\Gamma}_\parallel^\perp\frac{\partial v_\parallel}{\partial\theta} + u\frac{B_{0\varphi}}{B_0^2}\epsilon^{1/2}\Gamma_\parallel^\perp\right]
$$

$$
= -\frac{\epsilon^{1/2}}{B_0}(mB_{0\varphi} + k\tau' B_{0z})\frac{\partial P}{\partial\theta} + \frac{1}{\mu}\left[\epsilon^{1/2}(mB_{0\varphi} + k\tau' B_{0z})\frac{\partial b_\parallel}{\partial\theta} + \epsilon^{1/2}\frac{B_{0\varphi}^2}{B_0}b_r\right.
$$

$$
+\epsilon^{1/2}\tau'\frac{dB_0}{dr}b_r + \epsilon^{1/2}\frac{B_{0\varphi}^2}{B_0}\gamma_\parallel^\perp b_r + \frac{m}{B_0}\epsilon^{1/2}\gamma_\parallel^\perp\frac{\partial b_\parallel}{\partial\theta} + \tau' b_r\frac{\partial b_\parallel}{\partial\tau}
$$

$$
\left.+\epsilon^{1/2}\tau'\frac{B_{0\varphi}^2}{B_0^2}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)b_r b_\perp + \frac{k}{B_0}\epsilon^{1/2}\tau'\tilde{\gamma}_\parallel^\perp\frac{\partial b_\parallel}{\partial\theta}\right], \tag{B 3}
$$

$$
-\omega\bar{\rho}\epsilon^{1/2}\tau'\frac{\partial v_\perp}{\partial\theta} + \bar{\rho}\left[u\tau'\frac{\partial v_\perp}{\partial\tau} - \epsilon^{1/2}\tau'\frac{B_{0\varphi}^2}{B_0^2}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)uv_\parallel + \frac{m\epsilon^{1/2}}{B_0}\Gamma_\parallel^\perp\frac{\partial v_\perp}{\partial\theta}\right.
$$

$$
\left.+\frac{k}{B_0}\epsilon^{1/2}\tau'\tilde{\Gamma}_\parallel^\perp\frac{\partial v_\perp}{\partial\theta} + u\frac{B_{0z}}{B_0^2}\epsilon^{1/2}\Gamma_\parallel^\perp\right]
$$

$$
= -\frac{\epsilon^{1/2}}{B_0}(mB_{0z} - k\tau' B_{0\varphi})\frac{\partial P}{\partial\theta} + \frac{1}{\mu}\left[\epsilon^{1/2}(mB_{0\varphi} + k\tau' B_{0z})\frac{\partial b_\perp}{\partial\theta} + \epsilon^{1/2}\frac{B_{0\varphi}B_{0z}}{B_0}b_r\right.
$$

$$
-\epsilon^{1/2}\tau' B_{0\varphi}^2\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)b_r + \epsilon^{1/2}\frac{B_{0z}}{B_0^2}\gamma_\parallel^\perp b_r + \frac{m}{B_0}\epsilon^{1/2}\gamma_\parallel^\perp\frac{\partial b_\perp}{\partial\theta} + \tau' b_r\frac{\partial b_\perp}{\partial\tau}
$$

$$
\left.-\frac{\epsilon^{1/2}}{B_0^2}\tau' B_{0\varphi}^2\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)b_r b_\parallel + \frac{k}{B_0}\epsilon^{1/2}\tau'\tilde{\gamma}_\parallel^\perp\frac{\partial b_\perp}{\partial\theta}\right], \tag{B 4}
$$

$$
-\omega B_0\epsilon^{1/2}\tau'\frac{\partial b_r}{\partial\theta} = m\epsilon^{1/2}\frac{\partial}{\partial\theta}(u\gamma_\parallel^\perp - b_r\Gamma_\parallel^\perp)
$$

$$
-k\epsilon^{1/2}\tau'\frac{\partial}{\partial\theta}(b_r\tilde{\Gamma}_\parallel^\perp - u\tilde{\gamma}_\parallel^\perp) + \epsilon^{1/2}B_0(mB_{0\varphi} + k\tau' B_{0z}), \tag{B 5}
$$

$$
\begin{aligned}
-\omega\epsilon^{1/2}\tau'\frac{\partial b_\parallel}{\partial\theta} = &-\epsilon^{1/2}(mB_{0z} - k\tau' B_{0\varphi})\frac{\partial v_\perp}{\partial\theta} - \epsilon^{1/2}\tau'\frac{dB_0}{dr}u - \tau' B_0\frac{\partial u}{\partial\tau}\\
&-\epsilon^{1/2}\frac{B_{0z}^2}{B_0}u + \epsilon^{1/2}\frac{B_{0z}}{B_0^2}(b_r\tilde{\Gamma}_\parallel^\perp - u\tilde{\gamma}_\parallel^\perp)\\
&-\frac{\epsilon^{1/2}}{B_0}\frac{\partial}{\partial\theta}(v_\perp b_\parallel - v_\parallel b_\perp)(mB_{0z} - k\tau' B_{0\varphi})\\
&-\epsilon^{1/2}\tau'\frac{B_{0\varphi}^2}{B_0^2}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)(ub_\perp - b_r v_\perp) - \tau'\frac{\partial}{\partial\tau}(ub_\parallel - b_r v_\parallel),
\end{aligned} \tag{B 6}
$$

$$
\begin{aligned}
&-\omega\epsilon^{1/2}\tau'\frac{\partial b_\perp}{\partial\theta}\\
&= \epsilon^{1/2}(mB_{0\varphi}+k\tau' B_{0z})\frac{\partial v_\perp}{\partial\theta} + \epsilon^{1/2}\tau'\frac{B_{0\varphi}}{B_0}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right) - \epsilon^{1/2}\frac{B_{0\varphi}}{B_0^2}(b_r\tilde{\Gamma}_\parallel^\perp - u\tilde{\gamma}_\parallel^\perp)\\
&\quad + \frac{\epsilon^{1/2}}{B_0}\frac{\partial}{\partial\theta}(v_\perp b_\parallel - v_\parallel b_\perp)(mB_{0\varphi}+k\tau' B_{0z}) + \epsilon^{1/2}\tau'\frac{B_{0\varphi}^2}{B_0^2}\frac{d}{dr}\left(\frac{B_{0z}}{B_{0\varphi}}\right)(ub_\parallel - b_r v_\parallel)\\
&\quad -\tau'\frac{\partial}{\partial r}(ub_\perp - b_r v_\perp) + \epsilon^{1/2}\frac{B_{0\varphi}B_{0z}}{B_0}u, \qquad \text{(B 7)}
\end{aligned}
$$

$$
\left[u\tau'\frac{\partial}{\partial\tau} - (\omega\tau' - mv - k\tau' w)\epsilon^{1/2}\frac{\partial}{\partial\theta}\right]\frac{\bar{p}}{\bar{\rho}^\gamma} = 0, \qquad \text{(B 8)}
$$

$$
P = p + \frac{B_0}{\mu}b_\parallel + \frac{1}{2\mu}(b_r^2 + b_\parallel^2 + b_\perp^2), \qquad \text{(B 9)}
$$

where

$$
\Gamma_\parallel^\perp = v_\parallel B_{0\varphi} + v_\perp B_{0z}, \qquad \tilde{\Gamma}_\parallel^\perp = v_\parallel B_{0z} - v_\perp B_{0\varphi},
$$
$$
\tilde{\gamma}_\parallel^\perp = b_\parallel B_{0z} - b_\perp B_{0\varphi}, \qquad \text{(B 10)}
$$

References

Ballai, I. and Erdélyi, R. 1998 Resonant absorption of nonlinear slow MHD waves in isotropic steady plasmas – I. Theory. *Solar Phys.* **180**, 65.

Ballai, I., Ruderman, M. S. and Erdélyi, R. 1998a Nonlinear theory of slow dissipative layers in anisotropic plasmas. *Phys. Plasmas* **5**, 252.

Ballai, I., Erdélyi, R. and Ruderman, M. S. 1998b Interaction of sound waves with slow dissipative layers in anisotropic plasmas in the approximation of weak nonlinearity. *Phys. Plasmas* **5**, 2264.

Ballai, I., Erdélyi, R. and Goossens, M. 2000 Nonlinear theory of non-axisymmetric resonant slow waves in straight magnetic flux tubes. *J. Plasma Phys.* **64**, 579.

Davila, J. M. 1987 Heating of the solar corona by the resonant absorption of Alfvén waves. *Astrophys. J.* **317**, 514.

Erdélyi, R. 1997 Analytical solutions for cusp resonance in dissipative MHD. *Solar Phys.* **171**, 49.

Erdélyi, R. and Ballai, I. 1999 Resonant absorption of nonlinear slow MHD waves in isotropic steady plasmas. *Solar Phys.* **186**, 67.

Erdélyi, R. and Goossens, M. 1994 Viscous computations of resonant absorption of MHD waves in flux tubes by FEM. *Astrophys. Space Sci.* **213**, 273.

Erdélyi, R., Ballai, I. and Goossens, M. 2001 Nonlinear resonant absorption of fast magneto-acoustic waves due to coupling into slow continua in the solar atmosphere. *Astron. Astrophys.* **368**, 662.

Goossens, M. 1991 Magnetohydrodynamic waves and wave heating in nonuniform plasmas. In: *Advances in Solar System Magnetohydrodynamics* (ed. E. R. Priest and A. W. Hood). Cambridge University Press, p. 135.

Goossens, M. and Hollweg, J. V. 1993 Resonant behavior of MHD waves on magnetic flux tubes. IV – Total resonant absorption and MHD radiating eigenmodes. *Solar Phys.* **145**, 19.

Goossens, M. and Poedts, S. 1992 Linear resistive magnetohydrodynamic computations of resonant absorption of acoustic oscillations in sunspots. *Astrophys. J.* **384**, 148.

Goossens, M. and Ruderman, M. S. 1995 Conservation laws and connection formulae for resonant MHD waves. *Physica Scripta* **T60**, 171.

Goossens, M., Ruderman, M. S. and Hollweg, J. V. 1995 Dissipative MHD solutions for resonant Alfvén waves in 1-dimensional magnetic flux tubes. *Solar Phys.* **157**, 75.

Hollweg, J. V. 1988 Resonance absorption of solar p-modes by sunspots. *Astrophys. J.* **335**, 1005.

Hollweg, J. V. 1991 Alfvén waves. In: *Mechanism of Chromospheric and Coronal Heating* (ed. P. Ulmschneider, E. R. Priest and R. Rosner). Berlin: Springer-Verlag, p. 423.

Ionson, J. A. 1978 Resonant absorption of Alfvénic surface waves and the heating of solar coronal loops. *Astrophys. J.* **226**, 650.

Kuperus, M., Ionson, J. A. and Spicer, D. 1981 On the theory of coronal heating mechanisms. *Annu. Rev. Astron. Astrophys.* **19**, 7.

Lou, Y. Q. 1990 Viscous magnetohydrodynamic modes and p-mode absorption by sunspots. *Astrophys. J.* **350**, 452.

Narain, U. and Ulmschneider, P. 1990 Chromospheric and coronal heating mechanisms. *Space Sci. Rev.* **54**, 377.

Narain, U. and Ulmschneider, P. 1996 Chromospheric and coronal heating mechanisms II. *Space Sci. Rev.* **75**, 453

Ruderman, M. S. 2000 Interaction of sound waves with an inhomogeneous magnetized plasma in a strongly nonlinear resonant slow-wave layer *J. Plasma Phys.* **63**, 43.

Ruderman, M. S., Goossens, M. and Hollweg, J. V. 1997a Nonlinear theory of resonant slow waves in dissipative layers. *Phys. Plasmas* **4**, 75.

Ruderman, M. S., Hollweg, J. V. and Goossens, M. 1997b Nonlinear theory of the interaction of sound waves with an inhomogeneous magnetised plasma in the resonant slow wave layer. *Phys. Plasmas* **4**, 92.

Sakurai, T., Goossens, M. and Hollweg, J. V. 1991 Resonant behavior of MHD waves on magnetic flux tubes. I – Connection formulae at the resonant surfaces. *Solar Phys.* **133**, 227.

Zirker, J. B. 1993 Coronal heating. *Solar Phys.* **148**, 43.