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## Ambiguities in phase-shift analysis

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## 1 INTRODUCTION

One of the most important sources of information about the structure of particles, and the interactions between particles, is the scattering experiment. Much of what is now understood about the properties of particles, whether elementary or composite, has been learned from such experiments. Since the days of the classic experiment of Rutherford, in which the scattering of alpha particles from thin foils of different metals was measured, the experimental techniques have changed considerably, but the essence of the experiments has remained the same. A target is bombarded with a beam of incident particles, and the number of particles scattered in different directions is measured. The target, for obvious practical reasons, consists of long living particles, the incoming particles may be unstable, as long as they live long enough to give a beam of reasonable intensity at the target. Many of these unstable particles were in fact first discovered in scattering processes, and have to be produced in inelastic reactions.

If the interaction between target and incident particles is known (for example in scattering of particles from a given potential) one can calculate in principle, with the knowledge of quantummechanics, the scattering amplitude. This is a complex-valued function, which depends upon the energy of the incoming particle, and upon the scattering angle. The scattering amplitude can in turn be used to calculate the differential cross-section, and other measurable quantities, and thus the results of the scattering experiment can be predicted. In the analysis of experimental results this procedure is reversed. One first attempts to find the scattering amplitude from the differential cross-section and the constraints of unitarity, and once the amplitude has been obtained one tries to deduce from it what the interaction has been. The last step of obtaining the interaction from the scattering amplitude is usually called the inverse problem. In this thesis we shall consider certain aspects of the analysis of scattering data, i.e. the determination of the scattering amplitude from the differential cross-section and unitarity.

The differential cross-section determines essentially only the modulus of the scattering amplitude  $F$ . Although it is interesting to speculate about the theoretical possibility of measuring the phase of  $F$ , this phase is seldom accessible with present experimental techniques. Thus the phase of  $F$  must be obtained from the unitarity relation. In recent years the understanding of the nature of this constraint has greatly improved. In particular, sufficient conditions under which a unitary scattering amplitude is unique, once the differential

cross-section has been specified for all physical scattering angles  $\theta$ , have been given (M69, N68, At72). These results on uniqueness are valid for scattering of spinless particles in the energy-region below the first inelastic threshold, and even there only for a rather restricted class of differential cross-sections. Below the first inelastic threshold one can have isolated or discrete ambiguities, which can be constructed explicitly for certain cross-sections. Above the first inelastic threshold the constraints of unitarity are less severe, and one can have a continuum ambiguity, as well as discrete ambiguities. By a continuum ambiguity we mean that it is possible to change the scattering amplitude continuously, while each of the amplitudes constructed in this way corresponds to precisely the same differential cross-section. These continuous changes are subject to certain restrictions, besides unitarity, which we shall discuss in section III.4.

We have referred in the previous paragraph to an idealized situation. We have assumed that the differential cross-section is known for all scattering angles in the physical region without any experimental uncertainty whatsoever. In any actual experiment it will be impossible to obtain such a perfect knowledge of the differential cross-section. In practice the scattering amplitude is obtained from the experimental data by least-square fits. This introduces another kind of non-uniqueness because in general many fits can be found that are equivalent in a statistical sense. Also one has to choose a parametrization of the amplitude that reduces the number of parameters to a number that can be handled numerically. This introduces a bias in the analysis, since in fits to the data one will not obtain any solutions that cannot be represented by the chosen parametrization. Because of these practical limitations one cannot expect to find all amplitudes that correspond to the same differential cross-section by fits to the data, but within the chosen parametrization one can expect to find a large number of possible scattering amplitudes.

We shall consider in this thesis mainly the non-uniqueness that exists in the idealized case of perfect data. In particular we shall present methods for the construction of amplitudes that are analytic within a unifocal ellipse in the  $\cos\theta$ -plane, where  $\theta$  is the scattering angle in the centre-of-mass system, and that correspond to the same differential cross-section as a specified scattering amplitude. We shall also consider separately the smaller class of amplitudes that are polynomials in  $\cos\theta$ . We treat only the cases of scattering of spinless particles, and of spin 0 - spin  $\frac{1}{2}$  scattering, but generalizations to processes in which higher spins are involved are possible.

In chapter II we shall present a short introduction to phase-shift analysis and we gather those properties of scattering amplitudes that are employed in the rest of the thesis. The methods for the construction of ambiguities are given in chapter III. It has been known for a long time that certain

discrete ambiguities exist in phase-shift analysis. It is for instance a well-known fact that one can change the sign of the real part of the scattering amplitude for scattering of spinless particles to obtain a new amplitude that fits the data equally well. This reflection ambiguity, and similar ambiguities for spin 0 - spin  $\frac{1}{2}$  scattering, are given in section III.1. The presently known results on existence and uniqueness of a scattering amplitude corresponding to a given cross-section are summarized in section III.2. We then go on to consider polynomial amplitudes for scattering of spinless particles. Even though one knows that the scattering amplitude is certainly not a polynomial, it might be well approximated by an expansion in  $\cos\theta$  with a finite number of terms. Polynomial approximations of the amplitude are often used in the actual analysis of scattering data. We present a general method, first given by Gersten (Ge69) to obtain all polynomials of the same degree that have the same modulus. The scattering amplitudes that one can obtain by this method do not automatically satisfy unitarity. In the same section we construct examples of ambiguities in the energy-region below the first inelastic threshold. The method of obtaining continuum ambiguities is set up in such a way that inelastic unitarity is satisfied by construction. We present this method in section III.4. The method guarantees that the amplitudes one obtains are analytic in a unifocal ellipse in the  $\cos\theta$ -plane. In section III.5 we generalize these results to the case of spin 0 - spin  $\frac{1}{2}$  scattering.

The reason for the continuum ambiguity in the inelastic region is that there is in each partial wave an unknown contribution to the unitarity equation. It can be shown that the unitarity equation defines the absorptive part of the amplitude,  $A(\cos\theta)$ , as an implicit function of the inelasticities. In the same manner one can also consider the differential cross-section itself to be a parameter, and one can then show that  $A$  is an implicit function of both  $\sigma$ , the cross-section, and the inelasticities  $I_\ell$ . We employ this fact in chapter IV, to show that it is possible to change the differential cross-section continuously and to obtain, by an iterative procedure, the correspondingly altered amplitudes. In section IV.3 we consider applications of this method to polynomial amplitudes.

We have applied the methods of constructing discrete and continuum ambiguities to the case of scattering of alpha-particles. The results are given in chapter V. In alpha-alpha scattering there are no spin and isospin complications, so the methods of chapters III and IV can be applied. However, we do have to neglect the effect of the Coulomb interaction between the alpha particles. The Coulomb amplitude, in the non-relativistic form, has a branchpoint at  $\cos\theta=1$ , so that the full scattering amplitude, which is the sum of the Coulomb part and a modified nuclear scattering amplitude, is not analytic in a unifocal ellipse in the  $\cos\theta$ -plane. The implications of

neglecting the Coulomb contribution, and some methods of circumventing the complication of Coulomb interference, are discussed in section VI.1. In particular we show that the continuum and discrete ambiguities for amplitudes analytic in a unifocal ellipse in the  $\cos\theta$ -plane are resolved if the Coulomb interference term is treated correctly, and if the differential cross-section is known exactly in the physical region. Other methods that might be employed to resolve ambiguities are also considered in chapter VI. These methods require either more extensive measurements, particularly for the inelastic reactions, or more theoretical input.

To conclude this introduction we wish to make a remark about our notation. The scattering amplitude is a function of the energy of the incoming particles, and of the scattering angle. We shall usually denote the amplitude by  $F(k,z)$ , where  $k$  is the momentum in the centre-of-mass system (c.m.s), and  $z$  is the cosine of the scattering angle in the c.m.s. We shall often consider phase-shift analysis at a fixed energy, and we then suppress the explicit energy-dependence of  $F$  and the derived quantities like phase-shifts and partial waves in the equations. For convenience we use in some sections other arguments for  $F$  that are related to  $k$  and  $z$ . Such changes of arguments are not always mentioned in the text. We trust that this will not cause confusion to the reader.