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Single Spin Asymmetries from a Single Wilson Loop

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We study the leading-power gluon transverse-momentum-dependent distributions (TMDs) of relevance to the study of asymmetries in the scattering off transversely polarized hadrons. Next-to-leading-order perturbative calculations of these TMDs show that at large transverse momentum they have common dynamical origins but that in the limit of a small longitudinal momentum fraction x , only one origin remains. We find that in this limit, only the dipole-type gluon TMDs survive and become identical to each other. At small x , they are all given by the expectation value of a single Wilson loop inside the transversely polarized hadron, the so-called spin-dependent odderon. This universal origin of transverse spin asymmetries at small x is of importance to current and future experimental studies, paving the way to a better understanding of the role of gluons in the three-dimensional structure of spin-polarized protons.

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Scattering off protons in high-energy particle collisions such as those performed at RHIC and LHC can be described as scattering off quarks and gluons inside the proton. As the energy of the scattering process increases, the gluons play an increasingly important role, as reflected by a fast growing gluon density inside the proton. The limit of high gluon density is the one in which the gluons carry only a very small fraction x of the longitudinal momentum of the proton. It is expected on theoretical grounds that the gluon density will not increase without bound towards small x but rather that it will saturate. Because of the dominance of gluons over quarks and the expected gluon saturation, the description of scattering processes considerably simplifies in the small- x limit. A complication that remains in this limit though is that gluonic effects do not manifest themselves in the same way in all processes.

It has recently become clear that the transverse momentum distribution of gluons inside the proton is not a unique quantity. Different processes may probe different distributions and, thus, yield different answers. This has become apparent from studies of the unpolarized gluon distribution in the small- x regime [1–3] and independently from studies of spin effects in high-energy scattering processes. Scattering experiments involving a spin-polarized proton exhibit large asymmetries in the production of final state particles [4–12]. Theoretical studies of these still largely understood single spin asymmetries (SSAs) led to the insight that transverse-momentum-dependent distributions (TMDs) of both quarks and gluons are sensitive to the flow of the color charge of quarks and gluons in a process and, hence, that they are, in general, process specific, i.e., nonuniversal [13]. TMD studies of the color flow dependence are generally not performed in the high gluon density

region. In Refs. [2,3], the two types of treatments were connected for the case in which neither the proton nor the gluons are spin polarized. For transversely polarized protons, the connection between the TMD and small- x formalisms has, so far, not been made. This is our aim here.

We will study the (T -odd) gluon TMDs inside a proton that is polarized transversely to its momentum direction and consider the limit of high gluon density or small- x fraction. We then connect the results to those that arise in a small- x treatment and observe that the two pictures are fully compatible, despite the initial mismatch in the number of distributions. Surprisingly, unlike the unpolarized case, we find that there is, in fact, only one type of gluon correlation to consider in the transversely polarized case in the limit of small x , thereby reducing the high degree of nonuniversality [14] to a single, universal distribution. The distribution is linked to what has been discussed in the literature under the name of spin-dependent odderon. Although gluon-induced SSAs are likely smaller than valence quark ones, the universality of gluon effects in the transverse spin case is of high experimental interest, as it can be investigated at RHIC using collisions of polarized protons on heavy ions and possibly directly compared to data from proposed experiments at an electron-ion collider or a polarized fixed-target experiment at the LHC called AFTER@LHC.

The interplay of spin or TMD physics and small- x physics is a topical issue. Recent developments in this direction include modeling nuclear quark TMDs using quasiclassical methods [15–17], the study of linear gluon polarization in the small- x formalism [18,19], and the evolution of gluon distributions from moderate to low x [17,20–22]. Phenomenological studies of T -odd gluon

TMDs have been performed in Refs. [23–26] without including process dependence. Spin asymmetries in pA collisions have been investigated extensively [27–34]. The longitudinal proton spin distribution Δg or g_{1L} has been studied in the small- x regime in Refs. [35–37]. The transverse spin case turns out to be quite different.

This Letter is structured as follows. First, we present the calculation of the large transverse momentum tail of the gluon TMDs of relevance for single transverse spin asymmetries and consider the small- x limit. A reduction from three independent TMDs to just one is observed. Subsequently, we connect this distribution to the spin-dependent odderon distribution arising in small- x studies, finding full consistency among the results. We end with a discussion and conclusions.

The information on gluon TMDs inside a transversely polarized hadron (with spin vector S_T) is formally encoded in the following matrix element (for the sake of simplicity, we omit a soft factor in the properly defined gluon TMDs [38–40], as this does not affect the results of this work),

$$\Gamma^{\mu\nu[U,U']} = \frac{1}{xP^+} \int \frac{dy^- d^2y_T}{(2\pi)^3} e^{iky} \times \langle P, S_T | 2\text{Tr}[F_T^{\nu\mu}(0) U F_T^{\mu\nu}(y) U'] | P, S_T \rangle_{y^+=0}, \quad (1)$$

where U and U' are process-dependent gauge links in the fundamental representation. At leading power, this correlator can be parametrized by six independent tensor structures [41],

$$\begin{aligned} \Gamma^{\mu\nu} = & \delta_T^{\mu\nu} f_1^g - \left(\frac{2k_T^\mu k_T^\nu}{k_T^2} + \delta_T^{\mu\nu} \right) h_1^{\perp g} \\ & - \delta_T^{\mu\nu} \frac{\epsilon_{T\alpha\beta} k_T^\alpha S_T^\beta}{M} f_{1T}^{\perp g} - i \epsilon_T^{\mu\nu} \frac{k_T \cdot S_T}{M} g_{1T}^g \\ & - \frac{\tilde{k}_T^{\{\mu} S_T^{\nu\}} + \tilde{S}_T^{\{\mu} k_T^{\nu\}}}{2M} h_{1T}^g + \frac{\tilde{k}_T^{\{\mu} k_T^{\nu\}}}{k_T^2} \frac{k_T \cdot S_T}{M} h_{1T}^{\perp g}, \quad (2) \end{aligned}$$

where all six TMDs are functions of x and k_T^2 , $\epsilon_T^{\mu\nu} = \epsilon^{\rho\sigma\mu\nu} n_\rho p_\sigma$, with $\epsilon_T^{12} = 1$ and $\delta_T^{\mu\nu} = -g^{\mu\nu} + p^{\{\mu} n^{\nu\}}/p \cdot n$. We also used shorthand notations like $\tilde{k}_T^\nu = \epsilon_T^{\mu\nu} k_{T\mu}$. Note that the normalizations for gluon TMDs $h_1^{\perp g}$, h_{1T}^g , and $h_{1T}^{\perp g}$ are slightly different from the ones used in Refs. [41,42] because our results suggest they are a more natural choice.

The first two gluon TMDs f_1^g and $h_1^{\perp g}$ are the unpolarized and linearly polarized gluon distribution, respectively. Among the four transverse-spin-dependent gluon TMDs, the three T -odd gluon TMDs $f_{1T}^{\perp g}$, $h_{1T}^{\perp g}$, and h_{1T}^g are relevant for the single spin asymmetry studies. As mentioned, none of these TMDs is universal and should have a $[U, U']$ label, as in general, different processes probe matrix elements with different gauge links. Here we will restrict ourselves to the two most important cases, involving only single future

or past pointing staplelike gauge links denoted by $+$ and $-$, respectively. In the notation of Ref. [43], there are two T -odd combinations labeled with (f) and (d) , $\Gamma_{(f)}^{(T\text{-odd})} = (\Gamma^{[+,+\dagger]} - \Gamma^{[-,-\dagger]})/2$ and $\Gamma_{(d)}^{(T\text{-odd})} = (\Gamma^{[+,-\dagger]} - \Gamma^{[-,+\dagger]})/2$. For the unpolarized gluon distribution at small x , the first type is usually referred to as the Weizsäcker-Williams (WW) distribution, while the latter one is commonly known as the dipole gluon distribution [2,3]. Here we will also refer to TMDs with the superscript “ (f) ” as WW-type distributions and with a “ (d) ” as dipole-type gluon TMDs. The transverse moments of the (f) - and (d) -type functions are related to single gluon pole matrix elements with different color structures, f^{abc} and d^{abc} , respectively.

T -odd TMDs with more complicated link structures can, in principle, arise but do not in any currently known TMD-factorizing process. At small x , some processes can become effectively TMD factorizing, where additional distributions could enter [2]. These differ from the (f) - and (d) -type distributions by terms of subleading order in $1/N_c$, which can be calculated within a small- x formalism [31,33]. So if relevant at all, they can, to some extent, be related to the distributions considered here (also following the methods of Ref. [14]).

In analogy to the T -odd quark TMDs [44–46], all three T -odd gluon TMDs can be perturbatively calculated in the collinear twist-3 formalism at large transverse momentum. The hard coefficients entering in these expressions are usually different for different gauge links appearing in the gluon matrix element given in Eq. (1). Here we will present the results for the (f) - and (d) -type functions that are C even and C odd, respectively. The T -odd collinear twist-3 functions that appear in the large- k_T tail expressions are (chiral-even) quark-gluon and trigluon Qiu-Sterman functions [47–49], with matching C parity. Chiral-odd quark-gluon Qiu-Sterman functions are suppressed for gluon TMDs.

The perturbative calculation follows a similar procedure as in Ref. [44]; cf. Fig. 1. The WW-type gluon Sivvers function $f_{1T}^{\perp g}$ has been computed in the quark channel in terms of the quark-gluon Qiu-Sterman function $T_{F,q}$ in Ref. [50],

$$\begin{aligned} f_{1T}^{\perp g/q(f)}(x, k_T^2) = & C_1 \frac{M}{k_T^4} \int_x^1 \frac{dz}{z} \sum_{q+\bar{q}} \\ & \times \left\{ T_{F,q}(z, z) \frac{1 + (1 - \xi)^2}{\xi} \right. \\ & \left. - T_{F,q}(z, z - x) \frac{2 - \xi}{\xi} \right\}, \quad (3) \end{aligned}$$

where $\xi = x/z$ and $C_1 = (N_c/2)(\alpha_s/2\pi^2)$. The notation T_F is the same as in Ref. [44]. The $\sum_{q+\bar{q}}$ indicates that the sum runs over all quark flavors and antiflavors. Here, a factor $-g$ is included in the definition of the antiquark Qiu-Sterman function, such that it satisfies $T_{F,\bar{q}}(x_1, x_2) = T_{F,q}(-x_1, -x_2)$ [51]. The soft-gluon pole contribution (the first term within the

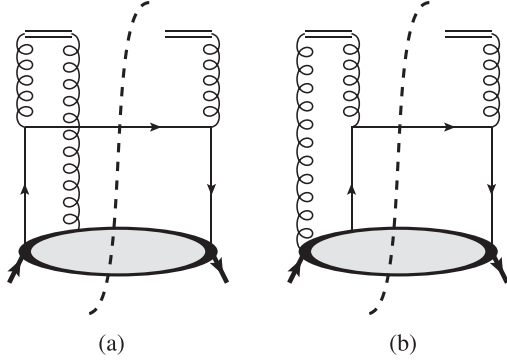


FIG. 1. Diagrams contributing to T -odd gluon TMDs at large transverse momentum in the flavor-singlet case. (a) Soft-gluon pole contribution. (b) Hard-gluon pole contribution. (Mirror diagrams are not shown.)

brackets) is generated by the diagram shown in Fig. 1(a), and the hard-gluon pole contribution (the second term) arises from Fig. 1(b). This expression can be related to the one in Ref. [52]. Throughout this Letter, we will neglect the contribution from the antisymmetric partner of the Qiu-Sterman function \tilde{T}_F [49], which becomes suppressed in the small- x regime, regardless of the gauge link structure, as it is antisymmetric in its two arguments (assuming it has no pole).

The gluon TMDs h_{1T}^g and $h_{1T}^{\perp g}$ can be calculated similarly at large transverse momentum. They turn out to possess the same perturbative tail $1/k_T^4$ behavior as the Sivvers function, only differing in the hard coefficients:

$$h_{1T}^{g/q(f)}(x, k_T^2) = C_1 \frac{M}{k_T^4} \int_x^1 \frac{dz}{z} \sum_{q+\bar{q}} \times \left\{ T_{F,q}(z, z) \frac{2-2\xi}{\xi} - T_{F,q}(z, z-x) \frac{2-\xi}{\xi} \right\}, \quad (4)$$

$$h_{1T}^{\perp g/q(f)}(x, k_T^2) = C_1 \frac{M}{k_T^4} \int_x^1 \frac{dz}{z} \sum_{q+\bar{q}} T_{F,q}(z, z) \frac{4-4\xi}{\xi}. \quad (5)$$

We note that the hard-gluon pole contribution to $h_{1T}^{\perp g}$ is absent.

We now extrapolate these results to the small- x limit. For the WW-type distributions, it is easy to see that in the small- x limit, the gluon TMDs $f_{1T}^{\perp g/q(f)}$ and $h_{1T}^{g/q(f)}$ vanish up to leading logarithm $\ln(1/x)$ accuracy, due to the cancellation among the soft-gluon and hard-gluon pole contributions. The same cancellation occurs at small x for the trigluon correlation contribution: $f_{1T}^{\perp g/g(f)} \approx h_{1T}^{g/g(f)} \approx 0$.

The case of $h_{1T}^{\perp g(f)}$ is different, however. Combining the small- x limit of the quark channel in Eq. (4) with the contribution of the gluon channel, it takes the form

$$h_{1T}^{\perp g(f)}(x, k_T^2) \approx C_1 \frac{M}{k_T^4} \frac{4}{x} \times \int_{x \rightarrow 0}^1 dz \left\{ \sum_{q+\bar{q}} T_{F,q}(z, z) + T_G^{(+)}(z, z) \right\}, \quad (6)$$

where $T_G^{(+)}$ is the C -even trigluon correlation [48,53,54]. This particular integral vanishes as a consequence of transverse momentum conservation, as it can be related (at tree level certainly [55] and the relation is stable under QCD corrections [51]) to the Burkardt sum rule for the first transverse momentum of the Sivvers TMD [56]. Therefore, for the $h_{1T}^{\perp g(f)}$ case, the leading logarithm contributions cancel out between the quark and gluon channels.

Now we consider the dipole case. Again all three TMDs can be dynamically generated by the Qiu-Sterman function, and possess the same perturbative tail $1/k_T^4$. The result for the gluon Sivvers function in the quark channel is

$$f_{1T}^{\perp g/q(d)}(x, k_T^2) = C_2 \frac{M}{k_T^4} \int_x^1 \frac{dz}{z} \sum_{q-\bar{q}} \times \left\{ T_{F,q}(z, z) \frac{1+(1-\xi)^2}{\xi} + T_{F,q}(z, z-x) \frac{2-\xi}{\xi} \right\}, \quad (7)$$

where $C_2 = [(N_c^2 - 4)/2N_c](\alpha_s/2\pi^2)$. The $\sum_{q-\bar{q}}$ indicates that in this C -odd case, the sum runs over all quark flavors minus antiflavors. Similarly, for the other two gluon TMDs, we find

$$h_{1T}^{g/q(d)}(x, k_T^2) = C_2 \frac{M}{k_T^4} \int_x^1 \frac{dz}{z} \sum_{q-\bar{q}} \times \left\{ T_{F,q}(z, z) \frac{2-2\xi}{\xi} + T_{F,q}(z, z-x) \frac{2-\xi}{\xi} \right\}, \quad (8)$$

$$h_{1T}^{\perp g/q(d)}(x, k_T^2) = C_2 \frac{M}{k_T^4} \int_x^1 \frac{dz}{z} \sum_{q-\bar{q}} \times T_{F,q}(z, z) \frac{4-4\xi}{\xi}. \quad (9)$$

It is worth noting that as compared to the WW-type distributions, the overall color factor is different, and the sign of the hard-gluon pole contributions is reversed. The complete expressions for the gluon channel (g/g) will be presented elsewhere. Here we only present the extrapolation to the small- x limit. In this limit, all three dipole-type T -odd gluon TMDs take the same form in both the quark and the gluon channel:

$$f_{1T}^{\perp g(d)} \approx h_{1T}^{g(d)} \approx h_{1T}^{\perp g(d)} \approx \frac{M^4}{k_T^4 x} \times \int_{x \rightarrow 0}^1 dz \left\{ C_2 \sum_{q-\bar{q}} T_{F,q}(z, z) + C_1 T_G^{(-)}(z, z) \right\}, \quad (10)$$

where $T_G^{(-)}$ is the C -odd trigluon correlation defined in Refs. [48,53,54]. The splitting kernel that appears in the above formula is identical to that for the ordinary unpolarized gluon distribution for the gluon-to-gluon channel. Therefore, from these large- k_T expressions, we conclude that the dipole-type T -odd gluon TMDs are not necessarily suppressed at small x with respect to the unpolarized gluon distribution (which grows very rapidly towards small x), but the WW-type T -odd gluon TMDs are. The additional $1/k_T^2$ suppression compared to the $1/k_T^2$ large- k_T tail of the unpolarized gluon distribution does not mean the TMDs are power suppressed at smaller k_T too.

The existence of leading logarithm contributions in the dipole case actually justifies a small- x treatment of T -odd gluon TMDs *without* the restriction of large k_T . Our starting point is the T -odd part of the *dipole*-type gluon TMD matrix element [we will suppress the label (d) from now on]:

$$\Gamma_{T\text{-odd}}^{\mu\nu}(x, k_T; S_T) = \frac{1}{xP^+} \int \frac{dy^- d^2 y_T}{(2\pi)^3} e^{iky} \times \left\{ \langle P, S_T | \text{Tr} [F_T^{+\nu}(0) U^{[+]} F_T^{+\mu}(y) U^{[-]\dagger} - F_T^{+\nu}(0) U^{[-]} F_T^{+\mu}(y) U^{[+]\dagger}] | P, S_T \rangle \right\} \Big|_{y^{\pm}=0}. \quad (11)$$

Next we approximate the exponential $e^{ik^+y^-}$ by 1. In Refs. [2,3], this is argued to be a good approximation as long as $k^+ = xP^+$ is very small, although corrections may affect the rapidity evolution, as recently discussed in Ref. [22]. After making this approximation, Eq. (11) can be reorganized as

$$\Gamma_{T\text{-odd}}^{\mu\nu}(x, k_T; S_T) = \frac{k_T^\mu k_T^\nu}{g^2 V x P^+} \int \frac{d^2 y_T}{(2\pi)^3} e^{ik_T y_T} \times \langle P, S_T | \text{Tr} [U^{[\square]}(0_T, y_T) - U^{[\square]\dagger}(0_T, y_T)] | P, S_T \rangle, \quad (12)$$

where $V = \int dy^-$, and $U^{[\square]}$ represents a rectangular Wilson loop with lightlike Wilson lines at transverse separation y_T . To arrive at this equation, we used translational invariance and

$$\partial_T^\mu U(y_T) = -ig \int_{-\infty}^{+\infty} dy^- \times U[-\infty^-, y^-; y_T] \partial_T^\mu A_+(y^-, y_T) U[y^-, \infty^-; y_T], \quad (13)$$

where $\partial_T^\mu A_+(y^-, y_T)$ is part of the gluon field strength operator $F_T^{+\mu}$. The $\partial_+ A_T^\mu(y^-, y_T)$ part corresponds to the transverse gauge link at light cone infinity, which can be neglected in a covariant gauge calculation [13]. The remaining part is power suppressed. One notices that $\text{Tr}[U^{[\square]}(0_T, y_T) - U^{[\square]\dagger}(0_T, y_T)]$ is, in fact, the dipole odderon operator [57]. The spin-dependent odderon has been considered in this way in Ref. [58] and in many studies of elastic scattering but without reference to TMDs; e.g., Refs. [59–61].

Next, we use that the matrix element of the odderon operator only has one possible S_T dependence. It follows that for a transversely polarized nucleon, the T -odd part of $\Gamma^{\mu\nu}$ can be parametrized by only one leading-twist tensor structure [58]:

$$\Gamma_{T\text{-odd}}^{\mu\nu}(x, k_T; S_T) = \frac{k_T^\mu k_T^\nu N_c \epsilon_T^{\alpha\beta} S_{T\alpha} k_{T\beta}}{2\pi^2 \alpha_s x M} O_{1T}^\perp(x, k_T^2), \quad (14)$$

where $O_{1T}^\perp(x, k_T^2)$ is identified as a spin-dependent odderon in Ref. [58]. This leads us to identify

$$\begin{aligned} & \frac{k_T^\mu k_T^\nu N_c \epsilon_T^{\alpha\beta} S_{T\alpha} k_{T\beta}}{2\pi^2 \alpha_s x M} O_{1T}^\perp(x, k_T^2) \\ &= -\delta_T^{\mu\nu} \frac{\epsilon_{T\alpha\beta} k_T^\alpha S_T^\beta}{M} f_{1T}^{\perp g} \\ & \quad - \frac{\tilde{k}_T^{\{\mu} S_T^{\nu\}} + \tilde{S}_T^{\{\mu} k_T^{\nu\}}}{2M} h_{1T}^g + \frac{\tilde{k}_T^{\{\mu} k_T^{\nu\}}}{k_T^2} \frac{k_T \cdot S_T}{M} h_{1T}^{\perp g}. \end{aligned} \quad (15)$$

In other words, the dipole-type T -odd gluon TMDs satisfy

$$x f_{1T}^{\perp g} = x h_{1T}^g = x h_{1T}^{\perp g} = \frac{-k_T^2 N_c}{4\pi^2 \alpha_s} O_{1T}^\perp(x, k_T^2), \quad (16)$$

which is the main result of this Letter. We have now obtained a consistent picture at small x involving only one independent TMD determined by the expectation value of the spin-dependent odderon. We conclude that this one universal function determined by the imaginary part of a closed Wilson loop should govern the single transverse spin asymmetries in $p^\uparrow p$ and $p^\uparrow A$ scattering at RHIC in the small- x regime. This description differs from SSAs involving the spin-independent odderon [16,29].

The spin-dependent odderon has been considered in Ref. [58] in the context of the McLerran-Venugopalan (MV) model [62]. The MV model describes the small- x distribution of gluons in the proton or nucleus as generated by a Gaussian distribution of color sources. The spin-dependent odderon can be obtained [58] by including terms

that are cubic in the color sources [57,63]. In Ref. [58], it is observed that in the extended MV model $O_{1T}^{\perp}(x, k_T^2)$ exhibits a node in k_T . Furthermore, our tail calculations suggest that $k_T^2 O_{1T}^{\perp}(x, k_T^2)$ should match onto a $1/k_T^4$ behavior at large k_T . About the x dependence, it was noted in Ref. [58] that the evolution of the odderon with increasing energy [64,65] suggests that the T -odd dipole gluon TMDs should fall off moderately with decreasing x as $x^{0.3}$ with respect to the unpolarized dipole gluon TMD (because the odderon has zero intercept; see, also, Ref. [66]). It should be feasible to test these expectations experimentally. For this purpose, one could study SSAs in a number of processes that in the small- x regime probe the dipole distributions; see, e.g., Ref. [3]. In $p^{\uparrow}A$ collisions, these are backward hadron production (as the odderon is C -parity odd, for gg -dominated scattering one should select final states that are not C even, i.e., $h^{\pm}X$ as opposed to $\pi^0 X$ or $\text{jet}X$), γ^* production, and γ^* jet production in the back-to-back correlation limit. BRAHMS data on SSAs in backward charged hadron production still allow for 10%-level asymmetries [10].

We end this Letter with some comments on the relation between the SSAs and parton orbital angular momentum. It is known that the large- x quark distribution in the transverse plane inside a transversely polarized proton is distorted [67]. This distortion is related to the nonzero orbital angular momentum of the quarks. As a consequence of this distortion, there will be a left-right asymmetric distribution of color sources resulting in an asymmetric distribution of gluons at small x . This explains the necessity of cubic source terms and the appearance of an odderon contribution in a transversely polarized proton. In Ref. [58], this idea was exploited to find a relation between the spin-dependent odderon size and the anomalous magnetic moments of the up and down quarks in the proton. Given these insights, it is now also natural to expect that the three dipole-type T -odd gluon TMDs for general x reflect features of the distortion in the transverse plane of the gluon distribution in a transversely polarized proton and as such are an indirect reflection of the presence of gluon orbital angular momentum.

In summary, we have calculated three leading-power T -odd gluon TMDs inside a transversely polarized hadron at large k_T and in the saturation regime. It has been found that the dipole-type T -odd gluon TMDs rise rapidly with decreasing x , whereas the WW-type ones are suppressed at small x . In deriving the latter, momentum conservation was seen to play an essential role. This aspect remains to be understood. The three dipole-type T -odd gluon TMDs become equal in the small- x limit and are determined by the spin-dependent odderon, which is given by the expectation value of a single Wilson loop. This leads to a surprisingly simple picture of transversely polarized hadrons at small x .

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