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*Published in:*

MED: 2009 17TH MEDITERRANEAN CONFERENCE ON CONTROL & AUTOMATION, VOLS 1-3

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*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2009

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Cao, M., Yu, C., & Anderson, B. D. O. (2009). Coordination with the leader in a robotic team without active communication. In *MED: 2009 17TH MEDITERRANEAN CONFERENCE ON CONTROL & AUTOMATION, VOLS 1-3* (pp. 252-257). IEEE (The Institute of Electrical and Electronics Engineers).

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# Coordination with the leader in a robotic team without active communication

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**Abstract**— We propose a coordination algorithm for robotic multi-agent systems with leader-follower structures so that when a leader moves with a constant velocity, its followers can compute the leader's velocity after measuring their distances to the leader for a finite number of times. One feature of the proposed algorithm is that no active communication is needed, and as a result, the algorithm becomes advantageous in the application of robotic sensor networks where energy efficient algorithms are highly desirable to maximize network lifespan. The algorithm makes use of the Cayley-Menger determinant which is a powerful tool from distance geometry. It is shown that the proposed algorithm has the potential to be applied to robotic swarms in a challenging scenario where each robot is installed with only range sensors and cannot measure the position of a target directly.

**Keywords:** autonomous agents, leader-follower structure, range measurement, Cayley-Menger determinant

## I. INTRODUCTION

Recent advances in sensor-equipped autonomous robotic platforms have enabled the evolution of sensing systems into large-scale, distributed, cooperative, mobile sensor networks. Correspondingly, there is an emerging trend worldwide to utilize robotic sensor networks to infrastructure security, environment and habitat monitoring, industrial sensing, traffic control and so on [1]. There has been much research in the control of robotic multi-agent systems for automating cooperative tasks, see e.g. [2]. In particular, for each robotic agent in a multi-agent team, various simple local cooperative strategies have been proposed in order to achieve the emergence of desired collective behavior in the group level [3], [4], [5].

In this context, it is of special interest to study information architectures within a robotic multi-agent system that can guarantee efficiency and adaptability when executing complex tasks in an unknown environment [6]. One popular choice of such information architectures is the so called leader-follower structure where a leader is chosen to guide its

fellow teammates to accomplish a given task. The advantage of the leader-follower structure is that while simple cooperative control strategies are utilized by the followers to keep the team adaptive and flexible in an complex and unknown environment, sophisticated control strategies and prioritized objectives can be designed into the leading agent. However, one shortcoming inherent in the leader-follower structure is that task-specific guiding information needs to be constantly propagated from the leader to its followers and this creates a heavy communication burden especially considering the stringent power constraints in sensor networks.

The goal of this paper is to design computational algorithms for the followers in a robotic team such that the leader's information can be inferred by the followers without active communication. To be more concrete, we consider an idealized but fundamentally important scenario where a leader is moving with a constant velocity and a follower is trying to figure out the leader's velocity. What makes this problem challenging is that in some of the practical applications, the follower may be installed with only range sensors for the consideration of cost and thus cannot easily localize a target in its own coordinate system. Furthermore, when no active communication is allowed in this scenario, the usual approach for the leader to constantly broadcast its position information is no longer an option. The approach that we propose to attack this problem is to let the follower repeatedly measure its distance to the leader at discrete time instants and then process the data using the Cayley-Menger determinant which is a convenient tool from distance geometry.

The rest of the paper is organized as follows. In Section II, we formulate the problem of coordination with the leader through repeated range measurements. The Cayley-Menger determinant and its related properties are introduced in Section III. We discuss the main coordination algorithm in Section IV and apply it to robotic swarms in Section V. Finally, concluding remarks are made in Section VI.

## II. PROBLEM FORMULATION

In this section, we formulate the problem of coordination without active communication that we want to study. We consider a leading agent  $l$  and a following agent  $f$  in the plane. Agent  $l$  is moving with a constant velocity  $v$  that is

Anderson is supported by the Australian Research Council under DP-0877562 and by National ICT Australia, which is funded by the Australian Government as represented by the Department of Broadband Communications and the Digital Economy and the Australian Research Council through the ICT Centre of Excellence program. Cao was supported by the Australian Research Council and NICTA during his visit to the Australian National University. Yu is supported by the Australian Research Council through an Australian Postdoctoral Fellowship under DP-0877562.

unknown to the follower  $f$ . Agent  $f$  cannot communicate with the leader  $l$  to acquire agent  $l$ 's position and can only measure the distance  $d(t)$  between itself and agent  $l$  through its range sensor. We assume that agents  $l$  and  $f$  are in generic positions; in other words, they do not coincide with each other in the beginning and agent  $f$ 's initial position is not in the line that is determined by agent  $l$ 's initial position and direction of movement. We also assume that agent  $l$  will be within the disc, which is centered at agent  $f$ 's initial position with radius to be agent  $f$ 's sensing range, for a sufficiently long time. Another assumption is that agent  $f$  always has a map of its own motion in its own coordinate system. The

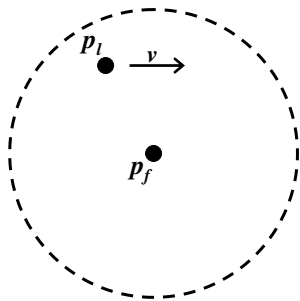


Fig. 1. The leader moving with constant velocity  $v$  is within the sensing range of the follower.

problem is to devise a computation algorithm for agent  $f$  that enables it to infer agent  $l$ 's velocity  $v$  after a minimum number of discrete measurements of  $d(t)$ .

There are a couple of remarks we want to make with respect to this formulation of the problem.

*Remark 1:* Algorithms that depend only on sensor range measurements are of particular importance in the design and application of robotic sensor networks [7], [8]. For the reader who is interested in more detail about range sensing technology in robotic systems, please refer to [9] and references therein.

*Remark 2:* In the formulation of the problem, we require that the leader  $l$  moves with constant velocity. This can be seen as an approximation of the leader's movement in a relative short period of time when the leader is doing some non-stationary but slow maneuvering. Also in the formulation of the problem, we have flexibility in designing the follower  $f$ 's motion. In fact agent  $f$  may stay stationary during the whole measurement and computation process, but as becomes apparent later in the paper, it is advantageous for agent  $f$  to carry out some controlled local movement when taking range measurements to solve the formulated problem efficiently.

*Remark 3:* It would be sufficient to solve the problem if we were to localize the moving leader  $l$  using range measurements. In fact some existing algorithms suggest that it is possible to achieve localization in this context [10], [7], [11]. However, this turns out to be unnecessary and there exists much simpler and computationally efficient approaches. In this paper, we will present a solution to the problem that does not involve localization computations and thus avoids solving

the associated nonlinear optimization problems. Of course, the calculations which are performed yield information that, together with measured data, would be enough to localize the moving leader if desired; but the calculations to do this would go beyond those required for our purposes.

The approach that we propose takes advantage of a powerful tool in distance geometry, called the Cayley-Menger determinant. This tool has been successfully applied by the authors to the sensor network localization problems [10]. In the next section, we give a brief review about its definition and main properties that are related to the computational approach that we propose later in the paper.

### III. CAYLEY-MENGER DETERMINANTS

We first look at the definition for the Cayley-Menger determinant.

#### A. Definition

The *Cayley-Menger Matrix* of two sequences of  $n$  points,  $\{\mathbf{p}_1, \dots, \mathbf{p}_n\}$  and  $\{\mathbf{q}_1, \dots, \mathbf{q}_n\} \in \mathbb{R}^m$ , is defined as

$$M(\mathbf{p}_1, \dots, \mathbf{p}_n; \mathbf{q}_1, \dots, \mathbf{q}_n) \triangleq \begin{bmatrix} d^2(\mathbf{p}_1, \mathbf{q}_1) & d^2(\mathbf{p}_1, \mathbf{q}_2) & \cdots & d^2(\mathbf{p}_1, \mathbf{q}_n) & 1 \\ d^2(\mathbf{p}_2, \mathbf{q}_1) & d^2(\mathbf{p}_2, \mathbf{q}_2) & \cdots & d^2(\mathbf{p}_2, \mathbf{q}_n) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ d^2(\mathbf{p}_n, \mathbf{q}_1) & d^2(\mathbf{p}_n, \mathbf{q}_2) & \cdots & d^2(\mathbf{p}_n, \mathbf{q}_n) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \quad (1)$$

where  $d(\mathbf{p}_i, \mathbf{q}_j)$ ,  $i, j \in \{1, \dots, n\}$  is the Euclidean distance between the points  $\mathbf{p}_i$  and  $\mathbf{q}_j$ . The *Cayley-Menger bideterminant* [12] of these two sequences of  $n$  points is defined as

$$D(\mathbf{p}_1, \dots, \mathbf{p}_n; \mathbf{q}_1, \dots, \mathbf{q}_n) \triangleq \det M(\mathbf{p}_1, \dots, \mathbf{p}_n; \mathbf{q}_1, \dots, \mathbf{q}_n) \quad (2)$$

This determinant is widely used in distance geometry theory [13], [12] which deals with Euclidean geometry in spaces where distance is defined and invariant. When the two sequences of points are the same,  $M(\mathbf{p}_1, \dots, \mathbf{p}_n; \mathbf{p}_1, \dots, \mathbf{p}_n)$  and  $D(\mathbf{p}_1, \dots, \mathbf{p}_n; \mathbf{p}_1, \dots, \mathbf{p}_n)$  are denoted for convenience by  $M(\mathbf{p}_1, \dots, \mathbf{p}_n)$  and  $D(\mathbf{p}_1, \dots, \mathbf{p}_n)$  respectively, and the latter is simply called a *Cayley-Menger determinant*.

The Cayley-Menger determinant provides another way of expressing the hyper-volume of a "simplex" by using only the lengths of the edges. A *simplex* of  $n$  points is the smallest  $(n - 1)$ -dimensional convex hull containing these points. The hyper-volume  $V$  of the simplex formed by the points  $\mathbf{p}_1, \dots, \mathbf{p}_n$  is given by [12]

$$V^2(\mathbf{p}_1, \dots, \mathbf{p}_n) = \frac{(-1)^n}{2^{n-1}((n-1)!)^2} D(\mathbf{p}_1, \dots, \mathbf{p}_n) \quad (3)$$

We can check equation (3) for the following low dimensional cases:

- For  $n = 2$ ,  $D(\mathbf{p}_1, \mathbf{p}_2) = 2d^2(\mathbf{p}_1, \mathbf{p}_2)$ , and  $V(\mathbf{p}_1, \mathbf{p}_2) = d(\mathbf{p}_1, \mathbf{p}_2)$ .
- For  $n = 3$ , the simplex is the triangle formed by  $\mathbf{p}_1$ ,  $\mathbf{p}_2$ , and  $\mathbf{p}_3$ . Then  $V(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$  is the area of this triangle. Let  $a$ ,  $b$ ,  $c$  be the lengths of the three edges of the

triangle, namely  $a = d(\mathbf{p}_1, \mathbf{p}_2)$ ,  $b = d(\mathbf{p}_2, \mathbf{p}_3)$ ,  $c = d(\mathbf{p}_3, \mathbf{p}_1)$ . Let  $s$  denote the semi-perimeter  $s = \frac{1}{2}(a + b + c)$ . Then from Heron's formula [14], we know that  $V(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \sqrt{s(s-a)(s-b)(s-c)}$ . Hence, it is easy to check that  $V^2(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \frac{-1}{16}D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ .

- For  $n = 4$ , the simplex is the tetrahedron formed by  $\mathbf{p}_1$ ,  $\mathbf{p}_2$ ,  $\mathbf{p}_3$ , and  $\mathbf{p}_4$ . We can obtain Euler's formula [15] relating the volume of a tetrahedron with its edge-lengths:  $V^2(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \frac{1}{288}D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)$ .

Now we present the main properties of the Cayley-Menger determinant.

### B. Properties of the Cayley-Menger determinant

The following theorem is a classical result on the Cayley-Menger determinant and is later generalized in [16].

*Theorem 1:* Consider an  $n$ -tuple of points  $\mathbf{p}_1, \dots, \mathbf{p}_n$  in  $m$ -dimensional space. If  $n \geq m+2$ , then the Cayley-Menger matrix  $M(\mathbf{p}_1, \dots, \mathbf{p}_n)$  is singular, namely

$$D(\mathbf{p}_1, \dots, \mathbf{p}_n) = 0 \quad (4)$$

A stronger statement can be made as follows in terms of the rank of the Cayley-Menger matrix.

*Theorem 2:* (Theorem 112.1 in [13]) Consider an  $n$ -tuple of points  $\mathbf{p}_1, \dots, \mathbf{p}_n$  in  $m$ -dimensional space with  $n \geq m+1$ . The rank of the Cayley-Menger matrix  $M(\mathbf{p}_1, \dots, \mathbf{p}_n)$  is at most  $m+1$ .

In fact, the rank of  $M(\mathbf{p}_1, \dots, \mathbf{p}_n)$  equals  $m+1$  if and only if at least  $m+1$  points of the  $n$  points are in generic positions. A similar statement made in terms of the cofactors of the Cayley-Menger determinant can be found in Corollary 1 of [16].

Using the main properties of the Cayley-Menger determinant, it is possible to describe the geometric relationships between relative distances by algebraic equations.

### C. Geometric relationships as algebraic equations

We consider four points  $p_0, p_1, p_2$  and  $p_3$  in the plane and the distances between them, as shown in Figure 2 below, are denoted by  $d_i$ ,  $i = 1, \dots, 6$ , respectively. From Theorem 1

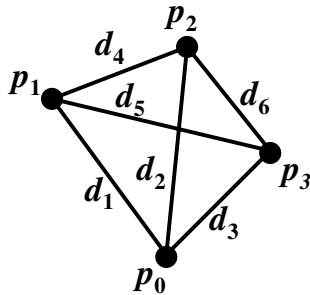


Fig. 2. Six distances between four points in the plane.

we know that  $D(\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = 0$ . Then the distances  $d_i$

must satisfy the following equation:

$$\det \begin{bmatrix} 0 & d_1^2 & d_2^2 & d_3^2 & 1 \\ d_1^2 & 0 & d_4^2 & d_5^2 & 1 \\ d_2^2 & d_4^2 & 0 & d_6^2 & 1 \\ d_3^2 & d_5^2 & d_6^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = 0 \quad (5)$$

The algebraic equation (5) provides a concise description about the geometric relationships for distances between points in the plane. This becomes essential in the development of the main algorithm in the next section. Similar algebraic expressions also exist for distances in three dimensional space. Since we have confined ourselves in the problem formulation in Section II to study two dimensional problems, we skip the results in three dimensional space in this paper.

## IV. MAIN ALGORITHM

In this section, we present the main algorithm to solve the leader-follower coordination problem using the Cayley-Menger determinant. For clarity of the discussion, we assume that the follower  $j$ 's range measurements are precise. The proposed algorithm consists of two steps. The first step is to compute the speed  $|v|$  of the leader; and the second step is to compute the direction of  $v$ .

### A. Computation of the speed $|v|$

It is obvious that agent  $f$  cannot determine  $|v|$  by just taking one range measurement. Then the questions are how many more range measurements are needed and when and how these additional measurements should be taken. By pure geometric arguments, one can easily see that it is still not sufficient to determine  $|v|$  by just taking two range measurements. Now we will show that it is possible for agent  $f$  to compute  $|v|$  by taking three range measurements. In fact, a more careful examination using tools in graph rigidity theory [6] reveals that three range measurements are indeed the minimum number of measurements required to have a unique solution of  $|v|$ . Since a detailed discussion on graph rigidity theory is beyond the scope of this paper, we have kept the discussion on the required minimum number of measurements in an intuitive level.

As indicated in Figure 3, we propose to let agent  $f$  measure, at a fixed position, the distances  $d_1$ ,  $d_2$  and  $d_3$  with respect to agent  $l$  at times 0,  $T$  and  $2T$  where  $T$  is some positive constant.

*Remark 4:* During the time interval  $[0, 2T]$ , agent  $f$  may remain stationary or hover around and come back to its initial position at  $T$  and  $2T$ . The latter situation applies to Dubins vehicle [17] type of mobile robots and those UAVs that cannot easily remain stationary in the air.

*Remark 5:* At the end of this sub-section, we will relax the constraint that the measurement times are equally spaced. Note that the motion of  $f$  is unconstrained between the measurement times. Note further that different motion requirements will be imposed on  $f$  in the second step of the algorithm when the direction of motion is determined.

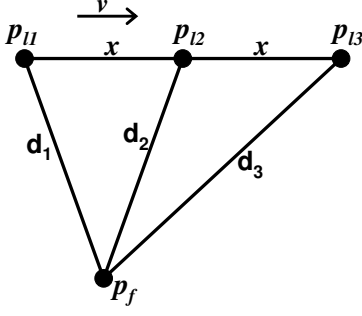


Fig. 3. The follower  $f$  measures the distances  $d_1$ ,  $d_2$  and  $d_3$  at times 0,  $T$  and  $2T$ .

Let  $x$  denote the distance traveled by agent  $l$  over a period of time  $T$ . Then agent  $l$ 's speed  $|v| = \frac{x}{T}$ . Let  $y = x^2$ ,  $l_1 = d_1^2$ ,  $l_2 = d_2^2$  and  $l_3 = d_3^2$ . Now consider the tetrahedron formed by  $\mathbf{p}_f$ ,  $\mathbf{p}_{l1}$ ,  $\mathbf{p}_{l2}$  and  $\mathbf{p}_{l3}$  as shown in Figure 3 and compare it with the tetrahedron shown in Figure 2. In view of equation (5), we can obtain the algebraic equation describing the geometric relationships between  $x$ ,  $d_1$ ,  $d_2$  and  $d_3$  as follows:

$$\det \begin{bmatrix} 0 & y & 4y & l_1 & 1 \\ y & 0 & y & l_2 & 1 \\ 4y & y & 0 & l_3 & 1 \\ l_1 & l_2 & l_3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = 0 \quad (6)$$

It can be computed that equation (6) is equivalent to

$$\begin{aligned} & -8y^3 + (8l_1 - 16l_2 + 8l_3)y^2 \\ & + (-2l_1^2 - 8l_2^2 - 2l_3^2 + 8l_1l_2 - 4l_1l_3 + 8l_2l_3)y = 0 \end{aligned}$$

There are three solutions to this cubic equation, which are  $y_1 = 0$  and  $y_2 = y_3 = \frac{l_1 - 2l_2 + l_3}{2}$ . Since  $x$  has to be positive, it must be true that

$$x = \sqrt{\frac{d_1^2 - 2d_2^2 + d_3^2}{2}} \quad (7)$$

Then there is a unique solution for speed  $|v|$  which is

$$|v| = \sqrt{\frac{d_1^2 - 2d_2^2 + d_3^2}{2}} / T \quad (8)$$

Equation (7) can be checked in special cases. For example, if agent  $f$ 's two range measurements at time 0 and time  $2T$  are equal to each other, i.e.  $d_1 = d_3$ , then we know that in Figure 3 the line determined by  $p_{l2}$  and  $p_f$  is perpendicular to the leader  $l$ 's linear trajectory determined by  $p_{l1}$  and  $p_{l3}$  because  $p_{l2}$  is the midpoint between  $p_{l1}$  and  $p_{l3}$ . Hence, in this case the value of  $x$  can be easily computed by using the Pythagoras' Theorem:

$$x = \sqrt{d_1^2 - d_2^2}$$

which agrees with equation (7) when  $d_1 = d_3$ .

In fact, the strategy for agent  $f$  discussed in this subsection can be further generalized. It is not needed for the agent  $f$  to measure its distances to the leader  $l$  at exactly the time instants 0,  $T$  and  $2T$ . Now assume agent  $f$  takes the

measurements in sequence at times  $T_1$ ,  $T_2$  and  $T_3$ . Then the distances traveled by agent  $l$  over  $[T_1, T_2]$  and  $[T_2, T_3]$  are  $(T_2 - T_1)|v|$  and  $(T_3 - T_2)|v|$  respectively. Again, let  $l_1 = d_1^2$ ,  $l_2 = d_2^2$  and  $l_3 = d_3^2$  and denote  $|v|^2$  by  $z$ . Then similar to equations (5) and (6), one can use the Cayley-Menger determinant again to write down an algebraic equation:

$$\det \begin{bmatrix} 0 & (T_2 - T_1)^2 z & (T_3 - T_1)^2 z & l_1 & 1 \\ (T_2 - T_1)^2 z & 0 & (T_3 - T_2)^2 z & l_2 & 1 \\ (T_3 - T_1)^2 z & (T_3 - T_2)^2 z & 0 & l_3 & 1 \\ l_1 & l_2 & l_3 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = 0 \quad (9)$$

From this equation, one can solve for  $z$  and then solve for  $|v|$ .

### B. Computation of the direction of $v$

In order to figure out the direction of movement of the leader, agent  $f$  has to perform some local movement and cannot remain stationary. Here we propose a possible maneuvering strategy that continues the discussion in the previous subsection where range measurements  $d_1$ ,  $d_2$  and  $d_3$  are taken at times 0,  $T$  and  $2T$ .

As indicated in figure 4, we require agent  $f$  to start a linear motion at time 0 with a given constant velocity  $s$ , take a measurement  $d_4$  at time  $T/2$  and then return to its initial position to take the planned measurement  $d_2$  at time  $T$ .

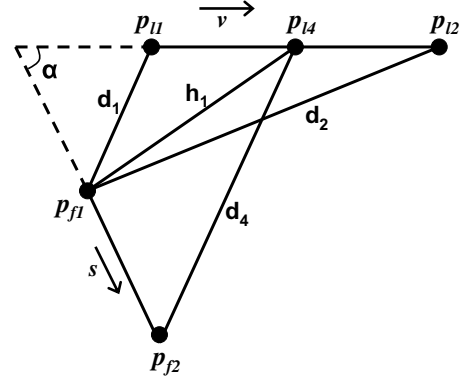


Fig. 4. Three measured distances  $d_1$ ,  $d_2$ ,  $d_4$  and one computed distance  $h_1$ .

After  $2T$ , we have obtained  $|v|$  using the algorithm discussed in the previous subsection. Then we can compute the distances  $|\mathbf{p}_{l1}\mathbf{p}_{l4}| = |\mathbf{p}_{l4}\mathbf{p}_{l2}| = |v|T/2$ . Now we examine the distances between the four points  $\mathbf{p}_{f1}$ ,  $\mathbf{p}_{l1}$ ,  $\mathbf{p}_{l4}$  and  $\mathbf{p}_{l2}$ . There is only one unknown  $|\mathbf{p}_{f1}\mathbf{p}_{l4}| \triangleq h_1$  which can be computed by solving the equation in the form of the Cayley-Menger determinant of the corresponding four points:

$$D(\mathbf{p}_{f1}, \mathbf{p}_{l1}, \mathbf{p}_{l4}, \mathbf{p}_{l2}) = 0.$$

Similar to the process of computing the value of  $x$  in (7), one can find that this equation gives us the solution

$$h_1 = \sqrt{\frac{d_1^2 + d_2^2}{2} - \frac{|v|^2 T^2}{4}}.$$

To find the direction of the velocity  $v$ , we need to solve for the value of the angle  $\alpha$  as indicated in Figure 4. We first note that in the triangle formed by the points  $\mathbf{p}_{f1}$ ,  $\mathbf{p}_{f2}$  and  $\mathbf{p}_{l4}$ , using the law of cosines, we have

$$\angle p_{f1}p_{f2}p_{l4} = \cos^{-1} \left( \frac{|s|^2 T^2 / 4 + d_4^2 - h_1^2}{|s| T d_4} \right)$$

and

$$\angle p_{f1}p_{l4}p_{f2} = \cos^{-1} \left( \frac{h_1^2 + d_4^2 - |s|^2 T^2 / 4}{2 h_1 d_4} \right).$$

Then applying the law of cosines to the triangle formed by the points  $\mathbf{p}_{f1}$ ,  $\mathbf{p}_{l1}$  and  $\mathbf{p}_{l4}$ , we have

$$\angle p_{f1}p_{l4}p_{l1} = \cos^{-1} \left( \frac{|v|^2 T^2 / 4 + h_1^2 - d_1^2}{|v| T h_1} \right)$$

Finally, we examine the triangle formed by the points  $\mathbf{p}_{f2}$ ,  $\mathbf{p}_{l4}$  and the intersection point of the lines  $\mathbf{p}_{f1}\mathbf{p}_{f2}$  and  $\mathbf{p}_{l1}\mathbf{p}_{l4}$ , we have

$$\alpha = \pi - \angle p_{f1}p_{f2}p_{l4} - \angle p_{f1}p_{l4}p_{l1}. \quad (10)$$

However, with only the value of  $\alpha$ , there is still a flip ambiguity for the direction of  $v$  with respect to the linear trajectory of the follower's movement. To see this, if we flip over the whole diagram in Figure 4 with respect to the line  $\mathbf{p}_{f1}\mathbf{p}_{f2}$ , all the computations still hold. To get rid of the flip ambiguity, the follower can take some range measurements while making a linear motion in a different direction in the time interval  $(T, 2T)$ . Then there must be a unique solution which fits all the range measurement data with respect to both of the two linear motion trajectories.

*Remark 6:* In fact, it is not necessary for the agent  $f$  to follow a linear trajectory in the second time interval  $(T, 2T)$ . Any motion that is not in the direction of  $s$  should be sufficient to get rid of the flip ambiguity in the direction of  $v$ .

The manoeuvring strategy discussed in this subsection is just one possible way to obtain the direction of  $v$ . There are other approaches and some of them may be especially simple in certain scenarios. For example, consider the case illustrated in Figure 5. When the agent  $f$  is moving with constant speed around a circle that contains its initial position, it may record  $k$ , the rate of change of the distances to the leader. If the measurements are made continuously in real time, it is possible to identify an angle, denoted by  $\beta$ , for which the value of  $k$  is the smallest at  $\beta$  and the largest at  $\pi + \beta$ . Then the direction determined by  $\beta$  in the follower  $f$ 's local coordinate system is the direction of motion of the leader  $l$ . Although such a simple scheme may become very useful in certain settings, the strategy making use of the Cayley-Menger determinant can be applied to a wider range of applications in general.

Combining the discussions in subsections IV.A and IV.B together, we have designed an algorithm for the follower  $f$  to obtain the velocity of the leader  $l$  using only range measurements without active communication. This algorithm

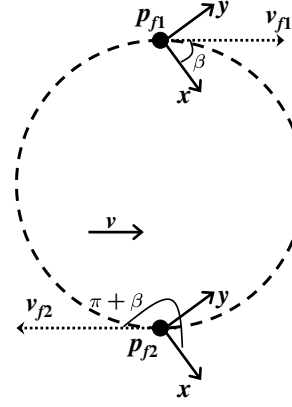


Fig. 5. Determine the direction of  $v$  by recording the rate of change of the distance to the leader.

is distributed because it does not need centralized coordination and only makes use of local information. Thus this algorithm can be easily applied to a scalable team of autonomous mobile agents. In the next section, we discuss briefly how the idea of coordination with the leader without active communication can be applied to design coordination rules for robotic swarms.

## V. APPLICATION IN ROBOTIC SWARMS

We now consider a team of mobile autonomous agents with a leader  $l$  that is moving with a constant velocity  $v$ . All the agents cannot communicate actively with one another. All the followers are equipped with range sensors and have no knowledge about the leader  $l$ 's velocity  $v$ . For a follower  $i$ , we say agent  $j$  is its neighbor if and only if agent  $j$  is within agent  $i$ 's sensing range. Thus the neighbor relationships in this robotic multi-agent system can be conveniently described by a directed graph where each vertex corresponds to an agent and there is a directed edge from vertex  $j$  to vertex  $i$  if and only if agent  $j$  is a neighbor of agent  $i$ . We say such a graph is connected if it is possible to reach all the vertices corresponding to the followers from the vertex corresponding to the leader by traversing directed edges of the graph. In this section, we want to show that, by adopting the strategy described in Section IV and under some mild assumption about the connectivity of the group, it is possible for all the followers to compute the velocity  $v$  after a finite time and as a result the whole group can move as a cohesive swarm with the same group velocity  $v$  in the end.

For the clarity of discussion, we assume that all the followers are initially stationary; and to keep the discussion general, we assume only a small subset of the followers can sense the leader directly. In order to figure out the velocity of its neighbors, each follower uses the strategy to follow two different straight lines over  $[0, T]$  and  $[T, 2T]$  and takes range measurements with respect to all its neighbors at time 0,  $T/2$ ,  $T$  and  $2T$  as suggested in Section IV. Note that the measurements at times 0,  $T$  and  $2T$  are made at the corresponding agent's initial position. We assume that all the agents are interconnected initially in such a way

that the neighbor relationships remain fixed during the local maneuvering period  $[0, 2T]$ . Suppose the computations of velocities are completed sufficiently fast, then at the end of this maneuvering period, leader  $l$ 's immediate followers can identify an agent that is moving with constant velocity for the whole period of  $[0, 2T]$ . Denote the set of these agents by  $\mathcal{F}_1$ . Then at  $2T$ , all the agents in  $\mathcal{F}_1$  know which neighboring agent is the leader as well as the velocity  $v$  and they start to move with the constant velocity  $v$ .

For all the followers that are not in the set  $\mathcal{F}_1$ , they start another maneuvering period at  $2T$ . Let  $\mathcal{F}_2$  be the set of agents who are immediate followers of  $\mathcal{F}_1 \cup \{l\}$  at  $2T$ . Then at  $4T$ , all the agents in  $\mathcal{F}_2$  can compute the velocity  $v$  and start to move with this velocity. If the whole group is connected all the time, then after a finite time, all the followers will be able to figure out  $v$  and move with the other agents in the group with the same velocity  $v$ . Hence, we have achieved a cohesive swarm of agents using only range measurements.

*Remark 7:* In the discussion, we have assumed that all the agents' clocks are synchronized. This requirement can be relaxed. We only need to have an upper bound for the differences between the rates of the agents' clocks. One only needs to make sure that  $T_m$ , the length of  $T$  measured by the slowest clock among the agents', should be less than  $2T_M$  where  $T_M$  is the length of  $T$  measured by the fastest clock among the agents'. If this is guaranteed, an agent will always be able to distinguish an agent moving with the desired velocity  $v$  from all the other fellow followers who are still doing local manoeuvring to figure out the velocity  $v$ . So the discussion in this section can be generalized to asynchronous situations.

In this coordination scheme, no location information is propagated, measured or computed. So we have achieved a strategy which not only requires no active communication, but needs no location information as well. It has been a challenging problem in the field of cooperative control of robotic teams to design coordination rules without using location information. The discussion in this section sheds some light on the possible ways to attack this problem by utilizing range measurements with tools in distance geometry.

## VI. CONCLUDING REMARKS

In this paper, we have discussed a coordination algorithm for autonomous robotic agents which enables a follower to compute the leader's velocity using only range measurements without active communication. The main idea is to utilize the Cayley-Menger determinant in distance geometry to express geometric relationships between distance measurements as algebraic equations. We also discussed briefly the potential to apply the proposed algorithm to a more general problem of coordinating robotic swarms with leader-follower information structures.

In the ongoing work, we are studying the coordination algorithms that take into account various measurement errors. Similar problems have been studied before in the context of sensor network localization. Tools in distance geometry

prove to be helpful in finding innovative solutions to this challenging problem. In the future, we will be interested in extending the results in this paper to the case where the leader moves with a slowly varying velocity. We will also try to study similar problems in three dimensional space.

## REFERENCES

- [1] C.-Y. Chong and S.P. Kumar. Sensor networks: Evolution, opportunities, and challenges. *Proceedings of the IEEE*, 91:1247–1256, 2003.
- [2] V. Kumar, N. E. Leonard, and A. S. Morse. *Cooperative Control*. Springer-Verlag New York, Inc., 2005.
- [3] W. Ren, R. W. Beard, and E. M. Atkins. Information consensus in multivehicle cooperative control: Collective group behavior through local interaction. *IEEE Control Systems Magazine*, 27:71–82, 2007.
- [4] Z. Lin, B. Francis, and M. Brouche. Local control strategies for groups of mobile autonomous agents. 49:622–629, 2004.
- [5] R. Olfati-saber, J. A. Fax, and R. M. Murray. Consensus and cooperation in multi-agent networked systems. *Proceedings of the IEEE*, 45:215–233, 2007.
- [6] B. D. O. Anderson, B. Yu, B. Fidan, and J. Hendrickx. Rigid graph control architectures for autonomous formations. *IEEE Control Systems Magazine*, 28:48–63, 2008.
- [7] P. Shiroma B. Grocholsky, E. Stump and V. Kumar. Control for localization of targets using range-only sensors. In *Proc. of the International Symposium on Experimental Robotics*, 2006.
- [8] M. Cao and A. S. Morse. Station keeping in the plane with range-only measurements. In *Proc. of the 2007 American Control Conference (ACC)*, pages 5419–5424, 2007.
- [9] M. Hebert. Active and passive range sensing for robotics. In *Proc. of the IEEE International Conference on Robotics and Automation (ICRA '00)*, pages 102 – 110, 2000.
- [10] M. Cao, B. D. O. Anderson, and A. S. Morse. Sensor network localization with imprecise distances. *System and Control Letters*, 55:887–893, 2006.
- [11] S. H. Dandach, B. Fidan, S. Dasgupta, and B. D. O. Anderson. A continuous time linear adaptive source localization algorithm, robust to persistent drift. *Systems and Control Letters*, 58:7–16, 2009.
- [12] G. M. Crippen and T. F. Havel. *Distance Geometry and Molecular Conformation*. John Wiley & Sons Inc., New York, 1988.
- [13] L. M. Blumenthal. *Theory and Applications of Distance Geometry*. Oxford University Press, 1953.
- [14] H. S. M. Coxeter. *Introduction to Geometry*, page 12. New York: Wiley, 2 edition, 1969.
- [15] D. M. Y. Sommerville. *Analytical Geometry of Three Dimensions*, page 36. Cambridge University Press, London, 1934.
- [16] L. Yang. Solving geometric constraints with distance-based global coordinate system. In *Proc. of the International Workshop on Geometric Constraint Solving*, 2003.
- [17] L. E. Dubins. On curves of minimal length with a constraint on average curvature and with prescribed initial and terminal positions and tangents. *American Journal of Mathematics*, 79:497C516, 1957.