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The Effects of Cartelization on Product Design

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Abstract

We consider the following model. First, two firms choose locations on a Hotelling line. Second, they play a repeated price-setting game, in which they may be able to collude. Transportation costs are quadratic. We show that if firms collude in the location stage, they choose locations that coincide with the social optimum, provided that the discount factor is high enough. If the discount factor is lower, the firms locate further apart. Furthermore, we show that if firms choose locations non-cooperatively, they both locate in the middle of the line, again provided that the discount factor is high enough. If the discount factor is lower, the firms locate further apart. Thus, with the possibility of a price cartel and a discount rate that is sufficiently high, Hotelling's principle of minimum differentiation is restored.

JEL Classification Codes: D43; L13; L41.

Keywords: Collusion; Product differentiation.

1   Introduction

In a seminal paper, Hotelling (1929) argued that firms tend to supply products that bear a close resemblance to each other. Hotelling considered a two-stage duopoly model in which consumers are uniformly distributed on a line of unit length, and firms first choose locations and then set prices. Using linear transportation costs for consumers, he solved for the

subgame perfect equilibrium[1] of this game and concluded that both firms choose to locate exactly in the middle of the line. Firms thus choose to produce identical products. This result was coined the principle of minimum differentiation by Boulding (1966). Hotelling argued that this principle readily follows from casual observation as well:

> "Buyers are confronted everywhere with an excessive sameness. [...] The tremendous standardisation of our furniture, our houses, our clothing, our automobiles and our education we due in part to the economies of large-scale production, in part to fashion and imitation. But over and above these forces is [...] the tendency to make only slight deviations in order to have for the new commodity as many buyers of the old as possible, to get, so to speak, between one's competitors and a mass of customers." (Hotelling 1929, pp. 54).

Exactly 50 years later, however, d'Aspremont et al. (1979) noted that there is a mistake in Hotelling's analysis. When locations are relatively close, a pure strategy equilibrium in prices does not exist. Firms then have an incentive to undercut their rival and capture the entire market, a possibility that Hotelling did not take into account.[2] To be able to find a clear-cut solution, these authors employ quadratic rather than linear transportation costs. Yet, under this assumption, they find that firms choose to locate at the endpoints of the line. Hence, we have maximum rather than minimum differentiation. This not only contradicts Hotelling's analysis, but also his casual observation.

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[1]Although, of course, the concept of subgame perfectness was only introduced almost half a century later (Selten, 1975).

[2]For a more technical discussion regarding the exact reason as to why an equilibrium in pure strategies in the price-setting stage of Hotelling's model does not always exist, see e.g. Economides (1984).

These models, and most of the subsequent literature, do assume however that firms will *compete* in prices after locations have been chosen. We introduce the possibility of a cartel to be formed after the location stage. In our model, we use quadratic transportation costs. Firms first choose locations, and then play a repeated price setting game. We show that when firms choose locations noncooperatively, the principle of minimum differentiation re-emerges, provided that the discount factor is high enough. Interestingly, this seems to imply that if Hotelling's casual observation was correct, we may conclude that the markets he referred to could be characterized as being collusive rather than competitive.

Admittedly, our paper is not the first to study the possibility of tacit collusion in a Hotelling framework. Yet, to the best of our knowledge, we *are* the first to study how the possibility of collusion affects the irreversible location choices of two firms that choose those locations noncooperatively. Chang (1991) and Ross (1992) study the stability of price collusion in the Hotelling model for given locations. They use quadratic transportation costs and delivered pricing, and employ grim trigger strategies. Hence, in these models, in equilibrium a firm sticks to an implicit cartel agreement if and only if both firms have always done so in the past. Häckner (1996) analyzes a similar model, but instead uses optimal punishment strategies in the sense of Abreu (1986). The case of linear transportation costs is studied by Rath (1998). Gupta and Venkatu (2002) use delivered pricing rather than mill pricing. A few papers endogenize the choice of location or product design. Häckner (1995) does so in a framework where firms can redesign in every period at negligible costs. In Chang (1992), relocation may also occur in every period, but in his model relocation is costly. The paper that perhaps is closest to ours is Gill (2002). In his model, firms make

an irreversible location choice at the beginning of the game, as they do in our model. Yet, Gill only considers the collusive location choice when a repeated price setting game follows. He does not consider noncollusive location choices, which is the main contribution of our paper.

The remainder of this paper is structured as follows. In section 2, we describe our model. Section 3 solves for the case in which firms explicitly collude in the location stage. This model merely serves as a benchmark that aides in the analysis of section 4. There, we consider the case of noncooperative location choices. We consider this to be the more interesting case; as location is a one-shot decision, it is not possible to tacitly collude in location choices, whereas it may be possible to tacitly collude in the price-setting stage. Section 5 concludes.

2 The Model

Our model has two stages. In the first stage, which we denote as $\tau = 0$, two firms choose locations on a Hotelling line of unit length. We denote the location of firm 1 by a_1, and that of firm 2 by $1 - a_2$, with $a_1, a_2 \in [0, 1]$. Without loss of generality, we assume that firm 1 is located to the left of firm 2, so $1 - a_1 - a_2 \geq 0$. In the case of symmetric locations, we will write $a_1 = a_2 = a$. In our model, location choices are irreversible. We thus assume that once a firm has chosen a location, or more generally, once a firm has designed its product, it becomes prohibitively expensive to change that design.

Consumers are uniformly distributed along the line and have unit demand. The firms (and their products) are ex ante identical. Transportation costs are quadratic: a consumer

located at x who chooses to buy from a firm located at a incurs transportation costs that equal $t(x-a)^2$. Marginal costs of production are denoted c, and are constant and equal among firms. The consumers' willingness-to-pay for the product equals v. We assume that v is sufficiently high, such that the market is always covered. For our purposes, it is sufficient to have $v > c + 4t$.

The second stage of the model is an infinitely repeated price-setting game, and consists of the periods $\tau = 1, \ldots, \infty$. Firms employ a common discount factor $\delta \in (0,1)$. The price set by firm i in period τ is denoted $p_{i\tau}$. The profits of firm i in period τ are denoted $\pi_i(p_{1\tau}, p_{2\tau}; a_1, a_2)$. Where this cannot yield confusion, we will often drop arguments and subscripts. The indifferent consumer is now located at z implicitly given by

$$p_1 + t\left(z - a_1\right)^2 = p_2 + t\left(1 - a_2 - z\right)^2.$$

Solving yields

$$z = \frac{1}{2}\left(a_1 + 1 - a_2\right) + \frac{p_2 - p_1}{2t\left(1 - a_1 - a_2\right)}. \tag{1}$$

Firm profits are given by

$$\pi_1(p_1, p_2; a_1, a_2) = z\left(p_1 - c\right),$$

$$\pi_2(p_1, p_2; a_1, a_2) = \left(1 - z\right)\left(p_2 - c\right). \tag{2}$$

For the price setting stage, we use the canonical tacit collusion model with grim trigger strategies (see e.g. Tirole 1988, pp. 245-6). Thus, if there is an implicit cartel agreement, a firm sticks to that agreement if and only if both firms have always done so in the past. Otherwise, both firms will choose to compete forever.

For the collusive agreement, we assume that firms choose prices that maximize joint profits. Thus, tacit collusion has firms setting monopoly prices p_1^m and p_2^m with

$$\{p_1^m(a_1, a_2), p_2^m(a_1, a_2)\} \in \arg \max_{p_1, p_2} \{\pi_1(p_1, p_2; a_1, a_2) + \pi_2(p_1, p_2; a_1, a_2)\}. \qquad (3)$$

Denote the per-period profits of firm i of sticking to this cartel agreement, and given the locations, as π_i^k. In what follows, we will show that, given a_1 and a_2, the prices p_1^m and p_2^m are uniquely determined. We can thus write $\pi_i^k(a_1, a_2) \equiv \pi_i(p_1^m(a_1, a_2), p_2^m(a_1, a_2))$. Competitive profits are denoted by $\pi_i^c(a_1, a_2)$. The maximum one-shot profits a firm can earn when defecting from the collusive agreement are denoted $\pi_i^d(a_1, a_2)$. Firm i will thus stick to the cartel agreement if and only if

$$\delta \geq \frac{\pi_i^d(a_1, a_2) - \pi_i^k(a_1, a_2)}{\pi_i^d(a_1, a_2) - \pi_i^c(a_1, a_2)} \equiv \delta_i^*(a_1, a_2). \qquad (4)$$

When this condition is satisfied for both firm 1 and firm 2, we have a stable cartel. We refer to such a cartel as one of *full collusion*. We thus have

Definition 1 *A fully collusive outcome consists of the prices (p_1^m, p_2^m) that satisfy (3). A cartel with full collusion is stable if (4) is satisfied for $i = 1, 2$.*

Yet, even when a cartel with full collusion is not stable, we may still have a stable cartel, but one at prices that do not maximize joint per-period profits. We will refer to such a situation as one of *constrained collusion*. The one-shot profits a firm can earn at most when defecting from the agreement are denoted $\pi_i^d(p_1, p_2, a_1, a_2)$. Firm i will stick to the cartel agreement if and only if

$$\delta \geq \frac{\pi_i^d(p_1, p_2; a_1, a_2) - \pi_i(p_1, p_2; a_1, a_2)}{\pi_i^d(p_1, p_2; a_1, a_2) - \pi_i^c(a_1, a_2)}. \tag{5}$$

We thus have

Definition 2 *There is a constrained collusive outcome if full collusion is not a stable cartel, but there is some $(\tilde{p}_1, \tilde{p}_2)$ such that (5) is satisfied for $i = 1, 2$, and moreover $\pi_i(\tilde{p}_1, \tilde{p}_2) > \pi_i^c$ for $i = 1, 2$.*

As a benchmark, we will first solve our model for the case in which firms explicitly collude in the location stage. Then, taking the analysis of collusion in the location stage as a starting point, we solve our model for the case in which locations are chosen nonco-operatively. Throughout our analysis, we will restrict attention to symmetric equilibria. For ease of exposition, we will write profits as a function of a in cases where locations are symmetric. For example, we will write $\pi_i^k(a)$ rather than $\pi_i^k(a_1, a_2)$ for cases in which $a_1 = a_2 = a$.

3 Model I: Collusion in the location stage

We first solve for the case in which firms choose their locations cooperatively. Hence, apart from tacit collusion in the repeated price-setting stage, we assume here that there is also explicit collusion in the location stage. Firms then choose locations as to maximize their profits in the price-setting game that follows.

We proceed as follows. First, for given symmetric locations a, we derive the competitive outcome, which allows us to determine $\pi^c(a)$. Second, we solve for the fully collusive cartel

prices (p_1^m, p_2^m). Then we derive the optimal defection from that cartel. This allows us to derive the cartel stability condition. Finally, given these results, we solve for the symmetric location a that maximizes joint profits, while also taking into account the possibility of constrained collusion.

Competitive outcome   Suppose that firms are located at a and $1-a$, with $0 \leq a \leq 1/2$. We look for the competitive equilibrium prices $p_1^c(a)$ and $p_2^c(a)$. From (1), the indifferent consumer z is located at

$$z = \frac{1}{2} + \frac{p_2 - p_1}{2t(1 - 2a)}.$$ (6)

Again, firm 1's profits are given by $z(p_1 - c)$ and firm 2's profits by $(1 - z)(p_2 - c)$. Taking firm i's FOC yields the reaction function for firm i :

$$p_i = \frac{1}{2}(c + p_j + t(1 - 2a)).$$ (7)

Imposing symmetry yields

$$p_i^c(a) = c + t(1 - 2a).$$

Corresponding profits are

$$\pi_i^c(a) = \frac{1}{2}t(1 - 2a).$$ (8)

Full collusion   We now solve for fully collusive prices in the case of symmetric locations. First note that we have effectively assumed that v is large enough such that firms always choose to cover the entire market. In the case of full collusion with symmetric locations, firms will thus choose identical prices that are as high as possible, but are such that all

consumers on the line are still willing to buy. Here, this implies that consumers located at the endpoints as well as consumers located in the middle are just willing to buy. Firms will thus set

$$p_i^m(a) = v - t \cdot \left(\max\{a, \frac{1}{2} - a\} \right)^2. \tag{9}$$

This can be seen as follows. Suppose that firms are located at some $a \geq 1/4$. That is, they are closer the middle of the line than they are to an endpoint of it. In that case, if firms set a price such that the consumer located at the endpoint is willing to buy, then consumers located in the middle are willing to buy as well, as their transportation costs are lower. Hence, the profit-maximizing price then is $p = v - ta^2$. Now suppose that firms are located at some $a < 1/4$. In that case, they are closer to the endpoints than they are to the middle of the line. Now, if they set a price such that consumers in the middle are just willing to buy, then the consumers located at the endpoint are willing to buy as well. This involves setting a price $p = v - t(\frac{1}{2} - a)^2$.

Cartel profits now equal

$$\pi_i^k(a) = \frac{1}{2} \left(v - t \cdot \left(\max\{a, \frac{1}{2} - a\} \right)^2 - c \right). \tag{10}$$

Optimal defection Now suppose that a firm defects from the tacit cartel agreement derived above. Without loss of generality, assume that this is firm 1. A defecting firm will always set a price such that he just captures the entire market, i.e. that the consumer at the opposite endpoint of the line just prefers purchasing from the defecting firm.[3] This

[3]Suppose this is not the case. The optimal reply to the collusive price set by firm 2 can be found by plugging p_2^m from (9) into the reaction function (7). But this is only the best reply if it yields $z \leq 1$, with z as defined in (6). Otherwise, we have a corner solution, and the best reply for firm 1 is indeed to set a

implies setting a price p_i^d such that

$$p_i^d + t(1-a)^2 = p_j^k + ta^2,$$

or

$$p_i^d = p_j^k - t(1-2a). \tag{11}$$

The profits from this defection are

$$\pi_i^d(a) = v - \left(\left(\max\{a, \tfrac{1}{2} - a\}\right)^2 + 1 - 2a\right)t - c. \tag{12}$$

Cartel stability Plugging (8), (10) and (12) into (4), stability of a cartel with full

collusion requires that

$$\delta \geq \frac{1}{2}\frac{v - \left(\left(\max\left\{a, \tfrac{1}{2} - a\right\}\right)^2 + 2 - 4a\right)t - c}{v - \left(\left(\max\left\{a, \tfrac{1}{2} - a\right\}\right)^2 + \tfrac{3}{2} - 3a\right)t - c} \equiv \delta^*(a). \tag{13}$$

Note that $\delta^*(a)$ is continuous and differentiable for all $a \in [0, \tfrac{1}{2}]$. If $a < \tfrac{1}{4}$, we have

$$\frac{\partial \delta^*(a)}{\partial a} = 2t\frac{4(v-c) + t(1-2a)^2}{(4(v-c) - t(1-2a)(7-2a))^2} > 0.$$

For $a \geq \tfrac{1}{4}$, we have

$$\frac{\partial \delta^*(a)}{\partial a} = 2t\frac{v - c - ta(1-a)}{(2v - 2c - t(3 - 6a + 2a^2))^2} > 0.$$

Hence, $\delta^*(a)$ is strictly increasing in a, which implies that the cartel becomes less stable as

firms move closer.[4] Intuitively, as firms move closer, it becomes easier to undercut one's

price such that it captures the entire market. From (7) we have $p_1^d = \tfrac{1}{2}(c + p_2^m + t(1-2a))$ which yields

$$z = \frac{1}{2} + \frac{p_2^m - c - t(1-2a)}{4t(1-2a)}.$$

We thus have that p_1^d as defined above is not an admissible solution if this z is larger than 1, or $p_2^m > c + 3t(1-2a)$. Using our assumption that $v > c + 4t$, this will always prove to be the case.

[4]Note that $\lim_{a \uparrow \frac{1}{4}} \frac{\partial \delta^*(a)}{\partial a} < \lim_{a \downarrow \frac{1}{4}} \frac{\partial \delta^*(a)}{\partial a}$. Hence the derivative has a discontinuous increase at $a = \tfrac{1}{4}$, which implies that δ^* is kinked at that point. This, however, has no effect for our analysis.

rival, as the amount by which one has to undercut ones rival in order to capture the entire market is now lower. This makes defecting from a cartel more attractive as firms move closer, and hence makes a cartel less stable.

Location choice We now solve for the collusive location choice of both firms. First, consider the case in which the condition for cartel stability is always satisfied with full collusion. Firms will then simply choose the location that maximizes cartel profits, as given by (10). It is easy to see that this yields locations $a = 1/4$. But now suppose that $\delta < \delta^*(1/4)$. With $\delta^*(a)$ increasing in a, the firms may still be able to achieve full collusion — but only at a lower value of a. Alternatively, firms can also choose to still locate at $a = 1/4$ (or any other location, for that matter) and settle for a cartel with constrained collusion. However, we can show that firms will always strictly prefer to locate where full collusion is still stable. We thus have the following result:

Theorem 1 *In the model with collusion in the location stage, the equilibrium location choice is given by*

$$a = \begin{cases} 0 & \text{if } \delta \leq \delta^*(0), \\ a^* & \text{if } \delta^*(0) < \delta < \delta^*\left(\frac{1}{4}\right), \\ \frac{1}{4} & \text{if } \delta \geq \delta^*\left(\frac{1}{4}\right), \end{cases}$$

with a^* the unique solution of $\delta = \delta^*(a)$.

Proof. The case $\delta \geq \delta^*(1/4)$ follows from the discussion above. For the other cases, suppose that firms locate at some a where full collusion is not stable. Rewriting (5), we can still have constrained collusion at symmetric prices p when

$$(1 - \delta)\,\pi_i^d(p; a) < \pi_i^k(p; a) - \delta\pi_i^c(a),$$

11

where we have simplified the notation to reflect symmetry. Obviously, $\pi_i^k(p;a) = \frac{1}{2}(p-c)$.

From (8), we have $\pi_i^c(a) = \frac{1}{2}(1-2a)t$. Initially, we assume that defecting entails capturing the entire market. If that is the case we have using (11) that $p_i^d = p - t(1-2a)$, so $\pi_i^d(p;a) = p - t(1-2a) - c$. Using these expressions, the inequality above reduces to

$$(1-2\delta)p < (2-3\delta)(1-2a)t + (1-2\delta)c.$$

Note from (13) that $\delta^*(1/2) = \frac{1}{2}$. With δ^* increasing, we have that $\delta < \delta^*(1/4)$ implies $\delta < 1/2$. We can thus rewrite the inequality above as

$$p < c + \frac{2-3\delta}{1-2\delta}(1-2a)t. \tag{14}$$

As the fraction is strictly positive for $\delta < 1/2$, the upper bound on p is decreasing in a, which implies that the highest possible constrained cartel profit is decreasing in a. Hence when $\delta = \delta^*(a)$ at some $a^* < 1/4$, then the result implies that maximum cartel profits are increasing in a for $a \leq a^*$ (where full collusion is still stable) and decreasing in a for $a > a^*$ (where only constrained collusion is feasible). Hence, firms will choose to locate at a^*. This establishes the theorem. ∎

Interestingly, we thus have that when cartel stability is not an issue, firms choose locations that are socially optimal. Since the market is covered, prices paid by consumers to firms are just tranfers that do not affect total welfare. Maximizing welfare then entails minimizing total transportation costs — which indeed implies having firms located at $1/4$ and $3/4$. In this set-up, we thus have that full collusion, in both locations and prices, yields the best possible outcome from a welfare point of view.

Of course, this policy conclusion is hard to swallow. Indeed, it is largely driven by one peculiar feature of the Hotelling model. Different from most other competition models, firms with monopoly power do not restrict output in the Hotelling model, which implies that there is no welfare loss from monopoly power. In our model, firms that collude in both stages of the game have an incentive to choose locations such that total transportation costs are as low as possible. The lower the transportation costs that consumers have, the higher the price that the cartel can charge. Hence, in the location stage our collusive duopoly has the exact same incentive as a social planner has.

4   Model II: Noncooperative location choices

We now solve the model for noncooperative location choices. To do so, we generalize our model and also consider asymmetric locations a_1 and a_2, where $a_1 \leq 1 - a_2$. This section is structured along the same lines as the previous one. We first solve for the competitive outcome, then for full collusion. We then solve for the optimal defection and finally we derive the equilibrium of the location stage.

Competitive outcome   Firm is profits are given by (2), using (1). Taking firm is FOC yields

$$\frac{1}{2}\left(a_i + 1 - a_j\right) + \frac{p_j - 2p_i + c}{2t\left(1 - a_i - a_j\right)} = 0,$$

Solving this system of two equations for prices we find the competitive outcome

$$p_i^c = c + t\left(1 - a_i - a_j\right)\left(1 + \frac{a_i - a_j}{3}\right),$$

for $i = 1, 2$ and $j \neq i$. In equilibrium, the indifferent consumer is located at

$$z = \frac{1}{2} + \frac{1}{6}(a_1 - a_2)$$

and equilibrium profits are

$$\pi_i^c = \frac{1}{18}(3 + a_i - a_j)^2(1 - a_i - a_j)t. \tag{15}$$

Full collusion   Now consider a joint-profit-maximizing cartel. Again, profit maximization requires that all consumers are served. Naturally, any profit-maximizing cartel has both firms serving their own "backyard". That is, firm 1 will serve any consumer that is located in the interval $[0, a_1]$, whereas firm 2 will serve any consumer located in $[1 - a_2, 1]$. To induce all these consumers to buy, both firms have to set a price that is such that the consumer located at the closest endpoint is just willing to buy. Thus

$$p_i = v - a_i^2 t. \tag{16}$$

Given these prices, any consumer located in $[0, 2a_1]$ is willing to buy from firm 1, whereas any consumer located in $[0, 1 - 2a_2]$ is willing to buy from firm 2. Hence, with $2a_1 \geq 1 - 2a_2$ the entire market is covered at these prices. Therefore, these prices maximize joint profits. Profits can be found by plugging prices from (16) into (1) and substituting into (2), which yields

$$\pi_i^k = \frac{1}{2}(1 - 2a_j)\frac{v - ta_i^2 - c}{1 - a_i - a_j}$$

We refer to this situation as case I.

Now suppose that the entire market is not covered at prices (16), so we have $2a_1 < 1 - 2a_2$. The jointly profit-maximizing solution then has one or both firms setting a lower price.

14

The profit-maximizing choice of prices now boils down to a profit-maximizing division of the market. That is, firms have to decide on some location $x \in [2a_1, 1 - 2a_2]$. They will both set a price that is such that the consumer located at x is just willing to buy. They will then set x such that joint profits are maximized. This implies

$$
\begin{aligned}
p_1 &= v - t(x - a_1)^2, \\
p_2 &= v - t(1 - a_2 - x)^2.
\end{aligned}
\tag{17}
$$

Joint profits can be written

$$
\begin{aligned}
\pi &= (p_1 - c)x + (p_2 - c)(1 - x) \\
&= v - t(x - a_1)^2 x - t(1 - a_2 - x)^2 (1 - x) - c.
\end{aligned}
$$

Taking the derivative with respect to x yields

$$
\frac{\partial \pi}{\partial x} = 3(1 - 2x) + 4a_1 x - 4a_2(1 - x) + a_2^2 - a_1^2.
$$

Setting this equal to zero yields

$$
\hat{x} = \frac{1}{2} \frac{3 - 4a_2 - a_1^2 + a_2^2}{3 - 2a_1 - 2a_2}.
\tag{18}
$$

Two cases can now occur. First, we may have that $\hat{x}$ as defined above falls strictly within the interval $[2a_1, 1 - 2a_2]$. If that is the case, then (17) are the prices that maximize joint profits. Both firms now set a price that is lower than the price such that their backyard is just served. We refer to this as case II. Second, we may have that the constrained x as defined in (18) is outside the admissible interval. Suppose that $\hat{x} < 2a_1$. Given that profits are strictly concave in x, joint profits are then maximized by setting $x = 2a_1$. Hence, firm

15

1 sets a price such that its backyard is just served, whereas firm 2 sets a price such that the remainder of the market is just covered. Naturally, with $\hat{x} > 1 - 2a_2$, we have the exact opposite. It is then profit-maximizing to set $x = 1 - 2a_2$: firm 2 just covers its backyard, whereas firm 1 serves the remainder of the market. These cases are just mirror images of each other. We refer to them as case III. More precisely, we refer to the first as case IIIa, and to the second as case IIIb. Summing up, using the definition of $\hat{x}$ yields the following areas in (a_1, a_2)−space:

- In area I we have $2a_1 > 1 - 2a_2$;

- in area IIIa we have

$$\frac{6}{7} - \frac{4}{7}a_2 - \frac{1}{7}\sqrt{(15 - 20a_2 + 9a_2^2)} < a_1 < 1 - a_2/2;$$

- in area IIIb we have

$$\frac{6}{7} - \frac{4}{7}a_1 - \frac{1}{7}\sqrt{(15 - 20a_1 + 9a_1^2)} < a_2 < 1 - a_1/2;$$

- area II consists of all other $(a_1, a_2) \in \left[0, \frac{1}{2}\right] \times \left[0, \frac{1}{2}\right]$.

— INSERT FIGURE 1 ABOUT HERE —

We have depicted the different areas in figure 1. We now have:

16

Result 1 *With asymmetric locations, the fully collusive prices $(p_1^m(a_1, a_2), p_2^m(a_1, a_2))$ are*

given by

$$(p_1^m, p_2^m) = \begin{cases} (v - ta_1^2, v - ta_2^2) & \text{in area I} \\ \left(v - t(\hat{x} - a_1)^2, v - t(1 - a_2 - \hat{x})^2\right) & \text{in area II} \\ \left(v - ta_1^2, v - t(1 - a_2 - 2a_1)^2\right) & \text{in area IIIa} \\ \left(v - t(1 - a_1 - 2a_2)^2, v - ta_2^2\right) & \text{in area IIIb.} \end{cases} \tag{19}$$

Fully collusive profits $(\pi_1^k(a_1, a_2), \pi_2^k(a_1, a_2))$ are given by

$$(\pi_1^k, \pi_2^k) = \begin{cases} \left(\frac{1}{2}(1 - 2a_2)\frac{v - ta_1^2 - c}{1 - a_1 - a_2}, \frac{1}{2}(1 - 2a_1)\frac{v - ta_2^2 - c}{1 - a_1 - a_2}\right) & \text{in area I} \\ \left((v - t(\hat{x} - a_1)^2 - c)\hat{x}, (v - t(1 - a_2 - \hat{x})^2 - c)(1 - \hat{x})\right) & \text{in area II} \\ \left(2a_1(v - ta_1^2 - c), (1 - 2a_1)(v - t(1 - a_2 - 2a_1)^2 - c)\right) & \text{in area IIIa} \\ \left((1 - 2a_2)(v - t(1 - a_1 - 2a_2)^2 - c), 2a_2(v - ta_2^2 - c)\right) & \text{in area IIIb.} \end{cases}$$

In these expressions, $\hat{x}$ is given by (18), and the areas are as defined above.

Optimal defection   Now consider the optimal defection from a fully collusive agreement.

Such a defection has the defecting firm capturing the entire market.[5] That implies

$$p_i^d = p_j^k + ta_j^2 - t(1 - a_i)^2. \tag{20}$$

Profits from defecting equal

$$\pi_i^d = p_i^d - c = p_j^k + ta_j^2 - t(1 - a_i)^2 - c. \tag{21}$$

For ease of exposition, and without loss of generality, we only consider the incentive that

firm 1 has to defect.

Result 2 *With asymmetric locations, defection profits for firm 1 are given by*

$$\pi_1^d(a_1, a_2) = \begin{cases} v - t(1 - a_i)^2 - c & \text{in areas I and IIIa} \\ v - t(1 - a_2 - \hat{x})^2 - t(1 - a_1)^2 - c & \text{in area II} \\ v - t(1 - 2a_2 - a_1)^2 + ta_1^2 - t(1 - a_2)^2 - c & \text{in area IIIb.} \end{cases}$$

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[5]See footnote 3.

17

Location stage   For the case of asymmetric locations, we refrain from explicitly deriving the cartel stability condition for all possible cases; this yields particularly nasty expressions, and is not necessary in what follows. We look for a symmetric equilibrium in the location stage. Thus, we look for symmetric locations $(a_1, a_2) = (a, a)$ which are such that no firm has an incentive to defect to some other location. We can therefore draw heavily on the analysis in section 3.

Theorem 2 *Consider the case in which full collusion is stable for every $(a_1, a_2) \in \left[0, \frac{1}{2}\right] \times \left[0, \frac{1}{2}\right]$. Then the unique symmetric equilibrium has $a = \frac{1}{2}$, that is, we have minimum differentiation.*

Proof.   Suppose we have a symmetric candidate equilibrium with $a \in \left[\frac{1}{4}, \frac{1}{2}\right)$. Using figure 1, we are then in area I. Within this area, we have

$$\frac{\partial \pi_1^k (a_1, a_2)}{\partial a_1} = \frac{1}{2} (1 - 2a_2) \left[\frac{v - c + a_1 t (2 - a_1 - 2a_2)}{(1 - a_1 - a_2)^2} \right] > 0.$$

Hence, provided that this will still yield a stable cartel in the price-setting stage, firm 1 wants to defect from this candidate equilibrium by choosing a location closer to the middle. Hence, this is not a Nash equilibrium in locations. Now consider a symmetric candidate equilibrium with $a < \frac{1}{4}$. We are then in area II. First, note from (18) that in this area

$$\frac{\partial \hat{x}}{\partial a_1} = \frac{(a_1 + a_2)^2 + (3 - 4a_2 - 3a_1)}{(3 - 2a_1 - 2a_2)^2} > 0$$

as $a_1, a_2 < \frac{1}{4}$. Within area II, we have

$$\frac{\partial \pi_1^k}{\partial a_1} = \left(v - t (\hat{x} - a_1)^2 - c \right) \frac{\partial \hat{x}}{\partial a_1} + 2t (\hat{x} - a_1) \hat{x} > 0.$$

Again, firm 1 wants to defect from this candidate equilibrium by choosing a location closer to the middle. The only possible symmetric equilibrium thus has $a = 1/2$. To see that this is indeed an equilibrium, note from figure 1 that any defection of one firm from $a = 1/2$ implies that the firms end up in area I. But we already showed that within area I a firm's cartel profits are increasing in its location. Hence any defection from $a = 1/2$ yields lower cartel profits. ∎

With a high enough discount factor, we thus have that the principle of minimum differentiation re-emerges. In that case, when firms make an irreversible location choice, and then play a repeated price-setting game, they choose to locate as close as possible — as Hotelling (1929) claimed in his analysis of the one-shot case.

Now consider the case in which the full cartel is not stable at $a = 1/2$, but it is at some $a < 1/2$. Denote the highest a for which this holds as a^*. Note that $\delta^*(1/2) = 1/2$. Hence, we must have $\delta < 1/2$. The analysis now becomes much more involved. In the appendix we prove the following result:

Theorem 3 *In the model with noncooperative location stages, the equilibrium location choices are given by*

$$
a = \begin{cases} 0 & \text{if } \delta \leq \delta^*(0), \\ a^* & \text{if } \delta^*(0) < \delta < \delta^*\left(\frac{1}{2}\right), \\ \frac{1}{2} & \text{if } \delta \geq \delta^*\left(\frac{1}{2}\right), \end{cases}
$$

with a^* the unique solution of $\delta = \delta^*(a)$.

Hence, both firms locate in the middle, provided that full collusion then yields a stable cartel. If that is the case, both firms choosing socially optimal locations is no longer an

equilibrium. Both firms have an incentive to locate closer to the middle, as this yields higher collusive profits for the defecting firm. The choice of locations is then a prisoners' dilemma. Now suppose that full collusion with firms located in the middle no longer constitutes a stable cartel. Consider the case where both firms are located such that a cartel with full collusion is just stable. Defecting towards the middle then implies that full collusion is no longer a stable cartel for the other firm. To restore cartel stability, the defecting firm has to give up so much profits that the defection is not profitable.

Interestingly, we have that if $\delta > \delta^*(1/4)$, firms are strictly better off when the discount factor δ decreases to some $\tilde{\delta} \in [\delta^*(1/4), \delta)$. Such a decrease would commit firms to locate further apart, which increases equilibrium profits of both. In the standard cartel model, a decrease in δ can never benefit firms, as it can only weaken cartel stability.

5   Concluding remarks

In this paper, we revisited Hotelling's (1929) claim that firms will choose to design products in such a manner that they resemble each other as closely as possible. By now, it has been widely established that in Hotelling's original specification with linear transportation costs this claim does not hold true. With quadratic transportation costs, competing firms will even choose to design products in such a manner that they resemble each other as little as possible. In our model, we studied the case in which firms make an irreversible location choice, but then play a repeated price-setting game in which they may collude. Most importantly, we showed that if firms are sufficiently patient Hotelling's original claim holds true. When firms are able to collude in prices, they will choose to design products in

such a manner that they resemble each other as closely as possible. When collusion is more difficult to sustain, firms will choose locations that are further apart, such that full collusion is just stable. We also showed that if firms are able to collude in the location stage as well, they will select the socially optimal locations, again provided that the discount factor is high enough.

Our results have implications for competition policy. When product design is endogenous, our results suggest that antitrust agencies should monitor more closely those industries where products are close substitutes. To paraphrase Hotelling (1929), whenever buyers are confronted with an excessive sameness, this may be due in part to the economies of large-scale production, in part to fashion and imitation. But over and above these forces is the tendency to try to capture cartel profits that are as high as possible.

Appendix: Proof of Theorem 3

For the analysis that follows, we need the following lemma:

Lemma 1 *For given a_1 and a_2, the cartel stability condition for firm 2 is relaxed if p_1 increases or if p_2 decreases, provided that p_1 and p_2 are such that both firms have a strictly positive market share, and that $p_1, p_2 > t$.*

Proof. For given a_1 and a_2, and dropping the arguments referring to locations, the cartel stability condition for firm 2 can be written

$$(1 - \delta) \pi_2^d - \pi_2 (p_1, p_2) < -\delta \pi_2^c. \tag{22}$$

The RHS of this inequality does not depend on prices. For the LHS, we have from (2) and (21)

$$\frac{\partial \left((1-\delta)\pi_2^d - \pi_2(p_1, p_2)\right)}{\partial p_1} = (1-\delta) - \frac{p_2}{2t(1-a_1-a_2)}.$$

This expression is negative iff $p_2 > 2t(1-a_1-a_2)(1-\delta)$. The condition that both firms have positive market share requires that $z < 1$. From (1) this implies

$$\frac{p_2}{2t(1-a_1-a_2)} < \frac{p_1}{2t(1-a_1-a_2)} - \frac{1}{2}(a_1+1-a_2) + 1.$$

This implies that indeed $p_2 > 2t(1-a_1-a_2)(1-\delta)$ if

$$\frac{p_1}{2t(1-a_1-a_2)} - \frac{1}{2}(a_1+1-a_2) + 1 > 1 - \delta,$$

which simplifies to

$$p_1 > t(1-a_1-a_2)(a_1-a_2+1-2\delta),$$

which is always satisfied if $p_1 > t$. Hence, increasing p_1 indeed relaxes (22). Similarly

$$\frac{\partial \left((1-\delta)\pi_2^d - \pi_2(p_1, p_2)\right)}{\partial p_2} = -\frac{1}{2}(1+a_2-a_1) - \frac{p_1-2p_2}{2t(1-a_1-a_2)}$$

This expression is positive iff $2p_2 > p_1 + t(1-a_1-a_2)(1+a_2-a_1)$. But given that $p_1 > t$ and $a_1, a_2 \leq \frac{1}{2}$, sufficient for this to hold is that $2p_2 > 2t$, which is always satisfied since $p_2 > t$. The result stated in the lemma then holds true. ∎

Note that the restriction that prices have to exceed t is mild: with firms located at the endpoints, the competitive price has $p = c + t$. It will be easy to see that, for our purposes, the constraint is never binding. We can now establish:

Lemma 2 *If $1/2 > a^* \geq 1/4$, defecting from (a^*, a^*) to some $\tilde{a} > a^*$ yields lower cartel profits for the defector than sticking to (a^*, a^*).*

Proof. Denote full collusion prices at (a^*, a^*) as (p^*, p^*). By construction, the cartel is just stable at (a^*, a^*) and (p^*, p^*). Consider a defection to some $\tilde{a} > a^*$ by firm 1. Since $a^* > 1/4$, the original situation had both firms just covering their backyard. Efficiency requires that, in the new situation, firm 1 at least serves its backyard as well. Hence the highest possible price firm 1 will set now equals

$$\tilde{p}_1 = v - \tilde{a}^2 t.$$

Cartel stability for firm 2 requires

$$(1 - \delta) \, \pi_2^d \, (p_1, p_2; a_1, a_2) \leq \pi_2 \, (p_1, p_2; a_1, a_2) - \delta \pi_2^c \, (a_1, a_2). \tag{23}$$

Note from (21) that in this case

$$\frac{d\pi_2^d}{da_1} = \frac{\partial p_1}{\partial a_1} + 2a_1 t = 0.$$

Hence, the LHS of (23) is unaffected if a_1 increases. We now consider the effect on the RHS. Take the prices (p^*, p^*) as a starting point. Consider the direct effect of the change in a_1 on the cartel profits of firm 2, while keeping prices constant. From (2), we then have:

$$\frac{\partial \pi_2 \, (p^*, p^*; a_1, a_2)}{\partial a_1} = -\frac{1}{2} \, (p^* - c).$$

For the effect on π_2^c we have from (15)

$$\frac{\partial \pi_2}{\partial a_1} = -\frac{1}{18} t \, (3 + a_2 - a_1) \, (5 - 3a_1 + a_2)$$

Note that this expression is always negative, but strictly bigger than $-t$. Also note that $p^* = v - t \left(\frac{1}{2} - a^*\right)^2$. Hence, with $v > c + 4t$, we necessarily have that the RHS of (23) is

23

decreasing in a_1 — even without taking into account that p_1 will decrease as well, which lowers π_2 even further. Hence, at prices $(\tilde{p}_1, p^*)$ the cartel is not stable for firm 2. To achieve cartel stability for firm 2, we need to increase p_1, or to decrease p_2. Increasing p_1 cannot be efficient, as it implies firm 1's backyard is no longer fully covered. Necessarily, we thus need to decrease p_2. By construction, we had

$$\pi_2\left(p^*; a^*\right) = (1 - \delta)\, \pi_2^d\left(p^*; a^*\right) + \delta \pi_2^c\left(a^*\right)$$

Since π_2^c is decreasing in a_1, while π_2^d is unaffected, for cartel stability for firm 2 we need, using (23), that $\tilde{p}_2$ decreases so much that

$$\pi_2(\tilde{p}_1, \tilde{p}_2; \tilde{a}, a^*) > \pi_2\left(p^*; a^*\right).$$

Firm 2 thus needs to achieve higher profits with a lower price. That implies that its market share has to increase. In turn this implies that the market share of firm 1 has to decrease. Since firm 1 also sets a lower price, the defection necessarily lowers its profits, which establishes the result. ∎

We also have:

Lemma 3 *In the case that $a^* < 1/4$, defecting from (a^*, a^*) to some $\tilde{a} > a^*$ yields lower cartel profits for the defector than sticking to (a^*, a^*).*

Proof. Denote full cartel prices at (a^*, a^*) as (p^*, p^*). By construction, the cartel is just stable at (a^*, a^*) and (p^*, p^*). Consider a cartel at locations $(\tilde{a}, a^*)$. This needs to satisfy

$$\pi_2\left(\tilde{p}_1, \tilde{p}_2; \tilde{a}, a^*\right) \geq (1 - \delta)\, \pi_2^d\left(\tilde{p}_1, \tilde{p}_2; \tilde{a}, a^*\right) + \delta \pi_2^c\left(\tilde{a}, a^*\right). \tag{24}$$

24

Consider the case in which $\tilde{p}_1 = \tilde{p}_2 = p^*$. We then have

$$\frac{\partial}{\partial a_1} \left((1-\delta) \pi_2^d + \delta \pi_2^c \right) = (1-\delta) 2t (1 - a_1) - \frac{\delta}{18} t (a_2 - a_1 + 3) (5 - 3a_1 - a_2).$$

It can be shown that this expression is decreasing in δ, a_1 and a_2. The lowest possible value is thus reached for $\delta = 1/2$, and $a_1 = a_2 = 1/4$. In that case, the expression simplifies to $5t/12$. Hence, we have that the RHS of (24) is always increasing in a_1. Combined with the fact that we already established that $\partial \pi_2 / \partial a_1 < 0$, this implies that at prices (p^*, p^*) and locations $(\tilde{a}, a^*)$, the cartel is no longer stable. It also implies that necessarily $\pi_2 (\tilde{p}_1, \tilde{p}_2; \tilde{a}, a^*) > \pi_2 (p^*, p^*; a^*, a^*)$. Denote the indifferent consumer at the constrained cartel as $\tilde{z}$. For cartel stability for firm 2, we thus need that $(\tilde{p}_2 - c)(1 - \tilde{z}) > \frac{1}{2}(p_2^* - c)$. There are two ways to achieve this. The first is to increase p_1. This hurts firm 1, as its profits are decreasing in p_1 at (p^*, p^*). The second is to decrease p_2. However, as we need $(\tilde{p}_2 - c)(1 - \tilde{z}) > \frac{1}{2}(p_2^* - c)$, this implies that we need $\tilde{z} < 1/2$. In that case, the market share of firm 1 decreases, whereas its price does not change. Hence, also in that case, profits of firm 1 decrease. ∎

The final lemma we need to establish our result is the following:

Lemma 4 *In the case that $a^* < 1/2$, defecting from (a^*, a^*) to some $\tilde{a} < a^*$ yields lower cartel profits for the defector than sticking to (a^*, a^*).*

Proof. By construction, the cartel is just stable at at (a^*, a^*). Suppose that firm 1 defects to some $\tilde{a} < a^*$. We will first establish that at locations $(\tilde{a}, a^*)$, a cartel with full collusion is stable. From the proof of Theorem 2 we then immediately have the result.

For cartel stability to be satisfied for firm 1, we need

$$\pi_1^k\left(\tilde{a}, a^*\right) - (1 - \delta)\,\pi_1^d\left(\tilde{a}, a^*\right) \geq \delta\pi_2^c\left(\tilde{a}, a^*\right). \tag{25}$$

Regardless of the case we are in, firm 1 profits with full collusion can be written $\pi_1^k = (p_1^m - c)\,y$, with y the market share of firm 1. We then have

$$\frac{d\pi_1^k}{da_1} = (p_1^m - c)\frac{dy}{da_1} + \frac{\partial p_1^m}{\partial a_1}y.$$

From (21),

$$\frac{d\pi_1^d}{da_1} = \frac{\partial p_2^m}{\partial a_1} + 2t\left(1 - a_1\right).$$

In area I, we thus have

$$\frac{d}{da_1}\left(\pi_1^k\left(a_1, a_2\right) - (1 - \delta)\,\pi_1^d\left(a_1, a_2\right)\right) = (p_1^m - c)\frac{dy}{da_1} - 2t\left(a_1\left(3 - \delta\right) - (1 - \delta)\right).$$

Here y can be found by plugging monopoly prices from Result into (1). This yields

$$\begin{aligned}
\frac{dy}{da_1} &= \frac{1}{2} - \frac{p_2 - p_1}{2t\left(1 - a_1 - a_2\right)} + \frac{1 - a_1}{1 - a_1 - a_2} \\
&= \frac{1}{2} - \frac{ta_1^2 - ta_2^2}{2t\left(1 - a_1 - a_2\right)} + \frac{1 - a_1}{1 - a_1 - a_2} \\
&= \frac{1}{2} + \frac{\left(1 + a_2^2 - a_1^2\right) + \left(1 - 2a_1\right)}{2\left(1 - a_1 - a_2\right)}
\end{aligned}$$

For the lhs of (25) to be increasing in a_1 we thus need

$$(p_1^m - c)\frac{dy}{da_1} > 2t\left(a_1\left(3 - \delta\right) - (1 - \delta)\right)$$

or

$$\left(v - ta_1^2 - c\right)\frac{dy}{da_1} > 2t\left(a_1\left(3 - \delta\right) - (1 - \delta)\right).$$

26

Note that from $v > c + 4t$, we have

$$\frac{1}{2}\left(v - ta_1^2 - c\right) > \frac{1}{2}\left(4t - ta_1^2\right).$$

For the inequality to hold, it is thus sufficient to have

$$\left(v - ta_1^2 - c\right)\left(\frac{\left(1 + a_2^2 - a_1^2\right) + \left(1 - 2a_1\right)}{2\left(1 - a_1 - a_2\right)}\right) > 2t\left(a_1\left(3 - \delta\right) - \left(1 - \delta\right)\right) - \frac{1}{2}\left(4t - ta_1^2\right)$$

or

$$\left(a_2^2 - a_1^2 - 2a_1 + 2\right)\left(v - c + ta_1^2\right) > -t\left(-4\delta a_1 + 12a_1 + 4\delta + a_1^2 - 8\right)\left(1 - a_2 - a_1\right).$$

The lhs is decreasing in a_1 and increasing in a_2. Within area I, the smallest value it can achieve is thus when $a_1 = \frac{1}{2}$ and $a_2 = \frac{1}{4}$. We then have that the lhs equals at least $\frac{13}{16}\left(v - ta_1^2 - c\right)$. Similarly, it can be shown that the rhs is increasing in a_1. The highest value it can reach is if $a_1 = \frac{1}{2}$. In that case the rhs equals $t\left(\frac{7}{4} - 2\delta\right)\left(\frac{1}{2} - a_2\right) < \frac{7}{8}t$. Thus, with $v > c + 4t$, the lhs is always larger than the rhs, which establishes that the lhs of 25 is increasing in a_1. We already have that the rhs is decreasing in a_1. That implies that an increase in a_1 makes the inequality stricter. But that implies that a decrease in a_1 relaxes the inequality. Thus, at a^*, when firm 1 defects to some $\tilde{a} < a^*$, its condition for cartel stability is still satisfied. The proof for case II goes along the exact same lines.

For firm 2, cartel stability requires that (24) is satisfied. We established in the proof of the previous lemma that, given prices (p^*, p^*), an increase in a_1 decreases the lhs, and increases the rhs. Hence, a decrease in a_1 weakens the inequality. Thus for firm 2, the agreement (p^*, p^*) is stable at the new locations. But at the new locations, p_1^m will only

increase, which further relaxes firm 2's cartel stability condition. Hence, full collusion yields a stable cartel at the new locations. This establishes the result. ∎
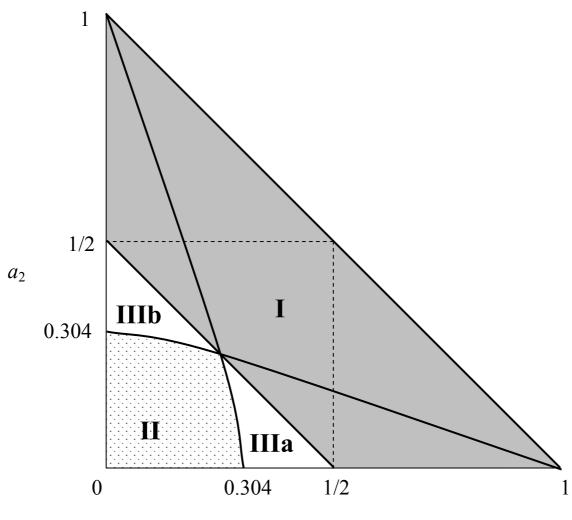
Combining these lemmas establishes theorem 3.

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Figure 1.
Feasible location choices and corresponding areas for joint profit maximization