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Abstract. Published input–output tables in constant prices are relatively scarce. Therefore, input–output tables often have to be deflated by the practitioners themselves. The method of double deflation is used predominantly for this purpose. The present paper shows that the double-deflation method is subject to aggregation problems. Necessary and sufficient conditions for the double-deflation method to provide the correct answers are derived. The conditions are found to be stringent and unlikely to be met in empirical cases. The results for aggregation in the case of double deflation are shown to be dual to the traditional results for aggregation in the case of a quantity model, which have been extensively discussed in the literature.

1 Introduction

For analyzing changes over time in the production structure of an economy, real figures are typically used. Nominal figures include a price component as well as a quantity component. Examination of the developments in the technical structure of production requires that the quantity component is singled out. Also for long-term multisectoral planning purposes, input–output (IO) tables in constant prices are indispensable. As another example, many studies in the field of economic growth seek to explain the behaviour of real GDP.

The first two examples above indicate that IO tables in constant prices are an important tool for planning issues and structural analyses, both within a multisectoral context. Unfortunately, tables in constant prices are often not readily available. As a consequence, planners and analysts are forced to estimate IO tables in constant prices themselves. In this paper we explicitly adopt the practitioner's (or user's) point of view. The method that is predominantly used for this purpose is the double-deflation method. It should be noted, however, that the United Nations (1973) originally proposed this method for the estimation of the value added (or GDP) in constant prices.

In the method of double deflation, the gross output and intermediate and final deliveries of each sector are deflated by the price index of this sector, under the assumption that each sector produces a single homogeneous good. The value added for each sector can then be obtained as the difference between the deflated gross output of this sector and the deflated intermediate inputs plus imports in constant prices. The single-good assumption appears to be rather crucial and very likely to be violated in empirical cases. For example, intermediate deliveries from sector i to sector j typically cover a basket of goods, the composition of which will differ from that of a basket delivered from sector i to another sector k. The price of each basket delivered by sector i will therefore be different, because of its different composition. Using a single price index for each basket will certainly affect the results.

The published IO tables in current prices, used by the practitioner in the deflation procedure, record data which have been aggregated. Therefore it seems relevant to

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investigate how this affects the estimates in constant prices. In the next section, necessary and sufficient conditions are derived for the total intermediate purchases in each sector to be estimated correctly. It is shown that these conditions can be met only when the IO table in current prices satisfies certain stringent restrictions. It is unlikely that an IO table at the usual level of aggregation satisfies these requirements. These results on the 'price side' of the model are dual to the traditional conditions for aggregation for the 'quantity side' of the model, as found in the literature. This dual relationship is examined in section 3, and the conditions are given for the case of a simultaneous analysis of the price side and the quantity side.

2 Aggregation effects of double deflation

In this section we adopt the viewpoint of the practitioner whose purpose it is to deflate an IO table containing aggregated data. It is shown that the double-deflation method is a correct approach for deflating such IO tables, only under unrealistically strong conditions. The assumption that each sector i produces exactly one product, with price p_i, is crucial. Intuitively speaking, the single product i is actually a basket of various products. The price p_i, therefore, is an average price, determined by the composition of basket i. The intermediate deliveries, z_{ij}, from sector i to sector j denote the value of the basket sold by sector i and used in sector j. It may be expected, however, that the composition of these baskets will differ across sectors j. As a consequence, the price paid for the baskets should also differ across sectors j. Thus, instead of p_i, sector-specific prices p_{ij} should be used.

Part of the problem encountered with deflating IO tables is thus an aggregation problem. This problem has received considerable attention particularly in the IO literature.[1] It is therefore somewhat surprising that the focus has been exclusively on the quantity side of IO models. In the discussion on aggregation issues, the price side seems to have been neglected (a rare exception is Olsen, 1993).

In what follows, we analyze the effects of aggregation upon the value-added vector when the IO table is deflated according to the double-deflation method. As a starting point, we take the following 'ideal', but hypothetical, situation. Suppose that we have an IO table in current prices, suppose that each of the n sectors produces exactly one good, and suppose that information about the prices (or price indexes) for these goods is available. Also for imports, full information (either sectoral imports in constant prices or deflators for each imported product) is assumed to be available. Applying the double-deflation method gives the n-element vector of values added in each sector, in constant prices.

Now suppose that the n sectors are aggregated into N $(<n)$ sectors. Calculating the aggregated N-element value-added vector in constant prices may proceed along two different lines. First, aggregation after deflation, which yields the correct answer and, second, deflation after aggregation, which usually leads to a different answer. The aggregation of n sectors into N aggregated sectors is called *price acceptable* (or *P*-acceptable) if both procedures always yield the same result. Below we derive necessary and sufficient conditions for price acceptability.

The IO table in current prices is given in figure 1. The $n \times n$ matrix $\mathbf{Z}$ denotes the intermediate deliveries, the vector $\boldsymbol{f}$ the final demands (private and government consumption and investment, and exports), $\boldsymbol{x}$ denotes the vector with sectoral outputs. The $k \times n$ matrix $\mathbf{M}$ gives the sectoral imports, where we have distinguished k different

(1) For recent contributions see, for example, Afrasiabi and Casler (1991), Aislabie and Gordon (1990), Cabrer et al (1991), De Mesnard and Dietzenbacher (1995), Dietzenbacher (1992), Howe and Johnson (1989), Howe and Stabler (1989), Oksanen and Williams (1992). See Kymn (1990) for an overview

$\mathbf{Z}$	f	x
$\mathbf{M}$		
$\boldsymbol{v}^{\mathrm{T}}$		
$\boldsymbol{x}^{\mathrm{T}}$		

Figure 1. Input–output table in current prices.

imported products[2], and $\boldsymbol{v}^{\mathrm{T}}$ is a row vector,[3] the elements of which give the value added in each sector.

The IO table in constant prices, according to the double-deflation method, is presented in figure 2. The subscript d (for deflated) is used to indicate that the corresponding matrices and vectors are in constant prices. Let p_i denote the ratio of the current price and the base-year price, for the product produced by sector i. Thus, $100p_i$ is the price index. The diagonal matrix with the elements of the vector $\boldsymbol{p}$ on its main diagonal is denoted by $\hat{\mathbf{p}}$. In the same way, r_j denotes the price ratio between the current import price and the base-year import price, for the imported product j.

In the double-deflation method the price indexes p_i and r_j are assumed to be

$\mathbf{Z}_{\mathrm{d}} = \hat{\mathbf{p}}^{-1}\mathbf{Z}$	$f_{\mathrm{d}} = \hat{\mathbf{p}}^{-1}f$	$x_{\mathrm{d}} = \hat{\mathbf{p}}^{-1}x$
$\mathbf{M}_{\mathrm{d}} = \hat{\mathbf{r}}^{-1}\mathbf{M}$		
$\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}$		
$\boldsymbol{x}_{\mathrm{d}}^{\mathrm{T}} = \boldsymbol{x}^{\mathrm{T}}\hat{\mathbf{p}}^{-1}$		

Figure 2. Input–output table in constant prices.

given.[4] The value-added vector $\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}$ is then obtained from the balancing equations. That is, the equality of the row sums and the column sums implies

$$\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}} = \boldsymbol{x}_{\mathrm{d}}^{\mathrm{T}} - \boldsymbol{e}_{(n)}^{\mathrm{T}}\mathbf{Z}_{\mathrm{d}} - \boldsymbol{e}_{(k)}^{\mathrm{T}}\mathbf{M}_{\mathrm{d}}\,, \tag{1}$$

where $\boldsymbol{e}_{(n)}$ denotes the n-element summation vector consisting entirely of ones.

The $n \times n$ matrix of input coefficients in current prices is defined as $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$, and the $k \times n$ matrix of import coefficients in current prices is defined as $\mathbf{B} = \mathbf{M}\hat{\mathbf{x}}^{-1}$. The coefficient matrices in constant prices are defined analogously as $\mathbf{A}_{\mathrm{d}}$ and $\mathbf{B}_{\mathrm{d}}$. Their relation to $\mathbf{A}$ and $\mathbf{B}$ is as follows:

$$\mathbf{A}_{\mathrm{d}} = \mathbf{Z}_{\mathrm{d}}\hat{\mathbf{x}}_{\mathrm{d}}^{-1} = \hat{\mathbf{p}}^{-1}\mathbf{Z}(\hat{\mathbf{p}}^{-1}\hat{\mathbf{x}})^{-1} = \hat{\mathbf{p}}^{-1}\mathbf{Z}\hat{\mathbf{x}}^{-1}\hat{\mathbf{p}} = \hat{\mathbf{p}}^{-1}\mathbf{A}\hat{\mathbf{p}}\,, \tag{2}$$

$$\mathbf{B}_{\mathrm{d}} = \mathbf{M}_{\mathrm{d}}\hat{\mathbf{x}}_{\mathrm{d}}^{-1} = \hat{\mathbf{r}}^{-1}\mathbf{M}(\hat{\mathbf{p}}^{-1}\hat{\mathbf{x}})^{-1} = \hat{\mathbf{r}}^{-1}\mathbf{M}\hat{\mathbf{x}}^{-1}\hat{\mathbf{p}} = \hat{\mathbf{r}}^{-1}\mathbf{B}\hat{\mathbf{p}}\,. \tag{3}$$

In the following, it appears more convenient from a notational point of view to use deflators instead of price indexes. That is, the deflator π_i is defined as the reciprocal price ratio, that is, $\pi_i = 1/p_i$. In the same way, the import deflator ρ_j $(j = 1, \ldots, k)$ is defined as $\rho_j = 1/r_j$. Consequently, equation (1) may be rewritten as

$$\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}} = \boldsymbol{\pi}^{\mathrm{T}}\hat{\mathbf{x}} - \boldsymbol{\pi}^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}} - \boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}\hat{\mathbf{x}} = [\boldsymbol{\pi}^{\mathrm{T}}(\mathbf{I}_{(n)} - \mathbf{A}) - \boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}]\hat{\mathbf{x}}\,, \tag{4}$$

where $\mathbf{I}_{(n)}$ denotes the $n \times n$ identity matrix.

(2) If the input–output table records only a single row of total imports, $k = 1$ and the matrix $\mathbf{M}$ becomes a vector.

(3) A superscript $^{\mathrm{T}}$ (for example, in $\boldsymbol{v}^{\mathrm{T}}$) is used to indicate transposition. As usual, vectors are column vectors.

(4) Alternatively, it may be assumed that, instead of $\boldsymbol{r}$, the matrix $\mathbf{M}_{\mathrm{d}}$ is given.

Next, the aggregation procedure is discussed. The n sectors are aggregated into $N\,(< n)$ larger sectors. Indexes $i, j\ (i, j = 1, \ldots, n)$ are used to indicate original sectors, indexes $I, J\ (I, J = 1, \ldots, N)$ are used for aggregate sectors. The aggregation of the IO table is obtained by using an $N \times n$ aggregator matrix $\mathbf{G}$. The aggregator matrix has the following form

$$\mathbf{G} = \begin{bmatrix} 1 & \ldots & 1 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 1 & \ldots & 1 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \ddots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 1 & \ldots & 1 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & 1 & \ldots & 1 \end{bmatrix}. \tag{5}$$

The typical element g_{Ij} (for $I = 1, \ldots, N$, and $j = 1, \ldots, n$) is defined as

$$g_{Ij} = \begin{cases} 1, & \text{if } j \in I, \\ 0, & \text{if } j \notin I. \end{cases} \tag{6}$$

The aggregation scheme as given by the matrix in equation (5) is as follows. The first aggregated sector consists of the first set of (say k_1) original sectors, the second consists of the second set of (say k_2) original sectors, etc.[5]

The aggregated IO table in current prices is given in figure 3(a). Applying the double-deflation method to the aggregated table in current prices [that is, figure 3(a)], yields figure 3(b) for the IO table in constant prices. The $N \times N$ matrices of input coefficients are given by $\bar{\mathbf{A}} = \bar{\mathbf{Z}}\hat{\bar{\mathbf{x}}}^{-1}$, and $\bar{\mathbf{A}}_\mathrm{d} = \bar{\mathbf{Z}}_\mathrm{d}\hat{\bar{\mathbf{x}}}_\mathrm{d}^{-1}$, and the $k \times N$ matrices of import coefficients by $\bar{\mathbf{B}} = \bar{\mathbf{M}}\hat{\bar{\mathbf{x}}}^{-1}$, and $\bar{\mathbf{B}}_\mathrm{d} = \bar{\mathbf{M}}_\mathrm{d}\hat{\bar{\mathbf{x}}}_\mathrm{d}^{-1}$. In the same way as equation (4) was derived, we now obtain

$$\bar{\boldsymbol{\nu}}_\mathrm{d}^\mathrm{T} = [\bar{\boldsymbol{\pi}}^\mathrm{T}(\mathbf{I}_{(N)} - \bar{\mathbf{A}}) - \boldsymbol{\rho}^\mathrm{T}\bar{\mathbf{B}}]\hat{\bar{\mathbf{x}}}. \tag{7}$$

$\bar{\mathbf{Z}} = \mathbf{GZG}^\mathrm{T}$	$\bar{\boldsymbol{f}} = \mathbf{G}\boldsymbol{f}$	$\bar{\boldsymbol{x}} = \mathbf{G}\boldsymbol{x}$
$\bar{\mathbf{M}} = \mathbf{MG}^\mathrm{T}$		
$\bar{\boldsymbol{\nu}}^\mathrm{T} = \boldsymbol{\nu}^\mathrm{T}\mathbf{G}^\mathrm{T}$		
$\bar{\boldsymbol{x}}^\mathrm{T} = \boldsymbol{x}^\mathrm{T}\mathbf{G}^\mathrm{T}$		

(a)

$\bar{\mathbf{Z}}_\mathrm{d} = \hat{\bar{\boldsymbol{\pi}}}\bar{\mathbf{Z}}$	$\bar{\boldsymbol{f}}_\mathrm{d} = \hat{\bar{\boldsymbol{\pi}}}\bar{\boldsymbol{f}}$	$\bar{\boldsymbol{x}}_\mathrm{d} = \hat{\bar{\boldsymbol{\pi}}}\bar{\boldsymbol{x}}$
$\bar{\mathbf{M}}_\mathrm{d} = \hat{\boldsymbol{\rho}}\bar{\mathbf{M}}$		
$\bar{\boldsymbol{\nu}}_\mathrm{d}^\mathrm{T}$		
$\bar{\boldsymbol{x}}_\mathrm{d}^\mathrm{T} = \bar{\boldsymbol{x}}^\mathrm{T}\hat{\bar{\boldsymbol{\pi}}}$		

(b)

Figure 3. Aggregated input–output table: (a) current prices; (b) deflated.

Next, we derive the relation between $\bar{\mathbf{A}}$ and $\mathbf{A}$.

$$\bar{\mathbf{A}} = \bar{\mathbf{Z}}\hat{\bar{\mathbf{x}}}^{-1} = \mathbf{GZG}^\mathrm{T}(\widehat{\mathbf{Gx}})^{-1} = \mathbf{GA\hat{x}G}^\mathrm{T}(\widehat{\mathbf{Gx}})^{-1} = \mathbf{GAH}^\mathrm{T}, \tag{8}$$

where $\mathbf{H}$ is defined as $\mathbf{H} = (\widehat{\mathbf{Gx}})^{-1}\mathbf{G\hat{x}}$. The matrix $\mathbf{H}$ has the same structure as $\mathbf{G}$, that is,

$$\mathbf{H} = \begin{bmatrix} * & \ldots & * & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & * & \ldots & * & \ldots & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \ddots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & * & \ldots & * & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & * & \ldots & * \end{bmatrix},$$

where a $*$ is used to indicate nonzero elements. The typical element h_{Ij} (for $I = 1, \ldots, N$,

[5] Note that for any aggregation scheme the aggregator matrix can be written as in equation (5) after a suitable renumbering of the original sectors.

and $j = 1, \ldots, n$) is defined as

$$h_{Ij} = \begin{cases} x_j\left[\sum_{i \in I} x_i\right]^{-1}, & \text{if } j \in I, \\ 0, & \text{if } j \notin I. \end{cases}$$

The elements h_{Ij} are weights, denoting the value of the output (x_j) in the original sector j as a fraction of the value of the output $\left(\sum_{i \in I} x_i\right)$ in the aggregate sector I. Note that these weights add up to one, that is $\sum_{j \in I} h_{Ij} = 1$, which implies $\mathbf{H}e_{(n)} = e_{(N)}$. Note also that $\mathbf{H}\mathbf{G}^{\mathrm{T}} = \mathbf{I}_{(N)}$.

Similar to equation (8), the relation between $\bar{\mathbf{B}}$ and $\mathbf{B}$ is given by

$$\bar{\mathbf{B}} = \bar{\mathbf{M}}\hat{\bar{\mathbf{x}}}^{-1} = \mathbf{M}\mathbf{G}^{\mathrm{T}}(\widehat{\mathbf{G}\mathbf{x}})^{-1} = \mathbf{B}\hat{\mathbf{x}}\mathbf{G}^{\mathrm{T}}(\widehat{\mathbf{G}\mathbf{x}})^{-1} = \mathbf{B}\mathbf{H}^{\mathrm{T}}. \tag{9}$$

Finally, we derive the relation between $\bar{\boldsymbol{\pi}}^{\mathrm{T}}$ and $\boldsymbol{\pi}^{\mathrm{T}}$. From figure 2 it follows that the vector $\boldsymbol{\pi}^{\mathrm{T}}$ of deflators satisfies $\boldsymbol{x}_{\mathrm{d}} = \hat{\boldsymbol{\pi}}\boldsymbol{x}$ or, equivalently, $\boldsymbol{\pi}^{\mathrm{T}} = \boldsymbol{x}_{\mathrm{d}}^{\mathrm{T}}\hat{\mathbf{x}}^{-1}$. In the same way, it follows from figure 3(b) that $\bar{\boldsymbol{\pi}}^{\mathrm{T}} = \bar{\boldsymbol{x}}_{\mathrm{d}}^{\mathrm{T}}\hat{\bar{\mathbf{x}}}^{-1}$. The elements of the vector $\bar{\boldsymbol{x}}_{\mathrm{d}}$ denote the outputs, in constant prices, in the aggregated sectors. Ideally, these are equal to the aggregation of the deflated outputs of the original sectors. Therefore, it is required that $\bar{\boldsymbol{x}}_{\mathrm{d}} = \mathbf{G}\boldsymbol{x}_{\mathrm{d}}$. Then, we find

$$\bar{\boldsymbol{\pi}}^{\mathrm{T}} = \bar{\boldsymbol{x}}_{\mathrm{d}}^{\mathrm{T}}\hat{\bar{\mathbf{x}}}^{-1} = (\mathbf{G}\boldsymbol{x}_{\mathrm{d}})^{\mathrm{T}}(\widehat{\mathbf{G}\mathbf{x}})^{-1} = \mathbf{x}_{\mathrm{d}}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}(\widehat{\mathbf{G}\mathbf{x}})^{-1} = \boldsymbol{\pi}^{\mathrm{T}}\hat{\mathbf{x}}\mathbf{G}^{\mathrm{T}}(\widehat{\mathbf{G}\mathbf{x}})^{-1}.$$

Or, equivalently,

$$\bar{\boldsymbol{\pi}}^{\mathrm{T}} = \boldsymbol{\pi}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}. \tag{10}$$

The deflator ($\bar{\pi}_I$) for the aggregate sector I is the weighted average of the deflators (π_i) of the original sectors as contained by this aggregate sector (that is, $i \in I$). The weights reflect the contribution of the original sector to the value of the output, in current prices, of the aggregate sector.

Now we are able to specify the aggregation problem. We are interested in the values added for each of the aggregate sectors. Aggregation of the deflated IO table (figure 2) yields $\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}$. Applying the double-deflation method to the aggregated IO table in current prices [figure 3(a)] yields $\bar{\boldsymbol{v}}_{\mathrm{d}}^{\mathrm{T}}$ as in equation (7) with the deflator vector $\bar{\boldsymbol{\pi}}^{\mathrm{T}}$ as given in equation (10). In general, aggregation after deflation (that is, $\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}$) yields a different answer than does deflation after aggregation (that is, $\bar{\boldsymbol{v}}_{\mathrm{d}}^{\mathrm{T}}$). The aggregation is called price acceptable (or P-acceptable) if equality between $\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}}$ and $\bar{\boldsymbol{v}}_{\mathrm{d}}^{\mathrm{T}}$ holds for *any* deflator vector $\boldsymbol{\pi}$.(6) Next we derive necessary and sufficient conditions for the aggregation to be P-acceptable.

Theorem 1. *The aggregation is P-acceptable if and only if* $\mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}} = \mathbf{A}\mathbf{H}^{\mathrm{T}}$.

Proof.

$$\boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}\mathbf{G}^{\mathrm{T}} = [\boldsymbol{\pi}^{\mathrm{T}}(\mathbf{I}_{(n)} - \mathbf{A}) - \boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}]\hat{\mathbf{x}}\mathbf{G}^{\mathrm{T}} = [\boldsymbol{\pi}^{\mathrm{T}}(\mathbf{I}_{(n)} - \mathbf{A}) - \boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}]\mathbf{H}^{\mathrm{T}}(\widehat{\mathbf{G}\mathbf{x}}),$$

and

$$\bar{\boldsymbol{v}}_{\mathrm{d}}^{\mathrm{T}} = [\bar{\boldsymbol{\pi}}^{\mathrm{T}}(\mathbf{I}_{(N)} - \bar{\mathbf{A}}) - \boldsymbol{\rho}^{\mathrm{T}}\bar{\mathbf{B}}]\hat{\bar{\mathbf{x}}} = [\boldsymbol{\pi}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\mathbf{I}_{(n)} - \bar{\mathbf{A}}) - \boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}\mathbf{H}^{\mathrm{T}}](\widehat{\mathbf{G}\mathbf{x}}).$$

Postmultiply both expressions by $(\widehat{\mathbf{G}\mathbf{x}})^{-1}$, then equality holds if and only if

$$[\boldsymbol{\pi}^{\mathrm{T}}(\mathbf{I}_{(n)} - \mathbf{A}) - \boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}]\,\mathbf{H}^{\mathrm{T}} = \boldsymbol{\pi}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\mathbf{I}_{(N)} - \bar{\mathbf{A}}) - \boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}\mathbf{H}^{\mathrm{T}}.$$

Or, equivalently,

$$\boldsymbol{\pi}^{\mathrm{T}}(\mathbf{I}_{(n)} - \mathbf{A})\mathbf{H}^{\mathrm{T}} = \boldsymbol{\pi}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\mathbf{I}_{(N)} - \bar{\mathbf{A}}). \tag{11}$$

(6) Note that equality may occur also when the aggregation is not P-acceptable. For example, for a specific choice of $\boldsymbol{\pi}$, equality can be shown to hold. This is the subject of Olsen's (1993) paper.

This equality holds for all π if and only if $(\mathbf{I}_{(n)} - \mathbf{A})\mathbf{H}^{\mathrm{T}} = \mathbf{H}^{\mathrm{T}}(\mathbf{I}_{(N)} - \bar{\mathbf{A}})$ which is equivalent to $\mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}} = \mathbf{A}\mathbf{H}^{\mathrm{T}}$. ■

This theorem states that the aggregation is P-acceptable if and only if the matrix $\mathbf{A}$ satisfies certain restrictions. If we consider the equality $\mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}} = \mathbf{A}\mathbf{H}^{\mathrm{T}}$ element-wise, the following condition must hold:

$$\sum_{j \in J} a_{ij} h_{Jj} = h_{Ii} \bar{a}_{IJ}, \qquad \forall i \in I, \tag{12}$$

which proves the following theorem.

Theorem 2. $\mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}} = \mathbf{A}\mathbf{H}^{\mathrm{T}}$ *if and only if the condition given in equation (12) holds for all* I, J $(I, J = 1, \ldots, N)$.

In order to get an impression of the meaning of the requirements in this theorem, we give an interpretation in terms of the underlying IO table. To this end, substitute $h_{Jj} = x_j \Big/ \sum_{j \in J} x_j$, $h_{Ii} = x_i \Big/ \sum_{i \in I} x_i$, $\bar{a}_{IJ} = \bar{z}_{IJ}/\bar{x}_J = \sum_{i \in I} \sum_{j \in J} z_{ij} \Big/ \sum_{j \in J} x_j$, and use $a_{ij}x_j = z_{ij}$. This yields the following corollary.

Corollary 1. *The condition in equation (12) holds if and only if*

$$\frac{1}{x_i} \sum_{j \in J} z_{ij} = \sum_{i \in I} \sum_{j \in J} z_{ij} \Big/ \sum_{i \in I} x_i, \qquad \forall i \in I.$$

This condition states that each sector i $(i \in I)$ sells the same percentage of its output to the aggregate sector J. For P-acceptability this must hold for all the aggregate sectors I and J.

The results provide conditions under which the sectoral value-added terms are correct when the double-deflation method is applied to an aggregated IO table. As all data in an IO table are somehow aggregated in the process of compilation, and as the conditions are necessary and sufficient we may also turn the argument around. The condition in corollary 1 then explicates what exactly is required by the assumption of producing a homogeneous good in each sector. Namely, each subsector of sector i sells the same percentage of its output to sector j, which must hold for all sectors i and j.

For issues of planning and for carrying out structural analyses at a multisectoral level, we are not so much interested in a deflation method as a means of estimating the value added in constant prices. In these cases, we are more interested in obtaining the matrix of intermediate deliveries in constant prices, as a reflection of the production structure. The following corollary expresses that P-acceptability is necessary and sufficient for obtaining the correct column sums of the intermediate deliveries, when the double-deflation method is applied.

Corollary 2. *The column sums of the matrix of intermediate deliveries are estimated correctly by the double-deflation method if and only if the aggregation is P-acceptable.*

Proof. The column sums for the case of aggregation after deflation yield

$$e_{(N)}^{\mathrm{T}} \mathbf{G}\mathbf{Z}_{\mathrm{d}}\mathbf{G}^{\mathrm{T}} = e_{(N)}^{\mathrm{T}} \mathbf{G}\hat{\pi}\,\mathbf{Z}\mathbf{G}^{\mathrm{T}} = e_{(n)}^{\mathrm{T}} \hat{\pi}\,\mathbf{Z}\mathbf{G}^{\mathrm{T}} = \hat{\pi}^{\mathrm{T}}\mathbf{Z}\mathbf{G}^{\mathrm{T}} = \pi^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}}\mathbf{G}^{\mathrm{T}}.$$

In the case of deflation after aggregation, the column sums are given by:

$$e_{(N)}^{\mathrm{T}} \hat{\bar{\pi}}\,\bar{\mathbf{Z}} = \bar{\pi}^{\mathrm{T}}\bar{\mathbf{A}}\hat{\bar{\mathbf{x}}} = \pi^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}}(\widehat{\mathbf{G}\mathbf{x}}).$$

Equality between the two expressions for the column sums is required to hold for all π, which implies $\mathbf{A}\hat{\mathbf{x}}\mathbf{G}^{\mathrm{T}} = \mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}}(\widehat{\mathbf{G}\mathbf{x}})$. Postmultiplying both sides by $(\widehat{\mathbf{G}\mathbf{x}})^{-1}$ and using $\hat{\mathbf{x}}\mathbf{G}^{\mathrm{T}}(\widehat{\mathbf{G}\mathbf{x}})^{-1} = \mathbf{H}^{\mathrm{T}}$ yields $\mathbf{A}\mathbf{H}^{\mathrm{T}} = \mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}}$. ■

Note that P-acceptability is necessary for the column sums of the intermediate deliveries to be correct (when the double-deflation method is applied). As such, P-acceptability can be viewed as a minimum requirement for the correct estimation of the intermediate deliveries themselves.

3 Duality of aggregation results

The results as derived in the previous section for price-acceptable aggregation are dual to the traditional results for quantity-acceptable aggregation. These primal results and their relation to the dual results of section 2 are discussed in this section.

Consider the following IO model:

$$\boldsymbol{x} = \mathbf{A}\boldsymbol{x} + \boldsymbol{f}. \tag{13}$$

When the final demand vector $\boldsymbol{f}$ is given, the output vector $\boldsymbol{x}$ is solved as $\boldsymbol{x} = (\mathbf{I}_{(n)} - \mathbf{A})^{-1}\boldsymbol{f}$. When we are interested only in the aggregate output vector, this yields $\mathbf{G}\boldsymbol{x} = \mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})^{-1}\boldsymbol{f}$, that is, aggregation after solution. On the other hand, an aggregate output vector $\bar{\boldsymbol{x}}$ may also be obtained according to solution after aggregation. The aggregated model is given by

$$\bar{\boldsymbol{x}} = \bar{\mathbf{A}}\bar{\boldsymbol{x}} + \bar{\boldsymbol{f}}, \qquad \text{with } \bar{\mathbf{A}} = \mathbf{G}\mathbf{A}\mathbf{H}^{\mathrm{T}}. \tag{14}$$

An arbitrary final demand vector $\boldsymbol{f}$ is aggregated into $\bar{\boldsymbol{f}} = \mathbf{G}\boldsymbol{f}$, and the solution of the aggregated model yields $(\mathbf{I}_{(N)} - \bar{\mathbf{A}})^{-1}\bar{\boldsymbol{f}} = (\mathbf{I}_{(N)} - \bar{\mathbf{A}})^{-1}\mathbf{G}\boldsymbol{f}$.

The aggregation is called Q-acceptable if both approaches yield the same result for any final demand vector $\boldsymbol{f}$.

Theorem 3. *(Hatanaka, 1952) The aggregation is Q-acceptable if and only if $\bar{\mathbf{A}}\mathbf{G} = \mathbf{G}\mathbf{A}$.*

Proof.

$$\mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})^{-1}\boldsymbol{f} = (\mathbf{I}_{(N)} - \bar{\mathbf{A}})^{-1}\mathbf{G}\boldsymbol{f}$$

must hold for any $\boldsymbol{f}$. Hence,

$$\mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})^{-1} = (\mathbf{I}_{(N)} - \bar{\mathbf{A}})^{-1}\mathbf{G},$$

which is equivalent to

$$(\mathbf{I}_{(N)} - \bar{\mathbf{A}})\mathbf{G} = \mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A}),$$

or $\bar{\mathbf{A}}\mathbf{G} = \mathbf{G}\mathbf{A}$, because $\mathbf{I}_{(N)}\mathbf{G} = \mathbf{G}\mathbf{I}_{(n)} = \mathbf{G}$. ■

As an alternative, Q-acceptability could have been defined by starting at the other end. That is, given $\boldsymbol{x}$, $\boldsymbol{f}$ is solved as $\boldsymbol{f} = (\mathbf{I}_{(n)} - \mathbf{A})\boldsymbol{x}$. Q-acceptability then requires that aggregation after solution [that is, $\mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})\boldsymbol{x}$] gives the same answer as solution after aggregation [that is, $(\mathbf{I}_{(N)} - \bar{\mathbf{A}})\mathbf{G}\boldsymbol{x}$]. Equality must hold for any vector $\boldsymbol{x}$.

The dual versions of systems (13) and (14) are $\boldsymbol{\pi}^{\mathrm{T}} = \boldsymbol{\pi}^{\mathrm{T}}\mathbf{A} + \boldsymbol{u}^{\mathrm{T}}$ and $\bar{\boldsymbol{\pi}}^{\mathrm{T}} = \bar{\boldsymbol{\pi}}^{\mathrm{T}}\bar{\mathbf{A}} + \bar{\boldsymbol{u}}^{\mathrm{T}}$. The vector $\boldsymbol{u}^{\mathrm{T}}$ consists of coefficients. Note that in section 2 we used the expression $\boldsymbol{u}^{\mathrm{T}} = \boldsymbol{v}_{\mathrm{d}}^{\mathrm{T}}\hat{\mathbf{x}}^{-1} + \boldsymbol{\rho}^{\mathrm{T}}\mathbf{B}$. Similar to the aggregation of the import coefficients, that is, $\bar{\mathbf{B}} = \mathbf{B}\mathbf{H}^{\mathrm{T}}$, we now have $\bar{\boldsymbol{u}}^{\mathrm{T}} = \boldsymbol{u}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}$. Acceptability requires that the aggregated solution of the full model, that is, $\boldsymbol{u}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}} = \boldsymbol{\pi}^{\mathrm{T}}(\mathbf{I}_{(n)} - \mathbf{A})^{-1}\mathbf{H}^{\mathrm{T}}$, equals the solution of the aggregated model, that is, $\bar{\boldsymbol{\pi}}^{\mathrm{T}}(\mathbf{I}_{(N)} - \bar{\mathbf{A}})^{-1} = \boldsymbol{\pi}^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}(\mathbf{I}_{(N)} - \bar{\mathbf{A}})^{-1}$. This is precisely the requirement in equation (11). The dual equation to $\bar{\mathbf{A}}\mathbf{G} = \mathbf{G}\mathbf{A}$ then yields $\mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}} = \mathbf{A}\mathbf{H}^{\mathrm{T}}$, as in theorem 1.

Element-wise equality of $\bar{\mathbf{A}}\mathbf{G}$ and $\mathbf{G}\mathbf{A}$ immediately gives the analogue of theorem 2.

Theorem 4. *(Ara, 1959)* $\bar{\mathbf{A}}\mathbf{G} = \mathbf{GA}$) *if and only if the following condition holds for all* $I, J\ (I, J = 1, ..., N)$.

$$\sum_{i \in I} a_{ij} = \bar{a}_{IJ}, \qquad \forall j \in J.$$

With $a_{ij} = z_{ij}/x_j$, and $\bar{a}_{IJ} = \sum_{i \in I} \sum_{j \in J} z_{ij} \Big/ \sum_{j \in J} x_j$, the condition in theorem 4 may also be expressed as

$$\frac{1}{x_j} \sum_{i \in I} z_{ij} = \sum_{i \in I} \sum_{j \in J} z_{ij} \Big/ \sum_{j \in J} x_j, \qquad \forall j \in J.$$

Thus, a necessary and sufficient condition is that each sector $j\ (j \in J)$ buys the same percentage of its inputs from the aggregate sector I. Note that this condition requires a special *input* structure as given by the *columns* in the IO table. The dual result in corollary 1, requires a special *output* structure as given by the *rows* in the IO table.

We conclude this section with two corollaries which give conditions for the aggregation to be price as well as quantity acceptable.

Corollary 3. *The aggregation is both P-acceptable and Q-acceptable if and only if* $\mathbf{AH}^{\mathrm{T}}\mathbf{G} = \mathbf{H}^{\mathrm{T}}\mathbf{GA}$.

Proof. ($\Rightarrow$) Q-acceptability implies that $\mathbf{H}^{\mathrm{T}}(\bar{\mathbf{A}}\mathbf{G}) = \mathbf{H}^{\mathrm{T}}\mathbf{GA}$. P-acceptability implies $(\mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}})\mathbf{G} = \mathbf{AH}^{\mathrm{T}}\mathbf{G}$.
($\Leftarrow$) Postmultiplying $\mathbf{AH}^{\mathrm{T}}\mathbf{G} = \mathbf{H}^{\mathrm{T}}\mathbf{GA}$ by $\mathbf{H}^{\mathrm{T}}$ yields $\mathbf{AH}^{\mathrm{T}}\mathbf{GH}^{\mathrm{T}} = \mathbf{H}^{\mathrm{T}}\mathbf{GAH}^{\mathrm{T}}$. The first term equals $\mathbf{AH}^{\mathrm{T}}$ because $\mathbf{GH}^{\mathrm{T}} = \mathbf{I}_{(N)}$, the second term equals $\mathbf{H}^{\mathrm{T}}\bar{\mathbf{A}}$ because $\mathbf{GAH}^{\mathrm{T}} = \bar{\mathbf{A}}$. This proves P-acceptability. Premultiplying $\mathbf{AH}^{\mathrm{T}}\mathbf{G} = \mathbf{H}^{\mathrm{T}}\mathbf{GA}$ by $\mathbf{G}$ gives $\mathbf{GAH}^{\mathrm{T}}\mathbf{G} = \mathbf{GH}^{\mathrm{T}}\mathbf{GA}$ or, equivalently, $\bar{\mathbf{A}}\mathbf{G} = \mathbf{GA}$, which proves Q-acceptability. ■

In IO models and applications thereof, the Leontief inverse $(\mathbf{I}_{(n)} - \mathbf{A})^{-1}$ plays a crucial role. In general, the Leontief inverse of the aggregated input matrix, that is $(\mathbf{I}_{(N)} - \bar{\mathbf{A}})^{-1}$, will be different from the aggregated Leontief inverse $\mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})^{-1}\mathbf{H}^{\mathrm{T}}$. This simple relationship (the inverse of the aggregate is the aggregate of the inverse) holds if the aggregation is both P-acceptable and Q-acceptable.

Corollary 4. *If the aggregation is both P-acceptable and Q-acceptable,*

$$(\mathbf{I}_{(N)} - \bar{\mathbf{A}})^{-1} = \mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})^{-1}\mathbf{H}^{\mathrm{T}}.$$

Proof. P-acceptability and Q-acceptability imply $\mathbf{AH}^{\mathrm{T}}\mathbf{G} = \mathbf{H}^{\mathrm{T}}\mathbf{GA}$, which is equivalent to $(\mathbf{I}_{(n)} - \mathbf{A})\mathbf{H}^{\mathrm{T}}\mathbf{G} = \mathbf{H}^{\mathrm{T}}\mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})$, or $\mathbf{H}^{\mathrm{T}}\mathbf{G} = (\mathbf{I}_{(n)} - \mathbf{A})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})$. Premultiplying both sides by $\mathbf{G}$ and postmultiplying by $\mathbf{H}^{\mathrm{T}}$ yields

$$\mathbf{GH}^{\mathrm{T}}\mathbf{GH}^{\mathrm{T}} = \mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})\mathbf{H}^{\mathrm{T}}.$$

Note that $\mathbf{GH}^{\mathrm{T}} = \mathbf{I}_{(N)}$, and $\mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})\mathbf{H}^{\mathrm{T}} = (\mathbf{I}_{(N)} - \bar{\mathbf{A}})$. Thus,

$$\mathbf{I}_{(N)} = \mathbf{G}(\mathbf{I}_{(n)} - \mathbf{A})^{-1}\mathbf{H}^{\mathrm{T}}(\mathbf{I}_{(N)} - \bar{\mathbf{A}}),$$

which proves the result. ■

4 Conclusions

In this paper we have analyzed the effects of aggregation when an IO table is estimated in constant prices by means of double deflation. In doing this, two possibilities exist. On the one hand, aggregation after deflation means that the original table in current prices is deflated first, after which the resulting table in constant prices is aggregated into a smaller table. On the other hand, deflation after aggregation implies that the original table in current prices is aggregated into a smaller table, which is then deflated. Usually the two approaches yield different answers. If aggregation after deflation is taken to provide the correct answer, deflation after aggregation thus induces aggregation errors.

Applying the double-deflation method results in estimates of the sectoral values added and the intermediate deliveries. Necessary and sufficient conditions were derived for the sectoral values added to be free of aggregation errors. The same conditions are also necessary and sufficient for the column sums of the intermediate deliveries to be error free. Under these conditions, the aggregation is termed price acceptable. The conditions were shown to be dual to the traditional results for acceptability of the aggregation in the quantity version of the IO model. Both sets of conditions are very strong and unlikely to be met in any practical case.

The consequences of this conclusion are relevant for empirical work. The double-deflation method is widely used by practitioners to estimate IO tables, and intermediate deliveries in particular, in constant prices. For this purpose they have to rely on published IO tables in current prices as a starting point. Typically, these published tables are (often highly) aggregated. This implies that deflation is actually a form of deflation after aggregation, which in turn implies that the results will suffer from aggregation errors.

Originally, double deflation was developed for estimating the value added in constant prices. Nowadays, practitioners predominantly use double deflation for estimating the intermediate deliveries in constant prices. In empirical studies, it has been our experience that published information is often readily available, precisely for the sectoral values added in constant prices. We observe (Dietzenbacher and Hoen, 1998) that the information required for applying double deflation plus the information with respect to sectoral values added in constant prices, exactly satisfies the requirements for applying the RAS method (Stone, 1963). This biproportional adjustment method is usually applied for updating IO matrices. The RAS method estimates the cells of a matrix, given its row and column sums, and given a full matrix for, say, an earlier year.(7)

We proposed using RAS (Dietzenbacher and Hoen, 1998), as a heuristic alternative for double deflation, for the purpose of estimating the intermediate deliveries in constant prices. In an empirical analysis, it is found that RAS deflation performs better than double deflation, in particular for the columns of the matrix of intermediate deliveries. This is not very surprising, because the RAS method is able to exploit the additional information that is available.

The empirical examination of RAS versus double deflation which was given in our paper (Dietzenbacher and Hoen, 1998) also sheds some light on the aggregation errors. Our calculations are based on two published 58-sector IO tables for the Netherlands in 1988. One is in current prices and the other in prices of 1987. The published table in constant prices is assumed to be the correct (or 'true') table, and provides the required information on the price indexes. Deflating the 58-sector table in current prices by means of double inflation yields an answer that differs from the correct answer. The column sums of the intermediate deliveries show absolute errors ranging from 0.22% to 20.35%, with a weighted average of 2.26%.(8) On the one hand, this deflated 58-sector table may be aggregated into a 12-sector table. The largest absolute error in a column sum then reduces to 4.62%, and the weighted average becomes 0.84%. On the other

(7) For an elaborate introduction to the issue of updating procedures in general and the RAS method in particular, see Miller and Blair (1985). A detailed discussion of the technical aspects regarding the existence and uniqueness of the solution can be found in Bacharach (1970) or Macgill (1977). For critical surveys and evaluations of the empirical performance of RAS, see Allen and Gossling (1975) or Lynch (1986), for example.

(8) Note that these errors can also be interpreted as aggregation errors. Assume that the statistical bureau has full information and deflates a 'super' table, the aggregated version of which is published. The reported deflation of the 58-sector table is then just a case of deflation after aggregation.

hand, the 58-sector table in current prices may be aggregated first, after which the 12-sector table in current prices is deflated. This yields 18.00% as the largest absolute error in a column sum and a weighted average of 2.13%. These errors may seem to be rather small, but it should be borne in mind that the price changes are with respect to only one year. The results clearly indicate that the absolute percentage error in the total intermediate deliveries is 60% smaller for aggregation after deflation than for deflation after aggregation.

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References

Afrasiabi A, Casler S D, 1991, "Product-mix and technological change within the Leontief inverse" *Journal of Regional Science* **31** 147–160
Aislabie C, Gordon M, 1990, "Input–output in practice: sector disaggregation in regional input–output tables" *Economic Systems Research* **2** 407–419
Allen R I G, Gossling W F (Eds), 1975 *Estimating and Projecting Input–Output Coefficients* (Input–Output Publishing Company, London)
Ara K, 1959, "The aggregation problem in input–output analysis" *Econometrica* **27** 257–262
Bacharach M, 1970 *Biproportional Matrices and Input–Output Change* (Cambridge University Press, Cambridge)
Cabrer B, Contreras D, Miravette E, 1991, "Aggregation in input–output tables: how to select the best cluster linkage" *Economic Systems Research* **3** 99–109
De Mesnard L, Dietzenbacher E, 1995, "On the interpretation of fixed input coefficients under aggregation" *Journal of Regional Science* **35** 233–243
Dietzenbacher E, 1992, "Aggregation in multisector models: using the Perron vector" *Economic Systems Research* **4** 3–24
Dietzenbacher E, Hoen A R, 1998, "Deflation of input–output tables from the user's point of view: a heuristic approach" *Review of Income and Wealth* **44** 111–122
Hatanaka M, 1952, "Note on consolidation within a Leontief system" *Econometrica* **20** 301–303
Howe E, Johnson C R, 1989, "Linear aggregation of input–output models" *SIAM Journal on Matrix Analysis and Applications* **10** 65–79
Howe E, Stabler J C, 1989, "Canada divided: the optimal division of an economy into regions" *Journal of Regional Science* **29** 191–211
Kymn K O, 1990, "Aggregation in input–output models: a comprehensive review, 1946–71" *Economic Systems Research* **2** 65–93
Lynch R C, 1986, "An assessment of the RAS method for updating input–output tables", in *Readings in Input–Output Analysis* Ed. I Sohn (Oxford University Press, New York) pp 271–284
Macgill S M, 1977, "Theoretical properties of biproportional matrix adjustments" *Environment and Planning A* **9** 687–701
Miller R E, Blair P D, 1985 *Input–Output Analysis: Foundations and Extensions* (Prentice-Hall, Englewood Cliffs, NJ)
Oksanen E H, Williams J R, 1992, "An alternative factor-analytic approach to aggregation of input–output tables" *Economic Systems Research* **4** 245–256
Olsen J A, 1993, "Aggregation in input–output models: prices and quantities" *Economic Systems Research* **5** 253–275
Stone R (Ed.), 1963 *Input–Output Relationships, 1954–1966, A Programme for Growth, Volume 3* (Chapman and Hall, London)
United Nations, 1973 *Input–Output Tables and Analysis* Studies in Methods, Series F, number 14, revision 1 (United Nations, New York)