ABSTRACT

Title of Document: Multi-level, Multi-stage and Stochastic Optimization Models for Energy Conservation in Buildings for Federal, State and Local Agencies

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Energy Conservation Measure (ECM) project selection is made difficult given real-world constraints, limited resources to implement savings retrofits, various suppliers in the market and project financing alternatives. Many of these energy efficient retrofit projects should be viewed as a series of investments with annual returns for these traditionally risk-averse agencies. Given a list of ECMs available, federal, state and local agencies must determine how to implement projects at lowest costs. The most common methods of implementation planning are suboptimal relative to cost.

Federal, state and local agencies can obtain greater returns on their energy conservation investment over traditional methods, regardless of the implementing organization. This

dissertation outlines several approaches to improve the traditional energy conservations models.

Any public buildings in regions with similar energy conservation goals in the United States or internationally can also benefit greatly from this research. Additionally, many private owners of buildings are under mandates to conserve energy e.g., Local Law 85 of the New York City Energy Conservation Code requires any building, public or private, to meet the most current energy code for any alteration or renovation. Thus, both public and private stakeholders can benefit from this research.

The research in this dissertation advances and presents models that decision-makers can use to optimize the selection of ECM projects with respect to the total cost of implementation. A practical application of a two-level mathematical program with equilibrium constraints (MPEC) improves the current best practice for agencies concerned with making the most cost-effective selection leveraging energy services companies or utilities. The two-level model maximizes savings to the agency and profit to the energy services companies (Chapter 2).

An additional model presented leverages a single congressional appropriation to implement ECM projects (Chapter 3). Returns from implemented ECM projects are used to fund additional ECM projects. In these cases, fluctuations in energy costs and uncertainty in the estimated savings severely influence ECM project selection and the amount of the appropriation requested. A risk aversion method proposed imposes a minimum on the number of "of projects completed in each stage. A comparative method using Conditional Value at Risk is analyzed. Time consistency was addressed in this chapter. This work demonstrates how a risk-based, stochastic, multi-stage model with binary decision variables at each stage provides a much more accurate estimate for planning than the agency's traditional approach and deterministic models.

Finally, in Chapter 4, a rolling-horizon model allows for subadditivity and superadditivity of the energy savings to simulate interactive effects between ECM projects. The approach makes use of inequalities (McCormick, 1976) to re-express constraints that involve the product of binary variables with an exact linearization (related to the convex hull of those constraints). This model additionally shows the benefits of learning between stages while remaining consistent with the single congressional appropriations framework.

Multi-level, Multi-stage and Stochastic Optimization Models for Energy Conservation in Buildings for Federal, State and Local Agencies

By

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Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park, in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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Chapter 1: Introduction

1.1. **Background**

1.1.1. Definition of Federal, State and Local Agencies

Federal agencies, as described in this work, are departments of the United States federal, government that operate within the Executive and Legislative branches or as independent establishments and government corporations. The Congress and President of the United States delegate specific authority to government agencies and establishes the goals towards which the agency must work. When the agency has rulemaking power, these agency rules or regulations have the power of federal law.

Agencies that operate within the federal executive departments include the President's cabinet-level departments and their subunits such as the Department of Defense and the Department of Housing and Urban Development (HUD).

The independent agencies of the federal government, such as General Services Administration (GSA), exercise limited independence from the President's control. The leadership of independent agencies are often appointed and usually agencies work together in groups, such as a commission, board or council. An example of this is the Federal Trade Commission (FTC), which is made up of three bureaus who mission is to protect consumers, and prevent anticompetitive business practices. Independent agencies, as well as state agencies and even local agencies often function like the federal government with the authority to legislate to and enforce agency regulations.

These agencies are responsible for all aspects of their mission and program including the operation and maintenance of their physical infrastructure and energy use.

1.1.2. Energy Consumption by Federal State and Local Agencies

Energy consumption is the amount of energy consumed in a process, system or an organization. Despite a consistent decline in energy consumption, the federal government has consistently been one of the largest consumer of energy using almost 1.2 quadrillion BTUs (British thermal units) per year from all fuel sources in the United States. The cost of meeting the federal government's facility energy costs had grown to $6.5 billion per year in 2007 (Energy, 2010). State and local governments spend an additional $10 billion a year on energy to provide public services and meet constituent needs.

Federal agencies report energy used in three end-use sectors:

- Buildings subject to statutory energy reduction requirements (goal buildings),
- Buildings excluded from the energy reduction requirements (goal-excluded facilities),
- Vehicles and equipment.

During FY 2014, federal agencies reported using 0.9 quadrillion British thermal units (Btu) or "quads" of delivered energy across the three end-use sectors.[1] In terms of primary energy, which also includes the energy used at utility plants to generate electricity and steam, federal agencies used 1.4 quads, which is approximately 1.4% of the 98.5 quads used in the United States.

[1] Primary energy refers to energy used at the source including fuel input to electric power plants. Delivered energy is not primary.

In FY 2014, 39% of all federal energy was consumed by federal facilities. Energy consumed in federal government facilities has generally been declining over the past four decades. However; the reduction stems from both the total square footage occupied by the federal government, which continues to fall from its peak in FY 1987, and from the energy consumed per square foot inside federal buildings, which has been declining since FY 1975 (EERE, 2016).

Because of its energy use and other activities, the federal government emits approximately 82 million metric tons of carbon dioxide equivalent ($MMTCO_2e$) of greenhouse gas (GHG) emissions. For those emissions targeted for reduction, the federal government reduced GHG emissions by 17.4%, from 51.4 million metric tons of $MMTCO_2e$ in FY 2008 to 42.4 $MMTCO_2e$ in FY 2014.

Figure 1-1 below provides a comprehensive accounting of the government's energy and water use, associated greenhouse gas emissions and other resource management data for FY 2014.

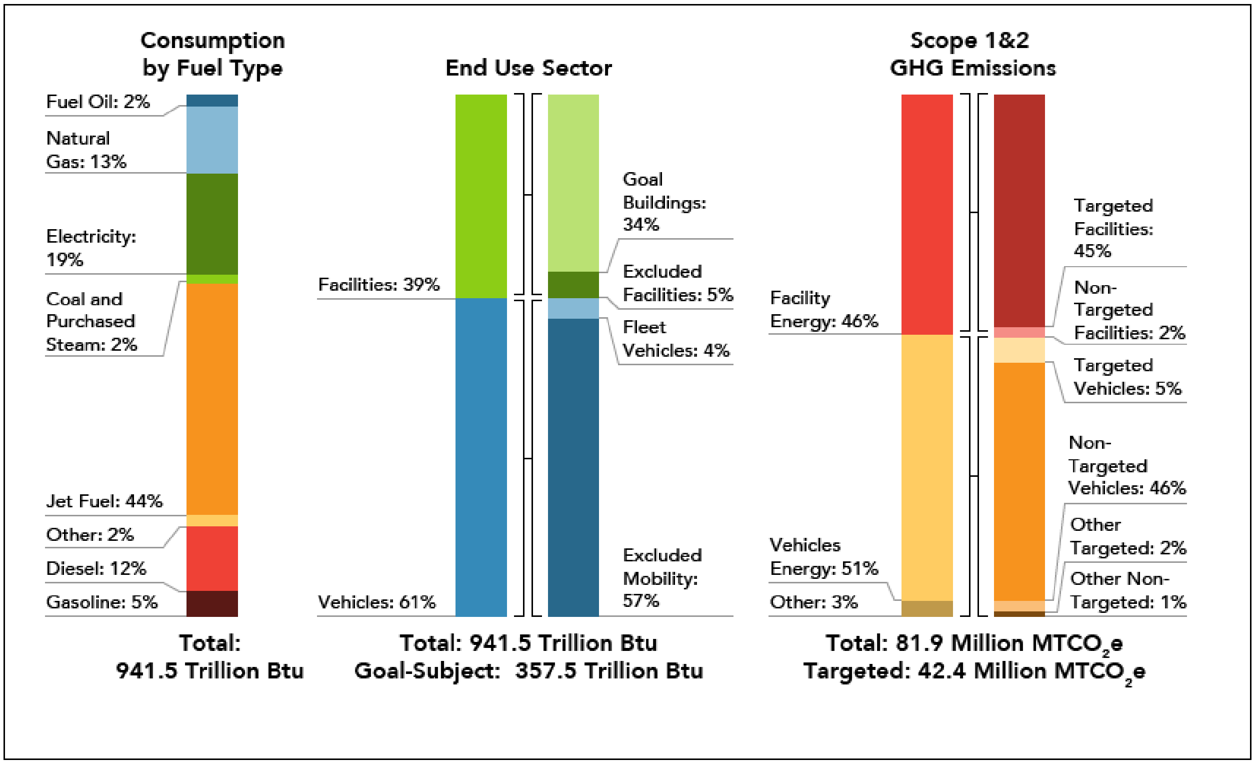

Figure 1-1: Total Energy Consumption by End Use Sector and Type, FY 2014 (EERE, 2016)

Energy conservation in the building end use sector is the primary focus of this research. The federal, state and local agencies have continually issued regulations mandating focus on energy conservation in these sectors. The following regulation are discussed based on the connection with energy use by these agencies.

1.1.3. Regulatory Drivers

In 1978, the United States Congress signed The National Energy Conservation Policy Act (NECPA) into law. This law is the basis for federal energy management goals and requirements in the United States. The overall purpose of the law was to promote the conservation and the efficient use of energy and water, and the use of renewable energy sources by the federal government. The resulting goals for energy performance were issued for federal buildings mandated and it was mandated that each agency apply energy conservation measures (ECMs) and improve the design for construction so that the energy consumption per gross square foot was reduced (Congress, 1978). The NECPA has been regularly updated and amended by subsequent laws and regulations. One such regulation is the Energy Independence and Security Act of 2007 (EISA 2007), which established energy management goals and requirements while also amending portions of the NECPA.

These Congressional Acts mandate specific goals and targets including:

- Reducing energy intensity (Btu/ft^2) by 15% by the end of FY 2010, compared to a FY 2003 baseline and by 30% by the end of FY 2015;
- Increasing renewable electric energy equivalent to at least 5% of total electricity use in FYs 2010-2012 and at least 7.5% in FY 2013 and beyond; at least half must come from sources developed after January 1, 1999; and
- Achieving a 20% reduction in vehicle fleet petroleum use by 2015.

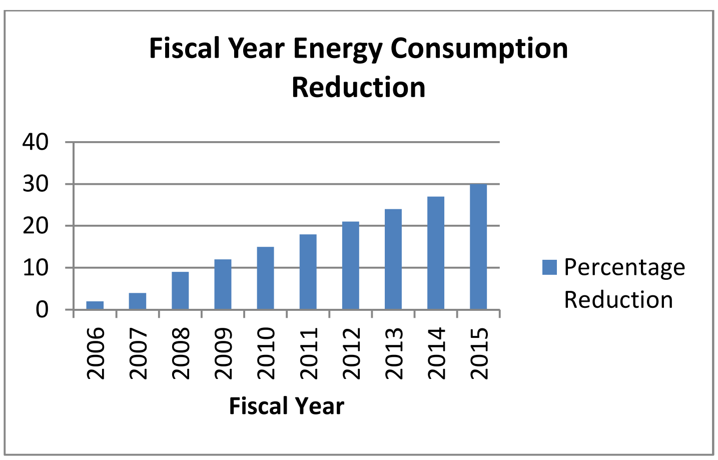

Figure 1-2: Required Reduction in Energy Consumption

Overall, federal agencies must enhance efforts towards sustainable buildings and communities. Specifically, agencies must implement high performance sustainable federal building design, construction, operation, management, maintenance, and deconstruction by ensuring all new federal buildings, entering the design phase in 2020 or later, are designed to achieve zero net energy by 2030.[2]

In FY 2014, federal agencies reported that buildings subject to the National Energy Conservation Policy Act's energy intensity reduction goals collectively decreased energy use per gross square foot (Btu/GSF) by 21% relative to FY 2003. This falls short of the 27% reduction requirement for FY 2014. Although the rate of reduction in energy intensity slowed in FY 2013 and FY 2014, federal agencies achieved the FY 2014 reductions despite less favorable climatic conditions; heating degree daysincreased by 25.5% between FY 2012 and FY 2014.[3]

While significant reductions in building energy intensity have been made, many more are required, while tougher challenges exist in funding energy conservation and renewable projects. Facility energy intensity fell short of the 27% goals of Executive Order 13423 and Energy Independence and Security Act to reduce energy intensity (Btu/GSF) with only a 21% reduction (Tremper, 2014). The remaining conservation opportunities will require ingenuity to both fund and implement the projects and thus provides an impetus for this dissertation.

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[2] A zero net energy building is one with zero net energy consumption taking in to account any energy generated by the building itself.
[3] A heating degree day is the difference between the daily mean (assumed to be lower than 65°F) temperature and 65° F.

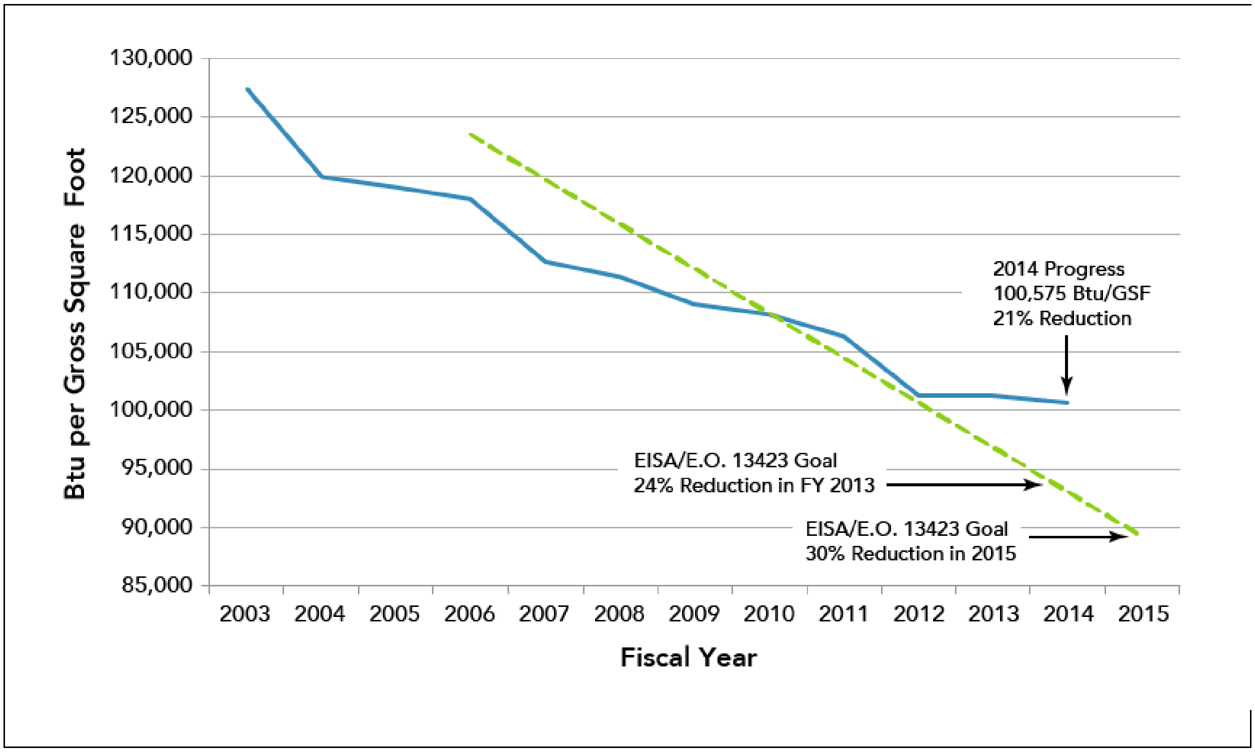

Figure 1-3: Energy Reduction Goals Not Being Met (Tremper, 2014)

President Obama signed Executive Order (EO) 13693, Planning for Federal Sustainability in the Next Decade on 19 March 2015. The goal of EO 13693 is to maintain federal leadership in sustainability and greenhouse gas emission reductions. Specifically, federal agencies shall promote building energy conservation, efficiency, and management by reducing agency building energy intensity measured in BTU/SF by 2.5% annually through year 2025. The order began in fiscal year 2016 and savings were to be measured against the baseline of the agency's building energy use in fiscal year 2015. Federal agencies are also required to ensure that a minimum percentage of the total building electric energy and thermal energy shall be clean energy, renewable electric energy or alternative energy of

- not less than 10 percent in fiscal years 2016 and 2017;
- not less than 13 percent in fiscal years 2018 and 2019;

- not less than 16 percent in fiscal years 2020 and 2021;
- not less than 20 percent in fiscal years 2022 and 2023; and
- not less than 25 percent by fiscal year 2025 and each year thereafter.

Regulations such as EO 13693 also exist on the state and commercial levels. For example, California's Title 24, 2016 Building Energy Efficiency Standards specify requirements for manufacturing, construction, and installation of certain systems, equipment, appliances and building components (Comission, 2015).

The central objective is clear; buildings must reduce their energy consumption. However, the secondary objective of lowering spending on energy, while adding the cost of implementing energy savings measures complicates the directives. Furthermore, low hanging fruit has been picked. The remaining programs have longer simple paybacks while many are renewable programs with little or no payback. These challenging problems require much more innovation to solve. The mandates make the implicit assumption that methods of reducing energy consumption and lowering energy spending are known with certainty, easily quantified and energy conservation projects are optimally selected.

1.2. Agency Approach

Faced with the multitude of requirements with the ultimate objectives of conserving energy and lowering spending, many agencies' facility and energy managers find themselves with a computational challenge. There is a clear understanding of the extent to which energy efficiency must be achieved but a clear path to achieving these goals has not been dictated. Fortunately, there is an industry standard for best practice (EnergyStar, 2013). The primary tool that the agency's decision-makers use is the energy audit. There are several types of audits, however; an American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) level II or III is most often used for planning and decision-making as defined and discussed in Chapter 2 (Kelsey & Pearson, 2011).

The audit is a comprehensive energy analysis and assessment of the building's energy-using components such that a list of energy conservation measures can be proposed with the following attributes:

- the proposed system or component description
- each measure's required investment
- the annual savings by fuel source
- the annual cost savings in dollars
- measure of such as simple payback ratio or savings to investment ratio

The energy auditors have assessed the regulatory requirements and conducted an audit that recommends the projects necessary to save the requisite energy. All projects must be completed. The agency's approach to implementing these projects has been the naïve method, which involves sorting by cost/ benefit then selecting until the budget has been depleted. They have not leveraged integer programming (or other optimization methods) that solves a resource allocation problem to choose a subset of projects to optimize savings (a "knapsack" problem) as discussed in Chapter 2. Specifically, the costs are the investment costs of each ECM project. The benefits are the annual savings realized from executing ECM projects in a previous stage.

1.3. Agency Options

Given this list of ECMs, the agency's decision-maker faces a series of strategic decisions. Each project, from which the energy manager or decision-maker must select, saves energy or annual energy costs and, in most cases, federal agencies have three options to fund these energy conservation projects

- Energy management programs funded by congressional appropriations[4]
- Private financing through energy savings performance contracts (ESPCs)
- Private financing through utility energy service contracts (UESCs)

Agencies must use these three funding sources in the most effective manner to maximize energy savings and minimize life-cycle cost.

During FY 2014, federal agencies used these three primary options for financing energy efficiency, water conservation, and renewable energy projects in buildings totaled approximately $1,712 billion. Congressional appropriations accounted for approximately $900.6 million. Energy savings performance contract awards by agencies resulted in approximately $706.6 million in project investment. Approximately $105.2 million in project investment came from utility energy service contracts (Danielson, 2015).

With the funding options available, the agency should select the timing of these projects as well as the implementing organization. In general, simpler projects can often be implemented with in-house resources and staff. Lower-cost projects can often be financed with internal operating

[4] A Congressional appropriation is a designation of money for specific use, here, Energy and Water programs, by House and Senate Appropriations Committees.

budgets. Using in-house budgets and resources may provide the best return on investment. These projects may also free up capital for further energy projects. Higher risk, more difficult or projects that require large capital investment can be performed by energy service companies per the guidance first issued in NECPA. However, the energy savings performance contracts completed by the energy services companies and utilities do not yield the cost savings that in-house projects do as the cost savings are shared with the energy services companies and utilities.

The largest opportunity for energy conservation lies in the creation of an implementation plan that contains the appropriate timing of each ECM project Optimization is needed to properly create a plan that maximizes the energy savings while identifying financing and firms available to implement the recommended measures. The appropriate method of achieving these goals has not been regulated. The standard method of creating this plan segments these agency and energy service company and/or utility decisions.

If working with an energy service performance contract or utility energy service contracts, the agency traditionally selects the projects with the quickest paybacks. Only the least desirable or most costly projects are left for energy services companies. As a result, many agencies select a subset of projects to complete internally only to find that the remaining projects can no longer be completed with a decent payback for firms in the market. Furthermore, in the current practice, the agency may leave a subset of projects incomplete assuming that energy services companies will complete them. The current selection process can generate a mix of selected projects that might not maximize the agency's share of the benefit (in dollars) of the energy saved. Given the profit-maximization objectives for of these firms, it is possible that the agency will have chosen to take care of projects themselves that will leave the remaining ones unattractive to energy services companies and therefore the whole set of projects will be incomplete. Thus, the current procedure may be ineffective as it does result in the completion of all projects.

If an agency is funding energy conservation through appropriations, the agency traditionally requests the total cost to execute all projects at the beginning of the program. The agency knows that energy costs and forecasted energy savings can vary and assumes a risk-averse position. As a result, agencies rarely ask for a smaller appropriation or plan to use existing savings to plan fund addition projects. The current procedure ensures that all projects are completed but may be overly conservative and cost-inefficient.

It is important to note that the agency does not get to keep the energy savings. The agency can save energy that saves money, which, in turn, should be used for energy-saving programs.

Project selection approaches that optimize the agency's value of the total energy saved continues to elude decision-makers. Many approaches of this type of problem have been studied however; few have been applied to energy conservation.

1.4. Dissertation Objectives

The dissertation shows that agencies can obtain greater returns on their energy conservation investment over traditional methods regardless of implementing organization. Innovative approaches to solving the agency project-selection problem allow for optimal resource allocation and the highest monetary savings and/or lowest investment required.

The objectives of this dissertation are threefold:

1. to introduce novel optimization models that improve the traditional approaches to increase returns on energy conservation investment
2. to model and find tractable solutions to a complex problem that have traditionally forced agencies to leverage inefficient heuristics in decision-making
3. to present options and practical solutions to a common yet complicated problem that can be customized for each federal and state or local government's budgets and risk appetite.

Throughout this work, a consistent set of data are used so that the applications are practical and results can be compared. The practical applications of the models are demonstrated using data from a college in the Southeastern, United States. In 2011, EMG, a third party engineering consulting firm, conducted an ASHRAE Level II Energy Audit of a college campus comprised of 38 buildings categorized residential, student, academic, and administration.[5] The campus covers over 1.04 million ft^2. There is one central boiler/chiller plant (physical plant) serving 11 of the 38 buildings, while the other 27 buildings are served by local systems.
EMG was contracted to perform a detailed energy audit and make energy saving recommendations on the physical plant and its connected 11 buildings. As part of the study, EMG reviewed the buildings' construction features, historical energy and water consumption with costs, envelope, heating ventilation and air conditioning (HVAC) equipment, heat distribution systems, lighting, and operating and maintenance practices. In the numerical

[5] Bill Champion was the Director of Asset Management Consulting at EMG at the time this dissertation was written.

examples, there are 48 ECMs with varying characteristics and project attributes as shown in Table A-1 of the Appendices.

In models where energy savings performance contracts or utility energy service contracts are leveraged, the firms' share of savings are varied, generally, between 60 and 80% and are taken from several firms (approximately 30) and market factors. Similarly, industry rules of thumb generally add approximately up to 20% more savings when a large energy services company or equipment-related contractor implements ECM projects of specific types. These arbitrary performance-related benefits attributed to specific implementers are applied through a matrix of quality factors.

1.5. Dissertation Organization

This balance of this dissertation is organized as follows. Chapter 2 presents an improved model for those agencies seeking to meet regulatory goals with private financing through energy savings performance contracts (ESPCs) and/or utility energy service contracts (UESCs). It is assumed that the agency will not seek Congressional appropriation.

In that chapter, the objectives of maximizing energy savings while minimizing costs are served by solving a two-level optimization problem. The agency has a budget, which is exhausted in a good-faith effort to make the best use of tax dollars. This budget rarely covers all possible projects and the agency seeks private financing through energy savings performance contracts and/or utility energy service contracts to complete the balance of projects whose expenses exceed the budget. The agency chooses projects and the utility or energy services companies to increase the energy savings, which also improves the monetary savings, thereby allowing the agency to complete more projects in house. The energy services companies compete on price and have specific internal rates of return. The selection process is modeled as a two-level, single-stage, life-cycle problem in which the upper-level player (agency) chooses a subset of projects to self-perform with its own budget. The upper level passes the balance of projects to the lower level, the energy services companies, and utility energy service contracts and/or outside firms. These firms compete for projects while seeking to maximize their own profits. In a shared-savings approach, these firms choose projects and share savings with the upper-level agency. The shared savings are added to the agency's budget for completing projects.

In Chapter 3, energy conservation through the implementation of energy-efficient retrofit projects can be viewed as a series of investments with annual returns. This chapter presents a model that assist agencies in meeting regulatory goals for buildings through funding projects by congressional appropriation. As in the two-level model from Chapter 2, returns can be used to fund additional projects. However, planning for energy conservation in later years ignoring the fluctuations in energy costs and uncertainty in the estimated savings severely affects project selection and initial appropriation requests. These impacts drive returns and influence the ability to implement future projects. This third chapter demonstrates how a risk-based, stochastic multi-period model with binary decision variables at each stage provides a much more accurate estimate for planning than traditional and deterministic models. This model is a one-level model as opposed to the one presented in Chapter 2. This approach accounts for uncertainties while determining the proper budget request that minimizes risk of the expected or average loss if the worst-case threshold is ever crossed.

Chapters 4 also presents a model for meeting regulatory goals for buildings through funding projects by congressional appropriation. Chapter 4 improves the multi-stage model by adding a longer planning horizon, which is consistent with the requirement of Executive Order 13693. By examining the length of the planning and realizing that the uncertainty is directly proportional to the length of the model's planning horizon, an improved rolling-horizon model that updates energy-saving yields between specific stages is proposed (a learning effect). This model is run using experimental cases showing its vast improvement in computational speed to solve, total stages required and total cost to implement all projects versus a fixed-horizon, multi-stage model.

Chapter 5 summarizes the work in the dissertation and provides some suggestions for future research directions.

Chapter 2: An Improved Strategic Decision-Making Model for Energy Conservation Measures

2.1. Introduction

The federal government has consistently been the largest consumer of energy using almost 1.2 quadrillion BTUs (British thermal units) per year from all fuel sources in the United States. The cost of meeting the federal government's facility energy costs had grown to $6.5 billion per year in 2007 (Energy, 2010). State and local governments spend an additional $10 billion a year on energy to provide public services and meet constituent needs.

In 1978, the United States Congress signed The National Energy Conservation Policy Act (NECPA) into law. This law is the basis for federal energy management goals and requirements in the United States. The overall purpose of the law was to promote the conservation and the efficient use of energy and water, and the use of renewable energy sources by the federal government. The resulting goals for energy performance were issued for federal buildings mandated and it was mandated that each agency apply energy conservation measures (ECMs) and improve the design for construction so that the energy consumption per gross square foot was reduced (Congress, 1978). The NECPA also gave federal agencies the authority to enter into shared-energy savings contracts with private-sector energy service companies (ESCOs). The NECPA has been regularly updated and amended by subsequent laws and regulations. One such regulation is the Energy Independence and Security Act of 2007 (EISA 2007), which established energy management goals and requirements while also amending portions of the NECPA.

These Congressional Acts mandate specific goals and targets including:

- Reducing energy intensity (Btu/ft^2) by 15 percent by the end of FY 2010, compared to a FY 2003 baseline and by 30 percent by the end of FY 2015;
- Increasing renewable electric energy equivalent to at least five percent of total electricity use in FYs 2010-2012 and at least 7.5 percent in FY 2013 and beyond; at least half must come from sources developed after January 1, 1999; and
- Achieving a 20 percent reduction in vehicle fleet petroleum use by 2015.

Overall, federal agencies must enhance efforts towards sustainable buildings and communities. Specifically agencies must implement high performance sustainable federal building design, construction, operation and management, maintenance, and deconstruction by ensuring all new federal buildings, entering the design phase in 2020 or later, are designed to achieve zero net energy by 2030. [6]

The central objective is clear; buildings must reduce their energy consumption. However, the secondary objective of lowering spending on energy, while adding the cost of implementing energy savings measures complicates the directives. Implicit to the mandates of reducing energy consumption and lowering energy spending is the assumption that both are known, easily measured and reported.

[6] A zero net energy building is one with zero net energy consumption. The total amount of energy used by the building on an annual basis is less than or equal to the amount of renewable energy created on site.

Faced with the multitude of requirements with the ultimate objectives of conserving energy and lowering spending, many agencies and property owners / managers find themselves with a computational challenge. There is a clear understanding of the extent to which energy efficiency must be achieved but a clear path to achieving these goals has not been dictated. Fortunately, there is an industry standard for best practice (EnergyStar, 2013). The primary tool that the agency's decision-makers use is the energy audit. There are several types of audits, however; an American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) level II or 3 is most often used for planning and decision-making (Kelsey & Pearson, 2011).

The audit is a comprehensive energy analysis and assessment of the building's energy-using components such that a list of energy conservation measures can be proposed with the following attributes:

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The energy auditors have assessed the regulatory requirements and conducted an audit that recommends the projects necessary to save the requisite energy. All projects must be completed. The agency's approach to implementing these projects has been the naïve method, which involves sorting by cost/ benefit then selecting until the budget has been depleted. They have not leveraged the integer programming approach that solves a resource allocation problem to choose a subset of projects to optimize savings (a "knapsack" problem).

Given this list of ECMs, the agency's decision-maker faces a series of strategic decisions. Each project from which the energy manager or decision-maker must select, saves energy or annual energy costs and, in most cases, both. Simpler projects can often be implemented with in-house resources and staff. Lower-cost projects can often be financed with internal operating budgets. Using in-house budgets and resources provide the best return on investment. These projects also free up capital for further energy projects. Higher risk, more difficult or projects that require large capital investment can be performed by Energy Service Companies per the guidance first issued in NECPA. The energy performance contracts or utility energy service contracts c do not yield the cost savings that in-house projects do as the cost savings are shared with the ESCOs and utilities. Neither organization is fond of taking on projects with very long paybacks. ESCOs generally have performance periods of 23 years while utilities prefer projects with paybacks of less than 10 years.[7]

Still, many technical challenges are faced by each organization. Inability for capacity expansion, building constraints and competing technologies provide key obstacles in today's energy projects. Risk of encountering the issues are often factored in as reductions to savings. In some cases, these cost to address these issues could exceed all overall savings.

The largest opportunity for energy conservation lies in the creation of the plan. Optimization is needed to properly create a plan that maximizes the energy savings while identifying financing

[7] The length of time that the ESCOs and utilities choose are generally a weighted average of the estimated useful lifetimes of the equipment installed.

and firms available to implement the recommended measures. The appropriate method of achieving these goals has not been regulated. The standard method of creating this plan segments these decisions. The agency traditionally selects the projects with the quickest paybacks. Only the least desirable or most costly projects are left for ESCOs. As a result, many agencies select a subset of projects to complete internally only to find that the remaining projects can no longer be completed with a decent payback for firms in the market. In contrast, many agencies allow the audit providers to choose the appropriate projects for them. This selection process may not be aligned with the agency's objectives.

Furthermore, in the current practice, the agency may leave a subset of projects incomplete assuming that ESCOs will complete them. The current selection process can generate a mix of selected projects that might not maximize the agency's share of the benefit (in dollars) of the energy saved. Most of the simpler projects have been completed in previous energy retrofit programs making remaining projects less attractive to both agency and firms. Given the profit maximization objectives for of these firms, it is possible that the agency will have chosen to take care of projects themselves that will leave the remaining ones unattractive to ESCOs and therefore the whole set of projects will be incomplete. Thus, the current procedure may be ineffective as it does result in the completion of all projects.

2.2. <u>Literature Review</u>

Project selection that optimizes the agency's value of the total energy saved continues to elude decision-makers. Many approaches of this type of problem have been studied however; none have been applied to energy conservation. The agency selection problem is a related to the classical knapsack problem, which is described below.

Dantzig described and demonstrated methods of solution to the knapsack problem (Dantzig, 1957). In this problem, for example, a person is planning a hike and has decided not to carry more than 70 lbs. of different items, such as a bedroll, Geiger counters, cans of food, etc. The hiker would like to maximize his /her benefit of these items while remaining below the weight limit. Dantzig noted that in these types of problems, extreme point solutions (to the corresponding linear program) might yield values that are neither one nor zero (which correspond to selection or omission of items). In Dantzig, it was noted that extensions to two or more limitations, for example, one on total weight and another on total volume could be done, but there would be a considerable increase in the amount of computational work. In the current context, the weights are the projects' costs and the weight limitation is the budget (Dantzig, 1957).

Markowitz wrote that the process of portfolio selection (similar to some extent to project selection) may be divided into two stages: observation and experience, leading to beliefs about the future performances and the relevant beliefs about future performance leading to the choice of portfolio (Markowitz, 1952). Selecting the highest anticipated return may leave projects undone and violate a key constraint. The current problem should incorporate constraints on the purchases, mainly that the Energy Manager cannot maximize the agency's share of savings without the profit maximization of lower-level firms such as ESCOs and Contractors.

In Gabriel et al., a multi-objective, integer-constrained optimization model with competing objectives for project selection was proposed in which probability distributions were used to describe uncertain costs (Gabriel, et al., 2006). That model was novel since it integrated multi-objective optimization, Monte Carlo simulation, and the Analytic Hierarchy Process.

In Asadia et al., the authors present a multi-objective optimization model to assist stakeholders in the definition of measures aimed at minimizing the energy use in the building in a cost effective manner while satisfying the occupants' needs and requirements (Asadia, et al., 2012). The set of retrofit actions in that study contained combinations of choices regarding windows, external wall insulation material, roof insulation material, and installation of solar collector to the existing building. Only one retrofit action from each four set of actions could be selected for the building retrofit. However, the model described in (Asadia, et al., 2012) incorporates many subjective attributes that make the quantification of value difficult.

A multi-criteria knapsack model was proposed to help designers to select the most feasible renovation actions in the conceptual phase of a renovation project in Alanne (Alanne, 2004). The paper asked which renovation actions should be selected in order to achieve the best possible improvement in the sustainability of the building that is to be renovated? In that paper, a multi-criteria knapsack model was to help designers to select the most feasible renovation actions in the conceptual phase of a renovation project using case analysis concerning a real, Finnish apartment building also has been presented. The additional criteria added some subjectivity as a feature of multi-criteria evaluation as to the model but the results were as expected. The additive knapsack model presented in that study was based on linear programming. Methods like Branch-and-Bound now make it possible to solve the integer problem in minutes or even in seconds. The problem faced by the agency discussed in this chapter is much more complex.

Gustafsson used a mixed-integer, linear programming (MILP) model to minimize the life-cycle cost of retrofits subject to minimum space heating requirements (Gustafsson, 1998). The author showed that a building' heating system could be described mathematically in the form of a MILP. The integrality constraint was very important because step increases (i.e., fixed charges or costs that do not vary with quantities over a finite ranges) in the cost functions could be defined but the author admitted that small changes in these steps might have resulted in different optimal solutions. That paper had many similarities to the current research, as it is one of the few to incorporate life-cycle costing in its evaluation of building retrofits. However, Gustafson's approach is vastly different from the research presented in this chapter. Gustafsson's optimization determines which measure to select based on the reductions to the overall cost of energy consumption, i.e., electricity, heating fuel and demand (kW savings). Because a savings in electricity may not lower the billed cost due to a higher demand charge, that measure would not be selected in Gustafsson (Gustafsson, 1998). The primary objective of the research here is energy savings with cost being a secondary consideration as well as a two-level optimization approach to more accurately model the ECM decision process.

Another paper, Caputoa et al., presented a methodology for optimal choice of safety measures in industrial plants. The methodology used a set of easy-to-compute ratings in a cost/benefit type fraction (Caputoa, et al., 2013). The problem of choosing a set of safety measures was then formulated as a linear program.

That knapsack, linear programming model solved for an optimal portfolio of safety measures complying with a limited budget. That linear program employed a simple additive weighting model. Single scores representing the utility of an option were merely added to the scores of the other selected safety measures in order to compute the overall utility.

Zhivov et al. (Zhivov, et al., 2012) described a net zero fossil fuel-based energy optimization process and illustrated it with an example based on the results of study conducted for a cluster of buildings at Fort Irwin, CA. The integrated optimization process consisted of several optimization problems solved in series beginning with the optimization of each building to achieve the most cost effective energy efficient optimization of the building envelope and building systems that use energy. Then, energy saving measures affecting the total building cluster were optimized, taking advantages of the diversification between energy intensities, scheduling, and waste energy streams utilization. The energy demands of the resulting optimized cluster required the smallest size renewable energy systems needed to make the building cluster net zero. The Zhivov et al. (Zhivov, et al., 2012) approach is a unique to energy conservation but is impractical in its objective. The optimization in Zhivov et al. (Zhivov, et al., 2012) essentially minimizes the energy needed in the future for a cluster of buildings by installing retrofits now. The results showed that those energy saving projects would reduce the energy at a very high cost. However, there are no cost constraints in the model. In this chapter, we seek to save the most energy by spending the lowest possible costs in a two-level model, which essentially leads to the smallest investment.

In Ma et al., a systematic approach to the proper selection and identification of the best retrofit options for existing buildings is presented (Ma, et al., 2012). That work highlights the generic building retrofit problem and key issues that are involved in building retrofit investment decisions. Ma et al. discuss major retrofit activities such as energy auditing, quantification of energy benefits, economic analysis, and measurement and verification (M&V) of energy savings (Ma, et al., 2012). However, the authors also discuss building performance assessment and risk assessment, all of which are essential to the success of a building retrofit project. An overview of the research and development as well as application of the retrofit technologies in existing buildings is also provided. While there is no optimization here, the aim of that work is to provide building researchers and practitioners with a better understanding of how to effectively conduct a building retrofit to promote energy conservation and sustainability.

Diakaki et al. investigated the feasibility of the application of multi-objective optimization techniques to the problem of the improvement of the energy efficiency in buildings, so that the maximum possible number of alternative solutions and energy efficiency measures may be considered (Diakaki, et al., 2008). The authors recognized that several measures were available for the improvement of the energy efficiency of the buildings and the quality of their indoor environment, and that the decision-maker has to compensate environmental, energy, financial and social factors in order to select among them. They noted that the problem of the decision-maker is characterized by the existence of multiple and in several cases competing objectives each of which should be optimized against a set of feasible and available solutions that is prescribed by a set of parameters and constraints that should be taken into account. The decision-maker is facing a multi-objective optimization problem that is usually approached through simulation and/or multi-criteria decision-making techniques that focus on particular aspects of the problem. Their results showed that no optimal solution exists for that problem due to the competing objectives of the involved decision criteria. A simple example is used to identify the potential strengths and weaknesses of the proposed approach, and highlight potential problems that may arise. In contrast, the current chapter limits the criteria and factors (energy and dollars saved) associated with the objective function in energy conservation measure selection.

The current chapter presents a two-level optimization problem formulation, which is a special case of a more general mathematical program with equilibrium constraints (MPEC). The optimization problem that selects the proper conservation measures at the upper-agency level and properly aligns with the service provider at the lower level has not been studied before and thus this constitutes novel research.[8]

One way to view the overall MPEC is as a two-level, knapsack problem in which the upper-level is the agency filling its knapsack (budget) with as many useful ECMs as possible, taking into account a lower-level set of providers as well. This chapter's focus lies in the intersection of energy conservation, the knapsack problem and two-level optimization (MPECs). While there is a multitude of work that has been done on each of these topics individually; the treatment of

[8] In the case of a lower-level solution set which is not a singleton, we have assumed that the lower-level players act in an "optimistic" fashion relative to the upper-level player as in Bard (Bard, 1998).

these three concepts together is new. A few studies discuss combining two of the topics but fall short of the full analysis.

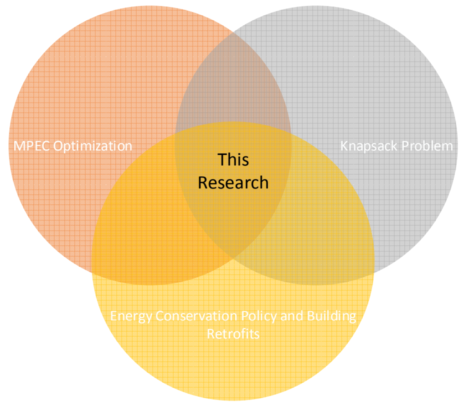

Figure 2-1: The Intersection of this Research

For example, Ye and Daoli discuss a two-level optimization problem however, not specific to energy conservation (Ye & Daoli, 2010). The authors discuss the classical approach to solving such a problem by replacing the lower-level problem by its Karush–Kuhn–Tucker (KKT) conditions and solving the resulting MPEC. In Ye and Daoli, the classical approach is not valid for a non-convex bi-level programming problem so the author uses a value function to derive optimality in a very specific case (Ye & Daoli, 2010). The current chapter builds on the classical approach by handling non-convexities through disjunctive constraints.

Another paper, Fortuny-Amat and McCarl, presents a formulation for two-stage decision making processes; this formulation is a mathematical programming problem (master) containing other multilevel programs in the constraints (subproblems) (Fortuny-Amat & McCarl, 1981). A two-level problem is analyzed in detail and a solution procedure is developed that replaces the subproblem by its KKT conditions and then further transforms it into a mixed integer quadratic programming problem by exploiting the disjunctive nature of the complementary slackness conditions. The authors note that mixed integer quadratic program could be solved directly by using disjunctive constraints or special ordered sets (SOS). One of the key differences between Fortuny-Amat and McCarl (Fortuny-Amat & McCarl, 1981) and the work in this chapter is that the latter includes additional complexity of the upper-level where integer programming was used, not to accommodate the complementary slackness conditions. In this chapter, there are more constraints at both levels; however; careful selection of variables made the solution tractable using disjunctive constraints. A final contrasting concept is that this novel approach to energy conservation highlights a significant improvement over a current common practice.

In Siddiqui and Gabriel, SOS type 1 (SOS1) variables are used and a new a new method for solving MPECs where the lower level is a complementarity problem is demonstrated (Siddiqui & Gabriel, 2013). An application of the method to an MPEC representing the United States natural gas market is given. The first formulation, based on SOS1 variables, when solved to optimality provides a global solution to the MPEC. The second, penalty-based formulation is used to heuristically obtain local solutions to large-scale MPECs. The advantage of these methods over disjunctive constraints for solving MPECs is that computational time is much lower, which is corroborated by numerical examples.

In Gabriel, et al., a Stackelberg game for a network-constrained energy market using integer programming is solved where there is a single leader and the independent system operator acts as the follower (Gabriel & Leuthold, 2010). The MPEC is reformulated as an MILP by using disjunctive constraints and linearization of bilinear terms. The MILP formulation gives the opportunity to solve the problems reliably and paves the way to add discrete constraints to the original MPEC formulation, which can be used in order to solve discretely-constrained mathematical programs with equilibrium constraints (DC-MPECs). This approach was applied to a three-node and a fifteen-node network model of electricity markets for the Western European grid.

This current chapter represents modeling efforts stemming from the convergence of the legal requirement for reduction of energy in government buildings, the desire for reducing costs and government spending, the advances in new energy savings technology and the large number of firms and financing methods available. This optimization is now needed more than ever because traditional funding methods have ended with the American Recovery and Reinvestment Act (ARRA) but still funding is still authorized and available through secondary sources. The amount of capital needed to fund these projects has grown because many of the no /low-cost Projects have all been completed in buildings across the United States. Exotic new programs and funding sources become available daily and competition for these funds continue to grow. Still, this funding only supplements activities required by law.

2.3. Methodology

A novel way to meet the objectives of maximizing energy savings while minimizing costs is by solving a two-level optimization problem (MPEC) as described earlier. The audit returns the set of maximum energy savings projects. Each agency has a budget that should be exhausted in a good faith effort to make the best use of tax dollars. However, this budget rarely covers all possible projects. If all projects recommended by the audit are not completed, then the regulatory requirements will not be met. Choosing an ESCO is a regulated way to complete the balance of projects whose expenses exceed the budget. The choices of projects and the right ESCO can increase the energy savings, which also improves the monetary savings, thereby allowing the agency to complete more projects in house.

This selection process is best modeled as a two-level problem in which the upper-level player (agency) chooses a subset of projects to self-perform with its own budget. The upper level passes the balance of projects to the lower level, the ESCOs / outside firms. The ESCOs compete for projects while seeking to maximize their own profit. In a shared-savings approach, the ESCOs choose projects and share savings with the upper-level agency. The shared savings are added to the agency's budget for completing projects.[9]

This strategic decision making-model is an improvement over the existing practice in which a single-level model minimizes the agency's capital outlay. The single-level model ignores the secondary object of earning the right to the saving generated by implementing energy conservation measures. In addition, the existing practice does not incorporate the ESCO's objectives nor does it predict the expected shared-shavings. The model presented here includes both objectives allowing the agency and ESCO's to work together. This collaboration makes it possible for agency to use the shared-savings to invest in additional projects that can be implemented in-house, thereby driving additional savings.

[9] It is important to note that the revenues are shared but the liability is owned by the ESCO.

The following is the notation, variables, and parameters used in the general statement of the ECM two-level problem.

Sets

F = set of firm types (ESCOs) with F = {1, 2,…n_f} where n_f = |F|

P = set of ECM projects with P = {1, 2,…n_p} where n_p = |P|

Main Primal Decision Variables

x a vector of n_p binary variables representing selection of the projects; variable =1 if selected by the agency (upper-level variables) to be implemented with the agency budget, =0 otherwise

q a two-dimensional set of n_p by n_f, non-negative variables representing the percentage of the projects selected by each ESCO firm (0-100%) (lower-level variables)

z a vector of n_p binary variables representing selection of the projects; variable =1 if selected by the agency (upper-level variables) to be implemented with third-party financing, =0 otherwise

Intermediate Variables

SSR The shared savings revenue returned to the agency

Parameters

η_p the estimated useful life in years of the equipment or retrofit
B the budget in dollars for the agency's in-house projects (capital, operating, stimulus, grants, etc.)
D the cost of financing: 1 plus the current interest rate
TF the present value of the investment dollars including financing cost needed by agency to implement all projects
MP_f a minimum profit requirement in dollars, set by firm f
α_p the estimated annual savings in KBTU achieved by implementing project p
θ_p the estimated annual savings in dollars achieved by implementing project p

φ_f the coefficient of the cost curve of firm f [10]
μ_p the percentage of project p's initial cost estimate associated with material, labor and equipment
δ_p the estimated annual percentage savings degradation after implementing project p
ε_p the present value factor of an estimated annual savings achieved by implementing project p
γ_p the estimated investment in dollars needed to implement project p
ν_{pf} the variable cost in dollars of project execution for firm f and project p
ω_{pf} the quality factor of project execution for firm f and project p
ζ_f the rate of shared savings (percentage) to the firm, f agreed upon by firm and agency
Δ the discount rate

$$\textbf{Note:}\ \boldsymbol{\varepsilon_p} = \frac{\mathbf{1} + \boldsymbol{\delta_p}}{\Delta - \boldsymbol{\delta_p}}\left[\mathbf{1} - \left(\frac{\mathbf{1} + \boldsymbol{\delta_p}}{\mathbf{1} + \Delta}\right)^{\boldsymbol{\eta}}\right] \tag{2a}$$

This equation above employs the Lifecycle Costing Methodology, which calculates the Uniform Present Value, must be used to properly account for the time value of the money (savings) with the concurrent decrease in efficiency of the implemented measure. The present value is used to properly scale the annual savings with the current financing needed for projects.

K_f the exponent associated with cost curve of firm f

The firms generally estimate their projects costs with a nonlinear curve. The costs are nonlinear and lower at small q. As the share of a project increases, so does the cost due to the need for managing shared savings through contracting, maintenance and verification. Thus, only K > 1 is considered.

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[10] Energy auditors provide estimates for project implementation which include labor, equipment, material and soft cost as well as overhead and profit. The firms generally estimate their projects costs with a nonlinear curve with one coefficient of φ_f.

Upper-Level Problem

The agency's annual savings maximizing problem is given in (2b):

$$\max_{x,z,q} Z = \sum_{p=1}^{n_p} \varepsilon_p \theta_p \cdot \left(\boldsymbol{x}(p) + \boldsymbol{z}(p)\right)$$
$$+ \sum_{p=1}^{n_p} \sum_{f=1}^{n_f} \left(\varepsilon_p \theta_p \omega_{p,f} - \mu_p \gamma_p\right) \cdot \left(1 - \zeta_f\right) \boldsymbol{q}(p,f) - D \cdot TF \tag{2b}$$

Subject to:

$$\sum_{p=1}^{n_p} \gamma_p \cdot \boldsymbol{x}(p) \le B + SSR \tag{2c}$$

$$SSR = \sum_{p=1}^{n_p} \sum_{f=1}^{n_f} \boldsymbol{q}(p,f) \cdot \left(\varepsilon_p \theta_p \omega_{p,f} - \mu_p \gamma_p\right) \cdot \left(1 - \zeta_f\right) \tag{2d}$$

$$\sum_{p=1}^{n_p} \gamma_p \cdot \boldsymbol{z}(p) = TF \tag{2e}$$

$$\boldsymbol{x}(p) + \boldsymbol{z}(p) + \sum_{f=1}^{n_f} \boldsymbol{q}(p,f) = 1 \ \forall p \tag{2f}$$

The energy auditor submits an estimate of the project investment that includes labor, material, equipment and overhead and profit. An example of a bid-ready project estimate is shown below in Table 2-1.

Lighting Energy Conservation Measure Project	
Material	\$750
Labor	\$100
Equipment	\$50
Overhead and Profit	\$100
Total	\$1,000

Table 2-1: Typical Lighting ECM Project

In this example, an energy audit reports that a lighting retrofit will require an estimated investment of \$1,000 (γ_p). An ESCO's profit should not include overhead and profit of \$100 because the ESCO's profit is included in the total shared savings $(\varepsilon_p\theta_p\omega_{p,f} - \mu_p\gamma_p)$.

The estimate prepared by the auditor assumes the projects will be completed by traditional contractors. The ESCOs are leveraging the performance contracting method where they are paid through shared savings. The shared savings includes the ESCO's overhead and profit. When applying the auditors' estimate to the shared savings amount, the overhead and profit must be removed to avoid double-counting. This is done by reducing the project cost by the portion of the project that estimated for labor, material, and equipment,(μ_p).

The quantity $(\varepsilon_p\theta_p\omega_{p,f} - \mu_p\gamma_p)$ is the total savings available to be shared. The rate of shared savings (percentage) to the firm as agreed upon by firm and agency is ζ_f. Therefore, the quantity $(1 - \zeta_f)$ is the agency's share of that savings.

Constraint (2c) indicates that the cost (initial investments) of the projects selected by the agency, to be performed in-house cannot exceed the in-house budget (B) plus the shared savings (SSR). Because only complete projects are taken on by the agency, there may be an amount of money left in the budget after projects are selected. In addition, the budget is augmented by the shared savings amount (SSR) which is defined in (2d). Constraint (2e) shows the cost invested in projects completed by the agency, but financed by other means (TF).

Constraint (2f) stipulates that projects can only be selected by either the agency or the firms and that all projects must be selected to meet the mandated requirement. Only projects that save energy or enable energy savings are considered. The agency must select complete projects while the firms can work with the agency or with each other to complete projects, i.e., this means that ***q****(p,f)* can be fractional as shown in equations (2g) and (2h).

$$\boldsymbol{x}(p), \boldsymbol{z}(p) \;\; are \;\; binary \tag{2g}$$

$$0 \le \boldsymbol{q}(p,f) \;\; \forall p \; and \; f \tag{2h}$$

Note that $\boldsymbol{q}(p,f) \le 1 \; \forall \, p$ and f is implied by (2f), (2g), and (2h).

<u>Lower-Level Problem</u>

The ESCOs compete for their profit-maximizing share of the projects not taken on by the agency's in-house staff. These heterogeneous firm types each represent different competencies and services for each project. These ESCOs fall into three separate categories, equipment-affiliated, utility-affiliated and non-utility energy services companies. For example, many firms are equipment-specific and are only capable of performing portions of projects within their competency.

Likewise, each firm has a different cost model. In general, each cost model has a fixed component of cost, which includes project management, contract administration and leasing. The variable component of their cost structure includes design, engineering, commissioning, maintenance, and verification (M&V). Each firm becomes capacity-constrained as higher percentages of projects are implemented due to resource limitations.

Adding to the heterogeneity of the firms, each ESCO category also has a quality level for each type of project. This quality factor allows for firms with specialized skills to generate higher savings when implementing projects within their competency.

Firm f's f's profit maximization objective function takes on the following form (2i).

ESCO / Firms' Profit-Maximizing Problem

$$\max_q \pi_f = \sum_{p=1}^{n_p} \Big[\boldsymbol{q}(p,f) \cdot \left(\varepsilon_p \theta_p \omega_{p,f} - \mu_p \gamma_p\right) \cdot \left(\zeta_f\right) - \left(\phi_f \left(\varepsilon_p \theta_p - \mu_p \gamma_p\right) \cdot \left(\boldsymbol{q}(p,f) + \boldsymbol{q}(p,f)^{\boldsymbol{K}_f}\right)\right) \Big] \tag{2i}$$

Subject to:

$$\sum_{p=1}^{n_p}\Big[\boldsymbol{q}(p,f)\cdot\left(\varepsilon_p\theta_p\omega_{p,f}-\mu_p\gamma_p\right)\cdot\left(\zeta_f\right)$$

$$-\left(\phi_f\left(\varepsilon_p\theta_p-\mu_p\gamma_p\right)\cdot\left(\boldsymbol{q}(p,f)+\boldsymbol{q}(p,f)^{\boldsymbol{K}_f}\right)\right)\Big] \;\geq MP_f \tag{2j}$$

$$\boldsymbol{q}(p,f)\leq 1 \quad \forall\, p\ in\ P \tag{2k}$$

$$\boldsymbol{q}(p,f)\geq 0\ \forall\, p\ in\ P \tag{2l}$$

The objective function (A-3a) quantity, $q(p,f)\cdot\left(\varepsilon_p\theta_p\omega_{p,f}-\mu_p\gamma_p\right)\cdot\left(\zeta_f\right)$ represents the revenue gained by the ESCO in the form of shared savings by taking on $\mathrm{q}(\mathrm{p},\mathrm{f})$ percent of project, p. The quantity $\left(\phi_f\left(\varepsilon_p\theta_p-\mu_p\gamma_p\right)\cdot\left(q(p,f)+q(p,f)^{K_f}\right)\right)$ represents the cost of implementing project, p by firm, f. The parameter (ϕ_f) is the percentage of the shared savings that is attributed to material, labor and equipment costs.

The lower-level optimization problems represents for $f=1..n_f$, firm types, not necessarily a single firm. Each firm type seeks to maximize profit as long as their internal rates of return met (minimum profit is achieved, see constraint (2j)). As such, it is assumed that projects will be selected by all firm types given the large number and variety of energy conservation project types required by the agency. If a firm type chooses not to select any projects or the number of projects available makes selection unattractive to the firms then the constraint (2j) should be removed to avoid infeasibility although without loss of generality, one can set this minimum profit just to be zero for feasibility reasons. Furthermore, this parameter suitable adjusted, can be used in a sensitivity analysis.

The approach to solving this two-level problem is to use the Karush-Kuhn-Tucker (KKT) optimality conditions, apply them to the lower-level optimization problems and insert them into the upper-level problem as additional constraints. In this way, the original two-level problem is reformulated as a single-level nonlinear optimization problem. In Appendix B, we show that under mild assumptions, these KKT conditions are both necessary and sufficient for optimality as well as the equivalent one-level problem to solve the MPEC given by (2b) and (2i).

As discussed previously, results of the energy audit are presented to the agency. The results contain specific attributes for each ECM recommended including the cost to implement the project and the projected savings. The agency selects an optimal subset of projects to implement itself given a fixed budget in a knapsack problem-like fashion. At the lower level, three types of ESCOs solve their profit-maximizing problems. The overall objective is to save the most energy possible by implementing the recommended ECMs at the lowest cost.

2.4. Practical Application

The model developed in Section 2.3, aligns the objective of saving energy while reducing cost with the understanding that the providers and financial agencies are also working to maximize profit. A practical application of the model is demonstrated using data from a college in the Southeastern, United States.

In 2011, EMG, a third party engineering consulting firm, conducted an ASHRAE Level 2 Energy Audit of a college campus comprised of 38 buildings categorized residential, student, academic, and administration. The campus covers a total of over 1.04 million ft^2.[11] There is one central boiler/chiller plant (physical plant) serving 11 of the 38 buildings, while the other 27 buildings are served by local systems.

EMG was contracted to perform a detailed energy audit and make energy saving recommendations on the physical plant and its connected 11 buildings. As part of the study, EMG reviewed the buildings' construction features, historical energy and water consumption with costs, envelope, heating ventilation and air conditioning (HVAC) equipment, heat distribution systems, lighting, and operating and maintenance practices.

EMG identified forty-eight energy conservation measures. The following paragraphs describe a typical ECM, "Decommissioning of Central Steam Boilers and Installation Individual High Efficiency Condensing Boilers."

The central boiler in the central utility plant currently serves nine buildings on campus. The steam from the boilers is piped to the individual buildings. The central plant currently has two inefficient Continental steam boilers and an aging chiller plant. A significant amount of energy is spent raising boiler temperature from 55°F to 220°F in order to evaporate the boiler feed water, instead of the normal 185°F to 220°F because more than 75% of condensate return is fresh, unheated water. Based on the observations and analyses, the audit proposes a new chiller plant along with new boilers with thermal operating efficiency of 92-96% in contrast to the current boiler thermal efficiency of 60%. The hot water circulation pumps and variable frequency drives will save additional electrical consumption. This project will also result in an

[11] Bill Champion is the Director of Asset Management Consulting at EMG.

annual water savings currently being drained into the city sewer due to lack of proper condensate return system.

The total savings annual saving for the ECM will be $80,023. The table below summarizes the attributes of this proposed ECM project.

	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Cost of Energy Saved ($)	Degradation / Escalation Rate	Estimated Useful Life	Payback Ratio
project 1	710,354	5,334,857	0.015	80,023.00	1.50%	30 years	8.877

Table 2-2: Typical ECM Project Attributes

There are 48 such ECMs with varying characteristics and project attributes. The model presented earlier is applied to the real data follows.

n_f =|3|
n_p =|48|
γ_p = 90%
Δ =3%
ζ_f shown in the third column of Table 1-4.
ω_{pf} shown in Table 1-5, below.

Below is a sample of the actual ECM data characteristics from the energy audit. Please see Appendix C for the complete dataset.

	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Cost of Energy Saved ($)	Degradation / Escalation Rate (%)	Estimated Useful Life (Years)	Payback Ratio (Years)
P	γ_p	α_p			δ_p	n	
project1	$ 710,354	5,334,857	$ 0.015	$ 80,023	-1.50%	30	8.88
project2	$ 637,975	1,849,047	$ 0.033	$ 61,019	-1.00%	23	10.46
project3	$ 468,071	1,768,079	$ 0.023	$ 40,666	-2.00%	30	11.51

project4	$ 40,368	445,600	$ 0.010	$ 4,456	-1.38%	30	9.06
project5	$ 8,557	213,025	$ 0.012	$ 2,556	-1.50%	15	3.35
project6	$ 15,328	124,584	$ 0.023	$ 2,865	-0.75%	9	5.35
project7	$ 55,207	287,971	$ 0.027	$ 7,775	-2.50%	15	7.10
project8	$ 59,355	416,045	$ 0.022	$ 9,153	-2.00%	15	6.48

Table 2-3: Sample of ECM Data in Practical Application

	Provider Type	**Shared savings % (to Firms)**
Firm (f)		ζ_f
1	Non-utility ESCO	67.5%
2	Utility Affiliated ESCO	70.0%
3	Equipment Affiliated ESCO	65.0%

Table 2-4: Practical Application Table of Firms

It should be noted that the firms' share of savings can vary, generally, between 60 and 80% and is dependent on several firms (approximately 30) and market factors. The numbers used in Table 2-4, represent typical firms in each of the ESCO types. Similarly, industry rules of thumb generally add approximately up to 20% more savings when a large ESCO or equipment-related contractor implements ECM projects of specific types. A sample of the quality factors are tabulated below in Table 2-5. Please see Appendix C for the complete dataset.

Baseline Project Quality, ω_{pf}		**Note, ω_{pf} =1 as executed by Agency**	
Project	Firm 1	Firm 2	Firm 3
project1	1.06	1.10	1.05
project2	1.06	1.01	1.05
project3	1.05	1.09	1.07
project4	1.06	1.01	1.09

project5	1.05	1.10	1.04
project6	1.10	1.15	1.05
project7	1.04	1.15	1.05
project8	1.10	1.08	1.03

Table 2-5: Sample of Practical Application Table of ESCO Quality Factors (ω_{pf})

2.5. Numerical Results

The mixed-integer nonlinear program (MINLP) shown in Appendix A and derived from the above discussions, was programmed in GAMS Rev 23.6 using a 64-bit MS Windows machine and the SBB solver. The model statistics included 1,465 single equations and 1,221 single variables with 435 binary variables, most supporting the disjunctive constraints (see Appendix A for a discussion of disjunctive constraints). The MINLP model was solved using a maximization format and arrived at an integer optimal solution after 355 branch and bound nodes were evaluated. For the practical example, with a budget of $200,000, the resulting value of the model is apparent given the additional $1,374,794 in savings to the agency as compared to the current method of sorting by payback and having ESCOs perform the balance (Industry Practice). The key component of the additional savings comes from the ability of the upper level agency to anticipate what the lower level will perform and use this feedback to plan for the shared savings. The upper level uses this shared savings to invest in implementing its own projects. These projects are those funded by the agency through shared savings from ESCOs.

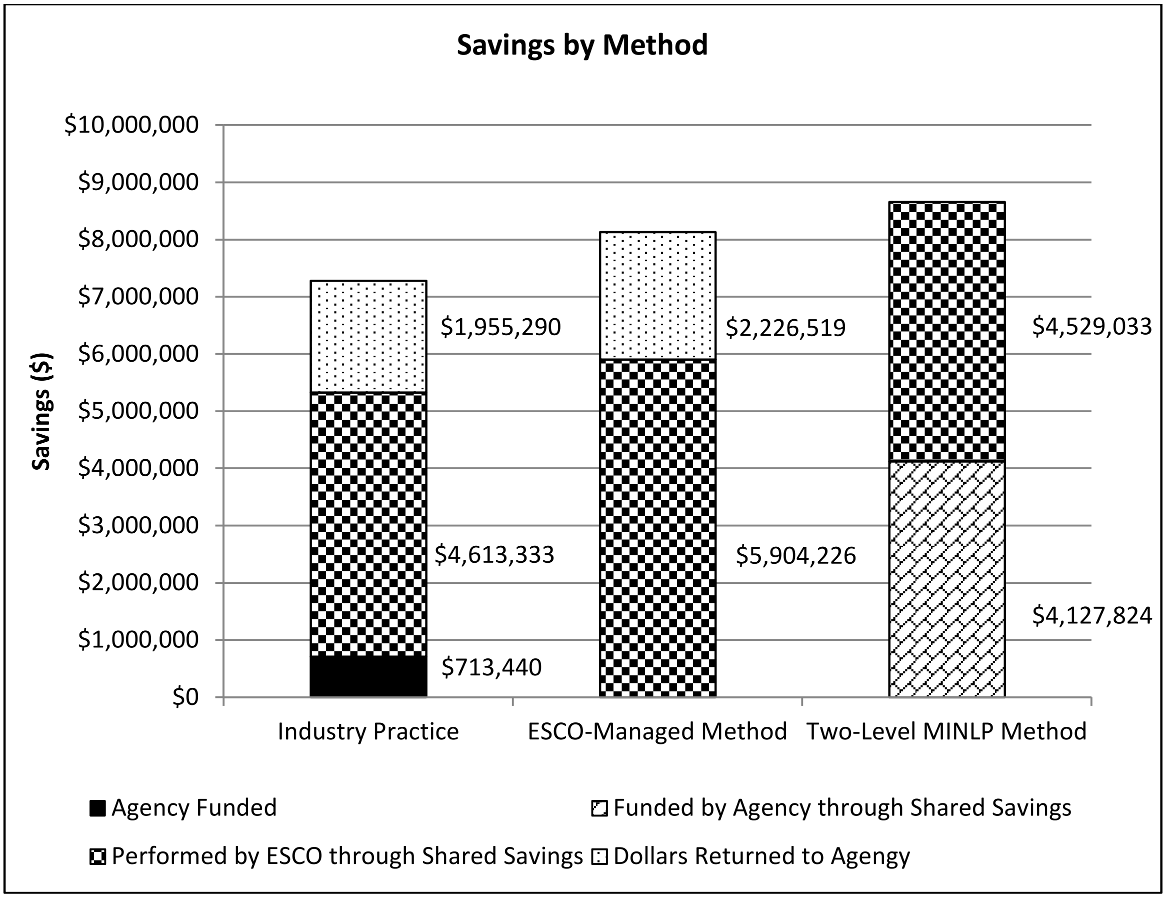

Figure 2-2: Comparison among Approaches

In Figure 2-2, the Industry Practice savings is composed of three quantities. The first of these quantities ($713,440) is the savings through agency funding which are achieved by implementing projects (5, 23, and 28) whose investments are below the budget (of $200,000). In this case, **x**(5), **x**(23) and **x**(28) =1 and all other **x**(p) =0. The second quantity ($4,613,333) represents the savings achieved through projects implemented by ESCOs (all other projects). The third quantity, ($1,955,290) is the savings in dollars returned to the agency from the ESCO. This quantity is the agency's share of the shared savings and is essentially a refund and not reinvested for funding additional projects. These quantities can be seen in the other two stacked bars in Figure 2-2 with the addition of the savings from projects funded by the agency through shared savings. For example, in the ESCO-managed method where the agency outsources 100%

of the projects, $5,904,226 represents the savings from projects implemented by ESCOs. The amount $2,226,519 is the savings in dollars returned to the agency from the ESCOs. In the Two-level MINLP Method, the amount $4,127,824 represents the agency's savings from projects completed with existing budget ($200,000) supplemented by the shared savings returned (now $1,303,978, not shown). The agency can use this refund combined with its initial budget to invest in additional projects that generate savings. It is important to note that ten projects were completed by the agency (see Figure 2-5), more than the three in the case of the Industry Practice.

The driver of the optimization is the federal regulation that requires the percentage reduction in energy savings. All of the projects must be completed to meet that objective. The agency's decision-maker needed only to select which projects would be completed "in-house" and which would be implement by an ESCO. Figure 2-3, below shows the "Industry Practice" selections made by the decision–maker. The standard practice instructs the agency to sort by payback and select projects until the budget is exhausted. In this case, the decision-make selects three projects to remain below the budget. The balance of the projects is given to the ESCOs for implementation. The shared savings from ESCO-completed projects are returned to the agency. However, having already selected the projects that maximize the payback (the single-level approach); the agency has no projects left to implement. This naïve or greedy approach leaves the agency with $1,955,290 in shared savings (returned to agency by the ESCOs), whereas the two-level approach gives the agency $1,303,978, which the agency uses to select the optimal project mix and generates $4,127,824 in savings. This gives the agency an additional $1,374,794 in cost savings. The individual projects selected by each approach are shown in Figures 2-3 to 2-5.

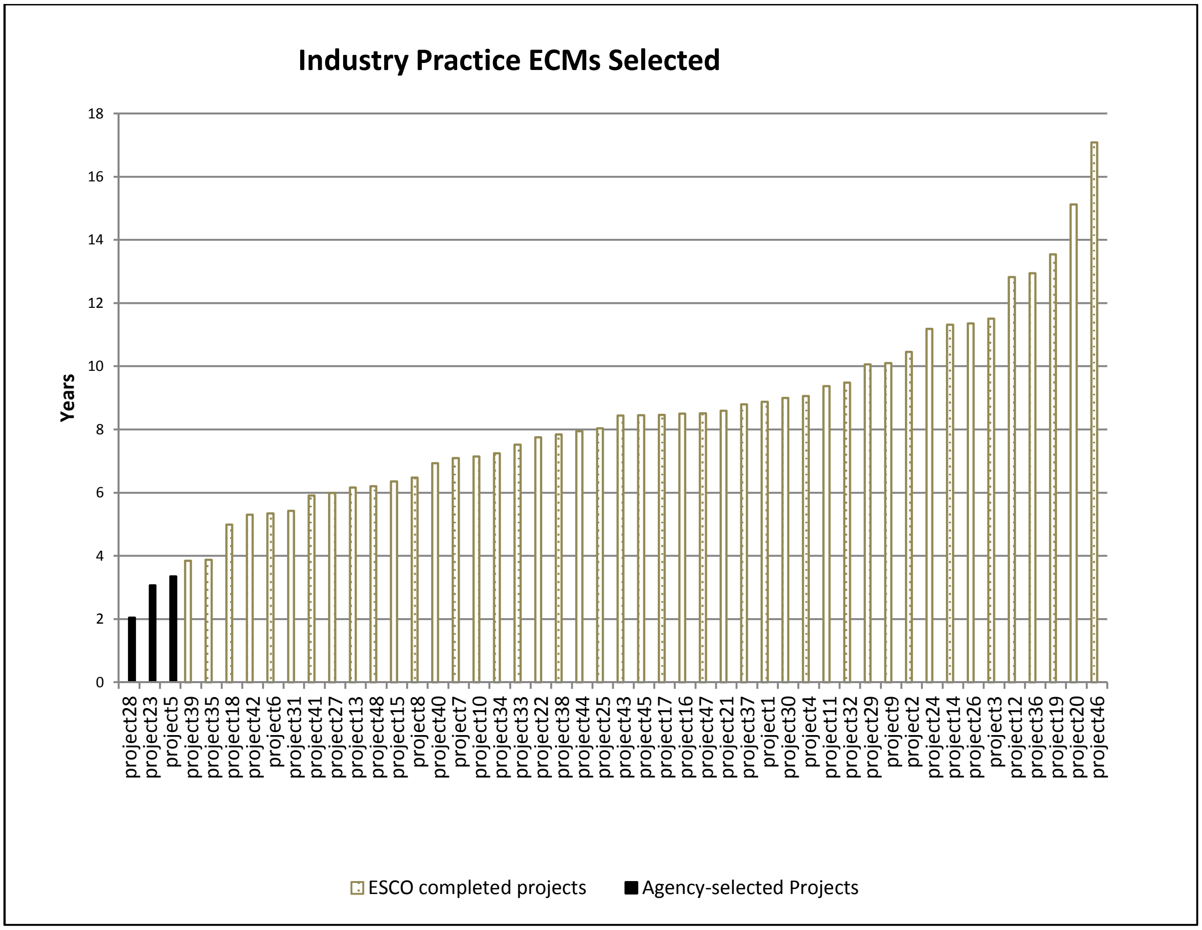

Figure 2-3: Lifecycle Savings of $7,282,063 to the agency achieved by selecting three projects and Shared savings returned by ESCOs from projects completed.

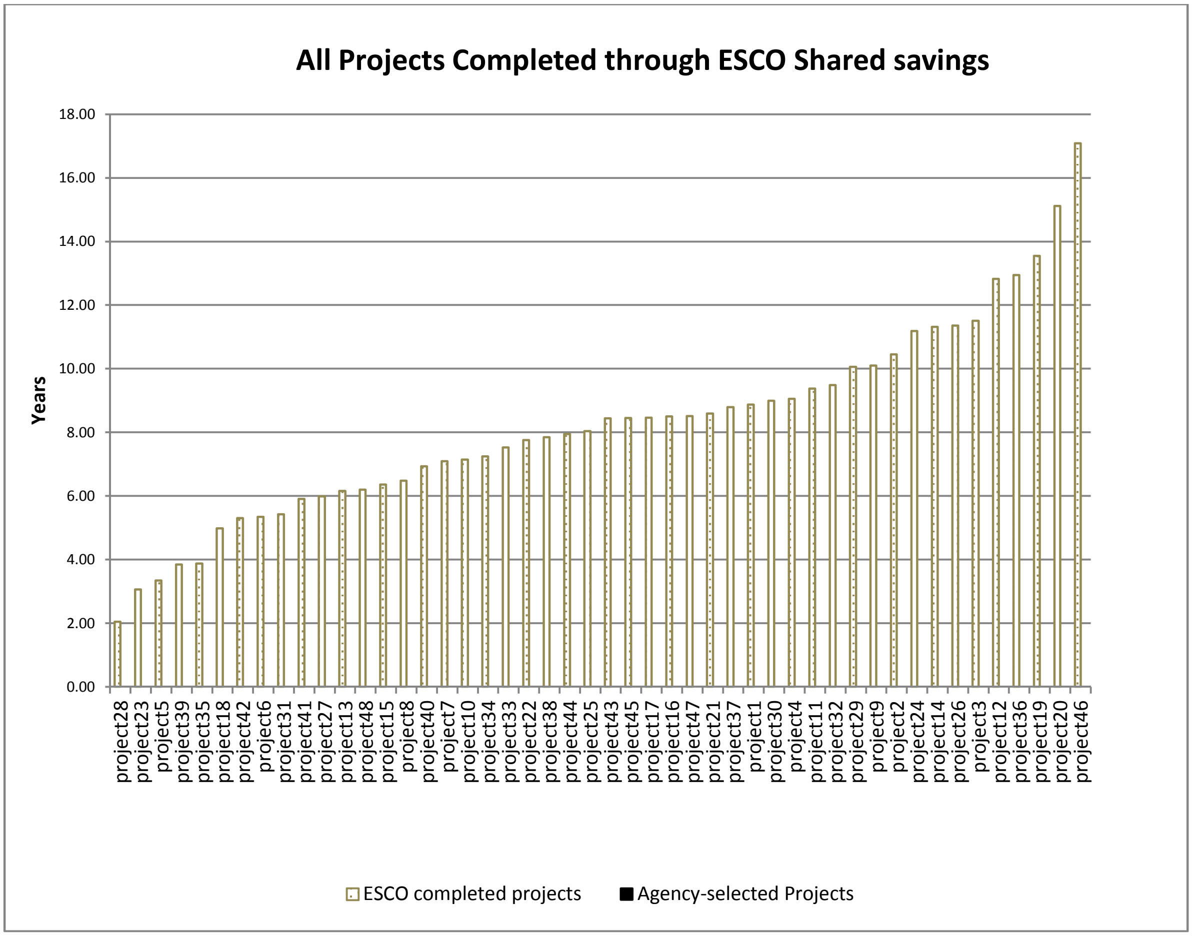

Figure 2-4: Lifecycle Savings to the agency of $8,130,745 through Shared savings from projects completed by ESCOs.

It should be noted that in the model in which ESCOs perform all projects, shown in Figure 2-4, the minimum profit constraint becomes binding for firm 3. Here, without the impact of the upper-level savings maximization, the lower-level profit is maximized by the two firm types that keep a highest percentage of the shared savings, as expected.

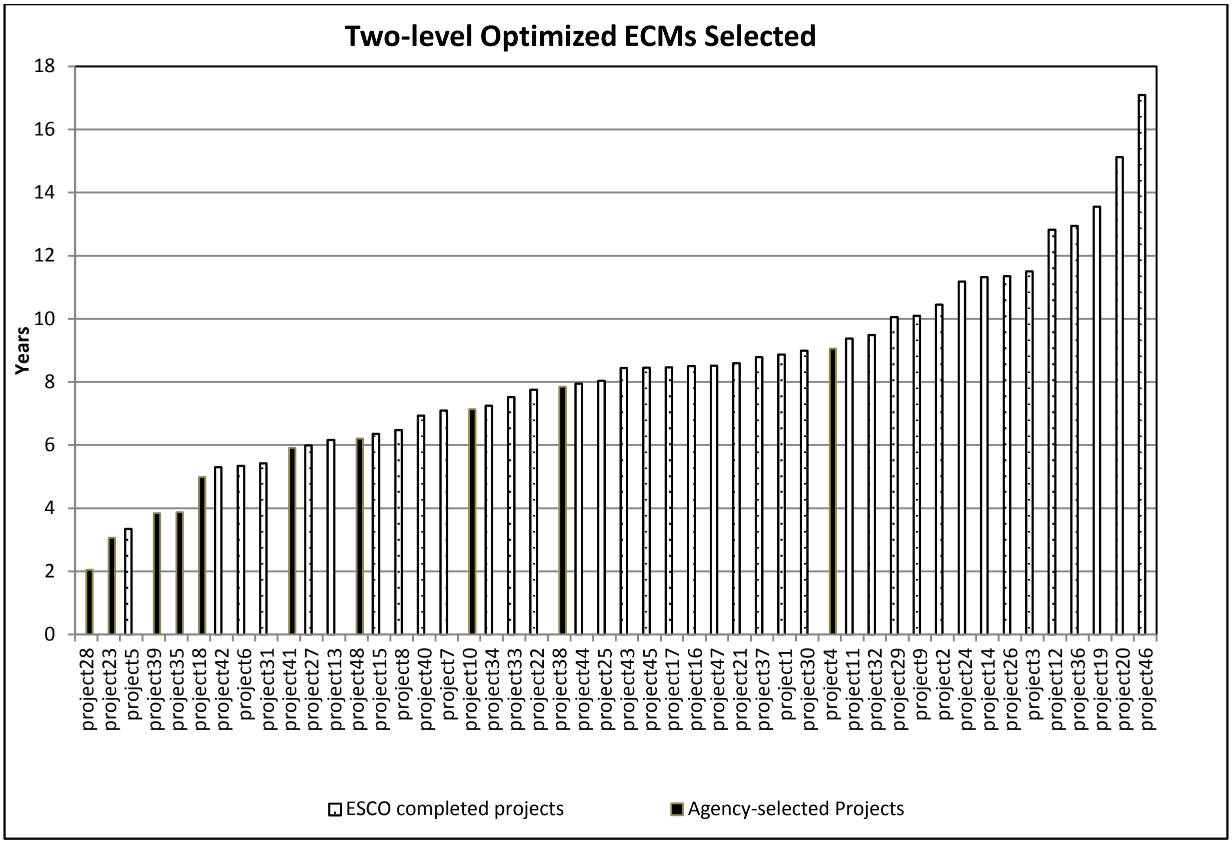

Figure 2-5: Two-level MINLP lifecycle saving to the agency of $8,656,857

Figure 2-5, above, shows the model presented in this chapter. This model provided the lifecycle saving to the agency of $8,656,857 through a combination of agency-completed projects and shared savings from projects completed by ESCOs.

The problem statement assumes that all projects need to be completed to meet the 30% consumption of energy requirement mandated by the regulation. No other projects would be completed. All projects have been evaluated in present value amounts to allow for comparison. This is modeled after the common practice of large capital and renovation projects, which generally take place once in several-year intervals. The estimated useful lives (EULs) of equipment and financing terms are usually between 10 and 30 years and often drive the same renovation patterns.

The practical application had an agency-operating budget of $200,000 for energy projects. The optimization yielded an optimal selection strategy to complete ECMs 4, 10, 18, 23, 28, 35, 38, 39, 41, and 48. None of the projects required financing which is the least-cost effective option. It should be noted that the lower level returns $1.3 M to the upper level for budgetary use (the shared amount that goes to the agency). Without this insight and communication afforded by the two-level problem, the upper level would have only completed ECM projects 5, 23 and 28. The balance of the projects is left for the execution by ESCOs. In this case, the project mix and the magnitude of the savings to share are lucrative enough for the ESCOs to complete the balance without hardship meeting the minimum profit levels).

The three types of ESCOs split the remaining projects for a maximum energy saved of 63.0 M KBTU annually and a total savings of $4,260,222 to the agency.

	Agency	**Firm 1**	**Firm 2**	**Firm 3**	**Totals**
	$\boldsymbol{x(p)}$	$\boldsymbol{q(p,1)}$	$\boldsymbol{q(p,2)}$	$\boldsymbol{q(p,3)}$	
Total Projects	**10**	9.61	26.277	2.113	
Total Profit		**$615,865**	**$1,848,516**	**$100,000**	

Table 2-6: Practical Application Results at $200K Budget

Please see Appendix C for the complete set of results showing project allocations by agency and/or firm.

2.6. Discussion

The results of the practical application show that the value of the optimization is almost $1.4M ($1,374,794 in savings to the agency as compared to the current method of sorting by payback and having ESCOs perform the balance (Industry Practice)). This optimal strategic value represents the lifecycle savings difference between the industry standard practice and the two-level optimized model presented, as realized by the agency.

The industry standard method instructs the agency's decision-maker to order projects by payback and select projects until the budget is exhausted. That method leaves as subset of projects for execution by ESCOs. In applying that method to this data, the project mix and the magnitude of the savings shared are lucrative enough for ESCOs to complete the balance without hardship. However, at higher initial budgets or higher minimum profit requirements by the ESCOs, the industry standard method runs the risk of leaving undesirable projects incomplete. This result is due to the luxury that the additional budget provided allows the agency to choose a larger set of the lucrative projects without regard for the ESCOs. Likewise, as the ESCOs' minimum-profit requirements increase, fewer of the less lucrative projects would be accepted. The federal requirements cannot be met if any projects are left undone.

In practice, once the agency identifies project scopes for outside firms, the ESCOs or utilities perform additional assessments that are similar to the level II or III energy audit as defined by ASHRAE. The firms assess their ability to achieve these savings targets and begin their contracting phase. More aggressive estimates of savings usually drive more terms, conditions and lengthy contract / negotiation phases.

It should be noted that the Industry Practice method provides a "rule of thumb" that is very easy to apply. While these methods save energy, much of the additional benefit of reducing consumption, namely lowering the cost of energy, are lost to ESCOs, improper assignment, and poor implementation strategies.

While regulation, stewardship and many other factors drive energy conservation, there has not been much direction on how to achieve high efficacy of those actions. Today, many of the model and tools are not readily available to the casual energy manager. As energy conservation becomes more integrated with building management, operations and finance, the level of sophistication will rise. The intrinsic benefit of teaching energy managers and in-house staff how to select and implement these projects is also essential for the long-term viability of building management energy and sustainability.

2.7. Conclusions

The current industry practice selects projects based on suboptimal criteria such as, payback, savings to investment ratio or ease of implementation. Once those projects are implemented, the agency seeks financing or EPCs for the balance of projects. This segmentation of the timing two decisions by the agency, the different objectives of agency and the ESCOs, and the inability of the leader, the agency, to leverage the knowledge of how the lower level firms will respond, make the entire process suboptimal. This suboptimal selection process can waste millions of taxpayer dollars through inefficient allocations while not providing any additional profit to the ESCOs. There is also the risk of the agency selecting too many of the profitable projects, thereby leaving only undesirable projects for ESCO. Many of these projects are currently being left undone while agencies struggle to meet their mandated conservation goals. The agency must then finance these projects, which is least cost effective option.

The two-level model maximizes savings to the agency and profit to the ESCO industry. While the EnergyStar guidance provides “rules of thumb” that may simplify the selection, this process does not make the best use of the dollars and options for project execution.

The benefits of the of the two-level optimization are apparent when comparing these results to both the standard practice and even a single level optimization problem. Giving the agency’s ability to select projects while evaluating the implementation and financing mechanisms available to them, make them the best stewards of taxpayers' money.

Chapter 3: Energy Conservation Project Selection using Risk-based, Multistage, Stochastic Programming

3.1. Introduction

In 2009, the United States Congress issued Executive Order (EO) 13514, "Federal Leadership in Environmental, Energy, and Economic Performance." EO 13514 introduced new greenhouse gas (GHG) emissions management requirements, expanded water reduction requirements for federal agencies, and addressed waste diversion, local planning, sustainable buildings, environmental management, and electronics stewardship. In addition, EO 13514 retained the energy reduction requirements of EO 13423, directing agencies to set a percentage target for reducing their Scope 1 and Scope 2 greenhouse gas (GHG) emissions in absolute terms by fiscal year (FY) 2020, relative to an FY 2008 baseline.

EO 13514 required that federal agencies must enhance efforts towards sustainable buildings and communities. Specifically agencies must implement high performance sustainable federal building design, construction, operation and management, maintenance, and deconstruction by ensuring all new federal buildings, entering the design phase in 2020 or later, are designed to achieve zero net energy by 2030.[12]

Federal agencies have been relying on Congressional appropriations to fund the energy projects needed to meet federal requirements. Supplemental funding options have included energy savings performance contracts, utility energy service contracts, power purchase agreements, and

[12] A zero net energy building is one with zero net energy consumption or in other words, the total amount of energy used by the building on an annual basis is less than or equal to the amount of renewable energy created on site.

energy incentive programs. This often means combining Congressional appropriations and project funding mechanisms (United States Department of Energy, 2013).

Agency energy and facility managers have the objective of conserving energy, with limited budgets in many buildings that require costly retrofits. The agency leverages the industry standard for best practice in order to identify potential projects (EnergyStar, 2013). The primary tool that the agency's decision-makers use is the energy audit. There are several types of audits, however, an American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) Level II or III is most often used for planning and decision-making (Kelsey & Pearson, 2011). A Level II audit is an energy building survey and energy analysis that assesses the energy use within the building. A Level II energy audit identifies and provides the savings and cost analyses of all practical measures that meet the agency's constraints and economic criteria, along with a discussion of any changes to operation and maintenance procedures. It also provides a listing of potential capital-intensive improvements that require more thorough data collection and engineering analysis and a judgment of potential costs and savings. A Level III energy audit is a detailed analysis of capital-intensive modifications that involves more detailed field data gathering and a more rigorous engineering analysis. It provides detailed project cost and savings calculations with a high level of confidence sufficient for major capital investment decisions (American Society of Heating, 2004).

The audit is an onsite assessment and comprehensive energy analysis of the building's energy-using components resulting in a list of proposed energy conservation measures (ECM) which include the following attributes:

- the proposed system or component description
- an estimate of the investment required to implement the measure
- an estimate of annual savings
- the annual cost savings in dollars
- a performance measure such as simple payback ratio or savings to investment ratio

A typical set of these measures are shown below in Table 3-1.

Project Description	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Savings ($)	Estimated Useful Life (years)	Payback Ratio (years)
Lighting project	50,000	625,000	0.015	9,375.00	30	5.33

Table 3-1: Simple Example of ECM Data in the Agency's Decision-making Process

The energy auditors have to assess the regulatory requirements and conduct an audit to recommend the projects necessary to save the requisite energy. All projects must be completed. Projects that do not fulfil specific "savings to investment" ratio criteria are not considered. The agency's approach to implementing these projects is ultimately risk-averse. The agency requests a budget in an early time period and seeks to fund the required energy conservation projects.

Given this list of ECMs, the agency's decision-maker faces key strategic decisions. The agency can chose to execute these energy conservation projects in-house with or with supplemental methods. The highest return is in executing the projects in-house. Each project that the energy manager or decision-maker implements in-house, saves energy or annual energy costs and, in most cases, both. The traditional approach used by the agencies is to complete the projects in-house with an initial capital outlay. However, projects that when implemented in-house, generate cost savings, also free up capital that can be used to fund additional energy projects. This practice is not common as most agencies are risk-averse and lack the ability to model uncertainty. The largest opportunity for energy conservation lies in the selection strategy. Optimization is needed to properly create a plan that maximizes the expected energy savings while identifying financing available to implement the recommended measures.

A preferred method of energy conservation project selection has not been regulated, however; a method that includes fluctuations in the energy prices and uncertainty in savings estimates would be much more accurate. Project selection that minimizes the agency's initial investment and leverages annual savings to fund future projects is complicated by selections that must be made today but realized in future years. Many approaches of this type of problem have been studied however; fewer have been applied to energy conservation.

Overall, the need for energy conservation is required, however; no formal approach can prescribed beyond the identification of measures that meet the reduction goals. This chapter presents a formal method to achieve the best implementation plan while including both uncertainties and risk. Again, this method

- is the application of stochastic optimization (as opposed to deterministic selection) to a common energy conservation problem to more realistically capture hedging effects,
- includes of a measure risk, beyond a single scalar variable, which is not considered mathematically in agency planning,
- is unlike portfolio theory, where securities can be excluded, minimizes risk even with a constraint requires that all projects be selected.

While none of previous works includes all three of these key points, it is beneficial to review the prior work that is related and/or has addressed some of these concepts.

3.2. Literature

The current work represents the convergence of stochastic programming, risk-based project selection and the importance of cost energy conservation issues. While stochastic programming is not a new or unstudied concept, but the inclusion of risk in the current energy conservation setting makes this novel. The literature review chosen reflects stochastic programming works in relevant applications. The literature pertaining to risk represents a much smaller subset of the field. Applications to this specific energy conservation problem were limited in the stochastic programming and risk literature.

3.2.1. Stochastic Programming with Risk Literature

Beginning with (Dantzig, 1955), (Beale, 1955), and (Charnes & Cooper, 1959) stochastic programming has grown into a very important subfield of mathematical programming with well-established theoretical developments. Research on algorithms and applications has also been very active, especially in recent years. There has been a growing number of specialists in the area, and knowledge is widespread among the leaders of the field. Research on algorithms and applications of stochastic programming, the study of procedures for decision-making under uncertainty over time, has been very active in recent years. There are many applications in areas such as production, supply chain and scheduling, gaming, environmental and pollution control, financial modeling, telecommunications, and electricity (Ziemba & Wallace, 2005). The current work furthers multistage stochastic programming with an application to energy conversation project selection.

The following papers (below) were specifically studied, while developing the current work because they include viable risk approaches, however; they lack the energy conservation application above. The ability to compare random outcomes based on the decision-makers risk preferences is imperative when modeling problems with uncertainty. The objective here is to model optimization problems that feature risk appetite as constraints. Risk measures including semi-deviations, quantiles (value-at-risk) and conditional value-at-risk (CVaR) and properties of risk measures, such as law invariance and coherence, were introduced (Artzner, et al., 1999). Shortly after, Rockafellar and Uryasev (2000) introduced an approach to optimizing a portfolio to reduce the risk of high losses. Value-at-Risk (VaR) had a role in the approach, but the emphasis is on Conditional Value-at-Risk (CVaR), which is known also as Mean Excess Loss, Mean Shortfall, or Tail VaR. By definition with respect to a specified probability level, the VaR of a portfolio is the lowest amount such that, with probability the loss will not exceed whereas the CVaR is the conditional expectation of losses above that amount. Although VaR is a very popular measure of risk, it has undesirable mathematical characteristics such as a lack of subadditivity and convexity.

Further, in Pflug (2000) it was proved that CVaR is a coherent risk measure having the following properties: transition-equivariant, positively homogeneous, convex, monotonic with respect to stochastic dominance of order 1, and monotonic with respect to monotonic dominance of order 2. Because a coherent risk measure of a stochastic convex function is also convex as shown in

(Ruszczynski & Shapiro, 2006), CVaR is more popular in stochastic convex optimization. In Sarykalin et al. (2008), the authors show that conditional Value-at-Risk has a clear engineering interpretation and measures outcomes that improve the overall objective. However, CVaR optimization can be represented via convex programming, in some cases as linear programming (i.e., for discrete distributions).

In Conejo et al. (2010), the authors presented several methods to avoid unfavorable outcomes in spite of favorable expected objective function values using risk functions. These risk functions assign a real number associated with the risk of that project to the random variable. These functions could be added to the objective function or the constraints. Upon review of several methods, the use of Conditional Value-at-Risk appears to be most applicable to the current work in this chapter due its advantages of being a coherent risk measure and its ability to quantify tails beyond the Value at Risk.

3.2.2. Project Selection Literature

Markowitz (1952) wrote that the process of portfolio selection (similar in some extent to project selection) may be divided into two stages: observation and experience, leading to beliefs about the future performances and the relevant beliefs about future performance leading to the choice of portfolio. This concept is used in the decisions-makers' problem, here in multiple stages. The experience leading to the beliefs about the future are of the focus of this current research. The current problem in which the agency must select projects is made difficult by beliefs on where energy prices and the uncertain outcome of planned work may be in the future.

In Raifee et al. (2014), the joint problem of project selection and project scheduling in an uncertain environment is formulated, analyzed, and solved by multistage stochastic programming. A general mathematical formulation that can address several versions of the problem is presented. A multi-period project selection and scheduling problem is introduced and modeled by multistage stochastic programs, which are effective for solving long-term planning problems under uncertainty. A set of scenarios and corresponding probabilities is applied to model the multivariate random data process (costs or revenues, available budget, chance of success). Then, due to computational complexity, a scenario tree of the resulted scenarios is constructed by scenario-reduction algorithms. Finally, assuming resources of the projects to be limited and renewable, the present worth of the profit of the projects is maximized. Eventually, a case study is introduced and solved, and the results are presented. The effectiveness of the proposed algorithm is shown by the numerical results.

Shi et al. (2011) addressed a risk-loaded, stochastic model evaluating objectives to optimize synergies among the various procurement means. This model was also able to produce optimal results in profit while mitigating risk. The implementation of this portfolio approach was based on a multistage stochastic programming model in which replenishment decisions were made at various stages along a time horizon. The replenishment quantities were determined by simultaneously considering the stochastic demand and the price volatility of the spot market. The model attempted to minimize the risk exposure of procurement decisions measured as conditional value-at-risk. The integrated framework proposed in the current work allows the various risks involved to be holistically considered and dealt with while the performance of the portfolio is measured in terms of the expected profit for specific timing and project selection.

In Huang (2008), variance, semivariance and probability approaches to risks are presented. An alternative definition of risk for portfolio selection and proposes a new type of model based on this definition. A hybrid algorithm is employed to solve the optimization problem in general cases. A model that integrates both severity levels of loss and the corresponding occurring probabilities of these losses is presented. While Huang's definition of risk presents a new model, it still relies upon symmetric distributions not found in the returns on energy projects. A symmetric distribution in these cases would imply that the likelihood of energy price increases are equivalent to price decreases. In the United States, energy prices increases have steadily increased annually.

Federal agencies have not been completely remiss in addressing building improvements and how to predict outcomes and include risk in the selection of energy conservation projects. In Committee (2012) the authors addresses the ways to identify and mitigate the risk incurred by not funded specific projects in any given year. The recommendations here are to ensure that the most critical requirements rise to the top of the funding requests and that the senior decision-makers are made aware of the implications of not funding these projects. The authors recommends the use of the Analytical Hierarchy Method (ASTM 1765-07e1) to allow for consideration in the decision-making criteria in the priority-setting process. The Committee's interpretation of risk involves ensuring the most important projects are selected as defined by the method. The model to be presented in this chapter removes this subjectivity by using reducing the multiobjective nature of the problem down to quantitative outcomes (total amount of savings achieved in dollars). It also includes constraints that state all project must be completed to meet energy consumption reduction goals with a risk-based objective function.

Real options were developed as a result of the dissatisfaction with traditional capital budgeting techniques such as the discounted cash flow (DCF) method of valuation. Stochastic methods and multiple scenarios have been used to deal with uncertain variables in the DCF, however; calculating the DCF, investors rest on a series of simplifying assumptions.

In the presence of certain types of uncertainty about the future costs and benefits of capital investments, investors have to estimate the likelihood of various future scenarios, calculate the DCF in each of these futures, and sum to find the average expected DCF across the possible futures. These real options are attributed to Myers, who first identified investments in real assets as mere options (Myers, 1977).

As in the current research, a real option is an opportunity with different value at different periods to undertake some business decision, typically an option to make, abandon, alter or switch a capital investment. For example, an opportunity to delay investment in a specific energy conservation project is a real option. Similarly, the agency has a single discrete investment opportunity despite fluctuations between stages. If using a real options approach, the annual savings that are used to fund additional projects could be modeled (approximated) as an implicit dividend (Dixit & Pindyck, 1994). An equivalent real options approach may be developed if constraints, such as the ability observe actual fluctuations to invest in later stages are removed are added to the real options approach.

However, in this dissertation, analytic solutions may not exist and it may not be possible to determine the partial differential equations describing the underlying stochastic processes particularly in risk averse cases (Trigeorgis, 1997). Limitations of the real options approach include the lack of a time derivative in discrete-time or continuous-time stochastic processes. In this dissertation, the key uncertain components are fluctuations in annual savings. The interaction and grouping of projects to generate annual savings to fund future projects would present a challenge in the real options approach since the nodes in the tree do not recombine. Specifically, nonanticipativity requires that values of the budget and the decision variables chosen at stage t, depend on the data available up to time t, but not future observations. This limitation makes the multistage stochastic model more attractive. Further, the ability to apply varying approaches to risk and allowable recourse actions provide a more flexible model for this application.

The value of the current work is the application of stochastic programming / risk modeling to energy conservation which, to our knowledge, an unstudied area. The proposed research provides significant value to agencies and energy managers as it allows more efficient selection with varying options for risk tolerance. The stakeholder that will benefit are federal agency decision-maker, energy managers and ultimately U.S. taxpayers.

3.3. Model

A way to meet U. S. energy independence and sustainability objectives by using existing savings to fund future projects while accounting for uncertainty in implementation yields and energy prices is presented here. The audits return the set of maximum energy savings projects. Each agency has a limited budget, which is requested then granted from tax dollars. If all projects recommended by the audit are not completed, then the regulatory requirements will not be met. The traditional approach used by the agencies Projects requires that all projects be completed with the initial capital outlay.

However, the choices of projects, the amount of annual savings, and the timing of selection can minimize the budget requested by the agency to complete and fund future projects.

This selection process can be modeled as a multistage stochastic problem where the agency has a single opportunity to request capital budget (C in later examples) at t=0 for projects and the timing of the selected energy conservation project to be implemented. In particular, the agency receives the budget at t =1 to implement projects and selects projects based on the belief of future energy prices and estimate of annual savings. The agency implements projects at times t= 1…N_T but without injecting additional capital budget. The agency's annual energy operating budget is fixed and does not account for energy savings pursued by the agency. Beyond the initial period (t=1), the agency see realize fluctuations in both energy price and energy savings forecasts (for example, in Table 3-1, the annual savings in dollars is the product of annual energy savings (KBTU) and the energy cost ($/KBTU).

In practice, after the initial period, the energy costs may change. Another uncertain factor is the annual energy savings. The energy auditors estimate these values without specific design conditions and/or knowledge of the interactive effects of other project implemented. Of the two factors (energy cost and energy savings forecast), the inaccuracy of the estimate of savings dominates (larger deviation from expectation) making the overall savings (energy savings in KBTU x energy rate in $/KBTU) generally less than anticipated. This problem allows the total savings to be modeled as a single random variable.[13]

As discussed previously, results of the energy audit are presented to the agency. The results contain specific attributes for each ECM recommended including the cost to implement the project and the projected savings. The agency selects an optimal subset of projects, today, to implement with some schedule of later projects in the future. The overall objective is to minimize the budget needed to complete all projects, while leveraging future energy savings when projected savings are uncertain.

The data for three-phase, five-project clarifying example is shown below in Table 3-2. Given the projects, the agency must request the capital budget now and implement the projects in the first phase. The balance of the projects must be implemented from the remaining capital budget (budget left over after implementing first-phase projects), an annual operating budget and the annual savings from the implemented project, to avoid the agency's cash flow falling below zero. The clarifying example is deterministic therefore, the annual savings are certain and singular (there are not multiple scenarios with associated probabilities). The decision variables ($\mathbf{x}_p$ and $\mathbf{y}_p^t$) are binary representing selection of the project p; equaling 1, or 0 otherwise. The resulting values for the binary decisions variables ($\mathbf{x}_p$ and $\mathbf{y}_p^t$) for each project representing selection are shown in Table 3-2.

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[13] Energy Rates have remained flat over the last 7 years with slight increases in electricity rates and slight reductions in natural gas (Administration, 2016).

	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Cost of Energy Saved ($)	Payback Ratio (Years)
project1	10,000	50,000	0.02	1,000	10.0
project2	20,000	45,000	0.04	1,800	11.1
project3	30,000	40,000	0.06	2,400	12.5
project4	40,000	35,000	0.08	2,800	14.3
project5	50,000	30,000	0.10	3,000	16.7

Table 3-2: Clarifying Example of ECM Data in the Agency's Decision-Making Process

With a deterministic optimization model, minimizing total cost to complete all projects, the agency determines that it should request $85,800 at t=0. From Table 3-2, it can be seen that the agency implements projects 1, 3 and 4 in the first phase (t=1) at a cost of $80,000. The agency ends up with a positive cash flow of $5,800 at the end of that phase. In phase two (t=2), the agency leverages the $5,800 surplus, receives $25,000 from the operating budget (O^t, as an exogenous factor) and $6,200 in savings, generated from projects implemented in the first phase. The total budget available for that phase is thus $37,000 (5,800 + 25,000 + 6,200). The agency uses the $37,000 and implements a project (project 2) at a cost of $20,000 at t=2. The agency leaves that phase (phase 2) with a positive cash flow of $17,000. In phase three, the agency leverages the $17,000 surplus, the $25,000 from the operating budget (exogenous factor) and the $8,000 ($6,200 + 1,800) in savings generated from projects implemented in the first two phases and uses that $50,000 to complete the final project (project 5) at a cost of $50,000. Key observations are

- The request in the first phase exceeds the cost of projects implemented in the first phase.
- The savings from those projects implemented in the final phase cannot be used to fund any additional projects.
- There is a strict constraint that does not allow the cash flow to go below zero in any phase.
- The cash flow at the final phase is zero.

	x_p	y_p^t	PHASE 1	PHASE 2	PHASE 3
Investment Cost ($)					
project1	1_1	0	10,000	0	0
project2	0	1_2^2	0	20,000	0
project3	1_3	0	30,000	0	0
project4	1_4	0	40,000	0	0
project5	0	1_5^3			50,000
Annual Cost of Energy Saved ($)					
project1	1_1	0	1,000	0	0
project2	0	1_2^2	0	1,800	0
project3	1_3	0	2,400	0	0
project4	1_4	0	2,800	0	0
project5	0	1_5^3		0	3,000
Cash Flow ($)					
Capital Budget (C, a one-time request)			85,800	0	0
Operating Budget (O^t, annually)			0	25,000	25,000
Saving from Previous Phase Projects			0	6,200	8,000
Surplus from Previous Phase			0	5,800	17,000
Total Budget for Phase (sum of Capital Budget, Savings and Surplus)			85,000	37,000	50,000
Total Invested in Implementing Projects (difference between Total Budget for Phase and Cost of Projects Implemented			(80,000)	(20,000)	(50,000)

Cash	**5,800.00**	**17,000**	**0**

Table 3-3: Agency Cash flow Statement with Selections by Phase

Figure 3-1, below, shows the source of the budget used in the clarifying example. Note the fixed operating budget and varying contributions from executed projects and prior period surpluses.

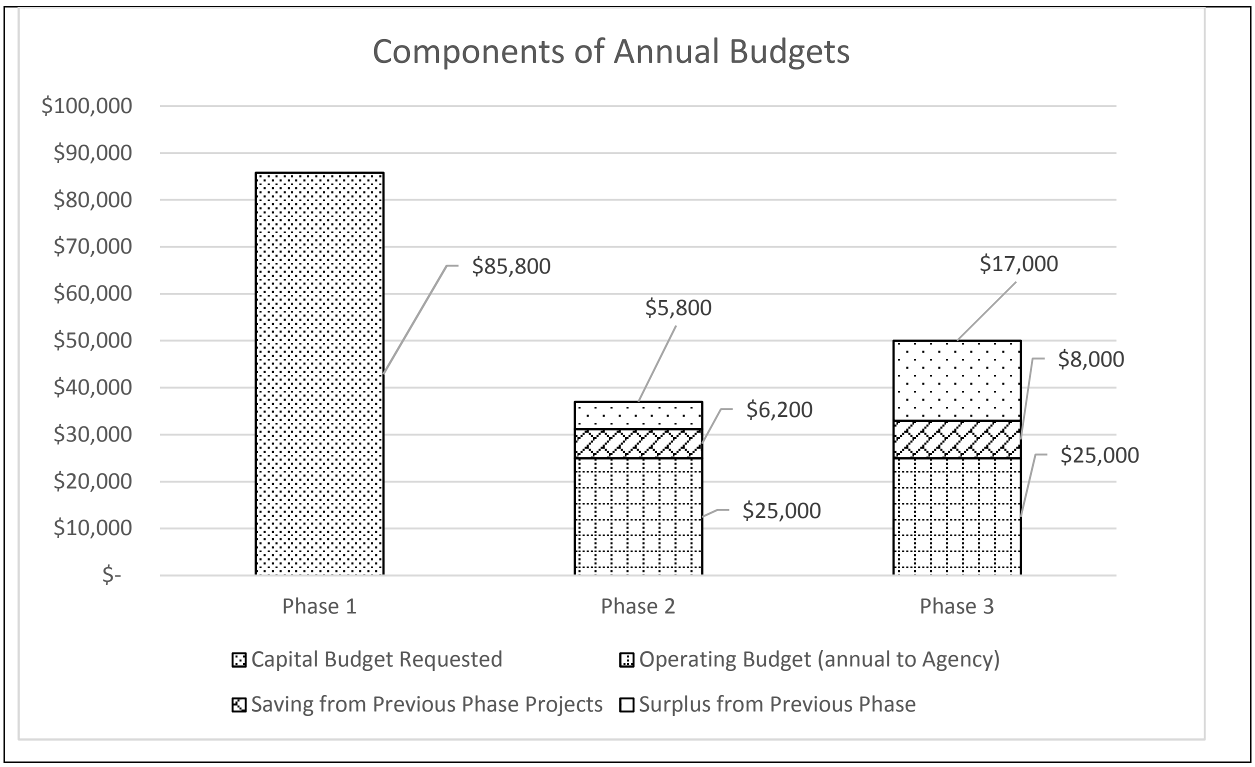


Figure 3-1: Components of Annual Budgets Available for Projects in the Clarifying Example

The safe-haven year project (project 5) does not contribute the $3,000 (the annual cost of energy saved by implementing project 5) to the annual budget. There is also no surplus value at t=3 in this clarifying example.

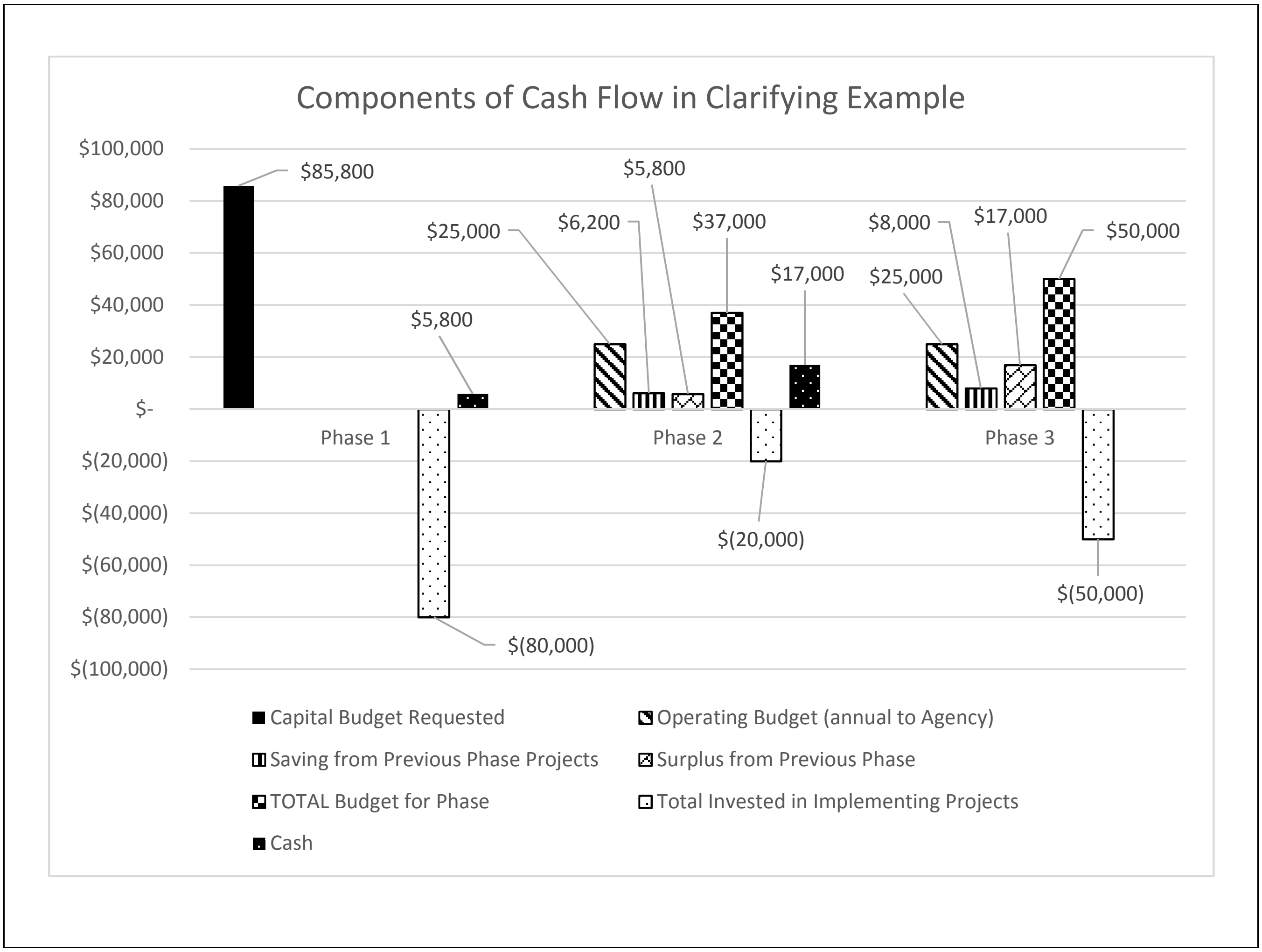


Figure 3-2: Components of Cash flow with Annual Cost of Projects

The clarifying example above does not include the stochastic yields recognized at each phase for the sake of simplicity. A much more realistic approach is model by adding scenarios, (ω) with associated probabilities, $\pi(\omega)$ as in Figure 3-3, below.

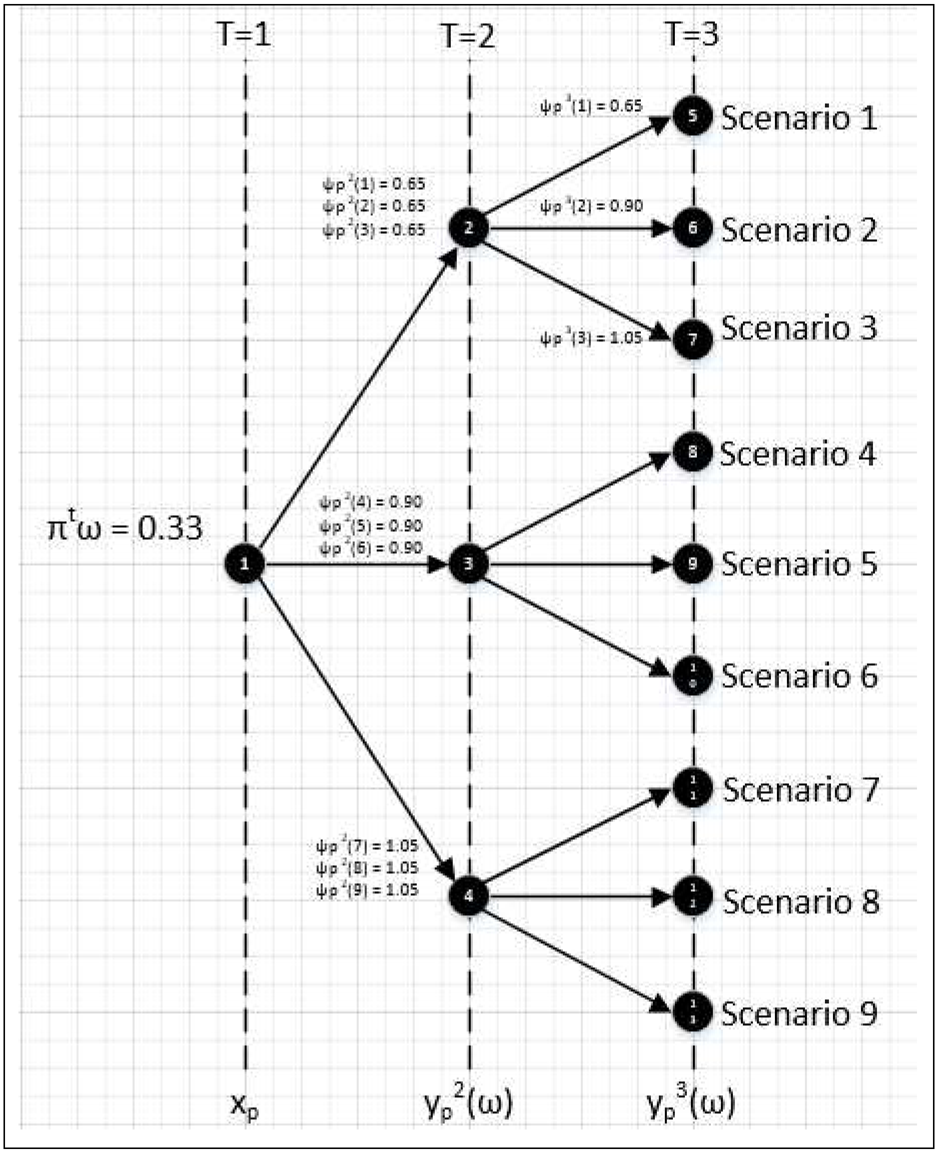


Figure 3-3: Adding Yield Scenarios to Clarifying Example

Below, we present a second clarifying example (a simplifying example). Adding stochasticity to the example above requires the addition of both $\psi_p^t(\omega)$, the yield of the annual savings at each stage as realized through each project's annual savings and $\pi(\omega)$, the probability of the discrete energy price at each stage t. For the simplifying stochastic example, a four-stage model best shows the nodes, leaves and scenarios.

	Probability, $\pi^t(\omega)$	Rate / Yield Factor, $\psi_p{}^t(\omega)$ [14]
Scenario 1 (ω_1)	0.33	0.65
Scenario 2 (ω_2)	0.33	0.95
Scenario 3 (ω_3)	0.33	1.25

Table 3-4: Probability of Annual Saving Fluctuation based Energy Price and Savings Yield for Simplifying Example

The probabilities and yields shown in Table 3-4 were derived from review of observation of approximately 120 maintenance and verifications (M&V) studies.

In addition, the second clarifying example requires nonanticipativity constraints where realizations of the stochastic processes and value of the decisions are the same up to stage t. Decisions on project selections at stage, t do not depend on the scenario realization Shapiro et al. (2009).

The resulting solution with and objective function of -$140,769 is shown below. The perfect information objective is -$138,870.

PHASE 1 (first stage variable – selection)	PHASE 2	PHASE 3	PHASE 4	Ω
	project2	project4		1
	project2	project4		2
	project2	project4		3

[14] The Rate / Yield Factor (x Annual Savings), $\psi_p{}^t(\omega)$ are arbitrary for illustrative purposes. A value of 0.65 means that the implementation of the project returns 65% of the estimated annual savings.

	project2		project4	4
	project2		project4	5
	project2		project4	6
	project2		project4	7
	project2		project4	8
	project2		project4	9
		project2 project4		10
		project2 project4		11
		project2 project4		12
			project2 project4	13
project1 project3 project5			project2 project4	14
			project2 project4	15
			project2 project4	16
			project2 project4	17
			project2 project4	18
		project2 project4		19
		project2 project4		20
		project2 project4		21
			project2 project4	22
			project2 project4	23
			project2 project4	24
			project2 project4	25
			project2 project4	26
			project2 project4	27

Table 3-5: Agency cash flow statement with selections by Stage

The stochastic optimization model presented in the current work includes risk. The method employed was Conditional Value-at-Risk (CVaR) as this approach resulted in a convex program (Conejo et al., 2010). As stated previously, the agency's unwillingness to fund future projects with savings is a direct result of the weak definition or interpretations of risk.

Projects selected at t=1:	Projects selected at t=2	Projects selected at t=3	Projects selected at t=4
$\mathbf{x}_1, \mathbf{x}_3, \mathbf{x}_4 = 1$ Project not selected at t=1 $\mathbf{x}_2, \mathbf{x}_5 = 0$	$\mathbf{y}_2^2(1) = 1$ $\mathbf{y}_2^2(2) = 1$ $\mathbf{y}_2^2(3) = 1$ $\mathbf{y}_2^2(4) = 1$ $\mathbf{y}_2^2(5) = 1$ $\mathbf{y}_2^2(6) = 1$ $\mathbf{y}_2^2(7) = 1$ $\mathbf{y}_2^2(8) = 1$ $\mathbf{y}_2^2(9) = 1$ Projects not selected at t=2 $\mathbf{y}_1^2(\omega), \mathbf{y}_3^2(\omega), \mathbf{y}_4^2(\omega), \mathbf{y}_5^2(\omega)$ $= 0$ for all ω in Ω **OR 0 otherwise**	$\mathbf{y}_4^3(1) = 1$ $\mathbf{y}_4^3(2) = 1$ $\mathbf{y}_4^3(3) = 1$ $\mathbf{y}_2^3(10) = 1$ $\mathbf{y}_4^3(10) = 1$ $\mathbf{y}_2^3(11) = 1$ $\mathbf{y}_4^3(11) = 1$ $\mathbf{y}_2^3(12) = 1$ $\mathbf{y}_4^3(12) = 1$ $\mathbf{y}_2^3(19) = 1$ $\mathbf{y}_4^3(19) = 1$ $\mathbf{y}_2^3(20) = 1$ $\mathbf{y}_4^3(20) = 1$ $\mathbf{y}_2^3(21) = 1$ $\mathbf{y}_4^3(21) = 1$	$\mathbf{y}_4^4(4) = 1$ $\mathbf{y}_4^4(5) = 1$ $\mathbf{y}_4^4(6) = 1$ $\mathbf{y}_4^4(7) = 1$ $\mathbf{y}_4^4(8) = 1$ $\mathbf{y}_4^4(9) = 1$ $\mathbf{y}_2^4(13) = 1$ $\mathbf{y}_4^4(13) = 1$ $\mathbf{y}_2^4(14) = 1$ $\mathbf{y}_4^4(14) = 1$ $\mathbf{y}_2^4(15) = 1$ $\mathbf{y}_4^4(15) = 1$ $\mathbf{y}_2^4(16) = 1$ $\mathbf{y}_4^4(16) = 1$ $\mathbf{y}_2^4(17) = 1$ $\mathbf{y}_4^4(17) = 1$ $\mathbf{y}_2^4(18) = 1$ $\mathbf{y}_4^4(18) = 1$ $\mathbf{y}_2^4(22) = 1$ $\mathbf{y}_4^4(22) = 1$ $\mathbf{y}_2^4(23) = 1$ $\mathbf{y}_4^4(23) = 1$ $\mathbf{y}_2^4(24) = 1$ $\mathbf{y}_4^4(24) = 1$ $\mathbf{y}_2^4(25) = 1$ $\mathbf{y}_4^4(25) = 1$ $\mathbf{y}_2^4(26) = 1$ $\mathbf{y}_4^4(26) = 1$ $\mathbf{y}_2^4(27) = 1$ $\mathbf{y}_4^4(27) = 1$

Table 3-6: Decision Variables for Simplifying Example

The following is the notation, variables, and parameters used in the general statement of the stochastic multistage problem.

<u>Sets</u>

p set of ECM projects with $P = \{1, 2, \ldots n_p\}$ where $n_p = |P|$

t a set of time periods T (typically years) = $\{1, 2, \ldots n_T\}$ where $n_T = |T|$, $J \equiv T$

<u>Main Primal Decision Variables</u>

$\mathbf{x}_p$ a first-stage binary variable representing selection of the project p; variable = 1, if selected by the agency to be implemented at t=1, = 0 otherwise

$\mathbf{y}_p^t(\omega)$ a t-stage binary recourse variable representing selection of the projects for scenario (ω); variable = 1, if selected by the agency to be implemented in stage t>1, = 0 otherwise

From the example above, the following are the variable values for x and y.

Intermediate Variables

$s(\omega)$ is a continuous non-negative variable equal to the maximum of CVaR and 0
η is an auxiliary variable related to CVaR
$B^t(\omega)$ the budget in dollars for implementing the agency's projects at stage $t > 0$

Parameters

C A scalar representing the capital budget requested at t=0 by the agency in dollars for implementing the agency's projects at stage t = 1
O^t the operating budget in dollars prescribed for the agency's at stage t
θ_p the estimated annual savings in dollars achieved by implementing project p, = energy savings in dollars equal to the product of annual energy savings (KBTU) and energy rate ($/KBTU)
ρ_t the minimum number of projects that can be completed in each year
γ_p the estimated investment in dollars needed to implement project p
ω the scenario with given probability, $\pi^t(\omega)$
$\psi_p^t(\omega)$ the yield of the annual savings at each stage as realized through each project's annual savings
$\pi(\omega)$ the probability of the discrete energy price at each stage t
β a weighting parameter between 0 and 1, (tradeoff between the risk-neutral and upper CVaR cost of projects). $\beta = 0$ is risk neutral, $\beta = 1$, risk averse at the given confidence level
α confidence level for CVaR

General Formulation

The specific objective function can be written as:

(3a)

$$\max_{x,y(\omega),\eta,s(\omega),B}\left(-C-\sum_{p=1}^{n_P}\gamma_p\boldsymbol{x_p}-\sum_{\omega\in\Omega}\pi(\omega)\sum_{p=1}^{n_P}\sum_{t=2}^{n_T}\gamma_p\boldsymbol{y_p^t}(\omega)\right)$$

Subject to:

$$\boldsymbol{x_p}+\sum_{\boldsymbol{t=2}}^{\boldsymbol{n_T}}\boldsymbol{y_p^t}(\omega)=1\ \forall p\in \mathrm{P},\forall\ \omega\in\Omega \tag{3b}$$

$$B^t(\omega)\geq 0\quad \forall t\in T,\forall\omega\in\Omega \tag{3c}$$

$$\sum_{p=1}^{n_p}\gamma_p\boldsymbol{y_p^t}(\omega)\leq\ B^t(\omega)\quad \forall t\in T,\forall\ \omega\in\Omega \tag{3d}$$

$$B^t(\omega)\ =\sum_{p=1}^{n_P}\sum_{j=2}^{t-1}\psi_p^t(\omega)\ \theta_p\left(\boldsymbol{x_p}+\boldsymbol{y_p^j}(\omega)\right)+O^t+B^{t-1}(\omega)-\sum_{p=1}^{n_P}\gamma_p\boldsymbol{y_p^{t-1}}(\omega)\ \forall t \in T,\forall\ \omega\in\Omega \tag{3e}$$

$$\sum_{p=1}^{n_P}\sum_{\omega=1}^{n_\omega}\boldsymbol{y_p^t}(\omega)\ \geq\ \rho\ \ \forall t\in T \tag{3f}$$

$$\begin{gathered}\boldsymbol{y_p^t}(\omega)\ =\ \boldsymbol{y_p^t}(\omega')\quad\forall\ \omega,\omega'\ \in\ \{1\ldots\Omega\}\\ B^t(\omega)\ =\ B^t(\omega')\ \ \forall\ \omega,\omega'\ \in\ \{1\ldots\Omega\}\end{gathered} \tag{3g}$$

$$B^t(\omega)\ =\ B^t(\omega')\ \ \forall\ \omega,\omega'\ for\ which\ \xi^t(\omega)=\xi^t(\omega'),t=2\ldots T$$

$$\boldsymbol{y}_p^t(\omega) \; = \; \boldsymbol{y}_p^t(\omega') \;\; \forall \; \omega, \omega' \; for \; which \; \xi^t(\omega) = \xi^t(\omega'), t = 2 \ldots T$$

$$where \; \xi^t(\omega) \;\; is \; a \; node \; a \; stage, t \; and \; scenario, \omega$$

$$\boldsymbol{x_p} \, , \boldsymbol{y_p^t}(\omega) \;\; binary \tag{3i}$$

This model allows agencies to prescribe their risk tolerance. A risk-neutral agency would choose β=0, whereas a risk-averse agency would choose β=1 in Equation 3a. Tolerances can be adjusted between the two approaches.

For equation 3g, the values of the decision variable $y_p^t(\omega)$ and variable $B^t(\omega)$ chosen at stage t, depend on the data ξ^tavailable up to time t, but not future observations. This is the basic requirement of nonanticipativity (Wets, 1974). Using the tree diagram, Figure 3-3, for the clarifying example, the information observed at T=3, nodes 5-7, must be the same as they are all successors of Node 2 at T=2.

In the later sections, we will contrast the current model with the traditional and deterministic approaches. In the traditional approach, the agency funds all projects in the first stage with the initial capital outlay.

$$C = \sum_{p=1}^{n_P} \gamma_p \boldsymbol{x_p}$$

$$\boldsymbol{x_p} = 1 \; \forall p \in \mathrm{P}$$

In the deterministic approach, the nonanticipativity constraints in 3g are relaxed (i.e. removed).

3.4. Case Study

The model described above aligns the objective of selecting projects to save energy at the lowest cost using future savings to complete more projects. A practical application of the model is demonstrated using data from an agency's campus of buildings in the southeastern United States.

Recall from Chapter 2, EMG, a decision intelligence and engineering consulting firm, conducted an ASHRAE Level 2 Energy Audit of a college campus comprised of 38 buildings categorized residential, student, academic, and administration.[15] The campus covers a total of over 1.04 million ft^2. EMG was contracted to perform this detailed energy audit and make energy saving recommendations on the physical plant and connected 11 buildings. As part of the study, EMG reviewed the buildings' construction features, historical energy and water consumption with costs, envelope, heating ventilation and air conditioning (HVAC) equipment, heat distribution systems, lighting, and operating and maintenance practices. EMG identified forty-eight energy conservation measures. The following paragraphs describe a typical ECM, "Decommissioning of Central Steam Boilers and Installation Individual High Efficiency Condensing Boilers."

There is one central boiler/chiller plant (physical plant) serving 11 of the 38 buildings, while the other 27 buildings are served by local systems. The central boiler in the central utility plant currently serves nine buildings on campus. The steam from the boilers is piped to the individual buildings. The central plant currently has two inefficient Continental steam boilers and an aging chiller plant. A significant amount of energy is spent in raising its temperature from 55°F to

[15] Bill Champion is the Director of Asset Management Consulting at EMG.

220°F in order to evaporate the boiler feed water, instead of the normal 185°F to 220°F because more than 75% of condensate return is fresh, unheated water. Based on the observations and analyses, the audit proposes a new chiller plant along with new boilers with a thermal operating efficiency of 92-96% in contrast to the current boiler thermal efficiency of 60%. The hot water circulation pumps and variable frequency drives will save additional electrical consumption. This project will also result in an annual water savings currently being drained into the city sewer due to lack of proper condensate return system.

The total savings annual saving for the ECM will be $80,023. The table below summarizes the attributes of this proposed ECM project.

	Investment Cost ($)	**Annual Energy Savings (KBTU)**	**Energy Rate ($/KBTU)**	**Annual Cost of Energy Saved ($)**	**Estimated Useful Life (years)**	**Simple Payback (years)**
project 1	710,354	5,334,857	0.015	80,023.00	30	8.877

Table 3-7: Typical ECM Project Attributes Revisited

In the numerical example, there are 48 such ECMs with varying characteristics and project attributes. The model presented earlier is applied to these data as follows.

n_p =|48|

ζ_p shown in the fourth column of Table 3-9

θ_p^T shown in the fifth column of Table 3-9.

$\Psi_p^t(\omega)$ the annual savings fluctuation at each stage is realized through each project's annual savings as shown in Table 3-8, below.

	Probability, $\pi^t(\omega)$	Rate / Yield Factor, $\psi_p^t(\omega)$[16]
Scenario 1 (ω_1)	0.33	0.65
Scenario 2 (ω_2)	0.33	0.90
Scenario 3 (ω_3)	0.33	1.05

Table 3-8: Probability of Annual Saving Fluctuation based Energy Price and Savings Yield at Each Stage, t

Below are the actual ECM data characteristics from the energy audit.

	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Cost of Energy Saved ($)	Estimated Useful Life (Years)	Payback (Years)
P	γ_p	α_p	ζ_p	θ_p	N	
project1	$ 710,354	5,334,857	$ 0.015	$ 80,023	30	8.88
project2	$ 637,975	1,849,047	$ 0.033	$ 61,019	23	10.46
project3	$ 468,071	1,768,079	$ 0.023	$ 40,666	30	11.51
project4	$ 40,368	445,600	$ 0.010	$ 4,456	30	9.06
project5	$ 8,557	213,025	$ 0.012	$ 2,556	15	3.35
project6	$ 15,328	124,584	$ 0.023	$ 2,865	9	5.35
project7	$ 55,207	287,971	$ 0.027	$ 7,775	15	7.10
project8	$ 59,355	416,045	$ 0.022	$ 9,153	15	6.48
project9	$ 84,738	559,247	$ 0.015	$ 8,389	30	10.10
project10	$ 188,994	801,565	$ 0.033	$ 26,452	40	7.14
project11	$ 142,377	660,074	$ 0.023	$ 15,182	30	9.38
project12	$ 186,520	440,470	$ 0.033	$ 14,536	30	12.83
project13	$ 165,932	2,243,077	$ 0.012	$ 26,917	15	6.16
project14	$ 169,521	650,787	$ 0.023	$ 14,968	20	11.33
project15	$ 95,238	554,558	$ 0.027	$ 14,973	15	6.36
project16	$ 220,871	1,366,652	$ 0.019	$ 25,966	15	8.51

[16] The Rate / Yield Factor (x Annual Savings), $\psi_p^t(\omega)$ are arbitrary for illustrative purposes.

project17	$ 201,577	793,782	$ 0.030	$ 23,813	30	8.46
project18	$ 119,351	724,725	$ 0.033	$ 23,916	23	4.99
project19	$ 152,286	488,525	$ 0.023	$ 11,236	30	13.55
project20	$ 95,631	632,278	$ 0.010	$ 6,323	30	15.12
project21	$ 53,495	518,592	$ 0.012	$ 6,223	15	8.60
project22	$ 276,920	1,551,851	$ 0.023	$ 35,693	20	7.76
project23	$ 94,078	1,135,237	$ 0.027	$ 30,651	20	3.07
project24	$ 228,071	784,038	$ 0.026	$ 20,385	15	11.19
project25	$ 236,862	2,103,902	$ 0.014	$ 29,455	10	8.04
project26	$ 438,530	1,678,580	$ 0.023	$ 38,607	23	11.36
project27	$ 558,439	3,212,065	$ 0.029	$ 93,150	12	6.00
project28	$ 84,237	2,054,672	$ 0.020	$ 41,093	10	2.05
project29	$ 18,149	138,751	$ 0.013	$ 1,804	26	10.06
project30	$ 64,378	420,774	$ 0.017	$ 7,153	20	9.00
project31	$ 387,393	2,743,397	$ 0.026	$ 71,328	15	5.43
project32	$ 266,812	937,263	$ 0.030	$ 28,118	25	9.49
project33	$ 185,099	2,236,000	$ 0.011	$ 24,596	20	7.53
project34	$ 205,145	1,664,432	$ 0.017	$ 28,295	10	7.25
project35	$ 195,433	3,599,559	$ 0.014	$ 50,394	23	3.88
project36	$ 184,600	750,238	$ 0.019	$ 14,255	28	12.95
project37	$ 110,377	1,045,732	$ 0.012	$ 12,549	23	8.80
project38	$ 252,736	1,533,356	$ 0.021	$ 32,200	37	7.85
project39	$ 157,354	2,043,132	$ 0.020	$ 40,863	18	3.85
project40	$ 247,218	1,573,358	$ 0.028	$ 44,054	20	5.61
project41	$ 256,421	1,806,445	$ 0.024	$ 43,355	25	5.91
project42	$ 152,886	2,399,913	$ 0.012	$ 28,799	28	5.31
project43	$ 455,000	2,448,183	$ 0.022	$ 53,860	36	8.45
project44	$ 473,225	3,500,838	$ 0.017	$ 59,514	33	7.95
project45	$ 127,011	883,506	$ 0.017	$ 15,020	14	8.46
project46	$ 492,782	1,802,085	$ 0.016	$ 28,833	32	17.09

project47	$ 266,790	1,010,352	$ 0.031	$ 31,321	10	8.52
project48	$ 115,006	741,117	$ 0.025	$ 18,528	20	6.21
Totals	**$ 10,402,698**	**66,372,316**		**$ 1,351,279**		

Table 3-9: ECM Data in Practical Application Revisited

3.5. Numerical Results

The mixed-integer program (MIP) shown and derived from the above discussions, was programmed in GAMS Rev 23.6 using a 64-bit MS Windows machine and the XPRESS solver. The model included 15,521 single equations and 16,672 single variables with 15,552 binary variables. The optimality tolerance percentage (*optcr*) was set to 0.0%.[17] The MIP model was solved using a minimization format and arrived at integer optimal solution in 2:03:16:348.

Resource usage is the amount of CPU time (in seconds) taken by the solver, as well as the time limit allowed for the solver. The *optcr* in this model is set at 0.0% which forces the application to solve to optimality. As such, the maximum CPU time (in seconds) was kept at the default value of 1,000. The iteration count and the iteration upper limit was set at 2,000,000,000. The results can be seen in Table 3-10, below.

Beta	Solver Status	Model Status	Objective Value	CPU Time (seconds)	Relative Gap
0.0	Normal Completion	Optimal	9,663,282	357.164	0.00

Table 3-10: Solve Summary

For the 48-project, 5-stage model, the minimum capital outlay to complete all projects was $9,663,282 (a cost) in the risk-neutral approach.

3.6. Discussion

[17] In general, *optcr* is not set for 0.0%. This was set for 0.0% in this case having that an exact solution was possible.

Recall that the traditional approach used by the agency requires that all projects be completed with the initial capital outlay. Table 3-10, above, shows the resulting model's objective function in the risk neutral approach. The results of the model show a vast improvement on the traditional approach.

In the traditional approach, the agency funds all projects in the first stage with the initial capital outlay.

$$C = \sum_{p=1}^{n_P} \gamma_p \boldsymbol{x_p}$$

$$\boldsymbol{x_p} = 1 \; \forall p \in \mathrm{P}$$

In the deterministic approach, the nonanticipativity constraints in 3g are relaxed.

There are three major observations from the results of this model. They are:

1. The results of the stochastic approach with risk spreading, using an annual minimum number of projects, reduces risk of a back end loaded shortfall.
2. The proposed approach exceeds the traditional and deterministic Approaches
3. Using CVaR in all cases except for the risk neutral case yields a time inconsistent policy. A more practical hedging approach spreads project implementation throughout the horizon.

Each of these results are discussed in detail below.

Key Result: The Current Approach Exceeds Traditional and Deterministic Approaches

The first key result of this approach is that its results (the objective function: total cost, and corresponding initial capital outlay) exceed those of both the deterministic approach and the agency's approach. The traditional agency approach does not allow for anticipated use of energy savings to fund future projects because of agency discomfort with risk. Adding risk to the model should serves as a mechanism for even the most risk-averse agencies to determine their acceptable tolerance.

In the traditional agency approach, the agency would be forced to fund all projects at a cost of \$10,402,698. This is both the total cost to complete all projects and the initial capital outlay requested at t=0. In contrast, the model presented in this chapter results in a complete funding cost of \$9,666,282 and an initial capital outlay of \$5,885,967.

The risk-neutral approach gives an optimal objective function value of \$9,663,282 for the 48-project, 5-stage model. The expected value deterministic solution is \$9,644,413. These quantities result in a value of the stochastic solution (VSS) of \$18,869. The Expected Value of Perfect Information (EVPI) is therefore \$476,735 for the risk-neutral scenario.

The risk-neutral approach of the current work is chosen for comparison because the agency's general approach to risk has been only to ensure that critical projects are included as early as possible. Agencies have interpreted that direction in many different ways resulting in an absolutely zero risk policy being adopted. This is the agency's traditional approach. The constraints in this work, specifically, that all projects must be completed and the five-year time horizon accounts for these agency practices.

In contrast, the deterministic approach requires less to complete all projects ($9,686,834) and the smallest capital outlay of less than $5.0M. This is found by relaxing the nonanticipativity constrains and is equivalent to the "perfect information" approach. Therefore, in many stages and scenarios the agency could be left without enough funding for projects thereby violating a key objective of the regulation (unacceptable solutions where equations 3b and 3c are not met). The current stochastic model requires less total cost to complete all projects but more initial capital than the deterministic approach. While the overall total cost to complete all projects in in these two approaches are close (within $25K), the agency's initial capital request and project selection vary greatly. This initial capital outlays vary by almost $900K.

These results, the traditional, deterministic and stochastic approaches vary by total costs to complete all projects, cost requested for initial capital to fund projects and the projected selected in initial phase (see Table 3-11, below).

	Traditional Agency Model: Projects Completed with Initial Capital Outlay	**Deterministic Approach: Projects Completed leveraging Expected Savings and Initial Capital Outlay**	**Stochastic Model: Projects Completed with Approach (Risk-neutral) with Recourse Savings and Initial Capital Outlay**
Total Cost of Projects	**$10,402,698**	**$9,644,413**	**$9,663,282**
Initial Capital Outlay Needed	**$10,402,698**	**$4,986,524**	**$5,885,967**
Initial Period Projects Selected	project1 project2 project3 project4 project5 project6 Project7 project8 project9 project10 project11 project12 project13 project14 project15 project16 project17 project18 project19 project20 project21 project22	project5 project6 project8 project10 project13 project15 project18 project22 project23 project25 project27 project28 project31 project33 project34 project35 project38 project39 project40 project41	project6 project7 project8 project10 project13 project15 project16 project17 project18 project21 project22 project23 project25 project27 project28 project31 project33 project34 project35 project37

	Project23 project24 project25 project26 project27 project28 project29 project30 project31 project32 project33 project34 Project35 project36 project37 project38 project39 project40 project41 project42 project43 project44 project45 project46 project47 project48	project42 project43 project44 project48	project38 project39 project40 project41 project42 project43 project44 project47 project48

Table 3-11: Agency's Traditional, Deterministic, and Risk Neutral Approaches

Table 3-11 above only show the first stage projects. As projects are executed, savings are realized in the following year and every year thereafter. An additional risk based model was developed to more closely at later time periods.

<u>The results of the stochastic approach with risk spreading, using an annual minimum number of projects, reduces risk of a back end loaded shortfall.</u>

In the model, costs and number of projects increase between years 2 and 5. This is a result of the model's use of annual savings to fund projects. Projects with the worst simple payback ratio are executed in stage 5 where their lower annual savings does not contribute to earlier budgets, which were used to fund projects. This result can be seen in Figure 3-4, below.

In order to account for more risk averse approaches, equation (3f) was added. This equation addresses the risk neutral behavior of delaying all possible projects while stockpiling annual savings. Results without the measure, where there is no minimum to the amount of projects implemented in each year, are shown below.

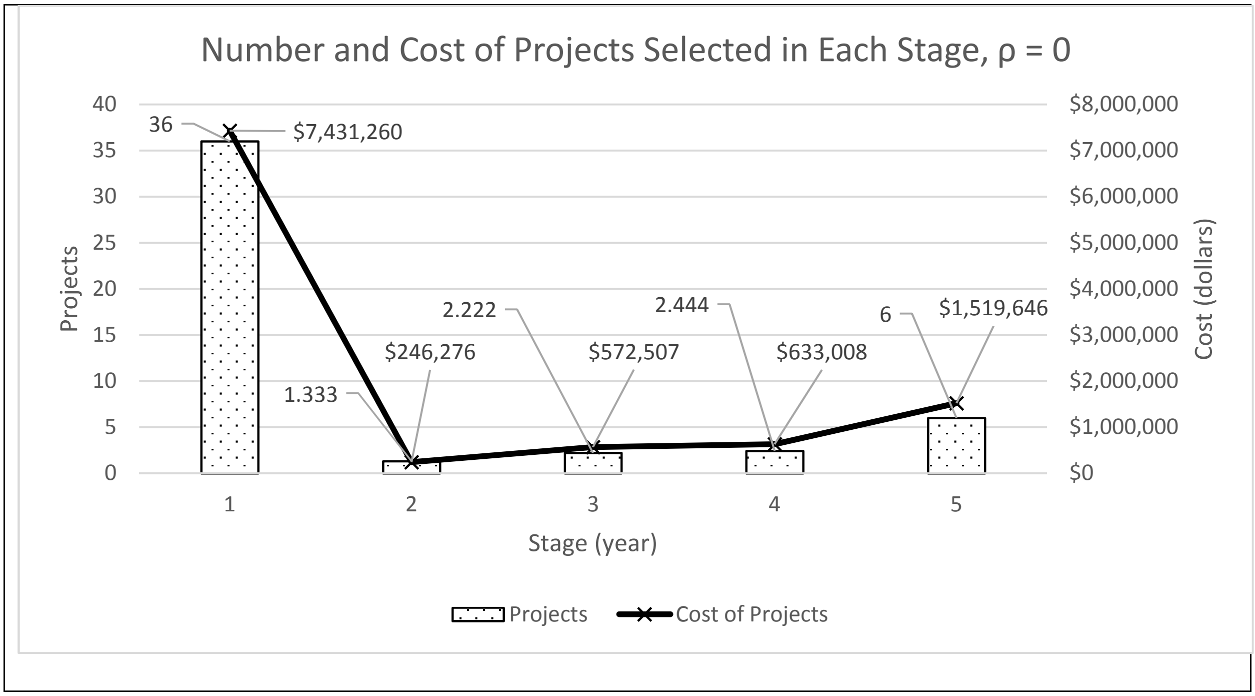


Figure 3-4, Project Selection by Stage without Risk Measure, ρ=0

The agency requests $7.4M capital and executes 36 projects for a cost of $7.4M. The agency then relies solely on annual savings and operating budgets over the next four stages. In the fifth stage, the agency will implement six projects (more projects than it has completed in the previous three stages). It may also my possible that the agency experiences a shortfall (the agency does not have enough budget to fund the remaining projects).

The annual minimum project model combats this concern. This model reduces risk by spreading projects throughout the horizon. The success of the overall energy program will not be as heavily weighted on the Stage 5 realizations. The agency will be aware much more quickly if a shortfall is experienced.

Results are shown below when the agency must complete at least five projects per year to reduce the risk of a delayed shortfall.

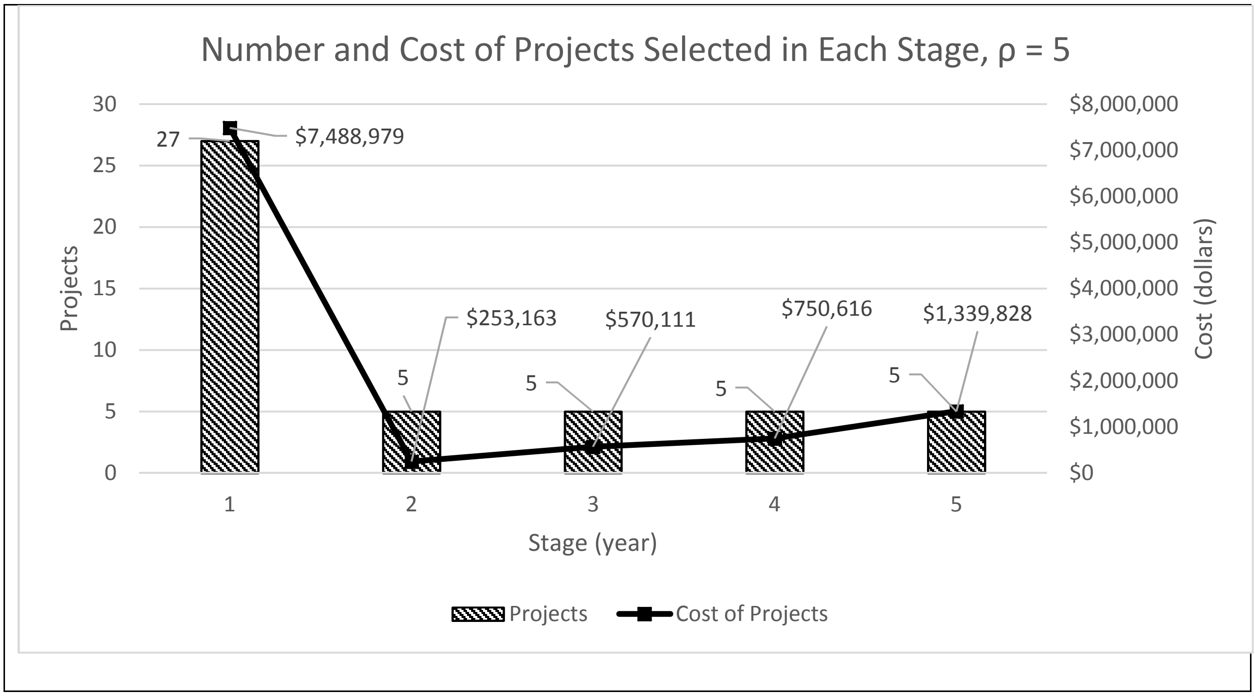


Figure 3-5, Project Selection by Stage with Five Project Minimum, ρ=5

This approach allows the Agency to minimize the risk of a shortfall in year 5, by spreading project throughout the time horizon.

In Figure 3-6, below illustrates an interesting result. Even the minimum projects are raised to ten for stages 2-5, the cost of project are preserved, that is, much higher cost are expending the first stage, followed by lower costs and increasing through the fifth stage.

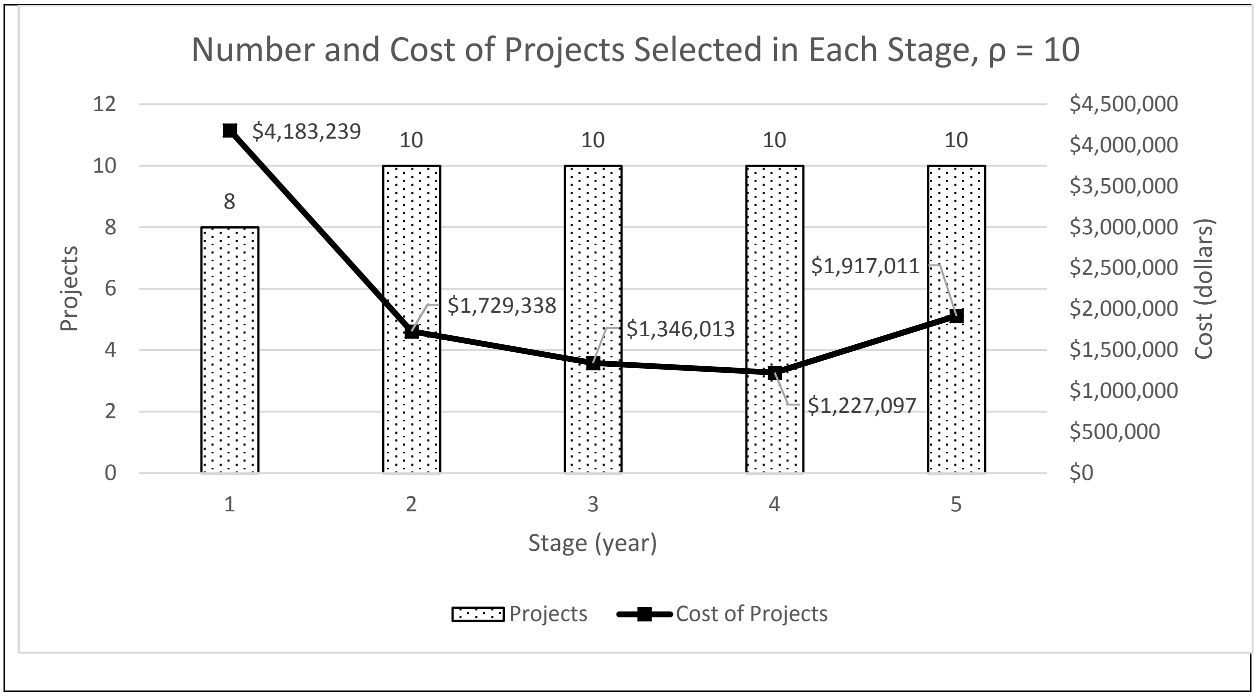


Figure 3-6, Project Selection by Stage with Minimum Number of Projects, ρ=10

The corresponding objective function (total cost to complete all projects), capital requesting and projects completing in first stage at several minimum projects are shown in figure 3-7, below.

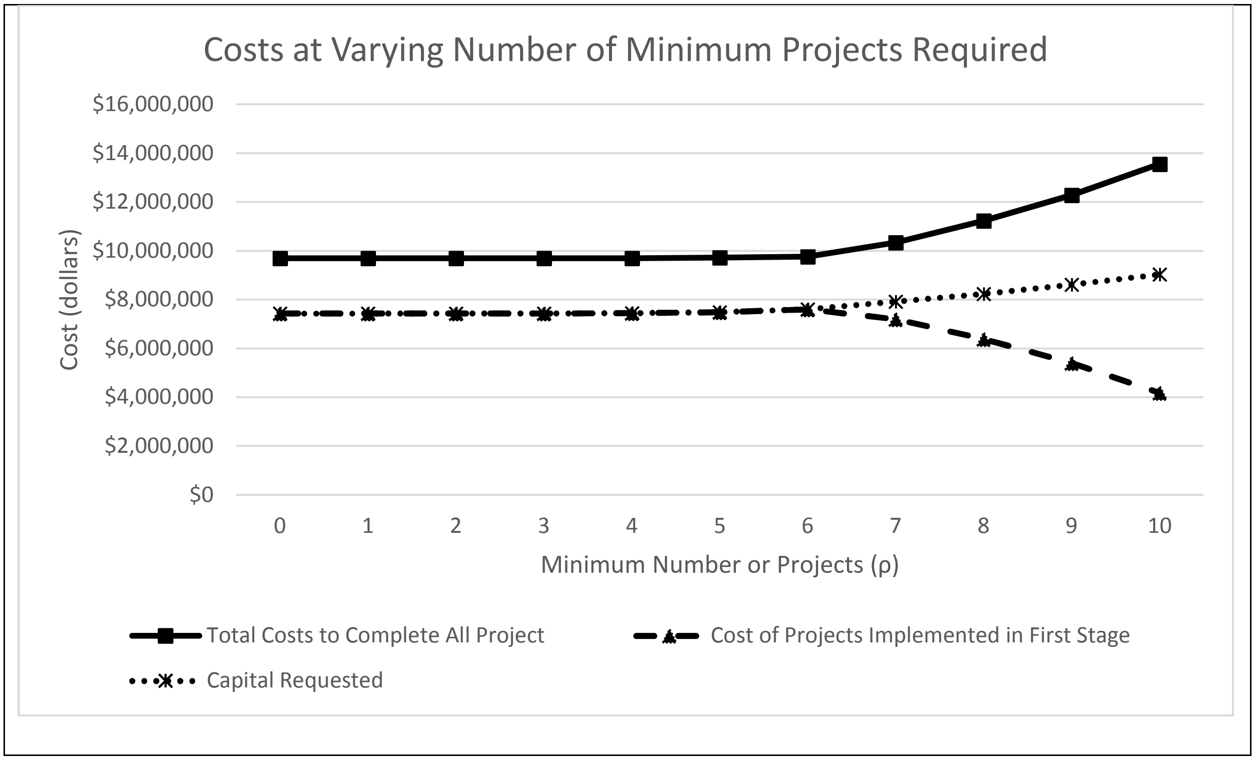


Figure 3-7, Cost at Varying Number of Minimum Projects Required

After the minimum number of projects is raised to six projects, the capital request exceeds the cost of projects complete in the first stage. It is at this level the agency must request capital to execute projects and save for projects that must be executed in later stage where annual savings will not suffice.

Using CVaR Provides Expected Results But May Yield A Time Inconsistent Policy

The CVaR model was run for comparison of risk measures.

The conditional Value-at-Risk can be written as

$$CVaR(A,x) = \max\left\{\eta - \frac{1}{1-\alpha}\ \varepsilon_\omega \max\{\eta - f(x,\omega), 0\}\right\}$$

$$\forall\ \alpha \in (0,1)$$

For a given α ∈ (0, 1), the conditional value-at-risk, CVaR, is defined as the expected value of the profit smaller than the (1 − α)-quantile of the profit distribution. If all profit scenarios are equiprobable, CVaR(α, x) is computed as the expected profit in the (1 − α) × 100% worst scenarios. This is also known as mean excess loss or average value-at-risk (Conejo et al, 2010).

Specifically in the CVaR model the objective function was written as:

$$\max_{x,y(\omega),\eta,s(\omega),B}(1-\beta)\left(-C - \sum_{p=1}^{n_P}\gamma_p \boldsymbol{x_p} - \sum_{\omega\in\Omega}\pi(\omega)\sum_{p=1}^{n_P}\sum_{t=2}^{n_T}\gamma_p \boldsymbol{y_p^t}(\omega)\right) - \beta\left(\eta - \frac{1}{1-\alpha}\sum_{\omega\in\Omega}\pi(\omega)\boldsymbol{s}(\omega)\right) \tag{3j}$$

and included constraint:

$$\eta - \left(-C - \sum_{p=1}^{n_P}\gamma_p \boldsymbol{x_p} - \sum_{\omega\in\Omega}\pi(\omega)\sum_{p=1}^{n_P}\sum_{j=2}^{n_J}\gamma_p \boldsymbol{y_p^j}(\omega)\right) \leq s(\omega)\ , \forall\omega \in \Omega, \forall t \in T \tag{3k}$$

$$s(\omega) \geq 0\ , \forall\ \omega \in \Omega \tag{3l}$$

The model derived from the above discussions, was programmed in GAMS Rev 23.6 using a 64-bit MS Windows machine and the XPRESS solver. The model included 15,521 single equations and 16,672 single variables with 15,552 binary variables. The optimality tolerance percentage (*optcr*) was set to 0%. The MIP model was solved using a minimization format and arrived at integer optimal solution in 2:12 (7,920 seconds) on average.

Resource usage is the amount of CPU time (in seconds) taken by the solver, as well as the time limit allowed for the solver. The *optcr* in this model is set at 0.00 which forces the application to solve to optimality. As such, the maximum CPU time (in seconds) was kept at the default value of 1,000. The iteration count and the iteration upper limit was set at 2,000,000,000. The results can be seen in Table 3-12, below.

The following results were given.

Beta	Solver Status	Model Status	Objective Value	CPU Time (seconds)	Relative Gap
0.0	Normal Completion	Optimal	9,663,282	357.164	
0.1	Normal Completion	Optimal	9,737,228	584.271	
0.2	Resource Interruption	Integer Solution	9,811,173	1000.512	0.000005
0.3	Normal Completion	Optimal	9,885,106	575.909	
0.4	Resource Interruption	Integer Solution	9,959,057	999.623	0.000002
0.5	Normal Completion	Optimal	10,032,997	584.271	
0.6	Resource Interruption	Integer Solution	10,106,946	1000.996	0.000006
0.7	Resource Interruption	Integer Solution	10,180,886	1000.684	0.000009
0.8	Normal Completion	Optimal	10,254,821	596.937	
0.9	Normal Completion	Optimal	10,328,760	480.701	
1.0	Normal Completion	Optimal	10,402,698	0.952	

Table 3-12: Risk-based Solve Summary

Minimizing only the Conditional Value-at-Risk (Beta=1.0), yields a higher capital outlay to complete all projects. The total budget required using this risk-averse approach is $10,402,698. The total cost needed to complete all projects increases between cost minimizing (expressed as maximization) Beta = 0.0 and Beta = 1.0 weightings as can be seen in table 3-11, above.

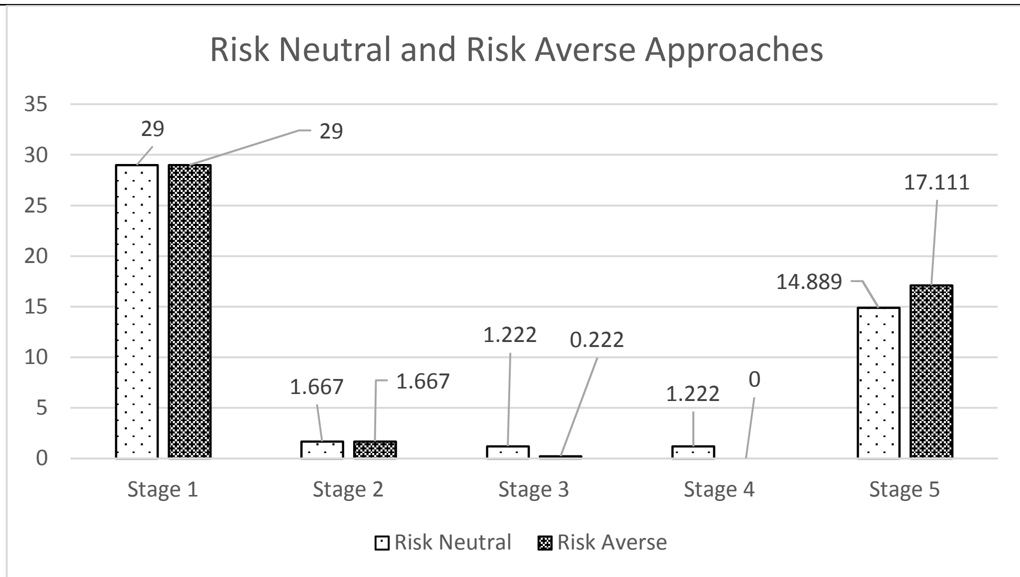


Figure 3-7: Number of Projects Implemented in Each Stage in Risk Neutral and Risk Averse Approaches

As expected, the number of projects selected for implementation generally increases in later years, beyond the initial time period, for both approaches. The key result is that risk (reducing risk by increasing the weight of the Conditional Value-at-Risk) requires a larger capital outlay and compounds the ability to fund future projects.

However, upon examination, the risk averse model appears to overestimate the risk. Evidence of this can be seen in the results of Table 3-13. The total investment cost of all projects are \$10,402,698. In the risk averse model, the objective function and the capital requested are both \$10,402,698. While this represents the most conservative approach possible, it completely discounts any annual savings. This result is similar to the traditional approach where the agency executes request the total investment cost of all projects immediately and possible to avoid the expected lower yields in future years. However, this model delays several projects and assumes savings to make up the balance. The gap between this model’s solution and an optimal one is at least \$850,000, the difference between total investment costs of all projects and all the annual savings at the lowest yield. The model returns an objective function that is at least 8% from optimal from a truly risk averse case.

It should be noted that number of projects appear similar, however; the projects selected vary greatly between the risk neutral approach and among all other approaches. This is illustrated in the table below.

	Traditional Agency Model: Projects Completed with Initial Capital Outlay	**Deterministic Approach: Projects Completed leveraging Expected Savings and Initial Capital Outlay**	**Stochastic Model: Projects Completed with Approach (Risk-neutral) with Recourse Savings and Initial Capital Outlay**	**Stochastic Model: Projects Completed with Approach (Risk-averse) with Recourse Savings and Initial Capital Outlay**
Total Cost of Projects	**\$10,402,698**	**\$9,644,413**	**\$9,663,282**	**\$10,402,698**
Initial Capital Outlay Needed	**\$10,402,698**	**\$4,986,524**	**\$5,885,967**	**\$6,990,197**
Initial Period Projects Selected	project1 project2 project3 project4 project5 project6 Project7 project8 project9 project10 project11 project12 project13 project14 project15 project16 project17 project18 project19 project20 project21 project22 Project23 project24 project25 project26 project27 project28 project29 project30 project31 project32 project33 project34 Project35 project36 project37 project38	project5 project6 project8 project10 project13 project15 project18 project22 project23 project25 project27 project28 project31 project33 project34 project35 project38 project39 project40 project41 project42 project43 project44 project48	project6 project7 project8 project10 project13 project15 project16 project17 project18 project21 project22 project23 project25 project27 project28 project31 project33 project34 project35 project37 project38 project39 project40 project41 project42 project43 project44 project47 project48	project1 project2 project3 project4 project9 project10 project11 project12 project14 project16 project17 project20 project21 project22 project24 project25 project26 project29 project30 project32 project33 project34 project36 project37 project38 project43 project44 project45 project47

	project39 project40 project41 project42 project43 project44 project45 project46 project47 project48			

Table 3-13: Agency's Traditional, Deterministic, Risk Neutral Approaches and Risk Averse but Time Inconsistent Approach

This suboptimal solution is most likely due to a time inconsistent policy. Time consistency is the requirement that that at every state of the system, the optimal decisions should not depend on scenarios which cannot happen in the future. This time consistency requirement is closely related to Bellman's principle used to derive dynamic programming equations (Shapiro, 2009). The standard risk neutral formulation of multistage stochastic programming problems satisfies this principle, however; in this case, the risk neutral formulation does not.

3.7. Conclusions

The option to choose all 48, any subset or no projects at all in the initial phase, gives the agency 248, over 281 trillion options at 5 phases, each with 3 possible realizations. The possibilities at each of the periods yield another billion scenarios in the tree after the initial capital outlay. It is important to note that the realizations of energy prices in the fifth year are not included in the model as there are no longer projects to fund in the fifth year.

The results of the practical application show that the value of the stochastic solution is limited by the number of constraints and recourse actions taken upon the realization of the random variable, which are energy prices and yields. The annual minimum project model provides positive results while allowing agencies to spread risk throughout the horizon rather than waiting for the Stage 5 where the lower saving projects are mostly to be encountered. If a shortfall is encountered, it will be earlier in the horizon. Meanwhile, in the CVaR model, risk aversion increases with Beta, and as a result, the capital outlay required increases. Risk-neutrality provides the lowest capital outlay but comes with a higher chance of a shortfall. A combination of taking on more risk with additional recourse actions may prove to be a more practical but complex model.

The traditional approach used by the agency requires that all projects be completed with the initial capital outlay. This does not allow the agency to accurately predict savings that could be used to fund future projects. However, the results of the deterministic model, is the key deterrent for agency's considering the use of future savings to fund projects. Using the attractive results of the deterministic model may leave the agency with a shortfall in later periods where capital budget cannot be requested. The agency is then forced to seek outside sources for project funding. This causes the agencies to assume risk-averse stances. This improvement provided by this work adds both stochasticity and allows the agency to select their risk tolerance.

These model proposed in the current work is preferred because savings can be used to fund additional programs while incorporating the seemingly random fluctuations in energy prices and incorporates proposed energy savings that may return lower estimates. It further expounds upon the advent of taking on uncertain outcomes with the inclusion a risk measure. The lower risk comes at higher costs.

In the practical applications presented in this chapter, the value of performing the optimization is compared to the agency's traditional approach by including the ability to leverage the existing savings and understanding the impact of the energy price and forecast of future savings. In this case, the optimized value to the agency is more realistic and superior to both the traditional and deterministic model.

Chapter 4: A Multistage Stochastic Energy Model with Rolling Horizons and Endogenous Learning

4.1. Introduction

The federal government buildings are one of the largest energy consumers in the world. In FY 2014, 39% of all federal energy was consumed by federal facilities. Energy consumed in federal government facilities has generally been declining over the past four decades. However; the reduction stems from both the total square footage occupied by the federal government, which continues to fall from its peak in FY 1987, and from the energy consumed per square foot inside federal buildings, which has been declining since FY 1975 (EERE, 2016). While significant reductions in building energy intensity have been made, many more are required, while tougher challenges exist in funding energy conservation and renewable projects. Facility energy intensity fell short of the 27% goals of Executive Order 13423 and Energy Independence and Security Act to reduce energy intensity (Btu/GSF) with only a 21% reduction (Tremper, 2014). The remaining conservation opportunities will require ingenuity to both fund and implement the projects. However, funding energy conservation continues to follow a lengthy multiple-year planning process.

There are many approaches to the implementation of an energy or renewable project but most comprehensive energy programs begin with an assessment of current consumption and energy conservation opportunities at the individual building level. The initial assessment is the ASHRAE Level 2 Energy Audit. The audit is an onsite assessment and comprehensive energy analysis of the building's energy-using components resulting in a list of proposed energy conservation measures (ECMs) which include the following attributes:

- the proposed system or component description
- an estimate of the investment required to implement the measure
- an estimate of annual savings
- an estimate of the annual cost savings in dollars
- a performance measure such as simple payback ratio or savings to investment ratio

A typical set of these measures are shown below in Table 1.

Project Description	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Savings ($)	Estimated Useful Life (years)	Payback Ratio (years)
Heating project	250,000	2,375,000	0.011	26,125.00	30	9.57
Lighting project	50,000	625,000	0.015	9,375.00	15	5.33

Table 4-1: Typical Energy Conservation Projects Attributes

The energy auditors determine the appropriate regulatory requirements as part of their scope of work in the contract with the agency. The energy auditors then conduct audits to recommend the projects necessary to save the required energy. Projects that do not meet specific savings-to-investment ratios are not considered. All reported projects must be completed. The agency requests a conservative budget from direct appropriated funding in the first stage and seeks to fund the required energy conservation projects. [18] (Note that a stage is a one-year time period in the current research.) agencies would greatly benefit from innovation and novel approaches to assist in project implementation, funding and timing.

The technical and financial performance of these projects are uncertain and often managed by a, “wait- -and-see” approach. Here we present more original approaches that request reasonable

[18] Financing energy projects through appropriations allows federal agencies to own their projects and immediately benefit from the cost savings. This type of financing should be an agency's first consideration in pursuit of its renewable energy goals given the hierarchy of action items in Executive Order 13693 (Obama, 2015).

budgets and allow for recourse actions. The savings from implemented projects are used for investment in future projects. However, anticipated energy savings, varying energy costs, and interaction between energy projects affect the ability of these models to predict future savings. A rolling-horizon model that updates the optimization model's inputs and optimal decision variable values for past stage is presented. This model is run in experimental cases showing its vast improvement over the fixed-horizon, multistage model. These improvements are:

- a reduction in the total number of stages required to implement all projects
- the total cost to implement all projects

the computational speed to solve a model with many decision and auxiliary variables

The remainder of this chapter is presented as follows. Section 2, discusses the current landscape project selection, stochastic optimization and rolling-horizon methodologies, as well as provides context and highlights novelty of the current research. Section 3 presents the model formulation and Section 4 applies the model to experimental yet practical examples. Sections 5 and 6 continue with discussion of the results and conclusions, respectively.

4.2. Literature and Context

A novel way to meet U. S. reduction and renewable goals is by using existing savings to fund future projects while accounting for uncertainty in implementation yields and energy prices. This requires selecting energy projects in a method that allow agencies to account for and reduce uncertainty associated with long planning horizons. An applicable method must address subadditivty and superadditivity of energy savings but be computationally solvable. Many of these concepts have been studied individually but this chapter considers them simultaneously resulting in an improved energy project selection model.

4.2.1. Project Selection

The goal of the current research is to develop a model that selects projects that optimizes the agency's value of the energy conservation program, minimizing the total cost of the program by maximizing the annual savings to fund additional projects. The model is developed to optimally select energy conservation projects and applied to project selection for energy conservation. In this approach, annual savings from projects selected in prior periods become investments in projects in future periods.

Markowitz illustrated that the process of portfolio selection, similar to project selection and thus relevant here, was based on available information and beliefs about the future performances of individual securities (projects) and their returns (Markowitz, 1952). In that work, the variance of expected return is minimized. By contrast, in the current research, the observations are made in the first stage while experience or realizations are made in later stages. The current research leverages annual savings in theses later stages from projects previously implemented which is analogous to the securities in Markowitz's work. The current problem also incorporates constraints on the cost of selecting projects, whereas the cost of the securities were not specifically limited in that earlier work.

Many approaches of this type of problem (a problem where projects must all "fit" into the program) have been studied however as applied to energy conservation, such research has to our knowledge not been overly active. The agency selection problem is related to the classical "knapsack problem". Dantzig described and demonstrated methods of solution to the knapsack problem (Dantzig, 1957). In this problem, for example, a person is planning a hike and has decided not to carry more than 70 lbs. of different items, such as a bedroll, Geiger counters, cans of food, etc. The hiker would like to maximize his / her benefit of these items while remaining below the weight limit. Dantzig noted that in these types of problems, extreme point solutions (to the corresponding linear program) might yield values that are neither one nor zero (which correspond to selection or omission of items). Since that original publication, the knapsack problem has become a classical formulation in operations research. A recent example of a project selection knapsack problem is (Gabriel, et al., 2006). In this paper, a multi-objective, integer-constrained optimization model with competing objectives for project selection was proposed in which probability distributions were used to describe uncertain costs. That model was novel since it integrated multi-objective optimization, Monte Carlo simulation, and the Analytic Hierarchy Process. The connection with the knapsack problem was that the budget for funding all the projects was the knapsack and the projects the items to go into the knapsack.

In (Asadia, et al., 2012) the authors present a multi-objective optimization model to assist stakeholders in the definition of measures aimed at minimizing the energy use in the building in a cost effective manner while satisfying the occupants' needs and requirements. However, the model described incorporates many subjective attributes, which make the quantification of value difficult. A multi-criteria knapsack model was proposed to help designers to select the most feasible renovation actions in the conceptual phase of a renovation project (Alanne, 2004).The additive knapsack model presented in that study was based on linear programming. The current research and the problem is much more complex. Gustafsson used a mixed-integer, linear programming (MILP) model to minimize the life-cycle cost of retrofits subject to minimum space heating requirements (Gustafsson, 1998). The author showed that a building's heating system could be described mathematically in the form of a MILP. The primary objective of the research here is energy savings with cost being a secondary consideration as well as a two-level optimization approach to model the ECM decision process more accurately. A two-level optimization approach is modeled in (Champion & Gabriel, 2015). However, in the current research, the budgets are funded by direct appropriation, which is best modeled by a single objective function. The above are just a small sample of some project selection papers that have relevance to the current work. For further details, see Models and Method for Project Management (Graves & Ringuest, 2003).

4.2.2. Stochastic Programming

When some aspects of the objective or constraints functions or other data in the problem are not known with certainty, stochastic optimization can be used (Vajda, 1972). Stochastic programming models can be of the recourse type where some here-and-now decisions are made at the current time period and other recourse (corrective) actions appear later (Birge & Louveaux, 1997). Alternatively, chance- constrained programs have no recourse but seek to optimize in the presence of probabilistic constraints. There are other variations of stochastic optimization such as stochastic dynamic programming, worse-case analysis, etc. (Puterman, 1994), (Birge & Louveaux, 1997).

Stochastic programming for project selection has been well-studied with early developments in (Dantzig, 1955), (Beale, 1955), and (Charnes & Cooper, 1959). In (Cano, et al., 2014), a decision supports system to manage energy sub-systems in a more robust energy-efficient and cost-effective manner is presented. In this paper, a two-stage stochastic model is proposed, where some first-stage decisions regarding investments in new energy technologies have to be taken before uncertainties are resolved. Later recourse (second-stage decisions) on how to use the installed technologies are taken once values for uncertain parameters become known, thereby providing a trade-off between long- and short-term decisions. Developments continue with application in many areas as production, supply chain, scheduling, gaming, financial modeling, telecommunications, and electricity (Ziemba & Wallace, 2005). The current work furthers applications in multistage stochastic programming concentrating on energy conservation project selection and building on the work in (Champion & Gabriel, 2015), (Yu, et al., 2003), (Shapiro, et al., 2009) .

4.2.3. Subadditivity and Superadditivity for Energy Conservation Measures

Subadditivity and superadditivity for energy conservation projects can be explained by the interactive effects of these projects in terms of energy savings or costs.[19] For example, an energy conservation project that retrofits lighting may decrease the electricity consumption but also reduce the heat gain to the building. This project, in return, makes the building's boiler work harder to provide the additional heat load to the building. The resulting savings of the projects together will be lower than if only one project was implemented. Using the typical set of these measures from Table 4-1, consider for example that the agency will implement projects expecting to spend the total cost of $300,000 and get repeating annual savings of $35,500.

[19] It should be noted that other factors such usage patterns and changes in rates based on time of day impact additivity. These are not modeled as this chapter as this is a strategic decision making model.

Project Description	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Savings ($)	Estimated Useful Life (years)	Payback Ratio (years)
Heating project	250,000	2,375,000	0.011	26,125	30	9.57
Lighting project	50,000	625,000	0.015	9,375	15	5.33
TOTAL Expected Additive Savings	**300,000**	**3,000,000**	**0.012**	**35,500**		**8.45**

Table 4-2: Additivity: No interactive effects on Energy Conservation Projects Attributes

However, in specific cases, the results of Table 4-2 overestimate the total annual savings. The agency may only realize annual repeating savings of $26,625 as shown in Table 4-3, below.

Project Description	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Savings ($)	Estimated Useful Life (years)	Payback Ratio (years)
Heating project	250,000	2,375,000	0.011	26,125	30	9.57
Lighting project	50,000	625,000	0.015	9,375	15	5.33
TOTAL Subadditive Savings	**300,000**	**2,250,000**	**0.012**	**26,625**		**11.27**

Table 4-3: Subadditivity: Interactive effects on Energy Conservation Projects Attributes

Similarly, superadditive effects of energy conservation projects are also possible. An example of this can be observed with the selection of an energy management system or controls projects in combination with higher-efficiency heat or cooling generation equipment. For example, a controls project has a long payback and, generally, is proposed for the existing generation equipment. A second project's scope could replace the existing generation equipment with newer, higher-efficiency equipment. The higher-efficiency heat or cooling generation equipment will use less energy when operating while the controls will minimize operating heating and cooling times based on demand for the load. The combined energy savings of these two projects will be greater than their individual savings. Table 4-4, provides an example of superadditivity in energy conservation project selection.

Project Description	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Savings ($)	Estimated Useful Life (years)	Payback Ratio (years)
Heating project	250,000	2,375,000	0.011	26,125	30	9.57
Controls project	100,000	50,000	0.015	750	15	133
TOTAL Subadditive Savings	**300,000**	**2,700,000**	**0.012**	32,400		**9.26**

Table 4-4: Superadditivity: Interactive effects on Energy Conservation Projects Attributes

In the current research, all projects are evaluated for subadditivity and superadditivity by comparing all combinations of projects selected in each stage. For example, Table 4-5, below gives attributes for four projects that are available for selection.

Project Description	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Savings ($)	Estimated Useful Life (years)	Payback Ratio (years)
Heating project	250,000	2,375,000	0.011	26,125	30	9.57
Lighting project	50,000	625,000	0.015	9,375	15	5.33
Insulation Project	150,000	125,000	0.09	11,250	50	13.33
Controls project	100,000	50,000	0.13	6,500	20	15.38

Table 4-5: Additivity: Conservation Projects Attributes for Simplifying Example Calculation

Table 4-6, below gives attributes and totals with no interactive effects for the three projects that are selected in this stage. Note that the total savings are simply the sum of savings for each project.

Project Description	Investment Cost ($)	Annual Savings ($)	$x_{project}$	Savings Generated ($)
			(=1 if selected)	(xproject * annual Savings)
Heating project	250,000	26,125	1	26,125
Lighting project	50,000	9,375	1	9,375
Insulation Project	150,000	11,250	0	0
Controls project	100,000	6,500	1	6,500
Additive Total				**42,000**

Table 4-6: Additivity of Projects Selected

Next consider possible subadditivity and super additivity for energy conservation annual savings. The comparison of each project for additivity is achieved by creating an alias for the set of projects, here called "project prime" or just "prime." Subadditivity or superadditivity is possible when both projects are selected or when $x_p = x_{p'} = 1$ for this stage. (Note, just two projects at a time are considered but one could imagine three or more relevative to subadditivity or superadditivity). The product of the pairwise comparison of the selection of one project and another becomes the binary variable for the possibility of subadditivity or superadditivity, with 0 turning off and 1, turning on. The combination of the product and the discount / premium ($K_{p,p'}$ matrix, below) determines subadditivity or superadditive effect on that project.

$$K_{p,p'} = \begin{bmatrix} 0 & .8 & 1 & 1.2 & .9 \\ .8 & 0 & .7 & 1 & 1 \\ 1 & .7 & 0 & 1.1 & 1 \\ 1.2 & 1 & 1.1 & 0 & 1 \\ .9 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Figure 4-1: Sample K matrix of Subadditive and Superadditive Multipliers

The sum of all "*Per Project Additivity*" is the total subadditivity or superadditivity for the projects selected in that stage. The total annual savings in this stage are \$43,306.25 (\$42,000 from Table 5 + \$1306.25 from Table 6) which is greater than the sum of the individual annual savings.

The key benefits of this approach are

- all projects are compared for the potential of subadditivity or subadditivity,
- subadditivity and superadditivity are addressed at the individual project level allow interactions to be additive with other projects,

- and this approach allows the comparison to be made and the discount and or premium to be calculated in each stage. Annual savings will become subadditive or superadditive as they are selected in later stages.

However, adding the interactive functionality to the model for the sake of realism, results in additional complexity. The complexity lies in Column J of Table 6, above. The product of variables makes the problem nonlinear and potentially harder to solve without some sort of exact linearization which is described next via the McCormick inequalities (McCormick, 1976).

4.2.4. McCormick Inequalities

Subadditivity of stochastic processes as discussed above are the key organizing principle driving problems of nearly intractable difficulty (Steele, 1997). Further solving models as proposed in the current research may not provide global optima with nonlinear models. McCormick developed a method for convex / concave relaxations of factorable functions that allow for vast improvements in goal finding and CPU speed to solve (McCormick, 1976). In some cases, like the one below, the linearization resulting from these inequalities is exact.

An alternative to McCormick's relaxation is McCormick's auxiliary variable method (AVM) that employs auxiliary variables for each factor involved. More precisely, instead of relaxing the functions, the nonconvex optimization problem is relaxed. The nonconvex problem is reformulated introducing auxiliary variables in such a way that the intrinsic functions are decoupled and can be relaxed one by one. A lower bound to the nonconvex problem is calculated via a relaxed NLP or linear program (Tsoukalas & Mitsos, 2014).

The current research leverages this AVM which addresses the problem of characterizing the convex envelope (the smallest convex set that covers a set of points) of the bilinear function as in (Sherali & Alameddine, 1990).[20] Specifically, $x_p \cdot x_{p'} \equiv V_{p,p'}$ which collapses the decision variables into a single variable allows multiplication by data and parameters, from Table 6, above. Additional constraints on the auxiliary variable, $V_{p,p'}$, establishing upper and lower bounds are discussed in Section 3.

4.2.5. Rolling-Horizon Approach

Budgeting for the entire planning horizon with perfect foresight can be overly optimistic. Perfect foresight assumes that the yields and budgets from savings are known for all stages, perfectly, when making the first-stage decisions. Models that assume perfect foresight of the time horizon have perfect information for the entire time horizon. In deterministic, perfect foresight models, parameters are assumed to be known with 100% certainty. Perfect foresight models while useful as base cases are less realistic than ones that allow for stochastic elements and/or some rolling-horizon foresight (Devine, et al., 2016).

In reality, energy project selection is often made under uncertainty with hedging of worst-case scenarios. Scenario-based models include non-anticipativity constraints (Birge and Louveaux, 1997) to ensure that the worst yields and the budgets scenarios are included and observed to be the same at all successor nodes. In the rolling-horizon approach which is more realistic than

---

[20] A bilinear function is a function of two variables that are linear with respect to each other, for example $f(x, y) = xy$.

perfect foresight, decisions are taken one stage at a time, realizing and possibly updating the parameters between these stages.

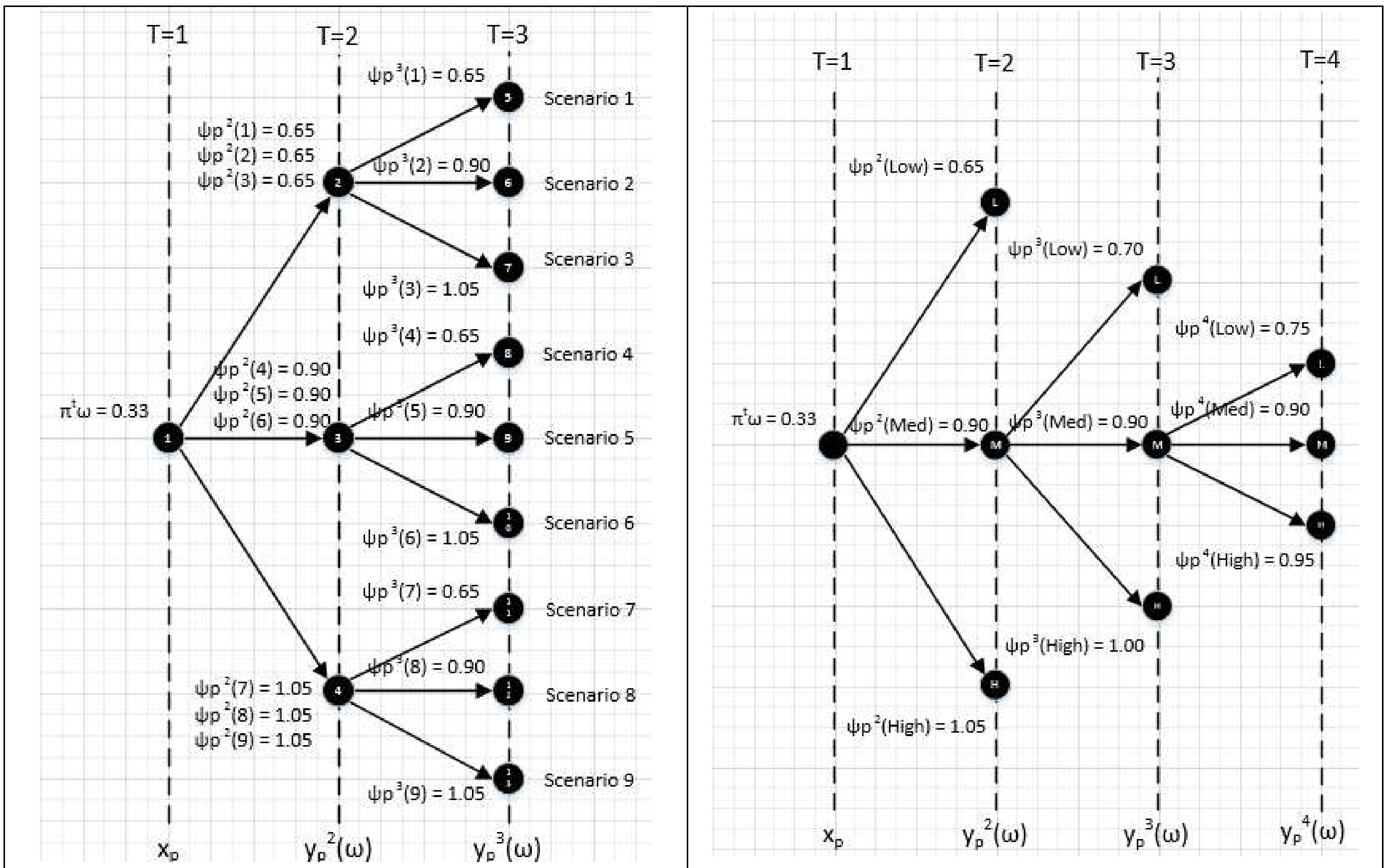
In a multistage problem, decisions made in the current stage influence the recourse decisions made in later stages. Rolling-horizon approaches solve multistage problems with a planning horizon of $n_T = |T|$, by looking at smaller rolling-horizons, $\bar{H}$ that models subsets of the full problem. Rolling-horizon models have been considered since at least (Baker, 1977). That experimental study was designed to investigate the efficiency of decisions obtained from optimizing a finite, multistage model and implementing those decisions on a rolling basis. The results of the study suggest that rolling schedules are quite efficient.

The typical practice with a rolling-horizon policy calls for establishing the “master schedule” for a certain number of future stages, known as the planning horizon, based on the currently available relevant information e.g. demand forecasts, available capacity, inventory and backlog records, etc. (As’ad & Demirli, 2010) , (Sethi & Sorger, 1991). This terminology is used in the current research. In the current research, the demand is established in the first stage by the auditor after a review of the applicable regulations. This is a one-time activity for the program and as such is not continually forecasted. Further, in this research, the model is updated as more data (project performance) and variables (budgets) become available.

A rolling-horizon approach as presented below only considers a smaller future set of stages and allows for learning in between each "roll" of the horizon. As such, the approach can be computationally quicker as well as more realistic. Additionally, such a rolling-horizon approach also allows for learning (in between rolls) for the decision-maker and thus can be used to model "endogenous probabilities". Endogenous uncertainty problems are described as discrete event dynamic systems where the underlying stochastic process depends on the optimization decisions (Pflug, 1990). Thus, for example, a scenario tree with probability p for one of the nodes really depends on the values of the optimal decision variables. As an example, consider (Dupacova, 2014), that describes project selection with endogenous variables for exploration of new oil fields. The possibility of investment for these projects may be initiated in each stage. The probability distributions of the uncertain characteristics of projects are discrete, within each scenario, but the capacity and delivery are realized only after the optimal decisions are made. In the current research, projects are also selected in each stage but the endogenous variables affect the returns determines future project selection similar to (Dupacova, 2014).

Stochastic programming models can be classified into two broad categories (Jonsbraten, 1998): exogenous uncertainty where stochastic processes are independent of decisions that are taken (e.g. demands, prices), and endogenous uncertainty where stochastic processes are affected by these decisions. Decisions can affect the stochastic processes by altering the probability distributions (type 1) or determining the timing when uncertainties in the parameters are resolved (type 2) (Goel & Grossman, 2006). A number of planning problems that involve very large investments at an early stage have endogenous (technical) uncertainty (type 2) that dominates the exogenous uncertainty (Gupta & Grossman, 2014). In the current research, the endogenous uncertainty is modeled because decisions regarding timing of projects selected severely impact the overall objective through the realization of the yields.

Devine, et al. (2016) present improvements that come with rolling-horizons for mixed complementarity problems (MCP) in the context of natural gas market equilibria (Devine, et al., 2016). For example, one advantage is that each roll is a separate solving of an MCP, which allows the opportunity to adjust inputs in between these rolls. For example, a new scenario tree for the next roll can be endogenously changed, by one or more players, based on a solution from the previous roll so that the model has endogenous probabilities. That novel approach is adopted in this research but is applied to multistage stochastic programming rather than MCPs.



a) Multistage stochastic model with all scenarios	b) Rolling-horizon model with fixed improvemnt

Figure 4-2: Comparison on Multistage Stochastic Program and of Rolling Horizon Approach

Figure 4-2, above illustrates the comparison between the models presented in the current research. In Figure 4-2a, the multistage stochastic model shows the probability of each scenario(ω), $\pi^t(\omega)$, and yield, $\psi_p^t(\omega)$. The yield is a low, medium or high return coefficient related to the estimated annual savings. In the multistage stochastic model, all decisions are made with information known in this first stage. At every stage beyond the first, the agency has a set of fixed recourse decisions, with fixed probability and yields. This results in three possible nodes at $t = 2$ and nine nodes at $t = 3$. The model also contains non-anticipativity, which does not allow the agency to anticipate what node they will arrive at before recourse actions are taken. The improvement in Figure 2b, comes from realizing the node from which to start the next roll and update with endogenous learning. In Figure, 4-2b, the agency makes a decision for the planning horizon, here four years, however, after the first year, then realizes the actual node, $\pi^t(\omega)$, and yield, $\psi_p^t(\omega)$. However, the model assesses its position in the tree, here, $\boldsymbol{y_p^t}(med)$ taken as a representative example because it represents the expected value. This results in three possible nodes at $t = 2$ and only three nodes at $t = 3$ and so forth. At each stage, the agency reruns the model assuming a horizon of $\overline{H}$. The resulting treatment of the $\pi^t(\omega)$ for each model are illustrated in Table 4-7, below. Note that the endogenous learning in model presented in Figure 4-2b, also updates the yields, $\psi_p^t(\omega)$ in each stage per Table 4-7, below.

	T=2		**T=3**		**T=4**
Figure 2a, Multistage Stochastic Model	$\pi^2(1) = 0.33$ $\pi^2(2) = 0.33$ $\pi^2(3) = 0.33$	$\psi_p^2(1) = 0.65$ $\psi_p^2(2) = 0.90$ $\psi_p^2(3) = 1.05$	$\pi^3(1) = 0.33$ $\pi^3(2) = 0.33$ $\pi^3(3) = 0.33$	$\psi_p^3(1) = 0.65$ $\psi_p^3(2) = 0.65$ $\psi_p^3(3) = 0.65$	Not Shown in Figure
	$\pi^2(4) = 0.33$ $\pi^2(5) = 0.33$ $\pi^2(6) = 0.33$	$\pi^2(4) = 0.33$ $\pi^2(5) = 0.33$ $\pi^2(6) = 0.33$	$\pi^2(4) = 0.33$ $\pi^2(5) = 0.33$ $\pi^2(6) = 0.33$	$\psi_p^3(4) = 0.90$ $\psi_p^3(5) = 0.90$ $\psi_p^3(6) = 0.90$	
	$\pi^2(7) = 0.33$ $\pi^2(8) = 0.33$	$\pi^2(7) = 0.33$ $\pi^2(8) = 0.33$	$\pi^3(7) = 0.33$ $\pi^3(8) = 0.33$	$\psi_p^3(7) = 1.05$ $\psi_p^3(8) = 1.05$	

	$\pi^2(9) = 0.33$	$\pi^2(9) = 0.33$	$\pi^3(9) = 0.33$	$\psi_p^3(9) = 1.05$		
Figure 2b, Rolling-horizon model	$\pi^2(low) = 0.33$ $\pi^2(med) = 0.33$ $\pi^2(high) = 0.33$	$\psi_p^2(low) = 0.65$ $\psi_p^2(med) = 0.90$ $\psi_p^2(high) = 1.05$	$\pi^3(low) = 0.33$ $\pi^3(med) = 0.33$ $\pi^3(high) = 0.33$	$\psi_p^3(low) = 0.70$ $\psi_p^3(med) = 0.90$ $\psi_p^3(high) = 1.00$	$\pi^4(low) = 0.33$ $\pi^4(med) = 0.33$ $\pi^4(high) = 0.33$	$\psi_p^4(low) = 0.75$ $\psi_p^4(med) = 0.90$ $\psi_p^4(high) = 0.95$

Table 4-7: Probability and Yields by Model Type

4.3. Model

The value of the current work is the novel application and combination of several concepts such as multistage stochastic programming and subadditivity and superadditivity of energy conservation projects using McCormick Inequalities (McCormick, 1976) at several stages to improve on the current industry practice. The agency seeks to minimize the total cost of implementing all the energy conservation projects that it is considering. The inclusion of the four key concepts are discussed later in this section.

The following is the notation, variables, and parameters used in the general statement of the stochastic multistage energy conservation model (SM-ECM).

Sets

$\boldsymbol{P}$ set of ECM projects with P = {1, 2,…n_p} where n_p = |P|, P' ≡ P
$\boldsymbol{T}$ a set of stages T (typically years) = {2,…n_T} where n_T = |T|, T' ≡ T
$\boldsymbol{\omega}$ set of scenarios with given probability, $\pi^t(\omega)$, ω = {1, 2,…n_ω} where n_ω = |Ω|

Main Primal Decision Variables

$\boldsymbol{x_p}$ a first-stage binary variable representing selection of the project p; variable = 1, if selected by the agency to be implemented at t=1, = 0 otherwise
$\boldsymbol{y_p^t}(\omega)$ a t-stage (T = {2,…n_T}) binary recourse variable representing selection of the projects for scenario (ω); variable = 1, if selected by the agency to be implemented in stage T = {2,…n_T} , = 0 otherwise

Intermediate Variables

$\boldsymbol{B^t}(\omega)$ the budget in dollars for implementing the agency's projects at stage t > 0
$\boldsymbol{V_{p,p'}}$ McCormick envelope auxiliary variable for initial stage variables $\mathbf{x}_p$ and $\mathbf{x}_{p'}$ $\boldsymbol{V_{p,p'}} = \boldsymbol{x_p x_{p'}}$
$\boldsymbol{W_p^t}(\omega)$ McCormick envelope auxiliary variable for first- and recourse-stage variables ($\mathbf{x}_p$ + $\mathbf{y}_p^t(\omega)$) and ($\mathbf{x}_{p'}$ + $\mathbf{y}_{p'}^t(\omega)$)

Parameters

C A scalar representing the capital budget requested through direct appropriation by the agency for implementing the agency's projects in the first stage, in dollars ($)
O^t the operating budget prescribed for the agency at stage t, in dollars ($)
θ_p the estimated annual savings in dollars achieved by implementing project p, the energy savings, in dollars ($) which is equal to the product of annual energy savings (KBTU) and energy rate ($/KBTU)
γ_p the estimated investment needed to implement project p in dollars ($)
$\psi_p^t(\omega)$ the yield of the annual savings at each stage as realized through each project's annual savings
$\pi^t(\omega)$ the probability of the discrete energy price at each stage t

$K_{p,p'}$ an n_p x n_p matrix for pairwise comparison and multipliers of yield

General Formulation

The formulation of the stochastic multistage energy conservation model (SM-ECM) is as follows. The objective function minimizes the total cost to complete all energy conservation measures. The objection function is composed of the following terms:

C : The capital budget requested through direct appropriation by the agency for implementing the agency's projects for the entire planning horizon, in dollars ($) (4a)

$$\sum_{p=1}^{n_P} \gamma_p \boldsymbol{x_p}$$: The estimated first-stage investment cost to implement the agency-selected projects (4b)

$$\sum_{\omega \in \Omega} \pi^t(\omega) \sum_{p=1}^{n_P} \sum_{t=2}^{n_T} \gamma_p \boldsymbol{y_p^t}(\omega)$$: The estimated second- and later-stage investment cost to implement the agency-selected projects (4c)

The agency, by choosing the timing of when it undertakes each energy conservation project, p, is trying to minimize the sum of these three terms. The terms are costs in the problem solved when the objective function is maximized and, as such, are negated when presented in the minimization form of the problem as shown below.

$$-C - \sum_{p=1}^{n_P} \gamma_p \boldsymbol{x_p} - \sum_{\omega \in \Omega} \pi^t(\omega) \sum_{p=1}^{n_P} \sum_{t=2}^{n_T} \gamma_p \boldsymbol{y_p^t}(\omega) \tag{4d}$$

The first constraint faced by the agency is that all projects must be selected.

$$\boldsymbol{x_p} + \sum_{\boldsymbol{t=2}}^{\boldsymbol{n_T}} \boldsymbol{y_p^t}(\omega) = 1 \ \forall p \in \mathrm{P}, \forall\, \omega \in \Omega \tag{4e}$$

The second constraint states that the nonnegative available budget at the time, t, for scenario, ω, $B^t(\omega)$ is the sum of:

$$\sum_{p=1}^{n_P} \sum_{t'=2}^{t-1} \psi_p^t(\omega)\, \theta_p \left(\boldsymbol{x_p} + \boldsymbol{y_p^{t'}}(\omega)\right)$$: The estimated savings for energy conservation measures from all prior stagess in dollars ($) (4f)

O^t : The operating budget in dollars ($) (4g)

$$B^{t-1}(\omega)$$: The budget from the previous stage in dollars ($) (4h)

$$-\sum_{p=1}^{n_P} \gamma_p \boldsymbol{y}_{\boldsymbol{p}}^{\boldsymbol{t-1}}(\omega) \;\; \forall t \in T, \forall\, \omega \in \Omega$$: the cost of projects implemented in the prior stage in dollars ($) (4i)

$$-\sum_{p=1}^{n_p}\sum_{p'=1}^{n_{p'}}\sum_{t\prime=2}^{t-1} \mathrm{K}_{p,p'} \cdot \psi_p^t(\omega)\, \theta_p \left[\left(\boldsymbol{x}_{\boldsymbol{p}} + \boldsymbol{y}_{\boldsymbol{p}}^{\boldsymbol{t\prime}}(\omega)\right)\left(\boldsymbol{x}_{\boldsymbol{p'}} + \boldsymbol{y}_{\boldsymbol{p'}}^{\boldsymbol{t\prime}}(\omega)\right)\right]\left(\boldsymbol{x}_{\boldsymbol{p}} + \boldsymbol{y}_{\boldsymbol{p}}^{\boldsymbol{t\prime}}(\omega)\right)$$: The subadditivity or superadditivity of the product of energy savings and rates of all projects chosen in dollars ($) (4j)

The following constraints state that the investment in any stage must below within the budget. The sum of all projects starting at t=2 must not exceed the budget.

$$\sum_{p=1}^{n_p} \gamma_p \boldsymbol{y}_{\boldsymbol{p}}^{\boldsymbol{t}}(\omega) \leq\; B^t(\omega) \qquad \forall t \in T, \forall\, \omega \in \Omega \tag{4k}$$

The following constraints enforce non-anticipativity of the model (Rockafellar & Wets, 1976).

$$\begin{gathered}
\boldsymbol{y}_{\boldsymbol{p}}^{\boldsymbol{t}}(\omega) \;=\; \boldsymbol{y}_{\boldsymbol{p}}^{\boldsymbol{t}}(\omega') \quad \forall\, \omega, \omega' \;\in\; \{1 \ldots \Omega\} \\
B^t(\omega) \;=\; B^t(\omega') \;\; \forall\, \omega, \omega' \;\in\; \{1 \ldots \Omega\} \\
B^t(\omega) \;=\; B^t(\omega') \;\; \forall\, \omega, \omega' \; for\ which\ \xi^t(\omega) = \xi^t(\omega'), t = 2 \ldots T \\
\boldsymbol{y}_p^t(\omega) \;=\; \boldsymbol{y}_p^t(\omega') \;\; \forall\, \omega, \omega' \; for\ which\ \xi^t(\omega) = \xi^t(\omega'), t = 2 \ldots T \\
where\ \xi^t(\omega)\;\; is\ the\ node\ at\ stage, t\ and\ scenario, \omega
\end{gathered} \tag{4l}$$

For equation 3j, the values of the decision variable $y_p^t(\omega)$ and variable $B^t(\omega)$ chosen at stage t, depend on the data ξ^tavailable up to time t, but not future observations. This is the basic requirement of non-anticipativity (Wets, 1974).

The budget must remain nonnegative (no loans).

$$B^t(\omega) \geq 0 \quad \forall t \in T, \forall\, \omega \in \Omega \tag{4m}$$

The initial budget is zero.

$$\begin{aligned} B^0(\omega) &= 0 \\ B^1(\omega) &= C \end{aligned} \tag{4n}$$

The decision variables are binary.

$$\boldsymbol{x_p}\,, \boldsymbol{y_p^t}(\omega) \;\; binary \tag{4o}$$

As discussed in Section 4-2, the model adds elements making this approach innovative yet practical for application.

4.3.1. Subadditivity and Superadditivity

The model above is modified to include the possibility of subadditivity or superadditivity. It begins by noting that the key impact is to the annual savings in the example of Table 4-2. In the model, the annual savings are repeating every year after project implementation (selections by $\boldsymbol{x_p}\ or\ \boldsymbol{y_p^t}(\omega) = 1$) and formulated in the budget equation as term (3j).

The term in (4j), above determines the budget by multiplying the yield $\psi_p^t(\omega)$ by the estimated annual saving θ_p if the project is chosen $\left(\boldsymbol{x_p} + \boldsymbol{y_p^j}(\omega)\right) = 1$. However, for subadditivity and superadditivity, we compare two projects p and p' at a time. Letting $K_{p,p'}$ be a $n_p\ by\ n_{p'}$ matrix of pairwise multipliers for yields, $\psi_p^t(\omega)$ equal to the energy annuals saving interactive effect.

The addition of subadditivity (as opposed to super-additivity) best models the practical approach to energy conservation measures. However, this change results in a mixed integer non-linear program (MINLP) which is computationally much more challenging and thus less likely that agencies will use it to find a global optimal solution. By contrast, the (exact) linearization via the McCormick inequalities shown below, is a computational tool that makes solving such problems easier.

4.3.2. McCormick Envelopes

In order to transform the nonlinearities introduced by the subadditivity of the model, we apply the auxiliary variable model (McCormick, 1976). This is achieved by letting $a = \left(\boldsymbol{x_p} + \boldsymbol{y_p^{t\prime}}(\omega)\right)$ and letting $b = \left(\boldsymbol{x_{p\prime}} + \boldsymbol{y_{p\prime}^{t\prime}}(\omega)\right)$ as discussed in Section 2 where p and p' are indices for two distinct projects.

The linearization of the product of the terms a and b is given as follows. First, let W=ab and note that W=1 if and only if a=1 and b=1.

Consider the following McCormick inequality constraints:

Let $W = a^2 b$ where

$$
\begin{aligned}
W &\geq 0 \\
W &\geq b + a - 1 \\
W &\leq b \\
W &\leq a
\end{aligned}
\qquad (4p)
$$

Note that in (4p) if a=1 and b=1 then the second inequality forces W to be greater than or equal to 1. The last two inequality provide 1 as an upper bound for W so taken together imply that W=1. Conversely, if W=0 then the second inequality shows at most one of a or b can be equal to

1 (other an infeasibility). The last two inequalities are still valid in this case. Thus, the nonlinear equation of W=ab has been exactly linearized in (4p).

The budget equation below replaces terms (4f) – (4j)

$$B^t(\omega) = \sum_{p=1}^{n_P}\sum_{t'=2}^{t-1} \psi_p^t(\omega)\,\theta_p \left(\boldsymbol{x_p} + \boldsymbol{y_p^j}(\omega)\right) + O^t + B^{t-1}(\omega)$$
$$- \sum_{p=1}^{n_P} \gamma_p \boldsymbol{y_p^{t-1}}(\omega) - \sum_{p=1}^{n_p}\sum_{p'=1}^{n_{p'}}\sum_{t'=2}^{t-1} \mathrm{K}_{p,p'} \cdot \psi_p^t(\omega)\,\theta_p \cdot W_p^t(\omega) \quad \forall t \in T, \forall\, \omega \in \Omega \tag{4q}$$

The following additional constraints are added.

$$\begin{gathered} W_p^t(\omega) \ \geq 0 \\ W_p^t(\omega) \geq \left(\boldsymbol{x_p} + \boldsymbol{y_p^t}(\omega)\right) + \left(\boldsymbol{x_{p'}} + \boldsymbol{y_{p'}^t}(\omega)\right) - 1 \\ W_{p'}^t(\omega) \leq \left(\boldsymbol{x_{p'}} + \boldsymbol{y_{p'}^{t'}}(\omega)\right) \\ W_p^t(\omega) \leq \left(\boldsymbol{x_p} + \boldsymbol{y_p^{t'}}(\omega)\right) \end{gathered} \tag{4r}$$

The terms and equations above complete the model in its entirety.

4.3.3. Rolling-Horizon

The rolling-horizon method involves making first-stage decisions, based on a stochastic forecast/estimation. At the beginning of the second stage, the first-stage decisions are apparent. In order to make these decisions, forecasts for additional stages into the future are required. In addition, existing forecasts can be revised or updated. This procedure repeats for every stage justifying the term rolling-horizon decision making for the practice. Here, the term "horizon" refers to the number of stages in the future for which the forecast is made. It is this horizon, that is "rolled over" each stage (Sethi & Sorger, 1991).

In this model and chapter the endogenous learning is applied as such.

$$\psi_p^t(low) = \begin{cases} \psi_p^{t-1}(low) * 1.00, & 0 \leq y_p^{t-1}(low) < 1 \\ \psi_p^{t-1}(low) * 1.04, & 1 \leq y_p^{t-1}(low) < 3 \\ \psi_p^{t-1}(low) * 1.06, & 3 \leq y_p^{t-1}(low) \end{cases} \tag{4s}$$

$$\psi_p^t(med) = \begin{cases} \psi_p^{t-1}(med) * 1.00, & 0 \leq y_p^{t-1}(med) < 1 \\ \psi_p^{t-1}(med) * 1.02, & 1 \leq y_p^{t-1}(med) < 3 \\ \psi_p^{t-1}(med) * 1.04, & 3 \leq y_p^{t-1}(med) \end{cases} \tag{4t}$$

$$\psi_p^t(high) = \begin{cases} \psi_p^{t-1}(high) * 0.98, & 0 \leq y_p^{t-1}(high) < 1 \\ \psi_p^{t-1}(high) * 1.00, & 1 \leq y_p^{t-1}(high) < 3 \\ \psi_p^{t-1}(high) * 1.01, & 3 \leq y_p^{t-1}(high) \end{cases} \tag{4u}$$

$$\begin{aligned} \psi_p^0(high) &= 0.00 \\ \psi_p^1(high) &= 0.65 \\ \psi_p^1(high) &= 0.90 \\ \psi_p^1(high) &= 1.05 \end{aligned} \tag{4v}$$

The energy conservation model described above is run as follows:

1. The rolling horizon $\overline{H}$ for a subset of the total time periods is specified.
2. The $\overline{H}-$ year model is run
3. At the end of year 1, first-stage decisions become input parameters for stage 2.[21]
    a. The budget is reduced by the cost of projects implemented in the previous phase.
    b. The budget is increased based on annual savings realized in year 1.
    c. The endogenous learning adjustment is applied to yields, $\psi_p^2(\omega)$ per the learning above.

[21] Recall that a stage is a one-year time period in the current research.

d. The probability of the yields, $\pi^2(\omega)$ can also be adjusted. In these cases, the uniform distribution will remain (there is an equal likelihood of all discrete yields).

4. The $\overline{H} -$ year 2, the model is rerun
   a. the budget is reduced by the of cost of project in the previous stage
   b. budget is increased based on annual savings realized in stage 1 and stage 2
   c. The endogenous learning adjustment to yields, $\psi_p^3(\omega)$
   d. The probability of the yields, $\pi^3(\omega)$ can also be adjusted. The uniform distribution will remain.
5. Repeat until the end of year $n_T - \overline{H}$ where the final $\overline{H}$ - year model is run or all projects are complete.

A flow chart for a 4-year rolling-horizon is shown below in Figure 4-3.

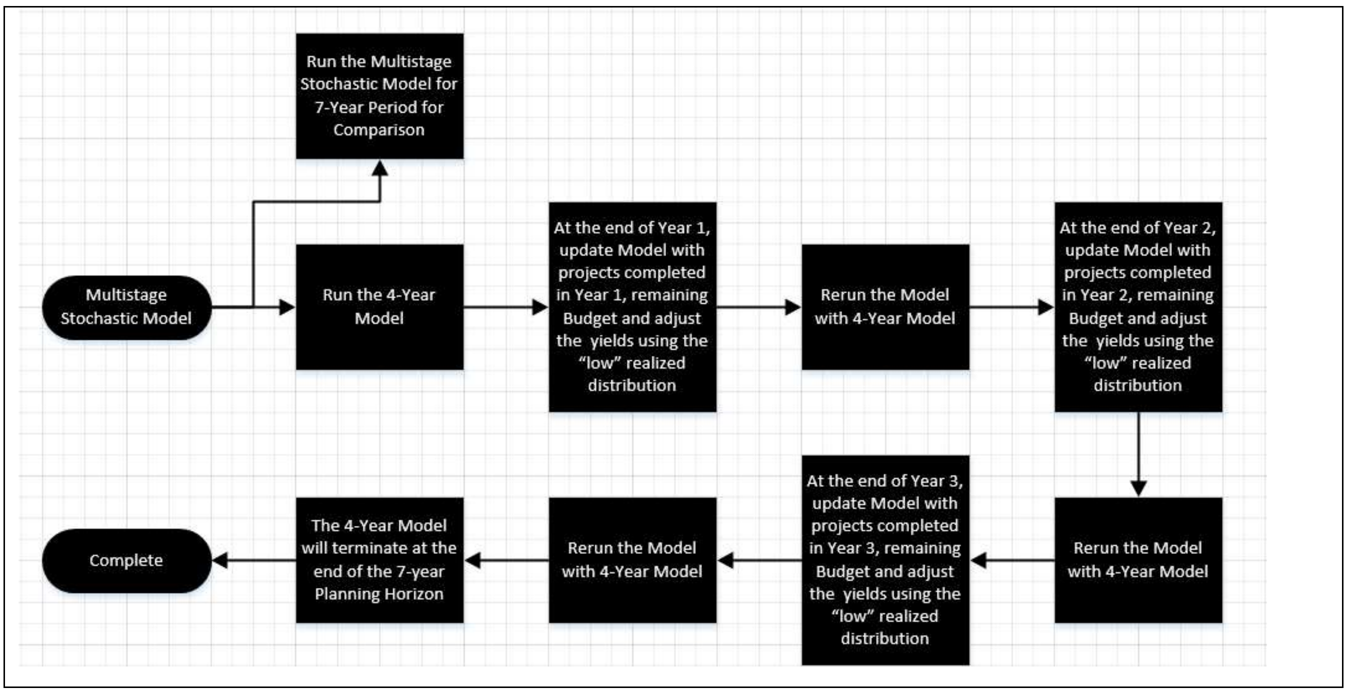


Figure 4-3: Rolling-Horizon Approach with Update Rules

4.4. <u>Experimental Example</u>

The model described above seeks the objective of selecting the lowest-cost energy program (all projects must be completed). The lowest-cost program will make the most efficient use of the annual savings realized by implementing projects in prior stages. A practical application of the model is demonstrated using data from an agency's campus of buildings in the southeastern United States (Champion & Gabriel, 2015).

In the numerical example, there are 48 ECMs with varying characteristics and project attributes as shown in Table D-1 of the Appendices. The model presented earlier is applied to these data as follows.

n_p =|48|, the total number of ECMs

ζ_p Energy Rate in \$/KBTU as shown in the third column of Table D-3 in the Appendices

θ_p^t Annual Cost of Energy Saved in dollars (\$) as shown in the fourth column of Table D-3 in the Apendices

$\psi_p^t(\omega)$ the static annual savings fluctuation at each stage is realized through each project's annual savings as shown in Table 4-8, below.

	Probability, $\pi^t(\omega)$	Rate / Yield Factor, $\psi_p^t(\omega)$[22]
Scenario 1 (ω_1)	0.33	0.65
Scenario 2 (ω_2)	0.33	0.90
Scenario 3 (ω_3)	0.33	1.05

Table 4-8: Probability of Annual Saving Fluctuation based Energy Price and Savings Yield at t=1

Endogenous learning (updates) were modeled using three possibilities for the distribution of yields. The "medium" of the low, medium and high discrete distribution (Probability $\pi^t(\omega)$) was kept constant in all cases. The details of the endogenous learning (Rate / Yield Factor changes) are shown in Table D-2 of the Appendices.

4.4.1. Multistage Results

[22] The Rate / Yield Factor (x Annual Savings), $\psi_p^t(\omega)$ are arbitrary for illustrative purposes.

The Multistage Stochastic 7-year MIP model was run for a 48-project model in GAMS Rev 23.6.5 on an x86 64bit Microsoft Windows machine. This model includes the subadditivity of the energy conservation savings in equation (4j), and the McCormick envelopes for variables $\boldsymbol{x_p}$ and $\boldsymbol{y_p^t}(\omega)$ in (4r). The reported model statistics are:

Blocks of Equations	6,618	Single Equations	2,895,041
Blocks of Variables	36	Single Variables	2,155,824
Non Zero Elements	9,994,223	Discrete Variables	210,000

Table 4-9, Multistage Stochastic Energy Program Results at the Planning Horizon

The resulting solver status was 1, "Normal Completion" with a model status of 8, "Integer Solution." The Resource Usage was 540.559 and the Iteration Count was 326,664. It should be noted that models with interactive effects affecting just 5 of the 48 projects took over 72 hours to solve and thus represents a large-scale instance of the problem described above given 48 total projects and 7 years considered.

The optimal objective function is $9,912,042 which satisfies the relative optimality tolerance of 0.0 . This means that the total cost to complete all projects from all sources except the annual budget is $9,912,042. The capital requested in the first year is $6,731,889, which funds 32 projects, leaving the balance of 16 projects to be funded through annual savings. The details of the results can be viewed in Table D-3 of the Appendices.

4.4.2. Rolling-Horizon Results

The summarized results of the experimental model at several horizons are shown below in Table 4-10. Detailed results are shown in Table D-4 in the Appendices.

		Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6	Stage 7
Multistage	Capital Outlay	$9,912,042	NA	NA	NA	NA	NA	NA
	Projects Completed	32 Projects	0 Projects	0 Projects	0 Projects	0 Projects	0 Projects	16 Projects
Case 1	Capital Outlay	$10,204,280	$1,234,959	All Projects Completed				
	Projects Completed	43 Projects	5 Projects					
Case 2	Capital Outlay	$10,204,280	$1,271,557					
	Projects Completed	43 Projects	5 Projects					
Case 3	Capital Outlay	$10,099,872	$2,087,085	$1,337,466	$1,314,253	All Projects Completed		
	Projects Completed	39 Projects	3 Projects	5 Projects	1 Project			
Case 4	Capital Outlay	$10,099,872	$2,191,840	$1,321,871	$1,406,161			
	Projects Completed	39 Projects	5 Projects	3 Projects	1 Project			
Case 5	Capital Outlay	$6,970,456	$939,454	NA	NA	NA	NA	NA
	Projects Completed	34 Projects	4 Projects	0 Projects	0 Projects	0 Projects	0 projects	10 Projects
Case 6	Capital Outlay	$6,970,456	$952,661	NA	NA	NA	NA	NA
	Projects Completed	34 Projects	4 Projects	0 Projects	0 Projects	0 Projects	0 projects	10 Projects

Table 4-10: Resulting Projects at "Med" Yield and Remaining Budget

Table 4-10 illustrates the comparison between the multistage stochastic model and the rolling-horizon model. The tradeoffs involving the shorter length of the horizon for the costs of the overall program are apparent. The endogenous learning was not impactful in the shortest horizons.

4.5. Discussion

The experimental examples were run for both two, four and six-year rolling-horizons. There were three major finding from the results of the practical application and the several cases observed. These are:

1. Early selection of projects by the rolling horizon approach limited the ability to spread projects throughout the planning horizon
2. The benefit of the year-over-year savings are lost in shorter horizons

3. The impact of the rolling-horizon length is greater than that of the endogenous learning
4. Subadditivity and superadditivity becomes intractable when over 5 projects interact
5. The rolling-horizon model only outperforms (requires a lower total cost than) the multistage models with longer horizons regardless of learning

Early selection of projects limited the ability to spread projects throughout the planning horizon. This is best observed in cases 1 and 2, where the first rolling horizons compressed the selection of all projects into two years. As a result, most projects were implemented in the first stage. The model could not anticipate additional rolls. All six cases were heavily influenced by the model's early selection of projects. This result is apparent in Tables 4-10 and D-3 where more projects were selected in the first stage than any other stage, in all cases. This result can be explained by observing the projects returns. Greater than 60% of the projects listed in Table D-1 have a simple payback greater than the planning horizon (7 years). This means that most projects could not fund themselves within the planning horizon, let alone a shorter one. The model selects projects early, as shorter horizons will not generate enough savings to fund many additional projects. If the projects could generate significant savings within the rolling horizon, their selections would be delayed and therefore available for later stages.

The benefit of the year-over-year savings are lost in shorter horizons. This key result is one of the disadvantages of the rolling-horizon approach. In many cases, the multistage model provides a better model as it spans the length of the planning horizon. The annual savings are cumulative over time. Likewise, compressed horizons do not allow for most of the learning to make a significant impact. Most projects are completed in the earliest stages prior to the endogenous learning taking effect, reducing the ability to provide a significant impact. A relatively smaller subset of projects benefit for the learning in the prior stages.

Cases 3 and 4 present the most profound and meaningful results. The rolling-horizon model implements all projects in four years (the years shorter than that of the planning horizon). In the fourth year, case 3 (no learning) completes one project (project 20). In the same year, Case 4 (endogenous learnings) completes one project as well (project 24). The learning allows larger projects to be selected in later stages even with the cumulative annuals savings of the multistage model.

Cases 5 and 6 provide the best results of all cases as these cases are only the only ones that outperform the multistage model. The cost of the overall program is $7,909,910 and saves the agency over 23%. This result is attributable to the 6-year model's ability to leverage five years of the repeated annual savings and the impact of endogenous learning.

An alternative application of the rolling-horizon was modeled in Case 7. The last roll should have been in year 2, however; allowing additional rolls have only improved the model. If allowed to extend beyond the planning horizon, the model will attempt to delay the final project's completion in order to take full advantage of year over year savings over implemented projects. This result violates the constraints of the model. If that project were forced to complete in the final year of the planning horizon (i.e. force project 5 to be completed in stage 7) then this model would be superior to cases 3 and 4.

The rolling-horizon model is superior to the multistage model in specific cases. The savings can be greater than 20% in these cases.

4.6. <u>Conclusion</u>

In this research, we introduced the concept of rolling-horizons with fixed improvement and with additivity. We incorporated McCormick's auxiliary variable model to make this problem solvable. The subadditivity and superadditivity provided challenges with regard to the size of the problem. This model was compared to several experimental cases to a multistage stochastic program for energy project selection.

While there are improvements to the results of the model from improved yield in each stage, the larger impacts to the objective were made by selecting the appropriate length of the horizon. The rolling-horizon selected should start with the length of the planning horizon and reduced until the objective exceeds that of a comparable multistage model. Shorter horizons will allow for more endogenous learning but in these application of this research, cumulative savings outweigh the ability to learn and better estimate yields.

This model provides great improvement over the comparable stochastic model in the longer rolling-horizons. The three main benefits are improved objective functions (greater than 20% lower cost to implement all projects), adaptability, which allow agencies to choose their risk tolerance and speed allowing subadditivity and superadditivity to be solved in less than a few hours. Federal, state and local agencies will greatly benefits from this model in their strategic decision making and energy project selection.

Chapter 5: Summary and Conclusions

The new approaches presented in the research give the agencies the ability to save millions of dollars while implementing more energy-conservation measures and paying for them most cost-efficiently. Each model is a vast improvement over the agencies' current approaches for each of the implementation and funding methods. It is hoped that agencies embrace the use of the novel optimization models and these practices become the default approaches.

The dissertation asserted that agencies could obtain greater returns on their energy conservation investments over traditional methods, regardless of funding and the particular implementing organization. The first objective of this dissertation was to introduce novel optimization models that provide improvements above the traditional approaches through increased returns on energy conservation investment. The traditional approach for agencies leveraging energy savings performance contracts and/or utility energy service contract is to sort by paybacks and implement ECM project until the budget is exhausted. The agency then turns to firms to complete the balance of ECM projects. The traditional approach for agencies completing all projects through congressional appropriation is to fund the full cost of projects without reinvestment of annual savings. The models and case studies in Chapters 2-4 demonstrate savings of over 6% in each case. These summary results are illustrated in Table 5.1 below.

	Two-Level Mathematical Program with Equilibrium Constraints	Multi-Stage Risk Model with minimum project per stage (($\rho = 0$)	Multi-Stage Risk Model with minimum project per stage (($\rho = 5$)	Rolling-horizon Method with Endogenous Learning
	Chapter 2	Chapter 3	Chapter 3	Chapter 4
Savings Increase Over Traditional Model	18.9%	6.83%	6.60	23.9%

Initial Capital Outlay Reduction	0.00%	28.5%[23]	28.0%	-17.2%[24]
CPU Run time (seconds)[25]	93	813	876	19

Table 5-1: Dissertation Results: Agency Savings of Models

The additional objectives of the dissertation were

1. to model and find tractable solutions to a complex problem that has traditionally forced agencies to leverage inefficient heuristics in decision making
2. to present options and practical solutions to a common yet complicated problem that can be customized for each federal, state or local government's budgets and risk appetites.

These objectives were met in Chapters 2-4 of the dissertation.

In Chapter 2, the current industry practice (traditional approach) of selecting projects based on suboptimal criteria such as, payback, savings to investment ratio or ease of implementation were discussed. Once those projects were implemented, the agency sought energy savings performance contracts or utility energy service contracts for the balance of projects. This separation of the two decisions by the agency, the different objectives of agency and the energy services companies, and the inability of the leader, the agency, to leverage the knowledge of how the lower level firms will respond, made the entire process suboptimal. The suboptimal selection process most often results in inefficient allocations while not providing any additional profit to the energy services companies. There was also the risk of the agency selecting too many of the

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[23] In the traditional approach the appropriation was equal to the initial capital outlay
[24] There was a 17.2% increase in the initial congressional appropriation sought in this model.
[25] Run times are in seconds and are the average of three runs.

profitable projects, thereby leaving only undesirable projects for energy services companies. Many of these projects were currently being left undone while agencies struggle to meet their mandated conservation goals. The agency must then finance these projects, which is a least cost-effective option.

The two-level model presented in Chapter 2 maximized savings to the agency and profit to the energy services industry. While the EnergyStar guidance provides “rules of thumb” that may simplify the selection, this process does not make the best use of the dollars and options for project execution.

The benefits of the of the two-level optimization were apparent when comparing these results to both the standard practice and even a single-level optimization problem. Giving the agency’s ability to select projects while evaluating the implementation and financing mechanisms available to them, made them the best stewards of taxpayers' money.

In Chapter 3, the traditional approach used by the agency required that all projects be completed with a single appropriation. The traditional, deterministic approach does not allow the agency to predict savings that could be used to fund future projects accurately. The results of deterministic models are the key deterrents for agencies considering the use of future savings to fund projects. Using results of the deterministic model may leave the agency with a shortfall in later periods where additional capital budget cannot be requested. In these cases, the agency is then forced to seek outside sources for project funding. This causes the agencies to assume risk-averse stances.

The proposed model in Chapter 3 added stochasticity in energy savings and allowed the agency to select their risk tolerance. The results of the proposed multistage, risk-loaded model showed a value of the stochastic solution (VSS) of $18,869 in a practical application. As risk-aversion increased, the required capital outlay (the total cost to complete all projects) increased. Risk-neutrality without the minimum project per year constraining provided the lowest capital outlay.

The model proposed in the Chapter 3 is preferred to the traditional model because savings can be used to fund additional programs while incorporating the seemingly random fluctuations in energy prices and addressing proposed energy savings that may return lower estimates. The lower risk is a tradeoff that comes at a higher cost.

In the practical application presented with this model, the value of the optimization is compared to the agency's traditional approach by including the ability to leverage the existing savings and understanding the impact of the energy price and forecast of future savings. In this case, the optimized value to the agency is more realistic and superior to both the traditional and deterministic model.

Chapter 4 introduced the concept of rolling horizons with endogenous learning and supplemented this model with subadditivity and superadditivity of energy savings. This model incorporated McCormick's auxiliary variable method to replace constraints involving the product of variables with an exact linearization for computational improvement. The subadditivity and superadditivity provided computational challenge, presumably with due to the nonconvexities in the problem. When comparing this model to an extended multi-stage model as in Chapter 3, there was great improvement in the objective function (the total cost implement all ECM projects) when the rolling-horizons greater than half the length of the planning horizons. The major findings from the results of the practical application and several cases observed. The rolling-horizon model outperformed the multistage models by 20% (20% less cost to implement all projects) with longer rolling horizons regardless of learning. However, there was a 17% increase in the initial congressional appropriation sought in this case as shown in Table 5.1.

While there is great innovation in the use endogenous learning, larger impacts to the objective function were made by selecting the appropriate length of the rolling horizon. The longer rolling horizons selected yielded better results (improved objective functions). Shorter horizons will allow for more endogenous learning but in this application of this research, cumulative savings achieved by projects implemented in prior stages outweighed the ability to learn and better estimate the yields.

The three main benefits of the rolling-horizon model were an improved objective function (greater than 20% lower cost to implement all projects as compared to a multistage stochastic model), adaptability, which allow agencies to choose their risk tolerance and speed allowing subadditivity and superadditivity of energy savings to be solved in less than a few hours. Federal, state and local agencies will greatly benefit from this model in their strategic decision-making and energy project selection. These methods but also have far-reaching implications for international agencies as well as commercial owners both domestic and abroad.

The methods and models presented in this research each have unique approaches and simulate the real-world challenges and the options available to the agencies. Each method is an improvement on what is currently being done today as the so-called best practice. These models can be easily implemented and provide immediate benefit to every agency that is consuming energy in buildings. It is proposed that these methods become the standard for federal, state and local ECM project selection before outside parties (ESCOs and utilities) adopt these approaches and assume the available savings.

6. Appendix

6.1. Appendix A

Upper-Level Problem

The agency's annual savings maximizing problem presented earlier, is repeated here and is given in (A-1):

$$\max_{x,z,q} Z = \sum_{p=1}^{n_p} \varepsilon_p \theta_p \cdot \big(\boldsymbol{x}(p) + \boldsymbol{z}(p)\big) + \sum_{p=1}^{n_p} \sum_{f=1}^{n_f} \big(\varepsilon_p \theta_p \omega_{p,f} - \mu_p \gamma_p\big) \cdot \big(1 - \zeta_f\big)\, \boldsymbol{q}(p,f) - D \ \cdot TF \tag{A-1a}$$

Subject to:

$$\sum_{p=1}^{n_p} \gamma_p \cdot \boldsymbol{x}(p) \leq B + \ SSR \tag{A-1b}$$

$$SSR = \sum_{p=1}^{n_p} \sum_{f=1}^{n_f} \boldsymbol{q}(p,f) \cdot \big(\varepsilon_p \theta_p \omega_{p,f} - \mu_p \gamma_p\big) \cdot \big(1 - \zeta_f\big) \tag{A-1c}$$

$$\sum_{p=1}^{n_p} \gamma_p \cdot \boldsymbol{z}(p) = TF \tag{A-1d}$$

$$\boldsymbol{x}(p) + \boldsymbol{z}(p) + \sum_{f=1}^{n_f} \boldsymbol{q}(p,f) = 1 \ \forall p \tag{A-1e}$$

$$\boldsymbol{x}(p), \boldsymbol{z}(p) \ \ are \ \ binary \tag{A-1f}$$

$$0 \leq \boldsymbol{q}(p,f) \;\; \forall p \; and \; f \tag{A-1g}$$

with $\boldsymbol{q}(p,f)$ solving the lower-level problem which is the solution set of the following optimization problems for ESCO firm *f (f=1,…,n$_f$)*.

6.2. Appendix B

Lower-Level Problem

ESCO / Firms' Profit-Maximizing Problem

$$\max_q \pi_f = \sum_{p=1}^{n_p} \Big[\big(\varepsilon_p\theta_p\omega_{p,f} - \mu_p\gamma_p\big)\cdot\big(\zeta_f\big)\boldsymbol{q}(p,f) - \Big(\phi_f\big(\varepsilon_p\theta_p - \mu_p\gamma_p\big)\cdot\big(\boldsymbol{q}(p,f) + \boldsymbol{q}(p,f)^{K_f}\big)\Big)\Big] \tag{B-2a}$$

Subject to:

$$\sum_{p=1}^{n_p} \Big[\big(\varepsilon_p\theta_p\omega_{p,f} - \mu_p\gamma_p\big)\cdot\big(\zeta_f\big)\cdot\boldsymbol{q}(p,f) - \Big(\phi_f\big(\varepsilon_p\theta_p - \mu_p\gamma_p\big)\cdot\big(\boldsymbol{q}(p,f) + \boldsymbol{q}(p,f)^{K_f}\big)\Big)\Big] \geq MP_f \tag{B-2b}$$

$$\boldsymbol{q}(p,f) \leq 1 \;\;\; \forall \; p \; in \; P \tag{B-2c}$$

$$\boldsymbol{q}(p,f) \geq 0 \; \forall \; p \; in \; P \tag{B-2d}$$

Note that:

- the quantity $\big(\varepsilon_p\theta_p\omega_{p,f} - \mu_p\gamma_p\big)\cdot\big(\zeta_f\big)\cdot\; q(p,f)$ represents the revenue gained by the ESCO in the form of shared savings by taking on $q(p,f)$ percent of project, p. For simplicity, let $c(p,f)$ be the shorthand for the objective function coefficient $\big(\varepsilon_p\theta_p\omega_{p,f} - \mu_p\gamma_p\big)\cdot\big(\zeta_f\big)$.
- the quantity $\Big(\phi_f\big(\varepsilon_p\theta_p - \mu_p\gamma_p\big)\cdot\big(q(p,f) + q(p,f)^{K_f}\big)\Big)$ represents the cost of implementing project, p where the parameter ϕ_f is the percentage of the shared savings

that is attributed to material, labor and equipment costs. For simplicity, let $d(p,f)$ be the shorthand for the objective function coefficient $\phi_f\left(\varepsilon_p\theta_p - \mu_p\gamma_p\right)$.

Then, the lower-level problem for firm f can be more succinctly written as follows with the λ values in parentheses the corresponding Lagrange multipliers to each constraint:

$$\min_q OBJ_f(q) \triangleq \sum_{p=1}^{n_p}\left[-c(p,f)\boldsymbol{q}(p,f) + d(p,f)\cdot(\boldsymbol{q}(p,f) + \boldsymbol{q}(p,f)^{\boldsymbol{K_f}})\right] \quad \text{(B-3a)}$$

Subject to:

$$g_{1f}(q) \triangleq MP_f - \sum_{p=1}^{n_p}\left[c(p,f)\boldsymbol{q}(p,f) - d(p,f)\cdot(\boldsymbol{q}(p,f) + \boldsymbol{q}(p,f)^{\boldsymbol{K_f}})\right] \quad \text{(B-3b)}$$

$$\leq 0 \ \ (\lambda_{1f})$$

$$g_{2pf}(q) \triangleq \boldsymbol{q}(p,f) - 1 \leq 0 \quad (\lambda_{2pf}) \quad \forall\, p\ in\ P \quad \text{(B-3c)}$$

$$g_{3pf}(q) \triangleq -\boldsymbol{q}(p,f) \leq 0 \quad (\lambda_{3pf}) \forall\, p\ in\ P \quad \text{(B-3d)}$$

The approach to solving this two-level problem is to use the Karush-Kuhn-Tucker (KKT) optimality conditions, apply them to the lower-level optimization problem and insert them into the upper-level problem as additional constraints. In this way, the original two-level problem is reformulated as a single-level nonlinear optimization problem. The KKT conditions for optimality of the lower-level problem are shown below noting that the Lagrange multiplier λ_{3pf} has been substituted away.

$$\boldsymbol{0} \leq \left[-c(p,f) + d(p,f) + d(p,f)\boldsymbol{K_f}(\boldsymbol{q}(p,f)^{\boldsymbol{K_f-1}})\right]$$

$$+ \left(\lambda_{1f}\right)\left[-c(p,f) + d(p,f) + d(p,f)\boldsymbol{K_f}(\boldsymbol{q}(p,f)^{\boldsymbol{K_f-1}})\right] +$$

$$\left(\lambda_{2pf}\right) \perp \boldsymbol{q}(\boldsymbol{p},\boldsymbol{f}) \geq \boldsymbol{0}, \forall \boldsymbol{p} \in \quad \text{(B-4a)}$$

$$\boldsymbol{P}, \boldsymbol{f} \in F$$

$$0 \leq -MP_f + \sum_{p=1}^{n_p} \left[c(p,f)\boldsymbol{q}(p,f) - d(p,f) \cdot \left(\boldsymbol{q}(p,f) + \boldsymbol{q}(p,f)^{\boldsymbol{K_f}}\right) \right] \perp \ (\lambda_{1f})$$

$$\geq \boldsymbol{0}, \forall \quad \text{(B-4b)}$$

$$\boldsymbol{f} \in F$$

$$0 \leq -\boldsymbol{q}(p,f) + 1 \quad \perp \quad \left(\lambda_{2pf}\right) \geq \boldsymbol{0}, \forall \boldsymbol{p} \in$$

$$\boldsymbol{P}, \boldsymbol{f} \in F \quad \text{(B-4c)}$$

The KKT optimality conditions are sufficient for solving problem (B-3) if the objective function is convex in the vector *q*, and each of the inequality constraint functions g_{1f} , g_{2pf} g_{3pf} g are convex in *q* (Bazaraa et al. (Bazaraa, et al., 2006)). To see that the objective function is convex, given that the first part $-c(p,f)\boldsymbol{q}(p,f)$ is linear, it suffices to check that the second part $d(p,f) \cdot (\boldsymbol{q}(p,f) + \boldsymbol{q}(p,f)^{K_f})$ is convex in *q*. But of course $d(p,f) \cdot \boldsymbol{q}(p,f)$ is linear as well so that only $d(p,f) \cdot \boldsymbol{q}(p,f)^{K_f}$ needs to be shown to be convex in *q*. Note that the Hessian matrix of the objective function in (B-3a) relative to the vector of variables *q* is just a diagonal matrix with diagonal entries given by the second derivative of $\boldsymbol{q}(p,f) \cdot \boldsymbol{q}(p,f)^{\boldsymbol{K_f}}$ relative to the scalar variable *q(p,f)* or just $K_f(K_f - 1)d(p,f) \cdot \boldsymbol{q}(p,f)^{\boldsymbol{K_f - 2}}$. Under Assumption B-1 shown below, this second derivative is nonnegative implying that the Hessian matrix of the objective function is positive semi-definite, equivalent to the objective function being convex (Bazaraa, et al., 2006).

Assumption B-1

The cost function for each firm f, $d(p,f) \cdot (\boldsymbol{q}(p,f) + \boldsymbol{q}(p,f)^{\boldsymbol{K_f}})$ *has the property that:*

a. $d(p,f) = \phi_f\left(\varepsilon_p\theta_p - \mu_p\gamma_p\right) \geq 0$ *for all p, f*
b. $K_f\left(K_f - 1\right)d(p,f) \cdot \boldsymbol{q}(p,f)^{\boldsymbol{K_f - 2}} \geq 0$ *for all p, f*

Condition a. is reasonable because the firms' costs are non-negative.

Condition b. is satisfied for example if $d(p,f)$ is nonnegative (condition a.) and $K_f \geq 1$ when the nonnegative variable *q(p,f)* takes on a positive value (if it's equal to zero no constraints are needed). Varying values of $K_f \geq 1$ were shown earlier.

Consequently, we have the following result.

Theorem B-1

Under Assumption B-1, the KKT conditions for problem (B-3) are sufficient for optimality.

Proof. Given the above analysis for the convexity of the objective function (B-3a), it suffices to check that each inequality constraint function g is convex. Since g_{2pf}, g_{3pf} are both linear, hence convex functions, only g_{1f} needs to be shown to be convex. However, $g_{1f}(q) \triangleq MP_f + OBJ_f$ which in light of MP_f being a constant, renders this constraint function convex given the above convexity analysis for OBJ_f.

QED

The next result is to show when these KKT conditions are also necessary. First, note that the linearity constraint qualification (CQ) often used to show necessity of the KKT conditions is invalid for (B-2b) given the polynomial form of the cost function (Bazaraa, et al., 2006). Likewise, the linear independence CQ may also fail at an optimal solution. To see this consider that at a solution q of (B-3), one or more of the constraints g_{1f}, g_{2pf}, g_{3pf} are may be binding. However, given the form of the functions, both g_{2pf} , g_{3pf} can't be binding at the same time. For sake of discussion and without loss of generality suppose that $g_{1f}(q) = 0, g_{2pf}(q) = 0$ for all *p, f* so that the set of binding indices $I = \{1,2\}$. In that case, the gradients of the binding indices are:

$$\nabla g_{1f}(q) = \begin{pmatrix} -c(1,f) + d(1,f) + d(1,f)K_f q(1,f)^{K_f - 1} \\ \vdots \\ -c(n_p,f) + d(n_p,f) + d(n_p,f)K_f q(n_p,f)^{K_f - 1} \end{pmatrix} = \begin{pmatrix} v(1,f) \\ \vdots \\ v(n_p,f) \end{pmatrix} = v$$

$$\nabla g_{2pf}(q) = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, p = 1, \ldots, n_p$$ with a 1 in the *p*th position. Clearly in this case we see that

$$\nabla g_{1f}(q) = v = \sum_{p} v(p,f) \nabla g_{2pf}(q)$$

invalidates the linear independence constraint qualification (Bazaraa, et al., 2006) However, as shown below, the Slater's constraint qualification (Bazaraa, et al., 2006) does hold for the problem at hand.

Slater's CQ

Consider the optimization problem

$$\min f(x)$$
$$s.t.\, g_i(x) \leq 0, i = 1, \ldots, m$$
$$h_j(x) = 0, j = 1, \ldots, l$$
$$x \in X$$

Then, for a local solution $\bar{x}$ let $I = \{i: g_i(\bar{x}) = 0\}$ be the binding set of indices.

Slater's CQ is then the following set of conditions:

1. *The set X is open.*
2. *Each g_i for $i \in I$ is pseudoconvex at $\bar{x}$.*
3. *Each g_i for $i \notin I$ is continuous at $\bar{x}$.*
4. *Each h_j for $j = 1, \ldots, l$ is quasiconvex, quasiconcave, and continuously differentiable at $\bar{x}$.*
5. *Each $\nabla h_j(\bar{x})$ for $j = 1, \ldots, l$ are linearly independent.*
6. *There exists an $x \in X$ such that $g_i(x) < 0, \forall i \in I$ and $h_j(x) = 0, j = 1, \ldots, l$*

Given the inequality-only form of the constraints for the lower-level problem (B-2), the fact that all the inequality constraint functions under Assumption B-1 are convex (hence pseudoconvex) and continuous, and that the set *X is R^n,* hence open, Slater's CQ reduces to the following:

1. *There exists an $x \in R^n$ such that $g_i(x) < 0, \forall i \in I$.*

The following assumption then leads to the result for necessity of the KKT conditions to the overall MPEC.

Assumption B-2

There exists a value of the vector q so that

1. *$0<q(p,f)<1$ for all p, f*
2. $\sum_{p=1}^{n_p}\left[c(p,f)\boldsymbol{q}(p,f) - d(p,f)\cdot(\boldsymbol{q}(p,f)+\boldsymbol{q}(p,f)^{\boldsymbol{K}_f})\right] > MP_f$

These conditions amount to saying that there is a strictly fractional assignment of the projects to the ESCOs where each firm makes more than the minimum profit. If this minimum profit is zero, then it just says that this fractional assignment is profitable for all firms. For the given data, this assumption holds for a minimum profit of $0.

With Assumptions B-1 and B-2, the overall equivalent formulation for the overall MPEC problem is then just the optimization problem (B-1) and conditions (B-4) inserted as constraints into the upper-level problem (B-4) as the KKT conditions (B-4) are both necessary and sufficient to optimality of the lower-level problem. If just Assumption B-1 is in force, then the KKT conditions are only guaranteed to be sufficient for optimality but still useful for the given approach.

The complementarity conditions given by " $\perp$" in (B-4) can be replaced by disjunctive constraints Fortuny-Amat and McCarl (Fortuny-Amat & McCarl, 1981), Gabriel and Leuthold, (Gabriel & Leuthold, 2010) using the following illustrative example.

Instead of

$$0 \leq -\boldsymbol{q}(p,f) + 1 \quad \perp \quad (\lambda_{2pf}) \geq \mathbf{0}$$

or equivalently

$$0 \leq -\boldsymbol{q}(p,f) + 1 \ , (\lambda_{2pf}) \geq \mathbf{0}, (-\boldsymbol{q}(p,f)+1)(\lambda_{2pf}) = \mathbf{0}$$

these conditions can be replaced by their disjunctive-constraints equivalent form

$$0 \leq -\boldsymbol{q}(p,f) + 1 \ \leq Mb \ , 0 \leq \lambda_{2pf} \leq M(1-b), b \in \{0,1\}$$

Here b is a binary variable and M is a large positive constant. Thus, the equivalent problem that was solved in this chapter was the original MPEC is (B-1) with the disjunctive-constraints form of the conditions (B-4) inserted as constraints.

6.3. Appendix C

Below are the actual ECM data characteristics from the energy audit.

	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Cost of Energy Saved ($)	Degradation / Escalation Rate (%)	Estimated Useful Life (Years)	Payback Ratio (Years)
P	γ_p	α_p			δ_p	n	
project1	$ 710,354	5,334,857	$ 0.015	$ 80,023	-1.50%	30	8.88
project2	$ 637,975	1,849,047	$ 0.033	$ 61,019	-1.00%	23	10.46
project3	$ 468,071	1,768,079	$ 0.023	$ 40,666	-2.00%	30	11.51
project4	$ 40,368	445,600	$ 0.010	$ 4,456	-1.38%	30	9.06
project5	$ 8,557	213,025	$ 0.012	$ 2,556	-1.50%	15	3.35
project6	$ 15,328	124,584	$ 0.023	$ 2,865	-0.75%	9	5.35
project7	$ 55,207	287,971	$ 0.027	$ 7,775	-2.50%	15	7.10
project8	$ 59,355	416,045	$ 0.022	$ 9,153	-2.00%	15	6.48
project9	$ 84,738	559,247	$ 0.015	$ 8,389	-1.50%	30	10.10
project10	$ 188,994	801,565	$ 0.033	$ 26,452	-1.00%	40	7.14
project11	$ 142,377	660,074	$ 0.023	$ 15,182	-2.00%	30	9.38
project12	$ 186,520	440,470	$ 0.033	$ 14,536	-1.38%	30	12.83
project13	$ 165,932	2,243,077	$ 0.012	$ 26,917	-1.50%	15	6.16
project14	$ 169,521	650,787	$ 0.023	$ 14,968	-0.75%	20	11.33
project15	$ 95,238	554,558	$ 0.027	$ 14,973	-2.50%	15	6.36
project16	$ 220,871	1,366,652	$ 0.019	$ 25,966	-2.00%	15	8.51
project17	$ 201,577	793,782	$ 0.030	$ 23,813	-2.00%	30	8.46
project18	$ 119,351	724,725	$ 0.033	$ 23,916	-1.38%	23	4.99
project19	$ 152,286	488,525	$ 0.023	$ 11,236	-1.50%	30	13.55
project20	$ 95,631	632,278	$ 0.010	$ 6,323	-0.75%	30	15.12
project21	$ 53,495	518,592	$ 0.012	$ 6,223	-1.50%	15	8.60
project22	$ 276,920	1,551,851	$ 0.023	$ 35,693	-0.75%	20	7.76

project23	$ 94,078	1,135,237	$ 0.027	$ 30,651	-2.50%	20	3.07
project24	$ 228,071	784,038	$ 0.026	$ 20,385	-2.00%	15	11.19
project25	$ 236,862	2,103,902	$ 0.014	$ 29,455	-1.38%	10	8.04
project26	$ 438,530	1,678,580	$ 0.023	$ 38,607	-1.50%	23	11.36
project27	$ 558,439	3,212,065	$ 0.029	$ 93,150	-1.50%	12	6.00
project28	$ 84,237	2,054,672	$ 0.020	$ 41,093	-1.50%	10	2.05
project29	$ 18,149	138,751	$ 0.013	$ 1,804	-2.00%	26	10.06
project30	$ 64,378	420,774	$ 0.017	$ 7,153	-1.50%	20	9.00
project31	$ 387,393	2,743,397	$ 0.026	$ 71,328	-1.50%	15	5.43
project32	$ 266,812	937,263	$ 0.030	$ 28,118	-2.00%	25	9.49
project33	$ 185,099	2,236,000	$ 0.011	$ 24,596	-2.50%	20	7.53
project34	$ 205,145	1,664,432	$ 0.017	$ 28,295	-1.00%	10	7.25
project35	$ 195,433	3,599,559	$ 0.014	$ 50,394	-2.50%	23	3.88
project36	$ 184,600	750,238	$ 0.019	$ 14,255	-1.38%	28	12.95
project37	$ 110,377	1,045,732	$ 0.012	$ 12,549	-1.50%	23	8.80
project38	$ 252,736	1,533,356	$ 0.021	$ 32,200	-1.50%	37	7.85
project39	$ 157,354	2,043,132	$ 0.020	$ 40,863	-2.00%	18	3.85
project40	$ 247,218	1,573,358	$ 0.028	$ 44,054	-2.50%	20	5.61
project41	$ 256,421	1,806,445	$ 0.024	$ 43,355	-2.00%	25	5.91
project42	$ 152,886	2,399,913	$ 0.012	$ 28,799	-2.00%	28	5.31
project43	$ 455,000	2,448,183	$ 0.022	$ 53,860	-1.38%	36	8.45
project44	$ 473,225	3,500,838	$ 0.017	$ 59,514	-2.50%	33	7.95
project45	$ 127,011	883,506	$ 0.017	$ 15,020	-1.00%	14	8.46
project46	$ 492,782	1,802,085	$ 0.016	$ 28,833	-1.50%	32	17.09
project47	$ 266,790	1,010,352	$ 0.031	$ 31,321	-1.00%	10	8.52
project48	$ 115,006	741,117	$ 0.025	$ 18,528	-2.50%	20	6.21
Totals	$ 10,402,698	66,372,316		$ 1,351,279			7.75

Table C-1: ECM Data in Practical Application

Baseline Project Quality, ω_{pf} **Note, ω_{pf} =1 as executed by Agency**			
Project	Firm 1	Firm 2	Firm 3
project1	1.06	1.10	1.05
project2	1.06	1.01	1.05
project3	1.05	1.09	1.07
project4	1.06	1.01	1.09
project5	1.05	1.10	1.04
project6	1.10	1.15	1.05
project7	1.04	1.15	1.05
project8	1.10	1.08	1.03
project9	1.09	1.04	1.01
project10	1.08	1.03	1.01
project11	1.10	1.09	1.01
project12	1.06	1.06	1.03
project13	1.05	1.10	1.01
project14	1.07	1.12	1.05
project15	1.04	1.10	1.04
project16	1.07	1.08	1.03
project17	1.06	1.15	1.03
project18	1.09	1.01	1.02
project19	1.10	1.09	1.01
project20	1.06	1.01	1.03
project21	1.15	1.10	1.04
project22	1.07	1.15	1.01
project23	1.09	1.10	1.05
project24	1.07	1.08	1.01

project25	1.06	1.06	1.05
project26	1.10	1.01	1.09
project27	1.05	1.05	1.07
project28	1.04	1.10	1.01
project29	1.15	1.08	1.03
project30	1.05	1.03	1.10
project31	1.01	1.10	1.15
project32	1.09	1.04	1.05
project33	1.09	1.08	1.10
project34	1.04	1.03	1.09
project35	1.01	1.01	1.01
project36	1.06	1.05	1.07
project37	1.06	1.10	1.12
project38	1.03	1.01	1.05
project39	1.04	1.09	1.10
project40	1.20	1.06	1.06
project41	1.11	1.06	1.07
project42	1.15	1.07	1.09
project43	1.10	1.15	1.10
project44	1.04	1.01	1.05
project45	1.07	1.13	1.02
project46	1.08	1.04	1.02
project47	1.01	1.03	1.04
project48	1.05	1.02	1.05

Table C-2: Practical Application table of ESCO Quality Factors (ω_{pf})

	Agency	Firm 1	Firm 2	Firm 3	Totals
	$x(p)$	$q(p,1)$	$q(p,2)$	$q(p,3)$	

Project 1			100%		100%
Project 2		100%			100%
Project 3			100%		100%
Project 4	100%				100%
Project 5				100%	100%
Project 6			100%		100%
Project 7			100%		100%
Project 8			100%		100%
Project 9		100%			100%
Project 10	100%				100%
Project 11			100%		100%
Project 12			100%		100%
Project 13			100%		100%
Project 14			100%		100%
Project 15			100%		100%
Project 16			100%		100%
Project 17			100%		100%
Project 18	100%				100%
Project 19			100%		100%
Project 20		100%			100%
Project 21		100%			100%
Project 22			100%		100%
Project 23	100%				100%
Project 24			100%		100%
Project 25			100%		100%
Project 26		100%			100%
Project 27			100%		100%
Project 28	100%				100%
Project 29		100%			100%

Project 30			0.356%	0.644%	100%
Project 31			0.921%	0.079%	100%
Project 32		100%			100%
Project 33			100%		100%
Project 34			100%		100%
Project 35	100%				100%
Project 36			100%		100%
Project 37			100%		100%
Project 38	100%				100%
Project 39	100%				100%
Project 40		100%			100%
Project 41	100%	0.61%		0.039%	100%
Project 42					100%
Project 43			100%		100%
Project 44			100%		100%
Project 45			100%		100%
Project 46		100%			100%
Project 47			100%		100%
Project 48	100%				100%
Total Projects	**10**	9.61	26.277	2.113	
Total Profit		**$615,865**	**$1,848,516**	**$100,000**	

Table C-3: Practical Application Results at $200K Budget

Objective Function	$9,691,951				
Capital Requested	$7,431,260				
low limit (ρ)	0				
Stage	1	2	3	4	5
Projects	36	1.333	2.222	2.444	6

Cost of Projects	$7,431,260	$246,276	$572,507	$633,008	$1,519,646
Objective Function	$9,691,951				
Capital Requested	$7,431,260				
low limit (ρ)	1				
Stage	1	2	3	4	5
Projects	36	1.333	2.222	2.444	6
Cost of Projects	$7,431,260	$246,276	$572,507	$633,008	$1,519,646
Objective Function	$9,692,199				
Capital Requested	$7,434,242				
low limit (ρ)	2				
Stage	1	2	3	4	5
Projects	34	2	2.444	2.778	6.778
Cost of Projects	$7,434,242	$253,163	$570,111	$750,616	$1,339,828
Objective Function	$9,694,605				
Capital Requested	$7,442,357				
low limit (ρ)	3				
Stage	1	2	3	4	5
Projects	32	3	3	3	7
Cost of Projects	$7,442,357	$249,460	$569,252	$630,950	$1,510,678
Objective Function	$9,699,283				
Capital Requested	$7,447,207				
low limit (ρ)	4				
Stage	1	2	3	4	6
Projects	27	4	4	4	4
Cost of Projects	$7,447,207	$259,952	$570,433	$681,170	$1,450,926
Objective Function	$9,714,788				
Capital Requested	$7,488,979				
low limit (ρ)	5				
Stage	1	2	3	4	5
Projects	27	5	5	5	5
Cost of Projects	$7,488,979	$253,163	$570,111	$750,616	$1,339,828

Objective Function	$9,758,849				
Capital Requested	$7,604,352				
low limit (ρ)	6				
Stage	1	2	3	4	5
Projects	24	6	6	6	6
Cost of Projects	$7,431,260	$269,036	$616,400	$836,582	$1,076,327
Objective Function	$10,340,195				
Capital Requested	$7,919,242				
low limit (ρ)	7				
Stage	1	2	3	4	5
Projects	20	7	7	7	7
Cost of Projects	$7,204,331	$461,673	$689,589	$922,142	$1,115,963
Objective Function	$11,226,727				
Capital Requested	$8,243,148				
low limit (ρ)	8				
Stage	1	2	3	4	5
Projects	16	8	8	8	8
Cost of Projects	$6,384,099	$799,424	$847,195	$1,014,415	$1,357,565
Objective Function	$12,277,923				
Capital Requested	$8,612,434				
low limit (ρ)	9				
Stage	1	2	3	4	5
Projects	12	9	9	9	9
Cost of Projects	$5,402,328	$1,219,183	$1,069,348	$1,107,323	$1,604,516
Objective Function	$13,549,186				
Capital Requested	$9,032,876				
low limit (ρ)	10				
Stage	1	2	3	4	6
Projects	8	10	10	10	10
Cost of Projects	$4,183,239	$1,729,338	$1,346,013	$1,227,097	$1,917,011

Table C-4: Objective Function, Capital Requested and Cost of Projects at Varying Projects Required.

6.4. Appendix D

Below are the actual ECM data characteristics from the energy audit.

	Investment Cost ($)	Annual Energy Savings (KBTU)	Energy Rate ($/KBTU)	Annual Cost of Energy Saved ($)	Estimated Useful Life (Years)	Payback (Years)
P	γ_p	α_p	ζ_p	θ_p	N	
project1	$ 710,354	5,334,857	$ 0.015	$ 80,023	30	8.88
project2	$ 637,975	1,849,047	$ 0.033	$ 61,019	23	10.46
project3	$ 468,071	1,768,079	$ 0.023	$ 40,666	30	11.51
project4	$ 40,368	445,600	$ 0.010	$ 4,456	30	9.06
project5	$ 8,557	213,025	$ 0.012	$ 2,556	15	3.35
project6	$ 15,328	124,584	$ 0.023	$ 2,865	9	5.35
project7	$ 55,207	287,971	$ 0.027	$ 7,775	15	7.10
project8	$ 59,355	416,045	$ 0.022	$ 9,153	15	6.48
project9	$ 84,738	559,247	$ 0.015	$ 8,389	30	10.10
project10	$ 188,994	801,565	$ 0.033	$ 26,452	40	7.14
project11	$ 142,377	660,074	$ 0.023	$ 15,182	30	9.38
project12	$ 186,520	440,470	$ 0.033	$ 14,536	30	12.83
project13	$ 165,932	2,243,077	$ 0.012	$ 26,917	15	6.16
project14	$ 169,521	650,787	$ 0.023	$ 14,968	20	11.33
project15	$ 95,238	554,558	$ 0.027	$ 14,973	15	6.36
project16	$ 220,871	1,366,652	$ 0.019	$ 25,966	15	8.51
project17	$ 201,577	793,782	$ 0.030	$ 23,813	30	8.46
project18	$ 119,351	724,725	$ 0.033	$ 23,916	23	4.99
project19	$ 152,286	488,525	$ 0.023	$ 11,236	30	13.55
project20	$ 95,631	632,278	$ 0.010	$ 6,323	30	15.12
project21	$ 53,495	518,592	$ 0.012	$ 6,223	15	8.60
project22	$ 276,920	1,551,851	$ 0.023	$ 35,693	20	7.76
project23	$ 94,078	1,135,237	$ 0.027	$ 30,651	20	3.07
project24	$ 228,071	784,038	$ 0.026	$ 20,385	15	11.19
project25	$ 236,862	2,103,902	$ 0.014	$ 29,455	10	8.04
project26	$ 438,530	1,678,580	$ 0.023	$ 38,607	23	11.36
project27	$ 558,439	3,212,065	$ 0.029	$ 93,150	12	6.00
project28	$ 84,237	2,054,672	$ 0.020	$ 41,093	10	2.05
project29	$ 18,149	138,751	$ 0.013	$ 1,804	26	10.06
project30	$ 64,378	420,774	$ 0.017	$ 7,153	20	9.00
project31	$ 387,393	2,743,397	$ 0.026	$ 71,328	15	5.43
project32	$ 266,812	937,263	$ 0.030	$ 28,118	25	9.49
project33	$ 185,099	2,236,000	$ 0.011	$ 24,596	20	7.53
project34	$ 205,145	1,664,432	$ 0.017	$ 28,295	10	7.25
project35	$ 195,433	3,599,559	$ 0.014	$ 50,394	23	3.88
project36	$ 184,600	750,238	$ 0.019	$ 14,255	28	12.95
project37	$ 110,377	1,045,732	$ 0.012	$ 12,549	23	8.80
project38	$ 252,736	1,533,356	$ 0.021	$ 32,200	37	7.85
project39	$ 157,354	2,043,132	$ 0.020	$ 40,863	18	3.85
project40	$ 247,218	1,573,358	$ 0.028	$ 44,054	20	5.61
project41	$ 256,421	1,806,445	$ 0.024	$ 43,355	25	5.91
project42	$ 152,886	2,399,913	$ 0.012	$ 28,799	28	5.31
project43	$ 455,000	2,448,183	$ 0.022	$ 53,860	36	8.45
project44	$ 473,225	3,500,838	$ 0.017	$ 59,514	33	7.95
project45	$ 127,011	883,506	$ 0.017	$ 15,020	14	8.46
project46	$ 492,782	1,802,085	$ 0.016	$ 28,833	32	17.09
project47	$ 266,790	1,010,352	$ 0.031	$ 31,321	10	8.52
project48	$ 115,006	741,117	$ 0.025	$ 18,528	20	6.21

Totals	**$ 10,402,698**	**66,372,316**		**$ 1,351,279**		

Table D-1: ECM Data in Practical Application

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