ABSTRACT

Title of Document:	INTEGRATED DYNAMIC DEMAND MANAGEMENT AND MARKET DESIGN IN SMART GRID
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Smart Grid is a system that accommodates different energy sources, including solar, wind, tidal, electric vehicles, and also facilitates communication between users and suppliers. This study tries to picture the interaction among all new sources of energy and market, besides managing supplies and demands in the system while meeting network's limitations. First, an appropriate energy system mechanism is proposed to motivate use of green and renewable energies while addressing current system's deficiencies. Then concepts and techniques from game theory, network optimization, and market design are borrowed to model the system as a Stackelberg game. Existence of an equilibrium solution to the problem is proved mathematically, and an algorithm is developed to solve the proposed nonlinear bi-level optimization model in real time. Then the model is converted to a mathematical program with equilibrium constraints using lower level's optimality conditions. Results from different solution techniques including MIP, SOS, and nonlinear MPEC solvers are compared with the proposed

algorithm. Examples illustrate the appropriateness and usefulness of the both proposed system mechanism and heuristic algorithm in modeling the market and solving the corresponding large scale bi-level model. To the best knowledge of the writer there is no efficient algorithm in solving large scale bi-level models and any solution approach in the literature is problem specific. This research could be implemented in the future Smart Grid meters to help users communicate with the system and enables the system to accommodate different sources of energy. It prevents waste of energy by optimizing users' schedule of trades in the grid. Also recommendations to energy policy makers are made based on results in this research. This research contributes to science by combining knowledge of market structure and demand management to design an optimal trade schedule for all agents in the energy network including users and suppliers. Current studies in this area mostly focus either in market design or in demand management side. However, by combining these two areas of knowledge in this study, not only will the whole system be more efficient, but it also will be more likely to make the system operational in real world.

INTEGRATED DYNAMIC DEMAND MANAGEMENT AND MARKET DESIGN IN SMART GRID

By

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Dedication

To my lovely parents, Zohreh and Akbar, who sacrificed their comfort for my dreams.

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Chapter 1: Introduction

Motivation and Background

Carbon Dioxide is one of the major reasons of global warming. Using electricity generated from fossil fuels means CO_2 is being released into the atmosphere. Fossil fuels not only are not environmentally friendly, but also have increasing prices due to their limited availability. A comprehensive interviews supplemented by academic literature (Sovacool, 2009), summarized the four most important mechanisms in promoting renewable energy and energy efficiency as: eliminating subsidies, altering electricity prices, forcing utilities to adopt renewable energies (FIT policies), and increasing the public information and supporting the low income families. In recent decade demand for green power has increased significantly. Currently about one third of US electricity customers have the option to purchase green power directly from marketer or utility company (Bird, Wüstenhagen et al., 2002). Contribution to environment and slowing global warming is not only about using more environmentally friendly sources of energy, but is also about energy consumption management. Schweitzer and Tonn (Schweitzer and Tonn, 2005) surveyed several energy efficiency projects and concluded that the least effective measures were related to carpools, interest reduction programs, procurement, and home energy rating systems. While the biggest opportunities were in improvements of thermal integrity of building shells and envelopes, electric equipment, and lighting, along with better energy fuels, and employment of energy management controls for shifting demands. Demand Response (DR) programs help consumers and environment through scheduling and assigning their consumptions to optimal time periods of day with more available supply capacities.

Different studies have discussed and examined importance and value of DR programs (Bushnell, Hobbs et al., 2009, Cappers, Goldman et al., 2010, Chao, 2010, Gottwalt, Ketter et al., 2011, Kowli, Negrete-Pincetic et al., 2010). Electricity DR programs mainly follow two approaches. One is minimizing consumption and the other is shifting demands from peak hours to non-peak hours. However, shifting demands without considering real time pricing, which is dependent on market, cannot make a significant contribution to managing demands. Research has shown the benefits of dynamic pricing in DR programs (Bushnell, Hobbs et al., 2009, Chao, 2010). A study on realistic generated load profiles in Germany showed peak demand is shifted from a peak time to another time if day ahead flat and hourly price is used in DR instead of Real Time Pricing (RTP). The study also depicted that DR with indirect participation of households through retailers has less incentive for users (Gottwalt, Ketter et al., 2011). Consequently it is important to have an integrated demand scheduling and real time pricing model for demand response management in the power grid. Although several approaches and techniques are introduced for defining RTP (Samadi, Mohsenian-Rad et al., 2010), there is a major issue in implementing RTP in DR programs due to communication and exchange of information between consumers and the market. Studies relate low penetration of the RTP in DR programs to poor marketing and limitations in technical assistance to help participants manage price volatility (Albadi and El-Saadany, 2008). Although enrolled customers in DR programs in USA helped with about 38,000 MW peak load reduction in 2008, an empirical study implied only a small share (less than 10%) of total potential peak reduction is in real time rate DR programs and concluded installing more interval meters for residential units and small commercials to inform the customers of the real time rates would overcome the issue (Cappers, Goldman et al., 2010).

Pricing rules are mainly defined according to the economy and structure of market. Energy market is usually designed in two steps: forward and spot market (Baldick, Helman et al., 2005). Forward market is a beforehand (usually day ahead) market in which long term contracts are set, while spot market is real time in which contingencies and uncertainties are carried out. Studies suggest that forward market reduces the risks, mitigates market power, reduces incentives for manipulating prices, and coordinates new investments (Ausubel and Cramton, 2010, Kamat and Oren, 2004). However, it is important that spot market sends reliable price signals to forward market (Kamat and Oren, 2004). Roles and market power in electricity market affect pricing rules. O'Neill (2009), divides market process into three stages. The first stage is market rules which are decided by voting with commission approval. The second stage is Independent System Operator (ISO) who allocates rights and costs of transmission and interconnection services. Finally, the third stage is about auction market including sellers and buyers in which same rights are assumed for expressing marginal cost and value for both sides. Monopoly market with one seller has the minimum social welfare while oligopoly market with few sellers and perfectly competitive market with several sellers result in higher social welfare because of their competitive environment (Nanduri and Das, 2009). Defined functions and hierarchy among different stages and level of competition in the system determine electricity market structure (Ralph and

Smeers, 2006). The structure defines whether pricing model would be based on demand or supply function, locational marginal prices (LMP), bidding offers, or other definitions.

The DOE & ORNL (Laboratory, 2008) concluded through their surveys that the most effective energy efficiency policies are combinations of policies addressing multiple issues. Having only feed-in-tariff (FIT), which is long term contracts or payments to renewable energy producers, without removing subsidies and implementing real time pricing interferes with the idea of users conserving energy or selling it back to the system. Moreover, removing subsidies on conventional energy sources only and not implementing more realistic prices in the system wouldn't help changing usage behaviors. That is because countries are not motivated enough to not use cheaper sources of international resources compared to domestic resources. They found giving information to people is important. Usually decisions on type of energy appliances are made by ones who are not paying the bills and have contradictory objectives to ones who pay the bills (like landlords). People just look at lump sum values of the bills and have no idea about details of their consumptions. That means implementing realistic prices should be followed by giving information to consumers. When Norway increased prices suddenly by 43% due to shortage of capacity in 2003, consumers only decreased their demands by 2.3% (Aune, 2007).

"*subsidies for electricity related fuels have existed at least since 1880s, alternative rate designs since the 1950s, feed in tariff, system benefits charges, and the rest since the 1970s and 1980s*" (Sovacool, 2009). Investing in FIT but not implementing more realistic prices does not motivate consumers to invest on new technologies. To implement new pricing rules, people need to receive information. Moreover, as long as consumers do not trust the source of information they do not change their behavior. And so persuasion is very difficult. "*So what seems to be lacking is not the availability of robust public policy mechanisms, but the political and social will to implement them*" (Sovacool, 2009). So all these factors are connected to each other like a cycle and lack of any of them would break the cycle.

Challenges discussed here could be resolved by implementing a system with two-way communication among participants in the power grid and market. Two-way communication facilitates information exchange and motivates interaction among all participants in the grid which may lead to shaving peak demands and managing storage capacities more efficiently. Participants in two-way communication could be individual costumers or retailers who help customers take most advantage of their money and resources. The main goal in this system would be improving the cost value of energy consumption and accommodation of environmentally friendly energy resources while providing financial benefit. This system is called Smart Grid. A Smart Grid would employ real-time, two-way communication technologies to allow users to connect directly with power suppliers and energy market. The U.S. National Institute of Standards and Technology (NIST) defines *"The term "Smart Grid" refers to a modernization of the electricity delivery system so it monitors, protects and automatically optimizes the operation of its interconnected elements – from the central and distributed generator through the high-voltage transmission network and the distribution system, to industrial users and building automation systems, to energy storage installations and to end-use consumers and their thermostats, electric vehicles,* *appliances and other household devices. The Smart Grid will be characterized by a two-way flow of electricity and information to create an automated, widely distributed energy delivery network. It incorporates into the grid the benefits of distributed computing and communications to deliver real-time information and enable the near-instantaneous balance of supply and demand at the device level."* (Von Dollen, 2009). The U.S. National Institute of Standards and Technology (NIST) also measures DR programs and consumer energy efficiency as the highest priorities in the overview of the Smart grid: *"Market information is currently not available to the customer domain. Without this information, customers cannot participate in the wholesale or retail markets. In order to include customers in the electricity marketplace, they need to understand when opportunities present themselves to bid into the marketplace and how much electricity is needed."*[1] Implementing DR programs in Smart Grid could be effective in improving capacity margins and providing attractive alternatives to generation resources' additions and transmission upgrades (Kowli, Negrete-Pincetic et al., 2010). The German industry group, BDI, has claimed demand side management technologies as the mainstream by 2015 in the roadmap of the transition from current energy infrastructure to Smart Grid (Block, Bomarius et al., 2008). They highlighted the development of *"applications and services implementing the coordination of the energy grid on the economic level"* as one of the main challenges in this way. This dissertation is focused on development of such an application. A novel energy system mechanism along with its mathematical model and solution algorithm for an integrated

[1] Report to NIST on the Smart Grid Interoperability Standards Roadmap, Page 95, Section 6.2.1

dynamic demand response scheme in spot market for Smart Grid is developed. Optimization and game theory frameworks are employed as a basis to support the formulations and analysis.

Problem Statement

This research develops an advanced model integrating an appropriate market structure with corresponding demand management system for Smart Grid. As the future of energy and electricity market in the world and USA (Energy, 2009), Smart Grid includes several decision makers at different levels whose strategies affect the system. A hybrid market is chosen in this study to model this system which has a decentralized and more sophisticated design, but a more favorable structure after failure of California design (Baldick, Helman et al., 2005). Spot market clearing procedure is designed as a model of hierarchal optimization between ISO and Big Generating Firms (BGFs). Based on a survey from eleven market modeling experts on priorities for future market model developments, system operator should be seen as a strategic agent (Neuhoff, Barquin et al., 2005).

To design an appropriate system for the future Grid, consideration of specific characteristics of Smart Grid is inevitable. First of all, there would be smart homes with smart appliances and meters in Smart Grid. Although some appliances in a house such as refrigerators, have strict time of use some others, such as dishwashers and dryers, have flexible time of use. Distribution of energy consumption among different appliances and devices in a household is demonstrated in Table 1. As it conveys, 78% of energy usage belongs to 12 main categories of appliances which mostly have demands with flexible time of use.

Table 1 Household appliances energy consumption[2]

Appliance	% of household energy consumption	kWh/year/household
Refrigerator	13.7	1,462
Air conditioning	16.0	1,446
Heating	10.1	3,524
Water heating	9.1	2,552
Lightening	8.8	940
Cloth dryer	5.8	1,079
Freezer	3.5	1,150
TV	2.9	313
Oven	2.9	314
Dishwasher	2.5	512
Computer	1.5	318
Washer	0.9	120
Residual	22.3	47,838

Using real time pricing information provided by smart meters to users, flexible demands could be shifted from peak periods to non-peak periods. Moreover, establishing Smart Grid will increase popularity of plug-in electric vehicles on the roads. Electrical vehicles should be able to be plugged into the network in different locations and time instances for charging or discharging electricity which affects electricity network stability drastically. Finally, subscribers to Smart Grid could have independent roles as consumer, supplier, storage owner or a combination of two or

[2] U.S. Energy Information Administration (July 2012)

three of these roles. That means they need to know whether they should consume, sell or store electricity at each time step. These unique characteristics introduce new decision making problems into energy systems. Subscribers to the system need to know not only when, but also where to buy/sell their extra energy for/from their appliances and vehicles from/to the grid in a way to meet their demands while being cost efficient. Some households, large schools, hospitals, companies or restaurants which have large solar panels on their roofs, or small wind turbines in their field might generate much more electricity than what they normally consume. This would be an option for them to sell the extra energy to the grid at the market price or they could store it for their future demands in their large scale batteries. Two-way communication in Smart Grid enables subscribers or their retailers to participate in the energy trade through smart meters which could be programmed beforehand or controlled by the retailer. Participants in the trade face a two dimensional decision making problem: location and schedule of their demands, supply, and storage based on their individual preferences and real time prices. Given the notice, dependency among these decision making problems due to elasticity of price to total supply and demand in the grid, a game approach should be incorporated in the model. So in the proposed model each participant looks for his best strategy in a non-cooperative game as his schedule of demand and supply to maximize his payoff based on his utility function and real time market price.

There are three main categories of decision makers in this problem (Figure 1): Independent System Operator (ISO), Big Generating Firms (BGF), and all Subscribers to the Demand Response program (SDR) in the system. Each decision maker has its own objective, constraints, limitations, and solves its own problem. However, each decision affects others' decisions.

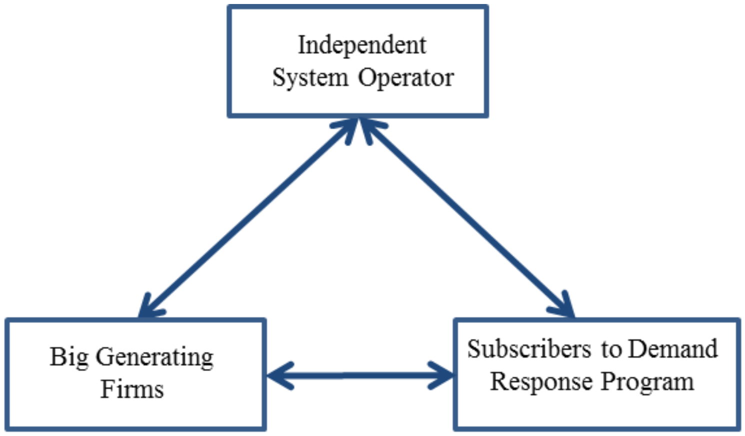

Figure 1 Decision makers in the system and their flow of influence

ISO operates the transmission and distribution network and clears market as an independent decision maker. ISO decides on amounts of allowable trades among all the participants in the market based on the specifications of the system and limitations of the network. This decision making problem will be referred to as ISO's decision problem in this study.

Big Generating Firms, BGF, are the firms owning large generating facilities or the ones capable of providing large amounts of supply to the system such as large heat engines, large wind turbines, etc. BGF's decisions can highly impact the system through availability of their generating capacity which determines market price. Their objective is to maximize their individual revenue while adhering to their capacity and network limitations. Since the number of BGFs is usually more than one, they compete against each other in an oligopoly market.

The last category includes majority of decision makers who subscribe to the Demand Response program in the system and participate in the trades directly through programming their smart meters or having a retailer to act on their behalf through controlling their smart meters and appliances. These players are referred to as SDR in this study. Their objective is to maximize their individual benefit, which here is defined as their individual welfare, while scheduling their demand and supply dynamically according to network limitations.

Modeling Smart Grid in its real size is almost impossible considering all the computational difficulties. In Chapter 3, an approach is proposed to deal with the large size and complexity of this system while solving the problem in real world size.

Research Context and Scope

The broad context for this research is design of an integrated market mechanism and demand management model in Smart Grid at a zonal level. In this research interaction among participants in the network under the influence of the market specifications and network characteristics is studied.

Current demand management studies are mostly appropriate for the old power grid (Albadi and El-Saadany, 2008). Few available studies that are related to Smart Grid (Chen, Kishore et al., 2011, Kiani Bejestani and Annaswamy, 2010, Mohsenian-Rad, Wong et al., 2010) do not consider and cover main characteristics of Smart Grid including location based decision making, two-way trades for each subscriber at each time step, incorporation of distributed storage capacities in the network, and most importantly including a market structure considering all these characteristics.

In this study a novel integrated dynamic market mechanism is used to propose a demand response model for location-based scheduling of demand, supply and storage of subscribers in the Smart Grid based on real time pricing and players' preference function in a game theoretic approach while the market is being managed by the system operator at a zonal level.

Computational tractability of optimization models and long term modeling capability of equilibrium models make their combination an appropriate approach to this problem. Few works attempted to solve multi player non cooperative games and they mostly used iterative methods (Nanduri and Das, 2009). In this study we attempt to solve large scale multi player non cooperative game through optimization techniques. The game among participants is modeled as an optimization problem for each individual to maximize its benefit, while the market structure is modeled as a one-leader multi-follower Stackelberg game through bi-level programming. Existence and validation of the approach is then proved through mathematical techniques. A decomposition based algorithm is used to develop a real time algorithm solution for the model in large scale. Mathematical proofs and solutions from the corresponding Mathematical Program with Equilibrium Constraints (MPEC), mixed integer, and SOS integer models are used to evaluate the proposed solution algorithm. Finally, the proposed model is evaluated on several case studies and sensitivity analysis is conducted on the parameters to show their effects on the decisions of the participants in the system.

The proposed model makes Smart Grid run easier and promotes a certain level of disaggregation in such a large system. The results of this research could be implemented in the future Smart Grid meters to help users and market operators communicate together and enable accommodation of different resources of energy in the network. It would help preventing waste of energy by optimizing users' schedule of trades in the grid. Moreover, recommendations to energy policy maker can be made upon some of the results. This research is mostly focused on the operation side of the demand response program in the future smart grid system. Therefore, the market design is not detailed in all its aspects.

Research Objectives

This research has the following main objectives:

- Propose a dynamic demand response model for location-based scheduling of demand, supply and storage of all subscribers in the Smart Grid which considers real time pricing and a preference function of the users in a game theoretic approach at a zonal level.
- Propose an integrated dynamic market equilibrium and demand response model for Smart Grid to manage and control the zonal energy market in the system.
- Develop a solution approach for the identified problems using mathematical techniques

Contributions

There are many future suggestions based on current game theory studies in both demand side management and micro grid distribution networks in Smart Grid which are summarized in (Saad, Han et al., 2012). This dissertation which is a combination of both micro-grid distribution and demand side management covers most of recommendations on future extensions to the state of the art mentioned in the current literature.

This dissertation will make the following contributions to the energy market and demand management state of the art:

- Designing a dynamic game for Demand Response program in the Smart Grid which contributes to the state-of-the-art through:
    - Location-based decision making
    - Two-way trades for each subscriber at each time step
    - Incorporating distributed storage capacities in the network
    - Joint scheduling and storage optimization
    - Considering multiple energy sources beside multiple consumers
    - Enabling customers to optimize tradeoff between waiting time and billing charges for using appliances
    - Considering strategic energy source whose objective is not aligned with that of the consumers
    - Applicability of the model for Smart Grid by considering its specific characteristics
- Designing an appropriate market mechanism to address current system's deficiencies
- Integrating market design and demand response program in a way to be applicable to Smart Grid at the zonal level
- Developing games with players with different and opposing strategies
- Developing solution algorithm for solving the proposed large scale Bi-level problem
- Conducting sensitivity analysis on exogenous parameters

Dissertation Outline

Following the introduction in Chapter 1 which presented the motivations, problem statement, scope, objectives and research contributions, Chapter 2 reviews the related research in the literature. Chapter 3 is dedicated to defining market mechanism and problem formulation. It presents assumptions, definitions, and the mathematical model for the problem. Chapter 4 introduces the solution methodology and validates the approach. It also describes the different evaluation methods to be used in this study. Chapter 5 documents different case studies and numerical results. And finally, Chapter 6 concludes the dissertation with a summary, conclusion, recommendations and future research path.

Chapter 2: Literature Review and Background

This dissertation is based upon combining two areas of research, energy market equilibrium and demand response modeling, which was rarely done previously. In this section these two areas of knowledge are reviewed and finally the only work related to combination of the two areas is discussed.

Market Equilibrium

Recently there has been a fair amount of literature devoted to finding market equilibrium. Market structure and regulations define the modeling approach for finding market equilibrium. Approaches mostly fit into one of these categories:

Cournot/Bertrand models, Supply Function Equilibrium (SFE), Conjecture Supply Function (CSF), Stackelberg, and Auction based models.

In Cournot/Bertrand models, agents compete on quantity/price strategies for the Nash equilibrium which is the best response of an agent to its opponents' strategy. Jing-Yuan and Smeers (1999) modeled an oligopolistic Cournot game for regulated transmission prices and generating firms and used Variational Inequality technique for solving it. Yao and Oren (2004) modeled a Stackelberg game for two settlement markets with forward market in the upper level and Cournot generating firms and Cournot ISO in the lower level while ISO decides on import/export quantities at nodes. They implemented two computational approaches: one based on penalty interior point algorithm (PIPA) and the other based on steepest descent approach. Yao and Willems (2005) capped forward prices and spot prices separately and concluded there is less incentive for generators to commit to forward contracts due to spot price caps, but there are more incentives due to forward price caps. They showed spot zonal prices decrease under both cap settlements compared to single settlement cases. Since this problem could be decomposed to generators, they concluded there is no importance of generators' ownership in the model. However, when transmission constraints were nonbinding, equilibrium gave uniform nodal prices that were systematically higher than Cournot equilibrium price corresponding to a single market with the aggregated system demand function. So they concluded that import/export variables for ISO as strategic variables are not satisfactory. To overcome the problem, Yao and Adler (2008) modeled a stochastic Stackelberg game for two settlement markets with forward market in the upper level. They assumed Cournot generating firms and Bertrand ISO in the lower level. Strategic variables for ISO were locational price premiums. They then solved the Equilibrium Problems with Equilibrium Constraints (EPEC) using their iterative approach on Belgian electricity network.

In SFE models firms compete over their offer curve strategies for the Nash equilibrium. Hobbs and Metzler (2000) proposed an oligopolistic game among supplier firms which was modeled as a MPEC. Leading firm in the upper level decided on the intercept of the bidding supply function and in the lower level ISO solved the quadratic problem for the single commodity Spatial Price Equilibrium (SPE) and linearized DC Optimal Power Flow (OPF) problem for all other firms to find their generation and demand quantity and transmission allowances. They used penalty interior point algorithm for solving the MPEC model. In their multi firm problem each firm solves its MPEC individually.

In CSF models, generating firms conjecture regarding their rival firm's sales' adjustment in response to price changes. CSF first used in power market in a work from Day and Hobbs (2002). This type of modeling is more flexible in large size transmission networks due to their smooth function; however, they have more behavioral parameters to calculate which should be estimated through empirical studies or sensitivity analysis. They claim CSF is a more realistic model for imperfection competitions for 3 main reasons: 1) CSF includes Cournot conjecture models as a special case. 2) Cournot models cannot be used when price elasticity in demand is zero. 3) Unlike supply function equilibrium models, CSF equilibriums can be used for large scale models.

In Stackelberg games one or more agents are leaders and other firms and agents will be followers. Decision on the hierarchy of decision makers in the model depends on assumptions and market organization defined in the problem (Ralph and Smeers, 2006). These problems are mostly modeled as EPEC models and leader decision problem is in the upper level while followers are in the lower level. Gabriel and Leuthold (2010) modeled an energy market with network constraints as a Stackelberg problem with a leading firm in the upper level and ISO as follower. The equilibrium constraints of the MPECs were converted to integer constraints and solved for a fifteen node network of the Western European grid. Pozo and Contreras (2011) formulated strategic bidding problem in pool based electricity market for joint price and quantity bids in multi agent, multi period and multi block games. They proposed finding multiple Nash equilibriums through an iterative procedure by adding a constraint emitting the hole containing previous solution. They linearized EPEC using strong duality for the lower level and used Fortuny-Amat representation and a binary expansion for the bilinear expression.

In auction based models, a bidding mechanism and market clearing procedure are designed. Gross and Finlay (1999) proposed a sealed bid auction model for determining the optimal bidding strategies of a bidder in a competitive electricity generation market. The only aspect they considered was generation strategy for generators. Attaviriyanupap and Kita (2005) proposed a bidding strategy for a day-ahead market. Bidding parameters for markets are determined through a non-convex optimization which they solved through evolutionary approach. They considered both single and double auctions. Double auction models for market mechanism in electricity and heat network has also been introduced in different studies (Block, Neumann et al., 2008, Block, Collins et al., 2009, Carsten, John et al., 2010). Zou (2009) designed a double auction mechanism to control the market power by transferring payments among participants based on their contributions to social welfare. Vytelingum and Ramchurn (2010) developed a market mechanism based on continuous double auction which manages congestion through pricing the electricity flow. They decided on quantities in an optimization approach and prices through a double auction. Wen and David (2001) assumed suppliers/large consumers bid a linear supply/demand function and maximized social welfare. They tried to find the optimal coefficients in the functions through stochastic optimization modeling using Monte Carlo approach. In another study, Lamparter and Becher (2010) proposed an agent-based double auction bidding mechanism for Smart Grid to maximize social welfare. Duan and Deconinck (2010) introduced a multi-agent model for the market in smart Grid considering all different agents in 3 phases: activation, negotiation and conclusion. They examined different types of auctions in the simulation such as Frist Price Sealed Bid (FPSB), Vickery, English, and Dutch.

There are also studies on energy market based on optimization models for one firm, and simulation based models (Marks, 2006, Ventosa, Baıllo et al., 2005) which do not necessarily look for an equilibrium in the market.

The studies on energy market equilibrium and their main characteristics are summarized in Table 2.

Demand Response Program

Users or system operators can support the grid by adjusting loads in time. Demand response refers to costumers' consumption adaptation to market either through shifting demands or reducing demands. Different demand response programs have been designed and implemented in energy and specifically electricity networks (Rahimi and Ipakchi, 2010).

Demand response programs can be divided into two main categories (Albadi and El-Saadany, 2008, Bollen, 2011): incentive based and price based programs. In incentive based programs customers receive either credit or money for reducing their consumption or shifting it. However in price based programs customers match their consumptions based on the prices in the system. Incentives can be either in form of curtailment programs or market based programs which can reward participants with bill credits, money rewards, or discounted rates. However, price based demand response programs are based on non-flat rates in the system during day which can be in different forms such as:

Table 2 Summary of literatures on energy market equilibrium modeling

Reference	*Dynamic*	*Stochastic*	*Network Constraints*	*Two Way Trade*	*Storage Capacity*	*Participants*	*Market Structure/ Model*	*Solution Approach*	*Example Size*
(Day, Hobbs et al., 2002)	×	×	√	×	×	ISO, BGFs, Arbitrager	CSF	LCP	13 nodes, 56 plants, 21 flow gates
(Yao, Adler et al., 2008)	√	√	√	√	×	ISO, BGFs	Cournot Suppliers-Bertrand ISO	EPEC-Iteratively solving MPECs	Belgian network (71 lines, 53 nodes)
(Pozo and Contreras, 2011)	√	√	×	×	×	ISO, BGFs	Stackelberg	EPEC-MILP	6 generators
(Yao, Willems et al., 2005)	√	√	√	√	×	ISO, BGFs	Cournot Suppliers-Cournot ISO	EPEC	Belgian network (71 lines, 53 nodes)
(Yao, Oren et al., 2004)	√	√	√	√	×	ISO, BGFs	Cournot Suppliers-Cournot ISO	EPEC-Iterative PIPA & RSM	
(Gabriel and Leuthold, 2010)	×	×	√	×	×	ISO, BGFs	Stackelberg	MPEC-MILP	Western European Grid (15 nodes)
(Hobbs, Metzler et al., 2000)	×	×	√	×	×	ISO, BGFs	SFE	MPEC-PIPA	30 nodes, 41 lines
(Jing-Yuan and Smeers, 1999)	√	×	√	×	×	Transmission, BGFs	Cournot	VI	4 nodes
(Block, Neumann et al., 2008)	×	×	×	√	×	Seller , Buyer	Double Auction	MILP-Heuristic	
(Carsten, John et al., 2010)	√	√	×	√	×	Seller, Buyer, Brokers	Double Auction	Simulation	
(Block, Collins et al., 2009)	√	√	×	√	√	Seller, Buyer, Brokers	Double Auction	Simulation	
(Duan and Deconinck, 2010)	√	×	×	√	×	Sellers, Buyers, Brokers	Multi Agent	Simulation	
(Gross, Finlay et al., 1999)	√	×	×	×	×	BGFs	Sealed Bid Auction	Optimization-Lagrangian Relaxation	
(Vytelingum, Ramchurn et al., 2010)	√	×	×	√	×	Sellers, Buyers	Double Auction	Optimization for Quantities-Algorithm for Prices	

(Attaviriyanupap, Kita et al., 2005)	√	√	×	√	×	BGFs, Big Consumers	Single Auction & Double Auction	Optimization-Evolutionary programming	
(Zou, 2009)	√	×	×	√	×	Sellers, Buyers	Double Auction	Mechanism Design-Algorithm	
(Wen and David, 2001)	×	√	×	√	×	BGFs, Big Consumers	Double Auction	Optimization-Monte Carlo	6 suppliers, 2 consumers
(Lamparter, Becher et al., 2010)	×	×	×	√	×	Sellers, Buyers	Double Auction	Mechanism Design-Agent Based	

- Time of use pricing: In this format there are higher prices during predefined expected peak periods.
- Critical peak pricing: In this format higher prices are in place during up to 15 extreme days of the year and flat rate is in place other times.
- Multi-tier prices: In this option prices are per kWh consumption over a certain consumption level or per carbon dioxide emission for a certain amounts of consumption.
- Real time pricing: In this system prices are fluctuating based on the real value of electricity in the market.

Benefits of non-flat pricing in DR programs are well known (Bushnell, Hobbs et al., 2009, Chao, 2010). Albadi and El-Saadany (2008) defined DR program benefits in four main categories: benefits to participants, market wide benefits, reliability and market performance. They divided DR costs for both participants and program owners to initial and running costs such as establishing technologies and inconvenience costs. Gottwalt and Ketter (2011) simulated a demand response program with realistically generated load profiles in Germany and used both day ahead flat and hourly prices. However, they concluded that peak demand is being shifted from a peak time to another time when real time pricing is not in effect.

DR programs schedule and assign consumptions in different time steps. Approaches toward modeling DR programs are optimization based, game based, agent based, and algorithm based modeling.

Optimization based models are mostly LP with one objective function. They model demand response for a user/users with a single resource of supply which could be a retailer or a utility company. Pedrasa and Spooner (2009, 2010) modeled appliances' scheduling through an optimization model with a fitness function for the users. They used Particle Swarm and Binary Particle Swarm Optimization method as a heuristic population based search technique for solving the model.

Mohsenian-Rad and Leon-Garcia (2010) proposed an LP optimization model for scheduling appliances with a tradeoff between minimizing waiting time cost and minimizing payments. They discussed different scenarios and adopted them into their model such as storage capacities in Plug in Hybrid Electric Vehicles (PHEV), and interruptible and uninterruptable residential loads. They depicted peak to average load ratio decreases drastically and claimed that as an incentive for utilities to deploy the model in large scale. They also showed that a combination of their designed scheduling and real time pricing would result in consumers' payments reduction.

Conejo and Morales (2010) proposed a LP robust optimization model for real-time demand response in Smart grid. They assumed price information is being communicated to the users hourly. Their model is dynamic on rolling horizon basis and uncertainty in price is considered. LP solver is used for solving the model. They showed users benefit from an increased utility by incorporating the proposed model.

Erol-Kantarci and Mouftah (2011) modeled a simple linear scheduling model for demand response and used heuristics for solving it.

Lujano-Rojas and Monteiro (2012) used predictions of electricity prices, energy demand, renewable power production and power purchase of the costumers to determine the optimal utilization of different appliances and electrical vehicles through negotiations between retailer and costumer. They showed that the number of combinations increases exponentially and so used genetic algorithm for solving the problem. They claimed their proposed model would reduce electricity bill 8-22% for a typical summer day for users.

Contrary to optimization models which only consider one objective, game-based models consider objectives of all players in the game. Game-based studies in DR programs mostly assume competition is among users of a single source of supply.

Mohsenian-Rad and Wong (2010) modeled demand response to minimize cost of generation in the system and users' payments minimization while distributing loads on time horizon to minimize peak to average load ratio (PAR). They showed that minimizing cost results in PAR reduction if the cost function is strictly convex and increasing. Their price function was proportional to total daily energy consumption of each user. However, in their proposed model network constraints, storage capacities, and location based decisions are not included.

Chen and Li (2010) using supply function equilibrium as pricing model, designed two demand response models for matching and shaping demands. They showed in a competitive market where customers are price takers, system achieves an efficient equilibrium that maximizes the social welfare, while in an oligopolistic market where customers are price anticipating and strategic, the system achieves a unique Nash equilibrium that maximizes another additive global objective function.

Wang and Kennedy (2010) designed a bidding mechanism among retailers and suppliers for demand response. Generators and retailers submit bids for the time horizon considering price elasticity matrix which is the change in electricity consumption at a scheduled hour due to a change in electricity price of that same hour or any other hour. Their bidding mechanism is carried out by an iterative market clearing algorithm.

Li and Chen (2011) proposed a demand response model in which users try to maximize their own benefit which leads to the retailer's benefit. They used an interesting approach for modeling different devices including electrical vehicles. They defined the real time price function and showed that everyone would benefit from the real time price.

Zhu and Basar (2011) found the Nash equilibrium for a stochastic model of demand response and scheduling of demands among large population. They introduced a multi-resolution stochastic differential game framework to capture macroscopic and microscopic interactions among a large population of players.

Chen and Kishore (2011) introduced a Stackelberg game among Energy Management Controller (EMC) in each home as follower and service provider as the leader based on a RTP model. Their RTP model is based on a retail price consisting of wholesale price (function of production) and the price gap which they designed to figure out the influence of the difference between the actual demand and available supply. They showed the model is beneficial for both reduction in consumption and consumers' payments.

Bu and Yu (2011) proposed a 4 stage Stackelberg model for decision making on amounts retailers should buy from the electricity pool, the price they should offer to the costumers, and finally the amounts costumers should buy. Retailers have the option of choosing supplies from cheap and uncertain supply or expensive and certain supply in the first two stages. They used a backward induction method to solve the game.

Li and Jayaweera (2011) proposed a two-level model in utility companies' level. In first stage the weighted sum of generation cost and operation delay cost for the utility is minimized as a convex optimization. In second stage using a game theory approach, a Vickrey auction is run for scheduling demands to maximize social welfare for the users. They showed truthful bidding is a weakly dominant strategy for all costumers in a one-time Vickrey auction. One of the disadvantages of this model is that demands for appliances are not specifically determined and they assumed demands are divisible. Samadi and Schober (2011) proposed a Vickrey Clarke Groves based mechanism for the scheduling model in (Mohsenian-Rad, Wong et al., 2010) to maximize social welfare in which users reveal their truthful utility functions.

Fan (2011) used a different approach and proposed a distributed framework for demand response and user adaption in smart Grid based on congestion pricing concept in Internet traffic control. Applying differential equation, they found game equilibrium and showed load can be shifted by users through pricing strategies.

There are few agent based models in energy management area. Vandael and Boucké (2010) proposed a multi agent model for demand management of the plug-in hybrid vehicles. Vytelingum and Voice (2010) proposed a game agent based model for optimizing storage devices strategies. They proved the Nash solution and empirically showed the convergence to the Nash solution. They discussed the learning curve of the agents and showed that when learning rate is low, the social welfare is maximum. Their game approach implied that when about 38% of the population owns storage devices, the social welfare is the maximum.

Algorithm based models are basically about randomly assigning consumptions. Lee and Park (2011a, 2011b) presented a genetic algorithm and an assignment heuristic for scheduling of appliances. A summary of the studies on Demand Response modeling is presented in

Table 3.

This dissertation is based upon combining the two areas of research, energy equilibrium and demand response modeling, which to the best of the author's knowledge has not been studied previously. The only study which is related (Kiani Bejestani and Annaswamy, 2010) modeled market equilibrium and demand response through a MLCP model considering ISO, consumers and generating companies. However, their approach has major differences from this study. Their model does not consider any hierarchy of decision making among decision makers and, more importantly, their demand response program is very general and only determines quantities and not schedule of demand/supply. Storage capability and location-based decision are not supported. They also assume exogenous bidding prices for participants and use LMP for market pricing. Sellers and buyers are separate entities. The study is mainly focused on market design, not demand management. Kiani and Annaswamy (2011) improved their work with a different approach to allow information exchange among dominant players such as real time price, congestion price, generation and consumption level and captured the dynamics of the real time market using the state-based game. They also investigated the stability of model under renewable energy and region of attraction uncertainty. Table 4 shows their work specifications and their differences from this research.

Table 3 Summary of literatures on demand response modeling

Reference	*Appliance Based*	*Network Constraints*	*Storage Capacity*	*Location Based*	*Stochastic-Probabilistic*	*Pricing Method*	*Model*	*Solution Approach*	*Example size*
(Lujano-Rojas, Monteiro et al., 2012)	√	×	×	×	×	RTP-time series based	Optimization-Enumeration	Genetic algorithm	1 user, 5 appliances, T=24
(Erol-Kantarci and Mouftah, 2011)	√	×	×	×	×	Exogenous tariffs	Optimization-LP	Heuristic	4 appliances, T=24, 10-210 days
(Fan, 2011)	×	×	×	×	×	RTP-Congestion pricing in Internet traffic control	Game	Differential equation approach	10 users
(Samadi, Schober et al., 2011)	√	×	×	×	×	Both approaches: price takers and price anticipating participants	Mechanism design-Vickrey Clarke Groves	Simulation	10 users, 10 appliances, T=24
(Zhu and Basar, 2011)	√	×	×	×	√	RTP-Demand function	Game	Multi resolution stochastic differential game framework	
(Li, Chen et al., 2011)	√	×	√	×	×	RTP-Game based	Game	Distributed algorithm based on	8 users, 6 appliances each, T=24

								gradient algorithm	
(Lee, Park et al., 2011a, Lee, Park et al., 2011b)	√	×	×	×	×		Scheduling algorithm	Heuristics-assignment approach & genetic algorithm	
(Chen, Kishore et al., 2011)	√	×	×	×	×	RTP-function of production	Game-Stackelberg	Backward induction	50 users, 3 appliances, one retailer, T=24, 10 min slots
(Bu, Yu et al., 2011)	×	×	×	×	×	RTP-Retailer decision variable	Game-Stackelberg	Backward induction	10 users
(Li, Jayaweera et al., 2011)	×	×	×	×	×	Bidding offers	Two level: Optimization & Vickrey auction mechanism	KKT optimality condition-Iterative algorithm	5000 users, 1 utility, T=24
(Mohseni an-Rad and Leon-Garcia, 2010)	√	×	√	×	×	RTP-simple weighted average price prediction model	Optimization-LP	LP techniques-Interior point method	1 user, 25 appliances, T=24, sensitivity to up to 10 users
(Conejo, Morales et al., 2010)	×	×	×	×	√	RTP(robust optimization)	Optimization-LP	Robust optimization for prices	1 user, T=24
(Pedrasa, Spooner et al., 2010)	√	×	√	×	×	Exogenous peak/non-peak pricing	Optimization-Fitness Func.	Heuristics-Particle Swarm Optimization	1 user, 4 appliances, T=24
(Vandael, Boucké et al., 2010)	×	×	×	×	×		Agent based	Multi agent system	Only PHEVs in the system
(Vyteling um, Voice et al., 2010)	×	×	√	×	×	RTP-Weighted moving average price prediction based on supply function	Game based agent based	Evolutionary game based heuristic	
(Mohseni an-Rad, Wong et al., 2010)	√	×	×	×	×	Proportional to their total daily energy consumption	Game based optimization	Iterative distributed algorithm	10 users, 10-20 appliances, T=24
(Chen, Li et al., 2010)	×	×	×	×	×	Competitive market/Oligo poly market (Demand function)	Game	Distributed demand response algorithm	10 users
(Wang, Kennedy et al., 2010)	×	×	×	×	×	RTP-Price elasticity matrix	Game-Bidding mechanism	Iterative market clearing algorithm	6 bus system with three retailers
(Pedrasa, Spooner	×	×	×	×	×		Optimization-Fitness Func.	Heuristics-Binary Particle	

et al., 2009)											Swarm Optimization	

Table 4 Comparison of this research's specification to current literature

Reference	*Dynamic*	*Stochastic*	*Network Constraints*	*Two Way Trade*	*Appliance based*	*Storage Capacity*	*Location Based*	*Participants*	*Pricing*	*Model*	*Solution Approach*	*Example Size*
(Kiani Bejestani and Annaswamy, 2010)	√	×	√	√	×	×	×	ISO, BGFs, Consumer Firms	RTP-Curtailm ent factor	Cournot Game-One Level	MLCP	IEEE 4 Bus, 2Users, 2BGFs
(Kiani and Annaswamy, 2011)	√	×	√	√	×	×	×	ISO, BGFs, Consumer Firms	RTP-LMP	Cournot Game-One Level	State based Game based on gradient play	IEEE 30 Bus
THIS RESEARCH	√	×	√	√	√	√	√	ISO, BGFs, DSRs	RTP-Inverse Supply Function	Cournot Game-Stackel berg	Heuristic-MILP	Large Scale

Chapter 3: Market Mechanism and Modeling

Problem Structure

Rules and structure of the market determines most of the interactions among participants in the market. In this chapter, the system mechanism, different layers of market, market players' role and all other assumptions made through this study are introduced.

Dealing with One Large and Complex System: Layers of Smart Grid

As mentioned earlier, smart grid in its real size is a very large and complex system. Dealing with this large and complex system in real time is a real challenge. One way to overcome this issue is to decompose the whole system into different layers while considering the connection among them. This study tries to capture these connections and picture the whole system as a puzzle with several pieces. It then target each piece and models it. Then model all pieces' relations and glue them together and complete the puzzle.

There are several national electric power markets in USA, such as California (CAISO), New England (ISO-NE), and PJM (Figure 2). Each national market has several control regions (Figure 3). To model the market in Smart Grid it is assumed that each control region contains several zones. Zones include all users willing to participate in the market directly such as households in addition to all the retailers having customers in that zone. Retailers act on behalf of their customers who are not

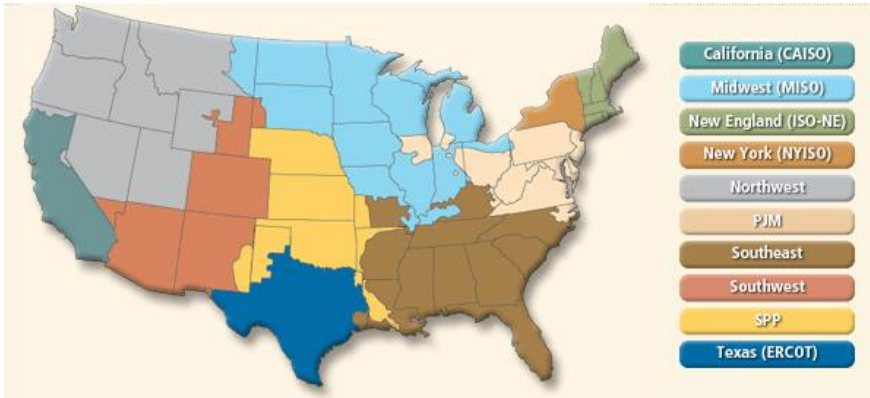

Figure 2 Electric power markets: national overview[3]

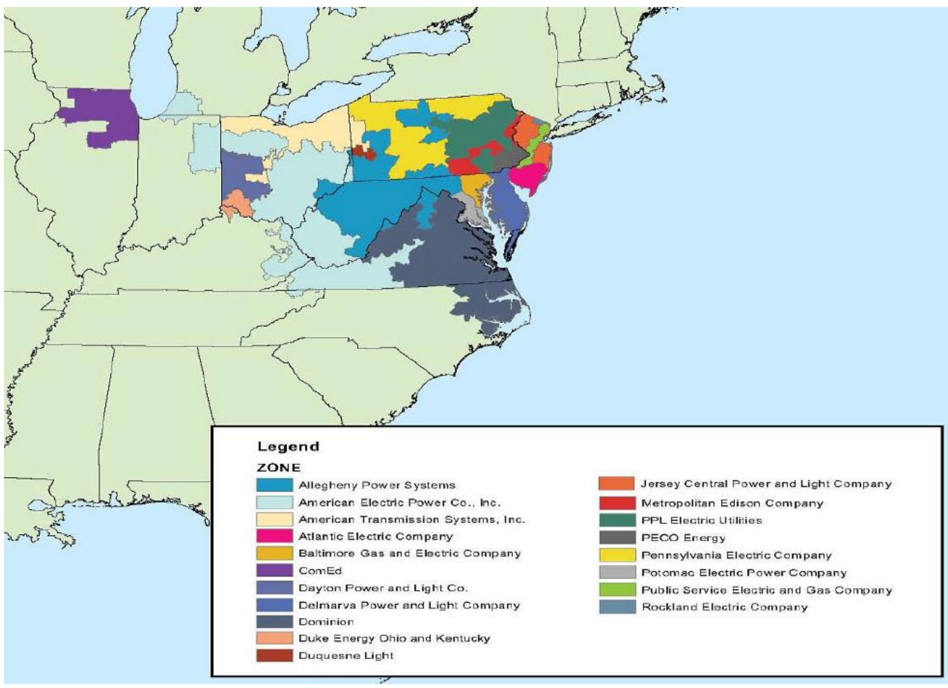

Figure 3 PJM control regions[4]

willing to participate in the market directly and have transferred responsibility of controlling their appliances and electricity trades to their retailers. Zone sizes may differ depending on regional characteristics and infrastructure. This market structure is

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[3] http://www.ferc.gov/

[4] http://www.ferc.gov/

not limiting and could be applied to current system if one assumes all households are required to choose a retailer to act on their behalf in the market.

Considering the above-mentioned structure, three main layers are introduced in this study for the Smart Grid: Zonal, Regional, and Cross Regionals. The first layer is in the Zonal layer in which households and local retailers are SDR players in the market. Generating companies and big suppliers such as BGE and PEPCO who are willing to provide electricity to that zone are playing as BGFs. Finally a Zonal ISO clears the market and manages the distribution network. At the second layer which is the Regional level, each Regional market operator such as PJM and CAISO would act as an ISO and clears and manages the transmission network. All suppliers and large generating firms which have participated in zonal markets which are covered by that Regional market would act as SDR while their supply commitments to zonal markets would become their responsibility and demands. All other Regional markets willing to import or export electricity to that specific Regional market have the role of BGFs in that market. Finally, in the third layer, Cross Regional market, a higher level entity such as government would play the role of ISO. Regional markets such as CAISO and PJM would act as SDRs in the system while their supply commitments would become their responsibilities. All other countries and markets willing to trade in the Cross Regional level with the above-mentioned market would act as BGFs in the system. This assumed general market structure and participants roles in the Smart Grid are summarized in Figure 4.

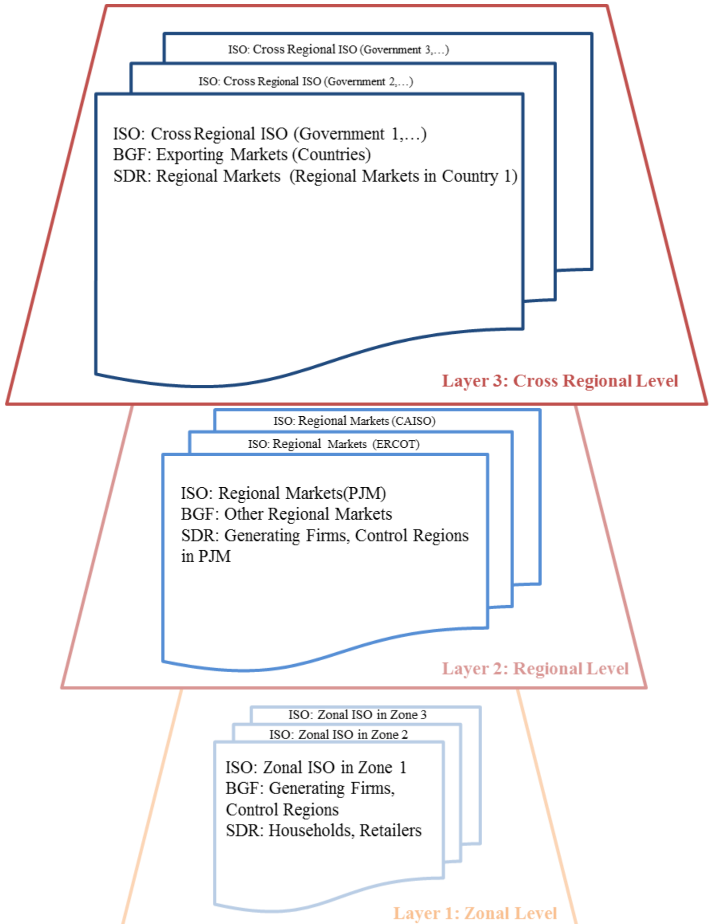

Figure 4 Electricity market structure and roles

This dissertation is focused on designing and modeling a market which reflects the interaction among all decision makers and their best strategies for their decision making problems at the Zonal level as the smallest piece of the puzzle. However, considering all the role adjustments defined above for the participants in different layers, without loss of generality, the model would be applicable to all other layers as well. Since the most difficult layer with the largest number of participants is the Zonal level, this study focuses on this layer of market. Thus, from this point on the word “market” refers to “Zonal market”. The market is modeled as a one-leader multi-follower Stackelberg game. Equilibriums of BGFs oligopoly and SDRs perfect competition in the lower level are modeled as optimization problems for each individual, while ISO's decision problem is in the upper level as the leader of the market.

Market Mechanism and Assumptions

Based on an extensive interview and survey among 93 institutions from all over the world during 3 years, Sovacool explored the most favored policy mechanisms for renewables and energy efficiency (Sovacool, 2009). Analysis of the results showed the most favorable mechanisms with the strongest support are eliminating subsidies, altering electricity prices, forcing utilities to adopt renewables, and increasing funding for renewable power through a national system benefit charge. The proposed mechanism and integrated demand response model in this study promote the most important recommended policy mechanisms in the mentioned study through different policies and rules. Following is the general market structure and assumptions in the mechanism.

An electricity distribution network consists of several nodes and lines connecting nodes together. The network may not be a complete graph, and electricity is transmitted between nodes through the links based on their capacity and limits and the network's specifications. Kamat and Oren (2004) provide a detailed literature review on models with transmission constraints besides transmission rights and pricing. Losses in transmission networks raise the nonlinearity issue in modeling and add significant complexity to the network problem (Chao and Peck, 1996). Hobbs and Drayton (2008) studied a one level Cournot game among generating firms and ISO, considering quadratic resistance losses and phase shifters in controllable DC lines. They showed the effectiveness of the considerations on prices and generation for both competitive and Cournot models through examples. They claimed that in the competitive solution, congestion is more important than losses due to increase of flows under competition for higher loads which worsens the congestion, while in the Cournot solution price differences because of losses are more important due to higher expenses to ISO which makes up expenses. So they concluded that in oligopoly models considering losses are more important than in competitive markets. Practical models of competition among power generators use simplified models of transmission costs and constraints in order to be tractable. However, linearized DC models are used in oligopoly models due to two main reasons: existence of solutions and also computational costs. Consequently, since in this study perfect competition is assumed, considering linearized lossless DC network is a justified assumption.

Subscribers, users and suppliers of electricity are all distributed over the network and located at nodes, which means there can be more than one participant at each node. Participants are either a BGF or a SDR with individual goals and limitations. Their strategies are influenced by the hierarchies of their decision levels in the system.

In the zonal layer of the market, SDRs are all Subscribers to Demand Response program who are willing to participate in the market through communication with the network and could be households, small companies, schools, or retailers. Any household or user in the system can choose to participate in the market directly with a smart meter or transfer the responsibility to a retailer by adopting their plans and services and give them the right for controlling their appliances. BGFs are Big Generating Firms who own the big generators all over the network and have the power of affecting prices considerably with their strategies. ISO is the Independent System

Operator who is responsible for clearing the market and managing the whole system and transmission network. This structure is not limiting and still allows a BGF to act as an ISO if it owns the distribution network. However, in general, allowing suppliers to be owners of distribution networks leads to unfair distribution of market power among suppliers which is not in the best interest of suppliers or consumers. So in order to eliminate the conflict of interests and also consider the social welfare for the society a third party should be in charge of managing the system which here is called ISO. This confirms the conclusion from (Neuhoff, Barquin et al., 2005) on seeing system operator as a strategic agent.

Since ISO manages the system with hierarchy, all other decision makers should follow its decision while competing with each other in the lower level. This type of market could be modeled as a one-leader multi-followers Stackelberg game in which ISO is the leader in the upper level and all other players are followers in the lower level while deciding on their strategies based on the leader's decision. Stackelberg games were first proposed in 1934. This type of formulation is mostly appropriate for games with sequential moves among players (Fudenberg and Tirole, 1991, Gibbons, 1992).

Figure 5 shows categories of decision makers and hierarchy of decision levels in the zonal market in Smart Grid.

At each time instance, a SDR can buy or generate electricity from the grid or his generator and consume or store it, and/or he can sell or reserve electricity from his generator or extra energy stored in his batteries or appliances to the grid. Each SDR can have several appliances, batteries or generators with different demands, storage

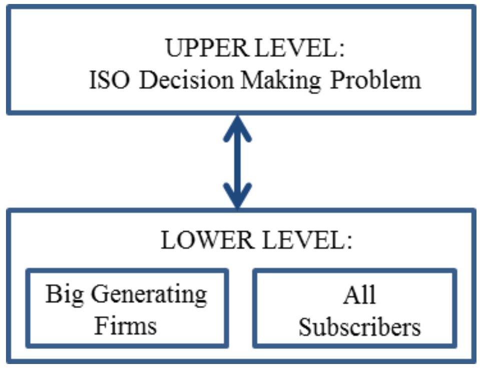

Figure 5 Market structure in Smart Grid

and generation capacity with a set of locations and time intervals at which they can be plugged into the network. For example, one can plug his electrical vehicle into the network both at his office when he is at work or at his house any time after he gets back. However, his preference value and cost may differ at each time and location. The problem for each SDR would be when, where, and how much to generate, consume, reserve, sell or buy electricity to or from the network at each time step during a time horizon in order to satisfy demands, generation and reserve constraints in addition to network limitations while competing with other players over maximizing his individual welfare. This is a dynamic non-cooperative Cournot game among SDRs on maximizing individual welfares through supply/demand scheduling strategies. Dynamic energy scheduling for appliances is on rolling horizon and demand shifting basis. To picture the scale of participants, it is possible to have each costumer participate in the market individually, or having one or several retailers in each zone acting on behalf of costumers while considering each household's payoff function individually, or having both options available to users. The model which is proposed is not limiting in this aspect and can cover any of these scenarios as discussed previously.

BGFs compete in an oligopoly game on their generation, storage and trade quantities as Cournot players to maximize their revenue while the price in the market is determined based on the market's inverse supply function. BGFs can be located anywhere in the grid and generation cost is a function of time and location. BGFs willing to provide supply to a zone may announce their maximum generating capacity assigned to that zone. The maximum capacity for each zone for each BGF can be determined based on stochastic demand estimation techniques which are out of the scope of this research.

ISO is operating the distribution network and is responsible for clearing the market. As a Cournot player, ISO decides on trades' (imports/exports) quantities for each player at each time instance and node of the network with hierarchy over all other players in the system. As an operator its goal is to motivate green energy consumption and reducing peak demands through rewarding local energy consumptions while meeting the networks' constraints. Each big generating firm has to pay a fee to system coordinator (ISO) for using the electricity network, while ISO gives credit to small users exporting electricity to network which is a function of location and time. It is worth mentioning that the ISO objective in the current system is either revenue or social welfare maximization. However, in this study it is assumed that a new system is designed in which the goal is not to collect revenue but it is to expand the use of green and distributed energy in the system. To do so the ISO, as a non-profit entity, is motivating users to use local electricity by minimizing its income from the defined process. Comparison of different ISO objective would be an interesting future work.

The overall problem is to solve the defined real time operational decision making problems simultaneously. At each point in time, each SDR announces its demand, supply, storage quantities, and utility function; each BGF announces its maximum generating and storage capacities dedicated to that zone; and ISO depicts the network's constraints. Output from the model would be the schedule of appliances for SDRs, schedule of generation and storage for BGFs and allowable trades (import/exports) in the Network.

In order to incentivize businesses, utilities and big generating firms to invest in renewables, the proposed model maximizes their revenue as decision makers of the system. Moreover, the defined market mechanism relies on unbundling generation, transmission, and distribution providers in order to eliminate power monopoly in the market and maximize social benefits. This would increase public participation in the system and results in a perfect competition with higher supply diversification. Additionally, considering each consumer's individual utility function in maximizing its individual welfare is a green light for users to engage in the market.

Based on experimental studies giving frequent feedback to consumer would help reducing demands (Becker, Seligman et al., 1979). The other aspect of the proposed mechanism is to inform consumers of energy efficiency and renewable sources. This will be possible through real time two way communications between users and suppliers and the grid. Real time data such as demand distributions and historical consumption trends would be among information available to all participants in the system. This would help consumers understand how their consumption behavior and decision affects their bills.

However, sending information without having real time prices implemented and appropriate models for benefiting from these information would only have slight behavioral effect (Geller, 1981). Even making use of green technology mandates or incentives without using real time prices would not send right signals to consumers and so would not be beneficial. In the proposed mechanism, there is no cap for electricity prices in the market. Prices are determined in real time based on available supply capacities in the market and would be a good signal of consumption and peak times during the day to consumers. On the other hand, eliminating caps will likely result in higher prices which may hurt lower income families and poorer households. Nodal price functions are a good reflection of socioeconomic characteristics of the area and their parameters may be set based on level of poverty in each area. Moreover, in order to protect these groups, this mechanism is capable of considering different solutions such as offering subset of concessions to lower income families and households. That may include discounts on the bills, special loans such as loans for upgrading their energy efficiency systems or tax credits for installing more efficient systems, rebates, or implementing feed-in-tariffs (FIT) for suppliers of green and renewable resources. These offers are not unrealistic and are currently being used in some states such as California and New York and some countries such as Denmark and Australia (Sovacool, 2009). Subsidies for electricity related fuels have existed since 1880. Using the proposed mechanism it is possible to use these subsidies in promoting distributed energies and green energy technologies instead of just reducing electricity prices through payments to big suppliers and transmission companies. Instead it provided loans to local suppliers and households. Even increasing prices without having all other aspects in consideration would not help (Aune, 2007). So in general, any successful mechanism should be a comprehensive one in order to overcome all the issues and deficiencies. The proposed mechanism in this study, as discussed above, covers all these aspects.

Here is a list of assumptions made throughout this study:

- The distribution network is a linearized lossless DC network.
- All players in the system are rational.
- Appliances could be only located at one node at each time step.
- BGFs are individually owned and their maximum dedicated capacities to each zone are estimated and known.
- Without loss of generality BGFs do not have demands. A BGF with demands could be divided into a BGF without demands and a SDR with demands only.
- Demands are less desirable as they delay. That is, consumption preference functions are assumed to be Gaussian functions. Although there is no penalty for not meeting demands, and it is assumed that if any demand request is not satisfied the problem would be infeasible, this assumption is not limiting. Considering the available extra supply capacities during off peak hours, results showed that the infeasibility issue is overcome. This assumption makes the lower level problem convex, which is necessary in proving solution existence.
- Generation cost for BGFs and SDRs, and reward fees are exogenous and depending on time and location.
- Market price is the inverse of the supply function and is assumed to be linear and decreasing with positive parameters. Since the market is modeled for each zone, the prices would be Zonal prices and all users in one geographical zone with similar socioeconomics behaviors have the same prices.

- SDRs have access to trade information through their smart meters.
- Dynamic problem is based rolling horizon with hourly steps.

Problem Formulation

It is of interest to see how market structure and rights in decision making would affect competition and demand management in the energy system. There may be questions regarding determining roles' eligibility on making decisions over some variables. i.e., why not ISO be the only entity deciding on the trade values. So, for the sake of comparison, the problem defined in previous section is formulated from two aspects. The first, which is more likely, is when trade amounts are decision variables for all decision makers. This means trade amounts are variables in both levels. The second is giving the right of decision making over trade amounts only to the ISO. This means trades are only variable in one level (upper level). In the first case, there is a complete competition over all decision variables in the lower level and participants in the market can make their own decisions over their trade values. However, in the second case, despite having a competition in the lower level, the ISO has more control over the system and controls one of the variables solely with respect to other participants' decisions. Comparing these two different policies in the market is of interest and will show if supervising a competition would make any difference in a system. These two perceptions make some differences in relations among participants in the system and also in the approach for solving the problem. In this section both concepts are formulated as One-Way and Two-Way models which refer to problems with trade variables in only upper level and with trade variables in both upper and lower level, respectively.

The general form of the bi-level model called Two-Way model would be as follows:

$max_x \quad f(x)$

$s.t. \quad x \in argmax\{g(x,y): (x,y) \in C(x,y)\}$

As opposed to the One-Way model which is

$max_x \quad f(x)$

$s.t. \quad x \in argmax\{g(y): y \in C(x,y)\}$

Preliminaries and Problem Parameters

Parameters and definitions used in this dissertation are as follows.

N: Set of nodes in the network

L: Set of lines in the network

T: Set of time steps in a time horizon

I: Set of SDRs

G: Set of BGFs

T_0: First time interval in the time horizon

A(i): Set of appliances of SDR i

N(i, a): Set of locations where appliance a of SDR i can be plugged into the network

$\text{Trequest}_{i,a}$: Initial time of request for $\text{DEM}_{i,a}$

$\text{TE}_{i,a,n}$: Earliest possible time for appliance a of SDR i to connect to network in node n

$\text{TL}_{i,a,n}$: Latest possible time for appliance a of SDR i to connect to network in node n

$\mathrm{V}_{i,a,t,n}$: Preference function for consuming one unit of electricity for appliance a of SDR i at node n at time t

$\mathrm{VCHAR}_{i,a,t}$: Preference function for storing one unit of electricity for appliance a of SDR i at time t

$\mathrm{C}_{i,a,t,n}$: Generation cost of one unit of electricity by appliance a of SDR i at node n at time t

$\mathrm{CR}_{i,a,t}$: Storage cost for one unit of electricity for appliance a of SDR i at time t

$\mathrm{DEM}_{i,a}$: Demand request for appliance a of SDR i

$\mathrm{CHAR}_{i,a,t}$: Charge request for appliance a of SDR i at time t

$\mathrm{Q}_{i,a}$: Generation capacity for appliance a of SDR i

$\mathrm{RCAP}_{i,a}$: Storage capacity for appliance a of SDR i

$\mathrm{R0}_{i,a}$: Initial storage quantity for appliance a of SDR i at T_0

$\mathrm{P}_{n,t}$: Nodal export fee/reward fee at time t

K_l: Thermal capacity of line l

$\mathrm{D}_{n,l}$: Power transfer distribution factor from node n to line l

QG_g: Generation capacity for generator g

RG_g: Storage capacity for generator g

$\mathrm{CG}_{g,t}$: Generation cost for one unit of electricity by BGF g at time t

$\mathrm{CRG}_{g,t}$: Storage cost for one unit of electricity for generator g at time t

GNODE_g: Node at which generator g is located

G_n: BGFs located on node n

Decision Variables

$\mathrm{dem}_{i,a,t,n}$: Consumption quantity for appliance a of SDR i in node n at time t

$\mathrm{q}_{i,a,t,n}$: Supply quantity for appliance a of SDR i in node n at time t

$\mathrm{r}_{i,a,t}$: Storage level for appliance a of SDR i at time t

$\mathrm{x}_{i,a,t,n}$: Trade (import(+) or export(−)) quantity for appliance a of SDR i in node n at time t

$\mathrm{qg}_{g,t}$: Generation quantity for generator g at time t

$\mathrm{rg}_{g,t}$: Storage level for generator g at time t

$\mathrm{xg}_{g,t}$: Export quantity for generator g at time t

$\mathrm{expo}_{n,t}$: Total nodal export by all SDRs from node n at time t

Interactions among all decision variables in the problem are shown in Figure 6, Figure 7, and Figure 8.

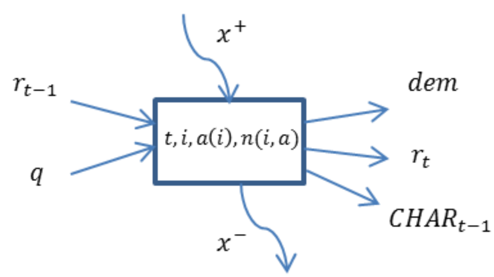

Figure 6 SDR's decision problem at each point of time for each appliance

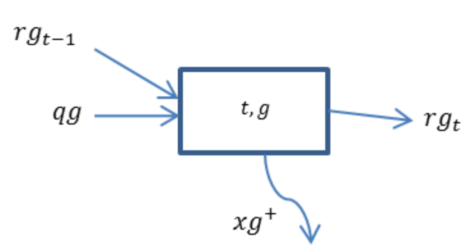

Figure 7 BGF's decision problem at each point of time

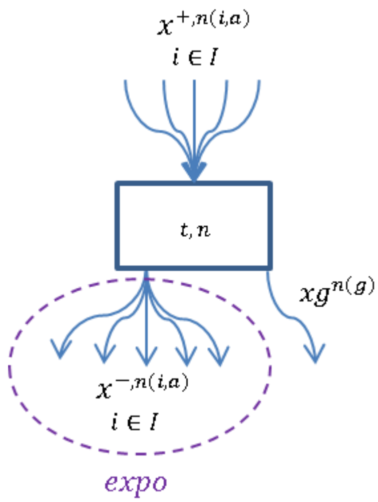

Figure 8 ISO's decision problem at each point of time at each node

Two-Way Model: Model with Trades in both levels

ISO Decision Problem

ISO's decision problem is as follows:

$$\text{Min.} \sum_{t} \sum_{n \in N} P_{n,t} \cdot \left(\sum_{(g \epsilon G | gnode(g) = n)} xg_{g,t} - expo_{n,t} \right)$$

(3. 1)

Subject to:

$$expo_{n,t} \leq 0 \quad ; \forall n, t$$

(3. 2)

$$expo_{n,t} \leq \sum_{i \in I} \sum_{a \epsilon A(i)} x_{i,a,t,n} \quad ; \forall n, t$$

(3. 3)

$$\sum_{i \epsilon I} \sum_{a \epsilon A(i)} \sum_{n \epsilon N(i,a)} x_{i,a,t,n} - \sum_{g \epsilon G} xg_{g,t} = 0 \; ; \; \forall t$$

(3. 4)

$$-K_l \leq \sum_{n \epsilon N} \left(\left(\sum_{i \epsilon I} \sum_{a \epsilon A(i)} \sum_{n \epsilon N(i,a)} x_{i,a,t,n} - \sum_{g \epsilon G | GNODE_g = n} xg_{g,t} \right) D_{n,l} \right) \leq K_l \; ; \; \forall l \epsilon L, t$$

(3. 5)

$$x_{i,a,t,n}\ free\ ;\ \forall i, a\epsilon A(i), n\epsilon N(i,a), t$$

(3. 6)

$$xg_{g,t} \geq 0\ ;\quad \forall g, t$$

(3. 7)

ISO is a governmental entity or a third party whose goal is to motivate energy demand management participation and green energy consumption. Consequently, ISO intends to encourage users to use local generators which are mostly non-fossil generators and to generate their own electricity as much as possible in order to benefit from disperse energy capacities. To do so system operator is charging BGFs for using lines while paying users who exports electricity. The charge and payment is based on the location and time of use. ISO minimizes total income from these fees and maximized total rewards (3. 1). Fees are assumed to be exogenous and nodal based. (3. 2) and (3. 3) are total amount of exports by all SDRs at any time from each node. BGFs may include their fees for use of lines in their costs while selling electricity to market. But eventually this would be reflected in the behavior of users by reducing use of fossil generated electricity.

(3. 4) is the flow conservation constraint. Equation (3. 5) is the thermal limit on lines in which K_l is thermal capacity of line l. x values are import/export quantities at each node which could be negative. Their PTDF and $D_{n,l}$ is the amount of flow increased/decreased on each link due to injection/withdrawal of electricity at each node. Since the flow could be in both directions on each line, this constraint should hold for both positive and negative values of the limits. Constraints (3. 6), and (3. 7)

indicate import and export variables in the grid in this decision making problem. The ISO model is a linear model with linear constraints.

BGF Decision Problem

In the lower level the first set of decision makers are the big generators and big suppliers. They compete with each other for supplying market's demand while maximizing their revenue.

$\forall g \epsilon G$:

$$\text{Max.} \sum_t ((Z - W \sum_{gg \epsilon G} xg_{gg,t}) xg_{g,t} - CG_{g,t} . qg_{g,t} - CRG_{g,t} . rg_{g,t}) \tag{3. 8}$$

Subject to:

$$qg_{g,t} - QG_g \leq 0 \quad ; \quad \forall t \tag{3. 9}$$

$$qg_{g,t} - xg_{g,t} - rg_{g,t} + rg_{g,t-1} = 0 \quad ; \quad \forall t \tag{3. 10}$$

$$rg_{g,t} - RG_g \leq 0 \quad ; \quad \forall t \tag{3. 11}$$

$$xg_{g,t}, \; rg_{g,t}, \; qg_{g,t} \geq 0 \; ; \forall t \tag{3. 12}$$

This model is solved for each BGF in the system. Solving for all BGFs optimal strategies, the Nash equilibrium (if exists) to the game will be found. Existence of equilibrium to the problem is discussed in Chapter 4. Equation (3. 8) is the objective function and is maximizing revenue which is the difference between generator's income and cost. Price in this market is assumed to be equal to the inverse supply function which is a linear function. As supply increases in the market, price decreases and so it has a negative slope. In this model, since zonal markets are considered, prices would be zonal prices. People living in one geographical zone have similar socioeconomics behaviors. So considering similar physical conditions in a zone such as severe weather conditions, and similar social economics behaviors, it is justified to assign each zone one price based on the zonal supply and demand function. A question may arise here: How realistic is using inverse supply function for setting the market price? There are different approaches in setting the market price. Each has its own advantages and disadvantages and is upon designer's preference. Inverse supply function is chosen here to capture the game among suppliers in the market. Using shadow prices in determining the real time price could be also interesting. However, this makes the problem nonlinear and even more complex with such a large size and dynamic nature. Studying shadow prices and comparison to current inverse supply function would be of interest and is left for future investigations.

Constraint (3. 9) is the generation capacity for each generator at each time step. (3. 10) is flow conservation constraint for each generator at each time step which includes all the generations, sales and difference in the storage level of the storage system. (3. 11) is the limit on storage capacity of the generators. Although large scale electricity storage is still not very efficient and economically justified, research path in this area of technology is very promising. Being one of the highlights of Smart Grid in the future (Von Dollen, 2009), storage capacities are considered in this modeling, though this assumption is not limiting at all. Finally (3. 12) are the variables in this decision making problem. Each generator is solving a quadratic programming problem. The objective function is concave while the feasible region is a convex hull due to linearity of all the constraints. So each generator will have a global solution for its concave objective function.

SDR Decision Problem

The last set of decision makers are all subscribers in the system. These are the ones who declared their interest in participating in the market and demand response program through smart meters. There could be retailers who themselves cover a set of subscribers and customers and act on behalf of them considering each individual payoff function. In this problem each subscriber tries to find his optimal schedule of demand and supply to meet all his demand and limitations while maximizing his individual welfare. Since SDRs follow ISO's decisions, their optimal schedule will also comply with network constraints.

$\forall i \epsilon I$:

$$\text{Max. } \sum_t \Big(\sum_{a\epsilon A(i)} \sum_{n\epsilon N(i,a)} \big(V_{i,a,t,n}.\, dem_{i,a,t,n} - C_{i,a,t,n}.\, q_{i,a,t,n} - \big(Z - W \sum_{g\epsilon G} xg_{g,t}\big).\, x_{i,a,n,t}\big) + \sum_{a\epsilon A(i)} \big(VCHAR_{i,a,t}.\, CHAR_{i,a,t} - CR_{i,a,t}.\, r_{i,a,t}\big)\Big) \tag{3. 13}$$

Subject to:

$$\sum_t \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} = DEM_{i,a} \quad ; \quad \forall a\epsilon A(i) \tag{3. 14}$$

$$r_{iat} - CHAR_{i,a,t} \geq 0 \quad ; \quad \forall a\epsilon A(i), t \tag{3. 15}$$

$$\sum_{n\epsilon N(i,a)} q_{i,a,t,n} - Q_{i,a} \leq 0 \quad ; \quad \forall a\epsilon A(i), t \tag{3. 16}$$

$$\mathrm{r_{iat} - RCAP_{i,a} \leq 0 \quad ; \quad \forall a \epsilon A(i), t}$$

(3. 17)

$$\sum_{n\epsilon N(i,a)} \mathrm{q_{i,a,T_0,n}} + \sum_{n\epsilon N(i,a)} \mathrm{x_{i,a,T_0,n}} - \mathrm{r_{i,a,T_0}} + \mathrm{R0_{i,a}} - \sum_{n\epsilon N(i,a)} \mathrm{dem_{i,a,T_0,n}} = 0 \quad ; \quad \forall a \epsilon A(i)$$

(3. 18)

$$\sum_{n\epsilon N(i,a)} \mathrm{q_{i,a,t,n}} + \sum_{n\epsilon N(i,a)} \mathrm{x_{i,a,t,n}} - \mathrm{r_{i,a,t}} + \mathrm{r_{i,a,(t-1)}} - \sum_{n\epsilon N(i,a)} \mathrm{dem_{i,a,t,n}} - \mathrm{CHAR_{i,a,t-1}} = 0 \quad ; \quad \forall a \epsilon A(i), t \neq T_0$$

(3. 19)

$$\mathrm{r_{i,a,t}, q_{i,a,t,n}, dem_{i,a,t,n} \geq 0 \quad ; \quad \forall a \epsilon A(i), n \epsilon N(i,a), t}$$

(3. 20)

$$\mathrm{x_{i,a,t,n}}\ free\ ; \forall \mathrm{i}, \mathrm{a \epsilon A(i), n \epsilon N(i,a), t}$$

(3. 21)

In the objective function (3. 13), each subscriber maximizes his individual welfare. Individual welfare is the benefit each subscriber gains minus all his costs. Utility function is defined as the value of the electricity consumption for the subscriber at each time instance. The utility function for each subscriber's energy consumption is assumed to be Gaussian and convex which means user's preference is to consume energy as close as possible to their time of request (Figure 9): $V_t = a.e^{-((t-T_{Request})/b)^2}$

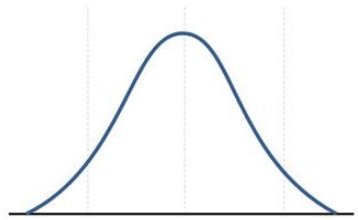

Figure 9 SDRs schematic consumption utility function

SDR's energy generation cost includes operational cost of devices such as solar panels and electric vehicles, plus device's depreciation cost. Storage cost is also assumed to be sum of operational and depreciation cost per unit of electricity storage in storage devices and batteries. The price that a buyer pays or a seller earns from trading electricity is the inverse supply function of the BGFs.

Equation (3. 14) assures all the demands are met. This is a hard constraint and may cause infeasibility if sufficient supply is not available in the network. It is possible to make it a soft constraint by changing the strict equality to less or equal constraint; however, this would cause semi-satisfied demands. Constraint (3. 15) is batteries' demand satisfaction. For example, one may want to have his electric vehicle being fully charged by a specific time, this constraint would consider this request.

Constraint (3. 16) is generation capacity limit of appliances. Constraint (3. 17) is the storage capacity constraint. Constraints (3. 18) and (3. 19) are flow conservation at each time step for each appliance. And finally, constraints (3. 20) and (3. 21) define variable conditions.

As it is clear in model, BGFs' variables are affecting SDRs' decisions, but that is not true in reverse. This results in a hidden hierarchy of BGFs over SDRs. That is because

SDRs' individual production and consumption levels are very small comparing to BGFs' level of production. This decreases their effect on BGFs' decision problem in general. However, to include their effect on the market behavior, indirect parameters and techniques such as demand functions and congestion pricing are suggested. In this study, demand and supply function is used to capture the behavior. Studying each individual's influence on the market makes would be interesting and is among future work.

Stackelberg Mathematical Model

The Nash-Cournot game defined in this problem has three sets of players: ISO, BGFs, and SDRs. Optimization problem for each player in the game is described before. In this section the one leader multi-follower Stackelberg game among ISO as the leader, and BGFs and SDRs as its followers is presented. In this problem, ISO minimizes its objective by decision over trade levels given the response of the followers. Summary of the Stackelberg game model is illustrated in Table 5.

In the main model of this study, Two-Way model, every participant shares the power of making decisions over trade amounts in the system. Here, the other model is formulated and evaluated with giving this power only to ISO. In other words, instead of having the market settle equilibrium through a competition, it gives one entity the power to control one of the variables for the whole system as an ISO. To evaluate such a system, the problem is modeled as a bi-level model in which ISO in the upper level maintains network constraints while trade amounts are variables only for ISO in the upper level and dictated to all other participants in the lower level. In reality, giving the right of decision making only to ISO would decrease level of supply diversification.

Table 5 Summary of one leader multi-follower Stackelberg game: Two-Way model

Level	*Hierarchy*	*Player*	*Objective*	*Main Variable*	*Equation(s)*
Upper Level	Leader	ISO	Min. Income	$x_{i,a,t,n}$,$xg_{g,t}$	(3. 1)-(3. 7)
Lower Level	Follower	BGFs	Max. Revenue	$qg_{g,t}$, $xg_{g,t}$, $rg_{g,t}$	(3. 8)-(3. 12)
	Follower	SDRs	Max.Individual Welfare	$dem_{i,a,t,n}$,$q_{i,a,t,n}$, $r_{i,a,t}$, $x_{i,a,t,n}$	(3. 13)-(3. 21)

One-Way Model: Model with Trades solely in ISO level

The One-Way model would be the same as Two-Way model with only deference in variable definitions. The One-Way model with modified variables is illustrated in Table 6.

Table 6 Summary of one leader multi-follower problem: One-Way Model

Level	*Hierarchy*	*Player*	*Objective*	*Main Variable*	*Equation(s)*
Upper Level	Leader	ISO	Min. Income	$x_{i,a,t,n}$,$xg_{g,t}$	(3. 1)-(3. 7)
Lower Level	Follower	BGFs	Max. Revenue	$qg_{g,t}$	(3. 8)-(3. 12)
	Follower	SDRs	Max. Individual Welfare	$dem_{i,a,t,n}$,$q_{i,a,t,n}$,$r_{i,a,t}$	(3. 13)-(3. 21)

Chapter 4: Solution Methodology and Validation

This chapter is dedicated to proving the existence of a solution to the model in order to validate the Stackelberg game proposed for integrated demand response and market equilibrium in Smart Grid (Table 5). First a brief background on bi-level models is provided and the proof follows. The solution approach for the proposed model is discussed afterward.

Background

"Multilevel optimization problems are mathematical programs which have a subset of their variables constrained to be an optimal solution of other programs parameterized by their remaining variables" (Vicente and Calamai, 1994). Multilevel optimization is mostly related to economical decision making problems such as Stackelberg problems in game theory. It has different applications, such as revenue management, congestion management, network design problems, principal agent problem, origin destination matrix estimation and many more in transportation, management, planning and engineering design. In this study it is applied in the area of energy sector. Bi-level programming is a generally complex problem and most of studies focus on solving simple and small cases.

Different methodologies in solving Bi-level problems can be categorized as following:

- Methods based on vertex enumeration: This technique is mainly based on searching through different possible nodes in the region. For instance, Bard (Bard, 1983) developed grid search algorithm which uses bi-criteria optimization concept for solving bi-level programming. Hansen and Jaumard (Hansen, Jaumard et al., 1992) developed a branch and bound algorithms for solving bi-level problems based on binding constraints of the followers. Bialas (Bialas, 1984) also proposed a vertex search algorithm in the accessible region, though his approach may result in local optimum.

- Methods based on Karush-Kuhn-Tucker (KKT) conditions: One of the ideas in solving bi-level problems is converting the bi-level to a one level problem by adding the optimality conditions of the lower level to the upper level. Adding the KKT conditions of the lower level to the upper level, there will be an MPEC problem which is a one level problem to solve. However, due to the non-convex and non-smooth characteristics of a MPEC, normally solving such a problem is a difficult and complex process (Ban, Liu et al., 2006). Judice and Faustino (Júdice and Faustino, 1992) used hybrid enumerative method to develop a sequential LCP algorithm for solving MPEC problem which performs well in medium sized problems, but does not guarantee solution in all cases. White and Anandalingam (White and Anandalingam, 1993) benefited from penalty function for satisfying the complementarity constraints. MIP techniques can be also used in solving the corresponding MIP of the MPEC such as benders decomposition (Gabriel, Shim et al., 2010). Audet and Savard (Audet, Savard et al., 2007) proposed a finite branch and cut algorithm for solving the linear bi-level programming based on new classes of valid cuts in the corresponding MIP problem. Moreover, Bard and Moore (Bard and Moore, 1990) reformulated the problem as a standard mathematical program by exploiting the follower's Kuhn–Tucker conditions. Then a branch and bound scheme suggested by Fortuny-Amat and McCarl (Fortuny-Amat and McCarl, 1981) is used to enforce the underlying complementary slackness conditions. The algorithm performs well in small problems with 100 variables.
- Methods based on heuristics: As the problem size increases solving bi-level problems get even more complex. Some studies used heuristic and meta-heuristic methods for dealing with this issue. (Sahin and Ciric, 1998) incorporated simulated annealing

method. However, it was inefficient in solving even small problems. (Hejazi, Memariani et al., 2002) developed a genetic algorithm, but they recognized the difficulty in generating feasible chromosomes. Finally, (Gendreau, Marcotte et al., 1996) developed a hybrid Tabu ascent algorithm based on penalty functions to find and improve the initial feasible solution which is a problem specific algorithm.

- Nonlinear approaches: One other technique is to see the optimality conditions of the lower level as a type of nonlinear constraints. Having this idea, nonlinear approaches can be used for solving the problem. (Fletcher* and Leyffer, 2004, Leyffer, 2003, Leyffer and Munson, 2010, Ralph* and Wright, 2004) used this approach in their papers.

There are great comprehensive surveys on bi-level programming problems and their solution techniques to which interested readers are referred (Colson, Marcotte et al., 2005, Dempe, 2002, Vicente and Calamai, 1994).

Two-Way model

MPEC Model

The game among participants and operator in Smart Grid is modeled as a one-leader multi-follower Stackelberg game. Stackelberg games are bi-level optimization models due to the hierarchy of decision making among players. One approach in solving Stackelberg problems is to substitute the lower level problems with their optimality conditions. This results in a one level MPEC problem. Convexity of BGFs' quadratic and SDRs' linear models makes the first order optimality condition and so Karush Kuhn Tucker optimality conditions (Bazaraa, Sherali et al., Cottle, Pang et al., 2009), both necessary and sufficient. As a result equilibrium for BGFs and SDRs' game could be found through solving KKT optimality conditions for all SDRs and BGFs together.

Dual variables (ε) for BGFs' constraints are shown in (4. 1) and (4. 3):

$$qg_{g,t} - QG_g \le 0 \quad ; \quad \forall g \epsilon G, t \;\; -- \; \varepsilon^1_{g,t} \tag{4. 1}$$

$$qg_{g,t} - xg_{g,t} - rg_{g,t} + rg_{g,t-1} = 0 \quad ; \quad \forall g \epsilon G, t \;\; -- \; \varepsilon^2_{g,t} \tag{4. 2}$$

$$rg_{g,t} - RG_g \le 0 \quad ; \quad \forall g \epsilon G, t \;\; -- \; \varepsilon^3_{g,t} \tag{4. 3}$$

KKT conditions for BGFs would be:

$$0 \le -Z + 2Wxg_{g,t} + W\sum_{gg\neq g} xg_{gg,t} - \varepsilon^2_{g,t} \perp xg_{g,t} \ge 0; \forall g \epsilon G, t \tag{4. 4}$$

$$0 \le CG_{g,t} + \varepsilon^1_{g,t} + \varepsilon^2_{g,t} \perp qg_{g,t} \ge 0; \forall g \epsilon G, t \tag{4. 5}$$

$$0 \le CRG_{g,t} + \varepsilon^2_{g,t+1}(t < T) - \varepsilon^2_{g,t} + \varepsilon^3_{g,t} \perp rg_{g,t} \ge 0; \forall g \epsilon G, t \tag{4. 6}$$

$$0 \le QG_g - qg_{g,t} \perp \varepsilon^1_{g,t} \ge 0 \quad ; \quad \forall g \epsilon G, t \tag{4. 7}$$

$$qg_{g,t} - xg_{g,t} - rg_{g,t} + rg_{g,t-1}(t > T0) = 0, \quad \varepsilon^2_{g,t} \;\; \text{free} \quad ; \quad \forall g \epsilon G, t \tag{4. 8}$$

$$0 \le RG_g - rg_{g,t} \perp \varepsilon^3_{g,t} \ge 0 \quad ; \quad \forall g \epsilon G, t \tag{4. 9}$$

And for SDRs model dual variables (β) are assigned as in (4. 10)-(4. 15):

$$\sum_t \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} = DEM_{i,a} \quad ; \quad \forall i\epsilon I, a\epsilon A(i) \quad -- \beta^1_{i,a}$$

(4. 10)

$$r_{iat} - CHAR_{i,a,t} \geq 0 \quad ; \quad \forall i\epsilon I, a\epsilon A(i), t|CHAR_{i,a,t} \text{ exists} \quad -- \beta^2_{i,a,t}$$

(4. 11)

$$\sum_{n\epsilon N(i,a)} q_{i,a,t,n} - Q_{i,a} \leq 0 \quad ; \quad \forall i\epsilon I, a\epsilon A(i), t|Q_{i,a} \text{ exists} \quad -- \beta^3_{i,a,t}$$

(4. 12)

$$r_{iat} - RCAP_{i,a} \leq 0 \quad ; \quad \forall i\epsilon I, a\epsilon A(i), t|RCAP_{i,a} \text{ exists} \quad -- \beta^4_{i,a,t}$$

(4. 13)

$$\sum_{n\epsilon N(i,a)} q_{i,a,T_0,n} + \sum_{n\epsilon N(i,a)} x_{i,a,T_0,n} - r_{i,a,T_0} + R0_{i,a} - \sum_{n\epsilon N(i,a)} dem_{i,a,T_0,n} = 0 \quad ; \quad \forall i\epsilon I, a\epsilon A(i) \quad -- \beta^5_{i,a}$$

(4. 14)

$$\sum_{n\epsilon N(i,a)} q_{i,a,t,n} + \sum_{n\epsilon N(i,a)} x_{i,a,t,n} - r_{i,a,t} + r_{i,a,(t-1)} - \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} - CHAR_{i,a,t-1} = 0 \quad ; \quad \forall i\epsilon I, a\epsilon A(i), t \neq T_0 \quad -- \beta^6_{i,a,t}$$

(4. 15)

Also to have all variables positive in the KKT condition the free trade variable is substituted by difference of two positive variable as $x = x^+ - x^-$. KKT conditions for SDRs would be as in (4. 16)-(4. 26):

$$0 \leq Z - W\sum_{g\epsilon G} xg_{g,t} + \beta^5_{i,a}\langle t = T_0\rangle + \beta^6_{i,a,t}\langle t \neq T_0\rangle \perp x^+_{i,a,t,n} \geq 0; \forall i, a\epsilon A(i), n, t$$

(4. 16)

$$0 \leq -Z + W\sum_{g\epsilon G} xg_{g,t} - \beta^5_{i,a}\langle t = T_0\rangle - \beta^6_{i,a,t}\langle t \neq T_0\rangle \perp x^-_{i,a,t,n} \geq 0; \forall i, a\epsilon A(i), n, t$$

(4. 17)

$$0 \leq -V_{i,a,t,n} + \beta^1_{i,a} - \beta^5_{i,a}\langle t = T_0\rangle - \beta^6_{i,a,t}\langle t \neq T_0\rangle \perp dem_{i,a,t,n} \geq 0; \forall i, a \epsilon A(i), n, t$$

(4. 18)

$$0 \leq C_{i,a,t,n} + \beta^3_{i,a} + \beta^5_{i,a}\langle t = T_0\rangle + \beta^6_{i,a,t}\langle t \neq T_0\rangle \perp q_{i,a,t,n} \geq 0 ; \forall i, a \epsilon A(i), n, t$$

(4. 19)

$$0 \leq CR_{i,a,t} - \beta^2_{i,a,t} + \beta^4_{i,a,t} - \beta^5_{i,a}\langle t = T_0\rangle + \beta^6_{i,a,(t+1)}\langle t = T_0\rangle - \beta^6_{i,a,t}\langle t \neq T_0\rangle + \beta^6_{i,a,(t+1)}\langle t \neq T_0\rangle \perp r_{i,at} \geq 0 ; \quad \forall i, a \epsilon A(i), t$$

(4. 20)

$$0 \leq r_{i,a,t} - CHAR_{i,a,t} \perp \beta^2_{i,a,t} \geq 0 \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 21)

$$0 \leq Q_{i,a} - \sum_{n \epsilon N(i,a)} q_{i,a,t,n} \perp \beta^3_{i,a,t} \geq 0 \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 22)

$$0 \leq RCAP_{i,a} - r_{i,a,t} \perp \beta^4_{i,a,t} \geq 0 \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 23)

$$\sum_t \sum_{n \epsilon N(i,a)} dem_{i,a,t,n} = DEM_{i,a} \quad , \quad \beta^1_{i,a} \quad \text{free} \quad ; \quad \forall i, a \epsilon A(i)$$

(4. 24)

$$\sum_{n \epsilon N(i,a)} q_{i,a,T_0,n} + \sum_{n \epsilon N(i,a)} (x^+_{i,a,t,n} - x^-_{i,a,t,n}) - r_{i,a,T_0} + R_{0_{i,a}} - \sum_{n \epsilon N(i,a)} dem_{i,a,T_0,n} = 0 \, , \ \beta^5_{ia} \quad \text{free} \, ; \ \forall i, a \epsilon A(i)$$

(4. 25)

$$\sum_{n \epsilon N(i,a)} q_{i,a,t,n} + \sum_{n \epsilon N(i,a)} (x^+_{i,a,t,n} - x^-_{i,a,t,n}) - r_{i,a,t} + r_{i,a,(t-1)} - \sum_{n \epsilon N(i,a)} dem_{i,a,t,n} - CHAR_{i,a,t-1} = 0 \, , \ \beta^6_{i,a,t} \ \text{free} \, ; \forall i, a \epsilon A(i), t \neq T_0$$

(4. 26)

Optimality conditions for the lower level games are now in complementarity form and could be added to the upper level problem as a new set of constraints (Chen, Hobbs et al., 2006). This would shape a Mathematical Problem with Equilibrium Constraints (MPEC) (Cottle, Pang et al., 2009) as summarized in (Table 7).

Table 7 Summary of one leader multi-follower Stackelberg game- MPEC Two-Way model

Type	*Hierarchy*	*Description*	*Equation(s)*
Objective Function	Leader	ISO Objective Function	(3. 1)
Constraints	Leader	ISO Constraints	(3. 2)-(3. 7)
	Follower	BGFs Optimality Conditions	(4. 4) -(4. 9)
	Follower	SDRs Optimality Conditions	(4. 16)-(4. 26)

Although the MPEC has a convex objective function, it is nonlinear due to the presence of complementarity constraints. Generally, finding MPECs' global solution is not easy using standard algorithms because of their non-convex feasible region. However, many researchers worked on these models to find an appropriate method for finding optimal solutions (Ferris and Pang, 1997, Gabriel and Leuthold, 2010, Leyffer and Munson, 2010). Recent developments are capable of computing local stationary points and make MPEC a tractable tool for solving large scale Stackelberg games (Chen, Hobbs et al., 2006, Leyffer and Munson, 2010). Although solvers such as NLPEC and NPATH are available for solving MPECs, the problem mostly gets computationally expensive and complex when it is large and so these solution approaches do not guarantee global optimality.

MIP Model

Many researchers worked on MPEC models to develop an appropriate method for finding optimal solutions (Ferris and Pang, 1997, Gabriel and Leuthold, 2010, Leyffer and Munson, 2010). One famous approach is using integer programming technique to convert the nonlinear MPEC model to MILP.

Computational tractability of optimization models makes them a favorable approach in solving large scale modeling (Ventosa, Baıllo et al., 2005). On the other hand, in games, equilibrium models are more appropriate since they can consider players' strategies in the system. If equilibrium models could be converted into optimization models then both fitness of models and computational complexities are in favor of the model. So in this section the proposed MPEC model is transformed into an integer programming model not only to evaluate the applicability of this approach in real world size implementation, but also to compare its results with the results of the solution algorithm which will be introduced in this study. Integer programming techniques are employed and the complementarity constraints are transformed into linear disjunctive constraints based on the technique introduced in (Audet, Hansen et al., 1997).

For any complementarity constraint such as (4. 27), using a dummy binary variable φ and a dummy large number M (as an upper bound for the constraint), it can be written as (4. 28)-(4. 30):

$$0 \leq f(x) \perp x \geq 0 \qquad (4.\ 27)$$

$$0 \leq f(x) \leq M.\varphi$$

(4. 28)

$$0 \leq \mathrm{x} \leq \mathrm{M}.(1-\varphi)$$

(4. 29)

$$\varphi \in \{0,1\}$$

(4. 30)

However, the associated MIP model has some disadvantages. Beside difficulties in solving the problem in large scale and real time, there would be many BigMs in the associated MIP model. Using the approach introduced in (Gabriel, Shim et al., 2010) some of the BigMs can be estimated. However, for those not being estimated, sensitivity analysis should be conducted. These constants make the problem unstable which makes MIP an unreliable approach.

BigMs for most of constraints are estimated based on their maximum limit or their marginal prices. In this study shadow prices on the capacities are assumed to be α times their usage cost. Some other bounds need more investigations. For instance, bound on $\mathrm{xg}_{\mathrm{g,t}}$ in (4. 32) can be estimated based on (4. 37).

Following the idea, complementarity constraints of BGFs and SDRs in the MPEC model introduced in Table 7 can be converted to linear disjunctive constraints using dummy binary variables δ and λ as shown below. The resulting problem is a large MIP model which can be solved using integer programming methodologies.

$$0 \leq -\mathrm{Z}+2\mathrm{Wxg}_{\mathrm{g,t}}+\mathrm{W}\sum\nolimits_{\mathrm{gg}\neq \mathrm{g}} \mathrm{xg}_{\mathrm{gg,t}}-\varepsilon^{2}_{\mathrm{g,t}} \leq \mathrm{M}.\delta^{1}_{g,t}\ ; \forall \mathrm{g} \epsilon \mathrm{G}, \mathrm{t}$$

(4. 31)

$$0 \leq \mathrm{xg_{g,t}} \leq (\mathrm{RG}_g + \mathrm{QG}_g).(1 - \delta_{g,t}^1)$$

(4. 32)

$$0 \leq \mathrm{CG_{g,t}} + \varepsilon_{\mathrm{g,t}}^1 + \varepsilon_{\mathrm{g,t}}^2 \leq \mathrm{M}.\delta_{g,t}^2 \;\; ; \;\; \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 33)

$$0 \leq \mathrm{qg_{g,t}} \leq \mathrm{QG_g}.(1 - \delta_{g,t}^2) \quad ; \quad \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 34)

$$0 \leq \mathrm{QG_g} - \mathrm{qg_{g,t}} \leq \mathrm{QG_g}.\delta_{g,t}^3 \quad ; \quad \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 35)

$$0 \leq \varepsilon_{\mathrm{g,t}}^1 \leq \alpha.\mathrm{CG_{g,t}}.(1 - \delta_{g,t}^3) \quad ; \quad \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 36)

$$\mathrm{qg_{g,t}} - \mathrm{xg_{g,t}} - \mathrm{rg_{g,t}} + \mathrm{rg_{g,t-1}}(\mathrm{t} > \mathrm{T0}) = 0 \quad ; \quad \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 37)

$$0 \leq \mathrm{CRG_{g,t}} + \varepsilon_{\mathrm{g,t+1}}^2(\mathrm{t} < \mathrm{T}) - \varepsilon_{\mathrm{g,t}}^2 + \varepsilon_{\mathrm{g,t}}^3 \leq \mathrm{M}.\delta_{g,t}^4 \;\; ; \;\; \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 38)

$$0 \leq \mathrm{rg_{g,t}} \leq \mathrm{RG_g}.(1 - \delta_{g,t}^4) \quad ; \quad \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 39)

$$0 \leq \mathrm{RG_g} - \mathrm{rg_{g,t}} \leq \mathrm{RG_g}.\delta_{g,t}^5 \quad ; \quad \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 40)

$$0 \leq \varepsilon_{\mathrm{g,t}}^3 \leq \alpha.\mathrm{CRG_{g,t}}.(1 - \delta_{g,t}^5) \quad ; \quad \forall \mathrm{g}\epsilon\mathrm{G,t}$$

(4. 41)

$$\delta^1_{g,t}, \delta^2_{g,t}, \delta^3_{g,t}, \delta^4_{g,t}, \delta^5_{g,t} \in \{0,1\} \quad ; \quad \forall g \epsilon G, t$$

(4. 42)

$$\varepsilon^2_{g,t} \quad \text{free} \quad ; \quad \forall g \epsilon G, t$$

(4. 43)

$$0 \leq z - w \sum_{g \epsilon G} xg_{g,t} + \beta^5_{i,a} \langle t = T_0 \rangle + \beta^6_{i,a,t} \langle t \neq T_0 \rangle \leq M. \lambda^1_{i,a,t,n}; \forall i, a \epsilon A(i), n, t$$

(4. 44)

$$0 \leq x^+_{i,a,t,n} \leq M. (1 - \lambda^1_{i,a,t,n})$$

(4. 45)

$$0 \leq -z + w \sum_{g \epsilon G} xg_{g,t} - \beta^5_{i,a} \langle t = T_0 \rangle - \beta^6_{i,a,t} \langle t \neq T_0 \rangle \leq M. \lambda^2_{i,a,t,n}; \forall i, a \epsilon A(i), n, t$$

(4. 46)

$$0 \leq x^-_{i,a,t,n} \leq M. (1 - \lambda^2_{i,a,t,n})$$

(4. 47)

$$0 \leq -V_{i,a,t,n} + \beta^1_{i,a} - \beta^5_{i,a} \langle t = T_0 \rangle - \beta^6_{i,a,t} \langle t \neq T_0 \rangle \leq M. \lambda^3_{i,a,t,n} ; \quad \forall i, a \epsilon A(i), n, t$$

(4. 48)

$$0 \leq dem_{i,a,t,n} \leq DEM_{i,a}. (1 - \lambda^3_{i,a,t,n}) ; \quad \forall i, a \epsilon A(i), n, t$$

(4. 49)

$$0 \leq C_{i,a,t,n} + \beta^3_{i,a} + \beta^5_{i,a} \langle t = T_0 \rangle + \beta^6_{i,a,t} \langle t \neq T_0 \rangle \leq M. \lambda^4_{i,a,t,n} ; \quad \forall i, a \epsilon A(i), n, t$$

(4. 50)

$$0 \leq q_{i,a,t,n} \leq Q_{i,a}. (1 - \lambda^4_{i,a,t,n}) ; \quad \forall i, a \epsilon A(i), n, t$$

(4. 51)

$$0 \leq CR_{i,a,t} - \beta^{2}_{i,a,t} + \beta^{4}_{i,a,t} - \beta^{5}_{i,a}\langle t = T_0\rangle + \beta^{6}_{i,a,(t+1)}\langle t = T_0\rangle - \beta^{6}_{i,a,t}\langle t \neq T_0\rangle + \beta^{6}_{i,a,(t+1)}\langle t \neq T_0\rangle \leq M.\lambda^{5}_{i,a,t}\ ; \quad \forall i, a \epsilon A(i), t$$

(4. 52)

$$0 \leq r_{i,at} \leq RCAP_{i,a}.(1 - \lambda^{5}_{i,a,t})\ ; \quad \forall i, a \epsilon A(i), t$$

(4. 53)

$$0 \leq r_{i,a,t} - CHAR_{i,a,t} \leq RCAP_{i,a}.\lambda^{6}_{i,a,t} \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 54)

$$0 \leq \beta^{2}_{i,a,t} \leq M.(1 - \lambda^{6}_{i,a,t})\ ; \ \forall i, a \epsilon A(i), t$$

(4. 55)

$$0 \leq Q_{i,a} - \sum\nolimits_{n \epsilon N(i,a)} q_{i,a,t,n} \leq Q_{i,a}.\lambda^{7}_{i,a,t} \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 56)

$$0 \leq \beta^{3}_{i,a,t} \leq \alpha.\sum\nolimits_{n \epsilon N(i,a)} C_{i,a,t,n}.(1 - \lambda^{7}_{i,a,t}) \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 57)

$$0 \leq RCAP_{i,a} - r_{i,a,t} \leq RCAP_{i,a}.\lambda^{8}_{i,a,t} \quad ; \quad \forall i, a \epsilon A(i), ts$$

(4. 58)

$$0 \leq \beta^{4}_{i,a,t} \leq \alpha.CR_{i,a,t}.(1 - \lambda^{8}_{i,a,t}) \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 59)

$$\sum\nolimits_{t} \sum\nolimits_{n \epsilon N(i,a)} dem_{i,a,t,n} = DEM_{i,a} \quad ; \quad \forall i, a \epsilon A(i)$$

(4. 60)

$$\sum_{n\epsilon N(i,a)} q_{i,a,T_0,n} + \sum_{n\epsilon N(i,a)}(x^{+}_{i,a,t,n} - x^{-}_{i,a,t,n}) - r_{i,a,T_0} + R_{0_{i,a}} - \sum_{n\epsilon N(i,a)} dem_{i,a,T_0,n} = 0\ ; \quad \forall i, a\epsilon A(i)$$

(4. 61)

$$\sum_{n\epsilon N(i,a)} q_{i,a,t,n} + \sum_{n\epsilon N(i,a)}(x^{+}_{i,a,t,n} - x^{-}_{i,a,t,n}) - r_{i,a,t} + r_{i,a,(t-1)} - \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} - CHAR_{i,a,t-1} = 0\ ; \quad \forall i, a\epsilon A(i), t \neq T_0$$

(4. 62)

$$\lambda^1_{i,a,t,n}, \lambda^2_{i,a,t,n}, \lambda^3_{i,a,t,n}, \lambda^4_{i,a,t,n}, \lambda^5_{i,a,t}, \lambda^6_{i,a,t}, \lambda^7_{i,a,t}, \lambda^8_{i,a,t} \in \{0,1\}\ ; \quad \forall i, a\epsilon A(i), n, t$$

(4. 63)

$$\beta^1_{i,a}, \beta^5_{i,a}, \beta^6_{i,a,t} \quad \text{free}\ ; \quad \forall i, a\epsilon A(i), t \neq T_0$$

(4. 64)

A summary of the complete MIP model for the Stackelberg game problem is shown in Table 8.

Table 8 Summary of one leader multi-follower Stackelberg game- MIP Two-Way model

Type	*Hierarchy*	*Description*	*Equation(s)*
Objective Function	Leader	ISO Objective Function	(3. 1)
Constraints	Leader	ISO Constraints	(3. 2)-(3. 7)
	Follower	BGFs Integer Opt. Conditions	(4. 31)-(4. 43)
	Follower	SDRs Integer Opt. Conditions	(4. 44)-(4. 64)

SOS Model

Another way to linearize the complementarity constraint of an MPEC is to use SOS1 variables (Siddiqui and Gabriel, 2013). A specifically ordered set of variables (SOS1), is a group of variables of which at most one member can have a nonzero value in the solution. A mixed integer solver is required to solve any model containing SOS1 variables since the solution process needs to impose mutual exclusively and so it implicitly defines an additional set of binary variables. However, the SOS1 variables do not have to take on integer solutions. One of the advantages of employing SOS technique to complementarity constraints instead of MIP technique is preventing unstable solutions in MIP model due to large number of BigMs.

Using newly defined variables π and δ, any complementarity constraint such as (4. 27) could be written as a group of linear constraints. Assume

$$\pi = \frac{x+f(x)}{2} \tag{4. 65}$$

$$\delta = \frac{x-f(x)}{2} \tag{4. 66}$$

$$\pi^2 = \delta^2 \tag{4. 67}$$

Since

$$0 \le x \ , \ f(x) \ge 0 \tag{4. 68}$$

It can be concluded that

$$\pi = |\delta| \tag{4. 69}$$

Moreover, based on theorem mentioned in (Beale, 1975), an absolute value of a variable can be written as combination of its positive and negative parts which can be interpreted as special order set variables of type 1 (SOS1). So if δ^+ and δ^- be SOS1 variables then the complementarity constraint (4. 27) can be written as;

$$f(x) \geq 0 \tag{4. 70}$$

$$x \geq 0 \tag{4. 71}$$

$$\pi = \delta^+ + \delta^- \tag{4. 72}$$

$$\pi = \frac{x + f(x)}{2} \tag{4. 73}$$

$$\delta^+ - \delta^- = \frac{x - f(x)}{2} \tag{4. 74}$$

Applying the above mentioned definition to complementarity constraints on the MPEC problem in Table 7, an SOS model will be produced. The corresponding SOS model can be solved using CPLEX commercial solver.

Existence of Solution

Before introducing the developed solution algorithm for the proposed model, it is necessary to validate the model and show whether the aforementioned Stackelberg game has any feasible solution or not. That is to check whether the game among different participants in the lower level has any feasible equilibrium which is also feasible in the upper levels' constraints.

In this section solution existence to the Nash-Cournot game among BGFs and SDRs shown in Table 7 is discussed. Ralph and Smeers (2006) overviewed application of Equilibrium Problems with Equilibrium Constraints (EPEC) in electricity markets and brought up non-existence and multiple solution existence in this type of models. Hu and Ralph (2007) studied Equilibrium Problems with Equilibrium Constraints (EPEC) which are multiple MPECs in bi-level game for restructured electricity market. They established sufficient conditions for existence of pure strategy Nash equilibriums for these categories of problems and showed the stationary conditions of an EPEC can be phrased as a complementarity problem for which solutions are the Nash stationary points. However, contrary to general MPECs, there are different theorems and lemmas on solution existence for Linear Complementarity Problems (LCP) (Cottle, Pang et al., 2009). LCP is a special case of MPEC with no equality constraints. These theorems will be borrowed in this study to prove the existence of equilibrium.

Existence of equilibrium for the lower level games does not necessarily guarantee a solution to the whole MPEC problem. Even if the feasible region of the lower level is non empty and has feasible equilibrium points in it, but still these feasible equilibrium sets may not be feasible in the network constraints which are in the upper level problem. To deal with this issue, for the sake of proving the existence of solution, network constraints are considered in the BGFs and SDRs' decision problems. This ensures the optimality conditions also will consider upper level's constraints. That means if solution exists to this problem, then it is justified to conclude that the main problem with upper level constraints also has feasible solution.

To transform the MPEC to an LCP, some modifications to the original problems are needed. The modifications do not make any changes to the main problem and the model still will have the same feasible region and objective function. To convert the MPEC model in Table 7 to an LCP model, equality constraints (4. 8), (4. 24), (4. 25), and (4. 26) should be transformed to inequality constraints which means their corresponding constraints in the original model (Table 5), would change. (3. 10) would be

$$qg_{g,t} - xg_{g,t} - rg_{g,t} + rg_{g,t-1} \geq 0 \quad ; \quad \forall g \epsilon G, t \; -- \varepsilon^{2-}_{g,t} t$$

(4. 75)

$$qg_{g,t} - xg_{g,t} - rg_{g,t} + rg_{g,t-1} \leq 0 \quad ; \quad \forall g \epsilon G, t \; -- \varepsilon^{2+}_{g,t} t$$

(4. 76)

And (3. 14) would be

$$DEM_{i,a} \leq \sum_t \sum_{n \epsilon N(i,a)} dem_{i,a,t,n} \quad ; \quad \forall i, a \epsilon A(i) -- \beta^{1-}_{i,a}$$

(4. 77)

$$\sum_t \sum_{n \epsilon N(i,a)} dem_{i,a,t,n} \leq DEM_{i,a} \quad ; \quad \forall i, a \epsilon A(i) -- \beta^{1+}_{i,a}$$

(4. 78)

And (3. 18) would be

$$\sum_{n \epsilon N(i,a)} q_{i,a,T_0,n} + \sum_{n \epsilon N(i,a)} (x^{+}_{i,a,t,n} - x^{-}_{i,a,t,n}) - r_{i,a,T_0} + R_{0_{i,a}} - \sum_{n \epsilon N(i,a)} dem_{i,a,T_0,n} \geq 0 \quad ; \quad \forall i, a \epsilon A(i) \;---- \beta^{5-}_{i,a}$$

(4. 79)

$$\sum_{n \epsilon N(i,a)} q_{i,a,T_0,n} + \sum_{n \epsilon N(i,a)} (x^{+}_{i,a,t,n} - x^{-}_{i,a,t,n}) - r_{i,a,T_0} + R_{0_{i,a}} - \sum_{n \epsilon N(i,a)} dem_{i,a,T_0,n} \leq 0 \quad ; \quad \forall i, a \epsilon A(i) \;---- \beta^{5+}_{i,a}$$

(4. 80)

And finally (3. 19) would be

$$\sum_{n\epsilon N(i,a)} q_{i,a,t,n} + \sum_{n\epsilon N(i,a)}(x^{+}_{i,a,t,n} - x^{-}_{i,a,t,n}) - r_{i,a,t} + r_{i,a,(t-1)} - \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} - CHAR_{i,a,t-1} \geq 0 \quad ; \forall i, a\epsilon A(i), t \neq T_0 \quad ---- \beta^{6-}_{i,a,t}$$

(4. 81)

$$\sum_{n\epsilon N(i,a)} q_{i,a,t,n} + \sum_{n\epsilon N(i,a)}(x^{+}_{i,a,t,n} - x^{-}_{i,a,t,n}) - r_{i,a,t} + r_{i,a,(t-1)} - \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} - CHAR_{i,a,t-1} \leq 0 \quad ; \forall i, a\epsilon A(i), t \neq T_0 \quad ---- \beta^{6+}_{i,a,t}$$

(4. 82)

Also network constraints (3. 4) and (3. 5) should be considered in taking the KKT conditions.

$$\sum_{i\epsilon I} \sum_{a\epsilon A(i)} \sum_{n\epsilon N(i,a)} x_{i,a,t,n} - \sum_{g\in G} xg_{g,t} \geq 0\ ;\ \forall t ---- \ \theta^{1-}_{t}$$

(4. 83)

$$\sum_{i\epsilon I} \sum_{a\epsilon A(i)} \sum_{n\epsilon N(i,a)} x_{i,a,t,n} - \sum_{g\in G} xg_{g,t} \leq 0\ ;\ \forall t ---- \ \theta^{1+}_{t}$$

(4. 84)

$$\sum_{n\epsilon N}((\sum_{i\epsilon I} \sum_{a\epsilon A(i)} \sum_{n\epsilon N(i,a)} x_{i,a,t,n} - \sum_{g\in G|GNODE_g=n} xg_{g,t})D_{n,l})+K_l \geq 0\ ;\ \forall l\epsilon L, t \ ---- \ \theta^{2}_{l,t}$$

(4. 85)

$$K_l - \sum_{n\epsilon N}((\sum_{i\epsilon I} \sum_{a\epsilon A(i)} \sum_{n\epsilon N(i,a)} x_{i,a,t,n} - \sum_{g\in G|GNODE_g=n} xg_{g,t})D_{n,l}) \geq 0\ ;\ \forall l\epsilon L, t \ ---- \ \theta^{3}_{l,t}$$

(4. 86)

Substituting the modified constraints in the original model, the model would be still linear and convex, and so KKT conditions are both necessary and sufficient. Taking

KKTs from the modified model, the Nash-Cournot game among the BGFs and SDRs would be in the form of an LCP as follows:

$$0 \leq -Z + 2W.xg_{g,t} + W\sum_{gg \neq g} xg_{gg,t} + \varepsilon_{g,t}^{2-} - \varepsilon_{g,t}^{2+} + \theta_t^{1-} - \theta_t^{1+} + \theta_{l,t}^{2}\sum_{l} D_{n,l} - \theta_{l,t}^{3}\sum_{l} D_{n,l} \perp xg_{g,t} \geq 0 \quad ; \quad \forall g\epsilon G, t \tag{4.87}$$

$$0 \leq CG_{g,t} + \varepsilon_{g,t}^{1} - \varepsilon_{g,t}^{2-} + \varepsilon_{g,t}^{2+} \perp qg_{g,t} \geq 0 \quad ; \quad \forall g\epsilon G, t \tag{4.88}$$

$$0 \leq CRG_{g,t} + \varepsilon_{g,t+1}^{2+}(t < T) - \varepsilon_{g,t+1}^{2-}(t < T) - \varepsilon_{g,t}^{2+} + \varepsilon_{g,t}^{2-} + \varepsilon_{g,t}^{3} \perp rg_{g,t} \geq 0; \forall g\epsilon G, t \tag{4.89}$$

$$0 \leq QG_g - qg_{g,t} \perp \varepsilon_{g,t}^{1} \geq 0 \quad ; \quad \forall g\epsilon G, t \tag{4.90}$$

$$0 \leq qg_{g,t} - xg_{g,t} - rg_{g,t} + rg_{g,t-1}(t > T0) \perp \varepsilon_{g,t}^{2-} \geq 0 \quad ; \quad \forall g\epsilon G, t \tag{4.91}$$

$$0 \leq -qg_{g,t} + xg_{g,t} + rg_{g,t} - rg_{g,t-1}(t > T0) \perp \varepsilon_{g,t}^{2+} \geq 0 \quad ; \quad \forall g\epsilon G, t \tag{4.92}$$

$$0 \leq RG_g - rg_{g,t} \perp \varepsilon_{g,t}^{3} \geq 0 \quad ; \quad \forall g\epsilon G, t \tag{4.93}$$

$$0 \leq \sum_{i\epsilon I}\sum_{a\epsilon A(i)}\sum_{n\epsilon N(i,a)} x_{i,a,t,n} - \sum_{g\in G} xg_{g,t} \perp \theta_t^{1-} \geq 0 \tag{4.94}$$

$$0 \leq -\sum_{i\epsilon I}\sum_{a\epsilon A(i)}\sum_{n\epsilon N(i,a)} x_{i,a,t,n} + \sum_{g\in G} xg_{g,t} \perp \theta_t^{1+} \geq 0 \tag{4.95}$$

$$0 \le \sum_{n\epsilon N}((\sum_{i\epsilon I}\sum_{a\epsilon A(i)}\sum_{n\epsilon N(i,a)} x_{i,a,t,n} - \sum_{g\in G|GNODE_g=n} xg_{g,t})D_{n,l}) + K_l \perp \theta^2_{l,t} \ge 0$$

(4. 96)

$$0 \le -\sum_{n\epsilon N}((\sum_{i\epsilon I}\sum_{a\epsilon A(i)}\sum_{n\epsilon N(i,a)} x_{i,a,t,n} - \sum_{g\in G|GNODE_g=n} xg_{g,t})D_{n,l}) + K_l \perp \theta^3_{l,t} \ge 0$$

(4. 97)

$$0 \le Z - W\sum_{g\epsilon G} xg_{g,t} + \beta^{5+}_{i,a}\langle t = T_0\rangle - \beta^{5-}_{i,a}\langle t = T_0\rangle + \beta^{6+}_{i,a,t}\langle t \ne T_0\rangle - \beta^{6-}_{i,a,t}\langle t \ne T_0\rangle +$$

$$\theta^{1+}_t - \theta^{1-}_t - \theta^2_{l,t}\sum_l D_{n,l} + \theta^3_{l,t}\sum_l D_{n,l} \perp xp_{i,a,t,n} \ge 0; \forall i, a\epsilon A(i), n, t$$

(4. 98)

$$0 \le -Z + W\sum_{g\epsilon G} xg_{g,t} - \beta^{5+}_{i,a}\langle t = T_0\rangle + \beta^{5-}_{i,a}\langle t = T_0\rangle - \beta^{6+}_{i,a,t}\langle t \ne T_0\rangle + \beta^{6-}_{i,a,t}\langle t \ne T_0\rangle -$$

$$\theta^{1+}_t + \theta^{1-}_t + \theta^2_{l,t}\sum_l D_{n,l} - \theta^3_{l,t}\sum_l D_{n,l} \perp xn_{i,a,t,n} \ge 0; \forall i, a\epsilon A(i), n, t$$

(4. 99)

$$0 \le -V_{i,a,t,n} + \beta^{1+}_{i,a} - \beta^{1-}_{i,a} - (\beta^{5+}_{i,a} - \beta^{5-}_{i,a})\langle t = T_0\rangle - (\beta^{6+}_{i,a,t} - \beta^{6-}_{i,a,t})\langle t \ne T_0\rangle \perp$$

$$dem_{i,a,t,n} \ge 0 ; \quad \forall i, a\epsilon A(i), n, t$$

(4. 100)

$$0 \le C_{i,a,t,n} + \beta^3_{i,a} + (\beta^{5+}_{i,a} - \beta^{5-}_{i,a})\langle t = T_0\rangle + (\beta^{6+}_{i,a,t} - \beta^{6-}_{i,a,t})\langle t \ne T_0\rangle \perp q_{i,a,t,n} \ge$$

$$0 ; \quad \forall i, a\epsilon A(i), n, t$$

(4. 101)

$$0 \le CR_{i,a,t} - \beta^2_{i,a,t} + \beta^4_{i,a,t} - (\beta^{5+}_{i,a} - \beta^{5-}_{i,a})\langle t = T_0\rangle + (\beta^{6+}_{i,a,(t+1)} - \beta^{6-}_{i,a,(t+1)})\langle t = T_0\rangle -$$

$$(\beta^{6+}_{i,a,t} - \beta^{6-}_{i,a,t})\langle t \ne T_0\rangle + (\beta^{6+}_{i,a,(t+1)} - \beta^{6-}_{i,a,(t+1)})\langle t \ne T_0\rangle \perp r_{i,at} \ge 0 ; \quad \forall i, a\epsilon A(i), t$$

(4. 102)

$$0 \le r_{i,a,t} - CHAR_{i,a,t} \perp \beta^2_{i,a,t} \ge 0 \quad ; \quad \forall i, a\epsilon A(i), t$$

(4. 103)

$$0 \leq Q_{i,a} - \sum_{n \epsilon N(i,a)} q_{i,a,t,n} \perp \beta^{3}_{i,a,t} \geq 0 \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 104)

$$0 \leq RCAP_{i,a} - r_{i,a,t} \perp \beta^{4}_{i,a,t} \geq 0 \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 105)

$$0 \leq \sum_{t} \sum_{n \epsilon N(i,a)} dem_{i,a,t,n} - DEM_{i,a} \perp \beta^{1-}_{i,a} \geq 0 \quad ; \quad \forall i, a \epsilon A(i)$$

(4. 106)

$$0 \leq DEM_{i,a} - \sum_{t} \sum_{n \epsilon N(i,a)} dem_{i,a,t,n} \perp \beta^{1+}_{i,a} \geq 0 \quad ; \quad \forall i, a \epsilon A(i)$$

(4. 107)

$$0 \leq \sum_{n \epsilon N(i,a)} q_{i,a,T_0,n} + \sum_{n \epsilon N(i,a)} (xp_{i,a,T_0,n} - xn_{i,a,T_0,n}) - r_{i,a,T_0} + R0_{i,a} -$$
$$\sum_{n \epsilon N(i,a)} dem_{i,a,T_0,n} \perp \beta^{5-}_{ia} \geq 0 \quad ; \quad \forall i, a \epsilon A(i)$$

(4. 108)

$$0 \leq - \sum_{n \epsilon N(i,a)} q_{i,a,T_0,n} - \sum_{n \epsilon N(i,a)} (xp_{i,a,T_0,n} - xn_{i,a,T_0,n}) + r_{i,a,T_0} - R0_{i,a} +$$
$$\sum_{n \epsilon N(i,a)} dem_{i,a,T_0,n} \perp \beta^{5+}_{ia} \geq 0 \quad ; \quad \forall i, a \epsilon A(i)$$

(4. 109)

$$0 \leq \sum_{n \epsilon N(i,a)} q_{i,a,t,n} + \sum_{n \epsilon N(i,a)} (xp_{i,a,t,n} - xn_{i,a,t,n}) - r_{i,a,t} + r_{i,a,(t-1)} -$$
$$\sum_{n \epsilon N(i,a)} dem_{i,a,t,n} - CHAR_{i,a,t-1} \perp \beta^{6-}_{i,a,t} \geq 0; \forall i, a \epsilon A(i), t \neq T_0$$

(4. 110)

$$0 \leq - \sum_{n \epsilon N(i,a)} q_{i,a,t,n} - \sum_{n \epsilon N(i,a)} (xp_{i,a,t,n} - xn_{i,a,t,n}) + r_{i,a,t} - r_{i,a,(t-1)} +$$
$$\sum_{n \epsilon N(i,a)} dem_{i,a,t,n} + CHAR_{i,a,t-1} \perp \beta^{6+}_{i,a,t} \geq 0 \, ; \forall i, a \epsilon A(i), t \neq T_0$$

(4. 111)

Now that the model is in LCP format, it is possible to continue with the proof of existence. Assume

$z = [z_1 \quad z_2]$

$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$

$M = \left[\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}\right]$

Then the modified Stackelberg model (4. 87)-(4. 111) could be written as $LCP^*(q, M)$ when:

$z_1^T = [\mathrm{xg}_{g,t}\ \mathrm{qg}_{g,t}\ \mathrm{rg}_{g,t}\ \varepsilon^1_{g,t}\ \varepsilon^3_{g,t}\ \varepsilon^{2-}_{g,t}\ \varepsilon^{2+}_{g,t}\ \theta^{1-}_t\ \theta^{1+}_t\ \theta^2_{l,t}\ \theta^3_{l,t}]$

$z_2^T =$

$[\mathrm{xp}_{i,a,t,n}\ \mathrm{xn}_{i,a,t,n}\ \mathrm{dem}_{i,a,t,n}\ \mathrm{q}_{i,a,t,n}\ \mathrm{r}_{i,a,t}\ \beta^2_{i,a,t}\ \beta^3_{i,a,t}\ \beta^4_{i,a,t}\ \beta^{1-}_{i,a}\ \beta^{1+}_{i,a}\ \beta^{5-}_{ia}\ \beta^{5+}_{ia}\ \beta^{6-}_{i,a,t}\ \beta^{6+}_{i,a,t}\ \theta^{1-}_t\ \theta^{1+}_t\ \theta^2_{l,t}\ \theta^3_{l,t}]$

$$q_1 = \begin{bmatrix} -Z \\ \mathrm{CG}_{g,t} \\ \mathrm{CRG}_{g,t} \\ \mathrm{QG}_g \\ \mathrm{RG}_g \\ 0 \\ 0 \\ \sum_{i\epsilon I}\sum_{a\epsilon A(i)}\sum_{n\epsilon N(i,a)} x_{i,a,t,n} \\ -\sum_{i\epsilon I}\sum_{a\epsilon A(i)}\sum_{n\epsilon N(i,a)} x_{i,a,t,n} \\ \sum_{n\epsilon N}(D_{n,l}\sum_{i\epsilon I}\sum_{a\epsilon A(i)}\sum_{n\epsilon N(i,a)} x_{i,a,t,n}) + K_l \\ -\sum_{n\epsilon N}(D_{n,l}\sum_{i\epsilon I}\sum_{a\epsilon A(i)}\sum_{n\epsilon N(i,a)} x_{i,a,t,n}) + K_l \end{bmatrix}$$

$$
q_2 = \begin{bmatrix}
Z \\
-Z \\
-\mathrm{V}_{\mathrm{i,a,t,n}} \\
\mathrm{C}_{\mathrm{i,a,t,n}} \\
\mathrm{CR}_{\mathrm{i,a,t}} \\
-\mathrm{CHAR}_{\mathrm{i,a,t}} \\
\mathrm{Q}_{\mathrm{i,a}} \\
\mathrm{RCAP}_{\mathrm{i,a}} \\
-\mathrm{DEM}_{\mathrm{i,a}} \\
\mathrm{DEM}_{\mathrm{i,a}} \\
\mathrm{R}_{0_{\mathrm{i,a}}} \\
-\mathrm{R}_{0_{\mathrm{i,a}}} \\
-\mathrm{CHAR}_{\mathrm{i,a,t-1}} \\
\mathrm{CHAR}_{\mathrm{i,a,t-1}} \\
-\sum_{\mathrm{g \in G}} \mathrm{xg}_{\mathrm{g,t}} \\
\sum_{\mathrm{g \in G}} \mathrm{xg}_{\mathrm{g,t}} \\
-\sum_{\mathrm{n \epsilon N}} \mathrm{D}_{\mathrm{n,l}} \sum_{\mathrm{g \in G | GNODE_g = n}} \mathrm{xg}_{\mathrm{g,t}} + \mathrm{K}_{\mathrm{l}} \\
\sum_{\mathrm{n \epsilon N}} \mathrm{D}_{\mathrm{n,l}} \sum_{\mathrm{g \in G | GNODE_g = n}} \mathrm{xg}_{\mathrm{g,t}} + \mathrm{K}_{\mathrm{l}}
\end{bmatrix}
$$

$M_1 =$

$$
\begin{bmatrix}
\begin{Bmatrix} 2W;\ g' = g \\ W; g' \neq g \end{Bmatrix} & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & \mathrm{D}_{\mathrm{n,l}} & -\mathrm{D}_{\mathrm{n,l}} \\
0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \begin{Bmatrix} 1;\ g' = g \\ -1; t' = t+1 | t < T \end{Bmatrix} & \begin{Bmatrix} -1;\ g' = g \\ 1; t' = t+1 | t < T \end{Bmatrix} & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & \begin{Bmatrix} -1;\ g' = g \\ 1; t' = t-1 | t > \mathrm{T}_0 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & \begin{Bmatrix} 1;\ g' = g \\ -1; t' = t-1 | t > \mathrm{T}_0 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\mathrm{D}_{\mathrm{n,l}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathrm{D}_{\mathrm{n,l}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
M_2 = \left[\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \begin{Bmatrix} 0; t = T_0 \\ -1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ 1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = \mathrm{t}, \mathrm{t} \neq T_0 \\ 1; t = \mathrm{t} - 1 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \begin{Bmatrix} 0; t = T_0 \\ 1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ -1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = \mathrm{t}, \mathrm{t} \neq T_0 \\ -1; t = \mathrm{t} - 1 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
D_{n,l} & -D_{n,l} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-D_{n,l} & D_{n,l} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}\right.
$$

$$
\left.\begin{matrix}
0 & 0 & 0 & 0 & -1 & 1 & -D_{n,l} & D_{n,l} \\
0 & 0 & 0 & 0 & 1 & -1 & D_{n,l} & -D_{n,l} \\
\begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ 1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ -1; t \neq T_0 \end{Bmatrix} & 0 & 0 & 0 & 0 \\
\begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ -1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ 1; t \neq T_0 \end{Bmatrix} & 0 & 0 & 0 & 0 \\
\begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = \mathrm{t} + 1 \\ 1; t = \mathrm{t}, t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = \mathrm{t} + 1 \\ -1; t = \mathrm{t}, t \neq T_0 \end{Bmatrix} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}\right]
$$

Cottle and Pang (2009) define a matrix $M \in R^{n\times n}$ to be:

a. Copositive if $y^T M y \geq 0$ for all $y \in R^n_+$

b. Strictly copositive if $y^T M y > 0$ for all nonzero $y \in R_+^n$

c. Copositive-plus if M is copositive and the following implication holds:

$[y^T M y = 0, y \geq 0] \Longrightarrow [(M + M^T)y = 0]$

d. Copositive-star if M is copositive and the following implication holds:

$[y^T M y = 0, My \geq 0, y \geq 0] \Longrightarrow [M^T y \leq 0]$

They showed if $M \in R^{n \times n}$ is strictly copositive, then for each $q \in R^n$, the $LCP(q, M)$ has a solution. And if $M \in R^{n \times n}$ is copositive star, then the following statements would be equivalent:

a. M is an S-matrix (Stiemke Matrix) ($LCP(q, M)$ is feasible for all choices of q)

b. M is a Q-matrix ($LCP(q, M)$ is solvable for all choices of q).

Using these theorems and the defined $LCP^*(q, M)$, solution existence is proved as follows.

Proposition 1

$LCP^*(q, M)$ has a solution and is feasible and solvable for all choices of q.

Proof:

If $y_1 \in R_+^{11*|G|}$, $y_2 \in R_+^{18}$, and $y = [y_1 \quad y_2] \in R_+^n$:

$$y^T M y = y^T \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} y = {y_1}^T M_1 y_1 + {y_2}^T M_2 y_2 \qquad \text{(P. 1)}$$

If two parts are separated then it is easy to show

$$y_1^T M_1 y_1 = 2W \sum_g {y_1^{1g}}^2 + W \sum_g \sum_{g' \neq g} y_1^{1g'} . y_1^{1g} \qquad \text{(P. 2)}$$

Based on assumptions made for the problem it is given that

$$\begin{Bmatrix} y_1 > 0 \\ W > 0 \end{Bmatrix}$$

(P. 3)

(P. 2) & (P. 3) $\Longrightarrow y_1^T M_1 y_1 > 0$

(P. 4)

If it is assumed that

$$\begin{Bmatrix} y_1 \geq 0 \\ M_1 y_1 \geq 0 \\ y_1^T M_1 y_1 = 0 \end{Bmatrix}$$

(P. 5)

$$\Longrightarrow y_1 = 0 \Longrightarrow M_1^T y_1 \leq 0$$

(P. 6)

On the other side, since M_2 is a skew-symmetric matrix for any y_2, it is clear that

$$y_2^T M_2 y_2 = 0$$

(P. 7)

and

$$M_2 + M_2^T = 0 \ \Longrightarrow (M_2 + M_2^T) y_2 = 0$$

(P. 8)

So if it is assumed that

$$\begin{Bmatrix} y_2 > 0 \\ M_2 y_2 \geq 0 \\ y_2^T M_2 y_2 = 0 \end{Bmatrix}$$

(P. 9)

Considering $M_2 = -M_2^T$

It can be concluded that $M_2^T y_2 \leq 0$ should always be true. So if $y > 0$, it can be concluded that

$$\text{(P. 4)\&(P. 7)} \Longrightarrow \begin{Bmatrix} {y_1}^T M_1 y_1 \geq 0 \\ {y_2}^T M_2 y_2 = 0 \\ y > 0 \end{Bmatrix} \Longrightarrow y^T M y > 0$$

(P. 10)

Based on (Cottle, Pang et al., 2009) and (P. 10), M is strictly copositive, and so for each $q \in R^n$, $LCP^*(q, M)$ has a solution.

Moreover, if

$$y^T M y = 0 \Longrightarrow {y_1}^T M_1 y_1 + {y_2}^T M_2 y_2 = 0$$

(P. 11)

but given (P. 7) it is concluded that

$${y_2}^T M_2 y_2 = 0 \ \Longrightarrow {y_1}^T M_1 y_1 = 0$$

(P. 12)

Given

$$W > 0 \Longrightarrow y_1 = 0$$

(P. 13)

As a result if $\begin{Bmatrix} My \geq 0 \\ y \geq 0 \end{Bmatrix}$ then

$$\Longrightarrow M_1 y_1 + M_2 y_2 \geq 0$$

(P. 14)

However, from (P. 13) it is known that

$$y_1 = 0 \Longrightarrow M_2 y_2 \geq 0$$

(P. 15)

Since M_2 is a skew-symmetric matrix so

$$\Longrightarrow M_2 y_2 \geq 0 \; and \; M_2 = -M_2^T \Longrightarrow M_2^T y_2 \leq 0 \Longrightarrow My \leq 0$$

(P. 16)

So M is copositive star which means M is both S and Q-matrix, and so $LCP^*(q, M)$ is feasible and solvable for all choices of q. □

Proposition 2

The MPEC Stackelberg game defined in Table 7 has at least a solution if feasible region is non-empty.

Proof:

In the MPEC model, ISO as the leader decides on the amount of trades (imports/exports) in the network. Different decisions of ISO affect the lower level $LCP^*(q, M)$ by varying q vector. According to Proposition 1, $LCP^*(q, M)$ is feasible and solvable for all choices of q and has at least a solution.

Since network constraints are added to the LCP of the lower level decision problems, it is shown that there is always at least one feasible equilibrium solution for every decision made by ISO which is also feasible in the ISO's constraints. So if ISO's constraints make a feasible region itself, then it definitely has a common area with feasible region of the lower level game, and so the MPEC would at least have one optimal solution for the linear objective function of the ISO. □

Despite proving existence of solution to general proposed MPEC, there is still an issue of having multiple solutions for the problem. Although $LCP^*(q, M)$ has a feasible solution for all q, the solution might not be unique and there might be multi-equilibria for the game. Studies are available on finding all equilibria in a game (Cottle, Pang et al., 2009, Day, Hobbs et al., 2002, Leyffer and Munson, 2010). However, existence of multiple solutions for the lower level game is a benefit for the model since it extends the feasible region for ISO. Choosing the right market equilibrium in case of multiple equilibriums in the Stackelberg model will be a major concern as this issue has been raised in some studies (Neuhoff, Barquin et al., 2005). However, the author believes this can be a plus to the model since at any time the operator can choose the best solution among multiple solutions based on the situation.

Solution Algorithm

Two main challenges are on the table solving the proposed Stackelberg model. First is the huge size of variables in real cases which increases dramatically with increase in the number of users and appliances and changes in network topology. This gets worse considering the dynamic nature of the problem. The problem is being solved in a rolling horizon manner and so should be solved in a matter of minutes in real cases. Second is the special shape of the feasible region. Heuristics are basically developed based on finding an initial feasible solution and then improving it. However, if the feasible region is small and has a special shape, then heuristics mostly behave weakly in finding the initial feasible point to start with. Considering the size and dynamic nature of the problem, the MPEC and the converted MIP model are both very complex and time consuming to solve. To overcome these mentioned challenges there is a need for developing a new algorithm to benefit from the special structure of the problem.

Bi-level programming problems are mathematical optimization modeling in which decision variables are divided into two sets and one of them being determined parametrically based on the other set. Although using bi-level programming is very useful and realistic in many real world problems, generally solving them is very complex and difficult. The complexity is even more sensible when the size of the problem increases. Complexity of the models and solution approaches mainly are dependent on how decision variables are divided between different levels of the model and how all of them are related to each other. Consequently, how to model a bi-level problem and benefiting from special structure of the model are the main keys toward solving them.

Previously, it was shown that the lower level optimization model is guaranteed to have at least one feasible solution for any solution of the upper level decision problem. This is one of the key bases on which to build the algorithm. In the first stage of the algorithm, ISO's decision problem is solved including all lower level constraints. Then total trades at each time and flow levels on lines are fixed. The fixed amounts are dictated as constants to all other participants in the system in the second stage. There are two main games in the lower level. One is among BGFs and the other is among SDRs. In the BGF's game, SDRs' trade values are set based on the ISO's solution. Then the algorithm solves BGF's problem for their best decisions through maximizing sum of their objectives in stage 2 while including ISO's constraints. BGFs decisions from stage 2 are then fixed and fed into the SDRs decision problem. In stage 3, each user's best action in the game assuming fixed BGFs' variables, total trades at each time and lines' flows is found through maximizing total sum of SDRs' objectives. To ensure feasibility of solutions for ISO's problem, ISO's constraints are included in stage 3. Moreover, to eliminate some of the bad solutions a cut is added based on the best bound of the solution. Finally, in stage 4, ISO's problem is resolved assuming all the lower level variables are fixed to search for the best possible solution and objective value. The flowchart of the developed algorithm and the algorithm itself are shown in Figure 10 and Table 9.

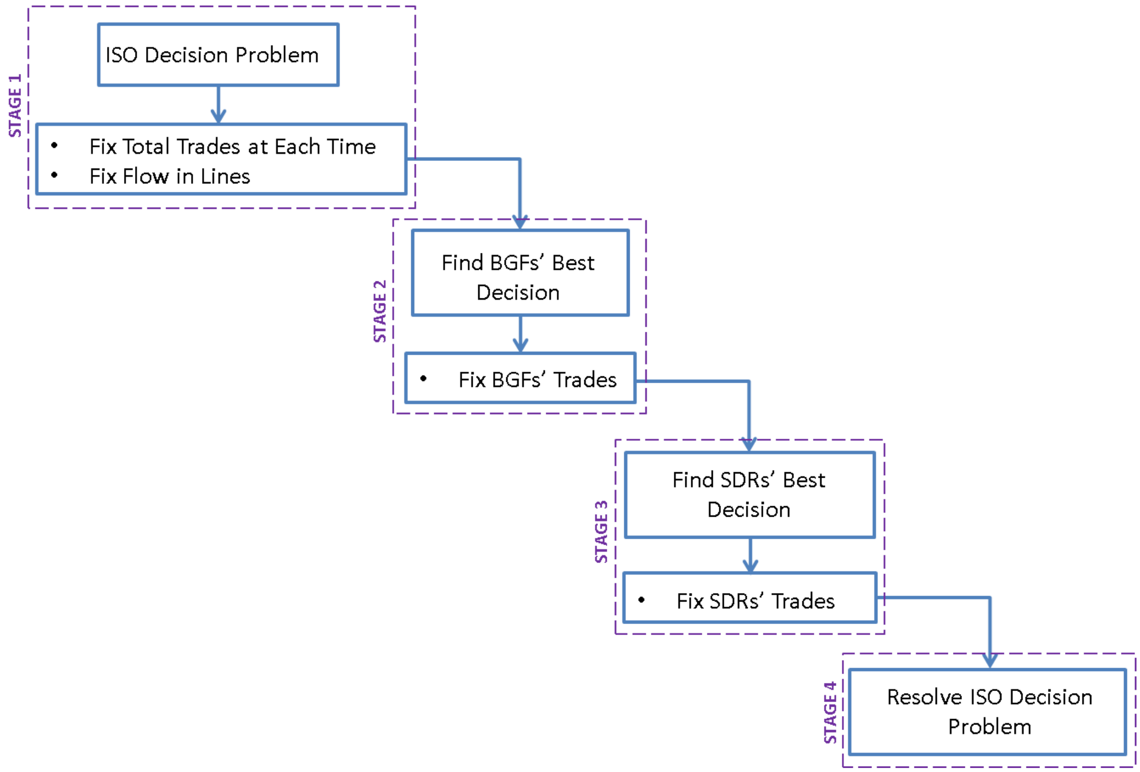

Figure 10 Flowchart for the solution algorithm for the Two-Way model

Benefiting from the special structure of the model and the assumptions, it can be shown the solution of the proposed algorithm is a feasible solution to the proposed Stackelberg model. Assume the following:

Case 1: is the MPEC shown in Table 7 with BGFs' and SDRs' KKT conditions included as the ISO's constraints. Optimal solution and objectives for this case are shown by x_1^*, ${x_g}_1^*$, ISO_1^*, BGF_1^*, SDR_1^* respectively.

Case 2: is the algorithm proposed for solving the Stackelberg game in Case 1 and shown in Table 9. Optimal solution and objectives for this case are shown as x_2^*, ${x_g}_2^*$, ISO_2^*, BGF_2^*, SDR_2^*.

Finally, assume solution set of x_2, ${x_g}_2$, ISO_2, BGF_2, SDR_2 is reached in the first stage of Case 2. In stage 2, sum of x_2 in (3. 4) and (3. 5) is fixed and then BGFs decision problem is solved for $Max. \sum_g BGF_g$ to find ${x_g}_2^*$, ${q_g}_2^*$, and ${r_g}_2^*$.

Table 9 Summary of the solution algorithm for the Two-Way model

Stage	*Input*	*Equation(s)*
1		Solve $\mathrm{Min.} \sum_t \sum_{n \in N} P_{n,t} . (\sum_{(g \epsilon G \mid gnode(g) = n)} xg_{g,t} - expo_{n,t})$ s.t.: (3. 2)-(3. 7) & (3. 9)-(3. 12) & (3. 14) -(3. 21)
2	$\bar{x}_{i,a,t,n}$	Solve $\mathrm{Max.} \sum_{g \epsilon G} \sum_t ((Z - W \sum_{gg \epsilon G} xg_{gg,t}) xg_{g,t} - CG_{g,t} . qg_{g,t} - CRG_{g,t} . rg_{g,t})$ s.t.: (3. 4)-(3. 5) & (3. 9)-(3. 12)
3	$\overline{xg}_{g,t}$, $\overline{expo}_{n,t}$	Solve $\mathrm{Max.} \sum_{i \epsilon I} \sum_t (\sum_{a \epsilon A(i)} \sum_{n \epsilon N(i,a)} (V_{i,a,t,n} . dem_{i,a,t,n} - C_{i,a,t,n} . q_{i,a,t,n} - (Z - W \sum_{g \epsilon G} xg_{g,t}) . x_{i,a,n,t}) + \sum_{a \epsilon A(i)} (VCHAR_{i,a,t} . CHAR_{i,a,t} - CR_{i,a,t} . r_{i,a,t}))$ s.t.: $\overline{expo}_{n,t} \leq \sum_{i \in I} \sum_{a \epsilon A(i)} x_{i,a,t,n}$ & (3. 4)-(3. 5) & (3. 14) -(3. 21)
4	$\bar{x}_{i,a,t,n}$, $\overline{xg}_{g,t}$	Solve $\mathrm{Min.} \sum_t \sum_{n \in N} P_{n,t} . (\sum_{(g \epsilon G \mid gnode(g) = n)} \overline{xg}_{g,t} - expo_{n,t})$ s.t.: (3. 2)-(3. 3)

Proposition 3

Set of ${x_g}_2^*$, ${q_g}_2^*$, ${r_g}_2^*$, and $\sum_g BGF_2^*$ in Case 2 is an equilibrium set for the BGFs' game in Case1.

Proof: (Proof by contradiction)

Assume to the contrary that the solution set is not an equilibrium in Case 1. That is having all players' with the same decisions; one player has a better move to play. Its move can be either through change of ${q_g}_2$, ${r_g}_2$ or ${x_g}_2$.

i. If it is due to a different value of ${q_g}_2$ and ${r_g}_2$, then new set of decisions for the player does not affect other decision makers' decision. Since its new decision does not have influence over other players' objective. This means there should be a better objective value for that player using the new set of ${q_g}_2$ and ${r_g}_2$. According to Bellman's principle of optimality, this results in a better total $\sum_g BGF_g$. However, it was assumed that $\sum_g BGF_2^*$ is maximized and no better solution should exist. So this is contradictory to the assumption.

ii. If it is due to different value of ${x_g}_2$, then in order to comply with constant sum of ${x_g}_2$ in constraints (3. 4) and (3. 5), other players should also change their strategy which is a contradiction as well.

So solution set $\sum_g BGF_2^*$, ${x_g}_2^*$, ${q_g}_2^*$, and ${r_g}_2^*$ is an equilibrium to the BGF's game in Case1. □

It can also be shown in the same approach that solution of stage 3 in Case 2 is an equilibrium in the SDR's game in Case 1.

Proposition 4

Objective value of stage 1 in Case 2 is a lower bound for the problem.

Proof:

In Case 2, the equilibrium in the lower level games are found upon a fixed constant fed into (3. 4) and (3. 5) in each game. This fixed constant is set by ISO, while in Case 1, for each set of equilibrium in the lower level there is a fixed constant for the (3. 4) and (3. 5) constraints. ISO is optimizing its objective over these feasible equilibrium sets. In stage 1 of Case 2, the best constant for (3. 4) and (3. 5) constraints for ISO's decision problem is found. Although this constant remains the same for the final equilibrium sets in stage 4, but export variable would change and so objective value will change. However, this value will never be better than the stage 1's objective value. So the objective in stage 1 can be used as the lower bound for the problem. □

One-Way model

Existence of Solution

Solution existence for the One-Way model can be proved in the same way as the Two-Way model. The only difference is that while $x_{i,a,t,n}$ and $xg_{g,t}$ are variables in the lower level of the Two-Way model, they are not in the lower level of the One-Way model. So as will be shown, there is no need to consider the upper level constraints in the lower level game to prove the solution existence. After modifying the equilibrium constraints of the One-Way problem in Table 6, and taking the KKT of the modified model, the LCP of the One-Way problem would be as follows:

$$0 \leq CG_{g,t} + \varepsilon^{1}_{g,t} - \varepsilon^{2-}_{g,t} + \varepsilon^{2+}_{g,t} \perp qg_{g,t} \geq 0 \quad ; \quad \forall g \epsilon G, t$$

(4. 112)

$$0 \leq CRG_{g,t} + \varepsilon^{2+}_{g,t+1}(t < T) - \varepsilon^{2-}_{g,t+1}(t < T) - \varepsilon^{2+}_{g,t} + \varepsilon^{2-}_{g,t} + \varepsilon^{3}_{g,t} \perp rg_{g,t} \geq 0; \forall g \epsilon G, t$$

(4. 113)

$$0 \leq QG_g - qg_{g,t} \perp \varepsilon^1_{g,t} \geq 0 \quad ; \quad \forall g \epsilon G, t$$

(4. 114)

$$0 \leq qg_{g,t} - xg_{g,t} - rg_{g,t} + rg_{g,t-1}(t > T0) \perp \varepsilon^{2-}_{g,t} \geq 0 \quad ; \quad \forall g \epsilon G, t$$

(4. 115)

$$0 \leq -qg_{g,t} + xg_{g,t} + rg_{g,t} - rg_{g,t-1}(t > T0) \perp \varepsilon^{2+}_{g,t} \geq 0 \quad ; \quad \forall g \epsilon G, t$$

(4. 116)

$$0 \leq RG_g - rg_{g,t} \perp \varepsilon^3_{g,t} \geq 0 \quad ; \quad \forall g \epsilon G, t$$

(4. 117)

$$0 \leq -V_{i,a,t,n} + \beta^{1+}_{i,a} - \beta^{1-}_{i,a} - (\beta^{5+}_{i,a} - \beta^{5-}_{i,a})\langle t = T_0 \rangle - (\beta^{6+}_{i,a,t} - \beta^{6-}_{i,a,t})\langle t \neq T_0 \rangle \perp dem_{i,a,t,n} \geq 0 ; \quad \forall i, a \epsilon A(i), n, t$$

(4. 118)

$$0 \leq C_{i,a,t,n} + \beta^3_{i,a} + (\beta^{5+}_{i,a} - \beta^{5-}_{i,a})\langle t = T_0 \rangle + (\beta^{6+}_{i,a,t} - \beta^{6-}_{i,a,t})\langle t \neq T_0 \rangle \perp q_{i,a,t,n} \geq 0 ; \quad \forall i, a \epsilon A(i), n, t$$

(4. 119)

$$0 \leq CR_{i,a,t} - \beta^2_{i,a,t} + \beta^4_{i,a,t} - (\beta^{5+}_{i,a} - \beta^{5-}_{i,a})\langle t = T_0 \rangle + (\beta^{6+}_{i,a,(t+1)} - \beta^{6-}_{i,a,(t+1)})\langle t = T_0 \rangle - (\beta^{6+}_{i,a,t} - \beta^{6-}_{i,a,t})\langle t \neq T_0 \rangle + (\beta^{6+}_{i,a,(t+1)} - \beta^{6-}_{i,a,(t+1)})\langle t \neq T_0 \rangle \perp r_{i,at} \geq 0 ; \quad \forall i, a \epsilon A(i), t$$

(4. 120)

$$0 \leq r_{i,a,t} - CHAR_{i,a,t} \perp \beta^2_{i,a,t} \geq 0 \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 121)

$$0 \leq Q_{i,a} - \sum_{n \epsilon N(i,a)} q_{i,a,t,n} \perp \beta^3_{i,a,t} \geq 0 \quad ; \quad \forall i, a \epsilon A(i), t$$

(4. 122)

$$0 \leq RCAP_{i,a} - r_{i,a,t} \perp \beta^{4}_{i,a,t} \geq 0 \quad ; \quad \forall i, a\epsilon A(i), t$$

(4. 123)

$$0 \leq \sum_{t} \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} - DEM_{i,a} \perp \beta^{1-}_{i,a} \geq 0 \quad ; \quad \forall i, a\epsilon A(i)$$

(4. 124)

$$0 \leq DEM_{i,a} - \sum_{t} \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} \perp \beta^{1+}_{i,a} \geq 0 \quad ; \quad \forall i, a\epsilon A(i)$$

(4. 125)

$$0 \leq \sum_{n\epsilon N(i,a)} q_{i,a,T_0,n} + \sum_{n\epsilon N(i,a)} x_{i,a,T_0,n} - r_{i,a,T_0} + R0_{i,a} - \sum_{n\epsilon N(i,a)} dem_{i,a,T_0,n} \perp$$

$$\beta^{5-}_{ia} \geq 0 \quad ; \quad \forall i, a\epsilon A(i)$$

(4. 126)

$$0 \leq -\sum_{n\epsilon N(i,a)} q_{i,a,T_0,n} - \sum_{n\epsilon N(i,a)} x_{i,a,T_0,n} + r_{i,a,T_0} - R0_{i,a} + \sum_{n\epsilon N(i,a)} dem_{i,a,T_0,n} \perp$$

$$\beta^{5+}_{ia} \geq 0 \quad ; \quad \forall i, a\epsilon A(i)$$

(4. 127)

$$0 \leq \sum_{n\epsilon N(i,a)} q_{i,a,t,n} + \sum_{n\epsilon N(i,a)} x_{i,a,t,n} - r_{i,a,t} + r_{i,a,(t-1)} - \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} -$$

$$CHAR_{i,a,t-1} \perp \beta^{6-}_{i,a,t} \geq 0; \forall i, a\epsilon A(i), t \neq T_0$$

(4. 128)

$$0 \leq -\sum_{n\epsilon N(i,a)} q_{i,a,t,n} - \sum_{n\epsilon N(i,a)} x_{i,a,t,n} + r_{i,a,t} - r_{i,a,(t-1)} + \sum_{n\epsilon N(i,a)} dem_{i,a,t,n} +$$

$$CHAR_{i,a,t-1} \perp \beta^{6+}_{i,a,t} \geq 0 \; ; \forall i, a\epsilon A(i), t \neq T_0$$

(4. 129)

Assume

$$z = [z_1 \quad z_2]$$

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$M = \left[\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}\right]$$

Then the modified One-Way Stackelberg model (4. 112)-(4. 129) could be written as $LCP^*(q, M)$ when:

$$z_1^T = \left[\mathrm{qg_{g,t}}\ \mathrm{rg_{g,t}}\ \varepsilon_{g,t}^{1}\ \varepsilon_{g,t}^{3}\ \varepsilon_{g,t}^{2-}\ \varepsilon_{g,t}^{2+}\right]$$

$$z_2^T = \left[\mathrm{dem_{i,a,t,n}}\ \mathrm{q_{i,a,t,n}}\ \mathrm{r_{i,a,t}}\ \beta_{i,a,t}^{2}\ \ \beta_{i,a,t}^{3}\ \beta_{i,a,t}^{4}\ \beta_{i,a}^{1-}\ \beta_{i,a}^{1+}\ \beta_{ia}^{5-}\ \beta_{ia}^{5+}\ \beta_{i,a,t}^{6-}\ \beta_{i,a,t}^{6+}\right]$$

$$q_1 = \begin{bmatrix} \mathrm{CG_{g,t}} \\ \mathrm{CRG_{g,t}} \\ \mathrm{QG_g} \\ \mathrm{RG_g} \\ -\mathrm{xg_{g,t}} \\ \mathrm{xg_{g,t}} \end{bmatrix}$$

$$q_2 = \begin{bmatrix} -\mathrm{V_{i,a,t,n}} \\ \mathrm{C_{i,a,t,n}} \\ \mathrm{CR_{i,a,t}} \\ -\mathrm{CHAR_{i,a,t}} \\ \mathrm{Q_{i,a}} \\ \mathrm{RCAP_{i,a}} \\ -\mathrm{DEM_{i,a}} \\ \mathrm{DEM_{i,a}} \\ \sum_{n\epsilon N(i,a)} \mathrm{x_{i,a,T_0,n}} + \mathrm{R0_{i,a}} \\ -\sum_{n\epsilon N(i,a)} \mathrm{x_{i,a,T_0,n}} - \mathrm{R0_{i,a}} \\ \sum_{n\epsilon N(i,a)} \mathrm{x_{i,a,t,n}} - \mathrm{CHAR_{i,a,t-1}} \\ -\sum_{n\epsilon N(i,a)} \mathrm{x_{i,a,t,n}} + \mathrm{CHAR_{i,a,t-1}} \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 0 & 0 & 1\,0 & -1 & 1 \\ 0 & 0 & 0\,1 & \begin{Bmatrix} 1;\ g' = g \\ -1;\ t' = t+1 \vert t < T \end{Bmatrix} & \begin{Bmatrix} -1;\ g' = g \\ 1;\ t' = t+1 \vert t < T \end{Bmatrix} \\ -1 & 0 & 0\,0 & 0 & 0 \\ 0 & -1 & 0\,0 & 0 & 0 \\ 1 & \begin{Bmatrix} -1;\ g' = g \\ 1;\ t' = t-1 \vert t > \mathrm{T_0} \end{Bmatrix} & 0\,0 & 0 & 0 \\ -1 & \begin{Bmatrix} 1;\ g' = g \\ -1;\ t' = t-1 \vert t > \mathrm{T_0} \end{Bmatrix} & 0\,0 & 0 & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ 1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ -1; t \neq T_0 \end{Bmatrix} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ -1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ 1; t \neq T_0 \end{Bmatrix} \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = t+1 \\ 1; t = t, t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = t+1 \\ -1; t = t, t \neq T_0 \end{Bmatrix} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = T_0 \\ 0; t \neq T_0 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\begin{Bmatrix} 0; t = T_0 \\ -1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ 1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} -1; t = t, t \neq T_0 \\ 1; t = t-1 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\begin{Bmatrix} 0; t = T_0 \\ 1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 0; t = T_0 \\ -1; t \neq T_0 \end{Bmatrix} & \begin{Bmatrix} 1; t = t, t \neq T_0 \\ -1; t = t-1 \end{Bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Proposition 5

$LCP^*(q, M)$ has a solution and is feasible and solvable for all choices of q.

Proof:

If $y_1 \in R_+^{6*|G|}$, $y_2 \in R_+^{12}$, and $y = [y_1 \quad y_2] \in R_+^n$:

$$y^T M y = y^T \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} y = {y_1}^T M_1 y_1 + {y_2}^T M_2 y_2$$

(P. 17)

Both $M_1$and M_2 are skew-symmetric matrices for any y_1 and y_2, and so it is clear that

$$y_1^T M_1 y_1 = 0$$

(P. 18)

$$y_2^T M_2 y_2 = 0$$

(P. 19)

And

$$M_1 + M_1^T = 0 \implies (M_1 + M_1^T)y_1 = 0$$

(P. 20)

$$M_2 + M_2^T = 0 \implies (M_2 + M_2^T)y_2 = 0$$

(P. 21)

So if it is assumed that

$$\begin{Bmatrix} y_1 > 0 \\ M_1 y_1 \geq 0 \\ y_1^T M_1 y_1 = 0 \end{Bmatrix}$$

(P. 22)

$$\begin{Bmatrix} y_2 > 0 \\ M_2 y_2 \geq 0 \\ y_2^T M_2 y_2 = 0 \end{Bmatrix}$$

(P. 23)

Considering $M_1 = -M_1^T$ and $M_2 = -M_2^T$

It can be concluded that $M_1^T y_1 \leq 0$ and $M_2^T y_2 \leq 0$ should always be true. So if $y > 0$, it can be concluded that

$$\text{(P. 22)\&(P. 23)} \implies \begin{Bmatrix} {y_1}^T M_1 y_1 = 0 \\ {y_2}^T M_2 y_2 = 0 \\ y > 0 \end{Bmatrix} \implies y^T M y = 0$$

(P. 24)

Based on (Cottle, Pang et al., 2009) and (P. 24), M is copositive.

Moreover, if

$$y^T M y = 0 \Longrightarrow {y_1}^T M_1 y_1 + {y_2}^T M_2 y_2 = 0$$

(P. 25)

given (P. 22)&(P. 23) it is known that

$${y_2}^T M_2 y_2 = 0 \ \& \ {y_1}^T M_1 y_1 = 0$$

(P. 26)

Since M_1 and M_2 are skew-symmetric matrices so

$$\Longrightarrow if\ M_1 y_1 \geq 0 \ \& \ y_1 \geq 0 \mid M_1 = -M_1^T \Longrightarrow M_1^T y_1 \leq 0$$

(P. 27)

$$\Longrightarrow if\ M_2 y_2 \geq 0 \ \& \ y_2 \geq 0 \mid M_2 = -M_2^T \Longrightarrow M_2^T y_2 \leq 0$$

(P. 28)

$$\Longrightarrow y^T M y = 0, My \geq 0, y \geq 0 \Longrightarrow \ M^T y \leq 0$$

(P. 29)

Based on (Cottle, Pang et al., 2009) and (P. 29), M is copositive star which means M is both S and Q-matrix, and so $LCP^*(q, M)$ is feasible and solvable for all choices of q. □

Proposition 6

The One-Way model has at least a solution if the feasible region is non-empty.

Proof:

In the MPEC model, ISO as the leader decides on the amount of trades (imports/exports) in the network. Different decisions of ISO affect the lower level $LCP^*(q, M)$ by varying q vector. According to Proposition 5, $LCP^*(q, M)$ is feasible and solvable for all choices of q and has at least a solution. So if ISO's constraints make a feasible region which has a common area with feasible region of the lower level game, the MPEC would at least have one optimal solution from the linear objective function of the ISO. □

Solution Algorithm

As proved in previous section the One-Way $LCP^*(q, M)$ is feasible and solvable for all choices of q. That means for all decisions of ISO, lower level problem has at least a feasible solution. Relying on this fact, the algorithm for solving the One-Way model is designed as described here.

The algorithm for solving the One-Way model has the same base as of the algorithm for the Two-Way model with few changes. For the One-Way algorithm, in stage 1, trade values are all fixed and fed into next stages. Then in stages 2 and 3, instead of having game among participants, a decision problem is solved through an optimization. Since trade values are fixed in stage 1, and ISO's variables wouldn't change in the process, there wouldn't be any need to stage 4. The algorithm's flowchart is shown in Figure 11.

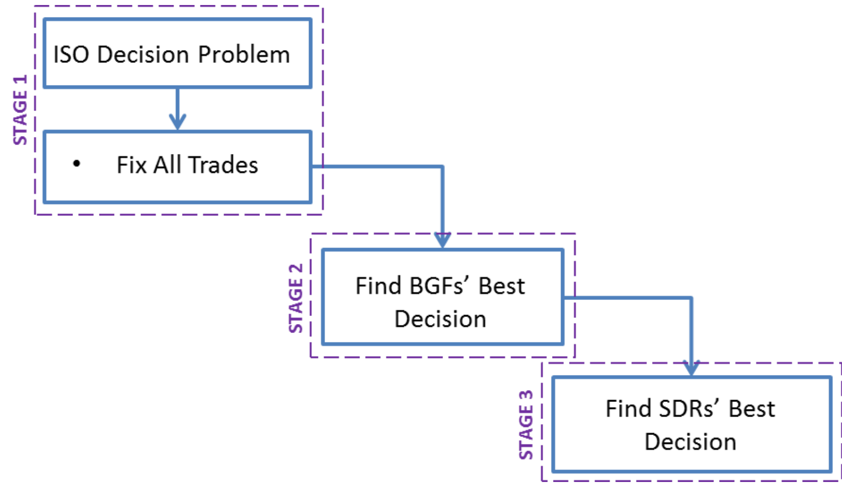

Figure 11 Flowchart for the solution algorithm for the One-Way model

Benefiting from special structure of the model and the assumptions, it can be shown the solution of the proposed algorithm is an optimal solution to the original One-Way model in Table 6. Assume the following:

Case 1: is an MPEC depicted in Table 6 with ISO's decision problem in the upper level and BGFs' and SDRs' KKT conditions included as the ISO's constraints. The set of optimal solutions and objectives for this case are shown by $x_1^*, {x_g}_1^*, ISO_1^*, BGF_1^*, SDR_1^*$. The feasible region for Case 1 is $f(BGF_{equ.}\left(x_{g_1}\right), SDR_{equ.}\left(x_1, x_{g_1}\right))$, while the feasible region of the lower level is continuous over the upper levels' variables based on what was discussed in the solution existence.

Case 2: is the algorithm shown in Figure 11. Set of optimal solutions and objectives for this case are shown as $x_2^*, {x_g}_2^*, ISO_2^*, BGF_2^*, SDR_2^*$. The feasible region for Case 2 is shown by χ and the set of all equilibriums for the lower level is $\Gamma(\chi)$.

Proposition 7

Case 1 and Case 2 give the same optimal objective value.

Proof:

Since decision variables of ISO are not variables in the lower level, ISO has hierarchy over BGFs and SDRs. Also some decision variables of the BGFs are not variables in SDRs' problem, which results in a hidden hierarchy of BGFs over SDRs. Case 1's feasible region is a subset of lower level's feasible equilibriums. On the other hand, Case 2's feasible region is larger compared to Case 1 and BGFs and SDRs feasible region is getting more restricted by setting the decisions from the upper level to fix values. So the feasible region of the lower level problem in Case 2 is a subset of the feasible region of the lower level in Case 1.

Now to the contrary assume $ISO_1^* \neq ISO_2^*$, then:

As mentioned earlier: $\forall x_2^*, {x_g}_2^* \in \chi : \Gamma(\chi) \neq \emptyset$

Solving the lower level problems for optimality the results would be:

$$\left(BGF_{equ.}\left({x_g}_2^*\right), SDR_{equ.}\left(x_2^*, {x_g}_2^*\right)\right) \in \Gamma(\chi)$$

Moreover: $f(BGF_{equ.}\left({x_g}_2^*\right), SDR_{equ.}\left(x_2^*, {x_g}_2^*\right)) \in$

$f(BGF_{equ.}\left({x_g}_1\right), SDR_{equ.}\left(x_1, {x_g}_1\right))$

So there will be only three possible cases:

$$\begin{Bmatrix} a)\ ISO_2^* > ISO_1^* \\ b)\ ISO_2^* < ISO_1^* \\ c)\ ISO_2^* = ISO_1^* \end{Bmatrix}$$

$a)\ If\ ISO_2^* > ISO_1^*$:

$\exists\ f(BGF_{equ.}\left({x_g}_2^*\right), SDR_{equ.}\left(x_2^*, {x_g}_2^*\right))$

$\in f(BGF_{equ.}\left({x_g}_1\right), SDR_{equ.}\left(x_1, {x_g}_1\right)) | ISO_2^* > ISO_1^*$

$\rightarrow ISO_1^*\ is\ not\ optimal\ \otimes$

$b)\ If\ ISO_2^* < ISO_1^*$:

$\exists\ (x_1^*, {x_g}_1^*)\ \in \chi | ISO_1^* > ISO_2^* \Rightarrow ISO_2^*\ is\ not\ optimal \otimes$

"a" and "b" are contradictions. So "c" should be true and so $ISO_2^* = ISO_1^*$. □

As shown in the proof, the objective value of the proposed algorithm and the main MPEC problem have to be the same. However, this may not be true for the solution variables. This is due to possible existence of multiple solutions at equilibrium.

Chapter 5: Results

To evaluate the proposed model and compare different solution methodologies with the proposed solution algorithm, several examples are designed and solved. Results are compared and presented in this chapter. Sensitivity analysis is then conducted on the exogenous parameters. Finally conclusions and recommendations are made based on the results.

Data Generation

To evaluate the model it is best to apply the model and solution algorithm on real world systems with real data. However, due to confidentiality and security concerns finding a complete network system with its real data is almost impossible. As a result, here network and demand data is generated randomly in MATLAB based on real data found from open sources. Then different scenarios on several electricity networks with different characteristics defined to evaluate the model.

Network data is borrowed from IEEE distribution test feeders[5]. Different networks with different sizes are used in generating examples. PTDF matrix for each network is calculated based on method explained in (Benjamin, 2012). To calculate the PTDF matrix for each case, networks are assumed to be built from one phase lines. That means the largest impedance is taken for the ones with more than one phase. Also, networks are assumed to be DC lossless and so R is set to be zero for all lines. Moreover, node

[5] http://ewh.ieee.org/soc/pes/dsacom/testfeeders/index.html

one in all networks is taken as the reference node and level of consumption is assumed to be the same among all nodes while generating PTDF matrices.

SDRs in the system are divided into four categories: commercial and building, residential, industrial with shifts and industrial with no shift users. Percentage of each category among users is shown in Table 10. The hourly demand distribution is estimated based on PJM dataset[6] in May 2011. Since the provided data is for the whole PJM territory with a population of about 61 million[7], the distribution is adjusted according to the size of each designed problem. The hourly distribution demand function is used for generating request times of demands for the users. For demand amounts, each appliance of users is assigned a random demand according to their category and their share of demand distribution in Table 10. For large user categories, industrials and commercials, some constant and uninterrupted demand is generated to cover the minimum demand they have for utilities in 24 hours. Each constant demand is set as a new appliance with an inflexible request time during the day. Two of these constant demands are assumed to be flexible and 1/3 of them could be shifted to other time spots as two appliances, which means these categories each have 26 appliances. The maximum number of registered appliances for the residential users is assumed to be 5. Appliances could connect to grid at different random nodes and time periods upon availability of SDRs.

[6] http://www.eia.gov/tools/faqs/faq.cfm?id=100&t=3

[7] http://www.pjm.com/about-pjm/who-we-are.aspx

Covering demands for electrical vehicles is one of the highlights of this model. It is assumed that 40% of the residential users have EV and their demand is generated based on EV hourly charging demand distribution function published by ECOtality[8] (a DOE Project). Electrical vehicles are taken to be an appliance with a battery capacity of 4-10 (kWh)[9].

Table 10 Percentage of Different Demand Category[10]

User Categories	*Percentage of users*
Commercial	0.20%
Shift industrial	0.30%
No shift idustrial	0.10%
Residential	99.40%

Nodal fees and rewards function is estimated based on price distribution provided by BGE for May of 2011 to comply with the time span used for demand distribution[11]. Prices are generated randomly and then smoothed to reduce the peaks and downs.

The utility function for satisfying demands is assumed to be Gaussian with its maximum at the request time of the demand (5. 1). As the time of meeting demand diverges from the requested time, the value of satisfying demand is decreasing.

$$V_t = a.e^{-((t-T_{Request})/b)^2} \qquad (5.\ 1)$$

Where “a” and “b” are randomly generated constants.

[8] http://www.theevproject.com/documents.php

[9] U.S. Department of Energy (July 2012)

[10] http://www.eia.gov/tools/faqs/faq.cfm?id=447&t=3

[11] http://www.eia.gov/tools/faqs/faq.cfm?id=100&t=3

Storage and all other data used in the examples are randomly generated based on information and data provided by U.S. Energy Information Administration[12], U.S. Department of Energy[13] and Electricity Storage Association[14] as follows.

Distributed storage in the market has maximum capacity of 100 (kWh) and assigned to 10% of the users. Storage cost for any type of battery is assumed to be 0.01-0.03 ($/kWh)[15]. 20% of SDRs assumed to have generation capacity. Distributed generation such as solar panels, or small wind turbines for SDRs are assumed to have capacity of about 50-1000 (kWh) and their generation cost is uniformly distributed on 0.03-0.09 ($/kWh). BGFs generation capacity is uniformly distributed on 5-1000 (MW), and their generating cost is about 0.009-0.050 ($/kWh)[16]. Their storage capacity is assumed to be 1% of their generation capacity with the cost of 0.01-0.03 ($/kWh). Slope and intercept for the inverse supply function for BGFs in the system are set to 0.000001 ($/kWh^2) and 0.05 ($/kWh) respectively.

To compare the complexity of the problem, different scenarios on different networks with different number of BGFs and DSRs are defined and compared as shown in Table 11. To evaluate the appropriateness of the proposed Stackelberg model and market structure, each scenario is solved with two different models of One-Way and Two-Way to show the difference between the two mechanisms defined in Chapter 3. Additionally,

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[12] http://www.eia.gov/

[13] http://www.eere.energy.gov/

[14] http://www.electricitystorage.org/about/welcome

[15] Electricity Storage Association (July 2012)

[16] U.S. Energy Information Administration (July 2012)

two more models are compared with the One-Way and Two-Way Stackelberg models. One is demand satisfaction with no Demand Response program for SDRs. In this model demands are assigned to the first available time spot after the request time of the demands. So SDRs' feasibility constraints are added to the ISO's decision problem, while BGFs are still competing in the lower level. This model is called NO DR model. The other model assumes that there is neither a demand response program among SDRS, nor there is any game among BGFs. Hence their constraints are all added to ISO's decision problem as a one level problem. This model is called NO DR-NO GAME. Comparing these two models with the Stackelberg game model would show benefits of the integrated demand response model and also the new market mechanism designed in this study.

Moreover, to evaluate the proposed solution algorithm, different solution methodologies in solving the Stackelberg model are compared. As discussed in Chapter 4, the proposed Stackelberg game can be converted to a MPEC model. Then the MPEC model can be solved through either nonlinear MPEC solvers in the market or through SOS, MIP or heuristic techniques. In next section, all these approaches are used and compared together and with the proposed algorithm in Chapter 4 which will be called Mona from this point. The corresponding MPECs, SOS, and MIPs are solved using GAMS 24.2.1 (NPATH, CPLEX, and Xpress solvers). All other algorithms, Mona, NO DR, and NO DR-NO GAME are coded and solved in Xpress optimization suite 7.1 on a computer with i7 CPU and 8.00 GB of RAM. All designed scenarios are shown in Table 11. Each set of network is used to generate different sets of scenarios with different number of users. Each scenario is then solved with 6 different algorithms:

Mona, SOS, MIP, MPEC, NO DR, NO DR-NO GAME with both One-Way and Two-Way models. Detailed results for all scenarios are presented in Appendix A. Results and computational statistics for all scenarios will be discussed in next section.

Table 11 Scenarios' Settings

Topology	*Node*	*Line*	*Model*	*Scenario*	*SDRs*	*BGFs*
Small	13	12	One-Way	1	5	1
				2	50	2
				3	100	2
			Two-Way	1	5	1
				2	50	2
				3	100	2
Medium	37	36	One-Way	4	10	1
				5	100	2
				6	1000	10
				7	5000	10
			Two-Way	4	10	1
				5	100	2
				6	1000	10
				7	5000	10
Large	122	117	One- Way	8	100	2
				9	1000	5
				10	5000	10
			Two-Way	8	100	2
				9	1000	5
				10	5000	10

Numerical Results

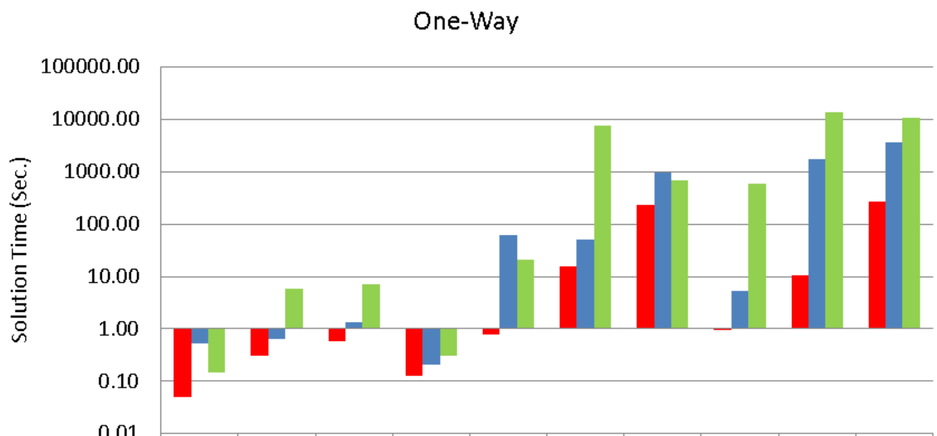
Basic computational statistics for different scenarios defined in Table 11 are summarized in Table 12. As number of users and suppliers increases, number of variables and constraints increases dramatically and so does solution time.

Results indicate that the MPEC solver is incapable of solving almost all problem instances other than Scenario 1 with only 5 users for local optimal (Table 13-Table 15). When user number increases to 1000 and more, SOS and MIP solvers also become unreliable and inefficient in most cases. However, Mona solves all scenarios to optimality or a good near optimal solution in a reasonable time, which makes it a reliable algorithm for solving both One-Way and Two-Way models compared to other solution methodologies.

Table 12 Computational Statistics and Objectives for Different Scenarios

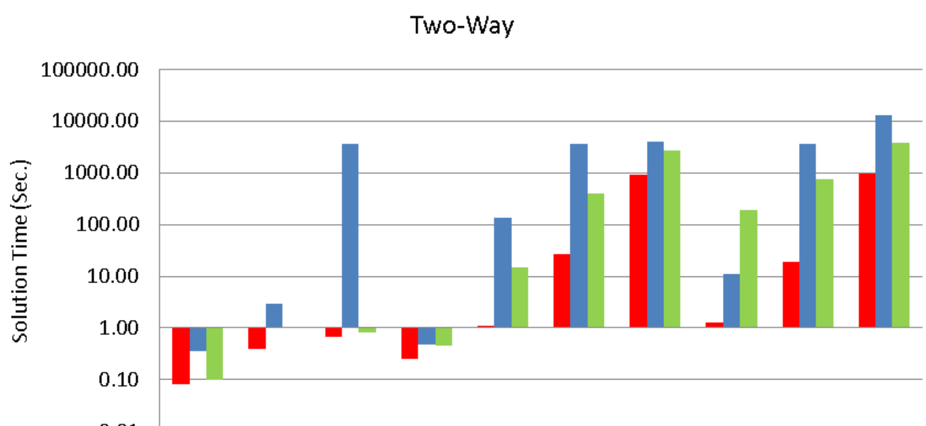
Topology	*Node*	*Line*	*Model*	*Scenario*	*SDRs*	*BGFs*	*Number of Constraints*	*Number of Variables*	*Solution Time (Sec) (Mona)*
Small	13	12	One-Way	1	5	1	3,142	1,449	0.05
				2	50	2	20,903	12,326	0.31
				3	100	2	39,488	23,636	0.59
			Two-Way	1	5	1	5,410	2,174	0.08
				2	50	2	50,526	21,495	0.39
				3	100	2	97,970	41,744	0.67
Medium	37	36	One-Way	4	10	1	7,995	3,611	0.13
				5	100	2	43,457	25,309	0.80
				6	1000	10	405,629	247,015	15.52
				7	5000	10	9,928,633	2,168,684	231.16
			Two-Way	4	10	1	22,367	5,456	0.25
				5	100	2	191,408	43,562	1.10
				6	1000	10	2,000,587	441,527	27.34
				7	5000	10	9,928,633	2,168,684	911.59
Large	122	117	One-Way	8	100	2	49,383	26,114	0.94
				9	1000	5	410,297	247,197	10.31
				10	5000	10	1,978,042	1,208,129	267.32
			Two-Way	8	100	2	503,799	44,410	1.26
				9	1000	5	5,381,282	442,287	19.28
				10	5000	10	26,734,009	2,170,724	982.62

To have an effective demand response model, the dynamic model needs to be run every 30 minutes. So it is important to have a solution algorithm which can find a solution in less than 30 minutes. Comparing solution times for both One-Way and Two-Way models, the algorithm Mona is performing better than any other solution algorithm in all scenarios (Figure 12 and Figure 13).



	1	2	3	4	5	6	7	8	9	10
Mona	0.05	0.31	0.59	0.13	0.80	15.52	231.16	0.94	10.31	267.32
SOS	0.54	0.65	1.36	0.21	60.50	50.55	952.06	5.42	1741.3	3624.4
MIP	0.15	5.76	7.16	0.30	20.94	7389.4	682.05	585.75	13689.	10910.

Figure 12 Solution time for different solution algorithms for One-Way Model



	1	2	3	4	5	6	7	8	9	10
Mona	0.08	0.39	0.67	0.25	1.10	27.34	911.59	1.26	19.28	982.62
SOS	0.36	2.96	3600.4	0.48	135.22	3605.7	4052.3	11.13	3611.1	12846.
MIP	0.10	0.00	0.84	0.45	15.21	406.20	2760.9	193.46	749.58	3870.2

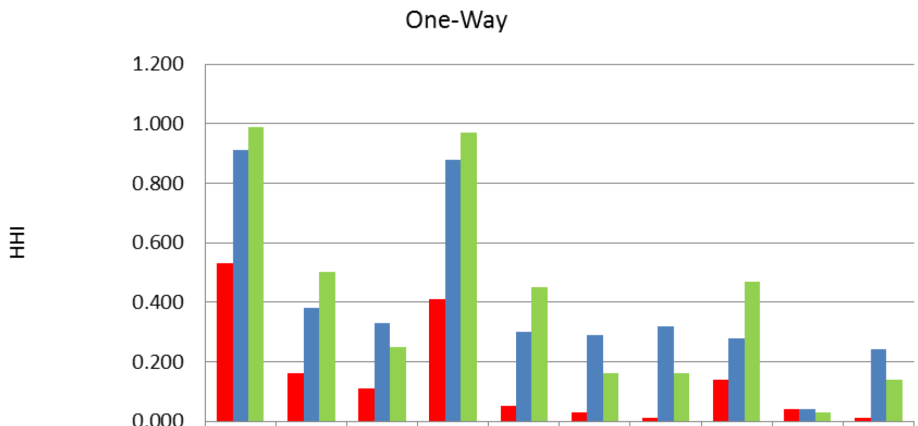
Figure 13 Solution time for different solution algorithms for Two-Way Model

To be able to evaluate the results and effect of the models on the market, it is important to investigate some market indicators. Market share, s_i, is an indicator of competition in a market and is a percentage of the market supplied by a specific entity. When market share is mostly assigned to a limited number of suppliers, it conveys that there is less competition in the market. As the market moves toward perfect competition, market share distributes among all suppliers more evenly and so their standard deviations decrease. Hirschman-Herfindhal Index (HHI) (Ventosa, Baıllo et al., 2005) is an indicator for market share evaluation and is defined in equation (5. 2). s_i is the market share of supplier i among N suppliers.

$$HHI = \sum_{i=1}^{N} s_i^2, \ \ \frac{1}{N} \leq HHI \leq 1$$

(5. 2)

As HHI increases, competition level in the market decreases. So small enough HHIs show that the market is not a monopoly. One of the goals in Smart Grid is to encourage participation of individuals in the market and motivate use of distributed energy sources in supplying demands. Here HHI is used to show whether a market is a monopoly or a competitive market and it also indicates the level of supply diversification in the market. Figure 15 illustrates HHI for different scenarios comparing the proposed Stackelberg model with NO DR and NO DR-No GAME. Lower HHI in the models with competition among suppliers (Mona and NO DR algorithms) compared to NO DR-NO GAME approach shows how implementing competition among suppliers can prevent the monopoly of supply in the market and motivate supply diversification. This is valid in both One-Way and Two Way models (Figure 14 and Figure 15). Results demonstrate the capability of the proposed Stackelberg model in distributing market power among players in the game which interprets perfect competition in the market. Distributed market share in a system not only prevents system from going toward monopoly, but also supports availability of supply in case of unpredicted disruptions. In this market, the system is not relying on a few sets of generating companies and so it would be more reliable. Interestingly, results confirm that as the number of generators and users increases in the market, there is more supply diversification in the system.



	1	2	3	4	5	6	7	8	9	10
Mona	0.530	0.160	0.110	0.410	0.050	0.030	0.010	0.140	0.040	0.010
NO DR	0.910	0.380	0.330	0.880	0.300	0.290	0.320	0.280	0.040	0.240
NO DR-NO GAME	0.990	0.500	0.250	0.970	0.450	0.160	0.160	0.470	0.030	0.140

Figure 14 Comparison of HHI index for different algorithms in One-Way model

	1	2	3	4	5	6	7	8	9	10
Mona	0.530	0.160	0.100	0.410	0.050	0.030	0.003	0.140	0.040	0.010
NO DR	1.000	0.640	0.550	0.890	0.180	0.330	0.240	0.210	0.040	0.340
NO DR-NO GAME	0.990	0.500	0.850	0.970	0.460	0.160	0.160	0.470	0.040	0.230

Figure 15 Comparison of HHI index for different algorithms in Two-Way model

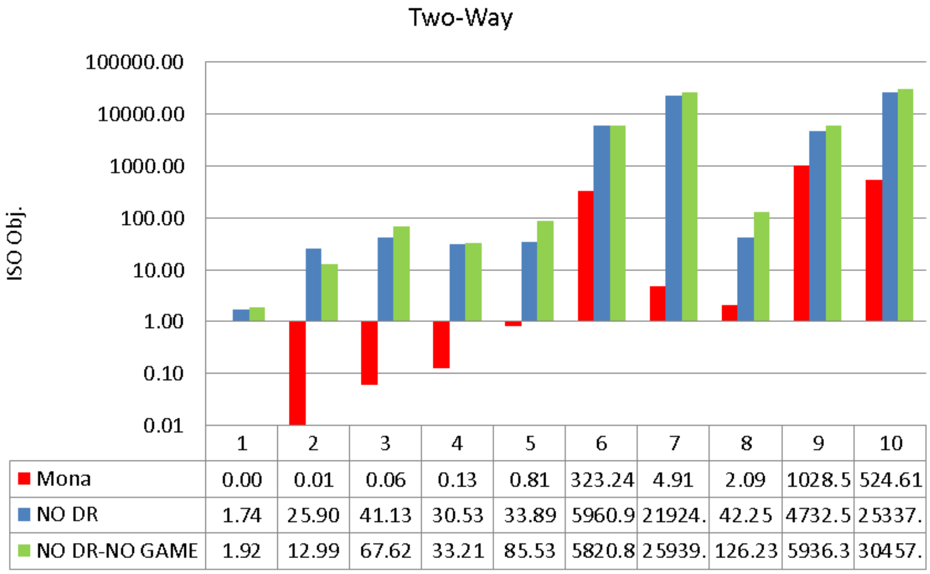
Increase of supply diversification in the market not only leads to better market indicators, it also results in use of more renewable energies and thus a better objective function value (Figure 16 and Figure 17). Results convey that having a competitive market without demand response is better than having none of the features, while implementing the integrated competitive demand response model is the best of all.

	1	2	3	4	5	6	7	8	9	10
Mona	0.00	0.01	0.06	0.13	0.63	322.71	4.91	2.09	1028.5	524.61
NO DR	1.66	18.13	34.44	26.40	35.74	5954.9	19722.	39.68	5448.2	23560.
NO DR-NO GAME	1.92	15.25	77.98	27.59	95.96	6622.5	26698.	146.68	6050.9	29982.

Figure 16 Comparison of ISO Obj. for different algorithms in One-Way model

Mohsenian-Rad and Wong (2010) used an index called PAR (5. 3), which is Peak to Average load Ratio, to indicate how loads are distributed during the time horizon.

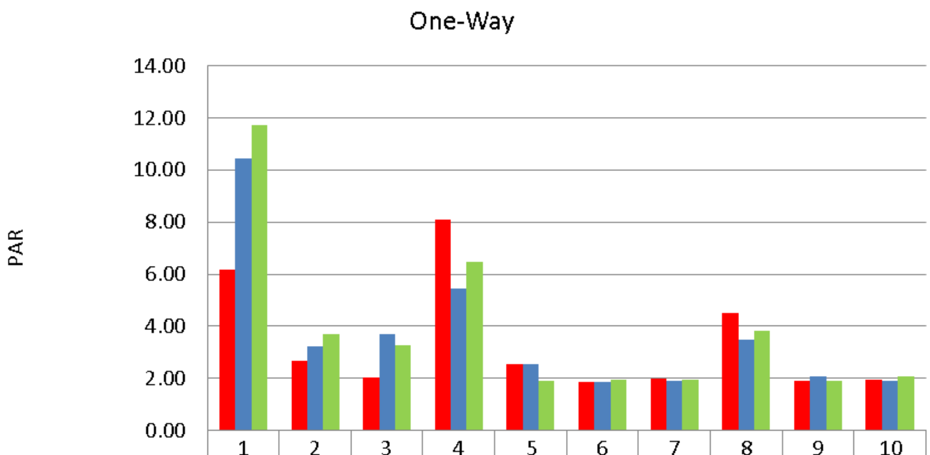
$$PAR = \frac{Load_{peak}}{Load_{average}} \tag{5. 3}$$



	1	2	3	4	5	6	7	8	9	10
Mona	0.00	0.01	0.06	0.13	0.81	323.24	4.91	2.09	1028.5	524.61
NO DR	1.74	25.90	41.13	30.53	33.89	5960.9	21924.	42.25	4732.5	25337.
NO DR-NO GAME	1.92	12.99	67.62	33.21	85.53	5820.8	25939.	126.23	5936.3	30457.

Figure 17 Comparison of ISO Obj. for different algorithms in Two-Way model

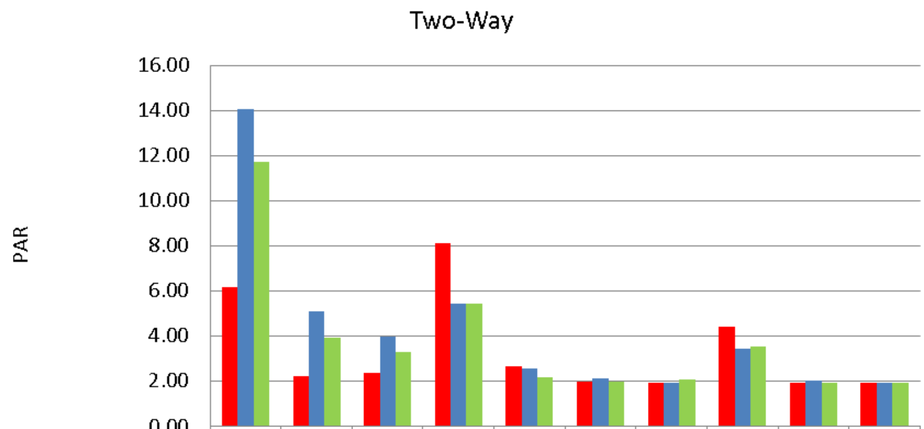
They minimized PAR in their objective function and claimed that as PAR decreases, consumption would get leveled and this would be beneficial to the system. However, this may not always be true, specifically when dynamic pricing is applied and idle capacities are available in the system. PAR might be a good indicator when limited constant capacities are available, but with integration of renewable energy and distributed generation capacities in the network, it is better to minimize cost instead of PAR while trying to keep load levels in acceptable range through applying real time prices based on total consumption in the market. This approach not only reduces costs, but also incentivizes users to consume during time periods with lower prices and more available capacities which may lead to lower PAR depending on parameters in the pricing function. In this study the objective function is to motivate use of local generators, while market price is directly dependent on total consumption in the market which encourages demands being shifted to more cost efficient time windows while using available capacities efficiently. Although Figure 18 and Figure 19 demonstrate better PAR index for algorithm Mona in most scenarios, it is not always guaranteed and it is dependent on generated data sets. Limited feasible time intervals for satisfying demands, lower inverse supply function’s parameters in some time windows, and availability of low cost distributed supplies in the network in specific time periods could be reasons for shifting demand toward specific time instances in different data sets which result in always better objective function but not necessarily a lower PAR indicator. However, as is shown in Figure 20, it is clear that the algorithm shifted demands from some peak windows to to off peak times of day.



	1	2	3	4	5	6	7	8	9	10
Mona	6.18	2.68	2.03	8.11	2.53	1.87	1.98	4.49	1.92	1.94
NO DR	10.44	3.24	3.68	5.44	2.55	1.88	1.93	3.47	2.06	1.93
NO DR-NO GAME	11.72	3.71	3.29	6.49	1.92	1.95	1.96	3.83	1.92	2.08

Figure 18 Comparison of PAR index for different algorithms in One-Way model

Among different solution methodologies applied to the Stackelberg model, as is shown in Figure 12, Figure 13, Figure 21, and Figure 22, Mona has better solution time compared to other methods for all scenarios. This is while it has a good objective value and almost the same HHI and PAR indices as other methods (Table 13-Table 15).



	1	2	3	4	5	6	7	8	9	10
Mona	6.18	2.24	2.37	8.11	2.68	2.00	1.94	4.44	1.92	1.96
NO DR	14.08	5.10	4.00	5.44	2.55	2.11	1.94	3.47	2.06	1.94
NO DR-NO GAME	11.72	3.92	3.29	5.44	2.18	2.00	2.07	3.56	1.96	1.95

Figure 19 Comparison of PAR index for different algorithms in Two-Way model

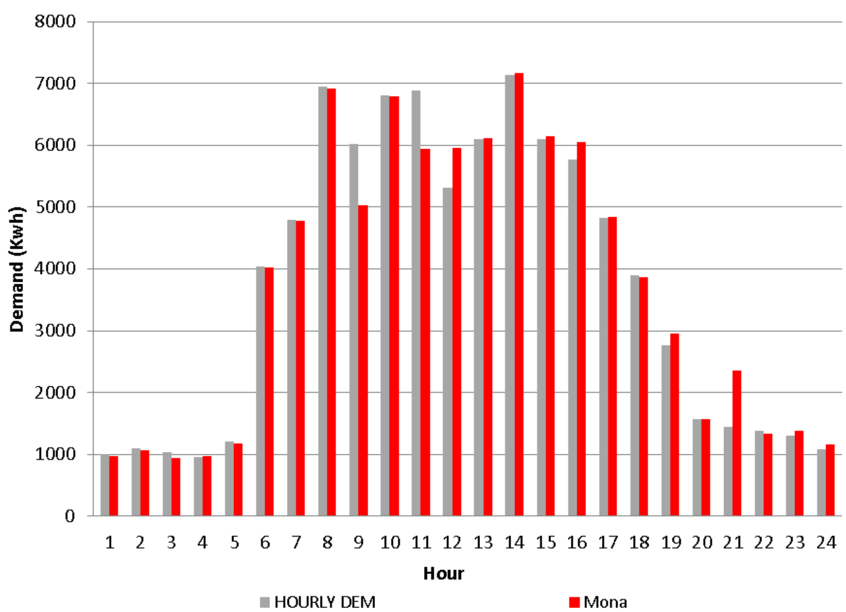
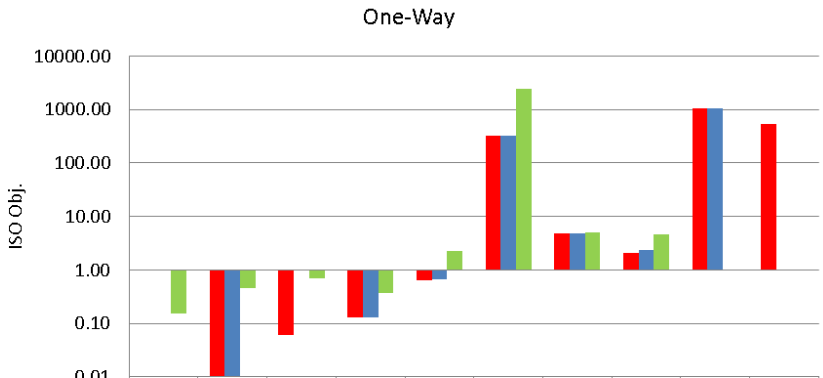


Figure 20 Demand distribution in Two-Way Model comparing to original hourly demands in Scenario 9



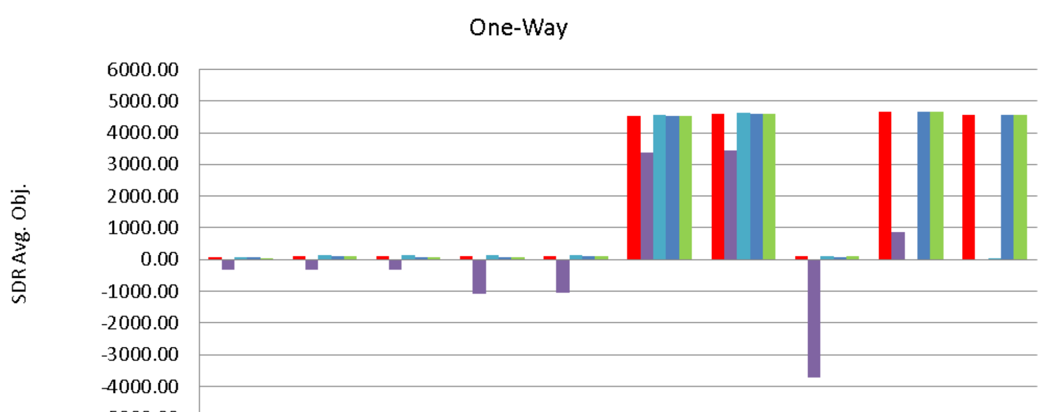
	1	2	3	4	5	6	7	8	9	10
Mona	0.00	0.01	0.06	0.13	0.63	322.71	4.91	2.09	1028.5	524.61
SOS	0.00	0.01	0.00	0.13	0.66	322.70	4.82	2.31	1069.9	0.00
MIP	0.15	0.45	0.68	0.37	2.25	2441.5	4.92	4.67	0.00	0.00

Figure 21 ISO objective for different solution methodologies in Two-Way Model

	1	2	3	4	5	6	7	8	9	10
Mona	0.00	0.01	0.06	0.13	0.81	323.24	4.91	2.09	1028.5	524.61
SOS	0.00	0.01	0.00	0.13	0.69	0.00	0.00	2.09	0.00	0.00
MIP	0.00		0.06	0.13	0.66	322.84	4.91	2.09	1028.5	0.00

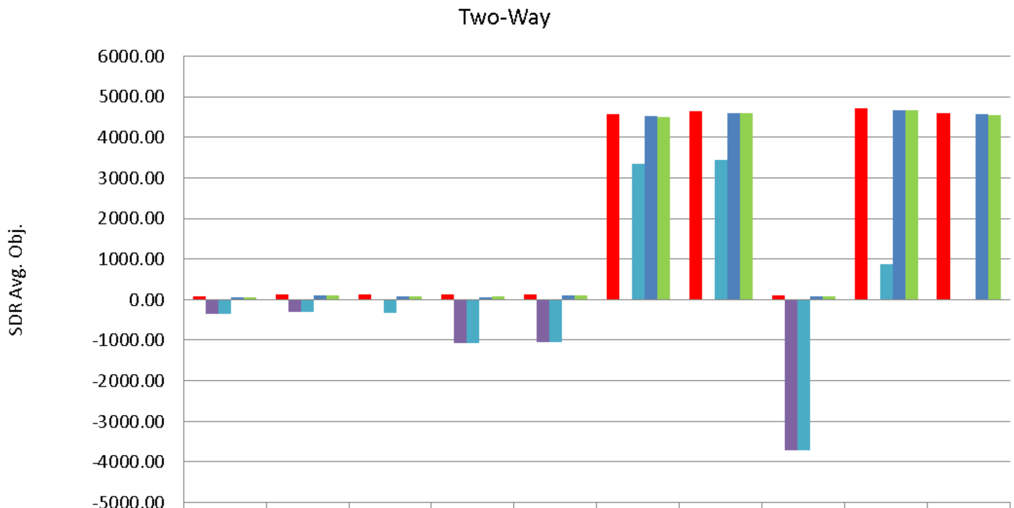
Figure 22 ISO objective for different solution methodologies in Two-Way Model

Moreover, Mona performs well in encouraging participants in the network. Average SDRs and BGFs' objective functions in Mona are considerably better than any of the other models (NO DR and NO DR-NO GAME) (Figure 23-Figure 26). It is obvious that having competition in the market is very important for BGFs to increase revenue. Also applying demand response model in a competetive market makes it more appealing for households and users to participate in the system. Comparing participants' avgerage objective in Mona with MIP and SOS is not a valid comparison due to the way the algorithm works. In the proposed algorithm by adding cuts in the second and third stages based on the best possible solution of the ISO, the algorithm maximizes all individual objectives. This results in finding the best set of equilibrium solutions for the lower level problem. However, in the MIP and SOS algorithms, the objective is only focused on optimizing the objective function and so comparison of the equilibrium sets is not a fair comparison. It is also necessary to mention that without having access to real data it is very difficult to estimate the users'utility function. Underestimation of the utilities is a reason for big differences in SDRs' average objective functions from different methods as are shown in Figure 23 and Figure 24.



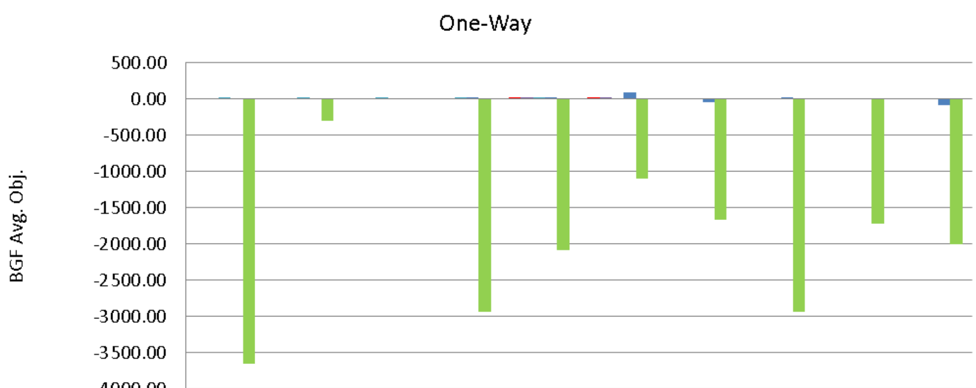
	1	2	3	4	5	6	7	8	9	10
Mona	70.86	111.10	102.61	115.09	107.13	4518.75	4595.73	93.72	4672.74	4562.70
SOS	-336.00	-308.00	-315.00	-1078.00	-1058.00	3359.00	3438.00	-3720.00	861.00	0.00
MIP	72.00	139.00	129.00	130.00	138.00	4568.00	4633.00	121.00	0.00	5.00
NO DR	62.00	97.00	88.00	67.00	97.00	4515.00	4592.00	85.00	4666.00	4558.00
NO DR-NO GAME	48.00	97.00	87.00	67.00	94.00	4529.00	4598.00	90.00	4667.00	4562.00

Figure 23 SDRs average objective in One-Way model



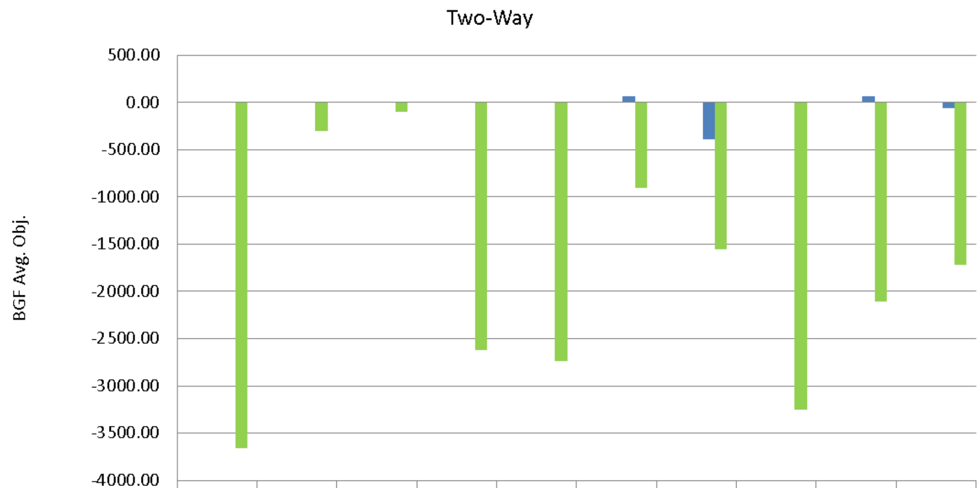
	1	2	3	4	5	6	7	8	9	10
Mona	71.00	135.00	128.00	118.00	125.00	4570.00	4637.00	99.00	4702.00	4600.00
SOS	-351.00	-304.00	0.00	-1078.00	-1061.00	0.00	0.00	-3723.00	0.00	0.00
MIP	-359.00	-309.00	-317.00	-1080.00	-1057.00	3351.00	3434.00	-3721.00	862.00	0.00
NO DR	50.00	99.00	82.00	66.00	96.00	4528.00	4590.00	85.00	4668.00	4558.00
NO DR-NO GAME	48.00	95.00	87.00	74.00	102.00	4507.00	4593.00	88.00	4668.00	4557.00

Figure 24 SDRs average objective in Two-Way model



	1	2	3	4	5	6	7	8	9	10
Mona	0.00	0.00	0.00	0.00	0.11	0.39	0.00	0.00	0.00	0.00
SOS	0.00	0.00	0.00	0.00	0.11	5.37	0.00	0.00	0.00	0.00
MIP	0.06	0.01	0.03	0.03	0.18	0.00	0.00	0.00	0.00	0.00
NO DR	0.00	0.00	0.00	0.12	1.53	91.67	-41.91	1.03	0.00	-87.15
NO DR-NO GAME	-3657.62	-301.52	0.00	-2942.96	-2083.67	-1096.81	-1672.45	-2943.68	-1718.28	-2006.26

Figure 25 BGFs average objective in One-Way model

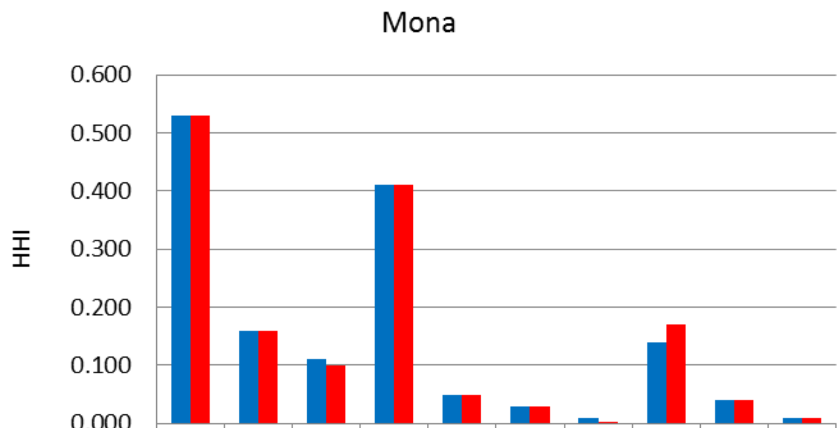


	1	2	3	4	5	6	7	8	9	10
Mona	0.00	0.00	0.00	0.00	0.27	0.61	0.00	0.00	0.00	0.00
SOS	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00
MIP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NO DR	0.00	0.00	0.00	0.12	1.15	62.34	-388.12	1.03	69.55	-57.19
NO DR-NO GAME	-3657.62	-301.52	-100.36	-2617.13	-2741.67	-898.68	-1558.27	-3254.26	-2104.69	-1722.57

Figure 26 BGFs average objective in Two-Way model

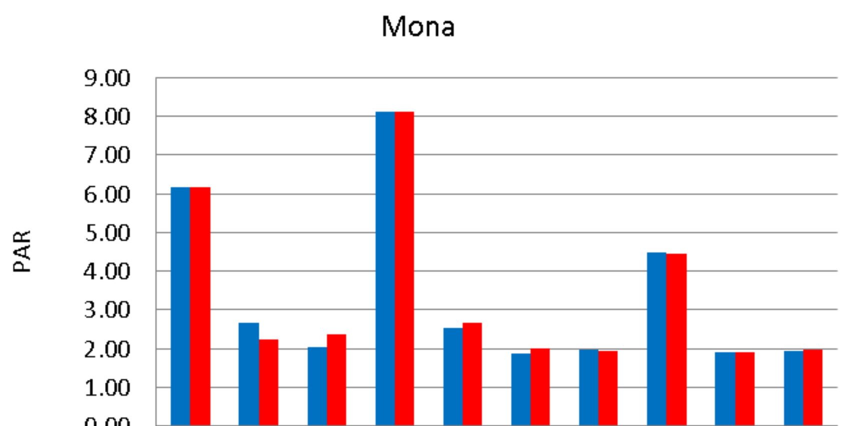
So Mona generally perfoms better from both aspects of solution methodology (compared to MPEC, MIP, and SOS) and modeling (comapred to NO Dr and NO DR-NO GAME).

An interesting result came from comparison of One-Way and Two-Way models. As it is clear in Figure 27 and Figure 28, the market share index is almost the same in both models. This conveys that although having a competition would improve the system, strict controling of one variable by an operator does not affect the competition level. That means, if the system is left to reach its equilibrium itself or let an operator to control one of the system variables individually does not make any differnce in market indices. However, if all users have the right of decision making over all variables, their objective values will be better. That is a system with perfect competition is more encouraging for participants than a semi-controled competetive market (Figure 29).



	1	2	3	4	5	6	7	8	9	10
One-Way	0.530	0.160	0.110	0.410	0.050	0.030	0.010	0.140	0.040	0.010
Two-Way	0.530	0.160	0.100	0.410	0.050	0.030	0.003	0.170	0.040	0.010

Figure 27 HHI index for One-Way and Two-Way models for all scenarios



	1	2	3	4	5	6	7	8	9	10
One-Way	6.18	2.68	2.03	8.11	2.53	1.87	1.98	4.49	1.92	1.94
Two-Way	6.18	2.24	2.37	8.11	2.68	2.00	1.94	4.44	1.92	1.96

Figure 28 PAR index for One-Way and Two-Way models for all scenarios

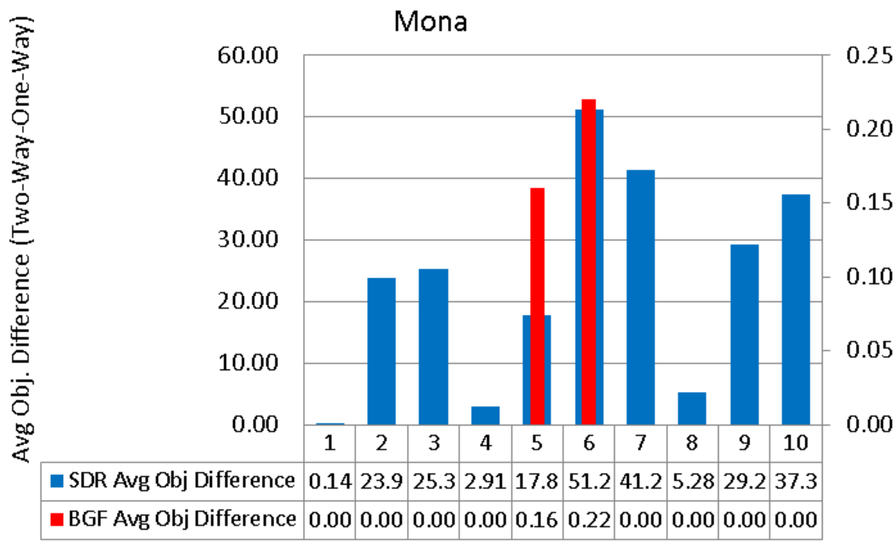


Figure 29 SDRs and BGFs average objective difference ((Two-Way Obj.) - (One-Way Obj.))

Sensitivity Analysis

Exogenous input in this problem is generated randomly based on real data. However, it is of interest to study how their values may affect solution results. To study these changes and also make appropriate policy recommendations, sensitivity analysis is conducted over several scenarios. In this section, results from one of the scenarios are discussed (scenario 9). In the sensitivity analysis several cases are compared to the base case. In each case a multiplier is applied to the studied base input. That is numbers on the horizontal axis on the sensitivity graphs are percentage multipliers of the base case in the sensitivity analysis.

Higher storage capacities in the system would make the decision problem more flexible. As is shown in Figure 30, increase of storage capacity gives the option to users to buy and store more electricity in the time of low prices and use during peak times. This results in lower objective function. On the other hand, having more storage capacity, users are more willing to buy from one specific provider with lower cost and store in advance instead of buying from a higher cost local generator. This would result in higher HHI and so less supply diversification in the market. However, this would encourage local users to install lower cost and more efficient storage and generation systems. The intercept between HHI and objective function would be a good estimate of average storage capacity to have in the system in order to have both good objective function and competitive market. This is the same for distributed generation capacities (Figure 31). Having larger local generation capacities, objective function gets better. However, the provider with lower cost would be able to supply more electricity in the system which results in higher supply diversification in the market. However, this again could be a good motive for local generators to reduce their initial costs and use more efficient generators. So motivating use of bigger generators and storage devices in households and local generators would result in a better objective function and eventually more efficient system.

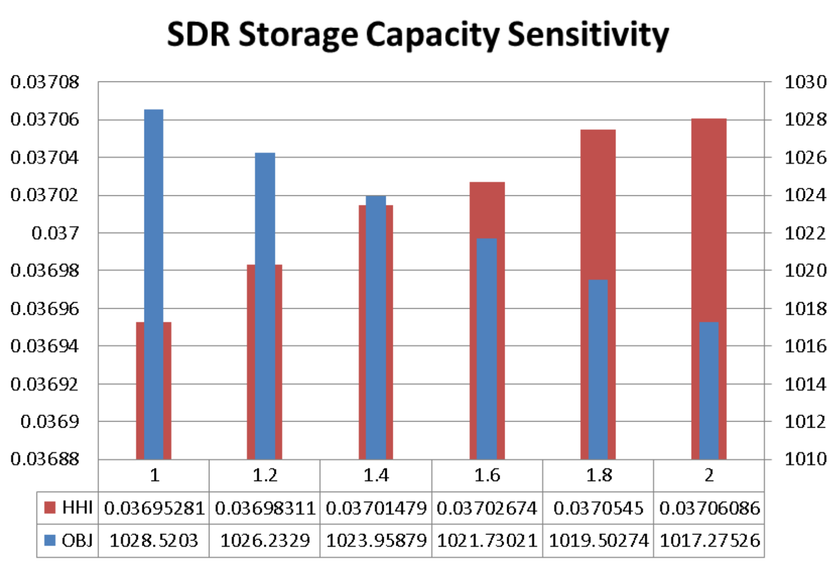
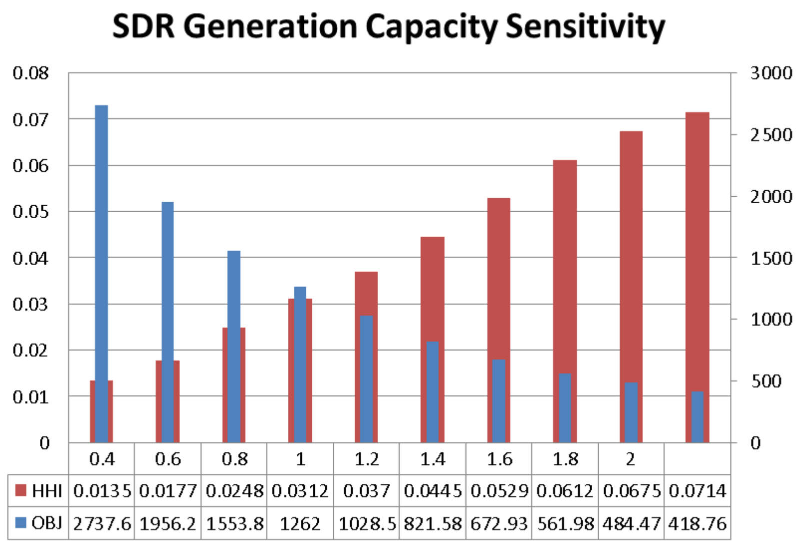


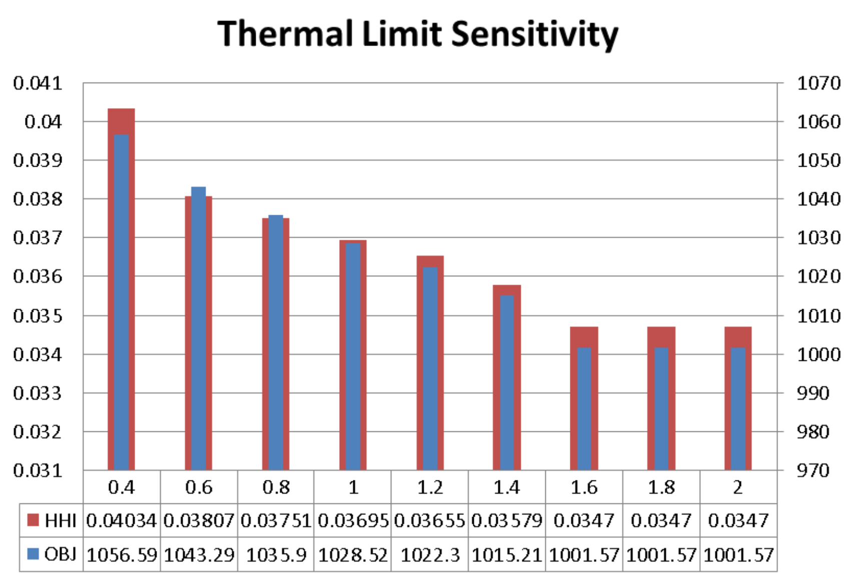
Figure 30 Sensitivity over SDRs' storage capacity-Scenario 9- Two-Way model



	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	
HHI	0.0135	0.0177	0.0248	0.0312	0.037	0.0445	0.0529	0.0612	0.0675	0.0714
OBJ	2737.6	1956.2	1553.8	1262	1028.5	821.58	672.93	561.98	484.47	418.76

Figure 31 Sensitivity over SDRs' generation capacity-Scenario 9- Two-Way model

However, larger storage capacities and genertors for local use are not efficient and economical yet. Improvements in the area of storage and generation technologies need to advance more. Consequently, responsible entities may want to invest in these area of research for development of more efficient and reliable storage and generation capacities.



	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
HHI	0.04034	0.03807	0.03751	0.03695	0.03655	0.03579	0.0347	0.0347	0.0347
OBJ	1056.59	1043.29	1035.9	1028.52	1022.3	1015.21	1001.57	1001.57	1001.57

Figure 32 Sensitivity over Thermal Limit-Scenario 9- Two-Way Model

Network limitation always makes problems more complex. Sensitivity analysis on thermal limits of lines demontrated that increase of thermal limit would result in better objective function and HHI. However, after a certain capacity, increaing the limit would not be beneficial. Though decreasing limits more than 40% of current limits would cause problem infeasibilty. So another good area of investment would be to increase the lines capacities in some parts of the network.

One of the ideas in demand response programs benefit from flexibility of users in their demands. The best case is when all demands are completely flexible (Relaxed), and can be met at any time of the day, and the worse case is when demands are very restricted (Restricted) and have to be satistied at the time of request. These two scenarios are compared with the base case in which demands are just flexible and have to be satisfied during a certain time window. Figure 33 illustrate results from these three cases. As it is expected, objective function, PAR and HHI are all better when demands are relaxed and have no time restrictions. However, when it comes to restricted case, the problem is not feasible anymore. That means with increase of public knowledge and encouraging people toward relaxing their time of use for some of their demands, both users and suppliers would benefit. So investment in increasing public knowledge could be another area of attention.

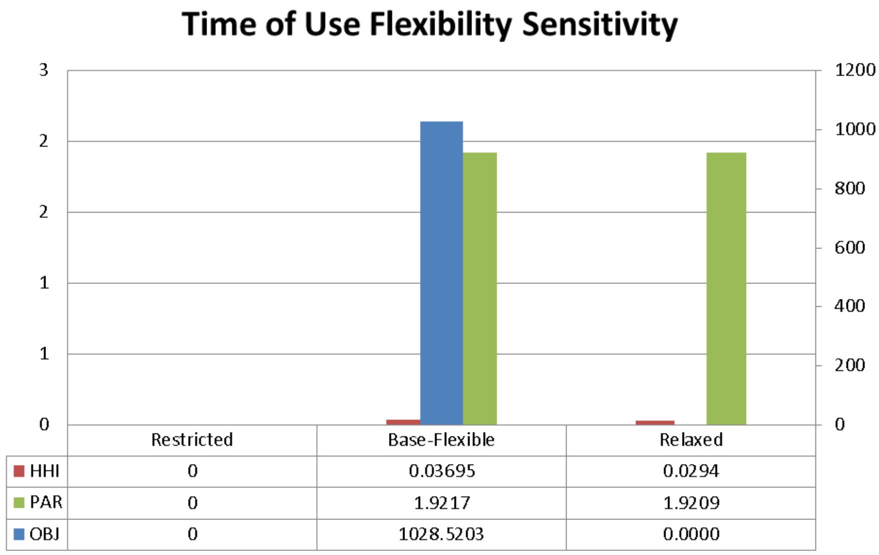


Figure 33 Sensitivity over Time of Use flexibility-Scenario 9- Two-Way model

Chapter 6: Conclusions and Future Work

Conclusions and Recommendations

Researchers showed that one of the best areas for energy efficiency opportunity is in installing energy management controls that shift time of electricity use (Schweitzer and Tonn, 2005). In this study Smart Grid in its real size is divided into three main tiers of zonal, regional and cross regional market. Then an appropriate market mechanism and an integrated dynamic Demand Response program and market equilibrium applicable to zonal level is proposed. The model promotes a certain level of disaggregation in such large systems. The problem is modeled as a one-leader multi-follower Stackelberg game. This research has contributed to the state of art not only in energy market and demand management modeling individually, but also extended the state of the art by combining these two areas of knowledge to develop an appropriate system applicable to the future of energy systems, Smart Grid. To the best of the author's knowledge, this is the first attempt in combining Demand Response scheduling and market equilibrium modeling. The algorithm proposed in this research could be implemented in the future Smart Grid meters to help users communicate with the system and enables the system to accommodate different sources of energy.

The proposed market mechanism has several important characteristics. It has good incentives for both public and private sectors to engage in the system. The proposed model would give users knowledge of their usage portfolio. That is instead of old fashioned lump sum electricity bill, they can understand how much they are paying for each appliance and when and what is the highest and lowest rates during day. This would give them a better sense of their usage behavior. Results showed higher individual welfare through applying the proposed integrated model, which is a good incentive for consumers to participate in the proposed mechanism and also motivate them to install renewable energy sources. Also the proposed model maximizes big suppliers' revenue as decision makers of the system which is a good incentive for them to participate in the system. Also unbundling generation, transmission, and distribution providers eliminates power monopoly in the market and maximizes individual benefits. Moreover, in the current systems big consumers pay less than small consumers while the proposed model is treating everyone the same which is another appealing feature for small users. Also the mechanism eliminates price cap in the system and considers real time nodal prices based on reverse supply function. Although this captures the elasticity of the market price but it may hurt the lower income families. In order to protect these groups, this mechanism is capable of considering different solutions such as offering subset of concessions to lower income families and household. Although the proposed mechanism is covering many different aspects since this research is mostly focused on the operation side of demand response program in the future system, the market design is not detailed in every aspects.

In the demand response modeling of the Smart Grid the hierarchal decision making problem among ISO, BGFs, and SDRs is modeled as a bi-level model. Two different types of hierarchy for the ISO are studied. One with complete power over trade values, and the other is a shared right of decision making with other users. Presuming that ISO has the hierarchy in decision making over the trade amounts in the network, while proving feasibility of the lower level equilibrium problem, an algorithm is then developed for solving the dynamic problem. The model is then transformed to a MPEC and then to a mixed integer problem. Commercial solvers and software is used to evaluate the proposed algorithm and mathematical models. The proposed integrated demand response and market equilibrium model in this study is applied to several different scenarios on several electricity networks with different characteristics.

Although the model can be transformed to MPEC model, MPEC algorithms are based on nonlinear techniques and so are not capable of solving large scale problems. The SOS algorithm is very time consuming and is not applicable in real size problems. Eventually, the MIP algorithm, is both time consuming and not reliable in large scale problems. However, in some cases it provides better solutions. There is a major issue with the MIP approach. That is the instability of the model. MIP's solution is very much dependant on the value of BigMs in the problem. With a little change in the value of BigM, the problem suddenly falls into infeasibility. As discussed in the MIP model section, determining BigM in solving the MIP is very crucial. Since the MIP model is unstable due to its sensitivity to BigMs' value, even if it provides a good solution in a reasonable time, the solution is not very reliable.

One of the specifications of the proposed algorithm, Mona, is its search direction. In the proposed algorithm the problem is being solved from top to bottom while optimization algorithms are mostly bottom to top. Bottom to top search means searching among feasible solutions, while in this algorithm search starts from the big picture which is the best bound.

Comparison of numerical solutions from the integrated demand response model applied through Mona or any of the solution methods (SOS, MIP, or MPEC) with the NO DR and NO DR-NO GAME models, shows better market indicators for the proposed mechanisms. This means better use of distributed energy sources in the system while implementing DR programs combined with competition in the market. Better HHI indicates that the proposed model would prevent monopoly in the system and also results in higher supply diversification. Moreover, the proposed model would distribute demand peaks to the best time of day with more available resources and less prices. Better individual welfare and higher revenues are good incentives for public and private sectors for acceptance of the proposed system among themselves. The model also facilitates use of storage capacities through managing their charging schedule during low price time periods which also results in more leveled demands during the day. Depending on the data sets, an average of 23% of demands are shifted during the day from peak times to off peak times applying the proposed model. This is a little better than 20% shifted demands through applying only time-of-use tariffs (Sovacool, 2009). Based on sensitivity analysis it is recommended that responsible entities invest in development of storage and renewable generation with higher capacities in order to have more efficient system. Also investment on network capacity improvement and increase of public knowledge would result in a better market share indicator and so better individual welfare for public and revenue for suppliers.

Finally, the author believes unbundling generation, transmission, and distribution providers is a necessity in order to make a market performs better in both aspects of system efficiency and incentivizing users. Moreover, expanding information availability to participants and trusting the system to converge to an equilibrium itself rather than forcing controls over some decisions would increase public participation which results in a better system performance.

Future Research Paths

This study opens a new path of research in the area of energy system modeling and management. Several studies can be done after this research which are introduced in this section as follows.

- System resiliency is an issue in electrical distribution and transmission networks. Modifying the proposed model to be resilient in case of unforeseen events is the next step of this study.
- It would be of interest to study the stochastic feature of demand and generation capacity in future. Though, due to dynamic nature of the studied problem it is not necessary to consider it in the model to have meaningful results.
- Implementing a good and accurate pricing function directly affects the output of the proposed system. Applying and evaluating different dynamic pricing functions in the proposed system such as shadow prices are among interesting future works.
- In the proposed system, SDRs influence market price indirectly through total market demand and supply. It would be interesting to study their direct effect on market pricing and compare the results with the current model.
- It is of interest to study different objective functions and attitudes for the ISO and compare the results in future.
- Using an accurate utility function would definitely result in better and more encouraging output. Implementing Smart Grid system, there will be a huge amount of data collected and available for investigation. One piece of information to gain from these data is estimating users' utility function. This can be captured from historical demand data to better understand users' demand behavior.
- This model is based on shifting and scheduling demands. However, considering demand reduction and semi satisfied demands in future would also be interesting.

- Although the examples are generated based on open sourced real data, it is always of interest to apply the model on real networks with real demand data.

Appendices

Appendix A:

Results from all scenarios are summarized in the following three tables.

Table 13 Results for Small Sized Network

Topology	*Node*	*Line*	*Model*	*Scenario*	*SDRs*	*BGFs*	*Algorithm*	*Solution Time (sec.)*	*ISO Obj.*	*Bound*	*Peak to Avg. Load (PAR)*	*Shifted Demand (%)*	*Market Share (HHI)*	*Market Share (SD)*	*BGF Obj. (Avg.)*	*BGF Obj. (SD)*	*SDR Obj. (Avg.)*	*SDR Obj. (SD)*	*Model Stat*	*gap*
Small	13	12	One-Way	1	5	1	Mona	0.05	0.00	0.00	6.18	72.62	0.530	0.2700	0.00	-	70.86	30.18	Optimal	-
							SOS	0.54	0.00	0.00	6.18	82.70	0.530	0.2700	0.00	-	-336.00	29.00	Optimal	-
							MIP	0.15	0.15	0.00	6.18	79.13	1.000	0.4100	0.06	-	72.00	33.00	Optimal	-
							MPEC	5.13	0.11	0.00	7.62	85.02	0.370	0.2000	0.03	-	-335.00	30.00	Local Optimal	-
							NO DR	0.06	1.66	0.00	10.44	89.21	0.910	0.3900	0.00	-	62.00	35.00	Optimal	-
							NO DR-NO GAME	0.04	1.92	0.00	11.72	89.21	0.990	0.4100	-3657.62	-	48.00	41.00	Optimal	-
				2	50	2	Mona	0.31	0.01	0.01	2.68	25.01	0.160	0.0500	0.00	0.00	111.10	64.57	Optimal	0%
							SOS	0.65	0.01	0.01	2.67	19.75	0.130	0.0500	0.00	0.00	-308.00	66.00	Optimal	0%
							MIP	5.76	0.45	0.00	2.02	21.27	0.180	0.0600	0.01	0.01	139.00	73.00	Optimal	-
							MPEC	2036.98	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.23	18.13	0.00	3.24	39.04	0.380	0.0800	0.00	0.00	97.00	67.00	Optimal	-
							NO DR-NO GAME	0.19	15.25	0.00	3.71	37.96	0.500	0.1000	-301.52	65.00	97.00	67.00	Optimal	-
				3	100	2	Mona	0.59	0.06	0.06	2.03	17.84	0.110	0.0300	0.00	0.00	102.61	69.69	Optimal	0%
							SOS	1.36	0.00	0.00	2.24	18.19	0.080	0.0300	0.00	0.00	-315.00	69.00	Optimal	-
							MIP	7.16	0.68	0.00	2.56	14.84	0.090	0.0300	0.03	0.04	129.00	76.00	Optimal	-
							MPEC	3665.71	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.40	34.44	0.00	3.68	31.15	0.330	0.0600	0.00	0.00	88.00	67.00	Optimal	-
							NO DR-NO GAME	0.35	77.98	0.00	3.29	31.94	0.250	0.0500	0.00	0.00	87.00	64.00	Optimal	-

Topology	*Node*	*Line*	*Model*	*Scenario*	*SDRs*	*BGFs*	*Algorithm*	*Solution Time (sec.)*	*ISO Obj.*	*Bound*	*Peak to Avg. Load (PAR)*	*Shifted Demand (%)*	*Market Share (HHI)*	*Market Share (SD)*	*BGF Obj. (Avg.)*	*BGF Obj. (SD)*	*SDR Obj. (Avg.)*	*SDR Obj. (SD)*	*Model Stat*	*gap*
			Two-Way	1	5	1	Mona	0.08	0.00	0.00	6.18	89.21	0.530	0.2700	0.00	-	71.00	30.00	Optimal	-
							SOS	0.36	0.00	0.00	6.35	99.30	0.530	0.2700	0.00	-	-351.00	39.00	Optimal	-
							MIP	0.10	0.00	0.00	6.51	50.93	0.530	0.2700	0.00	-	-359.00	47.00	Optimal	-
							MPEC	3.08	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.56	1.74	0.00	14.08	89.21	1.000	0.4100	0.00	-	50.00	43.00	Optimal	-
							NO DR-NO GAME	0.04	1.92	0.00	11.72	89.21	0.990	0.4100	-3657.62	-	48.00	41.00	Optimal	-
				2	50	2	Mona	0.39	0.01	0.01	2.24	21.30	0.160	0.0500	0.00	0.00	135.00	72.00	Optimal	0%
							SOS	2.96	0.01	0.01	2.59	18.02	0.200	0.0600	0.00	0.00	-304.00	68.00	Optimal	0%
							MIP	1.72	0.01	0.00	2.41	26.20	0.350	0.0800	0.00	0.00	-309.00	64.00	Optimal	-
							MPEC	95.02	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.27	25.90	0.00	5.10	43.83	0.640	0.1100	0.00	0.00	99.00	67.00	Optimal	-
							NO DR-NO GAME	0.23	12.99	0.00	3.92	36.47	0.500	0.1000	-301.52	65.00	95.00	67.00	Optimal	-
				3	100	2	Mona	0.67	0.06	0.06	2.37	12.56	0.100	0.0300	0.00	0.00	128.00	75.00	Optimal	0%
							SOS	3600.43	-	-	-	-	-	-	-	-	-	-	No Solution	-
							MIP	0.84	0.06	0.00	1.69	15.62	0.170	0.0400	0.00	0.00	-317.00	67.00	Optimal	-
							MPEC	244.97	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.43	41.13	0.00	4.00	31.09	0.550	0.0700	0.00	0.00	82.00	62.00	Optimal	-
							NO DR-NO GAME	0.33	67.62	0.00	3.29	31.94	0.850	0.0900	-100.36	141.94	87.00	64.00	Optimal	-

Table 14 Results for Medium Sized Network

Topology	*Node*	*Line*	*Model*	*Scenario*	*SDRs*	*BGFs*	*Algorithm*	*Solution Time (sec.)*	*ISO Obj.*	*Bound*	*Peak to Avg. Load (PAR)*	*Shifted Demand (%)*	*Market Share (HHI)*	*Market Share (SD)*	*BGF Obj. (Avg.)*	*BGF Obj. (SD)*	*SDR Obj. (Avg.)*	*SDR Obj. (SD)*	*Model Stat*	*gap*
Medium	37	36	One-Way	4	10	1	Mona	0.13	0.13	0.13	8.11	33.61	0.410	0.1800	0.00	-	115.09	61.79	Optimal	0%
							SOS	0.21	0.13	0.13	7.28	35.34	0.610	0.2300	0.00	-	-1078.00	67.00	Optimal	0%
							MIP	0.30	0.37	0.00	9.92	37.16	0.530	0.2100	0.03	-	130.00	56.00	Optimal	-
							MPEC	300.96	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.13	26.40	0.00	5.44	38.09	0.880	0.2800	0.12	-	67.00	42.00	Optimal	-
							NO DR-NO GAME	0.11	27.59	0.00	6.49	39.90	0.970	0.3000	-2942.96	-	67.00	36.00	Optimal	-
				5	100	2	Mona	0.80	0.63	0.63	2.53	20.61	0.050	0.0200	0.11	0.16	107.13	66.45	Optimal	0%
							SOS	60.50	0.66	0.63	2.15	19.47	0.060	0.0200	0.11	0.16	-1058.00	62.00	Integer Solution	5%
							MIP	20.94	2.25	0.00	2.39	17.15	0.070	0.0200	0.18	0.25	138.00	73.00	Optimal	-
							MPEC	3642.14	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.50	35.74	0.00	2.55	35.67	0.300	0.0500	1.53	2.16	97.00	64.00	Optimal	-
							NO DR-NO GAME	0.53	95.96	0.00	1.92	18.98	0.450	0.0700	-2083.67	1343.60	94.00	60.00	Optimal	-
				6	1000	10	Mona	15.52	322.71	322.71	1.87	1.58	0.030	0.0100	0.39	1.25	4518.75	71397.36	Optimal	0%
							SOS	50.55	322.70	322.70	1.89	1.48	0.020	0.0000	5.37	11.56	3359.00	71324.00	Optimal	0%
							MIP	7389.47	2441.59	0.00	0.00	0.00	0.000	0.0000	0.00	0.00	4568.00	71770.00	Unfinished	-
							MPEC	3857.73	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	4.86	5954.96	0.00	1.88	2.43	0.290	0.0200	91.67	174.07	4515.00	71232.00	Optimal	-
							NO DR-NO GAME	14.81	6622.54	0.00	1.95	2.03	0.160	0.0100	-1096.81	1125.89	4529.00	71670.00	Optimal	-
				7	5000	10	Mona	231.16	4.91	4.91	1.98	1.37	0.010	0.0000	0.00	0.00	4595.73	76372.76	Optimal	0%
							SOS	952.06	4.82	4.82	1.98	1.09	0.004	0.0000	0.00	0.00	3438.00	76355.00	Optimal	0%
							MIP	682.05	4.92	0.00	0.00	0.00	0.000	0.0000	0.00	0.00	4633.00	76313.00	Unfinished	-
							MPEC	35044.54	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	23.04	19722.69	0.00	1.93	1.74	0.320	0.0100	-41.91	164.14	4592.00	76281.00	Optimal	-
							NO DR-NO GAME	248.39	26698.45	0.00	1.96	2.08	0.160	0.0100	-1672.45	1406.26	4598.00	76330.00	Optimal	-

Topology	*Node*	*Line*	*Model*	*Scenario*	*SDRs*	*BGFs*	*Algorithm*	*Solution Time (sec.)*	*ISO Obj.*	*Bound*	*Peak to Avg. Load (PAR)*	*Shifted Demand (%)*	*Market Share (HHI)*	*Market Share (SD)*	*BGF Obj. (Avg.)*	*BGF Obj. (SD)*	*SDR Obj. (Avg.)*	*SDR Obj. (SD)*	*Model Stat*	*gap*
			Two-Way	4	10	1	Mona	0.25	0.13	0.13	8.11	35.11	0.410	0.1800	0.00	-	118.00	59.00	Optimal	0%
							SOS	0.48	0.13	0.13	7.28	38.22	0.610	0.2300	0.00	-	-1078.00	67.00	Optimal	0%
							MIP	0.45	0.13	0.00	8.33	37.67	0.510	0.2000	0.00	-	-1080.00	67.00	Optimal	-
							MPEC	1588.43	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.20	30.53	0.00	5.44	38.09	0.890	0.2800	0.12	-	66.00	42.00	Optimal	-
							NO DR-NO GAME	0.10	33.21	0.00	5.44	34.16	0.970	0.3000	-2617.13	-	74.00	61.00	Optimal	-
				5	100	2	Mona	1.10	0.81	0.63	2.68	16.47	0.050	0.0200	0.27	0.39	125.00	67.00	Optimal	30%
							SOS	135.22	0.69	0.63	2.60	16.77	0.050	0.0200	0.11	0.16	-1061.00	64.00	Integer Solution	11%
							MIP	15.21	0.66	0.00	2.09	20.09	0.070	0.0200	0.00	0.00	-1057.00	61.00	Optimal	-
							MPEC	243.85	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.51	33.89	0.00	2.55	35.01	0.180	0.0400	1.15	1.63	96.00	65.00	Optimal	-
							NO DR-NO GAME	0.50	85.53	0.00	2.18	21.67	0.460	0.0700	-2741.67	348.77	102.00	66.00	Optimal	-
				6	1000	10	Mona	27.34	323.24	322.71	2.00	1.73	0.030	0.0100	0.61	1.93	4570.00	71726.00	Optimal	0%
							SOS	3605.72	-	-	-	-	-	-	-	-	-	-	No Solution	-
							MIP	406.20	322.84	0.00	1.90	2.04	0.030	0.0100	0.00	0.00	3351.00	71114.00	Optimal	-
							MPEC	4177.31	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	5.22	5960.98	0.00	2.11	2.49	0.330	0.0200	62.34	116.34	4528.00	71605.00	Optimal	-
							NO DR-NO GAME	11.64	5820.87	0.00	2.00	2.86	0.160	0.0100	-898.68	634.83	4507.00	71059.00	Optimal	-
				7	5000	10	Mona	911.59	4.91	4.91	1.94	0.93	0.003	0.0000	0.00	0.00	4637.00	76316.00	Optimal	0%
							SOS	4052.34	-	-	-	-	-	-	-	-	-	-	No Solution	-
							MIP	2760.95	4.91	0.00	1.94	1.23	0.003	0.0000	0.00	0.00	3434.00	76314.00	Optimal	-
							MPEC	-	-	-	-	-	-	-	-	-	-	-	-	-
							NO DR	66.52	21924.74	0.00	1.94	1.92	0.240	0.0100	-388.12	1282.74	4590.00	76285.00	Optimal	-
							NO DR-NO GAME	236.44	25939.55	0.00	2.07	2.00	0.160	0.0100	-1558.27	1429.85	4593.00	76289.00	Optimal	-

Table 15 Results for Large Sized Network

Topology	*Node*	*Line*	*Model*	*Scenario*	*SDRs*	*BGFs*	*Algorithm*	*Solution Time (sec.)*	*ISO Obj.*	*Bound*	*Peak to Avg. Load (PAR)*	*Shifted Demand (%)*	*Market Share (HHI)*	*Market Share (SD)*	*BGF Obj. (Avg.)*	*BGF Obj. (SD)*	*SDR Obj. (Avg.)*	*SDR Obj. (SD)*	*Model Stat*	*gap*
Large	122	117	One-Way	8	100	2	Mona	0.94	2.09	2.09	4.49	16.03	0.140	0.0400	0.00	0.00	93.72	62.60	Optimal	0%
							SOS	5.42	2.31	2.09	4.20	17.07	0.150	0.0400	0.00	0.00	-3720.00	60.00	Integer Solution	11%
							MIP	585.75	4.67	0.00	4.55	15.13	0.150	0.0400	0.00	0.00	121.00	67.00	Optimal	-
							MPEC	3794.09	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.55	39.68	0.00	3.47	32.18	0.280	0.0500	1.03	1.09	85.00	61.00	Optimal	-
							NO DR-NO GAME	0.57	146.68	0.00	3.83	15.18	0.470	0.0700	-2943.68	893.29	90.00	64.00	Optimal	-
				9	1000	5	Mona	10.31	1028.52	1028.52	1.92	2.47	0.040	0.0100	0.00	0.00	4672.74	75566.20	Optimal	0%
							SOS	1741.35	1069.99	1028.04	1.92	2.44	0.030	0.0100	0.00	0.00	861.00	75565.00	Integer Solution	4%
							MIP	13689.96	-	-	-	-	-	-	-	-	-	-	Unfinished	-
							MPEC	4251.55	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	4.51	5448.28	0.00	2.06	3.89	0.040	0.0100	0.00	0.00	4666.00	75488.00	Optimal	-
							NO DR-NO GAME	8.07	6050.91	0.00	1.92	2.60	0.030	0.0100	-1718.28	1106.39	4667.00	75507.00	Optimal	-
				10	5000	10	Mona	267.32	524.61	524.61	1.94	1.06	0.010	0.0000	0.00	0.00	4562.70	73595.50	Optimal	0%
							SOS	3624.47	-	-	-	-	-	-	-	-	-	-	No Solution	-
							MIP	10910.98	0.00	0.00	0.00	0.00	0.000	0.0000	0.00	0.00	5.00	8.00	Unfinished	-
							MPEC	48968.62	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	77.86	23560.86	0.00	1.93	1.12	0.240	0.0100	-87.15	445.26	4558.00	73412.00	Optimal	-
							NO DR-NO GAME	228.39	29982.30	0.00	2.08	1.71	0.140	0.0100	-2006.26	2090.49	4562.00	73560.00	Optimal	-

Topology	***Node***	***Line***	***Model***	***Scenario***	***SDRs***	***BGFs***	***Algorithm***	***Solution Time (sec.)***	***ISO Obj.***	***Bound***	***Peak to Avg. Load (PAR)***	***Shifted Demand (%)***	***Market Share (HHI)***	***Market Share (SD)***	***BGF Obj. (Avg.)***	***BGF Obj. (SD)***	***SDR Obj. (Avg.)***	***SDR Obj. (SD)***	***Model Stat***	***gap***
			Two-Way	8	100	2	Mona	1.26	2.09	2.09	4.44	17.00	0.170	0.0400	0.00	0.00	99.00	61.00	Optimal	0%
							SOS	11.13	2.09	2.09	4.37	18.50	0.140	0.0400	0.00	0.00	-3723.00	61.00	Optimal	0%
							MIP	193.46	2.09	0.00	4.38	20.22	0.150	0.0400	0.00	0.00	-3721.00	62.00	Optimal	-
							MPEC	6063.09	-	-	-	-	-	-	-	-	-	-	Infeasible	-
							NO DR	0.58	42.25	0.00	3.47	32.18	0.210	0.0400	1.03	1.09	85.00	61.00	Optimal	-
							NO DR-NO GAME	0.56	126.23	0.00	3.56	16.07	0.470	0.0700	-3254.26	307.68	88.00	66.00	Optimal	-
				9	1000	5	Mona	19.28	1028.52	1028.52	1.92	2.53	0.040	0.0100	0.00	0.00	4702.00	75500.00	Optimal	0%
							SOS	3611.16	-	-	-	-	-	-	-	-	-	-	No Solution	-
							MIP	749.58	1028.52	0.00	1.92	2.46	0.040	0.0100	0.00	0.00	862.00	75569.00	Optimal	-
							MPEC	-	-	-	-	-	-	-	-	-	-	-	-	-
							NO DR	5.37	4732.54	0.00	2.06	3.42	0.040	0.0100	69.55	115.64	4668.00	75495.00	Optimal	-
							NO DR-NO GAME	9.26	5936.32	0.00	1.96	2.86	0.040	0.0100	-2104.69	1367.78	4668.00	75509.00	Optimal	-
				10	5000	10	Mona	982.62	524.61	524.61	1.96	0.89	0.010	0.0000	0.00	0.00	4600.00	73552.00	Optimal	0%
							SOS	12846.89	-	-	-	-	-	-	-	-	-	-	No Solution	-
							MIP	3870.23	-	-	-	-	-	-	-	-	-	-	No Solution	-
							MPEC	65213.77	-	-	-	-	-	-	-	-	-	-	N/A	-
							NO DR	96.63	25337.72	0.00	1.94	1.01	0.340	0.0100	-57.19	209.66	4558.00	73408.00	Optimal	-
							NO DR-NO GAME	476.15	30457.98	0.00	1.95	1.91	0.230	0.0100	-1722.57	1873.10	4557.00	73402.00	Optimal	-

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