ABSTRACT

Title of Document:	LOCATING CHARGING STATIONS FOR ELECTRIC VEHICLES IN RURAL AND URBAN NETWORKS
	Kiana Roshan Zamir, M. Sc., 2014
Directed By:	Professor Ali Haghani, Department of Civil and Environmental Engineering

Using new alternative fuels for motorized transportation vehicles has become increasingly popular with the growing concerns on the limitation of fossil fuels and environmental degradation. Introduction of numerous models of electric vehicles in the 21 century raised hope for replacing conventional internal combustion engine vehicles with these vehicles; however several barriers has adversely impacted the widespread adoption of these vehicles. Providing adequate number of charging stations and planning the layout of their infrastructure will help overcome some of the existing challenges. In this thesis, two formulations are presented for the optimal layout of these stations in rural and urban networks and the models are applied on two networks. For the rural model, the results indicate the solution is highly sensitive to the assumptions about the range of vehicles for which we are designing the layout. In the urban context, the decision about number and location of chargers is highly dependent on the probability threshold we choose for satisfying the demand.

LOCATING CHARGING STATIONS FOR ELECTRIC VEHICLES IN RURAL AND URBAN NETWORKS

By

Kiana Roshan Zamir

Thesis submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Master of Science
2014

Advisory Committee:
Professor Ali Haghami, Chair
Professor Paul Schonfeld
Associate Professor Lei Zhang

Dedication

To my mother, father, and brother; I couldn’t have done this without your inspiration, unconditional love, and encouragement.

Acknowledgements

First and foremost, I would like to express the deepest appreciation to my advisor Professor Haghani for the continuous support, encouragement, and patience during my study at UMD. Without his guidance, this thesis would not have been completed or written.

A special feeling of gratitude to my family and friends who have supported me throughout the process of my graduate study.

Table of Contents

List of Tables

List of Figures

Chapter 1: Introduction

Promoting electric vehicles (EVs) along other alternative fuel vehicles have gained interests among government officials and policy makers in recent years due to the advantages of these vehicles over the conventional internal combustion engine vehicles. The goal of this thesis is to develop a model to optimally locate charging facilities for electric vehicles.

In sections one and two of this chapter, we will go through some of the features of these vehicles and their infrastructures which make them different from the internal combustion engine vehicles. In the third section, their advantages and some of the challenges that these vehicles are facing are discussed. This will give us a framework for stating the problem in the following section. In the fifth and sixth section of this chapter the contributions and the structure of this thesis are presented.

1.1 Types of Electric Vehicles

There exist different types of vehicles that run with electricity, some of them are completely electric and they are known as all-electric vehicles (also called battery-electric vehicles (BEVs) and some are partially electric, including hybrid electric vehicles (HEVs) and plug-in hybrid electric vehicles (PHEV) [1].

HEVs get most of their power from the internal combustion engine and in most models their electric motors work as auxiliary power sources**.** Batteries of these vehicles are charged by internal combustion engines and through regenerative braking and they cannot be plugged to an electric power source to be charged. Unlike HEVs, PHEVs get most of their power from the electric motor and they use internal combustion engine as back-up**.** They have larger batteries compared to HEVs and unlike them; they can be plugged in to be charged. BEVs get all of their power from an electric motor and they do not have an internal combustion engine [1].

Here we use term PEVs when we are referring to both PHEVs and BEVs.

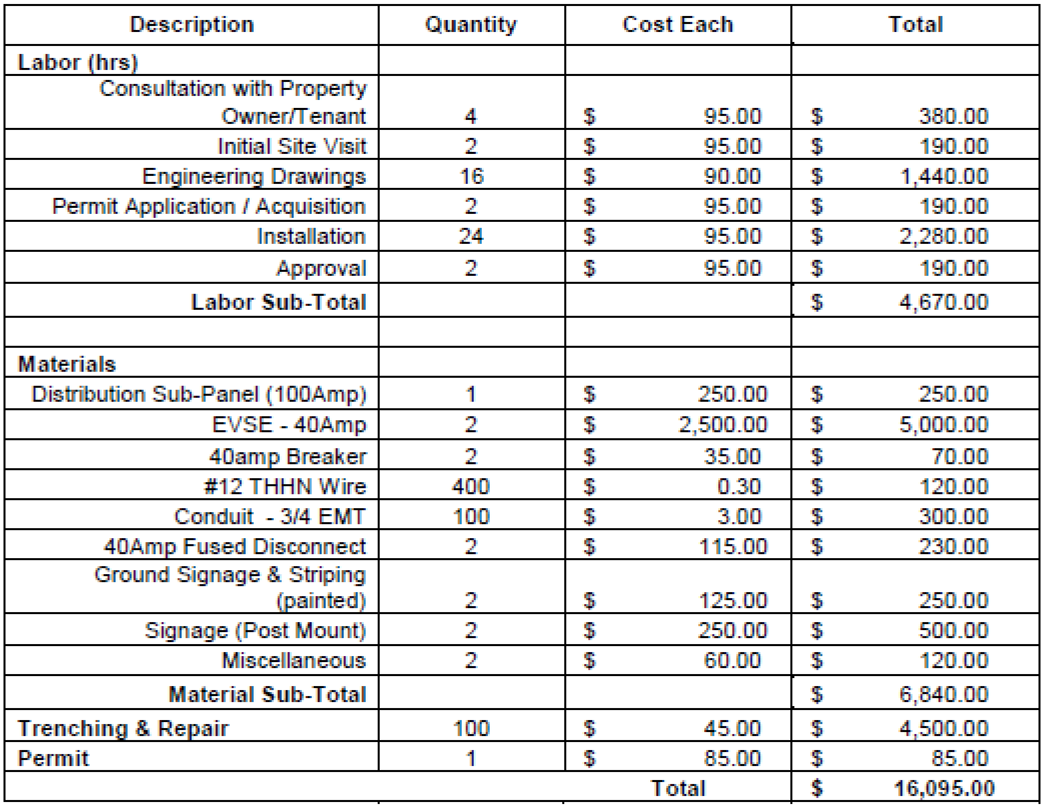
1.2 Types of Infrastructures

In terms of recharging, three types of infrastructure exist for PEVs: level 1, level 2, and level 3 (also called DC fast charging). Level 1 chargers use alternating current at110-120 Volts (V) 15Ampers (A) (12A useable) or 20A (16A useable) branch circuit. As a result, these chargers can provide relatively small amounts of power and charging the vehicle fully may take between 10 to 20 hours, depending on the size of battery. However due to availability of 120V outlets and their relatively low installation cost these type of chargers are still very common in residential areas [2].

Level 2 chargers use 240-280V alternating current, single-phase, 30-80A branch circuit. PEVs can be fully charged in 4 to 8 hours. These features make these chargers suitable for both residential and public places [2].

Level 3 chargers are for public places and they are very similar to the gas stations. They use 480V alternating current and a three phase circuit. PEVs can be 80% charged in less than 30 minutes. Although these chargers have the benefit of being very fast in terms of recharging the PEVs, their impact on the grid make the practicality of using these chargers in residential areas dubious [2]. It should also be noted that if they are used on a regular basis they can have negative impact on the life of the battery. Tables 1 and 2 provide cost estimation for installation of level 2 and level 3 chargers in public areas [3].

Table 1 Cost Estimation for Level 2 Charging Facility[1]



Description	Quantity	Cost Each	Total
Labor (hrs)			
Consultation with Property Owner/Tenant	4	$ 95.00	$ 380.00
Initial Site Visit	2	$ 95.00	$ 190.00
Engineering Drawings	16	$ 90.00	$ 1,440.00
Permit Application / Acquisition	2	$ 95.00	$ 190.00
Installation	24	$ 95.00	$ 2,280.00
Approval	2	$ 95.00	$ 190.00
Labor Sub-Total			$ 4,670.00
Materials			
Distribution Sub-Panel (100Amp)	1	$ 250.00	$ 250.00
EVSE - 40Amp	2	$ 2,500.00	$ 5,000.00
40amp Breaker	2	$ 35.00	$ 70.00
#12 THHN Wire	400	$ 0.30	$ 120.00
Conduit - 3/4 EMT	100	$ 3.00	$ 300.00
40Amp Fused Disconnect	2	$ 115.00	$ 230.00
Ground Signage & Striping (painted)	2	$ 125.00	$ 250.00
Signage (Post Mount)	2	$ 250.00	$ 500.00
Miscellaneous	2	$ 60.00	$ 120.00
Material Sub-Total			$ 6,840.00
Trenching & Repair	100	$ 45.00	$ 4,500.00
Permit	1	$ 85.00	$ 85.00
		Total	**$ 16,095.00**

1.3 Advantages and Challenges

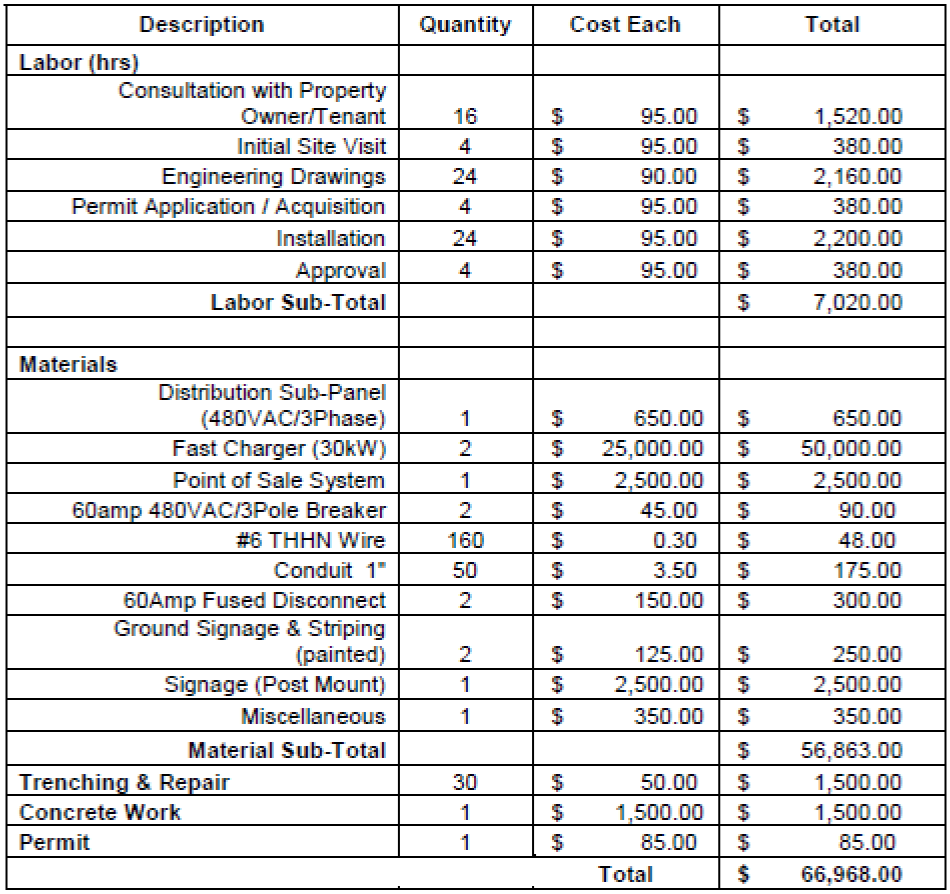
EVs have the advantages of being environmentally friendly and cleaner compared to vehicles that use fossil fuels. They reduce greenhouse gas emissions depending on the technology used for electricity generation and some of them (battery electric vehicles

---

[1] Source: Charging infrastructure deployment guidelines for the greater San Diego area (2010) prepared by electric transportation engineering corporation

(BEVs)) emit no tailpipe pollutants (zero-emission vehicle). They also have the benefit of reducing dependence on foreign petroleum and contribute to the nation's energy independence.

Table 2 Cost Estimation for Level 2 Charging Facility[2]



Description	Quantity	Cost Each	Total
Labor (hrs)			
Consultation with Property Owner/Tenant	16	$ 95.00	$ 1,520.00
Initial Site Visit	4	$ 95.00	$ 380.00
Engineering Drawings	24	$ 90.00	$ 2,160.00
Permit Application / Acquisition	4	$ 95.00	$ 380.00
Installation	24	$ 95.00	$ 2,200.00
Approval	4	$ 95.00	$ 380.00
Labor Sub-Total			$ 7,020.00
Materials			
Distribution Sub-Panel (480VAC/3Phase)	1	$ 650.00	$ 650.00
Fast Charger (30kW)	2	$ 25,000.00	$ 50,000.00
Point of Sale System	1	$ 2,500.00	$ 2,500.00
60amp 480VAC/3Pole Breaker	2	$ 45.00	$ 90.00
#6 THHN Wire	160	$ 0.30	$ 48.00
Conduit 1"	50	$ 3.50	$ 175.00
60Amp Fused Disconnect	2	$ 150.00	$ 300.00
Ground Signage & Striping (painted)	2	$ 125.00	$ 250.00
Signage (Post Mount)	1	$ 2,500.00	$ 2,500.00
Miscellaneous	1	$ 350.00	$ 350.00
Material Sub-Total			$ 56,863.00
Trenching & Repair	30	$ 50.00	$ 1,500.00
Concrete Work	1	$ 1,500.00	$ 1,500.00
Permit	1	$ 85.00	$ 85.00
		Total	**$ 66,968.00**

According to the Organization for Economic Co-operation and Development (OECD) and International Energy Agency (IEA), in 2009 the percentage share of oil demand for transportation sector was 57%. According to U.S. Energy Information Administration,

---

[2] Source: Charging infrastructure deployment guidelines for the greater San Diego area (2010) prepared by Electric Transportation Engineering Corporation

this number was 72% for US in 2013 and about 40% of total consumed petroleum was imported.

In addition, electric vehicles have other benefits such as having lower noise and better efficiency compared to conventional internal combustion engine vehicles which are relatively inefficient since the majority of energy is lost as heat during the conversion of fuel energy to propulsion. According to Tesla Company, drive efficiency of the Tesla Roadster is 88% which is almost three times more efficient than the conventional vehicle powered by internal combustion engine.

Curtin et Al. (2009) showed the equivalent recharging cost of PEVs would be $.75 per gallon which is about 79% less than conventional vehicles [7] assuming that:

1) 0. 24 kWh consumed per mile for PEV[3] [4],

2) 30 mile traveled per gallon of gas[4] [5], and

3) national average price of $0.1065 per kWh for residential electricity[5] [6].However, in terms of initial cost, PEVs are expensive due to high cost of their batteries.

Another challenge besides cost that prevents mass production of these vehicles is the limited range of these vehicles compared to conventional vehicles. Range anxiety which is defined as “users’ continual concern for being stranded with a fully discharged battery” (Tate et al. 2008) has been identified as one of the major drawbacks of these vehicles [8].

---

[3] Value calculated using Advisor modeling results for the full charge test, which simulates the all-electric mode. Specifically, 0.24 kWh / mile = 33.4 kWh / 1 gal gasoline * 1 gasoline gal equivalent / 142.1 miles

[4] Value represents the average 2005 fuel efficiency for a light-duty passenger car

[5] Value represents the annual average residential retail price of electricity for 2007

One survey of American consumers in 2011 shows that about 55 %, 45%, and 25% of consumers think that price, range, and charging time of these vehicles are major limitations of these vehicles [9]. According to 2012 Car Brand Perception survey, about 77 percent of people are concerned about the limited range of EVs [10].

Travel adaption including “using a substitute vehicle, choosing another mode such as public transportation or changing their travel plan such as canceling the trip” is needed when the length of trip is beyond the comfort level which is defined as minimum level of battery charge at which drivers are still comfortable driving the BEV) [11].

Looking at travel itineraries in which drivers use conventional internal combustion engines vehicles can aid us in developing realistic models for the driver’s behavior [11]. Based on a report by the U.S. Department of Transportation, over 50percent of all vehicle trips are less than 10 miles; however these trips only account for 28 percent of all household vehicle miles traveled. This report also shows that although less than one percent of all vehicle trips are above 100 miles, these trips account for nearly 15 percent of all household vehicle miles traveled [12]. This emphasizes the importance of planning for placing charging facilities in rural networks as well as in urban areas. Figure 1 shows the distribution of percent daily vehicle trips and percent of daily vehicle miles as a function of trip length based on the 2009 National Household Travel Survey.

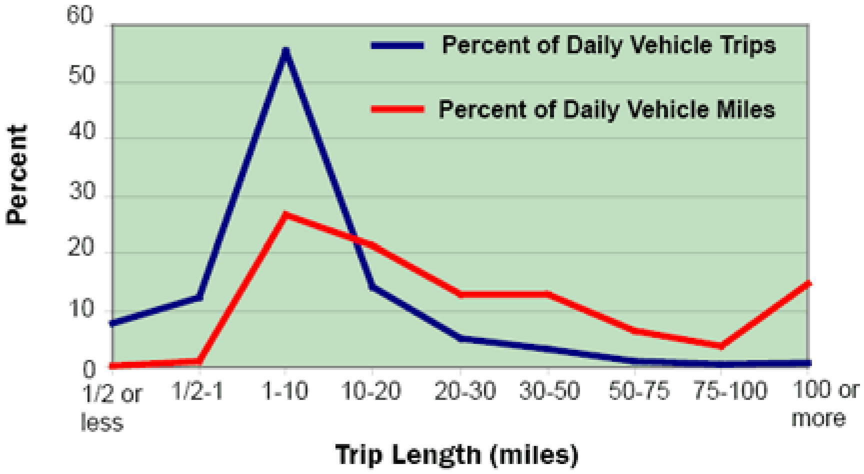


Figure 1 Distribution of Percent of Daily Vehicle Trips and Vehicle Miles[6]

Another major barrier that is explained in Traut et al. (2012), Melaina and Bremson (2008), Leiby and Rubin (2004), and Sperling (1990), that prevents widespread use of alternative fuel vehicles is the "chicken and egg" problem consisting of three stakeholders: consumers, manufacturers, and fuel providers. Manufacturers do not want to produce vehicles that do not have market; consumers do not want to buy vehicles that cannot be fueled easily, and fuel providers do not want to provide infrastructure for fuels that do not have considerable demand ([13],[14],[15], and [16]).

In addition, the inability to forecast charging demand and distribution of this demand add difficulty to power-grid planning and placing public charging facilities for these vehicles. Predicting the spatial pattern of EV ownership and identifying the households and neighborhoods that are most likely to own these vehicles can help planners provide charging facilities for these demands and consequently promote these vehicles [17].

---

[6] Source: U.S. Department of Transportation, Federal Highway Administration, Office of Highway Policy Information, National Household Travel Survey.

Chen et al. have shown that EVs ownership is higherin zones with morenumber of households and higher resident-worker densities. (This relationship is similar for internal combustion engine vehicles.) Distance to central business district (CBD) has a negative impact on EV ownership. Lower income households tend to be unwilling to buy EVs. EV ownership is also influenced by “neighbor effect” which suggests that nearby households impact each other’s EV ownership decision [17].

To overcome these hurdles and limitations and promote these vehicles many deployment efforts have been taken by government officials around the world such as giving financial incentives to consumers, giving access to restricted roadways, investment on research with the goal of developing high performance battery technologies, and providing facilities for charging EVs [19].

According to global EV outlook [19], by 2012 United States with 38% of global electric vehicle stock had the most EV owners, while Japan and France with 24% and 11% were in second and third place, respectively. As it is shown in figure 2, in terms of market share, Norway and Japan with 3% and 1% respectively had the highest market share in 2012 [19]. Figures 3 and 4 show world PHEV and BEV sales by country in 2012. Figures 5 and 6 show the alternative fuel vehicle in use and electric vehicle respectively in the United States from 1995 to 2010. Figure 7 shows PEV in use by model for the United States from 2010 to 2013.

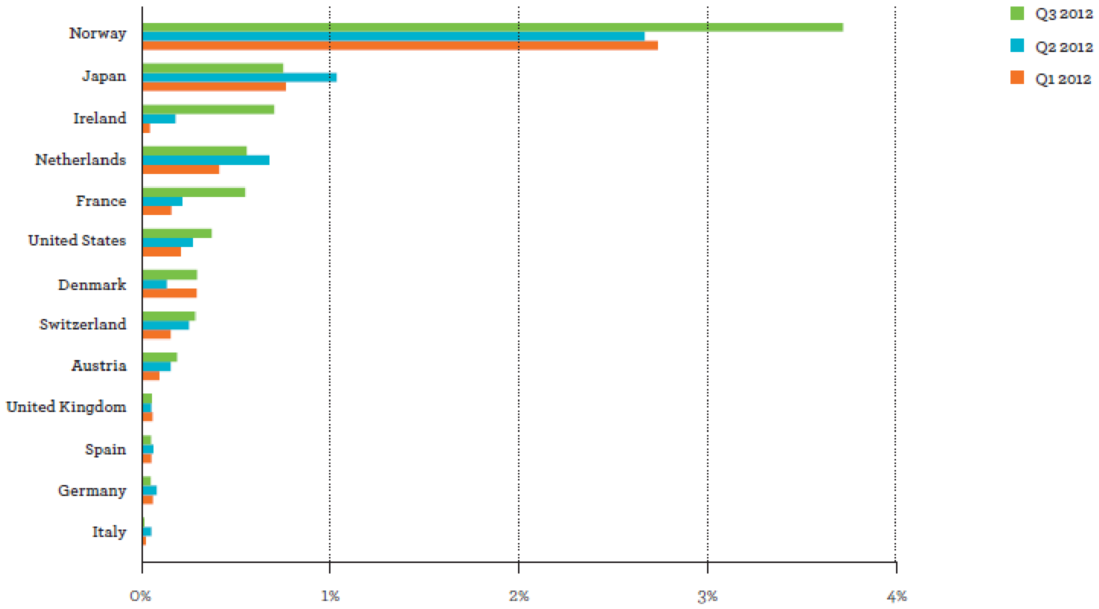


Figure 2 2012 EVs Sale as % of Total Passenger Vehicle Sales[7]

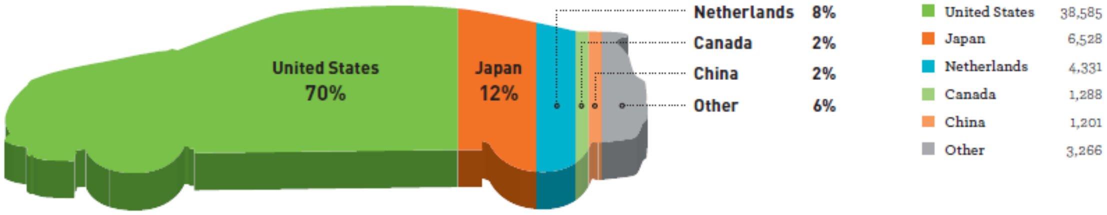


Figure 3 2012 World PHEV Sales, by Country[8]

---

[7] Source: Bloomberg New Energy Finance and Global EV Outlook 2013 - Understanding the Electric Vehicle Landscape to 2020. International Energy Agency

[8] Source: EVI, Mark Lines Database and Global EV Outlook 2013 - Understanding the Electric Vehicle Landscape to 2020. International Energy Agency

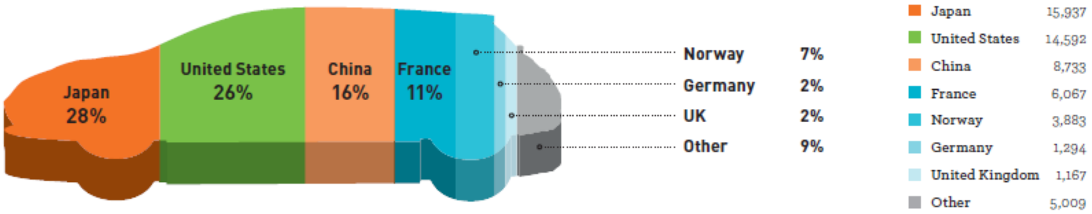


Figure 4 2012 World BEV Sales by Country[9]

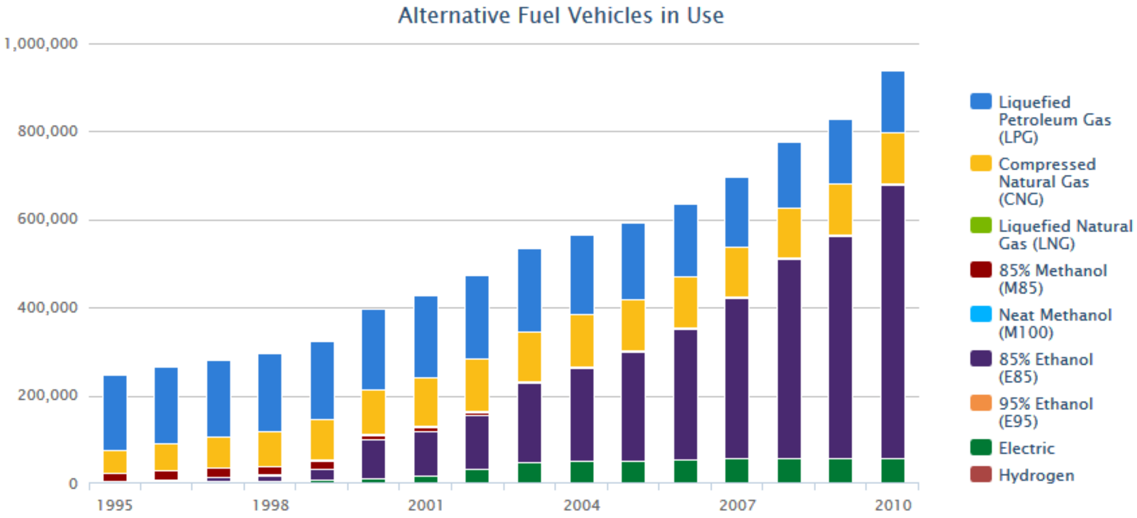


Figure 5 Alternative Fuel Vehicles in Use in the United States[10]

---

[9] Source: EVI, Mark Lines Database and Global EV Outlook 2013 - Understanding the Electric Vehicle Landscape to 2020. International Energy Agency

[10] Source: EIA’s Annual Energy Review

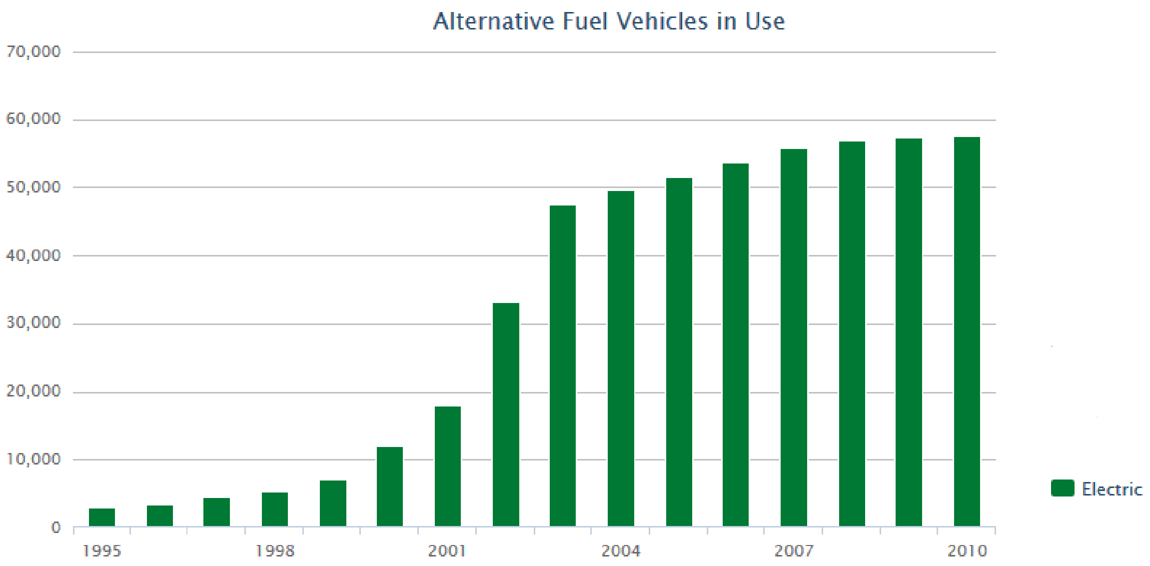


Figure 6 Electric Vehicles in Use in the United States[11]

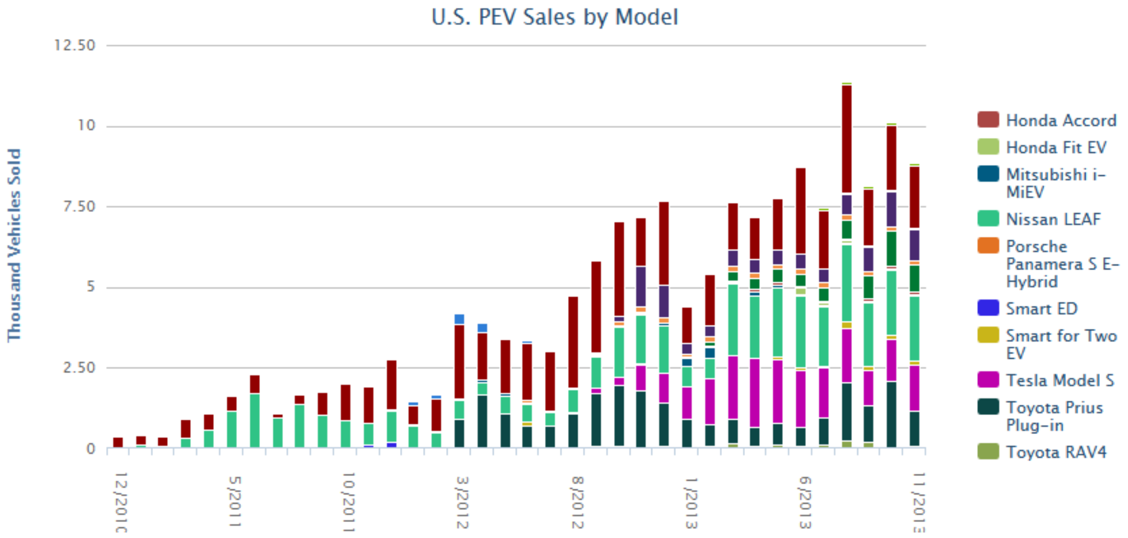


Figure 7 PEV sales by model in the United States[12]

---

[11] Source: EIA's Annual Energy Review

[12] Source: Hybridcars.com/market-dashboard.html

1.4 Motivation and Problem Statement

Lack of charging facilities is one of the major barriers that exist in increasing the adoption rate of alternative fuel vehicles [14]. Studies also show providing faster chargers will also help in increasing the adoption rate as a study in Netherland indicates that expected EV adopters tend to have relatively high valuation of time spend in charging stations. [18].

In this thesis, two individual models are presented for the optimal location of charging stations in urban and rural network addressing some of the challenges mentioned in the previous section such as limited range, uncertainty in demand, and proper combination of type of chargers. In the urban context, as the daily travel pattern shows, EV users do not need to recharge their car more than once. They can either do it at home, work, or school. (Probably the drivers who drive longer distances which need multiple stop for recharging with the purpose of doing their daily activity are not eager to buy EVs.) As a result in the urban context, only the optimal combination, sizing, and placement are discussed; also uncertainty in demand is accounted for in the model. However recharging the vehicle in multiple locations is necessary for longer trips which do not happen often, so for the rural network the proposed model finds the optimal location considering the fact that vehicle might need to stop at multiple charging facilities to be recharged.

1.5 Contribution

Several factors distinguish this study from previous studies in this area. First, a new formulation is proposed for placing charging stations in order to recharge longer trips.

The proposed formulation is more efficient than the formulation introduced by Kuby and Lim (2005) in terms of preparing input data [35]. In addition this formulation has the advantage of considering paths other than the shortest path (second, third, and … shortest path) to place charging facilities in optimal places along the path which is more conform to travel behavior of drivers.

In the urban context, a two stage stochastic model is developed which accounts for uncertainty in demands; also two types of charging facilities (level two and level three chargers (DC fast charging)) are considered for installation in the proposed model.

1.6 Structure of Thesis

In the first chapter of this thesis, some of the features of electric vehicles and their charging infrastructure in addition to the challenges they are currently facing were discussed. Moreover, the focus of this thesis and its contributions were discussed in this chapter as well. In the second chapter, some of the previous research and their methodologies are discussed. In the third chapter two models are presented for the layout of charging facilities in urban and rural networks. In the following chapter the models are applied in two networks and sensitivity analysis is performed on the input data. Finally, in Chapter 5, results presented in the previous chapter are interpreted and some suggestions for the future research are provided.

Chapter 2: Literature Review

There are quite a few papers in the of literature that focus on the siting and sizing of charging stations for electric and other alternative fuel vehicles. Many aspects of these vehicles such as their range, their impact on the grid, and their demand distribution along with many others have been addressed so far; however there are still some aspects that have not been fully addressed. Also, some of the methodologies introduced so far are not efficient and can be enhanced to become more efficient.

Problems involving location decisions have been well studied. Set covering location models, maximum covering location models, center models, and median problems along with their variants are some of the models that have been developed for facility location problems [20].

Set covering models try to minimize the total cost of opening facilities with the constraint that all of the demands need to be covered. Jia et al. (2012) used similar methodology and tried to minimize the sum of investment cost, operation cost, and user cost by locating charging station in a network of roads. Their results showed that the stations must be placed at the nodes or near the nodes since demands are highly concentrated at the nodes [21].

Church and ReVelle (1974) introduced Maximum covering location model. Unlike set covering models this model does not need to cover all demands. Based on the budget the number of facilities is fixed and the objective function tries to maximize the number of covered demands based on the weight of each demand node [22].

Several papers use this methodology. As an example, Xi et al. (2013) proposed two models for locating stations with charger level 1 or level 2 with the aim of maximizing the number of served EV and the amount of energy recharged by having constraints on the available budget. They apply their model on a case study in central-Ohio region [23]. Chen et al. (2013) used parking information from trip records in the Puget Sound Regional Council's 2006 household travel survey to estimate the future demand of EVs then a mixed integer programming problem was solved to locate the charging stations with the objective of minimizing users' access cost and unmet demand with limitation on the number of installed charging stations [24]

Gradual covering location model is a variant of maximal covering location model in which the optimal location of facilities also depends on their distance to candidate locations and the coverage is reduced depending on the distance of demands to the facilities [25].

Frade et al. (2011) used this model as the basis of proposing a model for locating charging stations in a neighborhood of Lisbon which has a mix of residential and business uses. They estimate demand for EV recharging based on modeling vehicle ownership and volume of employment for each census block respectively and using the forecast of European Commission of share of EVs by 2020 and 2030. Using this methodology they maximized the demand coverage within acceptable maximum distance [26].

Some studies used other methods such as topological techniques to allocate charging stations. Koyanagi et al. (2010) used Voroni and priority order circular diagram to make a blueprint for the installation of charging stations in Musashino City [27]. Ge et al. (2011) divided the study area into several zones with grid partition method and then used genetic algorithm to search the feasible region and determine the location and size of charging station with the aim of minimizing user loss going to the charging station [28].

Another widely use models in locating facilities such as charging stations are flow capturing location models (FCLM) which account for demands in the form of traffic flows. These models which were introduced by Hodgson (1990) and Berman et al. (1992) aim to locate facilities assuming that demands are flows going from preplanned origins to preplanned destinations ([29] and [30]). For these models the demands are captured if the flow passes the facility. Hodgson et al. (1994) compared the efficiency of exact, vertex, and greedy solutions for FCLM problem [31]. Berman et al. (1995) introduce other extensions of this model allowing for deviation in the original path (relaxing the assumption that flows only can traverse the shortest path to get to their destination) [32].

In this original flow capturing location model proposed by Hodgson (1990) there is no benefit in passing by a facility more than once [29]; however in some of the applications such as a model developed by Hodgson and Berman (1997) for locating billboards capturing flows even more than once will be beneficial, so they defined an objective function that considers the number of each additional viewing [33]. Locating inspection stations is another application of models which can capture flows more than once [34].

As currently available EVs have limited ranges compared to conventional vehicles, for rural network and between relatively far distance points they need to be recharged more than once. This feature is the major difference that distinguishes this problem from original FCLM.

Kuby and Lim (2005) introduce flow refueling location models (FRLM), which are different from FCLM in terms of conditions which consider a flow refueled. This model only considers a flow refueled if an adequate number of stations are with proper space along the path of origin-destination (O-D pair) [35]. Upchurch et al. (2009) also work on the capacitated version of flow refueling location model [36]. Equations 2.1 to 2.4 show the formulation developed by Kuby and Lim (2005) [35].

$$Max\ Z = \sum_{q \in Q} f_q y_q \qquad (2.1)$$

$$\sum_{h \in H} b_{qh} v_h \geq y_q \quad for\ all\ q\ \in Q \qquad (2.2)$$

$$a_{hk} x_k \geq v_h\ for\ all\ h \in H | a_{hk} = 1 \qquad (2.3)$$

$$\sum_{k \in K} x_k = p \qquad (2.4)$$

$$x_k, v_h, y_q \in \ \{0,1\}\ for\ all\ k, h, q \qquad (2.5)$$

Where:

Parameters:

$q = index\ of\ O - D\ pairs$

$Q = set\ of\ all\ O - D\ pairs$

$f_q = flow\ volume\ on\ \ O - D\ pair\ q$

$k = candidate\ \ facility\ location$

$K = Set\ of\ all\ potential\ facility\ locations$

$p = Number\ of\ facilities\ to\ be\ located$

$h = index\ of\ combinations\ of\ facilities$

$H = set\ of\ all\ potential\ facility\ combinations$

$a_{hk} = whether\ or\ not\ facility\ k\ is\ in\ combination\ h$

$b_{qh} = whether\ or\ not\ facility\ combination\ h\ can\ refuel\ O - D\ pair\ q$

Variables:

$y_q = whether\ or\ not\ flow\ is\ captured$

$x_k = whether\ or\ not\ open\ facility\ k$

$v_h = whether\ or\ not\ all\ the\ facilities\ in\ combination\ h\ are\ open$

The objective function (2.1) tries to maximize the refueled flow. Constraints (2.2) ensure that $y_q$is equal to 1 only if there exists at least one combination of facilities which all of the facilities in that combination are opened and they are capable of refueling that specific flow. Constraints (2.3) ensure that all of the facilities need to be opened in order that $v_h$is equal to 1 in constraint (2.2). Constraints (2.4) ensure that the number of built facilities is equal to p. Constraints (2.5) ensure variables are binary.

In order to calculate a_{hk} and b_{qh}an algorithm was developed in Kuby and Lim (2005) [35]. This algorithm first generates shortest path for all O-D pairs and a list of all possible combinations h of nodes on the path. After that, facility combinations which cannot refuel a vehicle for the given range on the path are removed. In the last step, algorithm also removes the feasible combinations which are superset of other combinations.

As this algorithm is computationally burdensome due to enumerating all possible combinations, Lim and Kuby (2010) used heuristic algorithms such as greedy-adding, greedy adding with substitution, and genetic algorithm to solve this problem again [37].

Kuby and Lim (2007) also extend FRLM by comparing two methods of adding candidate sites along arcs and including them as candidate locations for charging facilities in addition to the existing nodes [38].

Similarly to Berman et al. (1995), Kim and Kuby (2012) introduce deviation flow refueling location model (DFRLM) ([32] and [39]). This model relaxes the assumption that flows only can be refueled if the stations are located in the shortest path. The formulation used is similar to Kuby and Lim (2005) and needs to generate all feasible combinations of refueling [35].

Kim and Kuby (2013) develop a greedy based heuristic algorithm to solve (DFRLM) [40]. The advantage of this algorithm is that there is no need to generate all feasible combinations of refueling plus it does not need to generate all the deviation paths; however reaching to the optimal or near optimal solution is not guaranteed.

Although many aspects of developing layout for installing charging stations for EVs have been addressed so far, many others still remain such as considering uncertainty in the demand and decision about the combination of chargers; in addition some of the methods used are not efficient in terms of need for preprocessing the input data. The aim of this thesis is to fill some of the gaps mentioned above which exist in the literature.

Chapter 3: Methodology

3.1. *Locating charging station for electric vehicles in rural networks*

Compared to conventional vehicles, the main challenge of EV users while travelling long distances is their limited range. In relatively long trips, we do not need to consider type of chargers since DC fast charging stations and battery sweeping stations seems like the only plausible facilities to recharge EVs. The goal of this section is to develop a model for locating charging stations along the paths that connect origins and destinations (O-D pairs) for example cities with long distance from each other with the objective of maximizing the total flow recharged with the restriction on the number of facilities that can be opened. This is an alternate formulation for the problem described by Kuby and Lim (2005) [35].

Similarly to Kuby and Lim (2005) the assumption here is that if there is no facility at origin the vehicle starts its origin with half of its battery full and if there is no charging stations at its destination, then at least half of its battery must bebe full at the end of trip [35]. These assumptions insures that roundtrip could be repeated again and again without EV running out of fuel. Another assumption here is that the battery consumption is constant and it is proportional to distance traveled by EVs; also nodes are the only candidate locations for facilities. Also this model does not account for the effect of placing charging facilities on the demand.

To emphasize the importance of location decision for charging facilities along the rural network, an example is provided in figure 8 for a vehicle with range of 8. Table 3 shows the optimal location of charging facilities assuming that only O-D pair A-B has flow demand and A-1-2-3-B is the shortest path from A to B. As it is shown, depending on the range of the vehicle and the length of the section, optimal location of facilities changes in different cases. For larger networks with multiple origins and destinations we need to develop an optimization model for placing the stations.

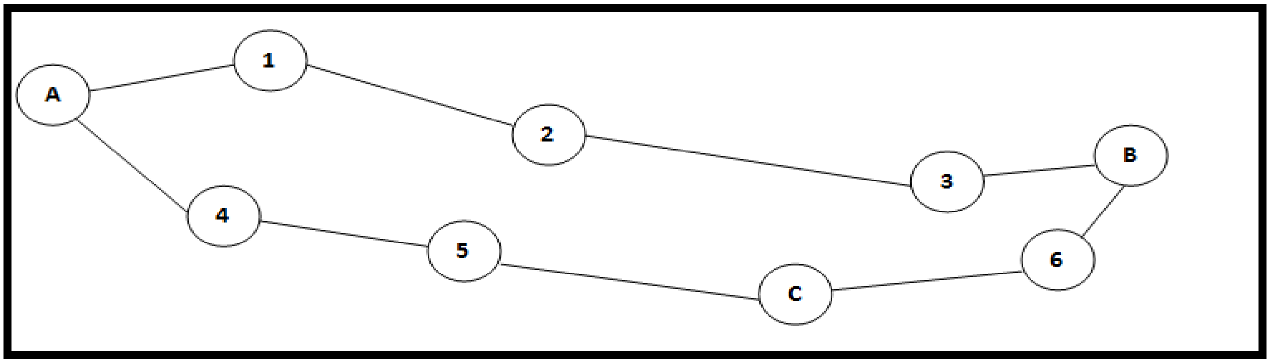


Figure 8 Example of Flow Refueling Location Model

Table 3 Optimal Location of Facilities

Case Number	Length of section A-1	Length of section 1-2	Length of section 2-3	Length of section 3-B	Combination of charging facilities
1	4	6	6	4	1,2,3
2	4	3	4	2	1,3
3	2	2	2	2	2
4	4	9	4	2	Not possible

3.1.1 Locating charging station along the shortest path

The formulation proposed for this the problem is as follows:

$$Max\ Z = zt_q * f_q \qquad (3.1)$$

$$B_{q1} = half\ range \qquad (3.2)$$

$$B_{qi} - l_{qi} + x_{k\in facility_q}(range - B_{qi}) = B_{qi+1} \quad for\ all\ q \in Q \text{ and } \mathrm{i} \in S_q \qquad (3.3)$$

$$B_{qi} \geq M(zt_q - 1) \quad for\ all\ \mathrm{q} \in Q \text{ and } \mathrm{I} \in S_q \qquad (3.4)$$

$$B_{qi} \leq range \qquad (3.5)$$

$$x_k, zt_q \in \{0,1\}\ for\ all\ k\ and\ q \qquad (3.6)$$

$$\sum_{k\in K} x_k = p \qquad (3.7)$$

Where:

Parameters:

$q = Index\ of\ O - D\ pairs$

$Q = Set\ of\ all\ O - D\ pairs$

$f_q = Flow\ volume\ on\ O - D\ pair\ q$

$i = Index\ of\ segments$

$k = Index\ of\ candidate\ facility\ locations$

$K = Set\ of\ all\ candidate\ facility\ locations$

$p = Maximum\ number\ of\ facilities\ to\ be\ located$

$facility_q = Candidate\ facilities\ along\ O - D\ pair\ q$

$S_q = Set\ of\ segments'\ index\ for\ each\ O - D\ pair\ q$

$M = Big\ number = Maximum\ length\ of\ shortest\ path\ among\ O - D\ pairs$

$l_{qi} = Length\ of\ section\ i\ \ for\ O - D\ pair\ q$

$range = Capacity\ of\ the\ EV\ battery\ in\ length\ unit$

Variables:

$zt_q = Binary\ variable\ indicating\ whether\ or\ not\ flow\ is\ captured for\ O - D\ pair\ q$

$x_k = Binary\ variable\ indicating\ whether\ or\ not\ to\ open\ facility\ k$

$B_{qi} = Remaining\ amount\ of\ battery\ at\ the\ beginning\ of\ section\ i\ for\ O - D\ pair\ q\ in\ length\ unit$

The objective function (3.1) is to maximize the flow that can be recharged with the opened facilities. Constraints (3.2) indicate that if there isn't any charging stations at origin EVs will start their path with half of their battery full. As it is shown in figure 9, each path from each origin to each destination is divided into segments where each segment connects one candidate facility to another one.

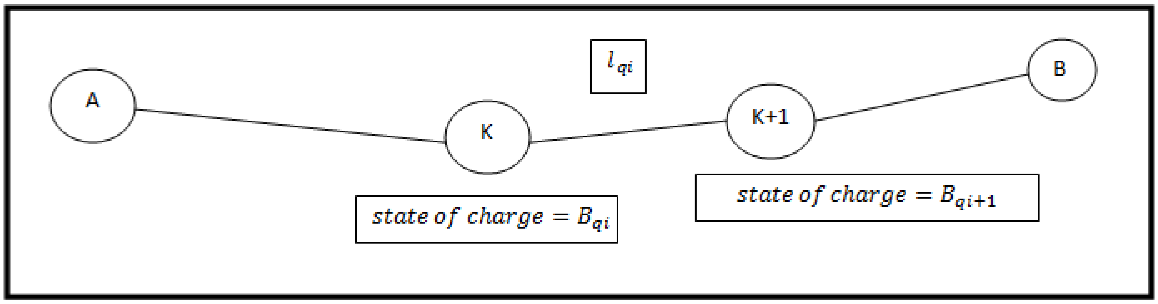


Figure 9 Formulation Description

For example in figure 9, k-k+1 is one of the segments in a path that connect origin A to destination B. B_{qi} is the initial stage of battery at the beginning of segment i and based on the assumption is half of the range of the vehicle for the first segment of all the O-D pairs. Constraints (3.3) show for each flow (q) and at each segment ($i \in S_q$) the initial stage of battery (B_{qi}) plus the amount of battery recharged in the station $x_k * (range - B_{qi})$ minus the length of trip (l_{qi}) is equal to the initial stage of battery in the next segment (B_{qi+1}). Constraints (3.4) indicate if B_{qi} is positive along the path for all segments existing in the path and stage of charge is at least equal to half of the battery size at the destination then the flow can be recharged along the whole path without running out of battery. Parameter M in constraints (3.4) can be any number larger than the longest path among the overall shortest paths. Constraints (3.5) show the maximum capacity of battery and constraints (3.6) make sure that the solutions are integral. Constraints (3.7) enforce the maximum number of opened facilities to be p. Term $x_{k \in facility_q} * B_{qi}$ in the constraints (3.3) which is multiplication of a binary variable and a continuous variable makes constraint (3.3) nonlinear. To linearize constraint (2) a new variable z_{qi} which is equal to multiplication of x_k and B_{qi} is introduced in the model and constraints (3.8) and (3.9) are added to the formulation. It should be noted that M should be a number equal or less than minimum of B_{qi} overall $q \in Q$ and $\mathrm{i} \in S_q$. Here, the longest path among shortest paths could be used to make sure this criterion is satisfied.

$$\mathrm{range} * x_k \geq z_{qi} \geq -Mx_k \qquad (3.8)$$

$$B_{qi} - (1 - x_k) * range \leq z_{qi} \leq B_{qi} - (1 - x_k) * -M \qquad (3.9)$$

3.1.2 Locating charging stations along the path allowing for deviation

The model described in the previous section assumes that people will use the shortest path going from their origin to their destination; however this is not probably the case for EV users as they are willing to deviate from the shortest path in order to be able to recharge their battery. For example in figure 8, we want to recharge demands going from A to B and A to C with maximum number of facility equal to 3 with the assumption that length of sections is equal to 4, 6, 6, 4, 4,7, 6, 2, and 2 for sections A-1, 1-2, 2-3,3-B,A-4,4-5,5-C,C-6,and 6-7. The shortest path going from A to B is A-1-2-3-B and the only path from A to C is A-4-5-C. There is no combination of facilities that can recharge both flows along their shortest path with only opening 3 facilities; however if demands choose to go to B from the path A-4-5-6-C-7-B then we can recharge both flows with opening just 3 facilities. The first model is adjusted and an index n is added to account for the paths other than the shortest path. This means that flows should not necessary traverse the shortest path and they can use a path that has deviation from the shortest path. Generating all the paths that connect O-D pairs is not practical and also not very realistic since people are not going to take routes which deviate significantly from the shortest path (they either are going to use an alternative vehicle, mode or cancel their trips); hence here the maximum allowable deviation is limited to a number. The formulation is revised as follows:

$$Maximize\ Z = zt_{qn} * f_q \quad (3.10)$$

$$B_{q1n} = half\ range \quad (3.11)$$

$$B_{qin} - l_{qin} + x_{k\in S_{qn}}\left(range - B_{qin}\right) = B_{qi+1n} \quad (3.12)$$

$$B_{qin} \geq M\left(zt_{qn} - 1\right) \qquad for\ all\ q\ and\ n \quad (3.13)$$

$$B_{qin} \leq range \quad (3.14)$$

$$\sum_{n} zt_{qn} \leq 1 \quad (3.15)$$

$$zt_{qn}, x_k \in \{0,1\}\ for\ all\ k, n\ and\ q \quad (3.16)$$

Where added parameters and variables are:

Parameters:

$l_{qin} = length\ of\ section\ i\ of\ path\ n\ for\ O - D\ pair\ q$

$n = index\ of\ \ paths\ for\ each\ O - D\ pair\ q$

Variables:

$zt_{qn} = Binary\ variable\ indicating\ whether\ or\ not\ flow\ is\ captured\ \ by\ nth$
$shortest\ path\ for\ O-D\ \ pair\ q$

$B_{qin} = Remaining\ amount\ of\ \ battery\ at\ the\ beginning\ of\ section\ i\ for\ nth$
$shortest\ path\ for\ O-D\ \ pair\ q\ in\ length\ unit$

All the constraints are similar to previous model except that constraint (3.15) is added to make sure flows are not captured more than once.

3.1.3. Maximizing Vehicle-Miles Traveled

The introduced objective function (maximizing the flow demand) is more appropriate for incentivizing people to buy EVs with the assumption that people are willing to buy EVs if they can complete more trips [36]. However in some cases the goal is to reduce emission or consumed gasoline [36]. If this is the case it is better to maximize vehicle-miles traveled instead of maximizing the flow recharged [36]. The following objective function (3.17) can be used for maximizing the vehicle-miles traveled.

$$Maximize\ z = zt_q * l_{sp} * f_q \quad (3.17)$$

3.1.4. Decay Function

To simulate the effect of deviation from the shortest path on the decision of people to whether or not make the trip, a decay function (3.18) similar to [39] is multiplied to the original objective function. Here we assume change in the flow is a linear function of deviation.

$$Decay\ function = f(Deviation) = \left(1 - \frac{Deviation}{\beta * l_{sp}}\right) \qquad (3.18)$$

Figure 10 shows change in the decay function value with respect to beta and the ratio between the length of deviation and shortest path.

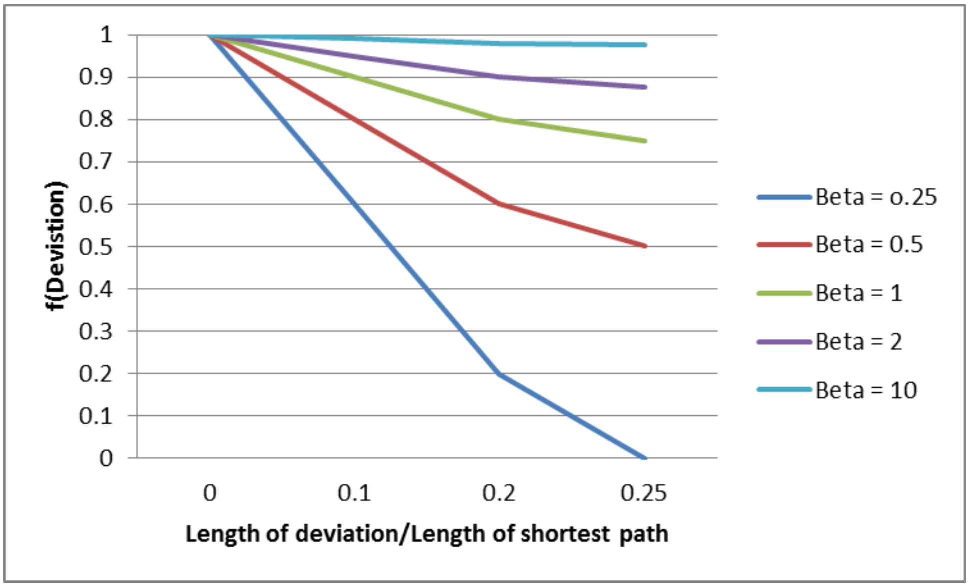


Figure 10 Change in Decay Function with Respect to Length of Deviation

3.1.5. Improving the solution time

There is no explicit way to describe convex hull for the NP-hard problems; however by using valid inequalities we can improve approximations of convex hull. The effect of two valid inequalities can be tested for this formulation.

1) Demands cannot be recharged along the paths with links that are longer than the range of vehicles.
2) The number of facilities in each path should be at least equal to the ceiling value of length of the path divided by the range in order to recharge that particular path

3.2 Locating charging stations for electric vehicles in urban networks

In the model for the urban networks, we want to place charger types 2 and 3 in parking lots and parking spaces located in downtown of city. Due to uncertainty in demand forecasting explained in the introduction, we want to place these chargers at multiple stages (here two stages are used). Chargers need to cover at least a certain percentage of demand for each of the lots and type of chargers. We have capacity restriction on total number of chargers as well as each type of charger that could be placed in a parking lot (The restriction on total number of chargers is based on the size of each parking lot and it is different for each parking lot. The restriction for each type of charger is based on the negative effect of each type of charger on the electrical grid and it has the same value for each parking but different value for each type of charger). We assume that the daily demand for each type of charger at each parking lot follows a Poisson distribution and the mean value does not change during each year. Also demands in one parking lot can get service from nearby charging facilities in other parking lots. It should be noted that if demand cannot be satisfied in its original destined parking lot a penalty will be included in the objective function. Finally, our objective is to minimize the penalty caused by getting service from other lots or in other words the deviation cost given that we have a fixed budget for placing charging stations.

To account for uncertainty in the future demand we assume that at each stage of planning we have three possible scenarios for demand of each parking facility and type of charger. For each of the facilities the scenarios are dependent, meaning that all of facilities will have either, low, medium, or high demand scenario at each stage. (This assumption is logical; however it’s not always true.) The decision tree is shown in figure11.

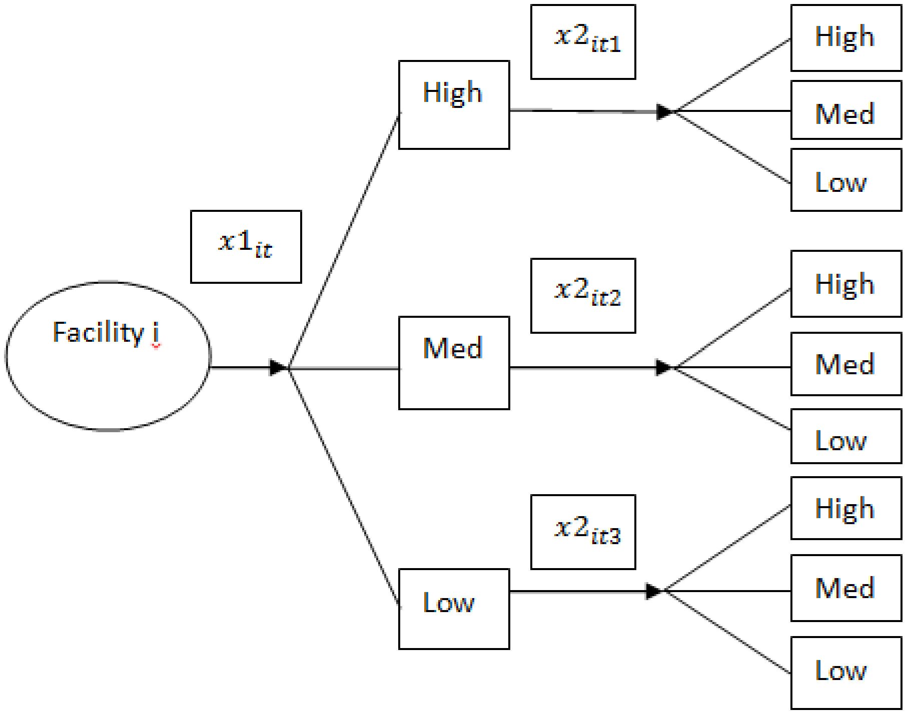


Figure 11 Decision Tree

Based on the problem description in the previous section, a formulation was developed for this problem as follow:

Objective function

Minimize

$$\begin{aligned}&\textstyle\sum_i \sum_t \sum_s (P1(s) * (1 - z1_{it}) * Mean\ demand_{it1s} * Penalty_t * Avg\ distance_{ti}\\&+ \textstyle\sum_i \sum_t \sum_{s'=1,2,3} (P2(s') * (1 - z2_{it1}) * Mean\ demand_{it2s'} * Penalty_t * Avg\ distance_{ti}\\&+ \textstyle\sum_i \sum_t \sum_{s'=4,5,6} (P2(s') * (1 - z2_{it2}) * Mean\ demand_{it2s'} * Penalty_t * Avg\ distance_{ti}\\&+ \textstyle\sum_i \sum_t \sum_{s'=7,8,9} (P2(s') * (1 - z2_{it3}) * Mean\ demand_{it2s'} * Penalty_t * Avg\ distance_{ti} \quad (3.19)\end{aligned}$$

Demand coverage

$$Probability\left(\sum_{i=cov_i} x1_{it} \geq \sum_{i=cov_i} D1_{its}\right) \geq \alpha \qquad for\ all\ i,t,s \quad (3.20)$$

$$Probability\left(\sum_{i=cov_i} x2_{it1} \geq \sum_{i=cov_i} D2_{its'}\right) \geq \alpha \qquad for\ all\ i,t,s' = 1,2,3 \quad (3.21)$$

$$Probability\left(\sum_{i=cov_i} x2_{it2} \geq \sum_{i=cov_i} D2_{its'}\right) \geq \alpha \qquad for\ all\ i,t,s' = 4,5,6 \quad (3.22)$$

$$Probability\left(\sum_{i=cov_i} x2_{it3} \geq \sum_{i=cov_i} D2_{its'}\right) \geq \alpha \qquad for\ all\ i,t,s' = 7,8,9 \quad (3.23)$$

Budget constraints

$Year\ 1\ cost = \sum_i \sum_t (z1_{it} * C_{instalation\ cost\ level\ t} + x1_{it} * C_t)$ (3.24)

$Year\ 2\ cost =$

$\sum_i \sum_t \sum_{s'} (P1(s) * (z2_{its} - z1_{it}) * C_{instalation\ cost\ level\ t} + P1(S) * (x2_{its} - x1_{it}) * C_t$ (3.25)

$Year1\ cost + Year\ 2\ cost/(1 + IRR) \leq Budget$ (3.26)

Capacity of each parking lot

$$(x1_{i2} + x1_{i3}) \leq Capacity_i \qquad \forall i \quad (3.27)$$

$$(x2_{i2s} + x2_{i3s}) \leq Capacity_i \qquad \forall i,s \quad (3.28)$$

$$x1_{i2}, x2_{i2s} \leq Capacity_{charger2} \forall\ i,s \quad (3.29)$$

$$x2_{i3}, x2_{i3s} \leq Capacity_{charger3} \quad \forall i,s \quad (3.30)$$

Other constraints

$$z2_{its} \geq z1_{it} \quad \forall i,s,t \quad (3.31)$$

$$x2_{its} \geq x1_{it} \quad \forall i,s,t \quad (3.32)$$

$$Capacity_i * z1_{it} \geq x1_{it} \qquad \forall i,t \quad (3.33)$$

$$Capacity_i * z2_{its} \geq x2_{its} \qquad \forall i,s,t \quad (3.34)$$

$$Mean\ \text{demand}_{it} * z1_{it} \leq x1_{it}\ \forall i,t \qquad (3.35)$$

$$Mean\ \text{demand}_{it} * z2_{its} \leq x2_{its}\ \forall i,s,t \qquad (3.36)$$

$$x1_{i2}, x1_{i3}, x2_{i2s}, x2_{i3s} \geq 0 \text{ and integer } \forall s \qquad (3.37)$$

$$z1_{i2}, z1_{i3}, z2_{i2s}, z2_{i3s} \in \{0,1\} \quad \forall s \qquad (3.38)$$

Parameters:

n = Candidate locations (Parking lots)

$Capacity_i = \ Capacity\ of\ parking\ lot\ i$

$Capacity_{charger2}$
$= Maximum\ number\ of\ charger\ type\ 2\ that\ can\ be\ installed\ in\ a\ facility$

$Capacity_{charger3}$
$= Maximum\ number\ of\ charger\ type\ 3\ that\ can\ be\ installed\ in\ a\ facility$

$C_t = \ Cost\ of\ charger\ type\ t$

$C_{installation\ cost\ for\ charger\ type\ t} = \ Installation\ cost\ for\ charger\ type\ t$

$Coverage_i = Set\ of\ parking\ lots\ that\ can\ cover\ demand\ \ in\ parking\ lot\ i$

$Penalty_t = Penalty\ for\ not\ installing\ charging\ facilities\ of\ type\ t$

$Budget = Total\ budget\ for\ the\ two\ stages$

$Avg\ distance\ _{ti}$
$= Average\ distance\ of\ parking\ lot\ i\ from\ other\ parking\ lots\ that\ can\ cover\ charger\ type\ t$

$Mean\ demand_{it1s}$
$= The\ mean\ demand\ for\ charger\ type\ t\ in\ parking\ lot\ i\ in\ year\ 1\ in\ scenario\ s$

$Mean\ demand_{it2s'}$
$= The\ mean\ demand\ for\ charger\ type\ t\ in\ parking\ lot\ i\ in\ year\ 2\ in\ scenario\ s'$

$D1_{its}$
$= Distribution\ of\ demand\ for\ charger\ type\ t\ in\ facility\ i\ in\ year\ 1\ in\ scenario\ s$

$D2_{its'}$
$= Distribution\ of\ demand\ for\ charger\ type\ t\ in\ facility\ i\ in\ year\ 2\ in\ scenario\ s'$

$IRR = Internal\ rate\ of\ return$

$P1(s) = probability\ of\ scenario\ s\ in\ year\ 1$

$P2(s') = probability\ of\ scenario\ s'\ in\ year\ 2$

$Scenario\ in\ year\ 1 = s = 1,2,3$

$Scenario\ in\ year\ 2 = s' = 1,2,3,4,5,6,7,8,9$

Variables:

$x1_{i2} = Number\ of\ charger\ \ type\ 2\ to\ install\ in\ facility\ i\ in\ year\ 1\ and\ integer$

$x1_{i3} = Number\ of\ charger\ type\ 3\ to\ install\ in\ facility\ i\ in\ year\ 1\ and\ integer$

$x2_{i2s} = Number\ of\ charger\ type\ 2\ to\ install\ in\ facility\ i\ in\ year\ 2\ in\ scenario\ s\ and\ inte$

$x2_{i3s} = Number\ of\ charger\ type\ 3\ to\ install\ in\ facility\ i\ in\ year\ 2\ in\ scenario\ s\ and\ integer$

$z1_{i2} = Whether\ or\ not\ install\ charger\ type\ 2\ in\ facility\ i\ in\ year\ 1\ and\ binary$

$z1_{i3} = Whether\ or\ not\ install\ charger\ type\ 3\ in\ facility\ i\ in\ year\ 1\ and\ binary$

$z2_{i2s} = Whether\ or\ not\ install\ charger\ type\ 2\ in\ facility\ i\ in\ year\ 2\ in\ scenario\ s\ and\ binary$

$z2_{i3s} = Whether\ or\ not\ install\ charger\ type\ 3\ in\ facility\ i\ in\ year\ 2\ in\ scenario\ s\ and\ binary$

$Year1\ cost = Cost\ of\ installation\ for\ year\ 1 = Budget\ soend\ in\ year\ 1$

$Year2\ cost = Cost\ of\ installation\ for\ year\ 2 = Budget\ spend\ in\ year\ 2$

As explained before, the objective function tries to minimize the deviation of demands from their original destination for using the chargers. Constraints (3.20) to (3.23) ensure that we can cover α percentage of demand for chargers type 2 & 3 at each parking lot at each stage of planning (probability that numbers of chargers are more than the demand for them is at least α percent). Assuming that demand at each parking facility has Poisson distribution and the demands are independent. We can linearize these constraints by writing the cumulative distribution and writing the inverse of the cumulative distribution. Constraint (3.26) is the budget constraint. Constraints (3.27) and (3.28) are for the capacity of each charging facility and constraints (3.29) and (3.30) are max number of each type of charger that can be installed in each facility. Constraints (3.31) and (3.32) indicate that we cannot remove the chargers that we already installed at the first stage. Constraints (3.33) and (3.34) ensure that the installation cost is included in the total cost. Constraints (3.35) and (3.36) ensure that if we put charging facilities in a parking lot, at least mean demand for that facility is satisfied (It also prevents the solver from generating illogical solutions). Constraints (3.37) and (3.38) ensure that the solver gives us integral solutions.

Chapter 4: Case Study

4.1 *Locating charging stations for electric vehicles in rural networks*

The proposed FRLM and DRFLM formulations are tested on a network shown in figure 12 which has been used in Berman and Simchi-Levi (1988), Hodgson (1990), and Kuby and Lim (2005 and 2007), Kim and Kuby (2012 and 2013) ([41],[29],[35],[38],[39], and [40]). The network has 25 nodes and 43 edges and the total number of O-D flows is 300. The flow demands between O-D pairs are randomly chosen from a number between 50 and 500, and the total flow demand is 79980. Given that the links' lengths are between 2 to 9, the model is tested assuming 3 different ranges 4, 8, and 12 for vehicles. All the models are coded and solved in Xpress IVE by the branch and bound method.

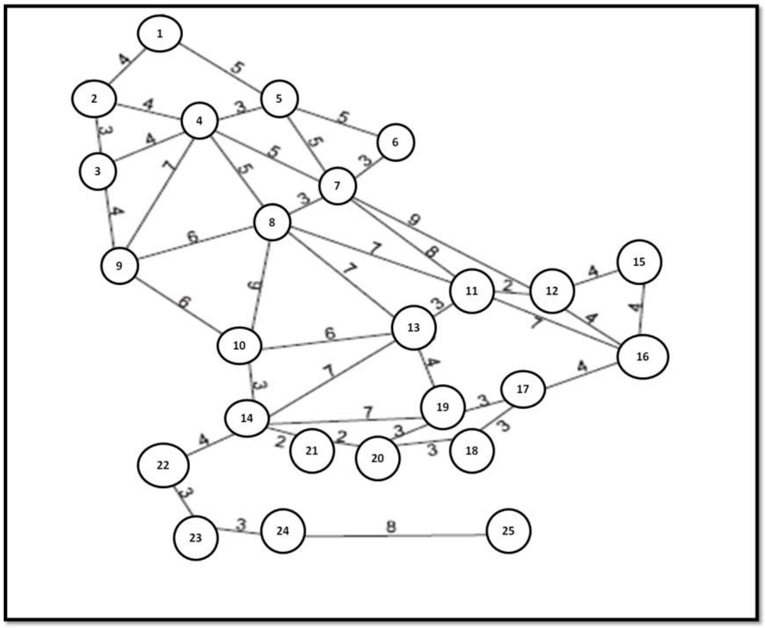


Figure 12 Rural Network

4.1.1 Locating charging station along the shortest path

First, the model is solved assuming the flows only use shortest path to go to their destinations. It should be noted that for some of the origin destination paths multiple shortest paths exist and if there are enough chargers along any of them then the flow is recharged.

Percent of flow recharged and running time are presented in table 4 for multiple shortest paths. As it is shown in table 4 and discussed before in Kuby and Lim (2005), the solution time increases by the increase in range [35]. This is intuitive since increase in range results in increase in the size of feasible region therefore the enumeration by branch and bound method increases and as a result the solution time increases.

Another observation is that the solution time for each of the instances has a trend of increasing and then decreasing. The increase is due to the change in the feasible region as a result of increasing the maximum number of facilities that can be opened and the decrease that happens after that is due to decrease in gap between the first integral solution and the upper bound.

With the assumption that electric vehicles have range of 12, opening 16 facilities is enough for recharging the total flow demand. However in the instances that the range of vehicles is 4 or 8 even opening all of the facilities cannot recharge all the flows. This is because some of the links in the network have lengths longer than 4 and 8. This emphasizes the importance of considering other candidate locations and not only conjunction nodes along the links similar to Kuby and Lim’s work (2007) [38].

Tables 5 and 6 show the sequence of opening facilities for electric vehicles with the range equal to 4. For instance, if the maximum number of facility that can be opened is 3, opening the candidate facilities in node 17, 18, and 20 will give us the optimal solution which means this combination can recharge maximum flow demand. The result shows that greedy algorithm will not give us an optimal solution since the optimal place of facilities change as the maximum number of facilities that can be opened increases. For example for p equal to 1 placing charging facility in node 21 will give us the optimal solution; however if we increase p to 2, node 21 is not in the optimal solution anymore.

Tables 7 and 8 show the optimal sequence of opening facilities for vehicles with the range of 8. Tables 9 and 10 show this number for vehicles with the range of 12. Comparing tables 5, 7, and 9 together and tables 6, 8, and 10 together it can be concluded the optimal place of charging facilities is highly dependent on the assumption about the range of vehicles.

Table 4 Percent of Flows Recharged and Solution Time for Multiple Shortest Paths

Maximum Number of Facilities						
	P=1	**P=2**	**P=3**	**P=4**	**P=5**	**P=6**
Percent of Flows Recharged						
Range =4	0.63	1.14	2.08	3.93	6.01	8.32
Range =8	2.64	7.27	12.67	18.78	25.19	32.43
Range =12	4.96	12.63	21.74	32.51	43.53	53.13
Solution Time (S)						
Range =4	0.2	0.5	1.7	3.9	3.0	3.7
Range =8	0.4	4.7	19.0	14.2	19.6	17.4
Range =12	0.5	41	97.1	61.6	161.0	23.9
Maximum Number of Facilities						
	P=7	**P=8**	**P=9**	**P=10**	**P=11**	**P=12**
Percent of Flows Recharged						
Range =4	11.03	13.68	16.60	18.99	21.31	23.49
Range =8	37.97	47.34	55.73	63.43	70.47	77.95
Range =12	62.57	70.41	74.96	82.08	88.36	93.05
Solution Time (S)						
Range =4	3.7	3.3	2.8	3.6	2.9	2
Range =8	23.5	18.9	10.2	7.6	6.9	6.6
Range =12	14.9	8.4	30.9	8.1	3.1	2.2
Maximum Number of Facilities						
	P=13	**P=14**	**P=15**	**P=16**	**P=17**	**P=18**
Percent of Flows Recharged						
Range =4	25.33	27.51	27.51	28.03	28.91	29.99
Range =8	84.90	86.12	92.87	94.08	94.72	95.94
Range =12	95.81	98.26	99.48	100	100	100
Solution Time (S)						
Range =4	2.3	0.3	0.6	0.9	0.7	0.6
Range =8	0.8	4	0.9	1.0	1.1	0.4
Range =12	2.0	0.8	0.5	0.3	-	-
Maximum Number of Facilities						
	P=19	**P=20**	**P=21**	**P=22**	**P=23**	**P=24**
Percent of Flows Recharged						
Range =4	31.0	31.58	31.58	32.02	32.60	32.60
Range =8	96.51	96.51	96.51	96.51	96.51	96.51
Range =12	100	100	100	100	100	100
Solution Time (S)						
Range =4	0.2	0.2	0.3	0.4	0.1	-
Range =8	0.1	-	-	-	-	-
Range =12	-	-	-	-	-	-

Table 5 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=4 for 1-12 Maximum Facilities

Opened Facilities	P=1	P=2	P=3	P=4	P=5	P=6	P=7	P=8	P=9	P=10	P=11	P=12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	1
11	0	0	0	1	0	0	0	0	0	0	1	1
12	0	0	0	0	1	0	0	0	0	1	0	0
13	0	0	0	1	0	0	0	0	0	0	1	1
14	0	0	0	0	0	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	1	1	1	1	1	1	1	1
17	0	0	1	0	1	1	1	1	1	1	1	1
18	0	1	1	0	1	1	1	1	1	1	1	1
19	0	0	0	1	0	0	1	1	1	1	1	1
20	0	1	1	1	1	1	0	1	1	1	1	1
21	1	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	1	1	1	1	1	1	1
23	0	0	0	0	0	0	1	1	1	1	1	1
24	0	0	0	0	0	0	0	0	1	1	1	1
25	0	0	0	0	0	0	0	0	0	0	0	0

Table 6 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=4 for 13-25 Maximum Facilities

Opened Facilities	P=13	P=14	P=15	P=16	P=17	P=18	P=19	P=20	P=21	P=22	P=23	P=24
1	0	0	0	0	0	1	1	1	1	1	1	-
2	0	0	1	0	1	1	1	1	1	1	1	-
3	0	0	0	1	1	1	1	1	1	1	1	-
4	0	0	0	1	1	1	1	1	1	1	1	-
5	0	0	0	0	0	0	1	1	1	1	1	-
6	0	0	0	0	0	0	0	0	1	1	1	-
7	0	0	0	0	0	0	0	0	0	1	1	-
8	0	0	0	0	0	0	0	0	0	1	1	-
9	0	0	0	0	0	0	0	1	1	0	1	-
10	0	1	1	1	1	1	1	1	1	1	1	-
11	1	1	1	1	1	1	1	1	1	1	1	-
12	1	1	1	1	1	1	1	1	1	1	1	-
13	1	1	1	1	1	1	1	1	1	1	1	-
14	1	1	1	1	1	1	1	1	1	1	1	-
15	1	1	1	1	1	1	1	1	1	1	1	-
16	1	1	1	1	1	1	1	1	1	1	1	-
17	1	1	1	1	1	1	1	1	1	1	1	-
18	1	1	1	1	1	1	1	1	1	1	1	-
19	1	1	1	1	1	1	1	1	1	1	1	-
20	1	1	1	1	1	1	1	1	1	1	1	-
21	0	0	0	0	0	0	0	0	0	0	0	-
22	1	1	1	1	1	1	1	1	1	1	1	-
23	1	1	1	1	1	1	1	1	1	1	1	-
24	1	1	1	1	1	1	1	1	1	1	1	-
25	0	0	0	0	0	0	0	0	0	0	0	-

Table 7 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=8 for 1-12 Maximum Facilities

Opened Facilities	P=1	P=2	P=3	P=4	P=5	P=6	P=7	P=8	P=9	P=10	P=11	P=12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	1	1	1	1	1
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	1	1	1
8	0	0	0	0	0	0	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	0	0	1	1
10	0	0	0	0	0	0	1	1	1	1	1	1
11	0	0	0	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	1	0	0	1	1	1	1
13	0	0	0	0	1	1	1	1	1	1	1	1
14	1	0	0	1	1	1	0	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	0	1	1	1	1	1	1	1	1	1	1	1
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0
20	0	1	1	1	1	1	1	1	1	1	1	1
21	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	1	0	0	0	1	0	0	0	0	0
23	0	0	0	1	1	1	0	1	1	1	1	1
24	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0

Table 8 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=8 for 13-25 Maximum Facilities

Opened Facilities	P=13	P=14	P=15	P=16	P=17	P=18	P=19	P=20	P=21	P=22	P=23	P=24
1	0	0	0	0	1	1	1	-	-	-	-	-
2	1	1	1	1	1	1	1	-	-	-	-	-
3	0	0	0	0	0	0	0	-	-	-	-	-
4	1	1	1	1	1	1	1	-	-	-	-	-
5	0	0	0	0	1	1	1	-	-	-	-	-
6	0	0	0	0	0	0	1	-	-	-	-	-
7	1	1	1	1	1	1	1	-	-	-	-	-
8	1	1	1	1	1	1	1	-	-	-	-	-
9	1	1	1	1	1	1	1	-	-	-	-	-
10	1	1	1	1	1	1	1	-	-	-	-	-
11	1	1	1	1	1	1	1	-	-	-	-	-
12	1	0	1	1	1	0	0	-	-	-	-	-
13	1	1	1	1	1	1	1	-	-	-	-	-
14	1	1	1	1	1	1	1	-	-	-	-	-
15	0	1	0	0	0	1	1	-	-	-	-	-
16	0	1	0	1	0	1	1	-	-	-	-	-
17	1	1	1	1	1	1	1	-	-	-	-	-
18	0	0	0	0	0	0	0	-	-	-	-	-
19	0	0	0	0	0	0	0	-	-	-	-	-
20	1	1	1	1	1	1	1	-	-	-	-	-
21	0	0	0	0	0	0	0	-	-	-	-	-
22	0	0	1	0	1	1	0	-	-	-	-	-
23	1	1	0	1	0	0	1	-	-	-	-	-
24	0	0	1	1	1	1	1	-	-	-	-	-
25	0	0	1	1	1	1	1	-	-	-	-	-

Table 9 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=12 for 1-12 Maximum Facilities

Opened Facilities	P=1	P=2	P=3	P=4	P=5	P=6	P=7	P=8	P=9	P=10	P=11	P=12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	1	1	1
3	0	0	0	0	0	0	0	0	1	1	0	0
4	1	0	0	0	0	1	1	1	1	0	1	1
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	1	0	1	1	1	1	0	1
8	0	1	1	1	0	1	0	0	0	1	1	1
9	0	0	0	0	0	0	0	0	0	0	1	1
10	0	0	0	0	1	0	1	1	1	1	0	0
11	0	0	1	1	0	1	0	0	0	0	1	0
12	0	0	0	0	0	0	1	1	1	1	0	1
13	0	0	0	0	1	0	1	1	1	1	1	1
14	0	0	1	1	0	1	0	0	0	0	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	1	1	1	1	1
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	1	0	1	0	0	0	0	0	0
20	0	0	0	0	1	0	1	1	1	1	1	1
21	0	1	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	1	0	1	1	1	1	0	0
23	0	0	0	0	0	1	0	0	0	0	1	1
24	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	1	1

Table 10 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=12 for 13-25 Maximum Facilities

Opened Facilities	P=13	P=14	P=15	P=16	P=17	P=18	P=19	P=20	P=21	P=22	P=23	P=24
1	0	0	0	0	-	-	-	-	-	-	-	-
2	1	1	1	1	-	-	-	-	-	-	-	-
3	0	0	0	0	-	-	-	-	-	-	-	-
4	1	1	1	1	-	-	-	-	-	-	-	-
5	0	1	1	1	-	-	-	-	-	-	-	-
6	0	0	0	0	-	-	-	-	-	-	-	-
7	1	1	1	1	-	-	-	-	-	-	-	-
8	1	1	1	1	-	-	-	-	-	-	-	-
9	1	1	1	1	-	-	-	-	-	-	-	-
10	0	0	0	0	-	-	-	-	-	-	-	-
11	1	1	1	1	-	-	-	-	-	-	-	-
12	1	1	1	1	-	-	-	-	-	-	-	-
13	1	1	1	1	-	-	-	-	-	-	-	-
14	1	1	1	1	-	-	-	-	-	-	-	-
15	0	0	1	1	-	-	-	-	-	-	-	-
16	0	0	0	0	-	-	-	-	-	-	-	-
17	1	1	1	1	-	-	-	-	-	-	-	-
18	0	0	0	0	-	-	-	-	-	-	-	-
19	0	0	0	0	-	-	-	-	-	-	-	-
20	1	1	1	1	-	-	-	-	-	-	-	-
21	0	0	0	0	-	-	-	-	-	-	-	-
22	0	0	0	0	-	-	-	-	-	-	-	-
23	1	1	1	1	-	-	-	-	-	-	-	-
24	0	0	0	1	-	-	-	-	-	-	-	-
25	1	1	1	1	-	-	-	-	-	-	-	-

4.1.2 Locating charging stations allowing for deviation

As discussed in chapter 3, accounting for paths other than the shortest path is a reasonable assumption when we want to recharge flows. In this section, the optimal location of charging facilities with this assumption is presented.

For the generation of kth shortest paths Yen's algorithm which is a loopless algorithm for the kth shortest path is used [42]. The output of this algorithm is used as input for the proposed optimization model. The kth shortest path algorithm has the ability to give all the existing paths from an origin to a destination; however putting a maximum deviation for generating the paths is more realistic and more consistent to the travelers' behavior. Travelers probably are not going to use paths that make their trips significantly longer. Here, the model is tested for two maximum deviations of 10% and 25%. Tables 11 and 12 show the solution times and percent recharged for the maximum allowable deviation of 10% and 25 % respectively.

Comparing tables 4, 11, and 12 shows that increasing the maximum deviation path has significant effect on solution times which is due to increase in number of feasible paths which connect O-D pairs.

The results indicate that although for the allowable maximum deviation of 10 percent and vehicle range of 4 and 8 all the flows cannot be recharged, the percent of rechargedflows increases from 32.6 to 34.2 for range of 4 and from 96.5 to 99.5 for range of 8. In terms of percent of flow recharged for maximum allowable deviation of 25 percent unlike the case in which only shortest path were accounted, we can see that allowing for maximum deviation of 25 percent can recharge all the O-D pairs flows for instance with range equal to 8. For range equal to 4 the percent of flows recharged increases from 32.6 to 38.31. The maximum gap between the case with deviation and without deviation is 16 percent which indicates the importance of considering deviation from the shortest path in the formulation. However, the minimum number of facilities needed for recharging all the flows for vehicles with range of 12 remains 16 and considering deviation does not have any effect on reducing the total number of facilities for recharging all the flows for this particular network.

Table 11 Percent of Flows Recharged and Solution Time for 10 Percent Maximum Deviation

Maximum Number of Facilities						
	P=1	**P=2**	**P=3**	**P=4**	**P=5**	**P=6**
Percent of Flows Recharged						
Range =4	0.63	1.14	2.08	3.93	6.01	8.32
Range =8	2.64	7.27	12.67	18.78	25.19	32.43
Range =12	4.96	13.34	22.80	34.40	46.05	57.12
Solution Time (S)						
Range =4	0.2	0.7	5.2	6.3	9.6	6.8
Range =8	3.1	7.1	14.5	18.6	22.2	25.8
Range =12	2.8	122.9	677.1	17.5	21.1	18.8
Maximum Number of Facilities						
	P=7	**P=8**	**P=9**	**P=10**	**P=11**	**P=12**
Percent of Flows Recharged						
Range =4	11.03	13.68	16.60	18.99	21.78	23.96
Range =8	38.83	48.20	56.58	64.28	72.23	80.34
Range =12	65.06	71.27	77.59	83.38	90.02	93.95
Solution Time (S)						
Range =4	6.8	6.3	5.6	3.8	3.3	4.2
Range =8	22.8	20.2	25.4	21.6	10.5	3.4
Range =12	19.0	21.4	13.4	11.9	4.4	2.1
Maximum Number of Facilities						
	P=13	**P=14**	**P=15**	**P=16**	**P=17**	**P=18**
Percent of Flows Recharged						
Range =4	26.39	28.57	28.57	29.09	29.97	31.05
Range =8	86.44	87.92	94.41	95.88	97.77	98.98
Range =12	96.41	98.66	99.48	100	100	100
Solution Time (S)						
Range =4	3.6	0.4	1.4	1.4	1.1	1.2
Range =8	1.3	4.5	1.0	1.3	1.4	0.4
Range =12	1.5	0.9	0.6	0.2	-	-
Maximum Number of Facilities						
	P=19	**P=20**	**P=21**	**P=22**	**P=23**	**P=24**
Percent of Flows Recharged						
Range =4	32.06	33.22	33.22	33.60	34.24	34.24
Range =8	99.55	99.55	99.55	99.55	99.55	99.55
Range =12	100	100	100	100	100	100
Solution Time (S)						
Range =4	0.8	0.3	0.6	0.6	0.3	-
Range =8	0.2	-	-	-	-	-
Range =12	-	-	-	-	-	-

Table 12 Percent of Flows Recharged and Solution Time for 25 Percent Maximum Deviation

Maximum Number of Facilities						
	P=1	**P=2**	**P=3**	**P=4**	**P=5**	**P=6**
Percent of Flows Recharged						
Range =4	0.63	1.14	2.08	3.93	6.36	8.32
Range =8	2.64	7.27	12.90	20.22	27.07	35.28
Range =12	4.96	13.87	26.79	40.94	56.46	69.30
Solution Time (S)						
Range =4	4.6	25.9	35.7	40.8	36	47.5
Range =8	5.8	34.9	74.3	121.4	229.2	316
Range =12	41.1	101.6	125.7	200.1	236.8	200.9
Maximum Number of Facilities						
	P=7	**P=8**	**P=9**	**P=10**	**P=11**	**P=12**
Percent of Flows Recharged						
Range =4	11.07	13.78	16.73	20.59	24.56	27.12
Range =8	46.89	56.24	63.94	71.34	78.61	83.91
Range =12	76.86	82.63	87.62	90.93	95.31	96.88
Solution Time (S)						
Range =4	42.2	46.5	54.5	39.8	36.7	36.3
Range =8	330.3	346.9	375.5	244	274.7	155.9
Range =12	235.1	302.2	375.0	393.5	231.4	144.7
Maximum Number of Facilities						
	P=13	**P=14**	**P=15**	**P=16**	**P=17**	**P=18**
Percent of Flows Recharged						
Range =4	29.74	32.29	32.29	32.81	33.69	34.83
Range =8	90.13	93.15	98.09	98.46	99.07	99.63
Range =12	98.42	99.26	99.63	100	100	100
Solution Time (S)						
Range =4	30.6	23.9	25.2	26.6	24.9	24.2
Range =8	110.3	135.4	24.8	27.6	26.5	31.1
Range =12	87.9	61.7	70.0	41.3	-	-
Maximum Number of Facilities						
	P=19	**P=20**	**P=21**	**P=22**	**P=23**	**P=24**
Percent of Flows Recharged						
Range =4	36.09	37.29	37.29	37.67	38.31	38.31
Range =8	100	100	100	100	100	100
Range =12	100	100	100	100	100	100
Solution Time (S)						
Range =4	22.7	20.6	21.2	21.1	20.4	19.9
Range =8	25.9	-	-	-	-	-
Range =12	-	-	-	-	-	-

Tables 13 and 14 represent the sequence of opening the facilities allowing for maximum 25 % deviation from shortest path for vehicles with range of 12. Comparing tables 13, 9, 14, and 10 shows the difference in chosen facilities when allowing for deviation. An additional observation is that multiple optimal solutions can exist in some of the cases. For instance for the case that we want to recharge all the demands (maximum opened facility of 16) the model which only uses shortest path suggests that a facility should be opened in node 23; however the model which allows for deviation suggest that instead of node 23, a facility at node 22 should be opened.

4.1.5 Optimal location of charging facilities with the objective of maximizing vehicle-miles traveled

This section represents the result of applying the model on the network in figure 12 with the vehicle-miles traveled as the objective function. Table 15 shows the solution time and percent of total vehicle-miles recharged for this objective function. In most cases the solution time is higher for the objective of vehicle-miles traveled compared to recharged demand especially for the case that ranges of vehicles are 8 or 12.

Table 13 Sequence of Opening the Facilities for 25 Percent Maximum Deviation for Range=12 for 1-12 Maximum Facilities

Opened Facilities	P=1	P=2	P=3	P=4	P=5	P=6	P=7	P=8	P=9	P=10	P=11	P=12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	1	1	1	0	0	0
4	1	1	0	1	0	1	0	0	0	1	1	1
5	0	0	0	0	0	0	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	1	0	0	0	0	0	0	0
8	0	0	1	1	0	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	0	1	1	1
10	0	0	0	0	1	1	1	1	1	0	0	0
11	0	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	1	1	1
14	0	0	0	0	0	0	0	0	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	1	1
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	1	1	0	1	0	1	1	0	0	0
20	0	0	0	0	1	0	1	1	0	1	1	1
21	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	1	1	1	1	0	0	0	0
23	0	0	0	0	0	0	0	0	1	1	1	1
24	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	1	1	1	1

Table 14 Sequence of Opening the Facilities for 25 Percent Maximum Deviation for Range=12 for 13-24 Maximum Facilities

Opened Facilities	P=13	P=14	P=15	P=16	P=17	P=18	P=19	P=20	P=21	P=22	P=23	P=24
1	0	0	0	0	-	-	-	-	-	-	-	-
2	1	1	1	1	-	-	-	-	-	-	-	-
3	1	1	1	0	-	-	-	-	-	-	-	-
4	0	0	0	1	-	-	-	-	-	-	-	-
5	1	1	1	1	-	-	-	-	-	-	-	-
6	0	0	0	0	-	-	-	-	-	-	-	-
7	1	1	1	1	-	-	-	-	-	-	-	-
8	1	1	1	1	-	-	-	-	-	-	-	-
9	0	0	1	1	-	-	-	-	-	-	-	-
10	1	1	0	0	-	-	-	-	-	-	-	-
11	1	1	1	1	-	-	-	-	-	-	-	-
12	0	0	1	1	-	-	-	-	-	-	-	-
13	0	0	1	1	-	-	-	-	-	-	-	-
14	1	0	1	1	-	-	-	-	-	-	-	-
15	0	0	0	1	-	-	-	-	-	-	-	-
16	1	1	0	0	-	-	-	-	-	-	-	-
17	0	0	1	1	-	-	-	-	-	-	-	-
18	1	1	0	0	-	-	-	-	-	-	-	-
19	1	1	0	0	-	-	-	-	-	-	-	-
20	0	0	1	1	-	-	-	-	-	-	-	-
21	0	1	0	0	-	-	-	-	-	-	-	-
22	0	1	1	1	-	-	-	-	-	-	-	-
23	1	0	0	0	-	-	-	-	-	-	-	-
24	0	1	1	1	-	-	-	-	-	-	-	-
25	1	1	1	1	-	-	-	-	-	-	-	-

Table 15 Percent of Total Vehicle Miles Traveled and Solution Time for Multiple Shortest Paths

Maximum Number of Facilities						
	P=1	**P=2**	**P=3**	**P=4**	**P=5**	**P=6**
Percent of Total VMT						
Range =4	0.11	0.32	0.80	1.96	3.03	4.85
Range =8	1.04	3.51	8.30	13.07	18.01	25.22
Range =12	2.31	9.95	17.67	27.46	38.51	48.38
Solution Time (S)						
Range =4	0.2	0.3	2	3.2	3.6	4.3
Range =8	0.8	9.2	11.5	51.7	19.5	42.6
Range =12	1.2	12.8	872.3	36.3	24.2	66.7
Maximum Number of Facilities						
	P=7	**P=8**	**P=9**	**P=10**	**P=11**	**P=12**
Percent of Total VMT						
Range =4	7.37	10.04	11.60	13.12	14.45	15.74
Range =8	32.29	40.58	48.35	55.69	63.40	70.83
Range =12	57.03	65.10	73.05	82.12	90.64	94.13
Solution Time (S)						
Range =4	2.8	2.1	2	2.4	1.2	0.7
Range =8	44.8	27.5	35.5	15.7	17	9.9
Range =12	20.5	345.4	21.1	11.1	5.8	2.4
Maximum Number of Facilities						
	P=13	**P=14**	**P=15**	**P=16**	**P=17**	**P=18**
Percent of Total VMT						
Range =4	17.04	18.34	18.34	18.48	18.81	19.21
Range =8	78.56	85.62	93.35	94.45	94.82	95.87
Range =12	96.23	98.60	99.70	100	100	100
Solution Time (S)						
Range =4	0.5	0.3	0.6	0.7	0.6	0.7
Range =8	6.8	7.7	0.5	1.1	0.9	0.4
Range =12	2.6	1.2	0.5	0.1	-	-
Maximum Number of Facilities						
	P=19	**P=20**	**P=21**	**P=22**	**P=23**	**P=24**
Percent of Total VMT						
Range =4	19.61	19.88	19.88	19.96	20.18	20.18
Range =8	96.24	96.24	96.24	96.24	96.24	96.24
Range =12	100	100	100	100	100	100
Solution Time (S)						
Range =4	0.2	0.2	0.2	0.3	0.1	-
Range =8	0.1	-	-	-	-	-
Range =12	-	-	-	-	-	-

Tables 16, 17, 18, 19, 20, and 21 show the sequence of opening the facilities for the model which tries to maximize the vehicle miles traveled. Comparing tables 4 and 15, 5 and 16, 6 and 17, 7 and 18, 8 and 19, 9 and 20, and 10 and 21 shows that the decision of opening the facilities as well as the outcome of the project is dependent on the objective function we choose. For instance with the assumption that range of vehicles is 12 and the maximum number of facilities that can be opened is 8, the optimal solution can cover 70.41 percent of total flow whereas it can only cover 65.10 percent of total vehicle miles. In terms of facilities that should be opened and are different for the two objectives, facilities 7, 10, 12 and 22 should be opened to maximize the total flows recharged, but in maximizing the vehicle miles traveled facilities 8, 11, 23, and 24 should be opened.

Table 16 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=4 for 1-12 Maximum Facilities (Maximizing Total VMT)

Opened Facilities	P=1	P=2	P=3	P=4	P=5	P=6	P=7	P=8	P=9	P=10	P=11	P=12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	1
11	0	0	0	1	1	0	0	0	0	0	1	1
12	0	0	0	0	0	0	0	0	0	1	0	0
13	0	0	0	1	1	0	0	0	0	0	1	1
14	0	1	1	0	0	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	1	1	1	1	1	1	1
17	0	0	0	0	0	1	1	1	1	1	1	1
18	0	0	0	0	1	1	1	0	1	1	1	1
19	0	0	0	1	1	0	0	1	1	1	1	1
20	0	0	0	1	1	1	1	1	1	1	1	1
21	1	0	0	0	0	0	0	0	0	0	0	0
22	0	1	1	0	0	1	1	1	1	1	1	1
23	0	0	1	0	0	0	1	1	1	1	1	1
24	0	0	0	0	0	0	0	1	1	1	1	1
25	0	0	0	0	0	0	0	0	0	0	0	0

Table 17 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=4 for 13-25 Maximum Facilities (Maximizing Total VMT)

Opened Facilities	P=13	P=14	P=15	P=16	P=17	P=18	P=19	P=20	P=21	P=22	P=23	P=24
1	0	0	1	0	0	1	1	1	1	1	1	-
2	0	0	0	0	0	1	1	1	1	1	1	-
3	0	0	0	1	1	1	1	1	1	1	1	-
4	0	0	0	1	1	1	1	1	1	1	1	-
5	0	0	0	0	1	0	1	1	1	1	1	-
6	0	0	0	0	0	0	0	0	1	1	1	-
7	0	0	0	0	0	0	0	0	0	1	1	-
8	0	0	0	0	0	0	0	0	0	0	1	-
9	0	0	0	0	0	0	0	1	1	1	1	-
10	0	1	1	1	1	1	1	1	1	1	1	-
11	1	1	1	1	1	1	1	1	1	1	1	-
12	1	1	1	1	1	1	1	1	1	1	1	-
13	1	1	1	1	1	1	1	1	1	1	1	-
14	1	1	1	1	1	1	1	1	1	1	1	-
15	1	1	1	1	1	1	1	1	1	1	1	-
16	1	1	1	1	1	1	1	1	1	1	1	-
17	1	1	1	1	1	1	1	1	1	1	1	-
18	1	1	1	1	1	1	1	1	1	1	1	-
19	1	1	1	1	1	1	1	1	1	1	1	-
20	1	1	1	1	1	1	1	1	1	1	1	-
21	0	0	0	0	0	0	0	0	0	0	0	-
22	1	1	1	1	1	1	1	1	1	1	1	-
23	1	1	1	1	1	1	1	1	1	1	1	-
24	1	1	1	1	1	1	1	1	1	1	1	-
25	0	0	0	0	0	0	0	0	0	0	0	-

Table 18 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=8 for 1-12 Maximum Facilities (Maximizing Total VMT)

Opened Facilities	P=1	P=2	P=3	P=4	P=5	P=6	P=7	P=8	P=9	P=10	P=11	P=12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	1	1	1	1	1	1	1
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	1	0	1
8	0	0	0	0	0	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	0	0	1	1
10	0	0	0	0	0	1	1	1	1	1	1	1
11	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	1	0	0	1	0	1	1	1	1
13	0	0	0	1	1	1	1	1	1	1	1	1
14	1	0	0	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	0	1	1	0	1	0	0	1	1	1	1	1
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0
20	0	1	1	0	1	0	0	1	1	1	1	1
21	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	1	0	0	0	0	0	0	0	0	0
23	0	0	0	1	1	1	1	1	1	1	1	1
24	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0

Table 19 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=8 for 13-25 Maximum Facilities (Maximizing Total VMT)

Opened Facilities	P=13	P=14	P=15	P=16	P=17	P=18	P=19	P=20	P=21	P=22	P=23	P=24
1	0	0	0	0	0	1	1	-	-	-	-	-
2	1	1	1	1	1	0	0	-	-	-	-	-
3	0	0	0	0	0	1	1	-	-	-	-	-
4	1	1	1	1	1	1	1	-	-	-	-	-
5	0	0	0	0	1	1	1	-	-	-	-	-
6	0	0	0	0	0	0	1	-	-	-	-	-
7	1	1	1	1	1	1	1	-	-	-	-	-
8	1	1	1	1	1	1	1	-	-	-	-	-
9	1	1	1	1	1	1	1	-	-	-	-	-
10	1	1	1	1	1	1	1	-	-	-	-	-
11	1	0	1	1	1	1	1	-	-	-	-	-
12	1	1	1	1	1	0	0	-	-	-	-	-
13	1	1	1	1	1	1	1	-	-	-	-	-
14	1	1	1	1	1	1	1	-	-	-	-	-
15	0	0	0	1	0	1	1	-	-	-	-	-
16	0	0	0	0	1	1	1	-	-	-	-	-
17	1	1	1	1	1	1	1	-	-	-	-	-
18	0	0	0	0	0	0	0	-	-	-	-	-
19	0	0	0	0	0	0	0	-	-	-	-	-
20	1	1	1	1	1	1	1	-	-	-	-	-
21	0	0	0	0	0	0	0	-	-	-	-	-
22	0	0	0	1	1	0	1	-	-	-	-	-
23	1	1	1	0	0	1	0	-	-	-	-	-
24	0	1	1	1	1	1	1	-	-	-	-	-
25	0	1	1	1	1	1	1	-	-	-	-	-

Table 20 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=12 for 1-12 Maximum Facilities (Maximizing Total VMT)

Opened Facilities	P=1	P=2	P=3	P=4	P=5	P=6	P=7	P=8	P=9	P=10	P=11	P=12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	1	1	1	1
3	0	0	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	0	1	1	1	1	1	1	1
5	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	1	0	0	0	0	0	0	1
8	0	1	1	0	0	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	1	1	1	1
10	0	0	0	1	1	0	0	0	0	0	0	0
11	0	0	1	0	0	1	1	1	1	1	1	0
12	0	0	0	0	0	0	0	0	0	0	0	1
13	0	0	0	0	1	0	0	1	0	1	1	1
14	0	0	1	0	0	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	1	0	0	1	1
18	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	1	1	0	1	0	0	0
20	0	0	0	1	1	0	0	0	0	1	1	1
21	0	1	0	0	0	0	0	0	0	0	0	0
22	0	0	0	1	1	0	0	0	0	0	0	0
23	0	0	0	0	0	1	1	1	1	1	1	1
24	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	1	1	1	1	1	1

Table 21 Sequence of Opening the Facilities for Multiple Shortest Paths for Range=12 for 13-25 Maximum Facilities (Maximizing Total VMT)

Opened Facilities	P=13	P=14	P=15	P=16	P=17	P=18	P=19	P=20	P=21	P=22	P=23	P=24
1	0	0	0	0	-	-	-	-	-	-	-	-
2	1	1	1	1	-	-	-	-	-	-	-	-
3	0	0	0	0	-	-	-	-	-	-	-	-
4	1	1	1	1	-	-	-	-	-	-	-	-
5	0	1	1	1	-	-	-	-	-	-	-	-
6	0	0	0	0	-	-	-	-	-	-	-	-
7	1	1	1	1	-	-	-	-	-	-	-	-
8	1	1	1	1	-	-	-	-	-	-	-	-
9	1	1	1	1	-	-	-	-	-	-	-	-
10	0	0	0	0	-	-	-	-	-	-	-	-
11	1	1	1	1	-	-	-	-	-	-	-	-
12	1	1	1	1	-	-	-	-	-	-	-	-
13	1	1	1	1	-	-	-	-	-	-	-	-
14	1	1	1	1	-	-	-	-	-	-	-	-
15	0	0	0	1	-	-	-	-	-	-	-	-
16	0	0	1	0	-	-	-	-	-	-	-	-
17	1	1	1	1	-	-	-	-	-	-	-	-
18	0	0	0	0	-	-	-	-	-	-	-	-
19	0	0	0	0	-	-	-	-	-	-	-	-
20	1	1	1	1	-	-	-	-	-	-	-	-
21	0	0	0	0	-	-	-	-	-	-	-	-
22	0	0	0	1	-	-	-	-	-	-	-	-
23	1	1	1	0	-	-	-	-	-	-	-	-
24	0	0	0	1	-	-	-	-	-	-	-	-
25	1	1	1	1	-	-	-	-	-	-	-	-

4.1.4 Optimal location of charging facilities considering the decay function in the objective

The model is also tested with decay function for maximum deviation of 25 percent and the value of beta was set to 0.5. Table 22 shows the solution time and percent of flows recharged considering a reduction in flow demand of O-D pairs if the flow is not using the shortest path.

Table 22 Percent of Flows Recharged and Solution Time for 25 Percent Maximum Deviation with Decay Function

Maximum Number of Facilities						
	P=1	**P=2**	**P=3**	**P=4**	**P=5**	**P=6**
Percent of Flows Recharged						
Range =4	0.63	1.14	2.08	3.93	6.31	8.32
Range =8	2.64	7.27	12.84	19.47	25.91	34.04
Range =12	4.96	13.29	25.74	38.99	54.23	66.20
Solution Time (S)						
Range =4	4.5	25.3	33.1	46.1	40.9	45.2
Range =8	32.5	51.3	95.1	149.7	253.9	390.5
Range =12	42.8	113.3	141.2	206.3	224.1	247.1
Maximum Number of Facilities						
	P=7	**P=8**	**P=9**	**P=10**	**P=11**	**P=12**
Percent of Flows Recharged						
Range =4	11.03	13.72	16.60	19.94	23.68	26.31
Range =8	45.30	54.38	62.28	69.98	77.27	82.90
Range =12	73.50	79.56	84.92	88.98	93.88	95.95
Solution Time (S)						
Range =4	48.6	45.2	50.3	40.9	34.7	32.2
Range =8	512.6	360.2	458.8	247.5	184.3	121.4
Range =12	285.6	331.6	310.6	407.6	105.2	92.1
Maximum Number of Facilities						
	P=13	**P=14**	**P=15**	**P=16**	**P=17**	**P=18**
Percent of Flows Recharged						
Range =4	28.92	31.47	31.47	31.99	32.87	33.95
Range =8	89.48	91.72	97.45	97.93	98.70	99.26
Range =12	97.64	98.99	99.51	100	100	100
Solution Time (S)						
Range =4	31.6	23.1	24.4	24.6	23.6	24
Range =8	98.1	93.9	48.6	43.7	44	43.6
Range =12	79.3	57	57.4	48.1	-	-
Maximum Number of Facilities						
	P=19	**P=20**	**P=21**	**P=22**	**P=23**	**P=24**
Percent of Flows Recharged						
Range =4	35.17	36.37	36.37	36.75	37.39	37.39
Range =8	99.75	99.75	99.75	99.75	99.75	99.75
Range =12	100	100	100	100	100	100
Solution Time (S)						
Range =4	20.9	19.6	20.3	20.4	19	19
Range =8	40	-	-	-	-	-
Range =12	-	-	-	-	-	-

4.1.3 Improving the solution time

Table 23 shows the solution time and the percentage change in the solution time by adding valid inequalities to the constraints for the instance in which range is equal to 12 and the maximum deviation is equal to 25 percent. As it is shown in the table, in all cases except 3 adding these inequalities decrease the solution time.

Table 23 Solution Time and Percentage Change in Solution Time by Adding Valid Inequalities for 25 Percent Maximum Deviation

Maximum Number of Facilities						
	P=1	**P=2**	**P=3**	**P=4**	**P=5**	**P=6**
Solution Time (S)						
Range =12	35.2	115.0	123.0	179.3	251.4	138.0
Percentage Change in Solution Time						
Range =12	14.4	-13.2	2.1	10.4	-6.2	31.3
Maximum Number of Facilities						
	P=7	**P=8**	**P=9**	**P=10**	**P=11**	**P=12**
Solution Time (S)						
Range =12	153.8	132.8	253	170.2	81.4	74.5
Percentage Change in Solution Time						
Range =12	34.6	56.1	32.5	56.7	64.8	48.5
Maximum Number of Facilities						
	P=13	**P=14**	**P=15**	**P=16**	**P=17**	**P=18**
Solution Time (S)						
Range =12	67.2	60	51.2	43.7	-	-
Percentage Change in Solution Time						
Range =12	23.5	2.8	26.9	-5.8	-	-
Maximum Number of Facilities						
	P=19	**P=20**	**P=21**	**P=22**	**P=23**	**P=24**
Solution Time (S)						
Range =12	-	-	-	-	-	-
Percentage Change in Solution Time						
Range =12	-	-	-	-	-	-

4.2 *Locating charging station for electric vehicles in urban networks*

To test the proposed formulation for urban networks, 100 random nodes from a network shown in figure 13 with 407 nodes and 1284 edges, were selected as candidate facilities and demand points (left figure). The red circles in the right figure show these nodes. Each of these parking facilities has a specific demand distribution for each type of chargers. (The assumption here is that the demand follows a Poisson distribution.) For first stage demand predictions, the expected values of demand (Poisson distribution parameters) are randomly generated in Microsoft Office Excel. They have values between 1 to 5 for charger type 2; and 1 to 3 for charger type 3 for medium demand. For the low and high demand, medium demands are increased and decreased by 25 percent and then these demands are adjusted to reflect the change in the demand for the second stage. Here, we assume that the maximum coverage length of trip (time) in network for charger type 3 is 10 minutes. For charger type 2, this number is increased to 20 minutes meaning that the demand in a facility for charger type 2 can be charged only in other facilities within its 20 minutes. The reason that coverage distance of two chargers is different is that these two chargers have relatively different charging time. For charger type 2 charging time is between 4 to 8 hours whereas this number is less than 1 hour for charger type 3, hence it is more probable that the maximum deviation that user of charger 3 are willing to make is far less than users of charger type 2. The penalty for deviation is assumed to be the product of value of time, planning horizon of the stages and the average number of vehicles that each of chargers will service on a daily basis. Value of time is assumed to be \$22 per hour, the planning horizon for each of the stages is assumed to be 1 year (total of two years for two stages), and the average daily number of vehicles that are serviced by charger type 2 is 2 and for charger type 3 is 12. (These numbers are calculated based on 12 hour daily service time and the average charging time of 6 and 1 hour for charger type 2 and 3 respectively). Installation cost is assumed to be $5,300 for type 2 charge and $9,900 for type 3 and installing each additional charger costs $3,200 for charger type 2 and $28,400 for charger type 3. (These numbers are estimated based on table 1 and table 2. However these numbers are not reliable and the true costs can be different than the assumed numbers.) Capacity of parking facilities for installing chargers are randomly generated in Microsoft Office Excel and have values between 20 to 36. Maximum number of chargers type 2 that can be placed in each charging facility is 18. This number is assumed to be 12 for charger type 3. In addition, the internal rate of return is assumed to be 5 percent.

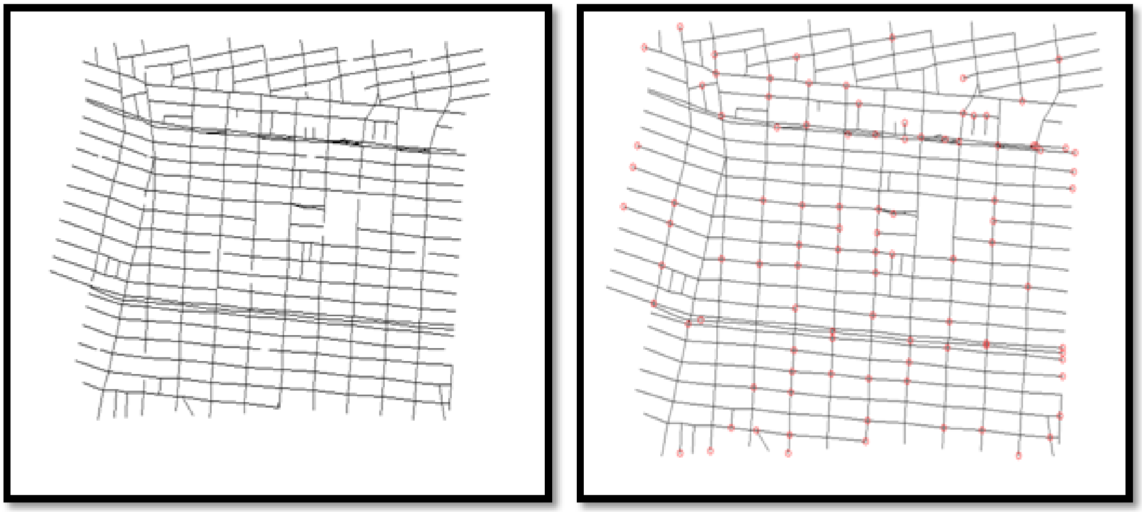


Figure 13 Urban Network

Tables 24, 25, 26, 27, 28, and 29 show the result of modeling for alphas of 10, 50, and 90 (for example alpha 10 indicates that with probability of 0.1, number of chargers are more than the number of demands) respectively and the indicated fixed budget. It should be mentioned solving for alpha equal of 90 percent is not very cost effective since this means with probability of 0.9 the demand will be less than the number of chargers installed, but here for the purpose of comparison the result for this value is also included. Overall, due to the structure of the objective function most of the chargers are placed in the first stage. The results also show that if the demand of first year is low we do not need to place any chargers at the beginning of second stage. This is because of the method used for generation of data from first year demand and the relationship between demand of first year and second year and can be changed with using different input data.

Table 24 Modeling Result (Number of Chargers and Objective Value) for Urban Network with Alpha Equal to 10 Percent

alpha =10 percent				
Budget ($)	7.5*10^6	8.5 *10^6	9.5*10^6	10^7
Solution Time (S)	11466.9	2.8	0.7	1.3
Deviation (Objective Function ($))	1.80477*10^8	7.69963*10^7	3.70948*10^7	2.27846*10^7
Number of Charger type 2 at stage 1	304	397	410	408
Number of Charger type 3 at stage 1	157	171	196	211
Number of Charger type 2 at stage 2 (L)	304	397	410	408
Number of Charger type 3 at stage 2 (L)	157	171	196	211
Number of Charger type 2 at stage 2 (M)	342	447	461	459
Number of Charger type 3 at stage 2 (M)	186	194	224	240
Number of Charger type 2 at stage 2 (H)	437	558	573	571
Number of Charger type 3 at stage 2 (H)	250	260	293	311

Table 25 Modeling Result (Number of Opened Facilities) for Urban Network with Alpha Equal to 10 Percent

alpha =10 percent				
Budget ($)	7.5*10^6	8.5 *10^6	9.5*10^6	10^7
Number of Opened Facility (type 2) at stage 1	53	94	100	99
Number of Opened Facility (type 3) at stage 1	49	63	73	78
Number of Opened Facility (type 2) at stage 2 (L)	53	94	100	99
Number of Opened Facility (type 3) at stage 2 (L)	49	63	73	78
Number of Opened Facility (type 2) at stage 2 (M)	53	94	100	99
Number of Opened Facility (type 3) at stage 2 (M)	50	64	74	79
Number of Opened Facility (type 2) at stage 2 (H)	54	94	100	99
Number of Opened Facility (type 3) at stage 2 (H)	57	71	80	85

Table 26 Modeling Result (Number of Chargers and Objective Value) for Urban Network with Alpha Equal to 50 Percent

alpha =50 percent				
Budget ($)	1.10*10^7	1.12*10^7	1.14*10^7	1.16*10^7
Solution Time (S)	2780.5	196.9	7.5	3.6
Deviation (Objective Function ($))	9.42488*10^7	5.42865*10^7	2.19783*10^7	2.567892*10^6
Number of Charger type 2 at stage 1	385	389	408	410
Number of Charger type 3 at stage 1	256	257	257	260
Number of Charger type 2 at stage 2 (L)	385	389	408	410
Number of Charger type 3 at stage 2 (L)	256	257	257	260
Number of Charger type 2 at stage 2 (M)	431	436	459	461
Number of Charger type 3 at stage 2 (M)	287	289	289	293
Number of Charger type 2 at stage 2 (H)	537	544	570	573
Number of Charger type 3 at stage 2 (H)	353	355	355	360

Table 27 Modeling Result (Number of Opened Facilities) for Urban Network with Alpha Equal to 50 Percent

alpha =50 percent				
Budget ($)	1.10*10^7	1.12*10^7	1.14*10^7	1.16*10^7
Number of Opened Facility (type 2) at stage 1	69	75	92	96
Number of Opened Facility (type 3) at stage 1	78	89	93	99
Number of Opened Facility (type 2) at stage 2 (L)	69	75	92	96
Number of Opened Facility (type 3) at stage 2 (L)	78	89	93	99
Number of Opened Facility (type 2) at stage 2 (M)	69	75	92	96
Number of Opened Facility (type 3) at stage 2 (M)	78	89	93	99
Number of Opened Facility (type 2) at stage 2 (H)	72	76	92	96
Number of Opened Facility (type 3) at stage 2 (H)	78	89	93	99

Table 28 Modeling Result (Number of Chargers and Objective Value) for Urban Network with Alpha Equal to 90 Percent

alpha =90 percent				
Budget ($)	1.52*10^7	1.55*10^7	1.57*10^7	1.6*10^7
Solution Time (S)	4582.5	604.8	55	1.8
Deviation (Objective Function ($))	1.40269*10^8	6.70056*10^7	3.07685*10^7	2.60557*10^6
Number of Charger type 2 at stage 1	458	472	481	502
Number of Charger type 3 at stage 1	389	389	390	391
Number of Charger type 2 at stage 2 (L)	458	472	481	503
Number of Charger type 3 at stage 2 (L)	389	389	390	391
Number of Charger type 2 at stage 2 (M)	510	525	538	561
Number of Charger type 3 at stage 2 (M)	430	430	432	433
Number of Charger type 2 at stage 2 (H)	634	646	659	681
Number of Charger type 3 at stage 2 (H)	512	512	514	519

Table 29 Modeling Result (Number of Opened Facilities) for Urban Network with Alpha Equal to 90 Percent

alpha =90 percent				
Budget ($)	1.52*10^7	1.55*10^7	1.57*10^7	1.6*10^7
Number of Opened Facility (type 2) at stage 1	52	77	84	100
Number of Opened Facility (type 3) at stage 1	69	82	91	99
Number of Opened Facility (type 2) at stage 2 (L)	52	77	84	100
Number of Opened Facility (type 3) at stage 2 (L)	69	82	91	99
Number of Opened Facility (type 2) at stage 2 (M)	53	77	84	100
Number of Opened Facility (type 3) at stage 2 (M)	69	83	91	99
Number of Opened Facility (type 2) at stage 2 (H)	55	78	85	100
Number of Opened Facility (type 3) at stage 2 (H)	73	85	92	100

Figure 14 shows the change in the objective function which is the total monetary value of deviation occurring during the two years of planning horizon with respect to the budget limit. As it is shown in the figure and can be understood from the formulation presented in the previous chapter, there is a tradeoff between the budget limit and the deviation. If the budget is increased, more facilities can be opened and as a result demands can use facilities that are more nearby. Figure 15 shows the relationship between the budget limit and solution time. The solution time is low for infeasible region and becomes high in the boundary between feasibility and infeasibility and again becomes low in the feasible region (actual trend is increasing and then decreasing for a fixed alpha; however in the figure just the solution time for budgets with feasible solution has been shown).

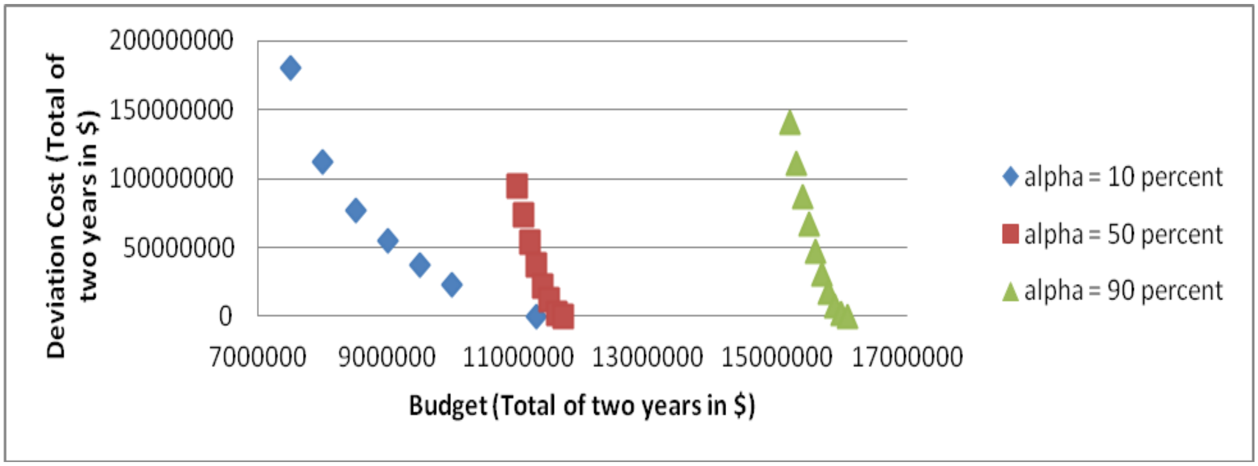


Figure 14 Tradeoff between Budget and Deviation

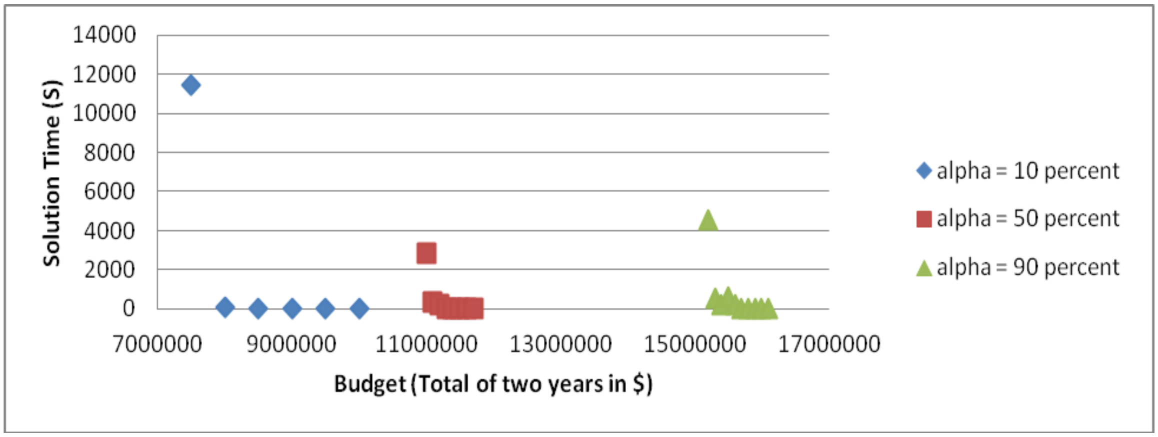


Figure 15 Solution Time Sensitivity

Chapter 5: Conclusions and Future Work

In the rural context, a new formulation for the FRLM and FRDLM was proposed in this thesis. The advantage of this formulation compared to previous model introduced by Kuby and Lim (2005) is that this model needs less preprocessing in terms of input data [35]. The formulation was tested on a small size network; however for further investigation of practicality of this model, it should be tested in a realistically sized network. Several different variants of the model were tested on the sample network. For example different objective functions, including maximizing vehicle-miles traveled and maximizing total flow accounting for reduction in the demand in the case of allowing for deviation from shortest path (decay function), were tested to explore the sensitivity of the solutions with respect to different objectives.

The results confirm that the sequence of opening the facilities is highly dependent on the parameters of the model such as range, max deviation, and maximum number of facilities and also the undertaken policy.

Allowing for deviation in the model resulted in choosing different set of candidate facilities. In solving the FRDLM, maximum deviation should be chosen with great caution since the results show that increasing the maximum deviation will greatly affect the solution time for the problem. The value chosen for this number is also important in terms of placing charging facilities in optimal locations. This number could be estimated based on the empirical data or stated preference surveys.

In terms of assumption about range of vehicles, the results indicate that optimal location of facilities is highly related to the range of the vehicles. This emphasizes the need for research on the tradeoff between the cost of making higher capacity batteries and cost of placing charging stations similar to Nie and Ghamami's work (2013) [43].

In this study, valid inequalities were added to the formulation to improve the solution time of the model. In most cases adding these inequalities results in better solution time; however more sophisticated methods such as cutting plane method should be used in the future to refine the feasible region of the problem and reduce the solution time especially for large size problems.

In the urban context, a two stage stochastic optimization model was presented with the objective of siting and sizing the charging facilities at two stages in order to minimize the deviation of users from their original destinations. Sensitivity analysis was performed to see the effect of change in the objective function with respect to budget limit and the probability of satisfying the predicted demand. For this model, several other factors can be considered in the objective function in the future. For example, the running cost and the revenue gathered from the users can be accounted for. These cost and revenue are not reflected in the proposed model since currently, the cost of using these facilities for the users is highly dependent on their location for example in most educational facilities the users can use these chargers for free in order to incentivize people to buy these vehicles.

Moreover, in including uncertainty in demand, the assumption is that in the future all the facilities will either have low, medium, or high demand; however this is not always the case and sometimes it is probable that some facilities will have high demand whereas others will have low or medium demand. In the future, more scenarios can be added to the problem to consider the possibility of having low, medium, and high demand at the same time.

Bibliography

1. Hybrid and plug-in electric vehicles (2014), Prepared by the National Renewable Energy Laboratory (NERL) a national laboratory of the U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy; NREL is operated by the Alliance for Sustainable Energy, LLC.

2. Morrow, K., Karner, D., & Francfort, J. (2008). Plug-in hybrid electric vehicle charging infrastructure review. *US Department of Energy-Vehicle Technologies Program*.

3. Vehicle charging infrastructure deployment guidelines for the greater San Diego area (2010) prepared by electric transportation engineering corporation.

4. EPRI, "Comparing the Benefits and Impacts of Hybrid Electric Vehicle Options for Compact Sedan and Sport Utility Vehicles," Palo Alto, CA: 2002. 1006892.

5. Bureau of Transportation Statistics, "National Transportation Statistics, 2006," Table 4-23, December 2006.

6. Energy Information Administration, "Table 5.3. Average Retail Price of Electricity to Ultimate Customers: Total by End-Use Sector, 1994 through November 2008," in Electric Power Monthly, Released February 13, 2009.

7**.** Curtin, R., Shrago, Y., & Mikkelsen, J. (2009). Plug-in Hybrid Electric Vehicles. *Reuters/University of Michigan, Surveys of Consumers.*

8. Tate, E. D., Harpster, M. O., & Savagian, P. J. (2008). *The electrification of the automobile: from conventional hybrid, to plug-in hybrids, to extended-range electric vehicles* (No. 2008-01-0458). SAE Technical Paper.

9. Carley, S., Krause, R. M., Lane, B. W., & Graham, J. D. (2013). Intent to purchase a plug-in electric vehicle: A survey of early impressions in large US cites. *Transportation Research Part D: Transport and Environment*, *18*, 39-45.

10. Bartlett, J. (2012). Survey: Consumers Express Concerns About Electric, Plug-In Hybrid Cars.6 *Consumer Reports*, January 30.

11. Dong, J., & Lin, Z. (2014). Stochastic Modeling of Battery Electric Vehicle Driver Behavior: The Impact of Charging Infrastructure Deployment on BEV Feasibility. In *Transportation Research Board 93rd Annual Meeting* (No. 14-5107).

12. U.S. Department of Transportation, Federal Highway Administration, Office of Highway Policy Information, National Household Travel Survey.

13. Traut, E., Hendrickson, C., Klampfl, E., Liu, Y., and Michalek, J. J. (2012). Optimal design 25 and allocation of electrified vehicles and dedicated charging infrastructure for minimum life 26 cycle greenhouse gas emissions and cost. *Energy Policy* **51**, 524-534.

14. Melaina, M., & Bremson, J. (2008). Refueling availability for alternative fuel vehicle markets: sufficient urban station coverage. *Energy Policy*, *36*(8), 3233-3241.

15. Leiby, P., & Rubin, J. (2004). Understanding the transition to new fuels and vehicles: lessons learned from analysis and experience of alternative fuel and hybrid vehicles. *The Hydrogen Energy Transition*, 191-212.

16. Sperling, D. (1990). *New transportation fuels: a strategic approach to technological change*. Univ of California Press.

17. Chen, T. D., Wang, Y., & Kockelman, K. M. (2014). Where Are the Electric Vehicles? A Spatial Model for Vehicle-Choice Count Data. In Transportation Research Board 93rd Annual Meeting (No. 14-1282).

18. Bockarjova, M., Rietveld, P., Knockaert, J., & Steg, L. (2014). Dynamic Consumer Heterogeneity in Electric Vehicle Adoption. In Transportation Research Board 93rd Annual Meeting (No. 14-1579).

19. Global EV Outlook 2013 - Understanding the Electric Vehicle Landscape to 2020. International Energy Agency. Retrieved 2013-04-20. *See pp. 14-15*.

20. Daskin, M. S. (2011). *Network and discrete location: models, algorithms, and applications*. John Wiley & Sons.

21. Jia, L., Hu, Z., Song, Y., & Luo, Z. (2012, March). Optimal siting and sizing of electric vehicle charging stations. In *Electric Vehicle Conference (IEVC), 2012 IEEE International* (pp. 1-6). IEEE.

22. Church, R., and C. ReVelle. The Maximal Covering Location Problem. *Papers in Regional Science,* Vol. 32, No. 1, 1974, pp. 101–118.

23. Xi, X., Sioshansi, R., & Marano, V. (2013). Simulation–optimization model for location of a public electric vehicle charging infrastructure. *Transportation Research Part D: Transport and Environment*, *22*, 60-69.

24. Chen, T. D., Kockelman, K. M., & Khan, M. (2013, January). The electric vehicle charging station location problem: a parking-based assignment method for Seattle. In *92nd Annual Meeting of the Transportation Research Board. Washington DC, USA*.

25. Berman, O., D. Krass, and Z. Drezner. The Gradual Covering Decay Location Problem on a Network. *European Journal of Operational Research,* Vol. 151, No. 3, 2003, pp. 474–480.

26. Frade, I., Ribeiro, A., Gonçalves, G., & Antunes, A. P. (2011). Optimal location of charging stations for electric vehicles in a neighborhood in Lisbon, Portugal. *Transportation research record: journal of the transportation research board*, *2252*(1), 91-98.

27. Koyanagi, F., & Yokoyama, R. (2010, August). A priority order solution of EV recharger installation by domain division approach. In *Universities Power Engineering Conference (UPEC), 2010 45th International* (pp. 1-8). IEEE.

28. Ge, S., Feng, L., & Liu, H. (2011, September). The planning of electric vehicle charging station based on Grid partition method. In *Electrical and Control Engineering (ICECE), 2011 International Conference on* (pp. 2726-2730). IEEE.

29. Hodgson, M. J. (1990). A Flow-Capturing Location-Allocation Model. *Geographical Analysis, 22*(3), 270-279.

30. Berman, O., Larson, R. C., & Fouska, N. (1992). Optimal location of discretionary service facilities. *Transportation Science*, *26*(3), 201-211.

31. John Hodgson, M., Rosing, K. E., Leontien, A., & Storrier, G. (1996). Applying the flow-capturing location-allocation model to an authentic network: Edmonton, Canada. *European journal of operational research, 90*(3), 427-443.

32. Berman, O., Bertsimas, D., & Larson, R. C. (1995). Locating discretionary service facilities, II: maximizing market size, minimizing inconvenience. *Operations Research*, *43*(4), 623-632.

33. Hodgson MJ, Berman O. A billboard location model. Geographical and Environmental Modeling 1997;1:25–45.

34. Hodgson MJ, Rosing KE, Zhang J. Locating vehicle inspection stations to protect a transportation network.Geographical Analysis 1996;28:299–314.

35. Kuby, M., & Lim, S. (2005). The flow-refueling location problem for alternative-fuel vehicles. *Socio-Economic Planning Sciences*, *39*(2), 125-145.

36. Upchurch, C., Kuby, M., & Lim, S. (2009). A Model for Location of Capacitated Alternative-Fuel Stations. *Geographical Analysis*, *41*(1), 85-106.

37. Lim, S., & Kuby, M. (2010). Heuristic algorithms for siting alternative-fuel stations using the Flow-Refueling Location Model. *European Journal of Operational Research*, *204*(1), 51-61.

38 . Kuby, M., & Lim, S. (2007). Location of alternative-fuel stations using the flow-refueling location model and dispersion of candidate sites on arcs. *Networks and Spatial Economics*, *7*(2), 129-152.

39. Kim, J. G., & Kuby, M. (2012). The deviation-flow refueling location model for optimizing a network of refueling stations. *international journal of hydrogen energy*, *37*(6), 5406-5420.

40. Kim, J. G., & Kuby, M. (2013). A network transformation heuristic approach for the deviation flow refueling location model. *Computers & Operations Research*, *40*(4), 1122-1131.

41. Berman O, Simchi-Levi D. A heuristic algorithm for thetraveling salesman location problem on networks. Oper Res1988;36:478e84.

42. Yen, J. Y. (1971). Finding the k shortest loopless paths in a network. *management Science*, *17*(11), 712-716.

43. Nie, Y. M., & Ghamami, M. (2013). A corridor-centric approach to planning electric vehicle charging infrastructure. *Transportation Research Part B: Methodological*, *57*, 172-190.