ABSTRACT

Title of Document: ACTIVE RELOCATION AND DISPATCHING OF HETEROGENEOUS EMERGENCY VEHICLES

Elham Sharifi, 2014

Directed By: Professor Ali Haghani

Department of Civil and Environmental Engineering

An emergency is a situation that causes an immediate risk to the property, health, or lives of civilians and can assume a variety of forms such as traffic accidents, fires, personal medical emergencies, terrorist attacks, robberies, natural disasters, etc. Emergency response services (ERSs) such as police, fire, and medical services play crucial roles in all communities and can minimize the adverse effects of emergency incidents by decreasing the response time. Response time is not only related to the dispatching system, but also has a very close relationship to the coverage of the whole network by emergency vehicles.

The goal of this dissertation is to develop a model for an Emergency Management System. This model will dynamically relocate the emergency vehicles to provide better coverage for the whole system. Also, when an emergency happens in the system the model will consider dispatching and relocation problem simultaneously. In addition, it will provide real-time route guidance for emergency vehicles. In summary, this model will consider three problems simultaneously: area coverage, vehicle deployment, and vehicle routing.

This model is event-based and will be solved whenever there is an event in the system. These events can be: occurrence of an emergency, change in the status of

vehicles, change in the traffic data, and change in the likelihood of an emergency happening in the demand nodes.

Three categories of emergency vehicle types are considered in the system: police cars, ambulances, and fire vehicles. The police department is assumed to have a homogeneous fleet, but ambulances and fire vehicles are heterogeneous. Advanced Life Support (ALS) and Basic Life Support (BLS) ambulances are considered, along with Fire Engines, Fire Trucks, and Fire Quints in the fire vehicle category.

This research attempts to provide double coverage for demand nodes by non-homogenous fleet while increasing the equity of coverage of different demand nodes. Also, the model is capable of considering the partial coverage in the heterogeneous vehicle categories. Two kinds of demand nodes are considered, ordinary nodes and critical nodes. Node demands may vary over time, so the model is capable of relocating the emergency fleet to cover the points with highest demand. In addition, an attempt is made to maintain work load balance between different vehicles in the system. Real-world issues, such as the fact that vehicles prefer to stay at their home stations instead of being relocated to other stations and should be back at their home depots at the end of the work shift, are taken into account.

This is a unique and complex model; so far, no study in the literature has addressed these problems sufficiently. A mathematical formulation is developed for the proposed model, and numerical examples are designed to demonstrate its capabilities. Xpress 7.1 is used to run this model on the numerical examples. Commercial software like Xpress can be used to solve the proposed model on small-size problems, but for large-size and real-world problems, an appropriate heuristic is needed. A heuristic method that can find good solutions in reasonable time for this problem is developed and tested on several cases. Also, the model is applied to a real-world case study to test its performance. To investigate the model’s behavior on a real-world problem, a very sophisticated simulation model that can see most of the details in the system has been developed and the real case study data has been used to calibrate the model. The results show that the proposed model is performing very well and efficient and it can greatly improve the performance of emergency management centers.

ACTIVE RELOCATION AND DISPATCHING OF HETEROGENOUS EMERGENCY VEHICLES

By

Elham Sharifi

Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park, in partial fulfillment of the requirements for the degree of Doctor of Philosophy
2014

Advisory Committee:
Professor Ali Haghani, Chair
Professor Martin Dresner
Professor Philip Evers
Professor Cinzia Cirillo
Professor Lei Zhang

Dedication

This Research is dedicated to my beloved parents,

my lovely husband Hadi and my adorable son Nickan.

Acknowledgements

First of all, I would like to express my sincere gratitude and appreciation to my advisor, Dr. Ali Haghani for all the help and support he provided me during these years. Without his invaluable guidance, comments and understanding the completion of this research would not have been possible. Also, I would like to extend my regards to my supervisory committee members, Dr. Martin Dresner, Dr. Philip Evers, Dr. Cinzia Cirillo and Dr. Lei Zhang for their invaluable suggestions and instructions through this research.

I offer my special thanks to my friend, Dr. Masoud Hamedi whose constructive suggestions and comments were remarkably helpful to the success of this dissertation. In addition, a special gratitude is acknowledged to Dr. Saini Yang whose outstanding dissertation was the main reference for this work. Some of her analyses have been used in this dissertation and it was a great help.

Also, I would like to express my deepest appreciation to my beloved parents and siblings. They supported me during my whole life and their unconditional love and encouragement made me who I am right now. Also, I am expressing my sincere love and gratitude to my lovely husband and my best friend, Hadi. He stood by my side all these years and this dissertation would never have been completed without his irreplaceable love, support and patience. At the end, I would like to extend my appreciation and my love to my adorable son, Nickan. He was the real inspiration that helped me work harder to complete this dissertation.

Also I am grateful to FICO Corporation for providing Xpress 7.1 for this research. This software added remarkable value to this dissertation.

Table of Contents

List of Figures

List of Tables

Chapter 1: Introduction

1.1 Emergency

An emergency is a situation that causes an immediate risk to the property, health, or lives of civilians. Emergencies can take a variety of forms like traffic accidents, fires, personal medical emergencies, terrorist attacks, robberies, natural disasters, etc (Yang, 2006). According to the National Highway Traffic Safety Administration (NHTSA), the number of fatalities from highway accidents has declined over the last 5 years as shown in Figure 1.1, but remains unacceptably high. In 2009, there were 30,000 fatal automobile accidents (Figure 1.1).

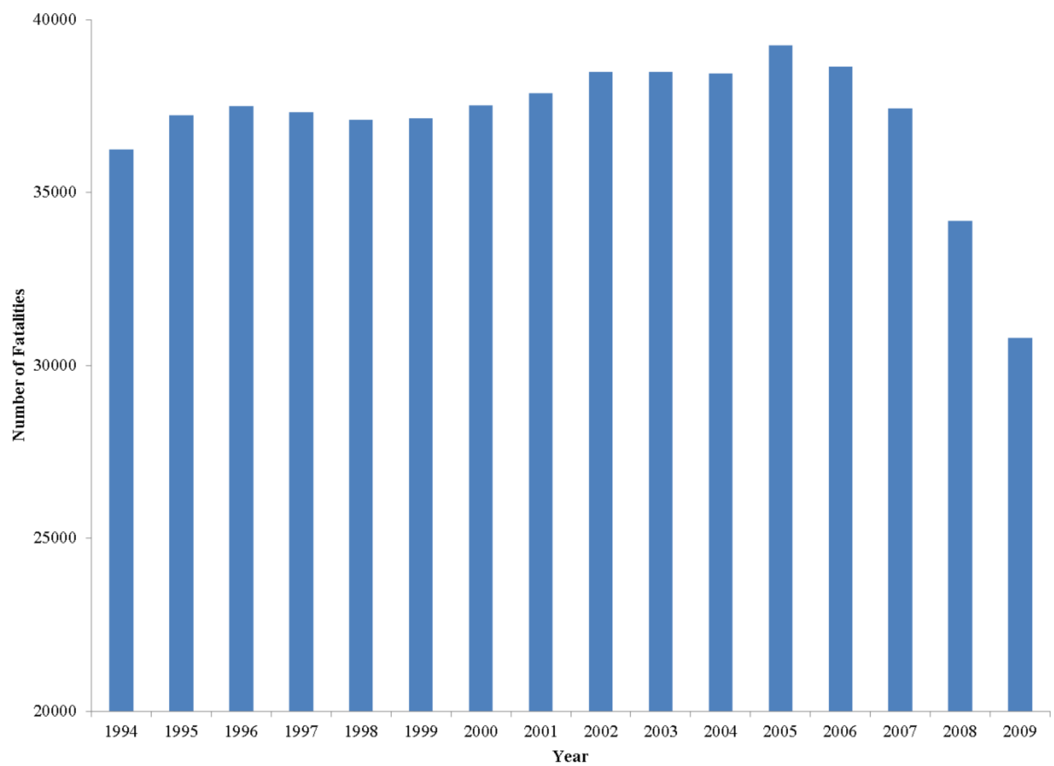

Figure 1.1 Fatal Crashes Trends in the U.S. from 1994 to 2009 (NHTSA)

Also, from the website of the U.S Fire Administration (USFA), it can be seen that in the United States in 2010, more than 1.3 million fires were reported, which caused 3,120 deaths, 17,720 injuries, and about 11.6 billion dollars in direct losses. The national fire death rate in 2008 was 12 deaths per million people. The District of Columbia had the highest fire-death rate in 2008, which was 32.2 deaths per million people.

Figures 1.2 and 1.3 show the trend for residential building fire deaths and residential building fire-dollar losses, respectively, from 2005 to 2009 in the U.S. Although the number of deaths declined, dollar losses rose to 7.3 billion dollars in 2009.

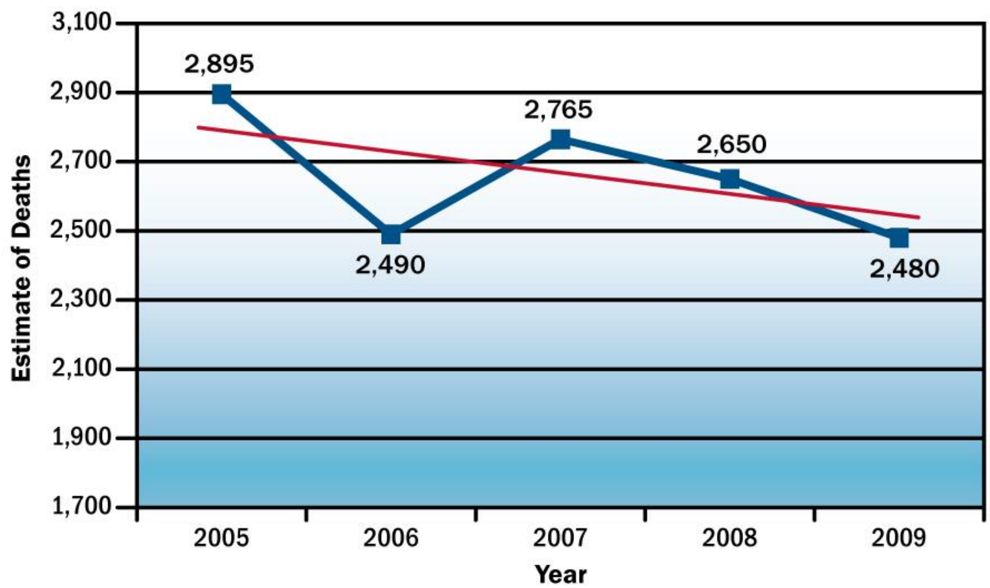

Figure 1.2 Residential Building Fire Deaths in the U.S. from 2005 to 2009
Source: http://www.usfa.fema.gov/

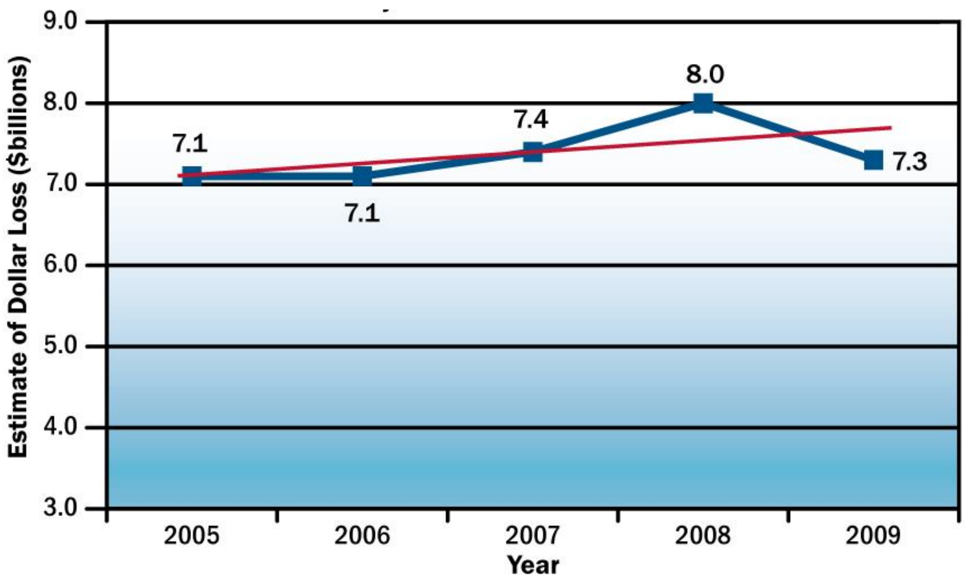

Figure 1.3 Residential Building-Fire Dollar Loss in the U.S. from 2005 to 2009
Source: http://www.usfa.fema.gov/

1.2 Emergency Response Centers

Emergency response centers such as police, fire, and medical centers play crucial roles in all communities. Their vital services can minimize the adverse effects of emergency incidents on civilians' life, health and property. Information is an essential component —for instance, traffic on the road network in the community, hospital vacancies, location and availability of emergency vehicles, and the characteristics of emergency incidents are four kinds of information without which these centers cannot be effective.

1.3 Emergency Call Center (911)

Usually the 911 call center has a room staffed by emergency personnel like police officers and firefighters, with several computer screens in front of them (see Figure 1.4). One screen is for displaying the pertinent information about the incoming call, such as telephone number and address, and another is for entering information on the emergency incident. Also, there is a screen showing the location of existing emergency vehicles in the system.

When a 911 center receives a call, 90% of the time the screen will display the address and phone number from which the call is being made. The 911 worker who answers the phone tries to get the basic information about the emergency. He will ask about the nature of the emergency in order to decide how many vehicles, and which types, should be dispatched to the scene.

Figure 1.4 Ramsey County 911 Call Center: ST. Paul, MN
Source: http://www.mcgough.com/

1.3 Emergency Response Time

There are different standards for measuring the effectiveness of an Emergency Response System; emergency-response time is one of the most important. The duration of an emergency can be divided into four phases (Yang et al., 2005):

1. Detection time: Time between the start of an emergency and the call to the 911 center.
2. Preparation time: Time between the call to the emergency center and the dispatch of emergency vehicles to the emergency site.
3. Travel time: Time required for emergency vehicles to reach the incident.
4. Treatment time: Time between the arrival of emergency vehicles at the scene and completion of the treatment.

Response time is interval between reception of the call at the emergency center and arrival of emergency vehicles at the site of the emergency (Figure 1.5).

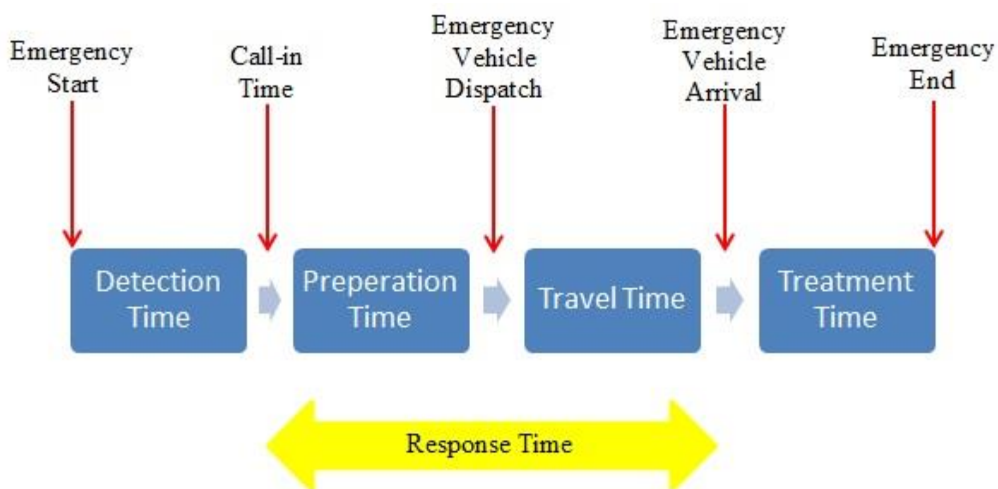

Figure 1.5 Emergency Response Time
Source: Yang et al., (2005)

Response time plays a crucial role in reducing the negative impacts of an emergency. The American Heart Association states that brain death starts to occur 4 to 6 minutes after cardiac arrest. This can be reversible if treated within a few minutes to restore a normal heartbeat. A victim's chances of survival are reduced by 7% to 10% with every minute that passes without advanced life-support intervention.

There is no official standard for response time in the United States, but the National Fire Protection Association's Standard 1710 (NFPA 1710), which is based on a combination of accepted practices and more than 30 years of research, establishes 5 minutes for the first-responder response time and 9 minutes for advanced life-support services; this objective should be met 90% of the time. Also, the NFPA 1710 states that the first fire engine company should arrive at a fire incident in 5 minutes, with full response in 9 minutes, 90% of the time (Yang, 2006).

1.4. Motivation for and Objectives of the Research

The importance of having an efficient emergency management system is undeniable. Many deaths, injuries, and loss of properties could be avoided by better planning for available resources and the execution of a better algorithm for dispatching emergency vehicles. Getting emergency vehicles to the emergency site in the required time is crucially important; sometimes decreasing response time by just several seconds can mean a victim’s survival.

The question is how to efficiently respond to these emergency incidents. In a greedy algorithm, one may decide to dispatch the closest vehicle to the incident. This algorithm may be useful when the system is not loaded, but definitely it will face substantial problems when the system is handling several emergency incidents at a time. It is obvious that by having better algorithms and operations, millions of lives and billions of dollars could be saved.

Response time is not only a function of the dispatching algorithm, but also has a very close relationship to the coverage of the area with emergency vehicles. If the system tries to satisfy coverage of the area with emergency vehicles, there will be an available vehicle nearby for future emergency incidents that can answer a call in an acceptable response time.

The goal of this dissertation is to develop a comprehensive relocation and dispatching model for emergency call centers or emergency management centers. This model can come up with the best relocation and dispatching algorithm based on real-time information about the status of the emergency-response fleet, traffic information, likelihood of emergency happening at the demand nodes and the status of emergency calls.

1.5. Contributions of the Research

Three categories of emergency vehicles are considered in the system: police, ambulance, and fire. The police department is assumed to have a homogeneous fleet, but ambulances and fire vehicles are heterogeneous. Two kinds of ambulances (Advanced Life Support and Basic Life Support) are considered in the model and three types of fire vehicles (Fire Engine, Fire Truck, and Fire Quint), for a total of six vehicle types. There is no dispatching model in the literature that considers non-homogenous vehicles. By having a heterogeneous dispatching algorithm, we can have better allocation of the available resources. For example if a fire quint is sent to an emergency scene it can perform the job of one fire engine and one fire truck.

Also, area coverage is an important part of this model and is addressed thoroughly. First of all, the model tries to cover the demand nodes within a predefined time (T_1minutes), which can be different for each vehicle type. Demand nodes that are not covered within T_1 minutes are, ideally, covered within T_2 minutes ($T_1 \leq T_2$). By having two specific times for coverage, equity is increased between the different demand nodes in the system. This part is also new to the literature. There are some researchers that assume adequate number of vehicles and try to cover all demand nodes within T_2 minutes and a specific percentage of demand nodes in T_1 minutes. However our model is more realistic, because most of the time vehicles are not sufficient to cover all demand nodes and in that case we are trying to increase equity between different demand nodes.

In addition two kinds of demand nodes are considered: ordinary demand nodes and critical demand nodes. Critical nodes are important demand nodes such as critical infrastructures, hospitals and schools, for which an emergency can have negative impacts on the performance of the whole system. Having two kinds of demand nodes is also new in the emergency vehicle coverage problem and this assumption increases the flexibility of applying different policies for different demand nodes.

Also, the model attempts to provide double coverage for ordinary nodes within T_2 minutes and double coverage for critical nodes within T_1minutes. There is no double coverage model in the literature that considers heterogeneous vehicles.

In addition the proposed model is capable of considering the benefits of partial coverage. For example in the fire vehicles category, a demand node will have full coverage if it is covered by one fire engine and one fire truck or one fire quint in the predefined time. However if it is only covered by either a fire engine or a fire truck the node will have partial coverage. Also, there is no model in the literature that addresses the full coverage and partial coverage together in vehicle relocation problem.

This model attempts to strike a work-load balance between different vehicles in the system. It is not desirable, for instance, for one vehicle to work 90% of the time and another one only 10% of the time. Therefore, the model attempts to balance the work load between vehicles which is new in the literature.

In addition, some real-life issues are taken into consideration, such as the fact that vehicles prefer to stay at their home stations instead of being relocated to other stations and that at the end of their work shifts they should be returned their home depots. Such a model is a unique and complex tool; to date, there is no study in the literature that has addressed this problem sufficiently.

1.6. Organization of the Dissertation

Previous work on emergency-fleet management will be reviewed in Chapter 2. The problem statement is described in Chapter 3 and the mathematical formulation of the model is presented. Chapter 4 shows a set of numerical problems that are solved with Xpress 7.1 software using the proposed model. These numerical examples are designed to demonstrate the capabilities of the proposed model. Then, to illustrate how the running time will increase by increasing the problem size, problems with different sizes are generated. The results show that by increasing the problem size, running time grows exponentially; commercial software such as Xpress is not suitable for this purpose. So, heuristic methods should be used to find near-optimal solutions in more reasonable time. In chapter 5, the developed heuristic method that has been coded in C^{++} language is introduced and explained in detail. Then its results are compared to optimal solutions. In chapter 6, a sophisticated simulation model that has been developed for this research and has been coded in C^{++} language is explained in detail. In chapter 7, the input analysis on the case study data on one of the counties in the Washington, DC metropolitan area is explained and the distributions of the different inputs are shown. In chapter 8, first the results of applying the proposed model on the case study is shown and then an extensive sensitivity analysis is applied on some important parameters in the model. Finally, in Chapter 9, the summary of this research is explained and some areas for future research are discussed.

Chapter 2: Literature Review

In this chapter a complete review of the emergency vehicle location problem and the emergency vehicle deployment problem will be presented.

2.1 Introduction

The goal of this dissertation is to improve the Emergency Vehicle Management System. One of the key effective measurements of the system is response time. Response time is not only related to the dispatching system, but also it has a close relationship to emergency vehicle coverage. In this dissertation, therefore, the proposed model relocates the emergency vehicles to provide better coverage for the whole system and also when an emergency happens in the system the model will consider dispatching and relocation problem simultaneously. Two areas in the literature will be reviewed, the emergency vehicle location problem and the emergency vehicle dispatching problem.

2.2 The Emergency Vehicle Location Problem

The models in the literature that address the emergency vehicle location problem can be classified in three main categories: deterministic models, probabilistic models, and dynamic models. Each model and the papers that used that kind of model are reviewed in the following subsections. Revelle et al. (1977), Batta et al. (1990), Marianov et al. (1995), Brotcorne et al. (2003), Goldberg (2004), Jia et al. (2005), Erkut et al. (2007, 2009), and Morohosi (2008) conducted comprehensive literature surveys on the emergency vehicle location and relocation problem, and Schilling et al. (1993) did a complete survey on coverage problems with facility location.

2.2.1 Deterministic Models

These models are usually used at the planning stage and ignore the stochastic nature of the emergency vehicles regarding their unavailability. Two of the early deterministic models were called location set covering problem (LSCP) proposed by Toregas et al. in 1971 and maximal covering location problem (MCLP) proposed by Church and ReVelle in 1974. These two models were early models for the static ambulance location problem. Several extensions of both models have been proposed in the emergency vehicle location literature. The location set covering problem (LSCP) determines the minimum number of ambulances needed to cover all demands, and the maximal covering location problem (MCLP) model attempts to maximize population coverage based on the number of available ambulances. In the latter model it is assumed that resources are limited, which is a true assumption in most cases.

These two models considered only one type of vehicles; also, they did not consider the case in which vehicles would be unavailable when they are handling an emergency incident. Therefore, some extensions of these models have been proposed in the literature to counter some of their shortcomings. Schilling et al. (1979) developed one of the first models to handle two types of vehicles. Their model is called the tandem equipment allocation model (TEAM) and can be applied to fire companies using two types of vehicles (fire engines and fire trucks). Also by a very small change it can be applied to ambulance location problem, because two types of vehicles are usually being used, the Basic Life Support (BLS) and Advanced Life Support (ALS) ambulances.

In the models that have been reviewed so far, some points may lose their coverage whenever some vehicles become busy. A strategy to handle this problem is trying to provide multiple coverage for demand nodes. In this case, if one vehicle becomes busy there is still another vehicle left to cover the area. Daskin and Stern (1981) extended MCLP and used a hierarchical objective to maximize the number of demand points that have multiple coverage.

Hogan and ReVelle (1986) presented two backup coverage models called BACOP1 and BACOP2. In BACOP1 they tried to cover all demand points with P ambulances, and at the same time they maximized the number of demands that are covered twice. In BACOP2 they used weights in the objective function for demands that are covered once and twice and attempted to maximize the total objective function with a total of P ambulances.

Eaton et al. (1986) extended Daskin's model and maximized the multiple coverage of demand in the predefined time with a minimum number of ambulances. They used a heuristic method to solve their model and used it on the Santo Domingo emergency medical system (EMS) in the Dominican Republic.

Haghani (1996) proposed two formulations for capacitated maximum covering facility location models. He is maximizing the covered demand by his first proposed model. In the second one he is maximizing the weighted covered demand

and minimizing the average distance from the uncovered demands at the same time. Also, he proposed two heuristic approaches to solve these two models.

Gendreau, Laporte and Semet (1997) proposed a model named the Double Standard Model (DSM). They use two coverage standards, r_1 and r_2, with $r_1 \leq r_2$, and they maximize the demands that are covered twice in r_1 minutes with P available ambulances when they are covering all the demand points in r_2 minutes and proportion α of the demand in $r_1$minutes.

As Brotcorne et al. (2003) note, the United States Emergency Medical Services Act of 1973 set 10 minutes for r_1 and 95% for α, but there is no value for r_2. However, by having the r_2 coverage constraint, the equity between demand nodes can be better addressed.

2.2.2 Probabilistic Models

These models reflect the fact that sometimes emergency vehicles are busy and are not always available to answer an emergency call. Chapman and White (1974) proposed the first probabilistic emergency model. They developed a probabilistic location set covering model called the maximum expected covering location problem (MEXCLP) and assumed that servers are not always available.

Daskin (1983) proposed a simplified version of maximum expected covering location problem for ambulances. He assumes that the ambulances are independent and the busy fraction of all ambulances is the same and equal to q.

Many researchers have used the MEXCLP model or a later extension of it in their research. Fujiwara et al. (1987) used this model in Bangkok, where they decreased fleet size from 21 to 15 and maintained the same performance level.

Goldberg et al. (1990) extended MEXCLP and considered stochastic travel times in their model, maximizing the expected number of calls covered within 8 minutes. They used their model on data from the city of Tucson, Arizona, and increased the number of calls covered in 8 minutes by 1% and the worst covering ratio of a zone from 24% to 53%.

Also, Repede and Bernardo (1994) developed a model called TIMEXCLP by extending MEXCLP. They considered variations in emergency vehicle speed throughout the day. They combined their model with a simulation module and applied it to Louisville, Kentucky data. The proportion of calls covered in 10 minutes was increased from 84% to 95% as a result, and response time decreased by 36%.

ReVelle and Hogan (1989) proposed two other probabilistic models formulating the maximum availability location problem (MALP I and MALP II). In MALP I they assume that the busy fraction is the same for all potential location sites, but in MALP II this assumption is relaxed and they estimate the busy fraction for each location site.

Estimating the busy fraction for each vehicle is a difficult task, since these values are the output of the models, but other researchers have addressed this. Some of them, like Larson (1974, 1975) and Burwell et al. (1992) used a hypercube analytical tool to come up with busy fractions. Others, like Jarvis (1975) and Fitzsimmons and Srikar (1982), used an iterative optimization algorithm; Davis

(1981) and Goldberg et al. (1990, 1991) used simulation models to find busy fractions.

Betta et al. (1989) developed an extended MEXCLP and called their model AMEXCLP. They relaxed the assumption that ambulances are independent and used a hypercube model to estimate busy fractions, which are assumed to be the same for the whole system.

Marianov and ReVelle (1994) proposed the queuing probabilistic location set covering problem (QPLSCP), in which the busy fraction is site-specific.

Ball and Lin (1993) extended the LSCP model to achieve a given reliability level. Their model, the Rel-P model, attempts to ensure that the probability of all vehicles' being unavailable to answer a call does not exceed a predefined value.

In the probabilistic models that have been discussed so far, only one type of vehicle is considered. Marianov and ReVelle in 1992 proposed a probabilistic fire protection siting model for fire vehicles. The demand points in their network are considered to be covered if they are covered by a fire engine and a fire truck.

Also, Mandell (1998) considered two types of vehicles (ALS and BLS) in their two-tiered model, which is called TTM. They assume that the probability of a call being served in a demand point i is related to the number h of ALS vehicles within r^A minutes of i, to the number k of ALS vehicles within r^B minutes of i, and to the number l of BLS vehicles within r^B minutes of i. Then they maximize the probability that the calls are served in the whole system.

Beraldi et al. (2009) modeled the problem of designing and planning emergency medical services as two-stage stochastic programming with probabilistic constraints. An exact solution method and three heuristics have been developed to solve this model, which considers only one type of vehicle.

Lei et al. (2009) considered the problem of optimal deployment of limited ERS units in a metropolitan area to cover critical infrastructures with time-dependent service demand and travel time in the system. They used their model in a case study in the city of El Paso, Texas, with 45 firefighting units and 23 ambulances among 34 fire stations to cover 138 critical transportation infrastructures, hospitals, and schools.

Sorensen et al. (2010) proposed the LR-MEXCLP, a hybrid model combining the maximum coverage objective of MEXCLP with the local busyness estimates of MALP and showed that LR-MEXCLP results in a modest but consistent service gains over both MEXCLP and MALP.

Ingolfsson et al. (2008) presented an optimization model for allocating a specified number of ambulances to stations to maximize expected coverage. Their model considers variation in pre-travel delay, variation in travel time, and uncertainty in ambulance availability. Only one type of vehicle, however, is considered.

Another group of probabilistic models uses queuing theory. These models reflect the fact that emergency vehicles operate as servers in a queuing with exponential arrival time, exponential service time, and c servers (M/M/C), which are not always available to answer an emergency call. Larson (1974) was one of the first to use this concept and developed a hypercube queuing model as a tool for urban emergency service facility location. In 1975 he developed a computationally efficient algorithm for a multi-server queuing system with distinguishable servers. In his models, one server was assigned to each call. (Yang, 2006)

Chelst and Jarvis (1979) and Chelst and Barlach (1981) extended the hypercube queuing model. In the former study, the probability distributions of travel time are estimated; in the latter, the assumption of only one server for each call is relaxed, and the model can capture the simultaneous response of two identical units dispatched to a call. However, calls for which there is no available server are lost, which is not the case in real-world emergency operations. (Yang, 2006)

In a two-server, two-customer system, Halpern (1977) demonstrated that a more accurate approximation of travel time is required for service time in a hypercube model and that the assumption that service time is independent of the locations of calls and dispatched units may not be realistic. (Yang, 2006)

Barker et al. (1989) developed an integer nonlinear mathematical programing model based on a multiple-channel queuing system to allocate ambulances to different sectors in a county in South Carolina. (Yang, 2006)

Marianov et al. (1996) proposed a queuing maximal availability model for emergency vehicle location that is an extension of the MALP model and is called Q-MALP. In their formulation, the probability of different servers' being busy is not independent, and they use queuing theory to find these probabilities.

Iannoni et al. (2007) extended the hypercube model to analyze emergency medical systems (EMSs) on highways. They assumed that emergency calls can be of different types and considered two types of vehicles in their system (rescue ambulances, medical vehicles). Also, their model can send multiple vehicles to incidents. They applied their model to a case study of an EMS operating on Brazilian highways. The same authors in 2009 combined adoption of the hypercube model with hybrid Genetic Algorithms to optimize the operation and configuration of EMSs on highways. They showed that the main performance measures, such as mean response time, fraction of calls not covered within a predefined time, and imbalance of ambulance workloads, could be improved by relocating ambulance bases and simultaneously determining the system's district sizes. In 2010 they proposed a series of simple and straightforward greedy heuristic algorithms to optimize large-scale EMS on highways.

Morabitoa et al. (2008) compared homogeneous servers with nonhomogeneous servers in a hypercube queuing model and found that, even when the degree of non-homogeneity of the servers is not significant, homogeneity may result in poor predictions of the actual operational characteristics of nonhomogeneous systems.

Geroliminis et al. (2009) developed a spatial queuing model (SQM) for locating emergency vehicles in urban networks while considering the probability that a server is busy. They assume that service rates may vary among servers and are dependent on incident characteristics. In 2011 they used a hybrid hypercube–Genetic Algorithm approach for deploying emergency response mobile units in an urban network.

In most of the models reviewed in this subsection, only one type of vehicle is considered, and only one model (TTM) considers two types.

In this dissertation, two types of ambulances and three types of fire vehicles are considered; therefore, the model developed is much more sophisticated than all previous models. The proposed model also accounts for many real-world issues that models currently in the literature are not able to accommodate.

2.2.3 Dynamic Models

These models have been developed recently to repeatedly relocate emergency vehicles throughout the system to provide better coverage for future demand and not leave areas unprotected. The first paper on this kind of model was written in 1974 by Kolesar and Walker, who used a dynamic model to relocate fire vehicles in New York City, which they solved by developing a heuristic algorithm.

Berman (1978) proposed a model for dynamic positioning of mobile servers on networks. He also extended his work on Markovian and stochastic networks in 1982, 1984, and 1985, in collaboration with colleagues, and they investigated the location and relocation of mobile facilities.

Carson et al. in 1990 worked on the relocation of a single ambulance on the Amherst campus of SUNY Buffalo as a case study and they relocated the ambulance with the moves of population throughout the day from classroom buildings to dining halls to dormitories and improved the level of service. (Yang, 2006)

Gendreau et al. (2001) developed an ambulance relocation model by expanding the Double Standard Model (DSM) they proposed in 1997. The result was their dynamic model, called the Dynamic Double Standard Model (DDSM). They maximized the demand covered at least twice in r_1 minus the cost for relocating the ambulances, while covering all the demand points in r_2 and proportion α of the demand in r_1. They used a fast parallel Tabu search heuristic to solve their model and implemented it on the Island of Montreal. They also in 2006 formulated the maximal expected coverage relocation problem for emergency vehicles. In considering the fact that the vehicles are not always available, they tried to relocate physicians' cars so as to have maximal expected coverage. They used CPLEX software to solve their model, which can be solved in a reasonable amount of time when there are few vehicles. This is the case for physicians' cars on the Island of Montreal. Their model is helping to decrease average response time and increase the proportion of calls that are covered in 8 minutes.

Sathe and Miller-Hooks (2005) proposed a model for location and relocation of response units for guarding critical facilities while considering double coverage, as well as probabilistic demand and travel times.

Nair and Miller-Hooks (2009) adopted Sathe and Miller-Hooks' 2005 model and proposed a multi-objective, probabilistic, integer program to investigate whether there are benefits in relocating EMS vehicles. They used concept of probabilistic models in dynamic models to develop their own. They compared it with a static location policy, and their research shows that with scarce resources, relocation is a better alternative for increasing the level of service and decreasing average response time.

Rajagopalana et al. (2008) proposed a set covering location model for dynamic redeployment of ambulances. They consider that the demand for ambulances fluctuates depending on the day of the week and time of day, and determine the minimum number of ambulances and their locations in each time interval while satisfying coverage requirements with a predefined reliability. They consider only one type of vehicle in their model.

Schmid et al. (2010) developed a multi-period double standard model by extending the DSM model and considered time-dependent variation in travel times. They attempted to optimize the tradeoff between coverage and the number of vehicle relocations using a Variable Neighborhood Search heuristic method to solve their model. Their model also considers only one type of vehicle.

Curtin et al. (2010) use Maximal Covering and Backup Covering Location Models to relocate available police vehicles to police patrol centers in order to have better system-wide coverage. A range of solutions that include tradeoff between maximal backup coverage and maximal coverage is presented. They used their model on the city of Dallas, Texas, and the results showed improvements in police efficiency.

2.3 The Emergency Vehicle Dispatching Problem

The most crucial role of a dispatcher is sending the required type(s) and numbers of emergency vehicles within each category to the emergency site based on the incident's characteristics. Also, these vehicles should reach the incident within the required time to be most effective in dealing with the emergency situation. When there are only a few emergencies in the system, this is easy and there is no need for sophisticated algorithms. The problem arises when the system is loaded with several emergencies. In that case, the dispatcher may decide to send the vehicles to more severe emergencies and let the minor ones be delayed. Clearly, having an effective dispatching system is essential in emergency fleet management. This kind of problem generally belongs to the Generalized Assignment Problem (GAP), which assigns n tasks to m machines in the best way. This problem is NP-complete and there have been several papers on this problem. As Yang (2006) states: some researchers, such as Ross et al. (1975), Martello et al. (1984), and Fisher (1986), have tried to find an exact solution for this kind of problem. Since it is NP-complete, finding an exact solution is very time-consuming; as a result, many studies have used heuristic methods. Some of them, such as Brown et al. (1985), Nulty et al. (1988), Trick (1992), Lorena et al. (1996), and Narciso et al. (1999) use linear relaxation. Others, such as Chu (1997) and Lorena (2002), use Genetic Algorithm heuristics. Catrysse et al. (1992) did a survey of studies using heuristic methods for the GAP.

Also, some studies exist that address the dispatching of emergency fleets specifically; Chaiken and Larson (1972) did a survey of methods for allocating urban emergency units.

Haghani et al. (2003) proposed a mathematical model for dispatching EMS. They used an integer model to minimize the total travel time, and considered a time-dependent shortest-path algorithm in their formulation. One of the deficiencies of their work is that only one type of vehicle is considered in their model. Also, they assumed that each emergency call requires only one vehicle, which may not be realistic in most cases.

Haghani et al. (2004) developed an integer programing model to deal with dispatching and used a dynamic shortest-path algorithm in their formulation. They also used a simulation model to test their model's performance.

The most complete dispatching model in the literature was developed by Yang et al. (2005). Their model can dispatch several emergency vehicles of each type to the emergency incident; they consider three categories of vehicles (police car, ambulances, and fire vehicles), but the fleets are homogenous in each category.

2.4 Simulation Models

This kind of model can provide information about the effect of a proposed policy change and show how the whole system works. Also, it can provide information on a wide range of variables like response times, workload of units, delays, and unit availability. In addition, they can be used as a tool to measure system performance.

Savas (1969) used a simulation model to test the impact of spatially repositioning ambulances on the reduction of travel times. He used his simulation model on New York City's emergency ambulance service.

Swersey (1970) used a simulation model to analyze the operations of the dispatch centers of the New York City Fire Department. Carter and Ignall (1970) used a simulation approach to compare a wide range of combinations of fire department allocation policies. Also, Adams and Barnard (1970) studied the value of an automated dispatch system for the San Jose, California, Police Department by using simulation models.

Fitzsimmons (1973) developed a Computerized Ambulance Location Logic model (CALL) for ambulance deployment and applied the model to data from the City of Los Angeles. Lubicz and Mielczarek (1987) used their simulation model to classify calls into several priorities. Goldberg et al. (1990) proposed a first-call, first-serve multi-server queuing system.

Haghani et al. (2003) developed an optimization model for a real-time dispatching strategy and they developed a simulation model to test their formulation and see how their optimization model performed.

Haghani et al. (2004) developed a simulation model and tested different dispatching strategies, such as the first-called, first-served strategy, nearest-origin assignment strategy, and real-time flexible assignment strategy. They used their model with these strategies under different circumstances, like different emergency arrival rates, route-change strategies, and dynamic travel times. Their model was implemented using data from the Arlington County, Virginia, Fire Department.

The deficiency of most of these models is that they only consider one type of vehicle, and in their simulation model they assume that each emergency incident requires only one emergency vehicle.

Yang et al. (2004) developed one of the most complete simulation models for EMS vehicle dispatching. This model is integrated in a Genetic Algorithm to solve an EMS location and assignment problem. Different emergency types, their response priorities, and the dispatching of multiple units are the characteristics of their model that made it more sophisticated and realistic compared to other models in the literature. Yang et al. (2005) proposed an optimization model for real-time dispatching and routing of emergency vehicles and developed a simulation model to check the model's performance.

Bjarnason et al. (2009) used a simulation optimization model called SOFER for placement of resources and emergency response.

2.5 The Emergency Vehicle Dispatching and Relocation Problem

More recently, researchers have addressed the dispatching and relocation of emergency vehicles simultaneously. They consider the fact that by dispatching some vehicles to emergency incidents, other areas of the system may be left without good coverage and, as a result, the response time for future calls from that area increases. In this situation, if the model simultaneously dispatches vehicles to the emergency scene(s) and relocates remaining emergency vehicles to provide better coverage of the entire area, total average response time will decrease and level of service will increase.

Lee (2010) used a dispatching algorithm based on a quantitative definition of preparedness of the area (proposed by Andersson et al. in 2007). The area is considered to be ill-prepared if there is not enough coverage for it. He showed that the consideration of preparedness in ambulance dispatching can provide significant benefits in reducing response time if it is combined with greediness in the conventional rule, to consider both current (greediness) and future (preparedness) calls. In this dispatching system, the calls have the same importance; no distinctions are made as to priority of calls.

Yang (2006) and Yang et al. (2005) were the first to address this kind of problem, with one of the most complex models in the literature. The model considers different categories of vehicles (police, fire, ambulance). Also, based on the severity of the emergency, the model will dispatch the appropriate types and required numbers of vehicle in each category to the incident; in addition, the model tries to have the vehicles arrive within the required time. Diversion of vehicles and reassignment to new destinations is also considered in the model. It also attempts to relocate vehicles throughout the system so as to provide better coverage. If the overall coverage for each type of vehicle is less than ρ_k, then a penalty would be added to the objective function. Yang used a Genetic Algorithm heuristic method to find good solutions for her model in the appropriate times. Also, she used a thorough simulation model to check the performance of her optimization model and used it on real-world data. One of the model's deficiencies is that vehicles in each category are homogenous. She does not consider the fact that if, for example, an Advanced Life Support ambulance gets to the incident sooner than a Basic Life Support ambulance, there is no need for the Basic Life Support ambulance; also, the Fire Quint can perform as both the Fire Engine and the Fire Truck.

The model in this research, in contrast, has the capability to consider a heterogeneous fleet. The coverage aspect of Yang's model is very simple; for instance, she only attempts to cover the area once, and whenever the overall coverage is less than ρ_k, a penalty is added to the objective function. The coverage criteria in the proposed model are sophisticated. We attempt to cover demand nodes within T_1minutes, and if some of the nodes are not covered within T_1 minutes, we seek to cover them within T_2 minutes. By this criterion, the equity between demand points is increased. We also attempt to have double coverage within T_1 minutes for critical nodes in the system and double coverage within T_2 minutes for ordinary nodes in the system. There is no model in the literature that considers heterogeneous fleet in double coverage problems. Also, this model is capable of considering partial and full coverage in heterogeneous categories which is new to the literature. In addition we consider two types of demand nodes: ordinary demand nodes and critical demand nodes which is new to the literature. Some other considerations related to the crews of these vehicles in the proposed model make it highly useful and suitable for application to real-world systems, which will be discussed in the next chapter.

Chapter 3: Problem Statement and Mathematical Formulation

In this chapter, first the problem and its properties will be described completely and then the assumptions addressed in the problem will be provided. The notations, parameters, and variables in the model will be described, and, at the end, the mathematical formulation will be presented. In the mathematical formulation, the objective function will be explained first, after which a brief description of every constraint is provided.

3.1 Problem Statement

Emergency response services such as police, fire, and medical services can minimize the negative effects of emergency incidents by decreasing the response time. Response time is not only related to the dispatching system, but also has a very close relationship to the area coverage of the network by emergency vehicles. The goal of this dissertation is to develop a model for an Emergency Management System. This model is a dynamic model that will relocate the emergency vehicles to provide better coverage for the whole system and also when an emergency happens in the system the model will consider dispatching and relocation problem simultaneously. In summary, this model will assist the emergency management centers in dispatching vehicles to emergency sites, and relocating vehicles to provide better coverage for the whole area. The characteristics and components of this problem are explained in this section.

3.1.1 The Dispatching Problem

Nowadays, because of advanced technologies like global positioning systems and powerful computers, interaction between different departments and components of a system is possible. For instance, most 911 call centers have access to the location of the emergency vehicles and their status all the time. Also, they can access traffic data on the network and information on hospital vacancies. When an emergency call comes to an emergency center, based on the emergency situation the dispatcher should decide the number of vehicles and types of vehicles that should be sent to the scene and also the required time within which those vehicles should get to the emergency location—the more severe the emergency, the less required time. Also, based on the location of the vehicles and their availability, the dispatcher will send them from different depots to the emergency site. Some of the vehicles, like ambulances, should take patients to hospitals after ambulance personnel are done working at the emergency site, so the dispatcher should send them to the nearest available hospital. In addition, when real-time traffic data are available, the dispatcher can provide the vehicles with the shortest-time route. Also, sometimes the dispatchers may decide to divert vehicles from their previous destinations to a new one. For example, if a vehicle is going to a station and another emergency happens it may be diverted to the emergency site. Another situation will be illustrated in Figure 3.1. Suppose that there are only two stations for the emergency vehicles in the system, and each one has only one vehicle. At time t an emergency happens at node 1 and the dispatcher sends the nearest vehicle, which is from station 2.

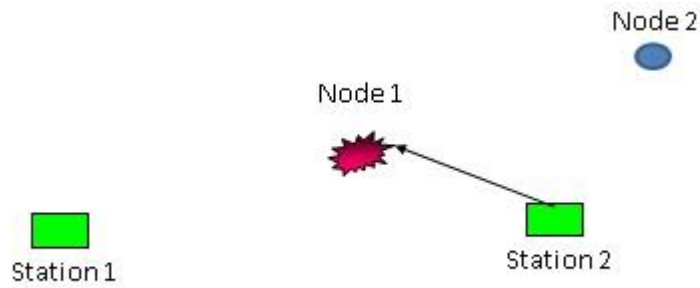

Figure 3.1 A Simple Example of a Dispatching Problem

Then, at time t+1, something else happens at node 2 in the system. At this time, the vehicle dispatched to node 1 is still en route and not at the emergency site yet. Then the dispatcher has two options: The first option is letting vehicle 2 continue to the previous task in node 1 and send the vehicle from station 1 to node 2 (Figure 3.2). The second option is diverting vehicle 2 to node 2 and dispatch the vehicle 1 to node 1 (Figure 3.3).

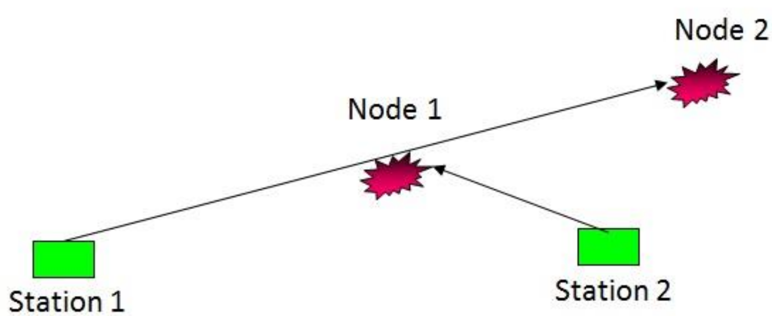

Figure 3.2 A Simple Example of a Dispatching Problem

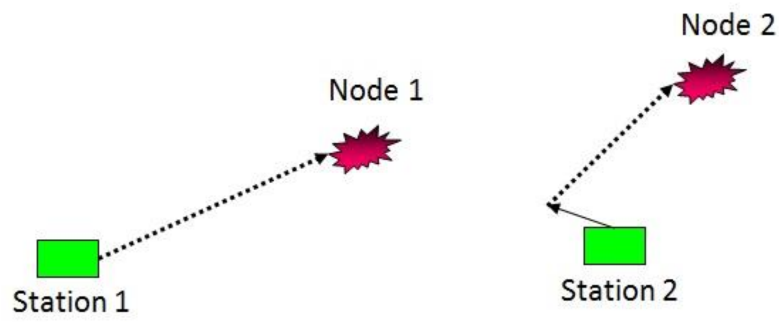

Figure 3.3 A Simple Example of a Dispatching and Diverting Problem

By comparing Figures 3.2 and 3.3, it can be seen that the total response time will be decreased by considering the diverting situation. It is a very good option when the emergency in node 2 is a severe one, but if the emergency in node 1 is very severe and the other one is less severe, the dispatcher may prefer to use Figure 3.2's situation. Therefore, by this simple example it is shown that having a good dispatching model is essential.

Another thing that should be mentioned here is that the diverting is not easy for drivers and may confuse them. Therefore, whenever there is only a slight positive impact on the whole system, diverting will not be considered. It is an option in the proposed model only when there is at least a minimum amount of benefit to the whole system. By this minimum threshold of benefit, the model avoids having too many diversions in the system.

3.1.2 The Coverage Problem

Response time is not only related to the dispatching system, but also to coverage of the whole area for future demands. When the vehicles in an area are busy that section is left without coverage for future demands, and in this situation the emergency fleet manager may decide to relocate the vehicles to have better coverage of the whole area for future demands. Several models for the coverage problem were investigated and explained in the literature review chapter. In this study, several criteria for coverage are considered.

The real street network is considered as a graph with n nodes and m directed links. If the network is detailed enough to include lots of nodes, this assumption is reasonable (Yang, 2006). These demand nodes belong to two categories, ordinary nodes and critical nodes. These two categories of demand nodes have different criteria for coverage.

The critical nodes are the ones that are strictly important to the system like hospitals and schools during school sessions. Also, each city has some infrastructures on which the city depends, such as subways that are also critical nodes. All other nodes in the system are ordinary nodes.

In this model two time limits for each type of vehicle are considered: T_1 minutes and T_2 minutes, with $T_1 \leq T_2$. The goal is to cover all demand nodes within T_1 minutes first, but if some are not covered within T_1 minutes, the goal is to cover them within T_2 minutes. By this assumption the equity between different nodes is increased. For instance, it is preferable to cover all nodes with a BLS ambulance in 5 minutes, but if some nodes are not covered with a BLS in 5 minutes, they should be covered in 8 minutes. Being covered in 8 minutes is better than being covered in 20 minutes. Also, the model attempts to have double coverage for ordinary nodes within T_2 minutes and for critical nodes within T_1 minutes. When the nodes are covered twice it can have a positive impact on the response time, because if one vehicle gets busy in an area there is still another vehicle left to cover future demands.

The coverage problem in this model can be explained in a simple example. Figure 3.4 shows a small network that has only 6 nodes and 3 stations. All nodes in this example are ordinary nodes and only one type of vehicle (BLS) is considered. There is only one vehicle in each staion and there is no emergency at time t in the system. Node 1 and node 2 are covered twice within T_1 minutes, and node 4 and node 5 are covered once within T_1 minutes and twice within T_2 minutes. Also nodes 3 and 6 are covered once within T_1 minutes.

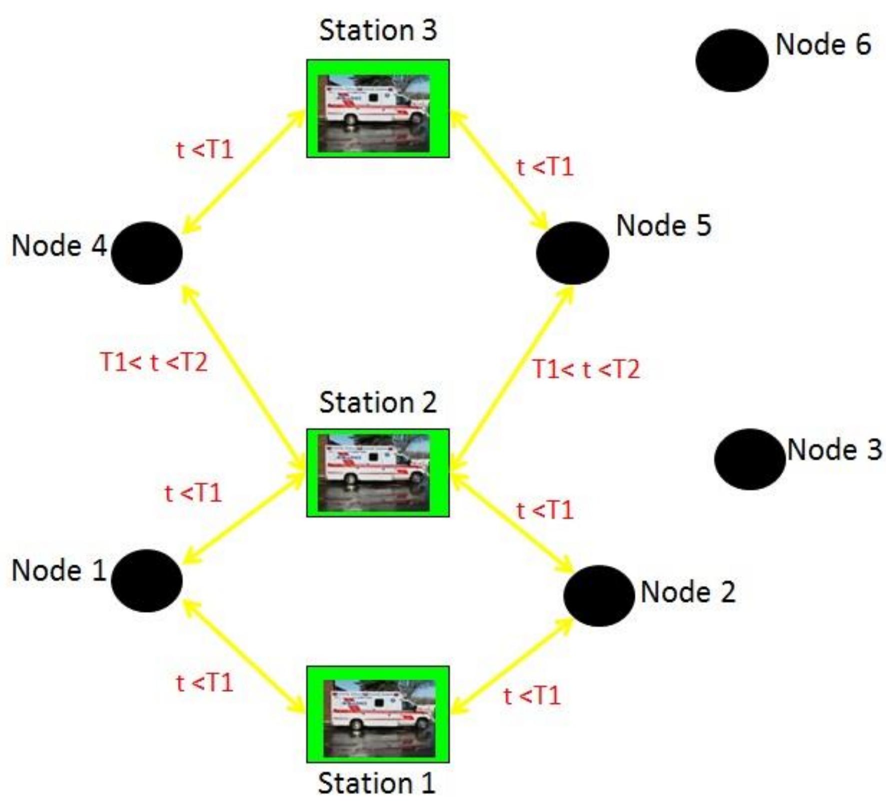

Figure 3.4 A Simple Example of a Coverage Problem

Suppose at time t+1 an emergency happens at node 3 and vehicle 2 from station 2 is dispatched to the emergency location at node 3 (Figure 3.5). In this situation, all the remaining nodes in the system are covered once within T_1 minutes, but if vehicle 1 from station 1 gets relocated to station 2, then nodes 1 and 2 will have the same situation and nodes 4 and 5 will have double coverage within T_2 minutes. If, in the near future, something happens—for example, at node 6—and vehicle 3 gets dispatched to that node, nodes 4 and 5 still have one coverage within T_2 minutes. Therefore, depending on the relocation cost of the vehicle, the likelihood of an emergency happening in those nodes, and other considerations, the relocation may take place.

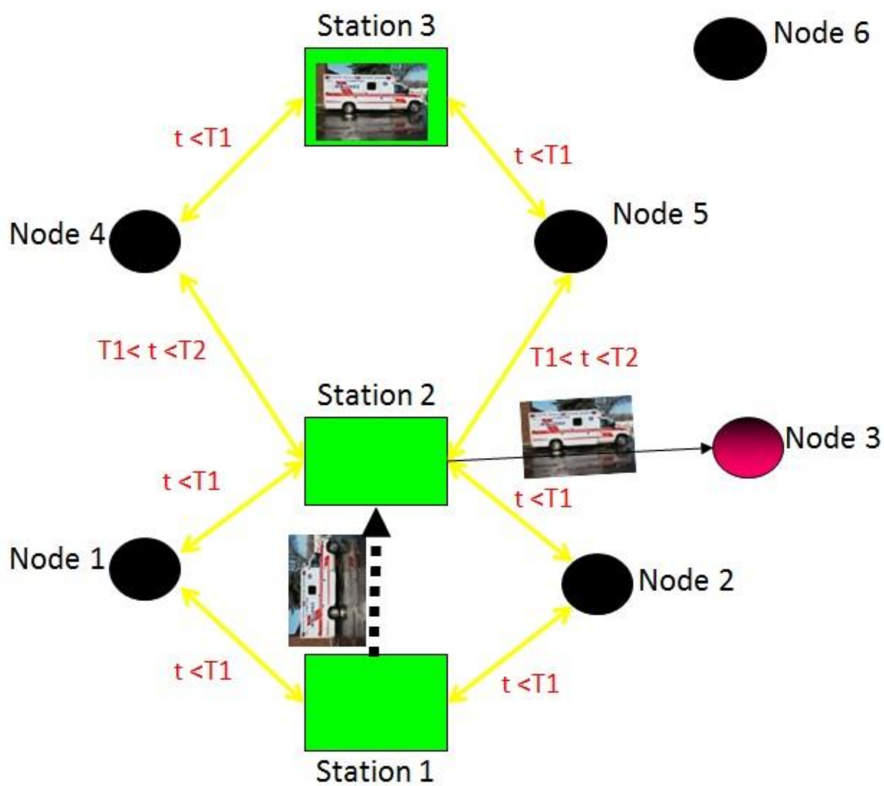

Figure 3.5 A Simple Example of a Dispatching and Relocation Problem

Another scenario that is helpful in illustrating the model's properties is the occurrence of two emergencies at the same time at node 3 and node 6. A vehicle from station 2 is dispatched to node 3 and a vehicle from station 3 is dispatched to node 6 (Figure 3.6). In this situation, node 1 and node 2 have one coverage within T_1 minutes, but node 4 and node 5 are not covered. If the vehicle from station 1 is relocated to station 2, then node 1 and node 2 will have the same situation; however, the situation will be improved for node 4 and node 5 and they will be covered within T_2 minutes. Therefore, by this relocation the model attempts to increase the equity between demand nodes.

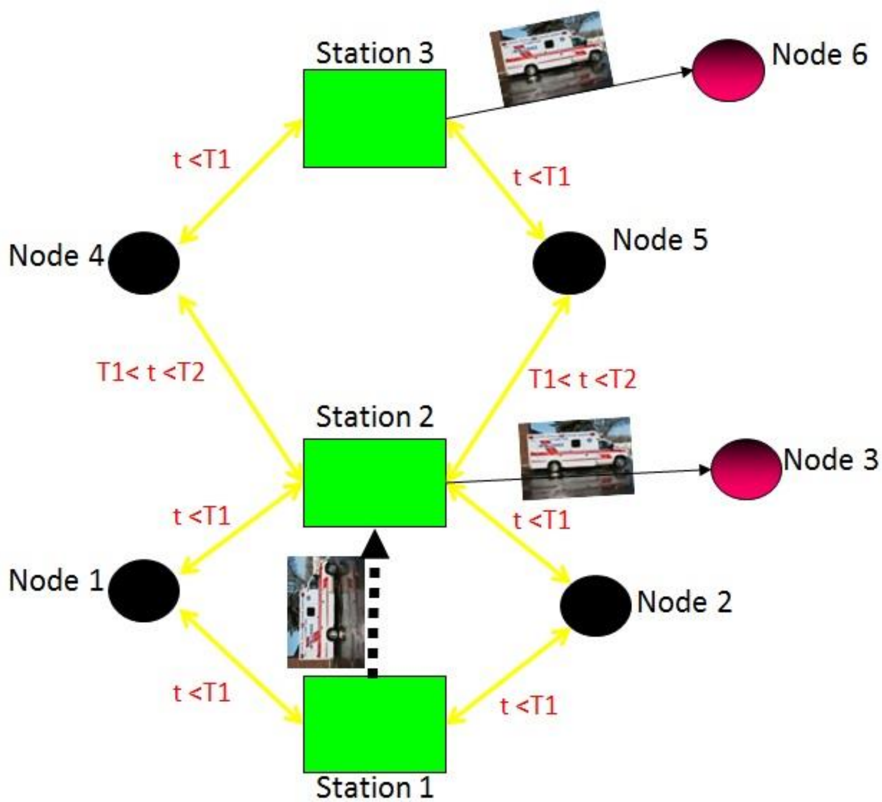

Figure 3.6 A Simple Example of a Dispatching and Relocation Problem

Also if, for example, two emergencies happen at the same time at node 3 and node 6—but the emergency at node 3 is not very severe—and the vehicle from station 1 can satisfy the required service time, the model will send a vehicle from station1 to node 3 and let vehicle 2 stays in station 2 to cover future demand nodes (Figure 3.7).

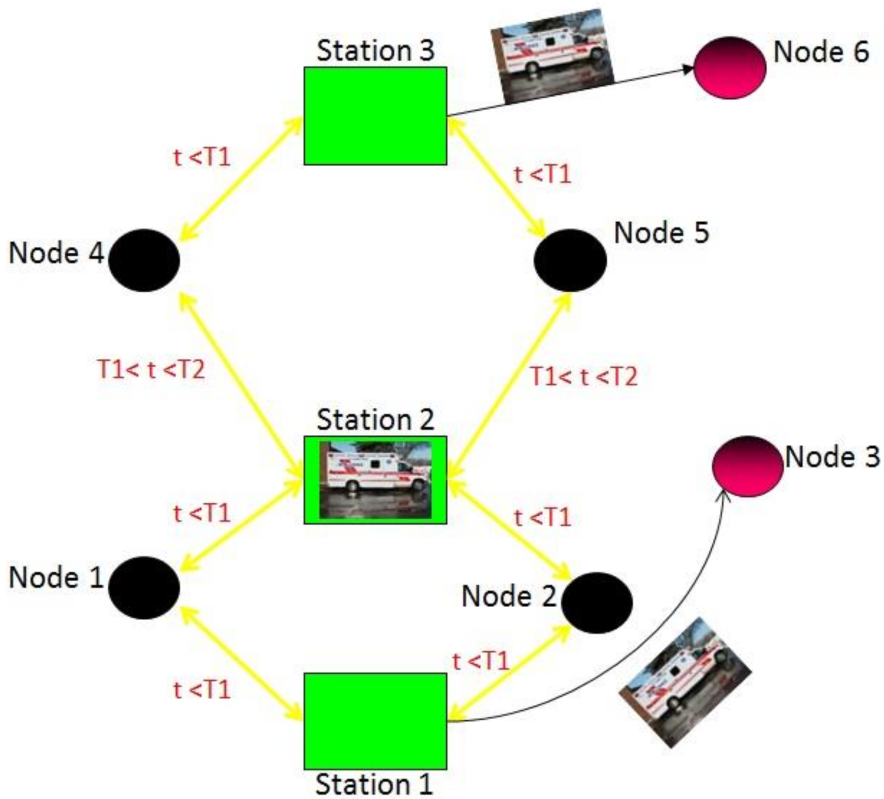

Figure 3.7 A Simple Example of a Dispatching and Coverage Problem

Another thing that should be mentioned is that the nodes do not have the same importance. Depending on the expected number of emergencies that can happen in each node, their importance varies. Also, the importance of each node can vary over time. For example, the downtown of a city can be very important in business hours but not that important overnight, and a school is very important during school hours but not very important at other times. So for each time step at which the model is solved, the importance of different demand nodes is input to the model, and, based on that, the model will decide whether to relocate its vehicles.

3.2 Characteristics of the Problem

3.2.1 Emergency Vehicle Fleet

Three categories of emergency vehicles are considered in this study:

- Police cars: Only one type of police car is considered.
- Ambulances: Two kinds of ambulances are considered:
    - Basic Life Support (BLS): Provides basic airway management (e.g., oxygen therapy), assistance with childbirth, automatic external defibrillation, etc. (NFPA 1710, 2001 Edition).
    - Advanced Life Support (ALS): Provides advanced airway management (e.g., intubation), advanced cardiac monitoring, drug therapy, etc. (NFPA 1710, 2001 Edition).
- Fire vehicles: Three types of vehicles are considered in this category:
    - Fire engine: Equipped with hose lines and water.
    - Fire truck: Equipped with ladders, rescue equipment, and other tools to support firefighting activities.
    - Fire quint: Recent additions to fire departments’ fleets, these fire trucks also carry a hose line and enough water to perform as both a fire truck and a fire engine.

Therefore, there are three categories in the emergency fleet and six types of vehicles. The police category is the only homogeneous fleet; the other two categories have heterogeneous fleets. This heterogeneous characteristic is important for both the dispatching and coverage problems.

3.2.2 Demand Nodes

As was explained earlier, the demand nodes in the system belong to two categories:

- Critical nodes: Hospitals, schools during school hours, important infrastructures like subways, etc. belong to this category. The model attempts to provide double coverage for these nodes within T_1 minutes.
- Ordinary nodes: All nodes other than critical nodes belong to this category; the model attempts to provide double coverage for these nodes within T_2 minutes.

The importance of the nodes in each category is not the same. Nodes that, based on historical data, have had more emergency incidents will have more importance in the system. In the model, the importance of each node is shown by integer numbers ranging from 1 to 4. One means that the expected number of emergencies in the node is not high, and 4 shows a high likelihood of emergencies in that node. Also, over different time steps the importance of the nodes can vary; for example, demand nodes that are near nightclubs will be more important during weekend nights, when the clubs are crowded. The model will relocate the vehicles in the system based on the importance of different demand nodes.

3.2.3 Emergency Calls

Emergency calls can have different categories, such as traffic accidents, fires, crimes, and medical emergencies (Yang, 2006). Based on the characteristic of the emergency, the dispatcher will choose the types of vehicles that should be sent to the scene. Also, based on the emergency's severity, the number of vehicles from each type and the required time for them to get to the scene will be defined. The more severe emergencies have shorter required response times.

3.3 Assumptions

In this section, the assumptions used in the study are explained.

3.3.1 Coverage Assumptions

As was mentioned before, there are two types of ambulances; Advanced Life Support and Basic Life Support ambulances (ALS and BLS). The ALS ambulances can perform the BLS ambulance's job. Usually the BLS gets to the scene sooner and does some preliminary work before the ALS arrives. A demand node is considered as fully covered if there is a BLS within T_B minutes and an ALS within T_A minutes such that $T_B \leq T_A$. For example, according to the NFPA 1710 guidelines, the BLS should get to the emergency scene in 5 minutes 90% of the time and the ALS should arrive in 9 minutes 90% of the time. In addition, in another situation the node is considered to be fully covered if there is an ALS ambulance within T_B minutes; because the ALS can perform the BLS ambulance's job, there is no need for the BLS. The full coverage situation for ambulances is shown in Figure 3.8.

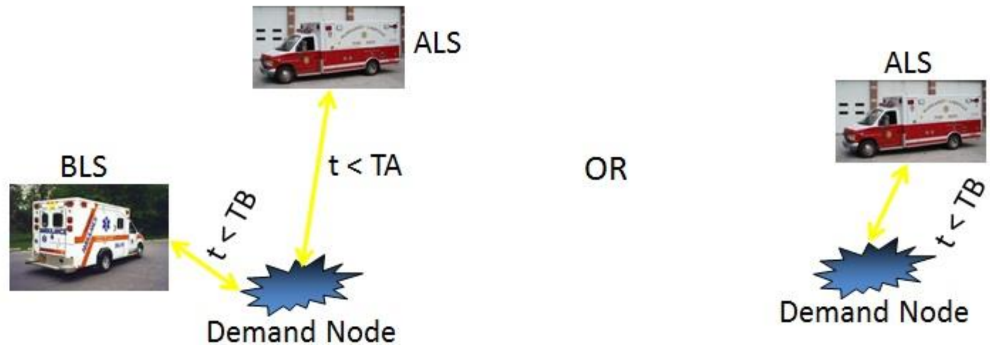

Figure 3.8 Demand Node Full Coverage with Ambulances

However if the node is only covered by one of these vehicles, it will be partially covered and it should benefit from the partial coverage. Partial coverage in the ambulance category happens if the node is covered either by a BLS ambulance within T_B minutes or by an ALS ambulance in t minutes (where $T_B \leq t \leq T_A$). In the latter case the ALS ambulance which is not within T_B minutes from the node cannot be considered as BLS ambulance in the coverage problem. The partial coverage situation for ambulances is shown in Figure 3.9.

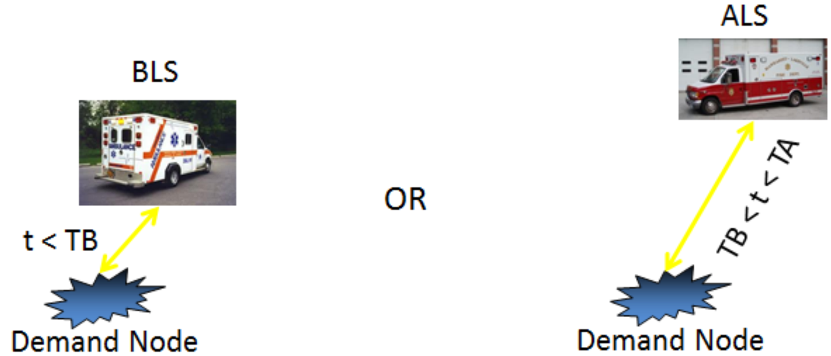

Figure 3.9 Demand Node Partial Coverage with Ambulances

Also, three types of fire vehicles are assumed in this study: fire engines, fire trucks and fire quints. As was explained earlier, the fire quint can perform as both a fire engine and a fire truck. So, a demand node is considered to be fully covered in the fire category if it is covered by both a fire engine and a fire truck within T_F minutes, or it is covered by a fire quint within T_F minutes. The full coverage by fire vehicles for a node is shown in Figure 3.10.

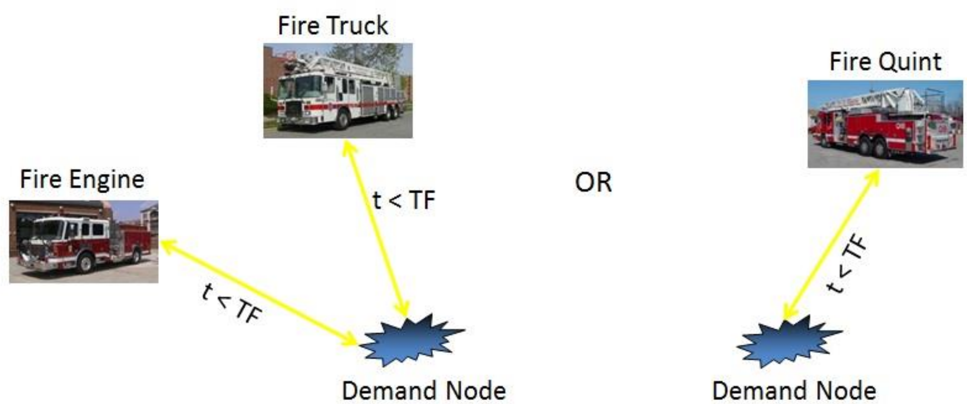

Figure 3.10 Demand Node Full Coverage with Fire Vehicles

However if the node is covered by either a fire engine or a fire truck, it will be partially covered and it should benefit from the partial coverage. The partial coverage situation for fire vehicles is shown in Figure 3.11.

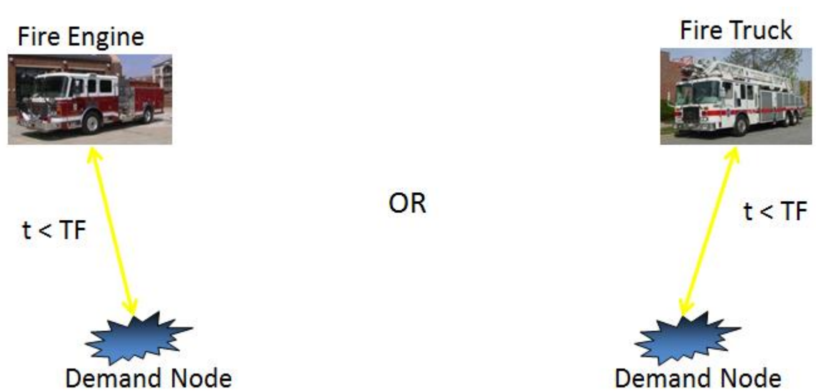

Figure 3.11 Demand Node Partial Coverage with Fire Vehicles

The coverage of the demand nodes by police cars is quite simple, because only one type of police car is considered in this study. The node is covered if the police car is within T_P minutes of the node. The coverage situation is shown in Figure 3.12.

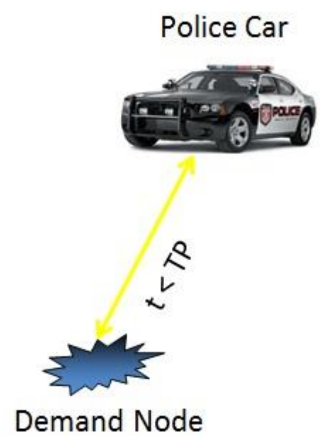

Figure 3.12 Demand Node Coverage with Police Cars

It should be mentioned that there are two time limits for each type; for example, there are $T1_F$ minutes and $T2_F$ minutes for fire vehicles and the nodes are, ideally, covered at least once within $T1_F$ minutes. If they are not covered within $T1_F$ minutes, the model attempts to cover them within $T2_F$ minutes. Also, the model attempts to provide double coverage for ordinary nodes within $T2_F$ minutes and for critical nodes within $T1_F$ minutes. For every type of vehicle, therefore, there are two time limits for coverage.

3.3.2 Dispatching Assumptions

In the dispatching problem, some practical assumptions related to the heterogeneous characteristics of the fleet are considered in this study. If an emergency happens at a node in the system and a BLS must arrive at the site within T_B minutes, if an ALS gets to the emergency location within T_B minutes then the need for a BLS is satisfied.

Also, when an emergency happens, the dispatcher defines the number of fire engines and fire trucks that should be sent to that location within T_F minutes. If a fire quint arrives at the location within T_F minutes, then it can do the job of one fire engine and one fire truck. These assumptions are required for real-world applications.

3.3.3 Availability of Emergency Vehicles

When a vehicle is on-site handling an emergency situation or when it runs out of its supply, it is not available for dispatching. For example, if an ambulance runs out of drugs it is no longer available and needs to return to a station to restock. When a vehicle runs out of resources, it must be sent to a station and should remain there until it is refilled; during this time, it is unavailable. To summarize, when emergency vehicles are at an emergency site they are unavailable. After they are done with the emergency scene, police cars are available because they do not need to recharge any supplies. Some vehicles, like ambulances have to restock their medications, so they are unavailable. In addition, ambulances may take patients to hospitals; they remain unavailable for dispatch to other destinations, as their only destination should be a hospital (Yang, 2006).

3.3.4 Divertible Emergency Vehicles

Vehicles on their way to an emergency site, hospital, or station are divertible. When the destination of a vehicle is a hospital, it can be diverted, but the new destination must be another hospital. For example, an ambulance is taking a patient to hospital A when the traffic data change and show congestion on the route to that hospital. In that case, the ambulance may be diverted to another hospital. Also, when the vehicle is out of resources and must go to a station to get recharged, it is divertible—but its destination has to be another station. As was mentioned before, because the diversion is difficult for the drivers and increases confusion, it is an option only when it has at least a predefined amount of benefit to the whole system (Yang, 2006).

3.3.5 Stations of Emergency Vehicles

Each vehicle category has its own stations. Sometimes different categories share the same station; for example, fire stations can be used by ambulances. Fire vehicles can use all fire stations in the system as their station and may be relocated to any one of them if the capacity of that station allows. Ambulances can use emergency rescue centers, hospitals, and fire stations as their station. Police cars can use every node in the network for their station and do not need to use police stations only, so police cars can be relocated to any node in the system to provide better coverage of the whole area.

Another assumption considered in this study is each station has a set capacity and, for example, a fire station cannot hold 10 fire engines when its capacity is only 3.

3.3.6 Dynamic Characteristic and Input Assumptions

The problem under investigation for this dissertation is a dynamic problem. At each time step it is assumed that the traffic data on the road network, the location of the vehicles and their status, the capacity of vehicle stations, the likelihood of emergencies happening at demand nodes, and information about the capacity of the hospitals are known as the input to the model. If an emergency happens at a node in the system, based on the emergency type and severity, the number of vehicles in each type and the required time to have those vehicles on site will be defined and the model is solved to decide which vehicles should be dispatched to emergencies and which vehicles should be relocated to provide better coverage of the whole area. Also, this model is event-based and will be solved whenever there is an event in the system. These events can be:

- Occurrence of an emergency: When an emergency happens, some vehicles need to be sent to the emergency site and others may be relocated to provide better coverage.
- Change in the status of vehicles: When the status of a vehicle changes, for example when it is finished at the emergency site and needs to get recharged, the model will be solved to send the vehicle to a station to get recharged. If it is finished with an emergency and becomes available, it needs to be sent to a station to provide better coverage.
- Change in the traffic data: When the traffic data change, the model needs to be resolved. For example, if a vehicle is taking a patient to a hospital and new traffic data show that due to congestion on the route to the previous

destination another hospital is closer; in that case the vehicle may be diverted to the other hospital.

- Change in the likelihood of an emergency happening in the demand nodes: When the importance of the demand nodes are changed, the model should be resolved to find better coverage for the system. For example, during the daytime more vehicles are needed near schools, but at night the importance of those nodes is decreased.

Therefore, whenever an event happens in the system, the model should be resolved to find a new solution.

3.3.7 Assumptions Related to Crews

3.3.7.1 Preference for Home Stations

It is obvious that the crews prefer to stay at their home station instead of being relocated to other stations. They usually keep their food and personal possessions there; sometimes they also have an assigned desk in their home station. The preference for their home station is considered in this study. The cost of relocating to stations other than the home station is higher than the cost of relocating to the home station. The coefficient of this cost is higher for fire vehicles, because their crews are the ones that prefer most to stay at their home station. This cost is lower for ambulances, because they are more flexible than the fire vehicles. In this study there is no such cost for police cars, because it is assumed that they can be relocated to any node in the system and therefore there is no preference for them.

3.3.7.2 End of the Shift

Some vehicles are relocated in our study over time and may end up in another station at the end of their work shift. Also when it is near the end of their shift it is better for them to not be assigned to a job if another vehicle can cover the emergency. Therefore, in this study the cost of assigning the crews that are near the end of their shifts to incoming jobs is higher than assigning those who are not close to the end of their shifts. By this assumption the model attempts to consider the crew's work hours as long as this does not interfere with the emergency incident. It means that if there is no vehicle in the required time, the vehicle assigned to crews near the end of their shifts should be dispatched to the emergency.

3.3.7.3 Workload Balance

One of the important characteristics of this model is that it attempts to maintain workload balance between different crews of the emergency vehicle fleet. It is assumed that the crews assigned to a vehicle always work together in that specific vehicle for that shift. In this case, the work hours of the crews in the same vehicle are the same. The cost of dispatching a vehicle to a job is higher if the workload of that vehicle is higher. The workload of vehicle i is defined as working hours so far for those crews divided by the total hours so far in the shift. For example, if the crews of vehicle i started working at 7 a.m., it is 11 a.m., and during this time this vehicle worked for 1 hour, the workload ratio of this vehicle is 0.25. It is preferable to dispatch a vehicle with a lower workload to a job instead of a vehicle with a higher workload.

3.4 Mathematical Formulation

As was explained in the problem statement section, two parts are considered in this model:

- Dispatching vehicles to emergencies,
- Relocating vehicles to provide better coverage for the whole area, and

In summary, the model has to take into account all of the following goals:

- Minimize the dispatching and relocation travel time and cost.
- Maximize the first coverage of the whole area within T_1 minutes.
- If some nodes are not covered within T_1minutes, try to cover them within T_2 minutes.
- Maximize the double coverage within T_2 minutes for ordinary nodes.
- Maximize the double coverage within T_1 minutes for critical nodes.
- Prefer to send vehicles to their home station.
- Prefer not to assign the job to the vehicles whose crews are near the end of their shifts.
- Try to maintain workload balance between vehicles.
- Ensure that emergencies can be serviced with the required types and numbers of vehicles.
- Ensure that vehicles arrive at the emergency scene within required time.
- Ensure that vehicle diversions occur with at least a minimal positive impact on the whole system performance.

The real-time dispatching and relocation of the emergency fleet is formulated as an integer-programming model based on the above objectives and assumptions. In the following subsections, the notations, coefficients, and variables used in the model will be introduced and the objective function and constrains are explained.

3.4.1 Notations

V Set of emergency vehicles in the system

K Set of emergency vehicle types in the system ($k = 1, \dots, 6$)

V_k Set of type k emergency vehicles in the system

K' Set of categorized emergency vehicle types in the system ($k' = 1, 2, 3$)

N_k Maximum number of emergency vehicle types in the system ($N_k = 6$)

N_{V_k} Maximum number of emergency vehicle type k in the system

V_k^{IS} Subset of type k emergency vehicles in V_k that are staying at the station with idle status

V_k^S Subset of type k emergency vehicles in V_k that are moving to a station

V_k^{SS} Subset of type k emergency vehicles in V_k that must go to a station to get recharged

V_k^{SSS} Subset of type k emergency vehicles in V_k that must stay in the station to get recharged

V_k^e Subset of type k emergency vehicles in V_k that are moving to an emergency site

V_k^{ee} Subset of type k emergency vehicles in V_k that are servicing an emergency

V_k^h Subset of type k emergency vehicles in V_k that are taking patients to hospitals after finishing on-site service

V_k^{hh} Subset of Type k Emergency Vehicles in V_k that are staying at the hospital

i Index of vehicles in set V_k , i = 1, 2, …, N_{V_k}

E Set of emergencies in the system

E^0 Set of emergency incidents in the system that are currently being serviced

E^1 Set of emergency incidents in the system that are waiting for service

j Index of emergencies in set E

S_k Set of emergency vehicle stations for type k

N_{S_k} Maximum number of emergency vehicle stations for type k in the system

s Index of stations in set S_k , s = 1, 2, …, N_{S_k}

H Set of hospitals

N_H Maximum number of hospitals in the system

h Index of hospitals in set H, h = 1, 2, …, N_H

P Set of ordinary nodes in the area

N_P Maximum number of nodes

p Index of nodes in set P, p = 1, 2, …, N_P

I Set of critical buildings and infrastructures in the system

N_I Maximum number of critical buildings and infrastructures in the system

l Index of critical buildings and infrastructures in set, l = 1, 2, …, N_I

3.4.2 Coefficients

T_{kj} Upper bound time for vehicle type k reaching emergency j

PT_{kj} Penalty for excess time in reaching emergency j by vehicle type k

PD_{kj} Penalty for deficiency of vehicle type k in emergency j

ω_s Minimum threshold of benefit for diverting a vehicle while going to a station (this means that the diversion will happen if the saving for the whole system is larger than ω_s).

ω_e Minimum threshold of benefit for diverting a vehicle while going to an emergency incident (this means that the diversion will happen if the saving for the whole system is larger than ω_e).

ω_h Minimum threshold of benefit for diverting a vehicle while going to a hospital (this means that the diversion will happen if the saving for the whole system is larger than ω_h).

$E_{K'p}$ Likelihood of an emergency at node p that would require category K' vehicles ($E_{K'p} = 1, \dots, 4$)

$E_{K'l}$ Likelihood of an emergency in Critical node l that would require category K' vehicles ($E_{K'l} = 1, \dots ,4$)

AA Benefit of ordinary node first coverage within T_1minutes

AB Benefit of ordinary node first coverage within T_2minutes

AC Benefit of ordinary node second coverage within T_2minutes

AD Benefit of critical node first coverage within T_1minutes

AE Benefit of critical node first coverage within T_2minutes

AF Benefit of critical node second coverage within T_1minutes

C_{kij} Cost of type k vehicle i to travel to emergency j, which is a function of travel time t_{kij}(t) , working hour ratio of the crews and whether or not it is the end of the working shift for the crews of vehicle i.

C_{kis} Cost of type k vehicle i to travel to station s, which is a function of travel time t_{kis}(t), working hour ratio of the crews, whether or not it's the end of the working shift for the crews of vehicle i, and the station s is the home station for vehicle i or not .

C_{kih} Cost of type k vehicle i to travel to hospital h, which is a function of travel time t_{kih}(t) only.

WHR_{ki} = $(working\ hours\ so\ far)_{ki}/(total\ hours\ so\ far)_{ki}$

Working hour ratio of the crews of vehicle i in type k

t_{kij} Predicted travel time for type k vehicle i to reach emergency j.

t_{kis} Predicted travel time for type k vehicle i to reach station s.

t_{kih} Predicted travel time for type k vehicle i to reach hospital h.

α_e , α_s , α_h Coefficients of the travel times in estimating travel costs

β_e , β_s Coefficients of the working hour ratios in estimating travel costs

γ_e , γ_s Coefficients of the end-of-shift indicator in estimating travel costs

θ_s Coefficients of the home-station indicator in estimating travel costs

N_{kj} Required number of type k vehicle for emergency j

$TC1_k$ First critical time (T_1 minutes) used for coverage by vehicle type k

$TC2_k$ Second critical time (T_2 minutes) used for coverage by vehicle type k

t_{ksp} Predicted travel time for vehicle type k from station s to ordinary node p.

t_{ksl} Predicted travel time for vehicle type k from station s to critical node l.

Cap_{ks} Capacity of s^{st} station for vehicle type k

Cap_h Vacancy of hospital h

M A large number

X_{kij}^{0} = 1 if type k vehicle i was dispatched to emergency j at the previous step,

= 0 otherwise;

X_{kih}^{0} = 1 if type k vehicle i was dispatched to hospital h at the previous step,

= 0 otherwise;

X_{kis}^{0} = 1 if type k vehicle i was dispatched to station s at the previous step,

= 0 otherwise;

δP_{ksp} = 1 if $t_{ksp} \leq TC1_k$, =0 otherwise;

γP_{ksp} = 1 if $t_{ksp} \leq TC2_k$, =0 otherwise;

δI_{ksl} = 1 if $t_{ksl} \leq TC1_k$, =0 otherwise;

γI_{ksl} = 1 if $t_{ksl} \leq TC2_k$, =0 otherwise;

δPAB_{sp} = 1 if $t_{3sp} \leq TC1_2$, =0 otherwise; (this means that if ALS from station s can get to the ordinary node p within BLS's first critical coverage time, the node is considered as covered by both ALS and BLS in the first critical time)

γPAB_{sp} = 1 if $t_{3sp} \leq TC2_2$, =0 otherwise; (this means that if ALS from station s can get to the ordinary node p within BLS's second critical coverage time, the node is considered as covered by both ALS and BLS in the second critical time)

δIAB_{sl} = 1 if $t_{3sl} \leq TC1_2$, =0 otherwise; (this means that if ALS from station s can get to the critical node l within BLS's first critical coverage time, the node is considered as covered by both ALS and BLS in the first critical time)

γIAB_{sl} = 1 if $t_{3sl} \leq TC2_2$, =0 otherwise; (this means that if ALS from station s can get to the critical node l within BLS's second critical coverage time, the node is considered as covered by both ALS and BLS in the second critical time)

3.4.3 Decision Variables

X_{kij} = 1 if the type k vehicle i is dispatched to emergency j at this time step,

= 0 otherwise;

X_{kih} = 1 if the type k vehicle i is dispatched to hospital h at this time step,

= 0 otherwise;

X_{kis} = 1 if the type k vehicle i is dispatched to station s at this time step,

= 0 otherwise;

XAB_{ij} = 1 if ALS i reaches emergency j within required time for BLS

= 0 otherwise;

D_{kj} = Deficiency of type k in emergency j

EXT_{kij} = 1 if travel time for type k vehicle i to emergency j is longer thanT_{kj},

= 0 otherwise;

R_{ki}^{e} = 1 if type k vehicle i is diverted while it is going to an emergency,

= 0 otherwise;

R_{ki}^{s} = 1 if type k vehicle i is diverted while it is going to a station,

= 0 otherwise;

R_{ki}^{h} = 1 if type k vehicle i is diverted while it is going to a hospital,

= 0 otherwise;

$ZPF^{1}_{k'p}$ = 1 if node p is covered at least once by the first vehicle type in category k'

in the first time,

= 0 otherwise;

$ZPS^{1}_{k'p}$ = 1 if node p is covered at least once by the second vehicle type in category k'

in the first time,

= 0 otherwise;

$ZP^{1}_{k'p}$ = 1 if node p is fully covered at least once by category k' in the first time,

= 0 otherwise;

$YPF^{1}_{k'p}$ = 1 if node p is covered at least once by the first vehicle type in category k'

in the second time and not covered in the first time,

= 0 otherwise;

$YPS^{1}_{k'p}$ = 1 if node p is covered at least once by the second vehicle type in

category k' in the second time and not covered in the first time,

= 0 otherwise;

$YP^{1}_{k'p}$ = 1 if node p is fully covered at least once by category k' in the second

time and not fully covered in the first time,

= 0 otherwise;

$YPF^{2}_{k'p}$ = 1 if node p is covered at least twice by the first vehicle type in

category k' in the second time

= 0 otherwise;

$YPS^2_{k'p}$ = 1 if node p is covered at least twice by the second vehicle type in

categoryk' in the second time

= 0 otherwise;

$YP^2_{k'p}$ = 1 if node p is fully covered at least twice by category k'in the second time,

= 0 otherwise;

$ZIF^1_{k'l}$ = 1 if critical node l is covered at least once by the first vehicle type in

category k' in the first time

= 0 otherwise;

$ZIS^1_{k'l}$ = 1 if critical node l is covered at least once by the second vehicle type in

category k' in the first time

= 0 otherwise;

$ZI^1_{k'l}$ = 1 if critical node l is fully covered at least once by category k' in the

first time

= 0 otherwise;

$YIF^1_{k'l}$ = 1 if critical node l is covered at least once by the first vehicle type in

category k' in the second time and not covered in the first time,

= 0 otherwise;

$YIS^1_{k'l}$ = 1 if critical node l is covered at least once by the second vehicle type in

category k' in the second time and not covered in the first time,

=0 otherwise;

$YI^1_{k'l}$ = 1 if critical node l is fully covered at least once by category k' in the

second time and not fully covered in the first time,

= 0 otherwise;

$ZIF^2_{k'l}$ = 1 if critical node l is covered at least twice by the first vehicle type in category k' in the first time,

= 0 otherwise;

$ZIS^2_{k'l}$ = 1 if critical node l is covered at least twice by the second vehicle type in category k' in the first time,

= 0 otherwise;

$ZI^2_{k'l}$ = 1 if critical node l is fully covered at least twice by category k' in the first time,

= 0 otherwise;

3.4.4 The Integer-Programming Model

The mathematical formulation of the problem is presented in this subsection. First the objective function will be presented and explained and then the constraints introduced.

3.4.4.1 Objective Function:

$$\text{Minimize } \sum_k \sum_i \sum_j C_{kij} \,.\, X_{kij} + \sum_k \sum_i \sum_s C_{kis} \,.\, X_{kis} + \sum_k \sum_i \sum_h C_{kih} \,.\, X_{kih}$$

$$+ \sum_k \sum_j PD_{kj} \,.\, D_{kj} + \sum_k \sum_i \sum_j PT_{kj} \,.\, \left|T_{kj} - t_{kij}\right| .\, EXT_{kij}$$

$$+\omega_e \sum_k \sum_{i \epsilon V_k^e} R_{ki}^e + \omega_s \sum_k \sum_{i \epsilon V_k^s} R_{ki}^s + \omega_h \sum_k \sum_{i \epsilon V_k^h} R_{ki}^h$$

$$- \sum_{k'} \sum_{p} \mathrm{AA}\,.\; E_{k'p}\,.\,(ZPF^{1}_{k'p} + \; ZPS^{1}_{k'p} + ZP^{1}_{k'p})$$

$$- \sum_{k'} \sum_{p} \mathrm{AB}\,.\; E_{k'p}\,.\,(YPF^{1}_{k'p} + YPS^{1}_{k'p} + YP^{1}_{k'p})$$

$$- \sum_{k'} \sum_{p} \mathrm{AC}\,.\; E_{k'p}\,.\,(YPF^{2}_{k'p} + YPS^{2}_{k'p} + YP^{2}_{k'p})$$

$$- \sum_{k'} \sum_{l} \mathrm{AD}\,.\; E_{k'l}\,.\,(ZIF^{1}_{k'l} + ZIS^{1}_{k'l} + ZI^{1}_{k'l})$$

$$- \sum_{k'} \sum_{l} \mathrm{AE}\,.\; E_{k'l}\,.\,(YIF^{1}_{k'l} + YIS^{1}_{k'l} + YI^{1}_{k'l})$$

$$- \sum_{k'} \sum_{l} \mathrm{AF}\,.\; E_{k'l}\,.\,(ZIF^{2}_{k'l} + ZIS^{2}_{k'l} + ZI^{2}_{k'l}) \qquad (3.1)$$

Equation (3.1) is the objective of this model and it is a minimization. The first row of this equation minimizes the weighted travel cost of vehicles to emergencies, to stations and to hospitals.

The second row of this equation minimizes the penalty if the number of vehicles reaching the emergency is less than the required number of vehicles or the time that the vehicles can be there is higher than the required time. In an ideal situation, the emergencies should get all the vehicles they need and the vehicles should get there within the required times, but in reality sometimes the system is very loaded and emergencies may not be serviced ideally. In that case, these penalties are added to the objective function.

The third row shows the minimum threshold of benefit for diverting the vehicles from their previous destinations. As was said before, because the diversion may confuse the crews, there should be a minimum level of positive impact on the system to warrant diversion, and in this row of the objective function these minimum levels of positive impact are defined. For example, this means that the total benefit of diverting a vehicle from a station to another destination should be more than ω_s on the whole system to let that diversion happen, because that diversion itself will increase the objective function by ω_s. In other parts of the objective function, the benefit should be more than ω_s to warrant diversion. By this minimum threshold of benefit for diversions, the model attempts to prevent having too many diversions.

The fourth, fifth and sixth rows of the objective function attempt to maximize the coverage for ordinary nodes in the system. The fourth row is for having first coverage within T_1minutes and the fifth row is for having first coverage within T_2 minutes if the node is not covered within T_1minutes. Sixth row attempts to provide double coverage for ordinary nodes within T_2minutes.

The seventh, eighth and ninth rows are the same as the fourth, fifth and sixth rows but for critical nodes. The only difference is that the ninth row attempts to provide double coverage for critical nodes within T_1 minutes instead of T_2 minutes because of the critical characteristics of these nodes.

In the coverage part, the model considers both partial and full coverage. For example if the ordinary node is covered by fire engine within T_1minutes, $ZPF^1_{k'p}$ gets value of 1, if the node is covered by fire truck within T_1minutes, $ZPS^1_{k'p}$ becomes 1, and if it is covered by both types of vehicles within T_1minutes, $ZP^1_{k'p}$ becomes 1 too to include the benefit of having full coverage.

One thing that should be mentioned here is how the travel cost to different destinations is calculated in this model. The following three equations illustrate how travel cost is estimated:

$$C_{kij} = \alpha_e \,.\, t_{kij} + \beta_e . WHR_{ki} + \gamma_e \,.\, (\text{whether it's the end of the shift for vehicle i or not}) \tag{3.2}$$

$$C_{kis} = \alpha_s \,.\, t_{kis} + \beta_s . WHR_{ki} + \gamma_s \,.\, (\text{whether it's the end of the shift for vehicle i or not}) + \theta_s \,.\, (\text{whether the s is the home station for vehicle i or not}) \tag{3.3}$$

$$C_{kih} = \alpha_h \,.\, t_{kih} \tag{3.4}$$

Equation (3.2) shows that the travel cost for type k vehicle i to emergency j is a function of travel time, working hour ratio of the crew for this vehicle, and whether or not it is the end of the shift for the crew.

Equation (3.3) is for travel cost to station s, which is a function of travel time, working hour ratio of the crew of this vehicle, whether or not it is the end of the shift for the crews, and whether or not station s is the home station for vehicle i. Also, equation (3.4) shows travel cost to a hospital, which is only a function of travel time.

3.4.4.2 Constraints

In this subsection the constraints for this model are provided, with a brief explanation for each.

As was mentioned in the notification, K' is the set of categorized emergency vehicle types in the system ($k' = 1,2,3$) and K is the set of emergency vehicle types in the system($k = 1,2,3,4,5,6$).

In this formulation it is assumed that:

$k' = 1$ represents the police category

$k' = 2$ represents the ambulance category

$k' = 3$ represents the fire vehicle category

And:

$k = 1$ represents police cars

$k = 2$ represents BLS ambulances

$k = 3$ represents ALS ambulances

$k = 4$ represents fire engines

$k = 5$ represents fire trucks

$k = 6$ represents fire quints

- Dispatching Constraints

$$\sum_j X_{kij} + \sum_{s_k} X_{kis} + \sum_h X_{kih} = 1 \qquad \forall\ k \in K\ \&\ i \in V_k \qquad (3.5)$$

Constraint (3.5) ensures that each vehicle is assigned to one and only one destination at each time step.

$$\sum_i X_{kij} + D_{kj} \geq N_{kj} \qquad \forall\ k \in \{1,3\}\ \&\ j \in E \qquad (3.6)$$

$$if \quad t_{3ij} \leq T_{2j} \quad \Longrightarrow \quad XAB_{ij} = X_{3ij} \qquad \forall\ i \in V_3\ \&\ j \in E \qquad (3.7)$$

$$if \quad t_{3ij} > T_{2j} \quad \Longrightarrow \quad XAB_{ij} = 0 \qquad \forall\ i \in V_3\ \&\ j \in E \qquad (3.8)$$

$$\sum_{i \varepsilon V_2} X_{2ij} + \sum_{i \varepsilon V_3} XAB_{ij} + D_{2j} \geq N_{2j} \qquad \forall\ j \in E \qquad (3.9)$$

$$\sum_{i \varepsilon V_4} X_{4ij} + \sum_{i \varepsilon V_6} X_{6ij} + D_{4j} \geq N_{4j} \qquad \forall\ j \in E \qquad (3.10)$$

$$\sum_{i\varepsilon V_5} X_{5ij} + \sum_{i\varepsilon V_6} X_{6ij} + D_{5j} \geq N_{5j} \qquad \forall\ j \in E \qquad (3.11)$$

Constraints (3.6) to (3.11) ensure that the emergencies are serviced by the required number of emergency vehicles. D_{kj} shows the deficiencies that exist in emergencies; if it is non-zero, a penalty will be added to the objective function. For police cars and ALS ambulances only equation (3.6) is needed, but for BLS and fire vehicles, more constraints are needed.

Constraints (3.7), (3.8), and (3.9) together define the number of deficiencies of BLS in the emergencies. Constraints (3.7) and (3.8) state that if an ALS ambulance gets to the emergency scene in the required time for a BLS ambulance, it can perform the job of a BLS. In that case, XAB_{ij} will be 1; otherwise, it is 0. Constraint (3.9) will calculate any BLS deficiency number(s) in emergencies.

Constraint (3.10) calculates the fire engine deficiency and states that if a fire quint arrives at the scene ($X_{6ij} = 1$), it can take care of a fire engine's job. Constraint (3.11) states the same thing for fire trucks, because a fire quint can perform as both a fire engine and a fire truck. Therefore, by this family of constraints the number of deficiencies in emergencies will be calculated, and if there are any, the penalty will be added to the objective function.

$$t_{kij} \,.\, X_{kij} - M.EXT_{kij} \leq T_{kj} \qquad \forall\ k \in K\ \&\ i \in V_k\ \&\, j \in E \qquad (3.12)$$

Constraint (3.12) ensures that emergency vehicles get to the emergency sites in the required time. If they don't arrive in time, this equation will be defined by $EXT_{kij} = 1$, and in that case the penalty will be added to the objective function.

$$1 - X_{kij} \,.X^0_{kij} \leq M.R^e_{ki} \qquad \forall\ k \in K\ \&\ i \in V^e_k\ \&\, j \in E \qquad (3.13)$$

$$1 - X_{kis} . X_{kis}^{0} \leq M . R_{ki}^{s} \qquad \forall \ k \in K \ \& \ i \in V_k^s \ \& \ s \in S_k \qquad (3.14)$$

$$1 - X_{kih} . X_{kih}^{0} \leq M . R_{ki}^{h} \qquad \forall \ k \in K \ \& \ i \in V_k^h \ \& \ h \in H \qquad (3.15)$$

These three constraints are for diversion in the model. As was discussed before, diversion will increase crews' confusion and is allowed to happen only if it has at least a minimum level of benefit to the whole system. Constraint (3.13) ensures that if a diversion occurs when a vehicle is heading to an emergency, R_{ki}^{e} will be 1 and it will add ω_e to the objective function. If the benefit of this diversion to the whole system is more than ω_e then the diversion will happen. Constraint (3.14) is for diversion from a station, and constraint (3.15) is for diversion from a hospital.

$$\sum_{h} X_{kih} = 1 \qquad \forall \ k \in K \ \& \ i \in V_k^h \qquad (3.16)$$

$$\sum_{s} X_{kis} = 1 \qquad \forall \ k \in K \ \& \ i \in V_k^{ss} \qquad (3.17)$$

Equation (3.16) states that a diversion can happen when a vehicle is heading to a hospital, but the new destination must be another hospital.

Equation (3.17) states that the diversion can occur for vehicles that have to go to a station to get recharged, but their destination must be another station.

$$X_{kis} = X_{kis}^{0} \qquad \forall \ k \in K \ \& \ i \in V_k^{sss} \ \& \ s \in S_k \qquad (3.18)$$

$$X_{kij} = X_{kij}^{0} \qquad \forall \ k \in K \ \& \ i \in V_k^{ee} \ \& \ j \in E^{0} \qquad (3.19)$$

$$X_{kih} = X_{kih}^{0} \qquad \forall \ k \in K \ \& \ i \in V_k^{hh} \ \& \ h \in H \qquad (3.20)$$

Constraint (3.18) ensures that if a vehicle has to remain at a station to get recharged, it must stay there until it has been recharged and cannot be dispatched to a job. Constraint (3.19) ensures that the vehicles that are responding to an emergency on site should stay at that location and continue their work; they cannot be dispatched to another location. Constraint (3.20) ensures the same thing for vehicles that have to remain at a hospital.

$$\sum_{k\in K'}\sum_{i} X_{kis} \leq Cap_{k's} \qquad \forall\ k' \in K'\ \&\ s \in S_{k'} \qquad (3.21)$$

$$\sum_{k}\sum_{i} X_{kih} \leq Cap_{h}(t) \qquad \forall\ h \in H \qquad (3.22)$$

Equation (3.21) states that the number of vehicles in each station cannot be more than the capacity of that station. Constraint (3.22) ensures that the number of patients sent to a hospital will be less than the number of vacancies at that hospital.

- Coverage Constraints for the Police Category

For the police category: $k = 1$ and $k' = 1$

$$\sum_{s\,\in\, s_k}\sum_{i} \delta P_{ksp}\ .X_{kis} - ZP^{1}_{k'p} \geq 0 \qquad k = 1,\ k' = 1\ \&\ \forall\, p \in P \qquad (3.23)$$

Equation (3.23) states that each ordinary node is or is not covered by a police car within T_1 minutes. If it's covered, a benefit will be added to the objective function.

$$\sum_{s\,\in\, s_k}\sum_{i} \gamma P_{ksp}\ .X_{kis} - YP^{1}_{k'p} \geq 0 \qquad k = 1,\ k' = 1\ \&\ \forall\, p \in P \qquad (3.24)$$

$$YP^{1}_{k'p} + ZP^{1}_{k'p} \leq 1 \qquad k = 1,\ k' = 1\ \&\ \forall\, p \in P \qquad (3.25)$$

Equations (3.24) and (3.25) state that the system will attempt to cover ordinary nodes that are not covered within T_1minutes, within T_2. Equation (3.25) states that ordinary nodes covered within T_1 minutes that are definitely covered within T_2 minutes as well will get the benefit of coverage within T_1minutes only.

$$\sum_{s \in s_k} \sum_{i} \gamma P_{ksp} \,.X_{kis} - YP^0_{k'p} - YP^2_{k'p} \geq 0 \quad k = 1,\ k' = 1\ \&\ \ \forall\, p \in P \quad (3.26)$$

$$YP^2_{k'p} \leq YP^0_{k'p} \qquad k = 1,\ k' = 1\ \&\ \ \forall\, p \in P \quad (3.27)$$

Constraint (3.26) states that each ordinary node will or will not be covered at least twice by a police car within T_2 minutes. If it is, a benefit will be added to the objective function.

Constraint (3.27) ensures that a node should be first covered once and then twice.

$$\sum_{s \in s_k} \sum_{i} \delta I_{ksl} \,.X_{kis} - ZI^1_{k'l} - ZI^2_{k'l} \geq 0 \qquad k = 1,\ k' = 1\ \&\ \ \forall\, l \in I \qquad (3.28)$$

$$ZI^2_{k'l} \leq ZI^1_{k'l} \qquad k = 1,\ k' = 1\ \&\ \ \forall\, l \in I \qquad (3.29)$$

$$\sum_{s \in s_k} \sum_{i} \gamma I_{ksl} \,.X_{kis} - YI^1_{k'l} \geq 0 \qquad k = 1,\ k' = 1\ \&\ \ \forall\, l \in I \qquad (3.30)$$

$$YI^1_{k'l} + ZI^1_{k'l} \leq 1 \qquad k = 1,\ k' = 1\ \&\ \ \forall\, l \in I \qquad (3.31)$$

Constraints (3.28) to (3.31) are for critical-nodes coverage by police cars. Equations (3.28) and (3.29) state that each critical node will or will not be covered at least once or at least twice within T_1 minutes. If it is, a benefit will be added to the objective function.

Constraints (3.30) and (3.31) ensure that the system attempts to cover critical nodes that are not covered within T_1 minutes, within T_2 minutes.

- Coverage Constraints for the Ambulance Category

For the ambulance category: $k' = 2$

- For BLS Ambulance: $k = 2$

- For ALS Ambulance: $k = 3$

Coverage constraints for the ambulance category are more difficult than coverage constraints for police cars. In this category a node is considered to be fully covered if there is one BLS within T_B minutes and one ALS within T_A minutes (for which $T_B \leq T_A$) or one ALS within T_B minutes from the node. Also a node is considered to be only partially covered if there is either a BLS within T_B minutes or an ALS in t minutes (where $T_B \leq t \leq T_A$) from the node.

$$\sum_{s \in s_3} \sum_{i \in V_3} \delta PAB_{sp} . X_{3is} + \sum_{s \in s_2} \sum_{i \in V_2} \delta P_{2sp} . X_{2is} - ZPF^1_{k'p} \geq 0 \quad k' = 2 \ \& \ \forall p \in P \quad (3.32)$$

$$\sum_{s \in s_3} \sum_{i \in V_3} \delta P_{3sp} \ . X_{3is} - ZPS^1_{k'p} \geq 0 \qquad k' = 2 \ \& \ \forall p \in P \quad (3.33)$$

$$\sum_{s \in s_3} \sum_{i \in V_3} \delta PAB_{sp} . X_{3is} + \sum_{s \in s_2} \sum_{i \in V_2} \delta P_{2sp} . X_{2is} - ZP^1_{k'p} \geq 0 \quad k' = 2 \ \& \ \forall p \in P \quad (3.34)$$

$$\sum_{s \in s_3} \sum_{i \in V_3} \delta P_{3sp} \ . X_{3is} - ZP^1_{k'p} \geq 0 \qquad k' = 2 \ \& \ \forall p \in P \quad (3.35)$$

Equation (3.32) states that ordinary nodes are or are not covered in the first critical time by BLS. If one ALS is within the BLS's first critical time from the node, then the first component of this equation becomes 1 and $ZPF^1_{k'p}$ can get 1 and shows that the node is covered by BLS ambulance.

Equation (3.33) states that ordinary nodes will or will not be covered by ALS in the ALS's first critical time.

Equations (3.34) and (3.35) define that the node is fully covered or not. If it is fully covered then $ZP^1_{k'p}$ becomes 1 and a benefit will be added to the objective function.

$$\sum_{s \in s_3} \sum_{i \in V_3} \gamma PAB_{sp} \, . X_{3is} + \sum_{s \in s_2} \sum_{i \in V_2} \gamma P_{2sp} \, . X_{2is} - YPF^{1}_{k'p} \geq 0 \quad k' = 2 \& \forall p \in P \quad (3.36)$$

$$YPF^{1}_{k'p} + ZPF^{1}_{k'p} \leq 1 \qquad k' = 2 \,\&\, \forall\, p \in P \quad (3.37)$$

$$\sum_{s \in s_3} \sum_{i \in V_3} \gamma P_{3sp} \, . X_{3is} - YPS^{1}_{k'p} \geq 0 \qquad k' = 2 \,\&\, \forall\, p \in P \quad (3.38)$$

$$YPS^{1}_{k'p} + ZPS^{1}_{k'p} \leq 1 \qquad k' = 2 \,\&\, \forall\, p \in P \quad (3.39)$$

$$\sum_{s \in s_3} \sum_{i \in V_3} \gamma PAB_{sp} \, . X_{3is} + \sum_{s \in s_2} \sum_{i \in V_2} \gamma P_{2sp} \, . X_{2is} - YP^{1}_{k'p} \geq 0 \quad k' = 2 \& \forall p \in P \quad (3.40)$$

$$\sum_{s \in s_3} \sum_{i \in V_3} \gamma P_{3sp} \, . X_{3is} - YP^{1}_{k'p} \geq 0 \qquad k' = 2 \,\&\, \forall\, p \in P \quad (3.41)$$

$$YP^{1}_{k'p} + ZP^{1}_{k'p} \leq 1 \qquad k' = 2 \,\&\, \forall\, p \in P \quad (3.42)$$

Constraints (3.36) and (3.37) define that ordinary nodes not covered in the first critical time will or will not be covered by BLS ambulances in the second critical time. If they are, a benefit will be added to the objective function. Constraints (3.38) and (3.39) are for coverage by ALS ambulances in the second critical time.

Equations (3.40), (3.41) and (3.42) ensure that if the node is fully covered by ambulances in the second critical time and not fully covered in the first critical time, the benefit will be added to the objective function.

$$\sum_{s \in s_3} \sum_{i \in V_3} \gamma PAB_{sp} \, . X_{3is} + \sum_{s \in s_2} \sum_{i \in V_2} \gamma P_{2sp} \, . X_{2is} - YPF^{0}_{k'p} - YPF^{2}_{k'p} \geq 0$$
$$k' = 2 \,\&\, \forall\, p \in P \quad (3.43)$$

$$YPF^{2}_{k'p} \leq YPF^{0}_{k'p} \qquad k' = 2 \,\&\, \forall\, p \in P \quad (3.44)$$

$$\sum_{s \in s_3} \sum_{i \in V_3} \gamma P_{3sp} \, . X_{3is} - YPS^{0}_{k'p} - YPS^{2}_{k'p} \geq 0 \qquad k' = 2 \,\&\, \forall\, p \in P \quad (3.45)$$

$$YPS^{2}_{k'p} \leq YPS^{0}_{k'p} \qquad k' = 2 \,\&\, \forall\, p \in P \quad (3.46)$$

$$\sum_{s\in s_3}\sum_{i\in V_3}\gamma PAB_{sp}\ .X_{3is}+\sum_{s\in s_2}\sum_{i\in V_2}\gamma P_{2sp}\ .X_{2is}-\ YP^0_{k'p}\ -\ YP^2_{k'p}\geq 0$$
$$k'=2\ \&\ \forall\ p\in P \qquad (3.47)$$

$$\sum_{s\in s_3}\sum_{i\in V_3}\gamma P_{3sp}\ .X_{3is}-\ YP^0_{k'p}\ -\ YP^2_{k'p}\ \geq 0 \qquad k'=2\ \&\ \forall\ p\in P \qquad (3.48)$$

$$YP^2_{k'p}\ \leq\ YP^0_{k'p} \qquad k'=2\ \&\ \forall\ p\in P \qquad (3.49)$$

Equations (3.43) and (3.44) state that ordinary nodes do or do not have double coverage by BLS ambulances in the second critical time. If they have at least double coverage, a benefit will be added to the objective function. Equations (3.45) and (3.46) are for determining the double coverage by ALS ambulances.

Constraints (3.47), (3.48) and (3.49) ensure that if the node has full double coverage, additional benefit will be added to the objective function.

$$\sum_{s\in s_3}\sum_{i\in V_3}\delta IAB_{sl}\ .X_{3is}+\sum_{s\in s_2}\sum_{i\in V_2}\delta I_{2sl}\ .X_{2is}-\ ZIF^1_{k'l}-\ ZIF^2_{k'l}\ \geq 0$$
$$k'=2\ \&\ \forall\ l\in I \qquad (3.50)$$

$$ZIF^2_{k'l}\ \leq ZIF^1_{k'l} \qquad k'=2\ \&\ \forall\ l\in I \qquad (3.51)$$

$$\sum_{s\in s_3}\sum_{i\in V_3}\delta I_{3sl}\ .X_{3is}-\ ZIS^1_{k'l}-\ ZIS^2_{k'l}\ \geq 0 \qquad k'=2\ \&\ \forall\ l\in I \qquad (3.52)$$

$$ZIS^2_{k'l}\ \leq ZIS^1_{k'l} \qquad k'=2\ \&\ \forall\ l\in I \qquad (3.53)$$

$$\sum_{s\in s_3}\sum_{i\in V_3}\delta IAB_{sl}\ .X_{3is}+\sum_{s\in s_2}\sum_{i\in V_2}\delta I_{2sl}\ .X_{2is}-\ ZI^1_{k'l}-\ ZI^2_{k'l}\ \geq 0$$
$$k'=2\ \&\ \forall\ l\in I \qquad (3.54)$$

$$\sum_{s\in s_3}\sum_{i\in V_3}\delta I_{3sl}\ .X_{3is}-\ ZI^1_{k'l}-\ ZI^2_{k'l}\ \geq 0 \qquad k'=2\ \&\ \forall\ l\in I \qquad (3.55)$$

$$ZI^2_{k'l}\ \leq ZI^1_{k'l} \qquad k'=2\ \&\ \forall\ l\in I \qquad (3.56)$$

This set of constraints states that critical nodes will or will not have first coverage and double coverage by ambulances in the first critical time. If they do, a benefit will be added to the objective function. Equations (3.50) and (3.51) are for BLS ambulances, equations (3.52) and (3.53) are for ALS ambulances and equations (3.54), (3.55) and (3.56) are for full coverage.

$$\sum_{s\, \in\, s_3} \sum_{i \in V_3} \gamma IAB_{sl} . X_{3is} + \sum_{s\, \in\, s_2} \sum_{i \in V_2} \gamma I_{2sl} \ . X_{2is} - \ YIF^1_{k'l} \geq 0 \quad k' = 2 \ \& \ \forall \ l \in I \quad (3.57)$$

$$YIF^1_{k'l} + \ ZIF^1_{k'l} \ \leq 1 \qquad\qquad k' = 2 \ \& \ \forall \ l \in I \qquad (3.58)$$

$$\sum_{s\, \in\, s_3} \sum_{i \in V_3} \gamma I_{3sl} \ . X_{3is} - \ YIS^1_{k'l} \ \geq 0 \qquad\qquad k' = 2 \ \& \ \forall \ l \in I \qquad (3.59)$$

$$YIS^1_{k'l} + \ ZIS^1_{k'l} \ \leq 1 \qquad\qquad k' = 2 \ \& \ \forall \ l \in I \qquad (3.60)$$

$$\sum_{s\, \in\, s_3} \sum_{i \in V_3} \gamma IAB_{sl} . X_{3is} + \sum_{s\, \in\, s_2} \sum_{i \in V_2} \gamma I_{2sl} \ . X_{2is} - \ YI^1_{k'l} \geq 0 \quad k' = 2 \ \& \ \forall \ l \in I \quad (3.61)$$

$$\sum_{s\, \in\, s_3} \sum_{i \in V_3} \gamma I_{3sl} \ . X_{3is} - \ YI^1_{k'l} \ \geq 0 \qquad\qquad k' = 2 \ \& \ \forall \ l \in I \qquad (3.62)$$

$$YI^1_{k'l} + \ ZI^1_{k'l} \ \leq 1 \qquad\qquad k' = 2 \ \& \ \forall \ l \in I \qquad (3.63)$$

This set of constraints states that critical nodes not covered by ambulances in the first critical time will or will not be covered in the second critical time. If they are, a benefit will be added to the objective function. Equations (3.57) and (3.58) are for BLS ambulances and equations (3.59) and (3.60) are for ALS ambulances.

Equations (3.61), (3.62) and (3.63) ensure that if the critical node is fully covered at least once in the second critical time and not fully covered in the first critical time, the benefit will be added to the objective function.

- Coverage Constraints for the Fire Vehicle Category

For the fire vehicle category: $k' = 3$

- For fire engines: $\mathrm{k} = 4$
- For fire trucks: $\mathrm{k} = 5$
- For fire quints: $\mathrm{k} = 6$

Coverage constraints for this category are also more difficult than for the police category. The fire department's fleet is heterogeneous, unlike the police category where the fleet is homogeneous. This makes these constraints more complicated. A node is considered as fully covered by fire vehicles if it is covered by both a fire engine and a fire truck or by a fire quint within the required time. Also, a node is considered as partially covered if it is only covered by either a fire engine or a fire truck.

$$\sum_{s \in s_6} \sum_{i \in V_6} \delta P_{6sp} \, . X_{6is} + \sum_{s \in s_4} \sum_{i \in V_4} \delta P_{4sp} \, . X_{4is} - ZPF^1_{k'p} \geq 0 \;\; k' = 3 \;\&\; \forall \, p \in P \quad (3.64)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \delta P_{6sp} \, . X_{6is} + \sum_{s \in s_5} \sum_{i \in V_5} \delta P_{5sp} \, . X_{5is} - ZPS^1_{k'p} \geq 0 \;\; k' = 3 \;\&\; \forall \, p \in P \quad (3.65)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \delta P_{6sp} \, . X_{6is} + \sum_{s \in s_4} \sum_{i \in V_4} \delta P_{4sp} \, . X_{4is} - ZP^1_{k'p} \geq 0 \quad k' = 3 \;\&\; \forall \, p \in P \quad (3.66)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \delta P_{6sp} \, . X_{6is} + \sum_{s \in s_5} \sum_{i \in V_5} \delta P_{5sp} \, . X_{5is} - ZP^1_{k'p} \geq 0 \;\; k' = 3 \;\&\; \forall \, p \in P \quad (3.67)$$

This set of constraints states that ordinary nodes will or will not be covered at least once within T_1 minutes by fire vehicles. Constraint (3.64) is for coverage by fire engines. The first component of this equation shows that a fire quint can do a fire engine's job. Constraint (3.65) is for fire truck coverage and shows that fire quint can do a fire truck's job as well. Therefore, in total the node is considered as fully covered if it is covered by both a fire engine and a fire truck or by a fire quint.

Equations (3.66) and (3.67) ensure that if the node is fully covered, the additional benefit will be added to the objective function.

$$\sum_{s\,\in\, s_6}\sum_{i\in V_6}\gamma P_{6sp}\ .X_{6is}+\sum_{s\,\in\, s_4}\sum_{i\in V_4}\gamma P_{4sp}\ .X_{4is}-YPF^{1}_{k'p}\geq 0\ k'=3\ \&\ \forall\, p\in P \quad (3.68)$$

$$YPF^{1}_{k'p}+\ ZPF^{1}_{k'p}\ \leq 1 \qquad k'=3\ \&\ \forall\, p\in P \quad (3.69)$$

$$\sum_{s\,\in\, s_6}\sum_{i\in V_6}\gamma P_{6sp}\ .X_{6is}+\sum_{s\,\in\, s_5}\sum_{i\in V_5}\gamma P_{5sp}\ .X_{5is}-YPS^{1}_{k'p}\geq 0\ k'=3\&\ \forall\, p\in P \quad (3.70)$$

$$YPS^{1}_{k'p}+\ ZPS^{1}_{k'p}\ \leq 1 \qquad k'=3\ \&\ \forall\, p\in P \quad (3.71)$$

$$\sum_{s\,\in\, s_6}\sum_{i\in V_6}\gamma P_{6sp}\ .X_{6is}+\sum_{s\,\in\, s_4}\sum_{i\in V_4}\gamma P_{4sp}\ .X_{4is}-YP^{1}_{k'p}\geq 0\ k'=3\ \&\ \forall\, p\in P \quad (3.72)$$

$$\sum_{s\,\in\, s_6}\sum_{i\in V_6}\gamma P_{6sp}\ .X_{6is}+\sum_{s\,\in\, s_5}\sum_{i\in V_5}\gamma P_{5sp}\ .X_{5is}-YP^{1}_{k'p}\geq 0\ k'=3\&\ \forall\, p\in P \quad (3.73)$$

$$YP^{1}_{k'p}+\ ZP^{1}_{k'p}\ \leq 1 \qquad k'=3\ \&\ \forall\, p\in P \quad (3.74)$$

This set of constraints state that ordinary nodes not covered within T_1 minutes will or will not be covered within T_2 minutes by fire vehicles. If they are, a benefit will be added to the objective function. Equations (3.68) and (3.69) are for fire engine coverage, equations (3.70) and (3.71) are for fire truck coverage and equations (3.72), (3.73) and (3.74) are for full coverage.

$$\sum_{s\,\in\, s_6}\sum_{i\in V_6}\gamma P_{6sp}\ .X_{6is}+\ \sum_{s\,\in\, s_4}\sum_{i\in V_4}\gamma P_{4sp}\ .X_{4is}-\ YPF^{0}_{k'p}-YPF^{2}_{k'p}\geq 0$$

$$k'=3\ \&\ \forall\, p\in P \quad (3.75)$$

$$YPF^{2}_{k'p}\ \leq\ YPF^{0}_{k'p} \qquad k'=3\ \&\ \forall\, p\in P \quad (3.76)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \gamma P_{6sp} \ .X_{6is} + \sum_{s \in s_5} \sum_{i \in V_5} \gamma P_{5sp} \ .X_{5is} - YPS^0_{k'p} - YPS^2_{k'p} \geq 0$$

$$k' = 3 \ \& \ \forall \ p \in P \quad (3.77)$$

$$YPS^2_{k'p} \leq YPS^0_{k'p} \qquad k' = 3 \ \& \ \forall \ p \in P \quad (3.78)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \gamma P_{6sp} \ .X_{6is} + \sum_{s \in s_4} \sum_{i \in V_4} \gamma P_{4sp} \ .X_{4is} - YP^0_{k'p} - YP^2_{k'p} \geq 0$$

$$k' = 3 \ \& \ \forall \ p \in P \quad (3.79)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \gamma P_{6sp} \ .X_{6is} + \sum_{s \in s_5} \sum_{i \in V_5} \gamma P_{5sp} \ .X_{5is} - YP^0_{k'p} - YP^2_{k'p} \geq 0$$

$$k' = 3 \ \& \ \forall \ p \in P \quad (3.80)$$

$$YP^2_{k'p} \leq YP^0_{k'p} \qquad k' = 3 \ \& \ \forall \ p \in P \quad (3.81)$$

This set of constraints attempts to cover ordinary nodes by fire vehicles at least twice within T_2 minutes. Equations (3.75) and (3.76) are for fire engine vehicles, equations (3.77) and (3.78) are for fire truck vehicles and equations (3.79), (3.80) and (3.81) are for full coverage.

$$\sum_{s \in s_6} \sum_{i \in V_6} \delta I_{6sl} \ .X_{6is} + \sum_{s \in s_4} \sum_{i \in V_4} \delta I_{4sl} \ .X_{4is} - ZIF^1_{k'l} - ZIF^2_{k'l} \geq 0$$

$$k' = 3 \ \& \ \forall \ l \in I \quad (3.82)$$

$$ZIF^2_{k'l} \leq ZIF^1_{k'l} \qquad k' = 3 \ \& \ \forall \ l \in I \quad (3.83)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \delta I_{6sl} \ .X_{6is} + \sum_{s \in s_5} \sum_{i \in V_5} \delta I_{5sl} \ .X_{5is} - ZIS^1_{k'l} - ZIS^2_{k'l} \geq 0$$

$$k' = 3 \ \& \ \forall \ l \in I \quad (3.84)$$

$$ZIS^2_{k'l} \leq ZIS^1_{k'l} \qquad k' = 3 \ \& \ \forall \ l \in I \quad (3.85)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \delta I_{6sl} \ .X_{6is} + \sum_{s \in s_4} \sum_{i \in V_4} \delta I_{4sl} \ .X_{4is} - ZI^1_{k'l} - ZI^2_{k'l} \geq 0$$

$$k' = 3 \ \& \ \forall \ l \in I \quad (3.86)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \delta I_{6sl} \ .X_{6is} + \sum_{s \in s_5} \sum_{i \in V_5} \delta I_{5sl} \ .X_{5is} - ZI^1_{k'l} - ZI^2_{k'l} \geq 0$$

$$k' = 3 \ \& \ \forall \ l \in I \qquad (3.87)$$

$$ZI^2_{k'l} \leq ZI^1_{k'l} \qquad k' = 3 \ \& \ \forall \ l \in I \qquad (3.88)$$

This set of equations defines whether or not critical nodes are covered at least once or at least twice by fire vehicles within T_1 minutes. Constraints (3.82) and (3.83) are for fire engines and constraints (3.84) and (3.85) are for fire trucks.

Equations (3.86), (3.87) and (3.88) ensure that if the critical node is fully covered, the additional benefit will be added to the objective function.

$$\sum_{s \in s_6} \sum_{i \in V_6} \gamma I_{6sl} \ .X_{6is} + \sum_{s \in s_4} \sum_{i \in V_4} \gamma I_{4sl} \ .X_{4is} - YIF^1_{k'l} \geq 0 \quad k' = 3 \ \& \ \forall \ l \in I \qquad (3.89)$$

$$YIF^1_{k'l} + ZIF^1_{k'l} \leq 1 \qquad k' = 3 \ \& \ \forall \ l \in I \qquad (3.90)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \gamma I_{6sl} \ .X_{6is} + \sum_{s \in s_5} \sum_{i \in V_5} \gamma I_{5sl} \ .X_{5is} - YIS^1_{k'l} \geq 0 \quad k' = 3 \ \& \ \forall \ l \in I \qquad (3.91)$$

$$YIS^1_{k'l} + ZIS^1_{k'l} \leq 1 \qquad k' = 3 \ \& \ \forall \ l \in I \qquad (3.92)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \gamma I_{6sl} \ .X_{6is} + \sum_{s \in s_4} \sum_{i \in V_4} \gamma I_{4sl} \ .X_{4is} - YIF^1_{k'l} \geq 0 \quad k' = 3 \ \& \ \forall \ l \in I \qquad (3.93)$$

$$\sum_{s \in s_6} \sum_{i \in V_6} \gamma I_{6sl} \ .X_{6is} + \sum_{s \in s_5} \sum_{i \in V_5} \gamma I_{5sl} \ .X_{5is} - YIS^1_{k'l} \geq 0 \quad k' = 3 \ \& \ \forall \ l \in I \qquad (3.94)$$

$$YI^1_{k'l} + ZI^1_{k'l} \leq 1 \qquad k' = 3 \ \& \ \forall \ l \in I \qquad (3.95)$$

This set of constraints states that critical nodes not covered within T_1 minutes by fire vehicles will or will not be covered within T_2 minutes. If they are, a benefit will be added to the objective function. Equations (3.89) and (3.90) show fire engine coverage, and equations (3.91) and (3.92) show fire truck coverage.

Constraints (3.93), (3.94) and (3.95) ensure that if the critical node is fully covered at least once in the second critical time and not fully covered in the first critical time, an additional benefit will be added to the objective function.

- Nonnegativity and Integrality Constraints:

$X_{kij}, X_{kis}, X_{kih}, EXT_{kij}, R_{ki}^{e}, R_{ki}^{s}, R_{ki}^{h}, XAB_{ij}$	Binary integer variables
$ZPF_{k'p}^{1}, YPF_{k'p}^{0}, YPF_{k'p}^{1}, YPF_{k'p}^{2}, ZIF_{k'l}^{1}, YIF_{k'l}^{1}, ZIF_{k'l}^{2}$	Binary integer variables
$ZPS_{k'p}^{1}, YPS_{k'p}^{0}, YPS_{k'p}^{1}, YPS_{k'p}^{2}, ZIS_{k'l}^{1}, YIS_{k'l}^{1}, ZIS_{k'l}^{2}$	Binary integer variables
$ZP_{k'p}^{1}, YP_{k'p}^{0}, YP_{k'p}^{1}, YP_{k'p}^{2}, ZI_{k'l}^{1}, YI_{k'l}^{1}, ZI_{k'l}^{2}$	Binary integer variables
D_{kj}	General integer variables

3.5. Summary

In this chapter the problem statement and its characteristics were explained first. Also the assumptions used in this research were introduced. Then the mathematical formulation that is developed for this problem was presented with a brief explanation of the objective function and constraints. Some numerical examples are conducted to show different features of the model and the results are shown in Chapter 4.

Chapter 4: Numerical Study

In this chapter a very small-size problem will be introduced and some scenarios will be examined to evaluate the features of the proposed mathematical model. Xpress 7.1 software is used to solve these numerical examples to find the optimal solution. Then, to illustrate how the running time will increase by increasing the problem size, different size problems are generated. The results show that by increasing the problem size, running time grows exponentially; commercial software such as Xpress is not suitable for solving the problem. Next chapter explains the heuristic method that has been developed to find near-optimal solutions in more reasonable time.

4.1 A Very Small-Size Problem

First, the different features of the proposed model will be shown for different scenarios on a very small-size problem, which is illustrated in Figure 4.1.

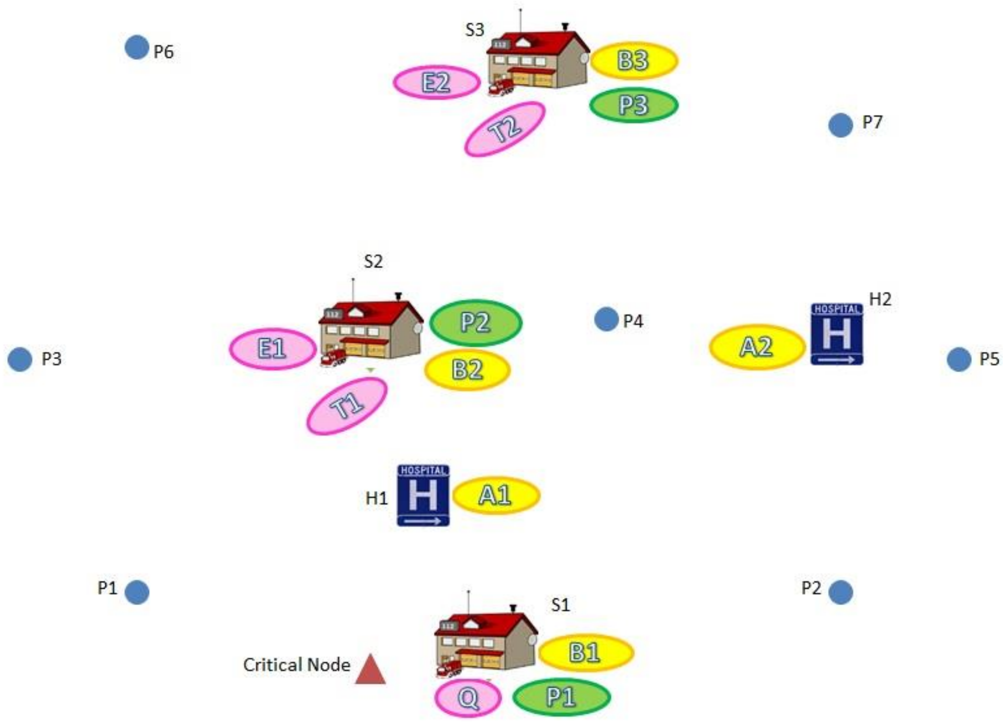

Figure 4.1 A Very Small-Size Problem Network and Characteristics

This problem has 7 ordinary demand nodes (shown as blue circles), 1 critical node (shown as a red triangle), 3 fire stations (shown as a building shape) and 2 hospitals (shown with an H). The oval figures represent the emergency vehicles in the system. Green ovals represent police cars; there are 3 police cars in the system. Yellow ones represent ambulances, with A signifying ALS ambulances and B signifying BLS ambulances; there are 2 ALS ambulances and 3 BLS ambulances in the system.

Pink ovals represent fire vehicles, with E signifying fire engines, T signifying fire trucks, and Q signifying fire quints; there are 2 fire engines, 2 fire trucks and 1 fire quint in the system.

First, it is assumed that all vehicles reside at their home stations. As shown in Figure 4.1, the home station for Q and B1 is fire station 1, the home station for E1, T1, and B2 is fire station 2, and the home station for E2, T2, and B3 is fire station 3. A1's home station is hospital 1, and A2's home station is hospital 2. P1, P2, and P3 are located at fire station 1, 2, and 3, respectively, but—as was mentioned in Chapter 3 police cars do not have any home station and can be relocated to any node in the system.

In this problem, fire vehicles can be relocated to other fire stations and therefore have three stations. Ambulances can use fire stations and hospitals and therefore have five stations. Police cars can use every node in the system and therefore have 13 stations. This sample problem is summarized in Table 4.1.

Table 4.1 The Small-Size Problem Specifications

Number of Ordinary Nodes	7
Number of Critical Nodes	1
Number of Police Cars	3
Number of BLS Ambulances	3
Number of ALS Ambulances	2
Number of Fire Engines	2
Number of Fire Trucks	2
Number of Fire Quints	1
Number of Stations for Police Cars	13
Number of Stations for Ambulances	5
Number of Stations for Fire Vehicles	3

4.1.1 Base Case

At first it is assumed that the vehicles have just started their working shift, that the working hour ratio is 0 for all of them, and that no vehicle is near the end of its shift. It is also assumed that all demand points in the system have the same likelihood of an emergency occurring, so $E_{k'p}$ and $E_{k'l}$ for all nodes are assumed to be 1.

The first and second critical times that are considered in the coverage problem are shown in Table 4.2.

Table 4.2 Assumptions for First and Second Critical Times for Coverage Criteria

Critical Time for Coverage Criteria (minute)	Police Cars	BLS Ambulances	ALS Ambulances	Fire Engines	Fire Trucks	Fire Quints
First Critical Time (T1)	5	5	8	5	5	5
Second Critical Time (T2)	9	9	12	9	9	9

As shown in Table 4.2, the first critical time for all types of vehicles except ALS is 5 minutes, and for ALS is 8 minutes. The second critical time is assumed to be 9 minutes for all types of vehicles except for ALS, which is 12 minutes. 5 and 9 minutes are important response times mentioned in NFPA guidelines. Also, it is assumed that these numbers can be changed to 8 and 12 minutes for ALS ambulances, because they can get to the incident later than BLS ambulances.

Therefore, based on these assumptions, the given network and characteristics of the problem, Xpress 7.1 is used to find the best location for the vehicles. The results show that police car P1 should be relocated to the critical node and police car P3 should be relocated to hospital H2 to maintain better coverage of the whole area. Figure 4.2 shows the software's optimal answer for providing better coverage of the area.

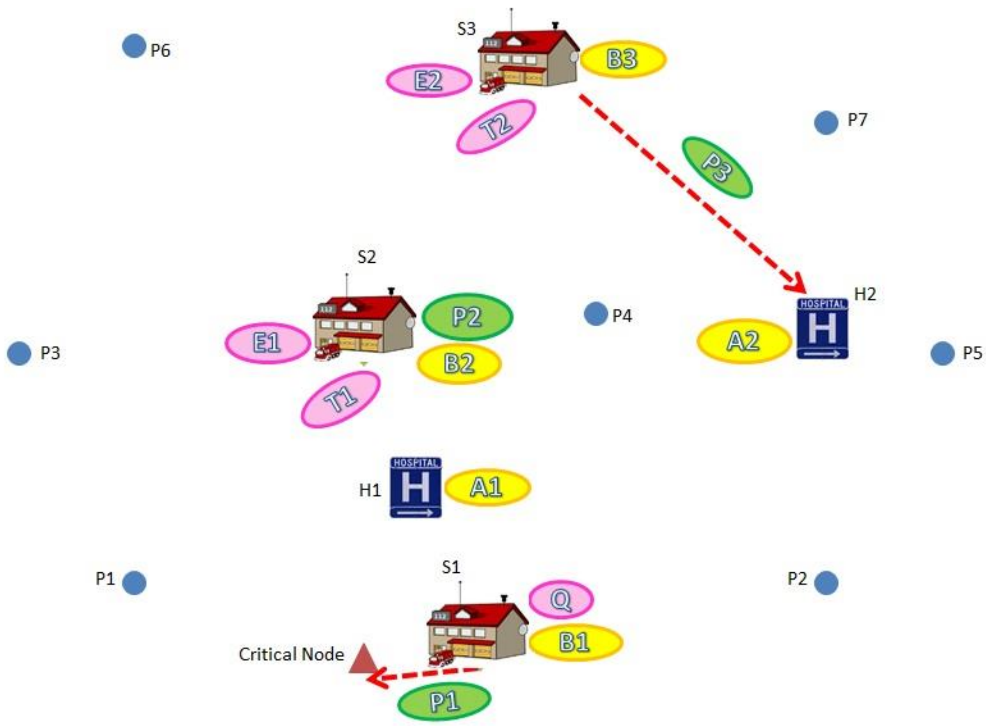

Figure 4.2 Relocation of P1 and P3 to Provide Better Coverage

Table 4.3 shows coverage of the ordinary nodes by different categories of vehicles after relocation of P1 and P3. All nodes except P6 are covered by the police and ambulance categories at least once within T_1 minutes and twice within T_2 minutes. P6 is only covered once within T_2 minutes.

P3, P4, and P7 are covered by the fire vehicle category at least once within T_1 minutes, and P1, P2, P5, and P6 are covered at least once within T_2 minutes. Also, all of the nodes except P3 have at least double coverage by fire vehicles within T_2 minutes.

Table 4.4 shows coverage of the critical node by different categories of vehicles. The critical node in the system is covered only once within T_1 minutes by all three categories.

Table 4.3 Full Coverage of the Ordinary Nodes after P1 and P3 Relocation

Nodes	**Police Category**			**Ambulance Category**			**Fire Vehicle Category**		
	First Coverage within T1	First Coverage within T2	Second Coverage within T2	First Coverage within T1	First Coverage within T2	Second Coverage within T2	First Coverage within T1	First Coverage within T2	Second Coverage within T2
P1	Yes	-	Yes	Yes	-	Yes	-	Yes	Yes
P2	Yes	-	Yes	Yes	-	Yes	-	Yes	Yes
P3	Yes	-	Yes	Yes	-	Yes	Yes	-	-
P4	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes
P5	Yes	-	Yes	Yes	-	Yes	-	Yes	Yes
P6	-	Yes	-	-	Yes	-	-	Yes	Yes
P7	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes

Table 4.4 Full Coverage of the Critical Node after P1 and P3 Relocation

Nodes	**Police Category**			**Ambulance Category**			**Fire Vehicle Category**		
	First Coverage within T1	First Coverage within T2	Second Coverage within T2	First Coverage within T1	First Coverage within T2	Second Coverage within T2	First Coverage within T1	First Coverage within T2	Second Coverage within T2
Critical Node	Yes	-	-	Yes	-	-	Yes	-	-

4.1.2 Scenario #1

In this scenario, it is assumed that two emergency incidents are occurring within the system, one in node P1 and the other one in node P3. Both need one police car, one BLS, one ALS, one fire engine, and one fire truck. As was mentioned in Chapter 3, if an ALS gets to the emergency in the required time for a BLS, there is no need for a BLS. Also, a fire quint can do the job of a fire engine and a fire truck.

The required time for the police car and BLS ambulance to both incidents is 5 minutes, the required time for the ALS ambulance is 8 minutes, and the time for the fire vehicles is 6 minutes. The dispatching and relocation of vehicles for this scenario is shown in Figure 4.3.

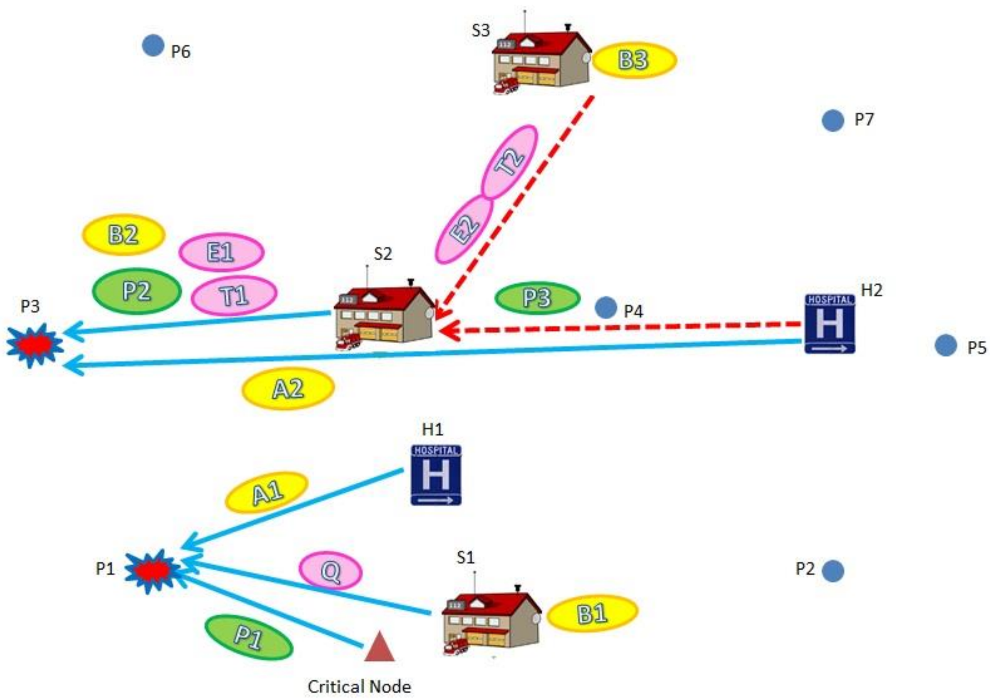

Figure 4.3 Dispatching and Relocation of Vehicles for Scenario #1

As demonstrated in Figure 4.3, P2, B2, E1, and T1 will be dispatched from S2 to the emergency in node P3 and A2 will be dispatched from hospital H2 to this emergency. A2 will arrive at this emergency site later than the required time, but other vehicles will get there in time.

Also, police car P1 will be dispatched from the critical node, Q will be dispatched from station S1, and A1 will be dispatched from hospital H1 to the emergency in node P1. A1 will arrive at the emergency in 4.5 minutes, so it arrives within the required time for BLS—which is 5 minutes—and there is no need for BLS at this emergency. Also, Q will serve this emergency as both a fire engine and a fire truck. All the vehicles will arrive at this emergency site within the required time.

Dispatching to emergency incidents is shown with solid blue lines and relocation of vehicles to provide better coverage is shown in dotted red lines. In this scenario, E2 and T2 will be relocated from station S3 to station S2 and P3 will be relocated from hospital H2 to station S2 to provide better coverage for future demand.

4.1.3 Scenario #2

In this scenario, we assume that the vehicles are finished working at emergency P3, E1 and T1 need to be sent to a station to be recharged, B2 should take a patient to a hospital, and A2 and P2 do not need to be recharged. A1, Q, and P1 are still dealing with emergency P1. Therefore, the model is solved with this new information to see where vehicles should be sent; the results are shown in Figure 4.4.

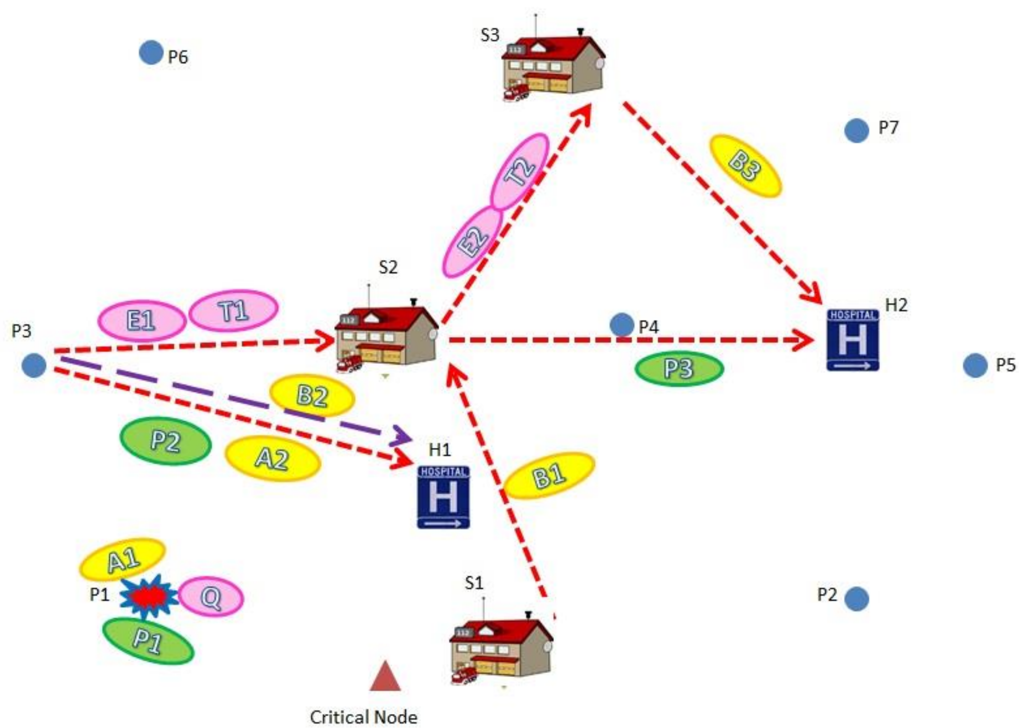

Figure 4.4 Relocation of Vehicles for Scenario #2

As seen in Figure 4.4, B2 is taking a patient to hospital H1 (long dashed purple lines represent vehicles taking patients to hospitals). P2 and A2 are sent to hospital H1, B1 is relocated to station S2, and P3 and B3 are relocated to hospital H2. E1 and T1 are sent to station S2, which is their home station, to get recharged, and E2 and T2 are sent back to their home station, S3.

4.1.4 Scenario #3

In this scenario, it is assumed that the vehicles are in the middle of their relocation when an emergency happens at node P6. This new emergency needs 1 police car in 5 minutes, 1 ALS in 8 minutes, 1 fire engine and 1 fire truck in 6 minutes. The model is run to see which vehicles should be sent to the new emergency incident; results are shown in Figure 4.5.

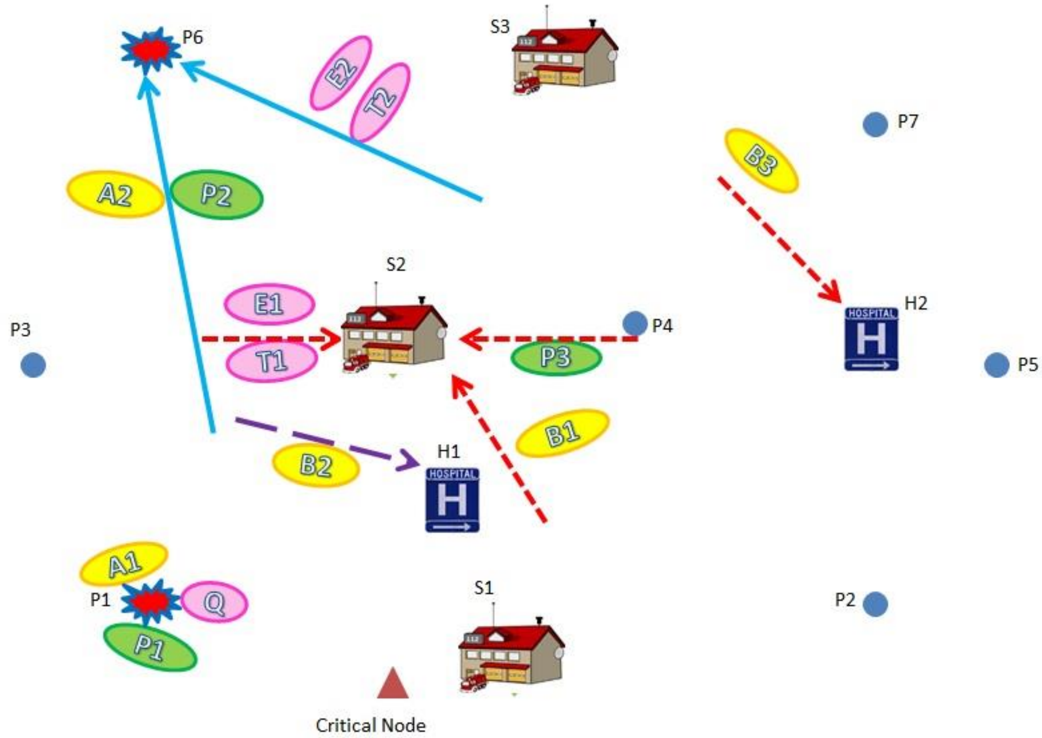

Figure 4.5 Dispatching and Relocation of Vehicles for Scenario #3

As demonstrated in Figure 4.5, E1, T1, B1, B2, and B3 will follow their previous assignments. A2, P2, E2, and T2 are reassigned to the emergency at node P6 en route to their previous destinations, and police car P3 is reassigned to station S2.

4.1.5 Scenario #4

In this scenario, it is assumed that all emergencies have been serviced and the vehicles are free to return to their stations. E2 and A2 need to be recharged. E1 and T1 have been recharged and are available to be dispatched, and B2 is finished with the patient at the hospital and is now free. The model is solved and optimal locations for vehicles are shown in Figure 4.6.

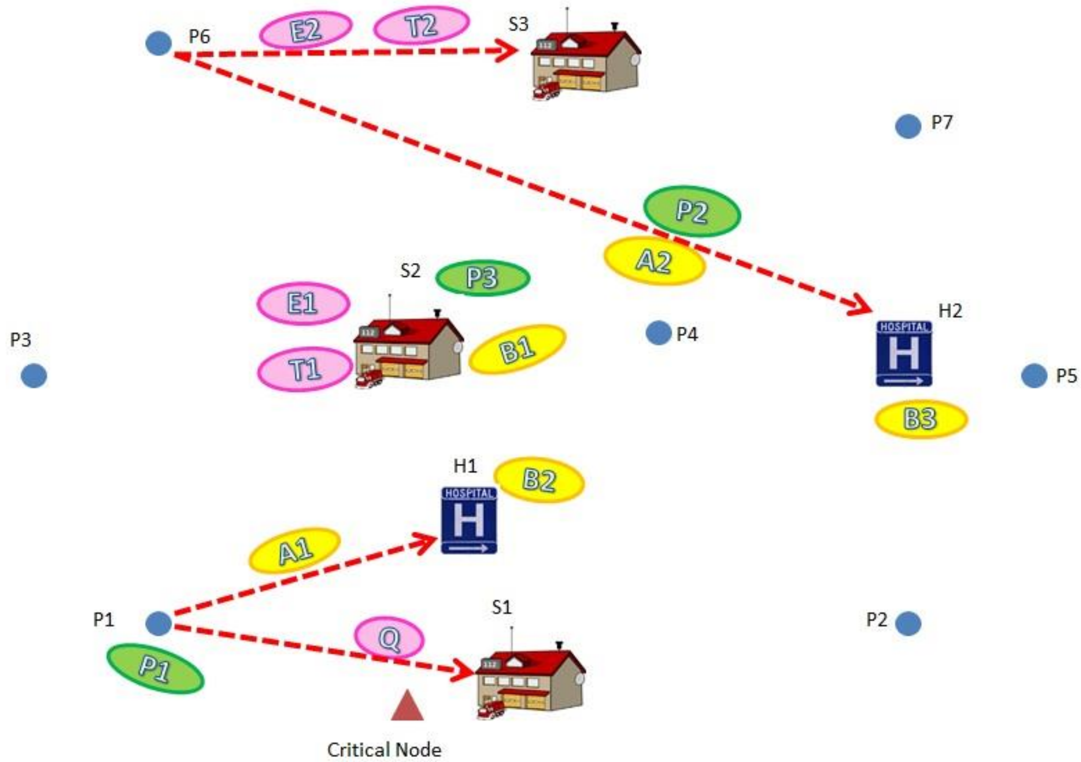

Figure 4.6 Relocation of Vehicles for Scenario #4

As shown in Figure 4.6, Q will be sent to station S1, A1 will be sent to hospital H1, E2 and T2 will be sent to station S3, and A2 and P2 will be sent to hospital H2. Police car P1 is choosing node P1 as its station, because police cars can reside at any point in the system.

4.1.6 Scenario #5

This scenario is identical to scenario #4 with only a small change. It is assumed that all the emergencies in the system have been serviced and the vehicles are free to return to their stations. E2 and A2 need to be recharged, E1 and T1 have been recharged and are available to be dispatched, and B2 is finished with the patient at the hospital and is now free. The only change is the likelihood of an emergency occurring in the nodes. In this scenario, it is assumed that the importance of node P6 has changed, and for some period it will be a 4 instead of a 1. This means that we expect more emergencies will occur in this node compared to other nodes in the system. For all other nodes in the system, $E_{k'p}$ and $E_{k'l}$ are 1, and only $E_{k'p}$ for node P6 is 4. With this new information the model is run, and the optimal locations for vehicles are shown in Figure 4.7.

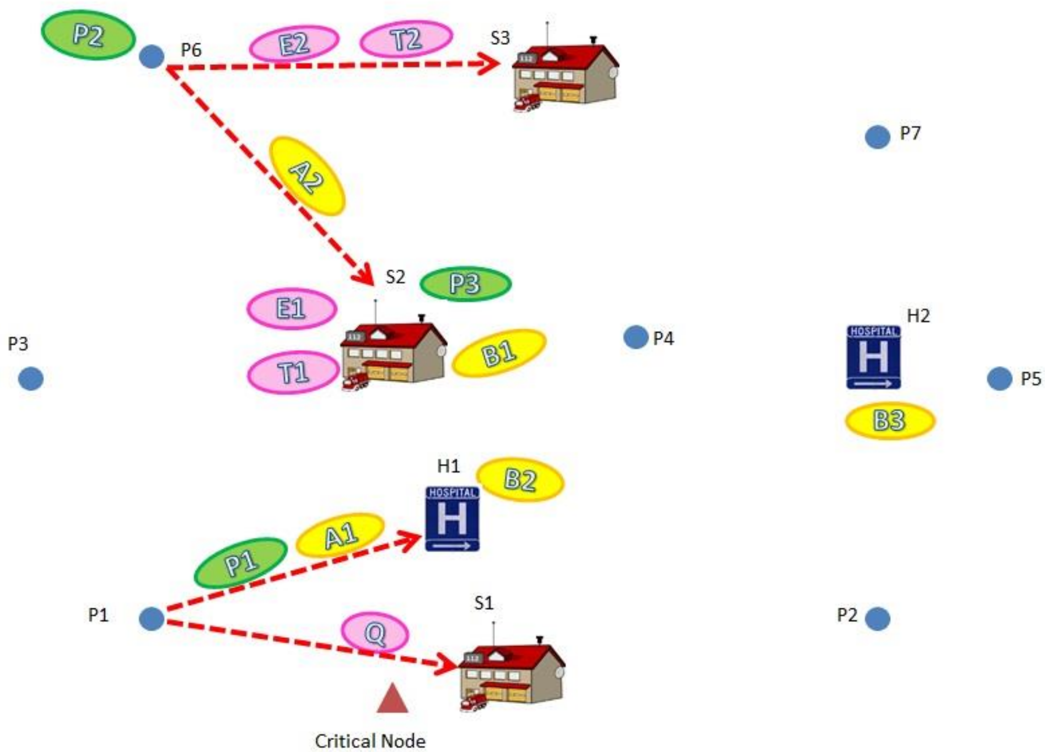

Figure 4.7 Relocation of Vehicles for Scenario #5

By comparing Figures 4.6 and 4.7, it can be seen that there have been some changes in the relocation of vehicles. Police car P1 will be sent to hospital H1 instead of remaining at node P1. Police car P2 will remain at node P6 instead of being relocated to hospital H2, and A2 will be sent to station S2 instead of to hospital H2.

Thus, the model is capable of taking into account the variation in likelihood of an emergency occurring, and it can relocate vehicles throughout the day based on demand variations.

4.1.7 Scenario #6

In the scenarios that have been shown so far, no vehicle was close to the end of its shift. In this scenario, we will demonstrate the model's capability to consider this aspect. B1 is very close to the end of its shift and prefers to return to its home station, S1. This new information is a new input to the model; and other inputs are similar to scenario #5. The solution of the model is demonstrated in Figure 4.8.

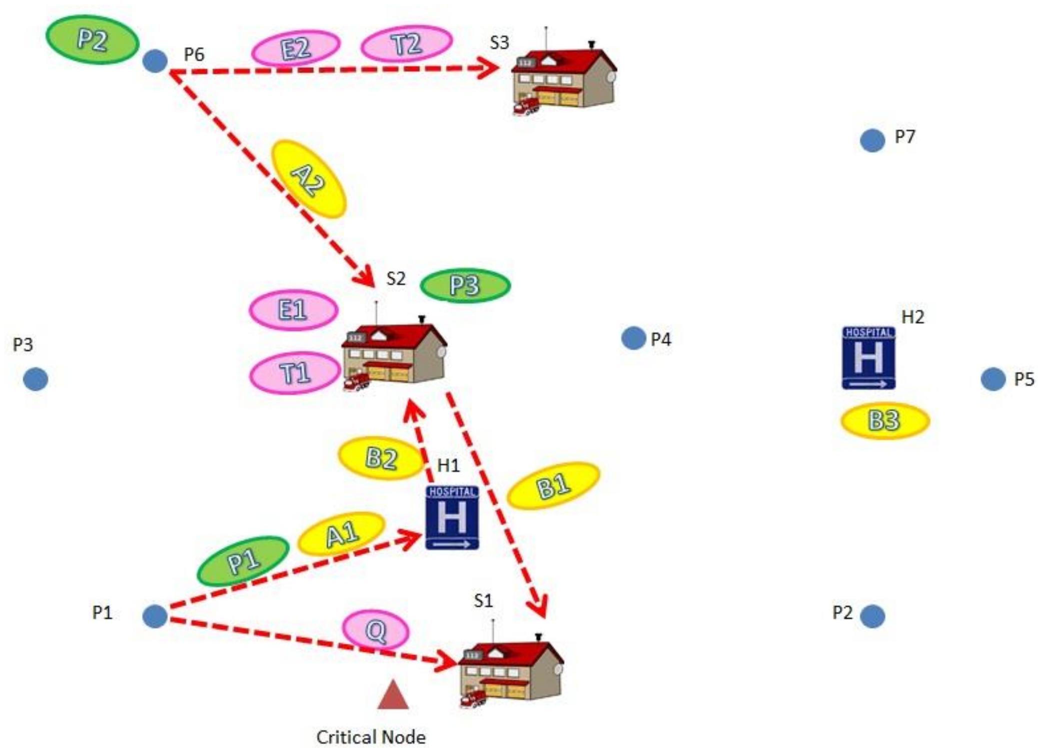

Figure 4.8 Relocation of Vehicles for Scenario #6

In this figure, it can be seen that vehicle B1—which is very close to the end of its shift—is relocated to its home station S1, and B2 is relocated to station S2 to provide better coverage for the whole system. Other relocations are similar to scenario #5.

4.1.8 Scenario #7

In this very simple scenario, the model's capability for attempting to maintain workload balance between the vehicles' crews is shown. In all the cases that have been studied so far, the vehicles' workload ratios were assumed to be the same. In this scenario, it is assumed that after all relocations from scenario #6 have been completed, an emergency happens at node P2. This emergency is not severe, and needs only one BLS ambulance within 9 minutes. The workload ratio for B1 is 0.1 and it is very close to its end of the working shift. The workload ratio for B2 is 0.3, and the workload ratio for B3 is 0.9. Also, the workload ratios for A1 and A2 are assumed to be 0.7. B1, B2, B3, A1, and A2 can all take care of the job at emergency P2, because they are all within 9 minutes of the emergency site. The model prefers to send the vehicle with a smaller workload ratio, which is B1. B1 is very close to the end of its working shift, however, so the model prefers to send the vehicle that has a smaller workload ratio and also is not as close to the end of its working shift. This is B2; the optimal solution of the model is shown in Figure 4.9.

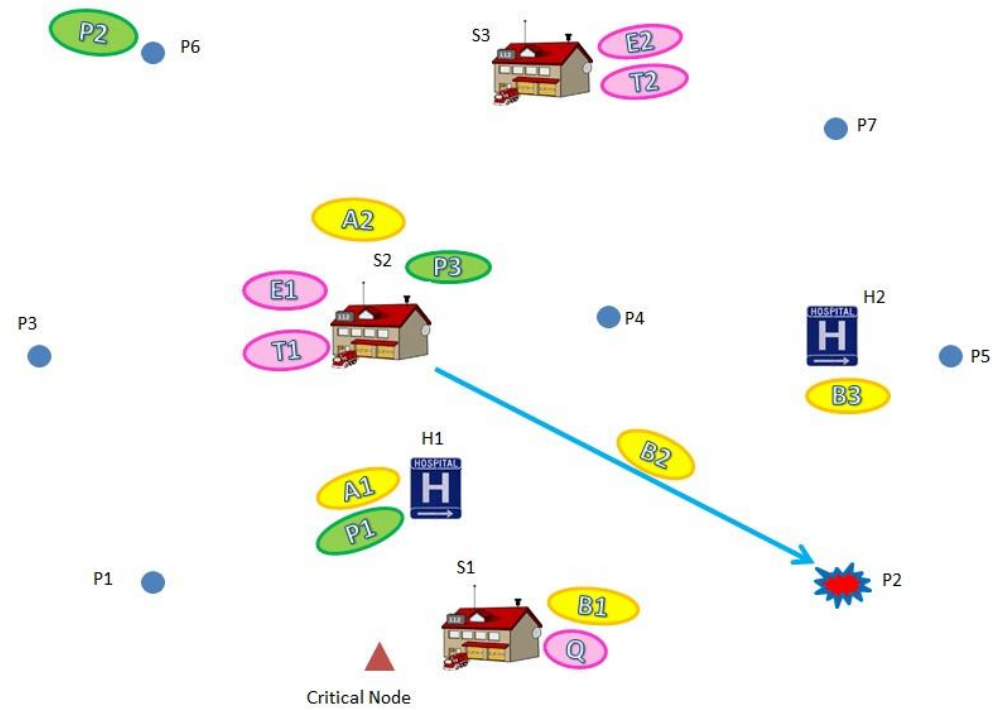

Figure 4.9 Dispatching of Vehicles for Scenario #7

4.2 Different Size Problems

Commercial software Xpress 7.1 was used to solve the model. For small-size problems, commercial solvers can find the optimal solution in a reasonable time, but when the size of the problem increases the running time increases exponentially; at some point, it is not practical to use commercial software. To see how the running time will increase by increasing the problem size, 14 cases with different sizes have been generated. The characteristics of these randomly generated cases are shown in Table 4.5.

Table 4.5 Characteristics of 14 Cases

Case	Number of Ordinary Nodes	Number of Critical Nodes	Number of Vehicles for Each Type	Number of Stations for Police Cars	Number of Stations for Other Categories	Number of Emergencies Waiting for Service
Case 1	50	5	5	5	5	1
Case 2	50	5	10	50	50	5
Case 3	500	50	10	50	50	5
Case 4	500	50	20	50	50	20
Case 5	500	50	30	50	50	20
Case 6	500	50	30	50	50	40
Case 7	500	50	30	100	100	10
Case 8	500	50	30	100	100	25
Case 9	500	50	30	400	50	25
Case 10	500	50	30	500	50	25
Case 11	1000	100	30	500	50	25
Case 12	1000	100	30	750	50	25
Case 13	1000	100	30	1000	50	25
Case 14	1500	100	30	1500	50	25

The model is solved with Xpress 7.1 for each case by one randomly generated set of data. The number of constraints and variables and the running time for each case are shown in Table 4.6.

Table 4.6 Number of Constraints and Variables and Running Time for 14 Cases

Case #	Number of Constraints	Number of Variables	Running Time (Sec) (Dispatching)	Running Time (Sec) (Dispatching and Coverage)
Case 1	2690	2132	0.0	0.1
Case 2	3654	5823	0.1	0.8
Case 3	26514	22338	1.2	24.2
Case 4	30529	30568	2.7	193.2
Case 5	32789	36608	4.2	58.5
Case 6	39529	45128	4.2	78.2
Case 7	29869	41348	8.5	120.3
Case 8	34924	47738	9.4	230.5
Case 9	34824	49238	7.9	257.2
Case 10	34924	52238	8.7	4918.3
Case 11	60324	70588	56.4	295.1
Case 12	60574	78088	51.1	494.9
Case 13	60824	85588	60.2	1439.9
Case 14	N.A.	N.A.	N.A.	N.A.

In Table 4.6, the number of constraints and variables for each case are shown. Also, there are two running times, one for dispatching only and the other for both dispatching and coverage. To find the dispatching running time, all the coefficients of demand nodes' coverage in the objective function are set to 0. In that case, the model will send vehicles only to emergencies and not try to relocate vehicles so as to provide better coverage for the whole area. As seen in this table, the running time for dispatching is at most 60.2 seconds for the 13 cases. In the last row there is no solution for dispatching vehicles, because Xpress was out of memory and could not even read the input file completely to calculate the number of variables and constraints.

The problem arises when the model wants to consider the coverage problem and relocate vehicles. In this situation, the running time for some cases like #10, 13 and 12 are unreasonable and for the last case, Xpress cannot find the optimal solution.

In designing these cases, the numbers of stations for different types of vehicles is assumed to be equal until case #8, for which the numbers of stations for all types of vehicles are assumed to be 100. This is not a realistic assumption; usually, the total number of stations for ambulances and fire vehicles is less than 50. Therefore, after case #8, the numbers of stations for all types of vehicles (except police cars) are limited to 50, which is a reasonable assumption. Police cars can reside at every point in the system, so the number of stations for this type of vehicle can increase with the increasing of the number of points.

4.3. Summary

In this chapter, a very small size problem was designed. A base-case scenario and 7 other scenarios for this small-size problem were generated to show the capabilities of the optimization model. These scenarios were solved by Xpress 7.1 and optimal solutions were discussed and shown in each scenario's subsection.

Then, to see how running times increase when the sample size is increased, 13 cases were generated. The number of ordinary nodes for these cases was between 50 and 1500, the number of critical nodes between 5 and 100, the number of vehicles between 5 and 30, the number of stations between 5 and 1500, and the number of emergencies awaiting vehicles between 1 and 40. In these cases, two running times were investigated, dispatching running time and dispatching and relocation running time. When the model considered the dispatching and coverage problem together, the running time could be too high to be practical; also, for some cases, Xpress cannot find an optimal solution. It is obvious, therefore, that heuristics algorithms are required to solve this optimization model for real-world problems in a reasonable time.

Chapter 5: Heuristic Method

As mentioned in the last chapter, commercial software like Xpress cannot find optimal solutions in a reasonable amount of time when the problem size increases. Developing a heuristic method that can find sound solutions in reasonable time is a must for the nature of this problem. In this chapter the developed heuristic method will be introduced and explained in detail; at the end of this chapter the results of the heuristic method will be compared to optimal solutions to demonstrate that the proposed heuristic works well.

5.1 Overall Explanation of the Heuristic Method

Figure 5.1 illustrates the heuristic method that has been developed for this research and has been coded in C^{++} language. As seen in Figure 5.1, at first, an initial solution for dispatching problem will be found. After that, several steps to improve the initial solution will be performed. Next, an initial solution for the coverage problem will be found and some improvements will be applied to the initial solution. Afterward, several improvements addressing the whole problem will be applied until a time limit is reached, at which point the solution will be reported. The details of the heuristic methods are explained in following sections.

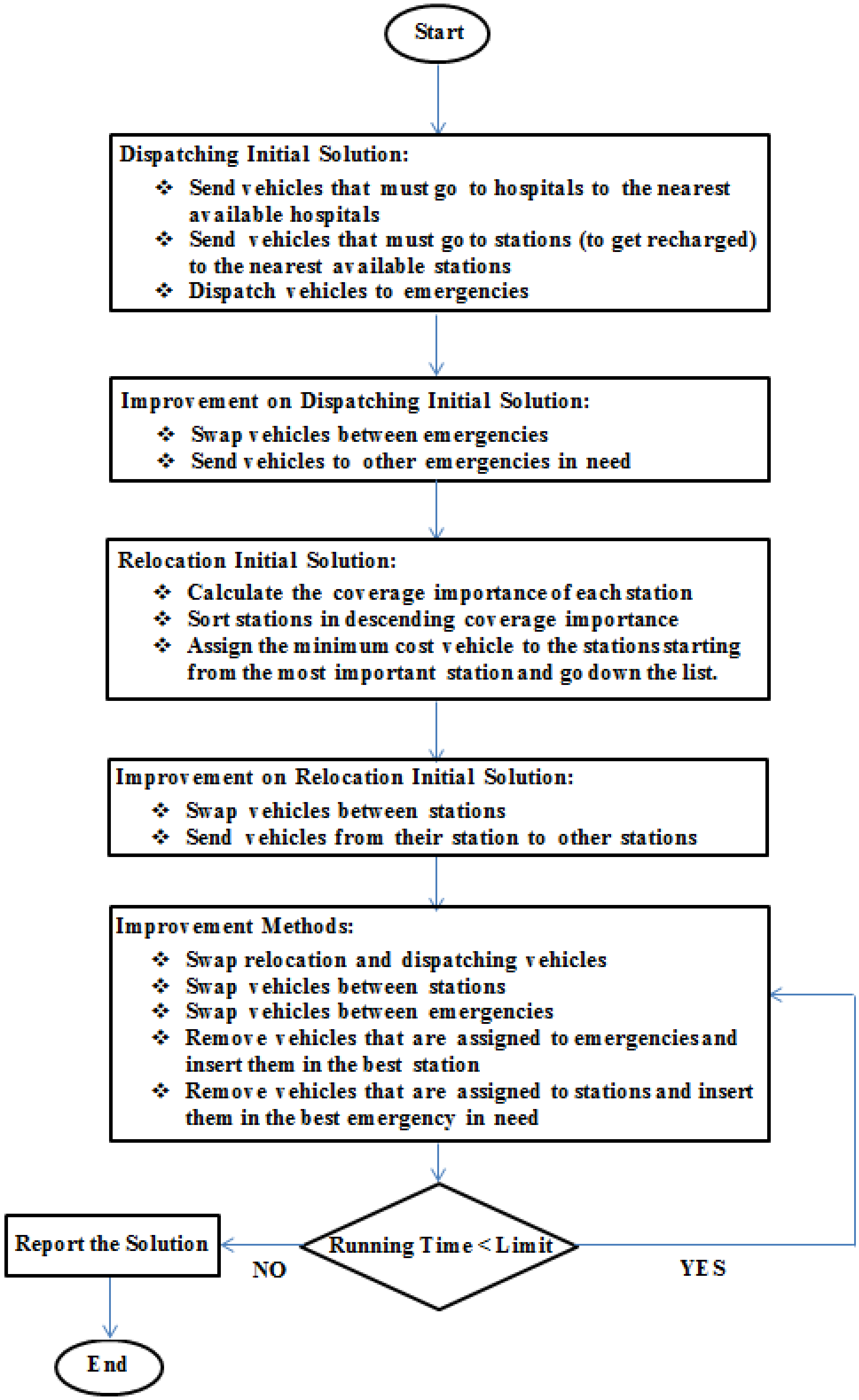

Figure 5.1 Heuristic Algorithm

5.2 Dispatching Initial Solution

Finding an initial solution for dispatching problem consists of three steps. First, the required vehicles must be sent to hospitals. Second, required vehicles are sent to stations to get recharged. Finally, the vehicles are dispatched to emergency locations.

5.2.1 Send Required Vehicles to Hospitals

Because some vehicles have to take patients to hospitals, the first step of the initial dispatching solution is to find the nearest available hospital for each one of these vehicles. The algorithm for this action is:

1. For all types of vehicles
    2. For each vehicle
        3. If the vehicle must go to hospitals
        4. Examine all available hospitals and choose the nearest one
        5. Send the vehicle to the nearest available hospital
        6. End if
    9. End for
10. End for

5.2.2 Send Required Vehicles to Stations to Get Recharged

Some vehicles deplete their resources after they handle an emergency. For example, an ambulance may be out of specific medications and must go to a station to get recharged. In this part, the nearest available station for each vehicle is identified and located, and the vehicle is sent to that station. The steps of this algorithm are:

1. For all types of vehicles
    2. For each vehicle
        3. If the vehicle must go to a station to get recharged
        4. Examine all available stations and choose the nearest one
        5. Send the vehicle to the nearest available station
        6. End if
    9. End for
10. End for

5.2.3 Dispatch Vehicles to Emergencies

This section provides an explanation for dispatching vehicles to emergencies. The dispatching algorithm for the various categories of emergency vehicles is different. The algorithm for initial dispatching of a police vehicle to an emergency, which is the simplest vehicle to dispatch, is called DIS1. When dispatching other categories, the DIS1 algorithm will be called.

5.2.3.1 Dispatching Initial Solution for Police Vehicles

The conceptual framework of the dispatching initial solution for police vehicle (DIS1) algorithm is:

1. Sort waiting emergencies in descending priority

- If there are emergencies in the same priority, sort them by their existing waiting time in the system in a non-increasing order

2. Select the first emergency on the list, until all the vehicles are dispatched, or all the emergencies are satisfied
3. Calculate the cost for sending available vehicles to this emergency
4. Assign the minimum cost vehicle to the emergency and make that vehicle unavailable, until the required number of vehicles are satisfied at the emergency
5. Remove the current emergency and go to step 2.

5.2.3.2 Dispatching Initial Solution for Ambulances

As previously mentioned, the ambulance category is assumed to have two types of vehicles - Advanced Life Support (ALS) and Basic Life Support (BLS). The conceptual framework of the dispatching initial solution for ambulances is:

1. Sort waiting emergencies in descending priority
    - If there are emergencies in the same priority, sort them by their existing waiting time in the system in a non-increasing order
2. Start with ALS vehicles
3. Select the first emergency on the list that needs ALS ambulances; continue until all the ALS vehicles are dispatched, or all the emergencies that need ALS are satisfied
4. Calculate the cost for sending available ALS vehicles to this emergency
5. Assign the minimum cost vehicle to the emergency and make that vehicle unavailable

6. If the current emergency needs a BLS and the ALS gets to the scene in the required time for BLS, decrease the number of needed BLS in this emergency by 1
7. Go to step 5 until the required number of ALS vehicles are satisfied at the emergency
8. Remove the current emergency and go to step 3
9. Call revised DIS1 algorithm for BLS ambulances

The ninth step is calling revised DIS1 algorithm. The term revised means that all available ALS vehicles are also considered in the pool of available BLS vehicles because they can do the job for BLS vehicles too.

5.2.3.3 Dispatching Initial Solution for Fire Vehicles

As explained in previous chapters, it is assumed that fire departments have three types of vehicles: Fire Engines (FE), Fire Trucks (FT), and Fire Quints (FQ). The conceptual framework of the dispatching initial solution for fire vehicles is:

1. Sort waiting emergencies in descending priority
    - If there are emergencies in the same priority, sort them by their existing waiting time in the system in a non-increasing order
2. Start with FE and FQ vehicles (because FQ can do the job for FE too)
3. Select the first emergency on the list, continue until all the FE and FQ vehicles are dispatched, or all the emergencies that need FE are satisfied
4. Calculate the cost for sending available FE and FQ vehicles to this emergency
5. Assign the minimum cost vehicle to the emergency and make that vehicle unavailable

6. If the current emergency needs a FT, too, and a FQ has been assigned to the emergency and it gets to the scene in the required time for FT, decrease the number of needed FT in this emergency by 1.
7. Go to step 5 until the required number of FE vehicles is satisfied at the emergency
8. Remove the current emergency and go to step 3.
9. Call revised DIS1 algorithm for FT vehicles

The ninth step is calling revised DIS1 algorithm. The term *revised* means that all available FQ vehicles are also considered in the pool of available FT vehicles because they can do the job for FT vehicles too.

5.3 Improvement on Dispatching Initial Solution

After finding an initial solution for the dispatching problem, improvement methods are applied to make the initial solution better. These improvement methods are:

- Swap vehicles between emergencies
- Send vehicles from emergencies to other emergencies in need

5.3.1 Swap Vehicles between Emergencies

In this section, one solution that is applied to save resources is to exchange the assignment of two vehicles assigned to different emergencies. At each step, the exchange that produces maximum saving will be chosen. The algorithm for this action is:

1. For k=1 to 6 (for all types of vehicles)

    2. For *V1*=1 to N_{V_k}

        3. For *V2*=1 to N_{V_k}

        4. If *V1* and *V2* are assigned to different emergencies

        5. Calculate Reassignment Saving

        6. End if

        7. End for

    8. Choose the *V2* that produces the maximum reassignment saving

    9. Change the assignment of *V1* and *V2*

    10. End for

11. End for

As explained in the section 3.4.1, N_{V_k} is the maximum number of type k emergency vehicles in the system.

Calculation of reassignment saving is more complicated for ALS and FQ vehicles. For these two types of vehicles, the previous role and the new role of the vehicles should be considered. For example, if two FQ vehicles get reassigned, then their role in the previous destination should be checked, whether they were working as either a FE or a FT or both a FE and a FT, and what their new role will be based on the time they can reach the new incident. These factors should figure in when calculating the reassignment saving.

5.3.2 Send Vehicles from Emergencies to Other Emergencies in Need

In this step, it is checked that by removing vehicles from their assigned emergencies and sending them to new emergencies in need, how much saving is achieved. The emergency that produces maximum saving will be chosen and that vehicle is assigned to this new emergency instead of the previous one selected in the initial solution. This process is reviewed for all vehicles assigned to emergencies in the initial solution.

The algorithm is:

1. For k=1 to 6 (for all types of vehicles)

    2. For *V1*=1 to N_{V_k}

        3. If *V1* is assigned to emergencies

            4. For all e in E^1

            5. Calculate Reassignment Saving

            6. End for

        7. Choose e that produces the maximum reassignment saving

        8. Change the destination of *V1* to emergency e

        9. Increase the number of needed vehicle k in the previous destination of vehicle *V1* by 1. (Check the previous role if *V1* is an ALS or a FQ)

        10. Decrease the number of needed vehicle k in emergency e by 1. (Check the new role if *V1* is an ALS or a FQ)

11. End if

12. End for

13. End for

As explained in the section 3.4.1, N_{V_k} is the maximum number of type k emergency vehicles in the system and E^1 is the set of emergency incidents in the system that are waiting for service.

In step 9 and 10 of this algorithm, if the vehicle is ALS or FQ, some precautions have to be taken into account. The previous role of the vehicle should be checked. For example, if *V1* is a FQ, and it was performing as both a FE and a FT, then the number of required FE and FT in the previous destination should be increased by one. Also, its role should be checked at the new destination and, if it is performing as both a FE and a FT, then the number of required FE and FT at the new destination should be decreased by 1.

Also, if the vehicle is an ALS or a FQ, the reassignment saving calculation will be more complicated, because the vehicle's previous role and new role will be important in the calculation.

5.4 Relocation Initial Solution

The first step in finding an initial solution for a relocation problem is calculating the coverage importance of each station. Based on how many ordinary demand nodes and critical demand nodes are covered in *T1* and *T2* minutes by each station, and the importance of the nodes, the coverage importance for each station is calculated. The algorithm is:

1. For k=1 to 6 (for all types of vehicles)

2. For s=1 to N_{S_k}

3. Calculate the coverage importance of the station

4. End for

5. End for

As explained in section 3.4.1, N_{S_k} is the maximum number of type k emergency vehicle stations in the system.

Also, it should be mentioned that the coverage importance of the station is more complicated for ALS and FQ vehicles. For example, we have to verify the ALS vehicles going to each station, how many nodes are going to have ALS, and how many nodes are going to have BLS. Also, partial coverage and full coverage for each station should be calculated in this case.

After identifying the coverage importance of each station, the following conceptual algorithm is used to find the initial solution for relocation problem.

1. For k=1 to 6 (for all types of vehicles)

2. Sort stations in descending coverage importance

3. Select the first station on the list, until all the available vehicles are assigned.

4. Calculate the cost for sending available vehicles to this station

5. Assign the minimum cost vehicle to the current station and make that vehicle unavailable.

6. Go to the next station in the list and go to step 4 until each station has a vehicle.

7. Go to step 3 until there is no available vehicle.

8. End for

5.5 Improvement on Relocation Initial Solution

The next step after finding an initial solution for relocation problem is to improve that initial solution. Two improvement methods have been applied to the relocation initial solution at this stage. These improvement methods are:

- Swap vehicles between stations
- Send vehicles from their stations to other stations

5.5.1 Swap Vehicles between Stations

In this section, the improvement method of saving by exchanging the assignment of two vehicles that are assigned to different stations is described. At each step, the exchange that produces maximum saving will be chosen. The algorithm for this action is:

1. For k=1 to 6 (for all types of vehicles)

2. For $V1$=1 to N_{V_k}

3. For $V2$=1 to N_{V_k}

4. If *V1* and *V2* are assigned to different stations

5. Calculate Reassignment Saving

6. End if

7. End for

8. Choose the *V2* that produces the maximum reassignment saving

9. Change the assignment of *V1* and *V2*

10. End for

11. End for

As explained in the section 3.4.1, N_{V_k} is the maximum number of type k emergency vehicles in the system.

5.5.2 Send Vehicles from Their Stations to Other Stations

In this step, how much can be saved is verified by removing vehicles from their assigned stations and sending them to other stations. Then the station that produces maximum saving, if there is any, will be chosen, and the vehicle gets assigned to this new one instead of the previous one selected in the initial solution. This process is checked for all vehicles assigned to stations in the initial solution. The algorithm is:

1. For k=1 to 6 (for all types of vehicles)

2. For *V1*=1 to N_{V_k}

3. If *V1* is assigned to stations

4. For s=1 to N_{S_k}

5. Calculate Reassignment Saving

6. End for

7. Choose *s* that produces the maximum reassignment saving

8. Change the destination of *V1* to station *s*

9. Adjust the coverage of the nodes in the network by this reassignment

9. End if

10. End for

11. End for

As explained in the section 3.4.1, N_{V_k} is the maximum number of type k emergency vehicles in the system and N_{S_k} is the maximum number of type k emergency vehicle stations in the system.

Calculation of saving based on reassignment is very complicated because the nodes around the previous station are going to lose coverage and the nodes around the new assigned station are going to gain coverage. These changes in the coverage of the demand nodes must be calculated and then the improvement in the objective function is calculated. Then, the cost saving of this action is evaluated. The calculation is more complicated when the vehicle is an ALS or a FQ; in that case partial coverage as well as full coverage of the demand nodes must also be considered.

5.6 Improvement Methods

So far, we have presented algorithms for constructing an initial solution for dispatching and relocation problem, sequentially. Also, some algorithms have been introduced to improve the constructed initial solutions.

Given an initial solution for the dispatching and relocation problems, we can apply several improvement methods. The dispatching problem and relocation problem are both considered together and the improvement methods are applied. These improvement methods are:

- Swap relocation and dispatching vehicles
- Swap vehicles between emergencies
- Swap vehicles between stations
- Remove vehicles that are assigned to emergencies and insert them in the best station
- Remove vehicles that are assigned to stations and insert them in the best emergency in need

The second and third improvement methods mentioned above are the same as 5.3.1 and 5.5.1 improvement methods respectively and they are not going to be explained in this section again. They are used at this stage again because, after swapping vehicles between relocation and dispatching problems, the solution is changed, and applying these two improvement methods may produce savings.

5.6.1 Swap Relocation and Dispatching Vehicles

This section explains how to save by exchanging the assignment of two vehicles when one of them is assigned to an emergency and the other is assigned to a station. First, each vehicle assigned to an emergency is considered and then vehicles assigned to stations are checked to see that how much improvement the objective function can have by exchanging the destination of those vehicles. At each step, the exchange that produces the maximum saving will be chosen. The algorithm for this action is:

1. For *k*=1 to 6 (for all types of vehicles)

    2. For *V1*=1 to N_{V_k}

        3. For *V2*=1 to N_{V_k}

            4. If *V1* is assigned to emergencies and *V2* is assigned to stations or Vice Versa

            5. Calculate Reassignment Saving

            6. End if

        7. End for

    8. Choose the *V2* that produces the maximum reassignment saving if there is any.

    9. Change the assignment of *V1* and *V2*

    10. End for

11. End for

As explained in the section 3.4.1, N_{V_k} is the maximum number of type k emergency vehicle in the system.

Calculation of reassignment saving is more complicated for ALS and FQ vehicles. The coverage factor would not change because in either case the station is going to get the same vehicle and the only difference is the cost of the travel for each of the two vehicles. However the emergency situation needs more consideration for these two types of vehicles. The role of the first vehicle at the emergency needs to be verified as well as what the role of the other vehicle will be at the emergency. For example, if two FQ vehicles get reassigned, then the role of the vehicle at the emergency needs to be confirmed, whether it was working as one FE or one FT, or both a FE and a FT, and also what the role of the other vehicle can be at the emergency based on the time it would reach the incident. Once these factors are known, then the reassignment saving can be calculated, which is quite complicated.

5.6.2 Remove Vehicles Assigned to Emergencies and Insert Them in the Best Station

In this part, each vehicle that is assigned to an emergency is checked to see how much savings is realized if it is removed from that emergency and is inserted in each station. The station that produces the maximum saving, if there is one, will be chosen and the vehicle is removed from the emergency and inserted in that station. The algorithm is:

1. For k=1 to 6 (for all types of vehicles)

2. For *V1*=1 to N_{V_k}

3. If *V1* is assigned to emergencies

4. For *s*=1 to N_{S_k}

5. Calculate Reassignment Saving

6. End for

7. End if

8. Choose *s* that produces the maximum reassignment saving if there is any.

9. Change the destination of *V1* to station *s*

10. Increase the number of needed vehicle *k* in the previous destination of vehicle *V1* by 1. (Check the previous role if *V1* is an ALS or a FQ)

11. Adjust the coverage of the nodes in the network by this reassignment

12. End for

13. End for

As explained in the section 3.4.1 N_{V_k} is the maximum number of type *k* emergency vehicles in the system and N_{S_k} is the maximum number of emergency vehicle stations for type *k* in the system.

In step 10 of the above algorithm it is explained that if the vehicle is ALS or FQ, then some precautions have to be taken into account. The previous role of the vehicle should be checked. For example, if *V1* is a FQ, and it was performing as both a FE and a FT, then the number of required FE and FT in the emergency should be increased by one. In addition, the reassignment saving calculation is more complicated if the vehicle is ALS or FQ, because in that case its previous role is important.

Also, the nodes around station *s* are going to have more coverage and their coverage must be adjusted. This coverage adjustment is more complicated when the vehicle is ALS or FQ, because the partial coverage as well as full coverage should be considered.

5.6.3 Remove Vehicles Assigned to Stations and Insert Them in the Best Emergency in Need

In this section, each vehicle that is assigned to a station is checked to see how much savings will be realized if it is removed from that station and is inserted in each emergency in need. The emergency that produces the maximum saving will be chosen if there is any and the vehicle is removed from the station to be inserted in that emergency. The algorithm is:

1. For *k*=1 to 6 (for all types of vehicles)

    2. For *V1*=1 to N_{V_k}

        3. If *V1* is assigned to stations

4. For all e in E^1

5. Calculate Reassignment Saving

6. End for

7. End if

8. Choose e that produces the maximum reassignment saving

9. Change the destination of *V1* to emergency e

10. Decrease the number of needed vehicle k in emergency e by 1. (Check the new role if *V1* is an ALS or a FQ)

11. Adjust the coverage of the nodes in the network by this reassignment

12. End for

13. End for

As explained in the section 3.4.1 N_{V_k} is the maximum number of type k emergency vehicles in the system and E^1 is the set of emergency incidents in the system waiting for service.

In step 10 of the above algorithm it is explained that if the vehicle is ALS or FQ, some precautions have to be taken into account. The new role of the vehicle should be checked. For example if *V1* is a FQ, and it is performing as both a FE and a FT, then the number of required FE and FT in the emergency should be decreased by one.

Also, the nodes around station *s* (the previous destination of the vehicle) are going to have less coverage according to this reassignment and their coverage must be adjusted. This coverage adjustment is more complicated when the vehicle is ALS or FQ, because the partial coverage as well as full coverage should be considered.

5.7 Heuristic Results

To see how the heuristic method is performing, its solution is compared to the optimal solution. For this purpose, eight categories were developed, and for each category four different cases were defined. These cases varied in the weight of the coverage problem and the weights were increased from very low numbers to high numbers. The characteristics of these cases are illustrated in Table 5.1. These characteristics are: number of ordinary nodes, number of critical nodes, number of stations for police cars, number of stations for other emergency service categories, number of vehicles for each type, number of emergencies waiting for service, number of constraints, number of variables, and coverage importance. The four cases in each category are only different in coverage importance and all other characteristics are the same.

Table 5.1 Characteristics of 32 Cases

Case #	Number of Ordinary Nodes	Number of Critical Nodes	Number of Stations for Police Cars	Number of Stations for other Categories	Number of Vehicles for Each Type	Number of Emergencies Waiting for Service	Constraints	Variables	Coverage Importance
1-1	100	10	5	5	5	10	6394	4990	Very Low
1-2									Low
1-3									Medium
1-4									High
2-1	100	10	20	5	5	10	6409	5065	Very Low
2-2									Low
2-3									Medium
2-4									High
3-1	500	50	50	50	10	20	30664	25690	Very Low
3-2									Low
3-3									Medium
3-4									High
4-1	1000	100	50	50	20	10	56074	47230	Very Low
4-2									Low
4-3									Medium
4-4									High
5-1	1000	100	50	50	20	20	60924	51280	Very Low
5-2									Low
5-3									Medium
5-4									High
6-1	1000	100	50	50	40	20	70644	65760	Very Low
6-2									Low
6-3									Medium
6-4									High
7-1	1000	100	100	50	20	20	60974	52280	Very Low
7-2									Low
7-3									Medium
7-4									High
8-1	1000	100	500	50	20	20	61374	60280	Very Low
8-2									Low
8-3									Medium
8-4									High

For each case, three different input files were randomly generated using three different random seed numbers. Each one of these examples was solved with both Xpress software and our heuristic method to compare the results. The running time for our heuristic was set to 30 seconds. The heuristic gap and Xpress running time are shown for each seed on each case in the Table 5.2. The numbers in black illustrate that Xpress could solve the problem optimally. For example, in case # 2-3 with seed 1, the heuristic gap is 2.7% and the running time for Xpress is 612.2 seconds, which is more than 10 minutes and is unreasonably high for the nature of our problem. This demonstrates that sometimes, even for the problems with small sizes, the commercial software cannot be used and a good heuristic method should be applied. Xpress was out of memory in some examples as it is shown in Table 5.2, like case # 3-4, so on those examples the heuristic cannot be compared to the optimal solution.

Also, in some cases, Xpress could not find optimal solutions even after a very long running time. In those cases the numbers are shown in red and the heuristic gap is showing deviation from the lower bound found with Xpress instead of the deviation from the optimal solution. Also, in those cases, after the heuristic gap, there is a number in parenthesis; this number is the gap between our heuristic solution and Xpress best solution. For example, at case # 5-3, in seed 3, the gap of our heuristic with the lower bound found with Xpress is 9.5% and the gap of our heuristic with best solution of Xpress after 3436.1 seconds is 4.8%. What is interesting here is that in just 30 seconds sometimes our heuristic can find better solution than Xpress in a very long running time. These cases are shown with blue numbers in parenthesis. For example one of these cases is case # 2-4 with seed 1. The heuristic gap from the lower bound found in Xpress is 33% but our heuristic solution is 15.5% better than the Xpress best solution after 86396.6 seconds.

Table 5.2 Heuristic Gap and Xpress Running Time of 32 Cases with 3 Different Random Seeds

Case #	Seed1		Seed2		Seed3	
	Heuristic Gap	Xpress Running Time	Heuristic Gap	Xpress Running Time	Heuristic Gap	Xpress Running Time
1-1	1.2%	0.5	0.0%	0.4	1.5%	0.2
1-2	1.3%	0.5	0.1%	0.6	1.5%	1.8
1-3	2.6%	15.4	0.8%	3.8	1.4%	167
1-4	19.9%	319.5	3.4%	18	7.0%	1.9
2-1	1.2%	0.2	0.0%	0.2	1.5%	0.6
2-2	1.3%	111.8	0.1%	0.4	1.5%	1.2
2-3	2.7%	612.2	0.8%	60.2	2.1%	10297.5
2-4	33.0% (-15.5%)	86396.6	6.8%	7.1	3.1%	3.1
3-1	6.3%	40.1	7.8%	27.9	2.0%	29.7
3-2	6.3%	31	5.1%	28.8	2.1%	160.3
3-3	7.6% (7.5%)	5735.4	8.0% (7.9%)	25001.2		Out of Mem.
3-4		Out of Mem.		Out of Mem.		Out of Mem.
4-1	0.5%	163.8	1.3%	175.9	0.5%	162.3
4-2	1.6%	220.8	1.9%	171.6	2.5%	163.5
4-3	0.2%	170.4	0.2%	204.2	0.2%	161.3
4-4	0.1%	184.7	0.1%	417.9	0.0%	157.4
5-1	0.1%	209	0.3%	212.6	0.1%	223.2
5-2	0.2%	224.2	0.4%	214	0.2%	226
5-3	4.9%	8750.3		Out of Mem.	9.5% (4.8%)	3436.1
5-4	0.8%	652.7	1.1% (0.8%)	3938.8		
6-1	18.5%	539.1	3.5%	532.9	4.3%	489.5
6-2	3.2%	633.7	1.0%	578.7	0.8%	549.8
6-3	0.6%	558.5	0.1%	532.5	0.1%	518.7
6-4	0.1%	571.1	0.0%	511	0.0%	493.1
7-1	0.1%	230.5	0.3%	216	0.2%	215.7
7-2	0.2%	232.4	0.4%	217.4	0.1%	224.7
7-3	5.9% (-11.6%)	3097.8		Out of Mem.	13.5% (8.4%)	3184.6
7-4	0.5%	6037	1.2% (0.8%)	1830.9	1.8% (1.5%)	2058
8-1	0.1%	268.8	0.3%	256.4	0.1%	255.6
8-2	0.3%	311.2	0.3%	263.6	0.3%	258.4
8-3	5.0% (3.6%)	2063.3		Out of Mem.	14.6% (-4.9%)	1351.7
8-4	1.7% (1.4%)	24296.1	0.9% (0.4%)	2007.7		Out of Mem.

The summary of Table 5.2 is shown on Table 5.3. In Table 5.3, average Xpress running time, maximum Xpress running time and average gap of our heuristic for 32 cases can be seen. There are some cases like case # 3-3, which have no results and they are the ones for which Xpress could not find the optimal solution or Xpress ran out of memory. The average Xpress running time for these cases can get as high as 8750.3 seconds and maximum Xpress running time can be as high as 10297.5

seconds. The average gap for lots of cases was less than 1%. For some cases it was around 5%. On two cases, the average gap was higher than 6%; case # 6-1 with an average gap of 8.8% and case # 1-4 with the average gap of 10.1%. The average gap of all the cases together was 1.99%, which is quite good.

Table 5.3 Average Xpress Running Time, Maximum Xpress Running Time and Average Gap for 32 Cases

Case #	Average Xpress Running Time (s)	Maximum Xpress Running Time (s)	Average Gap
1-1	0.4	0.5	0.9%
1-2	1.0	1.8	1.0%
1-3	62.1	167.0	1.6%
1-4	113.1	319.5	10.1%
2-1	0.3	0.6	0.9%
2-2	37.8	111.8	1.0%
2-3	3656.6	10297.5	1.9%
2-4	5.1	7.1	4.9%
3-1	32.6	40.1	5.4%
3-2	73.4	160.3	4.5%
3-3	N.A.	N.A.	N.A.
3-4	N.A.	N.A.	N.A.
4-1	167.3	175.9	0.7%
4-2	185.3	220.8	2.0%
4-3	178.6	204.2	0.2%
4-4	253.3	417.9	0.1%
5-1	214.9	223.2	0.2%
5-2	221.4	226.0	0.3%
5-3	8750.3	8750.3	4.9%
5-4	652.7	652.7	0.8%
6-1	520.5	539.1	8.8%
6-2	587.4	633.7	1.7%
6-3	536.6	558.5	0.3%
6-4	525.1	571.1	0.1%
7-1	220.7	230.5	0.2%
7-2	224.8	232.4	0.2%
7-3	N.A.	N.A.	N.A.
7-4	6037.0	6037.0	0.5%
8-1	260.3	268.8	0.2%
8-2	277.7	311.2	0.3%
8-3	N.A.	N.A.	N.A.
8-4	N.A.	N.A.	N.A.

The results show that the proposed heuristic works very well. In a very short time it can identify results that sometimes Xpress, after running for a very long time, cannot find.

So far, the running time for our heuristic was set to 30 seconds. A sensitivity analysis was performed on the running time to see whether a longer or shorter running time can be used or 30 seconds is suitable enough. For this purpose, in each one of the eight categories mentioned above, two cases were selected and heuristic was applied on one example. The running time was set to 1, 2, 5, 30, and 60 seconds in different runs and the objective function was compared in these different runs. The results are shown in Table 5.4.

Table 5.4 The Objective Function of Our Heuristic Method with Different Running Times

Case #	**Running Time**				
	1 Second	2 Seconds	5 Seconds	30 Seconds	60 Seconds
1-3	333799	333799	333799	333799	333799
1-4	120193	120193	120193	120193	120193
2-3	334596	334596	334596	334596	334596
2-4	113333	113333	113333	113333	113333
3-1	1547004	1547004	1547004	1547004	1547004
3-2	1538723	1538723	1538723	1538723	1538723
4-1	22443	22443	22443	22443	22443
4-2	-18339	-18339	-18339	-18339	-18339
5-2	167561	167553	167553	167553	167553
5-3	-116486	-116493	-116493	-116493	-116493
6-1	13380	13372	13372	13372	13372
6-2	-27679	-27684	-27684	-27684	-27684
7-3	-114697	-114770	-114770	-114770	-114770
7-4	-3636663	-3648056	-3648056	-3648056	-3648056
8-1	191035	191030	191030	191030	191030
8-3	-117788	-117796	-117796	-117796	-117796

Table 5.4 demonstrates that the objective functions in half of the cases remain exactly the same after one second running time, and there is no improvement after one second. In the other half of the cases, after two seconds, the objective functions remained the same and between one and two seconds, they had improvement. The percentage of improvement after one second running time can be seen in Figure 5.2 for those cases that show improvement.

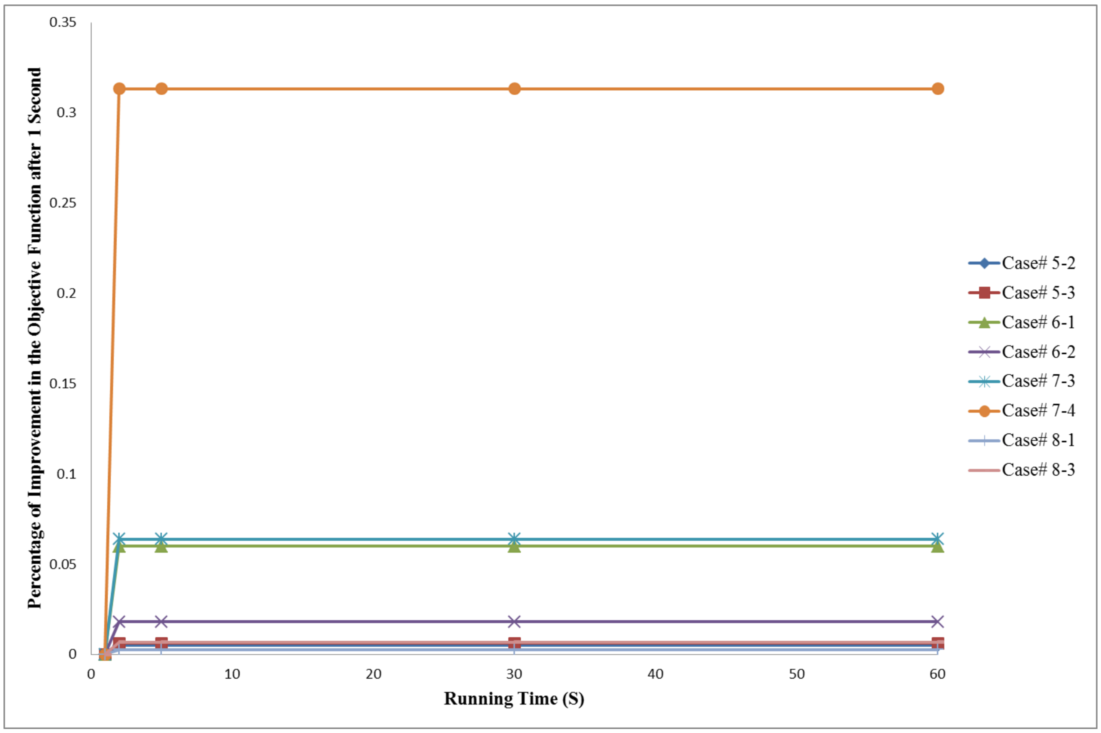

Figure 5.2 Objective Function Improvement after 1 Second Running Time

As it is obvious in the Figure 5.2, after two seconds the objective functions of all of the cases remain the same. To be on the safe side, it was decided to set the running time of our heuristic to five seconds instead of two seconds. So, there is no need to let the heuristic run for 30 seconds and five seconds running time can give solutions as good as 30 seconds or even 60 seconds running time.

5.8. Summary

In this chapter first the proposed heuristic method was explained in detail. Then to see how the heuristic method performed, several cases were generated and the results of the heuristic method were compared to Xpress optimal solutions. The comparisons illustrate that the heuristic method is very promising and it can find very good solutions in a very short time. Sometimes Xpress couldn't find optimal solutions after running for a very long time and also sometimes it was out of memory. In addition, in some examples, in a very short time our heuristic could find better solutions than Xpress after running for a long time. At the end of this chapter, a sensitivity analysis was performed on heuristic running time and it confirmed that five seconds running time is suitable enough for our heuristic.

Chapter 6: Simulation Model

In Chapter 5, the heuristic method that was developed for this research was explained in detail and it was shown that it is capable of finding very good solutions in a very short time. However, to see how the proposed model performs in a real-world case study, a simulation procedure is necessary. Because the framework of a simulation model that can mimic the entire operation of an emergency response system is complicated and unique, no existing simulation software was suitable for this purpose. As a result, a very sophisticated simulation model that can see most of the details in the system has been developed for this research and it has been coded in C^{++} language. In this chapter, this simulation model will be explained in detail and then in chapter 8, it will be applied to a real case study.

6.1 Conceptual Framework of the Simulation Model

Figure 6.1 illustrates the conceptual framework of the simulation model that is developed for this research. In this simulation model, travel times on links are randomly generated and all-to-all shortest travel times are calculated using Dijkstra's algorithm. The location of emergency calls, their type, and their severity are also randomly generated. At each time step, the status of the vehicles and their locations are updated. Additionally, the statuses of emergencies are also updated, reflecting whether they are completed or not and whether the emergency's needs are fully satisfied, or if it still needs more vehicles. The next event and its time are selected at this point, with events being accident arrival, accident removal, or travel time update. Based on the optimization model, the vehicles can be dispatched to emergencies or sent to stations if they are done at their current emergency, or they may take patients to the nearest hospitals if necessary. Next, vehicles that need to change their destinations or paths get reassigned. At this point, the statistics are updated and the simulation time gets updated to let the next event happen in the system. As a result, there are different modules in the simulation model. These modules are:

- Travel time module
- Emergency call module
- Vehicle module
- Emergency module
- Next event module
- Optimization module
- Reassign module
- Statistics module
- Simulation time module

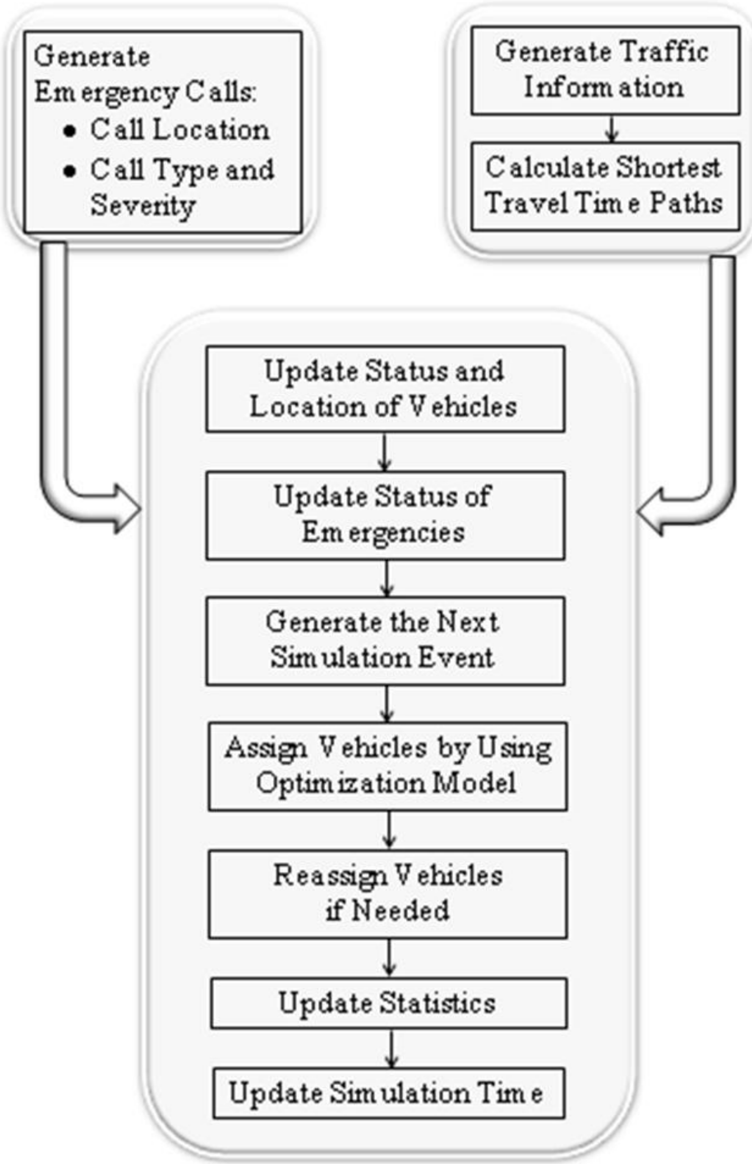

Figure 6.1 Conceptual Framework of the Simulation Model

6.1.1 Travel Time Module

It is assumed that the network of the area, the length of the links, and their free flow speed are known in advance. By knowing the length of the links and the free flow speed on those links, therefore, the travel time of the vehicles, if they travel with the free flow speed, can be calculated. Then at each time that the travel time updates, a coefficient between 0.8 and 1.2 is randomly generated for each link and the travel time of that link is multiplied by that coefficient. Using this procedure at the points when the travel time gets updated, the travel time is randomly generated. The assumption of travel time being between 0.8 and 1.2 of the free flow travel time is not unrealistic, because these vehicles can use their siren and travel very fast even when the roads are congested.

Whenever the travel time gets updated, Dijkstra's algorithm is used to calculate the shortest travel time paths. The travel time in this simulation model gets updated each five minutes, but the code is quite flexible and the five minutes can be changed. The minimum time that can be used for travel time update is one minute and it is better to not use increments less than one minute, because the running time of finding all to all shortest travel time paths by the implementation of Dijkstra's algorithm on 5000 nodes network (which is about the size of a real case) is around 30 seconds. It is therefore better that the time for travel time update not be less than one minute.

6.1.2 Emergency Call Module

It is assumed that the distribution of the accident arrival time and its mean arrival time are known in advance. It is also assumed that the emergencies can only happen at the demand nodes and the spatial distribution of the emergencies is known in advance. In addition, the emergency calls can belong to different categories and can have different severity and priority. As a result, the required number of vehicles in each type and the required time for them to be on the emergency incident will be defined based on the type and severity of the accident.

When it is the time for accident arrival, this module will randomly generate the location of the accident based on the spatial distribution. It will also randomly define the type and severity of the accident. As a result the number of needed vehicles of each type, as well as the required time for them to be on site, are defined.

6.1.3 Vehicle Module

In this module the vehicles are tracked and their location and their status are updated. At each time step the destination, the job and also the route that each vehicle takes is defined from the optimization module and the travel time module. So, when the simulation time gets updated, the location of the vehicle can be updated by knowing the time passed from the last event and also the route that each vehicle was taking to get to its destination. For example if the vehicle was assigned to an emergency in the last time step, and now another emergency has arrived in the system, the location of this vehicle is updated. If it has already reached the destination, its location is the location of that emergency and its status becomes busy, because when the vehicles reach the emergency they cannot get diverted to another emergency. If it is still en route, its location on the route is found. If at this exact time it is on a link between two nodes, its location is reported as the next point on that link, which is a reasonable assumption if the network is detailed enough. If it is the time that an emergency is finished, the vehicles that were handling that emergency become free. At each time step, therefore, this module is called to find the status and location of the vehicles.

6.1.4 Emergency Module

This module tracks the status of the emergencies in the system. When an emergency arrives in the system, based on the type and severity of that emergency, the number of vehicles in each type and the time required for those vehicles to reach the emergency, are defined from the emergency call module. This module checks to see whether the emergencies are fully satisfied or they are still waiting for vehicles, and also checks which emergency is still in the process and which one is finished and its vehicles are free.

6.1.5 Next Event Module

The events that can happen in this simulation model are:

- Travel time update
- Accident arrival
- Accident removal

The next event module checks the time for each one of these events and chooses the event that will happen sooner.

As was mentioned before, at each five-minute interval, travel time should be updated. Also, based on the accident inter-arrival time distribution, the time for the next accident arrival can be estimated.

For the accident removal, first of all it is assumed that the distribution of the service time of the accidents is known in advance. Also, based on the type and severity of the accidents, the service time required for each emergency can be estimated from the known distribution. The time for removal of each accident, therefore, can be estimated.

At each step, this module is called to define the next event and the time that event will happen in the system.

6.1.6 Optimization Module

This module calls the heuristic procedure. This module should be called whenever an event happens in the system and it finds the best destination for vehicles. When an accident arrives in the system, this module will send some vehicles to the emergencies and relocate others to provide better coverage. When an accident is removed from the system, this module will find the best stations for the free vehicles or may send them to other emergencies in need.

Also, after the travel time is updated, this module is called, because with the new travel time information another vehicle may be closer to the emergencies in need or another station may be closer to the vehicles seeking stations. In addition, based on the new travel time information the coverage importance of the stations can be updated and sending the vehicles to other stations may provide better coverage overall.

6.1.7 Reassign Module

At each time step, the destination of the vehicles is defined. Then in the next time step, when the optimization module is called, if the vehicles are still en route, they can be reassigned to other destinations. For example a vehicle that is available and is heading to a station can get reassigned to an emergency. Even a vehicle can get reassigned from one emergency to another emergency or even station. In summary the vehicles en-route to an emergency or a station can be reassigned. The vehicles that are out of supplies and need to be recharged can get reassigned, too; but their destination needs to be another station. Vehicles taking patients to hospitals can get reassigned but their new destination must be another hospital that is closer.

As was explained in chapter 3, the reassignment will confuse the drivers, so it will be considered in the model if it produces at least a minimum benefit to the whole system. Also, when the travel time gets updated, another route may have shorter travel time to the destination of a vehicle and in that case the vehicle gets reassigned to use another path instead of its previous one.

In this module, these cases are defined and they get reassigned to the new destination or even the new route.

6.1.8 Statistics Module

In this module the important statistics of the system are updated and saved. These important statistics are:

- Number of emergencies in the system
- Total response time to emergencies for each type of vehicle
- Average response time to emergencies for each type of vehicle
- Maximum response time to emergencies for each type of vehicle
- Number and percentage of vehicles that got to the scene later than required time for each type of vehicle
- Number and percentage of emergencies that got their first vehicle in each type in five minutes (five minutes is a critical time mentioned in the NFPA guidelines)
- Number and percentage of vehicles that got to the scene later than nine minutes for each type of vehicle (nine minutes is a critical time mentioned in the NFPA guidelines)
- Number of reassignment in the system

6.1.9 Simulation Time Module

This module is the last one in each time step and it will use the time selected in the next event module and set the simulation time to that time and let the next event happen in the system.

6.2. Summary

In this chapter, the unique and sophisticated simulation model that was developed for this research was explained. The simulation model has nine different modules and each one of them was explained in detail in this chapter. In chapter 7, the case study used for this research will be introduced and the input analysis done on the data to prepare it for the simulation model will be discussed.

Chapter 7: Case Study Characteristics

Real street network and real operational data are available on one of the counties in the Washington, D.C. metropolitan area. This case study data is used to calibrate the simulation model developed in this research. In this chapter, the input analysis, which was done on the data of this case study, is explained and the distributions of the different inputs are shown. These distributions are used in the next chapter to randomly generate the input for the simulation model to do sensitivity analysis. It should be mentioned that Yang (2006) worked with the same case study for her dissertation and she did input analysis on the data. As a result, there was no need to do the analysis again and in this research her input analysis is used.

7.1 Case Study Network

The existing network is consisted of 5496 nodes and 7325 directed links. The network is shown in Figure 7.1. In this region, there are 10 fire stations that are shown with red stars in Figure 7.1. These 10 stations have been used for fire vehicles in the analysis. It is also assumed that four hospitals exist in the network and the ambulance vehicles can use these four hospitals as well as 10 fire stations, so in total they are assumed to have 14 stations in the network.

Police cars can be relocated to any node in the system. As a result, they can have 5496 stations theoretically, but in the analysis usually the number of police stations has been limited to 100 or 200 random nodes in the network, which is a valid assumption and there is no need to consider all 5496 nodes as the police stations.

Figure 7.1 Case Study Network with Fire Stations
Source: Yang (2006)

The lengths of the links in the network are also shown in Table 7.1. By looking at this table, it can be seen that more than 96% of the links are shorter than 300 meters and the assumption of emergencies happening at the nodes is realistic when the network is detailed enough like this network.

Table 7.1 The Lengths of the Links in the Network
Source: Yang (2006)

Link Lengths (m)	Frequency	Cumulative Percentage
0-100	3941	53.8%
100-200	2586	89.1%
200-300	526	96.3%
300-400	110	97.8%
400-500	58	98.6%
500-600	33	99.0%
600-700	26	99.4%
700-800	7	99.5%
800-900	6	99.6%
900-1000	9	99.7%
> 1000	22	100.0%

7.2 Case Study Operational Data

The case study data is for ambulances and medical units on November and December of 2000. The data has 3029 records and each record is for one dispatched vehicle. There are 31 variables associated with each record, some important variables are: call type, vehicle identification number, call in time for the emergency, dispatching time of vehicles, and arrival time of the vehicles (Yang 2006).

7.2.1 Emergency Vehicles

Sixteen vehicle identification numbers have been reported in the data, so the fleet size of the ambulances is 16. There is no data showing the type of each vehicle, defining whether they are ALS or BLS ambulances. In the analysis of the case study, it is therefore assumed that 10 of them are BLS and six of them are ALS ambulances.

The data is only for ambulances, so based on the fleet number of ambulances it is assumed that the number of fleet for fire vehicles is the same and six of them are FE, six of them are FT, and four of them are FQ, totaling 16 fire vehicles.

Sixteen police vehicles seems to be low for a county, so it is assumed that the number of police vehicles is twice as much as number of ambulances and fire vehicles in the system and it is 32. The police vehicles are homogeneous, so they are all the same.

7.2.2 Number of Dispatched Vehicles

There are 3029 records of dispatched vehicles but only 2647 calls and it demonstrates that some calls get more than one vehicle. Yang (2006) categorized the emergency calls in four groups based on the number of dispatched vehicles each call got. The results are shown in Table 7.2.

Table 7.2 Categories of Calls Based on Number of Needed Vehicles
Source: Yang (2006)

Number of Dispatched Vehicles	Number of Calls	Percentage of Calls
1	2310	87.3%
2	299	11.3%
3	32	1.2%
>=4	6	0.2%

Table 7.2 shows that 87.3% of calls just need one vehicle, about 11.3% need two vehicles, 1.2% need three vehicles, and just 0.2% of calls need four vehicles.

7.2.3 Emergency Inter-arrival Time

Based on the analysis of 2646 inter-arrival time between 2647 calls with Arena Input Analyzer, Yang (2006) fitted five different distributions to the inter-arrival time between emergencies. The fitted functions and their squared error are shown in Table 7.3. It shows that the best fitted distribution is an exponential distribution $Exp(0.548)$, which has the minimum squared error.

Table 7.3 Five Best Distributions for Emergency Inter-Arrival Time
Source: Yang (2006)

Fitted Distribution	Squared Error
Exponential	0.00324
Lognormal	0.00395
Beta	0.006
Normal	0.109
Triangular	0.196

As a result, $Exp(0.548)$ distribution is used for emergency inter-arrival time in the simulation model. Yang (2006) graphed the real inter-arrival time data versus fitted distribution and the results can be seen in Figure 7.2.

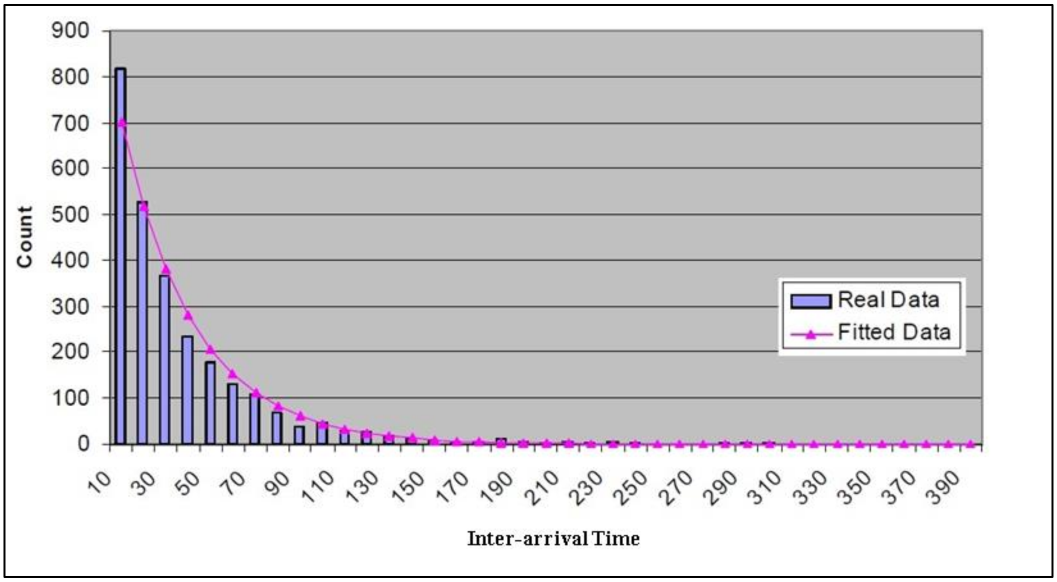

Figure 7.2 Comparison of Real Inter-arrival Time Data and Fitted Distribution
Source: Yang (2006)

7.2.4 Emergency Space Distribution

Another piece of information that is important in analyzing the case study is to see the space distribution of emergencies in the network. The emergencies seem to happen uniformly in the network and for that reason a uniform distribution of $U(1{,}5496)$ is used to randomly generate the location of the emergencies for simulation model in the case study. Yang (2006) graphed the real location of emergencies that happened in the system versus the fitted distribution and the results are shown in Figure 7.3.

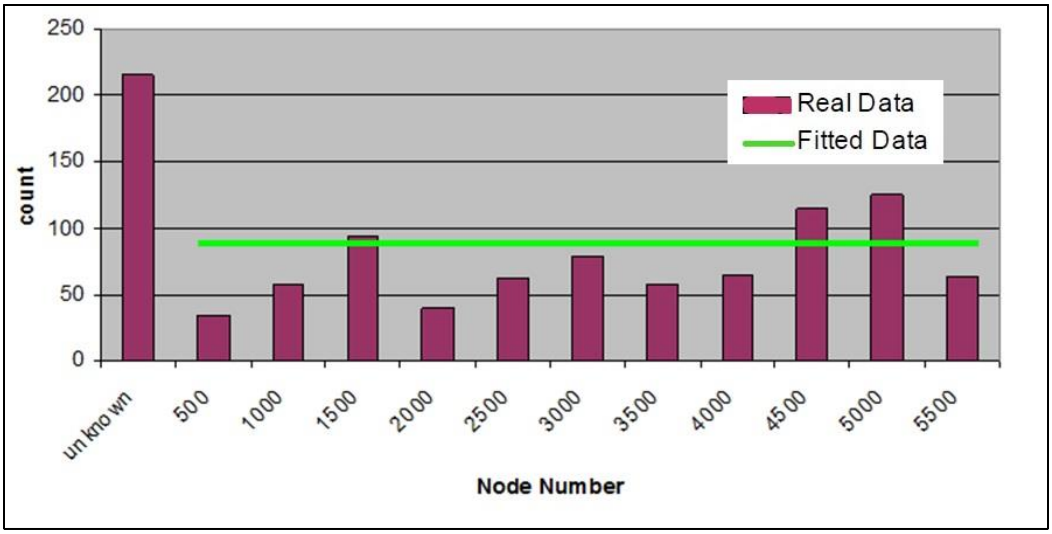

Figure 7.3 Comparison of Real Emergency Location and Fitted Distribution
Source: Yang (2006)

7.2.5 Emergency Service Time

For the service time, the time the vehicle arrives at the scene is subtracted from the time the vehicle departs the scene. Some calls are fake which means the vehicles are dispatched to the scene but it is a false alarm and then after a very short time the vehicles depart the scene. There are two peaks in the service time distribution graph and this illustrates that the service time distribution can be combination of several distributions. Yang (2006) analyzed the data for service time and she came up with the following conclusions:

- If further treatment is not needed for the emergency (fake calls), the service time distribution is lognormal and the probability of these cases is 20%.
- If further treatment is needed for the emergency, the service time will have normal distribution.

Yang (2006) also considered four different normal distributions for the calls that need further treatment. So, totally service time distribution can be estimated as the combination of five different distributions with P_i probability for each of them. These five distributions and their probability are (Yang (2006)):

$Type\ I : LOGN\ (2.7, 0.7) \qquad P_0 = 0.2$

$Type\ II : N\ (16\ , 7) \qquad P_1 = 0.13$

$Type\ III : N\ (57\ , 14) \qquad P_2 = 0.56$

$Type\ IV : N\ (85\ , 15) \qquad P_3 = 0.09$

$Type\ V : N\ (120\ , 40) \qquad P_4 = 0.02$

The first type is showing the fake calls; the other four normal distributions are for the calls that need further treatment. Type V is the most serious emergency, Type II is the mildest one, and Type I is a fake call. By combining these distributions the following equation for estimated pdf function of the service time can be obtained (Yang(2006)).

$$f(x) = P_0 * \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(lnx-\mu_1)}{2\sigma_1^2}} + \sum_{i=1}^{4} P_i * \frac{1}{x\sqrt{2\pi\sigma_i^2}} e^{-\frac{(x-\mu_i)}{2\sigma_i^2}} \qquad (7.1)$$

Figure 7.4 shows the real service time data against fitted distribution, which are very well matched.

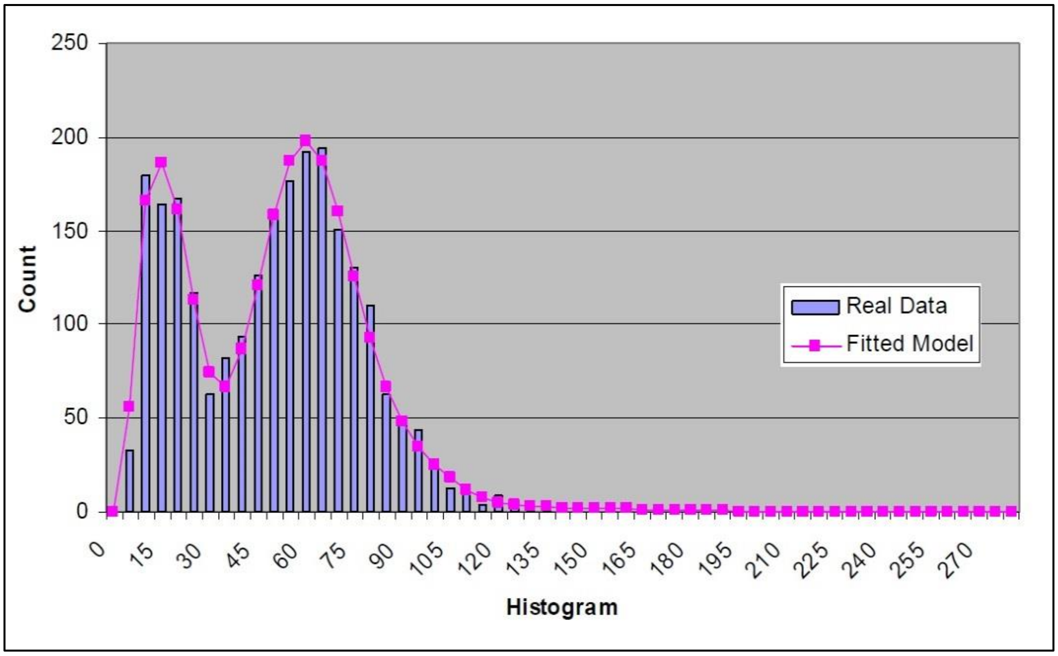

Figure 7.4 Comparison of Real Service Time Data and Fitted Distribution
Source: Yang (2006)

These five distributions are used to randomly generate the service time needed for each type of call. First based on the probabilities of these five distributions, one of these call types is generated and then based on the type of the call a random service time from the corresponding distribution is generated.

7.3. Summary

The input analysis on the real case study data was explained in this chapter. The analysis was done by Yang (2006) in a similar case study and in this research her input analysis has been used to randomly generate the required input data for the simulation model. In the next chapter the results of the simulation model in the case study will be shown and then sensitivity analysis will be done on some important parameters in the model.

Chapter 8: Case Study Results

The proposed model should be tested by using a simulation model on a real case study and if it shows improvements, it is safe to apply in real operations. To see how the proposed model performs in real operations, the simulation model is applied in the case study under investigation. In this chapter, first the results of applying the proposed model in the case study are explained and compared with dispatching models without coverage problem or with simpler coverage criteria. Then an extensive sensitivity analysis is performed on the parameters in the model to see how the model will react.

8.1 Proposed Model Results

As it was mentioned before, the distribution of the accident location, accident type and severity, accident inter-arrival time, and accident finish time are estimated based on the analysis of the real data and the data for the simulation model have been randomly generated using those distributions. Then to find the required output for each case, 10 different replications are used and simulation time is set to four days on each one of these replications. A different set of random seed numbers is used for each replication to come up with reliable and unbiased results.

In output analysis in simulation models, it is necessary to find the length of the warm-up period. When the warm-up period is passed, the system reaches the steady state and the output should be considered after this time. Yang (2006) performed the analysis to determine the warm-up period for this kind of problem and she showed that the warm-up period is less than 1300 minutes (less than a day) for the cases she considered. To be on the safe side, in this research, a one-day warm-up period is considered in each replication. It means that in each replication, the simulation is run for four days, but the results of the first day is not considered in the output analysis and the results from second day to the fourth day are taken into account. So, for each replication, 3-day results have been collected and totally each case is run for 30 days. (10 replications of 3-day runs)

It is assumed that there are 32 police vehicles, 10 BLS ambulances, six ALS ambulances, six fire engines, six fire trucks, and four fire quints in the system. With these numbers of vehicles, the simulation model is applied on the case study. Three approaches are considered and compared. The first one set all coefficients of coverage problem to zero, which means that the model is performing as only a dispatching problem and it is called dispatching (Dis) in the results. In the second approach only the simple coverage problem is added to the dispatching problem and in the results shown in this chapter it is called dispatching with simple coverage (DisSC). The last one combines the entire proposed coverage problem with dispatching problem. It is called dispatching with increased equity and double coverage (DisIEDC).

Some important statistics like “average response time,” “longest response time,” “percentage of vehicles arriving at the emergency later than the required time,” “percentage of emergencies which received their first vehicle in five minutes,” and “percentage of vehicles with response time greater than nine minutes” are calculated for these three approaches. The last two statistics are mentioned in NFPA’s standards. The results are shown in Table 8.1 and Table 8.2. Also, these statistics are calculated for each type of vehicle separately as it can be seen in Table 8.1 and Table 8.2. The only thing that should be mentioned here is that fire quint type does not exist in these tables, because when they perform as fire engines, they will be included in fire engine analysis and when they perform as fire trucks they will be included in fire truck analysis. If they perform as both fire engines and fire trucks, they will be included in the analysis of the both vehicles.

Table 8.1 shows “average response time” and “percentage of emergencies which received their first vehicles in five minutes.” Adding coverage to dispatching problem does not seem to have improvement on these two statistics. Sometimes these two statistics became even worse by adding coverage problem to the dispatching problem. The reason is that when the coverage problem is considered, the only goal is not to service emergencies in the system in the fastest way. Sometimes the model may send a far vehicle to an emergency if that emergency is not severe and prefer to keep other vehicles standing by for future demands. That’s why sometimes these two statistics show improvement and sometimes they become worse. On the other hand, Table 8.2 illustrates great improvement. In this table “longest response time,” “percent of vehicles arriving at the emergency later than the required time,” and “percent of vehicles with response time greater than nine minutes” are shown. On most of them, the DisIEDC model is better than DisSC model and DisSC model is better than Dis Model. For example, the longest response time for ALS ambulances is

10.54 minutes if only dispatching model is used, it decreases to 9.19 minutes if simple coverage is added to dispatching problem. It becomes 7.97 minutes if the full proposed coverage model is used with dispatching model, which is a significant improvement. Another statistic that is very important is the "percentage of vehicles arriving at the emergency later than the require time" and in all of the cases enormous improvement is obtained by using the full proposed model (DisIEDC).

A point that should be mentioned here is that the results shown in Table 8.1 and Table 8.2 are the average of 10 replications. For example the longest response time for fire engine with DisIEDC model is 7.74. This is not the maximum of longest response times of different replications, it is the average of the longest response times for those replications and that is why 0.1% of the vehicles have response times greater than nine minutes for that type of vehicle.

Table 8.1 Case Study Results by Applying 3 Different Models

Vehicle Type	Average Response Time (min)			Percent of Emergencies Received Their First Vehicle in 5 Minutes		
	Dis Model	DisSC Model	DisIEDC Model	Dis Model	DisSC Model	DisIEDC Model
Police Car	2.05	1.95	1.96	92.49%	92.49%	92.49%
BLS Ambulance	2.85	2.79	3.09	67.01%	69.70%	73.41%
ALS Ambulance	3.72	3.27	3.67	56.55%	62.95%	63.94%
Fire Engine	2.91	3.09	3.29	70.31%	70.88%	66.64%
Fire Truck	2.95	3.11	3.22	69.56%	71.38%	69.05%

Table 8.2 Case Study Results by Applying 3 Different Models (Continue)

Vehicle Type	Longest Response Time (min)			Percent of Vehicles Arrived at the Emergency Later than Required Time			Percent of Vehicles with Response Time Greater than 9 Minutes		
	Dis Model	DisSC Model	DisIEDC Model	Dis Model	DisSC Model	DisIEDC Model	Dis Model	DisSC Model	DisIEDC Model
Police Car	5.18	5.09	5.09	0.38%	0.13%	0.06%	0.00%	0.00%	0.00%
BLS Ambulance	7.25	7.48	7.62	5.38%	4.80%	3.41%	0.00%	0.00%	0.00%
ALS Ambulance	10.54	9.19	7.97	6.14%	3.83%	3.24%	2.32%	0.27%	0.00%
Fire Engine	11.38	7.92	7.74	5.89%	4.54%	3.79%	0.83%	0.41%	0.10%
Fire Truck	11.11	7.87	7.25	6.49%	4.99%	3.24%	1.03%	0.23%	0.23%

8.2 Sensitivity Analysis

To see how the model is working when the parameters are changing, an extensive sensitivity analysis is performed in this research. For the sensitivity analysis, the proposed model (DisIEDC) is used. The sensitivity analyses are done on some important parameters like "emergency inter-arrival time," "fleet numbers," "coverage benefit coefficients," "minimum threshold of benefit for diversion," and "cost of assigning vehicles to non-home stations;" the results are presented in this section.

8.2.1 Emergency Inter-arrival Time

Based on the analysis of the case study, the emergency inter-arrival time was about 32.8 minutes. As a result, it is important to see how the results will change if the load of the system increases or decreases. For example, it is interesting to see what the longest response time will be if the emergency inter-arrival time becomes 10 minutes or 60 minutes. The number of vehicles and all the other characteristics of the system are the same as before and just the emergency inter-arrival times have been changed. Tables 8.3 to 8.8 show the results for different emergency inter-arrival time.

Table 8.3 is presenting average response time for different emergency inter-arrival times for each type of vehicle. The graphs of the average response time can be seen in Figure 8.1. From Table 8.3 and Figure 8.1, it can be found out that when inter-arrival time is around 30 minutes, the average response time for almost all vehicles reaches a threshold and it does not decrease a lot after that. For some vehicles even at inter-arrival time around 20 minutes, the average response time becomes almost constant and it does not change that much after that. It shows that 30 minutes for some types of vehicles and 20 minutes for others are the thresholds for the system to transform from loaded system to less loaded one if we only consider the average response time. However for a more robust conclusion, other statistics have to be checked too.

Percentages of emergencies receiving their first vehicle in five minutes for different emergency inter-arrival times are shown in Table 8.4 and Figure 8.2. They show that more emergencies received their first vehicles in five minutes when the emergency inter-arrival time increases and the rate of increase is high until emergency inter-arrival time is around 30 minutes. After 30 minutes, most of the time they are still improving but with lower rates.

Table 8.3 Average Response Time with Different Emergency Inter-arrival Time

Vehicle Type	Average Response Time (min)					
	Emergency Inter-arrival Time (min)					
	10	20	32.8	40	50	60
Police Car	2.2	1.9	1.9	1.9	1.9	1.9
BLS Ambulance	3.4	2.9	2.9	2.8	2.8	2.8
ALS Ambulance	4.2	3.8	3.4	3.4	3.4	3.4
Fire Engine	3.6	3.0	3.1	3.1	3.1	3.0
Fire Truck	3.6	2.9	2.9	3.0	3.0	2.9

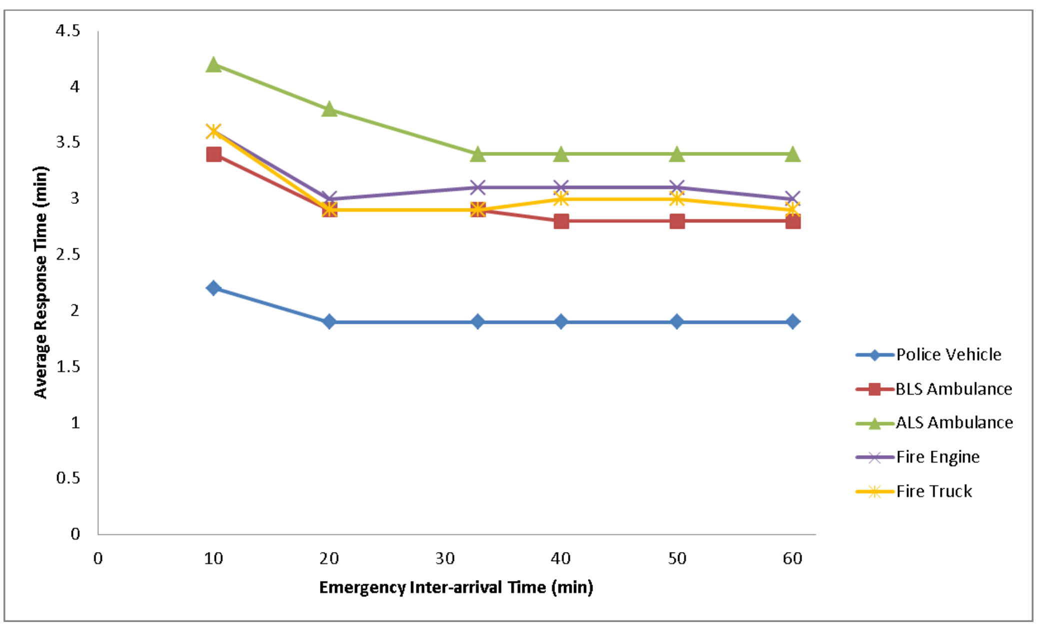

Figure 8.1 Average Response Time with Different Emergency Inter-arrival Time

Table 8.4 Percent of Emergencies Received Their First Vehicle in 5 Minutes with Different Emergency Inter-arrival Time

Vehicle Type	Percent of Emergencies Received Their First Vehicle in 5 Minutes					
	Emergency Inter-arrival Time (min)					
	10	20	32.8	40	50	60
Police Car	68.4%	76.6%	91.0%	91.5%	93.4%	98.2%
BLS Ambulance	41.3%	52.6%	65.6%	65.1%	67.1%	70.9%
ALS Ambulance	28.3%	41.9%	56.8%	56.7%	57.6%	61.5%
Fire Engine	41.6%	52.0%	66.0%	70.9%	72.1%	76.9%
Fire Truck	41.9%	53.3%	70.1%	70.6%	73.9%	79.8%

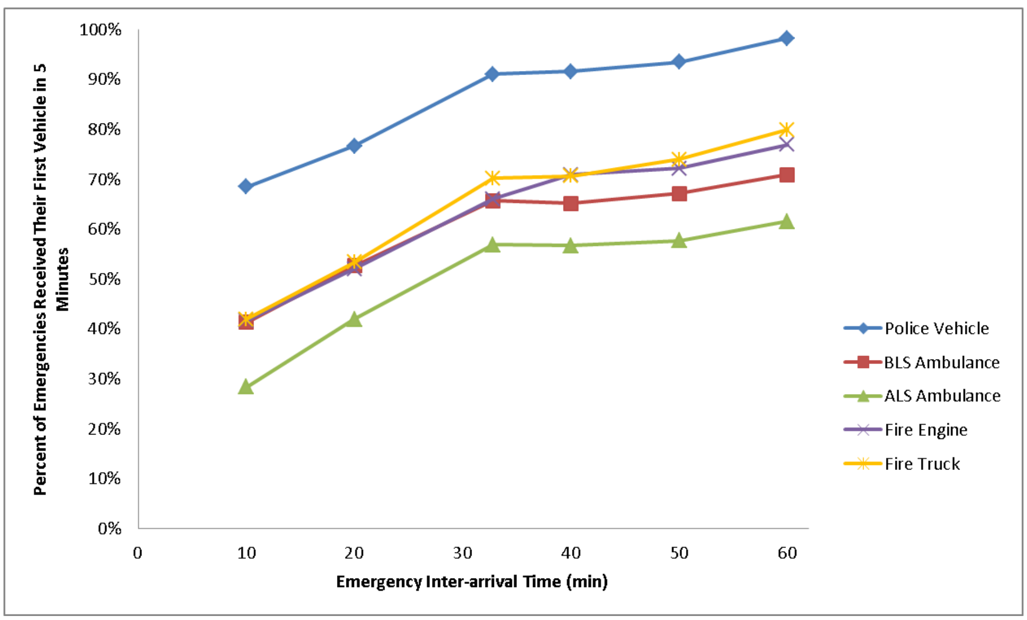

Figure 8.2 Percent of Emergencies Received Their First Vehicle in 5 Minutes with Different Emergency Inter-arrival Time

Longest response time for different emergency inter-arrival times is shown in Table 8.5 and Figure 8.3.

Table 8.5 Longest Response Time with Different Emergency Inter-arrival Time

Vehicle Type	Longest Response Time (min)					
	Emergency Inter-arrival Time (min)					
	10	20	32.8	40	50	60
Police Car	6.3	5.7	5.7	4.3	4.3	4.3
BLS Ambulance	8.1	7.9	7.9	7.6	7.6	7.6
ALS Ambulance	11.4	9.6	7.9	7.6	7.6	7.6
Fire Engine	8.5	7.1	7.1	7.1	6.6	6.6
Fire Truck	8.3	7.1	7.1	6.6	6.6	6.6

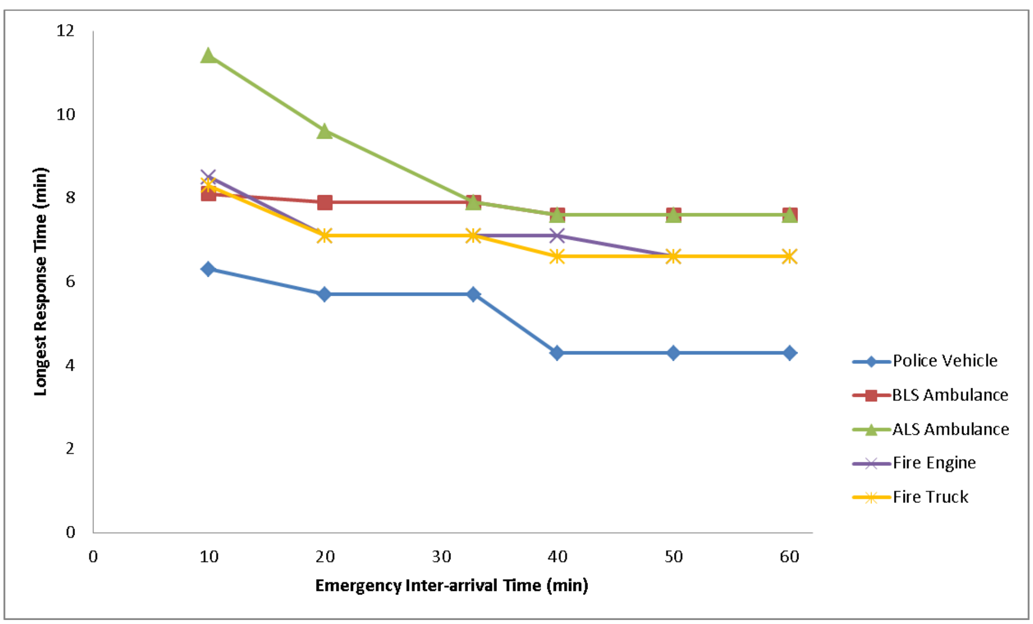

Figure 8.3 Longest Response Time with Different Emergency Inter-arrival Time

Table 8.5 and Figure 8.3, show that by increasing the emergency inter-arrival time, the longest response time decreases and it reaches a threshold after inter-arrival time reaches about 40 minutes.

In Table 8.6 and Figure 8.4, the percent of vehicles arriving at the emergency later than the required time for different emergency inter-arrival times is presented. They show that after emergency inter-arrival time reaches 40 minutes, the percentage of vehicles arriving later than the required time does not decreases a lot, sometimes it decreases but not that much. However, for most types of vehicles, it decreases a lot by increasing the emergency inter-arrival time before this time reaches 40 minutes.

Table 8.7 and Figure 8.5 show the percent of vehicles with response times greater than nine minutes. As it can be seen in Figure 8.5, the ALS ambulance has some vehicles with response time greater than nine minutes when the emergency inter-arrival time is less than 30 minutes and after that it has no vehicles with response time greater than nine minutes.

In the analyses of the case study, the emergencies could get enough vehicles and they did not have vehicle deficiencies. In these cases, when the system gets very loaded and, for example, every 10 minutes an emergency occurs in the system, it is important to see whether emergencies receive what they need or they have lack of vehicles sometime. So, the percent of vehicle deficiencies at the emergencies have been investigated with different emergency inter-arrival times, and the results are shown in Table 8.8 and Figure 8.6. They show that the ALS ambulance and Fire Engine types are the ones that have vehicle deficiencies at the emergencies.

Table 8.6 Percent of Vehicles Arrived at the Emergency Later than Required Time with Different Emergency Inter-arrival Time

Vehicle Type	Percent of Vehicles Arrived at the Emergency Later than Required Time					
	Emergency Inter-arrival Time (min)					
	10	20	32.8	40	50	60
Police Car	0.4%	0.0%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	11.5%	6.7%	5.7%	3.5%	2.7%	2.6%
ALS Ambulance	7.9%	5.9%	1.1%	1.3%	1.3%	1.1%
Fire Engine	6.6%	5.6%	4.5%	3.2%	3.2%	3.1%
Fire Truck	7.0%	5.8%	5.0%	3.6%	3.3%	2.7%

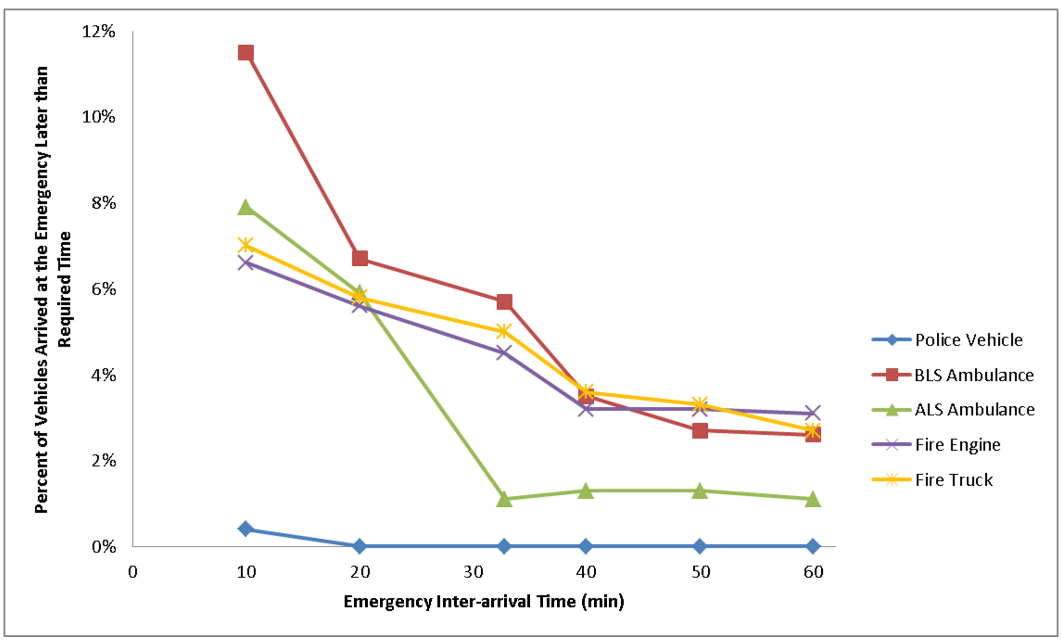

Figure 8.4 Percent of Vehicles Arrived at the Emergency Later than Required Time with Different Emergency Inter-arrival Time

Table 8.7 Percent of Vehicles with Response Time Greater than 9 Minutes with Different Emergency Inter-arrival Time

Vehicle Type	Percent of Vehicles with Response Time Greater than 9 Minutes					
	Emergency Inter-arrival Time (min)					
	10	20	32.8	40	50	60
Police Car	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
ALS Ambulance	3.0%	1.5%	0.0%	0.0%	0.0%	0.0%
Fire Engine	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Truck	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

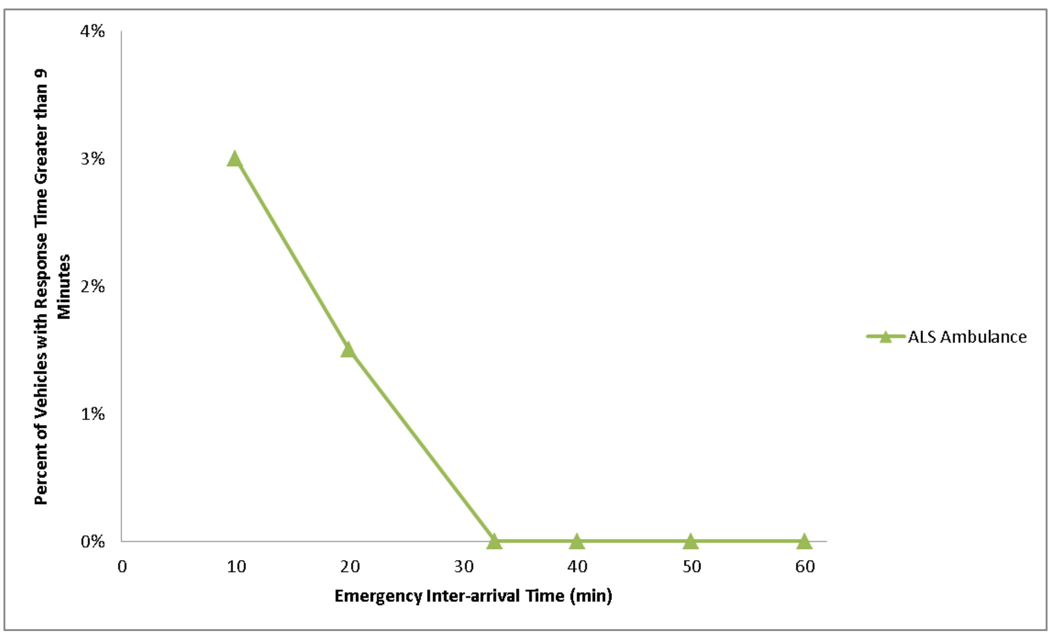

Figure 8.5 Percent of Vehicles with Response Time Greater than 9 Minutes with Different Emergency Inter-arrival Time

Table 8.8 Percent of Vehicle Deficiencies at the Emergencies for Different Emergency Inter-arrival Time

Vehicle Type	Percent of Vehicle Deficiencies at the Emergencies					
	Emergency Inter-arrival Time (min)					
	10	20	32.8	40	50	60
Police Car	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
ALS Ambulance	19.8%	8.1%	0.0%	0.0%	0.0%	0.0%
Fire Engine	0.6%	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Truck	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

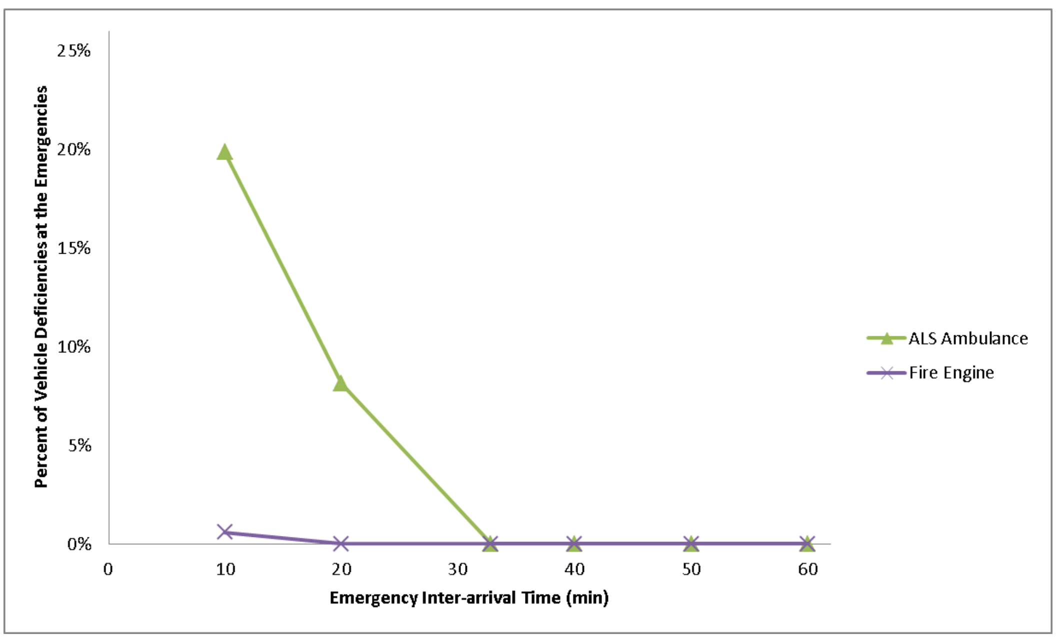

Figure 8.6 Percent of Vehicle Deficiencies at the Emergencies with Different Emergency Inter-arrival Time

Figure 8.6 shows that Fire Engine has a little vehicle deficiency when the emergency inter-arrival time is 10 minutes and after that it shows no vehicle deficiency, but the problem for ALS ambulance is quite serious. There are about 20% vehicle deficiencies when emergency inter-arrival time is 10 minutes and it reaches to about 8% when the emergency inter-arrival time is 20 minutes, which is still high. Twenty percent of vehicle deficiency means that if the total number of needed ALS ambulances in the emergencies that happened in the system was about 100 vehicles, the emergencies could get just 80 of them and some emergencies lacked the number of needed vehicles, which is not good at all. If the system, in reality, gets this loaded and the emergencies occur in less than 30 minutes, then adding the number of ALS ambulances is mandatory, otherwise the system will perform very poorly.

So, from all the results shown in this section, it can be concluded that if the emergency inter-arrival time is 40 minutes and above, the system performs very well. If it decreases to about 30 minutes, as it is the case right now in the case study, the system performs well. But less than 30 minutes emergency inter-arrival time is going to put the system in bad shape and for sure in that situation some vehicles have to be added to the system especially ALS ambulances.

8.2.2 Fleet Numbers

Another parameter that is worth looking at is fleet number of each vehicle type. It is obvious that with increasing the fleet number the results will improve and by decreasing the fleet number the results will deteriorate. It is interesting to see how the results will change by changing the fleet number of each type. In this section, six scenarios have been considered with different number of vehicles for each type. The number of vehicles for each scenario is shown in Table 8.9.

Table 8.9 Number of Vehicles for Each Scenario

Scenarios	**Vehicle Types**					
	Police	**BLS Ambulance**	**ALS Ambulance**	**Fire Engine**	**Fire Truck**	**Fire Quint**
#1	10	4	2	3	3	1
#2	15	6	3	5	5	3
#3	20	8	4	6	6	4
#4	25	10	6	8	8	6
#5	32	15	8	12	12	9
#6	40	20	10	15	15	10

Tables 8.10 to 8.15 show the results for different fleet numbers. Table 8.10 is presenting average response time for different scenarios with different number of vehicles for each type. The graphs of the average response time can be seen in Figure 8.7. In Table 8.10 and Figure 8.7, it can be seen that when the number of vehicles increases (number of vehicles increases when scenario# increases), most of the time the average response time decreases, which is expected, because there are more vehicles to serve the emergencies and also they can provide better coverage for future demands.

Percentages of emergencies receiving their first vehicle in five minutes for different fleet number scenarios are shown in Table 8.11 and Figure 8.8.

Table 8.10 Average Response Time with Different Fleet Number Scenarios

Vehicle Type	Average Response Time (min)					
	Scenarios					
	#1	#2	#3	#4	#5	#6
Police Car	3.7	3.4	3.3	3.3	3.2	3.2
BLS Ambulance	4.1	4.1	4.0	3.8	3.6	3.4
ALS Ambulance	4.4	4.4	4.2	3.6	3.4	2.8
Fire Engine	4.5	4.2	4.2	4.3	3.7	3.8
Fire Truck	4.3	4.4	4.4	4.2	4.1	3.7

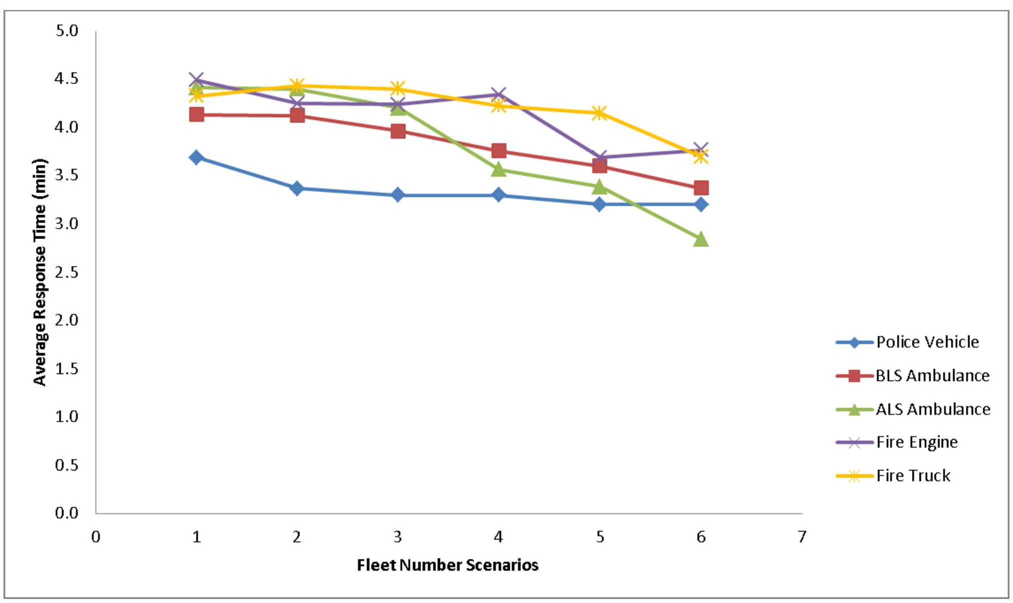

Figure 8.7 Average Response Time with Different Fleet Number Scenarios

Table 8.11 Percent of Emergencies Received Their First Vehicle in 5 Minutes with Different Fleet Number Scenarios

Vehicle Type	Percent of Emergencies Received Their First Vehicle in 5 Minutes					
	Scenarios					
	#1	#2	#3	#4	#5	#6
Police Car	58.7%	69.8%	80.7%	84.8%	92.2%	94.7%
BLS Ambulance	66.9%	72.9%	78.2%	84.6%	85.8%	89.8%
ALS Ambulance	54.3%	63.5%	72.4%	81.4%	84.1%	84.1%
Fire Engine	53.7%	64.3%	68.2%	73.3%	85.1%	84.9%
Fire Truck	48.4%	58.8%	70.7%	75.8%	83.2%	82.0%

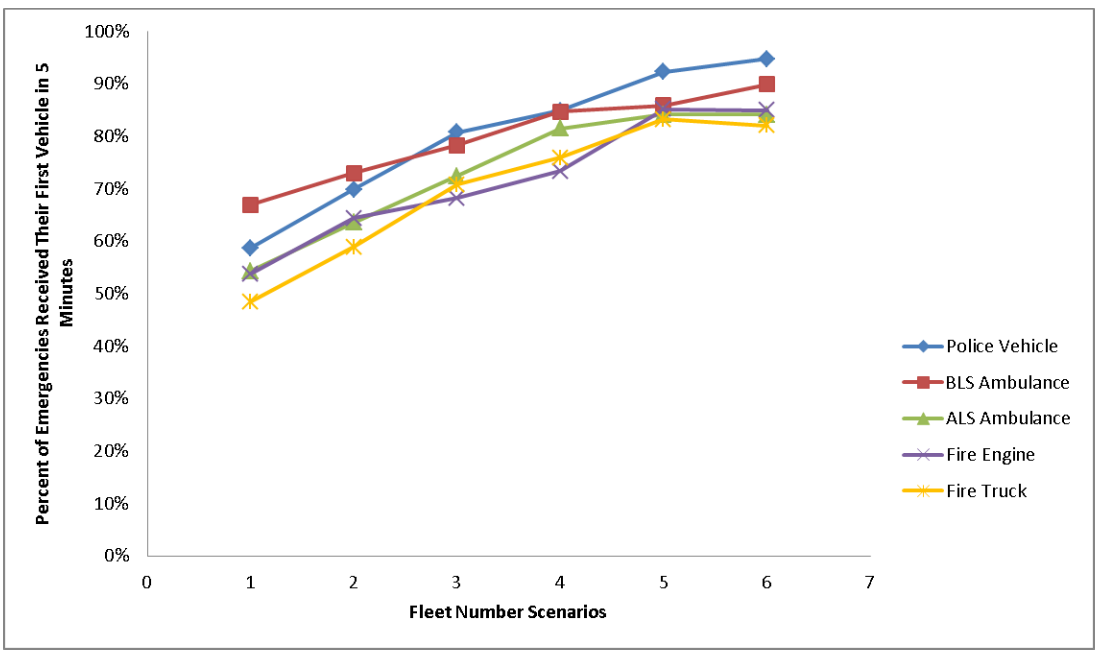

Figure 8.8 Percent of Emergencies Received Their First Vehicle in 5 Minutes with Different Fleet Number Scenarios

Table 8.11 and Figure 8.8 show that more emergencies receive their first vehicles in five minutes when the fleet number increases and the rate of increase is higher for the first couples of scenarios and then it diminishes for some types of vehicles after scenario #4 and for others after scenario #5.

Longest response time for different fleet number scenarios is shown in Table 8.12 and Figure 8.9. They show that overall by increasing the fleet number the longest response time decreases. However, there are some exceptions. For ALS ambulance the maximum longest response time happens at scenario #3 and the longest response time for scenario #1 and #2 is lower. Also, the same thing happens for Fire Engine at scenario #2. There exist vehicle deficiencies at emergencies for first couple of scenarios and that is the reason that maximum longest response time does not always happen at scenario #1 and sometimes it gets shifted to other scenarios. Since those maximum longest response times may have been skipped and the vehicles that produced the maximum longest response time may have never been sent to emergencies at lower scenarios, the maximum response times sometimes get shifted to other scenarios.

In Table 8.13 and Figure 8.10, the percent of vehicles arriving at the emergency later than the required time for different fleet number scenarios is presented. They show that by increasing the fleet number, the percentage of vehicles arriving later than the required time decreases a lot. For police vehicles, it reaches zero after scenario #1, for BLS and ALS, it reaches zero after scenario #3 and for Fire Engines and Fire Trucks it reaches zero after scenario #5.

Table 8.12 Longest Response Time with Different Fleet Number Scenarios

Vehicle Type	Longest Response Time (min)					
	Scenarios					
	#1	#2	#3	#4	#5	#6
Police Car	9.8	9.5	8.8	7.1	7.6	6.6
BLS Ambulance	9.3	9.0	7.9	7.8	7.6	7.6
ALS Ambulance	7.5	7.8	9.8	7.0	6.9	5.2
Fire Engine	8.8	9.7	8.3	7.8	7.6	7.6
Fire Truck	7.9	8.3	8.3	7.7	7.8	7.7

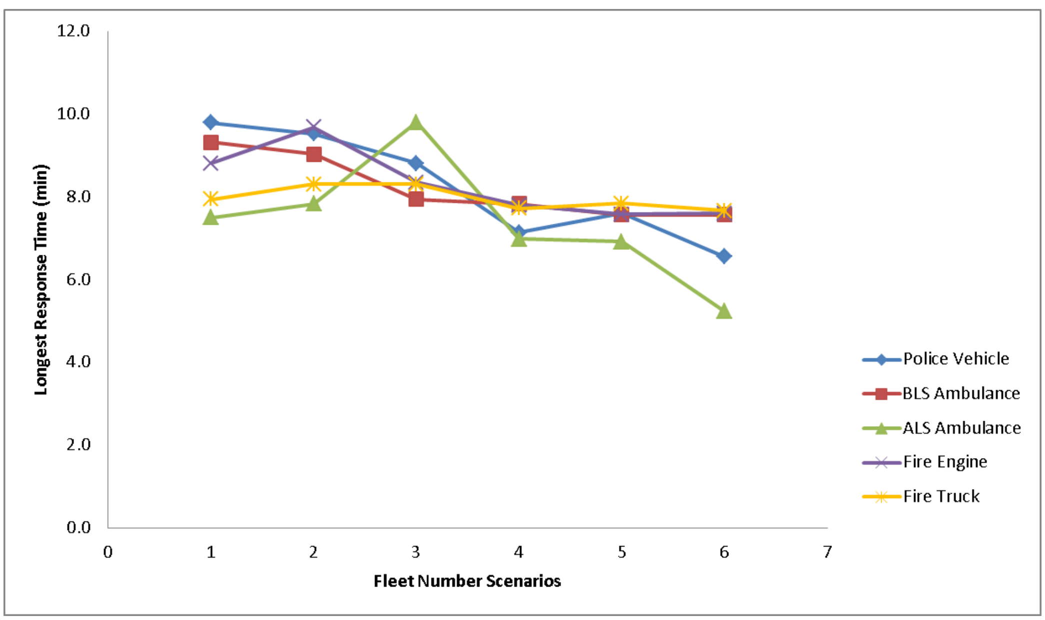

Figure 8.9 Longest Response Time with Different Fleet Number Scenarios

Table 8.13 Percent of Vehicles Arrived at the Emergency Later than Required Time with Different Fleet Number Scenarios

Vehicle Type	Percent of Vehicles Arrived at the Emergency Later than Required Time					
	Scenarios					
	#1	#2	#3	#4	#5	#6
Police Car	4.7%	0.0%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	5.9%	5.8%	1.9%	0.0%	0.0%	0.0%
ALS Ambulance	6.3%	3.8%	3.50%	0.0%	0.0%	0.0%
Fire Engine	16.3%	13.0%	7.4%	3.7%	1.9%	0.0%
Fire Truck	20.0%	12.0%	8.0%	4.0%	2.0%	0.0%

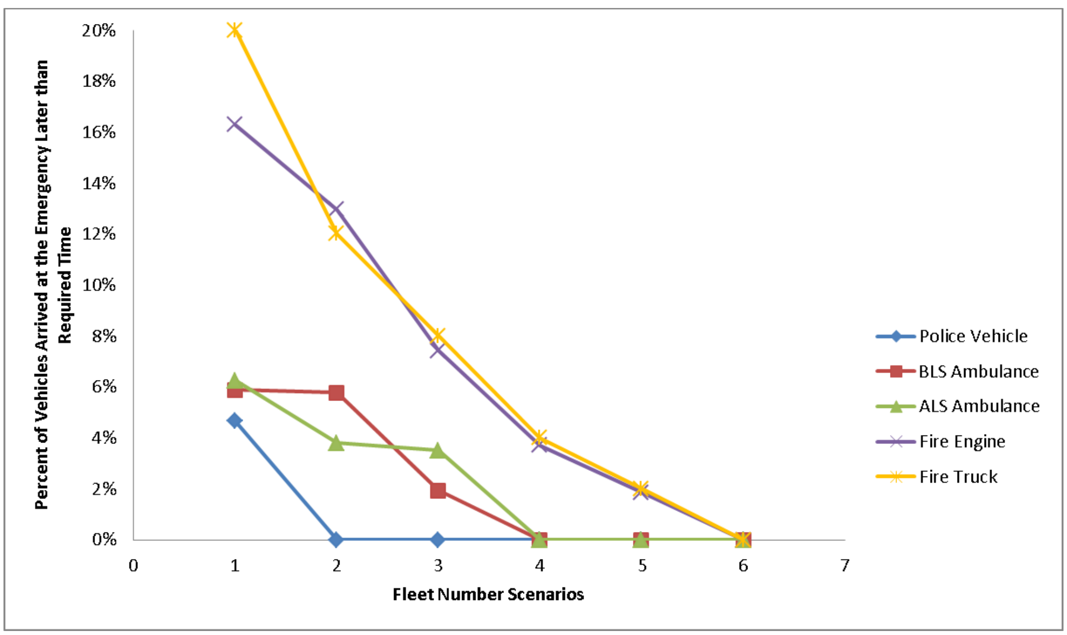

Figure 8.10 Percent of Vehicles Arrived at the Emergency Later than Required Time with Different Fleet Number Scenarios

Table 8.14 and Figure 8.11 show percent of vehicles with response times greater than nine minutes for different fleet number scenarios. They demonstrate that from scenario #4, none of the vehicles have response times greater than nine minutes. Before scenario #4, some types of vehicles have response times greater than nine minutes, but the percentage is very low. The maximum percentage for ALS happens in scenario #3 and for Fire Engine happens in scenario #2 instead of scenario #1 and it may happen because of the vehicle deficiencies in the first couple of scenarios.

It is also important to see that what are the vehicle deficiencies at emergencies. When the fleet number is very low, it is possible that there are vehicle deficiencies and emergencies cannot receive the required number of vehicles, which is very important. The percent of vehicle deficiencies at the emergencies have been investigated with different fleet number scenarios and the results are shown in Table 8.15 and Figure 8.12. They show that police vehicle type have deficiencies at scenario #1 and scenario #2 and after the number of police vehicles becomes 20 at scenario #3, there is no deficiencies for them. The BLS ambulance shows deficiency at scenario #1, but from scenario #2 it shows no deficiencies. However, the number of ALS in the system also helps BLS ambulances and when there are six BLS ambulances and three ALS ambulances in the system like scenario #2, no BLS ambulance deficiencies are found in the emergencies. ALS ambulances show no deficiencies from scenario #4, which has six ALS ambulances. Fire Engines and Fire Trucks show high percentage of deficiencies in scenario #1, but after that their deficiencies reach zero. So, the number of FE and FT is five and the number of FQ is three for scenario #2 and with these vehicles emergencies can receive their required number of fire vehicles.

Table 8.14 Percent of Vehicles with Response Time Greater than 9 Minutes with Different Fleet Number Scenarios

Vehicle Type	Percent of Vehicles with Response Time Greater than 9 Minutes					
	Scenarios					
	#1	#2	#3	#4	#5	#6
Police Car	2.3%	1.1%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	1.9%	1.5%	0.0%	0.0%	0.0%	0.0%
ALS Ambulance	0.0%	0.0%	3.1%	0.0%	0.0%	0.0%
Fire Engine	0.0%	1.9%	0.0%	0.0%	0.0%	0.0%
Fire Truck	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

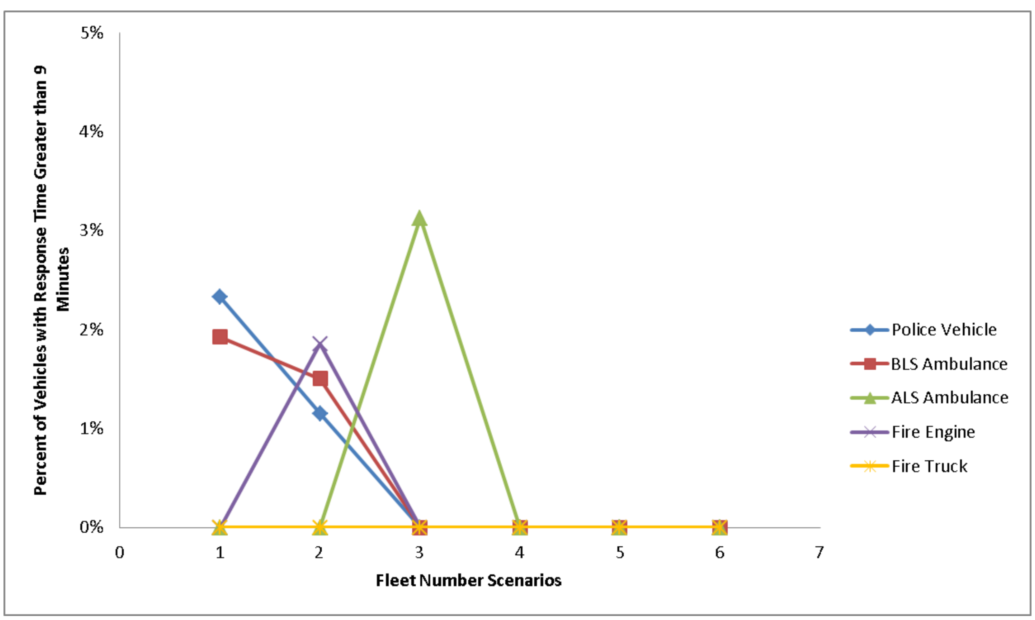

Figure 8.11 Percent of Vehicles with Response Time Greater than 9 Minutes with Different Fleet Number Scenarios

Table 8.15 Percent of Vehicle Deficiencies at the Emergencies for Different Fleet Number Scenarios

Vehicle Type	Percent of Vehicle Deficiencies at the Emergencies					
	Scenarios					
	#1	#2	#3	#4	#5	#6
Police Car	1.1%	1.1%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	1.9%	0.0%	0.0%	0.0%	0.0%	0.0%
ALS Ambulance	23.5%	11.8%	5.9%	0.0%	0.0%	0.0%
Fire Engine	20.4%	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Truck	20.0%	0.0%	0.0%	0.0%	0.0%	0.0%

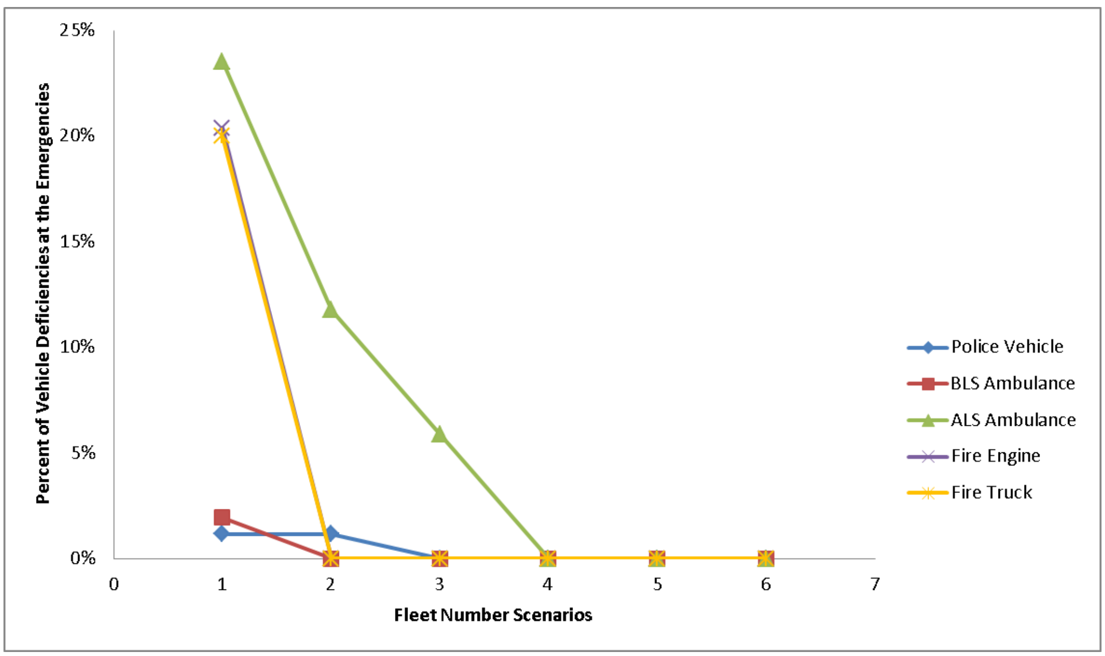

Figure 8.12 Percent of Vehicle Deficiencies at the Emergencies with Different Fleet Number Scenarios

As a result, in order to prevent vehicle deficiencies at emergencies, 20 police vehicles, six BLS ambulances, six ALS ambulances, five Fire Engines, five Fire Trucks, and three Fire Quints are required for the whole system. However for better results, like better average response time, better longest response time, or better other performance measures, number of vehicles should be increased as much as scenario #5 vehicle fleet or even scenario #6 vehicle fleet.

8.2.3 Coverage Benefit Coefficients

The coefficients of coverage benefit in the objective function are very important parameters in the system. If they are set to zero, the model will be changed to a dispatching only problem. If they are set to very high numbers, the model will prioritize coverage problem over dispatching problem. It is therefore important to see how results are changing when these parameters are changing. The parameters should be set to reasonable amounts. The goal is to service the emergencies well enough and at the same time if some emergencies are not very critical, their response can be delayed and good coverage can be provided for future demands.

As it was mentioned in section 3.4.2, the coverage benefit coefficients that exist in the model are:

AA Benefit of ordinary node first coverage within T_1 minutes

AB Benefit of ordinary node first coverage within T_2 minutes

AC Benefit of ordinary node second coverage within T_2 minutes

AD Benefit of critical node first coverage within T_1 minutes

AE Benefit of critical node first coverage within T_2 minutes

AF Benefit of critical node second coverage within T_1 minutes

The parameters, therefore, will be shown as (AA, AB, AC, AD, AE, AF). In the base case these coefficients are (2, 1, 0.5, 4, 2, 1). Then these base case numbers are changed to see how the results are changing.

Tables 8.16 to 8.21 show the results for different coverage benefit coefficients. Table 8.16 and Figure 8.13 present average response time for different coverage benefit coefficients. They show that overall base case and 100* base case scenarios provide better average response time. Some vehicles show better response time when the coefficients are set to 10,000 times base case, but this is because in that scenario, the system prioritizes coverage over dispatching and sometimes it does not send the required vehicles to emergencies and there are deficiencies at emergencies. As a result, it would not be a good scenario to choose.

Percentages of emergencies receiving their first vehicle in five minutes for different coverage benefit coefficients are shown in Table 8.17 and Figure 8.14. They do not show any significant difference between different scenarios, for some vehicles it gets better around base case but for some it does not. The ALS ambulance shows very bad results in the last scenario and it is because there are lots of deficiencies in emergencies in that scenario.

Longest response time for different coverage benefit coefficients is shown in Table 8.18 and Figure 8.15. They demonstrate that base case scenario and 100* base case scenario provide better longest response times. The BLS ambulance show a lot of decrease in longest response time in the last scenario and it is probably because the vehicle that produced longest response time has not been sent to emergency at all in this scenario and there are deficiencies in the system, which is why it shows better results.

Table 8.16 Average Response Time for Different Coverage Benefit Coefficients

Vehicle Type	**Average Response Time (min)**				
	Coverage Benefit Coefficients (AA, AB, AC, AD, AE, AF) Base Case Coefficients: (2,1,0.5,4,2,1)				
	0.0001* Base Case	**0.01*Base Case**	**Base Case**	**100*Base Case**	**10000* Base Case**
Police Car	3.1	3.1	3.1	3.1	3.1
BLS Ambulance	3.8	3.8	3.8	3.7	3.5
ALS Ambulance	3.5	3.5	3.6	3.5	4.3
Fire Engine	4.7	4.7	4.2	4.3	3.8
Fire Truck	4.7	4.7	3.9	4.1	4.2

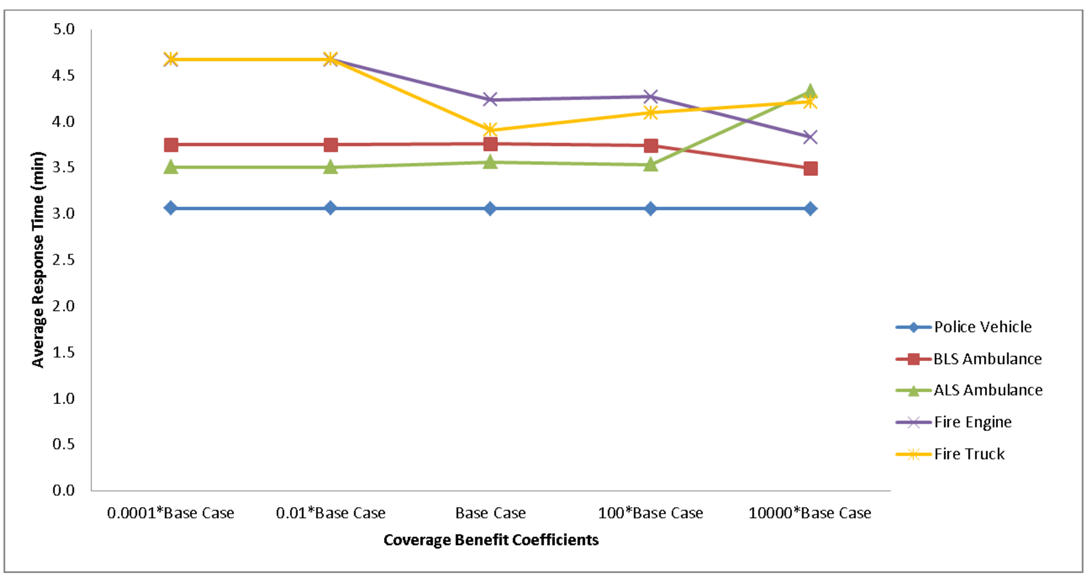

Figure 8.13 Average Response Time with Different Coverage Benefit Coefficients

Table 8.17 Percent of Emergencies Received Their First Vehicle in 5 Minutes for Different Coverage Benefit Coefficients

Vehicle Type	Percent of Emergencies Received Their First Vehicle in 5 Minutes				
	Coverage Benefit Coefficients (AA, AB, AC, AD, AE, AF) Base Case Coefficients: (2,1,0.5,4,2,1)				
	0.0001* Base Case	**0.01*Base Case**	**Base Case**	**100*Base Case**	**10000* Base Case**
Police Car	73.7%	71.7%	72.7%	72.7%	72.7%
BLS Ambulance	61.7%	61.7%	59.8%	59.8%	62.7%
ALS Ambulance	61.2%	61.2%	61.2%	61.2%	34.7%
Fire Engine	53.7%	53.7%	58.2%	56.0%	53.7%
Fire Truck	54.6%	54.6%	60.7%	57.0%	53.3%

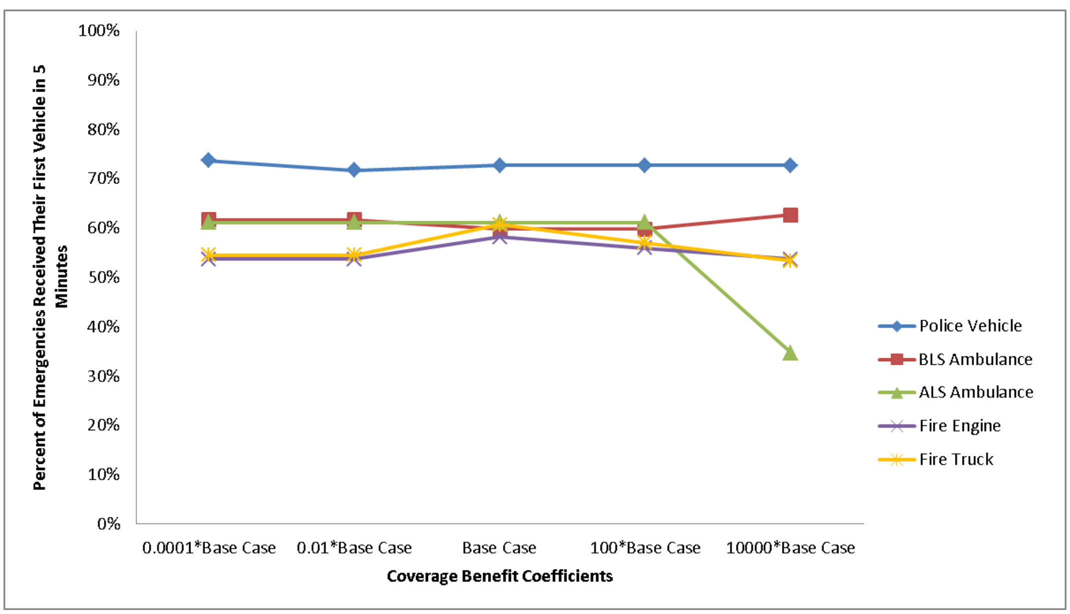

Figure 8.14 Percent of Emergencies Received Their First Vehicle in 5 Minutes with Different Coverage Benefit Coefficients

Table 8.18 Longest Response Time for Different Coverage Benefit Coefficients

Vehicle Type	Longest Response Time (min)				
	Coverage Benefit Coefficients (AA, AB, AC, AD, AE, AF) Base Case Coefficients: (2,1,0.5,4,2,1)				
	0.0001* Base Case	0.01*Base Case	Base Case	100*Base Case	10000* Base Case
Police Car	9.2	9.2	8.5	8.5	8.5
BLS Ambulance	10.0	10.0	9.3	9.3	7.8
ALS Ambulance	8.3	8.3	7.0	7.0	7.0
Fire Engine	9.3	9.3	7.6	7.5	8.4
Fire Truck	9.0	9.0	8.3	8.3	7.9

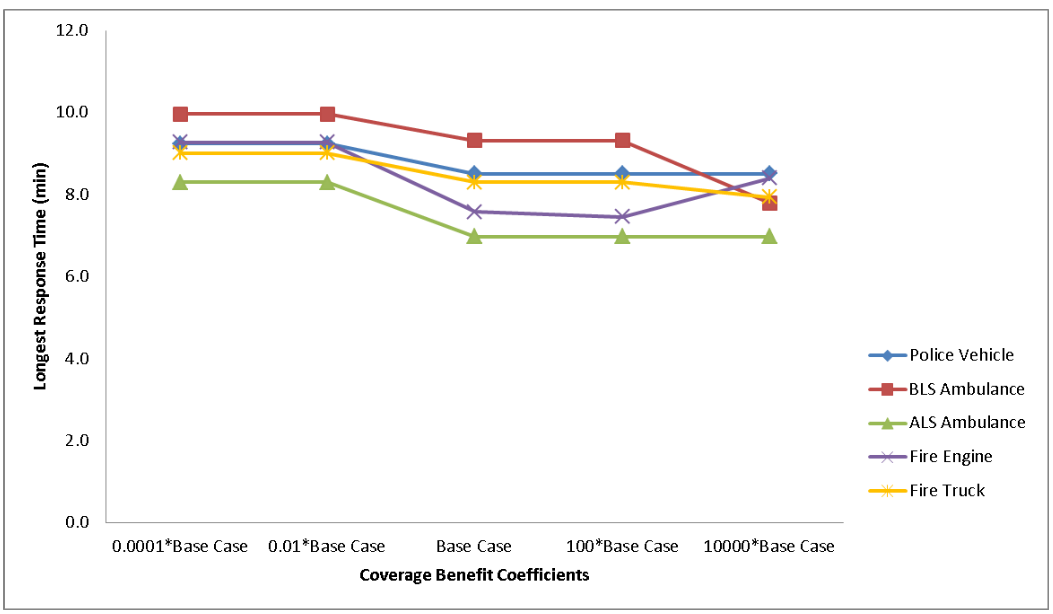

Figure 8.15 Longest Response Time with Different Coverage Benefit Coefficients

In Table 8.19 and Figure 8.16, the percent of vehicles arriving at the emergency later than the required time for different coverage benefit coefficients is presented. Again from Table 8.19 and Figure 8.16, it can be concluded that base case and 100*base case are the better scenarios. Some vehicles like Fire Engines show better results for last scenario, but as was explained earlier, probably because there are some deficiencies in the last scenarios and that is why the percentages got better for that scenario.

Table 8.20 and Figure 8.17 show percent of vehicles with response time greater than nine minutes for different coverage benefit coefficients. They show that the better results are for base case and 100*base case scenarios. The last scenario shows the best results here but the results are somehow fake because there are vehicle deficiencies for the last scenarios and the fact that there is no response time more than nine minutes would not make the last scenario more appealing.

When the coverage benefit coefficients are very high, the system prefers to provide good coverage for future and sometimes does not send vehicles to emergencies, which is against the goal of swift emergency response. An important performance measure to compare scenarios, therefore, would be analyzing the number of vehicle deficiencies for each scenario. Percent of vehicle deficiencies at the emergencies have been investigated with different coverage benefit coefficients, and the results shown in Table 8.21 and Figure 8.18 demonstrate that for the last scenario there are lots of deficiencies at emergencies.

Table 8.19 Percent of Vehicles Arrived at the Emergency Later than Required Time for Different Coverage Benefit Coefficients

Vehicle Type	**Percent of Vehicles Arrived at the Emergency Later than Required Time**				
	Coverage Benefit Coefficients (AA, AB, AC, AD, AE, AF) Base Case Coefficients: (2,1,0.5,4,2,1)				
	0.0001* Base Case	**0.01*Base Case**	**Base Case**	**100*Base Case**	**10000* Base Case**
Police Car	0.0%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%
ALS Ambulance	2.9%	2.9%	0.0%	0.0%	0.0%
Fire Engine	16.7%	16.7%	7.4%	7.4%	6.5%
Fire Truck	16.0%	16.0%	8.0%	8.0%	8.9%

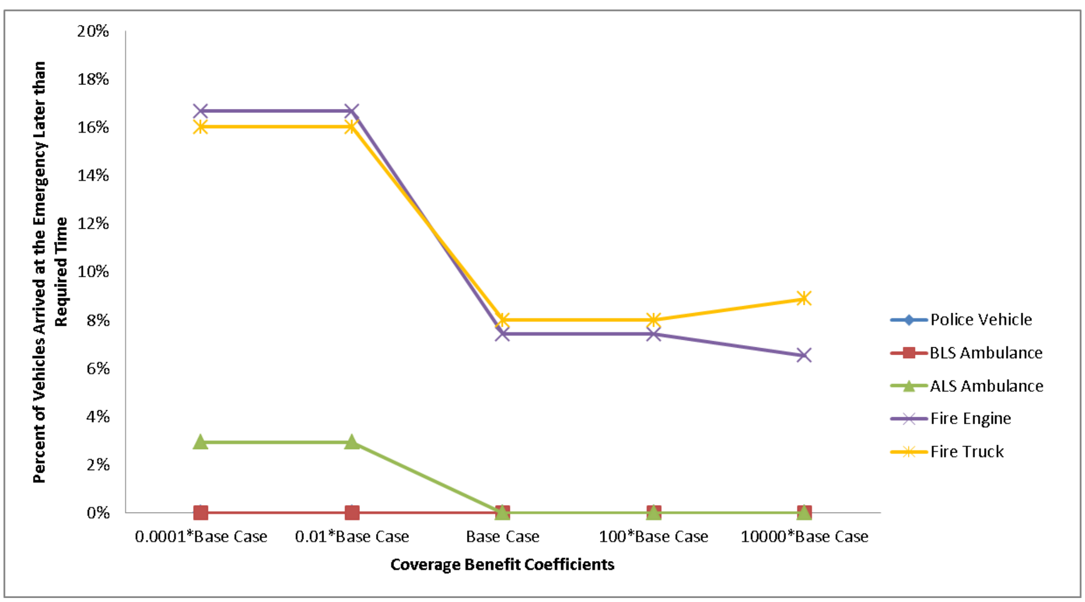

Figure 8.16 Percent of Vehicles Arrived at the Emergency Later than Required Time with Different Coverage Benefit Coefficients

Table 8.20 Percent of Vehicles with Response Time Greater than 9 Minutes for Different Coverage Benefit Coefficients

Vehicle Type	Percent of Vehicles with Response Time Greater than 9 Minutes				
	Coverage Benefit Coefficients (AA, AB, AC, AD, AE, AF) Base Case Coefficients: (2,1,0.5,4,2,1)				
	0.0001* Base Case	0.01*Base Case	Base Case	100*Base Case	10000* Base Case
Police Car	1.1%	1.1%	0.0%	0.0%	0.0%
BLS Ambulance	3.8%	3.8%	1.9%	1.9%	0.0%
ALS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Engine	5.6%	5.6%	0.0%	0.0%	0.0%
Fire Truck	2.0%	2.0%	0.0%	0.0%	0.0%

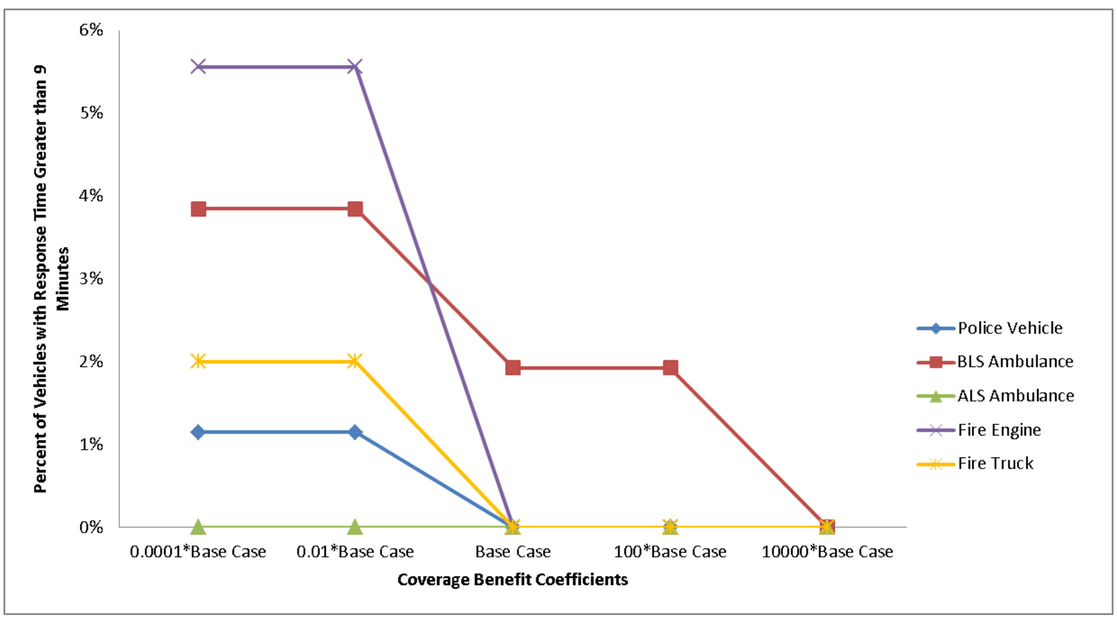

Figure 8.17 Percent of Vehicles with Response Time Greater than 9 Minutes with Different Coverage Benefit Coefficients

Table 8.21 Percent of Vehicle Deficiencies at the Emergencies for Different Coverage Benefit Coefficients

Vehicle Type	**Percent of Vehicle Deficiencies at the Emergencies**				
	Coverage Benefit Coefficients (AA, AB, AC, AD, AE, AF) Base Case Coefficients: (2,1,0.5,4,2,1)				
	0.0001* Base Case	**0.01*Base Case**	**Base Case**	**100*Base Case**	**10000* Base Case**
Police Car	0.0%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	0.0%	0.0%	0.0%	0.0%	7.3%
ALS Ambulance	0.0%	0.0%	0.0%	0.0%	52.9%
Fire Engine	0.0%	0.0%	0.0%	0.0%	14.8%
Fire Truck	0.0%	0.0%	0.0%	0.0%	10.0%

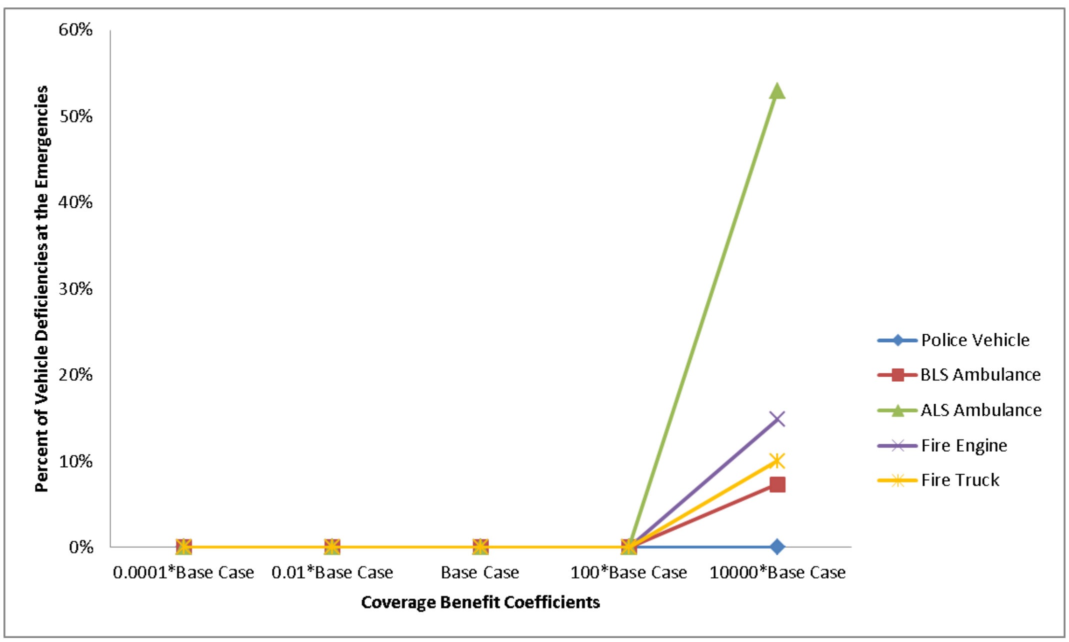

Figure 8.18 Percent of Vehicle Deficiencies at the Emergencies with Different Coverage Benefit Coefficients

So, from the analysis in this section, it can be concluded that the base case coefficients and 100* base case coefficients provide better results and the coverage benefit coefficients should be set to one of these numbers. If the coefficients are set to lower numbers, the dispatching problem will be very important and somehow the coverage problem will be ignored. Also, if the coefficients are set to higher numbers, the coverage problem will be highly prioritized over the dispatching problem and the system will perform poorly. As a result, the base case coefficients and 100* base case coefficients are reliable numbers to choose.

8.2.4 Minimum Threshold of Benefit for Diversion

In this model the vehicles can be diverted from their previous destinations to a new one if the whole system benefits from this action; however, diversion is not easy for drivers and it will confuse them. The diversion, therefore, is allowed in the system if it produces at least a minimum benefit to the whole system. The parameters that have been used in the model are:

ω_s: Minimum threshold of benefit for diverting a vehicle while going to a station

ω_e: Minimum threshold of benefit for diverting a vehicle while going to an emergency incident

ω_h: Minimum threshold of benefit for diverting a vehicle while going to a hospital

In the analysis of this section all three minimum thresholds of benefit have been set to the same amount and they have been changed from 30 seconds to 10 minutes to see how the results will change.

Tables 8.22 to 8.26 show the results for different minimum diversion benefit threshold. Average response time presented in Table 8.22 and Figure 8.19 show that by increasing the minimum threshold of benefit for diversion, the average response time is increasing. Overall, the 30-second threshold produces the best results.

Percentages of emergencies receiving their first vehicle in five minutes for different minimum diversion benefit thresholds are shown in Table 8.23 and Figure 8.20. However, a robust conclusion cannot be driven from them because some vehicles show better results at 30 seconds or one-minute threshold and others show better results with other threshold. This happens because when the vehicles get diverted it is not important for the whole system if some emergencies are going to get their vehicles in five minutes especially if they are not very severe ones. The goal is to provide better response to more severe ones.

Longest response time for different minimum diversion benefit threshold is shown in Table 8.24 and Figure 8.21. They show that the longest response time increases a little when threshold is increasing from 30 seconds to two minutes, but after two minutes it does not change in these cases.

Table 8.25 and Figure 8.22 show the percent of vehicles arriving at the emergency later than the required time for different minimum diversion benefit threshold. They demonstrate that just some Fire Engines and Fire Trucks arrive at the emergencies later than the required time and the percentage for other vehicles is zero for different scenarios under investigation. It can also be concluded that 30 seconds and one-minute thresholds provide better results.

Table 8.22 Average Response Time for Different Minimum Diversion Benefit Threshold

Vehicle Type	Average Response Time (min)				
	Minimum Diversion Benefit Threshold (min)				
	0.5	**1**	**2**	**5**	**10**
Police Car	3.1	3.3	3.3	3.3	3.3
BLS Ambulance	3.7	3.7	3.7	3.8	3.8
ALS Ambulance	3.5	3.6	3.6	3.5	3.5
Fire Engine	4.2	4.2	4.4	4.4	4.5
Fire Truck	3.9	3.9	4.1	4.3	4.2

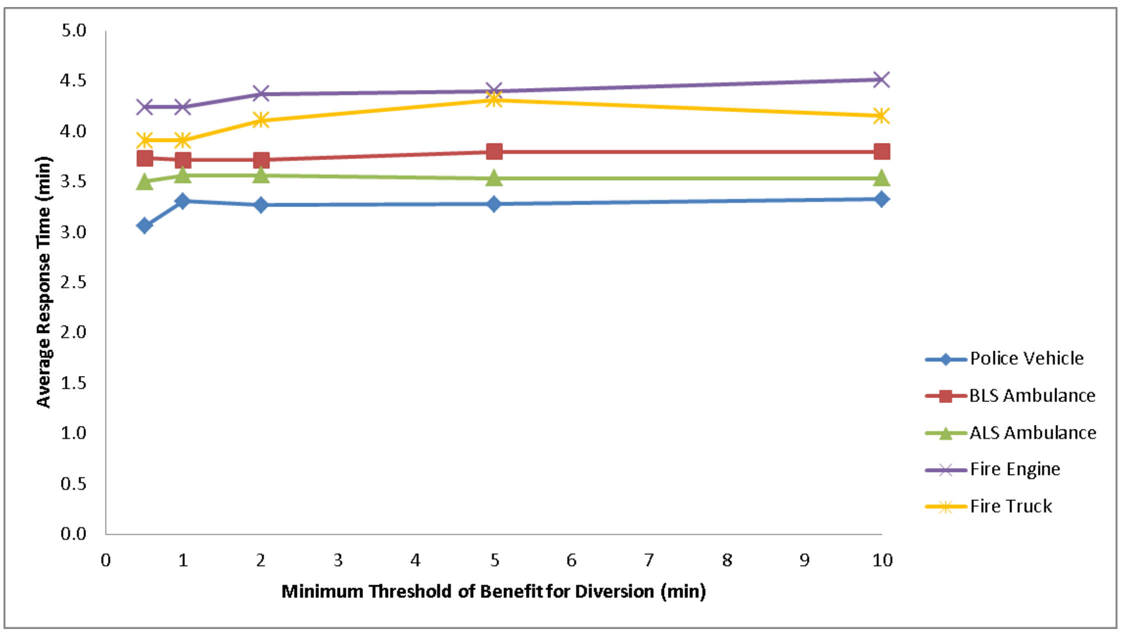

Figure 8.19 Average Response Time with Different Minimum Diversion Benefit Threshold

Table 8.23 Percent of Emergencies Received Their First Vehicle in 5 Minutes for Different Minimum Diversion Benefit Threshold

Vehicle Type	Percent of Emergencies Received Their First Vehicle in 5 Minutes				
	Minimum Diversion Benefit Threshold (min)				
	0.5	1	2	5	10
Police Car	72.7%	72.7%	70.8%	70.8%	69.8%
BLS Ambulance	60.8%	60.8%	59.8%	59.8%	59.8%
ALS Ambulance	61.2%	61.2%	61.2%	62.6%	62.6%
Fire Engine	58.2%	58.2%	56.0%	64.9%	56.0%
Fire Truck	60.7%	60.7%	58.3%	59.5%	59.5%

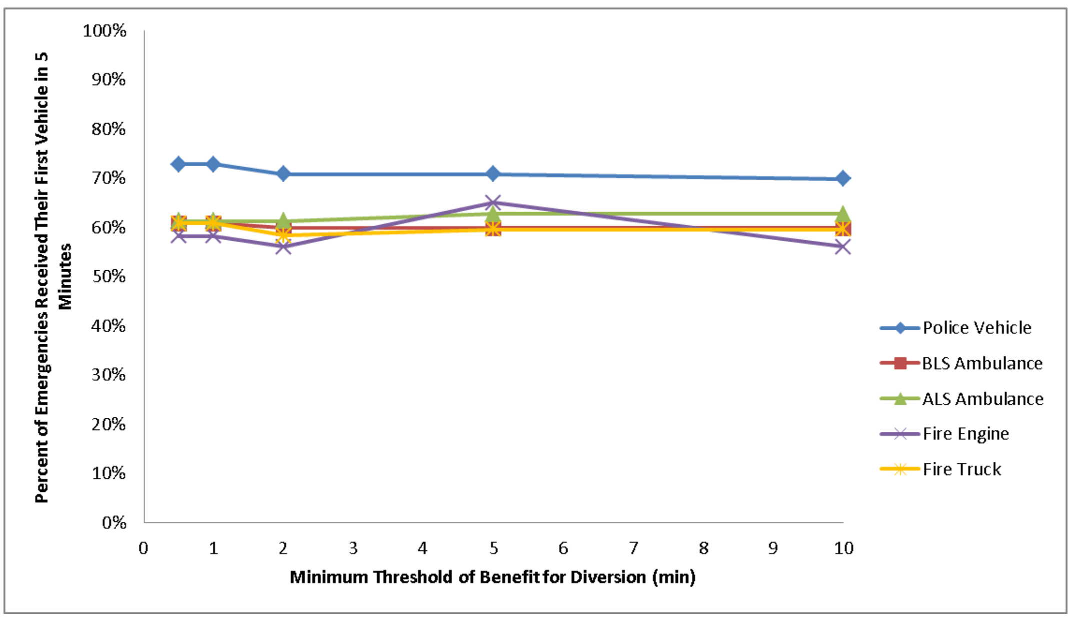

Figure 8.20 Percent of Emergencies Received Their First Vehicle in 5 Minutes with Different Minimum Diversion Benefit Threshold

Table 8.24 Longest Response Time for Different Minimum Diversion Benefit Threshold

Vehicle Type	Longest Response Time (min)				
	Minimum Diversion Benefit Threshold (min)				
	0.5	1	2	5	10
Police Car	8.5	9.8	9.8	9.8	9.8
BLS Ambulance	9.1	9.1	9.3	9.3	9.3
ALS Ambulance	6.8	7.0	7.0	7.0	7.0
Fire Engine	7.6	7.6	7.8	7.8	7.8
Fire Truck	8.3	8.3	8.3	8.3	8.3

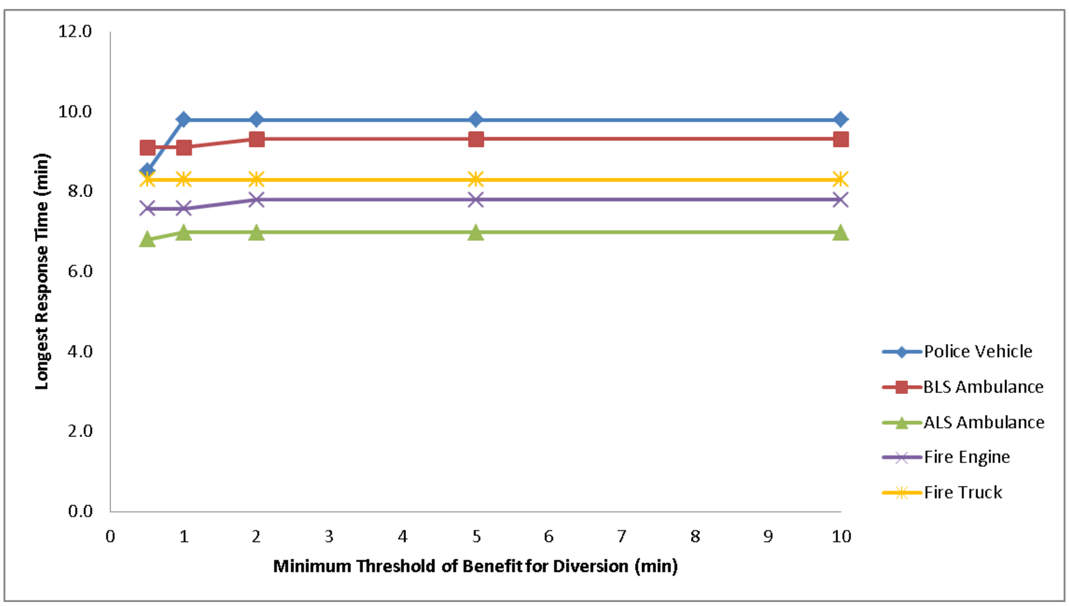

Figure 8.21 Longest Response Time with Different Minimum Diversion Benefit Threshold

Table 8.25 Percent of Vehicles Arrived at the Emergency Later than Required Time for Different Minimum Diversion Benefit Threshold

Vehicle Type	**Percent of Vehicles Arrived at the Emergency Later than Required Time**				
	Minimum Diversion Benefit Threshold (min)				
	0.5	**1**	**2**	**5**	**10**
Police Car	0.0%	0.0%	0.0%	0.0%	0.0%
BLS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%
ALS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Engine	7.4%	7.4%	7.4%	7.4%	9.3%
Fire Truck	8.0%	8.0%	8.2%	8.3%	8.3%

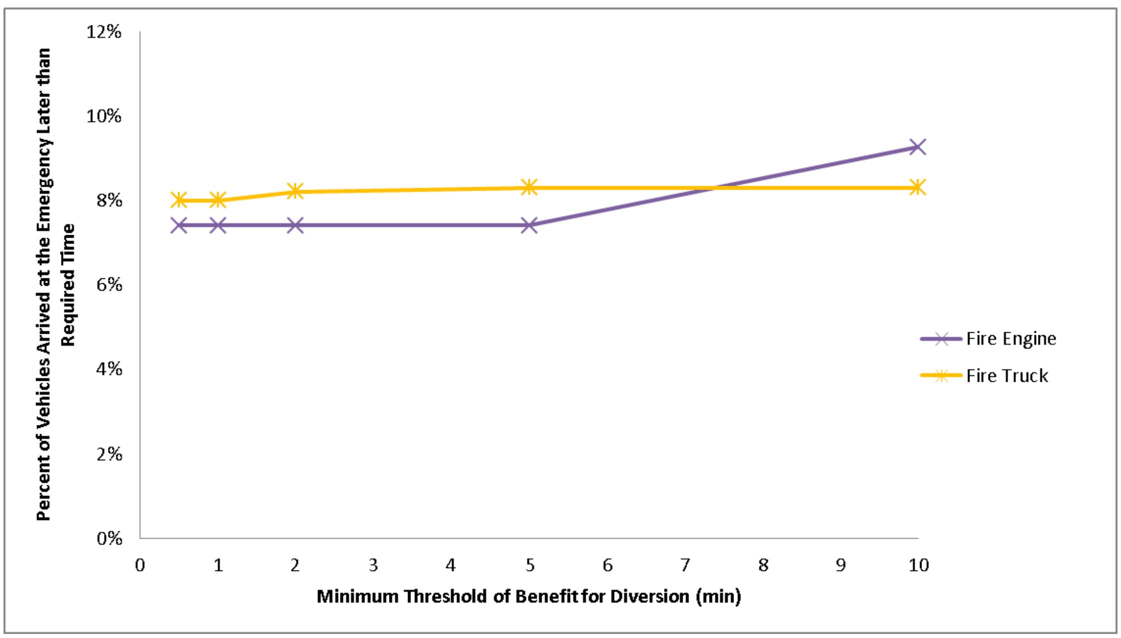

Figure 8.22 Percent of Vehicles Arrived at the Emergency Later than Required Time with Different Minimum Diversion Benefit Threshold

Table 8.26 and Figure 8.23 show percent of vehicles with response time greater than nine minutes for different minimum diversion benefit threshold.

Table 8.26 Percent of Vehicles with Response Time Greater than 9 Minutes for Different Minimum Diversion Benefit Threshold

Vehicle Type	**Percent of Vehicles with Response Time Greater than 9 Minutes**				
	Minimum Diversion Benefit Threshold (min)				
	0.5	**1**	**2**	**5**	**10**
Police Car	0.0%	1.1%	1.1%	1.1%	1.1%
BLS Ambulance	1.3%	1.3%	1.9%	1.9%	1.9%
ALS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Engine	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Truck	0.0%	0.0%	0.0%	0.0%	0.0%

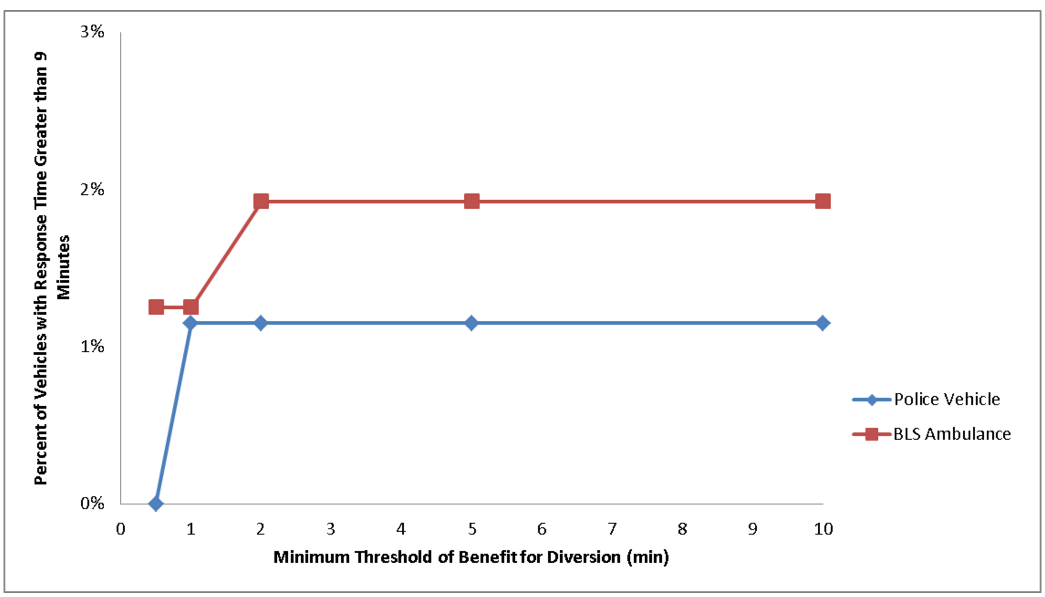

Figure 8.23 Percent of Vehicles with Response Time Greater than 9 Minutes with Different Minimum Diversion Benefit Threshold

As it can be seen in Table 8.26 and Figure 8.23, just police vehicles and BLS ambulances have vehicles with response time greater than nine minutes. The 30-second threshold produces the best results. For BLS ambulances, one-minute threshold is still the same, but there is a jump for police vehicles when the threshold changes from 30 seconds to one minute. After the two-minute threshold the results stay the same.

So, from the analysis in this section, it can be concluded that the 30-second threshold provides the best results and it is obvious, because there is more flexibility in the system and the vehicles can get diverted very easily. However, on the contrary it may confuse the drivers and it is not easy for them to change their destination very frequently. The one-minute threshold is still good and most of the time the results do not deteriorate a lot from 30 seconds to one minute. As a result, 30 seconds or one minute threshold is recommended in this model based on the sensitivity analysis.

8.2.5 Cost of Assigning Vehicles to Non-home Stations

It is obvious that drivers of emergency vehicles prefer to stay at their home station instead of being relocated to other stations. They usually keep their personal belongings at their home stations or even sometimes have an assigned desk at their home stations, so it is usually more comfortable for them to not get relocated to other stations. The cost of assigning to non-home stations is considered in equation 3.3, which is:

$$C_{kis} = \alpha_s \,.\, t_{kis} + \beta_s . WHR_{ki}$$

$$+ \gamma_s \text{ . (whether it's the end of the shift for vehicle i or not)}$$

$$+ \theta_s \text{ . (whether the s is the home station for vehicle i or not)} \quad (3.3)$$

θ_s is the cost for assignment to non-home stations. The police vehicles are quite flexible and they can get relocated to other stations easily, but ambulances and fire vehicles prefer to stay at their home station. In this section, the costs of assigning to non-home stations for all vehicles have been considered to be the same and they are changed to see how the results will change. This coefficient is changing from 0 to 10000 and the results are shown in Tables 8.27 to 8.31.

Table 8.27 and Figure 8.24 present the average response time. They show that overall by increasing the non-home station cost the average response time is increasing. However, it is not changing significantly when the cost is changing from 0 to 10 but when it gets to 1000 or 10000 the average response time is increasing significantly for some vehicles.

Percentages of emergencies receiving their first vehicle in five minutes for different non-home station costs are shown in Table 8.28 and Figure 8.25. A robust conclusion cannot be driven from them, because it fluctuates. However the last scenario shows worst case results.

Longest response time for different non-home station costs is shown in Table 8.29 and Figure 8.26. They demonstrate that the longest response time stays the same when the non-home station cost increases from 0 to 10, but for the last two scenarios it increases and for some vehicles this increase is very significant.

Table 8.27 Average Response Time for Different Non-home Station Costs

Vehicle Type	Average Response Time (min)				
	Non-home Station Cost				
	0	2	10	1000	10000
Police Car	3.1	3.1	3.1	3.4	4.0
BLS Ambulance	3.6	3.7	3.8	4.1	4.1
ALS Ambulance	3.6	3.6	3.6	3.6	3.6
Fire Engine	4.3	4.4	4.4	4.5	4.5
Fire Truck	4.0	4.1	4.1	4.1	4.3

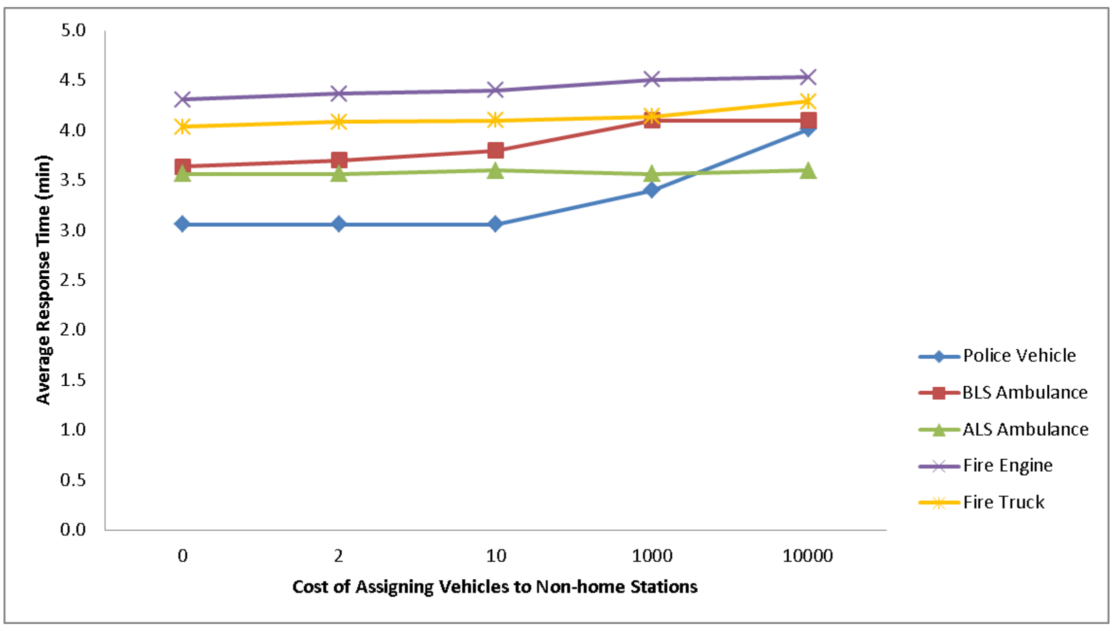

Figure 8.24 Average Response Time with Different Non-home Station Costs

Table 8.28 Percent of Emergencies Received Their First Vehicle in 5 Minutes for Different Non-home Station Costs

Vehicle Type	Percent of Emergencies Received Their First Vehicle in 5 Minutes				
	Non-home Station Cost				
	0	2	10	1000	10000
Police Car	87.7%	87.7%	83.8%	84.8%	78.0%
BLS Ambulance	70.8%	69.8%	68.8%	68.8%	68.8%
ALS Ambulance	71.2%	71.2%	71.2%	71.2%	71.2%
Fire Engine	70.4%	68.2%	67.1%	67.1%	66.0%
Fire Truck	72.0%	69.5%	68.3%	70.7%	68.3%

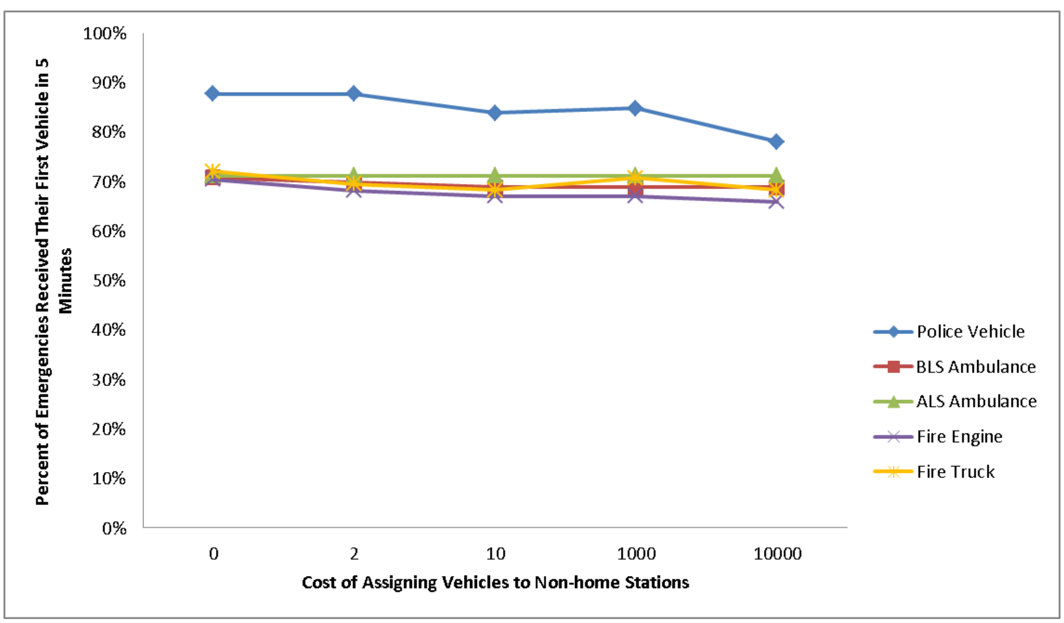

Figure 8.25 Percent of Emergencies Received Their First Vehicle in 5 Minutes with Different Non-home Station Costs

Table 8.29 Longest Response Time for Different Non-home Station Costs

Vehicle Type	Longest Response Time (min)				
	Non-home Station Cost				
	0	2	10	1000	10000
Police Car	9.8	9.8	9.8	10.1	14.8
BLS Ambulance	9.3	9.3	9.3	10.3	11.4
ALS Ambulance	7.0	7.0	7.0	7.0	7.0
Fire Engine	7.6	7.6	7.6	8.8	8.9
Fire Truck	8.3	8.3	8.3	8.5	8.7

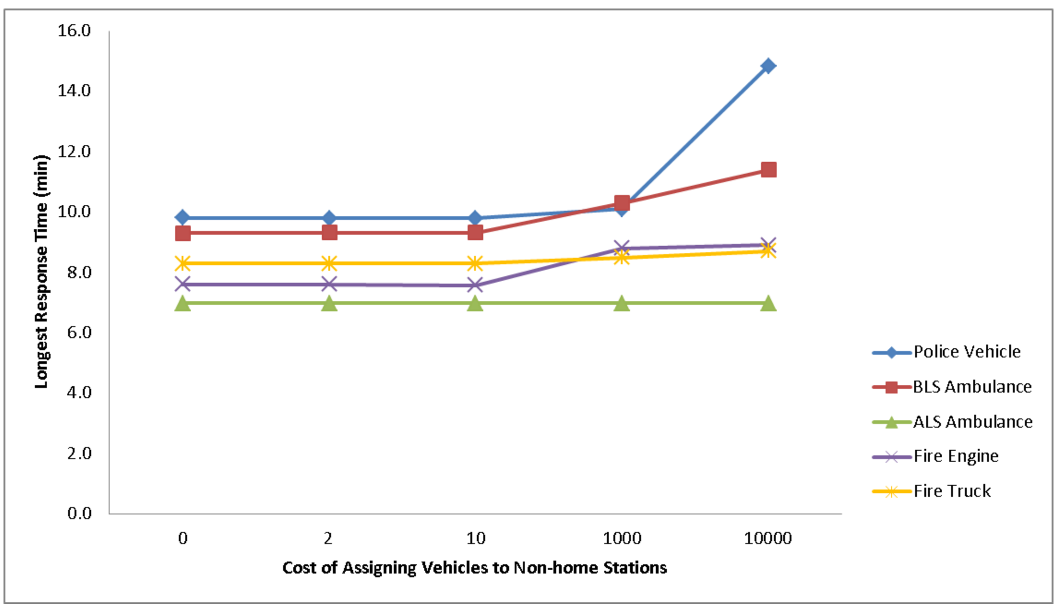

Figure 8.26 Longest Response Time with Different Non-home Station Costs

Table 8.30 and Figure 8.27 show the percent of vehicles arriving at the emergency later than the required time for different non-home station costs.

Table 8.30 Percent of Vehicles Arrived at the Emergency Later than Required Time for Different Non-home Station Costs

Vehicle Type	Percent of Vehicles Arrived at the Emergency Later than Required Time				
	Non-home Station Cost				
	0	2	10	1000	10000
Police Car	0.0%	0.0%	0.0%	1.2%	10.3%
BLS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%
ALS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Engine	7.4%	7.4%	7.4%	7.6%	7.8%
Fire Truck	8.0%	8.0%	8.0%	8.5%	9.0%

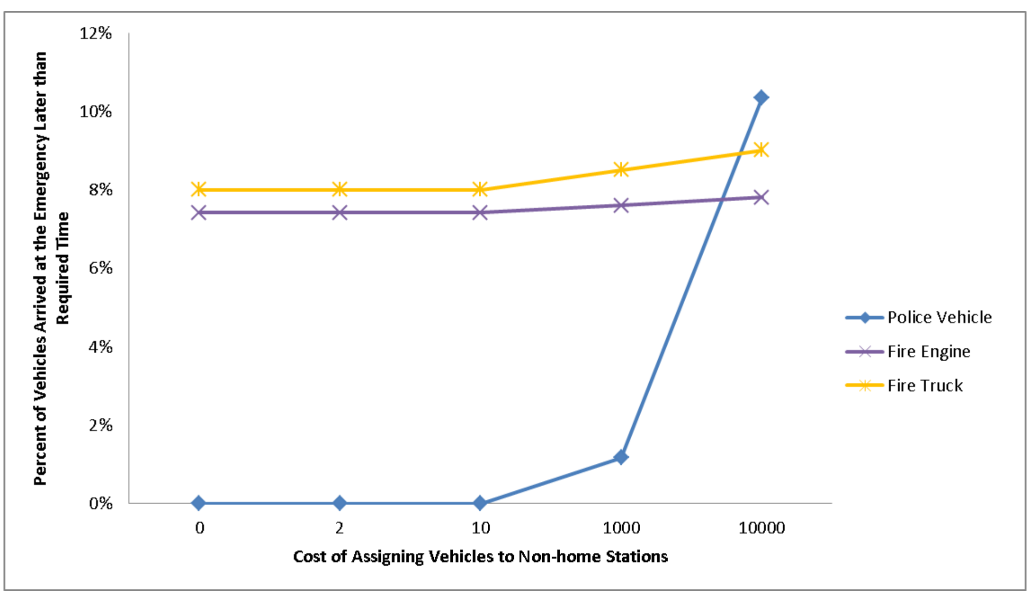

Figure 8.27 Percent of Vehicles Arrived at the Emergency Later than Required Time with Different Non-home Station Costs

As it can be seen in Table 8.30 and Figure 8.27, ALS ambulances and BLS ambulances always get to the emergency in required time for all scenarios. Police vehicles, Fire Engines, and Fire Trucks sometimes get to the scene later than the required time. The percentage of these vehicles reaching the emergency later than the required time stays the same for the first three scenarios and it increases when the non-home cost reaches 1000 or 10000. Police vehicles show a significant increase in the result when non-home cost is changing from 1000 to 10000.

Table 8.31 and Figure 8.28 show percent of vehicles with response times greater than nine minutes for different non-home station costs. They demonstrate that just police vehicles and BLS ambulances have vehicles with response times greater than nine minutes. Results for the first three scenarios almost stay the same, but they increase afterwards specially for the last scenario.

Table 8.31 Percent of Vehicles with Response Time Greater than 9 Minutes for Different Non-home Station Costs

Vehicle Type	Percent of Vehicles with Response Time Greater than 9 Minutes				
	Non-home Station Cost				
	0	2	10	1000	10000
Police Car	1.2%	1.1%	1.2%	1.3%	4.6%
BLS Ambulance	1.9%	1.9%	1.9%	2.2%	3.4%
ALS Ambulance	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Engine	0.0%	0.0%	0.0%	0.0%	0.0%
Fire Truck	0.0%	0.0%	0.0%	0.0%	0.0%

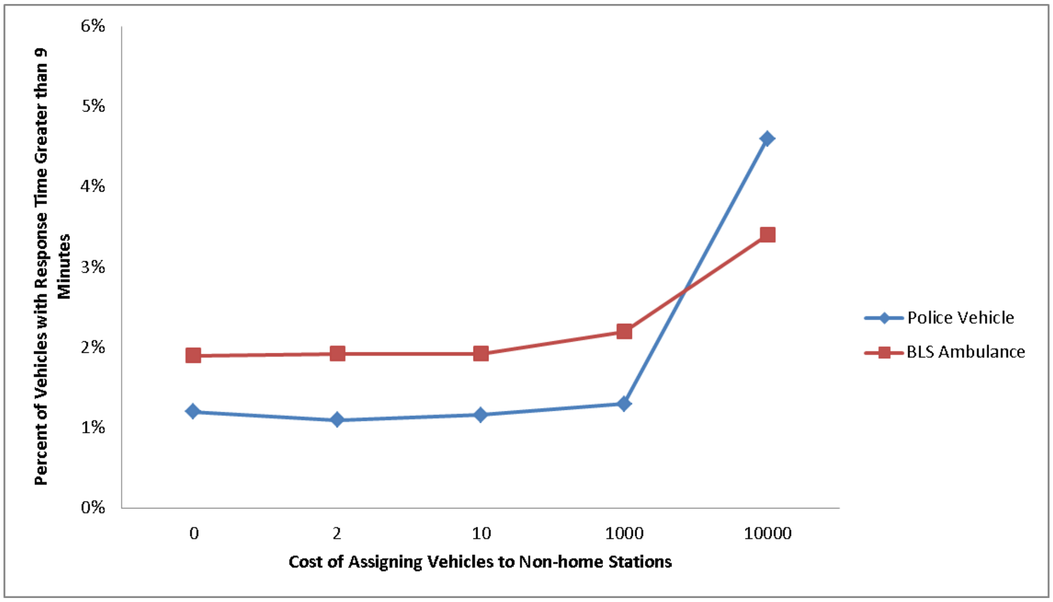

Figure 8.28 Percent of Vehicles with Response Time Greater than 9 Minutes with Different Non-home Station Costs

So, from the analysis in this section it can be concluded that the first three scenarios when the non-home cost increases from 0 to 10, the results somehow stay the same and there is not a lot of difference between them. On the other hand, the last two scenarios show poor results, especially the last one when the non-home station cost reaches 10000. The results are in the direction of what it is expected. When the non-home station cost is in the order of 10, it just forces the vehicles to go to their home stations even if they are closer to other stations and their home station is 10 minutes farther away in the case that two stations provide the same coverage. As a result, it is better not to send vehicles especially ambulances and fire vehicles to non-home stations just because the non-home stations are closer. However, when the non-home cost is in the order of 1000 or more, it forces vehicles to go to their home stations even if other stations provide much better coverage and this is not suitable from the coverage perspective. In the last scenario where the non-home cost is 10000, the vehicles are forced to just go to their home stations and that is the reason most of the time the results for this scenario are very poor. So, the non-home cost can be set to numbers around 10 and in that case the system tries to send vehicles to their home stations if the stations provide the same coverage somehow, but if sending the vehicles to non-home stations will improve the coverage of the whole network then the vehicles will be sent to the best station. Since police cars are very flexible, the non-home cost for them can be set to zero.

8.3. Summary

In this chapter, first the results of applying the proposed model on the case study was shown and compared with dispatching models without coverage problem or with simpler coverage criteria. The results confirm that the proposed model performs very well and most of the time it shows much improvement over other models. Then an extensive sensitivity analysis was performed on some important parameters in the model, and it was investigated that how the results will change by changing those parameters.

"Emergency inter-arrival time," "fleet numbers," "coverage benefit coefficients," "minimum threshold of benefit for diversion" and "cost of assigning vehicles to non-home stations" are the parameters that were investigated in this chapter.

Chapter 9: Summary, Conclusion and Future Research

9.1 Summary

In this research, the Emergency Vehicle Management System was studied. One of the key effective measurements of the system is response time. Response time is not only related to the dispatching system, but also it has a close relationship to emergency vehicle coverage. So, a comprehensive relocation and dispatching model for emergency call centers or emergency management centers was developed in this study. The proposed model relocates emergency vehicles to provide better coverage for the whole system and also when an emergency happens in the system the model will consider dispatching and relocation problems simultaneously. This model can come up with the best relocation and dispatching algorithm based on real-time information about the status of the emergency-response fleet, traffic information, likelihood of emergency happening at the demand nodes and the status of emergency calls.

Contributions of this research can be summarized as:

- Three categories of emergency vehicles are considered in the system: police, ambulance, and fire. The police department is assumed to have a homogeneous fleet, but ambulances and fire vehicles are heterogeneous. Two kinds of ambulances (Advanced Life Support and Basic Life Support) are considered in the model and three types of fire vehicles (Fire Engine, Fire Truck, and Fire Quint), for a total of six vehicle types. There is no dispatching model in the literature that considers non-homogenous vehicles.

- The model tries to cover the demand nodes within a predefined time (T_1 minutes), which can be different for each vehicle type. Demand nodes that are not covered within T_1 minutes are, ideally, covered within T_2 minutes ($T_1 \leq T_2$). By having two specific times for coverage, equity is increased between different demand nodes in the system. This part is also new to the literature.
- Two kinds of demand nodes are considered: ordinary demand nodes and critical demand nodes. Having two kinds of demand nodes is also new in the emergency vehicle coverage problem and this assumption increases the flexibility of applying different policies for different demand nodes.
- The model attempts to provide double coverage for ordinary nodes within T_2 minutes and double coverage for critical nodes within T_1 minutes. There is no double coverage model in the literature that considers heterogeneous vehicles.
- The proposed model is capable of considering benefit of partial coverage. There is no model in the literature that addresses the full coverage and partial coverage together in vehicle relocation problem.
- This model attempts to strike a work-load balance between different vehicles in the system and also tries to send vehicles to their home stations at the end of their work shifts, which are new in the literature.

- A new mathematical formulation is developed in this research that takes into account all the contributions mentioned above.
- A new heuristic model is developed for this problem to come up with good solutions in reasonable time.
- A new simulation model that can see most of the details in the system is constructed for this research.
- The simulation model is applied on a case study to check the performance of the proposed model.

9.2 Conclusion

The performance of only a dispatching problem (Dis) is compared with dispatching problem with simple coverage (DisSC) and dispatching problem with increased equity and double coverage (DisIEDC) which is the entire proposed model in this research. Adding coverage to dispatching problem does not seem to have improvement on "average response time" and "percentage of emergencies which received their first vehicles in five minutes." Sometimes these two statistics became even worse by adding coverage problem to the dispatching problem. The reason is that when the coverage problem is considered, the only goal is not to service emergencies in the system in the fastest way. Sometimes the model may send a far vehicle to an emergency if that emergency is not severe and prefer to keep other vehicles standing by for future demands. On the other hand, "longest response time," "percent of vehicles arriving at the emergency later than the required time," and "percent of vehicles with response time greater than nine minutes" illustrate great improvement. On most of them, the DisIEDC model is performing better than DisSC model and DisSC model is performing better than Dis Model. For example, the longest response time for ALS ambulances decreases 13% when simple coverage is added to the dispatching problem and it improves 24% when the full proposed model is used which is a significant improvement. Also, the “percentage of vehicles arriving at the emergency later than the required time” is a very important statistic and in all of the cases enormous improvement is obtained by using the full proposed model (DisIEDC). For example, the percentage of vehicles arriving at the emergency later than the required time for fire trucks decreases 23% when simple coverage is added to the dispatching problem and it improves 50% when the full proposed model is used. So overall, it can be concluded that adding coverage problem to dispatching problem shows enormous positive impact on the whole system. This benefit is more when the full proposed coverage problem (DisIEDC model) is used comparing with when a simple coverage problem (DisSC model) is used.

Also, to see how the model is working when the parameters are changing, an extensive sensitivity analysis is performed in this research. The parameters considered for sensitivity analysis on the case study and the results obtained from the analysis are as follows:

- Emergency Inter-arrival Time: If the emergency inter-arrival time is 40 minutes and above, the system performs very well. If it decreases to about 30 minutes, as it is the case right now in the case study, the system performs well. However less than 30 minutes emergency inter-arrival time is going to put the system in bad shape and for sure in that situation some vehicles have to be added to the system especially ALS ambulances.

- Fleet Numbers: In order to prevent vehicle deficiencies at emergencies, 20 police vehicles, six BLS ambulances, six ALS ambulances, five Fire Engines, five Fire Trucks, and three Fire Quints are required for the whole system. However for better results, number of vehicles should be increased.
- Coverage Benefit Coefficients: The base case coefficients (2, 1, 0.5, 4, 2, 1) and 100* base case coefficients provide better results and the coverage benefit coefficients should be set to one of these numbers. If the coefficients are set to lower numbers, the dispatching problem will be very important and somehow the coverage problem will be ignored. Also, if the coefficients are set to higher numbers, the coverage problem will be highly prioritized over the dispatching problem and the system will perform poorly.
- Minimum Threshold of Benefit for Diversion: 30-second threshold provides the best results and it is obvious, because there is more flexibility in the system and the vehicles can get diverted very easily. However, on the contrary it may confuse the drivers and it is not easy for them to change their destination very frequently. The one-minute threshold is still good and most of the time the results do not deteriorate a lot from 30 seconds to one minute.

- Cost of Assigning Vehicles to Non-home Stations: The non-home cost can be set to numbers around 10 and in that case the system tries to send vehicles to their home stations if the stations provide the same coverage somehow, but if sending the vehicles to non-home stations will improve the coverage of the whole network then the vehicles will be sent to the best station. Since police cars are very flexible, the non-home cost for them can be set to zero.

9.3 Future Research

Even though various issues related to real case situations have been considered, there are still some problems left that need to be addressed in future or some parts of this research that need improvements. In this section some recommendations for future studies are discussed.

9.3.1 Mathematical Formulation

The real-time dispatching and relocation of the emergency fleet is formulated as a deterministic integer-programming model. It is a dynamic model that should be solved at each time step. At each time step the model tries to send some vehicles to emergencies in need, if there are any in the system, and relocate other vehicles to provide good coverage for the next time step. Some parameters like travel time on the links for next time step and also the expected number of emergencies happening at different demand nodes for future time step are assumed to be known and are assumed to be deterministic. One improvement would be changing the model from deterministic one to stochastic one and apply it on a real case study and see how much improvement can be realized from the stochastic model.

Another approach that can be investigated for future studies is to divide the mathematical formulation into a bi-level problem. Now the objective function is the combination of dispatching problem and coverage problem. Another approach to tackle this problem would be using a bi-level problem; dispatching problem as the upper level and coverage problem as the lower level. This approach also needs to be checked on a case study to see how it performs and weather it is going to be better or worse than the proposed model.

9.3.2 Heuristic Method

In the heuristic method, some initial solutions are constructed first and then several improvement methods are used on those initial solutions to come up with the final solutions. These methods are greedy algorithms and at each time step they try to implement a change that produces the maximum saving, but we know that sometimes the non-greedy algorithms may produce better results. Also, other meta-heuristic methods like Tabu search or Genetic Algorithm can be analyzed for future studies. So far, in the examples considered in this research the proposed heuristic performs very well and also the five seconds running time is highly efficient, but there may exist some other heuristic approaches that can give better results than the proposed heuristic. This is an interesting path for future research.

9.3.3 Lower Bound

In this research the heuristic solutions were compared with the Xpress optimal solutions. This comparison showed good results and the errors were acceptable in the examples compared. However the comparison was done only on the examples that Xpress could handle. Xpress cannot solve large size problems even in a very long running time, so the comparison of heuristic method with optimal solutions is not done on the real size problems. The systematic approach to see how the heuristic method performs is to find a good lower bound for this problem that is missing in this research and is highly recommended for future studies. Lagrangian Relaxation and decomposition methods are very common methods for finding lower bounds. However these two methods may not be good enough and then some other methods that may be more complicated but more efficient can be investigated in future studies.

9.3.4 Simulation Model

We tried to develop the framework of a simulation model that can mimic the entire operation of an emergency response system. However there are some parts missing in the simulation model that can be considered as a recommendation for future studies. Some considerations related to the crews of these vehicles are ignored in the simulation model. For example, it was difficult to track the vehicles and see how much workload they have and enter their workload into the cost estimation. Also, the fact that when the crews are near their end of their shift they have to be sent to their home station instead of being relocated to other stations was ignored in the simulation analysis. These two issues were ignored, because it was difficult to consider the shift of the crews for each vehicle and see how much workload each vehicle has from the starting of the shift or check when their shift ends. Also, for simplification it was assumed that all types of vehicles are done at the same time in emergency sites, but this assumption may not be true. For example, an ambulance may finish its work sooner and return to a station, but a police vehicle may have to stay longer at the incident for investigation.

In addition, the number of needed vehicles and the required time they need to be at each type of emergency site are assumptions in this simulation model that need to be verified by emergency management or 911 centers.

9.3.5 Shortest Travel Time Algorithms

In the simulation model, travel times on links are randomly generated and all-to-all shortest travel times are calculated by implementation of Dijkstra's algorithm. Dijkstra is a very simple algorithm and there are lots of other shortest travel time algorithms that may perform better than Dijkstra. Investigation of different shortest travel time algorithms and choosing the best one for this kind of problem is an important path for future studies.

Another deficiency that exists in the simulation model is that random generation of travel time is not based on time of the day. For example during the peak hours, the travel time on links are usually higher compared to non-peak hours travel times, but in the simulation model, the random generation of peak hours is the same as non-peak hours.

9.3.6 Sensitivity Analysis

An extensive sensitivity analysis was done on some important parameters in the model. Something missing here and left for future studies is an economic analysis of the tradeoffs between the benefit of the proposed model and the operational cost. For example it is interesting to see what the benefit to cost ratio of increasing the fleet size for each type of vehicle would be.

9.3.7 Crew Scheduling

This kind of problem is in a very close relationship with crew scheduling problem. In this research, some preferences related to the crews of the emergency vehicles were considered, but combining this problem with crew scheduling problem will be a very interesting and difficult area that is left for future studies.

9.3.8 Step Wise Function for Vehicle Delay Penalty at Emergencies

In the objective function which is equation 3.1, there is a term for calculating the penalty if the vehicles arrive at the emergency scene later than the required time. This term is:

$$\sum_{k}\sum_{i}\sum_{j} PT_{kj} \,.\, \left|T_{kj} - t_{kij}\right| .\, EXT_{kij}$$

Penalty term which is PT_{kj} is only a function of the type of the vehicle and the type of the emergency and it is not related to how much delay the vehicle is going to have. Another area that is interesting for future research is changing this penalty to a step wise function based on how much delay the vehicle has. In that case the objective function will be non-linear and it will complicate the problem, but it is an interesting path to follow for future research.

Another thing that can be considered here is changing the penalty term in the heuristic method and make it step wise, in that case it will not make a lot of difficulties because in the heuristic method the objective function does not need to be linear.

References

1. Adams, R., & Barnard, S. (1970). *Simulation of Police Field Forces for Decision-Making in Resource Allocation.* Chicago: Law Enforcement Science and Technology III, IIT Research Institute, Port City Press.
2. Andersson, T., & Varbrand, P. (2007). Dicision Support Tools for Ambulance Dispatch and Relocation. *Journal of the Operational Research Society, Vol. 58, No.2*, 195-201.
3. Ball, M. O., & Lin, F. L. (1993). A Reliability Model Applied to Emergency Service Vehicle Location. *Operations Research, 41*, 18-36.
4. Barker, J., Clayton, E., & Taylor, B. (1989). A Non-linear Multi-criteria Programming Approach for Determining County Emergency Medical Service Ambulance Allocations. *Journal of Operational Research Society, Vol.40, No.5*, 423-432.
5. Batta, R., & Mannur, N. (1990). Covering-Location Models for Emergency SituationsThat Require Multiple Response Units. *Management Science, Vol. 36*, 16-23.
6. Batta, R., Dolan, J. M., & Krishnamurty, N. N. (1989). The Maximal Expected Covering Location Problem: Revisited. *Transportation Science, 23*, 277-287.
7. Beraldi, P., & Bruni, M. E. (2009). A Probabilistic Model Applied to Emergency Service Vehicle Location. *European Journal of Operational Research, 196*, 323-331.
8. Berman, O. (1978). *Dynamic Positioning of Mobile Servers on Networks.* Cambridge, MA.: Operations Research Center, Massachusetts Institute of Technology,.
9. Berman, O., & LeBlanc, B. (1984). Location-relocation of Mobile Facilities on a Stochastic Network. *Transportation Science, 18(4)*, 315-330.
10. Berman, O., & Odoni, A. R. (1982). Locating Mobile Servers on a Network with Markovian Properties. *Networks, Vol. 12*, 73-86.
11. Berman, O., Larson, R. C., & Chiu, S. S. (1985). Optimal Server Location on a Network Operating as an M/G/1 Queue. *Operations Research, 33 (4)*, 746–771.
12. Bjarnason, R., Tadepalli, P., Fern, A., & Niedner, C. (2009). Simulation-based Optimization of Resource Placement and Emergency Response. *Proceedings of the Twenty-First Innovative Applications of Artificial Intelligence Conference*, (pp. 47-53).
13. Brotcorne, L., Laporte, G., & Semet, F. (2003). Ambulance Location and Relocation Models. *European Journal of Operational Research, Vol. 147*, 451-463.
14. Brown, G. G., & McBride, R. (1985). Solving Generalized Networks. *Management Science, Vol. 20*, 1497-1523.
15. Burwell, T. H., McKnew, M. A., & Jarvis, J. P. (1992). An Application of a Spatially Distributed Queueing Model to an Ambulance System. *Socio-Economic Planning Sciences, 26*, 289-300.

16. Carson, Y. M., & Batta, R. (1990). Locating an Ambulance on the Amherst Campus of the State University of New York at Buffalo. *Interfaces, 20(5)*, 43-49.
17. Carter, G., & Ignall, E. (1970). A Simultion Model of Fire Department Operations. *IEEE System Science and Cybernetics, Vol. 5*, 282-293.
18. Catrysse, D., & Van Wassenhove, L. N. (1992). A Survey of Algorithms for the Generalized Assignment Problem. *European Journal of Operational Research,Vol. 60*, 260-272.
19. Chaiken, J. M., & Larson, R. C. (1972). Methods for Allocating Urban Emergency Units: A Survey. *Management Science, Vol. 19, No. 4*, 110-130.
20. Chapman, S. C., & White, J. A. (1974). Probabilistic Formulations of Emergency Service Facilities Location Problems. *ORSA/TIMS Conference.* San Juan, Puerto Rico.
21. Chelst, K., & Barlach, Z. (1981). Multiple Unit Dispatching in Emergency Services: Models to Estimate System Performance. *Management Science, Vol. 27, No. 12*, 1390-1409.
22. Chelst, K., & Jarvis, J. (1979). Estimating the Probability Distribution of Travel Times for Urban Emergency Service Systems. *Operations Research, Vol.27, No. 1*, 199-204.
23. Chu, P. C., & Beasley, J. E. (1997). A Genetic Algorithm for the Generalized Assignment Problem. *Computers and Operations Research*, 17-23.
24. Church, R. L., & Revelle, C. (1974). The Maximal Covering Location Problem. *Papers of the Regional Science Association, Vol. 32*, 101-118.
25. Curtin, K. M., Hayslett-McCall, K., & Qiu, F. (2010). Determining Optimal Police Patrol Areas with Maximal Covering and Backup Covering Location Models. *Netw Spat Econ*, 125-145.
26. Daskin, M. (1983). A Maximum Expected Covering Location Model Formulation Properties and Heuristic Solution. *Transportation Science, Vol. 17*, 48-70.
27. Daskin, M. S., & Stern, E. N. (1981). A Hierarchical Objective Set Covering Model for Emergency Medical Service Deployment. *Transportation Science, 15, 2*, 137-152.
28. Davis, S. (1981). Analysis of the Deployment of Emergency Medical Services. *Omega 9*, 655-657.
29. Eaton, D., Hector, M., Sanchez, V., Lantigua, R., & Morgan, J. (1986). Determine Ambulance Deployments in Santo Domingo, Dominican Republic. *Journal of the Operational Research Society, 113*.
30. Erkut, E., Ingolfsson, A., & Budge, S. (June, 2007). Maximum Availability Models for Selecting Ambulance Station and Vehicle Locations: A Critique.
31. Erkut, E., Ingolfsson, A., Sim, T., & Erdogan, G. (2009). Computational Comparison of Five Maximal Covering Models for Locating Ambulances. *Geographical Analysis 41*, 43-65.
32. Fisher, M. L., Jaikumar, R., & Wassenhove, L. N. (1986). A Multiplier Adjustment Method for the Generalized Assignment Problems. *Management Science, Vol. 32*, 1095-1103.

33. Fitzsimmons, J. (1973). A Methodology for Emergency Ambulance Deployment. *Management Science, Vol. 19, No. 6, 627*.
34. Fitzsimmons, J. A., & Srikar, B. (1982). Emergency Ambulance Location Using the Contiguous Zone Search Routine. *Journal of Operations Management, 2*, 225-237.
35. Fujiwara, O., Makjamroen, T., & Gupta, K. K. (1987). Ambulance Deployment Analysis: A Case Study of Bangkok. *European Journal of Operational Research, 31*, 9-18.
36. Gendreau, M., Laporte , G., & Semet, F. (1997). Solving an Ambulance Location Model by Tabu Search. *Location Science, 5*, 75-88.
37. Gendreau, M., Laporte, G., & Semet, F. (1999). Parallel Tabu Search Heuristic for Real-time Vehicle Routing and Dispatching. *Transportation Science 33*, 381-390.
38. Gendreau, M., Laporte, G., & Semet, F. (2001). A Dynamic Model and Parallel Tabu Search Heuristic for Real-time Ambulance Relocation. *Parallel Computing 27*, 1641-1653.
39. Gendreau, M., Laporte, G., & Semet, F. (2006). The Maximal Expected Coverage Relocation Problem for Emergency Vehicles. *Journal of Operational Research Society, 57*, 22-28.
40. Geroliminis, N., Karlaftis, M. G., & Skabardonis, A. (2009). A Spatial Queuing Model for the Emergency Vehicle Districting and Location Problem. *Transportation Research Part B, 43*, 798-811.
41. Geroliminis, N., Kepaptsoglou, K., & Karlaftis, M. G. (2011). A Hybrid Hypercube – Genetic Algorithm Approach for Deploying Many Emergency Response Mobile Units in an Urban Network. *European Journal of Operational Research 210*, 287-300.
42. Goldberg, J. (2004). Operations Research Models for the Deployment of Emergency Service Vehicles. *EMS Management Journal, 1(1)*, 20-39.
43. Goldberg, J., & Szidarovszky, F. (1991). Method for Solving Nonlinear Equations Used in Evaluating Emergency Vehicle Busy Probabilities. *Operations Research, Vol. 39*, 903-916.
44. Goldberg, J., Dietrich, R., Chen, J., & Mitwasi, M. (1990). Validating and Applying a Model for Locating Emergency Medical Vehicles in Tucson, AZ. *European Journal of Operational Research, No.49*, 308-324.
45. Haghani, A., (1996). Capacitated Maximum Covering Location Models: Formulations and Solution Procedures. *Journal of Advanced Transportation, Vol. 30, No. 3*, 101-136.
46. Haghani, A., Hu, H., & Tian, Q. (2003). An Optimization Model for Real-Time Emergency Vehicle Dispatching and Routing. *Presented at the 82nd annual meeting of the Transportation Research Board.* Washington, D.C.
47. Haghani, A., Tian, Q., & Hu, H. (2004). Simulation Model for Real-Time Emergency Vehicle Dispatching and Routing. *Transportation Research Record: Journal of the Transportation Research Board,No. 1882, TRB, National Research Council, Washington, D.C.,*, 176-183.

48. Halpern, J. (1977). The Accuracy of Estimates for the Performance Criteria in Certain Emergency Service Queuing Systems. *Transportation Science, Vol.11, No.3*, 223-241.
49. Hogan, K., & ReVelle, C. (1986). Concepts and Applications of Backup Coverage. *Management Science, Vol. 32*, 1434-1444.
50. Iannoni, A. P., & Morabito, R. (2007). A Multiple Dispatch and Partial Backup Hypercube Queuing Model to Analyze Emergency Medical Systems on Highways. *Transportation Research Part E 43*, 755-771.
51. Iannoni, A. P., Morabito, R., & Saydam, C. (2009). An Optimization Approach for Ambulance Location and the Districting. *European Journal of Operational Research, 195*, 528-542.
52. Iannoni, A. P., Morabito, R., & Saydam, C. (2010). Optimizing Large-Scale Emergency Medical System Operations on Highways Using the Hypercube Queuing Model. *Socio-Economic Planning Sciences*, 1-13.
53. Ingolfsson, A., Budge, S., & Erkut, E. (2008). Optimal Ambulance Location with Random Delays and Travel Times. *Health Care Management Science, Volume 11, Number 3*, 262-274.
54. Jarvis, J. P. (1975). *Optimization in Stochastic Service with Distinguishable Servers. In TR-19-75, Innovative Resource Planning Project in Urban Public Safety Systems.* Cambridge, MA.: Operations Research Center, Massachusetts Institute of Technology.
55. Jia, H., Ordonez , F., & Desso, M. (2005). A Modeling Framework for Facility Location of Medical Services for Large-Scale Emergencies. *Department of Industrial and Systems Engineering, University of Southern California, Los Angeles*.
56. Kolesar, P., & Walker, W. E. (1974). An Algorithm for the Dynamic Relocation of Fire Companies. *Operations Research 22*, 249-274.
57. Larson, R. (1974). A Hypercube Queuing Model for Facility Location and Redistricting in Urban Emergency Services. *Computer and Operations Research, Vol. 1*, 67-95.
58. Larson, R. (1975). Approximating the Performance of Urban Emergency Service Systems. *Operations Research, Vol.23, No.5*, 845-868.
59. Lee, S. (2010). The Role of Preparedness in Ambulance Dispatching. *Journal of the Operational Research Society*, 1-10.
60. Lei, H., Cheu , R. L., & Aldouri, R. (2009). Optimal Allocation of Emergency Response Service Units to Cover Critical Infrastructures with Time-Dependent Service Demand and Travel Time. *Transportation Research Record: Journal of the Transportation Research Board,No. 2137, Transportation Research Board of the National Academies, Washington, D.C.*, 74-84.
61. Lorena, L. A., & Narciso, M. G. (1996). Relaxation Heuristics for a Generalized Assignment Problem. *European Journal of Operational Research, Vol. 19, No.3*, 600-610.
62. Lorena, L. A., Narciso, M. G., & Beasley, J. E. (2002). A Constructive Genetic Algorithm for the Generalized Assignment Problem.

63. Lubicz, M., & Mielczarek, B. (1987). Simulation Modelling of Emergency Medical Services. *European Journal of Operational Research, No. 29*, 178-185.
64. Mandell, M. B. (1998). Covering Models for Two Tiered Emergency Medical Systems. *Location Science, 6*, 355-368.
65. Marianov, V., & ReVelle, C. S. (1992). The Capacitated Standard Response Fire Protection Sitting Problem: Deterministic and Probabilistic Models. *Annuals of Operations Research, 40*, 303-322.
66. Marianov, V., & ReVelle, C. S. (1992). A Probabilistic Fire-Protection Sitting Model with Joint Vehicle Reliability Requirements. *The Journal of the Regional Science Association International, 71, 3: 217-241*.
67. Marianov, V., & ReVelle, C. S. (1994). The Queueing Probabilistic Location Set Covering Problem and Some Extensions. *Socio-Economic Planning Sciences, 28*, 167-178.
68. Marianov, V., & ReVelle, C. S. (1996). The Queueing Maximal Availability Location Problem: A Model for the Sitting of Emergency Vehicles. *European Journal of Operations Research, 93*, 110-120.
69. Marinov, & Revelle, C. (1995). Siting Emergency Services, in Facility Location: ASurvey of Articles, Applications and Methods. *Drezner, Z(editor), Springer Series in Operations Research*, 199-222.
70. Martello, S., Pulleyblank, W. R., Toth, P., & Werra, D. d. (1984). Balanced Optimization Problems. *Operations Research Letters, Vol. 3*, 275-278.
71. Morabito, R., Chiyoshi, F., & Galvao, R. D. (2008). Non-homogeneous Servers in Emergency Medical Systems:Practical Applications Using the Hypercube Queueing Model. *Socio-Economic Planning Sciences 42*, 255-270.
72. Nair, R., & Miller-Hooks, E. (2009). Evaluation of Relocation Strategies for Emergency Medical Service Vehicles. *Transportation Research Record: Journal of the Transportation Research Board, No.2137, Washington, D.C.*, 63-73.
73. NFPA 1710. *Standard for the Organization and Deployment of Fire Suppression Operations.* Emergency Medical Operations, and Special Operations to the Public by Career Fire Departments, 2001 Edition, 1Batterymarch Park, PO Box 9101,Quincy, MA.
74. Nulty, W. G., & Trick, M. A. (1988). GNO/PC Generalized Network Optimization System. *O.R. Letters, Vol. 2*, 101-112.
75. Rajagopalan, H. K., Saydam, C., & Xiao, J. (2008). A Multiperiod Set Covering Location Model for Dynamic Redeployment of Ambulances. *Computers and Operations Research, 35*, 814-826.
76. Repede, J. F., & Bernardo, J. J. (1994). Developing and Validating a Decision Support System for Locating Emergency Medical Vehicles in Louisville, Kentucky. *European Journal of Operational Research, 75*, 567-581.
77. Revelle, C., & Hogan, K. (1989). The Maximum Reliability Location Problem and α-reliable p-center Problem: Derivatives of the Probabilistic Location Set Covering Problem. *Annuals of Operations Research*, 155-174.

78. ReVelle, C., Bigman, D., Schilling, D., Cohon, J., & Church, R. (1977). Facility Location: A Review of Context-free and EMS Models. *Health Services Res.*, 129-146.
79. Ross, G. T., & Soland, M. S. (1975). A Branch and Bound Algorithm for the Generalized Assignment Problem. *Mathematical Programming, Vol. 8,* 91-103.
80. Sathe, A., & Miller-Hooks, E. (2005). Optimizing Location and Relocation of Response Units in Guarding Critical Facilities. *Transportation Research Record, 1923,* 127-136.
81. Savas, E. (1969). Simulation and Cost-effectiveness Analysis of New York's Emergency Ambulance Service. *Management Science, Vol. 15, No.12*, B608-B627.
82. Schilling, D. A., Vaidyanathan, J., & Barkhi, R. (1993). A Review of Covering Problems in Facility Location. *Location Science, Vol. 1*, 25-55.
83. Schilling, D., Elzinga, D. J., Cohon, J., Church, R., & Revelle, C. (1979). The Team/Fleet Models for Simultaneous Facility and Equipment Siting. *Transportation Scienece, Vol.13, No.2*, 163-175.
84. Schmid, V., & Doerner, K. F. (2010). Ambulance Location and Relocation Problems with Time-Dependent Travel Times. *European Journal of Operational Research, 207*, 1293-1303.
85. Sorensen, P., & Church, R. (2010). Integrating Expected Coverage and Local Reliability for Emergency Medical Services Location Problems. *Socio-Economic Planning Sciences 44*, 8-18.
86. Swersey, A. (1970). Dispatching, Deployment and Relocation of Fire Engines. *Presented at the 37th National Meeting of Operations Research Society of America.* Washington, D.C.
87. Toregas, C., Swain, R., ReVelle, C., & Bergman, L. (1971). The Location of Emergency Service Facilities. *Operations Research, Vol. 19-6*, 1363-1373.
88. Trick, M. A. (1992). A Linear Relaxation Heuristic for the Generalized Assignment Problem. *Naval Research Logistics, Vol. 39*, 137-152.
89. Yang, S. (2006). Integrated Management of Emergency Vehicle Fleet. *Ph.D Dissertation, University of Maryland, College Park, MD.*
90. Yang, S., Hamedi, M., & Haghani, A. (2004). Integrated Approach for Emergency Medical Service Location and Assignment Problem. *Transportation Research Record: Journal of the Transportation Research Board, No. 1882, Washington, D.C.*, 184-192.
91. Yang, S., Hamedi, M., & Haghani, A. (2005). Online Dispatching and Routing Model for Emergency Vehicles with Area Coverage Constraints. *Transportation Research Record: Journal of the Transportation Research Board, No. 1923, Washington, D.C.*, 1-8.
92. *McGOUGH Website* (accessed February 2012) http://www.mcgough.com/projects.asp?p=0&c=Science/Technology&id=263
93. *National Highway Traffic Safety Administration*. (2011). Retrieved 2011, from NHTSA: http://www.nhtsa.gov/
94. *U.S. Fire Administration*. (2011). Retrieved 2011, from USFA: http://www.usfa.fema.gov/