ABSTRACT

Title of dissertation: ESSAYS ON THE COMPARISON OF PRODUCTION TECHNOLOGIES: APPLICATIONS TO MARYLAND DAIRY FARMS

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This dissertation proposes three new methodologies in empirical production economics for assessing technical change, production risks, and technological frontiers. Each methodology is demonstrated with an application to Maryland dairy operations, with an emphasis on comparing production technologies between confinement and management-intensive grazing (MIG) dairy systems. The rapid decline of small to medium scale dairies has made the study of alternative dairy production like MIG politically and socially important.

The first essay develops a regression-based approach to the Malmquist productivity index (MPI) decomposition that attributes production heterogeneity to technical change (i.e., shifts of technological frontiers) and technical efficiency change (i.e., shifts of technical efficiency). Unlike the conventional, producer-level decomposition measures, the proposed method obtains sample-level decomposition measures, for which the researcher can fully utilize unbalanced panel data and control for the influence of potentially-confounding non-production factors. The results find 1.3% and 0.6% annual technical change during 1995-2009 for confinement and grazers respectively. For both dairy systems, farm ownership and off-farm income are positively and negatively associated with technical efficiency respectively.

The second essay considers an empirical application of the state-contingent (SC) approach to production risks. In the context of agricultural production, uncertainty in the SC framework is defined over distinct weather events or market conditions, for which the producer is assumed to prepare a portfolio of SC production outcomes.

This study shows how production data over multiple years can be regarded as multiple draws from the contingent states of nature, by which SC technologies can be approximated by Data Envelopment Analysis (DEA). The results suggest that optimal production decisions for a moderate-to-maximally risk-averse producer have become riskier for the confinement system and less risky for the grazing system.

The third essay proposes a refinement of the DEA frontier approximation by integrating the concepts of technical, allocative, and scale inefficiencies. Technology is estimated in the form of a weighted-average of the benchmarking-frontiers that are associated with these inefficiency concepts. In the current dataset, the proposed method finds 7.5% to 9.2% higher mean-technical efficiency than the standard practice, indicating the increased discriminatory power in efficiency analysis.

ESSAYS ON THE COMPARISON OF PRODUCTION TECHNOLOGIES: APPLICATIONS TO MARYLAND DAIRY FARMS

by

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Table of Contents

List of Abbreviations

AE	allocative efficiency
AI	allocative inefficiency
ANOVA	analysis of variance
ARMS	Agricultural Resource Management Survey
C.TC	cross-period technical change (measurement)
CARA	constant absolute risk aversion
CES	constant elasticity of substitution
COLS	corrected ordinary least squares
CRRA	constant relative risk aversion
CRS	constant returns to scale
DDF	Difference in Distance-Functions (approach)
DEA	Data Envelopment Analysis
DGP	data generating process
FDH	free disposable hull
LMD	latent marginal demand
LMS	latent marginal supply
MAD	mean absolute deviation
MB	mean bootstrap (estimator)
MIG	management-intensive grazing
MPI	Malmquist Productivity Index
MRT	marginal rate of transformation
MTE	meta-technical efficiency
NIRS	non-increasing returns to scale
OLS	ordinary least squares
OS	outcome-states (approach)
PF	profit-function
PI	profit inefficiency
S.TC	specific-period technical change (measurement)
SAE	state-allocative efficiency
SC	state-contingent (approach)
SD	standard deviation
SE	scale efficiency
SFA	Stochastic Frontier Analysis
SI	scale inefficiency
SMB	shifted-mean bootstrap (estimator)
SSD	second-order stochastic dominance
TC	technical change
TE	technical efficiency
TEC	technical efficiency change
TGR	technology gap ratio
TI	technical inefficiency
TMDL	total maximum daily load
USDA	United States Department of Agriculture
VA	value-added
VRS	variable returns to scale
WCOLS	weighted COLS
WDEA	Weighted DEA (approach)
WTA	willingness to accept
WTP	willingness to pay

Chapter 1: Introduction

The US dairy industry has gone through significant structural changes, which has nearly halved the number of dairy farms from 1992 to 2007. Most of this decline is attributed to the disappearance of small-to-medium scale dairies (e.g., less than 200 cows) in the face of the competition against newly-emerged, very large-scale operations (e.g., over 1,000 cows). The search for economically-viable solutions for small to medium scale dairies has made the study on alternative dairy production politically and socially important.

In traditional dairy regions like the Northeast, some dairies have been experimenting with management-intensive grazing (MIG) techniques. Using a uniquely-long panel dataset of Maryland dairy operations over 15 years, Hanson et al. (2013) previously investigated the systematic differences in production decisions between MIG dairies (or "grazers" in below) and conventional confinement dairies (or "confinement"). The study concluded that financially, MIG is equally competitive and likely less risky. This dissertation utilizes the same dataset and expands on these results. The main findings include: varying rates of technical change and technical efficiency change for the two dairy systems, increased and decreased riskiness for confinement and grazers respectively, inefficient allocation of crop acreage between the two dairy systems, and tendencies for over-utilization of machinery under confinement and that of labor under MIG.

Compared to major crop productions, where large-scale operations have long dominated, the dairy sector presents a case of large production heterogeneity where many small- and medium-scale producers coexist with very large-scale producers. Given the diverse production practices across scales of operations, technical efficiency has been an important topic in the empirical literature on dairy production. In addition, the rapidly-increasing consumer interests in organic foods, grass-fed livestock products, and environmentally-sustainable production practices have created opportunities for smaller-scale producers to simultaneously pursue economic viability in addition to other goals of farm resource-managements. These factors motivate the comparison of confinement and MIG dairies, as described in chapter 2.

The main chapters 3, 4, and 5 develop new tools for empirical production economics in assessing technical change, production risks, and technological frontiers respectively. These methods are demonstrated with applications to Maryland dairy operations, with an emphasis on comparing the production technologies of the two dairy systems.

Chapter 3 links two strands of literature: one that pertains to the measurement of technical change and the other that analyzes the determinants of technical efficiency. The comparison of intertemporal productivity changes and the correlation between decisions and producer characteristics are integrally modeled as the analysis of between-frontier and within-frontier efficiency. Compared to the standard Malmquist Productivity Index (MPI) decomposition of efficiency scores by Data Envelopment Analysis (DEA), the proposed method allows for a full utilization of balanced panel data and the use of control variables in a regression framework.

Chapter 4 investigates a feasible empirical application of the state-contingent

(SC) perspective on production uncertainty, which assumes that the producer prepares a portfolio of SC outcomes for the states of nature. Under the assumptions of (i) no contingent-states change, (ii) no technical change, (iii) time-invariant SC portfolio decisions by the producer, and (iv) cross-sectionally homogeneous state realizations, balanced panel data can be viewed as cross-sectional data of the partially-revealed SC portfolio decisions. This allows for estimating SC technologies and simulating optimal portfolios under various risk preferences. The SC analysis is contrasted with the common approach that views a stochastic distribution of production outcomes as exogenously-determined production risks.

Chapter 5 extends a technological frontier approximation in DEA that integrates the concepts of technical, allocative, and scale inefficiencies. In contrast to the standard sequential estimations of these inefficiencies, the proposed method estimates them simultaneously using a weighted average of the associated benchmarking frontiers as a new technology approximation. Optimal weight selection is derived from well-known statistical properties of DEA estimators.

Chapter 6 draws brief conclusions, highlighting the main findings in empirical analyses and contributions of the proposed methodologies.

Chapter 2: The Background, Motivations, and Data

2.1 Overview

This chapter provides an overview of economic discussions on the US dairy sector and motivations for the study, followed by a brief description of the dataset used throughout the dissertation. The central focus in the US dairy literature is the increasing concentration of dairy production at large scale operations. A dominant perspective is that changes in technologies and production environments have enabled large-scale dairy operations to significantly benefit from scale economies while leaving many small-scale dairies unable to compete. Several of the most influential papers on the subject are briefly reviewed in below.

In regions like the Northeast and the Upper Midwest, management-intensive grazing (MIG) has emerged as a rediscovered dairy practice for small-scale dairies to potentially enhance their economic viability. MIG involves increased utilization of pasture as a main source of forage (by rotating cows through finely-partitioned plots on a daily basis), which greatly reduces the need for purchased or self-harvested feeds. While milk production per cow typically declines under MIG, the cost saved through reduced expenses in feed, labor, machinery, energy, and veterinary care could more than offset the lost milk sales revenues. Existing studies generally support the equal or higher competitiveness of MIG dairies. This dissertation utilizes unique panel data on Maryland dairies over a 15-year period to make relatively robust comparisons between MIG and conventional confinement dairy systems. The discussions on MIG in the following motivate the analyses presented in later chapters.

2.2 Trends in US Dairy Production and Scale Economies

The US dairy industry has been undergoing substantial consolidations in production. Table 2.1 contrasts the total number of farms and farm assets in the US dairy sector to the US agriculture as a whole in the four rounds of Agricultural Census from 1992 to 2007. While the total number of farms has been relatively stable in the US agriculture, the number of dairy farms has declined by 49% from 113,412 to 57,318 during this period. A similar decline has occured in the land acreage used for dairy production. In contrast, at the farm level, the monetary values of land and building structures and those of machinery and equipment have increased by 247% and 141% in the dairy sector, which is substantially faster than 122% and 88% for the corresponding national trends of increasing capital requirements and mechanization. Also, the composition of operational scales in table 2.2 suggests the rapid decrease in small-scale dairies and the steep increase in large-scale operations. By 2007, large-scale dairies with a herd size of 1,000 or higher accounted for only 3% of the dairy farms yet produced 42% of milk output in sales values (2007 US Census of Agriculture).

The increase in the scale and intensity of dairy operations is commonly attributed to a combination of economies of scale in the use of buildings, machinery, and other fixed factors of production and greater efficiency in the use of variable inputs (Kumbhakar et al., 1991; Tauer and Mishra, 2006; Mosheim and Lovell, 2009; Nehring et al., 2009). Using the data on 519 dairies from the USDA 1985 Farm Cost and Return Survey, Kumbhakar et al. (1991) estimated a stochastic Zellner-Revankar production function, a variant of Cobb-Douglas production function with a flexible returns to scale (RTS) structure, along with farm-specific technical and allocative efficiency measures. They found that large-scale farms were more cost efficient and also technically and allocatively more efficient, compared to medium- and small-scale farms.

Tauer and Mishra (2006) estimated stochastic unit-cost frontiers with 755 dairy operations in the Agricultural Resource Management Survey (ARMS) of year 2000. Variable and fixed cost frontiers per hundredweight (cwt) of milk were modeled with either quadratic or logarithmic function of herd size and state fixed effects. Economies of size and the correlation between farm size and technical inefficiency were found in the fixed cost equation but not in the variable cost equation. The fixed cost attributed to technical inefficacy was much larger than that of size inefficiency. For example, with a herd size 100, a farm would have incurred extra \$6.62 (per cwt) due to technical inefficiency and \$0.82 due to size inefficiency. The total fixed cost for this group, including these inefficiency costs, was estimated at \$10.86 (per cwt), compared to \$8.66, \$5.74, and \$3.55 for the farm groups with a herd size of 200, 500, and 1,000 respectively. The authors concluded that small-scale dairies could be competitive with large-scale operations if they were technically efficient.

Using the same ARMS 2000 data, Mosheim and Lovell (2009) estimated a variable cost function incorporating parameterized technical inefficiency and cost share equations for allocative inefficiencies. The authors analyzed variable costs and fixed costs of capital to study economies of scale from two sources: the elasticities of variable costs with respect to outputs (e.g., an elasticity smaller than one indicating increasing returns) and with respect to capital (e.g., a negative elasticity indicating decreasing variable costs in capital). On both accounts they found increasing returns to scale. The plot of a calculated average incremental cost curve for milk production visually illustrated a decreasing average cost curve for herd sizes ranging from 50 to 2000.

Lastly, Nehring et al. (2009) examined separate dairy production frontiers for conventional, confined operations, and pasture-based systems with ARMS 2003-2007 data sets.[1] By estimating system-specific distance functions by SFA, conventional operations of the largest category were found economically the most superior with respect to returns on assets, variable cost per cow, and technical efficiency. The elasticity of inputs to outputs was estimated at 0.65 for conventional dairies and 0.44 for pasture-based dairies, both suggesting the presence of scale economies.

While the above studies all point to increasing returns to scale in dairy production, several methodological weaknesses can be noted. First, some of the production frontiers may be misspecified. The Cobb-Douglas-like function in Kumbhakar et al. (1991) is estimated with three inputs of cows, capital, and labor with regional and farm-size dummies. This specification is more restrictive than the translog form used in Mosheim and Lovell (2009) or Nehring et al. (2009) and offers no clear explanations for including the arbitrary farm-size dummies and excluding feed (a major expense category in dairy production) from the input specification. Also, the stochastic cost frontiers of Tauer and Mishra (2006) that utilize herd size as both a covariate of the cost frontier and a determinant of technical inefficiency would in fact violate the basic

[1] Dairy-focused AMRS 2005 is used to predict dairy system determination for other ARMS data in 2003-2007.

assumption of SFA that technical inefficiency is independently distributed from the determinants of a frontier.

Second, despite the interests in identifying scale economies, the most crucial variables like capital stock or fixed cost may be poorly constructed. Mosheim and Lovell (2009) define capital K as $K = (\text{revenue} - VC)/(\text{cost of } K)$ where VC represents variable costs over feed, labor, and energy, implying that any measurement error in VC would propagate through the measurement error in capital K. Not only does the labor cost include some crude estimates for the opportunity costs of family labor, but also every farm is assumed to exactly break-even, by which every omitted cost component in VC is counted as a part of capital K.[2] In particular, given highly variable milk and feed prices, imposing such a zero-profit condition for a sample of a single production year seems unwarranted. In another example, the proxy for capital input in Kumbhakar et al. (1991) is constructed based on dairy machinery hours that consist of tractor-operation hours and horsepower-adjusted feed-machine hours, ignoring the substantial portion of capital such as land holdings or building structures.

Third, the relationship between variable costs and capital is inherently difficult to estimate. The methodology used in Mosheim and Lovell (2009) traces back to Caves et al. (1981), in which the cost structure for the railroad industry was estimated for exogenously-determined outputs of transportation services as a pseudo-public good. In contrast, in the case of dairy production, outputs are a part of the choice variables. Under standard economic theory, an increase in capital would be

[2] More natural construction seems $K = (\text{Total Cost} - VC)/(\text{cost of } K)$ without imposing restrictions on profits.

accompanied by an increase in outputs until the contribution of the new capital on variable costs completely dissipates. Thus, the contribution of capital on variable costs *for a given output level* may not be estimated from observational data if capital investments and outputs are simultaneously determined. The constant variable costs with respect to farm size found in Tauer and Mishra (2006)[3] is indeed sensible in a situation where producers of various scales of operations face a common set of factor prices.

Fourth, their production frontier estimations may be confounded with regional heterogeneity in production environments. Dairy production practices vary substantially across farms, including the conventional joint production of milk and feed crops, emerging large-scale operations with "dry lots" (confined and almost entirely relying on purchased feeds), and MIG operations. To what extent the regional compositions of these practices are attributable to regional differences in production environments such as climate, resource scarcity, market efficiency, and social institutions remains an open question. Without this knowledge, for instance, it is unclear to what extent the economy of scale in California or Idaho could be replicated in other states.

The previous studies try to account for unobserved regional heterogeneity via fixed effects (i.e., region-specific constants) at the state level (Tauer and Mishra, 2006), the regional level (Kumbhakar et al., 1991), and a single indicator for traditional dairy states (Mosheim and Lovell, 2009), which may be inadequate to properly accommodate producer adaptations to such heterogeneity. If production decisions follow fundamentally different data generating processes across regions, the distributional assumptions used in the above studies would be violated. Blayney (2002) and

[3]This result is also found when inefficiency specification is omitted.

McBride and Green (2009) suggest that the observed trends in scale compositions are likely due to favorable regional conditions such as the lower cost and availability of land, feed, and labor. In fact, some studies that focused on particular regions found no evidence of scale economies in production (in Wisconsin; Cabrera et al., 2010) and in the propensity to stay in business (in Connecticut; Foltz, 2004).

Simple budgetary statistics by scale and region are provided to supplement the above discussions. Tables 2.3 and 2.4 are taken from "Milk cost of production by size of operation" and "Milk cost of production by State" for years 2005 and 2010 by USDA Economic Research Service, which reported revenues and expenses per hundredweight (cwt) of milk as well as operational characteristics like average herd size and milk output per cow. In table 2.3, some indications for scale economies are discerned in the estimated opportunity cost of unpaid labor and the capital cost of machinery, both of which decrease substantially with herd size (e.g., $0.16 (per cwt) and $1.90 respectively at 1,000+ cows and $3.47 and $4.37 at 100-199 cows in 2010). These are typically considered part of fixed costs, which do not vary with the level of outputs and is spread over the total quantity of outputs. On the other hand, the influence of scale economies on variable costs is rather small (e.g., $8.98 (per cwt) for feed costs and $1.87 for hired labor at 1000+ cows and $11.18 and $1.25 respectively at 100-199 cows in 2010). Also, milk output per cow tends to be higher at larger scale operations (e.g., 23,019 (pounds/cow) at 1000+ cows and 18,925 at 100-199 cows in 2010) though the rate of increase declines for medium-scale operations (e.g., 19,840 at 200-499 cows and 22,546 at 500-999 cows).

Table 2.4 contains similar statistics grouped by State.[4] The price of milk, set by

[4]Due to space limitation, the top 5 states in milk sales are reported among the total of 23 states surveyed.

regional marketing orders, varies more by time than by State (see figures 2.1 and 2.2). In contrast, feed costs tend to vary by State in a given point in time (e.g., ranging from \$8.02 in WI to \$9.19 in PA in 2005; from \$8.23 in ID to \$11.06 in WI in 2010), likely reflecting regional differences in production practices and compositions of purchased and self-harvested feeds. The opportunity costs of unpaid labor and the capital cost of machinery, spread over the total output, dramatically vary by State (e.g., together \$1.76 (per cwt) in ID and \$10.68 in PA in 2010) and have significant influence on the overall financial performance. Also notable is relatively high variability in the average herd size and milk output per cow across States (e.g., 1,098 cows with 22,010 (pounds/cow) in ID and 73 cows with 19,701 (pounds/cow) in PA in 2010). It may be surprising to see that even the top dairy states are characterized with relatively small dairy farms on average except in California and later in Idaho in 2010. Overall, there is considerable regional heterogeneity in production practices, in which the structure of scale economies may be also different.

Figures 2.1 and 2.2 present the yearly fluctuations of milk price and feed cost for years 2005-2010. Around year 2008, the fear of food crises around the world had pushed food prices until financial downfall overtook the trend. Large-scale production in California that heavily depended on purchased feeds seems most severely affected during this period. Despite the large price fluctuations, the trends of increasing average herd size and milk output per cow were almost unchanged. This seems to suggest that dairy farms have very limited capabilities to adjust their production in response to price changes and hence the aggregate supply is nearly fixed in a short run. Some farms focusing on maximizing long-term profits might have utilized production contracts under predetermined prices or participating in futures markets to hedge market risks. While the period 2005-2010 may not be very representative, the volatile market conditions suggest that risk management has become a major component of dairy operation.

In many traditional dairy communities where smaller dairy operations are still prevalent but are one the decline, the debates on scale economies and survival of smaller farms have become an important policy issue. While the mechanism behind the on-going structural change in the industry, particularly the relationship between increased cost savings from scale economies and the shift in regional comparative advantages, is yet to be fully uncovered, some solutions are needed to mitigate the rapid and disproportional decline of small-scale dairies. This dissertation contributes to the discussion of whether and how the MIG dairy system can be a viable, alternative production model in these communities.

2.3 Management Intensive Grazing & Motivations for Studies

Management-intensive grazing (MIG) is gaining attention in the Northeast and Upper Midwest as an economically competitive practice for smaller-scale dairies. Studies using data from short term experiments and single-year farm records indicate that, while MIG systems produce less milk per cow, they can be equally or sometimes more profitable than confinement systems due to lower operating costs (Elbehri and Ford, 1995; Rust et al., 1995; Dartt et al., 1999; Soriano et al., 2001; Tucker et al., 2001; Gloy et al., 2002; White et al., 2002; Tozer et al., 2003; Fontaneli et al., 2005; Gillespie et al., 2009).[5] The recent analysis of Maryland dairy operations over a 15-

[5]Regional studies on the profitability of grazing relative to the confinement operation include field experiments in Minnesota (Rust et al., 1995), Virginia (Soriano et al., 2001), Mississippi (Tucker et al., 2001), North Carolina (White et al., 2002), and Pennsylvania (Tozer et al., 2003) and economic analyses on the data collected in Pennsylvania (Elbehri and Ford, 1995), Michigan (Dartt et al., 1999), and New York (Gloy

year period has augmented such findings (Hanson et al., 2013). Additionally, MIG is touted for its environmental sustainability with lower phosphorous run-offs (Bishop et al., 2005), reduced soil erosion (Digiacomo et al., 2001), and increased carbon sequestration (Guo and Gifford, 2002).

In a broad perspective, one can also see the development of the MIG dairy system as a form of a producer adaption to the trends in consumer attitudes and changing production environments. Consumer trends are a major driver for changing product characteristics and production practices in many sectors of agriculture. For example, the increased demand for diversity generally encourages a development of new products beyond the long standardized varieties of agricultural products. Also, some consumers seek higher food quality and food safety through assurances or certifications beyond the standards set by regulators. Non-traditional product attributes (e.g, environmental protection, animal welfare, local employments, fair-trade agreements) can influence the demand for agricultural products just as traditional characteristics like flavors and freshness do. Some of them are conducive to more flavorful or more nutritional products, as demonstrated with many organically- or locally-produced fruits and vegetables. Since the implementation of stringent food safety procedures, minimum quality standards, and mandatory labeling policies may be perceived as disguised protectionism and potentially develop into trans-border disputes, the oversight by regulators would likely be substituted over time with voluntary consumer-producer interactions, aided by information technology and social media. This scenario suggests that producers face higher incentives to improve quality and safety beyond the standardized grading scales when their products can be more easily and more distinc-

et al., 2002).

tively marketed to processors, retailers, or directly to consumers. Then, successful farms would seek to produce either standardized products most efficiently or to create additional values through product differentiations.

The dairy industry is no exception to emerging consumer concerns for potential effects of the modern agricultural production and consumption on health, environments, and local communities. Industrial-scale dairy operations relying on artificial hormones, antibiotics, and biotechnologies are increasingly met with varying consumer responses. Meanwhile, the demand for more "naturally" or "responsibly" produced milk seems to grow as the moral visions of producers resonate with those of consumers. Dairy producers need to be attentive not only to regulatory standards but to the trends in consumer preferences for product attributes such as local brands, organic or ecological certifications, unpasteurized (raw) milk, and additional health or nutritional benefits that allow for product differentiations. MIG operations seem well-suited for producers to experiment with enhancing multi-dimensional values of dairy products.

While future agricultural policies are highly uncertain, some changes may be postulated if the above consumer trends provide some guidance to policymaking. In a diverse agricultural-product market with an increased degree of product differentiation, it will be more difficult to subsidize producers directly through price supports or indirectly through input use for specific outputs. If such traditional subsidy programs are phased out, individual producers must assume increased responsibility in value creation and risk management. For the latter, some producers already hedge risks through private insurance schemes and futures markets, which weakens the case for continuing traditional insurance subsidies and income supports. Future policies may be alternatively focused on promoting efficient supply chains and fair competition by facilitating a network of contractual and institutional relationships between suppliers, producers, processors, and financial institutions with minimal distortionary influence on agricultural markets.

In dairy production, future prices of milk and feed will pose the biggest risks to many producers. Federal and State marketing orders have long been used to stabilize milk prices to partially offset negative production shocks. These policies may fail to keep pace with increasingly-volatile prices of feed and energy-intensive inputs. In theory, uncertainty over input and output prices can be hedged in futures markets, yet such hedging would be ineffective for short-term decisions due to the highly serially-correlated nature of future prices. The use of production contacts, as widely seen in poultry and hog industries, can shift a portion of risks to large-scale processors, which comes at the implicit cost of insurance to producers. An alternative is the MIG system that partially insulates producers from market risks as the intensive use of pasture greatly reduces the need for purchasing or producing feed. On the other hand, compared to the conventional dairy systems, it is more constrained by the cost and availability of land, which is subject to changes in non-dairy commodity or conservation programs.

Additionally, the advantage of scale economies that might have benefited the large-scale dairies may erode under a variety of environmental policy initiatives to improve water and air quality. There is a possibility that the Environmental Protection Agency (EPA) will start to regulate major agricultural producers as point-source polluters, especially highly-concentrated dairy operations for their waste disposal. In the Northeast, the dairy sector could be brought under water quality regulations for reducing nitrogen and phosphorous runoff into the Chesapeake Bay to meet the total maximum daily load (TMDL) of these pollutants. Regulations requiring dairy operations to internalize their pollution or to offset the associated damage will raise the comparative advantage of MIG operations relative to large-scale, confinement operations.

While many of the above topics are beyond the scope of this dissertation, this broad perspective provides a context for the following chapters that compare the conventional confinement operations and MIG operations for technical change, risks, and technological frontiers.

2.4 Data

This dissertation utilizes data derived from IRS Form 1040 Schedule F (farm income tax returns) of 63 dairy farmers who have been participating in the University of Maryland Extension. Twenty of these farmers have used an MIG system for all or part of the 15-year period covered by the study. The dataset contains a total of 580 unbalanced-panel observations on herd size, milk output, crop sales, cow sales, expenditures on numerous inputs, and profit for the period 1995-2009; some demographic information (e.g., age, education, family size and composition) and attitudes toward risks. As briefly summarized in table 2.5, the dataset contains on average 9 years of observations per farm, with 11 MIG farms and 28 confinement farms observed per year.

One unique aspect of the dataset is its relatively-long survey period. Most studies on MIG have used either single-year cross-section data from actual farm enterprises or experimental data collected over a relatively-short time period, limiting the inferences that can be drawn from them, especially in relation to long-term economic sustainability. In contrast, the unique panel of financial data, collected during a 15-year-long extension program, provides an opportunity to investigate relative performances of MIG operations with more robustness and more depth than previous studies. Since financial snapshots of dairy farms are heavily affected by milk and feed prices of the time, it is necessary to examine the long-term average and variability of income to make fair and robust assessments. Table 2.6 provides comparisons for various revenues and expenses from Hanson et al. (2013). MIG operations produce less milk per cow but also spend less on producing milk by reducing purchases of feed, fertilizers, chemicals, and other inputs. Lower costs contribute to lower variability of income, leading to lower downside income risk.

2.5 Tables

Table 2.1: Number of Farms and Farm Asset

Year	Farms	Land in farms (acres)	Ag. Revenue ($1000)	Land & Build. ($/farm)	Machine ($/farm)
(A) US Agriculture, Total					
1992	1,925,300	945,531,506	162,608,334	357,056	48,375
1997	1,911,859	931,795,255	196,854,649	449,748	57,678
2002	2,128,982	938,279,056	200,646,355	537,833	66,570
2007	2,204,792	922,095,840	297,220,491	791,138	88,357
% Δ	15%	-2%	83%	122%	83%
(B) Dairy cattle and milk production					
1992	113,412	38,133,176	20,008,977	377,865	92,624
1997	86,022	30,612,398	20,291,979	500,245	104,000
2002	72,537	27,351,777	22,737,525	845,790	155,271
2007	57,318	21,270,780	34,754,031	1,313,027	223,368
% Δ	-49%	-44%	74%	247%	141%

Based on Agricultural Census 1992-2007.

Table 2.2: Number of Farms by Herd Size

Herd Size	1992	1997	2002	2007	% Δ
20 to 49	49,418	33,137	21,974	16,344	-67%
50 to 99	41,813	33,488	25,465	18,986	-55%
100 to 199	14,062	12,602	10,816	8,975	-36%
200 to 499	4,652	4,881	4,546	4,307	-7%
500 to 999	1,130	1,379	1,646	1,702	51%
1,000 or more	564	878	1,256	1,582	180%
sub-total	111,639	86,365	65,703	51,896	-54%

Based on Agricultural Census 1992-2007 milk cow inventory.

Table 2.3: Milk Production Costs and Returns Per Hundredweight Sold, By Size Group

	2005						2010					
Item	50-99	100-199	200-499	500-999	1000+	all sizes	50-99	100-199	200-499	500-999	1000+	all sizes
Total revenue	17.65	17.23	17.29	16.60	16.56	17.03	19.58	18.86	18.64	18.07	16.98	18.07
Milk Sales	15.58	15.33	15.69	15.14	14.76	15.23	17.39	16.94	16.88	16.34	15.34	16.26
Total Cost	25.61	20.83	17.95	16.10	13.62	18.46	30.86	24.64	22.26	18.19	15.23	20.82
Total Feed Cost	9.01	8.30	8.14	8.26	7.48	8.14	11.70	11.18	11.05	9.73	8.98	10.16
Purchased feed	3.76	4.12	5.02	5.65	6.00	5.03	4.65	4.97	6.02	6.32	6.91	6.09
Self-Harvested feed	5.09	4.07	3.02	2.58	1.47	3.02	6.82	6.06	4.91	3.39	2.05	3.97
Grazed Feed	0.16	0.11	0.10	0.03	0.01	0.09	0.23	0.15	0.12	0.02	0.02	0.1
Hired labor	0.81	1.34	1.84	1.80	1.62	1.47	0.85	1.25	1.85	1.87	1.5	1.46
Opp. Cost of Unpaid Labor	6.11	3.13	1.34	0.54	0.17	2.30	6.92	3.47	1.42	0.5	0.16	2.19
Capital Cost of Machinery	4.58	3.89	2.55	2.03	1.66	2.83	6.18	4.37	3.49	2.42	1.9	3.28
Opp. Cost of Land	0.05	0.04	0.02	0.01	0.01	0.03	0.05	0.04	0.02	0.01	0	0.02
Net Profit	-7.96	-3.60	-0.66	0.50	2.94	-1.43	-11.28	-5.78	-3.62	-0.12	1.75	-2.75
Other Characteristics:												
Herd Size	69	133	295	666	2,086	155	68	135	312	699	2251	175
Output/cow (pounds)	17,075	18,185	19,455	20,707	20,191	18,951	17136	18925	19840	22546	23019	20620
3+ Milking Freq. (% farms)	3.58	6.25	23.25	44.24	43.24	7.02	2.59	8.53	30.5	59.8	55.3	9.42
bST Use(% of cows)	7.2	12.0	19.0	21.0	19.8	15.5	3.34	5.35	11.95	21.85	6.8	8.78
Organic Milk Rev. (%)	1.29	0.42	0.54	0.38	0.26	0.59	5.65	3.65	2.63	0.58	1.9	2.84

Based on "Milk cost of production by size of operation" by USDA ERS.

Monetary unit is in dollars.

Table 2.4: Milk Production Costs and Returns Per Hundredweight Sold, By State

	2010						2005					
Item	CA	ID	NY	PA	WI	All States	CA	ID	NY	PA	WI	All States
Total Revenue	15.64	16.57	17.57	17.66	17.68	17.03	16.23	16.81	19.42	20.39	18.2	18.07
Milk Sales	13.99	14.82	15.91	16.02	15.92	15.23	14.38	15.24	17.75	18.48	16.46	16.26
Total Cost	14.16	20.96	22.6	22.61	19.8	18.46	16.44	13.77	24.12	27.67	24.25	20.82
Total feed costs	8.17	8.94	8.86	9.19	8.02	8.14	9.29	8.23	10.94	10.87	11.06	10.16
Purchased feed	6.61	5.68	3.93	4.10	3.60	5.03	7.04	6.76	5.38	5.34	3.73	6.09
Self-harvested feed	1.51	3.17	4.83	4.97	4.32	3.02	2.22	1.44	5.39	5.4	7.23	3.97
Grazed feed	0.05	0.09	0.10	0.12	0.10	0.09	0.03	0.03	0.17	0.13	0.10	0.10
Hired labor	1.41	1.53	1.75	0.89	1.40	1.47	1.46	1.38	1.37	0.95	1.62	1.46
Opp. Cost of Unpaid Labor	0.39	3.17	3.63	4.22	3.30	2.30	0.39	0.34	2.67	5.96	3.19	2.19
Capital Recovery of Machinery	1.74	3.50	3.60	3.19	2.97	2.83	2.50	1.42	4.16	4.72	4.11	3.28
Opp. cost of Land	<0.01	0.01	0.02	0.05	0.03	0.03	<0.01	<0.01	0.02	0.03	0.03	0.02
Net Profit	1.48	-4.39	-5.03	-4.95	-2.12	-1.43	-0.21	3.04	-4.7	-7.28	-6.05	-2.75
Other Characteristics:												
Herd Size	980	134	119	85	92	155	999	1098	132	73	115	175
Output/cow (pounds)	19,973	17,969	18,835	19,069	19,581	18,951	22,113	22,010	19,564	19,701	20,072	20,620
3+ Milking freq. (% farms)	9.75	1.26	5.35	13.14	6.24	7.02	17.20	23.20	11.40	7.78	10.90	9.42
bST Use(% of cows)	94.0	10.0	20.0	19.0	16.0	24.0	5.0	<0.1	13.4	20.4	21.5	8.8
Organic Milk Rev. (%)	0.32	1.08	0.40	0.59	0.82	0.59	1.31	2.08	4.30	2.84	3.93	2.84

Based on “Milk cost of production by State” by USDA ERS, which calculates revenues and costs in 23 states.

Monetary unit is in dollars.

Table 2.5: Sample Characteristics: Summary

Item	Mean	S.D.	Min	Max
(A) Grazers				
years/farm	8.6	4.9	1	15
farms/year	10.9	3.1	4	15
cows	86	29	37	195
cwt milk	12320	5605	2670	42955
acres (total)	283	134	115	700
acres (pasture)	152	60	53	280
acres (crop)	132	108	0	600
(B) Confinement				
years/farm	8.7	4.7	2	15
farms/year	27.7	3.9	19	32
cows	116	70	22	468
cwt milk	22634	16114	3761	110668
acres (total)	338	160	90	845
acres (pasture)	50	39	0	141
acres (crop)	289	155	60	704

Sources: Hanson et al. (2011).

Table 2.6: Sample Characteristics: Differences Under Year Fixed Effects

	Item	Grazers	Confinement	Difference
(1)	Milk Output (cwt)	11545	22952	11,407***
(2)	Milk Sales ($)	180884	359980	179,096***
(3)	Crop Sales ($)	516	4516	4,000***
(4)	Cattle Sales ($)	15771	20599	4,828***
(5)	Other Income ($)	9940	25928	15,988***
(6)	Chemical ($)	1073	9293	8,220***
(7)	Custom Hire ($)	3600	11898	8,298***
(8)	Depreciation ($)	22521	35378	12,857***
(9)	Feed ($)	54476	104592	50,116***
(10)	Fertilizer ($)	3988	13300	9,312***
(11)	Fuel ($)	4513	9515	5,002***
(12)	Interests ($)	8872	13808	4,936***
(13)	Hired Labor ($)	3557	26192	22,635***
(14)	Maintenance ($)	11290	27057	15,767***
(15)	Seed ($)	4377	7812	3,435***
(16)	Veterinary ($)	5119	18236	13,117***
(17)	Supplies ($)	9788	18484	8,696***
(18)	Gross Income ($)	207118	411046	203,928***
(19)	Total Expense ($)	163308	361580	198,272***
(20)	Net Profit ($)	43811	49467	5,656

Sources: Hanson et al. (2013). Statistical significance: * 10%, ** 5%, *** 1%.

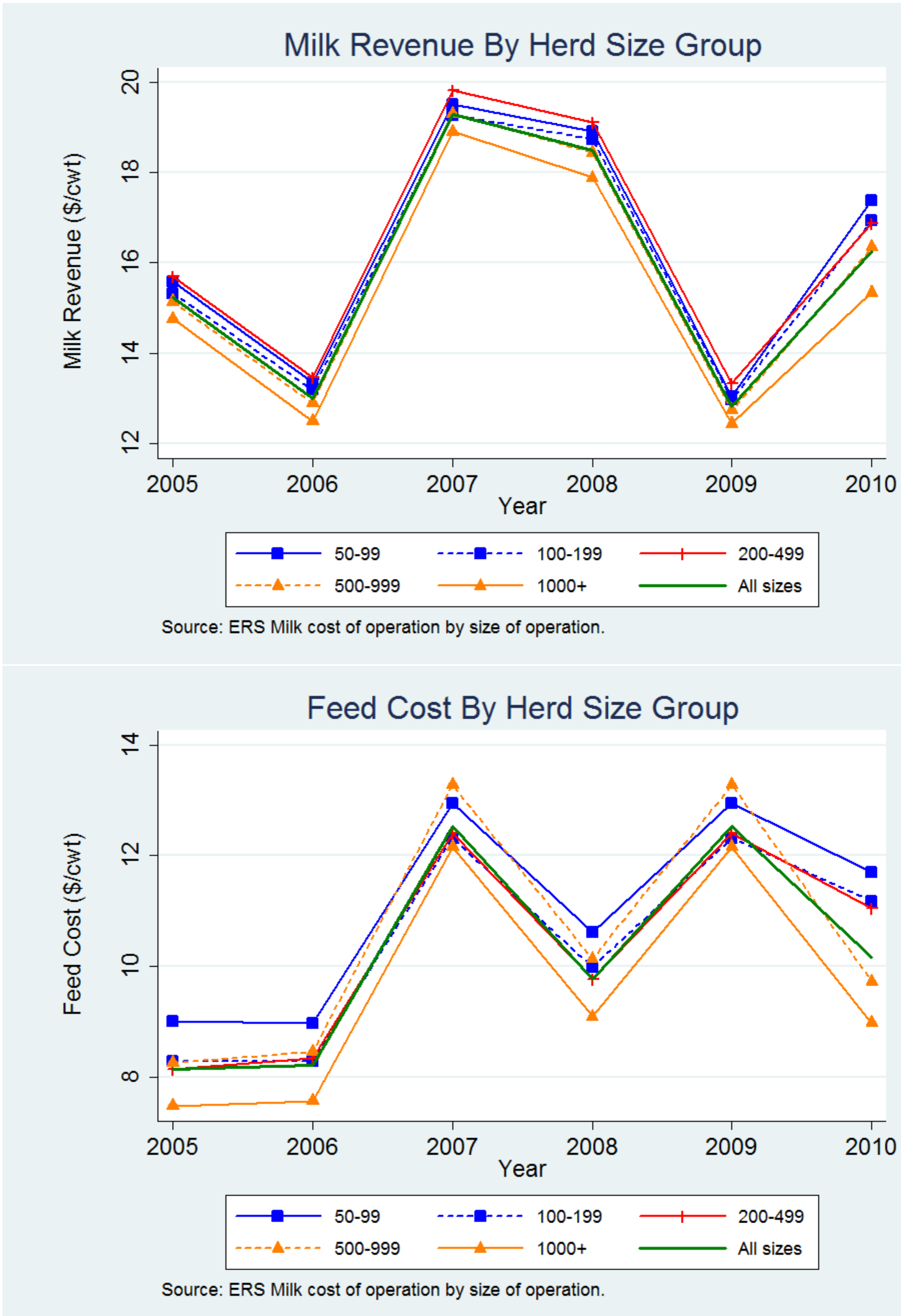

Figure 2.1: Milk Price and Feed Cost for 2005-2010 By Size

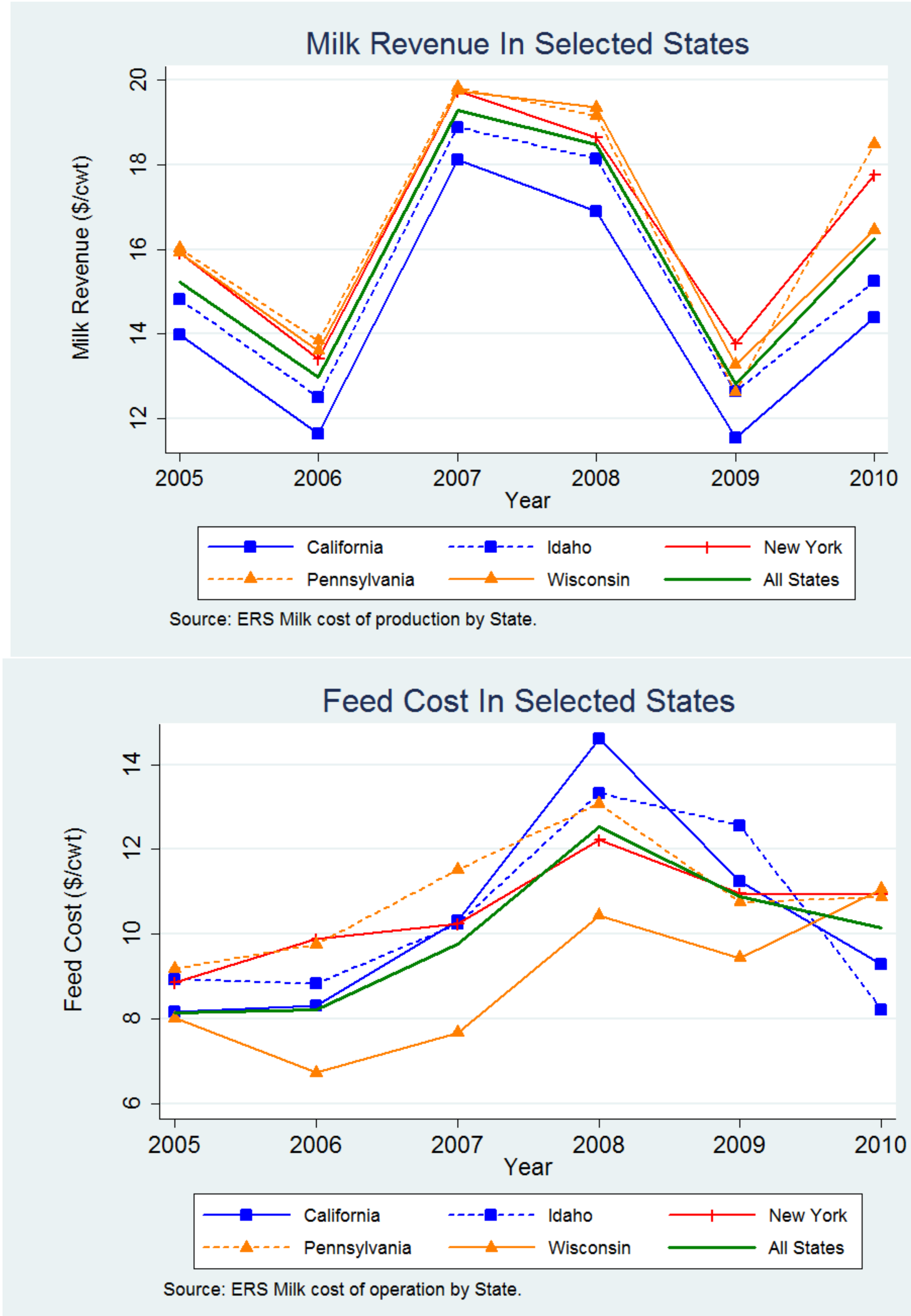

Figure 2.2: Milk Price and Feed Cost for 2005-2010 By Size

Chapter 3: A Difference in Distance-Functions (DDF) Approach to Production Heterogeneity: Application to Technical Change Measurement

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Abstract

This paper proposes a new approach to attributing observed production heterogeneity to the shift of a technological frontier (i.e., technical change) and the shift of technical efficiency (i.e., technical efficiency change) by extending the method of Malmquist Productivity Index (MPI) decomposition into a regression framework. The method, named Difference in Distance-Functions (DDF) approach, obtains decomposition measures at the sample level, allowing for these measures to be estimated from unbalanced panel data and to be identified while accounting for the influence of non-production factors. An empirical application using data on Maryland dairy operations during 1995-2009 finds an annual 0.60% MPI that decomposes into a 1.29% expansion in the technological frontier and a 0.69% decline in the mean technical efficiency. Farm ownership and off-farm income are associated with 4.48% higher and 5.78% lower technical efficiency respectively. The second version of the TC measurement, derived from hypothetical technical efficiency assessments in the spirit of the standard MPI calculations as opposed to a typical econometric formulation, yields an annual technical change of 2.17%. Increasing summer rainfall, winter temperature, and summer temperature by one standard deviation suggests 3.18%, 3.23%, and -3.46% shifts in the technological frontier respectively. The DDF approach appears to perform best among the alternative specifications which include the two-stage DEA of Simar and Wilson (2007), two-stage SFA, and pooled SFA.

Keywords: Data Envelopment Analysis, Technical Change, Productivity Index Decomposition, Sources of Frontier Gaps, Determinants of Technical Inefficiency, Agricultural Economics

JEL *Codes*: O33, C14, Q10

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3.1 Introduction

Understanding production heterogeneity is essential to policy-making for any given industry. This study analyzes production heterogeneity in two dimensions: *production contexts* such as the timing or the location of production decision and *non-production factors* such as non-input-output characteristics of the producer, the market, or the nature of production contexts. Research in the former is most frequently associated with assessing technical change, or the intertemporal shift of a technological frontier, while that in the latter with identifying the determinants of technical inefficiency. It is relatively simple to study the two dimensions of production heterogeneity simultaneously in a parametric frontier framework such as Stochastic Frontier Analysis (SFA). Yet, a typical application under Hicks-neutrality implies highly restrictive assumptions on the marginal rate of transformations (MRT's: i.e., shape of a frontier) and the trajectory of frontier shifts. In general, there is little theoretical ground for such explicit assumptions over the functional forms (i.e., technological frontier, technical inefficiency, and their intertemporal-shift structures), the distribution of the composite error (i.e., technical inefficiency and stochastic noise components), and the complex interactions between them.

On the other hand, for the nonparametric frontier framework such as Data Envelopment Analysis (DEA), there is no such integrated approach to production heterogeneity, and the literature reflects a disconnect between the two types of analyses. This study fills the gap.

On the first dimension of production heterogeneity above, a prominent nonparametric application is the decomposition of Malmquist Productivity Index (MPI) into technical change (TC) and technical efficiency change (TEC), first conceptualized by Nishimizu and Page (1982) and Caves et al. (1982a) and later adopted under DEA by Färe et al. (1994). MPI is a generalization of the T́ornqivist index that admits technical inefficiency (Färe et al., 1994) and is closely related to Fisher's productivity index and the like (e.g., see Grosskopf, 2003). The common MPI decomposition using DEA proceeds in two stages, where the researcher estimates time-specific technological frontiers in the first stage and analyzes the relationships among the technical efficiency measures against those frontiers in the second stage, yielding *producer-level* decomposition measures. Its advantage is the general treatment on the MRT's and their intertemporal structure, as demonstrated in empirical works such as Färe et al. (1994), Kumar and Russell (2002), Timmer and Los (2005), and Färe et al. (2006). Its disadvantage is that when summarized at the sample level, the overall TEC and TC in the existing method may be biased due to the failure to utilize the entire dataset (i.e., unbalanced panel data Kerstens and Van de Woestyne, 2014) or to account for potentially confounding, non-production factors. This study extends the MPI decomposition under a regression framework to overcome these drawbacks.

The second dimension of production heterogeneity, or the interaction mechanism between production decisions and non-production factors, is perhaps where the nonparametric frontier estimation is most frequently employed. Numerous studies have examined the determinants of technical inefficiency in the form of a two-stage DEA analysis (e.g., see Coelli, 2005; Fried et al., 2008), in which observed input-output bundles are first evaluated for technical efficiency and then the predicted technical inefficiencies are regressed on non-production factors (cast as the shifters for the underlying distribution of technical inefficiency). This procedure has been criticized for its lacking a clear relationship with the data generating process, and as such for the poor statistical inferences of its coefficient estimates in the second stage. Today, its leading statistical interpretation is found in Simar and Wilson (2007) where its use is still cautioned (e.g., Simar and Wilson, 2011).

This study develops a unified, nonparametric frontier framework to study the above two dimensions of production heterogeneity. The proposed Difference in Distance-Functions (DDF) approach is an analysis of between-frontier and within-frontier efficiency measures that can also estimate their intertemporal trends and correlations with non-production factors. It is a regression-based MPI decomposition as well as an extended two-stage DEA analysis under a multiple-frontier setting. Defining *sample-level* decomposition measures facilitates overcoming the above shortcomings of the standard MPI decomposition, while setting the marginal effects (of non-production factors) *proportional* to the level of technical inefficiency allows for a coherent statistical interpretation of the two-stage DEA estimation.

The most significant implication of the DDF approach is that the general methodology of MPI decomposition, rooted in the traditional economic theory of production such as Samuelson and Swamy (1974), Diewert (1976), and Caves et al. (1982b), is shown to be empirically more flexible and accessible. Indeed, this inherently general treatment is more widely applicable for analyzing production heterogeneity with respect to varying production contexts such as diverse geographical regions or different phases of policies and regulations.

The study proceeds as follows. Section 3.2 introduces the employed concept of frontier comparisons, describes the proposed DDF approach, and derives an additional technique to compare frontiers. An empirical application to Maryland dairy production is presented in section 3.3, followed by conclusions in section 3.4.[1]

3.2 The Model

3.2.1 Preliminaries

A production *technology* is a set of feasible input-output bundles, and the boundary of this set is referred to as a *technological frontier* or simply *frontier*. The time-specific technology in time period t is denoted by $F_t = \{\forall(\boldsymbol{x},\boldsymbol{y}) \in \mathbb{R}^L_+ \times \mathbb{R}^M_+\colon \boldsymbol{x}$ can produce $\boldsymbol{y}$ in time period $t\}$. For each period t, F_t is assumed to satisfy the following properties: (a) feasible inaction ($(\mathbf{0},\mathbf{0}) \in F_t$), (b) monotonicity ($(\boldsymbol{x},\boldsymbol{y}) \in F_t$, $(-\boldsymbol{x}',\boldsymbol{y}') \leq (-\boldsymbol{x},\boldsymbol{y}) \Rightarrow (\boldsymbol{x}',\boldsymbol{y}') \in F_t$), and (c) convexity ($(\boldsymbol{x},\boldsymbol{y}),(\boldsymbol{x}',\boldsymbol{y}') \in F_t$, $\lambda \in [0,1]$ $\Rightarrow \lambda(\boldsymbol{x},\boldsymbol{y}) + (1-\lambda)(\boldsymbol{x}',\boldsymbol{y}') \in F_t$). The union of such time-specific technologies is referred to as meta-technology $F = \cup_t F_t$, or a technology that envelops subsample-specific technologies (e.g., Bhattacharjee, 1955; Griliches, 1964; Salter, 1966; Krueger, 1968; Hayami and Ruttan, 1970).

Consider an empirical representation by dataset $\{(\boldsymbol{x}_{it},\boldsymbol{y}_{it})\}_{it\in\mathbb{IT}}$ for observation $it \in \mathbb{IT} = \{11,..1T,..,IT\}$ of producer $i \in \mathbb{I} = \{1,..,I\}$ and time period $t \in \mathbb{T} = \{1,..,T\}$. From a subsample of observations in $\mathbb{IT}(k) = \{it \in \mathbb{IT}|\ t = k\}$, F_t is approximated by DEA under non-increasing returns to scale (NIRS) (i.e., a free-

[1] Working paper version of this paper also contains (A) an appendix section on the joint modeling of a technological frontier and technical inefficiency, (B) a simple extension to group-specific MPI decompositions, related to (ODonnell et al., 2008; Chen and Yang, 2011), (C) additional illustrations for the second version of TC measurement (cross-period TC), (D) a supplementary discussion on the comparison of frontiers and mean efficiency levels, and (E) complete estimation tables. Estimation codes are also available upon request.

disposable convex hull including the origin);

$$\forall k \in \mathbb{T},\ \widehat{F}_k = \{(\boldsymbol{x}', \boldsymbol{y}') \in \mathbb{R}^L_+ \times \mathbb{R}^M_+ :\ \sum_{j \in \mathbb{IT}(k)} \lambda_j \leq 1,$$
$$\sum_{j \in \mathbb{IT}(k)} \lambda_j \boldsymbol{x}_j \leq \boldsymbol{x}',\ \sum_{j \in \mathbb{IT}(k)} \lambda_j \boldsymbol{y}_j \geq \boldsymbol{y}',\ \boldsymbol{\lambda} \in \mathbb{R}^{N_k}_+\}, \tag{3.1}$$

which also yields meta-technology $\widehat{F} = \cup_{t \in \mathbb{T}} \widehat{F}_t$.

For given decision $(\boldsymbol{x}_0, \boldsymbol{y}_0)$, the distance function of Farrell (1957) defines the output-oriented, radial technical efficiency (TE) against time-specific technology F_t and the meta-technical efficiency (MTE) against meta-technology F;

$$TE(\boldsymbol{x}_0, \boldsymbol{y}_0; t) = \inf\{\phi :\ (\boldsymbol{x}_0, \boldsymbol{y}_0/\phi) \in F_t\},$$
$$MTE(\boldsymbol{x}_0, \boldsymbol{y}_0; \mathbb{T}) = \inf\{\phi :\ (\boldsymbol{x}_0, \boldsymbol{y}_0/\phi) \in F\} \tag{3.2}$$

where efficiency measures TE and MTE represent the maximum, proportional expansions of outputs in the corresponding technologies F_t and F. Measures TE and MTE take values in $(0, 1]$ if the decision $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is technically feasible, with the value of 1 being fully technically-efficient. TE can be greater than 1 when the decision in a given time period outperforms the frontier of previous period. Estimates $\widehat{F}_t$ and $\widehat{F}$ allow for calculating feasible measures $\widehat{TE}(\boldsymbol{x}_0, \boldsymbol{y}_0; t)$ and $\widehat{MTE}(\boldsymbol{x}_0, \boldsymbol{y}_0; \mathbb{T})$. Note that specification (3.1) implies $\widehat{MTE}(\boldsymbol{x}_i, \boldsymbol{y}_i; \mathbb{T}) = \inf_{t \in \mathbb{T}}\{\widehat{TE}(\boldsymbol{x}_i, \boldsymbol{y}_i; t)\}$.[2]

The ratio of the two efficiency scores in (3.2) provides a technology gap ratio (TGR) that measures the relative distance between the meta-frontier and the subsample-specific frontier (Battese, 2002; Battese et al., 2004). TGR for period t at decision $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is;

$$TGR(\boldsymbol{x}_0, \boldsymbol{y}_0; t) = MTE(\boldsymbol{x}_0, \boldsymbol{y}_0; \mathbb{T}) / TE(\boldsymbol{x}_0, \boldsymbol{y}_0; t), \tag{3.3}$$

which represents the pseudo-technical efficiency of a subsample-specific frontier F_t relative to the meta-frontier F along the ray $(\boldsymbol{x}_0, \lambda \boldsymbol{y}_0)$, $\lambda \in \mathbb{R}$. Empirical applications include ODonnell et al. (2008),Chen and Song (2008), and Moreira and Bravo-Ureta

[2] $\widehat{F} = \cup_t \widehat{F}_t$ implies $\widehat{MTE}(\boldsymbol{x}_0, \boldsymbol{y}_0; \mathbb{T}) = \inf\{\phi :\ (\boldsymbol{x}_0, \boldsymbol{y}_0/\phi) \in \cup_t \widehat{F}_t\} = \inf_t\{\inf\{\phi :\ (\boldsymbol{x}_0, \boldsymbol{y}_0/\phi) \in \widehat{F}_t\}\} = \inf_t\{\widehat{TE}(\boldsymbol{x}_0, \boldsymbol{y}_0; t)\}$.

(2010).

Figure 3.1 depicts how decision $(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0})$ at point A is projected to meta- and time-specific technologies $\widehat{F} = \widehat{F}_{t1}$ and $\widehat{F}_{t0}$ at points B and C respectively. The projection of point A to the horizontal axis is denoted as point Q. The efficiency measures in (3.2) lead to $\widehat{TE}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t) = AQ/CQ$ evaluated against $\widehat{F}_{t0}$ and $\widehat{MTE}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; \mathbb{T}) = AQ/BQ$ evaluated against $\widehat{F}$, yielding $\widehat{TGR}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t) = CQ/BQ$.

The central idea in this study is to derive a measure of technical change from the intertemporal changes in TGR's. MTE, a productivity measure comparable across time periods, can be decomposed into the within-time technical efficiency $TE(.;t)$ (i.e., the distance between the observed decision and the time-specific frontier) and the between-time technological gap $TGR(.;t)$ (i.e., the distance between the time-specific frontier and the meta-frontier). Since the intertemporal shift of this productivity measure is analogous to the Malmquist productivity index (MPI), the intertemporal shifts in efficiencies $TE(.;t)$ and in frontier gaps $TGR(.;t)$ can be cast as alternative decomposition measures for technical efficiency change and technical change respectively. The next subsection briefly reviews the standard MPI decomposition and then describes this new decomposition method.

3.2.2 Regression-Based Malmquist Productivity Index and Technical Change Measurement

The Malmquist productivity index (MPI) by Caves et al. (1982a,b) compares the efficiency measures of observations from two different time periods, say $\{t0, t1\}$, using the frontier of either period as a baseline. The calculation involves efficiency assessments under hypothetical production contexts in the sense that decision $(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1})$ in period $t1$ is evaluated against the technology of period $t0$ or *vice versa*. The change in productivity with baseline of $t0$ or $t1$ is;

$$\begin{aligned} MPI_{t0}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= TE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t0)/TE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t0) \\ MPI_{t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= TE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t1)/TE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t1), \end{aligned} \tag{3.4}$$

which are commonly combined into the geometric mean of the two (Färe et al., 1994);

$$MPI_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) = [MPI_{t0}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) \cdot MPI_{t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1})]^{1/2}. \tag{3.5}$$

The MPI can be decomposed into technical efficiency change (TEC) and technical change (TC);

$$\begin{aligned} MPI_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= TEC_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) \cdot TC_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) \\ TEC_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= TE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t1)/TE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t0) \\ TC_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= \left(\frac{TE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t0)}{TE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t1)} \frac{TE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t0)}{TE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t1)} \right)^{1/2}. \end{aligned} \tag{3.6}$$

In these definitions, TEC is the ratio of technical efficiency measurements for the observed decisions in two time periods, where each decision is evaluated against the corresponding time-specific frontier. Also, TC is the geometric mean of the relative distances between the two frontiers along two rays $(\boldsymbol{x}_{t0}, \lambda_0 \boldsymbol{y}_{t0})$ and $(\boldsymbol{x}_{t1}, \lambda_1 \boldsymbol{y}_{t1})$, $\forall \lambda_0,\ \lambda_1 \in \mathbb{R}_+$.

Figure 3.1 illustrates these measurements in the single-input, single-output space. Two decisions in periods $\{t0,\ t1\}$ are plotted at points A and A'. Point A is projected to two time-specific frontiers $\widehat{F}_{t0}$ and $\widehat{F}_{t1}$ at points B and C, and so is point A' at points B' and C'. The projections of points A and A' to the input axis (i.e., zero-outputs) are labeled as Q and Q' respectively. According to (3.6), the MPI, TEC, and TC for the two decisions are;

$$\begin{aligned} \widehat{MPI}_{t0,t1} &= \left[\frac{A'Q'/C'Q'}{AQ/CQ} \frac{A'Q'/B'Q'}{AQ/BQ} \right]^{1/2}, \\ \widehat{TEC}_{t0,t1} &= \frac{A'Q'/B'Q'}{AQ/CQ},\ \widehat{TC}_{t0,t1} = \left[\frac{BQ}{CQ} \frac{B'Q'}{C'Q'} \right]^{1/2}. \end{aligned} \tag{3.7}$$

Sample-level summaries of MPI, TEC, and TC may be obtained by averaging the corresponding producer-level estimates. For example, given panel data on producers $i = 1, .., I$, the mean TC from time $t0$ to time $t1$ is $E[TC_{t0,t1}] = \frac{1}{I} \sum_i TC_{t0,t1}$ $(\boldsymbol{x}_{i,t0},\ \boldsymbol{y}_{i,t0},\ \boldsymbol{x}_{i,t1},\ \boldsymbol{y}_{i,t1})$. Estimates like this require balanced panel data over the time periods of comparison.

This study proposes a new decomposition method, named the Difference in Distance-Functions (DDF) approach, in which a sample-level MPI decomposition is derived from a regression analysis on estimated technical efficiencies. Specifically, the following system of linear equations are used to analyze estimated distance functions $\hat{q}_{it} \in \{\widehat{MTE}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T}),\ \widehat{TE}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t),\ \widehat{TGR}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)\}$ for the correlations with observation-specific characteristics $\boldsymbol{z}_{it}$, linearly-detrended time-specific characteristics $\boldsymbol{w}_t$, and time-fixed effects τ_t^q without constant terms[3];

$$q \in \{MTE,\ TE,\ TGR\},\ \ \ln \hat{q}_{it} = \tau_t^q + \boldsymbol{z}_{it}\boldsymbol{\alpha}^q + \boldsymbol{w}_t\boldsymbol{\gamma}^q + \varepsilon_{it}^q \tag{3.8}$$

where regression residuals ε_{it}^q are assumed orthogonal to covariates $\boldsymbol{z}_{it}$, $\boldsymbol{w}_t$. Under ordinary least squares (OLS), residuals ε_{it}^q and parameters $\boldsymbol{\theta}^q \in \{\boldsymbol{\alpha}^q,\ \boldsymbol{\gamma}^q,\ \tau_t^q\}$ are linearly decomposed;

$$\varepsilon_{it}^{MTE} = \varepsilon_{it}^{TE} + \varepsilon_{it}^{TGR},\ \ \boldsymbol{\theta}^{MTE} = \boldsymbol{\theta}^{TE} + \boldsymbol{\theta}^{TGR}. \tag{3.9}$$

For example, parameters τ_t^{MTE}, τ_t^{TE}, and τ_t^{TGR} are the time-t regression-means of MTE, TE, and TGR respectively under identity $\tau_t^{MTE} = \tau_t^{TE} + \tau_t^{TGR}$. Also, given the logarithmic transformation applied to distance function $\hat{q}_{it}$, marginal effects $\boldsymbol{\alpha}^q$ and $\boldsymbol{\gamma}^q$ represent proportional shifts in the dependent variable. The simplest case under $\boldsymbol{\alpha}^q = \boldsymbol{\gamma}^q = 0$ reduces to the analysis of variance (ANOVA) for estimating time-specific means.

The use of time-specific variables requires a few additional steps. First, to preserve the linear trends of MTE, TE, and TGR under τ_t^{MTE}, τ_t^{TE}, and τ_t^{TGR}, the linear time trends of these variables should be removed before being used in estimation (3.8). Second, due to the linearity of (3.8), parameters $\boldsymbol{\gamma}^q$ and τ^q need to be indirectly estimated from $\tilde{\tau}_t^q = \boldsymbol{\gamma}^q \boldsymbol{w}_t + \tau_t^q$ where $\tilde{\tau}_t^q$ is the initial estimate of τ_t^q without using variables $\boldsymbol{w}_t$.

Equation (3.8) provides a comparable framework between the standard MPI decomposition above and the following counterpart. Namely, regression-based MPI,

[3] $\tau_t^q \equiv \sum_{k=1}^T \tau_{t,k}^q \mathbb{1}_t(t=k)$ for time-period indicators $\mathbb{1}_t(t=s)$ that take the value of one if $t=s$ and zero otherwise.

TEC, and TC are the differences in time-specific means of MTE, TE, and TGR. For any two time periods $t0$, $t1 \in \{1, .., T\}$,

$$\begin{aligned}&\ln E[MPI_{t0,t1}] \equiv \tau_{t1}^{MTE} - \tau_{t0}^{MTE}, \\ &\ln E[TEC_{t0,t1}] \equiv \tau_{t1}^{TE} - \tau_{t0}^{TE}, \ \ln E[TC_{t0,t1}] \equiv \tau_{t1}^{TGR} - \tau_{t0}^{TGR}\end{aligned} \tag{3.10}$$

where $E[.]$ is the expectation operator over relevant observations. These sample-level measures are analogous to the ratio of means whereas the sample-averages of the standard producer-level counterparts in (3.6) are based on the means of ratios. The two sets of sample-level estimates differ in the method of aggregation but represent the same distance concepts defined among production decisions and frontiers.

Replacing time fixed effects in specification (3.8) with linear trends defines its close variant;

$$q \in \{MTE,\ TE,\ TGR\}, \ \ln \hat{q}_{it} = \beta_0^q + \beta_1^q\, t + \boldsymbol{z}_{it}\boldsymbol{\alpha}^q + \boldsymbol{w}_t\boldsymbol{\gamma}^q + \varepsilon_{it}^q \tag{3.11}$$

where parameter β_1^q yields the direct MPI decomposition given by;

$$\ln E[MPI_{t0,t1}] \equiv \beta_1^{MTE}, \ \ln E[TEC_{t0,t1}] \equiv \beta_1^{TE}, \ \ln E[TC_{t0,t1}] \equiv \beta_1^{TGR}. \tag{3.12}$$

The two sets of decompositions in (3.10) and (3.12) are conceptually equivalent except that specification (3.10) allows for the comparisons between any two periods while specification (3.12) summarizes them as a linear time-trend. Under OLS, point estimates of coefficients $\boldsymbol{\alpha}^q$ and $\boldsymbol{\gamma}^q$ are identical between (3.8) and (3.11).

The advantage of a regression-based MPI decomposition is twofold. First, the ratio-of-means-estimators in equations (3.10) do not require balanced panel data since the time-specific means are calculated without referencing to individual producers. This enables the proposed MPI decomposition to be estimated from unbalanced panel or repeated cross-section data. The capability of utilizing the data on firms that have dropped out of survey or newly emerged as industry leaders is crucial to study an industry with active entries and exits. Second, the measures of MPI, TEC, and TC can be refined by controlling for the potentially-confounding influence of non-production variables on production decisions. This also reveals correlations between the mean MTE, TE, and TGR and observation-specific characteristics such as demographics and skill sets of producers as well as time-specific shocks in weather and market conditions.

In specifications (3.8) and (3.11), DEA consistently identifies time-specific technological frontiers and associated technical efficiency measures in the first stage, while the OLS identifies the shifters of those frontiers and efficiencies in the second stage. Now, these two-stage procedures under DDF must also conform with the so-called separability condition of factors $\boldsymbol{z}_{it}$ from the production possibility F_t. To put it in econometric terms, variables $\boldsymbol{z}_{it}$ are assumed to shift the underlying distribution of technical efficiency $TE(\boldsymbol{x}, \boldsymbol{y}; t)$ without influencing time-specific technical feasibility F_t. This assumption is applicable for any two-stage estimation that eschews a single-stage, joint specification for a technological frontier and technical efficiency, simultaneously using variables $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$. A coherent two-stage estimation must (a) consistently estimate frontier F_t without variables $\boldsymbol{z}_{it}$ in the first stage and (b) cast the second-stage model of technical efficiency as an integral part of some implicit data generating process (DGP) for variables $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$.

In theory, item (a) always holds under the well-known consistency of DEA technology approximations. Empirically, there is no easy way to ensure that the status of full technical efficiency is statistically independent of variables $\boldsymbol{z}_{it}$; incidental correlations may exist without implying the causality. In many situations, it is safe to assume that variables like producer age, education, and experience may affect production decisions without influencing technical feasibility, which is defined at the industry level. In other situations, the concern for potential causality may be well-grounded, for example, when variables $\boldsymbol{z}_{it}$ can be seen as non-discretionary inputs such as weather conditions in agriculture that limit the technical feasibility at a large scale. If these factors are invariant across producers but vary with time, their influence on technological frontiers can be estimated as coefficients $\boldsymbol{\gamma}^{TGR}$. If observation-level variation is available, one can incorporate these variables into a technological specification.

For item (b), one way to ensure a coherent interpretation as the DGP is to employ a functional relationship between efficiency estimate $\widehat{TE}(.; t)$ and variables $\boldsymbol{z}_{it}$

that diminishes as the decision approaches the estimated frontier, or at the full technical efficiency. Conveniently, for specifications (3.8) and (3.11), the marginal effects of environmental factors $\boldsymbol{z}_{it}$ that are specified *proportional* to technical inefficiency must diminish with technical inefficiency and become zero at the full efficiency (i.e., $\ln \widehat{TE}(.;t) = \ln(1) = 0$). Thus, this built-in separability condition does not need to invoke a direct distributional assumption such as a truncated normal distribution in Simar and Wilson (2007).

The relations of specifications (3.8) and (3.11) to the existing methodologies are summarized in table 3.1. Notably, the equation under $q = TE$ is an OLS variant of the common two-stage DEA analysis on the determinants of technical efficiency. In particular, Simar and Wilson (2007)'s model employs a truncated regression using $\widehat{TE}_{it} < 1$ as a dependent variable. With the concepts of meta-frontier and technological gap, their model can be extended to the equations under $q = MTE$ and $q = TGR$.

In the single-output case, substituting first-stage DEA frontiers with SFA frontiers yields variants of the above DDF specifications. For instance, for a given subsample indexed by $\mathbb{IT}(k)$, consider a SFA estimation for $y_{it} = f_t(\boldsymbol{x}_{it})exp(-u_{it}+v_{it})$ under technical efficiency $exp(-u_{it}) \in (0, 1]$ and stochastic noise $exp(v_{it}) \in (0, \infty)$. Once time-specific frontiers $f_t(\boldsymbol{x}_{it})$ are estimated, the composite error term $exp(-u_{it}+v_{it}) = y_{it}/f_t(\boldsymbol{x}_{it})$, say $\widehat{TE}_{it}^{SFA}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)$, can be regressed on variables $\boldsymbol{z}_{it}$ (and $\boldsymbol{w}_t$) and time trends by an OLS or truncated regression in the form of (3.8) or (3.11). Such a two-stage SFA model can be consistent, despite the measurement error in the dependent variable.

Under the Hicks-neutrality of frontier shifts, one could also employ a single-stage, pooled SFA for a technological frontier, technical efficiency, and their time-trends and shifters. For instance, a specification based on Battese and Coelli (1995)'s efficiency-component frontier model is $y_{it} = A_{it} f(\boldsymbol{x}_{it})exp(-u_{it}+v_{it})$ for frontier shifter $A_{it} = exp(\beta_0^f + \beta_1^f\, t + \boldsymbol{z}_{it}\boldsymbol{\alpha}^f + \boldsymbol{w}_t\boldsymbol{\gamma}^f)$ and efficiency component $exp(-u_{it}) = exp(\beta_0^u + \beta_1^u\, t + \boldsymbol{z}_{it}\alpha^u + \boldsymbol{w}_t\boldsymbol{\gamma}^u)$. A similar specification under a nonparametric frontier may be pursued by adding an efficiency component to the corrected convex non-parametric least squares (C2NLS) model of Kuosmanen and Johnson (2008) (i.e., a single-stage quadratic programming problem for a DEA-like frontier and a technical efficiency component with additive and linear marginal effects of non-production factors).

Finally, confidence intervals for statistical inferences are obtained by bootstrapping as follows. Let parameters in equation (3.8) be denoted by $\widehat{\boldsymbol{\Theta}} \equiv [\hat{\boldsymbol{\theta}}^{MTE}\ \hat{\boldsymbol{\theta}}^{TE}\ \hat{\boldsymbol{\theta}}^{TGR}]$ where $\hat{\boldsymbol{\theta}}^{q} = [\hat{\boldsymbol{\alpha}}^{q}\ \ \hat{\boldsymbol{\gamma}}^{q}\ \ \hat{\tau}_t^{q}]$ for each $q \in \{MTE, TE, TGR\}$. Each bootstrap replication $b = 1, ..., B$ yields parameters $\widehat{\boldsymbol{\Theta}}^{*b}$. For given parameter $\hat{\theta}_j \in \widehat{\boldsymbol{\Theta}}$, the distribution of true deviation $\hat{\theta}_j - \theta_j$ can be approximated by the distribution of bootstrap deviation $\hat{\theta}_j^{*b} - \hat{\theta}_j$ through probabilistic bounds;

$$1 - a = Prob[\hat{\nu}_{j,a/2} \leq \hat{\theta}_j - \theta_j \leq \hat{\nu}_{j,1-a/2}] \approx Prob[\hat{\nu}_{j,a/2} \leq \hat{\theta}_j^{*b} - \hat{\theta}_j \leq \hat{\nu}_{j,1-a/2}] \quad (3.13)$$

where $\hat{\nu}_{j,x}$ is the x-percentile value in the distribution of bootstrap deviations $\{\hat{\theta}_j^{*b} - \hat{\theta}_j\}_{b=1}^{B}$. This yields $1 - a$ confidence interval $[\hat{\theta}_j - \hat{\nu}_{j,1-a/2},\ \ \hat{\theta}_j - \hat{\nu}_{j,a/2}]$.

Bootstrapping procedure incorporates implicit distributional assumptions on the error terms. Assuming $\varepsilon_{it}^{q} \sim (0, \sigma_t^{q})\ \forall t \in \mathbb{T}$ for equations $q \in \{MTE, TE, TGR\}$, this study implements the following procedure for each replication $b = 1, ..., B$.

(Bt1) For each observation $ik \in \mathbb{IT}$ in time period k, randomly draw error $\hat{\varepsilon}_{ik}^{TE,*b}$ (with replacement) from the empirical distribution of residuals $\{\hat{\varepsilon}_{it}^{TE}\}_{it\in\mathbb{IT}(k)}$ and define initial bootstrap-estimate $\widetilde{TE}_{ik}^{*b} = \hat{\tau}_k^{TE} + \boldsymbol{z}_{ik}\hat{\boldsymbol{\alpha}}^{TE} + \boldsymbol{W}_k\hat{\boldsymbol{\gamma}}^{TE} + \hat{\varepsilon}_{ik}^{TE,*b}$.

(Bt2) Construct frontiers $\widehat{F}_t^{*b}$ and $\widehat{F}^{*b}$ from pseudo-data $\{\boldsymbol{x}_{it}, \boldsymbol{y}_{it}(\widetilde{TE}_{it}^{*b}\ / \widehat{TE}_{it})\}_{it\in\mathbb{IT}}$ and obtain efficiency scores $\hat{q}^{*b} \in \{\widehat{MTE}^{*b}, \widehat{TE}^{*b}, \widehat{TGR}^{*b}\}$.

(Bt3) Estimate parameters $\hat{\boldsymbol{\Theta}}^{*b}$ by the regression analysis in (3.8).

Step (Bt1) requires that error term ε_{it}^{TE} is randomly distributed in a given period, allowing for potential heteroskedasticity across time. Steps (Bt2) and (Bt3) take the bootstrap data set as given and simply repeat the above two-stage estimation, in which the interdependence across equations $q \in \{MTE, TE, TGR\}$ is introduced through bootstrap frontiers $\widehat{F}_t^{*b}$ and $\widehat{F}^{*b}$.

3.2.3 Alternative TC Measure Using Cross-Period TGR

The basis of any TC estimation is the distance measurement between two frontiers, e.g.,, the mean distance between some *points* along these frontiers. In the above MPI decompositions, the basis of comparison is technical efficiency measurement, or the projections of observed input-output decisions toward the technological frontiers. In the equation for $q = TGR$ from specifications (3.8) and (3.11), each data point $(\boldsymbol{x}_{it}, \boldsymbol{y}_{it})$ is projected to meta-frontier $\widehat{F}$ and time-specific frontier $\widehat{F}_t$, yielding a between-frontier distance measure $\widehat{TGR}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)$. Now, another possibility is to project each data point to the meta-frontier $\widehat{F}$ and *all* time-specific frontiers $\widehat{F}_t$, $t \in \mathbb{T}$, yielding multiple distance measurements $\widehat{TGR}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)$ that involve cross-period technical efficiency assessments $\widehat{TE}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)$ against time-specific technologies $\widehat{F}_k$, $k = 1, .., T$.

The standard MPI, TC, and TEC in (3.6) employ such calculations. Recall that hypothetical efficiency scores are calculated for each production decision $(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}), t \in \{t0, t1\}$ (for given producer i) against each time-specific technology $\widehat{F}_t, t \in \{t0, t1\}$, regardless whether the decision took place in that period. In figure 3.1, four TGRs can be defined;

$$TGR(x_{i\,t0}, y_{i\,t0}; t0) = CQ/BQ, \;\; TGR(x_{i\,t1}, y_{i\,t1}; t1) = B'Q'/B'Q',$$
$$TGR(x_{i\,t1}, y_{i\,t1}; t0) = C'Q'/B'Q', \;\; TGR(x_{i\,t0}, y_{i\,t0}; t1) = BQ/BQ,$$

where the last two TGR measures involve cross-period evaluations for the decision of one time period at the frontier of another. These calculations amount to creating hypothetical "observations" that serve as additional sampling points for a frontier comparison.

In the absence of covariates, the TC in (3.10) from time $t0$ to $t1$ becomes;

$$\ln E[TC_{t0,t1}] = \ln E[TGR(x_{i\,t1}, y_{i\,t1}; t1)] - \ln E[TGR(x_{i\,t0}, y_{i\,t0}; t0)], \tag{3.14}$$

while another way to define TC is to use cross-period TGR's, say cross-period TC (C.TC);

$$\begin{aligned} \ln E[C.TC_{t0,t1}] = &\ln E[(TGR(x_{i\,t1}, y_{i\,t1}; t1)\, TGR(x_{i\,t0}, y_{i\,t0}; t1))^{1/2}] \\ &- \ln E[(TGR(x_{i\,t0}, y_{i\,t0}; t0)\, TGR(x_{i\,t1}, y_{i\,t1}; t0))^{1/2}]. \end{aligned} \tag{3.15}$$

When calculated for a given producer, C.TC measure is equivalent to the producer-specific TC measure in (3.6). For multiple time periods $\mathbb{T}$, C.TC measure is a geometric mean;

$$\ln E[C.TC_{t0,t1}] = \ln E[\,\Pi_{k\in\mathbb{T}} TGR(x_{i\,k}, y_{i\,k}; t1)^{1/T}] - \ln E[\,\Pi_{k\in\mathbb{T}} TGR(x_{i\,k}, y_{i\,k}; t0)^{1/T}] \tag{3.16}$$

where $\Pi_{k\in\mathbb{T}}$ denotes the multiplication operator across time index $k \in \mathbb{T}$.

Formally, the regression-based C.TC measure is obtained from a second-stage analysis. Cross-period TGR (C.TGR) is denoted by $\widehat{TGR}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)$ for the combination of production decisions $\{(\boldsymbol{x}_{it}, \boldsymbol{y}_{it})\}_{it\in\mathbb{IT}}$ and time-specific frontiers $\widehat{F}_k$, $k = 1, .., T$. Variables $\boldsymbol{z}_{it}$ are orthogonal to the C.TGR measures by construction and are hence omitted from the analysis. Then, the regression analyses of C.TGR parallel to equations (3.8) and (3.11) are[4];

$$\ln(\widehat{TGR}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)) = \tau_k^{TGR} + \boldsymbol{W}_k \boldsymbol{\gamma}^{TGR} + \varepsilon_{itk}^{TGR}, \tag{3.17}$$

$$\ln(\widehat{TGR}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)) = \beta_0^{TGR} + \beta_1^{TGR}\, k + \boldsymbol{W}_k \boldsymbol{\gamma}^{TGR} + \varepsilon_{itk}^{TGR} \tag{3.18}$$

where subscript itk represents the unit of observation at the C.TGR level. Intertemporal difference in τ_t^{TGR} yields a C.TC measure; $\ln E[\widehat{TC}_{t0,t1}] = \tau_{t1}^{TGR} - \tau_{t0}^{TGR}$ as shown in equation (3.10) or $\ln E[\widehat{TC}] = \beta_1^{TGR}$ as in equation (3.12). Statistical inferences can be made by a bootstrapping procedure based on implicit time-specific distributions $\varepsilon_{itk}^{TGR} \sim (0, \sigma_k^{TGR}), k \in \mathbb{T}$. The C.TC measure generally differs from the previous, specific-period TC measure ("S.TC" or simply "TC"), particularly when the underlying distribution of input-output systematically shifts from one period to another – possibly as a result of technical change or time-specific shocks that temporarily are correlated with the trend in observed decisions.

[4] The equations for $\ln \widehat{MTE}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})$, $\ln \widehat{TE}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)$ from (3.8) and (3.11) are omitted; equation MTE is redundant under the hypothetical observations, and equation TE no longer bears the interpretation for *technical efficiency*.

3.3 Application

3.3.1 Empirical Context & Data.

The proposed DDF approach is demonstrated with an analysis of Maryland dairy operations. This unbalanced panel dataset comprises revenues and expenses of 63 dairy farms during 1995-2009 (for details see Hanson et al., 2013). Two types of dairy systems are included in this dataset; conventional confinement dairies and management intensive grazing dairies, hereafter referred to as "confinement" and "grazers" respectively. The grazers manage pasture through the frequent rotation of cows between finely partitioned plots, by which they replace substantial portions of feed purchases and on-site crop production. Grazing operations tend to be smaller than their confinement counterparts in terms of both herd size and milk output per cow. The relative profitability of the two systems varies across years and producers, depending on the prices in relevant agricultural markets and the technical efficiencies of individual producers; on average there is no statistically significant difference in profits between the two (Hanson et al., 2013). These dairy systems may be directly comparable in budget analyses but not in production analyses that require the homogeneity of production inputs. Given the different breeds of cows utilized by the confinement and grazers, the following analysis examines the two systems separately.

Milk production is modeled with four inputs: herd size measured in the number of cows, capital equivalent (e.g., aggregate expenditure),[5] and crop and pasture acreages. The statistical properties of these inputs and milk output (hundredweight, cwt) are reported in table 3.2. Similarly, the average production practices are listed by dairy system and calender year in table 3.3. Given the relatively small sample size for DEA, the current application assumes no technological regress by imposing constraint $\widehat{F}_{t0} \subset \widehat{F}_{t1}$ for all $t0$, $t1 \in \mathbb{T}$ with $t0 \leq t1$; under new time-specific index set $\mathbb{IT}(k) = \{it \in \mathbb{IT} |\, t \leq k\}$, time-specific technology F_k is estimated from all decisions observed in period k or earlier.

Table 3.3 exhibits major trends in production decisions for the two dairy sys-

[5]Capital equivalent is a quasi-quantity aggregate-input defined as the total dairy expenditure deflated by an observation-specific production cost index. This cost index is a share-weighted average of cost indices for the corresponding itemized expenses. Price indices are obtained from National Agricultural Statistical Service at USDA.

tems. Average confinement dairy has nearly doubled its milk output from 15,338 cwt in 1995 to 30,399 cwt in 2009, for which the increase mostly stems from the increased scale of operation from 85 cows to 150 cows with a slight increase in output per cow from 183 cwt/cow to 199 cwt/cow. These increases have been matched by a similar increase in capital equivalent input from 255,522 to 504,675 with little changes in land acreages for crop production and pasture at around 300 acres and 50 acres respectively. In contrast, milk output for an average grazing operation has remained stable at around 13,000 cwt during the same period with a slight increase in herd size from 75 cows to 101 cows. Its milk output per cow has declined from 183 cwt/cow to 124 cwt/cow along with the reductions in land acreages from about 100 acres to about 130 acres for crop production and from about 170 acres to about 130 acres for pasture. Importantly, the assumption of equiproportional shifts in production frontiers, the common Hicks-neutral technical change assumption in a single-stage estimation, is unlikely to hold for these trends.

In the following, empirical results are reported in three parts. The first part discusses the estimates for DEA efficiency scores and regression-based MPI decompositions. The second part reports the estimates for the marginal effects of non-production factors. Observation-specific factors comprise the indicators for farm-ownership and off-farm income. Time-specific factors contain season-average rainfalls and temperatures. The third part summarizes current findings and compare results across alternative specifications.

3.3.2 DEA Scores & Regression-based MPI Decompositions.

Table 3.4 shows the summary of DEA results for the meta-technical efficiency (MTE) scores measured against the meta-frontiers, technical efficiency (TE) scores compared to year-specific frontiers, and technology gap ratios (TGR's). These efficiency scores are calculated separately for confinement and grazers under non-increasing returns to scale (NIRS). The parallel analyses under CRS obtain qualitatively and quantitatively similar results.

The median MTE is found at 0.820 for confinement and 0.797 for grazers, indicating that for given inputs, the producers of the median technical efficiencies achieve

82.0% and 79.7% of the maximum output levels relative to their meta-frontiers respectively. Similarly, the median TE estimates suggest that the median-producers among confinement and grazers respectively achieve 89.9% and 85.2% of the maximum output levels relative to their year-specific frontiers. The median (specific-period) TGR finds that the time-specific frontiers of confinement and grazers achieve 95.4% and 100% of the maximum output levels relative to their meta-frontiers at the medians of their technological gaps. Cross-period TGR (C.TGR) is similar to the specific-period TGR except that it is slightly smaller when compared at the 25th, 50th, and 75th percentiles of their distributions. The minimum of the C.TGR is estimated at 0.117 to 0.213, compared to 0.704 to 0.723 of the specific-period counterpart, indicating that some segments of the year-specific frontiers, examined only under C.TGR, are far less efficient than the meta-frontiers.

The second-stage analysis estimates the intertemporal mean differentials in productivity, efficiency, and frontiers, as represented by MTE, TE, and TGR, that provide estimates for regression-based MPI, TEC, and TC respectively. The regression coefficients are reported as point estimates, for which the above bootstrapping procedure provides statistical inferences.[6]

Table 3.5 contains the results for regression-based MPI decompositions in equations (3.8) and (3.10), relative to the 1995 baseline values. The means of MTE, TE, and TGR for confinement have changed respectively by 3.7%, -4.6%, and 8.7% in 2000; 5.2%, -7.5%, and 13.7% in 2005; and 8.7%, -8.9%, and 19.3% in 2009. On the other hand, the estimates for grazers indicate negative productivity change, which is mostly explained by negative TEC; from 1995 to 2009, grazers have experienced -11.7% MPI, -18.4% TEC, and 8.1% (statistically-insignificant) TC. These year-by-year estimates of TEC and TC are fitted to linear trends for the ease of interpretations. Figure 3.2 shows the point estimates, 95% confidence intervals, and fitted linear trends. The fitted trends are virtually equivalent to the linear-trends β_1^q under (3.11) as reported in columns (1) and (5) of table 3.6, or 0.60% MPI, -0.69% TEC, and 1.29% TC per year for confinement and -1.12% MPI, -1.67% TEC, and 0.56% (at the 10% statistical-significance level) TC per year for grazers.

[6] Accounting for cross-equation correlations may result in asymmetric confidence intervals around point estimates.

The estimates for cross-period TC (C.TC) are reported in the last two columns of table 3.5 and in the first column of table 3.8. They obtain 36.3% and 25.9% TC during 1995-2009 for confinement and grazers, resulting in the linear trends of 2.17% and 1.61% per year respectively. While these S.TC and C.TC measures provide substantially different interpretations of the data, it is not so surprising. The primary reason for such an apparent inconsistency is that these TC measures are directional, just as any other distance measure in an input-output space. In assessing technological progress, the S.TC considers the directions of comparisons that are likely the most relevant to producers in each time period. On the other hand, the C.TC indiscriminately accounts for technological progress in the directions of all observed decisions regardless of their timing. The S.TC focuses on the comparisons of the most productive, *observed production decisions* while the C.TC compares *estimated time-specific frontiers* in a time-invariant and more comprehensive set of directions.

3.3.3 Marginal Effects of Observation-specific & Time-specific Factors.

The marginal effects of non-production factors on MTE, TE, and TGR are reported in table 3.6, and the corresponding marginal effects on C.TGR in table 3.8. The results under the DDF approach by equations (3.11) and (3.18) are shown in columns (1) and (5) in both of these tables.

Observation-specific variables contain two indicator variables for farm ownership and off-farm income.[7] In the equation for MTE, the presence of farm ownership and off-farm income are associated with 5.66% higher and 5.73% (statistically insignificant) lower MTE respectively for confinement dairies. Correspondingly, the two variables indicate 10.17% higher and 6.30% lower MTE for grazers. These effects are mostly driven by the shifts in technical efficiency rather than the shifts in technological gaps; farm-ownership and off-farm income are correlated with 4.88% higher and 5.78% lower TE for confinement and 10.13% higher and 5.59% lower TE for grazers, leaving the correlations with TGR nearly zero or imprecise. These results are consistent with conventional economic theory. The owner-operator has higher incentives to work on his farm than a renter-operator whose profits are shared with the

[7] The sample means of the indicators for farm ownership and off-farm income are 0.77 and 0.07 respectively among confinement and 0.71 and 0.21 among grazers.

owner (i.e., avoiding a moral hazard problem).[8] The presence of off-farm income, an indication for a higher opportunity cost of the producer, prohibits full commitment to dairy operations, which would result in the lower optimal effort level at the margin than otherwise.[9]

Turning to time-specific factors, the marginal effects of seasonal-average rainfalls and temperatures are reported in the form of the prediction for a change in each variable by one standard deviation.[10] Generally, greater rainfall and higher temperatures are considered beneficial for dairy operations, with clear exceptions such as winter snowfall or summer droughts hindering plants' growth and cold winters or hot summers increasing animals' energy consumption. Estimated correlations are consistent with this conventional knowledge for economically significant marginal effects (e.g., 2% or higher). These effects, detected only among grazers, include 4.43% higher MTE and 5.66% higher TE for greater summer rainfall, and 2.27% higher MTE and 2.53% higher TE for greater spring temperature, and 3.22% higher TE for greater winter temperature. Other coefficient estimates are not entirely in agreement with the above predictions, but they are economically insignificant (e.g., less than 2%).

The effects on C.TGR are more pronounced and also similar between the two dairy systems, compared to the effects on single-period TGR. The increases in summer rainfall, winter temperature, and summer temperature (by one standard deviation) are associated with 3.18%, 3.23%, and -3.46% changes in the frontier-output level for confinement and, correspondingly, 0.54% (statistically insignificant), 1.57%, and -4.40% changes in the frontier-output level for grazers, as expected from the favorable conditions under moist summers, warm winters, and cool summers. The counterintuitive, negative correlations with spring rain for both systems seem to be explained by the relatively high profits in 1999 and low profits in 2009 that have coincided with the years of low and high spring rainfall respectively.

[8] Anecdotally, dairy producers tend to pay fixed-term cash rents to landlords and hence are less subject to the moral hazard problem than the case of crop producers. Alternatively, the renter-operator may be less willing to invest in on-site production facilities due to potential change of terms in his contract.

[9] The presence of off-farm income may also influence producer's decisions through reducing downside income risks, for which the effect on technical efficiency is theoretically ambiguous.

[10] The means (s.d.) of time-specific weather variables of seasonal rainfall and temperatures for winter, spring, summer, and fall during 1995-2009 are 9.08 (2.89), 11.32 (2.92), 11.83 (3.50), and 11.64 (4.76) for rainfall (inches) and 36.31 (2.60), 53.98 (1.66), 75.2 (1.6), and 57.27 (1.41) for temperatures (degrees Fahrenheit).

It may be noted that imposing the assumption of no technological regress would have introduced some measurement errors in estimating the marginal effects for time-specific variables. The observed positive impacts of the above weather variables are assumed to persist in the time-specific frontiers of subsequent periods and hence tend to be overrepresented. On the other hand, the observed negative impacts of these variables are partially or completely ignored in the concurrent and subsequent time-specific frontiers and hence tend to be underrepresented.

3.3.4 Summary & Comparisons Across Alternative Specifications

During 1995-2009, the average annual productivity change for confinement dairies is 0.60%, comprising -0.69% TEC and 1.29% TC, while that for grazers is -1.12%, comprising -1.67% TEC and 0.56% TC (at the 10% statistically-significance level). C.TC measurements alternatively indicate 2.17% TC and 1.61% TC per year for confinement and grazers. The increasing inefficiencies and positive technical change in both systems suggest that some producers have successfully adopted new technologies and improved their management while others have been struggling to keep up with these changes. The different magnitudes in these trends between the two systems indicate that the technological advances have benefited the majority of confinement operations but few grazing counterparts. This may be related to the fact that confinement dairy operations tend to follow fairly standardized production techniques of the industry (see e.g., Khanal et al. (2010) for recent technological adoption in the US dairy sector), while intensive grazing involves very localized production practices (due to local soil and micro-climate conditions that require experimentations by individual producers). Additionally, the analysis has obtained several results that are consistent with conventional economic theory and real-world production constraints in dairy operation.

The above results are contrasted with the estimates under alternative specifications listed in table 3.1. Tables 3.6 and 3.8 contain the parallel results under four different two-stage estimations, including DEA-OLS in columns (1) and (5), DEA-truncated regression in columns (2) and (6), SFA-OLS in columns (3) and (7), and

SFA-truncated regression models in columns (4) and (8).[11] Statistical inferences are provided by the same bootstrapping procedure adopted in the DDF approach, except that the two truncated regression models include Simar and Wilson (2007)'s bias correction technique for the first-stage frontier estimates.

The results are similar between the models that share the first-stage frontier estimation. That is, the second-stage marginal effects are similar between DEA-OLS and DEA-truncated regression models and between SFA-OLS and SFA-truncated regression models. Under the DEA-truncated regression model (columns (2) and (6)), annual MPI, TEC, and TC are estimated at 0.74%, -1.01%, and 1.75% for confinement and -1.69%, -2.41%, and 0.73% (statistically-insignificant) for grazers respectively, thus fairly similar to the previous DDF estimates. When the first-stage employs SFA, annual MPI, TEC, and TC are 0.46%, 0.56%, and -0.10% (statistically-insignificant) for confinement and -0.30% (statistically-insignificant), -1.75%, and 1.45% for grazers respectively under the SFA-OLS model, to which the SFA-truncated regression obtains qualitatively similar estimates. These SFA results would imply that positive productivity growth among confinement dairies is attributable to the increased technical efficiency rather than technical change (i.e., technological catch-up), while grazers' positive TC and negative TEC cancel out with each other, yielding no significant productivity change. Of these estimates, the findings under the DEA-based models are more in congruence with the conventional view of the positive TC for confinement and some negative TEC for both systems.

The influence of non-production factors shows mixed evidence for proper identification. By focusing on economically-significant marginal effects, the results reveal some similarities with the DDF estimates as well as counterintuitive correlations. Common findings with the DDF (columns (1) and (5)) include the positive effects of farm ownership on TE in columns (2), (3), and (6) and the negative effects of off-farm income on TE for confinement under the SFA models in columns (3) and (4). The same SFA models (columns (7) and (8)) find that for grazers, the farm ownership positively shifts the technological frontier by 21.65% and 18.27% respectively, which

[11] The first-stage SFA is estimated using the observations in a given year (without including other observations from preceding years). For grazers, observations in year 1995 are treated together with those in year 1996 due to insufficient sample size in 1995.

are probably incidental and rather suggest misspecification. For weather variables, summer rainfall is predicted to be positively correlated with TE in columns (3), (6), and (8), similar to the finding under the DDF in column (5). Autumn rainfall is predicted to be positively correlated with the frontier-output level in these SFA models. The negative correlations of spring and autumn rainfall with TE for confinement under the SFA models in columns (3) and (4) are likely spurious. Other marginal effects are either economically insignificant or inconsistent within the models sharing the first-stage frontier estimation by DEA or SFA.

In table 3.8, the marginal effects under C.TC vary between the models. The DDF findings of positive TC's for confinement and grazers (columns (1) and (5)) are supported respectively by the DEA-truncated regression model (column (2)) and the SFA-OLS model (column (7)), while other models detect no significant TC. Based on the relationships between dairy production and weather as noted above, three likely-spurious results are found for grazers: the positive correlations of summer temperature in columns (6) and (8), the negative correlations of summer rainfall in column (7), and the negative correlations of fall temperature in columns (6) and (8).

Table 3.7 shows the results under the single-stage, pooled SFA model. Cobb-Douglas technology is estimated along with proportional marginal effects of frontier shifters and technical-efficiency shifters. The table contains the two sets of results for the restricted model under α^u, $\gamma^u = 0$ and the full model. Technological parameters explain milk output from the relative input contributions of cows, capital, crop acre, and pasture acre, which are approximately 60%, 39%, 1% to and -2% for confinement and 65%, 36%, 3%, and -17% for grazers respectively in the full model. This appears reasonable, but for grazers the relative contributions of cows and capital to the output are sensitive to the inclusion of non-production variables.

Importantly, the highly dominant role of technical inefficiency relative to stochastic noise (i.e., parameter "Eta") suggests that the model identification depends crucially on the specification for the technical efficiency component. Indeed, all parameters (i.e., TC, TEC, and the shifters of frontier and technical efficiency) are sensitive to parametric restrictions of this component. The full model finds that confinement dairies have experienced neither significant TC nor TEC during 1995-2009, while intensive grazing dairies have become less productive on the technological frontier but become more technically efficient relative to that frontier, which is not consistent with any of the findings from the previous two-stage SFA models. Additionally, the marginal effects of non-production factors appear to suffer from spurious correlations.

Overall, the proposed DDF approach appears to perform best among the alternative specifications considered above. Replacing the second-stage OLS regression with a truncated regression (analogous to the model of Simar and Wilson (2007)) obtains similar estimates for the intertemporal trends in technological frontiers and technical efficiency. However, the resulting marginal effects of non-production factors appear less accurate, compared to the DDF estimates. This is particularly apparent in the equations for TGR and C.TGR, perhaps because truncated regressions discard the information that could be gathered from the fully-efficient decisions. Also, substituting the first-stage DEA with a SFA estimation yields drastically different estimation results. The TC and TEC estimates under two-stage SFA models do not conform with the conventional perspective on the US dairy industry. Relaxing the Cobb-Douglas technology assumption may improve estimation results. Lastly, the single-stage pooled SFA model for the same Cobb-Douglas specification is estimated under Hicks-neutral TC. The estimates for TC and TEC are sensitive to the specification of technical efficiency component, whose distribution is found to be more important than the distribution of stochastic noise. The model misspecification is clear from the ill-suited nature of Hicks-neutral TC for the data. The proposed DDF approach is both intuitive and conservative in its assumptions for the technological frontier, technical efficiency measurement, and the intertemporal relationships and also turns out to be the most reliable specification in this application. All the while, further assessments of the models' relative performance are outside the scope of this study and left for future works.

3.4 Conclusions

This study proposes a systematic treatment of contextual and non-production variables in comparing technological frontiers and technical efficiencies. This bridges the gap between the analysis of production contexts and the analysis of non-production factors in the nonparametric frontier literature. The concept of MPI decomposition, originally developed for producer-level technical change measures, is extended for sample-level measures in a regression framework. The generality of this concept suggests a wide range of potential applications in analyzing production heterogeneity for varying production contexts, including before and after policy intervention, in the form of mean context-specific frontier-outputs and mean efficiency levels.

The empirical study of production heterogeneity is no simple matter. The workhorse of modeling is the estimation of a technological frontier and technical efficiency from observed input-output decisions, to which incorporating information on production contexts or non-production factors poses serious challenges in the overall econometric identification. Despite the complexity of the problem, the proposed DDF approach employs an intuitive estimation procedure and based on fairly conservative assumptions. This simple methodology should afford increased attention to the specification choice of production variables, or the input-output space in which production heterogeneity is analyzed through distance measurements.

3.5 Tables and Figures

Table 3.1: Alternative Specifications For Technological Frontiers and Technical Efficiency

Models	Frontier	Efficiency	TC	M.E. in EC	Notes
(M1)	DEA	OLS	Non-Hicksian	Proportional	Proposed DDF Model
(M2)	DEA	Trunc.reg	Non-Hicksian	Linear$^\flat$	Simar and Wilson (2007)
(M3)	SFA	OLS	Non-Hicksian	Proportional	Variant of DDF Model
(M4)	SFA	Trunc.reg	Non-Hicksian	Linear$^\flat$	Variant of DDF Model
(M5)	Pooled C2NLS		Hicks-Neutral$^\sharp$	Linear	Kuosmanen and Johnson (2008)†
(M6)	Pooled SFA		Hicks-Neutral$^\sharp$	Proportional	Battese and Coelli (1995)

1. Marginal effects in the efficiency component (EC) typically enter in the estimation equation as linear or proportional shifts of the input-output distance measure. $\flat$ Either linear or proportional marginal effects may be admitted.

2. $\sharp$ It may be possible to alter a part of the frontier to be non-Hicksian TC.

3. $\dagger$ Admissible marginal effects for an additional efficiency component are likely linear effects.

Table 3.2: Summary Statistics

	Confinement (Obs. 314)				Grazers (Obs. 161)			
Variable	Mean	S.D.	Min	Max	Mean	S.D.	Min	Max
Milk (cwt)	24,145	17,577	3,761	110,668	12,442	5,573	2,670	42,955
Cow	122	76	22	468	87	29	37	195
Output Equiv.	369,033	289,506	56,331	1,917,846	199,108	85,553	59,487	696,891
Capital Equiv.	416,037	308,346	70,637	1,780,881	204,625	91,698	58,246	645,498
Total Acre	338	160	90	845	283	134	115	700
Crop Acre	289	155	60	704	132	108	0	600
Pasture Acre	50	39	0	141	152	60	53	280

Unit “cwt” stands for hundredweight (i.e., 100 pounds). Output equivalent is the gross income deflated by the observation-specific price index. Capital equivalent is the total cost of production, deflated by a farm production cost index. For more information on the dataset, see Hanson et al. (2013).

Table 3.3: Average Production Decisions By Dairy System and Year

Year	N.Obs	Output Eq.	Milk (cwt)	Cow	Capital Eq.	Tot.Acre	Cro.Acre	Pas.Acre
Confinement								
1995	21	222,173	15,338	85	255,522	328	273	55
2000	21	358,828	24,649	121	403,807	340	292	48
2005	22	397,284	27,628	137	498,483	348	297	51
2009	19	534,353	30,399	150	504,675	369	316	53
Grazers								
1995	4	183,251	13,534	75	207,129	368	195	173
2000	11	225,735	13,270	85	215,347	295	130	164
2005	12	181,581	11,076	84	181,681	254	109	144
2009	12	244,684	13,168	101	210,822	273	138	135

See supplemental materials for the values for complete years 1995-2009.

Table 3.4: Summary of DEA Efficiency and TGR Scores

	System	Summary Statistics					
		Min	25th	Median	Mean	75th	Max
A. Efficiency at meta-frontiers (MTE)							
(1)	Confinement	0.408	0.764	0.820	0.827	0.902	1.000
(2)	Grazers	0.362	0.698	0.797	0.796	0.927	1.000
B. Efficiency at year-specific frontiers (TE)							
(3)	Confinement	0.465	0.807	0.899	0.884	0.978	1.000
(4)	Grazers	0.362	0.715	0.852	0.822	0.951	1.000
C. Specific-Period Technology Gap Ratios (TGR)							
(5)	Confinement	0.704	0.903	0.954	0.937	0.988	1.000
(6)	Grazers	0.723	0.959	1.000	0.971	1.000	1.000
D. Cross-Period Technology Gap Ratios (C.TGR)							
(7)	Confinement	0.117	0.884	0.950	0.918	0.991	1.000
(8)	Grazers	0.213	0.922	0.993	0.935	1.000	1.000

1. Technical efficiencies (TE) and meta-technical efficiency (MTE) are measured against year-specific frontiers and meta-frontiers respectively. Technology gap ratio (TGR) is the ratio of those efficiency measurements (i.e., MTE/TE) at observation level.

2. Cross-Period TE measures include the pseudo-technical efficiency scores where observed input-output decisions are evaluated against frontiers of different time periods.

3. The results under Constant Returns to Scale are nearly identical.

Table 3.5: Regression-Based MPI decompositions

	MPI		TEC		TC		C.TC	
Year	Est.	95% CI	Est.	95% CI	Est.	95% CI	Est.	95% CI
Confinement								
1996	0.980‡	(0.980, 0.987)	0.977‡	(0.961, 0.994)	1.003	(0.990, 1.024)	1.021	(0.997, 1.049)
2000	1.037‡	(1.035, 1.050)	0.954‡	(0.912, 0.952)	1.087‡	(1.094, 1.146)	1.156‡	(1.141, 1.173)
2005	1.052‡	(1.046, 1.075)	0.925‡	(0.872, 0.931)	1.137‡	(1.132, 1.222)	1.254‡	(1.233, 1.277)
2009	1.087‡	(1.075, 1.117)	0.911‡	(0.835, 0.916)	1.193‡	(1.182, 1.322)	1.363‡	(1.333, 1.397)
Grazers								
1996	0.974‡	(0.966, 0.982)	0.988	(0.973, 1.058)	0.986†	(0.918, 0.998)	1.043‡	(1.017, 1.065)
2000	0.929‡	(0.908, 0.940)	0.898‡	(0.859, 0.948)	1.035	(0.981, 1.073)	1.102‡	(1.087, 1.118)
2005	0.889‡	(0.861, 0.916)	0.843‡	(0.796, 0.954)	1.055	(0.944, 1.106)	1.202‡	(1.173, 1.235)
2009	0.883‡	(0.834, 0.920)	0.816‡	(0.754, 0.926)	1.081	(0.964, 1.141)	1.259‡	(1.228, 1.295)

1. Statistical significance: ‡ $\alpha = 0.01$, † $\alpha = 0.05$, * $\alpha = 0.1$.

2. Regression-based MPI, TEC, TC, and C.TC are calculated relative to the coefficients (1.000) for year 1995.

3. The model controls for observation-specific factors (“$\boldsymbol{z}_{it}$”) and time-specific factors (“$\boldsymbol{w}_t$), or indicator variables for farmland ownership (for dairy operation) and off-farm income and season-average rainfalls and temperatures.

Table 3.6: Marginal Effect Estimates in the Second Stage across Alternative Specifications

	Confinement				Grazers			
First Stage:	DEA		SFA		DEA		SFA	
Second Stage:	OLS (1)	Trunc.♭ (2)	OLS (3)	Trunc. (4)	OLS (5)	Trunc.♭ (6)	OLS (7)	Trunc. (8)
Equation MTE								
Year (MPI/year)	0.60‡	0.74‡	0.46‡	0.79†	-1.12‡	-1.69‡	-0.30	-0.31
Farm ownership	5.66‡	6.43†	2.96‡	4.79	10.17‡	12.90‡	23.20‡	26.64‡
Off-farm income	-5.73	-4.74	-1.94‡	-0.91‡	-6.30‡	-6.03	2.50	4.98
Rainfall winter	-0.73‡	-0.87	-0.56	-0.91	1.15‡	2.54	1.74*	2.67
Rainfall spring	-0.12	-0.10*	-0.39‡	-0.75*	0.09	1.15	1.60†	1.37
Rainfall summer	0.32‡	0.30	1.61‡	2.53*	4.43‡	8.19*	2.19*	3.31
Rainfall fall	0.01	-0.17	-0.82†	-1.32‡	-1.92‡	-2.53	0.06	-0.59
Temp. winter	1.00‡	1.02	1.31	2.20	1.62‡	3.85	1.31	1.79
Temp. spring	-0.20†	-0.26	0.15	0.02	2.27‡	4.67	2.55‡	3.73
Temp. summer	-0.80‡	-0.79	0.26	0.07*	1.51‡	2.48	-1.31	-0.88
Temp. fall	-0.51‡	-0.64	-1.03	-1.17*	-0.22	-0.15	-0.85†	-0.60
Constant	21‡	-1458‡	-944‡	-1585*	-184‡	3011‡	513	446
Equation TE								
Year (TEC/year)	-0.69‡	-1.01‡	0.56‡	0.87†	-1.67‡	-2.41‡	-1.75‡	-6.90‡
Farm ownership	4.48‡	4.59†	2.51‡	3.55	10.13‡	14.09‡	1.55	8.37
Off-farm income	-5.78‡	-5.26	-5.05‡	-14.09‡	-5.59‡	-5.14	-2.48	-1.48
Rainfall winter	-0.42	-0.97	0.62	1.05	1.42*	3.38	-3.38†	-3.36
Rainfall spring	1.75‡	2.75*	-2.25‡	-7.70*	0.19	-1.52	-3.78	-3.84
Rainfall summer	-1.07‡	-1.61	4.35‡	9.96*	5.66‡	11.80†	4.00	28.48*
Rainfall fall	0.77‡	1.12	-2.87†	-9.41‡	-1.22	-2.54	-2.22	-10.77*
Temp. winter	-0.05	-0.71	-0.74	-3.96	3.22‡	9.70*	2.92	12.35
Temp. spring	-0.42†	-0.80	1.26	1.65	2.53‡	4.35	-1.42‡	4.68
Temp. summer	0.58	0.97	3.65	9.91*	0.46	-4.59	-3.46	6.38
Temp. fall	-0.65	-0.96	-1.68	-6.74*	0.73‡	2.27	5.82‡	7.57
Constant	1	2026‡	-1257‡	-1880*	-207*	4598‡	3416‡	12862‡
Equation TGR								
Year (TC/year)	1.29‡	1.75‡	-0.10	-0.08	0.56*	0.73	1.45‡	6.60‡
Farm ownership	1.18*	1.84	0.44	1.23	0.04†	-1.19	21.65‡	18.27‡
Off-farm income	0.06	0.52	3.12	13.18	-0.71	-0.90	4.97	6.46
Rainfall winter	-0.31	0.10	-1.18	-1.95	-0.27	-0.84	5.12†	6.03
Rainfall spring	-1.87‡	-2.85	1.87	6.95	-0.10	2.66	5.38	5.21
Rainfall summer	1.39‡	1.92	-2.74*	-7.44	-1.24‡	-3.60	-1.81	-25.17
Rainfall fall	-0.76‡	-1.29	2.05†	8.08†	-0.70‡	0.01	2.27	10.18*
Temp. winter	1.05†	1.73	2.05	6.16	-1.60‡	-5.85	-1.61	-10.56
Temp. spring	0.22	0.54	-1.10	-1.63	-0.26	0.32	3.97‡	-0.96
Temp. summer	-1.38	-1.76	-3.39	-9.84*	1.06‡	7.07	2.15	-7.25
Temp. fall	0.14	0.32	0.66	5.57	-0.95‡	-2.43	-6.67‡	-8.17
Constant	20	-3484‡	314	295	23	-1587	-2904‡	-12416‡

1. Statistical significance: ‡ α=0.01, † α=0.05, * α=0.1. ♭ Estimation adopts Simar and Wilson (2007)'s bias correction for DEA efficiency scores. Constant coefficients are rounded.

2. Marginal effects of these weather variables are reported for the change by one standard deviation.

Table 3.7: Pooled SFA Analysis Under Hicks-Neutral Technical Change

	M.E. (in Percentage) Confinement (N=314)				Grazers (N=161)			
	(1)		(2)		(3)		(4)	
	Estimate	S.D.	Estimate	S.D.	Estimate	S.D.	Estimate	S.D.
Frontier								
Intercept	274‡	(63)	186	(130)	69	(141)	411†	(127)
Year (TC/Year)	0.28*	(0.13)	0.96	(1.45)	-0.46	(0.39)	-0.68*	(0.28)
log(Cow)	63.77†	(3.70)	59.49‡	(4.64)	46.76‡	(8.52)	65.45‡	(8.27)
log(Capital)	34.02	(3.45)	39.41‡	(7.42)	59.92‡	(6.18)	35.93‡	(6.07)
log(Crop Acre)	3.85*	(1.64)	1.17	(6.96)	0.94	(0.76)	2.76‡	(0.72)
log(Pasture Acre)	-1.84	(0.41)	-1.90*	(0.96)	-19.00‡	(3.79)	-17.27‡	(2.15)
Farm ownership	3.75†	(1.32)	4.82.	(2.52)	0.58	(3.30)	-25.69‡	(4.76)
Off-farm income	-8.70‡	(2.20)	-22.48‡	(2.66)	-6.30.	(3.34)	-36.55‡	(4.54)
Rainfall winter	-0.51	(0.74)	0.01	(0.63)	0.91	(1.68)	1.65	(1.35)
Rainfall spring	0.12	(1.03)	-3.21	(2.61)	2.88	(2.36)	0.68	(1.83)
Rainfall summer	0.89	(1.14)	5.87‡	(1.66)	-0.67	(2.65)	-1.98	(1.98)
Rainfall fall	0.26	(0.68)	-3.49	(3.72)	-0.40	(1.58)	0.05	(1.30)
Temp. winter	1.51	(1.10)	2.65.	(1.50)	-0.56	(2.48)	-0.48	(2.04)
Temp. spring	-0.02	(0.73)	0.61	(0.69)	1.56	(1.62)	0.03	(1.44)
Temp. summer	-0.69	(1.21)	0.79	(0.67)	1.47	(2.92)	-0.47	(2.36)
Temp. fall	-0.44	(0.82)	-1.44.	(0.84)	-1.72	(1.90)	-0.99	(1.48)
Technical Efficiency								
Intercept	-17624	(39660)	-32	(63)	-934	(1502)	343	(412)
Year (TEC/Year)	-271.89	(622.23)	1.00	(4.42)	57.25	(88.60)	4.24‡	(0.94)
Farm ownership			2.24	(9.19)			-50.02‡	(7.00)
Off-farm income			-46.52	(33.36)			-6390	(95466)
Rainfall winter			0.91	(2.27)			1.90	(3.78)
Rainfall spring			-6.26‡	(1.82)			0.84	(5.64)
Rainfall summer			7.55†	(2.49)			-8.89	(6.12)
Rainfall fall			-5.05*	(2.28)			2.44	(3.89)
Temp. winter			3.04‡	(0.17)			-2.30	(6.57)
Temp. spring			0.85	(1.10)			-4.44	(4.72)
Temp. summer			-0.24	(1.38)			-2.77	(7.86)
Temp. fall			-0.43	(2.81)			-0.44	(4.13)
SFA parameters								
Sigma Square	16.850	(37.959)	0.018*	(0.008)	0.416	(0.597)	0.046‡	(0.008)
Eta	1.000*	(0.001)	0.998‡	(0.289)	0.967‡	(0.048)	0.923‡	(0.031)
Log Likelihood	278.2		283.48		66.35		95.74	
Mean TE	0.921		0.832		0.893		0.786	

1. Statistical significances: ‡ $\alpha = 0.01$, † $\alpha = 0.05$, * $\alpha = 0.1$. Intercepts and some large coefficient estimates are rounded.

2. Linear trends of weather variables are removed before they are used in the second-stage DEA regression analysis. Reported marginal effects of these weather variables are linearly extrapolated by multiplying with one standard deviation in each variable.

3. Output is log(Milk). "Sigma square (σ^2)" and "Eta (η)" define the joint distribution of technical inefficiency $u \sim N^+(\mu, \sigma_u^2)$ and stochastic noise $v \sim N(0, \sigma_v^2)$ where $\sigma_u^2 = \sigma^2\eta$, $\sigma_v^2 = \sigma^2(1-\eta)$.

Table 3.8: Marginal Effect Estimates in the Second Stage across Alternative Specifications (C.TC)

	Confinement				Grazers			
First Stage:	DEA		SFA		DEA		SFA	
Second Stage:	OLS (1)	Trunc. (2)	OLS (3)	Trunc. (4)	OLS (5)	Trunc. (6)	OLS (7)	Trunc. (8)
Equation Cross-Period TGR								
Year (C.TC/year)	2.17‡	1.42‡	-0.17	1.42‡	1.61‡	0.31	1.03‡	0.31
Rainfall winter	0.16	0.12	-0.03	0.12	0.14	0.01	0.21	0.01
Rainfall spring	-3.92‡	-0.51	-0.76	-0.51	-3.96‡	0.06	1.25*	0.06
Rainfall summer	3.18‡	-0.30	0.46	-0.30	0.54	-2.35	-2.39‡	-2.35
Rainfall fall	-1.40‡	0.31	-0.58	0.31	-0.69‡	-2.21	0.94	-2.21
Temp. winter	3.23‡	0.44	0.39	0.44	1.57‡	-3.82	-0.71	-3.82
Temp. spring	0.46	-0.07	-0.07	-0.07	-1.56‡	-1.56	-0.75	-1.56
Temp. summer	-3.46‡	-0.91	-0.16	-0.91	-4.40‡	7.54†	-1.50*	7.54†
Temp. fall	0.95‡	0.44	0.01	0.44	-0.55†	-4.32‡	-0.77	-4.32‡
Constant	30.43	-2820.34‡	332.13	-2820.34‡	286.54‡	-673.68	-1964.61‡	-673.68

1. Statistical significance: ‡ α=0.01, † α=0.05, * α=0.1.
2. Marginal effects of these weather variables are reported for the change by one standard deviation.

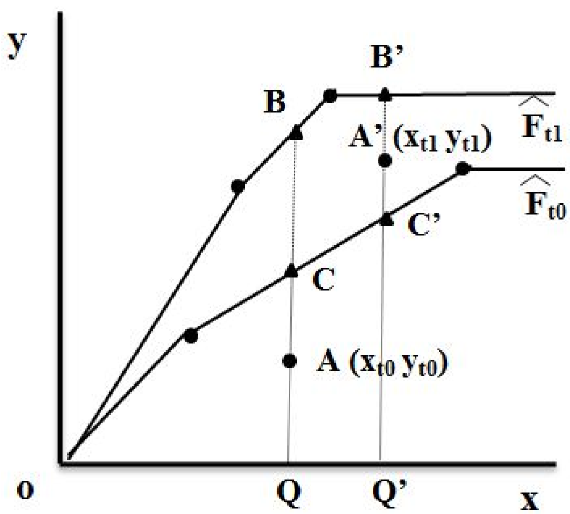

Figure 3.1: MPI Decomposition

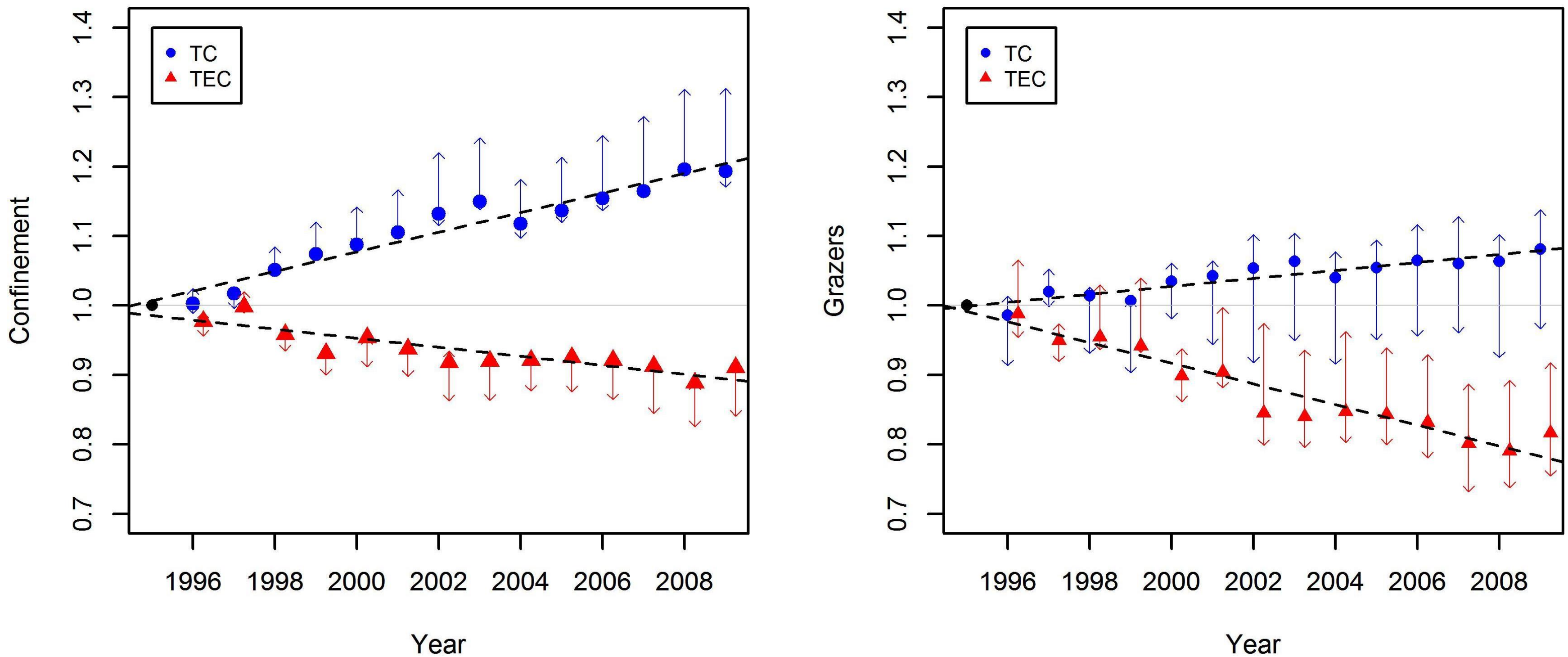

Figure 3.2: Technical Change and Technical Efficiency Change (Results for Table 3.5)

Chapter 4: Comparison of Production Risks in the State-Contingent Framework: Application to Balanced Panel Data

Kota Minegishi†

Abstract

In a balanced panel data setting, this article proposes an empirical application of the state-contingent (SC) framework for production uncertainty. The SC approach (e.g., Chambers and Quiggin, 2000) casts production decisions under uncertainty as the decision to select a portfolio of Arrow-Debreu SC outputs, scheduled to be delivered in the contingent states of nature. Under some stationarity assumptions on the SC decisions (i.e., no technical change, time-invariant states of nature, time-invariant SC portfolio decisions) and regularity assumptions on the data generating process (i.e., cross-sectionally homogeneous state realizations), SC technology can be estimated from balanced panel data that are framed as cross-sectional data of partially-revealed SC portfolio decisions. This allows one to simulate an optimal SC portfolio, determined by the interaction between the estimated SC technology and presumed risk preferences. In the application to Maryland dairy production data, the stochastic technologies of confinement and intensive-grazing dairy systems are compared. Of the two time intervals (years 2000-2004 and years 2006-2009) separately analyzed, the optimal production decision for a moderate-to-maximally risk-averse producer has become riskier for the confinement system and less risky for the grazing system. These contrasting trends appear directly related to the volatile milk prices, feed cost hikes, and increasing organic milk production during 2006-2009. The results from the 2006-2009 panel suggest that at the herd size of 100 cows, a risk-averse producer would prefer the grazing system to the confinement system for its reduced reliance on purchased feeds and rather stable organic milk prices.

Keywords: State Contingent Production, Uncertainty, Panel Data Analysis, Data Envelopment Analysis, Agricultural Economics

JEL *Codes*: D22, Q12, C44

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4.1 Introduction

In the absence of complete insurance markets, the producer bears risks under uncertainty. The extent of optimal risks depends on the production technology, nature of uncertainty, and the producer's attitude toward risks. The state contingent (SC) approach (e.g., Chambers and Quiggin, 2000) casts production decisions under uncertainty as the decision to select a portfolio of Arrow-Debreu SC outputs, scheduled to be delivered in the contingent states of nature. In the context of agricultural production, for example, uncertainty in the SC approach is defined as distinct weather events or market conditions, for which the producer prepares a portfolio of contingent production yields. This analytical framework is more general than a typical empirical specification that regards statistical errors for yield predictions as production uncertainty in the form of stochastic outcome-states (OS). However, empirical applications of the SC approach are scarce in production economics and primarily limited to the estimations of very specific stochastic technologies. The previous studies have proposed the estimations of output-cubical (i.e., non-substitutable SC outputs) (O'Donnell and Griffiths, 2006), state-specific (i.e., independent technologies across states) (ODonnell et al., 2009), single-input single-output CES (Shankar et al., 2010), and imputed SC-output production functions (Chavas, 2008). An exception is a survey-elicited *ex ante*-outputs technology (Chambers et al., 2014; Serra et al., 2014) utilizing a specifically-designed survey. Generally, the challenge is to specify *empirical* state-contingency, in which a stochastic technology should capture the technological relationships for the states that were realized while consistently handling those for the states that were never realized.

This study develops a simple empirical approach to adopting the SC framework in a balanced panel data setting. Under the assumptions of no technical change, time-invariant states of nature, time-invariant SC portfolio decisions (e.g., each producer using the same risk management practice over time), and cross-sectionally homogeneous state realizations (e.g., producers experiencing identical market and weather conditions in a given time period), production data over multiple time periods can be regarded as a sequence of draws from the scheduled SC decisions. By framing balanced panel data as cross-sectional data of partially-revealed SC portfolios, SC technology can be non-parametrically estimated by Data Envelopment Analysis (DEA), simply excluding the decisions for unrealized states. For this revealed subset of contingent states, one can then simulate an optimal SC portfolio determined by the interaction between the estimated SC technology and presumed risk preferences.

The rest of the study proceeds as follows. Section 4.2 reviews the SC approach for production decisions under uncertainty and discusses the scope of the proposed method. Section 4.3 formally presents comparable risk analyses under the SC and OS frameworks. Empirical application to Maryland dairy production in section 4.4 examines the riskiness associated with two types of dairy production systems. Section 4.5 concludes the study.

4.2 Background: The State Contingent Approach

Consider production decisions under uncertain outputs and output prices. Let $\Omega = \{1, 2, .., S\}$ be the index set for the contingent states of nature with associated probabilities $\boldsymbol{\pi} \equiv \{\pi_s\}_{s\in\Omega}$. For given non-stochastic input prices $\boldsymbol{w} \in \mathbb{R}^L_{++}$ and stochastic output prices $\boldsymbol{p} \in \mathbb{R}^{MS}_{++}$, the producer selects L-dimensional inputs $\boldsymbol{x} \in \mathbb{R}^L_+$ to schedule M-dimensional state-contingent (SC) outputs $\boldsymbol{z} \in \mathbb{R}^{MS}_+$ for the S states in Ω. Production technology is denoted by input set $\boldsymbol{X}(\boldsymbol{z}) = \{\boldsymbol{x} \in \mathbb{R}^L_+ : \boldsymbol{x}$ can produce $\boldsymbol{z}\}$ where $\boldsymbol{X}(\boldsymbol{z})$ is assumed to be (1) closed, (2) monotonic (i.e. freely-disposable inputs $\boldsymbol{x} \in \boldsymbol{X}(\boldsymbol{z})$, $\boldsymbol{x}' \geq \boldsymbol{x} \Rightarrow \boldsymbol{x}' \in \boldsymbol{X}(\boldsymbol{z})$ and freely-disposable outputs $\boldsymbol{z} \leq \boldsymbol{z}' \Rightarrow \boldsymbol{X}(\boldsymbol{z}) \subset \boldsymbol{X}(\boldsymbol{z}')$), and (3) convex (i.e. $\boldsymbol{x}, \boldsymbol{x}' \in X(\boldsymbol{z})$, $\lambda\boldsymbol{x} + (1-\lambda)\boldsymbol{x}' \in \boldsymbol{X}(\boldsymbol{z})$, $\forall\lambda \in [0, 1]$) and to satisfy (4) no free lunch (i.e. $\mathbf{0} \notin \boldsymbol{X}(\boldsymbol{z})$ for $\boldsymbol{z} \neq \mathbf{0}$, $\boldsymbol{z} \geq \mathbf{0}$) and no fixed cost (i.e. $\boldsymbol{X}(\mathbf{0}) = \mathbb{R}^L_+$) and (5) the concavity of the output correspondence (i.e. $\lambda\boldsymbol{X}(\boldsymbol{z}) + (1-\lambda)\boldsymbol{X}(\boldsymbol{z}') \subset \boldsymbol{X}(\lambda\boldsymbol{z} + (1-\lambda)\boldsymbol{z}')$, $\forall\lambda \in (0, 1)$).[1]

The production decisions can be equivalently stated for selecting a portfolio of SC revenues $\boldsymbol{r} \equiv \{\sum_{m=1}^{M} p_{ms}\ z_{ms}\}_{s\in\Omega}$ while accounting for revenue-cost function $C(\boldsymbol{r}; \boldsymbol{p}, \boldsymbol{w}) = \boldsymbol{w}\ \boldsymbol{x}(\boldsymbol{r}; \boldsymbol{p}, \boldsymbol{w})$ where optimal inputs $\boldsymbol{x}(\boldsymbol{r}; \boldsymbol{p}, \boldsymbol{w})$ depend on revenue requirement $\boldsymbol{r}$. $C(\boldsymbol{r}; \boldsymbol{p}, \boldsymbol{w})$ is homogeneous of degree zero in $(\boldsymbol{r}, \boldsymbol{p})$, bounded at zero from below by assumption (4), non-decreasing in $\boldsymbol{r}$ and non-increasing in $\boldsymbol{p}$ by (2), and convex in $\boldsymbol{r}$ by (5).

[1] The properties (1)-(4) represent the standard assumption for ensuring the duality between the input correspondence and the cost function, which makes duality theorems (e.g. McFadden, 1978; Färe, 1988) directly applicable. Property (5) can be equivalently stated for output set $\boldsymbol{Z}(\boldsymbol{x})$: $\mu\boldsymbol{Z}(\boldsymbol{x}) + (1-\mu)\boldsymbol{Z}(\boldsymbol{x}') \subset \boldsymbol{Z}(\mu\boldsymbol{x} + (1-\mu)\boldsymbol{x}')$, $\forall\mu \in (0, 1)$. Properties (3) and (5) together correspond to the convexity of the technology in both inputs and outputs.

Producer's risk preferences $W : \mathbb{R}^S \rightarrow \mathbb{R}$ are defined over SC incomes $\boldsymbol{y} = \boldsymbol{r} - C(\boldsymbol{r};\boldsymbol{p},\boldsymbol{w})\mathbf{1}^S$ where $\mathbf{1}^S$ denotes a S-dimensional vector of 1's and are assumed differentiable and generalized Schur-concave (i.e. weakly increasing in y_s for each state $s \in \Omega$ and weakly risk-averse in the sense that $W(\boldsymbol{y}) \leq W(\bar{y}\mathbf{1}^S)$ for $\bar{y} = \boldsymbol{\pi y}$).[2] If state probabilities $\boldsymbol{\pi}$ are unknown, perceived riskiness depends on *subjective* probabilities $\boldsymbol{\pi}_i$ of individual producer i.

The extent of risk aversion for $W(.)$ can be characterized with absolute risk premium $\rho(\boldsymbol{y};\boldsymbol{\pi})$ for portfolio $\boldsymbol{y} = \boldsymbol{r} - C(\boldsymbol{r};\boldsymbol{p},\boldsymbol{w})\mathbf{1}^S$ that measures the maximum willingness to pay (today with certainty) for its non-stochastic income $\boldsymbol{\pi y}\mathbf{1}^S$ at fair odds $\boldsymbol{\pi}$; $\rho(\boldsymbol{y};\boldsymbol{\pi}) = \max\{\rho : W((\boldsymbol{\pi y} - \rho)\mathbf{1}^S) \geq W(\boldsymbol{y})\}$.[3] This also defines certainty-equivalent income $e(\boldsymbol{y};\boldsymbol{\pi}) = \boldsymbol{\pi y} - \rho(\boldsymbol{y};\boldsymbol{\pi})$, or the minimum non-stochastic income to attain utility level $W(\boldsymbol{y})$. Figure 4.1 illustrates this risk premium $\rho(\boldsymbol{y}^0;\boldsymbol{\pi})$ as the distance between the utility level $W(\boldsymbol{y}^0)$ of stochastic incomes $\boldsymbol{y}^0 = \boldsymbol{r}^0 - C(\boldsymbol{r}^0)\mathbf{1}^S$ and the utility level $W(\boldsymbol{\pi y}^0\mathbf{1}^S)$ of non-stochastic income $\boldsymbol{\pi y}^0\mathbf{1}^S = (\boldsymbol{\pi r}^0 - C(\boldsymbol{r}^0))\mathbf{1}^S$. Imposing some properties of $\rho(\boldsymbol{y};\boldsymbol{\pi})$ along expansion paths in $\boldsymbol{y}$ yields the common characterizations of constant absolute risk-averseness (CARA: i.e., translation homotheticity) or constant relative risk-averseness (CRRA: i.e., radial homotheticity).[4] Additionally, under the property of *invariance* that refers to a preserved risk-ordering under the translation and radial expansion of SC incomes on equal-mean sets (Quiggin and Chambers, 2004),[5] Chambers et al. (2012) recently show that a variety of risk aversions specifications for the trade-off between the mean of stochastic income and its riskiness can be obtained.[6]

Formally, the producer chooses $\boldsymbol{r}$ to maximize $W(\boldsymbol{r} - C(\boldsymbol{r};\boldsymbol{p},\boldsymbol{w})\mathbf{1}^S)$. The first order conditions (FOC's) yield;

$$\text{FOC's:} \quad \forall s \in \Omega, \quad W_s - \sum_{t\in\Omega} W_t\, C_s \leq 0, \quad r_s \geq 0 \tag{4.1}$$

[2] Generalized Schur-concavity with respect to $\boldsymbol{\pi}$ refers to the condition $(W_s(\boldsymbol{y})/\pi_s - W_t(\boldsymbol{y})/\pi_t)(y_s - y_t) \leq 0$, $\forall s,t \in \Omega$ where $W_s(\boldsymbol{y}) \equiv \partial W(\boldsymbol{y})/\partial y_s$. These preferences can be defined for either known/objective or unknown/subjective probabilities $\boldsymbol{\pi}$.

[3] Relative risk premium is a monotonic transformation of absolute risk premium and takes the form $(1 - \rho(\boldsymbol{y};\boldsymbol{\pi}))^{-1}$.

[4] CARA: $e(\boldsymbol{y}+\lambda\mathbf{1}^S;\boldsymbol{\pi}) = e(\boldsymbol{y};\boldsymbol{\pi})+\lambda$, $\forall y \in \mathbb{R}^S$, $\forall\lambda \in \mathbb{R}$ and CRRA: $e(\lambda\boldsymbol{y};\boldsymbol{\pi}) = \lambda e(\boldsymbol{y};\boldsymbol{\pi})$, $\forall y \in \mathbb{R}^S$, $\forall\lambda \in \mathbb{R}_{++}$.

[5] Translation and radial invariance on equal-mean set $M(\mu)$ are defined as $\boldsymbol{y},\boldsymbol{y}' \in M(\mu)$, with $\boldsymbol{y} \succ \boldsymbol{y}' \Rightarrow e(\boldsymbol{y} + \delta\mathbf{1}^S) \geq e(\boldsymbol{y}' + \delta\mathbf{1}^S)$, $\forall\delta \in \mathbb{R}$ and $e(t\boldsymbol{y}) \geq e(t\boldsymbol{y}')$, $\forall t \in \mathbb{R}_+$ respectively.

[6] Also, invariant expected-utility class coincides with a family of mean-standard deviation (SD) preferences (Quiggin and Chambers, 2004).

where $W_s = \partial W/\partial y_s$ and $C_s = \partial C/\partial r_s$. Summing the FOC's across states ($\sum_{s\in\Omega} W_s$ $(1 - \sum C_s) \leq 0$) leads to *arbitrage condition* $\sum_{s\in\Omega} C_s(\boldsymbol{r};\boldsymbol{p},\boldsymbol{w}) \geq 1$, indicating that at the optimum the producer exhausts options to formulate a surely-profitable, marginal SC-revenue increase. Figure 4.2 illustrates the optimal SC revenue decision. The feasible SC revenue set for a given cost level is denoted by $\boldsymbol{R}(C(\boldsymbol{r},\boldsymbol{p},\boldsymbol{w})) = \{\boldsymbol{r'} \in \mathbb{R}^S_+ : C(\boldsymbol{r'};\boldsymbol{p},\boldsymbol{w}) \leq C(\boldsymbol{r};\boldsymbol{p},\boldsymbol{w})\}$ where the boundary represents an iso-cost curve of the underlying technology. Given the convexity of revenue-cost function $C(\boldsymbol{r},\boldsymbol{p},\boldsymbol{w})$ in $\boldsymbol{r}$, revenue set $\boldsymbol{R}(C_0)$ is convex for any cost level $C_0 \geq 0$.[7] Then, the optimal SC revenue $\boldsymbol{r}^0$ is located at the tangency between $W(\boldsymbol{r}^0 - C(\boldsymbol{r}^0;\boldsymbol{p},\boldsymbol{w})\mathbf{1}^S)$ and $\boldsymbol{R}(C(\boldsymbol{r}^0;\boldsymbol{p},\boldsymbol{w}))$ with a supporting hyperplane defined by *risk-neutral probabilities* $\boldsymbol{\pi}^* \equiv \{\pi^*_s\}_{s\in\Omega}$ (e.g., Yaari, 1969). These shadow probabilities $\boldsymbol{\pi}^*$ are defined with respect to the point of tangency in each producer's problem, apart from objective probabilities $\boldsymbol{\pi}$ or subjective probabilities $\boldsymbol{\pi}_i$.

Brief econometric discussion clarifies the scope of the proposed SC estimation that follows and highlights the issues associated with a common estimation strategy under the outcome-states (OS) representation of uncertainty. Suppose that the data are generated by the following production decisions; for contingent states of nature $\Omega = \{1, .., S\}$, each producer $i \in \{1, .., N\}$ chooses L inputs $\boldsymbol{x}_i \in \mathbb{R}^L_+$ and prepares a portfolio of 1-dimensional SC outputs $\{z_{i,s}\}_{s\in\Omega}$ for S states under given stochastic, technological frontier $f : \mathbb{R}^L_+ \times \Omega \to \mathbb{R}$. Under complete information, suppose that the true econometric specification is;

$$z_{i,s} = f(\boldsymbol{x}_i; s)\, exp(-u_i), \;\; \forall s \in \Omega \tag{4.2}$$

where $exp(-u_i) \in (0, 1]$ is technical efficiency of producer i. In practice, only one outcome $z_{i,s}$ in portfolio $\{z_{i,s}\}_{s\in\Omega}$ is observed, according to state realization s, so that specification (4.2) is inestimable with observational data.

In a balanced panel data setting, consider stationary SC decisions under no technical change, time-invariant states of nature, and time-invariant SC portfolio decisions for given fixed inputs.[8] Then, assuming homogeneous state realization $\tilde{s}(t) \in \widehat{\Omega}^{SC}$ for all producers in a given time period $t \in \{1, .., T\}$, this study frames observed outputs

[7] The convexity of $C(\boldsymbol{r};\boldsymbol{p},\boldsymbol{w})$ in $\boldsymbol{r}$ implies for any $\boldsymbol{r}$, $\boldsymbol{r}' \in R(C_0)$, $\forall\lambda \in [0, 1]$ $C(r;\boldsymbol{p},\boldsymbol{w})$, $C(r';\boldsymbol{p},\boldsymbol{w}) \leq C_0$ and $C(\lambda r + (1-\lambda)r';\boldsymbol{p},\boldsymbol{w}) \leq C_0$.

[8] In a multiple-input multiple-output specification, inputs can be modeled as either fixed over time, state contingent, or the mixture of the two.

over multiple time periods as a revealed subset of the portfolio, or $\{z_{i,\tilde{s}(t)}\}_{\tilde{s}(t)\in\hat{\Omega}^{SC}}$ under $\widehat{\Omega}^{SC} \subset \Omega$. This yields estimable frontier $\hat{f}^{SC}(.;\tilde{s}(t))$;

$$z_{i,\tilde{s}(t)} = \hat{f}^{SC}(\boldsymbol{x}_i;\tilde{s}(t))\, exp(-\hat{u}_i),\ \ \forall \tilde{s}(t) \in \widehat{\Omega}^{SC}. \tag{4.3}$$

Equation (4.3) differs from a typical econometric specification for (state-invariant) frontier $\hat{f}^{OS}(.)$ with (state-variant) stochastic error component $exp(v_{\tilde{s}(it)})$;

$$z_{\tilde{s}(it)} = \hat{f}^{OS}(\boldsymbol{x}_{it})\, exp(-u_{it} + v_{\tilde{s}(it)}),\ \ \forall \tilde{s}(it) \in \widehat{\Omega}^{OS} \tag{4.4}$$

where the empirical states of nature are represented by stochastic error $exp(v_{\tilde{s}(it)})$, or a distribution of outcome-states $\widehat{\Omega}^{OS}$. Whence the OS framework allows one to model a statistical relationship between inputs $\boldsymbol{x}_{it}$ and outputs $z_{\tilde{s}(it)}$ defined for a residual distribution over stochastic states $\widehat{\Omega}^{OS}$. However, it has long been recognized that such a representation of production uncertainty imposes, a priori, very restrictive patterns of technical substitution (e.g., Just and Pope (1978); Chambers and Quiggin (2002)).[9] Specifications like (4.4) do not account for the host of situations where the contribution of inputs $\boldsymbol{x}$ to SC outputs $\{z_s\}_{s\in\Omega}$ potentially varies across the states of nature. The key insight of the SC approach is that producers manage production *risks* through state-variant, input-output relationships, for example, by allocating resources to enhance *state-specific* output gains (e.g., applying fertilizers to boost yields under good weather conditions) or mitigate *state-specific* output losses (e.g., applying pesticides to protect plants from potential pest infestations). Analytically, the OS representation is a special case of the SC approach where the stochastic technology allows no technical substitutions across states. This study formally considers the sets of assumptions that lead to empirical specifications like (4.3) and (4.4) under the SC and OS frameworks respectively.

4.3 Comparison of Stochastic Technologies with Balanced Panel Data

This section considers the analyses of production risks using balanced panel data under each of the SC and OS frameworks of uncertainty. The decision-making

[9] Just-Pope technology specification partially relaxes this assumption by allowing the variance to depend on input variable (Just and Pope, 1978), capturing a certain inter-dependence between the inputs and the role of uncertainty.

of the standard SC approach in the previous section is adapted to a stylized empirical context for estimating a model of uncertain annual returns on fixed production assets.

4.3.1 Empirical Context

Consider a context of agricultural production where the producer, endowed with some fixed assets $\boldsymbol{x}^f$, selects a schedule of contingent input-output decisions $(\boldsymbol{x}^v, \boldsymbol{z})$ under uncertain weather outcomes and market conditions for the upcoming production season. The model in consideration casts production risks over multiple time periods as a repeated cycle of uncertainty resolutions, in which *predetermined* fixed assets $\boldsymbol{x}^f$ are regarded as "input" variables in the terminology of the previous section, and *state-contingent* input-output decisions $(\boldsymbol{x}^v, \boldsymbol{z})$ similarly as "output" variables. This allows some inputs in $\boldsymbol{x}^v$ to be committed before the resolution of uncertainty (e.g., applying herbicides to prevent weed infestations) and others to be brought in after the resolution (e.g., applying insecticides in response to an insect outbreak). Thus, the model does not require the classification of inputs $\boldsymbol{x}^v$ into such *ex ante* commitments or *ex post* responses, which largely depends on the situation at hand and may be ambiguous at best if the timing of input use is flexible in relation to the timing of contingent events.[10] The technology for this model is represented as $\Psi = \{(\boldsymbol{x}^f, [-\boldsymbol{x}^v\ \boldsymbol{z}]) \in \mathbb{R}_+^{L1} \times \mathbb{R}_+^{(L2+M)S} : \boldsymbol{x}_s^v$ can produce $\boldsymbol{z}_s$ in state $s \in \Omega$, given fixed assets $\boldsymbol{x}^f\}$.

For simplicity, the focus of the modeling is placed on a reduced-form, technological relationship, referred to as value-added (VA) SC technology $Y(\boldsymbol{x}^f) = \{\boldsymbol{y} \in \mathbb{R}^S$: a portfolio of SC incomes $\boldsymbol{y}$ for states Ω is producible, given fixed assets $\boldsymbol{x}^f\}$.[11] Since SC incomes $\boldsymbol{y} = \{\boldsymbol{p}_s \boldsymbol{z}_s - \boldsymbol{w}_s^v \boldsymbol{x}_s^v\}_{s\in\Omega}$ are simply the linear projection of SC input-outputs $(\boldsymbol{x}^v, \boldsymbol{z})$ at corresponding prices $(\boldsymbol{w}^v, \boldsymbol{p})$, it follows that $\boldsymbol{y} \in Y(\boldsymbol{x}^f) \Leftrightarrow (\boldsymbol{x}^v, \boldsymbol{z}) \in \Psi(\boldsymbol{x}^f)$ for technology Ψ conditionally on fixed assets $\boldsymbol{x}^f$. If, for instance, Ψ is closed, free-disposable/monotonic in $(\boldsymbol{x}, \boldsymbol{z})$, and convex in $(-\boldsymbol{x}, \boldsymbol{z})$ with feasible inaction, then $Y(\boldsymbol{x}^f)$ satisfies (VA1) closure, (VA2) monotonicity ($\boldsymbol{y}' \leq \boldsymbol{y} \in Y(\boldsymbol{x}^f) \Rightarrow \boldsymbol{y}' \in Y(\boldsymbol{x}^f)$, and $\boldsymbol{x}^{f\prime} \geq \boldsymbol{x}^f \Rightarrow Y(\boldsymbol{x}^{f\prime}) \supseteq Y(\boldsymbol{x}^f)$ for all $\boldsymbol{x}^f, \boldsymbol{x}^{f\prime} \in \mathbb{R}_+^{L1}$), (VA3) feasible inaction ($\boldsymbol{0}_S \in Y(\boldsymbol{x}^f)$ for all $\boldsymbol{x}^f \in \mathbb{R}_+^{L1}$), and (VA4) the convexity of input and output sets

[10] The potential input misclassification applies to the OS framework as well since specifying a distribution conditionally on regressors does not correctly account for the state-specific contributions of these inputs.

[11] The reduced-model through VA technology $Y(\boldsymbol{x}^f)$ is optional, but for the current dataset the reduction in the dimension of input-output specification is needed to conserve degrees of freedom. Also, it simplifies the comparison between the SC and OS frameworks in below.

$(\lambda Y(\boldsymbol{x}^f) + (1-\lambda)Y(\boldsymbol{x}^{f\prime}) \subseteq Y(\lambda \boldsymbol{x}^f + (1-\lambda)\boldsymbol{x}^{f\prime})$ for all $\lambda \in [0,1]$ and $\boldsymbol{x}^f, \boldsymbol{x}^{f\prime} \in \mathbb{R}_+^{L1})$.

The analysis seeks to compare the risks associated with distinct systems of production. Observed production decisions are partitioned based on employed system $g \in \{1,..,G\} = \mathbb{G}$ of technology $\Psi^g \subset \Psi$ that allows for distinct possibilities to arrange SC input-output decisions and are technically-evaluated against comparable peer-observations within the system, while the associated production risks can be compared both within and across the system. Each system $g \in \mathbb{G}$ of technology Ψ^g is assumed to be closed, monotonic, and convex with feasible inaction, and hence the associated VA technology $Y^g(\boldsymbol{x}^f)$ satisfies assumptions (VA1) through (VA4). Meanwhile, industry-level technology $\Psi = \cup_{g\in\mathbb{G}}\Psi^g$ can be non-convex if observed fixed assets $\boldsymbol{x}^f$ are systematically different across systems or potentially disjoint across them, for example, due to distinct characteristics of land and capital utilized under each system.

Consider a dataset containing decisions of N producers in T time periods, indexed by $i \in \{1,..,N\} = \mathbb{I}$ and $t \in \{1,..,T\} = \mathbb{T}$ respectively. Let $\mathbb{I}^g$ denote the index set for N_g producers employing system $g \in \mathbb{G}$. Each producer i employs single system $g \in \mathbb{G}$ throughout the time span $\mathbb{T}$, so that $\sum_{g\in\mathbb{G}} N_g = N$ and $\mathbb{I} = \cup_{g\in\mathbb{G}}\mathbb{I}^g$. Then, the analysis assumes the following.

A 1. ***No Contingent-State Change.*** *Contingent states Ω are time-invariant in $\mathbb{T}$.*

A 2. ***No Technical Change.*** *Each system $g \in \mathbb{G}$ of VA technology $Y^g(.)$ is time-invariant in $\mathbb{T}$.*

With assumptions A1 and A2, the above empirical context can be described as a repeated cycle of events defined as follows. In each time period $t \in \mathbb{T}$; each producer $i \in \mathbb{I}^g$ selects SC input-outputs $\{(\boldsymbol{x}^v_{it,s}, \boldsymbol{z}_{it,s})\}_{s\in\Omega} \in \Psi^g(\boldsymbol{x}^f_i)$ where subscripts it and s denote the index of producer-time periods it and the index of state s respectively; the Nature draws some state $s \in \Omega$ for the producer; the producer earns his SC income $y_{it,s} = \boldsymbol{p}_{t,s}\boldsymbol{z}_{it,s} - \boldsymbol{w}^v_{t,s}\boldsymbol{x}^v_{it,s}$ as scheduled.

Under full information (disregarding the availability of data) with assumptions A1 and A2, SC technology $\Psi^g(\boldsymbol{x}^f_0)$ for given $\boldsymbol{x}^f_0$ can be approximated by a free-disposable convex hull;

$$\widehat{\Psi}^{g,DEA}(\boldsymbol{x}_0^f) = \{(\boldsymbol{x}^{v\,\prime}, \boldsymbol{z}') \in \mathbb{R}^{(L2+M)S} : \forall s \in \Omega,\ \sum_{i\in\mathbb{I}^g}\sum_{t\in\mathbb{T}} \lambda_{it}\, \boldsymbol{z}_{it,s} \geq \boldsymbol{z}'_s,$$
$$\sum_{i\in\mathbb{I}^g}\sum_{t\in\mathbb{T}} \lambda_{it}\, \boldsymbol{x}^v_{it,s} \leq \boldsymbol{x}^{v\,\prime}_s,\ \sum_{i\in\mathbb{I}^g}\sum_{t\in\mathbb{T}} \lambda_{it}\, \boldsymbol{x}^f_i \leq \boldsymbol{x}^f_0,\ \sum_{i\in\mathbb{I}^g}\sum_{t\in\mathbb{T}} \lambda_{it} = 1,\ \boldsymbol{\lambda} \in \mathbb{R}_+^{N_g T}\} \tag{4.5}$$

or its reduced-form;

$$\widehat{Y}^{g,DEA}(\boldsymbol{x}_0^f) = \{\boldsymbol{y}' \in \mathbb{R}^S : \forall s \in \Omega,\ \sum_{i\in\mathbb{I}^g}\sum_{t\in\mathbb{T}} \lambda_{it}\, y_{it,s} \geq y'_s,$$
$$\sum_{i\in\mathbb{I}^g}\sum_{t\in\mathbb{T}} \lambda_{it}\, \boldsymbol{x}^f_i \leq \boldsymbol{x}^f_0,\ \sum_{i\in\mathbb{I}^g}\sum_{t\in\mathbb{T}} \lambda_{it} = 1,\ \boldsymbol{\lambda} \in \mathbb{R}_+^{N_g T}\} \tag{4.6}$$

where $\widehat{\Psi}^{g,DEA}(\boldsymbol{x}_0^f)$ and $\widehat{Y}^{g,DEA}(\boldsymbol{x}_0^f)$ are DEA approximations for SM-output and S-output technologies under variable returns to scale (VRS) respectively. The data requirement for estimating $\widehat{Y}^{g,DEA}(\boldsymbol{x}_0^f)$ is the set of fixed inputs $\boldsymbol{x}_i$ and SC-income portfolios $\{y_{it,s}\}_{s\in\Omega}$ of producer-time periods $it \in \mathbb{I}^g \times \mathbb{T}$.

So far, no assumption is made for producer's preferences (and his subjective probabilities). Specifying some *arbitrary* risk preferences $W(.)$ and predicted probabilities of states, say $\hat{\boldsymbol{\pi}}$, yields the desired risk comparison.

Remark 1. ***Analysis of Optimal SC Portfolios.*** *Given assumptions A1 and A2 and equation* (4.6)*, optimal portfolio can be simulated for each system $g \in \mathbb{G}$ of VA technology $\widehat{Y}^{g,DEA}(.)$ and ranked across systems, according to some risk preferences $W : \mathbb{R}^S \to \mathbb{R}$ and probability $\hat{\boldsymbol{\pi}}$.*

The feasibility of this analysis rests on the feasibility of a SC technology approximation like (4.6). The difficulty is to construct an empirical model for contingent states Ω, based on some available information on state realizations. On one hand, focusing on limited aspects of the states would fall short of representing the relevant state pace and likely misrepresent production risks. On the other hand, incorporating highly-detailed information on the states would create disjointedness in state realizations among observations and hence make a technology approximation like (4.6) infeasible. Given these complications, it is not surprising that empirical applications of the SC approach have been extremely rare in production economics. In below, additional assumptions are introduced to circumvent these challenges.

4.3.2 Cross-Sectional Analysis of SC Portfolios Under the SC Framework

An empirically-feasible SC risk analysis is proposed under two additional assumptions. One is that individual producers make time-invariant portfolio decisions during time span $\mathbb{T}$. This assumption is most naturally interpreted through the fixity of production assets that completely restricts the ability to adjust their SC portfolios in a short run.[12] The other is that the state realization is identical for all producers $i \in \mathbb{I}$ in a given time period. Then, each state realization can be indexed by time period t, or notationally state $s = \tilde{s}(t)$ represents a bundle of SC events in time t that are shared by producers. This allows one to frame balanced panel data on incomes as cross-sectional observations of partially-revealed SC portfolios.

Formally, the additional assumptions are stated as follows;

A-SC 1. ***Time-invariant SC Production Decisions.*** *During time span $\mathbb{T}$, producers are locked into time-invariant SC portfolio decisions (i.e., $\forall i \in \mathbb{I}$, $\forall a, b \in \mathbb{T}$, $\{y_{ia,s}\}_{s\in\Omega} = \{y_{ib,s}\}_{s\in\Omega}$).*

A-SC 2. ***Cross-sectionally Homogeneous State Realization.*** *In given time period $t \in \mathbb{T}$, the Nature draws identical state $s \in \Omega$ for all producers $i \in \mathbb{I}$.*

Assumption A-SC1 states that producers cannot alter their SC production schedules $\{\boldsymbol{p}_s \boldsymbol{z}_{it,s} - \boldsymbol{w}_s^v \boldsymbol{x}_{it,s}^v\}_{s\in\Omega} = \{y_{it,s}\}_{s\in\Omega}$ throughout time span $\mathbb{T}$. This is, in spirit, akin to the putty-clay model of technology (e.g. Bischoff, 1972; Fuss, 1978), in which the producer is initially presented with the opportunity to choose any input-output decision in the technology but becomes stuck with a specific decision once the decision is made. The putty-clay model reflects a stylized fact that the lumpiness of investment tends to severely limit producer's technical substitution possibilities in a short run. For example, the asset fixity for agricultural producers has long been considered as a major explanation for particularly slow downward supply curve responses in agricultural production (often referred to as irreversible supply), dating back to Galbraith and Black (1938), Johnson (1950), and Edwards (1959).[13]

[12] It is not uncommon to assume restrictive production sets at the producer level while regarding the envelop of these sets as an industry-level technological frontier; see the tradition since T. C. Koopmans (1952) and Houthakker (1955) and recent applications in, for example, Kortum (1997) and Jones (2005).

[13] Readers interested in empirical tests of asset fixity is directed to, for example, Chambers and Vasavada (1983).

In the current case, each schedule of SC input-output decisions $\{(\boldsymbol{x}^v_{it,s},\, \boldsymbol{z}_{it,s})\}_{s\in\Omega}$ should be regarded as a specific production technique accompanied with a set of contingent action plans for various risk scenarios. The fixity of the SC schedule does not imply that the producer makes identical, state-specific input-output decisions $(\boldsymbol{x}^v_{it,s}, \boldsymbol{z}_{it,s})$ in each period $t \in \mathbb{T}$ since his action depends on state realization s that is assumed to vary across time periods. Instead, it means that if the draws of states were identical for any two periods, the producer would act on the same input-output decision, according to the same schedule.

Note that this assumption is much more restrictive than the fixed effects assumption in panel-data econometrics, which invokes a time-invariant *conditional-mean* relationship between the observable and unobservable characteristics of a decision-maker. Under assumption A-SC1, the relationship between the observed SC decisions (for realized states of nature) and unobserved decisions (for unrealized states) of a producer is assumed to be strictly time-invariant. The deterministic and stronger version of the time-invariance assumption is necessary for modeling the exact technological relationships across states as opposed to some average characterizations at their conditional means. Assumption A-SC1 is most naturally interpreted as the strict time-invariance of risk management plans due to the underlying, short-run fixity of production assets. Then, the producer is stuck with a certain schedule of contingent actions $\{(\boldsymbol{x}^v_{it,s},\, \boldsymbol{z}_{it,s})\}_{s\in\Omega}$ over production cycles during $\mathbb{T}$, regardless the changes in the likelihoods of contingent states.

Assumption A-SC2 would be most sensible when producers operate in the same production environments – in terms of market conditions, regulations, and weather patterns – for example, due to geographical proximity. Then, the reference to period t can be interchangeably treated as the reference to state realization $s = \tilde{s}(t)$ under some mapping $\tilde{s}(.)$ from $\mathbb{T}$ to Ω, yielding an empirical set of state realizations $\widehat{\Omega}^{SC} \subset \Omega$. When appropriate, the representativeness of empirical state-space $\widehat{\Omega}^{SC}$ may be assessed based on the likelihoods of observed SC events like market prices and weather outcomes in historical records.

Some implications of assumptions A-SC1 and A-SC2 are noted in simple two-state examples. Figure 4.5 illustrates a case where SC decisions are in fact *time-variant* so that assumption A-SC1 is violated. Points $\boldsymbol{y}^A = (y^A_s,\, y^A_t)$ and $\boldsymbol{y}^B = (y^B_s,\, y^B_t)$ represent two SC portfolios $(y_{i1,s},\, y_{i1,t})$ and $(y_{i2,s},\, y_{i2,t})$ of producer i and time $k = 1, 2$ respectively. Portfolio $\boldsymbol{y}^A$ has a higher income in state s and a lower income in state t than portfolio $\boldsymbol{y}^B$, or $y_{i1,s} > y_{i2,s}$ and $y_{i1,t} < y_{i2,t}$. Of these, suppose that observed are $y_{i1,s}$ and $y_{i2,t}$ in time $k = 1, 2$, for which state realizations are modeled by assignments $s = \tilde{s}(k = 1)$ and $t = \tilde{s}(k = 2)$. This yields a prediction of single time-invariant SC portfolio $(y_{i1,s},\ y_{i2,t})$ depicted at $\boldsymbol{y}^C = (y^A_s,\ y^B_t)$, which overstates technical feasibility in its neighborhood. Alternatively, if observed decisions are instead $y_{i1,t}$ and $y_{i2,s}$ in time $k = 1, 2$, predicted portfolio $(y_{i1,t},\ y_{i2,s})$ at (y^A_t, y^B_s) would underestimate technical feasibility in this neighborhood.

In another example, figure 4.6 depicts a case where state-realizations are in fact cross-sectionally *heterogeneous* so that assumption A-SC2 is violated. This time, points $\boldsymbol{y}^A$ and $\boldsymbol{y}^B$ denote two time-invariant portfolios $(y_{i1,s},\ y_{i1,t}) = (y_{i2,s},\ y_{i2,t})$ and $(y_{j1,s},\ y_{j1,t}) = (y_{j2,s},\ y_{j2,t})$ of producers i and j respectively in time $k = 1, 2$. Suppose that observed decisions are $y_{i1,s}$ and $y_{i2,t}$ for producer i and $y_{j1,t}$ and $y_{i2,s}$ for producer j in time $k = 1, 2$. Under state assignments $s = \tilde{s}(k = 1)$ and $t = \tilde{s}(k = 2)$, time-invariant SC-income portfolios are predicted correctly as $(y_{i1,s},\ y_{i2,t})$ for producer i (at point $\boldsymbol{y}^A$) and incorrectly as $(y_{j1,t},\ y_{j2,s})$ for producer j (at point $\boldsymbol{y}^C$). When $y_{jk,s} < y_{jk,t}$, incorrect portfolio $\boldsymbol{y}^C$ would estimate a higher income in state s and a lower income in state t than correct portfolio $\boldsymbol{y}^B$, by which the frontier would be overestimated around $\boldsymbol{y}^C$ and underestimated around $\boldsymbol{y}^B$.

Thus, assumptions A-SC1 and A-SC2 compensate for the incomplete information on SC portfolio decisions at the risk of potential misrepresentations of a technology. Only by discarding any between-time variation in SC decisions, does assumption A-SC1 make it possible to study within-portfolio differences across states. Only by ignoring any within-time variation in realized states, does assumption A-SC2 allow one to compare between-portfolio differences across producers.

Under incomplete information with assumptions A1, A2, A-SC1, and A-SC2, the VA technology in (4.6) for empirical states $\widehat{\Omega}^{SC}$ becomes;

$$\begin{aligned}\widehat{Y}^{g,DEA}(\boldsymbol{x}^f_0; \widehat{\Omega}^{SC}) = \{&\boldsymbol{y}' \in \mathbb{R}^T : \forall s = \tilde{s}(t) \in \widehat{\Omega}^{SC},\ \sum_{i \in \mathbb{I}^g} \lambda_i\, y_{i,\tilde{s}(t)} \geq y'_s, \\ &\sum_{i \in \mathbb{I}^g} \lambda_i\, \boldsymbol{x}^f_i \leq \boldsymbol{x}^f_0,\ \sum_{i \in \mathbb{I}^g} \lambda_i = 1,\ \ \boldsymbol{\lambda} \in \mathbb{R}^{N_g}_+\},\end{aligned} \tag{4.7}$$

which is a DEA approximation for a T-output VRS technology. Its data requirement is a set of fixed inputs $\boldsymbol{x}_i$ and SC-income portfolios $\{y_{i,\bar{s}(t)}\}_{\bar{s}(t)\in\widehat{\Omega}^{SC}}$ of producers $i \in \mathbb{I}$ over time span $\mathbb{T}$.

The frontier approximation allows one to estimate technical efficiency (TE) for a given direction and a unit in the empirical state-space. For example, distance function $\widehat{D}^g(\boldsymbol{x}_0^f, \boldsymbol{y}_0) = \min\{\phi : \boldsymbol{y}_0/\phi \in \widehat{Y}^{g,DEA}(\boldsymbol{x}_0^f; \widehat{\Omega}^{SC})\} \in (0,1]$ measures the output-oriented, *radial* TE of portfolio $\boldsymbol{y}_0$ for given fixed assets $\boldsymbol{x}_0^f$ where the technical inefficiency represents the extent of the underexploited, *proportional* increase in SC incomes for states $\widehat{\Omega}^{SC}$. If the direction of the TE measurement is instead taken in the *uniform-increase* in state-space $\widehat{\Omega}^{SC}$, the associated technical inefficiency would measure the missed opportunity in *uniformly* increasing SC incomes for states $\widehat{\Omega}^{SC}$.

Under approximation (4.7), the SC incomes for unrealized states are seen as omitted variables in the input-output specification. Omitted variables do not induce bias in the estimation of a distance function like $\widehat{D}^g(\boldsymbol{x}_i^f, \boldsymbol{y}_i)$ but instead affect the efficiency calculation by limiting the direction of the TE measurements. For instance, the feasible radial TE measurement estimates the proportional increase in SC incomes for realized states $\widehat{\Omega}^{SC}$ but not those for unrealized states. Without a sound method to utilize information on the unrealized states, excluding these states from the efficiency evaluation seems sensible. The DEA frontier approximation is well-suited for this nonparametric treatment of an empirical state-space.

In summary, the SC risk analysis is feasible under empirical state-space $\widehat{\Omega}^{SC}$;

Remark 2. ***Cross-sectional Analysis of Optimal SC Portfolios.*** *Given assumptions A1, A2, A-SC1, and A-SC2 and equation* (4.7)*, optimal portfolio can be simulated for each system $g \in \mathbb{G}$ of VA technology $\widehat{Y}^{g,DEA}(.; \widehat{\Omega}^{SC})$ and ranked across $\mathbb{G}$ according to some risk preferences $W : \mathbb{R}^{\widehat{\Omega}^{SC}} \to \mathbb{R}$ and probability $\hat{\boldsymbol{\pi}}$ defined on $\widehat{\Omega}^{SC} \subset \Omega$.*

4.3.3 Pooled Analysis of Income Distribution Under the OS Framework

In contrast to the above SC approach, a standard practice is to analyze pooled observations across producers and time periods in what this study refers to as the outcome-state (OS) representation framework. The following highlights the linkage between the theoretical framework of SC decision-making in the previous section and the typical risk analysis under the OS framework. For consistency, the input-output specification in the previous SC framework (i.e., SC portfolio decision $\{y_{it,s}\}_{s\in\Omega}$ for given fixed assets $\boldsymbol{x}_i^f$) is kept unchanged. Notationally, each state realization is indexed by producer-time period it, or $s = \tilde{s}(it)$ where state $\tilde{s}(it) \in \widehat{\Omega}^{OS}$ represents a bundle of SC events that can vary across producers and time periods.

The assumption for the OS framework can be stated as the lack of technical substitutability of SC decisions across observations (Chambers and Quiggin, 2002). The assumption corresponds to "output-cubical" technology (e.g. Luenberger, 1995), represented by a free-disposable hull with a single vertex of portfolio $\boldsymbol{y}^g(\boldsymbol{x}^f) \in \mathbb{R}^S$ that weakly-dominates all feasible portfolio decisions (i.e., $Y^g(\boldsymbol{x}^f) = \{\boldsymbol{y'} \in \mathbb{R}^S : \boldsymbol{y}^g(\boldsymbol{x}^f) \geq \boldsymbol{y'}\}$). In a panel data analysis, this amounts to assuming the time-invariance and producer-invariance of the optimal portfolio decision for given inputs;

A-OS 1. *Output-Cubical Technology.* *For each system $g \in \mathbb{G}$, VA technology $Y^g(.)$ allows no substitutions of SC incomes across states (i.e., $\forall i, j \in \mathbb{I}$, $\forall a, b \in \mathbb{T}$, $\{y_{ia,s}\}_{s\in\Omega} = \{y_{jb,s}\}_{s\in\Omega}$).*

Casting production uncertainty as an exogenous distribution of SC incomes implies that the producer apparently plays no active role in formulating this portfolio, for example, due to technical restrictions. This is ensured under assumption A-OS1; conditionally on the differences in input levels, observed incomes $y_{\tilde{s}(it)}$ for $\forall it \in \mathbb{I}^g \times \mathbb{T}$ can be all traced back to a representative portfolio-decision $\{y_{\tilde{s}(it)}\}_{\tilde{s}(it)\in\widehat{\Omega}^{g,OS}}$.

Under incomplete information with assumptions A1, A2, and A-OS1, the VA technology in (4.6) for empirical states $\widehat{\Omega}^{g,OS}$ becomes;

$$\begin{aligned}\widehat{Y}^{g,DEA}(\boldsymbol{x}_0^f; \widehat{\Omega}^{g,OS}) = \{\boldsymbol{y'} \in \mathbb{R}^{N_g T} : \forall s = \tilde{s}(it) \in \widehat{\Omega}^{g,OS},\\ \lambda_{\tilde{s}(it)}\, y_{\tilde{s}(it)} \geq y'_s,\ \lambda_{\tilde{s}(it)}\, \boldsymbol{x}_i^f \leq \boldsymbol{x}_0^f,\ \boldsymbol{\lambda} \in \mathbb{R}_+^{N_g T}\}\end{aligned} \tag{4.8}$$

where each observation it is associated with distinct state realization $\tilde{s}(it)$, and its collection defines empirical states $\widehat{\Omega}^{g,OS}$. Specification (4.8) is a DEA approximation for a N_gT-output VRS technology estimated with N_gT observations, albeit it would be a free-disposable hull with single vertex $\{y_{\tilde{s}(it)}\}_{\tilde{s}(it)\in\widehat{\Omega}^{g,OS}}$ with no comparable peer observations.

In practice, the researcher instead employs a parametric-frontier estimation where the distribution of statistical errors is assumed to represent empirical state-space $\widehat{\Omega}^{g,OS}$. A typical stochastic frontier analysis (SFA) specification like equation

(4.4) is written as $y_{\tilde{s}(it)} = f^g(\boldsymbol{x}_i^f)\, exp(-u_{it} + v_{\tilde{s}(it)})$, for which model identification requires the statistical independence between frontier function $f^g(\boldsymbol{x}_i^f)$ and the joint distribution of technical efficiency $exp(-u_{it})$ and stochastic error $exp(v_{\tilde{s}(it)})$.[14] Such an estimation implies an output-cubical technology defined over some realized and other unrealized state-spaces, say $\widehat{\Omega}^{g,OS} = \widehat{\Omega}_R^{g,OS} + \widehat{\Omega}_U^{g,OS}$, such that;

$$\begin{aligned}\widehat{Y}^{g,SFA}(\boldsymbol{x}_0^f; \widehat{\Omega}^{g,OS}) = \{\boldsymbol{y}' \in \Omega : &\forall s = \tilde{s}(it) \in \widehat{\Omega}_R^{g,OS},\ f^g(\boldsymbol{x}_i^f)\, exp(v_{\tilde{s}(it)}) \geq y'_s, \\ &\forall s = \tilde{s} \in \widehat{\Omega}_U^{g,OS},\ f^g(\boldsymbol{x}_i^f)\, exp(\hat{v}_{\tilde{s}}) \geq y'_s\}, \end{aligned} \tag{4.9}$$

for which data requirement is a set of fixed inputs $\boldsymbol{x}_i$ and SC-income portfolios $\{y_{\tilde{s}(it)}\}_{\tilde{s}(it) \in \widehat{\Omega}^{g,OS}}$ of producer-time periods $it \in \mathbb{I}^g \times \mathbb{T}$.

Some implications of assumption A-OS1 are noted in simple two-state examples. Figure 4.7 considers a parallel case to figure 4.5 that the underlying decisions $(y_{i1,s},\ y_{i1,t})$ and $(y_{i2,s},\ y_{i2,t})$ of producer i in time $k = 1, 2$ (depicted at points $\boldsymbol{y}^A = (y_s^A,\ y_t^A)$ and $\boldsymbol{y}^B = (y_s^B,\ y_t^B)$) violate the *time-invariance* of SC decisions. Suppose that observed decisions are $y_{i1,s}$ and $y_{i2,t}$ in time $k = 1, 2$. Under state assignments $s = \tilde{s}(k = 1)$ and $t = \tilde{s}(k = 2)$, these observations can be interpreted as a vertex $(y_{i1,s},\ y_{i2,t})$ of the output-cubical technology in (4.8) (at point $\boldsymbol{y}^C = (y_s^A,\ y_t^B)$). Or, based on the average of the observations $(y_s^A + y_t^B)/2$ at $\boldsymbol{y}^D$ along with some predicted stochastic errors and technical inefficiencies, an SFA technology in (4.9) may yield non-stochastic portfolio $\boldsymbol{y}^{D\,*}$ (i.e. $f^g(\boldsymbol{x}_i^f)$) and stochastic portfolio $\boldsymbol{y}^E$ (i.e. $f^g(\boldsymbol{x}_i^f)\, exp(\hat{v}_{s(\tilde{it})})$) at full technical efficiency. Note that in this example, taking any portfolio among $\boldsymbol{y}^C$, $\boldsymbol{y}^D$, $\boldsymbol{y}^{D\,*}$, or $\boldsymbol{y}^E$ as a vertex of output-cubical technology would result in overestimating the technology around the vertex.

Figure 4.8 illustrates the case of having the same underlying decisions but observing decisions $y_{i1,s}$ and $y_{i2,s}$ of producer i in time $k = 1, 2$. Under state assignments $s = \tilde{s}(k = 1)$ and $t = \tilde{s}(k = 2)$ (instead of true assignments $s = \tilde{s}(k = 1) = \tilde{s}(k = 2)$), portfolios $\boldsymbol{y}^C$, $\boldsymbol{y}^D$, $\boldsymbol{y}^{D\,*}$, and $\boldsymbol{y}^E$ can be predicted similarly as in the previous example. Then, taking any of theses points as a vertex would underestimate the technology around the vertex in this case. Given the parallel roles of the time-invariance and

[14] See also Chambers and Quiggin (2002) for the relationships between the stochastic error structure an the restricted expansion path of a stochastic technology. The authors show that additive and multiplicative stochastic error structures impose constant absolute riskiness (CAR) and constant relative riskiness (CRR) respectively and that the Just-Pope technology specification relaxes these properties and allows for a flexible expansion path.

producer-invariance assumptions in defining empirical state space $\widehat{\Omega}^{g,OS}$, analogous examples can be constructed for violating *producer-invariant* SC decisions.

Thus, by discarding any differences in the underlying portfolio decisions conditionally on inputs, assumption A-OS1 makes it possible to reduce the analysis of SC portfolio decisions into the analysis of a stochastic income distribution across producers and time periods. Such an analysis can be additionally facilitated by certain parametric-distributional relationships among empirical states $\tilde{s}(it) \in \widehat{\Omega}^{g,OS}$.

The task of comparing different systems of technologies $\widehat{Y}^{g,SFA}(\boldsymbol{x}_0^f; \widehat{\Omega}^{g,OS})$, defined over different empirical-state spaces $\widehat{\Omega}^{g,OS}$ (which are not directly comparable), are undertaken through the ordered statistics of their optimal portfolios defined over a common domain, say, $\widehat{\Omega}^{OS}$. Under probabilistic sophistication of risk preferences, equally-probable states of nature can be regarded as interchangeable.[15] Then, the OS risk analysis is feasible under empirical state-spaces $\widehat{\Omega}^{OS}$.

Remark 3. ***Pooled Analysis of Optimal OS Portfolios.*** *Given assumptions A1, A2, and A-OS1 and equation* (4.9), *optimal portfolio can be simulated for each system* $g \in \mathbb{G}$ *of VA technology* $\widehat{Y}^{g,SFA}(.; \widehat{\Omega}^{g,OS})$ *and ranked across* $\mathbb{G}$ *according to some probabilistically-sophisticated preferences,* $W : \mathbb{R}^{\widehat{\Omega}^{OS}} \to \mathbb{R}$ *with probability* $\hat{\boldsymbol{\pi}}$ *defined on* $\widehat{\Omega}^{OS}$.

4.4 Application

4.4.1 The Data

Using data on Maryland dairy production, this section compares relative riskiness associated with two dairy production systems, or confinement and management-intensive grazing dairy practices. Confinement dairies tend to use more inputs like purchased feeds and machinery and produce more milk than grazers. The dataset was previously analyzed in Hanson et al. (2013), which found that on average the two groups earn about the same amount of income, but the confinement dairy was deemed riskier based on its higher variation in income.

This study separately analyzes two balanced-panel subsets for years 2000-2004 and years 2006-2009 of the original dataset. The 2000-2004 panel contains 11 grazers

[15] Following the definition of Machina and Schmeidler (1992), utility $W(.) : \mathbb{R}^{\Omega} \to \mathbb{R}$ is *probabilistically sophisticated* if $\forall \boldsymbol{y}, \boldsymbol{y}' \in \mathbb{R}^{\Omega}$ with the same cumulative distribution $\forall x$, $Pr(X \leq x; \boldsymbol{y}, \boldsymbol{\pi}) = Pr(X \leq x; \boldsymbol{y}', \boldsymbol{\pi})$ $\Rightarrow W(\boldsymbol{y}) = W(\boldsymbol{y}')$. In general, the probabilistic sophistication is not a particularly restrictive assumption.

and 20 confinement dairies, and in the 2006-2009 panel there are 11 grazers and 17 confinement counterparts. Table 5.1 reports the means and standard deviations of the relevant input and output variables by dairy system and production year, including milk output, herd size, crop acreage, and pasture acreage. These statistics, calculated within the two subsamples, are fairly representative of the whole dataset for the corresponding years.

For these dairies, the most important source of uncertainty was the fluctuation of milk and feed prices. Table A.1 shows the inflation-adjusted price indices from US Department of Agriculture as well as the sample-average milk prices calculated from the milk revenues and quantities in the data. The feed price index climbed from the baseline of 100 in 1990-1992 to a peak of 111-117 during 1996-1997, plateaued around 89-92 during 2001-2006, and spiked at 127-132 during 2008-2009 (due to the food crisis of 2008). The milk price index, which one would expect to follow the underlying trends in feed costs, frequently diverged from the movements of the feed price. Notably, milk price was rather volatile when the feed price was relatively stable.

Another source of uncertainty is weather. Reduced rainfall and lower temperatures generally limit forage production in dairy farming. Colder winters and hotter summers increase the physical stresses for cows and increase the costs associated with their dietary needs and veterinary cares than otherwise. During the two time intervals 2000-2004 and 2006-2009, the producers have experienced a dry autumn in 2001, a dry and warm winter in early 2002, and a dry spring in 2006, compared to their 20-year norms. No obvious weather shocks to milk outputs are observed in the data. It is likely because some inputs were adjusted to absorb major impacts of weather on milk production, impacting the cost of production.

4.4.2 Optimal Production Risks Under the SC Framework

Under the SC framework, the states of nature are indexed by calender year and assigned some probability distribution across states. While one could investigate the underlying likelihoods of relevant SC events in historical records, given the relatively short panel datasets at hand, this study simply assigns arbitrary probability distributions that represent four distinct probability-scenarios. To help interpret these scenarios, realized states are heuristically labeled as either “good,” “bad,” or “normal” states in terms of the average profit level in the sample. Specifically, in the 2000-2004 panel, years 2001 and 2004 are assumed to represent "good" states, years 2002 and 2003 "bad" states, and year 2000 a "normal" state. Similarly, in the 2006-2009 panel, years 2007 and 2008 are regarded as "good" states, and years 2006 and 2009 as "bad" states. These labels largely reflect the outcomes of market prices rather than those of weather conditions. The state probabilities in scenario P-1 (equal) are set all equally-likely (e.g., uniform state-probability of 0.2 for a 5-state case). Three other scenarios are derived by shifting these probabilities between good and bad states; probabilities are shifted from two bad to two good states respectively by 0.1 in P-2 (optimistic), shifted from two good to two bad states by 0.1 in P-3 (pessimistic), and shifted from the second worst and the second best to the worst and the best by 0.1 in P-4 (volatile).[16]

Value-added (VA) technologies for the two dairy systems are estimated separately, according to equation (4.7). The observed profits in relevant years are regarded as multi-dimensional SC outputs whereas herd size, crop acreage, and pasture acreage are used as short-term fixed inputs.[17] The assumption of constant returns to scale (CRS) is added to specification (4.7) (i.e., unrestricted $\sum_j \lambda_j$) to simplify optimal portfolio simulations across input-mixes in below.

Table 4.3 presents the summary of output-oriented technical efficiency and state-allocative efficiency scores (TE and SAE) by dairy system and balanced-panel sub-sample. In this application, the majority of decisions are found technically-efficient in most of the efficiency analyses. Confinement dairies are on average 80% and 85% technically efficient in the 2000-2004 and 2006-2009 panels respectively, indicating that the average confinement dairy was operating at the 80% and 85% SC profit-levels of the technically-efficeint peers. Grazers are on average 85% and 94% technically efficient in the panels of correspoding time periods. The rather high efficiency scores may be attributed to the relatively small degrees of freedom due to the small sample size and the selection effects by potential attrition of inefficient producers who might have gone out of operation at some point during the study periods.

Under each probability scenario of P-1 through P-4, SAE is measured as the ratio of the expected profit at a given SC portfolio decision (projected to the frontier)

[16]For example, probability scenarios for 2000-2004 are {0.2,0.2,0.2,0.2,0.2} in P-1 (equal), {0.2,0.3,0.1,0.1,0.3} in P-2 (optimistic), {0.2,0.1,0.3,0.3,0.1} in P-3 (pessimistic), and {0.2,0.3,0.1,0.3,0.1} in P-4 (volatile).

[17]Strictly speaking, these inputs have minor variations across production years (e.g., see table 5.1), so that the farm-averages of these variables are used.

to that of the state-allocatively efficient portfolio. For a given portfolio decision, its SAE score tends to vary across probability scenarios more substantially if the technology allows for a greater degree of technical substitution across states. At the sample level, higher prevalence of state-allocative inefficiency and its higher sensitivity across scenarios would indicate greater importance of formulating an SC portfolio with respect to state probabilities that effectively serve as the "prices" for SC incomes. For the 2000-2004 panels, the estimated SAE scores are very similar between the two dairy systems, averaging around 82% to 84%. For the 2006-2009 panels, these scores become higher among confinement (e.g., averaging at 91% to 96%) and lower among grazers (e.g., averaging at 76% to 88%). This is consistent with a view that confinement operations have become more homogenized over time, accompanied with a reduction in the SC substitutability, but intensive-grazing operations have become more diversified, accompanied with an increase in the SC substitutability, by which some grazers improved risk management through the means of production practices.

Optimal decisions along the two technological frontiers are simulated under three different risk-preference specifications: risk-neutral, maximin, and linear mean-MAD (mean absolute deviation) preferences. For each specification, the optimal portfolio of SC incomes is identified at the "tangency" between a technological frontier and an indifference curve.[18] The risk-neutral preferences (i.e., hyperplanes defined by state probabilities) maximize expected profits. Figure 4.3 illustrates how optimal expected-profits differ in revenue-mixes for "bad" state s and "good" state t under equal, optimistic, and pessimistic probability-scenarios (at points a, b, and c). The maximin preferences (i.e., Leontief utility: the most risk-averse case of increasing and concave preferences) maximize the minimum payment across states. The linear mean-MAD preferences for arbitrary risk-averseness (i.e., a special case of invariant preferences) maximize $\mu - k\phi$, given the trade-off between mean μ and MAD ϕ, in this case, for constant $k \in \{1/2, 1, 2\}$. Figure 4.4 depicts an optimal decision for maximin preferences at point a and that for a typical risk-averse utility at point b.[19]

In simulating optimal portfolios, short-term fixed assets $\boldsymbol{x}^f$ are set at the fol-

[18]Technically speaking, the indifference curves considered here are all non-smooth. These optimal decisions are easily estimated by linear programming. In future research, more general mean-MAD specification may be considered under quadratic programming.

[19]Indifference curves for linear mean-MAD preferences are characterized with kinked lines like those of maximin preferences but with larger angles. The associated optimal decisions differ from those of maximin preferences when the technological frontier is non-smooth like piecewise-linear DEA frontiers.

lowing ten input-mixes for confinement (c1-c10) and grazers (g1-g10). Holding herd size constant at 100 cows, the first 9 input-mixes are defined as the combinations of crop acreage in {200, 300, 400} and pasture acreage in {25, 50, 100} for confinement and the combinations of crop acreage in {50, 100, 150} and pasture acreage in {100, 150, 200} for grazers. Input-mixes c10 and g10 are chosen at the sample-averages in the corresponding subsamples. Under CRS, these input-mixes and the associated optimal portfolios can be scaled up or down by a constant proportion.

Table 4.4 presents the maximin and risk-neutral utility-levels at the optimal portfolio decisions.[20] The utility levels of confinement tend to vary with crop acreage while those of grazers tend to vary with pasture acreage. Also, the "good" states of years 2001 and 2004 positively impact the optimal utility of grazers, while the "bad" states of years 2006 and 2009 negatively affect the optimal utility of confinement. Under the representative case for input-mix c5 (100 cows, 300 crop acres, 50 pasture acres), the most risk-averse confinement producer would schedule the non-stochastic income of \$117k and \$62k at the frontiers of 2000-2004 and 2006-2009 technologies respectively. At input-mix g5 (100 cows, 100 crop acres, 150 pasture acres), the most risk-averse grazer would similarly earn the non-stochastic income of \$90k and \$84k at their frontiers respectively. The risk-neutral confinement producer with input c5 would obtain the expected income of \$127k in scenario P-1 (with ±\$4k in scenarios P-2 to P-4) and \$89k in P-1 (±\$12k in P-2 to P-4) respectively at the 2000-2004 and 2006-2009 frontiers, while the risk-neural grazer with input g5 would generate the expected income of \$121k in P-1 (±\$21k in P-2 to P-4) and \$88k in P-1 (±\$1k in P-2 to P-4) at their frontiers. It may be noted that these calculations do not account for the opportunity cost of fixed inputs.[21]

Similarly, tables 4.5 and 4.6 summarize the optimal, linear mean-MAD utility levels for selected input-mixes. While these utility levels ("$\mu - k\phi$") are not directly comparable to those of maximin or risk-neutral preferences in dollar terms, the expected value of optimal SC incomes ("μ") theoretically converges to that of the risk-neutral preferences as $k \to 0$ and to that of the maximin preferences as $k \to \infty$.

[20]For each of the point-estimate for utility levels, confidence interval (CI) can be calculated by a common bootstrapping method on efficiency score distributions (e.g., Simar and Wilson, 2000), which predicts a CI for a DEA frontier. To give some idea on the scale of precisions in this study, the length of the 95% CI measured in relation to the median uility-level is about 55% for maximin preferences, 50% for the risk-neutral preferences, and 47% for mean-MAD preferences.

[21] For example, input-mixes c5, c10, g5, and g10 would incur the cost of \$72k, \$86k, \$51k, and \$55k respectively at the rates of \$400/cow, \$70/crop-acre, and \$40/pasture-acre.

In the table, μ is generally close to the corresponding risk-neutral utility at $k = 0.5$ and maximin utility at $k = 2$ from table 4.4. At $k = 1$, the confinement producer with input-mix c5 obtains the expected income of \$127k in scenario P-1 (±\$10k in scenarios P-2 to P-4) with the associated MAD of \$8k (±\$8k in P-2 to P-4) at the 2000-2004 frontier and the expected income of \$76k in P-1 (±\$27 in P-2 to P-4) with the MAD of \$10k (±\$19 in P-2 to P-4) at the 2006-2009 frontier. Similarly, at $k = 1$, the grazer with input-mix g5 earns the expected income of \$118k in P-1 (±\$25k in P-2 to P-4) with the MAD of \$21k (±\$19k in P-2 to P-4) at the 2000-2004 frontier and the expected income of \$84k in P-1 (±\$3k in P-2 to P-4) with the MAD of less than \$1k (±\$2k in P-2 to P-4) at the 2006-2009 frontier. Overall, the optimal portfolios in these tables show how the optimal riskiness associated with each technology varies with producer risk-aversion and state probability scenario. The optimality of relatively low-risk portfolios under various degrees of risk-aversion and various probability scenarios (e.g., the 2000-2004 confinement frontier and the 2006-2009 grazing frontier) suggests that the uncertainty in SC incomes can be largely offset through production practices. Meanwhile, if a technology allows the producer to formulate portfolios of higher expected incomes at higher risks (e.g., the 2006-2009 confinement frontier and the 2000-2004 grazing frontier), taking higher risks is optimal under weak risk-aversion or an optimistic prospect for the uncertainty.

The results for optimal portfolios appear to reflect the factors that have been driving risks in dairy production. First, the optimal MAD levels tend to be higher for grazers than confinement in the 2000-2004 panel, and this tendency is reversed in the 2006-2009 panel, making the grazing system the lower-risk option. Second, such results in relative riskiness stem from the system-specific trends of increasing optimal riskiness for confinement and decreasing optimal riskiness for grazers. For confinement producers, larger-scale production and associated cost-savings under increased standardizations (e.g., Khanal et al., 2010; Winsten et al., 2010) appear to have increased the optimal riskiness in their SC decisions. Their incomes were highly influenced by the volatile milk prices in years 2006-2009 and the hikes of feed costs induced by the 2008 food crisis. On the other hand, grazers, who have been experimenting with their herd composition and pasture management in local production environments (e.g., Hanson et al., 2013), have increasingly insulated their operations from these market risks. The optimal decisions for grazers appear to have shifted toward low-market-risk practices through increased use of pasture to replace purchased feeds and more prevalent product differentiations under organic milk production.[22] As a result, at the scale of 100-cow dairy operations, the intensive-grazing system has become a preferred option to the confinement system under a relatively high risk-aversion or under a neutral-to-pessimistic prospect for market conditions.

4.4.3 Optimal Production Risks Under the OS Framework

In the outcome-state (OS) framework, production risks are analyzed in the pooled observations of production decisions across producers and production years. In particular, consider an estimation that decomposes a conditional profit distribution into a predicted frontier-output, a TE component, and a stochastic error component that represents outcome-states.[23] For the current dataset, common SFA estimations fail to produce sensible results due to the right-skewness of joint distributions for TE and stochastic error components (i.e., attributing nearly all deviations from the estimated frontier to technical inefficiency and none to stochasticity). For this decomposition, analogous estimation is specified under weighted COLS (WCOLS) as a convex combination of the OLS and COLS (corrected OLS, Greene, 1980).[24] The WCOLS specification for given weights $\lambda \in [0,1]$ placed on COLS and $1-\lambda$ on OLS is

$$y_{it} = f(x_{it};\boldsymbol{\beta}) + \varepsilon_{it} = [f(x_{it};\boldsymbol{\beta}) + \lambda\varepsilon_{max}] + [(1-\lambda)\varepsilon_{it}] + [\lambda(\varepsilon_{it} - \varepsilon_{max})] \qquad (4.10)$$

where $\varepsilon_{max} = \max_{it}\{\varepsilon_{it}\}$. WCOLS interprets the first component in brackets as the frontier-output, the second as stochastic error, and the third as technical inefficiency. This specification coincides with OLS as $\lambda \to 0$ and COLS as $\lambda \to 1$. Note that as parameter λ increases, the deviation is less attributed to stochastic error and more attributed to technical inefficiency and thus the increased frontier-output. Also, a higher variability of deviation ε_{it} tends to predict a larger constant shift in the frontier-output through greater ε_{max}. For given λ, the optimal portfolio is predicted as the

[22] Four out of eleven grazers in the dataset have become certified as organic milk producers in all or part of the 2006-2009 period, by which they earned the average of \$30/cwt or higher price in each year, compared to the milk prices of \$15.0, \$21,2, \$19.8, and \$13.7/cwt earned by non-organic grazers (and confinement dairies) correspondingly during years 2006-2009.

[23] One could additionally employ producer or year fixed effects, yet it would be unclear whether it assumes the fixity of production decisions or that of state realizations as well as how it relates to the distributional assumption.

[24] Alternatively, one may use some distributional assumption (e.g., modified OLS, Fried et al., 1993).

sum of the frontier-output for a given input-mix and the empirical distribution of stochastic errors.

Table 4.7 reports the summary statistics of optimal portfolios at two different input-mixes under WCOLS for $\lambda \in \{0, 0.05, 0.10\}$ as well as implied mean-SD utility-levels "$\mu - k\,\phi_{SD}$" for constant $k \in \{1/2, 1, 2\}$, or another special case of invariant preferences. The estimations are based on the OLS regression on the natural logarithm of profit on the natural logarithm of herd size.[25] At the 100-cow operational scale, the optimal portfolios for confinement have the expected income of \$71k at $\lambda = 0.05$ ($\pm$\$12k at $\lambda \in \{0, .10\}$) with the associated SD of \$58k ($\pm$\$3k at $\lambda \in \{0, .10\}$) for the 2000-2004 frontier and the expected income of \$52k at $\lambda = 0.05$ ($\pm$\$18k at $\lambda \in \{0, .10\}$) with the SD of \$78k ($\pm$\$4k at $\lambda \in \{0, .10\}$) for the 2006-2009 frontier. Similarly, the optimal portfolios for grazers consist of the expected income of \$72k at $\lambda = 0.05$ ($\pm$\$6k at $\lambda \in \{0, .10\}$) with the SD of \$43k ($\pm$\$3k at $\lambda \in \{0, .10\}$) for the 2000-2004 frontier and the expected income of \$64k at $\lambda = 0.05$ ($\pm$\$5k at $\lambda \in \{0, .10\}$) with the SD of \$37k ($\pm$\$2k at $\lambda \in \{0, .10\}$) for the 2006-2009 frontier. These portfolios can be ranked by mean-SD preferences for a given degree of risk aversion.

For a more general structure of risk-preferences, implicit utility levels can be ranked by a second-order stochastic dominance (SSD) test. For given λ, the test compares the optimal portfolios between confinement and grazers in each of the sub-panels I through IV defined for different operational scales of comparisons. When the minimum income in grazers' portfolio $F_G(:)$ is higher than that of confinement's $F_C(:)$, for instance, the null hypothesis is that $F_G(:)$ second-order stochastically dominates $F_C(:)$ (i.e. $H_0 : \int_{\underline{z}}^{z} F_C(t) - F_G(t)\,\mathrm{d}t \geq 0$ for all $z \in [\underline{z}, \bar{z}]$) against the alternative hypothesis of such a SSD relationship (i.e. $H_1 : \int_{\underline{z}}^{z} F_C(t) - F_G(t)\,\mathrm{d}t < 0$ for some $z \in [\underline{z}, \bar{z}]$). Following Barrett and Donald (2003), test statistic $\hat{S} = \sqrt{(N_G N_C)/(N_G + N_C)}\,\sup_z \left\{\int_{\underline{z}}^{z} F_C(t) - F_G(t)\,\mathrm{d}t\right\}$ for N_G and N_C sample observations is used as an estimate for the supremum of the cumulative difference between the two distributions. At the 100-cow operational scale, the test finds no SSD relationship for the 2000-2004 frontiers (sub-panel I) and the SSD of grazer's portfolio over confinement's for the 2006-2009 frontiers (sub-panel III). At the sample-average

[25]When included in the regression, the coefficients of crop acreage and pasture tend to be negative in many situations. Since such predictions are inconsistent with free-disposability, these variables are omitted from the regression.

operational scales, it finds the SSD of confinement's portfolio for the 2000-2004 frontiers (sub-panel II) and little evidence of SSD for the 2006-2009 frontiers (sub-panel IV).

The results under the OS framework are generally in agreement with those under the SC framework. However, the current WCOLS analysis depends on arbitrary parameter λ for weighting stochasticity against technical inefficiency in the optimal portfolio calculations, unlike the endogenous weight determinations under SFA. The above exercise, in turn, underscores the importance of distributional assumptions in representing production risks as outcome-states.

4.5 Conclusions

This study has proposed a simple application of the state-contingent (SC) approach in a balanced panel data setting, with an empirical analysis of Maryland dairy production data. The empirical challenge is to identify relevant heterogeneity in state realizations and model *ex ante* production decisions that were formulated to manage risks in those states. Assuming that SC decisions are time-invariant and that a subset of those decisions is observed under cross-sectionally homogeneous state realizations over multiple time periods, this study has proposed to analyze balanced panel data as cross-sectional observations of SC incomes. The results from separate panel analyses on years 2000-2004 and 2006-2009 suggest that the optimal portfolios for confinement and grazers have become riskier and less risky respectively. An analogous risk analysis under the outcome-state (OS) framework yields apparently similar results at some moderate degrees of stochasticity. Future research may be targeted at modifying the current assumptions for different data structures, extending optimal portfolio simulations for alternative risk preferences, or deriving formal statistical properties for the proposed SC risk analysis. More importantly, breakthroughs in the empirical analysis of production risks await novel methods for improved accommodations of the theoretical, SC production trade-offs in empirical settings. The value of adopting the SC approach increases with the extent that producers actively manage risks through the means of production.

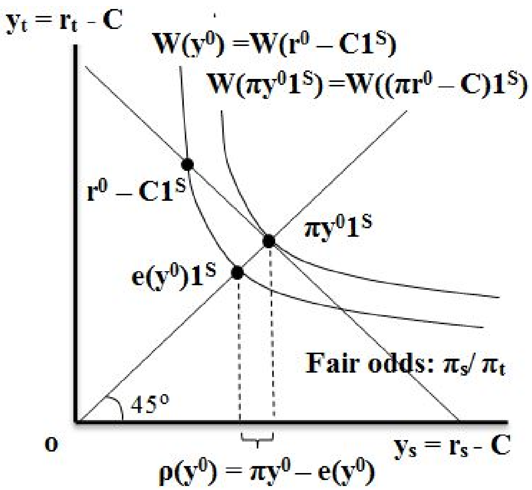

Figure 4.1: Risk Premium in State-Contingent Preferences

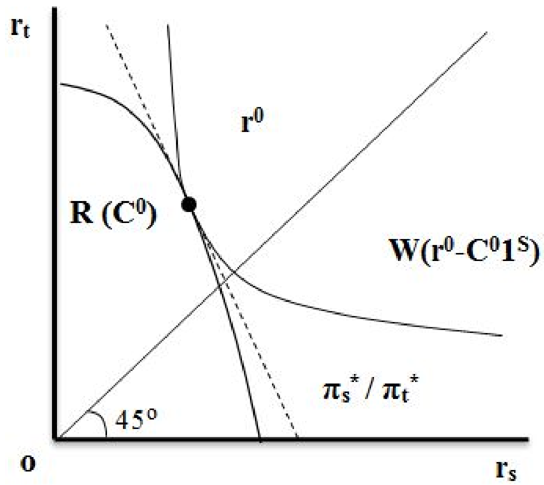

Figure 4.2: Optimal State-Contingent Revenues

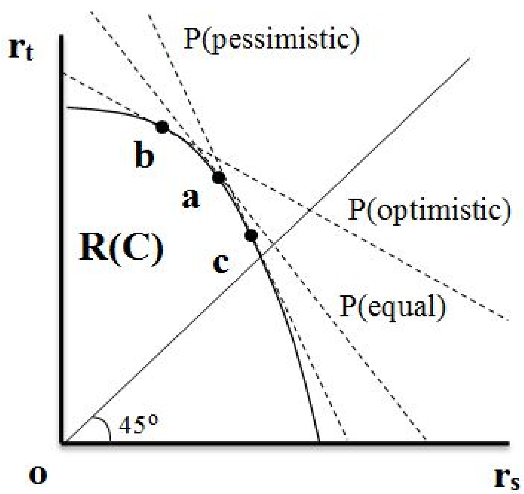

Figure 4.3: Optimal Decisions under Risk Neutral Preferences

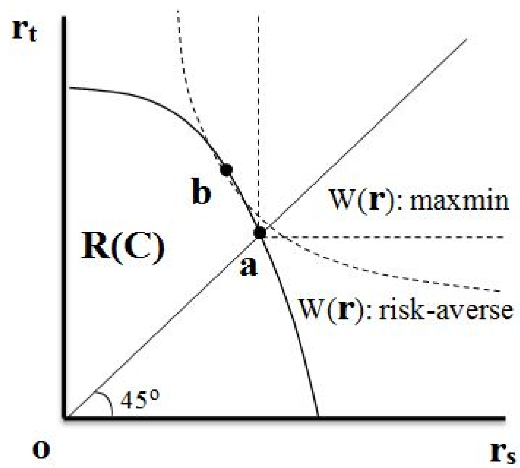

Figure 4.4: Optimal Decisions under Maxmin and Risk-averse Preferences

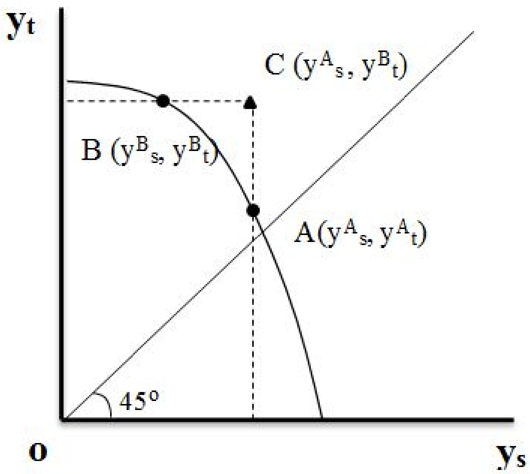

Figure 4.5: Violation of Assumption A-SC1

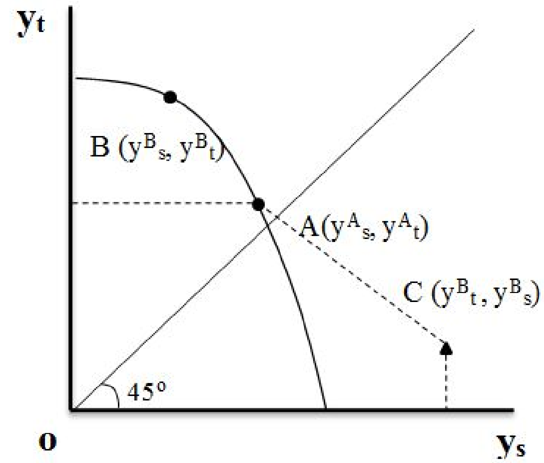

Figure 4.6: Violation of Assumption A-SC2

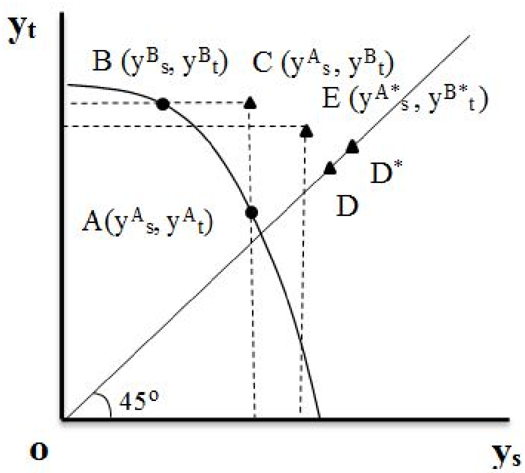

Figure 4.7: Violation of Assumption A-OS1 (1)

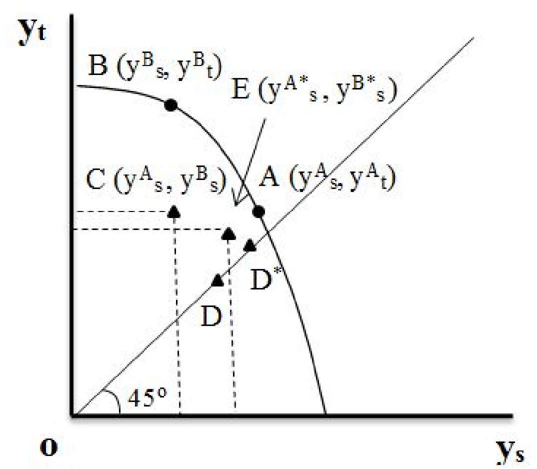

Figure 4.8: Violation of Assumption A-OS1 (2)

Table 4.1: Yearly Means and Standard Deviations of Variables By Dairy System and Subsample

	Confinement										Grazers									
	Profit		Milk		Herd size		Crop Ac.		Pasture		Profit		Milk		Herdsize		Crop Ac.		Pasture	
Year	μ	SD	μ	SD	μ	SD	μ	SD	μ	SD	μ	SD	μ	SD	μ	SD	μ	SD	μ	SD
A. Subsample Restricted to 2000-2004 Balanced Panel Data																				
2000	82	94	24.4	16.5	119	69	294	149	49	38	72	40	13.3	3.2	85	19	130	104	164	63
2001	112	124	24.4	16.5	121	72	294	149	49	38	79	50	12.5	3.7	85	23	130	104	164	63
2002	85	92	25.4	17.8	124	74	294	149	49	38	36	52	12.1	4.2	86	25	130	104	164	63
2003	56	81	24.3	17.8	126	78	294	149	48	38	28	50	12.1	4.1	90	29	130	104	164	63
2004	66	81	25.0	17.4	128	75	294	149	48	38	69	34	12.8	3.8	93	30	134	110	164	63
Avg.	80	94	24.7	17.2	124	74	294	149	48	38	57	45	12.5	3.8	88	25	131	105	164	63
Avg.*	72	88	23.0	16.1	117	69	294	154	48	38	56	45	12.3	3.8	86	26	129	104	162	64
B. Subsample Restricted to 2006-2009 Balanced Panel Data																				
2006	55	83	30.1	22.7	147	93	314	180	52	39	50	38	13.3	8.0	95	38	128	113	136	50
2007	108	145	31.9	23.4	154	97	319	184	54	42	66	59	13.0	8.4	98	39	128	113	136	50
2008	84	111	32.3	24.3	155	99	328	180	55	43	67	41	12.9	10.0	99	39	147	155	137	50
2009	33	52	32.0	23.5	158	102	328	180	55	43	52	39	13.5	10.4	105	39	150	166	137	50
Avg.	70	98	31.6	23.5	153	98	322	181	54	42	59	44	13.2	9.2	99	38	138	137	136	50
Avg.*	56	90	28.7	21.5	144	90	305	178	52	41	51	42	12.0	8.5	90	38	125	130	128	49

1. 2000-2004 analysis contains 11 grazers and 20 confinement dairies. 2006-2009 analysis contains 11 grazers and 17 confinement dairies. “Avg.*” is the average of the corresponding years from the full sample, including the observations dropped in the subsample analyses due to incomplete data. The full sample is unbalanced panel during 1995-2009 containing 19 grazers and 48 confinement dairies.

2. Profit variable ($1,000) is the reported total farm-related income minus the total expense, adjusted in 2009 dollars. Milk output (reported in 1,000 cwt = million pounds) accounts for generating about 87% of the total income. In a short run, herdsize (cows), crop acreage (acres), and pasture (acres) rarely change for a given balanced-panel sample.

Table 4.2: Prices Indices and Sample-Average Milk Price Received

				Inflation Adjusted		Avg. Milk Price	
Year	Dairy	Feed	Inflation	Dairy	Feed	Graz.	Conf.
1995	98	103	2.58	91	96	19.0	18.6
1996	114	129	2.83	103	117	20.7	20.4
1997	102	125	2.44	90	111	19.5	18.5
1998	119	110	1.43	104	96	20.7	20.7
1999	110	100	2.06	94	86	19.7	20.1
2000	94	102	3.40	78	85	17.0	16.8
2001	115	109	2.79	94	89	20.3	20.1
2002	93	112	2.06	75	90	16.0	16.5
2003	96	114	2.82	75	89	15.5	16.1
2004	123	121	3.46	94	92	20.1	19.6
2005	116	117	3.65	86	87	19.3	18.3
2006	99	124	3.61	72	90	17.5	15.1
2007	146	149	2.56	104	106	22.9	21.1
2008	140	191	3.99	97	132	23.0	20.1
2009	93	184	0.02	64	127	18.9	13.9

1. External sources: USDA's Indexes of Prices Received and Paid by Farmers, United States (1990-1992=100) and CPI for the Northeast. Annual inflation rate is calculated as the percentage change in the CPI from previous year.

2. Milk price ($) is inferred for each producer-year by dividing milk sales revenue by milk production quantity.

Table 4.3: Summary of Estimated Technical and State-Allocative Efficiencies

		A. 2000-2004 Analysis							B. 2006-2009 Analysis						
				Values at Percentiles							Values at Percentiles				
	Scores	Mean	S.D.	Min	25th	50th	75th	Max	Mean	S.D.	Min	25th	50th	75th	Max
Confinement															
(1)	TE	0.804	0.279	0.156	0.595	0.997	1.000	1	0.715	0.341	0.128	0.433	1.000	1.000	1
(2)	SAE: P-1	0.827	0.264	-0.069	0.788	0.936	0.981	1	0.958	0.172	0.297	0.899	0.900	0.999	1
(3)	SAE: P-2	0.824	0.261	0.007	0.775	0.930	0.995	1	0.931	0.164	0.375	0.857	0.876	1.000	1
(4)	SAE: P-3	0.828	0.270	-0.148	0.800	0.924	0.981	1	0.975	0.194	0.192	0.965	0.923	0.997	1
(5)	SAE: P-4	0.817	0.271	-0.137	0.760	0.906	0.975	1	0.907	0.178	0.236	0.856	0.875	0.965	1
Grazers															
(6)	TE	0.853	0.233	0.340	0.750	1.000	1.000	1	0.937	0.208	0.311	1.000	1.000	1.000	1
(7)	SAE: P-1	0.834	0.242	0.217	0.779	0.950	0.985	1	0.847	0.200	0.397	0.666	0.784	0.943	1
(8)	SAE: P-2	0.838	0.218	0.280	0.756	0.944	0.984	1	0.876	0.212	0.409	0.697	0.794	0.970	1
(9)	SAE: P-3	0.825	0.279	0.136	0.758	0.963	1.000	1	0.756	0.221	0.386	0.592	0.765	0.979	1
(10)	SAE: P-4	0.826	0.247	0.231	0.751	0.940	0.987	1	0.797	0.210	0.383	0.650	0.768	0.923	1

1. Probability scenarios: P-1 (Equal), P-2 (Optimistic), P-3 (Pessimistic), and P-4 (Volatile).

Table 4.4: Risk Comparisons in State-Contingent Framework, Maxmin & Risk-Neutrality

Fixed Inputs	A. 2000-2004 Analysis					B. 2006-2009 Analysis				
		Risk-Neutral					Risk-Neutral			
{cow, crop acre, pasture}	Maxmin	P-1	P-2	P-3	P-4	Maxmin	P-1	P-2	P-3	P-4
Confinement										
c1: {100, 200, 25}	98	115	119	118	117	55	89	104	71	90
c2: {100, 200, 50}	107	115	119	118	117	62	89	104	68	90
c3: {100, 200, 100}	107	115	119	118	117	62	89	104	68	90
c4: {100, 300, 25}	105	121	125	124	122	55	89	106	71	90
c5: {100, 300, 50}	117	127	130	131	126	62	89	106	68	90
c6: {100, 300, 100}	117	127	130	131	126	62	89	106	68	90
c7: {100, 400, 25}	105	121	125	124	122	55	90	108	71	91
c8: {100, 400, 50}	117	127	130	131	126	62	90	108	68	91
c9: {100, 400, 100}	117	127	130	131	126	62	90	108	68	91
c10: {sample avg.*}	141	154	158	158	153	92	136	159	107	137
Grazers										
g1: {100, 50, 100}	63	78	86	70	80	56	58	61	58	59
g2: {100, 50, 150}	79	109	121	97	112	80	84	86	85	86
g3: {100, 50, 200}	79	122	137	108	124	88	97	93	101	99
g4: {100, 100, 100}	73	95	103	86	98	61	65	71	63	66
g5: {100, 100, 150}	90	121	133	109	124	84	88	88	88	89
g6: {100, 100, 200}	90	122	137	109	124	88	97	93	101	99
g7: {100, 150, 100}	84	111	120	102	115	63	72	80	68	73
g8: {100, 150, 150}	90	121	133	110	124	84	88	88	88	89
g9: {100, 150, 200}	90	122	137	110	124	88	97	93	101	99
g10: {sample avg.*}	80	107	120	97	109	81	84	86	83	85

1. *Sample average input mixes for cow, crop acreage, and pasture acreage are {124,294,48} for 2000-2004 confinement, {88,131,164} for 2000-2004 grazers, {153,322,54} for 2006-2009 confinement, and {99,138,136} for 2006-2009 grazers.

2. Probability scenarios: P-1 (Equal), P-2 (Optimistic), P-3 (Pessimistic), and P-4 (Volatile).

Table 4.5: Risk Comparisons in State-Contingent Framework, Linear Mean-MAD (Confinement)

	A. 2000-2004 Analysis												B. 2006-2009 Analysis											
Fixed Inputs	P-1			P-2			P-3			P-4			P-1			P-2			P-3			P-4		
{cow, crop, past.}	$\mu - k\phi$	μ	ϕ	$\mu - k\phi$	μ	ϕ	$\mu - k\phi$	μ	ϕ	$\mu - k\phi$	μ	ϕ	$\mu - k\phi$	μ	ϕ	$\mu - k\phi$	μ	ϕ	$\mu - k\phi$	μ	ϕ	$\mu - k\phi$	μ	ϕ
Confinement																								
k=0.5																								
c2: {100,200,50}	112	115	6	116	118	4	109	112	6	112	116	7	71	82	21	88	104	32	65	71	10	75	89	28
c5: {100,300,50}	123	127	8	127	130	6	120	124	7	122	126	7	71	81	20	89	106	35	67	72	10	75	89	28
c8: {100,400,50}	123	127	8	127	130	6	120	124	7	122	126	7	71	81	20	89	108	38	67	72	10	75	89	28
c10: {sample av}	149	154	9	155	158	6	145	149	9	148	153	9	109	124	30	135	159	49	101	110	18	115	137	43
k=1																								
c2: {100,200,50}	109	115	6	114	118	4	107	107	0	109	116	7	64	73	10	73	102	29	62	62	0	68	78	10
c5: {100,300,50}	119	127	8	125	130	5	117	117	0	119	126	7	65	75	10	73	102	29	62	62	0	69	80	11
c8: {100,400,50}	119	127	8	125	130	5	117	117	0	119	126	7	65	75	10	73	102	29	62	62	0	69	80	11
c10: {sample av}	144	154	9	151	158	6	142	143	1	144	153	9	98	115	17	111	156	45	93	105	12	106	123	17
k=2																								
c2: {100,200,50}	107	107	0	109	118	4	107	107	0	107	107	0	62	62	0	62	65	1	62	62	0	62	65	1
c5: {100,300,50}	117	117	0	120	129	5	117	117	0	117	117	0	62	62	0	63	75	6	62	62	0	63	75	6
c8: {100,400,50}	117	117	0	120	129	5	117	117	0	117	117	0	62	62	0	63	75	6	62	62	0	63	75	6
c10: {sample av}	141	143	1	145	158	6	141	141	0	141	141	0	92	92	0	95	109	7	92	92	0	95	109	7

1. μ and ϕ represent mean and mean absolute deviation (MAD) respectively.

Table 4.6: Risk Comparisons in State-Contingent Framework, Linear Mean-MAD (Grazers)

	A. 2000-2004 Analysis												B. 2006-2009 Analysis											
Fixed Inputs	P-1			P-2			P-3			P-4			P-1			P-2			P-3			P-4		
{cow, crop, past.}	$\mu - k\,\phi$	μ	ϕ	$\mu - k\,\phi$	μ	ϕ	$\mu - k\,\phi$	μ	ϕ	$\mu - k\,\phi$	μ	ϕ	$\mu - k\,\phi$	μ	ϕ	$\mu - k\,\phi$	μ	ϕ	$\mu - k\,\phi$	μ	ϕ	$\mu - k\,\phi$	μ	ϕ
Grazers																								
k=0.5																								
g4: {100,100,100}	85	95	19	94	103	18	81	86	11	88	95	15	62	64	3	65	68	6	61	62	1	64	66	4
g5: {100,100,150}	109	121	24	123	133	21	99	109	21	110	124	28	85	87	3	86	87	2	86	87	2	87	88	2
g6: {100,100,200}	109	121	24	123	135	23	99	109	21	110	124	28	91	97	12	89	92	5	95	99	9	92	99	15
g10: {sample av}	96	106	21	109	118	20	90	97	14	98	109	22	82	83	2	83	85	4	81	82	0	83	84	2
k=1																								
g4: {100,100,100}	78	83	5	85	103	18	76	83	7	81	90	9	61	63	1	62	66	3	61	61	0	62	66	4
g5: {100,100,150}	97	118	21	112	133	21	91	93	2	97	118	21	84	84	0	85	87	2	85	87	2	85	88	2
g6: {100,100,200}	97	118	21	113	131	18	91	93	2	97	118	21	88	88	0	88	88	0	90	99	9	89	93	4
g10: {sample av}	87	92	5	100	116	16	84	92	8	90	92	3	81	81	0	82	84	2	81	82	0	82	83	1
k=2																								
g4: {100,100,100}	74	76	1	78	83	2	73	73	0	78	83	2	61	61	0	61	61	0	61	61	0	61	61	0
g5: {100,100,150}	91	93	1	97	118	11	90	90	0	92	93	1	84	84	0	84	84	0	84	84	0	84	84	0
g6: {100,100,200}	91	93	1	97	118	11	90	90	0	92	93	1	88	88	0	88	88	0	88	88	0	88	88	0
g10: {sample av}	82	92	5	87	92	3	80	80	0	87	92	3	81	81	0	81	81	0	81	81	0	81	81	0

1. μ and ϕ represent mean and mean absolute deviation (MAD) respectively.

Table 4.7: Risk Comparisons in Outcome-State Framework

	Mean	S.D.	$\mu - k\,\phi_{SD}$ k =0.5	k =1	k =2	Values at Percentiles Min.	25th	50th	75th	Max	SSD Test S-hat	P-val
A. 2000-2004 Analysis												
I. Comparison at the Same Input-level												
Confinement (cow=100)												
λ =0	59	61	28	-2	-63	-106	28	68	95	303	-	-
λ =0.05	71	58	42	13	-45	-85	41	80	105	303	-	-
λ =0.10	83	55	56	28	-26	-65	55	92	116	303	-	-
Grazers (cow=100)												
λ =0	66	46	43	21	-25	-78	45	67	90	184	59	0.103
λ =0.05	72	43	50	29	-15	-65	52	73	95	184	41	0.178
λ =0.10	78	41	57	37	-4	-52	59	79	100	184	27	0.276
II. Comparison at the Sample-Average Input-levels												
Confinement (cow=124)												
λ =0	84	61	53	23	-38	-81	53	93	120	328	$159^{\dagger}$	0.006
λ =0.05	96	58	67	38	-20	-60	66	105	130	328	$196^{\ddagger}$	< 0.000
λ =0.10	108	55	81	54	-1	-40	80	117	141	328	$234^{\ddagger}$	< 0.000
Grazers (cow=88)												
λ =0	57	46	34	12	-34	-87	36	58	81	175	-	-
λ =0.05	63	43	41	20	-24	-74	43	64	86	175	-	-
λ =0.10	69	41	48	28	-13	-61	50	70	91	175	-	-
B. 2006-2009 Analysis												
I. Comparison at the Same Input-level												
Confinement (cow=100)												
λ =0	33	82	-8	-49	-131	-137	-10	39	75	400	-	-
λ =0.05	52	78	12	-27	-105	-110	11	57	91	400	-	-
λ =0.10	70	74	33	-4	-78	-83	31	75	107	400	-	-
Grazers (cow=100)												
λ =0	59	39	40	20	-19	-4	35	54	92	149	$162^{\ddagger}$	< 0.000
λ =0.05	64	37	45	27	-10	3	41	58	94	149	$96^{\dagger}$	0.005
λ =0.10	68	35	51	33	-2	11	46	63	97	149	$61^{\dagger}$	0.033
II. Comparison at the Sample-Average Input-levels												
Confinement (cow=153)												
λ =0	70	82	29	-12	-95	-101	27	75	111	437	-	-
λ =0.05	88	78	49	10	-68	-74	47	93	127	437	-	-
λ =0.10	106	74	69	32	-42	-47	68	111	144	437	-	-
Grazers (cow=99)												
λ =0	59	39	39	20	-19	-5	34	53	91	148	$51\diamond$	0.099
λ =0.05	63	37	45	26	-11	3	40	58	94	148	31	0.225
λ =0.10	68	35	50	33	-2	10	46	63	97	148	18	0.345

1. Statistical significance of the SSD Test: $\ddagger$ $\alpha = 0.01$, $\dagger$ $\alpha = 0.05$, $\diamond$ $\alpha = 0.1$.

2. Income distributions are simulated using weighted COLS (WCOLS) in equation (4.10) as explained in the text.

3. $\mu - k\,\phi_{SD}$ represents the mean-SD (standard deviation) preference level for mean μ, SD ϕ_{SD}, and constant k.

4. SSD test (Barrett and Donald, 2003) compares income distributions of confinement and grazers for each input mix and value of λ. Reported $\hat{S}$-values are in the unit of 1,000's.

Chapter 5: Integrating Efficiency Concepts in Technology Approximation: A Weighted DEA Approach

Kota Minegishi†

Abstract

A method is developed to integrate the efficiency concepts of technical, allocative, and scale inefficiencies (TI, AI, and SI) into the variable returns to scale (VRS) frontier approximation in Data Envelopment Analysis (DEA). The proposed weighted DEA (WDEA) approach takes a weighted average of the profit, constant returns to scale (CRS), and VRS frontiers, so that the technical feasibility of a VRS frontier is extended toward scale- and allocatively-efficient decisions. A weight selection rule is constructed based on the empirical performance of the VRS estimator via the local confidence interval of Kneip et al. (2008). The resulting WDEA frontier is consistent and more efficient than the VRS frontier under the maintained properties of a data generating process. The potential estimation efficiency gain arises from exploiting sample correlations among TI, AI, and SI. An application to Maryland dairy production data finds that technical efficiency is on average 7.5% to 9.2% lower under the WDEA results than under the VRS counterparts.

Keywords: Data envelopment analysis, Technical Efficiency, Allocative Efficiency, Scale Efficiency, Production Economics

JEL *Codes*: D22, Q12, C44

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5.1 Introduction

The concept of optimality and the subsequent definition of inefficiency depends on the focus of benchmarking by a relevant "frontier" of decision possibilities. In the tradition of production economics, three concepts of optimality stand out. Technical inefficiency (TI) assesses the extent of feasible output expansions for given inputs (or input reductions for given outputs) relative to the technological frontier of input-output decisions. Allocative and scale inefficiencies (AI and SI) represent the extents of forgone opportunities by the misallocation of resources and the suboptimal scales of operations respectively, relative to the frontier of revenue maximization (or cost minimization) and the frontier of linear-homogeneous production process (i.e. constant returns to scale; CRS). Numerous empirical studies have analyzed TI while paying little attention to AI or SI.

However, the interconnections among the concepts of TI, AI, and SI suggest an opportunity to improve technological frontier estimations. Conceptually, TI is a gap between an input-output decision and a technological frontier, and AI and SI are the gaps between the technological frontier and its outer frontiers of different benchmarking focuses. Empirically, the pivotal role of a technological frontier implies that the most efficient estimation strategy entails a joint specification of the frontier and these inefficiency concepts. In the parametric frontier literature, such simultaneous estimations have been developed mainly by incorporating AI into the optimal factor demands from cost minimization (e.g., Yotopoulos and Lau, 1973; Schmidt and Lovell, 1979, 1980; Kumbhakar, 1989, 1997; Kumbhakar and Wang, 2006; Kumbhakar and Tsionas, 2011). For nonparametric frontier models like Data Envelopment Analysis (DEA), on the other hand, there is no coherent estimation technique that integrates these efficiency concepts. This gap in knowledge is partially filled in this article.

Inefficient DEA estimations manifest themselves in the form of a limited ability to discriminate individual TI measurements. Efficiency analysis with a small sample size tends to find an unexpectedly large number of observations being fully technically-efficient, a pervasive concern in the nonparametric frontier literature (e.g., Dyson et al., 2001; Podinovski and Thanassoulis, 2007). One strand of literature tackles this issue by applying direct value judgments (i.e. shadow price restrictions) based on perceived importance of inputs and outputs (e.g., Allen et al., 1997; Thanassoulis et al., 2004) or so-called assurance regions (e.g., Dyson and Thanassoulis, 1988; Thompson et al., 1990; Sarrico and Dyson, 2004; Podinovski, 2004a; Tracy and Chen, 2004; Khalili et al., 2010). Other lines of research incorporate additional knowledge about production processes or constrain the range of technological parameters so as to increase estimation efficiency. Examples include weak disposability of inputs or undesirable outputs (Chung et al., 1997; Scheel, 2001; Seiford and Zhu, 2002; Kuosmanen, 2005; Podinovski and Kuosmanen, 2011), non-discretionary factors (Ruggiero, 1998), unobserved decisions (Thanassoulis and Allen, 1998; Allen and Thanassoulis, 2004), selective linear homogeneity (Podinovski, 2004c; Podinovski and Thanassoulis, 2007), and prescribed producer trade-offs (Podinovski, 2004b). Following the second strand of literature, this article refines a variable returns to scale (VRS) frontier estimation by calibrating the degrees of technical substitution and linear homogeneity, based on sample-level properties of AI and SI respectively. The method is a variant of the DEA frontier bounds of Chambers and Quiggin (1998) and closely related to the allocative inefficiency bounds of Kuosmanen and Post (2001).

Namely, this study proposes a weighted DEA (WDEA) approach that estimates a technological frontier as a weighted average of the profit, CRS, and VRS frontiers. By integrating the concepts of TI, AI, and SI, it enhances the discriminatory power of DEA. An optimal weight selection rule is devised based on the empirical performance of the VRS estimator via the local confidence interval proposed by Kneip et al. (2008). The resulting WDEA frontier is consistent and more efficient than the VRS frontier under the maintained properties of a data generating process. The potential estimation efficiency gain arises from exploiting sample correlations among TI, AI, and SI.

In a single-input single-output (x-y) space, figure 5.1 illustrates the concept of WDEA for the relationships among the CRS, VRS and postulated technological frontiers (depicted as a solid-curve). The optimal projections of the decision at point A to the VRS and CRS frontiers are shown at points B and C, yielding the conventional measures of TI and SI as distances AB and BC respectively. WDEA postulates a technological frontier through a weighted average of the VRS and CRS frontiers (i.e., somewhere between the inner and outer frontier-approximations). The new TI and SI measurements under WDEA are distances $AD(> AB)$ and $DC(< BC)$ where point D denotes the projection of point A onto the WDEA technological frontier. A parallel refinement of the frontier approximation can be obtained using similar relationships among the profit, VRS, and postulated frontiers. Together, these refinements are formalized under a weighted-average of the profit, CRS, and VRS frontiers.

In the following, section 2 presents the WDEA approach, and section 3 applies the method to Maryland dairy production data, followed by conclusions in section 4.

5.2 Methods

Technical, allocative, and scale inefficiencies (TI, AI, and SI) are measured by the directional distance function of Chambers and Quiggin (1998). Its additive nature is notationally well-suited for describing the weighted average of these inefficiency concepts. The section consists of the descriptions of preliminary concepts, a weighted DEA (WDEA) approach, and a weight selection for WDEA.

5.2.1 Preliminaries

Notations and preliminary concepts are defined as follows. Technology T is a set of feasible input-output bundles, or $T = \{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^L_+ \times \mathbb{R}^M_+ : \ \boldsymbol{x} \text{ can produce } \boldsymbol{y}\}$ where

- A.1 T is closed.
- A.2 T satisfies free-disposability: $(\boldsymbol{x}, \boldsymbol{y}) \in T$ and $(-\boldsymbol{x}, \boldsymbol{y}) \geq (-\boldsymbol{x}', \boldsymbol{y}') \Rightarrow (\boldsymbol{x}', \boldsymbol{y}') \in T$.
- A.3 T is convex: $(\boldsymbol{x}, \boldsymbol{y}), (\boldsymbol{x}', \boldsymbol{y}') \in T \ \Rightarrow \forall \lambda \in [0,1], \forall (\lambda \boldsymbol{x} + (1-\lambda)\boldsymbol{x}', \lambda \boldsymbol{y} + (1-\lambda)\boldsymbol{y}') \in T$.

The boundary of a technology is referred to as *technological frontier*. T can be completely characterized by the directional distance function of Chambers and Quiggin (1998)[1] in the sense that $(\boldsymbol{x}, \boldsymbol{y}) \in T \Leftrightarrow D_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y) \geq 0$ where

$$D_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y) = \max\{b \in \mathbb{R} : (\boldsymbol{x} - b\boldsymbol{g}_x, \boldsymbol{y} + b\boldsymbol{g}_y) \in T\}. \tag{5.1}$$

Function $D_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y)$ measures the distance between point $(\boldsymbol{x}, \boldsymbol{y})$ and the frontier of technology T in direction $(-\boldsymbol{g}_x, \boldsymbol{g}_y)$, representing technical inefficiency (TI). As a special case, setting direction $(-\boldsymbol{g}_x, \boldsymbol{g}_y) = (-\boldsymbol{x}_0, \boldsymbol{0})$ yields an input-oriented, radial TI measurement, which is equivalent to Shephard's input distance function $\theta_V(\boldsymbol{x}_0, \boldsymbol{y}_0) =$

[1] The directional distance function is the technology-counterpart to the shortage function of Luenberger (1994).

$\max\{\theta : \boldsymbol{x}_0/\theta \in V(\boldsymbol{y}_0)\} \geq 1$ for the input set $V(\boldsymbol{y})$ associated with technology T. Similarly, setting direction $(-\boldsymbol{g}_x, \boldsymbol{g}_y) = (\boldsymbol{0}, \boldsymbol{y}_0)$ leads to an output-oriented, radial TI measurement, or the inverse of Farrell's output efficiency $\phi_Y(\boldsymbol{x}_0, \boldsymbol{y}_0) = \max\{\phi : \phi\boldsymbol{y}_0 \in Y(\boldsymbol{x}_0)\} \geq 1$ for the output set $Y(\boldsymbol{x})$ associated with T.[2]

Profit function $\pi_T(\boldsymbol{w}, \boldsymbol{p})$ attains the highest production value in technology T for given input-output prices $(\boldsymbol{w}, \boldsymbol{p}) \in \mathbb{R}_+^{L+M}$;

$$\begin{aligned}
\pi_T(\boldsymbol{w}, \boldsymbol{p}) &= \max_{\boldsymbol{x},\boldsymbol{y}}\{\boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x} :\ (\boldsymbol{x}, \boldsymbol{y}) \in T\} \\
&= \max_{\boldsymbol{x},\boldsymbol{y}}\{\boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x} + D_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y)(\boldsymbol{p}\boldsymbol{g}_y + \boldsymbol{w}\boldsymbol{g}_x)\}
\end{aligned} \tag{5.2}$$

where the second expression follows from the definition of directional distance function $(\boldsymbol{x} - D_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y)\boldsymbol{g}_x, \boldsymbol{y} + D_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y)\boldsymbol{g}_y) \in T$. The duality between the profit function and the directional distance function (Chambers et al., 1998) yields;

$$D_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y) = \min_{\boldsymbol{w}', \boldsymbol{p}'}\left\{\frac{\pi_T(\boldsymbol{p}', \boldsymbol{w}') - (\boldsymbol{p}'\boldsymbol{y} - \boldsymbol{w}'\boldsymbol{x})}{\boldsymbol{p}'\boldsymbol{g}_y + \boldsymbol{w}'\boldsymbol{g}_x}\right\}, \tag{5.3}$$

which shows that in the set of supporting hyperplanes for technology T, TI is obtained at the shadow values that evaluate the decision most favorably. For given market prices $(\boldsymbol{w}, \boldsymbol{p})$, profit inefficiency (PI) is defined as

$$D_{PF}(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y) = \frac{\pi_T(\boldsymbol{p}, \boldsymbol{w}) - (\boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x})}{\boldsymbol{p}\boldsymbol{g}_y + \boldsymbol{w}\boldsymbol{g}_x} \tag{5.4}$$

where the subscript PF refers to profit-function (PF) technology $T_{PF}(\boldsymbol{w}, \boldsymbol{p})$, which is cast as a hypothetical technology that envelopes T under linear techical substitutability and takes the form of a half-space bounded by the profit function, or $T_{PF}(\boldsymbol{w}, \boldsymbol{p}) = \{(\boldsymbol{x}, \boldsymbol{y}) :\ \boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x} \leq \pi_T(\boldsymbol{w}, \boldsymbol{p})\}$. Strictly speaking, the PI in (5.4) and the PF technology should be represented conditionally on $(\boldsymbol{w}, \boldsymbol{p})$, yet this notation is omitted by assuming fixed market prices. At direction $(-\boldsymbol{g}_x, \boldsymbol{g}_y) = (\boldsymbol{0}, \boldsymbol{g}_y)$ or $(-\boldsymbol{g}_x, \boldsymbol{g}_y) = (-\boldsymbol{g}_x, \boldsymbol{0})$, PF frontier reduces to a revenue or cost frontier for the associated revenue or cost function respectively. According to additive decomposition $PI = TI + AI$,[3] the difference between the PI in (5.4) and the TI in (5.3) defines

[2] $\theta_V(\boldsymbol{x}_0, \boldsymbol{y}_0) = 1/(1 - D_T(\boldsymbol{x}_0, \boldsymbol{y}_0; \boldsymbol{x}_0, \boldsymbol{0}))$ and $\phi_Y(\boldsymbol{x}_0, \boldsymbol{y}_0) = 1/(1 - D_T(\boldsymbol{x}_0, \boldsymbol{y}_0; \boldsymbol{0}, \boldsymbol{y}_0))$.

[3] The multiplicative decomposition in the form of $PI = TI * AI$ is referred to as a Nerlovian profit inefficiency measurement as it first appeared in Nerlove (1965). Its PI measure is given as $\pi(\boldsymbol{w}, \boldsymbol{p})/(\boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x})$.

allocative inefficiency (AI);

$$AI(\boldsymbol{x},\boldsymbol{y};\boldsymbol{g}_x,\boldsymbol{g}_y) = D_{PF}(\boldsymbol{x},\boldsymbol{y};\boldsymbol{g}_x,\boldsymbol{g}_y) - D_T(\boldsymbol{x},\boldsymbol{y};\boldsymbol{g}_x,\boldsymbol{g}_y). \tag{5.5}$$

Any input-output bundle on the frontier of T_{PF} is allocatively-efficient at prices $(\boldsymbol{w},\boldsymbol{p})$ and attains the maximum profit at $\pi_T(\boldsymbol{w},\boldsymbol{p})$.

The assumption of constant returns to scale (CRS) considers a hypothetical technology that envelops T under the linear homogeneity of input-output relationships, or $T_{CRS} = \cup_{\lambda\in\mathbb{R}_+}\lambda T$.[4] Let the profit function associated with T_{CRS} be $\pi_{CRS}(\boldsymbol{w},\boldsymbol{p}) = \max_{\boldsymbol{x},\boldsymbol{y}}\{\boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x} : (\boldsymbol{x},\boldsymbol{y}) \in T_{CRS}\}$, which equals 0 if $\pi_T(\boldsymbol{w},\boldsymbol{p}) \leq 0$ and ∞ if $\pi_T(\boldsymbol{w},\boldsymbol{p}) > 0$.[5] Assuming $\pi_{CRS}(\boldsymbol{w},\boldsymbol{p}) = 0$ (e.g., equilibrium outcome under perfect competition with free entry and exit), the corresponding pseudo-TI measurement under CRS, say TI(CRS), is;

$$D_{CRS}(\boldsymbol{x},\boldsymbol{y};\boldsymbol{g}_x,\boldsymbol{g}_y) = \min_{\boldsymbol{p}',\boldsymbol{w}'}\left\{\frac{\pi_{CRS}(\boldsymbol{p}',\boldsymbol{w}') - (\boldsymbol{p}'\boldsymbol{y} - \boldsymbol{w}'\boldsymbol{x})}{\boldsymbol{p}'\boldsymbol{g}_y + \boldsymbol{w}'\boldsymbol{g}_x}\right\}, \tag{5.6}$$

which is positive and bounded. According to additive decomposition $TI(CRS) = TI + SI$, the difference between the TI(CRS) in (5.6) and the TI in (5.3) defines scale inefficiency (SI);

$$SI(\boldsymbol{x},\boldsymbol{y};\boldsymbol{g}_x,\boldsymbol{g}_y) = D_{CRS}(\boldsymbol{x},\boldsymbol{y};\boldsymbol{g}_x,\boldsymbol{g}_y) - D_T(\boldsymbol{x},\boldsymbol{y};\boldsymbol{g}_x,\boldsymbol{g}_y). \tag{5.7}$$

Any decision on the frontier of T_{CRS} is scale-efficient.

5.2.2 Weighted DEA (WDEA) Approach

Turning to empirical efficiency measurements, the weighted DEA (WDEA) approach is presented below for input-output bundles $\{(\boldsymbol{x}_i,\boldsymbol{y}_i)\}_{i\in\mathbb{I}}$ with observations indexed by $\mathbb{I} = \{1...N\}$.

The DEA approximations under VRS and CRS are respectively the free-disposal

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[4] For input-output relationships specified in physical quantiles, linear-homogeneity is most suitably defined with respect to the origin. For those specified in monetary variables or qualitative indices, one may postulate a shifted CRS (SCRS) technology under a pseudo CRS-assumption around arbitrary point $(\boldsymbol{x}^o,\boldsymbol{y}^o)$ instead of the origin, or $T_{SCRS} = (\boldsymbol{x}^o,\boldsymbol{y}^o) + \cup_{\lambda\in\mathbb{R}_+}\lambda(T - (\boldsymbol{x}^o,\boldsymbol{y}^o))$. The associated profit function is $\pi_{SCRS}(\boldsymbol{w},\boldsymbol{p}) = \boldsymbol{p}\boldsymbol{y}^o - \boldsymbol{w}\boldsymbol{x}^o + \max_{\boldsymbol{x},\boldsymbol{y}}\{\boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x} : (\boldsymbol{x},\boldsymbol{y}) \in T_{SCRS}\}$ which equals $\boldsymbol{p}\boldsymbol{y}^o - \boldsymbol{w}\boldsymbol{x}^o$ if $\pi_T(\boldsymbol{w},\boldsymbol{p}) \leq \boldsymbol{p}\boldsymbol{y}^o - \boldsymbol{w}\boldsymbol{x}^o$ or ∞ if $\pi_T(\boldsymbol{w},\boldsymbol{p}) > \boldsymbol{p}\boldsymbol{y}^o - \boldsymbol{w}\boldsymbol{x}^o$.

[5] Under the CRS assumption, zero-profit is always feasible by the feasible inaction $(\mathbf{0},\mathbf{0}) \in T_{CRS}$ at $\lambda = 0$.

convex hull of data points (i.e., all convex combinations of data points and the points implied by free-disposability) and the free-disposal conical hull of data points (i.e., every point in $\widehat{T}_{VRS}$ and any scaler multiple of it), or

$$\widehat{T}_{VRS} = \{(\boldsymbol{x}', \boldsymbol{y}') : \sum_{j\in\mathbb{I}} \lambda_j \boldsymbol{y}_j \geq \boldsymbol{y}', \ , \sum_{j\in\mathbb{I}} \lambda_j \boldsymbol{x}_j \leq \boldsymbol{x}', \ \sum_{j\in\mathbb{I}} \lambda_j = 1, \ \boldsymbol{\lambda} \in \mathbb{R}^N_+\}, \tag{5.8}$$

$$\widehat{T}_{CRS} = \{(\boldsymbol{x}', \boldsymbol{y}') : \sum_{j\in\mathbb{I}} \lambda_j \boldsymbol{y}_j \geq \boldsymbol{y}', \ , \sum_{j\in\mathbb{I}} \lambda_j \boldsymbol{x}_j \leq \boldsymbol{x}', \ \boldsymbol{\lambda} \in \mathbb{R}^N_+\}. \tag{5.9}$$

$\widehat{T}_{VRS}$ corresponds to the smallest producible set satisfying assumptions A.1-A.3, while $\widehat{T}_{CRS}$ envelops $\widehat{T}_{VRS}$ under linear homogeneity. The estimates for TI and TI(CRS) by the directional distance function in (5.1) are $\widehat{D}_{VRS}(\boldsymbol{x}_0, \boldsymbol{y}_0; \boldsymbol{g}_x, \boldsymbol{g}_y) = \max\{b : (\boldsymbol{x}_0 - b\boldsymbol{g}_x, \boldsymbol{y}_0 + b\boldsymbol{g}_y) \in \widehat{T}_{VRS}\}$ and $\widehat{D}_{CRS}(\boldsymbol{x}_0, \boldsymbol{y}_0; \boldsymbol{g}_x, \boldsymbol{g}_y) = \max\{b : (\boldsymbol{x}_0 - b\boldsymbol{g}_x, \boldsymbol{y}_0 + b\boldsymbol{g}_y) \in \widehat{T}_{CRS}\}$ respectively. The dual problem for $\widehat{D}_{VRS}(\boldsymbol{x}_0, \boldsymbol{y}_0; \boldsymbol{g}_x, \boldsymbol{g}_y)$, corresponding to the dual representation in (5.3), is

$$\begin{aligned} \min\{\rho \in \mathbb{R} : \ \forall j \in \mathbb{I}, \ \boldsymbol{p}\boldsymbol{y}_j - \boldsymbol{w}\boldsymbol{x}_j \leq \boldsymbol{p}\boldsymbol{y}_0 - \boldsymbol{w}\boldsymbol{x}_0 + \rho, \\ \boldsymbol{p}\boldsymbol{g}_y + \boldsymbol{w}\boldsymbol{g}_x = 1, \ \boldsymbol{p} \in \mathbb{R}^M_+, \ \boldsymbol{w} \in \mathbb{R}^L_+\}, \end{aligned} \tag{5.10}$$

which minimizes TI-parameter ρ subject to the optimality of shadow value $\boldsymbol{p}\boldsymbol{y}_0 - \boldsymbol{w}\boldsymbol{x}_0 + \rho$ for decision $(\boldsymbol{x}_0, \boldsymbol{y}_0)$, given the feasibility constraints under $\widehat{T}_{VRS}$ and price normalization $\boldsymbol{p}\boldsymbol{g}_y + \boldsymbol{w}\boldsymbol{g}_x = 1$. The dual estimation for $\widehat{D}_{CRS}(\boldsymbol{x}_0, \boldsymbol{y}_0; \boldsymbol{g}_x, \boldsymbol{g}_y)$, corresponding to (5.6), is obtained by imposing additional constraint $\boldsymbol{p}\boldsymbol{y}_0 - \boldsymbol{w}\boldsymbol{x}_0 + \rho = 0$ in problem (5.10), as implied by condition $\pi_{CRS}(\boldsymbol{p}, \boldsymbol{w}) = 0$.

Profit-function (PF) technology is estimated as the half space bounded by $\widehat{\pi}_{VRS}(\boldsymbol{w}, \boldsymbol{p})$;

$$\begin{aligned} \widehat{T}_{PF} &= \{(\boldsymbol{x}, \boldsymbol{y}) : \ \boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x} \leq \widehat{\pi}_{VRS}(\boldsymbol{w}, \boldsymbol{p})\} \quad \text{where} \\ \widehat{\pi}_{VRS}(\boldsymbol{w}, \boldsymbol{p}) &= \max_{\boldsymbol{x},\boldsymbol{y}}\{\boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x} : \ (\boldsymbol{x}, \boldsymbol{y}) \in \widehat{T}_{VRS}\} = \max_{j\in\mathbb{I}}\{\boldsymbol{p}\boldsymbol{y}_j - \boldsymbol{w}\boldsymbol{x}_j\}. \end{aligned} \tag{5.11}$$

The conventional measures of technical, allocative, and scale inefficiencies are estimated as distances $\widehat{TI}_{VRS}$, $\widehat{AI}_{VRS}$, and $\widehat{SI}_{VRS}$ in (5.3), (5.5), and (5.7) respectively using frontier approximations (5.8), (5.9), and (5.11). The standard practice is to utilize technology approximation $\widehat{T}_{VRS}$, from which TI, AI, and SI are measured. While these estimates are consistent, a more efficient estimation can be devised under a simultaneous estimation of the technology and inefficiency concepts.

To this end, the current study proposes a weighted DEA (WDEA) approach for integrating the concepts of TI, AI, and SI into a technology approximation. Consider WDEA technology $\widehat{T}_{W(\alpha,\beta)}$ defined as the weighted average of $\widehat{T}_{VRS}$, $\widehat{T}_{PF}$, and $\widehat{T}_{CRS}$ for given weights $\{1-\alpha-\beta,\ \alpha,\ \beta\}$ respectively;[6]

$$\begin{aligned} \widehat{T}_{W(\alpha,\beta)} &\equiv (1-\alpha-\beta)\widehat{T}_{VRS} + \alpha\widehat{T}_{PF} + \beta\widehat{T}_{CRS} \\ &= \widehat{T}_{VRS} + \alpha(\widehat{T}_{PF} - \widehat{T}_{VRS}) + \beta(\widehat{T}_{CRS} - \widehat{T}_{VRS}), \end{aligned} \tag{5.12}$$

which expands the conventional producible set $\widehat{T}_{VRS}$ by the α-portion of the input-output space conventionally regarded as AI and the β-portion of the space conventionally regarded as SI.[7] The arbitrary weights α and β respectively generalize the extents of linear substitution and linear homogeneity assumptions in DEA. Consequently, $\widehat{T}_{W(\alpha,\beta)}$ includes the conventional DEA frontiers of PF, CRS, and VRS as special cases; $\widehat{T}_{W(0,0)} = \widehat{T}_{VRS}$, $\widehat{T}_{W(1,0)} = \widehat{T}_{PF}$, and $\widehat{T}_{W(0,1)} = \widehat{T}_{CRS}$. If weights α and β fall outside of range $[0,1]$, the technical feasibility can be defined as $\widehat{T}^{*}_{W(\alpha,\beta)} \equiv \widehat{T}_{W(\alpha,\beta)} \cup \widehat{T}_{VRS}$, so that the WDEA approximation of a technology is bounded from below by $\widehat{T}_{VRS}$.

For decision $(\boldsymbol{x}_0, \boldsymbol{y}_0)$, let the TI measured under WDEA technology $\widehat{T}_{W(\alpha,\beta)}$ be

$$\begin{aligned} \widehat{D}_{W(\alpha,\beta)}(\boldsymbol{x}_0,\boldsymbol{y}_0) &= (1-\alpha-\beta)\widehat{D}_{VRS}(\boldsymbol{x}_0,\boldsymbol{y}_0) + \alpha\widehat{D}_{PF}(\boldsymbol{x}_0,\boldsymbol{y}_0) + \beta\widehat{D}_{CRS}(\boldsymbol{x}_0,\boldsymbol{y}_0) \\ &= \widehat{TI}_{VRS}(\boldsymbol{x}_0,\boldsymbol{y}_0) + \alpha\widehat{AI}_{VRS}(\boldsymbol{x}_0,\boldsymbol{y}_0) + \beta\widehat{SI}_{VRS}(\boldsymbol{x}_0,\boldsymbol{y}_0), \end{aligned} \tag{5.13}$$

and let the associated AI and SI measures be

$$\widehat{AI}_{W(\alpha,\beta)}(\boldsymbol{x}_0,\boldsymbol{y}_0) = \widehat{D}_{PF,W}(\boldsymbol{x}_0,\boldsymbol{y}_0) - \widehat{D}_{W(\alpha,\beta)}(\boldsymbol{x}_0,\boldsymbol{y}_0), \tag{5.14}$$

$$\widehat{SI}_{W(\alpha,\beta)}(\boldsymbol{x}_0,\boldsymbol{y}_0) = \widehat{D}_{CRS,W}(\boldsymbol{x}_0,\boldsymbol{y}_0) - \widehat{D}_{W(\alpha,\beta)}(\boldsymbol{x}_0,\boldsymbol{y}_0) \tag{5.15}$$

where $\widehat{D}_{CRS,W}(\boldsymbol{x}_0,\boldsymbol{y}_0)$ and $\widehat{D}_{PF,W}(\boldsymbol{x}_0)$ are obtained by replacing $\widehat{T}_{VRS}$ in (5.9) and (5.11) with $\widehat{T}_{W(\alpha,\beta)}$ respectively. Note that at $\beta = 0$, the new AI measurement reduces

[6]While notation $(\boldsymbol{w}, \boldsymbol{p})$ is omitted, $\widehat{T}_{W(\alpha,\beta)}$ clearly depends on the market prices through the profit frontier along $\widehat{T}_{PF}$.

[7]Alternatively, one can define $\widehat{T}_{W(\alpha,\beta)}$ with a convex combination of arbitrary direction $(-\tilde{\boldsymbol{g}}_x, \tilde{\boldsymbol{g}}_y)$ instead of the current, radial orientation.

to;

$$\widehat{AI}_{W(\alpha,0)}(\boldsymbol{x}_i,\boldsymbol{y}_i)=(1-\alpha)(\widehat{D}_{PF}(\boldsymbol{x}_i,\boldsymbol{y}_i)-\widehat{D}_{VRS}(\boldsymbol{x}_i,\boldsymbol{y}_i))=(1-\alpha)\widehat{AI}_{VRS}(\boldsymbol{x}_i,\boldsymbol{y}_i). \tag{5.16}$$

Similarly, at $\alpha = 0$, the new SI measurement reduces to;

$$\widehat{SI}_{W(0,\beta)}(\boldsymbol{x}_i,\boldsymbol{y}_i)=(1-\beta)(\widehat{D}_{CRS}(\boldsymbol{x}_i,\boldsymbol{y}_i)-\widehat{D}_{VRS}(\boldsymbol{x}_i,\boldsymbol{y}_i))=(1-\beta)\widehat{SI}_{VRS}(\boldsymbol{x}_i,\boldsymbol{y}_i). \tag{5.17}$$

The next subsection considers an optimal weight selection for α and β. In the following, simplified notations are used for distance function $D_{T,i} \equiv D_T(\boldsymbol{x}_i,\boldsymbol{y}_i)$ and inefficiencies $TI_{T,i} \equiv TI_T(\boldsymbol{x}_i,\boldsymbol{y}_i)$, $AI_{T,i} \equiv AI_T(\boldsymbol{x}_i,\boldsymbol{y}_i)$, and $SI_{T,i} \equiv SI_T(\boldsymbol{x}_i,\boldsymbol{y}_i)$.

5.2.3 Weight Selection

Consider the following weight selection mechanism that proceeds in two steps. The first step makes some initial estimate $\widehat{D}_{T,i}$ at the observation level and the second step predicts sample-level relationships between this estimate $\widehat{D}_{T,i}$ and the conventional measures of TI, AI, and SI. Namely, the second step estimates optimal weights by minimizing least square errors of the form;

$$\{\hat{\alpha},\hat{\beta}\}=\underset{\alpha,\beta}{\operatorname{argmin}}\left\{\frac{1}{N}\sum_{i\in\mathbb{I}}\left(\widehat{D}_{T,i}-(\widehat{TI}_{VRS,i}+\alpha\widehat{AI}_{VRS,i}+\beta\widehat{SI}_{VRS,i})\right)^2\right\}, \tag{5.18}$$

which is the moment condition implied by equation (5.13) when $\widehat{D}_{W(\alpha,\beta),i}$ is substituted with some estimate $\widehat{D}_{T,i}$ from the first step. The remainder of this section describes the first-step estimation for $\widehat{D}_{T,i}$, discusses some properties of this weight selection, and provides a simple, illustrative example.

The conceptual underpinning for the first-step estimate $\widehat{D}_{T,i}$ draws on the subsample-bootstrap estimator proposed by Kneip et al. (2008). The authors showed that for a convex technology, the behavior of the VRS estimator can be analyzed through the relative frequency for observations to be located in a small neighborhood around the true frontier. Assuming a uniform density in the neighborhood, they derived an asymptotic distribution of this estimator. Given the equivalence between the asymptotic properties of (additive) directional distance functions and those of (multiplicative) radial inefficiency measures (Simar and Vanhems, 2012), the $1-a$ confidence interval for $\widehat{D}_{VRS,i}$ can be written as;

$$1-a = Pr(C_a \leq \widehat{D}_{VRS,i} - D_T \leq C_b) \approx Pr(C_a \leq \widehat{D}^*_{VRS,i} - \widehat{D}_{VRS} \leq C_b) \tag{5.19}$$

where C_a and C_b represent lower and upper critical values for the deviation, and $\widehat{D}^*_{VRS,i}$ is a bootstrap VRS estimator using $K(<N)$ observations sampled without replacement.[8] The critical values are substituted with estimates $\widehat{C}_a = \psi_{a/2,K}$ and $\widehat{C}_b = \psi_{1-a/2,K}$ where $\psi_{x,K} \leq 0$ denotes the x-quantile of the bootstrap distribution $\{K^{2/(L+M+1)}(\widehat{D}^{*,b}_{VRS,i} - \widehat{D}_{VRS,i})\}_{b=1}^{B}$ from B bootstrap replications. The concept behind the subsample-bootstrapping is that the distribution of the difference $\widehat{D}_{VRS,i} - D_T$ between the VRS estimator (in the sample) and the true value (in the universe) can be predicted from the distribution of the difference $\widehat{D}^*_{VRS,i} - \widehat{D}_{VRS}$ between the bootstrap-VRS estimator (in a subsample) and the VRS estimator (in the full sample), given the adjustments for the different rates of convergence under different sample sizes. Then, the confidence interval in (5.19) can be estimated as

$$[\widehat{D}_{VRS,i} - N^{-2/(L+M+1)}\psi_{1-a/2,K}, \quad \widehat{D}_{VRS,i} - N^{-2/(L+M+1)}\psi_{a/2,K}], \tag{5.20}$$

which reflects the accuracy of local VRS estimator $\widehat{D}_{VRS,i}$, predicted from the local sample density in the neighborhood. Let the mean of this confidence interval be referred to as mean bootstrap (MB) estimator (e.g., Simar and Vanhems, 2012), which makes upward adjustments to conventional TI estimate $\widehat{D}_{VRS,i}$;[9]

$$\widehat{D}_{MB,i} = \widehat{D}_{VRS,i} - \left(\frac{K}{N}\right)^{2/(L+M+1)} \frac{1}{B}\sum_{b=1}^{B}(\widehat{D}^{*,b}_{VRS,i} - \widehat{D}_{VRS,i}) \tag{5.21}$$

where $\widehat{D}^{*,b}_{VRS,i} - \widehat{D}_{VRS,i} \leq 0$. In effect, MB estimator tends to assume a larger producible set than $\widehat{T}_{VRS}$ everywhere along the frontier.

Two modifications to $\widehat{D}_{MB,i}$ are made before arriving at the proposed estimate for $\widehat{D}_{T,i}$. One modification is to correct for systematic bias in $\widehat{D}_{MB,i}$ with respect to the scales of operations. The bias arises from the potential inapplicability of TI

[8] Another approach would be the smooth-bootstrap method of Kneip et al. (2011).

[9] The mean can be replaced with the median or mode of distribution $\{K^{2/(L+M+1)}(\widehat{D}^{*,b}_{VRS,} - \widehat{D}_{VRS,i})\}_{b=1}^{B}$. Simulation study may be helpful to investigate these alternative estimators.

measurements under random subsamples (which may not contain reference observations for sufficiently small- or large-scale operations) and is systematically related to the direction of the TI measurement; small-scale decisions cannot be assessed for an output-oriented TI if comparably-small scale input-decisions are absent in the subsample, and similarly large-scale decisions cannot be evaluated for input-oriented TI if comparably-large scale output-decisions are absent. Using only the estimable cases of bootstrap-TI measurements would underestimate the bias-corrections in (5.21) for these decisions. By simultaneously employing input- and output-oriented MB estimators, denoted by $\widehat{D}^I_{MB,i}$ and $\widehat{D}^O_{MB,i}$ respectively, scale-neutral MB technology can be specified as;

$$\widehat{T}^N_{MB} = \{(\boldsymbol{x}', \boldsymbol{y}') : \sum_{j\in I} \lambda_j (y_j + \widehat{D}^O_{MB,i}) + \sum_{j\in I} \eta_j \boldsymbol{y}_j \geq \boldsymbol{y}', \sum_{j\in I} \lambda_j \boldsymbol{x}_j + \sum_{j\in I} \eta_j (\boldsymbol{x}_j - \widehat{D}^I_{MB,i}) \leq \boldsymbol{x}', \sum_{j\in I} \lambda_j + \sum_{j\in I} \eta_j = 1, \ \boldsymbol{\lambda}, \boldsymbol{\eta} \in \mathbb{R}^N_+\}, \tag{5.22}$$

which yields associated estimator $\widehat{D}^N_{MB,i} = \max\{b : (\boldsymbol{x}_i - b\boldsymbol{g}_x, \boldsymbol{y}_i + b\boldsymbol{g}_y) \in \widehat{T}^N_{MB}\}$ for direction $(\boldsymbol{g}_x, \boldsymbol{g}_y)$.

The other modification is to reduce the magnitude of upward adjustments in (5.21), such that some observed decisions can be regarded fully-technically efficient under the implied technology. This prevents the model from postulating a strictly larger technical feasibility than $\widehat{T}_{VRS}$. For constant $\bar{c} = E[\widehat{D}^N_{MB,i} - \widehat{D}_{VRS,i}]$, consider shifted-mean bootstrap (SMB) estimator $\widehat{D}_{SMB,i} = \max\{\widehat{D}^N_{MB,i} - \bar{c}, \ \widehat{D}_{VRS,i}\}$. In this construction, constant $\bar{c}$ is used to shift back MB estimator $\widehat{D}^N_{MB,i}$ by the mean difference between the above MB and VRS estimators. The lower bound for TI by $\widehat{D}_{VRS,i}$ is added to ensure that the associated technical feasibility is bounded from below by $\widehat{T}_{VRS}$. It may be noted that the magnitude of constant $\bar{c}$ directly affects the mean TI under SMB and hence the mean TI under WDEA.[10]

Hence, the proposed weight selection first estimates $\widehat{D}_{T,i}$ by $\widehat{D}_{SMB,i}$ and then weights $\hat{\alpha}$ and $\hat{\beta}$ by equation (5.18). These weights represent the sample-level relationships between locally-derived adjustments $\widehat{D}_{SMB,i} - \widehat{D}_{VRS,i}$ (i.e., predicted bias

[10] An alternative for $\bar{c}$ is to use the minimum of the difference $\widehat{D}^N_{MB,i} - \widehat{D}_{VRS,i}$ instead of the mean. Yet, given the relative inaccuracy in predicting $\widehat{T}^N_{MB}$, the use of the mean difference appears more reliable.

corrections for the conventional TI measure $\widehat{D}_{VRS,i}$) and the conventional measures of AI and SI. By accounting for the sample correlations among these inefficiency concepts, WDEA technology $\widehat{T}_{W(\hat{\alpha},\hat{\beta})}$ systematically extends conventional technology approximation $\widehat{T}_{VRS}$, or the smallest feasible set meeting assumptions A1-A3.

Some properties of the WDEA estimator are noted with respect to the following relationships between unobserved $D_{T,i}$ and its estimates by VRS, SMB, and WDEA;

$$\begin{aligned} VRS: &\quad D_{T,i} = \widehat{TI}_{VRS,i} + \varepsilon_{VRS,i}, \quad \varepsilon_{VRS,i} > 0 \\ SMB: &\quad D_{T,i} = \widehat{D}_{SMB,i} + \varepsilon_{SMB,i}, \quad E[\varepsilon_{SMB,i}] = 0 \\ WDEA: &\quad D_{T,i} = \widehat{TI}_{VRS,i} + \alpha\widehat{AI}_{VRS,i} + \beta\widehat{SI}_{VRS,i} + \varepsilon_{W(\alpha,\beta),i}, \quad E[\varepsilon_{W(\alpha,\beta),i}] = 0 \end{aligned} \tag{5.23}$$

where $\varepsilon_{VRS,i}$, $\varepsilon_{SMB,i}$, and $\varepsilon_{W(\alpha,\beta),i}$ are residual terms that close these identities. In the first equation, the well-known one-sided bias of the VRS estimator (i.e. $\varepsilon_{VRS,i} > 0$) implies mean-inconsistency $E[D_{T,i} - \widehat{TI}_{VRS,i}] = E[\varepsilon_{VRS,i}] > 0$, while it is asymptotically consistent, or $E[D_{T,i} - \widehat{TI}_{VRS,i}] \to 0$ for a sufficiently large sample (Banker et al., 1993). In the second equation, the SMB estimator is assumed to be consistent, so that $E[D_{T,i} - \widehat{D}_{SMB,i}] = E[\varepsilon_{SMB,i}] = 0$. Given this assumption, combining the second and the third equations to eliminate $D_{T,i}$ and using $\hat{\alpha}$ and $\hat{\beta}$ in (5.18) yields a consistent WDEA estimator;

Remark 4. *In* (5.18) *and* (5.23)*, if* $E[\varepsilon_{SMB,i}|\widehat{D}_{SMB,i}, \widehat{TI}_{VRS,i}, \widehat{AI}_{VRS,i}, \widehat{SI}_{VRS,i}] = 0$*, then the WDEA estimator is consistent, or* $E[\varepsilon_{W(\hat{\alpha},\hat{\beta}),i}] = 0$*.*

Turning to estimation efficiency, simple comparisons are noted;

Remark 5. *In* (5.23)*, if* $E[\varepsilon_{SMB,i}|\widehat{D}_{SMB,i}, \widehat{TI}_{VRS,i}] = 0$*, then the SMB estimator is more efficient than the VRS estimator in that* $E[(\varepsilon_{SMB,i})^2] \leq E[(\varepsilon_{VRS,i})^2]$ *where* $\varepsilon_{VRS,i} = (\widehat{D}_{SMB,i} - \widehat{TI}_{VRS,i}) + \varepsilon_{SMB,i}$*.*

Remark 6. *In* (5.18) *and* (5.23)*, if* $E[\varepsilon_{W(\hat{\alpha},\hat{\beta}),i}|\widehat{AI}_{VRS,i}, \widehat{SI}_{VRS,i}] = 0$ *and* $\hat{\alpha}$, $\hat{\beta} \geq 0$*, then the WDEA estimator is more efficient than the VRS estimator in that* $E[(\varepsilon_{W(\hat{\alpha},\hat{\beta}),i})^2] \leq E[(\varepsilon_{VRS,i})^2]$ *where* $\varepsilon_{VRS,i} = \alpha\widehat{AI}_{VRS,i} + \beta\widehat{SI}_{VRS,i} + \varepsilon_{W(\hat{\alpha},\hat{\beta}),i}$*.*

Remark 7. *In* (5.18) *and* (5.23)*, if* $E[\varepsilon_{SMB,i}|\widehat{D}_{SMB,i}, \widehat{TI}_{VRS,i}, \widehat{AI}_{VRS,i}, \widehat{SI}_{VRS,i}] = 0$ *and* $\hat{\alpha}$, $\hat{\beta} \leq 0$*, then the SMB estimator is more efficient than the WDEA estimator in* *that* $E[(\varepsilon_{SMB,i})^2] \leq E[(\varepsilon_{W(\hat{\alpha},\hat{\beta}),i})^2]$ *where* $\varepsilon_{SMB,i} = \alpha\widehat{AI}_{VRS,i} + \beta\widehat{SI}_{VRS,i} + (\widehat{TI}_{VRS,i} - \widehat{D}_{SMB,i}) + \varepsilon_{W(\alpha,\beta),i}$.

Remark 5 follows from $\widehat{D}_{SMB,i} - \widehat{TI}_{VRS,i} \geq 0$. Remark 6 similarly follows under $\alpha,\ \beta \geq 0$. Remark 7 states that under $\alpha,\ \beta \leq 0$, incorporating AI and SI into a technology estimation would be counterproductive. Meanwhile, there seems no simple condition that ensures higher efficiency of the WDEA estimator than the SMB counterpart.

The following example illustrates a process of constructing simplified versions of SMB and WDEA estimators. Consider a case of one-input, one-output production with a sample of 6 observations $(x_i, y_i),\ i = 1, .., 6$. Figure 5.3 depicts relative geometric locations of these observations, labeled A_1-A_6. Points A_1,A_2, and A_3 are technically-efficient under VRS but only point A_2 is technically-efficient under CRS. Points A_4, A_5, and A_6 are all technically-inefficient under VRS and are less efficient versions of points A_1,A_2, and A_3 respectively, such that $x_1 = x_4 < x_2 = x_5 < x_3 = x_6$ and $y_1 > y_4$, $y_2 > y_5$, $y_3 > y_6$. For the ease of illustration, consider the SMB estimator based solely on the output-oriented TI by $\widehat{D}^O_{MB,i}$ (without using the output-oriented TI by $\widehat{D}^I_{MB,i}$) and single WDEA weight $\hat{\beta} > 0$ (with $\hat{\alpha} = 0$). In relation to the total number of observations $N = 6$, the number of subsample observations is set at $K = 1$ for simplicity. At $K = 1$, bootstrap VRS frontier reduces to a free disposable hull (FDH), so that $\widehat{D}^O_{MB,i}$ can be described as the difference in outputs of two decisions. With a sufficient number of bootstrap replications $b = 1, .., B$, each data point is drawn at probability $p_i = 1/6$, and the mean bootstrap estimate $\widehat{D}_{MB,i}\ (1/B)\sum_b(\widehat{D}^{*,b}_{VRS,i} - \widehat{D}_{VRS,i})$ converges to its expected value. Then, without loss of generality, by treating observation index i interchangeably with bootstrap index $b = 1, .., 6$, the MB estimator for point A_1 can be described as

$$\begin{aligned}\widehat{D}_{MB,1} &= \widehat{D}_{VRS,1} - (K/N)^{2/(L+M+1)}[\tilde{p}_1(\widehat{D}^{*,b=1}_{VRS,1} - \widehat{D}_{VRS,1}) + \tilde{p}_4(\widehat{D}^{*,b=4}_{VRS,1} - \widehat{D}_{VRS,1})] \\ &= 0 - C_0[\frac{1}{2}(y_1 - y_1 - 0) + \frac{1}{2}(y_4 - y_1 - 0)] = C_0\ E_b[y_1 - y_b\ |b = 1, 4] \quad (5.24)\end{aligned}$$

where $C_0 = (K/N)^{2/L+M+1}$ is a constant ($(1/6)^{2/3}$ in this example), and $\tilde{p}_1 = \tilde{p}_4 = 1/2$ is a pseudo-probability defined conditionally on the feasible TI estimation under bootstrap replications $b = 1, 4$. [11] Similarly, those for points A_2 to A_6 are $\widehat{D}_{MB,2} =$

[11] The number of comparable points for calculating $\widehat{D}_{MB,i}$ increases in subsample size, making the issue

$C_0 E_b[y_2 - y_b | b = 1, 2, 4, 5]$, $\widehat{D}_{MB,3} = C_0 E_b[y_3 - y_b]$, $\widehat{D}_{MB,4} = (y_1 - y_4) + C_0 E_b[y_1 - y_b | b = 1, 4]$, $\widehat{D}_{MB,5} = (y_2 - y_5) + C_0 E_b[y_2 - y_b | b = 1, 2, 4, 5]$, and $\widehat{D}_{MB,6} = (y_3 - y_6) + C_0 E_b[y_1 - y_b]$. Thus, the local frontier levels are adjusted by C_0 times expected bootstrap deviation $E_b[y_i - y_b]$. By setting $\bar{c} = (1/3) \sum_{j=1,2,3} [\widehat{D}_{MB,j} - \widehat{D}_{VRS,j} | \widehat{D}_{VRS,j} = 0]$ and $\widehat{D}_{SMB,i} = \max\{(\widehat{D}_{MB,j} - \widehat{D}_{VRS,j} - \bar{c})/(1 + \bar{c}), \widehat{D}_{VRS,i}\}$ for $i = 1, .., 6$, optimal weight is estimated by $\hat{\beta} = Cov(\widehat{D}_{SMB,i} - \widehat{D}_{VRS,i}, \widehat{D}_{CRS,i} - \widehat{D}_{VRS,i})/Var(\widehat{D}_{CRS,i} - \widehat{D}_{VRS,i}) = Cov(\widehat{D}_{SMB,i} - \widehat{TI}_{VRS,i}, \widehat{SI}_{VRS,i})/Var(\widehat{SI}_{VRS,i})$ where $Cov(.)$ and $Var(.)$ denote the covariance and variance operators respectively.

Figure 5.4 sketches the SMB estimates (at points C_1, C_2, and C_3) and WDEA estimates (at points D_1, D_2, and D_3) from the above example. The SMB estimator yields no adjustment at A_1, a small expansion of technical feasibility at A_2, and a large expansion at A_3, according to the local performance of VRS estimators assessed by the bootstrapping process. The WDEA estimator consolidates these local adjustments into systematic frontier expansions from the VRS frontier toward the CRS frontier at weight $\hat{\beta}$, resulting in moderate expansions of technical feasibility at A_1 and A_3 with no expansion at scale-efficient point A_2. Weight $\hat{\beta}$ depends on the covariance between SMB's adjustments to TI (i.e., distances C_1A_1, C_2A_2, and C_3A_3 in figure 5.4) and conventional SI measures (i.e., distances B_1A_1, B_2A_2, and B_3A_3 in figure 5.3).

Hence, the proposed two-step weight selection is summarized as follows. The subsample bootstrapping in the first step links the performance of the VRS estimator to its local probability density. This yields a presumably-consistent adjustment at the observation level, yet its estimation efficiency depends on the nature of the data and the choice of subsample sizes that specifies the level of "locality."[12] The second-step regression summarizes this local adjustment into the sample correlations among TI, AI, and SI, all of which producers strive to minimize.

5.3 Results and Discussion

5.3.1 Data and Efficiency Measurements

Proposed WDEA approach is used to examine technical efficiencies and producer-specific input shadow values for Maryland dairy operations during 1995-2009. The

of non-estimable TI situations less important.

[12] The confidence interval in (5.21) can be sensitive to the choice of K (Kneip et al., 2008). The current application follows Simar and Wilson (2011)'s subsample-size selection that minimizes the volatility of the estimator.

dataset, previously analyzed in Hanson et al. (2013), contains unbalanced panel entries of production inputs and outputs. Each operation is categorized as either a conventional confinement dairy or management-intensive grazing dairy. The producer-year level observations represent 314 confinement operation-years and 164 grazing operation-years.

Dairy production is modeled with seven inputs; herd size (cows), hired labor, crop, animal care, machinery, crop acreage, pasture acreage, where the four items from "labor" to "machinery" are quasi-quantity inputs measured as the corresponding categorical expenses divided by observation-specific price indices.[13] Table 5.1 provides summary statistics of milk output and these inputs. In the sample, a typical confinement operation produces about twice as much milk as a typical grazing operation and utilizes a 40% bigger herd, 486% more labor, 160% more crop-production inputs, 93% more animal-care inputs, 76% more machinery, 119% more crop acreage, and 67% less pasture acreage. In the current application, the two dairy systems are separately analyzed for their efficiency measurements.

Table 5.2 reports a summary of input-oriented radial efficiency scores ranging from 0 to 1 with 1 being fully-efficient.[14] Listed items TE(VRS), SMB, and WDEA report technical efficiency estimates against a VRS technology, a shifted mean bootstrap (SMB) technology, and a weighted DEA (WDEA) technology respectively. Items SE(VRS) and AE(VRS) are the scale and allocative efficiency estimates under the VRS technology. The mean scores of TE, SE, and AE under VRS are 0.896, 0.965, and 0.787 for confinement and 0.931, 0.914, and 0.717 for grazers respectively. The relatively high SE scores are likely explained by the limited range of operational scales in the sample, ranging up to 468 cows for confinement and 195 cows for grazers; the inclusion of large-scale operations with over 1000 cows would lower these SE scores. The relatively low AE scores, on the other hand, suggest that in a short run these input mixes, often linked to long-term assets, are unlikely to be optimally allocated

[13]For example, the aggregate crop-related expenses for observation i is calculated as $p_i^{crop} = (seed_i/crop_i)p^{seed} + (fertilizer_i/crop_i)p^{fertilizer} + (chem_i/crop_i)p^{chem}$ where $crop_i = seed_i + fertilizer_i + chem_i$ for its total categorical expense for seeds, fertilizer, and chemicals. Price indices are obtained from Agricultural Statistical Service of USDA.

[14]Appendix A describes the market price estimation based on Kuosmanen et al. (2006) with some additional constraints. The estimated prices in table A.1 are used for obtaining allocative efficiency (AE) and WDEA. The primarily interest is the annual rental rates of dairy cows for confinement and grazers, estimated at \$575/cow and \$464/cow respectively. Under the expected 1.5 and 2.5 remaining-years of economically-viable milking for confinement and grazing cows, the culling value of \$500, and a 5% interest rate, these rental rates imply the present values of $\$575 + (\$575/2 + \$500)/1.05 \approx \$1325$, $\$464 + \$464/1.05 + (\$464/2 + \$500)/1.05^2 = \$1570$ respectively.

with respect to market prices.

The mean TE scores under SMB and WDEA are 0.812 and 0.814 for confinement and 0.865 and 0.861 for grazers respectively.[15] The optimal weights for WDEA are estimated at (α, β) = (0.451, -0.478) for confinement and (0.198, 0.004) for grazers. The negative estimate for β for confinement is explained by a positive correlation (0.283) between AI and SI, indicating that among confinement operations, 45.1% of the apparent allocative inefficiency and minus 47.8% of the apparent scale inefficiency under the VRS technology is attributed to the underestimation in technical inefficiency. The increased discriminatory power under WDEA leads to the decreased mean TE scores by 0.082 (i.e. 9.2%) and 0.069 (7.5%) for confinement and grazers respectively, compared to the VRS results. This is similar to the finding in Brissimis et al. (2010) that their SFA frontier estimation with incorporating AE decreased the TE by approximately 9%. The same explanation applies to both approaches; accounting for AE increases a linear substitutability of the predicted technological frontier and tends to lower the predicted TE for observed decisions. In a parametric model, moment conditions on AE can be used to augment a frontier estimation through distributional assumptions. In WDEA, the weight for linear substitution can be used to account for the correlation between the conventional AE and TE measures under a VRS frontier. Both approaches incorporate the concept of AE to enhance estimation efficiency.

5.3.2 Producer-Specific Shadow Values

Following Chambers and Färe (2008), the willingness to accept (WTA) and willingness to pay (WTP) for a change in input mixes are inferred by tracing the curvature along an estimated technological frontier. In theory, at an equilibrium in a frictionless economy, the WTA must be equal to the corresponding market price or higher, and the WTP must be equal to the market price or lower. The non-conformity of estimated shadow values to these theoretical predictions can provide insights into the state of factor markets or non-technological constraints for producers.

Figures 5.5 and 5.6 present the distributions of estimated WTA and WTP for confinement, plotted against the sample-proportion scaled from 0 to 1, and figures

[15] For SMB, the optimum sample sizes of input-oriented and output-oriented TE were found 157 and 267 for confinement and 121 and 129 for grazers. These values were searched from 10 equally-spaced values within the 40-90% (i.e. 40%,45%,...90%) of their sample sizes.

5.7 and 5.8 for grazers.[16] The WTA and WTP estimates vary substantially across observations. Under the VRS technology, the mean WTP for a cow (per year rental rate) is \$652 for confinement and \$344 for grazers, and the mean WTA \$4,082 and \$3,562 respectively. Similarly, under the WDEA technology, the mean WTP for a cow is \$283 for confinement and \$672 for grazers, and the mean WTA \$3,360 and \$3,425 respectively. Since the distributions of these shadow values are fat-tailed and contain extreme values, the averages are not be very informative. The distributions are alternatively studied with respect to their market rates in below.

Somewhat surprisingly, no systematic patterns are discerned in the difference between the VRS and WDEA results for WTA and WTP estimates. This may strike some readers as odd since the linear substitutability (as well as linear scalability) of a predicted technology would be systematically higher under WDEA than under VRS. In figures 1 and 2, for example, one may expect that WDEA yields a narrower range for the ratio of admissible shadow values, smaller gaps between WTA and WTP (e.g., the gap is infinite along a Leontief frontier and zero along a linear frontier), and generally smaller deviations of WTA and WTP from the market rates, compared to the VRS estimates. However, the relationships for the shadow values under VRS and WDEA frontiers in a simple two-dimensional diagram (e.g., see Appendix B) are generally inapplicable for higher dimensional input-output decisions.

The over-utilization and under-utilization of inputs are studied through the existence of unmet supply or demand for given market rate w_l^M for each input l, or latent marginal supply or demand (LMS or LMD) proportions defined as the sample proportions of observations $i \in \mathbb{I}$ satisfying $[WTA_{i,l} \leq w_l^M]$ or $[WTP_{i,l} \geq w_l^M]$ respectively. Under the VRS-technology specification, confinement operations exhibit high LMS proportions (i.e, 0.300 or above) for crop, animal, and machinery. Similarly, grazing operations show high LMS proportions for labor, animal, and crop acreage. The results under WDEA are generally similar, but the only high LMS proportion is for grazer's labor input. The over-utilization of these inputs may be caused by medium to long term investments in production assets and contractual agreements or the upward bias from subsidized dairy or crop production. Turning to under-utilization of

[16]The bottom figures in tables 5.6 and 5.8 contain the distributions of the estimated WTA and WTP for the bundle of a dairy cow and a crop acre. The WTA for the bundle is higher than the sum of WTA for individual inputs, and the WTP for the bundle is lower than the sum of WTP for individual inputs by the superadditivity of the directional derivative.

inputs, the VRS results find high LMD proportions for cow and crop acreage among confinement operations and no high LMD proportions among grazing operations. The WDEA results suggest high LMD proportions for labor and crop acreage among confinement. Confinement producer might not expand his herd when operational capacity is nearly full, cows of desired characteristics are scarce in the market, or operation expansions are put on hold for idiosyncratic reasons (e.g., uncertainty for family labor supply). Crop acreage appears over-utilized by grazers and under-utilized by confinement perhaps due to the inefficiency in land markets.

Additionally, the sensitivity of LMS and LMD proportions are summarized as unit-free elasticity measures with respect to market prices, or $|(dq/dp)/(q^M/p^M)|$ for LMS or LMD proportion q^M, market price p^M, and local slope dq/dp.[17] In contrast to the ideal factor market characterized with zero-LMS and zero-LMD with arbitrarily large elasticities, a poorly functioning factor market would exhibit a high LMS or LMD proportion with a very small elasticity. Panels B1 and B2 in table 5.3 report the calculated price elasticities of LMS and LMD across inputs and technology specifications. Among the inputs with high LMS proportions, highly-inelastic LMS (i.e. less than 0.300) are found in machinery (under VRS) for confinement and labor (under both VRS and WDEA) and crop acreage (under both VRS and WDEA) for grazers. These inputs appear systematically over-utilized since the extents of over-utilization depend little on their market rates. Capital-intensive confinement operations may face the difficulty in reversing their investments into machinery. Meanwhile, the apparent (and inexpensive) use of excess labor among grazers may potentially increase profits if it is related to organic milk production for sufficiently high price premiums. Among the inputs with high LMD proportions, highly-inelastic LMD values are identified for cows (under VRS), labor input (under WDEA), and crop acreage (under VRS and WDEA) among confinement. It may be worthwhile to investigate potential market failures or input distortions that prevent these dairies from expanding their herds and/or systematically hinder transfers of crop acreage from grazers to confinement operators.

[17] The slope is estimated by a linear regression with observations restricted to those of non-zero shadow values falling within the ± 5 percentile margins around the market price.

5.4 Conclusions

This article has developed a simple methodology that integrates the concepts of technical, scale, and allocative efficiencies into a nonparametric, technological-frontier estimation. The proposed Weighted DEA (WDEA) extends the standard VRS technological feasibility by estimating an optimal weighted average of the VRS, CRS, and profit frontiers. The proposed optimal weights minimize the sum of residual squares by regressing some initial adjustments for the VRS estimator on the conventional measures of scale and allocative inefficiencies. In the application to Maryland dairy data, the technical efficiency is on average 7.5% to 9.2% lower under WDEA, compared to the standard VRS estimates. Estimated producer-specific shadow values along the VRS and WDEA frontiers are generally similar with no obvious patterns of systematic differences. Considerations for alternative weight selection rules and rigorous statistical inferences are left for future research.

5.5 Tables and Figures

Table 5.1: Summary Statistics of Variables By Dairy System

	Mean	S.D.	Distribution				
			Min.	25th	50th	75th	Max.
Confinement							
Milk (cwt)	24,150	17,577	3,761	13,680	19,890	28,550	110,700
Cow (animal)	122	76	22	70	108	145	468
Labor†	35,020	50,010	0	2,064	18,500	45,540	281,300
Crop†	47,400	40,954	0	19,860	35,870	58,910	231,000
Animal Care†	181,200	129,666	32,390	97,320	137,900	210,100	746,900
Machinery†	148,900	101,620	22,060	87,530	129,700	182,100	806,300
Crop Acre (acre)	289	155	60	175	210	350	704
Pasture Acre (acre)	50	39	0	20	40	80	141
Grazer							
Milk (cwt)	12,440	5,573	2,670	9,467	11,550	14,550	42,950
Cow (animal)	87	29	37	70	81	97	195
Labor†	6,229	10,383	0	0	1,109	8,608	75,320
Crop†	18,250	17,932	0	6,188	12,240	24,770	107,200
Animal Care†	94,290	48,034	7,882	59,780	86,470	127,300	255,800
Machinery†	84,460	45,587	26,720	54,540	73,230	96,900	327,000
Crop Acre (acre)	132	108	0	30	150	180	600
Pasture Acre (acre)	152	60	53	96	130	207	280

1. Unbalanced panel data set on 1995-2009 contains 17 grazers and 29 confinement dairies with 5 dairies switching from confinement to grazing during the period, totaling 475 operation-year observations.

2. † Categorical expenses comprise the following: machinery ≡ custom hire + depreciation + fuel + rent + maintenance + utility, labor ≡ labor + employment benefit + pension, crop ≡ seed + chemicals + fertilizer, animal care ≡ feed + veterinary services.

Table 5.2: Summary of Efficiency Scores

	Mean	S.D.	Distribution				
			Min	25th	50th	75th	Max
Confinement							
TE (VRS)	0.896	0.095	0.589	0.821	0.905	0.996	1.000
SE (VRS)	0.965	0.058	0.677	0.962	0.989	0.999	1.000
AE (VRS)	0.787	0.095	0.525	0.729	0.788	0.853	1.000
SMB	0.812	0.170	0.428	0.670	0.848	0.985	1.000
WDEA	0.814	0.087	0.564	0.752	0.805	0.870	1.000
Grazers							
TE (VRS)	0.931	0.097	0.640	0.871	0.987	1.000	1.000
SE (VRS)	0.914	0.117	0.440	0.880	0.960	1.000	1.000
AE (VRS)	0.717	0.120	0.485	0.629	0.696	0.801	1.000
SMB	0.865	0.165	0.471	0.747	0.950	1.000	1.000
WDEA	0.861	0.090	0.623	0.820	0.878	0.926	1.000

1. TE (VRS), SMB, and WDEA are technical efficiency estimates. SE and AE are scale and allocative efficiency estimates respectively.

2. Weights (α, β) for WDEA, obtained in linear regressions, are (0.451, -0.478) for confinement and (0.198, 0.004) for grazers.

Table 5.3: Latent Marginal Supply and Demand (LMS and LMD) Proportions and Their Elasticities

Group	Technology	Cow	Labor	Crop	Animal	Machi.	Crop A.	Past. A.
A1. LMS Proportion								
Confinement	VRS	0.035	0.248	0.331	0.462	0.525	0.086	0.162
Confinement	WDEA	0.067	0.213	0.242	0.268	0.178	0.061	0.153
Grazers	VRS	0.075	0.472	0.130	0.323	0.242	0.311	0.248
Grazers	WDEA	0.075	0.460	0.174	0.280	0.224	0.273	0.124
A2. LMD Proportion								
Confinement	VRS	0.303	0.268	0.019	0.029	0.032	0.328	0.162
Confinement	WDEA	0.166	0.328	0.105	0.115	0.080	0.363	0.290
Grazers	VRS	0.180	0.043	0.093	0.037	0.043	0.062	0.180
Grazers	WDEA	0.273	0.130	0.143	0.081	0.087	0.106	0.211
B1. Elasticity of LMS								
Confinement	VRS	0.892	0.267	0.740	1.012	0.279	0.122	0.041
Confinement	WDEA	0.761	0.219	0.685	3.322	1.753	0.382	0.020
Grazers	VRS	0.396	0.071	0.449	0.674	0.771	0.075	0.063
Grazers	WDEA	0.280	0.115	0.479	0.760	1.223	0.118	0.095
B2. Elasticity of LMD								
Confinement	VRS	0.097	0.194	0.270	2.013	0.747	0.044	0.014
Confinement	WDEA	0.705	0.219	0.417	2.180	1.968	0.086	0.032
Grazers	VRS	0.154	0.005	0.098	1.080	0.952	0.047	0.020
Grazers	WDEA	0.101	0.347	0.184	0.629	0.618	0.134	0.043

1. Elasticity measures around the market prices are calculated through $|(dq/dp)/(q^M/p^M)|$ where p^M, q^M are the market price and the LMS or MUP proportion associated with WTP and WTA curves respectively, and dq/dp is the local slope estimate of those curves near the market valuation. The slops are estimated by linear regressions with observations restricted to those within the $\pm$ 5 percentile margins around the market price.

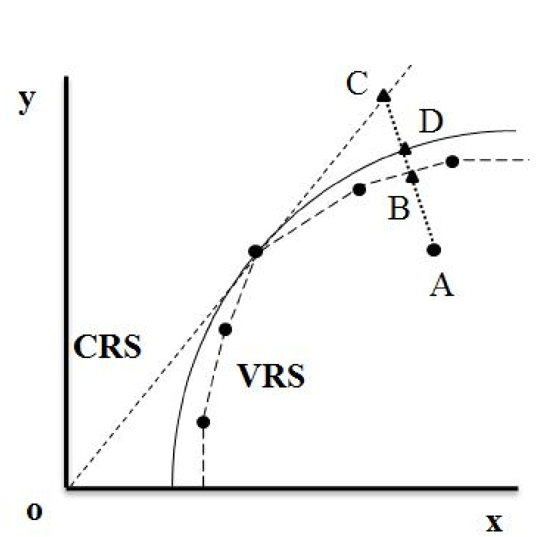

Figure 5.1: CRS, VRS, and Postulated Frontiers

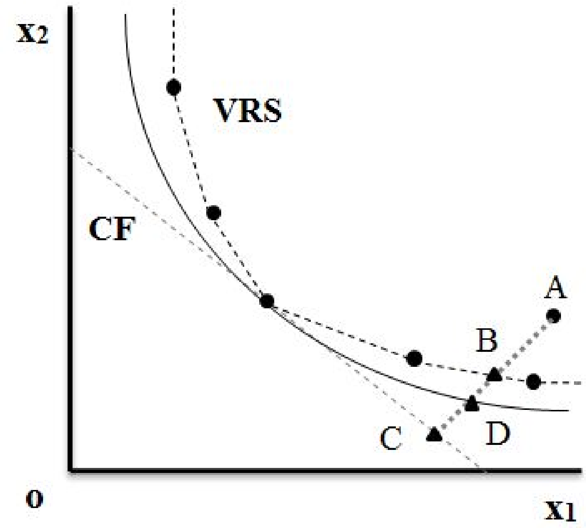

Figure 5.2: Cost, VRS, and Postulated Frontiers

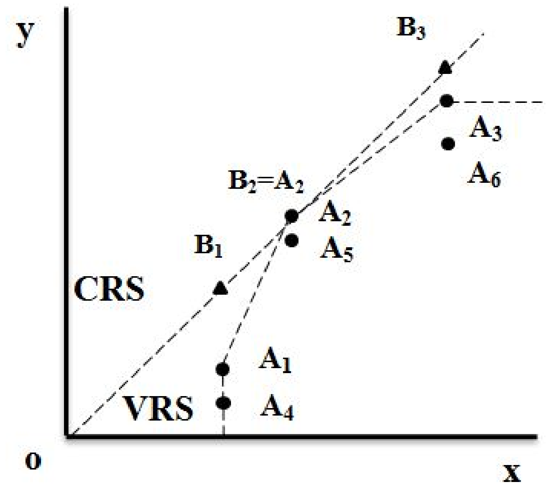

Figure 5.3: Example: SMB and WDEA (1/2)

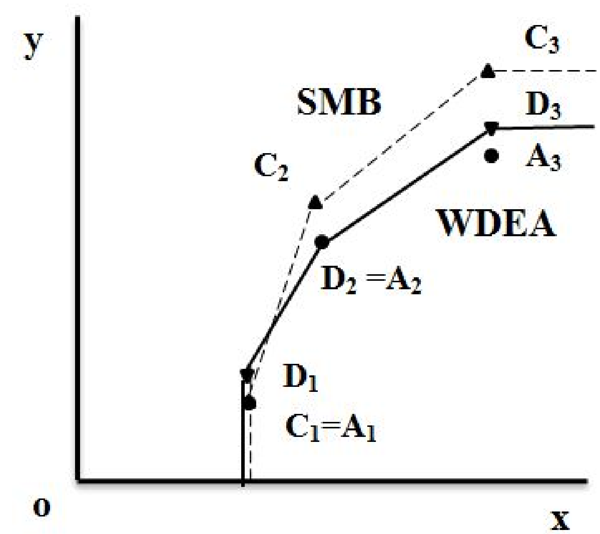

Figure 5.4: Example: SMB and WDEA (2/2)

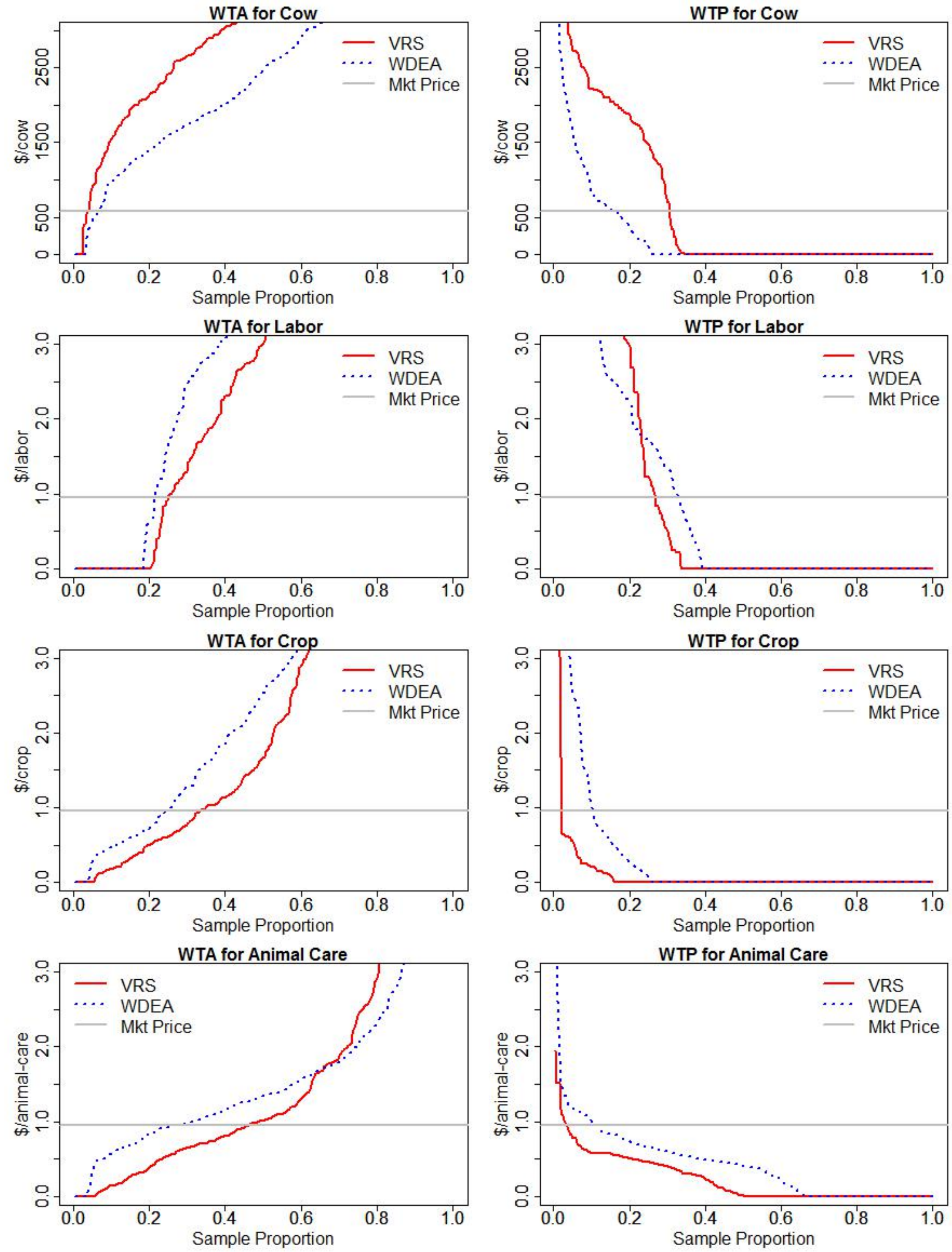

Figure 5.5: Shadow Values along VRS and WDEA Technological Frontiers (Confinement 1/2)

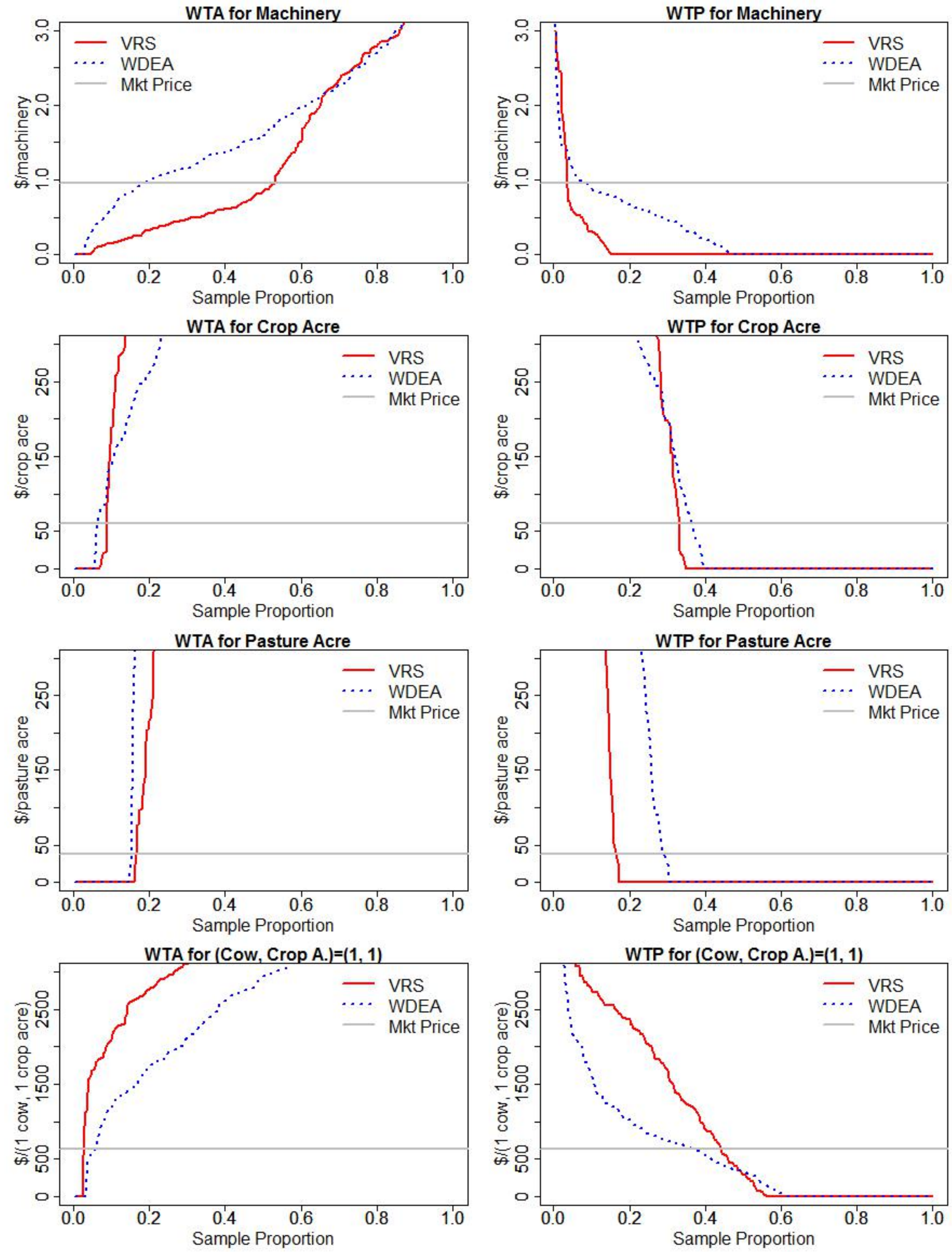

Figure 5.6: Shadow Values along VRS and WDEA Technological Frontiers (Confinement 2/2)

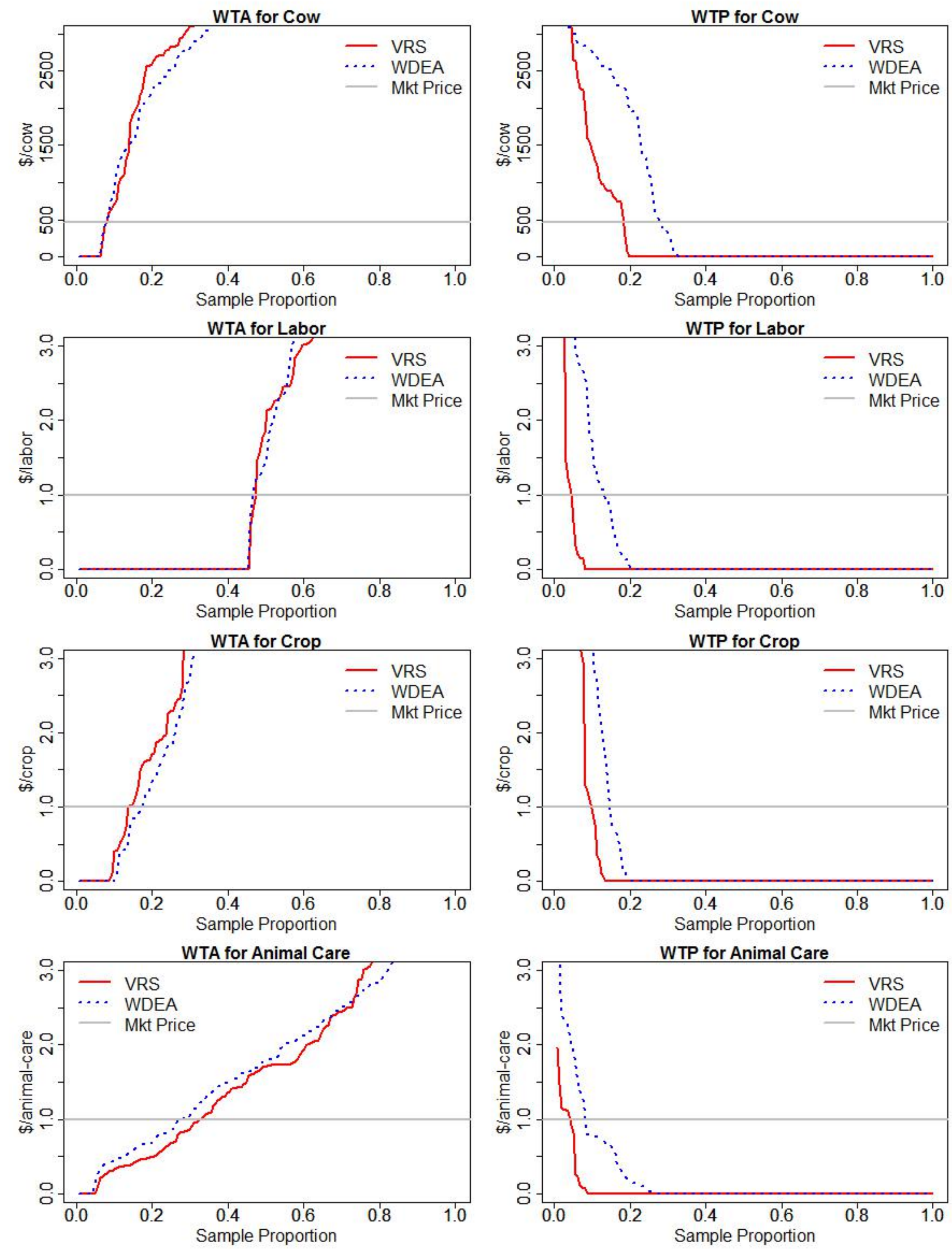

Figure 5.7: Shadow Values along VRS and WDEA Technological Frontiers (Grazers 1/2)

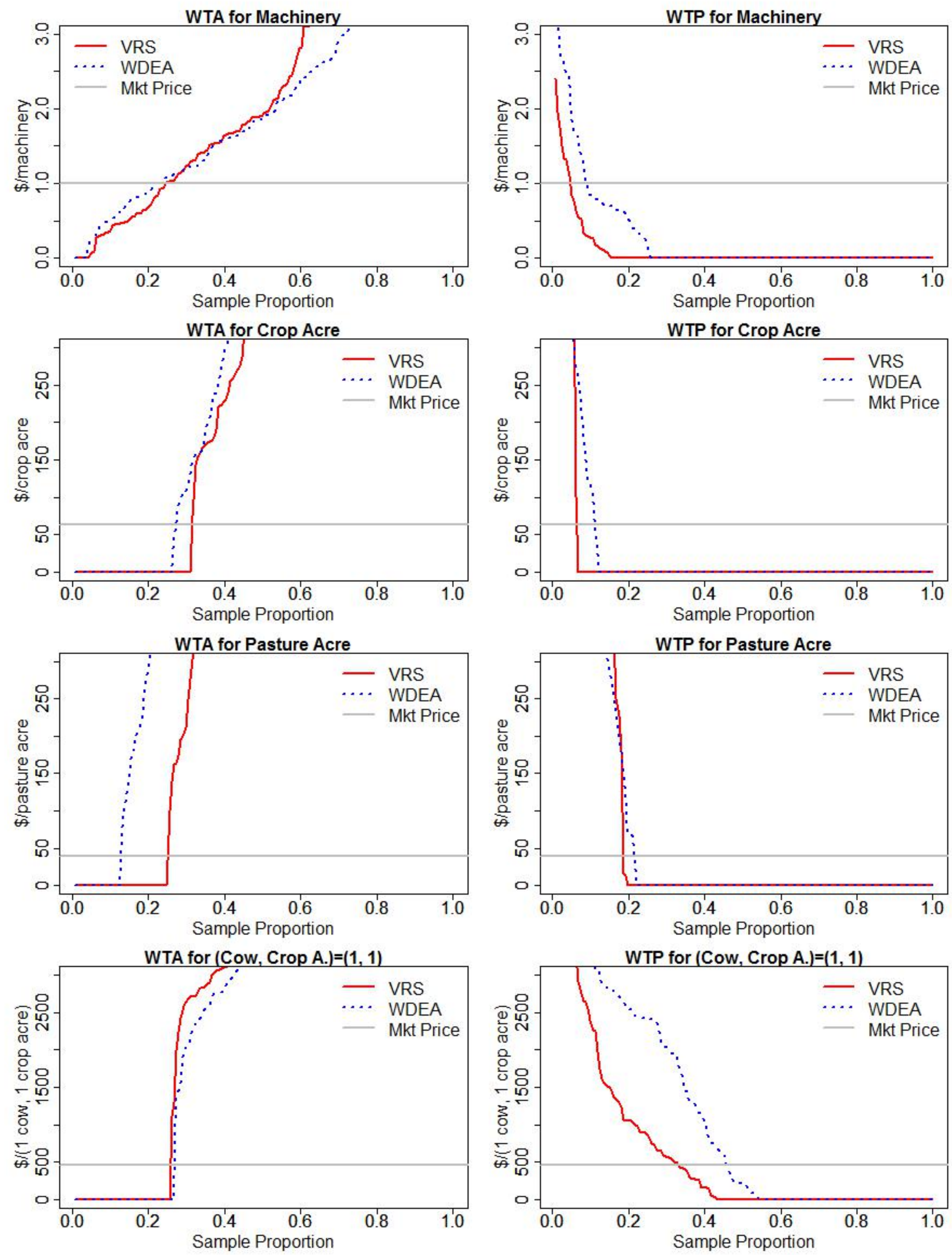

Figure 5.8: Shadow Values along VRS and WDEA Technological Frontiers (Grazers 2/2)

Appendix

A Estimations of Market-level and Producer-level Shadow Prices

This supplementary section describes two procedures employed in the application section; one for estimating unknown market-level prices of inputs (which are used to calculate AI and constructing WDEA frontiers) and the other for estimating producer-level shadow values (which are used to characterize VRS and WDEA frontiers).

Market-level input prices are estimated as shared shadow-values in a DEA setting. The linear programming problem proposed by Kuosmanen et al. (2006) combines (otherwise separately-estimated) DEA specifications of multiple producers into a single estimation problem for a common set of input prices that maximize the joint objective function subject to the (standard) DEA technological constraints;[18]

$$\max\{\sum_{j\in\mathbb{I}}\gamma_j : \forall j\in\mathbb{I},\ \gamma_j \le \boldsymbol{p}_j\boldsymbol{y}_j + f_j,\ \ \forall j,k\in\mathbb{I},\ \boldsymbol{p}_k\boldsymbol{y}_j - \boldsymbol{w}\boldsymbol{x}_j + f_k \le 0,$$
$$\boldsymbol{w}\left(\sum_{j\in\mathbb{I}}\boldsymbol{x}_j\right) \ge 1,\ \ \boldsymbol{p}\in\mathbb{R}_+^{MN},\ \boldsymbol{w}\in\mathbb{R}_+^{L},\ \boldsymbol{f}\in\mathbb{R}_+^{N}\} \tag{A.1}$$

where $\boldsymbol{p}_j$ is producer-specific shadow output values, f_j producer-specific scale parameter, and $\boldsymbol{w}$ the common input values across producers that are interpreted as market rates. To make the large scale linear programming problem manageable, this study estimates problem (A.1) as the average result of 100 subsample estimations, where each estimation uses 20 random observations in the sample.[19]

The current application additionally constraints the range of shadow values through incorporating market price information. Dairy production decision is modeled with milk output and the total of seven inputs including herd size, four categorical expenses (in labor, crop, animal, and machinery) divided by share-weighted price indices, and two types of land areas (for crop production and pasture). The common shadow-values of inputs are estimated by equation (A.1) with the following

[18] The common input price under a free-disposable hall (FDH) is; $\max\{\sum_j \gamma_j : \forall j,k,\gamma_j \le \boldsymbol{p}_{jk}\boldsymbol{y}_j + f_{jk},\ \ \forall j,k,\boldsymbol{p}_{jk}\boldsymbol{y}_j - \boldsymbol{w}\boldsymbol{x}_j + f_{jk} \le 0, \boldsymbol{w}(\sum \boldsymbol{x}_j) \ge 1,\ \ \boldsymbol{p}\in\mathbb{R}_+^{MN^2},\ \boldsymbol{w}\in\mathbb{R}_+^{L},\ \boldsymbol{f}\in\mathbb{R}_+^{N^2}\}$.

[19] When drawing each random subsample, the ratio of two groups of dairy farms was fixed at that of the sample.

constraints;

C1. $w_{labor} = w_{crop} = w_{animal} = w_{machine}$

C2. $\forall i \in \mathbb{I},\ 0.90\ (p_{milk,i}/p^M_{milk}) \leq w_{labor} \leq 1.10\ (p_{milk,i}/p^M_{milk})$

C3. $w_{pasture\_acre}/w^M_{pasture\_acre} = w_{labor},\ \ w_{crop\_acre}/w^M_{crop\_acre} = w_{labor}$

C4. $0.5\ w_{cow(conf.)} \leq w_{cow(graz.)} \leq 1.5\ w_{cow(conf.)}$

C5. $0.90(\sum_i \pi_i/C_i)/N \leq (\sum_i -f_i)/N \leq 1.10\ (\sum_i \pi_i/C_i)/N.$

Item C1 sets an identical shadow value for categorical expenses in labor, crop, animal, and machinery since these variables are originally in dollar terms that should be valued equally. Item C2 normalizes the price level with nominal milk price p^M_{milk}=\$18.74/cwt, so that the shadow value for one hundredweight of milk is worth about 18.74 times of the dollar-valued expense within $\pm$10% deviations. Similarly, item C3 normalizes the shadow rental rates for crop and pasture acreage by nominal rates $w^M_{pasture\_acre}$ =\$63.35 and $w^M_{crop\_acre}$ =\$39.02 per acre respectively, or the average rates in North Central Maryland during years 2008-2012.[20] In item C4, the rental rate for cow, estimated separately for two dairy systems of grazing and confinement, assumes the rate for grazers to fall within $\pm$50% of the rate for confinemnt. Finally, item C5 confines the average shadow profit $\sum_i(\boldsymbol{p}_i\boldsymbol{y}_i - \boldsymbol{w}\boldsymbol{x}_i)/N$ to $\pm$10% deviations from the sample-average returns to production costs. Once estimated, the shadow values can be converted back into dollar terms; for instance, the nominal shadow value for labor expenses is obtained as $w^N_{labor} = w_{labor} * (p^M_{milk}/\bar{p}_{milk,i})$ using the average shadow value $\bar{p}_{milk,i}$ of milk output.

Producer-specific shadow values are estimated as the marginal rate of transformations along a technological frontier. One difficulty is the non-uniqueness of the shadow values for inputs and outputs along a piecewise-linear DEA frontier, which admits infinitely many supporting hyperplanes at its kink points. Following Chambers and Färe (2008), a derivative-like concept for these hyperplanes are given by the supper differential of $D(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y)$ for a change of inputs from $\boldsymbol{x}$ to $\boldsymbol{x}^c \in \mathbb{R}^L$;

$$\begin{aligned}\partial D(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y) = \{\boldsymbol{\nu} \in \mathbb{R}^L_+ : & D(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y) + \boldsymbol{\nu}(\boldsymbol{x}^c - \boldsymbol{x}) \\ & \geq D(\boldsymbol{x}^c, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y), \forall \boldsymbol{x}^c \in \mathbb{R}^L\}. \end{aligned} \tag{A.2}$$

[20]These rental prices for crop and pasture acres are taken from the mean rents, across counties in North Central Maryland and years 2008-2012, of the corresponding items in USDA-NASS rental rate estimates.

While any member of the super differential could be interpreted as shadow values of inputs and outputs, its directional derivative is uniquely defined as

$$D'_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y, \boldsymbol{x}^c) = \lim_{\lambda \to 0+} \frac{D(\boldsymbol{x} + \lambda \boldsymbol{x}^c, \boldsymbol{y}, \boldsymbol{g}_x, \boldsymbol{g}_y) - D(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{g}_x, \boldsymbol{g}_y)}{\lambda}, \tag{A.3}$$

which is positive linear homogeneous and concave in $\boldsymbol{x}^c$ and satisfies $D'_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y, \boldsymbol{0}) = 0$.[21]

The most economically relevant shadow values are those for the willingness to pay (WTP) and the willingness to accept (WTA) for a marginal change in inputs. The authors show that under the directional change toward l-th unit vector $\boldsymbol{e}_l = [0..1..0]^T$ (i.e. $\boldsymbol{e}_l \boldsymbol{w} = w_l$), WTP for input l at decision $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is calculated as;[22]

$$\begin{aligned} \min\{w_l : & \forall j \in \mathbb{I},\ \boldsymbol{p}\boldsymbol{y}_j - \boldsymbol{w}\boldsymbol{x}_j + f \leq 0,\ \ \boldsymbol{w}\boldsymbol{g}_x \geq 1, \\ & \boldsymbol{p}\boldsymbol{y}_0 - \boldsymbol{w}\boldsymbol{x}_0 + f = 0,\ \ \boldsymbol{p} \in \mathbb{R}_+^M,\ \boldsymbol{w} \in \mathbb{R}_+^L,\ f \in \mathbb{R}\} \end{aligned} \tag{A.4}$$

where $\boldsymbol{g}_x$ is a bundle of inputs that normalizes shadow values (and $\boldsymbol{g}_x = \boldsymbol{x}_0$ for each observation 0 in this study). The estimated values are converted into dollar terms by, for example, $w^N_{labor,i} = w_{labor,i} * (p^M_{milk}/p_{milk,i})$ where $w_{labor,i}$ and $p_{milk,i}$ are the estimates from (A.4). Similarly, under directional change $-\boldsymbol{e}_l = [0..-1..0]^T$, willingness to accept (WTA) for input l is given by;[23]

$$\begin{aligned} \max\{w_l : & \forall j \in \mathbb{I},\ \boldsymbol{p}\boldsymbol{y}_j - \boldsymbol{w}\boldsymbol{x}_j + f \leq 0,\ \ \boldsymbol{w}\boldsymbol{g}_x \leq 1, \\ & \boldsymbol{p}\boldsymbol{y}_0 - \boldsymbol{w}\boldsymbol{x}_0 + f = 0,\ \ \boldsymbol{p} \in \mathbb{R}_+^M,\ \boldsymbol{w} \in \mathbb{R}_+^L,\ f \in \mathbb{R}\}. \end{aligned} \tag{A.5}$$

Table A.1: Market Prices Used For Calculating Allocative Efficiency

	Cow	Labor	Crop	Animal	Machi.	Crop A.	Past. A.	Avg.Profit
Confinement	574.83	0.955	0.955	0.955	0.955	60.52	37.28	79,970
Grazer	463.51	1.000	1.000	1.000	1.000	63.36	39.03	55,335

1. Based on Kuosmanen et al. (2006) with additional constraints.

[21] It is a support function of the super differential; $D'_T(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y, \boldsymbol{x}^c) = \inf\{\boldsymbol{\nu}\boldsymbol{x}^c : \boldsymbol{\nu} \in \partial D(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y)\}$.

[22] Dual problem: $\max\{\theta : \sum_{j\in\mathbb{I}} \lambda_j \boldsymbol{y}_j \geq \mu \boldsymbol{y}_0, \sum_{j\in\mathbb{I}} \lambda_j \boldsymbol{x}_j \leq \mu \boldsymbol{x}_0 - \theta \boldsymbol{g}_0 + \boldsymbol{e}_l, \sum_{j\in\mathbb{I}} \lambda_j = \mu, \boldsymbol{\lambda}\mathbb{R}_+^N, \mu\mathbb{R}_+, \theta\mathbb{R}_+\}$.

[23] Dual problem: $\min\{\theta : \sum_{j\in\mathbb{I}} \lambda_j \boldsymbol{y}_j \geq \mu \boldsymbol{y}_0, \sum_{j\in\mathbb{I}} \lambda_j \boldsymbol{x}_j \leq \mu \boldsymbol{x}_0 + \theta \boldsymbol{g}_0 - \boldsymbol{e}_l, \sum_{j\in\mathbb{I}} \lambda_j = \mu, \boldsymbol{\lambda}\mathbb{R}_+^N, \mu\mathbb{R}_+, \theta\mathbb{R}_+\}$.

B Relative Shadow Values Between VRS and WDEA: A Simple Case

Consider simple geometrical relationships of shadow values along VRS and WDEA frontiers, as depicted in figure B.1. In two-input space x_1-x_2, several observations (shown as circles) are connected by a piecewise-linear VRS frontier. Observation B^{AE} is also allocatively efficient and supporting a cost frontier. For two observations B and B', the intersections between the radial contractions (toward the origin) and the cost frontier are denoted as points C and C' respectively. The figure shows that decisions B and B' use too much input x_1 and too little x_2, compared to decision B^{AE}. Thus, for example, at decision B the implied shadow value of input x_2 to input x_1 is lower than those of market rates, or the slope of BB' is smaller than the slope of CC'.

Suppose that decision D between B and C and decision D' between B' and C' are the predicted technically-efficient decisions under WDEA with weights $\alpha > 0$ and $\beta = 0$. The slopes of segments BB', CC', and DD' correspond to the relative shadow values of input x_2 to input x_1 under the frontiers of VRS, cost, and WDEA respectively. The coordinates of decision $D = (x_1^D,\ x_2^D)$ are given by $x_l^D = x_l^B - \alpha(x_l^B - x_l^C) = (1-\alpha)x_l^B + \alpha x_l^C$ for input $l = 1,\ 2$. By nothing that $x_l^D - x_l^{D'} = (1-\alpha)(x_l^B - x_l^{B'}) + \alpha(x_l^C - x_l^{C'})$ for input $l = 1,\ 2$ on segment DD', it follows that the slope of BB' is smaller than the slope of DD';

$$\left|\frac{x_2^B - x_2^{B'}}{x_1^B - x_1^{B'}}\right| < \left|\frac{x_2^C - x_2^{C'}}{x_1^C - x_1^{C'}}\right| \quad \Rightarrow \quad \left|\frac{x_2^B - x_2^{B'}}{x_1^B - x_1^{B'}}\right| < \left|\frac{(1-\alpha)(x_2^B - x_2^{B'}) + \alpha(x_2^C - x_2^{C'})}{(1-\alpha)(x_1^B - x_1^{B'}) + \alpha(x_1^C - x_1^{C'})}\right|. \tag{B.1}$$

Similarly, if the slope of BB' were greater than the slope of CC', then the slope of BB' would be greater than the slope of DD'. This implies that the local marginal rate of substitution under WDEA (the slope of DD') is closer to the relative market rates (the slope of CC'), compared to that of VRS (the slope of BB'). Thus, WDEA's estimates for WTP or WTA of input x_1 at decisions D and D' are lower, and that of input x_2 higher than the VRS counterparts at decisions B and B'. While these results are fairly straightforward, such relationships become too complicated to derive simple characterizations in a higher dimensional input-output space.

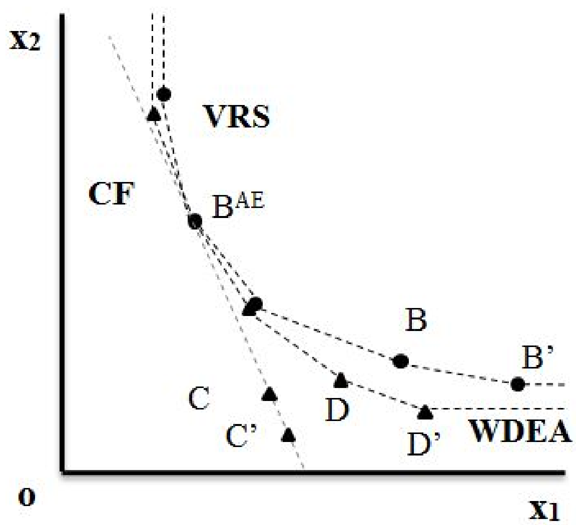

Figure B.1: Shadow Values Along Frontiers

Chapter 6: Conclusions

The composition of the US dairy industry has been shifting toward the high concentration of production by a small number of very large-scale dairy producers, who have been replacing small-to-medium-scale dairies. At the aggregate level, it appears that the dairy industry is simply following the suit of transformations that various grain and livestock producers have been put through and that are inevitable in many ways under ever-more sophisticated mechanical and chemical approaches to food production. At the producer level, however, a number of farms have been experimenting with combining dairy production and ecological approaches with farm resource management. Management-intensive grazing (MIG) demands an innovative approach to strike a balance between pasture production and the dietary needs of dairy cows. Such experiments are partly a natural response to the general trend in emerging consumer preferences for organic and local produce, environmentally- and socially-responsible production practices, and increased information flow between the producer and the consumer. Many opportunities seem to await innovative farmers who adapt to the increasingly complex food industry at it meets diverse consumer demands and accommodates various motivations behind their food choices. Another explanation for the emergence of MIG would be a technical response to uncertain production environments in terms of weather, markets, and future policies in agriculture and the environment. Innovations in MIG offer possibilities for greater input substitutability, a more diverse menu of risk management options, and easier entry and exit decisions with lower fixed costs than the conventional confinement dairy operations.

The main empirical findings in the thesis are the following. In chapter 3, the results indicate steady and modest technological progress for dairy production in the Northeast region, but on average more than half of the progress is unexploited by the producers. The production gaps between the technically efficient and the less efficient producers have increased, particularly among MIG farms. Increased efforts in disseminating the information on new production techniques would be helpful.

Chapter 4 finds evidence for increased production risks in confinement operations and decreased production risks in grazing operations for the selected study periods. During 2006-2009, when dairy profits fluctuated with volatile milk price and feed costs, MIG operations benefited from reduced reliance on purchased feed and other market inputs. Some MIG farms also commanded a consistently-high price premium through organic milk production. The results in chapter 5 suggest that machinery and (hired) labor are over-utilized by confinement and MIG dairies respectively, and crop acreage is under-utilized by confinement and over-utilized by MIG. Systematic over- or under-utilization of inputs calls for further investigations into the issue, which may require policy reforms that improve market efficiency and ameliorate production distortions.

Lastly, novelties and shortcomings of the proposed methodologies are noted along with some future directions for research. The difference in distance-functions (DDF) approach in chapter 3 is a Malmquist Productivity Index (MPI) decomposition under a regression framework. The method combines the advantages of flexible non-parametric functional forms (for technological frontiers and the shifts of those frontiers) and those of the parametric treatment of non-production factors (as regression covariates). The two versions of technical change measurements are proposed in the study: one similar to the econometric distance measurement and the other in the spirit of the standard MPI decomposition. The two technical change measurements merit further investigations. Chapter 4 develops a state-contingent (SC) risk analysis on balanced-panel data. This contributes to bridging the gap between the theoretical SC framework and the empirical risk analysis. One weakness is the simultaneous assumptions of no technical change and cross-sectionally homogeneous state realizations; a short-term time series of data are welcome for the former assumption but not for the latter. More suitable assumptions may be devised according to data availability. The weighted DEA (WDEA) approach in chapter 5 simultaneously utilizes several efficiency concepts in a new variant of technology approximation in DEA. One must recognize that the efficiency gain from imposing sample-level systematic relationships among these concepts may come at the expense of oversimplifications at the observation-level. A potential extension is to let a weight structure vary with certain characteristics of producers.

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