Abstract

Title of dissertation: THE RELATIONSHIP BETWEEN TEACHERS' MATHEMATICAL KNOWLEDGE AND THE MATHEMATICS ACHIEVEMENT OF STUDENTS IN GRADES FOUR AND FIVE

Jana E. Palmer, Doctor of Education, 2014

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The purpose of this study was to determine if there was a relationship between teacher mathematical knowledge (content and pedagogy) and the mathematics achievement of students in grades four and five. This study used a quantitative approach using Hierarchical Linear Modeling (HLM). Through a quantitative study based upon a teacher assessment of mathematics content and pedagogy and a student assessment entitled the Measures of Academic Progress (MAP), the researcher measured the teachers' mathematical content knowledge, mathematical pedagogy knowledge, and analyzed the data to determine if there was a relationship between teacher knowledge and student achievement. The assessments were based on the Maryland state curricular standards. All teachers involved in the study were considered generalists at the elementary level. Student achievement was measured through MAP. Through the use of the teacher knowledge assessment, the study provided valuable data that could be used to inform colleges providing training to pre-service teachers, principals, supervisors, and those providing professional development to elementary teachers. Additionally, the study could be used to inform teacher education and education policy efforts intended to strengthen and support teacher quality while improving the achievement of students in mathematics.

THE RELATIONSHIP BETWEEN TEACHERS' MATHEMATICAL KNOWLEDGE AND THE MATHEMATICS ACHIEVEMENT OF STUDENTS IN GRADES FOUR AND FIVE

by

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Dedication

Philippians 4:6-8

Be careful for nothing; but in everything by prayer and supplication with thanksgiving let your requests be made known unto God. And the peace of God, which passeth all understanding, shall keep your hearts and minds through Christ Jesus. Finally, brethren, whatsoever things are just, whatsoever things are pure, whatsoever things are lovely, whatsoever things are of good report; if there by any virtue, and if there be any praise, think on these things.

To my parents, David and Dawn Barnes, I am blessed to have grown up in a loving home with parents that prayed for me, believed in me, supported me, valued education, and provided me with their wisdom and guidance through every facet of my life. I was always told I could do anything I set my mind to do, if I worked hard. My dissertation and doctorate degree are a result of their belief, encouragement, and support. I am so blessed.

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Chapter One: Introduction

Overview/Research

"Teachers are the key to improving mathematics education…Regardless of the curriculum or the assessment process in a school district, the person in charge of adapting materials for a particular classroom and student is the teacher" (Burns, 2000, p. 3). A student's view of what it means to know and do mathematics is shaped at the elementary school level with the assistance of teachers. It is through each teacher's actions that every student can learn with understanding (Van de Walle, Karp, Lovin & Bay-Willams, 2014). "When teaching, mathematics teachers access their usable knowledge of mathematics content, as well as their knowledge of mathematics teaching and learning" (Campbell & Nishio, in press, p.1) Creating classrooms in which students problem solve, engage in positive struggle, and make connections with the content they are learning is very complex (Van de Walle et al., 2014). "Teachers who have developed a profound understanding of fundamental mathematics are better able to facilitate developing this understanding in their students, reveal and represent connections among and between topics, and encourage multiple ways of solving problems" (Lias, Krivak-Fowler, Holdan, maxwell, 2005-2006, p. 73).

In the United States, elementary teachers are, for the most part, generalists. Pre-service teacher education typically includes two or three courses in mathematics content and one course in the teaching of mathematics. Pre-service preparation programs generally provide future teachers with a breadth of mathematical content, which tends to eliminate the ability to help them develop a true mathematical understanding (Lias et al., 2005-2006). Furthermore, as generalists, elementary teachers are responsible for teaching all subjects, with particular attention to reading or language arts, in a self-contained classroom. "Traditionally, undergraduate pre-service programs prepare future elementary teachers to teach multiple subjects, often focusing on reading" (Gojak, 2012, p. 13). The pre-service background and general teaching responsibilities of elementary teachers do not typically furnish the continuous development of specialized knowledge that is needed for teaching mathematics today. "Upon graduation, teachers may know how to do some basic arithmetic and algebra skills, but lack real understanding of the important big ideas of elementary mathematics such as number sense, the base ten system, measurement and reasonableness" (Lias et al., 2005-2006). Despite the acknowledged importance of mathematics and teachers lacking adequate preparation, there are relatively few elementary teachers with a math specialist degree or an extensive math background to enable them to adequately support students in their learning of mathematics content.

Purpose of the Study

The purpose of this study was to determine if there was a relationship between teacher mathematical knowledge (content and pedagogy) and the mathematics achievement of students in grades four and five. This study used a quantitative approach using Hierarchical Linear Modeling (HLM). The methodology was based upon the research entitled, *The Relationship between Teachers' Mathematical Content and Pedagogical Knowledge, Teachers' Perceptions, and Student Achievement*, a study completed at the University of Maryland by Campbell and Nishio (in press).

Through a quantitative study based upon a teacher assessment of mathematics content and pedagogy and a student assessment entitled the Measures of Academic

Progress (MAP), the researcher measured the teachers' mathematical content knowledge, mathematical pedagogy knowledge, and analyzed the data to determine if there was a relationship between teacher knowledge and student achievement. The assessments were based on the Maryland state curricular standards. All teachers involved in the study were considered generalists at the elementary level. Student achievement was measured through MAP. Through the use of the teacher knowledge assessment, the study provided valuable data that could be used to inform colleges providing training to pre-service teachers, principals, supervisors, and those providing professional development to elementary teachers. Additionally, the study could be used to inform teacher education and education policy efforts intended to strengthen and support teacher quality while improving the achievement of students in mathematics.

Background

In this quantitative study of teacher knowledge, state assessment data was used to select four elementary schools with similar demographics. The demographics included: students receiving free and reduced meals, student enrollment, and geographic location. The principals of the four schools were asked to support the research by providing release time for teachers in grades four and five to complete content knowledge and pedagogical knowledge mathematics assessments. The data department in District A was asked to provide the teachers with the student data which they accessed through District A's database. The teachers returned both their completed assessment and student data directly to the researcher.

District A services more than 22,000 students in grades pre-kindergarten through grade twelve. Overall, the district consists of 46 schools; 27 elementary schools, seven middle schools, one middle-high school, seven high schools, one outdoor education center, one special education center, one technical high school, one school for the arts, and one evening high school. The demographics of the students in District A, based upon the 2012-2013 school year, are as follows: American Indian .002%; Asian .02%; Black/African American 11.8%; Hispanic/Latino .06%; White 74.4%; and two or more races .06%. At the elementary level, 19.9% of students are English language learners, and 27.7% are Special Education students. The number of students qualifying for free and reduced meals has increased from 32.2% in 1999 to 52.2% in 2012.

The measure of progress for students based upon the requirements of the *No Child Left Behind Act* of 2001(NCLB) has been determined in the state of Maryland through the use of the Maryland School Assessment (MSA). As shown in Table 1 and Table 2, the MSA mathematics data for District A students in grades four and five has improved. However, the improvement has not been significant. Overall, the scores have remained at about the same level of proficiency for approximately five years for all students, students receiving free and reduced meals (FARM), English language learners (ELL), students with disabilities (SWD) services, males, and females. The lack of significant improvement has been a cause for concern among administrators (school and central office based) and classroom teachers.

Table 1

Grade 5 Percentage of Students Scoring Proficient or Advanced on the Mathematics Portion of MSA by Subgroup

Year	All	FARMS	ELL	SWD	Male	Female
2004	62.4%	46.6 %	25.0 %	22.2 %	62.7 %	62.0 %
2006	72.9%	59.2 %	42.9 %	46.6 %	75.0 %	74.5 %
2009	85.7%	79.1 %	58.8 %	58.8 %	85.7 %	85.6 %
2012	86.6%	79.5 %	55.0 %	52.8 %	84.1 %	89.2 %

Table 2

Grade 4 Percentage of Students Scoring Proficient or Advanced on the Mathematics Portion of MSA by Subgroup

Year	All	FARMS	ELL	SWD	Male	Female
2004	78.1%	65.8 %	54.5 %	47.8 %	78.4 %	77.8 %
2006	89.6%	82.3 %	67.7 %	66.1 %	88.3 %	90.9 %
2009	93.8%	90.1 %	80.0 %	82.1 %	93.4 %	94.2 %
2012	90.5%	85.8 %	79.4 %	52.9 %	89.4 %	91.8 %

In 2004, District A began a math and reading mentoring, coaching, data analysis, and intervention initiative through the role of a Student Achievement Specialist (SAS) position in all elementary schools. In most schools, the teacher hired as an SAS had previously held the responsibility of reading specialist. Improving the mathematics content and pedagogy knowledge of the SAS was a priority through ongoing, job-embedded, and monthly county-wide professional development support. In 2012, the role of the SAS was refined and renamed to Lead Teacher (LT). The role of the LT was specifically designed to improve classroom instruction in all content areas through coaching and mentoring.

Professional development has been provided for elementary teachers through district wide initiatives, coaching, mentoring, and several different opportunities offered at the county level. Some of the professional development opportunities have been mandatory, while others have been optional. For the past two years, the elementary mathematics department has offered year long professional development institutes for one teacher from each grade level at each of the elementary schools. Each elementary school has also had the opportunity to provide professional development based upon the needs of the school and the required School Improvement Plan.

Statement of the Problem, Hypothesis, and Research Questions

"Although many studies demonstrate that teachers' mathematical knowledge helps support increased student achievement, the actual nature and extent of that knowledge – whether it is simply basic skills at the grade level they teach, or complex and professionally specific mathematical knowledge – Is largely unknown" (Ball, Hill, & Bass, 2005, p. 16). Current United States concerns involving the need to improve instruction in mathematics and science are similar to the concerns in 1957 with the challenge of Sputnik. The response to the Sputnik crisis was; "We see clearly what is broken (mathematics and science education and research), and we are going to fix it by taking the best first steps we can, and then by learning as we go along" (NMAP, 2008, pp. 12-13). As a result, the nation moved into an era of focus on science and engineering. The NMAP (2008), reemphasized the importance of further research in many areas of mathematics in order to achieve better results for students and to place a commitment of "learning as we go along" (NMAP, 2008, p. 13).

The research hypothesis for this study was: Teacher mathematical knowledge (content and pedagogy) would be positively related to the mathematics achievement of students in grades four and five.

The hypothesis was based upon research completed by Campbell and Nishio (in press). Their study indicated a significant relationship between upper-elementary teachers' mathematical content knowledge and their students' mathematics achievement (Campbell & Nishio, in press). It was determined that for each standard deviation increase in the content knowledge of the teachers, "the estimated mathematics achievement score of their students increased by 7.1%" (Campbell & Nishio, in press, p.26). Additionally, Hill, Schilling, and Ball (2004) identified mathematics content and pedagogy knowledge, developed a multiple choice assessment to measure the knowledge, and determined there was a significant relationship between teacher content knowledge and the achievement of students in grades one and three.

This study presented one research question with two sub questions.

Research Question:

To what extent does teacher mathematical knowledge (content and pedagogy) relate to the mathematics achievement of students in grades four and five?

Sub questions:

1. To what extent does teacher mathematical content knowledge correlate to the mathematics achievement of students in grades four and five?

2. To what extent does teacher pedagogy mathematical knowledge relate to the mathematics achievement of students in grades four and five?

As a first step, data was analyzed in District A to determine four elementary schools with similar demographics but different student achievement levels on MSA and MAP. The demographics included: students receiving free and reduced meals, student enrollment, and geographic location. Choosing four schools provided approximately 30 teachers to invite to take the assessment in order to have an adequate sample size to complete the quantitative data collection and analysis. The average class size in District A was 25 students per class. This provided the researcher with approximately 750 students whom each had three MAP scores throughout the 2012-2013 school year and one MSA score. A proposal requesting permission to complete the research in these four schools was submitted to District A's research and data analysis department. The principal of each school was contacted, requesting that they provide release time for their teachers in grades four and five to take the teacher knowledge assessment. The teacher knowledge assessment was designed to measure the understanding of mathematics content assessed in grades four and five and the understandings a teacher must have in order to teach that content (Campbell & Nishio, in press). The data department in District A provided the student level data connected to each teacher. This allowed the participants to remain anonymous to the researcher. The district had a student database linking student assessment scores of both MSA and MAP to teachers in each elementary school, which simplified the process of collecting the student data.

Pearson's correlation coefficients were used to determine if there was a relationship between teachers' content knowledge and pedagogical knowledge. Interclass correlation coefficients were used to measure the portion of total variance in student scores attributable to a teacher and student across time. Three components were analyzed using a three-level HLM model. The level one model measured student growth on MAP mathematics over time. The level two model controlled for student level variables of: gender, special education (SE), free and reduced meals (FARM), and English language learners (ELL). These were dichotomous indicators with a value of 0 indicating the student does not have the characteristic or 1 indicating the student does have the characteristic. The level three model predicted growth based upon teacher pedagogical knowledge and teacher content knowledge.

Potential Significance

The findings of this study add to the collective research on the need for teachers to have strong mathematical content knowledge and pedagogy to positively impact the achievement of elementary students. In addition, this study supported the recommendations of the NMAP (2008) to continue building capacity for more rigorous research in mathematics education to improve pre-service programs in an effort to better prepare teachers through developing a real understanding of the important content in mathematics. “Teachers who have developed a profound understanding of fundamental mathematics are better able to facilitate developing this understanding in their students, reveal and represent connections among and between topics, and encourage multiple ways of solving problems” (Lias et al., 2005-2006, p. 73).

Limitations

The findings of this study were limited to elementary teachers in District A teaching math in grades four and five. The teachers were currently employed as elementary teachers in a medium sized school district. In addition, many of the teachers had common experiences and professional development opportunities that may be different in other school districts. These commonalities included: the same mathematics curriculum and resources to support implementation and participation in similar professional development activities. Both commonalities were considered as a generalization of the findings of this study. Additionally, the researcher was an elementary principal and formerly an elementary mathematics supervisor for District A. The generalization of the findings to other populations should be utilized with caution.

Definition of Terms

Elementary Mathematics Specialist (EMS). A teacher hired to work with all teachers to improve the teaching and learning of mathematics instruction through ongoing, job-embedded professional development.

Measures of Academic Progress (MAP). Computerized adaptive assessments which provide educators with detailed data and information to assist teachers in meeting the individual needs of students. The assessments are aligned to national and state curricula and standards. They are currently used in District A three times per year for students in grades kindergarten through grade eight.

Maryland School Assessment (MSA). A yearly test of reading and mathematics achievement in the state of Maryland which meets the federal government requirements under the No Child Left Behind Act.

No Child Left Behind Act of 2001 (NCLB). NCLB is a United States Act of Congress to reauthorize the Elementary and Secondary Act of 1965. The federal government enacted Public Law 107-110 to ensure "all children have a fair, equal, and significant opportunity to obtain a high-quality education and reach, at minimum, proficiency on challenging state academic achievement standards and state academic assessments" (U.S. Education, 2001).

Common Core State Standards (CCSS). The standards define the knowledge and skills needed for students to be prepared to enter college and the workforce. They were created based upon a coordinated effort of the National Governors Association Center for Best Practices and the Council of the Chief State School Officers in order to provide a clear and consistent framework for teachers and parents (NGA, October 2011).

Mathematical Content Knowledge. According to Campbell and Nishio (in press), "Mathematical content knowledge is defined as knowledge related to or underlying the school mathematics content assessed in Grades 4 and 5" (p.6). This includes the knowledge of mathematical processes, procedures, concepts, and generalizations required to teach mathematics. This knowledge "includes knowledge of mathematical facts and procedures as well as knowledge of mathematical concepts and generalization" (Campbell & Nishio, in press, p.6).

Pedagogical Content Knowledge. Knowledge of teaching to best meet the needs of students by understanding the common conceptions and preconceptions students of different ages and background bring with them to the classroom. This is how the teacher presents the information so that students are able to learn (Ball, Thames, & Phelps, 2008). Campbell and Nishio (in press) defined pedagogical content knowledge as "knowledge of mathematics teaching and learning that teachers might draw on or use in instructional practice when teaching the mathematics content assessed on high-stakes assessments" (p.9).

Chapter Two: Review of Literature and Research

Introduction

The purpose of this study was to determine the relationship between teacher mathematical knowledge (content and pedagogy) and the mathematics achievement of students in grades four and five. This study focused on a quantitative approach to research using hierarchical linear modeling (HLM). The methodology was based on the research entitled, *The Relationship between Teachers' Mathematical Content and Pedagogical Knowledge, Teachers' Perceptions, and Student Achievement*, a study completed by Campbell and Nishio (in press).

Through a quantitative study based upon a teacher assessment of mathematics content and pedagogy and student Measures of Academic Progress (MAP) data, the researcher measured the teachers' mathematical content knowledge, mathematical pedagogy knowledge, and analyzed the data to determine if there was a relationship between teacher knowledge and student achievement. The student and teacher assessments were based upon the Maryland state curricular standards. Student achievement was measured through MAP. Through the use of the teacher knowledge assessment, the study provided valuable data that could be used to inform colleges providing training to pre-service teachers, principals, supervisors, and those providing professional development to elementary teachers. Additionally, the study could be used to inform teacher education and education policy efforts intended to strengthen and support teacher quality while improving the achievement of students in mathematics.

A review of literature on several topics was necessary to frame this study. The literature review was divided into four sections. Section one reviewed the current national research and recommendations for improving the teaching of elementary mathematics. The second section examined the mathematics preparation and training provided for pre-service elementary education majors and the training and background of current elementary mathematics teachers. The third section explored the research on quality professional development programs and grants to support the teaching and learning of elementary mathematics. The fourth section documented the research of utilizing elementary mathematics specialists in elementary schools to provide coaching, modeling, and on-site professional development for teachers.

National Research and Recommendations

"International and domestic comparisons show that American students have not been succeeding in the mathematical part of their education at anything like a level expected of an international leader" (NMAP, 2008, p. xii). In comparison to worldwide peers, American students achieve at mediocre levels in mathematics. On the "National Report Card" conducted by the National Assessment of Educational Progress, the scores of students in grades four and eight indicated positive trends showing significant progress. However, only 23% of students were proficient on the assessment in grade 12 and 32% of students were at or above the "proficient" level in grade eight. Furthermore, significant disparities exist in mathematics achievement between minority students and students coming from families with various incomes. Lastly, there has been an increasing need for students entering college to take remedial mathematics courses (NMAP, 2008).

Mathematics education provides individuals with career and college opportunities. "The National Science Board indicates that the growth of jobs in the mathematics-intensive science and engineering workforce is outpacing overall job growth by 3:1" (NMAP, 2008, p. xii). The ability to understand sophisticated, quantitative concepts assist countries with advancements in medicine, technology, commerce, defense, finance, and the ability to analyze prior failures and successes. Without leadership in mathematics and science, the safety of our nation, the quality of life, and prosperity are all cause for increasing concern. Therefore, the teaching of the education of mathematics needs to be improved in order for the United States to reach the ambitions and goals of our country. It is imperative that schools assist students to become problem solvers, innovators, and inventors. Students need to be able to think critically and logically to drive innovation in our country (National Governors Association (NGA), 2011). The steps to improve mathematics education is a journey requiring a commitment and coordination from educators at all levels including teachers, superintendents, school board members, principals, curriculum developers, researchers, textbook writers, and government officials at all levels (NMAP, 2008).

The NGA (2011) believes that education in Science, Technology, Engineering and Mathematics (STEM) is critical for advancements to occur in our country and for economic prosperity. A skilled work force is needed where discovery, innovation, and higher order thinking occur in order to compete in a global economy. Therefore, expanding the number of students interested in pursuing degrees in STEM areas is critical so that the United States does not fall behind other nations. Additionally, all students must have increased opportunities to become proficient in STEM fields. The problem solving and creative solutions required in these fields are applicable to all areas of life and support individuals in achieving economic prosperity (NGA, 2011).

Implementing the Common Core State Standards (CCSS) was one of the recommendations by the NGA (2011) to improve the opportunities for students to learn rigorous mathematics and science concepts. In 2008, governors and chief state school officials began the effort to create a common curriculum based upon research and evidence in the areas of mathematics and English language arts (NGA, October 2011). The standards were developed by teachers, national experts, and school administrators. The standards, released in June 2010, "define the knowledge and skills students should have along their K-12 progression so that they will graduate high school able to succeed in entry-level, credit bearing academic college courses and in workforce training programs" (NGA, 2011, p. 25).

The CCSS were designed to ensure focus and coherence in mathematics. "It is coherent because it supports large conceptual issues at the heart of K-12 mathematics, and considers how those concepts develop from grade to grade" (Schifter & Granofsky, 2012, p. 16). The interplay of the content standards and the Standards for Mathematical Practice, which are part of the CCSS, make the CCSS robust and different from standards of the past. The CCSS provide students with foundational skills in grades kindergarten through five to assist with helping students to build a deeper conceptual understanding of whole numbers, addition, subtraction, multiplication, division, fractions and decimals. A stronger understanding of these concepts provides the prerequisite knowledge needed to successfully master more complex standards at higher levels (NGA, 2011). The CCSS offer a common foundation to create a stronger educational system throughout our country. The curriculum implemented through skillful teaching can make the difference for student achievement (Ball & Forzani, 2011).

Assessments aligned with the CCSS are being designed to test a deeper knowledge of skills and the application of concepts. The Reinvestment Act of 2009 (ARRA), the Race to the Top Assessment Program, has provided funding for states to work together to create assessments that are valid, support the curriculum, inform instruction, provide accurate information in regards to what students know and are able to do, and measure the achievement of students based upon the important skills and knowledge needed to be successful in college or careers. Currently, assessments are being designed by two state coalitions for release in 2014-2015. The Partnership for Assessment of Readiness in College and Careers (PARCC) and the SMARTER Balanced Assessment Consortiums (SBAC) are developing assessments to provide a common measure of student progress and performance that can be compared across states. This also allows states to work together to create assessments and curriculum resources which assist financially and with the skills combined to create valid and reliable assessments (NGA, 2011).

In order for the curriculum and assessments to transform instruction in the classroom, investing in facilitating professional development is critical. Implementation of the CCSS presents many challenges. Educators will need to make significant changes in their instruction, assessments, teacher professional development programs, curriculum resources and materials to support instruction (NGA, October 2011). Teachers must understand the mathematics content and the conceptual challenges students encounter (Schifter & Granofsky, 2012). Teachers also need learning opportunities in how to best teach the Standards for Mathematical Practice and how to identify evidence of these in student work. In order to teach the CCSS well, teachers must have the opportunities and the professional development to change their instruction. “They cannot deliver the standards directly into students’ minds; there is extensive mathematical thought, practice, and peer collaboration that needs to happen” (Shifter & Granofsky, 2012, p. 20).

Teacher Training and Background

“The mathematics preparation of elementary and middle school teachers must be strengthened as one means for improving teachers’ effectiveness in the classroom” (NMAP, 2008, p. xxi). Ball and Forzani (2011) propose “a common core curriculum for teacher preparation” (p. 18). In order to improve teaching in the United States, a common curriculum focused on the knowledge and skills needed for aspiring teachers that can be assessed to determine readiness for independent practice, is needed. This training should directly support the specific content and preparation needed to support the demands of the actual work teachers do in the classroom. “Teachers must understand their subjects deeply and flexibly, and skillfully represent them in intellectually honest ways to a wide range of students” (Ball and Forzani, 2011, p. 20). Teachers need the opportunity to learn mathematics for teaching through pre-service teacher education programs, professional development, and support through coaching, mentoring, and model lessons in the classroom. It is critical that teachers know the content they are teaching and the connections of the content to the grade levels above and below (NMAP, 2008). United States educators have not come to an agreement regarding the most important knowledge and skills for pre-service students. This is a challenge that should be addressed in order to improve instruction. Ball and Forzani (2011) propose the identification of the high leverage practices, important skills, content knowledge, understandings, orientations, and commitments needed for effective teaching.

Many questions exist regarding the best way to train pre-service teachers so they are prepared to teach all students and to assist them in meeting the diverse needs of students. Which pre-service learning programs and experiences assist most with achievement growth? What support do teachers need, especially in low performing schools with students at risk of failure? What outcomes are measured and by whom? How can the teacher preparation programs assess student teachers to indicate their impact on student learning? What learning experiences have the greatest impact on student achievement and contribute to higher quality teacher preparation (Cave & Brown, 2010)? Further research is needed to determine the high leverage qualities of teacher education programs that have the greatest impact on the preparation of pre-service teachers insuring their readiness to step into a classroom and improve student achievement.

Clearly, there are many barriers to determining the best way to improve pre-training to help new teachers to be successful in the classroom. Students need teachers that are prepared to help them learn, able to manage a classroom, uncover questions and misconceptions, explain in ways that students understand, pose strategic questions designed to help new learners, communicate effectively with parents, and assess student work to plan strategically in order to meet the various needs of children. Teachers must also teach respect, assist students to develop habits that will help them in life, manage behaviors, and motivate students to learn. There are over 1,300 different teacher preparation programs in the United States, each with their own approach to training pre-service teachers. Students need teachers that have been prepared to teach effectively prior to the onset of their careers. Therefore, it is imperative that teachers have the requisite professional skills and knowledge to teach. "The current array of teacher preparation programs offers an unprecedented opportunity to move past trial and error and opinionated debate and to identify the key features of readiness for responsible practice and how it can be learned and assessed" (Ball & Forzani, 2010, p. 12).

In many undergraduate programs, teacher candidates are required to take two or three mathematics courses comprised of a methods course, college algebra, and mathematics for elementary teachers. These courses are often devised and taught in a procedural lecture. The courses provide a breadth of content, lacking the development of a true mathematical understanding. "This lack of understanding leads to teaching mathematics as a set of unrelated procedures, skills, and facts" (Lias et al., 2005-2006, p. 73). Additionally, in many teacher preparation programs, there is a disconnect between campus courses and field experiences. Zeichner (2010) argues for a more integrated approach and promotes linking the knowledge of the coursework and practice by having classroom teachers serve as adjunct professors. He believes professional development schools can assist with research and practice as universities and schools collaborate to effect change contributing to the learning of pre-service teachers (Zeichner, 2010).

Researchers agree (Ball, Hill, & Bass, 2005; Li, 2010; NMAP, 2008), that the key to improving mathematics instruction is improving the mathematical teaching knowledge of the classroom teacher. Increasing the emphasis on mathematics content would support elementary teachers and better prepare them to increase student achievement (Li, 2008). The methods courses teacher candidates take have a significant impact on the pedagogical practices used in the classroom. There is a critical need for teacher candidates to conceptually understand elementary mathematics, not simply to be able to complete procedural problems (Holmes, 2012). "To help students learn, teachers need to not only be able to do mathematics but they need to unpack the elements of that mathematics to make its features apparent to students" (Ball, Thames, & Phelps, 2008, p. 10). When teachers have developed a deep understanding of fundamental mathematics, they are better able to assist students in building their understanding, in making connections among and between content strands, and in providing opportunities for students to problem solve in multiple manners (Lias et al., 2005-2006). The knowledge needed to teach mathematics is multidimensional and can positively influence student learning (Ball et al., 2008). However, there is limited understanding and research to determine and agree upon the specific skills, dispositions, and knowledge of the best instructional methods that make a difference for learning (Ball et al., 2005; Cohen, 2007; Snow, 2002; NMAP, 2008). There is a great need in the United States to prepare teachers who are ready to meet the demands of the classroom. "It is time to lay down our resistance to acknowledging that teaching is hard work that many people need to learn to do well, and build a system of reliable professional preparation" (Ball & Forzani, 2011, p. 509).

"How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students' progress, and to make sound judgments about presentation, emphasis, and sequencing" (Ball et al., 2005, p. 14). The United States has made improvements in the past in the development of curriculum and standards in mathematics. However, this has not resulted in significant improvements in student achievement. Direct attention to the teaching of mathematics along with strong standards and a quality curriculum are needed. Many teachers lack a deep conceptual understanding of mathematics. According to Ball, Hill, and Bass (2005), studies over the last fifteen years indicate the mathematical knowledge of teachers is weak. In addition, the general population of most Americans has a very weak knowledge of mathematics. “We are simply failing to reach reasonable standards of mathematical proficiency with most of our students, and those students become the next generation of adults, some of them teachers” (Ball et al., 2005, p. 14). Bridging this gap is a big challenge.

Various arguments exist regarding the solution to this problem. Some argue that the solution is requiring additional coursework for teachers. Others believe that teachers need additional courses based upon the specific mathematics content they will be teaching in the classroom (Ball et al., 2005). While others believe there is simply a need to recruit teachers from highly selective colleges, where intelligence alone will be most effective at improving mathematics instruction. Questions exist with each of these solutions. What is the knowledge teachers need? Should teachers only know what they need to teach? Does there exist a “professional knowledge of mathematics for teaching, tailored to the work teachers do with curriculum, instruction, and students” (Ball et al., 2005, p. 16)? Building the knowledge needed for pre-service teachers in the area of elementary mathematics continues to be an area of need in our nation.

Learning Mathematics for Teaching

In the late 1970s, courses taken, degrees earned, or certification status were used to study the relationship between teachers’ mathematical knowledge and student achievement (Begle, 1979). However, there was little relationship found between these measures. By the mid 1980s, the question was reframed to determine how teachers’ content knowledge could contribute to the learning of students by concentrating on the knowledge needed to teach (Ball, Thames, & Phelps, 2008; Kennedy, 1997; Ma, 1999; Shulman, 1986; Wilson, Shulman, & Richert, 1987).

The idea of assessing pedagogical knowledge was a shift from the concept of determining teacher knowledge. The content knowledge needed to teach was considered different from how much knowledge teachers have. Student learning may result from teacher knowledge and the "interplay between teachers' content knowledge of students, their learning, and strategies for improving that learning" (Hill & Ball, 2004, p. 332). The specialized knowledge teachers need requires being able to assess student methods for solving problems and determining generalizations to other problems. Common knowledge of content involves computing accurately, solving word problems efficiently, and being able to identify what power of ten is equal to one. Hill and Ball (2004) argue that both specialized and common knowledge of mathematics is needed for teachers to be able to teach effectively.

In the early 1990s, researchers began to develop assessments that would be able to measure teachers' pedagogical knowledge of mathematics. These assessments included open ended and multiple-choice questions requiring teachers to explain their thinking involving the procedures and rules used to solve a particular problem (Hill & Ball, 2004). The questions were designed so that teachers had to solve the problem and explain their thinking. Additionally, teachers were required to construct concrete representations to correspond to the problem and were required to be able to develop a solution based upon the model. Using this type of measure, allowed researchers to begin to investigate "how teachers' mathematical knowledge, construed in these more pedagogically attuned ways, contributes to student achievement" (Hill, Schilling, & Ball, 2004, p. 333). Several scholars identified the mathematical content knowledge of teachers as an indicator of student achievement for tenth grade students (Rowan, Chiang, & Miller, 1997). Research has been conducted detailing what teachers know and do not know, but it has not been utilized to determine how the knowledge needed for teaching mathematics develops in teachers (Ma, 1999; Hill et al., 2004).

Additional research is needed to determine the effects of the solutions in teachers' mathematical knowledge to the achievement of their students (Ball et al., 2005). Very few studies have been able to successfully determine an appropriate mathematics curriculum that provides teachers with the mathematics needed to assist students with learning (Wilson & Berne, 1999). "Although many studies demonstrate that teachers' mathematical knowledge helps support increased student achievement, the actual nature and extent of that knowledge –whether it is simply basic skills at the grades they teach, or complex and professionally specific mathematical knowledge – is largely unknown" (Ball et al., 2005, p. 16). The NMAP (2008) recommended "more precise measures should be developed to uncover in detail the relationships among teachers' knowledge, their instructional skills, and students' learning" (p. 38). Additionally, the NMAP (2008) recommended measures "to identify the mathematical and pedagogical knowledge needed for teaching" (p.38).

Ball, Hill, and Bass (2005), have focused on research to determine what teachers do to support the instruction of students. This includes teaching in the classroom, as well as, "…planning those lessons, evaluating students' work, writing and grading assessments, explaining class work to parents, making and managing homework, attending to concerns for equity, dealing with the building principal who has strong views about the math curriculum, etc." (Ball et al., 2005, p. 17). These skills are all complex and involve a deep knowledge of mathematics including: basic skills, reasoning, communication, fluency, and mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001). Teachers must be able to complete traditional algorithms, but they also need to be able to explain and examine student work. They must be able to determine errors, and then assist with determining the source of the errors. They need to be able to answer questions from students and determine appropriate representations to best assist students in their understanding. Representation entails assisting students with making connections and careful advance thought about the choices within the representation. This requires additional mathematical understanding and skill (Ball et al., 2005).

"…Knowing mathematics for teaching demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably" (Ball et al., 2005, p. 21). Determining the errors students make requires teachers to deeply understand mathematics and mathematical reasoning. When teaching a concept or skill, teachers need to be able to think from the learner's perspective. For students to master mathematics, they need to learn the declarative knowledge based on the concepts and schemas while also acquiring procedural knowledge of skills and strategies (Cave & Brown, 2010). "The teacher has to think from the learner's perspective and to consider what it takes to understand a mathematical idea for someone seeing it for the first time" (Ball et al., 2005, p. 21). Students can struggle in multiple areas including: "concept comprehension, calculation, application strategies or problem solving skills" (Cave & Brown, 2010, p. 9). Students with learning difficulties and processing problems may experience additional challenges for teachers to diagnose. Additionally, ELL students may have linguistic difficulties. Teachers face many challenges when helping children to achieve at high levels in mathematics (Cave & Brown, 2010).

Teaching mathematics involves defining terms and vocabulary in an accurate manner based upon the level of students. Teachers must be able to provide definitions to students that are usable and connect to what students already know and understand. “In our research, we see repeatedly the need for teachers to have specialized fluency with mathematical language, with what counts as a mathematical explanation, and with how to use symbols with care” (Ball et al., 2005, p. 21).

The definition of mathematical knowledge for teaching and teacher quality varies among experts. Ball, Hill, and Bass (2005) developed a large-scale, survey-based measure to assess mathematical knowledge for teaching. They were seeking to answer two questions. First, “…is there a body of mathematical knowledge for teaching that is specialized for the work teachers do” (Ball et al., 2005, p. 22)? The second question was, “And does it have a demonstrable effect on student achievement” (Ball et al., 2005, p. 22)? They tested their hypothesis empirically by creating a large scale data base of over 250 multiple choice questions designed to measure teachers’ common knowledge and the knowledge needed specifically for teaching. The questions focused on the domains of number and operations and the domain of patterns, functions, and algebra. Through the assessment, they attempted to determine the common knowledge of mathematics that well-educated adults possess and the specialized knowledge that teachers needed in order to help students become proficient mathematicians. The teacher knowledge included the common misconceptions and common errors students make when completing their work.

Through their assessment and research, they were seeking to determine if success on their questions accurately reflected student achievement (Ball et al., 2005).

Ball, Hill, and Bass (2005) were involved with the *Study of Instructional Improvement* (SII) in which they used the scores of students on the Terra Nova standardized assessment, the socioeconomic background of students, and their teacher questionnaire to determine the relationship between teachers' knowledge of mathematics and the size of the gain in assessment scores of students. Their findings concluded that teachers' performance significantly predicted student achievement. Some argue with both the multiple choice method teacher assessment and the standardized assessment of the Terra Nova. "Others argue that teacher, and teacher learning, are such fine-grained complex endeavors that large-scale studies cannot probe or uncover anything worth measuring" (Ball et al., 2005, p. 45).

The NMAP (2008) specifically indicated the need for additional research to assist with the development of the relationship between teachers' knowledge of mathematics and student achievement. Through their continued research, Ball, Hill, and Bass and researchers at the University of Michigan have been statistically creating correlations at the elementary level regarding teachers' knowledge of mathematics and the achievement gains of their students. Additionally, they have trained others to use their assessments in order to replicate results and provide further research supporting the recommendation of the NMAP (2008).

Ball and Forzani (2010/2011) used several frameworks, including those by Charlotte Danielson, Lampers, and Lemov to identify specific practices determined to be fundamental to support student learning which they believe are essential in order to develop and build quality professional development for teachers. “By high leverage practices, we mean those practices at the heart of the work of teaching that are most likely to affect student learning” (Ball & Forzani, 2010, p. 43). These provide the foundation for the areas in which beginning teachers need to develop in order to become highly effective professionals. The University of Michigan has developed a pilot teacher education program built on nineteen high leverage practices specific to subjects and levels. The researchers expect the nineteen practices to develop and evolve as they complete their research over five years. “Identifying a set of practices that aims at complex outcomes for all students is a first step toward strengthening the teaching profession” (Ball & Forzani, 2010/2011, p. 45). They believe their research could serve as the foundation for the changes needed in teacher education. Additionally, their work could provide a framework for improving teaching by providing quality professional development and support to new and veteran elementary teachers (Ball & Forzani, 2010/2011).

Quality Professional Development to Support Teacher Content Knowledge and Pedagogy

The Elementary and Secondary Education Act (ESEA) became the federal No Child Left Behind Act (NCLB) when signed into law on January 8, 2002. The implications of this federal legislation impacted schools in every district based upon educational policy, with new testing requirements, accountability, and teacher quality provisions. The requirement of highly qualified teachers under NCLB has had a major impact on professional development, teacher preparation, and certification. Ensuring that all students have highly qualified teachers with the content knowledge and teaching skills to help all children achieve high academic standards is one of the primary goals of NCLB (Howard, Sraani, & Woods, 2009). Another goal of NCLB was for all students to meet a minimum proficiency by 2014, thus closing the achievement gaps that exist between minority students and those of lower incomes. In the Race to the Top (RTTT) legislation, the expectation for a teacher is different. RTTT encourages and rewards comprehensive reform and strategies to close the achievement gap. RTTT supports the increase of teacher effectiveness as determined through student assessment scores and new evaluation systems nationwide. A teacher is determined effective if students achieve at least one grade level in an academic year (Cave & Brown, 2010). Evidence supports that teacher quality is one of the largest factors impacting the learning of students in the classroom (Darling-Hammond, 2000; Pianta, Belsky, Bendergrift, Houts, & Morrison, 2008). The impact of an effective classroom teacher can close the achievement gap for students of poverty and minority students, regardless of parental involvement and educational attainment (Haycock, 2005).

There has been considerable funding and efforts expended throughout the United States over the past decade to improve the quality of mathematics teachers and their knowledge for teaching (Steven et al., 2009; Hill & Ball, 2004). The requirement to have a strong knowledge of content, as well as, the ability to think critically and make quick decisions daily in the classroom is vital to the education of each student (Steven, Harris, Aguirre-Munoz, & Cobbs, 2009). Teachers are expected to know and use curriculum materials and help students with various mathematical backgrounds succeed on challenging assessments. However, with the efforts to improve, there has been little success in determining if teachers are gaining content knowledge from professional development and, if they are, what features of the professional development are most effective. This is due to the lack of an assessment tool to measure teachers' content knowledge for teaching mathematics (Hill & Ball, 2004).

The West Texas Middle School Mathematics Partnerships funded through National Science Foundation (NSF) under the Mathematics and Science Partnership Program focused on providing professional enrichment opportunities for a cadre of teacher leaders. The activities focused on three attributes they felt were most desired in a mathematics teacher. First, they assisted middle school teachers by helping them develop a deep conceptual understanding of elementary mathematics. Second, they attempted to assist teachers develop the specific content knowledge needed to teach middle school mathematics. Third, they created professional development activities to assist with teaching mathematics effectively, in an effort to enhance the mathematics self-efficacy of the students in Texas (Steven et al., 2009). The teachers that were part of the study felt that the case study approach they used assisted with their development as a mathematics teacher. This study was limited by allowing the participants to be grouped according to their preferred learning style and based upon their perception of what they learned versus using an actual measurable objective.

In Oregon, a five year mathematics leadership institute was developed with funding from the NSF under the Mathematics and Science Partnership Program. This program was unique in the fact that teachers from grades kindergarten through grade twelve participated together. Teachers attended three, 3-week summer institutes and participated in ongoing professional development throughout the school year. Pre and post surveys were used to determine the content knowledge gained by the participants.

Both elementary and secondary teachers demonstrated significant gains in their content knowledge. However, their student data was inconclusive as to whether student achievement increased with increased teacher knowledge (Weaver & Dick, 2009).

The “Math in the Middle Institute Partnership” was developed in Nebraska with funding from the NSF to “build teachers’ capacities to improve mathematics learning for all students” (Heaton, Lewis, & Smith, 2009, p. 1). There were three components to this project: participants took twelve graduate level on-line courses and participated in a one or two week summer institute; mathematics learning teams were developed to assist teachers in aligning their teaching with state standards and to help in examining instructional and assessment practices in the classroom; and the third component was an action research initiative (Heaton et al., 2009). Similar to the programs in Texas and Oregon, this study lacked a direct correlation between the knowledge gained by the teachers and the improvement in student achievement.

In Oklahoma, NSF funding was used to develop a teacher preparation program requiring additional course work for elementary pre-service teachers. Evaluation of this program indicated enhanced learning for teachers that were part of the initiative. However, Oklahoma teachers were comprised of many veteran teachers with twelve to thirty years of experience preceding the additional coursework requirement. Therefore, they established professional development opportunities to meet the needs of veteran teachers. A two-week summer course was developed to build strong links between pedagogy and content knowledge. Teachers also spent time reflecting upon curriculum, evaluation methods, and classroom management. An important element of this program was the focus on building a professional learning community. An independent study has shown that the program was successful in increasing teacher knowledge (Howard, Sraani, & Woods, 2009). However, the program results lack a direct correlation to student achievement.

Howard, Sraani, and Woods (2009) claim three specific professional development approaches were found to be successful in their Oklahoma NSF funded program. First, in order for teachers to successfully use what they learn from professional development in their classrooms, they have to become motivated to use the ideas and strategies. Second, the content learned must directly connect to pedagogy and the scope and sequence of what is required in the classroom. Third, the program must allow for teachers to reflect upon their own learning and use the reflection to alter approaches and the way the content is shared with teachers. These three areas allowed teachers to take ownership of their learning and apply it in their classroom (Howard et al., 2009).

Blank and de las Alas (2009) have documented, through a meta analysis, the effects of well designed professional development on student achievement. They indicated that when teachers were provided with content-based professional development, in the area of mathematics, there were positive effects on student learning. They concluded that when teachers participated in professional development, their students outperformed students in classrooms where teachers were not part of the professional development (Blank & de las Alas, 2009). This direct link to student performance is missing in the Texas, Oregon, and Oklahoma studies.

Hill and Ball (2004) worked with the state of California to evaluate their mathematics Professional Development Institutes using measures of knowledge for teaching mathematics. The program began in 2000 and was designed as content-focused with extended learning opportunities for teachers. The program served over 23,000 kindergarten through grade twelve teachers within the first three years. The program was the largest content-focused professional development in the United States and required significant funding. Their results indicated that given quality professional development, elementary teachers could learn mathematics for teaching.

Mathematics Specialists to Build Teacher Content Knowledge and Pedagogy

The shift to the Common Core State Standards (CCSS) by 45 of the country's states has changed mathematics instruction nationwide (Common Core State Standards Initiative, 2012). Elementary mathematics specialists have assisted teachers, principals, and parents with regard to knowledge and skills needed to teach effectively. This includes Standards for the Mathematical Practice, the Mathematical Content Standards, professional development, and the way the new assessments align with curriculum and teaching. Mathematics specialists also have assisted with the resources to support this new curriculum. "The fact that many elementary teachers lack the knowledge to teach mathematics with coherence, precision, and reasoning is a systemic problem with grave consequences" (Wu, 2009, p. 14). With the increasing demands of the classroom teacher to know, understand and effectively teach the CCSS, the call for elementary mathematics specialists has become more important based upon the complexities of teaching elementary mathematics (Fennell, 2011).

In the early 1960's departmentalization of elementary schools became popular to ensure that teachers were able to focus upon content at a particular grade. In the 1970's, those teachers with an interest in mathematics used time before and after school to work as mentors to support mathematics instruction in their buildings. In 1981, The National

Council of Teachers of Mathematics (NCTM) recommended states create a teaching credential endorsement for mathematics specialists. In 1984, the NCTM president John Dossey again called for elementary mathematics specialists to support elementary teachers with developing and building content knowledge. In 1989, the National Research Council's "Everybody Counts" exclaimed the United States as one of few countries in the world to expect elementary teachers to become experts in all subjects. They suggested identifying teachers to become specialists in the areas of math and science (National Research Council, 1989).

Adding it Up (National Research Council, 2001), *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2001), *and The Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences, 2001) have all emphasized the need for elementary mathematic specialists to improve the content knowledge of teachers in order to support the achievement of students in the area of mathematics. Many researchers believe that strengthening elementary mathematics instruction requires concentrated, sustained support for elementary teachers to know content and pedagogy that promotes the teaching and learning of mathematics at a deeper understanding (Ball et al., 2005; Burns, 2000; Campbell & Malkus, 2009; Fennell, 2011; Larue-Davis, 2007). Elementary teachers are generalists and must teach all content areas. Having teacher leaders and mathematics specialists would enhance expertise for those elementary teachers responsible for teaching mathematics content and process standards (NCTM, 2001). The school-based mathematics specialists could coach and mentor teachers, provide professional development, co-teach, and provide interventions (National Research Council, 1989).

Across the country, many schools and districts use mathematics specialists to improve instruction and teaching of mathematics. However, little research exists based upon the effectiveness of mathematics specialists at the elementary level (NCTM, 2009). In 2002, McGrath and Rust (2002) completed a study to determine the effectiveness of departmentalization of mathematics on the achievement of students in grades five and six. They found no significant differences in the achievement based upon test data. However, a study in 2008 by Gerretson, Bosnick, and Schofield found that elementary mathematics specialists allowed teachers time to effectively plan and to have focused professional development. The teachers from the study reported gains in student achievement based upon the assistance from the mathematics specialists (Gerretson, Bosnick, & Schofield, 2008). Based upon the importance of teacher content knowledge, the NMAP report (2008) recommended completing additional research on the use of mathematics specialists in elementary schools. Despite the research supporting specialists, schools, districts, and states continue to use mathematics specialists to strengthen student performance (Fennell, 2011).

Many times, leaders of mathematics are appointed to positions without a certification or even the "proper vetting related to their content, pedagogical, and leadership knowledge and skills" (Fennell, 2011, p. 55). Fewer than fifteen states offer certifications for mathematics specialists. Virginia is the only state to require a Master's Degree for elementary mathematics specialists. Sponsored by the NSF, the Virginia program included courses in both mathematics content and pedagogy in order to obtain the mathematics specialization. In addition, Virginia has a research component which tracks the role and responsibility of elementary specialists and the impact of their work on student achievement (Campbell, 2007; Campbell & Malkus, 2009).

The Association of Mathematics Teacher Educators, the NCTM, the National Council of Supervisors of Mathematics (NCSM), and the Association of State Supervisors of Mathematics (ASSM) have issued a joint position statement on the use of elementary mathematics specialists in Pre-K to grade 6 environments (NCTM , 2009). "Elementary mathematics specialist professionals need a deep and broad knowledge of mathematics content, expertise in using and helping others use effective instructional practices, and the ability to support efforts that help all pre-K-6 students learn important mathematics" (NCTM, 2010, p. 1). Schools of higher education developing programs to train elementary mathematics specialists should focus upon content knowledge, pedagogical knowledge, and developing the knowledge and skills necessary for becoming an effective mathematics leader (NCTM, 2010).

As much of the country transitions to the CCSS, the need for elementary mathematics leaders increases in order to support teachers as they shift to mathematics content standards which require teachers to have a deeper understanding of content and pedagogy. Teachers also must understand how the practices and content will be assessed through the PARCC or SBAC assessments. Elementary mathematics specialists can support teachers with the shift to the more rigorous standards and practices of the CCSS through mentoring and coaching, providing professional development, assisting with curriculum and instruction, coordinating interventions and supporting professional learning communities (Fennell, 2011).

"Whether termed a specialist, coach, support teacher, or teacher leader, in many school districts today the intent is to place in a school a highly knowledgeable teacher, who frequently does not have responsibility for the instruction of a classroom of students, in order to advance instructional and programmatic change" (Campbell & Malkus, 2009, p. 1). The intent of the specialist is to provide professional development and support to teachers by assisting and sustaining the implementation of curriculum, instruction, and assessment while improving school-wide growth and change in regards to student learning and achievement. They support collaborative professional development, advocate for change, nurture performance, assist in advancing the thinking of teachers, as well as increase content knowledge and understanding (Campbell & Malkus, 2009).

Campbell and Malkus (2009) conducted a study on "Preparing Virginia's Mathematics Specialists," which was a school improvement effort that used a mathematics specialist to increase student achievement. In 2004, the (NSF) funded a project in Virginia schools which investigated the work and impact of full time mathematics coaches in elementary schools. The coaches were chosen by their school district and assigned to full time support in a school. They were required to complete coursework in mathematics content and in leadership. This substantial academic coursework was designed to "foster and support their transition to the position of whole-school elementary mathematics coach" (Campbell & Malkus, 2009, p. 23). The effects of coaching on student achievement data in grades three, four, and five as measured by the Virginia state standardized assessment were evaluated. The results indicated that simply placing a mathematics coach in a school did not increase student achievement. They found that in the second year of placements there was a positive impact in grade three and five. This increase continued in the third year of placement. "…at grade 4, the statistically significant positive impact of coaches on overall achievement was limited to those classrooms where the coach had a high level of engagement with a teacher…" (Campbell & Malkus, 2009, p. 18). This study found mathematics coaches had a positive effect on student achievement over a period of several years, especially in grades three and five. "A coach's positive effect on student achievement develops as a knowledgeable coach and the instructional and administrative staffs in the assigned school learn and work together" (Campbell & Malkus, 2009, p. 22).

Policy leaders in the Virginia school districts that were part of the NSF project affirmed their decision to deploy mathematics specialists at the elementary level after two years of implementation. For the participating schools, they found consistently stronger mathematics achievement, school satisfaction, and improved confidence from classroom teachers (Blount & Singleton, 2008). The research was based upon interviews with twelve policy leaders including: one school board member, three division superintendents, four superintendents for instruction, three mathematics supervisors, and one grants manager (Campbell & Malkus, 2009).

Horizon Research, Inc (HRI) was the external evaluator for the NSF grant with Virginia's "Preparing Virginia's Mathematics Specialists" project. As the evaluator, they observed the summer institute, surveyed and interviewed institute participants, and analyzed data in regards to: the impact on mathematics content knowledge; impact on participants' perception of their pedagogical content knowledge; and impacts on participants' perceptions of their leadership skills (Smith & Wickwire, 2009). They used data from pre and post course content assessments, a post institute questionnaire and interviews to indicate that the courses had positively affected the content knowledge of participants. The evaluation primarily used project-developed assessments to analyze the impact on content knowledge. However, the geometry assessment developed by the "Learning Mathematics for Teaching Project" based at the University of Michigan by Hill, Schilling, and Ball (2004) aligned with the institute course well enough to be considered a fair measurement. The increase in mean scores was significant with all courses having a positive effect on participants' mathematical content knowledge. Overall, the project has worked to develop participants' mathematical knowledge, pedagogical content knowledge, and leadership skills. The evaluation indicated that the model used in Virginia has positively and substantially impacted the participants. "Impact data indicated that the return is well worth the investment" (Smith & Wickwire, 2009, p. 138).

Conclusion

Improving the instruction of elementary mathematics is critical for the United States to reach the ambitious goals of our country (NGA, 2011). The steps to improving mathematics education are a journey requiring a commitment and coordination from educators at all levels (NMAP, 2008). Teacher professional development is critical to help teachers understand the math content and conceptual challenges students encounter as the transition to CCSS and PARCC occurs (Schifter & Granofsky, 2012). The need for teachers to know the content they are teaching is essential (NMAP, 2008). Teachers must have the opportunity to learn mathematics for teaching through pre-service teacher education programs, professional development, and support through coaching, mentoring, and model lessons in the classroom. Very few studies have been able to successfully determine an appropriate mathematics curriculum that provides teachers with the mathematics needed to assist students with learning (Wilson & Berne, 1999). Accordingly, the NMAP (2008) recommended “more precise measures should be developed to uncover in detail the relationships among teachers’ knowledge, their instructional skills, and students’ learning” (p. 38). In order to improve the instruction of mathematics in our country, further research is needed to determine: the most important content to teach, the best way to teach so that students achieve at high levels, the most effective professional development to assist teachers, and the most effective pre-service training to prepare our elementary teachers in the teaching of elementary mathematics.

There has been considerable funding and efforts expended throughout the United States over the past decade to improve the quality of mathematics teachers and their knowledge for teaching (Steven et al., 2009; Hill & Ball, 2004). However, with the efforts to improve, there has been little success in determining if teachers are gaining content knowledge from professional development, and if they are, what features of the professional development are most effective due to the lack of an assessment tool to measure teachers’ content knowledge for teaching mathematics (Hill & Ball, 2004). The purpose of this study was to examine the mathematical content and pedagogical knowledge of elementary teachers in order to determine the impact on the math achievement of students in grades four and five. This study should provide additional research to support the recommendations of the NMAP (2008) “to identify the mathematical and pedagogical knowledge needed for teaching” (p.38). In order for our country to be a leader in the areas of medicine, technology, commerce, defense, and finance, our schools and teachers must foster in students the critical practices to understand sophisticated, quantitative concepts. Students need to be able to think critically and logically to drive innovation in our country, to maintain safety and quality of life, and for our country to prosper (NGA, 2011).

Chapter Three: Methodology

Purpose of Study

The purpose of this study was to determine if there was a relationship between teacher mathematical knowledge (content and pedagogy) and the mathematics achievement of students in grades four and five. This study focused on a quantitative approach to research using hierarchical linear modeling (HLM). The methodology was based on the research entitled, *The Relationship between Teachers' Mathematical Content and Pedagogical Knowledge, Teachers' Perceptions, and Student Achievement,* a study completed by Campbell and Nishio (in press).

Through a quantitative study based on a teacher assessment and student Measures of Academic Progress (MAP) data, the researcher measured the teachers' mathematical content knowledge, mathematical pedagogy knowledge, and analyzed data in order to determine a relation of teacher knowledge to student achievement. Both teacher and student assessments were based upon the Maryland state curricular standards. The student assessments were measured through MAP, which is a computer adaptive, nationally normed assessment. "Every test item on MAP assessment corresponds to a value on the Rasch unit (RIT) scale, so educators gain a deep understanding of what a student knows" (N. E. Association, 2013). RIT measured student understanding, regardless of the grade level of the student. The RIT assists with measuring growth over time to indicate student progress. The teacher knowledge assessment measured content knowledge and pedagogy knowledge. The results of this study provided valuable data that could be used to inform colleges in the strategic development of their pre-service programs, as well as teachers, principals, supervisors, and those providing professional development to elementary teachers. Additionally, the study may be used to inform teacher education and education policy efforts to strengthen and support teacher quality while improving the achievement of students in mathematics.

The study investigated the need for teachers to have a strong mathematical content knowledge and have the ability to teach mathematics conceptually in order to positively impact the achievement of elementary students. Additionally, the research explored the need for pre-service programs to better prepare teachers through developing a conceptual understanding of the important content in mathematics. “Teachers who have developed a profound understanding of fundamental mathematics are better able to facilitate developing this understanding in their students, reveal and represent connections among and between topics, and encourage multiple ways of solving problems” (NMAP, 2008, p. 73).

This chapter presents information regarding the methodology of the study: the need for the research, the research rationale, the conceptual framework, the research questions, a design of the study that includes a discussion of the sample population, description of the assessment instrument, the process used in the quantitative study, methods of data analysis, handling of missing data, an explanation of both ethical issues and personal involvement with the researcher, and the limitations of the study.

Need for the Research

The National Mathematics Advisory Council (2008) called for “more precise measures” of both content knowledge and pedagogical knowledge for elementary teachers in order to determine the relationship with student achievement. At the elementary level, empirical evidence relating student achievement to teacher knowledge is inconsistent (Wayne & Youngs, 2003; Wilson, Floden, & Ferrini-Mundy, 2002). This quantitative study sought to provide additional information in response to the challenge of determining if there is a relation between precise measures of teacher knowledge and student achievement.

Research Rationale

Quantitative research was used to compare data in a systematic manner. Creswell, (1994) described quantitative research as using and analyzing data using statistical measures to explain phenomena. According to McMillan (2008) a correlation study investigates the relationships among two or more variables. In this type of design, relationships between and among facts are researched and analyzed. Correlational data recognizes trends and patterns in data, but does not determine cause and effect. Additionally, correlation research can be used to describe a phenomenon with correlations (McMillan, 2008).

The proposed statistical methodology for this study was hierarchical linear modeling (HLM). "HLM is a complex form of ordinary least squares (OLS) regression that is used to analyze variance in the outcome variables when the predictor variables are at varying hierarchical levels" (Woltman, Feldstain, MacKay, & Rocchi, 2012, p. 52). HLM allows for the simultaneous investigation of the relationship within hierarchical levels and across levels. This type of statistical analysis permits researchers to account for the nested structure of data often found in educational data sets. The structure of HLM allocates a nested statistical design in which students in the same class or school generally share common characteristics. HLM allows for the explanation of variance both within and between groups for students and schools to be examined (Byrk & Raudenbush, 1992).

"Hierarchical linear and nonlinear models (also called multilevel models) have been developed to allow for the study of relationships at any level in a single analysis, while not ignoring the variability associated with each level of the hierarchy" (Scientific Software International, 2005-2013). HLM is frequently used in educational sectors. "HLM simultaneously investigates relationships within and between hierarchical levels of grouped data, thereby making it more efficient at accounting for variance among variables at different levels than other existing analyses (Woltman et al., 2012, p. 53). The goal of HLM is to assess the correlated and interactive effects of personal background and social context of trajectories of individual development (Scientific Software International, 2005-2013).

"Prior to the development of HLM, hierarchical data was commonly assessed using fixed parameter simple linear repression techniques; however, these techniques were insufficient for such analyses due to their neglect of the shared variance" (Woltman et al., 2012, p. 52). The development of an algorithm to facilitate covariance component estimation was introduced in the early 1980s and assisted in the development of the widespread use of HLM to multilevel data analysis. Prior methods used to analyze nested data included disaggregation and aggregation which were simple linear regression techniques and did not account for the shared variance when using hierarchical data. The regression methods "resulted in the incorrect partitioning of variance to variables, dependencies in the data, and an increased risk of making a Type I error" (Woltman, et al. 2012, p. 53).

Conceptual Framework

This study used a multilevel HLM design (see figure 3.1). There were three levels to the model, the growth over time based upon the MAP scores of students, the student level and the teacher level. In the figure, the arrows indicate the relationships between student and teacher level variables.

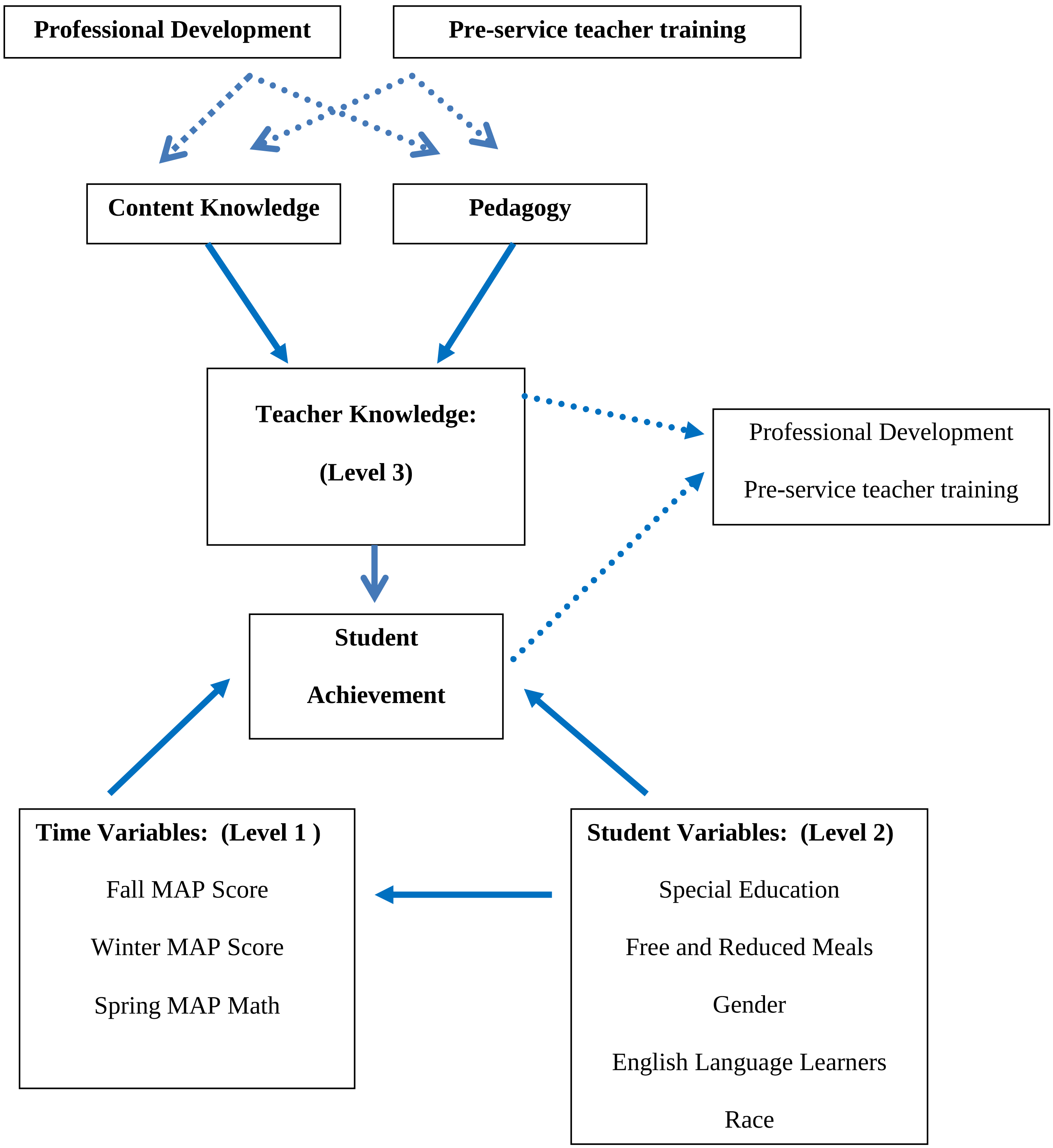

Figure 3.1: Conceptual Framework - The figure above demonstrates a conceptual model relating teacher knowledge of content and pedagogy to student achievement. This also shows that the study could be used to strengthen and support teacher quality through professional development and pre-service teacher training. The dotted arrows indicate items not examined in this study.

Research Questions

This study presented one research question with two sub questions:

Research question:

To what extent does teacher mathematical knowledge (content and pedagogy) relate to the mathematics achievement of students in grades four and five?

Sub questions:

1. To what extent does teacher mathematical content knowledge relate to the mathematics achievement of students in grades four and five?
2. To what extent does teacher pedagogy mathematical knowledge relate to the mathematics achievement of students in grades four and five?

Research Hypothesis:

The research hypothesis for this study was:

Hypothesis: Teacher mathematical knowledge (content and pedagogy) would be positively related to the mathematics achievement of students in grades four and five.

The hypothesis was based upon research completed by Campbell and Nishio (in press). Their study indicated a "significant relationship between upper-elementary teachers' mathematical content knowledge and their students' mathematics achievement (Campbell & Nishio, in press). It was determined that for each standard deviation increase in the content knowledge of the teachers, "the estimated mathematics achievement score of their students increased by 7.1%" (Campbell & Nishio, in press, p.26). Additionally, Hill, Schilling, and Ball (2004) identified mathematics content and pedagogy knowledge, developed a multiple choice assessment to measure the knowledge, and determined there was a significant relationship between teacher content knowledge and the achievement of students in grades one and three.

Design of Study

As a first step, data was analyzed in District A to determine four elementary schools with similar demographics but different student achievement levels on MSA and MAP. The demographics include: students receiving free and reduced meals, student enrollment, and geographic location. Choosing four schools provided approximately 30 teachers to invite to participate in order to have an adequate sample size to complete the quantitative data collection and analysis. The average class size in District A was 25 students per class. This provided the researcher with approximately 750 students who would each have three MAP mathematics scores throughout the 2012-2013 school year.

During the summer of 2013, a proposal requesting permission to complete the research in the four schools was submitted to District A's research and data analysis department. The principal of each school was contacted, requesting that he or she provide release time in September 2013 for teachers in grades four and five to take the assessments. The district data department provided anonymous, individual, student achievement and demographic data linked to individual teachers by the end of September 2013. The participants remained anonymous to the researcher by labeling teachers as: teacher A, teacher B, and teacher C; and students as: student 1, student 2, and student 3. District A had a student database linking student assessment scores of MAP mathematics scores to teachers in each elementary school simplifying the process of collecting the student data.

Hierarchical Linear Modeling (HLM) with a three-level random intercept design was used to analyze the data sets in order to explore the relationship of teachers' mathematical content knowledge and pedagogical knowledge on student achievement. The models incorporated factors for teachers' mathematical content knowledge and pedagogical knowledge; values for students' mathematics achievement scores as measured by fall MAP data, winter MAP data, and spring MAP data; and controlled for gender, students receiving free and reduced meals (FARM), students receiving special education services (SWD), students who qualify as English language learners (ELL), and race.

In this quantitative study, students were nested within a teacher. The three-level model as described in table 1, divided the variance of data into three levels. Level one was the growth in student achievement over time using fall, winter and spring MAP mathematics test scores for the 2012-2013 school year. Level two controlled for student-level variables: gender, SWD, FARM, ELL, and race. The purpose of including these variables in the analysis was to determine whether they were significantly related to the achievement of students after accounting for the variable differences. It would be difficult to determine how much of the variance in student achievement could be attributed to the teacher content and pedagogy knowledge and how much was related to the student variables (gender, SWD, FARM, ELL, and race) without the level two model. These variables were chosen because they match the requirements of No Child Left Behind with their purpose to increase the accountability for at risk groups of students in order to close the achievement gap between disadvantaged students and their more advantaged peers. The teacher level data (level three) was the teacher knowledge which was a combination of the mathematics content knowledge and mathematics pedagogy knowledge as measured through the teacher assessment.

Table 3

Three Level HLM Data Descriptions and Variable Names in the Data Sets

Descriptions	**Variable Names**
Growth over time variables (level 1) N = Approximately 1000	
Mathematics Achievement MAP	MAP
Fall 2012, Winter 2013, Spring 2013	
Student level variables (level 2) N = Approximately 1000	
Gender	FEMALE
Special With Disabilities	SWD
Free and Reduced Meals	FARM
English Language Learners	ELL
Race	RACE
Teacher level variables (level 3) N = Approximately 40	
Content Knowledge	CK
Pedagogy Knowledge	PK

In this study, teachers were asked to complete an assessment designed to measure teacher content knowledge and pedagogy knowledge (Appendix A). Participating teachers were asked to complete a consent form prior to taking the assessment (Appendix B). Permission to use the teacher assessments was granted from Dr. Patricia Campbell (Appendix C). The results of the assessments were analyzed to determine the relationship between teacher knowledge and their students' achievement as measured through MAP. The teacher assessment was designed to measure the understanding of content assessed in grades four and five and the understandings a teacher must have in order to teach that content. The mathematical content knowledge in this study referred to "knowledge related to or underlying the school mathematics content assessed in grades four and five" (Campbell & Nishio, in press, p. 6). This included the knowledge of facts, procedures, concepts, and generalizations. Pedagogical content knowledge, for the purpose of this study was defined as "knowledge of mathematics teaching and learning that teachers might draw on or use in instructional practice when teaching the mathematics content assessed on high-stakes assessments" (Campbell & Nishio, in press, p. 9).

Assessment Instrument

Researchers from the University of Maryland developed a framework specifying mathematical teacher knowledge topics aligned with assessed student achievement objectives shared across Delaware, Maryland, and Pennsylvania (Campbell & Nishio, in press). The content standards included: number and operations, algebra, geometry, measurement, data analysis, and probability. The research team at the University of Maryland was able to locate several items to measure teachers' content knowledge related to state assessed student content standards. To fill the gaps with the missing assessment items, they also developed assessment items. The items were designed to "measure teachers' deep understanding of the mathematics" (Campbell & Nishio, in press, p. 8). Teacher knowledge assessments were developed by Campbell and Nishio (in press) so that teachers could complete the assessments in one day. Therefore, the assessment was limited to 120 multiple choice items comprised of 80 content knowledge and 40 pedagogical content knowledge questions.

As a result of their literature review, four domains were determined to characterize the components of the pedagogical content knowledge assessment. Pedagogy content knowledge items were both found within a literature review and/or developed. The items were distributed across the content area: number and operations, geometry, measurement, probability, data analysis, patterns, functions and algebra as indicated in Table 4. The domains indicated components of pedagogical content knowledge (Ball, Lubienski, & Mewborn, 2001). Domain 1 was comprised of common student errors and misconceptions. Representations and contexts were represented by Domain 2. Domain 3 indicated developing a sense of order within mathematics. Domain 4 focused upon student interpretation of mathematics. The questions were limited to how a teacher chooses to teach mathematics. For example, one question asked, which of three contexts would be most useful when investigating volume, leading to the development of a formula. Another question asked about the prior knowledge needed for students to understand a concept. Yet another asked the teachers to determine a student misconception based on incorrectly solving a problem. Other questions assessed the teachers' ability to help students make connections, anticipate misconceptions, assess student understanding of a concept, develop a sequence for teaching a concept, determine where students lack skills and understanding, etc.

Table 4

Number of Pedagogy Content Knowledge Items Distributed Across Mathematical Content

Mathematical Content	Domain 1	Domain 2	Domain 3	Domain 4
Number and Operations	7	3	1	4
Geometry	2	0	1	2
Measurement	1	1	1	2
Probability	1	0	0	1
Data Analysis	2	1	0	3
Patterns, Functions and Algebra	3	0	2	2
Total	16	5	5	14

The content knowledge portion of the assessment included 80 items with questions focused upon number and operations, geometry, measurement, probability, data analysis, and algebra as indicated in Table 5. The items reflected a range of difficulty based upon Webb's level of depth of knowledge (2002). Level 1 items were recall in nature and presumed direct knowledge as in completing a simple algorithm, declaring a fact, definition, or vocabulary. Level 2 questions required the reader to apply information to an unfamiliar setting or solve a multi-step problem. Level 2 consisted of skills and concepts and included mental processing requiring students to determine how to solve a problem or activity. For an item to be considered level 3, it had to require reasoning, explanation of thinking, making connections, drawing conclusions and using concepts. Extending thinking to require complex reasoning and high cognitive demand was considered level four (Webb, 2002).

Table 5

Number of Content Knowledge Items Distributed Across Mathematical Content

Mathematical Content	Level 1	Level 2	Level 3
Number and Operations	10	15	5
Geometry	3	4	2
Measurement	4	5	2
Probability	1	2	1
Data Analysis	2	6	2
Patterns, Functions and Algebra	5	8	3
Total	25	40	15

The research team at the University of Maryland used released teacher content knowledge items modified to fit a multiple choice format and also created additional multiple choice items based upon their framework to include at least two items per content assessed. The items were screened and edited, placed into smaller subsets of items, then placed according to the mathematical content and grade band. Each subset was distributed to two mathematics educators and one mathematician for external vetting. The reviewed items were revised and rewritten and resulted in the possibility of 320 items for their pilot. Eight subsets of the assessments were completed by between 29 to 34 teachers, depending on each subset. As a result of classical test theory procedures that included: reliability, item difficulty, distracter analysis, and point-biserial correlation, 80 content knowledge items were finalized (Campbell & Nishio, in press).

In order to develop the pedagogical content knowledge items, a literature review was conducted by the team of researchers at the University of Maryland. The team was able to locate items addressing Webb's domain one items concentrating on student errors and misconception and domain four items focusing upon students' interpretations of mathematics. Additional items were developed to fill the gaps. The items were then screened and edited, sent to three mathematics educators and one school-district mathematics supervisor for external vetting. Items were also reviewed by attendees at the AERA, a Special Interest Group of Research in Mathematics Education. Forty–two upper elementary items were revised and piloted by 72 teachers. The items were exposed to classical test theory procedures. As a result, 40 items were determined for use in the pedagogical portion of the teacher assessment (Campbell & Nishio, in press).

After test administration, Campbell and Nishio (in press) completed exploratory factor analysis through item response theory (IRT) to examine the reliability of the assessment items. IRT considers the proportion of correct responses, the level of difficulty, and the relationship between the item and the construct being measured. The analysis confirmed that the assessment separately evaluated both teacher mathematical content knowledge and teacher mathematical pedagogical knowledge. The IRT indicted one inconsistent item in the elementary assessment that was removed prior to determining teacher scores. The empirical reliability value of the remaining 119 items was 0.932. The empirical reliability values for the content knowledge assessment items only was 0.925 and 0.704 for the pedagogical content knowledge.

Data Analysis

Descriptive and inferential statistics were used to analyze the quantitative data from the teacher assessments and student achievement data. Hierarchical Linear Modeling (HLM) with a three-level, random intercept model was used to determine the relationship between teachers' mathematical content knowledge and student achievement; and teachers' mathematical pedagogy knowledge and student achievement.

Pearson's correlation coefficients were used to determine a relationship between teachers' content knowledge and pedagogical knowledge. Interclass correlation coefficients were used to measure the portion of total variance in student scores attributable to a teacher and student across time. Three components were analyzed using a three-level HLM model. The level one model measured student growth on MAP mathematics over time. The level two model controlled for student level variables: gender, special education (SWD), free and reduced meals (FARM), and English language learners (ELL). These were dichotomous indicators with a value of 0 indicating the student did not have the characteristic or 1indicating the student did have the characteristic. The level three model predicted growth based upon teacher pedagogical knowledge and teacher content knowledge.

The level-1 model was: $\text{Math Map}_{ijk}=\pi_{0jk}+\pi_{1jk}*(TIME_{ijk})+e_{ijk}$

In this level-1 model, Math Map_{ijk} represented the mathematics achievement score of student i with teacher j, over time k; π_{0jk} is the average mathematics achievement score of students taught by teacher j, over time k; $\pi_{1jk}*(TIME_{ijk})$ represented the fall mathematics MAP score, the winter mathematics MAP score, and the spring mathematics

MAP score of student i, with teacher j, over time k; and e_{ijk} represented the random error for student i, with teacher j, over time k.

The level-2 model was: $\pi_{0jk} = \beta_{00k} + r_{0jk}$

$$\pi_{1jk} = \beta_{10k} + \beta_{11k}*(\text{Female}_{jk}) + \beta_{12k}*(\text{SWD}_{jk}) + \beta_{13k}*(\text{FARM}_{jk}) + \beta_{14k}*(\text{ELL}_{jk}) + \beta_{15k}*(\text{RACE}_{jk}) + r_{1jk}$$

The level-2 model controlled for several student variables. π_{0jk} was the average mathematics achievement of students taught by teacher j, over time k; β_{00k} represents the overall grand mean of average achievement of students with the average achievement of teachers; and r_{0jk} represented the random error of the average student taught by teacher j, over time k. π_{1jk} was the effect of time on MAP achievement in the classroom of teacher j over time k. β_{10k} was the student achievement on MAP overtime or the group effect on the slope. $\beta_{11k}*(\text{Female}_{jk})$ was the level–1coefficient of gender, $\beta_{12k}*(\text{SWD}_{jk})$ was the level-1 coefficient of Special Education, $\beta_{13k}*(\text{FARM}_{jk})$ was the level-1 coefficient of free and reduced meals, $\beta_{14k}*(\text{ELL}_{jk})$ was the level-1 coefficient of English language learners, $\beta_{15k}*(\text{RACE}_{jk})$ was the level-1 coefficient of Race, and r_{1jk} was the unique effect of the average of all students on achievement over time.

The teacher-level model (level 3) included variables in regards to teacher mathematical content knowledge and teacher mathematical pedagogy knowledge. HLM was completed to determine the degree to which teacher content knowledge and pedagogy knowledge explained the variance in student achievement.

The level - 3 model was: $\beta_{00k} = \gamma_{000} + u_{00k}$

$$\beta_{10k} = \gamma_{100} + \gamma_{101}\,(\text{PK}_k) + \gamma_{102}\,(\text{CK}_k)\,u_{10k}$$

$\beta_{12k} = \gamma_{120} + u_{12k}$

$\beta_{13k} = \gamma_{130} + u_{13k}$

$\beta_{14k} = \gamma_{140} + u_{14k}$

In this model β_{00k} represented the overall grand mean of average achievement of students with the average achievement of teachers, γ_{000} represented the random intercept treated as the function of the average intercept over all students or the grand mean of achievement for all students; u_{00k} was the random error over time for the average student score with average teacher score. β_{10k} was the student achievement on MAP overtime or the group effect on the slope; γ_{100} represented the common value of the random intercept for each student with each teacher over time; $\gamma_{101}(PK_k)$ was the teacher pedagogy knowledge, $\gamma_{102}(CK_k)$ represented teacher content knowledge; and u_{10k} represented the unexplained variance. γ_{110} - γ_{140} represented the average overall achievement scores of students over time and the average teacher assessment score based upon the variables of gender, SWD, FARM, ELL, and race respectively; u_{11k} - u_{14k} represented the random error of each variable (gender, SWD, FARM, ELL, and race).

Missing Data

As long as there were two out of the possible three pieces of student MAP data, the HLM regressions were used. This was a benefit to using HLM as the statistical measure. In multilevel regression, the missing data were estimated making use of incomplete data that does not bias estimates. Mean imputation was used based upon the average value substituted for the missing score (Newsom, 2013). The level two variables were able to be accessed through the district data department. The level three variables were based upon completion of the teacher assessments. Only teachers that agreed to complete the assessments were used in the research for this study.

Human Subjects

The researcher obtained approval from the University of Maryland's Human Subject Review Board, which is the Institutional Review Board (IRB). Additionally, approval from District A was also completed prior to the collection of school or student data.

Ethical Issues and Personal Involvement

The researcher conducted this study in the school district in which she was employed as an elementary school principal. The researcher did not directly supervise the teachers selected for this study to avoid the possibility of influence or bias. The researcher clarified to participants that the data collected through the assessments and surveys were used to determine how to best provide mathematics instruction for students. The information collected remained anonymous and not related to a specific school or teacher. There were not references to specific schools or teachers. The researcher obtained written consent from all participating teachers. Additionally, participants received detailed information in regards to the purpose of the research and their role in the study. The researcher emphasized her commitment to the anonymity of the teachers and schools involved.

Limitations of the Study

This study was limited to the correlation of the mathematical achievement of students to teachers' mathematical content and pedagogy knowledge. This study was not an exhaustive assessment of teacher knowledge. Additionally, this study was limited to teachers in District A who have either remained in his/her position from the 2012-2013 school year to 2013-2014 school year or if he/she has changed schools within the school system were still willing to take the teacher assessment. Although release time was provided for teachers to take the assessment, it was not mandatory. This may have limited the number of participants, requiring the researcher to expand the number of schools in the study. The study was limited to schools with a high proportion of students receiving free and reduced meals.

Summary

The researcher used a quantitative approach based upon the methodology in a study completed at the University of Maryland to determine if there was a relationship between teacher mathematical knowledge (content and pedagogy) and the mathematics achievement of students in grades four and five. The study focused upon approximately 30 elementary teachers in grades four and five in four schools with similar demographics. Teachers' mathematical content knowledge was assessed through the use of an 80- item assessment and teachers' mathematical pedagogical knowledge was assessed based upon an assessment with 39 items. The data was analyzed using HLM and correlated to the mathematical achievement of students based upon MAP data indicating growth over time and several student variables: Female, FARM, SWD, and ELL. The next chapter presents the quantitative findings of the study.

Chapter Four: Findings

Overview/Research

The math achievement of students is significantly impacted by elementary teachers as they are the key to improving math instruction for students regardless of the curriculum or the assessment process within a district (Burns, 2000). However, teaching mathematics through problem solving and making connections is very complex (Van de Walle, 2014). Teachers who have developed a deep understanding of mathematics content are better able to facilitate the progress of students who are able to make connections among and between topics and problem solve in multiple ways (Lias et al., 2005-2006).

The majority of elementary teachers in the United States are generalists and responsible for teaching all content areas in a self-contained classroom. As a result, pre-service programs generally provide future teachers with a breadth of mathematical content which does not promote a true conceptual understanding of mathematics (Lias et al., 2005-2006). Additionally, there are very few elementary teachers with math specialists' degrees or with an extensive background in mathematics. Therefore, elementary teachers struggle with both conceptual understanding and the pedagogy needed to teach mathematics.

This study focused on determining if there was a relationship between teacher mathematical knowledge and the mathematics achievement of students in grades four and five. This chapter presents the quantitative data analysis and findings in four sections. The first section is the introduction. In the second section, a description of the procedures used to design and implement the assessment is shared. The third section describes the participants of the study and how they were selected. The fourth section presents the quantitative data and statistical analysis of the teacher assessment and student MAP data. The last section is a summary of the chapter.

Assessment Design and Procedures

Through a quantitative study based upon a teacher assessment of mathematics including both content and pedagogy, and three student assessments using Measures of Academic Progress (MAP) data, the researcher measured the teachers' mathematical content knowledge, mathematical pedagogy knowledge, and analyzed the data to determine if there was a relationship between teacher knowledge and student achievement. The student and teacher assessments were based upon the Maryland state curricular standards. All teachers involved in the study were considered generalists at the elementary level. Student achievement was measured through MAP mathematics assessment.

In this quantitative study of teacher knowledge, state assessment data was used to select ten elementary schools with similar demographics. The demographics included: students receiving free and reduced meals, student enrollment, and geographic location. The principals of the schools were asked to support the research by providing release time for teachers in grades four and five to complete content knowledge and pedagogical knowledge mathematics assessments. An email was sent to the teachers in those schools requesting their participation. Once the teachers agreed, a consent form, the assessment, and a short survey were sent. The data department in District A was asked to provide the teachers with the student data which could be easily accessed through District A's database. The teachers returned their completed assessment, survey, and student data directly to the researcher

The research hypothesis for this study was:

Hypothesis: Teacher mathematical knowledge (content and pedagogy) would be positively related to the mathematics achievement of students in grades four and five.

This study presented one research question with two sub questions:

Research Question:

To what extent does teacher mathematical knowledge (content and pedagogy) relate to the mathematics achievement of students in grades four and five?

Sub questions:

1. To what extent does teacher mathematical content knowledge relate to the mathematics achievement of students in grades four and five?
2. To what extent does teacher pedagogy mathematical knowledge relate to the mathematics achievement of students in grades four and five?

Participants

In conducting the study, the researcher reviewed data from all schools in District A. Originally, four schools were chosen. Respondents were asked to participate based upon: teaching grades four or five during 2012-2013 school year and teaching mathematics to a heterogeneous group of students. However, there were only ten out of a total of 31 participants who initially volunteered to be part of the study. Therefore, the researcher added six more schools to ensure a large enough sample size to complete the study. There were 43 teachers in total asked to participate with a response rate of 18 or 41.8%. The email sent to teachers ensured teacher, student, and school confidentiality.

Once respondents agreed to participate, a consent form and the teacher assessment were sent for completion. At the end of the assessment was a very short survey asking information about the teacher.

Data regarding the personal characteristics of the eighteen respondents are displayed in Table 6. The respondents ranged in years teaching from three to forty with the majority (thirteen) having taught for between three and ten years. The teachers who participated taught in their current building between one and forty years. The largest group of teachers (twelve) taught in their current building between three and five years. The majority of the teachers held a Bachelor of Science degree in Elementary Education. Of the fifteen teachers with this degree, three had a focus on mathematics. The majority of the respondents (8) did not have a master's degree. One teacher had a master's degree with a mathematics focus and one was working on a master's degree in elementary mathematics leadership. Ten teachers had taken between one and four college level mathematics courses. Fourteen of the respondents had been part of the District's year long mathematics professional develop course which focused upon grade level mathematics content and pedagogy based upon a suggested scope and sequence.

Table 6

Background Information of Teachers Completing the Mathematics Assessment

Teacher Background Information	Number of Teachers (n=18)	Percentage of Teachers
Years Teaching		
3 to 5	6	33 %
6 to 10	7	38 %
11 to 15	3	17%

16 to 20	0	0 %
21 to 25	1	5 %
23 to 40	1	5 %
Years Teaching in Current Building		
1 to 3	6	33%
3 to 5	6	33 %
6 to 10	5	27 %
11 to 15	0	0 %
16 to 20	0	0 %
20 to 40	1	5 %
Undergraduate Degree		
Elementary Education: Math and Science	1	5 %
Elementary Education: Reading Specialization	1	5 %
Elementary Education: Middle School Math	1	5 %
Elementary Education: Early Childhood	2	11 %
Elementary Education: Minor in Mathematics	1	5 %
Elementary Education: Science Focus	1	5 %
Elementary Education	8	44 %
Accounting/Computer Science	1	5 %
Biology	1	5 %
Business	1	5 %
Graduate Degree		
No	8	44 %
Master of Teaching	2	11 %

Elementary Reading and Mathematics	1	5 %
Reading	1	5 %
Master's Equivalency in Science	1	5 %
Grades 1-8 Education	1	5 %
Working on Admin	1	5 %
Working on Elementary Mathematics Leadership	1	5 %
Master's Equivalency	2	11 %
Number of Mathematics Courses Taken		
1-2	7	38 %
3-4	3	17 %
5-6	4	22%
7-8	2	11 %
9-10	1	5 %
11-12	1	5 %
Participant in Year Long Mathematics Professional Development		
Did not Participate	3	17 %
2010-2011	5	27 %
2010-2011, 2011-2012	1	5 %
2011-2012	2	11 %
2011-2012; 2013-2014	4	22%
2012-2013	3	17%

Pearson correlations were completed based upon the teacher data and is indicated in Table 7. The Pearson correlation of teacher assessment to content was 0.974. This number is close to one, meaning there was a strong relationship between both variables. As the assessment scores increased, the content increased in value. The Sig. (2-tailed) value was 0.000. Because this was less than 0.01, it is considered statistically significant. The Pearson correlation of teacher assessment to pedagogy was 0.650. This number was close to one, meaning there was a strong relationship between both variables. As the assessment scores increased, the pedagogy increased in value. The Sig. (2-tailed) value was 0.004. Because this was less than 0.01, it is considered statistically significant. The overall assessment score was not significantly related to pedagogy knowledge, years teaching, years spent in one building, number of mathematics courses taken, or the college degree the teacher received. The content knowledge was not significantly related to pedagogy, years teaching, number of years in one building, number of courses taken, or the college degree the teacher received. Pedagogy knowledge was not significantly related to years teaching, years in one building, number of courses taken, or the college degree the teacher received. The years a teacher spent teaching were not related to years in one building, number of courses taken, or the college degree the teacher received. The years spent in one building were not related to number of courses taken, or the college degree the teacher received. The number of courses a teacher took were not related to, or the college degree the teacher received.

Table 7

Teacher Correlations

		Assessment	Content	Pedagogy	Years_Teach	Yr_Bldg	N_Courses	Institute
Assessment	Pearson Correlation	1	.974**	.650**	.057	-.104	.403	.050
	Sig. (2-tailed)		.000	.004	.821	.680	.097	.843
	N	18	18	18	18	18	18	18
Content	Pearson Correlation	.974**	1	.460	.129	.007	.452	.084
	Sig. (2-tailed)	.000		.055	.609	.978	.060	.741
	N	18	18	18	18	18	18	18
Pedagogy	Pearson Correlation	.650**	.460	1	-.209	-.431	.063	-.084
	Sig. (2-tailed)	.004	.055		.405	.074	.805	.740
	N	18	18	18	18	18	18	18
Years_Teach	Pearson Correlation	.057	.129	-.209	1	.815**	-.233	.246
	Sig. (2-tailed)	.821	.609	.405		.000	.352	.325
	N	18	18	18	18	18	18	18
Yr_Bldg	Pearson Correlation	-.104	.007	-.431	.815**	1	-.201	.144
	Sig. (2-tailed)	.680	.978	.074	.000		.424	.570
	N	18	18	18	18	18	18	18
N_Courses	Pearson Correlation	.403	.452	.063	-.233	-.201	1	-.243
	Sig. (2-tailed)	.097	.060	.805	.352	.424		.332
	N	18	18	18	18	18	18	18
Institute	Pearson Correlation	.050	.084	-.084	.246	.144	-.243	1
	Sig. (2-tailed)	.843	.741	.740	.325	.570	.332	
	N	18	18	18	18	18	18	18

**. Correlation is significant at the 0.01 level (2-tailed).

Student Information

Student demographic data, represented in Table 8 indicated those students to whom teachers taught mathematics during the 2012-2013 school year and for whom the MAP data were accessible for fall 2012, winter 2013, and spring 2013. These data included all students within the regular education classrooms who participated in the MAP testing. Students remained anonymous through the data collection process. The majority of the students (76%) were white. Of the 322 students, 64% were considered free and reduced meals students. Students with disabilities comprised 12% of the total students. Forty-five percent were female, and four percent were English language learners students.

Table 8

Demographics of Students Taught Mathematics by Participating Teachers

Demographics of Students	Grades 4 or 5 (n = 322)
Ethnicity	
African American	20%
White	76%
Multi	10%
Asian	2%
Hispanic	7%
American Indian	.3%
Middle Eastern	.3%
Free and Reduced Meals Students	64%
Students with Disabilities	12%
Female Students	45%
English language learners Students	4%

Pearson Correlations were completed on the student data and are shown in Table 9. FARM students were not related to SWD, female, or ELL students. The Pearson Correlation for FARM student to race was -0.268. This was considered statistically significant. The Sig (2-tailed) value was 0.000. Because this is less than 0.01, it was considered statistically significant. SWD were not related to ELL or race. The Pearson

Correlation for SWD and female was -0.112. The Sig. (2-tailed) was 0.046. This was considered statistically significant because it was less than 0.05. Female students were not related to ELL or race. The Pearson Correlation for ELL was -0.250. The Sig.(2-tailed) was 0.000 and was considered statistically significant because it was less than 0.05.

Table 9

Student Correlations

		FARM_mean	SWD_mean	Female_mean	ELL_mean	Race
FARM_mean	Pearson Correlation	1	.045	.084	.054	-.268**
	Sig. (2-tailed)		.422	.133	.333	.000
	N	322	322	322	322	322
SWD_mean	Pearson Correlation	.045	1	-.112*	-.029	.055
	Sig. (2-tailed)	.422		.046	.599	.323
	N	322	322	322	322	322
Female_mean	Pearson Correlation	.084	-.112*	1	.069	.035
	Sig. (2-tailed)	.133	.046		.214	.535
	N	322	322	322	322	322
ELL_mean	Pearson Correlation	.054	-.029	.069	1	-.250**
	Sig. (2-tailed)	.333	.599	.214		.000
	N	322	322	322	322	322
Race	Pearson Correlation	-.268**	.055	.035	-.250**	1
	Sig. (2-tailed)	.000	.323	.535	.000	
	N	322	322	322	322	322

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Quantitative Data

To determine if there was a relationship between teacher knowledge and student achievement, Hierarchical Linear Modeling (HLM) with a three-level model was used to account for students nested within a teacher and to show student growth on MAP overtime (Level 1). Level 1 variables included the three times the MAP was administered. The student level variables (Level 2) included controls for gender (female), students with disabilities (SWD), students receiving free and reduced meals (FARM), student with limited English (ELL), and race. The teacher level model (Level 3) included the teacher content knowledge and pedagogy knowledge as measured on the teacher assessment.

SSPS was used to calculate inferential and descriptive statistics. HLM–3 was used to complete the analysis of the data in order to answer the research question and sub questions. Additionally the data analysis was used to reject or accept the hypothesis: Teacher mathematical knowledge (content and pedagogy) correlate positively to the mathematics achievement of students in grades four and five.

Table 10 depicts the descriptive statistics. There were 916 MAP assessments analyzed based upon 322 students. The mean of the assessments from fall, winter, and spring was 208.82 with a standard deviation of 15.52. The scores ranged from 123 to 260 in point value. The level two variables included: race, students receiving free and reduced meals (FARM), students with disabilities (SWD), student gender, and English language learners students (ELL). The data for race was coded as: 0 for African American students, 1 for White students, 2 for multiracial students, 3 for Asian students, 4 for Hispanic students, 5 for American Indian students, and 6 for Middle Eastern students. The data were recoded as white versus all other races with white as 1 and all others as 0. The data indicating that a student was FARM, SWD, female, and ELL were dichotomous with 1 indicating the data represented the student and a 0 indicating the data did not represent the student. Level three data included teacher information: content knowledge, pedagogy knowledge, total assessment knowledge (combined content and pedagogy), years teaching, years in current building, number of college level mathematics classes taken, and if the teachers had participated in one of the year long mathematics professional development opportunities within District A.

The mean score for the teacher assessment was 70.61 with a standard deviation of 19.92. The range of scores on the overall assessment was 41 points to 91 points out of a possible 118 total points. Of the eighteen teachers, the mean score for content knowledge was 47.94 with a standard deviation of 12.78 points and a range of 25 to 68 points out of a total 79 points possible for content knowledge. The pedagogy portion of the assessment had 39 total points possible. The mean was 22.67 with a standard deviation of 3.82 and a range of 16 to 27 points. The eighteen teachers had a range of teaching experience between three and 40 years. The range of the years spent as a teacher in their current building was one to 40 years. The teachers took a range of classes, between one to 12 college level mathematics courses. Lastly, to indicate if teachers participated in the year long professional development, a dichotomous variable was used. One indicated that teacher participated and zero indicated the teacher did not participate. Seventy-eight percent of the teachers participated in the year long professional development.

Table 10

Descriptive Statistics

Level-1 Student RIT data over time				
Variable Name	**Number of Assessments**	**Mean**	**Standard Deviation**	**Range**
Time	916	1.01	0.81	0-2
Math RIT Score	916	208.82	15.52	123-260
		Level-2		
Variable Name	**N**	**Mean**	**Standard Deviation**	**Range**
Race	322	0.66	0.48	0-1
FARM (0 FARM; 1 not FARM)	322	0.64	0.48	0-1
SWD (0 SWD; 1 not SWD)	322	0.12	0.33	0-1
Female (0-Female; 1 – not Female)	322	0.45	0.50	0-1
English language learners (0-ELL; 1- not ELL)	322	0.04	0.20	0-1
	Level 3 –Teacher Knowledge Assessment			
Variable Name	**N**	**Mean**	**Standard Deviation**	**Range**
Assessment Total Score	18	70.61	14.92	41-94
Content Knowledge	18	47.94	12.78	25-68
Pedagogy Knowledge	18	22.67	3.82	16-27
Years teaching	18	9.72	8.88	3-40
Years in Current Building	18	6.28	8.73	1-40
Number of Mathematics Courses Taken	18	4.56	3.28	1-12
Participation in Yearlong Mathematics PD	18	0.78	.043	0-1

Using the HLM-3 program, a growth model was created to predict the outcome of the MAP student scores. The predictor variable was time (fall = 0, winter = 1, and spring

= 2 assessment scores) and was used to determine the outcome (MAP) as indicated in Table 11.

Table 11

Predicting the MAP Score Outcome

Level-1 Model
Y = P0 + P1*(TIME) + E
Y = MAP score
PO = intercept
P1*(TIME) = time slope

Table 12 presents the final estimate of the fixed effects indicating the mathematics growth of students. The estimated fall MAP score for all students in the sample was 204.55. The standard error was 1.54. The t-Test was used to determine if the intercept was different than 0. The p-value determined the probability level. A p-value of 0.05 or less was considered significant. The time slope indicated that for every testing period the MAP scored increased by 4.05 points. The p-value of the slope was <0.001. Overall, this indicates that there were significant changes in mathematics MAP scores across time.

Table 12

Final Estimate of Mathematics Growth Based upon MAP (Level1)

Fixed Effect	Coefficient	Standard Error	T-Ratio	Approximate df	P-Value
Fall MAP	204.55	1.54	132.41	17	<0.001
Time Slope	4.05	0.41	9.99	17	<0.001

Using the HLM-3 program, a growth model was created to predict the outcome of the MAP student scores. The level two model controlled for student demographics based upon students receiving free and reduced meals (FARM), students with disabilities (SWD), student gender (FEMALE), and English language learner students (ELL). Because of the relatively small sample size for teachers all of the level two variables were examined in separate models. The predictor variable was time (fall = 0, winter = 1, and spring = 2 assessment scores) and was used to determine the outcome (MAP) as indicated in Table 13. The level three model was used to determine if there was a relationship between teacher content knowledge and pedagogy knowledge with the achievement of students.

Table 13

Predicting the Relationship of Teacher Content and Pedagogy Knowledge to Student Achievement

Level- 2 and 3 Model
Level-2 Model
P0 = B00 + R0
P1 = B10 + B11*(ELL_MEAN) + R1
PO = intercept
B10 = change over time
B11*(ELL_MEAN) = Mean Score of ELL students
(ELL was replaced with SWD, FARM, and Female)
Level-3 Model
B00 = G000 + U00
B10 = G100 + U10
B11 = G110 + G111(CONTENT) + G112(PEDAGOGY) + U11
BOO = intercept
G110 = change over time
G111(CONTENT) = content knowledge of the teacher
G112(PEDAGOGY) = pedagogy knowledge of the teacher

To determine if the teacher's content knowledge and pedagogy knowledge was related to student achievement, HLM-3 was conducted. The results are displayed in Table 14. Teacher content knowledge and pedagogy knowledge were not significantly correlated, $r = .46$, $p = .055$. This moderate size of correlation shows that the two aspects of math assessment were measuring two different components. The overall intercept indicates that the estimated Fall MAP was 204.54. This Fall MAP score was significantly different from 0 (S.E. = 1.56, *t* (df = 17) = 131.26, $p<0.001$).

The coefficient for change over time was 4.07. This slope coefficient was significantly different from 0 (S.E. = 0.43, *t* (df =15) = 9.51, p = $p<0.001$). When determining if there was a relationship with teacher content knowledge and student achievement over time (controlling for teacher pedagogy knowledge), the coefficient was 0.02. This slope coefficient was not significantly different from 0 (S.E. = 0.03, *t* (df = 15) = 0.64, p = 0.54). When examining teacher pedagogy knowledge (controlling for teacher content knowledge) the coefficient was 0.12. This slope coefficient was also not significantly different from 0 (S.E. = 0.10, *t* (df = 15) = 1.18, p = 0.26). These results show that, contrary to the hypotheses, neither teacher content knowledge nor teacher pedagogy knowledge was related to growth in student math scores.

Table 14

Final Estimate of Teacher Content and Pedagogy Knowledge as the Predictor of Student Achievement (Level 3)

Fixed Effect	Coefficient	Standard Error	T-Ratio	Approximate df	P-Value
Fall MAP	204.54	1.56	131.26	17	<0.001
Change over time (MAP)	4.07	0.43	9.51	15	<0.001
Content	0.02	0.03	0.64	15	0.54
Pedagogy	0.12	0.10	1.18	15	0.26

Table 15 displays the data for the level two variable, free and reduced meal (FARM) students. The coefficient for FARM students was -5.30. The slope coefficient was significantly different from 0 (S. E. = -5.30, *t* (df = 265) = -3.01, p = 0.003). This coefficient shows that the initial math scores for FARM students was approximately 5.30 points lower than non FARM students. When determining if there was a relationship with teacher content knowledge and student achievement over time, the coefficient was -0.03. This slope coefficient was not significantly different from 0 (S.E. = 0.08, *t* (df = 15) = 0.39, p = 0.699). When controlling for teacher pedagogy knowledge, the coefficient was 0.11. The slope coefficient was not significantly different from 0 (S.E. = 0.25, *t* (df = 15) = 0.45, p = 0.66).

Table 15

Final Estimate of Teacher as the Predictor of Student Achievement (Level 2 FARM)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, π_0					
For INTRCPT2, β_{00}					
INTRCPT3, γ_{000}	207.901807	1.883160	110.400	15	<0.001
CONTENT, γ_{001}	0.017683	0.171955	0.103	15	0.919
PEDAGOGY, γ_{002}	0.074791	0.556334	0.134	15	0.895
For FARM_MEA, β_{01}					
INTRCPT3, γ_{010}	-5.304842	1.761764	-3.011	265	0.003
CONTENT, γ_{011}	-0.063749	0.155701	-0.409	265	0.683
PEDAGOGY, γ_{012}	0.188805	0.500540	0.377	265	0.706
For TIME slope, π_1					
For INTRCPT2, β_{10}					
INTRCPT3, γ_{100}	4.185658	0.647566	6.464	15	<0.001
CONTENT, γ_{101}	-0.015406	0.061200	-0.252	15	0.805
PEDAGOGY, γ_{102}	0.018623	0.185231	0.101	15	0.921
For FARM_MEA, β_{11}					
INTRCPT3, γ_{110}	-0.160022	0.872360	-0.183	15	0.857
CONTENT, γ_{111}	0.031417	0.079765	0.394	15	0.699
PEDAGOGY, γ_{112}	0.113514	0.251029	0.452	15	0.658

Table 16 indicates the coefficient for students with disabilities (SWD) was -16.17. The slope coefficient was significantly different from 0 (S.E. = 3.35, *t* (df = 15) = -4.82, p = <0.001). This coefficient indicates that the initial mathematics score for SWD was approximately 16.17 points lower than non SWD. When determining if there was a relationship with teacher content knowledge and student achievement over time, the coefficient as 0.19. This slope coefficient was not significantly different from 0 (S.E. = 0.14, *t* (df = 15) =1.36, p = 0.20). When controlling for teacher pedagogy knowledge the coefficient was 0.65. This slope was not significantly different from 0. (S.E. = 0.44, *t* (df = 15) = 1.48, p = 0.16).

Table 16

Final Estimate of Teacher as the Predictor of Student Achievement (Level 2 SWD)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, π_0					
For INTRCPT2, β_{00}					
INTRCPT3, γ_{000}	206.609157	1.358402	152.097	15	<0.001
CONTENT, γ_{001}	-0.017227	0.122304	-0.141	15	0.890
PEDAGOGY, γ_{002}	0.373168	0.408284	0.914	15	0.375
For SWD_MEAN, β_{01}					
INTRCPT3, γ_{010}	-16.171641	3.353903	-4.822	15	<0.001
CONTENT, γ_{011}	-0.065457	0.292282	-0.224	15	0.826
PEDAGOGY, γ_{012}	-0.663387	0.945650	-0.702	15	0.494
For TIME slope, π_1					
For INTRCPT2, β_{10}					
INTRCPT3, γ_{100}	4.164499	0.432706	9.624	15	<0.001
CONTENT, γ_{101}	0.004669	0.038580	0.121	15	0.905
PEDAGOGY, γ_{102}	-0.009810	0.127276	-0.077	15	0.940
For SWD_MEAN, β_{11}					
INTRCPT3, γ_{110}	-0.911180	1.555731	-0.586	15	0.567
CONTENT, γ_{111}	0.188580	0.138937	1.357	15	0.195
PEDAGOGY, γ_{112}	0.650566	0.440319	1.477	15	0.160

Table 17 indicates the coefficient for English language learner students was -13.91. The slope coefficient was not significantly different from 0 (S.E. = 9.44, t (df = 15) = -1.47, p = 0.16). The coefficient shows that the initial mathematics scores for ELL was approximately 13.91 points lower than non ELL. When controlling for the content knowledge of teachers, the coefficient was 0.12. The slope coefficient was significantly different from 0 (S.E. = 0.30, t (df = 15) = 0.38, p = 0.70). When determining if there was a relationship between teacher pedagogy knowledge and student achievement the coefficient was -0.69. The slope coefficient was not significantly different from 0 (S.E. = 0.97, t (df = 15) = -0.70, p = 0.49.

Table 17

Final Estimate of Teacher as the Predictor of Student Achievement (Level 2 ELL)

Fixed Effect	Coefficient	Standard Error	T-Ratio	Approximate df	P-Value
For INTRCPT1, π_0					
For INTRCPT2, β_{00}					
INTRCPT3, γ_{000}	205.048001	1.567020	130.852	15	<0.001
CONTENT, γ_{001}	0.023033	0.141382	0.163	15	0.873
PEDAGOGY, γ_{002}	0.249270	0.470835	0.529	15	0.604
For LEP_MEAN, β_{01}					
INTRCPT3, γ_{010}	-13.907172	9.438759	-1.473	15	0.161
CONTENT, γ_{011}	-0.146696	1.014596	-0.145	15	0.887
PEDAGOGY, γ_{012}	-0.167460	3.310717	-0.051	15	0.960
For TIME slope, π_1					
For INTRCPT2, β_{10}					
INTRCPT3, γ_{100}	4.003096	0.442150	9.054	15	<0.001
CONTENT, γ_{101}	0.017295	0.039501	0.438	15	0.668
PEDAGOGY, γ_{102}	0.095295	0.128939	0.739	15	0.471
For LEP_MEAN, β_{11}					
INTRCPT3, γ_{110}	1.372906	2.356932	0.582	15	0.569
CONTENT, γ_{111}	0.115709	0.302585	0.382	15	0.708
PEDAGOGY, γ_{112}	-0.683648	0.973244	-0.702	15	0.493

The coefficient for RACE (white) was 3.58 as shared in Table 18. The slope coefficient was not significantly different from 0 (S.E. = 1.78, *t* (df = 15) = 2.01, p = 0.06). This coefficient indicates that the initial mathematics score for RACE was approximately 3.58 points higher than non RACE. When controlling for the content knowledge of teachers, the coefficient was -0.03. The slope coefficient was not significantly different from 0 (S.E. = 0.07, *t* (df = 15) = -0.49, p = 0.63). When determining if there was a relationship between teacher pedagogy knowledge and student achievement the coefficient was -0.10. The slope coefficient was not significantly different from 0 (S.E. = 0.25, *t* (df = 15) = -0.40, p = 0.70).

Table 18

Final Estimate of Teacher as the Predictor of Student Achievement (Level 2 RACE)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, π_0					
For INTRCPT2, β_{00}					
INTRCPT3, γ_{000}	202.109680	1.720139	117.496	15	<0.001
CONTENT, γ_{001}	-0.034151	0.146399	-0.233	15	0.819
PEDAGOGY, γ_{002}	0.022037	0.518456	0.043	15	0.967
For RACE, β_{01}					
INTRCPT3, γ_{010}	3.578267	1.777753	2.013	15	0.062
CONTENT, γ_{011}	0.040415	0.151516	0.267	15	0.793
PEDAGOGY, γ_{012}	0.388729	0.535408	0.726	15	0.479
For TIME slope, π_1					
For INTRCPT2, β_{10}					
INTRCPT3, γ_{100}	3.851083	0.776410	4.960	15	<0.001
CONTENT, γ_{101}	0.028552	0.065032	0.439	15	0.667
PEDAGOGY, γ_{102}	0.172456	0.230937	0.747	15	0.467
For RACE, β_{11}					
INTRCPT3, γ_{110}	0.344456	0.858779	0.401	15	0.694
CONTENT, γ_{111}	-0.036090	0.074058	-0.487	15	0.633
PEDAGOGY, γ_{112}	-0.101680	0.252141	-0.403	15	0.692

Table 19 provides the coefficient for gender as 0.74. This slope coefficient was not significantly different from 0 (S. E. = 1.63, *t* (df = 15) = 0.46, p = 0.66). This coefficient indicates that the initial mathematics score for gender (female) was approximately 3.58 points higher than non gender. When controlling for the content knowledge of teachers, the coefficient was 0.09. This slope coefficient was not significantly different from 0 (S.E. = 0.07, *t* (df = 15) = 1.30, p = 0.22). When determining if there was a relationship with teacher pedagogy knowledge and student achievement, the coefficient was -0.54. This slope coefficient was significantly different from 0 (S.E. = 0.24, *t* (df = 15) = -2.31, p = 0.04).

Table 19

Final Estimate of Teacher as the Predictor of Student Achievement (Level 2 Gender)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, π_0					
For INTRCPT2, β_{00}					
INTRCPT3, γ_{000}	204.067834	1.842665	110.746	15	<0.001
CONTENT, γ_{001}	-0.028620	0.165837	-0.173	15	0.865
PEDAGOGY, γ_{002}	-0.012763	0.543746	-0.023	15	0.982
For FEMALE_M, β_{01}					
INTRCPT3, γ_{010}	0.741547	1.628417	0.455	15	0.655
CONTENT, γ_{011}	0.051154	0.145988	0.350	15	0.731
PEDAGOGY, γ_{012}	0.576807	0.486631	1.185	15	0.254
For TIME slope, π_1					
For INTRCPT2, β_{10}					
INTRCPT3, γ_{100}	4.145118	0.598068	6.931	15	<0.001
CONTENT, γ_{101}	-0.012471	0.053162	-0.235	15	0.818
PEDAGOGY, γ_{102}	0.296443	0.168883	1.755	15	0.100
For FEMALE_M, β_{11}					
INTRCPT3, γ_{110}	0.013098	0.791051	0.017	15	0.987
CONTENT, γ_{111}	0.092320	0.071315	1.295	15	0.215
PEDAGOGY, γ_{112}	-0.542910	0.235432	-2.306	15	0.036

Figure 4.1 provides a visual representation of the data for female versus male students based upon the pedagogy knowledge of the teacher. Overall females are increasing in their math scores over time. The simple slopes differ for females, but not males as indicated in Table 20. The slope in line 2 indicates females have more progress with teachers that have higher pedagogy knowledge. Females with teachers that have high pedagogy knowledge have more progress in math than females whose teacher is low in pedagogy. For male students (lines 1 and 3) the slope is not significantly different and the pedagogy knowledge of the teacher does not matter greatly in their performance.

Figure 4.1

Graph of Female students Versus Male Students based upon Teacher Pedagogy Knowledge

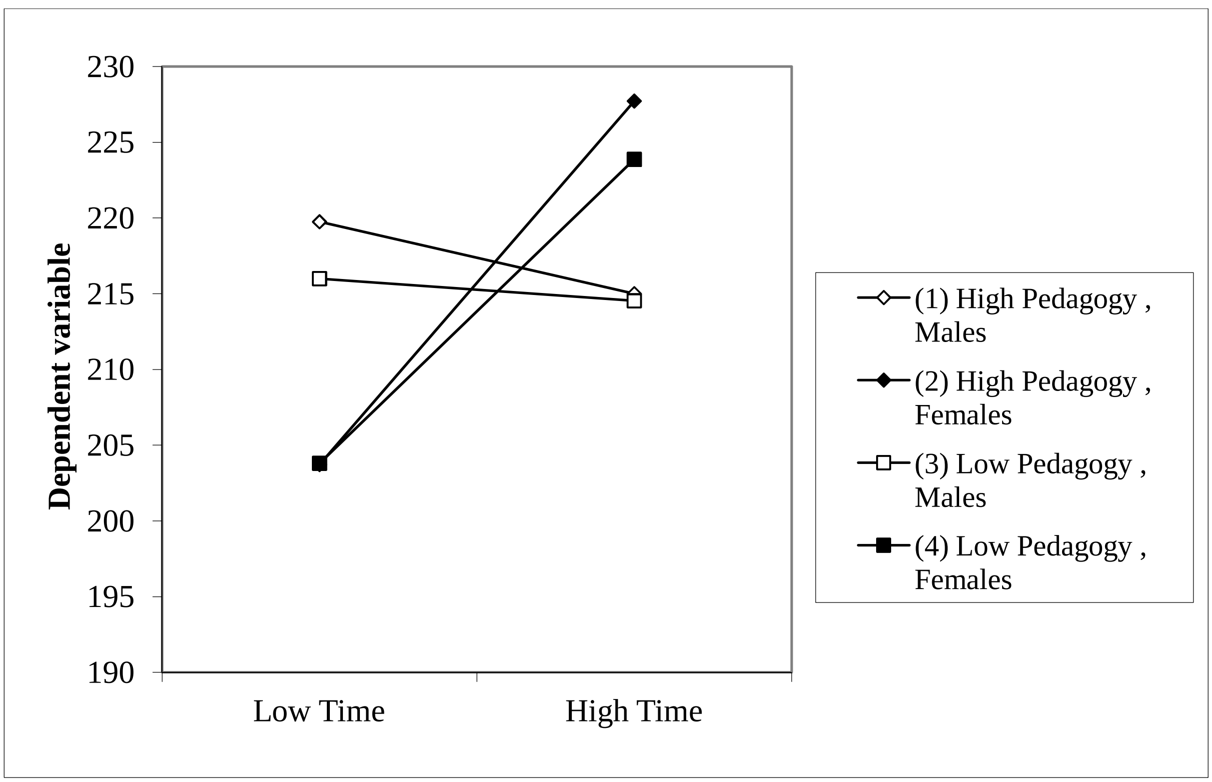

Table 20

Slope Difference Tests

Pair of slopes	t-value for slope difference	p-value for slope difference
(1) and (2)	-2.277	0.023
(1) and (3)	-1.277	0.202
(1) and (4)	-2.174	0.030
(2) and (3)	2.290	0.023
(2) and (4)	1.831	0.068
(3) and (4)	-2.256	0.025

Overall, FARM students scored 5.3 points less on the fall MAP test as compared to all other students. However, FARM students' math growth increased at about the same rate as other students. SWD scored 16.17 points less than the average student on the Fall Map test. Their math growth was at about the same rate as other students. All students had a growth coefficient of 4.05 overall with an average fall score of 204. This indicates that, for every testing period, there was an increase of approximately four points. The predicted winter score was 204 + 4.05 = 208.05.

Summary

In this chapter, the quantitative data analysis and the findings of this study have been presented. The schools were chosen based upon similar demographics and size. The teacher assessment data and survey information was collected through a voluntary process. Student data was collected and based upon the teachers that chose to participate. All information remained confidential throughout the research. In the next chapter, the researcher presents conclusions and recommendations based upon the findings of the study.

Chapter Five: Discussion, Conclusions, and Implications

Introduction

This study focused on determining if there was a relationship between teacher mathematical knowledge and the mathematics achievement of students in grades four and five. The purpose of this study was to explore the content knowledge and pedagogy knowledge of elementary teachers and determine if there was a relationship with the student MAP scores. This study used a quantitative approach using Hierarchical Linear Modeling (HLM). The methodology was based upon the research entitled, *The Relationship between Teachers' Mathematical Content and Pedagogical Knowledge, Teachers' Perceptions, and Student Achievement,* a study completed at the University of Maryland by Campbell and Nishio (in press).

This chapter is organized into four sections. The first section states the hypothesis. The second section presents the conclusions based upon the findings shared in chapter four. The third section provides recommendations for future practice based upon the findings. The last section shares recommendations for future research.

Hypothesis

The hypothesis stated teacher mathematical knowledge (content and pedagogy) would be positively related to the mathematics achievement of students in grades four and five.

Conclusions

The major hypothesis was not supported. Overall mathematics achievement of students was not found to be impacted by teacher content knowledge of pedagogy knowledge. However, when student demographic indicators were examined, the researcher found a significant three-way interaction between time tested, student gender, and teacher pedagogy knowledge. Probing this complex interaction showed that female students performed better over time based upon MAP scores with a teacher that was high in pedagogy than with teachers low in pedagogy knowledge. The other demographic variables were not found to be significant for content and pedagogy knowledge. These included: free and reduced meal students, English language learners, student with disabilities, and race. Although students with disabilities and free and reduced meals students did not significantly interact with either teacher content knowledge or pedagogy knowledge to predict growth in math achievement, both variables were significantly related to the initial math scores.

The findings indicated that the growth in math scores of free and reduced meal students and students with disabilities was not affected by either the content knowledge or pedagogy knowledge of the teacher. According to NCTM (2000) students with disabilities may need more time and accommodations to be successful in mathematics. These students may also need or benefit from additional resources. “Well documented examples demonstrate that all children, including those who have been traditionally underserved, can learn mathematics when they have access to high-quality instructional programs that support their learning” (NCTM, 2000, p. 14). Disadvantaged children and those with disabilities benefit greatly from significant human and material resources in schools and in classrooms. Mathematics performance can be improved with interventions that address “social, affective, and motivational factors” (NMAP, 2008, p. xix). However, there is a need to conduct experimental research based upon available interventions that support student achievement in mathematics to determine the effectiveness of the instruction support (NMAP, 2008).

Teacher professional development is an important component for achieving equity in elementary mathematics. "Teachers need help to understand the strengths and needs of students who come from diverse linguistic and cultural backgrounds, who have specific disabilities, or who possess a special talent and interest in mathematics" (NCTM, 2000, p. 14). Additionally, NMAP (2008) recommends that struggling students receive some explicit instruction in mathematics dedicated to ensuring that students conceptually understand the foundational skills for the mathematics they will encounter at their grade level. Although progress has been made with determining and understanding the difficulties students have with learning of concepts, procedures, and facts, research is still needed to understand the source of difficulty in areas of fractions and algebra (NMAP, 2008).

Teacher pedagogy knowledge did relate to the growth in math scores for female students. In a study completed by Beilock, Gunderson, Ramirez, and Levine (2010), they found that fear and anxiety about mathematics can impede mathematics achievement. This was especially true of female elementary school teachers. When female elementary teachers were math-anxious, their mathematics anxiety carried negative consequences for their female students. Since more than 90 percent of early elementary teachers in the United States are females, their anxiety has the potential to impact many students. Beilcok et al. (2010) completed their study with first and second grade female teachers by having them complete measures of mathematics anxiety while also assessing the mathematics achievement of the students. Their research found that there was no relation between a teacher's mathematics anxiety and student achievement at the beginning of the year. However, by the end of the year in the classroom with the teacher that was more math-anxious, the girls were more likely to believe the stereotype that boys were better in mathematics than girls. Of greater importance was the result that indicated the girls who believed that females had lower ability in mathematics, actually performed significantly lower than the boys in their mathematics achievement. The mathematics anxiety of an elementary teacher carries consequences for the achievement of girls in mathematics (Beilcok et al., 2010). The results of this study suggest that math pedagogy knowledge may be a way to overcome the stereotypes about math achievement that many female students have.

Palmer (2009) completed a study to evaluate the effectiveness of an alternative college level course to change the understanding and the anxiety associated with mathematics. The results indicated that when the teachers were aware of their own complex processes and strategies for problem solving in mathematics, they were better able to identify the mathematical subjectivities and change the attitudes of elementary mathematics teaching. This study highlighted a different pedagogical way to teach early childhood education students mathematics. Pedagogical practices were made visible and challenged the college level students to think deconstructively and gender-consciously. Teachers were also challenged to analyze their mathematics teaching practices in multiple ways through problem-based learning. They were taught to analyze their mathematical teaching practices in multiple ways to include the impact of gender bias. Palmer (2009) found that the attitudes and beliefs about mathematics changed for the students from the beginning of the class until the end. This indicated that, with a change in pedagogy, attitudes about mathematics can be changed to the positive which then can positively influence the achievement of female students.

In a study completed by Campbell and Nishio (in press), it was indicated that there was a significant relationship between upper-elementary teachers' mathematical content knowledge and their students' mathematics achievement. They also determined that for each standard deviation increase in the content knowledge of the teachers, "the estimated mathematics achievement score of their students increased by 7.1%" (Campbell & Nishio, in press, p.26). However, Campbell and Nishio (in press) found that the teacher mathematics pedagogy knowledge did not influence the students' mathematics achievement. In another study, Hill, Schilling, and Ball (2004) identified mathematics content and pedagogy knowledge, developed a multiple choice assessment to measure the knowledge, and determined there was a significant relationship between teacher content knowledge and the achievement of students in grades one and three.

Campbell and Nishio (in press) completed their study with both upper-elementary teachers and middle-grade teachers separately. Interestingly, the results of the middle-grade teachers indicated that for each standard deviation increase in teacher' pedagogy knowledge or overall knowledge (combined pedagogy and content knowledge) the estimated achievement of students increased by 22 percent of a standard deviation. This indicated that when middle-grades teachers understood more mathematics, students, on average, evidenced higher achievement in mathematics.

Although this study used the same teacher pedagogy and content knowledge assessment as the study completed by Campbell and Nishio (in press), the data from each study provided different results. Campbell and Nishio provided a monetary compensation of $350.00 for the teachers that volunteered to complete the assessments. The subjects of the study were from 23 districts and included 266 early career teachers of mathematics. The assessments and surveys were completed in a single setting, on a non-school day at a non-school site. Although Hierarchical Linear Modeling was used to determine the relationship in both studies, the student data was different. Campbell and Nishio (in press) used "students who completed their state's "regular" high-stakes measure as required under the No Child Left Behind federal regulations" (p.14). This study used student MAP scores based upon fall 2012, winter 2013, and spring 2013 for each student. The dissimilar use of student assessments could have been the reason for the variance in results between both studies.

In this study, 18 teachers volunteered, but were not provided compensation, to complete the assessment and survey. The assessments were sent to the teachers to be completed on their own time at their convenience. This could have been a reason for the difference in results between both studies. The teachers could have been provided with assistance on some of the questions, they could have researched how to solve the problems, and they could have worked together. Additionally, it was assumed that because the teachers agreed to participate, they probably had a level of comfort with the mathematics they were teaching since there was not an incentive to participate. The difference in the results of the two studies could be a consequence of the disparity between the setting for taking the assessments and surveys, the variance in the financial incentive and the variation in the sample size between both studies.

Furthermore, the sample size of students was significantly different between both studies. Campbell and Nishio (in press) had 6,413 students from grades four and five participating in the study. The demographics were: 35% white, 45.5% black/African American, 15% Hispanic, 3.7% Asian, .8% other, 14.5% special education, 5.1% English language learners, and 56.8% free and reduced meal students. In comparison, the sample size of this study included 322 students. Although the results of this study indicated a relationship between female students and pedagogy knowledge only, similar studies with a larger sample size indicated that the content knowledge of upper elementary teachers did make a difference in the achievement of students.

Recommendations for Future Practice

These results indicate the need for teachers, for female students at least, to have a strong pedagogy knowledge based upon the mathematics they teach. Teachers organize their classrooms based upon their own knowledge and beliefs about mathematics teaching and learning. They need substantial pedagogical knowledge in order to help students achieve in mathematics. When teachers have strong knowledge of pedagogy, they are able to assist students in building procedural and conceptual understanding to extend and challenge their thinking. Additionally, teachers are able to use their knowledge to make decisions about lessons, resources, manipulatives, classroom discussions, problem based lessons, and actions that occur during the learning process. When teachers have a limited knowledge of mathematics, teaching of discrete concepts occur in isolation instead of teaching that supports students in making connections between the standards, procedures, and practices. A deep understanding of mathematics helps teachers with the awareness of common misconceptions and is used to make daily instructional decisions that build students' conceptual understanding of the mathematics they are learning (Anthony and Walshaw, 2009).

Teachers with a sound knowledge of mathematics pedagogy are able to listen to students' justification of answers in a perceptive manner in order to effectively determine misconceptions, and have the ability to make on the spot decisions to assist student understanding of important mathematical ideas. These same teachers are able to teach, adapt, and modify plans and lessons to assist all students in becoming mathematically proficient (Anthony and Walshaw, 2009).

According to Kilpatrick, Swafford, and Findell (2001) a critical component to improving mathematics is the professional development and preparation of elementary mathematics teachers. Teaching for learning involves a significant reorientation of the acquisition of pedagogical knowledge. To be productive, teachers' investigation of student thinking needs to be secured by their own understanding of the standards and practices of mathematics. When teachers commit to understanding student thinking, classroom practices and pedagogy change significantly and result in student achievement (Wisconsin Center for Education, 2002). The knowledge and pedagogy needed to teach mathematics is specialized. "It includes an integrated knowledge of mathematics, knowledge of the development of students' mathematical understanding, and a repertoire of pedagogical practices that take into account the mathematics being taught and the student learning it" (Kilpatrick et al., 2001, p. 428.)

The implications of the research are that teachers need to learn these forms of knowledge to help them create and build connections. Teachers need to know and understand the curriculum with the connections between mathematical ideas and how to develop the understanding with students. Unpacking the standards and practices is critical for teachers to be able to help students develop a conceptual understanding of the content. “Teachers need not only mathematical proficiency but also the ability to use it in guided discussions, modifying problems, and making decisions about what matters to pursue in class and what to let drop” (Kilpatrick et al., 2001, p. 428).

Research has been mixed on the best way to approach professional development for elementary teachers. Improving mathematics instruction has been a priority in the United States. However, many elementary teachers continue to lack the mathematical skill to teach mathematics effectively and therefore improve mathematics instruction (NMAP, 2008). The current methodologies and pedagogy used to teach students mathematics, specifically the way mathematical knowledge is developed for elementary students in our country, is not working and there are many opportunities for improvement (Lias et al., 2005-2006). “How well teachers know mathematics is central to their capacity to use instructional materials wisely, to assess students’ progress, and to make sound judgments about presentation, emphasis and sequencing” (Ball, Hill, & Bass, 2005, p. 14). To improve instruction, teachers must have “access to high-quality materials, the support of parents, and ongoing, focused professional development” (Burns, 2000, p. 3).

Researchers, curriculum developers, textbook writers, policy makers, and school district officials have demonstrated great interest regarding how to provide effective training for elementary teachers so that the United States moves to the forefront in the area of mathematics. However, improving mathematics has been determined to be a complicated undertaking. “It relies on teacher understanding of math curriculum in addition to awareness of how children acquire mathematics concepts” (Chapman, Leonard, Burciaga & Jernigan, 2013, p. 191). The NMAP (2008) recommended further research to determine what constitutes teacher effectiveness and how to further develop programs to prepare elementary mathematics teachers.

This study investigated the need for teachers to have strong mathematical content knowledge and their ability to teach mathematics conceptually to positively impact the achievement of elementary students. Although the results of this study only indicated a relationship between female students and teachers with high pedagogy knowledge, the results of similar studies indicated that the content knowledge and pedagogy knowledge of teachers did have a positive relationship with the achievement of students (Campbell & Nishio, in press; Hill, Schilling & Ball, 2004). "Teachers who have developed a profound understanding of fundamental mathematics are better able to facilitate developing this understanding in their students, reveal and represent connections among and between topics, and encourage multiple ways of solving problems" (Lias et al., 2005-2006, p. 73).

Mathematics Specialists and Professional Development

Many school systems are currently exploring ways to ensure that students receive mathematics instruction from teachers who have a deep understanding of mathematics content and pedagogy. However, some educators still see mathematics instruction as less important at the elementary grade level. Major reports, including the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics; Adding It Up: Helping Children Learn Mathematics; The Final Report of the National Mathematics Advisory Panel;* and the Mathematical Education of Teachers, have provided a rationale for mathematics specialists at the elementary level to assist teachers with their pedagogy knowledge of elementary mathematics. There is a need for math specialists based upon the lack of pre-service background and the general teaching responsibilities of elementary teachers. Together, these create a need for teachers to have on-site support in developing their content and pedagogy knowledge needed to teach elementary mathematics effectively (Fennel, 2006).

The NMAP (2008) reported that, across the country, many schools and districts have implemented school-based mathematics specialists or mathematics coaches in an effort to improve instruction and student achievement in elementary mathematics. However, there is little research exploring the effectiveness of mathematics specialists. The roles and responsibilities of mathematics specialists and implementation models vary according to the state or school district. Additional research in this area would provide schools and district leaders with information in regards to the best way to help elementary teachers learn the content and pedagogy the mathematics curriculum demands in grades four and five.

The Association of Mathematics Teacher Educators, Association of State Supervisors of Mathematics**,** National Council Supervisors of Mathematics, and National Council of Teachers of Mathematics recommend the use of elementary mathematics specialists in pre-kindergarten through grade six schools. According to Fennel (2006) elementary mathematics specialists should be used in every elementary school to enhance the teaching, learning, and assessment of mathematics as an impetus to improve student achievement. Schools, districts, states or provinces, and institutions of higher education should work collaboratively to create advanced certification for elementary mathematics specialists and create rigorous programs to prepare elementary mathematic specialists. "We need elementary school mathematics specialists, elementary classroom teachers who know and understand mathematics and can effectively mentor their colleagues" (Fennell & Wray, 2012). Anecdotal evidence from programs throughout the United States supports the premise that coaching teachers is an effective method to improve teaching and learning of elementary mathematics (McGatha, 2009). “The available empirical and anecdotal evidence suggest that coaching is a promising professional development practice that can lead to improved teaching and learning. However, we need to continue to pursue research that can support these initial findings” (McGatha, 2009, p. 2).

The insights gained from this study indicated that teachers with a better understanding of the pedagogy of the mathematics they are teaching had a positive correlation with the achievement of female students. Because female students are comprised within the general education classroom along with males, teachers need to develop a deep understanding of the content and the pedagogy in order to best meet the needs of all students.

Limitations

For this study, the research identified a rather small sample of teachers (18) to explore if there was a relationship between teacher content and pedagogy knowledge and the mathematics achievement of students in grades four and five. The researcher used quantitative methods with the HLM-3 level model to determine if there was a relationship. The researcher also took into consideration the specific demographics of the students and teachers in the HLM analysis.

The biggest limitation of this study was the small number of teachers in District A that agreed to participate by completing the assessment. In order to build a deeper understanding of the correlation between teacher content and pedagogy knowledge and the achievement of students, future research should be conducted with a larger sample size. Additionally, because the research used a voluntary method to gain participants, many of the teachers had been part of year long mathematics professional development within District A and several of the participants either had or were pursuing degrees in the mathematics field. This indicated that the teachers choosing to participate had, for the most part, an appreciation and confidence for the mathematics they were teaching. Additionally, studies should investigate the relationship found between female students and the pedagogy knowledge of teachers. This study had 45 percent of 322 students that were female. This study also indicated additional research on the best methods to teach students with disabilities and free and reduced meals students was also needed, as neither content knowledge nor pedagogy knowledge correlated positively to the achievement of students with disabilities. Additionally, students with disabilities and free and reduced meal students scored significantly below students without disabilities and non-free and reduced meal students. "These teachers have many of the same mathematical and pedagogical weaknesses as regular classroom teachers and will benefit from participating in coaching-related experiences" (Campbell, Ellington, Haver, & Inge, 2013, p. 23).

Teacher pedagogy knowledge was assessed through 39 multiple choice test items and assessed only one aspect of the pedagogy knowledge of teachers. These questions were limited to how a teacher chooses to teach mathematics. For example, one question asked, which of three contexts would be most useful when investigating volume, leading to the development of a formula. Another question asked about the prior knowledge needed for students to understand a concept. Yet another asked the teachers to determine a student misconception based on incorrectly solving a problem. Other questions assessed the teachers' ability to help students make connections, anticipate misconceptions, assess student understanding of a concept, develop a sequence for teaching a concept, determine where students lack skills and understanding, etc. The assessment items did not consider the relationships that teachers build with students, the classroom environment developed by the teacher and the students, how to differentiate instruction for higher level student and for those that are struggling. The use of the multiple choice assessment measured only one aspect of teacher pedagogy and is a limitation of the study.

Furthermore, this study was limited to the use of MAP assessments to determine growth of students over time from fall 2012, winter 2013, and spring 2013. The results indicated that MAP did measure growth over time for students on average of about 4.05 points per testing session. Further research on the best assessment to measure student growth could be conducted.

As the transition to the Common Core State Standards continues in the United States, the need to create teacher assessments tied to the new standards for both content knowledge and pedagogy knowledge, with an emphasis on both content and the practices that promote understanding, will be needed to further develop ways to support teachers and students. As shared by Fennel (2011),few elementary teachers choose mathematics as their area of specialty. For these reasons, there is a need to further explore pre-service programs in order to better prepare teachers through creating pathways that assist pre-service teachers in developing a conceptual understanding of the important content in mathematics. In order to improve student understanding of mathematics, the knowledge of teachers must be enhanced (Campbell & Nishio, in press). More research is needed to determine how to better help elementary teachers build the knowledge needed to improve the mathematics instruction for elementary students. This research could be used to design new programs or update existing professional develop models to improve teaching and learning in elementary mathematics.

Summary

Research has indicated that the content knowledge of elementary mathematics teachers has a significant impact on the learning that occurs for elementary students (Campbell & Nishio, in press; Hill, Schilling & Ball, 2004). When teachers have a deep, conceptual understanding of the mathematics they are teaching, there is a positive effect on the learning of students. This study indicated that when teachers had a higher pedagogy knowledge of mathematics, female students performed better. However, this was not indicated in the scores of students overall. Additional research with a larger population of students and teachers would help to provide a better understanding of the influence that content and pedagogy knowledge play on the development of students' ability to understand and apply mathematics. Furthermore, determining how to best provide professional development for elementary teachers in both pre-service programs and for those teachers currently teaching has the potential to greatly influence and transform elementary mathematics education.

Appendix A: Cover Letter for Teacher Assessment

September 2, 2013

Dear Teachers,

I am a doctorate student from the University of Maryland. Currently I am studying the correlation between teacher mathematical knowledge (content and pedagogy) and the mathematics achievement of students in grades four and five. As part of my study, I am undertaking a dissertation project on the same topic. I need your input for the successful completion of the project. I am attaching a teacher content knowledge and pedagogy knowledge assessment.

The assessment has been designed by Dr. Patricia Campbell and Masako Nishio at the University of Maryland. The questions are all multiple choice. The results will be analyzed quantitatively and correlated to student MAP data. Individual responses will be anonymous. References to schools will be omitted in the research document.

I appreciate your time and for completing the assessment!

Sincerely,

Jana Palmer
Doctoral Candidate
University of Maryland

Appendix B: Consent to Participate

Project Title	*What is the correlation of teacher mathematical knowledge and the mathematics achievement of students in grades four and five?*
Purpose of the Study	*This research is being conducted by Jana Palmer at the University of Maryland, College Park. We are inviting you to participate in this research project because you taught fourth or fifth grade math during the 2012-2013 school year. The purpose of this research project is to determine the correlation between teacher mathematical knowledge (content and pedagogy) and the mathematics achievement of students in grades four and five.*
Procedures	*The procedures involve completing a teacher mathematics content knowledge and pedagogy knowledge assessment with 119 multiple choice items.*
Potential Risks and Discomforts	*There are no risks anticipated from participating in this research study.*
Potential Benefits	*There are no direct benefits from participating in this research. However, possible benefits include increased student achievement in the area of mathematics. We hope that, in the future, other people might benefit from this study through improved understanding of the training needed for pre-service teachers and professional development for elementary mathematics teachers.*

Confidentiality	*Any potential loss of confidentiality will be minimized by listing teachers as Teacher A and students as Student 1. The district will be labeled as District A. The names of the teachers and students will not be known to the researcher.* *If we write a report or article about this research project, your identity will be protected to the maximum extent possible. Your information may be shared with representatives of the University of Maryland, College Park or governmental authorities if you or someone else is in danger or if we are required to do so by law.*
Medical Treatment	*Not Applicable*
Compensation	*Not Applicable*
Right to Withdraw and Questions	*Your participation in this research is completely voluntary. You may choose not to take part at all. If you decide to participate in this research, you may stop participating at any time. If you decide not to participate in this study or if you stop participating at any time, you will not be penalized or lose any benefits to which you otherwise qualify.* *If you decide to stop taking part in the study, if you have questions, concerns, or complaints, or if you need to report an injury related to*

	the research, please contact the investigator: ***Jana Palmer*** *1 S. Clifton Drive, Williamsport, MD 21795* *301 766 8415*
Participant Rights	*If you have questions about your rights as a research participant or wish to report a research-related injury, please contact:* ***University of Maryland College Park*** ***Institutional Review Board Office*** ***1204 Marie Mount Hall*** ***College Park, Maryland, 20742*** ***E-mail: irb@umd.edu*** ***TeELLhone: 301-405-0678*** *This research has been reviewed according to the University of Maryland, College Park IRB procedures for research involving human subjects.*
Statement of Consent	*Your signature indicates that you are at least 18 years of age; you have read this consent form or have had it read to you; your questions have been answered to your satisfaction and you voluntarily agree to participate in this research study. You will receive a copy of this signed consent form.*

<table>
<tr><td></td><td colspan="2">If you agree to participate, please sign your name below.</td></tr>
<tr><td rowspan="3">Signature and Date</td><td>NAME OF PARTICIPANT

[Please Print]</td><td></td></tr>
<tr><td>SIGNATURE OF

PARTICIPANT</td><td></td></tr>
<tr><td>DATE</td><td></td></tr>
</table>

Appendix C: Permission to Use Teacher Assessment

Permission received through an email:

On 6/4/13 7:40 AM, Palmer, Jana wrote:

Dr. Campbell,

I am beginning my class this week in which I am required to write Chapter 3 of my dissertation. Is there any chance that I could get copies of the assessments from your study? They will not be used in any way to evaluate teachers or for use in firing teachers.

Thanks for considering!

Jana Palmer, Principal

Thu 6/6/2013 2:39 PM

Okay, but please at this time only use these as needed to work on your dissertation/dissertation proposal.

The attached zip file has five 24-item subtests. These are in the order that we administered them, with the teachers alternating completion of these with taking breaks, eating lunch, or completing other surveys.

These files do not identify which are the mathematical content items or which are the pedagogical content items. I can send you further information later about that and also an answer key later, but I leave town for Ireland tomorrow and cannot organize that for you now. So we will need to continue to be in touch. But this should get you started.

Good luck with your work on Chapter 3. I know this is when students often feel that this whole dissertation process is becoming real.

Pat Campbell

Patricia F. Campbell, Ph.D.

Associate Professor, Center for Mathematics Education

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College Park, MD 20742-1175

Telephone: 301-405-3129

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