

Mathematical modeling in the agro-industrial complex: basic problems and models construction

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Abstract. Currently, information technologies have been tightly integrated into agriculture. Since no computer calculations are possible without a powerful mathematical apparatus, the question arises about the possibility of modeling the processes occurring in agriculture with the help of modern achievements of science and technology. The present study is devoted to the existing methods of mathematical modeling in agriculture in relation to the applied aspects of agriculture. The aim of the research is to develop a critical approach to modern developments in the field of mathematical modeling and their place in agriculture. It is shown that the introduction of mathematical models based on modern scientific knowledge contributes to the optimization of agricultural processes and increasing the efficiency of any farm. Based on statistical studies, it is shown that among all branches of agriculture, mathematical methods are most often used in economic calculations, least of all, in calculations related to farm modeling. This is explained both by the complexity of modeling all processes occurring within a single farm, and by the loss of accuracy, which increases with the complexity of the system model. **Keywords:** Mathematical modeling, programming, smart agriculture, agro-industrial complex.

1 Introduction

The agro-industrial complex (AIC) is an intersectoral complex that unites economic sectors whose activities consist in the production and processing of agricultural raw materials and obtaining products from it that are brought to the end consumer. This is a system that includes sectors of the country's economy, including agriculture and industries that are closely interrelated with agricultural production; carrying out transportation, storage, processing of agricultural products, supplying them to consumers; providing agriculture with machinery, chemicals and fertilizers; serving the agricultural production.

Of course, modeling as a discipline develops synchronously with the science of mathematics, but modern research pays little attention to the general, structural analysis of trends in this area. Of course, the technological progress of recent years has contributed a lot

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to the creation of the concept of an “ideal farm”, where you can model both the initial data and track the process in any phase. Nevertheless, the question of the functionality of such models and the usefulness of their application remains open. [1, 2, 3]

The present study is devoted to the existing practices of mathematical modeling in agriculture in relation to the applied aspects of agriculture. On the basis of the fundamentals of the mathematical apparatus, the essence of the processes taking place in connection with the factual basis of the Earth sciences is revealed. The relevance of the research is justified by the need for a structural approach to the topic, on which there are many developments, but no independent general theory has yet been developed either within the existing Earth sciences or within any newly created discipline.

Thus, the aim of the study is to develop a critical approach to modern developments in the field of mathematical modeling and their place in agriculture. It is shown that the introduction of mathematical models based on modern scientific knowledge contributes to the optimization of agricultural processes and will contribute to increasing the efficiency of any farm. [4]

2 Materials and Methods

The methodological basis of the study consists of methods used in agriculture and mathematics.

Modeling is a method of cognition, consisting in the creation and study of models. The theory of substitution of original objects by a model object is called the theory of modeling. The following stages of modeling can be distinguished:

- problem statement;
- method detection;
- modeling;
- approval;
- check [5].

Agricultural activity in the scientific aspect is determined by three phases:

- the design phase (creation of the constructed model and its implementation plan);
- technological phase (model implementation and hypothesis testing);
- reflection phase (evaluation of the constructed system of new scientific knowledge, its correction, rejection of theory or construction of a new theory).

The cumulative nature of the development of scientific knowledge determines the existence of synergy, the relationship between the two applied sciences. It is obvious that the modeling stages essentially coincide with the knowledge-intensive phases of agricultural activity: there is a clear correlation, presented in Table 1.

Table 1. Phases and stages of modeling.

Stage	Phase
Problem statement	Design phase
Method detection	Technological phase
Modeling	
Approval	
Check	Reflection phase

The applied mathematical methods can be divided into two types:

1) optimal ones. These include: the simplex method (first published in 1949 by the American scientist J. Danzig); the method of potentials (the Soviet scientist L. V. Kantorovich was at the origin of its development in the 40s of the last century); the delta

method; the differential rent method; the Hungarian method (developed in 1931 by the Hungarian scientist B. Egervari) and others.

2) suboptimal ones. An example of this type can be the approximation or Vogel method, since it allows obtaining solutions that are close to optimal.

According to their capabilities, mathematical methods can be conditionally divided into groups:

a) universal ones, which allow solving problems of any type. For example, the simplex method;

b) special ones, solving problems of a certain type. Thus, transport problems are mainly solved by the distribution or potential method, and the assignment problem is implemented by the Hungarian method.

As for the research materials, modern big data sciences provide great opportunities for mathematical modeling. First of all, it is the Python programming language and the environment for working with it: PyCharm Community. Among the Python libraries, NumPy, SciPy, Matplotlib, Pandas, SymPy, Pint, Jupyter should be mentioned [5]. The Matlab software package provides great opportunities for modeling [6].

3 Results and Discussion

First of all, it is necessary to determine in which areas of agriculture mathematical modeling is most often used.

Certain statistics exist. A selection of articles in open access databases of scientific articles, such as Google Academy, Academia.org, ResearchGate, for the period 2019-2023, with the keywords “agriculture” and “mathematical modelling” was made. The results of the statistical sample are shown in Figure 1.

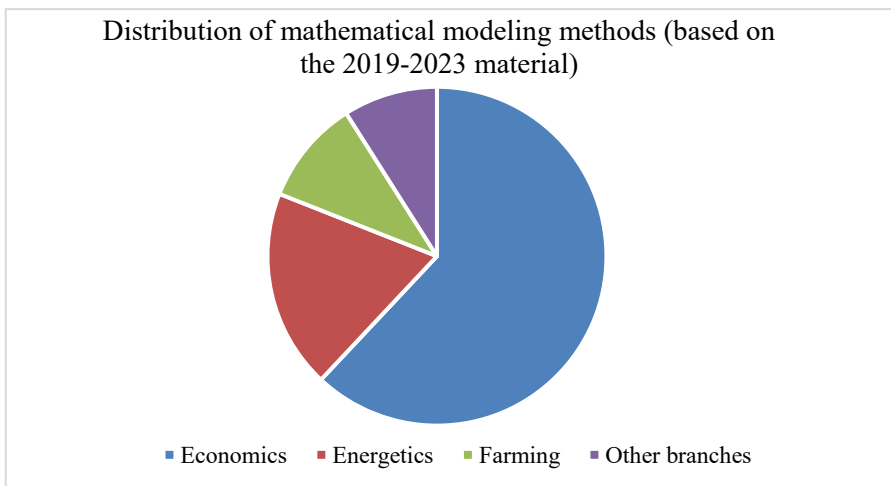


Fig. 1. Distribution of mathematical modeling methods, Source: drawn by the authors.

As it can be seen, the greatest number (62% of the total) of mathematical modeling methods were used in materials related to the economic indicators of the agro-industrial complex. A slightly smaller number of articles (19% of the total) used modeling in the energy sector of the agro-industrial complex. Modeling the processes of the entire farm was used by the authors of 10% of the articles in our sample, 9% of the articles relate to various branches of agriculture.

What could be the reason for such a distribution of articles? Determining the most appropriate distribution of production resources in order to maximize agricultural production

is the main task of the agro-industrial complex. In addition, economics as a science draws conclusions based on traditional methods of analyzing average (statistical) data and generalizing experience, which corresponds to one of the main criteria of any model: the model is ideal, that is, it does not take into account some factors, considering them insignificant within the framework of the problem.

As for energy, the energy equipment requires constant updating and optimization, which is why there is a constant recalculation of the distribution of capacities and the load of equipment. The advantage of any mathematical model is that it allows recalculating without using real-world objects, operating only with generalized algorithms, although they correspond in detail to real phenomena.

In the "Farming" section, we assign calculations related, for example, to determining the proportional ratio of the ingredients of various feeds and their impact on the functioning of the farm as a whole. In fact, this section is closely interrelated with the "Economy" section, but the priority in it is the calculation of the characteristics of some new object, and not economic indicators.

Economic and mathematical models are classified:

- according to the degree of aggregation of modeling objects (microeconomic, local, macroeconomic models);
- taking into account the time factor (static, dynamic models);
- taking into account the uncertainty factor (deterministic, stochastic models);
- for the purpose of creation and application (balance, econometric, optimization, simulation, network, queuing system models);
- by the type of mathematical apparatus (linear programming, non-linear programming; correlation and regression models; matrix models; network models; game theory models; queuing theory models, inventory management theory models).

For economic analysis, the most important model is the factor model. The group of factor models includes such models that, on the one hand, carry economic factors (the state of an economic object depends on them), and on the other hand, the parameters of the state of the object. The factor model is often represented by a linear or statistical function.

Balance models differ from the factor ones. The basis of the construction of this type of models is the balance method: the method of mutual comparison of material, financial and labor resources with the needs for them. If we consider the descriptions of the economic system, we can say that its balance sheet model is the management system. Each of these systems expresses the need for a balance between the manufactured economic models and the total need for this product. [7]

Finding the functional relationship between the parameters is one of the most important tasks of mathematical modeling: the correlation problem. The tasks of the balance model include, as a rule, tasks related to the optimization of a resource in order to track changes in the parameters of the system as a whole.

Optimization problems have been developed for arable land, fertilizers, irrigation, structures, the use of labor in crop production, herd structure and feeding rations of livestock. [8, 9, 10]

Next, we will consider several tasks for compiling mathematical models for different branches of the agro-industrial complex.

No. 1. Three crops are planted on an area of 22 hectares: cabbage, cucumbers and tomatoes. Growing cabbage allows making a profit (per 1 ha) in the amount of 80 thousand rubles, cucumbers, 50 thousand rubles, tomatoes, 65 thousand rubles. The costs of labor and material and monetary costs are shown in Table 2. Build a mathematical model for solving the problem.

Table 2. Background information

Indicators	Labor input per 1 hectare of sowing			Resource volume
	Cabbage	Cucumbers	Tomatoes	
A	1	2	3	4
Labor costs, hours	80	100	120	2520
Material and monetary costs, thousand rubles	60	40	50	1200

Here x_1 is the area of cabbage, ha; x_2 is the area of cucumbers, ha; x_3 is the area of tomatoes, ha. Thus, the total volume is:

- arable land areas:

$$x_1+x_2+x_3 \text{ (ha);} \quad (1)$$

- labor resources:

$$80x_1+100x_2+120x_3 \text{ (hours);} \quad (2)$$

- material and monetary costs:

$$60x_1+40x_2+50x_3 \text{ (th. rub.).} \quad (3)$$

Production is limited by the volume of resources: arable land = 22 (ha); labor resources = 2520 (hours); material and monetary costs = 1200 (thousand rubles). The profit should be maximum.

As a result, the model is obtained:

$$F = 80x_1+50x_2+65x_3 \rightarrow \max; \quad (4)$$

The following system is obtained (1):

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 22 \\ 80x_1 + 100x_2 + 120x_3 + x_5 = 2520 \\ 60x_1 + 40x_2 + 50x_3 + x_6 = 1200 \end{cases} \quad (5)$$

Here the variables x_4 , x_5 , x_6 are additional and show the underutilization of land resources, labor and material and monetary costs, respectively.

After solving the system of equations, it can be found out that the maximum profit can be 1600 thousand rubles. To do this, it is necessary that the sown area of cabbage is 20 hectares.

No. 2. Make an economic and mathematical model of optimization of the daily feeding ration for cows with an average live weight of 500 kg and an average daily milk yield of 14 kg of milk during the milking period. To ensure the desired productivity, it is necessary that the diet contains at least 11.6 kg of feed units, 1160 g of digestible protein, 81 g of calcium, 57 g of phosphorus and 520 mg of carotene. The dry matter in it should be no more than 14.9 kg.

The diet consists of compound feed, meadow hay, clover-thistle hay, barley straw, corn silage, potatoes and fodder beet. The content of nutrients in feed and their cost are presented in Table 3.

Table 3. Background information.

Feed	Feed units, kg	Digestible protein, g	Calcium, g	Phosphorus, g	Carotene, mg	Dry matter, kg	Cost of 1 kg of feed, rub.
A	1	2	3	4	5	6	7
Compound feed	0.9	112	15	13	-	0.87	19.5
Meadow hay	0.42	48	6	2.1	15	0.85	3.4
Clover-timothy hay	0.5	52	7.4	2.2	30	0.83	2.1
Barley straw	0.36	12	3.7	1.2	4	0.85	0.3
Corn silage	0.22	30	3.5	1.2	10	0.31	0.8
Potato	0.3	16	0.2	0.7	-	0.23	9.7
Fodder beetroot	0.12	9	0.4	0.4	-	0.13	2.1

In accordance with zootechnical requirements, individual groups of feed in the diet can vary within the following limits (% of the total number of feed units): concentrated from 10 to 15, coarse ones, from 16 to 19, juicy ones, from 50 to 60, root crops, from 10 to 15. The specific weight of straw in the rough feed group should be no more than 30%, and clover-timothy hay at least 40%, potatoes in the root crop group, no more than 20%.

The criterion of optimality is the minimum cost of the diet.

Let us define a list of variables. The amount of feed that can be included in the diet is denoted by:

x_1 - compound feed, kg;

x_2 - meadow hay, kg;

x_3 - clover-timothy hay, kg;

x_4 - barley straw, kg;

x_5 - corn silage, kg;

x_6 - potatoes, kg;

x_7 - fodder beetroot, kg;

x_8 - total nutritional value of the diet, kg.k. units.

Let us write down the system of restrictions in an expanded form:

1. Restrictions on the balance of nutrients in the diet:

feed units at least

$$0,9 x_1 + 0,42x_2 + 0,5x_3 + 0,36x_4 + 0,22x_5 + 0,3x_6 + 0,12x_7 = x_8; x_8 \geq 1,6 \quad (6)$$

digestible protein at least

$$112x_1 + 48x_2 + 52x_3 + 12x_4 + 30x_5 + 16x_6 + 9x_7 \geq 1160 \quad (7)$$

calcium at least

$$15x_1 + 6x_2 + 7,4x_3 + 3,7x_4 + 3,5x_5 + 0,2x_6 + 0,4x_7 \geq 81 \quad (8)$$

phosphorus at least

$$13x_1 + 2,1x_2 + 2,2x_3 + 1,2x_4 + 1,2x_5 + 0,7x_6 + 0,4x_7 \geq 57 \quad (9)$$

carotene at least

$$15x_2 + 30x_3 + 4x_4 + 10x_5 \geq 520 \quad (10)$$

2. Restriction on the content of dry matter in the diet:

$$0,87x_1 + 0,85x_2 + 0,83x_3 + 0,85x_4 + 0,31x_5 + 0,23x_6 + 0,13x_7 \leq 14,9 \quad (11)$$

3. Restrictions on the content of certain groups of feeds in the diet:

concentrated at least

$$0,9x_1 \geq 0,1x_8 \quad (12)$$

concentrated no more than

$$0,9x_1 \leq 0,15x_8 \quad (13)$$

rough at least

$$0,42x_2 + 0,5x_3 + 0,36x_4 \geq 0,16x_8 \quad (14)$$

rough no more than

$$0,42x_2 + 0,5x_3 + 0,36x_4 \leq 0,19x_8 \quad (15)$$

juicy at least

$$0,22x_5 \geq 0,5x_8 \quad (16)$$

juicy no more than

$$0,22x_5 \leq 0,6x_8 \quad (17)$$

tuberous roots at least

$$0,3x_6 + 0,12x_7 \geq 0,1x_8 \quad (18)$$

tuberous roots no more than

$$0,3x_6 + 0,12x_7 \leq 0,15x_8 \quad (19)$$

4. Specific gravity restriction:

specific gravity of straw in the group of roughage

$$x_4 \leq 0,3(x_2 + x_3 + x_4) \quad (20)$$

the specific weight of clover-timothy hay in the group of coarse

$$x_3 \geq 0,4(x_2 + x_3 + x_4) \quad (21)$$

the specific weight of potatoes in the group of tuberous roots

$$x_6 \leq 0,4(x_6 + x_7) \quad (22)$$

5. The condition of non-negativity of variables:

$$x \geq 0 \quad (j = \overline{1, 8}) \quad (23)$$

The objective function is the minimum cost of the ration:

$$Z = 19,5x_1 + 3,4x_2 + 2,1x_3 + 0,3x_4 + 0,6x_5 + 9,7x_6 + 2,1x_7 \rightarrow \min \quad (24)$$

4 Conclusion

Problem statement is the first and most important stage in building a model. From the above examples it can be seen that the mathematical model is reduced to the compilation of a system of deterministic equations; it follows that such a system is not a complete reflection of the real object on the basis of which the model is created. [11,16] It should be remembered about the difference between models intended for research and models intended for testing an object for use in real conditions [12,17].

Mathematical modeling provides the widest possibilities for solving a variety of problems related to the surrounding world, from the economic consequences of drought to the potential of biogas processing [13,14,18,19]. At the same time, it is necessary to take into account the errors that may arise when solving systems of high-order equations, and take into account the correlation of quantities, provided that its definition is correct. It is obvious that the development of a list of typical mathematical modeling tasks will contribute a lot to the introduction of advanced calculation methods both by classical methods and using Big Data technologies. [15]

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