ABSTRACT

Title of Document: RESILIENCE OF TRANSPORTATION INFRASTRUCTURE SYSTEMS: QUANTIFICATION AND OPTIMIZATION

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Transportation systems are critical lifelines for society, but are at risk from natural or human-caused hazards. To prevent significant loss from disaster events caused by such hazards, the transportation system must be resilient, and thus able to cope with disaster impact. It is impractical to reinforce or harden these systems to all types of events. However, options that support quick recovery of these systems and increase the system's resilience to such events may be helpful.

To address these challenges, this dissertation provides a general mathematical framework to protect transportation infrastructure systems in the presence of uncertain events with the potential to reduce system capacity/performance. A single, general decision-support optimization model is formulated as a multi-stage stochastic program. The program seeks an optimal sequence of decisions over time based upon the realization of random events in each time stage. This dissertation addresses three problems to demonstrate the application of the proposed mathematical model in

different transportation environments with emphasis on system-level resilience: Airport Resilience Problem (ARP), Building Evacuation Design Problem (BEDP), and Travel Time Resilience in Roadways (TTR). These problems aim to measure system performance given the system's topological and operational characteristics and support operational decision-making, mitigation and preparedness planning, and post-event immediate response. Mathematical optimization techniques including, bi-level programming, nonlinear programming, stochastic programming and robust optimization, are employed in the formulation of each problem. Exact (or approximate) solution methodologies based on concepts of primal and dual decomposition (integer L-shaped decomposition, Generalized Benders decomposition, and progressive hedging), disjunctive optimization, scenario simulation, and piecewise linearization methods are presented. Numerical experiments were conducted on network representations of a United States rail-based intermodal container network, the LaGuardia Airport taxiway and runway pavement network, a single-story office building, and a small roadway network.

RESILIENCE OF TRANSPORTATION INFRASTRUCTURE SYSTEMS: QUANTIFICATION AND OPTIMIZATION

By

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Chapter 1: Introduction

1.1. Motivation and research objectives

Transportation systems provide a network of options to support the mobility of people and goods. They connect businesses and support supply chains and services. Moreover, they offer accessibility to vital resources for daily activities and in emergency circumstances. In this latter case, these systems play a key role in survivor evacuation, rescue operations, and community reconstruction and recovery. These systems are exposed to risk from a multiplicity of hazards, ranging from natural events and technological failures to intentional malicious acts. Disruptions in the operation of these systems can have cascading impacts within the system and on other interconnected critical lifelines. In addition to the effects of direct damage to the physical transportation infrastructure, indirect damage to, for example, the economy and social systems may result.

The frequency of disasters, whether natural or human-made, has increased to an unprecedented level in the last decade (Guha-Sapir et al. 2011). Likewise, the impacts of such events on transportation infrastructure systems have intensified due to increased system complexity and interdependency, and urbanization in coastal and other disaster-prone areas. Hurricane Sandy (2012), Hurricane Irene (2011), the Japanese Tsunami (2011) and subsequent nuclear meltdown, the Sichuan Earthquake in China (2008), the Christchurch earthquake (2011) in New Zealand, the Minneapolis I-35W bridge collapse (2007), and Hurricane Katrina (2005) are only a few examples of recent devastating events. Their impacts illustrates how susceptible transportation systems with embedded infrastructure are to such events. Damage caused by Hurricane Sandy to the New York City transportation system amounted to $7.5 billion (New York News 2012). Hurricane Irene affected more than 500 miles of highways, 2000 miles of roadways, 200 miles of railways, and 300 bridges in Vermont (Lunderville 2011). The collapse of the I-35W Bridge over the Mississippi River imposed over $0.4 million in costs to daily trips alone due to traffic rerouting (Zhu et al. 2011).

Transportation infrastructure systems are also a common target of terrorist attacks, such as 9/11 attacks and the bombings in London (2005), Madrid (2004), and Mumbai (2006). In addition to resulting physical damage, these events have long-term socio-economic and psychological impacts. Furthermore, they affect traveler decisions. Gordon et al. (2007) concluded a 6% reduction in passenger trips and noted a big shift from public transit services to private automobiles during a two-year period following the 9/11 attacks.

While many societies have come to rely on transportation systems, these systems are operating at or near design capacity. They are aging and are faced with greater risk of attack, whether natural, accidental or human-induced. Because these systems have become quite complex, interdependent and interconnected, the possibility that a disruptive event to any one system will cascade into an event involving multiple systems is significant and can result in widespread failure or difficult recovery. Sustained loss of one or more of these lifelines can have catastrophic impact on the well-being of a society. Consequently, governments and agencies that own or operate these systems are reviewing their investment policies with goals of expanding system capacity, reducing risk of attack, and reducing susceptibility of infrastructure to damage given possible disruptions or disaster events (U.S. Department of Homeland Security Homeland Security, 2009). In addition, as it is impractical to reinforce or harden these systems to all types of events, efficient options to support quick recovery of these systems from such events are being considered.

To evaluate investment options that can be taken to reduce risk of failure and increase a system's ability to rebound from an attack, one must be able to quantify the innate ability of the system to cope with attack and its ability to adapt through the use of available resources. Numerous performance measures have been proposed in the literature for such quantification. These measures include various specifications of system reliability, vulnerability, robustness and flexibility, which describe the behavior of systems and their performance variations under different situations. They aim to quantify how well a system is expected to perform given the possibility of potential future events that affect system capacity.

Various definitions of such performance measures have been introduced in the literature. These definitions, however, are sometimes intertwined and often inconsistent. Moreover, the majority are qualitative in nature. As a result, it is often unclear to the agencies responsible for maintaining, expanding and protecting critical societal lifelines which measure or set of measures should be considered in evaluating these systems or potential investment options.This dissertation provides a comprehensive survey of the literature on performance measurement for transportation infrastructure systems under possible disaster occurrence. It further develops a mathematical framework for conceptualizing, categorizing, and quantifying such system performance measures. A single mathematical decision problem is developed based on the introduced framework for quantifying these measures congruously and maximizing their values. This problem is formulated generally as a two-stage stochastic programming model to capture the uncertain nature of disasters and their consequences. The model seeks the optimum allocatation of resources to pre-event mitigation and preparedness and post-event response actions given the realization of a single disaster scenario od many possible scneairos.

This general framework is customized for application to a variety of transportation systems, incliding freight networks, airport taxiway and runway pavement networks, roadway networks, and building environments. The characteristics of each system and their operations are captured. Application on a set of real world based case studies offer insights into the various performance measures, their relationships, and the relative importance of preparedness and response actions.

In the next section, specific problem classes addressed in this dissertation are discussed in detail. The main contributions of this dissertation are synopsized in Section 1.3. Formal definitions, as well as detailed description of mathematical approaches, models, and solution methodologies, are given in Chapters 3 through 6.

1.2. Specific problems addressed

1.2.1. A mathematical framework for quantifying and optimizing protective actions for civil infrastructure systems

In Chapter 3 of this dissertation, a comprehensive framework is addressed for conceptualizing, categorizing, and quantifying system performance measures in the presence of uncertain events, component failure, or other disasters with the potential to reduce system performance. The framework clarifies the interrelationships between notions of coping capacity, preparedness, robustness, flexibility, recovery capacity and resilience, previously espoused as independent measures, and provides a single mathematical decision problem for quantifying these measures congruously and maximizing their values.

Required solution methodologies are described for use in evaluating system performance in terms of these measures. Resulting solutions can be exploited to determine an optimal allocation of limited resources to preparedness and response options. A numerical transportation-related example is provided to illustrate its application. Results of this application offer insights into these various performance measures, their relationships, and the relative importance of preparedness and response actions. More details on this framework, the mathematical structure of the decision problem and solution methodologies are given in Chapter 3.

1.2.2. Resilience of airport runway and taxiway pavement networks

In Chapter 4 of this dissertation, the problem of assessing and maximizing the resilience of an airport's runway and taxiway pavement network under multiple potential damage-meteorological scenarios is addressed. The problem is formulated as a stochastic integer program with recourse and an exact solution methodology based on integer L-shaped decomposition is proposed for its solution. The formulation seeks an optimal allocation of limited resources to response capabilities and preparedness actions that facilitate them. The overall aim is to quickly restore post-event take-off and landing capacities to pre-event operational levels taking into account operational, budgetary, time, space, and physical resource limitations. Details, such as aircraft size impacts, reductions in capacity due to joint take-off and landing maneuvers on common runways or bidirectional flows on taxiways, potential for outsourcing repair work, and multi-team response, is incorporated.

The capabilities and applicability of the solution approach is demonstrated on an illustrative case study. Potential benefits to airport operators are described. These include for example, the tool's utility in suggesting equipment to have at the ready, identifying the critical pavement system components, and information to aid in prioritizing future facility developments.

1.2.3. Stochastic Models for Emergency Shelter and Exit Design in Buildings under System Optimum and User Equilibrium Conditions

In Chapter 5 of this dissertation, a bi-level, two-stage, binary stochastic program with equilibrium constraints and three variants, are presented that support the planning and design of shelters and exits along with hallway fortification strategies and associated evacuation paths in buildings. At the upper-level of this model, decisions are made regarding exit design, hallway fortification and the location of shelters, along with their size and level of protection, with the objective of minimizing the expected maximum endured risk over all scenarios. At the lower-level, the choice of evacuation routes by the users, following the upper-level design decisions, is modeled as a user equilibrium problem, where each individual seeks to minimize his/her risk exposure. Variants of the model involve both stochastic programming and robust optimization concepts under both user equilibrium and system optimal conditions. A multi-hazard approach is utilized in which the performance of a plan is tested given various possible future emergency scenarios. Piecewise linearization of travel time functions and a disjunctive constraints transformation method that converts the single-level equivalent math program with complementarity constraints to a mixed integer program are employed to eliminate nonlinearities in the model. Integer L-shaped decomposition is adopted for solution of all four variants. These approaches are compared on a case study involving a single-story building.

1.2.4. Travel time resilience of roadway networks in the presence of non-recurring disruptions

In Chapter 6 of this dissertation, a bi-level, three-stage stochastic mathematical program with equilibrium constraints (SMPEC) is proposed for quantifying and optimizing travel time resilience in roadway networks under nonrecurring natural or human-induced disaster events. At the upper-level, a sequence of optimal decisions is taken over pre-event mitigation and preparedness and post-event response stages of the disaster management life cycle. Appropriate preparedness and response actions that aim to preserve or restore capacity to damaged roadways are considered. Assuming semi-adaptive user behavior exists shortly after the disaster and after the implementation of immediate response actions, the lower-level problem is formulated as a Partial User Equilibrium, where only affected users are likely to rethink their routing decisions. An exact Progressive Hedging algorithm is presented for solution of a single-level equivalent, linear approximation of the SMPEC. A recently proposed technique from the literature that uses Schur’s decomposition with SOS1 variables in creating a linear equivalent to complementarity constraints is employed. Similarly, recent advances in piecewise linearization are exploited in addressing nonseparable link travel time functions. The formulation and solution methodology are demonstrated on an illustrative example.

1.3. Contributions

The main contributions of this dissertation are enumerated next.

1) Completion of an extensive literature review that archives, syntheses, and categorizes approximately 200 journal articles, conference proceedings and technical reports based on a host of criteria, including qualitative/quantitative concepts, measure employed/defined, assessment or management strategy used, and proposed mathematical methodology. This provides a framework for considering this body of literature, as well as similarities and differences in their coverage, approach and utility.
2) Development of a conceptual and mathematical framework for protection of infrastructure systems generically devised to permit consideration of a variety of applications, including, for example, applications arising in transportation, power grid, telecommunication, supply chain, and water supply networks.
3) Application of variants of stochastic programming approaches, including two- and three-stage stochastic programs, as well as the concepts of robust optimization. These models can be used to measure and optimize performance in different transportation environments in the presence of uncertain events in which a sequence of optimal decisions are taken over time based upon the evolution of uncertainty over time stages.
4) Application and adaptation of cutting edge OR techniques, especially linearization techniques particular to bi-level stochastic programs with

complementarity constraints to address inherent nonlinearities and nonconvexities.

5) Presentation of exact solution methodologies to address the specific mathematical properties of the considered problem classes based on concepts of primal and dual decomposition methods, including integer L-shaped decomposition, and progressive hedging.
6) Design and completion of extensive numerical experiments to illustrate the concepts and application of proposed stochastic programs and solution methods for their intended applications.

Chapter 2: Disaster Research in Transportation Infrastructure Systems: A Comprehensive Review

2.1. Introduction

Transportation infrastructure systems provide a network of options to support the mobility of people and goods. They connect businesses and support supply chains and services. Moreover, they offer accessibility to vital resources for daily activities and in emergency circumstances. In this latter case, these systems play a key role in survivor evacuation, rescue operations, and community reconstruction and recovery. These systems are exposed to risk from a multiplicity of hazards, ranging from natural events and technological failures to intentional malicious acts. Disruptions in the operation of these systems can have cascading impacts within the system and on other interconnected critical lifelines. In addition to the effects of direct damage to the physical transportation infrastructure, indirect damage to, for example, the economy and social systems may result.

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An increasing awareness of these issues has led to a growing body of literature on the subject of transportation systems performance in disaster. A marked and continued growth in journal articles, both qualitative and quantitative, on this topic followed the 1995 Kobe earthquake (also noted by Chang and Nojima 2001). The articles range in content from conceptual frameworks and performance metrics to strategies for improving preparedness and reducing the duration of time required for recovery. This chapter aims to provide a comprehensive overview of that portion of this literature which emphasizes performance evaluation in the presence of physical damage resulting from hazard impact.

The contributions of this work include: (1) an archive and synthesis of recent literature on the studied topic; (2) analysis and organization of approximately 200 journal articles, conference proceedings and technical reports based on a host of criteria, including qualitative/quantitative concepts, measure employed/defined, assessment or management strategy used, and proposed mathematical methodology; and (3) a framework for considering this body of literature, similarities and differences in their coverage, approach and utility. An additional benefit of this review is that it provides newcomers to the field with the background needed to contribute to the area, and enables the identification of gaps in the literature for which additional study is warranted.

2.2. Study Scope

An enormous number of works address the performance of transportation systems, and hundreds of these works consider aspects associated with disaster events involving these systems. This subject is rather general. The scope of this chapter, thus, was carefully chosen to provide insights into that portion of the literature pertaining to transportation system performance given damage to the physical infrastructure.

Articles that provide strategies for preparing for or responding to disaster events (e.g. evacuation planning, resource allocation), address humanitarian relief logistics, or focus on the effects of disaster on human well-being or the environment (e.g. air quality or ecology) are not included in this review. Additionally, studies on the material properties of transportation system components from a structural engineering perspective, such as modeling bridge fragility and road pavement cracking/distortion, are excluded. While several pioneering works from the late 1990s are included, this review primarily includes works published since 2000.

A variety of terms have been used to label events precipitating disaster. These include: hazard, threat, perturbation, and disruption event. They are referred to herein as "hazards" and are considered herein to fall within one of three categories: (1) natural climatic/geological events (e.g. earthquake, hurricane, flood, and tsunami); (2) operational and technological failures due to hardware/software degradation/error and human error (e.g. major traffic accidents); and (3) intentional malicious acts, such as terrorist attacks. The term "disaster" is used to describe an event in which such a hazard has caused extensive physical damage; the event is non-recurring and likely unanticipated, and its location, impact area and severity, cannot be predicted with certainty.

2.3. Overview of terminology

A variety of performance metrics have been proposed in the disaster literature for evaluating and analyzing disaster impacts on transportation systems. Selection of an appropriate disaster measure for the particular application is an important first step in system analysis. These measures can be generally categorized as: risk, vulnerability, reliability, robustness, flexibility (also known as agility and adaptability), survivability, and resilience. Other performance metrics, such as total travel time, throughput, economic loss and connectivity, that may also provide input in quantification of some of these measures, are considered and categorized herein as alternative measures of effectiveness (MOEs). Because authors use these terms in a variety of ways, and also sometimes introduce new terminology for similar concepts or do not define their chosen terminology, this review includes those works using alternative terminology under the most relevant of these categories. Where an author uses a measure that might be categorized under an alternative heading, the default is to assign that work based on the terminology adopted by the author.

2.3.1. Risk

Risk is a concept used to characterize the threat of a disaster event in terms of its likelihood of occurrence and consequences. Thus, risk is typically measured as with respect to the probability of an event arising and its corresponding effects (e.g. Basoz and Kiremidjian 1996). Often their product is taken. These two risk components must be derived through detailed location-specific probability and impact (e.g. likelihood of structural damage of varying levels, reduction in services, and health or environmental concerns) estimation. In the context of transportation system performance in disaster, risk can be a good measure when considering engineering failures related to a specific component, such as the collapse of a bridge; it may be impractical for use in networks consisting of many components. Thus, alternative measures may be preferred (Taylor et al. 2006).

2.3.2. Vulnerability

Vulnerability, like risk, considers the potential consequences of a disaster event on system performance. It captures a system's weaknesses or susceptibility to threats related to operational performance (e.g. Berdica 2002, Jenelius et al. 2006). Unlike risk, however, the probability of the disaster event is not accounted for (Jenelius et al. 2006). That is, vulnerability studies recognize that it may be difficult to predict the likelihood of very rare events for many systems, and expectations that incorporate such low probability events may not be very illuminating (Taylor et al. 2006). The concept of vulnerability can be vague and is often described qualitatively.

2.3.3. Reliability

Reliability is typically defined as the probability that a network remains operative (often a function of connectivity) given the occurrence of a disaster or disruption event (e.g. Scaparra and Church 2008, Balakrishnan et al. 2009). Variants with utility for transportation systems have been introduced that capture effects of disruption on performance level. Such a reliability measure might be, for example, the probability of a system performing within a satisfactory level of service under a disruption event (Wakabayashi and Iida 1992). One can view reliability as the complement of vulnerability, where the former considers remaining functionality and the latter potential loss or degradation (Berdica 2002 and D'Este and Taylor 2001). Concepts of reliability are used extensively in assessing telecommunication networks, electric power grids, and other engineered systems, where failures can be recurrent, and thus, their probability of occurrence may be significant and predictable.

2.3.4. Robustness

Robustness measures the ability of a system to continue in operation and, thus, maintain some level of functionality, even when exposed to disruption. Like reliability, it is a measure of strength rather than loss and can be seen as a complement to vulnerability (Jenelius et al. 2006, Snelder et al. 2012). For many works in the literature, robustness has been synonymous with reliability. Where they are distinguished from one another, it is that reliability considers probability of meeting a given level-of-service; whereas, robustness assesses remaining functionality for a given event. It might be noted that offering a high degree of reliability often requires a robust system. Robustness concepts have been applied to engineered systems (Nagurney and Qiang 2007), including computer systems and telecommunications, for example. In the context of transportation systems, this concept was initially applied to measure network-level impacts of node or link removal (e.g. Chang and Nojima 2001, Sakakibara et al. 2004, Scott et al 2006, Nagurney and Qiang 2007).

2.3.5. Robustness

Another relevant concept is flexibility (also known as adaptability or agility). It captures the inherent capacity of a system to cope with uncertainty. This concept is primarily used in manufacturing systems, where for example multipurpose system elements or processes enable adaptation to new circumstances, e.g. pooling resources to allow the same capacity to be used for production of a variety of products (Morlok and Chang 2004). This concept has been applied in the transportation arena. For example, Morlok and Chang (2004) measure system flexibility in terms of the transport system's ability to continue to accommodate traffic with existing capacity under demand uncertainty. Chen and Kasikitwiwat (2011) and Tomlin (2006) discuss flexibility with respect to supply uncertainties, e.g. possible degradation in the functionality of facilities, or other network nodes or links. Application to supply chain disaster management involves a general definition of flexibility as the ability to adapt and adjust to supply changes through contingency planning in the aftermath of disruptions. Flexibility can be viewed as the opposite of robustness, capturing the ability of the system to absorb changes with negative impact as opposed to the ability to endure these changes without adaptation (Faturechi and Miller-Hooks 2013).

2.3.6. Survivability

Survivability is a measure of whether or not a network can continue to perform its intended function given damage to network components (Mead et al. 2000). Morlok and Chang (2004) describe survivability as a supply-oriented concept aimed at measuring the fraction of system demand that can be met post-disruption. A main application area for survivability measures has been telecommunication networks. These networks are often partitioned hierarchically, rendering some components more important than others. Additionally, arc traversal times are considered to be trivial in comparison to time spent waiting to pass through network nodes. Thus, extension of specific survivability measures developed for this industry to transportation systems requires adaptation (Abdel-Rahim et al. 2007 and Du and Peeta 2012). This measure may be comparable to robustness.

2.3.7. Robustness

Resilience was initially conceptualized and applied in the context of ecological systems (Holling 1973). It is generally defined as a system's ability to resist and absorb the impact of disruptions (Bruneau et al. 2003). It builds on the strengths or weaknesses measured by risk, vulnerability, reliability, robustness and survivability (i.e. resistance) and adaptability measures, while also encapsulating the benefits of the system's ability to adapt to post-disaster circumstances as in flexibility measures. Resilience measures, thus, account for possible interventions that can aid in returning system performance to nearly pre-disaster levels. They can quantify the potential benefits of pre-disaster mitigation actions aimed at increasing the system's ability to cope with disaster impact and post-disaster adaptive actions that aim to restore functionality.

2.3.8. Summary

In Table 2-1, the most-agreed upon interpretations of these measures discussed in this section are given. Fig. 2-1 provides a schematic of their boundaries and interactions.

Table 2-1 Common Definitions of Common Performance Metrics

Measure	General definition
Risk	Combination of probability of an event and its consequences in terms of system performance
Vulnerability	Susceptibility of the system to threats and incidents causing operational degradation
Reliability	Probability of a system performing its intended purpose adequately post-disaster
Robustness	Ability to withstand or absorb disturbances and remain intact when exposed to disruptions
Flexibility	Ability to adapt and adjust to changes through contingency planning in the aftermath of disruptions
Survivability	Ability to withstand sudden disturbances to functionality while meeting original demand
Resilience	Ability to resist, absorb and adapt to disruptions and return to normal functionality

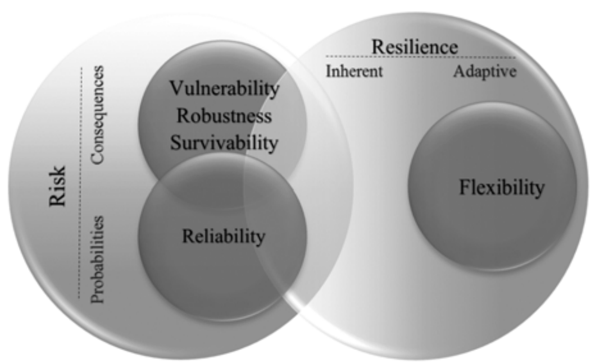


Figure 2-1 Disaster measures, their boundaries and interactions

2.4. Qualitative vs. quantitative approaches to assessing performance

The literature on disaster-related performance measurement can be categorized by whether qualitative descriptions are given or quantitative measures are defined. Such descriptions can provide insights into impact evaluation and management tactics. Quantitative measures, on the other hand, provide direct measurement that can be used to assess or predict disaster impact. Such measures can aid in the prioritization of mitigation, preparedness and adaptive actions.

Some quantitative measures have been implemented within software or other types of decision support tools. Table 2-2 provides an overview of the literature through this categorization approach, distinguishing those works in which mathematical models or quantification techniques are provided from those in which a tool employing such models or techniques is described. Mathematical models are further classified by whether they provide direct assessment or suggest decisions that can be used to alter system performance. Assessment includes component- and system-level performance, both of which allow for identification of critical system elements. The models that suggest decisions support management of these systems. Disaster management includes prioritization and optimization of pre- and post-disaster investment options with the aim of maximizing a system's coping capacity, reducing disaster losses, and/or restoring performance.

Table 2-2 Qualitative and quantitative publications in disaster research

Concept	Qualitative conceptualization	Quantitative approaches: Modeling – Assessment	Quantitative approaches: Modeling – Management	Quantitative approaches: Tool development
Risk	Basoz and Kiremidjian (1996), Cafiso et al. (2005), D'Andrea et al. (2005), Haimes et al. (2002), Homeland Security (2007)	Basoz and Kiremidjian (1996), Bensi et al. (2011), Chang et al. (2010), Chang et al. (2000), Chang and Nojima (2001), Cho et al. (2001), Dalziell and Nicholson (2001), Di Gangi and Luongo (2005), Gupta (2001), Ham et al. (2005), Kim et al. (2002), Kiremidjian et al. (2007), Murray-Tuite (2007, 2008, 2010), Na and Shinozuka (2009), Shiraki et al (2007), Stergiou and Kiremidjian (2010), Tatano and Tsuchiya (2008), Wang and Elhag (2007), Werner et al. (2000)	Chang et al. (2010), Kim et al. (2008), Murray-Tuite and Fei (2010), Shinozuka et al. (2003), Zhou et al. (2004)	Banerjee and Shinozuka (2009), Dalziell and Nicholson (2001), Eguchi et al (1997), Werner et al. (2008),
Vulnerability	Berdica (2002), Berle et al. (2011), D'Este and Taylor (2001)	Bell et al. (2008), Chen et al. (2007), D'Este and Taylor (2001), Ferber et al. (2007), Ibrahim et al. (2011), Jenelius et al. (2006), Knoop et al., (2012), Lownes et al. (2011), Luping and Dalin (2012), Luathep et al. (2011), Lu and Peng (2011), Murray-Tuite and Mahmassani (2004), Shimamoto et al. (2008), Sohn (2006), Sohn et al. (2003), Tampere et al. (2007), Taylor et al. (2006), Tu et al. (2012), Ukkusuri and Yushimito (2009), Yingfei et al. (2010)	Chang (2003), Lou and Zhang (2011), Mohaymany and Pirnazar (2007), Patidar et al. (2007), Viswanath and Peeta (2003)	Erath et al., (2008), Jenelius and Mattsson (2012), Taylor and Susilawati (2012),
Reliability	Bell (2000), Berdica (2002), Iida (1999), Nicholson and Du (1997), Nicholson (2003)	Al-Deek and Emam (2006), Andreas et al. (2008), Asakura (1999), Bell (2000), Bell and Iida (2001), Bell and Schmocker (2002), Chen et al. (2002), Chen et al. (2013), Chen and Eguchi (2003), Golroo et al. (2010), Iida (1999), Lam et al. (2008), Nicholson (2003), Sumalee and Watling (2008), Nojima (1999), Szeto (2011), Siu and Lo (2008), Wakabayashi and Iida (1992), Yin and Ieda (2001)	Augusti et al.(1998), Bin et al. (2009), Chootinan et al. (2005), Desai and Sen (2010), Dimitriou and Stathopoulos (2008), Golroo et al. (2010), Lo and Tung (2003), Lou and Zhang (2011), Sumalee and Kurauchi (2006), Poorzahedy and Bushehri (2005), Sanchez-Silva et al. (2005), Snyder and Daskin (2005), Park et al. (2007), Yin and Ieda (2002)	-

Table 2-2 (Continued)

Concept	Qualitative conceptualization	Quantitative approaches		
		Modeling		Tool development
		Assessment	Management	
Robustness	Berdica (2002), Nagurney and Qiang (2007), Snelder (2012)	Angeloudis and Fisk (2006), Derrible and Kennedy (2010), De-Los-Santos et al. (2012), Ip and Wang (2011), Moreira et al. (2009), Morohosi (2010), Nagurney and Qiang (2007,2009,2012), Snelder et al. (2012), Sakakibara et al. (2004), Sullivan et al. (2010), Scott et al. (2006)	Cappanera and Scaparra (2011), De-Los-Santos et al. (2012), Fan and Liu (2010), Faturechi and Miller-Hooks (under review), Huang et al. (2007), Liu et al. (2009), Laporte et al. (2010), Liberatore et al. (2011), Patriksson (2008), Santos et al. (2010), Scaparra and Church (2008,2012), Zhang and Levinson (2004)	-
Survivability	Abdel-Rahim et al. (2007), Mead et al. (2000)	Grubesic and Murray (2006), Matisziw and Murray (2009)	Abdel-Rahim et al. (2007), Chen et al. (2011), Du and Peeta (2012), Garg and Smith (2008), Peeta et al. (2010), Smith et al. (2007)	-
Flexibility	Chen and Kasikitwiwat (2011), Morlok and Chang (2004), Tomlin (2006)	Morlok and Chang (2004), Sun et al. (2006)	Faturechi and Miller-Hooks (2013)	-
Resilience	Bruneau et al. (2003), Caplice et al. (2008), Croope and McNeil (2011), Dorbritz (2011), Goodchild et al (2009), Mansouri et al. (2010), Ortiz et al (2008), Reggiani (2012), Ta et al. (2009)	Bekkem et al (2011), Berche et al. (2009), Cox et al (2011), Freckleton et al. (2012), Liu and Murray-Tuite (2008), Murray-Tuite (2006), Nguyen et al (2011), Omer et al. (2011), Vugrin et al. (2011), Zhang et al. (2009)	Chen and Miller-Hooks (2012), Faturechi et al. (under review), Faturechi and Miller-Hooks (2013), Losada et al. (2012), Miller-Hooks et al. (2012), Vugrin et al. (2010), Vugrin and Turnquist (2012)	Adams et al. (2012), Leu et al. (2010), Nair et al. (2010), Omer et al. (2011), Serulle et al. (2011)

2.5. Categorization by life-cycle phase

The disaster life-cycle is often described as having four phases: mitigation, preparedness, response, and recovery (e.g. Green 2002). The first two phases arise pre-disaster, when the disaster occurrence and its component- and system-level impacts can only be anticipated and actions can be developed for their mitigation. The latter two phases involve the implementation of post-disaster adaptive actions that aim to restore system performance to pre-disaster levels.

Mitigation efforts typically aim at reducing the probability of disaster occurrence or the level of its consequences. The aim of such efforts may be to reduce the probability of an attack (e.g. human-made) on the system or reduce the likelihood that an attack will cause a given level of damage (i.e. will have certain consequences). In the context of transportation systems, the primary mitigation strategies can be described as: (1) retrofitting system components, (2) expanding the system to include new links or nodes, (3) adding capacity to existing system elements, or (4) positioning resources for protective purposes. The concept of expansion as a mitigation strategy is fairly new, and its benefits are derived through added post-disaster residual capacities. Highway embankment, assignment of security teams, and bridge fortification, are some examples of mitigation strategies used to combat floods, terrorist attacks and earthquakes, respectively.

Preparedness strategies support quicker and more efficient response in a disaster's aftermath. Such strategies might include, for example, implementing awareness campaigns, training response teams, or pre-positioning equipment and/or other resources, such as fire extinguishers for firefighting, water pumps for use in floods, and salt spreaders for snow or ice handling.

Post-disaster emergency response includes short-term response actions in the aftermath of a disaster with the aim of restoring system performance. The first portion of this life-cycle phase is devoted to humanitarian relief operations, such as emergency rescue and medical service distribution (not covered in this study). This is followed by repair of damaged system components with the objective of restoring connectivity or increasing system throughput levels. Pavement crack repair, debris removal, and construction of temporary road mats, are some examples of response strategies.

Recovery, as the final phase of the disaster life-cycle, continues beyond emergency response, until actions to improve system performance are terminated. This phase may take months, even years, to accomplish; thus, requiring long-term planning. Short-term decisions taken in the response phase can impact the efficiency of the recovery phase (Baird 2010).

The reviewed literature is categorized by life-cycle phase and performance measure in Table 2-3.

Table 2-3 Summary of disaster management research based on life-cycle phases

Concept	References	Mitigation		Preparedness	Response	Recovery
		Retrofit	Expansion			
Risk	Chang et al. (2010), Kim et al. (2008), Shinozuka et al. (2003), Wang et al. (2008), Zhou et al. (2004)	✓				
Vulnerability	Mohaymany and Pirnazar (2007), Patidar et al. (2007), Viswanath and Peeta (2003)	✓				
	Lou and Zhang (2011)		✓			
Reliability	Augusti et al. (1998), Golroo et al. (2010), Poorzahedy and Bushehri (2005)	✓				
	Chootinan et al. (2005), Dimitriou and Stathopoulos (2008), Lo and Tung (2003), Lou and Zhang (2011), Park et al. (2007), Yin and Ieda (2002)		✓			
	Sanchez-Silva et al. (2005), Snyder and Daskin (2005)			✓		
	Desai and Sen (2010)	✓		✓		
	Bin et al. (2009), Sumalee and Kurauchi (2006)				✓	
Robustness	Cappanera and Scaparra (2011), Fan and Liu (2010), Liu et al. (2009) , Liberatore et al. (2011), Scaparra and Church (2008,2012)	✓				
	De-Los-Santos et al. (2012), Laporte et al. (2010), Patriksson (2008), Santos et al. (2010), Zhang and Levinson (2004)		✓			
	Huang et al. (2007)			✓		
	Faturechi and Miller-Hooks (2013)	✓	✓	✓		
Survivability	Du and Peeta (2012), Peeta et al. (2010)	✓				
	Abdel-Rahim et al. (2007), Chen et al. (2011), Garg and Smith (2008), Smith et al. (2007)		✓			
Flexibility	Faturechi and Miller-Hooks (2013)			✓	✓	
Resilience	Losada et al. (2012)	✓				
	Chen and Miller-Hooks (2012), Vugrin et al. (2010)				✓	
	Miller-Hooks et al. (2012)	✓		✓	✓	
	Faturechi et al. (under review)			✓	✓	
	Vugrin and Turnquist (2012)		✓	✓	✓	
	Faturechi and Miller-Hooks (2013)	✓	✓	✓	✓	
No specific concept	Barbarosoglu and Arda (2004), Chang (2003), Ferris and Ruszczynski (2000), Feng and Wang (2003), Karlaftis et al. (2007), Lambert and Patterson (2002), Lertworawanich (2012), Liu et al. (2008), Modarres and Zarei (2002), Yan and Shih (2009), Yan et al. (2012)				✓	
	Chen and Tzeng (2000), Mehlhorn (2009), Orabi et al. (2009), Sato and Ichii (1996)					✓

As illustrated in the histogram of Figure 2-2, reliability and robustness are common pre-disaster measures used in the literature, while most studies on post-disaster response and recovery do not involve any specific disaster measure. Furthermore, system resilience is the one measure chosen by the majority of studies to model both pre- and post-disaster actions simultaneously.

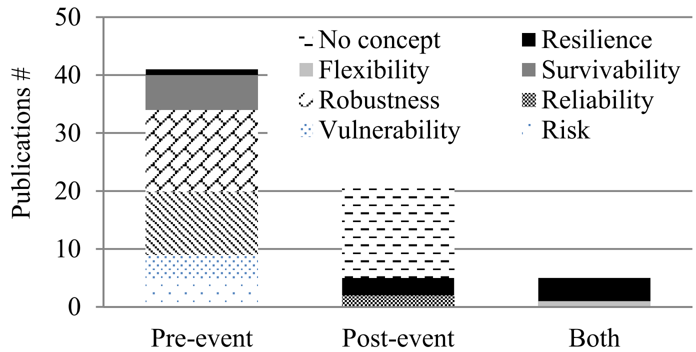


Figure 2-2 Number of disaster management publications in pre- and post-disaster phases

2.6. Categorization by MOEs

A variety of user- and supply-oriented MOEs have been developed in the literature. These differ depending on the transportation mode, such as intermodal ports, airports, highway networks and transit services, for which they were developed, and specific system objectives.

Two major categories of MOEs were identified: function and topological. Functional measures focus on serviceability of the transportation system as categorized by: travel time/distance, flow or throughput, and accessibility. Topological measures consider the transportation system as a pure network and characterize it based on concepts of graph theory. Measures such as connectivity, betweenness, and centrality fall into this category. These measures focus on the relative location of network nodes and links and their interconnections rather than operations.

In addition to functional and topological MOEs, a number of studies have been conducted on the estimation of economic losses due to disaster damage within transportation systems. However, it appears that no work in the literature presents or discusses quantitative economic measures for disaster management purposes.

Table 2-4 Categorization of publications based on the applied performance measure

Performance measure	Assessment	Management			
		Mitigation	preparedness	Response	Recovery
Topological measures	Andreas et al. (2008), Angeloudis and Fisk (2006), Asakura (1999), Bell and Iida (2001), Bell and Schmocker (2002), Berche et al. (2009), Chang et al. (2010), Chen and Eguchi (2003), Di Gangi and Luongo (2005), Derrible and Kennedy (2010), Ferber et al. (2007), Grubesic and Murray (2006), Iida (1999), Ip and Wang (2011), Matisziw and Murray (2009), Moreira et al. (2009), Murray-Tuite and Mahmassani (2004), Morohosi (2010), Sakakibara et al. (2004), Snelder et al. (2012), Sullivan et al. (2010), Scott et al. (2006), Tu et al. (2012), Wakabayashi and Iida (1992), Yingfei et al. (2010), Zhang et al. (2009)	Augusti et al. (1998), Balakrishnan et al. (2009), Du and Peeta (2012), Kim et al. (2008), Peeta et al. (2010)	-	Bin et al. (2009), Lertworawanich (2012)	-
Economic measures	Bensi et al. (2011), Cho et al. (2001), Dalziell and Nicholson (2001), Eguchi et al. (1997), Gupta (2001), Haimes et al. (2002), Ham et al. (2005), Kim et al. (2002), Na and Shinozuka (2009), Tatano and Tsuchiya (2008), Werner et al. (2000,2008),	-	-	-	-

Table 2-4 (Continued)

Performance measure		Assessment	Management: Mitigation	Management: preparedness	Management: Response	Management: Recovery
Functional measures	Travel time/distance	Basoz and Kiremidjian (1996), Bell (2000), Bell et al. (2008), Chang et al. (2000), Chang et al. (2010), Dalziell and Nicholson (2001), De-Los-Santos et al. (2012), Freckleton et al. (2012), Golroo et al. (2010), Ibrahim et al. (2011), Israeli and Wood (2002), Jenelius et al. (2006), Jenelius and Mattsson (2012), Kiremidjian et al. (2007), Knoop et al., (2012), Lam et al. (2008), Lownes et al. (2011), Lo and Tung (2003), Murray-Tuite (2006), Morohosi (2010), Nagurney and Qiang (2007,2009,2012), Omer et al. (2011), Stergiou and Kiremidjian (2010), Sumalee and Watling (2008), Suarez et al. (2005), Shimamoto et al. (2008), Shiraki et al (2007), Szeto (2011), Siu and Lo (2008), Ukkusuri and Yushimito (2009), Werner et al. (2000), Yin and Ieda (2001), Zhang et al. (2009)	Al-Deek and Emam (2006), Cappanera and Scaparra (2011), De-Los-Santos et al. (2012), Dimitriou and Stathopoulos (2008), Fan and Liu (2010), Golroo et al. (2010), Ieda (2002), Kim et al. (2008), Laporte et al. (2010), Losada et al. (2012), Liberatore et al. (2011), Liu et al. (2009), Lou and Zhang (2011), Lo and Tung (2003), Murray-Tuite and Fei (2010), Poorzahedy and Bushehri (2005), Scaparra and Church (2008, 2012), Shinozuka et al. (2003), Yin and Park et al. (2007), Zhang and Levinson (2004), Zhou et al. (2004),	-	Ferris and Ruszczynski (2000), Feng and Wang (2003), Lambert and Patterson (2002), Liu et al. (2008), Vugrin et al. (2010), Yan and Shih (2009)	Chen and Tzeng (2000), Orabi et al.(2009), Sato and Ichii (1996)
	Throughput/capacity	Adams et al. (2012), Bekkem et al (2011), Chang et al. (2010), Chen et al. (2002), Caplice et al. (2008), Chen et al. (2013), Cox et al (2011), Liu and Murray-Tuite (2008), Luping and Dalin (2012), Morlok and Chang (2004), Murray-Tuite (2006, 2010), Na and Shinozuka (2009), Nojima (1999), Sun et al. (2006), Sohn et al. (2003), Tampere et al. (2007), Vugrin et al. (2011)	Chen et al. (2011), Chootinan et al. (2005), Desai and Sen (2010), Faturechi and Miller-Hooks (2013), Garg and Smith (2008), Kim et al. (2008), Miller-Hooks et al. (2012),Smith et al. (2007)	Desai and Sen (2010), Faturechi et al. (under review), Miller-Hooks et al. (2012), Vugrin and Turnquist (2012)	Chen and Miller-Hooks (2012), Faturechi and Miller-Hooks (2013), Faturechi et al. (under review), Karlaftis et al. (2007), Miller-Hooks et al. (2012), Sumalee and Kurauchi (2006), Vugrin and Turnquist (2012), Yan et al. (2012)	-
	Accessibility	Chen et al. (2007), Chang and Nojima (2001), Chang et al. (2010), D'Este and Taylor (2001), Luathep et al. (2011), Lu and Peng (2011), Sohn (2006), Taylor and Susilawati (2012), Taylor et al. (2006)	Mohaymany and Pirnazar (2007), Santos et al. (2010), Viswanath and Peeta (2003)	Modarres and Zarei (2002), Sanchez-Silva et al. (2005)	Chang (2003)	Mehlhorn (2009)

Table 2-4 summarizes the literature by these three categories of MOEs: functional, topological and economic. The histograms in Figures 2-3 and 2-4 provide a graphical representation of the number of publications that falling under these categories. The figures indicate that travel time is the most utilized MOE. In the context of recovery, it is the predominant measure. Topological measures have been applied primarily in mitigation and response studies.

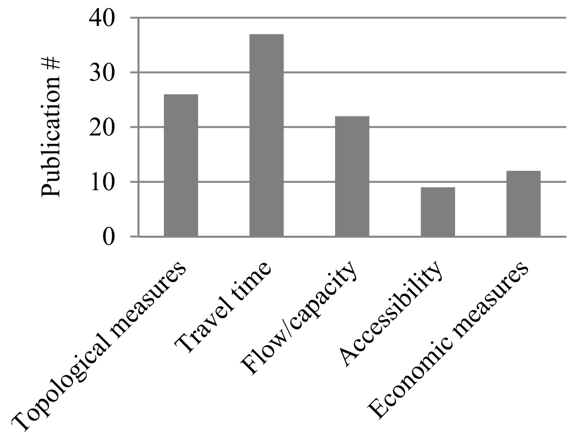


Figure 2-3 Number of disaster assessment publications on each MOE

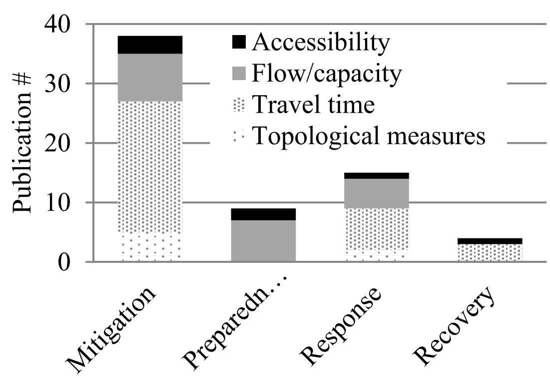


Figure 2-4 Number of disaster management publications on each MOE and life-cycle phase

2.7. Categorization by uncertainty modeling technique

The geographic location, severity and other impacts of a disaster event can at best be known *a priori* with uncertainty. Several different approaches have been applied within this literature for modeling possible disasters and their consequences. Such models are employed in providing input for system optimization and analysis. These approaches can be generally categorized as falling under scenario, simulation, probability distribution and worst-case performance-based techniques as given in Table 2-5. This table also includes those works that study a single historical disaster event.

Scenario-based techniques generate one or more hypothetical disaster scenarios; the probability of the scenario's occurrence is not regarded. Applications of these techniques generally consider a small set of component-level scenarios, e.g. failure of a road segment or a bridge. Before and after analysis are often conducted for comparison. Techniques that include targeted and coordinated attack scenarios aiming at the most important system components also fall in this category. Simulation techniques generate a wide range of scenarios for consideration. The scenarios are generated in proportion to the disruption or damage occurrence probabilities at the component-level. A distribution of system performance level over all considered scenarios can be generated. Other techniques that employ disruption or damage probability occurrences might use the probability distribution functions directly. Finally, optimization and game-theoretic modeling approaches, e.g. interdiction models, can be used to identify a worst-case performance that might results from damage to the system, where the damage may be given in terms of, for example, a number of link failures.

Table 2-5 Categorizing by disaster event modeling technique

Uncertainty modeling techniques	References
Scenario	Al-Deek and Emam (2006), Basoz and Kiremidjian (1996), Bell et al. (2008), Chang et al. (2000), De-Los-Santos et al. (2012), Fan and Liu (2010), Faturechi et al. (under review), Ferber et al. (2007), Freckleton et al. (2012), Feng and Wang (2003), Golroo et al. (2010), Gupta (2001), Ham et al. (2005), Ip and Wang (2011), Jenelius et al. (2006), Kim et al. (2002), Kiremidjian et al. (2007), Knoop et al., (2012), Liu et al. (2009), Liu and Murray-Tuite (2008), Luathep et al. (2011), Luping and Dalin (2012), Lu and Peng (2011), Murray-Tuite (2006, 2007, 2010), Nagurney and Qiang (2007,2009,2012), Nguyen et al (2011), Omer et al. (2011), Peeta et al. (2010), Shinozuka et al. (2003), Sohn et al. (2003), Shimamoto et al. (2008), Sumalee and Watling (2008), Sun et al. (2006), Sullivan et al. (2010), Scott et al. (2006), Tatano and Tsuchiya (2008), Taylor et al. (2006), Tu et al. (2012), Ukkusuri and Yushimito (2009), Vugrin et al. (2010, 2011), Vugrin and Turnquist (2012)
Simulation	Bell and Schmocker (2002), Bensi et al. (2011), Berche et al. (2009), Chang et al. (2010), Chang (2003), Chen et al. (2002), Chen and Miller-Hooks (2012), Cho et al. (2001), Dalziell and Nicholson (2001), Dimitriou and Stathopoulos (2008), Du and Peeta (2012), Faturechi and Miller Hooks (2013), Garg and Smith (2008), Kim et al. (2008), Miller Hooks et al. (2012), Morohosi (2010), Murray-Tuite (2008), Murray-Tuite and Fei (2010), Na and Shinozuka (2009), Nair et al. (2012), Nojima (1999), Patriksson (2008), Stergiou and Kiremidjian (2010), Shiraki et al (2007), Sumalee and Kurauchi (2006), Tampere et al. (2007), Werner et al. (2000), Zhou et al. (2004), Zhang and Levinson (2004)
Probability distribution	Andreas et al. (2008), Angeloudis and Fisk (2006), Asakura (1999), Augusti et al.(1998), Bin et al. (2009), Chen et al. (2007), Chootinan et al. (2005), Derrible and Kennedy (2010), Desai and Sen (2010), D'Este and Taylor (2001), Iida (1999), Lam et al. (2008), Moreira et al. (2009), Nicholson (2003), Park et al. (2007), Poorzahedy and Bushehri (2005), Sakakibara et al. (2004), Sanchez-Silva et al. (2005), Siu and Lo (2008), Wakabayashi and Iida (1992), Yin and Ieda (2001, 2002)
Worst-case performance	Bell (2000), Bell et al. (2008), Bell and Iida (2001), Chen et al. (2011), Cappanera and Scaparra (2011), Grubesic and Murray (2006), Huang et al. (2007), Ibrahim et al. (2011), Jenelius and Mattsson (2012), Laporte et al. (2010), Liberatore et al. (2011), Lim and Smith (2007), Lou and Zhang (2011), Losada et al. (2012), Lownes et al. (2011), Lo and Tung (2003), Matisziw and Murray (2009), Murray-Tuite and Mahmassani (2004), Scaparra and Church (2008, 2012), Smith et al. (2007), Snyder and Daskin (2005), Szeto (2011), Yan and Shih (2009), Yates and Lakshmanan (2011)
Historical scenario	Bekkem et al (2011), Chang (2000), Chang and Nojima (2001), Cox et al. (2011), Zhang et al. (2009)

2.8. Categorization by methodology

Mathematical models of system performance, for either assessment or management purposes, proposed in the literature can be considered as analytical, simulation, or optimization models. Those that address assessment are described in Table 2-6, while others addressing management are given in Table 2-7.

Analytical methods have been used to analyze potential failure states and risk classification based on disaster probabilities and consequences through different forms of logical structures, e.g. risk matrix, Event Tree Analysis (ETA), Fault Tree Analysis (FTA), and Failure Mode and Effects Analysis (FMEA). In disaster management, analytical models, specifically Analytical Hierarchy Process (AHP), have been applied for evaluating, ranking and prioritizing decision options through concepts of utility theory. These methods are not efficient for large-scale applications with a large number of possible failure states and candidate investment options (Wang et al. 2008, Murray-Tuite 2008).

Simulation methods, such as Monte Carlo simulation, are employed to generate a large sample of scenarios, each with a randomly selected damage state and probability of occurrence. These methods broadly allow generation of different combinations of degradation in the links or nodes. Simulation methods are also employed to evaluate the effectiveness of investment options by comparing system performance before and after expenditures are made. Such evaluation is made separately for an individual scenario; thus, related decisions may be suboptimal under other arising scenarios.

Table 2-6 Assessment methodologies

Methods		References	Description
Analytical methods	Risk matrix	Basoz and Kiremidjian (1996), FAA (2007)	• Ranking risk of system components with respect to disaster probability and consequences, from low to extreme risk
	ETA/FTA	Al-Deek and Emam (2006), Murray-Tuite (2007,2008)	• Representing probable states of system components using logical structures in the form of a tree
	FMEA	Bekkem et al. (2011), Caplice et al. (2008)	• Analyzing potential failure states and classifying risk
	Fuzzy inference approach	Freckleton et al. (2012), Serulle et al. (2011), Wang and Elhag (2007)	• Assessing vulnerability using linguistic terms such as High, Medium, and Low rather than precise numerical values
	Input-output analysis	Cho et al. (2001), Gupta (2001), Ham et al. (2005), Kim et al. (2002), Sohn et al. (2003), Tatano and Tsuchiya (2008)	• Modeling system losses, mostly economic, with respect component interconnections
	Bayesian analysis	Bensi et al. (2011), Murray-Tuite (2010)	• Real-time assessing of post-disaster system performance through evolving information
Simulation		Chen et al. (2002), Chen et al. (2013), Dalziell and Nicholson (2001), Kiremidjian et al. (2007), Knoop et al. (2012), Liu and Murray-Tuite (2008), Morohosi (2010), Murray-Tuite (2006), Na and Shinozuka (2009), Nojima (1999), Omer et al. (2011), Shinozuka et al. (2003), Shiraki et al (2007), Snelder et al. (2012), Suarez et al. (2005), Sumalee and Watling (2008), Tampere et al. (2007), Stergiou and Kiremidjian (2010), Vugrin et al. (2011), Werner et al. (2000)	• Generating a large number of disruption scenarios, useful for capturing failure dependencies of system components

Table 2-6 (Continued)

Methods		References	Description
Deterministic optimization	Graph-theoretic models	Abdel-Rahim et al. (2007), Andreas et al. (2008), Angeloudis and Fisk (2006), De-Los-Santos et al. (2012), Derrible and Kennedy (2010), Ferber et al. (2007), Ip and Wang (2011), Jenelius and Mattsson (2012), Moreira et al. (2009), Nagurney and Qiang (2007,2009,2012), Sakakibara et al. (2004), Scott et al. (2006), Shimamoto et al. (2008), Sullivan et al. (2010), Taylor et al. (2006), Tu et al. (2012), Yingfei et al. (2010), Wakabayashi and Iida (1992)	• Determining most critical nodes/links using graph theory concepts (e.g. connectivity); scenario-based, but no event probabilities included
	Game-theoretic models	Israeli and Wood (2002), Murray et al. (2007), Matisziw and Murray (2009), Ukkusuri and Yushimito (2009)	• Sequentially seeking to maximize and minimize transportation costs using a two-player game between a leader and follower for identifying worst-case performance as in interdiction problems; no event probabilities included
Stochastic optimization	Game-theoretic models	Bell (2000), Bell et al. (2008), Grubesic and Murray (2006), Ibrahim et al. (2011), Lownes et al. (2011), Murray-Tuite and Mahmassani (2004), Szeto (2011), Murray-Tuite and Fei (2010), Yates and Lakshmanan (2011)	• Incorporating in the game the uncertain characteristics of the transportation network due to disasters, where the leader seeks to maximize the expectation of transportation costs
	Markov chain models	Bell and Schmocker (2002), Nguyen et al. (2011)	• Modeling a set of failure states assuming Markovian transitions between states
	Utility-theoretic models	Asakura (1999), Chen et al. (2007), Lam et al. (2008), Luathep et al. (2011), Siu and Lo (2008), Sun et al. (2006), Yin and Ieda (2001)	• Using concepts of random utility theory to model stochastic user route choice under disruptions (Stochastic User Equilibrium)

Table 2-7 Management methodologies

Methods		References	Description
AHP		Modarres and Zarei (2002), Patidar et al. (2007), Wang et al. (2008)	• Prioritizing alternatives based on concepts of utility theory
Simulation		Chang (2003), Chen and Tzeng (2000), Sato and Ichii (1996), Zhang and Levinson (2004), Zhou et a. (2004)	• Evaluating management options under a large number of scenarios
Deterministic optimization		Feng and Wang (2003), Golroo et al. (2010), Karlaftis et al. (2007), Lambert and Patterson (2002), Lertworawanich (2012), Mehlhorn (2009), Mohaymany and Pirnazar (2007), Orabi et al. (2009), Viswanath and Peeta (2003), Vugrin et al. (2010), Yan and Shih (2009), Yan et al. (2012)	• Optimally selecting alternatives, e.g. resource allocation and reconstruction scheduling, regardless of event probabilities
Stochastic optimization	Game-theoretic models	Cappanera and Scaparra (2011), Lakshmanan (2011), Laporte et al. (2010), Lou and Zhang (2011), Losada et al. (2012), Liberatore et al. (2011), Scaparra and Church (2008, 2012), Smith et al. (2007), Yates and Snyder and Daskin (2005)	• Optimally selecting design alternatives under worst-case scenario through use of a multi-level defender-attacker game, where the defender makes decisions on network design in the upper–level and the attacker responds to these decisions in the lower-level
	Reliability-based constrained models	Bin et al. (2009), Chootinan et al. (2005), Desai and Sen (2010), Dimitriou and Stathopoulos (2008), Lo and Tung (2003), Park et al. (2007), Poorzahedy and Bushehri (2005), Santos et al. (2010), Sanchez-Silva et al. (2005), Sumalee and Kurauchi (2006), Yin and Ieda (2002)	• Optimally selecting design alternatives using stochastic network design with reliability requirements, e.g. chance constrained modeling
	Multi-stage stochastic programming	Barbarosoglu and Arda (2004), Chang et al. (2010), Chen et al. (2011), Chen and Miller-Hooks (2012), Du and Peeta (2012), Fan and Liu (2010), Faturechi et al. (in review), Faturechi and Miller-Hooks (2013), Ferris and Ruszczynski (2000), Garg and Smith (2008), Kim et al. (2008), Liu et al. (2008), Liu et al. (2009), Miller-Hooks et al. (2012), Nair et al. (2010), Peeta et al. (2010), Vugrin and Turnquist (2012)	• Optimizing sequence of alternative selection over time given realization of uncertain problem elements in each time stage
	Robust optimization	Huang et al. (2007), Laporte et al. (2010), Patriksson (2008)	• Optimally selecting alternatives to guarantee system performance under worst-case scenario; generating conservative and expensive solutions

2.9. Conclusions and insights

In this chapter, a comprehensive review of the literature addressing transportation system performance measurement given potential future disaster events is provided. Related publications were identified and categorized from a variety of perspectives. This categorization provides clarity through direct comparison of similarities, differences, intersections and interactions, permitting a deeper understanding of the topic. The review also aids in the identification of research challenges and gaps to be addressed in the future.

Although the literature was scoured the for all transportation environments, the vast majority of the scholarly literature related to disaster performance measures and transportation has focused on surface transportation as is reflected in this literature review. The review reveals that nearly 70% of publications on this topic reported in this literature address the assessment of the transportation system's ability to cope with disaster consequences. Publications including strategies for managing these systems in disaster are fewer in number, but growing. While decision-makers can benefit from techniques that consider interdependency of decisions in different stages of the disaster life cycle and multiple disaster scenarios, more than 90% of disaster management publications reviewed herein address only one component of the life-cycle. Although qualitative works of relevance were reviewed, much of the analysis provided herein focuses on quantitative efforts. Additional effort to categorize the qualitative studies on disaster assessment may be useful.

An uptick in papers explicitly considering uncertainty in future conditions can be noted from the review. More generally, an increase in articles that incorporate complexities of the real-world, including dependencies that contribute to system-level failure, is noticeable. In that vein, an increase can be noticed in the percentage of articles that consider system- rather than component-level performance. To consider these complexities, simulation is often required. Improved computational capabilities in recent years has also made sensitivity analysis possible on a larger scale, as evidenced by the increasing number of articles employing such approaches.

Chapter 3: A Mathematical Framework for Quantifying and Optimizing Protective Actions for Civil Infrastructure Systems

3.1. Introduction

All societies depend on a system of infrastructure for survival. The most advanced depend on this infrastructure to support a wide range of human activities. These infrastructure systems, often described in terms of the sectors of society that they affect, such as agriculture, finance, transportation, energy, water, healthcare, communications and defense, are crucial for public health, safety, security and economies.

While many societies have come to rely on these infrastructure systems, these systems are operating at or near design capacity. They are aging and are faced with greater risk of attack, whether natural, accidental or human-induced. Because these systems have become quite complex, interdependent and interconnected, the possibility that a disruptive event to any one system will cascade into an event involving multiple systems is significant and can result in widespread failure or difficult recovery. Sustained loss of one or more of these lifelines can have catastrophic impact on the well-being of a society. Consequently, governments and agencies that own or operate these systems are reviewing their investment policies with goals of expanding system capacity, reducing risk of attack, and reducing susceptibility of infrastructure to damage given possible disruptions or disaster events (U.S. Department of Homeland Security Homeland Security 2009). In addition, as it is impractical to reinforce or harden these systems to all types of events, efficient options to support quick recovery of these systems from such events are being considered.

To evaluate investment options that can be taken to reduce risk of failure and increase a system's ability to rebound from an attack, one must be able to quantify the innate ability of the system to cope with attack and its ability to adapt through the use of available resources. Numerous performance measures have been proposed in the literature for such quantification. These measures include various specifications of system reliability, vulnerability, robustness and flexibility, which describe the behavior of systems and their performance variations under different situations. They aim to quantify how well a system is expected to perform given the possibility of potential future events that affect system capacity.

Various definitions of such performance measures have been introduced in the literature. These definitions, however, are sometimes intertwined and often inconsistent. Moreover, the majority are qualitative in nature. As a result, it is often unclear to the agencies responsible for maintaining, expanding and protecting critical societal lifelines which measure or set of measures should be considered in evaluating these systems or potential investment options.

This chapter provides a comprehensive framework for conceptualizing, categorizing, and quantifying system performance measures, previously espoused as independent measures, in the presence of uncertain events, component failure, or other disruptions/disasters with the potential to reduce system capacity or performance from pre-event levels. The framework is structured from a supply-oriented perspective and assumes a constant demand for system capacity. It builds on concepts involving a system's innate ability to resist and recover from the negative consequences of events, and classes of mitigation and contingency actions designed to diminish damage impact.

This framework clarifies the intersection and overlap between notions of inherent characteristics of the system (coping capacity), preparedness, robustness, flexibility (or adaptability/agility), recovery capacity and resilience, and provides a common approach for their quantification. Building on the framework, a single decision problem is proposed that can be used to quantify these measures and determine optimal investment strategies so as to maximize their values. The formulation of the decision problem is generically devised to permit consideration of a variety of applications, including, for example, applications arising in power grid, transportation, telecommunication, supply chain, and water supply networks. A numerical example is provided to illustrate its application.

3.2. Related System Performance Measures In the Literature

Several system performance measures for assessing the coping capacity of a network that is subject to disaster or disruption have been proposed in the literature. These measures are applied within a variety of arenas ranging from transportation, water and other civil infrastructure systems to computer and supply chain networks. Key measures include vulnerability, reliability, robustness, flexibility (adaptability/agility) and resilience. No attempt is made herein to review all literature that discusses such measures. Rather, an overview of these measures with examples from the literature is given.

The most widely used of these performance concepts is that of vulnerability. Vulnerability typically expresses some notion of how susceptible a system is to malfunction or performance degradation in the event of an attack, natural or otherwise (Berdica 2002). Vulnerability relates directly to concepts of risk, which weight the susceptibility to performance degradation by the probability of attack. Because the concept of susceptibility can have many interpretations, exact definitions of vulnerability vary widely. Additional concepts, such as reliability, have been introduced to address this lack of specificity. Definitions of reliability, while varying, have in common that they aim to quantify the probability that the system will continue to function given a disruption event (Iida 1999) or measure system performance given a disruption (e.g. Berdica 2002).

Another related concept is that of system robustness, typically defined as the ability of a system to resist changes to its physical structure in response to a hazard event (e.g. Nagurney and Qiang 2007, Immers et al. 2004). Flexibility captures how a system adapts to significant internally- and externally-induced changes (Goetz and Szyliowicz 1997). Most of the network flexibility literature focuses on demand changes (e.g. Morlok and Chang 2004, and Chen and Kasikitwiwat 2011).

A number of works (e.g. Ukkusuri at al. (2007)) study the impact of system capacity expansion on system robustness and reliability. Others (e.g. Liu et al. 2009, and Peeta et al. 2010) consider retrofit actions that can be taken to reinforce existing infrastructure and improve system robustness. Numerous works (e.g. Huang et al. (2006) and Kondaveti and Ganz (2009)) develop techniques to support emergency response to disasters, but few works address planning for recovery efforts aimed at post-event restoration of system performance.

The concept of resilience has been introduced to measure not only the network's ability to absorb externally induced changes as in vulnerability, reliability and robustness measures, but also the network's ability to adapt to post-event circumstances, which can be likened to flexibility. This notion of resilience was initially conceptualized by ecologists in relation to stability of ecological systems in the presence of disruptions due to natural or anthropogenic causes and their ability to bounce back to a state of equilibrium (Holling 1973). It is discussed also in (Rose 2004) in the context of economic systems. Bruneau et al. (2003) define community resilience as the ability of a community to mitigate the effects of hazards and recover system performance so as to minimize life and economic loss. Cutter et al. (2008) discuss community resilience in terms of inherent and adaptive qualities. Additional works describe performance measures similar in concept to vulnerability or robustness under the name of resilience (e.g. Berche et al. 2009, Gutfraind 2010, Bekkem et al. 2011, Ip and Wang 2011, and Serulle et al. 2011).

Concepts of vulnerability, reliability, robustness and flexibility, or related concepts under headings of redundancy and adaptability, have been coupled to form a variety of resilience notions. Some of these notions involve pre- or post-event actions, such as preparedness, or actions taken in advance to improve resource availability, post-event recovery action implementation times and, ultimately, recovery time, to enhance resilience levels (Bruneau et al. 2003, Sheffi 2005, Murray-Tuite 2006, Caplice et al. 2008, McDaniels et al. 2008, Ortiz et al. 2008, Ta et al. 2009, and Cox et al. 2011). To illustrate, Bruneau et al. (2003) provide a qualitative measure of resilience whose components are robustness, redundancy, resourcefulness, and rapidity of response to disruption. Recently, Cox et al. (2011) adapted concepts from ecological system resilience to study passenger transportation systems. They discuss the potential role of vulnerability (defined in terms of robustness), wealth (given by system redundancy), and flexibility.

While numerous works discuss the concept of resilience, few provide the necessary methodology for its quantification. Murray-Tuite (2006) proposed quantitative measures for transportation system adaptability, safety, mobility and recovery, and applied a simulation-based method for their computation. Adams et al. (2010) applied the resilience framework of Caplice et al. (2008) to assess the resilience of ten high-risk segments along a U.S. interstate highway given knowledge of past events and their consequences. Nguyen et al. (2011) proposed four general mathematical formulations for the quantification of four criteria in the context of resilience of transportation networks: functionality degradation, recovery time, recovery speed and flexibility of the system.

A quantitative resilience measure for intermodal freight transport systems that seeks the maximum post-event expected fraction of demand that can be met in the aftermath of disruption was introduced by Chen and Miller-Hooks (2012). This measure incorporates both the innate coping capacity and effects of short-term adaptive actions on mitigating negative effects that can be taken post-event. They proposed a stochastic programming formulation of the problem and exact solution technique based on Benders decomposition, column generation and Monte Carlo simulation. This resilience concept was applied in Nair et al. (2010) to improve security at nodal facilities within intermodal freight networks. Miller-Hooks et al. (2012) extended this concept to include pre-event preparedness actions and investigated potential synergies between pre- and post-event investments to improve system resilience. They employ a two-stage stochastic program and propose an integer L-shaped method for its solution.

A variety of concepts have been proposed to address system performance under disruption from different perspectives. These concepts are often intertwined and the same term can be used with different definitions. The authors know of no prior work that has sought to provide a common framework with guidelines for creating consistent definitions of measures designed for assessing system performance under disaster/disruption as well as supportive roles of different classes of actions. This work seeks to fill this gap by defining important elements of infrastructure protection, positioning these elements within a single framework, showing how these elements can be combined to define the various performance measures, and clarifying connections between measures accordingly.

3.3. Framework for Infrastructure Performance Management

3.3.1. Infrastructure Protection Framework (IPF)

A single framework for understanding the various system performance measures discussed in previous sections is provided. This framework builds on concepts used in describing a system's innate capacity to endure natural and human-made disruptions and considering pre- and post-event actions to improve the system's performance. The former includes *coping capacity* characteristics (including ability to withstand stress, i.e. resistance, and/or excess in terms of redundancies and underutilized capacity), and the latter includes *retrofit*, *expansion, resource availability* and *response* activities that can be undertaken to mitigate the impact of the disaster event and increase inherent system qualities of resistance and excess. Together, these form a framework, referred to herein as the Infrastructure Protection Framework (IPF), and depicted in Figure 3-1.

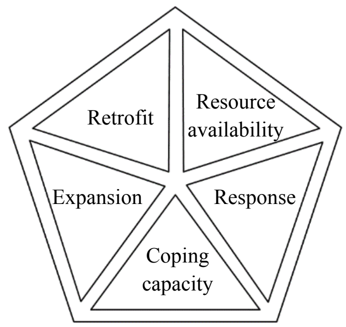


Figure 3-1 Infrastructure Protection Framework (IPF)

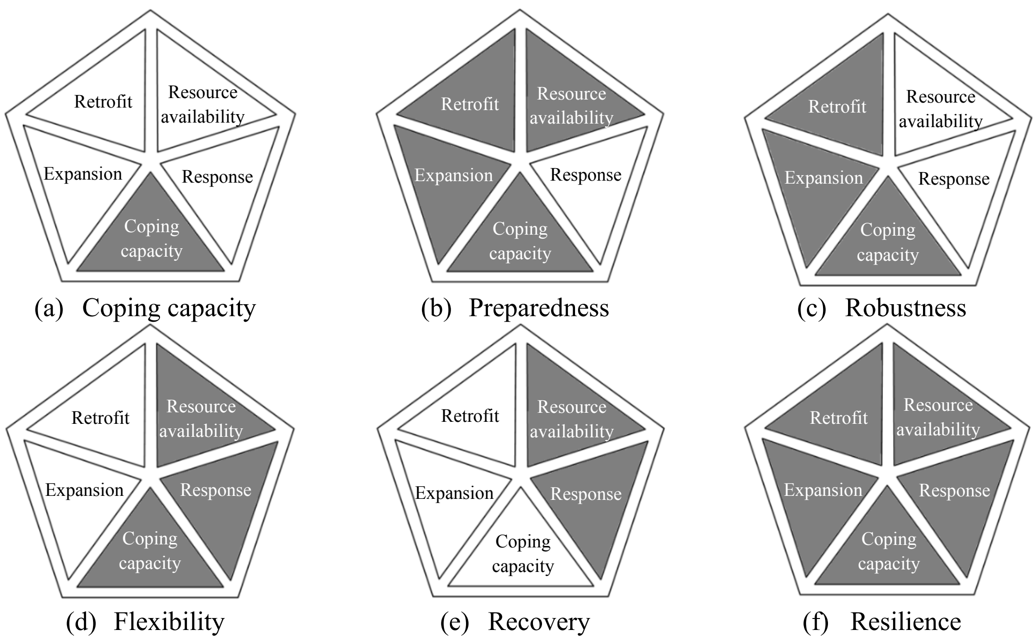


Figure 3-2 System Performance Measures Defined on IPF

Concepts of coping capacity, preparedness, flexibility, robustness, recovery capability and resilience are described in terms of elements of this framework as shown in Figure 3-2. These relationships are further illustrated in Table 3-1.

Figure 2 serves as a tool for understanding system performance measures designed for evaluating a system's disaster readiness, along with their interactions and differences. It can aid in assimilating relevant literature and choosing an appropriate measure for an application.

Table 3-1 Description of IPF components

Component	Description	Contributing factor in:	Examples
Coping capacity	Innate capability to resist disaster event through material strength and maintain functionality without intervention as well as built-in excess capacity and redundancies permitting post-disaster adaptation	Coping capacity, preparedness, robustness, flexibility, resilience	-
Expansion	Pre-event action to enhance network performance by increasing connectivity (e.g. adding redundancy) or capacity; aimed at reducing effects of disaster event by reducing marginal impact of loss; benefits for both pre-event and post-event performance.	Preparedness, robustness, resilience	Addition to roadway capacity
Retrofit	Pre-event actions to reinforce or harden system elements, diminishing likelihood or level of damage due to disaster event impact; does not affect pre-event system performance.	Preparedness, robustness, resilience	Drainage system for flood; fortifying bridges for earthquake
Resource availability	Pre-event actions aimed at supporting post-event response, including pre-positioning resources and contracting for response support; no pre-event performance benefits derived, but reduced response times and costs obtainable post-event if response actions taken.	Preparedness, flexibility, recovery, resilience	Fire extinguisher for arson; salt sparkler for snow; water pump for flood
Response	Post-event actions to quickly recover some portion of lost capacity and performance loss; benefits derived post-event only.	Flexibility, recovery, resilience	Temporary road mat construction; replacing components

Figure 3-2(a) describes inherent characteristics of the system that enable it to resist and absorb the impact of the disruption, i.e. its *coping capacity*. A *preparedness* measure in Figure 3-2(b) contains all actions that are taken prior to the disaster event, including those that have benefit only if response options are exercised (i.e. resource availability), as well as pre-existing qualities (i.e. coping capacity). Note that the costs of preparedness actions are incurred whether or not disaster occurs. Such a prepared system can be distinguished from one that is robust in that *robustness* can be described in terms of the system's innate coping capacity and pre-event actions taken to enhance system resistance under disruption as in Figure 3-2(c). A system that can withstand the impact of the disaster event is said to be robust.

A *flexibility* measure is defined in Figure 3-2(d). This definition, in contrast with robustness, accounts for a system's adaptive capabilities to respond to disruption. That is, flexibility measures the capability of the system to absorb system demand given reduced system offerings through post-event adaptive response actions. It draws on excess capacities that may exist through coping capacity. Measures of *recovery* in Figure 3-2(e) can be viewed as the converse of *preparedness*, where resource availability is established during the preparedness stages, but is exploited through response actions. These concepts further differ in that *preparedness* measures' focus on the existing coping capacity and system enhancements made through pre-event actions, while *recovery* measures include only post-event coping mechanisms to restore performance.

Finally, the framework of Figure 3-2(f) supports a concept of *resilience* that incorporates all elements of the system's inherent capabilities, disaster readiness and post-event response capability, i.e. its ability to both resist and adapt. With this conceptualization, *coping capacity*, *preparedness*, *robustness*, *recovery* and *flexibility* can all be seen facets of *resilience*. Note that one might argue for the inclusion of reliability, vulnerability or risk as important classes of related measures. Despite that reliability can be a measure of a system's inherent coping capacity, it is omitted here, because it is used to compute a probability, such as the probability that the system remains connected, or the probability that travel time/capacity remains within a desired range, rather than post-event expected performance level. Likewise, risk and vulnerability are measures of disruption probability and/or level of consequence. These measures capture potential losses and event likelihood rather than residual performance given event occurrence.

3.3.2. A common framework for performance measure qualification

a) Graphical representation through disruption profiles

A common framework for depicting and quantifying the performance measures considered herein is constructed using concepts of disruption profiles. Disruption profiles are used to display system behavior changes over time, beginning from the moment prior to disruption through the time at which the system is restored to its pre-event state or reaches an alternative desired state. The profile can be divided into distinct disruption and recovery periods, where the former refers to the duration of time from the moment the disruption takes place, t_0, until recovery begins, and the latter refers to the duration of time during which response actions are taken to recover performance. Bruneau et al. (2003) and Sheffi (2005) employed such disruption profiles to graphically depict system performance in the context of human communities and supply chain network resilience, respectively.

This disruption profile is employed herein (see Figure 3-3). It is a function of time, and hence denoted as $P(t)$, because it provides an indication of the system-wide performance level at each point over the time horizon. Its application enables insights into the impact of individual IPF components. Four performance functions, $P^u(t)$, $P^r(t)$ (or $P^e(t)$), $P^{re}(t)$, and $P^{res}(t)$, depict the performance of the same system given that select preparedness actions, i.e. no action (unprepared, u), retrofit (r), expansion (e), retrofit and expansion (re), and expansion, retrofit and resource availability (prepared, res), are taken. Preparedness actions lead to enhancements in system performance over time as shown in the figure. Let $Y = \{u, r, e, re, res\}$.

As in Figure 3-3, the period of time prior to and up to the very moment of a disruption event is referred to as the pre-event period, denoted by O. The moment just after the event arises, and the effects of the disruption on system performance begin, until the system experiences its lowest performance level as a consequence of the event is referred to as the post-event period. D is used within the nomenclature to refer to this period. Finally, the recovery period refers to the period beginning from the point in time when the system reaches its minimum performance level through the point at which a desired performance level is attained as a consequence of response actions. R is used within the nomenclature to refer to this period. $Z = \{O, D, R\}$.

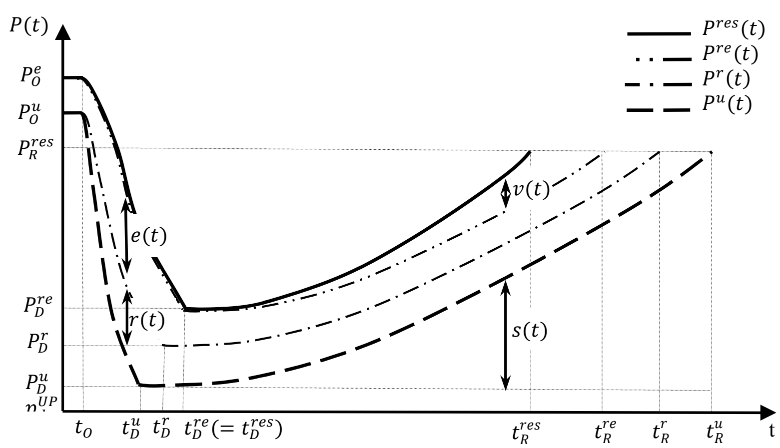


Figure 3-3 Graphical representation of IPF components

For the disruption period, let t_D^y be the point in time at which the system experiences its minimum performance level given, for $y \in Y$, and P_D^y be the corresponding minimum post-event performance level. Let $t_R^y, y \in Y$, be a point in time in the recovery period at which a desired performance level, referred to herein as the post-recovery performance level, is achieved. Figure 3-3 depicts this time for a desired performance level P_R^{res}.

Consider the unprepared system in Figure 3-3. The reduction in $P^u(t)$ during the period of disruption from t_O to t_D^u provides information about the system's inherent *coping capacity*. If no further action is taken, the system will continue to perform at the P_D^u level into the future. Likewise, for the other performance functions, the maximum achievable post-event performance level will be sustained if no additional response actions are taken.

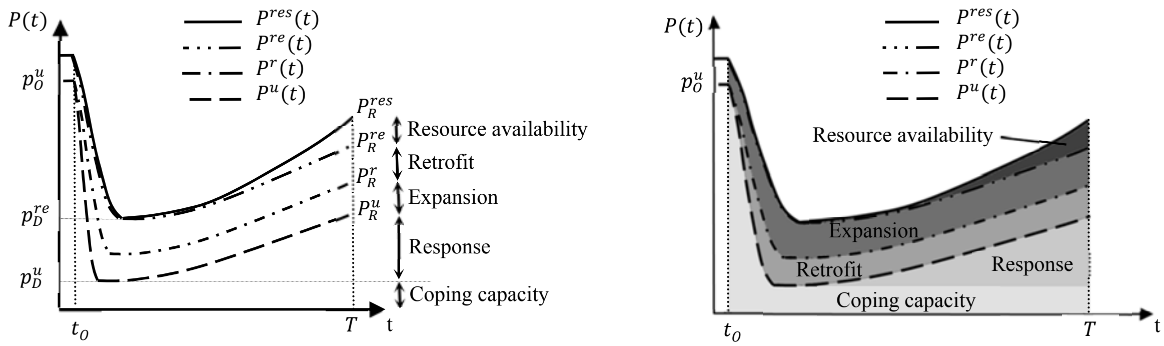
Improvements in a system's post-event performance can be attained through *retrofit* and *expansion* actions resulting in post-event performance P_D^{re}. Retrofit actions are intended to aid the system component(s) in withstanding a disaster event and may result in a higher minimum post-event performance level. That is $P^u(t) \leq P^r(t)$ for $t_O \leq t \leq t_D^u$, and $P_D^u \leq P_D^r$. Such actions will have no impact on pre-event performance. Expansion actions improve pre-event performance, i.e. $P_O^u \leq P_O^e$. Consequently, $P^r(t) \leq P^{re}(t)$ for $t_O \leq t \leq t_D^r$. Moreover, expansion and retrofit result in greater time to descend to this minimum, i.e. $t_D^{re} \leq t_D^u$. Note that $P^{res}(t) = P^{re}(t)$ for $t_O \leq t \leq t_D^{re}$, since investment made in resource availability has no effect on performance within the disruption period. Likewise, $t_D^{re} = t_D^{res}$. The performance improvements due to system retrofit and expansion actions are given in the figure by $r(t)$ and $\mathrm{e}(t)$, respectively.

The position on the y-axis and shape of performance function $P^{y}(t)$ over the recovery period, for $y \in Y$, depends on the value P_D^{y} and the impact of *response* actions taken to restore performance, $s(t)$. Retrofit and expansion actions, both of which are taken in advance of any disruption, do not have additional impact on system performance during the recovery period. $P^{u}(t) \leq P^{r}(t) \leq P^{re}(t)$, because $P_D^{u} \leq P_D^{r} \leq P_D^{re}$ for a given response action.

The impact of *resource availability* is evident only in the recovery period, as depicted in Figure 3-3. If resources are made available in advance to support recovery efforts, a higher performance level, $P^{res}(t)$, can be attained when considered at a specific point in time *t* during the recovery period, i.e. $P_R^{re}(t) \leq P_R^{res}(t)$, and a shorter duration of recovery period will be needed to attain a desired performance level, e.g. P_R^{res}. The vertical distance between $P^{re}(t)$ and $P^{res}(t)$, $v(t)$, indicates to what extent resource availability can improve the recovery process over time.

b) Point and period performance metrics

Using the concept of disruption profiles, mathematical equations can be derived for the studied system performance measures. Two conceptualization of the measures, point and period, are given. Let $T = t_R^{res}$. The contribution of the IPF components to the maximum post-recovery performance level P_R^{res} achieved at time T is called its point contribution (Figure 3-4(a)). Additional information about performance can be obtained by considering not only the maximum post-recovery achieved level, but also the rate at which the performance level drops or increases over time. The contribution of IPF components in characterizing performance over disruption and recovery periods can be captured by computing the relevant areas under the performance function curves, called period contribution, as depicted in Figure 3-4(b). The shaded areas illustrate the contributions of each IPF component over time period $[t_0, T]$.



(g) Point contribution of IPF components on post-recovery performance P_R^{res}

(h) Period contribution of IPF components over time period $[t_0, T]$

Figure 3-4 Point contribution of IPF components on post-recovery performance P_R^{res}

Point and period performance measures are computed from the summation of point estimates and areas under the curves, respectively, as mathematically formulated in Table 3-2. Let $\bar{I}^i_{T,B}$ and $I^i_{T,B}$ be point and period performance measures of i, $i \in$ {*coping capacity, preparedness, robustness, flexibility, recovery, resilience*}, for an available budget level B, and point in time T. Note that in the best case, the system will achieve a level of post-recovery performance equivalent to (possibly better than) its pre-event performance level. In the worst-case, a zero system performance level may be reached, indicating complete failure.

Table 3-2 Mathematical equations of point and period performance measures

Point performance measures		Period performance measures	
$\bar{I}_{T,B}^{Coping\ capacity} = \frac{P_D^u}{P_O^u}$	(1)	$I_{T,B}^{Coping\ capacity} = \frac{\int_{t_O}^{T}[P^u(t)-s(t)].dt}{p_O^u.(T-t_O)}$	(7)
$\bar{I}_{T,B}^{Preparedness} = \frac{P_R^{res}-(P_R^u-P_D^u)}{P_O^u}$	(2)	$I_{T,B}^{Preparedness} = \frac{\int_{t_O}^{T}[P^{res}(t)-s(t)].dt}{p_O^u.(T-t_O)}$	(8)
$\bar{I}_{T,B}^{Robustness} = \frac{P_D^{re}}{P_O^u}$	(3)	$I_{T,B}^{Robustness} = \frac{\int_{t_O}^{T}[P^{res}(t)-s(t)-v(t)].dt}{p_O^u.(T-t_O)}$	(9)
$\bar{I}_{T,B}^{Flexibility} = \frac{P_R^{res}-(P_D^{re}-P_D^u)}{P_O^u}$	(4)	$I_{T,B}^{Flexibility} = \frac{\int_{t_O}^{T}[P^u(t)+v(t)].dt}{p_O^u.(T-t_O)}$	(10)
$\bar{I}_{T,B}^{Recovery} = \frac{P_R^{res}-P_D^{re}}{P_O^u}$	(5)	$I_{T,B}^{Recovery} = \frac{\int_{t_O}^{T}[s(t)+v(t)].dt}{p_O^u.(T-t_O)}$	(11)
$\bar{I}_{T,B}^{Resilience} = \frac{P_R^{res}}{P_O^u}$	(6)	$I_{T,B}^{Resilience} = \frac{\int_{t_O}^{T} P^{res}(t).dt}{p_O^u.(T-t_O)}$	(12)

Considering the time dimension in assessing system period performance may also aid post-disaster response activity scheduling, permitting earlier gains in system performance levels. To further illustrate, consider Figure 3-5 in which a disruption event occurs at time 1. Capacity along the studied link is immediately reduced from 10 to 3 units. Two recovery options are available, both of which restore performance to 9 units of capacity by time 5. Using the point measures discussed in the previous section, these recovery options produce identical results. However, it can be noted that recovery option 1 restores capacity more quickly and, thus, may be preferred.

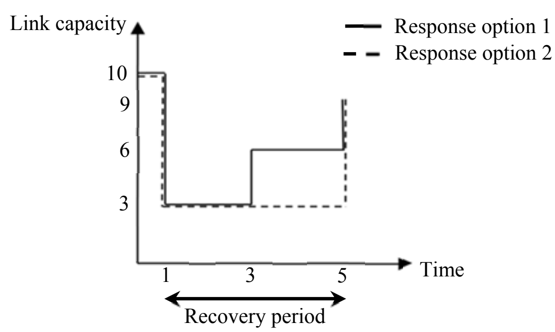


Figure 3-5 Comparison of point and period measurement

3.3.3. Contribution of Pre- and Post-event actions to infrastructure protection

While only preparedness, robustness, flexibility and resilience measures account for the coping capacity of the system, these measures, as well as recovery capability and flexibility, include the contributions of pre- and post-event actions that can be taken to prevent or mitigate the effects of disaster. The nature of these actions and related need for investment and implementation efforts are investigated in this section. This investigation requires the introduction of four variables, described in Table 3-3.

Table 3-3 Description of pre- and post-event action levels

Action type	Action level	Range	Description
Retrofit	β_a	[0,1]	• $\beta_a = 1$ is the level at which no disruption scenario can impact component a • Reduces disruption severity; no impact on event probabilities
Expansion	α_a	$[0, \infty)$	• Expansion strategies include: (1) expand capacity of existing component (2) add component • Theoretically ranges from 0 (no action) to infinity • $\alpha_a = 1$ for the first expansion strategy indicates an expansion of a component a equivalent to its starting capacity • For second expansion strategy one could think of relating expansion level to specific amount of component capacity to be constructed
Resource availability	γ_a	[0,1]	• $\gamma_a = 1$ means perfect resource availability in component a such that required resource for implementing responsive actions in the most efficient way are provided in advance
Response	$\lambda_a(\xi)$	$[0, \infty)$	• Reflecting restored system performance taking both resource availability and responsive actions • $\lambda_a(\xi)$ indicates the level of restoration related to the pre-event performance level of the component. In the event of complete failure, $\lambda_a(\xi) = 1$ would infer complete reconstruction. Note that a value greater than one is permitted as further enhancement may be desirable

The proposed action levels are continuous representations of actions with corresponding budgets and implementation times defined for each individual system component $a \in A$, given the set of all system components, A. In general, in a network representation of a system, the system components as designated herein will be represented by the links of the network (e.g. roadways or railways in a transportation network, pipelines in a water supply network, or gas lines in an energy network).

To assess system performance *a priori*, one must consider the prospect of numerous possible future disruptive event scenarios from a variety of potential sources. These sources might include natural weather events, accidental events, e.g. due to technological failure or a hazardous materials incident, or malicious acts. Each scenario will affect post-event performance differently. Which scenario will occur cannot be known *a priori* with certainty.

The post-event performance depends on the performance of the individual system components and their interactions. Furthermore, the components of the system that are impacted and the extent of impact depend on the specifics of the event. Let p_{Da}^{y} and p_{Ra}^{y} represent the post-event and post-recovery performance levels of component a, for $y \in Y$. p_{Da}^{y} and p_{Ra}^{y} are random variables, and $p_{Da}^{y}(\xi)$ and $p_{Ra}^{y}(\xi)$ are their corresponding post-event performance levels under disruption scenario ξ. Let Ω be the set of scenarios. Each scenario $\xi \in \Omega$ is defined and generated as a vector of random values $[p_{Da}^{u}(\xi)]_{a \in A}$, indicating post-event performance of all components of an unprepared system. Component performance after expansion, retrofit, and response can be calculated through equations (13)-(17) given corresponding action level decisions. Suppose that the effects of retrofit are linear to system performance. Then,

$$p_{Da}^{r}(\xi) = p_{Da}^{u}(\xi) + \beta_a \cdot [p_{Oa}^{u} - p_{Da}^{u}(\xi)]. \tag{13}$$

The distribution function of P_{Da}^{r} as a function of β_a (retrofit effort) is depicted in Figure 3-6 for a given component a in a retrofitted system. For $\beta_a = 0$, p_{Da}^{r} and p_{Da}^{u}

have equivalent distribution functions. When $\beta_a = 1, p_{Da}^r(\xi) = p_{Oa}^u, \forall \xi$, inferring that no disruption can impact the performance of component a.

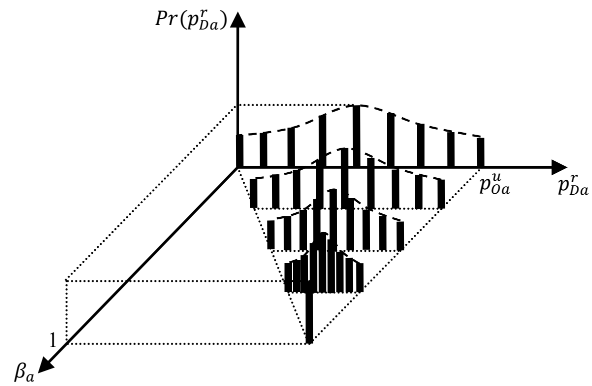


Figure 3-6 Discrete probability function of p_{Da}^r as a function of β_a

Component performance values between these extremes are derived from linear interpolation as is done in similar contexts (e.g. Liu et al. 2009, and Du and Peeta 2012). Retrofit does not impact the probability of event occurrence. However, with increasing retrofit level, the range on post-event performance narrows leading to higher expectation and lower variance. Moreover, the probability of higher post-event performance level increases. Thus, post-event component performance, and ultimately system performance, is decision-dependent.

The pre-event performance of component a expanded by level α_a is computed by equation (14). Post-event performance of the expanded component is presumed to be a linear function of the expansion level α_a in equation (15). This infers that any disaster impact on the operation of an existing component will similarly affect the expanded portion of the facility. Effects of new materials, etc. may be considered. The post-event performance of this retrofitted and expanded component for a given scenario ξ can be computed by equation (16) assuming that when a component is expanded the same level of retrofit is applied throughout.

$$p_{Oa}^{e} = (1 + \alpha_a) \cdot p_{Oa}^{u}, \quad (14)$$
$$p_{Da}^{e}(\xi) = (1 + \alpha_a) \cdot p_{Da}^{u}(\xi) \quad (15)$$
$$p_{Da}^{re}(\xi) = (1 + \alpha_a) \cdot p_{Da}^{r}(\xi) \quad (16)$$

For each component a and disruption realization ξ, post-recovery performance $p_{Ra}^{y}(\xi)$, for $y \in Y$, can be computed as in equations (17).

$$p_{Ra}^{y}(\xi) = p_{Da}^{y}(\xi) + \lambda_a(\xi) \cdot p_{Oa}^{u} \quad (17)$$

The cost and implementation time required for taking response actions is considered when determining the level of recovery action to execute. For a given component a, b_{β_a} and b_{α_a} give the costs of retrofit and expansion, respectively. The cost of resource availability, b_{γ_a}, is presumed to be linear in γ_a and independent of α_a and β_a. That is:

$$b_{\beta_a} = (1 + \alpha_a) \cdot b_{\beta_a}^{max} \cdot \beta_a, \quad (18)$$
$$b_{\alpha_a} = b_{\alpha_a}^{max} \cdot \alpha_a, \text{ and} \quad (19)$$
$$b_{\gamma_a} = b_{\gamma_a}^{max} \cdot \gamma_a, \quad (20)$$

where $b_{\beta_a}^{max}, b_{\alpha_a}^{max}$, and $b_{\gamma_a}^{max}$ are unit costs of retrofit, expansion and resource availability, respectively, in component a.. Level of retrofit is not included in equation (19) so that associated costs are applied only once in equations (18).

Let the implementation cost and time of post-event response actions in system component a, b_{λ_a} and q_{λ_a}, be defined as nonlinear functions of response and resource availability levels. Then,

$$b_{\lambda_a}(\xi) = \left[b_{\lambda_a}^{max} - \left(b_{\lambda_a}^{max} - b_{\lambda_a}^{min}\right) \cdot \gamma_a\right] \cdot \lambda_a(\xi), \text{ and} \quad (21)$$
$$q_{\lambda_a}(\xi) = \left[q_{\lambda_a}^{max} - \left(q_{\lambda_a}^{max} - q_{\lambda_a}^{min}\right) \cdot \gamma_a\right] \cdot \lambda_a(\xi), \quad (22)$$

where $b_{\lambda_a}^{max}$ ($q_{\lambda_a}^{max}$) and $b_{\lambda_a}^{min}$ ($q_{\lambda_a}^{min}$) are unit implementation costs (times) of response actions required to achieve the level of response equal to one, given zero and perfect level of resource availability, respectively. How the implementation cost and time functions change with level of response and resource availability is shown in Figure 3-7. With a higher level of resource availability, less effort is required to achieve a given performance level. Once the decision on action level is made, the corresponding specific action to take can be identified from a mapping of action level to implementation cost and time functions.

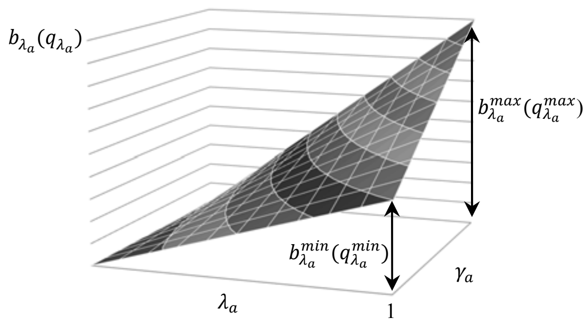


Figure 3-7 Implementation cost (time) of response actions as functions of resource availability and response levels

3.4. Infrastructure Protection Optimization

In this section, a general optimization program of this Infrastructure Protection Problem (IPP) is formulated to determine the maximum attainable system performance level using the point performance concept and identify an optimal investment in preparedness and responsive actions needed to achieve this level. It accounts for uncertainty in post-event performance of the system, since which event, if any, will be realized cannot be known in advance. The model selects the optimal retrofit, expansion and resource availability actions to take *a priori* (pre-event) so as to maximize system performance given that chosen responsive (recourse) actions will be taken post-event once the disaster circumstances are realized. Recourse options may also depend on the choice of preparedness actions.

The IPP exploits a network representation of the system infrastructure. Let $G = (N, A)$, where $N = \{1, \dots, n\}$ and $A = \{1, \dots, m\}$ are the set of nodes and links that connect the nodes. For instance, for a rail-based transport system, stations are represented by nodes and tracks along which trains travel are represented through the links. In a road transport network, nodes may represent demand source locations, like houses, businesses, or parking lots. At a higher level, these locations may represent an area, such as Traffic Analysis Zones (TAZ), entire towns or even countries. Alternatively, the nodes may merely denote a decision point between roadways. Links represent physical connections between nodes or actual roadways. In the electric power grid, generators, stations and consumers are taken to be the nodes of the network and power lines are represented by its links. Note that nodal components can be expanded accordingly.

3.4.1. The general model for IPP

The IPP is formulated as a nonlinear two-stage, stochastic program. Preparedness options are considered in the first-stage and remedial actions that can be taken in response to knowledge of the disaster scenario are determined in the second-stage in the form of recourse decisions. For simplicity, let $\alpha, \beta, \gamma,$ and λ be vectors of action levels in network links, and p and P be vectors of component- and system-level performance for all $y \in Y$ and $z \in Z$. The IPP is formulated generically, permitting its application in measuring and optimizing system performance with respect to any of the performance measures defined in Table 3-2.

(IPP)

First stage:

$$\max_{\alpha,\beta,\gamma} E_{\tilde{\xi}}[X(\xi)] \tag{23}$$

s.t.

$$\alpha, \beta, \gamma \in S \tag{24}$$

Second stage:

$$X(\xi) = \max_{\lambda,p,P} \bar{I}^{i}_{T,B}(\xi) \tag{25}$$

s.t.

$$q_\lambda(\xi) \le T, \tag{26}$$

$$b_\alpha + b_\beta + b_\gamma + b_\lambda(\xi) \le B, \tag{27}$$

$$\lambda(\xi) \in R^m_+, \tag{28}$$

$$H[\xi, p(\xi), P(\xi)] \le 0. \tag{29}$$

The objective function (23) seeks to maximize the expected system performance measure value given first-stage preparedness and second-stage recourse decisions for the set of possible disruption scenarios. $X(\xi)$ is the maximum value of the desired point performance measure $\bar{I}^{i}_{T,B}(\xi)$, $i \in$ {*coping capacity, preparedness, robustness, flexibility, recovery, resilience*}, for disruption scenario ξ, that can be attained given specified maximum recovery period duration and budgetary limitations, enforced through constraints (26) and (27), respectively, in the second stage.

Component-level action implementation cost and time variables in (26) and (27) are determined from equations (18)-(21). First-stage variables belong to the set $S = \{\alpha, \beta, \gamma\colon\ \beta, \gamma \le 1,\ \alpha, \beta, \gamma \in R^m_+\}$, enforcing non-negativity and retrofit and resource availability limits. Constraints (28) enforce non-negativity in second-stage decision variables. Finally, a general function H defined in constraints (29) describes the relationship between system-level performance P and component-level performance p, where p is a function of action level determined using equations (13)-

(17). A schematic of the how elements of the IPP contribute to the system performance level sought through its objective function is given in Figure 3-8.

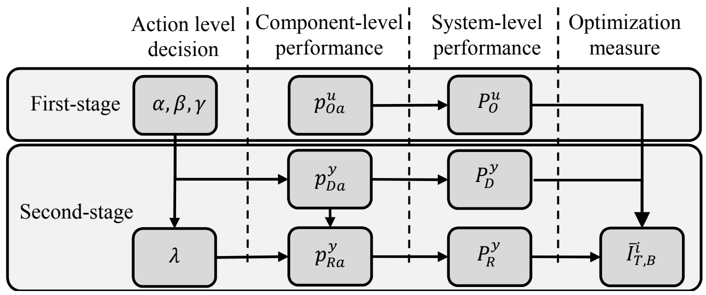


Figure 3-8 Schematic connecting IPP elements to its objective function

This formulation builds on a two-stage stochastic program introduced by Miller-Hooks et al. (2012) for the problem of measuring and maximizing resilience specific to intermodal freight transport. In that earlier work, preparedness and recovery actions are given by a set of discrete options, creating an integer stochastic program. The IPP expands on that program to provide a general model for measuring and maximizing not only resilience, but *coping capacity*, *preparedness*, *robustness*, *flexibility*, and *recovery* (Figure 3-2). It further permits its use over a wide array of applications. These performance measures are affected by the system's inherent characteristics and actions that can be taken through expansion, retrofit, resource availability and response actions, together comprising the components of the proposed IPF as described in Figure 3-2 and Table 3-1.

A more generic representation of actions is permitted through the use of continuous action variables related to expansion, retrofit, resource availability and response. This permits an abstract notion of action and, thus, an infinite set of choices in terms of action levels. In the prior work, a response action would be defined in a very specific way. For example, it might be exercising an option to borrow two gantry cranes from a competing stevedoring company at the same port or use of a pump to remove water from a flooded area. In the proposed IPP, the response action is given by an action level. Action level 0.5, thus, indicates that a response action of some type should be taken to increase the post-event performance of a specific system component by 50% of its pre-event performance level. Thus, if the component's performance is at 25% of its pre-event performance level, the resulting performance level of the component will be raised to 75% of its pre-event performance level. This use of continuous decision variables aids in clarifying the effects of budget limitations, permitted recovery period durations, and interaction effects between variable classes.

3.4.2. Mathematical structure of IPP

The general IPP is a nonlinear, stochastic program with nonlinear first and second-stage constraints and potentially nonlinear objective functions for both stages. The properties of the objective functions depend, in part, on the performance measure that is employed, which is a function of the application (e.g. a measure of connectivity needed to assess performance of computer systems or a measure involving travel time applicable for passenger transport systems). The mathematical properties of the IPP are explored in this section.

Several bilinear terms are employed within the constraints of the IPP, resulting in nonconvexity in the feasible region. Bilinear terms involving first-stage decision variables α and β appear in the budget constraint (27) and component-system relationship constraint (29), which uses link performance equations (16). Bilinear terms involving the multiplication of first- and second-stage decision variables, specifically γ and $\lambda(\xi)$, appear in constraints (26) and (27) which depend on equations (21) and (22). For a subset of performance measures, the bilinear terms can be eliminated from the model. For example, if one seeks the coping capacity of a system, all of the decision variables associated with preparedness and response actions will drop out of the model.

Table 3-4 synopsizes both the application of the IPP in terms of which decision variables or model parameters will be eliminated and which constraints will drop out as a consequence of variable elimination and general problem properties that result.

Whether or not additional nonlinearities or nonconvexities exist within the model depends on the specific application. That is, the specific form of the objectives and constraints depends on the performance specification and goals. Consider an application where the goal is to maximize throughput. The objective can be given as a linear function of flow with linear flow conservation and limitation constraints. Now, consider an alternative application where the objective is to minimize total travel time and travel time is a function of flow. Such a problem will be nonlinear, but convex. Last, consider an application seeking a user equilibrium solution as is typical in vehicular traffic applications. In such an application, the objective is identical to that of the system optimal problem, but the program will contain complementarity constraints needed to ensure that the solution assigns traffic such that no user can improve his/her travel time by unilaterally switching paths (Wardrop 1952). Such complementarity constraints introduce nonlinearities and nonconvexities. Additional constraints associated with the specific application will appear as part of constraints (29). Thus, the linearity or convexity of the program depends largely on the form these constraints.

Table 3-4 IPP mathematical structure for each optimization measure

Optimization measure	Eliminated variables/ parameters	Eliminated equations/constraints	IPP properties			
			Linearity/convexity	Linearity/convexity in 1st stage variables	Linearity/convexity in 2nd stage variables	Separability of 1st and 2nd stage variables
Coping capacity	$\alpha, \beta, \gamma, \lambda(\xi)$	All except $H(\xi, p, P) \leq 0$	✓	✓	✓	✓
Preparedness	$\lambda(\xi)$	(17), (21)~(22), (26)	-	-	✓	✓
Robustness	$\gamma, \lambda(\xi)$	(17), (20)~(22), (26)	-	-	✓	✓
Flexibility	α, β	(13)~(16), (17) for $l \in \{r, e, re, res\}$, (18)~(19)	-	✓	✓	-
Recovery	α, β, and $P_{Da}^{u}(\xi) = 0$	(13)~(16), (17) for $l \in \{r, e, re, res\}$, (18)~(19)	-	✓	✓	-
Resilience	-	-	-	-	✓	-

3.4.3. Solution methodology

The most general version of the IPP is nonlinear and nonconvex. Decomposition by stage results in a second-stage program that is linear in the level of response variable λ. Thus, for applications where constraints (29) are convex, the second-stage problem for fixed values of first-stage variables is convex and solution can be obtained using a generalized L-shaped method designed for nonlinear, stochastic programs. Such a method is based on generalized Benders decomposition (GBD) developed from concepts of Benders decomposition (Geoffrion 1972). In general, these methods decompose the program into stages that exploit a temporal (or sequential) relationship between decision variables. Solution first projects the problem onto first-stage variables and then applies a cutting plane technique to solve the resulting problem. Optimality cuts are also generated for inclusion in the first-stage program, iteratively producing a more restrictive first-stage problem. The approach iterates until convergence is achieved between solution values of the two stages. The projection of IPP onto α, β, and γ can be formulated as in (30).

$$\max_{\alpha,\beta,\gamma} \chi(\alpha,\beta,\gamma) \quad s.t.\ \alpha,\beta,\gamma \in S \cap \acute{S}, \tag{30}$$

where $\chi(\alpha,\beta,\gamma) = E_{\xi}\left\{\sup_{\lambda,p,P} \bar{I}^{i}_{T,B}(\xi) \ \ s.t.\ G[\xi,\alpha,\beta,\gamma,\lambda(\xi),p(\xi),P(\xi)] \le 0\right\}$ for $G[\xi,\alpha,\beta,\gamma,\lambda(\xi),p(\xi),P(\xi)] \le 0$ representing second-stage constraints (26)~(29). The additional restriction on the feasible set of first-stage variables is enforced by $\acute{S} = \{\alpha,\beta,\gamma: b_\alpha + b_\beta + b_\gamma \le B\}$. With $\acute{S}$, feasibility of $\chi(\alpha,\beta,\gamma)$ is guaranteed (complete recourse), i.e. $\acute{S} = \{\alpha,\beta,\gamma: G[\xi,\alpha,\beta,\gamma,\lambda(\xi),p(\xi),P(\xi)] \le 0$ for some $\lambda(\xi) \in R^{m}_{+}\}$. The problem (30) is decomposed into a master problem (MP) and a set of subproblems (SP), one for each disruption scenario realization, ξ:

$$(MP)\colon \max_{\alpha,\beta,\gamma,\theta} \theta \ \ s.t.\ \theta \le \chi(\alpha,\beta,\gamma),\ \ \alpha,\beta,\gamma \in S \cap \acute{S} \tag{31}$$

$$(SP)\colon \max_{\lambda,p,P} \bar{I}^{i}_{T,B}(\xi) \quad s.t.\ G[\xi,\alpha^{\upsilon},\beta^{\upsilon},\gamma^{\upsilon},\lambda(\xi),p(\xi),P(\xi)] \le 0 \tag{32}$$

The MP is an equivalent version of the original IPP (30) involving a set of optimality cuts given by $\theta \le \chi(\alpha,\beta,\gamma)$, where θ is an approximation of $\chi(\alpha,\beta,\gamma)$. In iteration υ, the solution value of the MP, θ^{υ}, provides an upper bound (UB) on the optimal solution value of (30). The values of α,β,γ are tentatively fixed to α^{υ}, β^{υ}, and γ^{υ}, i.e. the solution from the MP, in each SP. Thus, $\chi(\alpha^{\upsilon},\beta^{\upsilon},\gamma^{\upsilon})$, computed over solutions of the SPs, provides a lower bound (LB) on the optimal solution. Solutions to the SPs are used within a cutting plane technique to generate a new optimality cut for inclusion in the MP. The solution process continues until UB≤LB, at which point optimality is achieved. An overview of Benders-based decomposition methods is given in Figure 3-9.

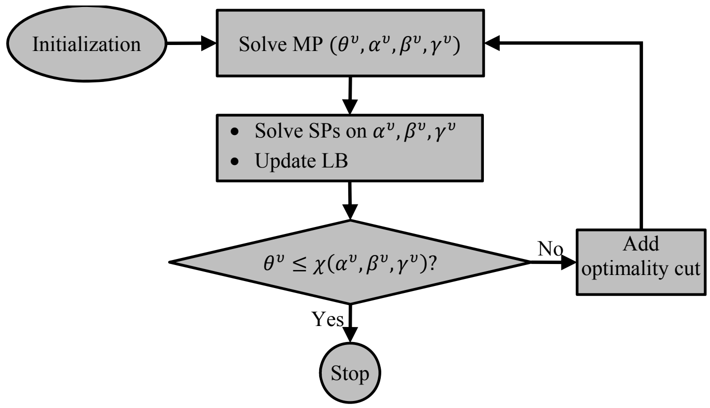


Figure 3-9 The general flowchart of Benders-based decomposition methods

For some measures, like preparedness and robustness, when constraints (29) are convex, the SPs will be convex since there are no bilinear terms. Thus, the conventional L-shaped method of Van Slyke and Wets (1969) designed for separable, linear and stochastic programs may also be applied. In this method, optimality cuts are generated by linear programming duality. Let second-stage constraints be represented by $G_1(\xi, \alpha, \beta, \gamma) + G_2[\lambda(\xi), p(\xi), P(\xi)] \leq G_3(\xi)$, where G_1 and G_2 are separated functions of first- and second-stage variables, respectively, and G_3 is a vector of modeling parameters that depend on disruption scenario ξ. The optimality cut is generated by replacing $\chi(\alpha, \beta, \gamma)$ in (31) by its dual objective function:

$$\theta \leq E_{\tilde{\xi}}\{\pi^v(\xi) \cdot [G_3(\xi) - G_1(\xi, \alpha, \beta, \gamma)]\}, \tag{33}$$

where $\pi^v(\xi)$ is the value vector of dual variables corresponding to second-stage constraints at iteration v.

If, on the other hand, constraints (29) are nonlinear but convex, the generalized L-shaped method, which exploits Lagrangian relaxation of $\chi(\alpha, \beta, \gamma)$ in (31), will be required:

$$\theta \leq E_{\tilde{\xi}}\{\bar{I}^i_{T,B}(\xi) + \sigma^v(\xi) \cdot G[\xi, \alpha, \beta, \gamma, \lambda^v(\xi), p^v(\xi), P^v(\xi)]\}, \quad \sigma^v(\xi) \geq 0, \tag{34}$$

where $\lambda^{\upsilon}(\xi), p^{\upsilon}(\xi), P^{\upsilon}(\xi)$, and $\sigma^{\upsilon}(\xi)$ are vectors delineating the optimal solution and corresponding optimal Lagrangian multipliers for the SPs associated with scenario ξ at iteration v.

For applications involving *flexibility*, *recovery* and *resilience*, bilinear terms $\gamma \cdot \lambda$ exist in equations (21)-(22), which ultimately feed into (30), resulting in nonseparability of decision variables in the SPs. Consequently, Benders-based decomposition methods fail to generate valid optimality cuts, and thus, will not guarantee convergence to the global optimum (Geoffrin 1972, Floudas et al. 1989). In fact, these methods require that a certain property (property P), in which the explicit form of the optimality cuts must be generated independent of first-stage variables, hold (Geoffrin 1972); however, this property does not hold where the variables are nonseparable, the case here. Local optimality is, however, achieved through the application of the generalized L-shaped method (Floudas et al. 1989, Bagajewicz and Manousiouthakis 1991). The quality of the solution depends largely on the starting values of first-stage decision variables. A multi-start version of the generalized L-shaped method may lead to improved local solutions. For the most difficult nonconvex programs with bilinear terms, for small problem instances, it is possible to obtain globally optimum solutions using, for example, a branch-and-reduce solution methodology as found in commercial software packages like BARON (Sahinidis and Mohit 2007). Alternatively, bilinear terms can be linearized as suggested in (McCormick 1976), where a linear relaxation of the bilinear terms using convex envelopes is proposed. The conventional L-shaped method can be applied to the relaxed problem to obtain approximate solutions.

For applications involving *coping capacity*, action variables are forced to zero. As variables are forced out of the program, some constraints drop out. Consequently, the stochastic program is decomposable by scenario. If constraints (29) are convex, and each scenario-dependent program will be a convex deterministic program, making the problem easy to solve. If, on the other hand, constraints (29) are nonconvex, solution of nonconvex, deterministic, scenario-dependent programs will be required. In both cases, however, decomposition by stage is not required.

Table 3-4 summarizes the properties of the IPP for each measure assuming convexity in constraints (29). Generally, when constraints (29) are nonconvex, dual decomposition methods can be applied. See, for example, (Rockafellar and Wets 1991; Caroe and Schultz 1999). Relying on concepts of column generation, these methods decompose the problem by scenario. Convexity is not required. Alternatively, convexification methods, including outer approximation techniques (e.g. Horst et al. 1992), can be applied; however, solutions obtained through such approximate methods do not guarantee locally or globally optimal solutions for the original problem.

3.5. Illustrative Numerical Example

The IPP can be applied to study the performance of many networked infrastructure-based systems. For a chosen application, p and P must be specified. To show how the proposed framework and modeling construct operates, the IPP framework is applied to the freight-rail problem class addressed in (Miller-Hooks et al. 2012), where the resilience concept involving inherent coping capacity, along with preparedness and adaptive actions was first introduced.

In the context of freight flows, p is a vector of link flows and P represents total throughput. Numerous applications, including passenger transport, water transport through pipes, and electricity supply through power grids, involve flow-based performance.

3.5.1. Specifying the example IPP

To specify the IPP for this application, constraints (29) must incorporate flow conservation, link capacity and demand limitation constraints. For background purposes, a generic path-based maximum flow (throughput) formulation is given by (T).

$$(T) \quad max \ \sum_{w\in W}\sum_{k\in K_w} f(k,w)$$

s.t.

$$\sum_{k\in K_w} f(k,w) \le d_w, \quad w \in W$$

$$x_a = \sum_{w\in W}\sum_{k\in K_w} \delta_{ak}^{w}. f(k,w), \quad \forall a \in A$$

$$0 \le x_a \le c_a, \quad \forall a \in A,$$

where $f(k,w)$ is the flow though path k between O-D pair w. W is the set of O-D pairs and K_w is the set of paths k connecting O-D pair w. The objective is to maximize the flow between all O-D pairs representing system-level performance while simultaneously limiting flow along all paths between a particular O-D pair w to the demand of that O-D pair, d_w. The component-level performance is captured in the vector of link flows, x_a the flow along link a, which is limited by the link's capacity, c_a. Path-link incidence indicators, δ_{ak}^{w} , are set to 1 if path k uses link a for shipping flow between O-D pair w, and zero otherwise. For $y \in Y$ and $z \in \{D, R\}$, let $f_z^y(\xi, k, w)$ be the path flow along path k between O-D pair w, and $x_{za}^{y}(\xi)$ and $c_{za}^{y}(\xi)$

be the flow and capacity for link a under scenario ξ. Constraints (29) in the IPP formulation are captured by linear constraints (35)-(37).

$$\sum_{k\in K_w} f_z^y(\xi,k,w) \le d_w, \quad \forall w \in W, \forall y \in Y, \forall z \in \{D,R\} \tag{35}$$

$$x_{za}^y(\xi) = \sum_{w\in W}\sum_{k\in K_w} \delta_{ak}^w . f_z^y(\xi,k,w), \quad \forall a \in A, \forall y \in Y, \forall z \in \{D,R\} \tag{36}$$

$$0 \le x_{za}^y(\xi) \le c_{za}^y(\xi), \quad \forall a \in A, \forall y \in Y, \forall z \in \{D,R\} \tag{37}$$

System-level performance is measured by $P_z^y(\xi) = \sum_{w\in W}\sum_{k\in K_w} f_z^y(\xi,k,w)$, for $y \in Y$ and $z \in \{D,R\}$. $P_O^u = \sum_{w\in W} d_w$, assuming that all demand can be served within level-of-service bounds prior to a disruption event. This term will appear in the denominator of the objective function. For performance in terms of resilience, these pieces together produce the following formulation.

(*IPP-T*)
$$\max_{\alpha,\beta,\gamma} E_{\xi}\left\{\max_{\lambda,f} \frac{\sum_{w\in W}\sum_{k\in K_w} f_R^{res}(\xi,k,w)}{\sum_{w\in W} D_w} \quad s.t.\ (26)-(29),(35)-(37)\right\}$$

s.t. $\alpha,\beta,\gamma \in S \cap \acute{S}$.

3.5.2. The network

The model from Section 3.5.1 and related versions of the IPP-T that seek to maximize coping capacity, preparedness, robustness, flexibility and recovery are demonstrated on the Double-Stack Container Network (shown in Figure 3-10), representing a rail freight network connecting 8 cities in the western United States (Morlok and Chang 2004). 17 O-D pairs are chosen for this case study. Five classes of disruption scenarios, based on scenarios developed in (Chen and Miller-Hooks 2012), are considered: bombing, earthquake, flooding, terrorist attacks, and intermodal attack. In practice, realizations of each scenario include a vector of post-event link capacities for the unprepared system: $[c_{Da}^{u}]_{a\in A}$. Correlation between link capacities under each scenario helps structure the event by type.

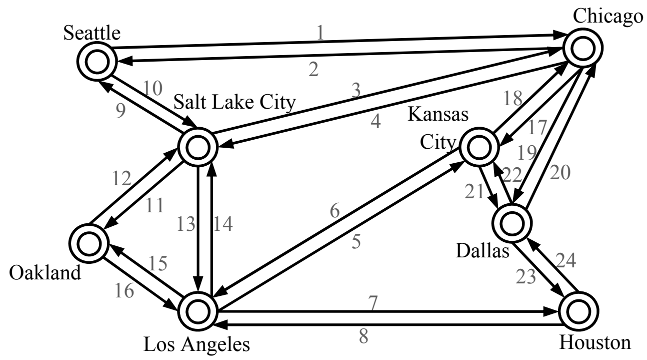


Figure 3-10 Double-stack container network (Morlok and Chang 2004)

Table 3-5 Values of modeling parameters

Link	Action implementation costs (×$1000)					Response action implementation time (days)	
	$b_{\alpha a}^{max}$	$b_{\beta a}^{max}$	$b_{\gamma a}^{max}$	$b_{\lambda a}^{min}$	$b_{\lambda a}^{max}$	$q_{\lambda a}^{min}$	$q_{\lambda a}^{max}$
1	78	1560	35	39	156	18	90
2	37	31	9	10	39	3	15
3	273	819	37	41	164	12	60
4	334	167	26	29	117	9	45
5	35	702	16	18	70	6	30
6	240	200	56	62	250	18	90
7	91	273	12	14	55	6	30
8	457	228	36	40	160	12	60
9	103	2067	47	52	207	15	75
10	105	87	25	27	109	9	45
11	351	1053	47	53	211	15	75
12	669	334	53	59	234	18	90
13	117	2340	53	59	234	18	90
14	210	175	49	55	218	18	90
15	475	1424	64	71	285	21	105
16	490	245	39	43	172	12	60
17	70	1404	32	35	140	12	60
18	168	140	39	44	176	15	75
19	273	819	37	41	164	12	60
20	279	139	22	24	98	9	45
21	68	1365	31	34	137	12	60
22	300	250	70	78	312	24	120
23	462	1385	62	69	277	21	105
24	446	223	35	39	156	21	105

The Monte Carlo method that captures this correlation structure described in (Chen and Miller-Hooks 2012) was adopted for use in generating scenario realizations, specifically 100 realizations for each scenario classification. B is set to \$400,000 and recovery period T is assumed to be two days. Parameter settings are given in Table 3-5. These parameters are hypothetical and are chosen only to illustrate the proposed concepts and solution methodologies.

3.5.3. Application of solution methodologies

As a result of convexity of constraints (35)-(37), for any measure considered herein and for a fixed set of first-stage variables, the second stage problem of IPP-T is convex and Benders-based decomposition methods can be employed. The solution methodology is implemented in GAMS calling CPLEX and BARON solvers for linear and nonlinear problems, respectively.

For *coping capacity*, IPP-T is decomposed by scenario, producing a set of linear programs, each of which can be solved using the linear solver in CPLEX. For *preparedness* and *robustness*, the conventional L-shaped method can be applied. Each nonlinear MP is solved by BARON, which guarantees a global optimum. A faster alternative with the capability of solving larger problem instances is to linearize bilinear terms $\alpha{\cdot}\beta$ in MP using the convex relaxation method of McCormick (1976). This permits solution using any linear solver. In McCormick's method, four constraints are introduced that restrict the variables in the bilinear term in relation to lower and upper bounds. Specifically, let $m_a = \alpha_a{\cdot}\beta_a$ represent the MP bilinear term for $a \in A$, and $\beta_a^L, \beta_a^U, \alpha_a^L,$ and α_a^U be lower and upper bounds of retrofit and expansion level variables. McCormick's constraints are formulated for each link a in (39).

$$\begin{aligned} m_a &\geq \beta_a^L \alpha_a + \alpha_a^L{\cdot}\beta_a - \beta_a^L \alpha_a^L, \\ m_a &\geq \beta_a^U \alpha_a + \alpha_a^U{\cdot}\beta_a - \beta_a^U \alpha_a^U, \end{aligned} \tag{39}$$

$m_a \leq \beta_a^U \alpha_a + \alpha_a^L \beta_a - \beta_a^U \alpha_a^L,$
$m_a \leq \beta_a^L \alpha_a + \alpha_a^U \beta_a - \beta_a^L \alpha_a^U, \qquad a \in A.$

Specifically in IPP, $\beta_a^L = \alpha_a^L$=0 for $a \in A$. According to feasibility set S, $\beta_a^U = 1$ for $a \in A$. Using constraints (27), in conjunction with definitions of (18) and (19), α_a^U can be defined by setting other action level variables equal to zero. Then, $\alpha_a^U = \frac{B}{b_{\alpha a}^{max}}$ for $a \in A$. Given these bounds, constraints (39) can be replaced by the following constraints (40). These constraints are added to the MP.

$$m_a \geq 0, m_a \geq \frac{B}{b_{\alpha a}^{max}} \beta_a - \frac{B}{b_{\alpha a}^{max}}, m_a \leq \alpha_a, m_a \leq \frac{B}{b_{\alpha a}^{max}} \beta_a, \qquad a \in A. \qquad (40)$$

Three different approaches are applied to solve the IPP-T associated with each of these measures. First, the extended version of each stochastic program, in which constraints are explicitly defined over all realizations, is solved by BARON permitting an optimality gap of 1%. To assess the applicability of the generalized L-shaped method, this method is applied with a starting point in which all first-stage variables are set to zero. As mentioned in Section 3.4.3, this approach can only guarantee locally optimal solutions for this application. Finally, bilinear terms are convexified using McCormick's method. This approach creates linearity and separability in the IPP-T. $\gamma \cdot \lambda(\xi)$ terms in the SPs are replaced by a vector $n(\xi)$ in which each element $n_a(\xi) = \gamma_a \cdot \lambda_a(\xi)$ for $a \in A$. Lower bounds of involved action level variables $\gamma_a^L = \lambda_a^L(\xi) = 0$. With respect to the feasibility set S, the upper bound of resource availability, γ_a^U, is set to be 1, i.e. $\gamma_a^U = 1$. According to constraints (26), a valid upper bound of response action level, $\lambda_a^U(\xi)$, is set to $\lambda_a^U(\xi) = \frac{T}{q_{\lambda a}^{min}(\xi)}$, assuming $\gamma_a = 1$, and the following linear constraints are added to the SPs.

$$n_a(\xi) \geq \lambda_a(\xi) + \frac{T}{q_{\lambda a}^{min}(\xi)} \cdot \lambda_a(\xi) - \frac{T}{q_{\lambda a}^{min}(\xi)}, n_a(\xi) \leq \lambda_a(\xi),$$

$$n_a(\xi) \leq \frac{T}{q_{\lambda a}^{min}(\xi)} \cdot \lambda_a(\xi), n_a(\xi) \geq 0, a \in A. \tag{41}$$

In the case of *resilience*, $\alpha \cdot \beta$ terms must also be convexified. This can be achieved through a similar replacement of terms and addition of bounding constraints (39) in the MP.

3.5.4. Numerical results

Results of the numerical experiments for competing solution methodologies are given in Figure 3-11. Solutions were obtained quickly and, thus, computation times are not reported. Analysis of the results shown in the figure indicates a system *coping capacity* of 56%. Thus, with no preparedness and response actions, the system has an expected throughput of just above half of its desired value. *Preparedness* and *robustness* measures are identical at 77%, indicating the value of preparedness actions. Their equivalence is expected, because these measures only differ in the inclusion of resource availability, which can only contribute to improved performance if response actions can be taken that exploit their existence. Greater improvement is obtained through response actions, including those that take advantage of resources made available through preparedness steps, as compared with other preparedness actions, as indicated by a *flexibility* value of 83%. 26% of system throughput is due to recovery actions taken in isolation, i.e. performance in terms of *recovery* is 26%. Thus, if complete system failure were to occur as a consequence of a disaster event, recovery actions could result in this level of throughput. Finally, taking all actions permissible, a resilience level of nearly 86% is achievable.

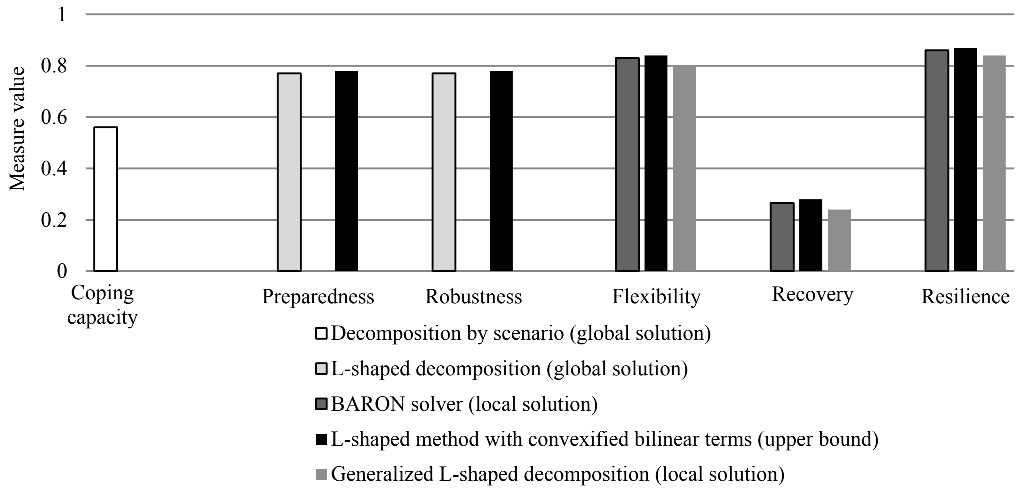


Figure 3-11 Comparison of results using different solution methodologies for each optimization measure

Modeling the action levels over a continuum permits sensitivity studies and enables insights that do not depend on the specific choice of potential available preparedness and response actions (i.e. a toolbox of pre- and post-event options) required in prior related works. For instance, the impact of budget and recovery time on resilience can be depicted, as in Figure 3-12, through resilience indifference curves, requiring continuous values of action level variables that contribute to resilience.

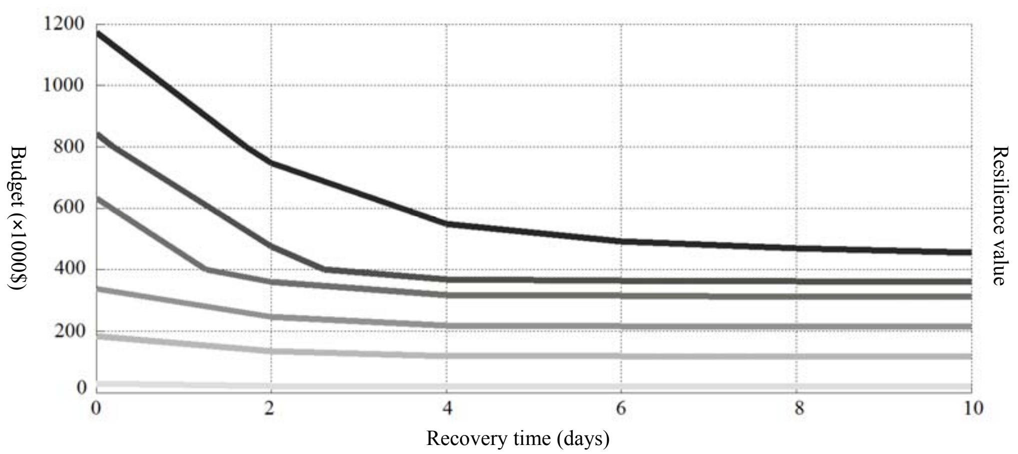


Figure 3-12 Resilience indifference curves for combinations of available budget and recovery period

3.6. Conclusions and Extensions

The proposed conceptualization of resilience and related measures, along with optimization framework and solution methods, provide a structure and needed tools for assessing system performance under potential future disruption scenarios. These further aid decision-makers with prioritization of preparedness and response actions and, thus, the development of investment strategies. Improvements in pre-event preparedness and post-event response capabilities aid in protecting the civil infrastructure and the people who live within it.

Chapter 4: Resilience of Airport Runway and Taxiway Pavement Networks

4.1. Introduction

Air transportation is one of the fastest growing modes of transportation globally. A recent market outlook (Boeing, 2012) forecasts a steady annual growth in demand of approximately 5%, implying that air traffic should double every 14 years. Currently, there are about 44 thousand airports worldwide, of which approximately one third have a paved network of runways and taxiways (CIA, 2012). The latter subset, which is the focus of this study, carries the vast majority of air passengers and cargo. In comparison with road or rail systems, airport pavement networks are compact in size and have a reduced degree of topological interconnectivity. Also, they provide service to 'vehicles' that are less tolerant to physical distress than other means of motorized transportation. As a direct consequence, the functionality of an entire airport may be impaired considerably even when a small part of its runway and taxiway network sustains damage. Pavement damage can be classified into four generic types, applicable to both asphalt and concrete pavements (FAA, 2007): (i) cracking - unplanned fracture lines traversing the surface, (ii) disintegration - breakup and fragmentation of the materials into small loose particles, (iii) distortion - permanent change in surface shape and elevations relative to original grades, and (iv) loss of skid resistance - increased surface slipperiness. The occurrence of any of the above distress types can lead to loss of serviceability and will require repair action to be taken before functionality is restored.

There are a myriad of events that might cause the aforementioned pavement damage types; these can also be classified into four broad categories: (i) extreme climate or geological events, (ii) random operational events, (iii) natural deterioration in combination with ill-timed maintenance, and (iv) intentional malicious acts such as terrorism or war. The first category encompasses those meteorological conditions not envisioned or not accounted for in the design of the facility. As an example, extreme high or low ambient temperatures (or fast transition between them) can cause airport pavements to abruptly buckle and crack or become locally distorted. When weather events include long dry spells or exceptional wet conditions, pavements founded on active soils can become severely cracked and distorted (McKeen, 1981). Snow/ice events negatively affect not only skid resistance - they can also induce cracking and distortion. In thunderstorms, when lightning strikes a concrete surface, spalling is usually the result (The Aberdeen group, 1984). Also included under this category are earthquakes, floods and tsunamis that can bring about crippling damage from any of the four abovementioned generic types.

Under random operational events, pavement damage can be caused by tire blowouts during takeoff or landing resulting in surface gouging (i.e. disintegration and distortion). Another probable ‘operational’ event can be a localized oil or fuel spill, which reduces the pavement skid resistance and produces disintegration (especially for asphalt pavements) leading to Foreign Object Debris (FOD) danger. Additionally, the need may arise to permit pavement overloading, i.e. allow aircraft operations with weights that far exceed the original design. This may come about in disaster relief or medical evacuation missions that demand special transport requirements, or during an emergency landing. In such cases, the sudden ‘abuse’ may be detrimental to the pavement network.

As for the third damage category, it is not uncommon in these economically constrained times to find runways and taxiways that have deteriorated to the verge of functionality loss, ‘setting the stage’ for subsequent unexpected shutdowns and unplanned demand for remedial actions to be taken. This situation usually arises when the natural pavement deterioration curve accelerates with age. Finally, acts of terrorism or war may involve targeted attacks on airports, with the aim of disabling the runway and taxiway systems to disrupt or completely disable takeoff and landing capabilities. This is usually ‘accomplished’ by cracking and distorting select/critical network components by means of explosives.

The economic impact of runway or full airport closure can be very significant. Specific impact estimates are given in (ARTBA, 2010) with respect to three major recent airport and airspace shutdown events. An 8-day shutdown of the Bangkok Airport in 2008 due to protests is reported to have cost the Thai economy over $8 billion. Incurred losses affected not only the well-being of stranded passengers, but specific industries with valuable, perishable cargo, as well as tourism to the area. $1.4 and $1.7 billion in revenue losses resulted from the three-day nationwide airport shutdown after the 9/11 terrorist attacks and multi-nation airport shutdown for the 2010 Eyjafjallajökull volcanic eruption in Iceland, respectively. On a smaller scale, daily runway shutdowns for pavement maintenance have significant local impact. For example, European airports shut down nearly 4 hours per month due to FOD incidents resulting from events such as oil spills or tire and engine damage. Direct costs per airport annually are on the order of $20 million. The cost to the top 300 European airports alone, including indirect costs due to, for example, fuel inefficiencies and delay costs, result in an estimated $12 billion in annual FOD-related expenses (McCreary, 2008). Additionally, cost estimates due to cancelling, ground holding and rerouting flights for only a one hour closure of runways at London's Heathrow Airport are between 700,000 and 1,250,000 euros (Pejovic et al., 2009). These estimates exclude additional substantial emissions costs due to increased fuel burn and other external costs.

The overall objective of this work is to transfer and apply the concept of resilience, as proposed in (Miller-Hooks, 2012) for rail-based intermodal cargo container networks, to the airport arena. Taking a multi-hazard perspective, resilience is measured in terms of the system's ability to provide for continuity of operations via existing attributes (topological and procedural) post-event. In this definition, the innate capability to resist and absorb disruption impacts through redundancies and underutilized capacity, the effects of adaptive actions that can be taken post-event, and the preparedness decisions that support these actions, are all integrated into the concept of resilience.

A plethora of works in the literature consider resilience, robustness, flexibility and other notions of network performance under disruption in the context of transportation and other critical civil infrastructure systems; see for example (Miller-Hooks et al., 2012; Bruneau et al., 2003; Amin and Horowitz, 2008; Shinozuka, 2009; Xu, 2009; Gopalakrishnin and Peeta, 2010). However, none of these consider disruptions to airport runway and taxiway systems. A number of decision support systems have also been developed in the literature for risk analysis of critical infrastructure systems in natural disasters, such as flooding (Multi Infrastructure Map for the Evaluation of the Impact of Crisis Scenarios (MIMESIS) (Rosato et al., 2011)), earthquake (e.g. Risks from Earthquake Damage to Roadway Systems (REDAR) (Werner et al., 2006)), human-caused malicious acts (e.g. Critical Infrastructure Protection Decision Support System (CIP-DSS) (Bush, 2005)), and "all-hazards" (e.g. Critical Infrastructure Protection Modelling and Analysis (CIPMA) (Australian Government, 2009)). These systems generally include a disaster scenario generator to create system inputs and support decisions by providing estimates of decision consequences and infrastructure risk to damage and failure. As compared to the above cited tools, the approach suggested herein offers greater specificity to the airport arena with a high level of mathematical rigor. In effect, the vast majority of works related to civilian airport disaster management focus on aviation security and vulnerability and do not address the physical infrastructure that supports take-offs and landings.

Perhaps the most related work in the literature to the problem at hand deals with military airfields under wartime situations. Wegner (1982) addressed the optimal sequencing of repair actions by a single team to damaged taxiways. A simple, ad-hoc path-based heuristic is proposed for use on a reduced network containing only damaged arcs that seeks the schedule that minimizes average time that aircraft located at parking areas are denied access to the runways. Several limiting assumptions were made, including the availability of only one repair team, suggesting that all repair actions be taken in series, and deterministically known repair times. Solution quality of the heuristic was studied through comparison with exact solutions obtained through branch-and-bound for 100 problem instances each associated with a single damage situation. Several works from the 90's describe pavement materials and procedures for Rapid Runway Repair (RRR) applications (Chang, 1990; Saroni, 1990). More recently, a methodology for computing the repair duration of a given Minimum Operating Strip (MOS) was proposed to aid in MOS selection, and thus, time to operation (Duncan, 2007). The Critical Path Method was suggested on an activity network representation of repair tasks to determine the repair duration.

In this chapter, the problem of evaluating and optimizing the resilience of a single airport's runway and taxiway network, referred to herein as the Airport Resilience Problem (ARP), is conceptualized and mathematically formulated as a two-stage stochastic integer program. The program captures complexities of modeling taxiway/runway capacities with bi-directional operations, optimal runway configuration selection under varying meteorological conditions, and minimum operating strip (MOS) restrictions, among other practical requirements (see Section 4.2). Novel modeling techniques and constraint specifications with applicability in airport ground traffic management beyond this emergency application and system-wide interactions are captured through a flow-based formulation. Budgetary, time, space, and physical resource limitations are also imposed. The program considers a myriad of potential future network disruption scenarios from multiple hazard classes based on the aforementioned distress-types and causal-categories, as well as their occurrence probabilities and potential consequences. Randomly arising meteorological conditions and their effects are also taken into consideration. An exact solution methodology based on concepts of integer L-shaped decomposition is proposed (Section 4.3). How the mathematical model and solution methodology might be embedded within a decision support tool is described (Section 4.4). The tool is subsequently applied to a specific case study on which its capabilities and applicability to the airport and pavement arenas are demonstrated (Section 4.5).

4.2. Formulation of the Airport Resilience Problem

The mathematical formulation of the ARP exploits a network representation of an airport's runway and taxiway pavement infrastructure. Let $Gr = (V \cup \{O, D\}, A = A_1 \cup A_2 \cup A_3)$ be an undirected graph, where O represents a supersource, i.e. the terminal, and D represents a supersink, i.e. the airways. A_1 and A_2 are sets of arcs (or links) representing taxiways and runways, respectively, and V is the set of vertices representing connections between these facilities. A_3 is the set of virtual arcs connecting physical network elements to the supersource and sink. While runway and taxiway arcs are undirected, any arc can only be used in one direction at a given point in time. Where appropriate, directed arc terminology is adopted.

The network is considered under a set of disruption events (i.e. network states) characterized by damage severity, type (climate/geological, operational, natural deterioration, and terrorism) and location, along with current meteorological conditions in terms of temperature, wind velocity, precipitation and visibility. The interrelationship between damage to the network and meteorological conditions is also considered. Damage may occur in multiple locations and its distribution over the pavement network depends on its cause. The likelihood of an event falling within any of these causal categories depends on the geographical characteristics of the airport.

Thus, each disruption event is equivalently a damage-meteorological scenario denoted by ξ.

A unique runway usage-pattern, in which particular runways operate in a pre-specified direction, called the runway configuration (Swedish, 1981), is selected through the specification of binary variables $\{\pi^g(\xi)\}_{g \in G}$, where G is the set of possible runway configurations that could be taken in different conditions and $\pi^g(\xi)$ indicates whether or not runway configuration g is 'selected' under damage-meteorological scenario ξ. Specification of the airport pavement network also involves runway and taxiway capacities on network flow rates. Flows are distinguished by aircraft size (small/large) and maneuver type (takeoff/landing). Capacities associated with each arc in A_1 and A_2 describe the rate at which aircraft can be served by the taxiways and runways, respectively. This rate depends on both meteorological conditions and facility use details.

The ARP seeks the optimal preparedness actions (resources that are made available) given all randomly generated 'disruptions', their probabilities of occurrence and the knowledge that the optimal recourse action will be invoked given available resources if an event is actualized. Mathematically, the solution takes the form of a two-stage integer stochastic program. Given that one of hundreds or thousands of potential disruption events may arise in the future, the first stage seeks optimal decisions pertaining to putting the appropriate personnel and agreements in place from which repair crews of skilled and certified workers (e.g. equipment operators, engineers, and electricians) will be formed and purchasing and prepositioning of heavy equipment (e.g. dump trucks, industrial tractors, towed sweepers). These decisions are taken *a priori*, i.e. prior to the occurrence (generation) of a disruption event. The second stage determines the post-event, i.e. *a posteriori*, recourse actions required to ameliorate damage impact once the event has occurred and damage assessment has been conducted. Thus, decisions taken *a priori* must support response actions needed for a host of damage situations.

Repair materials (e.g. aggregates, hot mix asphalt, Portland cement concrete, sealants) are assumed to be readily obtainable when needed. The choice of a repair action depends on a variety of factors, including: the damage type and extent, meteorological conditions, availability and cost of existing resources, available repair time, expected repair life and therefore willingness to tolerate long-term maintenance requirements, and willingness to restrict landing and/or take-off operations. The time interval required for implementing a chosen repair action depends on the type and dispersal of the damage, whether the task is undertaken using internal resources or if external resources (involving added start-up time) are used, and prevailing meteorological conditions. The latter conditions affect not only the repair time due to material properties and human/machine efficiency (Duncan, 2007), but may even necessitate selection of a different repair technology to cope with the situation. These conditions are accounted for in the model through repair-time multipliers. The solution is guided by an objective function that seeks the maximum expected post-event, post-repair taxiway and runway flow rates over all aircraft classes and runway maneuvers.

Nomenclature employed within the mathematical program is as follows:

A	=	set of links ($A = A_1 \cup A_2 \cup A_3$), where A_1 is the set of taxiways, A_2 is the set of runways, and A_3 is the set of added dummy links
G	=	set of runway configurations g
$A^c_{1,\acute{a}} \subset A_1$	=	subset of taxiways connected to a runway $\acute{a} \in A_2$ (entrance/exit taxiways)
$A^g_2 \subset A_2$	=	subset of runways that are active under configuration g

$I_{\acute{a}}$	=	set of segments of runway $\acute{a} \in A_2$
$I_{\acute{a}}^{a} \subset I_{\acute{a}}$	=	subset of segments of runway $\acute{a} \in A_2$ following (leading to) entrance (exit) taxiway $a \in A_{1,\acute{a}}^{c}$
O, D	=	super source and sink nodes
W	=	set of maneuver types $w \in W = \{arr, dep\}$ for arrival and departure maneuvers between nodes O and D, respectively
R	=	set of repair actions r
E	=	set of equipment types e
N_e	=	maximum number of equipment type e that could possibly be provided
M	=	maximum number of teams that might be deployed
S	=	set of aircraft classes (sizes) s
$y_{e,n}$	=	required storage space for n pieces of equipment type e
Y	=	total available storage space
$P^{g,w,s}$	=	set of active paths p for runway configuration g, maneuver type w and aircraft class s
$D^{w,s}$	=	original demand for maneuver type w and aircraft class s (arrival and departure demands)
F_a^{txc}	=	capacity envelop for taxiway $a \in A_1$ representing directional flow tradeoff
$c_a(\xi)$	=	capacity of taxiway $a \in A_1$ for both directions L and R under meteorological conditions of scenario ξ
F_g^{oc}	=	overall capacity envelop for runway configuration g representing total arrival and departure flow tradeoff
$F_{\acute{a}}^{rc}$	=	capacity envelop of runway $\acute{a} \in A_2$ representing arrival and departure flow tradeoff of that individual runway
C	=	large scalar
$\delta_{a,p}^{k,g,w,s}$	=	taxiway path-link incidence (=1 if path p for runway configuration g with maneuver type w for aircraft class s uses direction k of taxiway $a \in A_1$, and =0 otherwise)
$\acute{\delta}_{\acute{a},p}^{g,w,s}$		runway path-link incidence (=1 if path p for runway configuration g with maneuver type w for aircraft class s uses runway $\acute{a} \in A_2$, and =0 otherwise)
$\rho_{\xi,r}$	=	scenario-repair relationship parameter, which is set to 1 if repair action r can be taken under the meteorological conditions of scenario ξ, and 0 otherwise.
$\acute{\rho}_{e,r}$	=	equipment-repair relationship parameter, which is set to 1 if equipment e is needed for a team to take repair action r, and 0 otherwise.
b_m^{tm}	=	cost of employing m teams ($m = 0, 1, \dots M$)
$b_{e,n}^{eq}$	=	cost of providing n pieces of equipment type e
$b_{a,r}^{ex}(\xi), q_{a,r}^{ex}(\xi)$	=	implementation cost and time of repair action r by external resources in taxiway $a \in A_1$ under the meteorological conditions of scenario ξ, respectively
$b_{a,r}^{in}(\xi), q_{a,r}^{in}(\xi)$	=	implementation cost and time of repair action r by internal resources (employed teams and equipment) in taxiway $a \in A_1$ under the meteorological conditions of scenario ξ, respectively
$b_{\acute{a}_i,r}^{ex}(\xi), q_{\acute{a}_i,r}^{ex}(\xi)$	=	implementation cost and time of repair action r by external resources in segment i of runway $\acute{a} \in A_2$ under the meteorological conditions of scenario ξ, respectively
$b_{\acute{a}_i,r}^{in}(\xi), q_{\acute{a}_i,r}^{in}(\xi)$	=	implementation cost and time of repair action r by internal resources (employed teams and equipment) in segment i of runway $\acute{a} \in A_2$ under the meteorological condition of scenario ξ, respectively
$b_{a,r}^{mn}, b_{\acute{a}_i,r}^{mn}$	=	maintenance cost of taxiway $a \in A_1$ and segment i of runway $\acute{a} \in A_2$ if repair action r is taken, respectively
$T^{max}(\xi)$	=	maximum allowed repair time under scenario ξ
B	=	total budget
$\varphi_a(\xi), \Phi_a(\xi)$	=	pre- and post-repair damage state of taxiway $a \in A_1$ under scenario ξ (=1 if

functional, and =0 otherwise), respectively

$\varphi_{\acute{a}_i}(\xi), \Phi_{\acute{a}_i}(\xi)$ = pre- and post-repair damage state of segment i of runway $\acute{a} \in A_2$ under scenario ξ (=1 if functional, and =0 otherwise), respectively

$l^u_{\acute{a}_i}$ = length of runway segments i of runway $\acute{a} \in A_2$

$l^{min}_{w,s}$ = MOS requirements (minimum required length of runways for maneuver type w and aircraft class s to use that runway)

$l^w_{a,\acute{a}}(\xi)$ = length of consecutive of post-repair active segments of runway $a \in A_2$ following (leading to) entrance (exit) taxiway $a \in A^c_{1,\acute{a}}$ under scenario ξ

$\sigma_a^{w,s}(\xi)$ = binary variable indicating whether or not $l^w_{a,\acute{a}}(\xi)$ is longer than $l^{min}_{w,s}$ of maneuver type w and aircraft type s under scenario ξ (= 1 if longer, and = 0 otherwise)

Pre-event decision variables:

τ_m = binary variable indicating that m teams are employed (= 1 if exactly m teams are employed and = 0 otherwise)

$\gamma_{e,n}$ = binary variable indicating if n units of equipment type e are purchased (= 1 if provided and = 0 otherwise)

Post-event decision variables:

$\pi^g(\xi)$ = binary variable indicating whether or not runway configuration g is selected under scenario ξ (= 1 if selected, and = 0 otherwise)

$\lambda^{ex}_{a,r}(\xi), \lambda^{in}_{a,r}(\xi)$ = binary variable indicating whether or not repair action r is taken by external and internal (airport repair team and equipment) resources, respectively, on taxiway $a \in A_1$ under scenario ξ (= 1 if taken and = 0 otherwise) , respectively

$\lambda^{ex}_{\acute{a}_i,r}(\xi), \lambda^{in}_{\acute{a}_i,r}(\xi)$ = binary variable indicating whether or not repair action r is taken by external and internal resources, respectively, on segment i of runway $\acute{a} \in A_2$ under scenario ξ (= 1 if taken and = 0 otherwise), respectively

$f_p^{g,w,s}(\xi)$ = post-repair flow rate along path p for runway configuration g, maneuver type w and aircraft type s under scenario ξ

Airport Resilience Problem-ARP:

$$max \;\; E_{\tilde{\xi}}[Z(\xi)] \;\; \text{s.t. \{resource limitations: (4)-(8)\}}, \quad (1)$$

where

$$Z(\xi) = \; max \sum_{w,g,s} \sum_{p \in P^{g,w,s}} f_p^{g,w,s}(\xi) \quad (2)$$

s.t.

{Taxiway capacity estimation: (9)-(16)
Runway capacity estimation: (17)-(20)
Operational constraints: (21)-(24-1,2)
Runway configuration selection: (25)-(27)
Taxiway/runway segment post-repair damage states: (28)-(32)
Repair period limit: (33)-(36)
Budget and post-repair flow restrictions: (37)-(39)}

Details of the formulation are given next.

4.2.1. Objective

The objective of the ARP is to maximize resilience. Airport resilience, $\alpha_{B,T^{max}}$, can be mathematically defined as in (3) as the expected fraction of total pre-event demand in terms of arrival and departure flows that can be met post-repair with repair time limitation T^{max} and budget B.

$$\alpha_{B,T^{max}} = \frac{E_{\tilde{\xi}}[\sum_{w,g,s}\sum_{p\in P^{g,w,s}} f_p^{g,w,s}(\xi)]}{\sum_{w,s} D^{w,s}}. \tag{3}$$

Noting that the denominator is a constant, the denominator can be dropped from the objective function and reintroduced after solution of the ARP is obtained.

4.2.2. Resource limitations

Resources in terms of repair crews and equipment must be put into place in advance if they are to support repair operations. While a virtually infinite supply of personnel and equipment can be obtained, limitations on the availability of these resources are applied to align with reasonable practice in a civilian airport environment and work-space restrictions that if violated would hamper productivity. While space may be copious in many locations, space may be limiting for airports located in highly populated locations and for those located in close proximity to water or other physical barriers. In the first stage, space for equipment storage is restricted. Binary first-stage variables as well as space limitations for equipment storage are defined through first-stage constraints (4)-(8). Note that teams are assumed to be homogeneous and trained for all considered repair options.

$$\sum_m \tau_m \leq 1 \tag{4}$$

$$\sum_n \gamma_{e,n} \leq 1, \qquad \forall e \in E \tag{5}$$

$\sum_{e,n} y_{e,n}\gamma_{e,n} \leq Y$ (6)

$\tau_m \in \{0,1\},$ $\quad m = 1, \dots, M$ (7)

$\gamma_{e,n} \in \{0,1\},$ $\quad \forall e \in E, n = 1, \dots, N_e$ (8)

4.2.3. Taxiway capacity estimation

For a given scenario, the capacity of a taxiway is a function of its original capacity under the same meteorological situation and whether or not it is functioning. A taxiway may be functioning if it was never damaged or if it was damaged and repaired. Specification of the capacity requires information about meteorological conditions and is thus given as a function of scenario: for each taxiway $a \in A_1$, $c_a(\xi)$ represents capacity in terms of number of aircraft that can be served in one direction per unit time under meteorological conditions present under scenario ξ assuming no damage. The capacity is set to zero if the taxiway is damaged and not repaired under the given scenario. Considering a flow of aircraft with minimum headways, the capacity $c_a(\xi)$ can be calculated based on the taxiway speed and minimum separation requirements under meteorological conditions of ξ (adapted from (Clayton and Capozzi, 2004)):

$$c_a(\xi) = \frac{\bar{v}_a(\xi)}{\bar{d}_a^{min}(\xi)}, \quad \forall a \in A_1 \quad (9)$$

where $\bar{v}_a(\xi)$ and $\bar{d}_a^{min}(\xi)$ are average taxiing speed (meters per hour) and minimum separation requirement (meters), respectively, for taxiway $a \in A_1$ under meteorological conditions of scenario ξ. A taxiway that is used alternatively for movements in both possible directions will have lower capacity than one used in only one direction, since the taxiway needs to be entirely cleared before a second aircraft may enter from the other direction. Therefore, the hourly capacity in a chosen direction given a single aircraft movement in the opposite direction will be reduced by

$2l_a/\bar{d}_a^{min}(\xi)$ [$=\bar{v}_a(\xi)/\bar{d}_a^{min}(\xi) \cdot 2l_a/\bar{v}_a(\xi)$], where l_a is the length of the taxiway (meters).

Let $x_a^{k,g}(\xi)$ be the flow in direction $k \in \{k_+, k_-\}$ of taxiway a under configuration g.

$$x_a^{k,g} = \sum_{w,s} \sum_{p \in P^{g,w,s}} \delta_{a,p}^{k,g,w,s} f_p^{g,w,s}(\xi), \qquad \forall g \in G, a \in A_1, k \in \{k_+, k_-\} \quad (10)$$

where k_+ and k_- refer to left and right directions under meteorological conditions of scenario ξ. Taxiway flow in either direction is limited by lower and upper bounds, $0 \leq x_a^{k,g}(\xi) \leq c_a(\xi)$.

In circumstances when two or more aircraft coming from the same direction are scheduled to use the same taxiway one directly after the other, a small capacity reduction is incurred. However, when consecutive aircraft movements along a taxiway are in opposing directions this capacity reduction is significantly larger. The rate of reduction in capacity per aircraft movement diminishes with increasing number of such movements. This is captured through consideration of tradeoffs in opposing flows. The tradeoff between flows by direction of any taxiway link can be written as $x_a^{k_+,g}(\xi) = F_a^{txc}\left[x_a^{k_-,g}(\xi)\right]$. This tradeoff can be viewed graphically using a taxiway directional capacity envelope depicted in Figure 4-1.

This depiction assumes a symmetric, nonconvex, piecewise linear function $F_a^{txc}(\cdot)$; it further presumes independence in the operation of taxiways, as well as runways. To address the non-convexity of $F_a^{txc}(\cdot)$, a piecewise linearization method of Sherali (2001) is employed in the formulation. As shown in the figure, the range on possible values of $x_a(\xi)$ is partitioned into three non-overlapping segments, $0 \leq 1 \leq$

$c_a(\xi) - \frac{2l_a}{\bar{d}_a^{min}(\xi)} \le c_a(\xi)$ for flows in both directions. Let $\beta_{a,v}^1$ and $\beta_{a,v}^2$ be disaggregated convex-combination weights associated to left and right endpoints of affine segment $v \in \{1,2,3\}$, and $\psi_{a,v}$ be a binary variable indicating whether or not flow takes a value within that segment in taxiway a (=1 if flow takes a value within that segment, and =0 otherwise).

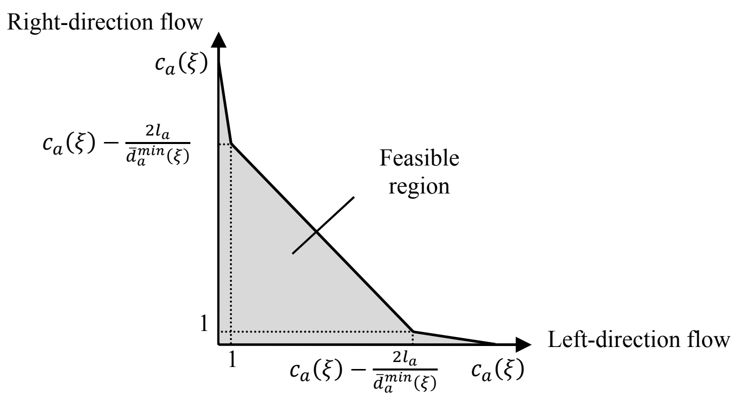


Figure 4-1 Directional capacity envelop in taxiways

Thus, $F_a^{txc}(\cdot)$ is given by constraints (11)-(15):

$$x_a^{k_+,g}(\xi) = \beta_{a,1}^2 + \beta_{a,2}^1 + [c_a(\xi) - \frac{2l_a}{\bar{d}_a^{min}(\xi)}](\beta_{a,2}^2 + \beta_{a,3}^1) + c_a(\xi)\,\beta_{a,3}^2, \quad \forall g \in G, a \in A_1 \quad (11)$$

$$x_a^{k_-,g}(\xi) \le c_a(\xi)\,\beta_{a,1}^2 + [c_a(\xi) - \frac{2l_a}{\bar{d}_a^{min}(\xi)}](\beta_{a,1}^2 + \beta_{a,2}^1) + \beta_{a,2}^2 + \beta_{a,3}^1 \quad \forall g \in G, a \in A_1 \quad (12)$$

$$\beta_{a,v}^1 + \beta_{a,v}^2 = \psi_{a,v}, \quad \forall a \in A_1, v \in \{1,2,3\} \quad (13)$$

$$\sum_{v=1}^{3} \psi_{a,v} = 1, \quad \forall a \in A_1 \quad (14)$$

$$\psi_{a,v} \in \{0,1\}, \quad \forall a \in A_1, v \in \{1,2,3\} \quad (15)$$

An advantage of this method to piecewise linearization is that it maintains a totally unimodular structure, allowing integrality constraints (15) to be relaxed (Sherali, 2001). A taxiway is assumed to function if it was never damaged or if it was damaged but repaired, i.e. $\Phi_a(\xi) = 1$; this is modeled through constraints (16).

$$x_a^{k,g} \leq c_a(\xi)\Phi_a(\xi), \qquad \forall g \in G, a \in A_1, k \in \{k_+, k_-\} \qquad (16)$$

4.2.4. Runway capacity estimation

Runway throughput rates are diminished when runways are used for both takeoffs and landings. They ultimately depend on the alternating pattern of maneuvers that is exercised. Likewise, minimum separation distances between aircraft using the runways, which directly affect runway flow rates, depend on the aircraft size mix and related wake vortex restrictions (Gilbo, 1993). Other physical impediments and operational dependencies, such as runway crossings, will further constrain flows. A common approach to modeling tradeoffs due to joint arrival-departure maneuvers on any runway and additional effects of capacity dependencies between runways is to use capacity envelopes. Capacity envelopes are given at the airport level and are estimated from historical data at the specific airport. The capacity envelopes are often convex and piecewise linear. They specify maximum effective arrival and departure flow rates achievable under chosen operating conditions by category (Visual Flight Rules (VFR), Marginal VFR (MVFR), Instrument Flight Rules (IFR), and Low IFR (LIFR)) and runway configuration, including runway dependencies (Gilbo, 1993). Noise and environmental ordinances may further limit capacities (Gilbo, 1993; FAA, 2004). Capacity envelopes are often developed from historical data at airports (e.g., (Frolow and Sinnott, 1989; Gilbo, 1993; Clayton and Capozzi, 2004), but optimization and statistical methods of estimation have also been proposed (Hu et al., 2007; Gilbo, 2003; Ramanujan, 2012; Jiang et al., 2011).

Let $\acute{x}^{g,w}(\xi)$ be the total flow of maneuver type $w \in \{arr, dep\}$ through the set of runways exploited for configuration g, A_2^g, under meteorological conditions of scenario ξ.

$$\acute{x}^{g,w}(\xi) \ = \sum_s \sum_{p \in P^{g,w,s}} \sum_{\acute{a} \in A_2} \acute{\delta}_{\acute{a},p}^{g,w,s} f_p^{g,w,s}(\xi) \quad \forall g \in G, w \in \{arr, dep\} \qquad (17)$$

Convex capacity envelopes of a runway configuration g can be given in terms of the total arrival flow as a function of total departure flow through active runways.

$$\acute{x}^{g,arr}(\xi) \ = F_g^{oc}[\acute{x}^{g,dep}(\xi)\,], \qquad \forall g \in G \qquad (18)$$

Typical airport-level capacity envelopes as produced by the FAA (2004) are presented in Figure 4-2 for a specific runway configuration under VFR and IFR conditions. As depicted in the figure, when the number of arrivals and/or departures is small, or when there are significantly more of one type than the other, runway capacities for both arrivals and departures remain at their highest levels. However, when there is significant mixing of both arrivals and departures, arrival and departure capacities decline. Because runway capacities are generally more restrictive than taxiway capacities, overall airport capacity is governed by the runway capacities.

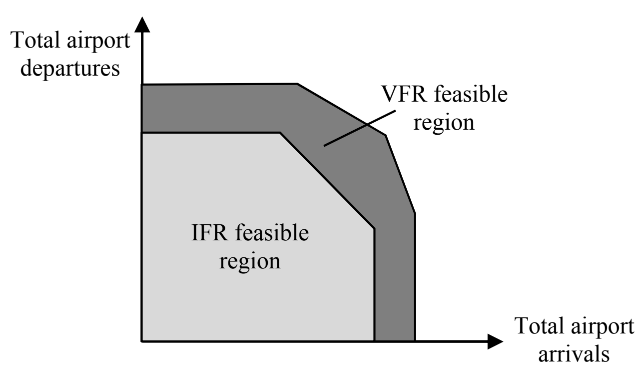


Figure 4-2 Typical airport-level capacity envelops given runway configuration (FAA, 2004)

In addition to imposing capacity restrictions at the airport-level, capacities of individual runways are further restricted. The individual runway capacity envelopes have similar form to that of the capacity envelope of a given runway configuration. Let $\acute{x}_{\acute{a}}^{g,w}(\xi)$ be the flow of maneuver type $w \in \{arr, dep\}$ in a runway $\acute{a}$ using configuration g under meteorological conditions of scenario ξ.

$$\acute{x}_{\acute{a}}^{g,w} = \sum_s \sum_{p \in P^{g,w,s}} \acute{\delta}_{\acute{a},p}^{g,w,s} f_p^{g,w,s}(\xi) \qquad \forall g \in G, \acute{a} \in A_2, w \in \{arr, dep\} \qquad (19)$$

The associated capacity limitations can be mathematically formulated as in (20).

$$\acute{x}_{\acute{a}}^{g,arr}(\xi) = F_{\acute{a}}^{rc}[\acute{x}_{\acute{a}}^{g,dep}(\xi)], \qquad \forall g \in G, \acute{a} \in A_2 \qquad (20)$$

where $F_{\acute{a}}^{rc}(\cdot)$ is the capacity envelope of an individual runway $\acute{a} \in A_2$ representing the arrival flow as a function of departure flow in an individual runway $\acute{a} \in A_2$ under meteorological conditions present under scenario ξ. The effects of disaster impact on runway operations are described in the operational constraints section.

4.2.5. Operational constraints

Capacities of individual runways given the occurrence of a disruption scenario must be modeled. A runway will be affected by any damage it incurs, but it may be partially or fully operational for certain purposes if an MOS remains or is restored due to the intelligent selection of repair actions within the runway when multiple segments have sustained damage. It is recognized that in civilian airports (unlike military applications) the use of a runway that has incurred damage to any portion is typically prohibited. Nonetheless, the concept of MOS may be relevant in extreme conditions and is, therefore, considered by the model for the sake of generality. It is easily annulled by equating the MOS with the full (pristine) runway length.

The capacity of a runway with given MOS for a given maneuver and aircraft class is nonzero only if the runway meets the minimum length requirement for that maneuver type and aircraft class. This is captured through constraints (21)-(29). The flow rate associated with a given maneuver type and aircraft class under a chosen configuration is permitted along an entrance (exit) taxiway $a \in A^c_{1,\acute{a}}$ only if the length of combination of post-repair active segments of the connected runway $\acute{a} \in A_2$ following (leading to) that taxiway, $l_{a,\acute{a}}(\xi)$, meets corresponding minimum length requirements, i.e. $\sigma^{w,s}_a(\xi) = 1$.

$$\sum_{p \in P^{g,w,s}} \delta^{k,g,w,s}_{a,p} f^{g,w,s}_p(\xi) \leq C\sigma^{w,s}_a(\xi), \quad \forall a \in A^c_{1,\acute{a}}, \acute{a} \in A_2, k \in \{k_+, k_-\}, g \in G, w \in W, s \in S \quad (21)$$

$$l^{min}_{w,s}\sigma^{w,s}_a(\xi) \leq l_{a,\acute{a}}(\xi) \quad \forall w \in W, s \in S, a \in A^c_{1,\acute{a}}, \acute{a} \in A_2 \quad (22)$$

$$\sigma^{w,s}_a(\xi) \in \{0,1\}, \quad \forall w \in W, s \in S, a \in A_1 \quad (23)$$

The functional runway length $l_{a,\acute{a}}(\xi)$ is calculated through the following nonlinear equation:

$$l_{a,\acute{a}}(\xi) = \sum_{o \in I^a_{\acute{a}}} [\prod^o_{j=i^*} \Phi_{\acute{a}_j}(\xi)] l^u_{\acute{a}_o} \quad \forall o \in I^a_{\acute{a}}, a \in A^c_{1,\acute{a}}, \acute{a} \in A_2 \quad (24)$$

where that i^* is the segment of set $I^a_{\acute{a}}$ at which taxiway a is connected to runway $\acute{a}$. The term $\prod^o_{j=i^*} \Phi_{\acute{a}_j}(\xi)$ in (24) is a source of nonlinearity. Let

$$\Phi_{\acute{a}_{i^*,i^*+1,\ldots,o}}(\xi) = \prod^o_{j=i^*} \Phi_{\acute{a}_j}(\xi), \quad \forall o \in I^a_{\acute{a}}, a \in A^c_{1,\acute{a}}, \acute{a} \in A_2$$

where variables $\Phi_{\acute{a}_{i^*,i^*+1,\ldots,o}}(\xi)$ indicate whether or not all binary variables, $\Phi_{\acute{a}_j}(\xi)$, for j from i^* to o, are equal to one. Hence, linearization can be achieved through replacement of constraints (24) by constraints (24-1) and (24-2).

$$l_{a,\acute{a}}(\xi) = \sum_{o \in I^a_{\acute{a}}} l^u_{\acute{a}_o} \Phi_{\acute{a}_{i^*,i^*+1,\ldots,O}}(\xi), \quad \forall o \in I^a_{\acute{a}}, a \in A^c_{1,\acute{a}}, \acute{a} \in A_2 \quad (24\text{-}1)$$

$$\Phi_{\acute{a}_{i^*,i^*+1,\ldots,o}} \leq \Phi_{a_j}(\xi), \quad \forall j \in \{i^*, i^*+1, \ldots, o\}, o \in I^a_{\acute{a}}, a \in A^c_{1,\acute{a}}, \acute{a} \in A_2 \quad (24\text{-}2)$$

4.2.6. Runway configuration selection

Constraints (25)-(27) are imposed in the model to select a single configuration under realized scenario conditions. These constraints further ensure that no flow can be shipped along paths that are not available given the chosen configuration.

$$\sum_{w,s} \sum_{p\in P^{g,w,s}} f_p^{g,w,s}(\xi) \leq C\pi^g(\xi), \qquad \forall g \in G \tag{25}$$

$$\sum_g \pi^g(\xi) = 1 \tag{26}$$

$$\pi^g(\xi) \in \{0,1\}, \qquad \forall g \in G \tag{27}$$

4.2.7. Taxiway/runway segment post-repair damage states

Post-repair damage states of taxiway and runway segments, $\Phi_a(\xi)$ and $\Phi_{\acute{a}_i}(\xi)$, are determined as functions of corresponding pre-repair damage states, $\varphi_a(\xi)$ and $\varphi_{\acute{a}_i}(\xi)$, as well as repair actions taken though constraints (28)-(32), respectively.

$$\Phi_a(\xi) = [1 - \varphi_a(\xi)]\{\sum_r \rho_{\xi,r} \cdot [\lambda_{a,r}^{in}(\xi) + \lambda_{a,r}^{ex}(\xi)]\} + \varphi_a(\xi), \qquad \forall a \in A_1 \tag{28}$$

$$\Phi_{\acute{a}_i}(\xi) = [1 - \varphi_{\acute{a}_i}(\xi)]\{\sum_r \rho_{\xi,r} \cdot [\lambda_{\acute{a}_i,r}^{in}(\xi) + \lambda_{\acute{a}_i,r}^{ex}(\xi)]\} + \varphi_{\acute{a}_i}(\xi) \qquad \forall \acute{a} \in A_2, i \in I_{\acute{a}} \tag{29}$$

$$\lambda_{a,r}^{in}(\xi), \lambda_{a,r}^{in}(\xi), \lambda_{\acute{a}_i,r}^{ex}(\xi)\lambda_{\acute{a}_i,r}^{ex}(\xi) \in \{0,1\}, \qquad \forall a \in A_1, \acute{a} \in A_2, i \in I_{\acute{a}}, r \in R \tag{30}$$

$$\sum_r \rho_{\xi,r} \cdot [\lambda_{a,r}^{ex}(\xi) + \lambda_{a,r}^{in}(\xi)] \leq 1, \qquad \forall a \in A_1 \tag{31}$$

$$\sum_r \rho_{\xi,r} \cdot [\lambda_{\acute{a}_i,r}^{ex}(\xi) + \lambda_{\acute{a}_i,r}^{in}(\xi)] \leq 1, \qquad \forall \acute{a} \in A_2, i \in I_{\acute{a}} \tag{32}$$

Repair actions that cannot be applied under the given meteorological conditions are precluded. The possibility of outsourcing one or more repair jobs is permitted through these constraints. Only one repair action, whether through internal or external resources, can be chosen for each taxiway/runway segment as guaranteed through constraints (31)-(32).

4.2.8. Repair period limit

The resilience of the airport pavement network, and thereby operational capacities, are evaluated at time T^{max}. Thus, repair actions can affect only capacities of those damaged segments that have been repaired by time T^{max}. T^{max} can be a function of the meteorological conditions that exist under each scenario; hence, repair implementation time constraints are imposed in the model's second stage. A range on T^{max} between four and 24 hours may be reasonable for given circumstances associated with civilian applications. In rapid runway repair applications associated with military operations, a duration limit of four to seven hours may be more appropriate (Duncan, 2007). The longer the duration required to attain reasonable airport runway and taxiway capacity rates comparable to pre-event rates, the greater the potential monetary losses due to forced cancellations and diversions. Repair time limitation is captured through second-stage constraints (33)-(36).

$$\sum_{a\in A_1,r} \rho_{\xi,r}\acute{\rho}_{e,r} q^{in}_{a,r}(\xi)\lambda^{in}_{a,r}(\xi) + \sum_{\acute{a}\in A_2,i,r} \rho_{\xi,r}\acute{\rho}_{e,r} q^{in}_{\acute{a}_i,r}(\xi)\lambda^{in}_{\acute{a}_i,r}(\xi) \leq T^{max}(\xi)\cdot\left[\sum_{n\in N_e} n\gamma_{e,n}\right], \quad \forall e \in E \quad (33)$$

$$\sum_{a\in A_1,r} \rho_{\xi,r} q^{in}_{a,r}(\xi)\lambda^{in}_{a,r}(\xi) + \sum_{\acute{a}\in A_2,i,r} \rho_{\xi,r} q^{in}_{\acute{a}_i,r}(\xi)\lambda^{in}_{\acute{a}_i,r}(\xi) \leq T^{max}(\xi)\cdot\left[\sum_{m} m\tau_m\right], \quad (34)$$

$$\sum_{r} \rho_{\xi,r} q^{ex}_{a,r}(\xi)\lambda^{ex}_{a,r}(\xi) \leq T^{max}(\xi), \quad \forall a \in A_1 \quad (35)$$

$$\sum_{r} \rho_{\xi,r} q^{ex}_{\acute{a}_i,r}(\xi)\lambda^{ex}_{\acute{a}_i,r}(\xi) \leq T^{max}(\xi), \quad \forall \acute{a} \in A_2, i \in I_{\acute{a}} \quad (36)$$

Constraints (33)-(34) restrict the use of each piece of available equipment to only one damage location at any point in time, and permit the simultaneous use of multiple pieces of the same equipment type at multiple damage sites when more than one team exists that can be allocated to repair activities. In constraints (35)-(36), the possibility of outsourcing one or more repair jobs is permitted.

4.2.9. Budget and post-repair flow restrictions

Budget constraints are included within the model to guarantee that the budget available for both preparedness and recovery actions is not exceeded. The budget can be allocated (through first stage decisions) to preparedness actions that support repair operations. All or some part of the budget may be kept in reserve to address damage post-event. If teams and equipment have been put in place as a preparedness strategy, they can service repair tasks. If all or some portion of the repairs will require external resources, a portion of the budget must be reserved for this purpose. This is imposed through the budget constraint (37), to be satisfied under each individual scenario.

$$\sum_h b_h^{tm}\tau_h + \sum_{e,n} b_{e,n}^{eq}\gamma_{e,n} + \sum_{a\in A_1,r}\rho_{\xi,r}\cdot\{[b_{a,r}^{ex}(\xi) + b_{a,r}^{mn}]\lambda_{a,r}^{ex}(\xi) + [b_{a,r}^{in}(\xi) + b_{a,r}^{mn}]\lambda_{a,r}^{in}(\xi)\} + \sum_{\acute{a}\in A_2,i,r}\rho_{\xi,r}\cdot\{[b_{\acute{a}_i,r}^{ex}(\xi) + b_{\acute{a}_i,r}^{mn}]\lambda_{\acute{a}_i,r}^{ex}(\xi)\,[b_{\acute{a}_i,r}^{in}(\xi) + b_{\acute{a}_i,r}^{mn}]\lambda_{\acute{a}_i,r}^{in}(\xi)\} \leq B \qquad (37)$$

Whether undertaken internally or externally, multiple repair options may exist for addressing certain damage. Consider, for example, that a pothole can be repaired through a temporary fill or by repaving a portion of the affected pavement segment. The duration of the repair and, thus, the long-term costs of addressing the damage, are included through an additional maintenance or replacement cost accounted for in the budget constraint.

Finally, arrival and departure flow rates by aircraft class s are restricted to be less than the corresponding pre-event demand through constraints (38). Flow is restricted to be non-negative in constraints (39).

$$\sum_{p\in P^{g,w,s}} f_p^{g,w,s}(\xi) \le D^{w,s}, \qquad \forall g \in G, w \in W, s \in S \qquad (38)$$

$$f_p^{g,w,s}(\xi) \ge 0, \qquad \forall g \in G, w \in W, s \in S, p \in P^{g,w,s} \qquad (39)$$

4.3. Solution Methodology

The ARP is formulated as a two-stage stochastic program with binary first-stage and binary and integer second-stage decision variables. To solve the ARP, an effective, exact solution methodology, the integer L-shaped method developed by Laporte and Louveaux (1993), is applied. This approach has been used to address a myriad of problems arising in a host of arenas.

The integer L-shaped method decomposes the original program into a master problem (MP) and set of subproblems (SPs) each of which relates to a realization of the network, i.e. a network state:

(MP)

$$max\ \theta \qquad (40)$$

s.t.

(4)-(6)

$$f\left(\gamma_{e,n}, \tau_m, \theta\right) \le 0 \qquad (41)$$

$$\sum_h b_h^{tm}\tau_h + \sum_{e,n} b_{e,n}^{eq}\gamma_{e,n} \le B \qquad (42)$$

$$0 \le \tau_m \le 1, \qquad m = 1, \dots, M \qquad (43)$$

$$0 \le \gamma_{e,n} \le 1, \qquad \forall e \in E, n = 1, \dots, N_e \qquad (44)$$

(SPs)

$$Z(\xi) = \max_{\lambda} \sum_{w,g,s} \sum_{p \in P^{g,w,s}} f_p^{g,w,s}(\xi) \quad \text{s.t.} \tag{45}$$

(10)-(39),

where θ in is the approximation of expected second-stage objective function in MP. Constraints (41) are optimality cuts generated during iterations. To ensure the feasibility of budget constraints of the SPs, constraint (42) is added to the MP to limit the allocated budget for provision of equipment and teams to be less than the total budget. With constraints (42), the solution of the MP always results in a feasible solution for the original problem. Decision variables, $\gamma_{e,n}$ and τ_m, are given through solution of the MP and are, therefore, fixed in each SP.

An optimality cut is a function of θ, absolute upper bound of subproblems UB, first-stage decision variables and expectation of second-stage objective functions. A tight upper bound can speed up the solution process. The method requires the problem to be bounded to a finite UB, which can be obtained from the total demand rate over all aircraft classes, i.e. $UB = \sum_{w,s} D^{w,s}$. Let χ_ω represent all first-stage variables for $\omega \in \Omega$, where $\Omega = \{\gamma_{e,n}\}_{\forall e,n} \cup \{\tau_m\}_{\forall m}$. The number of feasible first-stage solutions is limited due to their binary nature, here indexed by $\epsilon = 1,2, \dots, \mathrm{E}$; thus, the algorithm is guaranteed to converge in a finite number of steps. First, solution of the MP with relaxed integrality constraints, given in (43)-(44), continues through a Branch-and-Bound process until a feasible solution is achieved (binary solutions for all first-stage variables χ_ω). Let $\Omega_\epsilon = \{\omega | \chi_\omega = 1\}$, $\overline{\Omega}_\epsilon = \{\omega | \chi_\omega = 0\}$ and θ_ϵ^{SP} be the expectation over second-stage objective functions corresponding to feasible first-stage solution ϵ. Thus, (46) provides a valid optimality cut.

$$\theta \leq [UB - Q_{\epsilon}(\chi)](|\Omega_{\epsilon}| - 1) - [UB - Q_{\epsilon}(\chi)]\left[\sum_{\omega \in \Omega_{\epsilon}} \chi_{\omega} - \sum_{\omega \in \bar{\Omega}_{\epsilon}} \chi_{\omega}\right] + UB, \tag{46}$$

where $Q_{\epsilon}(\chi)$ is the expectation over second-stage objective functions corresponding to feasible first-stage solution ϵ.

Details of the integer L-shaped method specified for the ARP are outlined next following a similar structure to the description of the general integer L-shaped method presented by Laporte and Louveaux (1993). Let z be the objective value of the original problem and $\underline{z}$ be the lower bound of z:

Step 0: Set $\mu = 0$ and $\underline{z} = 0$. The value of θ is set to an absolute upper bound. A list is created that contains only a single pendant node corresponding to the initial subproblem.

Step 1: Select a pendant node in the list to specify the current problem; if the pendant node list is empty, stop.

Step 2: Set $\mu = \mu + 1$. Solve the current problem. If the current problem has no feasible solution, fathom the current node; go to *Step 1*. Otherwise, let $(\chi^{\mu}, \theta^{\mu})$ be an optimal solution.

Step 3: Check for integrality. If integrality is violated, create two new branches in which the most fractional variable is set to 0 or 1. Append the two nodes to the pendant node list, and go to *Step 1.*

Step 4: Solve the sub-problems and compute $Q(\chi^{\mu})$. $z^{\mu} = Q(\chi^{\mu})$. If $z^{\mu} > \underline{z}$, update lower bound $\underline{z} = z^{\mu}$.

Step 5: If $\theta^{\mu} \leq Q(\chi^{\mu})$, then fathom the current node and go to *Step 1*; otherwise, impose an optimality cut to the MP, and return to *Step 2*.

4.4. Framework for a decision support tool

The ARP is modeled with the fidelity needed to capture the many important operational considerations associated with airport taxiway and runway operations. To support airport operators in decisions pertaining to investment in preparedness and response, as well as other matters associated with creating a resilient airport pavement system, the model and solution methodology can be embedded within a decision support tool. A schematic overview of such a tool is depicted in Figure 4-3.

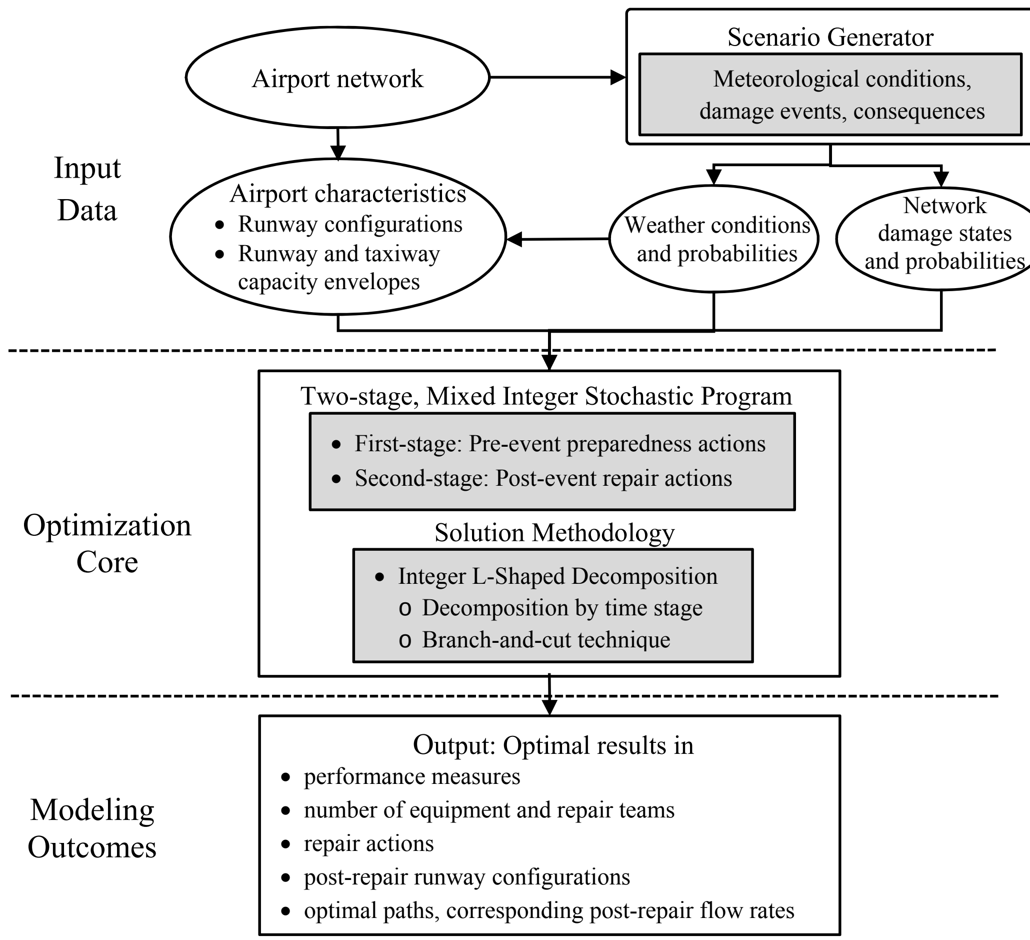


Figure 4-3 Schematic overview of the decision support framework.

As can be seen, the computational process is decomposed into three steps: (i) input data, including scenario generation; (ii) optimization core, consisting of the mathematical formulation and exact solution methodology described in Sections 2 and

3; and (iii) modeling outcomes. Details associated with the input and output steps are presented next. Two key input category requirements of the tool are stipulated: airport characteristics and a host of possible meteorological - damage scenarios. As means of expressing the interrelations between damage characteristics and meteorological conditions, conditional probabilities are employed in generating scenarios. The probability of each scenario is assumed to be known *a priori*. Thus, the probability of scenario ξ, $p(\xi)$, can be computed from conditional probability as in (47).

$$p(\xi) = p(\xi | disruption\ type) \cdot p(disruption\ type | meteorological\ condition) \cdot p(meteorological\ condition) \tag{47}$$

The scenario generation process is summarized in the flowchart of Figure 4-4. Using the computational results, performance indicators, such as measured post-repair capacity rates for take-offs and landings that can be achieved through the inherent coping capacity of the system and use of limited funds for preparedness and recovery actions, can be computed. Application of the tool culminates in equipment purchase or lease decisions, number of repair teams to train, repair actions to be taken for each generated scenario, post-repair taxiway and runway capacities in terms of potential flow rates and performance measures including coping capacity and resilience.

Given that the resilience of the airport pavement network is evaluated at time T^{max} (see Section 2), a range of T^{max} values between four and 16 hours is explored in the following section. Restricting T^{max} to less than a day reflects the criticality for the airport to provide operational continuity and low tolerance for shutdowns or delays. Additionally, this restriction precludes the model from considering major reconstruction or infrastructure enhancement projects as ‘legitimate’ coping options.

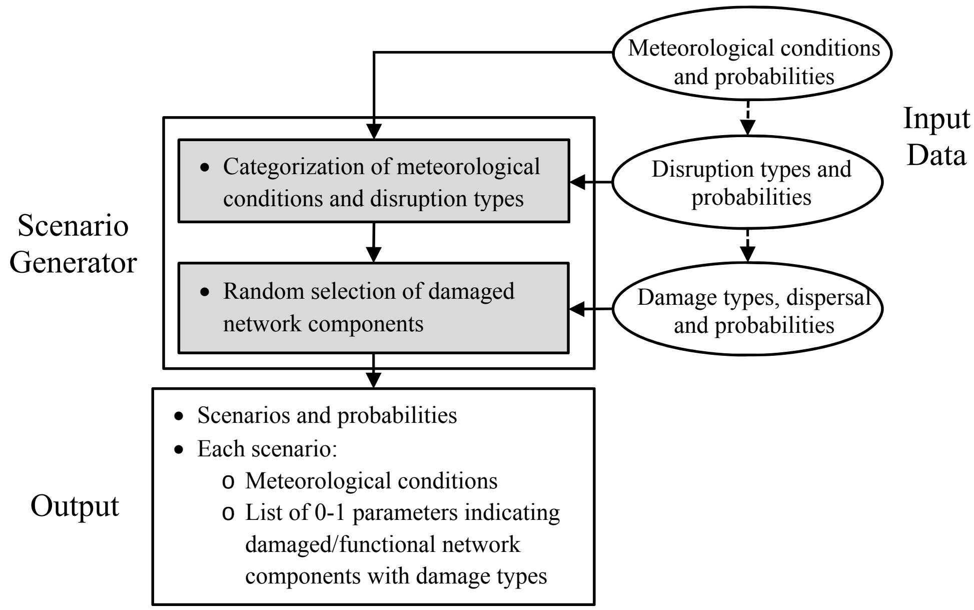


Figure 4-4 The flowchart of scenario generator

4.5. Illustrative case study

4.5.1. Case study details

The mathematical solution framework through its inclusion in the decision support tool is demonstrated in an application on a pavement network modeled from New York's LaGuardia Airport (LGA). Referring to Figure 4-5, the pavement network consists of two intersecting runways (04-22 and 13-31), each about 2,100 meters long, supported by an array of taxiways. The airport is represented with 68 nodes and 104 links, consisting of 10 dummy, 2 runway, and 92 taxiway links. The runway links are partitioned at 100 meter intervals with a shared segment at the crossing; the structural section for all segments is assumed asphaltic.

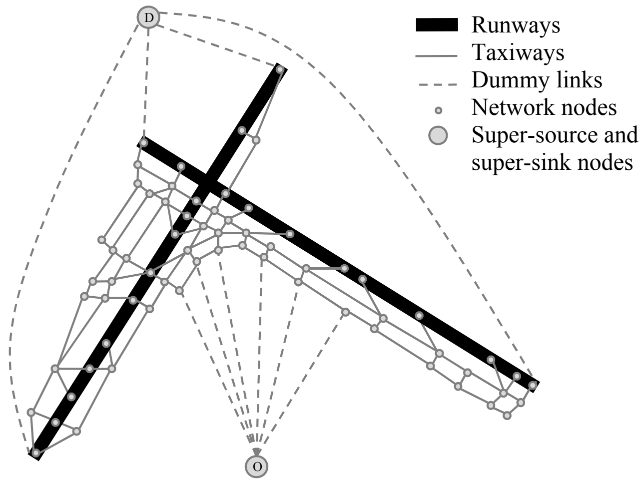
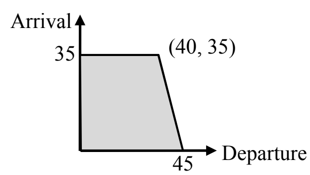
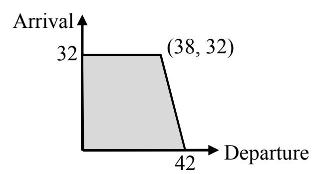


Figure 4-5 Study network representation.

It is presumed that the airport serves a fleet consisting of two aircraft classes like the Boeing 737 and Airbus A319, with average weights of 50 and 65 tons, respectively. The first (second) aircraft class requires 1700 (700) meters and 2000 (1800) meters for landing and takeoff, respectively. These values are specifically for LGA with nearly sea-level elevation under normal meteorological conditions. Landing length requirements should be increased by 15% for wet and slippery pavement conditions (FAA, 2006). The capacity of the airport taxiways is taken a computed maximum of 200 aircraft per hour based on 15 kilometers per hour average speed and a minimum separation distance of 100 meters. Taxiway links are categorized into three taxiway length classes, namely 50, 100, and 200 meters. These classes are used in computing capacity reduction in one direction due to flow in the opposite direction; taxiway directional capacity envelopes are captured by constraints (10)-(17).

In theory, there are 12 directional configurations by which the two runways can be utilized for takeoffs and landings: 04|04, 04|13, 04|31, 22|22, 22|13, 22|31, 13|13, 13|04, 13|04, 31|31, 31|04, and 31|22. In this notation, the first (left) number indicates runway threshold for arrival (landing) and the second (right) number indicates runway starting point for departure (takeoff). For this specific airport, the most commonly used runway configurations are 22|13 and 4|13 (FAA, 2004). Two additional configurations are considered for this case study as they may be preferable under certain disruption scenarios: 4|4 and 13|13. Table 4-1 provides information about the performance of the airport runways under each of these four possible directional configurations (developed from information from (FAA, 2004)). The capacity envelopes for configurations 4|4 and 13|13 also represent capacity envelopes of single runways 04-22 and 13-31, respectively. The capacity envelopes depend also on the visibility conditions in terms of Visual Flight Rules (VFR) and Instrument flight rules (IFR).

Table 4-1 Capacity envelopes for different runway configurations and visibility conditions

<table>
<tr><th rowspan="2">Runway configuration</th><th colspan="2">Capacity envelopes (aircraft per hour)</th></tr>
<tr><th>VFR</th><th>IFR</th></tr>
<tr><td>4|4
13|13</td><td>
</td><td>
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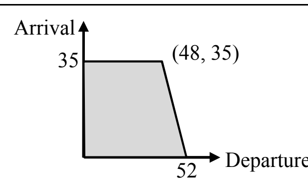
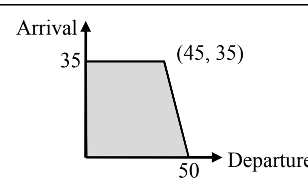
<tr><td>4|13
22|13</td><td>
</td><td>
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The meteorological conditions associated with the scenarios are selected in accordance with the airport location. For LGA, located in a bay area on the waterfront and sheltered from the North Atlantic Ocean by Long Island, six viable combinations of temperature, precipitation, and visibility levels were chosen (Fisk, 2012): (i) very hot, no precipitation, VFR (with probability 0.05); (ii) very hot, high precipitation, VFR (0.05); (iii) moderate temp., no precipitation, VFR (0.65); (iv) moderate temp., high precipitation, IFR (0.15); (v) very cold, no precipitation, VFR (0.05); and (vi) very cold, snow/ice, IFR (0.05). As can be seen, temperature level is categorized qualitatively as: very hot, moderate, and very cold. Precipitation intensity is also addressed qualitatively as either low or high. Visibility is categorized as: IFR and VFR.

Next, the scenario generator randomly produces ‘disruption events’ that lead to damage (out of the four generic types). Probabilities of disruption events in the following example are selected to be as realistic as possible. Nonetheless, they are given purely for demonstration purposes. As shown in Table 4-2, eight specific disruption events are considered in the case study, falling under one of three categories arising with given probabilities: extreme climatic or geologic event (flood, snow/ice, and extreme heat; probability 0.19), operational events (oil spill, overloading, and jet blast; probability 0.69), and intentional malicious acts (guided and unguided attacks; probability 0.12).

The fourth category mentioned in Section 4.1 dealing with natural deterioration (in combination with ill-timed maintenance) was excluded for the example. It is noted that the probabilities of the weather-related disruptions (i.e. first category) are determined through their conditional probabilities with respect to the six meteorological situations. Ten possible damage types resulting from the disruption events were identified and are presented in Table 4-2. The numbers in parentheses indicate the maximum number of affected segments. In this connection, the damaged segments used in the simulation were randomly generated between zero and the maximum number of affected segments (in each case) assuming a uniform probability density function. Overall, 36 possible combinations could be pooled from the 8 disruption events (Table 4-2) and 6 meteorological conditions, disregarding unlikely combinations, like snow/ice under the first meteorological condition (very hot, no precipitation). Ten scenarios were randomly generated from each of these 36 combinations, resulting in a total of 360 scenarios.

Table 4-2 Disruption events, probabilities and resulting damage

Disruption event		Probability	Alligator cracking	Block cracking	Transverse cracking	Jet Blast	Raveling	Rutting	Potholes	Single [illegible]	Slippery surface	Bleeding
Extreme climatic or geologic event	Flood	0.095	-	-	-	-	-	-	-	-	(20)	-
	Snow/ice	0.050	-	(15)	(30)	-	-	-	-	-	(5)	-
	Very hot	0.045	-	-	-	-	-	(2)	-	-	-	(4)
Operational events	Oil spill	0.280	-	-	-	-	(5)	-	-	-	-	-
	Overloading	0.220	(10)	-	-	-	-	-	-	-	-	-
	Jet Blast	0.190	-	-	-	(3)	-	-	-	-	-	-
Intentional malicious acts	Guided attack	0.060	-	-	-	-	-	-	(2*)	(3)	-	-
	Unguided attack	0.060	-	-	-	-	-	-	(10)	-	-	-

*Indicates that consecutive segments affected; otherwise, damage need not be adjoining.

Damage type linked to each disruption event was predicted based on the underlying governing mechanisms (Shahin, 2005; ASTM, 2011). For example, alligator cracking is a fatigue-related distress and hence linked to pavement overloading. Excessive rutting and bleeding are more likely to develop in asphalt pavements when surface temperatures are abnormally high causing bitumen expansion and loss of mix stability. Slippery surface conditions can be associated with ice/snow events.

Table 4-3 lists 19 types of machinery/tools that are required for damage repair; also listed are the annual lease and maintenance costs associated with each item. The latter were computed from hourly rates and number of working hours as suggested in (US Army Corps of Engineers, 2011). As a starting point, the airport is assumed to self-own one of each of the first 15 items on the list. The cost associated with self-owned equipment is disregarded in the model assuming they serve in non-emergency situations. To address their disaster-related costs, 10% of annual purchase, depreciation and maintenance costs are considered in the table for the additional four equipment items on the list, as well as for adding units from the self-owned items.

Table 4-3 Repair equipment

Identification #	Equipment	Cost ($/year)	Identification #	Equipment	Cost ($/year)
1	Small asphalt paver	8100	11	Tack coat sprayer	1800
2	Mechanical sweeper	8500	12	Seal injector/melter	1500
3	Small milling machine	6500	13	Crack chasing saw	2700
4	Small asphalt roller	1400	14	Small mixer	1350
5	Asphalt cutter	1700	15	Water pump	1350
6	Salt Sprinkler	2550	16	Large milling machine	9500
7	Snow shovel	4800	17	Vibratory roller	2210
8	Front loader	8160	18	Motor grader	25000
9	Backhoe	13600	19	Large asphalt paver	32000
10	Dump truck	11650			

It is further presumed that five repair crews can be assembled from existing employees with regular duties associated with non-emergency day-to-day operations. Table 4-4 defines the repair actions required for every damage type out of the possible ten (see Table 4-2) for a single taxiway or runway segment. In each case, the table identifies the equipment needed (from Table 4-3), the nominal repair time, and the nominal cost involved if the work is done with internal resources. The costs of repair actions with internal resources include associated operating costs of required equipment, e.g. fuel, FOG (filter, oil and grease), and tire wear, and cost of employing teams (US Army Corps of Engineers, 2011). Given a typical team of eight people and an hourly labor rate of $40, the hourly team cost for completing repairs is $320. Additional costs of $18,000 are associated with quarterly training, certifying repair crew personnel, and associated position backfill.

Table 4-4 Repair actions, implementation costs and execution times

Damage types to be repaired	Repaired Internally			Repaired externally		Weather-dependent multiplier for repair duration and costs					
	Equipment set requirement	Duration (hr)	Cost ($)	Duration (hr)	Cost ($)	1	2	3	4	5	6
Alligator cracking	1,2,4,8,10,15,16	5	2510	9	4267	1	10	1	10	1.5	2
Block cracking	2,11,12	2	736	6	1251	1	10	1	10	1	10
Transverse cracking	2,11,12	2	736	6	1251	1	10	1	10	1	10
Jet Blast	2,4,5,8,9,10,15	4	1912	8	3250	1	10	1	10	1.5	2
Raveling	2,4,5,8,9,10,15	4	1912	8	3250	1	10	1	10	1.5	2
Rutting	2,4,5,8,9,10,15	4	1912	8	3250	1	10	1	10	1.5	2
Array of small potholes	1,2,4,5,6,16	3	1407	7	2391	1	10	1	10	1.5	2
A single crater	1,2,3,4,5,6,7,15,16, 17,19	6	4374	10	7435	1	10	1	10	1.5	2
Slippery surface	2,4,14,15,18	1	461	5	783	1	1.5	1	1.5	2	10
Bleeding	2,4,5,6,13,17	3	1665	7	2830	1	10	1	10	1.5	2

The option of external repair is also included, in which case it can be noticed that the repair duration and cost are higher; equipment scarcity is not an issue if repairs are externally completed. Repair action costs using external resources are presumed to be 70% higher than repairs made through internal resources. Four hours more are added to the implementation duration to account for the time needed to arrange for the services and for the services to mobilize. Table 4-4 also includes weather-dependent multipliers, which are unitless numbers that depend on one of the six meteorological conditions; they multiply both the duration and costs (in each case) to represent escalation in costs and implementation times when repairing damage under sub-optimal weather conditions. A multiplier of 10 essentially means that the weather conditions preclude taking the associated repair action.

4.5.2. Tool output

Results from runs of the tool on the study airport are hereafter presented and discussed. While data are presented and analyzed, the main aim here is to demonstrate the tool's capabilities created through the proposed mathematical modeling and solution framework and types of general support that it can provide. Before producing the results, the model was first verified by checking consistency with capacity envelopes. That is, under normal operating conditions (i.e. meteorological condition 3 given probability-one of no disruption and zero budget for preparedness and recovery actions) the model produced the maximum total flow rate (i.e. number of takeoffs and landings) of 64 maneuvers per hour. This maximum flow rate was used as a benchmark for assessing subsequent performance metrics, such as coping capacity and resilience, and is consistent with FAA runway capacity estimates (FAA, 2004) for the study airport. It provides the denominator for the computation of resilience in equation (3).

The first decision question addressed using the tool deals with trade-offs between annual budget and T^{max}. For the case study, the outcome is depicted both numerically and graphically in Figure 4-6. The resilience indifference curves are plotted for a budget in the range of 0 to \$100,000 in combination with T^{max} in the range of 0 to 16 hours. To generate this figure, T^{max} was varied by four hour intervals (i.e. 0, 4, 8, 12, and 16) and only four budget levels were considered (0, \$25,000, \$50,000, and \$100,000). Intermediate levels of resilience shown were interpolated with a bilinear scheme. As can be seen, the resilience level ranged between a minimum of

0.54 (no budget, or short T^{max}, or both) and a maximum of 0.88 (maximal budget and long T^{max}).

The minimal value of 0.54 is the airport's coping capacity (akin to zero-budget resilience); it indicates that about 54% of the pre-event takeoff and landing flow rate can be achieved in expectation over the 360 random scenarios if no recovery actions are considered in evaluating the system's performance. The performance of the airport pavement network was investigated further for an annual budget of $25,000 and T^{max} of 8 hours. In this case, the performance improves to about 0.67 if repair actions can be taken externally and 0.68 if this is further supplemented by the use of existing equipment by teams that are trained through preparedness plans. When the system can avail itself of all preparedness and repair actions, a resilience level of 0.71 can be attained. Thus, the tool enables investigation of the airport's inherent capability to cope with and adapt to the considered random events within a specified time period and given available monetary resources. Moreover, the tool quantifies the trade-offs between resilience and level of T^{max} or the budget through sensitivity analysis. For example, given a chosen T^{max} of five hours, it can be seen from the figure that increasing the budget over and above about $35,000 does not induce a corresponding increase in resilience; thus, with such a figure, airport management can optimize associated resources.

The tool can also be used to study the frequency with which runways can be expected to operate with a MOS for each aircraft class. The impact on resilience level of repair opportunities can be quantified, and the likelihood that only one or no runways will operate given that some disruption event arises, with or without damage, can be ascertained. For the airport study location, with no budget for preparedness or response, the likelihood that all runways will operate (resilience of 1) is approximately 0.36 and that none will operate (resilience of 0) is 0.17. With a budget of \$50,000 and T^{max} of 8 hours, the former likelihood rises to around 0.48 and the latter decreases to roughly 0.08.

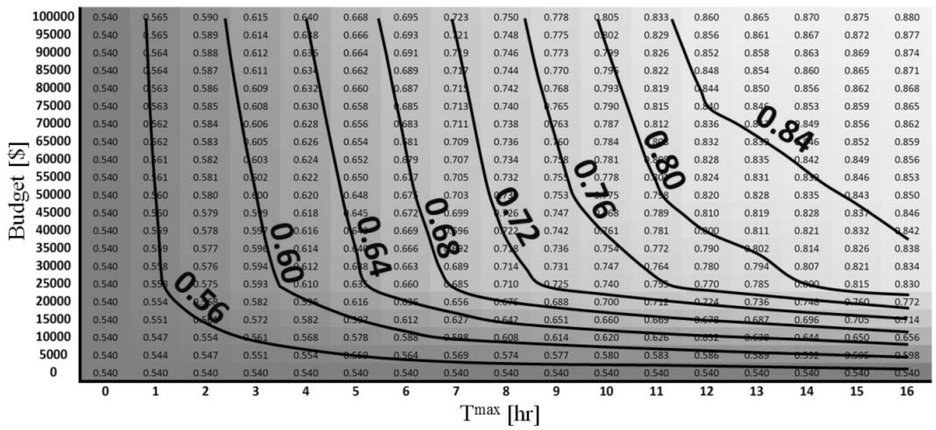
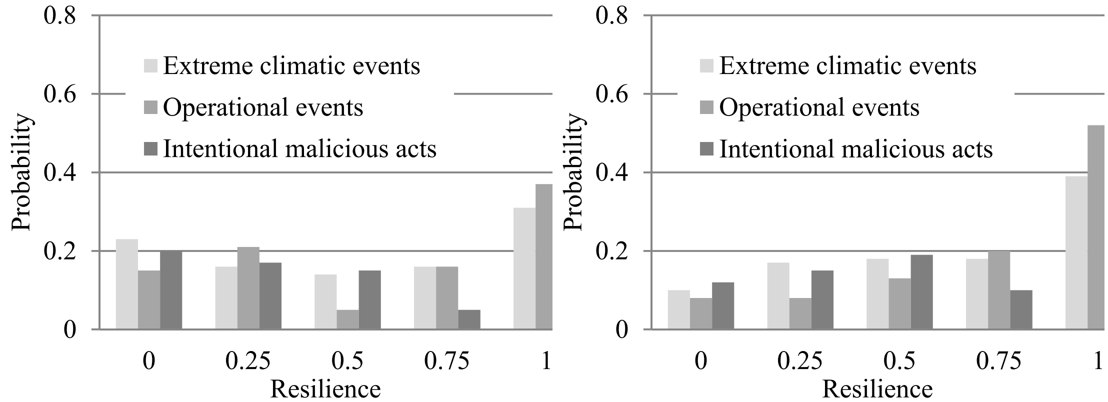


Figure 4-6 Resilience indifference curves for different combinations of budget and Tmax

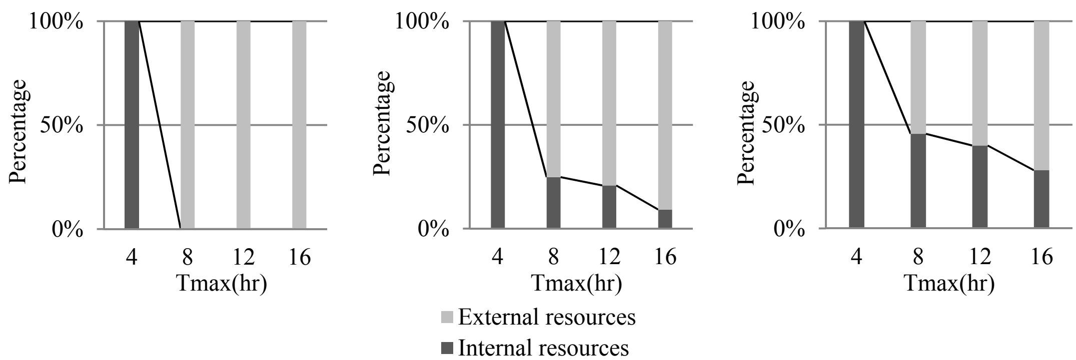
The probability of a given resilience level can be further investigated by causal category, an example of which is depicted in Figure 4-7. With no budget, the likelihood of the event resulting in a shutdown or low operating capacity (resilience 0 or 0.25) is highest for extreme climatic events. This is partially ameliorated when resources can be expended. Likewise, operational events are most easily absorbed. This is a positive finding for airport managers as these types of events have the greatest likelihood of occurrence.

Figure 4-8 depicts the proportion of the budget that is expended on internal and external resources for the purpose of completing repair actions for given budget and T^{max} levels. The figure shows the tradeoffs between the efficiencies associated with the use of internal resources and the ability to perform specific repair actions. For limited repair durations, e.g. T^{max}=4, and small budgets, there is insufficient time or funds to take advantage of external resources.



(a) No budget (coping capacity) (b) B=\$50000, T^{max}=8 hrs

Figure 4-7 Resilience probabilities under each disaster type for budget (B) and T^{max}combinations.



(a) Budget=\$25,000 (b) Budget=\$50,000 (c) Budget=\$100,000

Figure 4-8 Proportion of budget assigned to repair actions through internal and external resources

In connection with resource allocation, output from the model enables prioritization in the response to damage in infrastructure components. That is, it aids in the identification of key runways or taxiways within the pavement network. This is demonstrated in Figure 4-9, where the frequencies of chosen configurations over the possible scenarios are displayed for varying levels of budget and T^{max}. This depiction highlights the utility of runway configuration 04|04. This runway is the most likely configuration to be chosen by the model with and without repair action, and operates with more than 0.35 probability even when no repair actions are taken into consideration in performance evaluation. It can be observed from the network topology (Figure 4-5) that this runway has connectivity to redundant taxiways not available to the second runway. Additional insights can be garnered from these results; for example, the shorter T^{max}, the more likely that only one runway will be operating post-repair. That is, the model chooses to focus its resources on repairing one runway to support operations by both aircraft classes rather than only smaller aircraft on two runways. Likewise, the longer T^{max}, the higher the likelihood that configurations with two operating runways will be chosen post-repair. Thus, quantities computed by the tool can aid airport managers in ascertaining the criticality of airport pavement assets.

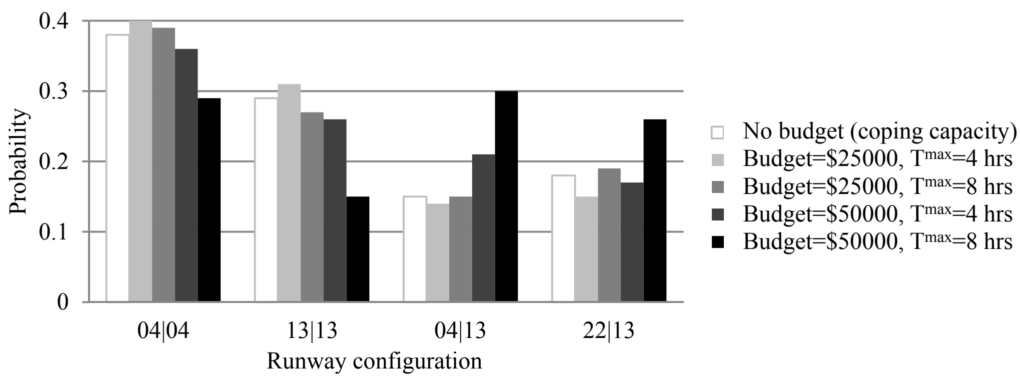


Figure 4-9 Runway configuration probabilities over scenarios.

In the immediate aftermath of a disruption, on average, the airport serves demand at 54% of its pre-disruption rate. With a budget of only $25,000 and T^{max} of 16 hours, this rate can be increased to 83% through outsourcing. When the budget is increased to $75,000 (which pays for two teams and one of each equipment type 12, 13, 16, and 17, along with costs associated with both internal and external repair operations), this rate can be further increased to 86%. A gain of only 2% in capacity is realized through an additional $25,000, where the optimal investment strategy is to train three teams that will use only existing equipment. This change in strategy can be explained by the large cost associated with training each team. It also illustrates how the discrete and combinatorial nature of the problem makes intuition about optimal solutions difficult without assistance of quantification methodologies

4.6. Summary and Remarks

The developed mathematical modeling and solution methodology techniques, and concepts for their inclusion in a decision support tool employing a scenario generator for multiple hazard classes, fills the need for a quantification methodology to assess the readiness of an airport to cope with damage to its paved network of runways and taxiways. Inherent uncertainties associated with disaster and disruption planning are explicitly recognized and directly addressed. The mathematical model differentiates small and large size aircraft. It further accounts for reductions in capacity due to joint take-off and landing maneuvers on common runways, reconfiguration strategies that consider bidirectional flows, weather effects, applicability and benefits of repair operations, implementation time and cost trade-offs related to conducting recovery actions taken in-house versus outsourcing, a range of disruption scenarios including those with multiple damage locations, multi-team response, limited equipment availability and a reasonable timeframe for conducting recovery operations. A mathematical tool with these capabilities makes it possible for optimal decisions to be derived for the large combinatorial and stochastic problem instances that are associated with real airports. It is also worth noting that, while it is not intended for use in real-time operations, solutions obtained for each scenario in the mathematical program's second state provide optimal recovery plans that can be used post-disaster once the disaster situation is understood.

Additional contributions are derived from insights gleaned from results of the tool's application on a case study based on the topology and meteorological conditions of an existing airport. To this end, the effectiveness of investment strategies that balance preparedness and recovery choices is explored in relation to the system's ability to cope with and adapt to unforeseen disruption scenarios, i.e. the system's resilience level. While the details of the optimal preparedness and response actions determined by the tool for the case study may have limited intrinsic value, they provide tangible examples of the tool's capabilities and potential utility in decision support in other comparable circumstances. For large airports, such as Baltimore Washington International (BWI) or San Francisco International (SFO) airport, both with 4 runways, if solution is difficult to obtain in a reasonable timeframe, one can reduce the combinations of actions that are permissible to greatly decrease solution time.

Airport managers may benefit from use of this tool and its methodologies in a variety of ways. Solutions from various what-if situations can be compared and trade-offs between investment approaches, use of limited internal resources and time can be assessed to evaluate proposed tactics for coping with major disruptions. Optimal solutions provide information about which equipment will be most important to have at the ready, how many repair crews should be on hand, and what outsourcing contracts to have in place. Benefits exist from merely considering the potential scenarios that might arise, but also from investigating which scenarios would be most problematic if realized and conducting a needs assessment for their resolution. In this regard, solutions from the tool would offer insights into not only what assets to pre-position, but where to locate them. Likewise, the tool can identify critical pavement system components and reveal system design weaknesses and other vulnerabilities. When appropriate external funds are made available for airport security, the tool can be used to analyze and back monetary requests. For example, it can quantify the potential performance benefits that could be attained in, say, less than 24 hours through the use of requested funds.

Chapter 5: Stochastic Models for Emergency Shelter and Exit Design in Buildings under Stochastic Optimum and User Equilibrium Conditions

5.1. Introduction and motivation

Regional evacuation studies have previously dealt with the problem of determining the optimal location and size of public shelters to which people can be evacuated in case of events such as floods and hurricanes. Studies on building evacuation, on the other hand, have mainly dealt with the question of how users can be evacuated as fast as possible to predefined building exits during an emergency. In practice, it might not be possible for all users to vacate a large or tall building in time. This may be true in particular in the case of disabled or elderly users. In other cases, it might be possible for the users to reach an exit, but this will not be the safest option because of the presence of internal hazards such as fire or smoke on the path of evacuation inside the building, or because of external hazards that originate outside the building.

A possible alternative is to evacuate building users to shelters inside buildings, which offer a certain level of protection. This policy is already being implemented in some countries, such as Singapore and Israel, where buildings are required to contain air-raid shelters in every dwelling or on every floor. As is standard in some countries, shelters have a protective envelope of 20-30 cm thick reinforced concrete walls and ceilings, as well as blast-proof doors and windows and an air-filtration system. They usually contain a single room that serves an additional purpose, such as a bedroom in an apartment or a conference room in an office building. In high-rise buildings, they are built one on top of another, sometimes with trap-doors and ladders that internally connect the shelters and can serve as an alternative evacuation route if staircases have become unusable. This creates a stable tower of shelters that will remain intact even if the rest of the building is heavily damaged. Such spaces have replaced the underground communal shelters that were originally built for this purpose in basements or even in public parks – serving several surrounding buildings. External communal shelters became less useful as buildings became higher, and the required time for evacuation decreased due to changing threats. This required shelters to be brought inside buildings and elevated to higher stories, so that they could be reached in time by evacuees. While the main purpose of existing shelters in buildings is to protect building users from missile attacks, they also offer protection during earthquakes. The possibility of using such shelters to protect users from additional hazards, such as fire or storms, is also considered herein.

While most shelters inside buildings are designed to house no more than a few dozen evacuees, local shelters, which serve an entire neighborhood, may house hundreds of evacuees. Such shelters are often located in public facilities, like schools or subway stations, and can serve the residents of buildings that do not contain internal shelters. Choice of where to locate these facilities depends on the type of hazard from which they are designed to protect. Regional evacuation may include even larger shelters, such as stadia that can house thousands of evacuees. The goal of this chapter is to develop mathematical models that supports the planning of shelters and evacuation paths in buildings designed to accommodate a limited number of people. The objective of these models is to ensure that evacuees are optimally protected during emergencies, both during the evacuation as well as after reaching their destinations. The objective function is therefore defined to minimize the risk to which evacuees may be exposed, rather than minimize evacuation time. The models support identification of the shelters to which a population should evacuate in various emergency scenarios, in light of possible hazards on the evacuation paths. Moreover, the models can aid in investigating if it is preferable for building users to evacuate to shelters inside the building, rather than to building exits. A network representation is used in the model to represent the layout of a building's circulation systems (i.e. the passageways along which building users can travel). A set of nodes may represent spaces inside buildings such as rooms and corridors. A set of links represents connections between these spaces. The movement of evacuees towards shelters is represented as flows on the links. The capacity of links and the risk exposure endured in traversing them may vary during emergencies as a result of structural failures or the spread of fire and smoke inside the building.

Different types of hazards may endanger a population's safety and require its evacuation. These may be natural (e.g. earthquakes), human-made (e.g. terror attacks), internal (e.g. fire) or external (e.g. hurricanes). Restricted construction budgets, and the difficulty to prepare evacuees for more than one evacuation procedure, imply the need to accommodate different hazards in a single solution. A multi-hazard approach is therefore adopted, in which the performance of a plan is tested under various possible future emergency scenarios. This chapter presents a solution for the problem of designing a single building so that its users can minimize their exposure to risk in an emergency situation involving building egress or sheltering. This problem is referred to as the Building Evacuation Design Problem (BEDP). To solve the BEDP, a bi-level, two-stage stochastic program is defined. The program falls under the class of Stochastic Mathematical Programs with Equilibrium Constraints (SMPECs).

At the upper-level of the proposed bi-level program, decisions are made regarding the location of shelters in the building, their size and level of protection, as well as the location of building exits, with the objective of minimizing the exposure of evacuees to risk over all scenarios. The uncertainty in the scenarios that will be realized is taken into account. It is assumed that construction costs are limited to a certain budget. This budget can be used for the planning of shelters that offer a high level of protection. Alternatively, the budget can be allocated for a partial fortification of sections of the hallways and staircases through which users evacuate to increase the level of protection that they offer, for widening hallways to increase their capacity, or for the construction of additional or redesign of existing building exits. The advantages of allocating the available budget for the construction of shelters can thus be weighed against the benefits of using it to add or redesign exits or to reduce the risks for evacuees on certain sections of the evacuation paths by fortifying or widening them.

At the lower-level of the program, the choice of evacuation paths by the users, following the upper-level decisions on the location of safe locations (shelters, fortified hallways) and exits, is modeled either as a User Equilibrium (UE) problem, or as a System Optimization (SO) problem. When modeled as a UE problem, it is assumed that users are homogenous, that they are perfectly informed of the conditions in the building or region, and choose a path with minimum risk. Evacuees will choose between evacuating to a specific shelter, evacuating to an exit, or staying in a partially fortified hallway. On the other hand, when the choice of evacuation paths is modeled as a SO problem, it is assumed that evacuees are assigned to an exit or shelter and told which path to use to reach that location. The SO approach uses the available system resources optimally, but requires command and control by a trained staff to direct the evacuees. It may require some evacuees to follow paths or take cover in shelters that are not necessarily optimal for them individually. The UE approach ensures that no evacuee can do better by taking an alternative decision, but requires that evacuees be familiar with the building and with the risks imposed by the hazard, in order to have full information about all alternatives.

Four variants of the BEDP are formulated using concepts of stochastic programming and robust optimization each under UE and SO conditions. UE models involve the bi-level formulation described previously. By recognizing that the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality, these models are reduced to equivalent single-level, two-stage stochastic integer programs. All variants are nonlinear. Using a disjunctive constraints transformation method and piecewise linearization, the models are linearized and integer L-shaped decomposition is proposed for solution of each of these mathematical programs. The capabilities of the modeling and solution techniques are illustrated on an office building using the original architectural plans. Trade-offs between system optimal and UE solutions and their implications in terms of their application, as well as in the use of stochastic programming versus robust optimization, are investigated.

5.2. Literature review

To the best of our knowledge, there have been no prior works in the literature that address optimal shelter and exit location in buildings. However, models with relevance to the BEDP have been developed in the literature for locating shelters in the context of regional evacuation problems. These works are reviewed next.

It appears that Sherali et al. (1991) were the first to study the shelter location problem for regional evacuation planning. They proposed a nonlinear mixed integer program to determine the shelter locations, resources allocations and assignment of evacuees to minimize evacuation time. They suggest a SO approach, which assumes that a central authority controls the flow of evacuees. The model uses a single given hazard scenario. A deterministic, multi-objective p-median problem formulation is proposed by Alcada-Almeida et al. (2009) for locating *p* shelters in a given area so as to minimize demand-weighted distance traveled, incurred risk and travel time associated with an evacuation. Similar deterministic and system optimal assumptions are made. Congestion is not considered.

Kongsomaksakul et al. (2005) proposed a bi-level programming model for determining locations and sizes of shelters that can be used by evacuees to minimize evacuation time in the event of a flood. The model is intended for pre-disaster planning. The upper-level problem determines the number and locations of shelters among a given set of potential locations, and the lower-level problem is a combined trip distribution and assignment problem. The inclusion of the lower-level problem allows evacuees to freely select their preferred shelters and choose the shortest route to their chosen shelters. Shelter selection behavior is modeled with a logit model, and a

Wardrop equilibrium is assumed to be reached. A genetic algorithm is employed to solve the problem. It is tested through a simulated flood scenario. Ng et al. (2010) also propose a bi-level programming model for regional shelter location, but optimize the shelter assignment in the upper-level problem, instead of assuming that evacuees themselves choose the shelters to which they will evacuate, as in Kongsomaksakul et al. (2005). A simulated annealing heuristic is proposed.

These earlier models all use a single given hazard scenario for locating shelters. Therefore, the identified solution may not be optimal for a wider range of hazard scenarios. Further, these models disregard the uncertain nature of disaster events. Kulshrestha et al. (2011) take into account uncertainty in demand for shelter capacity in a robust, bi-level program to determine the locations and sizes of shelters. As in Kongsomaksakul et al. (2005), it is assumed that the number of shelters, their locations and capacities are determined by a central authority, while the evacuees choose shelters and routes to access them. Although a set of possible demand scenarios is considered, other uncertainties regarding the type of hazard and the level of its severity are disregarded. An exact cutting plane algorithm is presented. Li et al. (2011) study sheltering network planning and operations for natural disaster preparedness and response with a two-stage stochastic program. In their study, the number of evacuees present at each origin at the start of the evacuation period (i.e. the evacuation demand) and transportation costs are assumed to be known only with uncertainty. In the first stage, the locations, capacities and resources required to supply the shelters are determined. In the second stage, the evacuees and resources are distributed to shelters under various disaster scenarios. With only continuous variables in the second stage, the L-shaped method can be employed. The proposed model and solution method were applied on a case study involving the Louisiana Gulf Cost.

Another paper that explicitly addresses the uncertainties inherent in disaster situations is by Li et al. (2012). They developed a scenario-based, bi-level stochastic program for optimal shelter location that considers a range of possible hurricane scenarios. The program seeks to minimize expected total travel time and unmet shelter demand under one of a host of possible disaster scenarios. Such scenarios differ in the area of impact. A dynamic user equilibrium is sought in the lower-level. Unlike earlier works, this work considers the possibility that evacuees will exit the area, and will not necessarily use the shelters. While Li et al. (2012) is the most relevant to this work, it only considers only a single type of hazard. Moreover, the problem is solved using a heuristic rather than exact solution methodology.

This literature is summarized in Table 5-1. The contributions of this chapter are, in light of existing relevant works: (1) a mathematical formulation to address shelter and exit design and location, possible fortification of hallways with reduced risk exposure, and selection of evacuation routes for buildings; (2) a multi-hazard approach with applicability to not only a multitude of disaster types, but simultaneous consideration of special and competing needs arising from these hazard types; (3) explicit consideration of risk exposure and its relation to the effects of user route choice on travel congestion; (4) simultaneous consideration of shelter and exit use; (5) a comparison of stochastic programming and robust optimization modeling; (6) an evaluation of the role of cooperative behavior and related need for command and control through a comparison of user equilibrium and system optimum formulation applications; and (7) an exact solution methodology that addresses problem nonlinearities for a set of complicated SMPECs and stochastic nonlinear programs (SNLPs).

Table 5-1 Synthesis of the related literature

Reference	SO vs. UE	What problem elements are stochastic	Optimization approach	Solution method	Hazard type	Application
Sherali et al. (1991)	SO	n/a	NLMIP	Generalized Benders & heuristic	Hurricane, flood	Geographic
Alcada-Almeida et al. (2009)	SO	n/a	Multi-objective p-Median program	Heuristic algorithm (nondominated solutions)	Generic	Geographic
Kongsomsaksakul (2005)	UE	n/a	Bi-level program	Genetic algorithm	Flood	Geographic
Ng & Park (2010)	UE	n/a	Bi-level program	Simulated annealing	Generic	Geographic
Kulshrestha et al. (2011)	UE	Number of evacuees	Bi-level RO	Cutting plane algorithm	Generic	Geographic
Li et al. (2011)	SO	Evacuation cost, number of evacuees	Two-stage SP	L-Shaped algorithm	Hurricane	Geographic
Li et al. (2012)	Dynamic UE	Evacuation capacity	Two-stage SP	Heuristic	Hurricane	Geographic
This work	Both	Evacuation risk exposure	Bi-level two-stage SP /RO	Integer L-shaped	Multi-hazard	Geographic & Building

5.3. Problem definition

5.3.1. Notation

In modeling the BEDP, a network representation $G = (N, A)$ of the building circulation system layout is used. A set of nodes N corresponds with locations inside the building, such as evacuation points of origin, transition points, candidate shelter locations, existing exits and candidate exit locations, as well as a supersink d. A set of links $A =$

$A_1 \cup A_2 \cup A_3$ connects these locations. A_1 is a subset of the links representing hallways, staircases, doorways and other passageways. A_2 is a subset of the links connecting existing and candidate shelters and fortified hallways, i.e. safe locations, to supersink d. Note that A_3 is subset of links similarly connecting existing and candidate emergency exits to supersink d. This network representation is illustrated in Fig. 5-1. The movement of evacuees in the circulation system is represented as flows along the links. The introduction of a supersink reduces the related network flow problem to that of a multi-source, single-sink problem.

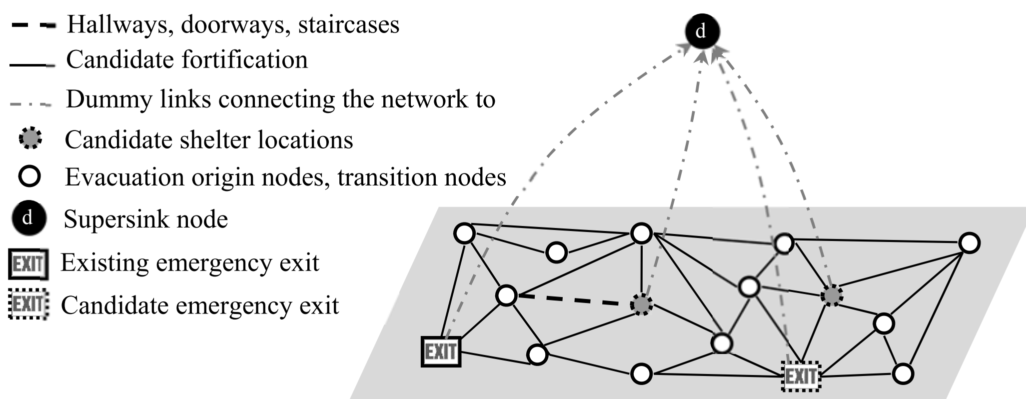


Figure 5-1 Building network representation scheme

The network is considered under a host of potential states (or scenarios) that might arise for a building under no-notice disaster events. Unlike disaster events with notice, such as a hurricane with two to three days advanced warning, notification of such a no-notice event in the context of buildings, perhaps provided by an alarm system, may entail only minutes. In this context, it is assumed that such notification provides information to the evacuees and building managers on the disaster type and possibly the location within the building (e.g. fire on a particular floor). This information may be imperfect, but can permit assessment of risk exposure associated with evacuee options, both in terms of safe locations and exits, as well as the paths that lead to these locations.

In the network representation, a particular state is given by the realization of parameters of link risk exposure functions. Risk exposure associated with a link consists of the likelihood of exposure while using the link and potential consequences. The longer the time spent en route to a safe location, the greater the likelihood of exposure. Thus, risk exposure is a function of travel time, which will depend not only on the link's length, but also on the number of people using it. It is assumed that the evacuees can assess risk exposure perfectly from the information they receive, and that all evacuees perceive risk identically. Risk associated with each safe location or an exit is also incorporated in the risk exposure functions. In the problem formulations proposed herein, evacuees choose or are guided to a safe location or exit with the goal of minimizing total risk exposure.

With this in mind, risk exposure associated with a link a is defined as a linear function of the link's flow-dependent travel time: $r_a[x_a(\xi)] = \alpha(\xi)t_a[x_a(\xi)] + \beta(\xi)$, where parameter $\alpha(\xi)$ converts the time it takes to evacuate through the hallways, staircases and doorways to risk exposure, and parameter $\beta(\xi)$ is a measure of the risk associated with staying in a shelter or hallway, or exiting the building. Both parameters are a function of the scenario. Different emergency scenarios, ξ, may induce different behaviors or decisions to reduce risk exposure. For example, when an internal hazard occurs (e.g. a fire event), exiting from the building will be of the highest priority; whereas, in the case of an external hazard (e.g. a storm), taking refuge within the building will provide protection. This is captured by parameter $\beta(\xi)$.

The BPR travel time function, originally used to estimate travel time on road networks, is adapted in the following form to estimate the evacuation travel time in a link $a \in A_1$, $t_a[x_a(\xi)]$, as a nonlinear function of link flow, $x_a(\xi)$ (see Schomborg et al. 2011). The travel time along link $a \in A_2 \cup A_3$ is also set to zero:

$$t_a[x_a(\xi)] = \begin{cases} t_a^0(\xi) + 0.15[\frac{x_a(\xi)}{c_a(\xi)}]^2, & \forall a \in A_1 \\ 0, & \forall a \in A_2 \cup A_3 \end{cases} \quad (1)$$

where t_a^0 and $c_a(\xi)$ are the freeflow travel time and capacity of link $a \in A_1$ under scenario ξ, respectively. The BPR function is generally formulated based on the velocity-density fundamental diagram for vehicle movement in road networks. Schomborg et al. (2011) argue that, in the context of macroscopic modeling, this function can also be utilized to estimate the pedestrian travel time using the parameter values adopted in equation (1).

Nomenclature used in the remainder of the chapter is provided next.

S	=	set of shelter/hallway fortification types
E	=	set of exit types/sizes
g_a^s	=	cost of fortification of type $s \in S$ in link $a \in A_2$
$\acute{g}_a^e$	=	cost of construction of exit type $e \in E$ in link $a \in A_3$
B	=	total budget for exit design and shelter/hallway fortification
p_a^s	=	capacity of shelter type $s \in S$, $a \in A_2$
q^o	=	number of evacuees originating at node $o \in N$
K_o	=	set of paths containing no cycles originating from node $o \in N$
$\delta_{a,k}^o$	=	link-path incidence matrix (=1 if link a belongs to path k originated from node o, and =0 otherwise)
Ξ	=	set of possible scenarios $\xi \in \Xi$

Pre-event variables:

y_a^s	=	binary variable indicating if fortification of type $s \in S$ is selected for application to link $a \in A_2$ (=1 if selected, and =0 otherwise)
$\acute{y}_a^e$	=	binary variable indicating if exit type $e \in E$ is selected for construction in link $a \in A_3$ (=1 if selected, and =0 otherwise)

Post-event variables:

$f_k^o(\xi)$	=	flow along path $k \in K_o$ from demand node o under scenario ξ
$x_a(\xi)$	=	flow along link $a \in A$ under scenario ξ

$t_a[x_a(\xi)]$ = travel time along link $a \in A$ under scenario ξ

$r_a[x_a(\xi)]$ = risk exposure associated with link $a \in A$ under scenario ξ; assumed to be a linear function of link travel time: $r_a[x_a(\xi)] = \alpha(\xi)t_a[x_a(\xi)] + \beta(\xi)$

$c_k^o(\xi)$ = risk exposure on path k, for $\forall k \in K_o, o \in N$

5.3.2. Problem formulations

Four BEDP formulations are presented. The programs use either Stochastic Programming (SP), which takes into account the expectation in performance over all future scenarios, or Robust Optimization (RO) with emphasis on the worst-case scenario imposing the highest evacuation risk exposure. The latter is a conservative approach, which may require a more expensive solution to attain the same level of risk exposure. Two of the models adopt a bi-level structure, where the evacuees choose their own routes to minimize their own risk exposure (taking a UE perspective). The remaining two models are single-level and assume the evacuees will follow system optimal instructions (taking a SO perspective). This latter perspective requires command and control for implementation. That is, users are commanded toward safe locations or exits that meet the system's goals and control is in place to ensure compliance (Feng and Miller-Hooks, 2012). These four programs are referred to by their acronyms: BEDP-SP-UE, BEDP-SP-SO, BEDP-RO-UE, and BEDP-RO-SO. The modeling specifications of these problems are summarized in Table 5-2.

Objectives that minimize the maximum or expected maximum risk exposure are proposed herein, because they indirectly address issues of equity and consider the protection of each individual. This differs from other network design formulations in the literature. For both emergency and nonemergency applications, it is common to minimize total travel time or other disutility measures.

Table 5-2 Modeling specifications for the proposed problems

Problem	Optimization approach	User behavior modeling	Modeling Structure	Objective
BEDP-SP-UE	SP	UE	• Bi-level o UL:1st stage decision on design/fortification options o LL: user response to UL decisions	*min* E[*max* evacuation risk]
BEDP-RO-UE	RO	UE		
BEDP-SP-SO	SP	SO	• Single-level (command and control)	*minmax* [evacuation risk]
BEDP-RO-SO	RO	SO		

a) BEDP-SP-UE

This BEDP-SP-UE problem is formulated as a bi-level, two-stage stochastic program with equilibrium constraints, a type of stochastic MPEC.

$$(BEDP-SP-UE)$$

Upper-level: $\min_{y} E_{\xi\in\Xi}[Z^U_{SP-UE}(\xi)]$ (2)

s.t.

$\sum_{a\in A_2}\sum_{s\in S} g^s_a y^s_a + \sum_{a\in A_3}\sum_{e\in E} \acute{g}^e_a \acute{y}^e_a \le B$ (3)

$\sum_{s\in S} y^s_a \le 1, \quad \forall a \in A_2$ (4)

$\sum_{e\in E} \acute{y}^e_a \le 1, \quad \forall a \in A_3$ (5)

$y^s_a, \acute{y}^e_a \in \{0,1\}, \quad \forall a \in A_2 \cup A_3, s \in S, e \in E$ (6)

where

$Z^U_{SP-UE}(\xi) = \min_{x} \max_{o\in O} u^o(\xi)$ (7)

Lower-level: $Z^L_{SP-UE}(\xi) = \min \sum_a \int_0^{x_a(\xi)} r_a(w)\, dw$ (8)

s.t.

$\sum_{k\in K_o} f^o_k(\xi) = q^o, \quad \forall o \in O$ (9)

$x_a(\xi) = \sum_{o\in O} \delta^o_{a,k} f^o_k(\xi), \quad \forall a \in A$ (10)

$x_a(\xi) \le \sum_{s\in S} p^s_a y^s_a, \quad \forall a \in A_2$ (11)

$x_a(\xi) \ge 0, \quad \forall a \in A$ (12)

$f^o_k(\xi) \ge 0, \quad \forall k \in K_o, o \in O$ (13)

At the upper-level, the problem is to determine the optimal location of exits, location and size of shelters to be constructed, and hallways to be fortified, as well as corresponding level of protection, aiming at minimizing the expectation of the worst-case risk exposure experienced by the evacuees over all scenarios. Construction costs are limited to an available budget in constraint (3). Constraints (4)-(6) ensure that only one type of fortification is constructed at any candidate location.

At the lower-level is a path-based capacitated user equilibrium problem with side-constraints adapted from Larsson and Patriksson (1995). Evacuees rationally seek to minimize their risk exposure, assuming that they have perfect information on the risks associated with the evacuation path choices under a given scenario ξ and the building design options (including the shelter capacities) determined at the upper-level.

Evacuees are assigned to paths through constraints (9). Link flows are defined in constraints (10) as the total flow in terms of evacuees traveling from any origin along any path containing that link. In constraints (11), flow is allowed through a link $a \in A_2$ if a shelter of any type $s \in S$ is constructed along that link. The flow is limited to the shelter's capacity, p_a^s. An infinite capacity is presumed for all exit doors $a \in A_3$. Non-negativity requirements for link and path flows are captured through constraints (12)-(13).

The formulation can be readily extended to permit shelter capacities as a function of hazard type. This is important in real applications, because the amount of space required per evacuee while sheltered depends on the amount of time the evacuee will remain in the shelter. The longer the required time, the greater the required space. Because it is morally difficult to restrict the number of evacuees to enter a shelter when it appears that there is more space, constructing shelters for the worst-case as is supported by the proposed objective functions is desirable.

b) BEDP-SP-SO

As an alternative modeling approach, safe locations, exits and evacuation routes are designed to support a system optimal flow of evacuees under the assumption that evacuees are directed in emergency situations by trained staff or through commands given electronically. Thus, it is presumed that the evacuees will follow the instructions they are provided. This problem is formulated as a single-level, nonlinear two-stage stochastic program.

$$(BEDP-SP-SO)$$

$$\min_{y} E_{\xi\in\Xi}[Z_{SP-SO}(\xi)] \text{ s.t. (3-6)} \tag{14}$$

where

$$Z_{SP-SO}(\xi) = \min_{x} \max_{o\in O} w^o(\xi) \tag{15}$$

s.t. (9-13)

$$f_k^o(\xi)\cdot[c_k^o(\xi) - w^o(\xi)] \le 0, \quad \forall k \in K_o, o \in N \tag{16}$$

As in the BEDP-SP-UE, the objective function is to minimize the expectation of the maximum evacuation risk exposure evacuees experience over all scenarios. $w^o(\xi)$ is defined as the worst (highest) evacuation risk exposure from node o. Through additional constraints (16), only the risk exposure of active paths from node o is used to determine $w^o(\xi)$. That is, the inequality $c_k^o(\xi) \le w^o(\xi)$ is imposed if $f_k^o(\xi) > 0$.

c) BEDP-RO-UE and BEDP-RO-SO

By focusing on the worst evacuation risk exposure under the worst-case scenario rather than on the expectation of worst risk exposure over all scenarios, this robust optimization model is even more conservative than the BEDP models that use stochastic programming (BEDP-SP-UE and BEDP-SP-SO). Scenario probabilities are not included in robust optimization. Two problems, BEDP-RO-UE and BEDP-RO-SO, are formulated using the UE and SO principles, respectively:

$$(BEDP-RO-UE)$$

Upper-level: $\min_{y} \max_{\xi \in \Xi} [Z^U_{RO-UE}(\xi)]$ s.t. (3-6) (17)

where

$$Z^U_{RO-UE}(\xi) = \min_{x} \max_{o \in O} u^o(\xi) \tag{18}$$

and the lower-level problem as given in (8-13).

$$(BEDP-RO-SO)$$

$$\min_{y} \max_{\xi \in \Xi} [Z_{RO-SO}(\xi)] \text{ s.t. (3-6)} \tag{19}$$

where

$$Z_{RO-SO}(\xi) = \min_{x} \max_{o \in O} w^o(\xi) \quad \text{s.t. (9-13), (16)} \tag{20}$$

Both formulations seek to minimize the maximum evacuation risk exposure over all scenarios.

5.4. Solving the BEDP variants

5.4.1. Complementarity constraints

a) BEDP-SP-UE and BEDP-RO-UE

A common approach to solving bi-level programs is, when possible, to eliminate the lower-level problem by incorporating the original lower-level constraints along with related KKT conditions (first-order optimality conditions) within the upper-level. This creates an equivalent single-level program. In the context of the BEDP-UE-SP and BEDP-UE-RO formulations, this includes constraints (9)-(13) and (21)-(24):

$$f_k^o(\xi) \cdot [\hat{c}_k^o(\xi) - u^o(\xi)] = 0, \quad \forall k \in K_o, o \in N \tag{21}$$
$$\hat{c}_k^o(\xi) - u^o(\xi) \geq 0, \quad \forall k \in K_o, o \in N \tag{22}$$
$$\lambda_a(\xi) \cdot [\sum_{s \in S} p_a^s \, y_a^s - x_a(\xi)] = 0, \quad \forall a \in A_2 \tag{23}$$
$$\lambda_a(\xi) \geq 0, \quad \forall a \in A_2 \tag{24}$$

Building on the work of Larsson and Patriksson (1995) who considered the capacitated assignment problem in which users selfishly seek to minimize their experienced disutilities, it is assumed that a generalized Wardrop equilibrium can be reached. In such an equilibrium, no evacuee can unilaterally switch routes and improve his/her disutility (risk exposure in the context of this work).

In constraints (21)-(24), $u^o(\xi)$ indicates the minimum risk exposure incurred by evacuees originating from node $o \in N$ under scenario ξ and $\hat{c}_k^o(\xi)$ is the generalized path risk exposure adapted from Larsson and Patriksson (1995):

$$\hat{c}_k^o(\xi) = c_k^o(\xi) + \sum_{a \in A_3} \delta_{a,k}^o \, \lambda_a(\xi), \quad \forall k \in K_o, o \in N. \tag{25}$$

In addition, $c_k^o(\xi) = \sum_{a \in A} \delta_{a,k}^o \, r_a[x_a(\xi)]$ is the risk exposure on path k, for $\forall k \in K_o, o \in N$, and $\lambda_a(\xi)$ is the Lagrange multiplier for link $a \in A_3$ associated with complementarity constraints (23). $\lambda_a(\xi)$ can be interpreted as the additional risk exposure that users passing through a saturated link are willing to endure to use the link (i.e. the link's shadow price).

In their compatible formulation, Larsson and Patriksson showed that the KKT conditions are both necessary and sufficient for optimality. Constraints (21) and (23) fo the KKT conditions fall under the class of complementarity constraints, and thus, are nonlinear. A transformation methodology, specifically a disjunctive constraints approach, initially introduced in (Fortuny-Amat and McCarl, 1981), is employed in which the introduction of binary variables converts these constraints into equivalent linear mixed-integer constraints.

The implementation of this methodology given in Wang and Lo (2010) is followed herein. Thus, constraints (13) are replaced by constraints (26)-(28):

$$L \cdot \varphi_k^o(\xi) + \varepsilon \le f_k^o(\xi) \le U \cdot [1 - \varphi_k^o(\xi)], \quad \forall k \in K_o, o \in N \tag{26}$$
$$L \cdot \varphi_k^o(\xi) \le \hat{c}_k^o(\xi) - u^o(\xi) \le U \cdot \varphi_k^o(\xi), \quad \forall k \in K_o, o \in N \tag{27}$$
$$\varphi_k^o(\xi) \in \{0,1\}, \quad \forall k \in K_o, o \in N \tag{28}$$

where L and U are very large negative and positive numbers, respectively, and ε is a very small positive number. Binary variable $\varphi_k^o(\xi)$ indicates whether or not path k from origin node o receives a flow, i.e. $\varphi_k^o(\xi) = 0$ resulting in $\hat{c}_k^o(\xi) = u^o(\xi)$ if $f_k^o(\xi) > 0$; $\varphi_k^o(\xi)$=1, otherwise.

Similarly, constraints (23) are replaced by constraints (29-31):

$$L \cdot \phi_a(\xi) + \varepsilon \le \lambda_a(\xi) \le U \cdot [1 - \phi_a(\xi)], \quad \forall a \in A_2 \tag{29}$$
$$L \cdot \phi_a(\xi) \le \sum_s p_a^s y_a^s - x_a(\xi) \le U \cdot \phi_a(\xi), \quad \forall a \in A_2 \tag{30}$$
$$\phi_a(\xi) \in \{0,1\}, \quad \forall a \in A_2 \tag{31}$$

where binary variable $\phi_a(\xi)$ indicates whether or not flow along link a reaches the link capacity. When the flow along link a reaches the link's capacity limitation, $\phi_a(\xi) = 0$, resulting in $\lambda_a(\xi) > 0$; and $\phi_a(\xi) = 1$, otherwise.

b) BEDP-SP-SO and BEDP-RO-SO

BEDP-SO-SP and BEDP-SO-RO do not involve UE constraints, and thus the need for the complementarity constraints described in the prior section is eliminated; they are, thus, single-level problems. However, complementarity constraints (16) are required to ensure that risk exposure is considered within the objective only for active paths. Thus, the programs are nonlinear. Again, a disjunctive constraints transformation approach is applied wherein constraints (32)-(34) replace constraints (16).

$$L \cdot \sigma_k^o(\xi) + \varepsilon \le f_k^o(\xi) \le U \cdot [1 - \sigma_k^o(\xi)], \quad \forall k \in K_o, o \in N \tag{32}$$
$$c_k^o(\xi) - w^o(\xi) \le U \cdot \sigma_k^o(\xi), \quad \forall k \in K_o, o \in N \tag{33}$$
$$\sigma_k^o(\xi) \in \{0,1\}, \quad \forall k \in K_o, o \in N \tag{34}$$

where $\sigma_k^o(\xi)$ is a binary variable indicating whether a path is active or not: $\sigma_k^o(\xi) = 0$ if $f_k^o(\xi) > 0$; and $\sigma_k^o(\xi) = 1$, otherwise

5.4.2. Piecewise linearization of the travel time function

For each link $a \in A_1$, the nonlinear travel time function is replaced by a piecewise linear function using a method presented by Sherali (2001) (also applied in (Farvaresh and Sepehri, 2011)). The first step of this technique is to bound link flow $x_a(\xi)$ by lower- and upper-bounds. One simple approach to setting these bounds is to use zero and total evacuation demand from all origin nodes, i.e. $0 \le x_a(\xi) \le \sum_{o \in O} q^o$, $\forall\, a \in A_1$. Next, this range is partitioned into I_a non-overlapping segments. Let the link flow $x_a(\xi)$ be represented as follows:

$$x_a(\xi) = \sum_{i=1}^{I_a} \dot{x}_{a,i-1} \pi_{a,i}^L + \dot{x}_{a,i} \pi_{a,i}^R, \quad \forall a \in A_1 \tag{35}$$

where $\dot{x}_{a,i-1}$ and $\dot{x}_{a,i}$ are link flow values at endpoints of segment i, and $\pi_{a,i}^L$ and $\pi_{a,i}^R$ are convex-combination weights of that segment such that equations (36) and (37) hold.

$$\pi_{a,i}^L + \pi_{a,i}^R = \theta_{a,i}, \quad \forall a \in A_1, i = 1,2,..,I_a \tag{36}$$
$$\sum_{i=1}^{I_a} \theta_{a,i} = 1, \quad \forall a \in A_1 \tag{37}$$

where

$$\pi_{a,i}^L, \pi_{a,i}^R \ge 0, \quad \forall a \in A_1, i = 1,2,..,I_a \tag{38}$$
$$\theta_{a,i} \in \{0,1\}, \quad \forall a \in A_1, i = 1,2,..,I_a \tag{39}$$

Then, the link travel time function can be replaced by the piecewise linear function given in (40).

$$t_a[x_a(\xi)] = t_a^0 + b_a . [\sum_{i=1}^{I_a} \dot{x}_{a,i-1}^2 \pi_{a,i}^L + \dot{x}_{a,i}^2 \pi_{a,i}^R], \quad \forall a \in A_1 \tag{40}$$

An advantage of this linearization method is that the matrix of coefficients in these added constraints (constraints (36)-(39)) is totally unimodular, making it possible to relax integrality constraints (39) (see Sherali (2001) for more details).

Given the above mathematical replacements, the nonlinear BEDPs are reformulated as SMIPs presented in Table 5-3.

Table 5-3 BEDPs reformulated as two-stage SMIPs

Problem	Objective function	Constraints					
		1st stage	2nd stage				
		Design decisions	Link/path flow assignment	UE CCs*	Capacitated link CCs*	Active path CCs *	Link travel time function linearization
		(3)-(6)	(9)-(13)	(26)-(28)	(29)-(31)	(32)-(34)	(35)-(40)
BEDP-SP-UE	$\min_{y} E_{\xi\in\Xi}\,[\min_{x} \max_{o\in O} u^o(\xi)]$	✓	✓	✓	✓	-	✓
BEDP-RO-UE	$\min_{y} \max_{\xi\in\Xi}[\min_{x} \max_{o\in O} u^o(\xi)]$	✓	✓	✓	✓	-	✓
BEDP-SP-SO	$\min_{y} E_{\xi\in\Xi}[\min_{x} \max_{o\in O} w^o(\xi)]$	✓	✓	-	-	✓	✓
BEDP-RO-SO	$\min_{y} \max_{\xi\in\Xi}[\min_{x} \max_{o\in O} w^o(\xi)]$	✓	✓	-	-	✓	✓

* CC: Complementarity Constraints

5.5. Solution methodology

The integer L-shaped method, introduced by Laporte and Louveaux (1993), is adopted to solve the four variants of the BEDP each having only binary decision variables in the first-stage as required by the procedure. This method is exact. It decomposes the original program into a master problem and set of subproblems representing second-stage problems $Z(\xi)$ for each scenario. Let $y \in Y = \{y_a^s, \acute{y}_a^e\}_{(a)\in A_2\cup A_3, s\in S, s\in E}$ represent all first-stage variables. The master problem is generally formulated as follows.

$$min\ \theta \tag{41}$$

s.t.

(3-5)

$$0 \le y \le 1 \tag{42}$$

$$f(y,\theta) \le 0 \tag{43}$$

where the objective is to minimize θ, an approximation of the expectation (maximum) of the second-stage objective functions $Z(\xi)$ over all scenarios $\xi \in \Xi$ for a general stochastic program or in robust optimization. Constraints (42) are relaxations of integrality constraints (6) for first-stage variables.

To solve the master problem, branch-and-bound steps are integrated within the procedure to obtain binary solutions at each iteration. The binary variables of these solutions are fixed in the subproblems. Optimality cuts (43) are iteratively generated and added to the master problem based on solution of the subproblems, creating a tighter feasible region. No feasibility cuts are required, since the master problem solution is always feasible for the subproblems.

Let Y^{ϵ} be the ϵth vector of feasible solutions, i.e. binary solutions from the master problem including the sets of 1's and 0's: $Y_1^{\epsilon} = \{r | y_r = 1\}$ and $Y_0^{\epsilon} = \{r | y_r = 0\}$. Valid optimality cuts are generated by (44).

$$\theta \geq \{\theta^{\epsilon} - LB\}\left[\sum_{r \in Y_1^{\epsilon}} y_r - \sum_{\epsilon \in Y_0^{\epsilon}} y_r\right] - \{\theta_{\epsilon} - LB\}(|Y_1^{\epsilon}| - 1) + LB, \qquad (44)$$

where $|Y_1^{\epsilon}|$ is the cardinality of the set Y_1^{ϵ}, and LB is a finite lower bound that can be set to zero in this problem. A tighter lower bound could significantly improve the solution time, however. One suggestion to find a better lower bound is to relax the budget constraint and solve the subproblems assuming best-quality shelters are constructed in all candidate locations.

Laporte and Louveaux (1993) proved that cuts given by (44), where $\theta^{\epsilon} = \theta_{SP}^{\epsilon} = E_{\xi \in \Xi}[Z(Y^{\epsilon}, \xi)]$ (i.e. the expectation over second-stage objective functions corresponding to first-stage feasible solutions Y^{ϵ}), are valid for stochastic programs.

Therefore, cuts (44) can be directly applied to solve both the BEDP-SP-UE and BEDP-SP-SO. In this work, these cuts are further modified for solving robust optimization versions: BEDP-RO-UE and BEDP-RO-SO.

Proposition 1. Let $\theta^{\epsilon} = \theta_{RO}^{\epsilon} = \max_{\xi \in \Xi}[Z(Y^{\epsilon}, \xi)]$ be the maximum second-stage objective function over all scenarios $\xi \in \Xi$ corresponding to first-stage feasible solutions Y^{ϵ}. Modified optimality cuts (46) are valid cuts for BEDP-RO-UE and BEDP-RO-SO.

$$\theta \geq \{\theta_{RO}^{\epsilon} - LB\}\left[\sum_{r \in Y_1^{\epsilon}} y_r - \sum_{\epsilon \in Y_0^{\epsilon}} y_r\right] - \{\theta_{RO}^{\epsilon} - LB\}(|Y_1^{\epsilon}| - 1) + LB. \quad (45)$$

Proof. The inequality $\sum_{r \in Y_1^{\epsilon}} y_r - \sum_{\epsilon \in Y_0^{\epsilon}} y_r \leq |Y_1^{\epsilon}|$ always holds; thus, the right-hand side of (46) takes a value less than or equal to LB. In the extreme case where $\sum_{r \in Y_1^{\epsilon}} y_r - \sum_{\epsilon \in Y_0^{\epsilon}} y_r = |Y_1^{\epsilon}|$, the right-hand side will be equal to θ_{RO}^{ϵ}. Therefore, the cuts (45) will never eliminate the globally optimal solution, and it is valid to impose them on first-stage solutions. □

Note that in numerical experiments described in Section 6, to improve the implementation time of the UE-based problems, the corresponding SO-based problems were solved first and their objective function values were used as the LB in optimality cuts (44) and (45).

The general algorithm of the integer L-shaped method (Laporte and Louveaux 1993) to solve the BEDPs is presented in the following. Let $\bar{Z}$ be the upper bound of the desired stochastic program or robust optimization model Z, and μ be the algorithm iteration number:

Step 0: Set $\mu = 0$, upper bound $\bar{Z} = \infty$. The value of θ is set to $-\infty$ or other absolute lower bound. A pendant node list is created that contains only a single pendant node corresponding to the initial subproblem.

Step 1: Select a pendant node in the list. Stop if the pendant node list is empty.

Step 2: Set $\mu = \mu + 1$ and solve the current problem. If the problem is infeasible, fathom the current node and go to *Step 1*. Otherwise, let (y^{μ}, θ^{μ}) be an optimal solution.

Step 3: Check for integrality. If violated, create two new branches in which the most fractional variable is set to 0 or 1. Append the two nodes to the pendant node list and go to *Step 1*.

Step 4: Given the first-stage solutions y^{μ}, solve the sub-problems $Z(y^{\mu}, \xi)$ for each scenario ξ. If the model is a stochastic program, calculate the expectation value over all scenarios, $Z(y^{\mu}) = \underset{\xi \in \Xi}{E}[Z(y^{\mu}, \xi)]$. Otherwise, if the model is of robust optimization models, calculate the corresponding maximum value over all scenarios, $Z(y^{\mu}) = \underset{\xi \in \Xi}{max}[Z(y^{\mu}, \xi)]$. If $Z(y^{\mu}) < \bar{Z}$, update upper bound $\bar{Z} = Z(y^{\mu})$.

Step 5: If $\theta^{\mu} \geq Z(y^{\mu})$, then fathom the current node and go to *Step 1*; otherwise, impose an optimality cut to the master problem, and return to *Step 2*.

5.6. Numerical example

5.6.1. Network representation

Numerical experiments were conducted using the design of an actual office building. The building has a reinforced concrete structure, and consists of two connected wings that surround an inner courtyard. In the original design of the building, each wing has a core containing a shelter. The layout of the building is illustrated in Fig. 5-2.

Two exits (E1 and E2) were already included in the initial building design. One additional emergency exit (E3) is also considered for incorporation in the design, and is represented by dashed lines. Seven locations are taken as candidates to fortify as shelters represented by dashed ovals (S1-S7). Four hallways (H1-H4) are already included in the building evacuation plan as relatively safe locations for evacuees in case of a hazard. One additional hallway, H5, is also considered in this example as a candidate for fortification. The network representation includes 75 links, as well as 15 dummy links that connect the locations of shelters, exits and fortified hallways to the supersink node.

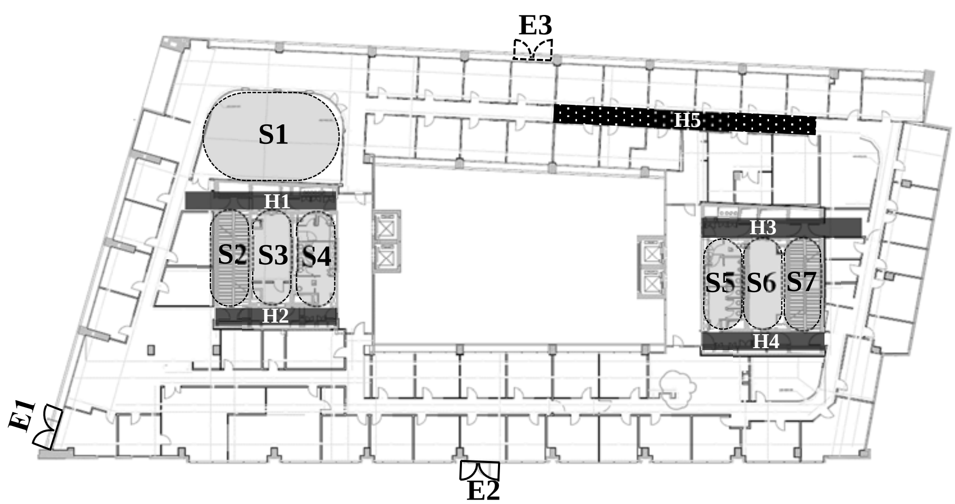


Figure 5-2 Office building layout

40 rooms in the building are considered evacuation origin nodes. The number of evacuees in these rooms is estimated based on their maximum occupancies from the National Fire Protection Association (NFPA) Life Safety Code (2009), and given in Table 5-4.

Table 5-4 Maximum occupancy of rooms in building

Room #	Max occ.	Room #	Max occ.	Room #	Max occ.	Room #	Max occ.
1	4	12	6	22	4	39	4
2	4	13	2	23	5	40	4
3	2	14	2	24	1	41	4
4	2	15	4	25	2	42	4
5	3	16	4	26	4	43	4
6	5	17	4	27	5	44	4
7	1	18	4	28	2	45	4
8	2	19	4	32	5	49	6
10	4	20	4	33	5	50	6
Total building occupancy =150 people							

5.6.2. Modeling parameters

In this example, only one fortification or construction type is considered for each location in terms of level of protection, cost and capacity. However, the general formulation of the optimization model allows different design options to be considered for any single location out of which one option can then be selected through the optimization. The costs and capacities (in terms of number of evacuees) of the design options are given in Table 5-5. These were estimated based on current average construction costs.

Table 5-5 Costs and capacities of design options

Design option	ID	Design cost ($)	Capacity
Shelter	S1	6,700	35
	S2	4,100	15
	S3	5,600	25
	S4	5,000	25
	S5	3,700	15
	S6	3,900	25
	S7	4,100	15
Unfortified hallway	H1	-	30
	H2	-	30
	H3	-	30
	H4	-	30
Hallway fortification	H5	3,600	40
Emergency exit	E3	2,200	-

Five disaster scenarios are generated assuming 20% occurrence probability of each: one scenario for an external malicious act which is likely to affect the whole building equally, and four scenarios for an internal fire in different parts of the building (north, south, west, and east). The stochastic nature of these scenarios is captured through parameters $\alpha(\xi)$ and $\beta(\xi)$ in the risk exposure function. $\alpha(\xi)$ represents the slope of the risk function line converting the evacuation time through passageways to a risk exposure value, and $\beta(\xi)$ represents the risk imposed by exiting the building or staying in a safe location.

To quantify the risk to which evacuees are exposed, a range of 0-100 points is considered, where 0 indicates no risk exposure and 100 indicates a maximum risk exposure (which can be interpreted as a high risk of death). To find risk equivalency of evacuation time, it is assumed that the maximum tolerable evacuation times is equal to a risk exposure of 100 points and occurs at 120 seconds for an external malicious act and 180 seconds for an internal fire. This results in $\alpha(\xi)$ values of 0.83 (=100/120) and 0.55 (=100/180), respectively. Moreover, given the range of 0-100, the risk exposure of using each individual evacuation option under different hazard types is estimated and is given in Table 5-6.

Table 5-6 Scenario-dependent values of parameter $\beta(\xi)$ in risk exposure function

Scenario	Evacuation option			
	Exit	Shelter	Unfortified hallway	Fortified hallway
External malicious act	100	5	30	10
Internal fire	0	20	100	40

The travel time function is divided into 20 linear segments with respect to link flow, and the function parameters for passageways $a \in A_1$, t_a^0 and c_a, are estimated from the Society of Fire Protection Engineers' (SFPE) Handbook (2002) based on passageway lengths, widths, and average speed of evacuees. These are presented in Table 5-7. Finally, four budget levels of \$0, \$7500, \$15,000 and \$42,000 (a sufficient budget for the construction of all the design options) are considered for experimental runs.

Table 5-7 Values of passageway travel time function parameters

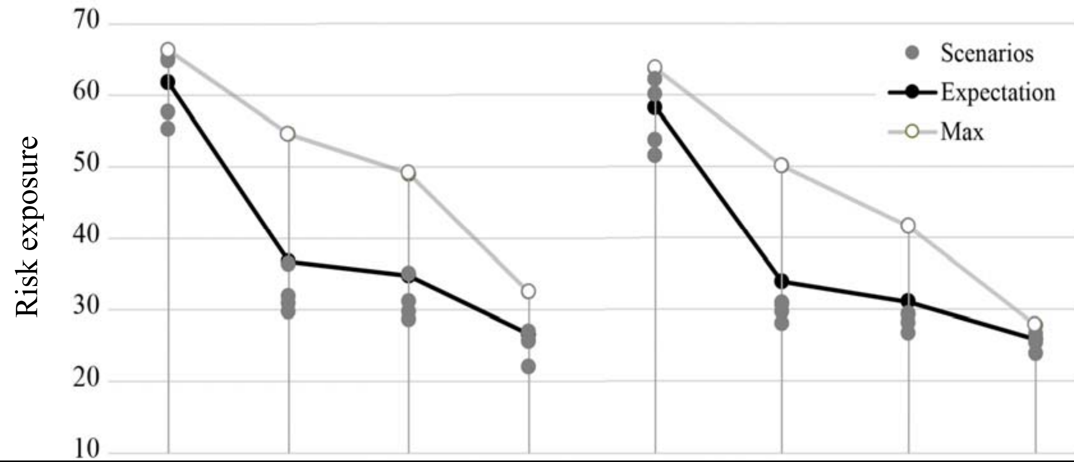
Link ID	Link type*	t_a^0 (s)	c_a (evac./s)	Link ID	Link type*	t_a^0 (s)	c_a (evac./s)	Link ID	Link type*	t_a^0 (s)	c_a (evac./s)
1	C	2.5	2	26	C	5.6	2	51	C	4.5	2
2	C	3.0	2	27	C	2.7	2	52	C	4.3	2
3	C	2.1	2	28	D	3.1	1	53	C	4.1	2
4	D	3.1	1	29	C	4.0	3	54	D	4.9	1
5	C	2.3	2	30	D	9.5	1	55	C	3.6	2
6	C	2.6	2	31	D	4.1	1	56	C	3.5	2
7	C	1.7	2	32	D	8.2	1	57	C	3.1	2
8	C	2.1	2	33	D	6.6	1	58	C	4.7	2
9	C	2.5	2	34	C	2.3	3	59	D	10.8	1
10	C	3.2	2	35	D	2.8	1	60	C	0.8	3
11	C	4.0	2	36	S	4.3	1	61	D	8.5	1
12	C	4.3	2	37	C	4.6	2	62	D	1.7	1
13	C	4.4	2	38	C	3.5	2	63	D	3.7	1
14	C	3.6	2	39	C	3.4	2	64	S	2.4	1
15	D	5.2	1	40	D	5.6	1	65	D	7.6	1
16	D	5.6	1	41	S	2.8	1	66	C	3.0	3
17	C	4.1	2	42	D	3.9	1	67	D	7.0	1
18	C	4.3	2	43	D	2.1	1	68	D	7.9	1
19	D	3.7	1	44	D	8.6	1	69	D	3.3	1
20	C	3.8	2	45	D	2.0	1	70	D	3.9	1
21	C	2.2	2	46	D	9.8	1	71	S	4.8	1
22	C	2.4	2	47	D	9.7	1	72	D	3.2	1
23	C	3.5	2	48	C	20.8	2	73	C	4.2	2
24	C	3.4	2	49	D	6.4	1	74	C	3.8	2
25	C	3.5	2	50	C	2.2	2	75	C	3.6	2

*D=Door, C=Corridor, S=Stairs

5.6.3. Experimental results

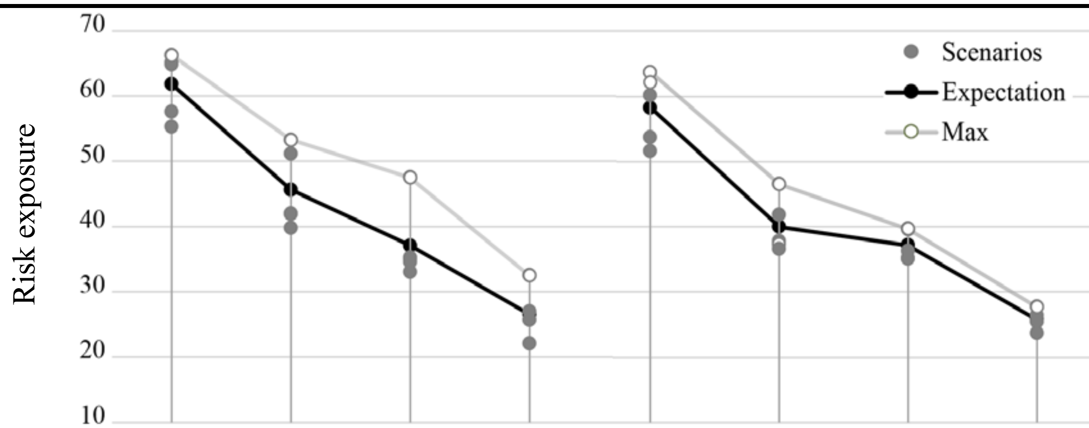
The SP (BEDP-SP-UE, BEDP-SP-SO) and RO (BEDP-RO-UE and BEDP-RO-SO) model results are reported in Tables 5-8 and 5-9, respectively. The RO and SP approaches lead to different design solutions. Scenarios with external hazards frequently give the worst results in terms of evacuation risk exposure. Under these scenarios, the RO design solutions are best, because they target these worst-case situations.

Table 5-8 SP run results



Problem	BEDP-SP-UE				BEDP-SP-SO			
Budget ($×1000)	0	7.5	15	42	0	7.5	15	42
Selected design options	-	S7, H5	S4, S7, H5, E3	All	-	S7, H5	S2, S7, H5, E3	All
Expected risk	61.8	36.7	34.7	26.6	58.3	33.8	31.0	25.8
Max. risk	66.3	54.6	49.1	32.5	63.7	50.1	41.6	27.7
Standard deviation	4.5	9.2	7.5	3.4	4.8	8.2	5.4	1.3

Table 5-9 RO run results



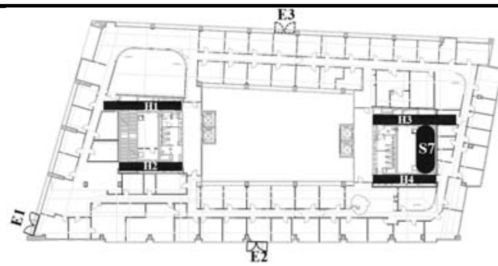
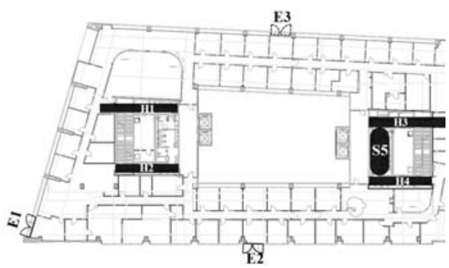
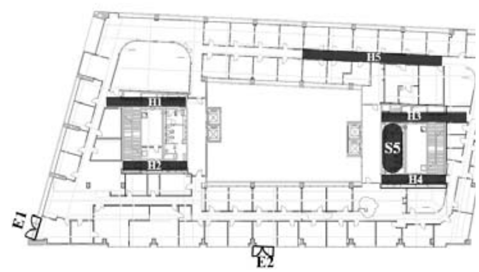
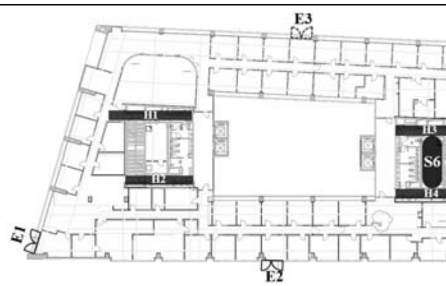
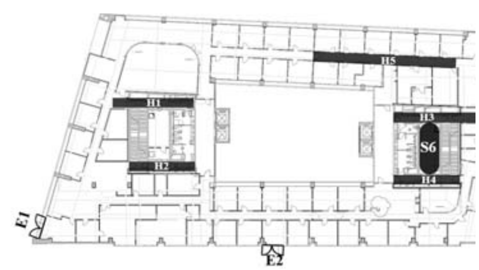
Problem	BEDP-RO-UE				BEDP-RO-SO			
Budget ($×1000)	0	7.5	15	42	0	7.5	15	42
Selected design options	-	S6, E3	S4, S5, H5, E3	All	-	S6, E3	S4, S6, H5	All
Expected risk	61.8	45.6	37.1	26.6	58.3	40.0	37.2	25.8
Max. risk	66.3	53.3	47.5	32.5	63.7	46.5	39.7	27.7
Standard deviation	4.5	5.5	5.3	3.4	4.8	3.7	2.1	1.3

As expected, modeling under SO results in slightly lower evacuation risk exposure compared to modeling under the UE condition for the same level of budget. This is also true in those cases in which the same optimal design solution was identified under SO or UE conditions. The difference in objective function values quantifies the benefits to the system of enforcing SO-derived routes and shelter/exit assignments. With a budget of $15,000, for example, the reduction in expected risk exposure achieved by enforcing the SO solution over allowing individuals the freedom to choose their own paths is approximately 12%.

Moreover, the maximum as well as the dispersion of risk data points over all scenarios (measured by standard deviation) diminishes through a RO approach. That is, RO modeling results in better solutions. Similar reduction in standard deviation is noted when comparing implementations with SO and UE conditions. That is, as expected, the SO solutions outperform the UE solutions. Of course, their practical implementation requires some level of support to ensure that evacuees adhere to directives.

The optimal design solutions were also determined under only internal fire scenarios given a budget of $7,500. The corresponding results are reported and compared with the design solutions under both internal and external scenarios in Table 8 and resulting designs are depicted in Table 5-10. Identical solutions are found for SPs under UE and SO conditions. However, a design shift is made from fortification of hallway 5 to construction of exit 3 for internal only scenarios. Evacuating out of the building through an emergency exit is the least desired option under the external malicious act scenario. When only an internally produced hazard is considered, evacuation from the building will produce best results. Such diametrically opposed optimal design solutions highlights the importance of pursuing a multi-hazard approach.

Table 5-10 Optimal design solutions under internal only scenarios vs. internal and external scenarios (budget= $7,500)

Problem	Hazard type	
	Internal	Internal & external
BEDP-SP-UE		
BEDP-SP-SO		
BEDP-RO-UE		
BEDP-RO-SO		

5.7. Conclusions

The mathematical program presented in this chapter allows the identification of building design solutions that ensure the safety of evacuees during emergencies. The program can be used to investigate different alternatives for the design of shelters, fortified hallways and exits in buildings, and permits exact solution that minimizes the exposure of evacuees to risks under various hazard scenarios. This solution requires a novel approach that differs from previous studies on building evacuation, which deal mainly with the analysis of a predefined building design, as well as previous studies on regional evacuation problems, which have focused on the minimization of evacuation time for a single type of hazard. The explicit consideration of risk exposure includes not only the time evacuees will spend in different locations in the building (which in turn depends on the length of the path traveled as well as on the number of people using that path), but also the level of protection from hazards that these locations provide.

This study follows a multi-hazard approach, in which different types of hazards are simultaneously taken into account when searching for an optimal solution. This can be crucial, since for each type of hazard a different solution may produce the best results, but eventually a single design solution must be chosen. All other relevant works in the literature consider only a single hazard class. Furthermore, the program allows the use of an objective function based on expectation, which gives weight to a range of hazard scenarios, or a more conservative RO approach, which focuses on the worst-case scenario in terms of evacuation risk exposure.

Finally, the program allows different types of user responses to be considered by embedding either SO or UE conditions. The SO approach assumes that evacuees will be guided by a trained staff person who is fully informed of the conditions in the building. This may be appropriate in certain types of buildings (e.g. train stations), in certain circumstances in which a building may be used (e.g. a concert or sporting event), and for certain types of events for which such information can be provided (e.g.

an internal fire). The UE approach assumes that fully informed evacuees will themselves choose their evacuation paths and destinations, and that the evacuees have full information about their options. This may be appropriate in buildings with which the evacuees are highly familiar (e.g. their home or workplace), and for certain types of events for which they have been repeatedly trained or which they have repeatedly experienced.

Chapter 6: Travel Time Resilience of Roadway Networks in the Presence of Non-recurring Disruptions

6.1. Introduction

More than 4 million miles of U.S. public roads serve approximately 90% of passenger transport in the country (BTS, 2013). Natural and human-caused hazards threaten this roadway network, and the possibility for significant economic loss due to damage to this network is significant. Damage caused by Hurricane Irene to the Vermont transportation network amounted to $65 million (Lunderville, 2012). The collapse of the I-35W Bridge over the Mississippi River interrupted more than 140,000 daily vehicular trips causing more than $0.4 million increase in daily passenger trip costs due to traffic rerouting (Zhu et al., 2010). The repair and reconstruction costs of transportation infrastructure systems after Hurricane Katrina were estimated to have exceeded $32 billion (Sundeen and Reed, 2006).

Transportation infrastructure systems are also attractive targets for malicious acts. Recent examples include bombings of passenger rail systems in London (2005), Madrid (2004), and Mumbai (2006). Since the 1990s, more than 25% of terrorist attacks have either targeted surface transportation systems or used them to provide access to other targets (Murray-Tuite, 2008). In addition to resulting physical damage, these events have long-term socio-economic and psychological impacts. Furthermore, they affect traveler decisions. Gordon et al. (2007) identified a 6% reduction in passenger trips and a sizable shift from public transit services to private automobiles during a two-year period following the 9/11 attack.

To prevent significant loss from disaster events, whether caused by a malicious act, an accident or technology failure, or nature, the transportation system must be resilient, and thus able to cope with disaster impact. Resilience is a measure of a network's ability to absorb disruption consequences (see for example: Bruneau et al., 2003; Rose, 2004; Sheffi, 2005; Cox et al., 2011; Chen and Miller-Hooks, 2012). Faturechi et al. (under review) provide a synthesis of approximately 200 works in the literature on resilience and other related measures, including coping capacity, robustness, flexibility and recovery, in the context of transportation. In addition to works that focus on resilience estimation, there are works that determine optimal pre-event mitigation or preparedness strategies (Losada et al., 2012), post-event response actions (Chen and Miller-Hooks, 2012; Vugrin et al., 2010) or both (Miller-Hooks et al., 2012) with the goal of maximizing resilience. A single paradigm for understanding and optimizing resilience and related measures that builds on the existence or nonexistence of possible actions that can be taken pre- or post-disaster is provided in (Faturechi and Miller-Hooks, 2013).

All prior works related to the maximization of resilience consider only applications in which resilience enhancing actions are chosen with the aim of achieving a system optimal solution. Such solutions inherently assume that the users of the system will follow the system optimal directives. For example, traffic might be centrally directed to use predetermined routes seeking a system optimum implementation. This is appropriate in many applications, such as in freight networks where the goods to be moved are not cognizant. Several relevant works involving network design under supply or demand uncertainty explicitly recognize the ability of people to make their own decisions regarding their path choice, often with the goal of selfishly maximizing their own utility functions. This is discussed in detail in (Nagurney and Qiang 2012). These works generally involve a bilevel program structure, where design decisions, such as capacity expansion of a network link, are taken at the upper level, while the response of travelers to the supply offerings is assessed at the lower level. Supply uncertainty typically arises from day-to-day incidents, like traffic accidents, that may cause degradation in network performance. The impact of demand uncertainty is typically measured through variations in travel speeds and, thus, travel times. Chen et al. (2011) provide an extensive review of this literature.

A few works in the literature employ a similar bilevel structure in addressing network design or enhancement problems in the context of disaster mitigation. Specifically, these works consider retrofit (Fan and Liu, 2010) and expansion (Lo and Tung, 2003; Dimitriou and Stathopoulos, 2008) actions with the aim of reducing the impact of potential disaster events on network performance. These works may be viewed as seeking to maximize system reliability or robustness through pre-event actions. Link capacities are only known with certainty post-disaster, however, these works build in the capacity uncertainty within the lower-level problem, where the system users take decisions only after the disaster scenario is realized. These earlier works suggest the use of inexact solution techniques in which the complicating complementarity constraints are relaxed or other heuristics methods. A general discussion on the properties of related problem classes and potential solution techniques can be found in (Patriksson, 2008).

In the earlier works where uncertainty in supply (e.g. link capacities) was considered in the lower-level, a UE is determined for each potential event scenario given upper-level decisions, and upper-level decisions are taken deterministically. Achieving a UE assumes fully-adaptive behavior by system users. The users are presumed to have perfect information about the state of the roadway network in choosing their paths. In the context of disaster events, this assumption might be valid only long after the event's initial occurrence at which time system users have enough information to adapt their travel behavior to the new situation, and a new UE is established. However, shortly after the event occurrence, such an assumption is likely erroneous. Despite the rich literature on travel behavior, modeling such behavior under disruption has received little attention and is conceptually complex (Zhu et al., 2010).

The subject of this lower-level problem is the period arising shortly after the occurrence of a disaster event in which short-term, contingency plans can be implemented. According to a user behavior survey of De Palma and Rochat (1999), users have high flexibility in their route choice shortly after the occurrence of an event. That is, user behavior is characterized as being semi-adaptive given limited information, including information on damage and completion of repairs, on network conditions (Iida et al., 2000). Thus, the lower-level problem is formulated as a Partial UE (PUE) traffic assignment problem. This concept of a PUE was introduced in Sumalee and Watling (2008).

This chapter incorporates user behavior in the measurement and maximization of travel time resilience for roadway networks given under a set of possible disaster scenarios. The problem of quantifying and optimizing travel time resilience (i.e. the

Travel Time Resilience Problem (TTRP)) is formulated as a bilevel, three-stage stochastic program with lower-level equilibrium constraints. Both upper- and lower-level problems involve capacity uncertainty. The upper-level includes a three-stage decision making process in which both pre- and post-event resilience enhancing actions may be taken. The decision process is informed by information that is revealed at each stage, as is compatible with the Disaster Management Life-Cycle (DMLC) (Waugh, 2000): (1) pre-event expansion and retrofit as mitigation options to enhance the coping capacity of the road network, (2) pre-event preparedness where resources are acquired and prepositioned shortly in advance of a predicted event occurrence to facilitate response actions, and (3) post-event short-term response actions taken post-disaster to restore network capacity, minimize the extent of damage, and/or protect the remaining facilities. A multi-hazard perspective is taken, whereby actions that may be effective in one scenario may be ineffective in another. An exact Progressive Hedging algorithm is presented for solution of a single-level equivalent to this bilevel, three-stage stochastic program.

Whether addressing day-to-day incident-induced traffic congestion or disaster events, including pre-event or both pre- and post-event actions for enhancing system performance, or employing a UE or PUE, these problems involving uncertainty in available system capacity in which user response to network supply decisions is captured can be mathematically modeled as Stochastic Mathematical Programs with Equilibrium Constraints (SMPECs). Thus, they are a type of Stochastic Network Design Problem (SNDP). In addition to its contributions to resilience measurement, this chapter extends the study of SNDPs (SMPECs) generally. Key to its contributions are its consideration of supply uncertainty in both upper- and lower-level problems, incorporation of a three-stage stochastic program in the upper-level to capture key relevant DMLC stages, use of a PUE in traffic assignment rather than a UE as is appropriate for the disaster-context, and application of cutting-edge linearization methodologies for dealing with complementarity constraints and nonlinear, nonseparable travel time functions.

The next section introduces the problem formulation. This is followed by description of the solution method in Section 6-3, and application on an illustrative example in Section 6-4.

6.2. Problem formulation

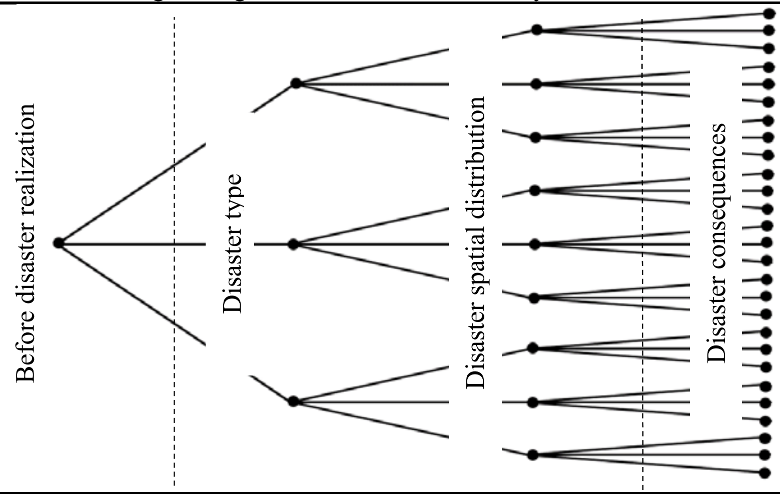
At the upper level of the proposed TTRP, mitigation, preparedness and response actions are chosen with information from the lower level about the resulting total travel time for all O-D pairs that can be expected given upper level choices. The upper level acts as the leader, determining the optimal supply decisions. The lower level acts as the follower, wherein affected system users selfishly determine their paths with knowledge of the upper-level decisions. The optimal solution to the bilevel problem results at a Stackelberg equilibrium (Gibbons, 1992).

The upper-level problem is a three-stage stochastic program that accounts for the occurrence of one of a set of potential disaster events, as well as the information that is revealed about these events and their consequences over time. In the first stage, possible disaster events and their consequences are known probabilistically. At the end of this stage, after some passage of time, certain attributes of the disaster event may be revealed, informing the second stage or equivalently creating a second information state. Here, this information includes the disaster event type and its temporal and spatial properties/distributions. The third information stage arises, revealing a final information state, once the event has occurred and quick assessment of the disaster region has been completed.

This information process can be captured through the concept of a scenario tree (Table 6-1) within which each node represents an information state and each link carries with it the probability of transitioning from one information state to the next over time (or stages). Decisions taken at each stage are depicted. The tree, thus, captures all possible outcomes, and each path from the root of the tree to a leaf (i.e. from the first-stage to the last) gives a possible scenario. Where a finite set of possible disaster scenarios is considered, the scenarios can be completely enumerated and are known *a priori*. Notation employed in describing the travel time resilience problem is given next, followed by the problem formulation.

A network representation of the roadway system is exploited. The network's topology is given by $G = (N, A)$, where N is the set of nodes and A is the set of links. Associated with each link is its travel time and capacity limitation, both of which are random variables. Network performance is measured under a set of possible disaster scenarios, each of which is defined by a disaster event type, affected links and its impact the travel time and capacity of these links.

Table 6-1 DMLC stages the given the level of uncertainty realization

Disaster event tree			
Information revealed	ξ^{1*}	ξ^2	ξ^3
DMLC stage	Mitigation	Preparedness	Response stage
Decision variables	$\boldsymbol{\gamma}^1$	$\boldsymbol{\gamma}^2(\xi^2)$	$\boldsymbol{\gamma}^3(\xi^3)$

* ξ^1 known deterministically from the start

6.2.1. A measure of travel time resilience in roadway networks

Total travel time, **tt**, to serve a given O-D demand is chosen as the system-level measure of performance. The disruption profile given in figure 6-1, graphically captures the variation of a roadway network's **tt** over the DMLC, from *pre-event* conditions in which a UE is reached until *recovery* is complete and a new UE state is established given new network conditions. Changes in the network conditions may result from long-term activities, such as reconstruction. Users adapt their travel behaviors to this new situation, creating a new UE. Immediately after the occurrence of a disaster event, i.e. *post-event (confusion)*, capacity is degraded and users may not be able to ascertain the disaster's impact on potential routes. Thus, they may be confused or indecisive, creating inefficiencies in the use of a network that is simultaneously performing below its norm. *Post-response*, travel times improve, reaching a PUE, as the network capacity is partially restored to a satisfactory level through the implementation of short-term repairs, and users have received limited information on network conditions and improvements.

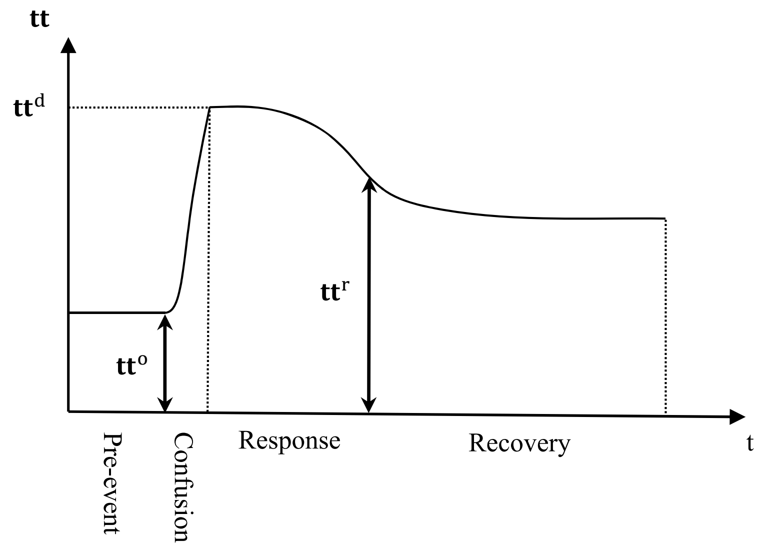


Figure 6-1 Travel time disruption profile for passenger traffic in a roadway network, where $\mathbf{tt^o}$, $\mathbf{tt^d}$, and $\mathbf{tt^r}$ are the total travel times at the end of the pre-event, confusion, and response stages

Total travel time is employed in assessing resilience, $R_{T,B}$, under a given budget, *B*, for taking mitigation, preparedness and response actions and given time allotted for response action implementation, *T*. As in equation (1), the reciprocal of total travel time achieved in reaching a PUE at the end of the response stage divided by the reciprocal of the total travel time achieved in a UE pre-event and pre-action is taken to quantify resilience.

$$R_{T,B} = \frac{(\mathbf{tt^r})^{-1}}{(\mathbf{tt^o})^{-1}} = \frac{\langle \mathbf{x^o}, \mathbf{t^o} \rangle}{\langle \mathbf{x^r}, \mathbf{t^r} \rangle} \tag{1}$$

6.2.2. Notation

The notation employed within the mathematical program is as follows:

Sets

A = set of links, a, in the roadway network

N = set of network nodes, n, representing roadway intersections and points of demand

W = set of origin-destination (OD) pairs w

K_w = set of paths k between OD pair $w \in$ W

S = set of disaster types $s \in S$

P = set of stages, $p = 1,2,3$, over which information is gained: $p = 1$ refers to a pre-event stage where the current state is known deterministically, but possible future disaster events in terms of type, location and consequences are known only probabilistically; $p = 2$ refers to a later point in time when the event type and location are known deterministically (either pre- or post-event), but the impact on the system is unknown, and the probability distribution of the event's potential or perceived consequences is updated; and $p = 3$ refers to the point in time after the event has occurred and all event characteristics, as well as the system state, are known deterministically.

Modeling parameters

$\{\xi^p\}_p$ = information process capturing the state of knowledge, ξ, about the system's current and future states at stage $p \in P$: ξ^1 captures pre-event conditions deterministically and probability distributions of future conditions; ξ^2 specifies event type and location, but updates the probability distributions of future conditions that are associated with the impact of the event on the system; ξ^3 specifies conditions of the network once the event is fully realized.

D = vector of OD travel demand, $\mathbf{D} = [\ldots, D_w, \ldots]^{\boldsymbol{T}}\ \forall\ w \in W$

$\mathbf{f}^{\mathrm{o}}(\xi^1)$ = known vector of pre-event path flows, $\mathbf{f}^{\mathrm{o}}(\xi^1) = [\ldots, f^o_{k,w}(\xi^1), \ldots]^{\boldsymbol{T}}\ \forall\ k \in K_w, w \in W$

$\mathbf{x}^{\mathrm{o}}(\xi^1), \mathbf{c}^{\mathrm{o}}(\xi^1)$ = known vectors of pre-event link flows and capacities, $\mathbf{x}^{\mathrm{o}}(\xi^1) = [\ldots, x^o_a(\xi^1), \ldots]^{\boldsymbol{T}}$ and $\mathbf{c}^{\mathrm{o}}(\xi^1) = [\ldots, c^o_a(\xi^1), \ldots]^{\boldsymbol{T}}$ $\forall a \in A$, respectively

$\mathbf{t}^{\mathrm{o}}(\xi^1)$ = vector of pre-event link travel times, $\mathbf{t}^{\mathrm{o}}(\xi^1) = [\ldots, t^o_a(\xi^1), \ldots]^{\boldsymbol{T}}\ \forall\ a \in A$, respectively

$\mathbf{c}^{\mathrm{d}}(\xi^3)$ = vector of post-event link capacities under information state ξ^3, $\mathbf{c}^{\mathrm{d}}(\xi^3) = [\ldots, c^d_a(\xi^3), \ldots]^{\boldsymbol{T}}\ \forall\ a \in A$

Δ = link-path incidence matrix, $\Delta = \left[\Delta_{a,k,w}\right]\ \forall\ a \in A, k \in K_w, w \in W$ ($\Delta_{a,k,w} = 1$ if path $k \in K_w$ uses link a, and = 0 otherwise)

$\boldsymbol{\Lambda}$ = OD pair-path incidence matrix, $\boldsymbol{\Lambda} = \left[\Lambda_{k,w}\right]$ for $\forall k \in K_w, w \in W$($\Lambda_{k,w} = 1$ if path k connects OD pair w and $\Lambda_{k,w} = 0$ otherwise)

$\boldsymbol{\Pi}(\xi^3)$ = post-event damage state matrix of paths under information

		state ξ^3,
		$\boldsymbol{\Pi}(\xi^3) = [\pi_{k,w}(\xi^3)]$ for $\forall k \in K_w, w \in W$ ($\pi_{k,w}(\xi^3) = 1$ if the path $k \in K_w$ is affected given ξ^3, and $\pi_{k,w}(\xi^3) = 0$ otherwise)
$\boldsymbol{\Theta}(\xi^3)$	=	disaster type matrix $\boldsymbol{\Theta}(\xi^3) = [\dots, \Theta_s(\xi^3), \dots]^{\boldsymbol{T}}$, where $\Theta_s(\xi^3) = 1$ if when reaching information state ξ^3 the disaster event that has occurred is of type s, $\Theta_s(\xi^3) = 0$ otherwise
B	=	available budget
T	=	response time

1st stage variables

$\boldsymbol{\gamma}^1(\xi^1)$	=	vector of first-stage action variables, $\boldsymbol{\gamma}^1(\xi^1) = [\boldsymbol{\gamma}^{1,\mathrm{e}}(\xi^1), \boldsymbol{\gamma}^{1,\mathrm{h}}(\xi^1)]^{\boldsymbol{T}}$ where $\boldsymbol{\gamma}^{1,\mathrm{e}}(\xi^1) = [\dots, \gamma_a^{1,e}(\xi^1), \dots]^{\boldsymbol{T}}$ is the vector of link capacity expansion levels $\forall\, a \in A$, and $\boldsymbol{\gamma}^{1,\mathrm{h}}(\xi^1) = [\dots, \gamma_{a,s}^{1,h}(\xi^1), \dots]^{\boldsymbol{T}}$ is the vector of link retrofit levels $\forall\, a \in A, s \in S$. Since ξ^1 is revealed from the start of the decision horizon, $\boldsymbol{\gamma}^1(\xi^1)$ is given as $\boldsymbol{\gamma}^1$ for simplicity.

2nd stage variables

$\boldsymbol{\gamma}^2(\xi^2)$	=	vector of disaster-specific link preparedness (resource availability – second stage) action levels given information state ξ^2, $\boldsymbol{\gamma}^2(\xi^2) = [\dots, \gamma_{a,s}^2(\xi^2), \dots]^{\boldsymbol{T}}$ $\forall\, a \in A, s \in S$

3rd stage variables

$\boldsymbol{\gamma}^3(\xi^3)$	=	vector of disaster-specific link response (third-stage) levels under information state ξ^3, $\boldsymbol{\gamma}^3(\xi^3) = [\dots, \gamma_{a,s}^3(\xi^3), \dots]^{\boldsymbol{T}}$ $\forall\, a \in A, s \in S$
$\mathbf{x}^{\mathrm{r}}(\xi^3), \mathbf{c}^{\mathrm{r}}(\xi^3)$	=	vectors of post-response link flows and capacities under information state ξ^3, $\mathbf{x}^{\mathrm{r}}(\xi^3) = [\dots, x_a^r(\xi^3), \dots]^{\boldsymbol{T}}$ and $\mathbf{c}^{\mathrm{r}}(\xi^3) = [\dots, c_a^r(\xi^3), \dots]^{\boldsymbol{T}}$ $\forall\, a \in A$, respectively
$\mathbf{t}^{\mathrm{r}}(\xi^3)$	=	vector of post-response link travel time as a function of link flow and capacity, $\mathbf{t}^{\mathbf{r}}(\xi^3) = [\dots, t_a^r(\xi^3), \dots]^{\boldsymbol{T}}$ $\forall\, a \in A$
$\mathbf{f}^{\mathrm{r}}(\xi^3)$	=	vector of post-response path flows under information state ξ^3, $\mathbf{f}^{\mathrm{r}}(\xi^3) = [\dots, f_{k,w}^r(\xi^3), \dots]^{\boldsymbol{T}}$ $\forall\, k \in K_w, w \in W$
$\boldsymbol{\tau}^{\mathrm{r}}(\xi^3)$		vector of post-response path travel times under information state ξ^3, $\boldsymbol{\tau}^{\mathrm{r}}(\xi^3) = [\dots, \tau_{k,w}^r(\xi^3), \dots]^{\boldsymbol{T}}$ $\forall\, k \in K_w, w \in W$
$\mathbf{u}^{\mathrm{r}}(\xi^3)$	=	vector of post-response shortest travel times under information state ξ^3, $\mathbf{u}^{\mathrm{r}}(\xi^3) = [\dots, u_w^r(\xi^3), \dots]^{\boldsymbol{T}}$ $\forall\, w \in W$

6.2.3. TTRP

The TTRP formulated as a bi-level, three-stage, stochastic, nonlinear program for maximizing travel time resilience of roadway networks is now presented. The program involves stochasticity in both upper- and lower-levels.

Upper-level problem:

$$\max_{\boldsymbol{\gamma}^1 \in \boldsymbol{\Gamma}^1} E_{\xi^2}\left\{\max_{\boldsymbol{\gamma}^2 \in \boldsymbol{\Gamma}^2(\boldsymbol{\gamma}^1,\xi^2)} E_{\xi^3|\xi^2}\left[\max_{\boldsymbol{\gamma}^3 \in \boldsymbol{\Gamma}^3(\boldsymbol{\gamma}^2,\xi^3)} R_{T,B}(\xi^3)\right]\right\} \tag{2}$$

s.t.

$$c_a^r(\xi^3) = \left(1+\gamma_a^{1,e}\right)\{c_a^d(\xi^3) + \textstyle\sum_s \Theta_s(\xi^3)[c_a^o - c_a^3(\xi^3)][\gamma_{a,s}^{1,h} + \gamma_{a,s}^3(\xi^3)]]\}, \forall a \in A \tag{3}$$

$$q_{a,s}^3(\xi^3) \le T, \forall a \in A, s \in S \tag{4}$$

$$\textstyle\sum_{a\in A}\{b_a^{1,e} + \sum_{s\in S}\Theta_s(\xi^3)\left[b_{a,s}^{1,h} + b_{a,s}^2(\xi^2) + b_{a,s}^3(\xi^3)\right]\} \le B \tag{5}$$

In the upper-level formulation, the first-stage feasibility set for mitigation actions is given by $\boldsymbol{\Gamma}^1 = \{\boldsymbol{\gamma}^1 \,\big|\, \underline{\gamma}_a^{1,e} \le \gamma_a^{1,e} \le \bar{\gamma}_a^{1,e}, 0 \le \gamma_{a,s}^{1,h} \le 1, \forall a \in A, s \in S\}$ with $\underline{\gamma}_a^{1,e}$ and $\bar{\gamma}_a^{1,e}$ for link a as lower- and upper-bounds on capacity expansion level, respectively. The retrofit variable $\gamma_{a,s}^{1,h}$ sets the desired fortification level for link a for each disaster type s. More than one retrofit action can be taken on the same link. $\gamma_{a,s}^{1,h}$ ranges between 0 and 1, where $\gamma_{a,s}^{1,h} = 1$ refers to the highest fortification level obtainable for link a such that no damage will be incurred as a consequence of the occurrence of a disaster of type s. The range on second-stage preparedness levels is defined by the feasibility set $\boldsymbol{\Gamma}^2(\boldsymbol{\gamma}^1, \xi^2) = \{\boldsymbol{\gamma}^2(\xi^2)\,|\,0 \le \gamma_{a,s}^2(\xi^2) \le 1, \forall a \in A, s \in S\}$, where $\gamma_{a,s}^2(\xi^2) = 1$ means all resources required to repair damage following a disaster of type s are provided in advance upon realizing information state ξ^2. $\boldsymbol{\Gamma}^3(\boldsymbol{\gamma}^2, \xi^3) = \{\boldsymbol{\gamma}^3(\xi^3)\,|\,0 \le \gamma_{a,s}^3(\xi^3) \le 1, \forall a \in A, s \in S\}$ is the third-stage response level feasibility set for information state ξ^3, where $\gamma^3_{a,s}(\xi^3) = 1$ infers that capacity along link a is restored to the pre-event capacity.

The objective function (2) seeks to maximize the expectation of network resilience over all possible scenarios given by each possible information state ξ^3. The numerator of the resilience measure $R_{T,B}(\xi^3)$, $\langle \mathbf{x}^{\mathrm{o}}, \mathbf{t}^{\mathrm{o}} \rangle$, is constant and scenario-independent, representing UE-based total travel time under pre-event, pre-action conditions. Thus, the objective seeks to minimize the post-response expected total travel time forming the denominator of $R_{T,B}(\xi^3)$, $\langle \mathbf{x}^{\mathrm{r}}(\xi^3), \mathbf{t}^{\mathrm{r}}(\xi^3) \rangle$. Thus, objective function (2) can be replaced by equation (6):

$$\min_{\boldsymbol{\gamma}^1 \in \boldsymbol{\Gamma}^1} E_{\xi^2} \left\{ \min_{\boldsymbol{\gamma}^2 \in \boldsymbol{\Gamma}^2(\boldsymbol{\gamma}^1, \xi^2)} E_{\xi^3|\xi^2} \left[\min_{\boldsymbol{\gamma}^3 \in \boldsymbol{\Gamma}^3(\boldsymbol{\gamma}^2, \xi^3)} \langle \mathbf{x}^{\mathrm{r}}(\xi^3), \mathbf{t}^{\mathrm{r}}(\xi^3) \rangle \right] \right\} \tag{6}$$

Post-response link capacity is defined in equations (3) as a function of the links' pre-action, pre-event capacity, as well as first-stage link expansion and retrofit decisions and third-stage link response decisions. The effects of decisions are presumed to be linear to the original link capacities. Inclusion of parameter $\Theta_s(\xi^3)$ ensures that the effects of specialized link retrofit and response actions have effects that are consistent with disaster type s and associated information state ξ^3. Second-stage link preparedness actions do not directly affect link capacity, and are not included in the equation.

Constraints (4) guarantee that, for each information state ξ^3, all response actions that are to be taken are completed before the end of the response period, i.e. by time T. The budget limitation is assured through constraint (5). Link expansion, retrofit, preparedness, and response costs used in constraint (5), as well as response implementation times of constraints (4) are as assumed to be functions of action level as described through constraints (7) - (11).

$$b_a^{1,e} = \bar{b}_a^{1,e} \cdot \gamma_a^{1,e}, \forall a \in A \quad (7)$$

$$b_{a,s}^{1,h} = \left(1 + \gamma_a^{1,e}\right) \cdot \bar{b}_{a,s}^{1,h} \cdot \gamma_{a,s}^{1,h}, \forall a \in A, s \in S \quad (8)$$

$$b_{a,s}^2(\xi^2) = \bar{b}_{a,s}^2(\xi^2) \cdot \gamma_{a,s}^2(\xi^2), \forall a \in A, s \in S \quad (9)$$

$$b_{a,s}^3(\xi^3) = \left(1 + \gamma_a^{1,e}\right)\left\{\bar{b}_{a,s}^3(\xi^3) - \left[\bar{b}_{a,s}^3(\boldsymbol{\xi}^3) - \underline{b}_{a,s}^3(\xi^3)\right]\gamma_{a,s}^2(\xi^2)\right\}\left[\frac{c_a^o - c_a^d(\xi^3)}{c_a^o}\right]\gamma_{a,s}^3(\xi^3), \quad (10)$$

$\forall a \in A, s \in S$

$$q_{a,s}^3(\xi^3) = \left(1 + \gamma_a^{1,e}\right)\left\{\bar{q}_{a,s}^3(\xi^3) - \left[\bar{q}_{a,s}^3(\xi^3) - \underline{q}_{a,s}^3(\xi^3)\right]\gamma_{a,s}^2(\xi^2)\right\}\left[\frac{c_a^o - c_a^d(\xi^3)}{c_a^o}\right]\gamma_{a,s}^3(\xi^3), \quad (11)$$

$\forall a \in A, s \in S$

where $\bar{b}_a^{1,e}$ and $\bar{b}_{a,s}^{1,h}$ are first-stage unit costs of expanding link a or, for a given disaster event of type s, retrofitting link a, respectively. The implications for retrofit costs of link expansion are captured in constraints (8). In constraints (9), $\bar{b}_{a,s}^2(\xi^2)$ denotes second-stage unit costs of link preparedness actions for a given disaster event of type s and information state ξ^2. Third-stage unit costs and times required for implementing response actions are defined in constraints (10) and (11), respectively. Both are functions of response and preparedness levels, wherein the effects of preparedness in advance of an event affect the efficiency of post-event response actions. $\underline{b}_{a,s}^3(\xi^3)$ [$\underline{q}_{a,s}^3(\xi^3)$] and $\bar{b}_{a,s}^3(\boldsymbol{\xi}^3)$ [$\bar{q}_{a,s}^3(\boldsymbol{\xi}^3)$] are post-disaster (having realized information state $\boldsymbol{\xi}^3$) costs [times] of complete reconstruction of link a. In the former, it is presumed that no preparedness actions were taken, while in the latter all appropriate preparedness actions were taken. Thus, cost or time for taking response actions accounts for related preparedness actions having been taken. Incurred costs or implementation times are also a function of damage-level. Thus, an additional terms,

$\frac{c_a^o - c_a^d(\xi^3)}{c_a^o}$, is included in constraints (9) and (1) to capture the increasing expense and effort required to address situations with higher damage levels.

The link flows needed to compute the objective function of the upper-level problem are determined through solution of the lower-level problem (12) for each information state ξ^3. The lower-level problem seeks a flow pattern that achieves a PUE given actions taken in solution of the upper-level problem. In a PUE, user behavior is characterized as semi-adaptive and assumes that only those who are affected are likely to reconsider their original route decisions.

Lower-level problem:

$$\min_{\mathbf{x}^{\mathrm{r}} \in \Omega_{\mathbf{x}}^{\mathrm{r}}(\xi^3)} \sum_a \int_0^{x_a^r(\xi^3)} t_a^r(v, c_a^r(\xi^3))\, dv, \tag{12}$$

where $\Omega_{\mathbf{x}}^{\mathrm{r}}(\xi^3) = \{\mathbf{x}^{\mathrm{r}}(\xi^3) | \mathbf{x}^{\mathrm{r}}(\xi^3) = \Delta\mathbf{f}^{\mathrm{r}}(\xi^3) + \Delta[\mathbf{I} - \mathrm{diag}\boldsymbol{\Pi}(\xi^3)]\mathbf{f}^{\mathrm{o}}, \mathbf{f}^{\mathrm{r}}(\xi^3) = \mathrm{diag}\boldsymbol{\Pi}(\xi^3)\mathbf{f}^{\mathrm{o}}, \boldsymbol{\Lambda}\mathbf{f}^{\mathrm{o}} = \mathbf{D}, \mathbf{f}^{\mathrm{r}}(\xi^3) \geq 0\}$ is a feasibility vector set of post-response link flows in which the diagonal matrix $\mathrm{diag}\boldsymbol{\Pi}(\xi) = \begin{bmatrix} \ddots & 0 & 0 \\ 0 & \pi_{k,w}(\xi^3) & 0 \\ 0 & 0 & \ddots \end{bmatrix}$, $\forall\, k \in K_w, w \in W$, and $\mathbf{I} = \begin{bmatrix} \ddots & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \ddots \end{bmatrix}$ is the identity matrix of the same size. The formulation is path-based and is adapted from (Sumalee and Watling (2008). The equation $\mathbf{x}^{\mathrm{r}}(\xi^3) = \Delta\mathbf{f}^{\mathrm{r}}(\xi^3) + \Delta[\mathbf{I} - \mathrm{diag}\boldsymbol{\Pi}(\xi^3)]\mathbf{f}^{\mathrm{o}}$, where $\mathbf{f}^{\mathrm{r}}(\xi^3) = \mathrm{diag}\boldsymbol{\Pi}(\xi^3)\mathbf{f}^{\mathrm{o}}$, defines the link flow as the summation of the post-response flows of affected paths using that link, as well as the pre-event flows of unaffected paths. That is the summation of post-response flows on affected paths between an OD pair w equals the total pre-event flow between that OD pair. Note that presuming the traffic demand adjusts to pre-event pattern in spite of unrepaired damage (Iida et al., 2000), a fixed demand vector, $\mathbf{D} = [\dots, D_w, \dots]^{\boldsymbol{T}}$ for $\forall w \in W$ is assigned to the network through $\boldsymbol{\Lambda}\mathbf{f}^{\mathrm{o}} = \mathbf{D}$.

6.3. Solving the TTRP

The bilevel TTRP (2)-(12) is solved by first reducing it to a single-level problem as is often done in the literature. To accomplish this, the lower-level problem is eliminated and corresponding Karush–Kuhn–Tucker (KKT) conditions are embedded within the upper-level problem. Larsson and Patriksson (1995) showed, in a similar context involving a bilevel program with a UE in the lower level, that use of the KKT conditions in place of the lower-level problem is both necessary and sufficient for optimality. Their proof can be directly extended to this application.

The resulting single-level program is a three-stage stochastic program with nonlinear objective and constraints, e.g. complementarity constraints. Obtaining a globally optimal solution to such a program is formidable. Thus, linear approximations are employed.

Complementarity constraints are transformed into mixed integer constraints through Schur's decomposition (Horn and Johnson, 1985) using Special Ordered Sets of Type 1 (SOS1) variables (Siddiqui and Gabriel, 2013). SOS1 variables are defined as a set of variables at most one of which can be non-zero. That is, they are employed to mathematically capture the "if-then" condition in UE constraints implying that a path can take flow only if it is the shortest path. Alternative methods use a disjunctive constraint approach (e.g. Wang and Lo (2010)), employing binary variables and exploiting the global optimality of MILP solutions. This type of approach requires extensive computational resources. Moreover, the corresponding solutions are highly sensitive to the value of a specific constant introduced in their mathematical formulation.

Moreover, cutting-edge linearization techniques from Vielma and Nemhauser (2011) are employed to handle non-separable continuous travel time functions, as well as nonlinear design decision terms. These transformations are described in the next two subsections. They lead to a single-level, three-stage Stochastic Mixed Integer Linear Problem (SMILP). An exact solution technique based on concepts of the Progressive Hedging Algorithm (PHA) initially introduced in (Rockefeller and Wets, 1991) is presented in Subsection 6.3.3.

6.3.1. Single-level TTRP

The single-level problem can be formulated encompassing the upper-level problem with addition of the UE constraints (14) and (15) given the feasibility set $\Omega_{\mathbf{x}}^{\mathrm{r}}(\xi^3)$ representing the lower-level problem:

$$\min_{\boldsymbol{\gamma}^1 \in \boldsymbol{\Gamma}^1} E_{\xi^2}\left\{\min_{\boldsymbol{\gamma}^2 \in \boldsymbol{\Gamma}^2(\boldsymbol{\gamma}^1,\xi^2)} E_{\xi^3|\xi^2}\left[\min_{\boldsymbol{\gamma}^3 \in \boldsymbol{\Gamma}^3(\boldsymbol{\gamma}^2,\xi^3), \mathbf{x}^{\mathrm{r}} \in \Omega_{\mathbf{x}}^{\mathrm{r}}(\xi^3)} \langle \mathbf{x}^{\mathrm{r}}(\xi^3), \mathbf{t}^{\mathrm{r}}(\xi^3)\rangle\right]\right\} \tag{13}$$

s.t.

(3)-(5)

$$\mathbf{f}^{\mathrm{r}}(\xi^3)^{\boldsymbol{T}}[\boldsymbol{\tau}^{\mathrm{r}}(\xi^3) - \mathbf{u}^{\mathrm{r}}(\xi^3)] = 0 \tag{14}$$

$$\boldsymbol{\tau}^{\mathrm{r}}(\xi^3) - \mathbf{u}^{\mathrm{r}}(\xi^3) \geq 0 \tag{15}$$

where, $\boldsymbol{\tau}^{\mathrm{r}}(\xi^3) = \Delta^{-1}\mathbf{t}^{\mathrm{r}}(\xi^3) = [\dots, \tau_{k,w}^r(\xi^3), \dots]^{\boldsymbol{T}}$, is the vector of post-response path travel time, and $\mathbf{u}^{\mathrm{r}}(\xi^3) = [\dots, u_w^r(\xi^3), \dots]^{\boldsymbol{T}}$ is the vector of post-response shortest path, for $\forall k \in K_w, w \in W$ given information state ξ^3. The resulting problem is a three-stage, nonlinear, nonconvex stochastic problem.

6.3.2. Linear approximations

a) Complementarity constraints

The approach for transforming complementarity constraints with linear components into an equivalent set of linear constraints using Schur's decomposition and SOS1 variables introduced in (Siddiqui and Gabriel, 2013) is employed here for handling the UE constraints (14). Specific to the TTRP, this decomposition of constraints (14) is given by equations (16)-(18).

$$\mathbf{G}^{\mathrm{r}}(\xi^3) = \frac{\mathbf{f}^{\mathrm{r}}(\xi^3)+[\boldsymbol{\tau}^{\mathrm{r}}(\xi^3)-\mathbf{u}^{\mathrm{r}}(\xi^3)]}{2}, \tag{16}$$

$$\mathbf{H}^{\mathrm{r}}(\xi^3) = \frac{\mathbf{f}^{\mathrm{r}}(\xi^3)-[\boldsymbol{\tau}^{\mathrm{r}}(\xi^3)-\mathbf{u}^{\mathrm{r}}(\xi^3)]}{2}, \tag{17}$$

$$\mathbf{G}^{\mathrm{r}}(\xi^3)\mathbf{G}^{\mathrm{r}}(\xi^3)^{\boldsymbol{T}} - \mathbf{H}^{\mathrm{r}}(\xi^3)\mathbf{H}^{\mathrm{r}}(\xi^3)^{\boldsymbol{T}} = 0, \tag{18}$$

where $\mathbf{G}^{\mathrm{r}}(\xi^3)$ and $\mathbf{H}^{\mathrm{r}}(\xi^3)$ are Schur's decomposition vector functions. Since $\mathbf{f}^{\mathbf{r}}(\xi^3), \boldsymbol{\tau}^{\mathrm{r}}(\xi^3) - \mathbf{u}^{\mathrm{r}}(\xi^3) \geq 0$, $\mathbf{G}^{\mathrm{r}}(\xi^3) \geq 0$. Thus, only the positive square root of $\mathbf{G}^{\mathrm{r}}(\xi^3) \cdot \mathbf{G}^{\mathrm{r}}(\xi^3)^{\boldsymbol{T}}$ is feasible and bilinear constraints (18) can be reformulated as in (19).

$$\mathbf{G}^{\mathrm{r}}(\xi^3) - |\mathbf{H}^{\mathrm{r}}(\xi^3)| = 0 \tag{19}$$

To eliminate the absolute value function, $|\mathbf{H}^{\mathrm{r}}(\xi)|$, constraints (19) are transformed through the introduction of SOS1 variables $\mathbf{H}^{\mathrm{r}^+}(\xi)$ and $\mathbf{H}^{\mathrm{r}^-}(\xi)$.

$$\mathbf{G}^{\mathrm{r}}(\xi) - [\mathbf{H}^{\mathrm{r}^+}(\xi) + \mathbf{H}^{\mathrm{r}^-}(\xi)] = 0. \tag{20}$$

b) Objective function

The objective (13) of the TTRP seeks to minimize the expectation of total travel time incurred along the shortest time paths over all O-D pairs. The objective requires the multiple of flow and travel time variables, and thus, is nonlinear. A technique for linearizing the objective function introduced by Wang and Lo (2010) is employed. This technique exploits common travel time properties of active paths for each O-D pair under UE conditions. Given fixed demand vector, $\mathbf{D} = [\dots, D_w, \dots]^{\boldsymbol{T}}$, and information state ξ, objective (13) can be replaced by (21).

$$\min_{\boldsymbol{\gamma}^1 \in \boldsymbol{\Gamma}^1} E_{\xi^2} \left\{ \min_{\boldsymbol{\gamma}^2 \in \boldsymbol{\Gamma}^2(\boldsymbol{\gamma}^1, \xi^2)} E_{\xi^3|\xi^2} \left[\min_{\boldsymbol{\gamma}^3 \in \boldsymbol{\Gamma}^3(\boldsymbol{\gamma}^2, \xi^3), \mathbf{x}^{\mathrm{r}} \in \Omega_{\mathbf{x}}^{\mathrm{r}}(\xi^3)} \langle \mathbf{D}, \mathbf{u}^{\mathrm{r}}(\xi^3) \rangle \right] \right\} \tag{21}$$

c) Link travel time function

The link travel time is estimated using the Bureau of Public Roads (BPR) function (equation (22)). Given post-response link flow and capacity random variables, for a given link $a \in A$ this function is a two-dimensional, nonseparable function.

$$t_a^r(\xi^3) = t_a^0(\xi^3) + m_a \left[\frac{x_a^r(\xi^3)}{c_a^r(\xi^3)} \right]^{n_a}, \quad a \in A \tag{22}$$

where $t_a^0(\xi^3)$ is the link free flow travel time, and m_a and n_a are BPR function parameters (herein, $m_a = 0.15$, and $n_a = 4$). In this chapter, a novel logarithmic piecewise linearization technique introduced by Vielma and Nemhauser (2011) for general multidimentional functions is applied herein in linearizing this link travel time function. It has been shown to outperform other existing piecewise linearization techniques (Vielma et al., 2010). The following describes the application of this technique for the TTRP.

Using this method, link flow and capacity variable domains are partitioned into segments. The travel time domain is thus defined by a two-dimensional flow-capacity domain. In general, any point in an n-dimensional domain can be uniquely represented by a convex combination of n +1 points (Carathéodory, 1911). For two-dimensions, three points, thus, are required, and therefore, the link travel time domain can be partitioned into non-overlapping triangles.

Any flow-capacity pair falls within a single triangle and is given by a convex combination of the associated triangle's corner-point coordinates. The link travel time values associated with the corner-points are directly calculated using equation (29). Vielma and Nemhauser's (2011) method identifies the *active* triangle containing the flow-capacity pair under consideration, and approximates the corresponding travel time through the convex combination of the travel time values at the corner-points of this active triangle. Binary variables and constraints are introduced to determine the active triangle. The number of variables and constraints is logarithmic in the number of segments, and the active triangle is determined through a binary branching scheme of a logarithmic depth in three steps.

Let $x_a^r(\xi^3)$ and $c_a^r(\xi^3)$ be represented by the vector of segments $\ddot{\mathbf{x}} = [\dots, \ddot{x}_{a,i}, \dots]$ and $\ddot{\mathbf{c}} = [\dots, \ddot{c}_{a,j}, \dots]$, for $\forall i, j \in V_a = \{0,1, \dots, v_a\}$, respectively, where v_a is a power of two. The domain of the corresponding travel time function will be $[0, v_a]^2$ with the segments represented within the matrix $\ddot{\mathbf{t}} = [\dots, t_a^r(\ddot{x}_{a,i}, \ddot{c}_{a,j}), \dots]$, for $\forall a \in A$. This domain is triangulated using the J_1 Union Jack triangulation approach (originally proposed by Todd (1977)).

Figure 6-1 graphically depicts the J_1 Union Jack triangulation of the two-dimensional domain of the link travel time where the domain $[0, v_a]^2$ is covered by copies of a 2×2 size square (highlighted in Figure 6-1), each encompassing 8 triangles. The entire domain is partitioned into $2{v_a}^2$ triangles, accordingly. As shown in Figure 6-2, there are groups of white and gray triangles such that each square contains one triangle of each color.

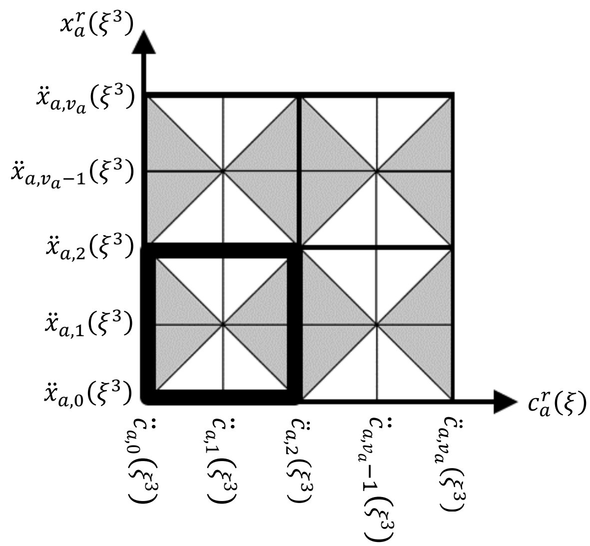


Figure 6-2 J_1 Union Jack triangulation of link travel time domain

$x_a^r(\xi^3)$ and $c_a^r(\xi^3)$ are formulated as convex piecewise-linear functions of $\ddot{x}_{a,i}$ and $\ddot{c}_{a,j}$ points, respectively, in equations (23)-(26).

$$x_a^r(\xi^3) = \sum_{i,j \in V_a} \ddot{x}_{a,i}\, \omega_{a,i,j}(\xi^3),\ \forall a \in A \tag{23}$$
$$c_a^r(\xi^3) = \sum_{i,j \in V_a} \ddot{c}_{a,j} \omega_{a,i,j}\, (\xi^3),\ \forall a \in A \tag{24}$$
$$\sum_{i,j \in V_a} \omega_{a,i,j}(\xi^3) \leq 1\,,\ \forall a \in A \tag{25}$$
$$\omega_{a,i,j}(\xi^3) \geq 0,\ \forall a \in A, i, j \in V_a \tag{26}$$

where $\omega_{a,i,j}(\xi^3)$ are convex combination weights under information state ξ^3. Accordingly,

$$t_a^r(\xi^3) = \sum_{i,j \in V_a} t_a^r(\ddot{x}_{a,i}, \ddot{c}_{a,j}) \omega_{a,i,j}(\xi^3),\quad \omega_{a,i,j}(\xi^3) \geq 0.. \tag{27}$$

In the first step, an independent SOS1 type branching is employed to select the active column of 1×1 size squares containing the active triangle. Let $\bar{V}_a = \{1, \dots, v_a\}$ be the set of columns in the link travel time domain for $a \in A$. A corresponding set is defined as $L_a = \{1, \dots, \log_2 v_a\}$ containing a logarithmic number of columns. Let $B_a: \bar{V}_a \to \{0,1\}^{\log_2 v_a}$ be a general bijective function with special structure such that $B_a(j)$ and $B_a(j+1)$ are allowed to be different in at most one vector element for $\forall j \in$

$V_a \backslash \{v_a\}$. Let $\sigma(B_a)$ be the support of vector B_a. SOS1 type branching is implemented on the logarithmic set L_a to find the active column.

$$\begin{aligned} &\sum_{j \in V_a} \sum_{i \in J_2^+(l,B_a,v_a)} \omega_{a,i,j}(\xi^3) \leq X_{a,l}(\xi^3), \ \ \forall a \in A, l \in L_a \\ &\sum_{j \in V_a} \sum_{i \in J_2^0(l,B_a,v_a)} \omega_{a,i,j}(\xi^3) \leq \left[1 - X_{a,l}(\xi^3)\right], \ \forall a \in A, l \in L_a \\ &X_{a,l}(\xi^3) \in \{0,1\}, \ \ \forall a \in A, l \in L_a, \end{aligned} \tag{28}$$

where $J_2^+(l, B_a, v_a) = \{j \in V_a : \forall i \in \bar{V}_a(j), l \in \sigma[B_a(i)]\}$ and $J_2^0(l, B_a, v_a) = \{i \in V_a : \forall i \in \bar{V}_a(j), l \notin \sigma[B_a(i)]\}$. Next, a similar SOS1 type branching is employed to select the active row of 1×1 size squares which contains the active triangle.

$$\begin{aligned} &\sum_{i \in V_a} \sum_{j \in J_2^+(l,B_a,v_a)} \omega_{a,i,j}(\xi^3) \leq C_{a,l}(\xi^3), \ \ \forall a \in A, l \in L_a \\ &\sum_{i \in V_a} \sum_{j \in J_2^+(l,B_a,v_a)} \omega_{a,i,j}(\xi^3) \leq \left[1 - C_{a,l}(\xi^3)\right], \ \forall a \in A, l \in L_a \\ &C_{a,l}(\xi^3) \in \{0,1\}, \ \ \forall a \in A, l \in L_a \end{aligned} \tag{29}$$

Given a square (i.e. equal number of columns and rows) in the link travel time domain $\bar{V}_a = \{1, \dots, v_a\}, a \in A$, can also be used to represent the rows. The *active* square is, thus, determined through the selection of *active* columns and rows.

In the final step, the *active* triangle is determined. A single binary variable, $I_a(\xi^3)$, for $a \in A$, is introduced in the following constraints to identify the color (grey or white) of the active triangle ($I_a(\xi^3) = 1$ if the active triangle is white, and $I_a(\xi^3) = 0$, otherwise).

$$\begin{aligned} &\sum_{i \in E_a} \sum_{j \in O_a} \omega_{a,i,j}(\xi^3) \leq I_a(\xi^3), \ \ \forall a \in A \\ &\sum_{i \in O_a} \sum_{j \in E_a} \omega_{a,i,j}(\xi^3) \leq [1 - I_a(\xi^3)], \ \ \forall a \in A \\ &I_a(\xi^3) \in \{0,1\}, \ \ \forall a \in A \end{aligned} \tag{30}$$

where $E_a = \{0,2, \dots, v_a\} \subset V_a$ and $O_a = \{1,3, \dots, v_a - 1\} \subset V_a$ are subsets of even and odd elements of V_a, $\forall a \in A$, respectively.

A schematic of the logarithmic three-step process for selecting the active triangle in a general travel time domain is given in Figure 6-3. Suppose that each axis is partitioned into two segments; that is, the domain contains 8 ($= 2 \times 2^2$) triangles half of which are white and the other half of which are gray as illustrated in the figure.

The gray triangle P is targeted as the active one. One binary variable (=$\log_2 2$) is introduced to select the active column, X_1, and another one to select the active row, C_1. Binary variable, I_1, is added and determines the triangle's color. First, the *active* column is selected by setting $X_1 = 0$. Setting $C_1 = 0$ in the second step, the *active* row is determined that when coupled with the first step column selection reveals the *active* square.

Finally, $I_1 = 0$ indicates the gray color of the active triangle and distinguishes it for the other white triangle in the active square. Note that the black areas in this figure indicate the union of triangles forbidden to be selected in the process based on the setting of the corresponding binary variables.

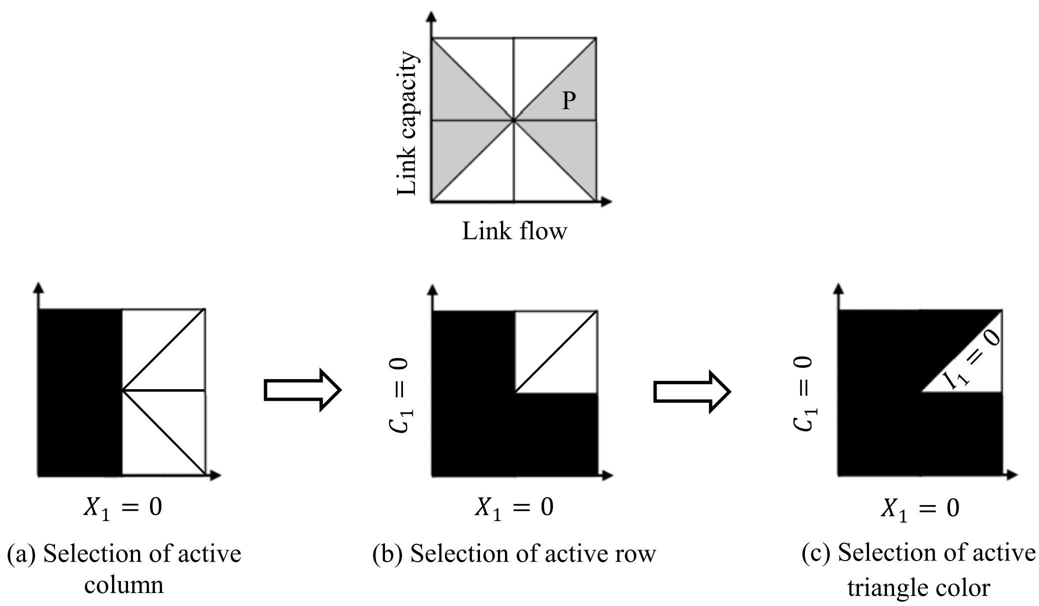


Figure 6-3 The schematic of the three-step process of active triangle selection in Vielma and Nemhauser (2011)'s logarithmic piecewise linearization method

Having v_a as a power of two involves no loss of generality. One might define a link travel time domain $[0, \max(v_a^x, v_a^c)]^2$ in which link flow and capacity are generally partitioned into v_a^x and v_a^c, respectively, with the following extra constraints for $\forall a \in A$:

$$\sum_{i=0}^{2^{\lceil log_2[max(v_a^x,v_a^c)]\rceil}-v_a^x} \sum_{j=0}^{2^{\lceil log_2[max(v_a^x,v_a^c)]\rceil}-v_a^c} \omega_{a,i,j}(\xi^3) \leq 0, \ \ \forall a \in A. \tag{31}$$

d) Design decision terms

There are bilinear action level terms $\gamma_a^{1,e} \cdot \gamma_{a,s}^{1,h}$ and $\gamma_a^{1,e} \cdot \gamma_{a,s}^{3}(\xi^3)$ in post-response link capacity equations (3) and link retrofit cost equations (8), as well as trilinear terms $\gamma_a^{1,e} \cdot \gamma_{a,s}^{2}(\xi^2) \cdot \gamma_{a,s}^{3}(\xi^3)$ expressed in response action time and cost equations (10)-(11).

The bilinear terms are approximated using the LP relaxation of their convex envelops introduced by McCormick (1976). Let first-stage variable $\varphi_{a,s}^1 = \gamma_a^{1,e} \cdot \gamma_{a,s}^{1,h}$ and third-stage variable $\phi_{a,s}^3(\xi^3) = \gamma_a^{1,e} \cdot \gamma_{a,s}^{3}(\xi^3) \ \ \forall a \in A, s \in S$. The convex relaxation of the first bilinear terms is implemented through change of variables in (3) and (8), and addition of constraints (32) as the outer-approximation of the rectangular feasible region $\left[\underline{\gamma}_a^{1,e}, \bar{\gamma}_a^{1,e}\right] \times \left[\underline{\gamma}_{a,s}^{1,h}, \bar{\gamma}_{a,s}^{1,h}\right]$ giving upper and lower bounds on $\gamma_a^{1,e}$ and $\gamma_{a,s}^{1,h}$, respectively.

$$\begin{aligned}
&\varphi_{a,s}^1 \geq \underline{\gamma}_a^{1,e}\gamma_{a,s}^{1,h} + \gamma_a^{1,e}\underline{\gamma}_{a,s}^{1,h} - \underline{\gamma}_a^{1,e}\underline{\gamma}_{a,s}^{1,h}, \ \ \forall a \in A, s \in S \\
&\varphi_{a,s}^1 \geq \bar{\gamma}_a^{1,e}\gamma_{a,s}^{1,h} + \gamma_a^{1,e}\bar{\gamma}_{a,s}^{1,h} - \bar{\gamma}_a^{1,e}\bar{\gamma}_{a,s}^{1,h}, \ \ \forall a \in A, s \in S \\
&\varphi_{a,s}^1 \leq \bar{\gamma}_a^{1,e}\gamma_{a,s}^{1,h} + \gamma_a^{1,e}\underline{\gamma}_{a,s}^{1,h} - \bar{\gamma}_a^{1,e}\bar{\gamma}_{a,s}^{1,h}, \ \ \forall a \in A, s \in S \\
&\varphi_{a,s}^1 \leq \underline{\gamma}_a^{1,e}\gamma_{a,s}^{1,h} + \gamma_a^{1,e}\overline{\beta}_{as} - \underline{\gamma}_a^{1,e}\bar{\gamma}_{a,s}^{1,h}, \ \ \forall a \in A, s \in S
\end{aligned} \tag{32}$$

Given boundaries $\underline{\gamma}_a^{1,e} \leq \gamma_a^{1,e} \leq \bar{\gamma}_a^{1,e}$ and $0 \leq \gamma_{a,s}^{1,h} \leq 1$, constraints (32) are reformulated as in (33).

$$\begin{aligned}
&\varphi_{a,s}^1 \geq \underline{\gamma}_a^{1,e}\gamma_{a,s}^{1,h}, \ \ \forall a \in A, s \in S \\
&\varphi_{a,s}^1 \geq \bar{\gamma}_a^{1,e}\gamma_{a,s}^{1,h} + \gamma_a^{1,e} - \bar{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S
\end{aligned} \tag{33}$$

$$\varphi^1_{a,s} \le \bar{\gamma}_a^{1,e}\gamma^{1,h}_{a,s}, \ \ \forall a \in A, s \in S$$
$$\varphi^1_{a,s} \le \underline{\gamma}_a^{1,e}\gamma^{1,h}_{a,s} + \gamma_a^{1,e} - \underline{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S$$

Similarly, $\underline{\gamma}_a^{1,e} \le \gamma_a^{1,e} \le \bar{\gamma}_a^{1,e}$ and $0 \le \gamma^3_{a,s}(\xi^3) \le 1$ produces constraints (34).

$$\begin{aligned}
&\phi^3_{a,s}(\xi^3) \ge \underline{\gamma}_a^{1,e}\gamma^3_{a,s}(\xi^3), \ \ \forall a \in A, s \in S \\
&\phi^3_{a,s}(\xi^3) \ge \bar{\gamma}_a^{1,e}\gamma^3_{a,s}(\xi^3) + \gamma_a^{1,e} - \bar{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S \\
&\phi^3_{a,s}(\xi^3) \le \bar{\gamma}_a^{1,e}\gamma^3_{a,s}(\xi^3), \ \ \forall a \in A, s \in S \\
&\phi^3_{a,s}(\xi^3) \le \underline{\gamma}_a^{1,e}\gamma^3_{a,s}(\xi^3) + \gamma_a^{1,e} - \underline{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S
\end{aligned} \tag{34}$$

A generalization of McCormick's relaxation method was proposed by Misener and Floudas (1995) for trilinear terms. Their generalized convex envelops are used to linearize trilinear terms $\gamma_a^{1,e} \cdot \gamma^2_{a,s}(\xi^2) \cdot \gamma^3_{a,s}(\xi^3)$. Let $\psi^3_{a,s}(\xi^3) = \gamma_a^{1,e} \cdot \gamma^2_{a,s}(\xi^2) \cdot \gamma^3_{a,s}(\xi^3), \forall a \in A, s \in S$ and replace the trilinear term in constraints (10)-(11) through a change of variables. Given $\underline{\gamma}_a^{1,e} \le \gamma_a^{1,e} \le \bar{\gamma}_a^{1,e}$, $0 \le \gamma^2_{a,s}(\xi^2) \le 1$, and $0 \le \gamma^3_{a,s}(\xi^3) \le 1$, additional constraints (35) are introduced.

$$\begin{aligned}
&\psi^3_{a,s}(\xi^3) \ge 0, \ \ \forall a \in A, s \in S \\
&\psi^3_{a,s}(\xi^3) \ge \gamma_a^{1,e} - \bar{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S \\
&\psi^3_{a,s}(\xi^3) \ge \underline{\gamma}_a^{1,e}\gamma^2_{a,s}(\xi^2) + \underline{\gamma}_a^{1,e}\gamma^3_{a,s}(\xi^3) - \bar{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S \\
&\psi^3_{a,s}(\xi^3) \ge \bar{\gamma}_a^{1,e}\gamma^2_{a,s}(\xi^2) + \underline{\gamma}_a^{1,e}\gamma^3_{a,s}(\xi^3) - \bar{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S \\
&\psi^3_{a,s}(\xi^3) \ge \underline{\gamma}_a^{1,e}\gamma^2_{a,s}(\xi^2) + \bar{\gamma}_a^{1,e}\gamma^3_{a,s}(\xi^3) - \bar{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S \\
&\psi^3_{a,s}(\xi^3) \ge \gamma_a^{1,e} + \bar{\gamma}_a^{1,e}\gamma_{as} + \bar{\gamma}_a^{1,e}\gamma^3_{a,s}(\xi^3) - 2\bar{\gamma}_a^{1,e}, \ \ \forall a \in A, s \in S
\end{aligned} \tag{35}$$

The original TTRP (2)-(12), a 3-stage SMPEC, is transformed into an equivalent three-stage SMILP:

$$\max_{\boldsymbol{\gamma}^1 \in \boldsymbol{\Gamma}^1} E_{\xi^2} \left\{ \max_{\boldsymbol{\gamma}^2 \in \boldsymbol{\Gamma}^2(\boldsymbol{\gamma}^1, \xi^2)} E_{\xi^3|\xi^2} \left[\max_{\boldsymbol{\gamma}^3 \in \boldsymbol{\Gamma}^3(\boldsymbol{\gamma}^2, \xi^3), \mathbf{x}^{\mathrm{r}} \in \Omega^{\mathrm{r}}_{\mathbf{x}}(\xi^3)} \langle \mathbf{D}, \mathbf{u}^{\mathrm{r}}(\xi^3) \rangle \right] \right\} \tag{36}$$

s.t.
(a) three-stage action decision constraints (3)-(5)
(b) UE constraints (20)
(c) Link travel time function linear constraints (23)-(30)

(d) LP relaxation of bilinear and trilinear action level terms (33)-(35)}

6.3.3. Progressive hedging algorithm (PHA)

Problem (36) contains pure continuous first- and second-stage variables and mixed integer third-stage variables. Benders-based decomposition methods, e.g. Disjunctive Decomposition-based branch-and-cut (D2-BAC) approach by Sen and Sherali (2006), are computationally intensive, and the Lagrangian-based decomposition method by Caroe and Schultz (1999) which might ordinarily be applicable will not guarantee a globally optimal solution for this problem class. Thus, an exact solution method that is based on concepts of progressive hedging (Rockefeller and Wets, 1991) is presented. This method decomposes the problem by scenario using Lagrangian decomposition. It is particularly attractive here, because it guarantees global optimality for problems with pure continuous first- and second-stage variables; convexity is not required.

In this approach, first- and second-stage variables, $\boldsymbol{\gamma}^1$ and $\boldsymbol{\gamma}^2(\xi^2)$, are converted into third-stage scenario-dependent variables, $\boldsymbol{\gamma}^1(\xi^3)$ and $\boldsymbol{\gamma}^2(\xi^3)$, respectively. This allows decomposition of the problem by third-stage information states (i.e. scenarios). The following non-anticipativity constraints are added to force $\boldsymbol{\gamma}^1(\xi^3)$ to take a single value $\tilde{\boldsymbol{\gamma}}^1$ over all third-stage information states, ξ^3, and to force $\boldsymbol{\gamma}^2(\xi^3)$ to take identical values $\tilde{\boldsymbol{\gamma}}^2(\xi^2)$ over those third-stage information states with identical type and spatial distribution, i.e. $\xi^3|\xi^2$.

$$\boldsymbol{\gamma}^1(\xi^3) - \tilde{\boldsymbol{\gamma}}^1 = 0 \quad (37)$$
$$\boldsymbol{\gamma}^2(\xi^3) - \tilde{\boldsymbol{\gamma}}^2(\xi^2) = 0 \quad (38)$$

Note that $\tilde{\boldsymbol{\gamma}}^1$ and $\tilde{\boldsymbol{\gamma}}^2(\xi^2)$ are vectors of unrestricted variables.

The PHA solves each scenario-specific problem (39) separately wherein non-anticipativity constraints are relaxed.

$$\min_{\boldsymbol{\gamma}^1 \in \boldsymbol{\Gamma}^1(\xi^3), \boldsymbol{\gamma}^2 \in \boldsymbol{\Gamma}^2(\xi^3), \boldsymbol{\gamma}^3 \in \boldsymbol{\Gamma}^3(\xi^3), \mathbf{x}^{\mathrm{r}} \in \Omega_{\mathbf{x}}^{\mathrm{r}}(\xi^3)} \langle \mathbf{D}, \mathbf{u}^{\mathrm{r}}(\xi^3) \rangle \tag{39}$$

s.t.

(a) three-stage action decision constraints (3)-(5)
(b) UE constraints (20)
(c) Link travel time function linearization constraints (23)-(30)
(d) LP relaxation of bilinear and trilinear action level terms (33)-(35)}

If non-anticipativity constraints (37) and (38) are met, identical solutions for all first- and second-stage variables regardless of the information state ξ^3 will be guaranteed and the problem is solved. However, this is rarely the case. If all first-stage variables are equal, then they are also equal to their expected value. Similarly for second-stage variables. Given the solutions of (39) for all scenarios ξ^3**,** the expected values of first- and second-stage variables are computed: $\hat{\boldsymbol{\gamma}}^1 = E_{\xi^3}[\boldsymbol{\gamma}^1(\xi^3)]$ and $\hat{\boldsymbol{\gamma}}^2(\xi^2) = E_{\xi^3|\xi^2}[\boldsymbol{\gamma}^2(\xi^3)]$, respectively. The deviation in their values from the expected value is measured. The optimal solution is obtained when the values converge to the expected value: $\|\boldsymbol{\gamma}^1(\xi^3) - \hat{\boldsymbol{\gamma}}^1\|, \|\boldsymbol{\gamma}^2(\xi^3) - \hat{\boldsymbol{\gamma}}^2(\xi^2)\| \leq \varepsilon$, where ε is a small number representing the convergence factor. In future iterations, the relaxed anticipativity constraints are penalized in the objective function through Lagrangian relaxation. This objective function is given in (40).

$$\min_{\boldsymbol{\gamma}^1 \in \boldsymbol{\Gamma}^1(\xi^3), \boldsymbol{\gamma}^2 \in \boldsymbol{\Gamma}^2(\xi^3), \boldsymbol{\gamma}^3 \in \boldsymbol{\Gamma}^3(\xi^3), \mathbf{x}^{\mathrm{r}} \in \Omega_{\mathbf{x}}^{\mathrm{r}}(\xi^3)} \langle \mathbf{D}, \mathbf{u}^{\mathrm{r}}(\xi^3) \rangle + \langle \boldsymbol{\theta}^1(\xi^3), \boldsymbol{\gamma}^1(\xi^3) - \hat{\boldsymbol{\gamma}}^1 \rangle + \langle \boldsymbol{\theta}^2(\xi^3), \boldsymbol{\gamma}^2(\xi^3) - \hat{\boldsymbol{\gamma}}^2(\xi^2) \rangle + \frac{\rho}{2} \|\boldsymbol{\gamma}^1(\xi^3) - \hat{\boldsymbol{\gamma}}^1\|^2 + \frac{\rho}{2} \|\boldsymbol{\gamma}^2(\xi^3) - \hat{\boldsymbol{\gamma}}^2(\xi^2)\|^2 \tag{40}$$

where $\boldsymbol{\theta}^{\mathbf{1}}(\xi^3)$ and $\boldsymbol{\theta}^{\mathbf{2}}(\xi^3)$ are the vectors of dual variables corresponding to non-anticipativity constraints (37)-(38), and $\rho \geq 0$ is a penalty parameter. The quadratic terms $\|\boldsymbol{\gamma}^1(\xi^3) - \hat{\boldsymbol{\gamma}}^1\|^2$ and $\|\boldsymbol{\gamma}^2(\xi^3) - \hat{\boldsymbol{\gamma}}^2(\xi^2)\|^2$ are nonseparable and complicate the solution process. Thus, these terms are replaced by related absolute terms $|\boldsymbol{\gamma}^1(\xi^3) - \hat{\boldsymbol{\gamma}}^1|$ and $|\boldsymbol{\gamma}^2(\xi^3) - \hat{\boldsymbol{\gamma}}^2(\xi^2)|$. These absolute terms are piecewise linear and can be expressed through the introduction of pairs of continuous nonnegative variable vectors: $\boldsymbol{\gamma}_+^1(\xi^3)$ and $\boldsymbol{\gamma}_-^1(\xi^3)$, and $\boldsymbol{\gamma}_+^2(\xi^3)$ and $\boldsymbol{\gamma}_-^2(\xi^3)$, respectively. Consequently, the objective function (40) is replaced by (41), a linear function.

$$\min_{\boldsymbol{\gamma}^1\in\boldsymbol{\Gamma}^1(\xi^3),\boldsymbol{\gamma}^2\in\boldsymbol{\Gamma}^2(\xi^3),\boldsymbol{\gamma}^3\in\boldsymbol{\Gamma}^3(\xi^3),\mathbf{x}^{\mathrm{r}}\in\Omega_{\mathbf{x}}^{\mathrm{r}}(\xi^3)} \langle \mathbf{D}, \mathbf{u}^{\mathrm{r}}(\xi^3)\rangle + \langle \boldsymbol{\theta}^1(\xi^3), \boldsymbol{\gamma}^1(\xi^3) - \hat{\boldsymbol{\gamma}}^1\rangle + \langle \boldsymbol{\theta}^2(\xi^3), \boldsymbol{\gamma}^2(\xi^3) - \hat{\boldsymbol{\gamma}}^2(\xi^2)\rangle + \frac{\rho}{2}[\boldsymbol{\gamma}_+^1(\xi^3) + \boldsymbol{\gamma}_-^1(\xi^3)] + \frac{\rho}{2}[\boldsymbol{\gamma}_+^2(\xi^3) + \boldsymbol{\gamma}_-^2(\xi^3)] \quad (41)$$

Thus problem (39) is given with its new objective function (41) and the following additional constraints.

$$\boldsymbol{\gamma}^1(\xi^3) - \hat{\boldsymbol{\gamma}}^1 = \boldsymbol{\gamma}_+^1(\xi^3) - \boldsymbol{\gamma}_-^1(\xi^3) \quad (42)$$
$$\boldsymbol{\gamma}^2(\xi^3) - \hat{\boldsymbol{\gamma}}^2(\xi^2) = \boldsymbol{\gamma}_+^2(\xi^3) - \boldsymbol{\gamma}_-^2(\xi^3) \quad (43)$$

At each iteration μ, the expected values of first- and second-stage variables are updated given the new solutions. The penalization terms $\boldsymbol{\theta}^1(\xi^3)$ and $\boldsymbol{\theta}^2(\xi^3)$ are revised as in (44) and (45).

$$\boldsymbol{\theta}_{\mu+1}^1(\xi^3) = \rho[\boldsymbol{\gamma}^1(\xi^3) - \hat{\boldsymbol{\gamma}}^1] + \boldsymbol{\theta}_\mu^1(\xi^3) \quad (44)$$
$$\boldsymbol{\theta}_{\mu+1}^2(\xi^3) = \rho[\boldsymbol{\gamma}^2(\xi^3) - \hat{\boldsymbol{\gamma}}^2(\xi^2)] + \boldsymbol{\theta}_\mu^2(\xi^3), \quad (45)$$

An overview of the PHA is depicted in the flowchart of Figure 6-4. Global convergence of the proposed PHA in finite time is assured. A proof is given in (Rockefeller and Wets, 1991) for similar two-stage problems with pure continuous first-stage variables can be directly extended to this problem with three stages.

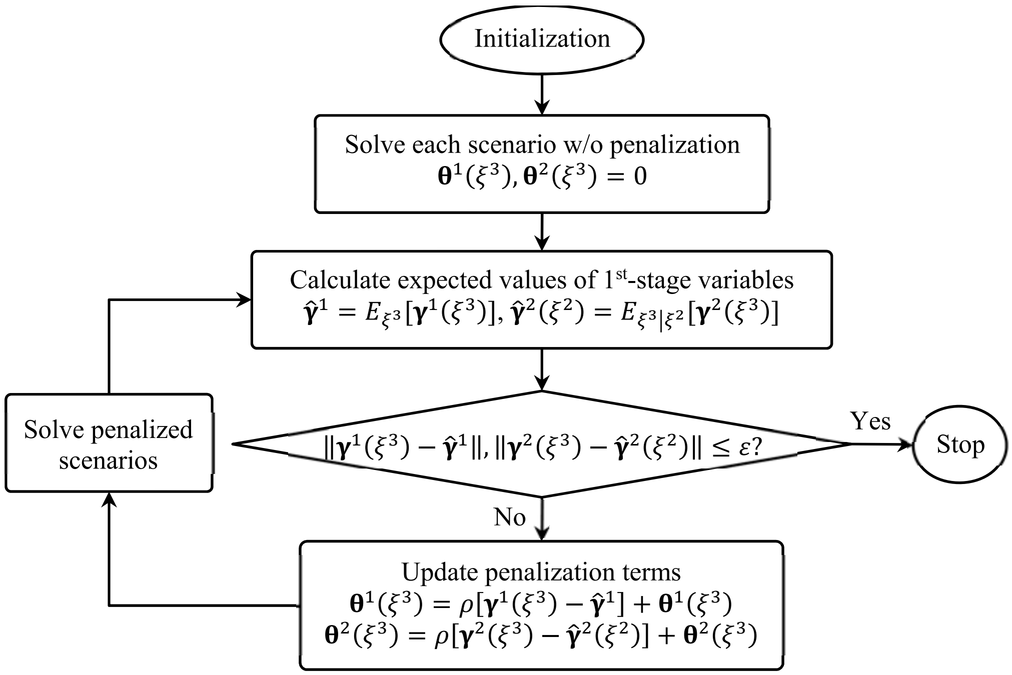


Figure 6-4 The PHA flowchart

6.4. Numerical experiment

The model and solution method are illustrated on a test network from (Suwansirikul et al., 1987). This network has 6 nodes and 16 links as shown in Figure 6-5 and has been used for similar purposes in many works (e.g. Li et al., 2012).

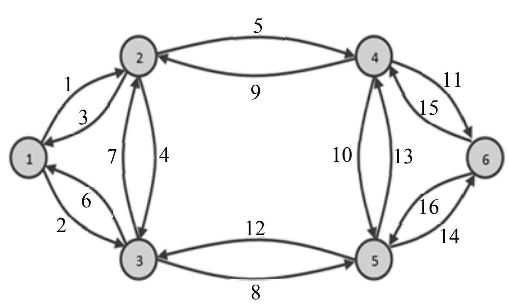


Figure 6-5 Test network (Suwansirikul et al., 1987)

The network is presumed to represent a roadway with highway bridges given by links 2, 5, 6 and 9. Four OD pairs are considered and the corresponding travel demand is reported in Table 6-2. The network data, including the values of link travel time function parameters from equation (22) are given in Table 6-3, and are identical to values suggested in (Suwansirikul et al., 1987). It is presumed that $m_a = 4$ for all links.

Table 6-2 The values of link travel time function parameters

OD ID	Origin	Destination	Travel demand
1	1	6	10
2	6	1	20
3	4	1	5
4	6	2	10

Table 6-3 The values of link travel time function parameters

Link ID	Link type	t_a^0	m_a	c_a^o
1	Roadway	1	10	3
2	Bridge	2	5	10
3	Roadway	3	3	9
4	Roadway	4	20	4
5	Bridge	5	50	3
6	Bridge	2	20	2
7	Roadway	1	10	1
8	Roadway	1	1	10
9	Bridge	2	8	45
10	Roadway	3	3	3
11	Roadway	9	2	2
12	Roadway	4	10	6
13	Roadway	4	25	44
14	Roadway	2	33	20
15	Roadway	5	5	1
16	Roadway	6	1	4.5

Three disaster categories (earthquake (s=1), flood (s=2) and malicious acts(s=3)) are considered in the numerical experiments. A specialized version of Monte Carlo simulation by Chang et al. (1994) is employed to generate disaster scenarios while addressing spatial dependencies that relate to each hazard (see Chen and Miller-Hooks (2012) for more details on the scenario generation process). 30 random scenarios are generated corresponding to each disaster type to capture possible consequences in terms of the level of damage to network links, i.e. 90 scenarios in all.

All network links except for bridges are candidates for capacity expansion with lower and upper bounds of 0 and 10 units, respectively. The bridge links (2, 5, 6 and 9) can be retrofitted for protection against earthquakes or specific malicious acts. Links 10, 11 and 13-16 at the eastern end of the network may be subject to flooding, and are candidates for related mitigation actions. Second-stage preparedness decisions are considered when flooding is predicted. When the event relates to an earthquake or malicious act, no preparedness actions will be available in this stage. Finally, response actions are considered as options for restoring capacity of all network links that may be affected by any disaster type. The unit action costs as well as unit implementation times of response actions are given in Table 6-4.

Table 6-4 Unit cost of actions

Link ID	Actions									
	Expansion	Retrofit			Preparedness			Response*		
	$\bar{b}_a^{1,e}$	$\bar{b}_{a,1}^{1,h}$	$\bar{b}_{a,2}^{1,h}$	$\bar{b}_{a,3}^{1,h}$	$\bar{b}_{a,s}^{2}$	$\bar{b}_{a,2}^{2}$	$\bar{b}_{a,3}^{2}$	$\bar{b}_{a,s}^{3}$ $(\bar{q}_{a,s}^{3})$	$\bar{b}_{a,s}^{3}$ $(\bar{q}_{a,s}^{3})$	$\bar{b}_{a,s}^{3}$ $(\bar{q}_{a,s}^{3})$
1	2	-	-	-	-	-	-	3.5 (7)	-	-
2	-	6	-	2	-	-	-	5.5 (11)	-	4 (8)
3	5	-	-	-	-	-	-	8 (16)	-	-
4	4	-	-	-	-	-	-	7 (14)	-	-
5	-	8	-	2	-	-	-	5 (10)	-	3 (6)
6	-	6	-	2	-	-	-	5 (10)	-	3 (6)
7	4	-	-	-	-	-	-	4.5 (9)	-	-
8	3	-	-	-	-	-	-	4 (8)	-	-
9	-	8	-	2	-	-	-	6 (12)	-	4 (8)
10	5	-	3	-	-	0.5	-	6.5 (13)	4.5 (9)	-
11	6	-	3.5	-	-	0.5	-	10 (20)	7.5 (15)	-
12	8	-	-	-	-	-	-	12 (24)	-	-
13	5	-	4	-	-	0.5	-	6 (12)	5 (10)	-
14	3	-	2	-	-	0.5	-	5.5 (11)	3.5 (7)	-
15	6	-	4	-	-	0.5	-	7.5 (15)	5 (10)	-
16	1	-	2	-	-	0.5	-	2.5 (5)	1.5 (3)	-

*Note that the perfect preparedness in advance is presumed to reduce response cost (implementation time) by 30%, i.e. $\underline{b}_{a,s}^3 = 0.7\bar{b}_{a,s}^3$ $(\underline{q}_{a,s}^3 = 0.7\bar{q}_{a,s}^3)$

Resilience indifference curves resulting from solution of this problem instance for different combinations of limited budget B and response time T are provided in Figure 6-6. As depicted in this figure, resilience is generally more sensitive to budget than to response time. However, when response times are short, resilience is almost unaffected by budget level. Likewise, when the budget is small, resilience is almost unaffected by a response time increase. In the former case, this is because time available to take action is too restrictive regardless of budget level. In the latter case, funds are unavailable to take additional actions.

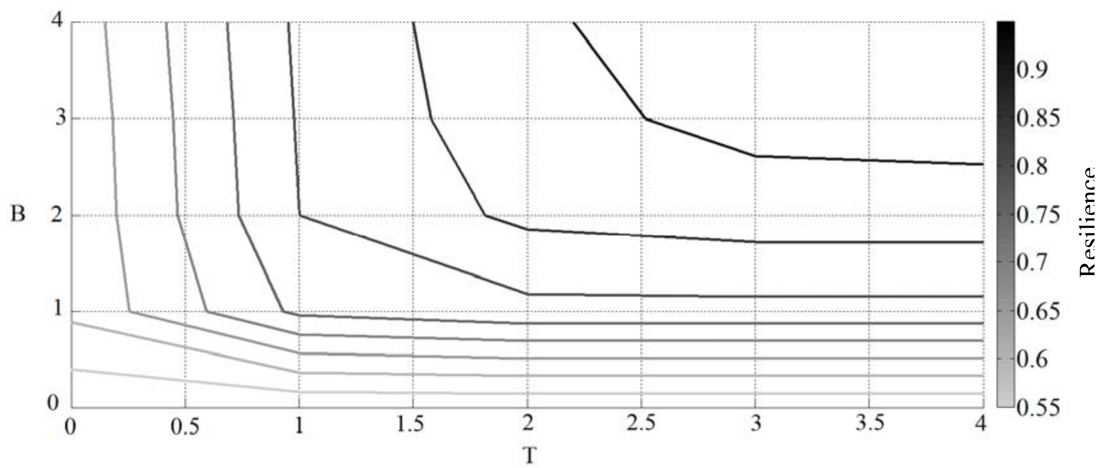


Figure 6-6 Resilience indifferent curves for the numerical experiment

Detailed results are given through plotting the cumulative distribution of network resilience in Fig. 6-7 for three sample strategies: $(B,T) = (0,0)$, $(B,T) = (3,0)$, and $(B,T) = (3,3)$. Each point in Fig. 6-7 represents the network resilience under a particular scenario, called point resilience. This concept with respect to resilience was introduced in Nair et al. (2010).

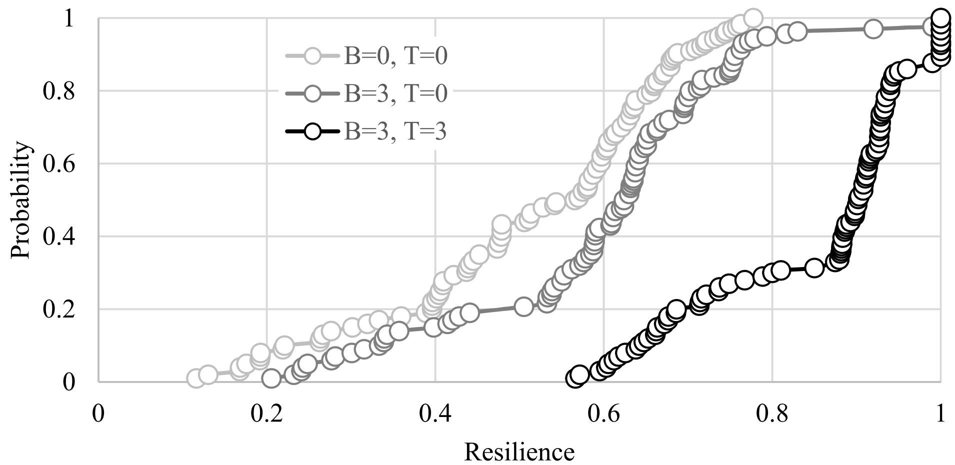


Figure 6-7 Cumulative distribution function of point resilience

This figure illustrates that the range and variance of the distribution decreases with larger budget and response time. Moreover, the resilience under the worst-case scenario, which occurs at the lowest probability level for each data set, improves with increasing budget and response time. For example, given $(B, T) = (0,0)$, the range of resilience values is between 0.12 and 1, while with subsequent increases in budget and response time in strategy $(B, T) = (3,3)$, the range reduces to between 0.56 and 1.

Additional runs were conducted assuming a UE could be achieved post-event and after the response time has elapsed. Results of these runs are compared in Figure 6-8 with those assuming only a PUE is obtained. From this comparison of results, it is seen that the expected total travel time (the numerator of the resilience measure) is slightly larger under a UE than under a PUE. Through further investigation into the results, it was found that for results from individual sampled scenarios, response actions were focused on different links under the UE and PUE assumptions. In particular, under a PUE, the target of the response was on only impacted arcs, while under the UE assumption, unaffected arcs were also improved.

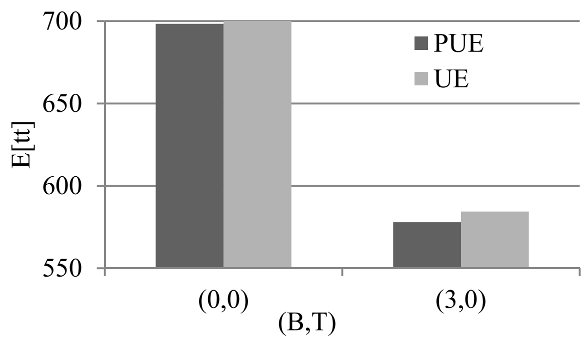


Figure 6-8 The network's total travel time under UE and PUE

6.5. Conclusions

This chapter proposes a novel stochastic network design formulation for maximizing travel time resilience for roadway networks. In particular, it targets freeway networks. The problem explicitly addresses the first three stages of the decision processes of the disaster management life cycle, specifically pre-event mitigation and preparedness, and post-event response. Decisions are taken at each stage based upon the evolution of uncertainty over the stages. The problem is formulated as a bilevel stochastic mathematical program with user equilibrium constraints. The three-stage decision process is embedded within the upper-level problem and user response to the upper-level decisions is modeled in the lower-level problem.

This problem differs from previous studies on stochastic transportation network design in which supply uncertainty is explicitly modeled in that these prior works have primarily addressed long-term mitigation planning applications. In these applications, a UE is an appropriate travel behavior model for estimating network travel times. In this application where this behavior seeks travel time estimates for the period immediately following a disaster and some quick response actions, a PUE is proposed. The PUE accounts for route choice decisions taken by the system users assuming that only affected users will have information on the disaster event's impacts and even these users will have limited information on network damage and repairs.

A multi-hazard approach is employed, and decisions are disaster event-dependent. Thus, mitigation actions may target different hazard scenarios even before the hazard event type is known. In fact, the model accounts for the varying benefits of any such action under different hazard classes. Preparedness decisions are taken only once the hazard class is known, but the specific event realization is uncertain. Response actions are designed for specific disaster event scenarios and are determined once the disaster scenario is realized.

Chapter 7: Conclusions and Extensions

7.1. Conclusions

This dissertation provided a general mathematical framework to protect transportation infrastructure systems in the presence of uncertain events with the potential to reduce system capacity/performance. The framework defined a number of disaster measures and clarified their boundaries and possible overlaps. These measures include, coping capacity, preparedness, robustness, flexibility, recovery capacity, and resilience. A single, general decision-support optimization model was formulated as a multi-stage stochastic program and captures the uncertain nature of disasters and their consequences. It seeks an optimal sequence of decisions over time based upon the realization of random events in each time stage. Exact (or approximate) solution methodologies based on concepts of decomposition, simulation, and cutting-edge linear approximation methods were presented for use in evaluating system performance in terms of these measures as well as optimally allocating the limited resources to mitigation, preparedness and response options.

This dissertation addressed three problems to demonstrate the application of the IPP in different transportation environments with emphasis on resilience and robustness: Airport Resilience Problem (ARP), Building Evacuation Design Problem (BEDP), and Travel Time Resilience in Roadways (TTR). These problems aimed at identifying opportunities to support system performance measurement, operational decision-making, preparedness planning, and immediate post-disaster actions, given their topological and operational characteristics. Potential benefits to transportation system operators were discussed in detail in Chapters 3 through 6, including, for example, the tool's utility in suggesting equipment to have at the ready and identifying the critical system components for prioritizing future facility developments.

The first problem, ARP, was formulated as a stochastic, integer program with recourse seeking to measure and optimize the resilience of airport runway and taxiway pavement networks under multiple potential damage-meteorological scenarios. The mathematical model and solution methodology were embedded within a decision support tool, along with a scenario generator for multiple hazard classes. The BEDP was formulated as a bilevel stochastic integer program with UE constraints for the robust design of shelters, fortified hallways and exits in buildings, and permited exact solution that minimizes the exposure of evacuees to risks under various hazard scenarios. Variants of the model involved both stochastic programming and robust optimization concepts under both user equilibrium and system optimal conditions, coupled with a multi-hazard approach to examine designs given various possible future emergency scenarios. Both the ARP and BEDP include binary first-stage and mixed integer second-stage variables, and the integer L-shaped decomposition was adapted to solve them.

Finally, the TTRP was formulated as a bilevel three-stage stochastic program. The upper-level problem included a three-stage decision on pre-event mitigation and preparedness, and post-event response, based upon information that is revealed at each stage. A specialized user equilibrium, PUE, was presented in the lower level to capture users' semi-adaptive behavior shortly after the event occurrence when short-term response actions are implemented and users have received limited information on network damage and completion of repairs. An approximation approach was presented involving an efficient piecewise linearization technique to address PUE constraints, and an exact solution algorithm was proposed based on concepts of progressive hedging for solution of the sequence of decisions over the three stages.

Numerical experiments were concluded on network representations of a United States rail-based intermodal container network, the LaGuardia Airport taxiway/runway pavement network, a single-story office building, and a small roadway network. The results illustrate the application of the proposed exact (approximate) solution techniques to solve small- and moderate-size problems to global optimality.

7.2. Extensions

A general mathematical framework along with three specialized problems, ARP, BEDP, and TTRP, have been addressed in this dissertation with emphasis on presenting exact solution techniques. These problems are all NP-hard. While potential application of these methods has been demonstrated, one might use the mechanism of the presented exact solution techniques to develop efficient heuristics for solving real-world size problems, particularly if decision makers need to make urgent decisions on post-disaster contingency plans. This research can be extended in several directions. Directions for future research are discussed in following section.

7.2.1. General IPP

The proposed IPP formulation has as its objective the maximization of system performance given budget and recovery period limitations. Alternative formulations may be considered in which costs are minimized and performance levels are satisfied.

Likewise, the recovery period duration can be minimized under performance and budget limitations. While either budget or recovery period parameters would become decision variables, program properties of convexity, linearity and separability would be unchanged. Thus, applicability of discussed solution methodologies would persist.

Additionally, for some applications, where a fixed number of actions or combinations of actions are to be considered, a discrete representation of the decision space might be required. In this case, formulation IPP would require integrality constraints and other adaptations. Appropriate solution methodologies would be needed. The conceptual framework, however, is developed in a general way and can be exploited regardless of the nature of action levels, whether discrete or continuous.

For applications where the implementation time depends on the price one is willing to pay, budget-related equations will need to incorporate cost-related decision variables. Period performance measures may be useful in operational decision-making, where it is necessary to schedule response actions required to restore system performance and benefits can be derived from early improvements or outperforming level-of service constraints. Consider IPP-T in which maximum throughput is sought through a freight rail network. To encourage solutions that also seek universally maximum throughput levels (i.e. throughput levels that are maximal at each point in time), problem dynamics must be explicitly considered. This is the subject of future work. Finally, it should be noted that concepts given herein provide only one approach to thinking about resilience and other related measures in a consistent framework. Other structures may be equally beneficial. In that regard, coping capacity can be further divided into resistance, i.e. the ability to endure when confronted by a stressor, and excess, i.e. the ability to respond to and absorb disruption impacts within the IPF.

7.2.2. ARP

Several assumptions were made in creating the ARP and solution methodology. It was presumed that those resources procured in the preparedness stage will be available for recovery and, thus, these resources will not be affected by the damage event. Additionally, benefits derived from specific ordering of repair actions, and possible precedence requirements, were not accounted for in the model. Further assumptions related to homogeneity in runway and taxiway materials were made. When multiple locations require similar equipment to complete a repair task, that setup times at the additional sites may be significantly reduced is not addressed. Likewise, the cost and/or time associated with the reconfiguration of runway direction in response to damage events is not included. This work can be extended to address many of these limitations. To assess the impact on airport pavement network resilience of large infrastructure enhancement projects, the model can be run multiple times, each time using a network topology consistent with the capital investment strategy. Given customer demand forecasts, the return on investment can be analyzed by comparison of the results. On a final note, while the tool was designed for civilian applications, the model can also be applied for military use, where decisions related to RRR and MOS need systematic and methodical support.

An alternative and more conservative modeling approach based on concepts of robust optimization might be considered. With such an approach, one would seek to maximize airport resilience under the worst-case scenario. The two-stage structure would be maintained; however, in place of the maximum expected flow over all scenarios, the maximum total flow under the scenario leading to the worst performance would be sought. This results in a stochastic program with deterministic objective function that takes a max-min form. The advantage of such a methodology is that scenario occurrence probabilities need not be known. Such an approach was developed and applied to the case study. For all tested combinations of Tmax and budget, the resulting resilience level was zero, however. This will arise with such an approach when even only one extreme scenario exists in which no demand can be met through repair actions taken within given budget and time limitations. Thus, such a conservative approach is limited in its general utility. Its application further demonstrates that concentrating all internal and external resources on improving conditions for the worst-case scenario may lead to insignificant improvements for the specific worst-case scenario and general performance.

7.2.3. BEDP

Though the presented BEDP is appropriate for supporting the design of buildings, an implementation of the UE approach for the actual management of evacuation events would require the development of a dynamic model in which link travel times are continuously reassessed, and of a sensor-based system that can capture in detail the movements of evacuees and provide in real-time information to each evacuee.

An implementation of the program in a case study of a geographical evacuation problem is planned as well. The use of a program that minimizes the exposure of evacuees to risk, through an explicit consideration of the level of protection that different evacuation routes and shelters provide, may constitute an improvement on previous geographical evacuation models that did not address such an objective. For example, in a flooding scenario, the risks of using different evacuation routes, depending on their location and elevation, can be considered when planning the location of emergency shelters.

Additional extensions may be desirable. For example, shelter capacities may be uncertain due to their multi-purpose use. That is, a shelter may be used for a community activity and, thus, filled to capacity at the time it is needed. Heterogeneity in the evacuee population is ignored herein. However, some evacuees may move more quickly than others. Some evacuees may put more weight on risk exposure from traveling in the corridors versus waiting for help in a shelter than other evacuees. Moreover, risk perception may vary by evacuee and may be imperfect. Thus, alternative models for handling risk may be appropriate. Individualized risk functions may be warranted, and a stochastic UE may be beneficial.

7.2.4. TTRP

A A number of assumptions were made in development of the TTRP that can be addressed in the future extensions. First, the proposed stochastic problem addressed exogenous uncertainties, where disaster event scenarios are generated independent from the decision process. In this study, the mitigation actions, particularly network link retrofit actions, are assumed to only impact disaster consequences in terms of the level of damage to the link and not the disaster probability. As an extension, one might model endogenous uncertainties in which scenario probabilities are updated based on decisions made at each information state. Secondly, only network supply/capacity uncertainties were taken into account and the travel demand pattern was assumed to be fixed and identical to the original pattern pre-event. Additionally, only travel time was considered in modeling user behavior. In reality, other factors, such as safety might also play a role in the route choice of the users, particularly under disaster events. It was assumed that all affected users are homogenous with respect to the evaluation function (a utility function) used in route selection in obtaining a PUE. Moreover, it was assumed that the users have perfect information on the damaged and repaired network links, and that they make decisions on their routes with the aim of selfishly minimizing their travel time. Alternative models may be of interest to capture uncertainty in user perception, heterogeneity in user route choice and other factors affecting users' decisions.

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