ABSTRACT

Title of Document: Optimization Models for
Improving Bus Transit Services

Myungseob Kim,
Doctor of Philosophy, 2013

Directed By: Professor Paul Schonfeld
Department of Civil and Environmental Engineering

To provide efficient public transportation services in areas with high demand variability over time, it may be desirable to switch vehicles between different types of services such as conventional services (with fixed routes and schedules) for high demand periods and flexible route services during low demand periods. Thus, this dissertation analyzes and compares conventional, flexible, and variable type bus service alternatives. Optimization formulations and numerical results show how the demand variability over time and other factors affect the relative effectiveness of such services. A model for connecting one terminal and one local region is solved with analytic optimization. Then, models are extended to consider multiple regions as well as multiple periods. Numerical results of problems for multiple regions and multiple periods are also discussed.

Secondly, a problem of integration of bus transit services (i.e., conventional and flexible services) with mixed fleets of buses is explored. A hybrid method

combining a genetic algorithm and analytic optimization is used. Numerical analyses confirm that the total system cost can be reduced by integrating bus services with mixed fleets and switching service types and vehicles over time among regions in order to better fit actual demand densities. The solution optimality and the sensitivity of results to several important parameters are also explored.

Thirdly, transit ridership may be sensitive to fares, travel times, waiting times, and access times. Thus, elastic demands are considered in the formulations to maximize the system welfare for conventional and flexible services. Numerical examples find that with the input parameters assumed here, conventional services produce greater system welfare (consumer surplus + producer surplus) than flexible services. Numerical analysis finds that conventional and flexible services produce quite acceptable trips with the zero subsidies, compared to various financially constrained (subsidized) cases. For both conventional and flexible services, it is also found that total actual trips increase as subsidies increase. When the cost is fully subsidized, conventional services produce 79.2% of potential trips and flexible services produce 81.9% of potential trips.

Several methods are applied to find solutions for nonlinear mixed integer formulations. Their advantages and disadvantages are also discussed in the conclusions section.

Optimization Models for Improving Bus Transit Services

By

Myungseob Kim

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2013

Advisory Committee:
Professor Paul Schonfeld, Chair/Advisor
Professor Ali Haghani
Professor Elise Miller-Hooks
Professor Robert Windle (Dean's Representative)
Associate Professor Cinzia Cirillo

Dedication

Mr. Youngjin Kim, who is my father, passed away in Feb 2013. I know he had hard times with his disease for last few years. I am extremely sorry that I could not take care of him while I studied at the University of Maryland, College Park (UMD). My mother also had hard times to take care of my father without me. I know that my parents dedicated their life to me. I love my parents and I dedicate this accomplishment to them.

I am sure that I could not finish my study without my wife, Mrs. Moonsil Chae. Without her endless supports, I cannot imagine having a masters and a Ph.D. degree at UMD in a relatively short time. She always cheered me up when I got depressed, and she was always supportive to my study. I am a very happy and lucky person to share my life with her. While I studied at UMD, my wife took care of our son, Minjae Kim, very well. I am very happy to see our child as a healthy big boy. I dedicate this work to my wife. Thank you very much and I love you.

My parents in law are always supportive to me. Their encouragements helped me move forward. I appreciate their supports, and I also thank you my sister and brother in law for their encouragements.

Acknowledgements

I am a happy person because I met my doctoral degree advisor, Professor, Paul Schonfeld. I sincerely appreciate his academic guidance for my study. He was and is always supportive and generous to me. Without his full supports, it is impossible for me to receive a masters and a Ph.D. degree within four years at the University of Maryland, College Park. I really appreciate his lessons and advices throughout my study. I will also miss playing tennis with you and group members. It was a great fun.

I think I am a very lucky person to have a great dissertation committee. All the comments and advices from the committee were very insightful and valuable to improve my dissertation. Thank you very much Drs. Ali Haghani, Elise Miller-Hooks, Cinzia Cirillo, and Robert Windle. I also thank you Dr. Robert Windle to be the dean's representative.

I thank you students in our research group members for valuable discussions, kindness networking and friendship. I also thank you Korean brothers and sisters (Dongwook Kim, Dr. Taehyeong (Tommy) Kim, Dr. Min-Wook Kang, Minseok Kim, Seonyeong Cheong, Woon Kim, Sung Yoon Park, Dahae Hwang, Hyoshin Park, and Hyeonmi Kim) in transportation program at UMD for their friendship.

Lastly, I met my first master's degree advisor, Dr. Eungcheol Kim, when I was a senior student with my age was 26. Dr. Eungcheol Kim is the person who introduced the area of transportation engineering to me. I appreciate his encouragements, supports, and valuable advices for not only professional careers but also my personal concerns.

Table of Contents

List of Tables

List of Figures

Chapter 1 Introduction

1.1. Background

Conventional bus operations are commonly provided in the urban mass public transportation. Conventional bus routes and timetables are preset, and buses operate on their fixed routes and fixed schedules. Conventional bus services are relatively economical when carrying many passengers during peak periods. However, their service quality is limited since passengers must somehow reach some predetermined stations, wait for a vehicle, possibly transfer several times, and then move from their exit stations to their destinations. Thus, conventional transit services are least disadvantaged in areas and time periods with high demand densities, which can sustain high network densities and service frequencies.

When ridership decreases bus operators typically adjust frequencies downward, thus increasing passenger wait times. This may further decrease ridership. Instead of changing conventional bus frequencies, providing a different type of bus service during off-peak periods which is more economical for low demands may be preferable for both bus operators and passengers. Some paratransit services can provide more flexible routes and schedules, including the possibility of door-to-door services. For instance, taxis provide great service flexibility, but at high unit costs (especially in labor cost per passenger-mile). Flexible bus services may be preferable in the low demand areas and periods. By integrating conventional and flexible services, bus transit passengers may experience overall improvements in transit services. The potential advantages of variable-type bus for integrated conventional and flexible bus operations have not been sufficiently explored.

The potential benefits of using variable operation types (or “modes”) and multiple fleets should theoretically increase when multiple dissimilar regions are considered, due to the increased variability of demand densities. Bus operating costs increase to some extent with bus sizes. Large buses are more economical at high demand densities, as average costs per passenger are relatively low. This leads us to consider the use of mixed bus fleets, consisting of vehicles of different sizes, which may more closely match demand variations.

The exploration of these potential benefits, especially combined with the integration of conventional and flexible bus services, has not been analyzed previously. Thus, the concept of mixed fleet variable type bus (MFV) operation in multiple regions is analyzed here. To provide efficient service, decision variables for bus sizes (i.e. large bus size and small bus size) and decision variables for bus operation characteristics (i.e. route spacing in region for conventional bus and service area in region for flexible bus) are optimized. Bus frequencies and fleets for both conventional and flexible services are also jointly optimized. An efficient solution method is required to provide good bus transit operational and managerial strategies.

Transit ridership may be sensitive to the elasticity of fares, in-vehicle times, waiting times, access times. To consider different qualities of service types, a system-wide welfare function, which is the sum of consumer surplus and producer surplus, should be formulated and optimized. Using elastic demand functions, various decision variables, which are fares on conventional and flexible services, bus sizes, headways and fleet sizes for both service types, route spacings for conventional services, and service areas for flexible services, are optimized here.

1.2. Objectives

Specific objectives for this dissertation are as follows:

1) Develop a multi-dimensional optimization model that integrates various types of

bus services, and finds good solutions for decision variables in nonlinear mixed integer problem formulations. Specifically, optimized values of vehicle sizes, bus network characteristics (e.g., route spacing for conventional bus, and service coverage for flexible bus), headways, and fleet sizes should be jointly optimized.

2) Extend models and formulations to consider different types of services as well as mixed vehicle fleets. Various operational alternatives, including single fleet conventional bus services, single fleet flexible bus services, mixed fleets conventional bus services, mixed fleets flexible bus services, and mixed fleets variable-type bus services, are analyzed. The models developed here should be capable of analyzing multiple regions and periods.

3) Develop an optimization model for maximizing social welfare. Demand functions with elasticity to service times and fares should be formulated and applied to welfare maximization problems. The objective functions for conventional and flexible services are taken as the sum of consumer surplus and producer surplus. Optimization models find solutions for fares, headways, fleet sizes, route spacings, service areas, and the number of zones for each region.

1.3. Scope

In this dissertation, the problem of designing routes and locating stations is assumed to be solved in advance with methods beyond the scope of this dissertation. The performance of flexible route services is analyzed with a tour distance approximation function, namely Stein's formula (1978), rather than with micro-level ridesharing algorithms.

For welfare problems, a linear elastic demand function is applied for both conventional and flexible services. The optimization problems that are solved in this dissertation are suitable for the planning stage. Real-time vehicle control strategies are outside the scope of this work.

The bus system analyzed here provides service from a major terminal (or Central Business District) to multiple regions. In Figure 1-1, a public bus system serves multiple regions connected to a central terminal. For each region, either conventional or flexible bus can be provided.

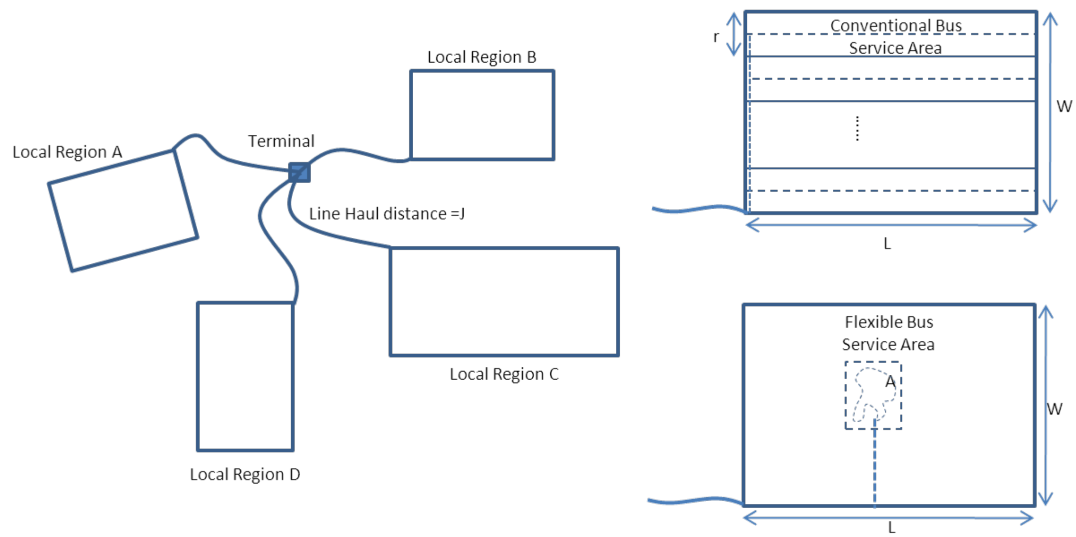


Figure 1-1 Multiple Local Regions and Different Types of Bus Services

1.4. Organization

In this dissertation, Chapter 2 reviews the relevant studies on integrating and coordinating bus transit services. Studies of transit welfare maximization are also reviewed. Chapter 3 explores the integration of conventional and flexible bus services between a terminal and a local region. Chapter 4 extends optimization models developed in Chapter 3 to cover multiple regions. Chapter 5 analyzes a bus transit integration with mixed fleets and explores benefits of sharing fleets (and switching vehicles over time) between conventional and flexible services. Chapter 6 considers demand elasticity in maximizing the welfare of conventional and flexible services. Two constrained optimization models are formulated and solved with a numerical solution approach. Chapter 7 summarizes findings, contributions and suggests extensions for future study.

Chapter 2 Literature Review

This chapter reviews relevant studies on bus transit operations, integration of various types of bus services, coordination of passenger transfers, and demand elasticity in the analysis of bus transit services.

2.1. Bus Transit Services

Kocur and Hendrickson (1982) design local bus services with demand equilibrium. They analyze bus service variables such as route spacing, headway, and fare, both with and without vehicle size constraints. They consider three objective functions, including profit maximization, maximization of a combination of net user benefit and operator profit, and maximization of net user benefit subject to a deficit constraint. They analytically optimize objectives and they find closed-form solutions. More specifically, they use calculus to find unconstrained optima, and use Lagrange multipliers to find optima when constraints are known to hold. They analyze a local area of 4 by 6 miles, and in this local area they assume an infinitely fine rectangular street grid. This means their models are not directly applicable to radial transit network. They also consider the sensitivity of ridership to some service characteristics.

The present dissertation analyzes the integration of conventional and flexible bus services. For conventional bus services, it uses some assumptions, such as parallel route spacings, regular bus stops, and rectilinear local regions that are similar to those of Kocur and Hendrickson (1982). However, it considers multiple regions while Kocur and Hendrickson consider only one local region within a city.

Salzborn (1972) studies the bus scheduling problem for a single route. The primarily objective is to minimize fleet size. This work seems theoretically inspired by Newell (1971). Salzborn also shows an application of the suggested method to a suburban railway system. For its secondary objective, his study minimizes the passenger waiting time by using a calculus of variation technique (Elsgolc, 1961). He applies a flow theory to this study by assuming that the vehicle movements can be considered continuous and time-dependent flows on the links of a network. This approach is more theoretical than applied.

Furth and Wilson (1981) study how frequencies should be set on bus routes. They also compare the theory and practice of bus frequency decisions in the 1980's. Furth and Wilson note policy headways, peak-load factor, revenue/cost ratio, and vehicle productivity as the most frequently used methods to set bus frequencies in practice (Attanucci et al, 1979). The objective is to maximize ridership by allocating buses with given constraints such as fares, routes, and subsidy. They approach this problem as a resource allocation problem in which limited resources (i.e., subsidies) can be allocated to maximize the ridership. They suggest an algorithm to solve this problem. Instead of minimizing cost only, they assume flat fares so that they approach a ridership maximization problem. Considering complex fare structures may be an interesting future research direction. They also note that an existing rule of thumb used in the transit industry may be less efficient than a formal model that uses a consistent objective, as expected.

Woodhull et al (1985) discuss two indicators, namely load factor and standee factor for bus transit scheduling. They provide a simple regression analysis for better understanding the effects of load factor and standee factor in transit scheduling. Sheffi and Sugiyama (1982) also study a bus scheduling problem with time-dependent demand and solve it with dynamic programming. They also consider many-to-many demand patterns in the formulations. The objective of this work is to minimize the total waiting time of passengers. This study is limited to one route and it does not optimize the bus size.

Furth and Day (1985) provide a short qualitative discussion on transit routing and scheduling for heavy demand corridors. They discuss four methods, namely short-turning, restricted zonal service, semi-restricted zonal service, and limited stop zonal service. They discuss the advantages and disadvantages of those four methods for local services.

In high demand corridors the ridership may be shared by several bus routes; thus, buses may operate on some overlapping routes. Han and Wilson (1982) study this problem of allocating buses among overlapping routes. They provide a deterministic mathematical formulation for solving this problem. Because they analyze overlapping routes with higher demands, they consider crowding levels for all patrons in the objective function. To solve the problem, they develop a two state heuristic which decomposes the problem into base allocation and surplus allocation. Although they do not guarantee optimal solutions, they show how a complex problem is solved relatively simply with approximations. They point out a major limitation which is that origins and destinations are pre-determined. They also do not consider the changes in passenger behavior due to overlapping routes.

Yan and Chen (2002) develop a deterministic scheduling model to analyze inter-city bus services with one-to-one demand patterns. They seek routing and timetable solutions with given demand, fleet size, and the related costs. Since inter-city bus services are direct trips and usually have long travel times, the authors leave out the users' waiting time by assuming they know the bus departure times in advance. Yan et al (2006) then extend a model to analyze bus routes and timetables with stochastic demand variation for inter-city travel. To solve this problem, they develop two heuristics, which are a link-based heuristic and a path-based heuristic. This work analyzes aggregated demands for inter-city (regional) transit planning.

Zhao and Zeng (2008) solve an optimization problem for transit network routing, headway, and timetable in a large network. The method is a metaheuristic that combines simulated annealing, tabu, and greedy search. They note that the solution quality is approximately proportional to the CPU time due to the stochastic nature of the problem. As they point out, we note that heuristics are widely used to solve large problems because of computational difficulties of finding the optimal solutions. Bus seating capacities are assumed rather than optimized. However, this paper is a useful reference on bus route designs.

Chang and Schonfeld (1991c) analyze conventional bus services with different demand conditions, namely steady fixed demand, cyclical fixed demand, steady equilibrium demand, and cyclical equilibrium demand. Chang and Schonfeld (1993) then study optimal bus service zones in conventional bus operations. They analytically optimize route spacings, headways, and zone size (i.e., length and width of rectilinear zone). They also specifically optimize an elongation ratio which is defined as zone length divided by zone width.

2.2. Bus Transit Integration

Several attempts have been made to jointly use conventional and flexible type bus services. Typically, flexible bus services provide Many-to-One and/or One-to-Many service with flexible route tours that operate on semi-fixed schedules. (The departure times from or arrival times at the One major trip generator are usually pre-determined and the tours may have cyclical schedules.) Conventional bus services operate with fixed routes and fixed schedules.

The relative advantages of conventional and flexible bus services are investigated using analytic optimization models in Chang (1990) and Chang and Schonfeld (1991a). They compare optimization models for conventional and subscription (i.e., flexible) bus operations. They analytically optimize vehicle sizes and headways and service design variables (i.e., route spacings and service area) for both conventional and flexible bus services connecting a terminal and a local region. They confirm that conventional bus service (with fixed routes and fixed schedule) is preferable to subscription bus service (which has demand responsive routes and flexible schedule) at high demand densities, and vice versa. Chang and Schonfeld (1991a) assumed that both conventional bus and flexible bus either collect passengers from a local service area OR distribute passengers to a local area. As a possible improvement, their model should include a controllable directional split factor, which would enable us to consider 2-directional demands in various proportions.

Chang and Schonfeld (1991b) analyze temporally integrated bus systems with a threshold demand analysis between conventional bus and subscription bus services. They compare three bus operations, namely conventional bus, flexible bus, and temporarily integrated bus services. They analytically optimize formulations and note that when demands vary over time, the integrated bus service has lower system costs than purely conventional or purely flexible bus services. The optimal bus size for temporarily integrated services is intermediate between the optimal bus sizes of conventional and flexible bus services. The suggested method is applied to bus services connecting a terminal and a local region with multiple time periods. Quadrifoglio and Li (2009) study the critical demand threshold for switching between fixed route bus and demand responsive services. They analytically derive a closed-form solution for the threshold demand, but do not optimize service frequencies or vehicle sizes.

Zhou et al (2008) develop a welfare maximization approach to compare various bus transit service types (i.e., conventional and flexible bus services) for a local region. To maximize the welfare objective, the formulation imposes financial constraints, i.e. that the operator cost should not exceed the sum of revenue and subsidy. They analytically optimize fares, headways, route spacings, and service areas for conventional and flexible bus services. They find that the break-even policy or low subsidy policy may be preferable for conventional bus services, but not for flexible bus services. The break-even policy causes a relatively large loss in social welfare for flexible bus services.

A different approach to reduce bus transit cost is to use different fleets of buses as the demand varies, with larger buses used at higher demand densities. Lee et al (1995) and Fu and Ishkhanov (2004) analyze the assignment of buses with dissimilar sizes (i.e. “mixed fleets”) to public transit operations.

Lee et al (1995) study mixed bus fleets in urban conventional bus operations for multiple routes. They consider bus operating cost, user in-vehicle cost, and user waiting cost, but not user access cost. They define the demand thresholds from the service cost formulation, which includes bus operating cost, user in-vehicle cost, and user waiting cost, to assign either large or small conventional buses. They propose a heuristic for optimizing vehicle sizes, headways, and fleets for a total cost formulation which includes capital cost. Their formulation is limited to many-to-one trips, although an extension to analyze many-to-many trips may be possible. Their work does not consider passenger transfers in a terminal. An important assumption of this study is that bus operating cost is a function of the bus size (i.e., $B = a+bS$).

Similarly, Fu and Ishkhanov (2004) study mixed fleet bus operations for paratransit services. They note that although larger vehicles have higher capacities, they do not automatically yield higher productivity because ridesharing may be limited by the time constraints of the clients. They use a program called FirstWin (TSS, 2003) to generate schedules and associated performance statistics for given demands. They have hard windows in their constraints. They propose a heuristic called scheduling, matching, allocation, and reduction (SMAR). This heuristic is fundamentally a greedy search procedure based on the idea of using as many small vehicles as possible without loss of productivity. This study offers some insights for possible ridesharing algorithms to be used by the flexible bus services with mixed fleets. Lee et al (1995) and Fu and Ishkhanov (2004) both confirm that when demand densities differ considerably over time or space, mixed fleets can reduce total system cost compared to single fleets because vehicles of different sizes may be matched to the operations for which they are most suited.

Besides the above studies, it is difficult to find studies that consider variations in service type as demand changes. Thus, it seems worthwhile to examine not only the relative advantages and disadvantages of conventional and paratransit services, but also to explore variable-type bus alternatives in which the service type changes in response to demand changes while using the same pool of resources (i.e. buses and drivers).

2.3. Coordination of Bus Transit Services

Kyte et al (1982) present a timed-transfer system in Portland, Oregon. They provide the history of planning, implementation, and evaluation of a timed transfer system which provides services since 1979. This system provides timed transfers to the suburban areas in which demands are low, and provides grid-type bus services to the higher demand regions. This paper also discusses the performances and results of the implemented system. They use two indicators, which are a successful meet and a successful connection, to analyze the transfer reliabilities. A successful meet is defined as all buses arriving as scheduled at a given time, and a successful connection is a direct transfer connection that results from two routes arriving as scheduled. The authors point out that weekday ridership increased by 40 percent after one year operation, and local trips using this system increased dramatically. However, the 40% increase of ridership resulted not only from a timed transfer system, but also from new route designs. Bakker et al (1988) similarly study a multi-centered time transfer system in Austin, Texas, and confirm that such a timed transfer system is particularly applicable for low density cities.

Abkowitz et al (1987) study timed transfers between two routes. They compare four policy cases, namely unscheduled, scheduled transfer without vehicle waiting, scheduled transfers with the lower frequency bus is held until the higher frequency vehicle arrives, and scheduled transfers when both buses are held until a transfer even occurs. In other words, this paper compares scheduled, waiting/holding, and double holding transfer strategies. They note that the effectiveness of timed transfers varies by route conditions. However, they find that the scheduled transfers are effective (over the unscheduled) when there is incompatibility between headways and the double holding strategy outperforms the other time transfer strategies when the headways on intersecting routes are compatible. This study also points out that slack time may be better built into the schedule so that vehicle holding does not cause significant delays to passengers.

Domschke (1989) explores a schedule coordination problem with the objective of minimizing waiting times. He provides a mathematical programming formulation which is generally applicable to a public mass transit network such as subways, trains and/or buses. The formulation is a quadratic assignment problem. With four routes and five transfer stations in a toy network, this paper considers heuristics and a branch and bound algorithm. The heuristics include a starting heuristic, which is based on rigid regret heuristic, and then a heuristic improvement procedure. Lastly, simulated annealing (SA) is applied to improve the solutions. For SA, the quality of the initial solution is important. He finds that problems with more than 20 routes cannot be solved by exact solution methods.

Knoppers and Muller (1995) provide a theoretical note on transfers in public transportation. Their main concerns are the transfer time needed and the probability of missed connection to minimize passengers' transfer time. They find that when the frequency on the connecting lines increases, the benefit of transfer coordination decreases. Muller and Furth (2009) try to reduce passenger waiting time through transfer scheduling and control. They provide a probabilistic optimization model, and discuss three transfer control types, namely departure punctuality control, attuned departure control, and delayed departure of connecting vehicles. They confirm that by increasing a buffer (slack), the probability of missing the connection decreases. However, a larger buffer increases the transfer time for people do not miss their connection. They also find that if the control policy allows a bus to be held to make a connection, the optimal schedule offset decreases.

Shrivastava et al (2002) first discuss existing algorithms for solving nonlinear mathematical programming, because transit scheduling problems are often nonlinear. The existing algorithms are generally gradient based, and require at least the first order derivatives of both objective and constraint functions with respect to the design variables. With the “slope tracking” ability, gradient-based methods can easily identify a relative optimum closest to the initial guess of the optimum design. However, there is no guarantee of locating the global optimum if the design space is known to be non-convex. In such case, exhaustive and random search techniques such as random walk or random walk with direction exploitation are quite useful. The main drawback with these methods is that they often require thousands of function evaluations, even for the simplest functions, to reach the optimum. They also note that genetic algorithms (GAs) are based on exhaustive and random search techniques, and are robust for optimizing nonlinear and non-convex functions. Thus, they apply a genetic algorithm (GA) to schedule coordination problems. The objective function includes waiting time, transfer time, and in vehicle time for users, and vehicle operating cost for operators. For a scheduling problem, they try to solve routing and scheduling simultaneously. The GA is designed with two substrings where one represents routes, and the other represents frequencies on those routes. By solving benchmark problems, they find that genetic algorithms provide better solutions than other heuristics. They also note that computational times are proportional to the pool size. Cevallos and Zhao (2006) also use a GA to solve a transfer time optimization problem for a fixed route system. Their main focus is efficient computational time.

Lee and Schonfeld (1991) study optimal slack times for coordinating transfers between rail and bus routes at one terminal. The transfer cost function is formulated as a sum of scheduled delay cost, missed connection cost for bus to train transfer, and missed connection cost for train to bus transfer. In their paper, the rail transit line is assumed to run on-time (no slack), and slack times for bus routes are to be optimized. Bus arrivals are assumed to vary independently from train arrivals so that the joint probabilities of arrivals may be obtained by simply multiplying the probabilities obtained separately from the bus and train arrivals distributions. Slack times are optimized analytically, and numerical results show that analytic optimization with simplifying assumptions is limited and difficult to solve for complex situations. Thus, they develop a numerical optimization method to find solutions efficiently.

Ting and Schonfeld (2005) extend Lee and Schofeld (1991)'s study. They explore bus service coordination among multiple transit routes in multiple hub networks. They analyze uncoordinated operations and coordinated operation, and compare the results. For uncoordinated operation, the formulation minimizes the total system cost which is sum of operating cost, user waiting cost, and user transfer cost. Transfer cost in uncoordinated operation is simply assumed to be the product of the average transfer waiting time and the total number of transfer passengers. For the coordinated operation, the transfer cost consists of slack-time cost, missed connection cost, and dispatching delay cost. Common headway and integer-ratio headway cases are optimized with a heuristic algorithm. Their algorithms and numerical results show when coordinated operations with integer-ratio headways are preferable over uncoordinated operation in terms of total cost. Simplifying assumptions of this work are that: 1) only one dispatching strategy is considered, which means vehicles do not wait for other vehicles that arrive behind schedules; 2) vehicle arrivals on a route are assumed to vary independently from those of other routes, so that the joint probabilities of arrivals may be obtained by simply multiplying the probabilities obtained separately from the two vehicle arrival distributions. A limitation of this work is that it does not ensure integer fleet size.

Chen and Schonfeld (2010) adapt the concept of bus transit coordination methods to freight transportation. They follow the main ideas of joint probabilities and transfer cost components from some previous transit studies (Lee and Schonfeld, 1991; Ting and Schonfeld, 2005). In this study, they propose two solution approaches, which are a genetic algorithm and sequential quadratic programming (SQP) to find good solutions for frequencies and slack times in intermodal transfers.

Chowdhury and Chien (2002) also study the coordination of transfers among rail and feeder bus routes. Their objective is to minimize total cost, including supplier, user costs, similarly to other studies. They explore various degrees of coordination such as full coordination, partial coordination, and no coordination. They also follow the assumption of joint probabilities of independent vehicle arrivals, and assume that trains operate on-time. Recently, Chowdhury and Chien (2011) extend a previous study by jointly optimizing bus size, headway, and slack time for timed transfer. They optimize bus size by assuming maximum allowable bus headways instead of minimum cost headways. Therefore, their optimized bus size may be overestimated. For solving this problem they apply Powell's algorithm (i.e., multi-variable numerical optimization). Unfortunately, they do not present enough details on the methodology to clarify how joint variables are optimized and how variables are constrained to be integer. Another limitation of this study is that although it finds optimized vehicle size jointly with other decision variables, such as headways and slack times, the vehicle size is optimized for only one time period. Optimizing vehicle size and required fleet size for daily demand or system-wide demand while finding headways and slack times for each time period is an opportunities for improvement.

2.4. Bus Transit Services with Demand Elasticity

In this subsection, we review papers of transit welfare objectives with demand elasticity. When considering the demand elasticity, formulations typically become maximization problems, presumably because it makes little sense to minimize costs if demand is elastic (and may be driven to zero). Kocur and Hendrickson (1982) optimize transit decision variables namely route spacing, headway, and fare, with demand elasticity. They assume a linear transit utility function rather than a logit form. The reasons for the linear utility approximation are that: it is analytically tractable; it is easily differentiated and manipulated; and it is convex within its upper and lower bounds. They consider wait time, walk time, in-vehicle time, fare, and auto time and cost in the demand model. They provide analytic closed form solutions, but this study is limited to a conventional bus service with a local region. Later, Imam (1998) extends Kocur and Hendrickson (1982)'s study by relaxing the linear demand function. Imam (1998) applies a log-additive demand function.

Chang and Schonfeld (1993) then consider time-dependent supply and demand characteristics for a transit welfare maximization problem. They have a linear demand function as in Kocur and Hendrickson (1982). Decision variables are route spacing, headways, and fare. Since this study considers multiple time periods, they optimize headways for multiple time periods. Their objective is to maximize consumer surplus and producer surplus. They solve this maximum welfare problem with alternative financial constraints, namely without any constraint, with a break-even constraint, and with subsidy. Their problem size extends to one local region and multiple periods. Solutions are obtained analytically with approximations. For the formulations with constraints, a Lagrange multipliers method is applied. The vehicle size is considered as an input, rather than a decision variable.

Zhou et al (2008) formulate welfare for conventional bus services and flexible bus services, but only for a system connecting a terminal to one local region in one period. They find solutions analytically because the formulation of a system that connects a terminal to one local region in one period is analytically tractable. Analyses of system welfare with larger problem sizes (i.e., multiple regions and multiple periods) for both conventional and flexible services are desirable. They analyze tradeoffs between subsidies and welfare, but do not provide detailed enough methods to duplicate their results.

Chien and Spasovic (2002) study a grid bus transit system with an elastic demand pattern. They optimize route spacings, station spacings, headways, and fare with the objective of maximum total operator profit and social welfare. The elastic demand is subtracted from the potential demand as in Chang and Schonfeld (1993), and the optimal solutions are found analytically. This work is applicable to conventional bus services.

Tsai et al (2013) find headway and fare solutions for a Taiwan High Speed Rail (THSR) line, with a maximum welfare objective. They consider elastic demand for the study, and apply a GA to obtain solutions. They compare solutions from a GA and solutions from a SSM (Successive Substitution Method). However, this study does not provide enough evidence on the global optimality of its solutions.

2.5. Review Summary

To date, the integration of different types of bus services with joint optimization of their decision variables is largely neglected in the literature. This dissertation will deal with several problems of integrating conventional and flexible bus services.

For the system welfare problems in bus transit systems, most of the literature covers conventional services. The problem size is constrained by the limits of analytic optimization. For conventional services, the solved problem size encompasses a local region with multiple periods. For flexible services, the solved problem size encompasses a local region and one period. With numerical solutions it seems desirable to consider problems with multiple regions as well as multiple periods for both conventional and flexible services.

Chapter 3 Variable-Type Bus Services for a Local Region

3.1. Problem Statement

Conventional public transit services (which include most bus and rail transit services) are characterized by their fixed routes and schedules. They can provide relatively high passenger-carrying capacities at relatively low average operating costs. However, their service quality is limited since passengers must somehow reach some predetermined stations, wait for a vehicle, possibly transfer several times, and then move from their exit stations to their destinations. Thus, conventional transit services are least disadvantaged in areas and time periods with high demand densities, which can sustain high network densities and service frequencies. Some paratransit services can provide more flexible routes and schedules, including the possibility of door-to-door service. Thus, taxis provide great service flexibility, but at high unit costs (especially in labor cost per passenger-mile). Flexible services, thus, can be preferable for low demand areas. Improvements in service quality as well as overall system costs may be achieved by integrating conventional and flexible services.

The potential advantages of variable-type bus for integrated conventional and flexible bus operations have not been sufficiently explored. Those potential advantages are the subject of this chapter, in which we seek to quantify them. This chapter modifies the cost function provided by Chang and Schonfeld (1991a). More specifically it (1) modifies the cost functions to reflect two-directional demands in round trip times, (2) develops an integrated model for variable-type bus services and (3) compares conventional, flexible and variable-type bus services under various assumed conditions. This model is intended for conceptual comparisons of services rather than detailed planning and operations.

3.2. Notation and Assumptions

Definitions and baseline values of variables are provided in Table 3-1.

Table 3-1 Notation and Baseline Values

Variable	Definition	Baseline Value
a	hourly fixed cost coefficient for operating bus service ($/veh hr)	30.0
a_i	hourly fixed cost coefficient for operating bus service at period i ($/veh hr)	30.0
a_c	fixed cost coefficient for bus ownership (capital cost) ($/veh day)	100.0
$\bar{a}$	weighted fixed cost coefficient defined in Table 2 ($/veh day)	-
A	service zone area(sq.mile)=LW/N'	-
A_{ik}	service zone area(sq.mile) for case k	-
b	hourly variable cost coefficient for operating bus service ($/seat	0.2

	hr))	
b_i	hourly variable cost coefficient for operating bus service at period i ($/seat hr)	0.2
b_c	variable cost coefficient for owning bus (capital cost) ($/day)	0.5
$\bar{b}$	weighted fixed cost coefficient defined in equation 54, 55, and 56 ($/veh day)	-
B	bus operating cost ($/veh hr), $(=a+bS_c, a+bS_s)$	-
B_c	bus operator cost for owning bus(capital cost) ($/veh hr)	-
C_c	service cost for conventional bus system ($/hr)	-
C_{ci}	service cost cost for conventional bus system at period i ($/hr)	-
C_{oc}	operating cost for conventional bus system ($/hr)	-
C_{os}	operating cost for flexible bus system ($/hr)	-
C_{pc}	capital cost for conventional bus system($/day)	-
C_{ps}	capital cost for flexible bus system ($/day)	-
C_s	service cost cost for flexible bus system ($/hr)	-
C_{sik}	flexible bus service cost for case k at period i ($/hr)	-
C_{si}	service cost for flexible bus system at period i ($/hr)	-
C_{tc}	total cost for conventional bus system ($/day)	-
C_{tc}	total cost for flexible bus system ($/day)	-
C_{uc}	user cost for conventional bus system ($/hr)	-
C_{us}	user cost for flexible bus system ($/hr)	-
C_{vc}	in-vehicle cost for conventional bus system ($/hr)	-
C_{vs}	in-vehicle cost for flexible bus system ($/hr)	-
C_{wc}	waiting cost for conventional bus system ($/hr)	-
C_{ws}	waiting cost for flexible bus system ($/hr)	-
C_{xc}	access cost for conventional system ($/hr)	-
d	bus stop spacing (miles)	0.2
D	equivalent average bus round trip distance for conventional bus service (= $2J/y+W/z+2L$),(miles)	-
D_c	distance of one tour of flexible bus service in local area (miles)	-
D_s	equivalent line haul distance for flexible bus service $(=(L+W)/z+2J/y)$, (miles)	-
F_c	fleet size for conventional bus system (vehicles)	-

F_{ci}	fleet size for conventional bus system in period i (vehicles)	-
F_{si}	fleet size for flexible bus system in period i (vehicles)	-
f	a controllable directional split factor	1.0
h_c	headway for conventional bus system (hrs/veh)	-
h_c^{max}	maximum allowable headway for conventional bus service (hrs/veh)	-
h_c^{opt}	optimized headway for conventional bus service (hrs/veh)	-
h_{ci}	headway for conventional bus system at period i (hrs/veh)	-
h_s	headway for flexible bus system (hrs/veh)	-
h_{si}	headway for flexible bus system in period i (hrs/veh)	-
h_{si}^{max}	maximum allowable headway for flexible bus service in period i (hrs/veh)	-
h_{sik}	Headway for flexible bus system for case k in period i (hrs/veh)	-
h_{si}^{opt}	optimized headway for flexible bus service in period i (hrs/veh)	-
h_{sik}^{max}	maximum allowable headway for flexible bus service for case k in period i (hrs/veh)	-
h_{sik}^{opt}	optimized headway for flexible bus service for case k in period i (hrs/veh)	-
i	period index	-
J	line haul distance (miles)	10.0
k	period index	-
l	load factor (passengers/seat)	-
l_c	load factor for conventional bus service (passengers/seat)	1.0
l_s	load factor for flexible bus service (passengers/seat)	1.0
L	length of service area (miles)	5.0
M	Equivalent average trip distance ($=J/y_c+W/2z_c+L/2$)	-
n	number of passengers in one collection tour	-
N	number of branched zones in conventional bus service	-
N'	number of service zones in flexible bus services	-
Q	round trip demand density (trips/sq.mile/hr)	-
Q_i	round trip demand density in period i (trips/ sq.mile/hr)	-
Q_p	demand density in peak time (trips/sq.mile/hr)	-
$\bar{Q}$	average round trip demand density at defined in equation 54, 55, and 56 (trips/sq.mile/hr)	-

R	round travel distance (miles)	-
r	route spacing	-
S_c	vehicle size for conventional bus service (seats/veh)	-
S_s	vehicle size for flexible bus service (seats/veh)	-
t_i	duration of period i	-
u	average number of passengers per stop point for flexible bus service	1.2
V_c	local service speed for conventional bus (miles/hr)	20
V_s	local service speed for flexible bus (miles/hr)	18
V_x	average access speed (mile/hr)	2.5
v_v	value of in-vehicle time ($/passenger hr)	5
v_w	value of wait time at bus stop ($/passenger hr)	12
v_x	value of access time ($/passenger hr)	12
W	width of service area (miles)	4.0
y	express speed/local speed ratio for conventional bus service	conventional bus = 1.8 flexible bus = 2.0
Y	term used in Table 3-1 and 3-2	-
z	non-stop ratio = local non-stop speed/local speed; same values as y	-
$Ø$	constant in the collection distance equation for flexible bus service	1.15
*	superscript indicating optimal value	-

The assumptions for both conventional bus and flexible bus are listed below.

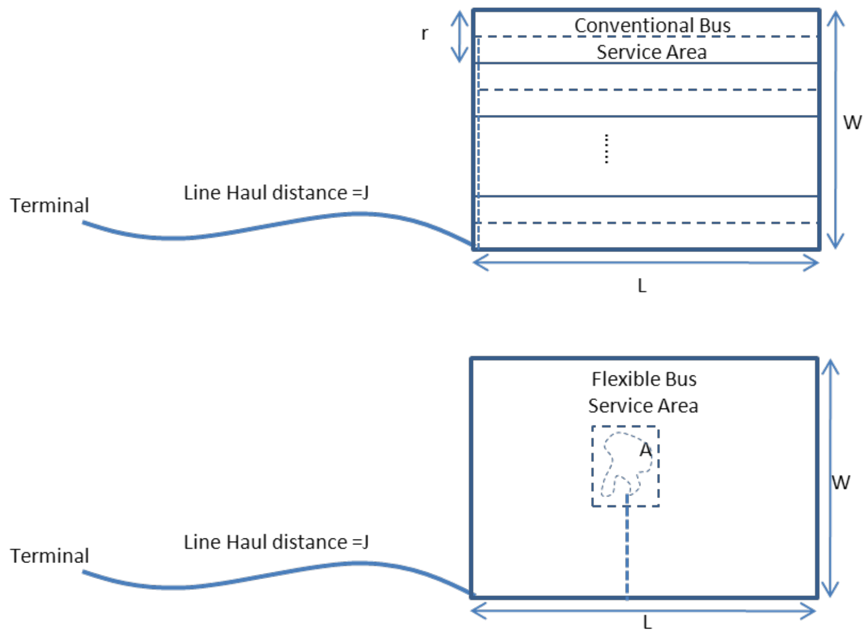


Figure 3-1 Conventional and Flexible Bus Services

3.2.1. For Both Conventional and Flexible Buses

- A rectangular service area of length L and width W (as shown in Figure 3-1) is J miles away from a transportation terminal at its nearest corner.
- The demand is fixed with respect to service quality and price.
- The demand is uniformly distributed over space within the service area and over time within each specified period.
- The vehicle size (S_c for conventional bus, S_f for flexible bus) is uniform throughout a system.
- The estimated average waiting time of passengers is equal to half the headway

(h_c for conventional bus, h_f for flexible bus).

- Vehicle layover time is negligible.
- Within the service area, the average speed (V_c for conventional bus, V_f for flexible bus) includes stopping times.
- External costs are assumed to be negligible.

3.2.2. For Conventional Bus Only

- The service area is divided into N parallel zones with a width r=W/N for conventional bus service, as shown in Figure 3-1. Local routes branch from the line haul route segment to run along the middle of each zone, at a route spacing r=W/N.
- A demand of Q trips/mile^2/hour, which is entirely channeled to (or through) the single terminal, is uniformly distributed over the service area.
- In each round trip, as shown in Figure 3-1, buses travel from the terminal a line haul distance J at non-stop speed yV_c to a corner of the service area, then travel an average of W/2 miles at local non-stop speed zV_c from the corner to the assigned zone, then run a local regionof length L at local speed V_c along

the central axis of the zone while stopping for passengers every d miles, and then reverse the above process in returning to the terminal..

3.2.3. For Flexible Bus Only

- The service area is divided into N' equal zones, each having an optimizable zone area A=LW/N'. The zones should be "fairly compact and fairly convex" (Stein, 1978).
- Buses travel from the terminal a line haul distance J at non-stop speed yV_f and an average distance (L+W)/2 miles at local non-stop speed zV_f to the center of each zone. They collect (or distribute) passengers at their door steps through a tour of n stops and length Dc at local speed V_f. The values of n and Dc are endogenously determined. Dc is approximated by Stein (1978), in which and for rectilinear space according to Daganzo (1984). To return to their starting point the buses retrace an average of (L+W)/2 miles at zV_f miles per hour and J miles at yV_f miles per hour.
- Buses operate on preset schedules with flexible routing designed to minimize each tour distance Dc.

- The tours are routed on the rectilinear street network.
- Tour departure headways are equal for all zones in the service area and uniform within each period.

3.3. Results of Chang and Schonfeld (1991a)

The formulation proposed by Chang and Schonfeld (1991a) considered one-way service (i.e. only collecting passengers OR distributing passengers) in which total demand density is Q trips/mile^2. Based on these assumptions, the analytic optimization results obtained for conventional bus and flexible bus services by Chang and Schonfeld (1991a) are presented in Table 3-2. For bus operating cost, a linear (i.e. $B=a+bS$) cost function was used (Jansson, 1980; Oldfield and Bly, 1988).

Table 3-2 Analytic Results from Chang and Schonfeld (1991a)

Conventional bus service		Flexible bus service	
Vehicle Size S_c	$\sqrt[3]{\frac{8a^2D^2LQ\mathrm{V_x}}{v_w v_x V_c^2 l_c^3}}$	Vehicle Size S_f	$\left(\frac{a^3D_f^3Qu}{v_w \varnothing^2 V_f l_f^3 \left(b+\frac{v_v l_f}{2}\right)^2}\right)^{1/5}$
Route Spacing r	$\sqrt[3]{\frac{8aDv_wV_x^2}{v_x^2LQV_c}}$	Service Area A	$\left(\frac{av_w^3V_f^3D_f^3u^{8/3}l_f^4}{\varnothing^4Q^{7/3}Y^{10/3}\left(b+\frac{v_v l_f}{2}\right)^2}\right)^{1/5}$

Service Cost (Conventional Bus)	$3LWQ\left(\frac{v_w v_x D}{8LQV_cV_x}\right)^{\frac{1}{3}}+\frac{bDLWQ}{l_cV_c}+\frac{v_vLWQM}{V_c}+\frac{v_xLWQd}{4V_x}$
Service Cost (Flexible Bus)	$LWQ\left[\frac{v_w a^2 D_f^2 Ø^2\left(b+\frac{v_v l_f}{2}\right)^2}{ul_f^2V_f^4Q}\right]^{1/5}+1.5LWQ\left[\frac{v_w a^2 Ø^2}{l_f^2V_f^4}\right]^{1/5}(\frac{Y^2}{uQ})^{1/3}$ $+\frac{LWQD_f\left(b+\frac{v_v l_f}{2}\right)}{V_f l_f}$
Note	$Y=\left[a^2 v_w Ø^2 V_f l_f^3\right]^{1/5}+\left[uD_f^3Q\left(b+v_v l_f/2\right)^3\right]^{1/5}$

D_c is approximated by Stein (1978),in which $D_c = Ø\sqrt{nA}$, and $Ø = 1.15$ for rectilinear space according to Daganzo (1984)

3.3.1. Total Cost including Capital Cost

When computing the total system cost for bus service, capital cost should be treated as another fixed cost. The capital cost C_p, is the cost to satisfy the peak period vehicle requirement. In equation (3.1), bus service cost is defined as the sum of bus operating cost C_o, user in vehicle cost C_v, user waiting cost C_w, and user access cost C_x:

Total cost = Capital cost + Bus operating cost + User cost (3.1)

Relation (3.1) can be rewritten as:

$$C_t = C_p + C_o + C_u = C_p + C_o + C_v + C_w + C_x \quad (3.2)$$

Analytic results with capital cost for conventional and flexible bus services are summarized in Table 3-3.

Table 3-3 Analytic Results with Capital Cost

Conventional bus service		Flexible bus service	
Vehicle Size S_c	$\sqrt[3]{\frac{8\bar{a}^2 D^2 L\bar{Q}V_x}{v_w v_x V_c^2 l_c^3}}$	Vehicle Size S_f	$\left(\frac{\bar{a}^3 D_f^3 \bar{Q} u}{v_w Ø^2 V_f l_f^3 \left(\bar{b} + \frac{v_v l_f}{2}\right)^2}\right)^{1/5}$
Routing Space r	$\sqrt[3]{\frac{8\bar{a}D v_w V_x^2}{v_x^2 L\bar{Q}V_c}}$	Service Area A	$\left(\frac{\bar{a} v_w^3 V_f^3 D_f^3 u^{8/3} l_f^4}{Ø^4 \bar{Q}^{7/3} Y^{10/3} \left(\bar{b} + \frac{v_v l_f}{2}\right)^2}\right)^{1/5}$
Total Cost (Conv. Bus)	$\frac{a_c DLWQ_p}{S_c{}^* V_c l_c} + \frac{b_c DLWQ_p}{V_c l_c} + \frac{D(a+bS_c{}^*)LW\sum_i^I Q_i t_i}{V_c S_c{}^* l_c} + \frac{v_v MLW \sum_i^I Q_i t_i}{V_c} + \frac{v_w lWS_c{}^* \sum_i^I t_i}{2r} + \frac{v_x (r+d)LW\sum_i^I Q_i t_i}{4V_x}$		
Total Cost (Flex. Bus)	$\frac{LWQ_p D_f (a_c + b_c S_f)}{V_f S_f l_f} + \frac{ØLWQ_p\,(a_c + b_c S_f)\sqrt{A/uS_f l_f)}}{V_f} + \sum_i^I \left\{ \frac{LWQ_i t_i D_f (a+bS_f)}{V_f S_f l_f} + \frac{ØLWQ_i t_i (a+bS_f)\sqrt{A/uS_f l_f}}{V_f} + \frac{v_v LWQ_i t_i D_f}{2V_f} + \frac{v_v LWQ_i t_i Ø\sqrt{AS_f l_f/u)}}{2V_f} + \frac{v_w LWS_f l_f t_i}{2A} \right\}$		
Note	$\bar{Q} = \frac{\sum_i^I Q_i t_i}{\sum_i^I t_i}, \bar{a} = \frac{a_c Q_p \sum_i^I a_i Q_i t_i}{\sum_i^I Q_i t_i}, \bar{b} = \frac{b_c Q_p \sum_i^I b_i Q_i t_i}{\sum_i^I Q_i t_i}, Y = \left[\bar{a}^2 v_w Ø^2 V_f l_f^3\right]^{1/5} + \left[uD_f^3 \bar{Q}\left(\bar{b} + v_v l_f/2\right)^3\right]^{1/5}$		

3.3.2. Limitations from Chang and Schonfeld (1991a) ‘s Study

Here, formulations in this chapter seek to overcome two main limitations in the bus service cost formulations of Chang and Schonfeld (1991a). First, they assume that trip demand for bus services is always one-directional (i.e. either all demand from terminal to local or local to terminal). The model is modified here by introducing a directional demand split factor, f. Second, they only consider the maximum allowable headway (required to satisfy demand) rather than an optimized headway. It seems preferable to optimize the headway for each period, which should be the minimum of (1) the maximum feasible headway which satisfies the demand and (2) the headway that minimizes total costs.

3.4. Cost Function Modification and Optimized Headways

Here a directional demand split factor, f, is introduced for conventional bus service only (because flexible service does not need a directional demand split factor unless passengers are collected and distributed in different tours) as well as provide optimized headway solutions for both conventional bus and flexible bus services. If f=1.0 all demand is one-directional. In other words, buses return without any passengers. Similarly, if f=0.5, then demand is equal in the two directions. In flexible service, since passengers are collected and distributed within the same tours, no directional split factor is needed. Therefore, if the demand density Q is assumed as the sum of both collected passengers and distributed passengers, the Chang and Schonfeld (1991a)'s flexible service cost functions are still applicable.

3.4.1. Conventional Bus Cost Formulation

As shown in Figure 3-1, buses travel from the terminal a line haul distance *J* at non-stop speed yV_c to a corner of the service area, then travel an average of *W/2* miles at local non-stop speed zV_c from the corner to the assigned zone, run a distribution segment of length *L* at local speed V_c along the central axis of the zone while stopping for passengers every *d* miles, and the reverse the process in returning. Therefore, the buses' average round trip time is:

$$R_c = \frac{2J}{yV_c} + \frac{W}{zV_c} + \frac{2L}{V_c} \tag{3.3}$$

This round trip time can be re-written as:

$$R_c = \left\{\frac{2J}{y} + \frac{W}{z} + 2L\right\} \Big/ V_c \tag{3.4}$$

In equation (3.4), the expression in parentheses represents an equivalent vehicle round trip distance, D.

The total cost of conventional bus service includes the operator cost C_{oc} and the user costs C_{uc}. To determine operator cost, the fleet size, F_c, which is the total vehicle round trip time divided by the headway is first determined. With the equivalent vehicle round travel distance *D*, a controllable directional split factor *f*, and conventional bus speed V_c. The required fleet size F_c is:

$$F_c = \frac{DW}{rh_cV_c} \qquad ,where \;\; D = 2J/y + W/z + 2L \tag{3.5}$$

The hourly conventional bus operator cost C_{oc} is the required fleet size multiplied by bus operating cost:

$$C_{oc} = F_cB \tag{3.6}$$

The bus operating cost B is formulated as:

$$B = a + bS_c \tag{3.7}$$

The required service headway h_c is:

$$h_c = \frac{S_cl_c}{rLfQ} \tag{3.8}$$

The operating cost C_{oc} can be reformulated by substituting equations (3.5), (3.7), and (3.8) into equation (3.6):

$$C_{oc} = \frac{D(a+bS_c)LWfQ}{l_cV_cS_c} \tag{3.9}$$

The hourly user cost for the conventional bus system C_{uc} is the sum of in-vehicle cost C_{vc}, waiting cost C_{wc}, and access cost C_{xc}:

$$C_{uc} = C_{vc} + C_{wc} + C_{xc} \tag{3.10}$$

The user in-vehicle cost for the conventional system can be formulated as

$$C_{vc} = v_vLWQt \tag{3.11}$$

The hourly in-vehicle cost for the conventional system is then:

$$t = \frac{J}{yV_c} + \frac{W}{2zV_c} + \frac{L}{2V_c} = \frac{M}{V_c} \quad ,where\ M=J/y + W/2z + L/2 \tag{3.12}$$

Then equation (3.11) can be written as:

$$C_{vc} = v_v LWQ \frac{M}{V_c} \tag{3.13}$$

It is assumed that the average waiting time is half the headway. Therefore, the hourly user waiting cost for conventional system C_{wc} is:

$$C_{wc} = v_w LWQ \frac{h_c}{2} = v_w LWQ \frac{S_c l_c}{2rLfQ} = \frac{v_w W S_c l_c}{2rf} \tag{3.14}$$

Since the spacing between adjacent branches of local bus service is *r*, and since service trip origins (or destinations) are uniformly distributed over the area, the average access distance to the nearest route is one-fourth of route spacing, *r/4*. Similarly, the access distance alongside the route to the nearest transit stop is one-fourth of the bus stop spacing, i.e., *d/4*. Therefore, the hourly access cost for the conventional bus system C_{xc} is:

$$C_{xc} = \frac{v_x LWQ(r+d)}{4V_x} \tag{3.15}$$

The total cost for the conventional system C_c is the sum of operating cost and user costs:

$$C_c = \frac{D(a+bS_c)LWfQ}{l_c V_c S_c} + \frac{v_v LWQM}{V_c} + \frac{v_w W S_c l_c}{2rf} + \frac{v_x LWQ(r+d)}{4V_x} \tag{3.16}$$

In equation (3.16), the optimizable variables are routing space *r* and vehicle size S_c, which are optimized by taking partial derivatives of C_c in equation (3.16). Setting the partial derivatives equal to zero and solving simultaneously, we obtain:

$$S_c^* = \frac{2f}{l_c}\sqrt[3]{\frac{a^2D^2LQV_x}{v_w v_x V_c^2}} \tag{3.17}$$

$$r^* = \sqrt[3]{\frac{8aDv_wV_x^2}{v_x^2LQV_c}} \tag{3.18}$$

The second derivatives of equation (3.16) with respect to vehicle size S_c and routing space *r* are positive for any reasonable inputs. Therefore, equations (3.17 and 3.18) yield the globally minimal total cost. From equations (3.17 and 3.18) it is found that that product of the optimized vehicle size and optimized route spacing is constant (i.e, $S_c^* \times r^* = (4faDV_x)/(l_c v_x V_c)$ = constant).

After optimizing vehicle size S_c^* and route spacing r^*, the headway h_c^*, which minimizes total cost C_c, is optimized. Optimal headway h_c^* should be the minimum of the maximum allowable headway and minimum cost headway. The maximum allowable headway h_c^{max} can be found by substituting equations (3.17) and (3.18) into equation (3.8).

$$h_c^{max} = \frac{S_c^* l_c}{r^* LfQ} \tag{3.19}$$

The optimized headway h_c^{opt} can be found from the total cost function, which is provided in equation (3.20), by setting its first derivative equal to zero. The second derivative is positive. Therefore, the optimized headway will yield the globally minimal total cost.

$$C_c = \frac{DW(a+bS_c)}{rV_c h_c} + \frac{v_v LWQM}{V_c} + \frac{v_w LWQh_c}{2} + \frac{v_x LWQ(r+d)}{4V_x} \tag{3.20}$$

The resulting minimum cost headway is:

$$h_c^{opt} = \sqrt{\frac{2D(a+bS_c^*)}{v_w r^* V_c LQ}} \tag{3.21}$$

Overall, the optimal headway h_c^* is then:

$$h_c^* = min\left\{\frac{S_c^* l_c}{r^* LfQ}, \sqrt{\frac{2D(a+bS_c^*)}{v_w r^* V_c LQ}}\right\} \tag{3.22}$$

After substituting equations (3.18) and (3.19) into equation (3.5), the optimal fleet size $F_c{}^*$ for the conventional bus system is:

$$F_c{}^* = \frac{DW}{r^* h_c^* V_c} \tag{3.23}$$

Therefore, the bus service cost based on the jointly optimized vehicle size S_c^*, route spacing r^*, and optimal headway h_c^* is:

$$C_c = \frac{DW(a+bS_c^*)}{r^* V_c h_c^*} + \frac{v_v LWQM}{V_c} + \frac{v_w LWQh_c^*}{2} + \frac{v_x LWQ(r^*+d)}{4V_x} \tag{3.24}$$

When computing total system cost for conventional bus service, the capital cost C_p should satisfy the peak period fleet size requirement. In equation (3.25), the bus service cost is the sum of bus operating cost C_o, user in vehicle cost C_v, user waiting cost C_w, and user access cost C_x.

Total cost = Capital cost + Bus service cost

= *Capital cost* + *Bus operating cost* + *User cost* (3.25)

Equation (3.25) can be expressed as:

$$C_t = C_p + C_o + C_u = C_p + C_o + C_v + C_w + C_x \tag{3.26}$$

Although the conventional bus cost is reformulated, the overall procedure for computing total cost with capital cost is basically similar to that in CS. The capital cost for conventional bus system should be computed based on peak-period demand. Therefore, capital cost C_{pc} for conventional bus service is:

$$C_{pc} = \frac{D}{V_c}\frac{W}{r}\frac{1}{h_p}B_c = \frac{D}{V_c}\frac{W}{r}\frac{rLfQ_p}{S_c l_c}B_c = \frac{D}{V_c}\frac{W}{r}\frac{rLfQ_p}{S_c l_c}(a_c + b_c S_c) \tag{3.27}$$

The total daily service cost for conventional bus service C_{tc} is formulated below. Subscript *i* denotes time periods in the following equations and t_i represents the number of hours in period *i*.

$$\mathrm{C_{tc}} = \mathrm{C_{pc}} + \sum_{\mathrm{i}}^{\mathrm{I}}\{\mathrm{C_{oci}} + \mathrm{C_{vci}} + \mathrm{C_{wci}} + \mathrm{C_{xci}}\} \tag{3.28}$$

Equation (3.28) can be rewritten as follows:

$$C_{tc} = \frac{a_c DLWfQ_p}{S_c V_c l_c} + \frac{b_c DLWQ_p}{V_c l_c} + \frac{D(a+bS_c)LWf\sum_i^I Q_i t_i}{l_c V_c S_c} + \frac{v_v MLW\sum_i^I Q_i t_i}{V_c} + \frac{v_w WS_c l_c \sum_i^I t_i}{2rf} + \frac{v_x(r+d)LW\sum_i^I Q_i t_i}{4V_x} \tag{3.29}$$

By simultaneously solving the derivatives of C_{tc} in equation (3.29) with respect to route space *r* and vehicle size S_c we find the optimal values of r^* and $S_c{}^*$:

$$r^* = \sqrt[3]{\frac{8\bar{a}Dv_wV_x^2}{v_x^2L\bar{Q}V_c}} \tag{3.30}$$

$$S_c{}^* = \frac{2f}{l_c}\sqrt[3]{\frac{\bar{a}^2D^2L\bar{Q}\mathrm{V_x}}{v_wv_xV_c^2}}, \tag{3.31}$$

$$where\ \ \bar{Q} = \frac{\sum_i^I Q_it_i}{\sum_i^I t_i}, \bar{a} = \frac{a_cQ_p+\sum_i^I a_iQ_it_i}{\sum_i^I Q_it_i}, \bar{b} = \frac{b_cQ_p+\sum_i^I b_iQ_it_i}{\sum_i^I Q_it_i}$$

Based on the optimized vehicle size $S_c{}^*$ and route spacing r^*, bus service cost for period *i* can be expressed as follows:

$$C_{ci} = \frac{DW(a+bS_c^*)}{r^*V_ch_{ci}} + \frac{v_vLWQ_iM}{V_c} + \frac{v_wLWQ_ih_{ci}}{2} + \frac{v_xLWQ_i(r^*+d)}{4V_x} \tag{3.32}$$

The optimized headway h_{ci}^{opt} for period *i* can be obtained by setting the first derivative of conventional bus service cost C_{ci} to zero.

$$h_{ci}^{opt} = \sqrt{\frac{2D(a+bS_c^*)}{v_wr^*V_cLQ_i}} \tag{3.33}$$

Therefore, the optimal headway h_{ci}^* for each period *i* is the minimum of h_{ci}^{max} and h_{ci}^{opt}:

$$h_{ci}^* = \min\{h_{ci}^{max}, h_{ci}^{opt}\} = min\left\{\frac{S_c^*l_c}{r^*LfQ_i}, \sqrt{\frac{2D(a+bS_c^*)}{v_wr^*V_cLQ_i}}\right\} \tag{3.34}$$

The optimal fleet size F_{ci}^* for each period depends on the optimal headway of that period:

$$F_{ci}^* = \frac{DW}{r^*V_ch_{ci}^*} \tag{3.35}$$

The capital cost should be determined from the peak period demand, which is denoted as period 1, as follows:

$$C_{pc}^* = \frac{DW}{r^*V_ch_{c1}^*}B_c = \frac{DW(a_c+b_cS_c^*)}{r^*V_ch_{c1}^*} \tag{3.36}$$

The bus service cost C_{ci} for each period i can be formulated using the optimal headway of that period. Therefore, the conventional bus service cost C_c for all periods can be expressed as:

$$C_c^* = \sum_i \left\{\frac{DW(a+bS_c^*)}{r^* V_c h_{ci}^*} + \frac{v_v LWQ_i M}{V_c} + \frac{v_w LWQ_i h_{ci}^*}{2} + \frac{v_x LWQ_i (r^*+d)}{4V_x}\right\} t_i \quad (3.37)$$

The total cost including capital cost can be found by substituting the optimal route spacing r^* and optimal vehicle size S_c^* into equation (3.29):

$$C_{tc}^* = \frac{DW(a_c+b_c S_c^*)}{r^* V_c h_{c1}^*} + \sum_i \left\{\frac{DW(a+bS_c^*)}{r^* V_c h_{ci}^*} + \frac{v_v LWQ_i M}{V_c} + \frac{v_w LWQ_i h_{ci}^*}{2} + \frac{v_x LWQ_i (r^*+d)}{4V_x}\right\} t_i \quad (3.38)$$

3.4.2. Flexible Bus Service Cost

When considering capital cost for Flexible Bus service, the optimized vehicle size S_f^* and vehicle service area A^* can be adopted from Table 3-2 from Chang and Schonfeld (1991a). In this section, headways for flexible bus service are optimized. Unlike Chang and Schonfeld (1991a) who only used the maximum allowable headway, the optimal headway should be the minimum of (1) the maximum allowable headway and (2) the minimum cost headway.

The maximum allowable headway h_{fi}^{max} for demand period i is a function of optimized vehicle size S_f^*, load factor l_f, service area A^*, and demand density Q_i:

$$h_{fi}^{max} = \frac{S_f^* l_f}{A^* Q_i} \tag{3.39}$$

From Table 3-2, the flexible bus service cost for period i C_{fi} can be rewritten as:

$$C_{fi} = \frac{LW\left(a+bS_f^*\right)(D_f+ØA^*\sqrt{\frac{Q_i h_{fi}}{u}})}{A^* V_f h_{fi}} + \frac{v_v LWQ_i(D_f+ØA^*\sqrt{\frac{Q_i h_{fi}}{u}})}{2V_f} + \frac{v_w LWQ_i h_{fi}}{2} \tag{3.40}$$

The optimized service headway h_{fi}^{opt} can be obtained by setting the first derivative equal to zero:

$$\frac{\partial C_{fi}}{\partial h_{fi}} = -\frac{LW\left(a+bS_f^*\right)D_f}{A^* V_f}\frac{1}{h_{fi}^2} - \frac{LW\left(a+bS_f^*\right)ØA^*\sqrt{\frac{Q_i}{u}}}{2A^* V_f}\frac{1}{\sqrt{h_{fi}^3}} + \frac{v_v LWQ_i ØA^*\sqrt{\frac{Q_i}{u}}}{4V_f}\frac{1}{\sqrt{h_{fi}}} + \frac{v_w LWQ_i}{2} = 0$$

(3.41)

Equation (3.41) is a quartic equation with respect to headway. In equation (3.41), by substituting $\frac{1}{\sqrt{h_{fi}}}$ into *t*, equation (3.41) becomes

$$-\frac{LW\left(a+bS_f^*\right)D_f}{A^* V_f}t^4 - \frac{LW\left(a+bS_f^*\right)ØA^*\sqrt{\frac{Q_i}{u}}}{2A^* V_f}t^3 + 0 \times t^2 + \frac{v_v LWQ_i ØA^*\sqrt{\frac{Q_i}{u}}}{4V_f}t^1 + \frac{v_w LWQ_i}{2} = 0$$

(3.42)

Equation (3.42) can be rewritten as:

$$At^4 + Bt^3 + Ct^2 + Dt^1 + E = 0 \tag{3.43}$$

,where $A = -\frac{LW\left(a+bS_f^*\right)D_f}{A^* V_f}, B = -\frac{LW\left(a+bS_f^*\right)ØA^*\sqrt{\frac{Q_i}{u}}}{2A^* V_f}, C = 0, D = \frac{v_v LWQ_i ØA^*\sqrt{\frac{Q_i}{u}}}{4V_f}, E = \frac{v_w LWQ_i}{2}$

To solve equation (3.44), the value of P, Q, R, S, T, and V must be computed using A, B, C, D and E.

$$P = \frac{B}{4A}$$

$$Q = \frac{2C}{3A}$$

$$R = C^2 - 3BD + 12AE$$

$$S = 2C^2 - 9BCD + 27AD^2 + 27EB^2 - 72ACE$$

$$T = -\frac{B^3}{A^3} + \frac{4BC}{A^2} - \frac{8D}{A}$$

$$V = \frac{\sqrt[3]{2}R}{3A\sqrt[3]{S+\sqrt{-4R^3+S^2}}} + \frac{\sqrt[3]{S+\sqrt{-4R^3+S^2}}}{3\sqrt[3]{2}A} \quad (3.44)$$

After finding the values of P, Q, R, S, T and V, the following results are obtained:

$$X1 = -P - \frac{1}{2}\sqrt{4P^2 - Q + V} - \frac{1}{2}\sqrt{8P^2 - 2Q - V - \frac{T}{4\sqrt{4P^2-Q+V}}}$$

$$X2 = -P - \frac{1}{2}\sqrt{4P^2 - Q + V} + \frac{1}{2}\sqrt{8P^2 - 2Q - V - \frac{T}{4\sqrt{4P^2-Q+V}}}$$

$$X3 = -P + \frac{1}{2}\sqrt{4P^2 - Q + V} - \frac{1}{2}\sqrt{8P^2 - 2Q - V + \frac{T}{4\sqrt{4P^2-Q+V}}}$$

$$X4 = -P + \frac{1}{2}\sqrt{4P^2 - Q + V} + \frac{1}{2}\sqrt{8P^2 - 2Q - V + \frac{T}{4\sqrt{4P^2-Q+V}}} \quad (3.45)$$

X1~X4 correspond to t ($=\frac{1}{\sqrt{h_{fi}}}$). Therefore, among the four solutions, the only feasible solution satisfying both t>0 and h_{fi} >0 is the optimized headway h_{fi}^{opt}.

The optimal headway h_{fi}^{*} is the minimum of the (1) maximum allowable headway and (2) optimized headway obtained by solving equation (3.42):

$$h_{fi}^{*} = min\left\{\frac{S_f^{*} l_f}{A^{*} Q_i}, h_{fi}^{opt}\right\} \quad (3.46)$$

Based on the optimal headway for period *i*, the required fleet size F_{fi}^{*} is

$$F_{fi}^{*} = \frac{LW(D_f+\varnothing A^{*}\sqrt{\frac{Q_i h_{fi}^{*}}{u}})}{A^{*}V_f h_{fi}^{*}} \tag{3.47}$$

Finally, the total cost C_{tf}^{*} which is the sum of capital cost and bus service cost for all periods can be expressed as:

$$C_{tf}^{*} = \frac{LW(a_c+b_c S_f^{*})(D_f+\varnothing A^{*}\sqrt{\frac{Q_i h_{f1}^{*}}{u}})}{A^{*}V_f h_{f1}^{*}} + \sum_i \left\{\frac{LW\left(a+bS_f^{*}\right)(D_f+\varnothing A^{*}\sqrt{\frac{Q_i h_{fi}^{*}}{u}})}{A^{*}V_f h_{fi}^{*}} + \frac{v_v LWQ_i(D_f+\varnothing A^{*}\sqrt{\frac{Q_i h_{fi}^{*}}{u}})}{2V_f} + \frac{v_w LWQ_i h_{fi}^{*}}{2}\right\} t_i \tag{3.48}$$

3.5. Variable-Type Bus Service using Conventional and Flexible Buses

Conceptually, conventional services using relatively large buses are expected to have lower cost per passenger trip than flexible services at higher demand densities, and vice versa. In this section, the demand boundary between conventional and flexible bus services is explored. Below this boundary, flexible services are chosen. Purely conventional and purely flexible service costs are also compared to variable-type services.

3.5.1. Integer Solutions

In the objective function shown in equation (3.50), only one service type (either conventional or flexible bus) is used in each period. The constraints in equations (3.50~3.53) are required to obtain integer values for the number of routes and fleet sizes per route.

$$C_t = \min\{C_{pc} + \textstyle\sum_i \left(C_{ci} + C_{fi}\right)t_i\} \tag{3.49}$$

Subject to

$$S_c{}^* = S_f{}^* = \text{integer} \tag{3.50}$$

$$\frac{\mathrm{W}}{\mathrm{r}}, \frac{\mathrm{LW}}{\mathrm{A}} = \text{integer} \tag{3.51}$$

$$F_{ci}{}^*, F_{fi}{}^* = \text{integer} \tag{3.52}$$

$$\frac{F_{ci}{}^*}{\mathrm{N}}, \frac{F_{fi}{}^*}{\mathrm{N}'} = \text{integer} \tag{3.53}$$

3.6. Numerical Evaluation: Base Case Analysis

In this section, bus operation costs (pure conventional bus service, pure flexible bus service, and variable-type bus service) are computed and compared. In this numerical analysis the cumulative demand distribution over time has four values, as shown in Figure 3-2.

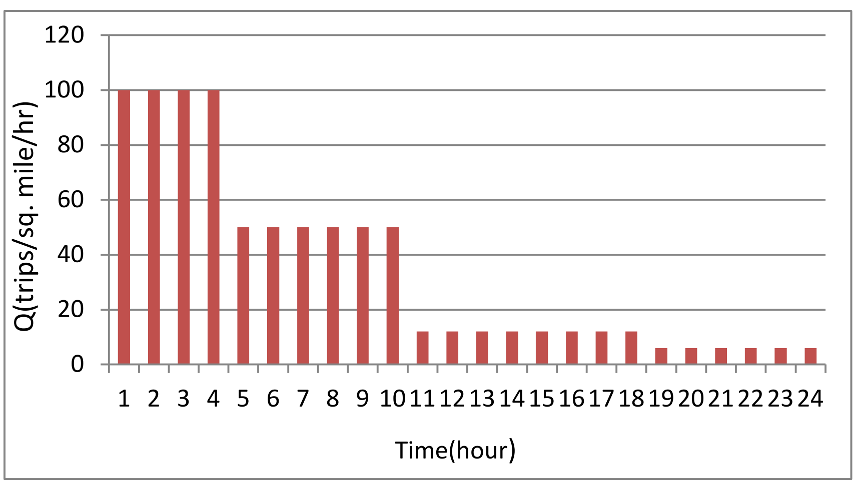


Figure 3-2 Demand Density Variations

For pure conventional service cost, equations (3.1~3.38) are used in this numerical example. For flexible service cost, optimized decision variables (optimized vehicle size and service area) can be found from Tables 3-2 and 3-3. The previous Chang and Schonfeld (1991a) study used the maximum allowable headway, without considering the minimum cost headway. Here, headways for flexible services are analytically optimized in equations (3.40~3.47).

3.6.1. Variable-Type Bus Service Boundary

For variable-type bus operation, the optimized bus size is usually determined by the conventional service requirements. As mentioned for constraint (3.52), integer values for both W/r and LW/A are required to obtain integer fleets. The resulting possible values of decision variables r and A are shown in Table 3-4.

Table 3-4 Possible Values of Decision Variables r and A

N, N’	r = W/N for conventional bus periods	A=LW/N’ for flexible bus periods
1	4	20
2	2	10
3	1.333	6.667
4	1	5

5	0.8	4
6	0.667	3.333
…	…	…

These values in Table 3-4 will be used to search for the minimum cost route spacing r^* for conventional bus and minimum cost service area A^* for flexible bus services.

3.6.2. Procedure for Finding Minimum Variable-Type Service Cost

In general, if our demand distribution has k periods, then there are (k+1) possible boundaries between periods (i.e. boundary 1… k+1) when service switches from one type to another. Thus our numerical example has five possible boundaries because it has four cumulative demand periods. Variable-type service is provided when $1 < k < 5$, while k = 1 means that service is always purely flexible service and k=5 implies purely conventional service in every period.

The computation procedures for variable-type service are as follows:

1) Set up boundary k =1.
2) Based on boundary, optimize decision variables, namely vehicle sizes and route spacing for conventional operations, or service area for flexible service.

3) Optimize headway for variable-type service.

4) Compute total cost using results in 2) and 3).

5) Change boundary k to k+1 (i.e. One more period has conventional service and the remaining periods have flexible service).

6) Continue 2) ~ 4) until the total cost starts increasing. Then the optimal boundary minimizes the total cost for variable-type service.

3.6.3. Results of Numerical Analysis

The results obtained with baseline inputs are provided in Table 3-1. The optimized pure conventional bus service costs $107,166/day, including capital costs and user time costs. To operate conventional bus service with the given demand density, 60 buses are required. The vehicle size is optimized with 40 seats/bus to satisfy all demand periods. While the local area route spacing is jointly optimized (subject to a constraint requiring an integer number of zones) at one mile.

Purely flexible bus results show that the optimized total cost for serving this demand is about $118,377/day, which is much costlier than the total cost of purely conventional bus services. The reason is that the optimized flexible services use many more zones (9 versus 4) and vehicles, but much smaller vehicles, than optimized conventional services. Moreover, purely flexible service requires more buses to cover peak demand since its optimized vehicles are smaller than for conventional bus. As shown in Table 3-5, flexible services require 108 buses in the peak period, which increases capital cost.

For variable-type services, the purely conventional bus size of 40 seats is used in all periods for both conventional and flexible operations as well as for capital cost computation. In this numerical analysis, it is found that variable-type services are preferable to purely conventional and flexible bus services. Therefore, conventional services are chosen in periods 1 and 2, and flexible services are chosen in periods 3 and 4, using the same bus size.

The variable-type service (using flexible service in period 3 and 4) reduces the total cost compared to both purely conventional and purely flexible services. Compared to purely conventional service, variable-type service saves $1,382/day. Similarly, variable-type service costs about $12,600/day less than purely flexible service.

Table 3-5 Numerical Results with Baseline Inputs

	Purely Conventional Service	Purely Flexible Service	Variable-type Service
S_c, S_f (seats/bus)	40	23	40

r, A	1	2.222	0.8	6.667
N	4	9	5	3
h1(hrs)	0.078	0.089	0.097	
h2(hrs)	0.146	0.177	0.194	
h3(hrs)	0.389	0.510		0.269
h4(hrs)	0.583	0.476		0.340
F1(vehicles)	60	108	60	
F2(vehicles)	32	54	30	
F3(vehicles)	12	18		12
F4(vehicles)	8	18		9
C1($/hr)	10,676.7	11,175.1	10,430.0	
C2($/hr)	5,822.7	8,111.1	5,798.3	
C3($/hr)	1,911.6	5,085.4		1927.3
C4($/hr)	1,171.8	1,824.9		1109.3
t1(hrs)	4	4	4	
t2(hrs)	6	6	6	
t3(hrs)	8	8		8
t4(hrs)	6	6		6
Cp($/day)	7,200.0	12,042.0	7,200.0	
TC($/day)	107,166.3	118,376.8	105,784.3	
% Change	1.290 %	10.64 %		

3.7. Numerical Evaluation: Sensitivity Analysis

Sensitivity analyses are conducted to explore the relative merits of conventional, flexible and variable-type bus services in different circumstances. Seven cases are presented below.

3.7.1. Case I - Directional Demand Split Factor

The directional demand split factor, *f*, is changed to 75% & 25% (vs. 100% & 0% in the baseline). In Table 3-6, the total costs of conventional and variable-type service in this case decrease compared to the baseline results in Table 3-5. In this case, variable-type service reduces total cost by 1.39% from purely conventional and 11.67% from purely flexible service, respectively. In this case I with *f*=0.75, a directional demand split factor can slightly reduce costs below the baseline case.

Table 3-6 Sensitivity Analysis Results for Directional Split Factor

	Purely Conventional Service	Purely Flexible Service	Variable Type Service	
S_c, S_f (seats/bus)	31	23	31	
r, A	1	2.857	0.8	6.667
N	4	7	5	3
h1(hrs)	0.078	0.066	0.097	
h2(hrs)	0.146	0.154	0.194	
h3(hrs)	0.389	0.334		0.269
h4(hrs)	0.583	0.487		0.340
F1(veh)	60	112	60	
F2(veh)	32	49	30	
F3(veh)	12	21		12
F4(veh)	8	14		9
C1($/hr)	10,568.7	11,481.7	10,322.0	
C2($/hr)	5,765.1	6,087.9	5,744.3	
C3($/hr)	1,890.0	1,990.4		1,897.1
C4($/hr)	1,157.4	1,220.0		1,087.7
t1(hrs)	4	4	4	

t2(hrs)	6	6	6	
t3(hrs)	8	8		8
t4(hrs)	6	6		6
Cp($/day)	6,930.0	12,488.0	6930.0	
TC($/day)	105,859.5	118,185.3	104,387.2	
% Change	1.39 %	11.67 %	-	

3.7.2. Case II - Load Factors

Maximum load factors for both conventional and flexible service are increased from 1 to 1.25 (implying that some standees are allowed). Table 3-6 shows the resulting costs. It is noted that the costs of purely conventional service in Table 3-7 are below the baseline case (Table 3-5). Similarly, purely flexible and variable-type services benefit from higher load factors. However, similarly to Case I, the effect of variable-type service is saving about 1.41% and 9.71% savings compared to purely conventional and purely flexible services, respectively.

Table 3-7 Sensitivity Analysis Results for Load Factors

	Purely Conventional Service	Purely Flexible Service	Variable Type Service	
S_c, S_f (seats/bus)	32	23	32	
r, A	1	3.333	0.667	6.667
N	4	6	6	3
h1(hrs)	0.078	0.068	0.117	
h2(hrs)	0.146	0.118	0.194	
h3(hrs)	0.389	0.340		0.209

h4(hrs)	0.583	0.494		0.340
F1(veh)	60	96	60	
F2(veh)	32	54	36	
F3(veh)	12	18		15
F4(veh)	8	12		9
C1($/hr)	10,580.7	11,391.4	10,247.3	
C2($/hr)	5,771.5	6,149.2	5,808.7	
C3($/hr)	1,892.4	1,940.2		1,896.5
C4($/hr)	1,159.0	1,176.9		1,090.1
t1(hrs)	4	4	4	
t2(hrs)	6	6	6	
t3(hrs)	8	8		8
t4(hrs)	6	6		6
Cp($/day)	6,960.0	10,704.0	6960.0	
TC($/day)	106,004.7	115,747.8	104514.3	
% Change	1.41 %	9.71 %	-	

3.7.3. Case III - Demand Variation

This Case explores the effect of very low demand density (i.e. Q1=10, Q2=5, Q3=1.2, Q4=0.6 trips/sq. mile, and equals to 10% of baseline value). Here the costs of purely conventional and flexible services are very close. With variable-type services, as shown in Table 3-8, the total cost is reduced by 3.19% and 4.06% from purely conventional and flexible services, respectively. It is interesting here that conventional service is only used during the highest demand period, leaving the other three periods to flexible service.

Table 3-8 Sensitivity Analysis Results for Demand Variation

	Purely Conventional Service	Purely Flexible Service	Variable-Type Service	
S_c, S_f (seats/bus)	17	16	17	
r, A	2.0	10.0	2.0	10.0
N	2	2	2	2
h1(hrs)	0.167	0.111	0.167	
h2(hrs)	0.292	0.198		0.253
h3(hrs)	1.167	0.472		0.472
h4(hrs)	1.167	0.944		0.944
F1(veh)	14	18	14	
F2(veh)	8	10		8
F3(veh)	2	4		4
F4(veh)	2	2		2
C1($/hr)	1,653.9	1,692.3	1,653.9	
C2($/hr)	935.4	930.2		903.0
C3($/hr)	353.2	317.7		318.7
C4($/hr)	210.0	192.8		193.4
t1(hrs)	4	4	4	
t2(hrs)	6	6		6
t3(hrs)	8	8		8
t4(hrs)	6	6		6
Cp($/day)	1,519.0	1,944.0	1519.00	
TC($/day)	17,832.1	17,992.9	17,262.8	
% Change	3.19 %	4.06 %		

3.7.4. Case IV - Time Period Variation

In the baseline case (Table 3-5) there are 4, 6, 8, and 6 hours, respectively, in periods 1, 2, 3 and 4. In Case IV the effect of higher demand variability is explored by changing those four periods to 2, 4, 4 and 14 hours. The results in Table 3-9 show that variable-type bus service now achieves much greater savings compared to the baseline case (Table 3-5). These savings are about 3.41% and 13.08% compared to pure services, while in the baseline (Table 3-5) variable-type bus service cost savings from purely conventional services are about 1.29 %.

Based on the sensitivity of results in these cases, it is found that significant advantages of variable-type bus service occur when there are long period of demand that is far below peak levels.

Table 3-9 Sensitivity Analysis Results for Service Time Variation

	Purely Conventional Service	Purely Flexible Service	Variable Type Service	
S_c, S_f (seats/bus)	40	21	40	
r, A	1.333	3.333	0.667	10.0
N	3	6	6	2
h1(hrs)	0.058	0.052	0.117	
h2(hrs)	0.117	0.105	0.194	
h3(hrs)	0.389	0.340		0.179
h4(hrs)	0.583	0.494		0.259
F1(veh)	60	120	60	
F2(veh)	30	60	36	
F3(veh)	9	18		12
F4(veh)	6	12		8
C1($/hr)	11,243.3	11,775.8	10,343.3	
C2($/hr)	5,971.7	6,202.7	5,866.3	
C3($/hr)	1,893.6	1,931.7		1,958.9

C4($/hr)	1,143.8	1,171.4		1,096.0
t1(hrs)	2	2	2	
t2(hrs)	4	4	4	
t3(hrs)	4	4		4
t4(hrs)	14	14		14
Cp($/day)	7,200.0	13,260.0	7,200.0	
TC($/day)	77,160.9	85,748.9	74531.6	
% Savings	3.41 %	13.08 %		

3.7.5. Case V - Operating Cost Parameters

In Case V, the sensitivity of total cost and other results to bus operating cost that is a linear function of the number of seats (i.e. B=a+bS) is examined. Here, the values of parameters a and b are increased by 50 % (i.e., a=45, b=0.3). The results in Table 3-10 show that we achieve the lowest total cost by providing variable-type service When the variable-type service is operated, the cost savings are 1.379% and 14.86% compared to purely conventional and flexible services, respectively.

Table 3-10 Sensitivity Analysis Results for Operating Cost Inputs

	Purely Conventional Service	Purely Flexible Service	Variable Type Service	
S_c, S_f (seats/bus)	50	27	50	
r, A	1	2.857	1	6.667
N	4	7	4	3
h1(hrs)	0.097	0.077	0.097	
h2(hrs)	0.194	0.154	0.194	
h3(hrs)	0.583	0.527		0.269

h4(hrs)	1.167	0.487		0.340
F1(veh)	48	98	48	
F2(veh)	24	49	24	
F3(veh)	12	14	12	
F4(veh)	4	14		3
C1($/hr)	11,510.0	13,381.0	11,510.0	
C2($/hr)	6,338.3	7,153.1	6,338.3	
C3($/hr)	2,215.6	2,357.3	2,175.6	
C4($/hr)	1,527.8	1,510.0		1,312.3
t1(hrs)	4	4	4	
t2(hrs)	6	6	6	
t3(hrs)	8	8	8	
t4(hrs)	6	6		6
Cp($/day)	6,000.0	11,123.0	6000.0	
TC($/day)	116,961.6	135,483.6	115348.9	
% Savings	1.379 %	14.86 %		

3.7.6. Case VI - Length of Service Region

In Case VI the service region length is increased by 20% (from 5 to 6 miles). It is found, for variable-type service, that conventional bus serves periods 1, 2, and 3; flexible bus only serves period 4. This result shows that as the local service region lengthens, the potential savings of variable-type service decrease because demand also increases, thus favoring conventional service. In Table 3-11, Period 3 in variable-type service is served by conventional service, unlike in the baseline case (Table 3-5).

Table 3-11 Sensitivity Analysis Results for Service Region Length

	Purely Conventional Service	Purely Flexible Service	Variable Type Service	
S_c, S_f (seats/bus)	45	25	45	
r, A	1	3.429	1	8
N	4	7	4	3
h1(hrs)	0.075	0.057	0.075	
h2(hrs)	0.141	0.122	0.141	
h3(hrs)	0.422	0.352	0.422	
h4(hrs)	0.633	0.511		0.358
F1(veh)	68	133	68	
F2(veh)	36	63	36	
F3(veh)	12	21	12	
F4(veh)	8	14		9
C1($/hr)	12,980.9	14,299.2	12,980.9	
C2($/hr)	7,045.3	7,524.1	7,045.3	
C3($/hr)	2,308.3	2,370.2	2,308.3	
C4($/hr)	1,414.6	1,435.0		1,351.4
t1(hrs)	4	4	4	
t2(hrs)	6	6	6	
t3(hrs)	8	8	8	
t4(hrs)	6	6		6
Cp($/day)	8,330.0	14,962.5	8,330.00	
TC($/day)	129,479.7	144,875.7	129,100.6	
% Savings	0.29 %	10.89 %		

3.7.7. Case VII - Line-haul Distance

In Case VII, the sensitivity to line-haul distance (from 10miles to 20miles) is analyzed. Here the ratio of line-haul distance/length of local area (i.e. J/L) is increased from 2 to 4. Table 3-12 shows variable-type service reduces total cost by 0.704% and 11.12% compared to purely services. By increasing line-haul distance (without changing demand), round trip time increases for both conventional and flexible service, favoring larger vehicles because bus operator wants to carry more passengers in round trip time. Thus, in Table 3-12, vehicle size for variable-type service is 50 seats/bus, but only 40 seats/bus in the baseline case (Table 3-4) is 40 seats/bus. With variable-type services, service costs are reduced in Periods 1 & 4 compared to purely conventional services. These service cost savings and capital cost savings allow variable-type service to outperform to pure services.

Table 3-12 Sensitivity Analysis Results for Line-haul Distance

	Purely Conventional Service	Purely Flexible Service	Variable Type Service	
S_c, S_f (seats/bus)	50	31	50	
r, A	1	4	0.8	10
N	4	5	5	2
h1(hrs)	0.096	0.066	0.123	
h2(hrs)	0.191	0.139	0.246	
h3(hrs)	0.431	0.405		0.239
h4(hrs)	0.574	0.523		0.324
F1(veh)	72	125	70	
F2(veh)	36	60	35	
F3(veh)	16	20		14
F4(veh)	12	15		10
C1($/hr)	14,269.3	15,954.5	14,037.3	

C2($/hr)	7,708.7	8,347.3	7,756.7	
C3($/hr)	2,488.9	2,546.6		2,539.7
C4($/hr)	1,507.8	1,523.0		1,422.5
t1(hrs)	4	4	4	
t2(hrs)	6	6	6	
t3(hrs)	8	8		8
t4(hrs)	6	6		6
Cp($/day)	9,000.0	14,437.5	8,750.0	
TC($/day)	141,287.5	157,850.3	140,292.1	
% Savings	0.704 %	11.12 %		

3.8. Chapter Summary

In Chapter 3, optimization models are developed for analyzing and integrating conventional services (having fixed routes and schedules) and flexible bus services. The optimization models are improved from those of Chang and Schonfeld (1991a). More specifically, the models developed in this chapter (1) reflect two-directional demands in round trips in conventional services, (2) optimize the flexible service headways rather than using maximum allowable headways, (3) develop an integrated model for variable-type bus services and (4) compare conventional, flexible and variable-type bus services which can switch between conventional and flexible service as the demand changes over time.

The above numerical analysis indicates that variable-type bus operation can reduce total cost compared to a purely conventional bus or purely flexible bus service. In our baseline case, variable-type service can reduce costs by about 1.29% compared to purely conventional service and about 10.64% compared to purely flexible service. Moreover, we present various sensitivity analyses to explore how major parameter changes affect the optimized results. In Case IV (when service periods are adjusted to increase the variability of demand over time), it is found that variable-type service can reduce costs by more than 3.41% and 13.08 %, respectively, compared to purely conventional and flexible services. These results confirm that such variable-type services are especially promising for systems whose demand (1) varies greatly over time and (2) straddles the threshold between conventional and flexible services.

To summarize, it is confirmed that conventional service with large buses is preferable when demand is high. Similarly, flexible service is less costly at relatively low demand. A public bus system alternating among these two service concepts based on demand variation and other conditions can be used to improve service efficiency.

Chapter 4 Integrating Bus Services for Multiple Regions

4.1. Problem Statement

In Chapter 3 a model for analyzing bus service in one region (i.e. connecting one terminal to one region) was presented. Here, some assumptions and notation are modified to analyze a more general system with multiple local regions as well as multiple periods. Also, an optimization method to deal with more generalized systems is proposed.

4.2. Assumptions

Henceforth, superscripts k and i correspond to region and time period, respectively, while subscripts c and f represent conventional and flexible service, respectively. Definitions, units and default values of variables are presented in Table 4-1.

Table 4-1 Notation

Variable	Definition	Baseline Value
a	hourly fixed cost coefficient for operating bus ($/bus hr)	30.0
a_c	fixed cost coefficient for bus ownership (capital cost) ($/bus day)	100.0
A^k	service zone area(mile^2)= L^kW^k/N'	-
b	hourly variable cost coefficient for bus operation ($/seat hr)	0.2
b_c	variable cost coefficient for owning bus (capital cost) ($/day)	0.5
d	bus stop spacing (miles)	0.2
D_c^{ki}	distance of one flexible bus tour in local region k and period i (miles)	-

D_f^k	equivalent line haul distance for flexible bus on k $(=(L^k+W^k)/z+2J^k/y)$, (miles)	-
D^k	equivalent average bus round trip distance for conventional bus on region k ($= 2J^k/y+ W^k /z+2 L^k$),(miles)	-
f	directional demand split factor	1.0
F^{ki}	fleet size for region k and period i (buses) subscript corresponds to (c = conventional, f=flexible)	-
h_c , h_c^{ki}	headway for conventional bus; for region k and period i (hours/bus)	-
h_f , h_f^{ki}	headway for flexible bus; for region k period i (hours/bus)	-
$h_{c\,max}^{ki}, h_{f\,max}^{ki}$	maximum allowable headway for region k and period i subscript: c = conventional, f=flexible	-
$h_{c\,min}^{ki}, h_{f\,min}^{ki}$	minimum cost headway for region k and period i subscript: c = conventional, f=flexible	-
$h_{c\,opt}^{ki}, h_{f\,opt}^{ki}$	optimized headway for region k and period i subscript: c = conventional, f=flexible	-
k ,i	index (k: route, i : period)	-
J^k	line haul distance of region k (miles)	-
l_c, l_f	load factor for conventional and flexible bus (passengers/seat)	1.0
L^k, W^k	length and width of local region k (miles)	-
M^k	equivalent average trip distance for region k ($=J^k/y_c+ W^k /2z_c+ L^k /2$)	-
n	number of passengers in one flexible bus tour	-
N, N'	number of zones in local region for conventional and flexible bus	-
Q^{ki}	round trip demand density (trips/mile^2/hr)	-
Q_t^{ki}	threshold demand density between conventional and flexible service (trips/mile^2/hr)	-
r^k	route spacing for conventional bus at region k (miles)	-
R_c^{ki}	round trip time of conventional bus for region k and period i (hours)	-
R_f^{ki}	round trip time of flexible bus for region k and period i (hours)	-
S_c , S_f	sizes for conventional and flexible bus (seats/bus)	-
S_l , S_s	sizes of larger and smaller buses in MFCS and MFFS service formulation	-
S_c^{ki}, S_f^{ki}	conventional and flexible bus sizes for region k and period i (seats/bus)	-
SC_c^{ki}, SC_f^{ki}	service cost for region k and period i subscript: c = conventional, f=flexible	-

TSC_c , TSC_f	total service cost over all routes and periods subscript: c = conventional, f=flexible	-
TC	total cost = service cost + capital cost over times and over regions	
t^{ki}	time duration for region k and period i	-
u	average number of passengers per stop for flexible bus	1.2
V_c^i	local service speed fo/r conventional bus in period i (miles/hr)	20 at i =1 30 at i = 2,3,4
V_f^i	local service speed for flexible bus in period i (miles/hr)	18 at i =1 25 at i = 2,3,4
V_x	average passenger access speed (mile/hr)	2.5
v_v, v_w, v_x	value of in-vehicle time, wait time and access time ($/passenger hr)	5, 12, 12
y	express speed/local speed ratio for conventional bus	conventional bus = 1.8 flexible bus = 2.0
z	non-stop ratio = local non-stop speed/local speed; same values as y	-
$\varnothing$	constant in the flexible bus tour equation (Daganzo, 1984) for flexible bus	1.15
$*$	superscript indicating optimal value; subscript: c = conventional, f=flexible	-

4.2.1. Assumptions for both conventional and flexible buses

All service regions, 1… k, are rectangular, with lengths L^k and widths W^k. These regions may have different line haul distances J^k (miles, in region k) connecting a terminal and each region's nearest corner.

- The demand is fixed with respect to service quality and price.
- The demand is uniformly distributed over space within each region and over time within each specified period.

- The bus sizes (S_c for conventional, S_f for flexible) are optimized based on their service coverage, and the optimized bus sizes are uniform throughout regions.
- The average waiting time of passengers is approximated as half the headway (h_c for conventional, h_f for flexible).
- Bus layover time is negligible.
- Within each local region k, the average speed (V_c^i for conventional bus, V_f^i for flexible bus) includes stopping times.
- External costs are assumed to be negligible.

4.2.2. Assumptions for conventional bus only

- The region k is divided into N^k parallel zones with a width $r^k=W^k/N^k$ for conventional bus, as shown in Figure 4-1. Local routes branch from the line haul route segment to run along the middle of each zone, at a route spacing $r^k=W^k/N^k$.
- Q^{ki} trips/mile2/hour, entirely channeled to (or through) the single terminal, are uniformly distributed over the service area.

- In each round trip, as shown in Figure 4-1, buses travel from the terminal a line haul distance J^k at non-stop speed yV_c^i to a corner of the local regions, then travel an average of $W^k/2$ miles at local non-stop speed zV_c^i from the corner to the assigned zone, then run a local region of length L^k at local speed V_c^i along the central axis of the zone while stopping for passengers every d miles, and then reverse the above process in returning to the terminal.

4.2.3. Assumptions for flexible bus only

- To simplify the flexible bus formulation, region k is divided into N'^k equal zones, each having an optimizable zone area $A^k = L^k W^k / N'^k$. The zones should be "fairly compact and fairly convex" (Stein, 1978).
- Buses travel from the terminal line haul distance J^k at non-stop speed yV_f^i and an average distance $(L^k+W^k)/2$ miles at local non-stop speed zV_c^i to the center of each zone. They collect (or distribute) passengers at their door steps through an efficiently routed tour of n stops and length D_c^{ki} at local speed V_f^i. D_c^{ki} is approximated according to Stein (1978), in which $D_c^{ki} = \emptyset\sqrt{nA^k}$, and $\emptyset$=1.15 for the rectilinear space assumed here (Daganzo, 1984). The values of

n and D_c^{ki} are endogenously determined. To return to their starting point the buses retrace an average of $(L^k+W^k)/2$ miles at zV_f^i miles per hour and J^k miles at yV_f^i miles per hour.

- Buses operate on preset schedules with flexible routing designed to minimize each tour distance D_c^{ki}.
- Tour departure headways are equal for all zones in the region and uniform within each period.

4.3. Bus Operation Costs and Optimal Headways

In terms of operation cost for conventional and flexible bus, I consider bus operating cost, user in-vehicle cost, user waiting cost, and user access cost. Since flexible bus provides door-to-door service, its user access cost is negligible. Detailed formulation derivations regarding conventional bus and flexible bus are provided in Chapter 3.

4.3.1. Conventional Bus Formulation and Optimal Headway

Conventional bus cost for region k and period i, SC_c^{ki}, includes operating cost, user in-vehicle cost, user waiting cost, and user access cost, as shown in Equation (4.1):

$$SC_c^{ki} = \frac{D^k W^k(a+bS_c\ )}{r^k V_c^i h_c^{ki}} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w L^k W^k Q^{ki} h_c^{ki}}{2} + \frac{v_x L^k W^k Q^{ki}(r^k+d)}{4V_x} \qquad (4.1)$$

Since multiple periods are considered for bus operations, the optimized headway should be the maximum allowable headway or the minimum cost headway, whichever is smaller. The maximum allowable headway for region k and period i is:

$$h_{c\,max}^{ki} = \frac{S_c l_c}{rL^k f Q^{ki}} \qquad (4.2)$$

The minimum cost headway can be obtained from the partial derivative of equation (4.1) with respect to headway;

$$h_{c\,min}^{ki} = \sqrt{\frac{2D^k(a+bS_c)}{v_w L^k r^k Q^{ki} V_c^i}} \qquad (4.3)$$

The optimal headway is then:

$$h_{c\,opt}^{ki} = \min\left\{\frac{S_c l_c}{r^k L^k f Q^{ki}}, \sqrt{\frac{2D^k(a+bS_c)}{v_w L^k r^k Q^{ki} V_c^i}}\right\} \qquad (4.4)$$

The optimized headway obtained in equation (4.4) applies for optimizing the conventional bus fleet size for region k and period i ($F_c^{ki} = \frac{D^k W^k}{r^k h_{c\,opt}^{ki} V_c^i}$). However, the resulting fleet size must be rounded off to an integer value. The modified headway h_c^{ki*} can be obtained with an integer value of fleet size ($h_c^{ki*} = \frac{D^k W^k}{r^k F_{c\,opt}^{ki} V_c^i}$).

The service cost for region k and in period i, is finally formulated by substituting the modified headway into equation (4.1).

$$SC_c^{ki*} = \frac{D^k W^k}{r h_c^{ki*} V_c^i}(a + bS_c) + v_v L^k W^k Q^{ki} \frac{M^k}{V_c^i} + v_w L^k W^k Q^{ki} \frac{h_c^{ki*}}{2} + \frac{v_x L^k W^k Q^{ki}(r+d)}{4V_x}$$

(4.5)

4.3.2. Flexible Bus Formulation and Optimal Headway

Similarly, flexible bus cost consists of bus operating cost, user in-vehicle cost and user waiting cost. Service cost for region k, in period i, SC_f^{ki}, is formulated as follows:

$$SC_f^{ki} = \frac{L^k W^k \left(a + bS_f\right)(D_f^k + \emptyset A^k \sqrt{\frac{Q^{ki} h_f^{ki}}{u}})}{A^k V_f^i h_f^{ki}} + \frac{v_v L^k W^k Q^{ki} (D_f^k + \emptyset A^k \sqrt{\frac{Q^{ki} h_f^{ki}}{u}})}{2V_f^i} + \frac{v_w L^k W^k Q^{ki} h_f^{ki}}{2}$$

(4.6)

Since multiple periods are considered, the optimized headway should be the maximum allowable headway or the minimum cost headway, whichever is smaller. The maximum allowable headway for region k and period i is:

$$h_{f\,max}^{ki} = \frac{S_f l_f}{A^k Q^{ki}} \qquad (4.7)$$

The minimum cost headway can be obtained from the partial derivative equation (4.6) with respect to the headway h_f^{ki}. An analytically optimized solution with respect to headway for a one region bus service is provided in the Chapter 3. However, since the partial derivation of equation (4.6) is difficult to solve analytically, we find it numerically using existing computing software (i.e., MATLAB). The minimum cost headway $h_{f\,min}^{ki}$ can easily be obtained using a function called *fminbnd* in MATLAB (version R2011b).

Thus, the optimized headway for flexible bus is:

$$h_{f\,opt}^{ki} = min\left\{\frac{S_f l_f}{A^k Q^{ki}}, h_{f\,min}^{ki}\right\} \tag{4.8}$$

The optimized fleet size for flexible bus is:

$$F_{f\,opt}^{ki} = \frac{L^k W^k (D_f^k + \emptyset A^k \sqrt{Q^{ki} h_{f\,opt}^{ki}/u})}{V_s A^k h_{f\,opt}^{ki}} \tag{4.9}$$

Equation (4.9), similarly to conventional bus fleet size ($F_{c\,opt}^{ki} = \frac{D^k W^k}{r^k h_{c\,opt}^{ki} V_c^i}$), must yield an integer value. The number of zones for flexible bus, $\frac{L^k W^k}{A^k}$, must have an integer value. Therefore, the remaining part of equation (4.9), $\frac{(D_f^k + \emptyset A^k \sqrt{Q^{ki} h_{f\,opt}^{ki}/u})}{V_s h_{f\,opt}^{ki}}$, should have an integer value. Since this part of equation is a function of headway, we round off fleet size to an integer value, and then check if the modified headway violates the maximum allowable headway. The modified headway corresponding to an integer fleet size should not exceed the maximum allowable headway. The modified headway denoted as h_f^{ki*} provides minimum total service cost with an integer fleet size.

Minimum service cost for flexible bus operation with an integer fleet is obtained by substituting the modified headway into equation (4.6):

$$SC_f^{ki*} = \frac{L^k W^k (a + b S_f)(D_f^k + \emptyset A^k \sqrt{Q^{ki} h_f^{ki*}/u})}{V_f^{ki} A^k h_f^{ki*}} + \frac{v_v L^k W^k Q^{ki}}{2} \frac{D_f^k + \emptyset A^k \sqrt{Q^{ki} h_f^{ki*}/u}}{V_f^{ki}} +$$

$$v_w L^k W^k Q^{ki} \frac{h_f^{ki*}}{2} \tag{4.10}$$

4.3.3. Capital Cost

After headways are optimized for each period, they and the round trip times determine fleet size. Thus, with optimized bus sizes, we have the required fleet matrix for each region and period. For capital cost, which is our fixed cost component, the required fleet size is the largest of the fleet sizes that are needed to serve any local region in any period (i.e. largest value among $\sum_{k=1}^{K} F^{k1}$, $\sum_{k=1}^{K} F^{k2}$, …, $\sum_{k=1}^{K} F^{kI}$). Here, the capital cost units are \$/day.

4.4. Total Cost Formulations

4.4.1. Single Fleet Conventional Bus (SFC)

For SFC, a single conventional bus size covers all regions. Since the number of zones can differ by regions, the number of unknown variables is k+1 (k=the number of regions). This bus size and the number of zones for each region must be optimized. Then, this integer number of zones for each region yields the route spacing in each region.

$$r^k = \frac{W^k}{N^k} \tag{4.11}$$

After vehicle size and route spacings are determined, headways and required fleets are analytically optimized using equations (4.2~4.6).

Total cost for SFC is formulated as

$$TC = f(S_c, N, r, F, h) = (a_c + b_c S_c)F + \sum_k \sum_i SC_c^{ki} t^{ki} \tag{4.12}$$

subject to

$\mathrm{S_c} = \text{integer} \;\forall\; 1, \dots S^{max}$

$N^k = \frac{W^k}{r^k} = \text{integer} \;\forall\; 1, \dots \frac{W^k}{r^{min}}$

$F^{ki} = \text{integer}$

$F^i = \sum_k F^{ki} \;\forall\; \mathrm{k} = 1, \dots, \mathrm{K}$

$\mathrm{F} \geq F^i \;\forall\; \mathrm{i} = 1, \dots, \mathrm{I}$

$0 \leq h^{ki} \leq \frac{S_c l_c}{r^k L^k f Q^{ki}}$

SC_c^{ki} *is provided in equation (5)*

4.4.2. Single Fleet Flexible Bus (SFF)

Similarly to SFC, SFF has the same number of decision variables, flexible bus size and the number of zones for each region. The difference from SFC is that the number of zones can be converted into service area (mi^2 / zone).

$$A^k = \frac{L^k W^k}{N'^k} \tag{4.13}$$

With vehicle size and service area values for regions, headway and fleet size are optimized with equations (4.7~4.10).

Total cost for SFF is formulated as

$$TC = f\left(S_f, N', A, F, h\right) = (a_c + b_c S_f)F + \sum_k \sum_i SC_f^{ki} t^{ki} \qquad (4.14)$$

subject to

$$\mathrm{S_f} = \text{integer} \; \forall \; 1, \dots S^{max}$$

$$N'^k = \frac{L^k W^k}{A^k} = \text{integer} \; \forall \; 1, \dots \frac{L^k W^k}{A^{min}}$$

$$F^{ki} = \text{integer}$$

$$F^i = \sum_k F^{ki} \; \forall \; \mathrm{k} = 1, \dots, \mathrm{K}$$

$$\mathrm{F} \geq F^i \; \forall \; \mathrm{i} = 1, \dots, \mathrm{I}$$

$$0 \leq h^{ki} \leq \frac{S_f l_f}{A^k Q^{ki}}$$

SC_f^{ki} *is provided in equation (10)*

4.4.3. Single Fleet Variable-Type Bus (SFV)

SFV switch the type of operation between conventional and flexible services, depending on the demand variability. The same size of vehicles is assumed to be used. Thus, the number of decision variables is 2k + 1. (k = the number of regions). The total cost formulation is shown as follows.

$$TC = (a_c + b_c S\ \ )F + \sum_k \sum_i \{\alpha^{ki} SC_c^{ki} + (1 - \alpha^{ki}) SC_f^{ki}\} t^{ki} \quad (4.15)$$

subject to

$$S \quad = \text{integer}\ \forall\ 1, \dots S^{max}$$

$$\alpha^{ki} = \begin{cases} 1\ if\ conventional\ service\ is\ chosen \\ 0\ if\ flexible\ service\ is\ chosen \end{cases}$$

$$N^k = \frac{W^k}{r^k} = \text{integer}\ \forall\ 1, \dots \frac{W^k}{r^{min}}$$

$$N'^k = \frac{L^k W^k}{A^k} = \text{integer}\ \forall\ 1, \dots \frac{L^k W^k}{A^{min}}$$

$$F^{ki} = \text{integer}$$

$$F^i = \sum_k F^{ki}\ \forall\ \text{k} = 1, \dots, \text{K}$$

$$\text{F}\ \geq F^i\ \forall\ \text{i} =\ 1, \dots, \text{I}$$

$$0 \leq h^{ki} \leq \frac{S_c l_c}{r^k L^k f Q^{ki}}$$

$$0 \leq h^{ki} \leq \frac{S_f l_f}{A^k Q^{ki}}$$

SC_c^{ki} is provided in equation (5)

SC_f^{ki} is provided in equation (10)

4.5. A Hybrid (Genetic Algorithm-Analytic) Optimization Approach

4.5.1. Number of Integer Variables

The number of integer variables varies based on the types of bus operations. SFC, for instance, has k+1 integer variables which are vehicle size and number of zones for each of k regions. Thus, if 4 regions are considered, we have 5 integer variables. Then, these integer variables should be used to analytically optimize headways and required fleet sizes over time. Similarly to SFC, SFF also has k+1 integer variables. For SFV, the number of integer variables is 2k+1 that is the vehicle size, the number of zones for conventional services, and the number of zones for flexible services.

4.5.2. Solution Approach

The problem formulations are non-linear mixed-integer problems, which are known to be NP-hard. For optimizing our three alternatives, a solution approach which combines analytic optimization with a genetic algorithm is proposed. To find a solution efficiently, the variables are split into two groups. k+1 integer decision variables (i.e., vehicle sizes, the number of zones for conventional bus, and the number of zones for flexible bus) are optimized by the GA, depending on the type of bus operations. Then, analytic optimization determines headways and required fleet sizes based on the values of decision variables provided by the GA. Thus, the GA and analytic optimization work iteratively in this hybrid solution approach. The detailed interactions between GA and analytic optimization are shown in Figure 4-1.

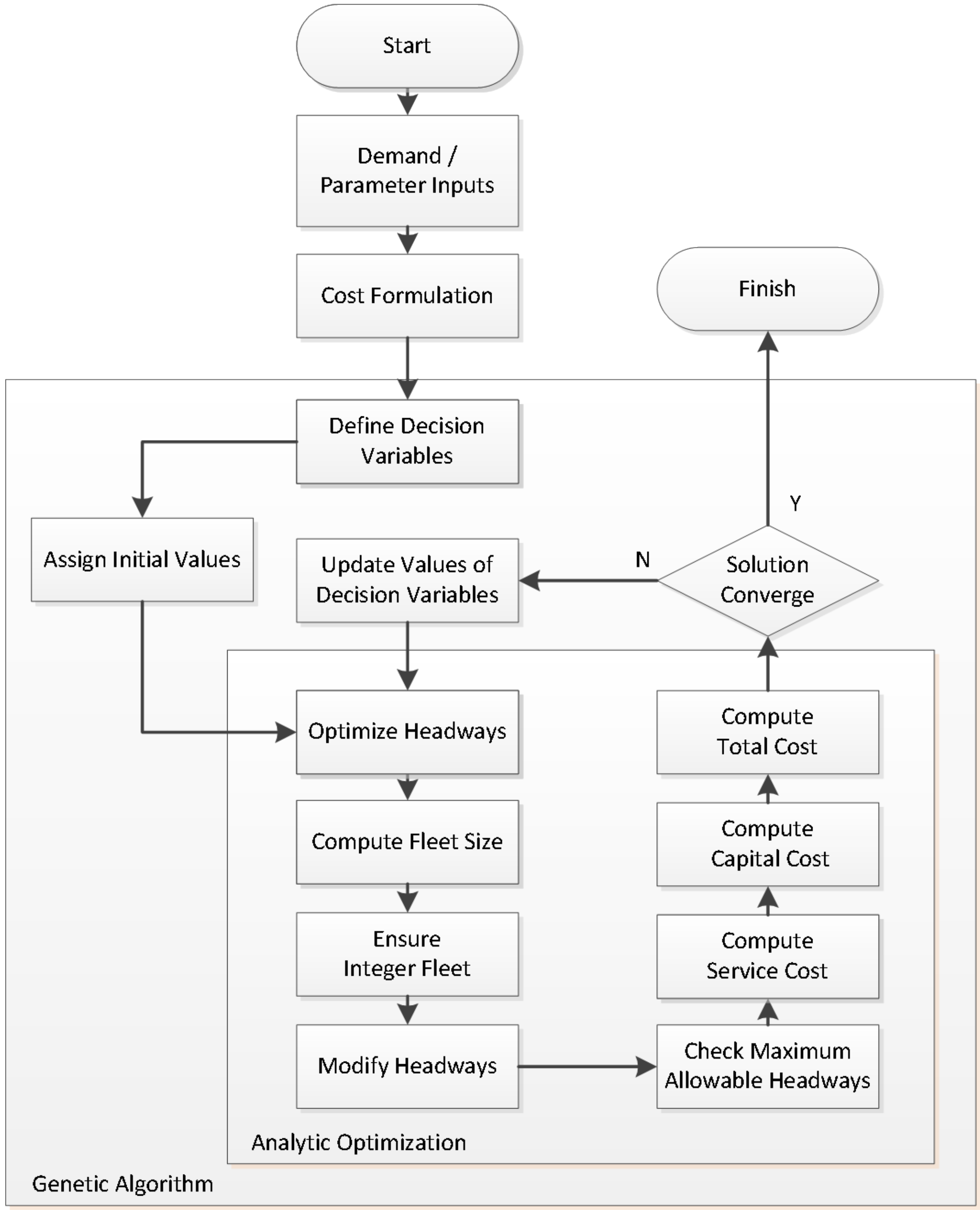


Figure 4-1 Graphical Description of Solution Approach

In this model, the role of GA is to find integer values for decision variables. To provide integer solutions, we use an Integer Genetic Algorithm (IGA), which is described below.

4.5.3. Integer Genetic Algorithm

Genetic Algorithms (GAs) are widely used for optimization problems. The GA concept was introduced by Holland (1975). A detailed implementation of GA may be found in Goldberg (1989). The way the variables are coded affects a GA's efficiency. Real Coded Genetic Algorithms (RCGAs), which use real numbers for encoding, have faster convergence towards optimal than binary and gray coded GAs (Deb, 2001; Deep et al, 2009). The details, such as Laplace crossover, Power mutation, truncation procedure for integer restrictions and constraint handling techniques, can be found in Deep et al (2009). Since this RCGA handles integer variables efficiently, we call this "Integer Genetic Algorithm (IGA), and use it to solve our nonlinear mixed integer formulations.

RCGAs attempt to minimize a penalty function, which includes a penalty term for infeasibility, rather than a normal fitness function. This penalty function is combined with binary tournament selection to select individual solutions for subsequent generations (Deb, 2000). According to Deb (2000), if the solution is feasible, the penalty function is the fitness function; however, if the solution is infeasible, the penalty function is the maximum fitness function among feasible solutions in the population, plus a sum of the constraint violations.

Since the method used here, which combines a GA and analytic optimization, is partially heuristic, it does not guarantee a global optimum. However, this hybrid solution approach can provide a near-optimal solution quickly. The proposed method is evaluated with numerical examples in the following section.

4.6. Numerical Evaluation: Base Case Analysis

4.6.1. Input Values

In the base numerical case, Four distinct local regions, each with four periods (i.e. K = 4 and I = 4) are considered. Demand, service time and line-haul distance are presented in Table 4-2. All other required input parameters are presented in Table 4-1.

Table 4-2 Demand, Service Time, and Line-haul Distance

Demand (trips/mile^2/hour)				
Period \ Region	A	B	C	D
1	70	80	60	55
2	30	35	40	40
3	10	15	30	15
4	5	7.5	10	5
Time(hours)				
Region	A	B	C	D

Period				
1	4	4	4	4
2	6	6	6	6
3	8	8	8	8
4	6	6	6	6
Region	A	B	C	D
Line-haul Distance (miles)	4	5	3	5
Length of Region (miles)	3	2	4	5
Width of Region (miles)	4	5	3	3

4.6.2. Optimization Results

The results of SFC for give base inputs are provided in Table 4-3. The optimized bus size is 30 seats. Route spacings are 0.75 or 1 miles, and required flee sizes vary from 4 buses to 24 buses by time periods. The total operation cost is 145,289.27 $/day, and the capital cost is 9,085 $/day. The total cost of SFC is then 154,374.27 $/day.

Table 4-3 SFC Results for Base Case

	Vehicle Size				Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus				A	B	C	D
	30				1	1	0.75	0.75
	Conventional Bus Headway (hours)				Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.141	0.154	0.153	0.144	18	20	17	24
2	0.169	0.206	0.158	0.153	10	10	11	15
3	0.338	0.294	0.173	0.255	5	7	10	9
4	0.422	0.411	0.347	0.459	4	5	5	5
	Conventional Bus Cost ($/hour)				Operation Cost × Time			

Region / Period	A	B	C	D	A	B	C	D
1	3,581.93	3,645.33	2,903.51	3,775.33	14,327.73	14,581.33	11,614.02	15,101.33
2	1,533.20	1,597.06	1,757.02	2,386.22	9,199.20	9,582.33	10,542.11	14,317.33
3	692.67	861.45	1,414.80	1,154.11	5,541.33	6,891.62	11,318.40	9,232.89
4	430.73	537.58	656.40	548.56	2,584.40	3,225.50	3,938.40	3,291.33
Total Operation Cost ($/day) = 145,289.27			Total Capital Cost ($/day) = 9,085.00			Total Cost ($/day) = 154,374.27		

For the flexible bus services, the optimized bus size is 19 seats, compared to the 30 seats for conventional services. Service areas are optimized to be 2.5 or 3 mile^2. Since the optimized bus size for SFF is smaller than that for SFC, SFF requires more buses. The total cost of SFF is 151,654.96 $/day.

Table 4-4 SFF Results for Base Case

	Vehicle Size				Service Area for Flexible Bus			
	Single Fleet Flexible Bus				A	B	C	D
	19				3	2.5	3	3
	Flexible Bus Headway (hours)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.090	0.094	0.098	0.115	38	37	32	41
2	0.139	0.156	0.119	0.129	16	15	18	25
3	0.295	0.240	0.138	0.228	7	9	15	13
4	0.379	0.421	0.266	0.459	5	5	7	6
	Flexible Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3,536.44	3,449.17	2,920.60	3,889.67	14,145.75	13,796.68	11,682.39	15,558.68

2	1,343.78	1,347.03	1,592.10	2,280.22	8,062.69	8,082.15	9,552.63	13,681.30
3	603.98	721.93	1,268.52	1,080.88	4,831.87	5,775.41	10,148.17	8,647.04
4	376.32	457.32	567.73	512.66	2,257.95	2,743.90	3,406.41	3,075.93
Total Operation Cost ($/day) =135,448.96			Total Capital Cost ($/day) =16,206.00			Total Cost ($/day) =151,654.96		

For the SFV, the optimized bus size is 25 seats. Period 1 is service by conventional service, and other periods are served by flexible type services as shown in Table 4-5. The total cost of SFV is 145,229.81 $/day, which is lower than either SFC of SFF.

Table 4-5 SFV Results for Base Case

	Vehicle Size	Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Single Fleet Variable-Type Bus	A	B	C	D	A	B	C	D
	25	0.8	1	0.75	0.6	6	5	4	5

	Conventional Bus Headway (hours)				Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.144	0.154	0.137	0.148	-	-	-	-
2	-	-	-	-	0.097	0.106	0.099	0.097
3	-	-	-	-	0.183	0.168	0.114	0.175
4	-	-	-	-	0.306	0.251	0.218	0.392
	Conventional Bus Fleet Assignment (buses)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	22	20	19	29	0	0	0	0
2	0	0	0	0	15	14	18	24
3	0	0	0	0	7	8	15	12

4	0	0	0	0	4	5	7	5
	Variable-Type Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3,518.45	3,625.33	2,886.96	3,802.33	14,073.82	14,501.33	11,547.83	15,209.32
2	1,393.14	1,365.74	1,628.51	2,346.75	8,358.86	8,194.47	9,771.06	14,080.49
3	569.14	684.24	1,285.39	1,050.21	4,553.13	5,473.91	10,283.11	8,401.66
4	341.80	405.88	554.03	474.26	2,050.82	2,435.28	3,324.15	2,845.56
Total Operation Cost ($/day) =135,104.81			Total Capital Cost ($/day) =10,125.00			Total Cost ($/day) =145,229.81		

Figure 4-2 compares the total cost of SFC, SFF, and SFV. It is notable that SFV has a total cost that is 6.30 % below SFC and 4.42% below SFF, as shown in Figure 4-3.

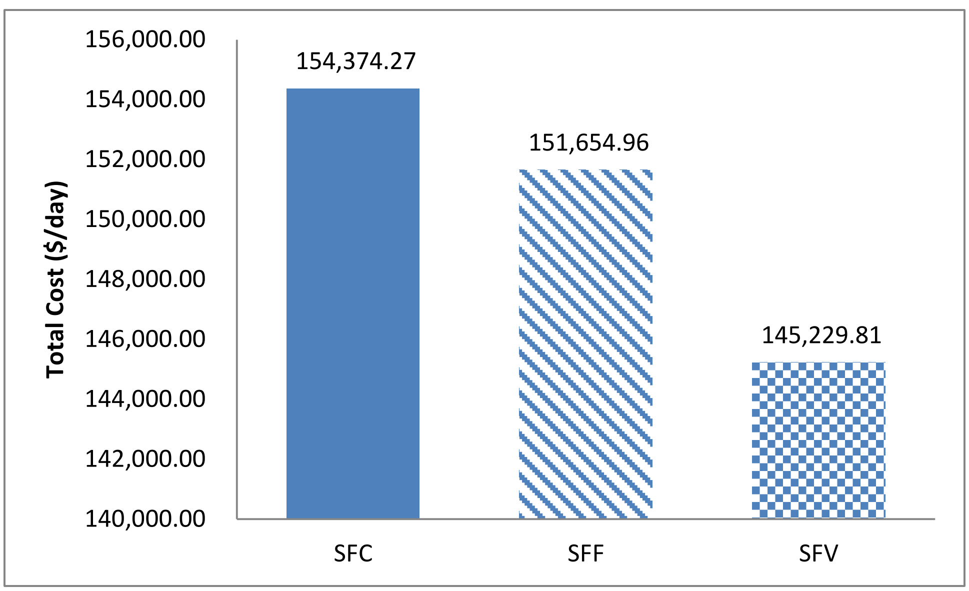


Figure 4-2 Total Costs of Base Case Study

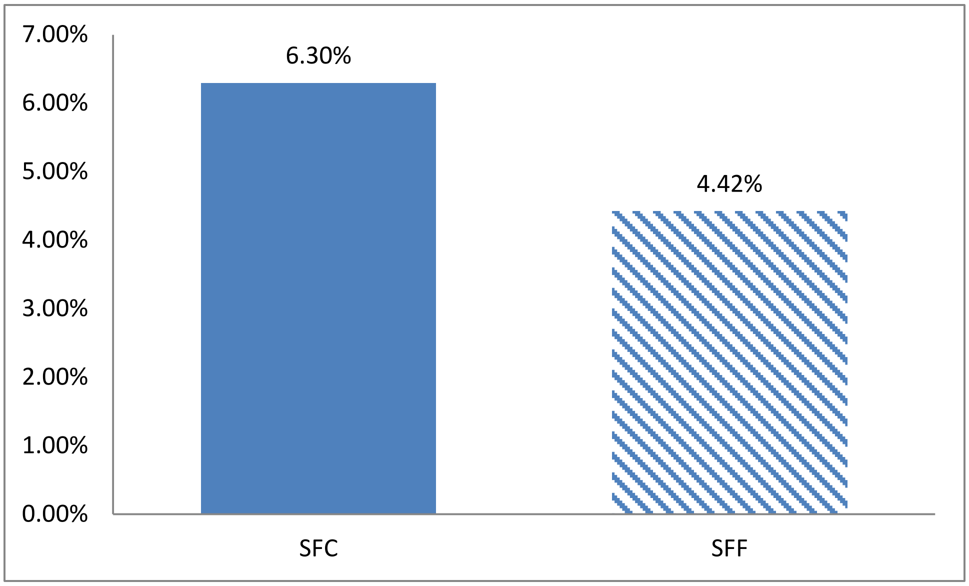


Figure 4-3 SFV Cost Savings for Base Case Study

4.7. Numerical Evaluation: Sensitivity Analysis

In this section, sensitivity analyses of demand and time variations, and capital cost with respect to the SFC, SFF, and SFV are explored.

4.7.1. Case I: Higher Demand Densities

Sensitivity Case I considers very high demand, which is 50 times higher than the baseline values. The demand inputs are shown in Table 4-6.

Table 4-6 Input Values for Sensitivity Case I

Demand (trips/mile^2/hour)				
Region / Period	A	B	C	D

1	3500	4000	3000	2750
2	1500	1750	2000	2000
3	500	750	1500	750
4	250	375	500	250

Figure 4-4 shows the total costs of SFC, SFF, and SFV for the first sensitivity case analysis. The total cost for SFC is 4,223,486.04 $/day. The total cost of SFF, which is 4,370,478.77 $/day is higher than the one of SFC.

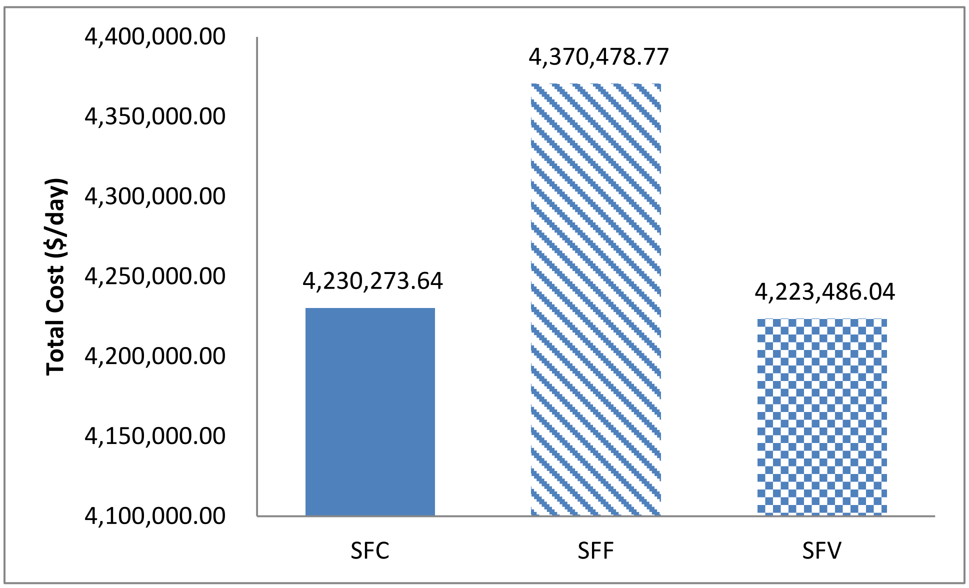


Figure 4-4 Total Costs of Sensitivity Case I

Figure 4-5 shows that the total cost of SFV is 0.16 % lower and 3.48% lower than SFC and SFF, respectively. It is also found that as demand densities are higher, the total costs of SFC and SFV converge.

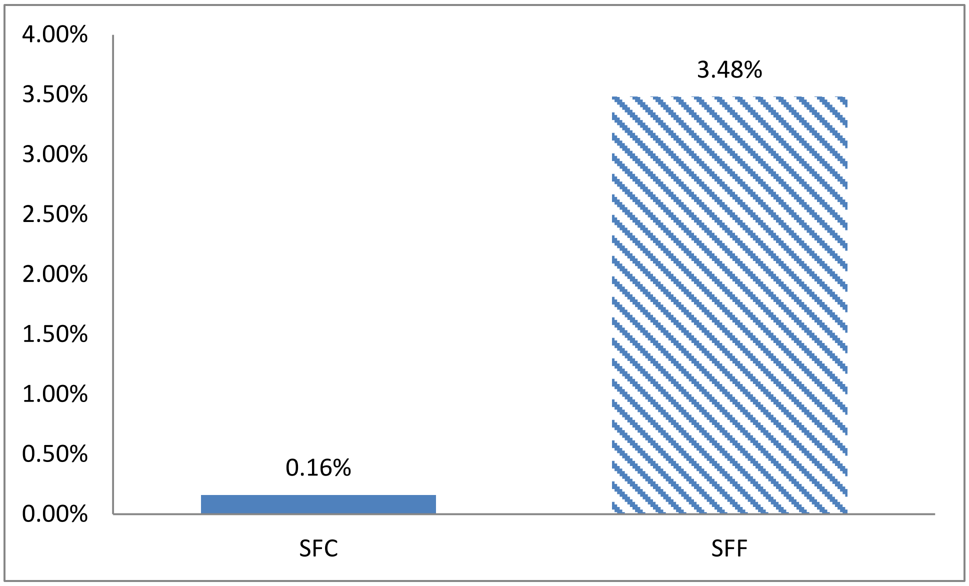


Figure 4-5 SFV Cost Savings for Sensitivity Case I

Details on results of SFC, SFF, and SFV are provided in Tables 4-7~ 4-9. For SFC (as shown in the results of Table 4-7), the optimized vehicle size is 50 seats/bus, which is the upper bound of the vehicle size variable. The route spacings for regions are all 0.5 miles, which are also lower boundary of the route spacing variable. These results confirm that as demand density increases, conventional services should use larger vehicles and smaller route spacings.

Table 4-7 SFC Results for Sensitivity Case I

	Vehicle Size				Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus				A	B	C	D
	50				0.5	0.5	0.5	0.5
	Conventional Bus Headway (hours)				Conventional Bus Fleet Assignment (buses)			
Region	A	B	C	D	A	B	C	D

Period								
1	0.010	0.012	0.008	0.007	532	494	468	711
2	0.022	0.029	0.013	0.010	152	144	208	345
3	0.061	0.060	0.017	0.026	55	68	156	130
4	0.089	0.086	0.050	0.078	38	48	52	44
	Conventional Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	109,710.00	108,022.62	91,260.00	127,909.36	438,840.00	432,090.47	365,040.00	511,637.43
2	38,100.00	38,527.13	48,280.00	71,352.66	228,600.00	231,162.78	289,680.00	428,115.94
3	14,284.24	18,198.92	36,660.00	27,896.79	114,273.94	145,591.37	293,280.00	223,174.36
4	8,056.67	10,226.25	13,420.00	10,490.81	48,340.00	61,357.50	80,520.00	62,944.85
Total Operation Cost ($/day) =3,954,648.64			Total Capital Cost ($/day) =275,625.00			Total Cost ($/day) =4,230,273.64		

SFF results are shown in Table 4-8 as follows. The vehicle size is optimized with the 40 seats/bus, which is lower than the optimized vehicle size of SFC. The service area is optimized with one mile^2, which is the upper boundary of the service area variable. As shown in Figure 4-4, SFF has a very high cost compared to SFC and SFV. This indicates that when the demand is high, SFF is not promising.

Table 4-8 SFF Results for Sensitivity Case I

	Vehicle Size				Service Area for Flexible Bus			
	Single Fleet Flexible Bus				A	B	C	D
	40				1	1	1	1
	Flexible Bus Headway (hours)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D

1	0.011	0.010	0.013	0.015	825	842	671	897
2	0.024	0.022	0.018	0.020	278	271	348	470
3	0.053	0.042	0.022	0.043	117	136	271	207
4	0.083	0.068	0.050	0.090	71	81	113	93
	Flexible Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	116,703.74	118,475.47	93,530.22	127,255.92	466,814.95	473,901.89	374,120.87	509,023.67
2	37,915.92	38,956.91	46,273.84	68,348.76	227,495.52	233,741.45	277,643.06	410,092.59
3	14,091.48	17,892.43	35,320.02	27,667.18	112,731.82	143,139.42	282,560.14	221,337.46
4	7,877.74	9,790.55	13,135.35	10,809.02	47,266.43	58,743.31	78,812.09	64,854.11
Total Operation Cost ($/day) =3,982,278.77			Total Capital Cost ($/day) =388,200.00			Total Cost ($/day) =4,370,478.77		

It is expected that when the demand varies over time and also over regions, SFV may have lower cost compared to cost of SFC or SFF. When the demand is too high, SFV has higher cost compared to SFC.

Table 4-9 SFV Results for Sensitivity Case I

	Vehicle Size	Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Single Fleet Variable-Type Bus	A	B	C	D	A	B	C	D
	50	0.5	0.5	0.5	0.5	-	-	1	-

	Conventional Bus Headway (hours)				Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.010	0.012	0.008	0.007	-	-	-	-
2	0.022	0.029	0.013	0.010	-	-	-	-
3	0.061	0.060	-	0.026	-	-	0.023	-
4	0.089	0.086	-	0.078	-	-	0.052	-

	Conventional Bus Fleet Assignment (buses)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	532	494	468	711	0	0	0	0
2	152	144	208	345	0	0	0	0
3	55	68	0	130	0	0	264	0
4	38	48	0	44	0	0	110	0
	Variable-Type Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	109,710.00	108,022.62	91,260.00	127,909.36	438,840.00	432,090.47	365,040.00	511,637.43
2	38,100.00	38,527.13	48,280.00	71,352.66	228,600.00	231,162.78	289,680.00	428,115.94
3	14,284.24	18,198.92	35,856.73	27,896.79	114,273.94	145,591.37	286,853.81	223,174.36
4	8,056.67	10,226.25	13,359.77	10,490.81	48,340.00	61,357.50	80,158.59	62,944.85
Total Operation Cost ($/day) =3,947,861.04			Total Capital Cost ($/day) =275,625.00			Total Cost ($/day) =4,223,486.04		

4.7.2. Case II: Lower Demand Densities

In this second case, low demand densities, which are 1/10 of base case demands, are considered. The input values for demand densities are provided in Table 4-10.

Table 4-10 Input Values for Sensitivity Case II

Demand (trips/mile^2/hour)				
Region / Period	A	B	C	D
1	7	8	6	5.5
2	3	3.5	4	4
3	1	1.5	3	1.5
4	0.5	0.75	1	0.5

The total costs of SFC, SFF, and SFV with lower demand densities are provided in Figure 4-6. It is found (in Figure 4-6 and Figure 4-7) that SFV approaches SFF when demand densities are very low. The cost of SFC is almost 12% above the cost of SFF and SFV. This case study shows that when demand densities are low and steady, flexible services are preferable to conventional services, and variable-type service is not economical.

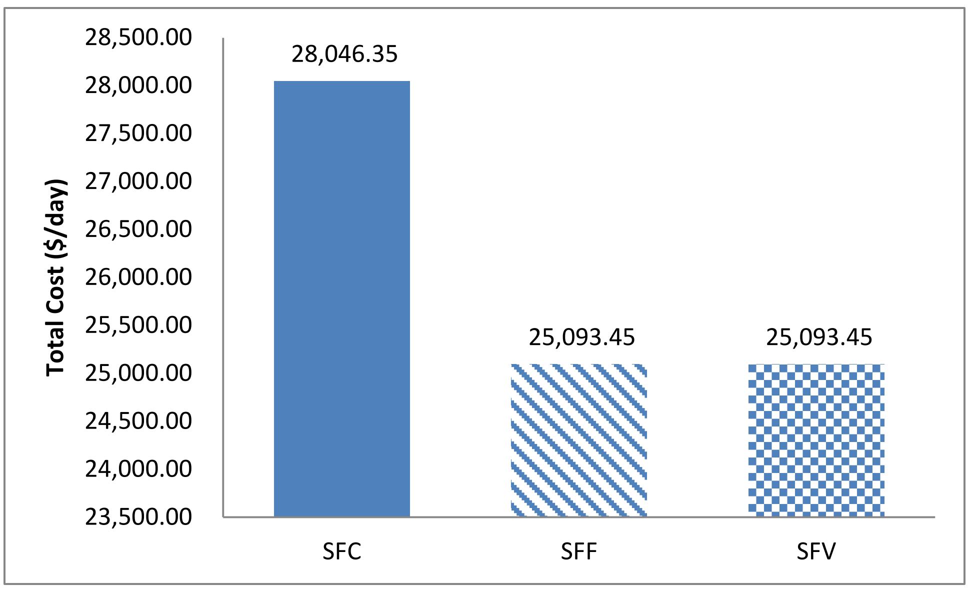


Figure 4-6 Total Costs of Sensitivity Case II

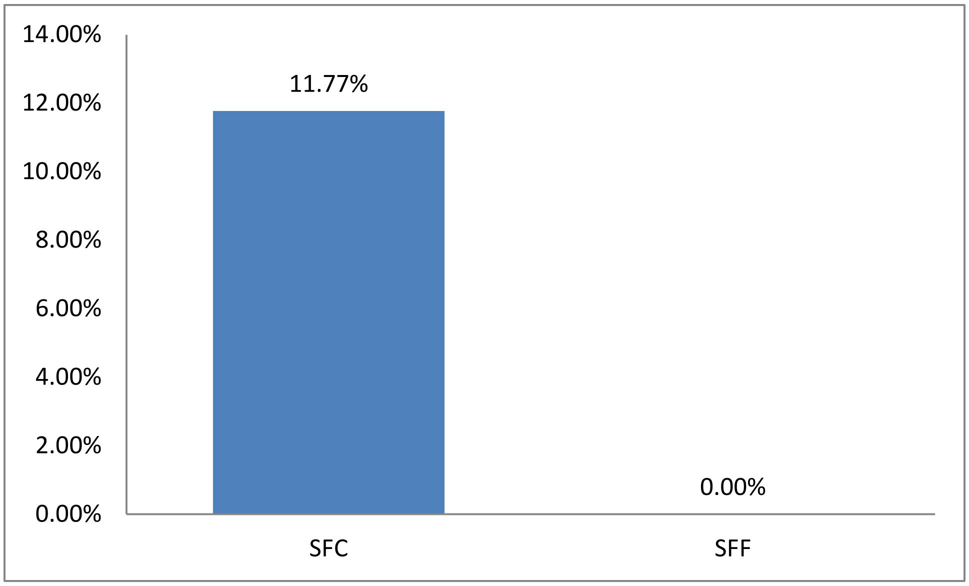


Figure 4-7 SFV Cost Savings for Sensitivity Case II

Results of SFC and SFF with a low demand input case are shown in Table 4-11 and Table 4-12, respectively. The optimized vehicle size of SFC is 15 seats/bus, and the optimized vehicle size of SFF is 14 seats/bus. As shown in Tables 4-11 and 4-12, headways of SFC exceed the headways of SFF. Due to the low demand density, SFC operates with low service frequencies.

Table 4-11 SFC Results for Sensitivity Case II

	Vehicle Size				Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus				A	B	C	D
	15				2	2.5	1.5	1.5
	Conventional Bus Headway (hours)				Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.317	0.308	0.325	0.344	4	4	4	5

2	0.422	0.411	0.433	0.383	2	2	2	3
3	0.844	0.822	0.433	0.574	1	1	2	2
4	0.844	0.822	0.867	1.148	1	1	1	1
	Conventional Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	614.86	642.53	500.28	629.84	2,459.44	2,570.13	2,001.12	2,519.37
2	281.24	295.87	324.72	420.29	1,687.44	1,775.23	1,948.32	2,521.73
3	135.15	168.52	260.04	212.32	1,081.17	1,348.13	2,080.32	1,698.53
4	84.07	100.76	128.88	107.61	504.44	604.55	773.28	645.63
Total Operation Cost ($/day) =26,218.85			Total Capital Cost ($/day) =1,827.50			Total Cost ($/day) =28,046.35		

SFF requires more capital cost but less operating cost than SFC. Thus, SFF provides lower cost solutions with a low demand density.

Table 4-12 SFF Results for Sensitivity Case II

	Vehicle Size				Service Area for Flexible Bus			
	Single Fleet Flexible Bus				A	B	C	D
	14				12	10	12	15
	Flexible Bus Headway (hours)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.138	0.162	0.182	0.167	8	7	6	8
2	0.244	0.242	0.257	0.178	3	3	3	5
3	0.731	0.800	0.225	0.434	1	1	3	2
4	0.569	0.628	0.674	0.744	1	1	1	1
	Flexible Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D

1	564.00	534.99	472.16	621.56	2,256.01	2,139.94	1,888.64	2,486.25
2	216.78	212.84	264.91	362.05	1,300.70	1,277.07	1,589.44	2,172.32
3	107.34	134.80	207.52	173.05	858.70	1,078.40	1,660.13	1,384.42
4	61.80	72.85	101.50	80.25	370.82	437.10	609.02	481.47
Total Operation Cost ($/day) =21,990.45			Total Capital Cost ($/day) =3,103.00			Total Cost ($/day) =25,093.45		

Table 4-13 shows that SFV results are identical to those of SFF. This confirms that SFV can converge to either pure SFC or SFF, depending on the demand variability.

Table 4-13 SFV Results for Sensitivity Case II

	Vehicle Size	Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Single Fleet Variable-Type Bus	A	B	C	D	A	B	C	D
	14	0.5	1	0.6	1.5	12	10	12	15

	Conventional Bus Headway (hours)				Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	-	-	-	-	0.138	0.162	0.182	0.167
2	-	-	-	-	0.244	0.242	0.257	0.178
3	-	-	-	-	0.731	0.800	0.225	0.434
4	-	-	-	-	0.569	0.628	0.674	0.744
	Conventional Bus Fleet Assignment (buses)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0	0	0	0	8	7	6	8
2	0	0	0	0	3	3	3	5
3	0	0	0	0	1	1	3	2
4	0	0	0	0	1	1	1	1
	Variable-Type Bus Service Cost ($/hour)				Operation Cost × Time			

Region / Period	A	B	C	D	A	B	C	D
1	564.00	534.99	472.16	621.56	2,256.01	2,139.94	1,888.64	2,486.25
2	216.78	212.84	264.91	362.05	1,300.70	1,277.07	1,589.44	2,172.32
3	107.34	134.80	207.52	173.05	858.70	1,078.40	1,660.13	1,384.42
4	61.80	72.85	101.50	80.25	370.82	437.10	609.02	481.47
Total Operation Cost ($/day) =21,990.45			Total Capital Cost ($/day) =3,103.00			Total Cost ($/day) =25,093.45		

4.7.3. Case III: Demand and Time Variation

This case considers lower demand densities and longer durations for low demand densities. The demand densities are 1/10 of base case demand inputs. Time periods are changed from 4, 6, 8, and 6 hours to 2, 4, 4, 14 hours, as shown in Table 4-14.

Table 4-14 Demand and Service Time for Sensitivity Case III

Demand (trips/mile^2/hour)				
Region / Period	A	B	C	D
1	7	8	6	5.5
2	3	3.5	4	4
3	1	1.5	3	1.5
4	0.5	0.75	1	0.5
Time(hours)				
Region / Period	A	B	C	D
1	2	2	2	2
2	4	4	4	4
3	4	4	4	4
4	14	14	14	14

The results in Figures 4-8 and 4-9 show that when demand densities are low and time periods are longer for low demands, SFF becomes preferable to SFC and the costs of SFF and SFV are the same. The cost of SFC is about 12% higher than SFF or SFV, as shown in Figure 4-9.

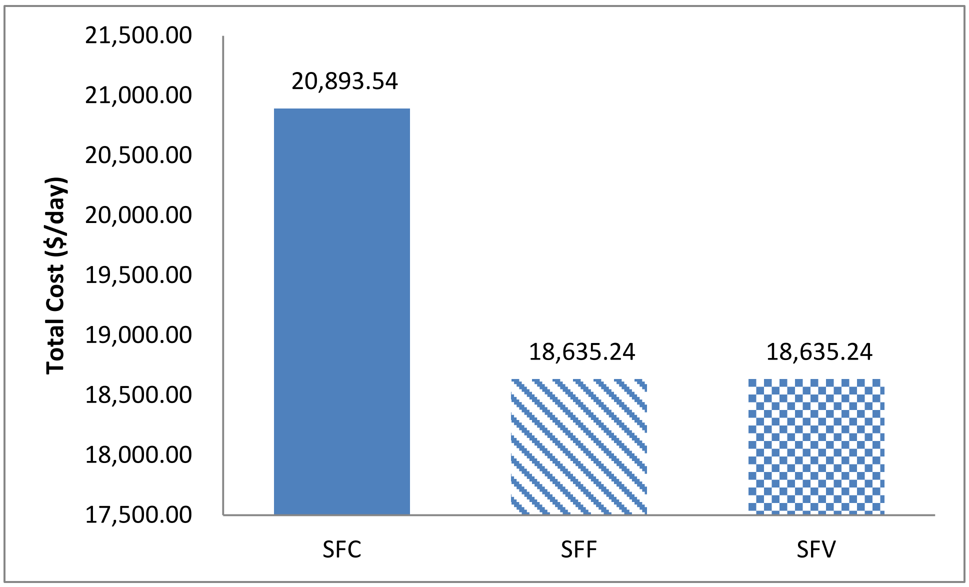


Figure 4-8 Total Costs of Sensitivity Case III

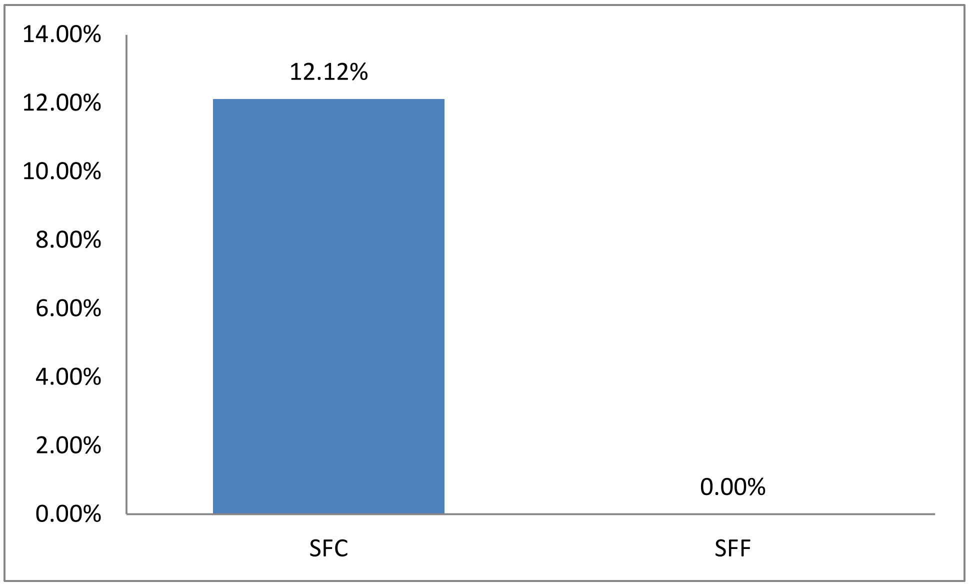


Figure 4-9 SFV Cost Savings for Sensitivity Case III

The results of SFC, SFF, and SFV are provided in Tables 4-15~4-17. Results of Case III are similar to those of Case II. SFV is expected to be the most cost effective operation when the demand varies over time and over regions. However, the demand variability is not significant, so that SFV reduces to SFF. The Base Case is a good example how SFV reduces the total cost when the demand varies over time as well as over regions.

Table 4-15 SFC Results for Sensitivity Case III

	Vehicle Size				Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus				A	B	C	D
	15				2	2.5	1.5	1.5
	Conventional Bus Headway (hours)				Conventional Bus Fleet Assignment (buses)			
Region	A	B	C	D	A	B	C	D

Period								
1	0.317	0.308	0.325	0.344	4	4	4	5
2	0.422	0.411	0.433	0.383	2	2	2	3
3	0.844	0.822	0.433	0.574	1	1	2	2
4	0.844	0.822	0.867	1.148	1	1	1	1
	Conventional Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	614.86	642.53	500.28	629.84	1,229.72	1,285.07	1,000.56	1,259.68
2	281.24	295.87	324.72	420.29	1,124.96	1,183.49	1,298.88	1,681.16
3	135.15	168.52	260.04	212.32	540.59	674.07	1,040.16	849.27
4	84.07	100.76	128.88	107.61	1,177.03	1,410.62	1,804.32	1,506.48
Total Operation Cost ($/day) =19,066.04			Total Capital Cost ($/day) =1,827.50			Total Cost ($/day) =20,893.54		

Table 4-16 SFF Results for Sensitivity Case III

	Vehicle Size				Service Area for Flexible Bus			
	Single Fleet Flexible Bus				A	B	C	D
	14				12	10	12	15
	Flexible Bus Headway (hours)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.138	0.162	0.182	0.167	8	7	6	8
2	0.244	0.242	0.257	0.178	3	3	3	5
3	0.731	0.800	0.225	0.434	1	1	3	2
4	0.569	0.628	0.674	0.744	1	1	1	1
	Flexible Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	564.00	534.99	472.16	621.56	1,128.01	1,069.97	944.32	1,243.13
2	216.78	212.84	264.91	362.05	867.13	851.38	1,059.63	1,448.21
3	107.34	134.80	207.52	173.05	429.35	539.20	830.07	692.21
4	61.80	72.85	101.50	80.25	865.26	1,019.91	1,421.04	1,123.43

Total Operation Cost ($/day) =15,532.24	Total Capital Cost ($/day) =3,103.00	Total Cost ($/day) =18,635.24

Table 4-17 SFV Results for Sensitivity Case III

	Vehicle Size	Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Single Fleet Variable-Type Bus	A	B	C	D	A	B	C	D
	14	1	0.5	0.75	1	12	10	12	15

	Conventional Bus Headway (hours)				Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	-	-	-	-	0.138	0.162	0.182	0.167
2	-	-	-	-	0.244	0.242	0.257	0.178
3	-	-	-	-	0.731	0.800	0.225	0.434
4	-	-	-	-	0.569	0.628	0.674	0.744
	Conventional Bus Fleet Assignment (buses)				**Flexible Bus Fleet Assignment (buses)**			
Region / Period	A	B	C	D	A	B	C	D
1	0	0	0	0	8	7	6	8
2	0	0	0	0	3	3	3	5
3	0	0	0	0	1	1	3	2
4	0	0	0	0	1	1	1	1
	Variable-Type Bus Service Cost ($/hour)				**Operation Cost × Time**			
Region / Period	A	B	C	D	A	B	C	D
1	564.00	534.99	472.16	621.56	1,128.01	1,069.97	944.32	1,243.13
2	216.78	212.84	264.91	362.05	867.13	851.38	1,059.63	1,448.21
3	107.34	134.80	207.52	173.05	429.35	539.20	830.07	692.21
4	61.80	72.85	101.50	80.25	865.26	1,019.91	1,421.04	1,123.43

Total Operation Cost ($/day) =15,532.24	Total Capital Cost ($/day) =3,103.00	Total Cost ($/day) =18,635.24

4.7.4. Case IV: No Capital Cost

When no capital cost is considered, possibly because it is subsidized or already paid, the optimization results tend to have large fleet sizes because there is no extra cost. The optimized bus size for SFC is 23 seats while SFF has 16 seats and SFV has 20 seats. Similarly to the base case study, SFV provides conventional services to the first period and flexible services to the other periods. The total cost of SFC is 144,352.96 $/day and the total cost of SFF is 134,891.93 $/day. The SFV has the lowest total cost, which is 134,310.49 $/day, as shown in Figure 4-10.

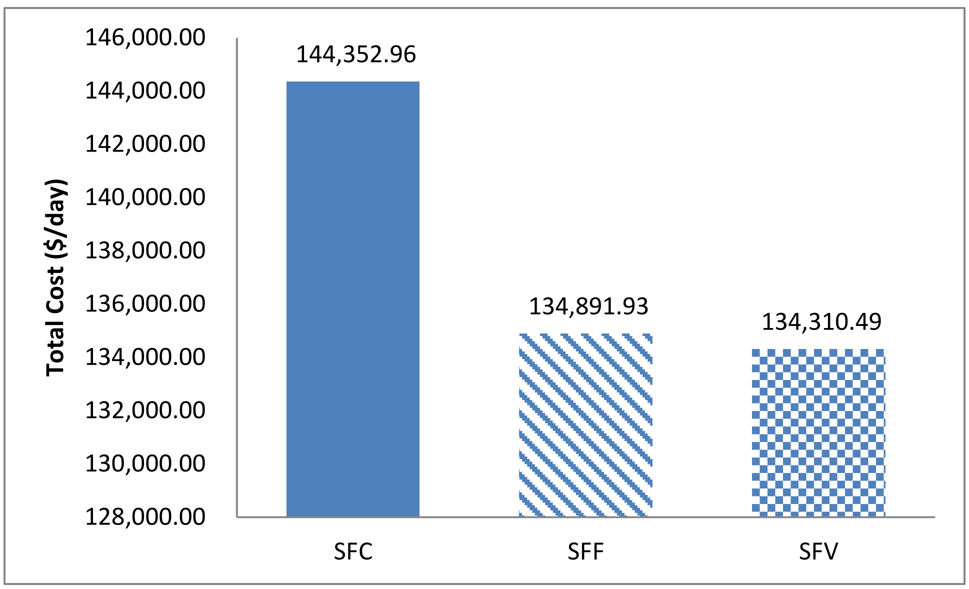


Figure 4-10 Total Costs of Sensitivity Case IV

Figure 4-11 provides total cost variations among SFC, SFF, and SFV. It is noted that the total cost of SFV is close to the total cost of SFF, so that without capital costs, flexible type services favor conventional bus services.

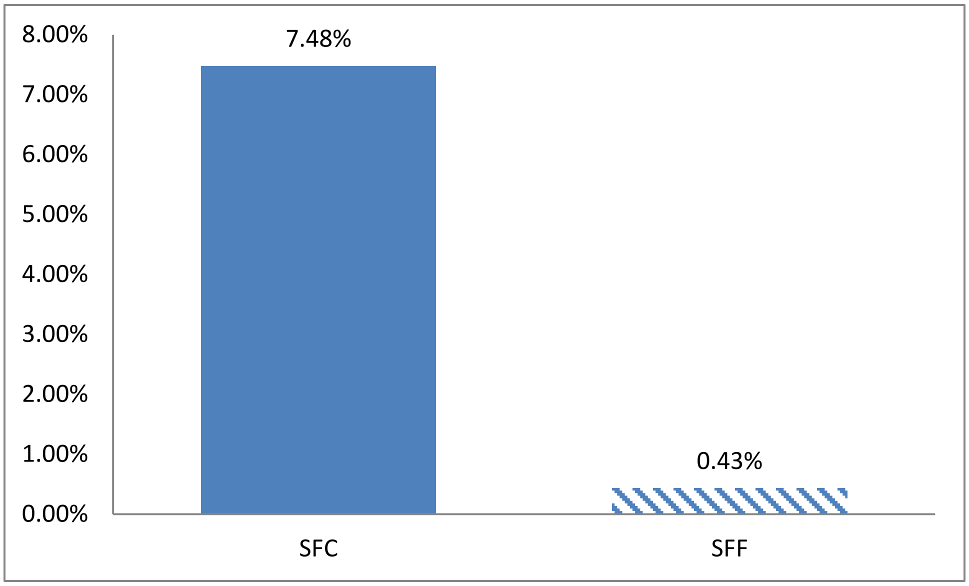


Figure 4-11 SFV Cost Savings for Sensitivity Case IV

When no capital cost is considered, the optimized vehicle sizes decrease. The optimized vehicle sizes are 23, 16, and 20 seats/bus for SFC, SFF, and SFV, respectively. However, the optimized vehicle sizes of the Base Case are 30, 19, and 25 seats/bus.

Table 4-18 SFC Results for Sensitivity Case IV

	Vehicle Size				Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus				A	B	C	D
	23				0.8	1	0.75	0.75
	Conventional Bus Headway (hours)				Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.132	0.140	0.124	0.111	24	22	21	31

2	0.192	0.187	0.144	0.153	11	11	12	15
3	0.352	0.294	0.173	0.255	6	7	10	9
4	0.528	0.411	0.289	0.459	4	5	6	5
	Conventional Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3,518.40	3,619.26	2,892.26	3,823.52	14,073.60	14,477.04	11,569.03	15,294.07
2	1,517.15	1,578.41	1,738.40	2,365.22	9,102.87	9,470.48	10,430.40	14,191.33
3	701.60	851.65	1,400.80	1,141.51	5,612.80	6,813.22	11,206.40	9,132.09
4	448.73	530.58	642.40	541.56	2,692.40	3,183.50	3,854.40	3,249.33
Total Operation Cost ($/day) =144,352.96			Total Capital Cost ($/day) =0			Total Cost ($/day) =144,352.96		

Table 4-19 SFF Results for Sensitivity Case IV

	Vehicle Size				Service Area for Flexible Bus			
	Single Fleet Flexible Bus				A	B	C	D
	16				3	3.333	3	3
	Flexible Bus Headway (hours)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.074	0.060	0.087	0.096	44	45	35	47
2	0.139	0.127	0.110	0.129	16	15	19	25
3	0.295	0.224	0.138	0.228	7	8	15	13
4	0.379	0.338	0.266	0.459	5	5	7	6
	Flexible Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3,546.16	3,576.37	2,906.89	3,895.61	14,184.62	14,305.48	11,627.55	15,582.45
2	1,334.18	1,320.51	1,578.83	2,265.22	8,005.09	7,923.05	9,472.97	13,591.30
3	599.78	690.51	1,259.52	1,073.08	4,798.27	5,524.10	10,076.17	8,584.64
4	373.32	423.46	563.53	509.06	2,239.95	2,540.74	3,381.21	3,054.33
Total Operation Cost ($/day) =134,891.93			Total Capital Cost ($/day) =0			Total Cost ($/day) =134,891.93		

Table 4-20 SFV Results for Sensitivity Case IV

	Vehicle Size	Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Single Fleet Variable-Type Bus	A	B	C	D	A	B	C	D
	20	0.667	0.833	0.6	0.6	6	5	4	5

	Conventional Bus Headway (hours)				Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.141	0.148	0.135	0.120	-	-	-	-
2	-	-	-	-	0.097	0.106	0.099	0.091
3	-	-	-	-	0.183	0.168	0.114	0.175
4	-	-	-	-	0.306	0.251	0.218	0.311

	Conventional Bus Fleet Assignment (buses)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	27	25	24	36	0	0	0	0
2	0	0	0	0	15	14	18	25
3	0	0	0	0	7	8	15	12
4	0	0	0	0	4	5	7	6

	Variable-Type Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3,515.93	3,585.73	2,902.20	3,868.43	14,063.73	14,342.93	11,608.80	15,473.72
2	1,378.14	1,351.74	1,610.51	2,320.99	8,268.86	8,110.47	9,663.06	13,925.96
3	562.14	676.24	1,270.39	1,038.21	4,497.13	5,409.91	10,163.11	8,305.66
4	337.80	400.88	547.03	460.48	2,026.82	2,405.28	3,282.15	2,762.88

Total Operation Cost ($/day) =134,310.49	Total Capital Cost ($/day) =0	Total Cost ($/day) =134,310.49

4.8. Chapter Summary

As an extension of Chapter 3, this chapter formulates SFC, SFF, and SFV between the main terminal and multiple regions. Since analytic optimization becomes intractable with multiple regions and periods, a hybrid solution approach, which jointly uses analytic optimization and genetic algorithm, is proposed for finding solutions. The base case results and sensitivity analyses show that SFV becomes preferable to purely SFC or SFF when demand densities vary over times and over regions. It is also shown that when demand densities are very high, SFV becomes identical to SFC. Similarly, SFV approaches SFF when demand densities are very low.

Chapter 5 Integrating Bus Services with Mixed Fleets

5.1. Problem Statement

The potential benefits of using variable operation types (or "modes") and mixture of vehicle fleets should theoretically increase when multiple dissimilar regions are considered, due to the increased variability of demand densities. To explore these potential benefits we analyze in this chapter the concept of mixed fleet bus operations in multiple regions. To provide efficient service, an optimization model is developed to optimize bus sizes (i.e. large bus size and small bus size) and decision variables for bus operation characteristics (i.e. route spacing in region for conventional bus, service area in region for flexible bus), headways, and required fleets.

The analyzed bus system provides service from a major terminal (or CBD) to multiple regions. In Figure 5-1, a public bus system serves multiple regions connected to a central terminal. For each region, either conventional bus or flexible bus can be provided. Assumptions for the system in Chapter 4.2 and cost formulations in Chapter 4.3 are used for mixed fleet formulations. Demand thresholds between various bus operations using mixed fleet services are formulated in this chapter.

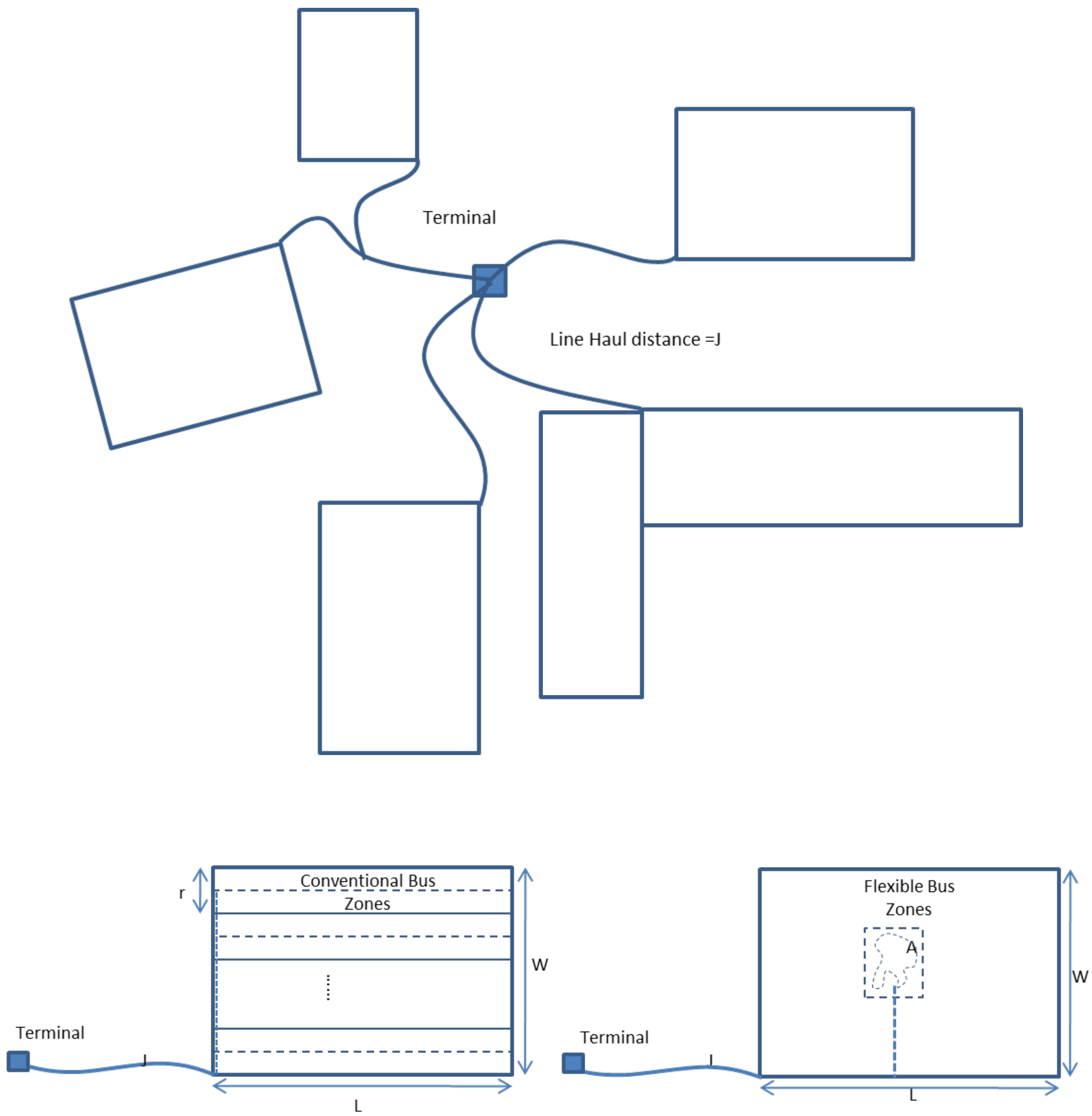


Figure 5-1 Local Regions and Bus Operations

5.2. Various Bus Services with Mixed Fleets

5.2.1. Mixed Fleets Conventional Bus Service (MFC)

MFC operates two sizes of conventional buses. To efficiently allocate demand between large and small buses, our approach identifies the threshold demand at which their costs are equal, using equation (4.1). To find the demand threshold, Q_t^{ki}, between these two, we first substitute the maximum allowable headway in equation (4.2) into conventional bus formulation in equation (4.1) to ensure acceptable headways. Then, equation (4.1) becomes equation (5.1).

$$SC_c^{ki} = \frac{D^k L^k W^k f Q^{ki}(a+bS_c)}{V_c^i S_c l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_c l_c}{2rf} + \frac{v_x L^k W^k Q^{ki}(r^k+d)}{4V_x} \tag{5.1}$$

Large conventional bus cost and small conventional bus cost are formulated in equations (5.2) and (5.3), respectively.

$$SC_l^{ki} = \frac{D^k L^k W^k f Q^{ki}(a+bS_l)}{V_c^i S_l\ l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_l\ l_c}{2rf} + \frac{v_x L^k W^k Q^{ki}(r^k+d)}{4V_x} \tag{5.2}$$

$$SC_s^{ki} = \frac{D^k L^k W^k f Q^{ki}(a+bS_s)}{V_c^i S_s\ l_c} + \frac{v_v L^k W^k Q^{ki} M^k}{V_c^i} + \frac{v_w W^k S_s\ l_c}{2rf} + \frac{v_x L^k W^k Q^{ki}(r^k+d)}{4V_x} \tag{5.3}$$

Then, equations (5.2) and (5.3) are set to be equal, and solved in terms of the threshold demand. The resulting threshold is:

$$Q_t^{ki} = \frac{v_w l_c^2 V_c^i S_l \ S_s}{2ar^k f^2 L^k D^k} \tag{5.4}$$

Thus, large buses are used when demand, Q^{ki}, exceeds the threshold, Q_t^{ki}, and small buses are used otherwise. After bus sizes are selected, the analytically optimized headway can also be found using equations (4.2~4.5). One interesting point from equation (5.4) is that the value of the passengers' in-vehicle time, v_v, does not affect the threshold demand.

5.2.2. Mixed Fleets Flexible Bus Service (MFF)

MFF operates two sizes of flexible buses. To find the demand threshold, Q_t^{ki}, between these two, we first substitute the maximum allowable headway in equation (4.7) into flexible bus formulation in equation (4.6) to ensure acceptable headways, and then set the cost of large and small flexible buses to be equal, using equation (4.6).

$$SC_f^{ki} = \frac{Q^{ki} L^k W^k \left(a + bS_f\right)(D_f^k + \emptyset A^k \sqrt{\frac{S_f l_f}{uA^k}})}{V_f^i S_f l_f} + \frac{v_v L^k W^k Q^{ki} (D_f^k + \emptyset A^k \sqrt{\frac{S_f l_f}{uA^k}})}{2V_f^i} + \frac{v_w L^k W^k S_f l_f}{2A^k} \tag{5.5}$$

Then, large flexible service cost and small flexible service cost are formulated in equations (5.6) and (5.7), respectively.

$$SC_l^{ki} = \frac{Q^{ki}L^kW^k(a+bS_l)(D_f^k+\emptyset\sqrt{\frac{A^kS_ll_f}{u}})}{V_f^iS_ll_f} + \frac{v_vL^kW^kQ^{ki}(D_f^k+\emptyset\sqrt{\frac{A^kS_ll_f}{u}})}{2V_f^i} + \frac{v_wL^kW^kS_ll_f}{2A^k} \tag{5.6}$$

$$SC_s^{ki} = \frac{Q^{ki}L^kW^k(a+bS_s)(D_f^k+\emptyset\sqrt{\frac{A^kS_sl_f}{u}})}{V_f^iS_sl_f} + \frac{v_vL^kW^kQ^{ki}(D_f^k+\emptyset\sqrt{\frac{A^kS_sl_f}{u}})}{2V_f^i} + \frac{v_wL^kW^kS_sl_f}{2A^k} \tag{5.7}$$

Now, equations (5.6) and (5.7) are set to be equal, and find the threshold demand in equation (5.8).

$$Q_t^{ki} = \frac{\frac{v_wl_f}{2A^k}(S_s-S_l)}{\left\{\frac{(a+bS_l)(D_f^k+\emptyset\sqrt{\frac{A^kS_ll_f}{u}})}{V_f^iS_ll_f} - \frac{(a+bS_s)\left(D_f^k+\emptyset\sqrt{\frac{A^kS_sl_f}{u}}\right)}{V_f^iS_sl_f} + \frac{v_v(D_f^k+\emptyset\sqrt{\frac{A^kS_ll_f}{u}})}{2V_f^i} - \frac{v_v\left(D_f^k+\emptyset\sqrt{\frac{A^kS_sl_f}{u}}\right)}{2V_f^i}\right\}} \tag{5.8}$$

For finding MFF headways in each period, Equations (4.7~4.10) are still applicable.

5.2.3. Mixed Fleets Variable Type Bus Service (MFV)

Anticipating that conventional bus has lower average cost than flexible bus at high demand densities, and vice versa, we find the threshold demand for region k in period i, above which conventional bus is preferable and below which flexible bus is preferable. This threshold is obtained in equation (5.9) by setting equations (5.1) and (5.5) to be equal:

$$Q^{ki} = \frac{\frac{v_w}{2}\left(\frac{S_f l_f}{A^k} - \frac{S_c l_c}{r^k f L^k}\right)}{\left\{\frac{D^k f\left(a+bS_c\right)}{V_c^i S_c l_c} + \frac{v_v M^k}{V_c^i} + \frac{v_x Q \quad (r^k+d)}{4V_x} - \frac{\left(a+bS_f\right)\left(D_f^k + \emptyset\sqrt{\frac{S_f l_f A^k}{u}}\right)}{V_f^i S_f l_f} - \frac{v_v\left(D_f^k + \emptyset\sqrt{\frac{S_f l_f A^k}{u}}\right)}{2V_f^i}\right\}} \tag{5.9}$$

5.3. Solution Method

Here, the same solution approach as we used in Chapter 4 is applied. However, the number of integer variables varies based on the mixed fleet operations with two bus sizes. Thus, MFC and MFF have k+2 integer variables. For instance, if 4 local regions are considered, the number of integer variables is 6. MFV requires up to 2k+2 integer variables because two different bus sizes (i.e. large conventional bus size and small flexible bus size) as well as the numbers of zones for both conventional bus and flexible bus are needed. Then, these integer variables are used to analytically optimize headways and required fleet sizes over time.

The detailed interactions between GA and analytic optimization are shown again in Figure 5-2.

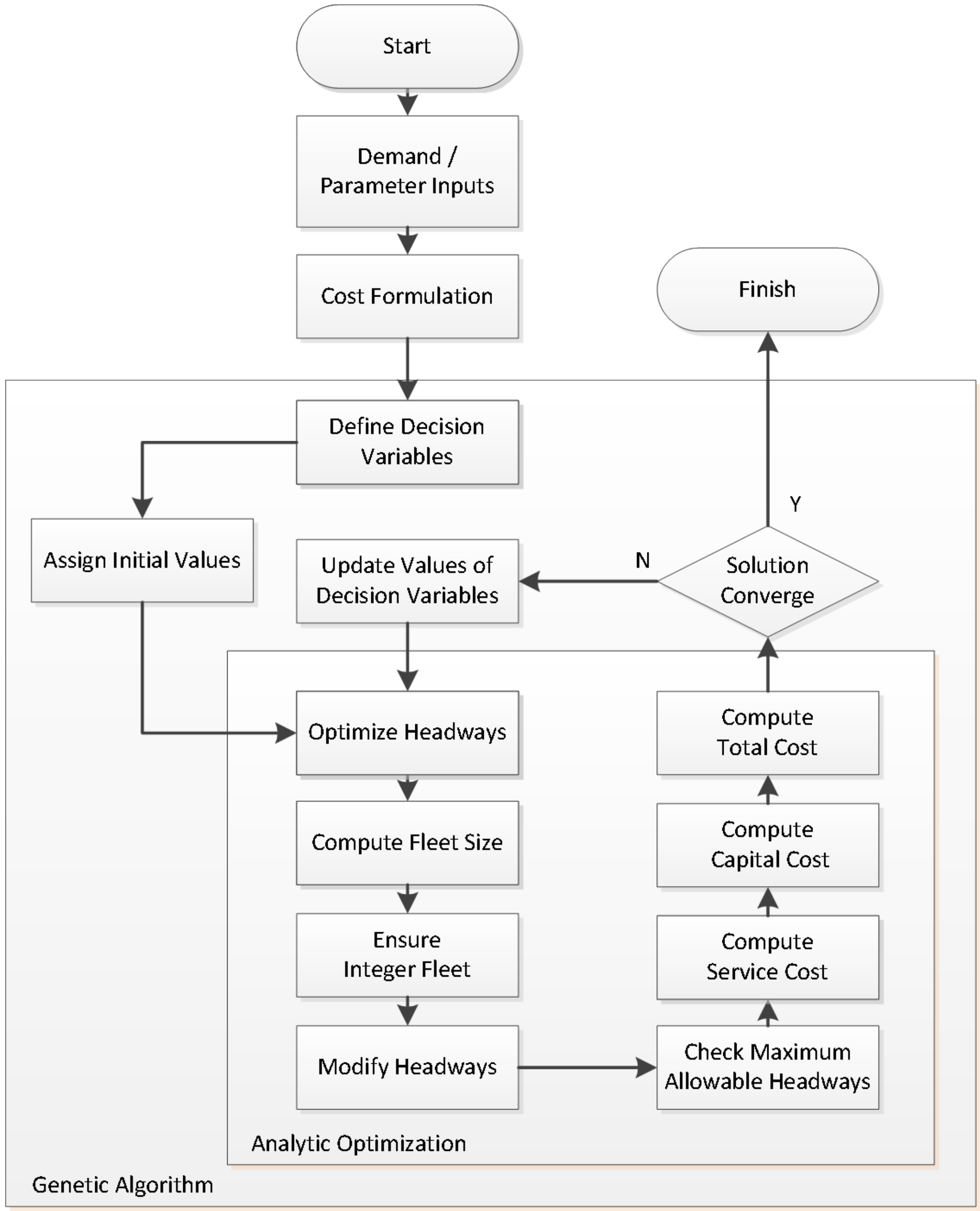


Figure 5-2 Graphical Description of Solution Approach (same as Figure 4-2)

5.4. Numerical Evaluation: Base Case Analysis

To confirm that the proposed method in Chapter 4 minimizes cost efficiently, a set of numerical evaluations is designed and compared to the cost of each bus operation (i.e., MFC, MFF, and MFV).

5.4.1. Input Values

The base numerical case has four distinct local regions, each with four periods (i.e. K = 4 and I = 4). Demand, service time and line-haul distance are presented in Table 5-1. All other required input parameters are presented in Table 4-1.

Table 5-1 Demand, Service Time, and Line-haul Distance

Demand (trips/mile^2/hour)				
Period \ Region	A	B	C	D
1	70	80	60	55
2	30	35	40	40
3	10	15	30	15
4	5	7.5	10	5
Time(hours)				
Period \ Region	A	B	C	D
1	4	4	4	4
2	6	6	6	6
3	8	8	8	8

4	6	6	6	6
Region	A	B	C	D
Line-haul Distance (miles)	4	5	3	5
Length of Region (miles)	3	2	4	5
Width of Region (miles)	4	5	3	3

5.4.2. Optimization Settings

The detailed IGA setting is provided in this section. As mentioned previously, IGA finds a solution using special creation, crossover, and mutation functions to enforce integer values (Deep et al, 2009). To insure integer decision variables, the population type should be “double vector” rather than “bit string” or “custom setting”. For optimizing our five bus operation alternatives, we must set a population size, elite counts, and the number of generations. Here we set a population size of 100, 10 elite counts, and up to 250 generations. To use this IGA algorithm, we must provide bounds for each decision variable. For both conventional and flexible buses we specify a range of 1 to 50 seats/bus. To optimize route spacings for conventional bus, we must first optimize the number of zones in each region. These optimized numbers of zones are convertible into route spacings. The minimum number of zones is set to be one; in this case, one conventional bus serves an entire local region. The minimum specified route spacing (0.5 miles here) determines the maximum number of zones for each region. Bounds are also needed for the service area of flexible bus. A minimum service area (one mile^2 is assumed here for all regions) determines the maximum number of zones for the various regions. The minimum number of zones is one, similarly to conventional bus; in this case, flexible buses serve the entire region (undivided into zones).

5.4.3. Base Case Results

The optimization results (i.e. total cost and solutions of decision variables such as vehicle size(s), and route spacings (or service areas)) are shown in Figure 5-3. Although the MATLAB user interface shown below does not provide the optimized headways and required fleets, the solutions of decision variables shown in Figure 5-3 are found by simultaneously evaluating optimized headways and required fleets. The detailed results are shown in Table 5-2. Figure 5-3 shows SFC results and, in its lower left, we note that SFC has 5 decision variables. The first decision variable corresponds to conventional bus size, and the second to fifth values are the numbers of zones in each region (4, 5, 4, and 4). These numbers of zones are transformable to route spacings, which are 1.0, 1.0, 0.75, and 0.75 miles for regions A, B, C, and D, respectively.

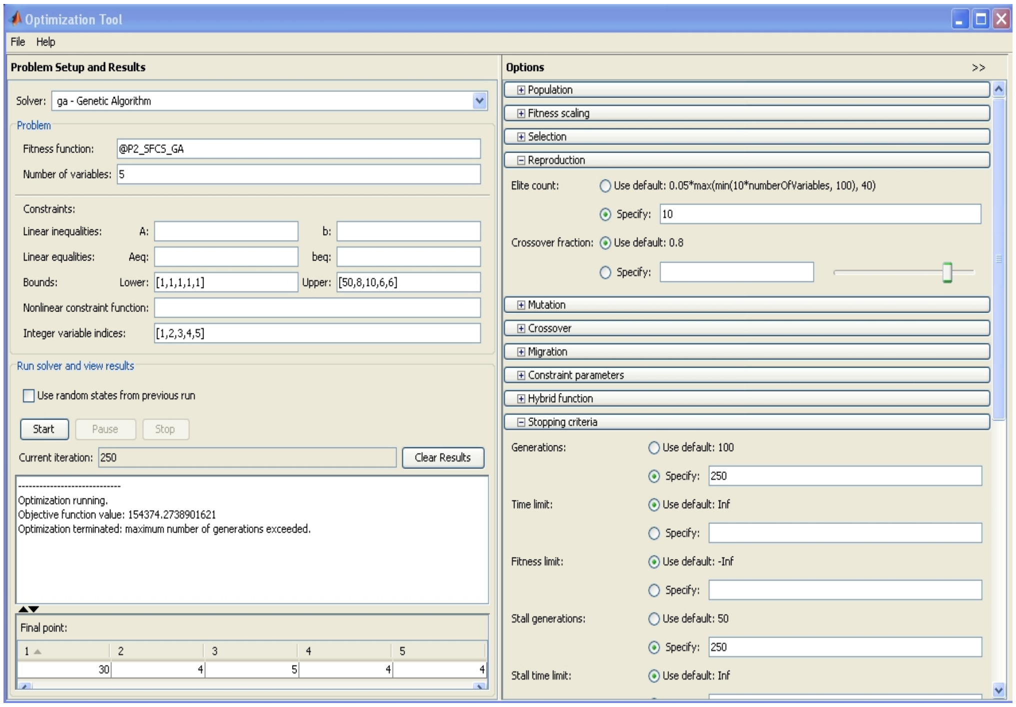


Figure 5-3 SFC Inputs and Results

The detailed results obtained with our hybrid approach, combining IGA and analytic optimization, are shown in Table 5-2, including vehicle sizes, route spacings, optimal headways, required fleets, and corresponding costs. For SFC capital cost is \$9,085/day and operation cost is \$ 145,289.27/day; thus, total cost is \$154,374.27/day.

Table 5-2 SFC Results for Base Case

	Vehicle Size				Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus				A	B	C	D
	30				1.00	1.00	0.75	0.75
	Conventional Bus Headway (hours)				Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.141	0.154	0.153	0.144	18	20	17	24

2	0.169	0.206	0.158	0.153	10	10	11	15
3	0.338	0.294	0.173	0.255	5	7	10	9
4	0.422	0.411	0.347	0.459	4	5	5	5
	Conventional Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3581.93	3645.33	2903.51	3775.33	14327.73	14581.33	11614.02	15101.33
2	1533.20	1597.06	1757.02	2386.22	9199.20	9582.33	10542.11	14317.33
3	692.67	861.45	1414.80	1154.11	5541.33	6891.62	11318.40	9232.89
4	430.73	537.58	656.40	548.56	2584.40	3225.50	3938.40	3291.33
Total Operation Cost ($/day) = 145289.27, Total Capital Cost ($/day) = 9085, Total Cost ($/day) = 154374.27								

For SFF, detailed results are shown in Table 5-3 The optimized flexible bus size is 19 seats/bus, and optimized service areas are 3.0, 2.5, 3.0, and 3.0 mile2/bus for regions A, B, C, and D, respectively. This 19 seat bus serves all regions as well as all time periods. Total cost is $151,654.96/day, which is slightly lower than for SFC, mainly because input parameters for line-haul distance and length of local region are relatively small. This is further explored in the sensitivity analysis section.

Table 5-3 SFF Results for Base Case

	Vehicle Size				Service Area for Flexible Bus			
	Single Fleet Flexible Bus				A	B	C	D
	19				3.00	2.50	3.00	3.00
	Flexible Bus Headway (hours)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D

1	0.090	0.094	0.098	0.115	38	37	32	41
2	0.139	0.156	0.119	0.129	16	15	18	25
3	0.295	0.240	0.138	0.228	7	9	15	13
4	0.379	0.421	0.266	0.459	5	5	7	6
	Flexible Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3536.44	3449.17	2920.60	3889.67	14145.75	13796.68	11682.39	15558.68
2	1343.78	1347.03	1592.10	2280.22	8062.695	8082.155	9552.63	13681.3
3	603.98	721.93	1268.52	1080.88	4831.873	5775.41	10148.17	8647.04
4	376.32	457.32	567.73	512.66	2257.946	2743.901	3406.406	3075.932
Total Operation Cost ($/day) = 135448.96, Total Capital Cost ($/day) = 16206, Total Cost ($/day) = 151654.96								

For mixed fleet conventional bus (MFC), two bus sizes are optimized with 40 and 27 seats/bus. It is noted that MFC's total cost (153,640.08$/day) is below that of SFC (154,374.27$/day, in Table 5-2). This result implies that, given significant demand variations, operating multiple sizes of buses can reduce capital cost and operation cost. With current input parameters, large conventional buses serve only period 1 in region D while all the other periods and regions are served by small conventional buses.

Table 5-4 MFC Results for Base Case

	Vehicle Size				Route Spacing for Conventional Bus			
	Large Conv. Bus		Small Conv. Bus		A	B	C	D
	40		27		1.00	1.00	0.75	1.00
	Large Conventional Bus Headway (hours)				Small Conventional Bus Headway (hours)			
Region	A	B	C	D	A	B	C	D

Period								
1	0.000	0.000	0.000	0.144	0.127	0.154	0.144	0.000
2	0.000	0.000	0.000	0.000	0.169	0.187	0.158	0.132
3	0.000	0.000	0.000	0.000	0.338	0.294	0.173	0.215
4	0.000	0.000	0.000	0.000	0.422	0.411	0.347	0.431
	Large Conventional Bus Fleet Assignment (buses)				Small Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0	0	0	18	20	20	18	0
2	0	0	0	0	10	11	11	13
3	0	0	0	0	5	7	10	8
4	0	0	0	0	4	5	5	4
	Mixed Fleet Conventional Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3571.00	3633.33	2892.00	3842.83	14284.00	14533.33	11568.00	15371.33
2	1527.20	1587.21	1750.42	2412.23	9163.20	9523.28	10502.51	14473.41
3	689.67	857.25	1408.80	1126.99	5517.33	6858.02	11270.40	9015.93
4	428.33	534.58	653.40	519.74	2570.00	3207.50	3920.40	3118.43
Total Operation Cost ($/day) = 144897.08, Total Capital Cost ($/day) = 8743, Total Cost ($/day) = 153640.08								

As noted for Table 5-5, sizes for flexible buses are below those of mixed conventional bus service (in Table 5-4) because flexible services are preferred for lower demand areas. Vehicle sizes are optimized with 22 and 17 seats/bus for larger and smaller flexible bus, respectively. In this MFF operation, large flexible bus is preferable for period 1 in regions A and B. The total cost of MFF's total cost is below that of Single

Fleet Flexible Bus (SFF) in Table 5-3.

Table 5-5 MFF Results for Base Case

	Vehicle Size				Service Area for Flexible Bus			
	Large Flex. Bus		Small Flex. Bus		A	B	C	D
	22		17		3.00	2.50	3.00	3.00
	Large Flexible Bus Headway (hours)				Small Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.097	0.105	0.000	0.000	0.000	0.000	0.094	0.101
2	0.000	0.000	0.000	0.000	0.139	0.156	0.110	0.129
3	0.000	0.000	0.000	0.000	0.295	0.240	0.138	0.228
4	0.000	0.000	0.000	0.000	0.379	0.338	0.266	0.459
	Large Flexible Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	36	34	0	0	0	0	33	45
2	0	0	0	0	16	15	19	25
3	0	0	0	0	7	9	15	13
4	0	0	0	0	5	6	7	6
	Mixed Fleet Conventional Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3559.10	3466.35	2907.78	3889.18	14236.40	13865.41	11631.12	15556.72
2	1337.38	1341.03	1582.63	2270.22	8024.29	8046.15	9495.77	13621.30
3	601.18	718.33	1262.52	1075.68	4809.47	5746.61	10100.17	8605.44
4	374.32	447.65	564.93	510.26	2245.95	2685.89	3389.61	3061.53
Total Operation Cost ($/day) = 135121.84, Total Capital Cost ($/day) = 16233 , Total Cost ($/day) = 151354.84								

MFV operation has up to 10 decision variables, namely conventional bus and flexible bus sizes, four route spacings and four service areas. Optimized vehicle sizes are somewhere between mixed fleet conventional buses and mixed fleet flexible buses. Except for MFV, Mixed Fleet Flexible Bus (MFF) operation is the least costly alternative. However, by considering different types of bus operations as well as different sizes of vehicles, MFV reduces total cost compared to MFF. Detailed results are presented in Table 5-6.

Table 5-6 MFV Results for Base Case

	Vehicle Size		Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Large Conv. Bus	Small Flex. Bus	A	B	C	D	A	B	C	D
	31	16	1.00	-	0.75	0.75	4.00	3.33	4.00	7.50

	Large Conventional Bus Headway (hours)				Small Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.141	0.000	0.153	0.150	0.000	0.060	0.000	0.000
2	0.000	0.000	0.000	0.153	0.125	0.127	0.092	0.000
3	0.000	0.000	0.000	0.000	0.240	0.224	0.114	0.135
4	0.000	0.000	0.000	0.000	0.404	0.338	0.218	0.298

	Large Conventional Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	18	0	17	23	0	45	0	0
2	0	0	0	15	15	15	19	0
3	0	0	0	0	7	8	15	12
4	0	0	0	0	4	5	7	5
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			

Region / Period	A	B	C	D	A	B	C	D
1	3585.53	3576.37	2906.91	3774.82	14342.13	14305.48	11627.62	15099.28
2	1330.98	1320.51	1593.94	2389.22	7985.91	7923.05	9563.62	14335.33
3	573.37	690.51	1258.39	1034.28	4586.97	5524.10	10067.11	8274.24
4	359.07	423.46	541.43	439.51	2154.44	2540.74	3248.55	2637.04
Total Operation Cost ($/day) = 134215.62, Total Capital Cost ($/day) = 11991 , Total Cost ($/day) = 146206.62								

5.4.3.1. Benefit of Sharing Fleets

When demands vary over time and among local regions, fleets can be shared among regions as well as time periods. For instance, fleets that are used for peak periods can also be used for other periods or other regions without additional capital costs. This sharing of fleets can significantly reduce capital costs; to realize such savings, we constrain vehicle size(s) to be consistent throughout regions and times. Table 5-7 shows that the cost of our integrated multi-zone approach can be significantly lower than the sum of four separately optimized costs.

Table 5-7 Comparison of Integrated and Separately Optimized Total Costs

	Vehicle Size		Total Cost	
Total Cost	Large Conv. Bus	Small Flex. Bus	Regional Cost	Total Cost
Integrated System	31	16	-	146,206.6
Region A only	30	16	32,745.0	149,437.8
Region B only	25	17	34,179.2	
Region C only	32	17	38,197.2	

Region D only	31	16	44,298.4	

5.4.4. Solution Reliability

Since a GA is heuristic and does not guarantee global optimality, the reliability of solutions is additionally compared with 20 time repetitive runs. .MFV is shown in Figure 5-4 since it is the most complex and computationally demanding among our five alternatives. 17 of 20 runs yield the same consistent minimized value while the remaining 3 runs yield a slightly costlier value (by less than 0.3%). SFC, SFF, MFC, and MFF yield results faster because their search boundaries are much smaller. Their results (not shown here) are also more consistent than those for MFV. The hybrid approach can find good solutions for these five alternative models.

Figure 5-5 shows the convergence of IGA. MFV, the most complex alternative, converges relatively quickly (i.e. in less than 50 generations). For each instance, IGA runs up to 250 generations to carefully check cost variations.

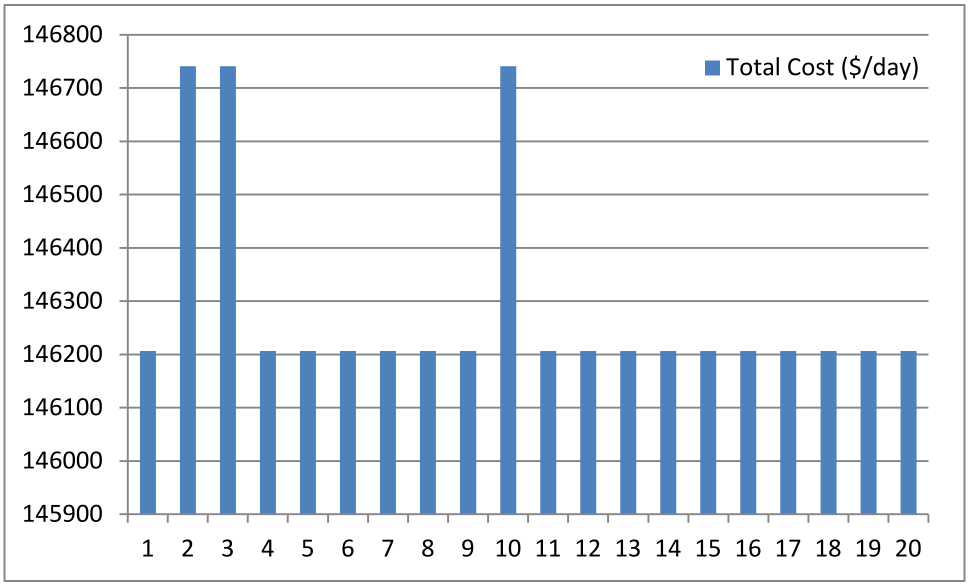


Figure 5-4 Reliability of IGA

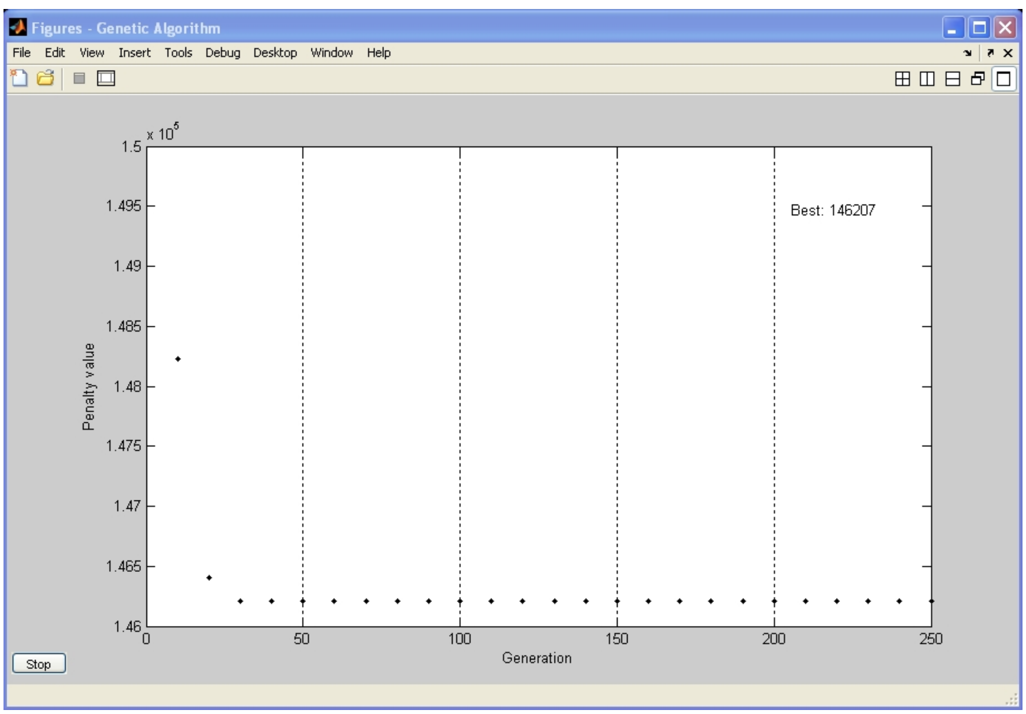


Figure 5-5 Convergence of IGA to the MFV

Figure 5-5 shows that solutions converge quickly. To assess the quality of the solution obtained from our partially heuristic hybrid approach without knowing the actual globally optimal solution a statistical test (Jong & Schonfeld 2003; Wang & Schonfeld 2012) is used here. One million random candidate solutions are generated, and compared with the solution provided by the hybrid algorithm. The computation time for generating a million random solutions was about 12.8 hours with a quad core processor (Intel(R) Core™ i7-3610QM CPU @ 2.30GHz). The best of the million candidate solutions is $147,563.32/day, which is 0.92% costlier than our hybrid solution. The average of the 10 best random solutions is $147,941.15/day, which is 1.17% costlier than our solution.

A small problem (a terminal connecting two local regions with four time periods) is additionally designed to check the solution quality by comparing solutions obtained with our method with the optimal solution obtained through complete enumeration. Since complete enumeration is only used to validate the solution quality of our approach, its computational time is not a great concern in this test. Input values for this complete enumeration are shown in Table 5-8. For the other input values, the Notation Table 5-1 is still applicable, and the same units are also applied.

Table 5-8 Input Values for a Complete Enumeration

Parameter		Region A	Region B
J		4	5
L		3	2
W		2	2
Q	Period 1	100	80
	Period 2	50	40
	Period 3	10	30
	Period 4	5	5

For this verification, a total of 185,856 candidate solutions (7~50 conventional bus seats * 7~50 flexible bus seats * up to 2 conventional bus zones for zone A * up to 2 conventional bus zones for region B * up to 6 flexible bus zones for region A * up to 4 flexible bus zones for region B) are compared.

Comparison of results with those of complete enumeration shows that they are identical. Only MFV results are compared (in Tables 5-9 and 5-10) because MFV has more optimizable variables and is more general than the other alternatives.

Table 5-9 Result Comparison

	Complete Enumeration	Our Method
Vehicle Size (Sc, Sf)	37, 15	37, 15
Conventional Bus Zones (A, B)	(2,2)	(2,2)
Flexible Bus Zones (A, B)	(2,1)	(2,1)
Total Cost	33452.64	33452.64

Table 5-10 Resulting Fleet Sizes

(Conv, Flex)		Region A	Region B
Periods	1	(10,0)	(0,18)
	2	(0,11)	(0,7)
	3	(0,3)	(0,5)
	4	(0,2)	(0,1)

With two local regions, it is found that our hybrid method finds identical solutions to those from complete enumeration. Thus, depending on the problem sizes, our proposed solution can provide optimal or near optimal solutions. Since MFV has a more complex formulation than the other alternatives, repeating such tests seems unnecessary for SFC, SFF, MFC, or MFF.

5.5. Numerical Evaluation: Sensitivity Analysis

This section explores the sensitivity of results to important input factors. From this analysis, it is be found how total cost and other optimized characteristics change from baseline values and vary among alternatives.

5.5.1. Sensitivity Case I: Demand

To explore how bus operations change mainly with demand density, demand inputs are multiplied by a factor of 10, as shown in Table 5-11.

Table 5-11 Demand Input

Demand (trips/mile^2/hour)				
Period \ Region	A	B	C	D
1	700	800	600	550
2	300	350	400	400
3	100	150	300	150
4	50	75	100	50

When demand increases tenfold, the optimized bus size becomes 50 seats, which is our upper bound for bus size. Similarly, optimized route spacings are at the specified lower boundary; hence, bus service has the maximum possible number of zones per region. Compared to the SFC Base Case in Table 5-2, headways noticeably decrease to satisfy the higher demand.

Table 5-12 Sensitivity of SFC to Demand

	Vehicle Size				Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus				A	B	C	D
	50				0.5	0.5	0.5	0.5
	Conventional Bus Headway (hours)				Conventional Bus Fleet Assignment (buses)			
Period \ Region	A	B	C	D	A	B	C	D
1	0.047	0.062	0.041	0.036	107	99	94	143
2	0.080	0.089	0.060	0.050	42	46	43	69
3	0.141	0.137	0.070	0.101	24	30	37	34

4	0.199	0.196	0.124	0.181	17	21	21	19
	Conventional Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	23872.54	24003.23	19700.34	27042.63	95490.17	96012.93	78801.36	108170.51
2	9341.14	9670.70	11093.40	15708.21	56046.86	58024.20	66560.37	94249.28
3	3948.00	4985.00	8721.84	6909.31	31584.00	39880.00	69774.70	55274.51
4	2382.63	2996.79	3639.43	2969.68	14295.76	17980.71	21836.57	17818.07
Total Operation Cost ($/day) = 921800.01, Capital Cost ($/day) = 55375.00, Total Cost ($/day) = 977175.01								

The SFF sensitivity results for a tenfold demand increase are shown in Table 10. Flexible bus size increases from 19 seats in the Base Case (Table 5-3) to 31 seats. Headways are significantly decreased from the Base Case, thus requiring more buses (Table 5-13). Additionally, optimized service areas for flexible bus are 1 mile^2 for all regions, which means that with higher demand, flexible bus operation is required to serve a smaller zone size than baseline case; thus, more zones are desirable. It is interesting that flexible bus size is optimized here at only 31 seats even though the bus size upper bound is 50 seats. Thus, even when demand densities increase tenfold, the optimized bus size below the "standard" 50 seat size is more effective for flexible bus operations.

Table 5-13 Sensitivity of SFF to Demand

	Vehicle Size				Service Area for Flexible Bus			
	Single Fleet Flexible Bus				A	B	C	D
	31				1	1	1	1
	Flexible Bus Headway (hours)				Flexible Bus Fleet Assignment (buses)			
Period \ Region	A	B	C	D	A	B	C	D
1	0.044	0.039	0.047	0.056	201	206	172	220
2	0.071	0.069	0.056	0.065	83	79	98	133
3	0.137	0.114	0.067	0.118	40	45	80	68
4	0.200	0.169	0.130	0.220	26	29	38	34
	Flexible Bus Cost ($/hour)				Operation Cost × Time			
Period \ Region	A	B	C	D	A	B	C	D
1	25073.58	25245.67	20305.20	27746.00	100294.34	100982.69	81220.81	110983.99
2	8998.98	9096.15	10676.35	15747.74	53993.91	54576.93	64058.10	94486.42
3	3796.69	4583.22	8387.95	7081.45	30373.54	36665.78	67103.61	56651.64
4	2309.17	2726.81	3544.91	3158.52	13855.04	16360.87	21269.44	18951.09
Total Operation Cost ($/day) = 921828.17, Capital Cost ($/day) = 82284.50, Total Cost ($/day) = 1014112.67								

Table 5-14 shows MFC sensitivity to a tenfold demand increase. As a result, demands in all regions are served by larger conventional buses. The demand threshold in equation (5.4) does not assign any periods in any regions to smaller conventional buses. The provision of two different sizes of conventional buses (MFC) at very high demand is superfluous. Therefore, the results in Table 5-14 are consistent with the results of SFC operation in Table 5-12. The one notable difference from Table 5-12 is that the smaller conventional bus size is optimized at 1 seat/bus, but never used for any service. It should also be noted that in the solution method (i.e., IGA), the bus sizes (integer values) are optimized within the range of from 1 to 50 seats/bus.

Table 5-14 Sensitivity of MFC to Demand

	Vehicle Size				Route Spacing for Conventional Bus			
	Large Conv. Bus		Small Conv. Bus		A	B	C	D
	50		1		0.5	0.5	0.5	0.5
	Large Conventional Bus Headway (hours)				Small Conventional Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.047	0.062	0.041	0.036	0.000	0.000	0.000	0.000
2	0.080	0.089	0.060	0.050	0.000	0.000	0.000	0.000
3	0.141	0.137	0.070	0.101	0.000	0.000	0.000	0.000
4	0.199	0.196	0.124	0.181	0.000	0.000	0.000	0.000
	Large Conventional Bus Fleet Assignment (buses)				Small Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	107	99	94	143	0	0	0	0
2	42	46	43	69	0	0	0	0
3	24	30	37	34	0	0	0	0
4	17	21	21	19	0	0	0	0
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	23872.54	24003.23	19700.34	27042.63	95490.17	96012.93	78801.36	108170.51
2	9341.14	9670.70	11093.40	15708.21	56046.86	58024.20	66560.37	94249.28
3	3948.00	4985.00	8721.84	6909.31	31584.00	39880.00	69774.70	55274.51
4	2382.63	2996.79	3639.43	2969.68	14295.76	17980.71	21836.57	17818.07

Total Operation Cost ($/day) = 921800.01, Capital Cost ($/day) = 55375.00, Total Cost ($/day) = 977175.01

When two flexible bus sizes are used for these four regions, period 1 in local regions B and D is served by the large flexible buses (33 seats). The small flexible bus is optimized at 24 seats/bus and optimized service areas are 1, 1, 1, and 1.07 mile^2 for local regions A, B, C, and D, respectively. These service area values are at or very near the lower bound. To serve higher demand with MFF, notable changes are made compared to the MFF Base Case results in Table 5-5; optimized bus sizes increase, while headways and service areas decrease. Therefore, larger fleets are also required.

Table 5-15 Sensitivity to MFF to Demand

	Vehicle Size				Service Area for Flexible Bus			
	Large Flex. Bus		Small Flex. Bus		A	B	C	D
	33		24		1	1	1	1.07
	Large Flexible Bus Headway (hours)				Small Flexible Bus Headway (hours)			
Period \ Region	A	B	C	D	A	B	C	D
1	0.000	0.041	0.000	0.055	0.034	0.000	0.040	0.000
2	0.000	0.000	0.000	0.000	0.070	0.067	0.055	0.056
3	0.000	0.000	0.000	0.000	0.137	0.114	0.066	0.110
4	0.000	0.000	0.000	0.000	0.200	0.169	0.126	0.208
	Large Flexible Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
Period \ Region	A	B	C	D	A	B	C	D
1	0	196	0	214	246	0	195	0

2	0	0	0	0	84	81	100	144
3	0	0	0	0	40	45	81	69
4	0	0	0	0	26	29	39	34
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	25034.94	25288.44	20130.05	27983.71	100139.76	101153.75	80520.21	111934.85
2	8881.76	8983.46	10536.96	15597.70	53290.54	53900.76	63221.77	93586.18
3	3740.69	4520.22	8274.78	6941.92	29925.54	36161.78	66198.24	55535.40
4	2272.77	2686.21	3490.08	3066.13	13636.64	16117.27	20940.45	18396.76
Total Operation Cost ($/day) = 914659.90, Capital Cost ($/day) = 97157.00 , Total Cost ($/day) = 1011816.90								

For MFV the total cost is $970,303.33/day with 50 seat conventional and 23 seat flexible buses. Flexible bus does not serve any passengers in region D. Conventional bus route spacings are optimized at 0.5 miles for all regions while flexible bus service areas are optimized as 3.0, 1.25, and 2.4 mile2 for region A, B, and C, respectively.

In the MFV Base Case, periods 3 and 4 in region D are served by flexible bus (in Table 5-6). However, with increased demand, as shown in Table 5-16, conventional bus serves all periods in region D. Also, period 1 in region B is now served by conventional bus instead of the flexible bus in MFV Base Case (Table 5-6). Basically, as demands increase, the optimized bus sizes, service frequencies, numbers of zones, and required fleets all increase.

Table 5-16 Sensitivity of MFV to Demand

	Vehicle Size		Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Large Conv. Bus	Small Flex. Bus	A	B	C	D	A	B	C	D
	50	23	0.5	0.5	0.5	0.5	3.0	1.25	2.4	-

	Large Conventional Bus Headway (hours)				Small Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.047	0.062	0.041	0.036	0.000	0.000	0.000	0.000
2	0.080	0.000	0.060	0.050	0.000	0.052	0.000	0.000
3	0.000	0.000	0.070	0.101	0.068	0.099	0.000	0.000
4	0.000	0.000	0.000	0.181	0.108	0.147	0.074	0.000

	Large Conventional Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	107	99	94	143	0	0	0	0
2	42	0	43	69	0	87	0	0
3	0	0	37	34	37	44	0	0
4	0	0	0	19	22	28	36	0

	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	23872.54	24003.23	19700.34	27042.63	95490.17	96012.93	78801.36	108170.51
2	9341.14	9029.42	11093.40	15708.21	56046.86	54176.49	66560.37	94249.28
3	3654.27	4440.55	8721.84	6909.31	29234.18	35524.37	69774.70	55274.51
4	2037.40	2594.18	3384.09	2969.68	12224.42	15565.08	20304.53	17818.07

Total Operation Cost ($/day) = 155071.00, Capital Cost ($/day) = 13853.50 , Total Cost ($/day) = 168924.50

5.5.2. Sensitivity Case II: Line-haul Distance

First, line-haul distances are increased by five miles, as shown in Table 5-17. This case explores how distances between the main terminal (or CBD) and local regions affect system costs for five alternative operations.

Table 5-17 Line-haul Distance by Region

Region	A	B	C	D
Line-haul Distance (miles)	9	10	8	10

When increasing line-haul distance, round travel times (i.e. distances) also increase. Therefore, optimized bus size (35 seats/bus) increases as well in order to serve more passengers per vehicle round trip. Higher frequency is more expensive when vehicle round trips are longer. This explains why route spacings are also equal or slightly increased compared to Base Case (Table 5-3). In this case, longer SFC line-haul distances mainly lead to larger buses and route spacings. However headways and fleet sizes do not change significantly. Detailed results are shown in Table 5-18.

Table 5-18 Sensitivity of SFC to Line-haul Distance

	Vehicle Size	Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus	A	B	C	D
	35	1	1.25	1	1
	Conventional Bus Headway (hours)	Conventional Bus Fleet Assignment (buses)			

Region / Period	A	B	C	D	A	B	C	D
1	0.166	0.170	0.139	0.127	22	21	20	27
2	0.221	0.217	0.155	0.163	11	11	12	14
3	0.405	0.341	0.186	0.253	6	7	10	9
4	0.607	0.477	0.309	0.456	4	5	6	5
	Conventional Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	4456.84	4575.67	3688.00	4646.72	17827.37	18302.67	14752.00	18586.89
2	1859.16	1934.78	2162.76	2856.60	11154.95	11608.68	12976.53	17139.62
3	838.58	1025.28	1725.87	1332.00	6708.62	8202.22	13806.93	10656.00
4	529.18	629.47	763.02	609.11	3175.07	3776.83	4578.13	3654.67
Total Operation Cost ($/day) = 176907.18, Capital Cost ($/day) = 10575, Total Cost ($/day) = 187482.18								

With increased SFF line-haul distance, both bus size and service areas increase to serve more passengers per bus round trip, since longer line-haul distances yield longer travel times, and thus higher operating cost. In Table 5-19, flexible bus size is optimized at 26 seats and optimized service areas are 4, 3.33, 4, and 3.75 mile^2 for regions A, B, C, and D, respectively. Optimized service areas are increased compared to Base Case results (in Table 5-3). It is found that changes in SFF line-haul distance do not seriously affect optimized headways and required fleets. Instead, bus size and service areas respond more sensitively.

Table 5-19 Sensitivity of SFF to Line-haul Distance

<table>
<tr><td rowspan="3"></td><td colspan="4">Vehicle Size</td><td colspan="4">Service Area for Flexible Bus</td></tr>
<tr><td colspan="4">Single Fleet Flexible Bus</td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td colspan="4">26</td><td>4</td><td>3.33</td><td>4</td><td>3.75</td></tr>
<tr><td></td><td colspan="4">Flexible Bus Headway (hours)</td><td colspan="4">Flexible Bus Fleet Assignment (buses)</td></tr>
<tr><td>Region / Period</td><td>A</td><td>B</td><td>C</td><td>D</td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>1</td><td>0.092</td><td>0.097</td><td>0.105</td><td>0.126</td><td>42</td><td>40</td><td>35</td><td>43</td></tr>
<tr><td>2</td><td>0.151</td><td>0.164</td><td>0.125</td><td>0.138</td><td>17</td><td>16</td><td>20</td><td>27</td></tr>
<tr><td>3</td><td>0.296</td><td>0.275</td><td>0.154</td><td>0.247</td><td>8</td><td>9</td><td>16</td><td>14</td></tr>
<tr><td>4</td><td>0.451</td><td>0.390</td><td>0.277</td><td>0.456</td><td>5</td><td>6</td><td>8</td><td>7</td></tr>
<tr><td></td><td colspan="4">Flexible Bus Cost ($/hour)</td><td colspan="4">Operation Cost × Time</td></tr>
<tr><td>Region / Period</td><td>A</td><td>B</td><td>C</td><td>D</td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>1</td><td>4641.98</td><td>4454.96</td><td>3889.00</td><td>4927.84</td><td>18567.92</td><td>17819.86</td><td>15556.01</td><td>19711.34</td></tr>
<tr><td>2</td><td>1697.63</td><td>1674.01</td><td>2068.75</td><td>2840.57</td><td>10185.77</td><td>10044.06</td><td>12412.50</td><td>17043.43</td></tr>
<tr><td>3</td><td>731.28</td><td>873.09</td><td>1634.54</td><td>1310.93</td><td>5850.21</td><td>6984.75</td><td>13076.32</td><td>10487.41</td></tr>
<tr><td>4</td><td>451.36</td><td>532.65</td><td>703.26</td><td>601.11</td><td>2708.15</td><td>3195.92</td><td>4219.59</td><td>3606.64</td></tr>
<tr><td colspan="9">Total Operation Cost ($/day) = 174169.88, Capital Cost ($/day) = 18080.0, Total Cost ($/day) = 189549.88</td></tr>
</table>

Results of MFC sensitivity to line-haul distance are shown in Table 5-20. Optimized conventional bus sizes increase (40 to 47 and 27 to 32 seat/bus) compared to the Base Case results (Table 5-4). Route spacings are either increased or equal to the Base Case. Optimized headways are similar to the Base Case results or slightly increased (i.e. reduced frequencies) based on the changes of bus size and route spacings.

Table 5-20 Sensitivity of MFC to Line-haul Distance

	Vehicle Size				Route Spacing for Conventional Bus			
	Large Conv. Bus		Small Conv. Bus		A	B	C	D
	47		32		1	1.25	1	1
	Large Conventional Bus Headway (hours)				Small Conventional Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.000	0.000	0.000	0.171	0.152	0.156	0.133	0.000
2	0.000	0.000	0.000	0.000	0.202	0.217	0.155	0.152
3	0.000	0.000	0.000	0.000	0.405	0.341	0.186	0.253
4	0.000	0.000	0.000	0.000	0.607	0.477	0.309	0.456
	Large Conventional Bus Fleet Assignment (buses)				Small Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0	0	0	20	24	23	21	0
2	0	0	0	0	12	11	12	15
3	0	0	0	0	6	7	10	9
4	0	0	0	0	4	5	6	5
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	4446.87	4564.76	3683.77	4654.96	17787.47	18259.02	14735.09	18619.83
2	1849.20	1928.18	2155.56	2845.56	11095.20	11569.08	12933.33	17073.33
3	834.98	1021.08	1719.87	1326.60	6679.82	8168.62	13758.93	10612.80
4	526.78	626.47	759.42	606.11	3160.67	3758.83	4556.53	3636.67
Total Operation Cost ($/day) = 176405.23, Capital Cost ($/day) = 10358.00, Total Cost ($/day) = 186763.23								

A similar finding is that for MFF bus sizes and service areas increase. Another point in comparison to MFC is that larger flexible buses are favored for period 1 in regions A and B (with MFC, and larger conventional buses are selected for period 1 in region D, Table 5-20). The main changes for MFF are increased bus size and service areas. With MFF, headways and fleet sizes are less sensitive to line-haul distances.

Table 5-21 Sensitivity of MFF to Line-haul Distance

	Vehicle Size				Service Area for Flexible Bus			
	Large Conv. Bus		Small Flex. Bus		A	B	C	D
	29		24		4	3.33	3	3.75
	Large Flexible Bus Headway (hours)				Small Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.098	0.107	0.000	0.000	0.000	0.000	0.129	0.116
2	0.000	0.000	0.000	0.000	0.151	0.164	0.146	0.138
3	0.000	0.000	0.000	0.000	0.296	0.275	0.176	0.247
4	0.000	0.000	0.000	0.000	0.451	0.390	0.347	0.456
	Large Flexible Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	40	37	0	0	0	0	35	46
2	0	0	0	0	17	16	21	27
3	0	0	0	0	8	9	17	14
4	0	0	0	0	5	6	8	7
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	4666.04	4474.86	3798.00	4914.45	18664.15	17899.43	15191.99	19657.78
2	1690.83	1667.61	2066.57	2829.77	10144.97	10005.66	12399.44	16978.63
3	728.08	869.49	1647.53	1305.33	5824.61	6955.95	13180.23	10442.61
4	449.36	530.25	736.97	598.31	2696.15	3181.52	4421.82	3589.84
Total Operation Cost ($/day) = 171234.80, Capital Cost ($/day) = 17888.50 , Total Cost ($/day) =								

189123.30

As in the MFV Base Case results (Table 5-6), it is notable that conventional bus does not serve local region B at all, as no route spacing for region B is provided. In this case, there are 9 optimized decision variables from the hybrid solution approach. Similarly to other alternatives such as MFC and MFF, the larger conventional buses serve when demand is higher, and smaller flexible buses serve the other periods. Bus sizes increase by 10 and 5 seats compared to the MFV Base Case (Table 5-6).

Table 5-22 Sensitivity of MFV to Line-haul Distance

	Vehicle Size		Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Large Conv. Bus	Small Flex. Bus	A	B	C	D	A	B	C	D
	41	21	1	-	1	1	6	3.33	4	7.5

	Large Conventional Bus Headway (hours)				Small Flexible Bus Headway (hours)			
Period \ Region	A	B	C	D	A	B	C	D
1	0.174	0.000	0.164	0.149	0.000	0.077	0.000	0.000
2	0.000	0.000	0.000	0.163	0.114	0.164	0.125	0.000
3	0.000	0.000	0.000	0.000	0.259	0.275	0.142	0.162
4	0.000	0.000	0.000	0.000	0.329	0.390	0.277	0.319

	Large Conventional Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
Period \ Region	A	B	C	D	A	B	C	D
1	21	0	17	23	0	48	0	0
2	0	0	0	14	17	16	20	0

3	0	0	0	0	7	9	17	13
4	0	0	0	0	5	6	8	6
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	4484.80	4476.65	3703.49	4635.26	17939.20	17906.60	14813.98	18541.04
2	1695.09	1658.01	2048.75	2873.40	10170.55	9948.06	12292.50	17240.42
3	697.16	864.09	1615.61	1253.55	5577.32	6912.75	12924.89	10028.37
4	413.05	526.65	695.26	528.51	2478.31	3159.92	4171.59	3171.04
Total Operation Cost ($/day) = 167276.54, Capital Cost ($/day) = 13207.0 , Total Cost ($/day) = 180483.54								

5.5.3. Sensitivity Case III: Value of Waiting Time

Waiting time value is important in this study because if affects the optimized headway, user cost and the threshold demands for mixed fleets operations such as MFC, MFF, and MFV. In this sensitivity analysis, the value of waiting time is increased by 40% from 12 to 16.8$/hour.

Table 5-23 shows how decision variables and costs of SFC operation change with higher values of waiting time. The detailed results of SFC are shown Table 5-23. Since waiting time is now more expensive, headways are reduced compared to SFC Base Case results, thus requiring more buses. Based on the changes of headways and fleet sizes, optimized bus size is reduced from 30 to 26 seats/bus. Route spacings also increase slightly.

Table 5-23 Sensitivity of SFC to Waiting Time

	Vehicle Size				Route Spacing for Conventional Bus			
	Single Fleet Conventional Bus				A	B	C	D
	26				1	1.25	1	1
	Conventional Bus Headway (hours)				Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.115	0.117	0.108	0.092	22	21	18	28
2	0.141	0.149	0.108	0.115	12	11	12	15
3	0.281	0.235	0.130	0.191	6	7	10	9
4	0.422	0.329	0.217	0.344	4	5	6	5
	Conventional Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3811.51	3953.87	3135.60	4073.39	15246.04	15815.47	12542.40	16293.57
2	1656.40	1737.10	1910.40	2581.78	9938.40	10422.62	11462.40	15490.67
3	764.40	932.57	1533.52	1231.63	6115.20	7460.53	12268.16	9853.07
4	488.33	578.28	692.40	577.39	2930.00	3469.70	4154.40	3464.33
Total Operation Cost ($/day) = 156926.95, Capital Cost ($/day) = 10057.00, Total Cost ($/day) = 166983.95								

The sensitivity of SFF results to waiting time value is shown in Table 5-24. Optimized headways are all reduced compared to the SFF Base Case results (Table 5-3) to provide higher bus frequencies because waiting time is more critical than the SFF Base Case, and this change results in larger fleets. Service areas are all increased, and vehicle size is re-optimized from 19 (Base Case) to 20 seats/bus. The results show that the optimized bus size for flexible bus (SFF) is less sensitive than for conventional bus (SFC in Table 5-23). With a higher value of waiting time, the optimization yields increased bus size, higher frequencies, more buses, and larger service areas.

Table 5-24 Sensitivity of SFF to Waiting Time

	Vehicle Size				Service Area for Flexible Bus			
	Single Fleet Flexible Bus				A	B	C	D
	20				4	3.33	4	3.75
	Flexible Bus Headway (hours)				Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0.070	0.075	0.073	0.092	40	38	35	43
2	0.106	0.117	0.086	0.100	17	16	20	27
3	0.202	0.193	0.105	0.176	8	9	16	14
4	0.304	0.270	0.183	0.319	5	6	8	7
	Flexible Bus Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3812.98	3690.67	3156.97	4150.08	15251.92	14762.68	12627.88	16600.33
2	1436.97	1432.60	1710.29	2434.33	8621.84	8595.60	10261.76	14605.95
3	637.27	765.46	1361.75	1154.94	5098.15	6123.72	10893.99	9239.55
4	399.45	474.85	602.18	544.06	2396.72	2849.09	3613.06	3264.35
Total Operation Cost ($/day) = 144806.58, Capital Cost ($/day) = 17160.00, Total Cost ($/day) = 161966.58								

Table 5-25 shows the sensitivity of MFC to waiting time value. Mainly, optimized headways are decreased to reduce waiting times; therefore, larger fleets are required compared to the Base Case (Table 5-4). With the changes in bus frequencies and fleets, the two conventional bus sizes are reduced from 40 to 35 and from 27 to 24 seats/bus. Route spacings are similar to the MFC Base Case results or slightly higher in order to increase passengers per bus round trip.

Table 5-25 Sensitivity of MFC to Waiting Time

	Vehicle Size				Route Spacing for Conventional Bus			
	Large Conv. Bus		Small Conv. Bus		A	B	C	D
	35		24		1	1.25	1	1
	Large Conventional Bus Headway (hours)				Small Conventional Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.000	0.000	0.000	0.123	0.110	0.117	0.098	0.000
2	0.000	0.000	0.000	0.000	0.141	0.149	0.108	0.115
3	0.000	0.000	0.000	0.000	0.281	0.235	0.130	0.191
4	0.000	0.000	0.000	0.000	0.422	0.329	0.217	0.344
	Large Conventional Bus Fleet Assignment (buses)				Small Conventional Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	0	0	0	21	23	21	20	0
2	0	0	0	0	12	11	12	15
3	0	0	0	0	6	7	10	9
4	0	0	0	0	4	5	6	5
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3802.18	3945.47	3132.48	4077.92	15208.73	15781.87	12529.92	16311.67
2	1651.60	1732.70	1905.60	2575.78	9909.60	10396.22	11433.60	15454.67
3	762.00	929.77	1529.52	1228.03	6096.00	7438.13	12236.16	9824.27
4	486.73	576.28	690.00	575.39	2920.40	3457.70	4140.00	3452.33

Total Operation Cost ($/day) = 156591.27, Capital Cost ($/day) = 9635.50, Total Cost ($/day) = 166226.77

The key change in MFF due to an increased waiting time value is the reduced headways; therefore, fleets sizes also increase. As with the SFF results (Table 5-24), it is noted that flexible bus size changes seem less sensitive than the conventional bus size changes.

Table 5-26 Sensitivity of MFF to Waiting Time

	Vehicle Size				Service Area for Flexible Bus			
	Large Felx. Bus		Small Flex. Bus		A	B	C	D
	20		17		4	3.33	3	3.75
	Large Flexible Bus Headway (hours)				Small Flexible Bus Headway (hours)			
Region / Period	A	B	C	D	A	B	C	D
1	0.070	0.075	0.000	0.000	0.000	0.000	0.087	0.082
2	0.000	0.000	0.000	0.000	0.106	0.117	0.097	0.100
3	0.000	0.000	0.000	0.000	0.202	0.193	0.117	0.176
4	0.000	0.000	0.000	0.000	0.304	0.270	0.224	0.319
	Large Flexible Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	40	38	0	0	0	0	35	47
2	0	0	0	0	17	16	21	27
3	0	0	0	0	8	9	17	14
4	0	0	0	0	5	6	8	7
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D

1	3812.98	3690.67	3064.12	4128.66	15251.92	14762.68	12256.48	16514.62
2	1426.77	1423.00	1700.73	2418.13	8560.64	8538.00	10204.36	14508.75
3	632.47	760.06	1366.53	1146.54	5059.75	6080.52	10932.20	9172.35
4	396.45	471.25	627.95	539.86	2378.72	2827.49	3767.67	3239.15
Total Operation Cost ($/day) = 144055.51, Capital Cost ($/day) = 17477.00 , Total Cost ($/day) = 161532.31								

The effects of increased waiting time on MFV are shown in Table 5-27. Period 1 in local region C and period 2 in local region D are now served by flexible bus while those periods are served by conventional bus in the MFV Base Case results (Table 5-6). Since conventional bus serves only period 1 in region A, which is the highest demand period, bus size is optimized to cover this single period. The optimized size for conventional bus increases by 4 seats while flexible bus size increases by 1 seat, from 16 to 17 seats/bus.

Table 5-27 Sensitivity of MFV to Waiting Time

	Vehicle Size		Route Spacing for Conv. Bus				Service Area for Flex. Bus			
	Large Conv. Bus	Small Flex. Bus	A	B	C	D	A	B	C	D
	35	17	1	-	-	1	6	3.33	4	5

	Large Conventional Bus Headway (hours)				Small Flexible Bus Headway (hours)			
Period \ Region	A	B	C	D	A	B	C	D
1	0.121	0.000	0.000	0.123	0.000	0.064	0.070	0.000
2	0.000	0.000	0.000	0.000	0.082	0.117	0.086	0.082

3	0.000	0.000	0.000	0.000	0.153	0.193	0.096	0.158
4	0.000	0.000	0.000	0.000	0.227	0.270	0.183	0.311
	Large Conventional Bus Fleet Assignment (buses)				Small Flexible Bus Fleet Assignment (buses)			
Region / Period	A	B	C	D	A	B	C	D
1	21	0	0	21	0	43	36	0
2	0	0	0	0	17	16	20	27
3	0	0	0	0	8	9	17	13
4	0	0	0	0	5	6	8	6
	Mixed Fleet Bus Service Cost ($/hour)				Operation Cost × Time			
Region / Period	A	B	C	D	A	B	C	D
1	3852.80	3684.00	3135.22	4077.92	15411.20	14735.98	12540.88	16311.67
2	1439.86	1423.00	1698.29	2428.33	8639.18	8538.00	10189.76	14570.00
3	604.76	760.06	1349.75	1116.07	4838.11	6080.52	10797.97	8928.53
4	366.96	471.25	597.38	512.84	2201.74	2827.49	3584.26	3077.04
Total Operation Cost ($/day) = 143272.33, Capital Cost ($/day) = 13615.00 , Total Cost ($/day) = 156887.33								

5.5.4. Summary of Sensitivity Analysis

Table 5-24 summarizes sensitivity results. For three important input parameters, namely demand, line-haul distance, and value of waiting time, are analyzed. Detailed results are presented above in Tables 5-3~5-23. The detailed sensitivity analysis results to in-vehicle time, access time, directional factor, and number of passenger per stop are omitted, but their results are summarized in Table 5-28.

When we analyze Base Case results, MFV reduces total cost by 5.29, 3.59, 4.84, and 3.40% compared to SFC, SFF, MFC, and MFF, respectively. Since Base Case input demands are relatively favorable to flexible bus operations, SFCS has the highest cost here.

Demand is an obviously important factor for public transit analysis. Thus, all demands are increased tenfold to explore resulting changes in system characteristics. Consequently total costs increase by 532~568% compared to the corresponding Base Case. MFV provides cost reductions about 4.32~4.10% compared to flexible bus operations such as SFF and MFF. It is also notable that MFC does not provide any cost reduction because all demands are assigned to the larger conventional bus by threshold demand in equation (5.4). This implies that when demand density exceeds some level, mixed conventional bus operation is no longer beneficial, unless perhaps unusually large buses could be operated. MFV and SFC have very small difference (0.70%) in total cost. This implies that flexible services are not preferable when demand is high. Nonetheless, providing different sizes of buses and different types of operations does reduce costs when demand densities vary greatly over time and over regions.

Line-haul distance directly affects travel time. When line-haul distance increases by 5 miles for all local regions, it is noted that total costs increase by about 21~25%

compared to the corresponding Base Case operations. More specifically, the costs of conventional bus operations such as SFC or MFC increase by 21.45~21.56 % while the costs of flexible bus operations by SFF or MFF increase by 24.95~24.99%. This line-haul distance change increases total cost significantly, and implies that conventional bus operations may be preferable with long line-haul distances. Still, MFV operation is the most promising alternative among MFV, SFC, SFF, MFC, and MFF. MFV reduces total cost about 3.36~4.78% compared to the other alternatives.

The directional factor f is also analyzed by setting it at 0.75 instead of 1.0 (Base Case input). In the other words, 75% of trips are in one direction, and the other 25% are in the opposite direction. This change yields a lower optimized bus size. Since this factor affects only conventional bus, it does not greatly reduce total cost. Reduced bus size is related to bus operating cost function (B=a+bS), and bus size may not affect to the bus operating cost significantly. Although we only have room here to present sensitivities to a few input parameters, further analyses of sensitivities to cost parameters are of interest. For SFC and MFC, about 1.08~1.10% of total cost can be saved compared to the baseline. MFV is the least cost alternative.

In these models the number of passengers per stop is only relevant to flexible bus operation. Rather than assuming 1.2 passengers per stop, we here assume 1.0; thus, flexible bus requires more bus stops. Hence, the total costs, which includes flexible bus, are 2.03~3.56% above Base Case values. Although flexible bus operation costs increase, MFV is still preferable to the other options; MFV decreases cost about 2.90~4.98%. With the same demand and fewer passengers per stop, flexible buses must stop more frequently; this also increases round-trip travel times. Therefore, flexible bus operations become less attractive with more frequent stops.

Table 5-28 Results Comparison among Various Sensitivity Inputs

	MFV	SFC	SFF	MFC	MFF
BASELINE	146206.6	154374.3	151655.0	153640.1	151354.8
Savings bet. MFV & alternatives		5.29%	3.59%	4.84%	3.40%
DEMAND* 10	970303.3	977175.0	1014112.7		1011816.9
Savings bet. MFV & alternatives		0.70%	4.32%		4.10%
Savings bet. Same Services	563.65%	532.99%	568.70%		568.51%
J+5	180483.5	187482.2	189549.9	186763.2	189123.3
Savings bet. MFV & alternatives		3.73%	4.78%	3.36%	4.57%
Savings bet. Same Services	23.44%	21.45%	24.99%	21.56%	24.95%
f=0.75 (was 1.0)	145617.9	152701.0	151655.0	151944.6	151354.8
Savings bet. MFV & alternatives		4.64%	3.98%	4.16%	3.79%
Savings bet. Same Services	-0.40%	-1.08%	0.00%	-1.10%	0.00%
u=1.0 (was 1.2)	149177.0	154374.3	156989.6	153640.1	156738.1
Savings bet. MFV & alternatives		3.37%	4.98%	2.90%	4.82%
Savings bet. Same Services	2.03%	0.00%	3.52%	0.00%	3.56%
v=7 (was 5, 40% up)	164279.6	167704.6	173325.4	166970.4	172979.7
Savings bet. MFV & alternatives		2.04%	5.22%	1.61%	5.03%
Savings bet. Same Services	12.36%	8.64%	14.29%	8.68%	14.29%

w=16.8 (was 12, 40% up)	156887.3	166984.0	161966.6	166226.8	161532.3
Savings bet. MFV & alternatives		6.05%	3.14%	5.62%	2.88%
Savings bet. Same Services	7.31%	8.17%	6.80%	8.19%	6.72%
x=16.8 (was 12, 40% up)	150176.0	170069.8	151655.0	169213.4	151354.8
Savings bet. MFV & alternatives		11.70%	0.98%	11.25%	0.78%
Savings bet. Same Services	2.71%	10.17%	0.00%	10.14%	0.00%

The sensitivities for three different time values, namely values of in-vehicle time, waiting time, and access time are also analyzed. The detailed results of sensitivity to waiting time value are shown in Tables 5-23~5.27. The brief summary results of these three analyses are also provided in Table 5-28. When the value of in-vehicle time increases from 5 to 7 ($/hour, 40 % up), total costs increase by 8.64~14.29%, respectively. The magnitudes of total cost increases (8.64~14.29%) are relatively small even when the in-vehicle time value increases by 40 %. As noted in Table 5-28, the increased value of in-vehicle time increases total costs of the flexible bus operation, as expected. This shows that flexible bus operations are sensitive to changes of in-vehicle time value. MFV reduces total cost by 1.61~2.04% compared to conventional bus (i.e. SFC and MFC). The MFV cost reduction compared to flexible bus (i.e. SFF, MFF) is about 5.03~5.22%.

With a waiting time value change, MFV reduces total cost by 2.88~6.05%. Total cost reductions in MFV are greater from SFC and MFC than other services (i.e., SFF or

MFF). As shown in Tables 5-20~5-24, a higher value of waiting time results in more bus frequencies, larger fleets, increased bus size(s), and increased route spacings (or service areas) to avoid expensive waiting cost. Compared to the Base Case results, the total costs increase by 6.72~8.19% when the waiting time value increases by 40%.

The last sensitivity analysis explores the effects of the value of access time. Since flexible bus operations are assumed to provide door-to-door services, the value of access time only affects access cost in conventional bus operations. As access time value increases by 40%, SFC and MFC costs increase by about 10.14~10.17% while flexible costs remain around Base Case results. Optimized route spacings are reduced in order to reduce access distances for passengers. This change increases fleet sizes; therefore, total costs increase although users' access cost decreases. MFV cost increases by 2.71%, relatively less than for SFC and MFC. The reason is that conventional bus in MFV serves only a small fraction of the entire demand. For the access time increase MFV can reduces cost by 11.70% and 11.25% from SFC and MFC, which is quite significant. However, the total cost gap between MFV and flexible services such as SFF and MFF is reduced significantly, since MFV cost is increased by the access time value while SFF and MFF total costs stay at Base Case values.

5.6. Chapter Summary

In Chapter 5, optimization models are developed to provide bus services to multiple regions while allowing the use of different vehicle sizes which may be switched to different regions in different periods. A hybrid solution method is proposed, which is combination of a genetic algorithm and analytic optimization.

To reduce total costs when demand and other factors vary over times and over regions, the Mixed Fleet Variable Type Bus (MFV) operation is preferable to alternatives. In order to compare the performances of MFV, we also compare four other types of bus operations, namely Single Fleet Conventional Bus (SFC), Single Fleet Flexible Bus (SFF), Mixed Fleet Conventional Bus (MFC), and Mixed Fleet Flexible Bus (MFF). For mixed fleet operations (i.e. MFC, MFF, and MFV), the demand thresholds between using large or small buses are analytically formulated using bus operation cost functions.

To solve these five different problems (nonlinear mixed integer problem formulations) efficiently, a hybrid solution approach is proposed, which combines an Integer Genetic Algorithm (IGA) and analytic optimization. Such a hybrid algorithm helps reduce the computation time because some variables (i.e. headways and resulting fleets) are optimized analytically.

To examine the quality of solutions, one million candidate solutions are generated and compared to the best solution found by the hybrid algorithm. It is found that the solution from the approach proposed in this chapter is superior to any of the million random solutions. A small problem (i.e., two regions with four periods) is additionally generated to obtain complete enumeration solutions. Our hybrid method finds the identical solution obtained through complete enumeration. This confirms that the proposed hybrid method yields solutions that are at least near-optimal.

As shown in Table 5-7, benefits of sharing fleets throughout the system are explored. To do that, common vehicle size(s) are optimized over all regions as well as periods. Through this numerical evaluation, it is found that the cost of an integrated multi-zone system is lower than the sum of separately optimized results. Numerical evaluation also shows that MFV can yield significantly lower costs than the other four alternatives. Other numerical cases and sensitivity analyses confirm that the proposed approach finds good solution quickly.

Chapter 6 Analyzing Bus Services with Demand Elasticity

6.1. Problem Statement

Transit riders may have different service preferences based on fares, travel times, and other factors. In this chapter, different service qualities and demand elasticities are considered in conventional and flexible service formulations. Total cost minimization is not a reasonable objective when the demand is elastic, since the demand can be driven toward zero in minimizing costs. Instead of minimizing total system costs, the objective in this chapter is to maximize the social welfare, which is the sum of consumer surplus (i.e. net user benefit) and producer surplus (i.e. profit). Optimizable decision variables include fares for conventional and flexible buses, route spacings for conventional bus services, service areas for flexible bus services, as well as headways, vehicle sizes and fleet sizes for both service types.

6.2. System Specifications and Assumptions

Chapter 4.2 addressed assumptions for analyzing a general system with multiple local regions as well as multiple periods. Some assumptions are modified from Sections

4.2.2 and 4.2.3 in order to consider elastic demand in the following formulations. Henceforth, superscripts k and i correspond to region and time period, respectively, while subscripts c and f represent conventional and flexible services, respectively. The definitions, units and default values of variables used in this chapter are presented in Table 6-1.

Table 6-1 Notation

Variable	Definition	Baseline Value
a	hourly fixed cost coefficient for operating bus ($/bus hr)	30.0
A^k	service zone area(mile^2)= L^kW^k/N'	-
b	hourly variable cost coefficient for bus operation ($/seat hr)	0.2
d	bus stop spacing (miles)	0.2
D_c^{ki}	distance of one flexible bus tour in local region k and period i (miles)	-
D_f^k	equivalent line haul distance for flexible bus on region k $(=(L^k+W^k)/z+2J^k/y)$, (miles)	-
D^k	equivalent average bus round trip distance for conventional bus on region k (= $2J^k/y+ W^k/z+2 L^k$),(miles)	-
d_s^{ki}	directional demand split factor	1.0
F^{ki}	fleet size for region k and period i (buses) subscript corresponds to (c = conventional, f=flexible)	-
h_c , h_c^{ki}	headway for conventional bus; for region k and period i (hours/bus)	-
h_f , h_f^{ki}	headway for flexible bus; for region k period i (hours/bus)	-
k ,i	index (k: region, i : period)	-
J^k	line haul distance of region k (miles)	-
l_c, l_f	load factor for conventional and flexible bus (passengers/seat)	1.0
L^k, W^k	length and width of local region k (miles)	-
M^k	equivalent average trip distance for region k (=(J^k/y_c+ $W^k/2z_c$+ $L^k/2$))	-
n	number of passengers in one flexible bus tour	-

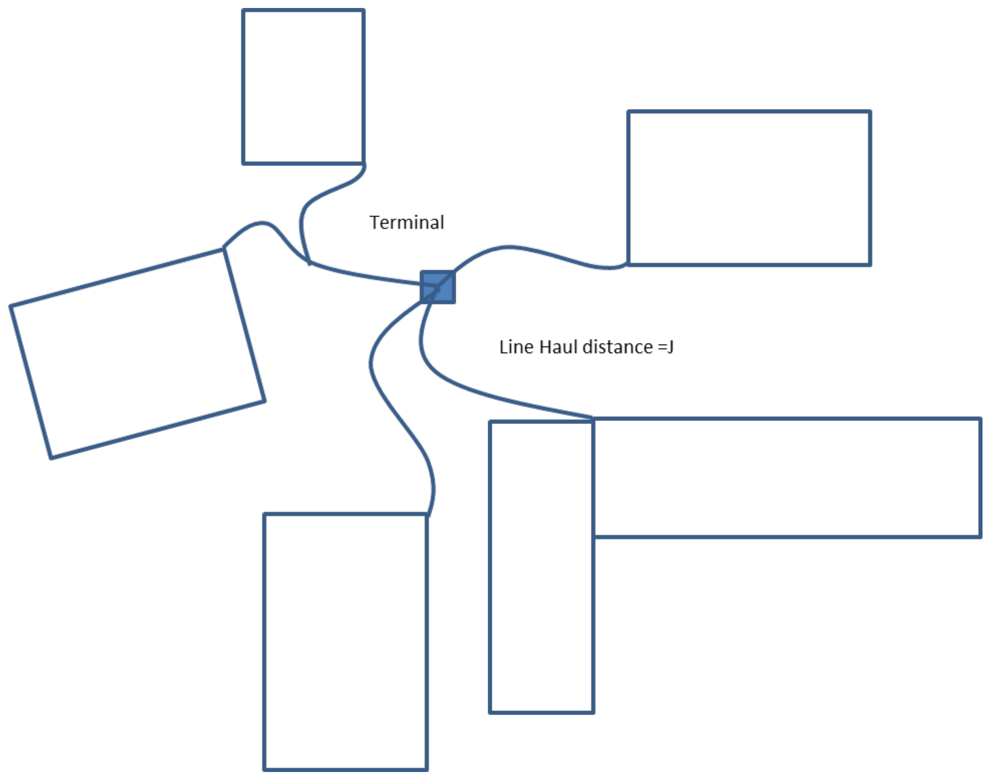
N_c^k, N_f^k	number of zones in local region for conventional and flexible bus	-
Q^{ki}	actual demand density (trips/hr)	-
q^{ki}	potential demand density (trips/mile^2/hr)	-
r^k	route spacing for conventional bus at region k (miles)	-
R_c^{ki}	round trip time of conventional bus for region k and period i (hours)	-
R_f^{ki}	round trip time of flexible bus for region k and period i (hours)	-
S_c , S_f	sizes for conventional and flexible bus (seats/bus)	-
t^{ki}	time duration for region k and period i	-
u	average number of passengers per stop for flexible bus	1.2
V_c^i	local service speed fo/r conventional bus in period i (miles/hr)	30
V_f^i	local service speed for flexible bus in period i (miles/hr)	25
V_x	average passenger access speed (mile/hr)	2.5
v_v, v_w, v_x	value of in-vehicle time, wait time and access time ($/passenger hr)	5, 12, 12
y	express speed/local speed ratio for conventional bus	conventional bus = 1.8 flexible bus = 2.0
Ø	constant in the flexible bus tour equation (Daganzo, 1984) for flexible bus	1.15
*	superscript indicating optimal value; subscript: c = conventional, f=flexible	-
Y_c^{ki}, Y_f^{ki}	total social welfare in region k and period i subscript: c = conventional, f=flexible	-
P_c^{ki}, P_f^{ki}	producer surplus (revenue – cost) in region k and period i subscript: c = conventional, f=flexible	-
R_c^{ki}, R_f^{ki}	revenue in region k and period i subscript: c = conventional, f=flexible	-
C_c^{ki}, C_f^{ki}	operating cost in region k and period i subscript: c = conventional, f=flexible	-
f_c, f_f	fares on the system ;subscript: c = conventional, f=flexible	-
G_c^{ki}, G_f^{ki}	consumer surplus in region k and period i subscript: c = conventional, f=flexible	
e_v, e_w, e_x, e_p	elasticity factors	0.35, 0.7, 0.7, 0.07
TUB_c^{ki}, TUB_f^{ki}	the total user benefit in region k and period i subscript: c = conventional, f=flexible	
Y_c , Y_f	the total welfare of system	

	subscript: c = conventional, f=flexible	
FS^{ki}	the amount of subsidy for region k and period i	

6.2.1. Common assumptions for conventional and flexible buses

All service regions, 1… k, are rectangular, with lengths L^k and widths W^k. These regions may have different line haul distances J^k (miles, in region k) connecting a terminal and each region's nearest corner.

- The demand is uniformly distributed over space within each region and over time within each specified period.
- The optimized bus sizes (S_c for conventional, S_f for flexible) are uniform throughout regions and time periods.
- The average waiting time of passengers is approximated as a constant fraction alpha of the headway (h_c for conventional, h_f for flexible). Alpha is usually assumed to be 0.5.
- Bus layover time is negligible.
- Within each local region k, the average speed (V_c^i for conventional bus, V_f^i for flexible bus) includes stopping times.
- External costs are assumed to be negligible.



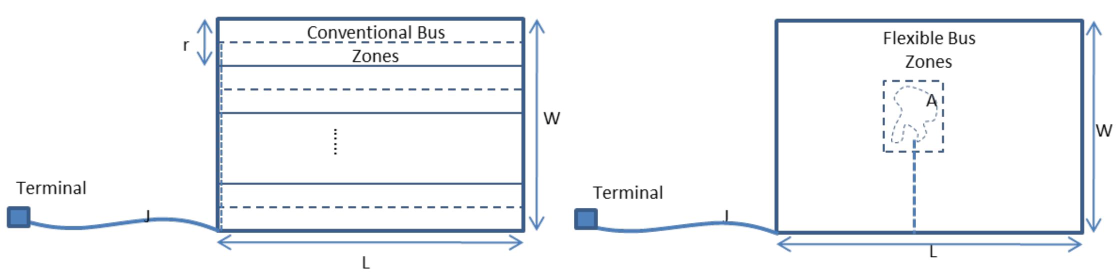


Figure 6-1 Local Regions and Bus Operations

6.3. Elastic Demand Functions and Operating Costs

6.3.1. Conventional Bus Service

In this section, the linear elastic demand function and the operating cost for conventional services are formulated. Chang and Schonfeld (1993) consider elastic demand for conventional bus services for one region and multiple periods. For conventional services, their elastic demand function is modified here to accommodate multiple regions as well as multiple periods. Assumptions from Section 4.2.2 are still applicable, and additional assumptions are introduced in the following sections when they are required.

6.3.1.1. Elastic Demand Function for Conventional Bus Service

The demand density may be sensitive to in-vehicle time, waiting time, access time, and the fare of the system. A linear demand function, Q_c^{ki}, in region k and period i is formulated as follows.

$$Q_c^{ki} = L^k W^k q^{ki} \left\{1 - e_w z_1 h_c^{ki} - e_x z_2 \frac{(r^k + d)}{V_x} - e_v \frac{M_c^k}{V_c^i} - e_p f_c\right\} \quad (6.1)$$

where z_1 = usually 0.5 for uniform passenger arrivals, uniform bus arrivals and sufficient bus capacity; z_2 = usually 0.25 for rectilinear network. The elastic demand function in equation (6.1) can be rewritten as:

$$Q_c^{ki} = L^k W^k q^{ki} \left\{K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c\right\} \quad (6.2)$$

where $K_c^k = 1 - e_x z_2 \frac{d}{V_x} - e_v \frac{M_c^k}{V_c^i}$

6.3.1.2. Conventional Bus Operating Cost

The conventional bus operating cost in region k and time period i is formulated below. Unit operating cost, B_c , is assumed to be a function of vehicle size (i.e., $B_c \; = a + bS_c$):

$$C_c^{ki} = B_c \; N_c^k \frac{D^k t^{ki}}{V_c^i h_c^{ki}} \tag{6.3}$$

6.3.2. Flexible Bus Service

6.3.2.1. Elastic Demand Function for Flexible Bus Service

The demand density of flexible bus services is affected by the in-vehicle time, waiting time and fare. Zhou et al (2008) considered a flexible service with elastic demand for only one time period and one region. Their solutions were obtainable with simple calculus since the problem was small. Here, the elastic demand function for flexible services is modified for multiple regions as well as multiple periods. The actual demand in region k and period i is formulated as:

$$Q_f^{ki} = L^k W^k q^{ki}\left\{1 - e_w z_1 h_f^{ki} - e_v M_f^{ki} - e_p f_f\right\} \tag{6.4}$$

where z_1 = usually 0.5 uniform passenger arrivals, uniform bus arrivals and sufficient bus capacity. Equation (6.4) can be rewritten as

$$Q_f^{ki} = L^k W^k q^{ki} \left\{ K_f^{ki} - e_w z_1 h_f^{ki} - e_v \frac{(\emptyset A^k \sqrt{\frac{q^{ki} h_f^{ki}}{u}})}{2V_f^i} - e_p f_f \right\} \tag{6.5}$$

where $K_f^{ki} = 1 - e_v \left(\frac{L^k + W^k}{2V_f^i y_f} + \frac{J^k}{V_f^i y_f} \right)$.

6.3.2.2. Flexible Bus Operating Cost

Flexible bus operating cost, C_f^{ki}, is formulated by multiplying unit bus operating cost, the number of zones in region k, and round travel time:

$$C_f^{ki} = B_f \ N_f^k \frac{(D_f^k + D_c^{ki}) t^{ki}}{V_f^i h_f^{ki}} \tag{6.6}$$

D_c^{ki} is the approximated flexible bus tour distance according to Stein (1978), in which $D_c^{ki} = \emptyset \sqrt{\mathrm{n} A^k}$, and $\emptyset$=1.15 for the rectilinear space assumed here (Daganzo, 1984). The service area, A^k, of flexible bus in region k is equal to $\frac{L^k W^k}{N_f^k}$. Thus, by substituting average tour distance D_c^{ki} into equation (6.6), the flexible bus operating cost in region k and time period i is estimated as:

$$C_f^{ki} \approx B_f \ N_f^k \frac{(D_f^k + \emptyset \sqrt{\mathrm{n} A^k}) t^{ki}}{V_f^i h_f^{ki}} = \frac{B_f \ N_f^k D_f^k t^{ki}}{V_f^i h_f^{ki}} + \frac{\emptyset B_f \ L^k W^k t^{ki} \sqrt{q^{ki} h_f^{ki} / u}}{V_f^i h_f^{ki}} \tag{6.7}$$

6.4. Welfare Maximization without Financial Constraints

For public transit services and in general, the social welfare is the sum of the consumer surplus and the producer surplus. In this section, social welfare functions are formulated for both conventional and flexible bus services.

6.4.1. Conventional Service Formulations

The welfare of conventional bus services, Y_c^{ki}, in region k and period i is the sum of the producer surplus, P_c^{ki} and the consumer surplus, G_c^{ki}:

$$Y_c^{ki} = P_c^{ki} + G_c^{ki} \tag{6.8}$$

The producer surplus P_c^{ki} is the total revenue R_c^{ki} minus the operating cost C_c^{ki} of the conventional bus service:

$$P_c^{ki} = R_c^{ki} - C_c^{ki} \tag{6.9}$$

The total revenue R_c^{ki} in region k and period i is the fare multiplied by the total demand density in region k and time period i:

$$R_c^{ki} = f_c Q_c^{ki} t^{ki} = f_c L^k W^k q^{ki} t^{ki} \left\{ K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c \right\} \tag{6.10}$$

where $K_c^k = 1 - e_x z_2 \frac{d}{V_x} - e_v \frac{M_c^k}{V_c^i}$

The producer surplus in equation (6.8) can be now rewritten as:

$$P_c^{ki} = f_c L^k W^k q^{ki} t^{ki} \left\{ K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c \right\} - B_c \ N_c^k \frac{D^k t^{ki}}{V_c^i h_c^{ki}} \tag{6.11}$$

Now, the consumer surplus G_c^{ki} is formulated for region k and time period i. The consumer surplus is the total user benefit minus the prices that actually transit users pay.

The total social benefit function can be obtained by using the willingness to pay function in equation (6.2). The fare in equation (6.2) is formulated as a function of the demand density, Q_c^{ki}:

$$f_c = \frac{1}{e_p}\left\{K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x}\right\} - \frac{Q_c^{ki}}{e_p L^k W^k q^{ki}} \tag{6.12}$$

The total user benefit is then obtained by integrating equation (6.12) over the demand density, Q_c^{ki}, which is expressed as:

$$\int f_c dQ_c^{ki} = \frac{1}{e_p}\left\{K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x}\right\} Q_c^{ki} - \frac{\left(Q_c^{ki}\right)^2}{2 e_p L^k W^k q^{ki}} \tag{6.13}$$

Equation (6.13) is rearranged by substituting the potential demand density q^{ki} using from equation (6.2). The total user benefit TUB_c^{ki} is now formulated as follows:

$$TUB_c^{ki} = \frac{Q_c^{ki} t^{ki}}{2 e_p}\left\{K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} + e_p f_c\right\} \tag{6.14}$$

The consumer surplus is now formulated as the total user benefit minus the fares that users actually pay to the conventional bus providers:

$$G_c^{ki} = \frac{L^k W^k q^{ki} t^{ki}}{2 e_p}\left\{K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c\right\}^2 \tag{6.15}$$

The total welfare in equation (6.8) that sums the producer surplus and consumer surplus in region k and period i is then expressed as:

$$Y_c^{ki} = f_c L^k W^k q^{ki} t^{ki}\left(K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c\right) - B_c \; N_c^k \frac{D^k t^{ki}}{V_c^i h_c^{ki}} + \frac{L^k W^k q^{ki} t^{ki}}{2 e_p}\left(K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c\right)^2 \tag{6.16}$$

The total welfare for the entire system is formulated as follows:

$$Y_c = \sum_{k=1}^{K}\sum_{i=1}^{I} Y_c^{ki} = \sum_{k=1}^{K}\sum_{i=1}^{I}\left(P_c^{ki} + G_c^{ki}\right)$$

$$Y_c = \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{f_c L^k W^k q^{ki} t^{ki}\left(K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c\right) - B_c\ N_c^k \frac{D^k t^{ki}}{V_c^i h_c^{ki}} + \frac{L^k W^k q^{ki} t^{ki}}{2e_p}\left(K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c\right)^2\right\} \tag{6.17}$$

Equation (6.17) can also be written as:

$$Y_c = \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{f_c L^k W^k q^{ki} t^{ki}\left(K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c\right)\right\} - \sum_{k=1}^{K}\sum_{i=1}^{I}\left(B_c\ N_c^k \frac{D^k t^{ki}}{V_c^i h_c^{ki}}\right) + \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{\frac{L^k W^k q^{ki} t^{ki}}{2e_p}\left(K_c^k - e_w z_1 h_c^{ki} - e_x z_2 \frac{r^k}{V_x} - e_p f_c\right)^2\right\} \tag{6.18}$$

The social welfare in equation (6.18) is maximized by optimizing the decision variables of vehicle size, fares, headways, fleet sizes, and route spacings (or the numbers of zones).

6.4.2. Flexible Service Formulations

The welfare of flexible bus services in region k and period i, Y_f^{ki}, is formulated as the sum of producer surplus P_f^{ki} and consumer surplus G_f^{ki}:

$$Y_f^{ki} = P_f^{ki} + G_f^{ki} \tag{6.19}$$

The producer surplus P_f^{ki} is computed by subtracting the flexible bus operating cost from the revenue of the flexible bus service:

$$P_f^{ki} = R_f^{ki} - C_f^{ki} \tag{6.20}$$

The total revenue of the flexible bus service in region k and period i, R_f^{ki}, is the flexible bus service fare multiplied by total demand density:

$$R_f^{ki} = f_f Q_f^{ki} t^{ki} = f_f L^k W^k q_f^{ki} t^{ki} \left(1 - e_w z_1 h_f^{ki} - e_v M_f^{ki} - e_p f_f\right) \tag{6.21}$$

Then, the producer surplus in region k and period i P_f^{ki} is:

$$P_f^{ki} = f_f L^k W^k q_f^{ki} t^{ki} \left(1 - e_w z_1 h_f^{ki} - e_v M_f^{ki} - e_p f_f\right) - \left(\frac{B_f \; N_f^k D_f^k t^{ki}}{V_f^i h_f^{ki}} + \frac{\emptyset B_f \; L^k W^k t^{ki} \sqrt{q_f^{ki} h_f^{ki} / u}}{V_f^i h_f^{ki}} \right) \tag{6.22}$$

The consumer surplus G_f^{ki} in region k and period i the total social benefit of the flexible bus services minus the price that flexible bus users actually pay. The total social benefit of the flexible bus service TSB_f^{ki} can be found by integrating the willingness to pay function:

$$TSB_f^{ki} = \int f_f dQ_f^{ki} = \int \left\{ \frac{1}{e_p} \left(K_f^k - e_w z_1 h_f^{ki} - e_v \frac{D_c^{ki}}{2V_f^i} \right) - \frac{Q_f^{ki}}{e_p L^k W^k q^{ki}} \right\} dQ_f^{ki}$$

$$TSB_f^{ki} = \frac{Q_f^{ki}}{e_p} \left(K_f^k - e_w z_1 h_f^{ki} - e_v \frac{D_c^{ki}}{2V_f^i} \right) - \frac{\left(Q_f^{ki}\right)^2}{2 e_p L^k W^k q^{ki}} \tag{6.23}$$

By substituting the potential demand density q^{ki} from equation (6.4), the total social benefit of the flexible bus in region k and period i TSB_f^{ki} becomes:

$$TSB_f^{ki} = \frac{Q_f^{ki} t^{ki}}{2 e_p} \left(K_f^k - e_w z_1 h_f^{ki} - e_v \frac{D_c^{ki}}{2V_f^i} + e_p f_f \right) \tag{6.24}$$

The consumer surplus of the flexible bus service is now formulated as the total social benefit minus the price that users actually pay to the flexible bus providers:

$$G_f^{ki} = \frac{L^k W^k q^{ki} t^{ki}}{2e_p}\left(K_f^k - e_w z_1 h_f^{ki} - e_v \frac{D_c^{ki}}{2V_f^i} - e_p f_f\right)^2 \tag{6.25}$$

The total welfare of the flexible bus service in region k and period i is now expressed as:

$$Y_f^{ki} = G_f^{ki} + P_f^{ki}$$

$$Y_f^{ki} = \frac{L^k W^k q^{ki} t^{ki}}{2e_p}\left(K_f^k - e_w z_1 h_f^{ki} - e_v \frac{D_c^{ki}}{2V_f^i} - e_p f_f\right)^2 + f_f L^k W^k q^{ki} t^{ki}\{1 - e_w z_1 h_f^{ki} - e_v M_f^{ki} - e_p f_f\} - \left(\frac{B_f\ N_f^k D_f^k t^{ki}}{V_f^i h_f^{ki}} + \frac{\emptyset B_f\ L^k W^k t^{ki}\sqrt{q^{ki} h_f^{ki}/u}}{V_f^i h_f^{ki}}\right) \tag{6.26}$$

The social welfare for the entire flexible bus services is formulated as follows:

$$Y_f = \sum_{k=1}^{K}\sum_{i=1}^{I} Y_f^{ki} = \sum_{k=1}^{K}\sum_{i=1}^{I}\left(G_f^{ki} + P_f^{ki}\right)$$

$$Y_c = \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{\frac{L^k W^k q^{ki} t^{ki}}{2e_p}\left(K_f^k - e_w z_1 h_f^{ki} - e_v \frac{D_c^{ki}}{2V_f^i} - e_p f_f\right)^2 + f_f L^k W^k q^{ki} t^{ki}\{1 - e_w z_1 h_f^{ki} - e_v M_f^{ki} - e_p f_f\} - \left(\frac{B_f\ N_f^k D_f^k t^{ki}}{V_f^i h_f^{ki}} + \frac{\emptyset B_f\ L^k W^k t^{ki}\sqrt{q^{ki} h_f^{ki}/u}}{V_f^i h_f^{ki}}\right)\right\} \tag{6.27}$$

Equation (6.27) can also be written as:

$$Y_f = \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{f_f L^k W^k q^{ki} t^{ki}\left(1 - e_w z_1 h_f^{ki} - e_v M_f^{ki} - e_p f_f\right)\right\} - \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{\frac{B_f\ N_f^k D_f^k t^{ki}}{V_f^i h_f^{ki}} + \frac{\emptyset B_f\ L^k W^k t^{ki}\sqrt{q^{ki} h_f^{ki}/u}}{V_f^i h_f^{ki}}\right\} + \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{\frac{L^k W^k q^{ki} t^{ki}}{2e_p}\left(K_f^k - e_w z_1 h_f^{ki} - e_v \frac{D_c^{ki}}{2V_f^i} - e_p f_f\right)^2\right\} \tag{6.28}$$

Equation (6.28) must be maximized by optimizing the decision variables of flexible bus size, fares, headways, fleet sizes, and service areas.

6.4.3. Solution Method: Purely Numerical Approach

The welfare formulations designed for both conventional and flexible services are nonlinear and they have both integer and continuous variables. In the literature, analytic optimization was applicable to problems of one region. The problem of multiple regions as well as multiple periods cannot be solved by analytic optimization. A numerical method (i.e., a genetic algorithm) is chosen to solve the proposed formulations. The genetic algorithm used in this chapter is called a real coded genetic algorithm (RCGA). Such an RCGA can efficiently handle integer variables.

6.4.4. Discussions with Numerical Example

In this section, a numerical analysis is designed to check formulations without financial constraints. For this case study, the maximum allowable headway constraints are enforced. The vehicle size (seats/bus) is one of the input values, and its sensitivity to the system welfare is also analyzed.

6.4.1.1. Input Values

For a numerical example, three local regions and four time periods are considered. The baseline input values are shown in Table 6-1. The potential demand densities, sizes of regions, and time periods are shown in Table 6-2. The minimum and maximum headways are assumed to be 3 and 60 minutes, respectively. The minimum and maximum fleet sizes can be obtained with headway boundaries. For the vehicle size inputs, 7, 10, 16, 20, 25, 35, and 45 seats are the acceptable values.

Regions A, B, and C have the same demand densities. However, the regional characteristics are different. Region A is 4 $mile^2$, region B is 12.25 $mile^2$, and region C is 25 $mile^2$. Therefore, the total demand in region C exceeds those in regions A or B, although the demand densities are the same. The same demand density inputs are assumed initially for all regions, in order to identify the effects of region size.

Table 6-2 Potential Demand, Service Time, Line-haul Distance, and Sizes of Regions

Demand (trips/$mile^2$/hour)			
Period \ Region	A	B	C
1	90	90	90
2	40	40	40
3	20	20	20
4	10	10	10
Time(hours)			

Period \ Region	A	B	C
1	4	4	4
2	6	6	6
3	8	8	8
4	6	6	6
Region	A	B	C
Line-haul Distance (miles)	6	6	6
Length of Region (miles)	2	3.5	5
Width of Region (miles)	2	3.5	5
Regional Area ($mile^2$)	4	12.25	25

6.4.1.2. Discussion of Results

Constrained optimization problems are solved with a real coded genetic algorithm (RCGA). The maximum allowable headways are enforced as constraints. The first order derivatives of the fare with respect to the total welfare formulations (i.e., equations (6.18) and (6.28)) are set to zero. Then, optimized fares found for both conventional and flexible services are zero. However, Figure 6-2 shows that optimized fares of conventional services with vehicle sizes 7 and 10 seats are non-zero. With these smaller vehicle sizes, optimized headways are less than three minutes, thereby violating the low boundary of the headway. In the other words, all demand cannot be served with the minimum 3 minutes headways. Thus, it is confirmed that the optimization model cannot find any feasible solutions with the conventional vehicle sizes of 7 or 10 seats because the input demand is too high. It is possible that if demand inputs were lower, conventional services with 7 or 10 seats buses could have solutions with optimized fares of zero. The flexible services also cannot find any feasible solutions for vehicle sizes of 7, 10, and 16 seats.

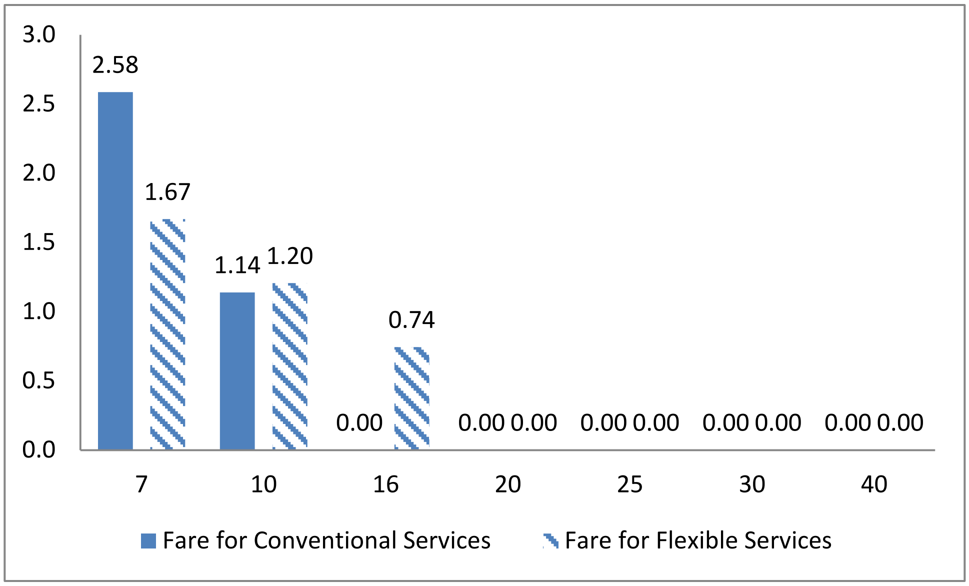


Figure 6-2 Optimized Fares with Vehicle Size Inputs

Figure 6-3 shows that the welfare is maximized (at 120219 $/day) when the conventional bus size is 25 seats. For the flexible services, the maximum welfare of flexible services is 113999 $/day with 20 seat vehicles.

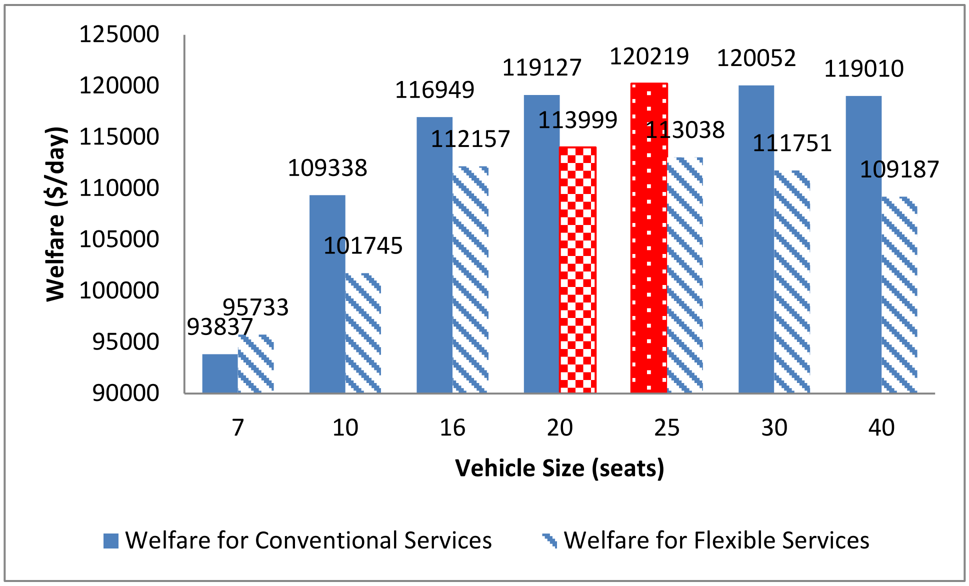


Figure 6-3 Welfare versus Vehicle Size Inputs

Table 6-3 shows the actual demand density results of conventional and flexible services. The actual demands of conventional services among regions A, B and C are very close (less than one trip/mile^2/hour). These results confirm that the size of regions does not significantly affect the demand elasticity. However, it is found that the actual trips for flexible services are higher than the actual trips for conventional services.

The main reason for higher actual trips in flexible services is that flexible services have door-to-door services, and hence zero access costs. However, conventional services include access times in the elastic demand function. Thus, it is noted that when demands are sensitive to the in-vehicle time, waiting time, and access time, flexible services are preferable to conventional services in terms of the total actual trips served.

Table 6-3 Demand with Elasticity

Demand (trips/mile²/hour)						
	Conventional Services			Flexible Services		
Period \ Region	A	B	C	A	B	C
1	74.21	73.59	73.35	77.01	75.86	74.20
2	32.53	31.74	31.47	33.68	32.83	32.30
3	15.81	15.26	15.36	16.05	16.32	15.98
4	7.22	6.72	7.30	7.85	7.55	8.10

Table 6-4 shows the optimized number of zones for conventional and flexible services. The numbers of zones for conventional services are two, four, and six for regions A, B, and C, respectively. The route spacings (which can be obtained by Width of region / Number of zones) are then 1.0, 0.875, and 0.833 miles for regions A, B, and C, respectively. The number of zones increases as the width of a region increases.

The sizes of regions A, B, and C are 4, 12.25, and 25 mile^2, respectively, as shown in Table 6-2. The numbers of zones for flexible services are one, three, and five for regions A, B, and C, respectively. Hence, optimized service areas for flexible services are 4.0, 4.08, and 5.0 mile^2 for regions A, B, and C, respectively. Additional zones increase operating costs. Thus, it is concluded that the optimal areas of flexible services with given inputs range between four and five square miles.

Table 6-4 Optimized Number of Zones

	Conventional Services			Flexible Services		
Period \ Region	A	B	C	A	B	C
Number of Zones	2	4	6	1	3	5
Route Spacings (mile)	1.0	0.875	0.833	-	-	-
Service Areas ($mile^2$)	-	-	-	4.0	4.08	5.0

The optimized headways for both conventional and flexible services are provided in Table 6-5. For conventional services, period 1, which has higher demand densities than other periods, has optimized headways of about four to six minutes. Optimized headways increase as demand densities decrease. It is also found that headways of flexible services are generally lower than headways of conventional services if they are compared in the same period and the same region. This indirectly explains why flexible services produce more actual trips than conventional services. For period 1, flexible service headways are slightly above 3 minutes, which is the minimum headway boundary.

For conventional services, the longest headway, which is about 31 minutes, is used for period 4 in region B. For flexible services, period 4 in region B has headways of about 21 minutes.

Table 6-5 Optimized Headways in Minutes

Headways (minute)						
	Conventional Services			Flexible Services		
Region / Period	A	B	C	A	B	C
1	5.89	6.24	4.86	3.53	3.59	3.17
2	7.85	10.41	9.72	6.77	7.68	6.60
3	11.78	15.61	12.96	13.54	10.52	9.84
4	23.56	31.22	19.44	18.43	21.03	10.83

Table 6-6 shows optimized fleet sizes for conventional and flexible services. As expected, period 1 requires larger fleet sizes than other periods. It is also noted that flexible services require much larger fleet sizes than conventional services. Conventional services require a total of 166 vehicles with 25 seats, while flexible services require 268 vehicles with 20 seats. Larger fleet sizes imply higher operating costs.

Table 6-6 Optimized Fleet Sizes

Fleet Sizes (buses)						
	Conventional Services			Flexible Services		
Region / Period	A	B	C	A	B	C
1	4	5	8	12	13	17
2	3	3	4	6	6	8
3	2	2	3	3	4	5
4	1	1	2	2	2	4
Total Fleet Size (buses)	166			268		

Table 6-7 provides costs and profits for conventional and flexible services. The analytically optimized fares using equations (6.18) and (6.28) are zero. Numerically optimized fares for both conventional and flexible services also show that fares are zero. Thus, without subsidy, revenues of conventional and flexible services in any periods are zero, and thus total revenues are also zero. Therefore, profits are simply negative values of costs in each period and each region. Costs and profits are shown per hour because each period has a different duration. As shown in Figure 6-7, the total cost of flexible services is about 52% higher than the total cost of conventional services.

Table 6-7 Costs and Profits for Conventional and Flexible Services

Cost ($/hour)						
	Conventional Services			Flexible Services		
Period \ Region	A	B	C	A	B	C
1	280.0	700.0	1680.0	408.0	1326.0	2890.0
2	210.0	420.0	840.0	204.0	612.0	1360.0
3	140.0	280.0	630.0	102.0	408.0	850.0
4	70.0	140.0	420.0	68.0	204.0	680.0
Total Cost ($/day)	31640			48144		
Profit ($/hour)						
	Conventional Services			Flexible Services		
Period \ Region	A	B	C	A	B	C

1	-280.0	-700.0	-1680.0	-408.0	-1326.0	-2890.0
2	-210.0	-420.0	-840.0	-204.0	-612.0	-1360.0
3	-140.0	-280.0	-630.0	-102.0	-408.0	-850.0
4	-70.0	-140.0	-420.0	-68.0	-204.0	-680.0
Total Profit ($/day)	-31640			-48144		

Table 6-8 shows results of the consumer surplus from conventional and flexible services. Period 1 in region A has the highest consumer surplus, which is 42705$/period for conventional services and 43692 $/period. It is found that the consumer surplus of flexible services exceeds the one for conventional services. The main reason is that with the elasticity, the actual trips for flexible services exceed those for conventional services. The main reason for the difference in actual trips is the access time factor, as already discussed. The total consumer surplus in flexible services is $162143/day while the total consumer surplus in conventional services is $151859/day.

Table 6-8 Consumer Surplus

Consumer Surplus ($/period)						
	Conventional Services			Flexible Services		
Region / Period	A	B	C	A	B	C
1	6994.1	21060.7	42705.5	7531.7	22381.2	43692.5
2	4534.1	13218.9	26523.8	4862.4	14148.1	27936.9
3	2854.9	8151.2	16842.9	2944.8	9321.6	18240.9
4	892.5	2370.6	5709.4	1056.3	2989.8	7037.4

Total ($/day)	151859	162143

From previously discussed results, it is found that flexible services have a larger consumer surplus than conventional services. Flexible services also have higher costs than conventional services, which explain why flexible services have a larger negative profit (i.e., loss) than conventional services.

Table 6-9 provides welfare results of conventional and flexible services for each period and region. It is noted that the total welfare of conventional services exceeds that of flexible services. For region A, the welfare of flexible services exceeds that of conventional services. For regions B and C, conventional services produce greater welfare than flexible services. As discussed, the higher cost of flexible service is the main reason why welfare is higher in conventional services than in flexible services. The total welfare difference between conventional and flexible services is about 5.45% (=120219/113999).

Table 6-9 Social Welfare

Welfare ($/period)						
	Conventional Services			Flexible Services		
Region / Period	A	B	C	A	B	C
1	5874.1	18260.7	35985.5	5899.7	17077.2	32132.5

2	3274.1	10698.9	21483.8	3638.4	10476.1	19776.9
3	1734.9	5911.2	11802.9	2128.8	6057.6	11440.9
4	472.5	1530.6	3189.4	648.3	1765.8	2957.4
Total ($/day)	120219			113999		

6.5. Welfare Maximization with Financial Constraint

In addition to vehicle capacity constraints, financial (i.e., subsidy) constraints are considered in this section. With various subsidy inputs, the resulting variations of fares, headways and fleet sizes are explored. To consider additional financial constraints, formulations for conventional and flexible services are modified.

The total welfare Y_c is the sum of the welfare for all time and all regions, shown in equation (6.29). The financial constraint is expressed in equation (6.30). The amount of subsidies is an input value. If zero subsidies are provided, the financial constraint simply becomes that the profit should be non-negative. The maximum allowable headway (service capacity) constraints are also applied in equation (6.31):

$$Maximize\ Y_c = \sum_{k=1}^{K}\sum_{i=1}^{I}\{Y_c^{ki}\} \quad (6.29)$$

subject to

$$\sum_{k=1}^{K}\sum_{i=1}^{I}\{P_c^{ki}\} + \sum_{k=1}^{K}\sum_{i=1}^{I}\{FS^{ki}\} \geq 0 \quad (6.30)$$

$$h_c^{ki} \leq h_{c,max}^{ki} = \frac{S_c l_c}{r^k W^k d_s^{ki} Q_c^{ki}} \quad (6.31)$$

6.5.1. Flexible Service Formulations

Flexible service formulations that consider financial constraints are provided in equations (6.32~6.34). The maximum allowable headway constraints for flexible services in equation (6.34) are different from those for conventional services in equation (6.31):

$$Maximize\ Y_f = \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{Y_f^{ki}\right\} \tag{6.32}$$

subject to

$$\sum_{k=1}^{K}\sum_{i=1}^{I}\left\{P_f^{ki}\right\} + \sum_{k=1}^{K}\sum_{i=1}^{I}\left\{FS^{ki}\right\} \geq 0 \tag{6.33}$$

$$h_f^{ki} \leq h_{f,max}^{ki} = \frac{S_f l_f}{A^k Q_f^{ki}} \tag{6.34}$$

6.5.2. Solution Method: Purely Numerical Approach

Welfare formulations for conventional and flexible services are highly nonlinear. In addition to the nonlinear objective functions, constraints are moved to the objective function with the Lagrange multiplier. Then, the objective function becomes more complex. Since objective functions are nonlinear and variables are continuous or integer, a real coded genetic algorithm (RCGA) is chosen to solve formulations. Fares for either conventional or flexible services are continuous variables, and fleet sizes are integer variables. Headways can be obtained from the optimized fleet sizes.

6.5.3. Discussion of Numerical Examples

In this numerical example, financial (subsidy) constraints are enforced in addition to maximum allowable headway constraints. Different input values for subsidy are considered through sensitivity analysis. As explained below, the sum of the total revenue and the total subsidy should be larger or equal to the total cost. If the total subsidy is zero, the total revenue minus the total cost (i.e. the profit) should be non-negative. The total subsidy is an input value so that unit subsidy ($/potential trip) is used to calculate the total amount of subsidies in this numerical analysis.

It is possible to jointly optimize vehicle sizes, numbers of zones, headways, and fleet sizes with a financial constraint. However, computation times are much longer and optimized vehicle sizes and numbers of zones are not significantly different from the ones in the financially unconstrained case. Thus, by using the optimized vehicle sizes and numbers of zones from financially unconstrained results, the complexity of financially constrained welfare formulations is reduced and converged solutions are found relatively quickly. It is also reasonable to think that route spacings of conventional services, service areas of flexible services, and vehicle sizes can be determined in an earlier planning level. The service providers (operators) may then want to re-optimize service frequencies and fares based on the subsidy.

Thus, headways, fleet sizes, and fares are optimized here with the various subsidy inputs. The value of route spacings for conventional services, service areas for flexible services, and vehicle sizes are adapted from the solution of the financially unconstrained optimization model. The main focus of this analysis is on exploring how optimized fares are changed with different financial constraints (i.e., subsidy). Results of conventional services will be discussed first, and then results of flexible services will be discussed.

6.5.3.1. Results for Conventional Services

Subsidy inputs are applied from zero to 1.2$/potential trip with 0.2$/potential trip increment. Table 6-10 provides detailed results for conventional services with various subsidy inputs.

For conventional services, seven sensitivity cases are considered, as shown in Figure 6-4. The amount of subsidy increases linearly. The total number of potential trips for the system with given inputs is 33825. Thus, when the unit subsidy is 1.0$/potential trip, the total subsidy is $33825/day.

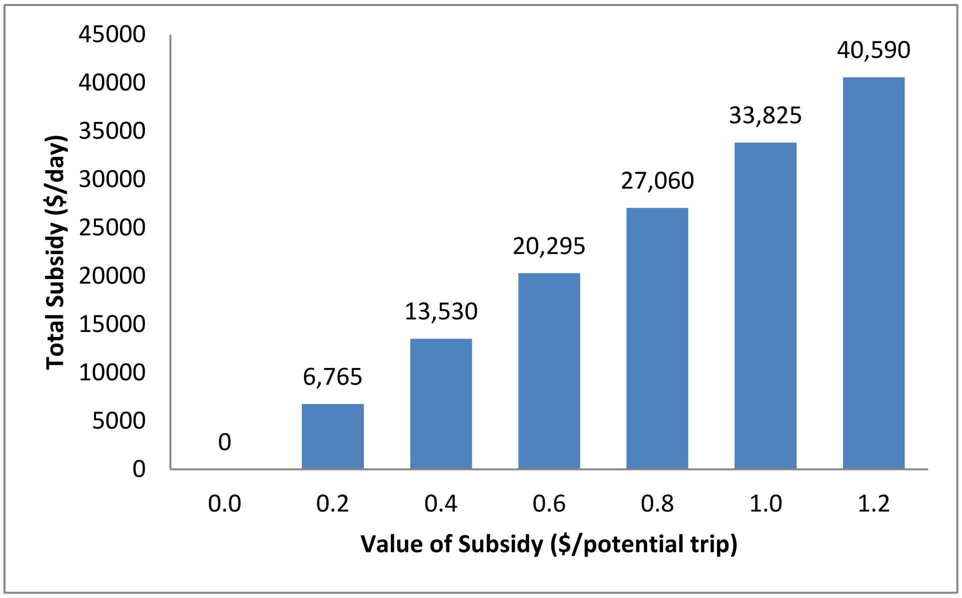


Figure 6-4 Total Amount of Subsidy

Figure 6-5 provides optimized fares of conventional services from various subsidy inputs. For the zero subsidy case, the fare for conventional services is 1.3$/actual trip. As the subsidies increases, the optimized fare decreases quite linearly. When the unit subsidy is about 1.0$/potential trip, the fare becomes zero, which means the total revenue is zero, and all the costs of bus operations are covered by the subsidy.

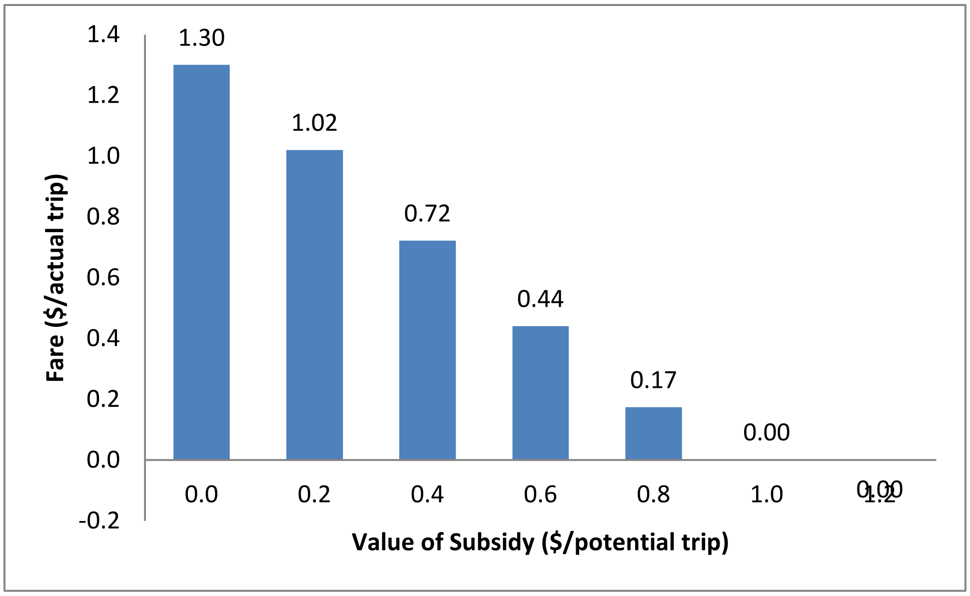


Figure 6-5 Fares for Conventional Services with Subsidies

Figure 6-6 provides profit results from various financial constraints (i.e., subsidy inputs). With the subsidy provision, the revenue decreases since the optimized fare decreases. Thus, the profit also decreases (as expected) because the revenue decreases. In the formulation the sum of the profit and subsidy can be either zero or positive. For the zero subsidy case result, the profit is positive, which means the optimized fare could have been slightly reduced to use this available budget.

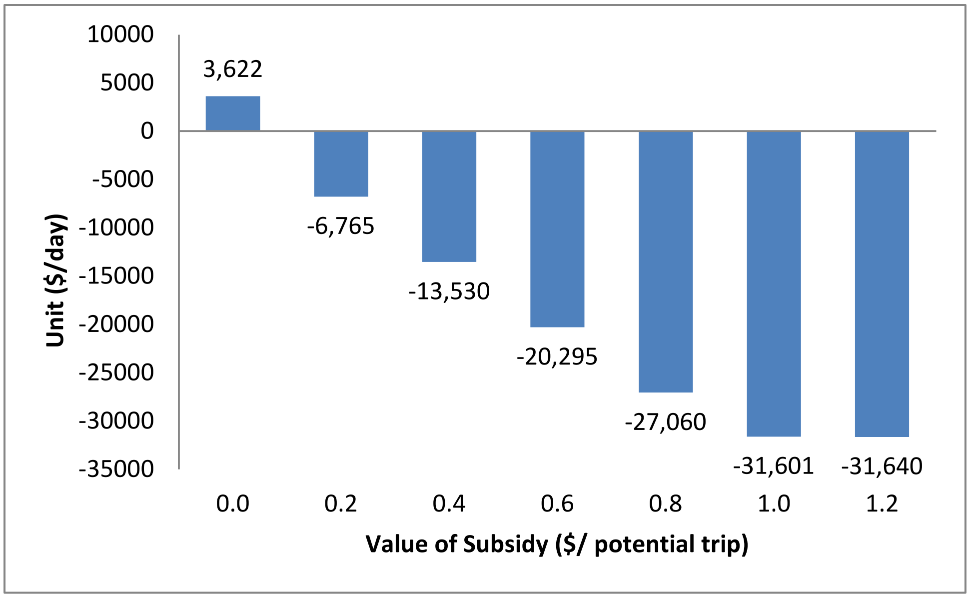


Figure 6-6 Profits of Conventional Services with Subsidies

Figure 6-7 shows total costs of conventional services for various subsidy inputs. It is interesting to note that the cost of the zero subsidy case is lower than other subsidy cases. From the 0.2 $/potential trip to 1.2$/potential trip, total costs are identical, which means, their resulting fleet sizes do not change over different subsidy inputs. It explains why fleet sizes and headways do not change significantly in conventional services with financial constraints while fares are changed.

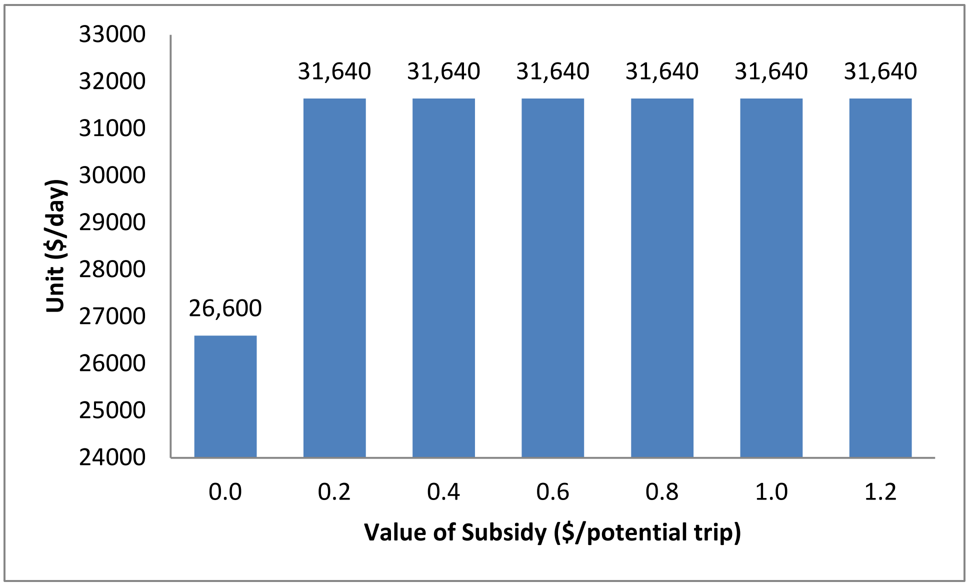


Figure 6-7 Total Costs of Conventional Services with Subsidies

The total consumer surplus in the zero subsidy case is $114699/day, as shown in Figure 6-8. Consumer surplus results of other subsidy cases show that the consumer surplus increases until the unit subsidy is 1.0$/potential trip. After that, the consumer surplus does not change significantly. When the unit subsidy is 1.2$/potential trip, the consumer surplus for the conventional services is 151859$/day. The consumer surplus difference between the subsidy inputs 1.0 and 1.2 $/potential trip is 39$/day, which is tiny if 39$ is divided by the total actual trips served per day. Thus, it can be confirmed that the consumer surplus converges beyond a unit subsidy of 1.0$/potential trip.

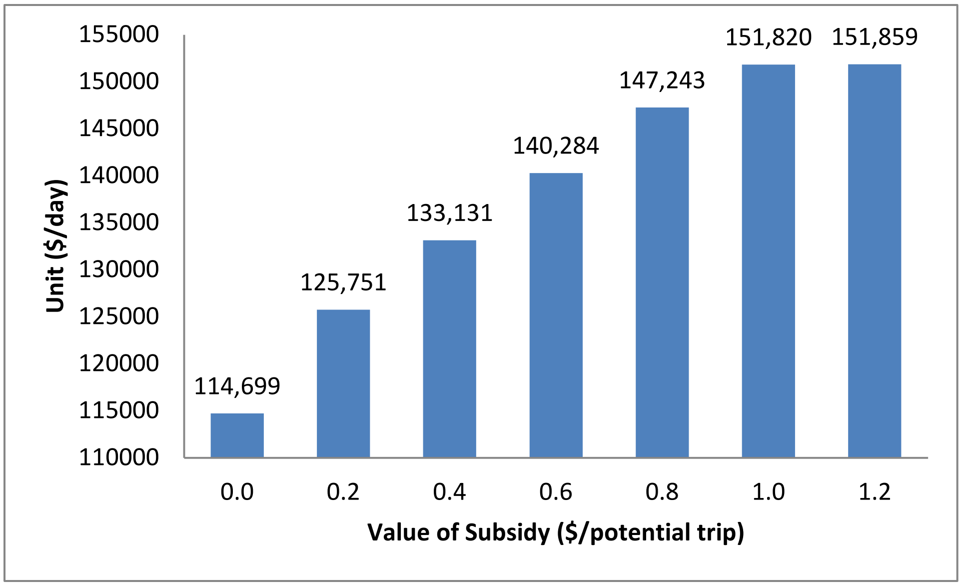


Figure 6-8 Consumer Surplus of Conventional Services with Subsidies

Figure 6-9 shows the social welfare results for conventional services. The welfare of the zero subsidy case is 118321$/day, while the maximum system welfare is found as 120219$/day from the unit subsidy of 1.0$/potential trip or more. As expected, when the total cost is fully covered by subsidies, the system welfare becomes identical to the one without financial constraints (discussed in the previous section). There is no unusual observation among comprehensive sets of sensitivity analyses. Thus, numerical results confirm that a RCGA used here finds good and consistent solutions although it does not guarantee the global optimality of solutions.

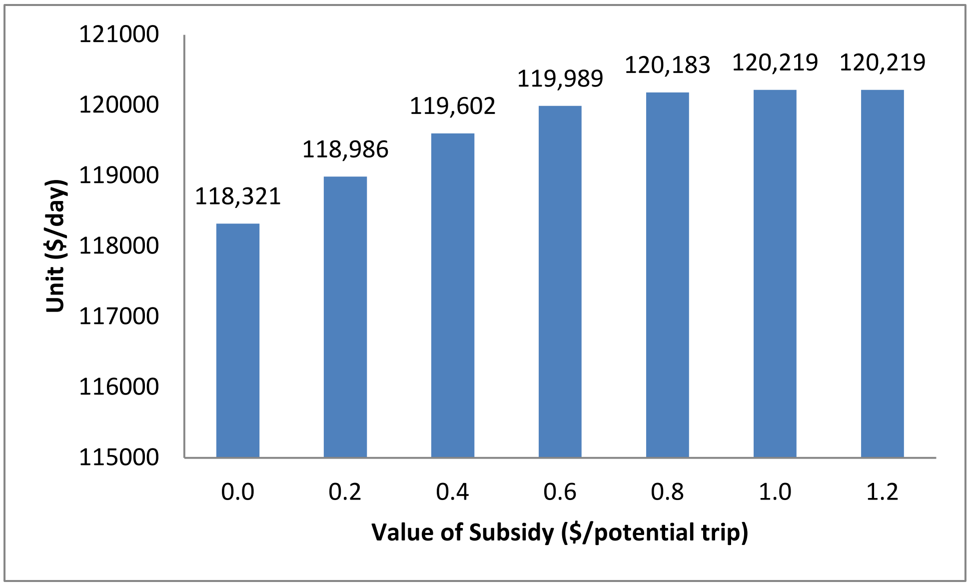


Figure 6-9 Total System Welfares of Conventional Services with Subsidies

Table 6-10 summarizes all results of conventional services with various subsidy input values. One further finding worth noting is that actual trips increase as the subsidy increases. For instance, the zero subsidy case serves about 68.7% of the total potential demand, but the fully subsidized case carries about 79.2% of the total potential demand.

Table 6-10 Results of Conventional Services with Financial Constraints

Unit Subsidy ($/trip)	0.0	0.2	0.4	0.6	0.8	1.0	1.2
Fare	1.30	1.02	0.72	0.44	0.17	0.00	0.00
Revenue	30221.6	24875.0	18110.4	11345.0	4580.0	38.8	0.0
Cost	26600.0	31640.0	31640.0	31640.0	31640.0	31640.0	31640.0
Profit	3621.6	-6765.0	-13529.6	-20295.0	-27060.0	-31601.2	-31640.0
Subsidy	0.0	6765.0	13530.0	20295.0	27060.0	33825.0	40590.0
Profit + Subsidy	3621.6	0.0	0.4	0.0	0.0	2223.8	8950.0
Consumer Surplus	114699.0	125751.3	133131.3	140283.9	147242.9	151819.8	151858.6
Welfare	118321.0	118986.3	119601.7	119988.9	120183.0	120218.6	120218.6

Total Actual Trips	23247	24381	25087	25754	26386	26793	26797
Total Actual Trips / Total Potential Trips	68.7%	72.1%	74.2%	76.1%	78.0%	79.2%	79.2%

6.5.3.2. Results for Flexible Services

Figures 6-10~6-15 provide results for flexible services with financially constrained cases (i.e., sensitivity analyses of subsidies with respect to welfares). Table 6-11 also provides details on these results.

Figure 6-10 shows optimized fares for flexible services with different subsidy inputs. In the zero subsidy case, the optimized fare is 1.91$/actual trip, which exceeds the optimized fare (1.30$/actual trip, Table 6-5) of conventional services with the zero subsidy case. The higher flexible service operating cost results in the higher flexible service fare. The optimized fares decrease as the subsidy increases. When the unit subsidy is 1.4$/potential trip, the optimized fare for flexible services is close to zero (three cents per actual trip). When the unit subsidy is 1.4$/potential trip, the total subsidy is 47355$/day, as shown in Figure 6-11. For conventional services (shown in Figure 6-5), the optimized fare becomes zero when the subsidy reaches 1.0$/potential trip. Thus, it is found that flexible services require larger subsidies than conventional services to cover all the operating cost.

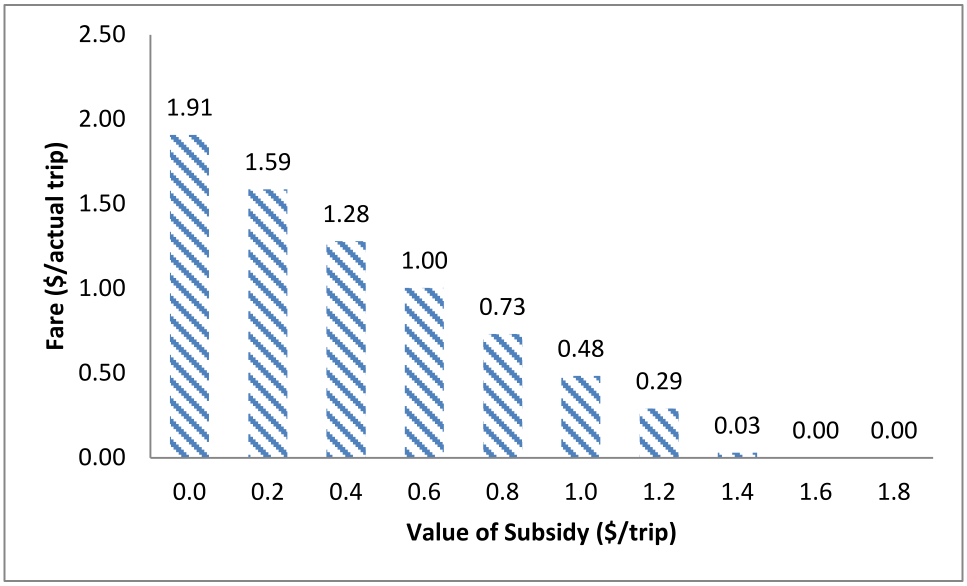


Figure 6-10 Fares for Flexible Services with Subsidies

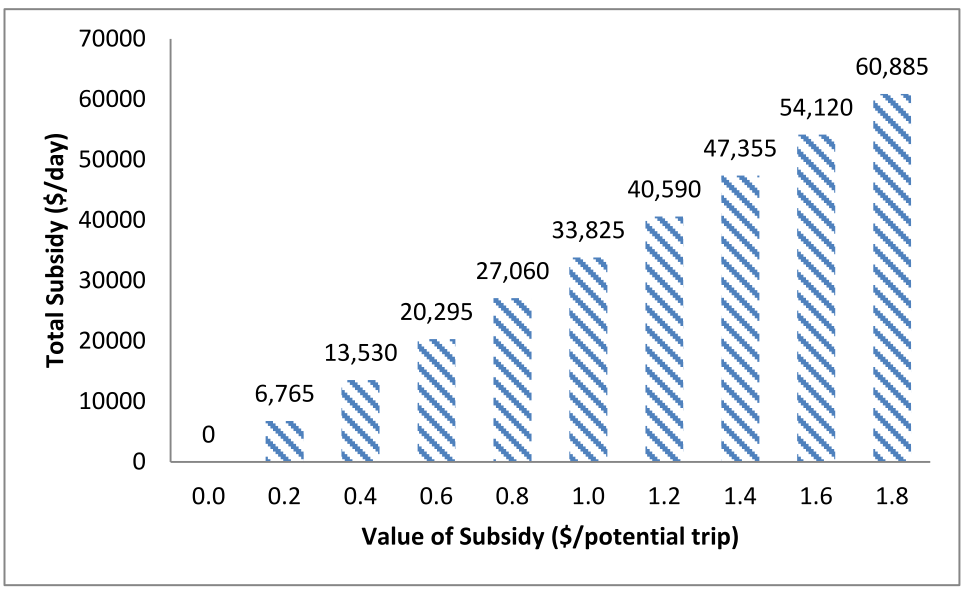


Figure 6-11Total Amount of Subsidy

Figure 6-12 provides results of the profit. For the zero subsidy case, flexible services have zero profit, as expected; this means the operating cost is exactly equal to the revenue. The profit decreases as the subsidy increases because the optimized fare decreases.

The cost of flexible service operation increases with the provision of subsidies since the financial subsidy allows providing more service frequencies. As shown in Figure 6-13, the total operating cost increases with the larger subsidy. The absolute value of the minimum profit in Figure 6-12 and the absolute value of the maximum cost in Figure 6-13 are identical (i.e., unit subsidy of 1.8$/actual trip). This can also explain why the optimized fare and revenue are zero.

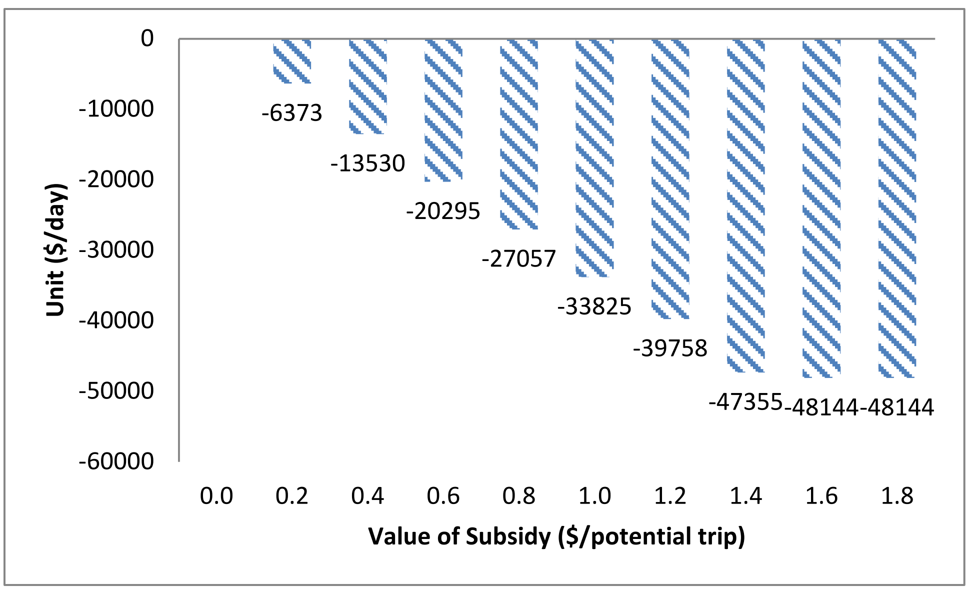


Figure 6-12 Profits of Flexible Services with Subsidies

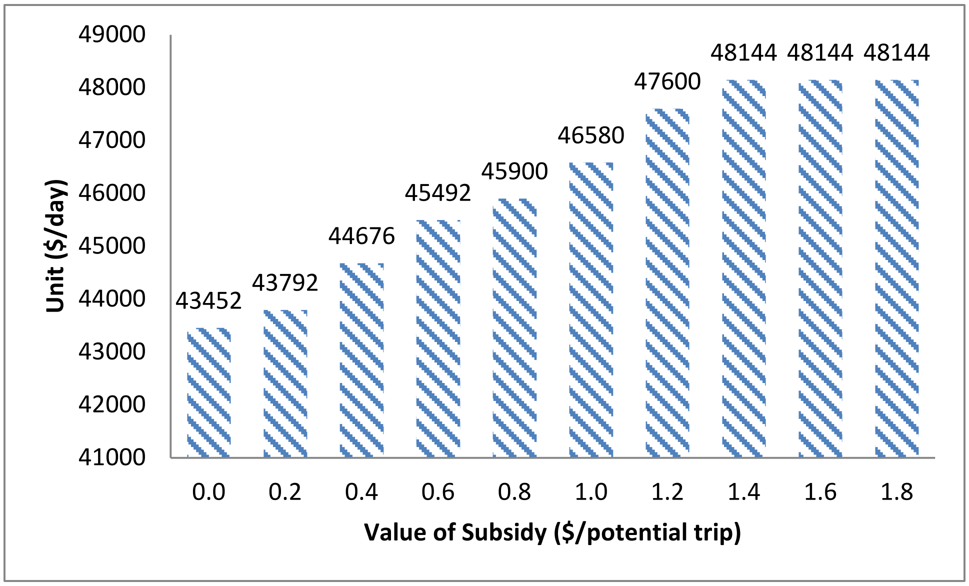


Figure 6-13 Costs of Flexible Services with Subsidies

The consumer surplus with the zero subsidy case is 109693$/day, as shown in Figure 6-14. The higher subsidies result in the reduced fare and increase in actual trips. Therefore, the consumer surplus increases as the subsidy increases. The maximum consumer surplus is 162143$/day when the unit subsidy is 1.6 $/potential trip or higher.

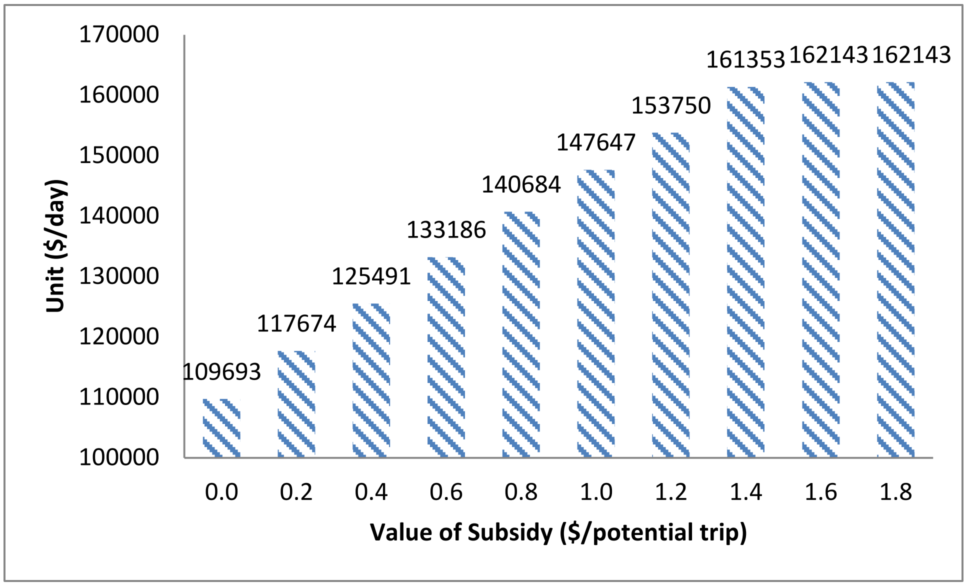


Figure 6-14 Consumer Surplus of Flexible Services with Subsidies

Figure 6-15 provides results of the system welfare for flexible services. The welfare in the zero subsidy case is 109693$/day, which is identical to the consumer surplus because the profit of the zero subsidy case is zero. The system welfare of flexible services converges to 113999$/day without exhausting the available subsidies.

Table 6-11 shows all detail results of flexible services. In Table 6-11 there is a row with "Profit + Subsidy". When the unit subsidy is 1.8 $/potential trip, the sum of the profit and the subsidy is positive, which means some budget is still available but unused. In the formulation, the sum of profit and subsidy is larger or equal to the cost. Therefore, the system does not have to use the entire budget. With this leftover amount of subsidies, the system welfare is not decreasing after reaching its maximum. Therefore, results confirm that unit subsidies beyond about 1.2$/potential trip yield no additional social benefits.

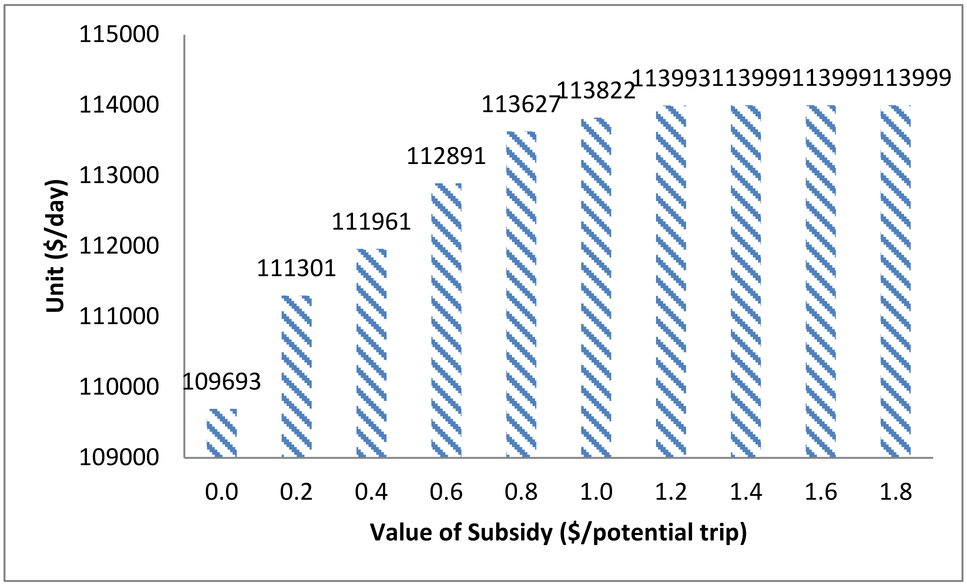


Figure 6-15 Total System Welfares of Flexible Services with Subsidies

In the zero subsidy case, 67.7% of the total potential demand yields actual trips. However, when the operating cost is fully subsidized, about 81.9% of the potential demand is served.

Table 6-11 Results of Flexible Services with Financial Constraints

Unit Subsidy ($/trip)	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
Fare	1.91	1.59	1.28	1.00	0.73	0.48	0.29	0.03	0.00	0.00

Revenue	43452.0	37419.1	31146.0	25197.0	18842.8	12755.2	7842.3	789.0	0.0	0.2
Cost	43452.0	43792.0	44676.0	45492.0	45900.0	46580.0	47600.0	48144.0	48144.0	48144.0
Profit	0.0	-6372.9	-13530.0	-20295.0	-27057.2	-33824.8	-39757.7	-47355.0	-48144.0	-48143.8
Subsidy	0.0	6765.0	13530.0	20295.0	27060.0	33825.0	40590.0	47355.0	54120.0	60885.0
Profit + Subsidy	0.0	392.1	0.0	0.0	2.8	0.2	832.3	0.0	5976.0	12741.2
Consumer Surplus	109692.6	117674.1	125490.7	133186.4	140684.4	147646.5	153750.4	161353.5	162143.5	162143.3
Welfare	109692.6	111301.2	111960.7	112891.4	113627.2	113821.7	113992.6	113998.5	113999.5	113999.5
Total Actual Trips	22774.8	23597.9	24359.6	25102.6	25803.4	26434.1	26977.1	27635.3	27702.9	27702.9
Total Actual Trips / Total Potential Trips	67.3%	69.8%	72.0%	74.2%	76.3%	78.1%	79.8%	81.7%	81.9%	81.9%

6.6. Chapter Summary

In this chapter, conventional and flexible services are formulated with the demand elasticity. The actual ridership is formulated as a linear function using elastic factors of the fare, in-vehicle time, waiting time, and access time. The welfare, which is sum of the consumer surplus and the producer surplus, are also formulated for multiple regions as well as multiple time periods for conventional and flexible services. Two constrained optimization models are analyzed. They have: 1) an objective of the maximum system welfare with the service capacity (maximum headway) constraints, and 2) an objective of the maximum system welfare with the service capacity and financial constraints, for both conventional and flexible services.

Objective functions (i.e., welfare functions) are highly nonlinear and decision variables include continuous and integer variables. Such nonlinear mixed integer formulations are known as NP-hard problems, and have no proven method for finding their exact optimum solution. Commercial optimization programs such as GAMS or LINGO are excluded because they only guarantee a local solution. Thus, a genetic algorithm, which is an iterative global solution search technique, is chosen to solve formulations.

In numerical examples, the fares, route spacings for conventional services, service areas for flexible services, headways and fleet sizes are optimized. The numerical examples show that the welfare of conventional services exceeds those of flexible services, with given input values. Numerical examples also explore the sensitivity of vehicle sizes and the sensitivity of the subsidies with respect to the social welfare for conventional and flexible services. For both conventional and flexible services, the actual trips increase as the subsidies increase.

For conventional services, the problem of one local region with multiple periods has been solved in previous studies. For flexible services, a problem with one local region and one period has also been solved in the literature. These were all solved with analytic optimization (and with approximations). This chapter extends the welfare problems of conventional and flexible services to the multiple regions and multiple periods.

Chapter 7 Conclusions and Future Studies

This dissertation analyzes several interesting problems in order to integrate bus transit systems. Contributions of this research are valuable for bus transit planning purposes. To be implemented realistically, further research is required, as discussed in the future studies section. Findings and contributions of this research are discussed below.

7.1. Findings and Contributions

7.1.1. Integrating Bus Services with Conventional and Flexible Buses

In Chapter 3, optimization models are developed for analyzing and integrating conventional bus services (having fixed routes and schedules) and flexible bus services (many-to-one or one-to-many demand patterns). Flexible services are formulated to pick-up or drop passengers concurrently. The optimization models are improved from those of Chang and Schonfeld (1991a). More specifically, (1) cost functions in conventional bus service are modified to reflect two-directional demands in round trips, (2) flexible service headways are optimized rather than using maximum allowable headways, (3) an analysis is presented that compares conventional, flexible and variable-type bus services which can switch between conventional and flexible service as the demand changes over time.

For a terminal connected to one local region, numerical analyses indicate that variable-type bus operations can reduce the total cost compared to purely conventional bus or purely flexible bus services. In the baseline case, variable-type services decrease costs by about 1.29% compared to purely conventional services and about 10.64% compared to purely flexible services. Moreover, various sensitivity analyses are used to explore how major parameter changes affect the optimized results. In Case IV (when service periods are adjusted to increase the variability of demand over time), it is found that variable-type services decrease costs by more than 3.41% and 13.08 %, respectively, compared to purely conventional and flexible services. These results confirm that such variable-type services are especially promising for systems whose demand (1) varies greatly over time and (2) straddles the threshold between conventional and flexible services.

If transit demand has heterogeneous characteristics, it may be desirable to separate demand with homogeneous patterns. The analysis of multiple regions is then required to handle such demand variability in the transit services coverage. Thus, Chapter 4 extends the Single Fleet Variable Type Service (SFV) to problems of multiple regions as well as multiple periods. Since analytic optimization is difficult to extend to problems with multiple regions as well as multiple periods, a combination of analytic optimization and a genetic algorithm is developed to find solutions. The base case results and sensitivity analyses show that SFV becomes preferable to Single Fleet Conventional Services (SFC) or Single Fleet Flexible Services (SFF) when demand densities fluctuate over times and over regions. It is also shown that when demand densities are very high, SFV provides conventional services to all regions in all periods because flexible services are not preferable when demand densities are high. Thus, SFV converges to SFC. Similarly, when demand densities are very low and have low variability, SFV provides all services with flexible type bus operations. Thus, SFV converges to SFF. It is therefore found that conventional services with large buses are preferable when demands are high. Similarly, flexible services are less costly at relatively low demands. A bus system alternating among these two service concepts based on demand variation and other conditions can be used to improve service efficiency.

7.1.2. Integrating Bus Services with Mixed Fleets

In Chapter 5, the optimization models are extended to analyze bus services to the multiple regions and periods with mixed fleets (containing different vehicle sizes). To reduce total costs when demand and other factors vary over times and over regions, the integration of conventional and flexible services with mixed fleets (i.e., Mixed Fleet Variable Type Bus (MFV)) is explored by comparing four alternatives, namely SFC, SFF, Mixed Fleet Conventional Bus (MFC), and Mixed Fleet Flexible Bus (MFF). For mixed fleet operations (i.e. MFC, MFF, and MFV), the demand thresholds between using large or small buses are analytically formulated using bus operation cost functions. Currently, no attempt for bus transit integration problems with mixed fleets exists in the literature.

For optimizing the decision variables, a hybrid solution method is proposed, which combines a genetic algorithm and analytic optimization. To examine the quality of solutions, one million random candidate solutions are generated and compared to the best solution found by the proposed hybrid algorithm. It is found that the solution obtained with the hybrid method proposed here is superior to any of the million random solutions. An additional small problem (i.e., two regions with four periods) is designed to obtain complete enumeration solutions. The proposed hybrid method also finds the solution obtained through complete enumeration. Thus, it may be concluded that the proposed hybrid method yields solutions that are at least near-optimal.

As shown in Table 5-7, the benefits of sharing fleets throughout the system are explored. Through numerical evaluations, it is found that the cost of an integrated multi-zone system is lower than the sum of separately optimized results. Numerical evaluations also show that MFV can yield significantly lower costs than the other four alternatives. Other numerical cases and sensitivity analyses confirm that the proposed approach finds very good solutions quickly.

7.1.3. Analyses of Social Welfare for Conventional and Flexible Services with Demand Elasticity

In Chapter 6, conventional and flexible services are formulated with demand elasticity. With demand elasticity, the social welfare, which is sum of the consumer surplus and the producer surplus, is the relevant objective function for both conventional and flexible services, for multiple regions as well as multiple time periods. Two constrained optimization models are formulated and discussed. They: 1) maximize social welfare with service capacity (maximum allowable headway) constraints, and 2) maximize welfare with service capacity and financial (subsidy) constraints, for both conventional and flexible services.

Objective functions (i.e., welfare functions) are highly nonlinear and decision variables include continuous and integer variables. Such nonlinear mixed integer formulations are known as NP-hard problems. There are no proven methods to find their exact optima. Commercial software such as GAMS or LINGO is excluded as solution approaches because they only guarantee finding a local solution. Thus, a read coded genetic algorithm, which is an iterative global solution search technique, is chosen to find solutions, even though it does not guarantee global optimality.

In numerical examples, the fares, route spacings for conventional services, service areas for flexible services, headways and fleet sizes are jointly optimized. The numerical examples show that the welfare of conventional services exceeds that of flexible services, with the given input values. The details on conventional and flexible services are discussed along with the sensitivity of vehicle sizes, and the sensitivity of the subsidies. For both conventional and flexible services, the total actual trips increase as the amount of subsidies increases. The input parameters for numerical analyses in Chapter 6 are mostly adopted from a previous paper by Chang and Schonfeld (1993). Further sensitivity analyses of input parameters for elastic demand functions and operating cost functions may be required to reflect current and future transit operations.

For conventional services, the problem of one local region with multiple periods has been solved in previous studies. For flexible services, a problem with one local region and one period has also been solved in the literature. These were all solved with analytic optimization (and with approximations). Chapter 6 extends the welfare problems of conventional and flexible services to the multiple regions and multiple periods.

7.1.4. Discussion of Solution Methods

This dissertation explores three different problem types that have all nonlinear objective functions with mixed integer variables. In Chapter 3, a purely analytic solution is proposed, which is able to find the globally optimal solution. The analytic optimization approach is fast and also insightful because it provides closed form solutions. However, analytic solutions become unreachable when problem become more complex (e.g., multiple regions and periods).

In the Chapter 6, all formulations are solved in one-stage, which means that all decision variables are found simultaneously. A real coded genetic algorithm is chosen to find solutions. There is no guarantee of finding the global optimum, but numerical analyses can confirm that a real coded genetic algorithm finds good solutions. Since all decision variables are optimized in one-stage, solutions converged with longer computational times compared to the computational time of a hybrid approach. To reduce computational times, specifically customized genetic algorithm operators for bus transit problems are desirable. General-purpose genetic algorithms such as a real coded genetic algorithm may only be feasible for planning purposes.

In Chapters 4 and 5, hybrid solution methods are proposed that combine analytic optimization and a genetic algorithm. The number of decision variables for a genetic algorithm is reduced by considering partly-analytic two-stage formulations. Once a genetic algorithm selects values for some decision variables, those are used to optimize other decision variables through analytic optimization. The numerical analyses discussed in Chapter 5 show that the solution obtained from the proposed hybrid method is superior to randomly generated one million candidate solutions. It is also confirmed that the hybrid method finds the exact globally optimal solution for a small problem. For larger problems, the two-stage solution method can provide good solutions more efficiently than purely numerical methods.

7.2. Future Research

This dissertation analyzes several interesting bus transit problems. However, it also leaves rooms for the further improvements, especially for more realistic implementations. Possible extensions are discussed as follows:

1) Problems in this dissertation are analyzed deterministically. To be more realistic and be ready for actual implementations, considerations of stochastic components are necessary. Examples are: 1) probabilistically distributed bus travel times due to congestion, incidents or other factors; 2) scheduled transfers coordination among vehicles; 3) stochastic variation of passenger arrivals and waiting times; and 4) consideration of dwell times as a function of passengers.
2) This dissertation assumes uniformly distributed demand within each region. In actual transit operations, the analysis of highly heterogeneous demand patterns may be desirable. Future studies may pursue such geographic detail.
3) Transit operators have information and control systems that collect origins and destinations of passengers before drivers operate their buses, especially for flexible services. Thus, the approximate Stein (1978) formula used in this

dissertation should be replaced with vehicle routing algorithms for real-time control. Many-to-many demand patterns should be also considered.

4) The complexity of switching service types and fleets in actual bus transit operations should be further explored. Research attempts of vehicle transitions (e.g., switching vehicles for different headways, vehicle transitions from regions to other regions, drivers scheduling and allocations) are not sufficiently explored. They are all interesting future study directions to make dissertation more realistic.

5) Studies of actual passenger responses (e.g., willingness to pay) to different service types are desirable.

6) In Chapter 4~6, formulations are NP hard problems. Thus, they are solved purely numerical or partially numerical approach. Therefore, the globally optimal solutions are unknown to these problems. Since methods used are partially or purely heuristic, some research attempt is required to fill the gap between the unknown global optima and the best solution from proposed methods. One possible attempt is: 1) break-down large problems into analytically tractable small problems, 2) sequentially or independently

optimize sub problems, and 3) compare obtained solutions. The attempts to estimate solution gaps should be useful for managing large transit data-driven optimization problems.

7) This dissertation analyzes several interesting topics in the context of bus transit integration. However, each topic is independently explored in separate chapters. It might be possible to integrate these in one analysis framework. More specifically, a joint optimization model that finds solutions for the integration decision using conventional and flexible services, while considering elastic demands for a general system (e.g., multiple terminals, multiple regions, and multiple analysis periods) should be useful but also quite challenging to develop.

8) Bus transit network design problems with realistic geographic information would be also an interesting topic. With recent technology developments, transit riders may easily obtain the bus arrival information. It would be worth considering how such information affects passenger arrivals at bus stops and overall travel times.

9) Optimization models developed in this dissertation may be applied to other

intermodal transportation system analyses. For instance, the elastic demand and welfare analyses can also be used for other intermodal transportation systems. This dissertation assumed a linear elastic demand function. An extension of linear demand curve for social welfare analyses may be another interesting study.

References

Abkowitz, M., Josef, R., Tozzi, J., and Driscoll, M. K. 1987, "Operational Feasibility of Timed Transfer in Transit Systems", *Journal of Transportation Engineering, Vol. 113*, No. 2, pp. 168-177.

Attanucci, J. P., Jaeger, L., and Becker, J. 1979, "Bus Service Evaluation Procedures: A Review", UMTA, U.S. Department of Transportation, Report UMTA-MA-09-7001-79-1.

Bakker, J. J., Calkin, J., and Sylvester, S. 1988, "Multi-Centered Time Transfer System for Capital Metro, Austin, Texas", *Transportation Research Record: Journal of the Transportation Research Board, No.1202*, pp. 22-28.

Baumgartner, D. S., Schofer, J. L., 2011, "Forecasting call-n-ride productivity in low-density areas", Transportation Research Board 90th Annual Meeting, online compendium, Jan. 23rd ~ 27th, 2011.

Becker, A. J., Teal, R. F., 2011, "Next generation general public demand responsive transportation", Transportation Research Board 90th Annual Meeting, online compendium, Jan. 23rd ~ 27th 2011.

Burkhardt, J. E. 2004, “Economic Benefits of Coordinating Human Service Transportation and Transit Services”, *Transportation Research Record: Journal of the Transportation Research Board, No.1887*, pp. 55-61.

Ceder, A. 1985, “Computer Application for Determining Bus Headways and Timetables”, *Transportation Research Record: Journal of the Transportation Research Board, No. 1011*, pp. 76-87.

Cevallos, F. and Zhao, F. 2006, “A Genetic Algorithm for Bus Schedule Syncronization”, *Applications of Advanced Technology in Transportation*, Proceedings of the 9th International Conference, Chicago, Illinois.

Chang, S. K., 1990. “Analytic optimization of bus systems in heterogeneous environments”, Ph.D. Dissertation, University of Maryland, College Park.

Chang, S. K. and Schonfeld, P. 1991a, “Optimization Models for Comparing Conventional and Subscription Bus Feeder Services”, *Transportation Science, Vol. 25*, No. 4, pp. 281-298.

Chang, S. K., Schonfeld, P. 1991b, “Integration of Fixed- and Flexible-Route Bus Systems”, *Transportation Research Record: Journal of the Transportation Research Board, No. 1308*, pp. 51-57.

Chang, S. K., Schonfeld, P. 1991c, “Multiple Period Optimization of Bus Transit Systems”, *Transportation Research Part B, Vol. 25*, No. 6, pp. 453-478.

Chang, S. K., Schonfeld, P. 1993, “Welfare Maximization with Financial Constraints for Bus Transit Systems”, *Transportation Research Record: Journal of the Transportation Research Board, No. 1395*, pp. 48-57.

Chen, C.-C. and Schonfeld, P. 2010, “Modeling and Performance Assessment of Intermodal Transfers at Cargo Terminals”, *Transportation Research Record: Journal of the Transportation Research Board, No. 2162*, pp. 53-62.

Chien, S. I.-J. and Spasovic, L. N. 2002, “Optimization of Grid Bus Transit Systems with Elastic Demand”, *Journal of Advanced Transportation*, Vol. 36, No. 1, pp. 63-91.

Chowdhury, M. S. and Chien, S. I.-J. 2002, “Intermodal Transit System Coordination”, *Transportation Planning and Technology*, Vol. 25, pp. 257-287.

Chowdhury, M. S. and Chien, S. I-J. 2011, “Joint Optimization of Bus Size, Headway and Slack Time for Efficient Timed Transfer”, *Transportation Research Record: Journal of the Transportation Research Board, No. 2218*, pp. 48-58.

Daganzo, C. F. 1984, “The Length of Tour in Zones of Different Shapes”, *Transportation Research, Vol. 18,* pp. 135-145.

Deb, K. 2000, “An Efficient Constraint Handling Method for Genetic Algorithms”, *Computer Methods in Applied Mechanics and Engineering, Vol. 186,* pp. 311–338.

Deb, K. 2001, *Multi-Objective Optimization using Evolutionary Algorithms*, John Wiley and Sons.

Deep, K., Singh, K. P., Kansal, M. L., and Mohan, C. 2009, “A Real Coded Genetic Algorithm for Solving Integer and Mixed Integer Optimization Problems”, *Applied Mathematics and Computation, Vol. 212,* No. 2, pp. 505–518.

Diana, M., Quadrifoglio, L., Pronello, C., 2009, “A methodology for comparing distances traveled by performance-equivalent fixed-route and demand responsive transit services”, *Transportation Planning and Technology*, Vol. 32, No. 4, pp. 377-399.

Domschke, W. 1989, “Schedule Syncronization for Public Transit Networks”, *OR Spectrum*, *Vol. 11*, pp. 17-24.

Elsgolc, I. E. 1961, *Calculus of Variations*, Pergamon Press.

Fu, L., and Ishkhanov, G. 2004, “Fleet Size and Mix Optimization for Paratransit Services”, *Transportation Research Record: Journal of the Transportation Research Board, No.1884*, pp. 39-46.

Furth, P. G. and Day, F. B. 1985, “Transit Routing and Scheduling Strategies for Heavy Demand Corridors”, *Transportation Research Record: Journal of the Transportation Research Board, No.1011*, pp. 23-26

Furth, P. G. and Wilson, H. M. 1981, “Setting Frequencies on Bus Routes”, *Transportation Research Record: Journal of the Transportation Research Board, No. 818*, pp. 1-7.

Goldberg, D. E. 1989, *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley, New York.

Han, A. F. and Wilson, N. H. M. 1982, “The Allocation of Buses in Heavily Utilized Networks with Overlapping Routes”, *Transportation Research Part B, Vol. 16,* No. 3, pp. 221-232.

Holland, J. H., 1975, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor.

Horn, M. E. T., 2002. “Multi-modal and demand-responsive passenger transport systems: a modeling framework with embedded control systems”, *Transportation Research Part A*, Vol. 36, pp. 167-188.

Imam, M. O. 1998, “Optimal Design of Public Bus Service with Demand Equilibrium”, *Journal of Transportation Engineering*, Vol. 124, No. 5, pp. 431-436.

Jansson, J. O. 1980, “A Simple Bus Line Model for Optimisation of Service Frequency and Bus Size”, *Journal of Transport Economic Policy, Vol. 14,* pp. 53-80.

Jong, J. C. and Schonfeld, P., 2003, “Evolutionary Algorithms for Optimizing 3 -dimensional Highway Alignments”, *Transportation Research Part B*, Vol. 37, No. 2, pp. 107-128.

Jung, J., Jayakrishnan, R., 2011, “High coverage point-to-point transit: study of path-based vehicle routing through multiple hub”, *Transportation Research Record 2218*, pp. 78~87.

Kim, M. and Schonfeld, P., 2012, “Conventional, flexible and variable-type services”, *Journal of Transportation Engineering*, Vol. 138, No. 3, ASCE, pp. 1~11.

Kim, M., Schonfeld, P., 2011, “Integration of fixed and flexible route public transportation systems”, TSC phase 2 report 2011-21, Univ. of Maryland, College Park, July 2011.

Kim, T., Haghani, A., 2011, “Model and algorithm considering time varying travel times to solve static multidepot dial-a-ride problem”, *Transportation Research Record 2218*, pp. 68~77.

Knowppers, P. and Muller, T. 1995, “Optimized Transfer Opportunities in Public Transport”, *Transportation Science, Vol. 29*, No. 1, pp. 101-105.

Kocur, G. and Hendrickson, C. 1982, “Design of Local Bus Service with Demand Equilibration”, *Transportation Science, Vol. 16*, No. 2, pp. 149-170.

Kyte, M., Stanley, K., and Gleason, E. 1982, “Planning, Implementing, and Evaluating a Timed-Transfer System in Portland, Oregon”, *Transportation Research Record: Journal of the Transportation Research Board, No.877*, pp. 23-29

Lee, K. K. Lee, Kuo, S. H. F. and Schonfeld, P. M. 1995, “Optimal Mixed Bus Fleet for Urban Operations.” *Transportation Research Record: Journal of the Transportation Research Board, No. 1503*, pp. 39-48.

Lee, K. K. T. and Schonfeld, P. 1991, “Optimal Slack Time for Timed Transfers at a Transit Terminal”, *Journal of Advanced Transportation, Vol. 25*, No. 3, pp. 282-308.

Luo, Y., Schonfeld, P., 2011a, “Online rejected-reinsertion heuristics for the dynamic multivehicle dial-a-ride problem”, *Transportation Research Record 2218*, pp. 59~67.

Luo, Y., Schonfeld, P., 2011b, “Performance metamodels for dial-a-ride services with time constraints”, Transportation Research Board 90th Annual Meeting, online compendium, Jan. 23rd ~ 27th, 2011.

Mathworks, 2013, “Online Documentation Center”, (http://www.mathworks.com/help/matlab/ref/roots.html?searchHighlight=roots).

MathWorks. 2011, MATLAB R2011b Documentation (http://www.mathworks.com)

Muller, T. H. J. and Furth, P. G. 2009, “Transfer Scheduling and Control to Reduce Passenger Waiting Time”, *Transportation Research Record: Journal of the Transportation Research Board, No.2112*, pp. 111-118.

Newell, G. F. 1971, “Dispatching Policies for a Transportation Route”, Transportation Science, Vol. 5, pp. 91-105.

Nourbakhsh, S. M., Ouyang, Y., 2011, “A structured flexible transit system for low demand areas”, *Transportation Research Part B*, Vol. 46, pp. 204-216.

Oldfield, R. H. and Bly, P. H. 1988, “An analytic Investigation of Optimal Bus Size”, *Transportation Research, Vol. 22,* pp. 319-337.

Quadrifoglio, L. and Li, Xiugang. 2009, “A Methodology to Derive The Critical Demand Density for Designing and Operating Feeder Transit Services”, *Transportation Research Part B, Vol. 43,* pp. 922-935.

Quadrifoglio, L., Dessouky, M. M., Ordonez, F., 2008, “Mobility allowance shuttle transit (MAST) services: MIP formulation and strengthening with logic constraints”, *European Journal of Operational Research*, Vol. 185, pp. 481-494.

Quadrifoglio, L., Hall, R. W., Dessouky, M. M., 2006, "Performance and design of mobility allowance shuttle transit services: bounds on the maximum longitudinal velocity", *Transportation Science*, Vol. 40, No. 3, pp. 351-363.

Salzborn, F. J. M. 1972, "Optimum Bus Scheduling", *Transportation Science*, *Vol. 6*, No. 2, pp. 137-148.

Sheffi, Y. and Sugiyama, M. 1982, "Optimal Bus Scheduling on a Single Route", *Transportation Research Record: Journal of the Transportation Research Board, No. 895*, pp. 46-52.

Shen, C., Quadrifoglio, L., 2011, "The coordinated decentralized paratransit system: formulation and comparison with alternative strategies", Transportation Research Board 90th Annual Meeting, online compendium, Jan. 23rd ~ 27th , 2011.

Shrivastava, P., Dhingra, S.L., and Gundaliya, P. J. 2002, "Application of Genetic Algorithm for Scheduling and Schedule Coordination Problems", *Journal of Advanced Transportation*, *Vol. 26*, No. 1, pp. 23-41.

Stein, D. M. 1978, "An Asymptotic Probabilistic Analysis of a Routing Problem", *Mathematics of Operations Research*, *Vol. 3*, pp. 89-101.

Teng, J., Chen, Y.-J., and Yang, X.-G. 2012, “Coordinated Optimization of Service Frequencies for Bus Corridor with Multiple Transfer Stations”, *Transportation Research Board 91st Annual Meeting*, Washington, D. C.

Ting, C-J. and Schonfeld, P. 2005, “Schedule Coordination in a Multiple Hub Transit Network”, *Journal of Urban Planning and Development, Vol. 131*, No. 2, pp. 112-124.

Tsai, F.-M., Chien, S., and Wei, C.-H. 2013, “Joint Optimization of Temporal Headway and Differential Fare for Transit Systems Considering Heterogeneous Demand Elasticity”, *Journal of Transportation Engineering*, Vol. 139, No. 1, pp. 30-39.

TSS Wireless Inc. 2003, *User’s Guide: FirstWin – A Tool for Routing and Scheduling Dial-A-Ride Paratransit Vehicles*, Los Angeles, California.

Wang, S. and Schonfeld, P., 2012, “Simulation-based Scheduling of Mutually Exclusive Projects with Precedence and Regional Budget Constraints”, *Transportation Research Record 2273*, pp. 1-9.

Woodhull, J., Simon, J., and Shoemaker, D. A. 1985, “Goals for Bus Transit Scheduling”, *Transportation Research Record: Journal of the Transportation Research Board, No. 1011*, pp. 72-76.

Yan, S. and Chen, H.-L. 2002, "A Scheduling Model and A Solution Algorithm for Inter-city Bus Carriers", *Transportation Research Part A, Vol. 36*, pp. 805-825.

Yan, S., Chi, C.-J., and Tang, C.-H. 2006, "Inter-city Bus Routing and Timetable Setting under Stochastic Demands", *Transportation Research Part A, Vol. 40*, pp. 572-586.

Zhao, F. and Zeng, X. 2008, "Optimization of Transit Route Network, Vehicle Headways, and Timetables for Large-scale Transit Networks", *European Journal of Operations Research*, *Vol. 186*, pp. 841-855.

Zhou, Y., Kim, H.S., Schonfeld, P., Kim, E., 2008, "Subsidies and Welfare Maximization Tradeoffs in Bus Transit Systems", *The Annals of Regional Science*, *Vol. 42*, pp. 643-660.