ABSTRACT

Title of Document: LONGITUDINAL ANALYSES OF ACHIEVEMENT GROWTH AND ASSOCIATED KINDERGARTEN FACTORS FOR SUBGROUPS OF CHILDREN WITH MATHEMATICS AND READING DIFFICULTIES

Ming-Yi Cho
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Directed By: Professor Margaret McLaughlin
Department of Counseling, Higher Education,
and Special Education.

The present study used data from the Early Childhood Longitudinal Study – Kindergarten Class of 1998-1999 (ECLS-K) to explore the performance profiles of children with difficulties in mathematics. Two issues were particularly addressed in the present study– the longitudinal manifestation of math difficulties and the differential influence of early predictors on math growth rates and fifth-grade achievement for children with different subtypes of MD. The first issue was investigated by considering the stability and patterns of subgroup change for children with MD, MD-RD, RD, and TA, as well as by examining the math and reading achievement trajectories of children in different achievement subgroups. The second issue was explored by investigating how the identified kindergarten predictors influence progress in learning math and whether the effects of these kindergarten predictors vary among children in different achievement subgroups. Two main findings emerged: (a) children with MD-RD differed from children with MD and children in the comparison groups in the patterns of subtype change over time, math and reading IRT scale scores, and math and reading achievement trajectories; and (b) children's demographic characteristics, learning-related skills, math and reading performance at kindergarten entry, class size, and instructional time were all significantly predictive of their later math achievement and progress. Among the identified kindergarten predictors, only the effects of socioeconomic status and initial math knowledge vary across children in different kindergarten achievement subgroups. Despite some study limitations, the results of the present study add to the knowledge of academic development for children with difficulties in mathematics and have implications on early identification and intervention for this population.

LONGITUDINAL ANALYSES OF ACHIEVEMENT GROWTH AND ASSOCIATED KINDERGARTEN FACTORS FOR SUBGROUPS OF CHILDREN WITH MATHEMATICS AND READING DIFFICULTIES

By

Ming-Yi Cho

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Advisory Committee
Margaret J. M. McLaughlin, Professor and Chair
Patricia A. Alexander, Professor
Jeffrey R. Harring, Associate Professor
Debra Neubert, Professor
Rebecca Silverman, Associate Professor

DEDICATION

To my mother, who inspired me to be a special education teacher and encouraged me to achieve my goals. You are always in my memories.

To my father, who always supports and believes in me.

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CHAPTER I

Introduction

Mathematics is a highly complicated and symbolic language that includes the concepts of quantity, structure, space, and change. It can provide individuals with ways to explore, investigate, and understand the world. Competency in mathematics is essential to the development of a well-rounded individual and has numerous applications in daily life. In school, the mathematical knowledge and skills students have obtained serve as a foundation for learning in other subjects; after students leave school, mathematical competence is found to be a crucial predictor for employment, income, and work productivity even after accounting for intelligence and reading skills (Rivera-Batiz, 1992). Although the importance of being mathematically literate is acknowledged, Lee, Grigg, and Dion (2007) found that only 39% of fourth-grade students in the United States are proficient in mathematics. For some children, struggles with mathematics may persist throughout their school years, even continuing into adulthood (Dowker, 2005). Previous studies focusing on school-age children (e.g., Badian, 1983; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Lewis, Hitch, & Walker, 1994) have indicated that about 3% to 10% of elementary school students demonstrate some form of difficulties in mathematics. Significantly, the estimated prevalence rates of mathematics difficulties are found to be increasing over the elementary school years but decreasing after children enter adolescence (Badian, 1983; Barbaresi et al., 2005). This trend shows that elementary school is a key stage for children's mathematics development.

During the past decades, studies addressing math difficulties for children in elementary school have accumulated. One primary research focus was the cognitive and academic characteristics of children with difficulties in mathematics (e.g., Andersson, 2008; Fuchs et al., 2005; Geary, Hoard, & Hamson, 1999). Another area of investigation for researchers in the field was how to improve the development in mathematical thinking or alter the trajectory of deficient math achievement for children with difficulties in mathematics (e.g., Badovski & Farkas, 2008). Despite the research efforts, however, many questions remain unanswered.

One of the areas in need of more clarification is whether children with comorbid math and reading difficulties (MD-RD) and children with only math difficulties (MD) represent two distinct groups. Previous studies have found that children with MD performed better than children with MD-RD in problem-solving tasks (Jordan, Hanich, & Kaplan, 2003); Fletcher (2005) also found that children with MD and those with MD-RD demonstrated two significantly different performance profiles. However, findings from some other studies showed no significant group difference between children with MD and MD-RD on mathematical tasks (e.g., Andersson, 2008; Geary, Hoard, & Hamson, 1999; Jordan & Hanich, 2000).

Meanwhile, the difference between children with and without difficulties in mathematics is also in need of research attention. Vukovic (2012) examined the growth of children with different subtypes of MD and typically achieving (TA) children on several cognitive skills associated with mathematics, and found that early numerical skills were a defining characteristic of math difficulties, specific or otherwise. Fletcher (2005) found that children with MD-RD and children with reading difficulties (RD) demonstrated similar cognitive profiles, which suggests that children with RD and MD-RD may possess same core attributes of both difficulties. Jordan, Kaplan and Hanich

(2002) found variation in the math and reading achievement trajectories of children with two subtypes of math difficulties (MD and MD-RD), reading difficulties (RD) and typically achieving (TA) children, and concluded that reading and math abilities may have different impacts on children's growth in math and reading.

Despite the increasing exploration on the distinct characteristics of children with difficulties in mathematics, most of the studies tend to be cross-sectional (e.g., Fuchs & Fuchs, 2002; Geary et al., 1999; Jordan & Hanich, 2001) or longitudinal but covering a short span of time (e.g., Jordan, Hanich, & Kaplan, 2003; Jordan, Kaplan, & Hanich, 2002; Silver, Pennett, Black, Fair, & Balise, 1999). It is still unclear whether children in the two MD groups, especially those who were identified in early grades, present distinct performance patterns and continue to exhibit such deficits in later grades. In addition, most of the extant studies focusing on MD subtypes were conducted using small local samples. Although the studies using small and local samples have contributed substantially to the current body of knowledge about math difficulties, studies conducted with large-scale, nationally representative samples may further broaden the current understanding of math difficulties; therefore, more such research is needed in the field.

In addition to investigating how math difficulties manifest longitudinally, researchers must examine the factors associated with the math achievement trajectories for children with difficulties in mathematics (Mazzocco, 2009). The investigation on predictors of math achievement has been an important area of research for children in general. During the past decades, the development of statistical methods of analyzing longitudinal data has also allowed researchers to examine the factors associated with math achievement growth. As a result, the number of research studies examining predictors of math growth for children in general has been increasing (Duncan et al., 2007). However, research on the math growth of children with difficulties in mathematics is still scarce. Among the limited extant studies, very few of them also addressed environmental influences, which play an important role in mathematical thinking and achievement of children with difficulties in mathematics (Mazzocco, 2009). Moreover, given the increasing attention on the need for early intervention, more research should be focusing on exploring the child and class factors in early school years that may be associated with the math growth and achievement for children with difficulties in mathematics as well as the differential effects of these predictors for children with different subtypes of MD.

In this regard, the current study aimed to address the two aforementioned pressing issues in the field. First, the study explored how math difficulties manifest over school years by examining the subgroup stability and change patterns, as well as the achievement growth trajectories for children with difficulties in mathematics. Second, the current study investigated the effects of kindergarten predictors on math achievement trajectories over the course of elementary school years in addition to examining how the effects of identified kindergarten predictors vary as a function of subtypes of math difficulties. In addition to children in the two MD subgroups, children with difficulties in reading (RD) and typically achieving children (TA) were also included in the first part of analyses for comparison purposes. To take advantage of a large-scale, nationally representative sample, the current study was conducted using the data from the Early Childhood Longitudinal Study, Kindergarten Class of 1998–99 (ECLS-K). Following is a synopsis of current knowledge regarding the two research topics of interest. A summary of the ECLS-K dataset and data analysis plan and the research questions are also presented.

Definition and Terminology of Mathematics Difficulties

Before reviewing the relevant research, it is necessary to briefly discussing the use of the term *mathematics difficulties* and its definition in the current study. Researchers have used different terms to refer to students struggling with mathematics, such as mathematics difficulty (e.g., Andersson, 2008; Jordan et al., 2003), mathematics disability (e.g., Fuchs & Fuchs, 2002; Geary et al., 1999; Mazzocco & Myers, 2003), arithmetic difficulties (e.g., Lewis, Hitch, & Walker, 1994), and dyscalculia (e.g., Badian, 1983; Kosc, 1974). In addition, no consensus has been formed on the cut-off score used for identifying students with difficulties in mathematics. For instance, Seethaler and Fuchs (2010) used the 15th percentile, Powell, Fuchs, Fuchs, Cirino, and Fletcher (2009) used the 25th percentile, Geary et al. (1999) used the 30th percentile, and Jordan et al. (2003) used the 35th percentile. Meanwhile, some researchers have used a strict cut-off score for mathematics disability and a lenient score for mathematics difficulty. For example, Murphy, Mazzoccom, Hanich, and Early (2007) used the 10th percentile to identify mathematics disability and between the 11th and 25th percentile for mathematics difficulty. It should be noted that the selection of cut-off scores is arbitrary, and the classification based solely on cut-off scores may not perfectly represent children with math difficulties (or disabilities), especially for students whose scores fall around the cut-off scores. Since differentiating math difficulty from disability was not the aim of the current study, a set of arbitrary cut-off scores—one standard deviation (SD) below the mean for achievement difficulties and one-half standard deviation (SD) below the mean for normal achievement—was determined after examining the score distributions of the whole sample from the ECLS-K study at the spring of kindergarten. Children identified as having math difficulties (MD) are those who scored below -1 SD on math and scored above -0.5 SD on reading; conversely, children with both math and reading difficulties (MD-RD) are those who scored below -1 SD on both math and reading achievement. These two MD groups are the target population of the present study. The term “math disabilities” is used in the present study only to refer to children who are identified through multiple measures involving mathematics achievement and intelligence tests in the reviewed studies.

Longitudinal Manifestation of Math Difficulties

As Dowker (2005) and Ginsburg (1997) pointed out, mathematics is composed of multiple types of cognitive processes and skills, and a child with difficulties in mathematics may demonstrate deficits in one area but not in another. Because of this multifaceted nature of mathematics, the question of interest for the researchers was whether a child identified as having difficulties in mathematics will continue to show the same learning problem later. Silver et al. (1999) found that only about half of the students with MD aged 9 to 13 years continued to present some types of arithmetic difficulties 19 months after the initial testing. Geary and colleagues (Geary, 1990; Geary, Brown, & Samaranayake, 1991; Geary, Hamson, & Hoard, 2000) found a group of “variable” children (Geary et al., 2000, p. 244) who met the identification criteria in either first or second grade but not in the other. Since the participants in the aforementioned studies were tested at only two time points, the questions remain as to how stable are math difficulties over a longer period and whether the stability differs between MD subtypes.

Also examining math difficulties from a developmental perspective, a group of researchers focused their studies on the achievement levels and trajectories of children with different subtypes of MD, as compared to those without difficulties in mathematics (i.e., TA and/or RD). Compared to their MD peers, children with MD-RD demonstrated pervasive difficulties in most of the areas of mathematics (Jordan & Hanich, 2000; Hanich et al., 2001); moreover, children with MD-RD also had a significantly lower average score and made slower progress in mathematics than did their peers with MD (Jordan et al., 2002). The results regarding math achievement growth between MD subgroups and non-MD groups were mixed. Some studies (e.g., Vukovic, 2012) found the math growth trajectories of children with MD and MD-RD were parallel to those of non-MD children; however, another study (Jordan et al., 2002) found that children with difficulties in mathematics progressed faster in mathematics than did their non-MD peers. The findings from Jordan et al. (2002) were particularly interesting because children with MD were found to score at the same level or higher than their RD peers in mathematics measures at the end of the study period; however, the same pattern was not found in reading achievement. That is, children with RD did not surpass their MD peers in reading achievement at the end of third grade. Therefore, the researchers concluded that children’s reading ability may influence their growth in mathematics, but mathematics ability does not influence children’s growth in reading. Since this particular study conducted by Jordan and colleagues (2002) observed participants across the second and third grade, it is still unclear whether similar performance patterns can be found for younger children and for a longer period of observation, such as the course of elementary school.

Kindergarten Predictors of Math Growth and Later Achievement

Kindergarten is typically the first step in children's formal schooling. The skills children have as they enter kindergarten and their early school experience constitute the foundations for their subsequent learning. The conceptual framework for this study, more fully described in Chapter 2, asserts that children's achievement growth is a function of both their individual skills and contextual factors. Based on this framework, and informed by the extant studies conducted using the ECLS-K dataset, the kindergarten predictors identified for the current study include children's individual skills in kindergarten (i.e., initial math and reading knowledge and learning-related skills) and class factors (i.e., class size, instructional time, and math instructional activities). Despite the potential importance of the kindergarten learning experience on math achievement growth for children with difficulties in mathematics, only one study (Bodovski & Farkas, 2007) examined the associations between kindergarten predictors and academic growth particularly for this population.

Therefore, the current knowledge regarding the influence of kindergarten experience on math achievement growth mostly applies to children in general. For instance, children's performance in mathematics at kindergarten entry has been identified as a strong predictor of their later mathematics achievement and growth (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Claessens, Duncan, & Engel, 2009; Duncan et al., 2007). The findings regarding the association between initial reading knowledge and later math achievement were mixed (Claessens et al., 2009; Duncan et al., 2007). Children's learning-related skills, which refer to student engagement and self-regulation, were found to be significant predictors of mathematics growth (Bodovski & Farkas, 2007; Classen et al., 2009; DiPerna, Lei, & Reid, 2007; Duncan et al., 2007; McClellan, Morrison, & Holmes, 2000; McClella, Acock, & Morrison, 2006). As for the effects of class variables on math achievement, small class size was found to have a positive association with kindergarten and later achievement (Milesi & Gamoran, 2006; Nye et al., 1999, 2000, 2002). Furthermore, kindergarten instructional activities were found to have a positive influence on math achievement (Milesi & Gamoran, 2006), and the time spent on instruction was also found to be a significant predictor of later mathematics achievement (Pianta, Belsky, Vandergrift, Houts, & Morrison, 2008; Milesi & Gamoran, 2006). Despite the accumulating research findings, it is still unclear whether these factors may contribute the same to math achievement growth for children with difficulties in mathematics.

In addition, the extant research studies on identified kindergarten predictors have mostly been conducted with children in grades K to 3. Little is known about whether the significant effects of identified kindergarten predictors can be extended beyond early elementary grades. Moreover, it is still unknown whether these kindergarten predictors behave differently for children with different subtypes of math difficulties (i.e., MD and MD-RD) and their typically achieving peers. Therefore, in the present study, children's math and reading achievement at kindergarten entry, learning-related skills, and class factors were examined in relation to mathematics growth from kindergarten through fifth grade for children in the MD, MD-RD, and TA groups.

ECLS-K Dataset and Data Analyses

Given the fact that previous studies mainly focused on small convenience samples and mostly targeted children in primary grades, the present study aimed to extend the investigations to address issues involving academic achievement and growth over the elementary school years. In this regard, the data used for answering the research questions were from the Early Childhood Longitudinal Study, Kindergarten Class of 1998–99 (ECLS-K). The ECLS-K is a longitudinal study involving a nationally representative sample of 21,260 students who began kindergarten in 1998. The study followed these students for nine years until the eighth grade. The data were collected in fall and spring semesters during the 1998–99 (kindergarten) and 1999–2000 (first grade) school years, and then in fall semesters during the 2001–02 (third grade), 2003–04 (fifth grade), and 2006–07 (eighth grade) school years. In the ECLS-K, children's academic performance was measured through direct assessments in reading and mathematics. In addition to the direct assessments of the sampled children's academic skills, the ECLS-K also collected information from the children's parents, teachers, and school administrators. The ECLS-K is designed to promote the understanding of children's development and experience in elementary school as well as how various child, home, classroom, school, and community factors relate to children's cognitive and social development. In this regard, the use of the ECLS-K provides a more comprehensive view of children in different academic groups (i.e., MD, MD-RD, RD, and TA) and their academic growth in elementary school. More detailed information regarding the dataset and analytical samples is provided in Chapter 3: Methodology.

Purpose of the Study

The purpose of the present study was two-fold. The first aim was to examine how children's math difficulties manifest over time by examining the stability and change of subgroups as well as the achievement growth trajectories of children with MD and MD-

RD. Children with RD and those who were typically achieving (TA) were also included in the analyses for comparison purposes. The second aim was to examine the associations between a set of identified kindergarten predictors, including a child's demographic characteristics, individual skills at kindergarten, and kindergarten class factors, and fifth-grade math achievement and math growth over the course of elementary school years. Also of interest was whether the effects of these identified kindergarten predictors vary between children with MD and those with MD-RD. Children in the TA group were also included in this series of analyses for comparison. By taking advantage of a large-scale, nationally representative, and longitudinal study like the ECLS-K, the results of the present study may add to the current understanding of the development of math difficulties, especially the difference that may be shown by children with different subtypes of math difficulties. The results regarding the effects of identified kindergarten predictors may also provide insights into the individual skills or class factors that may have positive associations with children's math growth and later math achievement. The findings may have implications on early identification and intervention for children with difficulties in mathematics.

Research Questions

Based on the above rationale, the present study was guided by the questions presented below.

Research Question 1: What is the extent of subgroup membership stability and change from kindergarten through fifth grade (i.e., kindergarten vs. first grade, first grade vs. third grade, and third grade vs. fifth grade)? Given membership change over time, what is the pattern of change across subgroups (e.g., MD to MD-RD; TA to MD)?

Given the findings from the literature on stability of subgroup membership, this researcher predicted that children with MD-RD would be the most stable among children with MD, RD, and MD-RD over the elementary school years (Silver et al., 1999). Some kindergarten children with RD can develop difficulties in mathematics and are therefore identified as having comorbid MD-RD in later grades. Since the results from Jordan et al. (2002) suggested that reading abilities influence children's mathematics performance but not vice versa, a smaller percentage of children with MD were expected to be classified later as comorbid MD-RD.

Research Question 2: To what extent do children's math and reading achievement and growth trajectories differ among children in different achievement subgroups (i.e., MD, MD-RD, RD, and TA)?

For achievement level at the end of fifth grade, children with MD-RD were expected to have the lowest score in mathematics and reading among the four groups at the end of elementary school. Meanwhile, the math achievement gap between children with MD and children with RD was expected to close at the end of fifth grade. For achievement growth over the elementary school years, children with MD and MD-RD were expected to progress faster than their RD and TA peers in mathematics; conversely, children with RD and MD-RD were predicted to progress faster than their MD and TA peers in reading. This researcher also expected that children with MD would score at the same level as or higher than their RD peers in mathematics achievement at the end of fifth grade. These hypotheses were primarily based on the findings of studies by Jordan and colleagues (2002, 2003).

Research Question 3: To what extent do children's demographics characteristics, initial reading and mathematics knowledge, learning-related skills, instructional time, class size, and math instructional activities predict growth and achievement level in mathematics performance over the course of the elementary school years? Does kindergarten achievement subgroup membership (i.e., MD, MD-RD, and TA) interact with the identified kindergarten variables to predict growth in mathematics over the elementary school years?

As for demographics variables, math growth was expected to be affected by children's SES and minority status, particularly for children with MD-RD. Children's learning-related skills were expected to have positive effects on children's math growth. However, this researcher acknowledged that the effects of initial reading and mathematics knowledge may be mediated by achievement subgroup membership given the anticipated high correlations between initial knowledge and achievement group membership. In addition, this researcher predicted that class size may not influence children's math growth, while instructional time and instructional activities in mathematics were expected to have some positive impacts on math growth, particularly for children with MD-RD.

Definition of Terms

Mathematics difficulties (MD)

Children with MD are those who have performance deficits in mathematics but not in reading achievement. They usually demonstrate below-average performance in mathematics achievement, and average or above average performance in reading achievement. Children's mathematics difficulties may be associated with educational, sociocultural, physical, or emotional factors. In the present study, children are identified by the researcher as having mathematics difficulties if their mathematics IRT scale scores are at, or below -1 SD of the sample mean, but their reading IRT scale scores are at, or above -0.5 SD in the ECLS-K dataset.

Reading difficulties (RD)

Children with RD are those who have performance deficits in reading but not in mathematics achievement. They usually demonstrate below-average performance in reading achievement, and average or above average performance in mathematics achievement. Children's reading difficulties may be associated with educational, sociocultural, physical, or emotional factors. In the present study, children are identified by the researcher as having reading difficulties if their reading IRT scale scores are at or below -1 SD, but their mathematics IRT scores are at or above -0.5 SD in the ECLS-K dataset.

Comorbid mathematics and reading difficulties (MD-RD)

Children with MD-RD are those who have performance deficits in both mathematics and reading achievement. They usually score below-average on mathematics and reading achievement tests. Children's mathematics and reading difficulties may be associated with educational, sociocultural, physical, or emotional factors. In the present study, children were identified as having comorbid mathematics and reading difficulties by the researcher if their IRT scores are at or below -1 SD on both mathematics and reading achievement tests in the ECLS-K dataset.

Typically achieving children (TA)

In comparison to children with MD or RD, typically achieving children are those who present average or above average performance in reading and mathematics. In this study, typically achieving children are identified as scoring at or above -0.5 SD on both reading and mathematics IRT scale scores.

ECLS-K

The Early Childhood Longitudinal Study, Kindergarten Cohort. ECLS-K is the longitudinal dataset focusing on children's learning experience from kindergarten through middle school (8th grade).

NCES

The National Center for Education Statistics. The primary federal entity for collecting and analyzing data related to education.

CHAPTER II

Literature Review

The purpose of the present study is to extend current understanding of students with MD and MD-RD from a developmental perspective. The research questions aim to explore how children with difficulties in mathematics progress over the course of elementary school; as well as what and the extent to which the identified kindergarten predictors associated with math achievement and growth for children with difficulties in mathematics. In this regard, the literature review for the present study focuses on two bodies of research: characteristics of children with difficulties in mathematics and kindergarten predictors of math achievement. In this chapter, the findings of reviewed studies are first summarized and presented and subsequently followed by a methodological review of the selected studies.

Review of Literature

Search Procedures

The search for research articles for this review involved several steps. A search was conducted to locate peer-reviewed journal articles through five electronic databases, including: Education Resources Information Center (ERIC), Education Research Complete, PsyARTICLES, Psychology and Behavioral Sciences Collection, and PsycINFO. For the first part of the literature review, regarding children with mathematics difficulties and how their difficulties manifest over time, the descriptors used for the online searches included various combinations of *math*, *mathematics*, *arithmetic*, and *difficulty*, *disability*, or *low achievement*, all of which pair with *characteristics*, *gender*, *socioeconomic status*, *identification*, and performance. For the second part of the review, regarding the individual and class factors in kindergarten associated with mathematics growth during the elementary school years for children with difficulties in mathematics, the descriptors used for the search include combinations of *mathematics, achievement, growth,* and *longitudinal,* all of which pair with *kindergarten, school, teacher, instruction, classroom* and *effect.* Since the present study was conducted using the data from the ECLS-K study, the studies conducted by using the ECLS-K dataset and focused on the predictors of math achievement were also included in this review.

Based on the literature obtained from the computerized searches, an additional search was conducted using the names of researchers whose work was relevant to the topic. Names searched included: David Geary, Laurie Hanich, Nancy Jordan, David Kaplan, and Michèle Mazzocco. Next, a manual search through the recent (2003-2013) issues of the relevant professional journals was conducted. Journals included in the search are: *Exceptional Children, Journal of Educational Psychology, Journal of Learning Disabilities, Journal of Special Education, Learning Disabilities Research and Practice,* and *The Elementary School Journal.* Finally, an ancestral search was also conducted through the references of the obtained articles.

Studies included in this review were selected based on the following criteria: (a) the study focused on elementary school age populations or some of the results reported included elementary school students; (b) the participants of the study included, but were not limited to, those who were identified as having mathematics difficulties or low-achieving in mathematics; (c) the main outcome variable(s) of the study were participants' performance scores on mathematical tasks; and (d) the study was published in a peer-reviewed journal. Based on the search criteria, only a limited number of studies were found focusing on the associations between individual or contextual factors and math growth for children with difficulties in mathematics. Therefore, some studies examining the effects of individual and contextual factors on achievement growth were also included for the current review.

Children with Mathematics Disabilities and Difficulties

Given the purpose of the present study, the research studies reviewed in this section were those targeting the demographic and achievement characteristics of children with difficulties in mathematics. In particular, the studies focusing on how children with difficulties in mathematics progress during the period of elementary school were included in this review.

For the purpose of clarification, the term “mathematics difficulties” in this literature review refers to children who show performance deficits in mathematics, which are usually determined by researchers applying particular cut points to children’s mathematics achievement. On the other hand, the term “mathematics disabilities” is used here to refer to children who are identified through multiple measures that may include mathematics achievement, intelligence tests and other diagnostic criteria in the reviewed studies. The original terms used by reviewed studies as well as the information regarding the study sample and identification criteria are presented in Table 1.

Prevalence and Demographics of MD

Prevalence of MD. Findings from prevalence studies of mathematics disability and difficulties provided information regarding the demographic characteristics of this population, risk factors, and comorbid conditions. The first known epidemiological study of mathematics disability was published by Kosc in 1974. A total of 375 fifth graders, including 199 boys and 176 girls, were randomly selected from 14 classes in 14 schools in Bratislava, Slovakia. Through a two-phase procedure, Kosc identified 24 students with mathematics disabilities, which is 6.4% of the total participants. As the first study of its kind, Kosc's study illustrated a framework for identification of mathematics disabilities. However, since the detailed information about classification criteria for the second phase were not described in this study, replication was an issue.

The prevalence of mathematics disability in the United States was not studied until almost a decade after Kosc's study was published. Badian (1983) examined the incidence of poor reading and mathematics among all students in grades 1 through 8 in a small town (*N*=1,476). The students were assessed using a norm-referenced achievement test and identified as having poor performance if they scored at or below the 20th percentile in reading, mathematics, or both achievement tests. Overall, the incidences of poor mathematics performance for grade 1-3, 4-6 and 7-8 were 5.5%, 5.2% and 8.9% respectively. The respective incidences of MD for three age groups were 3.5%, 3.0%, and 4.7%, while the incidences of comorbid MD-RD were 2.0%, 2.1%, and 4.2%, respectively.

Table 1

Descriptive Information of Reviewed Studies on Children with Mathematics Disabilities/Difficulties

Study	study sample				MD Term	Evaluation Criteria
	Sample N	MD N	Grade /Age	Gender (M/F)		
Andersson (2008)	182	MD: 41; MD-RD:50; RD:30; TA:61	Grade 3 & 4	87/95	Mathematics Difficulties	Raven's Standard Progressive Matrices Test and Word Ability Test **Receipt of Special Education**[a] IQ ≥ -1.5 SD
Badian (1983)	G1-3: 488 G4-6: 560 G7-8:428	MD:17; MD-RD:10 MD:=17; MD-RD:12 MD:20; MD-RD:18	Grade 1 to 8	N/A	Dyscalculia	SAT **≤ 20th percentile**
Barbaresi et al. (2005)	5,718	665	Age 7-19	417/248	Math Learning Disorder	WJ or Wide Range Achievement; WISC **Standard score < 90** IQ > 80
Desoete et al. (2004)	3,978	220	Grade 2 to 4	108/112	Mathematics Disabilities	Tests on number facts and a domain-specific test **≤ -2 SD**
Fuchs & Fuchs (2002)	40	MD:18; MD_RD:22	Grade 4	12/28	Mathematics Disabilities	Test of Computational Fluency; Comprehensive Reading Assessment Battery **< -1.5 SD** IQ ≥ 90; Have IEP goal on math
Geary et al. (2000)	84	MD-RD:16; MD:12; Variable:16	Grade 1 & 2	15/13	Mathematics Disabilities	WIAT; WJ-R **≤ 35th percentile** 80 ≤ IQ ≤ 120
Geary et al. (1999)	90	MD-RD:25; MD:15;	Grade 1	21/19	Mathematics Disabilities	WIAT; WJ-R **≤ 30th percentile** 80 ≤ IQ ≤ 120

Table 1. (continued)

Study	study sample				MD Term	Evaluation Criteria
	Sample N	**MD N**	**Grade /Age**	**Gender (M/F)**		
Gross-Tsur et al. (1996)	3.029	MD:140	Grade 5	65/75	Dyscalculia	Individual arithmetic battery **≤ mean score for normal children two grades younger** IQ≥80
Hanich et al. (2001)	210	MD:53; MD-RD:52;	Grade 2	112/98	Mathematics Difficulties	WJ-R **≤ 35th percentile for difficulties;** **≥ 40th percentile for normal achievement**
Jordan & Hanich (2000)	49	MD:10; MD-RD:10	Grade 2	27/21	Mathematics Difficulties	CTBS **≤ 35th percentile for difficulties;** **≥ 40th percentile for normal achievement**
Jordan, Kaplan, & Hanich (2002)	180	MD:46; MD-RD:42	G2 to 3	96/84	Mathematics Difficulties	WJ-R **≤ 35th percentile for difficulties;** **≥ 40th percentile for normal achievement**
Jordan, Hanich & Kaplan (2003)	180	MD:46; MD-RD:42	G2 to 3	96/84	Mathematics Difficulties	WJ-R **≤ 35th percentile for difficulties;** **≥ 40th percentile for normal achievement**
Kosc (1974)	375	24	Age 11	N/A	Dyscalculia	Individual psychological and neurological assessments **≤10th percentile** [b] IQ ≥ 90
Lewis et al., (1994)	1,056	MD:14; MD-RD:24	Age 9 to 10	18/20	Arithmetic Difficulties	GMT, SPAR, CPM **Standard scores < 85for difficulties; > 90 for normal performance** IQ > 90
Mazzocco & Myers (2003)	209	35	K to 3	103/106	Mathematics Disabilities	TEMA-2 **≤10th percentile**

Table 1. (continued)

Study	study sample				MD Term	Evaluation Criteria
	Sample N	**MD n**	**Grade /Age**	**Gender (M/F)**		
Morgan et al., (2009)	7,892	Fall-K:301 Spring-K:283 F&S-K: 404	K to 5	N/A	Mathematics Difficulties	Kindergarten math assessment[c] **≤ 10th percentile**
Silver et al. (1999)	80	MD:80	age 9 to 13	N/A	Arithmetic Disability	WRAT-Arithmetic, WJ-R Calculation, or WJ-R Applied Problems **Standard score < 90** WISC-R IQ > 90
Vukovic (2012)	203	MD: 19 MD-RD: 19	K to 3	110/93	Mathematics Difficulties	WRAT-3 Math; WJ-III LWID **≤ 25th percentile**; meet the criteria at least twice between kindergarten to third grade

Note: Cutoff criteria listed in bold type. MD = mathematics difficulties; SAT = Stanford Achievement Test (Madden et al., 1973); CTBS = Comprehensive Tests of Basic Skills (CTB/MacMillan, 1989); WJ-R = Woodcock-Johnson Psycho-Educational Battery-Revised, From A (Woodcock & Johnson, 1990); GMT = Group Mathematics Test (Young, 1970); SPAR = Spelling and Reading test (Young, 1976); CPM = Raven's Coloured Progressive matrices (Raven, Court & raven, 1984); TEMA-2 = Test of Early math Ability-second edition (Ginsburg & Baroody, 1990); WRAT-R = Wide Range Achievement Test-Revised (Jastak & Wilkinson, 1984); WISC-R = Wechsler Intelligence Scale for Children-Revised (Wechsler, 1974); WJ-III = Woodcock-Johnson third edition (Woodcock, McGrew, & Mather, 1999); LWID = Letter-Word Identification test of the Woodcock-Johnson third edition.

a. Participants were selected from children who were receiving special instruction; no cutoff criterion was applied.
b. This cutoff criterion was for screening participants. The criteria for identifying children as dyscalculia were not described in the study.
c. The math assessment was developed for the ECLS-K study.

Another large-scale epidemiological study in the United States was conducted by Barbaresi et al. (2005). Barbaresi and colleagues conducted a retrospective study to estimate the prevalence rate of mathematics disabilities for children from aged 7 up to age 19 during the period between the years of 1976 to 1982. The participants were identified as MD if they had documented learning problems in their educational and/or medical records, had normal or above-average intelligence (i.e., standard scores 80 or above), and scored below average on a norm-referenced mathematics achievement test (i.e., standard score of 90 or below). The cumulative incidence of math learning disorder at age 7, 9, 11, 13, 15, 17 and 19 were 2.1%, 7.1%, 10%, 11%, 12.5%, 13%, and 13.8%, respectively. It seems that more children were identified as having MD as they grew older, which also a trend was observed in the study by Badian (1983). The results also showed that prevalence rates increased drastically during the period of elementary school, and the rate of increase slowed down after children entered adolescence. In addition, by applying the same cutoff scores for children's reading achievement (i.e., a standard score 90 or below), the researchers found that about 65% of MD children identified in the study also had reading disorders. The ratio of children with MD to children with comorbid MD and RD was about 1:2.

A similar ratio between two MD groups was also found in another study conducted in England. Lewis et al. (1994) identified children with learning disabilities using cutoff scores of 90 or higher on a nonverbal intelligence test for normal intelligence and standard score 85 or lower on reading and mathematics achievement for academic difficulties. Among the 9 to 10 year olds who participated in the study, 1.3% of children were identified as having mathematics disabilities, and 2.3% of participants were identified as having comorbid mathematics and reading disabilities.

Overall, most of the studies found the prevalence of mathematic disabilities and difficulties to be around 3-10% among the elementary school population. As suggested by many researchers (i.e., Fuchs et al., 2005; Shalev et al., 2006), the variation of prevalence rates found in these studies might be attributed to the variety of identification criteria and mathematics measures used for identification. Another possible explanation for the broad range of prevalence rates is that all the reviewed studies were based on local samples and the resulting prevalence rates may represent the variations of children's academic performance in different areas. Nonetheless, one common trend observed in these studies is that the prevalence rate of math difficulties seemed to increase with grade level, especially in early grades, and remained unchanged after elementary school. This suggests that elementary school is a key stage for children to establish their fundamental knowledge and skills in mathematics, especially in the primary grades.

Gender. The literature on gender differences in children with mathematics difficulties showed mixed findings. Comparable numbers of boys and girls among students with mathematics disabilities and difficulties were found in prevalence studies that were conducted outside of the United States (Desoete et al., 2004; Gross-Tsur et al., 1996; Lewis et al., 1994); however, the prevalence studies done in the United States (Badian, 1983; Barbaresi et al., 2005) showed that boys were overrepresented in the group of students with mathematics difficulties. In particular, Barbaresi et al. (2005) found that not only were more male students identified as MD than their female peers at age 7, but also the gender ratio (male to female) increased dramatically during the elementary school years. Regardless of the identification method used, Barbaresi et al. found the gender ratio was approximately two boys to every one girl during elementary school years, which was consistent with the findings from Badian (1983).

Lachance and Mazzocco (2006) later conducted a longitudinal study to explore gender differences in mathematics for over 200 children during their primary grades (K-3). For the overall sample, the analyses showed a minimal or nonexistent gender difference during any single year of the study, or in any area of math skills. The results of growth curve modeling suggested that boys and girls performed at about the same level in kindergarten and had comparable math growth rates. On the other hand, among the low achievers (i.e., scored in the lowest quartile), boys and girls were equally represented in most of the mathematical measures across the years, except in the area of geometry and calculation. More boys were found scoring at the lower end of the Geometry subtest in kindergarten to the second grade. A gender difference also emerged in third grade with an overrepresentation of boys in the low achieving group on the WJ-R Math Calculation subtest.

The findings of Lachance and Mazzocco (2006) were inconsistent with the results from the studies of Badian (1983) and Barbaresi and colleagues (2005), in terms of gender ratio in children with MD. In addition to the mathematical measures and cutoff criteria used for identification, the different results could be due to the fact that all three studies were based on regional samples. Moreover, the participants in Lachance and Mazzocco's study were born much later, compared to those in the other two prevalence studies; the shrinking gender gaps in Lachance and Mazzocco's study could be a result from the changes in education and society during the past decade. On the other hand, Lachance and Mazzocco did find more third-grade boys in the low achieving group than girls, which led to a question whether the gender gap would increase if the researchers continued their investigation beyond the third grade. Therefore, more research is still in need to explore the longitudinal effects of gender on mathematics performance for children with mathematics difficulties in elementary school.

Socioeconomic Status. Socioeconomic status (SES) is typically based on some combinations of family income, level of poverty in the child's neighborhood, parental education level, and parental occupation. In *the National Mathematics Advisory Panel Reports*, released in 2008, children from low-income families performed substantially worse in mathematics than their peers from middle-income backgrounds, and the achievement difference in mathematical knowledge continued to widen throughout the school years. Particularly interested in the association between SES and mathematics achievement in primary grades, Jordan et al. (2006) found that children who received free or reduced lunch (hereafter low-income children) performed significantly worse than their peers in the middle-income group at the end of kindergarten on the overall number sense battery and on all of the subtests. However, children in these two income groups progressed at about the same rate on all but the story problems task, on which the low-income children showed almost no growth from the beginning to the end of the kindergarten year. Compared to their peers in the middle-income group, children in the low-income group were 17.29 times more likely to have low end-of-kindergarten performance and flat growth over the kindergarten year in the number sense.

As an extension of the previous study, Jordan et al. (2007) continued to track the children's number sense development after kindergarten for another two time points in first grade. Although children in two income groups were found to have similar growth rates in number sense in kindergarten (Jordan et al., 2006), children in low-income group showed significantly slower growth than their counterparts in middle-income group in this study, which suggested that the achievement gap in number sense between two income groups widened after children entered the first grade. In addition, about 55% of the children who fell into the group with low initial math achievement and flat growth were low-income children, as opposed to 30% in the group with median initial math achievement and steep growth and 13% in the group with high initial math achievement and flat growth. This finding indicated that low-income children were more likely to experience difficulties in mathematics and improve less during the primary grades. Jordan and colleges (Jordan et al., 2006; Jordan et al., 2007) provided evidence on the associations between SES and children's mathematics performance and growth in early grades. It was unclear, however, whether the influence of SES on math achievement continues beyond first grade. It also remains a question of whether the effect of SES on math achievement can be mediated by other individual and class factors during the elementary school years.

Overall, the prevalence studies revealed that 3% to 10% of the elementary school population suffers from mathematics disabilities in the United States (Badian, 1983; Barbaresi et al., 2005). Mixed results were found for the gender effects on mathematics for children with mathematics difficulties. Meanwhile, children from low-income families were likely to perform more poorly in overall and sub-areas of mathematics achievement. However, questions remain whether the associations between SES and low mathematics achievement can be mediated after years of school education. Therefore, further investigation regarding the effects of gender and socioeconomic status on mathematics performance and growth for children with mathematics difficulties beyond primary grades is needed.

Academic Achievement and Growth of Children with Difficulties in Mathematics

In recent years, researchers have recognized that children with mathematics disabilities/difficulties represent a heterogeneous group of disorders. In particular, MD children with and without comorbid RD may differ on their functional and cognitive profiles in mathematics. Mixing MD children with and without comorbid RD in research would lead to canceling out the differences between the two MD groups (Rourke and Strang, 1978). Therefore, it is necessary to examine children with different subtypes of MD separately. During the past decades, research had been accumulated on the cognitive profiles of children representing different subtypes of mathematics difficulties (i.e., Geary, Hoard, & Hamson, 199; Geary, Hamson & Hoard, 2000). However, there were fewer studies focusing on academic performance profiles of children with MD. In the following section, this researcher reviewed studies examining the academic performance profiles of elementary school children with different subtypes of difficulties in mathematics, including one study targeting children from kindergarten to third grade (Vukovic, 2012), two for second grade children (Hanich et al., 2001; Jordan & Hanich, 2000), two for second and third graders (Jordan, Hanich, & Kaplan, 2003; Jordan, Kaplan, & Hanich, 2002), one for third and fourth graders (Andersson, 2008), and one for fourth graders (Fuchs & Fuchs, 2002).

Vukovic (2012) investigated the relationships between math difficulties and measures of early numerical and cognitive skills by following a group of children from kindergarten to third grade. By examining how the effects of these numerical and cognitive skills on math achievement growth vary across children with MD, MD-RD and those were typically achieving (TA), Vukovic found that early numerical skills (including tasks of mathematics concepts and number series) were an defining feature of children with difficulties in mathematics (i.e., MD and MD-RD). That is, children in both MD groups performed significantly lower than typically achieving children on both the intercept (kindergarten performance) and growth of the early numerical skills. Between children with MD and MD-RD, math concept seemed to be the task that significantly differentiated the performance of the two groups; children with MD-RD performed significantly lower than children with MD on the math concept task in both kindergarten and third grade. In addition, early numerical skills were also found to influence growth in mathematics from kindergarten to third grade for children with difficulties in mathematics.

Jordan and Hanich (2000) examined the different performance in the mathematical thinking of second graders with MD, MD-RD, RD, and those who were typically achieving (TA). The findings showed that relative to TA children, children with MD-RD demonstrated weaknesses on all four tasks of mathematical thinking, including number facts, story problems, place value, and written calculation. On the other hand, children with MD showed weakness specifically on story problems and place value. Children with MD outperformed their peers in MD-RD group on story problems and written calculation. No significant differences were found between the RD and TA children on any of the tasks. Overall, the major distinction between children with MD-RD and those with MD was that the former presented pervasive deficiencies in mathematical thinking, and the latter demonstrated more specific deficits in story-problem solving.

Also focusing on children in early elementary grades, Hanich et al. (2001) investigated how children in the four achievement groups (i.e, MD, RD, MD-RD, TA) performed differently in seven mathematics tasks, including: exact calculation of arithmetic combination, story problems, approximate arithmetic, place value, calculation principles, forced retrieval of number facts, and written computation. The results showed that children in the two MD groups (i.e., MD and MD-RD) performed worse than their peers in the TA group on all of the tasks. Children with MD outperformed children with MD-RD on exact calculation of arithmetic combinations, calculation principles, and story problems, indicating that children with MD did have linguistic advantages over children with MD-RD, and MD children also performed better than MD-RD children on basic calculation. On the other hand, children in the RD group performed at about the same level as children in the TA group on all but one task (place value).

Extending the work of Hanich et al. (2001), Jordan et al. (2003) assessed the participants with the same seven mathematics tasks at four time points between second and third grades. By the end of third grade, MD-RD children performed significantly worse than their TA and RD peers, on almost all but two mathematics tasks (place value and written computation). Children with MD continued to outperform children with MD-RD on story problems and calculation principles to the end of the third grade. Another interesting finding was that no significant group differences were found in the achievement growth on all mathematics tasks. In other words, children in all achievement groups (i.e., MD, MD-RD, RD, TA) grew at about the same rate on all the mathematics tasks over second and third grades. Even children with MD-RD, who demonstrated pervasive difficulties on all the mathematics tasks in the beginning of the study, made progress in these mathematics tasks like their peers in the TA and RD groups.

Also part of the aforementioned two-year longitudinal project, Jordan et al. (2002) examined the effects of achievement group membership on overall achievement in mathematics and reading. There was no statistically significant difference on the level of achievement between children with MD and children with MD-RD on overall mathematics achievement, as well as the subtest of calculation and applied problems in the fall of second grade. However, children with MD grew significantly faster than their peers with MD-RD on the overall mathematics achievement test and two subtests. Meanwhile, the TA and RD children started with significantly higher scores than did MD-RD children; Meanwhile, the TA and RD children started with significantly higher average scores in math, but they also grew significantly slower than MD-RD children when the time-invariant predictors of gender, ethnicity, income, and IQ were held constant. As a result, by the end of the third grade, children with MD performed at about the same level as children with RD on Broad Mathematics and Applied Problems, and scored higher than RD children on Calculation.

On the other hand, the results on reading achievement showed that the TA and MD groups started with significantly higher scores and grew significantly slower than the MD-RD group on all three reading measures. No group difference was found between the RD and the MD-RD group in the achievement level at the beginning of second grade or in growth rates of any of the reading measures. In fact, by the end of third grade, the achievement gaps between children with difficulties in reading (i.e., MD-RD and RD) and those without difficulties in reading (i.e., TA and MD) remained apparent. RD children did not catch up to their peers with MD on reading performance as MD children did on mathematics performance. Given this observation, the researchers concluded that reading abilities have a significant impact on children's math growth, but math abilities did not have such influence on children's reading growth.

In addition, the researchers found that children without MD grew significantly slower than children in the two MD groups on math measures, and a similar pattern was also shown on reading measures, which indicated that children with better math/reading abilities actually made less progress over the period between second and third grade. The researchers attributed this phenomenon as a possible result of "instructional ceiling" (Jordan et al., 2002, p. 595). That is, the growth of children with strong reading (or mathematics) skills may reach a limit when the higher level of reading or mathematics skills had not been introduced in the curricula.

Focusing on older children in the third and fourth grades, Andersson (2008) examined performance profiles of achievement groups in eight areas of mathematics, including: arithmetic fact retrieval; written arithmetic calculation; one-step mathematic word problems; complex multi-step mathematic word problems; place value; calculation principles task; approximate arithmetic; and telling time. The results showed that both MD and MD-RD children performed significantly worse than TA children in all but one area, the place value task. The results indicated that both MD groups might be able to master the required skills for the place value task and to catch up to their typically achieving peers by the fourth grade. Also, there was no significant difference between the performance of MD and MD-RD children in all areas of mathematics, which was contrasted with findings from other studies (i.e., Fuchs & Fuchs, 2002; Jordan & Hanich, 2000; Hanich et al., 2001; Jordan et al., 2003). The different finding might be due to the fact that the word problem tasks used in this study required low linguistic skills but demanded high calculation and problem-solving competencies, and therefore the linguistic advantage of the MD children was eliminated.

In another study, Fuchs and Fuchs (2002) examined the performance profiles of fourth-grade students with MD or comorbid MD-RD on three word problem solving tasks. Students with MD-RD scored consistently lower than their peers with MD on both arithmetic operations and problem solving across all three tasks, which corroborated the observation of Jordan and Hanich (2000) that children with MD-RD showed more pervasive difficulties than their peers with MD. Moreover, the performance differences between children with MD and those with MD-RD were mediated by the types of problem-solving task and by performance dimension (i.e., arithmetic operations vs. problem solving). On arithmetic story problems, the differences between the two MD groups on arithmetic operation and problem solving were comparably large. In contrast, on complex story problems and real-world problem solving, MD children and MD-RD children performed comparably on operations, but the MD children outperformed the comorbid MD-RD group on the problem-solving dimension of these two tasks. This finding suggested that the advantage of the MD children over the MD-RD children regarding operational abilities was restricted to simple number fact operation. When given tasks that required more complex operational abilities, children with MD might experience similar difficulties to their MD-RD peers. Nevertheless, regardless of the complexity of the tasks, MD children still performed better than their MD-RD peers on problem solving tasks, which demanded better linguistic skills.

The findings from previous studies suggested that children with MD-RD were more impaired and had more pervasive mathematical difficulties than their peers with MD. The distinction between the two MD groups was not only in quantity (the areas children have difficulties in), but also in quality (the complexity of arithmetic operation). As children with MD, or comorbid MD-RD grew older, they seemed to perform at a similar level as their non-MD peers on tasks like place value or early numerical skills, but children in both MD groups still performed worse than their TA peers on the tasks associated with calculation and story problems. On the other hand, the findings regarding mathematics growth of children with mathematics difficulties and their non-MD peers were mixed, and the extant research only focused children during a short-term span (i.e., second to fourth grades). It remains unclear how the children with difficulties in mathematics and/or reading progress in mathematics and reading achievement over the course of elementary school. Furthermore, many reviewed studies employed small and localized samples, which limited the generalizability of the findings. Therefore, more findings from large-scale longitudinal studies over the elementary school years could add to our understanding of the mathematics performance and growth of children with mathematics disabilities.

Stability of Mathematics Difficulties over Time

As Dowker (2005) and Ginsburg (1997) pointed out, children might encounter difficulties in some areas of mathematics but not others. They can also “outgrow” or “grow into” some mathematics difficulties in their learning, and therefore, present different developmental patterns in mathematics. Silver, Pennett, Black, Fair and Balise (1999) identified a group of children who were aged 9 to 13 with arithmetic disabilities, and retested these children 19 months later. The researchers found that approximately half of the children continued to exhibit difficulties in arithmetic, as well as reading and/or spelling disabilities at second testing; about 20% of these children did not have any difficulties in arithmetic, reading or spelling anymore; the rest of children did not have difficulties in arithmetic but demonstrated disabilities in reading, spelling, or both at second testing.

The study by Silver et al. (1999) only examined children's achievement at two time points. The results could be biased by measurement errors, individual or environmental conditions at retesting. Mazzocco and Myers (2003) followed a group of children from kindergarten to third grade. Among children identified as having mathematics disabilities at any point during the study, about 37% of them were identified only at one of the grades; about 23% were identified at two of the grades; and about 26% met identification criteria at three of the grades. Most of the MD children were first identified at kindergarten (57%) and first grade (26%). Although many children were identified early, about 35% of children identified in kindergarten and 11% of children in first grade did not demonstrate difficulties in mathematics anymore during the study.

Using a large-scale, nationally representative sample for their analyses, Morgan, Farkas, and Wu (2009) investigated the achievement growth in mathematics for children with or without persistent MD. Based on their MD status in kindergarten, children were then classified into three MD groups and one non-MD group: MD in fall kindergarten only, MD in spring kindergarten only, MD in both spring and fall kindergarten, and non-

MD in either spring or fall kindergarten. Among the four groups, children with MD in both fall and spring of kindergarten displayed the lowest intercept (i.e., scores in spring of first grade) and lowest growth rates in math between the first and fifth grades; children with MD in spring kindergarten displayed substantially less proficiency in early mathematics skills and had the second-lowest growth rates among the groups; children with MD in fall kindergarten initially performed poorly, but they had growth rates similar to those of children with MD in spring kindergarten.

In addition to having the lowest intercept and growth rates, children with MD in both fall and spring of kindergarten also demonstrated persistent difficulties over the elementary school years. Approximately 70% of children in this persistent MD group met the criterion of MD again in first, third and fifth grade, whereas about 46% of children with MD in spring of kindergarten, about 24% of children with MD in fall of kindergarten, and 4% of non-MD children were identified as MD in the subsequent years. This study not only reflected the fact that many children identified as MD in both spring and fall kindergarten persistently experienced math difficulties throughout their elementary school years; it also addressed the significance and validity of MD identification, as early as in kindergarten.

These finding supported the comments by Dowker (2005) and Ginsburg (1997) that some children with MD could somehow overcome their difficulties. The results from Silver et al. (1999) echoed the findings of other researchers (e.g., Hanich et al., 2001; Jordan et al., 2003), that the children with comorbid reading and mathematics difficulties usually exhibited persistent deficits in mathematics. On the other hand, Mazzocco and Myers (2003) found that some children identified at kindergarten and first grade did not experience difficulties in mathematics in later grades. Morgan et al. (2009) also found that approximately 30% of children with persistent MD in kindergarten did not demonstrate MD later in first, third, or fifth grades. The findings from Morgan et al. are particularly informative since the study was conducted using the ECLS-K data set, which is composed of a nationally representative sample. However, although Morgan et al. examined children's mathematics growth over the course of elementary school, they did not distinguish MD children with and without RD in their study. More research on the stability or change of children's achievement group membership should be conducted.

In summary, from the findings of previous research, children with mathematics difficulties were found to be a heterogeneous group. Children with MD, specific or otherwise, seemed to be two distinct groups and presented different achievement patterns in mathematics and reading. Mixed results were found with regard to the achievement trajectories of these two MD groups, which might partly due to the various identification criteria and measures used for classification. In addition, most of the reviewed studies used small and regional samples, which limited the generalizability of study results. Finally, the studies reviewed were either cross-sectional research or longitudinal studies conducted within a short period of time. The results only presented a snapshot of children's academic performance in a short time frame. To extend and deepen our understanding of children with mathematics disabilities, a longitudinal study focusing on their achievement growth across elementary school years should be warranted (Silver et al., 1999). In particular, given the importance of identifying early signs and predictors of mathematics difficulties, the current study focused on kindergarten factors associated with children's mathematics learning and growth over the course of the elementary school years. In this regard, the following review focused on the individual and contextual factors associated with children's mathematics growth in elementary school

Kindergarten Predictors of Math Achievement and Growth for MD Children

Learning mathematics is a dynamic process, in which many factors interact and influence children's mathematics performance during the elementary school years. In the present study, one of the aims was to explore the effects of kindergarten predictors on math achievement growth, as well as whether the effects of these predictors vary as a function of kindergarten achievement subgroups.

The selection of kindergarten predictors included in the model for the present study was informed by Walberg's model of educational productivity and Ginsburg's developmental view on research of mathematics difficulties as well as the extant studies conducted using the ECLS-K dataset. Walberg (1984) proposed his model of educational productivity to address the factors influential in school learning. A total of nine theoretical constructs were categorized into three areas: aptitude, instruction, and social-psychological environment. Students' aptitude referred to (a) ability or prior achievement, (b) development, as indexed by developmental level of chronological age; and (c) motivation or self-concept. Instruction included (a) the amount of time students engage in learning, and (b) the quality of instruction. Factors in social-psychological environment included (a) the home, (b) the classroom, (c) the peers group outside the school, and (d) use of out-of-school time. According to Walberg, these three areas not only had direct impact on learning, but also influenced one another, and in turn were affected by the feedback on the amount of learning occurred. Studies that conducted based on Walberg's model of educational productivity (e.g., Koller, Baumert, Calusen, & Hosenfeld, 1999;

Thomas, 2000) confirmed that this model provided a great framework for examining the influences of students' aptitude, instruction they receive and their psychosocial environment on mathematics learning.

Based partially on the framework of Walberg's model of educational productivity, the conceptual model for the present study was developed from a developmental perspective to examine the math growth of children with difficulties in mathematics (Ginsburg, 1997). In addition, since the present study was conducted using data from the ECLS-K study, the selection of variables for the present study was also informed by the previous studies conducted using the same dataset. As a result, the conceptual model for the present study included three sets of variables – demographic characteristics, individual skills, and class factors (see Figure 1). The demographic characteristics included gender, socioeconomic status, and race. Individual skills focused on children's performance in kindergarten, including their initial mathematics and reading knowledge and learning-related skills. The class factors included in the present study were class size, instructional time and instructional activities in mathematics.

In this section, the previous studies focusing on the effects of identified kindergarten predictors were reviewed. Although the targeting population was children with difficulties in mathematics for the present study, the extant studies focusing on this population were scarce. Therefore, the studies focusing on children in general and examining the effects of identified predictors were also included. Table 2 presents the descriptive information of the review studies, which include one study on initial mathematics knowledge (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004), three studies examining both initial knowledge and learning-related skills (Bodovski & Farkas, 2007;

Claessens, Duncan, & Engel, 2009; Duncan et al., 2007), and three studies on learning-related skills (DiPerna, Lei, & Reid, 2007; McClellan, Acock, & Morrison, 2006; McClellan, Morrison, & Holmes, 2000).

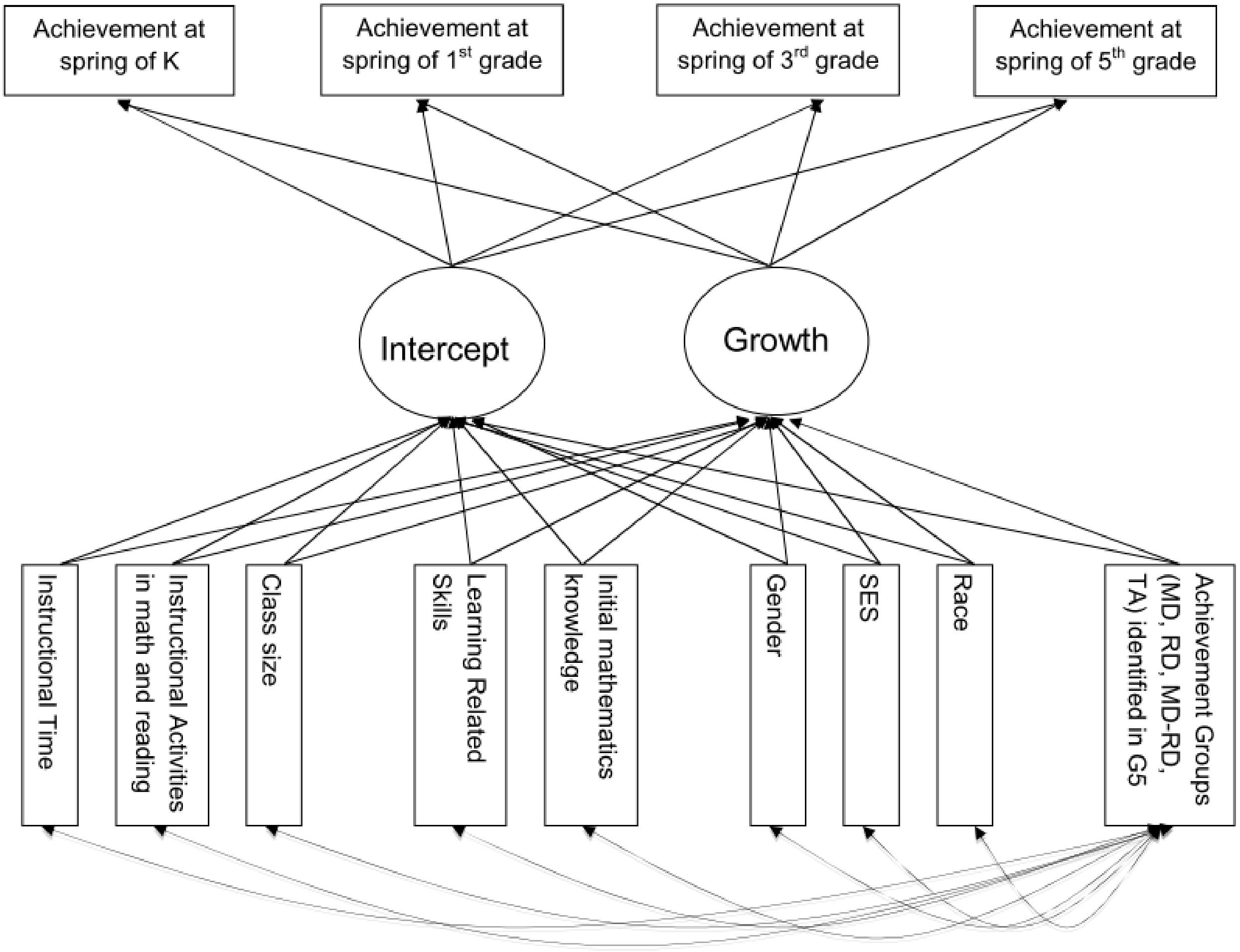

Figure 1. Path Diagram of Growth Curve Model with Achievement Groups, Demographic Variables, and Kindergarten Predictors.

Table 2

Descriptive Information of Studies of Identified Kindergarten Predictors

study sample			Methodology			
Study	**Sample N**	**grade/age**	**Study Design**	**Analysis**	**Predictor**	**Outcome Var.**
Aunola et al. (2004)	194	age 5-6	Empirical study	Growth Curve Analysis	IMK	K to 2 math
Duncan et al. (2007)		preschool to age13-14	Extant datasets (ECLS-K, NLSY, NICHD SECCYD, IHDP, MLEPS, BCS)	1. Regression analyses 2. Meta-analysis	IMK LRS	Math achievement (diff. for each dataset)
Bodovski & Farkas (2007)	13,043	grade K to 3	Extant dataset (ECLS-K)	Regression analysis	IMK LRS	Math achievement gains b/t waves
Claessens et al. (2009)	8,527	Grade K to 5	Extant dataset (ECLS-K)	Regression analysis	IMK LRS	1. 5th grade Math IRT scores 2. 5th grade teacher rated math competence
Diperna et al. (2007)	6,905	Grade K to 3	Extant dataset (ECLS-K)	Structural equation modeling	LRS	K to 3 math IRT scores
McClellan et al. (2000)	540	Grade K to 2	Empirical study	Regression analysis	LRS	1. School entrance math scores 2. math scores at 2nd grade
McClellan et al. (2006)	538	Grade K to 6	Empirical study	Growth Curve Analysis	LRS	K to 6th grade math scores
Nye et al. (1999)	6077	grade 3 to 8	Empirical study	Hierarchical Linear Models	small class	SAT CTBS (math, reading, science)

Table 2. (continued)

study sample				Methodology		
Study	**Sample N**	**grade/age**	**Study Design**	**Analysis**	**Predictor**	**Outcome Var.**
Nye et al. (2000)	4515	K to grade 3	Empirical study	Hierarchical Linear Models	small class	norm-referenced math and reading achievement
Nye et al. (2002)	N/A	Grade 1 to 3	Empirical study	Hierarchical Linear Models	small class by low achievement	SAT
Pianta et al. (2008)	791	54 months to grade 5	Non-empirical study (NICHD SECCYD)	Unstructured Latent Curve Model	Emotional Quality Quantity of exposure Instructional Quality Quantity by emotional quality Quantity by instructional quality	WJ-R: Letter-word Identificaiton(grade 1), Broad Reading (Grade 3 & 5), Applied problems, and Picture Vocabulary

Note: IMK = Initial mathematics knowledge; LRS = Learning-related skills; ECLS-K = Early Childhood Longitudinal Study-Kindergarten Cohort; NLSY = national Longitudinal Survey of Youth; NICHD SECCYD = National Institute of Child Health and Human Development Study of Early Child Care and Youth Development; IHDP = Infant Health and Development Program; MLEPS = Montreal Longitudinal-Experimental Preschool Study; BCS = British Birth Cohort Study (BCS); SAT = Stanford Achievement Test (Madden et al., 1973); CTBS = Comprehensive Tests of Basic Skills (CTB/MacMillan, 1989).

Initial Mathematics Knowledge

Initial mathematics knowledge is the informal mathematics knowledge that children develop in the home and in out-of-school contexts before they enter formal schooling (Ginsburg, 1997). Some researchers have suggested that children's initial mathematics knowledge can serve as a foundation for their mathematics learning in elementary school. Aunola et al. (2004) were particularly interested in the extent to which a child's counting ability, as well as their overall mathematics performance upon their entry into kindergarten predicted their mathematics growth in primary grades. A group of 194 children in Finland was tested on their mathematics achievement six times between kindergarten and second grade. The children's counting ability was measured by asking the children to count numbers forward and backward. The findings from a latent growth curve analysis showed a positive and statistically significant correlation ($r = .17, p < .05$) between a child's initial mathematics performance (intercept) and growth (slope). A child's counting ability at kindergarten entry was also found to be significant in predicting later children's mathematics growth ($\beta = .44, p < .001$). The findings indicated that the higher the level of overall math performance and counting ability that a child had at kindergarten entry, the faster the growth rate they displayed from kindergarten to second grade.

Also examining the associations between early skills and later academic achievement, Duncan and colleagues (2007) conducted two sets of analyses by using data from six longitudinal datasets. Regression analyses were first conducted across all taught subjects to examine the extent that initial math skills and attention skills/problems at school entry were predictive of later mathematics and reading achievement. The results indicated that initial skills were almost always a statistically significant predictor of subsequent mathematics achievement with standardized coefficients ranging from .29 to .53 for Grade 3, .12 and .19 for Grade 5, and .22 for children aged 13-14. In the case of attention skills and attention problems, although the regression coefficients were usually smaller than those of initial math achievement, more than half were statistically significant. The absolute values ranged from .09 to .14 for Grade 3 or age 8, .08 to .17 for Grade 5, and .09 for children aged 13-14. In the next stage, a meta-analysis of the standardized regression coefficients emerging from regression analyses of each study was conducted. The findings also revealed that early math skills were a powerful predictor for later mathematics achievement, with an average standardized coefficient of .33. The average regression coefficient for attention-related measures was .07, which was small but still significant.

By using data from a longitudinal study (i.e., ECLS-K), Bodovski and Farkas (2007) studied the effects of initial skill levels and student engagement in kindergarten on mathematics achievement gains in Grades K thru 3. The results of regression analyses showed that children who scored at or below 25th percentile of the study sample at kindergarten entry made slower math achievement progress (i.e., mathematics score differences between four data collection waves) than their peers who scored higher at school entry. Positive effects of school engagement on math achievement gains were also found at all survey waves, with effect sizes ranging from .13 to .17. The researchers also found negative interactions, in which the positive effect of school engagement on achievement gains was greater for low achieving children compared to their typically performing, or high achieving peers. In other words, the benefit of improving children's school engagement on math growth was more significant for low achieving children.

Also using ECLS-K dataset, Claessens and colleagues (2009) conducted a series of regression analyses on fifth grade mathematics achievement scores. Both math achievement and attention skills at kindergarten entry were found to be significant predictors ($\beta =. 346$ and $.171$, $p <. 01$) of a child's fifth grade math achievement in a full-control model; with early academic achievement, attention skills, and socio-emotional skills entered as predictors and background variables entered as controls. Moreover, when using teacher-reported mathematics competence in fifth grade as an outcome variable, a child's initial math performance and attention skills were nearly as predictive as they were for fifth grade test scores.

Learning-Related Skills

Learning-related skills (LRS) encompass behaviors, such as listening and following directions, participating group activities appropriately, staying on task, and organizing work materials (McClelland et al., 2000). Findings from studies using large-scale longitudinal datasets (i.e., Bodovski & Farkas, 2007; Claessens et al., 2009; Duncan et al., 2007) revealed that learning-related skills were a significantly positive predictor of children's later mathematics achievement and knowledge acquisition progress.

Similarly, DiPerna et al. (2007) used the ECLS-K dataset to explore predictive relationships between a child's classroom behavior at kindergarten entry and the improvement in their mathematics achievement between kindergarten and third grade. Approaches to learning, along with general knowledge and age at school entry, explained 41% of initial-status variance and 13% of growth-rate variance. After controlling for general knowledge and age, approaches to learning were found to be a significant predictor of mathematics growth, with standardized coefficient .19 for initial status and .10 for growth rate. Children who received higher ratings on approaches to learning were found scoring higher on their initial mathematics performance and demonstrated faster improvement in mathematics achievement.

In their empirical study, McClellan et al. (2000) found that learning-related skills were a significant predictor for mathematics achievement at kindergarten entry ($\beta = .117$, $p < .05$) and at the end of the second Grade ($\beta = .123$, $p < .05$). Learning-related skills accounted for a modest, but significant, amount of variance (1%) in mathematics at the beginning of kindergarten and at the end of second grade, after controlling for the influence of IQ, age at school entry, amount of preschool experience, parental educational level, ethnicity, and home literacy environment. McClellan et al. (2006) continued to observe the participants of the previous study, until they were in sixth grade and conducted a series of growth curve analyses. Similar to the results of McClellan et al. (2000), learning-related skills at kindergarten were found to be a significant predictor for kindergarten performance ($\beta = .17$, $p < .001$) and growth ($\beta = .19$, $p < .05$) in mathematics between kindergarten and second grade after controlling for the child's IQ, age in kindergarten, ethnicity, and maternal educational level. Between third and sixth grade, however, kindergarten learning-related skills predicted third-grade performance ($\beta = .24$, $p < .001$) but not the growth rate in mathematics achievement. Thus, these findings provided evidence of the associations between learning-related skills and mathematics achievement. In particular, learning-related skills had a significant effect on a child's math performance and growth in their early elementary school grades. It should be noted that the mathematics measures used before and after the third grade were different, which was the reason for conducting two separate analyses for kindergarten to the second grade and third grade to the sixth grade. Hence, it is unclear whether the two different sets of trajectories obtained were due to the children's development or were affected by the different measures used.

Based on the reviewed studies, children who scored higher in mathematics at kindergarten entry, or those who were equipped with better learning-related skills progressed faster than their peers with lower scores did in initial math knowledge or learning-related skills during the early elementary school years. The results also showed that the associations with later mathematics achievement or growth were greater for the initial math knowledge than for learning-related skills. Interaction effect was also found, in which the benefit of improving children's learning-related skills was greater for low-achieving children than for typically performing or high-achieving children. However, it is still unclear whether the association between the initial math knowledge, as well as learning-related skills and math growth beyond the third grade are different. The extant research is also limited by the extent to which the associations of these two kindergarten predictors of math skill acquisition in elementary school vary for children who experience difficulties in mathematics. Moreover, scarce evidence is available regarding whether the associations between these two kindergarten predictors and mathematics achievement growth may differ as a function of classroom context. Thus, in order to gain a greater understanding of the classroom context and how it may influence a child's learning and their mathematics growth in elementary school; the following section focuses on the studies of school and classroom effects.

Classroom Effects on Mathematics Growth

Students spend a significant amount of time in school once they begin their formal education. Therefore, researchers inevitably explore the influence of the classroom when investigating the associations between children's kindergarten experiences and their achievement growth. The literature review in this section focuses on the lasting effects of class size and instructional time in kindergarten classrooms on children's later achievement and growth over the elementary school years.

Class Size

Class-size reduction represents an important topic in research of school effectiveness of the past decades, and the long-term association between small class size and children's mathematics achievement signifies a particular interest of this literature review. A series of analyses conducted by Nye and colleagues used data from Project Star, a large-scale, four-year longitudinal study funded by the Tennessee General Assembly and conducted by the State Department of Education. Project Star, randomly assigned approximately 7000 kindergarteners and classroom teachers to one of three treatment conditions: small classes (13–17 students), regular classes (22–25 students), or regular classes with a full-time classroom aide. All participants remained in the same assigned class type from kindergarten through third grade.

Nye, Hedges, and Konstantopolos (2000) analyzed the Project STAR data and found the average effect of small classes statistically significant at every grade level between kindergarten and third grade. The respective effect sizes were 0.23, 0.30, 0.18, and 0.15 for mathematics and 0.23, 0.21, 0.23 and 0.18 for reading. The effect of small classes seemed comparable between reading and mathematics achievement and decreased over the years. Nye, Hedges, and Konstantopolos (2002) further examined the differential effects of small class size for low-achieving children. Children identified as low-achieving satisfied two different criteria; they scored below the median (low achieving) or below the first quartile (very low achieving) of kindergarten mathematics performance in their classes. Regardless of identification criteria adopted, the same patterns of small class effect on math and reading achievement groups emerged. In effect, low-achieving students benefited from small classes less than did their high-achieving peers in mathematics, but they benefited more from small class size than did high-achieving students in reading. These interaction effects between small classes and low achievement were not statistically significant, however, either in math or reading. It should be noted that despite insignificant results, the analyses did not absolutely rule out the possibility that the differential effects of small classes may exist. Meanwhile, methodological issues emerged during the experiment, such as attrition and participants switching between treatments, which could also create confounding bias. Whether the differential interaction effect between small class and math achievement exists warrants additional research into the issue.

Despite the previous efforts of Project STAR's evidence of small class effects on mathematics and reading achievement in primary grades, the lasting effect of small class size beyond third grade remained unclear. Nye, Hedges, and Konstantopoulos (1999) examined the achievement of students who participated in Project STAR for five years after the experiment ended, following these students from grades 4 to 8. They found the average small-class effect statistically significant and positive for mathematics and reading at grades 4, 6, and 8, with medium effects sizes of 0.126, 0.203, and 0.158 for mathematics and 0.112, 0.126, and 0.133 for reading, respectively. In addition, the small-class effect became larger the longer participants remained in small classes between kindergarten and grade 3. Researchers found the effect about twice as large for the students in small classes for all four years compared to the effect for those in small classes for one year only.

Findings by Nye et al. (2004) indicated the necessity of taking other classroom factors into consideration while studying classroom effects on achievement. To address this issue, Melesi and Gamoran (2006) used two waves of kindergarten data from the ECLS-K study to examine the effect of class size and instruction on achievement while taking other individual and classroom factors into consideration. The results from Hierarchical Linear Modeling (HLM) analyses showed no association between class size and achievement in either reading or mathematics, even after controlling for the type of kindergarten (full day vs. half day) and students' remedial status (i.e., percentage of children reading below class level, percentage of children with math skills below grade level, and percentage of children with diagnosed disability in each class). On the other hand, the study found instructional activities contributing to children's academic performance significantly. The estimates of two reading activities ("Whole-language" and "Phonics") in predicting kindergarten reading achievement were 0.080 and 0.181, and the estimates of two math activities ("Teaching for Understanding" and "Drill") in predicting math achievement were 0.074 and 0.078. The interaction effects of small class size and instructional activities, however, did not show any statistical significance in either reading or mathematics achievement, which indicated that instructional effects appeared equally consequential in small and large classes. Unlike the positive results obtained by Project STAR, Melesi and Gamoran (2006) found no significant effect of class-size reduction on students' reading and math achievement. The non-experimental nature of the ECLS-K dataset, where the students were not randomly assigned to classes as in Project STAR, may be associated with the different finding. Moreover, insufficient information on why some classes were smaller than others in the ECLS-K study made it hard to eliminate all the selection bias. Therefore, it is understandable that the small class effect found in this study was not as strong as those of a randomized trial.

Instructional Time

Pianta, Belsky, Vandergrift, Houts, & Morrison (2008) investigated the effect of quantity of instruction on achievement growth at first, third, and fifth grade. They found among the identified variables of classroom effects, quantity of exposure and the quality of emotional support scored as significant predictors of math and reading achievement. A higher level of exposure to mathematics instruction predicted enhanced mathematics achievement in third and fifth grade over and above the influences of child and family characteristics (i.e., family poverty, child gender, and prior math achievement). In addition, greater emotional support in fifth-grade classrooms also predicted better mathematics achievement in fifth grade. This study provided insight into the classroom effects; in particular, the results indicated that classroom factors contributed differently to mathematics and reading achievement.

Overall, the reviewed studies showed mixed results on small-class effects. The studies conducted based on the data from Project STAR showed strong effects of class size reduction in mathematics, whereas the study using a nationally representative sample like ECLS-K failed to detect such effects. The findings on interaction effects of small class and low achievement also proved inconclusive. Although one of the studies (Nye et al., 1999) examined the lasting effect of small class in kindergarten, evidence of long-term effect of class-size reduction remains scarce. Moreover, only a small number of studies researched the extent to which instructional activities in kindergarten predict later academic achievement. However, studies found instructional time and activities in kindergarten as significant predictors of later mathematics achievement. Future research should explore whether children's school experiences in kindergarten predict their later mathematics achievement and growth, especially for low-achieving children.

Methodological Review of Research Studies

In the previous sections, a synthesis of research was presented on the characteristics of children with mathematics difficulties, as well as kindergarten predictors associated with math achievement growth over the period of elementary school years. Subsequently, the methodological strengths and weaknesses of these studies were evaluated. Given the various types of literature sources included in this chapter, the guidelines for evaluation were established based on the quality indicators and criteria suggested by Gersten et al. (2005), Thompson, Diamond, McWilliam, Snyder, and Snyder (2005), and Isaac and Michael (1995). Each article was evaluated based on the following five criteria: (1) clear description of sample; (2) clear description of measures; (3) appropriate data analysis method; (4) clear description of attrition rate and missing data; and (5) information on practical significance. The detailed evaluation of each measure is presented below.

Description of Sample

Description of participants. Providing sufficient information about participants can help readers to identify the population to which study results may be generalized. As suggested by Gersten et al. (2005), the description of participants may include demographics (e.g., age, race, gender, socio-economic status, English language learner status, special education status), scores-related academic assessments, and percentage of students receiving free or subsidized lunch for participating schools. All of the reviewed studies included some, if not all, aforementioned information about participants. Participants' age, gender, and socio-economic status were usually reported, whilst some studies, particularly those conducted outside of US (e.g., Andersson, 2008; Aunola et al., 2004; and Lewis et al., 1994), did not report the racial composition of their sample. In addition, only a handful of studies reported special education status (i.e., Fuchs & Fuchs, 2002; Hanich et al., 2001; Jordan et al., 2003). Inclusion of information regarding participants' special education status is especially important in studies examining children with difficulties in mathematics and reading; since the children's special educational needs may also inform readers regarding children's difficulties in mathematics and/or reading achievement.

Description of sample selection. Clear description of the procedure for sample selection is helpful for readers to identify the population of participants to which results may be generalized. It is also informative for subsequent study replication. All of the reviewed studies provided sufficient information on sample selection—the location, targeted population, as well as materials, and criteria used for participants enrollment.

Moreover, flow charts were used if the sample selection process involved multiple stages and complex procedures (i.e., Barbaresi et al., 2005; Gross-Tsur et al., 1996).

Delineation of learning disabilities/difficulties definition. To ensure generalizability of study results, the researchers need to provide an operational definition of learning disabilities/difficulties. The information presented should include a clear criterion and the measures for identifying children with learning disabilities/difficulties. Gersten et al. (2005) recommended avoiding the using of school district-provided labels or identifying children simply by state criteria, since these classification labels provide limited information on the type or extent of disability a child may have. All reviewed studies focusing on children with mathematics difficulties included a description of criteria and measures used for identification. Four studies used school district-provided labels either for identification or as a prerequisite to screen participants for the study.

The participants in the study by Fuchs and Fuchs (2002) were students identified by the school district as having a learning disability and had a math goal on their Individualized Education Program (IEP). The participants in Andersson (2008) were selected only if they were receiving special instruction either in mathematics, reading, or both at the time of the study. Geary et al. (1999) and Geary et al. (2000) selected their study participants from a remedial program where children were referred by their teachers for additional instructional assistance in reading and mathematics. However, it has to be noted that using the receipt of special education service as a pre-requisite for sample selection may lead to identifying only children with more serious and apparent learning difficulties. Meanwhile, the additional instruction these children received may have a confounding effect on their achievement performance, which was not examined or controlled for in these studies.

Measures

To investigate phenomenon of interest, researchers collect data through a variety of measures. The quality of study findings partly depend upon the psychometric integrity of the data being analyzed in a given study. In addition to clearly identifying the measures used for specific purposes, the researchers should also provide adequate information pertaining to the reliability, validity, and standardization properties of instrumentation (Issac & Michael, 1995). Although many reviewed studies presented detailed description regarding the content of measures, most failed to provide crucial information on the reliability and validity of the measures. In particular, this is the case for the studies conducted by using an extant, large-scale dataset (e.g., Bodovski & Farkas, 2007; Claessens et al., 2009; Milesi & Gamoran, 2006). Cronbach's alphas for internal consistency of reliability were reported in some of the reviewed studies (e.g., Fuchs & Fuchs, 2002; Geary et al., 1999; Jordan et al., 2002; Jordan et al., 2003). Only one reviewed study (Fuchs & Fuchs, 2002) reported concurrent validity for the experimental tasks used in the study.

Data Analyses

There have been enormous advances in the area of statistical analysis in the past decade. The development of computerized statistical packages also enables researchers to approach research question in new ways. While there are numerous analytic strategies available, it is essential for researchers to choose techniques that are appropriate and aligned with research questions and hypotheses (Gersten et al., 2005). Most of the reviewed studies presented in this chapter seemed to meet this quality indicator. For instance, some researchers relied on ANOVA (analysis of variance) and MANOVA (multivariate analysis of variance) to compare the group performance on overall mathematics achievement as well as on a variety of math tasks (i.e., Andersson, 2008; Fuchs & Fuchs, 2002; Geary et al., 2000; Geary et al., 1999; Hanich et al., 2001; Jordan & Hanich, 2000; Jordan & Montani, 1997; Lachance & Mazzocco, 2006; Shalev et al., 1998). Post hoc tests were also used to further distinguish achievement difference between all groups. Regression analyses were conducted to estimate the extent to which mathematics achievement could be explained by specified predictors (i.e., Bodovski & Farkas, 2007; Claessens et al., 2009; Duncans et al., 2007; McClellan et al., 2000). To properly account for the level of aggregation at which class size occurs, some researchers used hierarchical linear models (HLM) for analyses (Milesi & Gamoran, 2006; Nye et al., 1999, Nye et al., 2000; Nye et al., 2002; Nye et al., 2004). In addition, growth curve analyses were used to examine children's achievement growth and the effects of associated predictors (Curby et al., 2009; Diperna et al., 2007; Jordan et al., 2002; Jordan et al., 2003; Jordan et al., 2006; McClellan et al., 2006; Morgan et al., 2009; Murphy et al., 2007; Pianta et al., 2008).

Despite the clear description of the statistical techniques used, only some of the studies provided rationale for their selection (e.g., Curby et al., 2009; Nye et al., 1999; Milesi & Gamoran, 2006; Pianta et al., 2008). Moreover, only one study (Jordan & Hanich, 2000) tested the theoretical assumptions underlying the procedures before conducting statistical analyses.

Attrition Rates and Missing Data

The attrition rate of a study refers to the percentage of participants who choose to withdraw from the study for any reason. The occurrence of missing data, although potentially due to participant attrition, can also stem from item nonresponse. Both attrition and missing data pose threats to a study's internal and external validity, particularly in the longitudinal studies. To ensure the internal and external validity of the studies, Gersten et al. (2005) suggested documenting overall attrition of participants and/ or missing data on key variables; and examined whether the remaining sample was comparable to the original one in order to ensure the representativeness of the analytic sample.

Among the reviewed studies, some experimental studies (e.g., Aunola et al., 2006; Lachance & Mazzocco, 2005; Jordan et al., 2002; Murphy et al., 2007) merely reported the number of participants who did not complete the study. For instance, Aunola et al. (2006) reported losing about 5% of participants during the two years of the study. Lachance and Mazzocco (2005) enrolled all kindergarteners (N = 249) from a school district, with 214 children still participating at the end of the four-year study with an attrition rate of 15%. Similarly Jordan et al. (2002) reported a total attrition rate of 13%, with the attrition rates for each achievement group ranging from 10% to 19%. The participant attrition for these studies may or may not have significant impact on the interpretation of the results. However, the researchers should not only document the attrition, but also to ensure the study groups remaining comparable at the conclusion of the study (Gersten et al., 2005).

Jordan et al. (2007) compared the participants' background characteristics at the beginning and the end of the study, and found the demographic composition of the groups virtually unchanged. Morgan et al. (2009) also conducted descriptive analyses to compare the full and analytical samples on a range of socio-demographic (i.e., gender, race, SES) and additional (i.e., disability status, math test scores at school entry) factors. By applying procedures to ensure the comparability of the analytic and full samples, the researchers strengthen the external validity of their studies. Another example of the impact of participant attrition was the study by Silver et al. (1999), where 204 children with MD were tested initially, and only 80 returned for the retesting 19 months later. The researchers found that fewer children with comorbid difficulties in mathematics and language at initial testing returned for the retesting. As a result, the generalization of their findings to children with comorbid difficulties may be misleading.

The development of statistical analysis packages provides another option for dealing with missing data. Morgan et al. (2009) used Hierarchical Linear Modeling (HLM) for their analyses, since the program allowed for the inclusion of any child who had missing data. McClelland et al. (2006) conducted a longitudinal study from kindergarten to sixth grade, whereby the sample size decreased from 538 to 260 due to attrition. They used Full Information Maximum Likelihood (FIML) estimation in *Mplus* to address the issue of attrition and missing data. This approach used all available data simultaneously in the estimation of a model, and produced parameter estimates that were less biased than other missing data strategies, such as listwise deletion, pairwise deletion and mean imputation. By using this strategy, McClellan and colleagues ensured sufficient variance was present on each variable in the remaining sample of participants. For studies with a large amount of missing data, adopting an appropriate strategy to deal with such issue would strengthen the power of the analyses and improve the generalization of the study results.

Practical Significance

Frequently, researchers examine the effects or associations of key variables by evaluating *statistical significance* of study results. Statistical significance refers to the unlikelihood that the observed results have occurred due to sampling error (Thompson et al., 2005). However, its use has been questioned in recent years. One major concern is that one may find statistical significance with seemingly very small differences as long as the sample size is large enough. As a result, the statistically significant finding is not necessarily important or meaningful in real-world context. *Practical significance*, as opposed to statistical significance, evaluates whether the results are noteworthy. To examine practical significance of study results, effect sizes are used to estimate the magnitude of an effect or association between two variables (Ferguson, 2009).

Given the importance of addressing practical significance in the study, Thompson et al. (2005) suggested that researchers should clearly identify the effect statistics they used and report effect sizes for each primary outcome. Among the reviewed studies, only four studies reported effect sizes for their results. Andersson (2008) conducted ANOVA analyses for group differences on eight mathematics tasks, and found significant group effects on all eight tasks, with effect sizes (η^2) ranging from .04 to .31. Bodovski and Farkas (2007) conducted regression analyses and found small effect (.03 to .04) of instructional time and medium effect (.13 to .17) of school engagement on children's mathematics score change in primary grades. Similarly, Fuchs and Fuchs (2002) found large effect sizes (.73 to 1.07) between children with MD and children with MD-RD on word problem solving, and relatively small to medium effect sizes (.19 to .46) between two groups on arithmetic operations for complex word problems. Hanich et al. (2001) also found significant group effects on the mathematics tasks, with effect sizes (η^2) ranging from .08 to 25.

In addition to using and reporting effect sizes, Thompson et al. (2005) also recommended explicitly and directly comparing study effects to those reported in related literature. However, none of the reviewed studies met this quality indicator.

Collectively, the reviewed studies presented clear description of study samples, measures, and data analysis techniques employed. There were also some common weaknesses. For instance, most authors did not provide a rationale for their selection of data analysis techniques. All but one reviewed studies failed to examine whether the theoretical assumption of the intended procedure was met before they conduct data analyses. Moreover, many studies did not address the impacts of attrition and missing data on their study results. There were only a few studies reported effect sizes. Finally, the comparisons of study effects were not reported in the reviewed studies. The evaluation of the strengths and weaknesses of the reviewed studies is informative and helpful to strengthen the methodology of the current study.

Conclusion

Although the research on children with MD has gained more attention during the past decade, the understanding of this phenomenon is still limited. The current knowledge about children with MD suggests that they are a heterogeneous group. In particular, children with MD and those with comorbid MD-RD present distinct performance profiles in mathematics. For instance, children with MD have advantage over their peers with MD-RD on word problem solving. In addition, children with MD, with or without RD, appear able to improve over time and perform at the same level as their peers without MD on some tasks, such as place value, but not on others. Mixed results are also found on the achievement growth rates of children with MD during the period of elementary school. The findings about mathematics difficulties are informative. However, the generalizability of these findings is questionable, since the extant studies were conducted with small and local samples. Another concern with many studies of children with MD is the absence of a more comprehensive model for analysis, where education-related factors, such as entry-level skills at onset of formal education, as well as the classroom variables, should be included.

The literature review on entry-level skills showed that both initial mathematics knowledge and learning-related skills were significant predictors of children's mathematics achievement in later elementary school grades. Further, children's initial mathematics knowledge was found to have greater lasting effects on mathematics achievement growth than their learning-related skills. The researchers also found that low-achieving children can benefit more from improving their learning-related skills than their high-achieving peers. On the other hand, only a few studies have investigated the associations between kindergarten classroom factors and children's later achievement; the results were mixed and inconclusive. The significant effects of small classes on mathematics achievement were found in some studies but not in the other. Teaching experience and teacher's education background did not predict children's achievement in primary grades. The instructional time and instructional activities, however, were found to be significant predictors of children's achievement at the end of kindergarten. Collectively, the reviewed studies present a preliminary portrait of the associations between children's kindergarten experience and their mathematics achievement in primary grades. However, it remains unclear whether the effects of these kindergarten predictors last beyond primary grades. More importantly, more research is needed to explore whether there are differential effects of these kindergarten predictors on achievement growth for children with MD or comorbid MD-RD.

The ELCS-K is a large-scale, longitudinal dataset containing rich descriptive information about children, their teachers, and schools. The reviewed studies using the ECLS-K show that both initial mathematics knowledge and learning–related skills are significant predictors of children's scores change and growth in early grades (Bodovski & Farkas, 2007; Claessens et al., 2009; Diperna et al., 2007). In kindergarten, reading and mathematics instructional activities predicted children's achievement. However, no significant effects of small class sizes were found (Milesi & Gamoran, 2006). By using the ECLS-K, the current study aimed to explore the lasting effects of identified kindergarten predictors (i.e., initial mathematics knowledge, learning-related skills, class size, instructional activities, and instructional time) on achievement growth over elementary school years. Specifically, the following three research questions were examined in the current study:

Research Question 1: What is the extent of subgroup membership stability and change from kindergarten through fifth grade (i.e., kindergarten vs. first grade, first grade vs. third grade, third grade vs. fifth grade)? Given membership change over time, what is the pattern of change across subgroups (e.g., MD to MD-RD; TA to MD)?

Research Question 2: To what extent do children’s math and reading achievement and growth trajectories differ among children in different achievement subgroups?

Research Question 3: To what extent do children’s spring kindergarten achievement group membership (i.e., TA, MD, and MD-RD), demographics characteristics, initial reading and mathematics knowledge, learning-related skills, instructional time, class size, and math instructional activities predict growth and level in fifth grade in mathematics performance over the course of the elementary school years? Does subgroup membership interact with the kindergarten variables to predict growth in mathematics over the elementary school years?

The detailed information regarding the data set and analytic sample, variables, and strategies for analyses for the current study is presented in Chapter 3: Methodology.

CHAPTER III

Method

The purpose of this study was to explore the group difference between children with math difficulties and/or reading difficulties. Also investigated were the effects of identified kindergarten child and class predictors on math achievement growth, as well as the extent to which the effects of these identified kindergarten predictors varied among kindergarten achievement subgroups. The methodology for the current study is described in detail in this chapter. The first section provides an overview of the ECLS-K data set, including its research design and sampling procedures. The second section describes the participants and variables that were extracted from the ECLS-K study and used in the present study. Finally, the third section outlines the data analytic strategies for handling and analyzing the data, including sampling weights, strategies for dealing with missing data, and the statistical procedures for analyzing the research questions.

The ECLS-K Data Set

Overview

The Early Childhood Longitudinal Study, Kindergarten Class of 1998-1999 (ECLS-K) is a multi-source, multi-method study that focuses on children's school experiences and development in elementary and middle school. Funded by the U.S. Department of Education's National Center for Education Statistics (NCES), the ECLS-K employed a multi-stage sampling design to select a nationally representative cohort of children, and followed them from kindergarten through eighth grade.

A total of 21,260 kindergartners throughout the nation participated in the ECLS-K study during the 1998-99 school year. The data collection was conducted first in the fall and spring of the children’s kindergarten year. The spring first-grade sample targeted all base-year respondents (i.e., respondents enrolled in fall or spring kindergarten), and the sample was freshened to include first-grade students who had not been enrolled in kindergarten during the base year. The sample of children from spring of the first grade was used for the next three waves of data collection: the spring of third grade, the spring of fifth grade, and the spring of eighth grade. As a result of the procedures used, the sample represents all children who were enrolled in kindergarten during the 1998-99 school year and in first grade during the 1999-2000 school year.

The ECLS-K study collected data using multiple methods from different sources, including direct and indirect assessments of the children’s academic, physical, and socio-emotional development; interviews with parents; and questionnaires completed by teachers and school administrators. The ECLS-K study describes children’s status at kindergarten entry, their transition into formal schooling, and their progress during the elementary and middle school years. The ECLS-K also provides rich data for researchers to use in associating a wide range of family, school, community, and individual variables with the children’s performance in school. Furthermore, the longitudinal nature of the ECLS-K study allows researchers to use kindergarten data as a baseline against which to gauge the children’s academic growth and to investigate the associations between developmental trajectories and variations in the children’s kindergarten experience. These characteristics make the ECLS-K an appropriate tool for the present study.

Participants

As indicated in the previous section, the participants of the ECLS-K study were sampled to represent children who were enrolled in kindergarten during the school year of 1998-99. For the current study, the children in the ECLS-K study were categorized into one of the five achievement groups - comorbid math and reading difficulties (MD-RD), math difficulties (MD), reading difficulties (RD), typically achieving (TA), and borderline group – based on their math and reading achievement IRT scale scored at the spring of kindergarten. The detailed information regarding the decision of cut-off criteria for achievement groups was described in the section of variables.

To address the research questions for the current study, two analytic samples were created using the ECLS-K data collected from five of the data collection waves: the fall of kindergarten, the spring of kindergarten, the spring of first grade, the spring of third grade, and the spring of fifth grade. The first analytic sample included children who were identified as MD-RD, MD, RD, or TA at the spring of kindergarten. This analytic sample included a total of 10,960 children, in which there were 1,073 children with MD-RD, 184 children with MD, 148 children with RD, and 9,555 typically achievement (TA) children. The second analytic sample included children who were identified as MD-RD, MD, RD or TA at the spring of kindergarten and continuously received math and reading achievement tests in the following waves of data collection until fifth grade. A total of 6,598 children were included in this analytic sample, including 499 children with MD-RD, 104 children with MD, 84 children with RD, and 5,762 children in the TA group at the spring of kindergarten. The cutoff scores used for identifying children of the four achievement groups were determined based on the score distribution of the full sample and described in detail in the section of variable "children's achievement group membership**".**

Variables

The variables used for the analyses include achievement variables (i.e., reading item response theory (IRT) scaled scores, reading proficiency probability scores, mathematics IRT scaled scores, math proficiency probability scores, and achievement group membership), demographic variables (i.e., gender, socioeconomic status (SES) and race/ethnicity), kindergarten child-level variables (i.e., initial reading knowledge, initial math knowledge, and learning-related skills), and kindergarten classroom-level variables (i.e., class size, instructional time, and instructional activities in reading and mathematics). These variables were derived from instruments including direct cognitive assessments in reading and mathematics, the teacher questionnaire, parent interviews and school records abstract. Appendix A described these instruments in more detail. Table 3 presents the source and the time points of data collection for the variables.

Table 3

Source and Data Collection Timeline of Selected Variables

				Data Collection Timeline				
Variable	Research Question	Source[a]	Composite	Fall-K	Spring-K	Spring G1	Spring G3	Spring G5
Math IRT scaled score	1 & 2	DA			x	x	x	x
Reading IRT scaled score	1 & 2	DA			x	x	x	x
Achievement group[a]	1 & 2	DA	x		x	x	x	x
Gender	3	PI, CR, FMS	x					x
Socioeconomic Status	3	PI	x					x
Race/Minority	3	PI, FMS	x					x
Initial Reading Knowledge	3	DA						
Initial mathematics knowledge	3	DA		x				
Learning-related skills	3	TQ	x		x			
Class size[a]	3	TQ	x		x			
Instructional time[a]	3	TQ			x			
Instructional activities[a]	3	TQ			x			

Note: DA=Direct assessment; PI=Parent interview; CR=Child record; FMS=Field management system; TQ=Teacher questionnaire.

a. Variables created by the researcher.

Achievement Variables

In the present study, children's reading and mathematics scores were used to establish achievement subgroups as well as to serve as dependent variables in the analyses. Both reading and math scores were obtained through direct child assessment at each wave of the data collection. The ECLS-K dataset provided several types of scores to represent children's performance in reading and mathematics. For the present study, the IRT scaled scores in reading and mathematics were used for creating achievement group variable as well as for serving as outcome variables in the multilevel models. In addition, the proficiency probability scores were used to describe how children in the four achievement groups performed in specific reading and math skills. The psychometric characteristics of the reading and math IRT scale scores in the ECLS-K are subsequently presented.

Characteristics of IRT scale scores. The ECLS-K assessed children directly through a two-stage procedure. First, all sampled children received the same routing test. Based on the child's performance on routing test, a second-stage form was determined for each sampled child with overlapping test items at different levels of difficulty (i.e., low, middle, and high). By means of assessment design within an IRT framework, , a common scale of achievement scores was established by incorporating the characteristics of test items (i.e., difficulty, discriminating ability, and "guess-ability" of each item) with children's response patterns to the items. Therefore, although children may receive different second-stage forms of test items, their IRT scores can still be calculated and compared within each wave of data collection.

In addition to ensuring that the achievement scores were comparable at each time point, the ECLS-K also generated scores that could be compared longitudinally. In the four waves of testing during kindergarten and first grade, the same sets of routing tests were used in each wave with alternative overlapping second-stage forms. The third and fifth grade assessment used the same overlapping two-stage design, but with more advanced sets of items. There were common test items shared between assessments in K-1 and the third grade, as well as between the third- and fifth-grade assessments. The obtained third and fifth grade scores were then re-estimated and calibrated to ensure that the scores were on a common scale and as a result could be compared longitudinally.

The ECLS-K examined the reliability and validity of direct cognitive assessments in reading and mathematics during their field testing. A group of education specialists examined the construct validity of test items by reviewing whether the test specifications of ECLS-K's reading and mathematics assessments matched national and state performance standards. The ECLS-K examined the concurrent validity of its test items by calculating the correlation between the scores of its reading and mathematics assessments and the Woodcock-McGrew-Werder Mini-Battery of Achievement (MBA; Woodcock, McGrew, and Werder, 1994; cited in Tourangeau et al., 2001). The convergent and discriminant validities were also investigated through examining the correlations between direct assessments and other indirect assessment in children's achievement and behaviors.

Reading IRT scale scores. The reading assessment was designed to measure basic skills (print familiarity, letter recognition, beginning and ending sounds, rhyming sounds, and word recognition), vocabulary (receptive and in-context vocabulary), and comprehension (listening comprehension, initial understanding, developing interpretation, personal reflection, and demonstrating critical stance).

For kindergarten and first grade children, the reading assessment focused on basic reading skills. For older children who were in the third and fifth grades, the emphasis of the assessment moved from basic skills to reading comprehension. This does not mean that children in the fifth grade were not tested on basic skills, or that younger children in kindergarten and first grade did not receive items on reading comprehension. The adaptive nature of the two-stage test design allowed children who did not perform well on their grade-specific routing tests to receive a second-stage form with a lower level of difficulty; and vice versa for children who performed well on their routing test to receive difficult test items at second stage. For the present study, the re-estimated IRT-scale scores were used for the longitudinal analyses (i.e., C1R3RSCL, C2R3RSCL, C4R3RSCL, C5R3RSCL, and C6R3RSCL). The scores ranged from 0 to 186.

The internal consistency coefficients (reliabilities of theta—latent ability for each child) for reading IRT scores were high (i.e., 0.91, 0.93, 0.96, 0.93, and 0.93 at the fall of kindergarten, spring of kindergarten, spring of first grade, spring of third grade, and spring of fifth grade, respectively). The validity for reading assessment was examined by evaluating the correlations between the ECLS-K reading items and established reading measures. For kindergarten and first grade reading assessment, the correlations with the Kaufman Test of Educational Achievement (KTEA) were over 0.80. For third and fifth grade reading assessments, the correlations between the reading items and the MBA reading assessment were 0.83 and 0.43, respectively. Since the correlation between two reading assessments in fifth grade seems low, generalizing the results from the ECLS-K data for fifth-grade children should be cautious.

Reading proficiency probability scores. Different from the reading IRT scale scores that describe the overall reading achievement, reading proficiency probability scores in the ECLS-K study provide information regarding the extent to which children master specific reading skills. The reading assessment in spring of kindergarten contained five proficiency levels, which represent a progression of skills and knowledge. A child who has mastered one of the higher levels is likely to correctly answer the items of lower levels. The five reading proficiency levels are: (1) recognizing upper- and lower-case letters of the alphabet by name; (2) associating letters with sounds at the beginning of words; (3) associating letters with sounds at the end of words; (4) recognizing sight words; and (5) reading words in context. The proficiency probability scores indicate the probabilities of a child mastering the specific reading skills. The proficiency probability scores can also be calculated to represent the performance of a subgroup of children on specific reading skills.

Mathematics IRT scores. The mathematics assessment was designed to measure skills in conceptual knowledge, procedural knowledge, and problem solving. The assessment addressed the following content strands: number sense, properties, and operations; measurement; geometry and spatial sense; data analysis, statistics, and probability; and pattern, algebra, and functions. Approximately half of the questions evaluated number sense, as well as number properties and operations. The remaining questions covered the remaining content strands. Manipulatives were available for children to use when solving some of the questions; paper and pencils were also provided for the appropriate parts of the assessment. For the current study, the re-estimated mathematics IRT scores from kindergarten to fifth grade were used (i.e., C1R3MSCL, C2R3MSCL, C4R3MSCL, C5R3MSCL, and C6R3MSCL). The scores ranged from 0 to 153.

The internal consistency coefficients of reliability for mathematics IRT scores were 0.89, 0.91, 0.92, 0.94, and 0.94 at the fall of kindergarten, spring of kindergarten, spring of first grade, spring of third grade, and spring of fifth grade, respectively. The intercorrelations between reading and mathematics assessment in the ECLS-K from kindergarten to fifth grade ranged from 0.74 to 0.77, which were consistent with estimates from other studies (Pollack, Atkins-Burnett, Najarian, & Rock, 2005). The coefficients of concurrent validity for mathematics IRT scores in the third and fifth grades were 0.84 and 0.80, respectively. No information on concurrent validity of kindergarten and first grade mathematics IRT scores were reported in the ECLS-K study.

Math proficiency probability scores. Similar to reading assessment, the math assessment in spring of kindergarten also contained five proficiency levels: (1) identifying some one-digit numerals, naming geometric shapes, and counting one-to-one up to ten objects; (2) identifying all one-digit numerals, recognizing patterns, counting beyond ten objects, and comparing objects using nonstandard units of length; (3) reading two-digit numerals, identifying the next number in a sequence, and naming the ordinal position of an object; (4) solving simple addition and subtraction problems; and (5) solving simple multiplication and division problems. These five proficiency levels also reflect a progression of skills, though the content of items in each level was more heterogeneous than the reading proficiency levels. Like the reading proficiency probability scores, the math proficiency probability scores were also used to describe children's performance on specific math skills.

Children's achievement group membership. Previous research suggests that the prevalence rate of children with difficulties in mathematics was about 2-10 % (Badian, 1983; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Lewis, Hitch, & Walker, 1994). Previous studies used various cutoff criteria to identify children with math difficulties, such as the 35th percentile (e.g., Jordan & Hanich, 2000; Jordan, Kaplan, & Hanich, 2002), the 15th percentile (e.g., Fuchs et al., 2008), and the 10th percentile (e.g., Mazzocoo & Myers, 2003). The cutoff score for achievement subgroups in the current study was determined by examining the score distribution of math and reading IRT scaled scores for the full sample. With reference to the prevalence rate of math difficulties, as well as the cutoff criteria used in the previous studies, the cutoff criteria used in current study was determined to be (1) lower than -1 SD for achievement difficulties and (2) above -0.5 SD for normal performance. To examine group differences between children with and without math difficulties, as well as the differences between the two MD groups (math difficulties with and without comorbid reading difficulties), this researcher categorized children in the full sample into four achievement groups based on aforementioned identification criteria. Figure 2 shows the cutoff scores and relations between achievement groups.

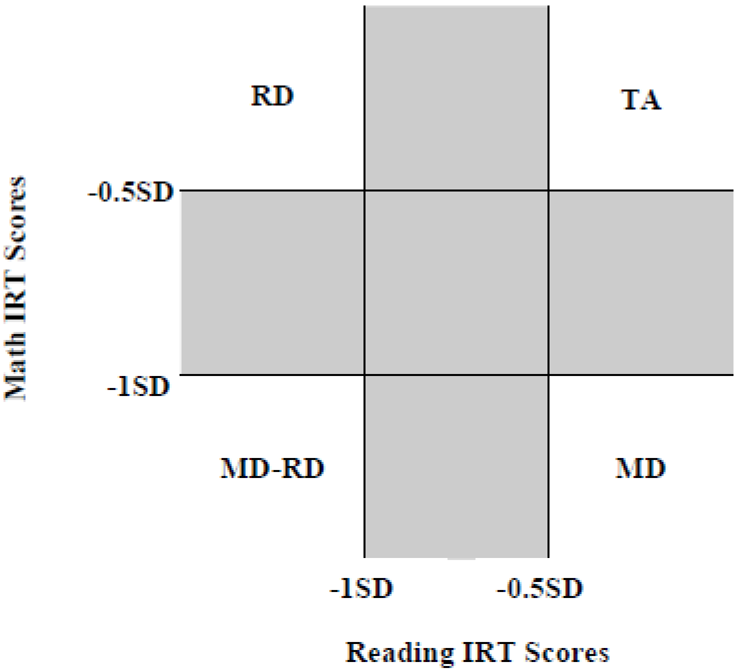

Figure 2. Composition of achievement subgroups based on math and reading IRT scale scores.

By applying the aforementioned criteria to the full sample, the prevalence rate of children with difficulties in mathematics (including children with MD and MD-RD) in spring of kindergarten was 7.4%, which was consistent with the estimates from the literature (e.g., Badian, 1983). In addition, the number of children with MD was significantly less than the number of children with MD-RD, which is not uncommon in the literature (e.g., Andersson, 2008; Geary et al., 1999). This observation suggests that a large proportion of children identified as having academic difficulties in the present study demonstrated performance deficits in both reading and mathematics. Meanwhile, it should be noted that about 40% of children in the full sample were excluded from analytic samples since these children scored between -1 SD and -0.5 SD in either one or both reading and mathematics assessments in spring of kindergarten. These excluded children showed borderline achievement performance (i.e., scored between -1 SD and -0.5 SD) in either or both reading and math assessments in spring of kindergarten, and they were not represented in the present study.

Demographic variables

All the variables in this section were used to describe the analytic sample and the achievement subgroups. In addition, children's demographic data also were used in the analyses of achievement growth in reading and mathematics.

Gender. The gender variable in current study was a dichotomous variable created by the ECLS-K study using data from various sources. First, the child's gender indicated in the parent interview was used. If the child's gender was missing from the parent interview, then the gender reported in the Field Management System (FMS) was used. If the child's gender indicated in the spring-kindergarten parent interview differed from the gender indicated in the fall-kindergarten parent interview, then the gender in FMS was used.

Race. In the ECLS-K, the race variable was created using data from parent interviews and the FMS. The categories of the variable included: White, non-Hispanic; Black or African American, non-Hispanic; Hispanic, race specified; Hispanic, no race specified; Asian; Native Hawaiian or other Pacific Islander; American Indian or Alaska Native; and more than one race specified, non-Hispanic. For the current study, the race categories developed by ECLS-K were combined into five categories: White, Black, Hispanic, Asian and Other races.

Socioeconomic status (SES). The SES variables used in the current study was created by the ECLS-K study. A continuous standardized composite variable, the SES variable was computed at the household level using data from the parent interviews in the spring of kindergarten in a zero z-score metric. The variables used to create the SES included: (a) father/male guardian's education; (b) mother/female guardian's education; (c) father/male guardian's occupation; (d) mother/female guardian's occupation; and (e) household income. More details regarding the missing data in the source of SES composite and imputation procedure were described in the Appendix A. The SES composite variable had a mean of zero and a standard deviation of one, and ranged from -4.75 to 2.75. Higher value indicates higher SES.

Kindergarten Child Individual Skills

Learning-related skills. Learning-related skills refer to children's behaviors that influence their participation and engagement in class activities. The variable of learning-related skills used in the current analysis was created by the ECLS-K study using a composite scale of six items in the Social Rating Scale (SRS), including the child's attentiveness, task persistence, eagerness to learn, learning independence, flexibility and organization.

Each sampled child was rated by his or her kindergarten classroom teacher on a scale of 1 = "Never" to 4= "Very often," or N/O if the teacher had no opportunity to observe such behavior in class. The ECLS-K created a composite variable of learning-related skills to represent the sampled child's overall approach to learning. The values of the composite variable (T2LEARN) ranged from 1 to 4. The correlations between learning-related skills and other two measures in SRS - social interaction and self-control

– were 0.70 and 0.66, which suggests satisfactory convergent validity. The split-half reliability for the scale was 0.89.

Initial mathematics knowledge. A child's initial mathematics knowledge indicates the ability and skills one was equipped with as they entered formal schooling. For the current study, a child's re-estimated mathematics IRT scaled score in the fall of kindergarten (C1R3MSCL) was used to represent their initial mathematics knowledge. The reliability of theta for the initial mathematics knowledge measure in the fall of kindergarten was 0.92. The correlation between mathematics and reading assessments in fall of kindergarten was 0.77, and the correlations between mathematics and other behavior measures (e.g., approaches to learning, self-control, social interaction) ranged from 0.22 to 0.28, which suggests good convergent and discriminant validities of the measure.

Initial reading knowledge. A child's initial reading knowledge indicates his or her reading performance upon entering kindergarten. For the current study, children's re-estimated reading IRT-scaled score in the fall of kindergarten (C1R3RSCL) was used. The reliability of theta for the initial reading knowledge measure in fall of kindergarten was 0.93. The concurrent validity, which was measured by correlating with KTEA reading assessment, was approximately 0.85.

Kindergarten Class Variables

Class size. The information regarding class size was not reported directly by the ECLS-K study. The variable of class size was created by this researcher through calculating the data from three questions in the fall kindergarten teacher questionnaire. The first question asked teachers about the number of children they had in their classes at different age levels (from three to nine years old). A second question asked about the number of children in each class belonging to different racial and ethnic groups. The third question asked about the number of girls and boys in each class. The arithmetic mean of the answers to these three questions was calculated to represent the class size in the current study.

Child's exposure to instruction. In the ECLS-K, teachers reported instructional time as the amount of time per day they conducted teacher-directed whole-class activities, teacher-directed small-group activities, and teacher-directed individual activities. The response categories for these three variables are: 1 = no time, 2 = half-hour or less, 3 = about one hour, 4 = about two hours, and 5 = three hours or more. The variable of instructional time was created by this researcher through calculating the sum of response values from three types of teacher-directed activities. Therefore, the score range of instructional time should be between 3 and 15.

Mathematics instructional activities. Information regarding math instructional activities in class was collected through a 17-item question in the spring kindergarten teacher questionnaire. The teachers were asked to report the frequency of instructional activities they conducted in class on a six-point Likert scale, with 1 representing "never" and 6 representing "daily". For the purpose of current study, it is necessary to reduce the large number of items relating to math instructional activities to a manageable number of variables. Before conducting exploratory factor analysis, the data were examined to ensure that the items of math instructional activities were factorable and all retained for factor analysis (see Appendix B).

Exploratory factor analysis was then conducted to construct factors of math instructional activities using principal component analysis with Varimax oblique rotation. Through examining eigenvalues, scree plot, and the amount of variance explained by the factors, four factors of math instructional activities were extracted. The total variance explained by the four factors retained in the analysis was 50.07%.

The items in each factor were also examined to ensure the interpretability of the extracted factors. As a result, one item (working on problems reflecting real-life situations) was removed from Factor 1 because this item did not seem to measure the same construct as the other four items (explaining how a problem is solved, working in small group or with a partner, working in achievement groups, and peer tutoring) in the factor. Therefore, the final four factors (see Table 4) represent four types of math instructional activities that have been practiced and researched by professionals in the field, including interactive activities (e.g., Rohrbeck, Ginsburg-Block, Fantuzzo, & Miller, 2005), manipulatives (e.g., Domino, 2010), practice activities (e.g., , Palardy & Rumberger, 2008), and integrated activities (e.g., Erdoğan & Baran, 2008). Cronbach's alpha coefficients of internal consistency for the four factors were .697, .725, .598, and .736, respectively. The results of factor analysis in the current study were similar to the findings from previous studies using the same ECLS-K study (Hausken & Rathbun, 2004; Wang, 2010). The scores of the factors were computed by taking the arithmetic mean of items for each factor scale, with ranges between 1 and 6. The average item scores for the four factors were then used as predictors and were incorporated into the multilevel models for further analysis.

Table 4

Factor loading for kindergarten mathematics instructional activities

Item	Factors of Math Instructional Activities			
	Factor 1	Factor 2	Factor 3	Factor 4
Factor 1: Interactive Activities				
Solving math problems in small groups or with a partner	**0.710**	0.222	0.198	0.091
Working in mixed achievement groups on math activities	**0.686**	0.154	-0.148	-0.056
Peer tutoring in mathematics	**0.637**	0.136	0.059	0.072
Explaining how a math problem was solved	**0.626**	0.132	0.171	0.131
Factor 2: Manipulative acitivities				
Working with geometric manipulatives	0.055	**0.818**	-0.045	0.101
Working with counting manipulatives to learn basic operations	0.237	**0.757**	0.112	0.059
Playing math-related games	0.273	**0.687**	-0.050	0.155
Working with rulers, measuring cups, spoons, or other measuring instruments	0.337	**0.441**	-0.056	0.248
Factor 3: Practice activities				
Doing math worksheets	-0.038	-0.014	**0.787**	-0.029
Doing math problems from their Textbooks	0.022	-0.003	**0.728**	-0.035
Completing math problems on the chalkboard	0.299	0.001	**0.647**	0.113
Factor 4: Integrated activities				
Using music to understand math concepts	0.103	0.152	0.012	**0.859**
Using creative movement or creative drama to understand math concepts	0.186	0.168	0.015	**0.850**

Data Analysis Plan

Sampling Weights

A sampling weight is the inverse of the probability of a case being selected; usually it is used to indicate the relative strength of an observation. When using unweighted data, each case is counted equally. With weighted data, on the other hand, each case is weighted according to its representation in the target population.

Since the ECLS-K is a complex dataset with an oversampling design and multi-source, multi-phase data collection, sampling weights were used to adjust the sample for unequal probability of selection and to adjust for the effects of non-response. In the ECLS-K, multiple weight variables were computed for different types of analyses or different sub-populations. For categorizing children into the achievement subgroups at the spring of kindergarten, first grade, third grade, and fifth grade, cross-sectional weights for these waves of data collection were used (C2CW0, C4CW0, C5CW0, and C6CW0). Since the first research question asked for comparison of subgroup change between any two adjacent waves between kindergarten and fifth grade, the longitudinal weights for these three comparison periods (i.e., C24CW0, C45CW0, and C56CW0) were used when computing the correlations and conducting chi-square tests. The second research question examined achievement growth over the course of elementary school, so the longitudinal weight C1_6FC0 was used. This longitudinal weight variable was created by the ECLS-K particularly for analyses involving a full sample of children across five rounds of data collection, including the fall of kindergarten, the spring of kindergarten, the spring of first grade, the spring of third grade, and the spring of fifth grade. All the sampling weight variables used in current study were normalized by dividing the average weight of all cases. In this case, the normalized weight variable was summed up to the actual number of cases in the analytic sample.

Missing Data

Missing data are a potential source of bias when analyzing large-scale survey data. The issue of missing data poses threats to a study's internal validity, specifically weakening statistical power, and increases the likelihood of committing Type II errors. Missing data also could negatively impact a study's external validity, in which the generalizability of a study's results to a target population may be brought into question (Croninger & Douglas, 2005).

Missing data in the ECLS-K. Data may be missing for several reasons. In the ECLS-K, missing values were coded as follows: (a) not applicable or legitimate skip (-1); (b) refused (-7); (c) don't know (-8); (d) not ascertained (-9); and (e) system missing (blank). The code for "not applicable" (-1) indicated that the respondent did not answer the question due to instructions to skip the question or to not participate in a particular section of the instrument. For instance, a "not applicable" code was used in the direct child assessment if a child did not take the assessment due to language barrier or a disability. A "not applicable" code also was used when the respondent was asked to skip the question because of a previous answer given. The "refused" code (-7) indicated that the respondent was unwilling to answer the question. The "don't know" code (-8) indicated that the respondent specifically told the interviewer that he or she did not know the answer to the question, or the respondent wrote "I don't know" on the self-administered questionnaires. For items where "don't know" was one of the options explicitly provided, the "don't know" response would be coded as indicated in the value labeled information rather than being coded as “-8”. The “not ascertained” code (-9) indicated that the respondent left an item blank that he or she should have answered. This code was used primarily for items of non-response for the school and teacher self-administered questionnaires. The “-9” code also was used when a direct assessment score was not ascertained or could not be calculated due to non-response. This code, along with the “refused” code and the “don’t know” code, indicated item non-response. System missing code (blank) indicated that an entire instrument or assessment was missing due to unit non-response. For instance, if a child’s parent did not participate in the parent interview, than all items from the parent interview would have a system missing code. In the current analyses, all the aforementioned coded values were considered missing data.

Missing data in analytic samples. Three missing data mechanisms, including missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR), are widely used to describe the relation between the probability of missing values and variables in the data. Suppose that the variables of interest for missing data analysis are the children’s fifth-grade mathematics IRT score and their SES. When the missingness of the data is MCAR, the probability of missing data on fifth-grade mathematics scores is unrelated to children’s SES, and is also unrelated to the value of observed fifth-grade math scores. Children with low SES are more likely to have missing data on fifth-grade math scores, but no correlation between the missingness and the scores of fifth-grade math assessment is indicative of MAR. The third type of mechanisms under which missing data occurs and the most problematic among the three, is MNAR. When the probability of missing data is related to a child’s fifth-grade math scores as well as to their SES, the missingness of the data is considered MNAR. In this case, the analysis would produce biased estimates of population parameters, and thus the external validity of study would be threatened. Even if the missingness is MCAR or MAR, a large amount of missing data may still weaken a study's statistical power. Since the strategies for handling missing data vary based on the missing data mechanism, it is essential to investigate the missing data mechanism so that an appropriate strategy for coping with missing data can be adopted before proceeding with further analysis for the research questions.

The purpose of missing data analysis was to examine the scope of missing data for the variables included in the current study, such as whether the missing data were scattered over many participants, or whether a few participants were missing data on many variables. In addition, the findings from missing data analysis provide insights regarding whether the results were biased by the missing data. Therefore, missing data analyses were conducted for the analytic samples in the current study.

The first analytic sample was established by including all children who were identified as in one of the four achievement groups at the spring of kindergarten. Some children may not have reading and/or math IRT scores in record for one or more subsequent waves of data collection. In this case, the missing data can have significant impacts on result interpretation when comparing the stability of subgroup membership change between adjacent waves of data collection. Therefore, missing data analysis for analytic sample 1 was conducted through chi-square tests to investigate whether significant differences exists between the sample and the dropped cases with regard to the achievement subgroup membership for each comparison period.

For analytic sample 2, Missing Value Analysis in SPSS 20.0 was used to investigate the amount and the patterns of missing data in each variable (Table 5). The amount of data missing in identified variables ranged from 0% to 9.1%. The descriptive statistics and observed patterns of missing data indicates that the missingness of the data might not be missing completely at random (MCAR), which can also be confirmed through Little's MCAR test (Little & Rubin, 2002). The results of Little's MCAR test showed a Chi-square value of 2380.660 with 1051 degrees of freedom (*df*) and a *p* value <.001. Based on the aforementioned evidence, one can conclude that the missingness of the data for current study was not MCAR. Further justification is required on whether the missingness is MAR or MNAR.

It is noted that there is no way to verify whether the missingness is MAR or MNAR (Enders, 2010). The major difference between MAR and MNAR is the existence of relation between the probability of missing data on a variable and the values of the variable itself. In a case of MAR, the missing values on variables are usually related to another measured variables. For instance, the variables with large amount of missing data (> 5%)--class size and instructional time--were collected from teacher's questionnaire, suggested that other measured variables, such as teacher nonresponse, might explain the missingness. In this regard, it seems reasonable to assume the missingness of analytic sample 2 is MAR.

A traditional approach to deal with missing data is to eliminate the variable or the cases with missing values from the study if the majority of missing data is exclusive to variables of debatable importance or a few cases in the sample. However, it is suggested

Table 5

Number of cases missing in each variable for analytic sample 2.

			Missing	
		N	Count	Percent
Achievement Variables	Spring-K Math IRT score	6,449	0	0
	Spring-K Reading IRT score	6,449	0	0
	Spring-first grade Math IRT score	6,445	4	0.1
	Spring-first grade Reading IRT score	6,445	4	0.1
	Spring-third grade Math IRT score	6,431	18	0.3
	Spring-third grade Reading IRT score	6,391	58	0.9
	Spring-fifth grade Math IRT score	6,423	26	0.4
	Spring-fifth grade Reading IRT score	6,417	32	0.5
	Achievement Group Membership	6,449	0	0
Demographic Variables	GENDER	6,449	0	0
	Race	6,443	6	0.1
	SES	6,322	127	2.0
Kindergarten Individual Variables	Initial Math Knowledge	6,383	66	1.0
	Initial Reading Knowledge	6,279	170	2.6
	Learning-related Skills			
Kindergarten Class Variables	Class Size	5,706	743	11.5
	Instructional Activities - Integrated Reading and Writing	6,189	260	4.0
	Instructional Activities - Reading Subskills	6,189	260	4.0
	Instructional Activities - Reading	6,187	262	4.1
	Instructional Activities - Problem-Solving Activities	6,165	284	4.4
	Instructional Activities - Paper and Pencil	6,169	280	4.3
	Instructional Activities - Manipulatives/Active Learning	6,166	283	4.4
	Instructional Time	5,740	709	11.0

Note: IRT = Item response theory; SES = socioeconomic status.

that the deletion methods (listwise or pairwise deletion) can only be used when the data are MCAR (Croninger & Douglas, 2005). Since the missingness of the data here is considered to be MAR, alternative strategies for coping with MAR include Full Information Maximum Likelihood and Maximum Likelihood via the EM algorithm, Bayesian Estimation, and multiple imputation. For the current study, multiple imputation strategy was used to deal with missing data in the analytic sample 2.

In multiple imputation (MI), a distribution of plausible values for each missing data point is estimated, and then a value is randomly selected from this distribution for imputation. The process is performed repeatedly, producing multiple imputed data sets. The researcher then conducts statistical analyses on each imputed data set, obtains multiple sets of results, and finally combines the results to produce parameter estimates for the overall analysis (Croninger & Douglas, 2005; Wayman, 2003). The primary advantage of multiple imputation is that this strategy maintains the overall variability in the population while preserving relations with other variables (Wayman, 2003). Moreover, multiple imputation can be used with virtually any kind of data and model (Allison, 2002). Given the aforementioned advantages, the missing values in the analytic dataset for research question 2 were therefore imputed using multiple imputation procedure in SPSS 20.0.

Analyses for Research Questions

This section describes the analysis strategies for each research question.

Research Question 1: What is the extent of subgroup membership stability and change from kindergarten through fifth grade (i.e., kindergarten vs. first grade, first grade vs. third grade, third grade vs. fifth grade)? Given membership change over time, what is the pattern of change across subgroups (e.g., MD to MD-RD; TA to MD)?

The same set of cutoff criteria for kindergarten identification (i.e., at or below -1SD for difficulties and at or above -0.5SD as normal performance) was applied to children's subsequent academic performance at the spring of first grade, spring of third grade, and spring of fifth grade. Based on their reading and mathematics IRT scores, children were classified into the groups of TA, RD, MD, or MD-RD at each semester of testing. Given the nominal nature of achievement group variable, the stability of subgroup membership was examined by calculating the Cramer's V correlations of subgroup membership between every two adjacent time points (i.e., spring of kindergarten vs. spring of first grade, spring of first grade vs. spring of third grade, spring of third grade vs. spring of fifth grade). The magnitude of the obtained correlation coefficients represented the strength of the relationship. A series of chi-square tests of independence was also conducted to examine the significance of subgroup change. Figures were used to present the patterns of subgroup membership change over the course of elementary school years.

Research Question 2: To what extent do children's math and reading achievement and growth trajectories differ among children in different achievement subgroups?

The purpose of this research question was to explore achievement trajectories and achievement level in fifth grade for children in different achievement subgroups. To estimate intra-individual growth trajectories and inter-individual differences in those growth parameters, a multilevel modeling (MLM) framework was adopted for the analyses.

In general, the phenomena of individual growth can be represented through a two-level hierarchical model where Level 1 examines intra-individual change as a function of time, and Level 2 represents inter-individual differences in initial status and growth rate (Raudenbush & Bryk, 2002). Selecting an appropriate mathematical model that adequately captures the interesting features of the repeated measures data is an important first step in these types of analyses. Because change in achievement does not necessarily conform to any theoretical pattern initially, examining each individual with several candidate functions is sensible (Cudeck & Harring, 2007). The form of change can be linear or nonlinear. The final functional form was selected based on three criteria proposed by Cudeck and Harring (2010). First, the model must fit the data reasonably well (i.e., the extent to which the model function accounts for the data). Second, the model must be appropriate or conform to the extent possible to the phenomena underlying the patterns of change (i.e., the shape and overall trajectory exhibit patterns of change as predicted). Finally, the model must have interpretable parameters in which the coefficients represent an interesting feature of the overall developmental trajectory.

For the current study, a generic form of the level 1 equation was written to express the achievement scores (i.e., reading or mathematics achievement in the present study) over time within an individual as follows:

$$Y_{ti} = \pi_{0i} + \pi_{1i}(a_{ti} - 5) + \pi_{2i}(a_{ti} - 5)^2 + \cdots + \pi_{pi}(a_{ti} - 5)^p + e_{ti} \tag{3.1}$$

where the errors e_{ti} are assumed to be independent and normally distributed with common variance σ^2. Here, Y_{ti} is the mathematics or reading achievement IRT score for student i at time t (t = 0, 1, 3, 5). The intercept parameter, π_{0i}, represents the average score (math or reading for person i at $a_{ti} = 5$ (i.e., spring of fifth grade), while π_{pi} is the growth trajectory parameter p for person i associated with the polynomial of degree P ($p = 0,\ldots,P$). Using Equation 3.1 as a template, a model for linear change can be written as

$$Y_{ti} = \pi_{0i} + \pi_{1i}(a_{ti} - 5)e_{ti} \tag{3.2}$$

At level 2 for the linear model, the simplest person-level model can be written as

$$\begin{aligned} \pi_{0i} &= \beta_{00} + r_{0i} \\ \pi_{1i} &= \beta_{10} + r_{1i} \end{aligned} . \tag{3.3}$$

Here, r_{0i} and r_{1i} are the level-2 random effects and are assumed to be normally distributed with covariance structure $\mathbf{T}$. That is,

$$\begin{pmatrix} r_{0i} \\ r_{1i} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{T}), \text{ where } \mathbf{T} = \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix},$$

and where $\tau_{00} = \mathrm{var}(r_{0i})$, $\tau_{11} = \mathrm{var}(r_{1i})$, and $\tau_{10} = \mathrm{cov}(r_{1i}, r_{0i})$.

It is possible that the longitudinal change in achievement is nonlinear. In the case of nonlinear change, several individual functions besides r-order polynomials can be considered and evaluated for model selection (see Seber & Wild, 1989; for a more comprehensive list of alternatives; and Cudeck & Harring, 2010; for nonlinear models for repeated measures data). To develop a model for nonlinear repeated measures data, Equation 3.1 can be modified as follows:

$$y_{ij} = f\left(\beta_{i1,\ldots,}\beta_{ip}x_{ij}\right) + e_{ij,} \qquad j = 1, \ldots, n_i \ , \tag{3.4}$$

where f is a mathematical function with at least one parameter entering in a nonlinear manner. The same criteria for model evaluation – fit, appropriateness, and interpretability – were used for evaluating and selecting an appropriate model.

Equation 3.2 is referred to as an unconditional model where no level-2 predictors for π_{0i} and π_{1i} have been introduced. For the current study, the analyses began with the fitting of an unconditional model, because the unconditional model provides baseline statistics and useful information for evaluating subsequent conditional models. For instance, the estimated mean intercept and mean growth rate were tested in the unconditional model to determine whether both parameters were necessary for describing the mean growth trajectory. The variances of individual growth parameters π_{0i} and π_{1i}, τ_{00} and τ_{11} also were tested for significance to determine whether there was significant variation in intercept or linear change across individuals (Raudenbush & Bryk, 2002).

After examining the unconditional model, identified predictors were introduced into the level-2 model. For example, a model with one predictor can be written as

$$\begin{aligned} \pi_{0i} &= \beta_{00} + \beta_{0AchGrp}X_{AchGrpi} + r_{0i} \\ \pi_{1i} &= \beta_{10} + \beta_{1AchGrp}X_{AchGrpi} + r_{1i} \end{aligned} \tag{3.5}$$

where $X_{AchGrpi}$ represents children's kindergarten achievement subgroups (i.e., TA, RD, MD, and MD-RD). In addition to conducting multilevel analyses, the math and reading growth trajectories for the four achievement subgroups were also investigated through graphical representation.

Research Question 3: To what extent do children's spring kindergarten achievement group membership (i.e., TA, MD, and MD-RD), demographics characteristics, initial reading and mathematics knowledge, learning-related skills, instructional time, class size, and math instructional activities predict growth and level in fifth grade in mathematics performance over the course of the elementary school years?

Does subgroup membership interact with the kindergarten variables to predict growth in mathematics over the elementary school years?

To examine the effects of identified kindergarten predictors on math achievement growth, another set of multilevel analyses was also conducted. The model can be written as

$$\pi_{0i} = \beta_{00} + \sum_{q-1}^{Q_0} \beta_{0q} X_{qi} + r_{0i}$$

$$\pi_{1i} = \beta_{10} + \sum_{q-1}^{Q_1} \beta_{1q} X_{qi} + r_{1i} \tag{3.6}$$

where X_{qi} represents a measured characteristic, such as children's achievement group membership (i.e., TA, MD and MD-RD), individual's background (i.e., gender, race and SES), children's kindergarten experience (i.e., initial mathematics and reading knowledge, and learning-related skills) or class variables (i.e., class size, instructional time and math instructional activities), β_{0q} represents the effect of X_q on the intercept parameter controlling for other explanatory variables in the model; β_{1q} represents the effect of X_q on the slope parameter again controlling for other explanatory variables in the model; and r_{pi} are random effects with mean of 0.

In addition to the main effects of identified predictors, also examined in this research question were the interaction effects of children's kindergarten achievement subgroup membership and other identified predictors. The purpose of interaction effects model was to investigate whether and the extent to which the effects of identified kindergarten predictors varied among children with MD-RD, MD, or children in the TA group. In the interaction effects model, Equation 3.5 can be modified to include these effects:

$$\pi_{0i} = \beta_{00} + \sum_{q-1}^{Q_0} \beta_{00q} X_{qi} + \sum_{q-1}^{Q_0} \beta_{01q} X_{Group} X_{qi} + r_{0i}$$
$$\pi_{1i} = \beta_{10} + \sum_{q-1}^{Q_0} \beta_{10q} X_{qi} + \sum_{q-1}^{Q_0} \beta_{11q} X_{Group} X_{qi} + r_{1i} \quad (3.7)$$

where X_{qi} represents a measured characteristic, including children's achievement group membership, individual's background (i.e., gender, race and SES), children's kindergarten experience (i.e., initial mathematics and reading knowledge, and learning-related skills) or class variables (i.e., class size, instructional time and math instructional activities); X_{Group} represents the three achievement subgroups (TA, MD, and MD-RD); β_{00q} and β_{10q} represent respectively the effect of X_q on the intercept and slope parameters controlling for other explanatory variables and interactions with achievement group variable in the model; β_{01q} and β_{11q} represents respectively the interaction effects of X_{qi} and X_{Group} on the intercept and slope parameters controlling for other variables in the model; and r_{pi} are random effects with mean of 0. The effect sizes were obtained through calculating the proportions of variance explained by the predictors on intercept, linear slope, and/or higher-order slope terms.

Summary

For the present study, the research questions were investigated by using the ECLS-K data collected from kindergarten to fifth grade. The first two research questions were designed to explore the difference among the four achievement subgroups (i.e., TA, RD, MD, and MD-RD) in terms of the subgroup stability and change over the elementary school years, as well as their math and reading achievement and growth trajectories. The third research question mainly focused on children with difficulties in mathematics and the effects of identified kindergarten predictors on mathematics achievement and growth. For the first research question, correlation analyses and chi-square tests were conducted; for the second and third research questions, multilevel modeling analyses were conducted The statistical package SPSS 20.0 was used to perform all the statistical procedures required for the current study.

CHAPTER IV

Results

This chapter contains a description of the procedure and results of missing data analyses as well as the descriptive information of the analytic samples. The findings of the research questions are then presented to show the stability of achievement group membership, the achievement growth trajectories for children in the four achievement subgroups, and the effects of kindergarten predictors and their interactions with kindergarten achievement subgroups on mathematics achievement growth for the identified students over the course of elementary school.

Missing Data Analyses and Analytic Samples

Analytical Sample and Dropped Cases for Research Question 1

The first research question was designed to explore whether children identified in kindergarten continue to remain in or change their achievement group membership in later elementary school years. Given the purpose of this research question, dropout cases could result in false interpretation. Therefore, chi-square tests of independence were conducted for each comparison period (i.e., fall of kindergarten vs. fall of first grade, fall of first grade vs. fall of third grade, fall of third grade vs. fall of fifth grade) to examine whether significant difference exists between analytic sample 1 and dropped cases in terms of their achievement group membership change. The results are presented in Table 6.

Table 6

Chi-square Tests for Missing Cases on Achievement Groups across Three Comparison Periods for Analytic Sample 1

		Dropped Cases		Remained Cases		Person	*p*
		n	%	n	%	χ^2	
K vs. 1st Grade							
Sample Size		9		10,413			
Achievement Groups							
	TA	4	0%	9,022	100%	28.984	.000
	RD	0	0%	133	100%		
	MD	2	1.1%	174	98.9%		
	MD-RD	3	0.3%	1,084	99.7%		
1st vs. 3rd Grade							
Sample Size		119		8,392			
Achievement Groups							
	TA	2	0%	4,960	100%		
	RD	1	0.6%	178	99.4%	990.422	.000
	MD	0	0%	134	100%		
	MD-RD	110	13.9%	681	86.1%		
	Borderline	6	0.2%	2,439	99.8%		
3rd vs. 5th Grade							
Sample Size		24		6439			
Achievement Groups							
	TA	4	0.1%	4,055	99.9%		
	RD	0	0%	161	100%	114.162	.000
	MD	0	0%	207	100%		
	MD-RD	18	2.8%	624	97.2%		
	Borderline	2	0.1%	1392	99.9%		

Note. The data were weighted by the normalized longitudinal weights C24CW0, C45CW0, and C56CW0 for comparison periods kindergarten vs. first grade, first grade vs. third grade, and third grade vs. fifth grade, respectively.

In all three comparison periods, significantly statistical differences were discovered between dropped cases and the analytical sample at the $\alpha = .05$ level (χ^2[3 *df*, *N*=10,442]=28.984, χ^2[4 *df*, *N*=8,511]=990.422, and χ^2[4 *df*, *N*=6,463]=114.162 for the three comparison periods, respectively). A similar disproportionate pattern was observed across all three comparison periods: children in the group of TA, RD, or MD tended to stay in the analytic sample, whereas children with MD-RD were more likely to drop out of the ECLS-K study. Given these findings, caution should be exercised when interpreting results regarding group membership change, since children with MD-RD were more likely to be underrepresented in analytic sample 1.

Table 7 and Table 8 present the demographic and achievement characteristics of children in the four achievement subgroups in analytic sample 1. Among the four kindergarten achievement subgroups in analytic sample 1, more boys than girls were found in the groups of RD and MD-RD, whereas similar percentages of boys and girls were found in the TA and MD groups. Over 60% of children with difficulties in mathematics (MD and MD-RD) were minority, as opposed to 30% in the TA group and 40% in the RD group. Children with difficulties in mathematics and/or reading also had significantly lower levels of socioeconomic status than their peers in the TA group. The investigation of children's achievement characteristics showed that children with RD and MD-RD were still struggling with the lower level of reading abilities, such as letter recognition and identifying beginning and ending sounds. Children with MD and MD-RD were also likely to struggle with the lower level of math tasks, such as comparing sizes and identifying the relative position of an object.

Table 7

Demographic Characteristics by Kindergarten Achievement Subgroups – Analytic Sample 1

	TA (n=9,555)		RD (n=148)		MD (n=184)		MD-RD (n=1,073)	
Variable	n	%	n	%	n	%	n	%
Gender								
Female	4,893	51.2	37	25.0	91	49.5	433	40.4
Race								
White	6,708	70.3	84	56.8	71	38.8	350	32.6
African American	1,043	10.9	13	8.8	52	28.4	349	32.5
Hispanic	1,130	11.8	35	23.6	42	23.0	277	25.8
Asian	296	3.1	3	2.0	6	3.3	15	1.4
Other	364	3.8	13	8.8	12	6.6	82	7.6
SES[a]	0.22	(0.76)	-0.31	(0.71)	-0.20	(0.76)	-0.57	(0.72)

Note. TA = typically achieving; RD = reading difficulty; MD = math difficulty; MD-RD = math and reading difficulties. The estimates were weighted by normalized weight variable C2CW0.

a. The reported values are group mean; group standard deviations are reported in parentheses.

Table 8

Descriptive Statistics of Math and Reading Achievement and Proficiency Probability Scores by Kindergarten Achievement Subgroups – Analytic Sample 1

	TA (n=9,555)	RD (n=148)	MD (n=184)	MD-RD (n=1,073)
	M (SD)	M (SD)	M (SD)	M (SD)
Reading Achievement				
Reading IRT Scale Score	47.08*	25.17*	37.13*	23.17
	(12.99)	(1.67)	(4.27)	(2.59)
Reading Proficiency Probability Score				
Level 1 - Letter Recognition	1.00*	0.60*	0.99*	0.43
	(0.01)	(0.16)	(0.01)	(0.23)
Level 2 - Beginning Sounds	0.90*	0.08*	0.74*	0.05
	(0.10)	(0.03)	(0.12)	(0.03)
Level 3 - Ending Sounds	0.72*	0.02*	0.41*	0.01
	(0.22)	(0.01)	(0.17)	(0.01)
Level 4 - Sight Words	0.24*	0	0.03*	0
	(0.30)	(0)	(0.11)	(0)
Level 5 - Word in Context	0.10*	0	0.01*	0
	(0.19)	(0)	(0.04)	(0)
Math Achievement				
Math IRT Scale Score	39.60*	30.91*	19.66*	16.71
	(9.75)	(3.63)	(1.31)	(2.97)
Math Proficiency Probability Score				
Level 1 - Count, Number, Shape	1.00*	1.00*	0.98*	0.88
	(0)	(0)	(0.02)	(0.18)
Level 2 - Relative Size	0.98*	0.94*	0.46*	0.26
	(0.03)	(0.03)	(0.11)	(0.17)
Level 3 - Ordinality, Sequence	0.81*	0.53*	0.02*	0.01
	(0.22)	(0.21)	(0.01)	(0.01)
Level 4 - Add/Subtract	0.27*	0.05*	<.001*	<.001
	(0.28)	(0.06)	(<.001)	(<.001)
Level 5 - Multiply/Divide	0.02*	<.001*	0	0
	(0.09)	(<.001)	(0)	(0)

Note. TA = typically achieving; RD = reading difficulty; MD = math difficulty; MD-RD = math and reading difficulties. The estimates were weighted by normalized weight variable C2CW0. *The mean difference between this group and MD-RD is significant at the 0.5 level.

Analytical Sample for Research Questions 2 and 3

The analytic sample for Research Questions 2 and 3 was established by including cases with non-zero longitudinal weight for spring of kindergarten, spring of first grade, spring of third grade, and spring of fifth grade (i.e., C1_6FC0 > 0). Although the sample size was large (*N* = 6,598), about 20% of cases were missing values on one or more independent variables. As discussed in Chapter 3, the missing data was not missing completely at random (MCAR), so case deletion was not an effective strategy to deal with missing data in this case. Therefore, a multiple imputation analytic strategy was adopted to address the issue of missing values. As for the number of imputations needed to yield valid results, Rubin (1987) recommended between three and five imputed datasets. However, Graham, Olchowski, and Gilreath (2007) argued that using a small number of imputations may lead to a decrease of statistical power. Based on the results of their simulation study, Graham et al. (2007) suggested that 20 imputations should be adequate for most situations. To determine an adequate number of imputations for the current study, an analysis to compare the results of a main effect model from datasets with five and with 20 imputations was conducted. Initially, two sets of data were compared by evaluating the estimates of relative efficiency, which indicates the adequacy of the number of imputations compared to an infinite set. Although the data with 20 imputations produced results with slightly better relative efficiency (see Appendix C for detailed information), the inferences obtained from the two sets of data were very similar. In addition, the processing time for running a model increased substantially when using the data with 20 imputations. Therefore, the data with five imputations were used for the subsequent analyses. Descriptive statistics of variables in the full sample and analytic sample 2 are presented in Table 9.

Table 10 and Table 11 present the demographic and achievement characteristics of the four achievement groups examined in analytic sample 2. Although the sample size of analytic sample 2 is much smaller than that of analytic sample 1, the demographic and achievement characteristics of the four kindergarten achievement groups in the two analytic samples were very similar.

Table 9

Descriptive Statistics of Variables in Full and Analytic Sample 2

	Full Sample N=17,093	Analytic Sample 2 N=6,598
Achievement Variables^{a}		
Spring-K Reading IRT score	40.40 (13.34)	44.64
Spring-first grade Reading IRT score	70.75 (22.23)	78.12
Spring-third grade Reading IRT score	117.17 (25.42)	124.55
Spring-fifth grade Reading IRT score	138.56 (23.40)	144.50
Spring-K Math IRT score	32.69 (11.45)	37.31
Spring-first grade Math IRT score	56.98 (16.84)	62.48
Spring-third grade Math IRT score	91.60 (21.64)	97.90
Spring-fifth grade Math IRT score	113.20 (21.60)	118.15
Demographic Variables		
GENDER^{b}		
Female	8,258 (48.3%)	3,301 (50%)
Male	8,817 (51.6%)	3,297 (50%)
Race^{b}		
White	9,864 (57.7%)	4,344 (65.8%)
African American	2,684 (15.7%)	821 (12.4%)
Hispanic	3,202 (18.7%)	939 (15.0%)
Asian	510 (3.0%)	185 (2.8%)
Other Races	791 (4.6%)	309 (4.7%)
SES^{a}	-0.03 (0.79)	0.14
Kindergarten Individual Variables^{a}		
Initial Reading Knowledge	29.18 (9.70)	31.75
Initial math Knowledge	22.53 (8.72)	25.59
Learning-related Skills	3.09 (0.69)	3.20

Table 9 (continued)

	Full Sample N=17,093	Analytic Sample 2 N=6,598
Kindergarten Class Variables[a]		
Class Size	20.12 (4.91)	20.06
Instructional Time	8.72 (1.70)	8.74
Math Instructional Activities		
Problem-Solving Activities	3.67 (1.13)	3.69
Paper and Pencil	3.01 (1.25)	2.95
Manipulatives/Active Learning	3.78 (0.85)	3.80

Note. The results of analytic sample 2 were pooled from the original dataset and five imputed datasets. No pooled estimate of standard deviation was reported by SPSS.
a. Values presented are the mean values of the variables, and the standard deviations are presented in parentheses.
b. Values presented are the counts, and the percentages are presented in parentheses.

Table 10

Demographic Characteristics by Kindergarten Achievement Subgroups – Analytic Sample 2

	TA (n=5,728)		RD (n=87)		MD (n=93)		MD-RD (n=499)	
Variable	n	%	n	%	n	%	n	%
Gender								
Female	2,979	52.01	20	22.99	44	47.31	211	42.28
Race								
White	4,142	72.31	47	54.02	36	38.71	150	30.06
African American	447	7.80	6	6.90	27	29.03	137	27.45
Hispanic	717	12.52	25	28.74	23	24.73	152	30.46
Asian	192	3.35	2	2.30	3	3.23	7	1.40
Other	224	3.91	9	10.34	4	4.30	52	10.42
SES[a]	0.25	(0.01)	-0.23	(0.07)	-0.15	(0.06)	-0.60	(0.03)

Note. TA = typically achieving; RD = reading difficulty; MD = math difficulty; MD-RD = math and reading difficulties. The estimates were weighted by normalized weight variable C2CW0.

a. The reported values are group mean; group standard deviations are reported in parentheses.

Table 11

Descriptive Statistics of Math and Reading Achievement and Proficiency Levels by Kindergarten Achievement Subgroups – Analytic Sample 2

	TA (n=5,728)	RD (n=87)	MD (n=93)	MD-RD (n=499)
	M (SD)	M (SD)	M (SD)	M (SD)
Reading Achievement				
Reading IRT Scale Score	47.02*	25.13*	36.34*	23.36
	(12.76)	(1.78)	(2.29)	(2.60)
Reading Proficiency Probability Score				
Level 1 - Letter Recognition	1.008	0.60*	0.99*	0.45
	(0.01)	(0.17)	(0.01)	(0.23)
Level 2 - Beginning Sounds	0.91*	0.08*	0.72*	0.05
	(0.10)	(0.03)	(0.11)	(0.04)
Level 3 - Ending Sounds	0.72*	0.02*	0.38*	0.01
	(0.22)	(0.01)	(0.14)	(0.01)
Level 4 - Sight Words	0.24*	0	0.02*	0
	(0.29)	(0)	(0.02)	(0)
Level 5 - Word in Context	0.10*	0	0.01*	0
	(0.18)	(0)	(0.01)	(0)
Math Achievement				
Math IRT Scale Score	39.90*	30.87*	19.88*	16.89
	(9.80)	(3.30)	(1.10)	(2.91)
Math Proficiency Probability Score				
Level 1 - Count, Number, Shape	1.00*	1.00*	0.99*	0.89
	(<.001)	(<.001)	(0.01)	(0.17)
Level 2 - Relative Size	0.98*	0.94*	0.47*	0.27
	(0.03)	(0.03)	(0.09)	(0.18)
Level 3 - Ordinality, Sequence	0.82*	0.53*	0.02*	0.01
	(0.22)	(0.20)	(0.01)	(0.01)
Level 4 - Add/Subtract	0.28*	0.04*	<.001*	<.001
	(0.28)	(0.05)	(<.001)	(<.001)
Level 5 - Multiply/Divide	0.03*	<.001*	0	0
	(0.10)	(<.001)	(0)	(0)

Note. TA = typically achieving; RD = reading difficulty; MD = math difficulty; MD-RD = math and reading difficulties. The estimates were weighted by normalized weight variable C2CW0. *The mean difference between this group and MD-RD is significant at the 0.05 level.

Stability and Subgroup Change for Children with Math Difficulties

The first research question aimed to explore whether and to what extent children identified as having difficulties in mathematics in the spring of kindergarten changed their subgroup membership over the course of the elementary school years. The first sub-question examined the overall stability of achievement subgroup identified in kindergarten. The second sub-question investigated and compared the change patterns of children with difficulties in mathematics (i.e., MD and MD-RD) and those without difficulties in mathematics.

Stability of Achievement Subgroup Membership over Time

The stability of achievement group membership was measured by using a contingency table with chi-square tests and Cramér's V coefficient for evaluating associations. Significant results from the chi-square tests (see Table 12) suggested the presence of association between children's achievement group membership at every pair of two adjacent time points. Cramér's V correlation coefficients for the three comparison periods ranged from .372 to .518, indicating a "relatively strong association (Rea & Parker, 1997, p. 203). Given the aforementioned findings, this researcher concluded that the associations of children's achievement subgroup membership between any two adjacent time points were strong.

Table 12

Stability of Achievement Group Membership over the Three Comparison Periods

Comparison Period	N	Cramer's V	χ^2 value	df[a]
K vs. 1st grade	10,413	.372	7238.786***	12
1st grade vs. 3rd grade	8,393	.409	6054.603***	16
3rd grade vs. 5th grade	6,439	.518	6906.412***	16

Note. The χ^2 values for three comparison periods are all significant at the .001 level.
a. Since no borderline group is identified in spring of kindergarten, the degrees of freedom (df) for K vs. 1st grade is (4-1)(5-1); the df values for the other two comparison periods are calculated as (5-1)(5-1). * = $p < .05$, ** = $p < .01$, ***= $p < .001$.

Patterns of Achievement Subgroup Change

Patterns of subgroup change for children in the four achievement subgroups between spring of kindergarten and first grade, between spring of first and third grade, and between spring of third and fifth grade were compared. Figure 3 through Figure 5 show the percentage of subgroup change for children in the four achievement subgroups. Overall, children with MD-RD were the most stable group among children with achievement difficulties (see Figure 3). Almost 80% of children with MD-RD continuously met the criteria of MD-RD in the following wave of data collection. Only a small number of children with MD-RD changed their achievement subgroup membership to RD (0.7%–2.4%), MD (0.6%–2.3%), or TA (0.6%–2.5%). In addition, about 15% to 20% of children with MD-RD did not meet identification criteria for any of the four achievement groups at the following wave of data collection. This group of children represented those who scored at the lower end of average achievement in either one or both academic areas and may be at risk of developing academic difficulties later; hence, these children were categorized as belonging to the borderline group. The finding regarding children with MD-RD indicated that children who experienced difficulties in both reading and mathematics were likely to continuously have difficulties in both areas. Even when some of these MD-RD children improved their performance in reading, math, or both, most of them still scored in the lower range of average achievement (i.e., borderline group).

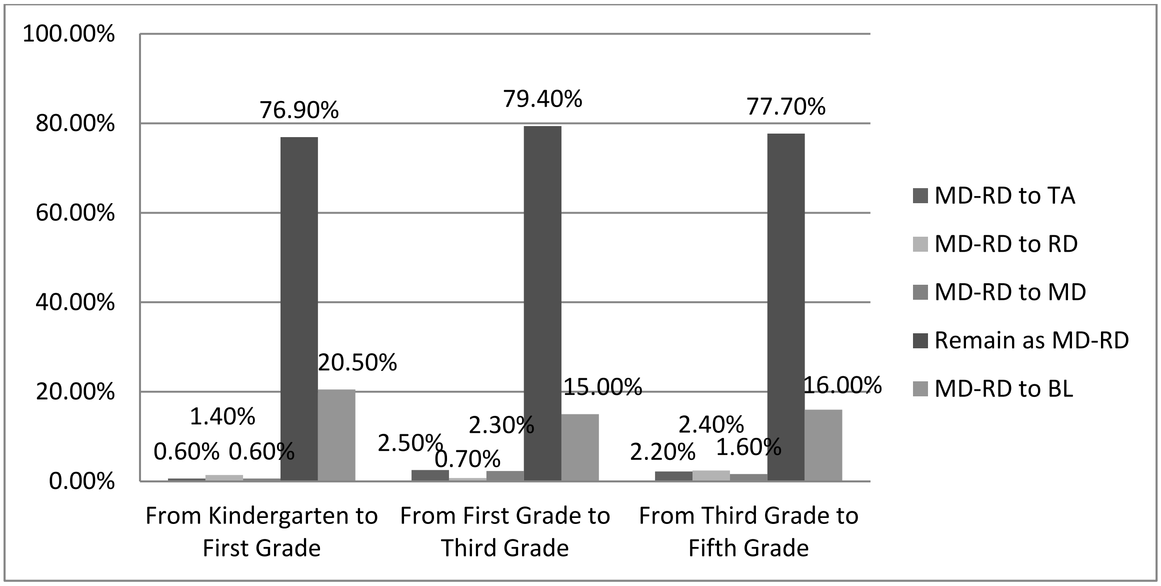

Figure 3. Percentages of subgroup change for children with MD-RD during three comparison periods.

The percentages of MD children who remained classified as MD in the following waves of data collection were between 15% and 26% across three comparison periods (see Figure 4). First grade seemed to be a turning point here: before first grade, a large percentage of MD children in kindergarten changed to the TA group at first grade; after first grade, only a small percentage of children with MD were identified as MD-RD at the following wave of testing. There were very few MD children who changed to the RD group in the following waves. Finally, about half of the MD children in each comparison period fell into the borderline group.

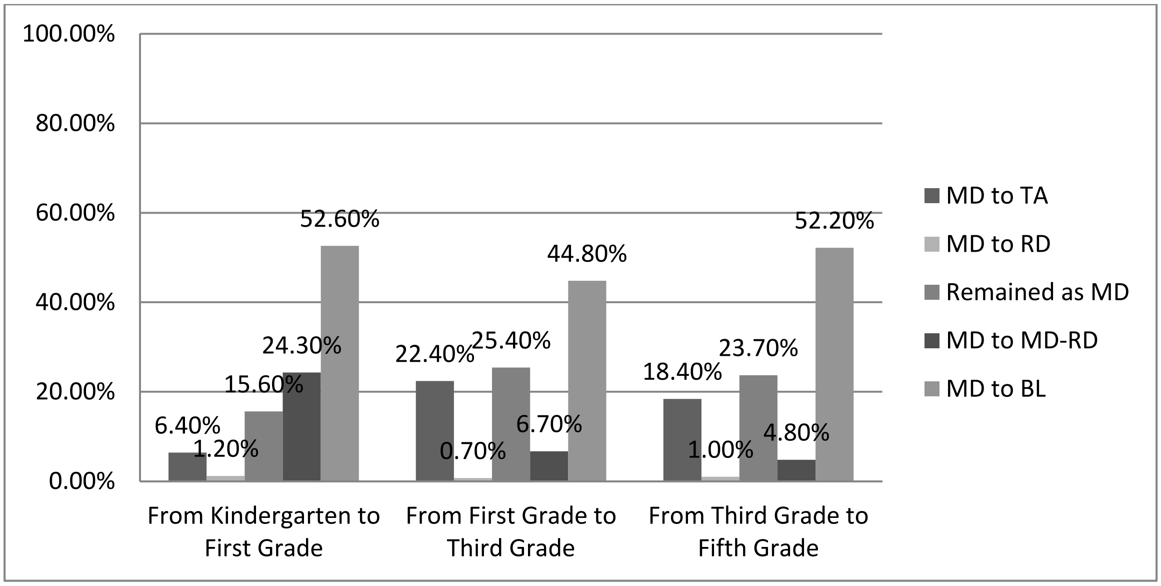

Figure 4. Percentages of subgroup change for children with MD during three comparison periods.

For children with RD, the turning point seemed to occur at third grade. Approximately 15% of children with RD remained in the RD group at first and third grade, but a larger percentage (i.e., 44%) of third-grade RD children stayed as RD at fifth grade (see Figure 5). Similar to the pattern found in children with MD, the percentages of RD children who changed to MD-RD decreased over time, suggesting that more children with RD developed comorbid difficulties in mathematics in early grades instead of later grades. In addition, almost none of the children with RD changed subgroup membership to MD during the comparison periods. A large group of children with RD were later identified as achieving normally in both reading and mathematics after first grade. Also similar to the findings of children with MD, a substantial number of children with RD were later identified as belonging to the borderline group.

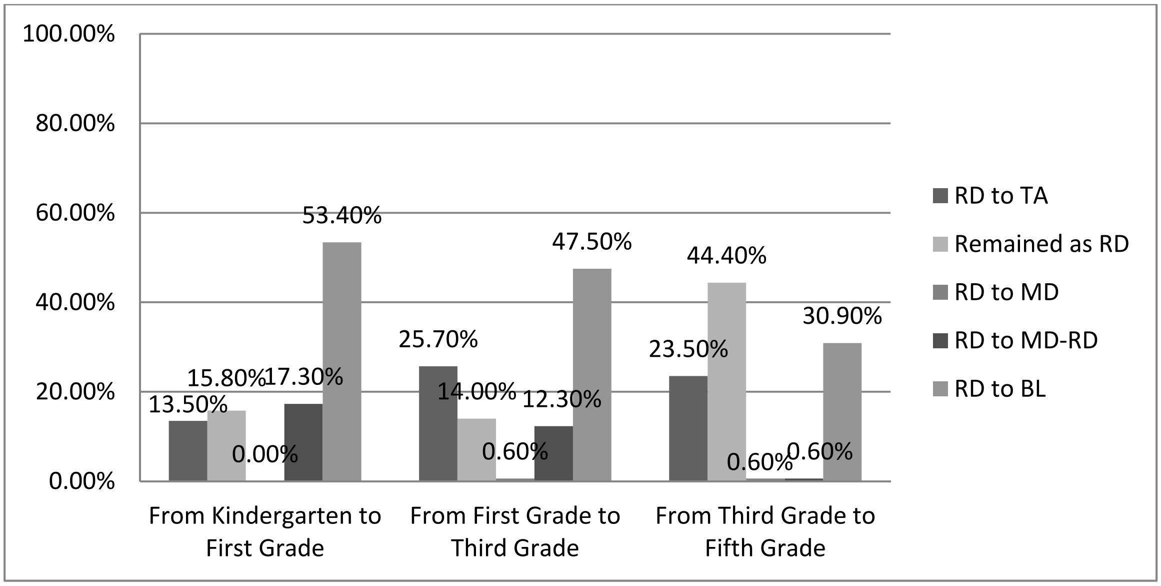

Figure 5. Percentages of subgroup change for children with RD during three comparison periods.

Figure 6 shows that a large and increasing percentage of children with TA continuously achieved normally in both reading and mathematics during the comparison periods. Less than 5% of TA children were identified as having RD, MD, or MD-RD at any following wave of data collection.

In summary, the stability of achievement subgroup membership was confirmed by the relatively strong correlations of subgroup membership between every pair of adjacent time points. The analyses on the patterns of subgroup change showed that children who performed poorly on both reading and mathematics in kindergarten were more likely to continue having difficulties in both subject areas in later elementary school years. On the other hand, children with RD or MD seemed to either remain in the same subgroup, or change to the borderline or the TA group later. Only a small proportion of children with RD or MD developed difficulties in the area in which they originally performed well.

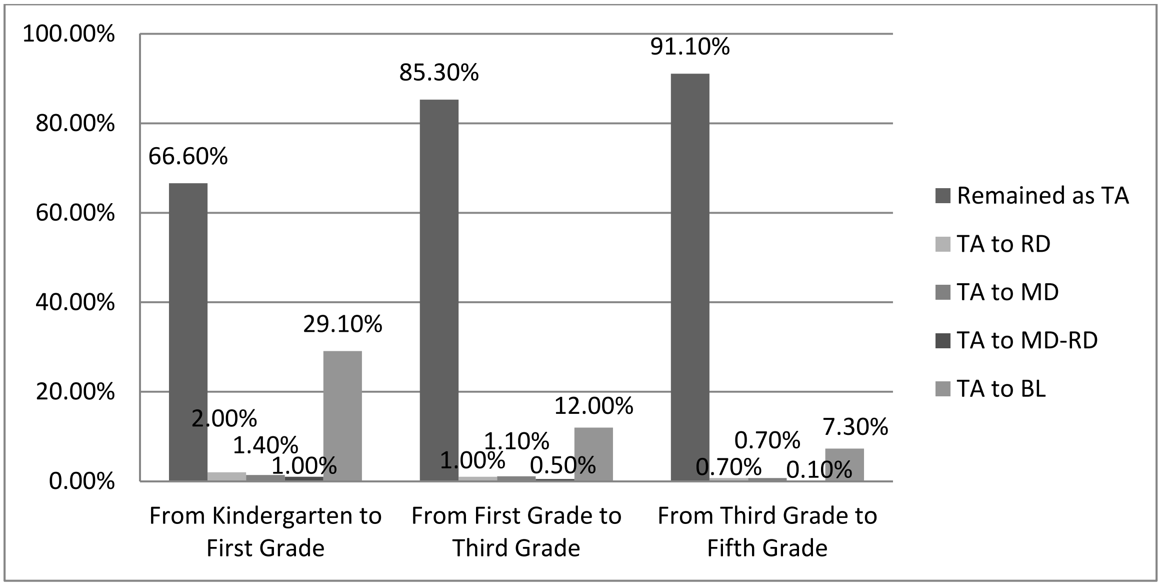

Figure 6. Percentages of subgroup change for TA children during three comparison periods.

Achievement Growth Trajectories of Children with Difficulties in Mathematics

The second research question was intended to examine the math and reading achievement growth trajectories of children with difficulties in mathematics and/or reading, as compared to the achievement trajectories of those without math difficulties. To investigate group difference on achievement trajectories, multilevel modeling analyses were conducted. The math and reading growth trajectories for children in each achievement subgroup were also examined through graphical representation. Descriptions of unconditional models for math achievement growth are presented below, followed by an examination of the difference of growth trends for children with and without math difficulties. The same investigation was also conducted for reading achievement growth for children in the four kindergarten achievement subgroups.

Math Achievement Growth Trajectories

Unconditional Models. As suggested by Singer and Willett (2003), two unconditional models were fitted to the data—the unconditional means model and the unconditional growth model—to check for systematic variation in the outcome worth further exploration.

Unconditional means model. The main purpose of fitting an unconditional means model to the data was to assess whether substantial variation existed within or between individuals. In addition, the magnitude of the intraclass correlation coefficient (ρ) provides information on the extent to which the total outcome variance can be explained by differences between individuals. The unconditional means model for math achievement found significant within-person variance (τ_e^2 = 1194.593, $p < .001$) and between-person variance (τ_0^2 = 79.325, $p < .001$), indicating sufficient variation at both levels to be potentially explained by other predictors. The estimate of the intraclass correlation coefficient (ρ) for math achievement was 0.062, indicating that about 6% of total variation in math achievement could be attributable to between-individual differences.

Unconditional growth model. By adding the time factor as the only level-1 predictor and no other predictors at level-2, the unconditional growth model established a baseline for the amount of variation in the data. A comparison of within-person variance between the unconditional means model (τ_e^2 = 1194.593) and the unconditional growth model (τ_e^2 = 79.746) showed that 93.3% of within-person variance was reduced in the unconditional growth model for math achievement, indicating that a large amount of within-person variance in the unconditional means model could be explained by the time factor added to the unconditional growth model. The fixed effects of intercept, linear, and quadratic growth terms were all highly significant (β_0 = 116.649, t = 517.74, $p < .001$ for intercept; β_1 = 11.973, t = 84.585, $p < .001$ for linear growth; β_2 = -0.627, t = -22.42, $p < .001$ for quadratic growth). Therefore, all three growth parameters in the models were retained for subsequent analyses. The between-person variances in the unconditional growth models for math achievement (i.e., τ_{00} = 231.586, τ_{11} = 0.882, and τ_{22} = 0.069) were all significant ($p < .001$). The finding suggested that there was sufficient between-individual variation left to be explained by other level-2 predictors, such as children's demographics, children's learning characteristics and behaviors, or certain classroom factors.

Covariance structures for the model. In order to better model the data with appropriate covariance structures for both level-1 and level-2 variance, models with different sets of covariance structures were compared based on the model fit indices and the number of parameters of the models. Table 13 provides information regarding the types of covariance structures adopted for the data. Table 14 presents the results of the model comparison. The results suggested that model 2 had the smallest deviance statistics and the most parameters and was therefore the best-fit model among the five examined models. As a result, the combination of homogeneous autoregressive structure for level-1 and unstructured matrix for level-2 was used for the subsequent analyses of mathematics achievement growth.

Table 13

Types of Model Covariance Structures

Type	Variance at each time point	Correlation between measurement times	Covariance matrix
Level-1 Structure			
Scaled Identify(ID)*	Constant	No correlation	$\sigma^2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Autoregressive, homogeneous (AR1)	Constant	Correlation gets less as time points get further apart	$\sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}$
Autogressive, heterogeneous (ARH1)	Different at each time point	Correlation decreases as time points get further apart	$\begin{bmatrix} \sigma_1^2 & \sigma_2\sigma_1\rho & \sigma_3\sigma_1\rho^2 & \sigma_4\sigma_1\rho^3 \\ \sigma_1\sigma_2\rho & \sigma_2^2 & \sigma_3\sigma_2\rho & \sigma_4\sigma_2\rho^2 \\ \sigma_1\sigma_3\rho^2 & \sigma_2\sigma_3\rho & \sigma_3^2 & \sigma_4\sigma_3\rho \\ \sigma_1\sigma_4\rho^3 & \sigma_2\sigma_4\rho^2 & \sigma_3\sigma_4\rho & \sigma_4^2 \end{bmatrix}$
Level-2 Structures			
Diagonal (DIAG)	Different at each time point	No correlation	$\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}$
Unstructured (UN)	Different at each time point	Different for each pair of time points	$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 \end{bmatrix}$

**Note.* Abbreviations in parentheses are the syntax codes for covariance structures in SPSS.

Table 14

Comparison of Fits of Unconditional Growth Model to Math Achievement in Elementary School

	Model Description	# of Parameter	Deviance Statistics
1	Level-1: Scaled Identify (ID) Level-2: Unstructured (UN)	10	216553.297
2	Level-1: Autoregressive, Homogenous (AR1) Level-2: Unstructured (UN)	11	210934.533
3	Level-1: Autoregressive, Homogenous (AR1) Level-2: Diagonal structure (DIAG)	8	219811.883
4[a,b]	Level-1: Autoregressive, Homogenous (ARH1) Level-2: Unstructured (UN)	14	
5[a,b]	Level-1: Autoregressive, Homogenous (ARH1) Level-2: Diagonal structure (DIAG)	11	

Note. All models examined here estimated fixed and random effects of intercept, linear, and quadratic terms of time.

a. The validity of model fit is uncertain since the final Hessian matrix is not positive finite.

b. The validity of model fit is uncertain because convergence has not been achieved.

Main effects of kindergarten achievement subgroups. The addition of the kindergarten achievement subgroup variable improved model fit significantly (Δ-*2LL* = 3207.722, $\Delta df = 9$, $p < .001$; see Table 15). The effect sizes (i.e., the proportion of variance explained by the kindergarten achievement subgroup variable) for the intercept, linear, and quadratic slopes were 0.33, 0.01, and 0.03, respectively. The results suggested that children with MD-RD scored significantly lower in mathematics than children with RD or MD, or those in the TA group at the spring of fifth grade. However, significant group difference on math growth rate was found only between children with MD-RD and those in the TA group. In addition, the mean math achievement trajectories of the four subgroups (see Figure 7) showed that the achievement gaps between children with difficulties in mathematics (i.e., MD and MD-RD) and those without math difficulties widened over the elementary school years.

Table 15

Multilevel Analysis Results of Math Achievement

	Model 1			Model 2		
	Estimate		SE	Estimate		SE
Fixed Effects						
Intercept	116.09	***	0.23	78.29	***	0.73
Ach. Group - TA				41.23	***	0.76
Ach. Group - RD				27.79	***	1.86
Ach. Group - MD				11.72	***	1.83
Linear slope	11.97	***	0.12	12.72	***	0.46
Ach. Group - TA				-0.85		0.48
Ach. Group - RD				1.90		1.18
Ach. Group - MD				-0.62		1.16
Quadratic slope	-0.56	***	0.02	0.23	*	0.09
Ach. Group - TA				-0.87	***	0.09
Ach. Group - RD				-0.12		0.23
Ach. Group - MD				-0.42		0.22
Random Effects						
Level-1 Variance						
AR1 Diagonal	31.36	***	0.57	31.48	***	0.57
AR1 rho	-0.99	***	0.01	-0.98	***	0.01
Level-2 Variance						
UN(1,1)	360.30	***	6.36	239.42	***	4.28
UN(2,1)	13.63	***	2.53	15.43	***	2.16
UN(2,2)	94.20	***	2.52	92.91	***	2.42
UN(3,1)	-6.00	***	0.49	-3.64	***	0.41
UN(3,2)	17.46	***	0.49	17.17	***	0.47
UN(3,3)	3.56	***	0.10	3.46	***	0.09
Model-Fit Criteria						
-2LL	210934.533			207726.811		

Note: Model 1: Unconditional growth model; Model 2: Conditional growth model with kindergarten achievement subgroups on all three growth parameters; * = $p < .05$, ** = $p < .01$, ***= $p < .001$.

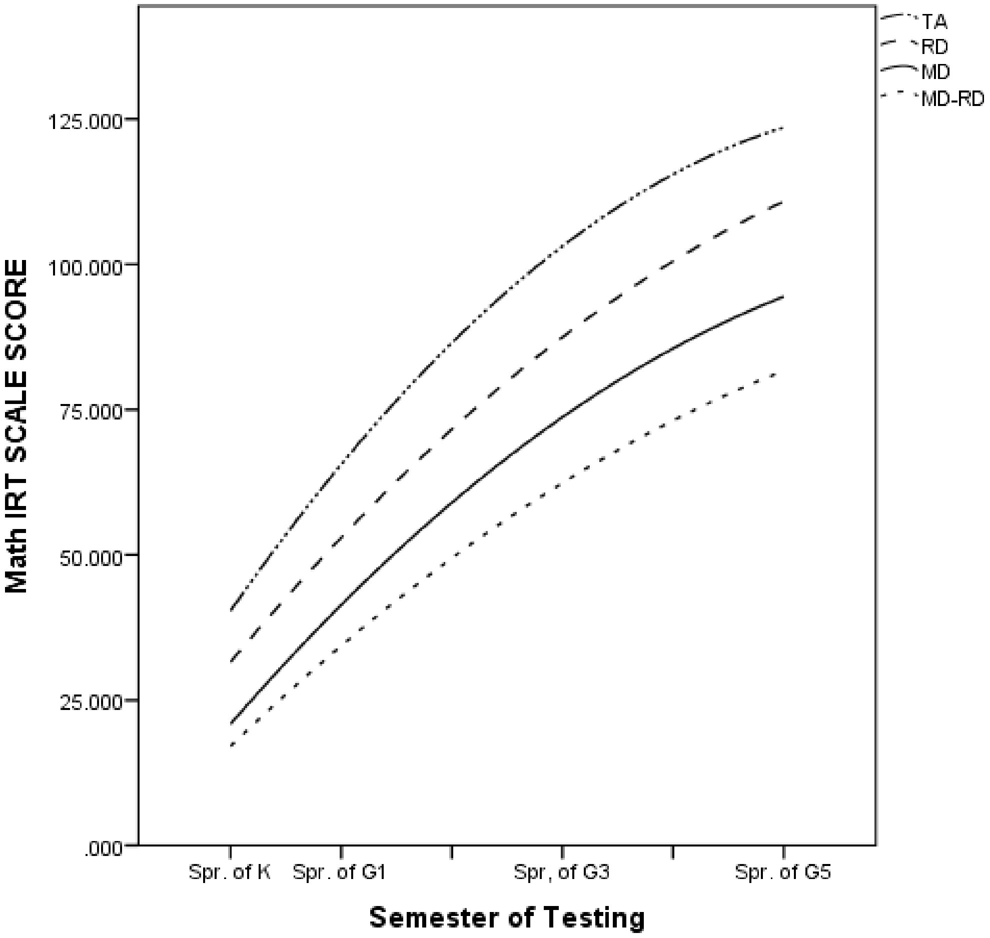

Figure 7. Growth trajectories of math achievement for children in the four achievement groups over elementary school years.

Reading Achievement Growth Trajectories

Unconditional models. The unconditional means models for reading achievement found significant within-person variance (τ_e^2 = 1864.712, p < .001) and between-person variance (τ_0^2 = 105.890, p < .001), indicating sufficient variation at both levels to be potentially explained by other predictors. The estimate of intraclass correlation coefficient (ρ) for math achievement was 0.053, indicating that about 5% of total variation in math achievement can be attributable to between-individual differences.

By adding the time factor as the only level-1 predictor and no other predictors at level-2, about 93.6% of within-person variance was reduced in the unconditional growth model, indicating that the time factors in the unconditional growth model explained a substantial amount of within-individual variance of reading achievement. The fixed effects of intercept, linear, and quadratic growth terms were all highly significant (β_0 = 142.766, t = 534.197, p < .001 for intercept; β_1 = 10.406, t = 59.929, p < .001 for linear growth; β_2 = -1.693, t = -49.407, p < .001 for quadratic growth). Therefore, all three growth parameters in the models were retained for subsequent analyses. The between-person variances in the unconditional growth models for reading achievement (i.e., τ_{00} = 315.4372, τ_{11} = 1.039, and τ_{22} = 0.148) were all significant (p < .001). The finding suggests that sufficient between-individual variation was left to be explained by other level-2 predictors, such as children's demographics, children's learning characteristics and behaviors, or certain classroom factors.

Covariance structures for the model. To identify appropriate covariance structures for level-1 and level-2 variance components, the model fit indices and the number of parameters of models with different sets of covariance structures were compared. Table 16 presents the results of the model comparison. The results suggest that model 2 had the smallest deviance statistics and the most parameters and was therefore the best-fit model among the five examined models. Therefore, the combination of homogeneous autoregressive structure for level-1 and unstructured matrix for level-2 was used to examine reading growth trajectories of children in the four kindergarten achievement subgroups.

Table 16

Comparison of Fits of Unconditional Growth Model to Reading Achievement in Elementary School

	Model Description	# of Parameter	Deviance Statistics
1	Level-1: Scaled Identify (ID) Level-2: Unstructured (UN)	10	221811.671
2	Level-1: Autoregressive, Homogenous (AR1) Level-2: Unstructured (UN)	11	220068.333
3	Level-1: Autoregressive, Homogenous (AR1) Level-2: Diagonal structure (DIAG)	8	227310.228
4[a,b]	Level-1: Autoregressive, Homogenous (ARH1) Level-2: Unstructured (UN)	14	
5[a,b]	Level-1: Autoregressive, Homogenous (ARH1) Level-2: Diagonal structure (DIAG)	11	

Note. All models examined here estimated fixed and random effects of intercept, linear, and quadratic terms of time.

a. The validity of model fit is uncertain since the final Hessian matrix is not positive finite.

b. The validity of model fit is uncertain because convergence has not been achieved.

Main effects of kindergarten achievement subgroups. By adding kindergarten achievement subgroup variable, the model fit to reading achievement data improved significantly (Δ-*2LL* = 3002.125, $\Delta df = 9$, $p < .001$; see Table 17). The effect sizes (i.e., the proportion of variance explained by the kindergarten achievement subgroup variable) for the intercept, linear and quadratic slopes were 0.33, 0.04, and 0.08. As expected, children with MD-RD had the lowest average reading score at the spring of fifth grade. However, children with MD-RD had increasing rates of growth and acceleration ($\beta_{1MDRD} = 17.97$, $p < .001$; $\beta_{2MDRD} = 0.60$, $p < .001$), whereas children in the other three groups had significant rates of deceleration ($\beta_{2TA} = -2.51$, $p < .001$; $\beta_{2RD} = -0.98$, $p = .001$; $\beta_{2MD} = -1.00$, $p = .001$). The results indicated that children with MD-RD made significantly faster progress than did children in the other three groups, when the time effect was accounted for in the model. On the other hand, by examining the mean reading achievement trajectories of the four achievement subgroups (see Figure 8), children with RD started with lower average reading scores than children with MD in kindergarten, but they progressed in a faster rate and surpassed their peers with MD in late elementary school years. This finding was somewhat opposite to the increasing achievement gaps between MD and non-MD groups found in math achievement growth.

In summary, when examining the group average of math achievement over time, the achievement difference between children with difficulties in mathematics (i.e., MD and MD-RD) and those without difficulties in mathematics (i.e., TA and RD) were increasing over the years. However, when taking the time factor into consideration, the results of multilevel modeling showed that children with MD-RD, though still performed the worst in mathematics among the four subgroups at the fifth grade, had similar math growth rates as their peers with RD and MD. The similar achievement growth pattern was not found in reading achievement. The investigation of children's mean reading achievement over time showed that, though children with MD-RD was the worst performing group in reading achievement across all the time points of testing, children with RD actually made faster progress and surpassed their peers with MD in reading achievement at third grade and continued performing better at fifth grade. On the other hand, the results from multilevel modeling analyses indicated that children with MD-RD had faster growth rates than their peers in the other three groups. However, children with MD-RD still had the lowest average reading scores among the four achievement groups at the spring of fifth grade.

Table 17

Multilevel Analysis Results of Reading Achievement Growth

Estimate	Model 1			Model 2		
	Estimate		SE	Estimate		SE
Fixed Effects						
Intercept	142.26	***	0.26	101.08	***	0.80
Ach. Group – TA				44.88	***	0.83
Ach. Group - RD				21.08	***	2.05
Ach. Group - MD				23.46	***	1.99
Linear slope	10.09	***	0.16	17.97	***	0.59
Ach. Group – TA				-8.65	***	0.62
Ach. Group - RD				-1.33		1.50
Ach. Group - MD				-3.02	*	1.48
Quadratic slope	-1.70	***	0.03	0.60	***	0.11
Ach. Group – TA				-2.51	***	0.12
Ach. Group - RD				-0.98	**	0.29
Ach. Group - MD				-1.00	**	0.29
Random Effects						
Level-1 Variance						
AR1 Diagonal	41.69	***	.75	41.77	***	0.75
AR1 rho	-0.98	***	.01	-0.98	***	0.01
Level-2 Variance						
UN(1,1)	430.21	***	7.63	290.30	***	5.21
UN(2,1)	-26.28	***	3.51	1.05		2.93
UN(2,2)	155.91	***	3.65	149.63	***	3.51
UN(3,1)	-16.42	***	.72	-8.60	***	0.57
UN(3,2)	29.40	***	.71	27.69	***	0.68
UN(3,3)	6.10	***	.14	5.63	***	0.14
Model-Fit Criteria						
-2LL	221840.745			218838.620		

Note: Model 1: Unconditional growth model; Model 2: Conditional growth model with kindergarten achievement subgroups on all three growth parameters; * = $p < .05$, ** = $p < .01$, ***= $p < .001$.

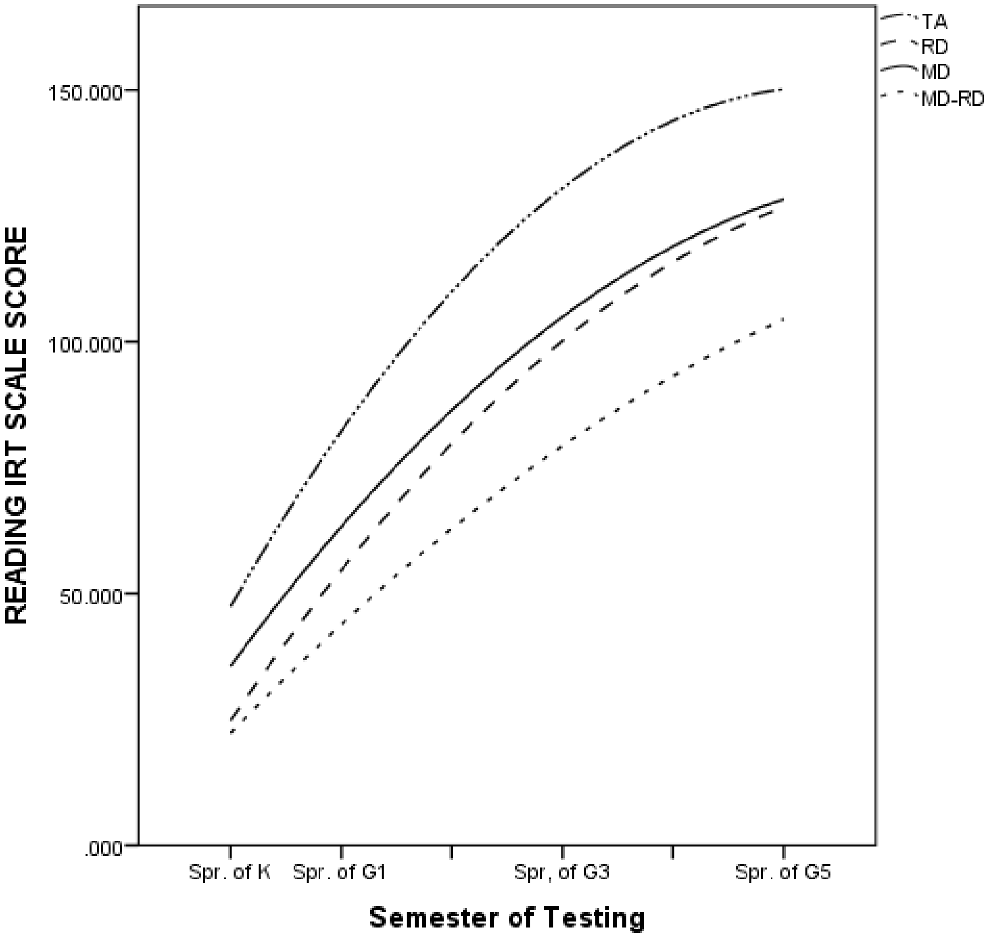

Figure 8. Growth trajectories of reading achievement for children in the four achievement groups over elementary school years.

Math Achievement Growth and Associated Kindergarten Predictors

The third research question investigated the effects of identified child and kindergarten class predictors, including children's kindergarten achievement subgroups (i.e., TA, MD, and MD-RD), child's demographic characteristics (i.e., gender, race, and SES), child's individual skills in kindergarten (i.e., learning-related skills and initial reading and initial math knowledge), and kindergarten class factors (i.e., class size, instructional time, and four types of math instructional activities). The question also examined whether the effects of these child and class variables vary among children in different kindergarten achievement subgroups.

Effects of Identified Child and Class Predictors

In this main effects model, all identified kindergarten predictors were added to examine their association with mathematics achievement and growth over the elementary school years. The deviance statistics for the main effects model was χ^2 =163424.319 with 62 degrees of freedom, which shows a significant improvement over the unconditional growth model (Δ-*2LL* = 47510.214, Δdf = 51, $p < .001$). Meanwhile, the proportions of variance explained by the main effects model in intercept, linear growth, and quadratic growth were 0.55, 0.06, and 0.08, respectively. The added kindergarten predictors explained a substantial amount of variance in fifth grade math achievement, as well as the linear and quadratic math growth.

Table 18 presents the results regarding the effects of kindergarten predictors on math achievement growth. The intercept (end-of-fifth grade math achievement), linear slope, and quadratic slope pertain to White boys with average scores in learning-related skills, initial reading and math performance, and all class factors in the MD-RD group from average SES families. The average fifth-grade mathematics IRT scale score for this group was β_0 =98.03, the linear slope was β_1 =8.54, and the quadratic slope was β_2 = -0.86. That is, children's math IRT scores increased over time from kindergarten to fifth grade; however, since the rate of acceleration was negative, the rate of score increase eventually slowed down.

Overall, all children's socioeconomic and demographic characteristics as well as individual variables in kindergarten were significantly associated with children's math scores at the spring of fifth grade as well as their linear and quadratic growth in mathematics, except the non-significant effect of SES on quadratic growth. Furthermore, class size, instructional time, and the time spent on *practice* and *integrated activities* were found to be significant predictors of fifth-grade math achievement. However, only the time spent on *practice activities* was significantly associated with math achievement growth. The results are described in detail in the following section.

Table 18

Main Effect Model in Math Achievement and Growth

Effects	Math		*t*	95% Confidence Interval	
	Estimate	SE		Lower Bound	Upper Bound
Fixed Effects					
Intercept	98.03 ***	0.77	127.24	96.52	99.55
Female	-4.61 ***	0.33	-13.80	-5.26	-3.95
Race – Black	-7.70 ***	0.61	-12.71	-8.89	-6.52
Hispanic	0.24	0.50	0.48	-0.74	1.21
Asian	2.49 ***	0.67	3.73	1.18	3.80
Other Races	-1.65 *	0.71	-2.31	-3.05	-0.25
SES	3.08 ***	0.26	11.96	2.57	3.59
Ach. Group - TA	22.44 ***	0.76	29.37	20.93	23.94
MD	5.77 ***	1.54	3.75	2.75	8.80
Learning-related Skills	3.81 ***	0.29	13.27	3.25	4.38
Initial Reading Knowledge	0.68 ***	0.03	25.67	0.63	0.74
Initial Math Knowledge	0.05 *	0.02	2.52	0.01	0.09
Class Size	0.12 **	0.03	3.47	0.05	0.18
Instructional Time	-0.23 *	0.09	-2.43	-0.41	-0.04
Math Instructional Activities					
Interactive Activities	0.00	0.16	0.01	-0.32	0.32
Manipulatives	-0.17	0.25	-0.66	-0.67	0.33
Practice Activities	-0.48 **	0.14	-3.37	-0.76	-0.20
Integrated Activities	0.34 *	0.14	2.35	0.05	0.62
Linear slope	8.54 ***	0.56	15.15	7.44	9.65
Female	0.84 **	0.25	3.36	0.35	1.33
Race – Black	0.00	0.46	0.00	-0.91	0.91
Hispanic	0.82 *	0.38	2.18	0.08	1.55
Asian	2.48 ***	0.51	4.89	1.49	3.47
Other Races	1.64 **	0.54	3.07	0.59	2.69
SES	0.35 *	0.17	1.99	0.01	0.69
Ach. Group - TA	2.95 ***	0.55	5.32	1.86	4.03
Ach. Group - MD	0.32	1.13	0.29	-1.90	2.55
Learning-related Skills	-0.62 **	0.23	-2.77	-1.07	-0.18
Initial Reading Knowledge	-0.21 ***	0.02	-10.75	-0.25	-0.17
Initial Math Knowledge	-0.02	0.02	-1.10	-0.05	0.01
Class Size	0.03	0.02	1.32	-0.02	0.08
Instructional Time	-0.06	0.08	-0.81	-0.22	0.09

Table 18 (continued)

Effects	Math		*t*	95% Confidence Interval	
	Estimate	SE		Lower Bound	Upper Bound
Math Instructional Activities					
Interactive Activities	-0.09	0.13	-0.68	-0.34	0.16
Manipulatives	-0.10	0.18	-0.58	-0.45	0.24
Practice Activities	0.15	0.11	1.37	-0.07	0.36
Integrated Activities	-0.09	0.11	-0.83	-0.30	0.12
Quadratic slope	-0.86 ***	0.11	-7.82	-1.07	-0.64
Female	0.30 ***	0.05	6.20	0.20	0.39
Race – Black	0.25 **	0.09	2.84	0.08	0.42
Hispanic	0.15 *	0.07	2.09	0.01	0.29
Asian	0.38 ***	0.10	3.97	0.19	0.57
Other Races	0.40 ***	0.10	3.89	0.20	0.60
SES	-0.04	0.03	-1.31	-0.11	0.02
Ach. Group - TA	0.08	0.11	0.74	-0.13	0.29
MD	-0.15	0.22	-0.67	-0.58	0.28
Learning-related Skills	-0.22 ***	0.04	-5.14	-0.31	-0.14
Initial Reading Knowledge	-0.04 ***	0.00	-10.65	-0.05	-0.03
Initial Math Knowledge	0.00	0.00	-0.77	-0.01	0.00
Class Size	0.00	0.00	0.40	-0.01	0.01
Instructional Time	0.00	0.01	-0.04	-0.03	0.03
Math Instructional Activities					
Interactive Activities	-0.01	0.03	-0.38	-0.06	0.04
Manipulatives	-0.03	0.03	-0.85	-0.10	0.04
Practice Activities	0.06 **	0.02	2.81	0.02	0.10
Integrated Activities	-0.03	0.02	-1.60	-0.07	0.01
Random Effects					
Within-person Var					
AR1 Diagonal	31.39 ***	0.56		30.30	32.49
AR1 rho	-0.98 ***	0.01		-0.99	-0.97
Between-person Var					
UN(1,1)	161.59 ***	2.95		155.81	167.38
UN(2,1)	30.72 ***	1.80		27.21	34.24
UN(2,2)	89.05 ***	2.25		84.64	93.46
UN(3,1)	0.32	0.33		-0.34	0.97
UN(3,2)	16.34 ***	0.43		15.49	17.19
UN(3,3)	3.26 ***	0.09		3.09	3.43

* = $p < .05$, ** = $p < .01$, ***= $p < .001$.

Main effects of socioeconomic and demographic characteristics. All three demographic characteristics—gender, race, and SES—were significant predictors of children's mathematics growth and achievement level at fifth grade, with the exception of children's SES on quadratic slope. When holding other predictors constant, girls were found to progress faster than boys over the elementary school years ($\beta_{1gender} = .84$, $p < .01$; $\beta_{2gender} = .30$, $p < .001$), but they still scored significantly lower in average than did boys in fifth grade ($\beta_{0gender} = -4.61$, $p < .001$). Children with higher SES in kindergarten were found to have higher scores in mathematics at fifth grade ($\beta_{0SES} = 3.08$, $p < .001$), and they had significantly higher linear growth in math ($\beta_{1SES} = 0.35$, $p < .05$). White children had lower average math score than Asian children, but had higher average scores than African American and children of the other races at the spring of fifth grade. In addition, compared to White children, children in the other racial groups made faster progress in mathematics over the elementary school years.

Main effects of individual variables. Children's learning-related skills and their initial math and reading knowledge were all positively associated with children's fifth-grade math performance. That is, children who had higher average scores in learning-related skills, initial math knowledge, and initial reading knowledge in kindergarten were likely to have higher math scores at fifth grade. On the other hand, children with higher scores on learning-related skills and initial reading knowledge made slower progress in mathematics ($\beta_{1LRS} = -0.62$, $p < .01$; $\beta_{2LRS} = -0.22$, $p < .001$; $\beta_{1IniRead} = -0.21$, $p < .001$; $\beta_{2IniRead} = -0.04$, $p < .001$). Children's initial math knowledge in kindergarten, however, was not a significant predictor of linear and quadratic growth in math achievement.

Main effects of classroom factors. Class size, instructional time, and math practice activities and integrated activities were significant predictors of children's fifth-grade math achievement. However, only the practice activities variable was significantly associated with math growth over the elementary school years.

Children who were in large classes and those who received less instructional time in kindergarten were predicted to score higher in fifth-grade math achievement. No significant association was found between children's class size and instructional time in kindergarten and their math growth. Among the four math instructional activities, the time spent on practice activities, which includes doing worksheets or completing math problems on textbooks or the chalkboard, was found to be negatively associated with fifth-grade math achievement ($\beta_{0practice} = -0.48, p < .01$), but positively associated with math growth ($\beta_{2practice} = 0.06, p < .01$). More time spent on integrated activities, including using creative activities or music to understand math concepts, was found to positively predict children's fifth-grade math score ($\beta_{0Integrated} = 0.34, p < .05$). However, integrated activities, interactive activities, and manipulatives were not significant predictors of children's math growth in elementary school.

The Interaction Effects of Achievement Subgroup and Kindergarten Predictors

To explore how children's kindergarten achievement subgroup membership moderates the effects of identified kindergarten predictors, the interaction effect model was established by including all the kindergarten predictors and interaction terms involving kindergarten achievement groups. The non-significant interaction terms were removed from the model. The model was then refitted with the remaining parameters. The final interaction model for math achievement contains interactions between three kindergarten achievement groups and children's SES, race, and children's initial math knowledge (see Table 19), in addition to the simple effects of all identified kindergarten predictors. The deviance statistics for the final interaction effect model was 163372.3 with 98 degrees of freedom, which is improved significantly from 163424.319 with 62 degrees of freedom of the main effect model (Δ-2LL = 52.019, $\Delta df = 36$, $p = .041$). The effect sizes (i.e., the proportions of explained residual variances) of the interaction terms were very small (0.0064, 0.0070, and 0.00673 for intercept, linear slope, and quadratic slope, respectively), which indicates that only a limited amount of variance was explained by the additional interaction terms in the model.

Table19

Interaction Effect Model in Math Achievement and Growth

Effects	Mathematics		t	95% Confidence Interval	
	Estimate	SE		Lower Bound	Upper Bound
Fixed Effects					
Intercept	102.28 ***	3.02	33.83	95.98	108.59
Female	-4.57 ***	0.33	-13.75	-5.23	-3.92
Race – Black	-8.59 ***	1.65	-5.22	-11.82	-5.36
Hispanic	2.49	1.61	1.55	-0.67	5.65
Asian	-4.89	3.63	-1.35	-12.03	2.25
Other Races	-5.16 *	2.09	-2.47	-9.26	-1.07
SES	3.43 **	1.11	3.10	1.14	5.72
Ach. Group - TA	18.18 ***	3.03	6.00	11.87	24.49
MD	-8.02	4.80	-1.67	-17.61	1.58
Learning-related Skills (LRS)	3.81 ***	0.29	13.27	3.25	4.37
Initial Math Knowledge	0.35	0.20	1.72	-0.06	0.76
Initial Reading Knowledge	0.69 ***	0.03	26.21	0.63	0.74
Class Size	0.11 **	0.03	3.38	0.05	0.18
Instructional Time	-0.22 *	0.09	-2.32	-0.40	-0.03
Math Instructional Activities					
Interactive Activities	0.00	0.16	0.03	-0.31	0.32
Manipulatives	-0.17	0.25	-0.65	-0.67	0.34
Practice Activities	-0.47 **	0.14	-3.33	-0.75	-0.19
Integrated Activities	0.33 *	0.15	2.26	0.04	0.62
Black* Ach. Group - TA	0.96	1.78	0.54	-2.52	4.44
Black* Ach. Group - MD	6.41	4.44	1.45	-2.35	15.17
Hispanic*Ach. Group - TA	-2.74	1.69	-1.62	-6.05	0.57
Hispanic*Ach. Group - MD	3.05	4.81	0.64	-6.60	12.71
Asian* Ach. Group - TA	7.71 *	3.70	2.08	0.44	14.98
Asian* Ach. Group - MD	6.15	7.01	0.88	-7.63	19.93
OtherRaces* Ach. Group - TA	4.06	2.22	1.83	-0.29	8.42
OtherRaces* Ach. Group -MD	12.34 *	5.53	2.23	1.50	23.19
SES* Ach. Group - TA	-0.45	1.10	-0.41	-2.70	1.80
SES* Ach. Group - MD	6.51 *	3.08	2.12	0.33	12.70
Initial Math Knowledge* TA	-0.30	0.21	-1.46	-0.72	0.12
Initial Math Knowledge* MD	-1.33 **	0.41	-3.25	-2.13	-0.52
Linear slope	12.14 ***	2.03	5.99	8.06	16.21
Female	0.87 ***	0.25	3.49	0.38	1.36
Race – Black	-1.69	1.28	-1.33	-4.19	0.81
Hispanic	0.42	1.23	0.34	-1.99	2.83
Asian	-3.02	2.79	-1.08	-8.50	2.46
Other Races	-1.64	1.56	-1.05	-4.70	1.42

Table 19 (continued)

Effects	Mathematics			t	95% Confidence Interval	
	Estimate		SE		Lower Bound	Upper Bound
SES	1.65	*	0.72	2.29	0.22	3.07
Ach. Group - TA	-0.70		2.03	-0.34	-4.78	3.39
MD	-2.91		3.51	-0.83	-9.87	4.04
Learning-related Skills(LRS)	-0.61	**	0.22	-2.76	-1.05	-0.18
Initial Math Knowledge	0.10		0.15	0.66	-0.21	0.40
Initial Reading Knowledge	-0.21	***	0.02	-10.60	-0.25	-0.17
Class Size	0.03		0.02	1.18	-0.02	0.08
Math Instructional Activities						
Interactive Activities	-0.08		0.13	-0.63	-0.33	0.17
Manipulatives	-0.08		0.18	-0.48	-0.43	0.26
Practice Activities	0.15		0.11	1.34	-0.07	0.36
Integrated Activities	-0.10		0.11	-0.92	-0.31	0.11
Black* Ach. Group - TA	1.91		1.37	1.40	-0.77	4.59
Black* Ach. Group - MD	2.32		3.11	0.75	-3.78	8.43
Hispanic*Ach. Group - TA	0.26		1.29	0.20	-2.27	2.78
Hispanic*Ach. Group - MD	3.22		3.14	1.03	-2.93	9.36
Asian* Ach. Group - TA	5.75	*	2.84	2.03	0.17	11.32
Asian* Ach. Group - MD	3.76		5.23	0.72	-6.49	14.01
OtherRaces* Ach. Group - TA	3.77	*	1.66	2.27	0.51	7.03
OtherRaces* Ach. Group -MD	5.45		4.32	1.26	-3.02	13.92
SES* Ach. Group - TA	-1.43		0.75	-1.92	-2.92	0.05
SES* Ach. Group - MD	1.04		2.05	0.51	-2.98	5.06
Initial Math Knowledge* TA	-0.12		0.15	-0.79	-0.42	0.18
Initial Math Knowledge* MD	-0.05		0.31	-0.16	-0.67	0.57
Quadratic slope	-0.35		0.39	-0.88	-1.14	0.45
Female	0.30	***	0.05	6.29	0.21	0.40
Race – Black	-0.02		0.25	-0.07	-0.50	0.46
Hispanic	0.06		0.24	0.27	-0.40	0.53
Asian	-0.41		0.53	-0.77	-1.45	0.64
Other Races	-0.08		0.30	-0.27	-0.67	0.51
SES	0.22		0.13	1.65	-0.04	0.48
Ach. Group - TA	-0.44		0.40	-1.11	-1.24	0.36
MD	-0.37		0.70	-0.54	-1.76	1.02
Learning-related Skills (LRS)	-0.22	***	0.04	-5.13	-0.31	-0.14
Initial Math Knowledge	0.01		0.03	0.27	-0.05	0.07
Initial Reading Knowledge	-0.04	***	0.00	-10.54	-0.05	-0.03
Class Size	0.00		0.00	0.28	-0.01	0.01
Instructional Time	0.00		0.01	0.05	-0.03	0.03

Table 19 (continued)

Effects	Mathematics		t	95% Confidence Interval	
	Estimate	SE		Lower Bound	Upper Bound
Math Instructional Activities					
Interactive Activities	-0.01	0.03	-0.33	-0.06	0.04
Manipulatives	-0.03	0.03	-0.74	-0.09	0.04
Practice Activities	0.06 **	0.02	2.78	0.02	0.10
Integrated Activities	-0.03	0.02	-1.69	-0.07	0.01
Black* Ach. Group - TA	0.31	0.26	1.17	-0.21	0.83
Black* Ach. Group - MD	0.22	0.61	0.37	-0.98	1.42
Hispanic*Ach. Group - TA	0.07	0.25	0.27	-0.42	0.55
Hispanic*Ach. Group - MD	0.51	0.61	0.83	-0.70	1.71
Asian* Ach. Group - TA	0.83	0.54	1.53	-0.23	1.89
Asian* Ach. Group - MD	0.48	1.00	0.48	-1.49	2.44
OtherRaces* Ach. Group - TA	0.55	0.32	1.73	-0.07	1.18
OtherRaces* Ach. Group -MD	0.67	0.83	0.80	-0.97	2.30
SES* Ach. Group - TA	-0.28 *	0.14	-2.06	-0.55	-0.01
SES* Ach. Group - MD	-0.09	0.40	-0.22	-0.88	0.71
Initial Math Knowledge* TA	-0.01	0.03	-0.36	-0.07	0.05
Initial Math Knowledge* MD	0.03	0.06	0.48	-0.09	0.15
Random Effects					
Within-person Var					
AR1 Diagonal	31.42 ***	0.56	0.00	30.33	32.52
AR1 rho	-0.98 ***	0.01	0.00	-0.99	-0.97
Between-person Var					
UN(1,1)	160.56 ***	2.94	0.00	154.80	166.31
UN(2,1)	30.22 ***	1.80	0.00	26.69	33.74
UN(2,2)	88.42 ***	2.25	0.00	84.01	92.84
UN(3,1)	0.25	0.33	0.46	-0.41	0.90
UN(3,2)	16.23 ***	0.43	0.00	15.38	17.08
UN(3,3)	3.24 ***	0.09	0.00	3.07	3.41

* = *p* < .05, ** = *p* < .01, ***= *p* < .001.

Among the significant findings, of particular interest to this researcher were the significant interactions between kindergarten achievement groups and children's SES and initial math knowledge. Significant simple effects of SES were found on intercept (β_{0SES} = 3.43, p =.005) and linear slope (β_{1SES} = 1.65, p = .024), indicating that children with MD-RD with higher SES in kindergarten were likely to have higher math scores in fifth grade. The current study also found that the effect of SES varied between kindergarten achievement subgroups. For example, the positive effect of SES was significantly larger for children with MD than for children with MD-RD ($\beta_{0SESxMD}$ = 6.51, p = .039) on intercept. In addition, compared to children with MD-RD, the effect of SES for TA children was significantly negative on math achievement growth ($\beta_{2SESxTA}$ = -0.28, p = .04). In addition to the interaction effects of SES, the current study also found the effect of initial math knowledge varied between children with MD and MD-RD. In particular, no significant effect of initial math knowledge was found for children with MD on either intercept or math growth; however, a negative effect of initial math knowledge was found for children with MD on intercept, indicating that children with MD who scored higher on math achievement at kindergarten entry were likely to have lower math scores at fifth grade.

In addition, an important yet surprising finding was the lack of significant interactions between kindergarten achievement subgroups and class factors. Despite some significant simple effects of class variables (i.e., the effects of class size, instructional time, and time spent on practice and integrated activities on fifth-grade math achievement) found in the model, the effects of class factors did not seem to vary for children in different achievement subgroups.

In summary, the results of the multilevel modeling analyses showed that girls had higher rates of math growth than boys, but they still performed worse than boys at the end of fifth grade in mathematics. Significant differences of fifth-grade math achievement and math growth over elementary school years were found between racial groups and between kindergarten achievement subgroups. Children's SES, learning-related skills, initial reading knowledge, and initial math knowledge were also positively associated with children's fifth-grade math achievement. The effects of these variables on math growth, however, were varied. Children's class size and the time they spent on doing integrated activities were positively predictive of their fifth-grade math achievement; the instructional time and the time spent on practice activities, however, were negatively associated with fifth-grade math scores. Among all the class factors, only the category of practice activities was significantly associated with math growth.

The lack of significant effects of class factors on math achievement and growth occurred not only in the main effects model, but also in the interaction effects model. The significant interaction effects were found only between children's kindergarten achievement subgroup and race, SES, and children's initial math knowledge, indicating that the effects of most identified kindergarten predictors were not varied across kindergarten achievement subgroups.

CHAPTER V

Discussion

The purpose of the present study was to explore, from a longitudinal perspective, the performance profiles of children with difficulties in mathematics. Using data from a large-scale, nationally representative, longitudinal sample, the current study was conducted to explore two issues – the longitudinal manifestation of math difficulties and the differential influence of early predictors on progress in learning math for children with different subtypes of MD. The first issue was investigated by considering the stability and patterns of subgroup change for children with MD, MD-RD, RD, and TA, as well as by examining the math and reading achievement trajectories of children in different achievement subgroups. The second issue was explored by investigating how the identified kindergarten predictors influence progress in learning math and whether the effects of these kindergarten predictors vary among children in different achievement subgroups. Two main findings emerged: (a) children with MD-RD differed from children with MD and children in the comparison groups in the patterns of subtype change over time, math and reading IRT scale scores, and math and reading achievement trajectories; and (b) children's demographic characteristics, learning-related skills, math and reading performance at kindergarten entry, class size, and instructional time were all significantly predictive of their later math achievement and progress. Though limited, the effects of some identified kindergarten predictors did vary across children in different kindergarten achievement subgroups.

In this chapter, the primary findings are discussed, followed by implications and study limitations. The chapter is divided into the following sections: (1) longitudinal manifestation of math difficulties, (2) kindergarten predictors of math progress, (3) implications, (4) study limitations, and (5) conclusion.

Longitudinal Manifestation of Math Difficulties

A major aim of the present study was to explore the difference in math learning between children with MD-RD and MD from a longitudinal perspective. The results showed distinct patterns of subgroup change between children in the two MD groups. More than 75% of children with MD-RD were identified in the same subgroup again at the following time point, whereas only 15 to 25% of children with MD maintained the same classification. In fact, 73% of children who were identified as MD-RD in kindergarten and remained in the sample still met the criteria for MD-RD at fifth grade, as opposed to 12% of children with MD who were identified as MD again at fifth grade.

Moreover, children's math achievement scores showed a difference in performance between the two MD groups. Compared to children with MD-RD, children with MD had significantly higher math scores in kindergarten and again in fifth grade. Although children in both MD groups had poor proficiency in lower levels of math achievement identified in the ECLS-K study, children with MD were more likely to reach mastery of lower math levels. These results confirm previous findings that children with MD-RD had more severe and persistent deficits in mathematics than their peers with MD (Hanich et al., 2001; Jordan et al., 2003; Silver et al., 1999).

In addition, the results of multilevel analyses showed no significant differences between children with MD-RD and children with MD in math development. The linguistic advantage of children with MD did not seem to influence their progress in mathematics. Some areas of mathematics, such as word problems and arithmetic combinations, are usually regarded as language-related, and children with MD usually have advantages in performance in these areas over their peers with MD-RD (Hanich et al., 2001; Jordan et al., 2003). However, in the present study, whereas children with MD scored significantly higher than children with MD-RD in mathematics at fifth grade, they progressed at a similar rate to children with MD-RD.

It was also found in the present study that children with MD-RD made significantly faster progress in reading than children in the other three achievement subgroups. Moreover, children with RD started with a similar level of reading achievement to children with MD-RD in kindergarten, but reached a similar level of reading achievement to children with MD at fifth grade. The findings on math and reading progress are contradictory to those of Jordan and colleagues (Jordan et al., 2002; Jordan et al., 2003), namely that children's reading abilities influence their progress in mathematics, but their math abilities do not influence their progress in reading.

The different findings here could be due to the measures of progress in math used by the ECLS-K study. Previous research (Andersson, 2008; Fuchs and Fuchs, 2002) suggests that the advantage that children with MD enjoy over children with MD-RD is reduced when the given tasks require low linguistic skills or demand high calculation and problem-solving competencies, which could be the case in the present study. In addition, the ECLS-K dataset used in this study does not allow controlling for intelligence, as Jordan and colleagues did in their studies, which may also account for different findings. Another possible explanation is that Jordan and colleagues adopted a more liberal criterion for identifying difficulties (below the 35th percentile), whereas the criterion for difficulty in this study was one standard deviation below the mean (approximately the

16th percentile). Finally, the population represented by the current study is clearly not identical to the population that was the focus of study for Jordan and colleagues.

In the present study, children with difficulties in mathematics at an early stage in their schooling were likely to go on demonstrating difficulties in mathematics throughout the subsequent years. However, only a small percentage of children with RD (i.e., about 15%) developed comorbid math difficulties in first and third grades. After third grade, almost none of the RD children changed subgroup to MD or MD-RD. Meanwhile, children in the TA group were found to remain as TA or fall into the borderline group that did not meet identification criteria for difficulties in math and/or reading difficulties during elementary school. The findings suggest that children in the RD or TA groups were unlikely to develop difficulties in mathematics in late elementary grade. This raises the question of whether children could develop late-emergent math difficulties. A supplemental analysis found that a group of children who were in the borderline group at third grade did meet the criteria for MD or MD-RD at fifth grade. However, the data did not allow for further examination. Since the mathematics skills and knowledge children have acquired in elementary school serve as a foundation for learning advanced math or science subjects, future research should also explore the issue of late-emerging math difficulties.

Effects of Kindergarten Predictors on Progress in Math

In the present study, children's demographic characteristics and learning-related skills in kindergarten were found to be significant predictors of their math learning and fifth-grade achievement level. Surprisingly, despite some class factors, including class size, instructional time, practice activities and integrated acitivities, that seemed significantly associated with children’s fifth-grade math achievement, only one class factor, practice activities, appeared to have any significant effect on math achievement growth. In addition, the interaction effects of children’s kindergarten achievement subgroups and identified kindergarten predictors were unexpectedly limited. The findings are discussed in detail in the following section.

Gender

In the present study, gender differences in math achievement emerged as early as in kindergarten. Girls were found to have significantly lower math scores than boys in kindergarten and fifth grade, after accounting for demographic characteristics and kindergarten predictors in the model. The present results confirm a previous finding (Jordan et al., 2006) that boys outperform girls in mathematics if income level, age, and reading ability are held constant.

Interestingly, even with lower average math scores, girls in the present study were outnumbered by boys in the MD and MD-RD groups in kindergarten and first grade. After third grade, the number of girls with difficulties in mathematics increased dramatically, especially in the MD group. However, the same pattern was not found for girls with difficulties in reading. The results contradicted previous claims that boys were more likely than girls to have difficulties in mathematics (Badian, 1983; Barbaresi et al., 2005) or that no gender difference in math achievement is found between kindergarten and third grade (Lachance & Mazzocco, 2006). The current finding raises the question of whether particular elements in the math curriculum or cognitive skills are associated with gender differences in learning mathematics, especially among children in later elementary grades.

Another interesting finding was the significantly higher math growth rates for girls, after accounting for the other kindergarten predictors in the model. Previous work by Aunola et al. (2004) found no gender differences in achievement level, and that boys had a higher growth rate in primary school math. The different finding obtained in the present study may be due to the fact that the other kindergarten predictors accounted for some score differences between the sexes. It would be interesting to explore further which kindergarten factors identified are associated with gender difference in learning mathematics.

Socio-economic Status (SES)

Previous studies by Jordan and colleagues (Jordan et al., 2006; Jordan et al., 2007) examined the influence of socio-economic status (SES) on growth in number sense for children between kindergarten and first grade. Holding background variables constant, Jordan and colleagues found that low-income children progressed more slowly on most of the number sense tasks than their middle-income peers. The present study extended the investigation into progress in learning math to fifth grade. The results were consistent with previous evidence that children with higher SES in kindergarten had higher math scores at fifth grade, and also made faster progress than their peers with average SES during their time at elementary school. The current results are particularly noteworthy since the significant predictive power of SES was still found after controlling for children's background characteristics, academic performance at kindergarten entry, and class factors. The associations between SES and children's demographic characteristics, academic skills at school entry, and learning environment have been documented in the literature (e.g., Aikens & Barbarin, 2008; Coley, 2002; Muijs, Harris, Chapman, Stoll, &

Russ, 2009). The findings are evidence of the significant influence of SES on math achievement and progress. In addition, they suggest that there may be some unidentified factors, other than those included in the present study, that can mediate the effect of SES on math achievement and growth.

Jordan and colleagues (2006) found that children from low SES families were more likely to fall into the low achieving group than their middle-income peers. The current results echoed these findings, showing that children with MD-RD in kindergarten had significantly lower scores in SES than their peers in the MD and TA group. The effect of SES was found to vary across kindergarten achievement groups. Specifically, children with MD-RD benefited more from a higher SES background than did their peers in the TA group, in terms of learning math. Meanwhile, the positive influence of SES on fifth-grade math achievement was more apparent for children with MD than for children with MD-RD. The results indicate that the advantages of higher SES and the accompanying access to greater educational and remedial opportunities were particularly beneficial to children with difficulties in mathematics, as compared to their TA peers.

Learning-related Skills

Learning-related skills include the ability to pay attention, to persist with a task, to be motivated, to work independently, and to show flexibility and organize one's work.

The positive associations between learning-related skills and math achievement have been established in the literature (Bodovski & Farkas, 2007; Claessens, Duncan, & Engel, 2009; Duncan et al., 2007; Li-Grining, Vortuba-Drzal, Maldonado-Carreno, & Haas, 2010; McClellan et al., 2006). The present study confirmed previous findings that children who started kindergarten with good learning-related skills were likely to perform well in mathematics in kindergarten and fifth grade.

Although children's learning-related skills in kindergarten positively predicted their fifth-grade math achievement, the present study also found negative associations between learning-related skills and children's math growth rates during elementary school years, which contradicts the results from previous studies using the same ECLS-K dataset (DiPerna, Lei, & Reid, 2007; Li-Grining et al., 2010). Since both previous studies examined the effects of learning-related skills with models that controlled only for children's demographic characteristics and early achievement, the different finding may be due to the more complex model adopted in the current study. That is, other predictors in the model, such as kindergarten achievement group, may moderate the effect of learning-related skills on math growth.

In addition, children in both MD groups in kindergarten scored significantly lower in learning-related skills than their TA peers; children with MD-RD also performed worse than children with MD. This finding is worrisome, since poor learning-related skills have been associated with low achievement scores and behavioral or emotional problems (Bronson, Tivnan, & Seppanen, 1995). In this regard, training or activities for enhancing learning-related skills should be beneficial to children with difficulties in mathematics.

Initial Math Knowledge

Children's math performance at school entry represents the mathematics skills they have developed in the preschool or out-of-school context before they enter formal schooling, and therefore serves as a foundation for mastering the new skills in the school years. Previous research (e.g., Aunola et al., 2004; Duncan et al., 2007) found positive associations between math performance at school entry and later math achievement. Thus, math performance at kindergarten entry is considered an indicator for school readiness (Duncan et al., 2007). In the present study, a positive link was found between math achievement at kindergarten entry and fifth-grade math achievement, confirming the previous findings (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Bodovski & Farkas, 2007; Claessens et al., 2009; Duncan et al., 2007). However, initial math knowledge did not significantly predict children's progress in math over the elementary school years. This result may be due to the addition of the variable of the kindergarten achievement subgroup in the model, since there were high correlations between initial math knowledge and kindergarten achievement subgroups. Nonetheless, the current study found that initial math knowledge was associated with higher levels of math achievement at fifth grade, even after accounting for other child and class factors in the model.

As expected, children in both MD groups had significantly lower math scores than their TA peers at the point of kindergarten entry. Children with MD-RD also scored significantly lower on initial math knowledge than children with MD. This result is also consistent with previous findings that children with MD-RD had more severe and persistent difficulties in mathematics than children with MD. One interesting finding is that the influence of initial math knowledge on later math achievement varied across kindergarten achievement subgroups. In particular, the effect of higher math performance at kindergarten entry was greater for children with MD-RD than for children with MD. Given the positive associations between initial math knowledge and later math achievement, as well as the benefit of improving initial math knowledge for children with

MD-RD, more math instructional activities should be considered in the preschool curriculum.

Class Size

Previous findings about the effects of class size on math achievement have been mixed and inconclusive. The results from some experimental studies (Nye, Hedges, & Konstantopoulos, 1999, 2000, 2001a, 2001b, 2002) indicated that smaller classes (with fewer than 20 students) had positive effects on academic performance, which continued for students who stayed in small classes. Yan and Lin (2005), using data from the ECLS-K study and controlling for children's background characteristics and prior math and reading achievement, found a small positive relationship between small class size (fewer than 17 students) and children's math gains during their kindergarten year. Also using the ECLS-K dataset, Melesi and Gamoran (2006) found no significant effects of class size on end-of-kindergarten math achievement when children's background characteristics, prior math achievement, class-level SES, and instructional activities were included in the model. While also controlling for child and class factors, the present study found a positive association between kindergarten class size and fifth-grade math achievement, indicating that kindergarten children in larger classes were likely to have a higher math score at fifth grade. In addition, class size showed no significant difference for kindergarten achievement subgroups. These findings seem counterintuitive, yet can be understandable, since insufficient information was provided on why some classes were smaller than others, and why some children were placed in small classes instead of large ones. In this regard, class size may indeed have a lasting positive effect, but this effect may equally be the result of some unidentified factors, such as children's intelligence or school SES.

Instructional Time

The present study found a significant negative association between instructional time and children's fifth-grade math achievement, indicating that children who spent more time on teacher-directed activities in kindergarten were likely to score low in fifth-grade math achievement. No significant effect of instructional time was found on math learning over the elementary school years. The result contradicted previous findings that the amount of time spent on math instruction had a positive influence on children's math achievement (e.g., Bodovski & Farks, 2007; Pianta et al., 2008). Unlike the previous studies, the present study examined the effect of instructional time, while controlling for prior achievement and other class factors other than children's demographic characteristics, so the different results may be due to the additional variables in the model. In addition, since the instructional time variable was extracted from teacher-reported data, response bias could be involved, that is, teachers reported the amount of time they perceived to have spent on instruction, rather than the time they actually spent in class. In addition, the amount of instructional time was found to make no significant difference among kindergarten achievement subgroups. In other words, although instructional time was found to have a negative association with fifth-grade math achievement, the effect was equal for all students.

Math Instructional Activities

Through factor analysis, the present study identified four types of math instructional activities in the ECLS-K dataset: interactive activities, manipulatives, practice activities, and integrated activities. Surprisingly, the present study found that neither interactive activities nor manipulatives were significant predictors of either progress in learning math or fifth-grade math achievement; however, the time spent on practice and integrated activities was significantly associated with fifth-grade math achievement, and practice activities had a small positive effect on math growth.

In the present study, interactive activities included activities in small or mixed-achievement groups, peer tutoring, or explaining solutions to a partner. The efficacy of these interactive activities has been well researched, yet the results are mixed (Rohrbeck, Ginsburg-Block, Fantuzzo, & Miller, 2005). Rohrbeck and colleagues conducted a meta-analysis to review studies examining the effects of peer tutoring, cooperative learning, and small group learning on elementary school children. The effect sizes of interactive activities in the literature reviewed ranged from -0.61 to 1.38, with a mean effect size of d=0.33, evidence that these activities enhanced academic achievement for elementary school children. In general, interactive activities such as small group or peer tutoring were considered to be academically effective and economical instructional strategies. Rohrbeck and colleagues also found that the interactive activities were most effective with disadvantaged children, such as urban, low-income, and minority ethnic students. In the present study, no significant association was found between interactive activities and children's math growth and fifth-grade achievement, which was unexpected. Moreover, there was no significant association between this type of instructional activity and kindergarten achievement subgroups.

Meanwhile, the use of manipulatives proved to have no significant effect on children's learning in this study. The activities clustered in the category of manipulatives study included working with measuring instruments, counting and geometric manipulatives, and playing math-related games. For some decades, the use of manipulatives has been recommended by the National Council of Teachers of Mathematics (NCTM), as a way to help children develop conceptual understanding of mathematical ideas. As a result, the efficacy of manipulatives has been the subject of a considerable body of math education research. Domino (2010) performed a meta-analysis of relevant studies conducted during the previous two decades, and found a medium effect size (d=0.50) of using manipulatives on improving math achievement for elementary school children. Thus, the finding in the present study that the use of manipulatives was not predictive of math achievement or growth was also unexpected. The insignificant results from the use of both interactive activities and manipulatives could be due to the five-point Likert scale used in the teacher questionnaires in the ECLS-K, which may not have allowed sufficient variation of response. The current finding also raises questions regarding the lasting effect of these two types of instructional activities, as they were not predictive of later math achievement.

Also examined in the current study were practice activities, which involved using worksheets, textbooks, or the chalkboard for solving problems in math class. Among the limited studied on the efficacy of this type of instructional activity, Palardy and Rumberger (2008) found a significant positive association between the use of worksheets and math achievement in kindergarten. In the present study, however, practice activities were found to have a significantly negative effect on fifth-grade math achievement, but a positive association with the rate of progress in math. Practice activities are designed to provide students with more opportunities to practice so that they can master the concepts and skills taught in class. However, having children working independently on worksheets or from textbooks may suggest less time for instruction or interactive activities. Therefore, the negative association found in the present study between the time spent on practice activities in kindergarten and fifth-grade math achievement may indicate the need to balance teacher direct instruction and child-centered activities.

The last type of math instructional activities was integrated activities, creative activities or music designed to help students understand math concepts. Previous research has found that music training can bolster children's cognitive development. For instance, Bilhartz, Bruhn, and Olson (2000) found a positive association between music instruction and spatial-temporal reasoning abilities in young children. Teachers are often encouraged to use music and movement to teach mathematics (e.g., Johnson & Edelson, 2003). However, there are few empirical studies on the effects of teaching math through music, creative movement, or drama. Among the scarce literature, a study conducted in Turkey (Erdoğan & Baran, 2008) found that children who received math instruction via musical activities twice a week for three months scored significantly better on standardized math achievement tests than those who did not. In the present study, a significantly positive association was found between the time spent on integrated activities in kindergarten and fifth-grade math achievement. No significant association was found in relation to progress in math. In addition, the effects of integrated activities did not vary across kindergarten achievement groups. In the context of the limited evidence on the efficacy of using music, movement, or drama to teach mathematical concepts, the current finding was able to provide some support for the use of this type of instructional activity.

In sum, the present study found significant associations between children's fifth-grade math achievement and their demographic characteristics and individual skills. Surprisingly, the significant effects of class size and instructional time, though existed, were counterintuitive. The lack of interaction effects of class variables and kindergarten achievement subgroups was also unexpected. It should be noted that the kindergarten data used in the present study were collected over a decade ago, and what was observed in the data may not reflect current practice in the kindergarten classrooms. Therefore, the results should be interpreted with cautions.

Implications

The findings of the present study provide insights into how children with difficulties in mathematics progress academically over the elementary school years, as well as the extent to which identified kindergarten predictors are associated with math achievement and development for children with difficulties in mathematics. There are several implications for practice.

First, children's kindergarten achievement subgroup membership was a significant predictor of their math development and math achievement at fifth grade, even after controlling for children's background characteristics, individual skills, and class factors. A large percentage of children with MD-RD identified in kindergarten were found to experience continuing difficulties in both math and reading achievement. These findings suggest that children who are at risk of difficulties in mathematics can be identified as early as in kindergarten. Children with difficulties in both math and reading are particularly in need of interventions that can help improve their math and reading proficiency.

Second, the current study found children's math and reading achievements at school entry were positively associated with their fifth-grade math achievement, indicating that enhancing children's reading skills and math knowledge should be an important and necessary focus in preschool education. In addition, learning-related skills were found to be positively associated with later math achievement. Further research should explore which particular learning-related skills are critical and how these skills can enhance children's math learning.

Aside from children's individual skills in kindergarten, children's socio-economic status was also a significant predictor of their math achievement and development, after controlling for the influence of other child and class factors. Children from low-income families may receive less quantitative and qualitative support for learning mathematics in their home environment than their middle-income peers (Jordan et al., 2006), and hence are more likely to be at risk of math difficulties. To help children from low-SES families, quality childcare and preschool programs should be provided, to help level the playing field as they enter formal schooling.

Study Strengths and Limitations

The present study examined the growing level of achievement of children with difficulties in mathematics during elementary school, extending prior investigations that typically focused on children in primary grades (e.g, Aunola et al., 2004; Diperna et al., 2007; Fuchs & Fuchs, 2005; Jordan et al., 2006). Most of the previous studies relied on small, convenient samples of children attending particular schools. By using a large, nationally representative sample, the present study was able to yield more accurate and population-based estimates of the effects of identified kindergarten predictors.

Several limitations should also be acknowledged for the current study. First, using arbitrary cutoff scores to identify children with special needs potentially resulted in imperfect classification, especially for those children whose scores were borderline (Francis et al., 2005). As a result, the current study excluded children who scored between -1SD and -0.5 SD on either or both math and reading IRT scale scores in kindergarten. A different set of cutoff criteria might have yielded different findings (Murphy, Mazzocco, Hanich, & Early, 2007).

Moreover, this study was correlational rather than experimental, and causal conclusions should not be drawn from the findings presented. In particular, the danger of omitted variable bias may be present, since some variables not captured by the ECLS-K, such as a child's IQ and classroom processes, may confound the detected effects.

Certain classroom predictors in the study, such as instructional time and math instructional activities, were reported by teachers using a four- or five-point Likert scale. The indistinct score difference between variables may make analysis difficult, which could explain the lack of significant effects for the variables of instructional activities. In addition, the results obtained from teacher questionnaires were subject to bias and may not reflect reality. To obtain a more accurate understanding regarding the effects of class variables, more research should be conducted through empirical studies.

In addition, it should be noted that the kindergarten data in the ECLS-K study were collected in the school year of 1998-99. Although the data provided rich descriptions regarding children's kindergarten experience, the information could be outdated and may not reflect what is delivered in the kindergarten classrooms nowadays. Therefore, cautions should be taken when drawing inferences from the present results.

Lastly, the current study included child and classroom predictors, yet teacher- or school-level effects were not taken into account in the model. Since the ECLS-K adopts a complex sampling design, participants were not randomly selected. Failure to take into account the organizational effects may lead to biased results. In view of these limitations, the current findings should be interpreted with caution.

Conclusion

The current study used data from the ECLS-K, a large-scale, longitudinal dataset, to examine how children with MD and those with comorbid MD-RD progressed over the course of elementary school, as compared to children without math difficulties. The findings suggest that children with MD-RD demonstrated more persistent and severe difficulties in mathematics than children with MD, as well as those in the RD and TA groups. On the other hand, children with MD-RD had a similar rate of development in math as their peers with MD, and they also progressed faster in reading than children without reading difficulties. As for the predictive power of identified kindergarten child and class factors, children's sex, race, SES, learning-related skills, math and reading achievement at kindergarten entry, as well as class size, instructional time, practice activities, and integrated activities, were all significant predictors of children's fifth-grade math achievement. However, in addition to children's demographic characteristics and the skills they acquired in kindergarten, only one class factor – practice activities –

significantly predicted children's math development. Moreover, only the effects of SES and initial math knowledge varied as a function of kindergarten achievement subgroups, indicating that most of the significant effects found on math achievement and rate of development were equal for children in all the kindergarten achievement subgroups. Despite its limitations, the present study did extend prior research by examining the rising levels of achievement of children with difficulties in mathematics between kindergarten and fifth grade. The findings have implications for early identification and intervention for children with difficulties in mathematics.

APPENDIX A

Technical Information for the ECLS-K study

Presented in this appendix was the technical information regarding research design and sampling procedures, as well as the instruments from where the variables for the present study are derived. More detailed information regarding the ECLS-K study in general can also refer to be found in the user's manual for the ECLS-K fifth-grade data files and electronic codebooks (Tourangeau, Nord, Lê, Pollack, & Atkins-Burnett, 2006)

Research Design and Sampling Procedure

The study design of the ECLS-K dataset is guided by a conceptual framework emphasizing the interrelationships among the child, the family, the school, and the community. The data were collected from four sources: the child, the child's parents/guardians, the child's teachers, and the schools that the child attended.

To establish the participants for the ECLS-K dataset, the NCES employed a multi-stage sampling design to select a nationally representative cohort of children attending kindergarten during the 1998-99 school year. In the base year, geographical areas consisting of counties or groups of counties were identified as the primary sampling units (PSUs). The second-stage sampling units were the public and private schools offering kindergarten programs within each PSU. The third- and final-stage sampling units were students within the schools selected at the earlier stages.

Data Collection

The data collection was conducted first in the fall and spring of the children's kindergarten year. While the spring first-grade sample targeted all base-year respondents (i.e., respondents enrolled in fall or spring kindergarten), the fall first-grade data collection included only a subsample of base-year PSUs. Additionally, the spring first-grade sample was freshened to include first-grade students who had not been enrolled in kindergarten during the base year, and therefore had not been included in the base-year kindergarten sample.

Children were assessed through various activities that measured their cognitive knowledge and skills (e.g., general knowledge, literacy, and mathematics skills) as well as non-cognitive skills (e.g., fine motor and gross motor coordination, and socio-emotional skills). All measures of the children's cognitive skills were conducted through untimed individually-administered assessments.

During the last two waves of data collection (i.e., the spring of their fifth- and eighth-grade years), children were asked to report on their experiences in and out of school. Data were also collected from parents each time their child was assessed, using computer-assisted telephone interviewing (CATI).

Data collected from teachers included the teacher's academic background, teaching practices, experience, and the classroom settings for the children who were sampled, as well as their evaluation of sampled children on a number of cognitive and non-cognitive skills. Teachers received self-administered questionnaires at each wave of data collection, except for the fall of first grade. Finally, school administers were asked to complete self-administered questionnaires in the fall of the students' kindergarten and first grade years. The administrators described the physical, organizational, and fiscal characteristics of their schools, as well as the learning environment and programs there.

Instrumentation

Direct cognitive assessments of students. The direct cognitive assessments were administered individually at all time points. In kindergarten and first grade, the direct cognitive assessment battery consisted of questions in three content areas: reading, mathematics, and general knowledge. The third and fifth grade cognitive assessment also included reading and mathematics sections, but the test for general knowledge was replaced by a measure of science, which focused on two sub-domains: social studies and science.

Prior to receiving direct cognitive assessments, children who had a non-English language background were given a brief language screening with the Oral Language Development Scale (OLDs). The OLDs measured children's listening comprehension, vocabulary, and ability to understand and produce language, and it was used as a language screening to determine whether participants had the adequate language ability to take the direct cognitive assessments in English. Children who passed the OLDs received the full direct assessment battery in English. Children who did not pass the OLDs, but whose native language was Spanish, received a Spanish translated form of the mathematics assessment, and an alternate Spanish version of the OLDs. The Spanish OLDs is similar in content to the English OLDs and measures the same constructs. If a child did not pass the OLDs, and his native language was not Spanish, then the child was excluded from the assessment.

To maximize the accuracy of measurement and reduce administration time, a two-stage cognitive assessment design was adopted for both reading and mathematics. The participants were given the same first-stage routing test, which consisted of 12 to 20 items with a broad range of difficulty. Based on their performance on the routing tests, a second-stage form was determined. The reading and mathematics second-stage tests had low, middle, and high difficulty forms, whereas the general knowledge assessment had two second-stage alternatives. By using the two-stage assessment approach, children received items that were appropriate for their ability levels to avoid boredom and frustration. Furthermore, this approach enabled the researchers to collect more information on children's cognitive performance within a limited time. There were several types of scores derived from direct cognitive assessment. For the purpose of current study, the IRT scale scores are used for analysis. More detailed information regarding IRT scores are presented in the variable section. In the following section, descriptions of the direct cognitive assessment in reading and mathematics are provided.

Reading assessment. The reading assessment was designed to measure basic skills (print familiarity, letter recognition, beginning and ending sounds, rhyming sounds, and word recognition), vocabulary (receptive and in-context vocabulary), and comprehension (listening comprehension, initial understanding, developing interpretation, personal reflection, and demonstrating critical stance). For kindergarten and first grade children, the focus of the reading assessment was on basic reading skills. For older children who were in the third and fifth grades, the emphasis of the assessment was moved from basic skills to reading comprehension. However, this does not mean that children in fifth grade were not tested on basic skills, or that younger children in kindergarten and first grade did not receive items on reading comprehension. With the adaptive nature of two-stage test design, children who did not perform well on their grade-specific routing tests were given a second-stage test with a lower level of difficulty.

Mathematics assessment. The mathematics assessment was designed to measure skills in conceptual knowledge, procedural knowledge, and problem solving. The following content strands were addressed in the assessment: number sense, properties, and operations; measurement; geometry and spatial sense; data analysis, statistics, and probability; and pattern, algebra, and functions. Approximately half of the questions of the assessment were on number sense and number properties and operations. The remaining questions were on the rest of the content strands. Manipulatives were available for children to use when solving some of the questions. Papers and pencils were also provided to the children for the appropriate parts of the assessment. The internal consistency coefficients for mathematics IRT scores in fall of kindergarten, spring of kindergarten, spring of first grade, spring of third grade, and spring of fifth grade were 0.89, 0.91, 0.92, 0.94, and 0.94 respectively.

Indirect assessment of students. The ARS was separated into three sections: reading, mathematics, and general knowledge (science and social studies for children in third grade and beyond). Teachers of the sample children were asked to rate each student’s skills, knowledge, and behaviors within each content domain on a five-point scale, with one representing “has not yet demonstrated skills,” and five representing “proficient in demonstrating skills and knowledge.” While the purpose of the direct cognitive assessment was to measure the products of children’s achievement, the ARS was designed to assess both the process and products of the children’s learning in school. In this regard, the ARS obtained information that could not be collected from the direct assessment due to practical constraints, such as children’s writing skills or the strategies that children used to solve problems. Furthermore, unlike the direct cognitive assessments, which were designed to measure children’s performance over time, the ARS was targeted to a specific grade level. Teachers were instructed to rate each child as compared to other children in the class. The reliability coefficients for ARS reading, mathematics, and general knowledge subscales in spring of kindergarten were 0.91, 0.93 and 0.94 respectively.

The SRS was designed to measure the frequency of the child demonstrating certain social skills or behaviors. The SRS was rated by both the teacher and parents on a four-point scale ranging from one, “Never,” to four, “Very Often.” While the constructs of the parent SRS were similar to the teacher SRS, the items in the parents’ SRS focused on what occurred outside of the school environment, and thus were different from those in the teacher SRS.

The teacher SRS was comprised of five subscales: Approaches to Learning, Self-Control, Interpersonal Skills, Externalizing Problem Behavior, and Internalizing Problem Behavior. The Approaches to Learning scale measured behaviors that affect children’s learning. The scale included six items that rated the child’s attentiveness, task persistence, eagerness to learn, learning independence, flexibility, and organization. The Self-Control scale, a measure of the child’s ability to control his behavior, included four items: respecting the property rights of others, controlling temper, accepting peer ideas for group activities, and responding appropriately to pressure from peers. The Interpersonal Skills scale had five items that rated the child’s skills in forming and maintaining friendships, getting along with people who are different, comforting or helping other children, expressing feelings, ideas, and opinions in positive ways, and showing sensitivity to the feelings of others. Children’s problem behaviors were measured by the scales of Externalizing and Internalizing Problem Behavior. On externalizing problem behaviors, five items were included to rate the frequency with which a child argues, fights, gets angry, acts impulsively, and disturbs ongoing activities. Another four items were adopted to measure the child's internalizing problem behaviors, including the apparent presence of anxiety, loneliness, low self-esteem, and sadness. The parent SRS had five corresponding scales with different names. In the current study, the Approaches to Learning scale are used as a measure for children's learning-related skills. The reliability coefficient for the Approaches to learning scale in spring of kindergarten was 0.89.

***Teacher questionnaire*. A** self-administered questionnaire was given to all kindergarten teachers, regardless of whether they taught a sample child. The teacher questionnaire was comprised of three distinct parts. The first section, Part A, asked about the characteristics of the kindergarten class and the children in the class. Part B included items regarding class organization, class activities, teachers' views on kindergarten readiness, teachers' views on the school's climate and environment, and general information about the teacher. Part C asked teachers to provide information on the academic and physical standing of the sampled child in the class, as well as to complete the ARS and the SRS for the sample child. For the present study, the variables of children's skills in kindergarten (i.e., initial mathematics knowledge and learning-related skills) and related class factors (i.e., class size, instructional time and instructional activities in reading and mathematics) were extracted from the fall and spring kindergarten teacher questionnaires.

Parent interviews. Parents or guardians of the sample children were asked to provide information regarding the child, the child's home environment, parent behavior (e.g., interactions with the child's teacher, activities with the sampled child), and family characteristics. Also included in the parent interview were questions about the family structure, child care use, household income, and child rearing practices. The interviews were conducted using a CATI program or computer-assisted personal interview for families without a telephone. Though the parent interviews were conducted primarily in English, parents who spoke other languages were provided with other alternatives. For instance, the questionnaire was translated into Spanish, and bilingual interviewers were trained to conduct the parent interview in either English or Spanish. The questionnaire for parent interviews also was translated into Chinese, Lakota, and Hmong languages, and administered using the same data collection procedures as were used with Spanish-speaking parents. The information from parent's interview was used by the ECLS-K study to create composites of demographic variables, such as gender and race.

School records abstract. Completed by school staff members, the school records abstract form was used to gather information about the sampled child's attendance record, report card, and IEP status. Also included in the form was the type of language or English proficiency screening that the school used, and whether the sampled child participated in Head Star prior to kindergarten.

APPENDIX B

Factorability of Kindergarten Math Instructional Activities

In the Spring Kindergarten Teacher Questionnaire A, teachers were asked to report the frequency of 17 mathematics instructional activities they have conducted in class (i.e., Q31). Ratings were given on a six-point Likert scale: 1 = never, 2 = once a month or less, 3 = two or three times a month, 4 = once or twice a week, 5 = three or four times a week, and 6 = daily.

Before conducting factor analysis, Pett, Lackey, and Sullivan (2003) suggest to access the characteristics of correlation matrix to ensure the available data is factorable. The first step is to investigate correlations between instructional activities to ensure that these activities were not correlated too highly (i.e., $\gamma \geq .80$) or insufficiently correlated ($\gamma < .30$) with one another. If the items are correlated too highly, it would be hard to determine the unique contribution of the activities to a factor; whereas, if the items are not correlated strongly enough, there are not much shared common variance, whereby there could be as many resulting factors as there are items (Pett et al., 2003). Correlations between math instructional activities ranged between .001 and .581 (see Table B1). Most of the correlations coefficients were in range between .05 and .40. These small but significant correlation coefficients showed weak correlations and may be an issue (Pett et al., 2003).

In addition to the correlations among instructional activities, several other measures are also suggested for evaluating the data. The determinant of a matrix—calculated by multiplying across the downward and upward diagonals of the correlation matrix—indicates whether the correlations between items are too strong or too weak for doing factor analysis. Ideally, the absolute value of determinant should be in the range between 0 and 1. Barlett's test of sphericity (Bartlett, 1950, cited in Pett et al., 2003) tests the null hypothesis of no relationship among the items. Serving as an indicator of sampling adequacy for overall scale, the Kaiser-Meyer-Olkin Test (KMO) can range between 0 and 1. According to Kaiser (1974, p. 35, cited in Pett et al., 2003), the size of KMO less than .60 is "unacceptable," in the .70 is "middling," in the .80s is "meritorious," and above .90 is "marvelous." Finally, a measure of sampling adequacy (MSA) can be computed for each individual item and also evaluated based on Kaiser's criteria.

According to the criteria of aforementioned measures, the math instructional activities are considered factorable, with the determinant of .032. The Barlett's tests of sphericity for mathematics is significant, thus the null hypothesis of no relationship is rejected. The KMO is .827 for mathematics, which indicates that the sample size is sufficient relative to the number of items in the scales. The MSAs for math instructional activities ranged from .625 to .918. Pett et al. (2003) suggested using .60 as a cutoff for removing low MSA item. As a result, all 17 math instructional activities in the ECLS-K study were deemed to be factorable and retained for factor analysis.

Item N0.	Q31A	Q31B	Q31C	Q31D	Q31E	Q31F	Q31G	Q31H	Q31I	Q31J	Q31K	Q31L	Q31M	Q31N	Q31O	Q31P	Q31
Q31A	1																
Q31B	.169**	1															
Q31C	.237**	.488**	1														
Q31D	.181**	.438**	.484**	1													
Q31E	.018*	.134**	.107**	.138**	1												
Q31F	.169**	.211**	.246**	.251**	.067**	1											
Q31G	.170**	.241**	.230**	.289**	.112**	.581**	1										
Q31H	.134**	.316**	.313**	.305**	.204**	.222**	.293**	1									
Q31I	.181**	.175**	.307**	.233**	.121**	.165**	.244**	.299**	1								
Q31J	.161**	.050**	.092**	.100**	-0.001	.083**	.095**	.065**	.104**	1							
Q31K	0.009	-.054**	.054**	-.065**	0.003	0.001	.019*	-.045**	.101**	-.073**	1						
Q31L	-.040**	-.072**	.022**	0.001	.103**	-0.011	0.003	0.01	.095**	-.117**	.357**	1					
Q31M	.057**	.039**	.142**	.094**	.117**	.108**	.119**	.095**	.283**	-.083**	.323**	.316**	1				
Q31N	.146**	.231**	.332**	.349**	.152**	.189**	.261**	.289**	.385**	.086**	.132**	.130**	.314**	1			
Q31O	.155**	.169**	.280**	.313**	.152**	.205**	.278**	.309**	.509**	.084**	0.004	.064**	.231**	.472**	1		
Q31P	.163**	.182**	.229**	.261**	.116**	.129**	.157**	.213**	.281**	.122**	-.024**	-.036**	.064**	.401**	.336**	1	
Q31Q	.114**	.167**	.261**	.281**	.107**	.195**	.214**	.192**	.272**	.045**	.076**	.079**	.189**	.454**	.338**	.390**	

APPENDIX C

Multiple Imputation Considerations

To determine an adequate number of imputations for the current study, the results of two main effect models from datasets with 5 and with 20 imputations were compared (Table C1 and C2, respectively). To determine which dataset to use for further analyses in the study, this researcher compared the estimates of parameters, inferences, and relative efficiency, which indicates the adequacy of the number of imputations compared to an infinite set. Although the dataset with 20 imputations produced results with slightly better relative efficiency, the inferences obtained from the two sets of data were very similar. Given the similar inferences, and substantial time required for running analyses using the dataset with 20 imputations, the dataset with 5 imputations was used for further analyses in the present study.

Table C1

Results from Main Effect Model in Math Achievement for Dataset with 5 Imputations.

Parameter	Estimate		S.E.	Fraction Missing Info.	Relative Increase Variance	Relative Efficiency
Fixed Effects						
Intercept	98.030	***	0.770	0.110	0.117	0.979
Female	-4.605	***	0.334	0.044	0.046	0.991
Race – Black	-7.703	***	0.606	0.003	0.003	0.999
Hispanic	0.237		0.498	0.017	0.017	0.997
Asian	2.493	***	0.668	0.050	0.051	0.990
Other Races	-1.649	*	0.713	0.038	0.039	0.992
SES	3.082	***	0.258	0.248	0.297	0.953
Ach. Group - TA	22.437	***	0.764	0.114	0.122	0.978
MD	5.774	***	1.542	0.070	0.073	0.986
Learning-related Skills (LRS)	3.812	***	0.287	0.040	0.041	0.992
Initial Reading Knowledge	0.683	***	0.027	0.111	0.118	0.978
Initial Math Knowledge	0.052	*	0.021	0.060	0.062	0.988
Class Size	0.116	**	0.033	0.133	0.144	0.974
Instructional Time	-0.229	*	0.094	0.025	0.025	0.995
Math Instructional Activities						
Interactive Activities	0.002		0.164	0.127	0.137	0.975
Manipulatives	-0.168		0.254	0.194	0.221	0.963
Practice Activities	-0.479	**	0.142	0.195	0.222	0.962
Integrated Activities	0.339	*	0.144	0.207	0.239	0.960
Linear slope	8.541	***	0.564	0.044	0.045	0.991
Female	0.841	**	0.250	0.021	0.021	0.996
Race – Black	0.000		0.462	0.020	0.020	0.996
Hispanic	0.817	*	0.375	0.010	0.010	0.998
Asian	2.480	***	0.507	0.018	0.019	0.996
Other Races	1.644	**	0.536	0.008	0.008	0.998
SES	0.346	*	0.174	0.022	0.023	0.996
Ach. Group - TA	2.947	***	0.554	0.030	0.030	0.994
MD	0.325		1.134	0.016	0.016	0.997
Learning-related Skills (LRS)	-0.624	**	0.225	0.103	0.109	0.980
Initial Reading Knowledge	-0.212	***	0.020	0.065	0.067	0.987
Initial Math Knowledge	-0.018		0.016	0.079	0.083	0.984
Class Size	0.032		0.024	0.049	0.051	0.990
Instructional Time	-0.064		0.078	0.201	0.230	0.961

Table C1 (continued)

Parameter	Estimate		S.E.	Fraction Missing Info.	Relative Increase Variance	Relative Efficiency
Math Instructional Activities						
Interactive Activities	-0.087		0.127	0.162	0.180	0.969
Manipulatives	-0.103		0.176	0.022	0.022	0.996
Practice Activities	0.148		0.108	0.195	0.222	0.962
Integrated Activities	-0.089		0.107	0.159	0.176	0.969
Quadratic slope	-0.855	***	0.109	0.068	0.071	0.986
Female	0.297	***	0.048	0.018	0.018	0.996
Race – Black	0.251	**	0.088	0.018	0.018	0.997
Hispanic	0.150	*	0.072	0.008	0.008	0.998
Asian	0.385	***	0.097	0.013	0.013	0.998
Other Races	0.400	***	0.103	0.009	0.009	0.998
SES	-0.043		0.033	0.008	0.008	0.998
Ach. Group - TA	0.079		0.108	0.057	0.058	0.989
MD	-0.147		0.220	0.036	0.036	0.993
Learning-related Skills (LRS)	-0.223	***	0.043	0.114	0.122	0.978
Initial Reading Knowledge	-0.040	***	0.004	0.065	0.068	0.987
Initial Math Knowledge	-0.002		0.003	0.076	0.079	0.985
Class Size	0.002		0.005	0.038	0.039	0.992
Instructional Time	-0.001		0.015	0.167	0.187	0.968
Math Instructional Activities						
Interactive Activities	-0.010		0.025	0.228	0.268	0.956
Manipulatives	-0.029		0.034	0.027	0.028	0.995
Practice Activities	0.061	**	0.022	0.281	0.346	0.947
Integrated Activities	-0.032		0.020	0.127	0.138	0.975

***p<.001; **p<.01; *p<.05

Table C2

Results from Main Effect Model in Math Achievement for Dataset with 20 Imputations.

Parameter	Estimate	S.E.	Fraction Missing Info.	Relative Increase Variance	Relative Efficiency
Fixed Effects					
Intercept	99.047 ***	0.791	0.080	0.086	0.996
Female	-4.605 ***	0.368	0.033	0.034	0.998
Race – Black	-7.703 ***	0.606	0.002	0.002	1.000
Hispanic	0.245	0.497	0.012	0.012	0.999
Asian	2.512 ***	0.671	0.037	0.038	0.998
Other Races	-1.742 *	0.713	0.028	0.028	0.999
SES	3.083 ***	0.262	0.182	0.219	0.991
Ach. Group - TA	22.527 ***	0.812	0.083	0.090	0.996
MD	5.774 ***	1.533	0.051	0.053	0.997
Learning-related Skills (LRS)	3.822 ***	0.297	0.029	0.030	0.999
Initial Reading Knowledge	0.683 ***	0.029	0.081	0.087	0.996
Initial Math Knowledge	0.062 *	0.022	0.044	0.045	0.998
Class Size	0.134 ***	0.037	0.097	0.106	0.995
Instructional Time	-0.232 *	0.095	0.018	0.019	0.999
Math Instructional Activities					
Interactive Activities	0.002	0.161	0.093	0.101	0.995
Manipulatives	-0.171	0.248	0.142	0.163	0.993
Practice Activities	-0.488 **	0.139	0.143	0.164	0.993
Integrated Activities	0.342 *	0.141	0.152	0.176	0.992
Linear slope	8.557 ***	0.562	0.033	0.033	0.998
Female	0.843 **	0.258	0.015	0.016	0.999
Race – Black	0.002	0.462	0.015	0.015	0.999
Hispanic	0.823 *	0.375	0.007	0.008	1.000
Asian	2.511 ***	0.509	0.014	0.014	0.999
Other Races	1.674 **	0.541	0.006	0.006	1.000
SES	0.346 *	0.174	0.016	0.017	0.999
Ach. Group - TA	2.983 ***	0.567	0.022	0.022	0.999
MD	0.327	1.147	0.012	0.012	0.999
Learning-related Skills (LRS)	-0.636 **	0.228	0.075	0.081	0.996
Initial Reading Knowledge	-0.213 ***	0.020	0.048	0.050	0.998
Initial Math Knowledge	-0.018	0.016	0.058	0.061	0.997
Class Size	0.033	0.027	0.036	0.037	0.998
Instructional Time	-0.069	0.078	0.147	0.170	0.993

Table C2 (continued)

Parameter	Estimate		S.E.	Fraction Missing Info.	Relative Increase Variance	Relative Efficiency
Math Instructional Activities						
Interactive Activities	-0.087		0.124	0.119	0.133	0.994
Manipulatives	-0.104		0.176	0.016	0.016	0.999
Practice Activities	0.153		0.106	0.143	0.164	0.993
Integrated Activities	-0.091		0.105	0.116	0.129	0.994
Quadratic slope	-0.864	***	0.109	0.050	0.052	0.998
Female	0.303	***	0.048	0.013	0.013	0.999
Race – Black	0.252	**	0.090	0.013	0.013	0.999
Hispanic	0.150	*	0.072	0.006	0.006	1.000
Asian	0.387	***	0.098	0.009	0.009	1.000
Other Races	0.413	***	0.107	0.006	0.006	1.000
SES	-0.043		0.037	0.006	0.006	1.000
Ach. Group - TA	0.081		0.109	0.041	0.043	0.998
MD	-0.149		0.221	0.026	0.027	0.999
Learning-related Skills (LRS)	-0.227	***	0.045	0.083	0.090	0.996
Initial Reading Knowledge	-0.041	***	0.005	0.048	0.050	0.998
Initial Math Knowledge	-0.002		0.003	0.056	0.059	0.997
Class Size	0.002		0.006	0.028	0.029	0.999
Instructional Time	-0.002		0.016	0.122	0.137	0.994
Math Instructional Activities						
Interactive Activities	-0.011		0.024	0.167	0.197	0.992
Manipulatives	-0.030		0.036	0.020	0.021	0.999
Practice Activities	0.067	**	0.027	0.207	0.255	0.990
Integrated Activities	-0.033		0.021	0.093	0.101	0.995

***p<.001; **p<.01; *p<.05

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