ABSTRACT

Title of dissertation:	MOBILE AD HOC NETWORKS- ITS CONNECTIVITY AND ROUTING OVERHEAD
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This dissertation focuses on a study of network connectivity and routing overhead in mobile ad-hoc networks (MANETs). The first part examines the smallest communication range needed for bi-directional connectivity of a network, called the critical transmission range (CTR), under a class of group mobility models. In the second part, we study the smallest communication range of the nodes necessary for no node isolation when trust constraints are introduced for one-hop connectivity between nodes. In the third part, under the assumption that nodes employ the CTR for network connectivity in MANETs, we study the overhead required for location service under geographic routing.

We begin with an investigation of the communication range of the nodes necessary for network connectivity, which we call bi-directional connectivity, in one dimensional case. Unlike in most of existing studies, however, the locations or mobilities of the nodes are correlated through group mobility: Nodes are broken into groups, with each group comprising the same number of nodes, and lie on a unit circle. The locations of the nodes in the same group are *not* mutually independent, but are instead *conditionally* independent

given the location of the group.

We examine the distribution of the CTR when both the number of groups and the number of nodes in a group are large. We first demonstrate that the CTR exhibits a parametric sensitivity with respect to the space each group occupies on the unit circle. Then, we offer an explanation for the observed sensitivity by identifying what is known as a *very strong threshold* and asymptotic bounds for CTR.

Related to the first part, we explore the communication range of the nodes necessary for no node isolation where the locations of nodes are mutually independent and uniformly distributed on a torus. However, unlike in our first study where the one-hop connectivity between two nodes depends only on their distance, one-hop connectivity of two nodes in this model is determined by both geometric and trust constraints. More specifically, in order to have a communication link between two nodes, they should be within a certain common communication range and satisfy trust requirements, i.e., the trust level of a node exceeds the required trust threshold of the other. Under this one-hop connectivity model, we find the smallest communication range needed so that no node will be isolated. While our analytical study focuses on the probability that no node will be isolated, our simulation results suggest that the probability of no node isolation and the probability of network connectivity behave very similarly.

In the third part of this dissertation, we study routing overhead due to location information collection and retrieval in MANETs employing geographic routing with no hierarchy. We first provide a new framework for quantifying overhead due to control messages generated to exchange location information. Second, we compute the minimum number of bits required on average to describe the locations of a node, borrowing tools from information theory. This result is then used to demonstrate that the expected overhead is $\Omega(n^{1.5}\ \log(n))$, where n is the number of nodes, under both proactive and reactive geographic routing, with the assumptions that (i) nodes' mobility is independent and (ii) nodes adjust their transmission range to maintain network connectivity. Finally, we prove that the minimum expected overhead under the same assumptions is $\Theta(n\ \log(n))$.

MOBILE AD HOC NETWORKS-
ITS CONNECTIVITY AND ROUTING OVERHEAD

by

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1. INTRODUCTION

1.1 Mobile Ad Hoc Networks

A mobile ad-hoc network (MANET) is a collection of mobile nodes that construct and maintain a network without a centralized authority. Unlike in a more traditional wired network (e.g., the Internet), there are no dedicated routers or switches responsible for forwarding packets. MANETs or multi-hop wireless networks (MHWNs) have attracted much interest from the networking community, due to their potential for numerous applications. In a traditional wired network, traffic generated by so-called end nodes is routed through the network by dedicated routers. However, in a MANET wireless nodes form and maintain the network and share the responsibility of routing packets from sources to destinations. Moreover, when (some of) nodes are mobile, the one-hop connectivity, hence topology, of the network varies with time. This requires the network protocols to cope with potentially frequent changes in network topology.

In this dissertation, we focus on two main issues in MANETs - network connectivity and routing overhead. In MHWNs, when a source wants to transmit data to a destination, there must exist at least one end-to-end route between the source and the destination. In order for a network to be able to provide such end-to-end routes between information sources and destinations, it should be connected. In addition, finding and maintaining end-to-end routes between nodes incurs overhead. In the second part, we examine how the overhead required for location service scales when geographic routing is employed.

1.2 Network Connectivity in MANETs

When information to be transferred by a MHWN cannot tolerate large delays, timely delivery of information demands that the network be able to find an end-to-end route between a source and a destination. In order for such an end-to-end route to exist when one is needed, the network should be connected (with a high probability). For this reason the issue of network connectivity enjoyed much attention in recent years.

In some cases, nodes may have access to a replenished energy source and interference between simultaneous transmissions may not be a concern (e.g., light traffic scenarios). In such cases, network connectivity can be dealt with by employing the largest transmit power at the nodes. In other cases, however, especially when some of the mobile nodes operate on batteries, this may not be an acceptable solution; it is likely to result in unnecessarily quick depletion of battery power. In these scenarios, it is in the interest of the battery powered nodes to use the minimum necessary transmit power, which will result in a smaller communication range between nodes, so as to conserve energy.

Another, perhaps, less obvious reason why nodes may want to employ smaller communication ranges through transmit power control stems from the study of network transport throughput: Gupta and Kumar showed in their seminal paper [9] that, in order for the nodes to maximize the network throughput, they should adopt the smallest communication range to maintain network connectivity. The basic intuition is that employing the smallest communication range allows for the maximum *spatial reuse* of the spectrum by minimizing the interference caused to nearby nodes.

A natural question that arises under these arguments is: "What is the smallest communication range needed for network connectivity?" In order to study the connectivity properties of MHWNs, researchers often represent the one-hop connectivity of the network as a random graph and investigate the connectivity of the graph. Study of connectivity property of random graphs dates back to late 1950's, starting with the pioneering work by Erdös and Rényi [5, 6].

More recently, another line of research more related to the connectivity of MHWNs examined various properties of *geometric* random graphs, including their connectivity (e.g., [1, 8, 11, 12, 15, 18, 20, 22]). We refer interested readers to a monograph by Penrose [17]. In a geometric random graph, one-hop connectivity between a pair of nodes is determined by the distance between them. In other words, there exists an edge between two nodes i and j if and only if their distance is smaller than some threshold γ. This threshold γ can be interpreted as a proxy to a common communication or transmission range of the nodes, which depends on the employed transmit power, in the context of MHWNs [49].

The one-hop connectivity model in geometric random graphs has been generalized in different ways, in order to capture, for instance, environmental factors in one-hop connectivity between a pair of nodes given transmit power (e.g., [44, 50]). In addition, Diaz et al. [4] studied the dynamic case where nodes move according to a mobility model similar to the Random Direction models [2, 16]. They computed the expected duration of a period during which a network remains connected or disconnected under the one-hop connectivity model of random geometric graphs.

1.2.1 Network connectivity in MANETs with group mobility model

Most of existing studies on connectivity of geometric random graph models focus on the scenarios where the locations of the nodes are independent of each other with identical spatial distribution (e.g., [1, 8, 11, 18, 20, 22]). The dynamic case studied in [4] also assumes independent and homogeneous node mobility. Unfortunately, when either of these assumptions is relaxed, little is known about the connectivity property of random graphs. In this dissertation, we take another step towards better understanding connectivity when nodes' mobility is correlated.

In chapter 2, we investigate how the smallest communication range needed for network connectivity, which we call a critical transmission range (CTR), behaves in simple one-dimensional cases, where nodes lie on a unit ring and nodes are clustered into groups with the same number of members. We examine the distribution of CTR as both the number of groups and the number of members in each group become large.

1.2.2 Node isolation in MANETs with trust constraints

In most of the studies on network connectivity, employing geometric random graph models, the one-hop connectivity between two nodes is determined solely by the distance between the two nodes (e.g., [1, 8, 11, 18, 20, 22]). In chapter 3, we introduce the concept of trustworthiness in one-hop connectivity.

In MANETs, packets are routed by individual nodes without the help of centralized infrastructure. Therefore, when a MANET is used to transfer sensitive information, only trusted nodes should be employed for routing the information. For this reason, researchers studied the issue of defining the *trustworthiness* of the nodes (e.g., [27]) and proposed new routing schemes that take into account the trustworthiness of the nodes for routing packets.

Similarly to [29], we model the trust relation between two nodes using two parameters, *trust level* and *trust threshold*; for any given node, its trust level reflects its trustworthiness and its trust threshold indicates the minimum trust level required of any other node before it trusts the node. In order for two nodes to establish a link between them, the trust level of each node must be larger than or equal to the trust threshold of the other node. We investigate how this new *trust* constraint for one-hop connectivity between nodes affects the smallest communication range necessary for network connectivity and no node isolation.

1.3 Expected Routing Overhead in MANETs Under Flat Geographic Routing

In MANETs, since nodes are assumed mobile, one-hop connectivity between nodes and the network topology can change over time. Consequently, underlying routing protocols are asked to cope with potentially frequent changes in network topology. Maintaining up-to-date information for routing packets requires exchange of control messages, incurring overhead.

Recently there has been much research on understanding the network transport throughput, or simply transport throughput, of multi-hop wireless networks: In their seminal paper [9] Gupta and Kumar investigated the transport throughput of static multi-hop wireless networks and showed that the transport throughput increases, at best, as $\sqrt{n}$ with an increasing number of nodes n, i.e., $O(\sqrt{n})$. This finding implies that per-node throughput decreases to zero as $n \to \infty$. Grossglauer and Tse [39] exploited the mobility of nodes and demonstrated that, if unbounded delays can be tolerated, under some technical conditions per-node throughput of $\Theta(1)$ can be achieved. To bridge the gap in the transport throughput between static networks and mobile networks, Sharma et al. [51] examined the trade-off between the transport throughput and delays that must be tolerated in order to achieve certain level of transport throughput. Other related works can be found in [36, 37, 43, 45].

In most of these studies, however, authors do not explicitly address the issue of routing overhead. To be more precise, they do not explain how necessary routing information is obtained and how much network resource (e.g., transport throughput) is required to obtain needed routing information in order to achieve claimed transport throughput. Therefore, in order to better understand the scalability of MANETs with an increasing number of nodes and to find out how to dimension them properly (e.g., bandwidth), one should examine how routing overhead scales in MANETs, in particular, in comparison to network transport throughput. A good understanding of routing overhead may also allow us to correctly identify critical bottlenecks and to deal with them more effectively.

To the best of our knowledge, the first serious attempt at an analytical study of protocol overhead was carried out by Gallager in [38]. There are also several recent analytical studies on routing overhead in MANETs, some of which we summarize here: Zhou and Abouzeid [56, 57] applied the tools from information theory to examine the overhead due to the changes in network topology under two-tier hierarchical routing. Their key idea is to model the time-varying network topology as a stochastic process and to evaluate the overhead required to describe the local network topology in subregions to cluster heads and to distribute the global ownership information to all cluster heads. Then, they studied the scaling laws of the memory requirement and routing overhead under three different physical scalings of the network.

In another study [33] Bisnik and Abouzeid formulated the problem of characterizing the minimum routing overhead as a rate-distortion problem. They considered geographic routing with location servers that have *known* locations and store location information of other mobile nodes, and investigated the information rate required to satisfy a prescribed squared-error distortion constraint. Viennot et al. [55] examined control overhead under both proactive and reactive routing, and suggested that control overhead is proportional to the square of the number of nodes in the network.

In this dissertation we take another step towards understanding routing overhead in MANETs: We assume that nodes employ flat geographic or position-based routing *without* designated location servers that maintain the location information of mobile nodes. Also, we focus on the scenario of practical interest where the network is connected with a high probability. To be more precise, we assume that the transmission range of the nodes is selected so that the network is connected with probability approaching one as the number of nodes grows. This issue of network connectivity has been studied in the first part of the dissertation.

The goal of our study is twofold: First, we aim to provide a new framework for studying routing overhead, especially for geographic routing, which can capture the differences that arise from the specific schemes employed to disseminate and acquire location information. To this end we develop a new framework, borrowing tools from information theory to compute the minimum average number of bits required to describe *approximated* locations of mobile nodes. Secondly, based on the proposed framework, we explore how routing overhead scales with the network size under different routing schemes. In particular, we focus on the routing overhead *only* due to *dissemination* and *acquisition* of location information, i.e,. location service.

1.4 Notation

In this section we describe the notation we will use throughout the dissertation.

N1. A function $a(n)$ is $O(b(n))$ if there exist $0 < c_1 < \infty$ and $n_1^\star < \infty$ such that, for all $n \geq n_1^\star$, we have $a(n) \leq c_1 \cdot b(n)$.

N2. A function $a(n)$ is $\Omega(b(n))$ if there exist $c_2 > 0$ and $n_2^\star < \infty$ such that, for all $n \geq n_2^\star$, we have $c_2 \cdot b(n) \leq a(n)$.

N3. A function $a(n)$ is $\omega(b(n))$ if for every $c > 0$, there exists $n^\star(c)$ such that, for all $n \geq n^\star(c)$, $c \cdot b(n) < a(n)$.

N4. A function $a(n)$ is $\Theta(b(n))$ if there exist $0 < c_3 < c_4 < \infty$ and $n_3^\star < \infty$ such that for all $n \geq n_3^\star$, we have $c_3 \cdot b(n) \leq a(n) \leq c_4 \cdot b(n)$. Note that $a(n) = \Theta(b(n))$ if and only if $a(n) = O(b(n))$ and $a(n) = \Omega(b(n))$.

N5. A function $a(n) \sim b(n)$ if $\lim_{n\to\infty}(a(n)/b(n)) = 1$.

2. NETWORK CONNECTIVITY WITH GROUP MOBILITY MODEL

In this chapter, we will investigate the communication range of the nodes necessary for network connectivity when the locations or mobilities of the nodes may be correlated through group mobility. The rest of the chapter is organized as follows: Section 2.1 explains the setup, mobility model and parametric scenario we introduce for carrying out asymptotic analysis. We provide a numerical example that demonstrates a parametric sensitivity of critical transmission range (CTR), which is defined as the smallest communication range of the nodes such that the network is connected, and summarize some of well known results for independent and identically distributed (i.i.d.) cases in Section 2.2. Main results are presented in Section 2.3. Simulation results are provided in Section 2.4 to validate our analysis.

2.1 Setup

We consider simple scenarios where nodes are placed on a unit ring [1]. Suppose that we are given a network consisting of N nodes that are placed on a unit ring, where $N \in \mathbb{N} := \{1, 2, 3, \ldots\}$. Two nodes i and j are said to be *immediate neighbors*, or simply *neighbors*, if and only if $D(i, j) \leq \gamma$, where $D(i, j)$ denotes the length of the shorter arc on the ring connecting the two nodes. There is a bi-directional (communication) link between two neighbors i and j, which we denote by $i \leftrightarrow j$.

Definition 2.1: A network is said to be *connected* if and only if it is possible to reach any node from any other node through a sequence of immediate neighbors. In other words, for every pair of nodes i and j, we can find $K \in \mathbb{N}$ and a sequence of nodes $i_1, i_2, \ldots, i_K$ such that

C1. $i_1 = i$ and $i_K = j$, and

C2. $i_k \leftrightarrow i_{k+1}$ for all $k = 1, 2, \ldots, K - 1$.

An example of a connected network is shown in Fig. 2.1

When the network is connected, in order for packets from node i to reach node j, they have to follow a sequence of intermediate nodes either in a clockwise direction or a counter-clockwise direction. In Fig. 2.1 packets from node i will follow a counter-clockwise (resp. clockwise) route to node j (resp. node k). In some cases, however, the packets may be routed only in one direction, but *not* in the other direction. When this

[1] We select a unit ring instead of a unit interval to avoid the boundary effects. However, simulation results show that the (distribution of the) communication range required for network connectivity is similar for both cases.

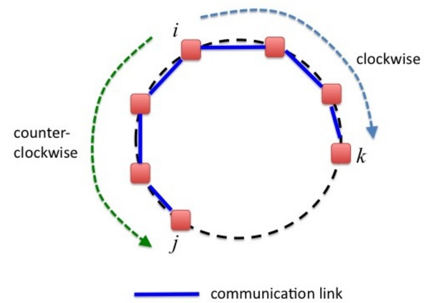

Fig. 2.1: An example of connected network.

happens, the one-hop connectivity of nodes does not form a complete ring and there is exactly one node that does not have a neighbor to its left (when all nodes are facing in the direction of the center of the disk). For instance, in Fig. 2.1 node k does not have a neighbor to its left and packets from node i cannot be routed clockwise to node j.

We focus on the case where the one-hop connectivity of the nodes forms a complete ring. In other words, every node has a neighbor to its left and a packet generated by any node can be routed to any other node by traversing a sequence of intermediate nodes both in clockwise and counter-clockwise directions. We call this *bi-directional connectivity*.

Obviously, bi-directional connectivity is a stronger condition than network connectivity in Definition 2.1; bi-directional connectivity implies 2-vertex connectivity (also called *biconnectivity*) and 2-edge connectivity [3]. In other words, the network would still be connected after removing any one node or a link in the network. It is obvious that the network in Fig. 2.1 is *not* bi-directionally connected. Unless stated otherwise, throughout the rest of the chapter, network connectivity refers to bi-directional connectivity. We will illustrate in Section 2.4, using numerical examples, that the communication ranges needed for the “usual” connectivity defined in Definition 2.1 and bi-directional connectivity do not differ significantly for large networks.

Given a network with $N \in \mathbb{N}$ nodes, the CTR of the network, denoted by $r^c(N)$, is defined to be the smallest communication range of the nodes such that the network is connected. Obviously, this CTR depends on the number of nodes in the network, N, and their exact locations, and computing the distribution of the CTR is challenging.

For this reason, researchers often turn to an asymptotic theory for $r^c(N)$ as the number of nodes N becomes large: Oftentimes, as the number of nodes grows (while keeping other parameters fixed), the distribution of the CTR concentrates over a (short) interval we can identify or approximate more easily. Following this spirit we are interested in examining how $\gamma^c(N)$ behaves as N increases. To this end, we introduce the following parametric scenario:

For each $n \in \mathbb{N}$, there are $N(n) \geq 1$ nodes in the network. These $N(n)$ nodes belong to $G(n)$ groups with each group consisting of $M(n) = N(n)/G(n)$ nodes, called the *members*. Let $\mathcal{G}^{(n)} := \{1, 2, \ldots, G(n)\}$ denote the set of groups and $\mathcal{M}^{(n)} := \{1, 2, \ldots, M(n)\}$ the set of members in a group. We assume that as $n \to \infty$, both the number of groups and the number of members in a group increase unbounded, i.e., $G(n) \to \infty$ and $M(n) \to \infty$.

2.1.1 Parametric Scenario

Given a network with $N \in \mathbb{N}$ nodes, the CTR of the network, denoted by $r^c(N)$, is defined to be the smallest communication range [2]of the nodes such that the network is connected. Obviously, this CTR depends on the number of nodes in the network, N, and their exact locations, and computing the distribution of the CTR is challenging.

[2] The communication range of a node is defined to be the maximum distance another node can be at, while maintaining a communication link with the node.

For this reason, researchers often turn to an asymptotic theory for $r^c(N)$ as the number of nodes N becomes large: Oftentimes, as the number of nodes grows (while keeping other parameters fixed), the distribution of the CTR concentrates over a (short) interval we can identify or approximate more easily. Following this spirit we are interested in examining how $\gamma^c(N)$ behaves as N increases. To this end, we introduce the following parametric scenario:

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Group Mobility Model

For each group $k \in \mathcal{G}^{(n)}$, there is a *virtual group leader* (VGL) $V_k^{(n)}$. [3] This VGL moves according to some stochastic mobility process on the unit ring. We denote the mobility process or trajectory of $V_k^{(n)}$ by $\mathbb{X}_k^{(n)} := \{X_k^{(n)}(t);\ t \in \mathbb{R}_+\}$, where $\mathbb{R}_+ := [0, \infty)$ and $X_k^{(n)}(t)$ is the location of $V_k^{(n)}$ at time t. Here, $X_k^{(n)}(t) \in [0, 1)$ denotes the length of the arc connecting some *fixed* reference point to $V_k^{(n)}$, moving clockwise on the unit ring.

The mobility process of the m-th node in the k-th group, denoted by $\mathbb{L}_{k,m}^{(n)} :=$

[3] The VGL $V_k^{(n)}$ is not a real node in the network. Instead, it is introduced to model the movement of the group.

$\{L_{k,m}^{(n)}(t);\ t \in \mathbb{R}_+\}$, can be written as a sum of two stochastic processes: [4]

$$\mathbb{L}_{k,m}^{(n)} = \mathbb{X}_k^{(n)} + \mathbb{Y}_{k,m}^{(n)}, \tag{2.1}$$

where $\mathbb{Y}_{k,m}^{(n)}$, $k \in \mathcal{G}^{(n)}$ and $m \in \mathcal{M}^{(n)}$, are identically distributed stochastic processes. We can interpret (2.1) as follows. While the process $\mathbb{X}_k^{(n)}$ describes the movement of (the VGL for) the group, the processes $\mathbb{Y}_{k,m}^{(n)}$ determine the movements of individual members in the group *relative to* the location of VGL. Note that this model is similar to the reference point group mobility model proposed in [13].

We consider the case where the movements of the members are constrained to an arc near the VGL. More precisely, the process $\mathbb{Y}_{k,m}^{(n)} := \{Y_{k,m}^{(n)}(t);\ t \in \mathbb{R}_+\}$ is limited to an interval $[0,\ d(n)] =: \mathbb{D}_g$ with $0 \leq d(n) \leq 1$, i.e., $Y_{k,m}^{(n)}(t) \in \mathbb{D}_g$ for all $t \in \mathbb{R}_+$. In this case the VGL is at the front of the group and the members follow the VGL, staying within $d(n)$. However, $\mathbb{D}_g$ can be any interval of length $d(n)$ without affecting the findings in this chapter.

We introduce the following assumptions on the mobility processes:

A1. The processes $\mathbb{X}_k^{(n)}, k \in \mathcal{G}^{(n)}$, and $\mathbb{Y}_{k,m}^{(n)}, k \in \mathcal{G}^{(n)}$ and $m \in \mathcal{M}^{(n)}$, are stationary and ergodic [7].

A2. The processes $\mathbb{X}_k^{(n)}, k \in \mathcal{G}^{(n)}$, are mutually independent and identically distributed. In addition, they yield a spatial distribution F_g with a continuous density f_g : $[0, 1) \to \mathbb{R}_+$, which is uniform over the unit ring, i.e., $f_g(x) = 1$ for all $x \in [0, 1)$.

A3. The processes $\mathbb{Y}_{k,m}^{(n)}$, $k \in \mathcal{G}^{(n)}$ and $m \in \mathcal{M}^{(n)}$, are mutually independent and are

[4] All additions are modulo one throughout the chapter.

also independent of $\mathbb{X}_k^{(n)}, k \in \mathcal{G}^{(n)}$. Moreover, they yield a spatial distribution F_m (with density f_m) uniform over the interval $\mathbb{D}_g$ with $f_m(y) = 1/d(n)$ for all $y \in \mathbb{D}_g$.

2.2 Connectivity of static graphs

As mentioned earlier, in order for a network to be able to provide an end-to-end route between arbitrary sources and destinations (when a connection is requested), the network should be connected most of the time. From the assumed ergodicity and stationarity of the mobility processes, this implies that the network sampled at some random time should be connected with high probability.

Suppose that we sample the network at time $t_s \in \mathbb{R}_+$. From the stated stationarity assumption, without loss of generality, we can assume $t_s = 0$. Furthermore, for notational simplicity we omit the dependence on time, e.g., we write $X_k^{(n)}$ in place of $X_k^{(n)}(0)$. Under assumptions A1 through A3, we can make following observations:

O1. The rvs $X_k^{(n)}, k \in \mathcal{G}^{(n)}$, are independent and uniformly distributed on the unit ring. Furthermore, $L_{k,m}^{(n)}, k \in \mathcal{G}^{(n)}$ and $m \in \mathcal{M}^{(n)}$, are uniformly distributed on the unit ring.

O2. The locations of members in the same group, $L_{k,m}^{(n)},\ m \in \mathcal{M}^{(n)}$, are *not* mutually independent when $d(n) < 1$. However, given $\{X_k^{(n)}, k \in \mathcal{G}^{(n)}\}$, the rvs $L_{k,m}^{(n)}, k \in \mathcal{G}^{(n)}$ and $m \in \mathcal{M}^{(n)}$, are *conditionally* independent. In particular, for each $k \in \mathcal{G}^{(n)}$, *given* $X_k^{(n)}$, the locations of the members in the k-th group, $L_{k,m}^{(n)}, m \in \mathcal{M}^{(n)}$, are conditionally independent rvs uniformly distributed on the arc $[X_k^{(n)},\ X_k^{(n)} + d(n)]$.

We note that when $d(n) = 1$, the locations of *all* the nodes are independent and uniformly distributed on the unit ring regardless of $\{X_k(n);\ k \in \mathcal{G}^{(n)}\}$ (so-called *i.i.d. case*). On the other hand, if $d(n)$ is very small, the locations $L_{k,m}^{(n)}, m \in \mathcal{M}^{(n)}$, are strongly correlated in that the members of a group are very close to each other. In this sense, $d(n)$ is used as a parameter to vary the degree of correlation in the locations of the members in a group.

2.2.1 An example of parametric sensitivity and motivation

Let us start with an example that illustrates the importance of understanding the role of correlation in nodes' locations. In particular, the following example highlights the sensitivity of the CTR with respect to the length $d(n)$ of $\mathbb{D}_g$ over some interval. Fig. 2.2 plots the probability that the network is connected as a function of the communication range γ of the nodes for a scenario where there are 100 groups $(G(n) = 100)$ and each group has 200 members $(M(n) = 200)$ for five different values of $d(n)$. The x-axis of the plot is $\log_{10}(\gamma)$, and the y-axis is the empirical probability (i.e., fraction of times the network was connected from 1,000 realizations).

What is surprising in this example is that, while three plots of the probability for $d(n) = 0.0001, 0.01$ and 0.03 are relatively close, the probability for $d = 0.08$ is very different from the first three; (loosely speaking) the required CTR is *more than an order of magnitude larger* when $d(n) = 0.03$ compared to when $d(n) = 0.08$. In particular, the median value of the CTR differs by a factor of more than 20! However, when $d(n)$ is further increased from 0.08 to 0.2, the change in probability is not nearly as significant.

A natural question that arises from this example is whether this is an atypical scenario or there is a more fundamental reason for this *parametric sensitivity* of the CTR to

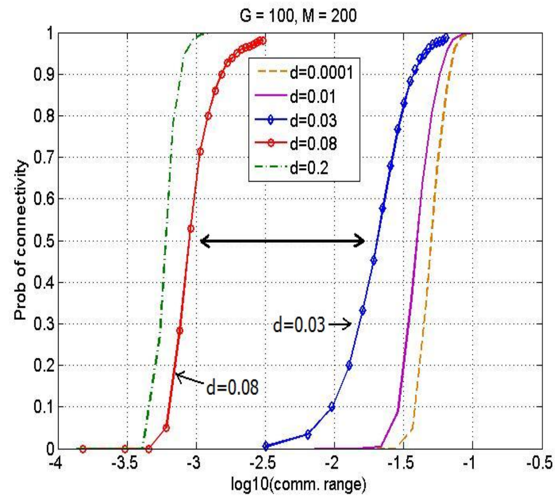


Fig. 2.2: Probability of network connectivity ($G = 100, M = 200$).

$d(n)$ (between 0.03 and 0.08). Furthermore, if this is not atypical, can we predict when and where it will occur? In this chapter, we will answer these questions. We will show that, indeed, this parametric sensitivity over an interval is not an isolated incident and will show up in many settings, and identify the range over which such parametric sensitivity will be displayed. We suspect that similar parametric sensitivity is likely to persist in high-dimensional cases as well. We will revisit this example in subsection 2.3.3.

2.2.2 I.i.d. cases and (very) strong threshold

Let $\mathbb{G}(G(n), M(n); \gamma)$ be the *geometric random graph* (GRG) representing the one-hop connectivity of the network with $G(n)$ groups and $M(n)$ members in each group (with a total of $N(n) = G(n) \times M(n)$ nodes), where each node employs a communication range of γ, according to the setup described in the previous section. We define

$$\mathbf{P}^{(n)}(\gamma) := \mathbb{P}\left[\mathbb{G}(G(n), M(n); \gamma) \text{ is connected}\right].$$

It is obvious that $\mathbf{P}^{(n)}(\gamma)$ is increasing in γ.

Consider the case where the locations of the $N(n)$ nodes are given by independent rvs uniformly distributed on the unit ring (i.e., $d(n) = 1$). In this case, the result by Han and Makowski [12] [5] tells us

$$\lim_{n\to\infty} \mathbf{P}^{(n)} \left(\frac{\log(N(n)) + \alpha_n}{N(n)} \right) = \begin{cases} 1 & \text{if } \alpha_n \to \infty \text{ (as } n \to \infty), \\ 0 & \text{if } \alpha_n \to -\infty \text{ (as } n \to \infty). \end{cases} \tag{2.2}$$

Here, α_n can increase (resp. decrease) to ∞ (resp. $-\infty$) *arbitrarily slow*. When (2.2) is true, Han and Makowski call

$$\gamma^{iid}(n) := \frac{\log(N(n))}{N(n)}, \ n \in \mathbb{N}, \tag{2.3}$$

a *very strong threshold* (VST).

The interpretation of a VST $\gamma^\star(n), n \in \mathbb{N}$, is that, for all sufficiently large n, if the communication range is set suitably larger than $\gamma^\star(n)$, the probability that a network is connected will be close to one. Similarly, if the communication range is set somewhat smaller than $\gamma^\star(n)$, the probability will be very small.

This sharp increase in the probability of network connectivity around the VST is called a *phase transition* in the literature, which often leads to a zero-one law (e.g., (2.2)). We point out that a VST may not exist in some cases (see [10] for an example).

[5] Although the authors of [12] consider a unit interval and the notion of network connectivity given in Definition 2.1, the same result is true for bi-directional connectivity we study in this chapter. This finding also follows directly from Theorem 2.2 in Section 2.3.

2.3 Main results

In this section we investigate how $\mathbf{P}^{(n)}(\gamma)$ changes as a function of the common communication range of the nodes, γ, as both $G(n)$ and $M(n)$ grow. Intuitively, we expect that, under the family of group mobility models described in Section 2.1.1, the CTR depends on $d(n)$ in relation to both $G(n)$ and $M(n)$. For instance, we know that the case $d(n) = 1$ is equivalent to the i.i.d. case discussed in subsection 2.2.2. On the other hand, if all members of a group are on top of each other, i.e., $d(n) = 0$ and $L_{k,m}^{(n)} = X_k^{(n)}$ for all $m \in \mathcal{M}^{(n)}$, the CTR would behave just as in the case when $G(n)$ nodes are independent and uniformly distributed on the unit ring.

In the following subsection we first discuss the cases for which we can identify a VST. Then, subsection 2.3.2 examines the remaining cases and provides asymptotic upper bounds (AUBs) and lower bounds (ALBs) to CTR for most of the remaining cases. [6] We only provide a proof for Theorem 2.1 in this dissertation. The proof of other results is similar in nature and can be obtained by modifying that of Theorem 2.1.

The following assumption is in place throughout this section:

Assumption 2.1: We assume $G(n) = \omega\left(\log^2(N(n))\right)$ and $M(n) = \omega\left(\log(N(n))\right)$.

Assumption 2.1 is introduced to ensure that $G(n)$ and $M(n)$ do not increase too slowly in relation to the total number of nodes in the network.

[6] We say that $\gamma(n),\ n \in \mathbb{N}$, is an AUB (resp. ALB) to CTR if $\mathbf{P}^{(n)}(\gamma(n)) \to 1$ (resp. $\mathbf{P}^{(n)}(\gamma(n)) \to 0$) as $n \to \infty$.

2.3.1 Very strong thresholds

Let us start with two cases for which we can "guess" the threshold from the results on i.i.d. cases summarized in subsection 2.2.2: Intuitively, on one hand, when $d(n)$ is very small and the locations of the nodes in a group are strongly correlated, we expect the CTR to behave similarly to the case with $d(n) = 0$. On the other hand, when $d(n)$ is large and the locations of the nodes in a group are weakly correlated, the distribution of CTR should be close to that of i.i.d. case with $N(n)$ nodes. Hence, a natural question is how small or large $d(n)$ needs to be in order for our intuition to provide the right answer. In the first two theorems, we provide a sufficient condition to these questions.

Theorem 2.1: Suppose $d(n) = o\left(\frac{1}{G(n)}\right)$. Then, $\gamma_1(n) = \log(G(n))/G(n),\ n \in \mathbb{N}$, is a VST, i.e.,

$$\lim_{n\to\infty} \mathbf{P}^{(n)}\left(\frac{\log(G(n)) + \alpha_n}{G(n)}\right) = \begin{cases} 1 & \text{if } \alpha_n \to \infty, \\ 0 & \text{if } \alpha_n \to -\infty. \end{cases} \tag{2.4}$$

The intuition behind Theorems 2.1 is as follows. If we compare the VST $\gamma_1(n)$ to $d(n)$, obviously, $d(n) = o\left(\gamma_1(n)/\log(G(n))\right)$. Hence, the members in different groups become clustered on short arcs they occupy that the presence of many nodes on each arc makes little difference in the required CTR. Consequently, we obtain the same VST from the i.i.d. case with $G(n)$ nodes.

Theorem 2.2: Suppose $d(n) = \omega\left(\frac{\log^2(N(n))}{G(n)}\right)$. Then, $\gamma_2(n) = \log(N(n))/N(n),\ n \in \mathbb{N}$, is a VST. In other words,

$$\lim_{n\to\infty} \mathbf{P}^{(n)}\left(\frac{\log(N(n))+\alpha_n}{N(n)}\right) = \begin{cases} 1 & \text{if } \alpha_n \to \infty, \\ 0 & \text{if } \alpha_n \to -\infty. \end{cases} \tag{2.5}$$

The finding in Theorem 2.2 is not as obvious as that of Theorem 2.1. One would expect $d(n) = \omega(\log(N(n))/N(n))$ to be *necessary* in order for $\gamma_2(n), n \in \mathbb{N}$, to be a VST. [7] However, it is not clear beforehand whether or not $d(n)$ can be allowed to decrease to zero, while retaining the VST of $\gamma_2(n)$, and if so, how quickly $d(n)$ may decrease. Theorem 2.2 tells us that $d(n) = \omega\left(\log^2(N(n))/G(n)\right)$, even though $d(n)$ may decrease to 0 as $n \to \infty$ from Assumption 2.1, is *sufficient* for the CTR to behave (asymptotically) as in the i.i.d. case with $N(n)$ nodes.

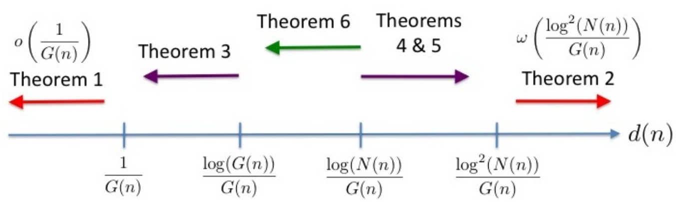


Fig. 2.3: Summary of results.

As shown in Fig. 2.3, Theorems 2.1 and 2.2 cover two extreme cases – namely $d(n) = o(1/G(n))$ and $d(n) = \omega\left(\log^2(N(n))/G(n)\right)$ – and leave out the case in the middle where $d(n)$ is both $\Omega(1/G(n))$ and $O(\log^2(N(n))/G(n))$. Unfortunately, we are not able to find a single VST for this case; in fact, we suspect that no such threshold exists. [8] Hence, we divide it into subcases to be studied separately. In addition, a VST is known

[7] In Section 2.4, we will provide a numerical example that hints $d(n) = \omega(\log(N(n))/N(n))$ is not sufficient for $\gamma_2(n)$ to be a VST.

[8] We base this comment on an observation that the VST provided in Theorem 2.3 below for a subregime does not appear to be large enough for another subregime as illustrated by a numerical example in Section 2.4.

only for one of the subcases, and only AUBs and ALBs to CTR are provided for most of the other subcases.

Theorem 2.3: Suppose that $d(n) = \beta \cdot \log(G(n))/G(n)$, where $0 < \beta < 1$. Then,

$$\begin{aligned}\gamma_3(n) &= \frac{\log(G(n))}{G(n)} - d(n) \\ &= (1-\beta)\,\frac{\log(G(n))}{G(n)},\ n \in \mathbb{N},\end{aligned} \tag{2.6}$$

is a VST. In other words,

$$\lim_{n\to\infty} \mathbf{P}^{(n)}\left(\frac{(1-\beta)\log(G(n)) + \alpha_n}{G(n)}\right) = \begin{cases} 1 & \text{if } \alpha_n \to \infty, \\ 0 & \text{if } \alpha_n \to -\infty. \end{cases}$$

The intuition behind Theorem 2.3 is that, when $d(n)$ is smaller than (but not negligible to) $\gamma_1(n)$ in Theorem 2.1, (roughly speaking) the effects of $d(n)$ to the CTR is a subtraction of $d(n)$ from the VST $\gamma_1(n)$. This is consistent with our finding in Theorem 2.1; the case $d(n) = o(1/G(n))$ can be viewed as a limiting case of Theorem 2.3 where $\beta \downarrow 0$, while the VST $\gamma_3(n) \uparrow \gamma_1(n)$ as a result of $d(n)$ being $o(\log(G(n))/G(n))$.

2.3.2 Asymptotic upper and lower bounds to CTR

As mentioned earlier, identifying a VST for the remaining cases (which are not covered by Theorems 2.1 through 2.3) is difficult. Here, we provide AUBs to CTR for most of the remaining cases and ALBs to some cases. We illustrate how good these AUBs are in the following section, using numerical examples.

Theorem 2.4: Suppose that $d(n) = \beta \, \frac{\log(N(n))}{G(n)}$, where $\beta > 1$. Then,

$$\lim_{n\to\infty} \mathbf{P}^{(n)}\left(\alpha \, \frac{\log(N(n))}{N(n)}\right) = 1$$

if $\alpha > \beta \, \log\left(\frac{\beta}{\beta-1}\right)$.

Define a mapping $f : (1, \infty) \to (1, \infty)$, where

$$f(\beta) := \beta \, \log\left(\frac{\beta}{\beta - 1}\right). \tag{2.7}$$

One can easily show that f is strictly decreasing and convex. Furthermore,

$$\lim_{\beta\uparrow\infty} f(\beta) = 1 \text{ and } \lim_{\beta\downarrow 1} f(\beta) = \infty. \tag{2.8}$$

This can be seen from Fig. 2.4.

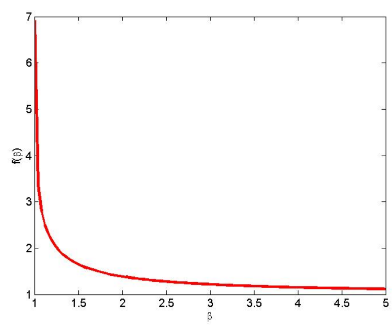


Fig. 2.4: Plot of $f(\beta) = \beta \log(\beta/(\beta - 1))$.

When $\beta \gg \log(N(n))$, we have $d(n) \gg \log^2(N(n))/G(n)$. Thus, intuitively one expects that the distribution of CTR is close to that of the case in Theorem 2.2 (when $d(n) = \omega(\log^2(N(n))/G(n))$) and, hence, is concentrated around the threshold $\gamma_2(n) = \log(N(n))/N(n)$ for all sufficiently large n. Indeed, for $\beta \gg 1$, $f(\beta) \approx 1$ from Fig. 2.4 and the AUBs are close to $\gamma_2(n)$.

In addition, when $\log(M(n)) = o\left(\log(G(n))\right)$, we have $\log(N(n))/G(n) = (\log(G(n)) + \log(M(n)))/G(n) \approx \log(G(n))/G(n)$. Thus, the result in Theorem 2.4 complements the finding in Theorem 2.3. Combined, these two theorems reveal an important observation that there is sudden change in the way the CTR scales as $d(n)$ increases from just below $\log(G(n))/G(n)$ to just above it; note that $\gamma_3(n) = \Theta(\log(G(n))/G(n))$, whereas the AUB in Theorem 2.4 is $O(\log(N(n))/N(n))$. For all sufficiently large n, which implies $M(n) \gg 1$, we have $\log(G(n))/G(n) \gg \log(N(n))/N(n)$.

Theorem 2.5: Suppose that $d(n) = \beta \, \frac{\log(N(n))}{G(n)}$, where $\beta > 1$, and $G(n) \geq M(n)$. Then,

$$\lim_{n \to \infty} \mathbf{P}^{(n)} \left(\frac{\log(N(n))}{N(n)} \right) = 0.$$

Note that Theorem 2.5 tells us that, under the conditions stated in the theorem, $\gamma_2(n) = \log(N(n))/N(n)$ is an ALB. Therefore, combined with the finding in Theorem 2.4, when $d(n) = \beta \cdot \log(N(n))/G(n)$ with $\beta > 1$ and $G(n) \geq M(n)$, *if* there exists a VST, it must lie between $\log(N(n))/N(n)$ and $f(\beta) \cdot \log(N(n))/N(n)$, where $f(\beta) \approx 1$ for large β (roughly speaking, for $\beta > 2$).

Furthermore, the finding in the theorem is not very surprising; intuitively, we believe that the required CTR tends to decrease with increasing $d(n)$, and $\gamma_2(n), n \in \mathbb{N}$, gives rise to a VST for the case $d(n) = 1$, i.e., i.i.d. case. Hence, we suspect that a VST for the case considered in Theorems 2.4 and 2.5, if one exists, should not be smaller than $\gamma_2(n)$.

Theorem 2.6: Suppose (i) $d(n) = \beta \, \frac{\log(N(n))}{G(n)}$ with $0 < \beta < 1$ and (ii) $M(n) = \omega(N(n)^{1-\beta+\epsilon})$ for some $\epsilon > 0$. Then,

$$\lim_{n \to \infty} \mathbf{P}^{(n)} \left(\gamma_6(n) \right) = 1,$$

where

$$\gamma_6(n) = \frac{((1-\beta)\,\beta\;\log(N(n)) + \alpha_n)\;\log(N(n))}{N(n)} \tag{2.9}$$

and $\alpha_n \to \infty$.

Note that the second part of the assumption in Theorem 2.6 is stronger than Assumption 2.1. Moreover, as $\beta \uparrow 1$, $\gamma_6(n) \approx \alpha_n \cdot \log(N(n))/N(n)$. This is consistent with the AUBs in Theorem 2.4, namely $\alpha \cdot \log(N(n))/N(n)$ with $\alpha > f(\beta) \uparrow \infty$ as $\beta \downarrow 1$ from (2.8).

Consider a special case where $\beta = 1/2$ and $M(n) = N(n)^{0.5+\epsilon}$ and $G(n) = N(n)^{0.5-\epsilon}$ for some $\epsilon > 0$. Then,

$$\begin{aligned} d(n) = \beta\,\frac{\log(N(n))}{G(n)} &\approx \frac{(1+2\epsilon)\;\log(G(n))}{G(n)} \\ &\approx \frac{\log(G(n))}{G(n)}, \end{aligned}$$

and

$$\begin{aligned} \gamma_6(n) &\approx \frac{(0.5\;\log(G(n)) + \alpha_n)\;\log(N(n))}{N(n)} \\ &\approx \frac{(\log(G(n)) + 2\alpha_n)\;\log(G(n))}{G(n)\cdot M(n)}. \end{aligned} \tag{2.10}$$

This provides us with a glimpse of how the scaling behavior of the CTR changes around $d(n) \approx \log(G(n))/G(n)$, i.e., as $d(n)$ crosses over from the regime considered in Theorem 2.3 (where $d(n) = \beta \cdot \log(G(n))/G(n)$ with $\beta < 1$) to the other side with $\beta \geq 1$. As one would suspect, the scaling behavior is quite different for $\gamma_3(n) = \Theta\left(\log(G(n))/G(n)\right)$ and the AUB $\gamma_6(n)$; since $\log(G(n)) = o(M(n))$ in (2.10), $\gamma_6(n)$ decreases faster than the VST $\gamma_3(n)$.

Our results in this section are summarized in Table 2.1.

Theorem	$d(n)$	$M(n)$	Tx Range	Type
1	$o\left(\frac{1}{G(n)}\right)$		$\frac{\log(G(n))}{G(n)}$	VST
2	$\omega\left(\frac{\log^2(N(n))}{G(n)}\right)$		$\frac{\log(N(n))}{N(n)}$	VST
3	$\beta\,\frac{\log(G(n))}{G(n)}$ with $0 < \beta < 1$		$\frac{\log(G(n))}{G(n)} - d(n)$	VST
4	$\beta\,\frac{\log(N(n))}{G(n)}$ with $1 < \beta$		$\alpha\,\frac{\log(N(n))}{N(n)}$ with $\alpha > \beta \cdot \log\left(\frac{\beta}{\beta-1}\right)$	AUB
5	$\beta\,\frac{\log(N(n))}{G(n)}$ with $1 < \beta$	$M(n) \leq G(n)$	$\frac{\log(N(n))}{N(n)}$	ALB
6	$\beta\,\frac{\log(N(n))}{G(n)}$ with $0 < \beta < 1$	$\omega(N(n)^{1-\beta-\epsilon})$, $\epsilon > 0$	$\frac{((1-\beta)\ \beta\ \log(N(n))+\alpha_n)\ \log(N(n))}{N(n)}$	AUB

Tab. 2.1: Summary of results (VST = very strong threshold, AUB = asymptotic upper bound, ALB = asymptotic lower bound)

2.3.3 Discussion on the numerical example in Fig. 2.2

Let us revisit the example provided in Fig. 2.2. For the given values $G(n) = 100$ and $M(n) = 200$, we have $\log(G(n))/G(n) = 0.0461$, $\log(N(n))/G(n) = 0.0990$ and $\log(N(n))/N(n) = 4.95 \times 10^{-4}$. Hence, when $d(n) = 0.03$, since $d(n) = \beta \cdot \log(G(n))/G(n)$ with $\beta = 0.65$, we can apply the finding in Theorem 2.3; it tells us that the phase transition should occur around $\gamma_3(n) = 0.0161 = 10^{-1.79}$. This is consistent with the plot for $d = 0.03$ in Fig. 2.2, where the median of CTR is approximately $10^{-1.73}$.

As $d(n)$ increases from 0.03 to 0.08 and then to 0.2, however, the (distribution of) CTR goes through a rather dramatic change: Note that $d(n) = 0.2 = \beta \cdot \log(N(n))/G(n)$ with $\beta = 2.02$. Hence, the finding from Theorem 2.4 and the plot of $f(\beta)$ in Fig. 2.4 indicate that the phase transition in probability should happen near or below the AUB given by $1.4 \cdot \log(N(n))/N(n) = 6.93 \times 10^{-4} = 10^{-3.159}$. Note that this is very close to the median of CTR for $d(n) = 0.2$ in Fig. 2.2.

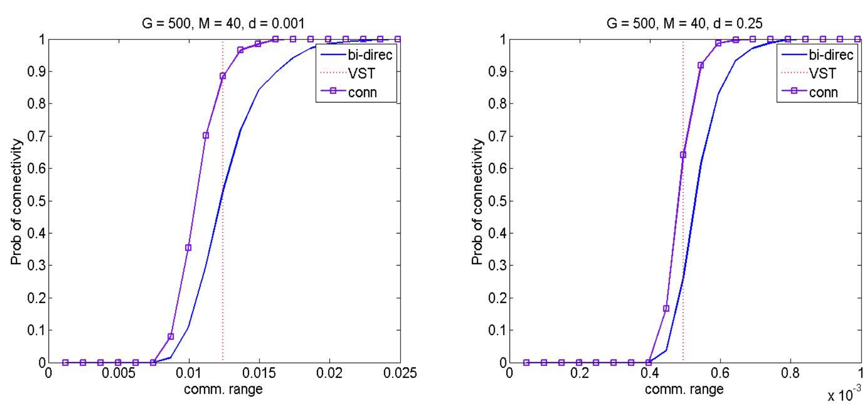
These illustrate that the parametric sensitivity exhibited by the (distribution of) CTR to $d(n)$ in Fig. 2.2 over an interval can be easily explained by our findings. Moreover, it suggests that such parametric sensitivity will exist in many, if not most, settings, possibly even in higher dimensions as well.

2.4 Numerical results

In this section we first present numerical examples that demonstrate the validity of VSTs found in subsection 2.3.1. Then, we show, using numerical examples, that the asymptotic bounds provided in Theorem 2.4 are relatively tight in many cases.

In these examples, there are 500 groups ($G = 500$) with 40 members in each group ($M = 40$) with a total of 20,000 nodes ($N = 20,000$). For each example, we generate 1,000 samples and compute the fraction of times the network is bi-directionally connected as a function of communication range. We also examine the fraction of times the network is connected according to the usual notion of connectivity in Definition 2.1.

Example 1: Fig. 2.5 plots the numerical results (solid blue curve) and the VST (dotted vertical red line) computed using (a) γ_1 for $d = 0.001$ and (b) γ_2 for $d = 0.25$. Note that $d = 0.001 < 1/G = 0.002$ in the first case, whereas $d = 0.25 > \log^2(500 \times 40)/500 = 0.196$ in the second case. The plots suggest that indeed the phase transition in the probability takes place around the provided VST for both cases, corroborating our findings in Theorems 2.1 and 2.2.



(a) (b)

Fig. 2.5: Example 1: VSTs in Theorems 2.1 and 2.2 ($G = 500,\ M = 40$). (a) $d = 0.001$, (b) $d = 0.25$

The figure also plots the probability of "usual" connectivity defined in Definition 2.1 (shown as purple line), which lies above the probability of bi-directional connectivity as expected. The plots reveal that it goes through a similar phase transition around the same threshold for both cases, as mentioned in Section 2.1.1.

Example 2: For this example, we vary d from 0.003 to 0.006 with an increment of 0.001, in order to see the effects of d on the CTR. Note that $\log(G)/G = 0.0124 > d$. It is clear from Fig. 2.6 that the phase transition in the probability of bi-directional connectivity occurs around the VST computed using γ_3 in (2.6). Moreover, as predicted by Theorem 2.3, the location of phase transition decreases linearly with d.

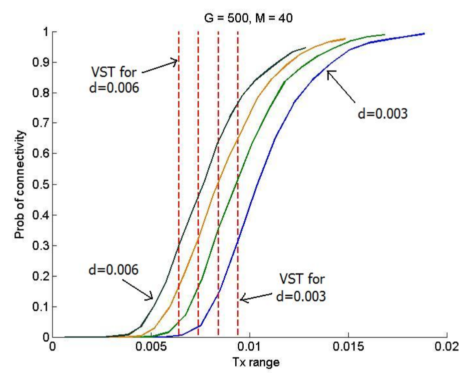


Fig. 2.6: Example 2: VSTs from Theorem 2.3 ($G = 500, M = 40$).

Example 3: Fig. 2.7 plots the probability of bi-directional connectivity (solid blue line) for three different values of d (d = 0.02, 0.03, and 0.04). Note that, for the given values of G and M, we have $\log(N)/G = 0.0198 < d$. Hence, we consider the regime in Theorems 2.4 and 2.5 in this example. We also plot $f(\beta) \cdot \log(N(n))/N(n)$ (dotted vertical red line) from Theorem 2.4, where $f(\beta)$ is defined in (2.7) and $\beta = d \times G/\log(N)$.

Fig. 2.7(a) suggests that when $\beta = 0.02/0.0198 \approx 1$, $f(\beta) \cdot \log(N(n))/N(n)$ tends

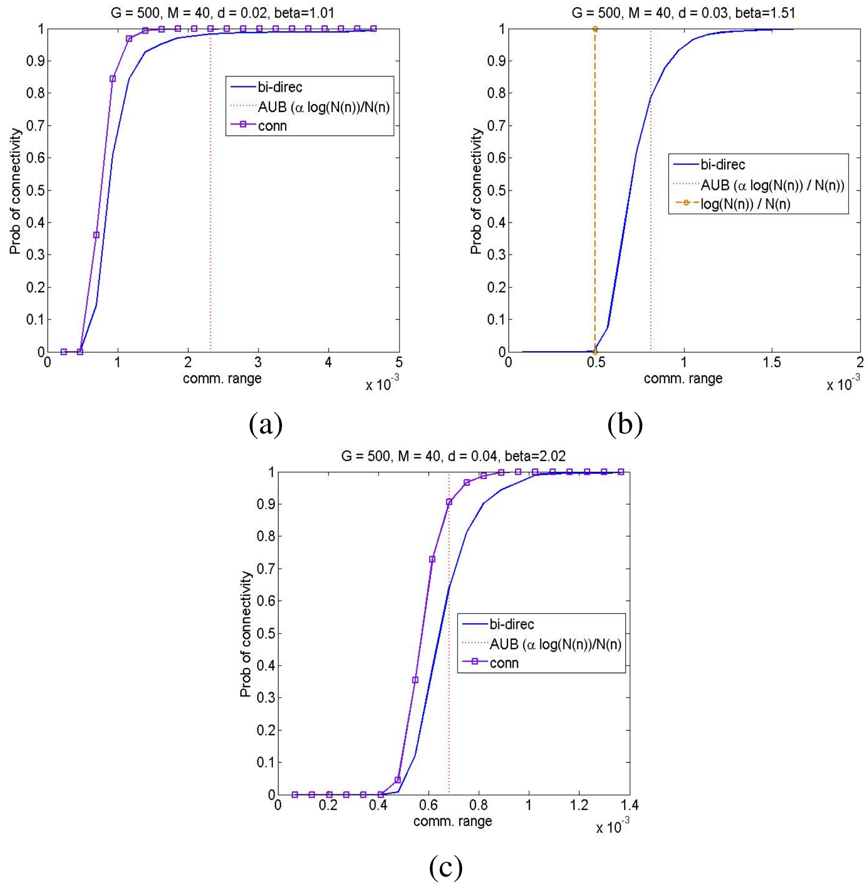


Fig. 2.7: Asymptotic upper bounds in Theorem 2.4 ($G = 500, M = 40$). (a) $d = 0.02$, (b) $d = 0.03$, (c) $d = 0.04$.

to overestimate where the phase transition happens. However, Figs. 2.7(b) and (c) show that when $\beta > 1.5$, $f(\beta) \cdot \log(N(n))/N(n)$ lies in the middle of phase transition, hinting that the AUB in Theorem 2.4 may provide a good estimate of a VST, if one exists.

Fig. 2.7(b) also plots $\log(N)/N$ (dotted vertical yellow line). The plot indicates that, although $d = 0.03 \gg \log(N)/N = 4.95 \times 10^{-4}$ (in fact, $d > \log(N)/G = 0.0198$), $\log(N)/N$ appears to underestimate the threshold, hinting that $d(n) = \omega(\log(N(n))/N(n))$ is not sufficient for $\gamma_2(n)$ to be a VST.

Example 4: In the final example we consider the regime studied in Theorem 2.6. Note that $d = 0.014 < \log(N)/G = 0.0198$ with $\beta = d \times G/\log(N) = 0.707$, and

$M = 40 > (500 \times 40)^{(1-0.707)} = 18.2$. Fig. 2.8 shows the plot of probability of bi-directional connectivity (solid blue line) and usual connectivity (purple line) as well as the AUB in (2.9).

There are two things to notice from the figure. First, the AUB provided in Theorem 2.6 does point to where the probability goes through a transition. Second, unlike in other cases, unfortunately the transition in probability takes place much slower for both notions of connectivity. In particular, while the probability increases rapidly at the beginning as in other cases, the tail of the distribution is much larger, especially for the bi-directional connectivity.

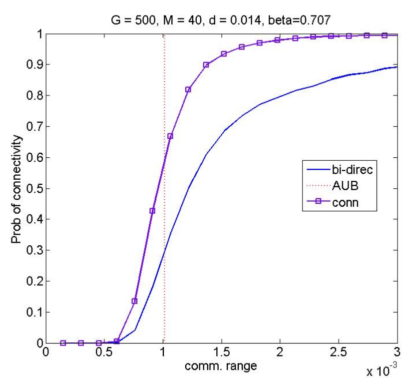


Fig. 2.8: Example 2: AUB from Theorem 2.6 ($G = 500, M = 40, d = 0.014$).

3. NODE ISOLATION WITH TRUST CONSTRAINT

In this chapter we will study the network connectivity when one-hop connectivity of two nodes is decided by both geometric and trust constraints. The rest of the chapter is organized as follows: Section 3.1 explains the setup, mobility model and parametric scenario. Main results are presented in Section 3.2 and numerical results are provided in Section 3.3.

3.1 Setup

In this section we first explain the assumed mobility model of nodes and the the one-hop connectivity of the random graph. Then, we describe the parametric scenario we assume for our asymptotic analysis as the number of nodes in the network increases.

3.1.1 Node mobility and one-hop connectivity

Suppose that for each $n \in \mathbb{N}$, there are $n \geq 1$, nodes in the network that move on a unit rectangle, which we denote by Ω, which is folded up into a torus. The *mobility process* or *trajectory* of node k where $k \in \mathcal{N}_n := \{1, 2, \ldots, n\}$ is denoted by $\mathbb{X}_k^{(n)} := \{X_k^{(n)}(t); t \in$

$\mathbf{R}_+\}$, where $\mathbf{R}_+ := [0, \infty)$. For $t \in \mathbf{R}_+$, the random variable (rv) $X_k^{(n)}(t) \in \Omega$ indicates the location of node k at time t. The locations of the n nodes at time $t \in \mathbf{R}_+$ are given by n independent rvs uniformly distributed on the torus.

At time $t \in \mathbf{R}_+$, we say that there exists a bi-directional (communication) link between two nodes j and k if and only if these two nodes satisfy the following two constraints.

First, the geometric constraint is that the two nodes should be within a certain communication range to have a link. In addition to the geometric constraint, we assume that each node $i \in \mathcal{N}_n$ communicates only with other nodes that are trustworthy. In order to model this, we introduce a trust constraint.

1. Geometric constraint- For $t \in \mathbf{R}_+$, we assume that nodes j and k can communicate if and only if their distance $D(X_j^{(n)}(t), X_k^{(n)}(t))$ satisfies

$$D(X_j^n(t), X_k^{(n)}(t)) = ||X_j^{(n)}(t) - X_k^{(n)}(t)||_2 \leq r(\xi) := \sqrt{\frac{\xi \log{(n)}}{\pi n}}. \tag{3.1}$$

2. Trust constraint- Let us introduce two arrays of rvs: Each node j has two variables $(\Theta_j^{(n)}, T_j^{(n)})$. The variable $T_j^{(n)}$ denotes node j's *trust level*, i.e., how much other nodes can trust node j. The variable $\Theta_j^{(n)}$ represents node j's *trust threshold*. In other words, node j would trust node k if and only if node k's trust level is higher than node j's trust threshold, i.e., $T_k^{(n)} \geq \Theta_j^{(n)}$. Hence, nodes j and k would trust each other if and only if

$$T_k^{(n)} \geq \Theta_j^{(n)} \text{ and } T_j^{(n)} \geq \Theta_k^{(n)}. \tag{3.2}$$

Throughout this chapter, we assume that $\{G_j^{(n)} = (\Theta_j^{(n)}, T_j^{(n)}), j \in \mathcal{N}_n\}$ are given by joint rvs with a common distribution $\mathbf{G^{(n)}}$ on $G \subset \mathbf{R}^2$.

Under the two aforementioned constraints, we will specify the one-hop connectivity of two nodes by the following definition.

Definition 3.1: Given the locations of n nodes and fixed $t > 0$, we define that node j is a *neighbor* of node k, which we denote by $j \leftrightarrow k$, if and only if (i) $D(X_j^{(n)}(t), X_k^{(n)}(t)) \leq r(\xi)$ and (ii) $T_j^{(n)} \geq \Theta_k^{(n)}$ and $T_k^{(n)} \geq \Theta_j^{(n)}$.

We denote the set of node *i*'s neighbors by $\mathbf{N}_i, i \in \mathcal{N}_n$. We say that a node is isolated if it does not have any neighbor, i.e., $\mathbf{N}_i = \phi$.

We adopt the same definition of the connected network in Definition 2.1 in Section 2.1. The network connectivity simply means that, given any two nodes in the network, we can find a sequence of intermediate nodes that can provide the end-to-end connectivity between the two nodes. It is clear that if there is an isolated node, then, the network is not connected.

3.1.2 Parametric scenario

We are interested in examining how the smallest communication range necessary for no node isolation scales as the number of nodes, n, increases. As we explained in the previous section, for each $n \in \mathbb{N}$, there are $n \geq 1$ nodes in the network. These n nodes move on Ω according to mobility processes $\mathbb{X}_k^{(n)} = \{X_k^{(n)}(t); t \in \mathbf{R}_+\}$, $k \in \mathcal{N}_n := \{1, 2, \ldots, n\}$. In order to make progress we introduce the following assumptions on the mobility processes:

A1. The processes are $\mathbb{X}_k^{(n)}$, $k \in \mathcal{N}_n$, are mutually independent;

A2. they are stationary and ergodic; and

A3. $\mathbb{X}_k^{(n)}$, $k \in \mathcal{N}_n$, yields a uniform spatial distribution.

Also, each node j has two variables $(\Theta_j^{(n)}, T_j^{(n)})$ which represent trust level and trust threshold, and $\{G_j^{(n)} = (\Theta_j^{(n)}, T_j^{(n)}), j \in \mathcal{N}_n\}$ are given by independent rvs with a common joint distribution $\mathbf{G^{(n)}}$ with a continuous density $\mathbf{g^{(n)}}$ on $G \subset \mathbf{R}^2$. We assume that the joint distribution is sufficiently smooth, which we capture by the following assumption.

A4. There exists $\kappa < \infty$ such that,

$$\left|\mathbf{g^{(n)}}(\bar{x_1}) - \mathbf{g^{(n)}}(\bar{x_2})\right| \leq \kappa ||\bar{x_1} - \bar{x_2}||_2$$

for all $\bar{x_1}, \bar{x_2} \in G$.

3.2 Main Results

In order for a network to be able to provide an end-to-end route between arbitrary sources and destinations, the network should be connected most of the time. Suppose that we sample the network at time $t_s \in \mathbf{R}_+$. From the stated stationary assumption, without loss of generality, we can assume $t_s = 0$. Furthermore, for notational simplicity we omit the dependence on time, e.g., we write $X_k^{(n)}$ in place of $X_k^{(n)}(0)$. Therefore, we examine the connectivity of the sampled *static* graph instead.

Let $\mathbb{G}(n; r)$ be the random graph representing the one-hop connectivity of the network with n nodes sampled at $t = 0$, where each node employs a common communication range of r.

We are interested in understanding how the probability of having an isolated node is affected by the trust constraint in (3.2) as n increases. As we explained in Section 3.1, link connection is decided by both locations and trustworthiness of two nodes.

Let us define $\mathbf{P}^{(n)}(r^{(n)}(\xi))$ to be the probability that none of n nodes is isolated with a common transmission range $r^{(n)}(\xi)$ where $r^{(n)}(\xi) = \sqrt{\xi \log{(n)}/\pi n}$, i.e.,

$$\mathbf{P}^{(n)}(r^{(n)}(\xi)) := \mathbb{P}\left[\mathbf{N}_i \neq \phi \text{ for all } i \in \mathcal{N}_n \text{ with a common transmission range } r^{(n)}(\xi)\right].$$

Since the locations of the nodes are mutually independent and uniformly distributed on Ω and $\{G_j^{(n)} = (\Theta_j^{(n)}, T_j^{(n)}), j =\in \mathcal{N}_n\}$ are given by joint rvs with a common distribution $\mathbf{G^{(n)}}$ on G, we will consider specific node 1 and node 2.

Suppose that we define

$$\psi^* := \inf_{\bar{g}\in G} P(\Theta_2^{(n)} \leq t_1 \text{ and } T_2^{(n)} \geq \theta_1 | G_1^{(n)} = \bar{g} = (\Theta_1^{(n)} = \theta, T_1^{(n)} = t)), \tag{3.3}$$

which is the infimum of the probability that node 1 and node 2 trust each other for given $(\Theta_1^{(n)} = \theta, T_1^{(n)} = t)$.

Next, let us define that

$$\phi := \sup_{\bar{g}\in G} P(\text{node 2 is not a neighbor of node } 1 | G_1^{(n)} = \bar{g})$$

Then, from the definition of ψ^*, ϕ can be expressed as

$$\begin{aligned}\phi &= 1 - \inf_{\bar{g}\in G} P(\text{node 2 is a neighbor of node } 1 | G_1^{(n)} = \bar{g}) \\ &= 1 - \pi r^{(n)}(\xi)^2 \cdot \psi^*.\end{aligned}$$

Then, from the following Lemma, we can show that when the common transmission range is larger than $\sqrt{\log(n)/(\psi^* \pi n)}$, the probability of no isolated nodes is very small for all sufficiently large n.

Lemma 3.1: Suppose that nodes employ a common transmission range $r^{(n)}(\xi)$. If $\xi > \psi^{*-1}$, the probability that there exists an isolated node decreases to zero as n increases. In other words,

$$\mathbf{P}^{(n)}(r^{(n)}(\xi)) \to 1 \text{ as } n \to \infty.$$

Also, we can show that for sufficiently large n, the probability of network connectivity is close to zero when the common transmission range is smaller than $\sqrt{\log(n)/(\psi^* \pi n)}$ in the following Lemma. Note that the probability of network connectivity is not larger than the probability that no node is isolated.

Lemma 3.2: Suppose that nodes employ a common transmission range $r^{(n)}(\xi)$. If $\xi < \psi^{*-1}$, the probability that there exists an isolated node goes to one as n increases. In other words,

$$\mathbf{P}^{(n)}(r^{(n)}(\xi)) \to 0 \text{ as } n \to \infty.$$

The proofs of Lemma 3.2 and Lemma 3.2 are provided in Appendix B.

3.3 Numerical Results

In this section, we provide a numerical example. In our example, in the network, there are $N = 200$ nodes that are mutually independent and uniformly distributed on the torus. We assume that each node i, $1 \leq i \leq 200$, has two variables (Θ_i, T_i) to represent trust relation of nodes. The trust level random variable T_i is uniformly distributed in $(\Delta, 1 + \Delta)$ and the trust threshold random variable Θ_i is uniformly distributed in $(0, 1)$. Also, Θ_i and T_i are independent. We assume that two nodes i and j are connected when they are within

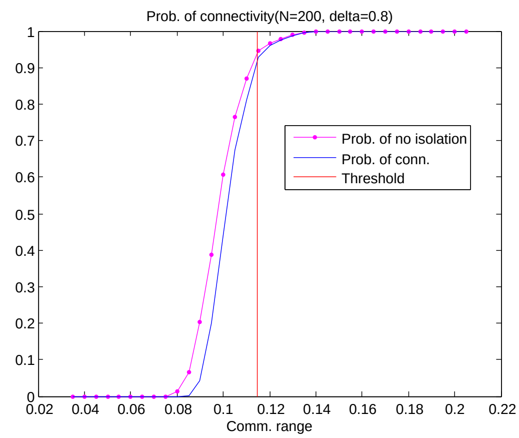


Fig. 3.1: Probability of network connectivity$(N = 200, \Delta = 0.8)$

the communication range and $T_i \geq \Theta_j$ and $T_j \geq \Theta_i$. Then, under this given distribution of trust level and trust threshold, we can derive that ψ^* is Δ^2.

We generated 1000 realization and computed the fraction of time the corresponding random graph is connected as the communication range of the nodes is varied. Fig. 3.1 plots the probability of network connectivity and the probability of no node isolation as a function of the communication range of the nodes (*x*-axis) when $\Delta = 0.8$. We also plot red vertical line at $x = 0.1148$ which comes from $\sqrt{\log{(200)}/(0.8^2 \cdot \pi \cdot 200)}$ to indicate where we expect the phase transition to occur. As the figure illustrates, indeed the probability of no node isolation increase sharply around the expected threshold. And, also, we can see that the probability of no node isolation behaves almost similar to the probability of network connectivity.

4. EXPECTED ROUTING OVERHEAD FOR LOCATION SERVICE IN MANETS UNDER FLAT GEOGRAPHIC ROUTING

In this chapter, we will study routing overhead due to location information collection and retrieval in mobile ad-hoc networks employing geographic routing. This chapter is organized as follows: Section 4.1 describes the problem we are interested in studying and provides a short summary of the results on network connectivity. Section 4.2 explains the mobility models, assumptions we introduce on mobility and the parametric scenario used to study the scaling law of expected routing overhead due to location service under different routing schemes. The minimum expected number of bits required on average to describe the approximated locations of a node is derived in Section 4.3, followed by a discussion on how expected routing overhead scales under proactive and reactive geographic routing schemes in Section 4.4. We study the minimum expected routing overhead and describe a scheme that achieves the same scaling order as the minimum expected routing overhead in Section 4.5. A discussion on our findings is provided in Section 4.6.

4.1 Setup

Throughout the chapter we use a discrete-time model and assume that time is divided into contiguous timeslots $t \in \mathbb{Z}_+ := \{0, 1, 2, \ldots\}$, where the duration of a timeslot is taken to be a unit time. Although the mobility of a node is continuous in real life, we approximate it using a discrete-time stochastic process and assume that the location of a node is fixed during a timeslot. This may be a reasonable assumption when a node is (quasi-)stationary much of the time and spends a relatively small fraction of time in transition between locations or if the duration of timeslot is small enough so that, with high probability, the location of a node does not change significantly over the duration of a single timeslot. However, with small probability, the location of a node may change significantly from one timeslot to next. A similar assumption is often introduced in the literature (e.g., [39, 56, 57]).

In a multi-hop wireless network, one-hop connectivity between nodes is likely to be maintained through exchange of control messages (e.g., HELLO messages) at the data link layer. For our analysis we model the one-hop network connectivity using a geometric random graph (GRG) [17]: Each node i is aware of and can communicate with all other nodes within its *communication* or *transmission* range γ (according to the Euclidean distance), which we call *immediate neighbors*, or simply *neighbors*, of node i. We say that there is a bi-directional link, or simply a link, between two neighbors.

The GRG model has been used extensively in the literature as an approximate model to one-hop connectivity of wireless networks (e.g., [8, 11, 12, 40, 20]). The transmission range γ in the GRG model is assumed to be determined by the transmit power employed by the nodes, channel propagation and the signal-to-noise ratio corresponding to a bit error rate constraint [8]. Under a channel loss model often used in the literature, the received power P_{rcv} is related to the transmit power P_{tx} and the distance d by

$$P_{rcv} = P_{tx} \cdot G_{tx} \cdot G_{rcv} \cdot L \cdot d^{-\alpha}, \tag{4.1}$$

where G_{tx} and G_{rcv} are the transmitter and receiver antenna gain, respectively, L accounts for system loss and other factors that may depend on the wavelength, and α is the path loss exponent [49]. If one requires that the received power $P_{rcv} \geq P_{\min}$ for some threshold $P_{\min}$, we must have

$$d \leq \left(\frac{P_{tx} \cdot G_{tx} \cdot G_{rcv} \cdot L}{P_{\min}} \right)^{1/\alpha} \tag{4.2}$$

and $P_{tx} \propto d^{\alpha}$. While our analysis is carried out under the GRG model, we will discuss how our results can be extended to different network connectivity models such as quasi unit disk model [44] and cost based model [50] in Section 4.6.

Throughout the chapter we assume that every node knows its immediate neighbors. In addition, when a packet reaches an immediate neighbor of its destination, the neighbor can deliver it to the destination in one-hop without any other information.

4.1.1 Geographic routing and overhead for location service

We assume that nodes are equipped with Global Positioning System (GPS) devices and know their positions, which are assumed accurate throughout. Each node is aware of exact locations of its immediate neighbors. [1] This can be done either by exchanging the

[1] In practice, for proper operation of geographic routing the location information of neighbors needs to be accurate *relative to* the transmission range of the nodes. However, for simplicity of exposition we assume that nodes know the exact locations of their neighbors.

GPS location information between one-hop neighbors (for example, by piggybacking it in HELLO messages) or by observing the received signal strength and angle in which signals arrive.

Nodes employ *geographic* (or *position-based*) routing; they route packets using location information of the destinations [53, 54]. It has been suggested [41, 46] that geographic routing leads to better performance in large multi-hop wireless networks than other routing schemes that do not exploit location information (e.g., destination-sequenced distance vector (DSDV) routing [47] or dynamic source routing (DSR) [42]). A main reason for the performance gain is that, while routing schemes such as DSDV require *global* topological information that can change frequently, geographic routing allows nodes to make *local* decisions based on the locations of their immediate neighbors and the destination, without having to learn end-to-end route information.

Obviously, for proper operation of geographic routing, the location information of the destination contained in packets must be accurate enough so that nodes can route them to their destinations using the destination ID and location information. However, more accurate location information requires more bits, hence, larger overhead. We are interested in the case where the provided location information of destinations is accurate enough so that multi-hop packet routing can be performed using the location information *without* having to flood the neighborhoods of destinations with packets, while minimizing the number of bits required to describe location information.

Our study aims at (i) developing a new framework for quantifying routing overhead in MANETs employing geographic routing and (ii) examining how the routing overhead (measured in the unit of bits$\times$meters per unit time proposed in [9]) required to *disseminate* and *acquire* location information of the nodes, scales with the number of nodes. We do not, however, concern ourselves with the delays experienced by messages. More precisely, we assume: (i) nodes can deliver their location information at timeslot $t \in \{1, 2, \ldots\} =: \mathbb{N}$, to any other nodes within the same timeslot (assuming network connectivity discussed in the following subsection); and (ii) assuming that nodes know where to access it, they can retrieve the location information of other nodes during the same timeslot. This implicitly assumes that the network has sufficient bandwidth to handle all overhead, including routing overhead, and to transport data in a timely manner. In practice, however, the delays incurred during dissemination and/or acquisition of location information can be non-negligible and cause inconsistency or staleness of location information.

Exchange of control messages to discover neighbors and to maintain links with them introduces additional overhead at the *data link* layer. However, we do not consider this overhead at the data link layer, including the overhead due to exchange of location information with immediate neighbors, because it does not depend on the adopted routing scheme. We refer interested readers to a study by Bisnik and Abouzeid in [33].

4.1.2 Network connectivity and critical transmission range

A primary function of a communication network is to enable exchange of information between nodes. When information is time-sensitive or cannot tolerate large delays, timely delivery of information demands that the underlying network be connected. In other words, there must exist an end-to-end path from a source to a destination (with a high probability) when such a path is desired. This is the scenario of interest we consider in this chapter.

Recently there has been much work on connectivity of a multi-hop wireless network (e.g., [8, 11, 12, 18, 20]). We refer interested readers to a monograph by Penrose [17]. In particular, Penrose [18] (and later by Santi [20]) proved the following result we will borrow: Suppose that $n, n \geq 1$, nodes are placed independently of each other, according to a common spatial density function f with connected and compact support $\mathbb{D}$ and smooth boundary $\partial\mathbb{D}$. Let γ be a common transmission range of the nodes. The network is said to be connected if, for *every* pair of nodes (i, j), we can find a sequence of links providing an end-to-end route between the two nodes.

Theorem 4.1 ([18, 20]): Define $f_\star := \inf_{\mathbf{x} \in \mathbb{D}} f(\mathbf{x})$ and assume $f_\star > 0$. The *minimum* common transmission range required for connectivity, denoted by $\gamma^\star(n)$, satisfies

$$\lim_{n \to \infty} \frac{n \, \pi \, \gamma^\star(n)^2}{\log(n)} = \frac{1}{f_\star} \qquad \text{with probability 1.} \tag{4.3}$$

4.2 Mobility model and parametric scenario

This section first describes the node mobility processes we consider, and then explains the parametric scenario we adopt to study how the *expected* routing overhead for location service increases with the network size. We define all the random variables (rvs) and stochastic processes of interest on some common probability space $(\Omega, \mathcal{F}, \mathbf{P})$.

4.2.1 Mobility model

Nodes move on a domain $[0, \overline{D}]^2 =: \mathbb{D}$. [2] As mentioned earlier, we approximate the mobility of the nodes using discrete-time processes; the *mobility process* or *trajectory* of a node i is given by a discrete-time stochastic process $\mathbb{L}_i = \{L_i(t); t \in \mathbb{Z}_+\}$, where $L_i(t) = (L_{i,x}(t), L_{i,y}(t)) \in \mathbb{D}$ specifies the *location* or *position* of the node at time t, using the Cartesian coordinate system. We assume that, at each timeslot $t \in \mathbb{N}$, the transition from $L_i(t-1)$ to $L_i(t)$ takes place at the *beginning* of the timeslot.

The steady-state spatial distribution of the nodes is assumed to yield a *continuous* density function $f : \mathbb{D} \to \mathbb{R}_+ := [0, \infty)$. For each $t \in \mathbb{Z}_+$, f^t denotes the joint density function of $(L_i(0), \ldots, L_i(t))$. We assume that there exist constants ξ_1 and ξ_2, $0 < \xi_1 \leq \xi_2 < \infty$, such that, for all $t \in \mathbb{Z}_+$ and for all $\underline{\ell}_t \in \mathbb{D}^{t+1}$,

$$0 < \xi_1^{t+1} \leq f^t(\underline{\ell}_t) \leq \xi_2^{t+1} < \infty \, , \tag{4.4}$$

i.e., for every finite t, the joint density function f^t is non-vanishing and is also upper bounded by ξ_2^{t+1} over $\mathbb{D}^{t+1}$. This implies that node's locations do not concentrate in some parts of the domain $\mathbb{D}$ over time. For example, a two-dimensional Brownian motion with reflection, starting with an appropriate initial condition and sampled periodically, satisfies this assumption. Removal of the assumption in (4.4) has a rather serious consequence on network connectivity (see [10] for an example). Its impact on expected routing overhead is discussed in more detail in Section 4.6.

[2] We assume a square region for convenience. However, similar results hold with any arbitrary compact, convex domain.

4.2.2 Parametric scenario

In order to study how the expected overhead scales with the number of nodes in the network, we consider the following parametric scenario with increasing n: For each fixed $n \in \mathbb{N}$, there are n nodes moving on the domain $\mathbb{D}$, and we denote the set of nodes by $N^{(n)} = \{1, 2, \ldots, n\}$. [3] We assume homogeneous mobility of the nodes. The mobility process of node $i \in N^{(n)}$, given by $\mathbb{L}_i^{(n)} := \{L_i^{(n)}(t); t \in \mathbb{Z}_+\}$, is assumed *stationary* and *ergodic*. Moreover, the mobility processes $\mathbb{L}_i^{(n)}, n \in N^{(n)}$, are *mutually independent*.

1. Connection requests: For each $i \in N^{(n)}$ and $t \in \mathbb{N}$, let $A_i^{(n)}(t)$ denote the number of requests arriving at the *other* nodes for a connection *to* node i at timeslot t. Without loss of generality, we assume $\{A_i^{(n)}(t); t \in \mathbb{N}\} =: \mathbb{A}_i^{(n)}$, are independent and identically distributed (i.i.d.) Bernoulli rvs with parameter $p^{(n)} > 0$. This implies that at most one other node will generate a connection request to node i, which is called the *source* of the connection, in each timeslot. We assume that the source is equally likely to be any of the remaining $n - 1$ nodes, independently of the past and the sources of other connection requests.

Each connection request arriving at its source needs the location information of its destination for geographic routing. We assume that connection requests arrive at their sources at the beginning of each timeslot $t \in \mathbb{N}$ *after* nodes move to their new locations $L_i^{(n)}(t), i \in N^{(n)}$. The connection request arrival processes $\mathbb{A}_i^{(n)}, i \in N^{(n)}$, are mutually independent and also independent of mobility processes $\mathbb{L}_i^{(n)}, i \in N^{(n)}$.

Since we are interested in studying how the routing overhead grows with the number of nodes, we assume that the average number of connection requests to each node per

[3] This is often called a *dense* network in the literature.

timeslot is fixed, i.e., $\mathbf{E}\left[A_i^{(n)}(t)\right] = p^{(n)} = p > 0$ for all $i \in N^{(n)}$ and all $n \in \mathbb{N}$. Because the source of a connection request to a node is equally likely to be any of the remaining $n-1$ nodes, it is clear that, for each fixed $n \in \mathbb{N}$, the number of connection requests that arrive at a node (as the source) in a timeslot is a binomial($n-1$, $\frac{p}{n-1}$) rv.

2. Transmission range: We are interested in the case where the nodes adjust their common transmission range to maintain network connectivity as discussed in subsection 4.1.2. Therefore, the transmission range of the nodes should be at least the CTR $\gamma^\star(n) = c^\star\sqrt{\log(n)/n}$ with $c^\star = 1/\sqrt{\pi f_\star}$ [20]. In their seminal paper on transport throughput [9], Gupta and Kumar showed that, in order to minimize interference to other simultaneous transmissions and to maximize transport throughput in a multi-hop wireless network, nodes should employ the *smallest* transmission range while maintaining network connectivity (i.e., the CTR $\gamma^\star(n)$).

In the subsequent sections we follow this finding by Gupta and Kumar [9] and assume that nodes employ a common transmission range of $\gamma^\star(n)$ to maximize transport throughput and keep the network connected with a high probability. [4]

Assumption 4.1: For each fixed $n \in \mathbb{N}$, the transmission range of the nodes is given by $\gamma^\star(n)$.

We will discuss how different choices of transmission ranges affect our findings in Sections 4.3 through 4.5.

[4] To ensure network connectivity with high probability for finite n, the transmission range should be set to $\beta^\star \cdot \gamma^\star(n)$, where $\beta^\star > 1$. However, for notational simplicity we omit $\beta^\star$ in the analysis. The omission of this constant $\beta^\star$ does not change our results.

4.3 Description of node locations

First, note that the location $L_i^{(n)}(t) \in \mathbb{D}$ of node i at time t is a two-dimensional *continuous* random vector for all $t \in \mathbb{Z}_+$. Therefore, they *cannot* be described exactly with a *finite* number of bits in general. Moreover, for the purpose of routing packets using location information, exact locations are not necessary and sufficiently accurate *approximations* of locations suffice. Hence, we are interested in finding out how accurate the location information contained in packets must be so as to allow successful routing of packets based on the provided location information.

The number of bits needed for approximated location information carried by packets for geographic routing is governed by the aforementioned required accuracy and the way location information is encoded. The first determines the *quantization level* to be selected for approximation. Bisnik and Abouzeid [33] utilized the rate distortion theory to compute the necessary information rate subject to a squared-error distortion constraint. This approach, however, may require that different quantization levels be used in different regions, depending on the spatial distribution, and allows for the possibility that the location information of nodes in an area of low spatial density is not accurate enough for successful delivery of packets.

We argue that a communication network should be able to deliver packets irrespective of nodes' locations. This is especially true when the spatial distribution of the nodes is not correlated with their communication needs. In this case, non-uniform approximation of location information demanded by rate distortion theory, which does not consider the communication needs, may compromise the communication with nodes in low spatial density areas and, hence, may be unsuitable.

In this section we investigate the minimum expected number of bits required per timeslot to specify approximated locations of a node to enable geographic routing. For the reason explained above, we assume that the selected quantization level for approximating node locations does *not* depend on their locations. Furthermore, as stated in subsection 4.2.2, we focus our study only on the case of practical interest where the network is connected with probability approaching one, by setting the common transmission range of the nodes to the CTR $\gamma^\star(n) = c^\star \sqrt{\log(n)/n}$.

Before stating our result, let us first briefly describe the class of packet routing schemes we consider. Packets carry both the destination ID and *approximated* location information. The encoding and decoding rules for approximated locations are assumed common knowledge.

1. The source of a packet encodes the location of its destination, which is approximated with a selected quantization level, using the common encoding rule and places the encoded location information in the packet.

2. A relay node that receives a packet first checks if the destination is an immediate neighbor. If so, it delivers the packet to the destination. If not, it decodes the approximated location of the destination using the common decoding rule. It then selects an immediate neighbor that is closest to the decoded approximated location as the next hop. Recall that the nodes are assumed to know the precise locations of their immediate neighbors.

It has been observed that as the network becomes dense, a greedy approach that either minimizes the distance or maximizes the forward progress to the destination works well [46]. However, when a greedy approach fails, other schemes, such as Greedy-Face-

Greedy (GFG) routing scheme [34], can be used to guarantee the delivery.

The following lemma states that the minimum expected number of bits needed on average to describe the approximated locations of a node for geographic routing approaches $\log(n)$ asymptotically as $n \to \infty$. This finding will be used to study how the expected overhead scales under proactive or reactive geographic routing (Section 4.4) and to derive the scaling law of minimum expected overhead (Section 4.5).

Lemma 4.1: The minimum expected number of bits required per timeslot to describe approximated locations of a node under Assumption 4.1, denoted by $m_{\text{loc}}(n)$, satisfies $m_{\text{loc}}(n) \sim \log(n)$.

Proof: We find lower and upper bounds for $m_{\text{loc}}(n)$ and show that both bounds are asymptotically $\log(n)$.

1. Lower bound: In order to find a lower bound for $m_{\text{loc}}(n)$, consider the following: Suppose that a quantization level of $4\gamma^\star(n)$ is selected for approximating locations and the domain $\mathbb{D}$ is divided into cells of length $4\gamma^\star(n)$, where $\gamma^\star(n) = c^\star\sqrt{\log(n)/n}$ is the CTR introduced in subsection 4.1.2, as shown in Fig. 4.1.

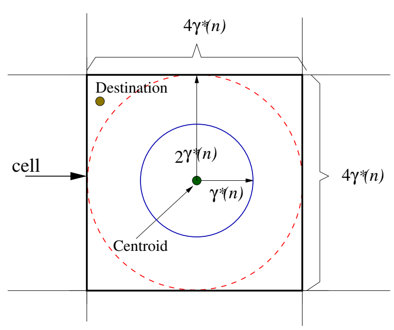


Fig. 4.1: Partition of $\mathbb{D}$ into cells of length $4\gamma^\star(n)$ on both sides.

Without loss of generality, we assume that the approximated locations of the nodes in a cell with the assumed quantization level are given by the centroid of the cell. This means that a relay node forwarding a packet to the destination shown in the figure will use the location of the centroid as the approximated location of the destination (after decoding the location using the common decoding rule). If none of relay nodes is an immediate neighbor of the destination (which is more likely than not), the packet will eventually enter the inner circle centered at the centroid with radius $\gamma^\star(n)$. Once this happens, the packet cannot be delivered to any node outside the outer (dotted) circle with radius $2\gamma^\star(n)$; the nodes inside the inner circle do not know the precise locations of the nodes outside the outer circle because they are not immediate neighbors. This implies that, without knowing a more precise location of the destination, the entire cell will need to be flooded with the packet before it can reach its destination. This tells us that the quantization level of $4\gamma^\star(n)$ is *not* accurate enough to prevent flooding of the packet.

Let us compute the expected number of bits required per timeslot to describe the locations using this *insufficient* quantization level of $4\gamma^\star(n)$. Under the stated assumptions on stationarity of the mobility processes and spatial density in (4.4) in subsection 4.2.1, the differential entropy rate of the mobility process [35, p.416]

$$h_\star := \lim_{T\to\infty} \frac{h(L_i^{(n)}(0), L_i^{(n)}(1), \dots, L_i^{(n)}(T-1))}{T} \tag{4.5}$$

exists and is bounded below (resp. above) by $-\log(\xi_2)$ (resp. $-\log(\xi_1) < \infty$).

For each $\Delta > 0$, let $L_{i,\Delta}^{(n)}(t)$ be an approximation of $L_i^{(n)}(t)$ with a quantization level Δ; $L_{i,\Delta}^{(n)}(t) = ((k_1+\frac{1}{2})\Delta, (k_2+\frac{1}{2})\Delta)$ if $L_i^{(n)}(t) \in [k_1\cdot\Delta,\ (k_1+1)\Delta)\times[k_2\cdot\Delta,\ (k_2+1)\Delta)$. Denote the *approximated* mobility processes by $\mathbb{L}_{i,\Delta}^{(n)} = \{L_{i,\Delta}^{(n)}(t); t \in \mathbb{Z}_+\}$. From the inherited stationarity of the approximated mobility processes, the entropy rate of $\mathbb{L}_{i,\Delta}^{(n)}$

$$H_{\Delta}^{(n)} := \lim_{T\to\infty} \frac{H(L_{i,\Delta}^{(n)}(0), L_{i,\Delta}^{(n)}(1), \ldots, L_{i,\Delta}^{(n)}(T-1))}{T} \tag{4.6}$$

exists [35, Thms 4.2.1 and 4.2.2, p.75]. In addition, the following equality holds [35, Thm 8.3.1, p.248]: For all $T \geq 1$,

$$\begin{aligned}
&\lim_{\Delta\downarrow 0}\left(\frac{H(L_{i,\Delta}^{(n)}(0), L_{i,\Delta}^{(n)}(1), \ldots, L_{i,\Delta}^{(n)}(T-1))}{T} + 2\log(\Delta)\right)\\
&= \frac{h(L_{i}^{(n)}(0), L_{i}^{(n)}(1), \ldots, L_{i}^{(n)}(T-1))}{T} \;.
\end{aligned} \tag{4.7}$$

Equations (4.5) through (4.7) imply that, for every $\nu > 0$, there exist $\Delta^*(\nu) > 0$ and $T^*(\nu) < \infty$ such that, for all $\Delta \leq \Delta^*(\nu)$ and $T \geq T^*(\nu)$, we have

$$\begin{aligned}
h_\star - 2\log(\Delta) - \nu \leq \frac{H(L_{i,\Delta}^{(n)}(0), \ldots, L_{i,\Delta}^{(n)}(T-1))}{T}&\\
\leq h_\star - 2\log(\Delta) + \nu \;.&
\end{aligned} \tag{4.8}$$

Substituting $\Delta(n) := 4\gamma^\star(n)$ in place of Δ yields

$$\begin{aligned}
&h_\star - 2\log(\Delta(n)) \pm \nu\\
&= h_\star - 2\log(4\gamma^\star(n)) \pm \nu = h_\star - 2\log\left(4\, c^\star\sqrt{\frac{\log(n)}{n}}\right) \pm \nu\\
&= \log(n) - \log(\log(n)) + (h_\star \pm \nu - 4 - 2\log(c^\star)) \;.
\end{aligned} \tag{4.9}$$

Since $h_\star \pm \nu - 4 - 2\log(c^\star)$ are fixed, it is clear from (4.9) that $h_\star - 2\log(\Delta(n)) \pm \nu \sim \log(n)$. Together with (4.8), this proves that, for *all* sufficiently large T,

$$\frac{H(L_{i,\Delta(n)}^{(n)}(0), \ldots, L_{i,\Delta(n)}^{(n)}(T-1))}{T} \sim \log(n) \;. \tag{4.10}$$

The left hand side of (4.10) is equal to the minimum expected number of bits we need per timeslot to *jointly* code the locations, $L^{(n)}_{i,\Delta(n)}(0), \ldots, L^{(n)}_{i,\Delta(n)}(T-1)$, [5] using an insufficient quantization level $\Delta(n)$. Hence, it serves as a lower bound to the number of bits we need, and (4.10) tells us that this lower bound increases (asymptotically) as $\log(n)$.

2. Upper bound: We can obtain an upper bound for $m_{\text{loc}}(n)$ following essentially the same argument used to find the lower bound: Recall that, in order to route a packet to a node i, it suffices to deliver the packet to any immediate neighbor within the transmission range $\gamma^\star(n)$ of node i. As in the previous case of lower bound, suppose that the domain $\mathbb{D}$ is divided into cells of length $\varsigma^{(n)}$, where $\varsigma^{(n)} := \sqrt{2}\gamma^\star(n)/3$. The approximated location of a node in a cell is given by the centroid of the cell. This is shown in Fig. 4.2.

A packet is relayed using the location of the centroid of the cell in which its destination lies. If none of relay nodes the packet traverses before it enters the cell is an immediate neighbor of its destination, it will eventually be relayed to a node in the same cell as the destination. [6] It is clear from Fig. 4.2 that, once a packet reaches any node in the same cell as the destination, the node will be able to deliver the packet directly to the destination because the distance between any two nodes in the same cell is bounded by $2\gamma^\star(n)/3$. Therefore, approximating locations with a quantization level of $\varsigma^{(n)}$ is *sufficient* to ensure successful delivery of packets using the approximated location information.

We proceed to compute the average number of bits needed per timeslot to approximate the locations using the quantization level $\varsigma^{(n)}$. From [35, Thm 8.3.1, p.248] and

[5] Joint coding of the locations of node i requires that, for each $t \in \mathbb{Z}_+$, the sequence of the locations $\{L^{(n)}_{i,\Delta(n)}(0), \ldots, L^{(n)}_{i,\Delta(n)}(t)\}$ be coded together, using a different coding scheme. As a result, such joint coding of node's locations will be difficult to implement in practice.

[6] Here we assume that there is a node in the cell with a high probability. We will revisit this issue in Section 4.5 and show that the probability that there is no node in the cell goes to zero as $n \to \infty$.

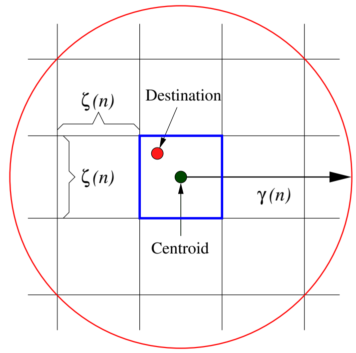


Fig. 4.2: Partition of $\mathbb{D}$ into cells with area of $\varsigma(n)^2$. ($\gamma^\star(n) = 3\,\varsigma(n)/\sqrt{2}$)

assumed stationarity of the mobility processes, we have

$$\lim_{\Delta \downarrow 0} H(L_{i,\Delta}^{(n)}(t)) + 2\log(\Delta) = h(L_i^{(n)}(t)) \qquad \text{for all } t \in \mathbb{Z}_+ \,.$$

Thus, for every $\nu > 0$, we can find $\Delta^\dagger(\nu) > 0$ such that, for all $\Delta \leq \Delta^\dagger(\nu)$,

$$\begin{aligned} h(L_i^{(n)}(t)) - 2\log(\Delta) - \nu \leq H(L_{i,\Delta}^{(n)}(t)) \\ \leq h(L_i^{(n)}(t)) - 2\log(\Delta) + \nu \,. \end{aligned} \tag{4.11}$$

Following the same steps in (4.9), after a little algebra

$$\begin{aligned} & h(L_i^{(n)}(t)) - 2\log(\varsigma^{(n)}) \pm \nu \\ & = h(L_i^{(n)}(t)) - 2\log\left(\frac{\sqrt{2}}{3} c^\star \sqrt{\frac{\log(n)}{n}}\right) \pm \nu \\ & = \log(n) - \log(\log(n)) \\ & \quad + (h(L_i^{(n)}(t)) \pm \nu - 2\log(\sqrt{2}/3) - 2\log(c^\star)) \\ & \sim \log(n) \,. \end{aligned} \tag{4.12}$$

Therefore, from (4.11) and (4.12) we find

$$H(L_{i,\varsigma(n)}^{(n)}(0)) \;\sim\; \log(n) \,. \tag{4.13}$$

Equation (4.13) suggests that, even when the locations of a node are coded *separately* at each timeslot, from the assumed ergodicity of mobility processes, the minimum *average* number of bits needed per timeslot to approximate node i's locations with a sufficient quantization level $\varsigma(n)$ is (asymptotically) $\log(n)$. Thus, from the lower and upper bounds in (4.10) and (4.13), respectively, one can conclude that the minimum expected number of bits needed per timeslot to describe the locations of a node satisfies $m_{\text{loc}}(n) \sim \log(n)$. ■

The above proof of Lemma 4.1 reveals the following interesting observation: In the calculation of $m_{\text{loc}}(n)$, node i's mobility determines the differential entropy of $L_i^{(n)}(t), t \in \mathbb{Z}_+$, and the differential entropy rate $h_\star$ of the mobility process $\mathbb{L}_i^{(n)}$. When the network size is small, the number of bits required to describe node i's locations is mostly governed by these differential entropy and entropy rate that depend on the details of the mobility processes. However, as the number of nodes n grows, in a dense network [7] $m_{\text{loc}}(n)$ is predominantly shaped by the required quantization level for describing the locations of nodes, which is in turn dictated by the CTR needed for network connectivity. As a result, the details of nodes' mobility become less important in a large, dense network, as long as the differential entropy of the locations of nodes and the differential entropy rate of the mobility processes, $h_\star$, are bounded, which is satisfied under the assumption in (4.4).

A similar result to Lemma 4.1 can be obtained for the cases where nodes are allowed to use different transmission ranges under the following assumption.

Assumption 4.2: Suppose that the nodes employ heterogeneous transmission ranges and

[7] A similar result can be obtained for extended networks with increasing domains.

that there exist constants $c_1, c_2 \in (0, \infty)$ and $0 < a_2 \le a_1 < \infty$ such that the transmission ranges of the nodes can be lower and upper bounded by $c_1 \cdot n^{-a_1}$ and $c_2 \cdot n^{-a_2}$, respectively, for all sufficiently large $n \in \mathbb{N}$.

Corollary 1: The minimum expected number of bits required per timeslot to describe approximated locations of a node under Assumption 4.2, denoted by $m^{\star}_{\text{loc}}(n)$, is $\Theta(\log(n))$.

The proof of the corollary is essentially the same as that of Lemma 4.1; we can show that the quantization level of $4 \cdot c_2 \cdot n^{-a_2}$ is not accurate enough, whereas $\sqrt{2} \cdot c_1 \cdot n^{-a_1}/3$ is a sufficient quantization level. These quantization levels give us asymptotic lower and upper bounds of $2 \cdot a_2 \cdot \log(n)$ and $2 \cdot a_1 \cdot \log(n)$, respectively, for $m^{\star}_{\text{loc}}(n)$. As we will see, this important observation allows us to relax Assumption 4.1 without voiding our findings in the following sections (Theorems 4.2 through 4.4).

4.4 Routing overhead under proactive and reactive geographic routing

In this section we examine how the expected routing overhead scales when proactive or reactive geographic routing is employed and address the issue of how to measure the total distance traveled by control messages. Recall that a geographic routing scheme is called a *proactive geographic* routing scheme if each node attempts to maintain consistent, up-to-date location information for *every* known destination in the network by flooding the network with *location update messages*. Similarly, a geographic routing scheme is said to be a *reactive geographic* routing scheme if location information is provided only when it is requested. When no location information of a desired destination is available at a source, the location information is discovered by flooding the network with a *location* *request message* until another node, possibly the destination itself, replies to the request with location information. We point out that these proactive or reactive *geographic* routing schemes are different from the traditional proactive or reactive routing algorithms that use topological information.

4.4.1 Routing overhead under proactive geographic routing

Suppose that location information of a node is forwarded to and stored at all other nodes within distance $\epsilon > 0$. If $\epsilon \geq \sqrt{2} \cdot \overline{D}$, the location information of every node is forwarded to *all* nodes in the network. This is because the distance between any two points in $\mathbb{D}$ is upper bounded by $\sqrt{2} \cdot \overline{D}$, i.e., $\sup_{\mathbf{x},\mathbf{y} \in \mathbb{D}} ||\mathbf{x} - \mathbf{y}|| = \sqrt{2} \cdot \overline{D}$, where $||\mathbf{x} - \mathbf{y}||$ is the Euclidean distance between $\mathbf{x}$ and $\mathbf{y}$. First, it is clear that, under our assumptions in subsection 4.2.2, at least $\log(n)$ (and at most $\log(n) + 1$) bits are required to identify the source of a message.

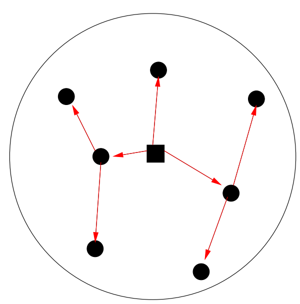


Fig. 4.3: Total distance traveled by a location message.

The total distance traveled by a location update message from a node, say node i, to all its neighbors within distance ϵ can be computed in different ways. In this chapter, we take the viewpoint that once a neighbor receives the location information of node i, it can serve as a surrogate source of the location information for other nodes. This is shown in Figure 4.3. It is more consistent with the operation of a multi-hop wireless network where each relay node is responsible for delivering a packet to the next hop, and thus each transmitter-receiver pair can be viewed as a source-destination pair for the purpose of exchanging location information.

If we count only the first copy that arrives at each node, the total distance traveled by a location update message to all the nodes within distance ϵ is given by the total length of a *spanning tree* constructed by the propagation of the message, which connects all the nodes within ϵ. Obviously, this distance is lower bounded by the total length of a *minimum* spanning tree (MST). In fact, by the definition of an MST, the total length of an MST is the minimum among all (reasonable) measures of the total distance connecting all neighbors.

Theorem 4.2: The minimum expected overhead required per timeslot under Assumption 4.1 for *disseminating* location information in proactive geographic routing is $\Omega(n^{1.5}\log(n))$.

proof: Let us first introduce a lemma that will be used in the proof of the theorem. Suppose $\{X_n; n \in \mathbb{N}\}$ is a sequence of rvs, where X_n is a binomial(n, p) rv with $0 < p < 1$.

Lemma 4.2: Define $Z_n^\alpha := \left(\frac{X_n}{n \cdot p}\right)^\alpha$, where $0 < \alpha \leq 1$. Then,

$$\lim_{n\to\infty} \mathbf{E}\left[Z_n^\alpha\right] = 1 \qquad \text{for all } \ 0 < \alpha \leq 1 \ .$$

proof: Let $Y_n := Z_n^1 = \frac{X_n}{n \cdot p}$. The strong law of large numbers [7, p.326] tells us that Y_n converges to 1 in mean square, i.e., $\mathbf{E}\left[|Y_n - 1|^2\right] \to 0$ as $n \to \infty$. Since $|Z_n^\alpha - 1| \leq |Y_n - 1|$ for all $0 < a \leq 1$, we have

$$\mathbf{E}\left[|Z_n^\alpha - 1|^2\right] \leq \mathbf{E}\left[|Y_n - 1|^2\right] \to 0 \qquad \text{as } \ n \to \infty,$$

which implies $Z_n^\alpha \to 1$ in mean square (clearly, $\mathbf{E}\left[(Z_n^\alpha)^2\right] \leq 1 + \mathbf{E}\left[(Y_n)^2 \cdot \mathbb{1}\{Y_n > 1\}\right] < \infty$ for all $n \in \mathbb{N}$).

Recall that convergence in mean square implies convergence in mean [7, p.310]. Hence,

$$\mathbf{E}\left[|Z_n^\alpha - 1|\right] \to 0 \quad \text{as } n \to \infty.$$

Theorem 3 [7, p.351] tells us that $Z_n^\alpha \to 1$ in mean *if and only if* $\mathbf{E}\left[Z_n^\alpha\right] \to 1$ as $n \to \infty$ (and, equivalently, $\{Z_n^\alpha; n \geq 1\}$ is uniformly integrable). This completes the proof of the lemma. ∎

We now proceed with the proof of Theorem 4.2. Steele [52] showed the following result on the total length of an MST with an increasing number of nodes: Suppose that nodes are placed independently of each other in accordance with distribution μ with compact support $\mathbb{S} \subset \mathbb{R}^2$. Let $M(n)$ denote the total length of an MST connecting the first n nodes. Then, with probability 1,

$$\lim_{n\to\infty} \frac{M(n)}{\sqrt{n}} = e^\star \int_{\mathbf{x}\in\mathbb{S}} \sqrt{g(\mathbf{x})}\, d\mathbf{x} \tag{4.14}$$

for some constant $e^\star$, where g is the density of the absolutely continuous part of μ. In other words, the total length of an MST is asymptotically proportional to $\sqrt{n}$.

From the assumed mutual independence and stationarity of the mobility processes $\mathbb{L}_i^{(n)}, i \in N^{(n)}$, the number of nodes in an area $\tilde{D} \subset \mathbb{D}$ at timeslot t is a binomial rv with parameter $(n, p_{\tilde{D}})$, where $p_{\tilde{D}} = \int_{\tilde{D}} f(\mathbf{y})\, d\mathbf{y}$. Let $d_\epsilon(\mathbf{x})$ denote the intersection of the mobility domain $\mathbb{D}$ and the disk centered at $\mathbf{x} \in \mathbb{D}$ with radius ϵ. Then, for every $\mathbf{x} \in \mathbb{D}$, Area$(d_\epsilon(\mathbf{x})) \geq \pi\, \epsilon^2/4$, hence $\xi_1\, \pi\, \epsilon^2/4 \leq \int_{d_\epsilon(\mathbf{x})} f(\mathbf{y})\, d\mathbf{y} \leq \xi_2\, \pi\, \epsilon^2$.

This observation, combined with the result by Steele in (4.14) and Lemma 4.2 with

$\alpha = 1/2$, suggests that the expected total length of an MST that connects all nodes in $d_\epsilon(\mathbf{x})$ is asymptotically proportional to $\sqrt{n}$ for all $\mathbf{x} \in \mathbb{D}$. Therefore, from the assumed ergodicity and mutual independence of the mobility processes, the average total distance traveled by a location update message of node i to its neighbors within a fixed distance ϵ is $\epsilon^2 \cdot \Omega(\sqrt{n})$.

Since (i) there are n nodes that move according to mutually independent mobility processes, (ii) each message requires at least $\log(n)$ bits to identify the source of the message, (iii) location information of the source needs asymptotically $\log(n)$ bits from Lemma 4.1, and (iv) the average total distance traveled by a location message is $\Omega(\sqrt{n})$, the expected routing overhead (measured in bits$\times$meters per unit time) for disseminating location information to local neighborhoods per timeslot under proactive geographic routing is $\Omega(n \cdot \log(n) \cdot \sqrt{n}) = \Omega(n^{1.5} \log(n))$. ■

4.4.2 Routing overhead under reactive geographic routing

As stated earlier we assume that, under reactive geographic routing, if location information is not available at a source when a connection request arrives, it generates a location request message and floods the network. When a node with the requested location information receives the request message, it generates a *location reply message* with the location information. In this subsection, we study the expected overhead due to the location request messages and location reply messages under reactive geographic routing.

In practice there may be additional overhead due to location recovery when a destination moves while the connection is active and the source does not know the new location of the destination. However, we do not study the overhead due to the recovery of location information while connections are still active. We will discuss this issue in section 4.6-C.

In order to make progress, we introduce following simplifying assumptions:

A1. Only the destination for which a location request is generated responds with a reply message;

A2. Location request messages reach the nodes in the order of increasing distance from their sources.

Assumption A2 implies that if a node that generates a reply message is at distance $\overline{d}$ from the source, the request message reaches all the nodes within distance $\overline{d}$ from the source.

Under our assumption in subsection 4.2.2 that $A_i^{(n)}(t)$ are i.i.d. Bernoulli rvs, at most one request is generated for a connection to node i in each timeslot. Thus, no other node will have cached up-to-date location of node i. However, when more than one node can generate a connection request to node i in a timeslot, it is possible that some other nodes that acquired node i's location information may cache the location information, and a reply can be generated by another node with cached location information. In this case, we can replace Assumption A1 with the following alternate assumption, without modifying our findings below:

A1a. Suppose that the location of a source generating a location request message is $\underline{\ell} \in \mathbb{D}$. Then, the location of the *closest* node that generates a reply message depends only on $\underline{\ell}$ and has distribution $M(\cdot, \underline{\ell})$.

Assumption A1a means that the distance to the closest node sending a reply does *not* depend on the number of nodes in the network. This may be reasonable when the location information of each node is available only at a limited number of other nodes, in particular in a small neighborhood around the node. When only the destination is allowed to generate a reply message, Assumption A1a holds by virtue of mutual independence of the mobility processes. Here, we assume that Assumption A1 (instead of Assumption A1a) is in place.

Theorem 4.3: The minimum expected overhead required per timeslot under Assumption 4.1 for location request and reply messages in reactive geographic routing is $\Omega(n^{1.5}\log(n))$.

proof: We examine the routing overhead that arises from location requests and replies separately. We first show that the expected overhead due to handling location requests is $\Omega(n^{1.5}\cdot\log(n))$, and then demonstrate that the expected overhead from location replies is $\Theta(n\cdot\log(n))$.

First, each location request message must have the ID of the destination, which requires at least $\log(n)$ bits. Second, analogously to the proactive geographic routing case, the total distance traveled by a location request message to all the nodes within the distance to the destination is lower bounded by the total length of an MST connecting the nodes. Under these assumptions, by conditioning on the distance to the destination and following the same argument used in the proof of Theorem 4.2, one can show that the expected total length of such an MST is $\Omega(\sqrt{n})$. Therefore, since location requests arrive at a rate of p at each node, the expected overhead for handling the request messages is $\Omega(n\cdot\log(n)\cdot\sqrt{n}) = \Omega(n^{1.5}\ \log(n))$.

Unlike location requests, location replies need not be flooded. [8] Also, because (i) the source of a request for the location information of node i is equally likely to be any of

[8] Replies can be routed back either by using the location information of the sources attached to the request messages or by maintaining a cache at intermediate nodes which temporarily stores all request messages received over a sliding time window along with the first nodes that forwarded them.

the other $n-1$ nodes, (ii) spatial density f does not vary with n, (iii) connection request processes $\mathbb{A}_i^{(n)}, i \in N^{(n)}$, are independent of the mobility processes, and (iv) the mobility processes $\mathbb{L}_j^{(n)}, j \in N^{(n)}$, are assumed *stationary* and *ergodic* and are also *mutually independent*, the *average* distance between the sources and the destinations (averaged over all timeslots and all source-destination pairs) is equal to the *expected* distance between a pair of randomly selected nodes. This expected distance is given by [9]

$$d_{\text{avg}} \equiv \int_{\mathbb{D}} \int_{\mathbb{D}} ||\mathbf{x} - \mathbf{y}|| \; f(\mathbf{x}) \, f(\mathbf{y}) \, d\mathbf{y} \, d\mathbf{x} > 0 \, . \tag{4.15}$$

The inequality follows from the assumption $\inf_{\mathbf{x} \in \mathbb{D}} f(\mathbf{x}) \geq \xi_1 > 0$ in (4.4) with $t = 0$. Note that d_{avg} does *not* depend on the number of nodes n. Since reply messages must carry the ID of the source and the location information of the destination (and the source), the overhead due to reply messages is $d_{\text{avg}} \cdot \Theta(n \cdot \log(n))$. Therefore, the overall routing overhead under reactive geographic routing is $\Omega(n^{1.5} \cdot \log(n))$. ∎

It is clear from the proof of Theorems 4.2 and 4.3 that the derived scaling laws for the expected overhead under proactive and reactive geographic routing do *not* change when Assumption 4.1 is replaced by Assumption 4.2. This is because the average number of bits in control messages remains $\Theta(\log(n))$ from Corollary 1.

In the following section, we will show that, compared to the minimum expected routing overhead, both proactive and reactive geographic routing suffers a penalty of at least $\sqrt{n}$ for flooding the network with either the location information of nodes (proactive geographic routing) or location request messages (reactive geographic routing). This also

[9] When Assumption A1a is in place instead and node i is a node with cached location information of the requested destination, (4.15) is replaced by $d_{\text{avg}} \equiv \int_{\mathbb{D}} (\int_{\mathbb{D}} ||\mathbf{x} - \mathbf{y}|| \; m(\mathbf{y}, \mathbf{x}) \, d\mathbf{y}) \; f(\mathbf{x}) \, d\mathbf{x} > 0$, where $m(\cdot, \mathbf{x})$ is the derivative of $M(\cdot, \mathbf{x})$.

hints that if we eliminate or reduce flooding of messages, we can alter the way routing overhead scales with the increasing network size.

4.5 Minimum expected routing overhead

In this section we examine how the *minimum* expected routing overhead from location service scales with the number of nodes under the assumptions stated in Section 4.2: For each fixed $n \in \mathbb{N}$, let us denote the minimum expected overhead required per timeslot for *disseminating* and *acquiring* location information under Assumption 4.1 by $R_{\min}(n)$. We prove that $R_{\min}(n) = \Theta(n \cdot \log(n))$ in two steps: First, we show that $R_{\min}(n)$ increases at least as $\alpha \cdot n \cdot \log(n)$ for some constant α, i.e., $R_{\min}(n) = \Omega(n \cdot \log(n))$. Second, we demonstrate that, for all sufficiently large n, the minimum expected routing overhead is upper bounded by $\beta \cdot n \cdot \log(n)$ for another constant β, proving $R_{\min}(n) = O(n \cdot \log(n))$. These two findings yield our claim that $R_{\min}(n) = \Theta(n \cdot \log(n))$.

Lemma 4.3: The minimum expected overhead for location service per timeslot under Assumption 4.1, $R_{\min}(n)$, is $\Omega(n \cdot \log(n))$.

proof: Let us first focus on a single connection request originating, say, at node $k \in N^{(n)}$, with node $i, i \neq k$, as the destination. First, *any* location message of node i must carry its ID and location. As mentioned earlier, a minimum of $\log(n)$ bits are needed to identify node i in the message, and from Lemma 4.1, the minimum expected number of bits required on average to describe the locations of node i asymptotically approaches $\log(n)$. Second, the *expected* distance the location message of node i must travel from node i to node k is given by d_{avg} in (4.15) and does not depend on n.

Summarizing these, (i) for the same reason provided before (4.15) in the proof of Theorem 4.3, the average distance the location messages have to travel from the destinations to the sources of connection requests equals $d_{\text{avg}} > 0$, and (ii) the expected number of bits in each location message required for both the ID and the location of a destination is $\Theta(\log(n))$. Since the average total number of connection requests in a timeslot equals $n \cdot p$, the minimum expected routing overhead required on average (in bits$\times$meters per unit time) for delivering location information from the destinations to the sources of connection requests is $\Theta(n \cdot \log(n))$. Obviously, the minimum expected routing overhead for location service cannot be smaller than the overhead required for transporting location information directly from the destinations to the sources. Hence, $R_{\min}(n) = \Omega(n \cdot \log(n))$.

■

Lemma 4.4: The minimum expected overhead for location service per timeslot under Assumption 4.1, $R_{\min}(n)$, is $O(n \cdot \log(n))$.

proof: In order to prove the lemma, it suffices to find a scheme under which the expected routing overhead per timeslot is upper bounded by $\beta \cdot n \cdot \log(n)$ for all sufficiently large n, for some finite constant $\beta > 0$. The scheme we describe here combines the features of both proactive and reactive geographic routing schemes in such a way we can avoid expensive *multi-hop* flooding of messages, by forming *virtual* location servers using the existing nodes. A similar idea of using existing nodes as location servers without knowing their identities was used by Li et al. [46].

A key idea is that we store the location information of each node i in a small region (relative to the transmission range) so that once a location request message for node i

reaches *some* node in the region, the node, if it does not have the location information of node i, can find another node with the location information *without* having to flood a multi-hop neighborhood. In this sense, the set of nodes in the region, which varies with time, *collectively* serve as a virtual location server on behalf of node i. Hence, individual nodes participate not only in routing packets, but also in providing location service for other nodes.

To this end, we choose a quantization level of $\varsigma^{(n)} = \sqrt{2}\gamma^{\star}(n)/3$ for approximating node locations [10] , divide the domain of mobility into *cells* of area $\mathcal{A}(n) = \varsigma^{(n)} \times \varsigma^{(n)}$, and store the location information of each node in a cell with a *known* coordinate. [11] The coordinate of the cell where the location information of node i resides, is computed using a hash function $h^{(n)} : N^{(n)} \to S_{h^{(n)}}$, where $S_{h^{(n)}}$ is the set of coordinates of the cells that hold location information. This allows us to skirt the problem of not having location servers with known or fixed locations. In addition, the hash functions can be designed to distribute the load of storing location information among the nodes. The hash functions $h^{(n)}$ are assumed common knowledge.

First, if we are to store location information in a cell, we must ensure that there is at least one node in the cell (with probability approaching 1 as $n \to \infty$) so that location information can be stored in the cell and be accessible to other nodes. It is obvious that $\mathcal{A}(n) = 2\, c^{\star 2}\ \log(n)/(9\, n) = \omega(1/n)$. From the assumption on the spatial distribution in (4.4) and mutual independence of the mobility processes, for the given cell size $\mathcal{A}(n)$ the

[10] Recall from the proof of Lemma 4.1 that $\varsigma^{(n)}$ is sufficient for enabling geographic routing.
[11] By storing location information in a cell, we mean storing it at one or more nodes in the cell.

probability that there is no node in a cell at timeslot $t \in \mathbb{Z}_+$ approaches zero as $n \to \infty$:

$$\mathbb{P}\left[\text{No node in a cell at timeslot } t\right] \leq (1 - \xi_1 \cdot \mathcal{A}(n))^n$$

For $\mathcal{A}(n) = 2\, c^{\star 2}\, \log(n)/(9\, n)$,

$$\begin{aligned}(1 - \xi_1 \cdot \mathcal{A}(n))^n &= \exp\left(n \cdot \log(1 - \xi_1 \cdot \mathcal{A}(n))\right) \\ &\to 0 \quad \text{as } n \to \infty\,.\end{aligned}$$

In the rest of the proof we describe how the location information is disseminated and retrieved by nodes and compute the overhead due to these operations.

1. Dissemination of location information: As mentioned earlier, under our scheme, we convey and store the location information of a node i in cell $h^{(n)}(i)$ that serves as a virtual location server for node i. Intermediate nodes route a location message of node i using the location of the cell $h^{(n)}(i)$ computed using the common hash function. [12] Unlike a unicast data packet that is routed to a specified destination node, however, a location update message of node i does not include a specific destination ID in the message. This is because node i is unlikely to know in advance which nodes are in cell $h^{(n)}(i)$. Instead, since there is a node in cell $h^{(n)}(i)$ (with probability approaching 1), when a location update message reaches *some* node in cell $h^{(n)}(i)$, the node stores the location information of node i and terminates the message without relaying it further. Obviously, the distance traveled by a location message of node i to any node in cell $h^{(n)}(i)$ is upper bounded by $\sqrt{2} \cdot \overline{D}$.

[12] The location of the cell is the same as the approximated location of a node in the cell, i.e., the centroid of the cell, as explained in Section 4.3.

A location update message of a node carries the node's ID and location information. Combining with our finding in (4.13) that the minimum average number of bits needed to describe the locations of a node with quantization level $\varsigma(n)$ is asymptotically $\log(n)$, we conclude that the routing overhead due to transporting the location information of the nodes to their respective cells that store their location information is $R_T(n) = \Theta(n \cdot \log(n))$.

2. Retrieval of location information: In order for a node j to access the location information of another node, say node i, node j first generates a location request with (i) its own ID and location information (with the same quantization level $\varsigma(n)$), and (ii) the ID of node i. The request message is then relayed by intermediate nodes using the location of the cell $h^{(n)}(i)$ computed from the ID of node i in the request message and the common hash function, until it reaches *some* node in cell $h^{(n)}(i)$.

When the request message arrives at a node in cell $h^{(n)}(i)$, one of following two events occurs: (i) If the node has the location information of node i, it generates a reply message, or (ii) if it does not, it broadcasts the request message to its neighbors in cell $h^{(n)}(i)$, all of which lie *within* its transmission range. In the latter event, since there is at least one node in the cell $h^{(n)}(i)$ with the location information of node i (with probability approaching 1), another node in cell $h^{(n)}(i)$ with the location information generates a reply message. Again, the reply message is heard by all other nodes in the cell because they are all within the transmission range, hence only a single reply message is generated.

In the case of second event, compared to the first, one additional broadcast transmission is required. However, there is no need to flood a *multi-hop* neighborhood in search of a node with location information (which is the case with reactive geographic routing).

The total distance traveled by the broadcast request message over the last hop to all the nodes in cell $h^{(n)}(i)$ can be computed as follows: From the assumed mutual independence of the mobility processes, the number of nodes in cell $h^{(n)}(i)$ is a binomial($n, \tilde{p}$) rv, where $\tilde{p}$ is the steady-state probability that a node is in cell $h^{(n)}(i)$. Recall that (i) from the assumption on spatial distribution in (4.4), $\tilde{p}$ is upper bounded by $\xi_2 \times$ area of a cell ($= \xi_2 \times 2\, c^{\star 2}\ \log(n)/(9 \cdot n)$) and (ii) the distance from the last relay node to any node in $h^{(n)}(i)$ that hears the message is bounded by the transmission range $\gamma^\star(n)$. Thus, the total expected distance from the last relay node to all nodes in $h^{(n)}(i)$ is upper bounded by

$$\begin{aligned} n \cdot \tilde{p} \cdot \gamma^\star(n) &\leq \frac{2\, \xi_2\, c^{\star 2}\ \log(n)}{9} \times c^\star \sqrt{\frac{\log(n)}{n}} \\ &= \frac{2\, \xi_2\, c^{\star 3}\ \log^{1.5}(n)}{9\, \sqrt{n}}\ . \end{aligned} \tag{4.16}$$

It is clear that (4.16) decreases to zero as $n \to \infty$. This tells us that the contribution from the last hop to the total expected distance traveled by a request message vanishes as $n \to \infty$, and that the total expected distance is $\Theta(1)$.

A reply message produced in response to a request message contains the IDs and location information of both nodes j and i. The reply message is then routed back to the source (i.e., node j), using the location information of node j copied from the request message. It is obvious that the expected distance traveled by a reply message is $\Theta(1)$.

Since a location request message generated by node j contains the IDs of both nodes j and i and the location information of node j with quantization level $\varsigma(n)$, the required expected number of bits in a location request is on average $b_{\text{req}}(n) \sim 3\log(n)$ from (4.13). Similarly, the expected number of bits required in reply messages for the IDs and location information of both nodes is on average $b_{\text{res}}(n) \sim 4\log(n)$. Hence, the expected number of bits needed for handling a single location request under our scheme is on average $b(n) = b_{\text{req}}(n) + b_{\text{res}}(n) \sim 7\log(n)$. Recall from Section 4.2 that each node generates route request messages at a rate of p requests per timeslot. Together with earlier findings on the expected total distance traveled by request and reply messages and their sizes, we conclude that the expected routing overhead incurred per timeslot due to retrieval of location information under our scheme is $R_A(n) = \Theta(n \cdot \log(n))$.

The minimum expected routing overhead $R_{\min}(n)$ is obviously not greater than the expected routing overhead incurred by our scheme, which is $R^{\star}(n) \equiv R_T(n) + R_A(n)$. Since $R^{\star}(n) = \Theta(n \cdot \log(n))$, we have $R_{\min}(n) = O(n \cdot \log(n))$. ∎

We note that nodes actively disseminate their location information to parts of the network under both proactive geographic routing (subsection 4.4.1) and our scheme in the proof of Lemma 4.4. However, there are some key differences between our scheme and both proactive and reactive geographic routing: First, proactive geographic routing floods and stores location information in the neighborhood around the nodes, whereas in our scheme the location information of a node is stored only in a small area (a cell) with a pre-assigned location that can be computed using its ID, *independently of its actual location*. Since nodes are mobile, unless sources are always close to selected destinations, promulgating location information to a small neighborhood around the nodes will be of limited use. Secondly, we limit the area to be flooded with a location request message to the same cell. In other words, only the cell in which the location information of a requested destination is stored, is flooded with the location request message. These simple features eliminate the need for unnecessary and expensive flooding of control messages in the network, resulting in lower overhead.

It is noteworthy that, in computing the routing overhead under both our scheme and proactive/reactive geographic routing, flooding of location information or request messages changes only the distances traveled by them, while the number of bits carried by them remains the same. Therefore, the disparity in expected routing overhead is caused only by larger distances traveled by control messages under proactive/reactive geographic routing (which demands higher resource expenditure by their transmissions). Therefore, this highlights the importance of modeling and accounting for the traveled distances; computing only the information rate required to model nodes' mobility and uncertainty in their locations (e.g., [33]) would *not* reveal this discrepancy in (the scaling law of) expected routing overhead under these schemes.

Theorem 4.4: The minimum expected overhead for location service per timeslot under Assumption 4.1, $R_{\min}(n)$, is $\Theta(n \cdot \log(n))$.

proof: The theorem follows from Lemmas 4.3 and 4.4. ■

Corollary 2: The minimum expected overhead for location service per timeslot under Assumption 4.2 is $\Theta(n \cdot \log(n))$.

This corollary follows from the observation that the average number of bits in control messages is still $\Theta(\log(n))$ under Assumption 4.2 (as a consequence of Corollary 1) and a minor modification of the proof of Lemma 4.4.

4.6 Discussion

Throughout this chapter we assumed geographic routing and a non-vanishing spatial density of the nodes while adopting the GRG model for one-hop network connectivity. In this section we first compare the expected overhead of geographic routing schemes to that of topology-based routing schemes. Then, we examine the effects of a vanishing spatial density and location recovery procedures on routing overhead. Finally, we consider a family of network connectivity models, which contains the GRG model as a special case, and show that our results still hold under the new models.

A. Geographic routing vs. topology-based routing: In Section 4.4 we showed that the expected routing overhead under proactive or reactive geographic routing is $\Omega(n^{1.5} \cdot \log(n))$. Here, we briefly discuss the same under a proactive or reactive routing scheme that uses *topological* information of the network (i.e., network connectivity) for routing decisions: Each node maintains and uses the next-hop information, for example, along a minimum-hop path, for each known destination through exchange of (local) topology information. We call these routing schemes *topological* routing schemes.

Proactive topological routing: Suppose that, under a proactive topological routing scheme, each node advertises the IDs of its neighbors along with its own ID to all other nodes within distance $\epsilon > 0$, which we call an *advertisement*. The information on immediate neighbors is the minimal amount of information needed to reconstruct the network topology and is the same information reported by regular nodes in [56, 57]. Given the assumptions in Section 4.2 and the transmission range $\gamma^\star(n)$, the expected number of neighbors of a node is $\Theta(\log(n))$. Thus, the average number of bits required for an advertisement containing the list of neighbors is $\Theta(\log(n)^2)$ because each ID requires on the average $\log(n)$ bits. From the proof of Theorem 4.2, we know that the expected total distance traveled by each advertisement is $\Omega(\sqrt{n})$. Therefore, the overall expected overhead due to advertisements per timeslot is $\Omega(n \cdot \sqrt{n} \cdot \log(n)^2) = \Omega(n^{1.5} \cdot \log(n)^2)$.

Zhou and Abouzeid [57] studied similar routing overhead under two-tier hierarchical proactive routing where regular nodes report the detailed local topology information to their cluster heads that maintain global ownership information. When the number of subregions M (with one cluster head per subregion) is fixed, the overall routing overhead is $\Omega(n^2 \cdot \log(n))$ under all three different physical scalings of the network they considered (Table IV in [57]). In particular, under the second physical scaling in which the communication range of the nodes is adjusted so that the expected number of neighbors of a node is $\Theta(\log(n))$, the routing overhead is $\Theta(n^{2.5})$. Furthermore, even when the number of subregions M is allowed to depend on n, one can show that the overall routing overhead is $\Omega(n^{1.5})$ under all three different physical scalings in [57] (and $\Theta(n^2/\sqrt{\log(n)})$ under the second physical scaling).

Reactive topological routing: Assume that routing information is discovered by flooding the network with a *route request message* under a reactive topological routing scheme, and Assumptions A1 and A2 in subsection 4.4.2 hold (with 'location request' replaced by 'route request'). Then, the overhead stemming from flooding of route request messages is $\Omega(n^{1.5} \cdot \log(n))$ by a similar argument in the proof of Theorem 4.3. As mentioned in the same proof, replies need not be flooded. Instead, they can be routed back to the source by maintaining a cache at intermediate nodes and temporarily storing all request messages with the IDs of the sources and the first nodes that forwarded them. Then, following a similar reasoning, one can show that the overhead due to reply messages is $\Theta(n \cdot \log(n))$, giving the overall routing overhead of $\Omega(n^{1.5} \cdot \log(n))$.

We also note that introducing virtual servers with *route information* to nodes (analogous to the virtual location server in the proof of Lemma 4.4) will be problematic in topological routing schemes. This is because, unlike in geographic routing where the *same* location information for node i can be provided to any node that wishes to communicate with node i, the end-to-end route information to node i varies from one node to another, depending on the position of the node in the network topology relative to that of node i. Hence, it is not obvious how one can reduce the routing overhead brought about by costly flooding of control messages.

B. Overhead due to location recovery in reactive geographic routing: As discussed in subsection 4.4.2, suppose that a destination of a connection moves while it is still active. In this case, unless the destination informs the source of its new location, the location information at the source will be outdated and the source will need to acquire the new location of the destination through a recovery process. If we assume that the recovery is performed by flooding a control message similar to the original location request message, then the additional overhead due to recovery will be comparable to the overhead incurred during the original location discovery process (through location request and reply messages). Thus, if we assume that connections need, on average, K recovery processes (per connection) while they are active, the expected routing overhead will scale by a factor of K, and the scaling law of the expected routing overhead will remain $\Omega(n^{1.5}\ \log(n))$.

In practice, however, the frequency of location recovery will depend on the details of an adopted routing scheme.

C. Different network connectivity models and choices of transmission ranges: While we modeled the network connectivity using an GRG model so far, our results can be generalized to other connectivity models: Given $n \in \mathbb{N}$ nodes in the network, let $\overline{\gamma}(n)$ be a *target* transmission range selected by the nodes. There exist constants $0 < \sigma_1 \leq 1 \leq \sigma_2 < \infty$ so that, given $\overline{\gamma}(n)$, (i) nodes i and j have a link if their distance $d(i,j) \leq \sigma_1 \cdot \overline{\gamma}(n)$, and (ii) they do not have a link if $d(i,j) > \sigma_2 \cdot \overline{\gamma}(n)$. When $\sigma_1 \cdot \overline{\gamma}(n) < d(i,j) \leq \sigma_2 \cdot \overline{\gamma}(n)$, however, we do not specify whether or not there exists a link between nodes i and j. Different rules, such as a probabilistic rule, can be applied to this case. The GRG model is a special case with $\sigma_1 = \sigma_2$. The interpretation of this family of models is that once nodes select a *target* transmission range, they should be able to communicate directly with other nodes that are well within the target range, whereas other nodes that are (much) farther away than the target range would not be directly reachable. Connectivity between nodes roughly target range away from each other, however, may depend on other factors, and we do not provide a specific rule for this case.

From Theorem 4.1 and the above rules, the minimum target transmission range required for network connectivity satisfies $\gamma^\star(n)/\sigma_2 \leq \overline{\gamma}(n) \leq \gamma^\star(n)/\sigma_1$. If this condition is met, following the proof of Lemma 4.1, one can show that the necessary quantization level for approximating location information is $\Theta(\gamma^\star(n))$ and Lemma 4.1 still holds: When the target range $\overline{\gamma}(n) = \Theta(\gamma^\star(n))$, the necessary quantization level changes at most by a constant factor from the GRG case. Thus, as mentioned in Sections 4.3 and 4.5, this does not affect the findings in Lemma 4.1 and, hence, Theorems 4.2 through 4.4.

Under a quasi unit disk graph (QUDG) model [44], presumably with fixed transmit power, there are two thresholds – $0 \leq \gamma_1 \leq \gamma_2 < \infty$, where $\gamma_1 = \tau \cdot \gamma_2$ for some $\tau \in [0, 1]$. (i) If the distance $d(i, j)$ between nodes i and j is at most γ_1, there is a link between i and j; (ii) if $d(i, j) > \gamma_2$, no link exists between them; and (iii) if $\gamma_1 < d(i, j) \leq \gamma_2$, there may or may not exist a link between them. It is obvious that, under suitable scaling of γ_1 and γ_2 (through transmit power control) as a function of n while maintaining network connectivity, the QUDG model is similar to the above model. Hence, our results are true under the QUDG model when $\tau > 0$.

Under a cost-based model (e.g. [50]), there is a cost function $c : \mathbb{R}_+ \to \mathbb{R}_+$ such that, (i) the cost at distance d is given by $c(d) \in [\varphi_1 \cdot d,\ \varphi_2 \cdot d]$, where $0 < \varphi_1 \leq \varphi_2 < \infty$, and (ii) nodes i and j have a link if and only if $c(d(i, j)) \leq c_{th}$ for some threshold c_{th}. The cost function c is not assumed monotonic in distance. It is clear that, given a threshold c_{th}, we can find an upper and lower bound on the maximum distance between two nodes that would permit a link between the nodes. Therefore, by selecting appropriate thresholds $c_{th}(n)$ as a function of the number of nodes n that would ensure network connectivity and following a similar reasoning as above, we can show that our results hold under this model as well.

5. CONCLUSION

5.1 Summary and Open Problems

In MANETs, in order to find a route promptly between arbitrary source and destination, the network should be connected and the study of network connectivity has attracted a lot of interest from researchers. In this dissertation, we opened the new chapter for the study of network connectivity by introducing correlation of nodes with the group mobility model and trustworthiness in one-hop connectivity. Also, when the network is connected, from the mobility of the nodes in MANETs, the pathes between a source and a destination vary over time. We studied the overhead incurred from maintaining up-to-date information for routing packets under geographic routing.

In this dissertation, we first investigated how the smallest communication range needed for network connectivity, which we call the critical transmission range, behaves in simple one-dimensional cases, where nodes lie on a unit ring. Unlike in most of previous studies, we relaxed the assumption that nodes' locations are independent; the nodes are clustered into groups with the same number of members. We demonstrated that the critical transmission range displays a form of parametric sensitivity with respect to the size of group which represents correlation between members in each group. In this study, while we focused on a simple one-dimensional case as a first step towards understanding the role of correlation in nodes' locations on network connectivity, it will inspire researchers to extend the study of network connectivity with correlated nodes to higher-dimensional cases.

In the second part of the dissertation, we the studied the presence of isolated nodes in the network when nodes' locations are given by mutually independent random variables uniformly distributed on a two dimensional torus and one-hop connectivity between two nodes is governed by not only a geometric constraint but also trust constraint. As expected, the trust constraint imposed on one-hop connectivity requires that nodes employ a large communication range in order to prevent isolated nodes. Even though we studied the common transmission range for no node isolation instead of network connectivity, we hope that we can show that the probability that there is no isolated node asymptotically converges to the probability that the network is connected with the additional trust constraint in one-hop connectivity.

In the third part, we studied the expected overhead due to exchange of location information under geographic routing when nodes employ a common transmission range to ensure network connectivity with a high probability. We focused on a scenario where packets can be routed to their intended destinations using only the ID and location information of the destinations without flooding the network with copies of packets. We showed that when nodes move independently, a minimum of $\log(n)$ bits are needed on average to describe the approximated location of each node, where n is the number of the nodes. Making use of this finding, we proved that the expected routing overhead is $\Omega(n^{1.5}\log(n))$ under both proactive and reactive geographic routing and the minimum expected routing overhead scales as $\Theta(n\log(n))$. As future works, we can consider the case when nodes' mobility is correlated, that may slow the growth of the expected routing overhead; the exact manner in which it will grow is likely to depend on many factors, including the details of correlation structure imposed on nodes' mobility as well as the selection of source-destination pairs.

APPENDIX

A. APPENDIX A

A.1 Proof of Theorem 2.1

We first introduce some notation and preliminary results that will be used to prove the theorem. If node j lies within the communication range to the left of node i (when facing in the direction of the center of the unit ring), we call node j a *left neighbor* (LN) of node i. With a little abuse of notation, for each $n \in \mathbb{N}$ (and fixed $G(n)$, $M(n)$ and $d(n)$),

- $I^{(n)}_{k,m}(\gamma)$, $k \in \mathcal{G}^{(n)}$ and $m \in \mathcal{M}^{(n)}$, is the indicator function of the event that node m in the k-th group does *not* have any LNs on the unit ring; and

- $C^{(n)}(\gamma) = \sum_{k \in \mathcal{G}^{(n)}} C^{(n)}_k(\gamma)$ denotes the total number of nodes without any LN, where $C^{(n)}_k(\gamma) = \sum_{m \in \mathcal{M}^{(n)}} I^{(n)}_{k,m}(\gamma)$.

Note that, according to these definitions, the event that the random graph $\mathbb{G}(G(n), M(n); \gamma)$ is bi-directionally connected is the same as the event $\{C^{(n)}(\gamma) = 0\}$, and $\mathbf{P}^{(n)}(\gamma) = \mathbb{P}\left[C^{(n)}(\gamma) = 0\right]$. Throughout the proof we will make use of this equality and investigate $\mathbb{P}\left[C^{(n)}(\gamma) = 0\right]$ in place of $\mathbf{P}^{(n)}(\gamma)$.

We borrow following results from [10] to simplify the proof of the theorem: Define $\mathbb{Z}_+ := \{0, 1, 2, \ldots\}$ to be the set of non-negative integers. Suppose $\{Z_n; n = 1, 2, \cdots\}$ is a sequence of $\mathbb{Z}_+$-valued rvs with finite second moment, i.e., $\mathbf{E}\left[Z_n^2\right] < \infty$, for every

$n = 1, 2, \ldots$. Then,

$$\lim_{n\to\infty} \mathbb{P}\left[Z_n = 0\right] = 1 \text{ if } \lim_{n\to\infty} \mathbf{E}\left[Z_n\right] = 0, \tag{A.1}$$

and

$$\lim_{n\to\infty} \mathbb{P}\left[Z_n = 0\right] = 0 \text{ if } \lim_{n\to\infty} \frac{\left(\mathbf{E}\left[Z_n\right]\right)^2}{\mathbf{E}\left[Z_n^2\right]} = 1. \tag{A.2}$$

Eq. (A.1) follows directly from Markov's inequality [7, p.311]. Eq. (A.2) can be easily shown using Cauchy-Schwarz inequality [7, p.65].

We now return to the proof of Theorem 2.1 and define

$$\tilde{\gamma}(n) := \frac{\log(G(n)) + \alpha_n}{G(n)}.$$

Here, when α_n increases (resp. decreases), it increases (resp. decreases) to ∞ (resp. $-\infty$) arbitrarily slow. In order to prove Theorem 2.1, we will first show

S1. $\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right] \to 0$ if $\alpha_n \to \infty$, and

S2. $\frac{\left(\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right]\right)^2}{\mathbf{E}\left[\left(C^{(n)}(\tilde{\gamma}(n))\right)^2\right]} \to 1$ if $\alpha_n \to -\infty$,

and then make use of (A.1) and (A.2), respectively.

Proof of S1: From the definition of $C^{(n)}(\gamma)$,

$$\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right] = N(n) \cdot \mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right], \tag{A.3}$$

and we focus on computing $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right]$. Without loss of generality (WLOG), suppose that the location of VGL $V_1^{(n)}$ is $X_1^{(n)} = 0$ and, hence, $L_{1,1}^{(n)} = Y_{1,1}^{(n)}$.

We denote the m-th node in the k-th group by the pair (k, m). Define event $A^{(n)}$ (resp. $B^{(n)}$) to be the event that node $(1, 1)$ has no LNs from group 1 (resp. from the other groups 2 through $G(n)$). Then,

$$\begin{aligned}\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right] &= \mathbf{E}\left[\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \;\middle|\; Y_{1,1}^{(n)}\right]\right] \\ &= \frac{1}{d(n)}\int_0^{d(n)} \left(\mathbb{P}\left[A^{(n)} \;\middle|\; y\right] \cdot \mathbb{P}\left[B^{(n)} \;\middle|\; y\right]\right)\, dy \\ &= \frac{\mathbb{P}\left[B^{(n)}\right]}{d(n)} \int_0^{d(n)} \mathbb{P}\left[A^{(n)} \;\middle|\; y\right]\, dy, \end{aligned} \tag{A.4}$$

where $\mathbb{P}\left[A^{(n)} \mid y\right]$ and $\mathbb{P}\left[B^{(n)} \mid y\right]$ are the conditional probability of $A^{(n)}$ and $B^{(n)}$, respectively, given $\{Y_{1,1}^{(n)} = y\}$,[1] and the second and third equalities follow from Assumptions A1 and A2 in subsection 2.1.1.

First, since $d(n) < \tilde{r}(n)$, it is easy to see that, given $Y_{1,1}^{(n)} = y$, in order for the event $A^{(n)}$ to be true, the other members in group 1 must lie in $[0, y)$. Hence, $\mathbb{P}\left[A^{(n)}|y\right] = (y/d(n))^{M(n)-1}$ for all $y \in [0, d(n)]$, and

$$\frac{1}{d(n)}\int_0^{d(n)} \mathbb{P}\left[A^{(n)} \;\middle|\; y\right]\, dy = \frac{1}{M(n)}. \tag{A.5}$$

Second, let $B_2^{(n)}$ be the event that node (1,1) does not have any LNs from group 2. Then, from Assumptions A1 through A3, $\mathbb{P}\left[B^{(n)}\right] = \left(\mathbb{P}\left[B_2^{(n)}\right]\right)^{G(n)-1}$. We compute $\mathbb{P}\left[B_2^{(n)}\right]$ by conditioning on $X_2^{(n)} = x_2$ and considering four cases as follows. WLOG, we assume $L_{1,1}^{(n)} = 0$:

Case 1. $0 \le x_2 \le \tilde{\gamma}(n) - d(n)$: The members of group 2 will lie in the interval $[x_2, x_2 + d(n)]$, where $x_2 + d(n) \le \tilde{\gamma}(n)$. Thus, because $[x_2, x_2 + d(n)] \subset [0, \tilde{\gamma}(n)]$, they will be all LNs of node (1,1), and $\mathbb{P}\left[B_2^{(n)}|X_2^{(n)} = x_2\right] = 0$.

Case 2. $\tilde{\gamma}(n) - d(n) < x_2 \le \tilde{\gamma}(n)$: The conditional probability $\mathbb{P}\left[B_2^{(n)}|X_2^{(n)} = x_2\right]$ is given by the probability that all members of group 2 are in the interval $(\tilde{\gamma}(n), x_2 + d(n))$.

[1] One should view these conditional probabilities as the limit of $\mathbb{P}\left[A^{(n)} \mid y < Y_{1,1}^{(n)} \le y + \delta\right]$ and $\mathbb{P}\left[B^{(n)} \mid y < Y_{1,1}^{(n)} \le y + \delta\right]$ when $\delta \to 0$.

From Assumption A3, this probability is given by $((x_2 + d(n) - \tilde{\gamma}(n))/d(n))^{M(n)}$.

Case 3. $\tilde{\gamma}(n) < x_2 < 1 - d(n)$: Since all members of group 2 lie in the interval $(\tilde{\gamma}(n), 1)$, $\mathbb{P}\left[B_2^{(n)} | X_2^{(n)} = x_2\right] = 1$.

Case 4. $1 - d(n) \le x_2 < 1$: The conditional probability $\mathbb{P}\left[B_2^{(n)} | X_2^{(n)} = x_2\right]$ is equal to the probability that all members in group 2 reside in $(x_2, 1)$. This probability is given by $((1 - x_2)/d(n))^{M(n)}$.

Integrating the above conditional probabilities over the corresponding intervals, we get

$$\begin{aligned}\mathbb{P}\left[B_2^{(n)}\right] &= \int_{\tilde{\gamma}(n)-d(n)}^{\tilde{\gamma}(n)} \left(\frac{x_2 + d(n) - \tilde{\gamma}(n)}{d(n)}\right)^{M(n)} dx_2 \\ &\quad + \int_{\tilde{\gamma}(n)}^{1-d(n)} 1 \; dx_2 + \int_{1-d(n)}^{1} \left(\frac{1 - x_2}{d(n)}\right)^{M(n)} dx_2 \\ &= 1 - \tilde{\gamma}(n) - \frac{M(n) - 1}{M(n) + 1} d(n). \end{aligned} \tag{A.6}$$

Recall that $X_2^{(n)}$ is uniformly distributed over $[0, 1)$ from observation O1 (in Section 2.2).

From the equality $\mathbb{P}\left[B^{(n)}\right] = \left(\mathbb{P}\left[B_2^{(n)}\right]\right)^{G(n)-1}$ and (A.6),

$$\begin{aligned}\mathbb{P}\left[B^{(n)}\right] &= \left(1 - \tilde{\gamma}(n) - \frac{M(n) - 1}{M(n) + 1} d(n)\right)^{G(n)-1} \qquad \text{(A.7)} \\ &= \exp\left((G(n) - 1) \cdot \log\left(1 - \tilde{\gamma}(n) - \frac{M(n) - 1}{M(n) + 1} d(n)\right)\right). \end{aligned}$$

Define $\Gamma(n) := \tilde{\gamma}(n) + \frac{M(n)-1}{M(n)+1} d(n)$. Then, because $\Gamma(n) \to 0$, we have $\log(1 - \Gamma(n)) = -\Gamma(n) + O\left(\Gamma(n)^2\right)$ and

$$\mathbb{P}\left[B^{(n)}\right] = \exp\left((G(n) - 1) \cdot \left(-\Gamma(n) + O\left(\Gamma(n)^2\right)\right)\right).$$

First, note that $G(n) \cdot d(n) = o(1)$ and $G(n) \cdot \tilde{\gamma}(n)^2 = o(1)$ because $G(n) = \omega(\log^2(N(n)))$ from Assumption 2.1. Hence, $G(n) \cdot \Gamma(n)^2 = o(1)$ because $d(n) =$

$o(\tilde{\gamma}(n))$. Therefore,

$$\begin{aligned}\mathbb{P}\left[B^{(n)}\right] &\sim \exp(-G(n)\cdot\tilde{\gamma}(n)) \\ &= \exp\left(-\left(\log(G(n))+\alpha_n\right)\right) \\ &= \frac{1}{G(n)}\exp(-\alpha_n). \end{aligned} \tag{A.8}$$

From (A.4), (A.5), and (A.8),

$$\begin{aligned}\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right] &\sim \frac{1}{M(n)}\times\frac{1}{G(n)}\exp(-\alpha_n) \\ &= \frac{1}{N(n)}\exp(-\alpha_n). \end{aligned} \tag{A.9}$$

Substituting (A.9) in (A.3), we obtain

$$\begin{aligned}\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right] &= N(n)\cdot\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right] \\ &\sim N(n)\cdot\frac{1}{N(n)}\exp(-\alpha_n) \\ &= \exp(-\alpha_n). \end{aligned} \tag{A.10}$$

Therefore, if $\alpha_n\to\infty$, $\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right]\to 0$. This completes the proof of S1.

Proof of S2: First, from the definition of $C^{(n)}(\tilde{\gamma}(n))$,

$$\begin{aligned}\mathbf{E}\left[\left(C^{(n)}(\tilde{\gamma}(n))\right)^2\right] &= \mathbf{E}\left[\left(\sum_{k=1}^{G(n)} C_k^{(n)}(\tilde{\gamma}(n))\right)^2\right] \\ &= G(n)\,\mathbf{E}\left[\left(C_1^{(n)}(\tilde{\gamma}(n))\right)^2\right] \\ &\quad + G(n)(G(n)-1)\,\mathbf{E}\left[C_1^{(n)}(\tilde{\gamma}(n))\cdot C_2^{(n)}(\tilde{\gamma}(n))\right] \\ &= N(n)\,\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right] \\ &\quad + N(n)(M(n)-1)\,\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\cdot I_{1,2}^{(n)}(\tilde{\gamma}(n))\right] \\ &\quad + N(n)M(n)(G(n)-1)\,\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\cdot I_{2,1}^{(n)}(\tilde{\gamma}(n))\right]. \end{aligned} \tag{A.11}$$

Recall that we already computed $\mathbf{E}\left[I_{1,1,}^{(n)}(\tilde{\gamma}(n))\right]$ and $\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right]$ in the proof of S1. Hence, in order to prove S2 using (A.11), we need to calculate $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \cdot I_{1,2}^{(n)}(\tilde{\gamma}(n))\right]$ and $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \cdot I_{2,1}^{(n)}(\tilde{\gamma}(n))\right]$. However, $\tilde{\gamma}(n) > d(n)$ immediately tells us that $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \cdot I_{1,2}^{(n)}(\tilde{\gamma}(n))\right] = 0$ because the two nodes are always within $\tilde{\gamma}(n)$ of each other (and, hence, one of them is a LN of the other). Thus, we only need to compute $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \cdot I_{2,1}^{(n)}(\tilde{\gamma}(n))\right]$.

Computation of $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \cdot I_{2,1}^{(n)}(\tilde{\gamma}(n))\right]$: Define $Q^{(n)}$ (resp. $R^{(n)}$) to be the event that no node from groups 1 and 2 (resp. from groups 3 through $G(n)$) is a LN of node (1,1) or (2,1). WLOG we assume that node (1,1) is at $L_{1,1}^{(n)} = 0$. Then, by conditioning on the location of node (2,1), $L_{2,1}^{(n)} = \ell_2$, and using observation O1 (in Section 2.2) that $L_{2,1}^{(n)}$ is uniformly distributed over [0, 1), we can write

$$\begin{aligned} &\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \cdot I_{2,1}^{(n)}(\tilde{\gamma}(n))\right] \\ &= \int_0^1 \left(\mathbb{P}\left[R^{(n)}|\ell_2\right] \cdot \mathbb{P}\left[Q^{(n)}|\ell_2\right]\right) \, d\ell_2, \end{aligned} \tag{A.12}$$

where $\mathbb{P}\left[R^{(n)} \mid \ell_2\right]$ and $\mathbb{P}\left[Q^{(n)} \mid \ell_2\right]$ are the conditional probabilities of $R^{(n)}$ and $Q^{(n)}$, respectively, given $\{L_{2,1}^{(n)} = \ell_2\}$. We will now identify an upper bound for $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \cdot I_{2,1}^{(n)}(\tilde{\gamma}(n))\right]$ by finding an upper bound to $\mathbb{P}\left[Q^{(n)} \mid \ell_2\right]$ and $\mathbb{P}\left[R^{(n)} \mid \ell_2\right]$.

(i) Upper bound for $\mathbb{P}\left[Q^{(n)} \mid \ell_2\right]$ – First, note that if $\ell_2 \in [1 - \tilde{\gamma}(n), \tilde{\gamma}(n)]$, nodes (1,1) and (2,1) will be immediate neighbors, hence, $\mathbb{P}\left[Q^{(n)}|\ell_2\right] = 0$. For the other case, we look for an upper bound to $\mathbb{P}\left[Q^{(n)}|\ell_2\right]$ using the following equality obtained by further conditioning on $X_1^{(n)}$ and $X_2^{(n)}$, i.e., the locations of VGLs $V_1^{(n)}$ and $V_2^{(n)}$. Note that, given $\{L_{1,1}^{(n)} = 0\}$ and $\{L_{2,1}^{(n)} = \ell_2\}$, $X_1^{(n)}$ and $X_2^{(n)}$ are uniformly distributed over $[1 - d(n), 1]$ and $[\ell_2 - d(n), \ell_2]$, respectively. Therefore,

$$\mathbb{P}\left[Q^{(n)}|\ell_2\right] = \frac{1}{d(n)^2}\int_{\ell_2-d(n)}^{\ell_2}\left(\int_{1-d(n)}^{1}\mathbb{P}\left[Q^{(n)}|\ell_2,x_1,x_2\right]\,dx_1\right)dx_2 \qquad \text{(A.13)}$$

Case 1. $d(n)+\tilde{\gamma}(n) < \ell_2 < 1-d(n)-\tilde{\gamma}(n)$: In this case because $X_2^{(n)} \in (\tilde{\gamma}(n), 1-d(n)-\tilde{\gamma}(n))$ and $X_1^{(n)} \in [1-d(n),1]$, it is clear that nodes from group 1 (resp. group 2) cannot be a LN of node (2,1) (resp. node (1, 1)). Therefore, in order for $Q^{(n)}$ to be true, nodes in group 1 (resp. group 2) should lie in $(x_1, 1)$ (resp. (x_2, l_2)). Using (A.13), we get

$$\begin{aligned}\mathbb{P}\left[Q^{(n)}|\ell_2\right] &= \frac{1}{d(n)}\left(\int_{1-d(n)}^{1}\left(\frac{1-x_1}{d(n)}\right)^{M(n)-1}dx_1\right)\\ &\quad\times\frac{1}{d(n)}\left(\int_{\ell_2-d(n)}^{\ell_2}\left(\frac{\ell_2-x_2}{d(n)}\right)^{M(n)-1}dx_2\right)\\ &= \frac{1}{M(n)^2}. \end{aligned} \qquad \text{(A.14)}$$

Case 2. $1-d(n)-\tilde{\gamma}(n) \le \ell_2 < 1-\tilde{\gamma}(n)$: In this case, while the nodes from group 2 cannot be a LN of node (1,1), some nodes from group 1 can be a LN of node (2,1) when $X_1^{(n)} < \ell_2+\tilde{\gamma}(n)$. Hence, we consider two subcases - $X_1^{(n)} \in [1-d(n), \ell_2+\tilde{\gamma}(n))$ and $X_1^{(n)} \in [\ell_2+\tilde{\gamma}(n), 1]$. Further, when $X_1^{(n)} \in [1-d(n), \ell_2+\tilde{\gamma}(n))$, for event $Q^{(n)}$ to be true, we need the nodes in group 1 to be in $(\ell_2+\tilde{\gamma}(n), 1)$, in order to avoid being a LN of node (1,1) or (2,1).

$$
\begin{aligned}
&\mathbb{P}\left[Q^{(n)}|\ell_2\right] \\
&= \frac{1}{d(n)}\left(\int_{1-d(n)}^{\ell_2+\tilde{\gamma}(n)}\left(\frac{1-\ell_2-\tilde{\gamma}(n)}{d(n)}\right)^{M(n)-1} dx_1\right. \\
&\qquad \left. + \int_{\ell_2+\tilde{\gamma}(n)}^{1}\left(\frac{1-x_1}{d(n)}\right)^{M(n)-1} dx_1\right) \\
&\quad \times \frac{1}{d(n)}\left(\int_{\ell_2-d(n)}^{\ell_2}\left(\frac{\ell_2-x_2}{d(n)}\right)^{M(n)-1} dx_2\right) \\
&= \frac{1}{M(n)^2}\left(\frac{1-\ell_2-\tilde{\gamma}(n)}{d(n)}\right)^{M(n)} \\
&\quad + \frac{\ell_2+\tilde{\gamma}(n)-1+d(n)}{d(n)\,M(n)}\left(\frac{1-\ell_2-\tilde{\gamma}(n)}{d(n)}\right)^{M(n)-1}
\end{aligned} \tag{A.15}
$$

One can show that (A.15) is decreasing in ℓ_2 between $1-d(n)-\tilde{\gamma}(n)$ and $1-\tilde{\gamma}(n)$. Hence, by substituting $\ell_2 = 1-d(n)-\tilde{\gamma}(n)$, we obtain an upper bound

$$
\mathbb{P}\left[Q^{(n)}|\ell_2\right] \le \frac{1}{M(n)^2}. \tag{A.16}
$$

Case 3. $\tilde{\gamma}(n) < \ell_2 \le d(n)+\tilde{\gamma}(n)$: This case is symmetric to case 2 above, and we obtain the same upper bound in (A.16).

(ii) Upper bound for $\mathbb{P}\left[R^{(n)} \mid \ell_2\right]$ **–** Let us first define $R_3^{(n)}$ to be the event that none of the nodes in group 3 is a LN of node (1,1) or (2,1). Then, from Assumptions A2 and A3,

$$
\mathbb{P}\left[R^{(n)} \mid \ell_2\right] = \left(\mathbb{P}\left[R_3^{(n)} \mid \ell_2\right]\right)^{G(n)-2}, \tag{A.17}
$$

where $\mathbb{P}\left[R_3^{(n)} \mid \ell_2\right]$ is the conditional probability of $R_3^{(n)}$ given $\{L_{2,1}^{(n)} = \ell_2\}$. We will now find an upper bound to $\mathbb{P}\left[R_3^{(n)} \mid \ell_2\right]$ by considering the same three cases above (in the calculation of an upper bound for $\mathbb{P}\left[Q^{(n)} \mid \ell_2\right]$). Keep in mind that $\mathbb{P}\left[R_3^{(n)} \mid \ell_2\right]$ is the probability that the nodes in group 3 lie outside $[0, d(n)]$ and $[\ell_2, \ell_2+d(n)]$.

Case 1. $d(n) + \tilde{\gamma}(n) < \ell_2 < 1 - d(n) - \tilde{\gamma}(n)$: Conditioning on the location of VGL $V_3^{(n)}$, i.e., $X_3^{(n)} = x_3$, we consider following six subcases.

1-i. $x_3 \in [0, \tilde{\gamma}(n) - d(n)]$ or $x_3 \in [\ell_2, \ell_2 + \tilde{\gamma}(n) - d(n)]$ – Since $[x_3, x_3 + d(n)] \subset [0, \tilde{\gamma}(n)]$ or $[x_3, x_3 + d(n)] \subset [\ell_2, \ell_2 + \tilde{\gamma}(n)]$ in this subcase, all members of group 3 will be LNs of either node (1,1) or (2,1). Hence, $\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = 0$.

1-ii. $x_3 \in (\tilde{\gamma}(n) - d(n), \tilde{\gamma}(n)]$ – In this subcase $x_3 + d(n) \leq \tilde{\gamma}(n) + d(n) < \ell_2$. Hence, in order for event $R_3^{(n)}$ to be true, all members of group 3 must lie in $(\tilde{\gamma}(n), x_3 + d(n)]$ and

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{x_3 + d(n) - \tilde{\gamma}(n)}{d(n)}\right)^{M(n)}.$$

1-iii. $x_3 \in (\tilde{\gamma}(n), \ell_2 - d(n))$ or $x_3 \in (\ell_2 + \tilde{\gamma}(n), 1 - d(n))$ – In this subcase, it is not possible for any member of group 3 to be a LN of node (1,1) or (2,1). Thus, $\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = 1$.

1-iv. $x_3 \in [\ell_2 - d(n), \ell_2]$ – In this subcase, we have $\tilde{\gamma}(n) < x_3 < x_3 + d(n) < 1 - \tilde{\gamma}(n)$, and the nodes in group 3 cannot be a LN of node (1,1). Thus, the event $R_3^{(n)}$ only requires that the members of group 3 lie in $[x_3, \ell_2)$ to avoid being a LN of node (2,1). Therefore,

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{\ell_2 - x_3}{d(n)}\right)^{M(n)}.$$

1-v. $x_3 \in (\ell_2 + \tilde{\gamma}(n) - d(n), \ell_2 + \tilde{\gamma}(n)]$ – Note that $\tilde{\gamma}(n) < x_3 < x_3 + d(n) < 1$ for this subcase. Consequently, the event $R_3^{(n)}$ that all members of group 3 are in the interval $(\ell_2 + \tilde{\gamma}(n), x_3 + d(n)]$ has probability

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{x_3 + d(n) - (\ell_2 + \tilde{\gamma}(n))}{d(n)}\right)^{M(n)}.$$

1-vi. $x_3 \in [1-d(n), 1]$ – First, note that $\ell_2+\tilde{\gamma}(n) < x_3 < x_3+d(n) < 1+\ell_2$. Hence, the members of group 3 cannot be a LN of node (2,1). The probability that all members of group 3 lie in $[x_3, 1)$ is equal to

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{1-x_3}{d(n)}\right)^{M(n)}.$$

Recall that $X_3^{(n)}$ is uniformly distributed over [0, 1) from observation O1. Taking these conditional probabilities for the six subcases and integrating them over the given intervals, we obtain

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2\right] = 1 - 2\tilde{\gamma}(n) - 2d(n) + \frac{4d(n)}{M(n)+1}. \tag{A.18}$$

Case 2. $1 - d(n) - \tilde{\gamma}(n) \le \ell_2 \le 1 - \tilde{\gamma}(n)$: We follow the same steps in case 1 and consider seven subcases by conditioning on $X_3^{(n)} = x_3$.

2-i. $x_3 \in [0, \tilde{\gamma}(n) - d(n)]$ or $x_3 \in [\ell_2, \ell_2 + \tilde{\gamma}(n) - d(n)]$ – By the same argument in subcase 1-i, $\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = 0$.

2-ii. $x_3 \in (\tilde{\gamma}(n) - d(n), \tilde{\gamma}(n)]$ – Following the same argument in subcase 1-ii,

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{x_3 + d(n) - \tilde{\gamma}(n)}{d(n)}\right)^{M(n)}.$$

2-iii. $x_3 \in (\tilde{\gamma}(n), \ell_2 - d(n))$ – Since no member of group 3 can be a LN of node (1,1) or (2,1) in this subcase, $\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = 1$.

2-iv. $x_3 \in [\ell_2 - d(n), \ell_2]$ – In this case, event $R_3^{(n)}$ demands that the members of group 3 be in $[x_3, \ell_2)$. Hence,

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{\ell_2 - x_3)}{d(n)}\right)^{M(n)}.$$

2-v. $x_3 \in [\ell_2 + \tilde{\gamma}(n) - d(n), 1 - d(n)]$ – Since all members of group 3 must lie in the interval $(\ell_2 + \tilde{\gamma}(n), x_3 + d(n)]$ for event $R_3^{(n)}$ to occur,

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{x_3 + d(n) - (\ell_2 + \tilde{\gamma}(n))}{d(n)}\right)^{M(n)}.$$

2-vi. $x_3 \in [1 - d(n), \ell_2 + \tilde{\gamma}(n)]$ – The probability that all members of group 3 lie in the interval $(\ell_2 + \tilde{\gamma}(n), 1]$ to avoid being a LN of node (1,1) or (2,1) is equal to

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{1 - (\ell_2 + \tilde{\gamma}(n))}{d(n)}\right)^{M(n)}.$$

2-vii. $x_3 \in (\ell_2 + \tilde{\gamma}(n), 1)$ – In order for $R_3^{(n)}$ to be true, all members in group 3 must be in $[x_3, 1)$. Thus,

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2, x_3\right] = \left(\frac{1 - x_3}{d(n)}\right)^{M(n)}.$$

Integrating the above conditional probabilities over the specified intervals, we have

$$\begin{aligned}\mathbb{P}\left[R_3^{(n)} \mid \ell_2\right] &= \ell_2 - \tilde{\gamma}(n) - d(n) \\ &+(\ell_2 + \tilde{\gamma}(n) + d(n) - 1)\left(\frac{1 - \ell_2 - \tilde{\gamma}(n)}{d(n)}\right)^{M(n)} \\ &+\frac{2d(n)}{M(n)+1}\left(1 + \left(\frac{1 - \ell_2 - \tilde{\gamma}(n)}{d(n)}\right)^{M(n)+1}\right).\end{aligned} \tag{A.19}$$

We can show that (A.19) is increasing in ℓ_2 between $1-d(n)-\tilde{\gamma}(n)$ and $1-\tilde{\gamma}(n)$. Hence, by substituting $\ell_2 = 1 - \tilde{\gamma}(n)$, we get an upper bound

$$\mathbb{P}\left[R_3^{(n)} \mid \ell_2\right] \le 1 - 2\tilde{\gamma}(n) - d(n) + \frac{2d(n)}{M(n)+1}. \tag{A.20}$$

Case 3. $\tilde{\gamma}(n) < \ell_2 \le d(n) + \tilde{\gamma}(n)$: This case is symmetric to case 2. Therefore, we have the same upper bound in (A.20).

Making use of (A.14), (A.16) - (A.18) and (A.20) in (A.12), we get

$$\begin{aligned}
&\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n)) \cdot I_{2,1}^{(n)}(\tilde{\gamma}(n))\right] \\
&= \int_{\tilde{\gamma}(n)}^{1-\tilde{\gamma}(n)} \left(\mathbb{P}\left[R^{(n)} \mid \ell_2\right] \cdot \mathbb{P}\left[Q^{(n)} \mid \ell_2\right]\right) \, d\ell_2 \\
&\leq \int_{d(n)+\tilde{\gamma}(n)}^{1-d(n)-\tilde{\gamma}(n)} \Lambda_1(n) \, d\ell_2 + 2\int_{\tilde{\gamma}(n)}^{d(n)+\tilde{\gamma}(n)} \Lambda_2(n) \, d\ell_2 \\
&= (1 - 2d(n) - 2\tilde{\gamma}(n)) \cdot \Lambda_1(n) + 2d(n) \cdot \Lambda_2(n), \qquad \text{(A.21)}
\end{aligned}$$

where

$$\Lambda_1(n) = \frac{1}{M(n)^2}\left(1 - 2\tilde{\gamma}(n) - 2d(n) + \frac{4\, d(n)}{M(n)+1}\right)^{G(n)-2} \qquad \text{(A.22)}$$

and

$$\Lambda_2(n) = \frac{1}{M(n)^2}\left(1 - 2\tilde{\gamma}(n) - d(n) + \frac{2\, d(n)}{M(n)+1}\right)^{G(n)-2}.$$

First, note that $(1 - 2d(n) - 2\tilde{\gamma}(n)) \to 1$. We will now show that

$$(A.21) \sim \frac{1}{N(n)^2}\exp(-2\alpha_n) \qquad \text{(A.23)}$$

by proving

$$\Lambda_1(n) \sim \frac{1}{N(n)^2}\exp(-2\alpha_n) \qquad \text{(A.24)}$$

and $d(n) \cdot \Lambda_2(n) = o\left(\Lambda_1(n)\right)$.

Clearly, $\Delta(n) := 2\tilde{\gamma}(n) + 2d(n) - 4d(n)/(M(n)+1) \to 0$. In addition, recall that $G(n) \cdot \tilde{\gamma}(n)^2 = o(1)$ because $\log^2(G(n))/G(n) = o(1)$ from Assumption 2.1. This implies that $G(n) \cdot \Delta(n)^2 = o(1)$ because $d(n) = o(\tilde{\gamma}(n))$. Hence, substituting $\Delta(n)$ in

(A.22) and multiplying both sides by $M(n)^2$,

$$M(n)^2 \cdot \Lambda_1(n) = (1-\Delta(n))^{G(n)-2} \tag{A.25}$$

$$= \exp\left((G(n)-2)\cdot \log(1-\Delta(n))\right)$$

$$= \exp\left((G(n)-2)\cdot \left(-\Delta(n) + O\left(\Delta(n)^2\right)\right)\right)$$

$$\sim \exp(-G(n)\cdot \Delta(n))$$

$$= \exp(-2\log(G(n)) - 2\alpha_n + o(1)) \tag{A.26}$$

$$\sim \frac{1}{G(n)^2}\exp(-2\alpha_n), \tag{A.27}$$

where $o(1)$ term in (A.26) represents $G(n)\cdot d(n)(4/(M(n)+1)-2)$. Dividing (A.25) and (A.27) by $M(n)^2$, we get

$$\begin{aligned}\Lambda_1(n) &\sim \frac{1}{M(n)^2\cdot G(n)^2}\exp(-2\alpha_n)\\ &= \frac{1}{N(n)^2}\exp(-2\alpha_n).\end{aligned}$$

Following the same steps, we obtain

$$\Lambda_2(n) \sim \frac{1}{N(n)^2}\exp(-2\alpha_n).$$

Since $d(n) = o(1/G(n))$, it follows that $d(n)\cdot\Lambda_2(n) = o\left(\Lambda_1(n)\right)$, and we get (A.23).

We plug in $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\cdot I_{1,2}^{(n)}(\tilde{\gamma}(n))\right] = 0$ and use (A.9), (A.10), and (A.23) in place of $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right]$, $\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right]$, and $\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\cdot I_{2,1}^{(n)}(\tilde{\gamma}(n))\right]$, respectively, in (A.11) to obtain

$$\lim_{n\to\infty}\frac{\left(\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right]\right)^2}{\mathbf{E}\left[(C^{(n)}(\tilde{\gamma}(n)))^2\right]} \tag{A.28}$$

$$\geq \lim_{n\to\infty}\frac{\exp(-2\alpha_n)}{\exp(-\alpha_n) + M(n)(G(n)-1)\frac{1}{N(n)}\exp(-2\alpha_n)}.$$

We multiply both the numerator and denominator by $\exp(2\alpha_n)$.

$$(A.28) \geq \lim_{n\to\infty} \frac{1}{\exp(\alpha_n) + \frac{M(n)(G(n)-1)}{N(n)}}$$

Note that $M(n)(G(n)-1)/N(n) = 1 - 1/G(n) \to 1$ as $n \to \infty$. Thus, if $\alpha_n \to -\infty$, $\exp(\alpha_n) \to 0$ and the limit in the right-hand side is equal to one. Since we know that $\mathbf{E}\left[\left(C^{(n)}(\tilde{\gamma}(n))\right)^2\right] \geq \left(\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right]\right)^2$, this implies that (A.28) is equal to one, completing the proof of S2.

A.2 Proof of Theorem 2.4

Basically, this theorem can be proved by showing that the given $d(n)$ and $\tilde{r}(n)$ satisfy the condition, (A.1), which follows directly from Markov's inequality [7, p.311]. The main stream of the proof is almost similar with the proof of Theorem 2.1. Hence, we will jump to (A.4) in the proof of Theorem 2.1 and derive new $\mathbb{P}\left[A^{(n)}\right]$ and $\mathbb{P}\left[B^{(n)}\right]$

First, because $d(n) > \tilde{r}(n)$, we compute $\int_0^{d(n)} \mathbb{P}\left[A^{(n)}|y\right] dy$ by considering two cases:

Case 1. $0 \leq y \leq d(n) - \tilde{\gamma}(n)$: The other members of group 1 will lie in the interval $[0, y] \cup [y + \tilde{\gamma}(n), d(n)]$. Thus,

$$\mathbb{P}\left[A^{(n)}|y\right] = \left(1 - \frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)-1}.$$

Case 2. $d(n) - \tilde{\gamma}(n) \leq y \leq d(n)$: The other members of group 1 will lie in the interval $[0, y]$. Thus,

$$\mathbb{P}\left[A^{(n)}|y\right] = \left(\frac{y}{d(n)}\right)^{M(n)-1}.$$

From the above, we get

$$\int_0^{d(n)} \mathbb{P}\left[A^{(n)}|y\right] dy = \int_0^{d(n)-\tilde{\gamma}(n)} \left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)-1} dy + \int_{d(n)-\tilde{\gamma}(n)}^{d(n)} \left(\frac{y}{d(n)}\right)^{M(n)-1} dy$$
$$= d(n)\left(\frac{1}{M(n)} + \left(1-\frac{1}{M(n)}\right)\left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)}\right). \quad \text{(A.29)}$$

Here, from given $d(n)$ and $\tilde{\gamma}(n)$, we know that

$$\left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)} = \left(1-\frac{\alpha}{\beta}\cdot\frac{1}{M(n)}\right)^{M(n)}$$

and

$$\lim_{n\to\infty}\left(1-\frac{\alpha}{\beta}\cdot\frac{1}{M(n)}\right)^{M(n)} = \exp\left(-\frac{\alpha}{\beta}\right). \quad \text{(A.30)}$$

From (A.30) and (A.29), we know that

$$\frac{1}{d(n)}\int_0^{d(n)} \mathbb{P}\left[A^{(n)}|y\right] dy = \exp\left(-\frac{\alpha}{\beta}\right) + o(1). \quad \text{(A.31)}$$

Second, let $B_2^{(n)}$ be the event that node (1,1) does not have any LNs from group 2. Then, from Assumptions A1 through A3, $\mathbb{P}\left[B^{(n)}\right] = \left(\mathbb{P}\left[B_2^{(n)}\right]\right)^{G(n)-1}$. We compute $\mathbb{P}\left[B_2^{(n)}\right]$ by conditioning on $X_2^{(n)} = x_2$ and considering four cases as follows. WLOG, we assume $L_{1,1}^{(n)} = 0$:

Case 1. $0 \le x_2 \le \tilde{\gamma}(n)$: The conditional probability $\mathbb{P}\left[B_2^{(n)}|X_2^{(n)} = x_2\right]$ is given by the probability that all members of group 2 are in the interval $(x_2 + d(n), \tilde{\gamma}(n))$. From Assumption A3, this probability is given by $((x_2 + d(n) - \tilde{\gamma}(n))/d(n))^{M(n)}$.

Case 2. $\tilde{\gamma}(n) < x_2 \le 1 - d(n)$: Since all members of group 2 lie in the interval $(x_2, x_2 + d(n))$, $\mathbb{P}\left[B_2^{(n)}|X_2^{(n)} = x_2\right] = 1$.

Case 3. $1 - d(n) < x_2 < 1 - d(n) + \tilde{\gamma}(n)$: Since all members of group 2 lie in the interval $(x_2, 1)$, $\mathbb{P}\left[B_2^{(n)}|X_2^{(n)} = x_2\right] = ((1 - x_2)/d(n))^{M(n)}$.

Case 4. $1-d(n)+\tilde{\gamma}(n) \leq x_2 < 1$: The conditional probability $\mathbb{P}\left[B_2^{(n)}|X_2^{(n)}=x_2\right]$ is equal to the probability that all members in group 2 reside in $(x_2,1)\cap(\tilde{\gamma}(n),x_2+d(n)-1)$. This probability is given by $((d(n)-\tilde{\gamma}(n))/d(n))^{M(n)}$.

Integrating the above conditional probabilities over the corresponding intervals, we get

$$\begin{aligned}
\mathbb{P}\left[B_2^{(n)}\right] &= \int_0^{\tilde{\gamma}(n)} \left(1-\frac{\tilde{\gamma}(n)-x}{d(n)}\right)^{M(n)} dx \\
&+ \int_{\tilde{\gamma}(n)}^{1-d(n)} 1\, dx + \int_{1-d(n)}^{1-d(n)+\tilde{\gamma}(n)} \left(\frac{1-x}{d(n)}\right)^{M(n)} dx \\
&+ \int_{1-d(n)+\tilde{\gamma}(n)}^{1} \left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)} dx \\
&= 1-\tilde{\gamma}(n)-d(n)\left(1-\left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)+1}\right) \\
&+\frac{2d(n)}{M(n)+1}\left(1-\left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)+1}\right) \\
&= 1-\tilde{\gamma}(n)-\frac{M(n)-1}{M(n)+1}\cdot d(n)\cdot\left(1-\left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)+1}\right). \qquad \text{(A.32)}
\end{aligned}$$

Recall that $X_2^{(n)}$ is uniformly distributed over $[0,1)$ from observation O1 (in Section **??**). From the equality $\mathbb{P}\left[B^{(n)}\right]=\left(\mathbb{P}\left[B_2^{(n)}\right]\right)^{G(n)-1}$ and (A.32),

$$\mathbb{P}\left[B^{(n)}\right]=\left(1-\tilde{\gamma}(n)-\frac{M(n)-1}{M(n)+1}\cdot d(n)\cdot\left(1-\left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)+1}\right)\right)^{G(n)-1} \qquad \text{(A.33)}$$

Define $\Gamma(n):=\tilde{\gamma}(n)+\frac{M(n)-1}{M(n)+1}\cdot d(n)\cdot\left(1-\left(1-\frac{\tilde{\gamma}(n)}{d(n)}\right)^{M(n)+1}\right)$. Then, (A.33) can be rewritten as

$$\mathbb{P}\left[B^{(n)}\right] = \exp\left((G(n)-1)\cdot \log\left(1-\Gamma(n)\right)\right). \tag{A.34}$$

From (A.30), we know that

$$\Gamma(n) \sim \tilde{\gamma}(n) + d(n)\cdot\left(1-\exp\left(-\frac{\alpha}{\beta}\right)\right). \tag{A.35}$$

Also, because $\Gamma(n)\to 0$, we have $\log\left(1-\Gamma(n)\right) = -\Gamma(n) + O(\Gamma(n)^2)$, which gives us that

$$\mathbb{P}\left[B^{(n)}\right] = \exp\left((G(n)-1)\cdot\left(-\Gamma(n)+O(\Gamma(n)^2)\right)\right).$$

From (A.35) and Assumption 1, we can check that $G(n)\cdot d(n) = o(1)$ and $G(n)\cdot d(n)^2 = o(1)$. Hence, $G(n)\cdot\Gamma(n)^2 = o(1)$ because $\tilde{\gamma}(n) = o(d(n))$. Therefore,

$$\begin{aligned}\mathbb{P}\left[B^{(n)}\right] &\sim \exp\left(-G(n)\cdot d(n)\cdot\left(1-\exp\left(-\frac{\alpha}{\beta}\right)\right)\right)\\ &= N(n)^{-\beta\left(1-\exp\left(-\frac{\alpha}{\beta}\right)\right)}.\end{aligned} \tag{A.36}$$

From (A.4), (A.31), and (A.36),

$$\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right] \sim N(n)^{-\beta\left(1-\exp\left(-\frac{\alpha}{\beta}\right)\right)}. \tag{A.37}$$

Substituting (A.37) in (A.3), we obtain

$$\begin{aligned}\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right] &= N(n)\cdot\mathbf{E}\left[I_{1,1}^{(n)}(\tilde{\gamma}(n))\right]\\ &\sim N(n)^{1-\beta\left(1-\exp\left(-\frac{\alpha}{\beta}\right)\right)}.\end{aligned}$$

Therefore, if $1-\beta\left(1-\exp\left(-\frac{\alpha}{\beta}\right)\right) < 0$, i.e., $\alpha > \beta\cdot\log\left(\frac{\beta}{\beta-1}\right)$, then, $\mathbf{E}\left[C^{(n)}(\tilde{\gamma}(n))\right] \to 0$. This completes the proof of Theorem 2.4.

B. APPENDIX B

B.1 Proof of Lemma 3.1

We use the first moment method to prove the lemma. For each $n \in \mathbb{N}$, we define following rvs: For every $i \in \mathcal{N}_n$,

$$I_i^{(n)} = \mathbf{1}\{\text{node } i \text{ is isolated}\},$$

and

$$Z^{(n)} = \sum_{i \in \mathcal{N}_n} I_i^{(n)}.$$

By its definition, $Z^{(n)}$ denotes the number of isolated nodes out of the n nodes. It is a simple exercise to show that

$$\lim_{n\to\infty} \mathbf{E}\left[Z^{(n)}\right] = 0 \text{ implies } \lim_{n\to\infty} \mathbb{P}[Z^{(n)} = 0] = 1.$$

Hence, we prove $\mathbf{E}\left[Z^{(n)}\right] \to 0$ as $n \to \infty$ and make use of this observation to prove the lemma.

First, since $I_i^{(n)}, i \in \mathcal{N}_n$, are identically distributed,

$$\mathbf{E}\left[Z^{(n)}\right] = n \cdot \mathbf{E}[I_1^{(n)}] = n \cdot \mathbb{P}[\text{node } 1 \text{ is isolated}].$$

We compute $\mathbb{P}[\text{node } 1 \text{ is isolated}]$ by conditioning on $T_1^{(n)}$ and $\Theta_1^{(n)}$.

$$\begin{aligned}\mathbb{P}[\text{node } 1 \text{ is isolated}] &= \int\int \mathbb{P}[\text{node } 1 \text{ is isolated}|G_1^{(n)} = g]dG(z) \\ &\leq \phi^{n-1} \\ &= \left(1 - \pi r^{(n)}(\xi)^2 \cdot \psi^*\right)^{n-1}. \end{aligned} \tag{B.1}$$

We substitute $\pi r^{(n)}(\xi)^2 = \xi \log(n)/n$ in (B.1).

$$\mathbb{P}[\text{node } 1 \text{ is isolated}] \leq \left(1 - \xi\psi^* \cdot \frac{\log(n)}{n}\right)^{n-1}. \tag{B.2}$$

Using the equality $(1-x)^k = \exp(k\log(1-x))$,

$$(B.2) = \exp\left((n-1)\log\left(1 - \xi\psi^* \cdot \frac{\log(n)}{n}\right)\right). \tag{B.3}$$

Recall that, for any $a > 0$,

$$\log\left(1 - \frac{a\log(n)}{n}\right) = -\sum_{k\in N}\frac{1}{k}\left(\frac{a\log(n)}{n}\right)^k,$$

and

$$\exp\left(n\log\left(1 - \frac{a\log(n)}{n}\right)\right) \sim \exp(-a\log(n)) = n^{-a}. \tag{B.4}$$

The relation (B.4) gives us

$$(B.3) \sim \exp\left(-\xi\psi^*\log(n)\right) = n^{-\xi\psi^*}.$$

Therefore, if $\xi\psi^* > 1$ or $\xi > \frac{1}{\psi^*}$, then, $n \cdot \mathbf{E}\left[I_1^{(n)}\right] \to 0$ as $n \to \infty$.

B.2 Proof of Lemma 3.2

For fixed τ, let $\sigma(\tau)$ denote the maximum number of disjoint disks with radius τ whose union is contained in a unit rectangle and $B(x, r)$ is the disk centered at x with radius r.

Moreover, for each $n \in \mathbb{N}$ and every $A \subset \Omega$, $\#N^{(n)}(A)$ denotes the number of nodes in A (out of n nodes).

Fix $\alpha > 1$ and choose $\epsilon < \gamma < \beta$ that satisfy

i. $\epsilon < \xi/4$;

ii. $\sqrt{\epsilon} + \sqrt{\xi} < \sqrt{\gamma}$.

For each $n \in \mathbb{N}$, $\sigma_n = \sigma(r^n(\beta))$ and $\{\mathbf{Y_j^{(n)}}; j = 1, 2, \cdots, \sigma_n\}$ such that $B(\mathbf{Y}_j^{(n)}, r^{(n)}(\beta))$ are disjoint and $\cup_{j=1}^{\sigma_n} B(\mathbf{Y}_j^{(n)}, r^{(n)}(\beta)) \subset \Omega$.

Define the following events.

$$\begin{aligned} E_j^{(n)} =& \{\#N^{(n)}(B(\mathbf{Y}_j^{(n)}, r^{(n)}(\epsilon))) = 1 \text{ and} \\ & \text{no other node in } B(\mathbf{Y}_j^{(n)}, r^{(n)}(\beta)) \text{ is a neighbor of the node in } B(\mathbf{Y}_j^{(n)}, r^{(n)}(\epsilon))\}. \end{aligned}$$

When $E_j^{(n)}$ is true, from the above assumption $\sqrt{\epsilon} + \sqrt{\xi} < \sqrt{\gamma} < \sqrt{\beta}$, the node in the smaller disk $B(\mathbf{Y}_j^{(n)}, r^{(n)}(\epsilon))$ is isolated. Therefore,

$$\mathbf{P}^{(n)}\left(r^{(n)}(\xi)\right) \leq \mathbb{P}[(\cup_{j=1}^{\sigma_n} E_j^{(n)})^c] = \mathbb{P}[\cap_{j=1}^{\sigma_n} E_j^{(n)^c}].$$

We prove that when $\xi < \frac{1}{\psi^*}$, $\sum_{n \in \mathbb{N}} \mathbb{P}[\cap_{j=1}^{\sigma_n} E_j^{(n)^c}] < \infty$. Then, the Borel-Cantelli lemma tells us that the event of no isolated nodes occurs only for finitely many $n \in \mathbb{N}$ with probability one. Let $N_j^{(n)} := \#N^{(n)}(B(\mathbf{Y}_j^{(n)}, r^{(n)}(\beta)))$ and $\mathcal{E} := \cap_{j=1}^{\sigma_n} E_j^{(n)^c}$, and define the following three events.

- $A^{(n)} = \cup_{j=1}^{\sigma_n}\{N_j^{(n)} \leq 1\}$;
- $B^{(n)} = \cup_{j=1}^{\sigma_n}\{N_j^{(n)} \geq \alpha\beta \log(n)\}$;;
- $D^{(n)} = \cap_{j=1}^{\sigma_n}\{2 \leq N_j^{(n)} < \alpha\beta \log(n)\}$.

We now show that $\mathbb{P}[\mathcal{E}^{(n)} \cap A^{(n)}]$, $\mathbb{P}[\mathcal{E}^{(n)} \cap B^{(n)}]$, and $\mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)}]$ are summable.

B.2.1 $\mathbb{P}[\mathcal{E}^{(n)} \cap A^{(n)}]$

In order to prove $\mathbb{P}[\mathcal{E}^{(n)} \cap A^{(n)}]$ is summable, we show that the upper bound $\mathbb{P}[A^{(n)}]$ is summable. Using a union bound,

$$\mathbb{P}[A^{(n)}] \leq \sum_{j=1}^{\sigma_n} \mathbb{P}[N_j^{(n)} \leq 1]. \tag{B.5}$$

First, for every $j = 1, 2, \cdots, \sigma_n$,

$$\mathbb{P}[\mathbf{X}_i^{(n)} \in B(\mathbf{Y}_j^{(n)}, r^{(n)}(\beta))] = \pi r^{(n)}(\beta)^2 = \frac{\beta \log(n)}{n} =: p_n.$$

For any $\delta > 0$, for all sufficiently large n, $(1 - p_n)^{-1} \leq 1 + \delta$. Therefore, we have that

$$\begin{aligned}\mathbb{P}[N_j^{(n)} \leq 1] &= \mathbb{P}[N_j^{(n)} = 0] + \mathbb{P}[N_j^{(n)} = 1] \\ &= (1 - p_n)^n + np_n(1 - p_n)^{n-1} \\ &\leq \exp(-np_n) + (1 + \delta)np_n \exp(-np_n) \\ &= \exp(-\beta \log(n))(1 + (1 + \delta)\beta \log(n)). \end{aligned} \tag{B.6}$$

Substitute (B.6) in (B.5), we have

$$\begin{aligned}\mathbb{P}[A^{(n)}] &\leq \sigma_n \exp(-\beta \log(n))(1 + (1 + \delta)\beta \log(n)) \\ &= \sigma_n n^{-\beta}(1 + (1 + \delta)\beta \log(n)).\end{aligned}$$

We know that $\sigma_n = \Theta(n/\log(n))$. Hence, if $\beta > 2$, $\mathbb{P}[A^{(n)}]$ is summable, i.e., $\sum_{n \in N} \mathbb{P}[A^{(n)}] < \infty$.

B.2.2 $\mathbb{P}[\mathcal{E}^{(n)} \cap B^{(n)}]$

Again, we prove that its upper bound $\mathbb{P}[B^{(n)}]$ is summable. Using a union bound,

$$\mathbb{P}[B^{(n)}] \leq \sum_{j=1}^{\sigma_n} \mathbb{P}[N_j^{(n)} \geq \alpha\beta \log(n)]. \tag{B.7}$$

First, we rewrite $N_j^{(n)}$ as a sum of independent Bernoulli rvs as follows:

$$N_j^{(n)} = \sum_{i=1}^{n} \mathbf{I}\{\mathbf{X}_i^{(n)} \in B(\mathbf{Y}_j^{(n)}, r^{(n)}(\beta))\}.$$

By Theorem 5.1 in [26],

$$\mathbb{P}[N_j^{(n)} \geq \alpha\beta \log(n)] \leq \mathbb{P}[Poisson(n\lambda_n) \geq \alpha\beta \log(n)]$$

where $Poisson(n\lambda_n)$ is a Poisson rv with parameter $n \cdot \lambda_n$, and

$$\begin{aligned} \lambda_n &= -\log\left(1 - \frac{\beta \log(n)}{n}\right) \\ &= \frac{\beta \log(n)}{n} + \sum_{k\geq 2} \frac{1}{k}\left(\frac{\beta \log(n)}{n}\right)^k. \end{aligned}$$

Thus,

$$n \cdot \lambda_n = \beta \log(n) + n \sum_{k\geq 2} \frac{1}{k}\left(\frac{\beta \log(n)}{n}\right)^k. \tag{B.8}$$

Using Proposition 1 in [25],

$$\begin{aligned} &\mathbb{P}[Poisson(n\lambda_n) \geq \alpha\beta \log(n)] \\ &\leq \left(1 - \left(\frac{n\lambda_n}{\alpha\beta \log(n) + 1}\right)\right)^{-1} \frac{(n\lambda_n)^{\alpha\beta \log(n)}}{(\alpha\beta \log(n))!} \exp(-n\lambda_n) \\ &=\leq \left(1 - \left(\frac{n\lambda_n}{\alpha\beta \log(n) + 1}\right)\right)^{-1} \frac{n^{\alpha\beta \log(n\lambda_n)}}{(\alpha\beta \log(n))!} \exp(-n\lambda_n). \end{aligned} \tag{B.9}$$

Making use of (B.8),

$$(B.9)\ \left(1 - \frac{1}{\alpha}\right)^{-1} \frac{n^{\alpha\beta \log(\beta \log(n))}}{(\alpha\beta \log(n))!} \exp(-\beta \log(n)). \tag{B.10}$$

By Stirling's formula,

$$(\alpha\beta \log(n))!\ n^{\alpha\beta \log(\alpha\beta \log(n))} \cdot n^{-\alpha\beta} \cdot \sqrt{2\pi\alpha\beta \log(n)}. \tag{B.11}$$

From (B.10) and (B.11), we have

$$(B.9)\ \left(1 - \frac{1}{\alpha}\right)^{-1} n^{-\alpha\beta\log(\alpha)+\beta(\alpha-1)}\sqrt{2\pi\alpha\beta\log(n)}. \tag{B.12}$$

From (B.7) and (B.12), a sufficient condition for summability of $\mathbb{P}[B^{(n)}]$ is $-\alpha\beta\log(\alpha)+\beta(\alpha-1) < -2$ or, equivalently,

$$\beta(\alpha(\log(\alpha)-1)+1) > 2. \tag{B.13}$$

One can show that the minimum of $\alpha(\log(\alpha)-1)+1$ is achieved by $\alpha = 1$ and, for all $\alpha > 1$, $\alpha(\log(\alpha)-1)+1 > 0$. Thus, for any fixed $\alpha > 1$, we can find sufficiently large β to satisfy (B.13).

B.2.3 $\mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)}]$

We first rewrite the probability $\mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)}]$ by conditioning on the possible values of $(N_1^{(n)}, N_2^{(n)}, \cdots, N_{\sigma_n}^{(n)}) : \mathbf{N}^{(n)} \in \Pi_{j=1}^{\sigma_n}\{2, 3, \cdots, \alpha\beta\log(n)\} =: \mathbb{N}_*^{(n)}$.

$$\mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)}] = \sum_{\mathbf{n}\in\mathbf{N}_*^{(n)}} \mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)} | \mathbf{N}^{(n)} = \mathbf{n}].$$

When $E_j^{(n)}$, $j = 1, 2, \cdots, \sigma_n$, holds, we denote the node in $B(\mathbf{Y}_j^{(n)}, r^{(n)}(\epsilon))$ by $i(j)$. We can compute the probability of the event that node $i(j)$ does not have any neighbor in $B(\mathbf{Y}_j^{(n)}, r^{(n)}(\beta)) \backslash B(\mathbf{Y}_j^{(n)}, r^{(n)}(\epsilon))$ by conditioning on $\Theta_{i(j)}^{(n)}$ and $T_{i(j)}^{(n)}$: Given $\Theta_{i(j)}^{(n)} = \theta$ and $T_{i(j)}^{(n)} = t$, the probability that another node in $B(\mathbf{Y}_j^{(n)}, r^{(n)}(\beta)) \backslash B(\mathbf{Y}_j^{(n)}, r^{(n)}(\epsilon))$, say node i^*, is not a neighbor of node $i(j)$, which we denote by $q_n(\theta, t)$, is equal to the sum of (i) the probability that $d(\mathbf{X}_{i(j)}^{(n)}, \mathbf{X}_{i^*}^{(n)}) > r^{(n)}(\xi)$ and (ii) the probability that $d(\mathbf{X}_{i(j)}^{(n)}, \mathbf{X}_{i^*}^{(n)}) \le r^{(n)}(\xi)$ but either $\Theta_{i^*}^{(n)}$ or $T_{i^*}^{(n)} < \theta$. Therefore, $q_n(\theta, t)$ can be expressed as

$$q_n(\theta,t) = P(d(\mathbf{X}_{i(j)}^{(n)}, \mathbf{X}_{i^*}^{(n)}) > r^{(n)}(\xi)) + P(d(\mathbf{X}_{i(j)}^{(n)}, \mathbf{X}_{i^*}^{(n)}) \le r^{(n)}(\xi)) \cdot P(\theta_{i^*}^{(n)} > t \text{ or } T_{i^*}^{(n)} < \theta)$$
$$= 1 - \frac{\xi}{\beta} + \frac{\xi}{\beta} \cdot P(\theta_{i^*}^{(n)} > t \text{ or } T_{i^*}^{(n)} < \theta). \quad \text{(B.14)}$$

Using (B.14), we know that

$$\mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)} | \mathbf{N}^{(n)} = \mathbf{n}, (\Theta_{i(j)}^{(n)} = \theta_j, T_{i(j)}^{(n)} = t_j), j = 1, 2, \cdots, \sigma_n]$$
$$= \prod_{j=1}^{\sigma_n} \mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)} | N_j^{(n)} = n_j, \Theta_{i(j)}^{(n)} = \theta_j, T_{i(j)}^{(n)} = t_j] \quad \text{(B.15)}$$
$$= \prod_{j=1}^{\sigma_n} \left(1 - n_j \cdot \frac{\epsilon}{\beta} \cdot q_n(\theta_j, t_j)^{n_j - 1}\right). \quad \text{(B.16)}$$

Equality in (B.15) follows from the observation that, once $\mathbf{N}^{(n)}$ is fixed, due to the assumed mutual independence of node locations, $\{E_j^{(n)^c} \cap D^{(n)}\}$, $j = 1, 2, \cdots, \sigma_n$, are conditionally independent.

We can now obtain an upper bound on $\mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)} | \mathbf{N}^{(n)} = \mathbf{n}]$ by integrating (B.16) over $(\Theta_{i(j)}^{(n)}, T_{i(j)}^{(n)})$, $j = 1, 2, \cdots, \sigma_n$: For $j = 1, 2, \cdots, \sigma_n$,

$$\int q_n(\theta_j, t_j)^{n_j - 1} dG = \int \left(1 - \frac{\xi}{\beta} + \frac{\xi}{\beta} \cdot P(\theta_{i^*}^{(n)} > t_j \text{ or } T_{i^*}^{(n)} < \theta_j)\right)^{n_j - 1} dG. \quad \text{(B.17)}$$

Now, let us define (θ^*, t^*) as

$$(\theta^*, t^*) = \arg_{(\theta_j, t_j)} \max P(\Theta_i^{(n)} > t_j \text{ or } T_i^{(n)} < \theta_j).$$

And, also we will define $S := B((\theta^*, t^*), s)$ which is the ball centered by (θ^*, t^*) with radius s. Then, from (**??**), the definition of ψ, within S we know that the maximum value of $P(\Theta_i^{(n)} > t_j \text{ or } T_i^{(n)} < \theta_j)$ is ψ which is achieved at the point, (θ^*, t^*). Now, within S we can think about the minimum value of $P(\Theta_i^{(n)} > t_j \text{ or } T_i^{(n)} < \theta_j)$ which can be expressed as $\psi - \epsilon'$. We know that as s goes to 0, ϵ' also goes to 0. From this definition of S and $\psi - \epsilon'$, an lower bound on (B.17) can be derived as

$$\begin{aligned}\int q_n(\theta_j, t_j)^{n_j-1} dG &= \int_S \left(1 - \frac{\xi}{\beta} + \frac{\xi}{\beta} \cdot P(\theta_{i^*}^{(n)} > t_j \text{ or } T_{i^*}^{(n)} < \theta_j)\right)^{n_j-1} dG \\ &\quad + \int_{S^c} \left(1 - \frac{\xi}{\beta} + \frac{\xi}{\beta} \cdot P(\theta_{i^*}^{(n)} > t_j \text{ or } T_{i^*}^{(n)} < \theta_j)\right)^{n_j-1} dG \\ &\geq \int_S \left(1 - \frac{\xi}{\beta} + \frac{\xi}{\beta} \cdot P(\theta_{i^*}^{(n)} > t_j \text{ or } T_{i^*}^{(n)} < \theta_j)\right)^{n_j-1} dG \\ &> \pi s^2 \cdot \left(1 - \frac{\xi}{\beta} + \frac{\xi}{\beta} \cdot (\psi - \epsilon')\right)^{n_j-1}. \qquad \text{(B.18)}\end{aligned}$$

From (3.3), (B.18) can be rewritten as

$$\begin{aligned}&\int q_n(\theta_j, t_j)^{n_j-1} dG > \pi s^2 \cdot \left(1 - \frac{\xi}{\beta} + \frac{\xi}{\beta} \cdot (1 - \psi^* - \epsilon')\right)^{n_j-1} \\ &= \pi s^2 \cdot \left(1 - \frac{\xi}{\beta} \cdot (\psi^* + \epsilon')\right)^{n_j-1} =: q_n. \qquad \text{(B.19)}\end{aligned}$$

Using the inequality (B.19), we obtain

$$\begin{aligned}\mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)} | \mathbf{N}^{(n)} = \mathbf{n}] &\leq \prod_{j=1}^{\sigma_n} \left(1 - n_j \cdot \frac{\epsilon}{\beta} \cdot q_n\right) \\ &\leq \exp\left(-\frac{\epsilon}{\beta} \sum_{j=1}^{\sigma_n} n_j q_n\right). \qquad \text{(B.20)}\end{aligned}$$

Let us consider the exponent in (B.20) without minus sign. Because $n_j \in \{2, 3, \cdots, \alpha\beta \log(n)\}$, we can get an lower bound on (B.20) as

$$\begin{aligned}
\frac{\epsilon}{\beta}\sum_{=1}^{\sigma_n} n_j q_n &\geq \frac{2\epsilon}{\beta}\sum_{j=1}^{\sigma_n} q_n \\
&= \frac{2\epsilon}{\beta}\cdot \pi s^2 \sum_{j=1}^{\sigma_n}\left(1-\frac{\xi}{\beta}\cdot(\psi^*+\epsilon')\right)^{n_j-1} \\
&> \frac{2\epsilon}{\beta}\cdot \pi s^2 \sum_{j=1}^{\sigma_n}\left(1-\frac{\xi}{\beta}\cdot(\psi^*+\epsilon')\right)^{\alpha\beta\log(n)} \\
&= \frac{2\epsilon}{\beta}\cdot \pi s^2 \cdot \sigma_n \left(1-\frac{\xi}{\beta}\cdot(\psi^*+\epsilon')\right)^{\alpha\beta\log(n)} \\
&= \frac{2\epsilon}{\beta}\cdot \pi s^2 \cdot \sigma_n \cdot \exp\left(\log(1-\frac{\xi}{\beta}(\psi^*+\epsilon'))\cdot\alpha\beta\log(n)\right) \\
&= \frac{2\epsilon}{\beta}\cdot \pi s^2 \cdot \sigma_n \cdot n^{\alpha\beta\log(1-\frac{\xi\psi^*}{\beta}-\frac{\xi\epsilon'}{\beta})} \\
&= \frac{2\epsilon}{\beta}\cdot \pi s^2 \cdot \sigma_n \cdot n^{\alpha\beta\log(1-\frac{\xi\psi^*}{\beta}-\frac{\epsilon''}{\beta})}. \qquad \text{(B.21)}
\end{aligned}$$

Then, from (B.20) and (B.21), an upper bound on $\mathbb{P}[\mathcal{E}^{(n)}\cap D^{(n)}|\mathbf{N}^{(n)}=\mathbf{n}]$ can be expressed as

$$\begin{aligned}
\mathbb{P}[\mathcal{E}^{(n)}\cap D^{(n)}|\mathbf{N}^{(n)}=\mathbf{n}] &\leq \exp\left(-\frac{2\epsilon}{\beta}\cdot\pi s^2\cdot\sigma_n\cdot n^{\alpha\beta\log\left(1-\frac{\xi\psi^*}{\beta}-\frac{\epsilon''}{\beta}\right)}\right) \\
&\leq \exp\left(-\frac{2\epsilon}{\beta}\cdot\pi s^2\cdot\sigma_n\cdot n^{\alpha\beta\log\left(1-\frac{\gamma\psi^*}{\beta}-\frac{\epsilon''}{\beta}\right)}\right).
\end{aligned}$$

Since $\sigma_n = \Theta(n/\log(n))$ and the above inequality does not depend on the value of $\mathbf{N}^{(n)}$, a sufficient condition for summability of $\mathbb{P}[\mathcal{E}^{(n)}\cap D^{(n)}]$ is

$$1+\alpha\beta\log\left(1-\frac{\gamma\psi^*}{\beta}-\frac{\epsilon''}{\beta}\right)>0,$$

or equivalently,

$$\gamma < \frac{\beta}{\psi^*}\left(1-\exp\left(-\frac{1}{\alpha\beta}\right)-\frac{\epsilon''}{\beta}\right). \qquad \text{(B.22)}$$

Note that, as $\beta \to \infty$,

$$\frac{\beta}{\psi^*}\left(1 - \exp\left(-\frac{1}{\alpha\beta}\right) - \frac{\epsilon''}{\beta}\right) \to \frac{1}{\alpha\psi^*} - \frac{\epsilon''}{\psi^*}.$$

Therefore, for sufficiently large β, we have

$$\frac{1-\epsilon''}{\alpha\psi^*} - \frac{\epsilon''}{\psi^*} < \frac{\beta}{\psi^*}\left(1 - \exp\left(-\frac{1}{\alpha\beta}\right) - \frac{\epsilon''}{\beta}\right).$$

Choose $\gamma = \frac{1-2\epsilon''}{\alpha\psi^*} - \frac{\epsilon''}{\psi^*}$. Then,

$$\gamma < \frac{1-\epsilon''}{\alpha\psi^*} - \frac{\epsilon''}{\psi^*} < \frac{\beta}{\psi^*}\left(1 - \exp\left(-\frac{1}{\alpha\beta}\right) - \frac{\epsilon''}{\beta}\right).$$

Since ϵ'' can be arbitrarily small and β can be arbitrarily large, we can choose ϵ'', γ, and β that satisfy the above inequalities, and the summability of $\mathbb{P}[\mathcal{E}^{(n)} \cap D^{(n)}]$ follows.

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