

ABSTRACT

Title of Document: THE INVENTORY ROUTING PROBLEM
WITH THIRD PARTY LOGISTICS

Hadi Sadrsadat, 2012

Directed By: Professor Ali Haghani
Department of Civil and Environmental
Engineering

There are two key planning issues in supply chain: inventory management and transportation. In this research, the inventory control and transportation of syrup concentrate and final products for one bottling company working for a beverage company is studied. Operation of most of beverage companies is based on a franchised distribution system. In this operation, syrup concentrate is produced by a beverage company and sold to bottlers. Bottlers, in turn, mix the syrup concentrate with different ingredients to produce various products and distribute them to retailers. Unsatisfied orders have several harmful effects on the bottling company. The bottling company may not satisfy all demands due to its small fleet size, which is not able to cover all deliveries in the right timeframe. One method for preventing missed orders is sending orders to some retailers in advance to hold for future use. This allows the fleet to be free to service the rest of the retailers. This policy is possible if those retailers have available capacity to keep products. Another way to deal with this

problem is by renting vehicles, which increases the fleet size. The last option for delivering to a retailer when the owned fleet is not able to do so is outsourcing shipping and/or warehousing. The bottling company contracts with a Third Party Logistics Provider (TPLP), who is responsible for delivery of final products to some of the bottler's retailers. Also, TPLPs can store commodities in their warehouses and deliver products to retailers at the right time if there is no available capacity in the bottler's warehouses.

This problem belongs to Inventory Routing Problem (IRP) with some new features such as options for rental vehicle and TPLPs. IRP is a well-studied problem in Operation Research but most of the studies take a single period into account. In contrast, the proposed model in this study includes several time steps in which a decision in one time step can affect future time steps. The proposed model is a multi-tier, multi-plant, multi-warehouse, and multi-product model which considers non-homogeneous fleet. No model in the literature considers all of these characteristics simultaneously.

In this research heuristic methods are developed to solve large problems for which optimization packages cannot find even a feasible solution. Two heuristic methods are proposed for this problem, which are based on fix-and-run algorithm. Three improvement phases are also developed to enhance the final solution of heuristics. The proposed heuristic methods in this research can find an appropriate feasible solution with only a small gap from an upper bound and in reasonable running time.

THE INVENTORY ROUTING PROBLEM WITH THIRD PARTY LOGISTICS

By

Hadi Sadrsadat

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Advisory Committee:
Professor Ali Haghani, Chair
Professor Paul M. Schonfeld
Professor Philip T. Evers
Professor Martin Dresner
Professor Cinzia Cirillo

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Chapter 1: Introduction

1.1. Supply Chain

The supply chain is a set of all activities that integrate suppliers, manufacturers, warehouses and stores to produce commodities in the right quantities and send them to the right locations at the right time to minimize total cost, while satisfying service-level requirements (Simchi-Levi, Kaminsky , & Simchi-Levi, 2003). This chain includes different management activities, such as purchasing, production, inventory control, and distribution. The main goals of supply-chain management are to organize the flow of raw materials and products in the network to all actors in the chain and minimize total costs, which include production, inventory and distribution.

There are two key planning issues in supply-chain management: inventory management and transportation. Inventory management includes activities such as production, ordering, holding, and shortage of products. Transportation includes shipping raw materials and final products between sources, factories, warehouses, and retailers.

Theoretically, there are some benefits in the integration of inventory control and transportation, and especially when product demand is high and the costs associated with inventory and transportation are considerable. The Inventory Routing Problem (IRP), which is the combination of inventory control and transportation, is the extension of the Vehicle Routing Problem (VRP), in which decisions about routing and inventory control are made simultaneously. However, there are some

major differences between these two problem categories. In the basic VRP, the target is finding the best routes to deliver (or pick up) predetermined amounts of products within specific time intervals while minimizing the objective function, which is usually the total cost. In this case, delivery quantities are known in advance.

However, in IRP, besides generating delivery routes for different vehicles, a producer must decide how much of each product to deliver to each customer. The producer has to consider many criteria, such as production rates at plants and inventory levels in warehouses. Another important difference between VRP and IRP is the planning horizon. Generally, IRP has a longer planning horizon than VRP, which is usually a single day problem (i.e., customers should be served by the end of the day). In contrast, with IRP a producer must decide about each day's delivery, which, in turn, influences what may happen in the future.

The process of assigning a set of customers to receive inventory from a specific location (the inventory allocation decision) is based on routing cost information and marginal profit for each customer in the set. On the other hand, the delivery cost for each customer depends on the location servicing it and the vehicle's route to that customer. This means that information about the assignment of customers to each location is required. This interrelationship between inventory management and transportation is the main reason researchers cite for integrating these two major supply-chain activities (Chien, Balakrishnan, & Wong, 1989). This study focuses on inventory control, routing, and their interaction.

A few commercial optimization-based packages integrate inventory management and transportation (Andersson, Hoff, Christiansen, Hasle, &

Lokketangen, 2010). However, it is not clear that the inventory and transportation models in these systems are truly integrated or executed sequentially (Andersson, Hoff, Christiansen, Hasle, & Lokketangen, 2010). Most industries solve these two models separately as follows. First, they solve the inventory-management problem and determine the delivery quantity and the time for each delivery. These outputs then become input for the transportation model to route vehicles to customers within the scheduled time windows.

In some industries, there is one player for both inventory management and transportation. In other words, these industries tend to jointly manage inventory management, transportation, and, sometimes, production (Andersson, Hoff, Christiansen, Hasle, & Lokketangen, 2010). On the other hand, some transportation companies are interested in managing their customers' inventories to improve service quality and to use their fleet better. Therefore, production companies can control their production more precisely by outsourcing inventory management and transportation. Thus, outsourcing inventory management and transportation may benefit both sides, depending on different factors such as each company's expertise and how they contract with each other. Most beverage companies such as Coca-Cola or Pepsi are examples of a company that outsources product transportation, which will be described in the next section.

1.2. A Beverage Company's Supply Chain

The managements of most beverage companies in North America have decided to decentralize their various operations to meet changing customer demands. As a result,

their operation is based on a franchised distribution system. In this operation syrup concentrate is produced by a beverage company and sold to bottlers. Bottlers, in turn, mix the syrup concentrate with different ingredients to produce various products and distribute them to retailers, vending machines, and restaurants (Wan, Evers, & Dresner, 2012).

As a result, the beverage company's supply chain includes three main tiers. In the first tier, factories produce syrup concentrate. Because their output is not ready to use, supplementary operations are needed to produce the final products (Wan, Evers, & Dresner, 2012).

The second tier is assigned to bottling companies, which receive syrup concentrate as raw material and mix it with other ingredients to produce the final products for different brands. Bottlers then fill bottles and cans with products and distribute them to consumption locations, which are in the third tier. Bottlers can store products for future demand. If there is no available transportation, products can be stored in the bottler's warehouse for delivery when transportation is again available (Wan, Evers, & Dresner, 2012).

The third tier, consumption locations, includes retailers, vending machines, and restaurants. Demand is defined at these locations and should be met at the right time. Some of these consumption locations, such as large retailers, have their own warehouses and can store products for future demand. However, small customers such as restaurants and vending machines, can only stock their current demand and future demand must be satisfied by the next delivery (Wan, Evers, & Dresner, 2012).

The vehicle fleet connects these three tiers. One vehicle can be routed to locations in different tiers. This depends on many factors, such as travel time between different points, vehicle capacity, and demand of consumption location. Different types of vehicles can comprise a fleet; for example, vehicles that transport syrup concentrate from plants (the first tier) to bottling companies (the second tier) must be specially equipped and differ from the vehicles that deliver the final products from bottling companies to retailers (the third tier). In addition, the fleet that carries final products from the second to the third tier is not homogeneous. For instance, large retailers (e.g., Costco, Wal-Mart, and Target) usually have large areas for unloading and maneuvering that makes delivery easy for large trucks, while small retailers like 7-Eleven and vending machines usually do not and must be serviced by smaller vehicles. Therefore, managing vehicles to meet the demands of different types of customers within their respective time windows and minimizing total cost is essential.

1.3. Problem Description

In this research, the inventory control and transportation of syrup concentrate and final products for one bottling company working for a beverage company is studied. As mentioned in the previous section, the bottling company receives syrup concentrate and mixes it with other ingredients to produce final products.

Final products are stored in warehouses until they are sent to their destination in the right time frame. Let's assume that there are some factories that belong to a beverage company and produce syrup concentrate. These are called "plants". Let's also assume that there are some locations operated by a bottling company that mix

syrup with other ingredients and store final products. These are called “bottlers” for the purposes of this study. Finally, assume that there are several consumption locations that are served by bottlers. These are called “retailers”. The main player in this problem is a bottling company that manages its bottlers and delivers final products to retailers. This company has its own nonhomogeneous fleet for transportation of syrup concentrate from plants to bottlers and final products from bottlers to retailers. The fleet is divided into two major groups. The first group delivers syrup concentrate to bottlers and travels only between plants and bottlers. The second group is responsible for delivery of final products to retailers, and therefore its vehicles work only between bottlers and retailers. Each of these two groups contains non-homogenous vehicles that are different in terms of speed, capacity, and equipment. For instance, some vehicles cannot service small retailers due to their size.

The bottling company knows retailers’ immediate future demands for a timeframe of several days. If the demand is not satisfied, a penalty is applied to the bottling company’s total cost based on quantity of demand that has not been met and shortage cost, which is not the same for all retailers. Retailers have the capability to store final products for future demand at a specific holding cost. Storage capacity and holding costs vary among retailers. Thus, if the company is not able to satisfy a retailer’s order because of shortage in fleet size or warehouse capacity, it has the opportunity to send products earlier to avoid the shortage penalty. In this case, the company has to pay the holding cost, which may be less than the penalty cost.

The same product may have different prices for different retailers. Price can be consistent for retailers in the same region, but it will not be the same for all retailers overall. Therefore, for each time step, the company must determine which retailer's demand should be satisfied when, for some reason, servicing all retailers is impossible.

One of the main reasons that the bottling company cannot meet the demands of some retailers is that its fleet size is too small to cover all deliveries in the right timeframe. Considering travel time between each pair of facility locations, service time (uploading in plants and bottlers and unloading in bottlers and retailers), and the limitations of daily business hours, some demands may be not satisfied. In this case, the bottling company has three alternatives:

1. Do nothing. The bottling company does not serve some retailers and pays the penalty.
2. Use rental vehicles. The company rents a vehicle or vehicles to deliver products to retailers whose demands cannot be satisfied by the current fleet. Rental vehicles cost the company more than owned vehicles; this includes fixed costs and operational costs. The bottling company has to operate rental vehicles, so routing and the delivery schedule for each rental vehicle is controlled by the bottling company.
3. Use Third Party Logistics Provider (TPLP). With this alternative which will be explained more in the next chapter, the company contracts with a logistics service provider which is called Third Party Logistics Provider (TPLP) to outsource the shipping of some products to some retailers.

The main difference between alternatives two and three is that the bottling company is responsible for routing rental vehicles, whereas with TPLP, vehicle routing is outside the authority of the bottling company. The bottling company asks TPLP to deliver products to retailers at a particular time. Although the bottling company is not in charge of routing for TPLP vehicles, all of the costs associated with these vehicles are included in the contract price and the bottling company pays them. The company, therefore, must decide which option is the best according to the costs and benefits associated with each.

Also, the bottling company may not have enough space in its warehouses to store its final products. In this case, in addition to storing products in retailers' warehouses, it can also store them in TPLP warehouses. In other words, the model considers warehousing outsourcing as an option that may benefit the bottling company.

Thus, the company must get raw material from plants, produce final products in bottlers, and ship them to retailers. It must also optimize its profit, taking into consideration inventory control in warehouses—both its own and retailers'—and, when necessary, store products in TPLP warehouses. Finally, it must deliver final products to retailers using different vehicles including its owned, rental, and TPLP.

1.4. Motivation for and Objective of the Research

The supply chain plays a key role in a company's profits and costs. At the same time, competition between companies in the same industry is steadily increasing. Sometimes the difference in quality between two rival companies' product is high and

a customer will wait for the better commodity to be available if it is out of stock. Most of the time, however, shoppers do not wait for a specific brand's product and instead take a similar one. Thus, it is so important that a company not run out of stock.

On the other hand, price is one of the primary factors affecting a buyer's choice of one commodity over another. Obviously, a company whose products are more expensive than another's—but of the same quality—will lose market share. Many different factors influence a commodity's price, such as the costs of raw materials, labor, equipment, inventory, and transportation. To achieve high market share, in addition to product quality and marketing, all the above costs should be optimized. This research focuses on two factors, inventory and transportation. It considers their benefits and costs and proposes a model to optimize both operations simultaneously.

In the real world, industries optimize inventory and transportation activities separately. They first solve inventory problems then use the results to address transportation problems. The final solution is not optimal, however, because the interaction between inventory control and transportation is ignored.

Fleet shortage is prevalent in many companies. This can cause some retailers to be out of stock and, if it happens again, may lead to loss of market share. There are two alternatives for avoiding this consequence: renting vehicles and contracting with other shipping companies as TPLPs. Either option increases total cost, but may benefit the company if the penalty for running out of stock is taken into account. This depends on many factors, such as the quantity and price of unshipped product, travel

time and distance between bottlers and retailers, penalty cost, and, of course, the costs associated with alternatives. Creating a model that takes all of these aspects into account is the main objective of this study.

1.5. Contributions of the Research

In this study, the inventory and routing of a bottling company working jointly with a beverage company is mathematically formulated. This problem is associated with IRP, with some new features such as options for rental vehicle and TPLPs. The IRP has been well studied in Operations Research but most of the studies have taken only a single period into account. In contrast, the model proposed in this study includes several time steps in which a decision in one time step can affect future time steps. The model is also multi-tier, multi-plant, multi-warehouse, and multi-product, with a nonhomogeneous fleet, which increases its complexity significantly. No model in the literature considers all of these characteristics simultaneously; for instance, rental vehicles and TPLP have not been included in the literature of IRP. In addition, in most of the current models trucks have not been tracked to simplify the problem. In other words, they figure out how many products should be delivered to different retailers. Even if they consider a vehicle's capacity, they do not take into account the cost of returning vehicles to the depot or providing a path for each vehicle. However, tracking vehicles helps managers of a supply chain to improve the quality of shipping and delivery. In this study, every vehicle is followed, which renders the model more adaptive to the real world than models that do not track vehicles.

One of the main goals of this research is to build an optimization model at the operational level. Inventory control and product transportation belong to the tactical and operational level, so the model must be able to capture realistic operational activities. Shortage in the fleet, retailer storage capacity, and fleet tracking are real-world factors that many industries face in their daily operations. By being more comprehensive, therefore, the proposed model is also more complicated than current models.

This research develops heuristic methods to solve large problems which cannot be solved by optimization packages. The optimization packages are not able to find even a feasible solution for large problems. Since this problem is not real-time, the running time of the heuristic is not the main concern; however, it should be fast enough to deliver a solution in reasonable amount of computation time. As a result, development of efficient heuristic methods to solve problem of this study is the other main contribution of this research.

1.6. Organization of the Dissertation

In the next chapter, previous works on IRP and TPLP is reviewed. The problem is described in more detail in Chapter 3 and the mathematical formulation of the model is presented. Chapter 4 presents a set of numerical studies to illustrate different capabilities of the model. In Chapter 5, the first heuristic algorithm to solve large problems of this study, which is based on Fix and Run algorithm is proposed. The numerical results to evaluate capabilities of the first heuristic method are presented in Chapter 5. Chapter 6 explains the second heuristic method to solve large problems,

which is also based on Fix and Run algorithm, and compares performances of two proposed heuristic algorithms. In Chapter 7, another heuristic method to solve this model, based on decomposition, is proposed and its results are discussed in details. Chapter 8 is dedicated to sensitivity analysis of some parameters of the model. Finally, a summary of this dissertation and some suggestions for future research are presented in Chapter 9.

Chapter 2: Literature Review

In this chapter, the supply chain (SC) is described and its literature is reviewed. The inventory-routing problem (IRP) is discussed in detail, and the gap between previous work and the proposed model is presented, after which the third-party logistics provider (TPLP) concept and its literature are reviewed. Finally, differences between other studies using the TPLP concept and this study are discussed.

2.1. Supply Chain

The SC begins with the extraction of raw materials and passes through producers, warehouses, and retailers to reach the final user. However, researchers from different fields have varying definitions for SC, and there is no unique definition in the literature (Tan, 2001). For instance, La Londe and Masters (1994) define SC as a set of firms passing materials forward, which is very general. Scott and Westbrook (1991) have defined SC in more detail, as a chain that connects different components of the production and supply process, from raw materials to the final product in a user's hands. This process includes several organizational boundaries. New and Payne (1995) also define SC this way.

The SC can be viewed from another perspective. The retailer's goal is to have products available for customers. From this standpoint, transportation and integrated logistics are also important SC activities. Logistics is the management of the flow of goods and services between suppliers and final users to satisfy customer demand.

This involves the integration of information, transportation, inventory, and warehousing. The end result, ideally, is that final products are shipped from manufacturers to retailers efficiently and take into account inventory replacement and the reduction of transportation costs.

Therefore, SC is an integrated process in which many actors— raw-materials producer, manufacturers, wholesalers, retailers, and transportation companies—must cooperate to produce the final commodity and distribute it among users (Eksioglu, 2002). Figure 2-1 shows Beamon’s (1998) definition of the SC process.

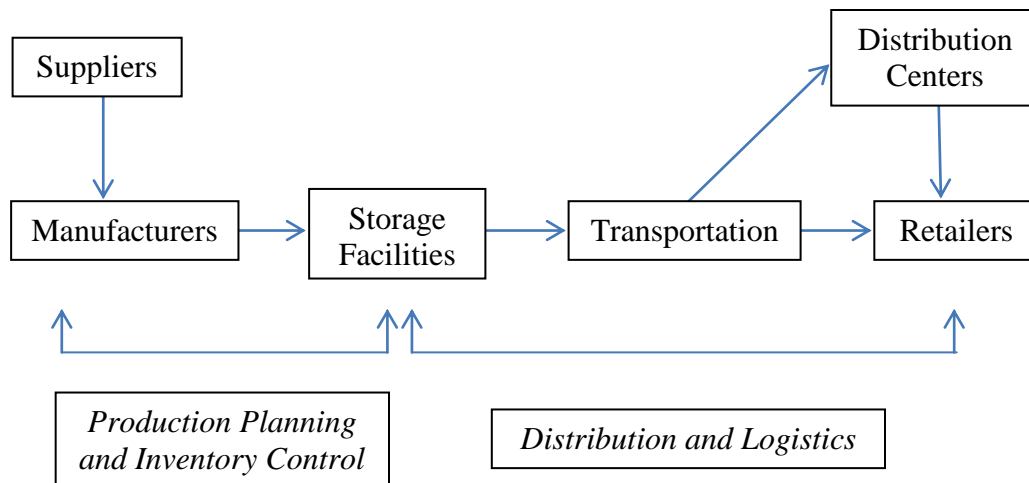


Figure 2-1 The supply chain process (Beamon, 1998)

The SC can be divided into two processes:

- 1- Production planning and inventory control
- 2- Distribution and logistics

The production planning and inventory control process includes the production and inventory of raw materials and final products. Extraction, inventory,

and storage of raw materials; production schedule; manufacturing-process design; and storage of the final product in warehouses are the main activities in this process.

The distribution and logistics process includes shipping the final product from warehouses to retailers and other consumption locations. In some industries, the final product is first transported to distribution centers to be distributed among retailers. In this case, inventory control in distribution centers' warehouses is also included in the process.

Each of the activities within the SC, such as inventory control or product delivery, has been extensively studied. However, most researchers tend to model the whole process and see the interaction between different activities. For instance, fleet size influences warehouse capacity. Similarly, warehouse capacity affects fleet size. Optimizing every element of the SC process individually, therefore, does not necessarily guarantee an optimal solution for the whole process, as the effects of different components on each other have not been considered.

Generally, researchers try to resolve the following issues:

1. Production capacity and related technology at each plant.
2. Number and locations of plants and warehouses.
3. Capacity of warehouses.
4. Delivery quantities for different retailers.
5. Inventory policies and controls in warehouses.
6. Routing for vehicles in the fleet to distribute products among retailers.

These can be divided into three main groups: facility location, inventory management, and routing. These groups belong to different levels of decision-

making, however, and some cannot be integrated due to incompatibility of planning horizon. At the strategic level, long-term decisions such as where to locate a facility, production technology, and plant capacity are made. These overarching SC decisions affect efficiency at lower levels (Schmidt & Wilhelm, 2000).

At the tactical level, midterm decisions such as material-flow management, which includes production levels at plants and inventory levels, are made (Schmidt & Wilhelm, 2000). Finally, at the operational level—which is the lowest level—, short-term decisions such as day-to-day production schedules and distribution management, including delivery routes, are made (Schmidt & Wilhelm, 2000).

This means, for instance, that selecting a facility location, which is a strategic-level decision, is not compatible with inventory and routing, which are operational activities. Since this study concentrates on activities at the operational level of the SC, facility location was excluded from the research design, which focuses only on inventory management and routing.

In the next section, a famous group of optimization problems that combine inventory and routing is introduced in more detail.

2.2. Inventory Routing Problem (IRP)

The Inventory Routing Problem (IRP), which is famous in operation research, is an extension of the Vehicle Routing Problem (VRP). The IRP, however, is concerned not only with delivery routes, but also determines delivery quantity for each customer as well as delivery time (Moin & Salhi, 2007). In the simplest form of IRP, there is a warehouse and some retailers. The retailers' demands are known, and a fleet of

specific vehicles is available to transport a commodity from warehouse to retailers in response to their demand and the inventory level at the warehouse. The goal is to determine the delivery route for each vehicle in the fleet and the delivery time and product quantity for each retailer while minimizing total cost and, at the same time, attempt to ensure that no retailer runs out of commodity. The total cost includes components such as travel time, transportation cost, and inventory cost.

Although inventory management and VRP seem to be independent from each other, there are benefits in integrating them, such as flexibility in service, better fleet usage, and reduction in total cost.

Both VRP and the inventory problem have been studied extensively, and many methods have been developed to solve them. In many cases, IRP is separated into inventory and VRP problems. In other words, the inventory problem is solved first in order to determine delivery quantity, and the output is used as data for the VRP. However, they are not in fact integrated and are solved independently. This means that the interaction between inventory and routing is lost in this approach. Most of the studies applying this approach to solve IRP adopt a two-stage solution method that follows one of two approaches: (Moin & Salhi, 2007)

1. They solve the routing problem first and then the inventory problem.
2. They solve the inventory control problem first (sometimes with approximate transportation cost), then aggregate customers with the same delivery time into one cluster and construct delivery routes for each cluster.

In both methods, modifying the data for one problem forces another problem to be resolved, and the algorithm iterates between obtaining a new set of routes and a new set of replenishments until a criterion for stopping has been met (Moin & Salhi, 2007).

There are currently few, if any, commercial-based systems in use that integrate inventory management and routing (Andersson, Hoff, Christiansen, Hasle, & Lokketangen, 2010). This method is used in road-based and maritime transportation more than in air and rail transportation. Even so, most companies in maritime transportation separate inventory management and ship routing and solve them consecutively.

Many different assumptions arise when inventory and routing are integrated. Unlike many other routing problems, for which the premises are well-known, almost every paper in this field presents new assumptions, generating yet another version of the problem (Andersson, Hoff, Christiansen, Hasle, & Lokketangen, 2010).

2.2.1. IRP Classifications

The IRP is used at the tactical and operational level, but can be categorized into three different groups according to the planning horizon. In some IRP studies the planning horizon is very short, which results in making at most one visit per retailer in the timeframe of the problem. These studies are grouped as the “instant” planning horizon. The instant planning horizon does not necessarily include only one step. If a planning horizon consists of more than one step, a retailer is visited in one of the periods and all retailers do not need to be visited in the same time step.

In the second group of IRP problems, a retailer can be visited more than once; however, the planning horizon is finite. In this group, which is called “finite,” a decision for one horizon is taken independently—yet it affects what happens in the next horizon. For instance, the inventory level at the beginning of each horizon depends on all decisions taken in previous horizons. Thus, interactions between the time before a horizon and after a horizon are ignored, although rolling-horizon method can be applied when decision-making for a timeframe longer than the planning horizon is needed.

The last group is the “infinite” planning horizon, which is used at the strategic level of decision making.

Inventory-management policies impact inventory levels at warehouses. In most industries, there is a minimum inventory level for a commodity, which can be either zero or greater than zero. But the inventory level may become negative due to lack of the commodity in plants or deficiency in transportation of the product. This failure to satisfy demand is called “stock out,” and requires emergency delivery to meet retailers’ demands.

The fleets used in SC can be divided into two groups: homogeneous and heterogeneous. In a homogeneous fleet, all vehicles have the same characteristics—such as speed, capacity, fixed cost, and operational cost—whereas in a heterogeneous fleet vehicles are different in some (or all) of these attributes. Having a heterogeneous fleet in IRP makes it more complicated, because using different vehicles leads to different total costs due to variation in vehicles’ characteristics.

In terms of solution technique, generally there are two approaches, theoretical and practical. Most of the papers in in the theoretical approach decompose the problem into two underlying problems, the inventory problem and the traveling-salesman problem (Moin & Salhi, 2007). In contrast, the practical approaches use heuristic or meta-heuristic methods to obtain near-optimal solutions (Moin & Salhi, 2007).

The IRP is a complicated problem. The optimal solution can be found only by considering few time steps. As a result, almost all methods available in the literature are heuristic and do not prove optimality. In next section, the IRP literature is reviewed.

2.2.2. IRP Literature

Bard and Nananukul (2010) model IRP with a single plant and a set of customers whose demands change over time. The problem belongs to a finite planning horizon, and the fleet is homogeneous. Customers have their own warehouses and are able to hold commodities for their future demand, although daily product distribution usually meets their requirements. A new methodology that combines exact and heuristic procedures within a branch and price algorithm was developed in this study. A column generation heuristic was embedded in the solution methodology to improve algorithm efficiency. The numerical results show that a problem with up to 50 customers and 8 time periods can be solved by this method in less than one hour.

Bard and Nananukul (2010) model differs from the proposed study in several significant ways. First of all, their model has only one level, which includes the plant

and its customers, with no distribution centers in between. Moreover, the model includes only a single plant and a homogeneous fleet. It also assumes that fleet size will always be large enough to distribute product among customers. In other words, a shortage in fleet size will never happen. Since trucks are always available, demand is always satisfied (i.e., production quantity is a decision variable and adapts to customer demand). Therefore, customers do not risk unsatisfied demand, which makes the problem easier than the model proposed in this study.

Jang, Jang, Chang, and Park (2002) propose a new method for designing a supply network with a global bill of material (BOM). They believe that to operate the supply network optimally, the following problems should be optimized simultaneously: (a) design for a network that includes all entities, from raw-material supplier to customer; (b) production assignment; (c) production planning; and (d) capacity planning for different producers. Their supply-network management system includes supply-network design optimization, planning for production and distribution operations, model management, and data-management modules. They use a Lagrangian relaxation and genetic algorithm for supply-network design and production/distribution modules.

Although these researchers (Jang, Jang, Chang, & Park, 2002) claim that all decisions should be optimized jointly, their model's framework shows that decisions are made sequentially and interactions between some decision variables are ignored. For instance, in the outbound model, the plant locations are determined according to the locations of warehouses and retailers. Then, in the inbound model, plant locations are used to find suppliers' locations. Thus, the effect of transportation cost between

suppliers and plants is ignored in the outbound model for optimizing plant location. In this approach, transportation cost is roughly calculated for each cluster of customers. This means that the locations of different facilities are determined based on minimizing approximate total cost, which includes transportation cost. Customer demand is not considered in this model, and the demands of a cluster of users are aggregated into required capacity for warehouses. Since routing was not included in the study, the fleet is not part of the formulation; neither is fleet size or vehicle capacity. Generally, this model belongs to the strategic level of IRP, and many operational details are not included.

Campbell and Savelsbergh (2004) developed a two-phase approach to solve IRP. They decomposed IRP into the inventory problem and VRP. In the inventory problem, the delivery schedule is optimized. In the VRP, delivery routes are determined. An integer model has been developed for the inventory problem, however, and some heuristics have been used in the second part. Numerical experiments have shown improvement in finding near-optimal solutions, and prove that the method is efficient for large-scale problems.

There are some major differences between Campbell and Savelsbergh's (2004) model and the model proposed in this study. For instance, Campbell and Savelsbergh's model optimizes IRP at the strategic level. Many details are not included in the formulation, such as vehicle capacity, unsatisfied demand, and shortage in fleet size. Also, there is no routing in their study, and a subset of customers is assigned to one vehicle. Thus, the ordering of customers along a path associated with each cluster is not determined.

Chandra (1993) developed an integrated model to determine replenishment quantity at warehouses and delivery routes to customers' locations. There are some similarities between his model and the model in this study. For example, both are multi-product and have a finite planning horizon of discrete time periods. Also, customer demand is deterministic. However, the model proposed in this study is more complex than Chandra's. For instance, the fleet is heterogeneous in this study, whereas in Chandra's model the fleet is homogeneous. Moreover, vehicles are not tracked in his model, and fleet size is infinite. In other words, assigning of commodities to customers is determined. There is no limitation on product distribution in terms of vehicle availability, which means that unsatisfied demand is not considered. In addition, Chandra's model has only one level and a single warehouse, while the model proposed in this study has two levels and is multi-plant and multi-bottler. Also, since Chandra's model assumes that vehicles are always available, rental vehicles and TPLPs—which are taken into account in the proposed study so as to avoid penalties—are not included in the model.

A heuristic method that integrates warehouse replenishment and distribution of commodities has been developed, and its results have been compared to the results of problems solved separately and sequentially. Improvement in total cost has been shown for small-case problems.

Lee, Bozer, and White III (2003) have studied IRP in a finite planning horizon with multi-supplier and capacitated vehicles. The problem was decomposed into two sub-problems: inventory control and vehicle routing. A linear model was used to find inventory level at warehouses for a set of given routes, then a heuristic method was

applied to improve the routes. The study found that the optimal solution is dominated by transportation cost. In the optimal solution, inventory cost is always less than the transportation cost.

This model (Lee, Bozer, & White III, 2003) is dynamic and multi-product, and all demand is deterministic. All characteristics of fleet vehicles, however, are identical. Also, fleet shortage is not allowed in this model, because fleet size is not limited. It is assumed that at least one vehicle is always available for each supplier. In addition, this problem is *one-to-one*, which means that different suppliers ship parts to a single plant. This model and its heuristic methods were applied to a small-case study with six suppliers and seven time steps. Numerical results showed that the gap between the optimal solution given by CPLEX and the proposed method was around 1.15 percent.

Kim and Kim (2000) considered a multi-period inventory/distribution planning problem with deterministic demand, one warehouse, multiple retailers, and a heterogeneous fleet. The objective function was to minimize total cost, which includes transportation cost and retailers' inventory-holding cost. A Lagrangian heuristic algorithm was developed to find a good feasible solution in a reasonable amount of computation time, and CPLEX was embedded in this methodology to solve some linear models.

This model (Kim & Kim, 2000) has only one warehouse and one product, and vehicles do not deliver to more than one retailer. Each vehicle makes a round trip between the warehouse and one retailer. This assumption makes the model exceptionally easy. Although the fleet is heterogeneous, there are many vehicles

available to distribute a product between retailers, so fleet shortage and unsatisfied demands do not occur in this model. Therefore, rental vehicles and TPLP are not included in the model.

Kim and Kim's (2000) results show that an approach developed for medium- and large-scale problems can obtain a good—but not optimal—solution with a gap of less than 2 percent in a reasonable amount of computational time. The authors claim that CPLEX 4.0 is not capable of achieving an optimal solution in less than two hours for any of the cases they have considered. In addition, they tested their model on a small problem (two vehicles, six retailers, and five time periods), and the gap was less than 1 percent. For these small problems, CPLEX 4.0 is able to find optimal solution in about two hours CPU time.

Yu, Chen, and Chu (2008) studied IRP with split delivery and fleet size constraints. They used a Lagrangian relaxation method to solve the problem, and the relaxed formulation was decomposed into inventory control and routing problems. These problems were solved by linear programming and a minimum cost flow algorithm, respectively. Despite the fact that some similarities exist between their study and the proposed study, such as customers' ability to store products or deterministic demand, there are some major differences between the two models. First, Yu et al.'s fleet is homogeneous. Also, vehicles are not tracked, and only the number of vehicles leaving the depot is counted to be less than fleet size. Second, there is a central depot, all facilities are on one level, and even though the fleet is limited, in their model is not clear as to what happens if the fleet size is too small to satisfy demands. In other words, although they have considered the fleet size, it is

also stipulated to be large enough to satisfy all demands. Numerical experiments show that for a scenario with 100 locations and five time steps, this hybrid approach can get a feasible solution with a gap between 5 and 8 percent in a reasonable amount of computation time. However, no optimal solution is addressed in this research, even for small cases, by using optimization packages.

Savelsberg and Song (2008) have studied the IRP with continuous moves. Their model covers a large geographic area in which delivery routes take several days. An integer programming algorithm was developed to solve small to medium-sized instances approximately. Then a local search procedure improved solutions by a randomized greedy heuristic. This model is single-product but multi-plant, and inventory is allowed at customer locations. The advantages of this model over Savelsberg and Song's include consideration of unsatisfied demand, rental vehicles, and use of TPLPs. The medium-sized problem, for which a feasible solution was found, has 10 time steps, 2 plants, 100 customers, and 3 vehicles. No optimal solution was presented in this study. All the results have gaps between 1 and 9 percent for medium-sized problems.

Gaur and Fisher (2004) worked on a periodic IRP for Albert Heijn B.V., which is a leading supermarket chain in the Netherlands. It included a periodic IRP, which means that customers' demands are repeated every τ periods. A heterogeneous fleet and only one distribution center were considered in the formulation.

In their model (Gaur & Fisher, 2004), fleet size is infinite and all demands are satisfied. On the other hand, inventory level is ignored in this model, and, as a result, it does not have the capability of considering product storage by customers for future

demand. In addition, the formulation defines two types of shipment, direct and shared; the latter refers to a cluster of customers, and its transportation cost is calculated by solving the traveling salesman problem for the cluster. However, the authors state that having at most two costumers in each cluster makes the model equivalent to a generalized minimum-weight-matching problem, which its algorithm is well known in the VRP literature. Thus, most clusters have at most two stores. Computational results show the heuristic algorithm developed in the study has improved Albert Heijn's quality of service. However, the gap between the model's best solution and the optimal solution was not addressed in the paper.

Dhaenens-Flipo and Finke (2001) modeled the production-distribution problem as a network flow problem. Their problem is multi-facility, multi-product, and multi-period. The model has a single plant, several warehouses, and several distribution centers. The focus of the model is to determine the best arrangement of production lines, which optimizes shipment from plant to warehouse and from warehouse to retailers. However, routing is not the concern here, and round trips between each pair of locations, instead of tours, are stipulated. Also, vehicles are not included in the model; the authors assume that vehicles will always be available to transport products from one location to another one. Therefore, options for use of rental vehicles and TPLPs are not considered. The model was tested in several cases, using CPLEX and since routing was not included in the research design, an optimal solution was found for some cases in a reasonable amount of computation time.

Rusdiansyah and Tsao (2005) considered IRP as it is experienced in vending-machine SCs working under a vendor-managed inventory (VMI) scheme. Their

model includes periodic IRP with time windows, and they have attempted to simultaneously optimize replenishment frequency for each retailer and build vehicle tours.

This model (Rusdiansyah & Tsao, 2005) considers only one product, vehicles are identical, and fleet size is not a constraint. Thus, delivery shortage is not possible. The problem includes a single plant with several retailers, which all are in one level. Since the model does not consider split delivery and there is no shortage in delivery, each retailer's demand is less than a vehicle's capacity. This assumption makes the model easy to solve. In addition, delivery quantity is not a decision variable in this model. When inventory level drops to zero, it is replenished to equal each retailer's demand. Therefore, inventory level also is not a decision variable here, and it is calculated according to total demand and frequency of replenishment. Steady consumption rate is another assumption in this model. No optimal solution is found, and a heuristic method is used in all solutions for several different cases. A comparison of solutions using the heuristic method and the best-known solutions shows that on average, there is a gap of 4.2 percent between them.

Bertazzi, Paletta, and Speranza (2002) studied a model in which a set of products is shipped from a single plant to several retailers by a vehicle with specific capacity and cost. In any replenishment, the retailer's inventory level should reach its maximum level. Thus, delivery quantity is not a decision variable. Also, like many other studies reviewed here, the number of vehicles is not a decision variable. The authors assume that there are numerous vehicles that can transport products to their destinations, and vehicle capacity is large enough to distribute products to all

retailers. Delivery shortage, therefore, is not considered. The authors developed a heuristic method to find a near-optimal solution. There is no clear mathematical formulation in the paper. Their main goal was to investigate the impact of different policies on total cost—for instance, how changing the holding costs of supplier and retailers affects operations, number of visits, and total cost—which is why an optimal solution was not found.

Lei, Liu, Ruszczyński, and Park (2006) have addressed the integrated production, inventory, and distribution routing problem (PIDRP). Their model includes several plants and distribution centers, the fleet is heterogeneous, and distribution centers can store products for future demand. A heuristic method is developed to solve the problem. In the first phase, routing is limited to direct shipment and tours are not generated. The second phase attempts to shift direct shipments to tours heuristically. The authors claim that by using this approach, the inventory problem and VRP are not optimized separately and production, inventory, and transportation are coordinated simultaneously.

This model (Lei, Liu, Ruszczyński, & Park, 2006) has only one level, and products are shipped from the top tier, which includes plants, to the lower tier, which includes distribution centers. Unsatisfied demand is not possible in this model—in other words; fleet size is large enough to meet all demands. Thus, rental vehicles and TPLPs are not considered. The heuristic method is applied to different cases, of which most are medium-sized problems, and solutions are compared with CPLEX solutions obtained in two hours of CPU time. The authors note that CPLEX did not find an optimal solution in two hours for most of the scenario cases.

2.2.3. Conclusions

Many studies have looked at IRP from different standpoints and various assumptions have been considered, which makes each paper a new version of IRP (Andersson, Hoff, Christiansen, Hasle, & Lokketangen, 2010). Most studies consider plant(s) in the upper tier and distribution centers/customers in the lower tier. In the proposed study, however, there are three tiers (plants, bottlers, and retailers) and there are multiple facilities in each tier, which makes the problem more complicated.

Another critical difference between the proposed study and previous research is fleet size. Many previous studies have assumed that fleet size will be large enough to handle all shipments. Delivery shortage, therefore, does not happen and at each facility location at least one vehicle is available to distribute products to customers or warehouses. In the proposed study, in contrast, fleet size and capacity are included. If the fleet is not able to satisfy all demands, there are two options available for SC management to meet demand and avoid stock out.

In all of the IRP publications reviewed, the objective has been minimizing cost; however, in the proposed study, the objective is to maximize profit. Therefore, some retailers may experience stock out with some products if it is more beneficial to service other retailers, which, in turn, depends on fleet size, rental-vehicle and TPLP costs, and different retailers' product price. Obviously, if the goal of one company is retaining customers by always having products available to retailers, it will have to rent vehicles, contract with TPLPs, or apply inventory policies when its fleet size is small. Defining different management policies is possible by choosing appropriate factors in an objective function which will be explained more in Chapter 3.

The IRP belongs to the NP-Hard class of linear models, for which running time goes up exponentially as the problem size grows. As the literature has shown, the medium-sized IRP has not yet been optimally solved, and various heuristic methods have been developed to find a feasible solution. This study proposes to develop an appropriate heuristic method for solving this problem. Finding a good feasible solution in a reasonable amount of computation time is another goal.

Since the TPLP is one of the main aspects of the proposed study, it will be explained in the next section. Publications in this area will be reviewed as well.

2.3. Third Party Logistics Provider

2.3.1. Introduction

Third-party logistics (TPL) also known as logistics outsourcing (e.g., Kenemeyer, Corsi, & Murphy, 2003) has recently received a lot of attention from industry (Marasco, 2008). Despite the fact that many companies in different industries use TPLPs for the management of all or parts of their logistics operations (Lieb & Bentz, 2004), one challenge has been the lack of a consistent definition of the TPL concept (Marasco, 2008). In some cases, TPL is the same as outsourcing of transportation and/or warehousing, while in other cases it includes more complicated outsourcing and encompasses the entire logistics process (Laarhoven, Berglund, & Peters, 2000). According to Lieb (1992), TPL involves “the use of external companies to perform logistics functions that have traditionally been performed within an organization. The functions performed by the third party can encompass the entire logistics process or selected activities within that process.”

Maloni and Carter (2006) cite three primary reasons for outsourcing logistics:

1. Reduction in cost due to expertise and economies of scale (Zineldin & Bredenlow, 2003; Wilding & Juriado, 2004).
2. Service-quality improvement (Greaver II, 1999; Lynch, 2004).
3. Buyers are able to concentrate on TPL providers' qualifications (Razzaque & Sheng, 1998; Boyson, Corst, Dresner, & Rabinovich, 1999).

The above, combined with other benefits, have made the outsourcing of logistics more attractive in the last two decades. For instance, the annual growth rate of TPL from 1995 to 2005 was between 5 and 10 percent (Ashenbaum, Martz, & Rabinovich, 2005).

2.3.2. TPLP Literature

Early studies of TPL focused on logistics-services users, while recent research has concentrated on third-party logistics providers. However, most of the publications in this area have investigated the strategic behavior of TPLPs (Marasco, 2008). For example, Yeung, Selen, Sum, and Huo (2006) investigated the effect of different strategic choices such as pure cost, pure differentiation, or a combination of the two on the financial performance of TPLPs in Hong Kong, and Carbone and Stone (2005) studied 20 leading European TPLPs' strategic behavior.

Some recent studies have focused on the use of information technology (IT) in this area. Lai, Ngai, and Cheng (2005) investigated IT usage in the TPLP industry in Hong Kong, and Evangelista and Sweeney (2006) researched its use in Italy. Both studies concluded that TPLPs have recognized the contributions of IT to their

improved performance, but small companies face many problems in using IT, due to lack of staff expertise and insufficient financial support.

Table 2-1, which shows the distribution of articles in different journals during the period 1989-2006 (Marasco, 2008), demonstrates that serious research on TPL started in the mid- to late 1990s. Lewis and Talalayevsky (2000) believe that implementation of IT influences TPL developments by allowing buyers and sellers of logistics services to communicate directly, thereby reducing coordination costs. As a result, IT supports outsourcing of logistics activities to TPLPs (Marasco, 2008). Most of the papers in this area have been published in the *International Journal of Physical Distribution & Logistics Management*, and the *Journal of Business Logistics*. Some of the most famous journals in transportation and operations research, such as *Transportation Science*, *European Journal of Operational Research*, and *Transportation Research*, encompass a small portion of all papers published during the past two decades.

Less than 7 percent of the papers shown in the Table 2-1 are concerned with the TPLP process, which includes partner selection, negotiation, contract design, operations planning, coordination, and monitoring. In fact, most of the papers have concentrated on behavioral characteristics of TPL relationships, such as trust, commitment, dependence, conflict, and equity (Marasco, 2008). For example, Moore and Cunningham III (1999) have found that trust and commitment are the main behavioral attributes that distinguish logistics alliances, and Lim (2000) developed the game theoretic model of how a contract should be designed to encourage TPLPs to reveal their true characteristics.

Table 2-1. Distribution of articles by journal in the period 1989-2006 (Marasco, 2008)

Journal	1989-1994	1995-2000	2001-2006	Total
<i>International Journal of Physical Distribution & Logistics Management</i>	4	13	23	40
<i>Journal of Business Logistics</i>	4	12	3	19
<i>The International Journal of Logistics Management</i>	1	7	4	12
<i>Transportation Journal</i>	-	3	9	12
<i>International Journal of Logistics: Research and Applications</i>	-	1	9	10
<i>Journal of Enterprise Information Management</i>	-	3	3	6
<i>Supply Chain Management: An International Journal</i>	-	-	6	6
<i>International Journal of Logistics Systems and Management</i>	-	-	5	5
<i>International Journal of Operations & Production Management</i>	1	-	3	4
<i>Transportation Research</i>	-	1	3	4
<i>Transport Logistics</i>	-	4	0	4
<i>Asia Pacific Journal of Marketing and Logistics</i>	-	1	2	3
<i>Journal of Supply Chain Management</i>	-	2	1	3
<i>European Journal of Operational research</i>	-	1	1	2
<i>Industrial Marketing Management</i>	-	1	1	2
<i>International Journal of Production Economics</i>	-	-	2	2
<i>Omega</i>	-	-	2	2
<i>European Journal of Purchasing & Supply Management</i>	-	-	1	1
<i>Harvard Business Review</i>	1	-	-	1
<i>Journal of Business & Industrial Marketing</i>	-	-	1	1
<i>Production and Operations Management</i>	-	1	-	1
<i>Transport Reviews</i>	-	1	-	1
<i>Transportation Quarterly</i>	-	1	-	1
<i>Transportation Science</i>	-	-	1	1
Other	-	3	6	9
Total	11	55	86	152

Meanwhile, Chen, Hum, and Sun (2001) considered three different third-party warehousing contracts with space commitments and adjustment options. Most of the papers in this area, therefore, are not in the scope of this study. They have focused on how to select TPLP's, contract with them, continue the contract, and use IT in operation. For instance, more than 90 percent of all publications have investigated various characteristics of TPL parties and their impact on the TPL arrangement; this is another reason most of the publications are not in operation research journals.

Zapfel and Wasner (2002) have mentioned that increased competition between transportation companies has encouraged small- and medium-sized TPLPs to cooperate with each other and form a hub-and-spoke system. In this system, each TPLP covers a region and several of them build a common network, providing service under the same name. Applying this system to parcel distribution was also investigated. In this model, therefore, all TPLPs are considered to be a single company and a mathematical formulation is provided to minimize cost. Constraints ensure that flow conservation is valid; however constraints on generating tours are ignored. Although the title of the paper refers to TPLP, its contributions are the same as other papers reviewed in the previous section. The problem has been defined for several TPLPs working together, which means that a mathematical formulation can be used for other companies, regardless of whether they are TPLPs or not.

Tyan, Wang, and Du (2003) evaluated freight consolidation policies in global third-party logistics. Freight consolidation is an approach that maximizes the utilization of expensive modes such as air transport by grouping smaller shipments into one large shipment to reduce the shipping cost and increase utilization of vehicle

capacity (Tyan, Wang, & Du, 2003). For example, truck consolidation happens when the shipment quantities are less-than-truckload (LTL). Although three different forms of consolidation exist—inventory consolidation, vehicle consolidation, and terminal consolidation (Hall, 1987)—Tyan, Wang, and Du, (2003) focused on only vehicle consolidation. The formulation proposed in their research minimizes the total cost of shipping some products from one location to another one. Inventory capacity and touring are not considered and the model attempts to achieve better utilization of different modes while total cost is minimized. Three freight-consolidation policies are investigated and their performances are compared.

Ko and Evans (2007) presented a mixed integer nonlinear model for the design of a dynamic integrated distribution network for TPLPs that considered forward and reverse flows simultaneously. This study dealt with warehousing and transportation operations. The model takes facility locations into account; however, tours are not generated. Also, vehicles are not included in the model, and all commodities are shipped via direct routes between plants and warehouses or warehouses and customers. The formulation in this study, like that of Zapfel and Wasner (2002), is TPLP manager problem. All facilities in this study (Ko & Evans, 2007) belong to different TPLPs; however, it can be assumed that they are owned by a single company and this company attempts to optimize the total cost of its facilities operations.

2.3.3. Conclusions

There has been little research on TPLPs, which is a new topic to which researchers have turned only recently. Most of the papers found in the literature have investigated the benefits of outsourcing and other issues such as different forms of contracting with TPLPs and consolidation of separate TPLPs, and few have included mathematical formulation.

In some of the papers that have presented a mathematical formulation, the TPLP manager's problem is defined and formulated. However in this research TPLP is presented as an option for a company to meet customer demand. The benefit of TPLP is not optimized; however, it is data to be entered into the model that helps a company be able to satisfy its demand. The approach of the proposed study, therefore, is completely different from other studies reviewed in the literature. Specifically, no study that integrates IRP and TPL exists in the current literature, to the best of the author's knowledge.

Chapter 3: Problem Description and Formulation

In this chapter, the problem is described completely and its properties are explained. All assumptions considered in this research are reviewed and all parameters and decision variables used in the model are explained. The mathematical formulation, objective function, and constraints are explained in the last section of the chapter.

3.1. Problem Description

Assume that there is a bottling company working jointly with a beverage company. The beverage company produces syrup concentrate in its plants. Syrup concentrate is the raw material and is not ready for drinking. Other processes must be applied to produce final products. These processes are done in facilities not belonging to the beverage company. The bottling company is responsible for these operations. Bottlers are locations at which the final processes are applied to syrup concentrate. The number and geographical distribution of bottlers vary among different bottling companies. Each bottler has its own production capacity and cost for different products.

Bottling companies sell their products through retailers. These can be big clients such as Costco, Target, Wal-Mart, and Giant, or smaller ones such as 7-Eleven, and/or restaurants in the region. Each of these locations has its own daily demand for different commodities. They order different products according to their demand and the bottling company must satisfy these orders at the right time.

Obviously, unsatisfied demand is a negative point for the bottling company and influences future orders. Therefore, the problem has two levels:

- 1- The upper level, which includes the beverage company plants and the bottling company's bottlers.
- 2- The lower level, which includes the bottling company's bottlers and its retailers.

The bottling company has its own fleet to transport syrup concentrate from plants to bottlers and final products from bottlers to retailers. The vehicles in the fleet are categorized into two groups according to their functionality. The first group contains vehicles equipped with the proper equipment to transport syrup concentrate from plants to bottlers. The second group includes vehicles that transport final products from bottlers to retailers. These vehicles do not have equipment for syrup transportation and are more useful for delivery of final products. Therefore, vehicles in one group cannot be used in the other group.

Vehicles within each group are heterogeneous. Vehicle capacity is the attribute that varies among all vehicles in each group. Other attributes of the vehicles, such as speed or transportation cost, can also be dissimilar.

Final products have different prices at different retailers. Therefore, meeting the demand of one retailer may benefit the bottling company more than meeting the demands of other retailers, depending on the price at each retailer and transportation cost from the bottlers to the retailers. Due to shortage in fleet size, vehicles have to make tours to visit retailers sequentially. Each vehicle can serve a limited number of retailers due to its capacity. Hence, the whole operation of the bottling company can

be described as follows. Syrup concentrate is produced at beverage company plants. The first group of vehicles transports syrup concentrate from plants to bottlers through several tours. Each tour connects one plant to bottlers in the upper level. The syrup concentrate is processed and final products are produced, after which the second group of vehicles distributes the final products among retailers through tours that start at bottlers and deliver to retailers.

Retailers have their own storage facilities, with particular capacity for each product, and can store commodities for future demand. The bottling company thus has the opportunity to store products in a retailer's warehouse by paying that retailer. If the bottling company does this, it may not need to visit that specific retailer again in the following days. Many factors, such as location of retailer, quantity of retailer's order in the following days, transportation cost, cost of storage at retailer, fleet size, and vehicle capacity affect this decision. For example, if a retailer is far from other retailers, it may be more beneficial to send it several orders in one delivery and pay a holding cost than to send one vehicle every day to meet daily orders. Holding costs vary among retailers.

There is a penalty for unsatisfied orders, which also varies among retailers. The main reason for not serving one retailer is that the number of vehicles in the second group is not sufficient for delivery to all retailers. Unsatisfied orders have several harmful effects on the bottling company, especially if they happen often. The biggest disadvantage is that customers may switch to another brand and threaten the bottling company's share in the market. Unsatisfied demand of a time step is transferred to the next time step. In other words, if the demand of a particular retailer

is not satisfied in one time step, the unsatisfied demand is added to the demand of that retailer in the next time step.

As mentioned, sometimes the fleet is not large enough to meet all retailers' orders. One method for preventing missed orders is sending orders to some retailers in advance to hold for future use. This allows the fleet to be free to service the rest of the retailers. This policy is possible if those retailers have available capacity to keep products, but may increase the bottling company's cost due to storage cost at retailers.

Another way to deal with this problem is by renting vehicles, which increases the fleet size. Rental vehicles have their own characteristics, such as speed and capacity which can differ from the bottling company's owned vehicles. Rental vehicles' cost, which includes fixed costs and operational costs, is definitely higher than owned vehicles' cost. A rental vehicle also needs routing and a path must be provided for it. Rental vehicles can be used at both levels. Unsatisfied orders can occur when bottlers do not have enough final products to distribute among all retailers or because the fleet size in the lower level is small. In this case, the bottling company can increase the fleet size in lower level by renting vehicles.

The last option for delivering to a retailer when the owned fleet is not able to do so is outsourcing shipping and/or warehousing. The bottling company contracts with TPLPs, who are then responsible for delivery of final products to some of the bottler's retailers. Also, TPLPs can store commodities in their warehouses and deliver products to retailers at the right time if there is no available capacity in the bottler's warehouses. Outsourcing of transportation can be contracted for more than one retailer and for several days. In fact, a TPLP can be in charge of shipping to some

retailers on one day and to other retailers on another. Therefore, a contract includes providing services to particular retailers for a specific day with determined cost. A difference between rental and TPLP options is that the bottling company is responsible for providing paths for rental vehicles, whereas a TPLP is responsible for its fleet. The bottling company determines product pick-up time from a bottler and delivery time to a retailer for the TPLP. Rental vehicles are used on both levels, while TPLP vehicles are used only in the lower level, for delivery to retailers.

Therefore, the bottling company maximizes its profit by considering all the above mentioned conditions. Different policies can be defined by choosing the right amount for different parameters in the objective function, which will be described in the following sections. The method selected determines how to distribute final products among retailers and how to deal with any shortage in the fleet or warehouses' capacity. Sending orders in advance, renting additional vehicles, contracting with TPLPs, and leaving some orders unsatisfied are options that must be considered in the decision-making process.

3.2. Modeling Approach

A mathematical formulation is suggested to model the problem described in section

3.1. The modeling approach has the following main characteristics:

- 1- The model belongs to IRP class which integrates inventory and routing to find the optimal solution.
- 2- The mathematical formulation models the problem at the operational level.
- 3- The model is multi-plant, multi-bottler, and multi-commodity.

- 4- The problem is modeled over a pre-determined planning horizon which is separated into several time steps.
- 5- The objective function is maximizing the profit of the bottling company considering all different costs and penalties.
- 6- The model has two levels and the flow of syrup concentrate and commodities goes down from the upper level to the lower level.
- 7- The production of each commodity type at each bottler is limited to its production capacity.
- 8- There is a heterogeneous fleet transporting syrup concentrate from plants to bottlers and products from bottlers to retailers.
- 9- Bottlers and retailers are able to store products for future orders or demands, respectively.
- 10- The model takes rental vehicles and contract with TPLPs into account as well as not satisfying orders when the bottling company faces shortage in fleet or warehouses' capacity.

3.3. Assumptions

The mathematical model proposed in this research has the following assumptions:

- 1- Since the problem is defined at the operational level, it is deterministic. All information about the bottlers, retailers, vehicles and the distribution network is known in advance.
- 2- The locations of all facilities are known and no decision is taken about them.

- 3- The time step is assumed to be one day.
- 4- Syrup concentrate is always available in the plants and they can store syrup concentrate as much as the model requires.
- 5- Vehicles that transport syrup concentrate from plants to bottlers cannot work in the lower level.
- 6- Vehicles that distribute final products among retailers cannot work in the upper level.
- 7- At both levels, every vehicle can do only one tour in each time step.
- 8- At the beginning of the first time step all vehicles working at the upper level are available in the plants and all vehicles transporting products at the lower level are available at the bottlers. The model figures out where each vehicle must be.
- 9- At both levels, each vehicle should return to its origin plant or bottlers at the end of each time step.
- 10- Inventory level at each retailer is calculated at the end of each time step.
- 11- Bottlers and retailers cannot hold syrup concentrate and final products more than their capacity allows.
- 12- If a retailer's order is not met in one time step, it is transferred to the next time step. In other words, it is assumed that consumers of that order do not satisfy their demands from other sources. A penalty is associated with not satisfying orders.
- 13- There are two types of rental vehicles, with each type assigned to one level.

- 14- The bottling company is allowed to outsource transportation of products in the lower level, but it cannot contract with a TPLP for transportation of syrup concentrate from plants to bottlers.
- 15- Every shipping TPLP contract includes providing service to one or more retailers in one time step.
- 16- Every warehousing TPLP contract determines the quantity of products that should be stored in TPLP warehouses as well as duration of storage.
- 17- The bottling company can consider more than one contract in daily operation.
- 18- Each retailer can receive its order with at most one TPLP in each time step.
- 19- If a retailer is assigned to a TPLP, all of its orders including all commodity types will be transported by TPLP.

3.4. Mathematical Formulation

In this section, all sets and parameters of the formulation are introduced. Decision variables of the mathematical model are then defined and finally, the objective function as well as the constraints are presented and explained in detail.

3.4.1. Sets

P: Set of all plants

W: Set of all bottlers

R : Set of all retailers

LN : Set of all network links

GVU : Set of all vehicles working in the upper level

GVW : Set of all vehicles working in the lower level

VU : Set of all rental vehicles ready to work in the upper level

VW : Set of all rental vehicles ready to work in the lower level

Su : Set of speed factors of vehicles working in the upper level

Sw : Set of speed factors of vehicles working in the lower level

S : Set of products

S_j : Set of different types of syrup concentrate

T : Set of time steps

TP : Set of all candidate third party contracts

L : Set of number of cost function steps of candidate third party contracts

USL : The maximum number of links outgoing from a node in the upper level

UEL : The maximum number of links incoming to a node in the upper level

LSL : The maximum number of links outgoing from a node in the lower level

LEL : The maximum number of links incoming to a node in the lower level

3.4.2. Parameters

Supply and Demand:

d_{ist} : Quantity of order of retailer i from commodity type s at time step t ,

$i \in R, s \in S, t \in T$

Pc_{sit} : Price of one unit of commodity type s at retailer i at time step t ,

$$s \in S, i \in R, t \in T$$

h_{sit} : Holding cost of one unit of commodity type s at retailer i at time step t ,

$$s \in S, i \in R, t \in T$$

Sh_{sit} : Shortage penalty of one unit of commodity type s at retailer i at time step t ,

$$s \in S, i \in R, t \in T$$

Tc_{st} : Transportation cost of one unit of commodity type s per mile at time step t ,

$$s \in S, t \in T$$

HC_{sit} : Storage capacity of commodity type s at facility location i at time step t ,

$$s \in S, i \in R \cup W, t \in T$$

Network:

c_{ij} : Travel time between locations i and j , $i, j \in R \cup W \cup P$

len_{ij} : Length of link between locations i and j , $i, j \in R \cup W \cup P$

$NS(l)$: Starting node of link l , $l \in LN$

$NE(l)$: Ending node of link l , $l \in LN$

$Lnu(i, k)$: The k^{th} outgoing link from node i in the upper level,

$$i \in P \cup W, k \in 1 \text{ to } USL$$

$Leu(i, k)$: The k^{th} incoming link to node i in the upper level, $i \in P \cup W, k \in 1 \text{ to } UEL$

$Lnw(i, k)$: The k^{th} outgoing link from node i in the lower level,

$$i \in R \cup W, k \in 1 \text{ to } LSL$$

$Lew(i,k)$: The k^{th} incoming link to node i in the lower level, $i \in R \cup W, k \in 1 \text{ to } LEL$

TLU : Loading time of trucks in the upper level

TLW : Loading time of trucks in the lower level

$TULU$: Unloading time of trucks in the upper level

$TULW$: Unloading time of trucks in the lower level

TT_u : The maximum business hours of trucks in the upper level

TT_w : The maximum business hours of trucks in the lower level

Owned and Rental Vehicles:

Cku_k : Loading capacity of vehicle k working in the upper level, $k \in GVU$

Ckw_k : Loading capacity of vehicle k working in the lower level, $k \in GVW$

$FCV_{u_{kt}}$: Fixed cost of rental vehicle k at time step t , which is working in the upper level, $k \in VU, t \in T$

$FCV_{w_{kt}}$: Fixed cost of rental vehicle k at time step t , which is working in the lower level, $k \in VW, t \in T$

$EPCV_{u_{kt}}$: Operational cost of rental vehicle k at time step t , which is working in the upper level, $k \in VU, t \in T$

$EPCV_{w_{kt}}$: Operational cost of rental vehicle k at time step t , which is working in the lower level, $k \in VW, t \in T$

SFu_k : Speed factor of vehicle k working in the upper level, $k \in GVU$

SFw_k : Speed factor of vehicle k working in the lower level, $k \in GVW$

Products:

a_s : Product converting factor. This factor converts different products to the unique volume unit, $s \in S$

λ_s : This factor converts syrup concentrate volume unit to volume unit of product type s , $s \in S$

SHC_{sit} : Storage capacity of syrup concentrate type s in bottler i at time step t , $s \in S, i \in W, t \in T$

CAP_{sit} : Production capacity of commodity type s in bottler i at time step t , $s \in S, i \in W, t \in T$

$PdCost_{is}$: Production cost of one unit of commodity type s in bottler i , $s \in S, i \in W$

Third Party Logistics:

CT_{sqi} : Cost of transportation of commodity type s by q level of contract i , $s \in S, q \in TP, i \in L$

NB_{ij} : Binary parameter indicating whether retailer j exists in contract i , $i \in R, j \in TP$

NU_i : Number of retailers in contract i , $i \in TP$

TLB_{sqi} : The maximum quantity of commodity type s can be shipped by TPLP with a cost equal to step q of cost function of contract i , $s \in S, q \in TP, i \in L$

WR_s : Holding cost of one unit of product type s per time step in TPLP warehouse

TLB

3.4.3. Decision Variables

Commodity Flow

Qu_{lskt}^{ij} : Quantity of commodity type on link shipped by vehicle from plant to bottler at time step , $i \in P, j \in W, l \cup LN, s \in S, k \in GVU, t \in T$

Qw_{lskt}^{ij} : Quantity of commodity type on link shipped by vehicle from bottler to retailer at time step , $i \in W, j \in R, l \cup LN, s \in S, k \in GVW, t \in T$

I_{sit} : Inventory level of commodity type at facility location at time step ,
 $s \in S, i \in R \cup W, t \in T$

e_{ist} : Shortage amount of commodity type at retailer at time step ,
 $i \in R, s \in S, t \in T$

DE_{ist} : Quantity of commodity type sold to customers in retailer at time step ,
 $i \in R, s \in S, t \in T$

DM_{ist} : The modified demand of commodity type in retailer at time step ,
 $i \in R, s \in S, t \in T$

Pd_{ist} : Quantity of commodity type produced in bottler at time step ,
 $i \in W, s \in S, t \in T$

Sc_{ist} : Quantity of syrup concentrate type stored in bottler at time step ,
 $i \in W, s \in S_1, t \in T$

Vehicle Flow

XU_{ijkt} : Binary variable equal to one if vehicle k visits location i directly after location j in the upper level and it leaves location i at time step t , otherwise it is zero,
 $i \in P \cup W, j \in P \cup W, k \in GVU, t \in T$

XW_{ijkt} : Binary variable equal to one if vehicle k visits location i directly after location j in the lower level and it leaves location i at time step t , otherwise it is zero,
 $i \in R \cup W, j \in R \cup W, k \in GVW, t \in T$

VZU_{kt} : Binary variable equal to one if a rental vehicle k is used in the upper level at time step t , $k \in VU, t \in T$

VZW_{kt} : Binary variable equal to one if a rental vehicle k is used in the lower level at time step t , $k \in VW, t \in T$

Uu_{kt} : Binary variable equal to one if the bottling company rents vehicle k at time step t for the upper level, $k \in VU, t \in T$

Uw_{kt} : Binary variable equal to one if the bottling company rents vehicle k at time step t for the lower level, $k \in VW, t \in T$

Third Party Logistics

TW_{smtq}^{ij} : Quantity of commodity type s shipped by third party's contract q picked up from bottler i at time step m and delivered to retailer j at time step t ,
 $i \in W, j \in R, s \in S, m, t \in T, q \in TP$

w_{qist} : Binary variable equal to one if level of a contract is selected for

transportation of commodity type in time step , $i \in TP, t \in T$

TN_{iqt} : Binary variable equal to one if an order of retailer is served by third party's

contract at time step , $i \in R, q \in TP, t \in T$

Z_{sqt}^{ij} : The cumulative quantity of commodity type over time steps which is shipped

from a bottler to a retailer at time step by third party's contract and needs to be stored in TPLP's storage, $i \in W, j \in R, s \in S, q \in TP, t \in T$

3.4.4. Objective Function

$$\begin{aligned}
Max \quad & \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} Pc_{sit} DE_{ist} - \sum_{i \in W \cup P} \sum_{j \in W \cup P} \sum_{k \in GVU} \sum_{t \in T} \alpha SFu_k C_{ij} XU_{ijkt} - \\
& \sum_{i \in W \cup R} \sum_{j \in W \cup R} \sum_{k \in GVW} \sum_{t \in T} \alpha SFu_k C_{ij} XW_{ijkt} - \sum_{i \in P} \sum_{j \in W} \sum_{l \in LN} \sum_{s \in S} \sum_{k \in GVU} \sum_{t \in T} \beta Qu_{lskt}^{ij} len_{ij} - \\
& \sum_{i \in W} \sum_{j \in R} \sum_{l \in LN} \sum_{s \in S} \sum_{k \in GVW} \sum_{t \in T} \beta Qw_{lskt}^{ij} len_{ij} - \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} e_{ist} Sh_{sit} - \\
& \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} I_{sit} h_{sit} - \sum_{k \in VU} \sum_{t \in T} FCV_{u_{kt}} U_{u_{kt}} - \sum_{k \in VW} \sum_{t \in T} FCV_{w_{kt}} U_{w_{kt}} - \sum_{k \in VU} \sum_{t \in T} EPCV_{u_{kt}} VZ_{u_{kt}} - \\
& \sum_{k \in VW} \sum_{t \in T} EPCV_{w_{kt}} VZ_{w_{kt}} - \sum_{q \in TP} \sum_{t \in L} \sum_{s \in S} \sum_{t \in T} CT_{sqi} w_{qist} - \sum_{i \in W} \sum_{j \in R} \sum_{q \in TP} \sum_{s \in S} \sum_{t \in T} Z_{sqt}^{ij} WR_s - \\
& \sum_{i \in W} \sum_{s \in S} \sum_{t \in T} Pd_{ist} PdCost_{is}
\end{aligned} \tag{3-1}$$

Equation (3-1) represents the objective function of the model which is the profit of the company. The first term is the revenue of the company obtained from selling final products. The second and the third terms calculate the fixed cost of transportation which is independent of volume of shipping. Travel time has been considered as a representative of this cost. is the value of time factor. The fourth and fifth terms represent shipping costs and is the shipping cost of one unit of final product per mile. It is assumed that transportation cost is the same for different final

products due to similarity in size and weight of different products. The sixth term is the shortage penalty and the seventh term calculates the inventory cost at retailers. The next two terms illustrate fixed rental cost of rental vehicles for the upper and lower levels, respectively. Likewise, the 10th and 11th terms present the operational cost of rental vehicles in the upper and lower levels. The 12th term is TPLP shipping cost and the 13th is TPLP warehousing cost. The last one calculates the production cost in different bottlers.

3.4.5. Constraints

Tour Generation Constraints

$$\sum_{j \in PUW} XU_{ijkt} \leq 1 \quad \forall i \in P \cup W, \forall k \in GVU, \forall t \in T \quad (3-2)$$

$$\sum_{j \in R} XW_{ijkt} \leq 1 \quad \forall i \in W, \forall k \in GVW, \forall t \in T \quad (3-3)$$

$$\sum_{j \in RUW} XW_{ijkt} \leq 1 - \sum_{q \in TP} TN_{iqt} \quad \forall i \in R, \forall k \in GVW, \forall t \in T \quad (3-4)$$

Constraints (3-2), (3-3), and (3-4) ensure that at each time step, each owned or rental vehicle can go to only one facility location after leaving plants, bottlers, or retailers. Constraint (3-2) is applied to plants and bottlers in the upper level, constraint (3-3) is applied to bottlers in the lower level, and constraint (3-4) is applied to retailers in the lower level. Moreover, based on constraint (3-4), if a retailer is assigned to a TPLP, all of its orders are shipped by the TPLP fleet; therefore, it is not visited by any owned or rental vehicles. This constraint is referred to assumption 19 in section 3.4.

$$\sum_{i \in PUW} XU_{ijkt} - \sum_{m \in PUW} XU_{jmkt} = 0 \quad \forall j \in W, \forall k \in GVU, \forall t \in T \quad (3-5)$$

$$\sum_{i \in RUW} XW_{ijkt} - \sum_{m \in RUW} XW_{jmkt} = 0 \quad \forall j \in R, \forall k \in GVW, \forall t \in T \quad (3-6)$$

Equations (3-5) and (3-6) force vehicles to leave the same facility location that they have entered. Equation (3-5) is valid for the upper level and equation (3-6) is applied to the lower level.

$$\sum_{j \in W} \sum_{t \in T} XU_{ijkt} - \sum_{j \in W} \sum_{t \in T} XU_{jikt} = 0 \quad \forall i \in P, \forall k \in GVU \quad (3-7)$$

$$\sum_{j \in R} \sum_{t \in T} XW_{ijkt} - \sum_{j \in R} \sum_{t \in T} XW_{jikt} = 0 \quad \forall i \in W, \forall k \in GVW \quad (3-8)$$

Equations (3-7) and (3-8) force each vehicle to return to its plant or bottler, respectively at the end of every time step.

$$\sum_{i \in P} \sum_{j \in WUP} XU_{ijkt} \leq 1 \quad \forall k \in GVU, \forall t \in T \quad (3-9)$$

$$\sum_{i \in W} \sum_{j \in WUR} XW_{ijkt} \leq 1 \quad \forall k \in GVW, \forall t \in T \quad (3-10)$$

Constraint (3-9) does not allow a vehicle to leave two separate plants at the same time step. Similarly, equation (3-10) does not allow a vehicle to leave two different bottlers in the same time step.

$$\sum_{k \in GVU} \sum_{t \in T} XU_{iikt} = 0 \quad \forall i \in W \quad (3-11)$$

$$\sum_{k \in GVW} \sum_{t \in T} XW_{iikt} = 0 \quad \forall i \in R \quad (3-12)$$

Constraints (3-11) and (3-12) do not allow a vehicle going from one facility location to the same facility location directly in the upper and lower levels, respectively.

$$TLU + \left(\sum_{i \in WUP} \sum_{j \in WUP} XU_{ikt} - 1 \right) \times TLUU + \sum_{i \in WUP} \sum_{j \in WUP} XU_{ikt} C_{ij} SFu_k \leq TT_u$$

$$\forall k \in GVU, \forall t \in T \quad (3-13)$$

$$TLW + \left(\sum_{i \in WUR} \sum_{j \in WUR} XW_{ikt} - 1 \right) \times TLUW + \sum_{i \in WUR} \sum_{j \in WUR} XW_{ikt} C_{ij} SFw_k \leq TT_w$$

$$\forall k \in GVW, \forall t \in T \quad (3-14)$$

Equations (3-13) and (3-14) calculate business hours of each vehicle in the upper and lower levels respectively, which includes loading time, unloading time, and travel time. According to these constraints, the business hours must not be greater than predetermined values for each level.

$$\sum_{i \in WUP} \sum_{t \in T} XU_{iJKt} = 0 \quad (3-15)$$

$$\sum_{i \in WUR} \sum_{t \in T} XW_{iJKt} = 0 \quad (3-16)$$

Constraints (3-15) and (3-16) do not allow a specific vehicle to visit a particular facility location. According to problem description, some vehicles are not able to deliver syrup concentrate/products to some bottlers/retailers due to their size. These constraints define these limits in the mathematical formulation.

Commodity Flow Constraints

$$I_{sit} = I_{si(t-1)} + Pd_{ist} - \sum_{l \in Lnw(i,k)} \sum_{k \in GVW} \sum_{m \in R} QW_{lskt}^{im} - \sum_{j \in R} \sum_{q \in TP} \sum_{m \in T} TW_{smtq}^{ij}$$

$$\forall i \in W, \forall s \in S, \forall t \in T \quad (3-17)$$

$$I_{sit} = I_{si(t-1)} + \sum_{l \in Lew(i,k)} \sum_{k \in GVW} \sum_{m \in W} QW_{lskt}^{mi} + \sum_{j \in W} \sum_{q \in TP} \sum_{m \in T} TW_{smtq}^{ji} - DE_{ist}$$

$$\forall i \in R, \forall s \in S, \forall t \in T \quad (3-18)$$

Equations (3-17) and (3-18) calculate inventory level at the end of each time step for different final products at every bottler in the upper level and every retailer in

the lower level, respectively. According to equation (3-17), the inventory of commodity type s at the end of time step t in a bottler i is equal to the inventory level at the end of time step $t-1$, plus the production quantity of commodity type s at the bottler i at time step t minus all final products shipped from the bottler i to the retailers at time step t by owned vehicles, rental vehicles, and TPLP vehicles.

Based on equation (3-18), the inventory level for commodity s at the end of time step t at a retailer i location is equal to the inventory level of the same product at the end of time step $t-1$ plus all quantities of commodity s that arrive at the retailer i location in time step t by owned, rental, and TPLP vehicles minus quantity of product s sold to the customers at the retailer i location at time step t .

$$\sum_{s \in S_1} \sum_{m \in P} \sum_{n \in W} a_s Q_{lskt}^{mn} \leq Cku_k \quad \forall i \in W \cup P, \forall k \in GVU, \forall l \in Lnu(i, k), \forall t \in T \quad (3-19)$$

$$\sum_{s \in S} \sum_{m \in W} \sum_{n \in R} a_s Q_{lskt}^{mn} \leq Ckw_k \quad \forall i \in W \cup R, \forall k \in GVW, \forall l \in Lnw(i, k), \forall t \in T \quad (3-20)$$

Constraints (3-19) and (3-20) illustrate the loading capacity of vehicles in each link in the upper and lower levels, respectively. Syrup concentrate in the upper level and final products in the lower level are converted to the unique volume unit by the factor a_s .

$$\sum_{l \in Lnu(i, k)} \sum_{m \in P} \sum_{n \in W} Q_{lskt}^{mn} = \sum_{l \in Leu(i, k)} \sum_{m \in P} \sum_{n \in W} Q_{lskt}^{mn} - \sum_{l \in Leu(i, k)} \sum_{m \in P} Q_{lskt}^{mi} \quad (3-21)$$

$$\forall i \in W, \forall k \in GVU, \forall s \in S, \forall t \in T$$

$$\sum_{l \in Lnw(i, k)} \sum_{m \in W} \sum_{n \in R} Q_{lskt}^{mn} = \sum_{l \in Lew(i, k)} \sum_{m \in W} \sum_{n \in R} Q_{lskt}^{mn} - \sum_{l \in Lew(i, k)} \sum_{m \in W} Q_{lskt}^{mi} \quad (3-22)$$

$$\forall i \in R, \forall k \in GVW, \forall s \in S, \forall t \in T$$

Equations (3-21) and (3-22) are flow conservation constraints in the upper and lower levels, respectively. For instance, at the lower level, the quantity of a

commodity that leaves a retailer i is equal to the quantity of that commodity that arrives at a retailer m minus the quantity of a commodity stored by retailer m .

$$\sum_{\substack{l \in Lnu(i,k) \\ i \neq m}} \sum_{n \in W} \sum_{k \in GVU} \sum_{s \in S} Qu_{lskt}^{mn} = 0 \quad \forall i, m \in P, \forall t \in T \quad (3-23)$$

$$\sum_{\substack{l \in Lnw(i,k) \\ i \neq m}} \sum_{n \in R} \sum_{k \in GVW} \sum_{s \in S} Qw_{lskt}^{mn} = 0 \quad \forall i, m \in W, \forall t \in T \quad (3-24)$$

According to constraint (3-23), a flow of syrup concentrate going from a plant to a bottler cannot pass through a plant. Similarly, based on constraint (3-24), a flow of a final product going from a bottler to a retailer cannot pass through a bottler.

$$\sum_{l \in Lnu(i,k)} Qu_{lskt}^{mn} = \sum_{l \in Leu(i,k)} Qu_{lskt}^{mn} \quad \forall i, n \in W (n \neq i), \forall m \in P, \forall k \in GVU, \forall s \in S, \forall t \in T \quad (3-25)$$

$$\sum_{l \in Lnw(i,k)} Qw_{lskt}^{mn} = \sum_{l \in Lew(i,k)} Qw_{lskt}^{mn} \quad \forall i, n \in R (n \neq i), \forall m \in W, \forall k \in GVW, \forall s \in S, \forall t \in T \quad (3-26)$$

Equations (3-25) and (3-26) force the flow to pass the location that is not its destination.

$$\sum_{l \in Lnu(i,k)} \sum_{m \in P} \sum_{n \in W} \sum_{k \in GVU} \sum_{s \in S} Qu_{lskt}^{mn} = 0 \quad \forall i \in W \cup R, \forall t \in T \quad (3-27)$$

$$\sum_{l \in Lnw(i,k)} \sum_{m \in W} \sum_{n \in R} \sum_{k \in GVW} \sum_{s \in S} Qw_{lskt}^{mn} = 0 \quad \forall i \in W \cup P, \forall t \in T \quad (3-28)$$

Since decision variables of flow (Qu and Qw) are defined over links, flow of syrup concentrate must be zero for all lower level links and flow of final products must be zero for all upper level links. According to constraints (3-27) and (3-28) all non-relevant flow decision variables become zero.

$$\sum_{l \in Leu(i,k)} \sum_{m \in P} \sum_{k \in GVU} Qu_{lskt}^{mi} - Sc_{ist} - \sum_{s_2 \in S} \frac{Pd_{ist}}{\lambda_{s_2}} = 0 \quad \forall i \in W, \forall s \in S_1, \forall t \in T \quad (3-29)$$

Constraint (3-29) makes a relation among quantity of syrup concentrate delivered to one bottler, quantity of commodity from different types produced, and quantity of syrup concentrate stored in that particular bottler. λ_{s_2} is the parameter indicates how many units of syrup concentrate s_1 are needed to produce one unit of commodity type s_2 .

$$Pd_{ist} \leq CAP_{sit} \quad \forall i \in W, \forall s \in S, \forall t \in T \quad (3-30)$$

$$Sc_{ist} \leq SHC_{sit} \quad \forall i \in W, \forall s \in S_1, \forall t \in T \quad (3-31)$$

Equation (3-30) limits the production quantity of commodity type s in a bottler at each time step to its production capacity. Constraint (3-31) also forces the model to store syrup concentrate s in each bottler less than its syrup storage capacity.

Shortage and Holding Amount Constraints

$$I_{sit} \leq HC_{sit} \quad \forall i \in W \cup R, \forall s \in S, \forall t \in T \quad (3-32)$$

$$DE_{ist} \leq DM_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T \quad (3-33)$$

$$e_{ist} + DE_{ist} = DM_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T \quad (3-34)$$

$$\mu e_{is(t-1)} + d_{ist} = DM_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T \quad (3-35)$$

$$I_{s i 0} = 0 \quad \forall i \in W \cup R, \forall s \in S \quad (3-36)$$

$$e_{i s 0} = 0 \quad \forall i \in R, \forall s \in S \quad (3-37)$$

Equation (3-32) emphasizes that the inventory level of commodity type s in facility location i at time step t should not exceed storage capacity. Safety stock is the

amount of inventory invested to response to fluctuations in demand to avoid running out of stock. Safety stock, therefore, can be considered as the minimum inventory level in bottlers and retailers. However, safety stock is assumed to be zero in this study; otherwise it can be easily added to the mathematical formulation.

Equation (3-33) shows that the delivery amount of final product to customers at each retailer cannot be more than its modified demand and equations (3-34) and (3-35) calculate the modified demand of commodity type k in retailer i at time step t based on its original demand and shortage amount of that commodity type in the same retailer at the same time step. According to constraint (3-34), shortage in a retailer's order happens when the delivery amount to customers is less than the modified demand of that retailer. Moreover, equation (3-35) indicates that the modified demand of commodity type k in retailer i at time step t is equal to the original demand of the same commodity type in the same retailer at time step t plus shortage amount of commodity type k in retailer i at time step $t-1$. μ is the parameter of the model showing the portion of unsatisfied demand transferred to the next time step. In this study, the value of μ has been considered to one, which means all unsatisfied demand is transferred to the next time step. Equations (3-36) and (3-37) set the inventory level and shortage amount of all commodity types at all facility locations at the beginning of planning horizon to zero.

Constraints of Relation between Different Decision Variables

$$\sum_{\substack{l \in Lnu(i,k) \\ NE(l)=j}} \sum_{m \in P} \sum_{n \in W} \sum_{s \in S} Qu_{lskt}^{mn} \leq M Xu_{ijkt} \quad \forall i, j \in W \cup P, \forall k \in GVU, \forall t \in T \quad (3-38)$$

$$\sum_{\substack{l \in Lnw(i,k) \\ NE(l)=j}} \sum_{m \in W} \sum_{n \in R} \sum_{s \in S} Qw_{lskt}^{mm} \leq M XW_{ijkt} \quad \forall i, j \in W \cup R, \forall k \in GVW, \forall t \in T \quad (3-39)$$

Constraints (3-38) and (3-39) relate flow and path decision variables in the upper and lower levels, respectively. In these two equations, M is a very large number. According to these constraints, flow of commodity can go from one facility location to another one, if a vehicle is available in the link that connects those two locations.

$$\sum_{i \in WUP} \sum_{j \in WUP} XU_{jikt} \leq M VZu_{kt} \quad \forall k \in VU, \forall t \in T \quad (3-40)$$

$$\sum_{i \in WUR} \sum_{j \in WUR} XW_{jikt} \leq M VZw_{kt} \quad \forall k \in VW, \forall t \in T \quad (3-41)$$

Constraints (3-40) and (3-41) find the number of time steps at which each rental vehicle is used, to calculate the operational cost of a rental vehicle. M is a very large number and VZu and VZw are binary decision variables representing whether a rental vehicle is used in time step t . Since VZu and VZw decrease objective function value, the model makes them equal to one when decision variables XU or XW are equal to one.

$$\text{If } VZu_{k(t+1)} - VZu_{kt} = 1 \Rightarrow Uu_{k(t+1)} = 1 \quad \forall k \in VU, \forall t \in T \quad (3-42)$$

$$\text{If } VZw_{k(t+1)} - VZw_{kt} = 1 \Rightarrow Uw_{k(t+1)} = 1 \quad \forall k \in VW, \forall t \in T \quad (3-43)$$

$$VZu_{k1} \leq M Uu_{k1} \quad \forall k \in VU \quad (3-44)$$

$$VZw_{k1} \leq M Uw_{k1} \quad \forall k \in VW \quad (3-45)$$

Equations (3-42) to (3-45) find the first time step at which a rental vehicle is used in the upper and lower levels, respectively. There are two different expenses associated with rental vehicles: fixed cost, and operational cost. Fixed cost is paid

once when the bottling company rents a vehicle and it is independent of rental duration. On the other hand, operational cost is a linear function of vehicle usage. So, figuring out which time step is the first one in which a rental vehicle is used is essential. Constraints (3-44) and (3-45) determine at which time steps a rental vehicle is used. According to equations (3-42) and (3-43), if a rental vehicle in two consecutive time steps is only used in the second one, the vehicle starts to work in the second time step. Since these equations cannot determine whether a rental vehicle starts to work on the first time step, equations (3-44) and (3-45) have been added to the model.

Third Party Logistics Provider Constraints

$$\sum_{i \in W} \sum_{s \in S} \sum_{m \in T} TW_{smtq}^{ij} \leq M TN_{jqt} \quad \forall j \in R, \forall q \in TP, \forall t \in T \quad (3-46)$$

$$\sum_{j \in W} \sum_{s \in S} \sum_{m \in T} NB_{iq} TW_{smtq}^{ji} \geq TN_{iqt} \quad \forall i \in R, \forall q \in TP, \forall t \in T \quad (3-47)$$

According to equations (3-46) and (3-47), flow of commodity is sent to a retailer by a TPLP if that retailer is supposed to get service from a TPLP. TN_{iqt} is the binary decision variable equal to one when a retailer receives its order by TPLP in time step . NB_{iq} is the parameter which is equal to one if contract serves retailer .

is a very large number and TW_{smtq}^{ij} represents the flow of commodity sent to a retailer by TPLP and it leaves a bottler at time step and arrives at retailer at time step . In other words, a package of products can be sent in time step but received at its destination at time step . It means TPLP stores the package for time steps. It may happen when the capacity of bottler-owned warehouses and retailer-

owned storage are not enough to keep all products. Despite the fact that TPLP storage increases the cost, which is considered in the objective function, it may increase profit, depending on TPLP warehousing cost and penalty for shortage in delivery. This means that the model is able to consider TPLP warehousing as well as TPLP transportation. According to equation (3-47) a retailer cannot be served by a TPLP if the retailer does not exist in its contract and based on equation (3-46) flow can be sent to a retailer by that particular TPLP if that retailer exists in its contract.

$$\sum_{m \in W} \sum_{n \in R} \sum_{p \in T} TW_{stp q}^{mn} - TLB_{sqi} \leq M w_{qist} \quad \forall q \in TP, \forall i \in L, \forall s \in S, \forall t \in T \quad (3-48)$$

Constraint (3-48) determines the value for decision variable needed in objective function. Cost function of TPLP is considered as a step function in this study. In other words, the cost of TPLP depends on the quantity of commodities shipped by TPLP but it does not change linearly. Figure 3-1 shows the typical step function considered for cost of TPLP.

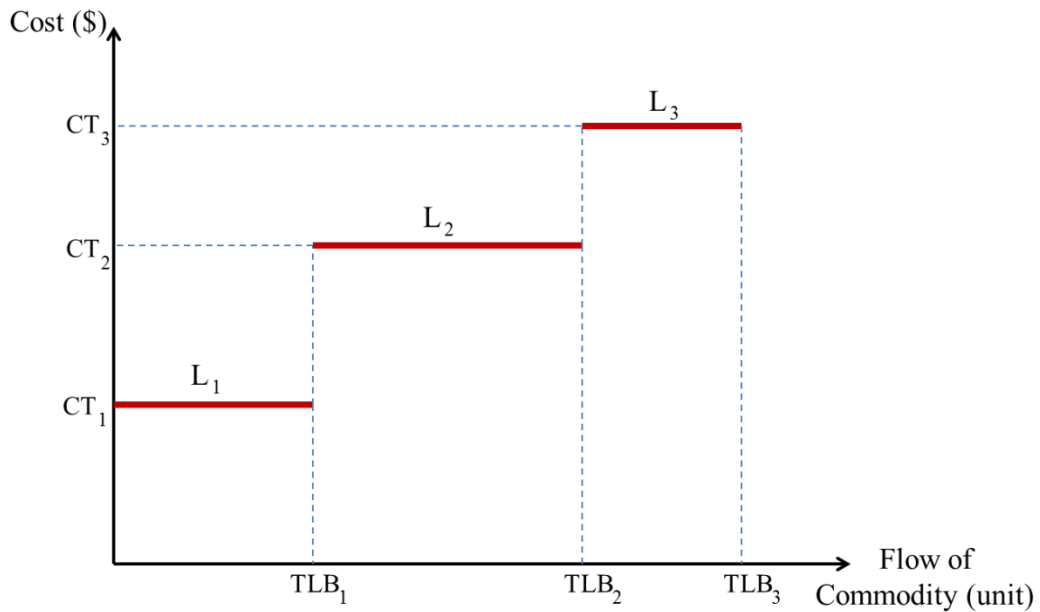


Figure 3-1 Stepwise cost function considered for TPLP

For example a cost of TPLP contract shown in Figure 3-1 has three levels.

The first level has cost of CT_1 for all shipping less than TLB_1 unit. If the volume of shipping is more than TLB_1 and less than TLB_2 , the cost of TPLP will be CT_2 . Upper and lower bounds of shipping for each level of contract cost function (or TLB_i) are parameters of the model. Therefore, equation (3-48) chooses the correct value for w_i binary variable used in objective function. According to constraint (3-48), if a step of a contract is selected, all steps lower than are selected, however the total cost must be equal to i^{th} step cost. Therefore, CT_i is defined as the incremental cost of the i^{th} step in compare to its last step cost.

$$\sum_{q \in TP} \sum_{i \in W} \sum_{j \in R} \sum_{s \in S} TW_{stmq}^{ij} = 0 \quad \forall t, m \in T \quad m < t \quad (3-49)$$

According to equation (3-49), TW is zero for all deliveries earlier than departure time.

$$Z_{sqm}^{ij} \geq \sum_{n \in T} (n - m) TW_{smnq}^{ij} \quad \forall i \in W, \forall j \in R, \forall q \in TP, \forall s \in S, \forall m \in T \quad (3-50)$$

Finally, equation (3-50) is the last constraint which calculates the cumulative quantity of product type s stored in TPLP's warehouses. Obviously, if a product is shipped and received in the same time step, Z becomes zero which means that nothing is stored by TPLP. Z is included in objective function and increases the cost.

Non-negativity and Integrality Constraints

$$\begin{aligned} & Qu_{lskt}^{mn}, \overline{Q}w_{lskt}^{mn}, I_{sit}, e_{ist}, DE_{ist}, DM_{ist}, \\ & TW_{stmq}^{ij}, Z_{sqm}^{ij}, Sc_{ist} \text{ and } Pd_{ist} \geq 0 \end{aligned} \quad \text{Real-valued decision variable}$$

$$\begin{aligned}
 & XU_{ijkt}, XW_{ijkt}, VZu_{kt}, VZw_{kt}, Uu_{kt}, Uw_{kt}, \\
 & TN_{it} \text{ and } w_{qist} \in \{0,1\}
 \end{aligned}$$

Binary integer variables

3.5. Summary

This Chapter introduced the mathematical formulation that is developed for the Inventory-Routing problem of a bottling company. Different details such as nonhomogeneous fleet, rental vehicles, and TPLP have been considered in the mathematical formulation. Table 3-1 presents the entire mathematical formulation proposed for the problem of this study. Limited numerical experiments are conducted to show different capabilities of the model and the results are reported in Chapter 4. This mathematical formulation which is limited to problems with three tiers and two levels can be applied to some other industries. For instance, auto industry has the same structure. Parts are shipped from the different suppliers in the first tier to car manufacturers in the second tier. Then, cars are sent to dealers in the third tier to sell to final customers. However, this structure has a limit that does not allow applying this model to some industries. According to the assumptions, the vehicles in the upper level cannot work in the lower level and vice versa. This model, therefore, cannot be applied to industries for which this assumption is not valid.

Table 3-1. The entire mathematical formulation of the problem of this study

$$\begin{aligned}
 & \text{Max} \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} P c_{sit} DE_{ist} - \sum_{i \in W \cup P} \sum_{j \in W \cup P} \sum_{k \in GVU} \sum_{t \in T} \alpha SF u_k C_{ij} XU_{ijkt} - \\
 & \sum_{i \in W \cup R} \sum_{j \in W \cup R} \sum_{k \in GVW} \sum_{t \in T} \alpha SF u_k C_{ij} XW_{ijkt} - \sum_{i \in P} \sum_{j \in W} \sum_{l \in LN} \sum_{s \in S} \sum_{k \in GVU} \sum_{t \in T} \beta Q u_{lskt}^{ij} len_{ij} - \\
 & \sum_{i \in W} \sum_{j \in R} \sum_{l \in LN} \sum_{s \in S} \sum_{k \in GVW} \sum_{t \in T} \beta Q W_{lskt}^{ij} len_{ij} - \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} e_{ist} Sh_{sit} - \\
 & \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} I_{sit} h_{sit} - \sum_{k \in VU} \sum_{t \in T} FCV u_{kt} U u_{kt} - \sum_{k \in VW} \sum_{t \in T} FCV W_{kt} U W_{kt} - \sum_{k \in VU} \sum_{t \in T} EPCV u_{kt} VZ u_{kt} - \\
 & \sum_{k \in VW} \sum_{t \in T} EPCV W_{kt} VZ W_{kt} - \sum_{q \in TP} \sum_{l \in L} \sum_{s \in S} \sum_{t \in T} CT_{sqi} W_{qist} - \sum_{i \in W} \sum_{j \in R} \sum_{q \in TP} \sum_{s \in S} \sum_{t \in T} Z_{sqt}^{ij} WR_s - \\
 & \sum_{i \in W} \sum_{s \in S} \sum_{t \in T} Pd_{ist} PdCost_{is} \\
 \\
 & \sum_{j \in PUW} XU_{ijkt} \leq 1 \quad \forall i \in P \cup W, \forall k \in GVU, \forall t \in T \\
 \\
 & \sum_{j \in R} XW_{ijkt} \leq 1 \quad \forall i \in W, \forall k \in GVW, \forall t \in T \\
 \\
 & \sum_{j \in RUW} XW_{ijkt} \leq 1 - \sum_{q \in TP} TN_{iqt} \quad \forall i \in R, \forall k \in GVW, \forall t \in T \\
 \\
 & \sum_{i \in PUW} XU_{ijkt} - \sum_{m \in PUW} XU_{jmkt} = 0 \quad \forall j \in W, \forall k \in GVU, \forall t \in T \\
 \\
 & \sum_{i \in RUW} XW_{ijkt} - \sum_{m \in RUW} XW_{jmkt} = 0 \quad \forall j \in R, \forall k \in GVW, \forall t \in T \\
 \\
 & \sum_{j \in W} \sum_{t \in T} XU_{ijkt} - \sum_{j \in W} \sum_{t \in T} XU_{jikt} = 0 \quad \forall i \in P, \forall k \in GVU \\
 \\
 & \sum_{j \in R} \sum_{t \in T} XW_{ijkt} - \sum_{j \in R} \sum_{t \in T} XW_{jikt} = 0 \quad \forall i \in W, \forall k \in GVW \\
 \\
 & \sum_{i \in P} \sum_{j \in W \cup P} XU_{ijkt} \leq 1 \quad \forall k \in GVU, \forall t \in T \\
 \\
 & \sum_{i \in W} \sum_{j \in W \cup R} XW_{ijkt} \leq 1 \quad \forall k \in GVW, \forall t \in T \\
 \\
 & \sum_{k \in GVU} \sum_{t \in T} XU_{iikt} = 0 \quad \forall i \in W
 \end{aligned}$$

Table 3-1 (Cont'd). The entire mathematical formulation of the problem of this study

$$\sum_{k \in GVW} \sum_{t \in T} XW_{ikt} = 0 \quad \forall i \in R$$

$$TLU + \sum_{i \in WUP} \sum_{j \in WUP} (XU_{ijkt} - 1) \times TLUU + \sum_{i \in WUP} \sum_{j \in WUP} XU_{ijkt} C_{ij} SFu_k \leq TT_u$$

$$\forall k \in GVU, \forall t \in T$$

$$TLW + \sum_{i \in WUR} \sum_{j \in WUR} (XW_{ijkt} - 1) \times TLWU + \sum_{i \in WUR} \sum_{j \in WUR} XW_{ijkt} C_{ij} SFw_k \leq TT_w$$

$$\forall k \in GVW, \forall t \in T$$

$$\sum_{i \in WUP} \sum_{t \in T} XU_{iJKt} = 0$$

$$\sum_{i \in WUR} \sum_{t \in T} XW_{iJKt} = 0$$

$$I_{sit} = I_{si(t-1)} + Pd_{ist} - \sum_{l \in Lnw(i,k)} \sum_{k \in GVW} \sum_{m \in R} Qw_{lskt}^{im} - \sum_{j \in R} \sum_{q \in IP} \sum_{m \in T} TW_{stmq}^{ij}$$

$$\forall i \in W, \forall s \in S, \forall t \in T$$

$$\sum_{s \in S_1} \sum_{m \in P} \sum_{n \in W} a_s Qu_{lskt}^{mn} \leq Cku_k \quad \forall i \in W \cup P, \forall k \in GVU, \forall l \in Lnu(i,k), \forall t \in T$$

$$\sum_{s \in S} \sum_{m \in W} \sum_{n \in R} a_s Qu_{lskt}^{mn} \leq Ckw_k \quad \forall i \in W \cup R, \forall k \in GVW, \forall l \in Lnw(i,k), \forall t \in T$$

$$\sum_{l \in Lnu(i,k)} \sum_{m \in P} \sum_{n \in W} Qu_{lskt}^{mn} = \sum_{l \in Leu(i,k)} \sum_{m \in P} \sum_{n \in W} Qu_{lskt}^{mn} - \sum_{l \in Leu(i,k)} \sum_{m \in P} Qu_{lskt}^{mi}$$

$$\forall i \in W, \forall k \in GVU, \forall s \in S, \forall t \in T$$

$$\sum_{l \in Lnw(i,k)} \sum_{m \in W} \sum_{n \in R} Qw_{lskt}^{mn} = \sum_{l \in Lew(i,k)} \sum_{m \in W} \sum_{n \in R} Qw_{lskt}^{mn} - \sum_{l \in Lew(i,k)} \sum_{m \in W} Qw_{lskt}^{mi}$$

$$\forall i \in R, \forall k \in GVW, \forall s \in S, \forall t \in T$$

$$\sum_{\substack{l \in Lnu(i,k) \\ i \neq m}} \sum_{n \in W} \sum_{k \in GVU} \sum_{s \in S} Qu_{lskt}^{mn} = 0 \quad \forall i, m \in P, \forall t \in T$$

$$\sum_{\substack{l \in Lnw(i,k) \\ i \neq m}} \sum_{n \in R} \sum_{k \in GVW} \sum_{s \in S} Qw_{lskt}^{mn} = 0 \quad \forall i, m \in W, \forall t \in T$$

Table 3-1 (Cont'd). The entire mathematical formulation of the problem of this study

$\sum_{l \in Lnu(i,k)} Qu_{lskt}^{mn} = \sum_{l \in Leu(i,k)} Qu_{lskt}^{mn} \quad \forall i, n \in W (n \neq i), \forall m \in P, \forall k \in GVU, \forall s \in S, \forall t \in T$	
$\sum_{l \in Lnw(i,k)} Qw_{lskt}^{mn} = \sum_{l \in Lew(i,k)} Qw_{lskt}^{mn} \quad \forall i, n \in R (n \neq i), \forall m \in W, \forall k \in GVW, \forall s \in S, \forall t \in T$	
$\sum_{l \in Lnw(i,k)} \sum_{m \in P} \sum_{n \in W} \sum_{k \in GVU} \sum_{s \in S} Qu_{lskt}^{mn} = 0 \quad \forall i \in W \cup R, \forall t \in T$	
$\sum_{l \in Lnu(i,k)} \sum_{m \in W} \sum_{n \in R} \sum_{k \in GVW} \sum_{s \in S} Qw_{lskt}^{mn} = 0 \quad \forall i \in W \cup P, \forall t \in T$	
$\sum_{l \in Leu(i,k)} \sum_{m \in P} \sum_{k \in GVU} Qu_{lskt}^{mi} - Sc_{ist} - \sum_{s_2 \in S} \frac{Pd_{ist}}{\lambda_{s_2}} = 0 \quad \forall i \in W, \forall s \in S_1, \forall t \in T$	
$Pd_{ist} \leq CAP_{sit} \quad \forall i \in W, \forall s \in S, \forall t \in T$	
$Sc_{ist} \leq SHC_{sit} \quad \forall i \in W, \forall s \in S_1, \forall t \in T$	
$I_{sit} \leq HC_{sit} \quad \forall i \in W \cup R, \forall s \in S, \forall t \in T$	
$DE_{ist} \leq DM_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T$	
$e_{ist} + DE_{ist} = DM_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T$	
$e_{is(t-1)} + d_{ist} = DM_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T$	
$I_{si0} = 0 \quad \forall i \in W \cup R, \forall s \in S$	
$e_{is0} = 0 \quad \forall i \in R, \forall s \in S$	
$\sum_{\substack{l \in Lnu(i,k) \\ NE(l)=j}} \sum_{m \in P} \sum_{n \in W} \sum_{s \in S} Qu_{lskt}^{mn} \leq M XU_{ijkt} \quad \forall i, j \in W \cup P, \forall k \in GVU, \forall t \in T$	
$\sum_{i \in W \cup P} \sum_{j \in W \cup P} XU_{jikt} \leq M VZu_{kt} \quad \forall k \in VU, \forall t \in T$	
$\sum_{i \in W \cup R} \sum_{j \in W \cup R} XW_{jikt} \leq M VZw_{kt} \quad \forall k \in VW, \forall t \in T$	

Table 3-1 (Cont'd). The entire mathematical formulation of the problem of this study

$If VZu_{k(t+1)} - VZu_{kt} = 1 \Rightarrow Uu_{k(t+1)} = 1$	$\forall k \in VU, \forall t \in T$
$If VZW_{k(t+1)} - VZW_{kt} = 1 \Rightarrow Uw_{k(t+1)} = 1$	$\forall k \in VW, \forall t \in T$
$VZu_{k1} \leq M Uu_{k1}$	$\forall k \in VU$
$VZW_{k1} \leq M Uw_{k1}$	$\forall k \in VW$
$\sum_{i \in W} \sum_{s \in S} \sum_{m \in T} TW_{smtq}^{ij} \leq M TN_{jqt}$	$\forall j \in R, \forall q \in TP, \forall t \in T$
$\sum_{j \in W} \sum_{s \in S} \sum_{m \in T} NB_{iq} TW_{smtq}^{ji} \geq TN_{iqt}$	$\forall i \in R, \forall q \in TP, \forall t \in T$
$\sum_{m \in W} \sum_{n \in R} \sum_{p \in T} TW_{stpq}^{mn} - TLB_{sqi} \leq M w_{qist}$	$\forall q \in TP, \forall i \in L, \forall s \in S, \forall t \in T$
$\sum_{q \in TP} \sum_{i \in W} \sum_{j \in R} \sum_{s \in S} TW_{stmq}^{ij} = 0$	$\forall t, m \in T, m < t$
$Z_{sqm}^{ij} \geq \sum_{n \in T} (n - m) TW_{smnq}^{ij}$	$\forall i \in W, \forall j \in R, \forall q \in TP, \forall s \in S, \forall m \in T$
$Qu_{lskt}^{mn}, Qw_{lskt}^{mn}, I_{sit}, e_{ist}, DE_{ist}, DM_{ist},$ $TW_{stmq}^{ij}, Z_{sqm}^{ij}, Sc_{ist} \text{ and } Pd_{ist} \geq 0$	Real-valued decision variable
$XU_{ijkt}, XW_{ijkt}, VZu_{kt}, VZW_{kt}, Uu_{kt}, Uw_{kt},$ $TN_{it} \text{ and } w_{qist} \in \{0, 1\}$	Binary integer variables

Chapter 4: Numerical Study

In this chapter, different sample problems are solved to evaluate the mathematical formulation proposed in this research. These experiments have been designed for small to large size problems so that they include different features of real-world problems. The main focus of this chapter is on small and medium size problems because interpreting the results is much easier for them. As it will be shown, large problems cannot be solved by commercial software. In this chapter, input data for the models is described first and then numerical results provided by Xpress 7.1 are presented in details.

4.1. Design of Sample Problems

The parameters provided in next sections are related to Scenario 1. Many of them will be changed in other scenarios to illustrate the effect of different parameters on the final solution as well as model capabilities.

4.1.1. Planning Horizon and Time Step

In this study, time-step length is assumed to be one day and the planning horizon varies over different sample problems, from 3 days for small problems to 5 days for large problem. Time steps shorter than one day are not appropriate for this problem, because in the real world, no retailer is met more than once every day. Although

shorter time steps help to capture more details in the model such as exact delivery time, it increases the number of parameters and decision variables so significantly that even small problems cannot be solved optimally.

4.1.2. Facility Locations

As mentioned in previous chapters, there are three tiers in this model: plants, bottlers, and retailers. Two plants, three bottlers, and six retailers have been considered, in the first scenario. In other scenarios more facility locations will be added to the model.

To determine the location of facilities, 11 locations of Giant and Wal-Mart stores in Maryland were chosen arbitrarily and divided among three tiers. These known locations are helpful in determining network parameters, which are described in the next section. Figure 4-1 shows locations of selected plants, bottlers, and retailers.

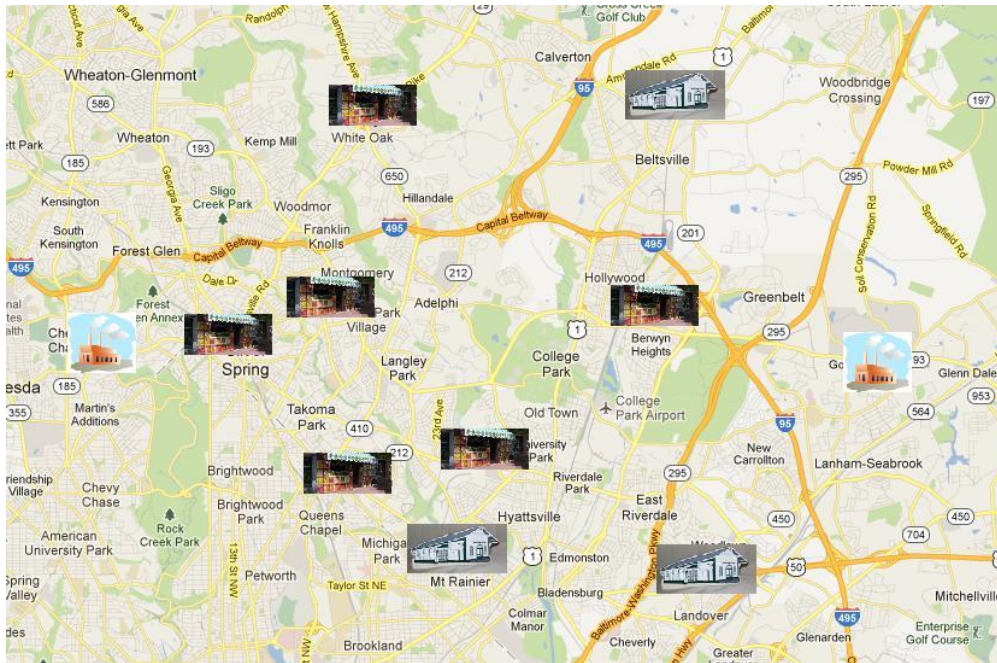


Figure 4-1 Delivery tours in both levels in the first time step in the first scenario

4.1.3. Network

The network of the problem contains 11 nodes representing plants, bottlers, and retailers, and also the links that connect each pair of locations. Therefore, the network has 121 links. The travel time for each link was obtained using Google Map. Travel times are triangular, which means that travel time for each pair of nodes is shorter than travel time for any other paths connecting those two nodes through other nodes. Table 4-1 represents the travel time matrix in minutes for this network. Nodes one to six represent retailers, nodes seven to nine represent bottlers, and nodes 10 and 11 represent plants. Since the time step is one day, all of these travel times should be divided by 1,440 to be used in the formulation.

Table 4-1. Network travel time (minutes)

Node	1	2	3	4	5	6	7	8	9	10	11
1	0	10.8	7.2	13.5	11.7	11.7	15.3	7.2	13.5	15.3	18.9
2	11.7	0	10.8	8.1	7.2	12.6	14.4	15.3	19.8	19.8	13.5
3	5.4	9.9	0	12.6	9.9	14.4	18.9	6.3	16.2	21.6	17.1
4	15.3	9	17.1	0	8.1	13.5	15.3	20.7	20.7	20.7	13.5
5	10.8	6.3	9.9	8.1	0	13.5	15.3	15.3	20.7	20.7	7.2
6	9.9	12.6	12.6	10.8	13.5	0	9	16.2	11.7	9.9	19.8
7	15.3	14.4	18	11.7	14.4	9.9	0	19.8	17.1	17.1	20.7
8	8.1	15.3	6.3	18	16.2	16.2	19.8	0	11.7	19.8	21.6
9	13.5	20.7	17.1	18	20.7	11.7	16.2	12.6	0	11.7	27
10	17.1	21.6	21.6	18.9	21.6	10.8	17.1	19.8	13.5	0	27
11	18	13.5	16.2	14.4	7.2	19.8	20.7	22.5	26.1	26.1	0

4.1.4. Supply and Demand

At this stage, there are two commodity types considered and results for problems with more commodity types will be shown in future applications. A pack of 24 bottles of two different types of soft drink is considered as a unit of final product in the first scenario.

As mentioned in previous chapters, price, order amount, holding cost, shortage penalty, and holding capacity vary over different retailers. Therefore, for every product and for each of these parameters a normal distribution with specific mean and standard deviation has been considered to generate corresponding values for each retailer. Table 4-2 presents the assumed parameters of normal distribution for different supply and demand input data.

Table 4-2. Parameters of normal distributions for different supply and demand input data

Input Data	Commodity Type	Mean	Standard Deviation
Price	1	24\$ per unit	0.4\$ per unit
	2	22\$ per unit	0.3\$ per unit
Order Amount	1	100 unit	15 unit
	2	105 unit	17 unit
Holding Cost	1	0.5\$ per unit	0.03\$ per unit
	2	0.5\$ per unit	0.03\$ per unit
Penalty Cost	1	3\$ per unit	0.30\$ per unit
	2	3\$ per unit	0.30\$ per unit
Holding Capacity at Bottlers	1	800 unit	30 unit
	2	800 unit	30 unit
Holding Capacity at Retailers	1	100 unit	10 unit
	2	100 unit	10 unit

Tables 4-3 and 4-4 illustrate the values for different input data generated according to the normal distribution parameters shown in Table 4-2.

Table 4-3. Values of different supply and demand input data

Data	Commodity Type	Price	Holding Cost	Penalty Cost	Holding Capacity
Retailer 1	1	24.34	0.52	2.75	113
	2	22.16	0.52	2.75	113
Retailer 2	1	24.00	0.49	3.00	103
	2	21.92	0.49	3.00	103
Retailer 3	1	23.90	0.46	3.25	100
	2	22.25	0.46	3.25	100
Retailer 4	1	23.90	0.53	3.08	103
	2	22.08	0.53	3.08	103
Retailer 5	1	23.79	0.49	3.08	97
	2	22.16	0.49	3.08	97
Retailer 6	1	24.00	0.46	2.62	113
	2	22.38	0.46	2.62	113
Bottler 1	1	0	0	0	792
	2	0	0	0	792
Bottler 2	1	0	0	0	825
	2	0	0	0	825
Bottler 3	1	0	0	0	808
	2	0	0	0	808

Table 4-4. Orders of different retailers in three time steps

Time Steps	Commodity Type	Time Step 1	Time Step 2	Time Step 3
Retailer 1	1	108	96	119
	2	108	96	119
Retailer 2	1	81	104	119
	2	100	108	108
Retailer 3	1	92	81	119
	2	108	92	113
Retailer 4	1	119	96	109
	2	109	127	101
Retailer 5	1	96	127	127
	2	114	119	91
Retailer 6	1	127	127	114
	2	101	109	127

4.1.5. Vehicles

As mentioned earlier, two groups of owned vehicles have been considered in the model. The first group works in the upper level and the second group distributes products in the lower level. Each group contains two different vehicle types in terms of capacity and speed. Similarly, two different groups of rental vehicles exist, one for each level, with two different vehicle types for each level. Besides capacity and speed, fixed cost and operational cost are also different for rental vehicle types. Table 4-5 presents the parameters of different vehicle types used in the model, which have been found from online data. The last column in Table 4-5 affects speed of vehicles in the fleet. Higher speed factor leads to lower speed for a truck.

Table 4-5. Different vehicle types' characteristics for the base case

Owned/ Rental	Level	Vehicle Type	Capacity (Unit)	Marginal Fixed Cost (\$/day)	Marginal Operational Cost (\$/day)	Speed Factor
Owned Vehicles	Upper Level	Type 1	590	0	0	1
		Type 2	635	0	0	2
	Lower Level	Type 1	440	0	0	1
		Type 2	440	0	0	2
Rental Vehicle	Upper Level	Type 1	620	49	16	1
		Type 2	610	54	17	3
	Lower Level	Type 1	440	99	16	1
		Type 2	450	103	18	3

All data in Table 4-5 are valid for Scenario 1. Some will be changed for other cases, which will be explained later.

In addition, the value of time and transportation costs that are used in the objective function are assumed as \$45/day and \$0.05/unit-mile, respectively.

4.1.6. Third Party Logistics Providers

Although the model allows for several candidate contracts that the manager of the bottling company can choose from, Scenario 1 has only two candidate contracts. Cost function of each contract has two steps. Characteristics of each candidate contract are shown in Table 4-6. These numbers were considered based on quotes from some TPLP websites.

Table 4-6. TPL candidate contracts' characteristics

Contract	Covered Retailers	Cost of the 1st Step (\$/day)	Cost of the 2nd Step (\$/day)	Upper bound of volume for the 1st step (unit)	Upper bound of volume for the 2nd step (unit)
Contract 1	5	155	513	760	1500
Contract 2	4 and 6	163	519	755	1500

4.2. Numerical Results

In this section, results obtained from solving different scenarios with Xpress 7.1 are presented and described in detail. The set of all scenarios includes the first scenario, whose parameters and input data were described in the previous section, and several other cases. In other scenarios, some parameters have been altered and the effects of these changes on the final solution are shown. Therefore, the results associated with the first scenario are presented first, followed by sections that present final solutions for the rest of the scenarios.

4.2.1. Scenario 1

As mentioned earlier, this scenario has three time steps (or three days), and all required information—such as supply and demand data, owned- and rental-vehicle data, network characteristics, and TPLP candidate contracts—has been introduced in previous sections. Table 4-7 presents general information for the final solution of Scenario 1.

Table 4-7. General output information for the optimal solution of Scenario 1

Criterion	Value
Number of Constraints	6320
Number of Decision Variables	62423
LP Relation Objective Function	48860.7
Best Bound for Objective Function	47877.4
Best Solution Objective Function	47827.7
Gap with the Best Bound (%)	0.009
Running Time (Seconds)	318.3

According to Table 4-7, the solver has found the optimal solution for Scenario 1 in reasonable running time. The gap in Table 4-7 represents the difference between objective function values of the best bound that is not feasible and the best feasible solution. Figures 4-2, 4-3, and 4-4 illustrate the tours of different vehicles and delivery quantity to each retailer in the first, second, and third time steps, respectively.

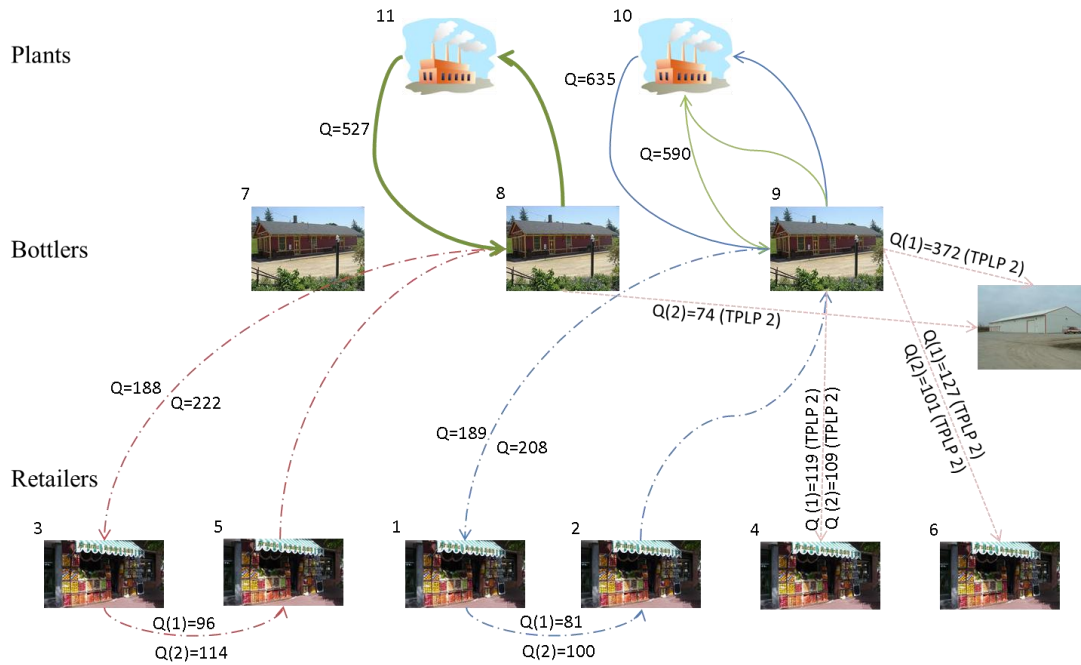


Figure 4-2 Delivery tours in both levels in the first time step in the first scenario

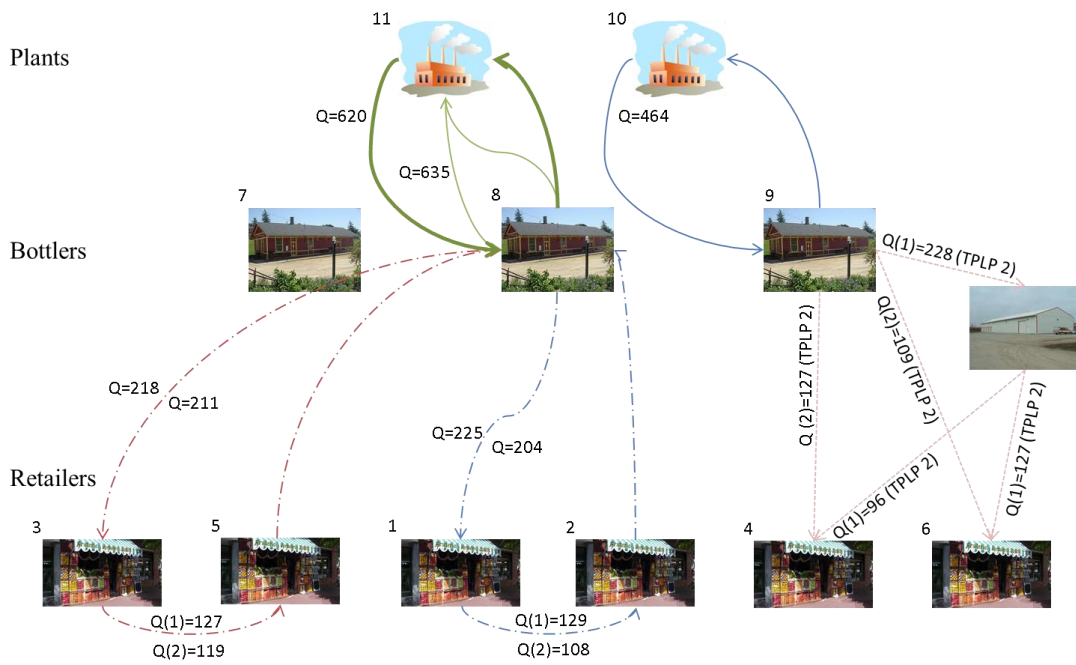


Figure 4-3 Delivery tours in both levels in the second time step in the first scenario

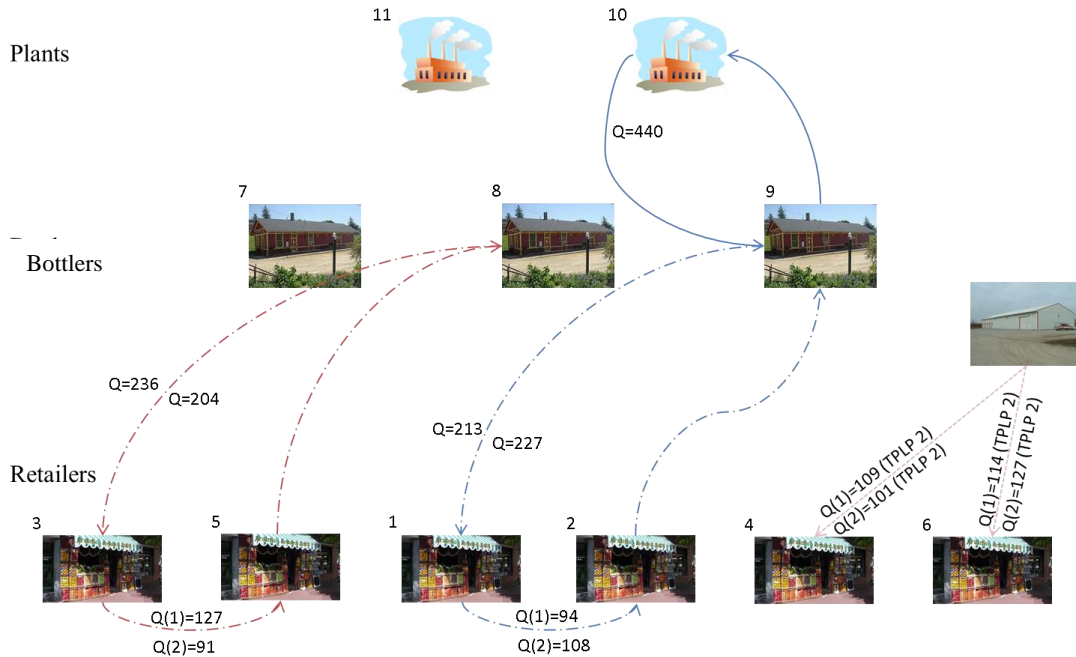


Figure 4-4 Delivery tours in both levels in the third time step in the first scenario

Different tours have been represented by different line types and colors to make them clear. Tours shown with ticker lines represent rental-vehicle paths. In the first and second time steps, a rental vehicle has been used in the upper level to transport more syrup concentrate to bottlers. According to the output of Xpress, in the optimal solution all orders have been satisfied without any penalty, and retailers 2 and 3 have stored commodity type 1 in the second time step for future demand. By this plan, the bottling company does not need to rent another vehicle for the third time step, because the owned trucks have used their capacity completely in the third time step and they are not able to deliver whole demands of retailers 2 and 3. In fact, the model has three options to deal with this deficiency in the fleet: storing a part of retailer 2 and 3's demands in the second time step, adding a rental vehicle to the fleet of the lower level, and outsourcing the delivery of the demands of retailers 2 and 3. The model has found the first option more beneficial. In addition, according to

Figures 4-3 and 4-4, in the upper level, vehicles transported bottler 8's orders for the second and third time steps together in the second time step. Since bottler 8 has enough capacity to store the products needed for delivery in the third time step with zero cost, this design model, therefore, sends more syrup concentrate to bottler 8 and saves the cost of upper-level tours in the third time step. Vehicles' capacities prohibit the model from sending all orders for the three time steps together in the first time step.

Owned vehicles and TPLP vehicles are enough for all deliveries in the lower level. The second TPLP, which fulfills the orders of retailers 4 and 6, has been selected for shipping final products to the designated retailers. Besides this TPLP, owned vehicles are able to deliver orders of the rest of the retailers; this is why the model does not elect to rent any other vehicles in the lower level. Retailers 4 and 6 are visited by TPLP vehicles in different time steps. Their first time step orders are satisfied by the second TPLP from bottler 9. The TPLP also picks up the orders for the second and third time steps from bottlers 8 and 9 and keeps them in its warehouse, which is shown in Figure 4-2. In the second time step, the TPLP delivers partial orders to retailers 4 and 6, and the balance comes from commodities stored in the TPLP warehouse. Again, some products are stored in the TPLP warehouse in the second time step for delivery to retailers 4 and 6 by TPLP in the third time step. Table 4-8 presents the load factor of different vehicles used in both levels in Scenario 1.

Table 4-8. Load factor of different vehicles used in Scenario 1

Owned/ Rental	Level	Capacity (Unit)	Time Step 1 Load Factor	Time Step 2 Load Factor	Time Step 3 Load Factor
Owned Vehicles	Upper	590	1.00	0.79	0.75
	Level	635	1.00	1.00	0.00
	Lower	440	0.90	0.98	1.00
	Level	440	0.93	0.98	1.00
Rental Vehicle	Upper Level	620	0.85	1.00	0.00

4.2.2. Scenario 2

This scenario has more retailers and a larger fleet size than Scenario 1. Scenario 2 includes 10 retailers, three owned and three rental vehicles in the upper level, and three owned and three rental trucks in the lower level. The rest of the characteristics, such as number of plants, number of bottlers, number of commodities, and number of candidate TPL contracts, are the same as in Scenario 1. Also, the same normal distributions have been used to generate input data for this scenario. Table 4-9 summarizes input data for Scenario 2.

Scenario 2 was solved by Xpress 7.1. Since there are more facility locations in this scenario—as well as more vehicles in both levels—showing all tours for each time step, with their commodity flow like that was shown in Scenario 1, makes each figure very disordered. Therefore, important outputs of this scenario are reported in Table 4-10.

Table 4-9. Input data for Scenario 2

Criterion	Value
Total Demand of Commodity Type 1 (unit)	3344
Total Demand of Commodity Type 2 (unit)	3262
Production Capacity of Commodity Type 1 (unit/day)	6080
Production Capacity of Commodity Type 2 (unit/day)	6080
Total Owned Fleet Capacity in the Upper Level (unit/day)	1920
Total Owned Fleet Capacity in the Lower Level (unit/day)	1300
Total Rental Fleet Capacity in the Upper Level (unit/day)	1840
Total Rental Fleet Capacity in the Lower Level (unit/day)	1270
Retailers That Can be Served by Contract 1	3, 4, and 10
Retailers That Can be Served by Contract 2	1, 2, 9, and 10

Table 4-10. General output information for the optimal solution of Scenario 2

Criterion	Value
Number of Constraints	19846
Number of Decision Variables	270319
LP Relation Objective Function (\$)	81541.3
Best Bound for Objective Function (\$)	80417.8
Best Solution Objective Function (\$)	80239.4
Gap with the Best Bound (%)	0.22
Running Time (CPU seconds)	19755.3
Unsatisfied Demand for Commodity Type 1 (unit)	0
Unsatisfied Demand for Commodity Type 2 (unit)	0
Commodity Type 1 Stored in Retailers' Storage (unit)	5
Commodity Type 2 Stored in Retailers' Storage (unit)	0
Number of Rental Vehicles Used in the Upper Level	1
Number of Rental Vehicles Used in the Lower Level	1
Number of TPLP Contracts Selected	2
Commodity Type 1 Delivered by TPLP (unit)	1698
Commodity Type 2 Delivered by TPLP (unit)	1594
Commodity Type 1 Stored in TPLP's Warehouse (unit)	413
Commodity Type 2 Stored in TPLP's Warehouse (unit)	536

Scenario 2 has more constraints and decision variables than Scenario 1. Table 4-10 shows that adding 4 retailers and 4 vehicles results in a large increase in the number of constraints and decision variables. It also confirms that the model is very sensitive to the number of facility locations and fleet size.

The solver has not found the optimal solution for this scenario in a reasonable amount of computation time; however, the gap between the best feasible solution and

the best bound is very small. According to the best feasible solution, all retailers receive their orders at the right time, and one retailer holds some products for future demand. Moreover, in addition to the owned fleet, one rental vehicle in each level has been used, although the model has decided to deliver some products by TPLP. According to the best solution, both TPLP contracts have been selected and they deliver almost 50% of all demands. Also, 12% of commodity type 1 demand and 16% of commodity type 2 demand have been stored in a TPLP warehouse. As a result, the model found shipping and warehousing outsourcing more beneficial than renting another vehicle or storing products in retailers' warehouses. Table 4-11 presents the load factor of different vehicles used in both levels. According to Table 4-11, some owned vehicles were not used for delivery because based on some assumptions, they were not able to visit some retailers or their travel time cost was very high in comparison to rental vehicles due to speed factors in Scenario 2.

Table 4-11. Load factor of different vehicles used in Scenario 2

Owned/ Rental	Level	Capacity (Unit)	Time Step 1 Load Factor	Time Step 2 Load Factor	Time Step 3 Load Factor
Owned Vehicles	Upper Level	620	0.96	1.00	1.00
		650	1.00	1.00	0.78
	Lower Level	650	1.00	1.00	0.00
		400	0.68	0.52	0.44
		450	1.00	0.00	0.57
		450	1.00	0.51	0.56
Rental Vehicle	Upper Level	590	1.00	0.99	0.83
	Lower Level	420	0.50	0.93	1.00

4.2.3. Scenario 3

This scenario is the same as Scenario 2, except that the number of retailers has increased to 15. It still has six trucks in the upper level and six trucks in the lower level for deliveries. All input data were generated based on normal distributions described in Section 4.1. Table 4-12 presents information about Scenario 3 input data.

Table 4-12. Input data for Scenario 3

Criterion	Value
Total Demand of Commodity Type 1 (unit)	4873
Total Demand of Commodity Type 2 (unit)	5435
Production Capacity of Commodity Type 1 (unit/day)	5550
Production Capacity of Commodity Type 2 (unit/day)	5550
Total Owned Fleet Capacity in the Upper Level (unit/day)	1640
Total Owned Fleet Capacity in the Lower Level (unit/day)	1250
Total Rental Fleet Capacity in the Upper Level (unit/day)	1560
Total Rental Fleet Capacity in the Lower Level (unit/day)	1180
Retailers That Can be Served by Contract 1	1, 3, 5, & 9
Retailers That Can be Served by Contract 2	3, 7, 9, 11, & 15

Some outputs of the best solution found by Xpress 7.1 are reported in Table 4-13. According to Table 4-13, increasing the number of retailers leads to a large increase in the number of constraints and decision variables. For instance, Scenario 3 has almost 700,000 decision variables, which put this problem in the category of medium size problems. Moreover, the total demand for both commodity types for all retailers is greater than fleet capacity in the lower level. Although the model has the opportunity to satisfy more demand with the TPLP option, it depends on TPLP contract cost and shortage penalty.

Table 4-13. General output information for the best solution of Scenario 3

Criterion	Value
Number of Constraints	39721
Number of Decision Variables	696819
LP Relation Objective Function (\$)	115097
Best Bound for Objective Function (\$)	113557
Best Solution Objective Function (\$)	113127
Gap with the Best Bound (%)	0.38
Running Time (CPU seconds)	50870.9
Unsatisfied Demand for Commodity Type 1 (unit)	462
Unsatisfied Demand for Commodity Type 2 (unit)	928
Commodity Type 1 Stored in Retailers' Storage (unit)	0
Commodity Type 2 Stored in Retailers' Storage (unit)	0
Number of Rental Vehicles Used in the Upper Level	3
Number of Rental Vehicles Used in the Lower Level	3
Number of TPLP Contracts Selected	2
Commodity Type 1 Delivered by TPLP (unit)	2099
Commodity Type 2 Delivered by TPLP (unit)	2068
Commodity Type 1 Stored in TPLP's Warehouse (unit)	0
Commodity Type 2 Stored in TPLP's Warehouse (unit)	0

The solver could not find the optimal solution; however, the gap between the best bound and the best solution is less than 0.5%. On the other hand, the running time was very high, which confirms that the problem is very sensitive to the number of facility locations and fleet size. The running time was more than double that of Scenario 2, but it was still in reasonable range.

According to Table 4-13, there are unsatisfied demands for both commodity types in the best solution. All rental vehicles have been used in the solution, and a large portion of deliveries are made by TPLP trucks. In other words, all owned and rental trucks in the lower level use their full capacity to deliver products to different retailers and the balance is delivered by TPLP. Therefore, the main reason for unsatisfied demand is insufficient capacity in the upper level to bring enough syrup concentrate to bottlers to produce more final products. Looking at the best solution confirms that all owned and rental trucks in the upper level have delivered syrup

concentrate to bottlers as much as their capacity allows, and the model does not have any other options for sending more syrup concentrate. The unsatisfied demand reported in Table 4-13 is cumulative. As mentioned in the Chapter 3, unsatisfied demand is transferred to the next time step. Table 4-14 presents the load factor of different vehicles used in both levels in Scenario 3.

Table 4-14. Load factor of different vehicles used in Scenario 3

Owned/ Rental	Level	Capacity (Unit)	Time Step 1 Load Factor	Time Step 2 Load Factor	Time Step 3 Load Factor
Owned Vehicles	Upper Level	550	1.00	1.00	1.00
		540	1.00	1.00	1.00
		550	1.00	1.00	1.00
	Lower Level	450	1.00	0.82	0.76
		400	0.79	0.96	0.74
		400	0.89	0.50	0.99
Rental Vehicle	Upper Level	450	1.00	1.00	1.00
		600	1.00	1.00	1.00
		510	1.00	1.00	1.00
	Lower Level	350	1.00	1.00	1.00
		450	0.91	0.56	0.47
		380	0.51	0.56	0.00

4.2.4. Scenario 4

In this scenario, the number of facility locations is the same as in Scenario 3; however, one commodity type has been added to the model. Moreover, the bottling company has one more owned truck in each level. As in Scenario 3, the bottling company has an option of three rental vehicles at each level in case its owned fleet is not large enough for deliveries. Table 4-15 presents Scenario 4 input data.

Table 4-15. Input data for Scenario 4

Criterion	Value
Total Demand of Commodity Type 1 (unit)	4593
Total Demand of Commodity Type 2 (unit)	4673
Total Demand of Commodity Type 3 (unit)	4647
Production Capacity of Commodity Type 1 (unit/day)	6080
Production Capacity of Commodity Type 2 (unit/day)	6080
Production Capacity of Commodity Type 3 (unit/day)	6080
Total Owned Fleet Capacity in the Upper Level (unit/day)	2400
Total Owned Fleet Capacity in the Lower Level (unit/day)	1800
Total Rental Fleet Capacity in the Upper Level (unit/day)	1500
Total Rental Fleet Capacity in the Lower Level (unit/day)	1050
Retailers That Can be Served by Contract 1	4,6,7,8,10,12,14
Retailers That Can be Served by Contract 2	3,4,5,12,14

Demand of commodity type 3 follows the normal distribution, with a mean of 102 units and standard deviation equal to 14 units. Also, its price across different retailers has a normal distribution, with a mean and standard deviation equal to 25 and 0.6 units. TPLP contracts cover more retailers in this scenario. Some outputs of the best solution found by Xpress 7.1 for this scenario are reported in Table 4-16. According to Table 4-16, this scenario has more than 1,100,000 decision variables. As a result, the model is also very sensitive to the number of commodity types. For this size problem, Xpress 7.1 was not able to find an optimal solution after 38000 CPU seconds. In total, more than 4300 units of products of different types have not been satisfied due to the lack of vehicle in the upper level. Similar to Scenario 3, all vehicles that are available in the upper level work with their full capacity, and the model does not have enough syrup concentrate to produce enough final products for all demand. In this case, the model has to leave unsatisfied the orders of some retailers, for which the loss is minimal. The interesting point in the solution is that the model has not used one of the rental vehicles in the lower level. Instead, it has found that outsourcing shipping is more beneficial. Since many retailers are covered by

TPLP contracts, this decision seems reasonable. Table 4-17 presents the load factor of different vehicles used in both levels in Scenario 4.

Table 4-16. General output information for the best solution of Scenario 4

Criterion	Value
Number of Constraints	61141
Number of Decision Variables	1189923
LP Relation Objective Function (\$)	144025
Best Bound for Objective Function (\$)	142583
Best Solution Objective Function (\$)	142503
Gap with the Best Bound (%)	0.06
Running Time (CPU seconds)	38038.1
Unsatisfied Demand for Commodity Type 1 (unit)	481
Unsatisfied Demand for Commodity Type 2 (unit)	3425
Unsatisfied Demand for Commodity Type 3 (unit)	476
Commodity Type 1 Stored in Retailers' Storage (unit)	0
Commodity Type 2 Stored in Retailers' Storage (unit)	0
Commodity Type 3 Stored in Retailers' Storage (unit)	0
Number of Rental Vehicles Used in the Upper Level	3
Number of Rental Vehicles Used in the Lower Level	2
Number of TPLP Contracts Selected	2
Commodity Type 1 Delivered by TPLP (unit)	2299
Commodity Type 2 Delivered by TPLP (unit)	1311
Commodity Type 3 Delivered by TPLP (unit)	2293
Commodity Type 1 Stored in TPLP's Warehouse (unit)	0
Commodity Type 2 Stored in TPLP's Warehouse (unit)	0
Commodity Type 3 Stored in TPLP's Warehouse (unit)	0

Table 4-17. Load factor of different vehicles used in Scenario 4

Owned/ Rental	Level	Capacity (Unit)	Time Step 1 Load Factor	Time Step 2 Load Factor	Time Step 3 Load Factor
Owned Vehicles	Upper Level	600	1.00	1.00	1.00
		600	1.00	1.00	1.00
		600	1.00	1.00	1.00
		600	1.00	1.00	1.00
	Lower Level	450	0.62	0.68	0.59
		450	0.70	1.00	0.93
		450	0.48	0.61	0.63
Rental Vehicle	Upper Level	500	1.00	1.00	1.00
		500	1.00	1.00	1.00
		500	1.00	1.00	1.00
	Lower Level	350	0.86	0.82	0.95
		350	0.85	0.79	1.00

4.2.5. Scenario 5

This scenario has several changes from previous scenarios. It covers three plants, four bottlers, and 20 retailers, which means a total of 27 facility locations. Moreover, four owned and four rental trucks can be used in the operation in each level. Finally, three TPLP contracts exist for delivery to some retailers. Table 4-18 shows Scenario 5 input data. According to Table 4-18, production capacity is higher than total demands but fleet size in the upper level is not large enough to transport all required syrup concentrate to bottlers. Unsatisfied demand, therefore, is expected to be seen in the solution. Some outputs of the best solution found by Xpress 7.1 for this scenario are reported in Table 4-19.

Table 4-18. Input data for Scenario 5

Criterion	Value
Total Demand of Commodity Type 1 (unit)	6197
Total Demand of Commodity Type 2 (unit)	6065
Total Demand of Commodity Type 3 (unit)	6161
Production Capacity of Commodity Type 1 (unit/day)	8080
Production Capacity of Commodity Type 2 (unit/day)	8080
Production Capacity of Commodity Type 3 (unit/day)	8080
Total Owned Fleet Capacity in the Upper Level (unit/day)	2460
Total Owned Fleet Capacity in the Lower Level (unit/day)	1875
Total Rental Fleet Capacity in the Upper Level (unit/day)	2460
Total Rental Fleet Capacity in the Lower Level (unit/day)	1845
Retailers That Can be Served by Contract 1	1,2,3,4,7,9,10,14,15,19
Retailers That Can be Served by Contract 2	1,3,11,14,19,20
Retailers That Can be Served by Contract 3	6,8,9,11,12,13,14,18

According to Table 4-19, Scenario 5 includes more than 150,000 constraints and 4,400,000 decision variables. Therefore, this scenario is a large problem. Xpress was not able to find the optimal solution for this scenario; however, the gap between the best solution and the best bound after 18 hours running was less than 0.5%, which is acceptable. As expected, there is unsatisfied demand in the best solution, mostly for

the second commodity type. The reason that the model preferred to produce commodity types 1 and 3 more than type 2 is that commodity type 2 has the lowest price among all products. Since the shortage penalty is the same for different product types, satisfying demand for products 1 and 3 is more profitable for the bottling company.

Table 4-19. General output information for the best solution of Scenario 5

Criterion	Value
Number of Constraints	152049
Number of Decision Variables	4,421,770
LP Relation Objective Function (\$)	183667
Best Bound for Objective Function (\$)	182504
Best Solution Objective Function (\$)	181661
Gap with the Best Bound (%)	0.46
Running Time (CPU seconds)	65167.9
Unsatisfied Demand for Commodity Type 1 (unit)	519
Unsatisfied Demand for Commodity Type 2 (unit)	6331
Unsatisfied Demand for Commodity Type 3 (unit)	470
Commodity Type 1 Stored in Retailers' Storage (unit)	0
Commodity Type 2 Stored in Retailers' Storage (unit)	0
Commodity Type 3 Stored in Retailers' Storage (unit)	0
Number of Rental Vehicles Used in the Upper Level	4
Number of Rental Vehicles Used in the Lower Level	0
Number of TPLP Contracts Selected	3
Commodity Type 1 Delivered by TPLP (unit)	5019
Commodity Type 2 Delivered by TPLP (unit)	2259
Commodity Type 3 Delivered by TPLP (unit)	4991
Commodity Type 1 Stored in TPLP's Warehouse (unit)	0
Commodity Type 2 Stored in TPLP's Warehouse (unit)	0
Commodity Type 3 Stored in TPLP's Warehouse (unit)	0

Another interesting point in this scenario is that the model has not used any rental vehicles in the lower level. Owned trucks and the TPLP fleet operate all deliveries. The reason is that TPLP contracts have cost functions with two steps, and the second step has a very high upper bound. For instance, the first TPLP ships more than 700 units and less than 3000 units with a flat rate. As a result, the model sends

full owned trucks, and the rest of the deliveries fall within the lower and upper bounds of the second step of the TPLP cost function. The model, therefore, does not need to rent any vehicles and prefers to use the maximum capacity of TPLP vehicles for deliveries. If the upper bound of the second step of the contract was significantly less than 3000 units, the model might use some rental vehicles in the lower level instead of outsourcing deliveries. Table 4-20 presents the load factor of different vehicles used in both levels in Scenario 5.

Table 4-20. Load factor of different vehicles used in Scenario 5

Owned/ Rental	Level	Capacity (Unit)	Time Step 1 Load Factor	Time Step 2 Load Factor	Time Step 3 Load Factor
Owned Vehicles	Upper Level	620	1.00	1.00	1.00
		650	1.00	1.00	1.00
		590	1.00	1.00	1.00
		600	1.00	1.00	1.00
	Lower Level	475	0.44	0.45	0.43
		450	0.71	0.44	0.50
		475	0.99	0.00	0.91
		475	0.00	0.00	0.45
Rental Vehicle	Upper Level	610	1.00	1.00	1.00
		590	1.00	1.00	1.00
		650	1.00	1.00	1.00
		610	1.00	1.00	1.00

4.2.6. Scenario 6

This scenario has a longer planning horizon compared to Scenario 5. In this scenario planning for five days is a goal and other characteristics of the model, such as number of facility locations, number of commodity types, fleet size, and number of TPLP contracts, are the same as Scenario 5. Table 4-21 displays input data for Scenario 6. Xpress 7.1 was not able to find a feasible solution for this scenario after

15 hours, which means that Scenario 6 is larger than Xpress’s capabilities. According to Table 4-21, Scenario 6 includes more than 8,800,000 decision variables and 300,000 constraints, which show that this scenario is a large problem.

Table 4-21. Input data for Scenario 6

Criterion	Value
Total Demand of Commodity Type 1 (unit)	10397
Total Demand of Commodity Type 2 (unit)	10245
Total Demand of Commodity Type 3 (unit)	10236
Production Capacity of Commodity Type 1 (unit/day)	8080
Production Capacity of Commodity Type 2 (unit/day)	8080
Production Capacity of Commodity Type 3 (unit/day)	8080
Total Owned Fleet Capacity in the Upper Level (unit/day)	2485
Total Owned Fleet Capacity in the Lower Level (unit/day)	1840
Total Rental Fleet Capacity in the Upper Level (unit/day)	2400
Total Rental Fleet Capacity in the Lower Level (unit/day)	1840
Retailers That Can be Served by Contract 1	4,7,8,15
Retailers That Can be Served by Contract 2	5,6,13
Retailers That Can be Served by Contract 3	4,7,10,11,12
Number of Constraints	300451
Number of Decision Variables	8,892,749

4.3. Summary

Six sample problems in this chapter were designed to test the proposed model and evaluate its capabilities. Problems varied from small to large. Figures 4-5 and 4-6 illustrate the difference in size of these sample problems in terms of number of constraints and decision variables.

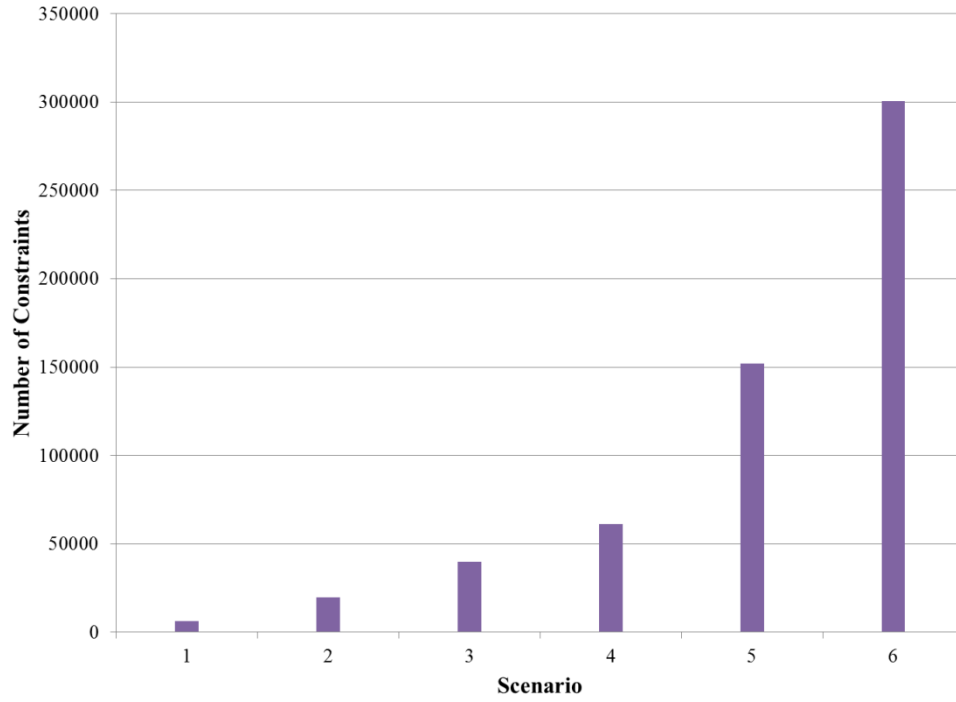


Figure 4-5 Number of constraints in different scenarios

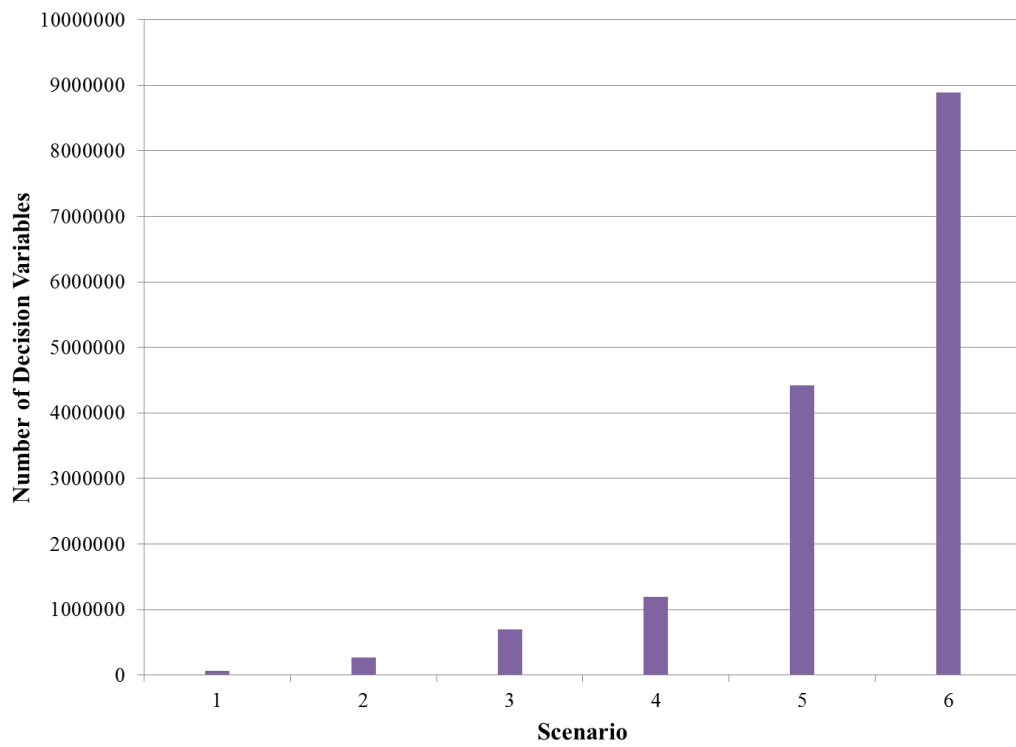


Figure 4-6 Number of decision variables in different scenarios

According to Figures 4-5 and 4-6, the model is very sensitive to some input data, such as number of facility locations, fleet size, and number of commodity types. An increase in any of these parameters increases the problem size significantly. The running time, therefore, also increases significantly. Figure 4-7 presents running time and the gap between the best solution and the best bound found by Xpress for each scenario.

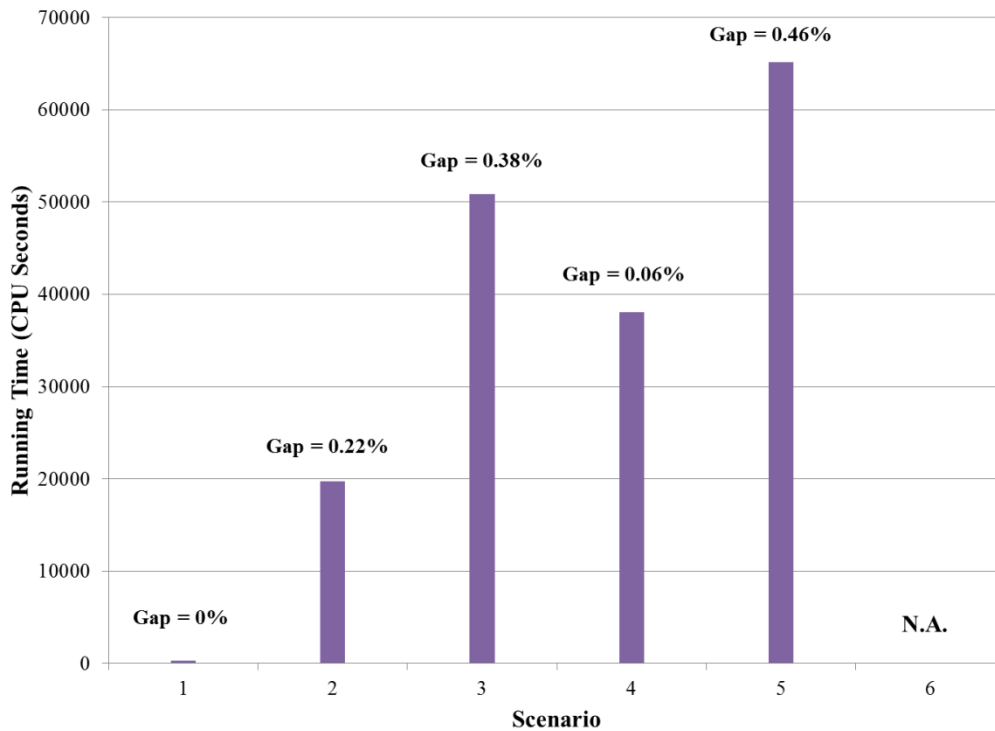


Figure 4-7 Running time and gap of the best solution with the best bound in different scenarios

Figure 4-7 shows that Xpress could solve only the first scenario, which is a small problem; for the rest of scenarios, the optimal solution was not found in a reasonable amount of computation time. However, the gap between the best solution and the best bound was less than 0.5%, meaning that the solution was very close to the optimal one. Finally, Scenario 6, which is the largest scenario, could not be solved

by Xpress; the software was not able to provide even a feasible solution for this scenario.

The results demonstrate that the model is able to take the different details of an operation into account, but medium and large size problems are not solved optimally in a reasonable amount of computational time. As mentioned earlier, IRP belongs to the NP-hard class of problems, for which running time increases exponentially when size increases. Even if the optimal solution is not a target, finding a feasible solution with a small gap from the best bound takes a long time for medium size problems with known solver packages. In addition, these known solvers are not able to deliver a feasible solution for large size or real-world problems. Therefore, developing a heuristic method that solves these problems optimally—or finds a good feasible solution with an appropriate gap in a short time—is essential.

Chapter 5: The First Heuristic Approach

In this chapter, the first heuristic algorithm proposed in this study to solve large-sized problems is described. As mentioned in Chapter 4, the mixed-integer model proposed in Chapter 3 was not solved by Xpress 7.1 for large-scale problems. The heuristic method, therefore, is needed to find a good feasible solution in a reasonable running time. Section 5.1 explains the Branch-and-bound algorithm used by Xpress to solve integer models. Section 5.2 describes the Fix-and-run algorithm in general, followed by Section 5.3, which describes the main challenges in applying the algorithm in this study. In Section 5.4, different steps of the proposed Fix-and-run algorithm are described in detail.

Section 5.5 introduces three improving phases applied to the final solution of the algorithm. In Section 5.6, more numerical analyses are performed to evaluate the robustness of this heuristic algorithm. Finally, Section 5.7 summarizes the efficiency of the first heuristic algorithm.

5.1. Branch-and-Bound Algorithm

The mathematical formulation proposed in Chapter 3 is a mixed-integer program, which means that continuous and integer decision variables exist in the model. All decision variables used in the model can be categorized into four groups:

- 1- The vehicle flow decision variables group, which includes owned and rental routing binary variables (XU and XW), fixed, and operational cost binary decision variables of rental trucks (VZu , VZw , Uu , and Uw).
- 2- The commodity flow decision variables group, containing flow of different commodity types in links (Qu , Qw , and TW), inventory level at different facility locations (I), shortage of each commodity type at retailer (e), stored products in TPLP warehouse (Z), modified demand (DM), and delivery amount to each retailer (DE). These decision variables are inherently integer; however, due to the integrity of retailers' orders, they become integer in final output even if they are defined as continuous decision variables.
- 3- The production decision variables group, which includes stored syrup concentrate (Sc) and production quantity (Pd). This group has the same character as the second group and can be defined as continuous decision variables; however, in the output they will be integer.
- 4- The third party contract decision variables group, which consists of the contract binary variable (w) and retailer visiting binary decision variable (TN).

Groups one and four make the model very complicated. The rest of the decision variables affect the size of the model without an increase in complexity of the model. The main focus of the heuristic algorithm, therefore, must be on binary decision variables. Xpress applies a modified branch-and-bound algorithm to solve these problems. In this method, all integer decision variables are relaxed and the

model is solved optimally with relaxed variables. In each iteration, a decision about one integer decision variable with decimal value in the optimal solution is made. According to this decision, the search domain is divided into two areas. In one of these two domains, the closest smaller integer number is assigned to that particular decision variable and, in the other, the closest larger number. The relaxed model, with a new value for the decision variable, is solved again and the same procedure applied to other integer decision variables if they have decimal values. The solution of relaxed models provides upper or lower bounds for the problem, depending on objective function. If a feasible solution is found by the algorithm that has integer values for all integer decision variables, the gap is calculated between its objective function and the best bound found by the model. The algorithm ends if decisions for all variables are made or a gap between an integer solution and a bound is ignorable.

The branch-and-bound algorithm, which is the most commonly used IP solution algorithm, is implemented in many commercial solvers. The main disadvantage of this method is its running time. In other words, its use can be expensive depending on the size of the problem and complexity of integer decision variables. The algorithm produces several intermediate and not optimal solutions; however, they are used to make the bound tighter. Most of the time, the branch-and-bound algorithm comes up with near optimal solutions quickly but it takes a long time to verify its optimality.

5.2. Fix-and-run Algorithm

The fix-and-run algorithm suggested by (Haghani & Oh, 1996) is a heuristic algorithm having a good performance for the vehicle routing problem (VRP). This method focuses mainly on relaxing integer decision variables; however, it makes decisions about them faster than the branch-and-bound. The main steps of the fix-and-run algorithm can be described as follows:

- 1- All integer decision variables are relaxed and the model is solved optimally.
- 2- The values of some integer variables are fixed in an orderly manner. The rest of the integer variables are relaxed and the model is solved again.
- 3- Step 2 continues iteratively until all integer variables are fixed.

The fix-and-run algorithm is similar to the branch-and-bound algorithm in terms of relaxing integer decision variables and making a decision based on the optimal solution found for the relaxed model. However, this algorithm is much faster than branch-and-bound because it fixes some integer variables at each iteration, while branch-and-bound separates the feasibility domain for one decision variable at each iteration. Moreover, fix-and-run ends after a limited number of iteration; however, the branch-and-bound needs to continue until either all branches terminate or a predetermined gap is found.

As a result, fix-and-run is an appropriate heuristic for models with a large number of integer decision variables, especially models that include VRP as one of the main parts of the problem. The study problem meets both criteria; however, it

has its own challenges in using fix-and-run as a heuristic method, which will be explained in the following sections.

5.3. Applying a Fix-and-Run Algorithm to the Proposed Model

The fix-and-run algorithm described in the previous section will be applied to the study problem as follows:

- 1- Divide the problem into several segments. Each segment includes one time step of the whole model.
- 2- Relax all integer decisions and solve the relaxed model optimally.
- 3- Fix some of the integer variables of the first time step and rerun the whole model while the rest of the integer variables of the first time step and the integer variables of other time steps are relaxed.
- 4- If all integer variables of the first time step have integer values, do the same process for the second time step; otherwise, fix some other integer variables and continue until all integer variables have integer values.
- 5- Do the same procedure for the next time step until all integer variables in all time steps are fixed.

The described algorithm presents some challenges when it is applied to the model. First, it needs Xpress as a solver of relaxed models. The algorithm, therefore, needs to communicate with Xpress to get the solution and give the new values for some variables. The code, which controls this communication and adds some new constraints continuously, must consider different details to be efficient; otherwise, a significant amount of running time is wasted for communication. Second, the model

has different groups of integer variables, such as vehicle routing, rental-vehicle cost, TPLP contract, and TPLP warehousing. Fixing some of these decision variables affects the others. As a result, the order of decision variables fixed in each time step is another challenge, which may impact the final solution. The different steps of the first fix-and-run algorithm proposed for this research are described in detail in Section 5.4.

5.4. Fix-and-Run Algorithm Steps (Heuristic 1)

As mentioned in the previous section, the order of fixing integer variables impacts the final solution. Therefore, it is important to determine which decision variable should be fixed first and which variable will be the next. In this section, the first fix-and-run algorithm developed for this problem will be explained step by step.

5.4.1. Step 1: Removing Unnecessary TPL Contracts

In the first step, all unnecessary TPL contracts are excluded from the solution. As described in previous sections, all integer variables are relaxed at the beginning and the relaxed model is solved optimally. The first decision variable for which a decision is made is the TPL contract binary variable. According to the mathematical formulation described in Chapter 3, the TPL contract binary variable (w) must satisfy the following constraint:

$$\sum_{m \in W} \sum_{n \in R} \sum_{p \in T} TW_{stpq}^{mn} - TLB_{sqi} \leq M w_{qist} \quad \forall q \in TP, \forall i \in L, \forall s \in S, \forall t \in T \quad (5-1)$$

According to equation 5-1, the left-hand side can be at least one to force w to take a non-zero value; otherwise, w is set to zero due to its impact on increasing total cost. Therefore, if w is forced to take a non-zero value, it must be greater than $1/M$. In a relaxed model, the solver is free to choose a very small value for TW . In this case, it can be a very small number that does not increase the total cost significantly. In the first step of the algorithm, therefore, all w decision variables with a value smaller than $1/M$ are fixed to zero. This procedure is applied to all time steps. In other words, all unnecessary TPL contract variables for every time step are excluded from the final solution. This assignment is fed to the model by adding some constraints to the model. Each constraint is defined for one w binary variable and assigns zero to that variable. These constraints will remain until the end of algorithm, and the algorithm is not able to change any of these decisions. Figure 5-1 shows a sample output of solving a relaxed model by Xpress with M equal to 1000. According to this figure, retailers 1, 2, and 4 are visited by TPLP; however, after applying the first step of heuristic 1, the TPL contract binary variables (w) associated with retailers 1 and 4 are set to zero because their values are less than 0.001, while no decision is made about the other contract variable.

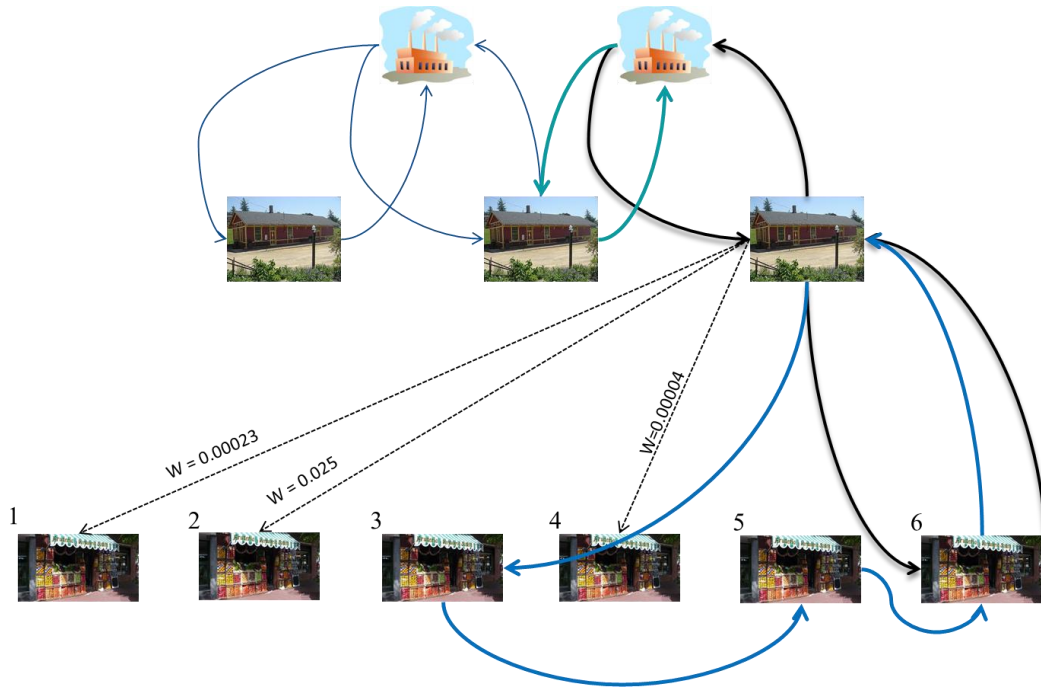


Figure 5-1 A sample of Xpress output for the relaxed model

5.4.2. Step 2: Fixing Selected TPL Contracts of the First Step

After fixing some w variables to zero in Step 1, the relaxed model with new constraints is solved using Xpress. In this step, a decision about TPL contract binary variable (w) for the first time step is made. The w variables of the first time step that exist in the optimal solution found by Xpress are considered. The w that has the maximum non-integer value is selected and is fixed to one. Therefore, one constraint is added to the relaxed model which assigns one to that particular w variable. The new relaxed model is solved again by Xpress optimally and the new optimal solution is analyzed for another non-integer w variable. If all w variables of the first time step are integers, Step 2 is finished; otherwise, another w variable is selected randomly and fixed to one and the related constraint is added to the relaxed model. This process continues until all w variables of the first time step have the value of either zero or

one. For instance, the delivery plan shown in Figure 5-2 can be the output of Xpress after the first step of heuristic 1 has been applied. The TPL contract binary variable for retailers 1 and 4 is equal to zero, and for retailer 2 has decimal value greater than a threshold. The algorithm, therefore, assigns retailer 2 to a TPLP and adds another constraint to the relaxed model, which sets the associated binary variable to one.

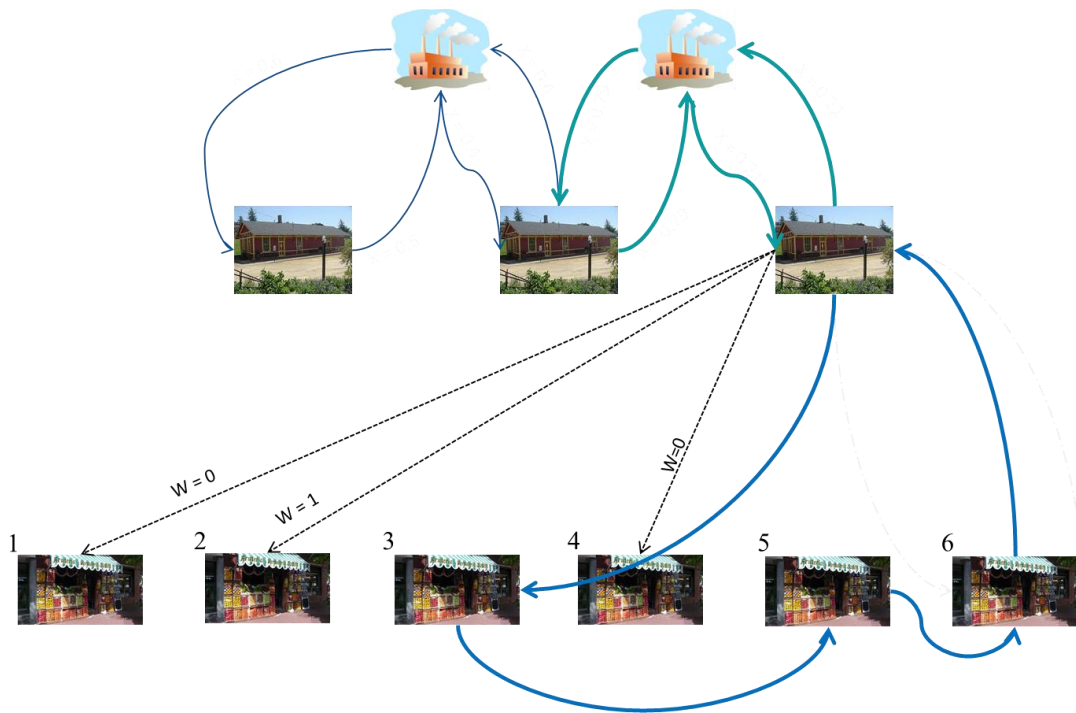


Figure 5-2 A sample of Xpress output after the first step of heuristic 1

5.4.3. Step 3: Fixing Retailers visited by TPLP

The heuristic algorithm has fixed TPLP contract binary variables in Step 1 and 2 for the first time step. As mentioned in Section 5.1, the retailer visiting binary decision variable (TN) is another integer variable whose values can be determined in this step. Since all w variables are fixed to either zero or one, the retailers visited by TPLP are

known. Moreover, it is clear which selected TPL contract serves each retailer. As a result, all information needed to fix TN is known after Step 2, and the algorithm fixes these binary variables in Step 3.

5.4.4. Step 4: Fixing Tours of Owned Vehicles in the Upper Level

In the first stage of Step 4, all rental vehicles of the upper level are excluded from the fleet and the model is solved by Xpress with this reduced fleet size in the upper level. Using rental vehicles for deliveries increases the total cost, which the model may cancel out by assigning very small numbers to their routing binary variable. This strategy, however, forces the model to use its owned fleet for all deliveries first. The delivery plan shown in Figure 5-3 can be an output after excluding rental trucks from the upper fleet. Since the focus of this step is on the the upper level, details of the lower level have not been included in the figure.

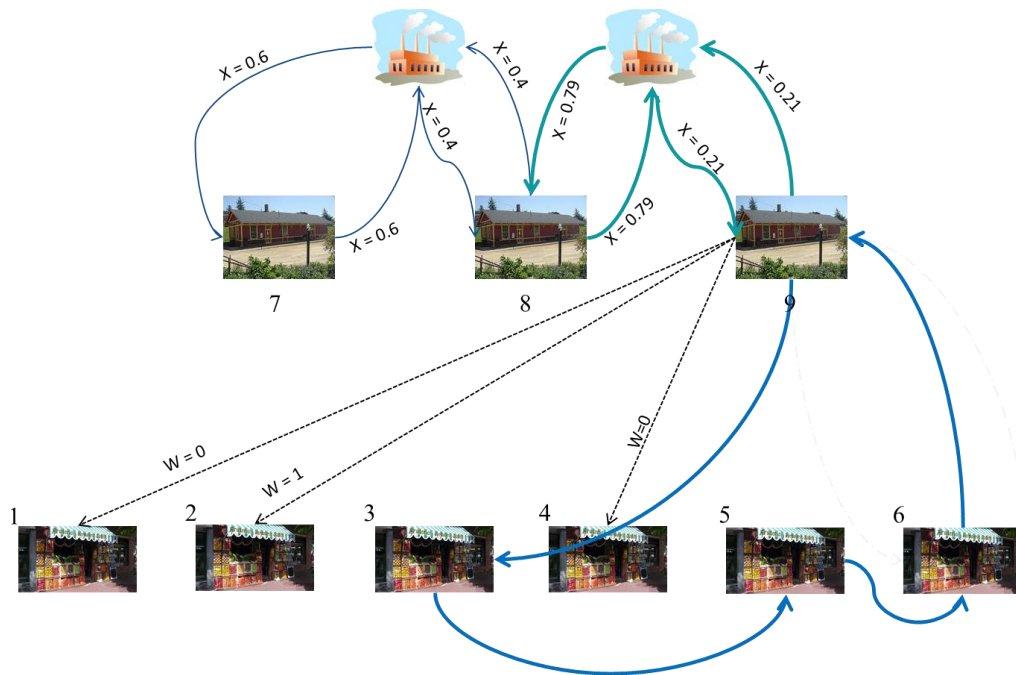


Figure 5-3 A sample of Xpress output after the third step of heuristic 1

According to Figure 5-3, the model has used two owned vehicles for transportation of syrup concentrate in the upper level. Each vehicle visits two bottlers through two separate tours. Moreover, the routing binary variable shown in the figure has a decimal value over all used links in the upper level. Therefore, the current solution is not feasible for the original model from two standpoints: First, the binary variable has non-integer value, and second, each truck does more than one trip per time step, which is inconsistent with the 7th assumption in Section 3.3.

In the next stage of Step 3, for every vehicle available in the solution of the upper level, one link among all the links used by different tours of that particular vehicle is chosen randomly and fixed to one. For instance, in the solution shown in Figure 5-3, which has used two vehicles in four tours, the routing binary variable of two links are fixed to one, as shown in Figure 5-4.

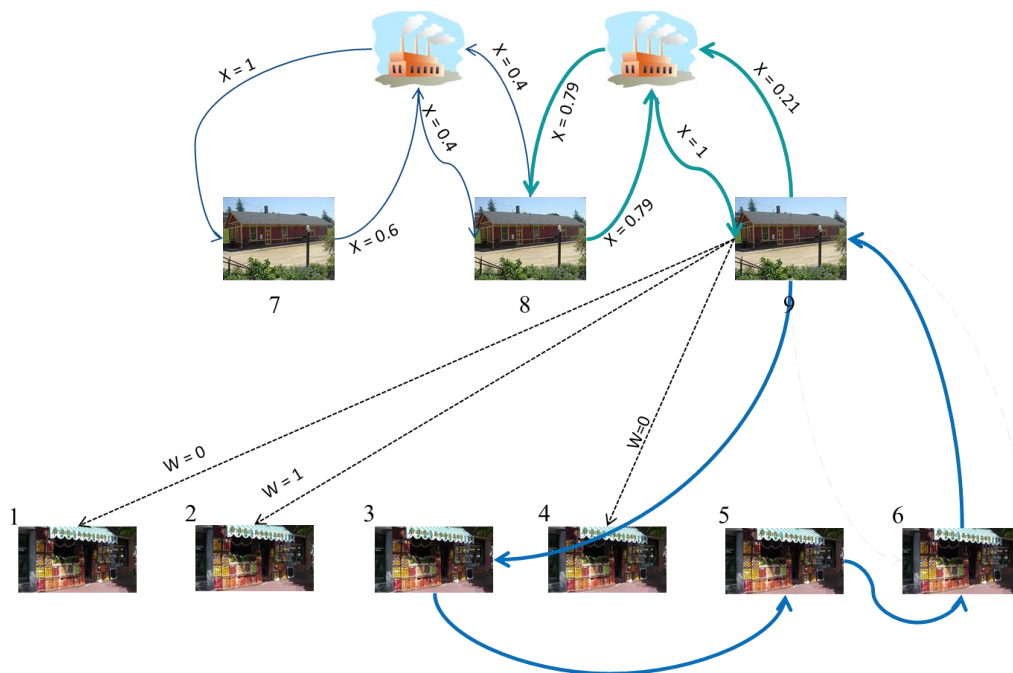


Figure 5-4 Fixing one link per owned vehicle in the upper level

As a result, some constraints are added to the model. The number of added constraint is equal to the number of used owned vehicles in the upper level. In each constraint, one link is fixed to one for one particular truck. The new relaxed model is solved again with Xpress and the solution is analyzed to see whether it has any other non-integer value for a routing binary variable in the upper level. In the new solution, more owned vehicles than in the previous solution may be used—or the same vehicles used—but the fixed links are definitely in the solution, and the number of tours may be decreased. For example, the decision made in Figure 5-4 means that the vehicle visiting bottlers 8 and 9 cannot have two tours anymore, because constraint 3-1 forces the summation of the routing variables of a vehicle leaving a plant to be less than or equal to one, and this vehicle has already a link with a value of 1 for its routing variable. Figure 5-5 presents a potential solution found by Xpress.

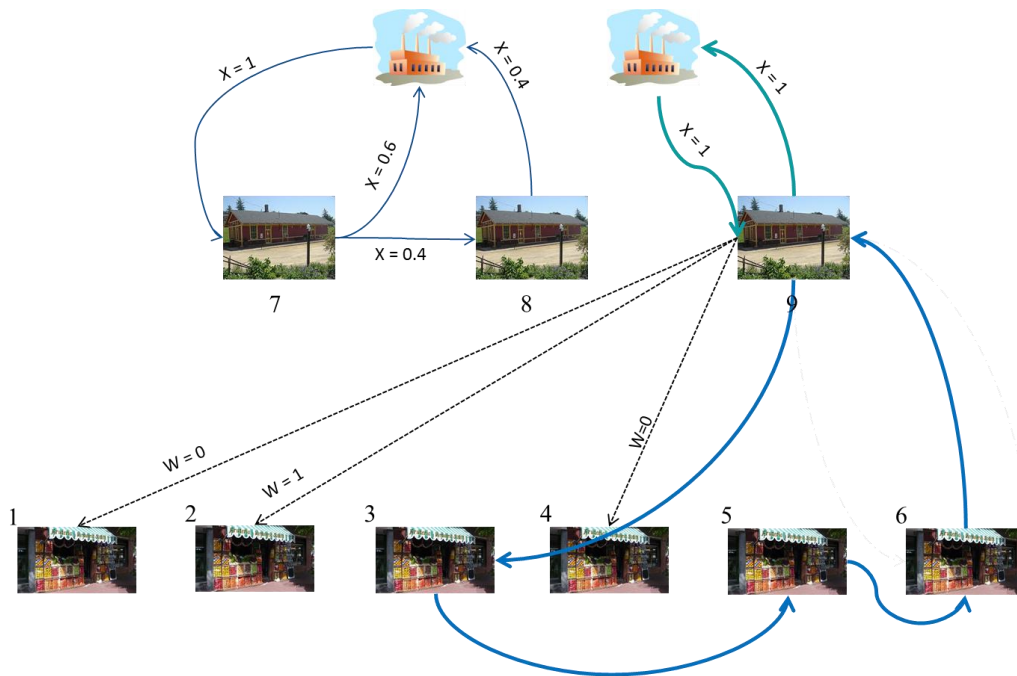


Figure 5-5 A sample solution of Xpress after fixing one link per owned vehicle in the upper level

According to Figure 5-5, one vehicle has a complete tour with an integer value for the routing binary variable; however, this step of the algorithm needs to be repeated until the model finds a complete and feasible tour for the other vehicle. If, during this stage, another unused owned vehicle is added to the fleet, the algorithm fixes links for that vehicle as well. The final output of Step 3 includes feasible tours of owned vehicles in the upper level.

5.4.5. Step 5: Fixing Tours of Owned Vehicles in the Lower Level

This step is identical to Step 4, but is applied to owned vehicles in the lower level. Rental vehicles, therefore, are excluded from the fleet first. The algorithm then fixes one link for every owned vehicle used in each iteration, and Xpress solves the new model with added constraints. The final output of this step is a feasible solution, with tours in the upper and lower levels operated by owned vehicles.

5.4.6. Step 6: Using other vehicles according to shortage and inventory level

In Step 6, another vehicle is added to the fleet if the model needs it. There are two parameters needed to make a decision as to whether adding another vehicle improves the profit or not. Shortage is the first parameter, which is defined as a summation of shortages of all products over all retailers in the first time step. The second parameter is inventory level, which is equal to summation of inventory levels of all commodity types over all bottlers in the first time step. According to shortage amounts and inventory levels, different scenarios happen as follows:

1- Shortage > 0 , Inventory level > 0 , and Shortage $<$ Inventory level

In this case, enough products have been produced by bottlers; however, the fleet of the lower level is not large enough to satisfy some orders. As a result, one vehicle must be added to the fleet of the lower level.

2- Shortage > 0 and Inventory level = 0

In this scenario, bottlers do not have enough syrup concentrate to produce more commodity. Consequently, some retailers' orders are not satisfied completely. In this case, a vehicle must be added to the fleet of the upper level.

3- Shortage > 0 , Inventory level > 0 , and Shortage $>$ Inventory level

In this case, not only the fleet size of the lower level is small for deliveries, but more syrup concentrate is needed to produce more final product at bottlers. Therefore, if a vehicle can be added to the fleet of the upper level, more syrup concentrate can be brought to bottlers; otherwise, adding another vehicle to the fleet of the lower level fills some unsatisfied demands.

4- If a vehicle is added to the upper level and the quantity of shortage is still positive and has not changed after adding that vehicle, the fleet in the lower level needs another vehicle to deliver products. This scenario happens after any of previous cases occur and the associated decisions have been made.

In any of these scenarios, the algorithm gives a priority to owned vehicles. In other words, if the algorithm needs to add a vehicle and if an unused owned vehicle is available, that vehicle is added. Otherwise, the algorithm adds the rental vehicle with the minimum fixed and operational costs. This policy is applied for both levels.

After a vehicle is added to the fleet, the relaxed model is executed again with the new fleet size. The solution may have infeasible tour(s) for the new vehicle. Then, Step 4 or 5 is executed repeatedly, depending on the level to which the vehicle is added, until the tour of that added vehicle becomes feasible. Step 6 is performed on the solution again and adds another vehicle to one of the levels if any of cases mentioned above become true. Again, Step 4 or 5 is executed. The loop is terminated when either all orders are satisfied or the algorithm uses all owned and rental trucks.

5.4.7. Step 7: Checking the Cost Binary Variables of Rental Vehicles

In Step 7, if a rental vehicle is used in the upper or lower level, the algorithm fixes its fixed and operational costs' binary variables. These variables are relaxed in the Xpress model, and since they increase the cost—which then decreases the company's profit—the model assigns a small number to them if it cannot take them as zero. Therefore, the algorithm fixes any costs' binary variables of rental vehicles to one if they are not zero in the solution found by Xpress.

5.4.8. Step 8: Executing the Same Procedures for the Next Time Step

After Step 7, the algorithm has found the feasible solution for the first time step. The algorithm considers the effects of future time steps on the current one and makes a decision based on those effects. The algorithm then runs the same procedures for the second time step while all variables of the first time step have been fixed. As a result, the running time of each step is decreased. The algorithm still takes the impact of

future time steps on the second one into account. After all steps are finished for the second time step, the solution is complete for the first two time steps and the algorithm continues running the procedures for the third time step. The algorithm keeps running until it finds the solution for the last time step. The entire algorithm of heuristic 1 can be described as follows:

Step 1: Set *Counter* = 0 and $t=1$. Relax all integer decision variables and solve the model optimally.

Step 2: Find all w decision variables in all time steps, which are less than $1/M$.

Step 3: Add a constraint for each w decision variable found in previous step to fix that variable to zero.

Step 4: Do the rest of algorithm for time step t .

Step 5: Find all w decision variable(s) greater than $1/M$ and less than one.

Step 6: If only one w is found, add a constraint to the model and fix that w to one. Run the new relaxed model and go to Step 5.

Step 7: If more than one w is found, find the w which has the maximum value. Add a constraint to the model to fix this w to one. Run the new relaxed model and go to Step 5.

Step 8: For each pair of a retailer and a TPL contract, check the associated TN decision variable (TN_i) to see if it is integer or not. If it is not integer go to Step 9, otherwise go to Step 12.

Step 9: If the set of *Fix_TN* includes another TN decision variable for the same retailer but with different TPL contract, add TN_i to *Exclude_TN*; otherwise add TN_i to *Fix_TN*.

Step 10: Add one constraint for each TN variable in Fix_TN and fix it to one.

Step 11: Add one constraint for every TN decision variable in $Exclude_TN$ and fix it to zero.

Step 12: Go to Step 8 and do the same process for another pair of a retailer and a TPL contract.

Step 13: If $Added_Rental_Upper$ set is empty and $Counter = 0$, exclude all rental vehicles from the fleet of the upper level; otherwise, if $Added_Rental_Upper$ set is not empty, add every vehicle k in $Added_Rental_Upper$ set to the fleet of the upper level.

Step 14: Solve the new relaxed model optimally.

Step 15: Find all non-integer XU in the optimal solution of the relaxed model.

Step 16: Find vehicles used in the optimal solution of the relaxed model.

Step 17: For every vehicle used, find one link used by that vehicle and has a non-integer XU . Add a constraint to the model to fix XU of the selected link to one.

Step 18: Solve the new relaxed model optimally.

Step 19: If there is any non-integer XU in the optimal solution, go to Step 15.

Step 20: If $Added_Rental_Lower$ set is empty and $Counter = 0$, exclude all rental vehicles from the fleet of the lower level; otherwise, if $Added_Rental_Lower$ set is not empty, add every vehicle k in $Added_Rental_Lower$ set to the fleet of the lower level.

Step 21: Solve the new relaxed model optimally.

Step 22: Find all non-integer XW in the optimal solution of the relaxed model.

Step 23: Find vehicles used in the optimal solution of the relaxed model.

Step 24: For every vehicle used, find one link used by that vehicle and has a non-integer XW . Add a constraint to the model to fix XW of the selected link to one.

Step 25: Solve the new relaxed model optimally.

Step 26: If there is any non-integer XW in the optimal solution, go to Step 22.

Step 27: Add one unit to *Counter*

Step 28: Calculate the *shortage(Counter)* in retailers and *inventory level(Counter)* at bottlers for every commodity type.

Step 29: If *Counter* is not equal to 1 and *shortage(Counter)*=*shortage(Counter-1)* and *shortage(Counter)*>0 go to Step 30; otherwise go to Step 31.

Step 30: If there is unused rental vehicles in the fleet of the lower level, find the rental vehicle with the minimum fixed and operational costs and add it to the *Added_Rental_Lower* set. Then, go to Step 20.

Step 31: If *shortage(Counter)*> 0 and *inventory level(Counter)*> 0 and *inventory level(Counter)*> *shortage(Counter)* go to Step 32; otherwise go to Step 33.

Step 32: If there is unused rental vehicles in the fleet of the lower level, find the rental vehicle with the minimum fixed and operational costs and add it to the *Added_Rental_Lower* set. Then, go to Step 20.

Step 33: If (*shortage(Counter)*> 0 and *inventory level(Counter)*= 0) or (*inventory level(Counter)*>0 and *shortage(Counter)*>0 and *shortage(Counter)*> *inventory level(Counter)*) go to Step 34; otherwise go to Step 36.

Step 34: If there is unused rental vehicles in the fleet of the upper level, find the rental vehicle with the minimum fixed and operational costs and add it to the *Added_Rental_Upper* set. Then, go to Step 13. Otherwise, go to Step 35.

Step 35: If there is unused rental vehicles in the fleet of the lower level, find the rental vehicle with the minimum fixed and operational costs and add it to the *Added_Rental_Lower* set. Then, go to Step 20.

Step 36: For every used rental vehicles in the upper level, add constraints to fix its variables of fixed and operational costs to one.

Step 37: For every used rental vehicles in the lower level, add constraints to fix its variables of fixed and operational costs to one. Run the new relaxed model.

Step 38: If a vehicle has not been used by the model in the upper level, add it to *Exclude_Upper* set. Add a constraint to the model for each vehicle in *Exclude_Upper* set to fix its XU over all links of the upper level to zero.

Step 39: If a vehicle has not been used by the model in the lower level, add it to *Exclude_Lower* set. Add a constraint to the model for each vehicle in *Exclude_Lower* set to fix its XW over all links of the lower level to zero.

Step 40: Solve the new relaxed model optimally.

Step 41: If the algorithm is in the last time step, it terminates; otherwise, set $t=t+1$ and go to the Step 4.

Figure 5-6 shows the flow chart of the proposed heuristic for the first time step.

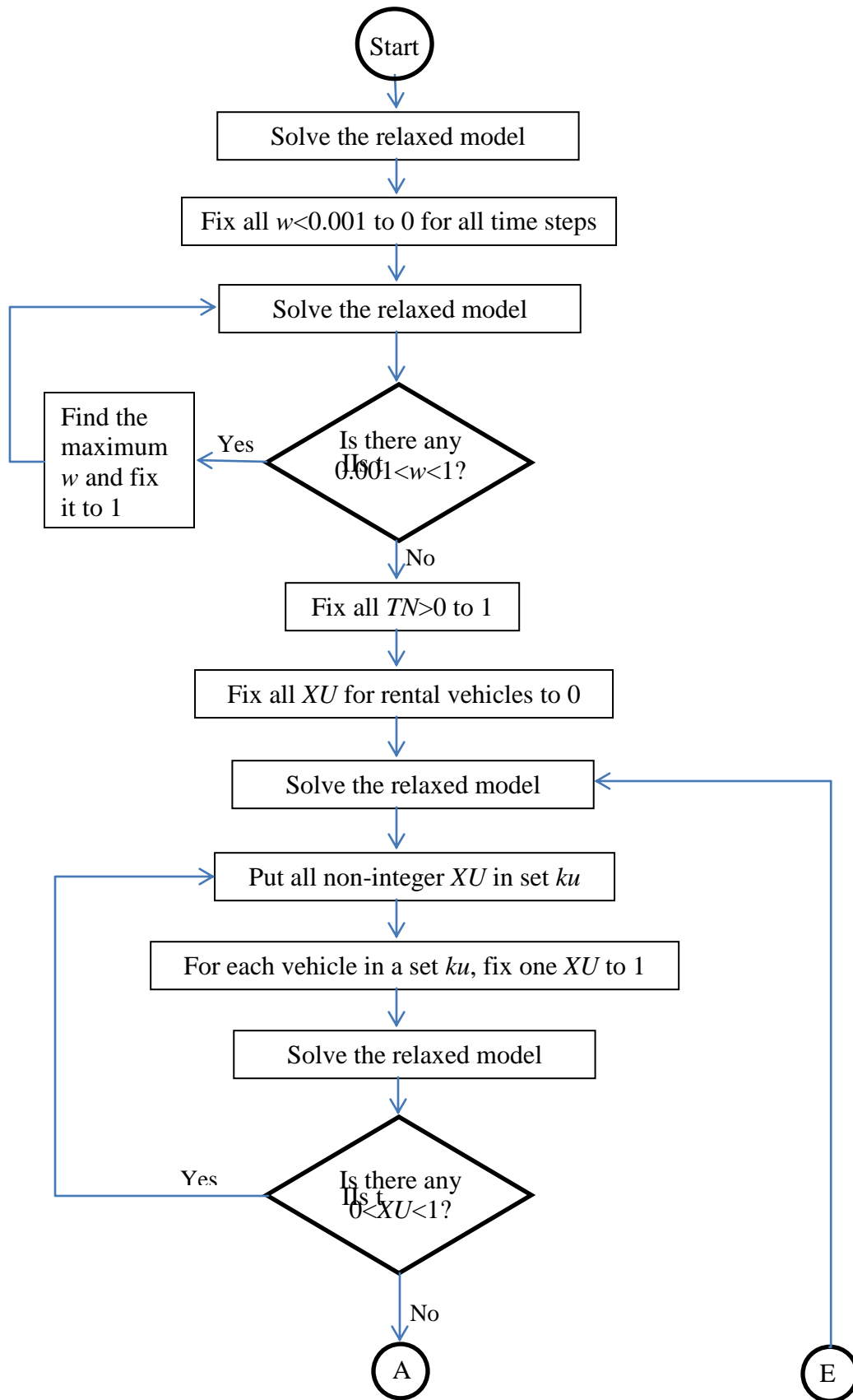


Figure 5-6 The flow chart of heuristic 1 in the first time step

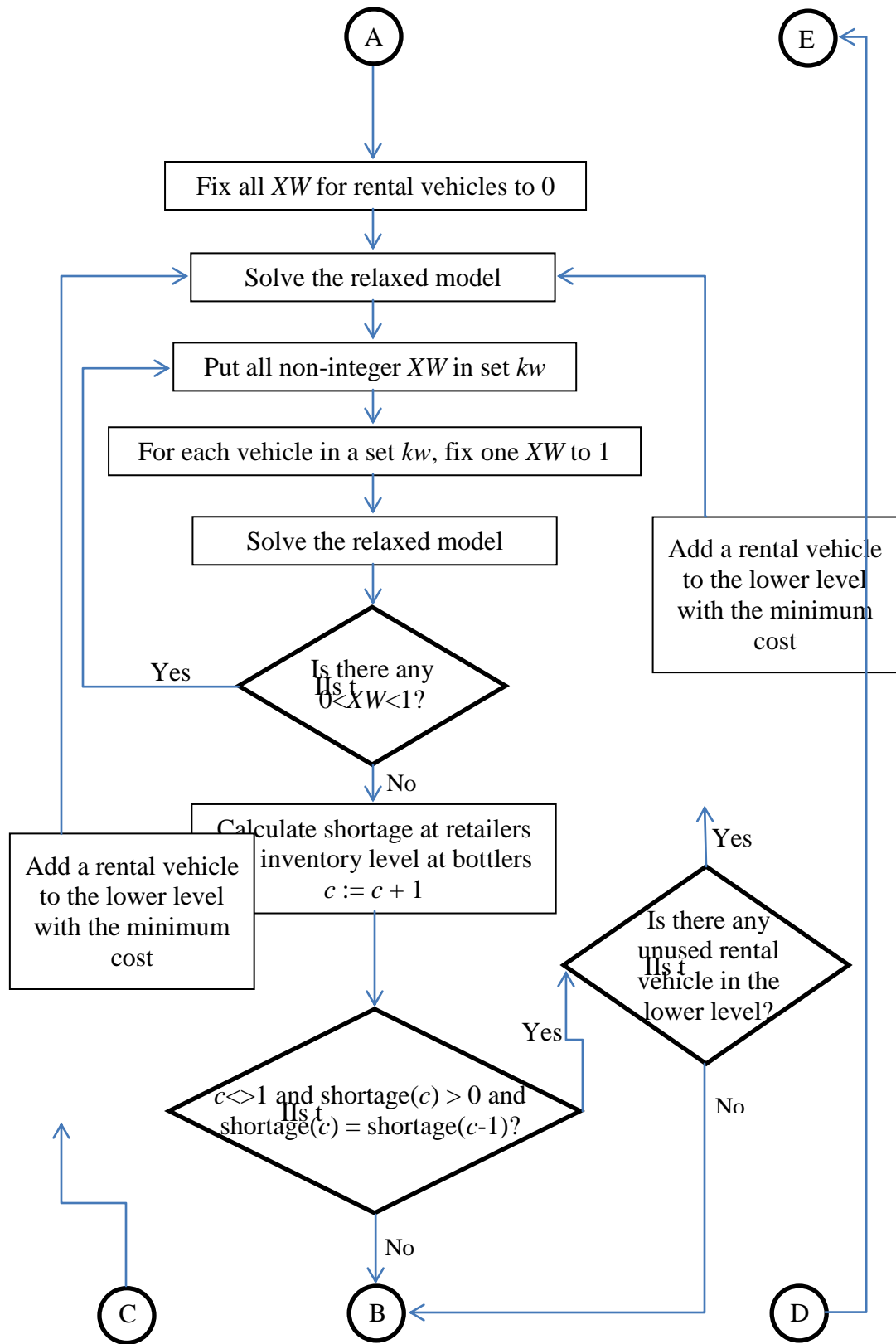


Figure 5-6 The flow chart of heuristic 1 in the first time step (Cont'd)

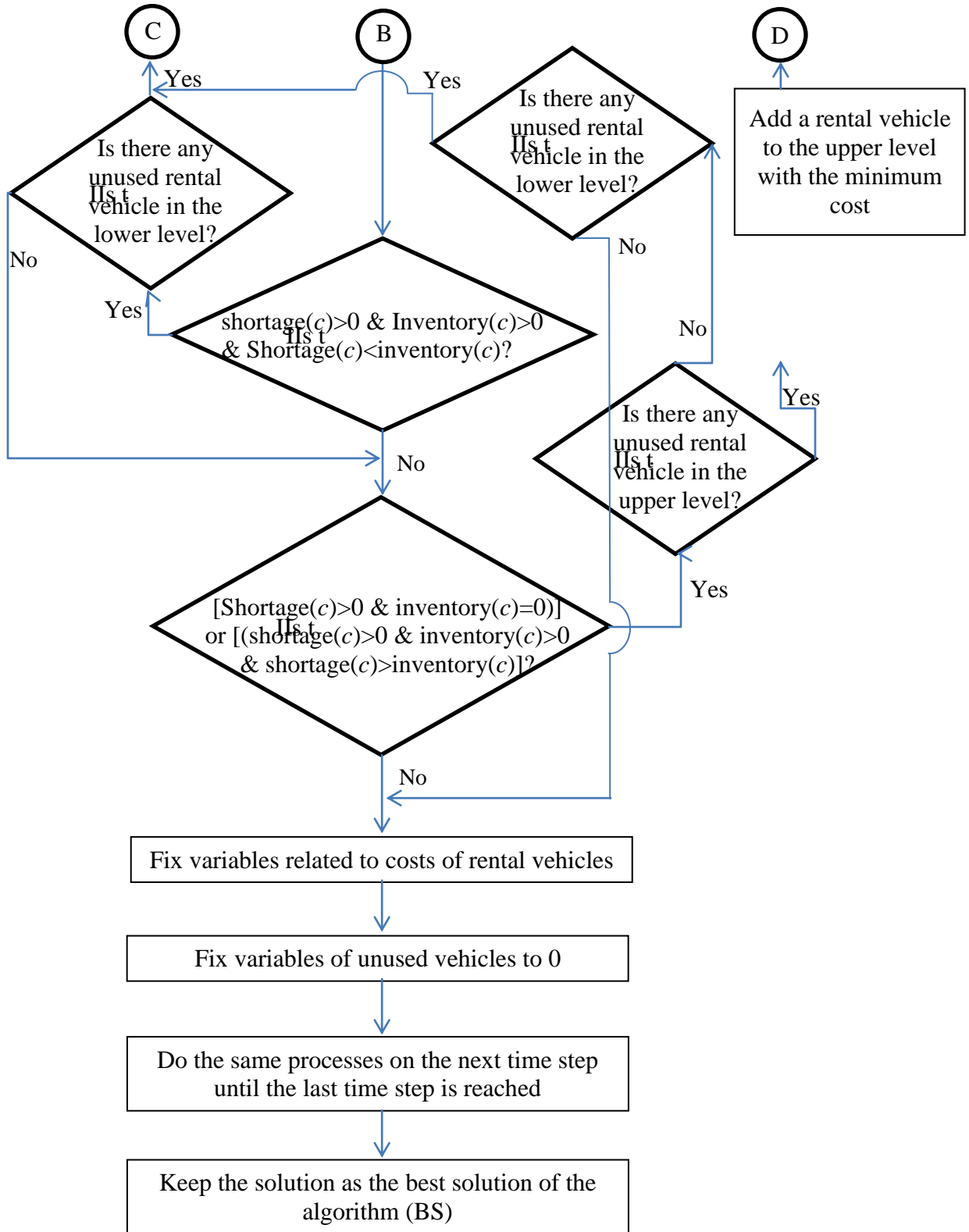


Figure 5-6 The flow chart of heuristic 1 in the first time step (Cont'd)

5.5. Improvement of a Solution

In developing the first heuristic described in Section 5.4, the target was considering all aspects so as to build a comprehensive heuristic method. The final solution found by heuristic 1, however, can potentially be improved. In this section, three different procedures are introduced that can improve the objective function of the final solution. These procedures reconsider TPL warehousing, avoiding redundant TPL contracts, and saving in tour cost. The performance of these improvement phases depends on the quality of the input data and final solution, which will be explained in detail later.

5.5.1. Phase 1: Reconsidering TPL Warehousing

In the first phase of improvement, TPL warehousing is analyzed to see whether it can lower the total cost. As mentioned in Chapter 3, the model has a capability of storing products in TPLP warehouses to save TPL shipping cost. If the original model with binary variables is solved, the difference between the cost of two separate deliveries by TPLP and the cost in the case of one delivery and storage in a TPLP warehouse is recognized correctly. In the proposed heuristic, the algorithm is free to pick any small numbers for binary variables. The mentioned difference, therefore, is canceled out, whereas the real objective function will decrease.

According to Step 3 of the algorithm described in Section 5.4.3, the algorithm fixes one TPL shipping contract at each iteration. The contract is selected randomly and its binary variable must have a positive value. As mentioned, the model may choose a very small number, which results in two separate deliveries instead of

storage in a TPLP warehouse. Therefore, in the first phase of improvement, the potential benefit of storing products in TPLP warehouses is analyzed.

If a TPL shipping contract is selected for two successive time steps, the binary variable associated with the second time step is fixed to zero. The new relaxed model is solved by Xpress optimally. Since it is assumed that all retailers visited by the deleted contract must be served with the same TPLP, the model has to store products in a TPLP warehouse.

The added constraint, which set the second contract binary variable to zero, is not consistent with the previous solution. Therefore, the current solution, which is the best one so far, is stored. In other words, all TPL contract binary variables except for the deleted contract are kept in the solution. Constraints associated with other contracts remain in the model. Other variables such as routing and commodity flow variables are relaxed, and the new solution is found by Xpress. In this solution, in addition to TPL binary variables, other variables may have non-integer values. Therefore, all steps after Step 2 are executed on the solution to find a feasible solution. Tours can be changed, in comparison to the previous feasible solution. Also, more or fewer rental vehicles can be used in the final solution. Eventually, the objective function of the final solution is compared to that of the best solution found before starting the improvement phases. If the new objective function shows improvement, the current solution is stored as the best solution; otherwise, the algorithm keeps the best solution, which means the fix-and-run algorithm ignores the final output of phase 1. Then the procedure is executed again for more savings. The

first phase of improvement is finished when it checks all successive selected TPL contracts. The algorithm of the first improvement phase can be described as follows:

Step 1: Set t = the last time step

Step 2: Generate $Temp_Set$ and $Temp_Var$ for different sets and decision variables of the current solution. Store elements of each set in its associated $Temp_Set$ and value of each variable in its associated $Temp_Var$. This phase uses $Temp$ sets and variables in the operation.

Step 3: If the same w variable has been selected in time steps t and $t-1$, go to Step 4; otherwise go to Step 9.

Step 4: If w exists in $Tabu_Set$, go to Step 8; otherwise go to Step 5.

Step 5: Add a temporary constraint to the model to fix w to zero in time step t and solve the new relaxed model optimally.

Step 6: Do Steps 13 to 37 of heuristic 1 with temporary sets and variables.

Step 7: Compare the obtained objective function with the objective function of the solution before starting the first improvement phase. If it has improved, keep the change and update all sets and variables. In fact, store $Temp_Sets$ in permanent sets and $Temp_Vars$ in permanent variables. Then, go to Step 2; otherwise, add w to the $Tabu_Set$ and go to Step 2.

Step 8: Check another w variable in time step t . If there is another w in time step t go to Step 3.

Step 9: Set $t=t-1$. If $t=1$ this phase is terminated; otherwise, go to Step 3.

5.5.2. Phase 2: Avoiding Redundant TPL Contracts

The main goal of this phase of improvement is the same as the first phase: saving on the cost of TPLP contracts. However, phase 2 is different from phase 1 in some details. In phase 1, the algorithm considers only contracts that have been selected for two successive time steps, on the second time step. For instance, if a contract is selected in time steps one and two, the algorithm relaxes the binary variable of time step two to determine whether it benefits the model. By this policy, the contract on time step 1 will definitely be in the final solution. Moreover, if a contract is selected only in one time step, it will not be checked by the first phase. As a result, all TPL contracts selected in the solution are not considered by phase 1.

Phase 2 of improvement analyzes all TPL contracts one by one to see whether there is any benefit in removing a contract from the solution. Another main difference between phase 1 and 2 is that in phase 2, if a contract is deleted from the solution, retailers served by that contract can be visited by owned or rental trucks. In other words, the algorithm can change the shipping plan significantly, meaning that some retailers which had been visited by TPLP trucks in the best solution can be served by owned or rental trucks. In fact, tours of owned and rental trucks may change substantially. Therefore, the operation of phase 2 can be summarized as follows: It keeps the current solution as the best solution. It then relaxes all routing and flow variables, as well as one of the selected TPL contracts. The rest of the selected TPL contracts remain in the model. Xpress solves the new relaxed model and generates new tours, which may visit retailers of the deleted TPL contract. All steps of the fix-and-run algorithm that fix routing variables and rental-vehicle cost variables are

executed to reach a feasible solution. If the objective function of the new solution is greater than that of the best solution, the best solution is replaced with the new one and all changes are accepted; otherwise, the deleted TPL contract is known as the crucial contract and will remain in the solution. Another TPL contract is then analyzed and the same procedure executed. Phase 2 is finished after analyzing all selected TPL contracts. The algorithm of the second improvement phase can be described as follows:

Step 1: Generate *Temp_Set* and *Temp_Var* for different sets and decision variables of current solution. Store elements of each set in its associated *Temp_Set* and value of each variable in its associated *Temp_Var*. Similar to Phase 1, this phase uses *Temp* sets and variables in the operation.

Step 2: Empty *Temp_Fix_TN* and *Temp_Exclude_TN*. In fact, the model is free to make decision for every *TN* variable.

Step 3: Pick one of the w variables has been selected in time steps t . If w exists in *Tabu_Set*, go to Step 7; otherwise go to Step 4.

Step 4: Add a temporary constraint to the model to fix w to zero in time step t and solve the new relaxed model optimally.

Step 5: Do Steps 8 to 37 of heuristic 1 with temporary sets and variables for all time steps.

Step 6: Compare the obtained objective function with the objective function of the solution before starting the second improvement phase. If it has improved, keep the change and update all sets and variables. In fact, store *Temp_Sets* in permanent

sets and *Tem_Vars* in permanent variables; otherwise, add w to the *Tabu_Set* and go to Step 1.

Step 7: Check another w variable. If there is no more w , the second phase of improvement is terminated; otherwise, go to Step 2.

5.5.3. Phase 3: Saving in Tour Cost

Heuristic 1 fixes one link for every vehicle in each iteration. The selected link for a vehicle carries products to the facility location at the end of the link in that particular iteration. In next iterations, the algorithm may find delivery to that facility location by another vehicle more beneficial. In other words, other links that end at the same facility location may be selected by the algorithm for other delivery vehicles.

Therefore, in the final solution of heuristic 1, more than one vehicle visits one location, but only one of them serves it. This situation happens in the final situation, because heuristic 1 selects links one by one and, as it gets close to the end, using all selected links for delivery is not optimal. As a result, vehicles move along some links without serving bottlers or retailers located at the end of the link. Figure 5-7 shows a sample of final output in which a vehicle visits retailers 1, 2, 3, and 4; however, it delivers to only retailers 1 and 4.

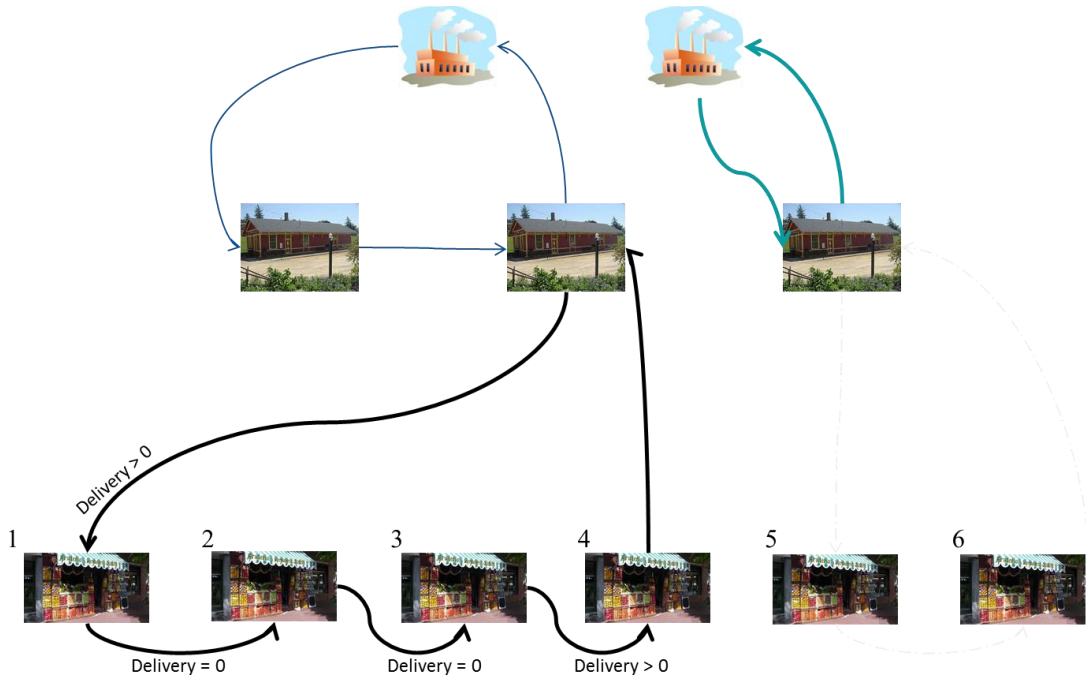


Figure 5-7 A sample solution of Xpress showing redundant links in the path of one vehicle

Since travel times of links are triangular, the model saves the tour cost if the vehicle goes from retailer 1 to retailer 4 directly. In this case, the truck skips redundant retailers in its path and visits only retailers for which it has products. This situation can happen in the upper level, where a truck can pass some bottlers with no delivery. Figure 5-8 shows the output of running phase 3 on the solution shown in Figure 5-7.

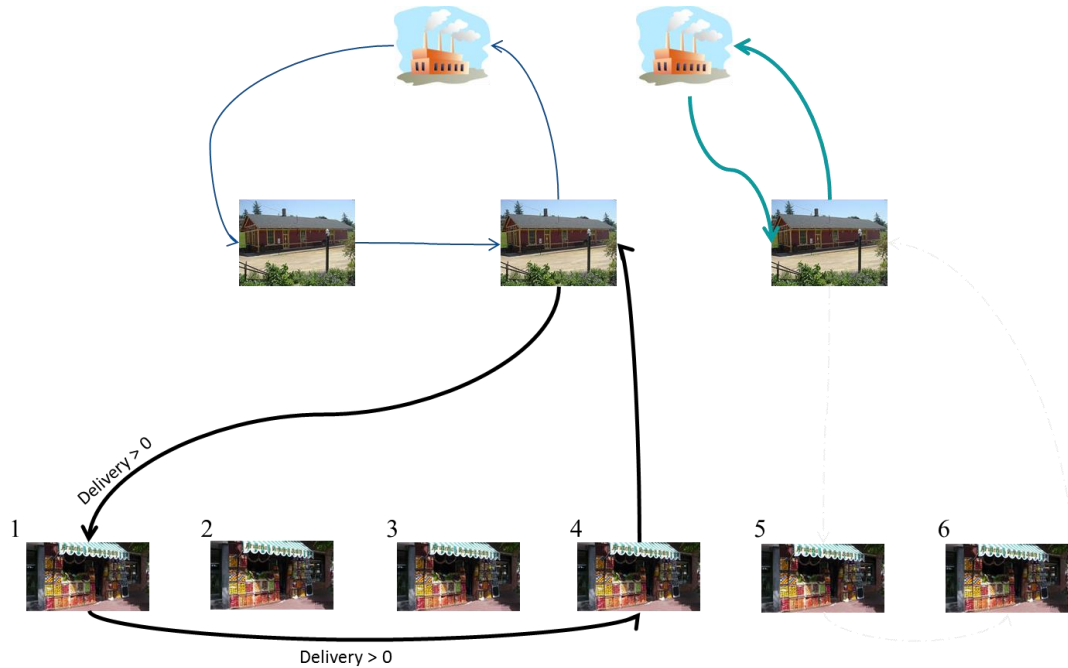


Figure 5-8 The output of applying phase 3 to the situation shown in Figure 5-7

The algorithm of the third improvement phase can be described as follows:

Step 1: Set $t = 1$.

Step 2: Set routing variable XU to one for all selected links in time step t in the current solution.

Step 2: Put all links passed by a vehicle k in separate set $Route_k$.

Step 3: Sort links of each set $Route_k$ based on the path of the vehicle k .

Step 4: If delivery of vehicle k to location j is zero, the link which ends at location j is deleted from the set $Route_k$. Also, the link which starts from location j is deleted from the set $Route_k$.

Step 5: Connect the origin node of the first deleted link to the destination node of the second deleted link.

Step 7: Repeat Step 1 to 5 for XW in time step t .

Step 8: Set $t = t+1$. If t is less than or equal to planning horizon length, repeat Steps 1 to 8 for the new time step; otherwise, go to Step 9.

Step 9: Compare the obtained objective function with the objective function of the solution before starting the third improvement phase. If it does not change, the third improvement phase is terminated; otherwise, go to Step 1.

5.6. Numerical Results

In this section, results obtained by solving different scenarios with heuristic 1 are presented and described in detail. First, several categories were designed, each having particular characteristics. Then several scenarios were generated in each category according to the characteristics of that category. Therefore, scenarios that are in one category have the same number of facility locations, fleet size, number of commodity types, number of candidate TPL contracts, and other major model characteristics; however, input data for scenarios in the same category are different.

5.6.1. Category 1

The first category is the smallest of all the categories. Table 5-1 illustrates the main characteristics of category 1. The smallest category of scenarios has 6,320 constraints and 62,423 decision variables. 10 scenarios were generated in this category, which have different input data—but the main characteristics are the same and equal to the amounts shown in Table 5-1. All input data for the 10 scenarios were generated randomly according to the assumed normal distribution introduced in

Chapter 4. Each scenario was solved by both Xpress 7.1 and heuristic 1. The results of different solvers on scenarios are reported in Table 5-2.

Table 5-1. Main characteristics of Category 1

Criterion	Value
Number of Plants	2
Number of Bottlers	3
Number of Retailers	6
Number of Commodity Types	2
Number of Available TPL Contracts	2
Number of Owned Trucks in the Upper Level	2
Number of Rental Trucks in the Upper Level	2
Number of Owned Trucks in the Lower Level	2
Number of Rental Trucks in the Lower Level	2
Number of Time Steps	3
Number of Constraints	6,320
Number of Decision Variables	62,423

Table 5-2. Comparison between results of Xpress and Heuristic 1 on Category 1 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 1 Objective Function (\$)	Gap of the Heuristic 1 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 1 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	49330	48875	0.92	0.92	771	43
2	49105	48259	1.72	1.82	9473	43
3	49330	48791	1.09	1.09	152	37
4	49356	49035	0.65	0.65	204	40
5	45088	44971	0.25	0.25	23	47
6	47877	47167	1.48	1.48	318	46
7	45726	45068	1.43	1.46	79040	53
8	45088	44486	1.34	1.34	98	45
9	44841	44484	0.79	0.79	30	45
10	46299	45951	0.75	0.91	1530	50

Xpress was not able to find the optimal solution for some scenarios. For these scenarios, the best solution found by Xpress is reported in Table 5-2, which may not be the optimal. The gap between the objective function of the solution found by

heuristic 1 and the best solution found by Xpress is reported in the fourth column, and the gap between the objective function of the heuristic 1 solution and the best bound found by Xpress can be found in the fifth column of Table 5-2. The best bound is the closest infeasible solution to the best solution. Obviously, for scenarios for which Xpress finds the optimal solution, the best bound and the best solution are equal to each other. In these cases, the gaps reported in the 4th and 5th columns in Table 5-2 are equal. The last two columns of Table 5-2 represent running times of Xpress and heuristic 1, respectively.

According to Table 5-2, Xpress could not find the optimal solution for scenarios 2, 7, and 10. In all scenarios, Xpress found better solutions than heuristic 1; however, the gap between heuristic 1's objective function and the best solutions found by Xpress is less than 2%. On the other hand, heuristic 1 found the solution in less than one minute, while the running time of Xpress was significantly longer.

Table 5-3 summarizes the information presented in Table 5-2. According to Table 5-3, the average gap between the heuristic 1 solution's objective function and the Xpress solution's objective function is around 1%, which is very small. In addition, Table 5-3 indicates the advantage of using the proposed heuristic, which is its short running time. Xpress needed to run 9100 seconds on average to find the optimal or near-optimal solution. Also, if Scenario 7 is not considered, Xpress still needed to run 1400 seconds on average; however, heuristic 1, after less than 45 seconds, found a solution that was very close to the Xpress solution.

Table 5-3. Criteria for comparison of Xpress and fix-and-run based on Category 1 scenarios

Criterion	Value
Average of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress (%)	1.07
Variance of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress	0.21
Average of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress (%)	1.04
Variance of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress	0.20
Average of Xpress Running Time (CPU seconds)	9164
Average of Xpress Running Time without considering Scenario 7 (CPU seconds)	1400
Average of the Heuristic 1 Running Time (CPU seconds)	45

Table 5-3 indicates that the proposed algorithm is efficient on small problems. The performance of this algorithm on larger problems is analyzed in the following sections.

5.6.2. Category 2

This category contains more retailers and larger fleet size than Category 1. As a result, the scenarios generated in this category are larger than scenarios for Category 1. Table 5-4 presents the main characteristics of Category 2. This category of scenarios has 19846 constraints and 270319 decision variables, which is three times larger than for the scenarios in Category 1. Similar to Category 1, 10 scenarios with differences in input data were generated in this category. Again, each scenario was solved by both Xpress 7.1 and heuristic 1, and the results are reported in Table 5-5.

Table 5-4. Main characteristics of Category 2

Criterion	Value
Number of Plants	2
Number of Bottlers	3
Number of Retailers	10
Number of Commodity Types	2
Number of Available TPL Contracts	2
Number of Owned Trucks in the Upper Level	3
Number of Rental Trucks in the Upper Level	3
Number of Owned Trucks in the Lower Level	3
Number of Rental Trucks in the Lower Level	3
Number of Time Steps	3
Number of Constraints	19,846
Number of Decision Variables	270,319

Table 5-5. Comparison of results of Xpress and Heuristic 1 on Category 2 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 1 Objective Function (\$)	Gap of the Heuristic 1 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 1 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	80170	78940	1.53	1.53	42615	438
2	79901	78405	1.87	1.90	21570	860
3	80122	78527	1.99	2.39	4482	599
4	80129	79125	1.25	1.53	9283	469
5	80239	79763	0.59	0.81	19755	389
6	81656	81042	0.75	0.90	9897	404
7	82998	82470	0.63	0.74	25387	382
8	82681	82101	0.70	0.70	36195	424
9	81235	79062	2.67	3.38	17306	433
10	80455	78234	2.76	3.47	61521	387

In this category, more scenarios than Category 1 were not solved by Xpress optimally. According to Table 5-5, Xpress was not able to find the optimal solution in 8 scenarios out of 10. Since the problems in Category 2 are larger than Category 1, the running times for Xpress and heuristic 1 increase compared to the previous category. Running time of heuristic 1 was, at most, less than 15 minutes, which is

short; however, in some cases, after 5 hours Xpress had still not found the optimal solution. Table 5-6 summarizes information presented in Table 5-5, which more clearly compares the performance of Xpress and the fix-and-run algorithm on Category 2.

Table 5-6. Criteria for comparison of Xpress and fix-and-run based on Category 2 scenarios

Criterion	Value
Average of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress (%)	1.71
Variance of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress	1.09
Average of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress (%)	1.47
Variance of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress	0.68
Average of Xpress Running Time (CPU seconds)	24801
Average of the Heuristic 1 Running Time (CPU seconds)	478

According to Table 5-6, the average gap between heuristic 1's objective function and the objective function of the best solution found by Xpress was 1.47%. The gap with the best bound found by Xpress was 1.71%. On the other hand, the average running time for heuristic 1 and Xpress was 478 and 24801 seconds, respectively. As a result, heuristic 1 found solutions that were close to Xpress results in a short time. In other words, the gap and running time are in a reasonable range, which shows the strength of the proposed heuristic.

5.6.3. Category 3

This category includes 15 retailers, but the rest of its main characteristics are the same as those of Category 2. Table 5-7 shows the main characteristics of Category 3. This category of scenarios has 39721 constraints and 696819 decision

variables, meaning that problems in this category are medium size. Similar to Category 1 and 2, 10 scenarios with different input data were generated in this category to be solved with Xpress 7.1 and heuristic 1. The results are reported in Table 5-8.

Table 5-7. Main characteristics of Category 3

Criterion	Value
Number of Plants	2
Number of Bottlers	3
Number of Retailers	15
Number of Commodity Types	2
Number of Available TPL Contracts	2
Number of Owned Trucks in the Upper Level	3
Number of Rental Trucks in the Upper Level	3
Number of Owned Trucks in the Lower Level	3
Number of Rental Trucks in the Lower Level	3
Number of Time Steps	3
Number of Constraints	39,721
Number of Decision Variables	696,819

Table 5-8. Comparison of results of Xpress and Heuristic 1 on Category 3 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 1 Objective Function (\$)	Gap of the Heuristic 1 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 1 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	118019	115040	2.52	3.01	38656	5027
2	89067	84750	4.84	5.86	12029	15255
3	115753	109276	5.59	6.05	53738	11330
4	118047	115864	1.85	2.30	29160	6456
5	122439	120294	1.75	2.01	18072	5569
6	113127	110855	2.00	2.37	50871	5054
7	109817	108639	1.07	1.67	25023	2751
8	111336	107411	3.52	3.89	29228	6386
9	121530	119689	1.51	2.00	13021	5711
10	109682	107916	1.60	1.60	15123	5849

In this category, only the last scenario was solved optimally by Xpress. This demonstrates that the complexity of the problem is sensitive to the number of facility locations in the lower level. Increasing the number of retailers from 10 to 15 not only increases the size of the problem significantly, but also makes the problem very complicated. As a result, the running times for Xpress and heuristic 1 increase remarkably in comparison to Categories 1 and 2. Table 5-9 summarizes the information reported in Table 5-8.

Table 5-9. Criteria for comparison of Xpress and fix-and-run based on Category 3 scenarios

Criterion	Value
Average of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress (%)	3.08
Variance of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress	2.76
Average of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress (%)	2.62
Variance of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress	2.33
Average of Xpress Running Time (CPU seconds)	28492
Average of the Heuristic 1 Running Time (CPU seconds)	6939

According to Table 5-9, the average gap between heuristic 1's objective function and the objective function of the best solution found by Xpress was 2.62%, and 3.08% with the best bound found by Xpress. The gap, therefore, is still in the acceptable range. In addition, the running time of heuristic 1 was much more than Category 2; however, it was much less than that of Xpress. As a result, the proposed heuristic performs well in this category.

5.6.4. Category 4

The number of facility locations in this category is the same as Category 3; however, its fleet size is larger and it has more commodity types than Category 3. Table 5-10 shows the main characteristics of Category 4. This category of scenarios contains 61141 constraints and 1,189,923 decision variables. In other words, the problem is medium size. Five scenarios with different input data were generated in this category to be solved with Xpress 7.1 and heuristic 1. The results are reported in Table 5-11.

Table 5-10. Main characteristics of Category 4

Criterion	Value
Number of Plants	2
Number of Bottlers	3
Number of Retailers	15
Number of Commodity Types	3
Number of Available TPL Contracts	2
Number of Owned Trucks in the Upper Level	4
Number of Rental Trucks in the Upper Level	3
Number of Owned Trucks in the Lower Level	4
Number of Rental Trucks in the Lower Level	3
Number of Time Steps	3
Number of Constraints	61,141
Number of Decision Variables	1,189,923

Table 5-11. Comparison of results of Xpress and Heuristic 1 on Category 4 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 1 Objective Function (\$)	Gap of the Heuristic 1 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 1 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	147348	145667	1.14	1.87	8333	20505
2	142034	141410	0.43	0.43	3501	13819
3	142503	139972	1.77	1.83	38038	25458
4	164400	162589	1.10	1.49	11870	13636
5	164721	164954	-0.14	0.17	10151	7201.8

In this category, only scenario 2 was solved optimally by Xpress. This shows that the complexity of the problem is sensitive to the number of commodity types as well as fleet size. The running time of heuristic 1 increases remarkably, in comparison to the previous categories. If running time for Xpress is compared to Category 3, it shows a decrease in running time despite the fact that the problems are larger than the problems in Category 3. This is because the models were stopped earlier than in the previous category: After the gap between the solution of Xpress and the best bound became less than 0.8%, the model was stopped—whereas models in previous categories were executed longer to find smaller gaps. The running time for heuristic 1 was much longer than in previous categories, confirming that the size of this category is significantly larger than previous categories. The main point of Table 5-11 is that in the last case, the proposed heuristic found a better solution than that found by Xpress. Table 5-12 summarizes the information reported in Table 5-11.

Table 5-12. Criteria for comparison of Xpress and Fix & Run based on Category 4 scenarios

Criterion	Value
Average of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress (%)	1.16
Variance of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress	0.64
Average of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress (%)	0.86
Variance of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress	0.53
Average of Xpress Running Time (CPU seconds)	14378
Average of the Heuristic 1 Running Time (CPU seconds)	16124

According to Table 5-12, the average gap between heuristic 1's objective function and the objective function of the best solution found by Xpress was less than 1%, and with the best bound found by Xpress was 1.16%. The gap, therefore, is very small, but the running time for heuristic 1 was much more than in Category 3. The

proposed heuristic needs to run, on average, more than 4 hours to find a good feasible solution. This is still acceptable, but would probably increase significantly for larger problems, which results in its becoming impractical.

5.6.5. Evaluation of Improvement Phases' Impact on Final Solutions

As mentioned in Section 5.6.4, the running time of the fix-and-run algorithm grows significantly as the problem size increases. The fix-and-run algorithm has two main components: general heuristic steps, which find a feasible solution, and improvement phases, which try to make a final solution better. In this section, the impact of improvement phases is analyzed, because the first two phases are especially time-consuming.

Categories 2, 3, and 4 were selected for this analysis. Two scenarios from Category 2, eight scenarios from Category 3, and all scenarios from Category 4 were considered for evaluation of the impact of improvement phases on the final solution. The objective function after each improvement phase, therefore, was considered as a base for calculations. Table 5-13 shows the results of this evaluation.

Table 5-13. Impact of different improvement phases on the objective function (%)

Category	Scenario	Phase 1	Phase 2	Phase 3
2	5	0.35	0.96	0.40
	6	0.12	0.42	0.46
3	1	0.03	0.00	1.43
	4	0.02	0.00	1.02
	5	0.00	0.00	0.36
	6	0.00	0.00	0.35
	7	0.13	0.00	0.14
	8	0.36	0.00	1.74
	9	0.07	0.00	0.41
	10	0.13	0.00	1.34
	4	1	0.00	0.00
2		0.10	0.00	0.88
3		0.23	0.05	1.04
4		0.05	0.00	0.31
5		0.00	0.00	0.01
Average		0.11	0.10	0.71

According to Table 5-13, the first and second phases improve the objective function 0.11% and 0.10%, respectively. Phase 3 has the best performance among the three phases, with 0.71% improvement of the objective-function. As mentioned earlier, the first two phases take a long time to execute; however, the output gets better, to around 0.1%. On the other hand, phase 3 is a very fast procedure with considerable improvement. As a result, the first two phases are only efficient for small problems, while the third phase has a remarkable effect on even large problems. In other words, applying the base of the fix-and-run algorithm and the third phase provides a quality solution in a reasonable amount of running time. For larger categories, therefore, the first and second phases of improvement can be eliminated.

5.6.6. Category 5

Category 5 has more retailers than Category 4. In addition, another TPL contract has been added to the model. Table 5-14 shows the main characteristics of Category 5.

Table 5-14. Main characteristics of Category 5

Criterion	Value
Number of Plants	2
Number of Bottlers	3
Number of Retailers	18
Number of Commodity Types	3
Number of Available TPL Contracts	3
Number of Owned Trucks in the Upper Level	4
Number of Rental Trucks in the Upper Level	3
Number of Owned Trucks in the Lower Level	4
Number of Rental Trucks in the Lower Level	3
Number of Time Steps	3
Number of Constraints	86,059
Number of Decision Variables	1,875,966

According to Table 5-14, this category of scenarios contains 86,059 constraints and 1,875,966 decision variables, which confirms that this category's problems are large size. Five scenarios with different input data were generated in this category to be solved with Xpress 7.1 and heuristic 1. The results are reported in Table 5-15.

Table 5-15. Comparison of results of Xpress and Heuristic 1 on Category 5 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 1 Objective Function (\$)	Gap of the Heuristic 1 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 1 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	126274	114358	9.44	10.17	34287	34689
2	147278	146294	0.67	1.54	61888	28574
3	157349	155822	0.97	1.46	32610	23522
4	144202	142855	0.93	1.48	24915	15837
5	145404	143457	1.34	1.80	41285	21806

In this category, except for the gap for scenario 1, which is high, other scenarios' gaps are less than 2% and, in some cases, less than 1%. Variances of gap between heuristic 1's objective function and the objective function of the best solution and the best bound are very high due to high value for gaps of scenario 1. Improvement of phases 1 and 2 was not executed on problems in this category. Empirically, they can reduce the gap, especially for the first scenario; however, they were removed from the framework for heuristic 1 algorithm for large problems due to their expensive running time. Table 5-16 summarizes information reported in Table 5-15.

Table 5-16. Criteria for comparison of Xpress and fix-and-run based on Category 5 scenarios

Criterion	Value
Average of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress (%)	3.29
Variance of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress	14.81
Average of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress (%)	2.67
Variance of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress	14.37
Average of Xpress Running Time (CPU seconds)	38997
Average of the Heuristic 1 Running Time (CPU seconds)	24885

According to Table 5-16, the average gap between heuristic 1's objective function and the objective function of the best solution found by Xpress is less than 3%, and with the best bound found by Xpress less than 3.5%. Therefore, the gap for this category of large problems is in an acceptable range. Heuristic 1 needed to run an average of 7 hours to find a feasible solution, while Xpress needed to run for a longer time.

5.6.7. Category 6

Category 6 has more facility locations in both levels and a larger fleet size than Category 5. Table 5-17 shows the main characteristics of Category 6.

Table 5-17. Main characteristics of Category 6

Criterion	Value
Number of Plants	3
Number of Bottlers	4
Number of Retailers	20
Number of Commodity Types	3
Number of Available TPL Contracts	3
Number of Owned Trucks in the Upper Level	4
Number of Rental Trucks in the Upper Level	4
Number of Owned Trucks in the Lower Level	4
Number of Rental Trucks in the Lower Level	4
Number of Time Steps	3
Number of Constraints	152,049
Number of Decision Variables	4,421,770

Similar to the previous category, five scenarios with different input data were generated to be solved with Xpress 7.1 and heuristic 1. The results are shown in Table 5-18.

Despite the fact that the size of problems in this category is larger than Category 5, the running time is shorter. The reason is that Xpress uses the dual simplex as a solver for large problems by default; however, dual simplex is not always efficient. Two tests on a relaxed problem in this category, in which the first one was solved by dual simplex and the second by primal simplex, show that dual simplex found the optimal solution after 600 seconds; primal simplex took 90 seconds. The solver, therefore, was changed manually for all problems in this category, which resulted in shorter running time.

Table 5-18. Comparison of results of Xpress and Heuristic 1 on Category 6 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 1 Objective Function (\$)	Gap of the Heuristic 1 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 1 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	176600	175932	0.37	1.60	43823	14161
2	182435	182312	0.07	1.17	30124	10232
3	184701	184845	-0.08	1.35	36585	11771
4	181661	180829	0.46	0.92	65168	6377
5	178979	178336	0.36	0.87	52403	6834

The gap for problems in this category is less than 2%, which is very promising. Moreover, the running time for heuristic 1 was remarkably shorter than that of Xpress. Xpress needed to run more than 10 hours while heuristic 1 took 4 hours at most. Table 5-19 summarizes the information shown in Table 5-18.

Table 5-19. Criteria for comparison of Xpress and fix-and-run based on Category 6 scenarios

Criterion	Value
Average of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress (%)	1.18
Variance of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress	0.09
Average of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress (%)	0.24
Variance of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress	0.05
Average of Xpress Running Time (CPU seconds)	45620
Average of the Heuristic 1 Running Time (CPU seconds)	9875

As Table 5-19 shows, the average gap between objective functions of heuristic 1 and Xpress is very small. Xpress found better solutions but in longer running times most of the time. In addition, heuristic 1 found a better solution than Xpress in Scenario 3. As a result, fix-and-run heuristic performance on this large-scale category is both good and fast.

5.6.8. Category 7

Category 7 has the same number of facility locations as Category 6 but with a longer planning horizon, covering 5 time steps instead of 3. The last two scenarios in Category 7 include two more vehicles in both levels. Table 5-20 shows main characteristics of Category 7.

Table 5-20. Main characteristics of Category 7

Criterion	Value
Number of Plants	3
Number of Bottlers	4
Number of Retailers	20
Number of Commodity Types	3
Number of Available TPL Contracts	3
Number of Owned Trucks in the Upper Level	4
Number of Rental Trucks in the Upper Level	4
Number of Owned Trucks in the Lower Level	4
Number of Rental Trucks in the Lower Level	4
Number of Time Steps	5
Number of Constraints	300,451
Number of Decision Variables	8,892,749
Number of Owned Trucks in the Upper Level (in the Last 2 Scenarios)	5
Number of Rental Trucks in the Upper Level (in the Last 2 Scenarios)	5
Number of Owned Trucks in the Lower Level (in the Last 2 Scenarios)	5
Number of Rental Trucks in the Lower Level (in the Last 2 Scenarios)	5
Number of Constraints (in the Last 2 Scenarios)	314,978
Number of Decision Variables (in the Last 2 Scenarios)	9,215,024

Problems in this category cannot be solved by Xpress due to their large size, as described in Section 4.2.6. A fix-and-run algorithm was tested to solve the same problem and results are shown in Table 5-21. It found a feasible solution after 4 days' running time for all scenarios. Table 5-22 summarizes the information shown in Table 5-21.

Table 5-21. Comparison of results of Xpress and Heuristic 1 on Category 7 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 1 Objective Function (\$)	Gap of the Heuristic 1 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 1 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	N.A.	268836	N.A.	3.61	N.A.	4 days
2	N.A.	272568	N.A.	3.79	N.A.	4 days
3	N.A.	276753	N.A.	3.31	N.A.	4 days
4	N.A.	390395	N.A.	3.87	N.A.	4.5 days
5	N.A.	398105	N.A.	3.63	N.A.	4.5 days

Table 5-22. Criteria for comparison of Xpress and fix-and-run based on Category 7 scenarios

Criterion	Value
Average of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress (%)	3.64
Variance of Gap between Heuristic 1 Solution and the Best Bound Found by Xpress	0.04
Average of Gap between Heuristic 1 Solution and the Best Solution Found by Xpress (%)	N.A.
Average of Xpress Running Time (CPU seconds)	N.A.
Average of the Heuristic 1 Running Time (CPU seconds)	4 days

According to Table 5-22, the average gap between heuristic 1's objective function and the objective function of the best bound found by Xpress is 3.64%. Since Xpress was not able to find a feasible solution or a bound for these problems, the objective function of the relaxed problem was considered as a base to calculate the gap. The real gap with the best bound, therefore, is definitely smaller than 3.64%, because the relaxed problem solution is not the best infeasible solution. Although the gap with the optimal solution is less than 3.64%, if it is considered as the gap it still is in an acceptable range and is small. As a result, the proposed heuristic performs better than Xpress in providing a good and accurate feasible solution for large problems with reasonable distance from the optimal solution. The only problem with heuristic 1

for large problems is its high running time. It needs to run for 4 days to find a solution for the whole planning horizon; however, the proposed heuristic can find a solution for the first time step after 1.5 days. Also, the solution for the second time step was found after 2.5 days. In other words, the algorithm does not necessarily need to run to the end to provide a solution for the first time step. It finds the solution for the first time step and then continues to the second time step. Therefore, it can deliver the solution of the first time step before starting execution of the second time step. This capability of the algorithm makes this method more practical for large problems.

5.7. Summary

In this chapter, a new heuristic algorithm based on a fix-and-run algorithm was proposed to solve the problem under study. This heuristic algorithm is based on relaxation of the original problem and fixing integer values for integer variables for one time step sequentially. After all integer variables of the time step have been fixed to integer values, the algorithm runs the same processes on the next time step. The main point of this method is that in the process of decision-making for one time step, its impact on future time steps is considered as well as the effect of future decision variables on current time-step variables. The algorithm runs faster when it gets close to the last time steps, since it has already fixed variables for previous time steps.

Moreover, three improvement phases were developed to increase the solution found by heuristic 1. Two are related to TPL contracts, and the third saves tour cost.

Finally, seven category problems in this chapter were designed to test the proposed heuristic and compare its results to those of Xpress. The sizes of the

categories varied from small to large problems, and each category included several scenarios. Scenarios for each category had the same characteristics, such as the number of facility locations, fleet size, the number of final products, and the number of time steps; however, input data for the parameters of the scenarios were different and were generated randomly, based on the normal distributions described in Chapter 4.

Figures 5-9 and 5-10 compare the performance of Xpress and heuristic 1 in different categories from the standpoints of gap and running time, respectively. According to Figure 5-9, on the average Xpress found better solutions than heuristic 1; however, it was not able to find feasible solutions for problems in Category 7, while heuristic 1 found good feasible solutions for them with acceptable gaps from the complete relaxed problem.

Figure 5-10 compares the running times of Xpress and the proposed heuristic in different categories. In all categories, heuristic 1 ran much faster than Xpress, and in most categories, it ended after a reasonable amount of computation time. As mentioned, Xpress did not find a feasible solution for problems in Category 7. The running times for heuristic 1 for problems of Category 7 were very high; however, it delivered a plan for the first time step after 1.5 days and for the second time step after 2.5 days.

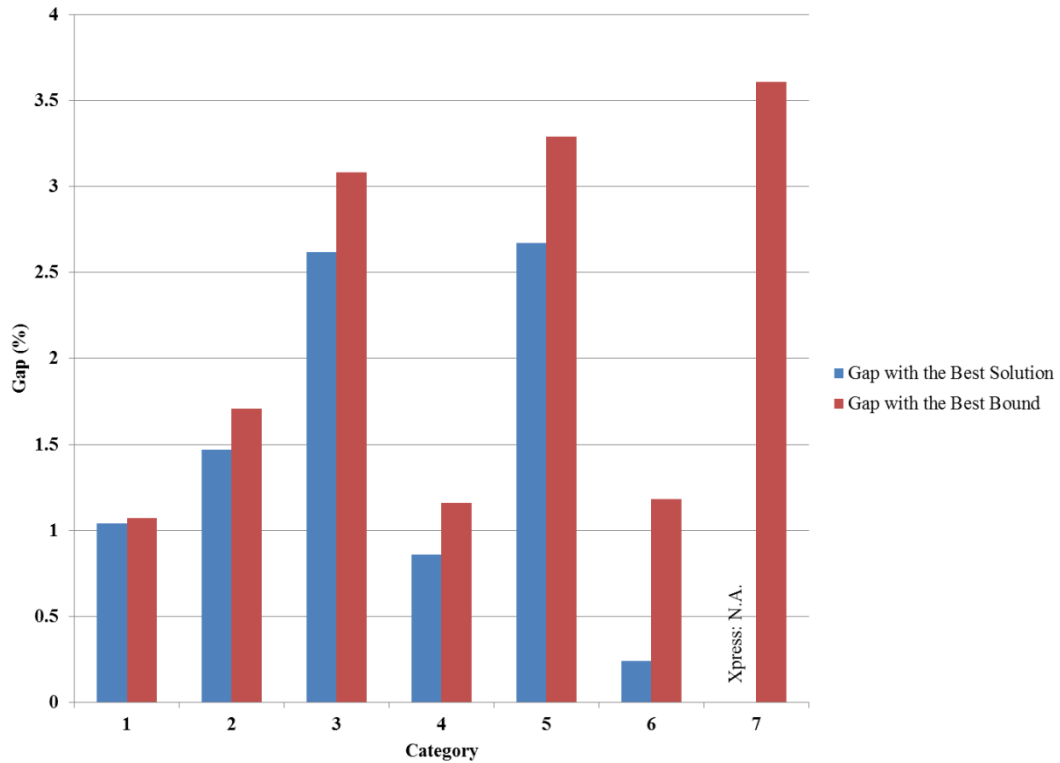


Figure 5-9. Average gap of the heuristic 1 solution with Xpress outputs in different categories

In other words, it is not necessary to wait 4 days to get a solution; the algorithm delivers a plan for the time steps gradually. This capability and the advantage of providing quality solutions, therefore, make the algorithm attractive for large problems. If heuristic 1 is changed to become faster, it will also be more practical for real-world problems. This change will be discussed in Chapter 6.

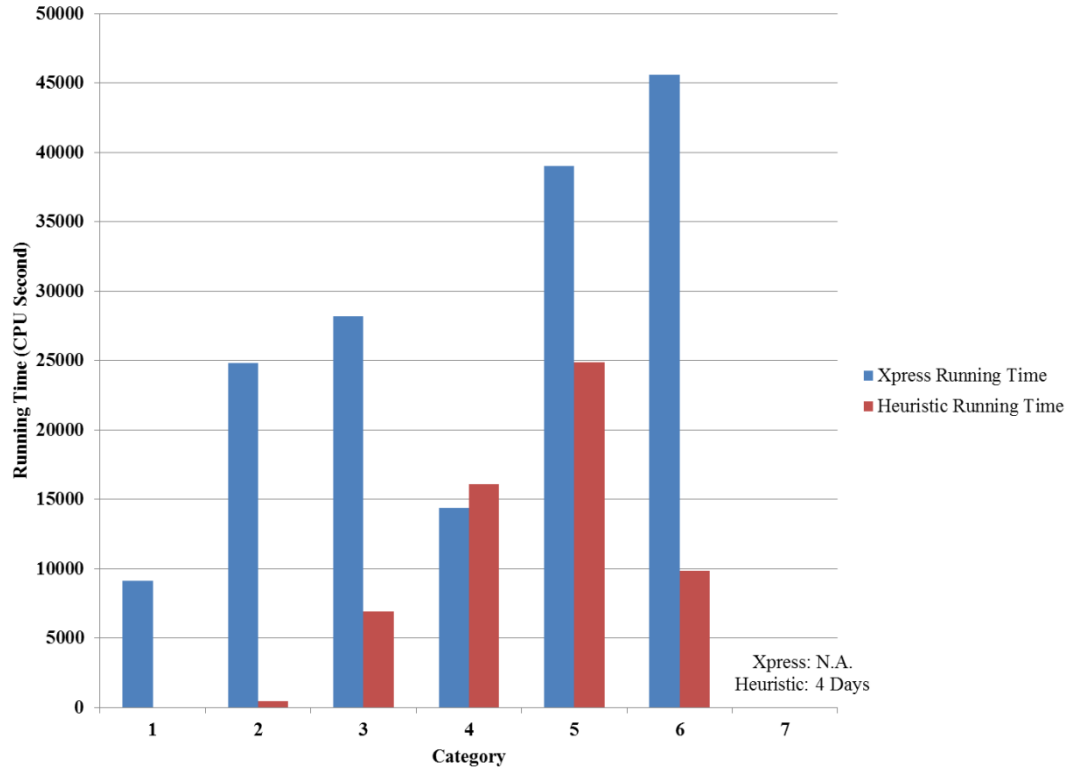


Figure 5-10. Average running time for heuristic 1 and Xpress in different categories

The variance of gap between the solution of heuristic 1 and the best solution or best bound found by Xpress indicated that in most of categories heuristic 1 provided consistent solutions. In other words, the difference between gaps in scenarios belonging to the same category is not significant except Category 5 that the first scenario has higher gap in compare to other scenarios. It confirms that heuristic 1 performs consistently over different scenarios with the same main characteristics.

Chapter 6: The Second Heuristic Approach

In this chapter, the second heuristic algorithm is proposed to solve the problem under study. This approach is based on the first heuristic method described in Chapter 5, with some changes to make it faster. The main problem with the first heuristic method is its long running time for some large problems. The algorithm provides a quality solution with a small gap from the upper bound in a reasonable amount of computation time, which for small and medium size problems is short. Some large problems can also be solved by heuristic 1 in a reasonable timeframe. For a particular category for which Xpress is not able to find even a feasible solution, however, the running time is long. Although heuristic 1 does not need to wait until the end to deliver distribution plans for earlier time steps, some parts of it have been changed in this chapter to improve the algorithm's running time. The final output is known as the second heuristic algorithm. Section 6.1 explains the changes that were made to the first heuristic approach. The results of applying the new heuristic to different categories are discussed in Section 6.2. Finally, Section 6.3 summarizes the advantages and disadvantages of the second heuristic approach.

6.1. Changes Applied to the First Approach

As mentioned, the new heuristic method is based on the first approach. The main framework, therefore, is the same as the first heuristic. Some steps of heuristic 1 have been changed, which are described in this section.

6.1.1. Fixing the Whole Path for Every Vehicle in Steps 4 and 5

In steps 4 and 5 of heuristic 1, in each iteration for every used vehicle only one link is chosen randomly to be fixed. The heuristic, therefore, spends a lot of time routing vehicles, but the final solution is accurate enough. In the second heuristic proposed in this study, these steps have been changed to make the new heuristic faster. In Step 4 of heuristic 2, all rental vehicles are first excluded from the fleet of the upper level. The new model is then solved by Xpress optimally. Figure 6-1 shows an output of Xpress for a problem.

According to Figure 6-1, two vehicles are used in the upper level; each one goes to bottlers through two separate tours, which is not feasible in the original problem. For every vehicle used, heuristic 2 recognizes the plant(s) visited. If a vehicle starts its tour from one plant, that plant is considered to be a source for the vehicle. If tours are begun from different plants, the plant of the link with a higher value for routing variable (XU) is considered to be a source. In addition, all bottlers visited by a vehicle are recognized and grouped in one category. The final path must go to all bottlers in the category. As a result, heuristic 2 finds the shortest tour connecting the source to bottlers in the category and returns to the same plant. The heuristic, therefore, fixes the routing variable of links on the tour to one. The measure

of the shortest-tour algorithm used in the heuristic is the summation of travel time costs and shipping costs.

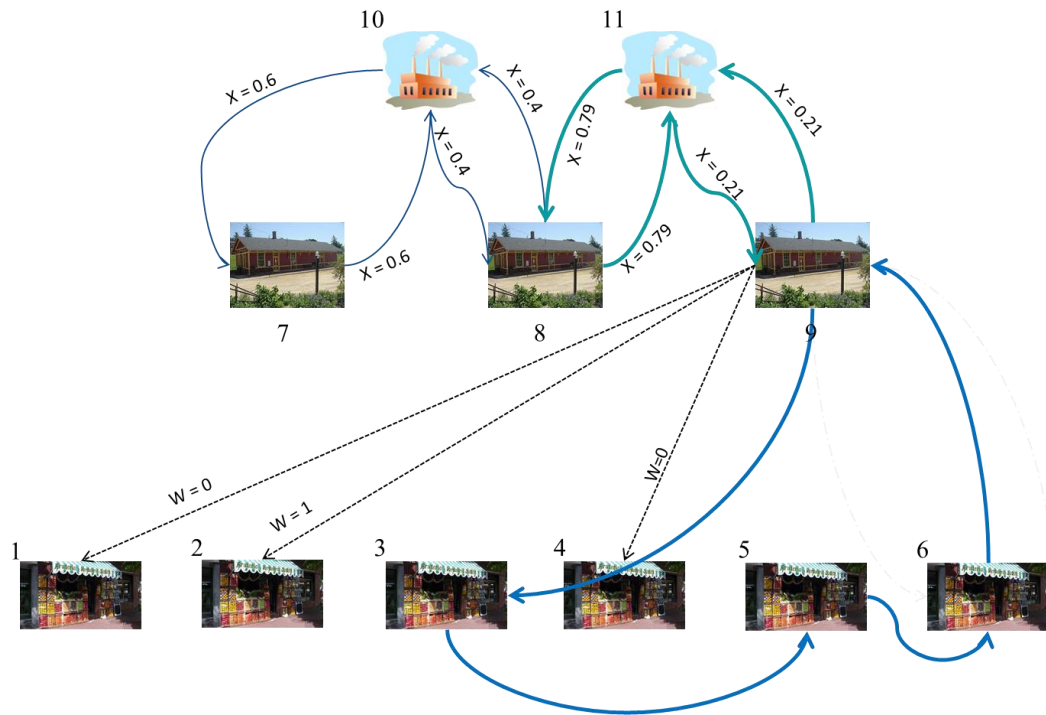


Figure 6-1 A sample of Xpress output after relaxing routing variables in heuristic 2

Therefore, heuristic 2 selects plant 10 as a source of vehicle 1 and plant 11 as a source of vehicle 2. In addition, vehicle 1 must visit bottlers 7 and 8 and vehicle 2 must visit bottlers 8 and 9. Figure 6-2 presents the output of Xpress after heuristic 2 fixes links for these two vehicles.

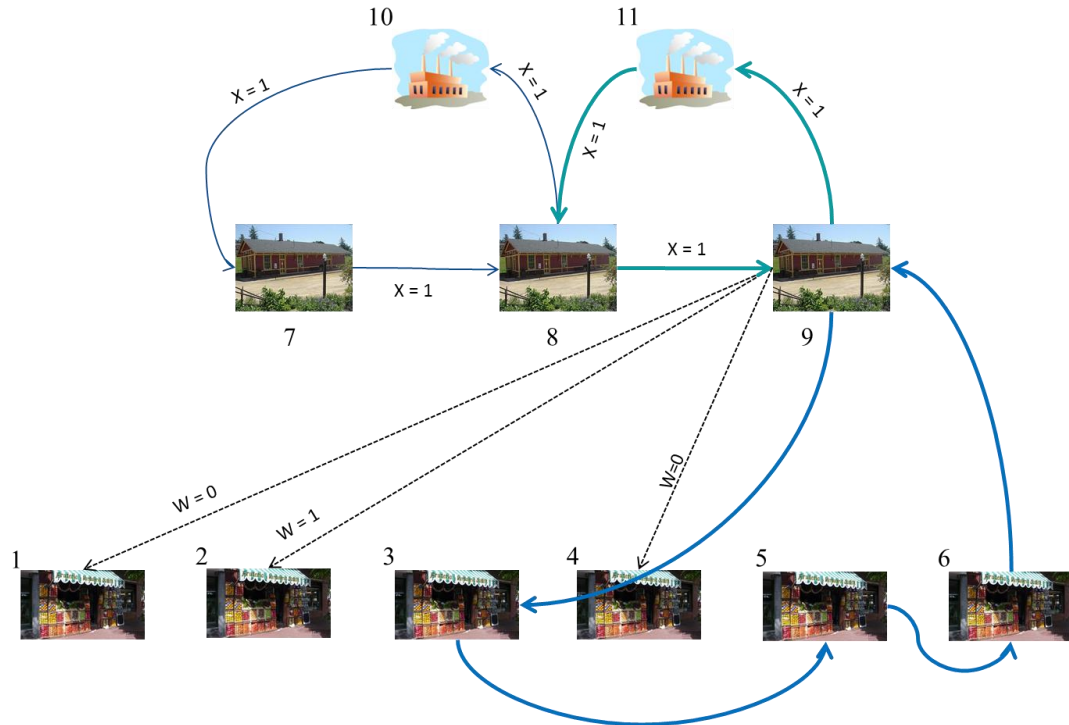


Figure 6-2 The output of Xpress after step 4 of heuristic 2

If a rental vehicle is added to the fleet of the upper level in the next steps, the same process is applied to find a tour for the rental vehicle. Moreover, Step 5 of heuristic 2 has the same changes as Step 4. In other words, for every owned or rental vehicle used in the lower level, a tour is designed in one iteration according to the solution of Xpress after relaxation of the routing variables. This change saves significant running time for the routing process, and since heuristic 2 repeats this process several times, the algorithm's running time is greatly decreased.

6.1.2. Sending more vehicles in Step 6

As explained in Section 5.4.6, heuristic 1 calculates shortages and inventory levels of a time step in Step 6 and, based on the values of these indexes, and according to

different scenarios, decides whether another vehicle should be added to either the upper or lower level. Only one vehicle, therefore, is added to the fleet in each iteration. Heuristic 2 decides the number of added vehicles more intelligently. It takes the shortage amounts and inventory levels, as well as average vehicle capacity, into account and calculates how many vehicles the model needs to serve unsatisfied demands. The number of vehicles needed in the upper level is calculated using Equation 6-1.

$$Vu_i = (X_i - Y_i) / Z_u \quad (6-1)$$

Where:

Vu_i : The number of needed vehicles in the upper level in time step i ;

X_i : The shortage of products for retailers in time step i ;

Y_i : The inventory levels for bottlers in time step i ; and

Z_u : The average capacity of vehicles in the fleet of the upper level.

The number of vehicles added to the upper level is then calculated based on Equation 6-2.

$$NU_i = \text{Min}(f(Vu_i), A_i) \quad (6-2)$$

Where:

NU_i : The number of vehicles added to the upper level in time step i ;

$f(Vu_i)$: The function that rounds up the value of Vu_i ; and

A_i : The number of available vehicles in the upper level in time step i .

Similarly, the number of vehicles needed in the lower level can be calculated according to Equation 6-3.

$$Vw_i = X_i / Z_w \quad (6-3)$$

Where:

Vw_i : The number of needed vehicles in the lower level in time step i ;

X_i : The shortage of products for retailers in time step i ;

Z_w : The average capacity of vehicles in the fleet of the lower level.

The number of vehicles added to the lower level is then calculated according to

Equation 6-4.

$$NW_i = \text{Min}(f(Vw_i), A_i) \quad (6-4)$$

Where:

NW_i : The number of vehicles added to the lower level in time step i ;

$f(Vw_i)$: The function that rounds up the value of Vw_i ; and

A_i : The number of available vehicles in the lower level in time step i .

As a result, heuristic 2 adds vehicles to the upper and lower levels according to Equations 6-2 and 6-4, respectively. The routing process for added vehicle(s) is the same as the process described in Section 6.1.1. The entire algorithm of heuristic 2 can be described as follows:

Step 1: Set $t=1$. Relax all integer decision variables and solve the model optimally.

Step 2: Find all w decision variables in all time steps, which are less than $1/M$.

Step 3: Add a constraint for each w decision variable found in previous step to fix that variable to zero.

Step 4: Do the rest of algorithm for time step t .

Step 5: Find all w decision variable(s) which are greater than $1/M$ and less than one.

Step 6: If only one w is found, add a constraint to the model and fix that w to one. Run the new relaxed model and go to Step 5.

Step 7: If more than one w is found, find the w which has the maximum value. Add a constraint to the model to fix this w to one. Run the new relaxed model and go to Step 5.

Step 8: For each pair of a retailer and a TPL contract, check the associated TN decision variable (TN_I) to see if it is integer or not. If it is not integer go to Step 9, otherwise go to Step 12.

Step 9: If the set of Fix_TN includes another TN decision variable for the same retailer but with different TPL contract, add TN_I to $Exclude_TN$; otherwise add TN_I to Fix_TN .

Step 10: Add one constraint for each TN decision variable in Fix_TN and fix it to one.

Step 11: Add one constraint for every TN decision variable in $Exclude_TN$ and fix it to zero.

Step 12: Go to Step 8 and do the same process for another pair of a retailer and a TPL contract.

Step 13: If $Added_Rental_Upper$ set is empty and $Counter = 0$, exclude all rental vehicles from the fleet of the upper level.

Step 14: Solve the new relaxed model optimally.

Step 15: Find all non-integer XU in the optimal solution of the relaxed model.

Step 16: Find vehicles used in the optimal solution of the relaxed model.

Step 17: For every vehicle used, find the first plant as a source of the vehicle.

Step 18: For every vehicle used, find all bottlers visited by different tours of the vehicle in the optimal solution.

Step 19: Find the shortest tour which starts from the source found in Step 17 and visits all bottlers found in Step 18. The criterion in the shortest tour is the summation of travel time and shipping costs.

Step 20: Add constraints to the model fixing routing variable (XU) of the selected links in Step 19 to one.

Step 21: Solve the new relaxed model optimally.

Step 22: If there is any non-integer XU in the optimal solution, go to Step 15.

Step 23: If *Added_Rental_Lower* set is empty and $Counter = 0$, exclude all rental vehicles from the fleet of the lower level.

Step 24: Solve the new relaxed model optimally.

Step 25: Find all non-integer XW in the optimal solution of the relaxed model.

Step 26: Find vehicles used in the optimal solution of the relaxed model.

Step 27: For every vehicle used, find the first bottler as a source of a vehicle.

Step 28: For every vehicle used, find all retailers visited by different tours of the vehicle in the optimal solution.

Step 29: Find the shortest tour which starts from the source found in Step 27 and visits all bottlers found in Step 28. The criterion in the shortest tour is the summation of travel time and shipping costs.

Step 30: Add constraints to the model fixing routing variable (XW) of the selected links in Step 29 to one.

Step 31: Solve the new relaxed model optimally.

Step 32: If there is any non-integer XW in the optimal solution, go to Step 25.

Step 33: Calculate the *shortage* in retailers and *inventory level* at bottlers for every commodity type.

Step 34: Calculate number of vehicles needed in the upper level according to equation 6-2.

Step 35: Calculate number of vehicles needed in the lower level according to equation 6-4.

Step 36: If $inventory\ level > 0$ and $shortage > 0$ and $inventory\ level = > shortage$ go to Step 37; otherwise go to Step 38.

Step 37: If there is unused rental vehicles in the fleet of the lower level, sort them decreasingly based on fixed and operational costs. Choose needed number of them calculated in Step 35 and add them to the *Added_Rental_Lower* set.

Step 38: If $shortage > inventory\ level$ and $inventory\ level > 0$ go to Step 39; otherwise go to Step 41.

Step 39: If there is unused rental vehicles in the fleet of the lower level, sort them decreasingly based on fixed and operational costs. Choose needed number of them calculated in Step 35 and add them to the *Added_Rental_Lower* set.

Step 40: If there is unused rental vehicles in the fleet of the upper level, sort them decreasingly based on fixed and operational costs. Choose needed number of them calculated in Step 34 and add them to the *Added_Rental_Upper* set.

Step 41: If $shortage > inventory\ level$ and $inventory\ level = 0$ go to Step 42; otherwise go to Step 44.

Step 42: If there is unused rental vehicles in the fleet of the upper level, sort them decreasingly based on fixed and operational costs. Choose needed number of them calculated in Step 34 and add them to the *Added_Rental_Upper* set.

Step 43: If there is unused rental vehicles in the fleet of the lower level, sort them decreasingly based on fixed and operational costs. Choose needed number of them calculated in Step 35 and add them to the *Added_Rental_Lower* set.

Step 44: If *Added_Rental_Upper* set is not empty go to Step 14. Otherwise, if *Added_Rental_Upper* set is not empty go to Step 24.

Step 45: For every used rental vehicles in the upper level, add constraints to fix its variables of fixed and operational costs to one.

Step 46: For every used rental vehicles in the lower level, add constraints to fix its variables of fixed and operational costs to one. Run the new relaxed model.

Step 47: If a vehicle has not been used by the model in the upper level, add it to *Exclude_Upper* set. Add a constraint to the model for each vehicle in *Exclude_Upper* set to fix its *XU* over all links of the upper level to zero.

Step 48: If a vehicle has not been used by the model in the lower level, add it to *Exclude_Lower* set. Add a constraint to the model for each vehicle in *Exclude_Lower* set to fix its *XW* over all links of the lower level to zero.

Step 49: Solve the new relaxed model optimally.

Step 50: If the algorithm is in the last time step, it terminates; otherwise, set $t=t+1$ and go to the Step 4.

Figure 6-3 shows the flow chart of the proposed heuristic for the first time step.

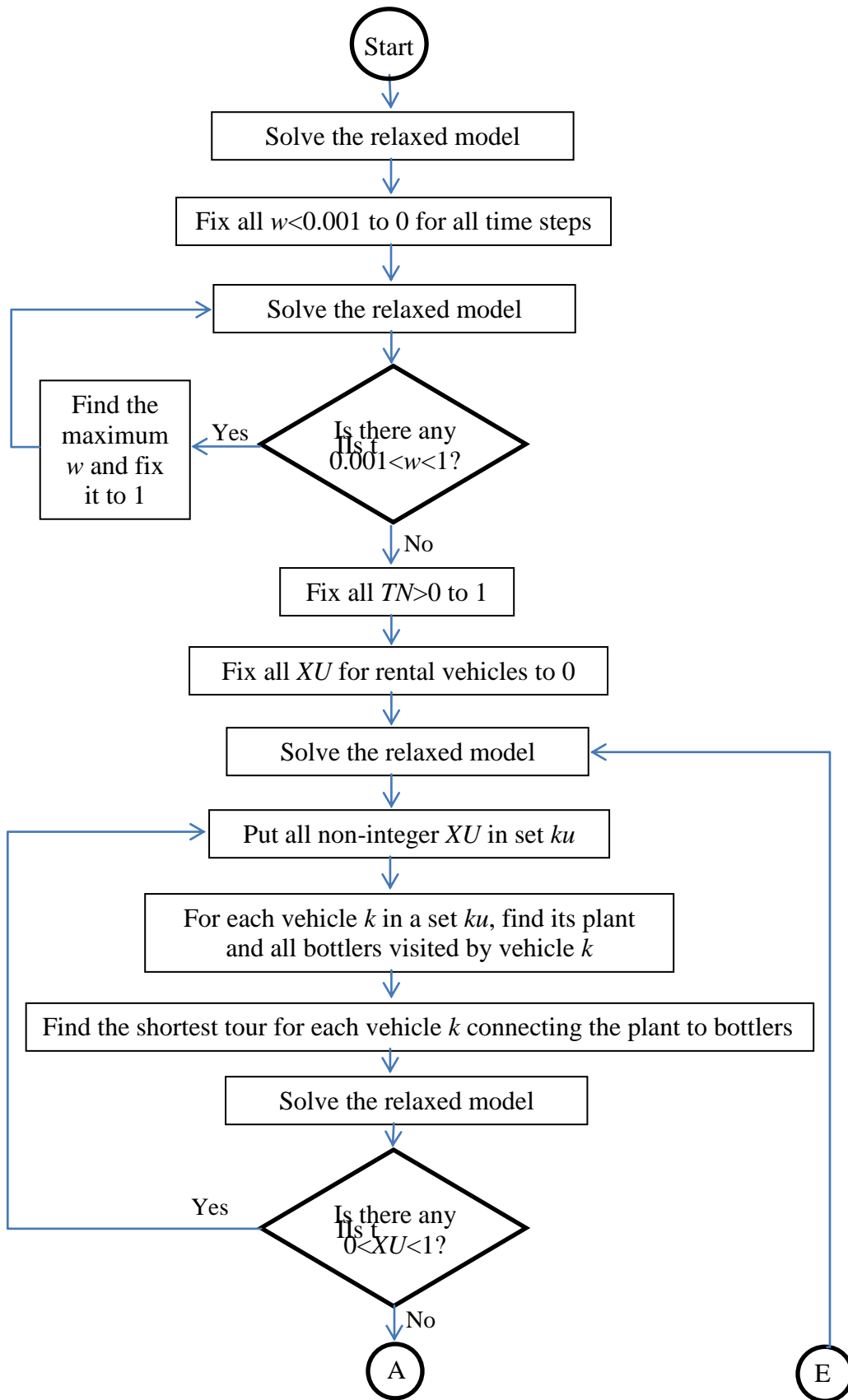


Figure 6-3 The flow chart of heuristic 2 in the first time step

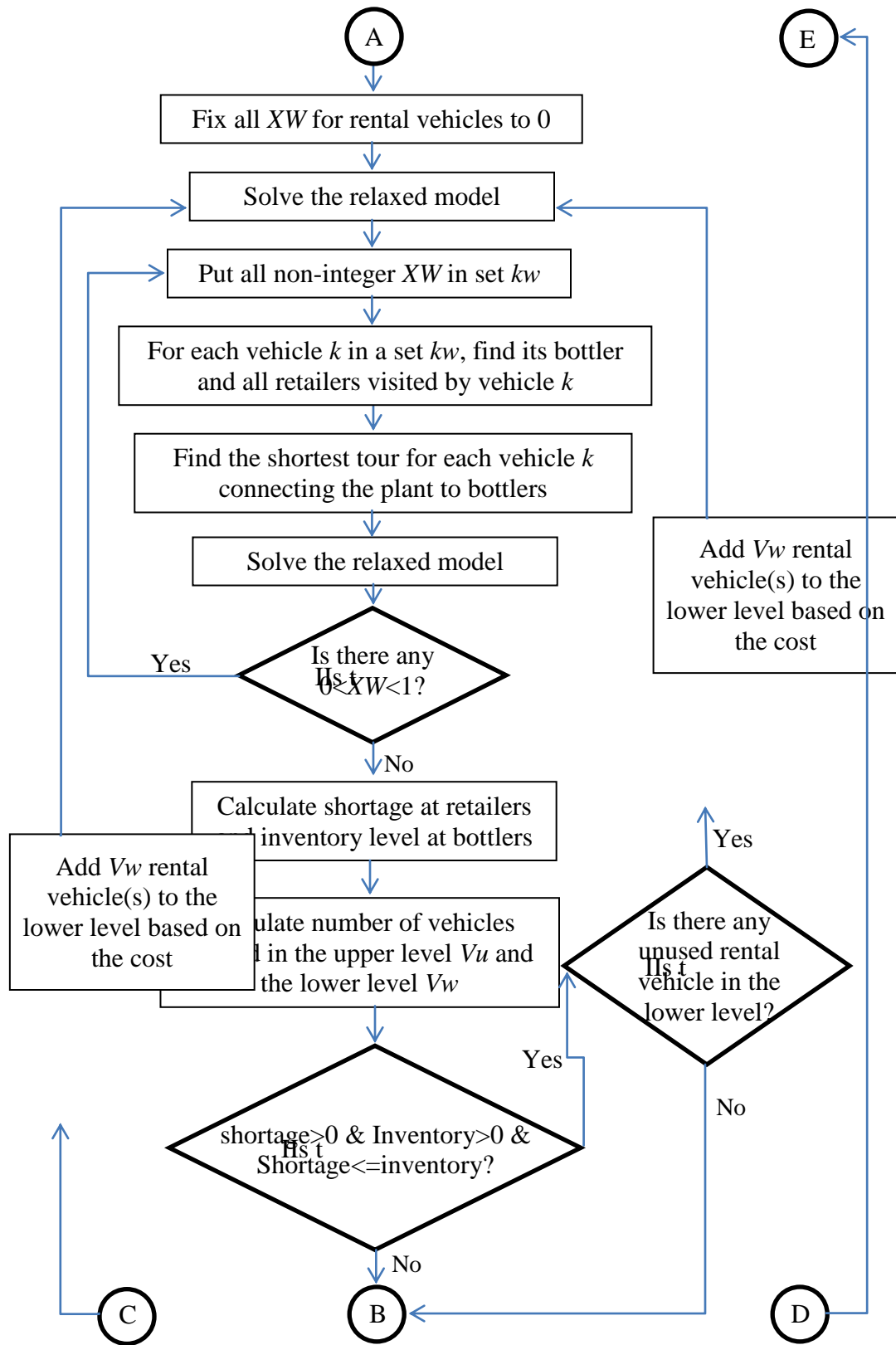


Figure 6-3 The flow chart of heuristic 2 in the first time step (Cont'd)

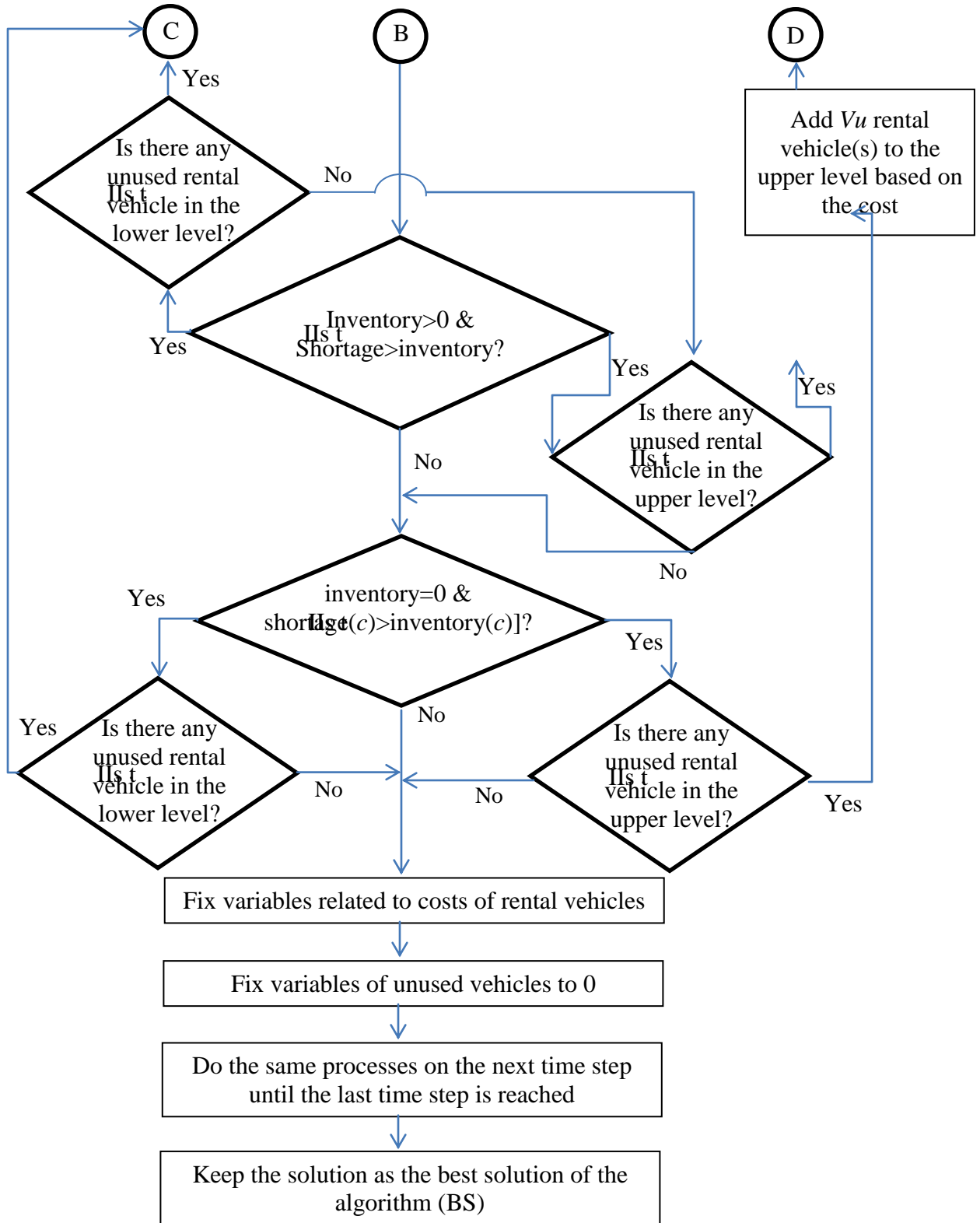


Figure 6-3 The flow chart of heuristic 2 in the first time step (Cont'd)

6.2. Numerical Results

In this section, the results obtained by solving different scenarios with heuristic 2 are presented and described in detail. As mentioned, for better comparison of heuristics 1 and 2, the same categories and scenarios are solved in this section. Therefore, in the next sections, heuristic 2's outputs are discussed but details of categories are not explained.

6.2.1. Category 1

The first category's scenarios were solved by heuristic 2; the results are presented in Table 6-1. Also, Table 6-2 summarizes the information presented in Table 6-1.

Table 6-1. Comparison of results of Xpress and Heuristic 2 on Category 1 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 2 Objective Function (\$)	Gap of the Heuristic 2 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 2 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	49330	48678	1.32	1.32	771	59
2	49105	47564	3.13	3.24	9473	19
3	49330	47382	3.94	3.94	152	33
4	49356	48704	1.32	1.32	204	58
5	45088	44971	0.25	0.25	23	69
6	47877	46312	3.27	3.27	318	63
7	45726	44759	2.11	2.14	79040	60
8	45088	44064	2.27	2.27	98	58
9	44841	44494	0.77	0.77	30	72
10	46299	45447	1.84	1.99	1530	53

Table 6-2. Criteria for comparison of Xpress and Heuristic 2 based on Category 1 scenarios

Criterion	Value
Average of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress (%)	2.05
Variance of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress	1.39
Average of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress (%)	2.02
Variance of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress	1.36
Average of Xpress Running Time (CPU seconds)	9164
Average of the Heuristic 2 Running Time (CPU seconds)	54

According to Tables 6-1 and 6-2, the average gap between the heuristic 2 solution's objective function and the Xpress solution's objective function is around 2%, which is higher than the gap for heuristic 1. This result is consistent with expectations because in the routing part of heuristic 2, all decisions are made simultaneously; however, in heuristic 1 the algorithm makes decisions gradually, based on the model's reaction to every decision that has been made. As Table 6-2 shows, the average running time for heuristic 2 is much less than that of Xpress; however, it is more than that of heuristic 1, which contradicts the expectation. There are two possibilities for this unexpected outcome: First, since the problems were small after a few links had been fixed in heuristic 1, the rest of the selected links in the solution of the model became integers and the heuristics found paths quickly. Second, the RAM of the machine running heuristic 2 was full, which affected the heuristic's speed and solution time. Despite this contradiction, heuristic 2 found appropriate solutions with acceptable gaps in reasonable running time.

6.2.2. Category 2

The results of solving scenarios in the second category are reported in Tables 6-3 and 6-4.

Table 6-3. Comparison of the results of Xpress and Heuristic 2 on Category 2 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 2 Objective Function (\$)	Gap of the Heuristic 2 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 2 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	80170	79042	1.40	1.40	42615	423
2	79901	77759	2.68	2.71	21570	269
3	80122	79120	1.25	1.65	4482	300
4	80129	78791	1.67	1.95	9283	501
5	80239	79483	0.99	1.22	19755	339
6	81656	80522	1.39	1.54	9897	414
7	82998	82062	1.13	1.23	25387	398
8	82681	81371	1.58	1.58	36195	367
9	81235	78971	2.79	3.49	17306	349
10	80455	78557	2.36	3.07	61521	419

Table 6-4. Criteria for comparison of Xpress and Heuristic 2 based on Category 2 scenarios

Criterion	Value
Average Gap between Heuristic 2 Solution and the Best Bound Found by Xpress (%)	1.98
Variance Gap between Heuristic 2 Solution and the Best Bound Found by Xpress	0.66
Average Gap between Heuristic 2 Solution and the Best Solution Found by Xpress (%)	1.72
Variance Gap between Heuristic 2 Solution and the Best Solution Found by Xpress	0.42
Average Xpress Running Time (CPU seconds)	24801
Average the Heuristic 2 Running Time (CPU seconds)	378

The gaps between heuristic 2 solutions and Xpress solutions were greater than the gaps of heuristic 1 and Xpress. The running time, however, improved in this category, as expected. This confirms that heuristic 2 sacrifices accuracy to find a

solution in a shorter time. The outcome of heuristic 2 is still acceptable and has a reasonable gap with Xpress results.

6.2.3. Category 3

Tables 6-5 and 6-6 show the results of applying the second heuristic to the scenarios of Category 3.

Table 6-5. Comparison of the results of Xpress and Heuristic 2 on Category 3 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 2 Objective Function (\$)	Gap of the Heuristic 2 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 2 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	118019	116709	1.11	1.61	38656	4200
2	89067	87475	1.79	2.84	12029	4380
3	115753	103173	10.87	11.30	53738	3762
4	118047	115176	2.43	2.88	29160	3974
5	122439	120602	1.50	1.76	18072	2077
6	113127	111795	1.18	1.55	50871	2328
7	109817	109369	0.41	1.01	25023	2950
8	111336	109523	1.63	2.01	29228	2544
9	121530	120336	0.98	1.47	13021	3315
10	109682	108498	1.07	1.07	12217	2946

Table 6-6. Criteria for comparison of Xpress and Heuristic 2 based on Category 3 scenarios

Criterion	Value
Average of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress (%)	2.75
Variance of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress	9.43
Average of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress (%)	2.30
Variance of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress	9.37
Average of Xpress Running Time (CPU seconds)	28492
Average of the Heuristic 2 Running Time (CPU seconds)	3248

According to Table 6-6, the gap between heuristic 2 and Xpress solutions as well as the bound has improved in comparison to heuristic 1. Moreover, the average running time for heuristic 2 is less than half of heuristic 1's mean running time. This is the first category in which heuristic 2 performs better than heuristic 1. The difference between running time for heuristic 2 and Xpress is noticeable in this category.

6.2.4. Category 4

Outputs for heuristic 2 and Xpress on this larger category are presented in Tables 6-7 and 6-8. This category includes 5 scenarios.

Table 6-7. Comparison of the results of Xpress and Heuristic 2 on Category 4 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 2 Objective Function (\$)	Gap of the Heuristic 2 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 2 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	147348	145018	1.58	2.31	8333	14274
2	142034	140355	1.18	1.18	3501	9889
3	142503	141544	0.67	0.73	38038	6254
4	164400	163853	0.33	0.72	11870	7279
5	164721	164838	-0.07	0.25	10151	6085

Table 6-8. Criteria for comparison of Xpress and Heuristic 2 based on Category 4 scenarios

Criterion	Value
Average of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress (%)	1.04
Variance of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress	0.61
Average of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress (%)	0.74
Variance of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress	0.43
Average of Xpress Running Time (CPU seconds)	14378
Average of the Heuristic 2 Running Time (CPU seconds)	8756

As mentioned in Section 5.6.4, the solver was stopped after finding a solution with a gap of less than 0.8% between the best solution and the best bound. As a result, the reported running time in Table 6-7 is not comparable to running times for other categories. Despite this fact, heuristic 2 has a lower running time compared to Xpress, and its average gap is around 1% with the best bound. Similar to results for Category 3, heuristic 2 found better solutions than heuristic 1 for scenarios in this category; however, their objective functions were very close to each other.

6.2.5. Category 5

Tables 6-9 and 6-10 present the outputs of heuristic 2 and Xpress for five scenarios in Category 5. In this category, the first and second improvement phases were eliminated from heuristic 2; thus, it contains only the third improvement phase. This policy will be applied to larger categories as well.

Comparison of the results provided by heuristics 1 and 2 shows that heuristic 2 had a smaller average gap than heuristic 1; however, in four scenarios heuristic 1 found better solutions. In other words, the output of heuristic 1 for the first scenario was far from that of the Xpress solution (the gap was more than 9%), which ruined the average gap. Similar to its performance in previous categories, heuristic 2 was much faster than heuristic 1.

Table 6-9. Comparison of the results of Xpress and Heuristic 2 on Category 5 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 2 Objective Function (\$)	Gap of the Heuristic 2 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 2 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	126274	122726	2.81	3.61	34287	11224
2	147278	145009	1.54	2.40	61888	11991
3	157349	154862	1.58	2.07	32610	9706
4	144202	142616	1.10	1.64	24915	9400
5	145404	142043	2.31	2.77	41285	10577

Table 6-10. Criteria for comparison of Xpress and Heuristic 2 based on Category 5 scenarios

Criterion	Value
Average of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress (%)	2.49
Variance of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress	0.56
Average of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress (%)	1.87
Variance of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress	0.46
Average of Xpress Running Time (CPU seconds)	38997
Average of the Heuristic 2 Running Time (CPU seconds)	10580

6.2.6. Category 6

Tables 6-11 and 6-12 present the outputs for heuristic 2 and Xpress for five scenarios of Category 5. In this category, heuristic 1 has better performance in terms of a gap with Xpress solutions; however, the average gap of heuristic 2 is less than 4%, which means that heuristic 2 has found good solutions. On the other hand, the average running time of heuristic 2 is almost half of heuristic 1. In other words, heuristic 2 delivers good solutions in a short time especially when it is compared to Xpress which needs to run more than 10 hours without finding an optimal solution.

Table 6-11. Comparison of the results of Xpress and Heuristic 2 on Category 6 scenarios

Scenario	Xpress Objective Function (\$)	Heuristic 2 Objective Function (\$)	Gap of the Heuristic 2 Objective Function with		Xpress Running Time (CPU seconds)	Heuristic 2 Running Time (CPU seconds)
			The Best Solution (%)	The Best Bound (%)		
1	176600	170574	3.41	4.60	43823	5967
2	182435	174194	4.52	5.56	30124	4832
3	184701	178886	3.15	4.53	36585	4926
4	181661	180722	0.52	0.97	65168	3791
5	178979	172230	3.77	4.27	52403	4048

Table 6-12. Criteria for comparison of Xpress and Heuristic 2 based on Category 6 scenarios

Criterion	Value
Average of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress (%)	3.99
Variance of Gap between Heuristic 2 Solution and the Best Bound Found by Xpress	3.08
Average of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress (%)	3.07
Variance of Gap between Heuristic 2 Solution and the Best Solution Found by Xpress	2.30
Average of Xpress Running Time (CPU seconds)	45620
Average of the Heuristic 2 Running Time (CPU seconds)	4712

6.2.7. Category 7

Category 7 is a category for which Xpress is not able to find a feasible solution. Heuristic 1 delivered a feasible solution with appropriate gap with the best bound but in long running times. Tables 6-13 and 6-14 show the performance of both heuristics on 5 scenarios of this category. According to Tables 6-13 and 6-14, heuristic 1 provided better solutions than heuristic 2 with a smaller gap with the objective function value of relaxed problems. Since Xpress could not find even a feasible solution for the scenarios in this category, the objective function value of a

relaxed problem was considered as an upper bound. Based on the outputs of other categories and scenarios, there was a better upper bound than a relaxed objective function. As a result, the real value of a gap reported in Table 6-13 is smaller.

Table 6-13. Comparison of the results of Heuristic 1 and Heuristic 2 on Category 7 scenarios

Scenario	Heuristic 1 Objective Function (\$)	Heuristic 2 Objective Function (\$)	Gap of Heuristic 1 with Relaxed Problem	Gap of Heuristic 2 with Relaxed Problem	Heuristic 1 Running Time (CPU seconds)	Heuristic 2 Running Time (CPU seconds)
1	268836	266011	3.61	4.62	4 days	41305
2	272568	268903	3.79	5.08	4 days	36960
3	276753	273169	3.31	4.56	4 days	41365
4	390395	387270	3.87	4.64	4.5 days	69467
5	398105	394026	3.63	4.61	4.5 days	67265

Table 6-14. Criteria for comparison of Heuristics 1 and 2 based on Category 7 scenarios

Criterion	Value
Average of Gap between Heuristic 1 Solution and the Relaxed Problem Objective Function (%)	3.64
Variance of Gap between Heuristic 1 Solution and the Relaxed Problem Objective Function	0.04
Average of Gap between Heuristic 2 Solution and the Relaxed Problem Objective Function (%)	4.70
Variance of Gap between Heuristic 2 Solution and the Relaxed Problem Objective Function	0.04
Average of the Heuristic 1 Running Time (CPU seconds)	4 days
Average of the Heuristic 2 Running Time (CPU seconds)	51272

On the other hand, heuristic 2's running time was significantly shorter than that of heuristic 1. This is the main advantage of heuristic 2, which provides good solutions in a reasonable amount of computation time. In other words, heuristic 2 sacrifices accuracy to find an appropriate solution in an acceptable running time.

6.3. Summary

In this chapter, another heuristic algorithm based on the first heuristic algorithm was proposed to solve the problem. Heuristic 2, which uses a fix-and-run algorithm as a background, is the same as heuristic 1 except for changes in some of the steps. The motivation for developing heuristic 2 was heuristic 1's long running time for large problems. Heuristic 2 solves problems significantly faster, which is its main advantage.

The same scenarios as those in Chapter 5 were solved by heuristic 2 to evaluate its performance. Figures 6-4 and 6-5 compare the performances of heuristics 1 and 2 in different categories from the standpoints of gap and running time, respectively. According to Figure 6-4, in four categories heuristic 1 provided better solutions than heuristic 2. As explained in Section 6.2.5, in four scenarios of Category 5 heuristic 1 delivered better solutions than heuristic 2; however, the gap for the solution of another scenario was very high, which caused the average gap of heuristic 1 to be higher than heuristic 2.

Moreover, the variance of gap between the solution of heuristic 2 and the best solution or best bound found by Xpress is acceptable in most of categories except Category 3. As a result, similar to heuristic 1, heuristic 2 performs consistently over different scenarios with the same main characteristics.

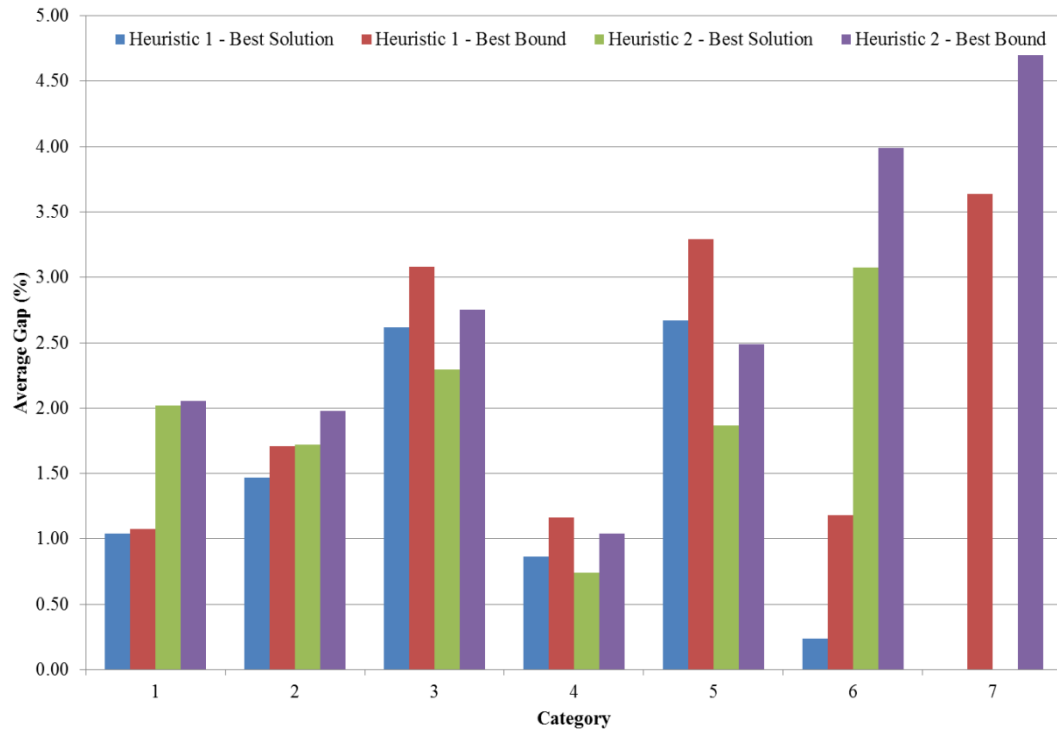


Figure 6-4 Average gaps of heuristics 1 and 2 solutions with Xpress outputs in different categories

Figure 6-5 shows the most important characteristics of heuristic 2. This heuristic is much faster than heuristic 1, which provides good solutions in acceptable running time. Since the average running times for heuristics 1 and 2 for Category 7 scenarios are significantly higher than other numbers in Figure 6-5, they have not been shown to scale. According to Figure 6-5, heuristic 2 was able to find appropriate solutions for the problems in Category 7. Xpress was not able to find feasible solutions for those problems, and heuristic 1 required around 4 days to deliver solutions. Also, the running times for Category 1 are not shown due to their small values.

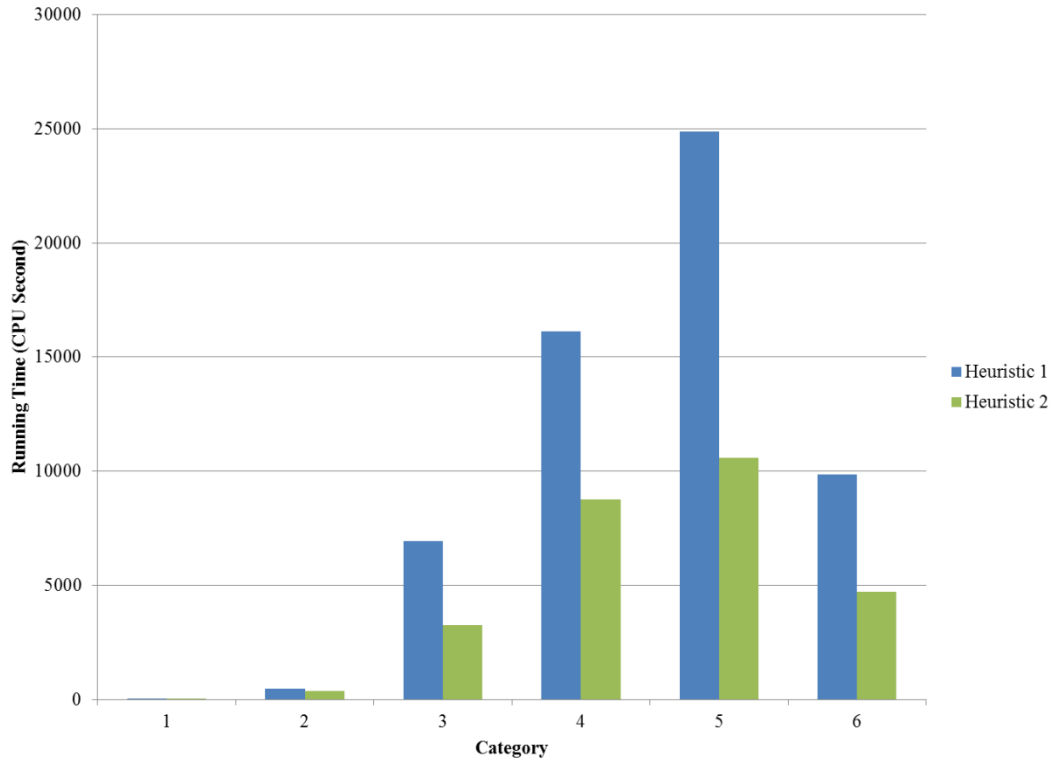


Figure 6-5 Average running time for heuristics 1 and 2 in different categories

As a result, heuristic 1 looks more accurate than heuristic 2, while heuristic 2 is sufficiently precise. If running time is not a concern, heuristic 1 is the better approach, since it is more likely to provide a better solution than heuristic 2. If a solution is necessary in a shorter time, heuristic 2 is the appropriate method to use; it delivers reliable results in a reasonable running time. In addition, both heuristics beat Xpress for very large-sized problems and provide a feasible solution, whereas Xpress is not able to do this.

Chapter 7: Level Decomposition

One of the characteristics of the research problem is integrating different activities of the supply chain. This problem consists of the transportation of raw material from sources (plants) to factories (bottlers), production of final products in factories (bottlers), and shipping of final commodities to consumption locations (retailers). The problem, therefore, includes two levels and integrating these levels increases the complexity of the problem significantly. Another approach to approximately solve this problem is to decompose the two levels and solve them separately. In other words, the model for the upper level is solved first. Some output of the first model becomes input for the second model, which solves the lower part of the original model. Besides the fact that this solution provides another heuristic approach, it demonstrates by how much the integration of levels will increase a company's profit. Section 7.1 provides more detail as to the mathematical formulation of the level-decomposition approach, and Section 7.2 presents the results of solving different problems with this method. Section 7.3 concludes with a discussion of the benefits and disadvantages of the level-decomposition approach.

7.1. Mathematical Formulation for Level Decomposition

In this section, the mathematical formulation for this approach is presented. Since this approach uses most of the constraints described in Section 3.4.5, only constraints that

have been changed are explained and the rest of constraints are shown with their equation in Section 3.4.5. Similarly, sets and parameters are the same as in Chapter 3, and the new parameters and decision variables are introduced.

7.1.1. The Upper-Level Model

In the upper-level problem, syrup concentrate must be delivered to bottlers to produce final products. Bottlers' demand is based on retailers' orders. Bottlers are not consumption locations; they play an intermediate role in the supply chain between plants and retailers. In the original model, this relationship between plants and retailers through bottlers was defined by connecting different levels. This information transmission from lower-level components to the upper-level model must be considered.

As a result, retailers' orders must be transferred to bottlers to be used in the upper-level model. Since there are multiple locations for bottlers, the assignment of retailers to bottlers affects the final objective function. The best measure for this assignment is the travel time between retailers and bottlers. In other words, a retailer's order is assigned to the demand of the closest bottler. Figure 7-1 shows the process of assignment of retailers to bottlers.

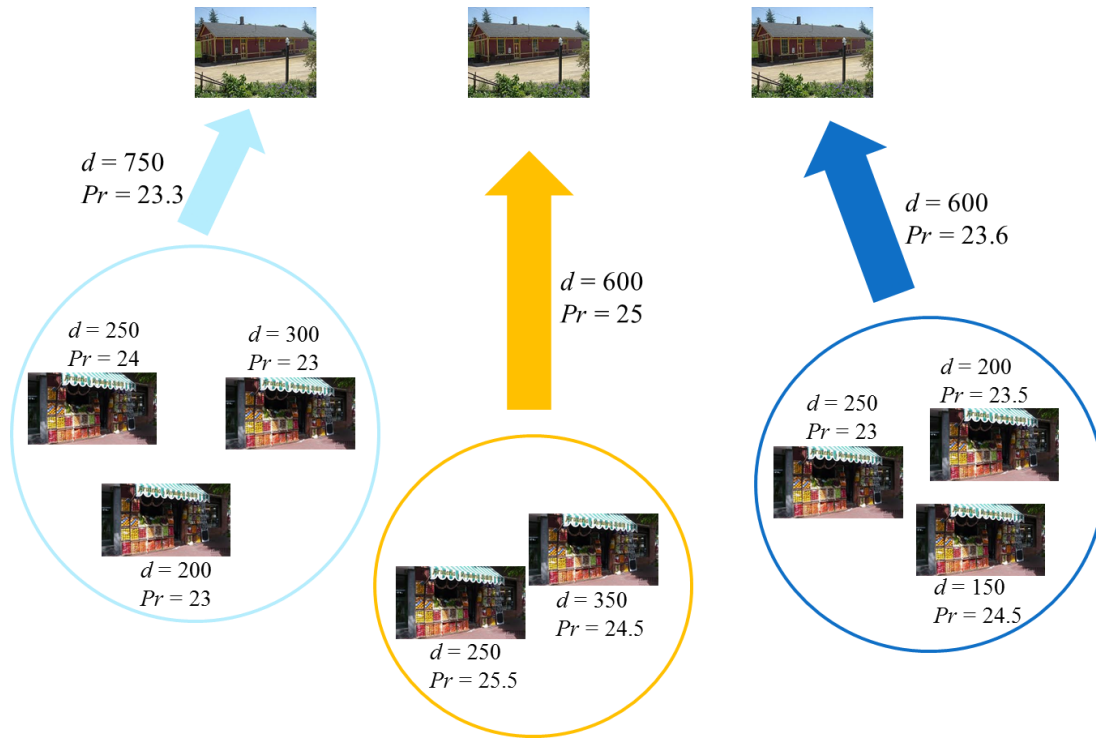


Figure 7-1. Assignment of retailers' orders to the demands of bottlers

In the upper-level model, the role of bottlers is the same as retailers in the original model. In other words, demand for different products, product price, and shortage penalties are defined for bottlers in the upper-level model. Bottlers' demand for and price of a commodity type are calculated as shown in Figure 7-1. Retailers assigned to a bottler are used as a base for demand, product price, and shortage-penalty calculation. The average price of a commodity type for retailers assigned to a bottler is considered to be the price of that particular product for the same bottler. Similarly, the same calculation is applied for a shortage penalty, whereas the original value of a bottler's warehouse capacity remains in the model. As a result, the mathematical formulation of the upper-level model can be written as flows:

$$\begin{aligned}
& Max \sum_{i \in W} \sum_{s \in S} \sum_{t \in T} Pc_{sit} DE_{ist} - \sum_{i \in W \cup P} \sum_{j \in W \cup P} \sum_{k \in GVU} \sum_{t \in T} \alpha SFu_k C_{ij} XU_{ijkt} - \\
& - \sum_{i \in P} \sum_{j \in W} \sum_{l \in LN} \sum_{s \in S} \sum_{k \in GVU} \sum_{t \in T} \beta Qu_{lskt}^{ij} len_{ij} - \sum_{i \in W} \sum_{s \in S} \sum_{t \in T} e_{ist} Sh_{sit} - \sum_{k \in VU} \sum_{t \in T} FCV u_{kt} U u_{kt} - \\
& \sum_{k \in VU} \sum_{t \in T} EPCV u_{kt} VZ u_{kt} - \sum_{i \in W} \sum_{s \in S} \sum_{t \in T} Pd_{ist} PdCost_{is} \quad (7-1)
\end{aligned}$$

s.t.

$$I_{sit} - I_{si(t-1)} - Pd_{ist} + DE_{ist} = 0 \quad \forall i \in W, \forall s \in S, \forall t \in T \quad (7-2)$$

$$\sum_{j \in PUW} XU_{ijkt} \leq 1 \quad \forall i \in P \cup W, \forall k \in GVU, \forall t \in T \quad (7-3)$$

$$\sum_{i \in PUW} XU_{ijkt} - \sum_{m \in PUW} XU_{jmkt} = 0 \quad \forall j \in W, \forall k \in GVU, \forall t \in T \quad (7-4)$$

$$\sum_{j \in W} \sum_{t \in T} XU_{ijkt} - \sum_{j \in W} \sum_{t \in T} XU_{jikt} = 0 \quad \forall i \in P, \forall k \in GVU \quad (7-5)$$

$$\sum_{i \in P} \sum_{j \in W \cup P} XU_{ijkt} \leq 1 \quad \forall k \in GVU, \forall t \in T \quad (7-6)$$

$$\sum_{k \in GVU} \sum_{t \in T} XU_{iikt} = 0 \quad \forall i \in W \quad (7-7)$$

$$TLU + \left(\sum_{i \in W \cup P} \sum_{j \in W \cup P} XU_{iikt} - 1 \right) \times TLUU + \sum_{i \in W \cup P} \sum_{j \in W \cup P} XU_{iikt} C_{ij} SFu_k \leq TT_u \quad \forall k \in GVU, \forall t \in T \quad (7-8)$$

$$\sum_{i \in W \cup P} \sum_{t \in T} XU_{iJKt} = 0 \quad (7-9)$$

$$\sum_{s \in S_1} \sum_{m \in P} \sum_{n \in W} a_s Qu_{lskt}^{mn} \leq Cku_k \quad \forall i \in W \cup P, \forall k \in GVU, \forall l \in Lnu(i, k), \forall t \in T \quad (7-10)$$

$$\begin{aligned}
\sum_{l \in Lnu(i, k)} \sum_{m \in P} \sum_{n \in W} Qu_{lskt}^{mn} &= \sum_{l \in Lnu(i, k)} \sum_{m \in P} \sum_{n \in W} Qu_{lskt}^{mn} - \sum_{l \in Lnu(i, k)} \sum_{m \in P} Qu_{lskt}^{mi} \\
&\forall i \in W, \forall k \in GVU, \forall s \in S, \forall t \in T \quad (7-11)
\end{aligned}$$

$$\begin{aligned}
\sum_{\substack{l \in Lnu(i, k) \\ i \neq m}} \sum_{n \in W} \sum_{k \in GVU} \sum_{s \in S} Qu_{lskt}^{mn} &= 0 \quad \forall i, m \in P, \forall t \in T \quad (7-12)
\end{aligned}$$

$$\sum_{l \in Lnu(i,k)} Qu_{lskt}^{mn} = \sum_{l \in Leu(i,k)} Qu_{lskt}^{mn} \quad \forall i, n \in W (n \neq i), \forall m \in P, \forall k \in GVU, \forall s \in S, \forall t \in T \quad (7-13)$$

$$\sum_{l \in Leu(i,k)} \sum_{m \in P} \sum_{k \in GVU} Qu_{lskt}^{mi} - Sc_{ist} - \sum_{s_2 \in S} \frac{Pd_{ist}}{\lambda_{s_2}} = 0 \quad \forall i \in W, \forall s \in S_1, \forall t \in T \quad (7-14)$$

$$Pd_{ist} \leq CAP_{sit} \quad \forall i \in W, \forall s \in S, \forall t \in T \quad (7-15)$$

$$Sc_{ist} \leq SHC_{sit} \quad \forall i \in W, \forall s \in S_1, \forall t \in T \quad (7-16)$$

$$I_{sit} \leq HC_{sit} \quad \forall i \in W, \forall s \in S, \forall t \in T \quad (7-17)$$

$$DE_{ist} \leq DM_{ist} \quad \forall i \in W, \forall s \in S, \forall t \in T \quad (7-18)$$

$$e_{ist} + DE_{ist} = DM_{ist} \quad \forall i \in W, \forall s \in S, \forall t \in T \quad (7-19)$$

$$e_{is(t-1)} + d_{ist} = DM_{ist} \quad \forall i \in W, \forall s \in S, \forall t \in T \quad (7-20)$$

$$I_{is0} = 0 \quad \forall i \in W, \forall s \in S \quad (7-21)$$

$$e_{is0} = 0 \quad \forall i \in W, \forall s \in S \quad (7-22)$$

$$\sum_{\substack{l \in Lnu(i,k) \\ NE(l)=j}} \sum_{m \in P} \sum_{n \in W} \sum_{s \in S} Qu_{lskt}^{mn} \leq M XU_{ijkt} \quad \forall i, j \in W \cup P, \forall k \in GVU, \forall t \in T \quad (7-23)$$

$$\sum_{i \in W \cup P} \sum_{j \in W \cup P} XU_{ijkt} \leq M VZu_{kt} \quad \forall k \in VU, \forall t \in T \quad (7-24)$$

$$If VZu_{k(t+1)} - VZu_{kt} = 1 \Rightarrow Uu_{k(t+1)} = 1 \quad \forall k \in VU, \forall t \in T \quad (7-25)$$

$$VZu_{k1} \leq M Uu_{k1} \quad \forall k \in VU \quad (7-26)$$

$$Qu_{lskt}^{mn}, I_{sit}, e_{ist}, DE_{ist}, DM_{ist}, Sc_{ist} \text{ and } Pd_{ist} \geq 0 \quad \text{Real-valued decision variable} \quad (7-27)$$

$$XU_{ijkt}, VZu_{kt}, Uu_{kt} \in \{0, 1\} \quad \text{Binary integer variables} \quad (7-28)$$

Equation 7-1 is the objective function of the upper level, calculating the profit of the company. As mentioned, in this model bottlers act like retailers did in the original model. Therefore, they have determined their order for each commodity type, and sell products based on the price that is equal to the average of the prices for the same retailers assigned to the bottler. As a result, the first term in Equation 7-1 shows the revenue received from selling the final products to bottlers. The second and third terms are the travel time costs and the shipping costs in the upper level, which are calculated the same as in the original model. The fourth term in Equation 7-1 is the shortage penalty cost at bottlers. The sixth and seventh terms indicate the fixed and operational costs of rental vehicles, and the last term is production cost. Since TPLPs are not used in the upper level, its objective function does not include the TPL cost.

Constraints of the upper-level model come from the original model, except constraint 3-17 which has been replaced by Equation 7-2. Variables of Qu and TW in Equation 3-17, which are lower-level decision variables, are replaced by DE , which has the same implicit meaning and function. The output of the upper-level model includes the syrup concentrate delivery plan in the upper level and delivery amounts to a group of retailers from each bottler (decision variable DE), which is an input to the lower-level model described in next section.

7.1.2. The Lower Level Model

The mathematical formulation of the lower-level model is shown below.

$$\begin{aligned}
& \text{Max} \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} P c_{sit} DE_{ist} - \sum_{i \in W \cup R} \sum_{j \in W \cup R} \sum_{k \in GVW} \sum_{t \in T} \alpha SF w_k C_{ij} XW_{ijkt} - \\
& \sum_{i \in W} \sum_{j \in R} \sum_{l \in LN} \sum_{s \in S} \sum_{k \in GVW} \sum_{t \in T} \beta Q w_{lskt}^{ij} len_{ij} - \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} e_{ist} Sh_{sit} - \sum_{i \in R} \sum_{s \in S} \sum_{t \in T} I_{sit} h_{sit} - \\
& \sum_{k \in VW} \sum_{t \in T} FCV w_{kt} U w_{kt} - \sum_{k \in VW} \sum_{t \in T} EPCV w_{kt} VZ w_{kt} - \sum_{q \in TP} \sum_{i \in L} \sum_{s \in S} \sum_{t \in T} CT_{sqi} w_{qist} - \\
& \sum_{i \in W} \sum_{j \in R} \sum_{q \in TP} \sum_{s \in S} \sum_{t \in T} Z_{sqt}^{ij} WR_s
\end{aligned} \tag{7-29}$$

s.t.

$$\begin{aligned}
Sol_DE_{ist} &= \sum_{l \in Lw(i,k)} \sum_{k \in GVW} \sum_{m \in R} Q w_{lskt}^{im} + \sum_{j \in R} \sum_{q \in TP} \sum_{m \in T} TW_{smtq}^{ij} \\
&\quad \forall i \in W, \forall s \in S, \forall t \in T
\end{aligned} \tag{7-30}$$

$$\sum_{j \in R} XW_{ijkt} \leq 1 \quad \forall i \in W, \forall k \in GVW, \forall t \in T \tag{7-31}$$

$$\sum_{j \in R \cup W} XW_{ijkt} \leq 1 - \sum_{q \in TP} TN_{iqt} \quad \forall i \in R, \forall k \in GVW, \forall t \in T \tag{7-32}$$

$$\sum_{i \in R \cup W} XW_{ijkt} - \sum_{m \in R \cup W} XW_{jmkt} = 0 \quad \forall j \in R, \forall k \in GVW, \forall t \in T \tag{7-33}$$

$$\sum_{j \in R} \sum_{t \in T} XW_{ijkt} - \sum_{j \in R} \sum_{t \in T} XW_{jikt} = 0 \quad \forall i \in W, \forall k \in GVW \tag{7-34}$$

$$\sum_{i \in W} \sum_{j \in W \cup R} XW_{ijkt} \leq 1 \quad \forall k \in GVW, \forall t \in T \tag{7-35}$$

$$\sum_{k \in GVW} \sum_{t \in T} XW_{iikt} = 0 \quad \forall i \in R \tag{7-12}$$

$$\begin{aligned}
TLW + \left(\sum_{i \in W \cup R} \sum_{j \in W \cup R} XW_{iikt} - 1 \right) \times TLUW + \sum_{i \in W \cup R} \sum_{j \in W \cup R} XW_{iikt} C_{ij} SF w_k \leq TT_w \\
\quad \forall k \in GVW, \forall t \in T
\end{aligned} \tag{7-36}$$

$$\sum_{i \in W \cup R} \sum_{t \in T} XW_{iJKt} = 0 \tag{7-37}$$

$$\begin{aligned}
I_{sit} = I_{si(t-1)} + \sum_{l \in Lew(i,k)} \sum_{k \in GVW} \sum_{m \in W} Q w_{lskt}^{mi} + \sum_{j \in W} \sum_{q \in TP} \sum_{m \in T} TW_{smtq}^{ji} - DE_{ist} \\
\quad \forall i \in R, \forall s \in S, \forall t \in T
\end{aligned} \tag{7-38}$$

$$\sum_{s \in S} \sum_{m \in W} \sum_{n \in R} a_s Q u_{lskt}^{mn} \leq C k w_k \quad \forall i \in W \cup R, \forall k \in GVW, \forall l \in Lnw(i, k), \forall t \in T \quad (7-39)$$

$$\sum_{l \in Lnw(i, k)} \sum_{m \in W} \sum_{n \in R} Q w_{lskt}^{mn} = \sum_{l \in Lew(i, k)} \sum_{m \in W} \sum_{n \in R} Q w_{lskt}^{mn} - \sum_{l \in Lew(i, k)} \sum_{m \in W} Q w_{lskt}^{mi} \quad (7-40)$$

$$\forall i \in R, \forall k \in GVW, \forall s \in S, \forall t \in T$$

$$\sum_{\substack{l \in Lnw(i, k) \\ i \neq m}} \sum_{n \in R} \sum_{k \in GVW} \sum_{s \in S} Q w_{lskt}^{mn} = 0 \quad \forall i, m \in W, \forall t \in T \quad (7-41)$$

$$\sum_{l \in Lnw(i, k)} Q w_{lskt}^{mn} = \sum_{l \in Lew(i, k)} Q w_{lskt}^{mn} \quad \forall i, n \in R (n \neq i), \forall m \in W, \forall k \in GVW, \forall s \in S, \forall t \in T \quad (7-42)$$

$$\sum_{l \in Lnu(i, k)} \sum_{m \in W} \sum_{n \in R} \sum_{k \in GVW} \sum_{s \in S} Q w_{lskt}^{mn} = 0 \quad \forall i \in W \cup P, \forall t \in T \quad (7-43)$$

$$I_{sit} \leq H C_{sit} \quad \forall i \in W \cup R, \forall s \in S, \forall t \in T \quad (7-44)$$

$$D E_{ist} \leq D M_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T \quad (7-45)$$

$$e_{ist} + D E_{ist} = D M_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T \quad (7-46)$$

$$e_{is(t-1)} + d_{ist} = D M_{ist} \quad \forall i \in R, \forall s \in S, \forall t \in T \quad (7-47)$$

$$I_{si0} = 0 \quad \forall i \in W \cup R, \forall s \in S \quad (7-48)$$

$$e_{is0} = 0 \quad \forall i \in R, \forall s \in S \quad (7-49)$$

$$\sum_{\substack{l \in Lnw(i, k) \\ NE(l) = j}} \sum_{m \in W} \sum_{n \in R} \sum_{s \in S} Q w_{lskt}^{mn} \leq M X W_{ijkt} \quad \forall i, j \in W \cup R, \forall k \in GVW, \forall t \in T \quad (7-50)$$

$$\sum_{i \in W \cup R} \sum_{j \in W \cup R} X W_{jikt} \leq M V Z w_{kt} \quad \forall k \in VW, \forall t \in T \quad (7-51)$$

$$\text{If } V Z w_{k(t+1)} - V Z w_{kt} = 1 \Rightarrow U w_{k(t+1)} = 1 \quad \forall k \in VW, \forall t \in T \quad (7-52)$$

$$V Z w_{k1} \leq M U w_{k1} \quad \forall k \in VU \quad (7-53)$$

$$\sum_{i \in W} \sum_{s \in S} \sum_{m \in T} T W_{smtq}^{ij} \leq M T N_{jqt} \quad \forall j \in R, \forall q \in TP, \forall t \in T \quad (7-54)$$

$$\sum_{j \in W} \sum_{s \in S} \sum_{m \in T} NB_{iq} TW_{smtq}^{ji} \geq TN_{igt} \quad \forall i \in R, \forall q \in TP, \forall t \in T \quad (7-55)$$

$$\sum_{m \in W} \sum_{n \in R} \sum_{p \in T} TW_{stpq}^{mn} - TLB_{sqi} \leq M w_{qist} \quad \forall q \in TP, \forall i \in L, \forall s \in S, \forall t \in T \quad (7-56)$$

$$\sum_{q \in TP} \sum_{i \in W} \sum_{j \in R} \sum_{s \in S} TW_{stmq}^{ij} = 0 \quad \forall t, m \in T, m < t \quad (7-57)$$

$$Z_{sqm}^{ij} \geq \sum_{n \in T} (n - m) TW_{smnq}^{ij} \quad \forall i \in W, \forall j \in R, \forall q \in TP, \forall s \in S, \forall m \in T \quad (7-58)$$

$$QW_{lskt}^{mn}, I_{sit}, e_{ist}, DE_{ist}, DM_{ist}, TW_{stmq}^{ij}, Z_{sqm}^{ij} \geq 0 \quad \text{Real-valued decision variable} \quad (7-59)$$

$$XW_{ijkt}, VZw_{kt}, Uw_{kt}, TN_{it} \text{ and } w_{qist} \in \{0, 1\} \quad \text{Binary integer variables} \quad (7-60)$$

Equation 7-29 presents the objective function of the lower-level model, which includes components of Equation 3-1 related to the lower level. Since TPLP is an option for shipping and warehousing in the lower model, it exists in the objective function of this level. Most of the constraints are the same as the constraints in the original model. Equation 7-30, which is a replacement for constraint 3-17 in the original model, makes a relation between the upper- and lower-level models. Sol_DE in the lower model is a constant and is equal to decision variable DE of the upper-level model. In other words, DE is found by the upper-level model and, according to its value, the lower model decides how to distribute the quantity of DE (known as Sol_DE) between retailers.

The output of the lower level model is a delivery plan between bottlers and retailers. The plan for the upper level is provided by the upper-level model. As a result, combining the results of the two models yields a plan for shipping and warehousing for the entire network; however, the company's profit is not equal to

summation of the objective functions of the two models. The company's total profit can be easily calculated based on components of the objective functions of the upper- and lower-level models. Use of this approach for different scenarios and comparison of its efficiency with the other heuristic methods proposed in this study are discussed in next section.

7.2. Numerical Results

In this section, the results obtained by solving different scenarios with the new heuristic approach described in this chapter are presented. As explained in the previous section, the level-decomposition approach divides the model into two separate problems. The upper-level model is solved first; one of its outputs becomes an input to the second model; solving the lower-level problem.

Some scenarios from the categories explained in Chapter 5 have been selected randomly to evaluate the performance of the level-decomposition approach. Table 7-1 shows the results of solving these scenarios with the level-decomposition approach.

According to Table 7-1, all upper-level models have been solved optimally, while most lower-level models have a gap with the best bound. This gap, for some of them, is considerable with regard to their long running time. Table 7-2 compares the performance of the level-decomposition method to that of Xpress, heuristic 1, and heuristic 2 on the same scenarios shown in Table 7-1.

Table 7-1. Solutions using the level-decomposition approach on some scenarios

Category	Scenario	Level Decomposition Objective Function (\$)	Upper Model Running Time (s)	Lower Model Running Time (s)	Upper Model Gap (%)	Lower Model Gap (%)
1	1	48709.6	0	3213	0	0
1	8	44433.9	0.3	850.7	0	0
2	5	79189	13.2	362.3	0	0
2	7	81664.6	9.9	101.8	0	0
2	9	80934.9	2.6	23525	0	0.03
3	3	114584.5	0.5	34773.1	0	0.63
3	7	109326.8	0.7	9524.1	0	0
3	9	121175	1.7	53805.4	0	0.22
3	10	108945.6	24.6	42048	0	0.16
4	4	163396	201.1	68496.2	0	0.19
4	5	162755	764.2	12834.5	0	0.41
5	1	111062	0.1	13272.2	0	0.86
6	1	166814	74	46164.5	0	0.61
7	1	256056	114	48174.3	0	0.95

Table 7-2. Comparison of the performances of different heuristic methods

Category - Scenario	Level Decomposition Objective Function (\$)	Xpress Objective Function (\$)	Heuristic 1 Objective Function (\$)	Heuristic 2 Objective Function (\$)	Xpress Improvement in compare to Decomposition Approach (%)
1-1	48709.6	49330	48875	48678	1.26
1-8	44433.9	45088	44486	44064	1.45
2-5	79189	80239	79763	79438	1.31
2-7	81664.6	82998	82470	82062	1.61
2-9	80934.9	81235	79062	78971	0.37
3-3	114584.5	115753	109276	103173	1.01
3-7	109326.8	109817	108639	109369	0.45
3-9	121175	121530	119689	120336	0.29
3-10	108945.6	109682	107916	108498	0.67
4-4	163396	164400	162589	163853	0.61
4-5	162755	164721	164954	164838	1.19
5-1	111062	126274	114358	122726	12.05
6-1	166814	176600	175932	170574	5.54
7-1	256056	N.A.	268836	266011	N.A.

According to Table 7-2, the objective function of the original model is better than the model using level decomposition approach. It shows that integrating levels is

more beneficial than decomposing levels. Moreover, the level-decomposition method performs better for small categories than large categories. In some scenarios, it found better solutions than heuristics 1 and 2—but in most cases, heuristics 1 and 2 performed better, especially in large problems. Moreover, the running time for the level-decomposition approach was much higher than for heuristics 1 and 2 and did not necessarily generate better solutions. This method solved category 7 scenarios that were not solved by Xpress; however, the gap with the relaxed problem objective function was more than 8%. Heuristic 2 delivers a solution with less than a 5% gap with the relaxed problem objective function and in shorter running time.

7.3. Summary

In this chapter, another heuristic method was developed to solve the problem. This heuristic decomposes the problem into two levels and solves them separately; however, an output of the upper-level model is considered as an input to the lower-level model. The main point of this method is to decrease the size of a problem by decomposition, which, for some large problems, makes it solvable with Xpress. However, the original model which integrates both levels finds a better solution with higher company's profit. Two mathematical formulations for the upper- and lower-level problems were proposed in this chapter, which are obtained by modification of the original model described in Chapter 3.

The numerical results of solving several problems with this method and comparison of its results with other heuristics' solutions indicate that although constraints and decision variables are categorized into two groups, making each

problem smaller than the original one, the gap with the best found solution is higher than that of heuristics 1 and 2, especially in large problems. Moreover, the running time for this method is higher than the other heuristics proposed in this study. Finally, the level-decomposition method uses the branch-and-bound algorithm as a solver for its mixed-integer models. Therefore, by increasing the size of categories, there is a good chance that it will not provide a feasible solution; however, heuristics 1 and 2 relax the model and can support larger problems. As a result, heuristics 1 and 2 perform better than the level-decomposition method especially for large problems.

Chapter 8: Sensitivity Analysis

Some parameters of the model have a significant impact on the solution and behavior of the model. In fact, if the values of these parameters change, the output of the model will change drastically. Choosing the correct values for these parameters, therefore, is essential—but, on the other hand, difficult. Moreover, the values of some parameters depend on different factors such as the economy, conditions of rivals, and even political issues. As a result, analysis of the sensitivity of a solution to these parameters provides a better vision of the model's potential outcome in different situations.

8.1. The Base Scenario

The base scenario is the model that is considered the benchmark for sensitivity analysis. Therefore, the results, after changing a parameter of the model, are compared to the solution of the base scenario. This scenario cannot be very large due to its long running time. On the other hand, considering a small base scenario misses the real effects of some parameters. Table 8-1 presents the main characteristics of the base scenario. In addition, Table 8-2 illustrates the output of Xpress for this problem.

Table 8-1. Main characteristics of the base scenario

Criterion	Value
Number of Plants	2
Number of Bottlers	3
Number of Retailers	10
Number of Commodity Types	1
Number of Available TPL Contracts	2
Number of Owned Trucks in the Upper Level	3
Number of Rental Trucks in the Upper Level	3
Number of Owned Trucks in the Lower Level	3
Number of Rental Trucks in the Lower Level	3
Number of Time Steps	3
Number of Constraints	13,874
Number of Decision Variables	149,006

Table 8-2. General output information for the best solution of base scenario

Criterion	Value
Objective Function (\$)	41522.4
Running Time (CPU Seconds)	41588
Total Unsatisfied Demand (unit)	0
Commodity Quantity Stored in Retailers' Storage (unit)	6
Number of Rental Vehicles Used in the Upper Level	1
Number of Rental Vehicles Used in the Lower Level	1
Number of TPLP Contracts Selected	1
Commodity Quantity Delivered by TPLP (unit)	1233
Commodity Quantity Stored in TPLP's Warehouse (unit)	0
Total Production of all Commodity Types in Bottlers (unit)	3191

According to Tables 8-1 and 8-2, the model has used one rental vehicle in each level and has satisfied all demands. One TPL contract has been selected for shipping only, and no commodity is stored in a TPLP warehouse. The base scenario includes more than 13,000 constraints and 149,000 decision variables, confirming that the problem is large enough to consider different factors. Sensitivity analysis on some parameters is discussed in the following sections.

8.2. Vehicle Capacity

Vehicle capacity is the first parameter selected for sensitivity analysis. Increase in fleet capacity may cause the model to use fewer vehicles for delivery. In fact, the number of tours in both levels may decrease due to increase in vehicle capacity. On the other hand, a decrease in fleet capacity may result in using more vehicles to operate more generated tours. The key point is that this change does not have a linear relation with increased (or decreased) capacity. In other words, it may be that although increasing fleet capacity incurs a certain cost, this change may bring benefits to the company that outweigh its expenses. It should be noted that the cost of increasing capacity (for example, by buying a new fleet) is a capital cost and is paid once; however, the benefits accrue to the company daily. As a result, the small amount of benefit to be gained from a longer planning horizon may compensate the large amount of capital cost. The aim of this section is to see how this model reacts to an increase or decrease in vehicle capacity.

Four scenarios are considered for sensitivity analysis of vehicle capacity. In two scenarios it is increased, and in two it is decreased. Each scenario was solved by Xpress 7.1 and its output for all scenarios is shown in Table 8-3.

As Table 8-3 shows, objective function increases as vehicle capacity increases. The result is consistent with what was expected because by increasing vehicle capacity, each truck delivers more syrup concentrate or final products, which results in generating fewer tours. The total cost, therefore, goes down, causing higher objective function. The total unsatisfied demand is equal to zero for four scenarios, while there is an unsatisfied demand in the case with the lowest vehicle capacity.

Table 8-3. Criteria from the best solution found by Xpress for different scenarios of vehicle capacity

Criterion	Scenarios				
	50% Decrease	25% Decrease	Original Value	25% Increase	50% Increase
Objective Function (\$)	32035.4	41121.1	41522.4	41771.1	41897.3
Total Unsatisfied Demand (unit)	884	0	0	0	0
Commodity Quantity Stored in Retailers' Storage (unit)	0	27	6	0	78
Number of Rental Vehicles Used in the Upper Level	3	3	1	1	0
Number of Rental Vehicles Used in the Lower Level	1	0	1	0	0
Number of TPLP Contracts Selected	2	2	1	1	1
Commodity Quantity Delivered by TPLP (unit)	1471	1893	1233	1233	1233
Commodity Quantity Stored in TPLP's Warehouse (unit)	0	493	0	390	449
Total Production of all Commodity Types in Bottlers (unit)	2730	3191	3191	3191	3191

This output should be analyzed with the number of rental vehicles used in both levels. When the capacity of vehicles is decreased, the model has used more rental vehicles, especially in the upper level. Three rental vehicles in the upper level are enough to compensate for vehicle capacity when it is decreased by 25%; however, in the lowest-capacity case, adding all three available rental vehicles cannot deliver enough syrup concentrate to bottlers to produce enough final products to fill retailers' orders. This is the main reason for unsatisfied demand in the first case, and explains the differences in TPLP shipping volumes in different scenarios. As expected, by decreasing vehicle capacity, TPLP's share in shipping of products in the lower level

is increased. In the first scenario, however, fewer final products are produced by bottlers. Therefore, there are fewer final products available for shipping, which affects TPLP's share of transportation in the lower level. Figure 8-1 shows the objective functions and number of rental vehicles used in both levels in different scenarios. The objective function drops significantly after the vehicle capacity is decreased by 50%.

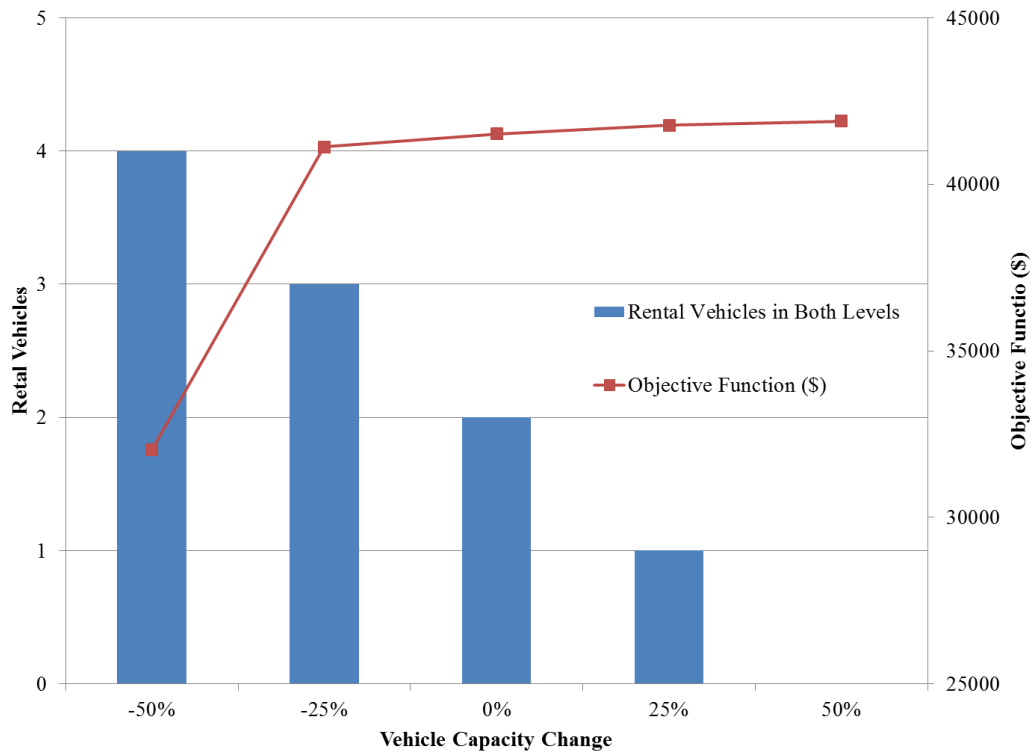


Figure 8.1 Sensitivity of objective function and number of rental vehicles used in both levels to vehicle capacity change

8.3. Inventory Cost

One of the factors affecting the solution is the inventory cost for retailers. The base scenario stores 6 units by retailers, showing that the current values for inventory costs do not allow the model to store more products by retailers. Increase in the inventory

cost may cause the model not to store even this small amount in retailers' warehouses, while a decrease in the cost may lower total cost and provide a solution with a higher objective function. Table 8-4 presents the output by Xpress for different scenarios. Three scenarios in addition to the base case have been considered in order to analyze sensitivity of the model to inventory cost.

Table 8-4. Criteria from the best solution found by Xpress for different scenarios for inventory cost

Criterion	Scenarios			
	50% Decrease	25% Decrease	Original Value	25% Increase
Objective Function (\$)	41527.9	41524.2	41522.4	41518.8
Total Unsatisfied Demand (unit)	0	0	0	0
Commodity Quantity Stored in Retailers' Storage (unit)	60	54	6	6
Number of Rental Vehicles Used in the Upper Level	1	1	1	1
Number of Rental Vehicles Used in the Lower Level	1	1	1	1
Number of TPLP Contracts Selected	1	1	1	1
Commodity Quantity Delivered by TPLP (unit)	1233	1233	1233	1233
Commodity Quantity Stored in TPLP's Warehouse (unit)	0	0	0	0
Total Production of all Commodity Types in Bottlers (unit)	3191	3191	3191	3191

According to Table 8-4, the objective function increases as the inventory cost decreases; however, output is the same from the standpoint of number of rental vehicle used, quantity of products delivered by TPLP, and unsatisfied demand. In cases with less inventory cost, the model stores more products by retailers, which is

the main reason for the objective-function improvement. The model keeps more products in retailers' warehouses at less cost to save the tours cost of future time steps. On the other hand, the last scenario, with higher inventory cost than the others, has the lowest objective function, while other outputs, such as the quantity of products stored in retailers' warehouses or number of rental vehicles used in both levels, are the same as the base scenario.

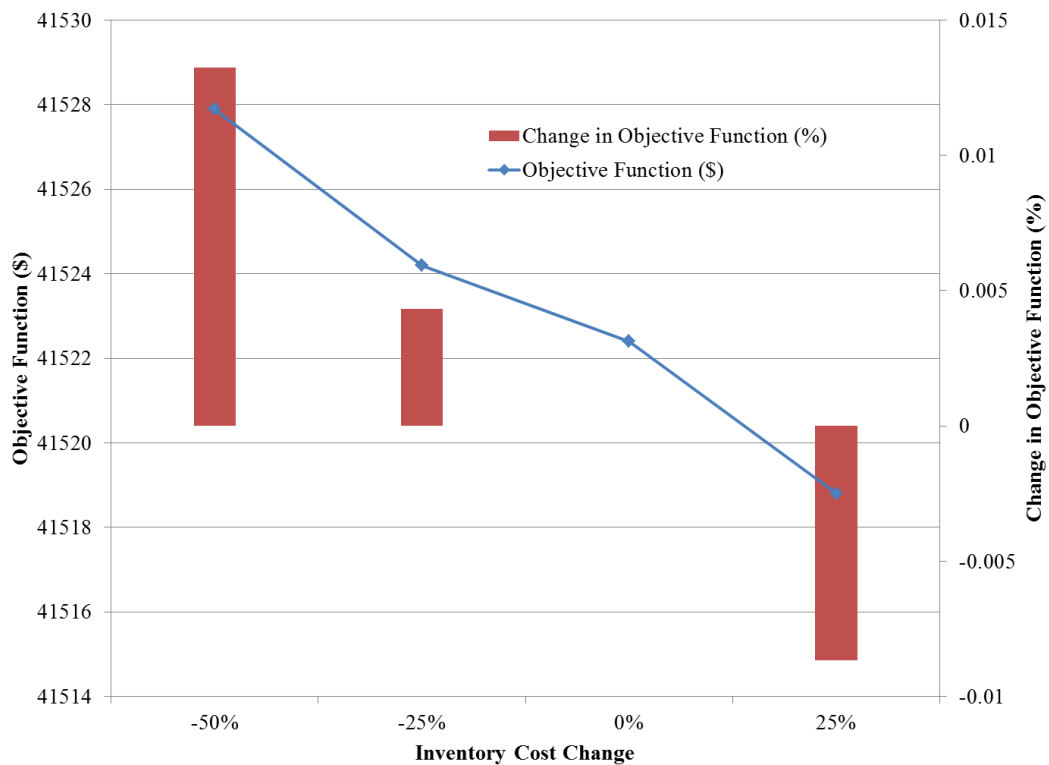


Figure 8.2 Sensitivity of objective function to inventory-cost change

According to Figure 8-2, the impact of an inventory cost decrease (or increase) of 50% on the objective value is less than 1%. This result is reasonable due to the small proportion of inventory cost in the total cost. Moreover, the different trend-line slopes in Figure 8-2 indicate that an increase of 25% in inventory cost affects the objective function more than a decrease of 25% in inventory cost.

8.4. Production Cost

In this section, sensitivity analysis of the production cost is discussed. Many factors influence production cost, such as syrup concentrate price, labor costs, equipment, and utility expenses. Tracking all factors is outside the scope of this study; however, their effects can be considered together as a change in the final cost of production.

Five scenarios, in addition to the base scenario, are considered in which each scenario has its own production cost. According to the Xpress results for these scenarios, most of the output components—such as unsatisfied demand, inventory level at retailers, number of rental vehicles used, and even production quantity—are the same in different scenarios. Figure 8-3 shows objective function value and its change over different scenarios.

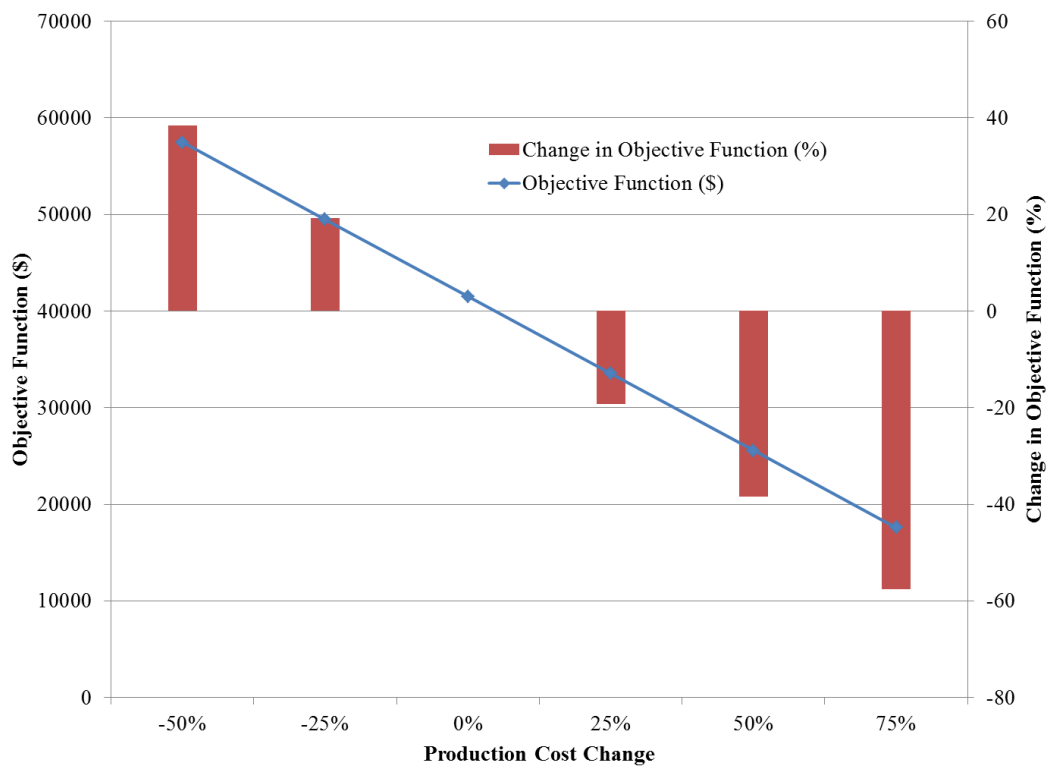


Figure 8.3 Sensitivity of objective function to change in production cost

According to Figure 8-3, objective function changes when production cost is increased or decreased. This suggests that objective-function change is less than production-cost variation. For instance, as the production cost decreases by 50%, the objective function improves by 38%. Similarly, the company's profit declines by 57% when production cost increases by 75%. Other factors also affect the company's profits, which cancel out a portion of the increase or decrease in production cost.

8.5. Contract and Warehousing Cost of TPLP

One of the most important factors that significantly affect the solution and delivery plan for the company is TPLP. The company can outsource shipping and warehousing to TPLP with certain costs. If the cost of the TPLP contract or TPLP warehousing changes, the company's profit will be altered due to its dependency on TPLP. Variation in the cost of TPLP contracts and holding costs may occur because of changes in the TPLP's policy, fuel prices, and market situation. In this section, the costs of TPLP contracts and of storing products in their warehouses are changed to see their impact on the objective function and the company's delivery plan. Table 8-5 presents Xpress solutions for different scenarios generated by this sensitivity analysis. In these cases, a change is applied to the cost of TPLP contracts and warehousing cost. In other words, in the case that experiences a 25% increase, the costs of contracts and warehousing by each TPLP are also increased by 25%.

Table 8-5. Criteria from the best solution found by Xpress for different scenarios of TPLP cost

Criterion	Scenarios				
	50% Decrease	25% Decrease	Original Value	25% Increase	50% Increase
Objective Function (\$)	41899	41680.8	41522.4	41412.2	41324.5
Total Unsatisfied Demand (unit)	0	0	0	0	0
Commodity Quantity Stored in Retailers' Storage (unit)	38	38	6	6	216
Number of Rental Vehicles Used in the Upper Level	1	1	1	1	1
Number of Rental Vehicles Used in the Lower Level	0	0	1	1	1
Number of TPLP Contracts Selected	2	2	1	1	1
Commodity Quantity Delivered by TPLP (unit)	1893	1893	1233	1091	1091
Commodity Quantity Stored in TPLP's Warehouse (unit)	226	226	0	307	97
Total Production of all Commodity Types in Bottlers (unit)	3191	3191	3191	3191	3191

As Table 8-5 shows, the number of selected TPLP contracts is increased as their price goes down. Moreover, the quantity of products shipped or stored by TPLP in scenarios with lower TPLP cost is higher than in cases with higher TPLP cost. In addition, TPLP cost affects the number of rental vehicle used in the lower level. As expected, by increasing TPLP deliveries, the model needs fewer rental vehicles in the lower level, such that in the first and second scenarios, the model found that the owned and TPLP fleets are enough for delivery. In cases with more expensive TPLP cost, the model found that outsourcing was still beneficial. Although it rented one vehicle in the lower level, it preferred delivery by TPLP to adding another rental

vehicle to the fleet of the lower level. Figure 8-4 presents the sensitivity of the objective function to TPLP shipping and warehousing cost variation.

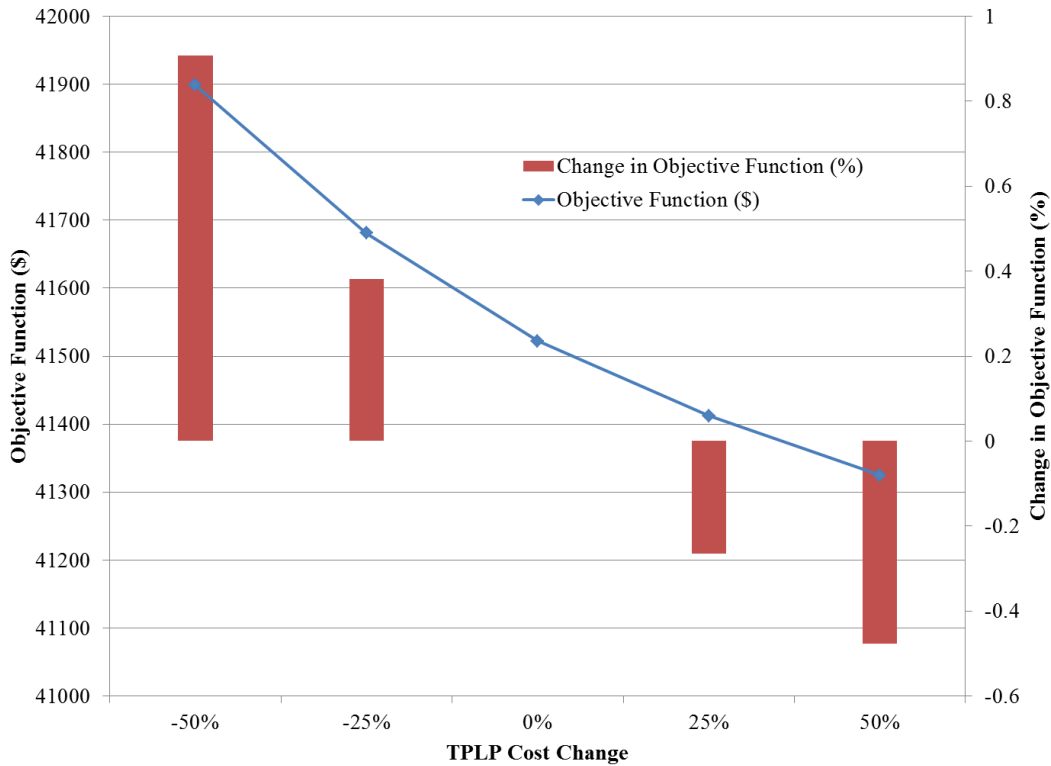


Figure 8.4 Sensitivity of objective function to TPLP shipping and warehousing cost change

According to Figure 8-4, the impact of TPLP cost on the company's profit is less than 1%. Variation in the cost of TPLP does not affect the objective function considerably, because TPLP is an option of the company and the company has other choices, such as owned and rental vehicles. When the cost of TPLP contracts is increased, the model can choose to use rental vehicles for delivery instead of outsourcing shipping. Similarly, the cost of using owned vehicles is less than the first scenario in this section, which has the minimum TPLP cost. The model, therefore, outsources shipping of products that are delivered by rental vehicles in the base scenario, while owned vehicles are used at their full capacity. As a result, the impact

of change in TPLP cost on objective function is less than the variation in TPLP cost due to availability of other options to replace TPLP.

8.6. Fuel Price

Fuel price is one of the most important factors affecting the company's profit and delivery plan. The history of fuel price in North of America over past 10 years shows that the price of gas and diesel fluctuates considerably due to several factors. As a result, sensitivity analysis of fuel price is essential and provides a useful insight on what happens if the price of fuel changes during the planning horizon. In the objective function of this study, shipping cost is separated from other costs related to transportation such as driver cost or fixed costs for rental vehicles. In this section, therefore, these objective-function items have been altered to reflect changes in fuel price.

Five scenarios have been considered for fuel price; it is assumed that fuel price does not change by more than 50% of its current value. In addition, a decrease in fuel price has been considered—which, however, may not happen. Table 8-6 shows Xpress outputs for different scenarios in this section.

Table 8-6. Criteria from the best solution found by Xpress for different scenarios of fuel price

Criterion	Scenarios				
	50% Decrease	25% Decrease	Original Value	25% Increase	50% Increase
Objective Function (\$)	42779	42126.8	41522.4	40937.5	40377.7
Total Unsatisfied Demand (unit)	0	0	0	0	0
Commodity Quantity Stored in Retailers' Storage (unit)	6	6	6	11	55
Number of Rental Vehicles Used in the Upper Level	1	1	1	2	1
Number of Rental Vehicles Used in the Lower Level	1	1	1	0	0
Number of TPLP Contracts Selected	1	1	1	2	2
Commodity Quantity Delivered by TPLP (unit)	1091	1233	1233	1699	1893
Commodity Quantity Stored in TPLP's Warehouse (unit)	307	0	0	779	226
Total Production of all Commodity Types in Bottlers (unit)	3191	3191	3191	3191	3191

As Table 8-6 shows, the objective function decreases by increase in the fuel price. Moreover, the model stores more products in retailers' warehouses in scenarios with higher fuel price to save on tours at future time steps. In addition, the number of rental vehicles used in the lower level is decreased as the fuel price is increased. More products are delivered by TPLPs to save on fuel because in these scenarios, the TPLP costs are the same as in the base scenario. This is the main reason that in scenarios with more expensive fuel, more TPLP contracts have been selected.

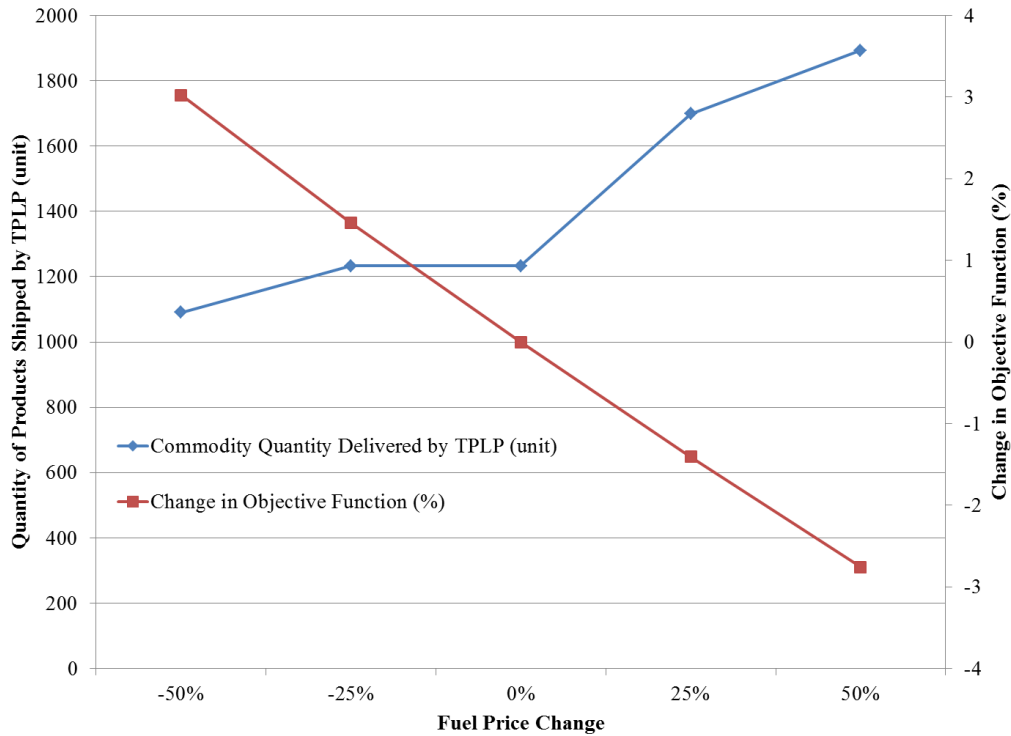


Figure 8.5 Sensitivity of objective function and quantity of products delivered by TPLP to change in fuel price

Figure 8-5 shows the effect of fuel price on objective function and TPLP delivery quantity. As shown, if the fuel price is decreased by 25%, TPLP ships the same amount of products—while a 25% increase in fuel price causes TPLPs to carry 466 units more, which is around 38% of TPLP delivery in the base scenario. The next step increase in the fuel price (from 25% to 50%) affects TPLP delivery quantity less than in the previous case. In the fifth scenario, the quantity of delivery by TPLP is increased by 11.4% compared to the fourth scenario. The objective function decreases linearly as fuel price increases, which is shown in Figure 8-5.

8.7. Product Price

Another factor that impacts the company's profit significantly is the products' price for retailers. Many factors affect the final price, including production cost and transportation cost; however, in this section it is assumed that these costs are fixed and that other factors are affecting the price of products; this section is designed to demonstrate the effect of product price on the company's profit and delivery plan. Since price has a significant impact on buyers and may cause customers to shift to another brand, it cannot vary widely. The change in product price, therefore, is limited to 25% more or less than the current value. Similar to previous cases, four scenarios in addition to the base scenario were generated and solved by Xpress; results are shown in Figure 8-6.

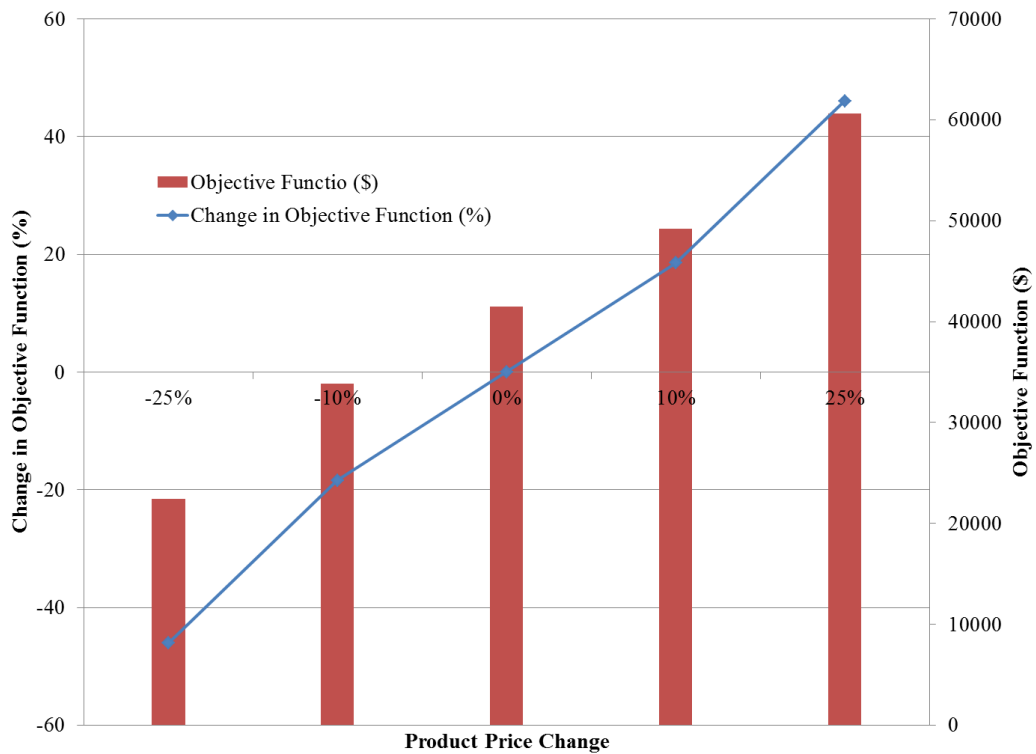


Figure 8.6 Sensitivity of objective function to change in product price

According to Figure 8-6, the percentage of objective-function change is more than product-price change. For instance, the price of product increases 25%, while the objective function goes up 46%. The reason is that increase in product price improves revenue; however, revenue is only one component of the objective function. In fact, other costs that are also components of the objective affect the objective function to become less than revenue. Therefore, an increase in product price increases revenue, and eventually the objective function equally. Since the final value of the objective function is compared to its previous value, variation in the objective function is more than product-price change.

8.8. The Value of Information

The model considers information about future time steps to find an optimal solution for the entire planning horizon. More accurate time-step data results in better final solution. Some of the information needed by the model is estimated based on historical data. As a result, variation in the data is possible due to lack of information about the future. Orders by retailers are the input of the model, which significantly affect the company's profit and the delivery plan. On the other hand, accuracy in retailers' orders is not 100% guaranteed, meaning that they can change during the operation. This section analyzes the value of information fed to the model to see how much they affect the company's profit if they vary.

Imagine that retailers' orders in the third time step increase by 10% during the operation. The delivery plan, therefore, does not change, and vehicles deliver products to retailers based on outputs of the model for the base scenario. As a result,

the added demand goes unsatisfied, and a shortage penalty is applied to the objective function. A total of 114 units of the product are not satisfied in different retailers. According to the shortage penalty at each facility location, the penalty is equal to \$348.33, which decreases the objective function from \$41522.4 to \$41174.7, which is equal to a 0.83% loss. If retailers' orders increase by 20%, the loss is equal to 1.66%, indicating that loss has a linear relation with changes in retailers' orders. This can be interpreted the value of information about retailers' orders. It helps managers of a beverage company considerably to face any uncertainty of information. In other words, preparing for a 10% variation in orders entails a certain cost, which should be compared to a 0.83% loss. Different management policies encountered in this case are not a concern of this study. The goal of this section is introducing the model's capability to provide valuable information that helps managers make appropriate decisions.

8.9. The Effect of a Longer Planning Horizon on the Objective Function

One of the main characteristics of this model is that it integrates several properties. For instance, this model is multi-level, multi-location, and multi-product. One question that arises is whether this integrity improves the objective function. Considering different factors definitely increases the complexity of the model; heuristic methods are required to solve large problems. As a result, if a factor does not improve the company's profit, it can be removed or simplified in the model to make the main model simpler.

In Chapter 7, levels of the model were decomposed and two separate models were generated. The analyses performed in Chapter 7 indicated that considering two levels together in the model led to a better solution with higher profit. Even for large problems, for which Xpress was not able to find a feasible solution, the proposed heuristics provided a better solution than the third method, which decomposed the model.

This section analyzes the impact of planning horizon length on the final output. A longer planning horizon forces the model to consider more details and opportunities to improve the quality of the final solution; however, it may not necessarily be successful. Considering a longer planning horizon improves the company's profit when the decisions of earlier time steps can assist in the operation of later time steps. To make this clear, an example is designed that has the same number of facility locations, number of time steps, and fleet size as the base scenario; however, its retailers' orders are different. This scenario is solved using two different approaches: In the first approach, the mathematical formulation proposed in Chapter 3 is applied to the entire planning horizon. The model used in this approach is called Model 1. In the second method, the problem is decomposed to two models:

1. This model includes the first and second time steps of the original model and is called Model 2.
2. This model includes only the third time step of the original model and is called Model 3.

The proposed mathematical formulation in this study is applied to both Models 2 and 3 and they are solved by Xpress. The key point in this approach is that

some outputs of Model 2 must be considered as input to Model 3. For instance, if the final solution of Model 2 has unsatisfied demand, it must be added to the demands of Model 3. Similarly, the inventory level in facility locations must be transferred from Model 2 to Model 3. Table 8-7 presents the output of these two approaches applied to the scenario.

According to Table 8-7, the company's profit in the second approach, which breaks up the planning horizon, is decreased by 35% in comparison to the first approach. The reason is that a large portion of demands in the third time steps are not satisfied in the second approach. In fact, orders of the third time step are more than bottlers' production capacity; however, Model 1 sends more products to retailers in time step 2 and stores them for the third time step order, while Model 2—which does not consider the third time step—satisfies the demand of the first and second time steps. As a result, commodities produced in the third time step can meet a portion of retailers' orders, although bottlers use their production capacity fully.

This example shows the impact of a longer planning horizon on the problem, especially when demands change in later time steps significantly. The longer planning horizon allows the model to make ready for future demands by storing in retailers' warehouses in earlier time steps. It depends on the inventory cost, shortage penalty, and shipping cost. The model selects different policies for different scenarios for these costs; however, if the planning horizon gets shorter, the model may not be able to meet future demands appropriately.

Table 8-7. Comparison of considering the entire planning horizon and breaking it up

Criterion	The First Approach	The Second Approach			Change in 2 nd Approach (%)
	Model 1	Model 2	Model 3	Combined	
Objective Function (\$)	70532.3	26722.7	18737.3	45460	-35.54
Unsatisfied Demand in Time Step 1 (unit)	0	0	N.A.	0	0
Unsatisfied Demand in Time Step 2 (unit)	0	0	N.A.	0	0
Unsatisfied Demand in Time Step 3 (unit)	220	N.A.	1800	1800	+718.18
Commodity Quantity Stored in Retailers' Storage in Time Step 1 (unit)	594	77	N.A.	77	-87.03
Commodity Quantity Stored in Retailers' Storage in Time Step 2 (unit)	1373	0	N.A.	0	-100.00
Commodity Quantity Stored in Retailers' Storage in Time Step 3 (unit)	0	N.A.	0	0	0
Production Quantity in Time Step 1 (unit)	1820	1170	N.A.	1170	-35.71
Production Quantity in Time Step 2 (unit)	1820	890	N.A.	890	-51.09
Production Quantity in Time Step 3 (unit)	1820	N.A.	1820	1820	0

As the number of time steps increases, the complexity of the problem grows drastically, because all decision variables have an index of time. On the other hand,

considering a longer planning horizon improves the quality of the solution. There is a tradeoff, therefore, between the accuracy of the solution and the complexity of the problem. The nature of the input data affects the final solution significantly. For instance, if input data have a strong variation during a planning horizon, the model should consider more time steps to react to variation; otherwise, dividing a planning horizon into shorter segments provides an acceptable solution in reasonable running time.

Table 8-8 presents the performance of heuristics 1 and 2 on the same problem. As shown in Table 8-8, these heuristics have found better solutions than the second approach proposed in this section. The heuristics consider the entire planning horizon and send more products in earlier time steps. This opportunity is not available for an approach that uses a shorter planning horizon. Figure 8-7 illustrates the differences between the objective functions of different approaches and the objective function of Xpress. The running time for each method is also shown in Figure 8-7. Although heuristics did not provide a solution as good as that of Xpress, they found solutions that were much better than the approach that divides the planning horizon into several pieces. Also, heuristics found solutions in shorter execution time.

Table 8-8. Comparison of the method breaking down the planning horizon and heuristics

Criterion	The First Approach	The Second Approach	Heuristic 1	Heuristic 2
Objective Function (\$)	70532.3	45460	58684.5	63105.6
Unsatisfied Demand in Time Step 1 (unit)	0	0	219	61
Unsatisfied Demand in Time Step 2 (unit)	0	0	0	0
Unsatisfied Demand in Time Step 3 (unit)	220	1800	850	588
Commodity Quantity Stored in Retailers' Storage in Time Step 1 (unit)	594	77	205	521
Commodity Quantity Stored in Retailers' Storage in Time Step 2 (unit)	1373	0	516	703
Commodity Quantity Stored in Retailers' Storage in Time Step 3 (unit)	0	0	0	0
Production Quantity in Time Step 1 (unit)	1820	1170	1190	1820
Production Quantity in Time Step 2 (unit)	1820	890	1820	1820
Production Quantity in Time Step 3 (unit)	1820	1820	1820	1452

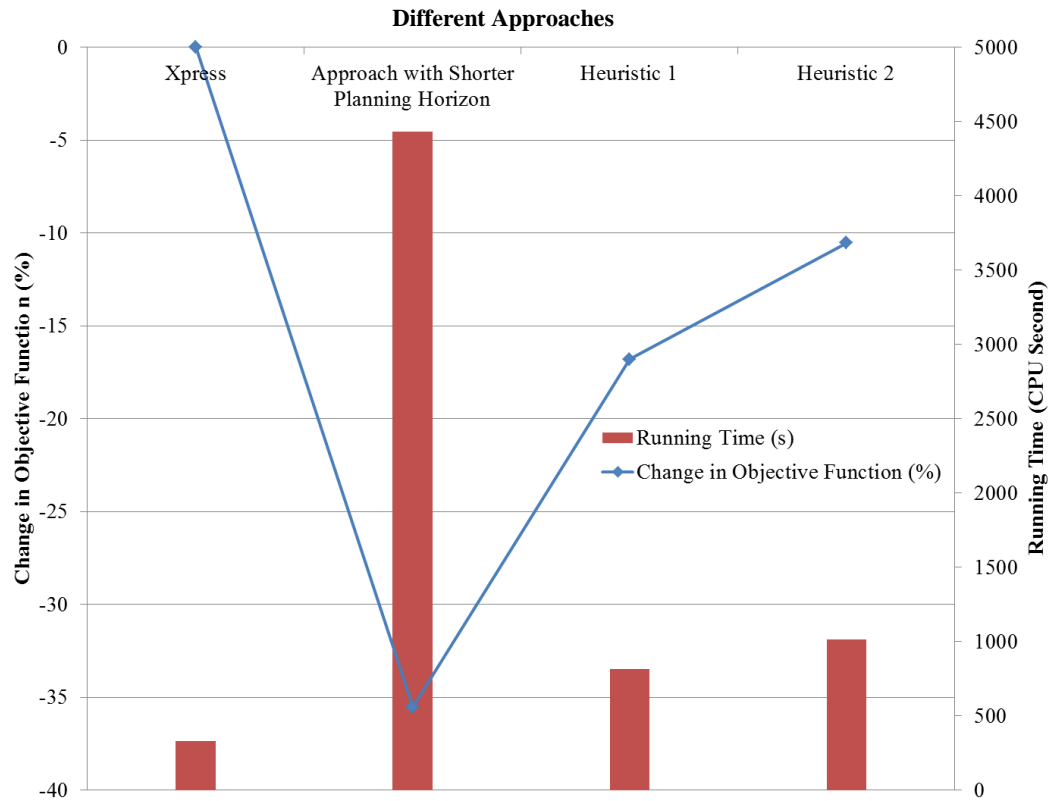


Figure 8.7 Comparison of variation in the objective function and running time in different approaches used to solve the problem

8.10. Summary

In this chapter, sensitivity of the model to various parameters and input data was analyzed. Analyses were performed to illustrate the model's capabilities to react to variation in input data. Different components of input data, such as vehicle capacity, inventory cost, production cost, TPLP cost, fuel price, and product price for retailers, were selected and changed to show their impact on the final solution. Besides their effect on the objective function, their influence on other decision variables was discussed in detail. For instance, an increase in vehicle capacity causes the model to select fewer rental vehicles due to the larger quantity of products delivered by owned

vehicles, and an increase in fuel price forces the model to outsource more shipping to TPLPs.

The useful information gained in each section of this chapter is the response of the objective function to a change in each parameter. In other words, a change in one element of the input data increases or decreases the objective function. Some changes are desirable because they improve the company's profit; however, their effect on the objective function may not be found beneficial in comparison to the cost of the change. In other words, a 50% change in one parameter entails a certain cost, while it improves the company's profit by an amount that is less than the capital cost needed for the change. Since the model is defined in an operational level, saving in costs or improvement in the objective function should be considered over a very long period to compare with the capital cost. For instance, a 25% increase in the capacity of vehicles due to replacing current vehicles with new ones increases the company's profit by about 0.6% in a 3-day operation. The cost of fleet renovation is expensive; however, if a 0.6% savings is realized every 3 days for the lifetime of a vehicle, it may be beneficial. As a result, the analyses in this chapter provide valuable information that will help the managers of a beverage company in the decision-making process.

Another section of this chapter yielded valuable information by analyzing retailers' orders in detail. Uncertainty about future information, such as changes in retailers' orders, must be considered in the daily operation of the company. If the variation in orders is reported to the beverage company soon enough, the model can be executed again with the new data to update the delivery plan; however, sometimes

these changes are received late, in which case the company may have to pay a penalty for unsatisfied demands. This analysis can be defined as a finding of a value of information. This value for some parameters is very high, forcing managers to respond to changes as much as possible—while for some others, they may prefer to keep the original plan. For instance, a variation of 10% in retailers' orders reduces the company's profit by 0.83%. Interpretation of this value depends on many factors that are beyond the scope of this study, but this model is capable of delivering this information for different parameters.

The last part of this chapter analyzed the impact of a long planning horizon on the final solution. The number of time steps affects running time significantly. If a model with a long planning horizon is not solved by Xpress, due to the size of the problem and number of constraints and decision variables, it may be solved by optimization packages after dividing it into smaller models; each one covers a small part of the planning horizon. In fact, the time-step index has the highest influence on model size of all indexes. As a result, dividing a planning horizon into several segments and running the model on each segment is an approach to resolve the impossibility of executing large-sized problems with optimization packages. This approach can be practical if storing products in retailers' warehouses is not beneficial. In other words, by breaking down the planning horizon, the model loses the possibility of sending more products to retailers in advance and keeping them in retailers' warehouses for their future demand. As shown in Section 8.10, in some cases this policy brings lots of benefits to the company; by dividing the planning horizon, the company loses this opportunity. This method can reduce the company's

profit by 35%, which is a considerable amount. As a result, running the model on a longer planning horizon provides more benefits for the company, and if Xpress and other optimization packages are not able to solve the problem, the proposed heuristic methods can find a good delivery plan with an acceptable gap with the upper bound.

Heuristics 1 and 2 can be executed once in a week to determine distribution plan for the following week; however, if there is a potential fluctuation in orders of retailers, it can be run every day. In this case, the rolling horizon method is applied to the heuristic such that every day the model is solved by new and updated information.

Chapter 9: Summary, Conclusions, and Recommendations for Future Research

9.1. Summary

In this research, the inventory control and transportation of syrup concentrate and final products for one bottling company working for a beverage company is studied. Inventory management and transportation are two key planning issues in supply-chain management. Inventory management includes activities such as production, ordering, holding, and shortage of products. Transportation includes shipping raw materials and final products between sources, factories, warehouses, and retailers.

Theoretically, there are some benefits in the integration of inventory control and transportation, and especially when product demand is high and the costs associated with inventory and transportation are significant. Inventory-allocation decisions are based on routing-cost information and marginal profit for each customer in the set. On the other hand, the delivery cost for each customer depends on the location of the facility servicing it and the vehicle's route to that customer. This interrelationship between inventory management and transportation is the main reason researchers cite for integrating these two major supply-chain activities.

In the problem for this study, the bottling company has its own nonhomogeneous fleet for transportation of syrup concentrate from plants to bottlers and final products from bottlers to retailers. The bottling company knows retailers'

immediate future demands for a timeframe of several days. If the demand is not satisfied, a penalty is applied to the bottling company's total cost based on quantity of unmet demand and shortage cost. Retailers can store final products for future demand at a specific holding cost.

The bottling company may not satisfy all demands due to small fleet size, which is not able to cover all deliveries in the right timeframe. In this case, in addition to the do-nothing alternative, the bottling company has other options, such as sending more products to retailers in advance, adding rental vehicles to the fleet, and outsourcing delivery of some retailers to TPLPs. Moreover, TPLPs are able to store products if the bottling company finds it beneficial.

Thus, the company must get raw materials from plants, produce final products at bottlers' factories, and ship them to retailers. It must also optimize its profit, taking into consideration inventory control in warehouses—both its own and retailers'—and, when necessary, store products in TPLP warehouses. Finally, it must deliver final products to retailers using different vehicles, including owned, rental, and TPLP.

The mathematical formulation to solve the problem was presented in Chapter 3 and considers all details and opportunities and must be solved by optimization packages. This linear model, which is a mixed-integer program, maximizes the company's profit subject to several constraints. The performance of the model was verified in Chapter 4 by solving different scenarios. These scenarios were generated to show the model's capabilities in different situations. Results confirmed the accuracy of the model and its performance under different conditions. Outputs of Xpress from these scenarios also demonstrate that the model is very complex and that

even medium size scenarios cannot be solved optimally. Moreover, the model is very sensitive to some parameters, such as length of planning horizon, fleet size, and number of commodity types. The model's complexity, therefore, increases significantly as these parameters grow. As a result, not only is Xpress incapable of solving large problems; it is also unable to deliver a feasible solution. Therefore, development of a heuristic method to find a good feasible solution in reasonable running time is essential. Chapters 5 and 6 proposed two heuristic methods for this problem, which are based on fix-and-run algorithm. Three improvement phases were also developed to enhance the final solution of heuristics. Moreover, performance of the two proposed heuristics was verified by solving several categories of scenarios. According to the results, heuristic 1 generally finds a better solution than heuristic 2; however, heuristic 2 is much faster than heuristic 1. Solutions of both heuristics have acceptable gaps from either an optimal solution or upper bound.

In the real world, retailers are clustered into groups and each group is assigned to one bottler. Although retailers are grouped based on their distances from bottlers, the output is not the optimal necessarily. In the mathematical formulation and heuristic methods proposed in this study, the model assigns retailers to bottlers based on their contributions to the objective function. In other words, retailers are connected to bottlers according to different factors considered in the objective function. The final assignment, therefore, is more beneficial than clustering which happens in the real world and is based on distance only.

Chapter 7 proposed another method to solve large problems approximately, which is based on decomposition of the model's levels. This method is much easier

than the proposed heuristics; however, the accuracy of the heuristics and running time are superior to level decomposition approach. Moreover, a branch-and-bound algorithm is used in level decomposition method, which causes it to be incapable of solving larger problems; heuristics, however, are able to find good feasible solutions for them. Finally, in Chapter 8, the model's sensitivity to some parameters and input data is analyzed. The outputs of these analyses will be valuable for managers of the bottling company as they make decisions in different situations.

9.2. Conclusions

The problem of this study is IRP with some new features, such as options for rental vehicle and TPLPs. Moreover, the model proposed in this study includes several time steps in which a decision in one time step can affect future time steps. The model is also multi-tier, multi-plant, multi-warehouse, and multiproduct, with a nonhomogeneous fleet, which increases its complexity significantly.

Numerical results indicated that the model is very sensitive to some parameters such as fleet size, number of commodity types, and length of planning horizon. In fact, increase in any of these parameters grows the complexity and running time exponentially. Moreover, tracking of vehicles and commodity flow in the network brings more complexity to the model. For this purpose, many constraints must be added to the formulation as well as defining a lot of indices in decision variables. However, they are inevitable if capacity of vehicles is considered in the model. In addition, having more than one facility location in each tier makes the model complicated especially when heuristic algorithms are developed. In fact, the

flexibility of the model is expanded drastically, which makes control of these flexibilities very difficult in developing a heuristic.

Different approaches to develop a heuristic can be considered. In some methods, the entire formulation is used by a heuristic and assumptions such as relaxation of binary variables help the heuristic run faster. On the other hand, some methods decompose the model and solve smaller problems sequentially. Two of these decomposition techniques were studied in this research: decomposition of levels and planning horizon. Both approaches found solution worse than heuristics proposed in this research. In other words, integrating different aspects and considering the entire model in developing the heuristic provides better solution than other heuristics which separate the problem into several smaller models.

Analyzing the sensitivity of the model to its parameters showed that the model is very sensitive to production cost, product price, and vehicle capacity. For instance, 50% increase in production cost decreases the profit of the company by 30%. Also, it showed that change in fuel price impacts less on the profit of the company and more on the quantity of commodity shipped by TPLP.

Finally, the value of information was studied in this research. Since most of needed information for this problem is predicted for the near future, there is a high chance of error in prediction. For instance, demand of retailers can be different from the original estimation. This research calculated the loss due to incorrect predictions. Reaction to this loss depends on management policies and is out of scope of this study; however, calculation of potential loss or gain is valuable information considered in this study.

9.3. Recommendations for Future Research

There are some recommendations for future studies which are described in this section.

1- Considering Stochastic Parameters

Some parameters used in this study have a stochastic nature, such as retailer demand and travel time of the links. In other words, finding exact values for such parameters is difficult. As shown in the sensitivity analyses, variation in retailers' orders affects the objective function, which was explained as a value of information. A more accurate approach is to consider this variation in the mathematical formulation. In this study, it was assumed that all information that the model needs is known; however, in some parameters and input data, uncertainty is inherent. Adding the concept of stochasticity to the model improves the quality of the solution and provides a more robust solution for the problem. This suggestion will increase the complexity of the model significantly.

2- Development of Other Heuristic Methods

The heuristic methods proposed in this study use Xpress as a background solver: They make decisions and add new constraints to the model, which is solved by Xpress optimally. The strength of these heuristics, therefore, is limited to Xpress capabilities. For instance, Xpress limits the number of constraints and decision variables, even for relaxed models. The heuristics, therefore, cannot solve models

which pass those limits. This is a weak point for the heuristics. In addition, the fix-and-run algorithm, which is at the core of the heuristic methods, fixes integer variables time step by time step. Therefore, when applying this algorithm to time step t , decision variables of future time steps are relaxed. As a result, if there are large variations in the input data in the future time steps, the algorithm may not find a good solution to the problem. Modification of the fix-and-run algorithm or finding another heuristic method may be useful in solving this problem. Meta-heuristic methods, such as Tabu Search, Genetic Algorithm, and Ant Colony, can be independent from Xpress and may resolve the second weak point of the heuristics proposed in this research.

3- Reformulation in Mathematical Formulation

The mathematical formulation proposed in this research includes many decision variables, most of which concern the flow of commodities in links. These decision variables are naturally integer; however, because demands are integer, they can be assumed to be continuous. Since one of the goals of this study is tracking vehicles and the capacity of vehicles has been considered, the number of flow-decision variables increases rapidly as the model's parameters such as the number of facility locations, the fleet size and the number of time steps increase. Analysis of the proposed mathematical formulation is recommended, to see whether there is any way to reformulate this model with fewer decision variables while holding constant the model's functionality and capabilities.

4- Considering TPLPs in the Upper Level

In the current study, TPLPs exist in the lower level; however, they can be used in the upper level to transport syrup concentrate between plants and bottlers. The same TPLP, in other words, could contract with the bottling company for delivery in both levels.

5- Considering Tradeoff between Fuel and TPLP Costs

In the sensitivity analysis section of this research, the fuel price was changed while the cost of TPLP was constant. In the reality, there is a tradeoff between these two costs. In fact, by increase in the fuel price, cost of TPLP contract increases and it affect the delivery distribution plan. Considering this tradeoff between costs of fuel and TPLP contract is another recommendation for the future research.

6- Considering Different Values for Transferred Unsatisfied Demand Parameter

As mentioned in Chapter 3, this research assumed the unsatisfied demand is transferred to the next time step completely. μ is the parameter indicating the portion of unsatisfied demand transferred and it can vary from zero to one. Different values of μ impact on the final solution and the objective function. Another research, therefore, can focus on possible values of μ and their effect on the objective function.

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