Optimization Criteria and Design of Few-Mode Erbium-Doped Fibers for Cladding-Pumped Amplifiers

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Abstract: We propose a novel optimization method that combines two design criteria to reduce the differential modal gain (DMG) in few-mode cladding-pumped erbium-doped fiber amplifiers (FM-EDFAs). In addition to the standard criterion that considers the mode intensity and dopant profile overlap, we introduce a second criterion that ensures that all doped regions have the same saturation behavior. With these two criteria, we define a figure-of-merit (FOM) that allows the design of MM-EDFAs with low DMG without high computational cost. We illustrate this method with the design of six-mode erbium-doped fibers (EDFs) for amplification over the C-Band targeting designs that are compatible with standard fabrication processes. The fibers have either a step-index or a staircase refractive index profile (RIP), with two ring-shaped erbium-doped regions in the core. With a staircase RIP, a fiber length of 29 m and 20 W of pump power injected in the cladding, our best design leads to a minimum gain of 22.6 dB while maintaining a DMG_{max} under 0.18 dB. We further show that the FOM optimization achieves a robust design with low DMG over a wide range of variations in signal power, pump power and fiber length.

1. Introduction

With the unrelenting need to transfer large amounts of data, optical fiber communication networks have experienced tremendous growth over the past decade. As we approach the fundamental Shannon limit of standard single-mode fiber transmission, space-division multiplexing (SDM) technology is increasingly considered for delivering high-capacity [1-3]. SDM can be accomplished either by increasing the number of cores in a single fiber strand or by multiplying channels through transmission on different modes of a few-mode fiber (FMF), the latter technique being also referred to as mode division multiplexing (MDM) [4-6]. To support long haul transmission [7], FMFs require amplification to compensate for the inevitable background loss of silica-based fibers. Few-mode SDM communication systems must thus be able to rely on optical amplifiers that deliver high performance for all channels.

Although erbium-doped fiber amplifiers (EDFAs) are a well-established optical amplification technology [8, 9], the design of SDM amplifiers is challenging. Such amplifiers must be able to simultaneously amplify all modes with high gain, while providing low differential modal gain (DMG) and low noise figure (NF). The architecture of a few-mode erbium-doped fiber amplifier (FM-EDFA) is akin to that of a single-mode EDFA, where the signal to be amplified is injected into the doped core of the erbium-doped fiber (EDF). However, in FM-EDFAs, the gain must be balanced across both signal wavelengths and modes, this means that both differential spectral gain (DSG) and DMG should be considered when designing a FM-EDFA [10]. To achieve these goals, the design of few-mode erbium-doped

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fibers (FM-EDFs) entails the optimization of two key parameters, namely the erbium doping profile and the refractive index profile.

With uniform doping in the core, amplifier performance depends on the overlap of the signal mode intensity and pump mode intensity, as determined by the refractive index profile and injection conditions. Some amplifier designs optimize the pump modal content to shape the pump transverse intensity distribution and produce a population inversion in the EDF that renders the small signal gain equal for the various signal modes. This is possible if the amplified mode closely matches the pump intensity profile [11-13]. This approach is difficult to implement as the coupling efficiency of the pump to the various modes must be precisely controlled. Selective excitation of pump modes is challenging and complex to implement experimentally. For example, spatial light modulators (SLM) can be used but they have low damage threshold. Consequently, in this paper, we consider cladding pumping that can be implemented using a high-power low-cost multimode pump laser [14].

Tailoring of the refractive index profile (RIP) of the fiber to adjust the signal mode field distribution is the second approach proposed in the literature to minimize DMG. For example, the FM-EDF proposed in [15] has a dip at the center of its RIP that is engineered to modify the intensity distribution of the LP_{01} mode. By adjusting the depth of the refractive index dip, the LP_{01} mode profile resembles more closely the LP_{11} mode and LP_{21} profiles, which minimizes the DMG between these spatial modes. The first experimental demonstration of a FM-EDFA was accomplished with a fiber having a simple uniformly doped core, yielding a DMG between 5 and 10 dB [15]. Subsequently, Wada et al. demonstrated an EDF with a dip in the centre of its RIP and obtained a gain, in the L-band, of about 20 dB for five modes with a 3 dB DMG using an LP_{01} pump mode [16]. This approach can lead to substantial splice loss unless mode-field adaptors are used. Pushing this approach further leads to the design of a ring-core fiber in which the central dip reaches a refractive index that is equal, or even lower, than that of the cladding. Since all the signal spatial modes will have a similar overlap factor with the erbium-doped core, the ring-core fiber amplifier provides similar gain for all guided modes [17-22].

In another approach, several interesting fibers with tailored erbium doping profiles (EDP) were proposed to reduce the DMG of FM-EDFAs. A fiber with central dips in both the EDP and the RIP was proposed and fabricated by Jung et al. [15]. In this fiber, the erbium ion concentration increased from the center to the edge of the fiber core. Wakayama et al. further extended the doped region in the cladding to provide more gain to the higher order fiber modes when using cladding pumping, which decreased DMG to 3.3 dB. Such a cladding-pumped six-mode EDFA was experimentally demonstrated and a DMG of less than 3.3 dB was obtained over the C-band [23]. Similarly, by intentionally oversizing the fiber core to allow a greater number of modes than required, the desired modes are more confined in the doped core, which lowers the DMG. In [24], a 10-mode EDFA was demonstrated with a fiber having a 24 μm core diameter and a 2 dB DMG was reported. This method also requires mode field adaptors to reduce splice loss.

In most of this previous work, the criteria used in the fiber design optimization targeted equal overlap of the signal modes with the EDP, which amounts to equalizing the small signal gain. When the EDFA is operated in saturation, as in a fully loaded scenario, this condition is no longer valid. To find a more general solution, other optimization techniques, such as genetic algorithms, can be used. If all possible variations of the erbium doping profile and refractive index of the fiber are examined, an excessively large number of possible designs must be simulated. The spectral gain and noise figure (NF) of each mode are usually computed using coupled differential equations accounting for the erbium population rate equations of a two-level model, and the pump, signal and amplified spontaneous emission (ASE) mode intensity propagation. These coupled equations are solved using algorithms such as the 4th order Runge-Kutta method. In addition to the high computational costs when examining all possible designs, we do not gain physical insights [25-27]. For example, a one-ring EDP design was optimized to minimize the DMG in a three-mode EDF using a full propagation model [25]. In [26], a

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genetic algorithm is proposed to examine an enlarged two-layer EDP of a cladding-pumped fiber. The algorithm minimized the difference in the signal mode overlap with the doped regions and, simulations of the resulting 6-mode cladding pumped EDF design predicted an average gain of 24.8 dB and a DMG of less than 0.64 dB over the C-Band.

In this paper, we present a novel design method that is based on a novel figure of merit (FOM) combining two optimization criteria to achieve low-DMG in cladding-pumped FM-EDFAs that are fully loaded. This FOM requires only the computation of the mode intensity profile, the total intensity profile, and their overlap with the EDP. This significantly decreases the total computational time required to find a solution, as compared to a Runge-Kutta method-based solution. Our method allows exploration of a large number of fiber designs. Furthermore, the low DMG property of these designs is robust with respect to variations in pump power, fiber length and input signal power.

The paper is organized as follows: in section 2, we present the optimization criteria and method. We start with the standard mode intensity-EDP overlap criteria to which we add a second criteria that takes saturation of the EDP into account. We then detail the simulation method, which is based on generating RIP, calculating mode intensity profiles, computing overlaps with different EDP, and finally evaluating the FOM. In section 3, we apply the method to a step-index refractive index profile and, in section 4, we compare the results to a staircase RIP with a raised refractive index region near the center of the fiber core. Finally, we examine the robustness of both designs in section 5 and a conclusion follows.

2. Optimization criteria and design method

To reduce the computation time, we propose a FOM combining two criteria that do not require solution of the full simulation model of EDF. The optimization of these two criteria is necessary to achieve low DMG in cladding-pumped FM-EDFAs. We assume that cladding pumping results in a uniform pump intensity distribution over the fiber cross-section. The cladding diameter used for all the designs presented in this paper is the standard 125 µm that is mechanically and optically compatible with current fiber technologies. Under the assumption of uniform power distribution, the required pump power directly scales with the cladding-to-core area ratio. Uniform pump power distribution is a usual assumption in the modeling of cladding pumped amplifiers [28-30] that was validated by comparing experimental measurements to simulation results in the case of a 100 µm cladding in [30]. Thus, as long as the assumption holds, it is possible to reduce the necessary pump power by reducing the cladding diameter, e.g. the pump power would be reduced by a factor of nearly two for a 90 µm cladding diameter. In the examples we examine, we restrict our demonstration to designs that are simple and compatible with EDF fabrication that uses the modified chemical vapor deposition technique (MCVD) with solution doping. Firstly, we present the standard criterion and show that two fibers meeting this criterion can have very different DMG. We then introduce the second criterion and discuss the physical insight on which it is based. Lastly, we combine these criteria in a new FOM and present the steps in the numerical optimization technique. Throughout our discussion, the DMG is defined as

$$DMG(\lambda) = \max_{m \in M}(G(m_i, \lambda) - G(m_j, \lambda), \quad (1)$$

where the gain G, in dB, is a function of the wavelength λ and of the LP mode order $m_j \epsilon M$; the maximization is taken over modes with $j \neq i$. The DMG represents the largest modal gain difference at each wavelength. The value DMG_{max} is defined as the maximum of DMG(λ) over the C-band. With our optimization, we aim to minimize DMG_{max}.

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2.1 First optimization criterion

Equalizing modal gain in FM-EDFAs is commonly based on the calculation of the overlap between each mode and the doped regions [26] and, as such, it corresponds to optimizing the design under a small signal gain condition. Dividing the fiber core into rings, as represented in Fig. 1., the criterion is defined as

$$Z_1 = \max_{\lambda_i}\left(\frac{\max\limits_{m \in M}\left(\sum_{k=1}^{K} \Gamma_{s,(\mathrm{m},\lambda_i,k)}\rho_{Er,(k)}\right)}{\min\limits_{m \in M}\left(\sum_{k=1}^{K} \Gamma_{s,(\mathrm{m},\lambda_i,k)}\rho_{Er,(k)}\right)}\right), \tag{2}$$

where $\Gamma_{s,(m,\lambda_i,k)}$ is the distribution of the m$^{\text{th}}$ mode intensity over a specific region k of the fiber, λ_i is the wavelength and $\rho_{Er,(k)}$ is the erbium ion concentration over the same k region of the fiber. In the present case, each k region is defined as a 0.1 µm thick ring (annular regions), in the fiber core.

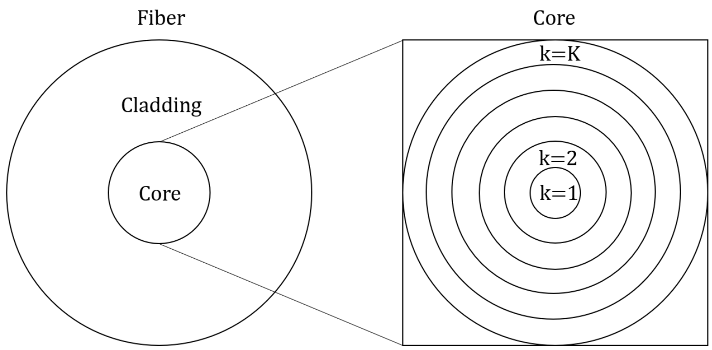

Fig. 1. Fiber core divided into K rings (annular regions), each ring k characterized by its erbium ion concentration $\rho_{Er,(k)}$ and mode intensity distribution $\Gamma_{s,(\mathrm{m},\lambda_i,k)}$.

This criterion ensures that the different signal modes have access to a similar quantity of erbium ions for amplification, and therefore that a low DMG can be obtained. However, when used alone, this criterion is not sufficient since it does not consider the fact that the saturation of the erbium ion population in each ring is a function of the total signal intensity in that ring. To illustrate that this criterion alone is not sufficient, we examine the EDP profiles EDF-A and EDF-B shown in Fig. 2. The fibers have the same step-index RIP but two different EDP. The fiber mode profiles, simulated with COMSOL, considering the step-index RIP with a core index, n_{core}, of 1.449, a numerical aperture, NA, of 0.12 and a core radius of 10 µm, are displayed in Fig. 2. The graph also shows the total signal intensity, I_k, defined as the sum of the intensity of all modes in a given region k.

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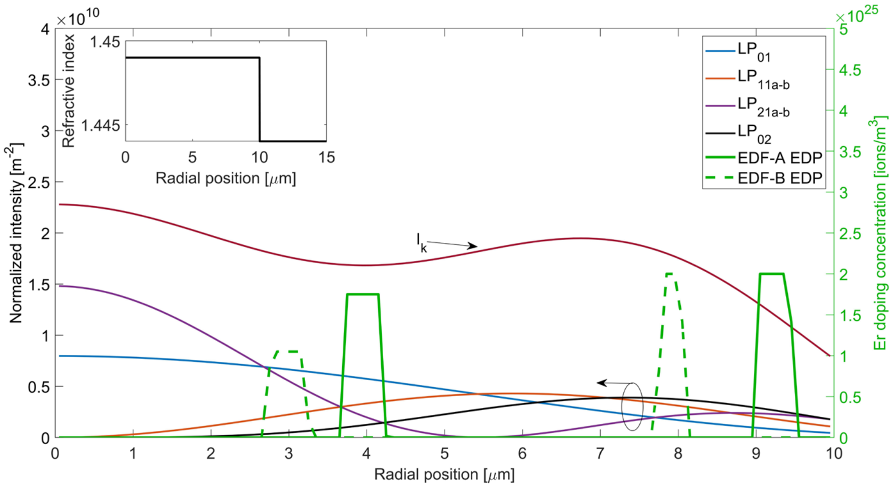

Fig. 2. The mode profile distributions and the total signal intensity of EDF-A and EDF-B across the fiber core. The green solid and dashed lines are the two-ring erbium doping profile of EDF-A and EDF-B respectively. The inset shows the RIP used for both designs.

Now, calculating Z_I for both profiles, we obtain a value of 1.012 for EDF-A and 1.022 for EDF-B, indicating that EDF-A should provide better gain equalization between the modes considering the more equal overlap of each mode with the erbium ion distribution. We now simulate both EDF using the full simulation model with the two-level erbium ion model and propagation equations that are solved with a 4th Runge-Kutta method. The simulations consider the radial dependency of the signal power and population inversion across the fiber core as indicated by the summation over k in the equations presented in [27]. In the simulations, we neglect clustering and use typical absorption and emission cross-sections of aluminosilicate erbium doped fibers. We also consider that the LP_{11a}, LP_{11b}, LP_{21a} and LP_{21b} are excited with equal power and propagate independently, i.e. the coupling between the modes is neglected. These mode pairs thus result in a power distribution with no azimuthal dependence. This model is described in more detail in [27] and the simulation parameters can be found in Table 1. In this configuration, the EDFA is cladding-pumped and we use 36 wavelength channels per mode, which amounts to a total input power of -9 dBm per mode or -1.22 dBm total input signal power. That is, we are examining performance outside the small signal gain region. The resulting gain and DMG, calculated over the C-band, are shown in Fig. 3. and Fig. 4.. Results reveal that EDF-B has much lower DMG, with DMG_{max}=0.464 dB, than EDF-A, with DMG_{max}=2.065 dB. EDF-A and EDF-B respectively display a maximum noise figure of 3.96 dB and 4.12 dB over the C-band. In these simulations, the optimal length is the fiber length for which the minimal gain over the C-band for all the modes is the highest. We used the minimum gain to determine the optimal fiber length since, when optimizing for gain flatness, the process would sometimes lead to fiber designs with low DMG accompanied by very low gain values.

Table 1. EDP parameters sweeping ranges for the step-index RIP design method

Parameter	Value
λ_p	976 nm
P_p	20 W
P_s	-9 dBm/mode
λ_s	1530-1565 nm (1 nm step)

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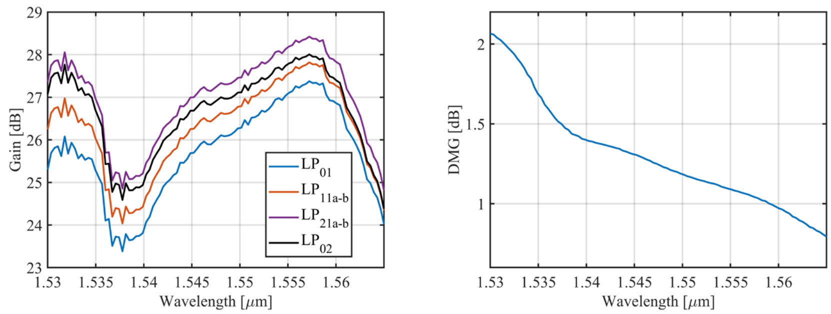

Fig. 3. Gain (left) and DMG (right) over the C-band in EDF-A for the six modes LP_{01}, LP_{11a}, LP_{11b}, LP_{21a}, LP_{21b} and LP_{02} (P_p=20 W, P_s=-9 dBm/mode, fiber length = 23 m).

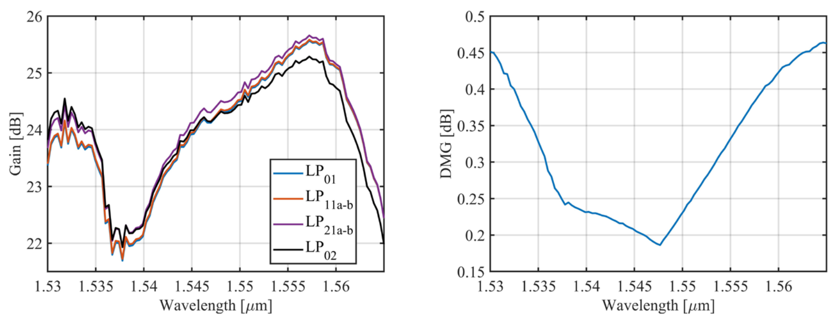

Fig. 4. Gain (left) and DMG (right) over the C-band in EDF-B for the six modes LP_{01}, LP_{11a}, LP_{11b}, LP_{21a}, LP_{21b} and LP_{02} (P_p=20 W, P_s=-9 dBm/mode, fiber length = 30 m).

The difference in DMG observed between the two FM-EDF can be understood when examining the distribution of the total signal intensity in the core as represented by the red curve in Fig. 2. In the case of EDF-A, there is an important difference between the total signal intensity overlapping with the two doped regions. This means that, as the signal propagates along the doped fiber, the two doped rings will have different inversion levels. Because the overlap of each mode with the two rings are different, the modes will experience different gain saturation, resulting in an increased DMG with respect to the small signal criterion. In the case of EDF-B, the total signal intensity at the position of both doped rings is similar and, consequently, the doped rings will experience the same level of gain saturation. This means that all modes will also saturate similarly since how the mode interaction with the doped region is divided between the two rings no longer introduces a difference in the modal gain. It is thus necessary to place erbium doping at locations that experience similar level of total signal intensity in order to achieve low DMG.

2.2 Second optimization criterion

The results presented in the previous section highlight the need for a second optimization criterion, Z_2, that will take saturation of the population inversion into account to correctly estimate if a fiber design, with a specific RIP and EDP combination, will provide a low DMG. We are considering cladding-pumped FM-EDFAs and assume a uniform distribution of the pump power over the whole cladding area. We here further assume that the amplifier has uniform input signal conditions, which means that all signal modes are present with the same

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number of channel wavelengths and the same input power. We now define a new optimization criterion that will direct solutions towards designs for which the erbium doped regions will overlap with similar levels of total signal intensity, for example by placing erbium doping in the region where the total signal intensity, I_k, is flatter, and preferably equal (see Fig. 2.). The criterion is defined as

$$Z_2 = \frac{\max\limits_{k \in EDP}\left(I_{(k,\rho \neq 0)}\right)}{\min\limits_{k \in EDP}\left(I_{(k,\rho \neq 0)}\right)}, \tag{3}$$

where $I_{(k,\rho \neq 0)}$ is the total signal intensity over a specific k region of the fiber where erbium doping is present.

To achieve the ideal DMG of 0 dB, a few mode EDF must thus meet the following three conditions: it has a Z_1=1, i.e. all the modes have the same overlap with erbium ion population; it has Z_2=1, i.e. the total signal intensity is uniform over the doped region such that saturation will be uniform for all modes; and the same signal power spectrum is injected for each guided mode in the fiber (although the absolute power level can change provided that the change is uniform across channels). Respecting these conditions, the signal power spectrum will be modified for each mode exactly the same way between z_i and $z_j=z_i+\Delta z$, starting with z_i=0, such that the three conditions remain true at any other longitudinal position along the fiber. Now, evaluating the Z_2 criterion for EDF-A and EDF-B, we respectively obtain 1.582 and 1.053, which shows that the signal intensity inside the doped regions of EDF-B is more uniform than in EDF-A, thereby explaining the previously demonstrated DMG difference between these fibers.

Thus, considering that a fiber design that respects the conditions mentioned above could theoretically lead to a DMG of 0 dB, we propose a novel FOM and design method that targets these solutions. Optimization of fiber designs with this method involves parameter sweeping of combinations of RIP and EDP, but is much less computationally demanding than using the full simulation model with Runge-Kutta solutions for each possible combination of parameters.

2.3 Figure-of-merit

Our goal is to examine a wide variety of designs by sweeping parameters describing the RIP and EDP of the fiber. The criteria Z_1 and Z_2 for each of these designs are evaluated from overlaps of the EDP with the mode intensity profiles and total intensity profile. Then, we choose the optimal design between all the possible designs for which Z_1 and Z_2 were calculated by calculating the figure of merit Z_{tot}, which is defined as

$$Z_{tot} = (Z_1 - 1)(Z_2 - 1). \tag{4}$$

We examine below the usefulness of this FOM that minimizes both Z_1 and Z_2. In the following method, we calculated Z_1 and Z_2 individually to gain more physical insight on the designs. However, one could also simply use Z_{tot} as the sole optimization criteria to obtain a low DMG FM-EDF, without first calculating the Z_1 and Z_2 values.

2.4 Design method

As an example, in order to target designs compatible with common fabrication processes based on MCVD with solution doping, we focus on designs that have two-doped ring regions in the core. Furthermore, we consider at most two different values of refractive index in the core, one

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near the center of the core and one around. Although challenging to fabricate, fiber designs with similar small features were successfully fabricated with a MCVD-based manufacturing process in [31].

Such a generic profile is presented in Fig. 5. and the overall design process is illustrated by the schematic in Fig 6. Despite its simplicity, the choice of the two-ring EDP offers great flexibility and provides a large number of explorable designs.

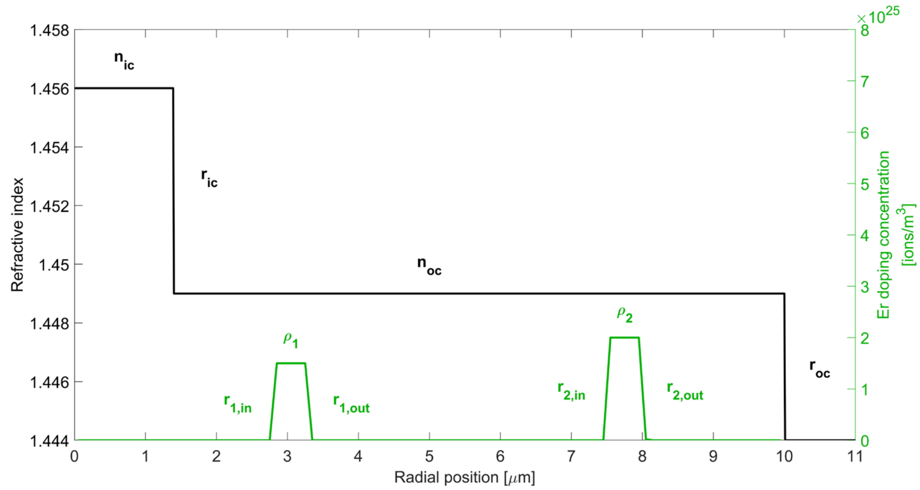

Fig. 5. EDP (green) and RIP (black) with the optimization parameters used in the simulations.

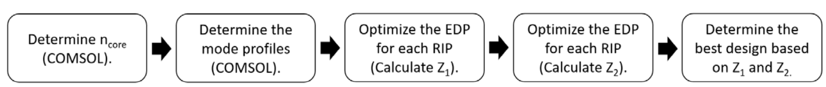

Fig. 6. Schematic of the methodology used to design low-DMG few-mode EDFs.

The method consists of five steps. The first step consists of defining the RIP such that the fiber core supports only the desired number of modes. We wish to explore a space of RIPs defined by a set of four parameters, namely the inner core index and radius (n_{ic} and r_{ic}), and the outer core index and radius (n_{oc} and r_{oc}) as represented in Fig. 6. We first choose to limit our investigation to a fixed outer core radius, r_{oc}=10 µm, as it is a reasonable value for fabrication purposes, power coupling and bending loss. The second step consists of calculating the mode profiles using COMSOL while sweeping values of n_{ic} and r_{ic}. For every parameter set (n_{ic}, r_{ic}), we need to determine the highest possible n_{oc}. This value is found by identifying the n_{oc} for which the effective index of the lowest order unwanted mode is at least 10^{-5} lower than the index of the cladding ($n_{eff} \approx n_{clad} - 10^{-5}$); thus the mode is not guided. We fix n_{oc} to this value. The mode profiles calculated with the parameter values (n_{oc}, $r_{oc,}$ n_{ic} and r_{ic}) are stored and can then be used to compare different EDPs. In the third and fourth steps, we thus calculate the values of the Z_1 and Z_2 criteria by sweeping the values of the two-doped ring parameters, which are ρ_1, $r_{1,in}$, $r_{1,out}$, ρ_2, $r_{2,in}$ and $r_{2,out}$. Finally, in the fifth step, we determine the optimized design by evaluating Z_{tot}.

In the next sections, we use of this method to find the optimal design for two different types of RIP. Firstly, in section 3, we consider a simple step-index fiber with only one refractive index value in the core (r_{ic}=0 µm). Then, in section 4, we consider a raised index region at the center of the fiber core and examine fiber designs with staircase RIP. It is worth noting that the

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choice to restrict the design of the 6-mode FM-EDFA to a two-ring erbium doping structure is simply a design choice. The method could be applicable to the exploration of a larger parameter space that would encompass a wider variety of designs. The initial parameter values of the two-ring structure in Table 2 were determined by coarse parameter sweeping. A similar approach could be developed for the case of core pumping but it would require redefining the optimization criteria to consider the different overlap of the various pump modes with the doped regions.

3. Application of the design method for a step-index fiber

As a first example, in this section, we design a two-ring erbium doped 6-mode FM-EDFA with a step-index RIP using the method previously described. The results show the benefits of using the FOM. We follow the steps outlined in Fig. 6.

3.1 Determination of n_{oc}

The goal of this step is to adjust the RIP such that only 6 modes are guided. Since the chosen design is a step-index fiber (r_{ic}=0 μm), and considering that we have set r_{oc}=10 μm, we only need to choose the n_{oc} value that will result in 6 guides modes (LP_{01}, LP_{11a}, LP_{11b}, LP_{21a}, LP_{21b} and LP_{02}), it is set to n_{oc}=1.449.

3.2 Calculating the mode profiles

The mode profiles are calculated using *COMSOL* although, for this simple step-index RIP, the analytical expression of the mode profiles could also be used. The resulting mode profiles have been presented earlier in Fig. 2.

3.3 Determining the optimal design using the figure-of-merit Z_{tot}

We now calculate Z_1, Z_2, and Z_{tot} for a wide variety of EDPs with two doped rings located in the fiber core. Based on previous simulation results and limiting erbium doping concentrations to achievable values, the sweeping range for each parameter are the presented in Table 2.

Table 2. EDP parameters sweeping ranges for the step-index RIP design method.

Parameter	Range	Step
ρ_1	0.10 to 2.00 ×10^{25} ions/m^3	0.05× 10^{25} ions/m3
$r_{1,in}$	2.00 to 4.00 μm	0.02 μm
$r_{1,out}$	$r_{1,in}$ + 0.5 μm	
ρ_2	2·10^{25} ions/m^3	
$r_{2,in}$	5.00 to 9.00 μm	0.02 μm
$r_{2,out}$	$r_{2,in}$ + 0.3 to $r_{2,in}$ + 0.7 μm	0.01 μm

To examine if the optimization criteria Z_1 and Z_2, and the FOM Z_{tot} are good predicators for low DMG, we performed full simulations of the resulting fiber designs under the scenario presented in Table 1, i.e. we solved the radially resolved population rate equations and the signal/pump propagation equations through numerical simulations for each design. In Fig. 7., we present a color map of DMG_{max} as a function of Z_1 and Z_2, while the relationship between DMG_{max} and the FOM Z_{tot} is shown in Fig. 8. Therefore, in Fig.7., each circle represents a different EDP that was generated and evaluated in the optimization process. The DMG_{max} value on the colormap that was capped to maximum of 2.5 dB to better illustrate the change in the

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low DMG_{max} values (some EDP led to DMG up to 8.65 dB). All the 526 data obtained from the simulations is present in Fig. 7., but data with DMG values above 2.5 dB are assigned the same color in order to enhance the discrimination between the lower DMG values on the color scale. Fig. 7. shows the importance of evaluating both criteria. To choose the optimal design between all the generated EDP, we now turn to Z_{tot} as the FOM and we obtain the fiber labeled as EDF-B, with the RIP and EDP profiles shown in Fig. 2. As discussed earlier, this design achieved 21.7 dB of minimal gain and less than 0.46 dB of DMG over the C-band for a fiber length of 30.0 m. EDF-B is characterized by Z_{tot}=0.0012.

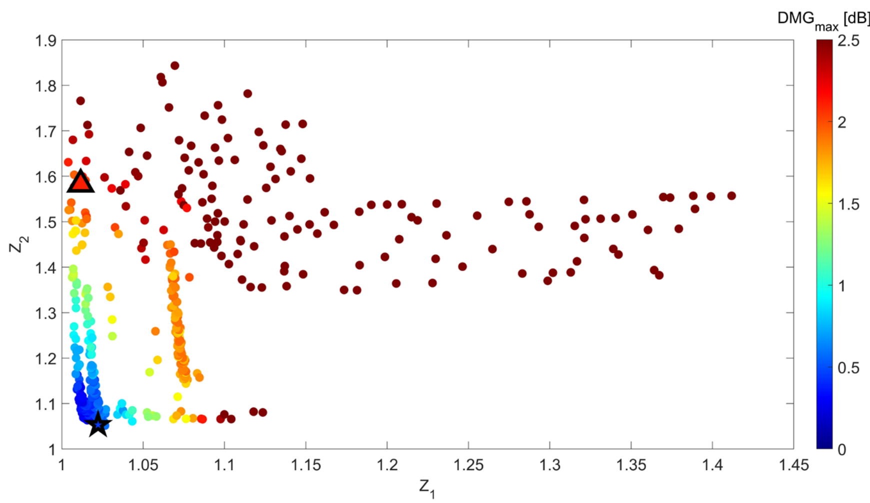

Fig. 7. Color map of the DMG_{max} as a function of both optimization criteria in the case of a step-index refractive index profile (EDF-A and EDF-B are respectively represented by a black triangle outline and a black star outline filled with their appropriate color from the DMG color scale).

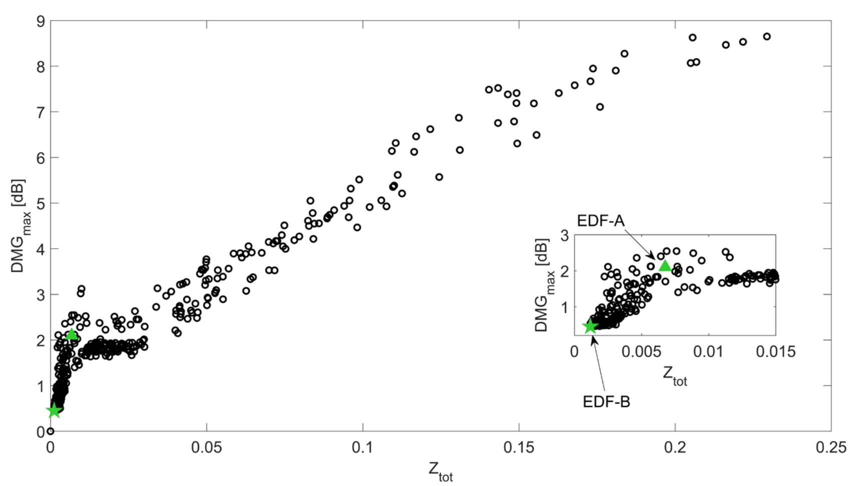

Fig. 8. DMG_{max} as a function of the FOM Z_{tot} for a step-index refractive index profile. The inset zooms on the low DMG_{max} region (EDF-A is shown by a green triangle and EDF-B are shown by a green star).

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4. Results for a staircase index fiber

We now examine if more complex RIP can be used to further reduce the DMG using the proposed optimization technique. Since we are targeting simple designs compatible with standard fiber fabrication techniques, we add only one additional step to the refractive index profile, resulting in the staircase profile shown in Fig. 5. The sweeping ranges of the different optimization parameters of the RIPs and EDPs are presented in Table 3. An optimized fiber, EDF-C, with the RIP and EDP shown in Fig. 9. left, is obtained. The optimization led to a RIP with a r_{ic} of 1.75 µm and a n_{ic} of 1.453, and an EDP with the first doped ring having an erbium ion concentration of 1.2×10^{25} ions/m^3 located at r=3 µm, and a second doped ring with a higher erbium doping concentration of 2.0×10^{25} ions/m^3 located at r=8 µm. Fig. 9. right displays the six mode profiles as well as the total intensity, which shows that the total intensity in both doped rings is very similar in this optimum design.

Table 3. EDP and RIP parameters sweeping ranges for the staircase RIP design method.

Parameter	Range	Step
ρ_1	0.10 to $2.00\cdot10^{25}$ ions/m^3	$0.05\cdot10^{25}$ ions/m^3
$r_{1,in}$	2.00 to 4.00 µm	0.02 µm
$r_{1,out}$	$r_{1,in}$ + 0.5 µm	
ρ_2	$2\cdot10^{25}$ ions/m^3	
$r_{2,in}$	5.00 to 9.00 µm	0.02 µm
$r_{2,out}$	$r_{2,in}$ + 0.3 to $r_{2,in}$ + 0.7 µm	0.01 µm
r_{step}	0.5 to 2.5 µm	0.25 µm
n_{step}	1.450 to 1.460	0.001

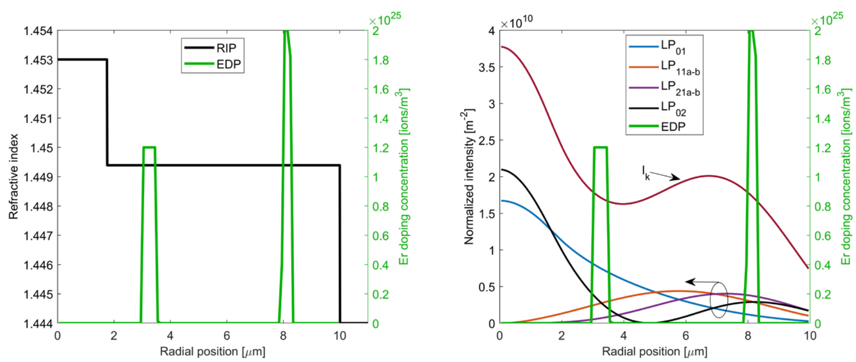

Fig. 9. The staircase RIP and the two-ring EDP (left), and the simulated mode profiles plotted with the EDP (right), of the optimized design EDF-C.

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This fiber design is now simulated with the full EDFA model described previously under the scenario presented in Table 1, and the results are shown in Fig. 10. In this case, the optimization technique led to a combination of RIP and EDP that achieved 22.6 dB of minimal gain and reached a low DMG_{max} of 0.18 dB over the C-band. The optimal fiber length was of 29 m and the maximum noise figure was of 4.08 dB. This fiber reached Z_{tot}=0.0006. It should be noted that we also investigated designs with a depressed index region at the center of the fiber core and it did not lead to improved performance.

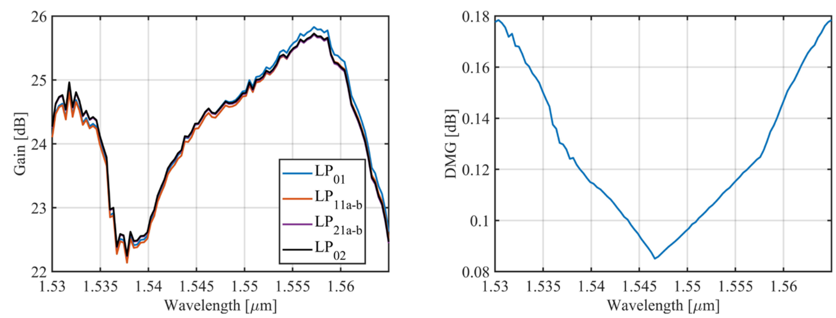

Fig. 10. Gain (left) and DMG (right) for the LP_{01}, LP_{11a}, LP_{11b}, LP_{21a}, LP_{21b} and LP_{02} modes of the few mode EDF design EDF-C, which has a staircase RIP (P_p=20 W, P_s=-9 dBm/mode, fiber length = 29 m).

Fig. 11 shows the spectral gain and DMG of fiber EDF-C when the fiber length is chosen for optimum gain flatness instead of the highest minimum gain. With a length of 27.3 m, we obtain a minimum gain of 21.95 dB and a DMG_{max} of 0.1820 dB.

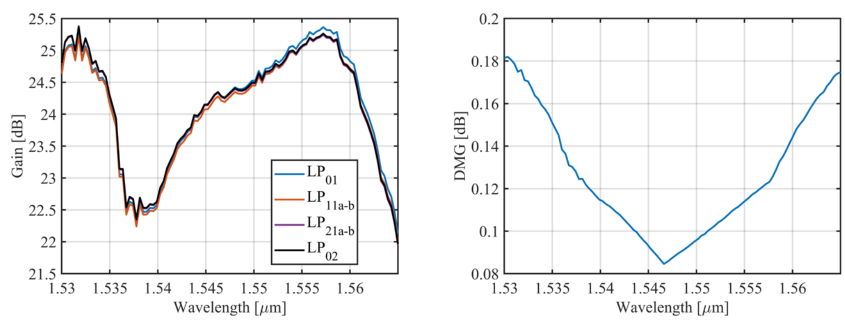

Fig. 11. Gain (left) and DMG (right) for the LP_{01}, LP_{11a}, LP_{11b}, LP_{21a}, LP_{21b} and LP_{02} modes of the few mode EDF design EDF-C, which has a staircase RIP and where the length has been optimized for gain flatness (P_p=20 W, P_s=-9 dBm/mode, fiber length = 27.3 m).

5. Sensitivity to the operation point

We now examine the earlier claim that the DMG_{max} variations will be minimum when the operating point of the amplifier changes. This property follows from the similar level of total signal intensity in the doped regions. If we compare the FOM value Z_{tot} for EDF-B and EDF-C (0.0012 vs 0.0006) we see that EDF-C will perform better than EDF-B. However, both designs are robust with regards to operating conditions because Z_2 was minimized in both cases. In Fig.12., we plot the variations of DMG_{max} over the C-band as a function of fiber length for both designs, which shows almost no DMG_{max} variations as the fiber length deviates from the optimal fiber lengths represented by the dash-dotted lines. Although the inversion varies along the fiber length, all signal modes interact with erbium doped regions that are similarly inverted,

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which preserved the low DMG_{max} property. By a similar argument, the resulting designs also maintain a low DMG_{max} for a wide range of pump power and signal power. The variations of DMG_{max} as a function of the pump power injected in the cladding is presented in Fig. 13. DMG_{max} slightly increases with pump power, but so does the minimum gain, with the result that the ratio $DMG_{max}/Gain_{min}$ in linear scale is approximately constant at 0.007197 ± 0.000004 for EDF-B and 0.005915 ± 0.000004 for EDF-C, over a pump power range from 15 to 30 W. The DMG_{max} as a function of the modal signal input power is shown in Fig. 14. It is seen that the DMG_{max} decreases as the input signal power per mode increases but, as the minimum gain also decreases, the ratio $DMG_{max}/Gain_{min}$ is 0.00672 ± 0.00006 for EDF-B and 0.00555 ± 0.00005 for EDF-C over an input signal mode power range from -25 dBm to -5 dBm. In Fig. 12., 13. and 14., the parameter value corresponding to the optimized design are indicated by the vertical dash-dotted lines, while other parameters are held constant to the values used in the optimization, i.e. 20 W of pump power, -9 dBm/mode of signal power and the optimum fiber length. Fig. 13. and Fig. 14. both show the increased robustness of the EDF-C design to changes in the operation point of an amplifier.

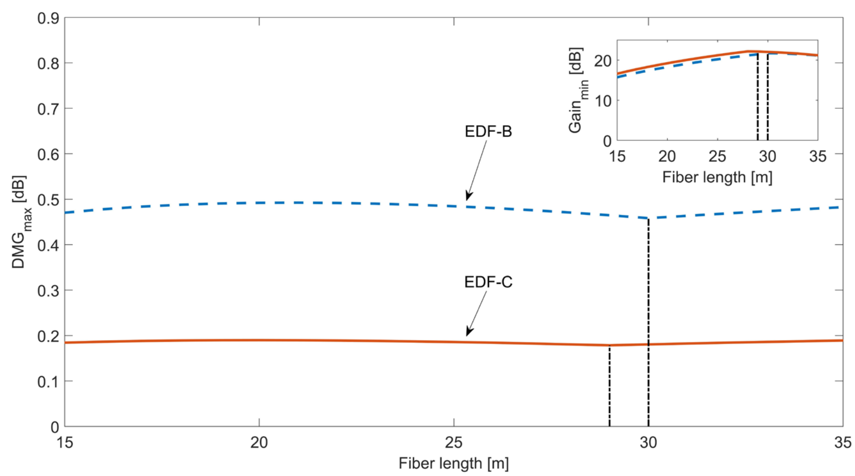

Fig. 12. DMG_{max} (over the C-band) as a function of the fiber length for EDF-B (dashed blue) and EDF-C (solid red). The dash-dotted vertical lines show the optimal length (P_s=20 W and P_s=-9 dBm/mode). The inset shows the variation in $Gain_{min}$.

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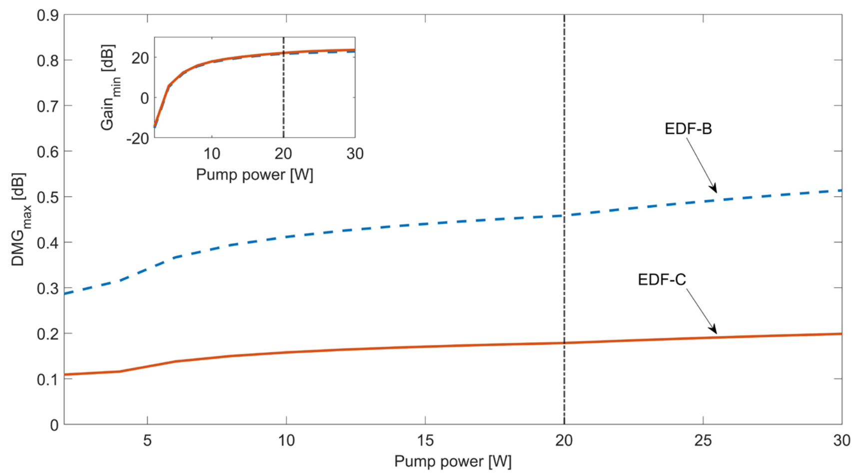

Fig. 13. DMG$_{max}$ (over the C-band) as a function of the pump power for EDF-B (blue dashed) and EDF-C (red solid). The dash-dotted vertical line indicates the pump power used to optimize the design (P_s=-9 dBm/mode, fiber length = 30 m for EDF-B and fiber length = 29 m for EDF-C). The inset shows the variation in Gain$_{min}$.

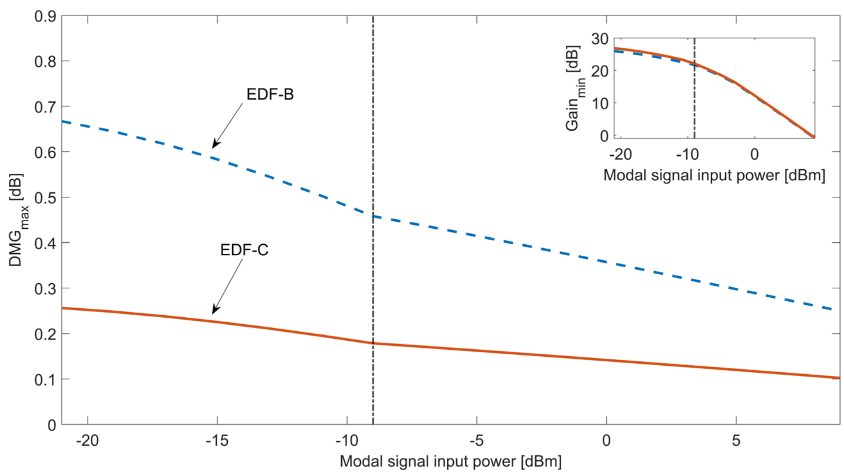

Fig. 14. DMG$_{max}$ (over the C-band) as a function of the signal input power in each mode for EDF-B (blue dashed) and EDF-C (red solid). The dash-dotted line shows the input signal power in each mode used to optimize the two fiber designs (P_p=20 W, fiber length = 30 m for EDF-B and fiber length = 29 m for EDF-C). The inset shows the variation in Gain$_{min}$.

6. Conclusion

We proposed novel FOM and design method based on two optimization criteria that results in low DMG in cladding-pumped few-mode EDFAs. This method has low computational cost, which allows to explore a wide variety of possible erbium-doping profiles. The interest of this method was validated by finding the best designs with two-doped ring regions in six-mode

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EDFs. Even if a two-doped ring 6-mode FM-EDFA was used as an example in this paper, the method in itself is not restricted to this fiber design. The design of a first EDF, with a simple step-index refractive index profile, showed that the usual criteria based on equal overlap of the fiber mode profile with the erbium doping is not sufficient to identify the optimum designs. By using the proposed FOM, we can identify the fiber designs that will provide the lowest DMG. The best design of a two-ring erbium doped few-mode EDF was achieved with the staircase refractive index profile for which a DMG_{max} of 0.18 dB and a minimal gain of 22.6 dB were obtained. The novelty of this FOM is that it considers the local saturation of the erbium ion population by the total signal intensity of all modes, which impacts the optimal distribution of the erbium doping in the core. With this method, fiber design optimization can be performed without having to resort to full EDFA simulations. Lastly, the use of this novel FOM also has the advantage of identifying designs that will maintain a low DMG for a very wide range of signal power, pump power and fiber length values. We believe that this method greatly simplifies the design of erbium-doped fibers for cladding-pumped few mode EDFAs.

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Data availability. All the data presented in this paper are not publicly available but could potentially be shared upon request.

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