ABSTRACT

Title of dissertation:	ESSAYS IN MACROECONOMICS AND INTERNATIONAL FINANCE
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	Doctor of Philosophy, 2012
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This document is a collection of essays in two issues of interest in macroeconomics and international finance. Chapter 2 introduces price promotions in a monetary DSGE model where consumers differ in their price sensitivity and look for promotions, and where firms choose their regular and promotional prices as well as the frequency of promotions. In this environment, regular and promotional prices coexist, firm-level prices show rigidity in the form of inertial reference values from which weekly prices temporarily deviate, and promotions provide a new channel of price adjustment in the face of shocks. As a result, the economy displays near neutrality with respect to monetary shocks, with an impact response of output equal to one third of the one obtained in a model with no promotions. This result contrasts sharply with those of similar studies which, using alternative rationales for price

promotions, find that price promotions do not fundamentally alter the real effects of monetary shocks. Chapter 3 studies the currency substitution phenomenon and develops a two-currency model that introduces "dollarization capital" as a means to capture the economy's accumulated experience in using the foreign currency. The model is able to generate a low-inflation-high-substitution equilibrium consistent the data, and explains 1/6 of the gap between the observed currency substitution ratios and those generated by a model with no dollarization capital dynamics. The model, however, does not generate asymmetries in the relationship between inflation and currency substitution before and after high inflation episodes. Therefore, Chapter 4 presents a simple framework that creates non-linearities between inflation and currency substitution. The model has two consumers who can differ in their distance from money exchange points provided by the financial sector, who decides whether or not to pay a fixed cost necessary to install these exchange points. In this environment, a sequence of episodes of high and moderate inflation may push the financial sector into expanding the number of available money exchange points, therefore permanently reducing the consumers' cost of using the foreign currency and decreasing the inflation threshold at which households are willing to substitute foreign for domestic currency.

ESSAYS IN MACROECONOMICS AND

INTERNATIONAL FINANCE

by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2012

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ACKNOWLEDGEMENTS

Above all I want to thank the Lord Jesus Christ, my savior, for making it possible for me to earn this degree. Without Him I could do nothing. Without Him I would be lost. Jesus, I have no words to tell you how much I appreciate all you have done for me.

I have had a great dissertation committee. John Shea was willing to read every piece of work that I sent to him (even when he was not at UMD and was busy working at the Treasury Department!). Boragan Aruoba has been very patient, supportive and helpful. He is a terrific advisor, inspiring and always willing to spend time with his students. Sanjay Chugh is the person that first got me interested in monetary economics. His macro course was outstanding; my discussions with him were the seed of my job market paper. Carlos Vegh has been an inspiration, a mentor and a friend (gracias!). I also thank professor P.K. Kannan from the Robert H. Smith School of Business for accepting the invitation to be the Dean's Representative in my committee. To all of you, thank you! I wish you all the best in your professional and personal lives.

I would also like to thank Carmen Reinhart for all her support during her time at Maryland. Working as a research assistant for her and Carlos Vegh was an

extremely rewarding experience.

Also, I'd like to thank Enrique Mendoza, Anton Korinek and Pablo D'Erasmo for all the helpful discussions that we had during the time of preparation of my job market paper.

I also thank Tom Powers, Gerry Caprio, Peter Montiel, Peter Pedroni, Jon Bakija and Dukes Love from the Center for Development Economics and the Department of Economics at Williams College.

I thank my family (Joaquin, Maricarmen and Viviana) for their love and support, and Carlos Echazu (closest thing I have to a blood brother) for always being there.

I thank Avy, Carola, Cristina, Daniela, Chris, Danny, Ethan and Zoe, my Virginian family. I've had some really good days during my time in Maryland, but the ones I spend with Ethan and Danny were among the best.

Five years ago I met two guys looking for a roommate. Today I see them as my brothers. Cesar and Gabo, aguante el Chateau!!!

I thank Juandi and Caro for being like they are: two of the best people I've met in my life.

I thank Pablo Cuba for his friendship (he is always willing to help somebody in need). I'm glad he came to Maryland and I expect him to be one of the best macroeconomists this university has produced.

Finally, I thank Tim and Maria Belen, Teresa and Greg, and Abby for being such good friends.

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Chapter 1

Introduction

This document is a collection of essays On two different issues of interest in macroe-conomics and international finance. The second chapter presents a study on the role of sale-induced high frequency price fluctuations in monetary economics. The third and fourth chapters focus on the phenomenon of currency substitution observed in several developing countries and transition economies.

Chapter 2 is motivated by recent microeconomic price evidence that shows that price promotions may have a significant bearing on the measured frequency of price changes. This finding raises the question of whether micro price flexibility caused by the presence of price promotions translates into macro neutrality to mon-etary shocks. The issue is addressed by introducing price promotions in a dynamic stochastic general equilibrium (DSGE) environment where agents differ in their de-

gree of price sensitivity and face a time cost when looking for promotions. On the production side, firms optimally choose their regular and promotional prices as well as the frequency of promotions.

In this framework, regular and promotional prices coexist and promotions provide a new source of firm-level price flexibility. After introducing Calvo-stickiness in regular prices, individual price series behave similarly to those reported in recent empirical studies: regular prices are sticky; prices including promotions change very frequently, and realized prices show rigidity in the form of inertial reference values from which weekly prices temporarily deviate. In the aggregate, the economy displays near neutrality with respect to monetary shocks with a response of output that, on impact, is equal to one third of the one obtained from a standard Calvo model, a result that shows the importance of further exploration of firm-customer interactions in macroeconomic models.

Chapter 2 is organized in five sections. The first one presents the introduction. The model is developed in the second section. The solution method, parameters and steady state values are discussed in the third section. The fourth section presents results on the model's micro price behavior and the macroeconomic implications of price promotions, particularly on the responses to monetary shocks. The fifth section concludes.

Chapter 3 studies the phenomenon of currency substitution; that is, the use of foreign currency as a means of payment, as frequently observed in several developing countries and emerging economies. In several countries, currency substitution increases with inflation during high inflation periods and reaches its peak once the

economy has been stabilized, showing high persistence in the subsequent periods in spite of the downward trend in inflation. The chapter focuses on two examples of this particular phenomenon: Bolivia and Peru.

The chapter develops a perfect foresight two-currency model with exogenous production and transaction costs that is used to generate persistence in currency substitution. The model introduces "dollarization capital" as a means to capture the economy's accumulated experience in using the foreign currency. When parameters are set to replicate observed pre-hyperinflation ratios in the two selected economies, the model is able to generate a low-inflation-high-substitution equilibrium consistent with the post-stabilization behavior observed in various emerging markets and developing countries. The dollarization capital model explains from 1/6 of the gap between the observed currency substitution ratios and the ones generated by a model with no dollarization capital dynamics. As a result of quick and sizable changes in domestic currency real balances, the predicted post-stabilization currency substitution ratios adjust quickly in their path towards a low-inflation equilibrium, a finding that suggests focusing future modeling efforts on exploring plausible mechanisms that may introduce post-stabilization inertia in domestic currency holdings.

Chapter 3 is organized in five sections. After the introduction, the second section describes the pre and post high-inflation pattern of currency substitution in Bolivia and Peru. The theoretical model is presented in the third section and solved in the fourth section. The fifth section concludes.

One of the limitations of the model analyzed in Chapter 3 is that it is not able to generate an asymmetric relationship between inflation and currency substitution

before and after high inflation episodes. This behavior is at odds with the data in several partially dollarized economies. Therefore, Chapter 4 explores a simple model that generates non-linearities in the relationship between currency substitution and inflation.

The chapter presents a simple three agent model with two consumers who can differ in their distance, and therefore walking cost, from money exchange points provided by the third agent, the financial sector, who is the only one that possesses the technology to exchange domestic for foreign currency. Money is used for transaction purposes and agents have an incentive to hold foreign currency balances for high enough levels of inflation. The possibility of substitution, however, depends on whether or not the financial sector pays a fixed cost necessary to install money exchange points where consumers can buy currency exchange services. In this environment, a sequence of episodes of high and moderate inflation may push the financial sector into expanding the number of available money exchange points, therefore permanently reducing the consumers' cost of using the foreign currency and decreasing the inflation threshold at which households are willing to substitute foreign for domestic currency.

From a theoretical point of view, the work presented in Chapter 4 constitutes an effort to provide a micro foundation for the emergence and functioning of network externality effects that have been explored as a possible explanation for high levels of currency substitution even after several years of low inflation.

Chapter 4 is organized in three sections. The first section presents the introduction. The second section presents a simple three agent model used to study the

consumer's optimal currency substitution behavior and the financial sector's optimal

dollarization capital investment decision. The third section concludes.

Chapter 2

A Monetary Model With Price Promotions

Recent microeconomic price evidence shows that price promotions may have a significant bearing on the measured frequency of price changes, a finding that raises the question of whether micro price flexibility translates into macro neutrality to monetary shocks. This chapter addresses this issue by introducing price promotions in a dynamic stochastic general equilibrium (DSGE) environment where agents differ in their degree of price sensitivity and face a time cost when looking for promotions. On the production side, firms optimally choose their regular and promotional prices as well as the frequency of promotions. Under this setting, regular and promotional prices coexist and promotions provide a new source of firm-level price flexibility. After introducing Calvo-stickiness in regular prices, individual price series behave similarly to the ones reported in recent empirical studies: regular prices are sticky;

prices including promotions change very frequently, and realized prices show rigidity in the form of inertial reference values from which weekly prices temporarily deviate. In the aggregate, the economy displays near neutrality with respect to monetary shocks with a response of output that, on impact, is equal to one third of the one obtained from a standard Calvo model, a result that shows the importance of further exploration of firm-customer interactions in macroeconomic models.

The exercise of studying price promotions in a macroeconomic context is motivated by recent empirical work showing that the treatment of price sales may have an important effect on the estimated frequency of individual price changes. An example of this is the influential work of Bils and Klenow (2004), who analyzed the frequency of price changes in different components of the US Consumer Price Index (CPI), finding a median time between changes of 4.3 months. This estimate, which includes sale related price changes, is well below previous ones obtained using less comprehensive datasets. Given that the degree of price stickiness and, more specifically, the probability of price changes is an important element for the quantitative solution of New Keynesian models, the result has important implications for macroeconomic modeling. For example, when discussing Michael Woodford's *Interest and Prices*, Green (2005) mentions that "at the very least [Bils and Klenow's] study shows a need to recalibrate Woodford's model." Indeed, Bils and Klenow's result has become a calibration reference used in well known DSGE studies such as Altig, Christiano, Eichenbaum and Linde (2005) and Golosov and Lucas (2007).

Following Bils and Klenow's seminal work, recent research by Nakamura and Steinsson (2008) and Klenow and Krystov (2008) uses a more detailed dataset (the

US CPI Research Database) and finds that the estimated time between price changes is sensitive to the inclusion of sale prices. Particularly, Nakamura and Steinsson find median durations between 4.4 and 4.6 months when including sales, but between 8 and 11 months when excluding sales.

This new evidence on price setting has been complemented by recent studies using higher frequency micro price datasets. Eichenbaum, Jaimovich and Rebelo (2011) analyze a new weekly scanner dataset for a major US retailer, finding that nominal rigidities take the form of inertia in reference prices with weekly prices fluctuating around reference values that tend to remain constant over extended periods of time. Klenow and Malin (2010) find that the reference price phenomenon documented by Eichenbaum et. al. (2011) extends to most items in the non-shelter CPI.[1]

These results suggest that promotional prices have a large impact on measures of the frequency of price changes, but also point to the need to explore the potential macroeconomic implications of price promotions.[2] In the words of Klenow and

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[1]The shelter component of the CPI is composed of the following items: (i) Rent of primary residence, (ii) Lodging away from home, (iii) Housing at school (excluding board), (iv) Other lodging away from home including hotels and motels, (v) Owners' equivalent rent of primary residence and (vi) Tenants' and household insurance.

[2]This also seems to be the case outside the US. For example, Dhyne et. al. (2006) note that the statistical treatment and report of sale prices may be an important factor in explaining the observed differences in the frequency of price adjustment across euro-area countries. Cavallo (2009), meanwhile, uses price scraped data for four Latin-American countries, finding that, in most cases, measured price durations are considerably increased by the exclusion of sales. Data scraping is the activity of extracting data from output intended for the screen or printer of a computer

Krystov (2008), "sales are not ... [just] a discount from the regular price; they have a life of their own." In relation to this point, Mackowiak and Smets' (2008) survey paper on micro price evidence presents some interesting comments on the macro-modeling implications of sales in micro price data. First, an optimizing model of sales will, in general, predict that the magnitude, frequency and duration of sales respond to macroeconomic shocks. The practice of excluding all sale-related price changes from macro models may therefore be unjustified. Second, sale-related price changes in the Bureau of Labor Statistics (BLS) dataset are at least as sensitive to inflation as are regular price changes (Klenow and Willis, 2007). Third, even if price promotions are caused by shocks orthogonal to the macroeconomy, the presence of sales may matter for the response of prices to macroeconomic shocks.

The first papers that incorporate sale prices into a macro model are Kehoe and Midrigan (2007, 2008). These papers modify the Golosov and Lucas (2007) menu cost model by explicitly including a motive for sales, allowing firms to pay for temporary price changes at a lower menu cost than that paid when changing their price permanently. The authors then use their sales model as a data generating process and evaluate the effects of fitting a simple Calvo model (with no sales motive in it) in order to match the moments generated by the sales model. They consider two cases: one including temporary price changes (i.e. sales) from the price series generated by the benchmark sales model, and one excluding them. They find that excluding sales overstates the real effects of monetary shocks by almost 70%, and including sales produces only 1/9 of the real effects of monetary policy in the bench-

(human-readable output) rather than from original files or databases (www.pcmag.com).

mark model. Kehoe and Midrigan's exercise highlights the importance of explicitly developing a motive for temporary price changes in monetary models.

More recently, Guimaraes and Sheedy (2011) and Kehoe and Midrigan (2010) focus on whether sale-related and temporary price changes translate into macroeconomic neutrality in a monetary DSGE environment. Guimaraes and Sheedy (2011) develop a framework in which sale prices are not linked to the existence of menu costs. They construct an environment with differentiated goods and differentiated brands for each good, with consumers willing to substitute between brands for some products, but not for others. Firms are not able to price discriminate between consumers, and therefore choose to offer a regular price at some moments in time and a sale price at others. If a Calvo-type rigidity is introduced in regular prices, the model generates individual time series that behave similarly to those studied by Eichenbaum et al. (2011). When studying the effects of monetary shocks, the authors find that output impulse responses are very similar to those of a model with no sales, implying that micro price flexibility does not translate into macro flexibility and, therefore, sales are essentially irrelevant for monetary analysis. This result is due to the fact that in their model, sales are strategic substitutes; that is, a firm's incentive to have a sale is decreasing in the number of other firms having a sale.

Continuing with their earlier work, Kehoe and Midrigan (2010) build an extension of the standard menu cost model adding features that make the predictions of the modified environment consistent with the micro price data. In their model, prices are subject to both permanent and temporary shocks; retailers set a list price and a posted price and face two fixed costs: one when changing the list price, and

another when the posted price temporarily differs from the listed one. In this environment, firms tend to change list prices in order to offset permanent productivity shocks and monetary shocks and temporarily deviate from the list price in order to offset transitory shocks. Given the menu cost friction on list prices, firms may recur to deviations from the listed price as a channel for offsetting transitory monetary shocks. However, since these deviations are temporary, this only allow firms to react temporarily to a monetary shock, so that economy presents a high degree of non neutrality even in the presence of frequent temporary price changes. Kehoe and Midrigan also show that this result does not arise from strategic interactions as in Guimaraes and Sheedy (2011) but solely from the special nature of temporary price changes in their model.

A common characteristic of Kehoe and Midrigan (2007, 2008, 2010) and Guimaraes and Sheedy (2011) is that households have no choice over how often they look for price promotions. Sales are therefore exogenous from the consumer's point of view. Letting consumers choose the time spent looking for price promotions creates a new intratemporal margin that has important implications for the model's response to aggregate shocks. This is the line of work followed in this document.[3]

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[3]Two other models that can deliver sticky regular prices with frequent sales are Nakamura and Steinsson (2009) and Albrecht et. al. (2010). Nakamura and Steinsson develop a habit persistence model where firms face a time inconsistency problem, having the incentive to promise low prices in the present and raise them once that households have developed a consumption habit. In this context, price stickiness can arise endogenously and optimally as a partial commitment device, with firms choosing a price cap and discretionally setting prices only when they are below the cap. Albrecht et. al. (2010), on the other hand, model shopping as a search process in a framework

The model presented here analyzes the behavior of a monopolistically competitive economy in which firms face different market demands and maximize profits by setting a regular price and a promotional strategy composed of a discount price and a promotion frequency. Following Banks and Moorthy (1999), the model allows for consumer heterogeneity by having agents with different price elasticities of demand, and introduces a time cost of looking for price promotions. In this context, it is possible to make an explicit distinction between regular and promotional prices: regular prices are always available to anyone, while promotional prices are only available when offered and only to those who look for them. The approach here thus differs from previous work in the area in at least two important aspects. First, the definition of regular and promotional prices differs from previous work. Second, consumers choose optimally the time spent looking for price promotions.

In this setting, heterogeneity in price sensitivity and the assumption that households must pay a time cost in order to take advantage of price promotions are enough to (i) ensure that household time choices interact with firm promotional strategies, creating an environment in which regular and promotional prices coexist and (ii) generate a new source of price flexibility through movements in promotional prices.

similar to that used in the job search literature. The authors show that, in a model with a semi-durable good and consumer heterogeneity, there is an equilibrium pattern in which firms choose to post a high price for a number of periods followed by a single period sale. The sale rationales developed in these papers have not been introduced in a more general monetary DSGE framework yet, so their macroeconomic implications remain to be explored.

Given this framework, the model generates a large degree of micro-level price movement even in the absence of macroeconomic shocks, with firm-level realized prices fluctuating between the regular and promotional values in response to the firm's optimal choice of the fraction of time at which its product will be offered on promotion and to consumers' optimal choices of the time invested looking for those promotions. However, even though not every price movement is the result of a macroeconomic shock, the model's rationale for promotions has effects on how the economy reacts to monetary shocks. When extended in order to allow for Calvo-stickiness in regular prices, the model generates individual price series that behave in a way consistent with recent empirical studies: (i) regular (non-promotional prices) are sticky; (ii) prices including promotions change very frequently; (iii) realized prices show rigidity in the form of inertia in reference prices around which weekly prices fluctuate.

When studying the effects of monetary shocks, the model displays output responses well below the ones obtained with a standard Calvo model calibrated to match the frequency of regular price changes. In this case, sales are important for macroeconomics since their existence creates a dynamic pricing arrangement in which the market can outsmart the Calvo fairy. More generally, this result points to the importance of further exploration on how consumer behavior and firm-customer interactions are designed in macroeconomic models.

The remainder of the chapter is organized as follows. The model is developed in the second section. The solution method, parameters and steady state values are discussed in the third section. The fourth section presents results on the model's

micro price behavior and the macroeconomic implications of price promotions, particularly on the responses to monetary shocks. The fifth section concludes.

2.1 The Model

The model consists of households, firms and a government. Money is introduced as an argument of the utility function; households purchase goods for consumption, supply labor, spend time looking for price promotions and hold money and bonds, while firms hire labor and produce and sell a continuum of differentiated consumption goods produced in a monopolistically competitive market.

Households differ in the price elasticity parameter of the Dixit-Stiglitz consumption aggregator. Given this heterogeneity in consumption, each firm's price setting decision includes a choice of a regular price, and a promotional strategy consisting of a promotional price and a promotion probability (the fraction of time that the good is offered at promotion prices).

2.1.1 Households

There is a measure one of infinitely lived households divided in two possible types: measure h of high price elasticity consumers (HECs), and measure $1-h$ of low price elasticity consumers (LECs).

High Elasticity Consumers

High elasticity households maximize the expected present discounted value of utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t^h\right) + \phi\left(\frac{m_t^h}{P_t^h}\right) + v\left(1 - n_t^h - s_t^h\right) \right] \tag{2.1}$$

where $u(.)$, $\phi(.)$, and $v(.)$ are strictly increasing and strictly concave, m^h are

nominal balances, n^h is the fraction of time spent working and s^h is the fraction of

time spent looking for price promotions. c^h is a Dixit-Stiglitz composite consumption

good defined as

$$c_t^h = \left[\int\limits_0^{\theta^h(f_t,s_t^h)} (c_{jt}^{hp})^{\frac{\varepsilon-1}{\varepsilon}} dj + \int\limits_{\theta^h(f_t,s_t^h)}^1 (c_{jt}^{hr})^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.2}$$

where the parameter $\varepsilon > 1$ governs the price elasticity of demand for individ-

ual goods, $\theta^h(f_t, s_t^h)$ is the fraction of goods purchased at promotion prices (with

demanded quantities $\{c_{jt}^{hp}\}$), and $1 - \theta^h(f_t, s_t^h)$ is the fraction of goods acquired at

regular prices (with demanded quantities $\{c_{jt}^{hr}\}$). I do not consider any additional

frictions between consumers and firms such as matching frictions, so $\theta^h(f, s_t^h)$ is

given by $f_t \cdot s_t^h$ where f_t is the fraction of time that a firm offers its product at

promotional prices.[4]

Notice that even though a firm can sell a good at promotional and regular

prices simultaneously (this will become clear when presenting the problem of the

firm), in each period of time a consumer can acquire a specific good at either the

regular or promotional price, but not both. Therefore, ε in (2.2) has the same

interpretation as it would in the standard Dixit-Stiglitz consumption aggregator

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[4]For simplicity, and anticipating a symmetric equilibrium, in the consumer's problem I assume

$f_{j,t} = f_t$ for every firm j.

and captures the elasticity of substitution between differentiated goods.

The budget constraint for the household is

$$P_t^h w_t^h n_t^h + m_{t-1}^h + (1 + i_{t-1})B_{t-1}^h + \Pi_t + T_t = P_t^h c_t^h + m_t^h + B_t^h \qquad (2.3)$$

where P_t^h is the price of the consumption composite, w_t^h is the real wage received for labor services, B_t^h is the amount of nominal bonds, T_t is a nominal transfer from the government, and Π_t are nominal profits from the differentiated firms that are assumed to be owned by the households.

From the first order conditions for c_{jt}^{hp} and c_{jt}^{hr}:

$$c_{jt}^{hp} = \left(\frac{p_{jt}^p}{P_t^h} \right)^{-\varepsilon} c_t^h \qquad (2.4)$$

$$c_{jt}^{hr} = \left(\frac{p_{jt}^r}{P_t^h} \right)^{-\varepsilon} c_t^h \qquad (2.5)$$

Then, the relationship between quantities of a good bought at regular and promotional prices is given by

$$\frac{c_{jt}^{hp}}{c_{jt}^{hr}} = \left(\frac{p_{jt}^r}{p_{jt}^p} \right)^{\varepsilon} \qquad (2.6)$$

Replacing (2.4) and (2.5) in (2.2), the price index for aggregate consumption is given by

$$P_t^h = \left[\int_0^{\theta^h(f,s_t^h)} (p_{jt}^p)^{1-\varepsilon} dj + \int_{\theta^h(f,s_t^h)}^1 (p_{jt}^r)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \qquad (2.7)$$

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Notice that using the demand equations (2.4) and (2.5) and the price aggregator (2.7) it can be shown that

$$\int_0^{\theta^h(f_t,s_t^h)} p_{jt}^p c_{jt}^{hp} dj + \int_{\theta^h(f_t,s_t^h)}^1 p_{jt}^r c_{jt}^{hr} dj = P_t^h c_t^h \tag{2.8}$$

The first order conditions for n_t^h, m_t^h and B_t^h yield the standard intratemporal and intertemporal optimality conditions:

$$\frac{v'(1 - n_t^h - s_t^h)}{u'(c_t^h)} = w_t^h \tag{2.9}$$

$$\frac{u'(c_t^h) - \phi'(\frac{m_t^h}{P_t^h})}{P_t^h} = \beta E_t \frac{u'(c_{t+1}^h)}{P_{t+1}^h} \tag{2.10}$$

$$\frac{u'(c_t^h)}{P_t^h} = \beta E_t \frac{(1 + i_t)u'(c_{t+1}^h)}{P_{t+1}^h} \tag{2.11}$$

The introduction of a time cost in order to take advantage of price promotions, on the other hand, creates a new intratemporal optimal margin. The first order condition for s_t^h is

$$
\begin{aligned}
v'(1 - n_t^h - s_t^h) = {} & \left(\frac{\varepsilon}{\varepsilon - 1}\right) c_t^{h \frac{1}{\varepsilon}} \theta_s^h(f, s_t^h) \left(c_{\theta^h t}^{hp} {}^{\frac{\varepsilon-1}{\varepsilon}} - c_{\theta^h t}^{hr} {}^{\frac{\varepsilon-1}{\varepsilon}} \right) u'(c_t^h) \\
& + \theta_s^h(f, s_t^h) \left(\frac{p_{\theta^h t}^r c_{\theta^h t}^{hr} - p_{\theta^h t}^p c_{\theta^h t}^{hp}}{P_t^h} \right) u'(c_t^h)
\end{aligned}
\tag{2.12}
$$

Equation (2.12) implies that the high elasticity consumer will choose optimal s_t^h by equating the utility cost incurred when looking for promotions and the

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marginal utility gain derived from spending an extra unit of time looking for promotions. The right hand side of (12) has two components. The first one comes from a re-composition of the consumption bundle. From (2.6), $c_{\theta^h t}^{hp}{}^{\frac{\varepsilon-1}{\varepsilon}} - c_{\theta^h t}^{hr}{}^{\frac{\varepsilon-1}{\varepsilon}}$ is greater than zero, implying an increase in overall consumption coming from higher quantities of additional goods (a fraction $\theta_s^h(f, s_t^h)$ of them) now available at promotional prices. The second component is negative and corresponds to the utility loss because of higher spending after increasing the consumer's time searching for promotions. When a good previously consumed at regular prices is found at a lower price as a result of additional time looking for price promotions, the consumer increases her consumption in a quantity such that her new expenditure on that good is greater than previously. This is a direct result of the elastic demand assumption typical in monopolistic competition models (since ε is greater than one, $p_{\theta^h t}^r c_{\theta^h t}^{hr} - p_{\theta^h t}^p c_{\theta^h t}^{hp}$ is negative).

Combining equations (2.9) and (2.12), we have

$$
\begin{aligned}
w_t^h \;=\; & \left(\frac{\varepsilon}{\varepsilon-1}\right) c_t^{h\frac{1}{\varepsilon}} \theta_s^h(f, s_t^h) \left(c_{\theta^h t}^{hp}{}^{\frac{\varepsilon-1}{\varepsilon}} - c_{\theta^h t}^{hr}{}^{\frac{\varepsilon-1}{\varepsilon}} \right) \\
& + \theta_s^h(f, s_t^h) \left(\frac{p_{\theta^h t}^r c_{\theta^h t}^{hr} - p_{\theta^h t}^p c_{\theta^h t}^{hp}}{P_t^h} \right)
\end{aligned}
\tag{2.13}
$$

The left hand side of (2.13) is the relative price of leisure in terms of forgone labor income, while the right hand side is the relative price of leisure in terms of forgone gains from additional time spent looking for price promotions. Then, in equilibrium, consumers optimally choose their labor and shopping time allocations by equating the two alternative opportunity costs of an additional unit of leisure.

High Elasticity Consumers

Low elasticity consumers differ from high elasticity ones only in the price elasticity parameter of their consumption aggregator. Their consumption aggregator is given by

$$c_t^l = \left[\int_0^{\theta^l(f_t, s_t^l)} (c_{jt}^{lp})^{\frac{\eta-1}{\eta}} dj + \int_{\theta^l(f_t, s_t^l)}^1 (c_{jt}^{lr})^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \tag{2.14}$$

with $\varepsilon > \eta > 1$.

The LEC set of optimality conditions is analogous to the HEC one but changing "h" by "l" superscripts and ε by η when needed. The LEC price aggregator is given by

$$P_t^l = \left[\int_0^{\theta^l(f, s_t^l)} (p_{jt}^p)^{1-\eta} dj + \int_{\theta^l(f, s_t^l)}^1 (p_{jt}^r)^{1-\eta} dj \right]^{\frac{1}{1-\eta}} \tag{2.15}$$

2.1.2 Firms

Cost Minimization

There is a measure one continuum of firms indexed by j. Firm j minimizes costs $w_t N_{jt}$, where w_t is the real wage and N_{jt} is total labor hired by the firm, subject to a production restriction summarizing the available technology where output, c_{jt}, is a function of labor and a productivity shock: $c_{jt} = e^{z_t} N_{jt}$, with $E(z_t) = 0$.

From the firm's cost minimization problem, real marginal cost is given by $mc_{jt} = mc_t = w_t/e^{z_t}$.

Price Setting Decision

Flexible Regular Prices. The firm's price setting problem consists of choosing p_{jt}^r, p_{jt}^p and f_{jt} to maximize nominal profits:[5]

$$
\begin{aligned}
\Pi_{jt} \; = \; & p_{jt}^p h \theta^h(f_{jt}, s_t^h) c_{jt}^{hp} + p_{jt}^r h \left[1 - \theta^h(f_{jt}, s_t^h)\right] c_{jt}^{hr} \\[2mm]
& + p_{jt}^p (1-h) \theta^l(f_{jt}, s_t^l) c_{jt}^{lp} + p_{jt}^r (1-h) \left[1 - \theta^l(f_{jt}, s_t^l)\right] c_{jt}^{lr} \\[2mm]
& - mc_{jt} P_t \{ h \theta^h(f_{jt}, s_t^h) c_{jt}^{hp} + h \left[1 - \theta^h(f_{jt}, s_t^h)\right] c_{jt}^{hr} \\[2mm]
& + (1-h) \theta^l(f_{jt}, s_t^l) c_{jt}^{lp} + (1-h) \left[1 - \theta^l(f_{jt}, s_t^l)\right] c_{jt}^{lr} \}
\end{aligned} \tag{2.16}
$$

subject to the demand equations (2.4), (2.5), and their LEC counterparts.

The first order conditions for p_j^r, p_j^p and f_{jt} are

$$
h \left[1 - \theta^h(f_{jt}, s_t^h)\right] c_{jt}^{hr} + (1-h) \left[1 - \theta^l(f_{jt}, s_t^l)\right] c_{jt}^{lr} \; =
$$

$$
\left(1 - \frac{P_t}{p_{jt}^r} mc_t \right) \left\{ h[1 - \theta^h(f_{jt}, s_t^h)] \varepsilon c_{jt}^{hr} + (1-h)[1 - \theta^l(f_{jt}, s_t^l)] \eta c_{jt}^{lr} \right\} \tag{2.17}
$$

$$
h \theta^h(f_{jt}, s_t^h) c_{jt}^{hp} + (1-h) \theta^l(f_{jt}, s_t^l) c_{jt}^{lp} \; =
$$

$$
\left(1 - \frac{P_t}{p_{jt}^p} mc_t \right) \left[h \theta^h(f_{jt}, s_t^h) \varepsilon c_{jt}^{hp} + +(1-h) \theta^l(f_{jt}, s_t^l) \eta c_{jt}^{lp} \right] \tag{2.18}
$$

---

[5]A real world justification for having firms choosing f_{jt} may come from the existence of marketing agreements in which a producer specifies the number of times in which a retailer can put the good on sale in a given period (i.e. 20 days in the next three months). Then, the exact moment of promotion is chosen by the retailer and the producer's decision ends up being about the fraction of time that the good is on sale. Alternatively, the firm may have multiple retail outlets and may put the unit on sale at only a fraction of its stores.

$$\left(p_{jt}^p - mc_t P_t\right) \left[h\theta_f^h(f_{jt}, s_t^h)c_{jt}^{hp} + (1-h)\theta_f^l(f_{jt}, s_t^l)c_{jt}^{lp}\right] =$$

$$\left(p_{jt}^r - mc_t P_t\right) \left[h\theta_f^h(f_{jt}, s_t^h)c_{jt}^{hr} + (1-h)\theta_f^l(f_{jt}, s_t^l)c_{jt}^{lr}\right] \qquad (2.19)$$

Equations (2.17) and (2.18) equate the marginal revenue obtained from a variation in demand because of a change in a specific price (regular or promotional), with the marginal production cost of satisfying that marginal demand.

Equation (2.19), on the other hand, comes from the fact that firms choose the fraction of time that they put their good on promotion. The left hand side is the marginal profit obtained from demand at promotional prices when increasing f_{jt}, while the right hand side is the marginal profit obtained from demand at regular prices when increasing f_{jt}.

Further intuition can be gained after some rearrangement in order to re-express (2.19) as

$$h\theta_f^h(f_{jt}, s_t^h) \left(p_{jt}^p c_{jt}^{hp} - p_{jt}^r c_{jt}^{hr}\right) + (1-h)\theta_f^l(f_{jt}, s_t^l) \left(p_{jt}^p c_{jt}^{lp} - p_{jt}^r c_{jt}^{lr}\right) =$$

$$h\theta_f^h(f_{jt}, s_t^h)mc_t P_t \left(c_{jt}^{hp} - c_{jt}^{hr}\right) + (1-h)\theta_f^l(f_{jt}, s_t^l)mc_t P_t \left(c_{jt}^{lp} - c_{jt}^{lr}\right) \qquad (2.20)$$

The two terms in the left hand side of (2.20) corresponds to the marginal revenue coming from increasing the probability with which the firm offers its good on promotion. $\theta_f^h(f_{jt}, s_t^h) = s_t^h$ and $\theta_f^l(f_{jt}, s_t^l) = s_t^l$ are the additional fractions of consumers that find the good at promotional prices after the firm increases f_t, and $p_{jt}^p c_{jt}^{hp} - p_{jt}^r c_{jt}^{hr}$ and $p_{jt}^p c_{jt}^{lp} - p_{jt}^r c_{jt}^{lr}$ correspond to the additional revenue coming

from these new purchases.[6] On the other hand, the two terms in the right hand side of the equation correspond to the marginal cost associated with increasing f_t, with $c_{jt}^{hp} - c_{jt}^{hr}$ and $c_{jt}^{lp} - c_{jt}^{lr}$ representing the change in production needed to satisfy the higher demand coming from additional consumers able to acquire the good at promotional prices.

Notice that the standard monopolistic competition model is nested in the model presented here. To see this, take (2.17) and let h tend to 1. The equation reduces to:

$$\frac{p_{jt}^r}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) mc_{jt} \tag{2.21}$$

With no consumer heterogeneity ($h = 1$), $f = 0$ and therefore $\theta^h = 0$. Regular prices then are the only ones in the economy and P_t^h in (2.7) is reduced to

$$P_t^h = \left[\int_0^1 (p_{jt}^r)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \tag{2.22}$$

With $h = 1$, $P_t = P_t^h$, and (2.21) corresponds to the pricing optimality condition of a model with monopolistic competition and flexible prices.

Sticky Regular Prices. For the exercises performed in the following sections, it will prove useful to amend the model by including a nominal rigidity in the regular price. For this purpose, it is assumed that regular prices are subject to Calvo staggering. There may be reasons to consider ridigity in promotional prices

---

[6]Since $\varepsilon > 1$, revenue from sales of good j at promotional prices is greater than revenue from sales at regular prices, so the terms $p_{jt}^p c_{jt}^{hp} - p_{jt}^r c_{jt}^{hr}$ and $p_{jt}^p c_{jt}^{lp} - p_{jt}^r c_{jt}^{lr}$, are positive.

as well.[7] However, there is evidence that sale prices are less sticky than regular ones (Klenow and Kryvtsov (2008)) and allowing for rigidities only on regular prices is a simple way of capturing this fact. Also, stickiness in regular prices suffices to generate the type of reference price rigidity found in recent studies on micro price data like Eichenbaum et al. (2011) and Klenow and Malin (2010), and facilitates comparison with Guimaraes and Sheedy (2011) who also impose a Calvo rigidity only in regular prices and who argue that their exercise will provide an upper bound for price flexibility in the aggregate if there is rigidity in promotional prices in reality.

When the regular price is sticky, the price setting problem of the firm becomes

$$
\max_{p_{jt}^r, p_{jt}^p, f_{jt}} E_t \sum_{s=t}^{\infty} \rho^{s-t} \Xi_{t+s/t} \{ p_{js}^p h \theta^h(f_{js}, s_s^h) \left(\frac{P_s^h}{p_{js}^p} \right)^{\varepsilon} c_s^h
$$

$$
+ p_{js}^p (1-h) \theta^l(f_{js}, s_s^l) \left(\frac{P_s^l}{p_{js}^p} \right)^{\eta} c_s^l
$$

$$
+ p_{jt}^{r*} h \left[1 - \theta^h(f_{js}, s_s^h) \right] \left(\frac{P_s^h}{p_{jt}^{r*}} \right)^{\varepsilon} c_s^h
$$

$$
+ p_{jt}^{r*} (1-h) \left[1 - \theta^l(f_{js}, s_s^l) \right] \left(\frac{P_s^l}{p_{jt}^{r*}} \right)^{\eta} c_s^l \}
$$

$$
- mc_{js} P_s \{ h \theta^h(f_{js}, s_s^h) \left(\frac{P_s^h}{p_{js}^p} \right)^{\varepsilon} c_s^h
$$

$$
+ (1-h) \theta^l(f_{js}, s_s^l) \left(\frac{P_s^l}{p_{js}^p} \right)^{\eta} c_s^l
$$

$$
+ h \left[1 - \theta^h(f_{js}, s_s^h) \right] \left(\frac{P_s^h}{p_{jt}^{r*}} \right)^{\varepsilon} c_s^h
$$

$$
+ (1-h) \left[1 - \theta^l(f_{js}, s_s^l) \right] \left(\frac{P_s^l}{p_{jt}^{r*}} \right)^{\eta} c_s^l \} \tag{2.23}
$$

Where ρ is the probability of not being able to change the regular price in the period, and $\Xi_{t+1/t}$ is the firm's stochastic discount factor for nominal payoffs.

---

[7]For example, firms could be constrained in their ability to change their promotional strategy because of contractual pricing arrangements between producers and retailers.

The first order conditions for p_{jt}^p and f_{jt} are the same as in the flexible price scenario, and the first order condition for p_{jt}^{r*} is

$$p_{jt}^{r*} \left\{ (\varepsilon - 1)E_t \sum_{t=s}^{\infty} \rho^{s-t} \Xi_{t+s/t} h \left[1 - \theta^h(f_{js}, s_{js}^h)\right] c_{js}^{hr} \right\}$$

$$+ p_{jt}^{r*} \left\{ (\eta - 1)E_t \sum_{t=s}^{\infty} \rho^{s-t} \Xi_{t+s/t}(1-h) \left[1 - \theta^l(f_{js}, s_{js}^l)\right] c_{js}^{lr} \right\} =$$

$$\varepsilon E_t \sum_{t=s}^{\infty} \rho^{s-t} \Xi_{t+s/t} h \left[1 - \theta^h(f_{js}, s_{js}^h)\right] c_{js}^{hr} mc_{js} P_s$$

$$+ \eta E_t \sum_{t=s}^{\infty} \rho^{s-t} \Xi_{t+s/t}(1-h) \left[1 - \theta^l(f_{js}, s_{js}^l)\right] c_{js}^{lr} mc_{js} P_s \tag{2.24}$$

Notice that when $h = 1$ (and therefore $f_t = 0$) there is only one type of consumer in the economy and $P_t = P_t^h$. In this case this last equation collapses to

$$p_{jt}^{r*} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{t=s}^{\infty} \rho^{s-t} \Xi_{t+s/t} c_{js}^{hr} P_s mc_{js}}{E_t \sum_{t=s}^{\infty} \rho^{s-t} \Xi_{t+s/t} c_{js}^{hr}} \tag{2.25}$$

Which is the standard optimal pricing equation in a model with Calvo rigidities and no capital.

Further details on the solution of the sticky regular price case are provided in Appendix 1.

2.1.3 The Government

The Government Budget Constraint

Each period, the government satisfies the following budget constraint:

$$(B_t - B_{t-1}) + (M_t - M_{t-1}) = T_t + i_{t-1} B_{t-1} \tag{2.26}$$

where T_t are lump sum transfers.

Monetary Policy

It is assumed that the Central Bank reacts to inflation and output growth according to the following interest rate rule:

$$\left(\frac{1+i_t}{1+i}\right) = \left\{\left[\left(\frac{e^{\pi_t}}{e^{\overline{\pi}}}\right)^{\psi_1}\left(\frac{y_t}{y_{t-1}}\right)^{\psi_2}\right]^{1-\rho_R}\left(\frac{1+i_{t-1}}{1+i}\right)^{\rho_R}\right\}e^{\sigma_R\xi_{R,t}} \qquad (2.27)$$

where π_t is the inflation rate, y_t is total output, π and i are the steady state inflation and interest rates, and $\xi_{R,t}$ is a monetary policy shock. This specification corresponds to the one used by Aruoba and Schorfheide (2011) with the exception that these authors also introduce a time varying inflation target as part of the interest rate feedback rule.

2.1.4   The Economy's Resource Constraint

The economy's resource constraint can be recovered combining the consumers' budget constraints, the government balanced budget equation and the expression for firms' profits. Pre-multiplying the HEC's budget constraint (2.3) by h and the LEC's budget constraint by $1-h$ and adding up we have:

$$h\left[P_t^h w_t^h n_t^h + m_{t-1}^h + (1+i_{t-1})B_{t-1}^h + \Pi_t + T_t - P_t^h c_t^h - m_t^h - B_t^h\right] = (1-h)\left[P_t^l w_t^l n_t^l + m_{t-1}^l + (1+i_{t-1})B_{t-1}^l + \Pi_t + T_t - P_t^l c_t^l - m_t^l - B_t^l\right] \qquad (2.28)$$

Using equation (2.8) and its analogue for low elasticity consumers and applying symmetry yields:

$$h \left\{ P_t^h w_t^h n_t^h + m_{t-1}^h + (1 + i_{t-1}) B_{t-1}^h + \Pi_t + T_t \right\}$$

$$- h \left\{ \theta^h(f_t, s_t^h) p_{jt}^p c_{jt}^{hp} + \left[1 - \theta^h(f_t, s_t^h) \right] p_{jt}^r c_{jt}^{hr} + m_t^h + B_t^h \right\} =$$

$$(1 - h) \left\{ P_t^l w_t^l n_t^l + m_{t-1}^l + (1 + i_{t-1}) B_{t-1}^l + \Pi_t + T_t \right\}$$

$$- (1 - h) \left\{ \theta^l(f_t, s_t^l) p_{jt}^p c_{jt}^{lp} + \left[1 - \theta^l(f_t, s_t^l) \right] p_{jt}^r c_{jt}^{lr} + m_t^l + B_t^l \right\} \tag{2.29}$$

Define the economy's labor income in nominal terms as

$$P_t w_t N_t = h n_t^h P_t^h w_t^h + (1 - h) n_t^l P_t^l w_t^l \tag{2.30}$$

where $N_t = \int_0^1 N_{jt} dj = h n_t^h + (1 - h) n_t^l$.

Finally, plugging equations (2.26), (2.30) and the expressions for firms' profits and marginal costs into (2.29), we have:

$$\begin{aligned}
e_t^z \left[h n_t^h + (1 - h) n_t^l \right] &= h \theta^h(f_t, s_t^h) c_t^{hp} + h \left[1 - \theta^h(f_t, s_t^h) \right] c_t^{hr} \\
&\quad + (1 - h) \theta^l(f_t, s_t^l) c_t^{lp} \\
&\quad + (1 - h) \left[1 - \theta^l(f_t, s_t^l) \right] c_t^{lr} \tag{2.31}
\end{aligned}$$

where (2.31) is the economy's resource constraint.

2.1.5 Characterizing the Economy-wide Price Level

The final equilibrium condition is the relationship between the economy wide price level, P_t, and the two types of prices faced by consumers, p_t^p and p_t^r. In order to find this, define the economy's total consumption expenditure as

26

$$P_t c_t = h P_t^h c_t^h + (1-h) P_t^l c_t^l \tag{2.32}$$

where

$$c_t = \int_0^1 c_{jt} = h\theta^h(f_t, s_t^h)c_t^{hp} + h[1-\theta^h(f_t, s_t^h)]c_t^{hr} + (1-h)\theta^l(f_t, s_t^l)c_t^{lp} + (1-h)[1-\theta^l(f_t, s_t^l)]c_t^{lr}$$

Using equation (2.8) and its analogue for the LEC's case, we have:

$$
\begin{aligned}
P_t c_t &= h \left(\int_0^{\theta^h(f_t, s_t^h)} p_{jt}^p c_{jt}^{hp} dj + \int_{\theta^h(f_t, s_t^h)}^1 p_{jt}^r c_{jt}^{hr} dj \right) \\
&+ (1-h) \left(\int_0^{\theta^l(f_t, s_t^l)} p_{jt}^p c_{jt}^{lp} dj + \int_{\theta^l(f_t, s_t^l)}^1 p_{jt}^r c_{jt}^{lr} dj \right)
\end{aligned}
\tag{2.33}
$$

Assuming flexible regular prices and applying symmetry, we have:

$$
\begin{aligned}
P_t c_t &= h\theta^h(f_t, s_t^h)c_t^{hp}p_t^p + h\left[1 - \theta^h(f_t, s_t^h)\right] c_t^{hr}p_t^r \\
&+ (1-h)\theta^l(f_t, s_t^l)c_t^{lp}p_t^p + (1-h)\left[1 - \theta^l(f_t, s_t^l)\right] c_t^{lr}p_t^r
\end{aligned}
\tag{2.34}
$$

Rearranging, we have:

$$P_t = \varphi_t p_t^p + (1 - \varphi_t)p_t^r \tag{2.35}$$

where

$$\varphi_t = \frac{h\theta^h(f_t, s_t^h)c_t^{hp} + (1-h)\theta^l(f_t, s_t^l)c_t^{lp}}{h\theta^h(f_t, s_t^h)c_t^{hp} + h\left[1 - \theta^h(f_t, s_t^h)\right]c_t^{hr} + (1-h)\theta^l(f_t, s_t^l)c_t^{lp} + (1-h)\left[1 - \theta^l(f_t, s_t^l)\right]c_t^{lr}}$$

Equation (2.35) defines P_t as a time-varying weighted average of p_t^p and p_t^r, with the weights of each price given by the ratio of goods purchased at that price with respect to the total quantity of goods consumed in the economy. From (2.35) we can also write

$$\frac{1}{mc_t} = \varphi_t\left(\frac{p_t^p}{P_t mc_t}\right) + (1-\varphi_t)\left(\frac{p_t^r}{P_t mc_t}\right) \tag{2.36}$$

where, from the firm's first order conditions for p_t^p and p_t^r, the optimal real markups associated with each price are given by

$$\frac{p_t^p}{P_t mc_t} = \frac{h\theta^h(f_t, s_t^h)\varepsilon c_t^{hp} + (1-h)\theta^l(f_t, s_t^l)\eta c_t^{lp}}{h\theta^h(f_t, s_t^h)(\varepsilon - 1)c_t^{hp} + (1-h)\theta^l(f_t, s_t^l)(\eta - 1)c_t^{lp}}$$

and

$$\frac{p_t^r}{P_t mc_t} = \frac{h\left[1 - \theta^h(f_t, s_t^h)\right]\varepsilon c_t^{hr} + (1-h)\left[1 - \theta^l(f_t, s_t^l)\right]\eta c_t^{lr}}{h\left[1 - \theta^h(f_t, s_t^h)\right](\varepsilon - 1)c_t^{hr} + (1-h)\left[1 - \theta^l(f_t, s_t^l)\right](\eta - 1)c_t^{lr}}$$

Then, equation (2.36) defines the economy-wide markup as a time-varying weighted average of the real markups associated with the two prices available in the economy.

2.1.6 Stochastic Processes

The stochastic process for productivity z_t is given by

$$z_{t+1} = \rho_z z_t + \xi_t^z; \qquad \xi_t^z \sim i.i.d.(0, \sigma_z) \qquad (2.37)$$

Finally, it is assumed that the monetary policy shock $\xi_{R,t}$ is serially uncorrelated with mean zero and variance one. Then, the autoregresive component in monetary shocks is given by the interest rate smoothing parameter ρ_R in (2.27).

2.1.7 Equilibrium

An equilibrium for this economy is defined as a set of allocations $\{c_t^h, c_t^{hr}, c_t^{hp}, c_t^l, c_t^{lr},$

$c_t^{lp}, n_t^h, n_t^l, N_t, s_t^h, s_t^l, f_t\}$, portfolio choices $\{m_t^h, m_t^l, B_t^h, B_t^l, M_t, B_t\}$, prices $\{P_t, P_t^h, P_t^l,$

$p_t^r, p_t^p, w_t, w_t^h, w_t^l, i_t\}$, and transfers $\{T_t, \Pi_t\}$ such that:

i) $\{c_t^{hr}, c_t^{hp}, n_t^h, s_t^h, m_t^h, B_t^h\}$ solve the HEC utility maximization problem.

ii) $\{c_t^{lr}, c_t^{lp}, n_t^l, s_t^l, m_t^l, B_t^l\}$ solve the LEC utility maximization problem.

iii) $\{p_t^r, p_t^p, f_t\}$ are such that firms maximize profits.

iv) Aggregation satisfies equations (2.2), (2.7), (2.14) , (2.15), (2.30) and

(2.35).

v) The government runs a balanced budget.

vi) Markets clear: $N_t = hn_t^h + (1-h)n_t^l$, $B_t = hB_t^h + (1-h)B_t^l$, $M_t = hm_t^h + (1-h)m_t^l$.

2.1.8 Price Discrimination in General Equilibrium

As observed in the model's equilibrium conditions, this economy has the feature that both regular and promotional prices coexist at the same time for each firm. The

key modeling element behind this result is the fact that consumers must spend time looking for price promotions. This is formally stated in the following propositions:

PROPOSITION 1: *Given strictly increasing utility in consumption and leisure, in a general equilibrium with monopolistic competition ($\varepsilon > 1$, $\eta > 1$) and with consumers looking for price promotions ($s_t^h \in (0,1)$, $s_t^l \in (0,1)$), firms always price discriminate by setting a regular price greater than the promotional one ($p_{jt}^r > p_{jt}^p$).*

Proof: Suppose not. That is, let $p_{jt}^r \leq p_{jt}^p$. Consider the HEC case; using the demand equations (2.4) and (2.5), equation (2.12) can be re-expressed as:

$$v'(1 - n_t^h - s_t^h) = u'(c_t^h)\theta_s^h(f_t, s_t^h)\left(\frac{1}{\varepsilon - 1}\right)c_t^h\left(\frac{p_{jt}^{p\ 1-\varepsilon} - p_{jt}^{r\ 1-\varepsilon}}{P_t^{h^{1-\varepsilon}}}\right) \qquad (2.38)$$

Given $\varepsilon > 1$ and the assumption that $u(\cdot)$ is strictly increasing, $p_{jt}^r \leq p_{jt}^p$ implies that the right hand side of equation (2.38) is lower or equal to zero, so (2.38) is satisfied only when $v'(\cdot) \leq 0$, a contradiction that violates the assumption that $v(.)$ is strictly increasing.

PROPOSITION 2: *Given strictly increasing utility in consumption and leisure, in a general equilibrium with monopolistic competition ($\varepsilon > 1$, $\eta > 1$) and with consumers looking for price promotions ($s_t^h \in (0,1)$, $s_t^l \in (0,1)$), firms follow an active promotional policy by making the promotional price available with a positive probability $f_t > 0$.*

Proof: analogous to the one for PROPOSITION 1.

In this model, then, firms will always price discriminate; that is, they will set two different prices for the same good ($p_{jt}^r > p_{jt}^p$), and they will always follow an

active promotional strategy ($f_{jt} > 0$), making their promotional price available for consumers willing to spend the time needed to find a promotion and take advantage of the lower price.

The intuition behind this result is that there is a cost associated with finding a price promotion, and that cost is paid only by consumers and not by firms. If, in a given period, a firm decides not to offer promotions ($f_{jt} = 0$), it will charge only its regular price at every moment of time within that period without distinguishing between consumption demands. However, recognizing that looking for discounts is costly for consumers and that only a fraction of them will access price promotions, the firm can increase profits by making the promotional price available with a given probability ($f_{jt} > 0$) and therefore costlessly distinguishing between two different types of demand: demand from consumers that find the price promotion and demand from consumers that don't.[8] This follows the basic principle of using promotions for price discrimination purposes studied in the Industrial Organization and Marketing literature: when a firm wants to use a price promotion as a price discrimination tool, it should try to make it easy for some consumers to access the promotion, but harder for others; that is, the promotion must impose some cost on which consumers differ, rewarding the people willing to bear that cost (see Banks and Moorthy (1999) and Lu and Moorthy (2007)).

---

[8]The firm can price discriminate even if it puts its good on promotion all the time ($f_t = 1$). The reason for this is that since consumers do not spend all of their time looking for promotions ($s_t^h < 1$, $s_t^l < 1$), only fractions $\theta^h(f_t, s_t^h)$ and $\theta^l(f_t, s_t^l)$ of the firm's good are sold at promotional prices, with the complement transacted at the regular price.

The most salient real-world example of price promotions associated with consumer time costs is the use of coupons.[9] Coupons are certainly an important promotional vehicle in the United States, but there are several other promotional arrangements that are associated with different types of consumer costs; examples of these are mail-in rebates, cash-back offers, loyalty cards, reward programs and discount clubs.[10] [11] The mechanism developed here can be thought of as a reduced form

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[9]There are several types of time costs associated with the usage of coupons beyond the standard "cut and clip" costs: (i) in order to take advantage of the promotion, the consumer may need to spend time looking for the coupon (i.e. going over mailed fliers and newspapers, or browsing the internet); (ii) it may be necessary to prepare shopping lists so the coupon may be used in a future shopping trip before its expiration date; (iii) coupon redemption may be conditioned on some information exchange with the consumer (i.e. logging into a retailer's web page and providing contact information, or participating in an on-line consumer survey).

[10]According to NCH Marketing Services Inc., in 2009, 311 billion coupons were distributed by consumer packaged-goods companies in the United States, 3.2 billion of these were redeemed, and the redemption value reached 3.5 billion dollars. Also, a recent consumer study by Inmar Inc. reports that 75% of the participants in a shopping habits survey conducted by the Food Marketing Institute stated that coupons had at least some influence on their decision to buy a product. Inmar also reports that in the first half of 2009, 68% of US households participating in Nielsen's Homescan Consumer Panel used at least one coupon.

[11]An interesting fact about some of these promotional vehicles is that they are not used only by grocery stores and supermarkets, which have received most of the attention in recent studies on the implications of micro price data for macroeconomic modeling. Mail-in rebates, for example, are widely used by home appliance providers and electronics retail stores, with offers ranging from small-ticket items like computer accessories to big-ticket ones like home entertainment systems. Different types of cash-back options are also very common among automobile dealers. On the other hand, loyalty cards, typical in supermarkets and grocery stores, are also common among

capturing a broader fact with interesting implications when modeling price discrimination in monetary models: not every price promotion is realized, and whether a consumer ends up receiving a discount or not often depends endogenously on how much time, effort and money she is willing to invest in taking advantage of the offered promotion. This could be the case even for on-shelf promotions in the sense that even though there is no explicit consumer cost associated with them, a shopper may need to spend some time and attention comparing prices and brands before finally picking the products offered on discount.[12]

Thus, it is the fact that price promotions are costly for the consumer that ensures the coexistence of the two prices in the model economy. This will allow for a large amount of firm level transaction price variation over time even in the absence of shocks, a result consistent with Nakamura's (2008) finding of a high amount of idiosyncratic price variation in product categories with temporary sales, suggesting that retail prices may vary largely as a consequence of dynamic pricing strategies on

book, music and electronics retailers, while service providers like banks and airlines often offer loyalty programs associated with their products. Finally, discount clubs include warehouse clubs that sell grocery store and supermarket products, but also providers of big-ticket items like home furnishings. Also, wholesale clubs like Costco and BJ's sell a variety of non-supermarket household goods, including TVs, books, clothes, etc.

[12]Sometimes this may require shopping trips to different locations in order to compare on-shelf promotions between retailers. When the product is not offered at a physical location, taking advantage of price promotions may require spending time browsing the internet looking for the best available deals. Think, for example, about buying a plane ticket using kayak.com instead of visiting a specific airline, or finding discount lodging at hotwire.com instead of contacting a particular hotel.

the part of retailers or manufacturers. In the model, this dynamic pricing strategy consists of firms offering costly promotions and choosing the fraction of time at which they are available, but being indifferent about the exact moment of time at which a promotion is realized. Consider for example a case with no shocks and zero inflation in which firms choose to offer their goods on promotion 50% of the time. Panel a) in Figure 2.1 shows a sample time path for the price strategy of an individual firm that randomly chooses when to offer a price promotion, subject to ensuring that the the promotion is available half of the time. The regular price will be offered all the time, while the promotional price is offered only at intervals. Panel b) depicts the time series of the minimum price available, which is equal to p_t^r when no promotion is offered and p_t^p when a promotion is in place.

The following sections show how this type of strategic combined pricing, with nominal rigidity in regular prices, helps in generating price behavior similar to that documented in the recent microeconomic evidence on price-setting, and how this pricing has important implications on the way real variables respond to monetary shocks.

Among recent research on sale prices in monetary models, the Kehoe and Midrigan (2007, 2008, 2010) framework is an example of a model in which prices fluctuate only in response to stochastic shocks and not to randomization decisions made for price discrimination purposes. In their setup, every period a firm can either pay a large fixed cost and change their price permanently or pay a smaller cost and change their price only for one period. Kehoe and Midrigan's models, therefore, require time varying shocks and price rigidities in order to generate temporary

Figure 2.1: FIRMS PRICES IN THE ABSENCE OF MACRO SHOCKS

a) Regular and Promotional Price

b) min (Regular Price, Promotional Price)

movements away from the regular price.

Closer in spirit to the model presented here is the work of Guimaraes and
Sheedy (2011). They develop a model in which households consume a continuum
of heterogeneous goods and a continuum of heterogeneous brands for each good.
Consumers have different price sensitivities with respect to different sets of goods,
being loyal to specific brands for a given measure of products (that is, they do not
receive utility from consuming any alternative brand) but being bargain hunters
willing to substitute between different brands for the rest of the goods. Each firm

produces a brand specific product and faces a measure of loyal consumers and a measure of bargain hunters. Firms do not know the type of any individual consumer, and cannot practice price discrimination directly. Instead they follow the strategy of holding periodic sales in order to target the two types of consumers at different moments of time. This is done by randomizing the timing of sales after choosing a fraction of time (i.e. a number of moments within a given period) at which the firm charges a lower price. At any one point in time, all consumers recive the same price.

As in the framework presented in this document, Guimaraes and Sheedy's (2011) setup is able to generate price fluctuations even in the absence of the standard aggregate or idiosyncratic shocks included in DSGE models, as a result of the dynamic pricing strategy followed by firms that target different types of customers. In contrast to Guimaraes and Sheedy, however, the model developed in this paper does feature price discrimination at many points in time as part of the firm's optimal dynamic pricing strategy: for a fraction f_t of the time, both p_t^r and p_t^p coexist and transactions are made using both prices. This paper's framework thus allows for a distinction between price-quote and realized-price series. As I will show below, when combined with a nominal rigidity in the definition of regular prices, the model delivers individual price series with the type of rigidity recently found in studies on reference prices in micro price data, but with different macroeconomic implications from those found by Guimaraes and Sheedy (2011).

2.2 Solution Method and Parameter Values

The model is solved around a zero-inflation steady state applying Schmitt-Grohe and Uribe's (2004a) local approximation algorithm. I assume the following functional forms:

$$u(c_t^h) = log(c_t^h)$$

$$u(c_t^l) = log(c_t^l)$$

$$\phi(\tfrac{m_t^h}{P_t^h}) = \alpha log \left(\tfrac{m_t^h}{P_t^h}\right)$$

$$\phi(\tfrac{m_t^l}{P_t^l}) = \alpha log \left(\tfrac{m_t^l}{P_t^l}\right)$$

$$v(1 - n_t^h - s_t^h) = \Psi log \left(1 - n_t^h - s_t^h\right)$$

$$v(1 - n_t^l - s_t^l) = \Psi log \left(1 - n_t^l - s_t^l\right)$$

There are fourteen parameters in the model: the intertemporal discount factor, β; the constants pre-multiplying the money and leisure utility terms, Ψ and α; the measure of high elasticity consumers, h; the elasticities of substitution for HECs and LECs, ε and η; the probability of a firm not being able to change its regular price, ρ; the interest rate smoothing parameter, ρ_R; the inflation and output growth weights in the Central Bank's interest rate rule, ψ_1 and ψ_2; and the parameters governing the stochastic processes for productivity and monetary policy, σ_z, ρ_z and σ_R.

The length of each period is one week, so the discount factor β is set at $0.96^{1/52}$. The Calvo parameter, ρ, is set at 0.974, implying that, on average, firms change their regular prices every 9 months (39 weeks); this time between changes is in the middle range of the estimates reported by Nakamura and Steinsson (2008). The values for the productivity stochastic process, ρ_z and σ_z, correspond to the weekly counterparts

of the quarterly values used in Hansen (1985), while the parameters for the interest rate rule and the monetary policy stochastic process are taken from Aruoba and Schorfheide (2009): $\psi_1 = 1.7$, $\psi_2 = 0.86$, $\rho_R = 0.61^{1/13}$, $\sigma_R = 0.0036/13$.

The values of Ψ, α, ε, η and h are chosen to match five steady state calibration targets: average hours worked of $1/3$, a weekly velocity of money of 0.15, a ratio of time spent looking for promotions with respect to hours worked of $1/4$, a probability with which goods are offered on promotion of 0.7, and an average gross markup of 1.4.

The target value for total hours worked, $hn^h + (1 - h)n^l$, is standard in the literature. The value for money velocity was computed using the formula $V = GDP/(52 \cdot M1)$, where $M1$ is the weekly seasonally adjusted series for M1 of the Board of Governors of the Federal Reserve System, and GDP is annual nominal Gross Domestic Product. The 1975:1-2009:9 average is 0.15, close to the weekly analogue of the quarterly value calculated by Aruoba and Schorfheide (2009) using sweep-adjusted $M1$ data.

Time spent looking for price promotions is the only type of shopping time in the model. Therefore the target for the shopping/working time ratio is taken from US time surveys with information on daily time spent purchasing goods and services, and is close to the value used in Arseneau and Chugh (2007).[13]

The targeted markup is within the range of estimated values for the US and

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[13]The American Time Use Survey (ATUS) provides nationally representative estimates of how and where Americans spend their time. Data files are available from 2003 to 2009 and can be accessed at http://www.bls.gov/tus/.

close to values used in trade and macroeconomic studies such as Bernard et al. (2003) and Biblie et al (2007).

The choice of the target value for f requires more elaboration. In the model, f is the fraction of time firms offer their goods at promotional prices. Moreover, this is the only type of price promotion considered in the framework developed in Section 2. Given the model's definition of promotional prices as only available when offered and only to those who look for them, the empirical counterpart of f should correspond to a type of promotion that is not always offered and that requires some search effort from the consumer in order to be realized.

The James M. Kilts Center at the University of Chicago Booth School of Business provides data useful for constructing this target. The Booth School provides several publicly available datasets on weekly store-level transaction prices for over 100 stores operated by Dominick's Finer Foods (DFF), which have been used in various macroeconomic studies such as Golosov and Lucas (2007), Midrigan (2008) and Kehoe and Midrigan (2007,2008). One of these datasets, the Customer Count Files, has information on total coupon redemption figures by DFF defined product-category. Since sale coupons are a natural example of the type of promotion discussed in Section 2, I use this database to construct a proxy for the probability that firms offer their goods at promotional prices in a given period.

I proceed as follows: first, for each product category I construct a dummy variable, d_t, equal to 1 if at least one coupon was redeemed in a given week (that is, if the coupon redemption figure is different from zero). Next, I compute the fraction of time at which category k was offered on promotion:

$$\widehat{f}_k = \sum_t d_t / (number \, of \, weeks).$$

Finally, I compute the average across product categories:

$$\overline{f} = \sum_k \widehat{f}_k / (number \, of \, good \, categories).$$

The underlying assumption is that if a promotional price is available in a given week, then there is at least one person that takes advantage of it (alternatively, at least one coupon is redeemed). Therefore, I assume that if the coupon redemption figure in a given week is zero ($d_t = 0$), then no coupons were offered that week.

When considering a sample of all categories for which coupons were redeemed at least once in the sample period, the sales-weighted value for $\overline{f}$ is 0.87. When considering all product categories, the value is 0.7. Appendix 2 provides more details on these calculations.

Of course, it would be ideal to perform a similar calculation for a broader range of goods and sectors. However, in spite of this limitation, and given the lack of richer data on non-shelf price promotions for other sectors and goods, the above described calculation provides an empirical benchmark consistent with the theoretical definition of f and with the model's definition of promotional prices.

Table 2.2 presents the model's steady state under this parameterization, where $\widehat{p^p}$ and $\widehat{p^r}$ are relative prices with respect to the economy-wide price and a^h and a^l are HEC and LEC real balances with respect to each consumer's effective price index.

HECs represent 9% of the economy's population and spend a large fraction of their time working, and therefore are able to consume more than LECs:

$$\theta(f, s^h) c^{hp} + \left[1 - \theta(f, s^h)\right] c^{hr} = 0.84$$

$$\theta(f, s^l) c^{lp} + \left[1 - \theta(f, s^l)\right] c^{lr} = 0.28$$

As expected, HECs spend a lower fraction of their time looking for price promotions. HECs are very price sensitive, and whey they find a promotion they substitute a lot, buying very large quantities of a good when it is found on sale, so they are able to take advantage of price discounts even when spending only a very small fraction of time looking for them. LECs, on the other hand, substitute less when a good is on sale, and need to spend a higher fraction of time looking for promotions in order to take advantage of price discounts.

Under the current parameterization, the firm sets an optimal discount of 31%, very close to the values reported by Nakamura and Steinsson (2008), 30%, and Klenow and Kryvtsov (2008), 25%. The discount price is available 70% of the time, but only 6% of total transactions are realized at promotional prices. This value is close to the expenditure-weighted fraction of price quotes that are sales reported by Nakamura and Steinsson (2008), 7.4%.

2.3 The Workings of the Model

2.3.1 Individual Price Setting Behavior

A first step in evaluating the performance of the model is to assess its ability to generate individual price series similar to those reported in recent microeconomic studies of price setting. As mentioned in Section 1, a recently documented feature of the CPI data is that prices including sales tend to change roughly every quarter, while prices excluding sales tend to remain unchanged for two to three quarters

Table 2.1: Parameter Values

Parameter		Value	Comment
Intertemporal discount factor	β	$0.96^{1/52}$	Length of a period = 1 week
Fraction of HECs	h	0.09	To match $f = 0.7$
Scale parameter leisure utility	Ψ	0.21	To match $n = 1/3$
Scale parameter money utility	α	0.005	To match $Velocity = 0.15$
HEC elasticity of substitution	ε	20.26	To match $s/n = 1/4$
LEC elasticity of substitution	η	2.44	To match $Markup = 1.4$
Calvo parameter for regular prices	ρ	0.974	Implies a time between changes of 39 weeks
Interest rate smoothing parameter	ρ_R	$0.61^{1/13}$	Weekly analogue ofAruoba and Schorfheide's (2009) quarterly value
Inflation weight interest rate rule	ψ_1	1.7	From Aruoba and Schorfheide (2009)
Output growth weight interest rate rule	ψ_2	0.86	From Aruoba and Schorfheide (2009)
Standard deviation for monetary policy shocks	σ_R	0.0036/13	Weekly analogue ofAruoba and Schorfheide's (2009) quarterly value
Productivity autoregressive coefficient	ρ_z	$0.95^{1/13}$	Weekly analogue of Hansen's (1985) quarterly value
Standard deviation for productivity innovations	σ_z	0.007/13	Weekly analogue of Hansen's (1985) quarterly value

(Nakamura and Steinsson, 2008; Klenow and Krystov, 2008; Klenow and Malin, 2010). Also, new evidence from high frequency scanner data has found that nominal rigidities take the form of inertia in reference (modal) prices, with weekly prices fluctuating around a reference value that tends to remain constant over extended periods of time (see Kehoe and Midrigan (2007, 2008), and especially Eichenbaum et al., (2011)). This point is illustrated in Figure 2.2, which displays the time series

Table 2.2: Steady State Values

|	$\pi = \pi^h = \pi^l = 0$	
HEC	LEC	Firm
$c^{hr} = 0.05$	$c^{lr} = 0.25$	$\widehat{p^r} = 1.11$
$c^{hp} = 88.9$	$c^{lp} = 0.63$	$\widehat{p^p} = 0.77$
$n^h = 0.93$	$n^l = 0.27$	$discount = 0.31$
$s^h = 0.01$	$s^l = 0.09$	$f = 0.70$
$\theta^h(f, s^h) = 0.009$	$\theta^l(f, s^l) = 0.06$	$\theta(f, s) = 0.06$

behavior of four selected DFF items. From the figure one can see that there seems to be an inertial reference price (i.e. the most observed price in a given time window) and prices tend to return to this reference value after having deviated from it.

The model is able to replicate this type of behavior in prices, because it differentiates not only between regular and promotional prices, but also between price-quotes (like the ones recorded in CPI data) and realized transactions prices (like the ones recorded in scanner data). To see this, Figure 2.3 presents individual price simulations of the sticky regular price version of the model.

Panel a) presents the regular and promotional price series of an individual firm for a simulation of 250 weeks.[14]. The graph depicts the optimal regular price that the firm would set each period were it able to do so. However, the firm is subject to a Calvo friction and it is not able to adjust its regular price every period, so

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[14]The series is constructed by iterating forward the relative price optimal decision rules of the linearized model for 250 periods, and then by simulating the behavior of an individual firm. Each period, there is a probability $1 - \rho$ that the firm will be able to set its price at the optimal price level $p^r_{jt} = p^{r*}_t$, and a probability ρ that the firm's price will not change, $p^r_{jt} = p^r_{jt-1}$.

Figure 2.2: PRICE EVOLUTION IN FOUR SELECTED PRODUCTS
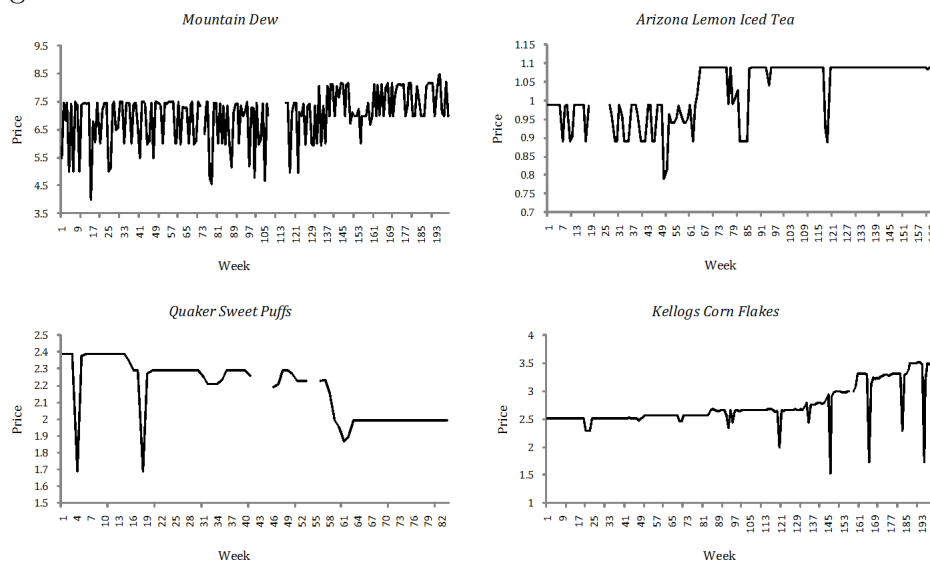
Source: Elaborated using Chicago Booth's price data on DFF specific items.

its actual regular price, depicted by the dotted line, tends to remain unchanged for several periods, adjusting to the optimal value every time that the firm is allowed to change its price.

Panel b) presents the evolution of the two prices available to consumers. The regular price is available in any period. The promotional price, however, is not always available, since the firm optimally chooses the fraction of time at which the price is offered. That is, in each period there is a probability f_t that the lowest available price will correspond to p_{jt}^p and a probability $1 - f_t$ that the only available price will be p_{jt}^r.

Consider the exercise of constructing a price quote series reporting the lowest available price in each period. In this case, the series will change almost every period, with frequent fluctuations between the regular and promotional price and with occasional shifts to the regular one. The series of price quotes is presented in

Figure 2.3: SIMULATED PRICE FOR AN INDIVIDUAL FIRM

a) Regular Prices *(optimal: solid line, realized: dashed line)* and Optimal Promotional Prices *(circle markers)*

b) Available Prices *(Regular: dashed line, Promotional: solid line)*

c) Minimum Available Price

d) Realized Price
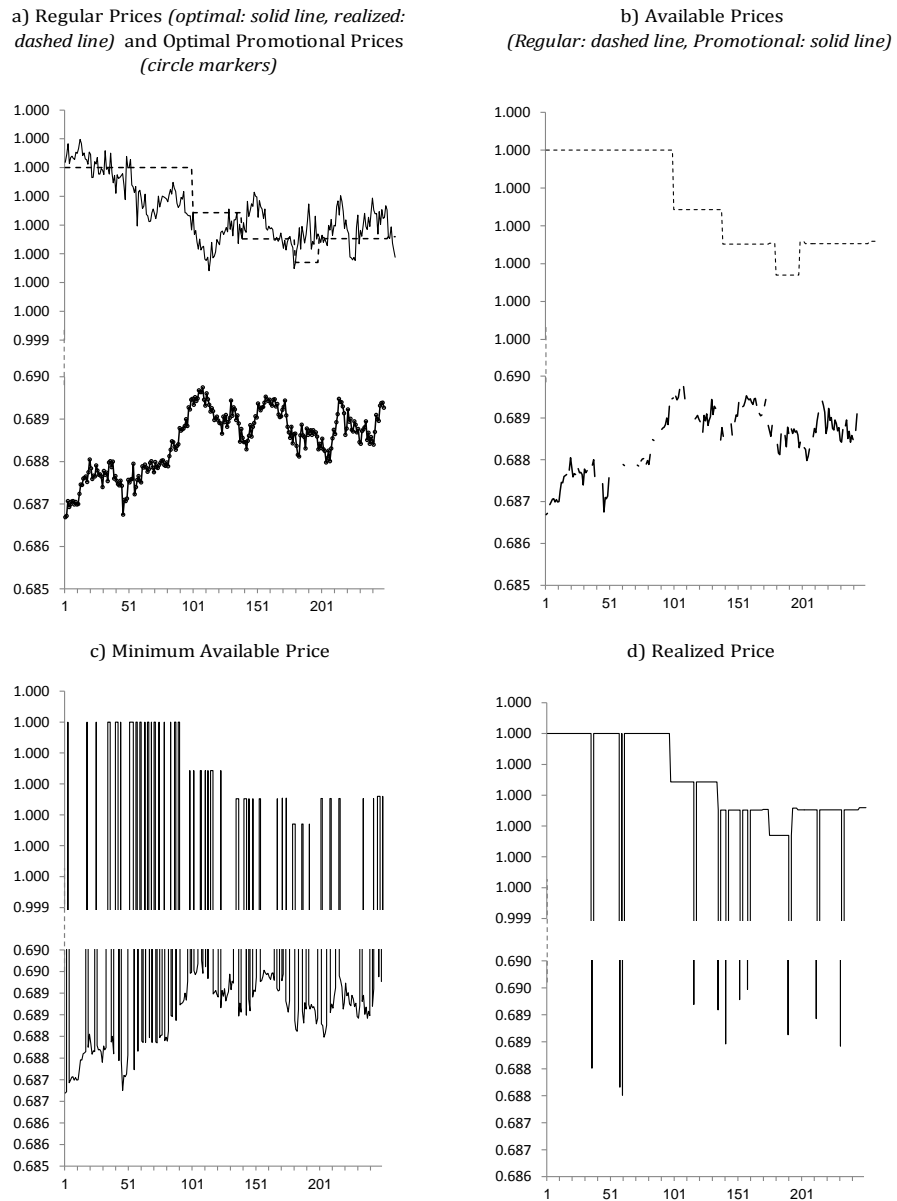
panel (c).

The price series described in panel (c), however, corresponds to quoted prices

Figure 2.4: SIMULATED PRICE FOR AN INDIVIDUAL FIRM: THE CASE OF
POSITIVE STEADY STATE INFLATION
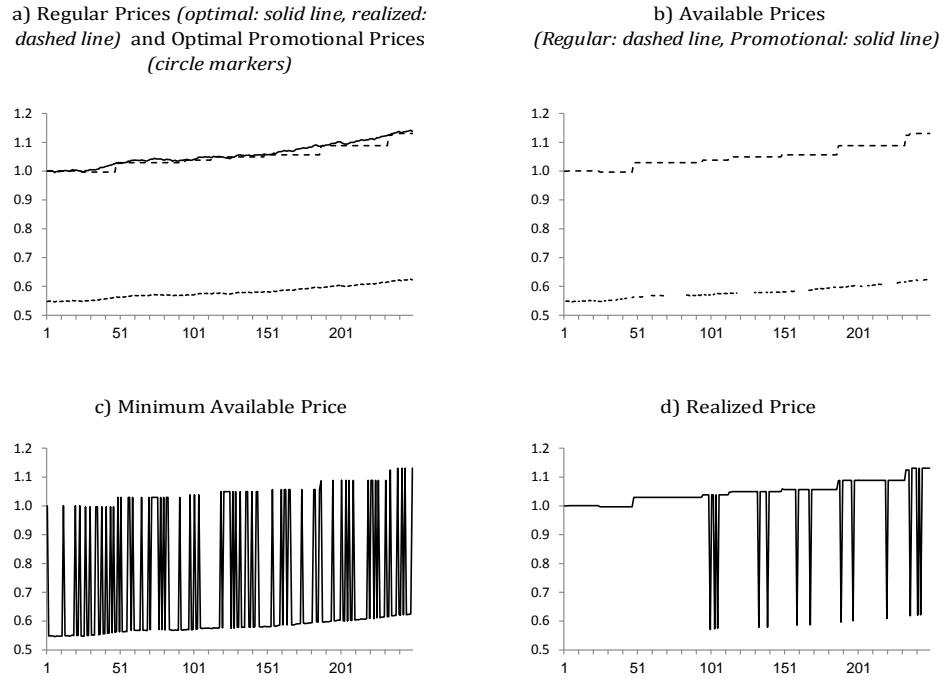
and not to the ones resulting from actual transactions. Each period, there is a probability θ_t that the good will be purchased at the promotional price p_j^p, and a probability $1 - \theta_t$ that it will be purchased at the firm's regular price.[15] The series of realized prices is presented in panel (d), showing the type of rigidity reported by Eichenbaum et. al. (2011). Realized prices show rigidity in the form of inertial reference values to which prices tend to return after having deviated from them.[16]

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[15]$\theta_t = h\theta_t^h(f_{jt}, s_t^h) + (1-h)\theta_t^l(f_{jt}, s_t^l)$

[16]In terms of constructing the underlying price series from observed data, this exercise is more consistent with the use of transaction-based datasets like supermarket scanner data than with price quotes like those recorded by BLS visitors. For example, think about a grocery store that uses loyalty cards in a way such that every week some of its products are offered on promotion and

The model with zero steady state inflation is able to replicate the behavior of observed individual time series as long as these do not trend because of inflation pressures. Since some individual price series seem to show an upward trend (see for example the Kellogg's Corn Flakes example in Figure 2.2), the model was also solved around a 0.05 percent weekly steady state inflation rate; this value corresponds to an annual cumulative inflation rate of 3 percent (the post-Volker average in the US).[17] The simulated price series are presented in panels (a) to (d) of Figure 2.4.

The model, then, is able to deliver price-quote and realized-price series consistent with the microeconomic evidence. Because of the Calvo-type rigidity, the time series behavior of an individual firm's regular price will be highly inertial. When including price promotions, on the other hand, prices will change very frequently. This behavior is consistent with the recent evidence on CPI price quotes that finds

their price tags display both the regular and promotional price. According to the BLS, a sale price must be temporarily lower than the regular price, be available to all consumers, and be usually identified by a sign or statement on the price tag (Klenow and Krystov, 2008). In the specific case of loyalty cards, discounts are reported as only if the outlet confirms that more than 50% of its customers use these cards (Nakamura and Steinsson, 2008). So, in this example, if the percent of customers using loyalty cards is lower to 50%, a price-quoting visitor will not even record the sales. But even if the sales were recorded, the visitor could gather data on both the regular and promotional price but she would have no information on the number of transactions closed at each price. A rich enough scanner dataset, on the other hand, would record both the regular and the promotional price as well as whether the loyalty card was used at the time of each transaction.

[17]The pattern of steady state allocations when the model is solved around a positive inflation rate is similar to the one observed in the zero inflation case but with a higher equilibrium discount, 0.45.

that the median frequency of price changes is sensitive to the inclusion or exclusion of sales (Nakamura and Steinsson, 2008; Klenow and Krystov, 2008; Klenow and Malin, 2010).

2.3.2 Does Micro Flexibility Translates Into Higher Neutrality at the Macro Level?

Figure 2.5 presents the responses of output to a monetary shock in the model with price promotions and sticky regular prices and in a standard model with no consumer heterogeneity and sticky prices. The latter model with no price promotions was calibrated to replicate the steady state hours, velocity and markup targets presented in Table 2.1 and with the same Calvo parameter used in the price promotions model.

The model with price promotions presents a smaller and less persistent output response, with an impact deviation equal to one third of the one obtained with the standard Calvo model.

Figure 2.6 helps in further understanding the intuition behind this result. Given the introduction of a New Keynesian interest rate rule, the model assumes that the impact of monetary policy on output operates through the real interest rate. There is thus a mapping between the time paths of output and the real interest rate. Panel (a) depicts the responses of these variables for both models. Real variables in the promotions model follow the same pattern as those of the Calvo model, but with smaller and less persistent responses.

Panel (b) presents the responses of the relative regular and promotional prices,

$\widehat{p_t^{r*}}$ and $\widehat{p_t^p}$, and the optimal promotional discount $((p_t^{r*} - p_t^p)/p_t^{r*})$.The graph also shows the average price response constructed using φ_t and $1 - \varphi_t$ as weights for the promotional regular price components respectively. After a shock to the interest rate, firms react mainly by reducing their promotional price and therefore increasing their optimal discount. Thus it is the possibility of adjusting their promotional price that provides firms with an alternative channel of adjustment when facing monetary shocks.

Finally, panel (c) presents the responses of the probability of price promotions and the size of the promotion-economy (the share of real activity sold at promotional prices, φ_t in equation 2.35) in response to a monetary policy shock. The graph shows that firms use the size of the promotional discount and not the frequency of promotions as a shock absorbing mechanism. Given the contraction in demand after the increase in the real interest rate, the share of the promotion economy also decreases and firms react by reducing the fraction of time at which they offer price promotions.[18]

Why is it that f_t decreases after a shock to the interest rate? To understand this, look at the two panels of Figure 2.7 which depict the time allocations of the two types of consumers. The shock affects consumers' time-allocation decisions asymmetrically.

HECs consumers take much more advantage from price promotions that LECs, and when they find a promotion they substitute large quantities of goods at regular prices for good at promotional prices. Now that the size of the price discount has

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[18]The complete set of the models impulse response functions is presented in Appendix A.4.

increased, they do not need to spend as much time as they used to looking for price promotions (they find less promotions but at larger discounts and they take full advantage of them) and they increase they labor time so they can have more resources dedicated to larger substitution of regular-price consumption for promotional-price consumption. Since LECs show a much lower degree of substitution when they find a good on promotion, they have an incentive to increase the time they spent looking for price promotions (they do not substitute as much as HECs do, so they need to find more promotions in order to take advantage of the increase in the price discount) and decrease their labor time.

How does this affect the firms' decisions about f_t? Since firms cannot distinguish between the two types of consumers, they price-discriminate by choosing a fraction of time at which they offer their goods at promotional prices. As mentioned in previous sections, in doing so, the firms take advantage of that fact that promotions are costly in the sense that consumers have to spent time looking for promotions in other to take advantage of them. In a perfect price discrimination environment, the firms would be able to distinguish between types of consumers and charge a high price to LECs and a low price to HECs. In the model presented here, the firm could achieve perfect price discrimination if HECs looked for promotions all of their time ($s_h = 1$) and HECs never looked for promotions ($s_l = 0$). In that case, by setting $f_t = 1$ the firm could completely distinguish between HECs and LECs and charge optimal perfect price discrimination prices (see Appendix A.3 for a formal treatment of this issue).

A firm, then, has a larger incentive to discriminate (setting a higher f_t) when

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s_t^h/s_t^l increases. But, as explained in previous paragraphs, this is exactly the opposite of what happens after a nominal shock when, given the asymmetric response of prices (the increase of the price discount) HECs decrease s_t^h and LECs increase s_t^l. Firms, therefore have less incentive to price-discriminate and reduce f_t.

The first panel in Figure 2.8 compares the output response of the model with price promotions presented in Figure 2.5 with several responses of the Calvo model calibrated to different times between price changes. No Calvo response overlaps exactly with the one of the model with price promotions. The reason for this is that the model with price promotions combines features of both flexible and sticky price models; the output response is small on impact but it dies out at a lower rate than those of the Calvo model.

What is, then, the equivalent response in a Calvo model? The second panel of the figure presents two responses with the same cumulative impact on output. From the point of view of the overall effect of an interest rate shock on output, the model with price promotions calibrated to a time between regular price changes of nine months is equivalent to a Calvo model calibrated to a time between price changes of one month. The dynamics, however, are very different; the response of the Calvo model shows a much higher initial impact but dies out almost twice as fast as the response of the model with price promotions.

The near neutrality result contrasts with that obtained by Guimaraes and Sheedy (2011), who find that even though their model delivers individual price paths similar to ones observed in the data, aggregate prices adjust by little in response to monetary shocks, which therefore have large effects on real variables. Since the

flexibility of prices at the micro level due to sales does not translate into flexibility at the macro level, the authors conclude that sales are essentially irrelevant for monetary policy analysis.

The key element behind Guimaraes and Sheedy's (2011) non-neutrality result is that in their model sales are strategic substitutes; that is, a firm's incentive to have a sale is decreasing in the number of other firms having a sale. For example, after a particular shock, an individual firm may have an incentive to have a sale and increase profits coming from bargain hunters. However, if other firms follow the same strategy, targeting bargain hunters' demand becomes a more competitive task, performed at the cost of charging a lower price to loyal customers willing to acquire the good at high prices. The firm then may find it more profitable to not have a sale, targeting only its demand from loyal customers.

Guimaraes and Sheedy's (2011) rationale for sales, then, implies a real rigidity constraining firms' promotional strategies. As explained in an earlier version of their paper (Guimaraes and Sheedy, 2008), this strategic substitution implies that, following an expansionary monetary shock, an individual firm has an incentive to decrease sales, therefore increasing its average price. However, if all other firms pursue the same course of action, bargain hunters' demand would be relatively neglected, increasing the returns to targeting non-loyal consumers. Then, in equilibrium, firms do not adjust sales by much in response to aggregate shocks, and monetary policy has large real effects.

Guimaraes and Sheedy's framework thus provides a well structured IO rationale for sales, consistent with micro price evidence on large amounts on price

variation due to idiosyncratic factors, and is able to generate realistic individual price series with inertial regular prices and frequent deviations from reference values. However, their model implies little adjustment in sales in response to aggregate shocks, which ends up being critical for their results on monetary non-neutrality.

Is there any evidence on the response of firms' promotional strategies to aggregate shocks? Chevalier et. al. (2003) use DFF data to study the behavior of prices in periods of high demand. They identify eight categories of products that a priori would seem susceptible to seasonal demand shifts, captured by dummy variables taking the value of 1 on determined holidays, as well as by temperature variables capturing weather-induced demand cycles. One of their exercises corresponds to evaluating the response of product advertising (price promotion) to demand peaks. The authors cite earlier evidence showing that high-demand items are more likely to be put on sale, and then regress the percentage of category revenues accounted for by items advertised by Dominick's against the holiday and temperature high-demand variables. They find that, in general, seasonally peaking items are significantly more likely to be placed on sale.[19]

Even though Chevalier et. al. (2003) focus only on a small number of product categories (due to data limitations), their study finds evidence that sale promotions do respond to aggregate shocks, providing some ground for exploring alternatives

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[19]Chevalier et. al. argue that, if market share can be viewed as a proxy for high demand, then the work of Nelson et. al. (1992) and Hosken and Reiffen (2001) shows a positive correlation between high-demand and the probability of being put on sale. Hosken and Reiffen (2004) also presents evidence that high-demand items are more likely to be placed on sale.

to Guimaraes and Sheedy's dynamic pricing model.[20]

Figure 2.5: OUTPUT RESPONSE TO A MONETARY POLICY SHOCK
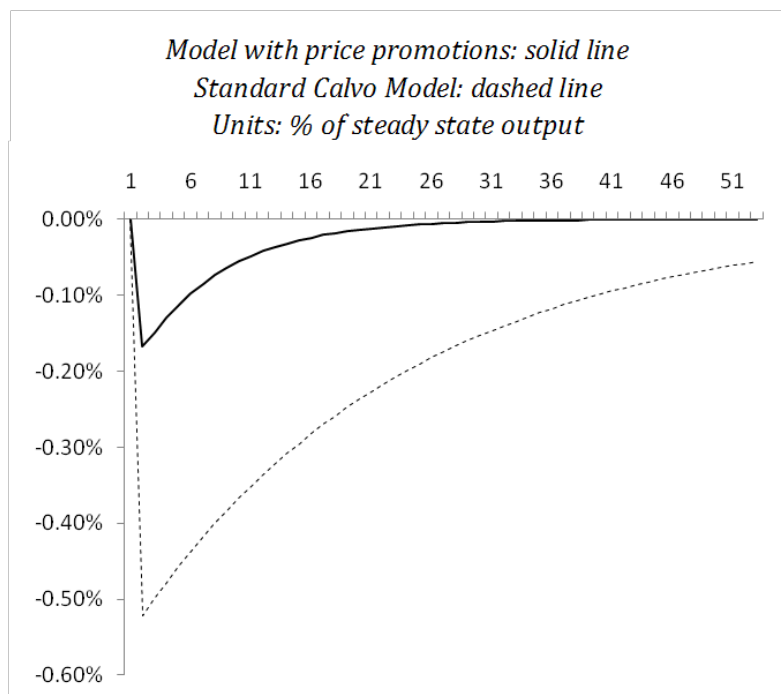
In the model presented in this document, firms have the alternative of offering regular and promotional prices at the same time, and randomize between moments of time in which only the regular price is available and moments of time in which both prices are available, implying a large amount of price fluctuations even in a deterministic scenario as in Guimaraes and Sheedy (2011). Also, as in Guimaraes and

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[20]Working with a more comprehensive dataset (bimonthly observations of the CPI Research Database), Klenow and Willis (2007) find that sales seem to respond to macro information at least as much as regular price changes. Given the difficulty to identifying aggregate nominal shocks for the economy, the authors focus on the expected change in prices based only on current information about inflation. For this purpose, they regress regular and sale individual prices against a measure of cumulative inflation since the previous price change, finding that sales are at least as responsive to recent inflation as are regular price changes.

Figure 2.6: IMPULSE RESPONSE ANALYSIS

(a) Output and Real Interest Rate Responses

real int. rate response model with promotions

real int. rate response Calvo Model

output response model with promotions

output response Calvo model
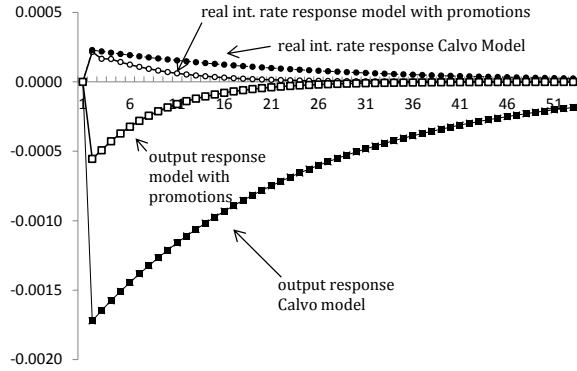
(b) Relative Prices and Price Discount in the Model with Promotions
(% deviations from steady state values)

- price discount
- relativel regular price
- relative promotional price
- average price response
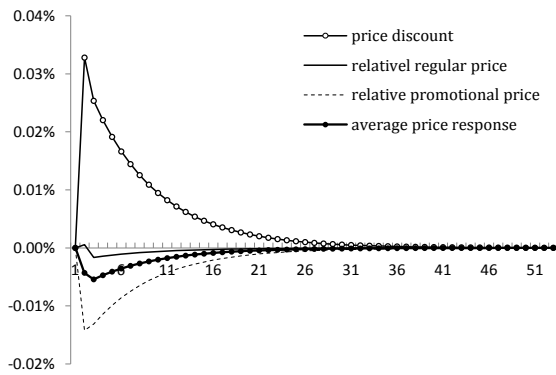
(c) Size of the Promotion-Economy, Price Discount and Fraction of Time at Which Goods are Offered on Promotion
(% deviations from steady state values)

price discount

φ_t

f_t
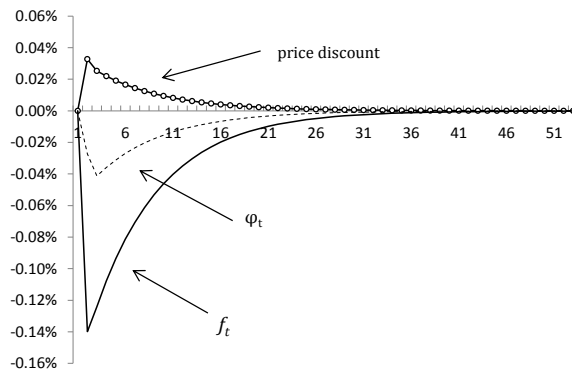
Figure 2.7: TIME ALLOCATIONS

HEC time allocations
(% deviations from steady state values)

LEC time allocations
(% deviations from steady state values)
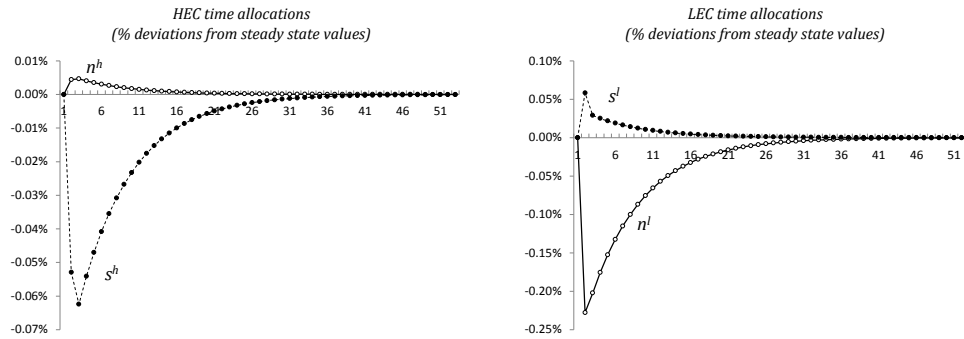
Figure 2.8: THE RESPONSE IN THE MODEL WITH PRICE PROMOTIONS COMPARED THAT OF A CALVO MODEL CALIBRATED TO DIFFERENT TIMES BETWEEN CHANGES

(a) Output Responses
(Model with price promotions: solid line
Standard Calvo Model: dashed line
Units: % of steady state output)

(b) Two Equivalent Responses in Terms of Cumulative Effects
(Units : % of steady state output)
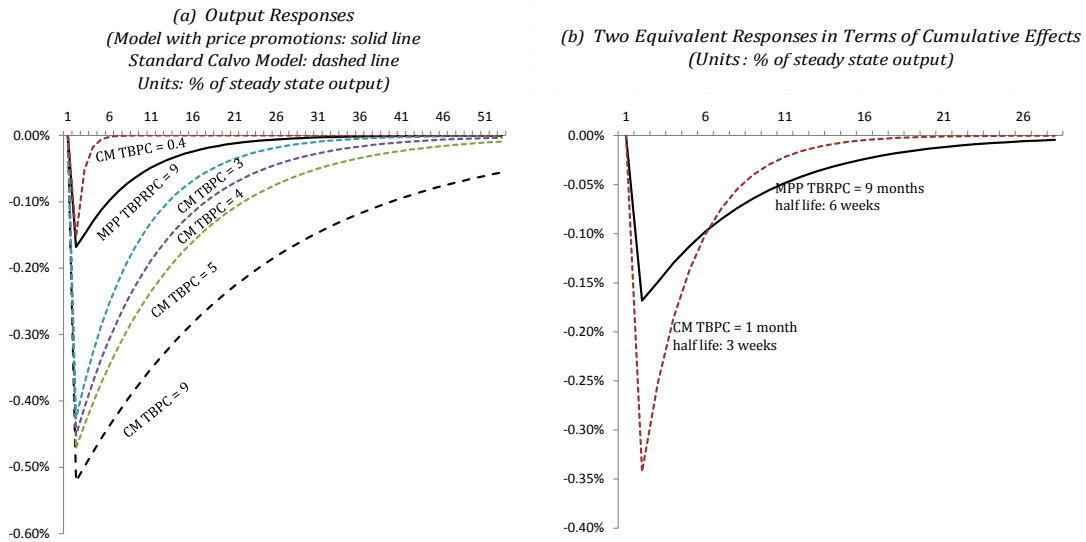
Sheedy's case, the inclusion of a nominal rigidity in regular prices allows the model to generate price series showing the type of reference price rigidity recently documented in the literature, providing also the opportunity of distinguishing between price quotes and realized prices. As regards effects of monetary policy, however, the type of price discrimination embedded in this paper provides a new source of price

flexibility that undermines the effect of nominal rigidities. When a monetary shock hits the economy and firms are not able to change their regular prices, they have the option of changing their promotional strategy (the promotional price and the fraction of time at which this is offered) in order to modify their average sale price.

The key assumption behind this result is that the model's rationale for sales relies on the use of price promotions for price discrimination, rather than on the assumption of customer loyalty. When consumers are given the possibility of endogenously determining the effort they want to invest to take advantage of price promotions, a new margin is created on the household side of the economy and, in a general equilibrium setting, firm's and custumers' choices interact, generating the coexistence of regular and promotional prices in a way such that the effects of nominal rigidities in regular prices are offset by changes in promotional strategies. In this framework, sale prices end up being important for macroeconomics since their existence gives the market a way to outsmart the Calvo fairy.

More generally, this result points out the importance of how consumer behavior is modelled. Indeed, since different approaches to modeling firm-customer relationships may perform similarly in replicating features of the microeconomic data but differ in their macro implications, further exploration of the interaction between firms and consumers may be a promising task for future research in macroeconomics.[21]

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[21] This line of research includes a wide range of possibilities such as modeling habit persistence in differentiated goods as in Nakamura and Steinsson (2009), allowing for switching costs when consumers change sellers as in Kleshchelski and Vincent (2007), and modeling shopping as a search

2.4 Conclusion

This paper has introduced time looking for price promotions as part of the endogenous set of consumer choices. When price promotions are associated with a consumer time cost, the economy features the coexistence of regular and promotional prices. The exercise is relevant not only because of the recent evidence on micro price setting showing that the inclusion or exclusion of sale prices has bearing on the estimated frequency of price changes, but also because the rationale behind sale prices may affect the way in which real variables respond to aggregate shocks.

When the model is complemented with a nominal rigidity in regular prices, individual price series behave in a way consistent with several features of the data: there is a large amount of price variation not associated with aggregate prices, price quotes rarely change when price promotions are excluded, and they change very often when price promotions are included. Moreover, the model allows a distinction between quoted and realized prices, with realized price series showing the type of rigidity recently found in studies using scanner data: there is inertia in reference prices, with frequent fluctuations around the reference price.

Regarding whether price rigidity at the firm level translates into monetary non-neutrality at the macro level, the results show that the type of price promotion considered in the model generates a new source of price flexibility that offsets the effect of nominal rigidity in regular prices. The economy thus features a dynamic

process as in Arsenau and Chugh (2007), Hall (2007), Levin and Yun (2008), and Albrecht et. al. (2010).

pricing arrangement in which the market is able to outmaneuver the Calvo fairy. This result is in contrast with the one recently obtained by Guimaraes and Sheedy (2011), showing the importance of how consumer behavior and firm-customer interactions are treated in macroeconomic models.

Finally, the results suggest that gathering more empirical evidence on the relationship between price promotions and aggregate prices would be fruitful. How many goods in the economy are associated with price promotions? How many of these correspond to "off-shelf" discounts? What types of customer costs are associated with accessing these promotions and how big are these costs? How much would these discounts affect estimates of the frequency of price changes? The answers to these questions could shed further light on the type and degree of price rigidity observed in the economy and on how this rigidity relates to aggregate fluctuations.

Chapter 3

A Quantitative Analysis of Currency Substitution

3.1   Introduction

A striking feature of various emerging and developing economies is the high level of currency substitution observed even after several years of low inflation. This paper focuses on two examples of this particular phenomenon: Bolivia and Peru. Data on the ratio of foreign currency deposits with respect to broad money, used as a proxy for total currency substitution, shows that the relationship between currency substitution and inflation is not the same before and after high inflation episodes. In both countries currency substitution increases with inflation during high inflation periods and reaches its peak once the economy has been stabilized, showing high persistence in the subsequent periods in spite of the downward trend in inflation.

This persistence is also observed in other Latin American countries, as well as in many transition economies[1]. Given that conventional monetary general equilibrium models cannot generate the hysteresis observed in the data, Uribe (1997) and Reding and Morales (2004) develop network externalities models that display low-inflation - high-substitution steady states. Particularly, Uribe's model proposes that the cost of buying goods with foreign currency is decreasing in the economy's accumulated experience in transacting in the foreign currency. He refers to this accumulated experience as the economy's "dollarization capital".

This chapter explores the quantitative implications of incorporating Uribe's dollarization capital idea in a simple endowment economy model in which domestic and foreign currency balances are held because of their services in reducing transaction costs. Particularly, the model is used to generate a low inflation-high-substitution equilibrium and to compare the predicted currency substitution ratios to those observed in the data as well as those obtained using a model in which the dollarization capital mechanism is not present.

When parameters are chosen as to replicate pre-hyperinflation currency substitution levels, the model performs well in predicting high levels of currency substitution during high inflation episodes, generating substitution ratios close to the ones observed in the data. The model also displays inertia in real foreign currency holdings during the transition towards a low-inflation equilibrium and generates higher levels of currency substitution than a model with no dollarization capital dynamics.

The model generates low inflation currency substitution ratios that are be-

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[1]See Savastano (1996), Valev (2007) and Feige (2003).

tween 40% and 50% of those observed in post inflation stabilization periods. Although the predicted currency substitution ratios are below the observed ones, the model is able to explain between 1/6 of the gap between the observed ratios and the ones generated by a model without dollarization capital dynamics.

The fact that the predicted low-inflation currency substitution ratios are below the ones observed in the data is a result of sizable post-stabilization impact effects on domestic currency holdings, a finding that suggests the need for future work on mechanisms that could induce inertia in domestic currency balances. Particularly, it would be interesting to study a model that displays uncertainty with respect to future inflation, probably including a devaluation risk probability as in Mendoza and Uribe (1999, 2000).

The rest of the chapter is organized as follows. The second Section describes the pre and post high-inflation pattern of currency substitution in Bolivia and Peru. The theoretical model is presented in the third section and solved in the fourth section. The fifth section concludes.

3.2   Currency Substitution in Two Selected Economies

3.2.1   Currency substitution and inflation

The first challenge in analyzing currency substitution in developing countries is constructing a relevant measure of the use of foreign currency in the domestic economy, often a difficult task because of the lack of data on the amount of foreign currency

held by households outside the financial system. Following Savastano (1996), a useful proxy for the total level of currency substitution in the economy is the ratio of foreign currency deposits ($FCDs$) to broad money. This measure, labeled FCD/BM^*, is represented by the dark gray areas in the left hand panels of Figure 3.1.[2] The graphs also depict the behavior of the inflation rate (in logarithmic scale for Bolivia and Peru).

In the Bolivian case, this measure of currency substitution is not available for the 1981-1984 period when the central government imposed restrictions on the use of foreign currency in the financial system, forcing the conversion of all bank liabilities into domestic currency. Therefore, this measure does not provide information on the behavior of currency substitution during the hyperinflation period of the early eighties. When observing the 1985-2007 period, it is interesting to notice that the substitution ratio reached its peak in 1995 after several years of reductions in the annual inflation rate, and decreased after that, even in the 2002-2008 period during which the inflation rate gradually rose.

Peruvian currency substitution follows a similar behavior. The substitution ratio was already high before 1988, with a 50% average for 1980-1987. The sharp decrease observed between 1984 and 1987 is related to the temporary prohibition

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[2]The broad money measure varies depending on the monetary aggregates classification used by each Central Bank. In the case of Bolivia the measure is constructed as the ratio of FCDs to M3', the broadest monetary aggregate used by the Bolivian Central Bank, which includes short term and long term deposits denominated in local and foreign currency. In the cases of Peru , the measure is computed as the ratio of quasi-money denominated in foreign currency to total money and quasi-money.

of U.S. dollar denominated deposits (Rojas-Suarez, 1992). As in the Bolivian case, restrictions on $FCDs$ limit the capacity of this measure to reflect the relationship between inflation and currency substitution during hyperinflation episodes. During the post-restrictions period (1990 onwards) currency substitution remained high, achieving its peak in 1993 and showing a slow downward trend after that.

Again following Savastano (1996), data limitations due to restrictions on FCDs in the domestic financial system in Bolivia and Peru can be addressed by combining the FCD/BM^* substitution measure with information on foreign currency deposits held abroad (da). This new measure, labeled $(FCD+da)/(BM^*+da)$, is represented by the light gray areas in the left hand panels of Figure 3.1. The measure was constructed using data on bank net foreign asset positions available at the Bank of International Settlements (BIS), taking the series of liabilities of BIS reporting banks against each country, converting them into domestic currency and adding them to the numerator and denominator of FCD/BM^*.[3]

The new and broader measure provides a clearer picture of the currency substitution - inflation relationship before and after hyperinflationary periods. In Bolivia, the broader substitution ratio was already high in 1980 (38%) and increased to 56% at the peak of the hyperinflation episode in 1985, the same year in which FCD restrictions were eliminated. Currency substitution continued rising even after the hyperinflation period ended, reaching a maximum of 83% in 1995. The reduction in the distance between the two substitution measures during the 1988-2008 pe-

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[3]BIS Banking Statistics. Table 6A: External positions of reporting banks vis--vis individual countries, vis--vis all sectors.

Figure 3.1: CURRENCY SUBSTITUTION IN BOLIVIA AND PERU
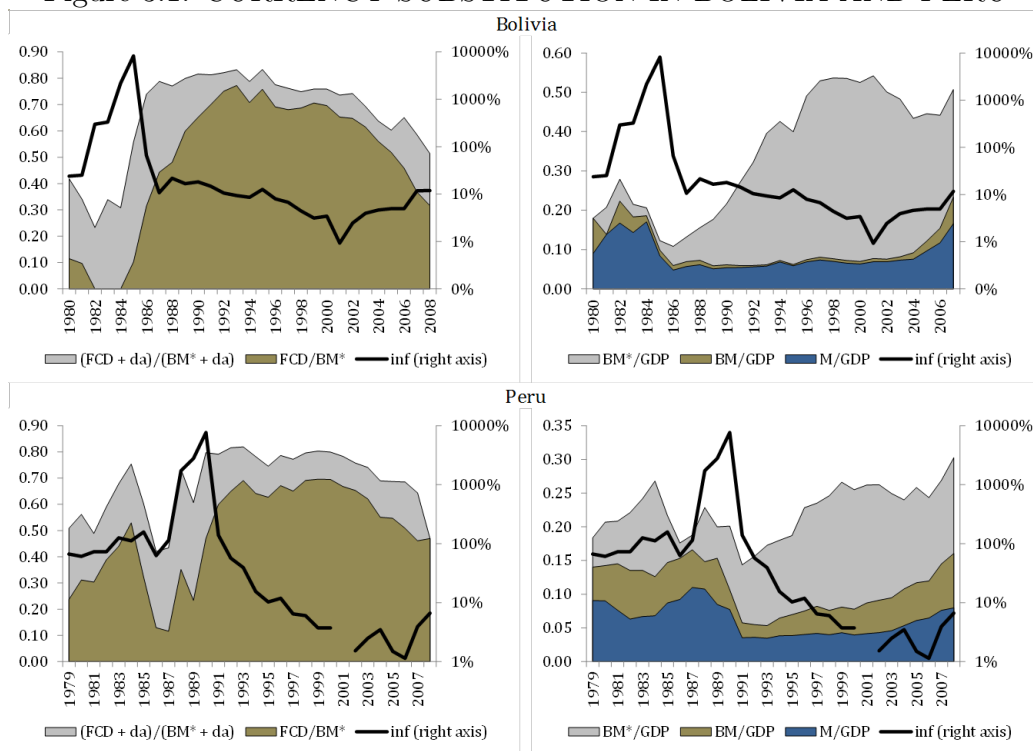
riod shows that as inflation was controlled and continually reduced, domestic agents changed the composition of their portfolios from assets held abroad to assets held in the domestic banking sector, but with little change in the currency composition of their monetary assets, relying heavily on $FCDs$.

A similar pattern is observed in Peru, where both inflation and the broader substitution measure were already high in the pre-hyperinflation period. The broader ratio increased with inflation in the 1987-1990 period, and reached its peak in 1993, by which time inflation already presented a marked downward trend. As in the Bolivian case, once FCD restrictions were eliminated (1990) the portfolio composition shifted towards domestically held deposits, but with the currency composition still leaning towards U.S. dollar denominated monetary assets.

Thus, the left-hand side panels in Figure 3.1 summarize the first main stylized fact that motivates the quantitative work presented later in the document: *while dollarization increases quickly with inflation during high-inflation periods, it does not quickly revert once inflation has been reduced and does not present a clear co-movement with inflation in the post-stabilization years.*

3.2.2 Inverse velocities of money

This section describes the behavior of the Bolivian and Peruvian financial systems by constructing different inverse velocities of money as proxies of financial deepness. Each of the right-hand side panels in Figure 3.1 presents inflation (in logarithmic scale) and the inverse of three money velocity ratios: i) M/GDP, money over GDP, ii) BM/GDP, broad money over GDP, and iii) BM^*/GDP, broad money including foreign currency deposits over GDP.[4]

The top right panel shows a sharp decline in monetization ratios during the Bolivian hyperinflation years (1984-1985); once that inflation was controlled and FCD restrictions lifted, the inverse velocities of money in domestic currency remained relatively low for several years, while BM^*/GDP increased strongly until reaching a peak in 2001 (54%). The domestic currency ratios (M/GDP and BM/GDP) increased in the 2004-2008 period, but this was not joined by a decrease in BM^*/GDP which increased from 43% in 2004 to 51% in 2007.

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[4]Bolivia: M/GDP = M1/GDP, BM=M2/GDP, BM* = M3'/GDP. Peru: M/GDP = Money/GDP, BM = (Money + Quasimoney)/GDP, BM* = (Money + Quasimoney in domestic and foreign currency)/GDP. See Appendix 2.

In the case of Peru, there was a marked reduction in the three monetary ratios during the hyperinflation period, followed by a strong increase in BM^*/GDP after inflation was controlled and FCD restrictions eliminated. While both domestic currency inverse velocities followed an increasing trend, they remained well below BM^*/GDP which reached its historical peak of 30% in 2008.

Overall, the right-hand side panels in Figure 3.1 summarize the second main stylized fact of interest in this document: *if the inverse velocities of money presented in Figure 3.1 are understood as proxies of financial deepness, then the graphs show that after high inflation episodes, the financial system recovery relies strongly on the use of the foreign currency. After the sharp decrease in monetization ratios, the post-hyperinflation financial sectors were rebuilt on the basis of the U.S. dollar instead of the national currency.*

The next section presents a simple perfect foresight model that seeks to capture the above mentioned stylized facts by introducing an accumulation externality in the use of the foreign currency.

3.3   A Model of Currency Substitution

The stylized facts analyzed in the previous section show that the relationship between currency substitution and inflation follows different patterns in pre-stabilization and post-stabilization periods, a type of asymmetry that is not captured by standard monetary general equilibrium models.

Motivated by the high degree of persistence in currency substitution observed

in several developing countries, Uribe (1997) develops a cash in advance model in which the cost of buying goods with foreign currency is decreasing in the economy's accumulated experience in transacting in foreign currency, a concept that Uribe calls the economy's *dollarization capital*. Uribe's framework delivers the possibility of an equilibrium in which the foreign currency circulates even in a low inflation steady state.

Building on Uribe's work, Reding and Morales (2004) replace the exogenous dollarization capital assumption by modeling endogenous network externality effects in which the size of the foreign currency network increases with the number of agents whose foreign currency balances are larger than a given threshold. Their model also renders the possibility of high-substitution - low-inflation equilibria by combining a learning effect (the acquisition of experience in the use of an alternative currency) with a size effect (the reduction in transaction costs caused by an increase in the number of agents with significant foreign currency balances).

Both of these works present heterogeneous agent models with complex dynamics and are not solved computationally. This section develops a simple representative agent monetary model incorporating Uribe's (1997) dollarization capital idea; the model is then solved using numerical methods.

3.3.1   Preferences

Consider a perfect foresight endowment economy with income $y_t = y$ in every period and lifetime utility stream given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{3.1}$$

where β is the inter-temporal discount factor, $u'(c_t) > 0$ and $u''(c_t) < 0$.

3.3.2 Monetary Policy

The economy is open. It follows a predetermined exchange rate regime and the law of one price holds at any moment in time. That is, the nominal exchange rate S_t is given by $S_t = p_t/p_t^*$, where p_t and p_t^*, are the domestic and international price levels respectively. Therefore inflation is given by $\pi_t = \varepsilon_t + \pi^*$ where ε is the policy-determined devaluation rate and π^* is international inflation.

Each period the government injects money in the economy through a transfer $T_t = M_{t+1} - M_t$, where M_t are nominal money balances. The transfer is such that it satisfies the law of one price and the model equilibrium conditions that will be derived below.

The agent uses her resources either in consumption (c_t) or in acquiring domestic and foreign currency balances carried to the next period (m_t, m_t^*).

3.3.3 Transaction Costs

Money enters into the model because of its transaction cost reduction services. Each period the agent must cover a transaction cost given by

$$\phi\left(c_t, \theta(\tfrac{m_t}{p_t}), \theta^*\left(\tfrac{m_t^*}{p_t^*}, d_t\right)\right); \ \phi_c > 0, \phi_\theta < 0, \phi_{\theta^*} < 0$$

where $\frac{m_t}{p_t}$ and $\frac{m_t^*}{p_t^*}$ are domestic and foreign real money balances respectively.[5]

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[5]Notice that m_t^* are nominal foreign currency balances expressed in the foreign currency, $m_t^* S_t$

The transaction cost reduction services provided by domestic currency real balances are given by $\theta(\frac{m_t}{p_t})$ with $\theta' > 0$, while the cost reduction services derived from from holding foreign currency real balances are given by $\theta^* \left(\frac{m_t^*}{p_t^*}, d_t\right)$, where d_t is the economy's accumulated experience in using foreign currency (the dollarization capital) and $\theta_{m_t^*/p_t^*}^* > 0$, $\theta_d^* > 0$.

Under this structure, the agent's budget constraint in period t is given by

$$
\begin{aligned}
y + \frac{m_{t-1}}{P_t} + \frac{m_{t-1}^*}{P_t^*} + T_t \;=\; & c_t + \frac{m_t}{P_t} + \frac{m_t^*}{P_t^*} \\
& + \; \phi \left(c_t, \theta \left(\frac{m_t}{P_t}\right), \theta^* \left(\frac{m_t^*}{P_t^*}, d_t\right) \right)
\end{aligned}
\tag{3.2}
$$

3.3.4 Dollarization Capital

I assume that the transaction cost reduction services provided by foreign currency depend not only on the amount of foreign currency real balances, but on the accumulated experience using this currency. The dollarization capital depreciates at an exogenous rate $(\tilde{\delta})$ and its accumulation is an increasing function of the current level of currency substitution.

$$
d_{t+1} = (1 - \tilde{\delta})d_t + G \left(\frac{\frac{m_t^*}{P_t^*}}{\frac{m_t^*}{P_t^*} + \frac{m_t}{P_t}} \right)
\tag{3.3}
$$

with $G'(.) > 0$ and $G''(.) < 0$.

are foreign currency balances expressed in domestic currency and $\frac{m_t^* S_t}{p_t} = \frac{m_t^* p_t}{p_t p_t^*} = \frac{m_t^*}{p_t^*}$ are foreign currency balances expressed in real terms.

3.3.5 Maximization Problem and Competitive Equilibrium

Letting $a_t = \frac{m_t}{p_t}$ and $a_t^* = \frac{m_t^*}{p_t^*}$, the consumer's problem is maximizing (3.1) subject to (3.2) and (3.3), with first order conditions given by

$$\beta \frac{u'(t+1)}{1 + \phi_c(t+1)} = (1 + \pi_{t+1})[\frac{u'(t)}{1 + \phi_c(t)}(1 + \phi_\theta(t)\theta'(t)) \\ + \gamma_t G'(t) \frac{a_t^*}{(a_t^* + a_t)^2}] \tag{3.4}$$

$$\beta \frac{u'(t+1)}{1 + \phi_c(t+1)} = (1 + \pi_{t+1}^*)[\frac{u'(t)}{1 + \phi_c(t)}(1 + \phi_{\theta^*}(t)\theta_{a^*}^*(t)) \\ - \gamma_t G'(t) \frac{a_t}{(a_t^* + a_t)^2} \tag{3.5}$$

$$\gamma_t = \beta \left[\gamma_{t+1}\left(1 - \tilde{\delta}\right) - \frac{u'(t+1)}{1 + \phi_c(t+1)}\phi_{\theta^*}(t+1)\theta_d^*(t+1) \right] \tag{3.6}$$

where γ_t is the multiplier on the dollarization capital accumulation equation, the expression (t) is used to denote that all the arguments of a given function are evaluated at period t, $\pi_t = p_t/p_{t-1}$ and $\pi_t^* = p_t^*/p_{t-1}^*$. Finally, when imposing market clearing in the money market, that is $M_t = m_t$, equation (3.2) is reduced to

$$\frac{m_t^*}{p_t^*} - \frac{m_{t-1}^*}{p_t^*} = y - c_t - \phi(t)$$

or

$$a_t^* - \frac{a_{t-1}^*}{1 + \pi_t^*} = y - c_t - \phi(t) \tag{3.7}$$

Notice that if the international price is normalized to one, $\pi_t^* = 0$ and this last equation is reduced to

$$a_t^* - a_{t-1}^* = [y - \phi(t)] - c_t \qquad (3.8)$$

that is, an increase in real foreign currency balances held by domestic agents is financed through a trade balance surplus, with the trade balance defined as the difference between output net of transaction costs, $y - \phi(t)$, and consumption.

Then, the model's *competitive equilibrium* is defined as a list of sequences for consumption and dollarization capital $\{c_t, d_t\}$, portfolio choices $\{m_t, m_t^*\}$, prices $\{p_t, p_t^*\}$, and policy variables $\{M_t, \varepsilon_t\}$ such that: i) Equations (3.2), (3.3), (3.4), (3.5) and (3.6) solve the representative agent's consumption maximization problem, ii) the monetary market clears $(M_t = m_t)$ and iii) the Law of One Price is satisfied in every period.

The intuition behind the first order conditions of the model is better understood by considering first the case of a model without dollarization capital. The equilibrium conditions of such a model correspond to equations (3.4), (3.5) and (3.11), setting $\gamma_t = 0$ and normalizing d_t to one:

$$\beta \frac{u'(t+1)}{1 + \phi_c(t+1)} \left(\frac{1}{1 + \pi_{t+1}} \right) = \frac{u'(t)}{1 + \phi_c(t)} (1 + \phi_\theta(t)\,\theta'(t)) \qquad (3.9)$$

$$\beta \frac{u'(t+1)}{1 + \phi_c(t+1)} \left(\frac{1}{1 + \pi_{t+1}^*} \right) = \frac{u'(t)}{1 + \phi_c(t)} (1 + \phi_{\theta^*}(t)\,\theta_{a^*}(t)) \qquad (3.10)$$

$$a_t^* - \frac{a_{t-1}^*}{1 + \pi_t^*} = y - c_t - \phi(t) \tag{3.11}$$

The left hand side of (3.9) is the marginal utility consuming one less unit of the good in period t and using those resources to increase domestic real balances at $t + 1$. In $t + 1$ these extra real balances are used for consumption, and the marginal utility of this future consumption is properly discounted by domestic inflation and the marginal increase in transaction costs $\phi_c(t + 1)$. This expression is equated to the marginal utility of consuming the good in period t (right hand side of (3.9)), adjusted by the increase in transaction costs caused by additional consumption and by the opportunity cost of not carrying an extra unit of domestic real balances to the next period. This opportunity cost equals the marginal reduction in the transaction cost from carrying domestic real balances from period t to period $t + 1$ (remember that $\phi_\theta(t)$ is negative).

A similar interpretation is given to equation (3.10), where the left hand side is the marginal utility of carrying a unit of foreign currency real balances to the next period, while the right hand side is the marginal utility of consuming a unit of the good in the current period, including the increase in transaction costs caused by additional current consumption and by having one less unit of foreign real balances carried to the next period.

Combining (3.9) and (3.10):

$$\frac{1 + \phi_{\theta^*}(t) \, \theta_{a^*}(t)}{1 + \phi_\theta(t) \, \theta'(t)} = \frac{1 + \pi_{t+1}}{1 + \pi_{t+1}^*} \tag{3.12}$$

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Equation (3.12) summarizes how the agent's currency portfolio composition is affected by domestic and foreign inflation in a model with no dollarization capital. When $\pi_t = \pi_t^*$, the opportunity cost (in transaction cost reduction terms) of not carrying a unit of domestic real balances $(\phi_\theta(t)\,\theta'(t))$ is equal, at an optimum, to the opportunity cost of not carrying a unit of foreign real balances $(\phi_{\theta*}(t)\,\theta_{a*}(t))$. When $\pi_{t+1} > \pi_{t+1}^* \geq -1$ however, $1 + \phi_{\theta*}(t)\theta_{a*}(t) > 1 + \phi_\theta(t)\theta'(t)$.

Thus, in the absence of dollarization capital dynamics, discrepancies between domestic and foreign inflation are immediately translated into changes in the current composition of the agent's real balances portfolio.

When endogenous accumulation of dollarization capital is introduced in the model, the equivalent of equation (3.12) is found by combining (3.4) and (3.5):

$$\frac{\frac{u'(t)}{1+\phi_c(t)}\left(1 + \phi_{\theta*}(t)\,\theta_{a*}^*(t)\right) - \gamma_t G'(t)\,\frac{a_t}{(a_t^*+a_t)^2}}{\frac{u'(t)}{1+\phi_c(t)}\left(1 + \phi_\theta(t)\,\theta'(t)\right) + \gamma_t G'(t)\,\frac{a*_t}{(a_t^*+a_t)^2}} = \frac{1 + \pi_{t+1}}{1 + \pi_{t+1}^*} \tag{3.13}$$

This last expression implies new trade-offs not present in (3.12). When $\pi_{t+1} = \pi_{t+1}^*$, (3.13) is reduced to:

$$\frac{u'(t)}{1 + \phi_c(t)}\left[\phi_{\theta*}(t)\,\theta_{a*}^*(t) - \phi_\theta(t)\,\theta'(t)\right] = \frac{\gamma_t G'(t)}{a_t^* + a_t} \tag{3.14}$$

where this last expression equates the marginal value of increasing the dollarization capital by holding a higher proportion of foreign real balances (right-hand side of 3.14) with the forgone marginal utility gain from reducing transaction costs by holding domestic balances over foreign ones and then spending the proceedings on consumption (left-hand side of 3.14, recalling that $\phi_{\theta*}(t)\,\theta_{a*}^*(t)$ and $\phi_\theta(t)\,\theta'(t)$

are negative).

In the absence of dollarization capital dynamics, the term in square brackets on the left-hand side of (3.14) would be zero when $\pi_{t+1} = \pi_{t+1}^*$ (see equation 3.12), but now it is positive at an optimum. That is, the model with dollarization capital dynamics requires higher marginal transaction cost reduction services from holding domestic currency balances than would be required in a framework without dollarization capital ($\mid \phi_\theta(t)\,\theta'(t) \mid > \mid \phi_{\theta^*}(t)\,\theta_{a^*}(t) \mid$).

This framework thus introduces inertia through a substitution motive not present in the model without dollarization capital. Suppose that dollarization (the proportion of foreign real balances with respect to total real balances , $a_t^*/(a_t^* + a_t)$) decreases. Then, for a given $\{\gamma_t, c_t\}$ pair, this would imply an increase in the right hand side of (3.14) because of the strict concavity of $G(t)$. This, in turn, implies broadening the gap between $\mid \phi_\theta(t)\,\theta'(t) \mid$ and $\mid \phi_{\theta^*}(t)\,\theta_{a^*}(t) \mid$ (left hand side of (3.14)), implying an increase in a_t^* and a decline in a_t.

3.4   IV. Solving the Model

3.4.1   Parametrization and Solution Method

The model is solved for the following functional forms:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$\phi(c_t, \theta(t), \theta^*(t)) = \left[\frac{c_t}{\theta(t)\theta^*(t)}\right]^\varphi c_t; \ \varphi > 0$$

$$\theta(t) = (a_t)^\eta; \ \eta \in (0, 1)$$

$$\theta^{*}(t) = (d_t a_t^{*})^{1-\eta}$$

$$G(t) = \left[\frac{a_t^{*}}{a_t^{*}+a_t}\right]^{\omega}; \ \omega \in (0,1)$$

where $a_t = \frac{m_t}{P_t}$ and $a_t^{*} = \frac{m_t^{*}}{P_t^{*}}$.

The functional form for the transaction cost term is similar to that used in works like Mendoza and Uribe (1997,1999,2000) but replacing m_t in the denominator by $m_t^{\eta}(d_t m_t^{*})^{1-\eta}$.

Table 3.1: Parameter Values

Parameter	Value			Source	Calibration Target
		Bolivia	Peru		
Discount factor	$\beta \in (0,1)$	0.96	0.96	Standard value in yearly models	-
Risk Aversion Coefficient	$\sigma > 0$	1.5	1.5	Reinhart and Vegh (95)	-
Inflation Rate	ε	0.24	0.93	Country Data	-
Exponential term: transaction cost function	$\varphi > 0$	0.081	0.139	Calibrated	$\frac{c}{y} = \frac{C}{GDP}$
Exponential term: cost reduction services provided by the domestic currency	$\eta \in (\frac{1}{2},1)$	0.927	0.911	Calibrated	$\frac{a}{y} = \frac{NCSR}{GDP}$
Exponential term: Dollarization Capital accumulation function	$\omega \in (0,1)$	0.689	0.490	Calibrated	$\frac{a^{*}}{y} = \frac{DCSR-NCSR}{GDP}$
Dollariation Capital depreciation rate	$\widetilde{\delta} \in (0,1)$	0.521	0.777	Calibrated	$d = 1$

C = Private Consumption

NCSR = Numerator in Currency Substitution Ratio (CSR)

DCSR = Denominator in CSR

Given the predetermined exchange rate regime and normalizing $p*$ to one, the inflation rate is given by the devaluation rate, $\varepsilon_t = \varepsilon$, taken as a policy parame-

Table 3.2: Model Results and Calibration Targets

| | The Bolivian pre-hyperinflation CSR | | The Peruvian pre-hyperinflation CSR | |
	Model	Data	Model	Data
a/y	0.17	0.17	0.14	0.14
a^*/y	0.11	0.11	0.20	0.20
c/y	0.5	0.7	0.5	0.6
d	1.0	-	1.0	-
CSR	0.39	0.38	0.60	0.59

ter that determines each steady state. The solution method consists of linearizing equations (3.3), (3.4), (3.5), (3.6) and (3.11) around alternative steady states using Schmitt-Grohe and Uribe's (2004) perturbation algorithm.

Regarding the values chosen for each parameter, y is normalized to one, while β is set at a standard value of 0.96. σ is set at 1.5, in the middle of the range used in Mendoza's (1991) study on real business cycles in small open economies; this value is also within the range estimated by Reinhart and Vegh (1995). Two sets of values for φ, ω, η and $\widetilde{\delta}$ are chosen to replicate the pre-hyperinflation currency substitution ratio in Bolivia and Peru.

The values for $\{\varphi, \omega, \eta, \widetilde{\delta}\}$ are jointly determined to match four calibration targets: first, a consumption-output ratio (c/y) equal to the observed consumption-GDP ratio in the pre-hyperinflation period (1980-1981); second, a foreign real balances–output ratio (a^*/y) equal to the observed $(FCD + da)/NGDP$ ratio in the pre-hyperinflation period, where $NGDP$ stands for nominal GDP, FCD stands for foreign currency deposits and da stands for deposits held abroad; third, a domestic real balances-output ratio (a/y) equal to the observed $(BM^* - FCD)/GDP$ ratio in the pre-hyperinflation period, where BM^* is the broad money measure used in

Section II; fourth, a dollarization capital value equal to one.

By setting the above mentioned targets for a^*/y and a/y, the model's ratio of foreign monetary assets to output ($\frac{a^*}{y} = \frac{sm^*}{py}$) corresponds to the ratio between the numerator of the currency substitution measure used in Section II and nominal GDP (that is, the ratio of foreign currency monetary assets to nominal GDP), while the model's ratio of domestic monetary assets to output ($\frac{a}{y} = \frac{m}{py}$) corresponds to the ratio of the difference between the denominator and numerator terms of the currency substitution measure used in Section II to nominal GDP (that is, the ratio of domestic currency monetary assets to nominal GDP). Given these targets, the model's ratio of foreign real monetary assets to total monetary assets ($a^*/(a^* + a)$) corresponds to the observed currency substitution measure discussed in Section II ($(FCD + da)/(BM^* + da)$).

The target for d_t is chosen to generate some similarity between the model's pre-hyperinflation equilibrium and that obtained in a model with no dollarization capital dynamics.

In the two cases the value for η is restricted to be greater than $1/2$ in order to ensure that domestic real balances are greater than foreign currency when $\pi_{t+1} = \pi^*_{t+1}$ and when there is no dollarization capital. To see this notice that under the current functional forms equation (3.12) implies $\frac{a_t}{a^*_t} = \frac{\eta}{1-\eta}$ when $\pi_{t+1} = \pi^*_{t+1}$, and $\eta > 1/2$ ensures $a_t \dot{\iota} a^*_t$.

The four sets of parameter values are reported in Table 3.1. Table 3.2 presents the observed and model-generated values for each calibration target.

Table 3.3 shows the performance of the model in matching the observed degree

of currency substitution during hyperinflation episodes.

Table 3.3: Currency Substitution Ratios in High-inflation Periods

	Model	Data 1*	Data 2**
Setting parameters as to replicate:			
The Bolivian pre-hyperinflation CSR			
CSR	0.70	0.56	0.74
d	1.50	-	-
The Peruvian pre-hyperinflation CSR			
CSR	0.74	0.80	0.79
d	1.10	-	-

* At the peak of the high-inflation period.

** One period after the hyperinflation.

The average value of Bolivian inflation for the 1982-1985 period was 2700%. When inflation is set at this value, the result is a 53% increase in dollarization capital and a currency substitution ratio of 70%, a little higher than the average between the ratio observed at the peak of the hyperinflation and the one observed one period after the hyperinflation.

In the case of Peru, the average inflation rate for the hyperinflation period (1988-1991) was 4000%. When the model is solved for this inflation rate, the dollarization capital level increases by 12% with respect to the pre-hyperinflation period and the currency substitution ratio is 74%, seven to eight percentage points below the observed ratio.

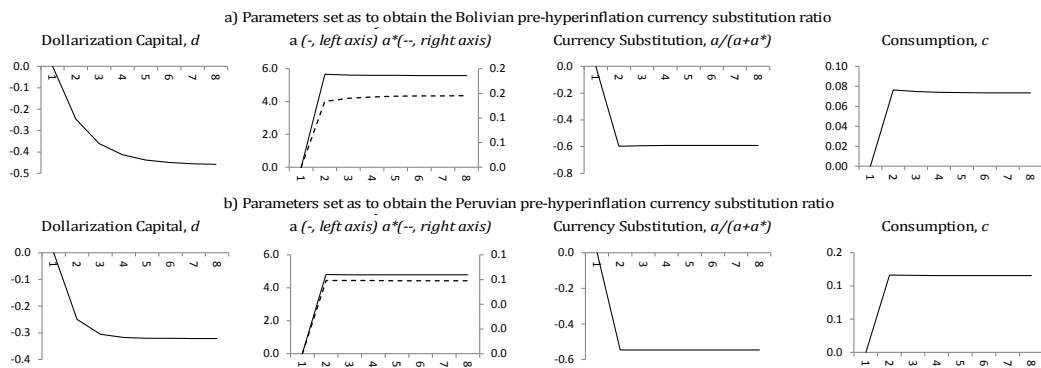## 3.4.2 Transitional Dynamics from High to Low inflation

As shown in Table 3.3, the model performs well in terms of generating high dollarization ratios for large values of inflation. The model can also be used to study

transitional dynamics from a high-inflation steady state to a low-inflation one. Figure 3.2 presents impact and transitional effects of reducing inflation from 2700% (the Bolivian post-hyperinflation value) to 24% (the Bolivian pre-hyperinflation value), and from 4000% (the Peruvian post-hyperinflation value) to 90% (the Peruvian pre-hyperinflation value). All series are expressed as percentage deviations from the original steady state.

The adjustment in dollarization capital is slow, taking more than three years in every case, and the rapid adjustment in currency portfolios is mainly due to sharp changes in domestic currency balances. The much slower and less sizable adjustment in foreign currency balances is directly related to the inertia observed in dollarization capital dynamics. Currency substitution dynamics are shown in the third graph in each panel, with a high impact effect explained by the quick adjustment in domestic currency balances.

Since money enters the model through a transaction cost function, the reduction in inflation has a positive effect on consumption and therefore utility.

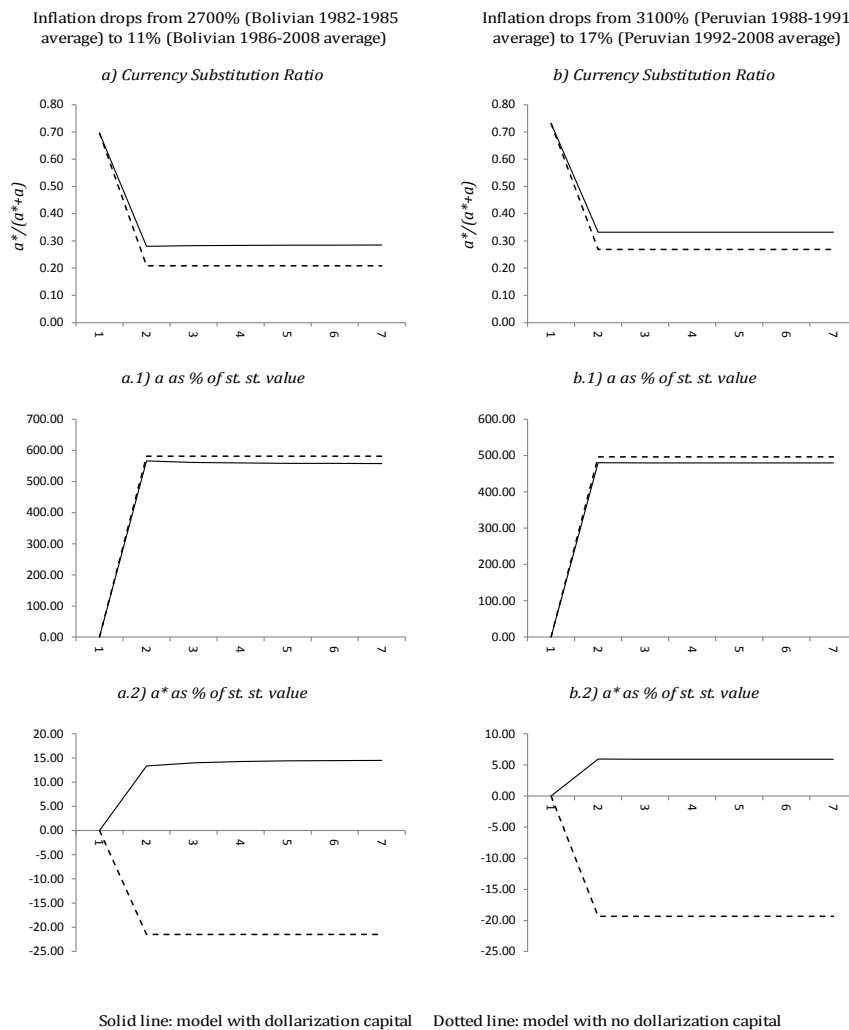Figure 3.2: TRANSITIONAL DYNAMICS

80

How does the model perform compared to one in which there is no dollarization capital? This question is answered by analyzing currency substitution transitional dynamics from the high-inflation steady state to one in which inflation equals the post-stabilization average. Panel a) in Figure 3.3 corresponds to a reduction in inflation from 2700% (the Bolivian hyperinflation average) to 11% (the Bolivian post stabilization average), while panel b) corresponds to a reduction in inflation from 4000% (the Peruvian hyperinflation average) to 17% (the Peruvian post stabilization average). Solid lines are used for the model with dollarization capital, while dashed lines are used for the model with no dollarization capital[6]. In all cases the initial value is set at the high inflation steady state in the model with dollarization capital.

As expected, the dollarization capital model delivers higher post-stabilization ratios than those obtained with the model without dollarization capital.4. Panels a.1) to b.2) show the transition paths of domestic and foreign real balances. In the model with no dollarization capital foreign balances fall with respect to their levels in the previous equilibrium while they increase when dollarization capital is added to the model. However, as mentioned earlier, in both models the dynamics of the currency substitution ratio are not driven by the adjustment in foreign balances but by quick and very sizable variations in domestic balances.

The currency substitution ratios of the model with dollarization capital are below the observed ratios, but still represent a large increase with respect to a more conventional model with no dollarization capital dynamics. The ratios generated by the dollarization capital model are between 40% and 50% of the observed ratios,

---

[6]Dollarization capital dynamics are shut down by setting $\widetilde{\delta} = 1$ and $\omega = 0$

Figure 3.3: TRANSITION WITH AND WITHOUT DOLLARIZATION CAPITAL

Inflation drops from 2700% (Bolivian 1982-1985 average) to 11% (Bolivian 1986-2008 average)

Inflation drops from 3100% (Peruvian 1988-1991 average) to 17% (Peruvian 1992-2008 average)

a) Currency Substitution Ratio

b) Currency Substitution Ratio

a.1) a as % of st. st. value

b.1) a as % of st. st. value

a.2) a* as % of st. st. value

b.2) a* as % of st. st. value

Solid line: model with dollarization capital   Dotted line: model with no dollarization capital

explaining 1/6 of the gap between the observed currency substitution ratios and those obtained with a model with no dollarization capital.

This is seen in Table 3.4, where the first three columns in each panel present the post-stabilization currency substitution ratios obtained by models with and without dollarization capital, as well as the observed ratios. The fourth column presents the ratio between the difference in predicted currency substitution ratios with and without dollarization capital and the difference between the observed ratio and the

Table 3.4: Currency Substitution Ratios in High-inflation Periods

	Setting parameters as to replicate:				
	The Bolivian pre-hyperinflation CSR				
	(i)	(ii)	(iii)		
	Model	Model (*no*	Data	$\frac{(i)}{(iii)}$	$\frac{(i)-(ii)}{(iii)-(ii)}*$
	(*dol capital*)	*dol capital*)	(86-08)		
CSR	0.29	0.21	0.74	0.40	0.15
	The Peruvian pre-hyperinflation CSR				
	(i)	(ii)	(iii)		
	Model	Model (*no*	Data	$\frac{(i)}{(iii)}$	$\frac{(iii)-(ii)}{(i)-(ii)}*$
	(*dol capital*)	*dol capital*)	(92-08)		
CSR	0.37	0.30	0.74	0.50	0.16

* Gap between (*ii*) and (*iii*) explained by (*i*)

one generated by the model without dollarization capital.

Overall, the dollarization capital framework delivers high post-stabilization substitution ratios using a very simple structure, and leaving ample room for further additions to the model dynamics. These additions could focus on generating inertia in the adjustment process for domestic currency holdings in a way consistent with the behavior of the inverse velocities of money discussed in Section II. An interesting line of work in this regard would be the introduction of uncertainty about the future value of inflation, by assigning a policy reversal probability as part of the agent's information set as in Mendoza and Uribe (1999,2000)[7].

[7]In the predetermined exchange rate regime environment of the present document, this feature could be added to the model by introducing a post-stabilization time-dependent devaluation probability as in Mendoza and Uribe (2000), or by making this probability an endogenous variable conditioned on foreign reserves as in Mendoza and Uribe (1999).

3.5   Conclusion

The present document has developed a simple endowment-economy monetary model able to generate high levels of currency substitution in low inflation environments. The introduction of a dollarization capital variable to capture the economy's accumulated experience in the use of foreign currency generates a motive for holding foreign currency balances even when inflation is low.

When applied to high-inflation equilibria, the model is able to generate currency substitution ratios very close to those observed during hyperinflation episodes, and it presents inertia in foreign currency real balances during the adjustment towards a low-inflation steady state.

In terms of the model's predicted post-hyperinflation dollarization levels, these are still below the observed ratios but represent a sizable increase with respect to those obtained using a more standard framework with no dollarization capital. The dollarization capital model generates post-hyperinflation currency substitution ratios that are between 40% and 50% of the observed ratios and is able to explain 1/6 of the gap between the observed substitution ratios and those generated by a model with no dollarization capital dynamics.

Further model building efforts could focus on introducing frictions that reduce the size of impact effects on post-stabilization domestic currency holdings. An important step in this direction may be the introduction of devaluation risk dynamics along the lines of Mendoza and Uribe (1999, 2000).

Chapter 4

Investment Activity in the Financial Sector and the Relationship between Inflation and Currency Substitution

4.1   Introduction

As shown in the previous chapter, the dollarization capital model generates a higher steady state currency substitution ratio than that obtained with a model with no dollarization capital in it. There are, however, three important limitations that affect the model's ability to study currency substitution dynamics. First, the dollarization capital variable was introduced in an ad hoc manner as a stock variable entering the transaction cost function. Second, the model has a representative household, and

the dollarization capital variable is optimally chosen by the consumer taking into account an accumulation law of motion in which dollarization capital increases with the currency substitution ratio. In this environment, the consumer fully internalizes the effect of currency substitution on the dollarization capital variable, leaving no room for network externality effects like the ones developed in Uribe (1997) and Reding and Morales (2004). Third, the model is not able to generate an asymmetric relationship between inflation and currency substitution before and after high inflation episodes. This behavior is at odds with the data in several partially dollarized economies.

This chapter presents a framework that provides a micro foundation for a quantitative model able to deal with the above mentioned limitations. Developing and solving such a model is beyond the scope of this chapter. The chapter rather focuses on exploring the theoretical elements needed to generate different inflation-substitution patterns before and after high inflation episodes. The main idea is that in an environment in which consumers show some heterogeneity in their ability to access the financial sector's money exchange services, a sufficiently high level of inflation can induce the financial sector to make a fixed cost investment that decreases this heterogeneity by permanently reducing money exchange costs.

More formally, this chapter presents a simple model of two consumers who use money to buy a consumption good. The consumers can hold either domestic or foreign currency, but they do not have the technology to transform domestic balances for foreign ones, so to do so they must use a money exchange point owned by the financial sector. The only difference between consumers is that they may be located

at different distances from the nearest exchange point, and they obtain disutility from walking that distance. The financial sector provides only one service, money exchange, and decides on whether or not to construct money exchange points at a given fixed cost. The financial sector makes its optimal exchange point construction decision anticipating the consumers' substitution decisions under different levels of inflation. For sufficiently high levels of inflation, the financial sector will construct new exchange points, changing consumers' distance to the closest exchange point and therefore permanently reducing currency substitution costs.

It could be said then that in this framework the dollarization capital variable corresponds to the number of exchange points set by the financial sector. This seems like a plausible example of the type of investment decisions made by the financial sector in developing countries as agents increase their usage of foreign currency. Think, for example, of a bank in a high-inflation developing country deciding on whether to offer money exchange services in some or all of its agencies. Provided that there are no legal restrictions, the bank may also decide to offer foreign currency high liquidity services (i.e. checking accounts denominated in dollars), an action that may be associated with different types of one-time costs, such as training personnel in the use of the foreign currency and setting up accounting and software systems that handle both currencies. For an example directly related to the concepts of distance and accessibility, think of banks investing in ATM machines and deciding whether to provide ATM services only in domestic currency or allowing them to work in foreign currency as well. The financial system may be willing to pay these investment costs during periods of high inflation (or of high inflation expectations) with

the purpose of taking advantage of the increase in household's demand for foreign currency. However, this investment permanently reduces household's costs in terms of accessing foreign currency services and therefore induces currency substitution at lower levels of inflation in the future.

The combination of consumer heterogeneity and fixed costs in exchange point construction generate inflation regions with different intensities in terms of investment in new exchange points. Particularly, starting from a state in which there are no exchange points in the economy, an episode of high inflation will push the financial sector into action in terms of expanding the economy's dollarization capital, by installing one exchange point which will be more closely located to one consumer than to the other. For a sufficiently high inflation rate, both consumers will be willing to walk to the exchange point, therefore eliminating any incentive of the financial sector to invest in a second exchange point. If in subsequent periods inflation declines to a level that is low enough to discourage the more distant consumer from walking to the exchange point, the financial sector will have an incentive to invest in a second money exchange point. This decision will locate both consumers at the same distance from an exchange point, eliminating heterogeneity in terms of access to financial services, and lowering the inflation threshold at which the second consumer is willing to substitute foreign for domestic currency. Thus, the model presented here is able to generate an equilibrium in which the relationship between inflation and currency substitution is path dependent and differs before and after periods of high inflation.

From a theoretical point of view, the work presented here constitutes a step

along the lines of Guidotti and Rodrigues (1992), Uribe (1997) and Reding and Morales (2004) in terms explaining non-linearities in the relationship between inflation and currency substitution. Guidotti and Rodrigues (1992) present a model in which agents may choose the currency in which purchases are denominated, but they must pay a cost every time they decide to switch the currency denomination of their transactions. Switching is optimal when the inflation rate is sufficiently high; when inflation drops back to a low level, inaction becomes optimal, generating a low-inflation - high-currency-substitution equilibrium.

Uribe (1997) remarks that Guidotti and Rodriguez (1992) ignores the possibility that hysteresis in currency substitution is likely to involve network effects. Therefore, he develops a model in which the economy's accumulated experience in using a foreign currency as a means of payment reduces the cost of buying goods with foreign currency. Chapter 3 of this dissertation studies a model similar to Uribe's but solved computationally. Uribe models this accumulated experience in using a foreign currency as a dollarization capital variable that is strictly increasing in the degree of dollarization of the economy.

Reding and Morales (2004) depart from Uribe's approach by not relying on a dollarization capital variable with exogenous dynamics, but rather trying to model explicitly the network effects related to the use of foreign currency. They assume agents are heterogeneous with respect to the transaction cost they face when they wish to use foreign currency as a means of payment. In turn, this cost is decreasing in the size of a foreign currency network, comprised of all agents whose balances in dollars are higher than an exogenous threshold.

The framework presented here can be seen as an effort to provide a micro foundation for the emergence and functioning of such a foreign currency network. This is done by considering an aspect not studied in the papers described so far: the role of the financial sector in developing the economy's dollarization capital. When thinking about the experience of several partially dollarized developing countries, it seems that the emergence of a network of foreign currency users is highly affected by the decision of the financial sector to provide services in foreign currency. As mentioned earlier, this is very likely to be associated with different types of investment costs (some of them fixed). In the model presented here, a network of foreign currency users emerges only after the financial sector has made the proper sunk investment.

In this context, and in contrast to Uribe (1997), the economy's dollarization capital corresponds to actual physical capital (the number of available exchange points), and, in contrast to Reding and Morales (2004), the currency substitution network is not composed of agents that have foreign currency holdings above some exogenous level, but by the financial sector and those agents that have access to money exchange services. Further, whether an agent has access to financial sector services or not depends on the consumer's endogenous inflation threshold for currency substitution and on the financial sector's endogenous optimal investment decision.

Network expansion effects, then, are the result of interactions between optimal decisions on the consumer side and on the financial sector side. Consumers define their substitution behavior based on the level of inflation and the availability

of exchange points (the level of dollarization capital), and do not internalize the impact that their demand for currency exchange services may have on the financial sector's decision to expand the number of exchange points. The financial sector makes its optimal investment decision based on the potential demand that it would face if consumers had access to the proper level of dollarization capital (the profit maximizing number of exchange points).

The rest of this chapter is organized as follows. The second section presents a simple three agent model used to study the consumer's optimal currency substitution behavior and the financial sector's optimal dollarization capital investment decision. The third section concludes.

4.2 A simple three-agent model

4.2.1 The environment

The world consists of a local economy given by a horizontal space of measure one, as depicted in the top panel of Figure 4.1, and an external economy (the rest of the world). Prices in the local economy are not necessarily the same as those of the rest of the world. There are three agents in the local economy: a financial sector that provides only one service, money exchange, and two consumers that start each period located on positions one and two. If these consumers want to acquire foreign currency they need to walk to the closest money exchange point constructed by the financial sector. There can be at most two exchange points, and these can only be

located in positions A and B.
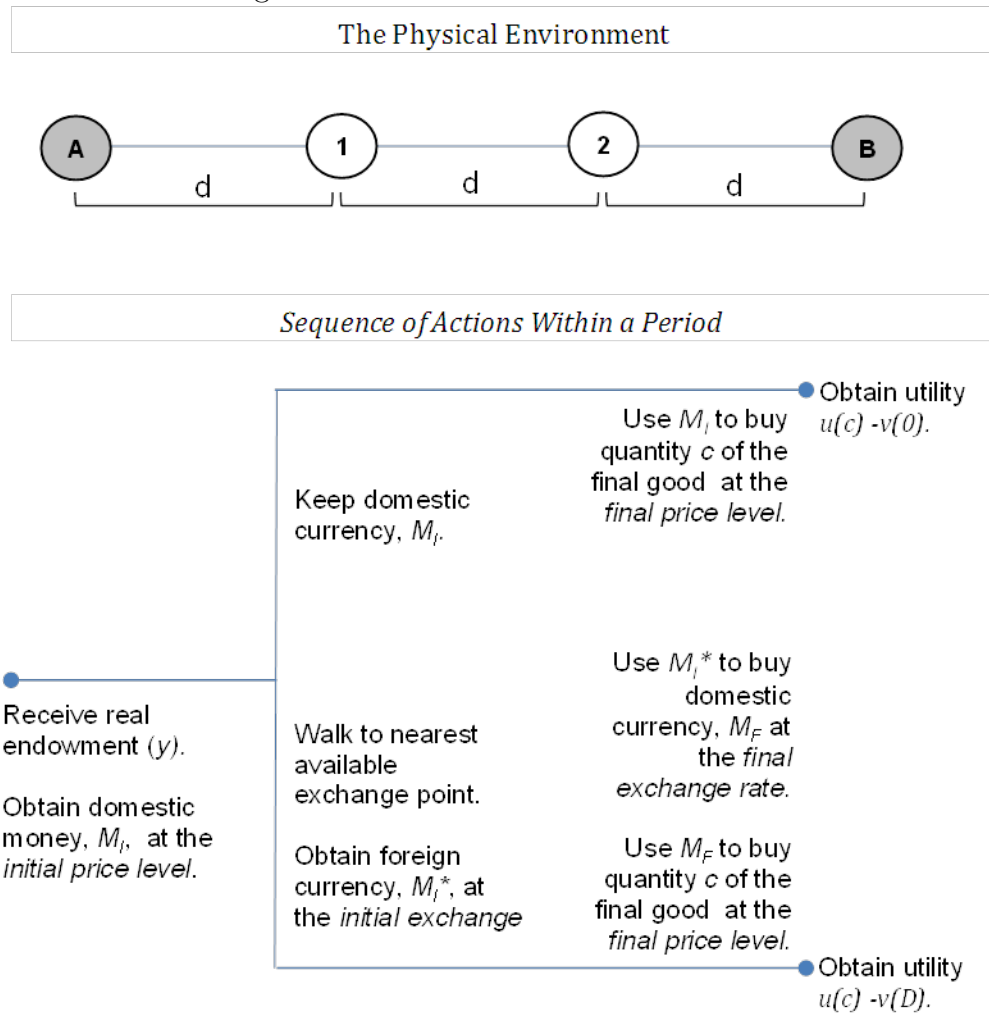
4.2.2 Consumer's problem

Each period consumers derive utility from consumption and disutility from walked distance. Preferences then are given by

$$u(c) - v(D)$$

Where $u(\cdot)$ and $v(\cdot)$ are strictly increasing Bernoulli utility functions, c is

consumption and D is walked distance. The consumer's problem is an intratemporal one, so time subscripts are ignored for simplicity.

Each period starts with an initial price level (P_I) and finishes with a final price level (P_F). At the beginning of each period, each consumer receives a real endowment y. As mentioned above, consumers receive utility from consumption and it is assumed that they need domestic money to buy the consumption good. That is, consumers cannot directly change their endowment y for consumption, but need money in order to perform the transaction. Therefore, after receiving their real endowment, consumers change it for nominal balances; at the end of the period, these balances are used to buy the consumption good. For simplicity, it is assumed that there is no savings technology; therefore, at the end of each period all currency balances must be exchanged for the consumption good. Then, the consumer's problem consists solely of choosing the currency denomination of the nominal balances that she holds within the period.

The sequence of actions is clarified by the graph presented in the second panel of Figure 4.1. The consumer starts the period with a real endowment y, which is immediately exchanged for domestic money at the initial price level. If the consumer has access to the money exchange services provided by the financial sector (that is, if there is a money exchange point located either at position A or B or both), the consumer decides whether to keep these domestic currency balances or change them for foreign currency. If she chooses not to substitute, then the consumer gets to the end of the period with the initial amount of domestic money holdings and uses them to finance consumption at the end of period price level. In this case the consumer's

utility is given by $u(c) - v(0)$.

If the consumer instead decides to hold foreign currency balances, she needs to walk to the closest available money exchange point (located at distance $D = d$ or $D = 2d$). There she pays a real fee f in order to change her domestic currency balances for foreign ones that she holds until the end of the period, when these are once again changed for domestic money and used to finance consumption. In this case the consumer's utility is $u(c) - v(D)$.

Let ξ be the set of positions where the financial sector has built a money exchange point. Then, the consumers' substitution decision can be analyzed under three case: no exchange points, $\xi = \{\oslash\}$, one exchange point, $\xi = \{A\}$ or $\xi = \{B\}$, and two exchange points, $\xi = \{A, B\}$.

Case 1. No Exchange Points Available, $\xi = \{\oslash\}$

This is a trivial problem since none of the consumers has a choice on the denomination of her currency holdings.

At the beginning of each period, each consumer receives the real endowment y, which she changes for money holdings $M_I = yP_I$, where P_I is the beginning-of-period domestic price level. At the end of the period, these money holdings are used to finance real consumption $c = \frac{M_I}{P_F}$, where P_F is the end-of-period domestic price level. Define gross inflation, $1 + \pi = \frac{P_F}{P_I}$. Then, real consumption in terms of the original endowment will be given by $c = \frac{yP_I}{P_F} = \frac{y}{1+\pi}$ and the consumer's utility in the period is $u(\frac{y}{1+\pi}) - v(0)$.

Case 2 One exchange point available, $\xi = \{A\}$ or $\xi = \{B\}$

Consider the case in which there is a money exchange point available at point A.
Let's start by considering agent one, who is closest to A. After exchanging its real
endowment y for money balances $M_I = yP_I$, the agent must decide whether to
substitute M_I for foreign currency or not. As derived in Case 1, if she decides not
to substitute her utility will be $u(\frac{y}{1+\pi}) - v(0)$.

If she wants to substitute M_I for foreign currency, she first needs to walk to the
exchange point, deriving disutility $v(d)$, and then pay a real fee γy, $\gamma \in (0,1)$, before
obtaining $M^* = S_I^{-1}(M - \gamma y P_I)$, where $S_I = \frac{P_I}{P_I^*}$ is the initial nominal exchange
rate and P_I^* is the begeining-of-period foreign price level. Then, foreign currency
holdings are given by $M^* = (1-\gamma)yP_I^*$. At the end of the period M^* is changed
for domestic currency at the end-of-period exchange rate and real consumption is
$c = \frac{M^* S_F}{P_F} = \frac{(1-\gamma)y}{1+\pi^*}$, where the consumer's utility is $u(\frac{(1-\gamma)y}{1+\pi^*}) - v(d)$.

Let s_1 be a variable that indicates whether the consumer substitutes or not:

$$
s_1 = \begin{cases} 1 & if\ consumer\ 1\ substitutes \\ \\ 0 & otherwise \end{cases}
$$

Then, consumer 1's currency substitution problem is

$$U = \max_{s_1}[U^{subst}, U^{no\,subst}] \tag{4.1}$$

$$U^{subst} = u(\frac{(1-\gamma)y}{1+\pi^*}) - v(d)$$

$$U^{no\,subst} = u(\frac{y}{1+\pi}) - v(0)$$

Consumer 2's problem is analogous, but with a lower utility payoff when sub-

stituting local for domestic currency balances:

$$U = \max_{s_2}[U^{subst}, U^{no\,subst}] \tag{4.2}$$

$$U^{subst} = u(\frac{(1-\gamma)y}{1+\pi^*}) - v(2d)$$

$$U^{no\,subst} = u(\frac{y}{1+\pi}) - v(0)$$

Then, in each period consumers one and two will substitute domestic for foreign currency holdings as long as

$$u(\frac{(1-\gamma)y}{1+\pi^*}) - v(d) > u(\frac{y}{1+\pi}) - v(0) \tag{4.3}$$

and

$$u(\frac{(1-\gamma)y}{1+\pi^*}) - v(2d) > u(\frac{y}{1+\pi}) - v(0) \tag{4.4}$$

For a given level of foreign inflation, $\overline{\pi^*}$, these two conditions implicitly define two inflation thresholds for currency substitution:

$$u(\frac{(1-\gamma)y}{1+\overline{\pi^*}}) - v(d) > u(\frac{y}{1+\pi_1}) - v(0) \tag{4.5}$$

$$u(\frac{(1-\gamma)y}{1+\overline{\pi^*}}) - v(2d) > u(\frac{y}{1+\pi_2}) - v(0) \tag{4.6}$$

For simplicity, consider the case of linear utility $u(c) = c$, $v(D) = D$. Then the substitution thresholds for inflation are

$$\pi_1 > \frac{y(1 + \overline{\pi^*})}{(1 - \gamma)y - d(1 + \overline{\pi^*})} - 1 \tag{4.7}$$

$$\pi_2 > \frac{y(1 + \overline{\pi^*})}{(1 - \gamma)y - 2d(1 + \overline{\pi^*})} - 1 \tag{4.8}$$

As expected, the inflation threshold increases with the cost of accessing the money exchange services provided by the financial sector, and therefore agent two, who has a longer distance to walk to the nearest money exchange point, has a greater threshold than that of agent one.

Case 3. Two exchange points available, $\xi = \{A, B\}$

In this case both consumers are at distance d from the nearest money exchange point, and therefore they face the same problem as consumer one in the $\xi = \{A, B\}$ case, with both consumers willing to substitute foreign for domestic currency when $\pi >= \pi_1$.

Figure 4.2 shows the relationship between inflation and currency substitution in the three cases $\xi = \{\oslash\}$, $\xi = \{A\}$ or $\xi = \{B\}$, and $\xi = \{A, B\}$.

Define the economy's end of period currency substitution ratio (CSR) as the ratio between foreign currency holdings evaluated at S_F and total money holdings before consumption. When there are no exchange points available, this ratio is zero for every level of inflation. When there is one exchange point available, only one consumer substitutes domestic for foreign currency when inflation is greater than π_1 but smaller than π_2. Then, before consumption, this consumer holds $S_F M^*$ money

holdings, while the other one holds M_I. Therefore, the currency substitution ratio is given by $\frac{S_F M^*}{S_F M^* + M_I}$. When inflation is above π_2, both consumers hold $S_F M^*$, so the economy's currency substitution ratio is $\frac{2 S_F M^*}{2 S_F M^*} = 1$. π_1 and π_2 correspond to levels of inflation at which either one or both consumers are indifferent between holding domestic or foreign currency and therefore also admit any convex combination of these two options. For simplicity, from now on the possibility of a convex combination between the two currencies is ruled out by assuming that consumers fully switch to the foreign currency as soon as inflation hits an indifference threshold.

Finally, when two money exchange points are available both consumers have the same inflation threshold and substitute domestic for foreign currency every time that $\pi \geq \pi_1$.

4.2.3 The financial sector's investment problem

Each period, the financial sector's problem consists in deciding whether to install new exchange points or not. The maximum of installed exchange points is two and each one has a real cost of installation of $F \in (0, \gamma y)$. Every time that a consumer approaches a money exchange point, the financial sector charges a real fee of γy.

It is assumed that once that an exchange point has been installed, it does not depreciate. Therefore, when no exchange point is available, $\xi = \{\oslash\}$, the financial sector decides between not investing in any exchange points, installing one exchange point at cost F, and installing two exchange points paying a cost of $2F$. When there is only one exchange point available to consumers, $\xi = \{A\}$ or $\xi = \{B\}$, the financial

Figure 4.2: CURRENCY SUBSTITUTION AND INFLATION
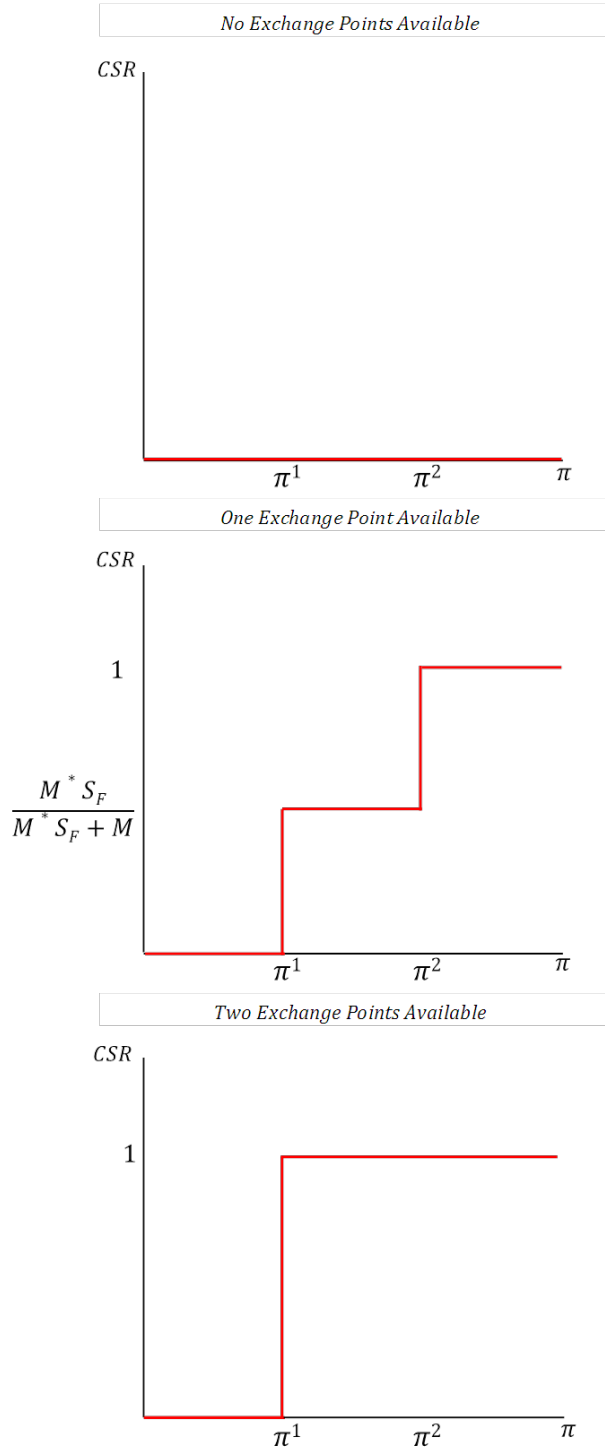
sector's problem consists of choosing whether not to invest in a new exchange point

or installing it at cost F. Finally, when $\xi = \{A, B\}$ the financial sector has no

investment decision.

The financial sector makes its investment decision before the consumers' currency substitution decision. Then, this corresponds to a sequential game in which the financial sector moves first, deciding on the number of installed exchange points. This in turn is observed by the consumers, who make their currency substitution decision taking into account the availability of money exchange services.

It is easier to start by considering a one-round game in which the financial sector makes its investment decision, and the game ends with the consumers' currency substitution choice after observing the behavior of the financial sector.

A one period investment problem

Case 1. Initial state: no exchange points available $\xi = \{\oslash\}$ The firm designs its investment strategy by anticipating the consumers' behavior under different levels of inflation. If inflation is below π_1, none of the consumers requires money exchange services and the financial sector profits are $\Pi = 0$ when it decides not to set up any new exchange points, $\Pi = -F$ when it installs one new exchange point, and $\Pi = -2F$ when it installs two new exchange points. Its optimal decision then is not to install any exchange points.

When $\pi_1 \leq \pi < \pi_2$, only one consumer holds foreign currency when $\xi = \{A\}$ or $\xi = \{B\}$ and both consumers hold foreign currency balances when $\xi = \{A, B\}$. Then, the financial sector makes profits $\Pi = 0$ when it decides not to set up any new exchange points, $\Pi = \gamma y - F$ when it installs one exchange point, and $\Pi = 2(\gamma y - F)$ when it installs two new exchange points. Its optimal decision then is to install two

exchange points.

When $\pi \geq \pi_2$, both consumers hold foreign currency as long as there is at least one money exchange point available. Then, the financial sector profits are $\Pi = 0$ when no new exchange points are installed, $\Pi = 2\gamma y - F$ when one new exchange point is installed, and $\Pi = 2(\gamma y - F)$ when two ne exchange points are installed. Then, the financial sector's optimal decision is to install only one new exchange point.

Case 2. Initial state: one exchange point available $\xi = \{A\}$ or $\xi = \{B\}$ As in the previous case, when $\pi < \pi_1$ the financial sector's optimal decision is not to install the new money exchange point.

When $\pi_1 \leq \pi < \pi_2$, the financial sector makes profits $\Pi = \gamma y$ when it doesn't install the new exchange point, and $\Pi = 2\gamma y - F$ when it decides to install it. Its optimal decision then is to install the new money exchange point.

When $\pi \geq \pi_2$, both consumers hold foreign currency as long as there is at least one money exchange point available. Therefore, the financial sector makes $\Pi = 2\gamma y$ when the new exchange point is not installed, and $\Pi = 2\gamma y - F$ when it is installed. Then, the financial sector's optimal decision is not to install the new exchange point.

Figure 4.3 depicts the relationship between the inflation rate and the number of existing and new money exchange points.

Inflation, availability of exchange points and currency substitution   Suppose that inflation follows a stochastic process such that $p(\pi = \bar{\pi})$ is the probability

Figure 4.3: EXCHANGE POINTS AND THE INFLATION RATE
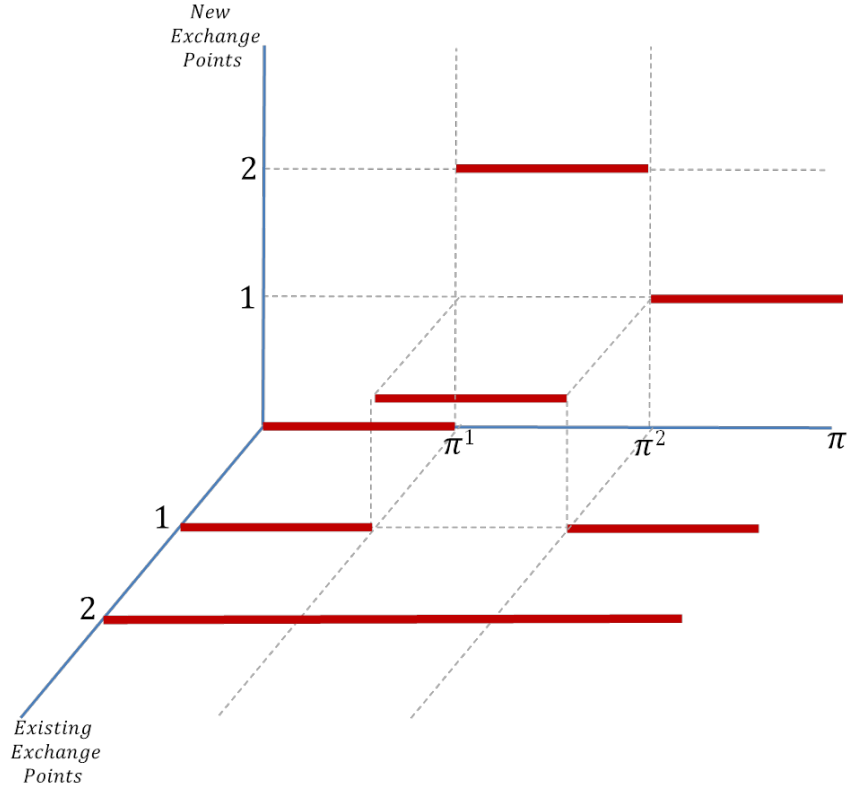
of inflation taking the value $\bar{\pi}$. Table 4.1 shows the probabilities governing the tran-
sition between exchange-technology states when each state is given by the number
of available money exchange points. From the matrix it can be seen that, if the
one period game solved above is played repeatedly, a sufficiently high number of
realizations in which $\pi \geq \pi_1$ will lead the economy to a point in which both agents
face the lowest possible cost in order to access the financial sector services (that is,
both agents are located at distance d from a money exchange point) and in which
both agents have the same substitution threshold for inflation (π_1).

Let *no substitution*, $CSR = 0$, *partial substitution*, $CSR = \frac{S_F M^*}{S_F M^* + M_I}$, and

complete substitution, $CSR = 1$, denote the three possible currency substitution

outcomes of the model. Table 4.2 shows the probability of each outcome under the different states given by the number of installed money exchange points. When the dollarization capital is at its maximum, $\xi = \{A, B\}$, the economy does not display any partial substitution, and currency holdings are either completely denominated in domestic currency (with probability $P(\pi < \pi_1)$) or fully dollarized (with probability $P(\pi \geq \pi_2)$).

Table 4.1: Inflation Probabilities and the State of Exchange Points

		Probability of new state		
		$\xi = \{\emptyset\}$	$\xi = \{A\}$ or $\xi = \{B\}$	$\xi = \{A, B\}$
Previous state	$\xi = \{\emptyset\}$	$P(\pi < \pi_1)$	$P(\pi \geq \pi_2)$	$P(\pi_1 \leq \pi < \pi_2)$
	$\xi = \{A\}$ or $\xi = \{B\}$	0	$P(\pi < \pi_1) + P(\pi \geq \pi_2)$	$P(\pi_1 \leq \pi < \pi_2)$
	$\xi = \{A, B\}$	0	0	1

Table 4.2: Inflation Probabilities and Currency Substitution

		Probability of		
		No substitution	Partial substitution	Complete substitution
Available Exchange Points	$\xi = \{\emptyset\}$	1	0	0
	$\xi = \{A\}$ or $\xi = \{B\}$	$P(\pi < \pi_1)$	$P(\pi_1 \leq \pi < \pi_2)$	$P(\pi \geq \pi_2)$
	$\xi = \{A, B\}$	$P(\pi < \pi_1)$	0	$P(\pi \geq \pi_1)$

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A forward-looking financial sector

Solving the financial sector's investment problem when it takes into account that consumers live infinitely and repeat their currency substitution problem each period is cumbersome and the different equilibria depend on the stream of inflation realizations. For simplicity here I present two cases, one when inflation is between π_1 and π_2 in every period and one when inflation is above π_2 in very period.

Case 1. $\pi_t = \pi^h \geq \pi_2$ **for every** t Suppose the economy starts with $\xi = \{\oslash\}$. In period 1, the financial sector has three options: invest in one new exchange point, invest in two new exchange points, and not to invest.

If the financial sector invests in two new exchange points at $t = 1$, then it makes $\Pi = 2(\gamma y - F)$ in the first period and $\Pi = 2\gamma y$ in every period after that. So the present discounted value of the financial sector profit stream is $\left(\frac{1}{1-\delta}\right) 2\gamma y - 2F$, where $\delta \in (0,1)$ is the financial sector's time discount factor. This value is greater than that obtained when the financial sector decides not to invest at $t = 1$ and instead installs both exchange points in the second period, $\left(\frac{\delta}{1-\delta}\right) 2\gamma y - \delta 2F$, and greater than that obtained when no investment is made in $t = \{1,2\}$ and both exchange points are installed in the third period, $\left(\frac{\delta^2}{1-\delta}\right) 2\gamma y - \delta^2 2F$ and so on.

If the financial sector invests in only one exchange point at t=1, then it makes $\Pi = 2\gamma y - F$ in the first period and $\Pi = 2\gamma y$ every period after that, so the present discounted value of its profit stream is $\left(\frac{1}{1-\delta}\right) 2\gamma y - F$, greater than those obtained if the investment is made in period two, $\left(\frac{\delta}{1-\delta}\right) 2\gamma y - \delta F$, or in period 3, $\left(\frac{\delta^2}{1-\delta}\right) 2\gamma y - \delta^2 F$, and so on.

The financial sector could also invest in the first exchange point at period $t = 1$, and in the second one at $t = 2$, making present discounted profits equal to $\left(\frac{1}{1-\delta}\right) 2\gamma y - F - \delta F$. However, postponing investing in the second exchange point for one period delivers a higher profit present discounted value, $\left(\frac{1}{1-\delta}\right) 2\gamma y - F - \delta^2 F$. Postponing investing in the second exchange point for two periods yields an even higher present discounted value, $\left(\frac{1}{1-\delta}\right) 2\gamma y - 2F$, and so on. Since inflation is above π_2 in every period, both consumers are willing to substitute and demand money exchange services from the financial sector who each period receives real fees of $2\gamma y$. This income stream is not changed by the installation of the second exchange point, so it not optimal to incur additional investment costs.

After considering all possible options, the financial sector's optimal investment decision when the inflation path is $\{\pi^h, \pi^h, \pi^h, ...\}$ is to install only one exchange point at $t = 1$.

Case 2. $\pi_t = \pi^m$, $\pi_1 < \pi^m \leq \pi_2$ **for every** t When the financial sector invests in two new exchange points at $t = 1$, it makes discounted profits $\left(\frac{1}{1-\delta}\right) 2\gamma y - 2F$, greater than those obtained if the two exchange points are simultaneously installed at any future period. If the financial sector invests in only one exchange point and does so at $t = 1$, the present discounted value of profits is $\left(\frac{1}{1-\delta}\right) \gamma y - F$, greater than that obtained if only one exchange point is installed in any period in the future.

If the financial sector invests in one exchange point in period $t = 1$ and in another in period $t = 2$, the present discounted value of profits is $\gamma y + \left(\frac{\delta}{1-\delta}\right) 2\gamma y - (1 + \delta)F$, greater than that obtained if the two exchange points are separately

installed at any two different periods in the future.

After considering all possible options, the financial sector's optimal investment decision when the inflation path is $\{\pi^m, \pi^m, \pi^m, ...\}$ is to install two exchange points in the first period.

**An example of the path of inflation and currency substitution over time**

After the analysis of the problems of the two consumers and the financial sector it is easy to think about some possible cases in which the co-movement between currency substitution and inflation will not be the same before and after a period of high inflation.

Think for example of a case in which before period T inflation remains at $\pi^l < \pi_1$, then it increases to $\pi^h \geq \pi_2$ at $t = T$, and decreases to $\pi^m \in [\pi_1, \pi_2)$ from $t = T + 1$ onwards. Assume also that there are no exchange points installed before $t = T$.

Using the analysis described in the previous section, it can be shown that the financial sector's optimal investment decision when the inflation path is $\{..., \pi_{T-1} = \pi^l, \pi_T = \pi^h, \pi_{T+1} = \pi^m, \pi_{T+2} = \pi^m, ...\}$ is not to invest before $t = T$, invest in one exchange point at $t = T$ and invest in the second one at $t = T+1$. The second panel of Figure 4.4 shows the time path of economy's dollarization capital (the optimal number of available exchange points).

The third panel of the figure depicts the time path of currency substitution associated with the given pattern of inflation. Before $t = T$, the currency substitution ratio is zero in every period; at $t = T$, the financial sector installs one exchange

Figure 4.4: PATHS OF INFLATION AND CURRENCY SUBSTITUTION
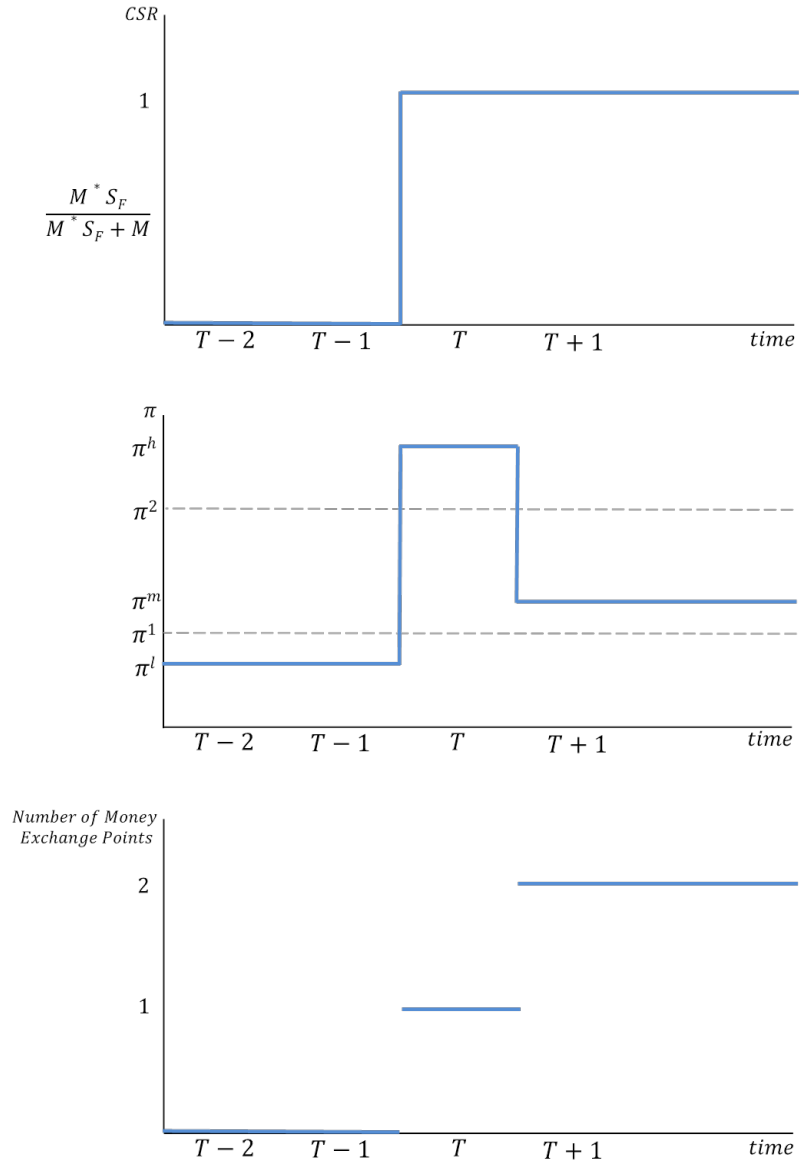
point and both consumers are willing to use its services, so all money balances in the economy are dollarized and the currency substitution ratio is 1. Finally, since a second exchange point is installed at $t = T + 1$, both consumers are at distance d from an exchange point, and an inflation rate lower than π^h but greater or equal than π_1 is enough to guarantee that the currency substitution ratio remains at 1.

The main message of the simple framework presented so far is that, in the

presence of fixed costs associated with the investment decisions of the financial sector, an episode of high inflation may push the financial sector into action in terms of expanding the economy's dollarization capital, therefore reducing (maybe permanently) the foreign currency usage costs faced by consumers and decreasing the inflation threshold at which households are willing to substitute domestic for foreign currency.

4.3   Conclusion

The framework presented here constitutes a step forward in developing a micro foundation for the presence of network effects and dollarization capital dynamics in models of currency substitution. The dollarization capital variable corresponds to physical capital used for the provision of money exchange services, and the network of foreign currency users is composed of the financial sector and those consumers who have access to the financial sector's money exchange services. Whether a consumer belongs or not to the network depends on endogenously determined inflation thresholds for currency substitution and on the endogenously determined optimal level of dollarization capital.

The combination of fixed costs associated with investment in money exchange services and consumer heterogeneity in terms of access to financial services provides an environment in which the relationship between currency substitution and inflation is not linear. Particularly, the model is able to deliver asymmetries in the relationship between inflation and currency substitution before and after high infla-

tion episodes. Starting from a level in which there are no exchange services available in the economy, a period of high inflation followed by one of moderate inflation will push the financial sector to expand the economy's dollarization capital, permanently reducing the consumer's cost of using foreign currency and decreasing the inflation thresholds at which they are willing to substitute foreign for domestic currency.

A natural step for future research is to embed the mechanism explored here into a fully-specified DSGE environment, providing a tool for a more comprehensive empirical analysis of the currency substitution phenomenon in Developing Countries and Transition Economies.

Appendix A

Appendix to Chapter 1

A.1 Calvo stickiness in regular prices

Optimal regular price and price aggregators

When the regular price is sticky, firm j's pricing decision problem is

$$
\max_{p^r_{jt}, p^p_{jt}, f_{jt}} E_t \sum_{s=t}^{\infty} \rho^{s-t} \Xi_{t+s/t} \{ p^p_{js} h \theta^h(f_{js}, s^h_s) \left(\frac{P^h_s}{p^p_{js}} \right)^{\varepsilon} c^h_s + p^p_{js}(1-h)\theta^l(f_{js}, s^l_s) \left(\frac{P^l_s}{p^p_{js}} \right)^{\eta} c^l_s
$$

$$
+ p^{r*}_{jt} h \left[1 - \theta^h(f_{js}, s^h_s) \right] \left(\frac{P^h_s}{p^{r*}_{jt}} \right)^{\varepsilon} c^h_s + p^{r*}_{jt}(1-h)\left[1 - \theta^l(f_{js}, s^l_s) \right] \left(\frac{P^l_s}{p^{r*}_{jt}} \right)^{\eta} c^l_s \}
$$

$$
- mc_{js} P_s \{ h \theta^h(f_{js}, s^h_s) \left(\frac{P^h_s}{p^p_{js}} \right)^{\varepsilon} c^h_s + (1-h)\theta^l(f_{js}, s^l_s) \left(\frac{P^l_s}{p^p_{js}} \right)^{\eta} c^l_s
$$

$$
+ h \left[1 - \theta^h(f_{js}, s^h_s) \right] \left(\frac{P^h_s}{p^{r*}_{jt}} \right)^{\varepsilon} c^h_s + (1-h)\left[1 - \theta^l(f_{js}, s^l_s) \right] \left(\frac{P^l_s}{p^{r*}_{jt}} \right)^{\eta} c^l_s \} \tag{A.1}
$$

Eliminating infinite sums in the regular price optimality condition

Rearranging the first order condition for p_t^{r*}:

$$p_{jt}^{r*}\left\{(\varepsilon-1)E_t\sum_{t=s}^{\infty}\rho^{s-t}\Xi_{t+s/t}h\left[1-\theta^h(f_{js},s_{js}^h)\right]c_{js}^{hr}\right\}$$

$$+p_{jt}^{r*}\left\{(\eta-1)E_t\sum_{t=s}^{\infty}\rho^{s-t}\Xi_{t+s/t}(1-h)\left[1-\theta^l(f_{js},s_{js}^l)\right]c_{js}^{lr}\right\}=$$

$$\varepsilon E_t\sum_{t=s}^{\infty}\rho^{s-t}\Xi_{t+s/t}h\left[1-\theta^h(f_{js},s_{js}^h)\right]c_{js}^{hr}mc_{js}P_s$$

$$+\eta E_t\sum_{t=s}^{\infty}\rho^{s-t}\Xi_{t+s/t}(1-h)\left[1-\theta^l(f_{js},s_{js}^l)\right]c_{js}^{lr}mc_{js}P_s \tag{A.2}$$

The numerator and denominator terms in the right hand side of (A.2) have current and forward looking variables. Following Schmitt-Grohe and Uribe (2004b,2005) this expression can be rewritten in a recursive fashion by defining four auxiliary variables: x_t^a, x_t^b, x_t^c and x_t^d.

Working with the four terms in (A.2) , define

$$P_tx_t^a=p_{jt}^{r*}(\varepsilon-1)E_t\sum_{t=s}^{\infty}\rho^{s-t}\Xi_{t+s/t}h\left[1-\theta^h(f_{js},s_{js}^h)\right]\left(\frac{p_{jt}^{r*}}{P_s^h}\right)^{-\varepsilon}c_t^h \tag{A.3}$$

$$P_tx_t^b=p_{jt}^{r*}(\eta-1)E_t\sum_{t=s}^{\infty}\rho^{s-t}\Xi_{t+s/t}(1-h)\left[1-\theta^l(f_{js},s_{js}^l)\right]\left(\frac{p_{jt}^{r*}}{P_s^l}\right)^{-\eta}c_t^l \tag{A.4}$$

$$P_tx_t^c=\varepsilon E_t\sum_{t=s}^{\infty}\rho^{s-t}\Xi_{t+s/t}h\left[1-\theta^h(f_{js},s_{js}^h)\right]\left(\frac{p_{jt}^{r*}}{P_s^h}\right)^{-\varepsilon}c_t^hP_smc_s \tag{A.5}$$

$$P_tx_t^d=\eta E_t\sum_{t=s}^{\infty}\rho^{s-t}\Xi_{t+s/t}(1-h)\left[1-\theta^l(f_{js},s_{js}^l)\right]\left(\frac{p_{jt}^{r*}}{P_s^l}\right)^{-\eta}c_t^lP_smc_s \tag{A.6}$$

Where

- $P_t x_t^a$ is the present discounted nominal marginal revenue of serving the HEC's demand at regular prices until the next price change.

- $P_t x_t^b$ is the present discounted nominal marginal revenue of serving the LEC's demand at regular prices until the next price change.

- $P_t x_t^c$ is the present discounted nominal marginal cost of serving the HEC's demand at regular prices until the next price change.

- $P_t x_t^d$ is the present discounted nominal marginal cost of serving the LEC's demand at regular prices until the next price change.

After some algebra it can be shown that

$$x_t^a = \left(\frac{p_{jt}^r}{P_t^h}\right)^{-\varepsilon} h \left[1 - \theta^h(f_{jt}, s_{jt}^h)\right] c_t^h(\varepsilon - 1)\frac{p_{jt}^{r*}}{P_t} + E_t \rho \Xi_{t+1/t} \frac{p_t^{r*}}{p_{t+1}^{r*}} \frac{P_{t+1}}{P_t} x_{t+1}^a \qquad (A.7)$$

$$x_t^b = \left(\frac{p_{jt}^r}{P_t^l}\right)^{-\eta} (1 - h) \left[1 - \theta^h(f_{jt}, s_{jt}^h)\right] c_t^l(\eta - 1)\frac{p_{jt}^{r*}}{P_t} + E_t \rho \Xi_{t+1/t} \frac{p_t^{r*}}{p_{t+1}^{r*}} \frac{P_{t+1}}{P_t} x_{t+1}^b \qquad (A.8)$$

$$x_t^c = \left(\frac{p_{jt}^r}{P_t^h}\right)^{-\varepsilon} h \left[1 - \theta^h(f_{jt}, s_{jt}^h)\right] c_t^h \varepsilon mc_t + E_t \rho \Xi_{t+1/t} \frac{P_{t+1}}{P_t} x_{t+1}^c \qquad (A.9)$$

$$x_t^d = \left(\frac{p_{jt}^r}{P_t^l}\right)^{-\eta} (1 - h) \left[1 - \theta^h(f_{jt}, s_{jt}^h)\right] c_t^l \eta mc_t + E_t \rho \Xi_{t+1/t} \frac{P_{t+1}}{P_t} x_{t+1}^d \qquad (A.10)$$

Then, (A.2) can be expressed as

$$x_t^c + x_t^d = x_t^a + x_t^b \qquad (A.11)$$

Now the first order condition for regular prices can be expressed in a recursive framework and the firm's optimal pricing conditions are given by equations (2.18), (2.19) and (A.12) , and the expressions for P_t^h and P_t^l are given by

$$x_t^c + x_t^d = x_t^a + x_t^b \qquad \text{(A.12)}$$

$$P_t^h = \left[\theta^h(f, s_t^h)(p_t^p)^{1-\varepsilon} + \rho \left[1 - \theta^h(f, s_t^h) \right] (\overline{p_{t-1}^r})^{1-\varepsilon} + (1-\rho) \left[1 - \theta^h(f, s_t^h) \right] (p_t^{r*})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \qquad \text{(A.13)}$$

$$P_t^l = \left[\theta^l(f, s_t^l)(p_t^p)^{1-\eta} + \rho \left[1 - \theta^l(f, s_t^l) \right] (\overline{p_{t-1}^r})^{1-\eta} + (1-\rho) \left[1 - \theta^l(f, s_t^l) \right] (p_t^{r*})^{1-\eta} \right]^{\frac{1}{1-\eta}} \qquad \text{(A.14)}$$

where $\overline{p_t^r}$ is the average regular price across firms

$$\overline{p_t^r} = \int_0^1 \left(\frac{p_t^r}{P_t^l} \right) dj \qquad \text{(A.15)}$$

and can be expressed recursively as

$$\overline{p_t^r} = \rho \overline{p_{t-1}^r} + (1-\rho) p_t^{r*}$$

Othe equilibrium conditions

Since not all firms are able to set their price optimally every period, symmetry cannot be applied in the same fashion as in the flexible price case, and several equations of the model need to be modified.

The resource constraint and economy wide price equations are now expressed

as

$$e_t^z \left[h n_t^h + (1-h) n_t^l \right] = h\theta^h(f_t, s_t^h) c_{jt}^{hp} + h \left[1 - \theta^h(f_t, s_t^h) \right] d_t^{h1} c_t^h$$

$$+ (1-h)\theta^l(f_t, s_t^l) c_{jt}^{lp} + (1-h) \left[1 - \theta^l(f_t, s_t^l) \right] d_t^{l1} c_t^l \quad (A.16)$$

$$P_t = \frac{h\theta^h(f_t, s_t^h) c_{jt}^{hp} + (1-h)\theta^l(f_t, s_t^l) c_{jt}^{lp}}{c_t} p_t^p$$

$$+ \frac{h \left[1 - \theta^h(f_t, s_t^h) \right] d_t^{h2}}{c_t} P_t^h$$

$$+ \frac{(1-h) \left[1 - \theta^l(f_t, s_t^l) \right] d_t^{l2}}{c_t} P_t^l \quad (A.17)$$

where

$$c_t = h\theta^h(f_t, s_t^h) c_{jt}^{hp} + h \left[1 - \theta^h(f_t, s_t^h) \right] d_t^{h1} c_t^h + (1-h)\theta^l(f_t, s_t^l) c_{jt}^{lp} + (1-h) \left[1 - \theta^l(f_t, s_t^l) \right] d_t^{l1} c_t^l$$

The consumption aggregators are given by

$$c_t^h = \left(\frac{\theta^h(f, s_t^h) c_t^{hp \frac{\varepsilon-1}{\varepsilon}}}{1 - [1 - \theta_t^h(f, s_t^h)] d_t^{h2}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (A.18)$$

$$c_t^l = \left(\frac{\theta^l(f, s_t^l) c_t^{lp \frac{\eta-1}{\eta}}}{1 - [1 - \theta_t^l(f, s_t^l)] d_t^{l2}} \right)^{\frac{\eta}{\eta-1}} \quad (A.19)$$

And the s_t^h and s_t^l first order conditions are

$$v'(1 - n_t^h - s_t^h) = \left(\frac{\varepsilon}{\varepsilon - 1} \right) c_t^{h \frac{1}{\varepsilon}} \theta_s^h(f, s_t^h) \left(c_{\theta^h t}^{hp \frac{\varepsilon - 1}{\varepsilon}} - d_t^{h2} c_t^{h \frac{\varepsilon - 1}{\varepsilon}} \right) u'(c_t^h)$$
$$- \theta_s^h(f, s_t^h) \left(\frac{p_{\theta^h t}^p c_{\theta^h t}^{hp}}{P_t^h} - d_t^{h2} c_t^h \right) u'(c_t^h) \tag{A.20}$$

$$v'(1 - n_t^l - s_t^l) = \left(\frac{\eta}{\eta - 1} \right) c_t^{l \frac{1}{\eta}} \theta_s^l(f, s_t^l) \left(c_{\theta^l t}^{lp \frac{\eta - 1}{\eta}} - d_t^{l2} c_t^{l \frac{\eta - 1}{\eta}} \right) u'(c_t^h)$$
$$- \theta_s^l(f, s_t^l) \left(\frac{p_{\theta^l t}^p c_{\theta^l t}^{hp}}{P_t^l} - d_t^{l2} c_t^l \right) u'(c_t^l) \tag{A.21}$$

Where

$$d_t^{h1} = \int_0^1 \left(\frac{p_t^r}{P_t^h} \right)^{-\varepsilon} dj$$

$$d_t^{l1} = \int_0^1 \left(\frac{p_t^r}{P_t^l} \right)^{-\eta} dj$$

$$d_t^{h2} = \int_0^1 \left(\frac{p_t^r}{P_t^h} \right)^{1-\varepsilon} dj$$

$$d_t^{l2} = \int_0^1 \left(\frac{p_t^r}{P_t^l} \right)^{1-\eta} dj$$

And dh1, d_t^{h1}, d_t^{h2}, d_t^{l1} and d_t^{l2} can be expressed recursively as

$$d_t^{h1} = \rho \left(\frac{1}{e^{\pi_t^h}} \right)^{-\varepsilon} d_{t-1}^{h1} + (1 - \rho) \left(\frac{p_t^{r*}}{P_t^h} \right)^{-\varepsilon} \tag{A.22}$$

$$d_t^{l1} = \rho \left(\frac{1}{e^{\pi_t^l}} \right)^{-\eta} d_{t-1}^{l1} + (1 - \rho) \left(\frac{p_t^{r*}}{P_t^l} \right)^{-\eta} \tag{A.23}$$

$$d_t^{h2} = \rho \left(\frac{1}{e^{\pi_t^h}} \right)^{1-\varepsilon} d_{t-1}^{h2} + (1 - \rho) \left(\frac{p_t^{r*}}{P_t^h} \right)^{1-\varepsilon} \tag{A.24}$$

$$d_t^{l2} = \rho \left(\frac{1}{e^{\pi_t^l}} \right)^{1-\eta} d_{t-1}^{l2} + (1-\rho) \left(\frac{p_t^{r*}}{P_t^l} \right)^{1-\eta} \qquad (A.25)$$

In order to reduce the size of the state space, the model is solved around a zero-inflation steady state, ignoring the evolution of d_t^{h1}, d_t^{h2}, d_t^{l1} and d_t^{l2}.

In order to reduce the size of the state space, the model is solved around a zero-inflation steady state, ignoring the evolution of d_t^{h1}, d_t^{h2}, d_t^{l1} and d_t^{l2}.

A.2   Looking for f in the data

The James M. Kilts Center at the University of Chicago Booth School of Business has made available several datasets on weekly store-level transaction prices for over 100 stores operated by Dominick's Finer Foods (DFF) for the $1989 - 1997$ period.

The customer count file includes information about in-store traffic. The data is store specific and on a daily basis. The customer count data refers to the number of customers visiting the store and purchasing something. Also in the customer count file is a total dollar sales and total coupons redeemed figure, by DFF defined department (product category). The figures are compiled daily from the register/scanner receipts.[1]

The average fraction of time at which a good is offered on coupon promotions is calculated through the following steps:

1. Aggregate the coupon redemption figure by store and by week (all stores with less than 52 time observations were dropped).

---

[1]The data is available at http://research.chicagobooth.edu/marketing/databases/dominicks/ccount.aspx

2. Construct the following dummy variable for each product category:

$$d_t = \begin{cases} 1 & \text{if coupon redemption} \neq 0 \\ 0 & \text{if coupon redemption} = 0 \end{cases}$$

Then, the underlying assumption is that if a promotion is available in a given week, then there is at least one person that takes advantage of it (alternatively, at least one coupon is redeemed). Therefore, it is assumed that if the coupon redemption figure in a given week is zero ($d_t = 0$), then no coupons were offered that week.

3. Compute $\widehat{f_k} = \sum_t d_t / (number\ of\ weeks)$, the fraction of time that good k was offered on promotion.

4. Compute $\overline{f} = \sum_k \widehat{f_k} / (number\ of\ good\ categories)$, the average fraction of time at which goods are offered on promotion.

If the underlying assumption in step 2 is wrong, then $\overline{f}$ is a lower bound for f.

Table A1 shows the results for all the product categories for which coupons were offered at least once in the sample time range. Table A2 presents the mean value across products and stores.

Table A.1: Fraction of Weeks by Department

Fraction of weeks that goods are offered on promotion, mean across stores (by DFF department)										
Grocery	Meat	Fish	Produce	Bulk	Salad Bar	Floral	Deli	Pharmacy	General Merchandise	Video
0.998	0.724	0.162	0.688	0.086	0.283	0.410	0.912	0.594	0.938	0.323

Table A3 presents the mean value across products and stores when including all product categories.

Table A.2: Fraction of Weeks: Sample I

Sample I: Product Categories for Which Coupons are Offered

Fraction of weeks that goods are offered on promotion, mean across products and stores

		Sales-weighted mean	
Unweighted mean across products and stores	Simple average across stores	Weighting stores by size using customer traffic	Weighting stores by size using total sales
0.56	0.88	0.86	0.87

Table A.3: Fraction of Weeks: Sample II

Sample II: All Product Categories

Fraction of weeks that goods are offered on promotion, mean across products and stores

		Sales-weighted mean	
Unweighted mean across products and stores	Simple average across stores	Weighting stores by size using customer traffic	Weighting stores by size using total sales
0.31	0.66	0.65	0.65

A.3 The Firm's Problem Under Perfect Price Discrimination (PPD)

For simplicity, here I present the case of flexible prices.

If the firm could distinguish between consumer types, its profit maximization problem would be:

$$
\max_{p_{jt}^{h,PPD},p_{jt}^{l,PPD}} \Pi_{maxjt} = h p_{jt}^{h,PPD} c_{jt}^{h,PPD} + (1-h) p_{jt}^{l,PPD} c_{jt}^{l,PPD}
$$
$$
- mc_{jt} P_t \{ h c_{jt}^{h,PPD} + (1-h) c_{jt}^{l,PPD} \} \qquad \text{(A.26)}
$$

subject to

$$
c_{jt}^{h,PPD} = \left(\frac{p_{jt}^{h,PPD}}{P_t^{h,PPD}} \right)^{-\varepsilon} c_t^{h,PPD} \qquad \text{(A.27)}
$$

$$c_{jt}^{l,PPD} = \left(\frac{p_{jt}^{l,PPD}}{P_t^{l,PPD}} \right)^{-\varepsilon} c_t^{l,PPD} \qquad (A.28)$$

where $c_{jt}^{k,PPD}$, $k = h, l$ are quantities consumed by consumers under perfect price discrimination and $p_{jt}^{k,PPD}$, $k = h, l$ are the prices charged by the firm.

Solving the problem, the firm's optimal price choices are:

$$\frac{p_t^{h,PPD}}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) mc_t \qquad (A.29)$$

$$\frac{p_t^{l,PPD}}{P_t} = \left(\frac{\eta}{\eta - 1} \right) mc_t \qquad (A.30)$$

These same optimal prices can be recovered from the firms problem with price promotions with $s_t^h = 1$ and $s_t^l = 0$ when the firm chooses $f_t = 1$. To see this, consider the optimality conditions from the firm's problem in the model with price promotions solved in Section 2.1.2 are:

$$h \left[1 - \theta^h(f_{jt}, s_t^h) \right] c_{jt}^{hr} + (1 - h) \left[1 - \theta^l(f_{jt}, s_t^l) \right] c_{jt}^{lr} =$$
$$\left(1 - \frac{P_t}{p_{jt}^r} mc_t \right) \left\{ h[1 - \theta^h(f_{jt}, s_t^h)] \varepsilon c_{jt}^{hr} + (1 - h)[1 - \theta^l(f_{jt}, s_t^l)] \eta c_{jt}^{lr} \right\} \qquad (A.31)$$

$$h\theta^h(f_{jt}, s_t^h) c_{jt}^{hp} + (1 - h)\theta^l(f_{jt}, s_t^l) c_{jt}^{lp} =$$
$$\left(1 - \frac{P_t}{p_{jt}^p} mc_t \right) \left[h\theta^h(f_{jt}, s_t^h) \varepsilon c_{jt}^{hp} + (1 - h)\theta^l(f_{jt}, s_t^l) \eta c_{jt}^{lp} \right] \qquad (A.32)$$

$$\left(p_{jt}^p - mc_t P_t\right) \left[h\theta_f^h(f_{jt}, s_t^h)c_{jt}^{hp} + (1-h)\theta_f^l(f_{jt}, s_t^l)c_{jt}^{lp} \right] =$$

$$\left(p_{jt}^r - mc_t P_t\right) \left[h\theta_f^h(f_{jt}, s_t^h)c_{jt}^{hr} + (1-h)\theta_f^l(f_{jt}, s_t^l)c_{jt}^{lr} \right] \tag{A.33}$$

when $s_t^h = 1$ and $s_t^l = 0$, these collapse to

$$h(1 - f_{jt})c_{jt}^{hr} + (1-h)c_{jt}^{lr} =$$

$$\left(1 - \frac{P_t}{p_{jt}^r}mc_t\right)\left\{ h(1 - f_{jt})\varepsilon c_{jt}^{hr} + (1-h)\eta c_{jt}^{lr} \right\} \tag{A.34}$$

$$c_{jt}^{hp} = \left(1 - \frac{P_t}{p_{jt}^p}mc_t\right)\varepsilon c_{jt}^{hp} \tag{A.35}$$

$$\left(p_{jt}^p - mc_t P_t\right) h f_{jt}c_{jt}^{hp} = \left(p_{jt}^r - mc_t P_t\right) h f_{jt}c_{jt}^{hr} \tag{A.36}$$
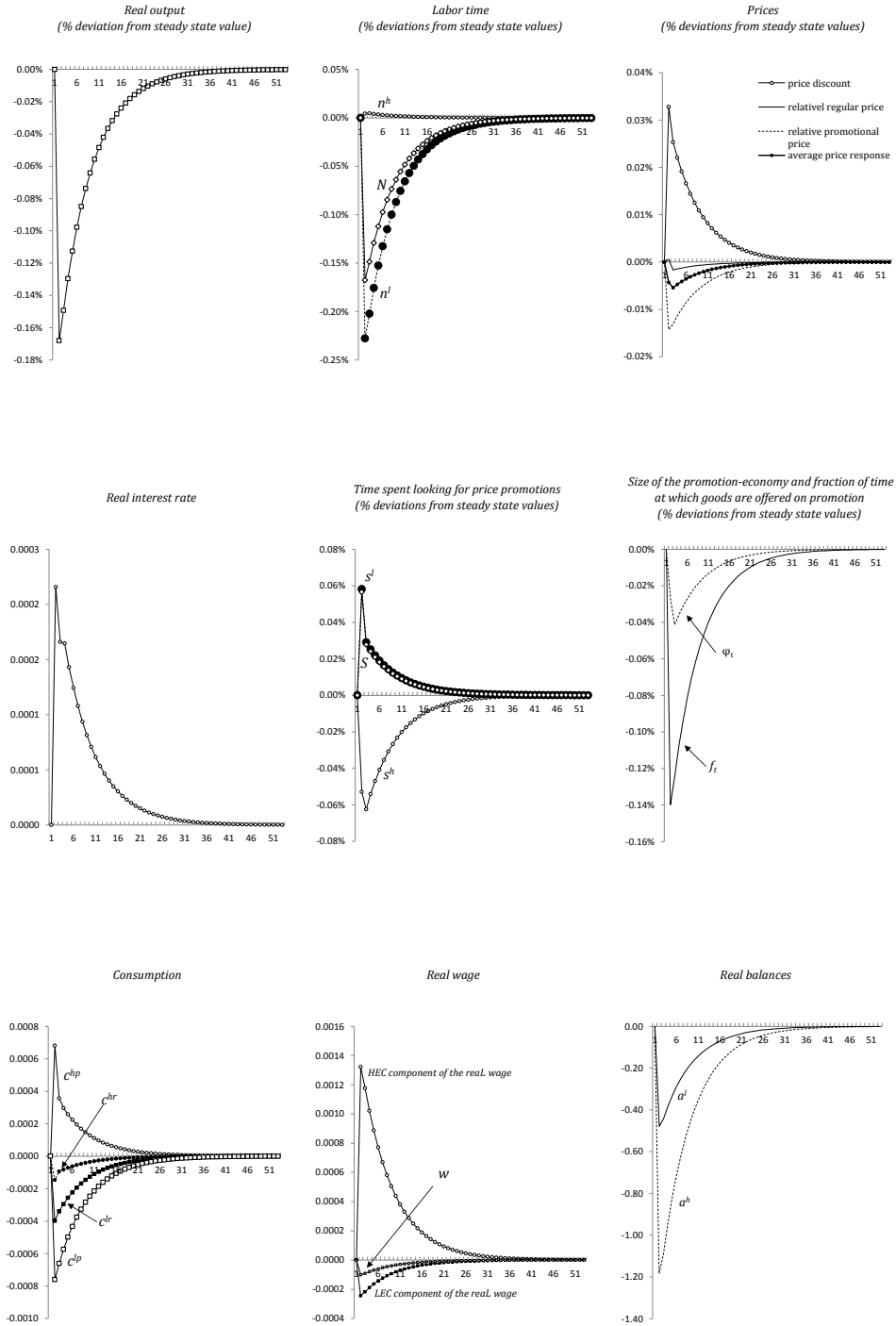
from (A.36) $p_{jt}^r > p_{jt}^p$ and rearranging (A.35)

$$\frac{p_t^p}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right)mc_t = \frac{p_t^{h,PPD}}{P_t} \tag{A.37}$$

Finally, when the firm sets $f_t = 1$ and rearranging (A.34)

$$\frac{p_t^r}{P_t} = \left(\frac{\eta}{\eta - 1}\right)mc_t = \frac{p_t^{l,PPD}}{P_t} \tag{A.38}$$

A.4 Monetary Model with Price Promotions: Impulse Response Functions

Real output
(% deviation from steady state value)

Labor time
(% deviations from steady state values)

Prices
(% deviations from steady state values)

Real interest rate

Time spent looking for price promotions
(% deviations from steady state values)

Size of the promotion-economy and fraction of time at which goods are offered on promotion
(% deviations from steady state values)

Consumption

Real wage

Real balances

Appendix B

Appendix to Chapter 2

Table B.1: Data Series Used in the Document

Series	Country	Source	Data Span
Nominal GDP	Bolivia	BNIS	1980-2008
Nominal GDP	Peru	CRBP	1979-2008
Private Consumption	Bolivia	BNIS	1980-2008
Private Consumption	Peru	IMF-IFS	1979-2008
Foreign Currency Deposits	Bolivia	BCB	1980-2008
Foreign Currency Deposits	Peru	CRBP	1979-2008
External Positions of BIS Reporting Banks (liabilities)	Bolivia	BIS	1980-2008
External Positions of BIS Reporting Banks (liabilities)	Peru	BIS	1979-2008
Money Aggregates	Bolivia	BCB	1980-2008
Money Aggregates	Peru	CRBP	1979-2008
Consumer Price Index	Bolivia	BCB	1979-2008
Consumer Price Index	Peru	CRBP	1978-2008

BNIS: Bolivian National Institute of Statistics

BIS: Bank of International Settlements

CBC: Central Bank of Bolivia

CRBP: Central Reserve Bank of Peru

IMF-IFS: International Monetary Fund, International Financial Statistics

Table B.2: Money Aggregates

Bolivia	
M1	Currency in circulation plus sight deposits denominated in national currency
M1'	M1 plus sight deposits denominated in foreign currency deposits
M2	M1 plus saving deposits denominated in national currency
M2'	M1' plus saving deposits denominated in foreign currency
M3	M2 plus time deposits and other assets denominated in national currency
M3'	M2' plus time deposits and other assets denominated in foreign currency
Peru	
Money	Currency in circulation plus sight deposits denominated in national currency
Quasi money in domestic currency	Saving and time deposits and other assets denominated in domestic currency
Quasi money in foreign currency	Saving and time deposits and other assets denominated in foreign currency

Table B.3: Inflation Periods

Bolivia	*Years*	*Annual inflation (period average)*
Pre-hyperinflation	1980-1981	24,5%
Hyperinflation	1982-1985	2743%
Post-hyperinflation	1986-2008	11,3%
Peru	*Years*	*Annual inflation (period average)*
Pre hyperinflation	1979-1987	93,9%
Hyperinflation	1988-1991	4049%
Post hyperinflation	1992-2008	17,4%

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