ABSTRACT

Title: **DYNAMIC RIDESHARE OPTIMIZED MATCHING PROBLEM**

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This dissertation develops a Dynamic Rideshare Optimized Matching (DROM) model and solution that is aimed at identifying suitable matches between passengers requesting rideshare services with appropriate drivers available to carpool for credits and HOV lane privileges. DROM receives passengers and drivers' information and preferences continuously over time and maximizes the overall system performance subject to ride availability, capacity, rider and driver time window constraints, and detour and relocation distances while considering users' preferences. The research develops a spatial, temporal, and hierarchical decomposition solution strategy that leads to the heuristic solution procedure. Three-Spherical Heuristic Decomposition Model (TSHDM). Quality and validity tests for the TSHDM algorithm are done by comparison of results between the exact and implemented algorithm solutions and major sensitivity analyses using the technique of Regression Analysis on all of the related parameters in the model are conducted to thoroughly investigate the properties of the proposed model and solution algorithm. A case study is constructed to analyze the model and TSHDM behaviors on a road network of northwest metropolitan area of Baltimore city. The study shows that however DROM is a very complicated and challenging problem from both mathematical

formulation and solution algorithm perspectives, it is possible to implement a dynamic rideshare system using appropriate technical tools and social networking media. Major sensitivity analysis conducted on several parameters and variables affecting the model shows that most influencing factors for the rate of success in the rideshare system are, in order of importance: number of participating drivers, number of stops, area size, and number of participating riders. The study also shows rate of success for the rideshare system is highly dependent to the matched routes connecting directly points of origin and destination for participating riders and also increasing the number of connections from one to two which requires two consecutive change of rides for a rider has the least impact on the rate of success.

DYNAMIC RIDESHARE OPTIMIZED MATCHING PROBLEM

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To Azadeh and Jiwan

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I dedicate this dissertation to my wife Azadeh - whose love, patience, understanding and one decade devotion buoy me in any storm - and to Jiwan, my greatest accomplishment.

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Chapter 1: Introduction

1.1. Motivation

The price of oil again began rising in February 2009 and it reached the two-year record level of $117.12 a barrel in May 2011 (Figure 1). The analysts predict that oil prices will continue to rise and consumers' budgets will be more under pressure. The world economy is running fast out of the cheap oil that has powered the economic development since the 1950s [Londarev and Baláž, 2005]. The problems related to traffic congestion and environmental pollutions in big cities are increasing [Slack et al., 2006].

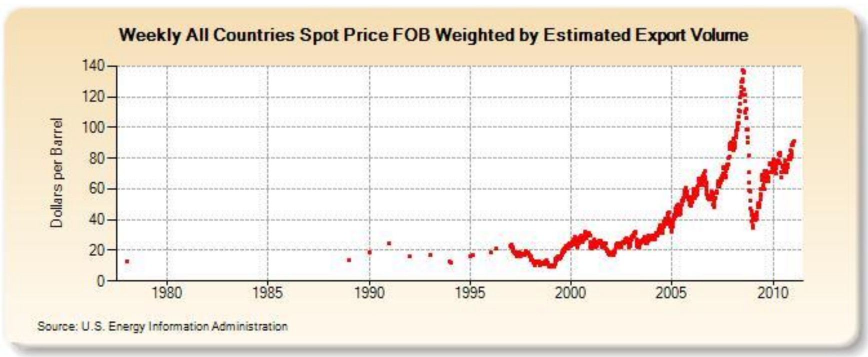


Figure 1: Weekly all countries oil price FOB weighted
Source: U.S. Energy Information Administration

In the face of the increasing price of transportation fuel cost and the worsening effects of traffic congestion and environmental pollutions, wise usage of personal automobiles are gaining more attraction. Rideshare is a solution for car travel reduction aiming to bring together travelers with similar itineraries and time schedules. Ridesharing has generated much interest in recent years with media coverage (the Wall Street Journal [Saranow, 2006], Time [Sayre, 2006], Newsweek [Levy, 2007], Business Week [Walters, 2007], ABC News [Bell, 2007], The NY Times [Wiedenkeller, 2008], USA Today [Jesdanun, 2008], and NBC4 News [McPeek, 2011], among many others.). Mean occupancy rates of personal vehicle trips (the average number of travelers per vehicle trip) in the united states is 1.6 persons per vehicle mile (Table 1) ranging from 1.14 for work-related trips to 2.05 for social or recreational trips and weekday trips have a weighted (by Miles travelled in trip) occupancy of 1.5 compared to 2 people per vehicle mile on weekend trips [BTS, 2001].

Table 1: Vehicle Occupancy per Vehicle Mile by Daily Trip Purpose

	Mean	SE
All personal vehicle trips	1.63	0.012
Work	1.14	0.007
Work-related	1.22	0.020
Family/personal	1.81	0.016
Church/school	1.76	0.084
Social/recreational	2.05	0.028
Other	2.02	0.130

NOTE: SE = standard error.

SOURCE: The 2001 National Household Travel Survey, daily trip file, U.S. Department of Transportation.

The large travel demand for personal car transportation together with low occupancies leads to traffic congestion that is an increasingly important issue in many urban areas with rapid population and economic growth. Congestion has gotten worse in regions of all sizes in the United States. In 2007, congestion caused urban Americans to travel 4.2 billion hours more and to purchase an extra 2.8 billion gallons of fuel for a congestion cost of $87.2 billion which is an increase of more than 50% over the previous decade (Table 2). This was a decrease of 40 million hours and a decrease of 40 million gallons, but an increase of over $100 million from 2006 due to an increase in the cost of fuel and truck delay [UMR 2009]. An effective ridesharing system that encourages the travelers to share their personal car could be an effective countermeasure against traffic congestion with reducing personal car travel demand.

In the United States more than 87% of commuters travel in private vehicles which accounts for a daily sum of 166 million Miles and single occupancy vehicles make up a big portion (77%) of the travels (Table 3), resulting in inefficient use of the transportation infrastructure [CIAIII, 2006] and giving a big opportunity for developing a rideshare system.

Table 2: Major Findings for 2009 (The Important Numbers for the 439 U.S. Urban Areas)

Measures of ……	**1982**	**1997**	**2006**	**2007**
… Individual Traveler Congestion				
Annual delay per peak traveler (hours)	14	32	37	36
Travel Time Index	1.09	1.2	1.25	1.25
"Wasted" fuel per peak traveler (gallons)	9	21	25	24
Congestion Cost (constant 2007 dollars)	$290	$621	$758	$757
Urban areas with 40+ hours of delay peak traveler	1	10	27	23
…The Nation's Congestion Problem				
Travel delay(billion hours)	0.79	2.72	4.2	4.16
"Wasted" fuel per peak traveler (gallons)	0.5	1.82	2.85	2.81
Congestion Cost (constant 2007 dollars)	$16.70	$53.60	$87.10	$87.20
… Travel Needs Served				
Daily travel on major roads (billion vehicle-Miles)	1.68	2.93	3.79	3.82
Annual Public transportation travel(billion person-Miles)	38.8	42.6	53.4	55.8
…Expansion Needed to to Keep Today's Congestion Level				
Lane-Miles of freeways and major streets added every year	15,500	16,532	15,032	12,676
Public transportation riders added every year(million)	3,456	3,876	3,779	3,129
… The Effect of Some Solutions				
Travel delay saved by				
Operational treatments (million hours)	7	116	307	308
Public transportation (milliion hours)	290	455	622	646
Congestion costs saved by				
Operational treatments (billions of 2007 dollars)	$0.02	$2.30	$6.40	$6.50
Public transportation (billions of 2007 dollars)	$6.30	$9.30	$13.10	$13.70

Travel Time Index (TTI): The ratio of travel time in the peak period to travel time at free-flow conditions. Peak Traveler: The extra time spent traveling at congested speeds rather than free-flow speeds divided by the number of persons making a trip during the peak period. Wasted Fuel: Extra fuel consumed during congested travel. Vehicle-Miles: Total of all vehicle travel. Expansion Needed: Either lane-Miles or annual riders to keep pace with travel growth and maintain congestion.
Source: 2009 Urban Mobility Report, Texas Transportation Institute, the Texas A&M University System, July 2009

Single occupancy vehicles commute a daily sum of 127 million Miles [CIAIII, 2006]. Composite national average driving cost per mile is 54.1 cents including average fuel, routine maintenance, tires, insurance, license and registration, loan finance charges and depreciation costs [AAA 2008]. Table 4 presents a more detailed breakdown by Miles driven and vehicle type. Therefore, a successful ridesharing program that increases the occupancy of vehicles may result in a significant saving in driving costs on the roadway system.

Table 3: Mode share Trends, 2000-2004

Mode	Census 2000 128,279,228*	2000 ACS 127,731,766*	2001 ACS 128,244,898*	2002 ACS 128,617,952*	2003 ACS 129,141,982*	2004 ACS 130,832,187*
	Percent					
Private vehicle	87.88	87.51	87.58	87.81	88.20	87.76
Drive alone	75.70	76.29	76.84	77.42	77.76	77.68
Carpool	12.19	11.22	10.74	10.39	10.44	10.08
Transit	4.57	5.19	5.07	4.96	4.82	4.57
Bus	2.50	2.81	2.79	2.71	2.63	2.48
Streetcar	0.06	0.07	0.06	0.06	0.06	0.07
Subway	1.47	1.57	1.51	1.45	1.44	1.47
Railroad	0.51	0.55	0.54	0.56	0.53	0.53
Ferry	0.03	0.04	0.04	0.04	0.04	0.03
Taxi	0.16	0.16	0.13	0.14	0.12	0.12
Motorcycle	0.11	0.12	0.12	0.11	0.11	0.15
Bike	0.38	0.44	0.42	0.36	0.37	0.37
Walk	2.93	2.68	2.55	2.48	2.27	2.38
Other	0.70	0.85	0.87	0.82	0.72	0.81
Work at home	3.26	3.21	3.38	3.46	3.50	3.84
All	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**

*Total workers
Note: ACS excludes group quarters population.

Source: Commuting in America III: The Third National Report on Commuting Patterns and Trends, 2006, Transportation Research Board, 2006

Table 4: Driving cost by Miles driven and vehicle type

miles per year	10,000	15,000	20,000
small sedan	55.1 cents	42.1 cents	35.7 cents
medium sedan	71.9 cents	55.2 cents	46.9 cents
large sedan	85.8 cents	65.1 cents	54.8 cents
composite average *	71.0 cents	54.1 cents	45.8 cents

Source: 2008 Your Driving Costs, American Automobile Association

Private automobile is also the most pollutant transportation mode [Hensher, 2008]. Transportation is a significant source of greenhouse gas (GHG) emissions. In 2003, the transportation sector accounted for about 27 percent of total U.S. GHG emissions and it was predicted to continue increasing rapidly, reflecting the anticipated impact of factors such as economic growth, increased movement of freight by trucks and aircraft, and continued growth in personal travel. About 81 percent of transportation GHG emissions in the United States came from "on-road" vehicles. Personal transport accounted for 62 percent of total transportation emissions (35 percent for passenger cars and 27 percent for light-duty trucks including SUVs, minivans and pickup trucks and less than 1 percent for motorcycles). Heavy-duty vehicles including trucks and buses were responsible for 19 percent of total transportation emissions. (Figure 2) [Transportation GHG Emissions Report, 2006].

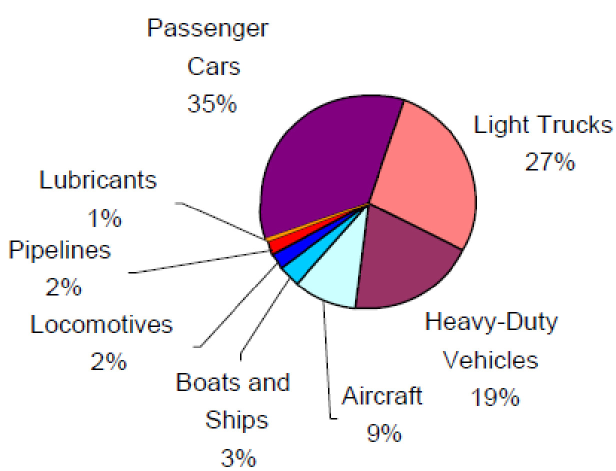


Figure 2. 2003 Transportation Greenhouse Gas Emissions, by Source
Source: U.S. Environmental Protection Agency, 2005. Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990–2003. Washington, DC, Table 2-9.

Ridesharing with increasing the rate of occupancy per vehicle represents an opportunity to decrease the cost and undesirable impacts of traffic congestion, fuel consumption, and pollution. Although several organized ridesharing projects have been attempted, successful ridesharing systems are still in short supply. Certainly, in order to be widely adopted, ride-sharing must be easy, safe, flexible, quick to respond and economical and must be able to compete with one of the greatest advantages of private car usage, i.e., immediate access to door-to-door transportation. [Agatz et al., 2010].

Dynamic ridesharing (also called real-time ridesharing) is a form of carpooling system that provides rides for single, one-way trips. Dynamic ridesharing differs from regular carpooling and vanpooling in that ridesharing is arranged on a per trip basis rather than for trips made on a regular basis [Casey et al., 2000]. Traditional carpooling, however, is too limiting to accommodate the unconventional schedules of today's rideshare demand, where many commuters will only respond to flexible commuting options [Levofsky et al., 2001]. Some of the transportation agencies have been working on innovative technologies to provide this flexibility. The focus has been on the concept of "smart travelers" riding in "smart vehicles". "Smart", in the most advanced sense, means that both the people and vehicles are continuously connected via wireless communications and the "smart traveler" is a person who has access to real-time and reliable information in order to make travel decisions [Schweiger et al., 1994].

In a dynamic ridesharing system, individuals submit requests for a ride to an operations center or central database, either by telephone, e-mail, or direct input to a system residing on the Internet. The database of trips that have been offered by registered drivers is searched by the ride-matching software to see if any match the approximate time and destination of the trip request. A request may be made for any destination or time of day, but matches are more likely to be found for travel in peak periods and in principal commute directions. Requests for ride-matches can be made well in advance or close to the time when the ride is desired. A return trip would be a separate trip request and could be matched with a different driver. The ITS element in dynamic ridesharing is the automation of the trip request matching and arrangement process, which allows trips to be arranged on short notice. This can be done by either the traveler using the Internet or by a customer service representative at a transit agency call center. The technology involved is rideshare software and possibly the Internet [Casey et al., 2000]. Dynamic ridesharing benefits both drivers and passengers. Passengers benefit by having an alternative when their usual mode is unavailable, and by possibly eliminating the need for an additional car for occasional use. Dynamic ridesharing is particularly valuable when public transportation is not an option. Drivers benefit by having someone to share the cost of the trip (although this may not always happen) or to gain enough passengers to qualify for high occupancy vehicle (HOV) lanes and reduce the travel time of their trip [Casey et al., 2000].

Dynamic ridesharing could combat the increase in the numbers of vehicle trips, levels of Vehicle Miles Traveled, and amounts of congestion on the road. According to the United States Department of Transportation, 17% of the growth of VMT in the United States between 1983 and 1990 was caused by a decrease in vehicle occupancy – accounting for far more than the 13% increase due to population growth [Surface Transportation Policy Project. 1999]. But addressing this growth through traditional means is difficult because only 11% of the United States urban population lives within one-quarter mile of a transit stop with non-rush hour frequency of 15 minutes or less [National Science and Technology Council, 1999]. Dynamic ridesharing, in contrast, has the potential to reduce each of these factors; 35% of participants in a Bellevue Smart

Traveler project focus group [Haselkorn et al. 2005] and 50% of respondents to a Hawaii Department of Transportation study [Flannelly and McLeod, 2000] expressed a willingness to use such a service if it were available. The failure of the experiment in the large (open to the general public) dial-a-ride, door-to-door transit service in San Jose, CA, showed the great potential that door to door services have in attracting users. The transit system abolished less than six months after it opened because it was more successful in luring riders than its originators expected it to be [Lindsey 1975]. An expensive U.S. average $13 per-ride cost, however, prohibits conventional dial-a-ride service from becoming a viable option for a large number of trips [John A. Volpe National Transportation Systems Center (U.S.), 2000]. Recent technological advances in internet based communication devices such as PDAs, smart phones, and wireless laptops could be key enablers to increase popularity of dynamic ridesharing. According to comScore report, 234 million Americans subscribed to mobile phone plans in January 2010. Of these, 42.7 million owned Internet-accessible smart phones, which represented an 18 percent increase over the three months ended in October.

1.2. Definition and Features of Real-Time Ridesharing

Dynamic ridesharing also known as dynamic carpooling, real-time ridesharing, ad-hoc ridesharing, and instant ridesharing has been defined differently by different scholars. An early effort to increase the industry's knowledge and adoption of successful applications of advanced technologies defined dynamic car-pooling as "a mode of transportation that is ready when you are. They are multipurpose and can be arranged either in real-time or close to it (near term). Participants pre-qualify and are put into a database. Upon receipt of a trip enquiry, the database is searched for others who are traveling in the same direction at the same time. Participants can not only use this database to arrange for carpools to and from work, but also to a shopping center, medical facility or any other trip generator" [Schweiger et al., 1994] . One of the other first definitions proposed was developed in preparation for a field operational test in Sacramento, CA in 1994 that defined dynamic ridesharing as "a one-time rideshare match obtained for a one-way trip either the same day or the evening before" [Kowshik et al., 1996]. Another trial in 1997 which was aimed to test the concept of dynamic rideshare matching services using Internet and e-mail at the University of Washington in Seattle defined dynamic ridesharing as "two or more people sharing a single trip, without regard to previous arrangements or history among the individuals involved. In comparison to traditional ride-matching services, which focus on commuters traveling to and from the same origins and destinations on fixed schedules, a dynamic ridesharing system must be able to match random trip requests at any time. Thus, the system must be able to match potential carpoolers quickly to respond to same-day trip requests, as well as the more traditional commute trips" [Dailey et al., 1997]. 'dynamicridesharing.org' defines dynamic ridesharing as "A system that facilitates the ability of drivers and passengers to make one-time ride-matches close to their departure time, with sufficient convenience and flexibility to be used on a daily basis" [Kirshner, 2008]. A recent definition proposed for dynamic ridesharing described it as "an automated system that facilitates drivers and riders to share one-time trips close to their desired departure times" and characterized it by the following features: Dynamic, independent private entities, cost sharing, non-recurring trips, prearranged, and automated matching [Agatz, et al., 2010]. Another recent work suggests real-time ridesharing as "A single or recurring rideshare trip with no fixed schedule, organized on a one-time basis, with matching of participants occurring as little as a few minutes before departure or as far in advance as the evening before a trip is scheduled to take place” [Amey, 2010].

All definitions emphasize that dynamic ridesharing is occasional in their nature and has no fixed amount of advanced notice required for establishing the shared trip. For the purposes of the study presented in this dissertation, real-time ridesharing is defined as: “A non-recurring multipurpose rideshare trip which is prearranged on a per trip basis on a short-notice to establish shared trips close to the desired departure times and locations of the participants to gain HOV lanes privileges or share the cost of the trip. ”

1.3. Research Problem Statement

This research project develops a Dynamic Rideshare Optimized Matching (DROM) model and solution that identifies suitable matches between passengers requesting rideshare services with appropriate drivers available to carpool for credits and HOV lane privileges. The optimization model seeks for optimal matched routes that maximize the overall system performance subject to ride availability, capacity, and rider and driver time window constraints while considering users’ preferences. DROM receives passengers and drivers information and preferences continuously over time and assigns passengers to drivers with respect to proximity in time and space and compatibility of characteristics and preferences among the passengers, drivers and passengers onboard. DROM is a core component of a typical real time rideshare system shown in Figure 3.

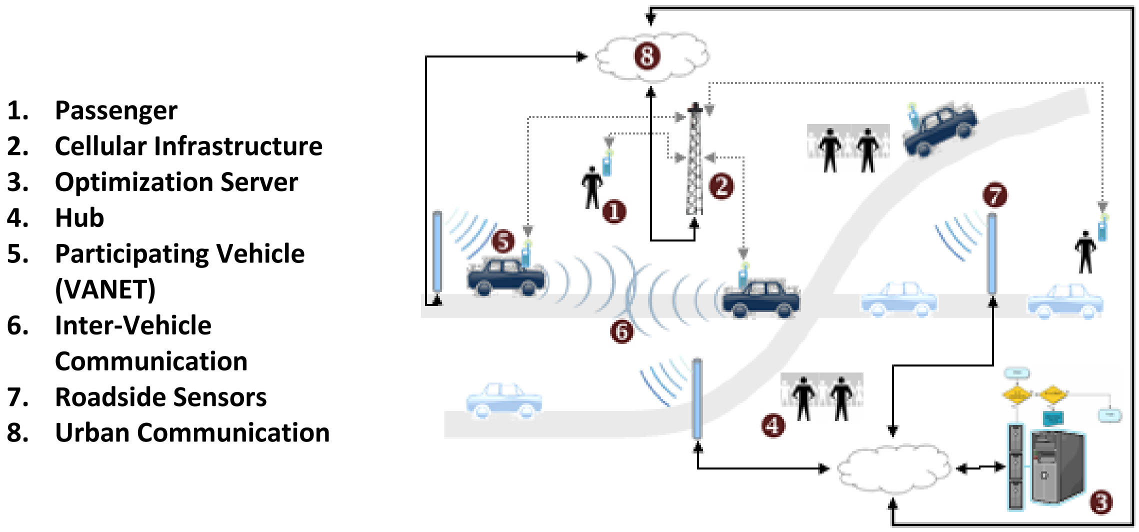


Figure 3: Real-time rideshare components

The present study will be concerned with the following research questions:

1. How has the transportation research communities responded to development of optimization models to increase popularity of Dynamic ridesharing in the era of emerging key enablers?
2. Is it possible to rely on commercial solvers for obtaining optimal solutions in a reasonable computing time? If not so,
3. How to develop an efficient solution algorithm for solving the optimization model proposed in this study?
4. How do the characteristics of model and system environment impact the solutions?
5. How do the model and solution behave in a real road network?

1.4. Organization of the Dissertation

The research presented in this dissertation is carried out in the following order: Chapter 2 reviews the state-of-the-art of dynamic ridesharing projects and research. At the end of the chapter a summary of reviews is presented. Chapter 3 defines the problem of dynamic rideshare optimized matching. Chapter 4 presents the MIP mathematical formulation for the problem. Chapter 5 tests the model and validity of the solutions. Chapter 6 develops an efficient heuristic solution method for DROM problem, namely Three-Spherical Heuristic Decomposition Model. Chapter 7 analyzes the sensitivity of model parameters with the most influencing on results. Chapter 8 presents a case to illustrate the DROM and evaluate the solution approach on a real non-virtual road network. Chapter 9 concludes the research and suggests directions for future research.

1.5. Contribution of the Dissertation

A viability analysis for dynamic rideshare system that examined both theoretical concepts and actual implementation of a dynamic rideshare system in Los Angeles [Hall and Qureshi, 1997] concluded that in theory dynamic ridesharing is a viable concept and a user should be successful to find a ride-match but in practice the story is different and at best one can expect a one in five chance of someone offering a ride. In another study, A GIS approach analysis to identify common clusters of commuters in University of Toronto [Sarraino et al., 2008] found that during morning commute hours (7:00–10:30am), 1,461 of 3,030 drive trips (48%) were suitable for ridesharing based on residential proximity and similar residential departure times. A similar study in Massachusetts Institute of Technology suggested that between 50% and 77% of the commuting population could rideshare on a maximum-effort day that is significantly higher than the 8% of the MIT community that currently choose to rideshare [Amey, 2011]. A simulation study in Metro Atlanta showed that the use of sophisticated optimization methods substantially increases the likelihood to find the ride-matches and also that dynamic ridesharing has potential for success in large U.S. metropolitan areas [Agatz et al., 2010]. While technological advances have greatly eased the communication and reputation systems and social network tools have tackled the fear of sharing a ride with strangers, the development of optimization algorithms for matching the participant in real-time and ultimately increasing the rate of participation in the ridesharing system has been largely ignored by transportation research community. This research is the first of its kind that:

1- Develops an optimization model for real-time rideshare matching problem.

2- Develops an optimization model with negotiating policies for rideshare preference matches for rideshare matching problem.

3- Develops a decomposition based solution strategy and algorithm to solve dynamic rideshare matching problem.

Chapter 2: Review of the Literature

The aim of the following is to review the state-of-the-art of dynamic ridesharing projects and research.

2.1. Bellevue Smart Traveler

The goal of the Bellevue Smart Traveler (BST) project was to design and test a traveler information center (TIC) prototype in downtown Bellevue, Washington, east of Seattle that is an area with concentrated employment facilities and a high percentage of single occupancy vehicle (SOV) commuters. The idea was to provide the participant with convenient off-site access to the TIC's information including up-to-the-minute traffic congestion information, transit information, and carpool/vanpoo1 ride-matches using a telephone, and/or a hand-held alpha-numeric pager.

The user population was employees of downtown Bellevue companies taking part in the BST demonstration project. Registered users had access to pagers in addition to the phone-based system and would be tracked to determine how they used the system and whether or not the system was effective in encouraging their use of HOV transportation options. The registration application acquired information such as: full name, gender, employer, Washington state driver's license number, work address, home address, work phone number, home phone number (public or private), work days, work hours, preferred arrival time to work, preferred departure time from work, schedule flexibility (in terms of time), preferred pickup points (three of them, selected from a list, in ranked order), smoking preference, gender preference (exclusive and nonexclusive), willingness to be a driver (how often, how many seats available), and willingness to be a rider (how often).

For ridesharing purposes, registered users were divided into “ride groups”. All registered users were working in a four square block area of downtown Bellevue but lived throughout the Puget Sound area. Hence, ride groups were based on where users lived so that each ride group was consisting of users that commute to and from the same general areas to increase the potential for successful dynamic ride-matches; each ride group had enough users so that a reasonable number of ride matches were possible. However, each ride group was not so large to prevent overflow of information for riders looking for rides. Ride groups covered a small enough geographical area so that drivers and riders could meet and be dropped off at convenient locations. The formation of ride groups was based on zip codes and preferred pick-up/drop-off points (as specified on the application). The TIC was tested and demonstrated over a five-month period (from late November 1993 to late April 1994). During that time, 53 users were registered. Of the registered users, 48 formed three ride groups: 23 from areas south of Bellevue, 10 from areas east of Bellevue, and 15 from areas north of Bellevue. Members from the ride groups offered 509 rides and only six ride-matches were logged. Results from the usage patterns and various surveys that were conducted suggested that participants liked the idea of dynamic ridesharing, the presentation of the information and the technology. However, for various reasons they were either unable or unwilling to form ride-matches. Some of the reasons were: insufficient rideshare choices due to the limited size of rideshare groups, being uncomfortable getting into someone else’s car, limited time saving incentives due to lack of HOV lanes in the Bellevue area, and technology limitations that reduced the effectiveness of pager delivery. Another possible reason for failure of the project may have been the inconvenience of the rideshare service. The system did not actually match the riders. When users received potential matches from their ride groups, they were left to coordinate the trip.

The BST project conclusions suggested that rideshare group is a new social entity and more work is needed to determine (1) how to encourage ride acceptance and (2) the dynamics of a viable ride group. Incentives such as management support and encouragement could have played a stronger role. Placing the BST TIC on the Internet would help people more easily obtain and respond to rideshare information [Haselkorn et al., 1995]. Since participants were placed in location-based ride groups, trips were limited to work and home, with time of the trip as the sole variable. For maximum benefits, dynamic ride-matching systems need to allow both location and time to vary to enable matching for work and non-work trips [Dailey et al., 1997].

2.2. Los Angeles Smart Traveler Field Operational Test

The Los Angeles Smart Traveler Field Operational Test (FOT) was one of the largest and most comprehensive Automated Rideshare Matching System (ARMS) experiments to date that operated only from July 1994 to September 1994 and it was limited to the approximately 68,000 people. The purpose of the study was to evaluate the performance and effectiveness of the Advanced Traveler Information System (ATIS). This project was implemented in Los Angeles as part of the new technology demonstrations being carried out by the California Advanced Public Transportation Systems Group (CAPTS) at Caltrans District 7. It was designed as a field operational test of three different media approaches for providing traveler information: fully automated telephone systems; automated multi-media touch screen kiosks; and PC via modem. The information included: transit routes, fares and services; traffic conditions on the freeways; and ride-matching information for ridesharing on both frequent and one time occasions.

Survey results indicated a high degree of user satisfaction for the kiosks that provided a new medium for obtaining pre-trip traveler information, yet the overall usage rate was low (an average of 25 transactions per day), relative to the cost of providing the kiosk service. Low usage combined with high capital and operating costs yielded a total cost per use of approximately $2.00 (over a five-year lifetime of the kiosk). Kiosks placed in office locations had the lowest usage while kiosks placed in Union Station in downtown Los Angeles and shopping malls had the highest usage. This finding suggests that the kiosks may be used more for non-work related trip information when users have more time, such as for shopping trips or by tourists.

Smart Traveler Automated Ride-matching Service (ARMS) allowed users to use their touch tone phone to find rideshare partners. It was designed to provide individuals with lists of potential compatible rideshare partners for either regular carpooling or an occasional emergency ride home. As with the kiosks, the service was available in both English and Spanish. For the purposes of finding either regular rideshare partners or a once only ride, those using the system used the touch tone phone to enter changes in preferred travel times. They received a computer generated list of people to contact who live and work near them with similar schedules. The user could then choose to call some or all of the people on the list, or record a message that Smart Traveler would automatically deliver to potential carpool partners, allowing them to call the individual back if they were interested in sharing a ride. The ARMS was found to have very little usage (34 persons per week). From a small telephone survey of ARMS users it was concluded that most users used the service to seek regular ridesharing opportunities and not the featured one-time ride service. The researchers concluded that there is not enough interest in ARMS to justify its cost of operation.

The modem service was found to have significant usage. In a period of 35 weeks a total of 83,155 uses were recorded (on an average weekday there were circa 400 uses per day). These levels of use indicated that there was indeed a demand for the service. This component of the ATIS system did not have the multi-modal component at the time of evaluation and instead only reported Caltrans congestion information. Usage was found to be higher in the mornings and evenings, consistent with commuter trip planning [Giulian et al., 1995].

In order to use ARMS, individuals had to be registered with Commuter Transportation Services. There is no way to know how many matches where actually made because users were not required to report them. The evaluation concluded that the market for “one-day-only” rides was very limited because of participants’ concerns over safety [Golob and Giuliano, 1996].

2.3. Sacramento Rideshare Matching Field Operational Test

A real-time rideshare matching field operational test evaluation was conducted in Sacramento, California which began in late 1994 and terminated in 1995 with the participation of the Federal Transit Administration, Caltrans, PATH, Sacramento Rideshare, and U.C. Davis Institute of Transportation Studies. The service was not automated, but operator-based. Users answered questions over the telephone about origin and destination locations, purpose of trip, etc. Trip matches were made by sorting from database orientation and destination zip codes, and then prioritizing by the closeness of desired trip times. Three hundred and sixty people (from a database of 5,000 who expressed interest in carpooling) registered as drivers willing to offer on-demand rides. The rate of match was very low and from the ten requests made for dynamic ridesharing, only one potential match was made, and it is not known if the match was secured.

The final report concluded there were several reasons for the poor performance of the program including poor marketing of the service and personal security concerns. As part of the system design, user needs were assessed through a review of literature and focus group discussions. Six user needs were identified: background screening; information security; matching and system reliability; system access; flexibility; and, a compensation scheme. The users needed flexible ridesharing arrangements that would allow users with non-identical origins and destinations to be matched as well as a reliable system that would be able to generate a large number of potential matches for any given trip [Kowshik et al., 1996].

2.4. Coachella Valley TransAction Network

Commuter Transportation Services, Inc. (CTS) developed the Coachella Valley TransAction Network (TAN) in 1994 as a pilot test for providing information on transit and ridesharing. The project was similar to the Los Angeles Smart Traveler project, in that real-time traffic and transit information and rideshare information were provided to over 700,000 registrants throughout the Riverside area via four stand-alone commuter information kiosks. During the seven-month test period, more than 21,510 people accessed the kiosk system. Approximately one-third of them accessed information on ridesharing and only 256 printouts were rideshare match lists. The project was expensive to implement and usage was low. CTS concluded that kiosks were probably not the best medium for obtaining real-time rideshare information and recommended it not be included in future models [Haselkorn et al., 1995].

2.5. Seattle Smart Traveler

Seattle Smart Traveler (SST) project was part of a larger Intelligent Transportation System Field Operational Test conducted by the Washington State Department of Transportation, the University of Washington, King County Metro, and five private sector partners from 1995 to 1997. This project was designed to test the concept of automated dynamic rideshare matching using the Internet and electronic mail at the University of Washington in Seattle [Dailey et al., 1999]. The SST project defined dynamic ridesharing as “two or more people sharing a single trip, without regard to previous arrangements or history among the individuals involved” and addressed the differences between dynamic ridesharing with traditional ride-matching services, which focus on commuters traveling to and from the same origins and destinations on fixed schedules, as “a dynamic ridesharing system must be able to match random trip requests at any time” [Federal Transit Administration, 1996]. User group was limited to faculty, students, and staff from the University of Washington. The SST was designed to respond to the request of three types of matches: regular commute trips, additional regular trips, and occasional trips. A user entered the origin, destination, day of week, departure time, and arrival time for each trip type. The system then identified potential matches using a search structure containing four levels of detail. To provide flexibility in the matching of trips, a time range or window was used for both the requested departure and arrival times. The SST automatically generated and sent an e-mail message with this information if the user desired or the participant could call the potential matches [Federal Transit Administration, 1996].

The evaluation report found that faculty and staff made up 68% of users, with students comprising the remaining 32%. Approximately 700 ride-matches were requested during the 15-month test period, of those 150 potential matches generated, and At least 41 matches actually made. It was possible that more ride-matches were made, as there was no requirement that actual trips be reported [Casey et al., 2000]. SST suggested that the relationship between the number of users and the number of carpools formed was quadratic, i.e., rate of carpooling would increase with the number of users. It also suggested that carpooling has the potential to have a larger effect on traffic demand management (TDM) if large groups of people participate. Further, SST suggested that a web-based ride-match system can be as effective as traditional ride-matching. SST suggested the following quantitative relationships between numbers of users, matches, and carpools. SST estimated the number matches expected (*Tm*) given U users is:

$$T_m(U) = \frac{\propto^2 U^2 - U}{2} P_m \tag{1}$$

And, the actual number of carpools (Cp) is:

$$C_p = \beta T_m(U) \tag{2}$$

Where $\propto$ and β are constant coefficients, P_m is the probability for a pair of trips matching assuming: (1) the probability of trips matching is approximately constant across the population of trips, (2) the relationship between the number of users and the number of trips is linear, and (3) the relationship between matches and actual carpools is linear [Dailey et al., 1999]. The SST project identified some issues that may have limited the use of the system. First, the project was implemented before the real boom in Internet use. Second, the developing technology for the dynamic ride-matching capabilities was somewhat cumbersome. Third, the SST had been viewed by some targeted users as a temporary endeavor. Fourth, there were no sufficient incentives to encourage greater ridesharing. Finally, there were safety concerns regarding sharing rides with strangers. Although the test ended in June 1997, the SST continued to operate for a few years later even though no staff was assigned to the project. Without staff support, the database was not updated or purged of former users [Turnbull, 1999].

The SST system is no longer operational; however, an offline demonstration of the project can be viewed by following the SST link: http://sst.its.washington.edu/sst/.

2.6. Missoula Ravalli Transportation Management Association

The Missoula Ravalli Transportation Management Association (MRTMA) operated a ridesharing program in Missoula, Montana using GeoMatch information system for matching new applicants with existing carpools. GeoMatch is a geographic based system that matches people with carpools, vanpools, and provides transit information. The program runs on personal computers using the Microsoft Access database software. Rideshare requests were provided by telephone and generating a match list usually took about four minutes. The rideshare program was in operation from 1997 and had over 300 names in the carpool database by September 2000. During that time period, it formed 30 regular carpools and four vanpools and received three to five rideshare request calls per week, one to two of those were one-time rides [Casey, 2000].

2.7. King County Metro's Regional Ride-match System

King County is located in Washington State and comprises 2,134 square Miles with more than 1.8 million people. Major cities include Seattle and Bellevue. Washington

State’ s Commute Trip Reduction (CTR) Act that was passed in 1991 and reauthorized in 2006, as a part of the Washington Clean Air Act, required major employers to reduce drive-alone commuting by their employees and provided a regulatory framework for measuring employer success. Since passage of the CTR Act, King County Metro Transit has worked closely with major employers to design products and programs to help them meet the CTR goals. Almost all these efforts focus on working with employers to reach employees and providing tools and incentives to employees to use alternatives like busing, carpooling, biking, telecommuting, and compressed work schedules [Travel Behavior, Environmental, and Health Impacts of Community Design and Transportation Investment, 2005].

King County Metro with about 1,300 transit coaches and more than 700 vans in its vanpool fleet and a well-integrated bicycle support program, has incorporated special event ride-matching into its regional rideshare program, *rideshareonline*, that is a self-serve, public, internet-based rideshare matching service in association with regional carpool/vanpool providers [Cooper, 2007]. RideshareOnline.com instantly matches registered commuters with carpool or vanpool partners with a similar daily commute in the area. Users enter their commuting times and locations and can instantly see a list of ride-matches to whom they may e-mail a rideshare request anytime for everything from carpools, vanpools, SchoolPools and biking to work, to one-time special events like ballgames, concerts and conferences [King County Metro Transit, 2010].

2.8. Redmond Transportation Management Association’s Ride-match system

Redmond is the seventh most populous city in King County and the fifteenth most populous city in the State of Washington, with a residential population of over 46,000. It encompasses an area of over 16.6 square Miles. The city is well known as a center of technology and the location for a number of known high-tech and biomedical companies such as Microsoft, Nintendo, AT&T Wireless, and Medtronic Physio-Control. The Greater Redmond Transportation Management Association (GRTMA) has established an automated ride-matching system for carpools and vanpools on the Internet. *RideQuest* is an employer and geographic information system (GIS) based system with the database accessible by SQL Server. Registered users enter a street address or a nearby intersection, and the software produces a map showing that location for verification by the registrant. Then the request is entered into the database along with information on the users travel needs and preferences such as whether they wish to drive or ride, ride with smokers or non-smokers, or ride with employees of specific companies. People are matched based on their origin address and final destination with a numbering system of the best match to least potential match [Knapp, 2005].

The system can send automatic emails to other registered commuters who may be able to rideshare. A map showing the requestor's location and the location of potential matches is displayed on the screen together with their names and methods of contacting them. Individuals can change their information at any time or remove themselves from the system if they have found satisfactory ridesharing arrangements, moved, changed jobs, etc. Every three months, e-mails are automatically sent to all registrants asking for their continued interest in participation. Non-respondents are automatically removed along with those responding in the negative. An early version of the system was tested in April 1999 with 1,200 registrants. There are no statistics available on carpool formation [Casey, 2000].

GRTMA promotes the program using posters, post cards, email and the web site and has a variety of promotions throughout the year to encourage people to register in the rideshare system including a trip to Hawaii, and a 12-oz Starbucks Coffee beverage free. Vanpool drivers don't have to pay the monthly vanpool fare and they also receive up to 40 personal use Miles on the van [Knapp, 2005].

2.9. Minerva Dynamic Ridesharing System

Aegis Transportation Systems developed a system called MINERVA in Oregon that takes advantage of ATHENA smart traveler system. ATHENA was developed in City of Ontario, California, with funding from the FTA in 1994, but the project was abandoned in 1996 due to a turnover of the city council. The ATHENA project differed from other dynamic ridesharing programs in that trip requestors would not receive a list of potential drivers, and would not have to contact trip providers to arrange travel. Instead, a central computer would arrange the match and advise the rider and driver of pickup points, times, and fares. The ATHENA project incorporated a central database that interfaced with personal digital assistants (PDA's) and hand held devices that have messaging and GIS capabilities. Interested parties would pre-register with ATHENA. Once registered, all ATHENA drivers would receive a PDA for their car, and all potential passengers would also use telephone-based information systems and other computer and communications technologies to integrate these new personalized transportation services with conventional transit (e.g. bus, rail, ferry), paratransit (e.g. taxi, shuttle, dial-a-ride), and ridesharing (e.g. carpool, vanpool, buspool) modes to develop more cost-effective public transportation systems. Market research studies indicate that this approach would reduce vehicle trips and vehicle Miles traveled (VMT) per capita significantly, at a low cost to taxpayers. MINERVA used “smart” technology including cellular phones, palmtop computers, and wireless data communications to provide low-cost alternatives to transportation in low-density areas and low travel corridors. MINERVA took the ATHENA concept one step further. MINERVA integrated the smart traveler system with other online information services—home shopping, telebanking, e-mail, and interactive games—in an attempt to reduce the need for some trips altogether [Levofsky et al., 2001].

The Oregon State legislature committed $1.5 million to the project, with additional commitments of $3 million in matching funds from local pilot sites, and $1 million in in-kind support from private management consulting outfits. A dozen Oregon cities expressed their interest in piloting MINERVA [Victoria Transport Policy Institute, 2010]. Both ATHENA and MINERVA did not progress beyond the developmental stage and were never implemented. However, their Internet and GIS components formed the basis of many ridesharing programs in use today [Chan and Shaheen, 2011].

2.10 Online Ride-Matching and Traveler Information Services

With respect to the fact that most of the dynamic ride-matching applications and pilot tests of the 1980s and 90s failed to provide enough users to consistently create a successful instant ridesharing match, next generation of the most dynamic ridesharing focused on more reliable strategies to encourage ridesharing including online ride-matching and traveler information services. Before 1999, the websites for ride-matching applications were either simple pages listing agency contact information, online forms for users to email the agency to receive a match list, or online notice boards for users to manually post or search carpool listings [Bower, 2004]. Between 1999 and 2004, private software companies began developing ride-matching platforms. Although it became much easier to find ride-matches in a larger online database, the carpools still suffered from the same inflexibility drawback as traditional carpools. Online ride-matching programs tended to be more static and inflexible and best suited for commutes with regular prearranged schedules and were not competitive enough to compete with the flexibility that private auto travel offered [Chan and Shaheen, 2011].

In another attempt, on July 2000, the Federal Communications Commission designated a uniform "511" as the traveler information telephone number to make real-time traveler information more widely available for local, regional, and state agencies across the U.S. including carpool and/or vanpool information services [Profiles of 511 Traveler Information Systems Update, 2009].

2.11. Dynamic Ridesharing in the era of Internet Enabling Technologies

From 2004 to the present, dynamic ridesharing programs have taken advantage of the incentive strategies that encourage ridesharing such as HOV lanes, and park-and-ride efforts and they have integrated Internet enabling technologies such as World Wide Web, Smart phones, Global Positioning System (GPS), Data Repository, Automated Financial Transactions, and social networking. To the best of the author's knowledge, there are approximately 33 notable applications and software platforms that offer ridesharing services. However, the systems typically serve as platforms that bring users together, rather than as active mechanisms that generate rideshare plans and provide fair payments [Kamar and Horvitz, 2009]. A brief description for those applications and software platforms is given below.

- *Aktalita* is an application currently under development that combines the Web, a geospatially enabled database, and a Java enabled cellphone to provide real-time carpooling between drivers and passengers. When a driver is about to travel or a passenger needs a ride, they enter an offer or request to the system via the web or Java enabled cellphone. The system then queries its geospatial database to attempt to match passenger and driver, and notifies them for further negotiation [http://www.aktalita.com].
- *AlterNetRides.com* works nationwide but also can be tailored for a community. It is completely automated, a person can become a member, set up a ride and be viewing others wanting to rideshare in just minutes [http://alternetrides.com/].
- *Avego* is a proprietary application for Apple iPhone. It uses GPS technologies and presents an intuitive user interface. The application relies on a proprietary service called Futurefleet, on which no implementation details are given [http://www.avego.com].
- *Carpoolconnect.com* matches up carpooling commuters based on similar commutes defined by home and work zip codes [http://carpoolconnect.com/].
- *Carpoolworld.com* uses the commuter's precise latitude and longitude coordinates to find the best matches for their trip among the other commuters in the database, based on exactly how close together they live and exactly how close together they work [http://www.carpoolworld.com/].
- *Carpool.ca* is available via the internet and uses home and destination locations, driving route and other personal information to help commuters identify potential carpool partners. This self-serve system has various levels of security, limiting

individual access to personal rideshare information while providing rideshare program administrators broader access. The program includes a built in CO2 savings calculator [http://www.carpool.ca/].

- *Carriva* is a proprietary solution using phone calls as communication system and a fixed price of 0.10€/km. It has got 1118 active users [https://www.carriva.org/MFC/app].
- *Carticipate* is a proprietary iPhone application that integrates with Facebook. It has an interface looking like Google Maps mobile. It is an experiment in social transportation. According to the website, it is available in 59 countries [http://www.carticipate.com].
- *Commuter Register* is a multimedia publication that provides listings of car and vanpools, transit routes and schedules, Park and Ride lots, and articles and helpful travel tips focusing on employee commute matching [http://www.2plus.com/].
- *Divide The Ride* is a static, web-based solution organized around children and family activities. Families invite other trusted families to join their group. Groups get notifications when a ride is needed [http://www.dividetheride.com/].
- *Ecolane DRT* and *Ecolane Dynamic Carpool* are two ridesharing software offered by Ecolane Company integrated with Nokia touchscreen device. Among the features, they declare that the device is capable of real-time data communication, reports of arrivals and departures with time information, device locking mechanisms, GPS location and direction, mileage tracking, and detailed trip information. It is a completely web-based, turn-key scheduling and dispatching solution with user interfaces that are accessed securely using a standard web-browser using seamless

integration with multiple Mobile Data Terminal (MDT) and Automatic Vehicle Location (AVL) platforms. It enables commuters to overcome the biggest obstacles of traditional carpooling today - irregular working schedules and finding a carpool partner. Commuters are able to select if they want to rideshare in as little as 15 minutes and create an instant carpool with the mobile phone or web-based applications. The Ecolane Dynamic Carpool software communicates the needs of both drivers and passengers, and automatically matches potential carpoolers based on digital maps, individual profiles, user groups, and user ratings [http://www.ecolane.com/].

- *eCommuter* is an internet-based technology application specializing in Real-Time Internet traveler solutions. It is the first-to-market in the category of Internet ride-matching that gives commuters the power to find their own partners for sharing a carpool or vanpool to work [http://www.ecommuter.com].
- *eRideShare.com* is a free service for connecting travelers going the same way. According to Yahoo and Google it is the leading carpool/ridesharing website and has been recognized as "Best of the Net" by About.com. The site has over 17,000 commuters and travelers throughout the US and Canada [http://www.erideshare.com/].
- *Flinc* comes from Germany and is a dynamic carpooling application system that can be used on smart phones or online. This application utilizes the location based capabilities of mobile phones and navigational software to connect passengers and drivers, offering a customer to customer (C2C) interaction both for ride coordination and financial interaction. The system analyzes real-time traffic and brings riders and

drivers together, eliminating the need for coordination methods such as phone calls, emails, or text messaging. Passengers can identify available seats in cars belonging to drivers in their network and send a request to be picked up at their location. The driver, after confirming the pickup, receives instructions via the navigation software and arrives to pick up the passenger [http://www.flinc.org/world/].

- *GoLoco* is a proprietary web application that also relies on Facebook. It uses a private payment system and coordinates carpool and vanpools for work, campus, religious and group events [http://goloco.org/].
- *Goose Networks* is a web-based ride-matching service that allows commuters to connect with each other for flexible, one-way carpools or for regular recurring trips. Users simply input their commute schedule online; existing matches are immediately shown and built-in email and SMS text message notifications help keep users informed of new options as they become available [http://www.goosenetworks.com].
- *GreenRide Connect Metro* has two employer and campus editions that combine a user-friendly interface with rich content management with multi-tiered administration. Social network integration, content management, GIS capabilities, employer management, single-trip matching, raffle management, cluster mapping, savings tracking (energy, economic, environmental), comprehensive and exportable reports, vanpool management are all available through the suite of GreenRide solutions [http://www.greenride.com/].
- *Hover* (High Occupancy Vehicles in Express Routes) is a casual carpooling system that was inspired in Auckland, New Zealand when a city manager observed that "if everyone shared a ride one day a week there would be 20% less traffic". Hover

creates a community of rideshare commuters who share benefit of the savings through the own credit system. Each time a driver provides a ride, he receives one credit from each passenger and each time a rider takes a ride, he uses one credit. The members are approved after security check and with two personal references. Hover uses RFID technology to identify members and cars and operates to agreed destinations. In morning, each member drives or walks to a Hover park that is a secure and safe place to leave the cars. Each Hover park, along the route to destination, has parking areas set up for agreed destinations. In general they require about 100 participants from one Hover park to a given destination area to keep the waiting times to a workable level and form a trip with at least 3 people. In the evening, participants make their way either on foot or by car to a Hover Port. Riders from the morning might wait at a Hover Point, like a bus stop. Drivers going by these Hover Point will stop and pick up riders and take them to the Hover Port. At the Hover Port passengers will get out of the car they came in and change to a car that is going back to their Hover park. On exiting the Hover Park, the system recognizes driver and passengers and distributes credit points. It also offers a guaranteed back-to-home system, by using taxis [http://www.hoverport.org/].

- *iCarpool* is a static, online and custom branded hosted solution for employers and regional public agencies with interactive maps, privacy protection, high precision trip matching and support for all trip types such as daily commute, one time trips or real time (dynamic carpool) trips. The application also supports - multiple modes such as carpool, vanpool, bike, walk and transit, integrated GIS data such as park-and-ride lots, bike routes, multi modal trip calendar and integrated incentives provided by employers or regional public agencies. Matching criteria includes social relationships, but no details are given [http://www.icarpool.com].

- *KOMOTOR TDM* management system offered by Base Technologies is a comprehensive, web-based total TDM service that combines ride-matching, management, measurement, and reporting tools in one product site [http://www.basetech.com/].
- *MyCasualCarpool.com* helps users find others with similar daily commuting patterns and create rideshare lots using only resources available in virtually every residential neighborhood [http://www.MyCasualCarpool.com].
- *NuRide Network* is the incentive-based ride network that rewards people every time they share a ride. Through the NuRide Network®, individuals can easily arrange individual ridesharing trips for work or pleasure and earn rewards for every confirmed trip they take. Unlike a traditional carpool, NuRide is flexible and casual with users being able to share a single ride without any ongoing commitments. The Miles-based reward points can be redeemed for gift cards, gift certificates and other rewards from their corporate sponsors [http://www.nuride.com].
- *Pathway EnRoute* is a turnkey solution to enable and track carpooling and vanpooling both public, and across organizations in Metropolitan Toronto and Ontario [http://www.carpoolzone.ca] and British Columbia [http://www.online.ride-share.com] in Canada. Pathway EnRoute claims that it has the most sophisticated route-based ride-matching available today as it not only finds passengers whose endpoints the driver pass by, but also the Pathway EnRoute Search Engine finds driver routes that pass by the driver endpoints and claims that this second category

typically accounts for increase in up to 50% of matches, and other systems are blind to these matches. Additional EnRoute Search Engine features include: instantly adding and dragging waypoints by clicking on maps for editing a route, viewing multiple routes together with capability of switching between routes without page reloads, and continually searching for matches and automatically sending notifications of suitable matches even after users log off. Pathway Rewards include: a calendar-based incentive tracking system and an online emergency ride home service [http://www.pathwayintelligence.com/].

- *Piggyback* is an Android application using a step-by-step approach (maximum one user input at each application screen) and makes wide use of graphical representations instead of text. When a driver and passengers are matched their compatibility is shown, represented with stars (0 to 5) and categorized as friendliness, reliability, driving skills and car. After the ride, the feedback system lets the user set the points for the aspects listed above. The application also lets the users plan rides using a static carpooling approach [http://www.piggybackmobile.com/].
- *Ridegrid* is another under-development proprietary application that uses mobile Internet and location technology to enable individuals to obtain rides to and from any location, spontaneously. RideGrid works by dynamically combining routes and evaluates the change required in a driver's route such that it passes through the desired source and destination of a compatible rider, and brokers the agreement [http://www.highregardsoftware.com/ridegrid-dynamic-ridesharing.html].
- *RideNow* is a Web and cell-phone ("Interactive Voice Response") interface system with parking space incentives for instant ridesharing where each ride-match request is

the basis for potentially new carpool arrangements. The system can give users a ride-match within 10 minutes [http://www.ridenow.org/].

- *RidePro* is an integrated desktop and web based rideshare information solution. It is a client-server, menu-driven, Windows®-based application with integrated GIS mapping that sends rideshare match report directly to an e-mail message. The web interface allows the public to create their own registrations and run their own match reports in a secure, confidential environment. The web component uses the same database as the local area network interface. Both interfaces support matching to carpools, vanpools, park-and-ride lots, public transit, telecommute centers, day care centers, bike partners, and bike routes [http://www.ridepro.net/].

- *RideshareOnline.com* is a Seattle-based online ride-matching system. Registered users enter their work location and the starting point of their commute that is either a home address or a nearby intersection and they enter their weekly work schedule and any daily variations. They can instantly see a list of rideshare matches to whom they may email a rideshare request [http:// www.rideshareonline.com/].

- *RideShark* is an online map-based rideshare solution that enables registrants to find rideshare partners based on customized search criteria that includes ride-matching based on a regional, TMA or secure cluster or private organization. *RideShark* utilizes Geographic Information System (GIS) technology from Microsoft MapPoint [http://www.RideShark.com].

- *RM 21* is the proprietary route-based carpool /vanpool ride-matching software system to power the Chicago Area Transportation Study in northeastern Illinois. This system departs from typical “mile-radius” searching by allowing users to chart their travel path. This path is then used to find matches of varying quality as determined by sameness of route, closeness of schedule, and matching of individual preferences [http://www.ShareTheDrive.org/].

- *Visual BACSCAP 2007* is a user-friendly transportation program designed by the Marketing Institute at Florida State University College of Business for use by commuter assistance programs. The primary function of the program is to provide commuters with information regarding pools. An online demo of the program, EzRide, can be viewed at http://nctr.cob.fsu.edu/ezridedemo [http://www.tmi.cob.fsu.edu/vbacscap/download07.htm].

- *VivaCommute* is a web-based commuter rideshare services for all geographical locations in Canada and the United States. This web-based application matches people who travel the same route and share the same driving schedule. The system uses nearest neighbor logic [http://www.vivacommute.com/].

- *Zimride.com* combines Google Maps and optional social network integration and a proprietary route-matching algorithm. Zimride has partnered with 50 U.S. colleges, universities, and companies that each has its own network of members. In addition to each network’s website, Zimride also uses the Facebook platform to attract public users [http://www.zimride.com/].

2.12. Behavioral Analysis studies

2.12.1. Gender

Results from rideshare and carpool formation analysis in the literature suggest that there is evidence for differences between men and women in participating in shared rides and socio-demographics are one of the influencing factors that affect trip choices. For example, Nazem et. al. (2011) suggests that women are more sensitive than are men to transfer of all types, especially transfers between two different modes and women are more sensitive to walking time in transfers whereas men are more sensitive to access and egress times in public transit [Nazem et al., 2011]. Another study suggests that one of the strongest characteristics affecting the willingness to use a toll road is being female. [Yan et al., 2002]. In a gender-based analysis of work trip mode choice of commuters in suburban Montreal, Canada, the authors conclude that women and men should be modeled separately for trip mode choice analysis and they suggest that women are less likely to choose public transit and more likely to choose to rideshare than men; and women are less time-sensitive in commuting than men are due to the fact that women commute shorter distances and make more trips owing to their pivotal household responsibilities [Patterson et al., 2005].

In the literature, three are two dominant hypotheses: household responsibility and entrapment. The household responsibility theory advises that females tend to commute less than males because of their larger share of child care and domestic responsibilities [Sermons et al., 2001]. Entrapment hypothesis suggests that females tend to be constrained to a smaller travel area due to household responsibility and the sort of employment available to females [Cristaldi et al., 2005]. However, another study, suggests that females are more likely to form carpools [Bulinug et al., 2009]. The other studies suggest that females are more likely to carpool because they face greater mobility constraints than males [washbrook et al., 2006], [Fergusen 1995]. Furthermore, an investigation on the nature and motivation of public response to Yorkshare ridesharing schemes in Britain, suggest that males tend more to participate as drivers while females tend more to participate as passengers and female passengers are more preferred by male and by female drivers as well. [Bonswall et al., 1984].

2.12.2. Age

One of the other socio-demographic factors that influences rideshare participation is difference between age cohorts on trip choices. Nazem et al. (2011) suggests that compared to adults, elderly people are less sensitive to the number of transfers and elderly people tolerate the commute and waiting times much better than the young and adult commuters [Nazem et al., 2011]. Yan et al. (2002) suggests that middle age commuters are more likely to use a toll road [Yan et al., 2002]. Although, the literature suggests that while participation in carpooling increases across the age profile up to 54 years of age [Winn, 2005] and the likelihood of achieving a successful outcome increases with age [Bulinug et al., 2009], the elderly commuters are less likely than others to participate in carpool formation through the deployment of a web-based carpool formation application [Ferguson, 1997], [Baldassare et al., 1998].

2.12.3. Age and Gender

The literature suggests different trip choices for combination of age and gender. For example, elderly men commuters are more sensitive than women are to transit access and egress times [Nazem et al., 2011]. Bonswall (1984) suggests that people in carpooling arrangements prefer to travel with people of their own age cohort and that females are particularly reluctant to give lifts to, or to pool with, men over 50. The study also revealed that drivers are likely to be males aged 30-50 and riders are likely to be female under 30 [Bonswall et al., 1984].

2.12.4. Trip distance

Deakin et al. (2010) in a recent research conducted to assess the potential for dynamic ridesharing for travel to downtown Berkeley, California, and the University of California, Berkeley, campus, suggests that most commuters tend to go slightly out of their way or wait a short time to obtain or offer a ride. The commuters who travel less than a mile or two are less interested in dynamic ridesharing than those who travel farther because of the excessive time required to make the connection and accommodate a pickup and drop off [Deakin et al., 2010]. Bonswall, et al. (1984) suggests that there is a positive correlation between journey length and likelihood of rideshare requests and proposes that the longer one's journey is, the more attractive will ridesharing appear due to the smaller contact costs (diversions to pick up a passenger, waiting for one's partner, etc.) relative to the cost savings as well as the greater absolute cost savings of ridesharing [Bonswall et al., 1984].

2.12.5. Time to match up

In a research conducted to assess the potential for dynamic ridesharing for travel to downtown Berkeley, California, and the University of California, Berkeley, campus, it is concluded that the 10 min added time for a match was too high for many of the users and a shorter time, 3 to 5 min maximum, cuts matches down considerably. [Deakin et al., 2010].

2.12.6. Occupancy preferences

Although effective usage of empty car seats that leads to increased occupancy rates is the primary objective for dynamic rideshare systems, there are influencing factors to have preferences on the number of people sharing a ride including the safety issues, seating space of the participating car as well as the personal preferences of riders and drivers such as convenience. Besides, the transportation demand management (TDM) policy strategies and promotions such as minimum requirement of HOV lanes and free or reduced-price access to high-occupancy toll (HOT) influence the occupancy. In HOV lanes in Interstate 84 Westbound, the average occupancy is 2.11 persons per automobile and in Interstate 91 Southbound, the average occupancy is 2.10 persons per automobile in HOV lanes [High Occupancy Vehicle Lane Report 2010].

2.12.7. Other preferences

One of the advantages of dynamic ridesharing is that a user can also find trips that fit his/her unique needs. For example, if a rider has a pet, he should find a pet-friendly driver. If he does not smoke, he should find a non-smoking driver. Kowshik et al. (1993) suggest that rideshare users are quite demanding of what they desire and system functionality of rideshare needs to address the user needs [Kowshik et al., 1993]. For the purposes of this research, smoking habits or preferences and pet friendliness have been considered as unique user needs. It is reasonable to assume that people are less likely to share a ride with a smoker or with a person who is commuting with a pet.

2.13. Summary of Reviews

Before 2004, almost all pilot test projects shared a number of common characteristics. All but the Seattle project were abandoned for low usage. They all suffered from a small number of requests for rides and a smaller number of matches made. This failure could be attributed to how each was designed. Commuter behavior is important to understanding what happened.

There were many reasons for low dynamic ridesharing before 2004. The deficiencies in the number of users' participation in the programs was mainly because of the lack of awareness of the ridesharing programs, insufficient incentives to encourage people to rideshare, safety concerns about sharing rides with strangers, and inflexibility of the existing rideshare programs. Other contributors in the low dynamic ridesharing included lack of funding for the systems' operation, lack of institutional support and incentives, time consuming process to receive a match list, and then burdensome attempts to make contact with possible drivers with no guarantee that a match would be made. After 2004 technological and computing advances help to overcome many of the potential obstacles. Internet-enabled technologies such as World Wide Web, Smart phones, Global Positioning System (GPS), Data Repository, Automated Financial Transactions, social networks, and automated ride-matching software are enabling technologies for ridesharing to organize rides in real time either a few minutes before the trip takes place or while the trip is occurring with passengers picked up and dropped off along the way.

Moreover, there has been significant growth and overall success with the strategy of partnerships between ride-matching software companies and the large-scale clients. This partnership strategy has gained more users and is most suited for commuters with regular schedules. Many public agencies and companies have started promoting ridesharing by providing incentives. The rise of social networks has enabled ridesharing companies to better address the security concerns of sharing a ride between potential riders and drivers and their friends [Chan and Shaheen, 2011].

There are many applications and software platforms that offer dynamic ridesharing services. However, the systems typically serve only as platforms that bring users together, rather than as active mechanisms that generate rideshare plans. All of the underlying systems use some form of algorithm to match riders and passengers. Some of the algorithms do so based only on origin and destination, while some of the newer algorithms match drivers and passengers based on the commonality of their travel route. The review of the literature revealed that development of optimization algorithms for real-time matching of participants has been largely ignored by transportation research community that has recently started pondering the value of optimization for tackling the problem. For example, Agatz et al. (2010) considered the problem of matching drivers and riders in the dynamic setting. In their research a simulation study based on 2008 travel demand data from metropolitan Atlanta was presented. The simulation results indicated that the use of sophisticated optimization methods instead of simple greedy matching rules substantially increases the likelihood that rideshare matches can be found for users, and improves the performance of ridesharing systems through larger overall system travel cost savings.

In another research carried out earlier Teodorovic and Dell'Orco (2005) explored the possible applications of collective bee intelligence in solving non-deterministic combinatorial problems. In that respect, they introduced Fuzzy Bee System (FBS) where the agents used fuzzy logic rules in their approximate reasoning. Bee System is an improved genetic algorithm depending on the behavior of bees proposed by Sato and

Hagiwara (1997). Teodorovic and Dell'Orco (2005, 2008) tested the performance of FBS on a ride-matching problem which aimed to constitute routing and scheduling of the vehicles and passengers by minimizing the total distance travelled by all participants, minimizing the total delay, or making relatively equal utilization of vehicles. They defined the problem as making "…routing and scheduling of the vehicles and passengers for the whole week in the best possible way". It was assumed that at the beginning of each week the following information for all participants in the program was available: vehicle capacity, origin and destination locations as well as the fixed desired departure and/or arrival time information for every day in a week that person was ready to participate in ridesharing. They collected data for 97 rideshare demands in a small city in southeastern Italy. Although, there were no theoretical results that could support the proposed approach, they indicated that preliminary results were very promising.

Amey et al. (2011) studied real-time ridesharing to identify, highlight, and discuss the potential benefits of, and challenges to, real-time ridesharing. They suggested that to have a successful ridesharing, a series of economic, behavioral, institutional, and technological challenges needs to be overcome [Amey, 2011].

The Avego's iPhone app is one example of real-time services that combines GPS-enabled real-time ride-matching with payment transactions to match a driver with riders searching for a ride along the same route. (http://www.avego.com).

Deakin (2010) conducted a research to assess the opportunities and challenges for dynamic ridesharing for travel to downtown Berkeley, California, and the University of California, Berkeley, campus. The study suggests that about one-fifth of commuters who drive alone to the campus are willing to use dynamic ridesharing at least occasionally.

The study also suggests that financial incentives and carpool parking subsidies greatly increase willingness to dynamic ridesharing [Deakin et al., 2010].

Xing et al., (2009) introduces a spontaneous ridesharing concept for short distance travel within metropolitan areas and presented a multi-agent-based simulation system, Smize ridesharing system. The Smize ridesharing system comprises two classes of software agents:user agents and supportive agents. User agents include Driver agents (DriverAgent) which advertises and manages new ridesharing opportunities from drivers and passenger agents (PassengerAgent) which searches and negotiates ride-sharing agreements for passengers looking for a transport opportunity. The community of user agents relies on service of an appropriate number of routing agents (RoutingAgent) for the calculation of drive and walk routes and a single administrative agent (NodeAgent) which maintains and provides access to a database of drive route information of active vehicles as well as ridesharing preferences of their drivers [Xing et al., 2009].

Chapter 3: Problem Definition

This research considers a Dynamic Rideshare Optimized Matching Problem (DROM) which attempts to spontaneously identify suitable matches between passengers requesting rideshare services with relevant drivers available to carpool for credits and HOV lane privileges. DROM receives registered passengers and drivers information and preferences continuously over time. At any time t the set of passengers in the system P_t is partitioned as $P_t = \mathrm{O}_t \cup \mathrm{W}_t$ where O_t is the set of onboard passengers already started their service and have not left the system at t and W_t is the set of passengers in the system waiting for a ride. The set of drivers D_t is further partitioned as $\mathrm{D}_t = \mathrm{D}_t^{s_0} \cup \mathrm{D}_t^{s_1} \cup \mathrm{D}_t^{s_2} \cup \ldots$ where $\mathrm{D}_t^{s_j}$ is the set of drivers in the system with j seats available for passengers to be assigned. For those drivers belonging to the subset $\mathrm{D}_t^{s_0}$, it means that there is no more seat available for passengers to be assigned. As time Δ elapses, new request for rideshare from passengers and drivers arrive and also some passengers and drivers depart the system. This addition and deletion needs to be incorporated into the existing carpool paths or new carpool created to handle them. Thus, at any time $\mathrm{t} + \Delta$, $\mathrm{P}_{t+\Delta}$=P_t + $\mathrm{P}_{t,t+\Delta}$- $\mathrm{P'}_{t,t+\Delta}$ where $\mathrm{P}_{t,t+\Delta}$ is the set of newly arrived passengers and $\mathrm{P'}_{t,t+\Delta}$ are the passengers left or gave up the system in the time slot (t, t+Δ). $\mathrm{P'}_{t,t+\Delta}$ is further partitioned as $\mathrm{P'}_{t,t+\Delta} = \mathrm{P'}^{\text{given up}}_{t,t+\Delta} \cup \mathrm{P'}^{\text{left}}_{t,t+\Delta}$. Likewise, $\mathrm{D}_{t+\Delta}$=D_t + $\mathrm{D}_{t,t+\Delta}$- $\mathrm{D'}_{t,t+\Delta}$, where $\mathrm{D}_{t,t+\Delta}$ and $\mathrm{D'}_{t,t+\Delta}$ are the set of newly added drivers and the drivers who left the system in time slot (t,t+Δ).

For each passenger $i \in P_t$ there are origin and destination points v_O^i and v_D^i. At first, DROM looks into the passenger's origin and destination points and examines the latest status of the system attempting to find an available driver who will visit the passenger's origin and destination points or will pass by nearby points. As soon as the system matches the passenger with an available driver, pickup and drop off points of passenger $i \in P_t$ will be set as the interest points that will be visited by the driver denoted by v_p^i and v_d^i. Associated with each passenger $i \in P_t$ is a requested time to start the service at the origin point denoted by t_O^i . The system may assign a different time for pickup that is denoted by t_p^i. Likewise, for each driver $j \in D_t$ there is an origin-destination pair point denoted by (v_O^j, v_D^j). Associated with each driver $j \in D_t$ is a departing time t_O^j from origin, the number Q^j of places available on the vehicle, the origin to destination route in the form of successive nodes as well as personal information and ridesharing preferences. In additions, for each pick up and drop off route per passenger $i \in P_t$ there would be a credit b_{ij} assigned to the driver $j \in D_t$.

Along with the aforementioned information, DROM takes in other personal information and ridesharing preferences for each passenger $i \in P_t$ and driver j $\in D_t$ and attempts to match passengers with the available drivers. Table 5 shows the most relevant information and preferences. Multiple competing or non-competing objectives can be defined for DROM including minimizing the total vehicle Miles traveled, minimizing the total delay time and minimizing the total travel time to name a few. For the interests of this research, maximizing the total number of matches in a given period of time is considered as the objective function that accounts for the ultimate goal of maximizing the rate of participating in the rideshare program. DROM asks to assign passengers to drivers and to identify the feasible routes to be driven by the drivers in order to:

- Maximize the overall system performance

Such that:

- Seating capacity is satisfied.
- The number of connections for each passenger is less than a predetermined parameter.
- Passenger time widows are met.
- Detour distance for each driver is less than a predetermined parameter.
- Relocating distances for riders are less than a predetermined parameter.
- Ridesharing preferences are secured.
    - Age matching preferences are satisfied.
    - Gender matching preferences are met.
    - Smoking matching preferences are secured.
    - Pet restrictions are met.
    - Preferences on maximum number of people sharing a ride are met.

Table 5: Personal information and ridesharing preferences

Information/preferences	Passenger p_i	Driver d_k	Type of input
Gender	☐	☐	Male or Female
Age	☐	☐	Young, Middle, Elderly
Smoker	☐	☐	Yes or No
Number of passengers	☐	-	1,2,…
Number of seats	-	☐	1,2,…, Q_k
Pet friendly	☐		Yes or No
Pet restriction		☐	Yes or No
Smoke restriction		☐	Yes or No
Flexible with relocating to a nearby walking distance point	☐		Yes or No
Flexible with detour		☐	Yes or No
Flexible with reconnection	☐		Yes or No

Chapter 4: Problem Formulation

This section presents the MIP mathematical formulation for DROM. First, some notations used through the formulation are presented and followed with the definition of internal decision variables and decision variables used in the model and then the model considered in this research is stated.

4.1. Notation

At each time t:

P_t: the set of passengers in the system at time t; $i \in P_t$;

O_t : the set of passengers on board at time t ; $O_t^j \subset O_t$

O_t^j: the set of passengers on board the vehicle belonging to driver $j \in D_t$

D_t: the set of passengers at time t

For passenger $i \in P_t$:

v_O^i: origin point of passenger $i \in P_t$; $v_O^i \in V^i$

v_D^i: Destination point of passenger $i \in P_t$; $v_D^i \in V^i$

t_O^i : requested time to start the service by passenger $i \in P_t$ at the origin point

V^i: the set of nodes to be visited by passenger $i \in P_t$ en route from origin to destination in the form of successive nodes

For driver $j \in D_t$:

v_m^j: origin/current point of driver $j \in D_t$; $v_m^j \in V^j$

t_O^j: departing time of driver j from his/her origin,

Q^j: the number of seats available on the vehicle belonging to driver $j \in D_t$

V^j: the set of nodes to be visited by driver $j \in D_t$ en route from origin to destination in the form of successive nodes

For point of interest $v \in V$; $V = V^i \cup V^j$:

$\|D\|$: distance matrix

D_{v_m, v_n} : distance of travel between two successive points $v_m, v_n \in V$

$t^j_{v^i_O, v^j_m}$: the travel time between origin point of the rider, v^i_O and point of interest v^j_m for driver j

$t^i_{v^j_m, v^{j'}_k}$: walking time for passenger from v^j_m to $v^{j'}_k$.

Personal information and ridesharing preferences for each passenger i and driver j:

Q^i, Q^j: the favorable car occupancy defined by passenger i, and driver j.

a^i : numerical value for the age class of passenger $i \in P_t$ defined as 1 for Young, 2 for Middle age, or 3 for Elderly.

η^i: set of age preferences for passenger $i \in P_t$ which would be specified by selecting one of the 7 possible combinations: {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

η^i_m : the mth element of the age preference set of passenger $i \in P_t$

a^j : numerical value for the age class of driver $j \in D_t$ defined as 1 for Young, 2 for Middle age, or 3 for ElderlyElderly.

η^j: set of age preferences for driver $j \in D_t$ which would be specified by selecting one of the 7 possible combinations: {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

η^j_m : the mth element of the age preference set of passenger $j \in P_t$

g^i: numerical value for the gender of the passenger $i \in P_t$ defined as 1 for Male and 2 for Female.

g^j: numerical value for the gender of driver $j \in D_t$ defined as 1 for Male and 2 for Female.

∂^i: set of gender preferences for passenger $i \in P_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

∂^i_m : the mth element of the gender preference set of passenger $i \in P_t$.

∂^j: set of gender preferences for driver $j \in D_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

∂^j_m : the mth element of the gender preference set of driver $j \in D_t$.

s^i: numerical value for the smoking class of passenger $i \in P_t$ defined as 1 for smoker and 2 for non-smoker.

Ω^i: set of smoking preferences for passenger $i \in P_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

μ^i_m: the mth element of the smoking preference set of passenger$i \in P_t$

s^j: numerical value for the smoking class of driver $j \in D_t$defined as 1 for Smoker and 2 for Non-smoker.

Ω^j: set of smoking preferences for driver $j \in D_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

μ^j_m : the mth element of the smoking preference set of driver $j \in D_t$

p^i: numerical value for the pet tendency class of passenger $i \in P_t$ defined as 1 for Pet friendly and 2 for Non- pet friendly.

ω^i: set of pet tendency preferences for passenger $i \in P_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

ζ^i_m: the mth element of the pet preference set of passenger $i \in P_t$

p^j: numerical value for the pet tendency class of driver $j \in D_t$ defined as 1 for Pet friendly and 2 for Non- pet friendly.

ω^j: set of pet tendency preferences of driver $j \in D_t$ which would be specified by selecting one of the 3 possible combinations: {1}, {2}, {1,2}

ζ_m^j: the mth element of the pet preference set of driver $j \in D_t$

Other input parameters:

δ **:** a predetermined parameter for maximum number of connections per passenger

γ : a predetermined time parameter for maximum waiting time of a rider

β : a predefined distance parameter for maximum detour distance of a driver

φ : a predefined distance parameter for maximum relocation distance of riders

M : a large positive numerical value.

4.1.1 Internal Decision variables

$$\rho_{ijt} = \begin{cases} 1; & riding\ preferences\ of\ passenger\ i \in P_t\ matches\ driver\ j \in D_t\ and\ all \\ & the\ passengers\ on\ board\ i' \in O_t^j\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$A_t^{ij} = \begin{cases} 1; & age\ pref.\ of\ pass.\ i \in P_t\ matches\ the\ age\ class\ driver\ j \in D_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$A_t^{ji} = \begin{cases} 1; & age\ pref.\ of\ driver\ j \in D_t\ matches\ age\ class\ pass.\ i \in P_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$A_t^{ii'} = \begin{cases} 1; & age\ pref.\ pass.\ i \in P_t\ matches\ age\ class\ pass.\ on\ board\ i' \in O_t^j\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$A_t^{i'i} = \begin{cases} 1; & age\ pref.\ pass.\ on\ board\ i' \in \mathrm{O}_t^{\mathrm{j}}\ matches\ age\ class\ pass.\ i \in P_t\ at\ time\ t \\ 0; & Otherwise \end{cases}$$

$$G_t^{ij} = \begin{cases} 1; & gender\ pref.\ for\ passenger\ i \in P_t\ matches\ driver\ j \in D_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$G_t^{ji} = \begin{cases} 1; & gender\ pref. of\ driver\ j \in D_t\ matches\ pass.\ i \in W_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$G_t^{ii'} = \begin{cases} 1; & gender\ pref. for\ pass.\ i \in P_t\ matches\ pass. on\ board\ i' \in \mathrm{O}_t^{\mathrm{j}}\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$G_t^{i'i} = \begin{cases} 1; & gender\ pref. of\ pass. on\ board\ i' \in \mathrm{O}_t^{\mathrm{j}}\ matches\ pass. i \in P_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$S_t^{ij} = \begin{cases} 1; & smoking\ pref.\ passs. i \in P_t\ matches\ smoking\ habit\ driver\ j \in D_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$S_t^{ji} = \begin{cases} 1; & smoking\ pref. driver\ j \in D_t\ matches\ smoking\ habit\ pass. i \in P_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$S_t^{ii'} = \begin{cases} 1; & smoking\ pref. pass.\ i \in P_t\ matches\ pass. on\ board\ i' \in \mathrm{O}_t^{\mathrm{j}}\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$S_t^{i'i} = \begin{cases} 1; & smoking\ pref. pass. on\ board\ i' \in \mathrm{O}_t^{\mathrm{j}}\ matches\ pass. i \in P_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$P_t^{ij} = \begin{cases} 1; & pet\ pref.\ pass. i \in P_t\ matches\ riding\ rules\ driver\ j \in D_t\ at\ time\ t \\ 0; & otherwise \end{cases}$$

$$P_t^{ji} = \begin{cases} 1;\ pet\ pref.\ \ driver\ j \in D_t\ \ matches\ \ pass.\ i \in P_t\ at\ time\ t \\ 0;\ otherwise \end{cases}$$

$$P_t^{ii'} = \begin{cases} 1;\ pet\ pref.\ \ pass.\ i \in P_t\ \ matches\ \ pass.\ on\ board\ i' \in \mathrm{O}_t^j\ at\ time\ t \\ 0;\ otherwise \end{cases}$$

$$P_t^{i'i} = \begin{cases} 1;\ pet\ pref.\ \ pass.\ on\ board\ i' \in \mathrm{O}_t^j\ \ matches\ \ pass.\ i \in P_t\ at\ time\ t \\ 0;\ otherwise \end{cases}$$

$$\rho_{ijt}^A = \begin{cases} 1;\ age\ pref. pass.\ i \in P_t\ \ matches\ driver\ j \in D_t\ and\ pass.\ on\ board\ i' \in\ \mathrm{O}_t^j\ at\ time\ t \\ 0;\ otherwise \end{cases}$$

$$\rho_{ijt}^G = \begin{cases} 1;\ gen. pref.\ \ pass.\ i \in P_t\ \ matches\ driver\ j \in D_t\ and\ pass.\ on\ board\ i' \in\ \mathrm{O}_t^j\ at\ time\ t \\ 0;\ otherwise \end{cases}$$

$$\rho_{ijt}^S = \begin{cases} 1;\ sex\ pref.\ \ pass.\ i \in P_t\ \ matches\ driver\ j \in D_t\ and\ pass. on\ board\ i' \in\ \mathrm{O}_t^j\ at\ time\ t \\ 0;\ otherwise \end{cases}$$

$$\rho_{ijt}^P = \begin{cases} 1;\ pet\ pref.\ pass.\ \ P_t\ matches\ driver\ j \in D_t\ and\ tpass.\ on\ board\ i' \in\ \mathrm{O}_t^j\ at\ time\ t \\ 0;\ otherwise \end{cases}$$

$$\rho_{ijt}^M = \begin{cases} 1;\ occu. pref.\ \ pass.\ i \in P_t\ \ matches\ driver\ j \in D_t\ and\ pass. on\ board\ i' \in\ \mathrm{O}_t^j\ at\ time\ t \\ 0;\ otherwise \end{cases}$$

$P^1_{ijv_p^j v_d^j k v_p^k v_d^k}$ and $D^1_{ijv_p^j v_d^j k v_p^k v_d^k}$: travel time for rider $i \in P_t$ and total travel time for driver $j \in D_t$ and driver $k \in D_t$, respectively to share a one-connection ride for the rider with pickup point $v_p^j \in V^j$, connection point $v_d^j \in V^j, v_p^k \in V^k$ and drop-off point $v_d^k \in V^k$.

$P^2_{ijv_p^j v_d^j k v_p^k v_d^k l v_p^l v_d^l}$ and $D^2_{ijv_p^j v_d^j k v_p^k v_d^k l v_p^l v_d^l}$: travel time for rider $i \in P_t$ and total travel time for driver $j \in D_t$,driver $k \in D_t$, and driver $l \in D_t$ respectively to share a two-connection ride for the rider with pickup point $v_p^j \in V^j$, the first connection point $v_d^j \in V^j, v_p^k \in V^k$ and the second connection point $v_d^k \in V^k, v_p^l \in V^l$ and drop-off point $v_d^l \in V^l$.

4.1.2 Decision variables

$$PS^1_{ijtv_m^j} = \begin{cases} 1; \ pass.\, i \text{ could be picked up by driver } j \text{ at point } v_m^j \text{ for the first leg of journey} \\ \quad \text{with respect to proximity in spa} \\ 0; \ otherwise \end{cases}$$

$$PS^2_{ijtv_m^j} = \begin{cases} 1; \ pass.\, i \text{ could be picked up by driver } j \text{ at point of origin } v_O^i \text{ for the first leg of journey} \\ \quad \text{when driver j makes a detour at point } v_m^j \text{, with respect to proximity in space} \\ 0; \ otherwise \end{cases}$$

$$PT^1_{ijtv_m^j} = \begin{cases} 1; \ pass.\, i \text{ could be picked up by driver } j \text{ at point } v_m^j \text{ for the first leg of journey} \\ \quad \text{with respect to proximity in time} \\ 0; \ otherwise \end{cases}$$

$$PT^2_{ijtv_m^j} = \begin{cases} 1; \ pass.\, i \text{ could be picked up by driver } j \text{ at point of origin } v_O^i \text{ for the first leg of journey} \\ \quad \text{when driver j makes a detour at point } v_m^j \text{, with respect to proximity in time} \\ 0; \ otherwise \end{cases}$$

$$P_{ijtv_m^j t} = \begin{cases} 1; \ \text{at least one of two binary variables } P^1_{ijtv_m^j t} \text{ and } P^2_{ijtv_m^j t} \text{ is equal to 1} \\ 0; \ otherwise \end{cases}$$

$$DS^1_{ijtv_m^j} = \begin{cases} 1; \ pass.\, i \text{ could be dropped off by driver } j \text{ at point } v_m^j \text{ for the last leg of journey} \\ 0; \ otherwise \end{cases}$$

$$DS^2_{ijtv_m^j} = \begin{cases} 1; \ pass.\,i\ could\ be\ dropped\ off\ by\ driver\ j\ at\ point\ v_m^j\ for\ the\ last\ leg\ of\ journey \\ \quad with\ respect\ to\ proximity\ in\ space \\ 0;\ otherwise \end{cases}$$

$$DS_{ijt} = \begin{cases} 1;\ \text{at least one of two binary variables } DS^1_{ijtv_m^j} \text{ and } DS^2_{ijtv_m^j} \text{ is equal to 1} \\ 0;\ otherwise \end{cases}$$

$$CS^1_{jv_k^{j'}j'v_m^j t} = \begin{cases} 1;\ pass.\,i\ \ leaves\ driver\ j\ at\ point\ v_m^j\ to\ walk\ to\ point\ of\ connection\ with\ driver\ j' \\ 0;\ otherwise \end{cases}$$

$$CS^2_{jv_k^{j'}j'v_m^j t} = \begin{cases} 1;\ driver\ j\ makes\ a\ detour\ at\ point\ v_m^j\ to\ pickup/drop\ off\ passenger\ i\ at\ point\ v_k^{j'} \\ \quad which\ is\ en\ route\ point\ for\ driver\ j' \\ 0;\ otherwise \end{cases}$$

$$C_{ijv_m^j j'v_k^{j'}t} = \begin{cases} 1;\ \text{at least one of the two binary variables } C^1_{ijv_m^j j'v_k^{j'}t} \text{ and } C^2_{ijv_m^j j'v_k^{j'}t} \text{ equals 1} \\ 0;\ otherwise \end{cases}$$

$t^j_{v_m^j}$: the time at which driver j meets point of interest v_m^j

$$RF_{ijv_m^j} = \begin{cases} 1;\ \text{pass.}\,i \text{ could be picked up by driver } j \text{ at point } v_m^j \text{ or when driver } j \text{ makes a detour} \\ \text{at point } v_m^j \text{ to pick up the passenger at the point of origin.} \\ 0;\ otherwise \end{cases}$$

$$RL_{ijv_m^j} = \begin{cases} 1;\ \text{pass.}\,i \text{ could be dropped off by driver } j \text{ at point } v_m^j \text{ or when driver } j \text{ makes a detour} \\ \text{at point } v_m^j \text{ to drop off the passenger at the point of origin.} \\ 0;\ otherwise \end{cases}$$

$$RFL_{ij} = \begin{cases} 1;\ \text{passenger } i \text{ could be picked up and dropped off by driver } j \\ 0;\ otherwise \end{cases}$$

$$RFL_{ij} = \begin{cases} 1; \text{ passenger } i \text{ could be picked up by driver } j \text{ and dropped off by driver } j' \\ 0; \; otherwise \end{cases}$$

$T^1_{ijv^j_m t}$: added time to travel time of driver j between points of interest v^j_m and v^j_{m+1} due to making a detour to pick up passenger i at his/her point of origin.

$T^2_{ijv^j_m t}$: added time to travel time of driver j between points of interest v^j_m and v^j_{m+1} due to making a detour to drop off passenger i at his/her destination.

$T^3_{ijv^j_m t}$: added time to travel time of driver j between points of interest v^j_m and v^j_{m+1} due to making a detour to pick up and drop off passenger i at his/her destination.

$T^4_{ijv^j_m j' v^{j'}_k t}$: added time to travel time of driver j between points of interest v^j_m and v^j_{m+1} due to making a detour to pick up passenger i at connection point $v^{j'}_k$ belonging to driver j'.

$T^5_{ijv^j_m j' v^{j'}_k t}$: added time to travel time of driver j between points of interest v^j_m and v^j_{m+1} due to making a detour to pick up passenger i at connection point $v^{j'}_k$ belonging to driver j' and then drop off the passenger at his/her destination.

$$R^0_{ijv^j_p v^j_d} = \begin{cases} 1; \; a \; 0 \; connection \; route \; with \; pickup \; point \; v^j_p \in V^j, drop-off \; point \; v^j_d \in V^j \\ \quad between \; rider \; i \in P_t and \; driver \; j \in D_t \\ 0; \quad Otherwise \end{cases}$$

$$R^1_{ijv^j_p v^j_d k v^k_p v^k_d} = \begin{cases} 1; \; a \; 1 \; connection \; route \; with \; pickup \; point \; v^j_p \in V^j, connection \; point \; v^j_d \in V^j, v^k_p \in V^k \\ \quad drop-off \; point \; v^k_d \in V^k \;\; between \; rider \; i \in P_t, driver \; j \in D_t, driver \; k \in D_t \\ 0; \quad Otherwise \end{cases}$$

$$R^2_{ijv_p^j v_d^j k v_p^k v_d^k l v_p^l v_d^l} = \begin{cases} 1; & a\ 2\ connection\ route\ with\ pickup\ point\ v_p^j \in V^j, first\ connection\ point\ v_d^j \in V^j, \\ & v_p^k \in V^k, second\ connection\ point\ v_d^k \in V^k, v_p^l \in V^l, drop-off\ point\ v_d^l \in V^l \\ & between\ rider\ i \in P_t, driver\ j \in D_t\ , driver\ k \in D_t, driver\ l \in D_t \\ 0; & Otherwise \end{cases}$$

4.2. Constraints

4.2.1. Proximity in time and space

Once an event such as receiving a request for rideshare from a driver or passenger, or establishing a rideshare after making a match between a driver and passenger, or even a drop-off or pickup triggers the system, a set of successive points, v_1^j, $v_2^j, v_3^j, \dots, v_n^j$ will be defined for each driver j. The set of points includes the unmet points before the trigger and every updates resulting from the trigger such as adding pickup and drop off points for passengers assigned to driver j or deleting the pickup or drop off points of passengers who are picked up or dropped off at the corresponding points. DROM is not responsible to find the best path to meet the points as it is assumed that drivers have the relevant technologies or experiences to find their own best path. . The origin and destination points for each passenger i requesting a rideshare in the system are denoted by v_O^i and v_D^i.

When the point of origin for passenger i is one of the points to be visited by driver j or it is within an acceptable walking distance, φ, from a point to be visited by driver j, or the point of origin for passenger i is within the acceptable detour distance, β, from the current location of driver j , driver j could be a promising driver to pick up passenger i for passenger i's first leg of journey with respect to proximity in space assuming all other conditions satisfied.

$$PS_{ijtv} = \begin{cases} 1 & ; \quad D_{v_O^i, v_m^j} \leq \varphi \ or \ D_{v_m^j, v_O^i} + D_{v_O^i, v_{m+1}^j} \leq D_{v_m^j, v_{m+1}^j} + \beta; \ v_m^j, v_{m+1}^j \in V^j \\ 0 & ; \quad otherwisez \end{cases}$$

(3)

Where v_O^i is the point of origin for passenger *i*. $D_{v_O^i, v_m^j}$ and $D_{v_m^j, v_{m+1}^j}$ are respectively the relocating distance for passenger i and the distance of travel independent of time between two successive nodes m and m+1 to be visited by driver j.

A driver could be a promising driver to pick up a passenger for his first leg of journey with respect to proximity in time assuming all other conditions satisfied.

$$PT_{ijtv} = \begin{cases} 1 & ; \quad t_O^i + t^i_{v_O^i, v_m^j} \leq t^j_{v_m^j} \leq \gamma_i + t_O^i + t^i_{v_O^i, v_m^j} \ or \\ & \quad t_O^i \leq t^j_{v_m^j} + t^j_{v_m^j, v_O^i} \leq \gamma_i + t_O^i; \ v_m^j, v_{m+1}^j \in V^j \\ 0 & ; \quad otherwise \end{cases} \tag{4}$$

When the point of destination for passenger i is one of the points to be visited by driver j or it is within an acceptable walking distance, φ, from a point to be visited by driver j, or the point of destination for passenger i is within an acceptable detour distance, β, from the location points of driver j, driver *j* could be a promising driver to drop off passenger *i* for last leg of passenger *i*'s journey with respect to proximity in distance assuming all other conditions satisfied.

$$DS_{ijtv} = \begin{cases} 0 \ or \ 1 & ; \quad D_{v_D^i, v_m^j} \leq \varphi \ or \ D_{v_m^j, v_D^i} + D_{v_D^i, v_{m+1}^j} \leq D_{v_m^j, v_{m+1}^j} + \beta; \ v_m^j, v_{m+1}^j \in V^j \\ 0 & ; \quad otherwise \end{cases}$$

(5)

Where v_D^i and $D_{v_D^i, v_m^j}$ are the original point of destination and the relocating distance for passenger i, respectively. $D_{v_m^j, v_{m+1}^j}$ is the distance of travel independent of time between two successive points v_m^j , v_{m+1}^j.

4.2.2. Origin-Destination route related constraints

When the first and last leg of journey for passenger i are shared with driver j, it means that driver j could be assigned to give a ride to passenger i for the entire trip from his or her origin to his or her destination without a need for reconnection. i.e.:

$$R_{ijtv} = \begin{cases} 0 \ or \ 1 & ; \quad P_{ijt} = DS_{ijt} \ AND \ P_{ijt} \neq 0 \\ 0 & ; \quad otherwise \end{cases} \tag{6}$$

When the first and last leg of journey for passenger i are not shared with driver j, it means that we need a feasible connection between the origin and destination of passenger i. Let's suppose that the first leg of journey for passenger i is shared with driver j and the last leg is shared with driver j'. At time t, the successive visiting points for driver j after he picks up passenger i are: $R_t^j = \{v_m^j, \dots, v_{m+n}^j\}$ and the remaining successive points for driver j' before he drops off passenger i are $R_t^{j'} = \{v_k^{j'}, ,\dots, v_{k+l}^{j'}\}$. From the viewpoint of proximity in space, passenger i would like to change the ride from driver j to driver j' when there is a node en route for driver j within φ Miles walking distance of a node to be visited by driver j'. Alternatively, passenger i can change ride when driver j makes a detour with no more than β_j Miles to drop off the passenger at a point to be visited by driver j'. or else when driver j' can make a detour with no more than $\beta_{j'}$ Miles to pick up the passenger who already left driver j at a point visited by driver j, i.e., $R_t^{jj'} = R_t^j \cap R_t^{j'} \neq \emptyset$. When a rider is picked up and dropped off by the same driver, then there is a zero-connection or direct route for the rider. i.e.

$$R^0_{ijv_p^j v_d^j} = \begin{cases} 1 & ; \quad PS_{ijtv_p^j} = PT_{ijtv_p^j} = DS_{ijtv_d^j} = 1 \ for \ i \in P_t, j \in D_t; \ v_p^j, v_d^j \in V^j, v_d^j \geq v_p^j \\ 0 & ; \quad otherwise \end{cases} \tag{7}$$

Each zero-connection route $R^0_{ijv_p^j v_d^j}$ is characterized with rider and driver travel times denoted by $P^0_{ijv_p^j v_d^j}$ and $D^0_{ijv_p^j v_d^j}$ respectively and calculated in terms of non-zero pick up and drop off variables.

If a rider is picked up by a driver and is dropped off by another driver, and there is a connection point to be visited by the either of the drivers, then there is a one-connection route for the rider who is flexible with one connection along the route. i.e.,

$$R^1_{ijv_p^j v_d^j k v_p^k v_d^k} = \begin{cases} 1 \quad ; \quad PS_{ijtv_p^j} = PT_{ijtv_p^j} = CS_{ijv_d^j k v_p^k} = CT_{ijv_d^j k v_p^k} = DS_{iktv_d^k} = 1 \\ \quad or\ i \in P_t;\ j,k \ \in D_t;\ v_p^j, v_d^j \in V^j, v_d^j \geq v_p^j;\ v_p^k, v_d^k \in V^k, v_d^k \geq v_p^k \\ 0 \quad ; \quad Otherwise \end{cases}$$

(8)

Each one-connection route $R^1_{ijv_p^j v_d^j k v_p^k v_d^k}$ is characterized with rider and total driver travel times denoted respectively by $P^1_{ijv_p^j v_d^j k v_p^k v_d^k}$ and $D^1_{ijv_p^j v_d^j k v_p^k v_d^k}$ and calculated in terms of non-zero pick up and drop off variables.

Finally, if a rider is flexible with more than one connection and can be picked up by a driver and would be dropped off by another driver, and there is a connection point between the second driver and a third driver who will meet the second driver in a connection point to be visited, then there is a two-connection route for the rider. i.e.,

$$R^2_{ijv_p^j v_d^j k v_p^k v_d^k l v_p^l v_d^l} = \begin{cases} 1 \quad ; \quad PS_{ijtv_p^j} = PT_{ijtv_p^j} = CS_{ijv_d^j k v_p^k} = CT_{ijv_d^j k v_p^k} = CS_{ijv_d^k k v_p^l} = CT_{ijv_d^k k v_p^l} = DS_{iktv_d^l} = 1 \\ \quad for\ i \in P_t;\ j,k \ \in D_t;\ v_p^j, v_d^j \in V^j, v_d^j \geq v_p^j;\ v_p^k, v_d^k \in V^k, v_d^k \geq v_p^k;\ v_p^l, v_d^l \in V^l, v_d^l \geq v_p^l \\ 0 \ ; \quad Otherwise \end{cases}$$

(9)

For simplicity and without loss of generality, it is assumed that riders are not interested in more than two connections before they reach to their destination.

4.2.3. Continuity constraints

When a passenger is matched with a driver and the passengers on board with respect to the matching preferences, the original route of driver and arrival times may change. There are a few possibilities:

1) when there is a detour to pick up and/or drop off the passenger without connection, $P^2_{ijv^j_m t} = 1 \; and/or \; DS^2_{ijtv^j_m t} = 1$ and

$C_{ijv^j_m j' v^{j'}_k t} \neq 1 \; for \;\; j \in D_t; \; v^j_m \in V^j; v^{j'}_k \in V^{j'}$

2) when there is a connection, $C_{ijv^j_m j' v^{j'}_k t} = 1 \; for \;\; j \in D_t; \; v^j_m \in V^j; v^{j'}_k \in V^{j'}$, and

3) when none of the above situations happens,

$$t^j_{v^j_{m+1}} = t^j_{v^j_m} + t^j_{v^j_m, v^j_{m+1}} \qquad for \;\; j \in D_t; \; v^j_m \in V^j \tag{10}$$

4.2.4. Ridesharing preferences constraints

The rideshare system considers matching between drivers and passengers when their age, gender, smoking, and pet preferences match.

4.2.4.1. Age preferences matching formulation

The rideshare system considers matching between drivers and passengers when their age preferences matches. That is,

$$\rho^A_{ijt} = \begin{cases} 1; \; age\; pref.pass.\, i \in P_t \;\; matches\; driver\; j \in D_t \; and\; pass.\, on\; board\; i' \in \; \mathrm{O}^{\mathrm{j}}_{\mathrm{t}} \; at\; time\; t \\ 0; \; otherwise \end{cases} \tag{11}$$

To formulate the constraints, it is assumed that age attitude of all individuals denoted by ξ is decomposed into 3 classes: Young, Middle age and Elderly. Numerical values 1, 2, 3 are defined for the classes as 1 for Young, 2 for Middle age, and 3 for Old. Each person defines the age preferences for the individuals with whom he/she will share the trip. The preferences which are denoted by η would be specified by selecting one of the 7 possible combinations. Table 6 and Figure 4 show the classification for age attribute and preferences.

Table 6: Age attitude and preferences classification
Individual A (passenger /passenger on board/driver)

Age Attitude	Age Preference
1, 2, or 3	{1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}

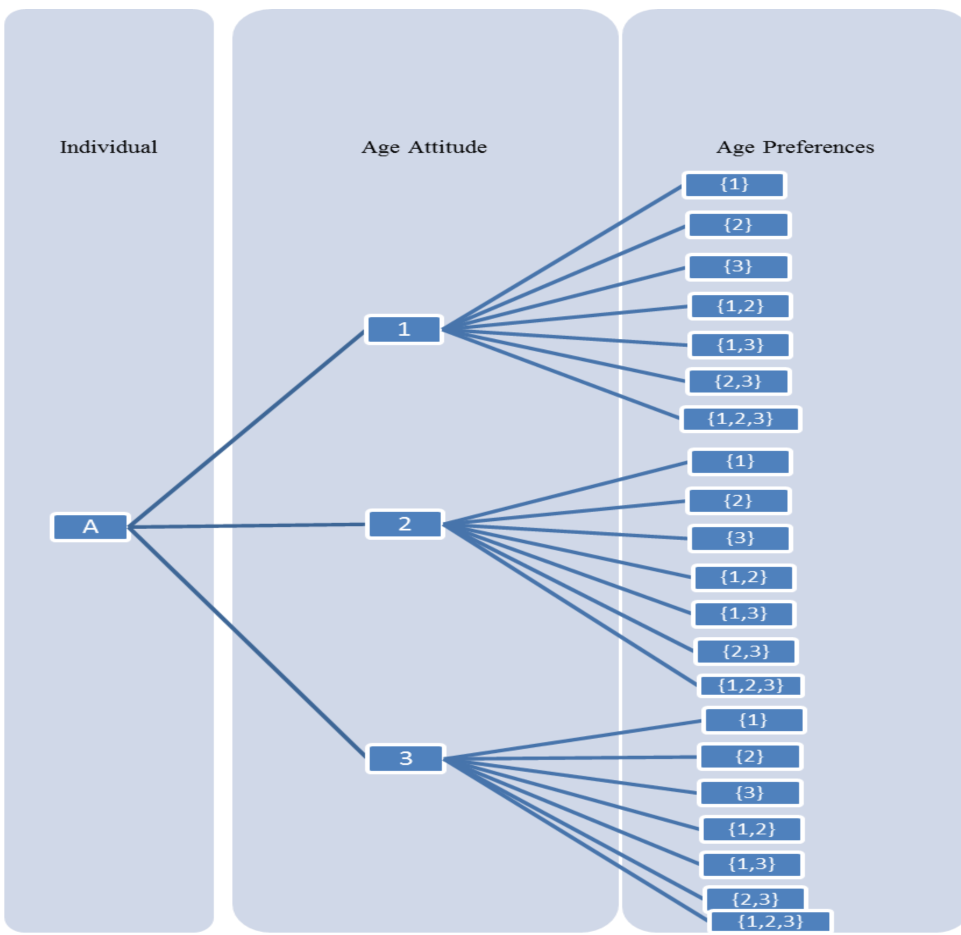


Figure 4: Age attitude and preferences classification

In order to have a better understanding of how the age preferences matching constraints should respond to the rideshare request from a passenger within a specific age category, following cases are defined with examples and the accuracy of the results are examined.

Case 1: There is a Young rider i ($a^i = 1$) requesting a rideshare and is willing to share ride only with Young individuals (i.e., $\eta^i = \{1\}$). Driver j is a Young person ($a^j = 1$) willing to give a ride to Young and Middle age individuals (i.e., $\eta^j = \{1,2\}$), and there are two passenger on boards, one is a Young passenger i' ($a^{i'} = 1$) whose age preference is Young $\eta^{i'} = \{1\}$), and another one is a Young passenger i''($a^{i''} = 1$) with no age preferences (i.e., $\eta^{i''} = \{1,2,3\}$). In this case, rider i can share the ride with passenger i' , passenger i'' and driver j.

Case 2: There is a Young rider i ($a^i = 1$) requesting a rideshare and is willing to share ride only with Young individuals (i.e., $\eta^i = \{1\}$). Driver j is a Young person ($a^j = 1$) willing to give a ride to Young and Middle age individuals (i.e., $\eta^j = \{1,2\}$), and there is one Middle age passenger on board i' ($a^{i'} = 2$) with no age preference(i.e., $\eta^{i'} = \{1,2,3\}$). In this case passenger i is not willing to share the ride with passenger i' and driver j.

Case 3: There is a Young passenger i ($a^i = 1$) requesting a rideshare and is willing to share the ride only with Young individuals (i.e., $\eta^i = \{1\}$). Driver j is a Middle age person ($a^j = 2$) willing to give a ride to Young and Middle age individuals (i.e., $\eta^j = \{1,2\}$). There is one Middle age passenger on board i' ($a^{i'} = 2$) who prefers sharing the ride with Middle and Elderly ages individual, i.e., $\eta^{i'} = \{2,3\}$). In this case, passenger i will not share the ride with passenger i' and driver j.

For each passenger i pending to be assigned to driver j, there are four decision checks:

1) **Passenger - Driver age matching check**: does the age preference of passenger $i \in W_t$ match with the age of driver $j \in D_t$. That is,

$$A_{tv}^{ij} = \begin{cases} 1 & ; \quad Age\ pref.\ of\ pass.\ \ i \in W_t\ matches\ with\ the\ age\ of\ driver\ j \in D_t \\ 0 & ; \quad otherwise \end{cases} \tag{12}$$

2) **Driver - Passenger age matching check**: does the age preferences of the driver match with the age of the passenger. That is,

$$A_{tv}^{ji} = \begin{cases} 1 & ; \quad age\ pref.\ of\ driver\ j \in D_t\ matches\ with\ the\ age\ of\ pass.\ \ i \in W_t \\ 0 & ; \quad otherwise \end{cases} \tag{13}$$

3) **Passenger - Passenger onboard age matching check**: does the age preference of passenger $i \in W_t$ match with the age of the passengers on board $i' \in \mathrm{O}_t^j$.

$$A_{tv}^{ii'} = \begin{cases} 1 & ; \quad age\ pref.\ of\ \ pass.\ \ i \in W_t\ matches\ age\ of\ pass.\ \ on\ board\ i' \in \mathrm{O}_t^j \\ 0 & ; \quad otherwise \end{cases} \tag{14}$$

4) **Passenger onboard - Passenger age matching check**: do the age preferences of the passengers on board $i' \in \mathrm{O}_t^j$ match with the age of passenger $i \in W_t$.

$$A_{tv}^{i'i} = \begin{cases} 1 & ; \quad age\ pref.\ \ pass.\ \ on\ board\ i' \in \mathrm{O}_t^j\ matches\ with\ age\ of\ pass.\ \ i \in W_t \\ 0 & ; \quad otherwise \end{cases} \tag{15}$$

If the answers to all four above mentioned questions are positive, then passenger $i \in W_t$ is considered to be assigned to driver $j \in D_t$ with respect to the age and age preferences criterion. Figure 5 shows age matching relations.

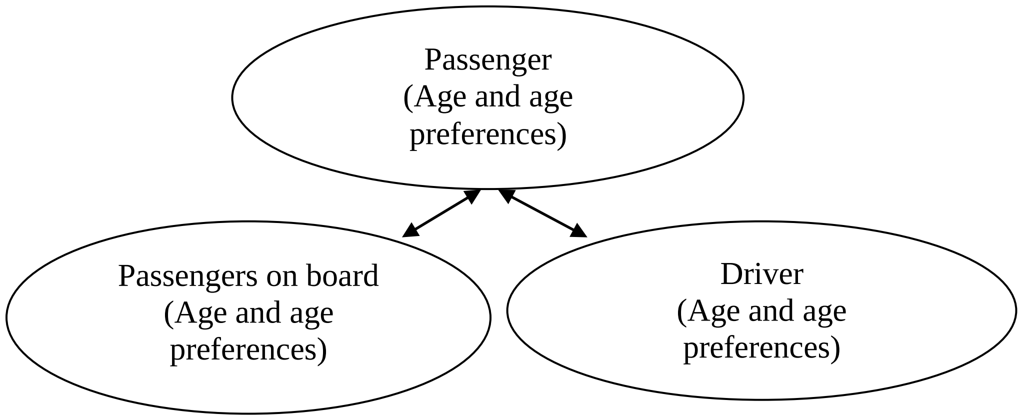


Figure 5: Age matching relations

In other words,

$$\rho_{ijtv}^{A} \leq A_{tv}^{ij}.A_{tv}^{ji}.A_{tv}^{ii'}.A_{tv}^{i'i} \tag{16}$$

4.2.4.2. Gender preferences matching formulation

The rideshare system considers matching between drivers and passengers when their gender preferences matches. That is,

$$\rho_{ijt}^{G} = \begin{cases} 1; & gen.pref.\ pass. i \in P_t \ matches\ driver\ j \in D_t\ and\ pass. on\ board\ i' \in \ O_t^j\ at\ time\ t \\ 0; & otherwise \end{cases} \tag{17}$$

To formulate the constraints, it is assumed that gender denoted by g is decomposed into 2 classes: Male and Female. Numerical values 1 and 2 are assigned to each class as 1 for Male and 2 for Female. Each person defines the gender preferences for the individuals whom he/she will share the trip. The preferences which are denoted by ∂ would be specified by selecting one of the 3 possible combinations. Table 7 and Figure 6 show the classification for gender attribute and preferences.

Table 7: Gender attitude and preferences classification

Individual A (passenger /passenger on board/driver)

Gender	Gender Preference
1 or 2	{1}, {2}, {1,2}

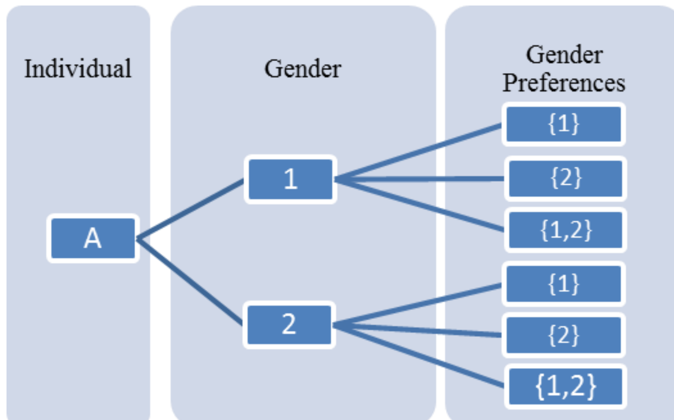


Figure 6: Gender attitude and preferences classification

In order to have a better understanding of how the gender preferences matching constraints should respond to the rideshare request from a passenger within a specific gender category, following cases are defined with examples and the accuracy of the results are examined.

Case 1: There is a Male rider i ($g^i = 1$) requesting a rideshare and willing to share the ride only with Male individuals (i.e., $\partial^i = \{1\}$). Driver j is a Male person ($g^j = 1$) with no gender preferences (i.e., $\partial^j = \{1,2\}$). There are two Male passengers on board i' and i'' ($g^{i'} = g^{ii'} = 1$) with no gender preferences $\partial^{i'} = \partial^{i''} = \{1,2\}$). In this case, individual i can share the ride with passengers i' and i'' and driver j.

Case 2: There is a Female individual i ($g^i = 2$) requesting a rideshare and willing to share the ride only with Female individuals (i.e., $\partial^i = \{2\}$). Driver j is a Female person ($g^j = 2$) with no gender preferences (i.e., $\partial^j = \{1,2\}$), and there is one Male passenger on board i' ($g^{i'} = 1$) who has no gender preference $\partial^{i'} = \{1,2\}$). In this case individual i is not willing to share the ride with passenger i' and driver j.

For each passenger $i \in W_t$ pending to be assigned to driver $j \in D_t$, there are four decision checks:

1) **Passenger - Driver gender matching check**: does the gender preference of passenger $i \in W_t$ match with the gender of driver $j \in D_t$. That is,

$$G_{tv}^{ij} = \begin{cases} 1 & ; \quad gender\ pref.\ for\ passenger\ i \in W_t\ \ matches\ with\ driver\ j \in D_t \\ 0 & ; \quad otherwise \end{cases} \tag{18}$$

2) **Driver - Passenger gender matching check**: does the gender preferences of driver $j \in D_t$ match with the age of passenger $i \in W_t$. That is,

$$G_{tv}^{ji} = \begin{cases} 1 & ; \quad gender\ pref.of\ driver\ j \in D_t\ matches\ with\ the\ pass.\ \ i \in W_t \\ 0 & ; \quad otherwise \end{cases} \tag{19}$$

3) **Passenger - Passenger onboard gender matching check**: does the gender preference of passenger $i \in W_t$ match with the gender of passengers on board $i' \in \mathrm{O}_t^j$.

$$G_{tv}^{ii'} = \begin{cases} 1 & ; \quad gender\ pref.for\ pass.\ \ i \in W_t\ matches\ with\ pass.on\ board\ i' \in \mathrm{O}_t^j \\ 0 & ; \quad otherwise \end{cases} \tag{20}$$

4) **Passenger onboard - Passenger gender matching check**: do the gender preferences of the passengers on board $i' \in \mathrm{O}_t^j$ match with the gender of passenger $i \in W_t$.

$$G_{tv}^{i'i} = \begin{cases} 0\ or\ 1 & ; \quad gender\ pref.of\ pass.on\ board\ i' \in \mathrm{O}_t^j\ matches\ with\ pass.i \in W_t \\ 0 & ; \quad otherwise \end{cases} \tag{21}$$

If answers to all four above mentioned questions are positive, then the passenger is considered to be assigned to driver $j \in D_t$ with respect to the gender and gender preferences criterion. Figure 7 shows gender matching relations.

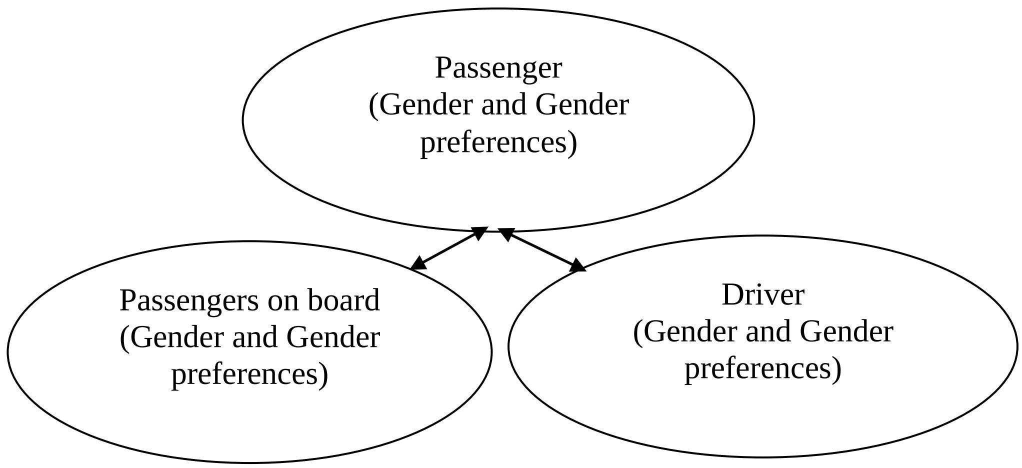


Figure 7: Gender matching relations

In other words,

$$\rho^{G}_{ijtv} \leq G^{ij}_{tv}.G^{ji}_{tv}.G^{ii'}_{tv}.G^{i'i}_{tv} \tag{22}$$

4.2.4.3. Smoking preferences matching formulation

The rideshare system considers matching between drivers and passengers when their smoking preferences match. That is,

$$\rho^{S}_{ijt} = \begin{cases} 1; & sex\ pref.\ pass. i \in P_t\ \ matches\ driver\ j \in D_t\ and\ pass. on\ board\ i' \in O^{j}_{t}\ at\ time\ t \\ 0; & otherwise \end{cases} \tag{23}$$

To formulate the constraints, it is assumed that smoking tendency denoted by s is decomposed into 2 classes: Smoker and Nonsmoker. Numerical values 1 and 2 are assigned to each class as 1 for Smoker and 2 for Nonsmoker. Each person defines the smoking preferences for the individuals with whom he/she will share the trip. The preferences which are denoted by Ω would be specified by selecting one of the 3 possible combinations. Table 8 and Figure 8 show the classification for smoking attribute and preferences.

Table 8: Smoking attitude and preferences classification
Individual A (passenger /passenger on board/driver)

Smoker	Smoking Preference
1 or 2	{1}, {2}, {1,2}

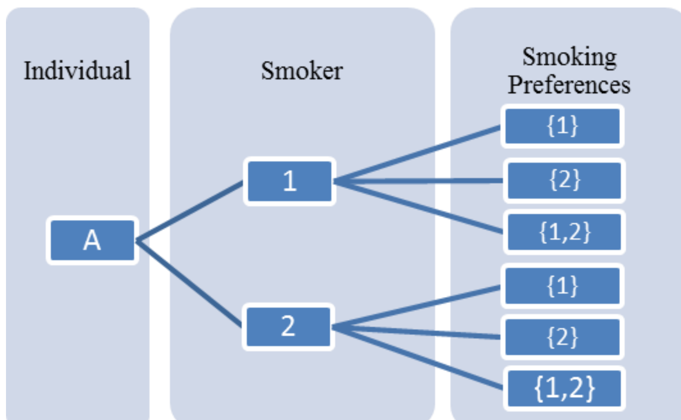


Figure 8: Smoking attitude and preferences classification

In order to have a better understanding of how the smoking preferences matching constraints should respond to the rideshare request from a passenger within a specific smoking category, following cases with examples are defined and the accuracy of the results are examined.

Case 1: There is a Smoker passenger i ($s^i = 1$) requesting a rideshare and prefers sharing the ride with Smoker individuals or individuals who have no preference on smoking (i.e., $\Omega^i = \{1,2\}$). Nonsmoker driver j ($s^j = 2$) has no disagreement with sharing a ride with Smokers (i.e., $\Omega^j = \{1,2\}$). There is one Nonsmoker passenger on board i' ($s^{i'} = 2$) who has no preferences on smoking (i.e., $\Omega^{i'} = \{1{,}2\}$). In this case, individual i can share the ride with passenger i' and driver j.

Case 2: There is a Smoker passenger i ($s^i = 1$) with no smoking preference (i.e., $\Omega^i = \{1{,}2\}$). Driver j is a Nonsmoker ($s^j = 2$) and disagrees with smoking in the car (i.e., $\Omega^j = \{2\}$). There is one nonsmoker passenger on board i' ($s^{i'} = 2$) who prefers sharing the ride with Nonsmokers (i.e., $\Omega^{i'} = \{1\}$). In this case, individual i will not share the ride with passenger i' and driver j.

For each passenger i pending to be assigned to driver j, there are four decision checks:

1) **Passenger - Driver smoking matching check**: does the smoking preferences of the passenger match with the smoking of the driver. That is,

$$S_{tv}^{ij} = \begin{cases} 1 & ; \quad smoking\ pref.passenger\ i\ \ matches\ smoking\ habit\ driver\ j \\ 0 & ; \quad otherwise \end{cases} \tag{24}$$

2) **Driver - Passenger smoking matching check**: does the smoking preferences of the driver match with the passenger. That is,

$$S_{tv}^{ji} = \begin{cases} 1 & ; \quad smoking\ preferences\ driver\ j\ matches\ the\ passenger\ i \\ 0 & ; \quad otherwise \end{cases} \tag{25}$$

3) **Passenger - Passenger onboard smoking matching check**: does the smoking preference of the passenger match with the passengers on board.

$$S_{tv}^{ii'} = \begin{cases} 1 & ; \quad smoking\ pref.pass.\ \ i\ \ matches\ pass.on\ board\ i' \\ 0 & ; \quad otherwise \end{cases} \tag{26}$$

4) **Passenger onboard - Passenger smoking matching check**: do the smoking preferences of the passengers on board match with the passenger.

$$S_{tv}^{i'i} = \begin{cases} 0 \text{ or } 1 & ; \quad smoking\ pref. pass. on\ board\ i'\ matches\ pass. i \\ 0 & ; \quad otherwise \end{cases} \tag{27}$$

If the answers to all four above mentioned questions are positive, then the passenger is considered to be assigned to the driver with respect to the smoking and smoking preferences criterion. Figure 9 shows smoking matching relations.

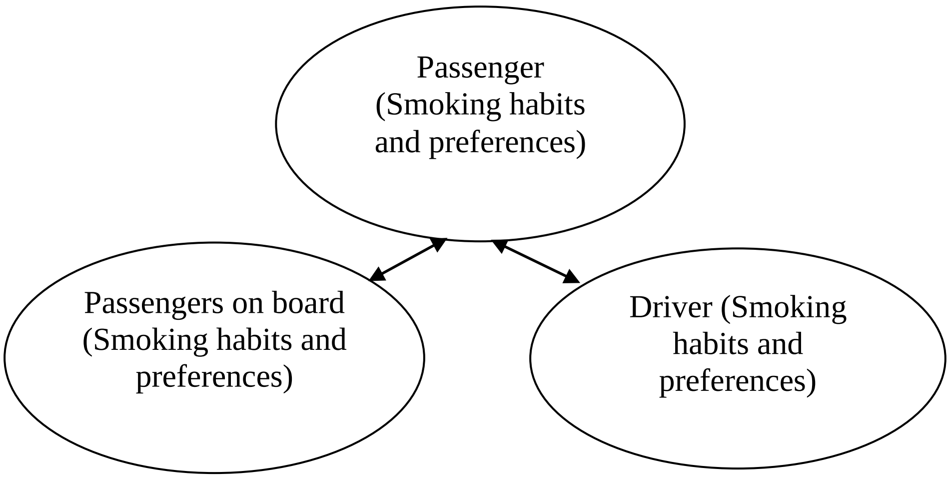


Figure 9: Smoking matching relations

In other words,

$$\rho_{ijtv}^{S} \leq S_{tv}^{ij}.S_{tv}^{ji}.S_{tv}^{ii'}.S_{tv}^{i'i} \tag{28}$$

4.2.4.4. Pet restrictions preferences matching formulation

The rideshare system considers matching between drivers and passengers when their pet restriction preferences match. That is,

$$\rho_{ijt}^{P} = \begin{cases} 1; \ pet\ pref.\ pass.\ P_t\ matches\ driver\ j \in D_t\ and\ tpass. on\ board\ i' \in O_t^j\ at\ time\ t \\ 0; \ otherwise \end{cases} \tag{29}$$

To formulate the constraints, it is assumed that pet policy p is decomposed into 2 classes: Friendly and Unfriendly. Numerical values 1 and 2 are assigned to each class as 1 for Friendly and 2 for Unfriendly. Each person defines his/her own pet policy preferences for the individuals with whom he/she will share the trip. The preferences which are denoted by ω would be specified by selecting one of the 3 possible combinations. Table 9 and Figure 10 show the classification for pet attribute and preferences.

Table 9: Pet attitude and preferences classification

Individual A (passenger /passenger on board/driver)	
Pet Friendly	Pet policy preferences
1 or 2	{1}, {2}, {1,2}

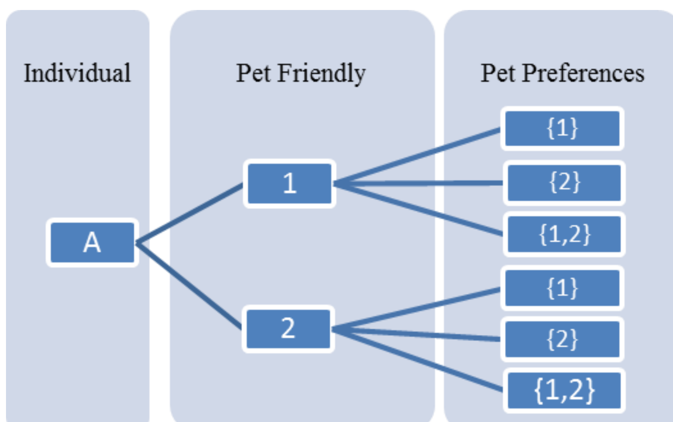


Figure 10: Pet attitude and preferences classification

In order to have a better understanding of how the pet preferences matching constraints should respond to the rideshare request from a passenger within a specific pet category, following cases are defined with examples and the accuracy of the results are examined.

Case 1: There is a Pet friendly passenger i ($p^i = 1$) who prefers sharing the ride with pet friendly individuals (i.e., $\omega^i = \{1\}$) or individuals who have no restriction on sharing the ride with a pet(i.e., $\omega^{i} = \{1,2\}$). Driver j is a Pet friendly individual ($p^{j} = 1$) with no disagreement with pets in the car (i.e., $\omega^{j} = \{1,2\}$). There is one Pet friendly passenger on board i' ($p^{i'} = 1$) with no preferences for sharing a ride with a pet (i.e., $\omega^{i'} = \{1,2\}$). In this case, individual i can share the ride with passenger i' and driver j.

Case 2: There is a Pet friendly passenger i ($p^{i} = 1$) with no preference on pet (i.e., $\omega^{i} = \{1,2\}$). Driver j ($p^{j} = 2$) disagrees with pets on car (i.e., $\omega^{j} = \{2\}$), then the individual i will not share the ride with driver j.

For each passenger i pending to be assigned to driver j, there are four decision checks:

1) **Passenger - Driver pet friendliness matching check**: does the pet preferences of the passenger match with the pet policy of the driver. That is,

$$P_{tv}^{ij} = \begin{cases} 1 & ; \quad pet\ preferences\ for\ passenger\ i\ \ matches\ with\ the\ policy\ of\ driver\ j \\ 0 & ; \quad otherwise \end{cases} \tag{30}$$

2) **Driver - Passenger pet friendliness matching check**: does the pet preferences of the driver match with the passenger. That is,

$$P_{tv}^{ji} = \begin{cases} 1 & ; \quad pet\ preferences\ of\ driver\ j\ matches\ with\ the\ passenger\ i \\ 0 & ; \quad otherwise \end{cases} \tag{31}$$

3) **Passenger - Passenger onboard pet friendliness matching check**: does the pet preference of the passenger match with the passengers on board.

$$P_{tv}^{ii'} = \begin{cases} 1 & ; \quad pet\ \ pref.for\ pass.\ \ i\ \ matches\ with\ pass.on\ board\ i' \\ 0 & ; \quad otherwise \end{cases} \tag{32}$$

4) **Passenger onboard - Passenger pet friendliness matching check**: do the pet preferences of the passengers on board match with the passenger.

$$P_{tv}^{i'i} = \begin{cases} 0 \; or \; 1 & ; \quad pet \; pref. for \; pass. on \; board \; i' \; matches \; with \; pass. i \\ 0 & ; \quad otherwise \end{cases}$$

(33)

If the answers to all four above mentioned questions are positive, then the passenger is considered to be assigned to the driver with respect to the pet preferences criterion. Figure 11 shows age matching relations. In other words,

$$\rho_{ijtv}^{P} \leq P_{tv}^{ij} . P_{tv}^{ji} . P_{tv}^{ii'} . P_{tv}^{i'i} \tag{34}$$

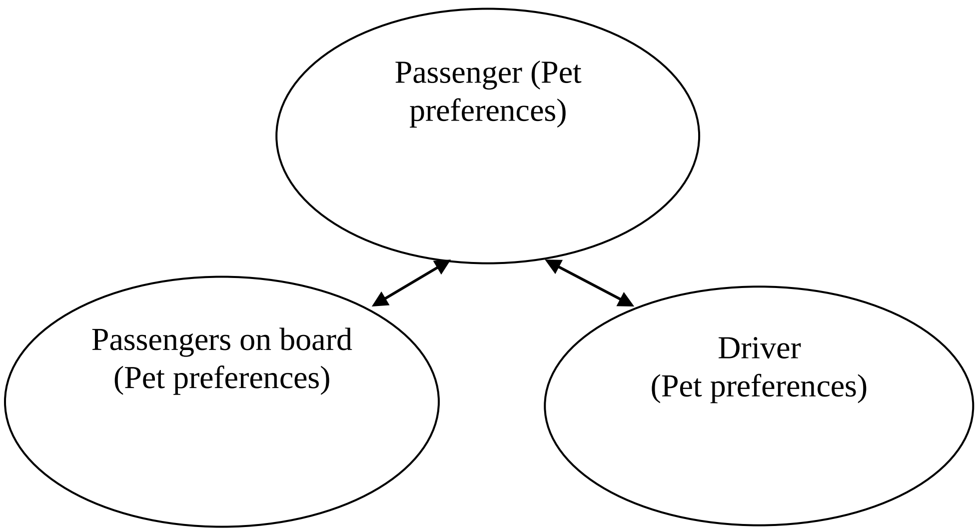


Figure 11: Pet matching relations

4.2.4.5. Maximum occupancy preferences constraints

The rideshare system considers matching between drivers and passengers when the maximum number of passengers on board policy defined by the driver and passengers are not violated. By definition, passengers are individuals traveling alone or a person traveling with his/her pet. In the latter case, a pet is considered to be a passenger.

$$\rho_{ijt}^{M} = \begin{cases} 1; \; occu. pref. \; pass. i \in P_t \; matches \; driver \; j \in D_t \; and \; pass. on \; board \; i' \in O_t^j \; at \; time \; t \\ 0; \; otherwise \end{cases}$$

As earlier defined, Q_{jtv} is the available space on vehicle j at time t at point of interest v and Q_j is the available seat on vehicle defined by the driver j that is equal to the seat capacity of the vehicle at its origin point of departure. Each passenger defines his/her favorable maximum number of people with whom to share the ride. Let Q^i, Q^j and $Q^{i'}$ be the favorable maximum number of people sharing the ride defined by passenger i , driver j, and passenger on board i' respectively. That is,

$$\rho^M_{ijtv} = \begin{cases} 0 \; or \; 1 & ; \quad Q^i \geq Q_j - Q_{jtv} + 1 \; and \; Q^j \geq Q_j - Q_{jtv} + 1 \; and \; Q^{i'} \geq Q_j - Q_{jtv} + 1 \\ 0 & ; \quad otherwise \end{cases} \tag{35}$$

equivalently:

$$\rho^M_{ijtv} = \begin{cases} 0 \; or \; 1 & ; \quad Min\{Q^i, Q^j, Q^{i'}\} \geq Q_j - Q_{jtv} + 1 \\ 0 & ; \quad otherwise \end{cases} \tag{36}$$

If all five above mentioned preferences are met, then the passenger is considered to be assigned to the driver, i.e.,

$$5\rho_{ijtv} \leq \rho^A_{ijtv} + \rho^G_{ijtv} + \rho^S_{ijtv} + \rho^P_{ijtv} + \rho^M_{ijtv} \text{ for } i \in W_t;\; j \in D_t\,;v \in V \tag{37}$$

4.3. Formulation of the objective function

As mentioned earlier, DROM assigns passengers to drivers and identifies feasible routes for drivers to maximize the total number of matching in a given planning horizon while the total passenger and driver travel times are minimized. The objective function for the model is:

$$Maximize \quad w_1.\sum_{i\in P_t}\sum_{j\in D_t}\sum_{v_p^j\in R_t^j}\sum_{v_d^j\in R_t^j\,|v_d^j\geq v_p^j}\left(M - P^0_{ijv_p^j v_d^j} - D^0_{ijv_p^j v_d^j}\right).R^0_{ijv_p^j v_d^j}$$

$$+ w_2.\sum_{i\in P_t}\sum_{j\in D_t}\sum_{v_p^j\in R_t^j}\sum_{v_d^j\in R_t^j\,|v_d^j\ge v_p^j}\sum_{k\in D_t|k\neq j}\sum_{v_p^k\in R_t^k}\sum_{v_d^k\in R_t^k\,|v_d^k\ge v_p^k}\Big(M - P^1_{ijv_p^jv_d^jkv_p^kv_d^k} - D^1_{ijv_p^jv_d^jkv_p^kv_d^k}\Big).R^1_{ijv_p^jv_d^jkv_p^kv_d^k}$$

$$+ w_3.\sum_{i\in P_t}\sum_{j\in D_t}\sum_{v_p^j\in R_t^j}\sum_{v_d^j\in R_t^j\,|v_d^j\ge v_p^j}\sum_{k\in D_t|k\neq j}\sum_{v_p^k\in R_t^k}\sum_{v_d^k\in R_t^k\,|v_d^k\ge v_p^k}\sum_{l\in D_t|l\neq k\neq j}\sum_{v_p^l\in R_t^l}\sum_{v_d^l\in R_t^l\,|v_d^l\ge v_p^l}\Big(M - P^2_{ijv_p^jv_d^jkv_p^kv_d^klv_p^lv_d^l} - D^2_{ijv_p^jv_d^jkv_p^kv_d^klv_p^lv_d^l}\Big).R^2_{ijv_p^jv_d^jkv_p^kv_d^klv_p^lv_d^l} \tag{38}$$

where M is a large positive value and w_1, w_2, w_3 are weighting factors. The first, second and third terms in the objective function maximize the number of all possible zero, one, and two connection routes respectively with minimum total rider and driver travel times.

The mathematical model for DROM is summarized as follows:

$$\begin{aligned}
\textit{Maximize}\quad & w_1.\sum_{i\in P_t}\sum_{j\in D_t}\sum_{v_p^j\in R_t^j}\sum_{v_d^j\in R_t^j\,|v_d^j\ge v_p^j}\Big(M - P^0_{ijv_p^jv_d^j} - D^0_{ijv_p^jv_d^j}\Big).R^0_{ijv_p^jv_d^j}\\
& + w_2.\sum_{i\in P_t}\sum_{j\in D_t}\sum_{v_p^j\in R_t^j}\sum_{v_d^j\in R_t^j\,|v_d^j\ge v_p^j}\sum_{k\in D_t|k\neq j}\sum_{v_p^k\in R_t^k}\sum_{v_d^k\in R_t^k\,|v_d^k\ge v_p^k}\Big(M - P^1_{ijv_p^jv_d^jkv_p^kv_d^k} - D^1_{ijv_p^jv_d^jkv_p^kv_d^k}\Big).R^1_{ijv_p^jv_d^jkv_p^kv_d^k}\\
& + w_3.\sum_{i\in P_t}\sum_{j\in D_t}\sum_{v_p^j\in R_t^j}\sum_{v_d^j\in R_t^j\,|v_d^j\ge v_p^j}\sum_{k\in D_t|k\neq j}\sum_{v_p^k\in R_t^k}\sum_{v_d^k\in R_t^k\,|v_d^k\ge v_p^k}\sum_{l\in D_t|l\neq k\neq j}\sum_{v_p^l\in R_t^l}\sum_{v_d^l\in R_t^l\,|v_d^l\ge v_p^l}\Big(M - P^2_{ijv_p^jv_d^jkv_p^kv_d^klv_p^lv_d^l} - D^2_{ijv_p^jv_d^jkv_p^kv_d^klv_p^lv_d^l}\Big).R^2_{ijv_p^jv_d^jkv_p^kv_d^klv_p^lv_d^l}
\end{aligned} \tag{39}$$

$$D_{v_O^i,v_m^j} \le \varphi + M\left(1 - PS^1_{ijtv_m^jt}\right) \qquad i\in P_t;\ j\in D_t;\ v_m^j\in V^j \tag{40}$$

$$t^j_{v_m^j} \le \gamma_i + t_O^i + t^i_{v_O^i,v_m^j} + M\left(1 - PT^1_{ijtv_m^jt}\right) \qquad i\in P_t;\ j\in D_t;\ v_m^j\in V^j \tag{41}$$

$$t^j_{v_m^j} \ge t_O^i + t^i_{v_O^i,v_m^j} - M\left(1 - PT^1_{ijtv_m^jt}\right) \qquad i\in P_t;\ j\in D_t;\ v_m^j\in V^j \tag{42}$$

$$D_{v_m^j,v_O^i} + D_{v_O^i,v_{m+1}^j} \le D_{v_m^j,v_{m+1}^j} + \beta + M(1 - PS^2_{ijtv_m^jt}) \qquad i\in P_t;\ j\in D_t;\ v_m^j, v_{m+1}^j\in V^j \tag{43}$$

$$t^j_{v_m^j} + t^j_{v_m^j,v_O^i} \le \gamma_i + t_O^i + M\left(1 - PT^2_{ijtv_m^jt}\right) \qquad i\in P_t;\ j\in D_t;\ v_m^j\in V^j \tag{44}$$

$$t^{j}_{v^j_m} + t^{j}_{v^j_m, v^i_O} \geq t^i_O - M\left(1 - PT^2_{ijtv^j_m t}\right) \qquad i \in P_t;\ j \in D_t;\ v^j_m \in V^j \qquad (45)$$

$$2P^1_{ijtv^j_m t} \leq PT^1_{ijtv^j_m t} + PS^1_{ijtv^j_m t} \qquad i \in P_t;\ j \in D_t;\ v^j_m \in V^j \qquad (46)$$

$$2P^2_{ijtv^j_m t} \leq PT^2_{ijtv^j_m t} + PS^2_{ijtv^j_m t} \qquad i \in P_t;\ j \in D_t;\ v^j_m \in V^j \qquad (47)$$

$$\textstyle\sum_j \sum_{v^j_m} P^1_{ijtv^j_m t} + P^2_{ijtv^j_m t} \leq 1 \qquad i \in P_t;\ j \in D_t;\ v^j_m \in V^j \qquad (48)$$

$$P^1_{ijtv^j_m t} + P^2_{ijtv^j_m t} \geq P_{ijtv^j_m t} \qquad i \in P_t;\ j \in D_t;\ v^j_m \in V^j \qquad (49)$$

$$D_{v^i_D, v^j_m} \leq \varphi + M\left(1 - DS^1_{ijtv^j_m}\right) \qquad i \in P_t;\ j \in D_t;\ v^j_m \in V^j \qquad (50)$$

$$D_{v^j_m, v^i_D} + D_{v^i_D, v^j_{m+1}} \leq D_{v^j_m, v^j_{m+1}} + \beta + M(1 - DS^2_{ijtv^j_m}) \qquad i \in P_t, j \in D_t, v^j_m, v^j_{m+1} \in V^j \qquad (51)$$

$$\textstyle\sum_{v^j_m \in V^j} DS^1_{ijtv^j_m} + \sum_{v^j_m \in V^j} DS^2_{ijtv^j_m} \leq 1 \qquad i \in P_t;\ j \in D_t;\ v^j_m \in V^j \qquad (52)$$

$$DS_{ijt} = \textstyle\sum_j DS^1_{ijtv^j_m} + \sum_j DS^2_{ijtv^j_m} \qquad i \in P_t;\ j \in D_t;\ v^j_m \in V^j \qquad (53)$$

$$P_{ijt} \leq DS_{ijt} + M\left(1 - R_{ijtv}\right) \qquad i \in P_t;\ j \in D_t \qquad (54)$$

$$P_{ijt} \geq DS_{ijt} - M\left(1 - R_{ijtv}\right) \qquad i \in P_t;\ j \in D_t \qquad (55)$$

$$R_{ijtv} \leq P_{ijt} \qquad i \in P_t;\ j \in D_t \qquad (56)$$

$$D_{v^j_m, v^{j'}_k} \leq \varphi + M\left(1 - CS^1_{ijv^j_m j' v^{j'}_k t}\right) \qquad i \in P_t, j \in D_t;\ v^j_m \in R^j_t;\ ;\ v^{j'}_k \in R^{j'}_t \qquad (57)$$

$$D_{v^j_m, v^{j'}_k} + D_{v^{j'}_k, v^j_{m+1}} \leq D_{v^j_m, v^j_{m+1}} + \beta_j + M(1 - CS^2_{ijv^j_m j' v^{j'}_k t}) \qquad i \in P_t, j \in D_t;\ v^j_m, v^j_{m+1} \in R^j_t \qquad (58)$$

$$t^{j'}_{v^{j'}_k} \leq \gamma'_i + t^j_{v^j_m} + t^i_{v^j_m, v^{j'}_k} + M\left(1 - CT^1_{ijv^j_m j' v^{j'}_k t}\right) \qquad i \in P_t, j \in D_t;\ v^j_m \in R^j_t; v^{j'}_k \in R^{j'}_t \qquad (59)$$

$$t^{j'}_{v^{j'}_k} \geq t^j_{v^j_m} + t^i_{v^j_m, v^{j'}_k} - M\left(1 - CT^1_{ijv^j_m j' v^{j'}_k t}\right) \qquad i \in P_t, j \in D_t;\ v^j_m \in R^j_t;\ v^{j'}_k \in R^{j'}_t \qquad (60)$$

$$t^{j'}_{v^{j'}_k} \leq \gamma'_i + t^j_{v^j_m} + t^j_{v^j_m, v^{j'}_k} + M\left(1 - CT^2_{ijv^j_m j' v^{j'}_k t}\right) \qquad i \in P_t, j \in D_t;\ v^j_m \in R^j_t;\ v^{j'}_k \in R^{j'}_t \qquad (61)$$

$$t^{j'}_{v^{j'}_k} \geq t^j_{v^j_m} + t^j_{v^j_m, v^{j'}_k} - M\left(1 - CT^2_{ijv^j_m j' v^{j'}_k t}\right) \qquad i \in P_t, j \in D_t;\ v^j_m \in R^j_t;\ v^{j'}_k \in R^{j'}_t \qquad (62)$$

$$2C^1_{ijv^j_m j' v^{j'}_k t} \leq CT^1_{jijv^j_m j' v^{j'}_k t} + CS^1_{ijv^j_m j' v^{j'}_k t} \qquad i \in P_t, j \in D_t;\ v^j_m \in R^j_t;\ v^{j'}_k \in R^{j'}_t \qquad (63)$$

$$2C^2_{ijv^j_m j' v^{j'}_k t} \leq CT^2_{ijv^j_m j' v^{j'}_k t} + CS^2_{ijv^j_m j' v^{j'}_k t} \qquad i \in P_t, j \in D_t;\ v^j_m \in R^j_t;\ v^{j'}_k \in R^{j'}_t \qquad (64)$$

$$C^1_{ijv^j_m j' v^{j'}_k t} + C^2_{ijv^j_m j' v^{j'}_k t} \geq C_{ijv^j_m j' v^{j'}_k t} \qquad i \in P_t, j \in D_t;\ v^j_m \in R^j_t;\ v^{j'}_k \in R^{j'}_t \qquad (65)$$

$$\begin{aligned} t^j_{v^j_{m+1}} = {} & t^j_{v^j_m} + t^j_{v^j_m, v^j_{m+1}} + \left(t^j_{v^j_m, v^i_O} + t^j_{v^i_O, v^j_{m+1}} - t^j_{v^j_m, v^j_{m+1}}\right) T^1_{ijv^j_m t} + \\ & \left(t^j_{v^j_m, v^i_D} + t^j_{v^i_D, v^j_{m+1}} - t^j_{v^j_m, v^j_{m+1}}\right) T^2_{ijv^j_m t} + \left(t^j_{v^j_m, v^i_O} + t^j_{v^i_O, v^i_D} + t^j_{v^i_D, v^j_{m+1}} - \right. \\ & \left. t^j_{v^j_m, v^j_{m+1}}\right) T^3_{ijv^j_m t} + \textstyle\sum_{j'} \sum_{v^{j'}_k} \left(t^j_{v^j_m, v^{j'}_k} + t^j_{v^{j'}_k, v^j_{m+1}} - t^j_{v^j_m, v^j_{m+1}}\right) T^4_{ijv^j_m j' v^{j'}_k t} + \\ & \textstyle\sum_{j'} \sum_{v^{j'}_k} \left(t^j_{v^j_m, v^{j'}_k} + t^j_{v^{j'}_k, v^i_D} + t^j_{v^i_D, v^j_{m+1}} - t^j_{v^j_m, v^j_{m+1}}\right) T^5_{ijv^j_m j' v^{j'}_k t} \end{aligned} \qquad i \in P_t,\ j \in D_t,\ v^j_m \in R^j_t \qquad (66)$$

$$RFL_{ijj'} + RFCL_{ijj'} \leq 1 \qquad i \in P_t, j, j' \in D_t \tag{67}$$

$$C_{ijj'} \leq \sum_{v_m^j} \sum_{v_k^{j'}} C_{ijv_m^j j' v_k^{j'}} \qquad i \in P_t, j, j' \in D_t \tag{68}$$

$$RL_{ij} \leq \sum_{v_m^j} RL_{ijv_m^j} \quad RF_{ij} = \sum_{v_m^j} RF_{ijv_m^j} \qquad i \in P_t, j \in D_t \tag{69}$$

$$3\, RFCL_{ijj'} \leq RF_{ij} + C_{ijj'} + RL_{ij} \qquad i \in P_t, j, j' \in D_t \tag{70}$$

$$\sum_{v_m^j} RF_{ijv_m^j} \leq \sum_{v_m^j} RL_{ijv_m^j} + M\left(1 - RFL_{ij}\right) \qquad i \in P_t, j \in D_t \tag{71}$$

$$\sum_{v_m^j} RF_{ijv_m^j} \geq \sum_{v_m^j} RL_{ijv_m^j} - M\left(1 - RFL_{ij}\right) \qquad i \in P_t, j \in D_t \tag{72}$$

$$RFL_{ij} \leq \sum_{v_m^j} RF_{ijv_m^j} \qquad i \in P_t, j \in D_t \tag{73}$$

$$RFL_{ij} + RFCL_{ijj'} = RFRFCL_{ij} \qquad i \in P_t, j \in D_t \tag{74}$$

$$DS'^{2}_{ijv_m^j t} = 1 - DS^{2}_{ijv_m^j t} \qquad i \in P_t, j \in D_t;\ v_m^j \in R_t^j \tag{75}$$

$$P'^{2}_{ijv_m^j t} = 1 - P^{2}_{ijv_m^j t} \qquad i \in P_t, j \in D_t;\ v_m^j \in R_t^j \tag{76}$$

$$4T^{1}_{ijv_m^j t} \leq P^{2}_{ijv_m^j t} + DS'^{2}_{ijv_m^j t} + RFRFCL_{ij} + \rho_{ijtv} \qquad i \in P_t, j \in D_t;\ v_m^j \in R_t^j \tag{77}$$

$$4T^{2}_{ijv_m^j t} \leq P'^{2}_{ijv_m^j t} + DS^{2}_{ijv_m^j t} + RFRFCL_{ij} + \rho_{ijtv} \qquad i \in P_t, j \in D_t;\ v_m^j \in R_t^j \tag{78}$$

$$4T^{3}_{ijv_m^j t} \leq P^{2}_{ijv_m^j t} + DS^{2}_{ijv_m^j t} + RFL_{ij} + \rho_{ijtv} \qquad i \in P_t, j \in D_t;\ v_m^j \in R_t^j \tag{79}$$

$$4T^{4}_{ijv_m^j j' v_k^{j'} t} \leq C^{2}_{jv_k^{j'} j' v_m^j t} + DS'^{2}_{ijv_m^j t} + RFCL_{ijj'} + \rho_{ijtv} \quad i \in P_t, j \in D_t;\ v_m^j \in R_t^j;\ v_k^{j'} \in R_t^{j'} \tag{80}$$

$$4T^{5}_{ijv_m^j j' v_k^{j'} t} \leq C^{2}_{jv_k^{j'} j' v_m^j t} + DS^{2}_{ijv_m^j t} + RFCL_{ijj'} + \rho_{ijtv} \quad i \in P_t, j \in D_t;\ v_m^j \in R_t^j;\ v_k^{j'} \in R_t^{j'} \tag{81}$$

$$x^{\partial,j} X_t^{\partial,ij} = \sum_m \eta_m^{\partial,i} x_m^{\partial,ij} \text{ and } \sum_m x_m^{\partial,ij} \leq 1 \qquad \partial = 1, 2, 3, 4;\ i \in P_t, j \in D_t \tag{82}$$

$$x^{\partial,i} X_t^{\partial,ji} = \sum_m \eta_m^{\partial,j} x_m^{\partial,ji} \text{ and } \sum_m x_m^{\partial,ji} \leq 1 \qquad \partial = 1, 2, 3, 4;\ i \in P_t, j \in D_t \tag{83}$$

$$x^{\partial,i'} X_t^{\partial,ii'} = \sum_m \eta_m^{\partial,i} x_m^{\partial,ii'} \text{ and } \sum_m x_m^{\partial,ii'} \leq 1 \qquad \partial = 1, 2, 3, 4;\ i \in P_t;\ i' \in O_t^j \tag{84}$$

$$x^{\partial,i} X_t^{\partial,i'i} = \sum_m \eta_m^{\partial,i'} x_m^{\partial,i'i} \text{ and } \sum_m x_m^{\partial,i'i} \leq 1 \qquad \partial = 1, 2, 3, 4;\ i \in P_t;\ i' \in O_t^j \tag{85}$$

$$2\left(1 + \left|O_t^j\right|\right)\rho_{ijt}^{\partial} \leq X_t^{\partial,ij} + X_t^{\partial,ji} + \sum_{i' \in O_t^j}\left(X_t^{\partial,ii'} + X_t^{\partial,i'i}\right) \qquad i \in P_t, j \in D_t, i' \in O_t^j, \partial = 1, \ldots, 4 \tag{86}$$

where Eq. 39 is the objective function for DROM that maximizes the total number of matching in a given planning horizon while the total passenger and driver travel times are minimized. Constraints 40-49 secure that a pickup is feasible. While constraints 40-42 deal with pickup type 1 (riders walks to the point of pickup), constraints 43-45 are dedicated to pick up type 2 (driver makes a detour to pick up the rider at his/her point of origin). Constrains 46-47 secures that both of time and space feasibilities for pickup are met and constraints 48-49 secures that a rider is not picked up by either type 1 and type 2. Constraints 50-53 secure that a drop off is feasible. Constraint 50 deals with drop-off type 1, i.e., riders walks to his/her point of destination after he/she leaves a driver. Constrain 51 deals with drop-off type 2, i.e., driver detours to drop-off the rider at his/her point of destination. Constraints 52-53 secures that a rider is not dropped off by either type 1 and type 2. Constraints 54-56 identify zero connection routes. Constraints 57-65 secure that a connection is feasible. Constrain 57 deals with feasibility of a connection type 1, i.e., the rider walks to the connection point, in terms of proximity in space and Constraint 58 corresponds to feasibility check of space for connection type 2, i.e., the driver detours to make possible a connection for the rider. Constraints 59-60 and constraints 61-62 secure proximity in time for connection type 1 and type 2, respectively. Constraints 63-64 secure proximity in time and space for a connection and Constraint 65 secures that a rider connects to his/her next ride by either type or type 2. Constraint 66 updates the arrival times. Constraints 67-70 and constraints 71-74 identify one connection routes and to connections routes for a rider, respectively. Constraints 75-81 identify details of a matched route for a rider, i.e., the types of pickup, connection and drop off which are determined to update the arrival times. Constraints 82-86 identify preference matches for riders, drivers and riders on board. Constraint 82 and Constraint 83 checks the match between the rider and the driver, and between the driver and the rider, respectively. Constraint 84 and Constraint 85 checks the match between the rider and the rider on board, and between the rider on board and the rider, respectively. Constraint 86 secures that all the preferences match for a perfect match between the rider, the driver and every rider on the board.

Chapter 5: Model Testing and Validation

In this section numerical examples are given to test the model and check for the validity of solutions. Through the first numerical example, the problem is elaborated and feasibility checks for pickup, drop off and connections are illustrated and negotiating policies are extracted to make the assignment possible. This numerical example includes one rideshare request from a rider and there are two available drivers in the system. One of the drivers has already started his journey and has one passenger on board while the other one has not yet started his journey. The second numerical example which is a medium size problem includes requests from 14 riders in an area of 400 square Miles where 14 drivers are available. This numerical example is designed to check for quality of solutions and efficiency of the mathematical model.

5.1. Numerical Example 1:

In time interval 9:30 AM to 9:45 AM, there is a request for rideshare from an individual. The time requested for service is 9:45 with maximum 10 minutes waiting time. This person is Young, Male, Smoker and Pet friendly. He is willing to share the ride with Young and male individuals. He has no disagreement with sharing a ride with smokers and prefers sharing the ride with pet friendly individuals. The maximum occupancy for sharing the ride defined by this passenger is 3. The (x, y) coordinates for the passenger's origin and destination points are (10, 15) and (15, 15), respectively. He is flexible with maximum relocating distance of 0.5 Miles.

There are two drivers available in the system: Driver j has already started his journey and has one passenger on board. Driver j is a Young, Male, Nonsmoker, and Pet friendly person who is willing to give a ride to Young and Middle age, Male or Female riders. He has no disagreement with smoking in the car or giving a ride to a pet. The maximum occupancy defined by this driver is 4. At 9:30 a.m. he is at point (5, 8) and his path en route is (7, 9), (10, 14), (12, 12), (15, 16). He is flexible with maximum detour distance of 1.5 Miles. The passenger on board is Young, Male, Nonsmoker and Pet friendly who has no age, gender, smoking or pet preferences. The maximum occupancy defined by this passenger is 4.

Driver j' is a Young, Male, Nonsmoker, and Pet friendly person who is willing to give a ride to young and middle age Male or Female individuals. He has no disagreement with giving a ride to Smokers or pets in the car. The maximum number of people sharing the ride defined by the driver is 4 . He will start his journey at 9:40 and has defined his path as (9, 9.5), (10.1, 15.2), (14, 16). Maximum acceptable detour distance for this driver is 1.5 Miles. Figures 12 and 13 show the map and all the rideshare preference matching relationships that need to be checked for this numerical example.

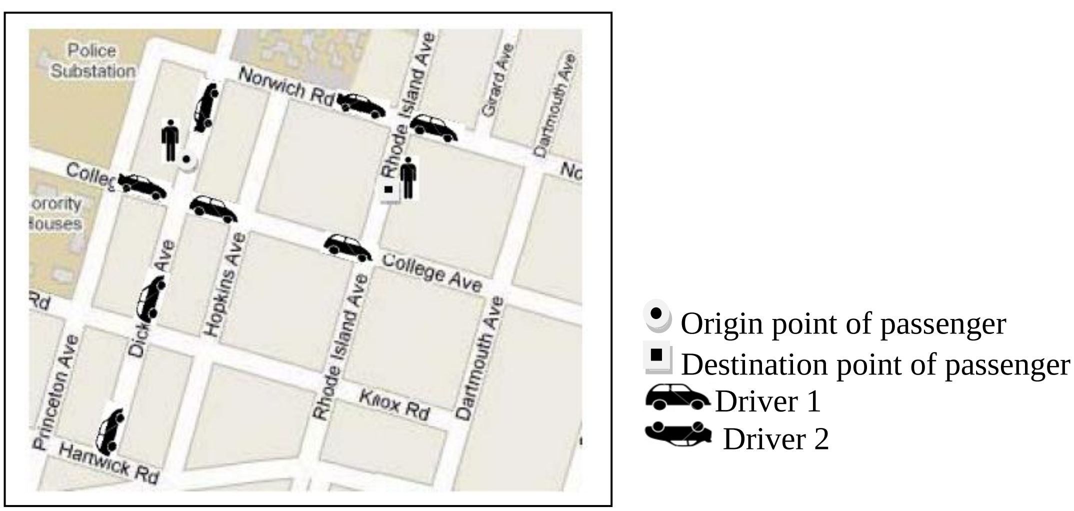


Figure 12: The map for the numerical example.

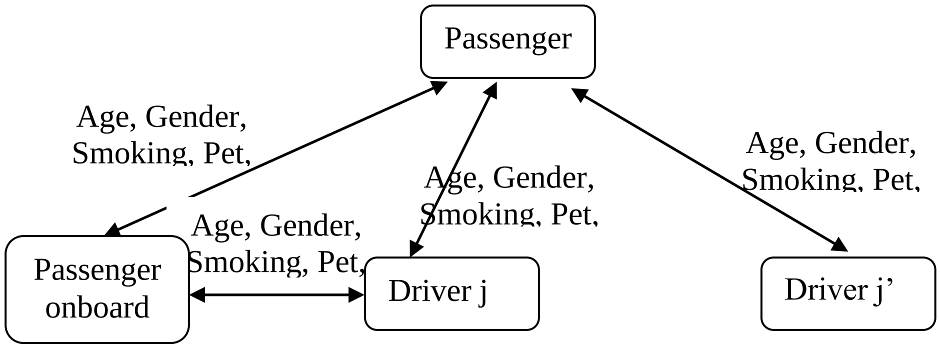


Figure 13: Rideshare preference matching relationships

Table 10 and 11 show the results for preferences match examination between the rider, drivers and riders on boards.

Table 10: Characteristics-Preferences check results

	Rider's Characteristics				
Preferences	Age	Gender	Smoking	Pet	Occupancy
Driver j	1	1	1	1	1
rider on board	1	1	1	1	1
Driver j'	1	1	1	1	1

Table 11: Preferences-Characteristics check results

	Rider's Preferences				
Characteristics	Age	Gender	Smoking	Pet	Occupancy
Driver j	1	1	1	1	1
rider on board	1	1	1	1	1
Driver j'	1	1	1	1	1

Figure 14 shows the feasibility check for pickup and droop off. It reveals that driver j' can pick up the rider with respect to proximity in space if the rider walks to the second point to be visited by this driver, or when driver j' makes a detour from his current location to the origin point of rider. Alternatively he can pick up the rider when he makes a detour from his second point to be visited to the origin point of the rider. With respect to proximity in time, driver j' can pick up the rider when he makes a detour from his first or second point to be visited by the driver to the origin point of the rider.

Feasibility check for proximities shows that driver j’ can pick up the rider if he makes a detour from his current point or the second point to be visited by the driver. Likewise, Driver j can pick up the rider with respect to proximity in time if he makes a detour from one of his second, third or fourth nodes to be visited to the point of origin of the rider though there is no way to pick up the rider with respect to proximity in space. Although Driver j cannot pick up the rider, he can drop him off at his fifth node to be visited and he will later walks to his destination.

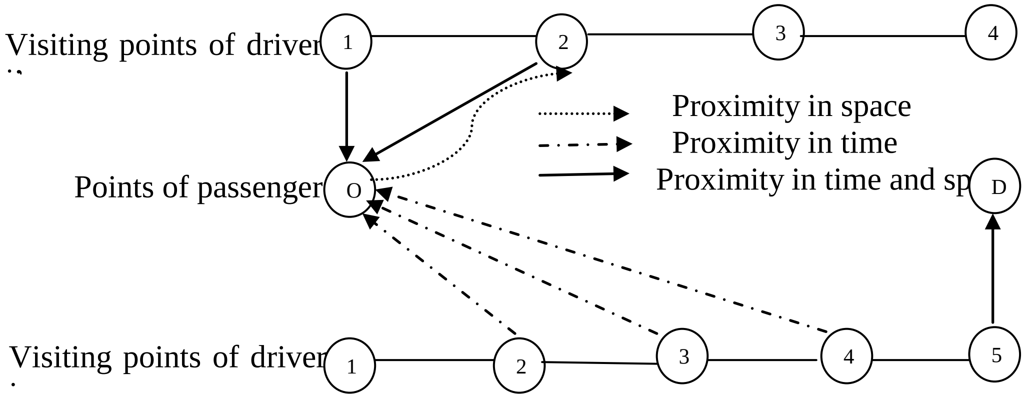


Figure 14: Proximity in time and space relationships for pick up/ drop off the passenger

According to what has been discussed so far, driver j’ can pick up the rider and driver j can drop him off. To have a feasible solution, there should be a feasible connection between two drivers. Figure 15 shows the feasibility check for connections. It reveals that there is no feasible connection between the two drivers.

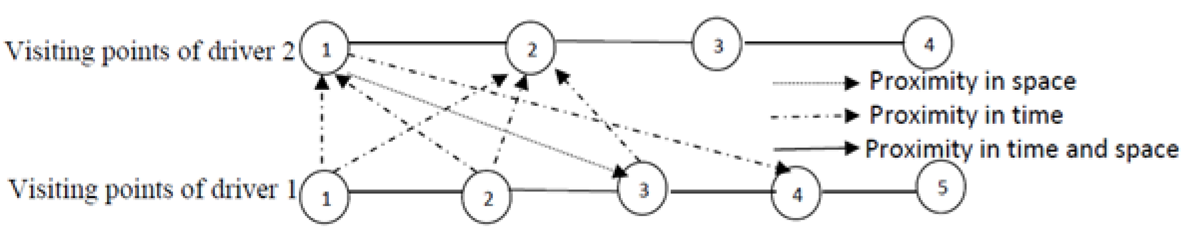
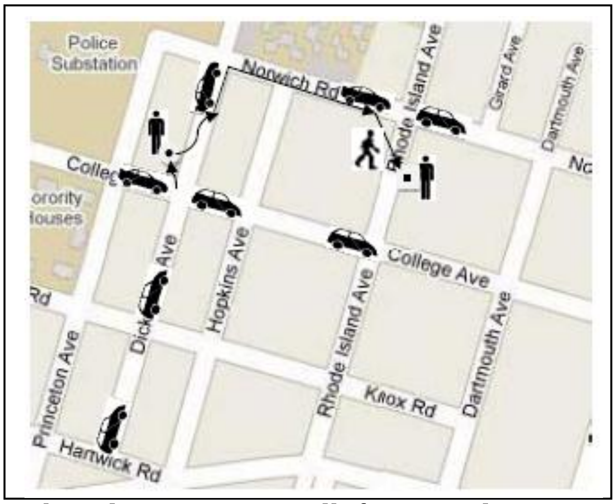
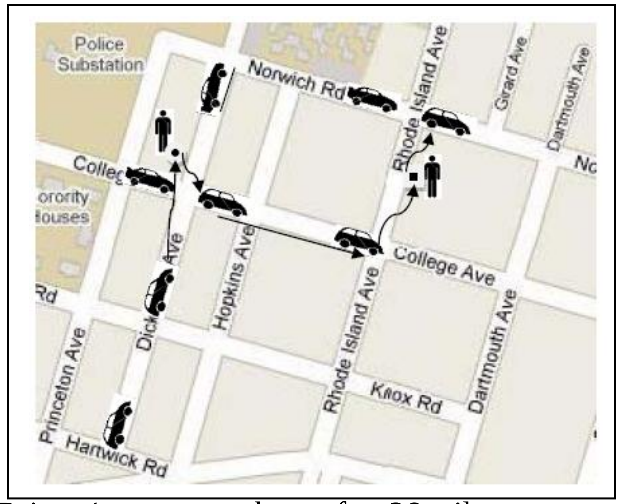


Figure 15: Proximity in time and space relationships for connects

Therefore, the passenger may start his first leg of journey with driver j’ and ends his last leg with driver j, but since there is no feasible connection between drivers, it is concluded that there is no rideshare possible for the passenger. Now, if the passenger accepts walking for .91 Miles more, the model will result in a feasible solution. Likewise, if driver j accepts to detour for .36 Miles more to pick up the passenger, he can pick up the passenger at his point of origin and drop him off at his destination after a detour. Alternatively, if driver j' accepts to detour for .5 Miles more, he can connect the passenger to driver j who will drop the passenger at his destination after making a detour. Figure 16 shows the compromise solutions.

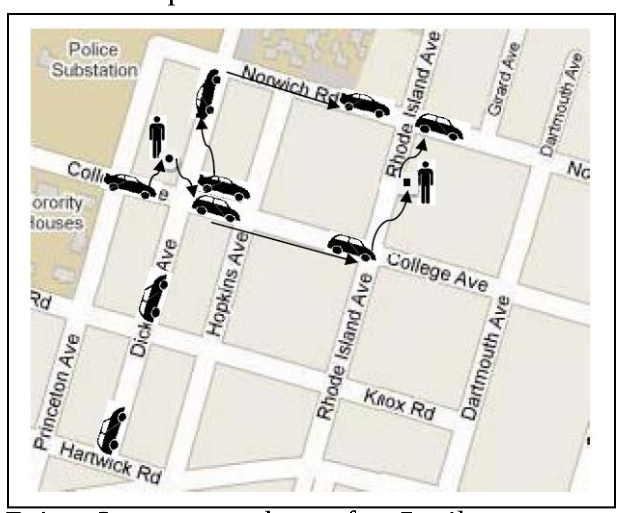


The rider accepts to walk for .91 miles more.



Driver 1 accepts to detour for .36 miles more.

Origin point of passenger
Destination point of passenger
Driver 1
Driver 2



Driver 2 accepts to detour for .5 miles more.

Figure 16: Compromise solutions.

5.2. Numerical Example 2

This is a medium size problem including requests from 14 riders at the same time in an area of 400 square Miles where 14 drivers are available and the path for each driver consists of 5 nodes. All the input parameters of the model are generated randomly including coordinates, requested time for service, waiting times, connection flexibilities, relocating distances, detour distances, seat capacities and preferences and characteristics. Figure 17 shows the screenshot for the input generator of the problem.

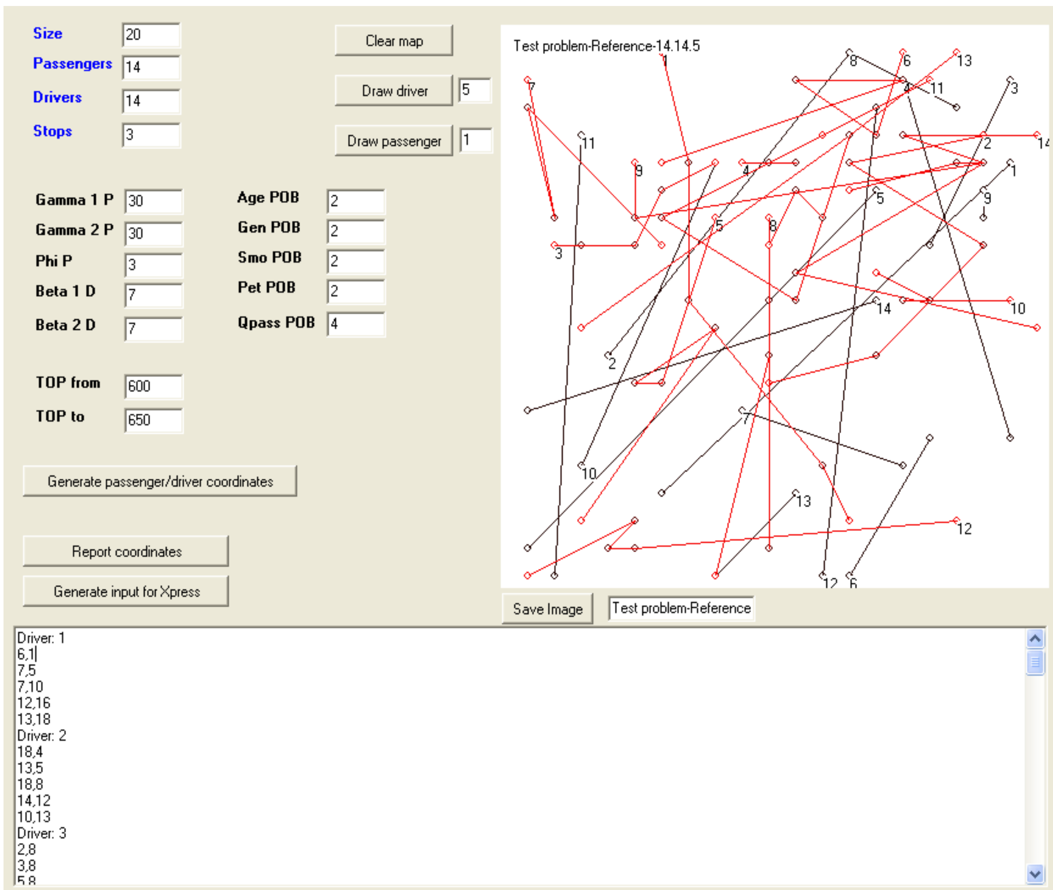


Figure 17: Screenshot for the input generator of the rideshare problem.

The problem is coded and solved using Xpress-IVE Version 1.21.02. The problem is expressed in a matrix with 484,134 rows (constraints) and 433,006 columns (variables). It took more than 6 hours for an Intel Core i7 CPU at 2.93 GHz speed to solve the problem. Following is a brief description of some of the matching results.

Figure 18 shows a zero connection route for passenger 13 and driver 12. Driver 12 makes a detour at node 1 at time 9:45 to pick up passenger 13 at time 10:09. Waiting time for the passenger is 4 minutes. Additional driving distance to pick up is 0 Miles. Driver 12 makes a detour at origin point of passenger 13 at time 10:09 to drop-off the passenger at his destination at time 10:25. Additional driving distance to drop off is 0 Miles.

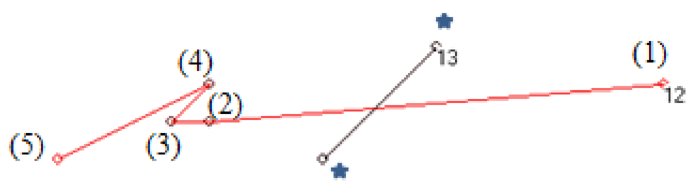


Figure 18: A zero connection route for rider 13 and driver 12

Figure 19 shows one connection routes for passenger 7 through drivers 8 and 1. The model resulted in 3 routes. Here are the details for those three routes:

1- Driver 8 makes a detour at node 2 at time 09:57 to pick up passenger 7 at time 10:13. Waiting time for the passenger is 6 minutes. Additional driving distance to pick up is 0 Miles. Driver 8 makes a detour at origin point of passenger 7 at time 613 to connect the passenger to driver 1 at node 3 at time 10:29. Additional driving distance for driver 8 to make a connection is 3 Miles. Driver 1 drops off the passenger at node 5 at time 10:57. The passenger walks for 2 Miles to reach to his destination at time 11:21.

2- Route_1 Connection(7,8,2,2,1,4,5) ===> Driver 8 makes a detour at node 2 at time 09:57 to pick-up passenger 7 at time 10:13. Waiting time for the passenger is 6 minutes. Additional driving distance to pick up is 0 Miles. Driver 8 makes a detour at origin point of passenger 7 at time 10:13 to connect the passenger to driver 1 at node 4 at time 10:25. Additional driving distance for driver 8 to make a connection is 0 Miles. Driver 1 drops off the passenger at node 5 at time 10:57. The passenger walks for 2 Miles to reach to his destination at time 11:21.

3- Route_1 Connection(7,8,2,3,1,4,5) ===> Driver 8 makes a detour at node 2 at time 10:57 to pick up passenger 7 at time 10:13. Waiting time for the passenger is 6 minutes. Additional driving distance to pick up is 0 Miles. Driver 8 makes a detour at node 3 at time 10:33 to connect passenger 7 at time 10:45 to driver 1 at node 4. Additional driving distance for driver 8 to make a connection is 0 Miles. Driver 1 drops off the passenger at node 5 at time 10:57. The passenger walks for 2 Miles to reach to his destination at time 11:21.

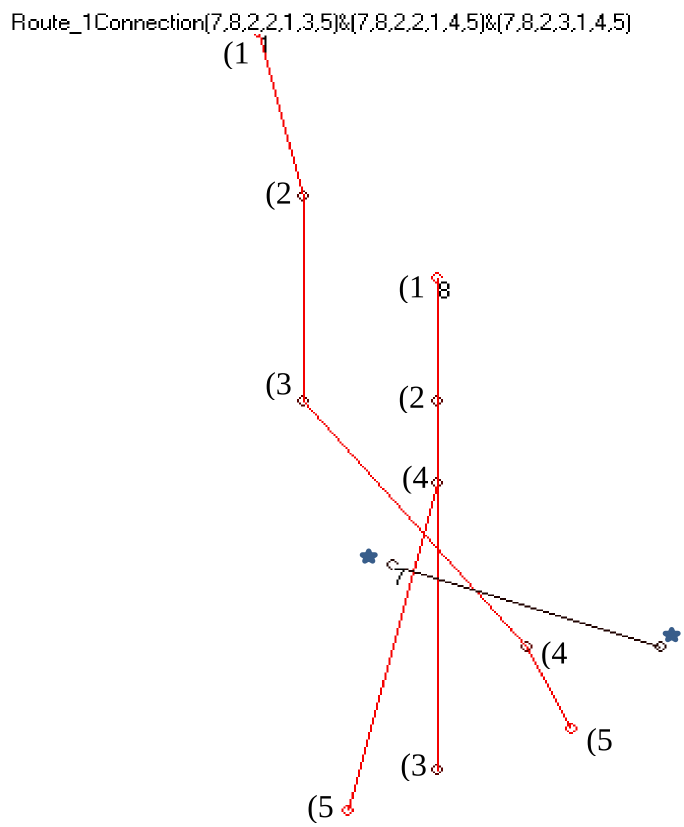


Figure 19: One connection routes for passenger 7 through drivers 8 and 1

Figure 20 shows a one connection route for passenger 10 through drivers 5 and 11. Driver 5 makes a detour at node 4 at time 10:25 to pick up passenger 10 at time 10:53. Waiting time for the passenger is 23 minutes. Additional driving distance to pick up is 1 Miles. Driver 5 makes a detour at origin point of passenger 10 at time 10:53 to connect the passenger to driver 11 at node 3 at time 11:33. Additional driving distance for driver 5 to make a connection is 6 Miles. Driver 11 makes a detour at node 4 at time 11:33 to drop off the passenger at time 11:53. Additional driving distance to drop off is 0 Miles.

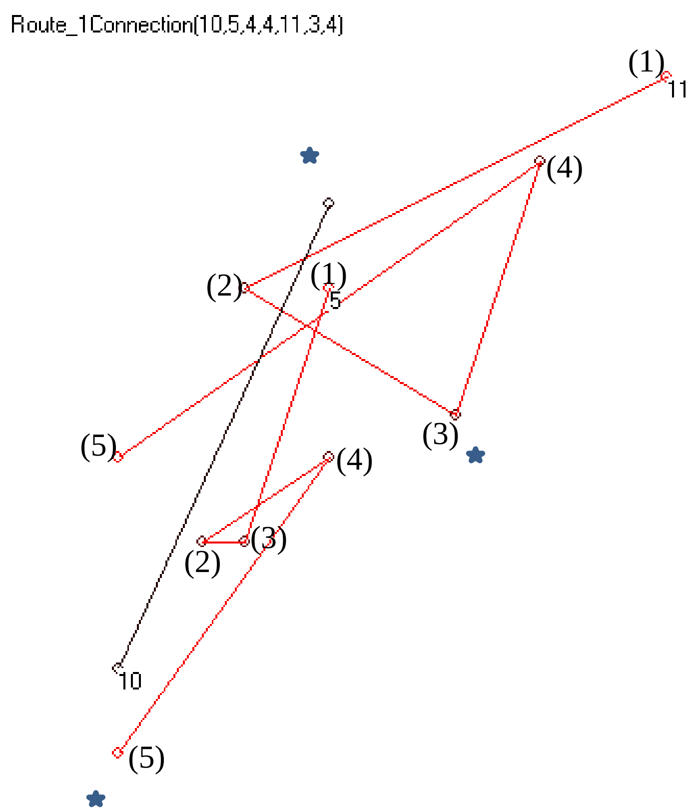


Figure 20: A one connection route for passenger 10 through drivers 5 and 11

Figure 21 shows a two connection route for passenger 11 through drivers 3, 1 and 5. Driver 3 makes a detour at node 2 at time 09:57 to pick up passenger 11 at time 10:13. Waiting time for the passenger is 11 minutes. Additional driving distance to pick up is 5 Miles. Driver 3 drops off the passenger at node 3 at time 10:33. The passenger walks for 2 Miles to connect to driver 1 at node 3 at time 10:57. Waiting time for the passenger is 0 minutes. Driver 1 makes a detour at node 3 at time 10:57 to connect the passenger to driver 5 at node 4 at time 11:01. Additional driving distance for driver 1 to make a connection is 0 Miles. Driver 5 drops off the passenger at node 5 at time 677. The passenger walks for 2 Miles to reach to his destination at time 701.

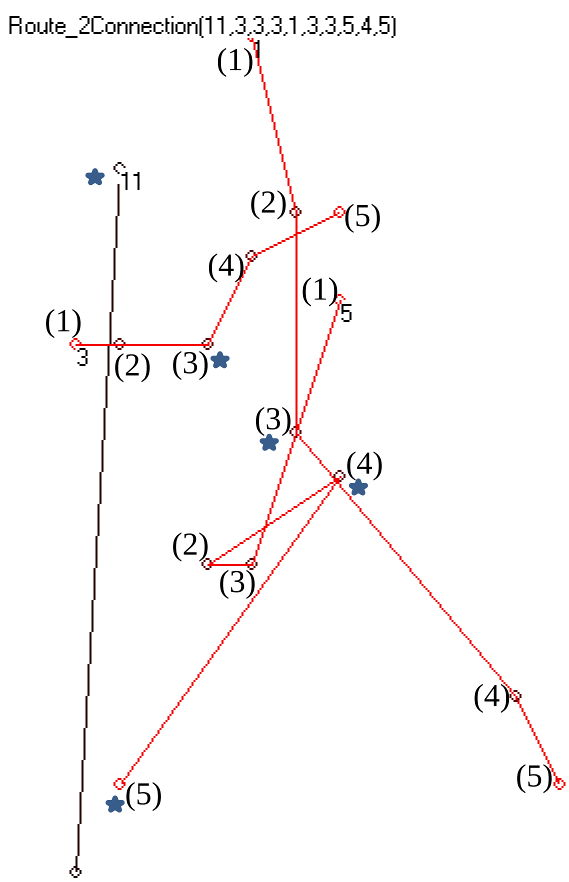


Figure 21:   A two connection route for passenger 11 through drivers 3, 1 and 5

The two connection model is infeasible with respect to the personal preferences and characteristics of the rider and drivers. The Model is able to generate negotiating policies. Figure 22 show the recommendation for negotiations to make the rideshare generated by DROM possible:

Age Preferences of passenger 11 and driver 3 doesn't match.
To have a feasible route, driver 3 needs to be flexible sharing a ride with age class of passenger 11 which is 1
To have a feasible route, passenger 11 needs to be flexible sharing a ride with age class of passenger on board which is 2 and passenger on board needs to be flexible sharing a ride with age class of passenger 11 which is 1
Gender preferences of passenger 11 and driver 3 doesn't match.
To have a feasible route, passenger 11 needs to be flexible sharing a ride with gender class of driver 3 which is 2
To have a feasible route, passenger 11 needs to be flexible sharing a ride with gender class of passenger on board which is 2
Pet preferences of passenger 11 and driver 3 doesn't match.
To have a feasible route, driver 3 needs to be flexible sharing a ride with pet class of passenger 11 which is 2
To have a feasible route, passenger on board needs to be flexible sharing a ride with pet class of passenger 11 which is 2
Smoking preferences of passenger 11 and driver 3 doesn't match.
To have a feasible route, passenger 11 needs to be flexible sharing a ride with smoking class of driver 3 which is 2
To have a feasible route, passenger on board needs to be flexible sharing a ride with smoking class of passenger 11 which is 2
Age Preferences of passenger 11 and driver 1 doesn't match.
To have a feasible route, driver 1 needs to be flexible sharing a ride with age class of passenger 11 which is 1
Gender preferences of passenger 11 and driver 1 doesn't match.
To have a feasible route, driver 1 needs to be flexible sharing a ride with gender class of passenger 11 which is 2
Pet preferences of passenger 11 and driver 1 doesn't match.
To have a feasible route, driver 1 needs to be flexible sharing a ride with pet class of passenger 11 which is 2
Age Preferences of passenger 11 and driver 5 doesn't match.
To have a feasible route, driver 5 needs to be flexible sharing a ride with age class of passenger 11 which is 1
Gender preferences of passenger 11 and driver 5 doesn't match.
To have a feasible route, driver 5 needs to be flexible sharing a ride with gender class of passenger 11 which is 2
Pet preferences of passenger 11 and driver 5 doesn't match.
To have a feasible route, driver 5 needs to be flexible sharing a ride with pet class of passenger 11 which is 2

Figure 22: Recommendation for negotiations generated by DROM

Chapter 6: Three-Spherical Heuristic Decomposition Model

The preliminary results of the optimization model suggest that the computational burden associated with the increasing size of the participants and visiting points of interests makes it impossible to rely on commercial solvers for obtaining optimal solutions in a reasonable computing time. This section develops an efficient solution algorithm for solving the optimization model. To develop an efficient heuristic solution method for the DROM problem, a comprehensive understanding of the structure of the problem is essential. The structure of rideshare problem can be viewed in two dimensions, i.e. the decisions involved and the constraints under which the decisions must be made. Major decisions involved in the DROM problem are as follows.

- Pickup: which rider should be picked up by a driver and how and at what time and location.
- Drop-off: which rider and how should be dropped off by a driver and how and at what time and location.
- Connection: which drivers should be connected and how and at what time and location to change the ride for a rider.
- Arrival time: what time each driver meets the points to be visited.

As mentioned in section 4, these decisions are interrelated. Constraints mainly come from two sources, the ridesharing to be done and matching preferences of the riders and drivers. Rideshare-related constraints require that rideshare must be properly performed. The following rideshare-related constraints are identified:

- Space proximity: ensures that pickup, drop off and connection points are within an acceptable walking distance for riders or within an acceptable detour distance for drivers.
- Time proximity: ensures that pickup is not earlier than the time requested by riders, waiting time for pickup, drop off and connections are not violating the predefined waiting times and connections are feasible with respect to proximity of arrival times for the drivers involved in the connection.
- Route: ensures that there is a route connecting pickup point and drop off point for each rider. The route can be a direct link which means rider receives a ride from only one driver who picks up the rider at the requested pickup point or at a convenient point close enough for walking from the requested pickup point and drops the rider off at the requested drop-off point or at a convenient walking distance from the destination point. The route also can be a multiple-link route which means rider receives multiple rides from multiple drivers who collaborate to connect pickup and drop-off points for the rider. In this research, multiple-link routes are called *n*-connection routes and *n* is selected to be 0, 1 or 2 for simplicity without loss of generality.
- Continuity: ensures that changes are made in original routes and arrival times after any changes in routes established by drivers.
- Rideshare preference – related constraints ensure that riders, riders on board, and drivers' preferences match before any rideshare is made.

Now we can start to derive a decomposition solution method by carefully considering the relationships between the decisions and constraints. This decomposition strategy leads to the heuristic solution procedure, Three-Spherical Heuristic Decomposition Model (TSHDM). The four types of decisions are interrelated and ideally need to be considered simultaneously. It is also reasonable to assume that most riders have a higher priority for less number of connections along their routes. To account for that widely accepted fact, TSHDM decomposes the problem to a three-level hierarchical problem. The first level is assigned to direct links as the riders have more preference on the zero connection routes compared to the routes with one or two connections. The solution strategy is searching at level one for all possible direct links connecting the pickup and drop off points for riders. The next step would be chasing at level two for one connection routes and after that for the routes with two connections at level three. When a route in lower priority level is established, arrival times updated, and points to be visited along the new route updated, the iterative solution strategy starts over to search for new rideshares to be established in the higher priority levels. The other considerations of the structure of rideshare problem suggest an attractive way of further decomposition. At each level, decisions can be made sequentially. At level one, problem decomposes into five subsequent sub-problems corresponding each to a decision in mind, i.e., pickup and drop-off with respect to proximities in time and space, route finding, establishing a route and updating of arrival times. At level two, problem decomposes into six subsequent sub-problems corresponding each to a decision in mind, i.e., pickup, drop-off and connection with respect to proximities in time and space, route finding, establishing a route and updating of arrival times. Finally, at level three, the problem would be decomposed into seven sub-problems namely pickup, drop-off, first and second connections with respect to proximities in time and space, route finding, establishing a route and updating of arrival times. To address the objective of minimizing the total travel time as well as maximizing the number of matched routes, THSDM considers a combined objective function that maximizes the number of matched routes and at the same time minimizes the combined total travel time of riders and drivers. For all the zero-connection iterations of the algorithm, the objective function is:

$$Max \sum_{i \in \hat{P}} \sum_{j \in \hat{D}} \sum_{m,n \in N} (M - t^{i}_{m^p_j, m^d_j} - t^{j}_{m^p_j, m^d_j}) \; X^0(i, j, m^p_j, m^d_j) \tag{87}$$

where $t^{i}_{m^p_j, m^d_j}$ and $t^{j}_{m^p_j, m^d_j}$ are the travel time for rider $i \in \hat{P}$ and driver $j \in \hat{D}$ respectively when m^p_j and m^d_j are the pickup and drop-off points for driver $j \in \hat{D}$. $X^0(i, j, m^p_j, m^d_j)$ is the binary variable that equals 1 when there is an established matched route with zero connection for rider $i \in \hat{P}$ and driver $j \in \hat{D}$ when m^p_j and m^d_j are the pickup and drop-off points for driver $j \in \hat{D}$. $\hat{P}$ and $\hat{D}$ are the set of riders and drivers with feasible pickup and drop-off which are not previously matched with the established routes respectively .

For all the one-connection iterations of the algorithm, the objective function is:

$$Max \sum_{i \in \hat{P}} \sum_{j \in \hat{D}} \sum_{m^p_j, n^c_j, n^c_k, m^d_k \in N} (M - t^{i}_{m^p_j, n^c_j, k, n^c_k, m^d_k} - t^{j}_{m,n m^p_j, n^c_j, k, n^c_k, m^d_k}) \; X^1(i, j, m^p_j, n^c_j, k, n^c_k, m^d_k) \tag{88}$$

where $t^{i}_{m^p_j, n^c_j, k, n^c_k, m^d_k}$ and $t^{j}_{m,n m^p_j, n^c_j, k, n^c_k, m^d_k}$ are the travel time for rider $i \in \hat{P}$ and driver $j \in \hat{D}$ respectively when m^p_j is the pickup point for driver $j \in \hat{D}$, n^c_j and n^c_k are the connection points for driver $j \in \hat{D}$ and driver $k \in \hat{D}$ and m^d_k is the drop-off point for driver $k \in \hat{D}$. $X^1(i, j, m^p_j, n^c_j, k, n^c_k, m^d_k)$ is the binary variable that equals 1 when there is an established matched route with one-connection for rider $i \in \hat{P}$ and driver $j \in \hat{D}$ when

m_j^p is the pickup point for driver $j \in \widehat{D}$, n_j^c and n_k^c are the connection points for driver $j \in \widehat{D}$ and driver $k \in \widehat{D}$ and m_k^d is the drop-off point for driver $k \in \widehat{D}$. $\widehat{P}$ and $\widehat{D}$ are respectively the set of riders and drivers with feasible pickup and drop-off who are not previously matched with the established routes.

For all the two-connection iterations of the algorithm, the objective function is:

$$Max \sum_{i \in \widehat{P}} \sum_{j \in \widehat{D}} \sum_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d \in N} (M - t^i_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d} - t^j_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d})\ X^2\left(i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d\right) \tag{89}$$

where $t^i_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d}$ and $t^j_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d}$ are the travel time for rider $i \in \widehat{P}$ and driver $j \in \widehat{D}$ respectively when m_j^p is the pickup point for driver $j \in \widehat{D}$, n_j^{c1} and n_k^{c1} are the first connection points for driver $j \in \widehat{D}$ and driver $k \in \widehat{D}$; n_k^{c2} and n_l^{c2} are the second connection points for driver $k \in \widehat{D}$ and driver $l \in \widehat{D}$ and m_l^d is the drop-off point for driver $l \in \widehat{D}$. $X^2\left(i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d\right)$ is the binary variable that equals 1 when there is an established matched route with two-connection for rider $i \in \widehat{P}$ and driver $j \in \widehat{D}$ when m_j^p is the pickup point for driver $j \in \widehat{D}$, n_j^{c1} and n_k^{c1} are the first connection points for driver $j \in \widehat{D}$ and driver $k \in \widehat{D}$; n_k^{c2} and n_l^{c2} are the second connection points for driver $k \in \widehat{D}$ and driver $l \in \widehat{D}$ and m_l^d is the drop-off point for driver $l \in \widehat{D}$. $\widehat{P}$ and $\widehat{D}$ are respectively the set of riders and drivers with feasible pickup and drop-off who are not previously matched with the established routes. Figure 23 shows the solution strategy of TSHDM followed by the step by step description for the algorithm.

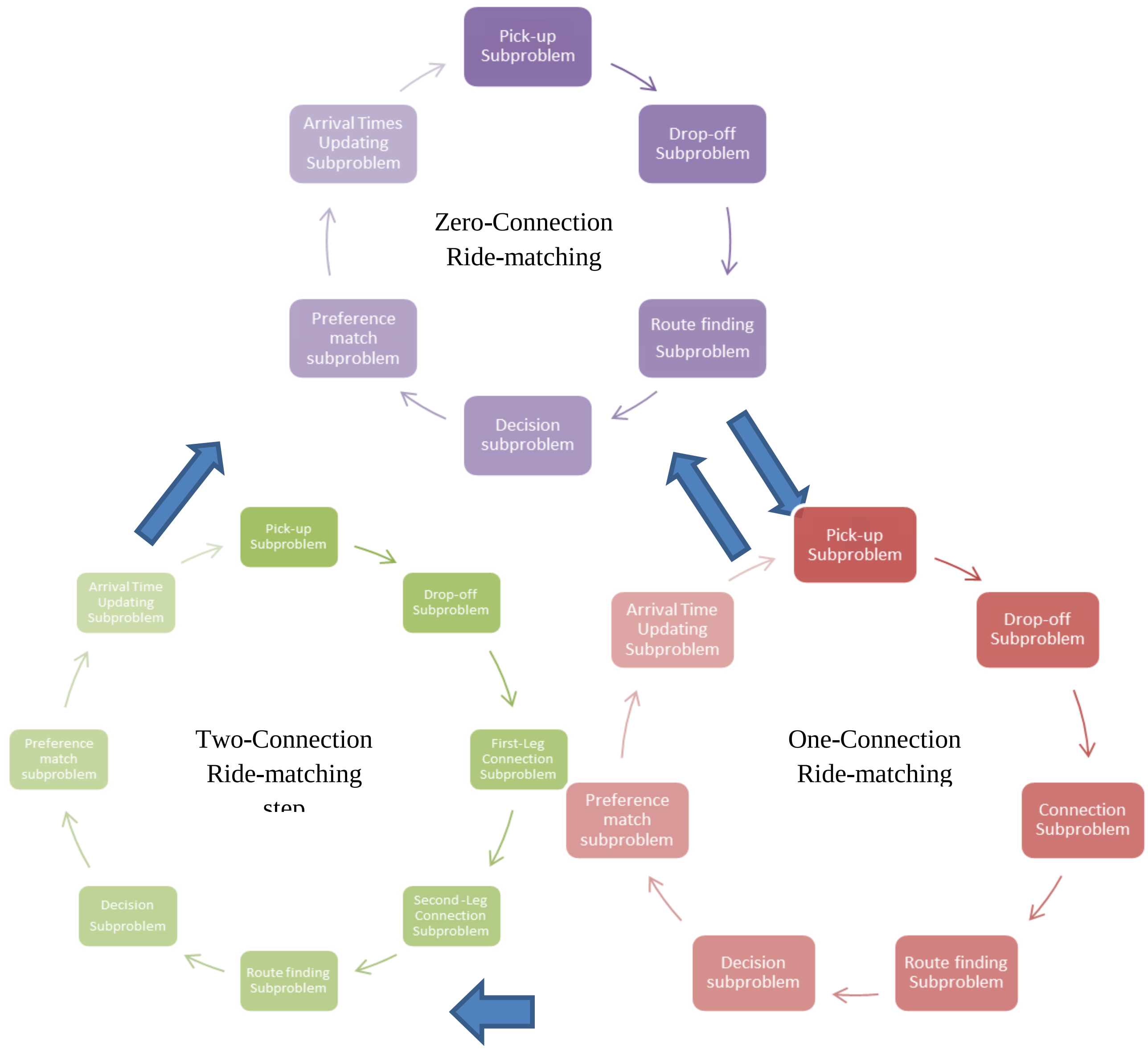


Figure 23: The solution strategy for TSHDM

6.1. TSHDM Algorithm

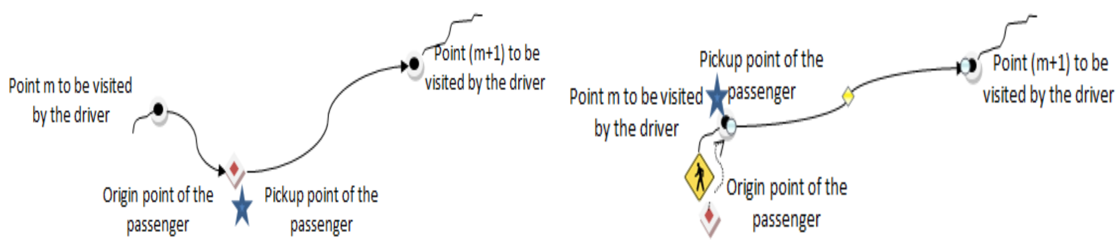
The following provides the step by step TSHDM algorithm which is entirely coded and solved in Xpress-IVE version 1.21.02 64 bit on a system with Intel® Core™ i7 CPU 870@ 2.93 GHZ processor:

0. Beginning of TSHDM algorithm:
1. List the participating riders and drivers.
2. I=1
3. **Zero-connection step:**
    a. Iteration I:
4. Pickup sub-problem:
    4-1. identify promising pick up drivers for each rider (type 1 and type 2 are illustrated in Figure 24)
    4-1.1. space feasibility check for pickup:

$$PS_{ijtv} = \begin{cases} 0 \text{ or } 1 \quad ; \quad D_{v_0^i, v_m^j} \leq \varphi \text{ or } D_{v_m^j, v_0^i} + D_{v_0^i, v_{m+1}^j} \leq D_{v_m^j, v_{m+1}^j} + \beta; \ v_m^j, v_{m+1}^j \in V^j \\ 0 \quad ; \quad otherwise \end{cases} \tag{90}$$

    4-1.2. time feasibility check for pickup:

$$PT_{ijtv} = \begin{cases} 0 \text{ or } 1; \ t_0^i + t'_{v_0^i, v_m^j} \leq t'_{v_m^j} \leq \gamma_i + t_0^i + t'_{v_0^i, v_m^j} \text{ or } t_0^i \leq t'_{v_m^j} + t'_{v_m^j, v_0^i} \leq \gamma_i + t_0^i; \ v_m^j, v_{m+1}^j \in V^j \\ 0 \quad ; \quad otherwise \end{cases} \tag{91}$$



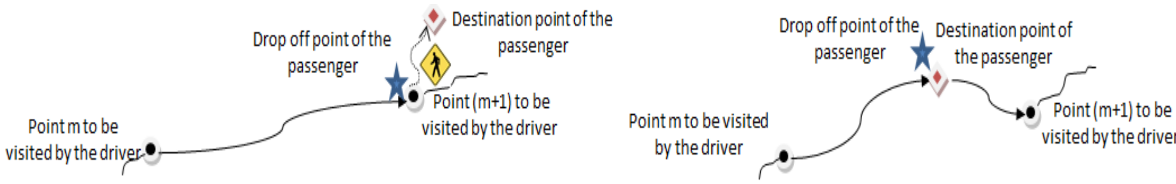
Type 2: driver makes a detour to pick up the rider

Type 1: rider walks to the point to be visited by the driver to be picked up

Figure 24: Feasibility check for pickup

    4-2. make a short list of the rider candidates whose feasibility checks for pickup are positive.
    4-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for pickup are positive.
5. Drop-off sub-problem:
    5-1. identify promising drop-off drivers for each rider (type 1 and type 2 are illustrated in Figure 25)
    5-1.1. feasibility check for pickup:

$$DS_{ijtv} = \begin{cases} 0 \text{ or } 1 \;; & D_{v_D^i, v_m^j} \le \varphi \text{ or } D_{v_m^j, v_D^i} + D_{v_D^i, v_{m+1}^j} \le D_{v_m^j, v_{m+1}^j} + \beta;\ v_m^j, v_{m+1}^j \in V^j \\ 0 \;; & otherwise \end{cases} \tag{92}$$



Type 1: rider leaves the driver at a point to be visited by the driver and walks to the destination point

Type 2: driver makes a detour to drop-off the rider at the destination point

Figure 25: Feasibility check for drop-off

5-2. make a short list of the rider candidates with positive drop-off feasibility checks
5-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for drop-off are positive.

6. Route finding sub-problem:
6-1. for every rider in the short list, check whether there is a same driver in his/her short lists of pickup and drop off drivers.

$$R^0_{ijv_p^jv_d^j} = \begin{cases} 1 \;; & PS_{ijtv_p^j} = PT_{ijtv_p^j} = DS_{ijtv_d^j} = 1 \text{ for } i \in P_t, j \in D_t;\ v_p^j, v_d^j \in V^j, v_d^j \ge v_p^j \\ 0 \;; & otherwise \end{cases} \tag{93}$$

6-2. for riders whose pickup-drop off feasibility checks is positive, make a list of zero connection routes.
6-3. for every route in the zero connection routes pool, calculate the associated characteristics including travel time for the rider and driver involved.

7. Decision sub-problem:
7-1. for the zero connection routes pool, solve the following assignment decision problem:

$$Max \sum_{i \in \hat{P}} \sum_{j \in \hat{D}} \sum_{m,n \in N} \left(M - t^i_{m_j^p, m_j^d} - t^j_{m_j^p, m_j^d}\right) X^0\left(i, j, m_j^p, m_j^d\right) \tag{94}$$

$$\sum_{j \in \hat{D}} \sum_{m,n \in N} X^0\left(i, j, m_j^p, m_j^d\right) = 1 \; ; \; i \in \hat{P} \tag{95}$$

$$\sum_{i \in \hat{P}} \sum_{m,n \in N} X^0\left(i, j, m_j^p, m_j^d\right) \le Q^j \; ; \; j \in \hat{D} \tag{96}$$

$$X^0\left(i, j, m_j^p, m_j^d\right) = 0 \text{ or } 1; \; i \in \hat{P}, j \in \hat{D}, m, n \in N \tag{97}$$

Eq. 94 is the objective function that maximizes the number of matched routes and minimizes the combined total travel time of riders and drivers simultaneously. Constraint 95 secures that there is only one route for every rider in the short list and constraint 96 secures that the driver has the enough seating capacity to offer a ride. Constraint 97 indicates that zero connection route decisions are binary variables.

7-2. make a list of assigned riders and drivers

8. Preference match sub-problem:

   8-1. for every rider, driver and riders on the board who has contributed in building a rideshare solution, do the following five decision checks.

      8-1.1. rider-driver matching check:

$$x^{\partial,j} X_t^{\partial,ij} = \sum_m \eta_m^{\partial,i} x_m^{\partial,ij} \text{ and } \sum_m x_m^{\partial,ij} \leq 1 \text{ for } \partial = 1, 2, 3, 4 \tag{98}$$

      8-1.2. driver-rider matching check:

$$x^{\partial,i} X_t^{\partial,ji} = \sum_m \eta_m^{\partial,j} x_m^{\partial,ji} \text{ and } \sum_m x_m^{\partial,ji} \leq 1 \text{ for } \partial = 1, 2, 3, 4 \tag{99}$$

      8-1.3. rider-rider onboard matching check:

$$x^{\partial,i'} X_t^{\partial,ii'} = \sum_m \eta_m^{\partial,i} x_m^{\partial,ii'} \text{ and } \sum_m x_m^{\partial,ii'} \leq 1 \text{ for } \partial = 1, 2, 3, 4 \tag{100}$$

      8-1.4. rider onboard-rider matching check:

$$x^{\partial,i} X_t^{\partial,i'i} = \sum_m \eta_m^{\partial,i'} x_m^{\partial,i'i} \text{ and } \sum_m x_m^{\partial,i'i} \leq 1 \text{ for } \partial = 1, 2, 3, 4 \tag{101}$$

      8-1.5. perfectly matched solution check:

$$2(1 + |\mathrm{O}_t^j|)\rho_{ijt}^{\partial} \leq X_t^{\partial,ij} + X_t^{\partial,ji} + \sum_{i' \in \mathrm{O}_t^j} (X_t^{\partial,ii'} + X_t^{\partial,i'i}) \; for \; i \in P_t, j \in D_t, i' \in O_t^j, \partial = 1, \ldots, 4 \tag{102}$$

   8-2. make a list of perfectly matched solutions and negotiating policies for imperfectly matched solutions based on the results for steps 8-1.1. to 8-1.4.

9. Arrival times updating sub-problem:

   Update the arrival times according to the following patterns:

   a. there is no detour to pick up and/or drop off the rider
   b. there is detour to pick up the rider
   c. there is a detour to drop-off the rider
   d. there is a detour to pickup and drop-off the rider

$$t^j_{v^j_{m+1}} = \begin{cases} t^j_{v^j_m} + t^j_{v^j_m, v^j_{m+1}} \; ; \; if \;\; P^2_{ijv^j_m t} = DS^2_{ijv^j_m t} = 0 \; ; \; j \in D_t ; \; v^j_m \in R^j_t ; \; v^{j'}_k \in R^{j'}_t \\ t^j_{v^j_m} + t^j_{v^j_m, v^i_O} + t^j_{v^i_O, v^j_{m+1}} \; ; \; if \; P^2_{ijv^j_m t} = 1 \; ; \; j \in D_t ; \; v^j_m \in R^j_t ; \; v^{j'}_k \in R^{j'}_t \\ t^j_{v^j_m} + t^j_{v^j_m, v^i_D} + t^j_{v^i_D, v^j_{m+1}} \; ; \; if \; DS^2_{ijv^j_m t} = 1 \; ; \; j \in D_t ; \; v^j_m \in R^j_t ; v^{j'}_k \in R^{j'}_t \\ t^j_{v^j_m} + t^j_{v^j_m, v^i_O} + t^j_{v^i_O, v^i_D} + t^j_{v^i_D, v^j_{m+1}} \; ; \; if \; P^2_{ijv^j_m t} = DS^2_{ijv^j_m t} = 1 \; ; \; j \in D_t ; \; v^j_m \in R^j_t ; \; v^{j'}_k \in R^{j'}_t \end{cases} \tag{103}$$

10. update the list of riders and drivers

    10-1. remove the assigned riders from the list.

    10-2. update seating capacity of drivers from the list.

11. I=I+1 and go back to step 3.

12. Repeat steps 3 to 11 until there is no more zero connection routes. Go to step 13.

13. One-connection step:

   a. Iteration I:

14. Pickup sub-problem:

    14-1. identify promising pick up drivers for each rider (type 1 and type 2)

       14-1.1. space feasibility check for pickup.

       14-1.2 time feasibility check for pickup.

14-2. make a short list of the rider candidates with positive pickup checks

14-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for pickup are positive.

15. Drop-off sub-problem:

15-1. identify promising drop-off drivers for each rider (type 1 and type 2)

15-1.1. feasibility check for drop-off.

15-2. make a short list of the rider candidates with positive drop-off checks.

15-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for pickup are positive.

16. Connection sub-problem:

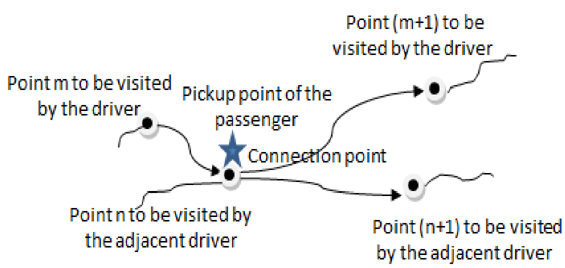
16-1. identify promising connection drivers for each rider (type 1 and type 2 are illustrated in Figure 26)

16-1.1. space feasibility check for connection:

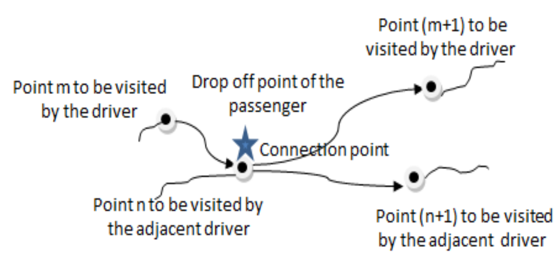
$$CS_{ijv_m^j j' v_k^{j'}} = \begin{cases} 0 \; or \; 1 \quad ; & D_{v_m^j, v_k^{j'}} \leq \varphi \, ; j \in D_t ; \; v_m^j, v_{m+1}^j \in R_t^j ; \; ; \; v_k^{j'} \in R_t^{j'} \; or \\ & Dr_{v_m^j, v_k^{j'}} + D_{v_k^{j'}, v_{m+1}^j} \leq (D_{v_m^j, v_{m+1}^j} + \beta_j) ; \; j \in D_t ; \; v_m^j, v_{m+1}^j \in R_t^j ; \; ; \; v_k^{j'} \in R_t^{j'} \\ 0 \quad ; & otherwise \end{cases} \tag{104}$$

16-1.2. time feasibility check for connection:

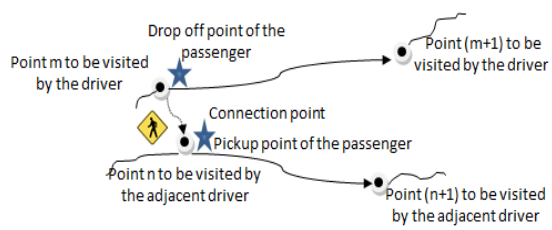
$$CT_{ijv_m^j j' v_k^{j'}} = \begin{cases} 0 \; or \; 1 \quad ; & \begin{matrix} t_{v_m^j}^j + t_{v_m^j, v_k^{j'}}^i \leq t_{v_k^{j'}}^{j'} \leq \gamma_i' + t_{v_m^j}^j + t_{v_m^j, v_k^{j'}}^i \quad or \\ t_{v_m^j}^j + t_{v_m^j, v_k^{j'}}^j \leq t_{v_k^{j'}}^{j'} \leq \gamma_i' + t_{v_m^j}^j + t_{v_m^j, v_k^{j'}}^j \end{matrix} \; ; \; j \in D_t ; \; v_m^j \in R_t^j ; \; ; \; v_k^{j'} \in R_t^{j'} \\ 0 \quad ; & otherwise \end{cases} \tag{105}$$



(a) driver makes a detour to pick up the rider at the point to be visited by the next



(b) driver makes a detour to drop off the rider at the point to be visited by the next



(c) rider leaves the driver at a point to be visited by the driver and walks to the point to be visited by the next

Figure 26: Feasibility checks for a connection

16-2. make a short list of the rider candidates with positive connection feasibility checks.

16-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for connection are positive.

17. Route finding sub-problem:

17-1. for every rider in the short list, check whether there is a same driver in his/her short lists of pickup, connection and drop off drivers.

$$R^1_{ijv_p^j v_d^j k v_p^k v_d^k} = \begin{cases} 1 \;\; ; \;\; PS_{ijtv_p^j} = PT_{ijtv_p^j} = CS_{ijv_d^j kv_p^k} = CT_{ijv_d^j kv_p^k} = DS_{iktv_d^k} = 1\,;\; i \in P_t;\; j,k \in D_t; \\ \qquad\qquad v_p^j, v_d^j \in V^j, v_d^j \geq v_p^j;\; v_p^k, v_d^k \in V^k, v_d^k \geq v_p^k \\ 0; \quad Otherwise \end{cases} \tag{106}$$

17-2. for riders whose pickup-connection-drop off feasibility checks are positive, make a list of one connection routes.

17-3. for every route in the one connection routes pool, calculate the associated characteristics including travel time for the rider and drivers involved.

18. Decision sub-problem:

18-1. for the one connection routes pool, solve the following assignment decision problem:

$$Max \sum_{i \in \hat{P}} \sum_{j \in \hat{D}} \sum_{m_j^p, n_j^c, n_k^c, m_k^d \in N} \left(M - t^i_{m_j^p, n_j^c, k, n_k^c, m_k^d} - t^j_{m,nm_j^p, n_j^c, k, n_k^c, m_k^d}\right) X^1\left(i, j, m_j^p, n_j^c, k, n_k^c, m_k^d\right) \tag{107}$$

$$\sum_{j \in \hat{D}} \sum_{m_j^p, n_j^c, n_k^c, m_k^d \in N} X^1\left(i, j, m_j^p, n_j^c, k, n_k^c, m_k^d\right) = 1\,;\; i \in \hat{P} \tag{108}$$

$$\sum_{i \in P} \sum_{m_j^p, n_j^c, n_k^c, m_k^d \in N} X^1\left(i, j, m_j^p, n_j^c, k, n_k^c, m_k^d\right) \leq Q^j\,;\; j \in \hat{D} \tag{109}$$

$$\sum_{i \in P} \sum_{m_j^p, n_j^c, n_k^c, m_k^d \in N} X^1\left(i, j, m_j^p, n_j^c, k, n_k^c, m_k^d\right) \leq Q^k\,;\; k \in \hat{D} \tag{110}$$

$$X^1\left(i, j, m_j^p, n_j^c, k, n_k^c, m_k^d\right) = 0 \text{ or } 1;\; i \in \hat{P}, j \in \hat{D}, m_j^p, n_j^c, n_k^c, m_k^d \in N \tag{111}$$

Eq. 107 is the objective function that maximizes the number of matched routes and minimizes the combined total travel time of riders and drivers simultaneously. Constraint 108 secures that there is only one route for every rider in the short list and constraints 109 and 110 secure that the two drivers contributing in the rideshare have the enough seating capacity to offer rides. Constraint 111 indicates that one connection route decisions are binary variables.

18-2. make a list of assigned riders and drivers

19. Preference match sub-problem:
    19-1. for every rider, driver and riders on the board who has contributed in building a rideshare solution, do the following five decision checks.
        19-1.1. rider-driver matching check.
        19-1.2. driver-rider matching check.
        19-1.3. rider-rider onboard matching check.
        19-1.4. rider onboard-rider matching check.
        19-1.5. perfectly matched solution check.
    19-2. make a list of perfectly matched solutions and negotiating policies for imperfectly matched solutions based on the results for steps 19-1.1. to 19-1.5.
20. Arrival times updating sub-problem:
    Update the arrival times according to the following patterns:
    a. there is detour to pick up the rider
    b. there is a detour to drop-off the rider
    c. there is a detour to pickup and drop-off the rider
    d. there is a detour connection to pick up or drop-off the rider
    e. there is a detour connection to pick up and a detour to drop-off the rider

$$t^{j}_{v^{j}_{m+1}} = \begin{cases} t^{j}_{v^{j}_{m}} + t^{j}_{v^{j}_{m},v^{j}_{m+1}} ; \; if \;\; P^{2}_{ijv^{j}_{m}t} = C^{2}_{jv^{j'}_{k}j'v^{j}_{m}} = DS^{2}_{ijv^{j}_{m}t} = 0 \; ; \; j \in D_t ; \; v^{j}_{m} \in R^{j}_{t} ; \; v^{j'}_{k} \in R^{j'}_{t} \\ t^{j}_{v^{j}_{m}} + t^{j}_{v^{j}_{m},v^{i}_{O}} + t^{j}_{v^{i}_{O},v^{j}_{m+1}} ; \; if \; P^{2}_{ijv^{j}_{m}t} = 1 \; ; \; j \in D_t ; \; v^{j}_{m} \in R^{j}_{t} ; \; v^{j'}_{k} \in R^{j'}_{t} \\ t^{j}_{v^{j}_{m}} + t^{j}_{v^{j}_{m},v^{i}_{D}} + t^{j}_{v^{i}_{D},v^{j}_{m+1}} ; \; if \; DS^{2}_{ijv^{j}_{m}t} = 1 \; ; \; j \in D_t ; \; v^{j}_{m} \in R^{j}_{t} ; v^{j'}_{k} \in R^{j'}_{t} \\ t^{j}_{v^{j}_{m}} + t^{j}_{v^{j}_{m},v^{i}_{O}} + t^{j}_{v^{i}_{O},v^{i}_{D}} + t^{j}_{v^{i}_{D},v^{j}_{m+1}} ; \; if \; P^{2}_{ijv^{j}_{m}t} = DS^{2}_{ijv^{j}_{m}t} = 1 \; ; \; j \in D_t ; \; v^{j}_{m} \in R^{j}_{t} ; \; v^{j'}_{k} \in R^{j'}_{t} \\ t^{j}_{v^{j}_{m}} + t^{j}_{v^{j}_{m},v^{j'}_{k}} + t^{j}_{v^{j'}_{k},v^{j}_{m+1}} ; \; if \;\; C^{2}_{jv^{j'}_{k}j'v^{j}_{m}} = 1 \; ; \; j \in D_t ; \; v^{j}_{m} \in R^{j}_{t} ; \; v^{j'}_{k} \in R^{j'}_{t} \\ t^{j}_{v^{j}_{m}} + t^{j}_{v^{j}_{m},v^{j'}_{k}} + t^{j}_{v^{j'}_{k},v^{i}_{D}} + t^{j}_{v^{i}_{D},v^{j}_{m+1}} ; \; if \;\; C^{2}_{jv^{j'}_{k}j'v^{j}_{m}} = DS^{2}_{ijv^{j}_{m}t} = 1 \; ; \; j \in D_t ; \; v^{j}_{m} \in R^{j}_{t} ; v^{j'}_{k} \in R^{j'}_{t} \end{cases} \tag{112}$$

21. update the list of riders and drivers
    10-1. remove the assigned riders from the list.
    10-2. update seating capacity of drivers from the list.
22. I=I+1 and go back to step 3.
23. Repeat steps 3 to 22 until there are no more assignments for zero and one connection routes. Go to step 24.
24. **Two-connections step:**
    a. Iteration I:
25. Pickup sub-problem:
    25-1. identify promising pick up drivers for each rider (type 1 and type 2)
        25-1.1. space feasibility check for pickup.
        25-1.2. time feasibility check for pickup.
    25-2. make a short list of the rider candidates with positive pickup checks.
    25-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for pickup are positive.

26. Drop-off sub-problem:
    26-1. identify promising drop-off drivers for each rider (type 1 and type 2)
        26-1.1. feasibility check for drop-off.
    26-2. make a short list of the rider candidates whose feasibility checks for drop-off are positive.
    26-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for pickup are positive.
27. First-leg connection sub-problem:
    27-1. identify promising first-leg drivers for each rider (type 1 and type 2)
        27-1.1. space feasibility check for first leg connection:
        27-1.2. time feasibility check for first leg connection:
    27-2. make a short list of the rider candidates whose feasibility checks for the first leg connection are positive.
    27-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for the first connection are positive.
28. Second-leg connection sub-problem:
    28-1. identify promising second-leg drivers for each rider (type 1 and type 2)
        28-1.1. space feasibility check for second leg connection:
        28-1.2. time feasibility check for second leg connection:
    28-2. make a short list of the rider candidates whose feasibility checks for the second leg connection are positive.
    28-3. make a short list of the driver candidates for each rider in the short list whose feasibility checks for the second connection are positive.
29. Route finding sub-problem:
    29-1. for every rider in the short list, check whether there is a same driver in his/her short lists of pickup, first leg connection, second leg connection and drop off drivers.

$$R^2_{ijv_p^j v_d^j k v_p^k v_d^k l v_p^l v_d^l} = \begin{cases} 1 \quad ; \quad PS_{ijtv_p^j} = PT_{ijtv_p^j} = CS_{ijv_d^j kv_p^k} = CT_{ijv_d^j kv_p^k} = CS_{ijv_d^k kv_p^l} = CT_{ijv_d^k kv_p^l} = DS_{iktv_d^l} = 1 \\ \quad for\ i \in P_t;\ j,k \in D_t;\ v_p^j, v_d^j \in V^j, v_d^j \geq v_p^j;\ v_p^k, v_d^k \in V^k, v_d^k \geq v_p^k;\ v_p^l, v_d^l \in V^l, v_d^l \geq v_p^l \\ 0 \quad ; \quad Otherwise \end{cases} \tag{113}$$

    29-2. for riders whose pickup-first connection-second connection-drop off feasibility checks are positive, make a list of two connection routes.
    29-3. for every route in the two connection routes pool, calculate the associated characteristics including travel time for the rider and drivers involved.
30. Decision sub-problem:
    30-1. for the two connection routes pool, solve the following decision problem:

$$Max \sum_{i\in\hat{P}}\sum_{j\in\hat{D}}\sum_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d \in N} \left(M - t^i_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d} - t^j_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d}\right) X^2\left(i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d\right) \tag{114}$$

$$\sum_{j\in\hat{D}}\sum_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d \in N} X^2\left(i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d\right) = 1;\ i \in \hat{P} \tag{115}$$

$$\sum_{i\in\hat{P}}\sum_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d \in N} X^2\left(i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d\right) \le Q^j\ ;\ j \in \hat{D} \tag{116}$$

$$\sum_{i\in\hat{P}}\sum_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d \in N} X^2\left(i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d\right) \le Q^k\ ;\ k \in \hat{D} \tag{117}$$

$$\sum_{i\in\hat{P}}\sum_{i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d \in N} X^2\left(i,j,m_j^p,n_j^{c1},k,n_k^{c1},n_k^{c2},l,n_l^{c2},m_l^d\right) \le Q^l\ ;\ l \in \hat{D} \tag{118}$$

$$X^1\left(i,j,m_j^p,n_j^c,k,n_k^c,m_k^d\right) = 0 \text{ or } 1;\ i \in \hat{P}, j \in \hat{D}, m_j^p, n_j^c, n_k^c, m_k^d \in N \tag{119}$$

Eq. 114 is the objective function that maximizes the number of matched routes and minimizes the combined total travel time of riders and drivers simultaneously. Constraint 115 secures that there is only one route for every rider in the short list and constraints 116 to 119 secure that the three drivers contributing in the rideshare have enough seating capacity to offer the rides. Constraint 119 indicates that two connection route decisions are binary variables.

30-2. make a list of assigned riders and drivers

31. Preference match sub-problem:

 31-1. for every rider, driver and riders on the board who has contributed in building a rideshare solution, do the following five decision checks.

  31-1.1. rider-driver matching check.
  31-1.2. driver-rider matching check.
  31-1.3. rider-rider onboard matching check.
  31-1.4. rider onboard-rider matching check.
  31-1.5. perfectly matched solution check.

 31-2. make a list of perfectly matched solutions and negotiating policies for imperfectly matched solutions based on the results for steps 31-1.1. to 31-1.5.

32. Arrival times updating sub-problem:

 Update the arrival times according to the following patterns:

  a. there is detour to pick up the rider
  b. there is a detour to drop-off the rider
  c. there is a detour to pickup and drop-off the rider
  d. there is a detour connection to pick up or drop-off the rider
  e. there is a detour connection to pick up and a detour to drop-off the rider

33. Update the list of riders and drivers
    33-1. remove the assigned riders from the list.
    33-2. update seating capacity of drivers from the list.
34. I=I+1 and go back to step 3.
35. Repeat steps 3 to 34 until there are no more zero, one and two connection routes.
36. End of TSHDM algorithm.

6.2. Illustrative examples:

6.2.1. Numerical Example 60*40*360

This numerical example is included to illustrate concepts presented in the heuristic solution strategy of TSHDM. It includes sixty passengers, forty drivers and there are a set of six points to be visited for each driver in an area size of 100 square Miles. Figure 27 shows the map for this problem. For each driver there is an original route which connects the eight points to be visited by the driver and it is shown in red line in the map. The numbers next to the first node of each red line show the drivers' Identification numbers as well as the direction of their move and their current location in the system. Riders are shown in the map with black lines that connect the origin and destination points for each of the riders. The numbers next to the black line show the origin points and identification number for riders.

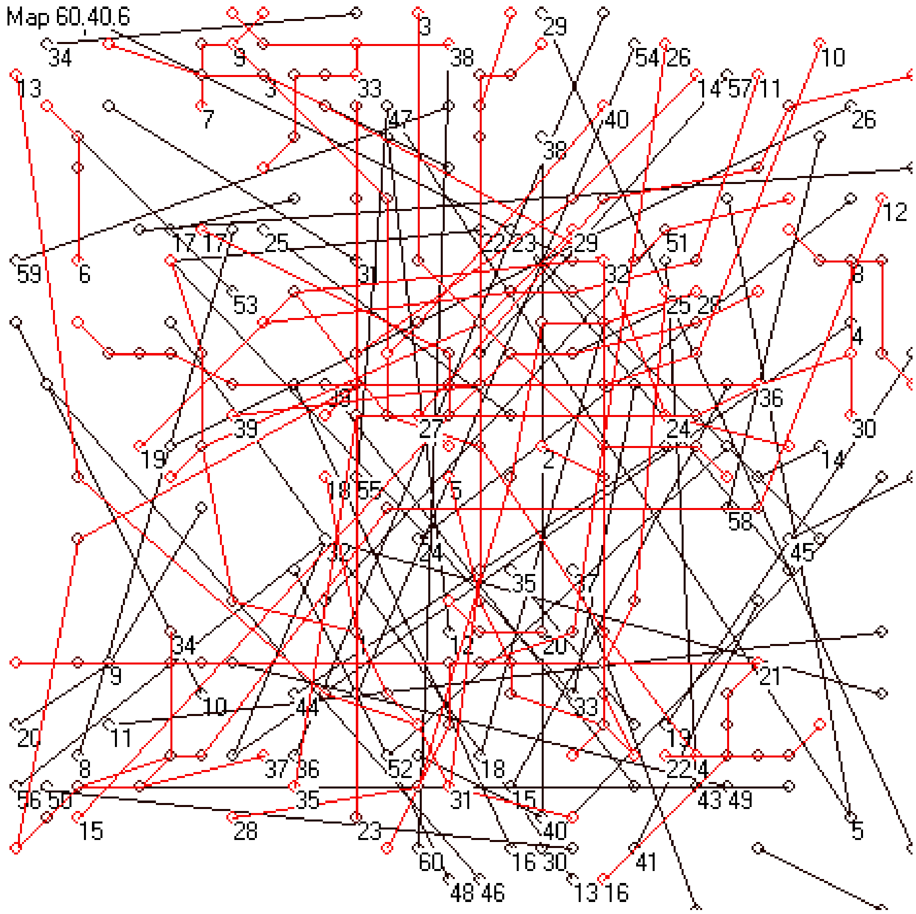


Figure 27: Map of the 60.40.360 test problem

Other input parameters are all generated randomly using a test problem generator. It is assumed that maximum waiting time for each rider to be picked up at the origin point or connection points is 20 minutes. Moreover, it is assumed that each driver is flexible with maximum diversion of 5 Miles from its original route to pick up or drop off a rider or to make a connection with other drivers to transfer a rider. In additions, the maximum relocation distance for riders is assumed to be 5 Miles. For the research purposes of this study, maximum waiting time, maximum detour and relocation distances are exaggerated to increase the chance of rideshare matches. Current time of the system is assumed to be 10:00 a.m. and all the rideshare service requests are within half an hour from 10:00 to

10:30. Seat capacity for each car is assumed to be 5 and it is assumed that only one seat of each car is occupied at the current time of the system. In addition, it is assumed that there is no restriction on the rideshare preferences of the entire participants to allow the system to have the maximize likelihood of the rideshare matches. Table 12 shows the summary of results for this problem.

Table 12: The summary of results for illustrated example 60.40.360

Iter	Matched established routes	Acc. Time (sec.)	# routes
1	0_Connection (p_i, d_j, n_p, n_d): (1,19,1,3); (2,1,1,6); (3,3,1,1); (6,35,1,1); (9,23,2,2); (12,31,2,4); (14,25,1,1); (15,28,2,6); (16,4,1,5); (18,24,1,2); (24,27,1,3); (26,36,1,2); (28,11,2,2); (31,7,1,2); (32,34,1,2); (34,33,2,3); (35,21,1,1); (39,2,2,6); (40,26,4,5); (45,12,1,1); (48,18,1,2); (53,6,2,4); (55,39,2,2); (57,29,1,2); (58,30,1,6); (59,9,1,2)	.054	26
2	0_Connection (p_i, d_j, n_p, n_d): (17,33,3,3); (21,12,1,1); (49,23,1,2); (52,35,4,5)	.073	4
3	0_Connection (p_i, d_j, n_p, n_d): (54,12,2,3)	.089	1
4	0_Connection (p_i, d_j, n_p, n_d): (4,12,5,5)	1.061	1
5	1_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, n_d)$: (38,26,1,3,13,4,5); (43,4,2,3,28,3,5); (60,20,3,6,31,2,5)	10.02	3
6	0_Connection (p_i, d_j, n_p, n_d): not found	12.65	0
	1_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, n_d)$: not found	14.86	0
8	2_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, d_{j''}, n_c^{d_{j''}}, n_d)$: (56,23,1,2,26,3,4,13,5,6)	29.32	1
9	0_Connection (p_i, d_j, n_p, n_d): not found	32.11	0
10	1_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, n_d)$: not found	37.29	0
11	2_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, d_{j''}, n_c^{d_{j''}}, n_d)$: not found	43.15	0

In Table 12,

p_i: passenger i $(i: 1, \ldots, 60)$,

d_j: first-link driver j $(j: 1, \ldots, 40)$,

$d_{j'}$: second-link driver j' $(j': 1, \ldots, 40)$,

$d_{j''}$: third-link driver j'' $(j'': 1, \ldots, 40)$,

n_p: pickup node,

n_d, drop off node,

$n_c^{d_j}$: first-connection node for driver j $(j: 1, \ldots, 40)$,

$n_c^{d_{j'}}$: second-connection node for driver j' $(j': 1, \ldots, 40)$,

$n_c^{d_{j''}}$: third-connection node for driver j'' $(j'': 1, \ldots, 40)$.

As the table shows, the heuristic algorithm established 32 zero-connection matched routes in 1.061 seconds, 3 one-connection matched routes in 14.867 seconds and 1 two-connection route in 43.15 seconds. 28 drivers (70% of the drivers) contributed to the solution and the system was successful to find rideshare solutions for 36 passengers, i.e., rate of successful matches for the system was 53%. The share of zero-connection routes in the 36 rideshare matches was 88% while the share for one-connection and two-connection routes were 8.3% and 2.7%, respectively.

The system has resulted in 32 zero-connection routes. Further we discuss some of the solutions that might be of interest to present capabilities of the algorithm. For example, rideshare routes (45,12,1,1), (21,12,1,1), and (54,12,2,3) are sharing the same driver, i.e. Driver 12 has contributed in the establishment of 3 rideshares for three passengers 45, 21, and 54. Figure (28-a) shows the origin and destination points for the three passengers and the current location of the driver and his original route and the 6 points to be visited along the route. Details show that Driver 12 has to make a detour at his current location to pick up Passenger 45 at his point of origin and drop him off at his destination (Figure 28-b). This driver goes to the origin point of Passenger 21 to pick him up at his origin and takes him to his destination (Figure 28-c) and then heads to his 2nd point to be visited on his original route and makes a detour at that point to pick up Passenger 54 at his point of origin and drops him off at his point of destination (Figure 28-d). Driver 12 heads up toward his 3rd point to be visited on the original route and continues along the route to meet his 4th, 5th, and 6th points to be visited (Figure 28-e). Figure (28-f) shows the updated route for Driver 12 as well as the routes of journey for Passenger 45, Passenger 21, and Passenger 54.

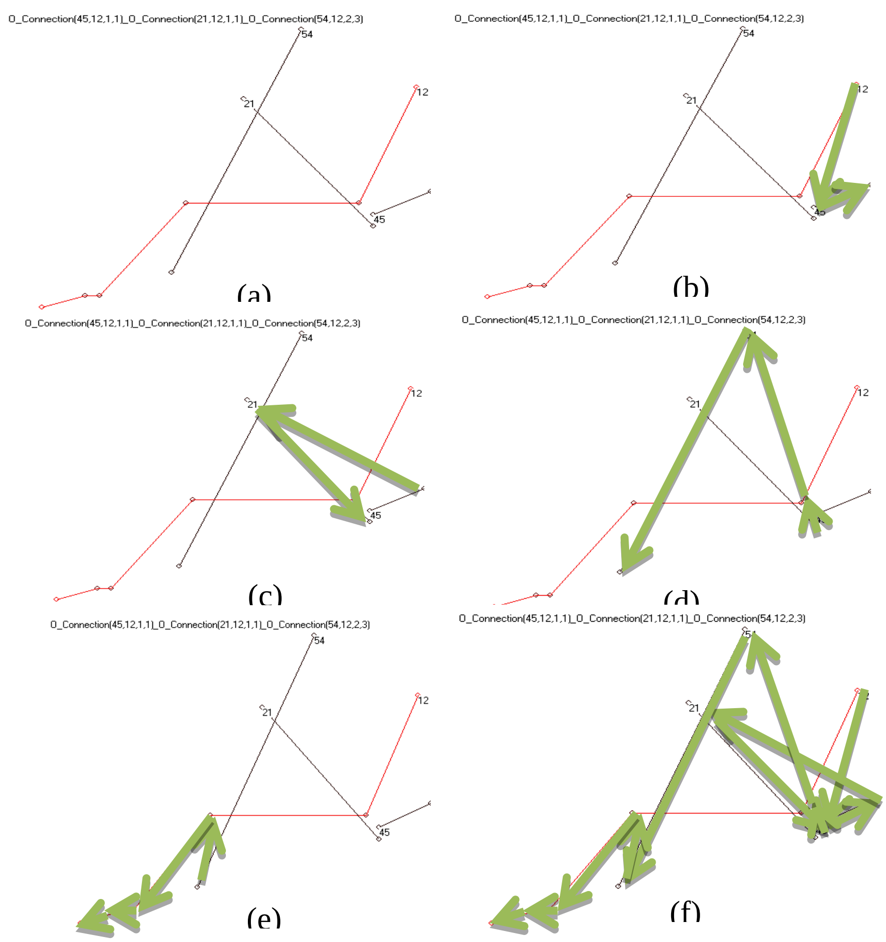


Figure 28: Course of events for Driver 12, Passenger 45, Passenger 21, and Passenger 54

The system has resulted in 3 one-connection route. For example, rideshare route (43, 4, 2, 3, 28, 3, 5) is established when Driver 43 and Driver 28 have contributed to make a rideshare connecting the origin and destination points of Passengers 4. Figure (29-a) shows the origin and destination of the passenger and the current locations of the two drivers along with their original routes and the 6 points to be visited along the routes. Details reveal that Driver 43 makes a detour at his current location to pick up Passenger 4 at his point of origin and then heads to his second point to be visited. At the same time Driver 28 is on way to his 2nd point to be visited (Figure 29-b). Driver 43 meets Driver 28 at the 3rd point to be visited for both drivers (connection point) where Passenger 4 changes his drive (Figure 29-c). Driver 43 heads toward the next points along his original route and Driver 28 visits his 4th and 5th original points and makes a detour at the 5th point to drop off the passenger at his destination (Figure 29-d) and then heads toward his 6th point to be visited (Figure 29-e). Figure 29-f shows the updated routes for Driver 43 and Driver 28 as well as the route of journey for Passenger 4.

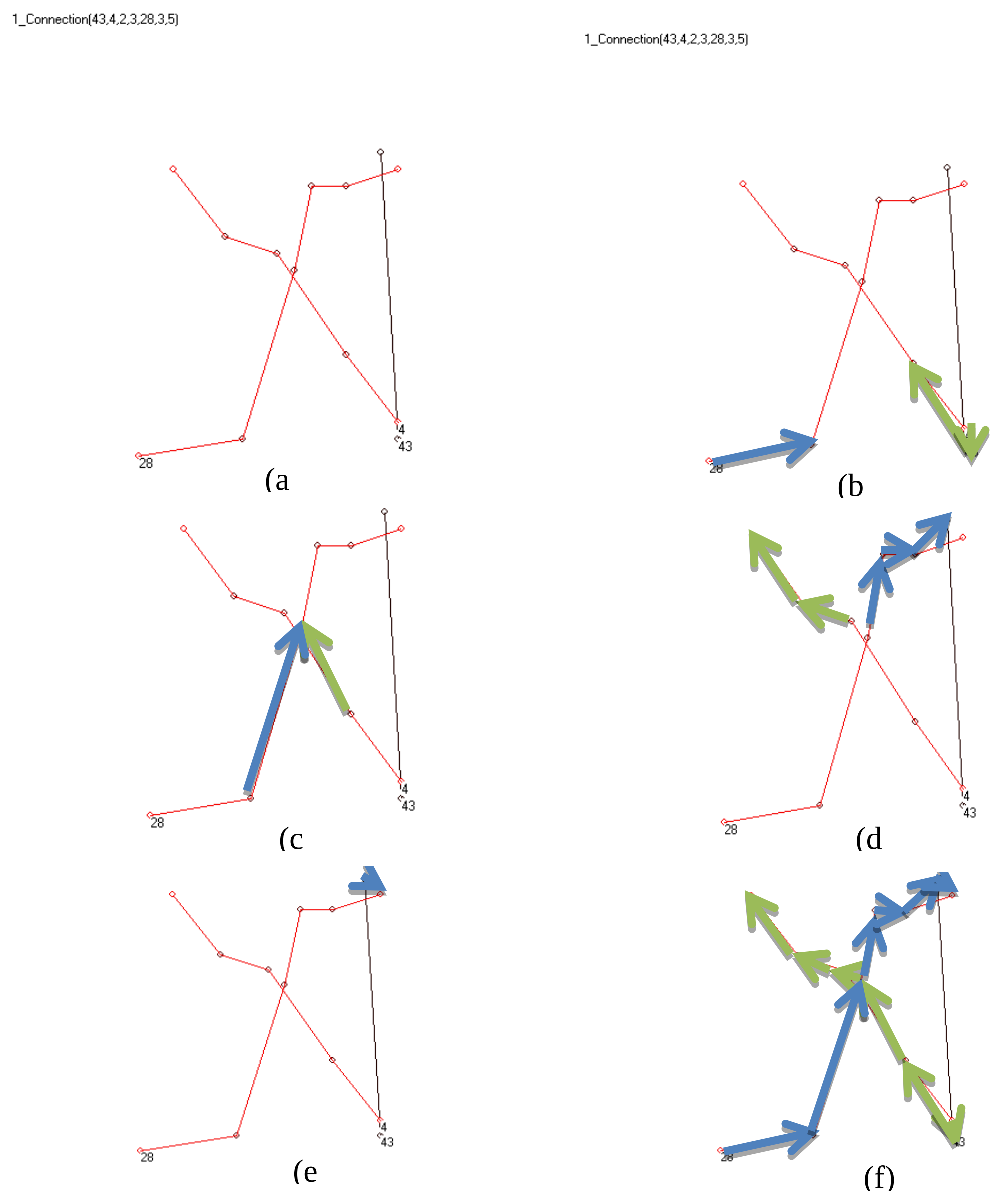


Figure 29: Course of events for Driver 43, Driver 28 and Passenger 4

The system has generated a two-connection route (56, 23, 1, 2, 26, 3, 4, 13, 5, 6). For this route, Driver 23, Driver 26 and Driver 13 have contributed in the establishment of rideshare for Passenger 56. Figure (30-a) shows the origin and destination points for the passenger and the current locations of the three drivers along with their original routes and the 6 points to be visited along the routes. Details reveal that Driver 23 makes a detour at his current location to pick up Passenger 56 at his point of origin and then heads to his second point to be visited. At the same time Driver 26 and Driver 13 are on their way to the 2nd points to be visited (Figure 30-b). Driver 23 makes a detour at his second point to be visited to meet with Driver 26 at his 3rd point to be visited (the first connection point) where Passenger 56 changes his ride. At this time Driver 13 is driving along his original route (Figure 30-c). Driver 23 heads toward his 3rd point to be visited and Driver 26 heads toward his 4th point and makes a detour at that point to meet Driver 13 at his 5th point to be visited (the second connection point). At this location passenger 56 changes his ride for the second time (Figure 30-d). Driver 26 goes to his 5th point and continues along the original route to reach to his 6th point to be visited. Driver 13 Continues along his original to his 6th point to be visited and drops the Passenger at that point and at the same time Driver 23 continues along his original point to meet his 5th and 6th points to be visited. Passenger 56 starts walking toward his destination after he leaves Driver 13 at his 6th point to be visited (Figure 30-e). Figure (30-f) shows the updated routes for Driver 23, Driver 26 and Driver 13 as well as the route of journey for Passenger 56.

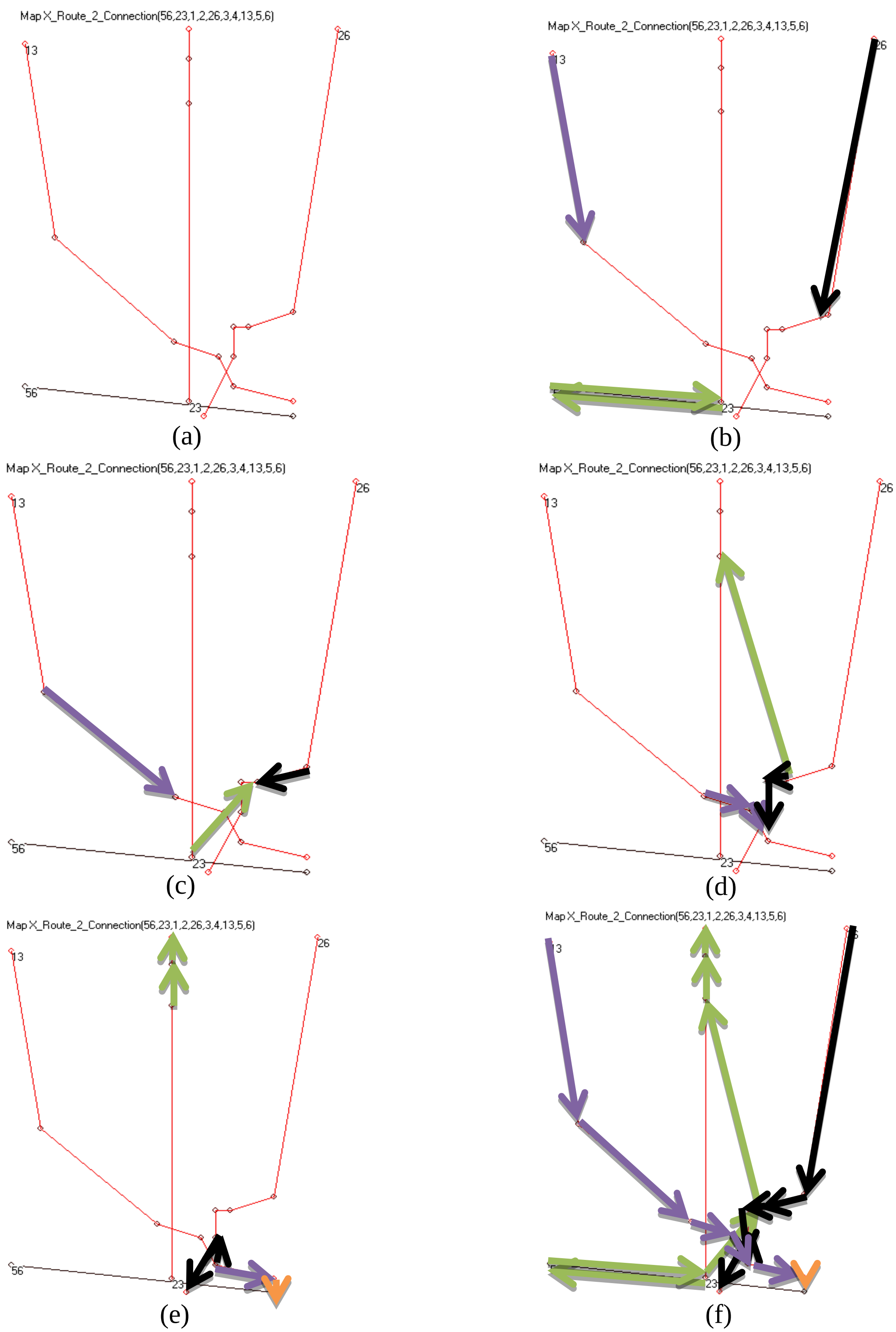


Figure 30: Course of events for Driver 23, Driver 26, Driver 13, and Passenger 56

6.2.2. Numerical Example 50*60*460

This numerical example is included to further illustrate concepts presented in the TSHDM heuristic solution strategy and to compare some of its results with those of Numerical Example 60*40*360. It includes fifty passengers (16% less than the number of passenger in Numerical Example 60*40*360), sixty drivers (50% more than the number of drivers in Numerical Example 60*40*360) and there are a set of six points to be visited for each driver in an area with the size of 100 square Miles. Figure 31 shows the map for this problem.

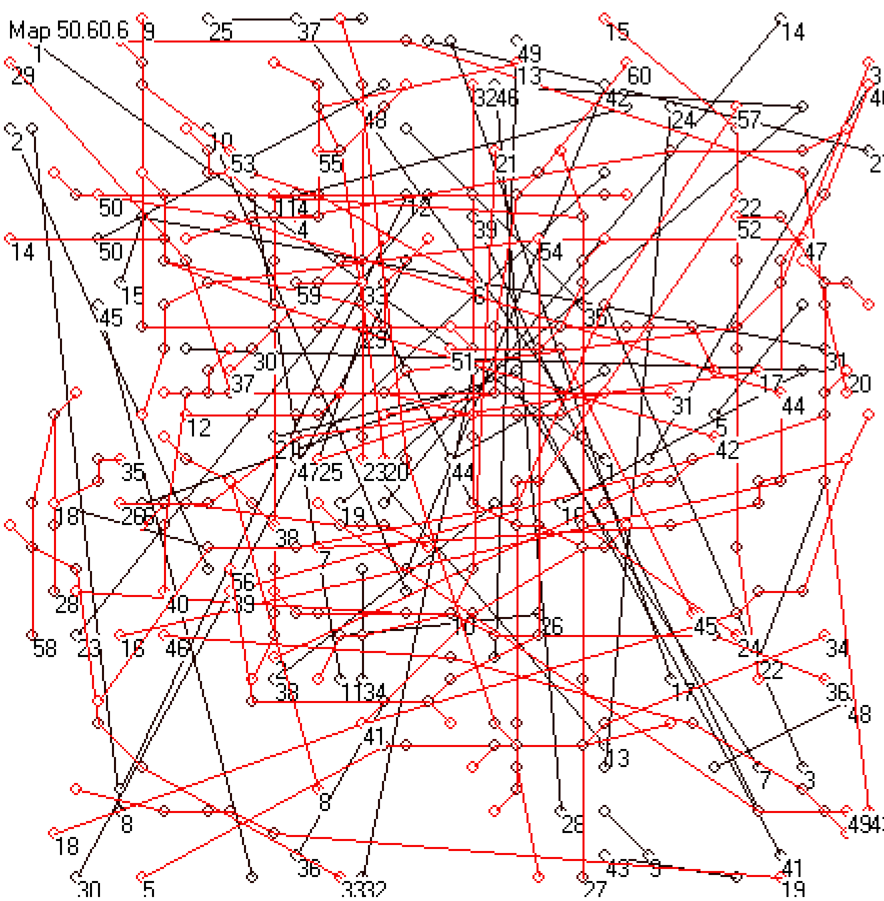


Figure 31: Map of the 50*60*460 test problem

Other input parameters are all generated randomly using a test problem generator. It is assumed that maximum waiting time for each rider to be picked up at the origin point or connection points is 15 minutes (25% less than the waiting time in Numerical Example 60*40*360). Moreover, it is assumed that each driver is flexible with maximum diversion of 4 Miles from its original route to pick up or drop-off a rider or to make a connection with other drivers to transfer a rider (20% less than the detour distance in Numerical Example 60*40*360). In additions, the maximum relocation distance for riders is assumed to be 4 Miles (20% less than the detour distance in Numerical Example 60*40*360) which is large enough to increase the chance of rideshare match. Current time of the system is assumed to be 10:00 a.m. and all the rideshare service requests are within half an hour from 10:00 to 10:30. Seat capacity for each car is assumed to be 5 and it is assumed that only one seat of each car is occupied at the current time of the system. In addition, it is assumed that there is no restriction on the rideshare preferences of the entire participants to allow the system to have the maximum likelihood of rideshare matches. Table 13 shows the summary of results for the illustrated example.

Table 13: The summary of results for illustrated example 50*60*460

Iter	Matched established routes	Acc. Time (sec.)	# routes
1	0_Connection (p_i, d_j, n_p, n_d): (4,60,1,1); (5,52,1,4); (6,51,1,2); (7,42,1,1); (9,34,1,3); (10,21,1,3); (15,50,1,1); (19,4,1,4); (21,23,1,2); (25,1,1,1); (26,36,3,5); (29,26,2,2); (33,7,1,1); (34,10,1,2); (35,24,3,3); (36,2,1,1); (41,45,1,2); (42,57,1,3); (44,25,4,5); (46,15,1,2); (48,49,2,2); (49,54,1,2); (50,55,2,4)	.062	23
2	0_Connection (p_i, d_j, n_p, n_d): (16,15,4,4); (18,57,4,4)	.087	2
3	0_Connection (p_i, d_j, n_p, n_d): Not found	.092	0
5	1_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, n_d)$: (38,27,2,4,57,2,3)	12.31	1
6	0_Connection (p_i, d_j, n_p, n_d): (2,57,5,5)	14.11	1
7	0_Connection (p_i, d_j, n_p, n_d): Not found	15.93	0
8	1_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, n_d)$: not found	18.86	0
9	2_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, d_{j''}, n_c^{d_{j''}}, n_d)$: Not found	45.79	0

In Table 13,

p_i: passenger $i(i: 1, \dots, 50)$,

d_j:First-link driver j $(j: 1, \dots, 60)$,

$d_{j'}$: Second-link driver j' $(j': 1, \dots, 60)$,

$d_{j''}$: Third-link driver j'' $(j'': 1, \dots, 60)$,

n_p: pickup node,

n_d, drop off node,

$n_c^{d_j}$: First-connection node for driver j, $(j: 1, \dots, 60)$,

$n_c^{d_{j'}}$: Second-connection node for driver j' (j': $1, \ldots, 60$),

$n_c^{d_{j''}}$: Third-connection node for driver j'' (j'': $1, \ldots, 60$).

As the table shows, the heuristic algorithm established 25 zero-connection matched routes in 0.092 seconds, 1 one-connection matched routes in 12.31 seconds, an additional zero-connection route in 14.11seconds and it was not successful to find two-connection route and the solution was completed in 45.79 seconds. 24 drivers (40% of the drivers) contributed to the solution and the system was successful to find rideshare solutions for 27 passengers, i.e., rate of successful matches for the system was 54%. The share of zero-connection routes in the 27 rideshare matches was 96% while the share for one-connection and two-connection routes were 4% and 0%, respectively. In comparison with Numerical Example 60*40*360, running time for the algorithm increased to 45.79 from 43.15 (6.1% increase) and number of rideshare matched routes decreased to 27 from 36 (25% decrease) while the rate of successful matches for the system slightly improved to 54% from 53% (1.8% increase). Although all parameters were generated randomly, there were 16% less riders, 50% more drivers, 25% less maximum waiting time, 20% less detour distance, and 20% less relocation distance compared with Numerical Example 60*40*360. It was expected to have a less rate of successful matches with tightening the problem (tighter waiting time, relocation and detour distances) and decreasing the number of riders but the system slightly improved the Success rate which seems to be the result of increasing the number of drivers. The sensitivity analysis of parameters in the model will be discussed in more details in the next section.

The system has resulted in 26 zero-connection routes. For example, rideshare routes (46, 15, 1, 2), (16, 15, 4, 4) that implies Driver 15 has contributed in the establishment of 2 rideshares for two passengers 46 and 15. Figure (32-a) shows the origin and destination of the two passengers and the current location for the driver and his original route with the 6 points to be visited along the route. Details reveal that Driver 15 has to make a detour at his current location to pick up Passenger 46 at his point of origin, continues to his second original point to be visited and makes a detour at that point to drop the passenger at his destination and continues to reach to his 3rd point to be visited (Figure 32-b). This driver heads toward his 4th point to be visited and at the same time Passenger 16 walks to that point to meet the driver (Figure 32-c). Driver 15 makes a detour at that point to drop-off Passenger 16 at his point of destination and proceeds his 5th and 6th points to be visited (Figure 32-d). Figure (32-e) shows the updated route for Driver 12 as well as the routes of journey for Passenger 46 and 16.

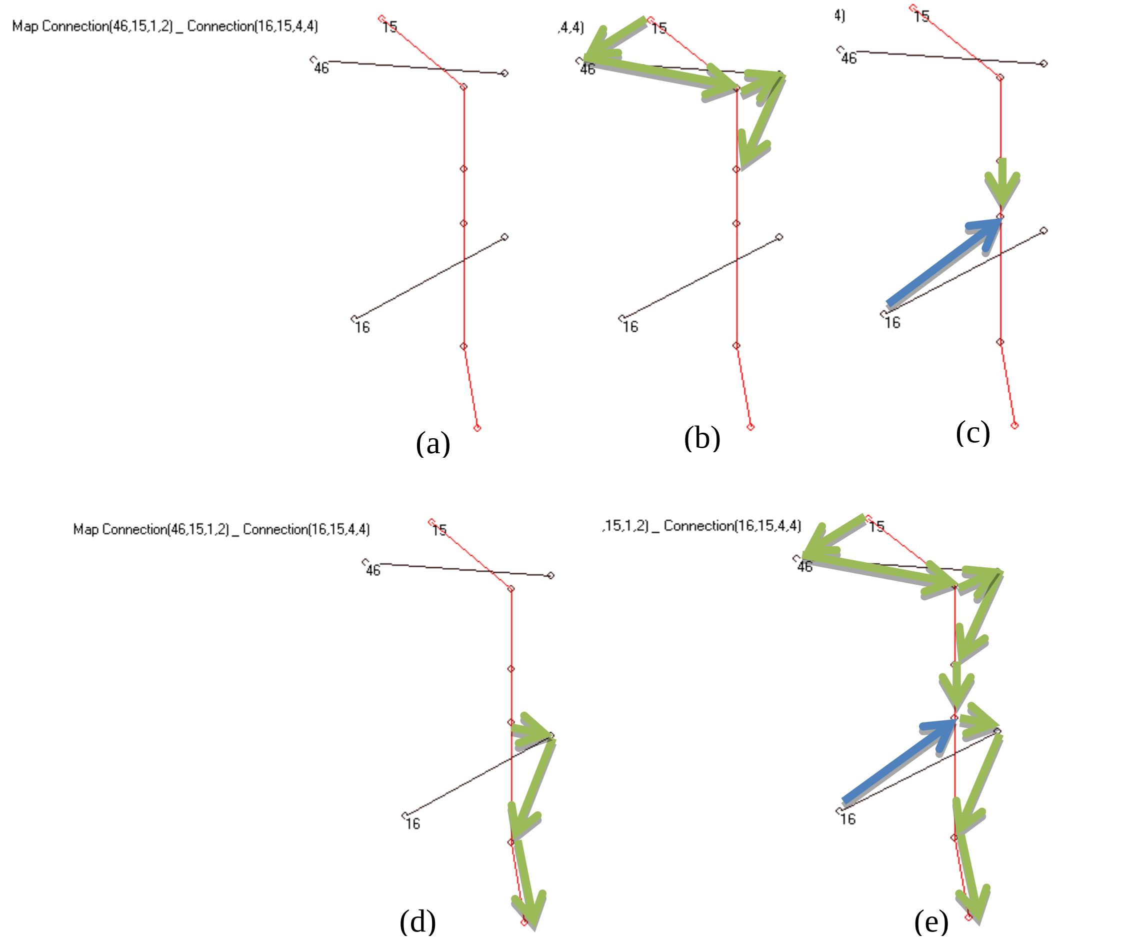


Figure 32: Course of events for Driver 15, Passenger 46, and Passenger 16

The system has generated a one-connection route (38, 27, 2, 4, 57, 2, 3). For this route, Driver 27 and Driver 57 have contributed in the establishment of rideshare for Passengers 38. Figure (33-a) shows the origin and destination points for the passenger and the current locations of the two drivers along with their original routes and the 6 points to be visited along the routes. Details reveal that Driver 27 makes a detour at his 2nd point to be visited to pick up Passenger 38 at his point of origin and then heads toward his third point to be visited. The interesting thing that happens here is neither this driver makes an additional short detour of about 0.15 mile before arriving to his 3rd point to drop off the passenger at his destination, nor the passenger walks for a about 0.3 from that point (third point to be visited by Driver 27) to his destination. A deeper look at the problem shows that this passenger has shown no flexibility with relocation distance (his randomly generated input for maximum relocation distance is 0) and the randomly generated maximum detour distance for the driver is 2.5 Miles which already 2.37 Miles of that has been used to pick up the passenger (Figure 33-b). Driver 27 makes a detour at his third point to be visited to meet with Driver 57 at his 2nd point to be visited (the connection point) where Passenger 38 changes his ride (Figure 33-c). Driver 27 proceeds toward his next points to be visited along the original route and Driver 57 heads toward his 3rd point and makes a detour at that point to drop-off the passenger at his destination and continues toward his 4th and 5th point along the original route (Figure 33-d). At the point, Driver 57 contributes in a zero-connection route. He makes a detour to pick up Passenger 2 at his origin and drops him off before heading to his 6th point to be visited (Figure 33-e). Figure (33-f) shows the updated routes for Driver 27 and Driver 57 as well as the routes of journey for Passenger 38 and passenger 2.

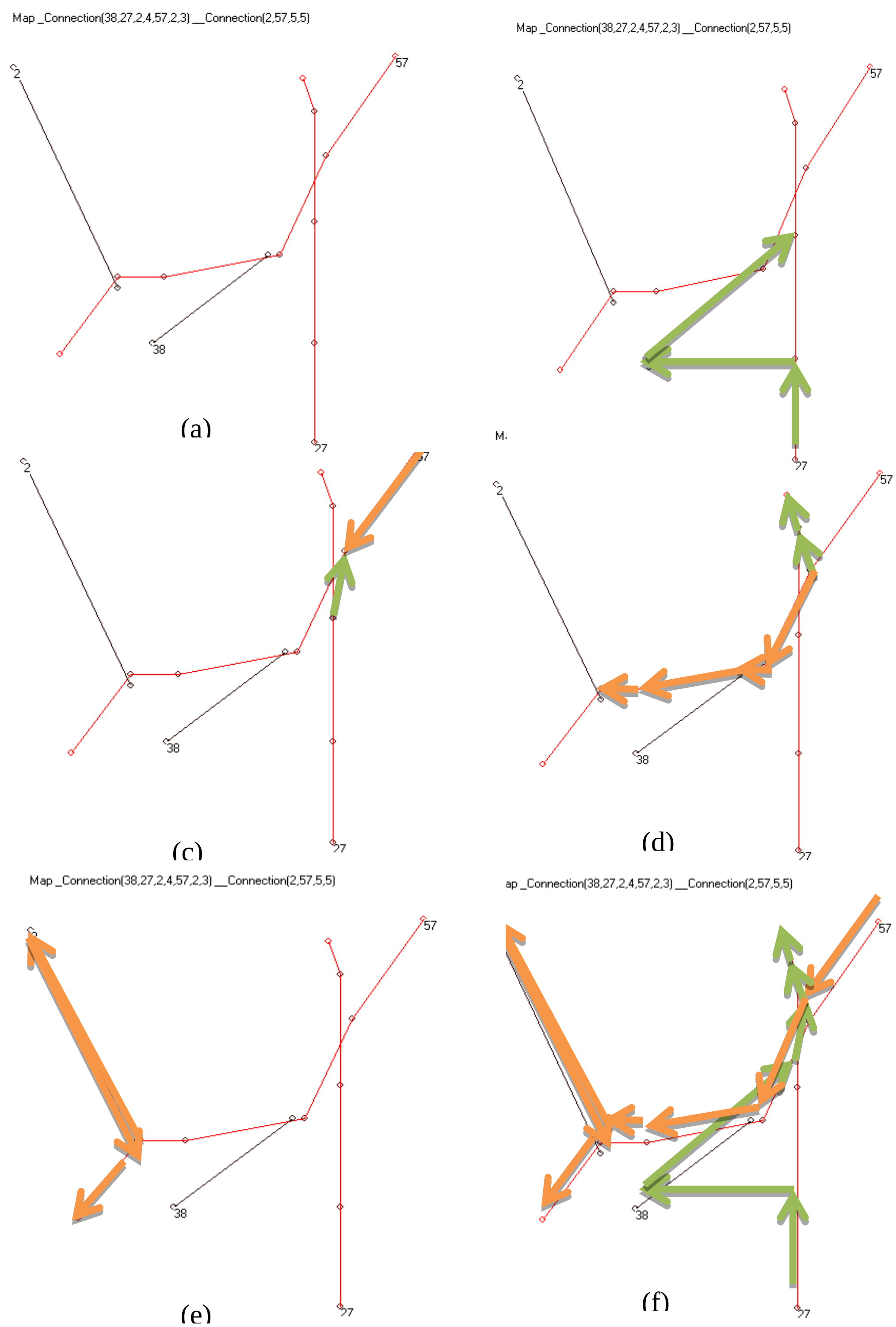


Figure 33: Course of events for Driver 27, Driver 57, Passenger 38, and Passenger 2

6.3. Model Validation

There are varieties of validation techniques that have been used by researchers in the literature of computational models. One of the validation techniques is verification which is about the determination of validity of a computational model relative to a set of real data using graphical or statistical methods [Carley, 1996]. For the purposes of this study, verification is a necessary step in moving the model to the next levels which are a thorough analysis of the sensitivity of the model with respect to its parameters as well as dealing with real world problems such as the one that will be later provided in the case study.

To verify the results of TSHDM for the rideshare optimization model, a set of 20 randomly-generated problems are solved using the heuristic approach and external and internal results are compared with the exact solutions. The exact solutions are generated using Xpress-IVE version 1.21.02. The size of problems are ranging from toy size problems with 2 passengers, 2 drivers and 3 points to be visited for each driver to medium size problem with 20 passengers, 20 drivers and 7 points to be visited for each driver.

Table 14 shows the verification results. As shown in the table, TSHDM generates very good quality solutions with little loss of total number of matched routes and provides enormous computational time savings.

Table 14: Model verification results

Test problem	#P	#D	#N	#constraints	#variables	Exact solutions							TSHDM solutions							Time saving (%)	% Difference in travel time	# Route loss
						Run time (sec.)	#0-con. routes	#1-con. routes	#2-con. routes	Total # of routes	Perfectly matched routes	Compromise routes	Run time	#0-con. routes	#1-con. Routes	#2-con. routes	Total # of routes	Perfectly matched routes	Compromise routes			
1	2	2	3	516	517	0.0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0.000	0	0
2	3	2	4	1344	1400	0.3	0	0	0	0	0	0	0.1	0	0	0	0	0	0	66.66	0	0
3	2	3	4	1824	1816	0.4	1	0	0	1	0	1	0.1	1	0	0	1	0	1	75.00	0	0
4	3	3	4	2748	2718	227.1	1	0	0	1	0	1	0.1	1	0	0	1	0	1	99.95	0	0
5	3	4	4	3642	4918	541.7	1	1	0	2	1	1	0.1	1	1	0	2	1	1	99.98	0	0
6	4	3	4	3634	3620	885.6	1	1	0	2	0	2	0.1	1	1	0	2	0	2	99.98	0	0
7	4	4	4	6192	5960	1202.4	2	0	0	2	1	1	0.1	2	0	0	2	1	1	99.99	0	0
8	5	4	5	12880	12150	5042.8	2	1	0	3	1	2	0.2	2	1	0	3	1	2	99.99	0	0
9	10	10	3	42340	39490	7706.3	3	1	0	4	1	3	0.4	3	1	0	4	1	3	99.99	0	0
10	10	10	5	150870	137710	16331	3	1	1	5	2	3	1.1	3	1	1	5	2	3	99.99	0.5	0
11	10	10	6	271810	239320	19140	4	1	0	5	1	4	1.2	4	1	0	5	1	4	99.99	0	0
12	10	10	10	573920	743048	74593	5	1	1	7	3	4	2.3	5	1	0	6	3	3	99.99	2.4	1
13	11	11	7	512083	443828	54238	4	2	1	7	3	4	2.1	4	2	1	7	3	4	99.99	0	0
14	12	12	6	467724	410784	22079	4	3	0	7	4	3	1.8	4	3	0	7	4	3	99.99	0	0
15	13	13	6	593710	520936	76562	6	2	0	8	3	5	2.3	6	2	0	8	3	5	99.99	1.82	0
16	14	14	5	484134	433006	23277	4	2	1	7	3	4	1.9	4	2	1	7	3	4	99.99	0	0
17	14	14	6	740502	649208	85953	6	2	1	9	4	5	3.1	6	2	1	9	4	5	99.99	0	0
18	15	15	5	502650	454740	51980	6	1	0	7	3	4	2.0	6	0	0	6	2	4	99.99	1.3	1
19	20	20	5	1183280	1066220	*	*	*	*	*	*	*	6.5	8	1	1	9	4	6	100.00	n/a	n/a
20	20	20	7	*	*	*	*	*	*	*	*	*	6.9	9	2	0	11	5	6	100.00	n/a	n/a

* Express Optimization software was unable to report results within 24 hours running

The results indicate that TSHDM is validated at the pattern as well as the value levels for total number of routes, number of zero-connection routes, number of one-connection and two-connection routes, number of perfectly matched routes and compromised route solutions. For example, using exact algorithm, test problem 4 with 3 riders, 3 drivers and 4 points to be visited resulted in 1 compromised zero-connection matched route which is the global optimal solution for the problem and is exactly the same result gained for the problem solved by THSDM. While the exact algorithm took 227.1 second to solve the 2748 constraints by 2718 variables problem, THSDM solved the problem in 3 iterations and .1 seconds. Figure 34 shows the convergence behavior for the test problem 4 using the exact algorithm.

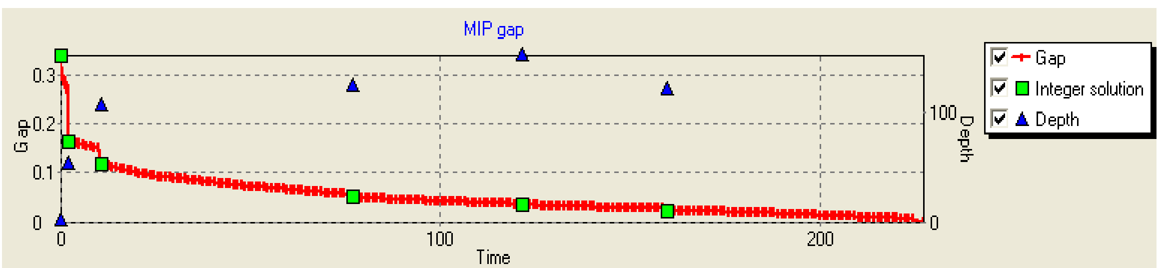


Figure 34: MIP search graph for Test problem 4 (Problem 3*3*4)

Considering the solution gained for test problem 4, one rider is added to the problem and results for the 4 riders, 3 drivers and 4 points to be visited (problem 6) are compared using the global optimal method and THSDM. As expected, the increase in number of riders resulted in 1 more compromised matched route which is a one-connection route. Results for both methods are the same except for the computational running time that THSDM shows more than 99.98% computational time saving. Figure 35 shows the convergence behavior for the test problem 6 using the exact algorithm.

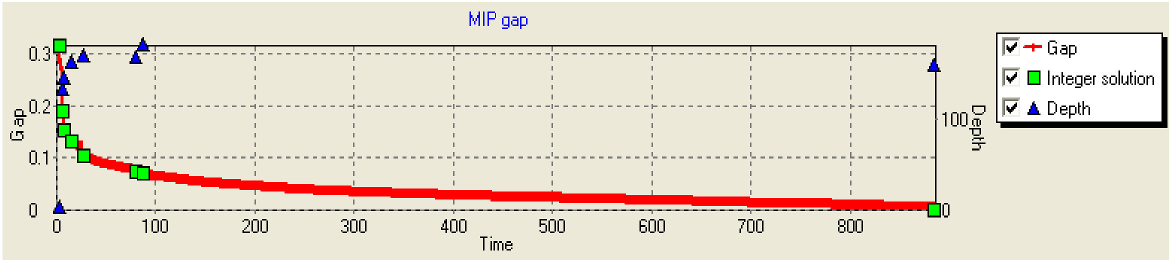


Figure 35: MIP search graph for Problem Test problem 6 (4*3*4)

Table 14 also shows that although the exact algorithm is unable to solve Test Problem 19 with 20 riders, 20 drivers and 5 points to be visited for each driver within 24 hours running of the algorithm, TSHDM solves the problem in 6.5 seconds. Xpress-IVE is also unable to determine the number of constraints and variables for Test Problem 20 with 20 riders, 20 drivers and 7 points to be visited for each driver. The results also support the accuracy of preference matching module of the heuristic algorithm. For all 20 test problems, the comparison between exact algorithm and THSDM indicates that solutions are completely consistent in terms of completely matched solutions and compromise matched solutions.

The only two inconsistencies observed for the 20 test problems are the number of routes generated by THSDM for Test Problem 12 and Test Problem 18. The maximum number of matched routes generated by the exact algorithm for Test Problem 12 with 10 riders, 10 drivers and 10 points to be visited for each driver is 7 and THSDM has lost 1 two-connection route for the problem. Likewise, THSDM has lost 1 one-connection route for test Problem 18 with 15 riders, 15 drivers and 5 points to be visited for each driver.

Chapter 7: Sensitivity Analysis

In this section, model parameters with the most influence on results are identified through a 'sensitivity analysis' that serves to guide future research efforts. There are also other reasons to conduct sensitivity analyses for TSHDM including the need to determine which inputs contribute most to output variability, and once the model is in use, what consequences are to be expected from changing given input parameters. There are more than a dozen sensitivity analysis methods available in the literature. For the purpose of this research, the technique of Regression Analysis is used. Regression analysis appears to be the most comprehensive technique which is commonly utilized to build response surfaces to approximate complex models and is relatively easy to perform with commercially available software [Hamby, 1994]. Regression analysis allows the sensitivity ranking to be determined based on the relative magnitude of the regression coefficient. This technique has been used for sensitivity analysis in several investigations (Iman et al., 1981a; Iman et al., 1981b). The value of regression coefficient is indicative of the amount of influence the parameter has on the results of model. As the parameters have different units and relative magnitudes, a standardization process is warranted.

To conduct regression analysis, a substantial computational effort is undertaken which comprises of 224 numerical examples with different combinations of area sizes, points to be visited, number of riders, and number of drivers. Due to stochastic behavior of the problem, for a given combination of area size, number of points to be visited, number of riders, and number of drivers each numerical example is run for three times and the best solution with the greatest total number of matched routes is presented. Tables A-1 through A-22 in Appendix A show the computational results. Each table includes number of riders (#Pt), number of drivers (#Dt), number of points to be visited (#Stops), number of matched routes with zero connection (#0-con.), cumulative running time to find all routes with zero connection (CPU Time (sec.)), percentage of matched routes with zero connection (0-con. perc.), number of matched routes with one connection (#1-con.), cumulative running time to find all routes with one connection (CPU Time (sec.)), percentage of matched routes with one connection (1-con. perc.), number of matched routes with two connection (#2-con.), cumulative running time to find all routes with two connection (CPU Time (sec.)), percentage of matched routes with two connections (2-con. perc.), total number of matched routes (Total Routes), and rate of successful matches (Success Rate). Each table also presents the median, mode and Mean of all statistics for a given combination of area size, number of riders and number of drivers participating in the rideshare program.

For each area size, aggregated computational results are presented (Tables A-17 and A-18 in Appendix A). As the tables for aggregated computational results show that a large number of combinations have no mode, Mode has been excluded from the further investigation. Tables A-19 and A-20 in Appendix A show the aggregated computational results of median and mean for area size 25 square Miles respectively. Likewise, Tables A-21 and A-22 show the aggregated computational results of median and mean for area size 100 square Miles respectively. Furthermore, examination of the relationship between mean and median of the data sets show that two statistics of mean and median are highly correlated. Pearson's correlation coefficient for mean and median of area size of 25 square Miles is 0.996 and the coefficient for the area size of 100 square Miles is 0.867. Considering the high correlation between the mean and median, the statistic of interest for discrete variables such as the number of matched routes would be median while for the continuous variables such as running time the mean would be the appropriate statistic for further consideration. Figure 36 shows the median number of matched routes versus the number of stops. Each curve represent the median of matched routes for a given combination of number of riders (#P), number of drivers (#D) and the area size (Area).

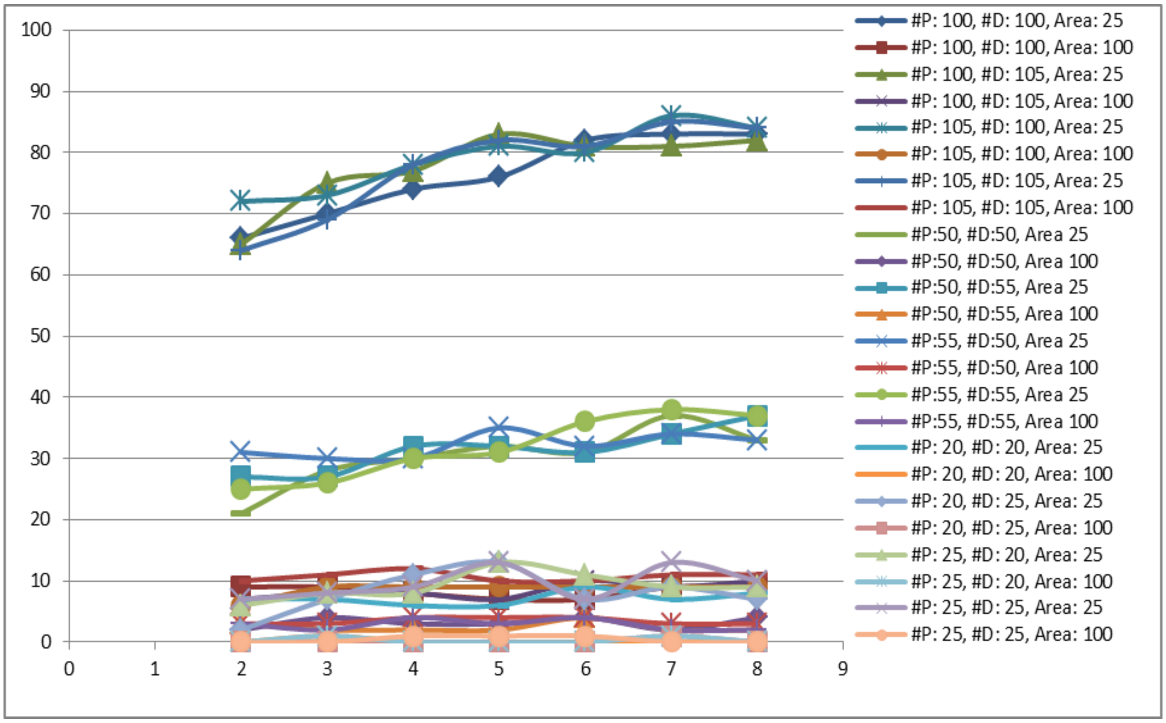


Figure 36: Median number of matched routes versus number of stops

To have a better understanding of the relationships between the median of the number of matched routes and the number of stops, Figure 36 is split into two separate figures in terms of area size. Figure 37 shows the relationships for area size of 25 square Miles. Given a combination of the number of riders and drivers, it appears that increasing the number of stops results in increase of the median number of matched routes for a small area size of 25 square Miles. It also shows that the number of riders and drivers participating in the program has a significant impact on the relationship. The greatest impact is in the crowded network where the density of participating riders and drivers are high. The impact decreases with any reduction in the density of participating riders and drivers.

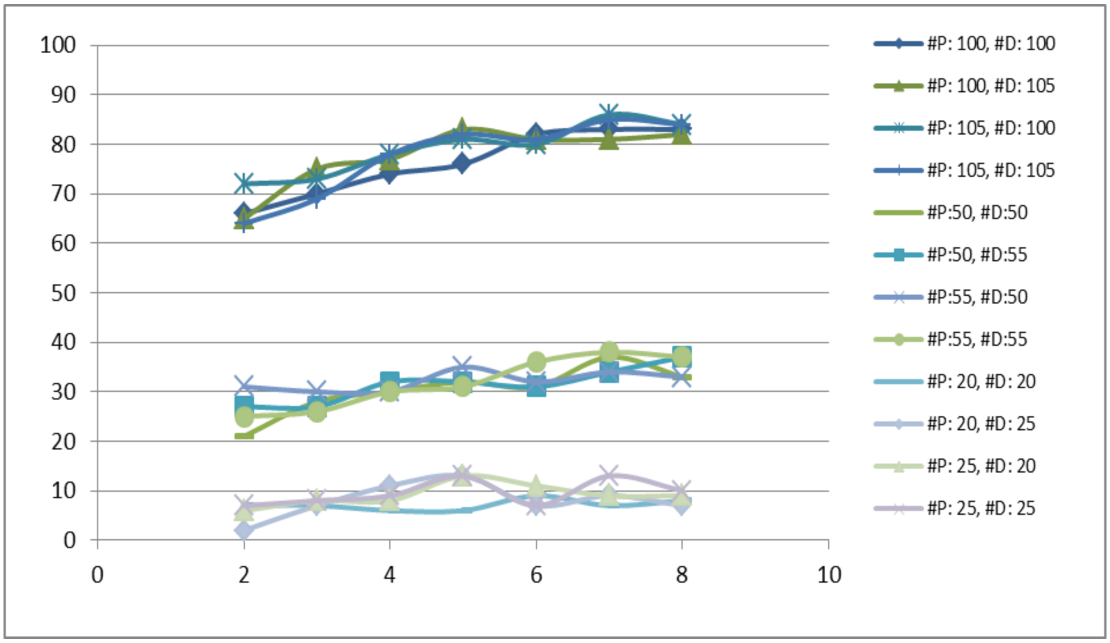


Figure 37: Median number of matched routes versus number of stops (Area size: 25)

Figure 38 shows the relationships for the area size of 100 square Miles. Given a combination of the number of riders and drivers, it appears that increasing the number of stops results in a insignificant increase in the median number of matched routes for a large area size of 100 square Miles where the density of participating riders and drivers are low. It also shows that the number of riders and drivers participating in the program have a meaningful impact on the relationship. The impact decreases significantly with reduction in the density of participating riders and drivers.

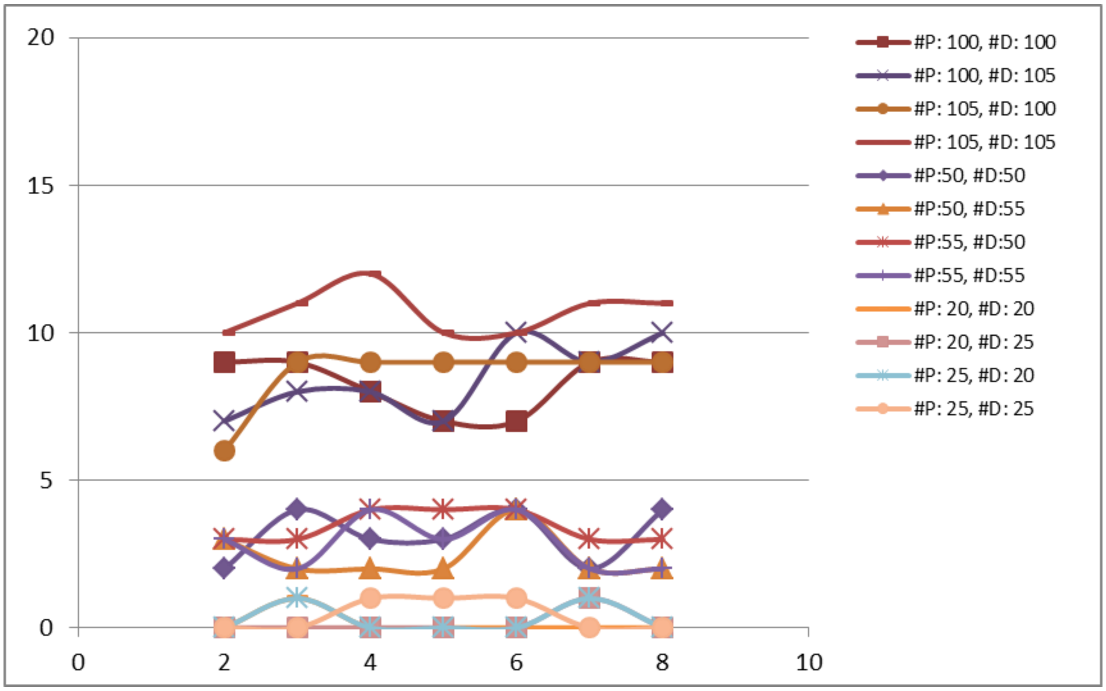


Figure 38: Median number of matched routes versus number of stops (Area size: 100)

TSHDM is to be run repeatedly and its efficiency becomes important. Generally, we associate efficiency with the time it takes the algorithm to run, although there are other resources to conserve, such as the amount of storage space taken, the amount of traffic generated on a network, and the amount of data transaction. For TSHDM, however, it is the running time that determines its suitability for practical purposes. To examine the running time behavior of the algorithm, its behavior on the computational effort for the 224 numerical examples is depicted on two separate figures in terms of the area size. Figures 39 and 36 show the mean CPU running time in seconds versus the number of stops respectively for area size 25 square Miles and 100 square Miles. Each curve represent the mean CPU running time for a given combination of the number of riders (#P), the number of drivers (#D) and the area size (Area).

Figure 39 shows the relationships for the area size of 25 square Miles. Given a combination of the number of riders and drivers, it appears that increasing the number of stops results in an increase in the mean running time for a small area size of 25 square Miles where the density of participating riders and drivers are high. It also shows that the number of riders and drivers participating in the program has a significant impact on the relationship. The greatest impact is in the crowded network where the density of participating riders and drivers is high. The impact increases with the increase in the density of participating riders and drivers.

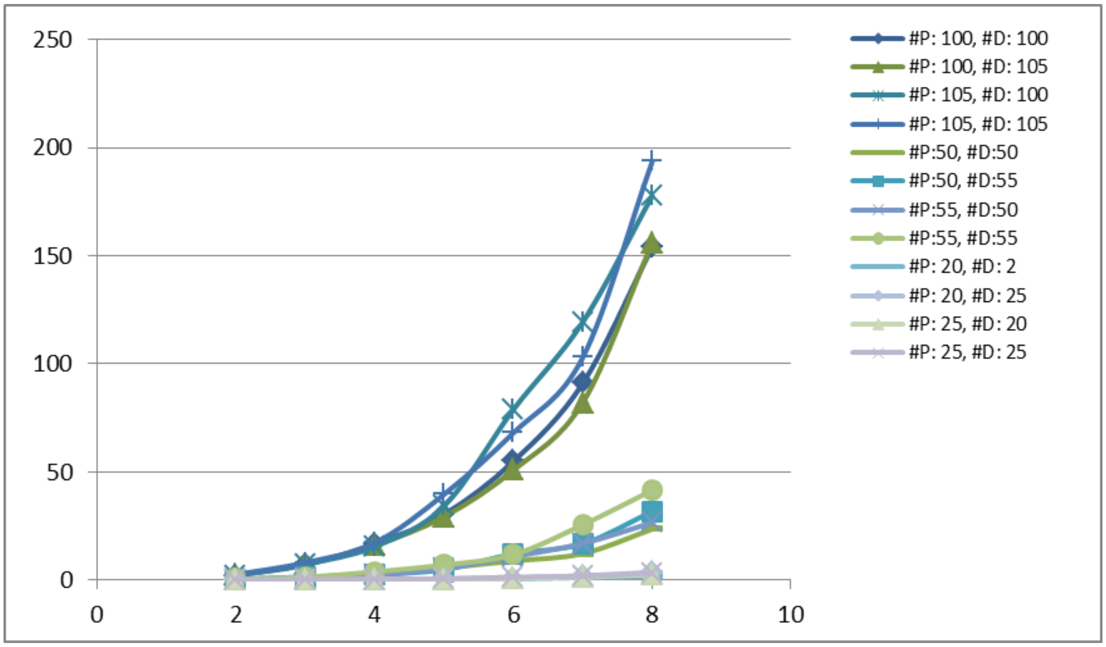


Figure 39: Mean CPU running time versus number of stops (Area size: 25)

Likewise, Figure 40 shows the relationships between Mean CPU running time in seconds versus the number of stops for the area size of 100 square Miles. The behavior for the area size of 100 square Miles resembles the behavior for the area size of 25 square Miles which accordingly increasing the number of stops results in an increase in the mean CPU running time. It also shows that area size is an important factor in terms of running time for the algorithm. The range of running time for the area size of 100 square Miles is

0 to 306 seconds which is wider than the range of 0 to 193 seconds for the area size of 25 square Miles.

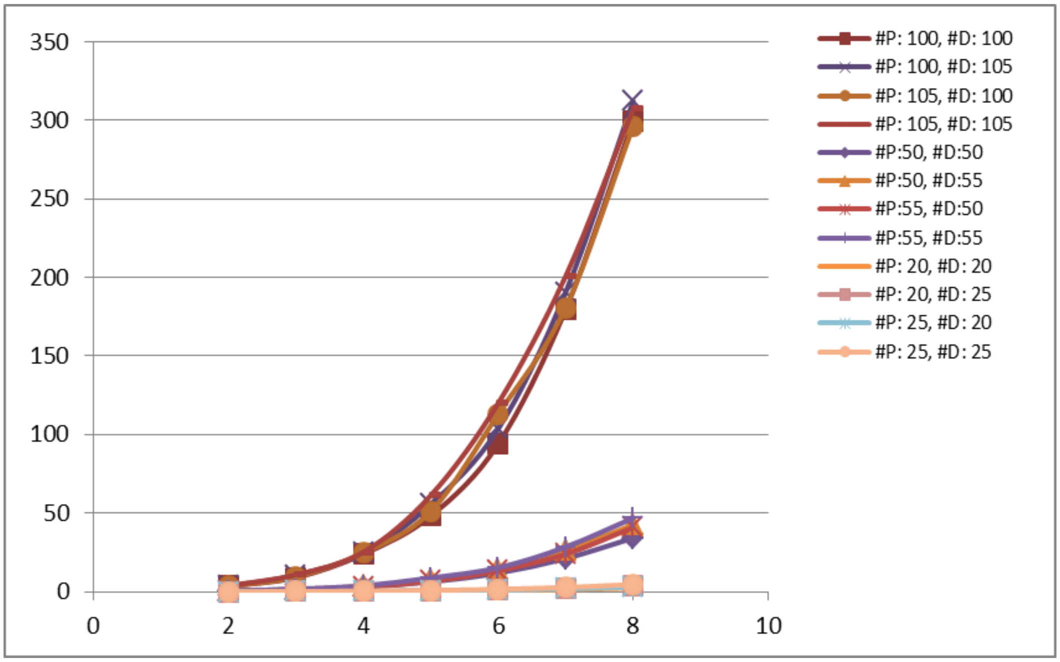


Figure 40: Mean CPU running time versus Number of stops (Area size: 100)

To have a deeper and closer understanding of the relationships between the parameters involved in the problem as well as finding the most influencing factors in the rate of success and running time of the algorithm, the following regression analyses have been conducted using SPSS Statistics ver. 17.0 software package:

1- Regression analysis (Dependent Variable: Rate of Success vs. Independent Variables: #D)
2- Regression analysis (Dependent Variable: Rate of Success vs. Independent Variables: #P)
3- Multiple Regression analysis (Dependent Variable: #0-connection routes vs. Independent variables: #P, #D ; Dense network , Area size: 25 square Miles

4- Multiple Regression analysis (Dependent Variable: Rate of Success vs. Independent Variables: #P, #D, #Stops while Area size is a fixed value, 25 sqM)
5- Multiple Regression analysis (Dependent Variable: CPU Running Time vs. Independent Variables: #0-connection routes, #1-connection routes, #2-connection routes while Area size is a fixed value, 25 sqM)
6- Multiple Regression analysis (Dependent Variable: Rate of Success vs. Independent Variables: #P, #D, #Stops, Area size)
7- Multiple Regression analysis (Dependent Variable: CPU Running Time vs. Independent Variables: #P, #D, #Stops, Area size)
8- Multiple Regression analysis (Dependent Variable: Rate of Success vs. Independent Variables: #0-connection routes, #1-connection routes, #2-connection routes)

The following presents details of regression analysis for two of the aforementioned analyses conducted in this research and results for the other six analyses are presented in Appendix B.

7.1. Multiple Regression Analysis for Rate of success

To examine the relationship between the mean Rate of Success and the number of points to be visited by each driver, the area size, the number of drivers and the number of riders participating in the rideshare program, a multiple linear regression analysis is conducted where the mean Rate of Success is the dependent variables and the number of points to be visited by each driver (#Stops), the area size (Area), the number of drivers (#D) and the number of riders (#P) participating in the rideshare program are independent variables. The data set includes all the 224 observations in the set of numerical examples.

Summary output for the multiple linear regression model is presented in Table 15. The high value of R square (R Square: 0.919, Adjusted R Square: 0.918) and the low value of Standard Error (.08280) suggest that the regression model explains the variation in the Rate of Success.

Table 15: Summary output for the multiple linear regression analysis of Rate of Success versus #Stops, Area, #D, #P

Model		R	R Square	Adjusted R Square	Std. Error of the Estimate
dimension0	1	.959	.919	.918	.08280

The high values of t-statistics as well as the low value of p-value for the coefficients of the model also suggest that variable coefficients are statistically significant for the multiple linear regression model. Table 16 shows the summary ANOVA analysis for the multiple linear regression analysis. Considering the unstandardized coefficients (B's) of dependent variables, the regression analysis results in the following multiple linear regression model:

$$Y = .518 + (-7.350E\text{-}5)\, X1 + (-7.350E\text{-}5)\, X2 + (-.007)\, X3 + (.015)\, X4 \qquad (120)$$

where #Rate of Success is denoted by Y, and X1, X2, X3, and X4 are representing #P, #D, Area, and #Stops in the model. The standardized coefficients (Beta) for the independent variables suggest that:

- A one S.D. change in #P produces a predicted change of -.008 S.D.'s in the mean Rate of Success, net of other variables, i.e., increase of #P does not have a significant impact on rate of success.
- A one S.D. change in #D produces a predicted change of .297 S.D.'s in the mean Rate of Success, net of other variables, i.e., the rate of success increases as the number of participating drivers increases.

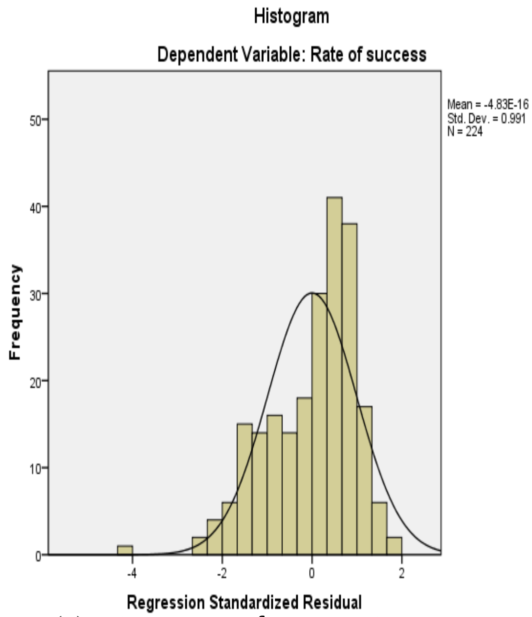
- A one S.D. change in Area Size produces a predicted change of -.908 S.D.'s in the mean Rate of Success, net of other variables, i.e., the rate of success decreases as the area size increases.
- A one S.D. change in #Stops produces a predicted change of (.101) S.D.'s in the rate of success, net of other variables, i.e., the rate of success increases as the number of stop points increases..
- #D is substantially more important than #Stops, and Area size and those are more important than #P in determining level of mean Rate of Success.
- More than 90 percent of the variation in mean Rate of Success is explained in order of importance by #D, #Stops, Area, #P.

Table 16: Summary ANOVA analysis for the regression analysis of Rate of Success versus #Stops, Area, #D, #P

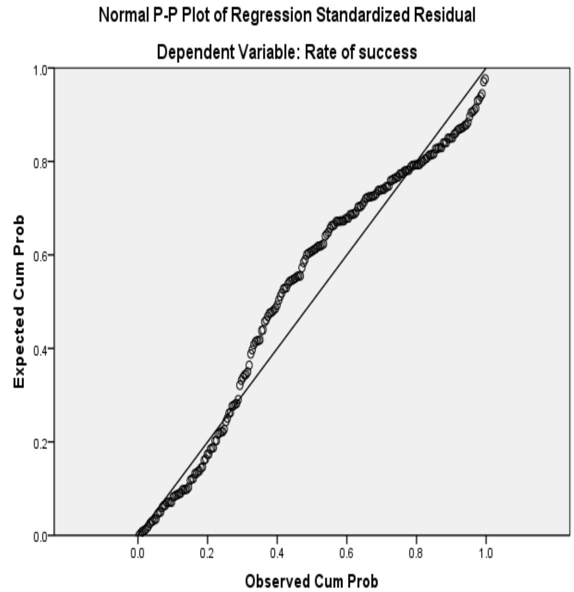
	Sum of Squares	df	Mean Square	F	Sig.
Regression	17.143	4	4.286	625.078	.000
Residual	1.502	219	.007		
Total	18.645	223			

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	.518	.021		25.277	.000	.478	.559
	#P	-7.350E-5	.000	-.008	-.167	.867	-.001	.001
	#D	.003	.000	.297	6.443	.000	.002	.004
	Area	-.007	.000	-.908	-47.366	.000	-.007	-.007
	#Stops	.015	.003	.101	5.261	.000	.009	.020

To assess the normality of the residuals, the P-P plot and histogram of residuals are examined. Figure (41-a) is a histogram of the residuals with a normal curve superimposed. The residuals look close to normal. Figure (41-b) is also a plot of the residuals versus predicted mean Rate of Success. The pattern shown here indicates no problems with the assumption that the residuals are normally distributed at each level of mean Rate of Success and constant in variance across levels of mean rate of Success.



(a) Histogram of Regression Standardized Residual



(b) Normal P-P Plot of Regression Standardized

Figure 41: Residual analysis for Rate of Success versus #Stops, Area, #D, #P

7.2. Multiple Regression Analysis of CPU Time (sec))

To examine the relationship between the mean CPU running time and the number of points to be visited by each driver, the area size, the number of drivers and the number of riders participating in the rideshare program, a multiple linear regression analysis is conducted where the mean CPU running time is the dependent variables and the number of points to be visited by each driver (#Stops), the area size (Area), the number of drivers (#D) and the number of riders (#P) participating in the rideshare program are independent variables. The data set includes all the observations extracted from the 224 numerical examples. Summary output for the multiple linear regression model is presented in Table 17. The relatively high value of R square (R Square: 0.512, Adjusted R Square: 0.504) and the relatively low value of Standard Error (37.84213) suggest that the regression model explains the variation in mean CPU running time.

Table 17: Summary output for the multiple linear regression analysis of CPU running time versus #Stops, Area, #D, #P

Model		R	R Square	Adjusted R Square	Std. Error of the Estimate
dimension0	1	.716	.513	.504	37.84213

The high values of t-statistics as well as the low value of p-value for the coefficients of the model also suggest that variable coefficients are statistically significant for the multiple linear regression model. Table 18 shows the summary ANOVA analysis for the multiple linear regression analysis. Considering the unstandardized coefficients (B's) of dependent variables, the regression analysis results in the following multiple linear regression model:

$$Y= -101.408 + (.436)\, X1 + (.556)\, X2+ (.164)\, X3 + (12.002)\, X4 \qquad (121)$$

where CPU Running Time is denoted by Y, and X1, X2, X3, and X4 are representing #P, #D, Area, and #Stops in the model. The standardized coefficients (Beta) for the independent variables suggest that:

- A one S.D. change in #P produces a predicted change of .246 S.D.'s in the mean CPU Running Time, net of other variables, i.e., running time increases as the number of participating riders increases.
- A one S.D. change in #D produces a predicted change of .314 S.D.'s in the mean CPU Running Time, net of other variables, i.e., running time significantly increases as the number of participating drivers increases.

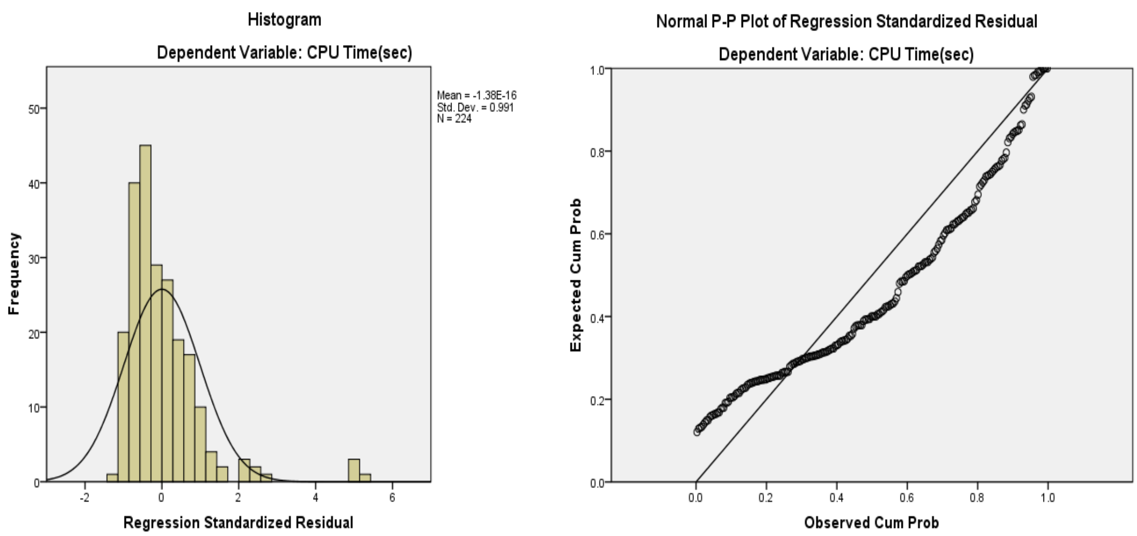
- A one S.D. change in Area Size produces a predicted change of .115 S.D.'s in the mean CPU Running Time, net of other variables, i.e., running time increases as the area size increases.
- A one S.D. change in #Stops produces a predicted change of .448 S.D.'s in the CPU Running Time, net of other variables, i.e., running time significantly increases as the number of stop points increases.
- #Stops and #D are substantially more important than #P and Area size in determining level of CPU Running Time.
- More than 50 percent of the variation in mean CPU running time is explained in order of importance by #Stops, #D, #P and Area size.

Table 18: Summary ANOVA analysis for the regression analysis of CPU running time versus #Stops, Area, #D, #P

	Sum of Squares	df	Mean Square	F	Sig.
Regression	330272.560	4	82568.140	57.658	.000
Residual	313613.884	219	1432.027		
Total	643886.444	223			

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	-101.408	9.36		-10.824	.000	-119.872	-82.943
	#P	.436	.201	.246	2.167	.031	.039	.832
	#D	.556	.201	.314	2.766	.006	.160	.952
	Area	.164	.067	.115	2.431	.016	.031	.297
	#Stops	12.002	1.26	.448	9.494	.000	9.511	14.494

To assess the normality of the residuals, the P-P plot and histogram of residuals are examined. Figure (42-a) is a histogram of the residuals with a normal curve superimposed. The residuals look relatively close to normal. Figure (42-b) is also a plot of the residuals versus predicted mean CPU running time. The pattern shown here indicates no problems with the assumption that the residuals are normally distributed at each level of mean CPU running time and constant in variance across levels of mean CPU running time.



(a) Histogram of Regression Standardized Residual

(b) Normal P-P Plot of Regression Standardized

Figure 42: Residual analysis for CPU running time versus #Stops, Area, #D, #P

Table 19 presents the summary of results for sensitivity analysis using technique of regression analysis.

Table 19: Summary of results for sensitivity analysis using regression analysis

Technique	Dependent Variable	Independent Variables	Results
Regression Analysis	Rate of Success	#D	Rate of success significantly increases as the number of participating drivers increases.
Regression Analysis	Rate of Success	#P	Rate of success significantly increases as the number of participating riders increases.
Multiple Regression Analysis	#0-connection routes	#P, #D	1- Number of zero connection routes significantly increases as the number of participating riders increases when other input parameters remain unchanged. 2- Number of zero connection routes significantly increases as the number of participating drivers increases. 3- #P is more important than #D in determining level of number of zero connection routes.
Multiple Regression Analysis	Rate of Success	#P, #D, #Stops	1- Rate of success slightly decreases as the number of participating riders increases when other input parameters remain unchanged. 2- Rate of Success significantly increases as the number of participating drivers increases. 3- Rate of success significantly increases as the number of stop points increases. 4- #D is substantially more important than #Stops and #P in determining Rate of Success. 5- More than 80 percent of the variation in Rate of Success is explained in order of importance by #D, #Stops, #P.
Multiple Regression Analysis	CPU Running Time	#0-connection routes, #1-connection routes, #2-connection routes	1- CPU running time increases as the number of zero connection routes increases. 2- CPU running time significantly increases as the number of one connection routes increases. 3- CPU running time increases as the number of two connection routes increases. 4- #1-connection routes is substantially more important than #0-connection routes and #2-connection routes in determining level of mean CPU Running. 5- More than 60 percent of the variation in mean CPU Running Time is explained in order of importance by #1-connection routes, #2-connection routes, and #0- connection routes.
Multiple Regression	Rate of Success	#P, #D, #Stops, Area	1- Increase in #P does not have a significant impact on rate of success when other parameters remain unchanged.

Technique	Dependent Variable	Independent Variables	Results
Analysis		size	2- The rate of success increases as the number of participating drivers increases. 3- The rate of success decreases as the area size increases. 4- The rate of success increases as the number of stop points increases. 5- #D is substantially more important than #Stops, and Area size and those are more important than #P in determining level of mean Rate of Success. 6- More than 90 percent of the variation in mean Rate of Success is explained in order of importance by #D, #Stops, Area, #P.
Multiple Regression Analysis	CPU Running Time	#P, #D, #Stops, Area size	1- Running time increases as the number of participating riders increases. 2- Running time significantly increases as the number of participating drivers increases. 3- Running time increases as the area size increases. 4- Running time significantly increases as the number of stop points increases. 5- #Stops and #D are substantially more important than #P and Area size in determining level of CPU Running Time. 6- More than 50 percent of the variation in mean CPU running time is explained in order of importance by #Stops, #D, #P and Area size.
Multiple Regression Analysis	Rate of Success	#0-connection routes, #1-connection routes, #2-connection routes	1- Rate of success significantly increases as the number of zero connection routes increases. 2- Rate of success increases as the number of one connection routes increases. 3- Rate of success slightly increases as the number of two connections routes increases. 4- #0-connection routes and #1-connection routes are substantially more important than #2-connection routes in determining Rate of Success. 5- Around 75 percent of the variation in Rate of Success is explained in order of importance by #0-connection routes, #1-connection routes, and #2-connection routes. 6- #2-connection routes have the least effect on the rate of success while it makes up more than .25 of mean total running time of the algorithm. 7- #1-connection routes and #2-connection routes decreases as the #0-connection routes increases.

Chapter 8: Case Study

This section presents a case to illustrate the DROM as well as an evaluation for the efficient solution approach, TSHDM on a real non-virtual road network. The road network of Baltimore city is chosen for the case study. There are different ways to travel into and throughout Baltimore. Maryland Transit Administration (MTA) manages public transit in the greater Baltimore region and has a range of options for commuting in the city including Light Rail which operates from Hunt Valley through downtown and ends at BWI or Glen Burnie, Metro Subway which runs between Owings Mills and John Hopkins Hospital, with a number of stops in the downtown area, local buses which serve the city and Baltimore County, commuter buses which are express lines to run from Laurel, Columbia, Bel Air, and Havre de Grace to downtown Baltimore, the MARC which is a commuter rail system with three lines to service West Virginia, Frederick, Washington DC, Baltimore, and Perryville, with stops in between. For ridesharing, the Baltimore City Rideshare program promotes commuter alternatives that reduce congestion, such as ride matching and priority parking for carpoolers. A key component of the Rideshare program is Commuter Choice Maryland which is an incentive program to encourage Marylanders who normally drive to work to switch to transit or vanpools. [http://www.baltimorecity.gov]. To benefit from the incentives, an employer must have at least 20 employees signed up to be part of Commuter Choice Maryland program [www.commuterchoicemaryland.com/ridesharing.htm]. Although the Baltimore metropolitan area is only No. 17 in population but it ranks fifth in the nation in the average number of hours automobile commuters are delayed during peak periods. On average, Commuters endure 50 hours of delay each year which places Baltimore behind only Chicago, Washington DC, Los Angeles and Houston, according to the Texas Transportation Institute's Urban Mobility Report [Schrank and Lomax, 2009]. According to the report's Commuter Stress Index, for the city size, the commuting traffic is not as bad as other cities of comparable size. In Baltimore metropolitan area, it takes 25 percent longer to commute at peak times than at non-peak hours, compared with 54 percent in the worst metro region, Los Angeles. That earned Baltimore a 25th-place ranking, but it's still a deterioration from 2008 [Dresser, 2011]. According to the 2010 Census, the city has a total area of 92.052 square Miles (238.41 km^2), of which 80.944 square Miles (209.64 km^2) is land and 11.108 square Miles (28.77 km^2) is water. The total area is 12.07 percent water [2010 census]. Figure 43 shows the geographical area of Baltimore city.

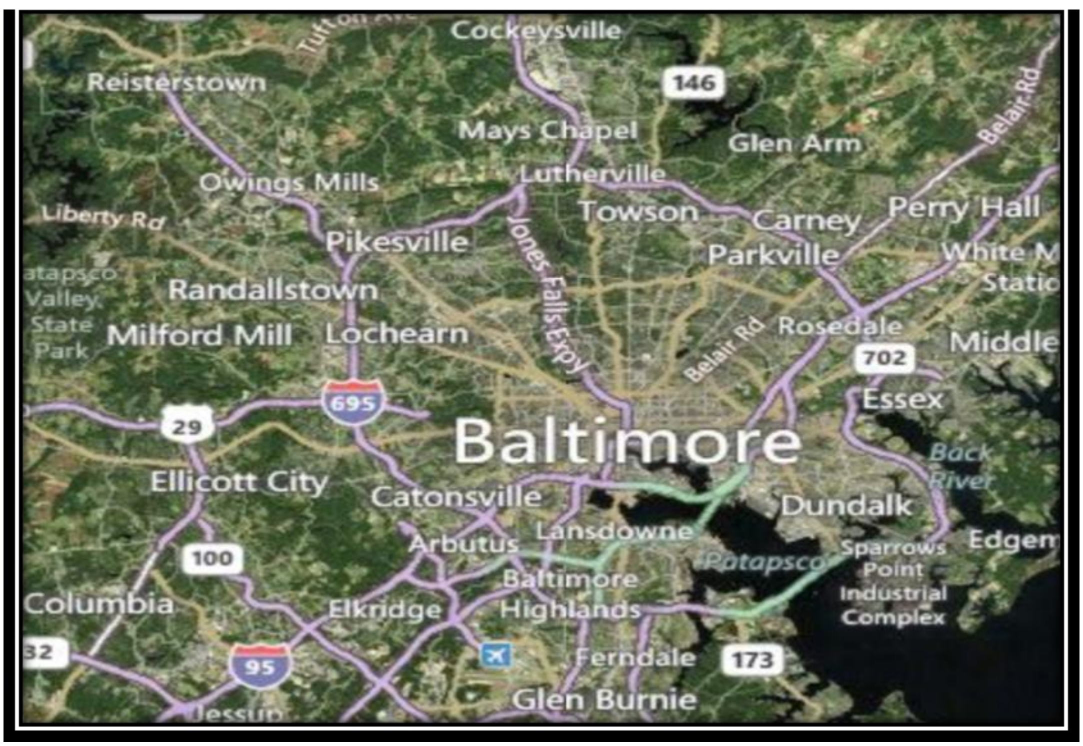


Figure 43: Geographical area of Baltimore city

The area of study is located northwest of metropolitan Baltimore city and has a total area of 27.960 square Miles (5.540 Miles by 5.047 Miles). Some of the major roadways in Maryland including MD45, MD139, MD129, MD26, MD140, MD126, MD122 and U.S. Route 1 and Interstate 83 (Jones Falls Expressway) cross the area of study. The southeast of the area is Downtown Baltimore which is the densest business, residential, tourist, and cultural destination in the region and the northwest of the region is mostly a residential area. Figure 44 shows the geographical metropolitan area boundary for the case study.

Figure 44: Geographical metropolitan area boundary for the case study

In order to simulate the rideshare system the area is represented by a road network which comprises of 9433 links and 11875 nodes. Figure 45 shows the road network of the area of study and Figure 46 shows the nodes on the network. Each node is an intersection of at least 2 links and could be a point of demand for rideshare service.

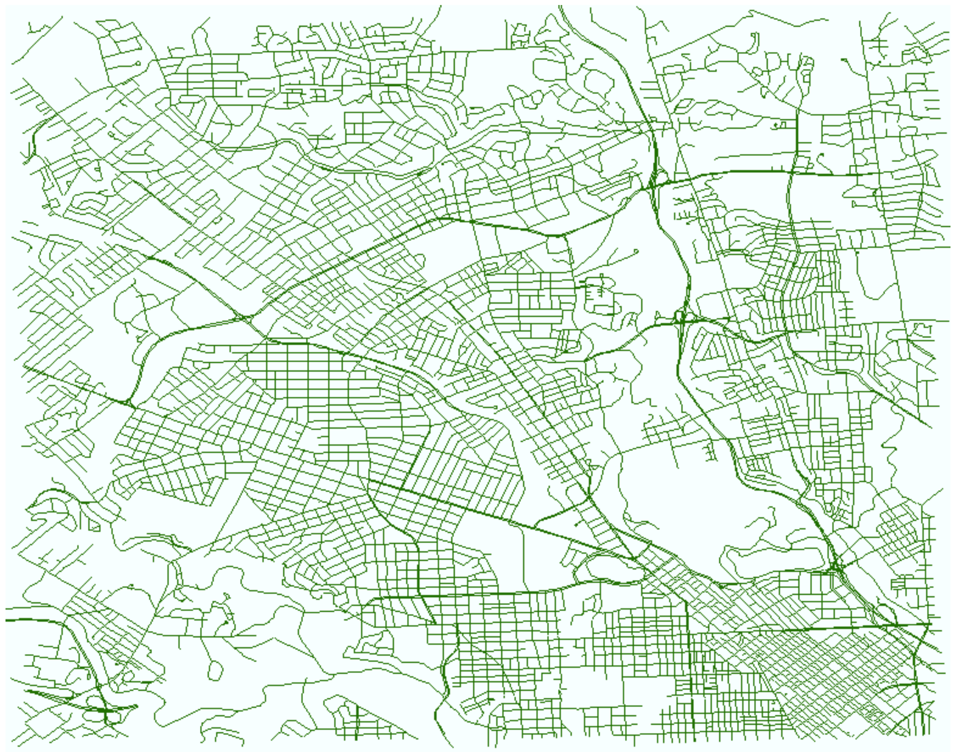


Figure 45: Road network of the area for the case study

Figure 46: The node spots for the area road network

At the beginning of the simulation, it is assumed that there are twenty drivers in the system. For each driver there is an original route connecting the initial points to be visited defined by the driver. The points to be visited for each driver are assumed to be three random points in the area size of 27.960 square Miles. As it was already mentioned, it is assumed that each driver defines a set of initial successive points to be visited when he/she signs in the system. For example, the first driver may have defined three initial points to be visited. Figure 47 shows the three initial points to be visited defined by the first driver.

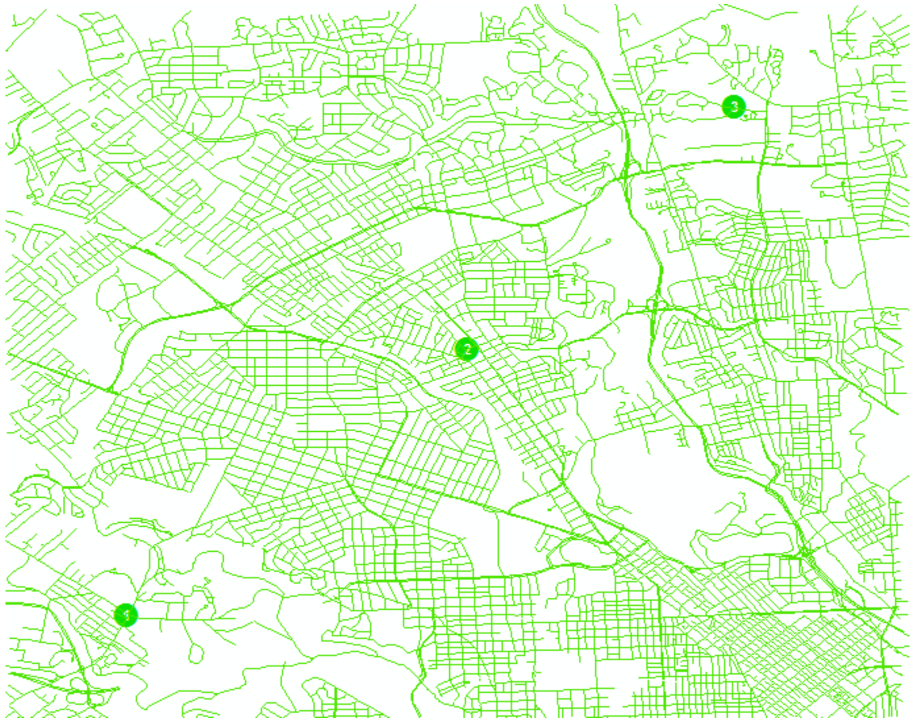


Figure 47: The initial points and route for the first driver in the system

The system assigns the shortest route in the network that connects the initial points. The Network Analyst functions of ArcGIS 10.1 are used to find the shortest path connecting the nodes for the driver. Figure 48 shows the route connecting the initial points to be visited by the first driver. The route is labeled with Driver 1. The numbers on the route indicate the three initial points to be visited successively by the driver.

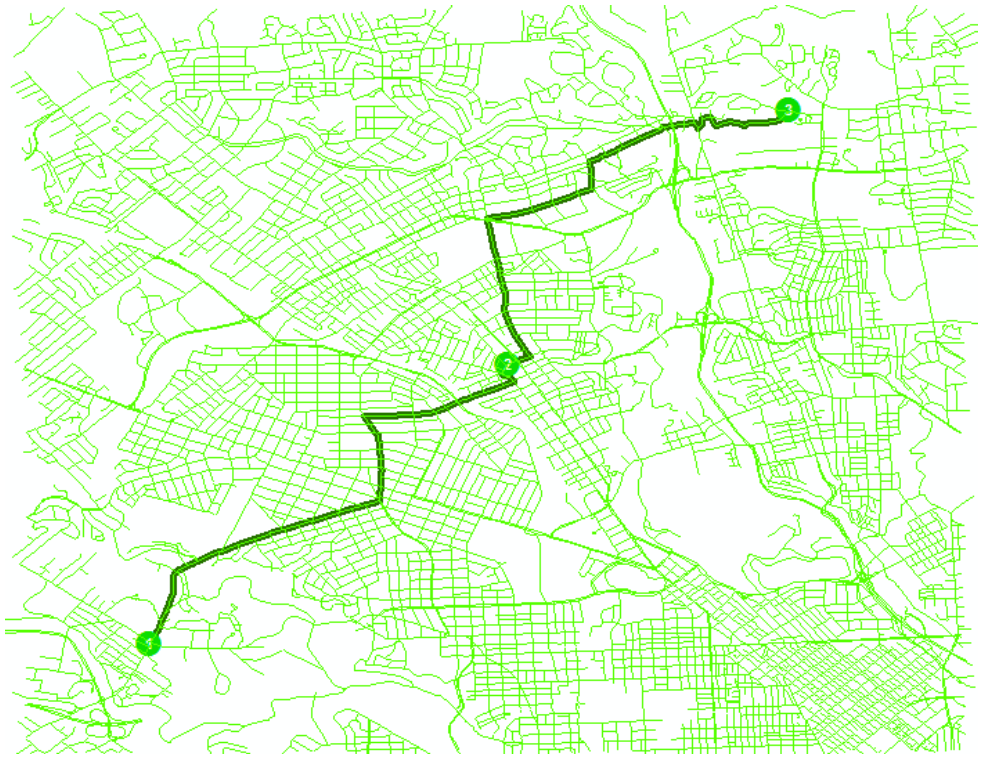


Figure 48: The shortest route for the first driver in the system

Likewise, the other drivers have inputted their initial points to be visited and the Network Analyst functions of ArcGIS 10.1 have returned the shortest paths connecting the nodes for drivers. Without loss of generality, it is assumed that every driver inputs 2 to 6 initial points to be visited.

Considering the fact that increasing the number of points to be visited for each driver yields improvement in the likelihood of ride matches, the .2 Miles buffer along the route for each driver is formed using the ArcMap Buffer Wizard functions of ArcGIS 10.1 to add a number of more appropriate points along the initial route within the buffer to the initial set of points to be visited by each driver. Figure 49 shows the .2 buffer along the route for the first driver.

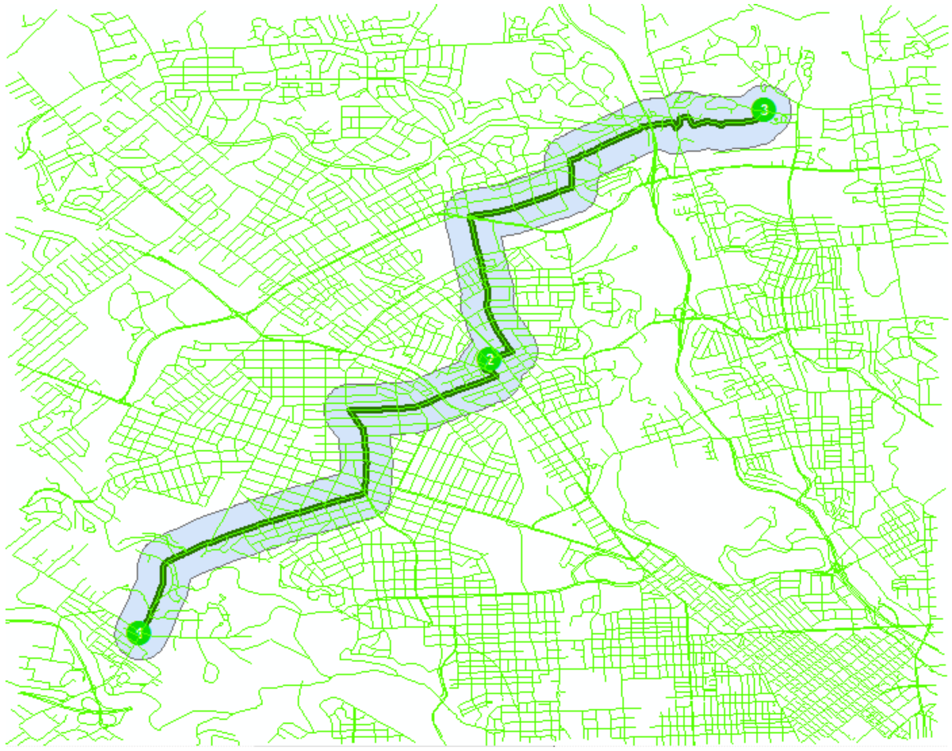


Figure 49: The buffer along the route and the set of points to be visited by the first driver

For simplicity and without loss of generality, it is assumed that the number of points to be visited for each driver adds up to eight points including the initial points defined by the driver and nearby appropriate points within the .2 Miles buffer along the route. These points could be popular spots with stopping facilities to pick up, drop off or making connection with other drivers. Figure 50 shows the set of eight points to be visited within .2 mile buffer along the route for the first driver.

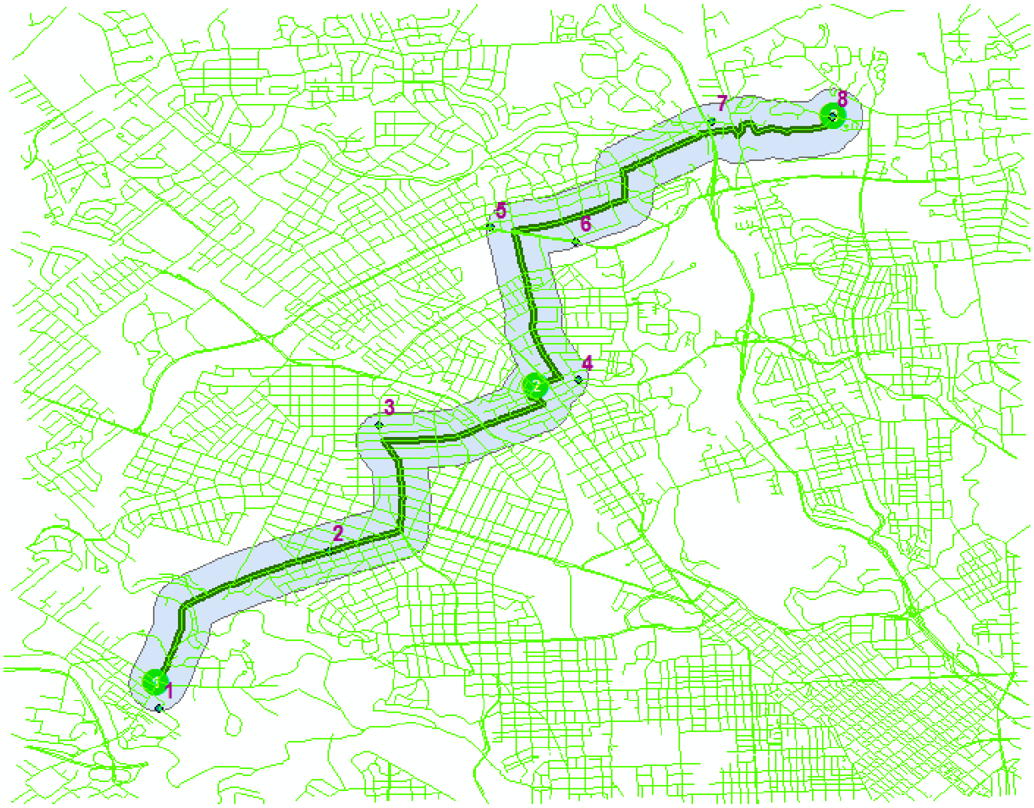


Figure 50: The set of points to be visited within buffer along the route for the first driver

The procedure has been undertaken for all twenty drivers in the system. Figure 51shows the .2 mile buffer along the routes and the set of points to be visited for each driver. The routes are labeled with Driver and an identification number for each driver.

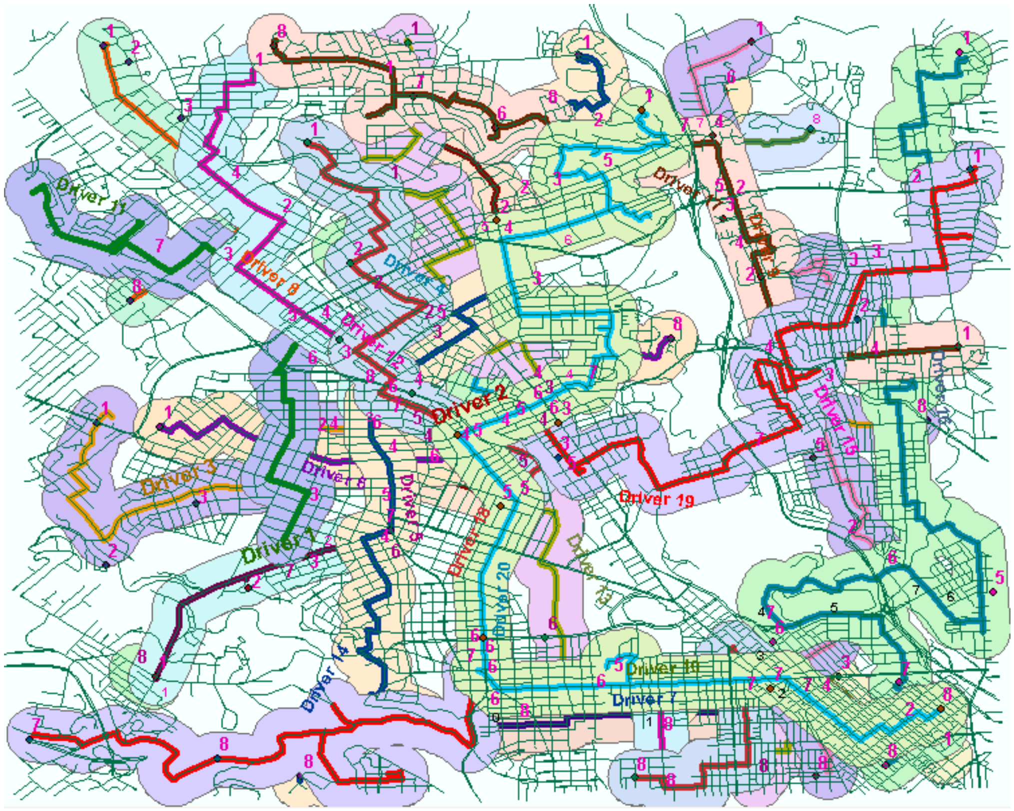


Figure 51: The .2 buffer along the routes and the set of points to be visited for each driver

The new set of points to be visited may result in a slight change of the initial route for each driver. The modified shortest path connecting the new set of nodes for each driver is found using the Network Analyst functions of ArcGIS 10.1. The modified routes assigned for each driver in the system is shown in Figure 52. Although, these routes would be used in the system as the basis to find ride matches, they are not necessarily the final routes for the drivers. While initial points defined by the drivers have to be visited, the other auxiliary points added by the system may or may not be visited at the final route solution for each driver.

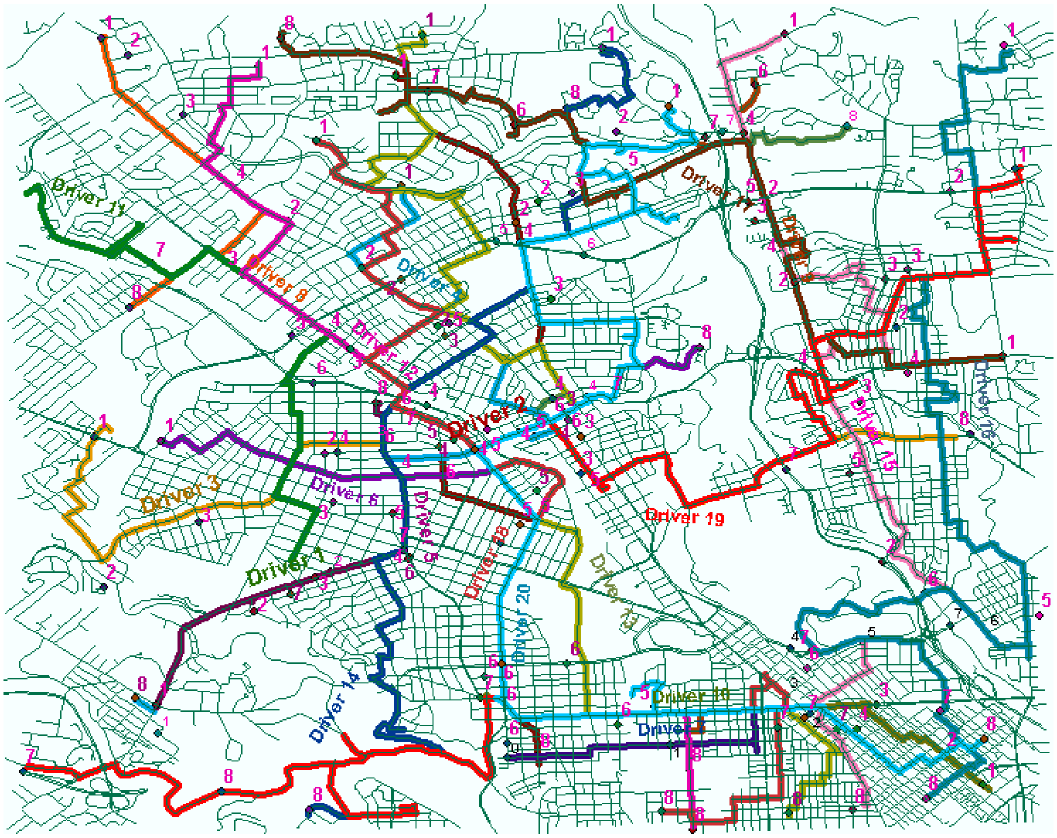


Figure 52: The modified routes and the set of points to be visited for each driver

The other input for the drivers include the drivers' characteristics, preferences and driving input parameters. To generate input information for the drivers and riders, the following behavioral facts and results from the study of the literature are considered:

Gender

1. There are differences between men and women in participating in rideshare trips.
2. Females are more sensitive than are males to transfer of all types, especially the transfers between two different modes.
3. Females are more sensitive to walking time in transfers.
4. Males are more sensitive to access and egress times in public transit.
5. Females are more willing to use toll roads.
6. Females and males should be modeled separately for trip mode choice analysis.

7. Females are less likely to choose public transit and more likely to choose to rideshare than males.
8. Females are less time-sensitive in commuting than are males.
9. Females commute shorter distances and make more trips.
10. Females tend to commute less than males.
11. Females tend to be constrained to a smaller travel area.
12. Females are more likely to form carpools.
13. Males tend more to participate as drivers.
14. Females tend more to participate as passengers.
15. Female passengers are more preferred by male and by female drivers.

Age

16. Compared to the adults, elderly people are less sensitive to the number of transfers.
17. Elderly people tolerate the commute and waiting times much better than the young and adult commuters.
18. Middle age commuters are more likely to use a toll road.
19. Participation in carpooling increases across the age profile, up to 54 years of age.
20. Likelihood of achieving a successful outcome increases with age.
21. Elderly commuters are less likely than others to participate in carpool formation through the deployment of a web-based carpool formation application.

Age and Gender

22. Elderly men commuters are more sensitive than women to access and egress times.
23. People in carpooling arrangements prefer to travel with people of their own age cohort.

24. Females are particularly reluctant to give lifts to, or to pool with, men over 50.

25. Drivers are likely to be males aged 30-50.

26. Riders are likely to be female under 30.

Trip distance

27. Most commuters tend to go slightly out of their way or wait a short time to obtain or offer a ride.

28. The commuters who travel less than a mile or two are less interested in dynamic ridesharing.

29. There is a positive correlation between journey length and likelihood of rideshare requests.

30. The longer one's journey the more attractive is ridesharing.

Time to match up

31. The 10 min added time for a match was too high for many of the users and a shorter time, 3 to 5 min maximum, cuts matches down considerably.

Occupancy preferences

32. The average occupancy is about 2.10 persons per automobile

Other preferences

33. People are less likely to share a ride with a smoker or with a person who is commuting with a pet.

Considering the aforementioned behavioral facts and results, 60% of drivers are assumed to be male (1) and 40% are assumed to be female drivers (2). The age distribution of drivers is assumed to be 35% for young drivers (1), 55% for middle age drivers, and 10% for elderly drivers (3). It is assumed that 10% of drivers are smoker (1) and 90% of the driver pool is made up of nonsmoker drivers (2). Moreover, it is presumed that only 5% of drivers offer a ride when they have a pet onboard (1) and 95% have no pet in the automobile (2). Figure 53 shows the distribution of the drivers in terms of age, gender, smoking habits and pet friendliness combinations and Table 20 shows the distribution of drivers in the driver pool size of 20 for this case study.

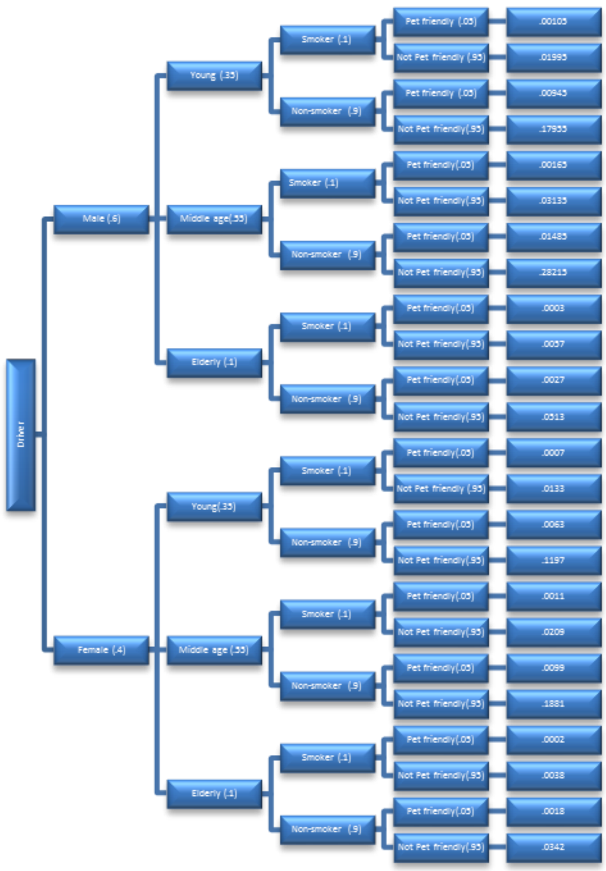


Figure 53: The characteristics distribution of driver pool

Table 20: The distribution of drivers in the pool for the case study.

M/F	**Y/M/E**	**S/NS**	**P/NP**	**# D**
M	Y	S	NP	1
M	Y	NS	NP	3
M	M	S	NP	1
M	M	NS	P	1
M	M	NS	NP	6
M	E	NS	NP	1
F	Y	NS	NP	2
F	M	NS	NP	5

The literature also suggests that most commuters tend to go slightly out of their way or wait a short time to offer a ride and a diversion more than a mile or two from the original route makes the participant less interested in dynamic ridesharing. The input for maximum flexibility for detouring to pick up or drop-off a rider as well as the maximum flexibility for detouring to make a connection for drivers are generated randomly and assumed to be less than 2 Miles. Table 21 shows the input parameters including maximum flexibility for detouring to pick up or drop off a rider (beta1D), maximum flexibility for detouring to make a connection (beta2D), maximum seat capacity (Qdrive), available seat capacity (Qavail) as well as the drivers' characteristics including age characteristic (Agechar), gender characteristic (Genchar), smoking habit (Smochar), and pet friendliness (Petchar).

For rideshare preferences input, due to the triple behavioral facts that female passengers are more preferred by male and by female drivers, people in carpooling arrangements prefer to travel with people of their own age cohort, and females are particularly reluctant to give lifts to, or to pool with, men over 50, it is assumed that 50% of female drivers are reluctant to give lift to elderly people and 20% of females drivers are reluctant to give lift to male riders. It's also assumed that non-smoker drivers are reluctant to give lift to smoker ride and not pet friendly drivers are reluctant to give lift to riders who travel with pet. Table 22 also shows the drivers' rideshare preferences including age, gender, smoking, and pet preferences.

Table 21: The drivers' input parameters

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16	D17	D18	D19	D20
beta1D	1.9	0.5	1.8	0	0.2	0	0.2	2	0.2	0.7	0	1.6	2.0	0.2	2	0.5	1.7	1.3	2.0	1.9
beta2D	0	1.9	1	1.5	0.7	0.6	0.8	0.5	0.1	1.3	1.5	1.2	1.8	1	0.1	0.7	1.5	1.8	1.2	0.7
Qdrive	4	3	4	2	2	4	3	3	3	4	3	3	2	2	4	3	2	3	3	2
Qavail	3	3	4	2	2	4	3	3	3	4	3	3	2	2	4	3	2	3	3	2
Agechar	1	1	2	2	2	2	1	1	1	1	1	2	2	2	2	2	2	2	2	3
Genchar	2	2	2	2	2	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1
Smocha	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2
Petchar	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2

Table 22: The drivers' rideshare preferences

	Age Preferences			Gen. Preferences		Smo. Preferences		Pet Preferences	
D1	1	2	3	0	2	0	2	0	2
D2	1	2	0	1	2	0	0	0	2
D3	1	2	3	1	2	0	0	0	2
D4	1	2	0	0	2	0	2	0	2
D5	1	2	3	1	2	0	0	0	2
D6	1	2	0	1	2	0	2	0	2
D7	1	2	3	1	2	1	2	0	2
D8	1	2	3	1	2	0	2	0	2
D9	1	2	3	1	2	0	2	0	2
D10	1	2	3	1	2	0	2	0	2
D11	1	2	3	1	2	0	2	0	2
D12	1	2	3	1	2	0	2	0	2
D13	1	2	3	1	2	0	2	1	2
D14	1	2	0	1	2	0	2	0	2
D15	1	2	3	1	2	0	2	0	2
D16	1	2	3	1	2	0	2	0	2
D17	1	2	3	1	2	0	2	0	2
D18	1	2	3	1	2	0	2	0	2
D19	1	2	3	1	2	0	2	0	2
D20	1	2	3	1	2	0	2	0	2

It is also assumed that at the beginning of the simulation, there are fifty riders who have requested the rideshare service. Figure 54 shows the point of pickup demands for the riders. The numbers beginning with letter "O" indicate the origin points and identification number for riders. Although the points of origin are all generated randomly, they are mostly allocated in the residential northwest area of study.

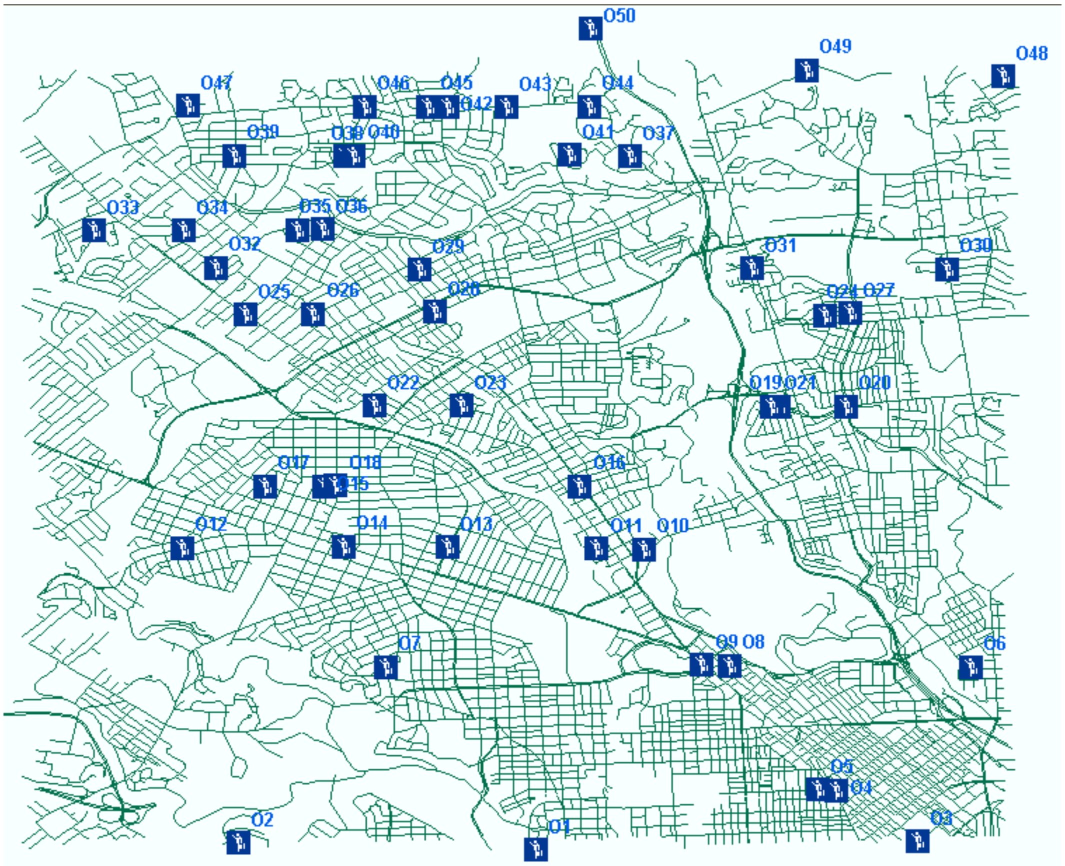


Figure 54: The point of pickup demands for the riders

Figure 55 also shows the point of drop-off demands for the riders. The numbers beginning with letter "D" indicate the destination points and identification number for riders. Although the points of destination are all generated randomly, they are mostly allocated in the Downtown Baltimore at the southeast area.

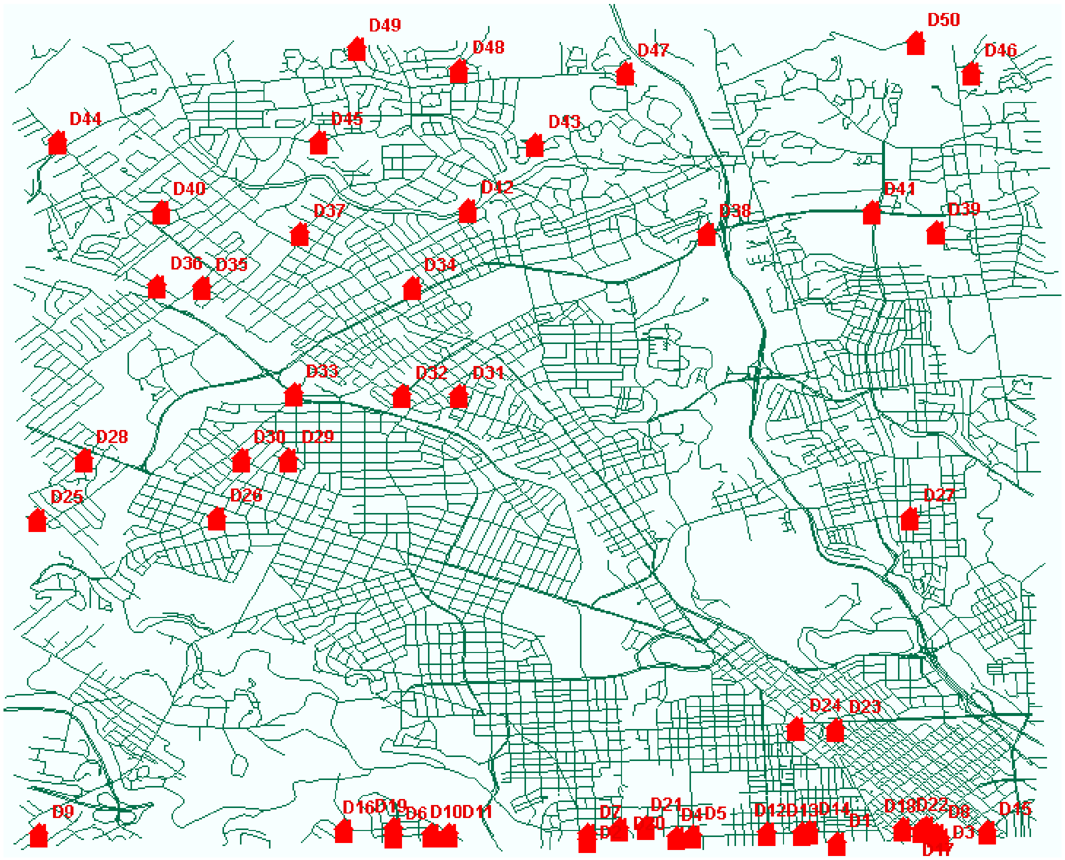


Figure 55: The point of drop-off demands for the riders

Considering the aforementioned behavioral facts and results, 40% of riders are assumed to be male (1) and 60% are assumed to be female riders (2). The age distribution of riders is assumed to be 50% for young riders (1), 40% for middle age riders, and 10% for elderly riders (3). It is assumed that 10% of the riders are smoker (1) and 90% of the rider pool is made up of nonsmoker riders (2). Moreover, it is presumed that only 5% of riders request a ride while they are traveling with a pet (1) and 95% have no pet to bring in the automobile (2). Figure 56 shows the distribution of the riders in terms of age, gender, smoking habits and pet friendliness combinations and Table 23 shows the distribution of riders in the rider pool size of 50 for this case study.

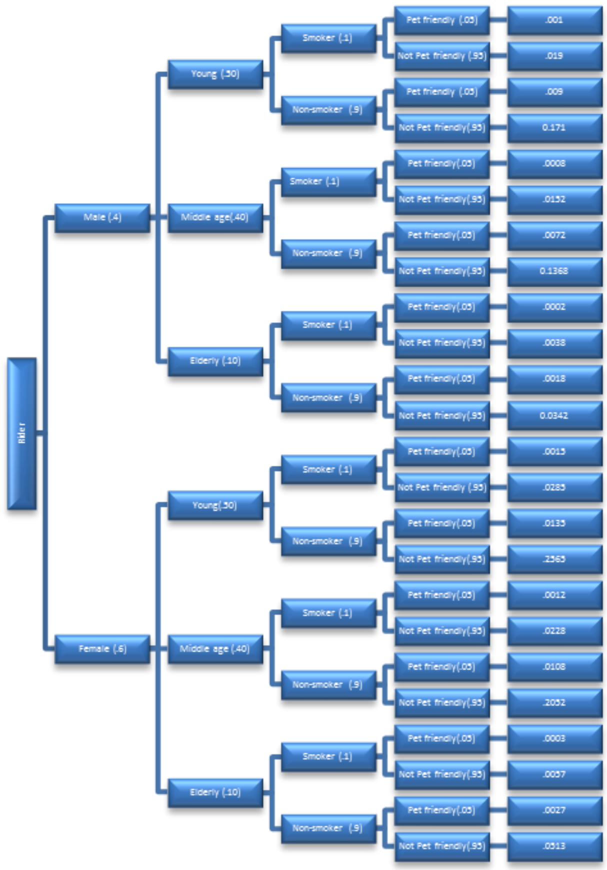


Figure 56: The characteristics distribution of rider pool

Table 23: The distribution of riders in the pool for the case study.

M/F	Y/M/E	S/NS	P/NP	P(P)	# P
M	Y	S	NP	0.019	1
M	Y	NS	NP	0.171	9
M	M	S	NP	0.0152	1
M	M	NS	NP	0.1368	7
M	E	NS	NP	0.0342	2
F	Y	S	NP	0.0285	2
F	Y	NS	P	0.0135	1
F	Y	NS	NP	0.2565	13
F	M	S	NP	0.0228	1
F	M	NS	NP	0.2052	10
F	E	NS	NP	0.0513	3

The literature also suggests that most commuters tend to go slightly out of their way or wait a short time to catch a ride and a diversion more than a mile or two walking distance makes the participant less interested in dynamic ridesharing. The literature also suggests that elderly people tolerate the commute and waiting times much better than the young and adult commuters and females are more sensitive to walking time in transfers as well. It is assumed that maximum waiting time for riders to be picked up at the origin point or connection points is 25 minutes for elderly people and 20 minutes for young and middle age riders. In additions, the input for maximum flexibility for relocation to be picked up or dropped-off as well as the maximum flexibility for relocating to make a connection for riders are generated randomly and assumed to be less than 2 Miles for male riders and 1.5 Miles for female riders. For the research purposes of this study, maximum relocation distances are exaggerated to increase the chance of rideshare matches. Current time of the system is assumed to be 10:00 a.m. and all the rideshare service requests are within half an hour from 10:00 to 10:30. Table 24 shows the input parameters including maximum flexibility for relocating to be picked up or dropped off (Gamma1P), maximum flexibility for relocating to make a connection (Gamma2P), time of rideshare request (TOP), maximum flexibility for waiting time (PhiP) as well as the riders' characteristics including age characteristic (AgecharP), gender characteristic (GencharP), smoking habit (SmocharP), and pet friendliness (PetcharP). For rideshare preferences input, due to the behavioral facts that people in carpooling arrangements prefer to travel with people of their own age cohort, and females are particularly reluctant to pool with men over 50, it is assumed that 25% of female riders are reluctant to pool with elderly people and 20% of females riders are reluctant to catch a ride from male drivers. It's also assumed that non-smoker riders are reluctant to pool with a smoker driver and riders who are not pet friendly are reluctant to pool with pets. Table 25 also shows the riders' rideshare preferences.

Table 24: The riders' input parameters

Gamma1P	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
	3.1	16	3.1	5.8	12.1	2.7	6.9	19.3	8.4	16.1	4.7	0.4	19.2	8.7	9.7	16	0.7	18.6	13.6	5.5	14.1	10.4	19.1	13.8	14.2
	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50
	18.8	5.2	15	8.1	2.4	15.5	19.5	0.6	0.7	17.2	18.1	4.6	8.2	10.3	4.5	14.5	13.6	5.5	13	1.5	5	3.5	8.7	13.1	5.9
Gamma2P	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
	7.8	7.6	5.9	2.5	5.4	16.6	12.4	5.8	15.2	5.7	2.5	3.6	17.6	0.1	11	2.6	16.7	5.3	0.4	10.7	1.3	19.5	11.6	3.3	7
	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50
	2.2	9	0.9	14.8	11.5	14.1	10	7.8	3.1	0.9	2.6	2.4	17.7	6.4	16.5	1.9	17.3	9.2	17.2	19	13	14.1	0.2	6.2	7.2
TOP	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
	1018	1010	1027	1018	1030	1016	1025	1028	1013	1030	1025	1007	1023	1026	1020	1022	1027	1028	1001	1019	1007	1027	1001	1010	1012
	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50
	1023	1015	1025	1001	1005	1027	1025	1004	1012	1008	1014	1003	1010	1010	1018	1003	1012	1027	1004	1004	1010	1006	1016	1004	1012
PhiP	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
	.6	.9	0	1.5	1.6	1.7	1	1	.3	.8	.9	.5	1.5	.7	1.6	1.4	1.1	1.7	.7	.6	.5	.4	.1	.2	1.3
	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50
	.6	1.5	1.2	1.4	.2	.2	1.1	.6	0	.9	1.4	.3	1.3	.2	1.2	.8	.8	1.0	.3	1.2	1.1	.5	.3	0	1.2
GencharP	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2
	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
AgecharP	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	1	1	1	1
	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50
	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	3	3	3
SmochaP	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
	1	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	1	1	2	2
	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50
	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2
PetcharP	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24	P25
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2
	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36	P37	P38	P39	P40	P41	P42	P43	P44	P45	P46	P47	P48	P49	P50
	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Table 25: The riders' rideshare preferences

	Age Preferences			Gen. Preferences		Smo. Preferences		Pet Preferences	
P1	1	2	3	1	2	1	2	0	2
P2	1	2	3	1	2	0	2	0	2
P3	1	2	3	1	2	0	2	0	2
P4	1	2	3	1	2	0	2	0	2
P5	1	2	3	1	2	0	2	0	2
P6	1	2	3	1	2	0	2	0	2
P7	1	2	3	1	2	0	2	0	2
P8	1	2	3	1	2	0	2	0	2
P9	1	2	3	1	2	0	2	0	2
P10	1	2	3	1	2	0	2	0	2
P11	1	2	3	1	2	1	2	0	2
P12	1	2	3	1	2	0	2	0	2
P13	1	2	3	1	2	0	2	0	2
P14	1	2	3	1	2	0	2	0	2
P15	1	2	3	1	2	0	2	0	2
P16	1	2	3	1	2	0	2	0	2
P17	1	2	3	1	2	0	2	0	2
P18	1	2	3	1	2	0	2	0	2
P19	1	2	3	1	2	0	2	0	2
P20	1	2	3	1	2	0	2	0	2
P21	1	2	3	1	2	0	2	0	2
P22	1	2	0	0	2	1	2	0	2
P23	1	2	3	0	2	1	2	1	2
P24	1	2	3	1	2	0	2	0	2
P25	1	2	0	1	2	0	2	0	2
P26	1	2	0	1	2	0	2	0	2
P27	1	2	3	1	2	0	2	0	2
P28	1	2	3	1	2	0	2	0	2
P29	1	2	3	1	2	0	2	0	2
P30	1	2	0	1	2	0	2	0	2
P31	1	2	3	0	2	0	2	0	2
P32	1	2	3	1	2	0	2	0	2
P33	1	2	3	1	2	0	2	0	2
P34	1	2	3	1	2	0	2	0	2
P35	1	2	0	0	2	0	2	0	2
P36	1	2	3	1	2	0	2	0	2
P37	1	2	3	1	2	1	2	0	2
P38	1	2	3	1	2	0	2	0	2
P39	1	2	3	1	2	0	2	0	2
P40	1	2	3	1	2	0	2	0	2
P41	1	2	3	0	2	0	2	0	2
P42	1	2	3	1	2	0	2	0	2
P43	1	2	3	1	2	0	2	0	2
P44	1	2	3	1	2	0	2	0	2
P45	1	2	3	1	2	0	2	0	2
P46	1	2	0	1	2	0	2	0	2
P47	1	2	3	1	2	0	2	0	2
P48	1	2	3	1	2	0	2	0	2
P49	1	2	0	1	2	0	2	0	2
P50	1	2	3	1	2	0	2	0	2

Other input in the model include:

1. the distance matrix (Distance) which is a $|D| \times |N-1|$ matrix with the distance between two successive nodes to be visited for each driver;
2. the driving time matrix (DrivingTime) which is a $|D| \times |N-1|$ matrix with the driving time between two successive nodes to be visited for each driver;
3. the distance matrix (DistanceOP) which is a $|D| \times |N| \times |P|$ matrix with the distance between the origin point of each rider from each point to be visited for each driver;
4. the driving time matrix (DrivingTimeOP) which is a $|D| \times |N| \times |P|$ matrix with the driving time between the origin point of each rider from each point to be visited for each driver;
5. the walking time matrix (WalkingTimeOP) which is a $|D| \times |N| \times |P|$ matrix with the driving time between the origin point of each rider from each point to be visited for each driver;
6. the distance matrix (DistanceDP) which is a $|D| \times |N| \times |P|$ matrix with the distance between the destination point of each rider from each point to be visited for each driver;
7. the driving time matrix (DrivingTimeDP) which is a $|D| \times |N| \times |P|$ matrix with the driving time between the destination point of each rider from each point to be visited for each driver;
8. the walking time matrix (WalkingTimeDP) which is a $|D| \times |N| \times |P|$ matrix with the driving time between the destination point of each rider from each point to be visited for each driver;

9. the driving time matrix (DrivingTimeODP) which is a $|D| \times |P|$ matrix with the driving time between the origin and destination points of each rider for each driver;
10. the distance matrix (DistanceDD) which is a $|D| \times |N| \times |D| \times |N|$ matrix with the distance between every point to be visited for each driver and every point to be visited for the other drivers.
11. the driving time matrix (DrivingTimeDD) which is a $|D| \times |N| \times |D| \times |N|$ matrix with the driving time between every point to be visited for each driver and every point to be visited for the other drivers.
12. the walking time matrix (WalkingTimeDD) which is a $|D| \times |N| \times |D| \times |N|$ matrix with the walking time between every point to be visited for each driver and every point to be visited for the other drivers.

where $|D|$ is the cardinality of the set of drivers; $|N|$ is the maximum cardinality of the set of points to be visited for the drivers; and $|P|$ is the cardinality of the set of riders. For the case study, $|D|$, $|N|$ and $|P|$ are 20, 8, and 50 respectively.

Although walking speeds can vary greatly depending on factors such as height, weight, age, terrain, surface, load, culture, effort, and fitness, the average human walking speed is about 3.1 Miles per hour. Specific studies have found pedestrian walking speeds ranging from 2.8 mph to 2.95 mph for older individuals to 3.2 mph to 3.3 mph for younger individuals [TranSafety, 2009], [Asspelin, 2005].

For the purposes of the case study and without loss of generality, the walking speed is assumed to be a random number in the range of 2.8 mph to 2.95 mph for older pedestrian and 3.2 mph to 3.3 mph for younger pedestrians. All the other distances and driving times input matrices are generated using Network Analyst extension tools and Hawth's Analysis Tools which are extensions for ArcGIS and work based on the shortest routes connecting the nodes.

The case study is solved using TSHDM approach. Table 26 shows the summary of results for the case study. As shown, it took 6 iterations and 8.237 second to generate 10 ride matches of which nine are matches with zero connection routes and the other one is a match with a one connection route.

Table 26: The summary of results for the case study

Iter	Matched established routes	Acc. Time (sec.)	# routes
1	0_Connection (p_i, d_j, n_p, n_d): (1,18,6,6); (3,20,7,7); (21,1,7,7); (23,13,6,7); (29,12,5,6); (40,17,7,8); (41,3,6,6)	.0191	7
2	0_Connection (p_i, d_j, n_p, n_d): (24,3,7,7); (27,13,7,8)	.0226	2
3	0_Connection (p_i, d_j, n_p, n_d): not found	.0265	0
4	1_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, n_d)$: (9,20,2,3,19,3,7)	6.302	1
5	0_Connection (p_i, d_j, n_p, n_d): not found	6.951	0
6	2_Connection $(p_i, d_j, n_p, n_c^{d_j}, d_{j'}, n_c^{d_{j'}}, d_{j''}, n_c^{d_{j''}}, n_d)$: Not found	8.237	0

Table 27 shows the summary of computational results with more details. Eight of the drivers and 10 riders contributed in the rideshare matches. While seven of the routes are perfectly matched, 3 matches are not perfect and need to be negotiated with the potential drivers and riders to form a carpool.

Table 27: The summary of computational results for the case study

Driver			change of arrival times to the final destination (mm:ss)					shares ride with rider	rider		Pick up type	Connection type	Drop off type	Perfect match solution	Compromise negotiable solution			
			Iteration			total changes			travel time (mm:ss)	wait time (mm:ss)								
#	travel time (mm:ss)	travel distance (m)	1	2	4	travel time (mm:ss)	travel distance (m)								Age	Gen	Smo	Pet
1	22:16	8.2	6:28			06:28	1.7	21	04:37	03:47	2		2			•		
3	47:49	16.8	10:0	06:16		16:18	1.8	41	08:14	14:44	2		2	✓				
								24	03:40	11:08	2		2	✓				
12	35:21	10.7	3:51			03:51	1.5	29	28:08	02:21	2		1	✓				
13	33:25	9.3	11:1	03:50		14:59	2	23	18:08	10:40	2		2				•	
								27	04:19	03:46	2		2					•
17	28:54	8.5	8:11			08:11	1.6	40	03:55	02:48	2		2	✓				
18	25:48	8.7	6:55			06:55	1.2	1	04:15	03:47	2		2				•	
19	33:20	13.2			02:56	02:56	1	9	17:58	02:22		1	2	✓				
20	31:08	11.6	1:13		04:13	05:26	2	3	00:03	02:02	2		2	✓				
								9	05:27	01:42	2	1		✓				

Following discussion is presented to provide more insight into the solution approach:

1. **Driver 18 and Rider 1:**

The system resulted in a compromise zero-connection route for imperfect match between Driver 18 and Rider 1.The original points to be visited by the driver are points number 1, 2 and 8 and earlier the system has come up with a set of 5 additional points within the .2 Miles buffer to be added to the set of points to be visited by Driver 18, namely points number 3, 4, 5, 6, and 7. Figure 57 shows the three initial points as well as the additional five points for Driver 18. The origin (O1) and destination point (D1) for Rider 1 are also shown in the figure.

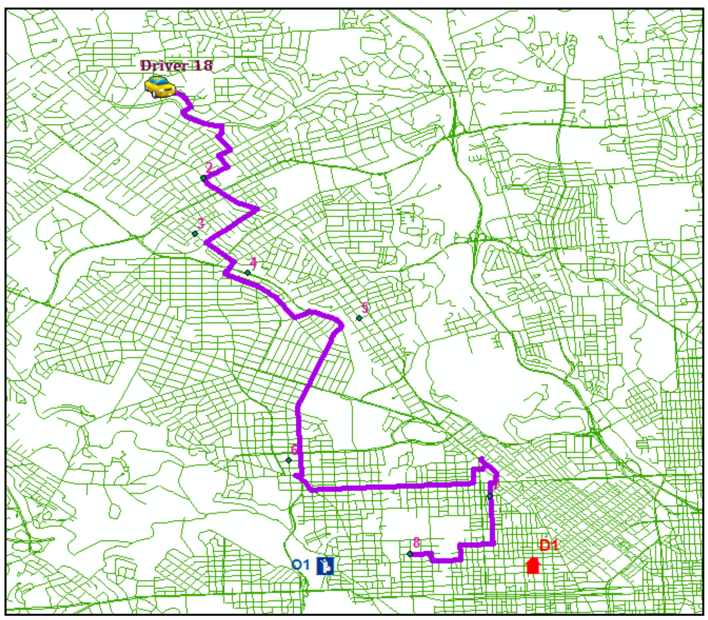


Figure 57: The initial route for Driver 18 and the O-D points for Rider 1.

According to the solution, Driver 18 makes a detour at his 6^{th} point to be visited to pick up (type 2) Rider 1 at his point of origin and takes him directly to his destination (type 2) and then changes his route toward his 8^{th} point to be visited. Figure 58 shows the course of actions and updated route for Driver 18 compared with his original route.

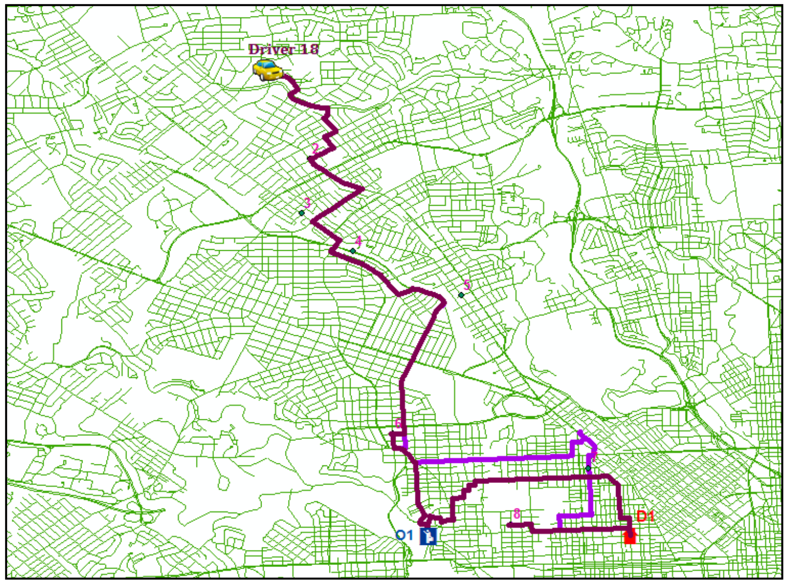


Figure 58: The updated (maroon) route compared with the original (purple) one for Driver 18 matched with Rider 1.

The system also shows that the match between Driver 18 and Rider 1 is not complete. While Driver 18 is a male, middle age and nonsmoker driver that doesn't allow pet in his car, Rider 1 is a male, middle age, and smoker who is reluctant to pool with pets. The input information for rideshare preferences show that Driver 18 is reluctant to offer a lift to a smoker rider. The system has generated a negotiating policy to make the pool possible. Accordingly, either Rider 1 has to agree that he will not smoke in the car or Driver 18 has to accept the smoker rider. Either of the agreement results into a perfect matched route between Driver 18 and Rider 1 and subsequently Driver 18 adopts an updated route. The updated route includes the origin point of Rider 1, O1, and his destination point, D1. Figure 59 shows the updated route for Driver 18. For the next run, the updated route includes the origin point of Rider 1, O1, and the destination point for Rider 1, D1. Figure 63 shows the updated route with 5 initial points to be visited for the next run. At the next trigger of the system, besides to the 5 initial points to be visited for the next run, the .2 mile buffer gives the additional points to be added to the set of points to be visited by driver 18 and the points already met by the driver would be removed for the set.

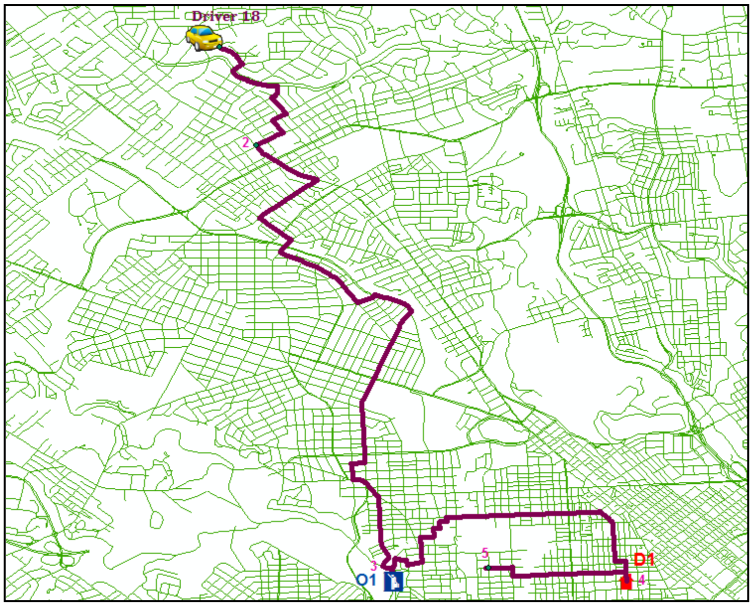


Figure 59: The updated route for Driver 18

The other input parameters that would be updated for the nexr run includes:

i. The maximum detour distance for Driver 18 changes to .1 Miles because 1.2 Miles of the 1.3 Miles maximum flexibility to pick up or drop off for this driver has already been used in this run.

ii. The maximum waiting time for Rider 1 changes to 04:33 because 03:47 minutes of the the 07:80 minutes maximum waiting time for this rider has been already used in this run. i.e., any future changes in the route of Driver 18 that results in more than 04:33 minutes change in arrival time of Rider 1 would be discarded. For the next run, the system will add an arrival time window with width 04:33 for Driver 18 to his arrival in the origin point of Rider 1, O1.

iii. The seating capacity for Driver 18 is updated.

iv. All travel time and distance matrices are updated accordingly.

2. **Driver 13, Rider 23 and Rider 27:**

The system resulted in a two compromise zero-connection routes for imperfect matches between Driver 13, Rider 23 and Rider 27. The original points to be visited by the driver were points 1, 2 and 8 and earlier the system came up with a set of 5 additional points within the .2 Miles buffer to be added to the set of points to be visited by Driver 13, namely points 3, 4, 5, 6, and 7. Figure 60 shows the three initial points as well as the five additional points for Driver 13. Origins (O23, O27) and destination points (D23, D27) for Rider 23 and Rider 7 are also shown in the figure.

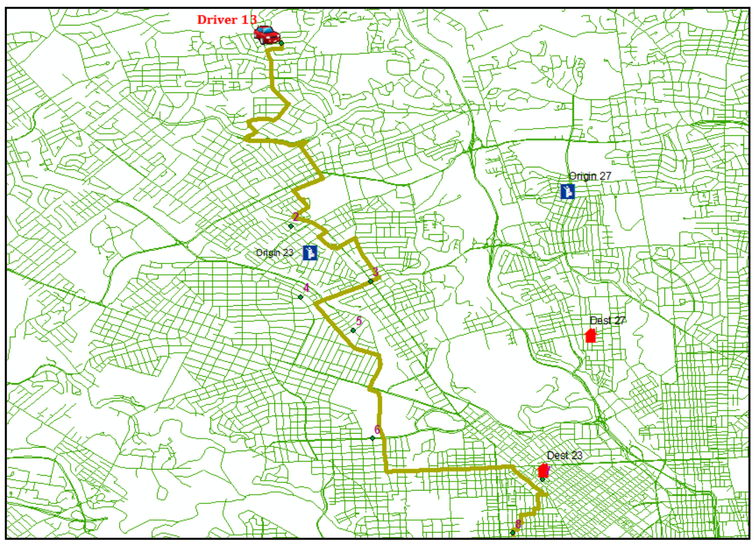


Figure 60: The initial route for Driver 13 and O-D points for Rider 23 and Rider 27.

According to the solution, Driver 13 makes a detour at his 2nd point to be visited to pick up (type 2) Rider 23 and then changes his route toward the origin point of Rider 27 to pick her up at her point of origin (type 2) and continues directly toward the destination point of Rider 27 to drop her off at her point of destination (type 2) and then continues toward the destination point of Rider 3 to drop her off at her point of destination (type 2) and then goes to his 8th point to be visited. Figure 61 shows the updated course of actions Driver 13 compared with his original one.

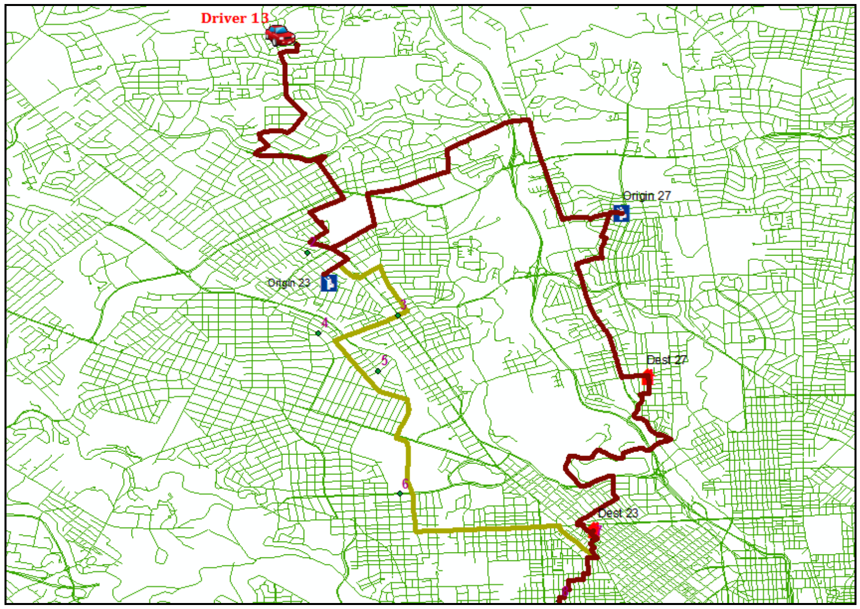


Figure 61: The updated (maroon) route compared with the original (yellowish-green) route for Driver 13 matched with Rider 23 and Rider 24.

The system also shows that the match between Driver 13, Rider 23 and Rider 27 is not complete. While Driver 13 is a male, middle age and nonsmoker driver that welcomes pet in his car, Rider 23 is a female, young, and smoker who has requested a ride with her pet and Rider 27 is a female, young and nonsmoker who is reluctant to pool with a pet. The input information for rideshare preferences show that Driver 13 is reluctant to offer a lift to a smoker rider which makes it an imperfect match with Rider 23 who is a smoker rider. In additions, Rider 27 is reluctant to pool with a pet. The system has generated a negotiating policy to make the pool possible. Accordingly, either Rider 23 has to agree that she will not smoke in the car and either Rider 23 has to agree not to bring her pet in the car or Rider 27 agrees to pool with a pet. Either of the agreement results into a perfect matched route for Driver 13, Rider 23 and Rider 27 and subsequently Driver 13 adopts an updated route. The updated route includes the O-D points for Rider 23 (O3, and D23) and Rider 27 (O27, and D27). Figure 62 shows the updated route for Driver 13. For the next run of the system, the updated route includes the origin point of Rider 23, O23, the destination point for Rider 23, D23 as well as the origin point of Rider 27, O27, the destination point for Rider 27, D27. Figure 62 shows the updated route with 7 initial points to be visited for the next run. At the next trigger of the system, besides the 7 initial points to be visited for the next run, the .2 mile buffer gives the additional points to be added to the set of points to be visited by driver 13 and the points already met by the driver would be removed from the set.

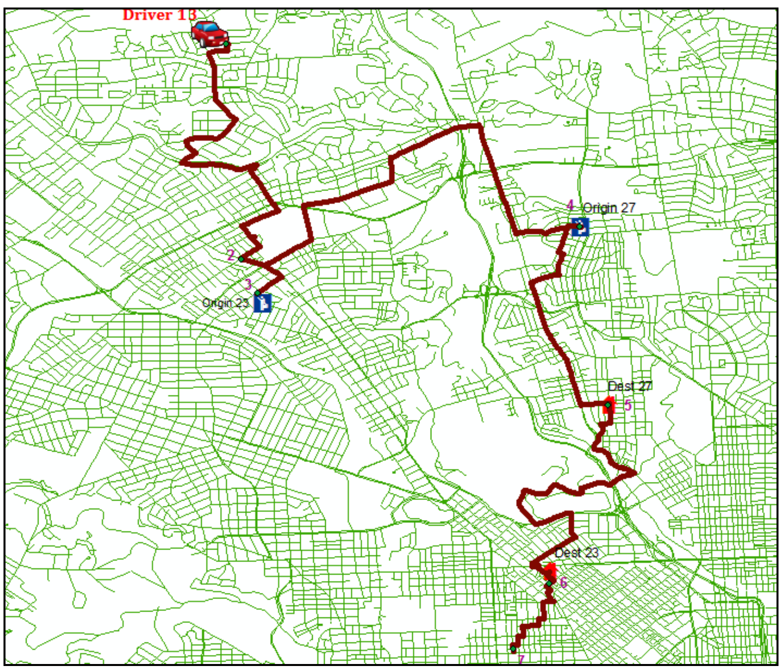


Figure 62: The updated route for Driver 13

The other input parameters that would be updated for the nexr run include:

i. The maximum detour distance for Driver 13 changes to 0 mile because all of his flexibility to pick up or drop off for this driver has already been used.

ii. The maximum waiting time for Rider 23 changes to 08:21 because 10:40 minutes of the the 19.1 minutes of maximum waiting time for this rider has been already used in this run. i.e., any future changes in the route of Driver 13 that results in more than 08:21 minutes change in arrival time of Rider 23 would be discarded. For the next run, the system will add an arrival time window with width 08:21 for Driver 13 to his arrival in the origin point of Rider 23, O23. Likewise, The maximum waiting time for Rider 27 changes to 01:34 because 03:46 minutes of the the 05:20 minutes maximum waiting time for this rider has been already used in this run. That means any future changes in the route of Driver 13 that results in more than 01:34 minutes change in arrival time of Rider 27 would be discarded. For the next run, the system will add an arrival time window with width 01:34 for Driver 13 to his arrival in the origin point of Rider 23, O23.

iii. The seating capacity for Driver 13 is updated.

iv. All travel time and distance matrices are updated accordingly.

3. Driver 17 and Rider 40:

The system resulted in a perfectly matched zero-connection route for Driver 17 and Rider 40.The original points to be visited by the driver are points 1, 2 and 8 and earlier the system has come up with a set of 5 additional points within the .2 Miles buffer to be added to the set of points to be visited by Driver 17, namely points 1, 2, 6, 7, and 8. Figure 63 shows the five initial points as well as the additional three points for Driver 17. The origin (O40) and destination point (D40) for Rider 1 are also shown in the figure.

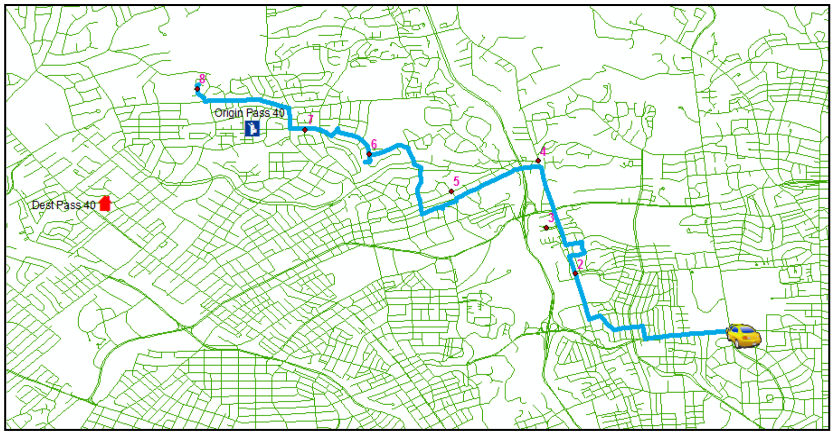


Figure 63: The initial route for Driver 17 and the O-D points for Rider 40.

According to the solution, Driver 17 makes a detour at his 7th point to be visited to pick up (type 2) Rider 40 at his point of origin and takes him directly to his destination (type 2) and then changes his route toward his 8th point to be visited. Figure 64 shows the course of actions and updated route for Driver 17 compared with his original route.

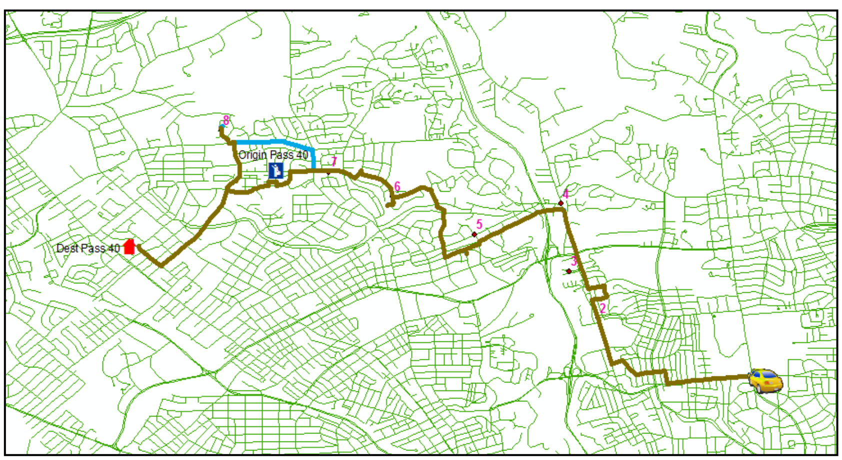


Figure 64: The updated (yellowish-green) route compared with the original (blue) route for Driver 17 matched with Rider 40.

The system also shows that match between Driver 17 and Rider 40 is complete and subsequently Driver 17 adopts an updated route. The updated route includes the origin point of Rider 40, O40, and the destination point for Rider 40, D40. Figure 65 shows the updated route for Driver 17. For the next run of the system, the updated route includes the O-D points for Rider 40. Figure 65 shows the updated route with 7 initial points to be visited for the next run. At the next trigger of the system, besides the 7 initial points to be visited for the next run, the .2 mile buffer gives the additional points to be added to the set of points to be visited by driver 17 and the points already met by the driver would be removed from the set.

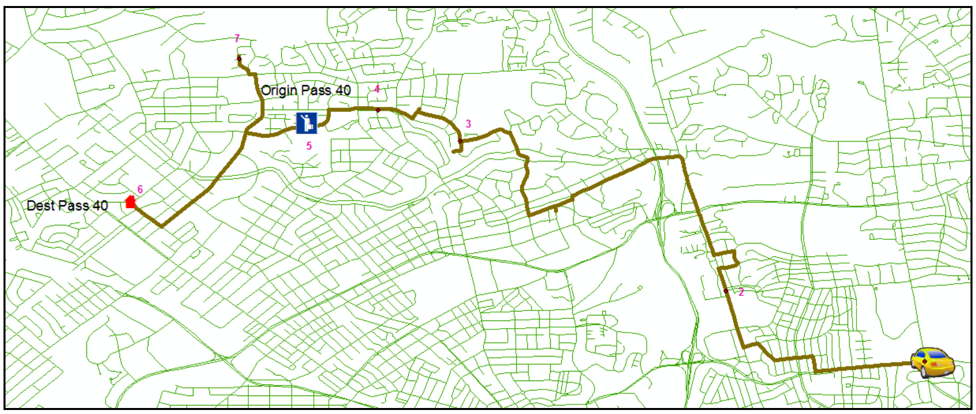


Figure 65: The updated route for Driver 17

The other input parameters that would be updated for the nexr run includes:

i. The maximum detour distance for Driver 17 changes to 0.1 mile because 1.6 mile of his 1.7 Miles flexibility to pick up or drop off for this driver has already been used.

ii. The maximum waiting time for Rider 40 changes to 02:02 because 02:48 minutes of the the 4:50 minutes of maximum waiting time for this rider has been already used. That means any future changes in the route of Driver 17 that results in more than 02:02 minutes change in arrival time of Rider 40 would be discarded. For the next run, the system will add an arrival time window with width 02:02 for Driver 17 to his arrival in the origin point of Rider 40, O40.

iii. The seating capacity for Driver 17 is updated.

iv. All travel time and distance matrices are updated accordingly.

4. Driver 19, Driver 20, Rider 9, Rider 3

The system resulted in a perfectly matched one-connection route for Driver 19, Driver 20 and Rider 9 as well as one perfectly matched zero-connection route for Driver 19, Driver 20 and Rider 3. The original points to be visited by Driver 19 are points 1, 2, 3, 7 and 8 and earlier the system has come up with a set of 3 additional points within the .2 Miles buffer to be added to the set of points to be visited by Driver 19, namely points 4, 5, and 6. The original points to be visited by Driver 20 are points 1, 2, 4, and 8 and earlier the system has come up with a set of 4 additional points within the .2 Miles buffer to be added to the set of points to be visited by Driver 20, namely points 3, 5, 6 and 8. Figure 66 shows the five original points and the additional three points for Driver 19 as well as the 4 original points and the additional 4 points to be visited by Driver 20. The origin (O9) and destination point (D9) for Rider 9 as well as the origin (O3) and destination point (D3) for Rider 3 are also shown in the figure.

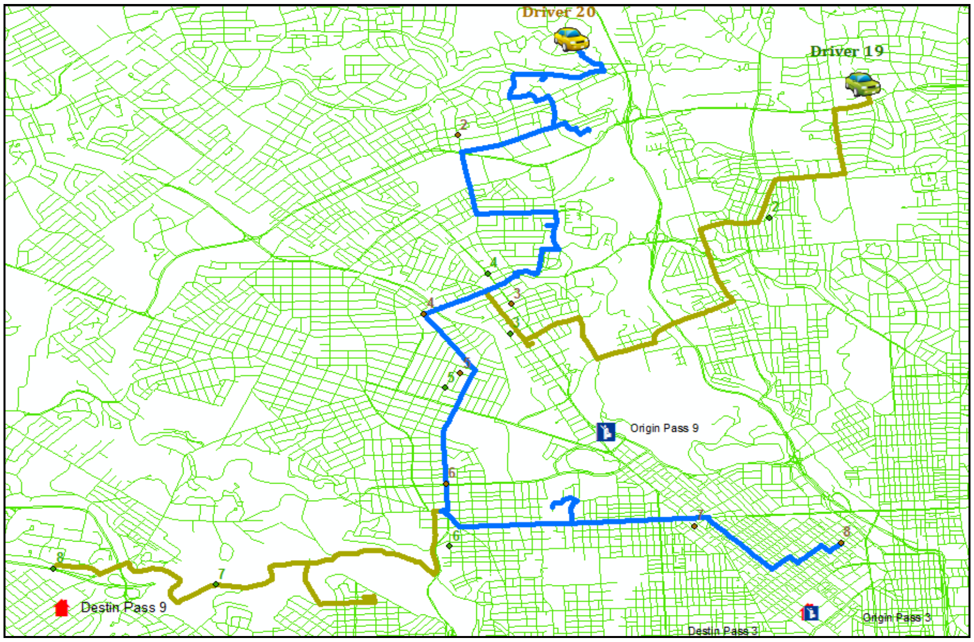


Figure 66: The initial routes for Driver 19, Driver 20 and the O-D points for Rider 9 and Rider 3.

According to the solution, Driver 20 makes a detour at his 2nd point to be visited to pick up (type 2) Rider 9 at his point of origin and then makes a detour to drop him off at the3rd point to be visited by Driver 19 (connection type 2) and then changes his route toward his own 4th point to be visited. Driver 20 continues his way toward his 7th point to be visited and then makes a detour to pick up (type 2) and drop off (type 2) Rider 3 before he arrives in his 8th point to be visited. At the same time, Driver 19 picks up Rider 9 at his 3rd point to be visited and continues to his 7th point to be visited. At that point, he makes a change in his route to drop off Rider 9 at his destination (type 2) and then goes toward his 8th point to be visited. Figure 67 shows the course of actions and updated routes for Driver 19 and Driver 20 compared with their original routes.

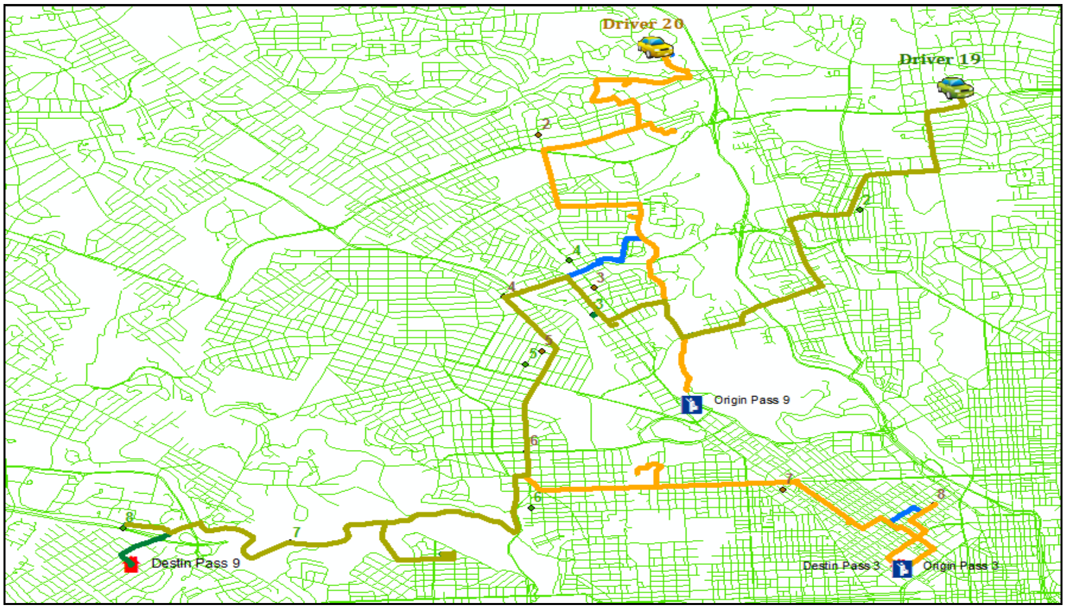
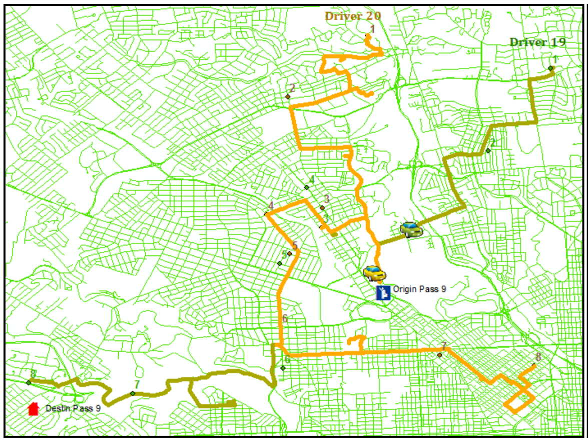
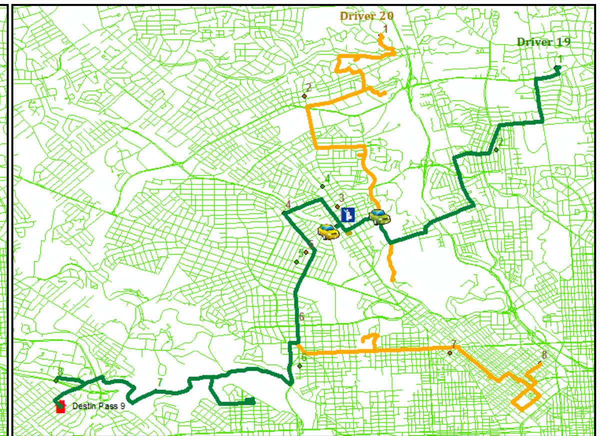


Figure 67: The updated (orange) route compared with the original (blue) route for Driver 20 matched with Rider 9 and Rider 3; the updated (dark green) route compared with the original (light green) route for Driver 19 matched with Rider 9.

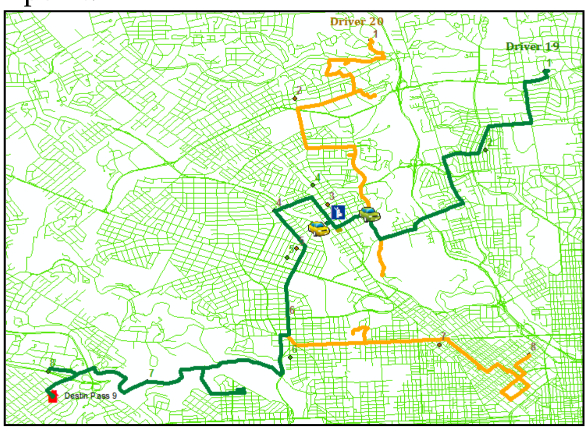
The courses of actions are shown step by step in Figure 68.



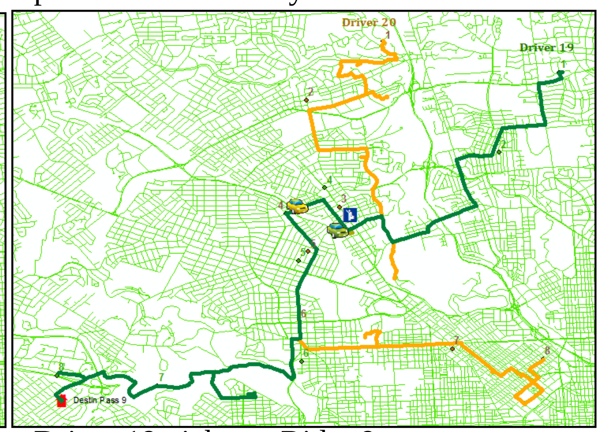
Driver 20 picks up Rider 9 at his origin point.



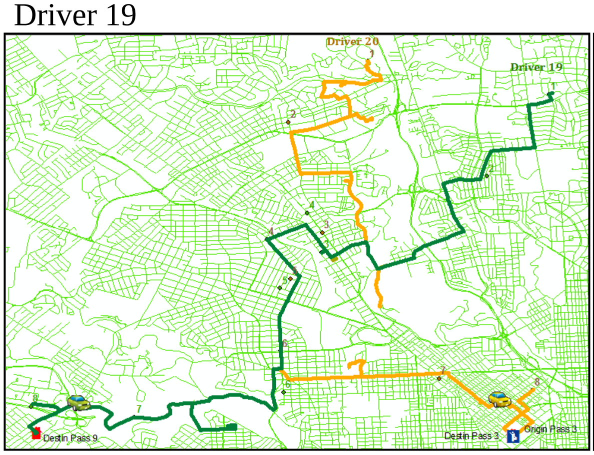
Driver 20 drops off Rider 9 at the 3rd point to be visited by Driver 19.



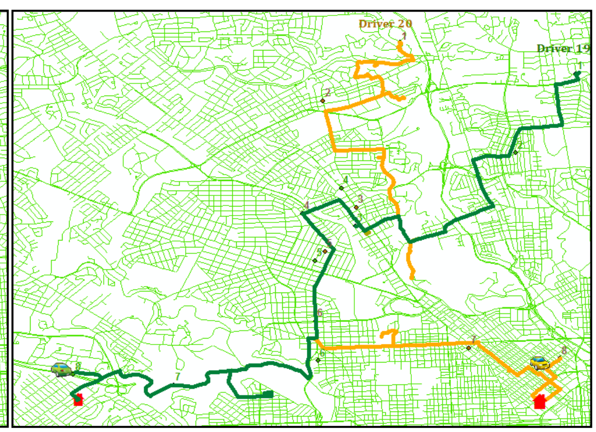
Rider 9 idles for 01:42 to be picked up by Driver 19



Driver 19 picks up Rider 9



Driver 19 detours to drop off Rider 9; Driver 20 detours to pick up Rider 3 and later will drop him off



Driver 19 and Driver 20 head toward their destination.

Figure 68: The course of events for Driver 19, Driver 20, Rider 9 and Rider 3.

Figure 69 provides a closed look at the matched route for Rider3 and Driver 20. Driver 20 picks up Rider 3 at his origin point and drops him off at his destination after 0.1 mile ride and then Driver 20 continues to reach to his destination at 10:33. The results also show that arrival time for Driver 19 at his destination is 10:31.

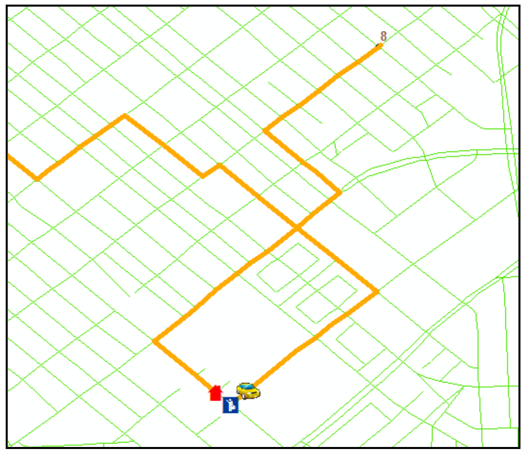


Figure 69: Bird view map for connection route between Driver 19 and Rider 3

The other input parameters that would be updated for the next run include:

i. The maximum detour distance for pick up for Driver 19 changes to 0 Miles because 2 Miles of the 2 Miles maximum flexibility to pick up or drop-off for this driver has already been used in this run.

ii. The maximum detour distance for connection for Driver 19 changes to .2 mile because 1 Miles of the 1.2 Miles maximum flexibility for connection for this driver has already been used in this run.

iii. The maximum detour distance for for pick up and drop- off for Driver 20 changes to .3 Miles because 1.6 Miles of the 1.9 Miles maximum flexibility to pick up or drop-off for this driver has already been used in this run.

iv. The maximum waiting time for pick up for Rider 9 changes to 07:00 minutes because 01:42 minutes of the the 08:42 minutes maximum waiting time for this rider has been already used in this run.

v. The maximum waiting time for connection for Rider 9 changes to 12:58 minutes because 02:22 minutes of the the 15:2 minutes maximum waiting time for connection for this rider has been already used in this run. That means for any future changes in the route of Driver 19 the time window width of 12:58 minutes should be considered.

vi. The maximum waiting time for Rider 3 changes to 01:08 because 02:02 minutes of the the 03:10 minutes maximum waiting time for this rider has been already used in this run.

vii. The seating capacity for Driver 19 and Driver 20 are updated.

viii. All travel time and distance matrices are updated accordingly.

The rideshares characteristics distribution for the case study is also analysed and its result is shown in Table 28.

Table 28: Rideshares characteristics distribution for results of the case study

Carpool		Characteristic								
		Gender		Age			Smoking Habit		Pet Friendliness	
		Male	Female	Young	Middle age	Elderly	Smoker	Nonsmoker	welcome	No pet
1	Driver 1		✓	✓				✓		✓
	Rider 21	✓				✓		✓		✓
2	Driver 3		✓		✓			✓		✓
	Rider 41		✓		✓			✓		✓
	Rider 24		✓	✓				✓		✓
3	Driver 12	✓			✓			✓		✓
	Rider 29		✓	✓				✓		✓
4	Driver 13	✓			✓			✓	✓	
	Rider 23		✓	✓			✓		✓	
	Rider 27		✓	✓						✓
5	Driver 17	✓			✓			✓		✓
	Rider 40		✓		✓			✓		✓
6	Driver 18	✓			✓			✓		✓
	Rider 1	✓			✓		✓			✓
7	Driver 19	✓			✓			✓		✓
	Rider 9	✓			✓			✓		✓
20	Driver 20	✓				✓		✓		✓
	Rider 9	✓			✓			✓		✓
20	Driver 20	✓				✓		✓		✓
	Rider 3	✓			✓			✓		✓
distribution		60%	40%	25%	60%	15%	10%	90%	10%	90%

Chapter 9: Summary, Conclusions and Directions for Future Research

This dissertation developed a Dynamic Rideshare Optimized Matching (DROM) model and solution that was aimed at identifying suitable matches between passengers requesting rideshare services with appropriate drivers available to carpool for credits and HOV lane privileges. For the purposes of the study presented in this research, real-time ridesharing was defined as a non-recurring multipurpose rideshare trip which is prearranged on a per trip basis on a short-notice to establish shared trips close to the desired departure times and locations of the participants to gain HOV lanes privileges or share the cost of the trip. DROM received passengers and drivers' information and preferences continuously over time and assigned passengers to drivers with respect to proximity in time and space and compatibility of characteristics and preferences among the passengers, drivers and passengers onboard. The optimization model maximized the overall system performance subject to ride availability, capacity, rider and driver time window constraints, and detour and relocation distances while considering users' preferences. The ridesharing preferences considered in the model are: age, gender, smoking as well as the maximum number of people sharing a ride. Computational burden associated with the increasing size of the participants and visiting points of interests showed that it was impossible to rely on commercial solvers for obtaining optimal solutions. The research developed an efficient solution algorithm for solving practical size problems. The idea was to develop a decomposition approach that decomposed the problem into a series of smaller solvable sub-problems. The decomposition was spatial, temporal, and hierarchical by carefully considering the relationships between the decisions and constraints. The decomposition strategy led to the heuristic solution procedure, Three-Spherical Heuristic Decomposition Model (TSHDM). Quality and validity tests for the TSHDM algorithm were done by comparison of results between the exact and implemented algorithm solutions. A major sensitivity analysis on all of the related parameters in the model was conducted to thoroughly investigate the properties of the proposed model and solution algorithm. To conduct regression analysis, a substantial computational effort was undertaken which comprised of 224 numerical examples with different combinations of area sizes, points to be visited, number of riders, and number of drivers. Due to stochastic behavior of the problem, for a given combination of area size, number of points to be visited, number of riders, and number of drivers each numerical example was run for three times and the best solutions with the greatest total number of matched routes were presented. The most influencing parameters of the model were identified through a sensitivity analysis using the technique of Regression Analysis. A case study was constructed to analyze the model behavior in the practical and real size scale operations as well as an evaluation for the efficient solution approach, TSHDM on a real non-virtual road network. The road network of Baltimore city was chosen for the case study.

This study showed that using appropriate technical tools and social networking media, it is possible to implement a dynamic rideshare system. The study showed that DROM is a very complicated and challenging problem from both mathematical formulation and solution algorithm perspectives. The review of the literature revealed that while technological advances have greatly eased the communication and reputation systems and social network tools have tackled the fear of sharing a ride with strangers, the development of optimization algorithms for real-time matching of the participant and ultimately increasing the rate of participation in the ridesharing system has been largely ignored by transportation research community. This research was the first of its kind to develop an optimization algorithm for real-time rideshare matching problem. Another main contribution of this research was that it develops an optimization algorithm with negotiating policies for rideshare preference matches. This study has successfully developed and implemented a decomposition solution approach that solves the problem in a very reasonable time which is suitable for solving large scale problems. While there could be other ways to define success rate in a dynamic rideshare system, for the purpose of this research, it was defined as the fraction or percentage of successful rideshare matches, i.e., the ratio of the number of matched riders to the ideal number of possible matches. The ideal number of matches is defined as the minimum of the number of riders and the total available seats. Major sensitivity analysis conducted on several parameters and variables affecting the model showed that most influencing factors for the rate of success in the rideshare system are, in order of importance: number of participating drivers, number of stops, area size, and number of participating riders. The study also showed rate of success for the rideshare system is highly dependent to the matched routes that directly connect points of origin and destination for the participating riders with no transfer and connection and also the study showed that increasing the number of connections from one to two which requires two consecutive change of rides for a rider has the least impact on the rate of success.

One of the drawbacks of dynamic ridesharing is the fear of sharing a ride with strangers. To deal with safety and security concerns of participating users, it is proposed that all users should be registered and there should be a registration and screening system in place to reduce the concerns and ultimately increase the likelihood of the success of the rideshare system by increasing the number of participants.

It is also notable that there is a positive correlation between journey length and likelihood of dynamic rideshare requests, i.e. the longer is one's journey, the more attractive is dynamic ridesharing. While carpooling targets commuters, the demand for dynamic ridesharing tends to be prearranged, long-distance and nonrecurring trips with more flexible travel schedules. Long-distance rideshare trips have the privilege of lower marginal cost and lower participants' out of pocket costs.

Future research directions include conducting a simulation study using TSHDM and based on a real travel demand data to identify and highlight the benefits and effects of, as well as the economic, behavioral, institutional, and technological challenges to, real-time ridesharing. Another area for further research is linking between travel demand management strategies and main results for the sensitivity analysis of this research that identified the most influencing factors to increase the rate of success for a dynamic rideshare system. Creating and field testing a smart phone app that combines GPS technologies with TSHDM algorithm to real-time ride-matching drivers and riders is another direction for further research.

Another area for further research is investigating the challenges and opportunities of centralized rideshare matching service compared to distributed rideshare matching mechanism. If the rideshare information and communication technologies are reliable enough that secure an effective mechanism to communicate between the users, distributed real-time rideshare matching system could also be a reliable alternative.

The review of the literature suggests that land use and trip purposes play a significant role in rideshare formation and the journey's distance and time have a stronger influence on inter-household ridesharing, Another direction for future research could be analyzing the likelihood of inter-household rideshare formation compared with intra-household ridesharing.

Appendix A: Computational Results

Table A-1: Computational results- #P: 20, #D:20

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
20	20	25	2	7	0.040	100.00	0.65	0	0.059	0.00	0.30	0	0.061	7	0.00	0.04	0.35
			3	6	0.039	85.71	0.28	1	0.117	0.14	0.55	0	0.142	7	0.00	0.17	0.35
			4	6	0.040	100.00	0.21	0	0.140	0.00	0.52	0	0.192	6	0.00	0.27	0.30
			5	6	0.054	100.00	0.14	0	0.281	0.00	0.61	0	0.376	6	0.00	0.25	0.30
			6	8	0.079	88.89	0.12	1	0.515	0.11	0.65	0	0.669	9	0.00	0.23	0.45
			7	6	0.080	85.71	0.07	1	0.983	0.14	0.74	0	1.227	7	0.00	0.20	0.35
			8	8	0.160	100.00	0.09	0	1.451	0.00	0.71	0	1.815	8	0.00	0.20	0.40
Median				**6.000**	**0.054**	**100.000**	**0.143**	**0.000**	**0.281**	**0.000**	**0.606**	**0.000**	**0.376**	**7.000**	**0.000**	**0.201**	**0.350**
Mode				**6.000**	**0.040**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**7.000**	**0.000**	**#N/A**	**0.350**
Mean				**6.714**	**0.070**	**94.331**	**0.222**	**0.429**	**0.507**	**0.057**	**0.583**	**0.000**	**0.640**	**7.143**	**0.000**	**0.196**	**0.357**
20	20	100	2	0	0.013	#DIV/0!	0.25	0	0.047	#DIV/0!	0.67	0	0.051	0	#DIV/0!	0.08	0.00
			3	1	0.026	100.00	0.24	0	0.082	0.00	0.50	0	0.112	1	0.00	0.27	0.05
			4	0	0.026	#DIV/0!	0.12	0	0.164	#DIV/0!	0.65	0	0.213	0	#DIV/0!	0.23	0.00
			5	0	0.026	#DIV/0!	0.06	0	0.351	#DIV/0!	0.76	0	0.426	0	#DIV/0!	0.18	0.00
			6	0	0.026	#DIV/0!	0.03	0	0.691	#DIV/0!	0.84	0	0.791	0	#DIV/0!	0.13	0.00
			7	0	0.054	#DIV/0!	0.04	0	1.276	#DIV/0!	0.86	0	1.420	0	#DIV/0!	0.10	0.00
			8	0	0.039	#DIV/0!	0.02	0	2.141	#DIV/0!	0.91	0	2.322	0	#DIV/0!	0.08	0.00
Median				**0.000**	**0.026**	**#DIV/0!**	**0.062**	**0.000**	**0.351**	**#DIV/0!**	**0.763**	**0.000**	**0.426**	**0.000**	**#DIV/0!**	**0.127**	**0.000**
Mode				**0.000**	**0.026**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#DIV/0!**	**#N/A**	**0.000**
Mean				**0.143**	**0.030**	**#DIV/0!**	**0.109**	**0.000**	**0.679**	**#DIV/0!**	**0.739**	**0.000**	**0.762**	**0.143**	**#DIV/0!**	**0.152**	**0.007**

Table A-2: Computational results - #P: 20, #D: 25

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
20	25	25	2	2	0.026	100.00	0.32	0	0.059	0.00	0.40	0	0.081	2	0.00	0.28	0.10
			3	7	0.054	100.00	0.35	0	0.117	0.00	0.42	0	0.152	7	0.00	0.23	0.35
			4	11	0.079	100.00	0.30	0	0.176	0.00	0.37	0	0.263	11	0.00	0.33	0.55
			5	12	0.398	92.31	0.55	1	0.620	0.08	0.31	0	0.720	13	0.00	0.14	0.65
			6	7	0.079	100.00	0.07	0	0.761	0.00	0.62	0	1.095	7	0.00	0.31	0.35
			7	6	0.106	66.67	0.05	3	1.592	0.33	0.71	0	2.099	9	0.00	0.24	0.45
			8	5	0.133	71.43	0.04	1	2.702	0.14	0.69	1	3.721	7	0.14	0.27	0.35
Median				**7.000**	**0.079**	**100.000**	**0.300**	**0.000**	**0.620**	**0.000**	**0.417**	**0.000**	**0.720**	**7.000**	**0.000**	**0.274**	**0.350**
Mode				**7.000**	**0.079**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**7.000**	**0.000**	**#N/A**	**0.350**
Mean				**7.143**	**0.125**	**90.058**	**0.241**	**0.714**	**0.861**	**0.079**	**0.501**	**0.143**	**1.162**	**8.000**	**0.020**	**0.258**	**0.400**
20	25	100	2	0	0.026	#DIV/0!	0.37	0	0.059	#DIV/0!	0.45	0	0.071	0	#DIV/0!	0.17	0.00
			3	0	0.026	#DIV/0!	0.19	0	0.105	#DIV/0!	0.56	0	0.142	0	#DIV/0!	0.26	0.00
			4	0	0.026	#DIV/0!	0.08	0	0.257	#DIV/0!	0.74	0	0.314	0	#DIV/0!	0.18	0.00
			5	0	0.040	#DIV/0!	0.06	0	0.539	#DIV/0!	0.77	0	0.649	0	#DIV/0!	0.17	0.00
			6	0	0.040	#DIV/0!	0.03	0	1.065	#DIV/0!	0.83	0	1.237	0	#DIV/0!	0.14	0.00
			7	1	0.093	100.00	0.04	0	1.931	0.00	0.85	0	2.160	1	0.00	0.11	0.05
			8	0	0.076	#DIV/0!	0.02	0	3.567	#DIV/0!	0.91	0	3.840	0	#DIV/0!	0.07	0.00
Median				**0.000**	**0.040**	**#DIV/0!**	**0.062**	**0.000**	**0.539**	**#DIV/0!**	**0.769**	**0.000**	**0.649**	**0.000**	**#DIV/0!**	**0.170**	**0.000**
Mode				**0.000**	**0.026**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#DIV/0!**	**#N/A**	**0.000**
Mean				**0.143**	**0.047**	**#DIV/0!**	**0.114**	**0.000**	**1.075**	**#DIV/0!**	**0.729**	**0.000**	**1.202**	**0.143**	**#DIV/0!**	**0.157**	**0.007**

Table A-3: Computational results - #P: 25, #D: 20

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
25	20	25	2	6	0.040	100.00	0.52	0	0.065	0.00	0.32	0	0.077	6	0.00	0.17	0.24
			3	8	0.043	100.00	0.33	0	0.095	0.00	0.41	0	0.127	8	0.00	0.25	0.32
			4	8	0.053	100.00	0.20	0	0.188	0.00	0.51	0	0.266	8	0.00	0.29	0.32
			5	12	0.090	92.31	0.19	1	0.354	0.08	0.54	0	0.487	13	0.00	0.27	0.52
			6	11	0.105	100.00	0.13	0	0.596	0.00	0.60	0	0.816	11	0.00	0.27	0.44
			7	9	0.116	100.00	0.08	0	1.195	0.00	0.70	0	1.534	9	0.00	0.22	0.36
			8	9	0.201	100.00	0.08	0	2.080	0.00	0.72	0	2.621	9	0.00	0.21	0.36
Median				**9.000**	**0.090**	**100.000**	**0.185**	**0.000**	**0.354**	**0.000**	**0.542**	**0.000**	**0.487**	**9.000**	**0.000**	**0.252**	**0.360**
Mode				**8.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**8.000**	**0.000**	**#N/A**	**0.320**
Mean				**9.000**	**0.093**	**98.901**	**0.216**	**0.143**	**0.653**	**0.011**	**0.543**	**0.000**	**0.847**	**9.143**	**0.000**	**0.240**	**0.366**
25	20	100	2	0	0.013	#DIV/0!	0.21	0	0.047	#DIV/0!	0.56	0	0.060	0	#DIV/0!	0.23	0.00
			3	1	0.040	100.00	0.30	0	0.117	0.00	0.58	0	0.132	1	0.00	0.11	0.04
			4	0	0.014	#DIV/0!	0.05	0	0.199	#DIV/0!	0.73	0	0.254	0	#DIV/0!	0.22	0.00
			5	0	0.026	#DIV/0!	0.05	0	0.433	#DIV/0!	0.76	0	0.537	0	#DIV/0!	0.19	0.00
			6	0	0.040	#DIV/0!	0.04	0	0.878	#DIV/0!	0.83	0	1.014	0	#DIV/0!	0.13	0.00
			7	1	0.079	100.00	0.04	0	1.649	0.00	0.86	0	1.835	1	0.00	0.10	0.04
			8	0	0.066	#DIV/0!	0.02	0	2.714	#DIV/0!	0.89	0	2.961	0	#DIV/0!	0.08	0.00
Median				**0.000**	**0.040**	**#DIV/0!**	**0.049**	**0.000**	**0.433**	**#DIV/0!**	**0.757**	**0.000**	**0.537**	**0.000**	**#DIV/0!**	**0.135**	**0.000**
Mode				**0.000**	**0.040**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#DIV/0!**	**#N/A**	**0.000**
Mean				**0.286**	**0.040**	**#DIV/0!**	**0.103**	**0.000**	**0.862**	**#DIV/0!**	**0.744**	**0.000**	**0.970**	**0.286**	**#DIV/0!**	**0.153**	**0.011**

Table A-4: Computational results - #P: 25, #D: 25

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
25	25	25	2	7	0.054	100.00	0.52	0	0.088	0.00	0.32	0	0.105	7	0.00	0.16	0.28
			3	7	0.048	87.50	0.24	1	0.147	0.13	0.51	0	0.195	8	0.00	0.25	0.32
			4	9	0.087	100.00	0.23	0	0.271	0.00	0.49	0	0.376	9	0.00	0.28	0.36
			5	12	0.124	92.31	0.18	1	0.503	0.08	0.56	0	0.681	13	0.00	0.26	0.52
			6	7	0.159	100.00	0.11	0	1.139	0.00	0.67	0	1.462	7	0.00	0.22	0.28
			7	12	0.159	92.31	0.08	1	1.463	0.08	0.67	0	1.957	13	0.00	0.25	0.52
			8	10	0.199	100.00	0.06	0	2.656	0.00	0.74	0	3.326	10	0.00	0.20	0.40
Median				**9.000**	**0.124**	**100.000**	**0.182**	**0.000**	**0.503**	**0.000**	**0.557**	**0.000**	**0.681**	**9.000**	**0.000**	**0.246**	**0.360**
Mode				**7.000**	**0.159**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**7.000**	**0.000**	**#N/A**	**0.280**
Mean				**9.143**	**0.119**	**96.016**	**0.204**	**0.429**	**0.895**	**0.040**	**0.564**	**0.000**	**1.157**	**9.571**	**0.000**	**0.232**	**0.383**
25	25	100	2	0	0.013	#DIV/0!	0.16	0	0.059	#DIV/0!	0.57	0	0.081	0	#DIV/0!	0.27	0.00
			3	0	0.026	#DIV/0!	0.15	0	0.128	#DIV/0!	0.59	0	0.172	0	#DIV/0!	0.26	0.00
			4	1	0.053	100.00	0.13	0	0.328	0.00	0.66	0	0.415	1	0.00	0.21	0.04
			5	1	0.066	100.00	0.08	0	0.656	0.00	0.74	0	0.801	1	0.00	0.18	0.04
			6	1	0.079	100.00	0.05	0	1.298	0.00	0.80	0	1.521	1	0.00	0.15	0.04
			7	0	0.066	#DIV/0!	0.02	0	2.445	#DIV/0!	0.87	0	2.727	0	#DIV/0!	0.10	0.00
			8	0	0.093	#DIV/0!	0.02	0	4.177	#DIV/0!	0.90	0	4.563	0	#DIV/0!	0.08	0.00
Median				**0.000**	**0.066**	**#DIV/0!**	**0.083**	**0.000**	**0.656**	**#DIV/0!**	**0.735**	**0.000**	**0.801**	**0.000**	**#DIV/0!**	**0.182**	**0.000**
Mode				**0.000**	**0.066**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#DIV/0!**	**#N/A**	**0.000**
Mean				**0.429**	**0.057**	**#DIV/0!**	**0.088**	**0.000**	**1.299**	**#DIV/0!**	**0.732**	**0.000**	**1.469**	**0.429**	**#DIV/0!**	**0.180**	**0.017**

Table A-5: Computational results - #P: 50, #D: 50

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
50	50	25	2	21	0.160	100.00	0.36	0	0.340	0.00	0.40	0	0.447	21	0.00	0.24	0.42
			3	27	0.235	96.43	0.22	1	0.686	0.04	0.43	0	1.054	28	0.00	0.35	0.56
			4	29	0.371	96.67	0.16	0	1.263	0.00	0.38	1	2.352	30	0.03	0.46	0.60
			5	26	0.411	81.25	0.07	3	2.925	0.09	0.44	3	5.668	32	0.09	0.48	0.64
			6	28	0.623	90.32	0.07	1	5.265	0.03	0.52	2	8.943	31	0.06	0.41	0.62
			7	35	0.978	94.59	0.08	1	7.227	0.03	0.51	1	12.342	37	0.03	0.41	0.74
			8	27	1.008	81.82	0.04	6	17.691	0.18	0.71	0	23.647	33	0.00	0.25	0.66
Median				**27.000**	**0.411**	**94.595**	**0.079**	**1.000**	**2.925**	**0.032**	**0.443**	**1.000**	**5.668**	**31.000**	**0.027**	**0.411**	**0.620**
Mode				**27.000**	**#N/A**	**#N/A**	**#N/A**	**1.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**#N/A**	**0.000**	**#N/A**	**#N/A**
Mean				**27.571**	**0.541**	**91.583**	**0.143**	**1.714**	**5.057**	**0.053**	**0.483**	**1.000**	**7.779**	**30.286**	**0.031**	**0.373**	**0.606**
50	50	100	2	2	0.093	100.00	0.20	0	0.398	0.00	0.67	0	0.456	2	0.00	0.13	0.04
			3	4	0.159	100.00	0.14	0	0.901	0.00	0.65	0	1.146	4	0.00	0.21	0.08
			4	3	0.247	100.00	0.08	0	2.406	0.00	0.71	0	3.049	3	0.00	0.21	0.06
			5	2	0.319	66.67	0.05	1	5.195	0.33	0.77	0	6.307	3	0.00	0.18	0.06
			6	4	0.517	100.00	0.04	0	10.109	0.00	0.81	0	11.915	4	0.00	0.15	0.08
			7	2	0.557	100.00	0.03	0	18.697	0.00	0.86	0	21.132	2	0.00	0.12	0.04
			8	4	0.742	100.00	0.02	0	30.876	0.00	0.88	0	34.202	4	0.00	0.10	0.08
Median				**3.000**	**0.319**	**100.000**	**0.051**	**0.000**	**5.195**	**0.000**	**0.773**	**0.000**	**6.307**	**3.000**	**0.000**	**0.152**	**0.060**
Mode				**2.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**4.000**	**0.000**	**#N/A**	**0.080**
Mean				**3.000**	**0.376**	**95.238**	**0.081**	**0.143**	**9.797**	**0.048**	**0.763**	**0.000**	**11.172**	**3.143**	**0.000**	**0.156**	**0.063**

Table A-6: Computational results - #P: 50, #D: 55

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
50	55	25	2	27	0.161	100.00	0.34	0	0.344	0.00	0.39	0	0.473	27	0.00	0.27	0.54
			3	23	0.259	85.19	0.17	3	0.875	0.11	0.40	1	1.538	27	0.04	0.43	0.54
			4	30	0.413	93.75	0.16	2	1.578	0.06	0.45	0	2.612	32	0.00	0.40	0.64
			5	31	0.544	96.88	0.11	1	2.948	0.03	0.49	0	4.928	32	0.00	0.40	0.64
			6	26	0.796	83.87	0.07	3	7.020	0.10	0.51	2	12.178	31	0.06	0.42	0.62
			7	29	0.928	85.29	0.05	4	11.232	0.12	0.61	1	16.883	34	0.03	0.33	0.68
			8	31	1.340	83.78	0.04	3	17.188	0.08	0.50	3	31.830	37	0.08	0.46	0.74
Median				**29.000**	**0.544**	**85.294**	**0.110**	**3.000**	**2.948**	**0.081**	**0.488**	**1.000**	**4.928**	**32.000**	**0.029**	**0.402**	**0.640**
Mode				**31.000**	**#N/A**	**#N/A**	**#N/A**	**3.000**	**#N/A**	**#N/A**	**#N/A**	**0.000**	**#N/A**	**27.000**	**0.000**	**#N/A**	**0.540**
Mean				**28.143**	**0.634**	**89.823**	**0.134**	**2.286**	**5.883**	**0.071**	**0.477**	**1.000**	**10.063**	**31.429**	**0.030**	**0.389**	**0.629**
50	55	100	2	3	0.123	100.00	0.22	0	0.488	0.00	0.64	0	0.570	3	0.00	0.14	0.06
			3	2	0.183	100.00	0.12	0	1.172	0.00	0.66	0	1.494	2	0.00	0.22	0.04
			4	2	0.265	100.00	0.08	0	2.714	0.00	0.71	0	3.448	2	0.00	0.21	0.04
			5	2	0.405	100.00	0.05	0	6.302	0.00	0.76	0	7.710	2	0.00	0.18	0.04
			6	4	0.572	100.00	0.04	0	12.431	0.00	0.81	0	14.646	4	0.00	0.15	0.08
			7	2	0.704	100.00	0.03	0	23.463	0.00	0.86	0	26.449	2	0.00	0.11	0.04
			8	2	0.862	100.00	0.02	0	38.786	0.00	0.89	0	42.517	2	0.00	0.09	0.04
Median				**2.000**	**0.405**	**100.000**	**0.052**	**0.000**	**6.302**	**0.000**	**0.765**	**0.000**	**7.710**	**2.000**	**0.000**	**0.151**	**0.040**
Mode				**2.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**2.000**	**0.000**	**#N/A**	**0.040**
Mean				**2.429**	**0.445**	**100.000**	**0.079**	**0.000**	**12.194**	**0.000**	**0.763**	**0.000**	**13.833**	**2.429**	**0.000**	**0.158**	**0.049**

Table A-7: Computational results - #P: 55, #D: 50

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
55	50	25	2	31	0.145	100.00	0.35	0	0.293	0.00	0.35	0	0.415	31	0.00	0.30	0.56
			3	30	0.239	100.00	0.22	0	0.656	0.00	0.39	0	1.065	30	0.00	0.38	0.55
			4	29	0.358	96.67	0.15	1	1.521	0.03	0.49	0	2.393	30	0.00	0.36	0.55
			5	31	0.518	88.57	0.10	2	3.008	0.06	0.46	2	5.435	35	0.06	0.45	0.64
			6	26	0.677	81.25	0.06	5	6.997	0.16	0.58	1	10.962	32	0.03	0.36	0.58
			7	30	0.928	88.24	0.05	3	11.045	0.09	0.59	1	17.055	34	0.03	0.35	0.62
			8	29	1.101	87.88	0.04	3	18.826	0.09	0.66	1	26.688	33	0.03	0.29	0.60
Median				**30.000**	**0.518**	**88.571**	**0.095**	**2.000**	**3.008**	**0.057**	**0.486**	**1.000**	**5.435**	**32.000**	**0.029**	**0.362**	**0.582**
Mode				**31.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**30.000**	**0.000**	**#N/A**	**0.545**
Mean				**29.429**	**0.566**	**91.800**	**0.139**	**2.000**	**6.049**	**0.061**	**0.503**	**0.714**	**9.145**	**32.143**	**0.021**	**0.357**	**0.584**
55	50	100	2	3	0.139	100.00	0.25	0	0.486	0.00	0.62	0	0.556	3	0.00	0.13	0.05
			3	3	0.199	100.00	0.15	0	1.042	0.00	0.64	0	1.318	3	0.00	0.21	0.05
			4	4	0.292	100.00	0.10	0	2.434	0.00	0.70	0	3.063	4	0.00	0.21	0.07
			5	4	0.358	100.00	0.05	0	5.371	0.00	0.76	0	6.601	4	0.00	0.19	0.07
			6	3	0.477	75.00	0.04	1	11.337	0.25	0.81	0	13.348	4	0.00	0.15	0.07
			7	3	0.655	100.00	0.03	0	21.057	0.00	0.85	0	23.924	3	0.00	0.12	0.05
			8	2	0.819	66.67	0.02	0	37.328	0.00	0.89	1	40.936	3	0.33	0.09	0.05
Median				**3.000**	**0.358**	**100.000**	**0.054**	**0.000**	**5.371**	**0.000**	**0.759**	**0.000**	**6.601**	**3.000**	**0.000**	**0.151**	**0.055**
Mode				**3.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**3.000**	**0.000**	**#N/A**	**0.055**
Mean				**3.143**	**0.420**	**91.667**	**0.091**	**0.143**	**11.294**	**0.036**	**0.754**	**0.143**	**12.821**	**3.429**	**0.048**	**0.155**	**0.062**

Table A-8: Computational results - #P: 55, #D: 55

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
55	55	25	2	25	0.173	100.00	0.32	0	0.386	0.00	0.40	0	0.538	25	0.00	0.28	0.45
			3	25	0.243	96.15	0.17	1	0.920	0.04	0.48	0	1.417	26	0.00	0.35	0.47
			4	24	0.428	80.00	0.11	4	2.254	0.13	0.47	2	3.923	30	0.07	0.43	0.55
			5	26	0.649	83.87	0.09	3	4.294	0.10	0.52	2	7.067	31	0.06	0.39	0.56
			6	29	0.913	80.56	0.08	7	8.409	0.19	0.63	0	11.875	36	0.00	0.29	0.65
			7	32	1.097	84.21	0.04	3	12.977	0.08	0.46	3	25.657	38	0.08	0.49	0.69
			8	28	1.300	75.68	0.03	6	24.301	0.16	0.55	3	41.615	37	0.08	0.42	0.67
Median				**26.000**	**0.649**	**83.871**	**0.092**	**3.000**	**4.294**	**0.097**	**0.478**	**2.000**	**7.067**	**31.000**	**0.065**	**0.392**	**0.564**
Mode				**25.000**	**#N/A**	**#N/A**	**#N/A**	**3.000**	**#N/A**	**#N/A**	**#N/A**	**0.000**	**#N/A**	**#N/A**	**0.000**	**#N/A**	**#N/A**
Mean				**27.000**	**0.686**	**85.781**	**0.121**	**3.429**	**7.649**	**0.101**	**0.501**	**1.429**	**13.156**	**31.857**	**0.042**	**0.379**	**0.579**
55	55	100	2	3	0.105	100.00	0.18	0	0.491	0.00	0.67	0	0.578	3	0.00	0.15	0.05
			3	2	0.185	100.00	0.11	0	1.252	0.00	0.64	0	1.673	2	0.00	0.25	0.04
			4	4	0.305	100.00	0.08	0	2.960	0.00	0.70	0	3.782	4	0.00	0.22	0.07
			5	3	0.424	100.00	0.05	0	7.488	0.00	0.80	0	8.822	3	0.00	0.15	0.05
			6	4	0.570	100.00	0.04	0	13.069	0.00	0.82	0	15.241	4	0.00	0.14	0.07
			7	2	0.729	100.00	0.03	0	25.121	0.00	0.86	0	28.504	2	0.00	0.12	0.04
			8	2	0.915	100.00	0.02	0	42.892	0.00	0.90	0	46.847	2	0.00	0.08	0.04
Median				**3.000**	**0.424**	**100.000**	**0.048**	**0.000**	**7.488**	**0.000**	**0.801**	**0.000**	**8.822**	**3.000**	**0.000**	**0.150**	**0.055**
Mode				**2.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**2.000**	**0.000**	**#N/A**	**0.036**
Mean				**2.857**	**0.462**	**100.000**	**0.072**	**0.000**	**13.325**	**0.000**	**0.769**	**0.000**	**15.064**	**2.857**	**0.000**	**0.159**	**0.052**

Table A-9: Computational results - #P: 100, #D: 100

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
100	100	25	2	66	0.689	100.00	0.30	0	1.475	0.00	0.35	0	2.272	66	0.00	0.35	0.66
			3	64	1.352	91.43	0.19	5	3.803	0.07	0.34	1	7.138	70	0.01	0.47	0.70
			4	65	2.665	87.84	0.16	6	8.869	0.08	0.37	3	16.903	74	0.04	0.48	0.74
			5	71	3.216	93.42	0.11	3	15.629	0.04	0.41	2	30.468	76	0.03	0.49	0.76
			6	76	4.266	92.68	0.08	4	26.639	0.05	0.41	2	55.001	82	0.02	0.52	0.82
			7	72	5.874	86.75	0.06	4	53.166	0.05	0.52	7	91.129	83	0.08	0.42	0.83
			8	65	7.333	78.31	0.05	13	122.967	0.16	0.75	5	154.066	83	0.06	0.20	0.83
Median				**66.000**	**3.216**	**91.429**	**0.106**	**4.000**	**15.629**	**0.049**	**0.407**	**2.000**	**30.468**	**76.000**	**0.026**	**0.467**	**0.760**
Mode				**65.000**	**#N/A**	**#N/A**	**#N/A**	**4.000**	**#N/A**	**#N/A**	**#N/A**	**2.000**	**#N/A**	**83.000**	**#N/A**	**#N/A**	**0.830**
Mean				**68.429**	**3.628**	**90.062**	**0.135**	**5.000**	**33.221**	**0.064**	**0.449**	**2.857**	**50.997**	**76.286**	**0.036**	**0.416**	**0.763**
100	100	100	2	9	0.597	100.00	0.17	0	2.958	0.00	0.68	0	3.463	9	0.00	0.15	0.09
			3	9	1.008	100.00	0.11	0	6.857	0.00	0.65	0	9.025	9	0.00	0.24	0.09
			4	7	1.686	87.50	0.07	1	18.777	0.13	0.71	0	24.002	8	0.00	0.22	0.08
			5	7	2.148	100.00	0.04	0	38.879	0.00	0.76	0	48.225	7	0.00	0.19	0.07
			6	7	2.997	100.00	0.03	0	79.174	0.00	0.81	0	94.078	7	0.00	0.16	0.07
			7	8	4.629	88.89	0.03	0	147.503	0.00	0.80	1	179.538	9	0.11	0.18	0.09
			8	8	5.583	88.89	0.02	0	259.179	0.00	0.85	1	299.698	9	0.11	0.14	0.09
Median				**8.000**	**2.148**	**100.000**	**0.045**	**0.000**	**38.879**	**0.000**	**0.762**	**0.000**	**48.225**	**9.000**	**0.000**	**0.178**	**0.090**
Mode				**7.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**9.000**	**0.000**	**#N/A**	**0.090**
Mean				**7.857**	**2.664**	**95.040**	**0.068**	**0.143**	**79.047**	**0.018**	**0.751**	**0.286**	**94.004**	**8.286**	**0.032**	**0.181**	**0.083**

Table A-10: Computational results - #P: 100, #D: 105

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
100	105	25	2	65	0.702	100.00	0.29	0	1.614	0.00	0.37	0	2.434	65	0.00	0.34	0.65
			3	69	1.419	92.00	0.19	5	3.755	0.07	0.31	1	7.595	75	0.01	0.51	0.75
			4	69	2.109	89.61	0.13	6	8.202	0.08	0.37	2	16.275	77	0.03	0.50	0.77
			5	75	3.481	90.36	0.12	4	12.599	0.05	0.31	4	29.417	83	0.05	0.57	0.83
			6	77	4.495	95.06	0.09	3	25.916	0.04	0.42	1	50.933	81	0.01	0.49	0.81
			7	77	6.803	95.06	0.08	1	40.482	0.01	0.41	3	82.185	81	0.04	0.51	0.81
			8	72	8.713	87.80	0.06	8	95.285	0.10	0.56	2	155.974	82	0.02	0.39	0.82
Median				**72.000**	**3.481**	**92.000**	**0.118**	**4.000**	**12.599**	**0.048**	**0.375**	**2.000**	**29.417**	**81.000**	**0.024**	**0.496**	**0.810**
Mode				**69.000**	**#N/A**	**95.062**	**#N/A**	**#N/A**	**#N/A**	**#N/A**	**#N/A**	**1.000**	**#N/A**	**81.000**	**#N/A**	**#N/A**	**0.810**
Mean				**72.000**	**3.960**	**92.843**	**0.136**	**3.857**	**26.836**	**0.049**	**0.393**	**1.857**	**49.259**	**77.714**	**0.023**	**0.471**	**0.777**
100	105	100	2	7	0.530	100.00	0.15	0	2.960	0.00	0.69	0	3.518	7	0.00	0.16	0.07
			3	8	1.163	100.00	0.11	0	8.095	0.00	0.66	0	10.539	8	0.00	0.23	0.08
			4	8	1.591	100.00	0.06	0	19.025	0.00	0.71	0	24.701	8	0.00	0.23	0.08
			5	7	2.477	100.00	0.04	0	45.008	0.00	0.77	0	55.585	7	0.00	0.19	0.07
			6	9	3.885	90.00	0.04	1	87.048	0.10	0.80	0	103.631	10	0.00	0.16	0.10
			7	9	4.429	100.00	0.02	0	165.114	0.00	0.84	0	190.586	9	0.00	0.13	0.09
			8	9	5.729	90.00	0.02	0	267.802	0.00	0.84	1	312.191	10	0.10	0.14	0.10
Median				**8.000**	**2.477**	**100.000**	**0.045**	**0.000**	**45.008**	**0.000**	**0.765**	**0.000**	**55.585**	**8.000**	**0.000**	**0.160**	**0.080**
Mode				**9.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**7.000**	**0.000**	**#N/A**	**0.070**
Mean				**8.143**	**2.829**	**97.143**	**0.064**	**0.143**	**85.007**	**0.014**	**0.758**	**0.143**	**100.107**	**8.429**	**0.014**	**0.178**	**0.084**

Table A-11: Computational results - #P: 105, #D: 100

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
105	100	25	2	72	0.783	100.00	0.36	0	1.533	0.00	0.34	0	2.191	72	0.00	0.30	0.69
			3	71	1.286	97.26	0.17	1	3.569	0.01	0.31	1	7.443	73	0.01	0.52	0.70
			4	73	2.174	93.59	0.14	3	7.769	0.04	0.36	2	15.575	78	0.03	0.50	0.74
			5	71	2.996	87.65	0.09	6	17.480	0.07	0.42	4	34.283	81	0.05	0.49	0.77
			6	71	5.052	88.75	0.06	5	34.141	0.06	0.37	4	78.970	80	0.05	0.57	0.76
			7	72	5.743	83.72	0.05	10	64.226	0.12	0.49	4	119.420	86	0.05	0.46	0.82
			8	71	7.957	84.52	0.04	10	116.079	0.12	0.61	3	177.676	84	0.04	0.35	0.80
Median				**71.000**	**2.996**	**88.750**	**0.087**	**5.000**	**17.480**	**0.063**	**0.368**	**3.000**	**34.283**	**80.000**	**0.036**	**0.490**	**0.762**
Mode				**71.000**	**#N/A**	**#N/A**	**#N/A**	**10.000**	**#N/A**	**#N/A**	**#N/A**	**4.000**	**#N/A**	**#N/A**	**#N/A**	**#N/A**	**#N/A**
Mean				**71.571**	**3.713**	**90.786**	**0.131**	**5.000**	**34.971**	**0.061**	**0.414**	**2.571**	**62.223**	**79.143**	**0.032**	**0.456**	**0.754**
105	100	100	2	6	0.517	100.00	0.15	0	2.890	0.00	0.69	0	3.437	6	0.00	0.16	0.06
			3	9	1.061	100.00	0.11	0	7.430	0.00	0.67	0	9.502	9	0.00	0.22	0.09
			4	8	1.845	88.89	0.07	1	19.482	0.11	0.71	0	24.887	9	0.00	0.22	0.09
			5	9	2.492	100.00	0.05	0	41.418	0.00	0.76	0	51.197	9	0.00	0.19	0.09
			6	7	3.647	77.78	0.03	0	86.882	0.00	0.74	2	113.010	9	0.22	0.23	0.09
			7	9	4.769	100.00	0.03	0	158.795	0.00	0.85	0	180.900	9	0.00	0.12	0.09
			8	8	5.383	88.89	0.02	1	266.936	0.11	0.88	0	296.220	9	0.00	0.10	0.09
Median				**8.000**	**2.492**	**100.000**	**0.049**	**0.000**	**41.418**	**0.000**	**0.737**	**0.000**	**51.197**	**9.000**	**0.000**	**0.191**	**0.086**
Mode				**9.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**9.000**	**0.000**	**#N/A**	**0.086**
Mean				**8.000**	**2.816**	**93.651**	**0.066**	**0.286**	**83.404**	**0.032**	**0.757**	**0.286**	**97.022**	**8.571**	**0.032**	**0.177**	**0.082**

Table A-12: Computational results - #P: 105, #D: 105

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
105	105	25	2	64	0.768	100.00	0.29	0	1.814	0.00	0.39	0	2.657	64	0.00	0.32	0.61
			3	68	1.366	98.55	0.17	0	3.990	0.00	0.32	1	8.122	69	0.01	0.51	0.66
			4	74	2.933	94.87	0.17	3	9.215	0.04	0.36	1	17.220	78	0.01	0.46	0.74
			5	69	3.736	84.15	0.09	9	22.130	0.11	0.46	4	39.737	82	0.05	0.44	0.78
			6	73	5.237	90.12	0.08	5	36.508	0.06	0.46	3	68.170	81	0.04	0.46	0.77
			7	76	6.789	89.41	0.07	8	59.752	0.09	0.51	1	103.093	85	0.01	0.42	0.81
			8	71	8.129	84.52	0.04	7	126.980	0.08	0.61	6	193.686	84	0.07	0.34	0.80
Median				**71.000**	**3.736**	**90.123**	**0.094**	**5.000**	**22.130**	**0.062**	**0.459**	**1.000**	**39.737**	**81.000**	**0.014**	**0.443**	**0.771**
Mode				**#N/A**	**#N/A**	**#N/A**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**1.000**	**#N/A**	**#N/A**	**#N/A**	**#N/A**	**#N/A**
Mean				**70.714**	**4.137**	**91.661**	**0.129**	**4.571**	**37.198**	**0.055**	**0.447**	**2.286**	**61.812**	**77.571**	**0.028**	**0.423**	**0.739**
105	105	100	2	10	0.597	100.00	0.16	0	3.161	0.00	0.68	0	3.790	10	0.00	0.17	0.10
			3	11	1.161	100.00	0.11	0	8.227	0.00	0.66	0	10.784	11	0.00	0.24	0.10
			4	12	1.764	100.00	0.07	0	19.282	0.00	0.70	0	25.127	12	0.00	0.23	0.11
			5	8	2.532	80.00	0.04	2	50.603	0.20	0.78	0	61.306	10	0.00	0.17	0.10
			6	9	4.003	90.00	0.03	0	94.427	0.00	0.75	1	119.974	10	0.10	0.21	0.10
			7	10	5.874	90.91	0.03	0	166.058	0.00	0.80	1	200.853	11	0.09	0.17	0.10
			8	10	7.028	90.91	0.02	1	272.096	0.09	0.86	0	307.166	11	0.00	0.11	0.10
Median				**10.000**	**2.532**	**90.909**	**0.041**	**0.000**	**50.603**	**0.000**	**0.754**	**0.000**	**61.306**	**11.000**	**0.000**	**0.175**	**0.105**
Mode				**10.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**10.000**	**0.000**	**#N/A**	**0.095**
Mean				**10.000**	**3.280**	**93.117**	**0.066**	**0.429**	**87.693**	**0.042**	**0.747**	**0.286**	**104.143**	**10.714**	**0.027**	**0.187**	**0.102**

Table A-13: Computational results - #P: 50, #D: 25

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
50	25	25	2	12	0.080	100.00	0.46	0	0.152	0.00	0.42	0	0.173	12	0.00	0.12	0.24
			3	17	0.133	100.00	0.35	0	0.281	0.00	0.39	0	0.375	17	0.00	0.25	0.34
			4	27	0.225	100.00	0.34	0	0.468	0.00	0.37	0	0.659	27	0.00	0.29	0.54
			5	20	0.225	95.24	1.00	1	1.041	0.05	0.57	0	1.429	21	0.00	0.27	0.42
			6	19	0.305	79.17	0.10	4	2.094	0.17	0.59	1	3.022	24	0.04	0.31	0.48
			7	24	0.358	96.00	0.09	1	3.077	0.04	0.68	0	4.005	25	0.00	0.23	0.50
			8	19	0.371	90.48	0.05	2	5.944	0.10	0.73	0	7.625	21	0.00	0.22	0.42
Median				**19.000**	**0.225**	**96.000**	**0.342**	**1.000**	**1.041**	**0.040**	**0.571**	**0.000**	**1.429**	**21.000**	**0.000**	**0.252**	**0.420**
Mode				**19.000**	**0.225**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**21.000**	**0.000**	**#N/A**	**0.420**
Mean				**19.714**	**0.242**	**94.412**	**0.342**	**1.143**	**1.865**	**0.050**	**0.536**	**0.143**	**2.470**	**21.000**	**0.006**	**0.242**	**0.420**
50	25	100	2	1	0.048	100.00	0.30	0	0.137	0.00	0.54	0	0.163	1	0.00	0.16	0.02
			3	1	0.065	100.00	0.18	0	0.282	0.00	0.62	0	0.353	1	0.00	0.20	0.02
			4	0	0.060	#DIV/0!	0.08	0	0.639	#DIV/0!	0.72	0	0.804	0	#DIV/0!	0.21	0.00
			5	0	0.083	#DIV/0!	1.00	0	1.412	#DIV/0!	0.79	0	1.687	0	#DIV/0!	0.16	0.00
			6	0	0.111	#DIV/0!	0.03	0	2.843	#DIV/0!	0.83	0	3.307	0	#DIV/0!	0.14	0.00
			7	1	0.190	100.00	0.03	0	5.149	0.00	0.86	0	5.751	1	0.00	0.10	0.02
			8	0	0.175	#DIV/0!	0.02	0	8.935	#DIV/0!	0.91	0	9.639	0	#DIV/0!	0.07	0.00
Median				**0.000**	**0.083**	**#DIV/0!**	**0.075**	**0.000**	**1.412**	**#DIV/0!**	**0.787**	**0.000**	**1.687**	**0.000**	**#DIV/0!**	**0.159**	**0.000**
Mode				**0.000**	**#N/A**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**#DIV/0!**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#DIV/0!**	**#N/A**	**0.000**
Mean				**0.429**	**0.105**	**#DIV/0!**	**0.234**	**0.000**	**2.771**	**#DIV/0!**	**0.752**	**0.000**	**3.101**	**0.429**	**#DIV/0!**	**0.150**	**0.009**

Table A-14: Computational results - #P: 25, #D: 50

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
25	50	25	2	8	0.071	100.00	0.28	0	0.199	0.00	0.49	0	0.257	8	0.00	0.23	0.32
			3	11	0.108	100.00	0.20	0	0.370	0.00	0.48	0	0.548	11	0.00	0.33	0.44
			4	12	0.191	92.31	0.15	1	0.854	0.08	0.53	0	1.242	13	0.00	0.31	0.52
			5	14	0.217	100.00	0.09	0	1.385	0.00	0.51	0	2.289	14	0.00	0.39	0.56
			6	16	0.367	94.12	0.10	1	2.427	0.06	0.54	0	3.819	17	0.00	0.36	0.68
			7	14	0.502	82.35	0.06	1	5.457	0.06	0.56	2	8.803	17	0.12	0.38	0.68
			8	11	0.626	68.75	0.04	3	11.711	0.19	0.63	2	17.505	16	0.13	0.33	0.64
Median				**12.000**	**0.217**	**94.118**	**0.096**	**1.000**	**1.385**	**0.059**	**0.534**	**0.000**	**2.289**	**14.000**	**0.000**	**0.331**	**0.560**
Mode				**11.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**17.000**	**0.000**	**#N/A**	**0.680**
Mean				**12.286**	**0.297**	**91.075**	**0.130**	**0.857**	**3.200**	**0.055**	**0.536**	**0.571**	**4.923**	**13.714**	**0.035**	**0.334**	**0.549**
25	50	100	2	2	0.065	100.00	0.25	0	0.224	0.00	0.61	0	0.262	2	0.00	0.15	0.08
			3	1	0.095	100.00	0.15	0	0.511	0.00	0.64	0	0.651	1	0.00	0.22	0.04
			4	0	0.099	0.00	0.05	0	1.229	0.00	0.60	1	1.886	1	1.00	0.35	0.04
			5	1	0.192	100.00	0.06	0	2.757	0.00	0.76	0	3.366	1	0.00	0.18	0.04
			6	2	0.269	100.00	0.04	0	5.286	0.00	0.80	0	6.245	2	0.00	0.15	0.08
			7	2	0.362	100.00	0.03	0	9.768	0.00	0.84	0	11.142	2	0.00	0.12	0.08
			8	2	0.435	66.67	0.02	1	16.781	0.33	0.88	0	18.512	3	0.00	0.09	0.12
Median				**2.000**	**0.192**	**100.000**	**0.052**	**0.000**	**2.757**	**0.000**	**0.762**	**0.000**	**3.366**	**2.000**	**0.000**	**0.154**	**0.080**
Mode				**2.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**2.000**	**0.000**	**#N/A**	**0.080**
Mean				**1.429**	**0.217**	**80.952**	**0.086**	**0.143**	**5.222**	**0.048**	**0.734**	**0.143**	**6.009**	**1.714**	**0.143**	**0.180**	**0.069**

Table A-15: Computational results - #P: 75, #D: 50

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
75	50	25	2	36	0.217	100.00	0.33	0	0.471	0.00	0.39	0	0.650	36	0.00	0.28	0.48
			3	41	0.410	95.35	0.25	2	1.052	0.05	0.40	0	1.611	43	0.00	0.35	0.57
			4	36	0.543	85.71	0.13	5	2.492	0.12	0.45	1	4.307	42	0.02	0.42	0.56
			5	36	0.740	83.72	0.08	5	5.482	0.12	0.53	2	8.954	43	0.05	0.39	0.57
			6	36	1.193	81.82	0.06	4	10.210	0.09	0.47	4	19.308	44	0.09	0.47	0.59
			7	45	1.590	93.75	0.07	2	14.201	0.04	0.56	1	22.558	48	0.02	0.37	0.64
			8	46	2.172	92.00	0.07	4	22.967	0.08	0.62	0	33.320	50	0.00	0.31	0.67
Median				**36.000**	**0.740**	**92.000**	**0.083**	**4.000**	**5.482**	**0.080**	**0.467**	**1.000**	**8.954**	**43.000**	**0.021**	**0.370**	**0.573**
Mode				**36.000**	**#N/A**	**#N/A**	**#N/A**	**2.000**	**#N/A**	**#N/A**	**#N/A**	**0.000**	**#N/A**	**43.000**	**0.000**	**#N/A**	**0.573**
Mean				**39.429**	**0.981**	**90.336**	**0.142**	**3.143**	**8.125**	**0.071**	**0.489**	**1.143**	**12.958**	**43.714**	**0.026**	**0.369**	**0.583**
75	50	100	2	2	0.135	100.00	0.19	0	0.608	0.00	0.67	0	0.707	2	0.00	0.14	0.03
			3	2	0.231	100.00	0.12	0	1.500	0.00	0.67	0	1.895	2	0.00	0.21	0.03
			4	6	0.377	100.00	0.08	0	3.539	0.00	0.70	0	4.505	6	0.00	0.21	0.08
			5	4	0.517	100.00	0.05	0	8.413	0.00	0.78	0	10.126	4	0.00	0.17	0.05
			6	5	0.695	83.33	0.04	1	16.330	0.17	0.82	0	19.124	6	0.00	0.15	0.08
			7	3	0.874	100.00	0.03	0	29.869	0.00	0.86	0	33.740	3	0.00	0.11	0.04
			8	1	1.063	100.00	0.02	0	52.398	0.00	0.90	0	57.264	1	0.00	0.08	0.01
Median				**3.000**	**0.517**	**100.000**	**0.051**	**0.000**	**8.413**	**0.000**	**0.780**	**0.000**	**10.126**	**3.000**	**0.000**	**0.146**	**0.040**
Mode				**2.000**	**#N/A**	**100.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**0.000**	**#N/A**	**2.000**	**0.000**	**#N/A**	**0.027**
Mean				**3.286**	**0.556**	**97.619**	**0.076**	**0.143**	**16.094**	**0.024**	**0.770**	**0.000**	**18.194**	**3.429**	**0.000**	**0.154**	**0.046**

Table A-16: Computational results - #P: 50, #D: 75

#Pt	#Dt	Area	#Stops	#0-con.	CPU Time (sec)	0-con. perc	CPU-time (perc)	#1-con.	CPU Time (sec)	1-con. Perc.	CPU-time (perc)	#2-con.	CPU Time (sec)	Total Routes	2-con. Perc.	CPU-time perc.	Success Rate
50	75	25	2	29	0.245	100.00	0.28	0	0.555	0.00	0.35	0	0.878	29	0.00	0.37	0.58
			3	30	0.373	93.75	0.15	1	1.233	0.03	0.34	1	2.493	32	0.03	0.51	0.64
			4	30	0.651	88.24	0.12	2	2.838	0.06	0.39	2	5.623	34	0.06	0.50	0.68
			5	29	1.085	85.29	0.10	4	6.391	0.12	0.48	1	10.962	34	0.03	0.42	0.68
			6	26	1.272	74.29	0.05	6	15.344	0.17	0.55	3	25.759	35	0.09	0.40	0.70
			7	33	1.669	89.19	0.05	1	17.733	0.03	0.47	3	33.850	37	0.08	0.48	0.74
			8	36	2.143	92.31	0.05	2	25.108	0.05	0.55	1	41.590	39	0.03	0.40	0.78
Median				**30.000**	**1.085**	**89.189**	**0.099**	**2.000**	**6.391**	**0.051**	**0.475**	**1.000**	**10.962**	**34.000**	**0.031**	**0.417**	**0.680**
Mode				**29.000**	**#N/A**	**#N/A**	**#N/A**	**1.000**	**#N/A**	**#N/A**	**#N/A**	**1.000**	**#N/A**	**34.000**	**#N/A**	**#N/A**	**0.680**
Mean				**30.429**	**1.063**	**89.009**	**0.113**	**2.286**	**9.886**	**0.065**	**0.449**	**1.571**	**17.308**	**34.286**	**0.045**	**0.437**	**0.686**
50	75	100	2	2	0.176	100.00	0.17	0	0.883	0.00	0.69	0	1.030	2	0.00	0.14	0.04
			3	2	0.300	100.00	0.11	0	2.160	0.00	0.67	0	2.774	2	0.00	0.22	0.04
			4	4	0.488	100.00	0.07	0	5.157	0.00	0.70	0	6.646	4	0.00	0.22	0.08
			5	1	0.723	100.00	0.05	0	12.158	0.00	0.77	0	14.869	1	0.00	0.18	0.02
			6	1	0.938	33.33	0.03	1	25.598	0.33	0.77	1	32.037	3	0.33	0.20	0.06
			7	2	1.248	100.00	0.02	0	44.987	0.00	0.86	0	50.872	2	0.00	0.12	0.04
			8	2	1.248	100.00	0.02	0	44.987	0.00	0.86	0	50.872	2	0.00	0.12	0.04
Median				**2.000**	**0.723**	**100.000**	**0.049**	**0.000**	**12.158**	**0.000**	**0.769**	**0.000**	**14.869**	**2.000**	**0.000**	**0.182**	**0.040**
Mode				**2.000**	**1.248**	**100.000**	**0.025**	**0.000**	**44.987**	**0.000**	**0.860**	**0.000**	**50.872**	**2.000**	**0.000**	**0.116**	**0.040**
Mean				**2.000**	**0.731**	**90.476**	**0.068**	**0.143**	**19.419**	**0.048**	**0.760**	**0.143**	**22.729**	**2.286**	**0.048**	**0.172**	**0.046**

Table A-17: Aggregated computational results - Area size: 25 sqM

#P	#D	Stat.	#0-co.	CPU Time(sec)	0-con. Perc.	CPU -time	#1-con.	CPU Time(sec)	#1-con.	CPU -time	#2-con.	CPU Time	Total Routes	2-con. perc	CPU -time	Success Rate
20	20	Med	6.00	0.05	100.00	0.14	0.00	0.28	0.00	0.61	0.00	0.38	7.00	0.00	0.20	0.35
20	20	Mod	6.00	0.04	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	7.00	0.00	#N/A	0.35
20	20	Mea	6.71	0.07	94.33	0.22	0.43	0.51	0.06	0.58	0.00	0.64	7.14	0.00	0.20	0.36
20	25	Med	7.00	0.08	100.00	0.30	0.00	0.62	0.00	0.42	0.00	0.72	7.00	0.00	0.27	0.35
20	25	Mod	7.00	0.08	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	7.00	0.00	#N/A	0.35
20	25	Mea	7.14	0.12	90.06	0.24	0.71	0.86	0.08	0.50	0.14	1.16	8.00	0.02	0.26	0.40
25	20	Med	0.00	0.04	#DIV/0!	0.05	0.00	0.43	#DIV/0!	0.76	0.00	0.54	0.00	#DIV/0!	0.13	0.00
25	20	Mod	0.00	0.04	#DIV/0!	#N/A	0.00	#N/A	#DIV/0!	#N/A	0.00	#N/A	0.00	#DIV/0!	#N/A	0.00
25	20	Avg	0.29	0.04	#DIV/0!	0.10	0.00	0.86	#DIV/0!	0.74	0.00	0.97	0.29	#DIV/0!	0.15	0.01
25	25	Med	9.00	0.12	100.00	0.18	0.00	0.50	0.00	0.56	0.00	0.68	9.00	0.00	0.25	0.36
25	25	Mod	7.00	0.16	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	7.00	0.00	#N/A	0.28
25	25	Mea	9.14	0.12	96.02	0.20	0.43	0.90	0.04	0.56	0.00	1.16	9.57	0.00	0.23	0.38
50	50	Med	27.00	0.41	94.59	0.08	1.00	2.93	0.03	0.44	1.00	5.67	31.00	0.03	0.41	0.62
50	50	Mod	27.00	#N/A	#N/A	#N/A	1.00	#N/A	0.00	#N/A	0.00	#N/A	#N/A	0.00	#N/A	#N/A
50	50	Mea	27.57	0.54	91.58	0.14	1.71	5.06	0.05	0.48	1.00	7.78	30.29	0.03	0.37	0.61
50	55	Med	29.00	0.54	85.29	0.11	3.00	2.95	0.08	0.49	1.00	4.93	32.00	0.03	0.40	0.64
50	55	Mod	31.00	#N/A	#N/A	#N/A	3.00	#N/A	#N/A	#N/A	0.00	#N/A	27.00	0.00	#N/A	0.54
50	55	Mea	28.14	0.63	89.82	0.13	2.29	5.88	0.07	0.48	1.00	10.06	31.43	0.03	0.39	0.63
55	50	Med	30.00	0.52	88.57	0.10	2.00	3.01	0.06	0.49	1.00	5.44	32.00	0.03	0.36	0.58
55	50	Mod	31.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	30.00	0.00	#N/A	0.55
55	50	Mea	29.43	0.57	91.80	0.14	2.00	6.05	0.06	0.50	0.71	9.14	32.14	0.02	0.36	0.58
55	55	Med	26.00	0.65	83.87	0.09	3.00	4.29	0.10	0.48	2.00	7.07	31.00	0.06	0.39	0.56
55	55	Mod	25.00	#N/A	#N/A	#N/A	3.00	#N/A	#N/A	#N/A	0.00	#N/A	#N/A	0.00	#N/A	#N/A
55	55	Mea	27.00	0.69	85.78	0.12	3.43	7.65	0.10	0.50	1.43	13.16	31.86	0.04	0.38	0.58
100	100	Med	66.00	3.22	91.43	0.11	4.00	15.63	0.05	0.41	2.00	30.47	76.00	0.03	0.47	0.76
100	100	Mod	65.00	#N/A	#N/A	#N/A	4.00	#N/A	#N/A	#N/A	2.00	#N/A	83.00	#N/A	#N/A	0.83
100	100	Mea	68.43	3.63	90.06	0.14	5.00	33.22	0.06	0.45	2.86	51.00	76.29	0.04	0.42	0.76
100	105	Med	72.00	3.48	92.00	0.12	4.00	12.60	0.05	0.37	2.00	29.42	81.00	0.02	0.50	0.81
100	105	Mod	69.00	#N/A	95.06	#N/A	#N/A	#N/A	#N/A	#N/A	1.00	#N/A	81.00	#N/A	#N/A	0.81
100	105	Mea	72.00	3.96	92.84	0.14	3.86	26.84	0.05	0.39	1.86	49.26	77.71	0.02	0.47	0.78
105	100	Med	71.00	3.00	88.75	0.09	5.00	17.48	0.06	0.37	3.00	34.28	80.00	0.04	0.49	0.76
105	100	Mod	71.00	#N/A	#N/A	#N/A	10.00	#N/A	#N/A	#N/A	4.00	#N/A	#N/A	#N/A	#N/A	#N/A
105	100	Mea	71.57	3.71	90.79	0.13	5.00	34.97	0.06	0.41	2.57	62.22	79.14	0.03	0.46	0.75
105	105	Med	71.00	3.74	90.12	0.09	5.00	22.13	0.06	0.46	1.00	39.74	81.00	0.01	0.44	0.77
105	105	Mod	#N/A	#N/A	#N/A	#N/A	0.00	#N/A	0.00	#N/A	1.00	#N/A	#N/A	#N/A	#N/A	#N/A
105	105	Mea	70.71	4.14	91.66	0.13	4.57	37.20	0.06	0.45	2.29	61.81	77.57	0.03	0.42	0.74
50	25	Med	19.00	0.23	96.00	0.34	1.00	1.04	0.04	0.57	0.00	1.43	21.00	0.00	0.25	0.42
50	25	Mod	19.00	0.23	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	21.00	0.00	#N/A	0.42
50	25	Mea	19.71	0.24	94.41	0.34	1.14	1.87	0.05	0.54	0.14	2.47	21.00	0.01	0.24	0.42
25	50	Med	12.00	0.22	94.12	0.10	1.00	1.39	0.06	0.53	0.00	2.29	14.00	0.00	0.33	0.56
25	50	Mod	11.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	17.00	0.00	#N/A	0.68
25	50	Mea	12.29	0.30	91.08	0.13	0.86	3.20	0.05	0.54	0.57	4.92	13.71	0.03	0.33	0.55
75	50	Med	36.00	0.74	92.00	0.08	4.00	5.48	0.08	0.47	1.00	8.95	43.00	0.02	0.37	0.57
75	50	Mod	36.00	#N/A	#N/A	#N/A	2.00	#N/A	#N/A	#N/A	0.00	#N/A	43.00	0.00	#N/A	0.57
75	50	Mea	39.43	0.98	90.34	0.14	3.14	8.12	0.07	0.49	1.14	12.96	43.71	0.03	0.37	0.58
50	75	Med	30.00	1.09	89.19	0.10	2.00	6.39	0.05	0.47	1.00	10.96	34.00	0.03	0.42	0.68
50	75	Mod	29.00	#N/A	#N/A	#N/A	1.00	#N/A	#N/A	#N/A	1.00	#N/A	34.00	#N/A	#N/A	0.68
50	75	Mea	30.43	1.06	89.01	0.11	2.29	9.89	0.07	0.45	1.57	17.31	34.29	0.04	0.44	0.69

Table A-18: Aggregated computational results - Area size: 100 sqM

#P	#D	Stat.	#0-co.	CPU Time(sec)	0-con. Perc.	CPU -time	#1-con.	CPU Time(sec)	#1-con.	CPU -time	#2-con.	CPU Time	Total Routes	2-con. perc	CPU -time	Success Rate
20	20	Median	0.00	0.03	#DIV/0!	0.06	0.00	0.35	#DIV/0!	0.76	0.00	0.43	0.00	#DIV/0!	0.13	0.00
20	20	Mode	0.00	0.03	#DIV/0!	#N/A	0.00	#N/A	#DIV/0!	#N/A	0.00	#N/A	0.00	#DIV/0!	#N/A	0.00
20	20	Mean	0.14	0.03	#DIV/0!	0.11	0.00	0.68	#DIV/0!	0.74	0.00	0.76	0.14	#DIV/0!	0.15	0.01
20	25	Median	0.00	0.04	#DIV/0!	0.06	0.00	0.54	#DIV/0!	0.77	0.00	0.65	0.00	#DIV/0!	0.17	0.00
20	25	Mode	0.00	0.03	#DIV/0!	#N/A	0.00	#N/A	#DIV/0!	#N/A	0.00	#N/A	0.00	#DIV/0!	#N/A	0.00
20	25	Average	0.14	0.05	#DIV/0!	0.11	0.00	1.07	#DIV/0!	0.73	0.00	1.20	0.14	#DIV/0!	0.16	0.01
25	20	Median	9.00	0.09	100.00	0.19	0.00	0.35	0.00	0.54	0.00	0.49	9.00	0.00	0.25	0.36
25	20	Mode	8.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	8.00	0.00	#N/A	0.32
25	20	Mean	9.00	0.09	98.90	0.22	0.14	0.65	0.01	0.54	0.00	0.85	9.14	0.00	0.24	0.37
25	25	Median	0.00	0.07	#DIV/0!	0.08	0.00	0.66	#DIV/0!	0.74	0.00	0.80	0.00	#DIV/0!	0.18	0.00
25	25	Mode	0.00	0.07	#DIV/0!	#N/A	0.00	#N/A	#DIV/0!	#N/A	0.00	#N/A	0.00	#DIV/0!	#N/A	0.00
25	25	Mean	0.43	0.06	#DIV/0!	0.09	0.00	1.30	#DIV/0!	0.73	0.00	1.47	0.43	#DIV/0!	0.18	0.02
50	50	Median	3.00	0.32	100.00	0.05	0.00	5.20	0.00	0.77	0.00	6.31	3.00	0.00	0.15	0.06
50	50	Mode	2.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	4.00	0.00	#N/A	0.08
50	50	Mean	3.00	0.38	95.24	0.08	0.14	9.80	0.05	0.76	0.00	11.17	3.14	0.00	0.16	0.06
50	55	Median	29.00	0.54	85.29	0.11	3.00	2.95	0.08	0.49	1.00	4.93	32.00	0.03	0.40	0.64
50	55	Mode	2.00	0.40	100.00	0.05	0.00	6.30	0.00	0.76	0.00	7.71	2.00	0.00	0.15	0.04
50	55	Mean	2.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	2.00	0.00	#N/A	0.04
55	50	Median	3.00	0.36	100.00	0.05	0.00	5.37	0.00	0.76	0.00	6.60	3.00	0.00	0.15	0.05
55	50	Mode	3.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	3.00	0.00	#N/A	0.05
55	50	Mean	3.14	0.42	91.67	0.09	0.14	11.29	0.04	0.75	0.14	12.82	3.43	0.05	0.16	0.06
55	55	Median	3.00	0.42	100.00	0.05	0.00	7.49	0.00	0.80	0.00	8.82	3.00	0.00	0.15	0.05
55	55	Mode	2.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	2.00	0.00	#N/A	0.04
55	55	Mean	2.86	0.46	100.00	0.07	0.00	13.32	0.00	0.77	0.00	15.06	2.86	0.00	0.16	0.05
100	100	Median	8.00	2.15	100.00	0.04	0.00	38.88	0.00	0.76	0.00	48.23	9.00	0.00	0.18	0.09
100	100	Mode	7.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	9.00	0.00	#N/A	0.09
100	100	Mean	7.86	2.66	95.04	0.07	0.14	79.05	0.02	0.75	0.29	94.00	8.29	0.03	0.18	0.08
100	105	Median	8.00	2.48	100.00	0.04	0.00	45.01	0.00	0.77	0.00	55.59	8.00	0.00	0.16	0.08
100	105	Mode	9.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	7.00	0.00	#N/A	0.07
100	105	Mean	8.14	2.83	97.14	0.06	0.14	85.01	0.01	0.76	0.14	100.11	8.43	0.01	0.18	0.08
105	100	Median	8.00	2.49	100.00	0.05	0.00	41.42	0.00	0.74	0.00	51.20	9.00	0.00	0.19	0.09
105	100	Mode	9.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	9.00	0.00	#N/A	0.09
105	100	Mean	8.00	2.82	93.65	0.07	0.29	83.40	0.03	0.76	0.29	97.02	8.57	0.03	0.18	0.08
105	105	Median	10.00	2.53	90.91	0.04	0.00	50.60	0.00	0.75	0.00	61.31	11.00	0.00	0.17	0.10
105	105	Mode	10.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	10.00	0.00	#N/A	0.10
105	105	Mean	10.00	3.28	93.12	0.07	0.43	87.69	0.04	0.75	0.29	104.14	10.71	0.03	0.19	0.10
50	25	Median	0.00	0.10	#DIV/0!	0.06	0.00	1.88	#DIV/0!	0.69	0.00	2.60	0.00	#DIV/0!	0.27	0.00
50	25	Mode	0.00	#N/A	#DIV/0!	#N/A	0.00	#N/A	#DIV/0!	#N/A	0.00	#N/A	0.00	#DIV/0!	#N/A	0.00
50	25	Mean	0.43	0.12	#DIV/0!	0.21	0.00	3.69	#DIV/0!	0.66	0.00	4.77	0.43	#DIV/0!	0.26	0.01
25	50	Median	2.00	0.23	100.00	0.04	0.00	3.68	0.00	0.67	0.00	5.18	2.00	0.00	0.27	0.08
25	50	Mode	2.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	2.00	0.00	#N/A	0.08
25	50	Mean	1.43	0.25	80.95	0.07	0.14	6.96	0.05	0.64	0.14	9.24	1.71	0.14	0.29	0.07
75	50	Median	3.00	0.61	100.00	0.04	0.00	11.22	0.00	0.68	0.00	15.58	3.00	0.00	0.26	0.04
75	50	Mode	2.00	#N/A	100.00	#N/A	0.00	#N/A	0.00	#N/A	0.00	#N/A	2.00	0.00	#N/A	0.03
75	50	Mean	3.29	0.65	97.62	0.06	0.14	21.46	0.02	0.68	0.00	27.99	3.43	0.00	0.27	0.05
50	75	Median	2.00	0.85	100.00	0.04	0.00	16.21	0.00	0.67	0.00	22.88	2.00	0.00	0.29	0.04
50	75	Mode	2.00	1.47	100.00	0.02	0.00	59.98	0.00	0.75	0.00	78.26	2.00	0.00	0.23	0.04
50	75	Mean	2.00	0.86	90.48	0.05	0.14	25.89	0.05	0.67	0.14	34.97	2.29	0.05	0.28	0.05

Table A-19: Aggregated computational results - Area size: 25 sqM (Median)

#P	#D	#0-con.	CPU Time (sec)	0-con. Perc.	CPU Time perc	#1-con.	CPU Time (sec)	#1-con. Perc	CPU time perc	#2-con.	CPU Time (sec)	Total routes	2-con. perc	CPU time perc.	Success Rate
20	20	6	0.05	100.0	0.14	0.00	0.28	0.00	0.61	0.00	0.38	6.00	0.00	0.20	0.35
20	25	7	0.08	100.0	0.30	0.00	0.62	0.00	0.42	0.00	0.72	7.00	0.00	0.27	0.35
25	20	9	0.09	100.0	0.19	0.00	0.35	0.00	0.54	0.00	0.49	9.00	0.00	0.25	0.36
25	25	9	0.12	100.0	0.18	0.00	0.50	0.00	0.56	0.00	0.68	9.00	0.00	0.25	0.36
50	50	27	0.41	94.59	0.08	1.00	2.93	0.03	0.44	1.00	5.67	31.00	0.03	0.41	0.62
50	55	29	0.54	85.29	0.11	3.00	2.95	0.08	0.49	1.00	4.93	32.00	0.03	0.40	0.64
55	50	30	0.52	88.57	0.10	2.00	3.01	0.06	0.49	1.00	5.44	32.00	0.03	0.36	0.58
55	55	26	0.65	83.87	0.09	3.00	4.29	0.10	0.48	2.00	7.07	31.00	0.06	0.39	0.56
100	100	66	3.22	91.43	0.11	4.00	15.63	0.05	0.41	2.00	30.47	76.00	0.03	0.47	0.76
100	105	72	3.48	92.00	0.12	4.00	12.60	0.05	0.37	2.00	29.42	81.00	0.02	0.50	0.81
105	100	71	3.00	88.75	0.09	5.00	17.48	0.06	0.37	3.00	34.28	80.00	0.04	0.49	0.76
105	105	71	3.74	90.12	0.09	5.00	22.13	0.06	0.46	1.00	39.74	81.00	0.01	0.44	0.77
50	25	19	0.23	96.00	0.34	1.00	1.04	0.04	0.57	0.00	1.43	21.00	0.00	0.25	0.42
25	50	12	0.22	94.12	0.10	1.00	1.39	0.06	0.53	0.00	2.29	14.00	0.00	0.33	0.56
75	50	36	0.74	92.00	0.08	4.00	5.48	0.08	0.47	1.00	8.95	43.00	0.02	0.37	0.57
50	75	30	1.09	89.19	0.10	2.00	6.39	0.05	0.47	1.00	10.96	34.00	0.03	0.42	0.68

Table A-20: Aggregated computational results - Area size: 25 sqM (Mean)

#P	#D	#0-con.	CPU Time (sec)	0-con. Perc.	CPU Time perc	#1-con.	CPU Time (sec)	#1-con. Perc	CPU time perc	#2-con.	CPU Time (sec)	Total routes	2-con. perc	CPU time perc.	Success Rate
20	20	6.71	0.07	94.33	0.22	0.43	0.51	0.06	0.58	0.00	0.64	7.14	0.00	0.20	0.36
20	25	7.14	0.12	90.06	0.24	0.71	0.86	0.08	0.50	0.14	1.16	8.00	0.02	0.26	0.40
25	20	9.00	0.09	98.90	0.22	0.14	0.65	0.01	0.54	0.00	0.85	9.14	0.00	0.24	0.37
25	25	9.14	0.12	96.02	0.20	0.43	0.90	0.04	0.56	0.00	1.16	9.57	0.00	0.23	0.38
50	50	27.5	0.54	91.58	0.14	1.71	5.06	0.05	0.48	1.00	7.78	30.29	0.03	0.37	0.61
50	55	28.1	0.63	89.82	0.13	2.29	5.88	0.07	0.48	1.00	10.06	31.43	0.03	0.39	0.63
55	50	29.4	0.57	91.80	0.14	2.00	6.05	0.06	0.50	0.71	9.14	32.14	0.02	0.36	0.58
55	55	27.0	0.69	85.78	0.12	3.43	7.65	0.10	0.50	1.43	13.16	31.86	0.04	0.38	0.58
100	100	68.4	3.63	90.06	0.14	5.00	33.22	0.06	0.45	2.86	51.00	76.29	0.04	0.42	0.76
100	105	72.0	3.96	92.84	0.14	3.86	26.84	0.05	0.39	1.86	49.26	77.71	0.02	0.47	0.78
105	100	71.5	3.71	90.79	0.13	5.00	34.97	0.06	0.41	2.57	62.22	79.14	0.03	0.46	0.75
105	105	70.7	4.14	91.66	0.13	4.57	37.20	0.06	0.45	2.29	61.81	77.57	0.03	0.42	0.74
50	25	19.7	0.24	94.41	0.34	1.14	1.87	0.05	0.54	0.14	2.47	21.00	0.01	0.24	0.42
25	50	12.0	0.22	94.12	0.10	1.00	1.39	0.06	0.53	0.00	2.29	14.00	0.00	0.33	0.56
75	50	36.0	0.74	92.00	0.08	4.00	5.48	0.08	0.47	1.00	8.95	43.00	0.02	0.37	0.57
50	75	30.0	1.09	89.19	0.10	2.00	6.39	0.05	0.47	1.00	10.96	34.00	0.03	0.42	0.68

Table A-21: Aggregated computational results - Area size: 100 sqM (Median)

#P	#D	#0-con.	CPU Time (sec)	0-con. Perc.	CPU Time perc	#1-con.	CPU Time (sec)	#1-con. Perc	CPU time perc	#2-con	CPU Time (sec)	Total routes	2-con. perc	CPU time perc.	Succ ess Rate
20	20	0	0.03	#DIV/	0.06	0.00	0.35	#DIV	0.76	0.0	0.43	0.00	#DIV/	0.13	0.00
20	25	0	0.04	#DIV/	0.06	0.00	0.54	#DIV	0.77	0.0	0.65	0.00	#DIV/	0.17	0.00
25	20	9	0.09	100.0	0.19	0.00	0.35	0.00	0.54	0.0	0.49	9.00	0.00	0.25	0.36
25	25	0	0.07	#DIV/	0.08	0.00	0.66	#DIV	0.74	0.0	0.80	0.00	#DIV/	0.18	0.00
50	50	3	0.32	100.0	0.05	0.00	5.20	0.00	0.77	0.0	6.31	3.00	0.00	0.15	0.06
50	55	29	0.54	85.29	0.11	3.00	2.95	0.08	0.49	1.0	4.93	32.00	0.03	0.40	0.64
55	50	3	0.36	100.0	0.05	0.00	5.37	0.00	0.76	0.0	6.60	3.00	0.00	0.15	0.05
55	55	3	0.42	100.0	0.05	0.00	7.49	0.00	0.80	0.0	8.82	3.00	0.00	0.15	0.05
100	100	8	2.15	100.0	0.04	0.00	38.88	0.00	0.76	0.0	48.23	9.00	0.00	0.18	0.09
100	105	8	2.48	100.0	0.04	0.00	45.01	0.00	0.77	0.0	55.59	8.00	0.00	0.16	0.08
105	100	8	2.49	100.0	0.05	0.00	41.42	0.00	0.74	0.0	51.20	9.00	0.00	0.19	0.09
105	105	10	2.53	90.91	0.04	0.00	50.60	0.00	0.75	0.0	61.31	11.00	0.00	0.17	0.10
50	25	0	0.08	#DIV/	0.08	0.00	1.41	#DIV	0.79	0.0	1.69	0.00	#DIV/	0.16	0.00
25	50	2	0.19	100.0	0.05	0.00	2.76	0.00	0.76	0.0	3.37	2.00	0.00	0.15	0.08
75	50	3	0.52	100.0	0.05	0.00	8.41	0.00	0.78	0.0	10.13	3.00	0.00	0.15	0.04
50	75	2	0.72	100.0	0.05	0.00	12.16	0.00	0.77	0.0	14.87	2.00	0.00	0.18	0.04

Table A-22: Aggregated computational results - Area size: 100 sqM (Mean)

#P	#D	#0-con.	CPU Time (sec)	0-con. Perc.	CPU Time perc	#1-con.	CPU Time (sec)	#1-con. Perc	CPU time perc	#2-con	CPU Time (sec)	Tota l rout es	2-con. perc	CPU time perc.	Succ ess Rate
20	20	0.14	0.03	#DIV/	0.11	0.00	0.68	#DIV	0.74	0.0	0.76	0.14	#DIV/	0.15	0.01
20	25	0.14	0.05	#DIV/	0.11	0.00	1.07	#DIV	0.73	0.0	1.20	0.14	#DIV/	0.16	0.01
25	20	9.00	0.09	98.90	0.22	0.14	0.65	0.01	0.54	0.0	0.85	9.14	0.00	0.24	0.37
25	25	0.43	0.06	#DIV/	0.09	0.00	1.30	#DIV	0.73	0.0	1.47	0.43	#DIV/	0.18	0.02
50	50	3.00	0.38	95.24	0.08	0.14	9.80	0.05	0.76	0.0	11.17	3.14	0.00	0.16	0.06
50	55	2.00	#N/A	100.0	#N/A	0.00	#N/A	0.00	#N/	0.0	#N/A	2.00	0.00	#N/	0.04
55	50	3.14	0.42	91.67	0.09	0.14	11.29	0.04	0.75	0.14	12.82	3.43	0.05	0.16	0.06
55	55	2.86	0.46	100.0	0.07	0.00	13.32	0.00	0.77	0.0	15.06	2.86	0.00	0.16	0.05
100	100	7.86	2.66	95.04	0.07	0.14	79.05	0.02	0.75	0.29	94.00	8.29	0.03	0.18	0.08
100	105	8.14	2.83	97.14	0.06	0.14	85.01	0.01	0.76	0.1	100.1	8.43	0.01	0.18	0.08
105	100	8.00	2.82	93.65	0.07	0.29	83.40	0.03	0.76	0.29	97.02	8.57	0.03	0.18	0.08
105	105	10.0	3.28	93.12	0.07	0.43	87.69	0.04	0.75	0.29	104.1	10.7	0.03	0.19	0.10
50	25	0.43	0.10	#DIV/	0.23	0.00	2.77	#DIV	0.75	0.0	3.10	0.43	#DIV/	0.15	0.01
25	50	1.43	0.22	80.95	0.09	0.14	5.22	0.05	0.73	0.14	6.01	1.71	0.14	0.18	0.07
75	50	3.29	0.56	97.62	0.08	0.14	16.09	0.02	0.77	0.0	18.19	3.43	0.00	0.15	0.05
50	75	2.00	0.73	90.48	0.07	0.14	19.42	0.05	0.76	0.1	22.73	2.29	0.05	0.17	0.05

Appendix B: Regression Analysis Results

B.1. Regression analysis (Rate of Success vs #D)

To examine the relationship between the rate of success for the rideshare system and the number of drivers participating in the program, a linear regression analysis is conducted where the rate of success is the dependent variable and the number of drivers is the independent variable. The data set includes 16 observations extracted from the 224 numerical examples. To obtain the 16 observations, the mean Rate of Success for each given #D is calculated by averaging over #P, #Stops and Area Size. Table B-1 shows the summary output for the regression analysis. R square statistic is a measure of the extent to which the total variation of the dependent variable is explained by the regression. It is obviously important if one wishes to use the model for predictive or forecasting purposes. The high value of R square (R Square: 0.928, Adjusted R Square: 0.922) and the low value of Standard Error (0.045) suggest that the regression model explains the variation in the Rate of Success well.

Table B-1: Summary output for the regression analysis of Rate of Success versus #D

Regression Statistics	
Multiple R	0.963
R Square	0.928
Adjusted R Square	0.922
Standard Error	0.045
Observations	16.00

The high value of *t*-statistics as well as the low value of *p*-value for the coefficients of the model also suggest that Intercept, i.e., 0.287 and coefficient of number of drivers' variable (#D), i.e., 0.005, are statistically significant for the linear model. Table B-2 shows the summary ANOVA analysis for the regression analysis.

Table B-2: Summary ANOVA analysis for the regression analysis of Rate of Success versus #D

	Df	*SS*	*MS*	*F*	*Significance F*
Regression	1.000	0.369	0.369	179.31	0.000
Residual	14.000	0.029	0.002		
Total	15.000	0.398			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.287	0.024	11.87	0.000	0.235	0.339
#D	0.005	0.000	13.39	0.000	0.004	0.006

Therefore, it is concluded that rate of success in rideshare program significantly increases as the number of participating drivers increases.

B.2. Regression analysis (Rate of Success vs. #P)

To examine the relationship between the rate of success for the rideshare system and the number of riders participating in the program, a linear regression analysis is conducted where the rate of success is the dependent variable and the number of riders is the independent variable. The data set includes 16 observations extracted from the 224 numerical examples. To obtain the 16 observations, the mean Rate of Success for each given #P is calculated by averaging over #D, #Stops and Area Size. Table B-3 shows the summary output for the regression analysis. The high value of R Square (R Square: 0.767 and Adjusted R Square: 0.751) and the low value of Standard Error (0.081) suggest that the regression model explains the variation in the Rate of Success well.

Table B-3: Summary output for the regression analysis of Rate of Success versus #P

Regression Statistics	
Multiple R	0.876
R Square	0.767
Adjusted R Square	0.751
Standard Error	0.081
Observations	16.00

The high value of *t*-statistics as well as the low value of *p*-value for the coefficients of the model also suggest that Intercept, i.e., 0.313 and coefficient of number of riders' variable (#P), i.e., 0.005, are statistically significant for the linear model. Table B-4 shows the summary ANOVA analysis for the regression analysis.

Table B-4: Summary ANOVA analysis for the regression analysis of Rate of Success versus #P

	df	*SS*	*MS*	*F*	*Significance F*	
Regression	1.000	0.306	0.306	46.210	0.000	
Residual	14.000	0.093	0.007			
Total	15.000	0.398				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.313	0.043	7.224	0.000	0.220	0.406
#P	0.005	0.001	6.798	0.000	0.003	0.006

Therefore, it is concluded that rate of success in rideshare program significantly increases as the number of participating riders increases.

B.3. Multiple Regression Analysis (Predictors: Constant, #D, #P and Dependent Variable: #0-connection routes; Dense network, Area size: 25 sqM)

To examine the relationship between the median number of zero connection routes and number of drivers and number of riders participating in the program in a dense network with area size 25 square Miles, a multiple linear regression analysis is conducted where the median number of zero connection routes is the dependent variable and number of drivers (#D) and number of riders (#P) are the independent variables. The data set includes 16 observations extracted from the 224 numerical examples. To obtain the 16 observations, the median #0-connection routes for each given combination of #P and #D at the area size of 25 Square Miles is calculated by averaging over #Stops. Table B-5 shows the descriptive statistics for the multiple linear regression analysis.

Table B-5: Descriptive statistics for the multiple linear regression analysis of #0-connection routes versus #P, #D; Dense network, area size: 25 sqM

Variable	Mean	Std. Deviation	N
#0-connection routes	32.5000	24.24321	16
#P	56.8750	31.24500	16
#D	56.8750	31.24500	16

The correlation analysis of the model suggests there are high correlations between the Median number of Zero connection routes and number of drivers and riders. Pearson correlation coefficient between #0-connection routes and #P is .984 and the correlation coefficient between #0-connection routes and #D is 0.954. Summary output for the multiple linear regression model is presented in Table B-6. The high value of R square (R Square: 0.988, Adjusted R Square: 0.986) and the low value of Standard Error (2.875) suggest that the regression model explains the variation in the Rate of Success well.

Table B-6: Summary output for the multiple linear regression analysis of #0-connection routes versus #P, #D; Dense network, area size: 25 sqM

Model		R	R Square	Adjusted R Square	Std. Error of the Estimate
dimension0	1	.994	.988	.986	2.87426

The high values of t-statistics as well as the low value of p-value for the coefficients of the model also suggest that variable coefficients are statistically significant for the multiple linear regression model. Table B-7 shows the summary ANOVA analysis for the regression analysis. Considering the unstandardized coefficients (B's) of dependent variables, the regression analysis results in the following multiple linear regression model:

$$Y= -12.278 + 0.520\ X1 + .267\ X2 \qquad \text{(B-1)}$$

where median #0-connection routes is denoted by Y, and X1 and X2 are representing #P and #D in model. The standardized coefficients (Beta) for the independent variables suggest that:

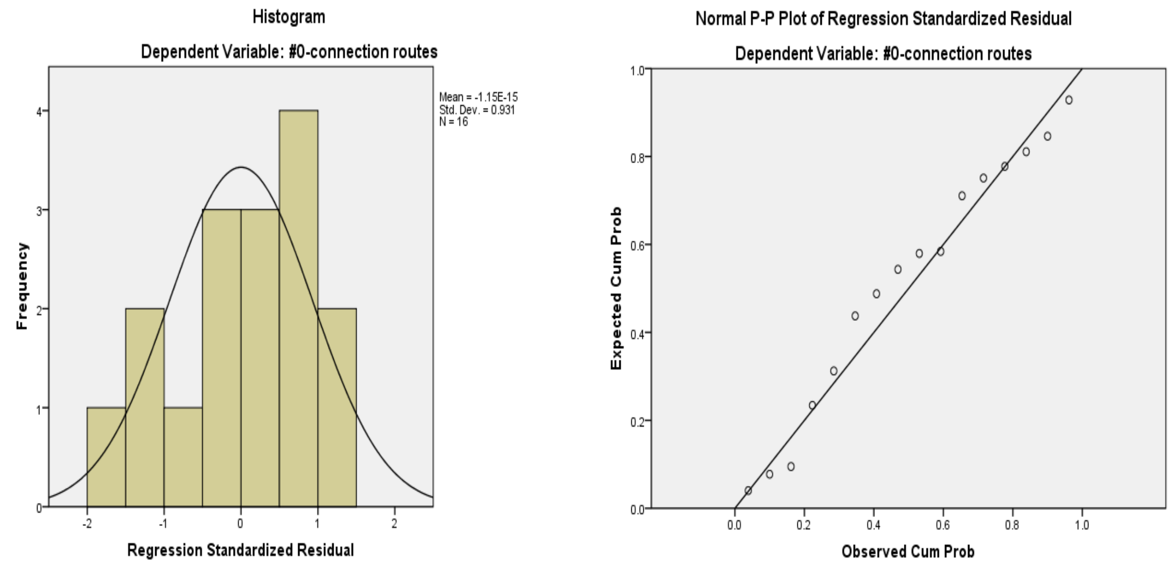
- A one S.D. change in #P produces a predicted change of 0.67 S.D.'s in the median number of zero connection routes in the dense and small network of area size 25 square Miles, i.e., number of zero connection routes significantly increases as the number of participating riders increases when other input parameters remain unchanged.
- a one S.D. change in #D produces a predicted change of .344 S.D.'s the median number of zero connection routes in the dense and small network of area size 25 square Miles, i.e., number of zero connection routes significantly increases as the number of participating drivers increases when other input parameters remain unchanged.
- #P (number of participating riders) is more important than #D (number of participating drivers) in determining level of median number of zero connection routes in the dense and small network of area size 25 square Miles.

Table B-7: Summary ANOVA analysis for the regression analysis of #0-connection routes versus #P, #D; Dense network, area size: 25 sqM

	Sum of Squares	Df	Mean Square	F	Sig.
Regression	8708.603	2	4354.301	527.069	.000[a]
Residual	107.397	13	8.261		
Total	8816.000	15			

Model		Unstandardized Coefficients		Standardized Coefficients			Correlations		
		B	Std. Error	Beta	t	Sig.	Zero-order	Partial	Part
1	(Constant)	-12.278	1.558		7.880	.000			
	#P	.520	.057	.670	9.101	.000	.984	.930	.279
	#D	.267	.057	.344	4.677	.000	.954	.792	.143

To assess the normality of the residuals, the P-P plot and histogram of residuals are examined. Figure (B1-a) is a histogram of the residuals with a normal curve superimposed. The residuals look close to normal. Figure (B1-b) is also a plot of the residuals versus predicted median number of zero connection routes. The pattern shown here indicates no problems with the assumption that the residuals are normally distributed at each level of median number of zero connection routes and constant in variance across levels of median number of zero connection routes.



(a) Histogram of Regression Standardized

(b) Normal P-P Plot of Regression Standardized

Figure B1: Residual analysis for number of zero connection routes versus #P, #D at the area size 25 sqM.

B.4. Multiple Regression Analysis (Predictors: (Constant), #Stops, #D, #P and Dependent Variable: Rate of success; area size 25 sqM)

To examine the relationship between the mean rate of success and the number of drivers, the number of riders and the number of points to be visited by each driver in a small dense network with the area size of 25 square Miles, a multiple linear regression analysis is conducted where the mean rate of success (Rate of Success) is the dependent variable and the number of drivers (#D), the number of riders (#P), and the number of points to be visited by each driver (#Stops) are the independent variables. The data set includes 112 observations extracted from the 224 numerical examples. The 112 observations are associated with the Rate of Success for each given combination of #P, #D and #Stops at the area size of 25 Square Miles. Table B-8 shows the descriptive statistics for the multiple linear regression analysis.

Table B-8: Descriptive statistics for the multiple linear regression analysis of Rate of Success versus #P, #D, #Stops; area size: 25 sqM

Variable	Mean	Std. Deviation	N
Rate of success	.5733	.16791	112
#P	56.8750	30.38881	112
#D	56.8750	30.38881	112
Area	25.0000	.00000	112
#Stops	5.0000	2.00899	112

The correlation analysis of the model suggests there are high correlations between the mean rate of success and the number of drivers and riders and a relatively important correlation between the mean rate of success and the number of points to be visited by each driver. Pearson correlation coefficient between Rate of success and #P is .757, the correlation coefficient between Rate of success and #D is 0.835, and the correlation coefficient between Rate of success and #Stops is .339. Summary output for the multiple linear regression model is presented in Table B-9 The high value of R square (R Square: 0.813, Adjusted R Square: 0.807) and the low value of Standard Error (.07368) suggest that the regression model explains the variation in the Rate of Success well.

Table B-9: Summary output for the multiple linear regression analysis of Rate of Success versus #P, #D, #Stops; area size: 25 sqM

Model		R	R Square	Adjusted R Square	Std. Error of the Estimate
dimension0	1	.901	.813	.807	.07368

The high values of t-statistics as well as the low value of p-value for the coefficients of the model also suggest that variable coefficients are statistically significant for the multiple linear regression model. Table B-10 shows the summary ANOVA analysis for the multiple linear regression analysis. Considering the unstandardized coefficients (B's) of dependent variables, the regression analysis results in the following multiple linear regression model:

$$Y = .169 + (-6.783E\text{-}5)\,X1 + (0.005)\,X2 + (0.028)\,X3 \qquad \text{(B-2)}$$

where Rate of Success is denoted by Y, and X1, X2, and X3 are representing #P, #D, and #Stops in the model. The standardized coefficients (Beta) for the independent variables suggest that:

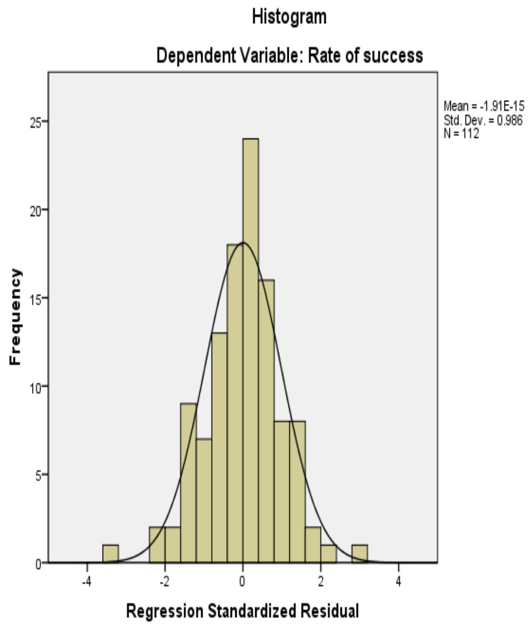
- Interpretation 1: A one S.D. change in #P produces a predicted change of -.012 S.D.'s in the mean of Rate of Success in the dense and small network of area size 25 square Miles, net of other variables, i.e., rate of success slightly decreases as the number of participating riders increases when other input parameters remain unchanged.
- Interpretation 2: a one S.D. change in #D produces a predicted change of .846 S.D.'s in the mean of Rate of Success in the dense and small network of area size 25 square Miles, net of other variables, i.e., rate of success significantly increases as the number of participating drivers increases when other input parameters remain unchanged..
- Interpretation 3: A one S.D. change in in #Stops produces a predicted change of 0.339 S.D.'s in the mean of Rate of Success in the dense and small network of area size 25 square Miles, net of other variables, i.e., rate of success significantly increases as the number of stop points increases when the other input parameters remain unchanged..
- Interpretation 4: #D is substantially more important than #Stops and #P in determining the level of Rate of Success in the small network of area size 25 square Miles.
- Interpretation 5: More than 80 percent of the variation in Rate of Success is explained in order of importance by #D, #Stops, #P.

Table B-10: Summary ANOVA analysis for the regression analysis of Rate of Success versus #P, #D, #Stops; area size: 25 sqM

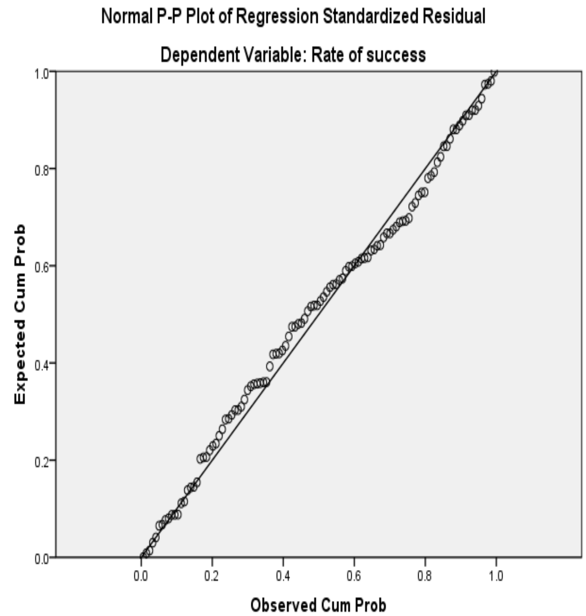
	Sum of Squares	Df	Mean Square	F	Sig.
Regression	2.543	3	.848	156.149	.000
Residual	.586	108	.005		
Total	3.130	111			

Model		Unstandardized Coefficients		Standardized Coefficients			95.0% Confidence Interval for B		Correlations		
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound	Zero-order	Partial	Part
1	(Constant)	.169	.023		7.30	.000	.124	.215	.124	.215	
	#P	-6.783E-5	.001	-.012	-.123	.903	-.001	.001	-.001	.001	.75
	#D	.005	.001	.846	8.46	.000	.004	.006	.004	.006	.85
	#Stops	.028	.003	.339	8.10	.000	.021	.035	.021	.035	.39

To assess the normality of the residuals, the P-P plot and histogram of residuals are examined. Figure (B2-a) is a histogram of the residuals with a normal curve superimposed. The residuals look close to normal. Figure (B2-b) is also a plot of the residuals versus predicted mean rate of success. The pattern shown here indicates no problems with the assumption that the residuals are normally distributed at each level of median number of zero connection routes and constant in variance across levels of mean rate of success.



(a) Histogram of Regression Standardized Residual



(b) Normal P-P Plot of Regression Standardized

Figure B2: Residual analysis for rate of success versus #P, #D, #Stops at the area size 25 sqM.

B.5. Regression (Predictors: (Constant), #2-connection routes, #0-connection routes, #1-connection routes and Dependent Variable: CPU Time(sec); area size: 25 sqM)

To examine the relationship between the mean CPU running time and the median number of zero connection routes, the median number of one connection routes, and the median number of two connection routes, a multiple linear regression analysis is conducted where the mean CPU running time (CPU Time(sec)) is the dependent variable and the median number of zero connection routes (#0-connection routes), the median number of one connection routes (#1-connection routes), and the median number of two connection routes (#2-connection routes) are independent variables. The data set includes 112 observations extracted from the 224 numerical examples. The 112 observations are associated with the CPU running time for each given combination of #0 connection routes, #1 connection route and #2 connection route at the area size of 25 Square Miles.

Summary output for the multiple linear regression model is presented in Table B-11. The high value of R square (R Square: 0.620, Adjusted R Square: 0.609) and the relatively low value of Standard Error (23.049) suggest that the regression model explains the variation in the CPU running time.

Table B-11: Summary output for the multiple linear regression analysis of CPU Time (sec) versus #0-connection routes, #1-connection routes, #2-connection routes; area size: 25 sqM

Model		R	R Square	Adjusted R Square	Std. Error of the Estimate
dimension0	1	.787[a]	.620	.609	23.04977

The high values of t-statistics as well as the low value of p-value for the coefficients of the model also suggest that variable coefficients are statistically significant for the multiple linear regression model. Table B-12 shows the summary ANOVA analysis for the multiple linear regression analysis. Considering the unstandardized coefficients (B's) of dependent variables, the regression analysis results in the following multiple linear regression model:

$$Y= -11.724+ (.285)\ X1 + (5.928)\ X2 + (7.139)\ X3 \qquad \text{(B-3)}$$

where CPU Running Time is denoted by Y, and X1, X2, and X3 are representing #0-connection routes, #1-connection routes, and #2-connection routes in the model. The standardized coefficients (Beta) for the independent variables suggest that:

- Interpretation 1: A one S.D. change in #0-connection routes produces a predicted change of .185 S.D.'s in the mean of CPU running time in the dense and small network of area size 25 square Miles, net of other variables, i.e., CPU running time increases as the number of zero connection routes increases.

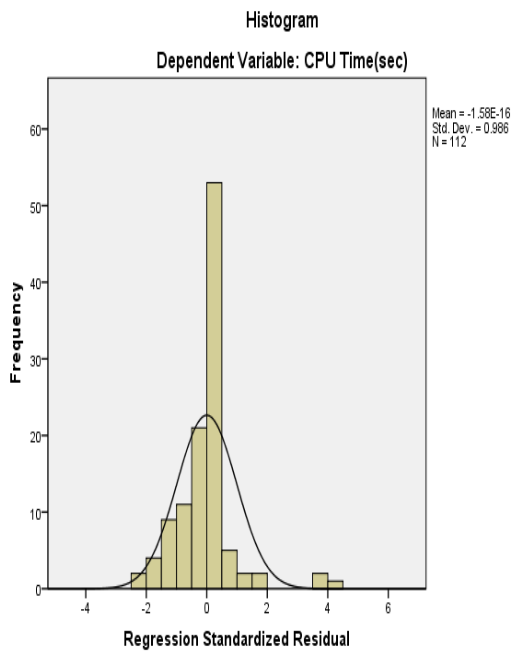
- Interpretation 2: A one S.D. change in #1-connection routes produces a predicted change of .429 S.D.'s in the mean of CPU running time in the dense and small network of area size 25 square Miles, net of other variables, i.e., CPU running time significantly increases as the number of one connection routes increases.
- Interpretation 3: A one S.D. change in #2-connection routes produces a predicted change of .286 S.D.'s in the mean of CPU running time in the dense and small network of area size 25 square Miles, net of other variables, i.e., CPU running time increases as the number of two connection routes increases.
- Interpretation 4: #1-connection routes is substantially more important than #0-connection routes and #2-connection routes in determining level of mean CPU Running Time in the small network of area size 25 square Miles.
- Interpretation 5: More than 60 percent of the variation in mean CPU Running Time is explained in order of importance by #1-connection routes, #2-connection routes, and #0- connection routes.

Table B-12: Summary ANOVA analysis for the regression analysis of CPU Time (sec) versus #0-connection routes, #1-connection routes, #2-connection routes; area size: 25 sqM

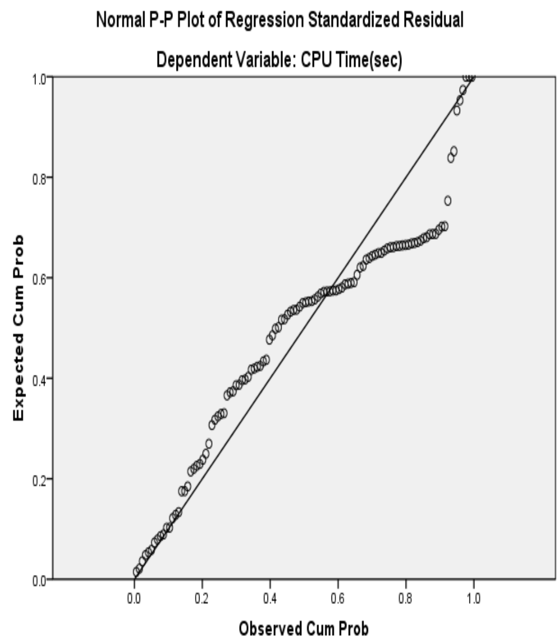
	Sum of Squares	Df	Mean Square
Regression	93576.395	3	31192.13
Residual	57379.509	108	531.292
Total	150955.905	111	

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-11.724	3.745		-3.131	.002
	#0-connection routes	.285	.118	.185	2.424	.017
	#1-connection routes	5.928	1.172	.429	5.058	.000
	#2-connection routes	7.139	2.138	.286	3.339	.001

To assess the normality of the residuals, the P-P plot and histogram of residuals are examined. Figure (B3-a) is a histogram of the residuals with a normal curve superimposed. The residuals look close to normal. Figure (B3-b) is also a plot of the residuals versus predicted mean CPU running time. The pattern shown here indicates no problems with the assumption that the residuals are normally distributed at each level of mean CPU running time and constant in variance across mean CPU running time.



(a) Histogram of
Regression Standardized



(b) Normal P-P Plot of
Regression Standardized

Figure B3: Residual analysis for CPU Time (sec) versus #0-connection routes, #1-connection routes, #2-connection routes at the area size 25 sqM.

B.6. Multiple Regression Analysis (Predictors: (Constant), #2-connection routes, #0-connection routes, #1-connection routes and Dependent Variable: Rate of success)

To examine the relationship between the mean Rate of Success and the median number of zero connection routes, the median number of one connection routes, and the median number of two connection routes, a multiple linear regression analysis is conducted where the mean Rate of Success is the dependent variable and the median number of zero connection routes (#0-connection routes), the median number of one connection routes (#1-connection routes), and the median number of two connection routes (#2-connection routes) are independent variables. The data set includes all the observations extracted from the 224 numerical examples. Table B-13 shows the descriptive statistics for the multiple linear regression analysis.

Table B-13: Descriptive statistics for the multiple linear regression analysis of Rate of Success versus #0-connection routes, #1-connection routes, #2-connection routes

Variable	Mean	Std. Deviation	N
Rate of success	.3113	.28915	224
#0-connection routes	18.1964	22.64023	224
#1-connection routes	1.2098	2.19573	224
#2-connection routes	.5848	1.17597	224

The correlation analysis of the model suggests there are high correlations between the mean rate of success and the median number of zero connection routes, the median number of one connection routes, and the median number of two connection routes. Pearson correlation coefficient between rate of Success and #0-connection routes is .856, the correlation coefficient between Rate of Success and #1-connection routes is .687, and the correlation coefficient between Rate of Success and #2-connection routes is .626. Summary output for multiple linear regression is presented in Table B-14. The high value of R square (R Square: 0.748, Adjusted R Square: 0.744) and the low value of Standard Error (.146) suggest the regression model explains the variation well.

Table B-14: Summary output for the multiple linear regression analysis of Rate of Success versus #0-connection routes, #1-connection routes, #2-connection routes

Model		R	R Square	Adjusted R Square	Std. Error of the Estimate
dimension0	1	.865	.748	.744	.14617

The high values of t-statistics as well as the low value of p-value for the coefficients of the model also suggest that variable coefficients are statistically significant for the multiple linear regression model. Table B-15 shows the summary ANOVA analysis for the multiple linear regression analysis. Considering the unstandardized coefficients (B's) of dependent variables, the regression analysis results in the following multiple linear regression model:

$$Y= .113+ (.009)\ X1 + (.021)\ X2+ (.005)\ X3 \qquad \text{(B-4)}$$

where mean rate of success is denoted by Y, and X1, X2, X3, and X4 are representing #0-connection routes, #1-connection routes, and #2-connection routes in the model.

Table B-15: Summary ANOVA analysis for the regression analysis of Rate of Success versus #0-connection routes, #1-connection routes, #2-connection routes

	Sum of Squares	df	Mean Square	F	Sig.
Regression	13.944	3	4.648	217.537	.000
Residual	4.701	220	.021		
Total	18.645	223			

Model		Unstandardized Coefficients		Standardized Coefficients			95.0% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	.113	.013		9.011	.000	.088	.138
	#0-connection routes	.009	.001	.729	14.560	.000	.008	.011
	#1-connection routes	.021	.007	.161	2.953	.003	.007	.035
	#2-connection routes	.005	.013	.022	.416	.678	-.020	.031

The standardized coefficients (Beta) for the independent variables suggest that:

- A one S.D. change in #0-connection routes produces a predicted change of .729 S.D.'s in the mean Rate of Success, net of other variables, i.e., rate of success significantly increases as the number of zero connection routes increases.

- A one S.D. change in #1-connection routes produces a predicted change of .161 S.D.'s in the mean Rate of Success, net of other variables, i.e., rate of success increases as the number of one connection routes increases.
- A one S.D. change in #2-connection routes produces a predicted change of .022 S.D.'s in the mean Rate of Success, net of other variables, i.e., rate of success slightly increases as the number of two connections routes increases.
- #0-connection routes and #1-connection routes are substantially more important than #2-connection routes in determining Rate of Success.
- Around 75 percent of the variation in Rate of Success is explained in order of importance by #0-connection routes, #1-connection routes, and #2-connection routes.
- #2-connection routes with very low standardized coefficient of .022 and high significant level of .678 has the least effect on the model. It also makes up more than .25 of mean total running time of the algorithm. Table B-16 shows the CPU-time percentage for #0-connection routes, #1-connection routes, and #2-connection routes.

Table B-16: CPU-time percentage for #0-connection routes, #1-connection routes, and #2-connection routes

	N	Mean	Std. Deviation	Variance
0-connection routes	224	.1294	.13754	.019
1-connection routes	224	.6218	.17106	.029
2-connection routes	224	.2567	.12543	.016
Valid N (listwise)	224			

To examine how the removal of #2-connection routes affects the model, a regression analysis has been conducted. By the removal of #2-connection routes in model, R square remains almost the same and the coefficients for remaining variables slightly change. Table B-17 shows the summary output after removal of #2-connection from the model.

Table B-17: Summary output for the multiple linear regression analysis of Rate of Success versus #0-connection routes and #1-connection routes

Model		R	R Square	Adjusted R Square	Std. Error of the Estimate
dimension0	1	.865	.748	.745	.14590

Table B-18 shows the summary output for the ANOVA analysis for the multiple linear regression analysis after removing #2-connection from the model.

Table B-18: Summary ANOVA analysis for the regression analysis of Rate of Success versus #0-connection routes, #1-connection routes

Model		Unstandardized Coefficients		Standardized Coefficients			95.0% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	.113	.013		9.019	.000	.088	.138
	#0-connection routes	.009	.001	.735	15.553	.000	.008	.011
	#1-connection routes	.023	.006	.172	3.640	.003	.010	.035

- There should be complementary relationships between #0-connection routes, #1-connection routes and #2-connection routes. To check the relationship between #0-connection routes, #1-connection routes, and #2-connection routes a correlation analysis is conducted. Table B-19 shows the Summary output for the correlation analysis. As the table shows by the increase of #0-connection routes, #1-connection routes and #2-connection routes decreases.

Table B-19: Summary output for the correlation analysis of #0-connection routes, #1-connection routes, and #2-connection routes

Model			#2-connection routes	#0-connection routes	#1-connection routes
1	Correlations	#2-connection routes	1.000	-.323	-.494
		#0-connection routes	-.323	1.000	-.416
		#1-connection routes	-.494	-.416	1.000

	Covariances	#2-connection routes	.000	-2.654E-6	-4.566E-5
		#0-connection routes	-2.654E-6	4.087E-7	-1.910E-6
		#1-connection routes	-4.566E-5	-1.910E-6	5.151E-5

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