

ABSTRACT

Title of Document: THE EVERYDAY MATHEMATICAL EXPERIENCES AND UNDERSTANDINGS OF THREE, 4-YEAR-OLD, AFRICAN-AMERICAN CHILDREN FROM WORKING-CLASS BACKGROUNDS

Grace Marie R. Benigno, Ph.D., 2012

Directed By: Dr. Patricia F. Campbell,
Department of Curriculum and Instruction

This qualitative study examined the everyday mathematical experiences and understandings of three, 4-year-old, African-American children from working-class backgrounds. The study drew on Street, Baker, and Tomlin's (2005) broad, ideological model of mathematics as "social" and their analytic concepts of mathematical events (units of analysis consisting of occurrences of mathematical activity) and mathematical practices (patterned uses, meanings, and ways of engaging in mathematics). Mathematical events were examined through four interrelated dimensions that were adapted for this study, constituting the mathematics (content) in and the social aspects (purpose and setting, values and beliefs, and social relations) situating the children's mathematical activities. Characteristics of mathematical events were determined through an analysis across the children's mathematical events.

Adapted naturalistic observation methods were used to yield data specifying children's everyday mathematical events within their homes, informal day care setting, and other familiar contexts. An iterative analytic process using inductive analytic procedures was employed to examine and interpret children's mathematical events and to determine characteristics of these events.

The three children each engaged in distinct, spontaneous mathematical experiences and understandings that reflected their unique family lives, individual predispositions, and knowledge development. For example, the values of one mother gave rise to many contexts fostering her daughter's nominal, ordinal, and numeric meanings for number. Findings indicated mathematical understandings that are not typically recognized in early childhood mathematics education research and practice and portrayed conditions that fostered children's meaningful engagement in and learning of mathematics. The children's everyday mathematical events tended to: emerge from their intrinsic motivation, involve their pursuit of goal-directed activities or interest in mathematics for its own sake, and promote their purpose-oriented verbal interactions with others.

Recognizing the unique, interrelated, and complex social aspects that underlie and support young children's everyday mathematical experiences and understandings, broadening what counts as evidence of mathematical thinking in early childhood, and creating conditions in formal settings that reflect characteristics of children's everyday mathematical events can foster children's continued meaningful engagement in and development of mathematical thinking in early childhood learning environments.

THE EVERYDAY MATHEMATICAL EXPERIENCES AND
UNDERSTANDINGS OF THREE, 4-YEAR-OLD, AFRICAN-AMERICAN
CHILDREN FROM WORKING-CLASS BACKGROUNDS

By

Grace Marie R. Benigno

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2012

Advisory Committee:
Professor Patricia F. Campbell, Chair
Professor Daniel I. Chazan
Professor Lawrence Clark
Professor Ann R. Edwards
Professor Mia A. Smith-Bynum

© Copyright by
Grace Marie R. Benigno
2012

Dedication

to Mama

and in loving memory of Daddy

Acknowledgements

First and foremost, I thank God for guiding and carrying me throughout this dissertation process and for granting me not only the capacity but also more importantly the perseverance to complete this journey. I also thank God for blessing me with a loving family and so many others who truly cared about me, cheered me on, and willingly gave their time, efforts, feedback, guidance, participation, support, encouragement, prayers, and love to help me achieve this work.

To my parents, Edward and Gloria Benigno, I give my deepest and eternal love, gratitude, and appreciation. This accomplishment is as much yours as it is mine. Thank you so much for your unconditional love; for instilling in me the values of faith, family, and education; and for always being there to support me no matter what. Without your love and support, I would not be where I am today. To Mama, words cannot express my eternal gratitude and appreciation for everything you have done and given, too much to list here, to support me throughout this journey. Thank you so much for your prayers, understanding, encouragement, generosity, and patience especially during those times that were most difficult for me. I could not have done this without you. To Daddy, thank you for showing me how to treat others with dignity and respect, for encouraging me to continue to strive toward my goals even in the final days before your passing, and for interceding for me in prayer ever since. In remembering you, I also found strength to persist. May you rest in eternal peace.

To my siblings, thank you so much for your love, prayers, support, and encouragement. To my sister, Carolyn Benigno, out of everyone, you were the one literally by my side, both celebrating the ups as well as enduring the many grueling

downs with me, for much of this journey. For this, I give you my eternal heartfelt appreciation and gratitude. Thank you so much for your generosity, patience, and encouragement. I would not have been able to complete this dissertation without your support. To my brother, John Benigno, thank you for making our family outings so much fun, for encouraging me to resume my fitness activities, and for supporting me in my solo and our joint athletic adventures. Our family time, whether at dinner or during a training run, always means a great deal to me, and it offered me a much-needed and appreciated break from the work of dissertating. To my sister, Veronica Baechler, thank you for reminding me to try to keep this task in perspective and for your efforts to help me move forward with my writing. To our beloved Pomeranian, Foxie, thank you for those times you kept me company when I was writing. Your precious little face always helped to uplift my spirit whenever I felt discouraged. May you rest in peace always with Lolo.

To my advisor and dissertation chair, Dr. Patricia F. Campbell, words cannot express my heartfelt gratitude for everything you have done to support and guide me throughout this process. Thank you so much for your patience and understanding, for the time and care you gave in providing me feedback and editing my work, for always believing in me even when I did not believe in myself, and for pushing me to keep moving forward even when I thought I could no longer do so. Thank you also for recognizing the importance of this study and for your unwavering commitment and efforts to see to it that I completed this dissertation. I know this accomplishment would not be possible without you. You are a true inspiration, and I thank you again from the bottom of my heart.

To the members of my dissertation committee, thank you so much for your time, efforts, insight, feedback, and encouragement. To Dr. Daniel Chazan, thank you for helping me clarify what it was I wanted to study when I came to you with ideas that made sense in my head but that I had not yet figured out how to articulate clearly to others. Thank you too for pushing me to think deeply about what I meant by mathematics in this study. To Dr. Ann Edwards, thank you for your feedback on an earlier paper from this research and for encouraging me to think further about and to convey in my writing even more so “who were these children” in my study in the context of their home and everyday lives and how their everyday mathematical activities related to these experiences. To Dr. Lawrence Clark, thank you for your steady encouragement from the time you first arrived at the University of Maryland to the day I submitted my dissertation to my committee for your review. To Dr. Mia Smith-Bynum, thank you for encouraging me to trust in my own scholarship and to be confident in my work regarding African-American children and their families.

To the former members of my dissertation committee who served earlier during the proposal stage, thank you for the contributions you made to this work and to my growth as a scholar. To Dr. Marvin Lynn, thank you pushing me to think critically about issues of race in education, for adopting me as one of your own students, and for your continued support and encouragement. To Dr. Aisling Leavy, thank you for supporting my capacity to engage in research concerning issues of equity and social justice in mathematics education. To Dr. Judith Torney-Purta, thank you for helping to inspire me to enter the field of education.

To the families who participated in this study, I will never be able to adequately thank you for your time and willingness to open your homes and lives to me, for your trust and your hospitality. You and your children are at the very heart of this work. Thank you for allowing me the opportunity to share a glimpse of your lives with others and to present accounts of your children's everyday mathematical experiences. I am eternally grateful for the opportunity you have given me to contribute to scholarship in a critical area. I tried to do your families justice and to represent you honestly and with the dignity that you deserve. I hope you can sense my love, care, and concern for your children's well being, and for other young children like them, and that I captured their capacity to think mathematically in their everyday lives. This dissertation would not be possible without your participation and I will be forever grateful for what you have allowed me to do.

To Ms. Franklin (pseudonym), the participating children's day care provider, I extend my sincere, heartfelt gratitude to you. You were a true blessing to me. Because of you, I was able to find families to participate in this study. Thank you so much for your willingness to help me, for trusting me, for opening your day care and home to me, for all your efforts to help me gain the trust and seal the participation of families, and for your friendship and continued support throughout the data collection process and beyond. This dissertation would not be possible without your support and efforts.

To the Mid-Atlantic Center for Mathematics Teaching and Learning and the Ford Foundation Dissertation Fellowship Program, thank you for the financial support you provided me during my doctoral program and for affording me meaningful opportunities to further develop and conceptualize this study.

To my dearest friends, extended family and other loved ones, colleagues, and countless others, thank you so much for your prayers, support and encouragement. I will be forever grateful for your contributions, both large and small, to my success in completing this dissertation.

Table of Contents

Dedication.....	ii
Acknowledgements.....	iii
Table of Contents.....	viii
List of Tables.....	xii
List of Figures.....	xiii
Chapter 1: Introduction.....	1
A Focus on Early Childhood Mathematics Education.....	1
Early Childhood Mathematics Education Discourse Regarding Deficits and Differences.....	3
Examining Young Children’s Everyday Mathematical Experiences and Understandings.....	8
Theoretical Lens.....	12
Research Questions.....	16
Limitations.....	20
Chapter 2: Review of Literature.....	22
Setting the Scene.....	22
Overview of Chapter.....	24
Equity in School Mathematics.....	24
Disparities in Student Achievement.....	24
Defining Equity in Mathematics Education.....	25
Problematizing Mainstream Attempts To Address Equity.....	27
Approaching Research on Equity in Early Childhood Mathematics Education from a Sociopolitical Perspective.....	28
Social/Situated Theories of Learning.....	33
Critiquing Dominant Views of Learning.....	34
Learning As Embedded in Social, Cultural and Political Contexts.....	37
Learning in Context.....	39
Conceptualizing the Mathematical Experiences of Children from Underserved Groups Across Contexts.....	42
Research on Young Children’s Everyday Mathematical Experiences and Understandings.....	45
Chapter 3: Methodology.....	53
Participants.....	55
Sources.....	56
Securing Participants.....	57
Defining Mathematical Activity.....	58
Mathematical Events.....	59
Characteristics of Mathematical Events.....	65
Observing From Within, Approximating an Essence of Everydayness, and Negotiating Pathways In the Field.....	67
Procedure.....	70

Analysis.....	72
Presentation of Findings	74
Chapter 4: Toya.....	76
Capturing Images of Toya’s Everyday Life.....	76
At the Day-Care “School	76
Kleenex, Notes, and Talk.....	77
Homes Sweet Homes	78
Getting Her Hair Done.....	78
Nicole, Toya’s Mother	79
A Note on Nicole’s Parenting Practices, From Ms. Franklin’s Perspective	80
Toya at Home.....	81
Snapshots of Reflections of Mathematics: Perspectives from Toya’s Mother, Nicole.....	86
“Math was not a good experience for me.”.....	87
“Nothing other than the basic math.”.....	88
“Whenever she asks for money she has to tell me what it is or... how much is it. I make her do that.”.....	89
Addressing In-the-Field Methodological Concerns And Applying the Adapted Analytic Framework:	91
An Illustration.....	91
“She may ask for a cookie... I might say, ‘Okay, you can get such and such amount of cookies.’”.....	98
What Counts “That’s Maybe Math” for Toya: Making Sense of Nicole’s Perspectives.....	99
Interpreting a Collage of Toya’s Everyday Mathematical Events.....	101
Conceptualizing a Wider View of Number As Mathematical	102
Composition.....	105
Content: Number as Label	106
Content: Number to Order	121
Content: Expressing Number Words Orally In a Descending Count Sequence.....	127
Content: Number to Enumerate	132
Content: Number to Signify Discernable Attributes of a Physical Quantity or Entity.....	149
Characterizing Toya’s Everyday Mathematical Events.....	167
Nurturing Understandings and Various Meanings of Small numbers	168
Arising within Practices Valued By Toya’s mother, Nicole.....	172
Emerging from Toya’s Intrinsic Motivation Or Goal-Directed Interests	176
Involving Toya in Verbal Interactions.....	181
Final Remarks	185
Chapter 5: Anika.....	186
Impressions of Anika and Capturing Glimpses of Her Everyday Life.....	186
At Day Care	186
A Reluctant, Picky Eater with a Love for Sweets.....	187
Displaying a Range of Temperaments.....	188
Noticing and Participating in Adults’ Everyday Verbal Interactions, Cultural Interests, and Tasks.....	189

Christina, Anika’s Mother	191
Christina’s Home	193
Obstacles to Observation Visits	194
Glimpses of Anika’s Everyday Life at Home	194
Candid Reflections of Mathematics: Perspectives from Anika’s Mother, Christina	198
"I was always doing it wrong."	198
"...we count...add and subtract..."	200
Money	201
Interpreting a Collage of Anika’s Everyday Mathematical Events	202
Conceptualizing Meanings of Difference and Similarity as Mathematical Content	202
Composition	205
Content: Observing and Identifying Differences And/or Similarities - Between Two Like Objects	206
Content: Observing and Identifying a Similarity And Difference - Between an Object and an Image	221
Content: Observing and Identifying a Similarity Between Two Unlike Objects	226
Content: Sorting Fruit Snacks into Same-Kind Groups	228
Content: Identifying Images that are the Same	230
Characterizing Anika’s Everyday Mathematical Events	233
Focusing upon Familiar Entities In Everyday Contexts	234
Intrinsically Motivated	235
Verbal Interactions	237
Employing Content for Its Own Sake	238
A Final Observation	239
Chapter 6: Aaron	241
Impressions of Aaron and Capturing Glimpses of His Everyday Life	241
At Day Care	241
At Home	243
Aaron the Observer	253
Keisha, Aaron’s Mother	254
Aaron Sr., Aaron’s Father	257
Aaron’s Parents’ Perspectives on Mathematics In their Everyday Lives	259
At Work	259
At Home	260
A Formal Perspective of Mathematics	262
Home Visits with Aaron Ended	264
Interpreting a Collage of Aaron’s Everyday Mathematical Events	264
Content: Expressing Number Words Orally In a Count Sequence	265
Content: Number to Enumerate Objects/Images, When Prompted with, “How Many?”	274
Content: Number to Enumerate Objects/Images, Spontaneously	285
Content: Employing the Number Word Two As a Descriptive Narrative Device	295

Content: Number Words and Symbols as Distinct from Letter Names and Symbols	299
Content: Geometric Thinking/Spatial Reasoning	303
Characterizing Aaron’s Everyday Mathematical Events	314
Arising Spontaneously from Aaron’s Observant Nature or Aaron’s Interest in Reading Books/Telling Stories	314
Verbal Interactions	316
Focus on Number with Notable Evidence of Emerging Geometric and Spatial Awareness	318
Two	319
Final Remarks	320
Chapter 7: Analysis of Mathematical Events and Characteristics of these Events Across All Three Children	322
Mathematical Content and Understandings	323
Distinctions Across the Children	323
Broadening Conceptualizations of Evidence of Mathematical Thinking in Young Children: A Commonality Across the Children.....	326
Purposes and Settings, Values and Beliefs	328
Distinctions Among Contexts and Values That Prompted Mathematical Events	328
Contexts for Mathematical Events at Day Care: Commonalities and Distinctions Across Children	334
Social Relations and Characterizing Children’s Mathematical Thinking.....	337
Child-Driven Mathematical Events	338
Child-and-Other-Driven Mathematical Events	344
Adult-Driven Mathematical Events	345
Distinctions in Children’s Mathematical Thinking.....	348
Common Characterizations of Children’s Everyday Mathematical Events	350
Some Final Thoughts	352
Chapter 8: Discussion and Implications	356
Addressing Sociopolitical Themes of Inquiry in Mathematics Education	357
Understanding the Everyday Mathematical Experiences of Young African-American Children.....	357
Broadening What Counts As Evidence of Young Children’s Mathematical Thinking.....	361
Re-conceptualizing Early Childhood Mathematics Teaching and Learning Practices	364
Examining the Role of Parents in Young Children’s Mathematical Development	369
Limitations	373
Additional Recommendations for Future Research.....	376
References.....	381

List of Tables

Table 1	Summary of Child and Parent Participants.....	55
Table 2	Summary of Frequency of Observation Visits.....	71

List of Figures

Figure 1	Examples of Mathematical Content and Activities	61
----------	---	----

Chapter 1: Introduction

A Focus on Early Childhood Mathematics Education

Attention to early childhood mathematics education has been on the rise for more than a decade as evidenced in the work of professional organizations and national and state policy agendas. For example, in 2000, the National Council of Teachers of Mathematics (NCTM) released its updated *Principles and Standards for School Mathematics (PSSM)* (NCTM, 2000) which featured newly added standards for prekindergarten. That same year, the Conference on Standards for Prekindergarten and Kindergarten Mathematics Education convened to coordinate efforts among experts from diverse fields to address the lack of consistency of standards and guidelines in mathematics education at the early childhood level. The work from this conference culminated in the publication, *Engaging Young Children in Mathematics: Standards for Early Childhood Mathematics Education (EYCM)* (Clements, Sarama, & DiBiase, 2004). This publication presents 17 recommendations addressing the following five major themes: standards in early childhood education; mathematics standards and guidelines; curriculum, learning, teaching, and assessment; professional development; and implementation and policy.

In 2002, the National Association for the Education of Young Children (NAEYC) and the NCTM released a joint position statement, “Early Childhood Mathematics: Promoting Good Beginnings,” in which they affirmed that “high-quality, challenging, and accessible mathematics education for 3- to 6-year-old children is a vital foundation for future mathematics learning” (NAEYC & NCTM, p. 1). This joint position statement, which was later updated in 2010, includes

recommendations for both classroom practice and for early childhood decision makers in venues beyond the classroom, including program developers and policy makers. In a focused update of the *PSSM*, the NCTM in 2006 released *Curriculum Focal Points* that identified three important mathematical topics for each grade level from prekindergarten through grade 8. In addition to the surge in attention paid to early childhood mathematics education in particular, attention to early childhood education in general also appeared evident during this period as the U.S. Department of Education, National Center for Educational Statistics launched the Early Childhood Longitudinal Study (e.g., U.S. Department of Education, National Center for Educational Statistics, 2004) commissioning three longitudinal studies to examine child development, school readiness, and early school experiences.

In 2007, the National Research Council (NRC) established the Committee on Early Childhood Mathematics in order to: review the extant research literature on this topic and to develop and determine evidenced-based mathematics learning objectives for preschool children; offer recommendations for curriculum, instruction, and teacher education for achieving the identified learning goals; and suggest implications for policy, practice, and research (NRC, 2009). In their primary recommendation, this committee proposed that, “A coordinated national early childhood mathematics initiative should be put in place to improve mathematics teaching and learning for all children ages 3 to 6,” (NRC 2009, p. 3). The resulting report, *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity (MLEC)* (NRC, 2009) has already demonstrated a broad impact on policy and classroom practice. Specifically, the Common Core State Mathematics Standards (Common Core State Standards

Initiative, 2010), adopted by 45 out of the 50 states in the U.S. as of December 2011, incorporates the *MLEC* recommendation that mathematics instruction in early childhood should focus only on two content areas, with more time spent on the content of number followed by the content of geometry, spatial sense, and measurement.

Each of these policy and standards documents concerning early childhood mathematics education reflects two primary themes. The first theme involves promoting excellence in mathematics teaching and learning, beginning in the early years. According to these documents, the goals of enhancing student learning and increasing student achievement in mathematics serve to position the U.S. and its citizens to compete successfully in both international comparisons of mathematics achievement and in a global economy. The second theme involves equity, in particular, closing the reported mathematics “achievement gap,” in which African Americans, Latinos, Native Americans, and students from low-income backgrounds consistently underperform in school mathematics assessments compared to their White, Asian, and more affluent peers.

Early Childhood Mathematics Education Discourse Regarding

Deficits and Differences

As attention to early childhood mathematics education intensified amidst broader, continued mainstream mathematics education attempts to improve student learning and achievement for “all,” a growing body of researchers seeking to examine issues of social justice and racism in mathematics education began to employ sociopolitical perspectives (Gutiérrez, 2010; Valero, 2004) to inform their work.

These researchers have drawn from or across, for example: Latcrit theory (Gutiérrez, 2010); critical race theory, critical theory, and poststructural theory (Stinson, 2009); and sociological and critical theory frameworks (Martin, 2009b). Researchers adopting a sociopolitical stance aim to examine taken-for-granted assumptions and practices in mathematics education (Gutiérrez, 2010; Valero, 2004) and to understand the mathematical experiences of students from groups who have been underserved historically in school mathematics. The term “underserved groups” refers to African Americans, Latinos/as, Native Americans, English language learners, and individuals from working-class and poor backgrounds.

Gutiérrez and Dixon-Román (2011) problematized mainstream mathematics education’s preoccupation with “the achievement gap,” or what Gutiérrez (2008) terms “gap gazing,” where discourse focuses on examining, addressing, and discussing race-, class-, or language-comparative disparities in student mathematics achievement. Within the discourse of gap gazing, White, middle-class students’ experiences and their mathematical behavior and achievement are normalized as the standard by which all students are judged or are expected to meet (Gutiérrez & Dixon-Román, 2011; Martin, 2009a). Gutiérrez and Dixon-Román (2011) contend, “Most researchers and practitioners fail to question the underlying assimilationist goal and the ways in which framing the problem as an achievement gap supports deficit thinking and negative narratives about marginalized students” (p. 23). This interpretation is particularly salient for most students of color as Martin (2009a, 2009b) asserts that the practice of focusing on racial disparities in mathematics education, without theorizing race as a social construct, has resulted in uncontested

racial hierarchies of mathematics ability, in which African-American, Latino/a, and Native-American students are positioned at the bottom. These sociopolitical perspectives must be recognized when exploring the ways in which young children, their families, and their home experiences might be portrayed and positioned within discourse addressing equity and in efforts to improve early childhood mathematics education.

The characterizations of the quality of everyday mathematical experiences and understandings of young children from traditionally underserved groups portrayed in early childhood mathematics education policy documents sway between deficit and difference perspectives. The *MLEC* (NRC, 2009) presents a deficit perspective contending that,

... although virtually all young children have the capability to learn and become competent in mathematics, for most the potential to learn mathematics in the early years of school is not currently realized. This stems from a lack of opportunities to learn mathematics either in early childhood settings or through everyday experiences in homes and in communities. This is particularly the case for economically disadvantaged children, who start out behind in mathematics and will remain so without extensive, high-quality early mathematics instruction. (pp. 1-2)

This statement presumes that before entering elementary school, young children should acquire a particular set of mathematical skills and understandings in order to learn and succeed in school mathematics. It implicates socioeconomic status and social class as responsible for either promoting or hindering children's opportunities

to learn mathematics in their everyday settings.

The *MLEC* positions young children's everyday mathematical experiences or opportunities to learn mathematics as being either favorable or deficient with respect to the mathematical aspect of these experiences (i.e., the mathematical understandings children develop). The *MLEC* seems to pay little attention to the other aspects of young children's everyday mathematical experiences and understandings, beyond formal mathematical content, that interact to shape and support children's opportunities to learn mathematics in their homes and other everyday settings.

The *MLEC*'s deficit perspective contrasts with the difference perspectives employed in the *EYCM* and in the NAEYC and the NCTM's joint position statement. The following is the first recommendation for early childhood mathematics education in the *EYCM*.

Equity is a major concern in mathematics education at all levels. There is an early developmental basis for later achievement differences in mathematics: Children from different sociocultural backgrounds may have different foundational experiences. Programs need to recognize sociocultural and individual differences in what children know and in what they bring to the educational situation. Knowledge of what children bring should inform planning for programs and instruction.

(Clements et al., 2004, p. 13)

The *EYCM* recommendation states a need to examine what we know about children and the mathematical knowledge and experiences they bring to the school context, while recognizing that children have different sociocultural backgrounds and

experiences. Furthermore, this recommendation calls attention to the need to understand children's sociocultural backgrounds and experiences and to use this knowledge to inform mathematics instruction. This perspective does not appear to position children from particular groups as being deficient or lacking the capacity to learn school mathematics. Instead, this perspective suggests that children may have different experiences that shape what they know and bring to the educational situation. The recommendation suggests that researchers and educators need to recognize, understand, and build upon what these experiences are. With such knowledge, children's experiences that might have otherwise been dismissed or overlooked may inform the context and culture of early childhood mathematics programs.

The joint position statement of the NAEYC and the NCTM (2002/2010) recommends that early mathematics classroom practice "build on children's experience and knowledge, including their family, linguistic, cultural, and community backgrounds; their individual approaches to learning; and their informal knowledge" (p. 4). The NAEYC and the NCTM assert that children have "varying" experiences and understandings that reflect their individual backgrounds and that teachers "must know as much as they can about such differences and work to build such bridges between children's varying experiences and new learning" (p. 4). This recommendation suggests that all children's experiences and understandings are important and valuable resources that can inform instruction and promote student learning of mathematics.

I propose that early childhood mathematics education focus on understanding rather than positioning young children's everyday mathematical experiences and understandings. Without a deep knowledge of children's everyday mathematical experiences and understandings and the sociocultural context in which these developed, early childhood mathematics education is ill prepared to foster and maximize the mathematical development of children from different backgrounds, in particular those from underserved groups.

*Examining Young Children's Everyday Mathematical
Experiences and Understandings*

As the *EYCM* and the joint position statement of the NAEYC and the NCTM suggest, children from diverse sociocultural backgrounds have different mathematical experiences and develop understandings that have meaning and value within each child's particular social and cultural context. Research that examines the everyday mathematical experiences and understandings of young children from underserved groups and that gives particular attention to the social and cultural context situating these may inform early childhood mathematics education discourse and teaching and learning practices.

In order to conduct such research, it is useful to consider Street, Baker, and Tomlin's (2005) ideological model of mathematics as "social" and their analytic concepts of mathematical events and mathematical practices (see Theoretical Lens section below). Through their lens children's mathematical experiences are conceptualized through a broader perspective of social, that includes the settings and purposes, values and beliefs, and social and institutional relations, that together with

mathematical content form the complex milieu and meanings in which children's mathematical experiences and understandings are situated. Conceptualizing children's mathematical experiences and understandings in this way can also be supported through ecological (e.g., Bronfenbrenner, 1979; García-Coll et al., 1996), ecocultural (e.g., Gallimore, Goldenberg, & Weisner, 1993; Weisner, 2002), and sociocultural (e.g., Rogoff, 2003; Vygotsky, 1978) perspectives on child and human development and a proposed person-centered cultural psychology (Gjerde, 2004) perspective. As such, Street et al.'s ideological model of mathematics as social and their analytic concepts provide theoretical tools with which to conceptualize and examine a rich picture of young children's everyday mathematical experiences and knowledge that in turn can be used to inform early childhood mathematics education practices.

Little is known about young children's everyday mathematical experiences in the extant literature. According to Anderson, Anderson and Thauberger (2008), "Interest in the home and childcare experiences of preschool children is a relatively recent development in early childhood mathematics research" (p. 102). As such, few studies have examined young children's everyday mathematical experiences and understandings as these naturally unfold in children's day-to-day activities at home or in other everyday settings (Anderson et al., 2008; Anderson & Gold, 2006; Guberman, 1999, 2004; Tudge & Doucet, 2004). Thus, although research asserts that all young children engage in mathematical activities and develop mathematical understandings, which are also referred to as informal or everyday mathematics, in their everyday experiences prior to formal schooling (e.g., Clements et al., 2004; Ginsburg, Cannon, Eisenband, & Pappas, 2006; NRC, 2009), when researchers refer

to young children's everyday mathematical activities in the context of implications for early childhood mathematics education practice, they typically provide brief anecdotal examples, such as when young children identify their age with their fingers, to illustrate these experiences rather than citing evidence from research.

While it is important to reconceptualize how we interpret young children's everyday mathematical experiences and understandings, it is equally important to design research that actually focuses on young children's naturally occurring mathematical experiences and understandings that emerge from their day-to-day activities at home and other familiar, informal settings. The majority of studies that have examined young children's early mathematical thinking and/or their everyday mathematical experiences have employed instead methods such as using researcher-created activities to assess children's early mathematical understandings (e.g., Ginsburg & Russell, 1981), observing children's spontaneous mathematical activities in formal day care settings (e.g., Ginsburg, Inoue, & Seo, 1999), relying upon mothers' self-reports of children's mathematical activities (e.g., Saxe, Guberman, & Gearhart, 1987), or focusing on children's everyday experiences at home with academic-centered mathematical activities (e.g., Tudge & Doucet, 2004).

Most studies that have attempted to understand the early mathematical understandings and/or experiences of young children before they entered school have not employed methods, such as naturalistic observation, that specifically aim to capture and understand children's everyday, spontaneous experiences with mathematics. By observing children in their day-to-day settings and activities, insights can be gained about the social aspects, broadly conceived, that shape and

give meaning to children's everyday mathematical experiences and understandings. A generative research endeavor would examine young children's naturally occurring, everyday mathematical experiences and understandings as situated within the social and cultural contexts that give meaning to these events.

Of the limited extant literature available, in addition to Street et al.'s (2005) research (see also Baker, Street, & Tomlin, 2006), Walkerdine (1988) and Anderson and Gold's (2006) studies reflect the type of research that focuses on understanding rather than positioning the mathematical experiences and understandings of young children from underserved groups and provide insight into how this knowledge can inform school mathematics practice.

Walkerdine (1988), drawing on Foucault's analysis of discourses of "truth," analyzed home speech data of young children from working-class families and examined the ways in which words such as "more" and "less," which hold taken-for-granted relational meanings in school mathematics discursive practices, can hold different meanings and associations in the everyday, home social practices of young children. As such, some children experienced difficulty on pedagogic assessments where a contrastive relationship between two words was presumed. For example, children did not identify the opposite of "more" as "less," but rather "no more" as the opposite of "more" as these words were used within their home discursive practices around regulation of food consumption.

Anderson and Gold (2006) examined the home-to-school numeracy practices and mathematical identities of four children attending an urban preschool in order to begin to address persistent disparities in student achievement in this setting. Drawing

on a perspective of mathematics as a situated social practice, Anderson and Gold observed children engaging in mathematical activities both in their preschool setting and at home and examined the ways in which meanings from these experiences traveled with children across these two contexts. By observing children in their everyday settings and by interpreting the contexts that shaped and gave meanings to events, notable insights were generated. Anderson and Gold argued that further research of this kind is necessary “for understanding the complex implications of sociocultural factors in underachievement” (p. 284).

In order to move toward more socially just practices in early childhood mathematics education that emphasize understanding rather than positioning children’s experiences and understandings, and given the extant literature, further research is needed to examine and understand the everyday mathematical experiences and understandings of young African-American children and of children from other underserved groups. As asserted by the Diversity in Mathematics Education Center for Learning and Teaching (2007), “Focusing on what students *are* doing opens conceptions of mathematical competence to the possibility of accommodating the diversity in students’ ways of knowing and paths to learning” (p. 413).

Theoretical Lens

Street et al. (2005) conducted a longitudinal study of children from different socioeconomic backgrounds in their first 3 years of schooling in order to understand disparities in student mathematics achievement. These researchers observed the children’s mathematical experiences at school and in their home settings. In order to examine the children’s experiences in both settings, Street et al. developed an

ideological model of mathematics¹ as social (see also Baker, Street & Tomlin, 2003; Baker et al., 2006), reflecting their perspective that mathematics is a social construction and that “relations of power” infuse all aspects of mathematical activity. Street et al.’s proposed ideological model of mathematics as social is distinct from other common notions of social in mathematics education including sociocultural perspectives (e.g., Vygotsky, 1978; Lerman, 2000), social in terms of interactions or conversations between students in mathematics classrooms, and social-as-pedagogic as when parents talk to their young children about mathematics that is embedded in their everyday activities.

Street et al.’s perspective of social entails the context, values and beliefs, and social and institutional relations, in which mathematical activity is situated. Street et al. define these aspects of social as follows:

- Context: the setting that is, “the framing of those occasions” entailing the use of mathematics and the “purposes for that use of mathematics” (p. 22);
- Values and beliefs: “the ways individuals’ beliefs, values and epistemologies affect” the mathematical practices adopted by that individual (p. 22); and
- Social and institutional relations:
 - Social relations: the “positions, roles, and identities of individuals in relation to others” in terms of mathematics (p. 21);
 - Institutional relations: involves “control over content [mathematics],

¹ Street et al. (2005) view mathematics and numeracy as synonymous interchangeable terms (e.g., mathematical events, numeracy events) in their research. I only employ the term mathematics when describing their work and when drawing upon their theoretical perspectives in this study.

management of context and invoking of values and ideology exercised by different institutions and roles” (p. 22).

Street et al. (2005) contend that this model of mathematics as social provides an alternative way of understanding student attainment in school mathematics. “Instead of viewing low achievement in terms of deficit the model accepts social notions of difference and multiple practices; it seeks to represent and build upon informal numeracy practices and ‘funds of knowledge’ (Moll, 1992)” (p. 18).

Street et al.’s (2005) perspective of mathematics as social is reflected in their research through two analytic concepts, mathematical events and mathematical practices, which are defined as follows:

- Mathematical events: units of analysis consisting of observed occasions in which mathematical activity “is integral to the nature of the participants’ interactions and their interpretive processes” (Baker et al., 2003, p. 12); and
- Mathematical practices: patterned uses and meanings of mathematics as situated in particular contexts (Baker et al., 2006) that emerge through an analysis of a set of mathematical events and that reflect the following four interrelated dimensions of mathematical practices – content, context (i.e., purpose and setting), values and beliefs, and social and institutional relations.

As indicated above, in order to operationalize the uses and meanings of mathematics in particular contexts, Street et al. (2005) identified four dimensions.

The first dimension is content, and it refers to the “activities, techniques, procedures

and processes” (p. 21) of mathematics in which individuals engage. The remaining three dimensions are the aspects that frame Street et al.’s perspective of social (context, values and beliefs, and social and institutional relations) as defined above. Although the four interrelated dimensions are addressed individually for analytic purposes, Street et al. contend that, “These dimensions are not seen as distinct separate aspects. They are always in interplay with each other” (p. 21).

In their study, Street et al. (2005) asserted that the “disjunctions” they found between the children’s home mathematical practices and school mathematical practices could account for some students’ “disengagement” in school mathematics, and they argued that future examinations of disjunctions between children’s home and school mathematical practices may be generative in terms of revealing how school mathematical practices can either hinder or promote student attainment. Street et al. encouraged researchers not only to use their conceptual framework but also, from a theoretical perspective, to modify and adapt their analytic concepts as appropriate for the local site(s) and purpose of the research.

In my interpretation of Street et al.’s (2005) analytic concepts, the four interrelated dimensions of content, context (i.e., purpose and setting), values and beliefs, and social and institutional relations can also be reflected in mathematical events. That is, a mathematical event can be examined with respect to these four interrelated dimensions. Also, it seems presumed that an individual’s constructed meanings regarding each of the four dimensions can also be explicated in analysis. For example, an analysis of the dimension of content in a mathematical event can identify both what mathematical content was employed in that moment as well as

what mathematical understandings an individual appeared to demonstrate in the event.

Finally, although Street et al. contend that their ideological model of mathematics as social is broader than and distinct from common notions of social in mathematics education, when they stated, “by ‘social’, we do not mean just the social theories of learning in the sense often meant by ‘sociocultural’ theorists following Vygotsky” (p. 15). Thus, they seem to suggest that their perspective could include an analysis of learning that occurs through sociocultural activity or through some other means of development. For example, when analyzing a mathematical event with respect to the dimension of social relations, a researcher can employ a sociocultural perspective (e.g., Rogoff, 2003) to examine an individual’s learning or development of mathematical content in everyday contexts that appeared evident in the event.

Research Questions

The purpose of this study was to examine and interpret the everyday mathematical experiences and understandings of three, 4-year-old, African-American children from working-class family backgrounds. In order to frame research questions specifying the meaning of everyday mathematical experiences and understandings, I drew on Street et al.’s (2005) broad, ideological model of mathematics as “social” and their analytic concepts of mathematical events and mathematical practices (see also Baker et al., 2003; Baker, et al., 2006), thereby informing my study and my analysis of children’s everyday mathematical experiences and understandings. For the purposes of this study, I employed the phrase “characteristics of mathematical events,” or similar phrases (e.g., “characteristics of these events”), to represent the

analytic characterization of patterned uses of or shared mathematics within and across events.

A mathematical event was a unit of analysis consisting of an occasion wherein mathematical activity was “integral to the nature of the participants’ interactions and their interpretive processes” (Baker et al., 2003, p. 12). Characteristics of mathematical events referred to the patterned uses and meanings of mathematics (Baker et al., 2006) within and across particular events and consistent ways of engaging in mathematics as situated in particular contexts. Mathematical events were examined in terms of each of the following four interrelated dimensions that I adapted for this study and that constituted the mathematics in and the social aspects situating children’s mathematical activity: content, purpose and setting, values and beliefs, and social relations. Characteristics of mathematical events were determined through an analysis of these four, adapted interrelated dimensions across a set of mathematical events.

In this qualitative study, I employed adapted naturalistic observation (Angrosino, 2005) methods of the children in their homes, informal day care, and other everyday settings in order to characterize their typical activities and dispositions and to record observation data of mathematical events. For each child, I identified and examined a collection of mathematical events and determined characteristics of these events.

In my process of examining the collection of mathematical events identified for each child, I adapted the meanings of the four interrelated dimensions in order to capture and discuss the various child, parent/caregiver, family, cultural, and/or other

ecological influences and meanings that I interpreted as contributing to, promoting, or shaping these mathematical events. The following paragraphs summarize how I adapted each of the four interrelated dimensions for this study.

Content referred to the identified mathematical content that appeared evident within an event as well as an analysis of the mathematical understandings that a child appeared to demonstrate.

Purpose and setting typically concerned the time, location, and activity/practice/episode in which a mathematical event took place and the child's content-related behaviors that transpired, indicating the underlying purpose that appeared to give rise to the content. Where appropriate, purpose and setting also concerned pertinent background information and insights regarding the activity, practice, or episode in which a mathematical event occurred.

Values and beliefs included the values and beliefs of a parent, family, or caregiver, such as those that fostered the activity, practice, or episode in which mathematical meanings (content) emerged or were employed, those that promoted the nature of social relations that occurred around a given content, or those that indicated a particular perspective concerning mathematics. Values and beliefs also referred to the values, dispositions, and inclinations demonstrated by a child.

Social relations concerned: the nature of verbal and social interactions that took place between a child and a significant other; the initiation, invoking, and management of the content, content-related behaviors, values and beliefs, or activity/practice/episode in which a mathematical event took place; possible motivators that appeared evident in prompting a child to employ content or to engage

in content-related behaviors; and/or an analysis of conditions wherein development or learning of mathematical content appeared evident.

I determined characteristics of mathematical events for each child by examining my interpretations of the above four, adapted interrelated dimensions across the child's collection of identified mathematical events.

In addition, I engaged in guided conversations with the children's parents in order to gather and report insights about their own experiences and perceptions regarding mathematics and to inform where appropriate my interpretation of their children's mathematical events and characteristics of these events.

The following set of research questions guided this study:

1. What mathematical events did three African-American children, who were approximately 4 years of age and from working-class family backgrounds, engage in within the context of their home or other everyday settings and activities?
 - (a) What mathematical content constituted these mathematical events?
What mathematical understandings did the children demonstrate in these mathematical events?
 - (b) What contexts framed these mathematical events? That is, in what settings (e.g., physical locations, activities, routine practices) did these mathematical events take place and for what purposes or meanings did the children employ or engage in the mathematical content identified in these mathematical events?

- (c) What and whose values and beliefs did these mathematical events instantiate?
 - (d) What social relations shaped the content, managed the context, and/or invoked the values and beliefs that helped frame these mathematical events?
2. What characterized the collection of significant mathematical events in which each child engaged within the context of her/his home or other everyday settings and activities?
 3. What insights emerged from an analysis across the children's collections of significant mathematical events and the characteristics of these events?

In each of Chapters 4, 5, and 6, I present a case of one child in which I address Research Questions 1 and 2. An analysis across these cases addressing Research Question 3 is presented in Chapter 7.

Limitations

This study reflects two limitations regarding data collection and analysis. While I encouraged parents, other caregivers, and children to act as they normally would and to carry on with their typical activities during my visits, at times, my presence did affect their behavior and choice of activities. In the analysis that follows I noted whenever it seemed as though a parent was engaging in an activity for my benefit or for my approval. In addition, as I was the main instrument of analysis for this study, I acknowledge that the nature of my observations and how I interpreted the data was filtered through my understandings and beliefs regarding mathematics, my perspectives concerning mathematics education discourse and practices, and my

empathy² for the families who participated in this study.

² According to Angrosino (2005), “‘Empathy’ ... should be interpreted in a political sense; that is, the researcher takes on a commitment to the community’s agenda” (p. 745).

Chapter 2: Review of Literature

Setting the Scene

Mainstream education research and policy narratives continue to position African-American students as deficient in school mathematics compared to their White peers and, similarly, students who are eligible to receive free or reduce-price lunch as performing below their more affluent peers. For example, data in the National Center for Education Statistics' (NCES) *The Nation's Report Card: Mathematics 2011* (NCES, 2011) indicated that African-American students had the lowest mathematics average scale scores across racial groups in grades 4 and 8. The positioning of African-American students as well as Latino students against their (normalized) White peers is further evidenced in the language used to describe some key findings in the report, for example at grade 4, "There were no significant changes in the White – Black or White –Hispanic score gaps from 2009 to 2011" (p. 1).

Not only are African-American children and children from low-SES backgrounds negatively portrayed while they are in school, but similar deficit perspectives appear in reports (e.g., Lee & Burkham, 2002) regarding young children from these and other underserved groups, their families, and their home experiences before they even start formal schooling with respect to the mathematical knowledge that children from underserved groups have prior to school and the quality of their home experiences in preparing them from school mathematics.

As attention to early childhood mathematics education continues to rise, the deficit positioning of young children from low-SES backgrounds and their home experiences persists. For example, the *MLEC* (NRC, 2009), a report that already has

demonstrated widespread influence in state kindergarten mathematics objects, asserted that children from low-SES backgrounds lack adequate opportunities to learn mathematics through their everyday experiences and that, “These children on average, demonstrate lower levels of competence with mathematics prior to school entry, and the gaps persist or even widen over the course of schooling” (p. 2).

A few studies (Anderson & Gold, 2006; Street et al., 2005; Walkerdine, 1988) have revealed evidence of how taken-for-granted assumptions in school mathematics promoted practices that did not recognize alternative meanings in the experiences and mathematical understandings that young children from underserved groups brought to school. Although some efforts to improve early childhood mathematics education have emphasized the importance of recognizing and building upon not only young children’s early mathematical knowledge, but also their family, cultural, linguistic, and community experiences as well (Clements et al., 2004; NAEYC & NCTM, 2002/2010), there appears to be a lack of research that examines the social aspects (Street et al., 2005) – purpose and setting, values and beliefs, and social relations – that situate the everyday mathematical experiences of young children, in particular those from underserved groups.

Given mainstream education narratives that continue to position African-Americans at the bottom of the racial hierarchy of mathematical ability (Martin, 2009b), the portrayal of young children from low-SES backgrounds and the quality of their home experiences as deficient in current early childhood mathematics education discourse, the increased attention to early childhood mathematics education, and the lack of research that examines the everyday mathematical experiences and

understandings of children from underserved groups, this study focused on 4-year-old African-American children from working-class families before the children experienced any formal schooling. In this way, the study characterized the everyday mathematical experiences and understandings of the participating children prior to formal schooling. It is these prior knowledge and experiences that seem to be the focus of early childhood mathematics education discourse, regardless of perspective (i.e., deficit or different) employed.

Overview of Chapter

This chapter provides a review of literature that includes the following: equity in school mathematics and how different perspectives have attempted to address this issue, social/situated perspectives on learning, broader approaches to conceptualizing learners from underserved groups, and research on young children's mathematical experiences and thinking in everyday contexts.

Equity in School Mathematics

Disparities in Student Achievement

The persistent disparities in mathematics achievement across race and class lines on standardized assessments have been well documented in the mathematics education research literature (Secada, 1992; Tate, 1997). While the use of standardized assessments as a measure of student achievement has been critiqued and questioned in their use as a proxy for student ability in mathematics (Secada, 1992), researchers have examined other indicators or trends of student achievement in mathematics to illustrate disparities across race and class in school mathematics. For example, trends in tracking students by ability and similarly, patterns of students who

enroll in advanced placement mathematics courses (Oaks & Lipton, 1996) reflect similar disparities. That is, non-Asian minority children and poor children are consistently overrepresented in lower-tracked mathematics classrooms.

The persistence of these inequities has resulted in efforts to reform school mathematics. Approaches to examining and responding to these concerns vary in terms of contextualizing student achievement disparities and conceptualizing the notion of equity. For the purpose of this review, I distinguish perspectives that reflect mainstream or “success for all” approaches that focus on change within the school mathematics context from those perspectives that are “sociopolitical” and perceive the issue of equity in school mathematics as a broader social issue, one that reflects larger systemic, societal differences in power and privilege.

Defining Equity in Mathematics Education

From the mainstream. The notion of equity appears as a priority in school mathematics reform. The *PSSM* (NCTM, 2000) and the *EYCM* (Clements et al., 2004) both placed equity, literally, first in their recommendations for school mathematics programs. While the NCTM does not specifically define equity it does state the following, “Equity... demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students” (NCTM, 2000, p. 12). That is, all students regardless of background should have the opportunity to and be supported in learning mathematics. The major concern is that all children develop mathematical understandings, in both content and process, as perceived by the NCTM.

From a sociopolitical perspective. Scholars taking on sociopolitical perspectives argue that inequities in school mathematics reflect social inequalities and injustices (Apple, 1996; Ladson-Billings, 1997; Secada, 1989; Stanic, 1989; Tate, 1997). From a sociopolitical perspective, disparities in student achievement are the result of not only the practice of school mathematics, but of the larger social and political contexts and arrangements in which schools exist. According to Secada (1989), “Equity in mathematics education...should be construed as a check on whether or not the actions taken in teaching mathematics to students and the social arrangements resulting from those actions are just” (p. 24). This characterization or definition of equity in mathematics education suggests that school mathematics practices must be examined in terms of student outcomes, not merely in terms of achievement on standardized tests, but also in terms of the implications such practices have on perpetuating the academic and social positions of marginalized groups.. In addition, working toward equity in learning school mathematics from this perspective emphasizes the use of mathematics to enable students not only to participate in the economic sector but more importantly, to examine critically social injustices in their local community and broader settings (Frankenstein, 1990; Ladson-Billings, 1997; Secada, 1989; Tate, 1995).

From a sociopolitical stance the discipline of mathematics and school mathematics are value laden (Bishop, 2002; Civil, 2002; Gonzalez, Andrade, Civil, & Moll, 2001). That is, particular forms of mathematical knowledge and its practice as well as perceptions of how mathematics should be taught and learned are legitimized over others and therefore must be critically reexamined (Stanic, 1989). In this sense,

sociopolitical scholars are willing to question taken-for-granted assumptions of school mathematics practice, its nature, and its worth (Stanic, 1989).

Problematizing Mainstream Attempts

To Address Equity

A review of education reform documents from the early-1980s through the early millennium (e.g., National Commission on Excellence in Education, 1983; NRC, 1989; NCTM, 1989, 2000) reveals a transition in rationale underlying the emphasis of “mathematics for all” expressed in these documents. Earlier documents emphasized mathematics for all from a perspective of preparing a work force that could support the U.S. in competing in an increasingly technological global economy. Secada (1989) referred to this as “enlightened self-interest.” That is, reform efforts arose primarily, not as a result of concerns for social justice, but as the result of U.S. concerns over its position in the global economy. Reform agendas centered on changing school mathematics practice by preparing better teachers of mathematics and rethinking and improving instruction.

By 2000, the rationale underlying school mathematics reform, from the NCTM perspective, focused less on preparing a future work force toward one where all students are given equitable opportunities to learn and develop deep understandings of mathematics. In addition to developing conceptual and procedural knowledge of content, the notion of teaching mathematics for all included an emphasis on fostering students’ mathematical processes such as reasoning and communicating. NCTM’s vision of school mathematics reform included a call for student-centered instruction, meaningful problem solving, and professional

development for teachers. More generally, this reform effort centered on teaching mathematics for understanding to all and on creating change within the schooling context.

Despite these reform efforts aimed at teaching mathematics for all, student disparities across race and class persist. According to Stanic (1989),

if the inequity in school mathematics is in any way a result of unjust inequalities in the wider society, then it is naïve to assume that equity in school mathematics will result in a reform effort that focuses on schools and is not linked to reform of the wider society. (p. 61)

In this sense, the problem of disparities in student attainment in school mathematics is deeply entrenched in the social, economic, and political institutional practices of the U.S. Thus it is not enough to create change within the schooling context alone.

Meaningful progress toward equity in school mathematics may benefit more so from efforts that also consider and examine the larger social and political milieu in which schools exist.

Approaching Research on Equity in Early Childhood

Mathematics Education from a Sociopolitical Perspective

Researchers have applied a range of theoretical perspectives to examine and understand the persistent disparities in student mathematics achievement across race and class. There are three prominent types of perspectives that seemed to emerge in the literature – deficit, cultural difference, and sociopolitical theories. For the purpose of this review, I will focus on the latter two as these perspectives support a sociopolitical perspective on research on equity in early childhood mathematics

education. In particular, I will provide an overview of both perspectives and then I will focus on those features that seem most salient in characterizing each perspective in terms of (a) how each understands the mathematics education of students from underserved groups, attending specifically on issues regarding student achievement, and (b) what recommendations, if any, each perspective can offer to address equity concerns in early childhood mathematics education.

Cultural difference perspectives. These theories contend that many poor and minority children have been underserved in schools and thus have persistently underachieved in mathematics because an incongruous relationship exists between minority and poor students' cultures and home experiences and the dominant culture (i.e., White, middle-class culture) they face in school (Banks, 1988/1994). From this perspective, researchers further argue that the culture of school mathematics privileges some students over others (Bernstein, 1975; Delpit, 1988; Lareau, 2000; Lubienski, 2000; Zevenbergen, 2001). Zevenbergen (2001) using Bourdieus' concept of habitus, that is "the embodiment of culture...[that] provides the lens through which the world is interpreted" (p. 202), summarized this point,

The structuring practices of the field of mathematics values particular practices over others so that those students for whom there is a greater synergy between the home practices and the school practices, and hence habitus, the greater the chance of being constructed as an effective participant and learner of mathematics. (p. 203)

From a cultural difference perspective, disparities in student mathematics achievement across race and class result from the "greater synergy" that exists

between the home practices and experiences of White, middle-class children and the practices employed in school mathematics settings (Lubienski, 2000; Zevenbergen, 2001).

Researchers who ascribe to this perspective acknowledge that the incongruous cultural relationship between school practices and students from underserved groups stems from larger societal inequities located well beyond the classroom environment (Delpit, 1988; Lubienski, 2000; Zevenbergen, 2001). While cultural difference perspectives may advocate for larger social reform as a necessary vehicle for transforming inequities in schools, these perspectives primarily focus analysis and change efforts on the interactions that take place within the classroom context. More specifically, much of the literature reflecting cultural difference perspectives examines the nature of classroom discourse and how children's understanding of and facility with dominant language and communicative patterns used in schools influences their achievement and positionality in school settings (Bernstein, 1975; Delpit, 1988; Heath, 1983; Lareau, 2000; Lubienski, 2000; Zevenbergen, 2001).

In focusing on classroom interactions, researchers applying cultural difference perspectives suggest that early childhood mathematics teachers undertake changes in their practice to accommodate the needs and experiences of diverse students. More specifically, researchers recommend that teachers become knowledgeable of their students' understandings and home or community interaction practices and incorporate these into students' school mathematics experiences. By understanding the cultural contexts and discourse practices of students and integrating these

meaningfully into classroom mathematics activities, schools can afford more students, particularly those from underserved groups, not only increased positive attainment in school mathematics but also the opportunity to participate and be successful in a White, middle-class dominated society (Delpit, 1988; A. B. Powell, personal communication, December 5, 2003).

While deficit perspectives view the home experiences and cultures of children from underserved groups as obstacles to learning that may be overcome through acculturation into dominant school culture, cultural difference perspectives instead suggest that diverse students' cultures should be considered as resources that can help inform and thus shape instructional practices (Moschkovich, 1999).

Sociopolitical perspectives. Sociopolitical and cultural difference perspectives both agree that students from underserved groups struggle in school as a result of systemic social and school structures that have historically privileged White, economically advantaged, and, as some researchers may argue, male students. Also, sociopolitical and cultural difference perspectives both advocate that teachers must understand and apply their students' diverse home experiences and knowledge within the mathematics classroom context. However, unlike cultural difference perspectives, which focus analysis and change primarily on teaching practice within classroom settings (i.e., at the micro-level), sociopolitical perspectives consider a broader, macro-level lens for analysis and change based on the presumption that schooling is a political act and therefore change in schools has larger societal implications. Sociopolitical perspectives therefore advocate for agency among teachers, students, and researchers to challenge the status quo regarding student achievement and

attainment in school mathematics and to link this critique toward action that addresses larger societal inequities (Villegas, 1988/1994). Sociopolitical perspectives aim to transform both schools and society with the ultimate goal of creating social justice (Freire; 2002; hooks, 1994; Ladson-Billings, 1995; Ladson-Billings & Tate, 1995).

From sociopolitical perspectives, structural inequities in society influence school organization, classroom practices, and student outcomes. As a consequence, schools privilege White, middle class students while marginalizing students from underserved groups. In turn, schooling inequities perpetuate social injustice. That is, sociopolitical perspectives recognize the symbiotic relationship between school change and social change (Anyon, 1997). Given the persistent low achievement of poor and minority students on standardized mathematics assessments, sociopolitical perspectives contend that the practice of mathematics education perpetuates the marginalization of these groups within the larger society. Low performance in mathematics often translates into fewer opportunities within school to learn substantive mathematics and limited employment options in the overall work force.

Secada (1995) argued that, in the context of mathematics education research, "... equity must develop a critical dimension ... equity-based analyses should draw on multiple disciplines and voices [in order to] create ways of talking about research and reform that allow for more textured analysis of phenomena" (p. 160-161).

Research that recognizes and examines the complex nature underlying the everyday mathematical experiences and understandings of young children from undeserved groups appears to offer one layer of analysis that can contribute to the broader issue of equity in school mathematics.

Some researchers ascribing to sociopolitical perspectives have called for research on equity in mathematics education to consider situated or social theories of learning (Boaler, 2000a, 2000b; Ladson-Billings, 1997; Stanic, 1989). For example, according to Boaler (2000a), “Situated [or social] perspectives may be helpful for researchers who would like to move mathematics education away from discriminatory practices that produce more failures than successes toward something considerably more equitable and supportive of social justice” (p. 118). Likewise, Ladson-Billings (1997) described mathematics teaching and learning as encompassing “complex situations” (p. 204). She therefore suggested that scholars consider the situated nature of learning in order to fill the “research void” to better understand and address issues of equity in mathematics education (Ladson-Billings, 1997, p. 204). Sociopolitical perspectives therefore support the use of situated, social, or sociocultural learning theories as appropriate lenses with which to conduct research that seeks to understand the home mathematical experiences of children from underserved groups before they enter school.

Social/Situated Theories of Learning

There are a range of insights in how researchers framed their analysis of learning as a socially, contextualized phenomenon. Authors described their theories of situated learning through concepts such as emergent practices (Cobb, Jaworski & Presmeg, 1996; Cobb & Yackel, 1996/2004; Saxe 1988, 1991; Saxe & Burmudez, 1996); learning through apprenticeship, enculturation, and participation (Brown, Collins, & Duguid, 1989; Kirshner & Whitson, 1997; Lave, 1988; Lave & Wenger,

1991; Rogoff, 1990, 2003; Vygotsky, 1978), and learning as a sociocultural process (Crawford, 1996; Forman, 1996, 2003).

The work reviewed encompassed research that described learning more generally as well as studies that focused on learning within a mathematics education context specifically. From this literature on social or situated perspectives of learning, I identified two themes that seemed particularly generative in conceptualizing the current study. One theme concerned a critique of dominant views of learning (e.g., individual and cognitive perspectives) while the other theme portrayed learning as embedded in social, cultural, and political contexts that shape (but do not determine) and are in turn shaped by individual and collective understandings and meaning-making. A discussion of these two themes along with supporting research follows.

Critiquing Dominant Views of Learning

Social or situated perspectives of learning critique the assumptions that underlie psychological approaches (i.e. Western traditions) to thinking and development. These assumptions include an emphasis on studying individual cognition and the privileging of abstract thinking over other modes of thought. For example, research on learning from psychological perspectives includes methods of examining individual children's understandings in unfamiliar, contrived experimental settings. Claims resulting from such studies presume generalizability across children regardless of their social and cultural histories. In this sense, an emphasis on the individual within a psychological learning perspective does not consider a broader lens of children's cognitive development in relation to their socially contextualized experiences (Kirshner & Whitson, 1997). By emphasizing the individual in cognition,

psychological perspectives inherently devalue learning and intuitive thinking that emerge through children's lived experiences in favor of "higher-level" abstract, rational, and decontextualized contemplative activity (Kirshner & Whitson, 1997; Walkerdine, 1997).

The works of Lave and Walkerdine illustrate a critique of dominant, individual psychological perspectives of learning with respect to developing mathematical understandings in particular. Lave (1997) argued that the culture of school mathematics is one of cultural transmission wherein the learning of mathematics becomes an exercise of accumulating factual knowledge in context-free settings. Schools therefore represent an institutional site for decontextualized learning and the passive acquisition of abstract knowledge to be generalized across real-world situations. This description of the culture of school mathematics reflects a psychological perspective of learning.

Lave (1997) contended that school mathematics teaching practices and learning activities do *not* promote context-embedded cognition, active meaning making, and intuition. Rather than constructing knowledge and making sense of the underlying principles of mathematics, children unintentionally acquire understandings of a school mathematics culture that include the beliefs that doing mathematics means following procedures of symbolic manipulation and working independently to complete textbook exercises of increasing difficulty (Brown et al., 1989; Lave, 1997). In the cases of many students from underserved groups, they experience school mathematics as a culture of failure (Lave, 1997).

Walkerdine (1988, 1997) framed her critique of dominant views of learning and related it to school mathematics through a discussion of differences in relations of signification and modes of regulation that reflected individual meaning making in diverse practices or contexts. According to Walkerdine, children make meaning of the world around them based on their personal experiences, their positions within these experiences, and the meanings they associate with these experiences. Furthermore, children's sense making experiences include an emotional level of understanding that is associated with particular discourse practices and activities.

In Walkerdine's (1988, 1997) research with young children from different economic backgrounds, she found that children from working-class families constructed meanings of *more* and *less* differently than their middle-class counterparts. The children from middle-class homes constructed meanings of *more* that were associated with comparisons of quantities. However, working-class children constructed meanings of *more* that were associated with a practice on their mothers' part to regulate their children's consumption of commodities (e.g., food). In the cases of many working-class children, the meanings of *more* and its opposite *no more* also conjured up negative emotions or feelings.

Given the children's different experiences with and meanings of *more* and the inclusion and presumed relational meanings of *more* and *less* typically employed in early school mathematics classrooms, Walkerdine argued that these findings have important implications for school mathematics. School mathematics practices, in the tradition of psychological views of learning, not only privilege abstract and logical reasoning but also in turn suppress emotional and intuitive thinking (1997). The

subjective meanings of children's experiences and practices are not valued and are therefore marginalized in the school context.

According to Walkerdine (1997), "The 'truth' about mathematical performance contains the fears and fantasies about those calculating others [i.e., girls, Black children, and working-class children] who threaten the very dominance of the government of reason" (p. 70). Furthermore, as Heckman and Weissglass (1994) suggested,

... to effectively teach and effectively learn in situated cognition and/or cognitive apprenticeships, many traditional values and beliefs must make way for more democratic concepts in which assumptions of the dominant culture are questioned and cultures of the students are respected. (p. 31)

That is, to consider learning as a socially, contextualized phenomenon requires a critique of deeply entrenched assumptions upheld by psychological perspectives of individual cognition. By doing so, social or situated perspectives of learning offer "a break from the hegemony of individual psychology" (Kirshner & Whitson, 1997, p. 6).

*Learning As Embedded in Social,
Cultural and Political Contexts*

According to the literature on social or situated perspectives of learning, learning is embedded in social, cultural, and political contexts that shape (but do not determine) and interact with individual and collective understandings and meaning making. That is, children's life experiences and the practices and activities in which

they engage form the contexts in which children learn and make sense of their experiences and the world in which they live. Furthermore, by making sense of and interacting within multiple systems, children can subsequently reshape these contexts.

An analysis of children's learning from a social or situated perspective takes into account the rich, contextualized experiences of children, from the more immediate settings and practices that children experience in their everyday lives to the larger, social and political arenas in which their experiences are situated. Thus, multiple aspects of children's learning and meaning making interact and are legitimized in children's understandings. Children's beliefs, understandings, and content knowledge are considered alongside their intuitions, emotions, and practice-based (or practical) knowledge as situated within their immediate settings and the broader contexts in which they live. Because the construct of learning is itself embedded in social, cultural and political contexts, what is meant by and valued in learning is socially, culturally, and politically defined. As such, an examination of learning must also consider these broader contexts (Kirshner & Whitson, 1997).

As learners make sense of and interact within multiple systems, they can subsequently reshape these contexts. That is, the social, cultural, and political contexts in which children live are dynamic entities that evolve through individual and collective sense-making, participation and practice within these settings. Thus, learning is a social, cultural, and political process by which individuals and collectives participate in understanding, sustaining, and redefining the contexts of their lived experiences.

By presupposing individuals' agency to influence and subsequently change the broader contexts of one's experiences, a social or situated perspective also accounts for differences that occur among children's learning. Therefore, while learning is embedded within contextualized experiences, children make sense of these experiences and their world in individual, unique ways that are not fully determined by the social, cultural or political contexts in which they live.

Social or situated perspectives of learning recognize that individual differences matter (Boaler, 2000a) and relate individual learning to the evolving contexts in which learning occurs (Cobb, 2000). According to Lerman (2000), "the greatest challenge in research in mathematics education from the 'social' is to develop accounts that bring together agency, individual trajectories, and the cultural, historical and social origin of the ways people think, behave, reason, and understand the world" (p. 36). Thus, although social or situated learning perspectives call for a critique of dominant, psychological views of learning, these perspectives do not suggest an all-out abandonment of individual cognition. Rather, social or situated perspectives of learning attempt to bring together and understand the relationship between the sense-making individual and collective and the complex, multiple contexts in which they live.

Learning in Context

The notion that learning is embedded in social and cultural contexts reflects the work and ideas of several researchers. For example, Rogoff (1990) described the process of children's learning as an "apprenticeship in thinking." In particular Rogoff (1990) stated that, "children's cognitive development is embedded in the context of

social relationships and sociocultural tools and practice” (p. 7-8). Brown et al. (1989), proposed similar ideas within the concept of “cognitive apprenticeship.” Brown et al. (1989) argued that “cognitive apprenticeship” could be a useful framework to develop children’s learning by engaging students meaningfully in the domain of mathematics within a mathematics classroom context. From this perspective, teachers need to create opportunities and provide guidance in the social atmosphere of a classroom setting for students to participate in authentic activities and use meaningful tools that reflect the principles and practices of mathematics (Brown et al., 1989).

According to Rogoff (1990), “thinking and learning are functional efforts by individuals to solve specific problems of importance in their culture, and that developmental courses vary in their goals rather than having a universal endpoint to which all should aspire” (p. 116). The work of Carraher, Carraher, and Schliemann (1985/2000), Saxe (1988, 1991), and Saxe and Bermudez (1996) supports Rogoff’s (1990) discussion of social relationships, sociocultural tools and practice, solving specific problems, and cultural goals. Carraher et al. (1985/2000) and Saxe (1988) observed school-age children who were selling candy in the streets of Brazil using informal procedures that they developed to solve mathematics problems within their selling practice (e.g., buying candy to sell, pricing candy for sale, calculating sums of purchases, handling currency). Being able to mentally compute in the practice of selling candy served as an important functional role if sellers were to profit financially from this activity. In their research, Carraher et al. (1985/2000) and Saxe (1988) found that although the sellers were rather adept at solving problems within the context of their selling practice, these strategies did not transfer to formal school

contexts as the sellers were not able to solve similar computational problems as traditionally presented in school mathematics settings.

Lave's (1988, 1997) research on "knowledge in practice" also demonstrated both the robust nature of learning through apprenticeship as well as the idea of learning as a context-embedded, dilemma-driven functional effort. Lave's (1997) description of the process of becoming a master tailor reflected learning through apprenticeship. In a different study, her accounts of Weight-Watcher participants (Lave, 1988, 1997) and their facility in accurately measuring and calculating quantities of food demonstrated how a dilemma or goal-driven need, in this case to lose weight, provided a meaningful context in which participants developed reliable and efficient procedures for portioning their food intake. These examples of research exemplify how cognition is embedded in social and cultural contexts.

As mentioned previously, social/situated perspectives contend that learning is embedded in social contexts and that individuals and collectives play an active role in making sense of and shaping the local and larger contexts in which they participate and live. Learning thus becomes a social, cultural, and political process by which individuals and collectives participate in sustaining or redefining the contexts of their lived experiences.

The idea of agency of individuals and collectives within this perspective draws from the work of Cobb (2000), Cobb et al. (1996), and Cobb and Yackel (1996/2004). According to these researchers and their work in the local context of a mathematics classroom community, school mathematics practice was viewed as an emerging phenomenon that was co-constructed by the participants in the community

(i.e, teacher and students). That is, the classroom setting was not considered to be a place wherein students would be fully enculturated into a static practice of mathematics. Cobb and fellow researchers cautioned that a perspective of enculturation seems to suggest assimilation, thus characterizing sociocultural theories of learning as deterministic. Rather than viewing students as passive participants in acquiring mathematics classroom culture, students were active members in the mathematics classroom community and therefore could shape the context in which they were learning (Cobb et al., 1996). Thus, individual and collective learning were embedded in the practice of school mathematics while the context of mathematics practice emerged through both an individual and a collective process of participation.

*Conceptualizing the Mathematical Experiences of
Children from Underserved Groups Across Contexts*

For young children from underserved groups, the experiences they have at home or in their communities versus what they encounter in formal, early childhood settings may be quite different in many aspects, not just with respect to mathematics. Children from underserved groups, and in particular children of color, must learn to navigate between these contexts, interpreting and managing the different practices and meanings that are situated in each and negotiating their developing sense of identity(ies) across their terrains of experience.

Research indicates that racial socialization plays an important role in promoting the cognitive development of children of color (García-Coll et al., 1996; Harrison, Wilson, Pine, Chan, & Buriel, 1990). Caughy, O'Campo, Randolph, and Nickerson (2002) found that African-American parents who engaged their children in

rich racial socialization practices at home had preschool children who demonstrated greater amounts of factual knowledge and more developed problem-solving skills. Cognitive measures in this study included a mental subtest involving triangles and an achievement subtest on arithmetic. Murrell (2009) contends, “To be both African American and a school achiever means that developing a healthy (i.e., ego integrated) racial identity is not an option but a necessity” (p. 101).

Much of the research in mathematics education that focuses on understanding the school mathematical experiences of African-American students, especially those who are considered successful in mathematics, involves students from upper-elementary through post-secondary educational contexts and oftentimes examines constructs of identity (e.g., Martin, 2000, 2006; Spencer, 2009) and socialization, both in and out of the mathematics classroom (e.g., Martin 2000, 2006). This body of work suggests that the school mathematics experiences of successful African-American students involved more than just being afforded high-quality mathematics instruction.

Murrell (2009) asserts,

Learning attainment and school success is really a whole-being enterprise requiring attention to how the social and cultural climate supports or negates development of African American learners (e.g., Grantham & Ford, 2003; Mercado, 2001; Nir, 2001; Oyserman, Bybee, & Terry, 2003; Rodriguez, Bustamente-Jones, Pang, & Park, 2004; Roth, 2004; Rowley & Moore, 2002).
(p. 101)

As Murrell (2009) suggests, in order to attend to the social and cultural climate, and therefore foster the mathematical development of African-American students, teachers must: recognize how African-American students are positioned in school, and more specifically in mathematics education practices; understand how African-American students make sense of their experiences in school, at home, or in their communities, and how they navigate across these climates; and develop an awareness of and sensitivity to the identity(ies) that African-American students develop in the climate of school and in their home communities.

Anderson and Gold's (2006) pilot study of the home-to-school numeracy practices and mathematical identities of four children attending an urban preschool has begun to shed light into how the construct of identity may be employed with young children. Drawing on a perspective of mathematics as a situated social practice, the researchers observed children engaging in mathematical activities both in their preschool setting and at home and examined the ways in which meanings from these experiences traveled with children across these two contexts.

Findings from Anderson and Gold's (2006) study revealed that adults from each setting perceived children's engagement in mathematical activities according to expectations relative to a given context and that these adults constituted children's identities differently in ways that reflected the social context in which such meanings were formed. For example, in the case of a child named Danny, at home he developed (cheating) double-counting strategies for playing the game *Chutes and Ladders* that his grandmother encouraged and viewed positively as the goal for this activity was

for Danny to win over his grandmother. In this setting, Danny's grandmother viewed him as being "very smart" when he employed these strategies.

However when Danny brought his practice of double counting while playing *Chutes and Ladders* from home to school, where the teacher enforced a no-cheating policy, Danny's strategies were viewed as inappropriate, and his teacher identified him as "a cheater." Because the teacher was not aware of Danny's practice and experiences of playing this game at home, she positioned him as a cheater rather than drawing on, and thus affirming, the mathematical understandings he brought from home and developing with him strategies to participate in the game at school in ways that were in accordance with classroom expectations regarding cheating.

Anderson and Gold (2006) developed vignettes from their observation data and used these for teacher professional development and praxis. The researchers argued that further research of this kind is necessary "for understanding implications of sociocultural factors in underachievement" (p. 284). This study further underscores the need to examine and understand children's mathematical experiences and knowledge in the context(s) in which these are situated.

Research on Young Children's Everyday

Mathematical Experiences and Understandings

Few studies conducted in the U.S. have directly investigated young children's informal mathematical activities and understandings as they naturally occurred in their everyday settings before they entered school (Guberman, 1999, 2004; Tudge & Doucet, 2004). Some studies that have attempted to understand the mathematical experiences and understandings of children, including children from underserved

groups, before they entered school either: occurred in formal daycare settings (Ginsburg et al., 1999; Seo & Ginsburg, 2004), relied on mothers' self-reports of children's home practices that involved early mathematical concepts such as counting (Saxe et al., 1987), or focused on children's experiences with academic-centered mathematics activities in their home settings (e.g., Tudge & Doucet, 2004).

Beyond the field of mathematics education, other educational researchers concerned with equity issues have called for studies that address the role of family and home experiences of young children and the relationship of these to children's learning in school. In particular, Slaughter and Epps (1987/1994), in raising their concerns about the home environment and academic achievement of African-American students, stated "While we have some idea of what would be beneficial to children's early reading, we have little idea of how these processes manifest themselves in the roles enacted by culturally and socially different families in their natural settings" (p. 82). Given the research to date, a similar statement can be made regarding our knowledge of children's early mathematics experiences and understanding in the context of economically and racially diverse family home environments.

Ginsburg and his research associates examined the mathematical activity of 4- and 5-year-old children in formal day-care settings (Ginsburg et al., 1999; Ginsburg, Lin, Ness, & Seo, 2003; Seo & Ginsburg, 2004). While these studies were not situated in the home settings of children, findings from this collection of research did begin to shed light on the frequency and nature of young children's informal mathematical activity. Based on data from naturalistic observation methods during

children's free play in formal day-care settings, children's activities were described using the following researcher-created mathematical content codes:

Classification: Systematic arrangement in groups according to established criteria,

Magnitude: Statement of magnitude or comparison of two or more items to evaluate relative magnitude,

Enumeration: Numerical judgment or quantification,

Dynamics: Exploration of the process of change or transformation,

Pattern and Shape: Exploration of patterns and spatial forms, and

Spatial Relations: Exploration of positions, directions, and distances in space.

(Seo & Ginsburg, 2004)

Although these children were observed in formal day-care settings where an adult/teacher was present, researchers argued that children spontaneously engaged in mathematical activity of a variety of types without the prompting of an adult or through explicit instruction (Ginsburg et al., 1999). Researchers observed children spontaneously engaging in activities, including playing with familiar objects (e.g., blocks, Legos) typically found in day-care settings, in which children's mathematical understandings naturally emerged. Thus, the context in which children engaged in mathematical activity helped shape the kinds of experiences and practices that occurred.

Participants in these studies somewhat varied. In one study, researchers observed low-income, African American, and Latino students (Ginsburg et al., 1999). In another project, researchers compared the mathematical activity and complexity of

activities between American and Chinese children from low- and middle-class backgrounds (Ginsburg et al., 2003). In the most recent study the following four groups were compared: low-income African-American and Latino students; middle-income African-American, Latino, and White students; and children from upper-income families, all of whom were White (Seo & Ginsburg, 2004).

In general, all three studies reported that the 4- and 5-year-old participating children engaged in significant amounts of mathematical activity and therefore suggested that children, including lower-income and non-Asian minority children, come to school with adequate amounts of informal mathematical knowledge necessary to be successful in school mathematics (Ginsburg et al., 1999). In the two comparative studies, no differences were found across groups with regard to the following: the occurrence of mathematical activity, the relative frequency of different types of activity, and the level of complexity of different types of mathematical activities. The only exception reported that Chinese children engaged in considerably greater amounts of pattern and shape mathematical activity as compared to American children (Ginsburg et al., 2003).

In all three studies, children overwhelmingly engaged the most in mathematical activities involving patterns and shape. Data of frequency of mathematical activity from the two more recent studies reflect similar trends: following pattern and shapes, children engaged the next most in magnitude and enumeration, followed by dynamics and spatial relations, with the least activity involving classification. With the exception of pattern and shapes, this trend did not appear in the earliest of these three studies, involving low-income African American

and Latino children. In this study, wherein slightly different content codes were employed, children engaged in the following mathematical activities in order of decreasing frequency: pattern and shapes, dynamics, relations, classification, enumeration.

The set of studies conducted by Ginsburg and his research associates presents a limitation with respect to understanding the everyday mathematical experiences of young children. These studies focused solely on young children's mathematical experiences in formal day-care settings. It can be argued that the materials (e.g., blocks, Legos) typically present in these settings are intended to foster children's mathematical development, even through children's spontaneous play with these objects. Because children's home or other familiar everyday settings are diverse and vary widely, well beyond the availability of educational resources, observations of children's everyday mathematical activities in these settings would most likely yield findings distinct from those found in these studies.

While Ginsburg et al. (1999, 2003) claimed that children engaged in significant amounts of mathematical activity such that children enter school with sufficient amounts of informal mathematical knowledge to be successful in school mathematics, in light of research that reports persistent disparities in student mathematical achievement across race in class (NCES, 2011; Tate, 1997) occurring as early as before students enter kindergarten (Lee and Burkam, 2002), Ginsburg, Klein, and Starkey (1998) also posited that some children come to school with developmentally immature, informal mathematical knowledge. This argument presumed that differences in children's mathematical development occur to the extent

to which families can provide their children with particular activities and discourse practices that will help them succeed in school mathematics. The tension between these empirical findings and assumptions requires further examination beyond the scope of this review.

Tudge and Doucet (2004) observed 3-year-old children's mathematical activities in their home and everyday settings. Tudge and Doucet recruited African-American and White children from working-class and middle-class families through community-based approaches to participate in this study. Researchers observed children's mathematical and literacy activities over a total of 18 hours in the span of one week. Naturalistic observations and time-sampling methods were used to collect data of activities occurring in the children's homes, child-care settings, someone else's home, and other public settings.

In particular, Tudge and Doucet's (2004) study focused on observing children's mathematical activities within one of two categories: academic lessons and play with academic objects. The first category involved "attempts to impart or elicit information ... involving competencies in mathematics" (p. 26) and the second involved playing with objects that appeared to have academic relevance, such as shapes, with no attempts to explicitly impart or elicit information.

Overall, Tudge and Doucet (2004) found that children engaged far less in mathematical activities as compared to literacy activities. The researchers contended that their findings seemed to reflect the U.S. culture's greater emphasis on literacy in the early years. Of the mathematical activities that Tudge and Doucet observed, a great deal of individual variation occurred, but could not be explained by race or

class. Tudge and Doucet also found that children who spent more time in formal day-care settings did not necessarily engage in more mathematical activities as compared to children who did not attend these settings.

Tudge and Doucet's (2004) research limited the conceptualization of children's mathematical activities to those that focused on academic lessons and play with academic objects. By limiting the conceptualization of children's mathematical activities to these two academic-focused categories, occurrences of children's mathematical activities beyond these categories were not documented and examined. From a sociopolitical stance, which examines taken-for-granted assumptions such as the privileging of formal or academic mathematics, it is critical that future research efforts aimed at examining the everyday mathematical experiences and understandings of young children consider a broader conceptualization of what constitutes evidence of children's mathematical thinking in everyday settings.

In a European study that focused on two preschool-aged, White children and their mothers, researchers compared the conversational interactions of each mother-child dyad around mathematical activities (Aubrey, Bottle, & Godfrey, 2003). The mothers were found to have two distinct pedagogical styles. One mother talked about mathematical ideas embedded in the everyday activities in which her child engaged, and the other mother only talked about mathematical ideas in activities in which learning mathematics was the sole purpose of the activity. Aubrey et al. (2003) found insufficient evidence to report how these parenting styles influenced each child's performance as they entered their first years of school. By focusing analysis of children's mathematical activities solely as observed in the conversational

experiences of the two children, Aubrey et al. suggested that other means of understanding children's mathematical experiences in their home settings are needed, particularly those that examine the context of and cultural tools present in children's home settings. Similar to Street et al.'s (2005) notion of mathematics as social (see Chapter 1), Aubrey et al. further contended that research should also consider children's perceptions of mathematics within their social interactions with their parents as well as how children value different kinds of mathematical activities.

As this review of literature indicates, we still have much to learn with respect to young children's everyday, naturally occurring mathematical experiences and understandings as situated within their home or other familiar, informal settings. Robust evidence from research on social or situated perspectives of learning demonstrates that learning mathematics is a social process that is shaped, but not determined by, the social and cultural context in which children's mathematical activities take place. Future research can draw on Street et al.'s (2005) ideological model of mathematics as social and their analytic concepts to develop analytic tools with which to conceptualize and examine young children's everyday mathematical activities including the complex and broad social aspects (i.e., purpose and setting, values and beliefs, social and institutional relations) that situate these experiences and the content that constitutes the mathematics in these activities.

Chapter 3: Methodology

The purpose of this study was to examine and interpret the everyday mathematical experiences and understandings of three, 4-year-old, African-American children from working-class family backgrounds. In this qualitative study, I employed adapted naturalistic observation (Angrosino, 2005) methods, visiting the children in their homes, informal day care, and other everyday settings in order to collect observation data of children's mathematical activities and to gather information to characterize the children's typical activities and dispositions. I drew on Street et al.'s (2005) broad, ideological model of mathematics as "social" and their analytic concept of a mathematical event (unit of analysis consisting of an occurrence of mathematical activity) (see also Baker et al., 2003; Baker, et al., 2006) in order to frame research questions specifying the meaning of everyday mathematical experiences and understandings.

I identified a collection of mathematical events for each child and examined these in terms of the four adapted interrelated dimensions of content, purpose and setting, values and beliefs, and social relations that constitute the mathematics (content) of an event and the three social aspects (purpose and setting, values and beliefs, and social relations) that situate this content. I then determined characteristics of mathematical events for each child from an analysis of these four dimensions across the child's collection of mathematical events.

In addition, I conducted guided conversations with the children's parents to gather and report insights about their own experiences and perceptions regarding

mathematics and to inform where appropriate my interpretation of their children's mathematical events and characteristics of these mathematical events.

The following set of research questions guided this study:

1. What mathematical events did three African-American children, who were approximately 4 years of age and from working-class family backgrounds, engage in within the context of their home or other everyday settings and activities?
 - (a) What mathematical content constituted these mathematical events?
What mathematical understandings did the children demonstrate in these mathematical events?
 - (b) What contexts framed these mathematical events? That is, in what settings (e.g., physical locations, activities, routine practices) did these mathematical events take place and for what purposes or meanings did the children employ or engage in the mathematical content identified in these mathematical events?
 - (c) What and whose values and beliefs did these mathematical events instantiate?
 - (d) What social relations shaped the content, managed the context, and/or invoked the values and beliefs that helped frame these mathematical events?
2. What characterized the collection of significant mathematical events in which each child engaged within the context of her/his home or other everyday settings and activities?

3. What insights emerged from an analysis across the children’s collections of significant mathematical events and the characteristics of these events?

Observation data for this study was collected from March to July 2006.

Analysis of data took place from that time to 2011.

Participants

Three African-American children, their parents, and their informal home day-care provider, Ms. Franklin, participated in this study. I assigned pseudonyms to all the participants. Table 1 provides a summary of the children, their gender, the month when each child turned 4 years of age in 2006, and their parents. The children are listed in the order in which I met them and secured their parents’ consent to participate in this study. At the time of this study, the children had not yet experienced any formal schooling or attended an early childhood center with an educational agenda.

The families shared some similarities. Each family lived in a rental apartment property located in an urban, lower-income, predominantly African-American community located in the Washington, D.C. metropolitan area. Toya’s mother and

Table 1

Summary of Child and Parent Participants

Child	Gender	Month of Fourth Birthday	Parents
Toya	Female	February	Nicole (Mother)
Anika	Female	March	Christina (Mother)
Aaron	Male	July	Keisha (Mother) and Aaron Sr. (Father)

Aaron's parents lived in the same neighborhood as Ms. Franklin's home day care while Anika's mother lived about 1.5 miles away. At the time of this study, the parents were all in their 20s, had near similar educational backgrounds, and worked in occupations that afforded them a working-class (Gilbert, 1998) income and lifestyle. In particular, Nicole and Christina had graduated high school, Aaron Sr. had earned a General Educational Development credential, and Keisha had completed approximately two semesters of course work at a 2-year community college.

Sources

After a number of unsuccessful attempts of trying to find participants through a familiar contact or chain of contacts, I began cold calling informal, home-day-care providers who were registered as licensed home day-care providers in the hopes that someone would agree to work with me and to help me enlist children/parents to participate in my study.

In order to increase my chances for success, I first identified a predominantly African-American, lower-income neighborhood within the same general area in which I grew up. The elementary school population within this neighborhood was 98% African-American students with 91% of the students receiving free and reduced-priced meals. I presumed these indicators reflected more broadly the demographics of the families living in the identified neighborhood, including families who might use a day-care provider within their community. I also knew that it was important that I had some initial level of credibility and trust with potential day-care providers and families. Even though I am a person of color (Filipino) I felt that if potential participants knew that I had grown up within the local area, this could increase their

level of comfort in interacting with me and possibly encourage them to consider participating in my study.

Once I identified the neighborhood, I used the regional childcare website to search for and to identify contact information for licensed home day-care providers in this neighborhood. From the cold calls I placed to the providers on my list, Ms. Franklin graciously agreed to help me. Ms. Franklin is African-American and at the time of the study she was in her mid-50s, had spent her entire life in the region, and lived in the house where she ran her day care for about 25 years. She reported that she had been working in childcare for over 30 years, but had only had her license since the mid-1990s. Ms. Franklin spoke candidly about her aversion to mathematics, even stating, “I just avoid it” (January 9, 2006 – Visit with Ms. Franklin). Although Ms. Franklin did not have an educational agenda for her day care, she expressed that she felt more confident about providing activities for young children that would support their reading compared to their mathematical development.

Securing Participants

A few weeks after I first spoke with Ms. Franklin, I began to make regular visits to her home day care in order to begin to familiarize myself with and to establish a rapport with the children at the day care, their parents, and Ms. Franklin’s neighbors. At the time of data collection, Toya, Anika and Aaron were the only 4-year-olds attending Ms. Franklin’s day care. Toya had been attending the day care for over a year while Anika and Aaron were newly arrived to the day care, beginning near the time I began visiting Ms. Franklin’s day care. In addition to these children,

two infants and three toddlers regularly attended the day care as did three elementary-aged children who came for after-school care.

Given Toya's mother's long trusting relationship with Ms. Franklin and given Ms. Franklin's endorsement of me, Nicole readily agreed to participate in my study. Subsequently, Anika's mother and then Aaron's parents consented. Coincidentally, I learned that Nicole, Christina, and I all attended the same high school, though at different times. Nicole had already agreed to participate in the study once we learned of this commonality. I believe that this shared high-school affiliation might have played some role in encouraging Christina to participate as we discovered this shared experience prior to her consent. Finally, Aaron's father directly informed me that if it were not for the facts that I was a person of color and that I was familiar with the local community, he would not have agreed to participate.

While I did not offer the children, parents, or Ms. Franklin any compensation for their participation in this study, over the course of my observation visits I occasionally brought them little gifts (e.g., a box of barquillos/Filipino wafer rolls) to show my appreciation for their time and willingness to open their homes to and to share their lives with me.

Defining Mathematical Activity

I drew on Street et al.'s (2005) broad, ideological model of mathematics as "social" and their analytic concept of mathematical events (see also Baker et al., 2003; Baker, et al., 2006) to inform my study and my analysis of children's everyday mathematical experiences and understandings. For the purposes of this study, I employed the phrase "characteristics of mathematical events," or similar phrases

(e.g., “characteristics of these events”), to represent the analytic characterization of patterned uses of or shared mathematics within and across events. In the following I discuss how I operationalized these concepts in my study.

Mathematical Events

According to Baker et al. (2003), a mathematical event is a unit of analysis consisting of an occasion wherein mathematical activity “is integral to the nature of the participants’ interactions and their interpretive processes” (p. 12). In this study, I examined mathematical events in terms of the following set of four adapted interrelated dimensions that constituted the mathematics in and the social aspects situating mathematical activity: content, purpose and setting, values and beliefs, and social relations.

Content. The dimension of content referred to the identified mathematical content that appeared evident in an event as well as an analysis of the mathematical understandings that a child appeared to demonstrate. To determine if an observed activity constituted a mathematical event, I employed the following procedures. I began by drawing on the existing research regarding young children’s mathematical understandings (e.g., Clements et al., 2004; Ginsburg & Seo, 1999), as well as other research literature concerning children’s everyday activities that could support their cognitive development (e.g., Cohen, Stern, & Balaban, 1997; Hansen, 2005) and that were typically considered mathematical in nature. This review provided me with an initial set of content topics and activities for identifying mathematical events and coding data. As needed, I later revised and fine-tuned codes based on fieldwork observations and subsequent analysis of the data.

From the outset, I presumed that I might observe occurrences that I would later contend were mathematical. As such, I allowed for this flexibility in my consideration and analysis of potential mathematical events. In the discussion of findings that follows in Chapters 4 through 6, wherever I identified a content as mathematical that seemed to not be addressed in the extant mathematics education literature, I explained my rationale for identifying it as such. Figure 1 identifies examples of mathematical content and activities that I considered mathematical in nature. If I observed an occurrence that reflected an item on this list then I identified that occurrence as a mathematical event.

Because I did not want to privilege traditional school mathematics topics and therefore miss recognizing other occasions wherein mathematical activity might have take place, I also considered Bishop's (1988) characterization of six universal fundamental activities that are "necessary and sufficient for the development of mathematical knowledge" (p. 182). According to Bishop (1988), these are:

Counting. The use of a systematic way to compare and order discrete phenomena. It may involve tallying, or using objects or string to record, or special number words or names ...

Locating. Exploring one's spatial environment and conceptualising and symbolising that environment, with models, diagrams, drawings, words or other means ...

Measuring. Quantifying qualities for the purposes of comparison and ordering, using objects or tokens as measuring devices with associated units or 'measure-words'...

Number: interpreting, identifying, and employing verbal (i.e., number words) and symbolic (i.e., numerals) representations of elements from the set of natural numbers (i.e., 0, 1, 2, 3, 4...) to signify nominal, ordinal, and/or numeric meanings

- Nominal meanings: number to label
- Ordinal meanings: number to indicate
- Numeric meanings:
 - Expressing number words orally in a count sequence without reference to discrete objects or events (i.e., rote counting or expressing a number word list)
 - Demonstrating concepts of enumeration (i.e., determining a quantity's numerosity), including developing understandings of one-to-one correspondence, cardinality, as well as enumeration strategies such as subitizing, using explicit number words to assign to each item in a quantity while simultaneously physically keeping track of each item in the quantity (e.g., pointing to elements and saying "This is one, this is two..."), and/or counting objects or events with or without one-to-one correspondence and/or cardinality
 - Employing number words to describe or interpreting numerals to identify attributes of physical quantities or entities including relative magnitude of a given attribute across a particular classification of objects
 - Numeric part-total relationships
 - Operations (e.g., adding and subtracting)

Descriptive terms for quantity (e.g., a lot, some, a little, more, less)

Magnitude (e.g., statement of magnitude or judgment of relative magnitude, big/little)

Classification, sorting, identifying differences and similarities of attributes across objects, matching

Exploration and creation of patterns

Geometric thinking (e.g., playing with formal shapes, composition and decomposition of formal shapes and everyday objects, and manipulating real life objects to fit together)

Spatial sense: relative position (e.g., under, over, in front, behind) and navigation or way-finding

Measuring

Putting together jigsaw or board puzzles

Play with board games (e.g., board games that encourage counting or one-to-one correspondence)

Identifying money or pretending to use money and assigning it a numeric value

Talk about time (e.g., asking what time it is)

Figure 1. Examples of mathematical content and activities.

Designing. Creating a shape or design for an object or for any part of one's spatial environment. It may involve making the object, as a 'mental template', or symbolising it in some conventionalised way ...

Playing. Devising, and engaging in, games and pastimes, with more or less formalised rules that all players must abide by ...

Explaining. Finding ways to account for the existence of phenomena, be they religious, animistic, or scientific ... (p. 182-183)

Bishop's (1988) fundamental activities provided an additional lens with which to consider the children's everyday activities as mathematical.

While the examples of mathematical content and of activities that foster mathematical development presented in Figure 1 were based on research focusing on children's early mathematical knowledge and everyday experiences, I did not limit the identification of mathematical activities to those in which the children directly engaged. In this way, I allowed for data collection to include the purposeful mathematical activities of parents or other significant others that were conducted in the presence of or were intended to engage a child and could therefore support the mathematical development of that child. I asserted that such activities could serve as funds-of-knowledge (Moll & González, 2004) for the child.

In addition to identifying the mathematical content evidenced in an event, I also interpreted the child's understanding of that content. In order to interpret the child's mathematical thinking, I examined evidence from the event such as the child's behavior, talk, or engagement with objects and I drew upon my own knowledge of young children's mathematical thinking.

Purpose and setting. This dimension typically concerned the time, location, and activity/practice/episode in which a mathematical event took place and the child's content-related behaviors that transpired, indicating the underlying purpose that appeared to give rise to the content. In this way, when discussing mathematical events, purpose and setting served to set the scene in which a mathematical event occurred. In the chapters that follow, sometimes excerpts from field notes or audio transcriptions data that were included in the findings are sufficient to capture the purpose and setting. Otherwise, I provide additional notes for the reader to clarify this dimension.

Where appropriate, purpose and setting also concerned pertinent background information and insights regarding the activity, practice, or episode in which a mathematical event occurred. For example, if a mathematical event took place at day care during free play, I provided background information regarding what this period typically involved, and I also described the scene in which the mathematical event took place.

Values and beliefs. This dimension included the values and beliefs of a parent, family member, or caregiver, such as those that fostered the activity, practice, or episode in which mathematical meanings (content) emerged or were employed, those that promoted the nature of social relations that occurred around a given content, or those that indicated a particular perspective concerning mathematics.

I interpreted parent's values and beliefs based on evidence that suggested that a value or belief was salient or important for that parent. Oftentimes, values were instantiated through practices that appeared significant in the family's day-to-day

activities. For example, in observing Toya in her home or other everyday settings outside of day care, her Aunt Pam and cousin Ricky were typically present, and I learned from Nicole that she, Pam, and their mother all shared in the responsibility for caring for Toya and Ricky. From these observations, it appeared that Nicole valued maintaining family relationships and the importance of supporting kin. In some cases, parents directly expressed to me that a practice was important to them, and evidence from observation visits confirmed this value. For example, Keisha informed me that she enjoyed and valued reading and emphasized this practice with Aaron. This value of reading was evidenced in the collection of library books that Keisha regularly maintained for and read with Aaron.

Values and beliefs also referred to the values, dispositions, and inclinations demonstrated by a child. These were typically evidenced by observed patterns in a child's behavior. For instance, on one of my home visits with Aaron, he repeatedly urged his mother to read to him and he spent a significant time "reading" his books.

Social relations. This dimension concerned: the nature of verbal and social interactions that took place between a child and a significant other; the initiation, invoking, and management of the content, content-related behaviors, values and beliefs, or activity/practice/episode in which a mathematical event took place; possible motivators that appeared evident in prompting a child to employ content or to engage in content-related behaviors; or analysis of conditions wherein development or learning of mathematical content appeared evident.

For this dimension I noted, for example, if a child initiated a mathematical event and whether or not social interactions were involved. If social interactions were

involved I described the nature of these and examined to what extent these interactions appeared to support the child's engagement in mathematical meanings. For instance, when Nicole overheard Toya inform me that she wore a size 4 pair of shoes because she was 4-years-old and her shirt size was also a 4, Nicole interjected into our conversation and tried to clarify with Toya that her shoe size was not a 4 but rather a 10. A verbal exchange over this issue ensued between Toya and Nicole and I interpreted this interaction including the manner in which each person addressed the other and what this meant in terms of the mathematical meanings regarding number and size that were being employed in this episode.

For each child, I identified a collection of mathematical events that emerged as most salient from an examination of the data. I analyzed each of these events, in general, in terms of the set of four adapted interrelated dimensions of content, purpose and setting, values and beliefs, and social relations. My analysis of each dimension within an event was based on my interpretations of the event itself as well as my interpretations of the event within the broader context of my understandings about the child, the child's family practices, and the child's everyday experiences. My interpretations of each of these dimensions across a child's set of mathematical events were then subsequently analyzed to determine characteristics of these events.

Characteristics of Mathematical Events

For this study, I determined a set of characteristics of mathematical events for each child through an analysis of my interpretations of each of the adapted dimensions of content, purpose and setting, values and beliefs, and social relations

(Street et al., 2005) across the collection of mathematical events that emerged as significant for each child. For example, I examined my interpretations of the dimension of purpose and setting across a child's collection of mathematical events to determine if any themes emerged with regard to this dimension. Any salient findings that resulted from this analysis then became identified as a characteristic of the mathematical events for that child.

Although I determined the characteristics of the children's home or everyday mathematical events through inductive analytic procedures of the data (i.e., my interpretations of content, purpose and setting, values and beliefs, and social relations for each mathematical event identified), findings from Street et al.'s (2005) research on home and school mathematical practices provided some insight regarding what seems to characterize children's home or everyday mathematical activities. In their study, Street et al. found that children's home mathematical activities tend to be:

- Local in that they refer to local situations;
- Bottom-up in that they are concerned with problems sited or chosen in a local setting;
- Purposeful in terms of organizing lives or addressing domestic situations;
- Solution driven with the activity and practices derived from the home's ways of doing and engaging in tasks;
- Occurring through dialogue and negotiation between child and others; and
- Reflective of the children's intrinsic motivation. (2005, p. 32)

*Observing From Within, Approximating an Essence of Everydayness, and
Negotiating Pathways In the Field*

In this study I employed adapted naturalistic observation methods, recognizing my presumption that my presence alone would influence, in some way, the dynamics of the spaces in which I was welcomed and the social interactions in which I became a part. As such, I sought to record observation data of children's everyday mathematical events as they naturally occurred in their day-to-day activities at home, informal day care setting, and other familiar contexts while I assumed the roles of an informal visitor or guest in the children's homes and a helper at Ms. Franklin's day care.

I chose to assume these roles as I believed that in doing so, I could encourage the parents, Ms. Franklin, and the children to be not only more comfortable with me in general, but also to be more at ease with having me in their spaces. My assumption was that, to the extent possible, they would carry on as they normally would, including maintaining their daily routines and common activities, natural behaviors, and ways of interacting with one another, more so than if I had tried to observe formally from a far. By assuming the role of informal visitor or helper, I attempted to observe from within the everyday experiences and activities of the children. This approach also helped me to establish a sense of rapport with the children and their families.

In my role as helper at Ms. Franklin's day care I assisted in tasks such as serving lunch, cutting and maintaining the stock of paper towels, and managing children during excursions outside the day care. Given the informal nature of the day

care, there were several periods of unstructured time that did not require adult assistance, and therefore I was able to observe the children engaging in mathematical events from a distance. During these periods, I oftentimes sat with my field journal in hand in a chair near the center of the day care space and the children soon began to recognize that the chair was my spot. The children would occasionally come over to me to see what I was writing or to ask to write in my journal.

While visiting with the families in their homes, in order to keep the atmosphere casual, I tried to act as an informal guest in the same way I would if I were visiting friends and their families in their homes. This meant that I did interact with the families, but I tried to do so mainly within the context of their typical activities. There were times, however, when I did feel that parents seemed focused on entertaining me as a formal guest and, when cognizant of that, I did note these occasions in my discussion of the findings.

By assuming the roles of helper and informal guest I became, in a sense, a part of the children's everyday experiences during the period of data collection. In doing so, the children's interactions with me were also considered in my analysis. Because of the nature of this study, however, I attempted to refrain from initiating or introducing any mathematical activities or ways of speaking around mathematical activities that I did not observe as already being part of the participants' natural, everyday behaviors or settings. In the analysis that follows for all three children, where appropriate, I discuss the extent to which my involvement in the children's everyday activities may have shaped or contributed to the mathematical events observed.

In my roles as an informal guest in the children's homes and as a helper at day care, I sought to approximate an essence of everydayness for this study.

Approximating an essence of everydayness, on the one hand, acknowledges that my presence in some way colored aspects of the children's observed everyday experiences. On the other hand, in my approach to data collection I attempted to minimize my potential influence on the children's everyday activities, particularly during moments wherein I observed them spontaneously engaging in mathematical events in the course of their daily activities and in their interactions with familiar others.

When negotiating the path in which my participation would proceed in the field in response to mathematical events I observed, I relied on the following to help guide me: (a) a priori concerns regarding my overall aim toward approximating an essence of everydayness, and therefore preserving, to the extent possible, the candid nature of the observed activities, practices, episodes and responses of the participants; and (b) in-the-moment, context-based decisions informed both by my observations and interpretations of participants' behavioral/verbal engagement and understandings at a given point in time, as well as my ongoing observations and evaluations of their prior mathematical events and understandings. For instance, if a child initiated and engaged me in discourse that I evaluated as mathematical, I typically interacted with the child accordingly, following her/his lead per se.

When reflecting on and analyzing the data, I found some moments wherein the pathways I chose to follow during data collection did not appear to serve best my efforts to approximate an essence of everydayness or to capture participants' candid

thoughts and actions that seemed most pertinent to this study. As such, I address these concerns where appropriate in my analysis of the children's mathematical events and in my discussion of the children's parents' comments and reflections.

I revisit and illustrate these methodological concerns of observing children's mathematical events from within their everyday experiences and activities, approximating an essence of everydayness, and negotiating my pathways as a researcher during data collection in my first case-study discussion, that of Toya.

Procedure

I employed adapted naturalistic observation methods in order to record observation data of children's everyday mathematical events. I collected observation data from March through July 2006. Ms. Franklin maintained an open-door policy with me that allowed me flexibility in scheduling my observation visits in this setting. I worked with parents to identify days and times when I could visit and observe in their homes or during other family activities in settings outside of their homes.

The number of visits I completed and the number of hours I observed each child in different settings varied along with the amount and quality of the data I collected across each child. For example, I completed more visits with Toya in her home and other everyday settings outside of day care compared to the other children. My observations of Toya also reflected a greater range of her typical activities over a course of a week, more so than the observations I was able to conduct with Anika and Aaron. In addition, although I completed 14 hours of observation with Toya and Aaron in each of their homes, a significant portion of my initial visits with Aaron at

home seemed to center around Aaron’s parents’ efforts to engage me as their (formal) guest. Table 2 presents the frequency of observation visits completed for each child.

Depending on the setting in which I observed the children, I employed slightly different methods to document my observations of what occurred. When I observed the children in their informal day care setting, I recorded jottings in a field journal and completed field notes afterwards. Alternatively, I utilized audio-recordings to capture talk and speech during my observational visits with the children in their homes and other everyday settings (e.g., grocery store) and later transcribed these. To a lesser extent, I also recorded jottings in a field journal when possible during observations in these settings and completed field notes afterwards. In addition to maintaining written accounts and audio-recordings of visits, I also tried to capture events visually through the use of digital photography.

During initial observation visits, I conducted guided conversations with parents and Ms. Franklin about where they lived, their experiences in school mathematics and general education background, their employment, their perceptions

Table 2

Summary of Frequency of Observation Visits

Location	Toya		Anika		Aaron	
	Visits	Hours	Visits	Hours	Visits	Hours
Home	5	14	4	9	3	14
Other	3	9	-	-	-	-
Day Care	5	16	3	9	5	16
	13	39	7	18	8	30

and use of mathematics both at work and at home, and their perceptions regarding when they thought their children engaged in or had engaged in mathematical activities in their everyday experiences. I also asked the parents about their children's typical activities over the course of the week. When possible, I followed-up informally with parents regarding inquiries that arose for me during a visit or from previous observation visits.

Analysis

I followed a four-tiered analytic process, using inductive analytic procedures, to address the first two research questions for each child. First, I utilized a qualitative research software program in order to code data from field notes and transcriptions of audio-recordings with a focus on identifying mathematical events. I created codes based on the collection of content topics and activities prepared prior to data collection, insights gained from observing the children in their everyday settings, and patterns that emerged through the process of coding the data. Codes indicated not only mathematical content but also reflected the nature of an event indicating, for example, who initiated a mathematical event (e.g., adult, child, or significant other), whether a mathematical event was spontaneous or prompted, if the child experienced a mathematical event directly or indirectly, or if a mathematical event seemed formal (academic) in nature. I then sorted the data based on codes and examined this data for themes. In each child's case, I identified content areas that seemed to emerge as salient in this initial sorting.

In the second-tier of my analytic process, I created a new document for analysis that included all of the events included in the content areas that I identified

from the first tier of analysis. This document included the raw data from those events. Each mathematical event was labeled and organized based on content area. As appropriate for each mathematical event, I included additional notes to clarify the content observed or to clarify the child's understanding or use of the content in an event. I then analyzed the events collected and annotated in this document, in particular examining events for robustness of content evidenced. From this analysis, I identified a set of mathematical events within a particular content area(s) that emerged as most significant for each child. My examination and interpretation of the children's everyday mathematical experiences and understandings in this study focused on these collections of mathematical events.

In the third-tier of analysis, I created a new document to examine the set of mathematical events identified in the previous tier of analysis. Specifically, I constructed a table where for each mathematical event I addressed each of the four adapted interrelated dimensions of content, purpose and setting, values and beliefs, and social relations. My analysis of each dimension within an event was based on my interpretations of the event itself as well as my interpretations of the event within the broader context of my understandings about the child, the child's family practices, and the child's everyday experiences. Where appropriate, I drew on existing literature to inform or further my interpretations. In this process I addressed Research Question 1.

In the fourth-tier of analysis, I used the table that I constructed in the previous tier of analysis to examine across the mathematical events. I paid particular attention to examining each of the four adapted dimensions of content, purpose and setting,

values and beliefs, and social relations across the events. Through this analysis, I identified themes that appeared to emerge as significant across a set of mathematical events. In order to verify the salience of these themes, I constructed a new table wherein for each mathematical event I determined whether or not a given theme characterized that event. Where appropriate, I also noted additional statements for clarification. These resulting themes constituted the characteristics of each child's mathematical events. In this process I addressed Research Question 2.

Finally, in order to address Research Question 3, I examined the documents that I created in the third and fourth tier of analysis in order to determine themes that appeared to emerge across the three children. I also examined across the children's case reports to determine additional insights.

Presentation of Findings

The three chapters that follow each present one child as a case. I begin each case report introducing the child and the child's parent(s), describing my understandings of the child's everyday life, including the child's typical activities at day care as well as the child's routine experiences and family life. I also describe the child's home and provide additional insights regarding, for example, my experiences gaining the consent of the child's parent(s) and any other unique circumstances that might help inform the reader about a particular case. I then present and discuss the parent(s)'s reflections regarding the parent(s)'s experiences in school mathematics as well as the parent(s)'s perspectives regarding if and in what ways the child engaged in or used mathematics in the child's everyday experiences. The introduction of each

case report thus forms a background in which to situate a discussion of the child's everyday mathematical experiences and understandings.

The next section of each case report focuses on the collection of significant mathematical events identified for the child. I discuss these events with respect to the four adapted interrelated dimensions of content, purpose and setting, values and beliefs, and social relations that together constitute the mathematics and the social aspects situating these events.

In the final section of each case report I present and discuss characteristics of the child's mathematical events.

I discuss findings from the analysis across the three children's collections of mathematical events and the sets of characteristics of these events in Chapter 7.

Chapter 4: Toya

Capturing Images of Toya's Everyday Life

At the Day-Care "School"

I first met Toya at Ms. Franklin's day care in late December, 2 months prior to her fourth birthday. By that time, Toya had been regularly attending Ms. Franklin's day care, or going to 'school' as Toya put it, for about 1 year. Toya was petite in size, light-skinned in complexion, and always came to day care well dressed in coordinated outfits complete with shoes to match. Her hair was typically done in either multiple ponytails or cornrows, created in various designs and adorned with colorful ball ponytail holders, barrettes or beads. On the days that Toya attended Ms. Franklin's day care, she arrived at 7:30 in the morning and her mother, Nicole, picked her up around 5 o'clock in the afternoon. While at day care, Toya took pleasure in playing educational games on the computer, putting together 12-piece board puzzles, participating in organized arts and crafts projects, helping to pick up and to put away toys, entertaining herself in various activities during free play, and drawing spontaneously. Toya, like all the children at the day care, also enjoyed singing and dancing along to the videotape *Kindercise with the Kinderman* (Taylor, 1998).

Toya exhibited relatively low-maintenance eating and sleeping habits at day care. She typically finished all her food, including any second helpings requested, during breakfast, lunch and snack times. For naptime, Toya slept well and often woke from her nap last, resting peacefully on her cot amidst the lively play of now-awake youngsters and the arrival of older children for after-school care. Ms. Franklin documented Toya's daily activities, including what she ate, the duration of her nap, as

well as her behavior for the day on a bear-shaped form that she sent home with Toya in the afternoons. Ms. Franklin also used the “daily bear” to indicate upcoming events (e.g., field trips to the library) and to communicate particular notices to Toya’s mother Nicole (e.g., requesting more handy-wipes for Toya).

Kleenex, Notes, and Talk

Toya had allergies since she was an infant. As a result, Toya seemed habitually congested, her nose often runny, and occasionally she missed coming to day care or left day care early so that her mother, Nicole, could take her to see a physician. I frequently observed Toya blowing her nose and then using her handi-wipes to clean her face or to wipe her hands afterward.

Early on in my day care visits, Toya noticed and seemed to take interest in my activity of periodically writing in my field journal. Toya inquired quite regularly, over the course of my visits with her, about my jottings. Taking pause from whatever activity she was doing at the time, Toya often came to me, pointed to the writing she observed in my journal, and asked me, “What’s that say?” Each time she asked, sometimes even referring to the same jottings more than once, I read, paraphrased, or repeated the notes I had scribbled down.

Sometimes, I had difficulty deciphering Toya’s enunciation of words and understanding portions of her speech. At times, I requested Toya to repeat what she said or I discretely asked other adults (e.g., her mother or Ms. Franklin) to interpret Toya’s comments. To my knowledge, my difficulty understanding Toya’s speech was not due to culture or region-based language patterns or usage as I also observed Toya’s mother and Ms. Franklin periodically clarifying with Toya what she said as

well. Thus, my difficulty in deciphering some of Toya's talk was likely due to her still-developing speech and her capacity for enunciating particular sounds and words. Despite genuine apologies for my difficulty in understanding her speech, Toya sometimes did get annoyed and, on a few occasions, she demonstrated her frustration toward me with comments such as, "You don't understand me!" and with refusals to repeat herself. However, moments later Toya seemed no longer frustrated and she resumed her interaction and verbal communication with me.

Homes Sweet Homes

In a given month, Toya spent about 2 weeks living with her paternal grandparents and the other 2 weeks living with her mother, Nicole, and her family. Because Nicole maintained a close relationship with her mother and her older sister Pam, Nicole's only sibling, whenever Toya stayed with Nicole, Toya also spent significant amounts of time with her Auntie Pam and Pam's son Ricky, who was just a year older than Toya, and her maternal grandmother, whom she called "mama." Toya and her cousin Ricky often spent weekends together, staying overnight either at Nicole's, Pam's, and/or their mama's apartment. During those weeks when Toya lived with Nicole, she attended Ms. Franklin's day care. I only observed Toya in her everyday contexts when she was under the care of her mother. While Toya was Nicole's only child, Toya had two siblings through her father. She had a half-brother who was about 4 years her senior and a half-sister who was about 2.5 years her junior.

Getting Her Hair Done

When Toya returned to Nicole's care after staying with her paternal grandparents, Toya at times talked to me about her aunt, the teen-aged daughter of

Toya's paternal grandparents, who helped maintain and "do" (e.g., comb, plait, or style in ponytails, cornrows, or braids) Toya's hair. Nicole claimed little skill in the art of doing hair. During those weeks when she had responsibility for Toya, if needed, Nicole brought Toya to her sister Pam's apartment to do Toya's hair. Nicole washed Toya's hair and Pam did Toya's hair in a style that Nicole could easily maintain.

Nicole, Toya's Mother

I first met Toya's mother, Nicole, at Ms. Franklin's day care when she came to pick up Toya one afternoon. I experienced both positive and encouraging initial interactions with Nicole. She was friendly, approachable, laughed a lot, and seemed genuinely receptive to talking to me about this project. Ms. Franklin had been, as she put it, "warming the oven" for me with Nicole, explaining to Nicole who I was and my interest in observing Toya for a university-related assignment. Ms. Franklin's endorsement carried substantial weight with Nicole as she viewed Ms. Franklin as a trusted and respected member of the community. During my first home visit with Toya, I asked Nicole why she agreed to participate in this project, given as I put it, "Most parents would have said, 'Heck no!'" Nicole replied, "Ms. Franklin told me, I just; well for one, I can see if she had you there [i.e., at the day care] then it was okay. And she did say you were really nice. And she said she had a good feeling about you from the beginning," (March 22, 2006 – Home Visit with Toya).

Nicole was medium-brown skinned and she wore glasses. She typically wore her hair back in a simple ponytail and for a number of weeks she had it done in thin, individual plaits. When Nicole arrived to pick up Toya in the afternoons, she usually came dressed in her standard work outfit – a black, polo-type shirt with a bit of red

trim on the collar and dark pants or jeans. Nicole typically stayed to visit for a while with Ms. Franklin before leaving with Toya to go home for the evening to their two-bedroom apartment, located in the same neighborhood as Ms. Franklin's day care/home. Nicole shared this space with her significant other, Alan. Nicole lived in this neighborhood for part of her childhood and attended the local elementary school. Nicole's mother lived in an apartment nearby in the same community, while Nicole's sister Pam lived in an apartment located about a 25-minute drive away. Pam occasionally worked at Ms. Franklin's day care, either as a substitute or as an assistant whenever Ms. Franklin needed the additional help.

Nicole turned 28-years-old during the data collection period. She immediately entered the work force upon graduation from high school, initially working in retail settings and later finding employment through a temporary work agency. Through this agency, Nicole secured the position she held at the time of this study. Nicole had been working in the mailroom of a patent law firm located in Washington, D.C. for 5 years. She worked a full-time schedule (business hours, Monday through Friday), and by choice, some half-days on Saturdays as well.

A Note on Nicole's Parenting Practices,

From Ms. Franklin's Perspective

In our initial discussions and on a few occasions throughout this study, Ms. Franklin expressed her opinion that Nicole was a "good" mother in that she often "worked with" Toya at home, reinforcing activities and skills that Ms. Franklin introduced at day care to help prepare Toya for school. Ms. Franklin did not award this praise lightly, as she suggested that working with a child at home did not seem to

be a common practice among parents she encountered in her recent years as a day care provider. At the beginning of my first home visit with Toya, I observed two examples of Nicole working with Toya in the way that Ms. Franklin described. Nicole helped Toya complete a worksheet on addition that Toya started earlier that day at day care. The worksheet displayed pictures of objects to be counted and totaled and provided lines on which to write the symbolic representation that matched the given picture. After completing this activity, Nicole reviewed Toya's memory of her five senses, a topic that I observed Ms. Franklin working on with Toya in day care several weeks before.

Toya at Home

Weekdays. On a typical weekday morning, Nicole left early for work and took public transportation into the city, leaving Alan responsible for overseeing Toya's morning routine. As reported by Toya this included: waking up, washing her face, brushing her teeth, getting dressed, putting on her coat, and going to Ms. Franklin's. Alan used Nicole's car to take Toya to Ms. Franklin's and then drove to the Metro station to take the train into work, leaving the car at the station for Nicole to pick up when she returned from work via the Metro in the afternoon. Toya ate breakfast at day care.

On weekday evenings, Nicole picked up Toya from Ms. Franklin's at approximately 5 p.m. After visiting with Ms. Franklin, Nicole and Toya drove to the Metro station to pick up Alan before returning to Nicole's apartment. Once at home, Toya often retreated to her room, plopped down in her child-sized Dora the Explorer saucer chair, and watched Disney Channel television programs (e.g., *That's so Raven*,

The Suite Life of Zack and Cody, High School Musical) on her pink, Disney Princess-themed combination television/DVD set. There were three television sets in Nicole's apartment, one in each bedroom and the largest one in the living room. During my visits, all three television sets seemed to remain on throughout the late afternoon, evening hours. Toya sometimes had a snack or a drink while watching television, and occasionally she took a break from watching television to play with toys in her room. Toya had many more toys than could fit in her toy box, a large, plastic green bin located near the center of her room, that toys frequently remained scattered across her bedroom floor.

Toya occasionally played with a Dora the Explorer video game that she connected to her television set. At times, Toya left her room to explore activity elsewhere in the apartment. For example, Toya sat with her mother in the living room while Nicole watched television, talked on the phone, and/or opened mail; or Toya engaged with Alan and climbed on his back as he watched television in the bedroom he shared with Nicole. In addition to these activities, in the afternoon or early evening, Toya typically talked to her "grandma" and/or "mama" (her paternal and maternal grandmothers respectively) on the telephone.

Nicole prepared Toya's dinner and Toya took her seat, propped up by two thick telephone books, at the circular table located in the dining area of the kitchen. Toya said grace over her meal and she typically ate dinner alone. Nicole and Alan oftentimes ate their dinners afterward in the living room while watching television.

After dinner, Toya either spent time in the living room with her mother and/or Alan as they watched television (e.g., *Wheel of Fortune*) or she watched television or played alone in her room.

Nicole informed Toya when it was time for her to brush her teeth. Toya enjoyed brushing her teeth, often spending about 10 minutes doing so. Nicole then prepared a bath for Toya. While Toya soaked and played with her toys in the bathtub, Nicole selected Toya's outfit, including shoes, for the next day and laid these items out in the living room. Nicole also supervised Toya while she bathed and made sure Toya washed all parts of her body. After Toya finished bathing, Nicole towel-dried Toya; applied moisturizer onto Toya's skin; and helped dress Toya in, as Nicole described, her "nighties" and "undies."

Sometimes, Nicole read a book to Toya at bedtime. Toya selected stories from her own collection of books or from a set of books her mother borrowed from the local library. Toya often requested to have a snack, of which cookies were her favorite choice, before she went to sleep. Toya also preferred to have a drink of water by her bedside at night. Depending on Toya's allergy symptoms, Nicole had Toya blow her nose one or two times and applied Vicks VapoRub ointment under Toya's nose and on her chest. Toya fell asleep in her racecar-shaped bed, watching either television shows (e.g., Disney Channel programs) or a children's video (e.g., Blue's Clues). Nicole thought that Toya probably napped for long periods at day care because she would stay up late watching television as opposed to falling right to sleep with the television on.

Weekends. On the weekends when Toya lived with Nicole, Toya's cousin Ricky was typically present, joining Toya after day care on Friday afternoons. Toya and Ricky spent their weekends together either with Nicole, Pam, and/or their mama. On those Saturday mornings when Toya and Ricky stayed with Nicole, Nicole prepared for them a hot breakfast, such as pancakes and sausage, and then, if Nicole chose to work that day, Nicole took the children to work with her for about 4 hours.

Nicole drove to work on Saturdays and did not mind working on weekends as this afforded her additional money and, since she could bring the children to work with her, she did not have to worry about finding or paying for childcare. As volunteered by Toya, "Ricky, always come in there [her mother's job] with me. And, and, and we can write all day. We can, we can, um, write and we can play..." (March 22, 2006 – Home Visit with Toya). Nicole reported that in order to help keep the children occupied while she worked, she gave Toya and Ricky paper on which they could write or, as I subsequently observed on a Saturday visit with Toya at Nicole's job, Nicole directed Toya and Ricky to complete tasks in activity books that they brought along with them from home. Helping Toya respond to items in activity books reflected another example of what Ms. Franklin described as Nicole working with Toya at home on school-like tasks.

In addition to keeping Toya and Ricky occupied with materials with which they could write and permitting them some level of unsupervised play within the mailroom, Nicole also brought Toya and Ricky along with her to hand-deliver mail to a number of offices located in various areas and on different floors throughout the building. Sometimes when Toya and Ricky joined Nicole at work on Saturday

mornings, they also met with their near-aged cousin Marcus, whose mother also worked at the law firm and often chose to work on Saturdays as well.

Nicole reported the kinds of activities that Toya and her cousin Ricky typically did on Saturdays after spending the morning with Nicole at work. These included: spending time at Nicole's mother's apartment (e.g., playing in the room their mama set aside just for them, enjoying games on mama's computer, and, weather permitting, swimming at the pool); going to Chuck E. Cheese's; watching the latest children's movie at a local theatre complex; or playing outside at a nearby park.

When describing Toya's typical Sunday activities, Nicole stated, "It depends. If we have to go to the store, Sunday, like, it's the day I usually go... We don't really do anything... on Sundays. We kind of sit around and lounge around on Sundays. That's all we usually do, lounge around on Sundays." (June 2, 2006 – Home Visit with Toya). As Nicole stated, shopping trips, for example to Shopper's Food Warehouse or Target, frequently took place on Sundays. Oftentimes, Pam (Nicole's sister/Ricky's mother) and Ricky joined Nicole and Toya on these shopping trips and other weekend activities as well.

Nicole loved shopping at Target. In particular, she was a big fan of their selection of young girls' clothing. Nicole purchased Toya's well-stocked wardrobe as coordinated outfits and she also maintained Toya's shoe collection, keeping them fresh-looking and sparkling-like-new. Once at day care, I observed Toya, in an effort to maintain the condition of her shoes, using a handy-wipe to clean the dirt from her white, K-Swiss Classic sneakers. This episode illustrated one example in which I

observed Toya independently engaging in an activity that reflected a practice established and valued by Nicole.

Snapshots of Reflections of Mathematics:

Perspectives from Toya's Mother, Nicole

Throughout my first home visit with Toya and her mother, Nicole, I infused in my conversations with Nicole questions regarding Nicole's school mathematics experiences and her perceptions of mathematics in her and Toya's everyday life. I encouraged Nicole and Toya to carry on as they normally would so that I could begin to develop a sense of their daily experiences and ways of interacting at home. In keeping with this goal, I avoided imposing a formal interview onto their activities, and instead, I attempted to embed questions relevant to my study into my conversations with Nicole as these naturally evolved over the course of my visit.

I tried to involve myself in the spaces and activities in which Toya seemed to typically occupy and assume on a regular weekday evening with her mother. For example, I joined both Nicole and Toya in the kitchen as Nicole prepared Spaghettios for Toya's dinner, and I offered to help in various tasks as I normally would if I were visiting a friend's home. I used the opportunity of being in the same space as Toya and Nicole to talk with Nicole and get to know her better (e.g., where she grew up, what schools she attended, her work experiences) and to continue to establish and develop my rapport with her. Through our conversations, I learned that Nicole and I attended the same high school, although not at the same time.

During what seemed to be appropriate moments in our interactions, I raised particular questions with the purpose of understanding the following: Nicole's

experiences in school mathematics, her perceptions regarding if and how mathematics was part of her everyday life, and her reflections regarding if and when Toya used mathematics or engaged in mathematics in her daily experiences. At times my efforts to phrase or lead into particular questions seemed awkward. Despite this methodological challenge, Nicole seemed quite gracious in trying to interpret my questions, and she offered her candid responses to my inquiries accordingly.

The following reflections shared by Nicole (some appearing as direct quotations in headings below) provide additional insights within which to situate my analysis of Toya's everyday mathematical events. I present my interpretation of Nicole's reflections afterward.

“Math was not a good experience for me.”

Nicole immediately responded with this statement when I inquired about her experiences in school mathematics. The conversation continued,

Nicole: In high school, math wasn't my favorite subject.
Grace: It wasn't your best,
Nicole: It wasn't my best subject. It probably was my worst subject.
Grace: Why do you say that?
Nicole: It just, it wasn't all that interesting to me and I guess I probably didn't try as hard because it wasn't interesting.
(March 22, 2006 – Home Visit with Toya)

In terms of the mathematics she had in high school, Nicole recalled taking an algebra class and one that she described “like a starting calculus class or something.” In further describing her mathematical experiences in high school, Nicole stated,

Nicole: I just really didn't want to be bothered because I felt like I just I didn't get it. Like I just wasn't getting it.
Grace: Okay.
Nicole: And so that probably, like I had a negative feeling like going to class everyday.
Grace: Oh really?

Nicole: So that probably,
Grace: Have you always felt that way about math?
Nicole: I don't think I felt, cause I don't remember having problems with math in grade school. In you know, elementary school.
Grace: In middle school?
Nicole: In elementary or even middle school.
(March 22, 2006 – Home Visit with Toya)

When I asked Nicole to further explain her experiences in and feelings toward mathematics in elementary school, Nicole did not provide any specific details. Instead, she shared a more general, brief reaction, “I just prob- [sic], just probably pretty much liked it”, adding a moment later, “I think because it prob- [sic], it was easier for me then.” It therefore seemed that, Nicole’s “problems” in and “negative” feelings toward school mathematics surfaced during high school when Nicole encountered mathematics classes that she found to be difficult, and consequently, less interesting.

“Nothing other than the basic math.”

Nicole stated this response to my inquiry regarding if and in what ways she felt she used mathematics in her everyday life, now that she was no longer in school.

The conversation continued,

Grace: Like what?
Nicole: Like just multiplying, adding, subtracting.
Grace: When do you use, when do you use that kind of stuff?
Nicole: At work everyday, because I have, like when I get to work in the morning, I get to work at 6:30 in the morning. When I get there, there's like at least a hundred faxes on the computer waiting for me so, and then I have to count up all the faxes that I do for the day, count them up at the end of the day, and count up all the pages of each fax and add that altogether; and, what else do I do? At the end of the month I do a, um, a report where I have to average out the, the faxes for the whole month. So I do a little of that. Not other than that though.
Grace: And then what about here in your home. Like,
Nicole: What do I do here?

Grace: Do you, do you think you do anything that has to do with math here in your home?
Nicole: No, I want nothing to do with math. [Nicole and Grace laugh]. No. Not really.
Grace: Wow.
Nicole: Nope. Unless I'm helping her [Toya] with something [e.g. a mathematics worksheet sent home from day care].
(March 22, 2006 – Home Visit with Toya)

“Whenever she asks for money she has to tell me what it is or... how much is it. I make her do that.”

When I changed the focus of the questions to ask specifically about Toya and if and in what ways she encountered mathematics in her daily experiences, Nicole wondered aloud, “What does she do that's maybe math?” She then immediately mentioned the above statement of ‘making’ Toya identify money. Upon hearing this response, I shared with Nicole a relevant event I observed one afternoon at the day care when Nicole arrived to pick up Toya. This event appeared in my journal as:

Toya: Can I have a dollar?
[Nicole was holding a \$20 bill.]
Nicole: This is twenty dollars. See, the two and zero?
[Nicole let Toya hold the \$20 bill. Nicole then displayed a single \$1 bill.]
Nicole: See the one?
[They traded bills.]
(February 15, 2006, Day Care Visit with Toya)

I also mentioned an event that occurred early on in this initial home visit with Toya and Nicole. As soon as Toya and I first entered her room that evening, she found a dollar on her bed and showed it to me. When I asked Toya if she knew what it was, she replied, “It’s a dollar.”

I later prompted Nicole again to think of any other examples of when she thought Toya used or encountered mathematics in her everyday life. Just as Nicole

began to speak, her thought was cut short as Toya lamented, “Ho!!” and literally limped into our conversation claiming that she broke her ankle. After Nicole’s immediate assessment of the situation as a case in which Toya was being “dramatic,” she noticed Toya hiding a coin in her hand. The following exchange then transpired,

Coin

Nicole: What is that?
[Referring to a coin that Toya had found and was now hiding in her hand.]

Toya: I'm not telling!

Nicole: You're not telling?

Toya: Yes.

Nicole: Do you know?

Toya: No.

Nicole: Let's. Look at it. Let's see.

Toya: No-o-o! Nope, I'm not seeing it.

Nicole: You're not seeing it?

Toya: No.

Nicole: Where is it?

Grace: The drama is all over her little face. [Nicole and Grace laugh.]

Toya: You can't see it.

Nicole: Okay, well it's mine anyway. So you don't have to [indiscernible word.]

Toya: 'kay, see. [Toys showed a dime in her hand.]

Nicole: Well, how much is it? What,

Toya: I-I dunno.

Nicole: Okay, see! Big girls,

Toya: [Indiscernible comment as there is cross talk with Nicole. The only decipherable word is "money".]

Nicole: Big girls would know!

Grace: Oo! [Laughed.]

Nicole: [After a brief pause, and noticing that Toya was reaching into a glass bowl on the kitchen counter for some candy, Nicole asked in a milder tone,] How much does the candy cost?

Toya: Eight dollars.

Nicole: Eight dollars? Do you have eight dollars?

Toya: Yes.

Nicole: Okay. Where's the eight dollars? [Nicole extended her hand as if to request the money. Toya handed her the coin and proceeded to grab a cylindrical container of candy from the glass bowl.]

Nicole: [Referring to the small pieces of candy in the cylindrical container,] Not all of them. Some.

Toya: [Indiscernible phrase. Closes glass bowl.]

Nicole: Some. [In a firmer tone,] Just a few. [In reaction to Toya's method of dispensing the small bits of round candy,] No, not like that! Pour 'em in your hand. [She then said under her breath something that sounded like, 'You're something else.']

Toya: No I'm not. Is that enough?
[Nicole gave a non-verbal, affirmative reply.]

(March 22, 2006 – Home Visit with Toya)

Before I resume my discussion regarding Nicole's reflections of when she thought Toya engaged in mathematics in her everyday life, I illustrate below the three methodological concerns, introduced in Chapter 3, pertinent to this study: observing (and analyzing) mathematical events as situated in the participants' everyday settings and activities, approximating an essence of everydayness, and negotiating methodological pathways during data collection. I use the above coin episode along with the interaction between Nicole and myself that occurred just moments before (i.e., Nicole's report of making Toya identify money and me sharing with Nicole my observations of Toya related to money) to demonstrate how I addressed these concerns during these particular episodes. Taking the coin episode as an example of data, I also demonstrate how I applied the four adapted dimensions of content, purpose and setting, values and beliefs, and social relations as an analytic framework with which to examine the participants' mathematical events. I will return to discussing Nicole's reflections afterward.

Addressing In-the-Field Methodological Concerns

And Applying the Adapted Analytic Framework:

An Illustration

Observing and analyzing mathematical events. As the coin episode unfolded before me, I observed Toya engaging in mathematical activity. Upon examining the

observation and transcript data from this episode, I determined that three distinct mathematical events took place, all of which Nicole initiated. I briefly discuss each one below in terms of the adapted dimensions of content, purpose, values/beliefs, and social relations (Street et al., 2005) that framed each event.

Each event is identified by the mathematical content employed by the participants. I further elaborate on how the content appeared to be instantiated in the event and I also provide analysis regarding the mathematical understandings that Toya appeared to demonstrate. In terms of the setting of these events, they all took place during a single episode in Nicole's kitchen when Toya spontaneously interacted with her mother concerning the coin that Toya held and the small bits of candy Toya noticed and sought to consume. The dimension of setting is normally examined along with purpose, however, since each of these events occurred in the same setting as previously described, I did not include this dimension below. I continue to discuss each mathematical event below with respect to the remaining dimensions of purpose, values and beliefs, and social relations.

1. Content: Recognizing and Identifying Forms of Money

- a. Evidence of Content and Mathematical Understandings: Nicole asked Toya to identify the value of the coin that Toya was hiding in her hand. While Toya did not know the name or value of the coin, she did realize it was some form of money as she later gave the coin to her mother to 'purchase' the candy. Also, when Nicole asked Toya how much the candy cost, Toya's response consisted of a monetary amount, "Eight dollars," indicating her understanding that number words are

used to identify money;

- b. Purpose: Nicole's initial question, "What is that?" may have been in response to Toya hiding the coin and wanting Toya simply to identify and produce what she was hiding. However, Nicole's persistent efforts for Toya to identify the coin seemed to support Nicole's reported practice of 'making' Toya identify money whenever she asked for some in order develop Toya's capacity to identify different forms of money. Nicole's inquiry regarding the cost of the candy may have been one last attempt to encourage Toya to identify the coin;
- c. Values/Beliefs: The persistence with which Nicole tried to get Toya to identify the coin along with her reported practice of 'making' Toya identify money when she asks for some seemed to demonstrate the importance Nicole placed in teaching Toya about money and it's purpose in everyday life (e.g., to purchase goods);
- d. Social relations: Toya's efforts in hiding the coin seemed to indicate that the coin was of value to her. She did not want to show the coin or share it with her mother. Toya's behavior, however, prompted Nicole to ask Toya about it and to show the coin to her. Toya's resistance and unwillingness to respond accordingly to her mother fueled the verbal exchange, leading Nicole to taunt and question Toya about her status as a "big" girl. Toya's non-compliant behavior demonstrated her temperament to challenge Nicole while their back-and-forth banter suggested that the nature of this verbal exchange seemed typical in

their day-to-day interactions. In an effort to coalesce her intent to have Toya identify the coin with Toya's desire for the candy, Nicole engaged Toya in a role-play moment to lure Toya into 'paying' for the candy with the coin. By accepting Toya's "eight dollars" to pay for the candy, Nicole indicated to Toya that her response was appropriate even though it was not accurate. Nicole may have been distracted and concerned by Toya's dispensing of a large quantity of the candy, and therefore in that moment chose to address this issue rather than inform Toya that the coin was actually worth ten cents. Nicole's inquiry of "how much" also reflected a school-like practice in that she posed a known-answer question (Rogoff, 2003) to Toya;

2. Content: Relative Magnitude

- a. Evidence of Content and Mathematical Understandings: Nicole employed the word "big" in her statement, "Big girls would know." Nicole seemed to emphasize an evaluative sentiment that implied, according to Nicole's expectations of Toya's understandings of money, that since Toya did not know the value of the coin, that she was not a big girl, hence, she was a little girl. Nicole's use and implication of the comparative terms big/small may have referred to multiple developmental aspects of Toya, including her age, physical size, and knowledge of money. It was unclear whether and to what extent Toya understood this evaluative statement;

- b. Purpose: Nicole's use and tone of the word big not only appeared to poke at Toya's maturity/development but Nicole's comment also seemed aimed at addressing Toya's initial non-compliant behavior and prompting Toya to identify the coin;
 - c. Values/Beliefs: Nicole's taunting of Toya suggested that Toya, as is typical for other young children, expressed a desire to be regarded as a big, and not little, girl. As such, Nicole's comment seemed to exploit Toya's preference to assume the status of a big girl in an effort to get Toya to name the coin;
 - d. Social Relations: Nicole's comment curtailed Toya's uncooperative behavior in response to her mother's initial inquiries about the coin. Toya then turned her attention to the bowl, intending perhaps to dodge further questioning from her mother about the coin. However, Nicole persisted in her efforts for Toya to identify the coin by attempting to capitalize on Toya's attention on the candy and asking Toya "how much" it cost; and
3. Content: Describing a Relative Amount of a Quantity of Objects
- a. Evidence of Content and Mathematical Understandings: Nicole employed non-numerical words/phrases "not all," "some," and "a few" to indicate an approximate quantity of candy. It might appear that Toya did not understand Nicole's descriptions given her mother's repeated emphasis regarding the relative

quantity of candy Toya could take. Rather, Toya's behavior most likely indicated her desire to acquire as much candy as possible despite her mother's directions;

- b. Purpose: Nicole employed non-numerical words/phrases "not all," "some," and "a few" to indicate to Toya the appropriate approximate quantity of candy she was permitted to have;
- c. Values/Beliefs: Nicole's language, tone and emphases regarding how much candy, particularly when she said, "just a few," demonstrated Nicole's intent for Toya to dispense and eat only a very small quantity of the pieces of candy.
Regulating and limiting Toya's intake of sweets/candy therefore seemed important to Nicole;
- d. Social relations: Although Toya attempted to act on her own accord with the candy, Nicole asserted her parental control and management of Toya's consumption of sweets as demonstrated in her tone and emphases of how much candy Toya could have and, eventually, Toya complied.

Approximating an essence of everydayness and negotiating researcher pathways in the field. Although I observed these three mathematical events as they naturally unfolded before me in the coin episode, coincidentally occurring amidst me asking Nicole to share examples of when she thought Toya used or encountered mathematics in her everyday life, I neither shared my insights regarding nor prompted Nicole to recognize the various mathematical content embedded in this episode.

Despite the focus of our conversation just prior to the coin episode, this did not appear to raise Nicole's consciousness to consider more broadly what could constitute mathematical activities in Toya's daily experiences. Even after a few minutes following the coin episode, when I resumed my conversation with Nicole regarding in what ways Toya engaged in mathematics in her everyday life, Nicole made no mention of noticing anything that just occurred as being mathematical. By taking this methodological path of not prompting Nicole to reflect upon the events in the coin episode, I aimed toward approximating an essence of everydayness and let the coin episode naturally dissolve. I did not want to influence Nicole's perceptions about what activities or concepts constituted something mathematical, which in turn, could shape Toya's subsequent activities.

In contrast to how I managed my observations of and my participation during and in response to the coin episode, I followed a different methodological path in response to Nicole when she noted, as described above, how whenever Toya asked her for money she 'makes' Toya tell her what it is. Since Nicole initiated her response indicating the content of money, it seemed natural and appropriate for me to respond to her comment by sharing two prior observations of Toya that involved displaying and naming paper monetary currency. As such, I affirmed Nicole's assertion that when she 'makes' Toya identify money that this was an example of when Toya did something that was "maybe math." Given the flow of the conversation and that Nicole had already perceived identifying money as being mathematical, the methodological path that I chose to follow seemed appropriate and not disruptive toward my goal to approximate an essence of everydayness.

I now return back to my discussion regarding Nicole's perceptions of when Toya engaged in mathematical activities in her everyday life.

"She may ask for a cookie... I might say, 'Okay, you can get such and such amount of cookies.'"

Shortly following the coin episode, I shifted Nicole's attention back to the conversation we were having before Toya limped into the scene, asking her if there were any other examples she could think of in which Toya engaged in mathematics. Nicole responded with the above cookie comment and further explained,

... and then she'll count it out and make sure she has the right amount... Yeah, she'll say, "Mommy, is this such and such amount of cookies?" [Nicole paused briefly and then sighed as if to indicate she had no further examples to share regarding when she perceived that Toya engaged mathematics.] Yeah. (March 22, 2006 – Home Visit with Toya)

In my visits with Toya at home, I learned from Nicole that Nicole's mother often baked cookies for Toya and that Toya enjoyed snacking on these before bedtime.

Nicole recognized the use of mathematics in the everyday experience of managing Toya's consumption of cookies. In her cookie example, Nicole explained that she stated a certain number word to describe how many cookies Toya could have and, in turn, Toya counted the cookies and repeated the given number word as she showed the set of cookies to her mother to verify that she constructed the appropriate numerosity of cookies. Given Nicole's report, the number of cookies that Nicole had in mind when sharing this example and in her actual practice of managing Toya's intake of cookies, seemed more than likely to be a small number (e.g., three or less). As such, Toya's reported activity of counting her cookies seemed to promote her developing understandings of enumeration, particularly with respect to small

numbers. Nicole's desire to limit Toya's sweets also supports the claims that Nicole employed small number words in this context and that Toya interpreted these number words through her response of constructing the appropriate numerosity of cookies.

What Counts "That's Maybe Math" for Toya:

Making Sense of Nicole's Perspectives

Nicole's perspectives of when she thought Toya did something "that's maybe math" in her everyday life seemed to focus on activities in which both of the following two criteria were evident: (1) Toya requested to have something (i.e., money or a cookie), and (2) Nicole and/or Toya employed number words to describe the value of an item or the numerosity of a quantity of objects (i.e., identifying the value of a particular monetary bill or coin or stating the specific numerosity of cookies that Toya could have).

This analysis seems consistent with my observation that Nicole did not appear to recognize any of the events that took place during her spontaneous exchange with Toya in the coin episode as occasions wherein Toya engaged in something "that's maybe math." For example, while Nicole attempted to "make" Toya identify the coin (which was consistent with her cited example of requiring that Toya identify money when she requests to have some), Nicole chose not to use this opportunity to state to Toya the actual name or value of the coin, perhaps because Toya did not ask for the coin in this particular episode.

Later in the coin episode, when Nicole watched as Toya proceeded to take a quantity of the candy she retrieved from the glass jar, Nicole firmly responded to Toya's actions by using comparative, non-numerical descriptors to emphasize the

relative amount of candy she was permitted to have, using phrases such as “not all”, “some”, “a few.” That is, Nicole used non-numerical expressions to convey to Toya the notion of relative quantity size as opposed to employing number words that would otherwise indicate the specific numerosity of pieces of candy Toya could have. While the underlying value (i.e., Nicole’s desire to limit sweets for Toya) and the mathematical activity (i.e., describing quantity) involved in regulating Toya’s consumption of sweets were evidenced in both the candy event and Nicole’s cookie example, Nicole observed a mathematical quality only in the latter, in which she employed number words to identify a specific numerosity of the quantity of cookies Toya could eat.

It may be that Nicole’s perception of, or framework for identifying, events wherein Toya did something “that’s maybe math” rested primarily on the extent to which an activity or event reflected a formal perspective of mathematics. For the purposes of this study, a *formal perspective of mathematics* refers to a perspective of mathematics that is characterized by a preeminence of number (e.g., rote counting, enumerating physical quantities, representing numbers symbolically) and numeric operations (e.g., addition, subtraction, multiplication, and division) involving concrete objects, number words, or symbolic representations with or without numerals. From this perspective, number and operations are viewed as the two concepts most associated with school mathematics and, consequently, are often perceived as constituting the domain of mathematics more generally.

A formal perspective of mathematics, not only seems to reflect Nicole’s perceptions regarding Toya’s experiences “that’s maybe math,” but this view also

seemed to underlie Nicole's observations about her own everyday mathematical experiences both at work and at home. When asked when she used or encountered mathematics at work, Nicole stated that she used "the basic math," and then listed the following activities that focused on number and numeric operations: counting the number of pages in each fax she sent, adding to determine the total number of pages in faxes she sent for each day, and finding the average number of pages in faxes she sent per day at the end of every month. Nicole reported the only occasions in which she perceived she engaged in mathematics at home was when she helped Toya complete school-like worksheets, that often involved computation, sent home from day care or in workbooks that Nicole purchased. However, when asked about examples of when Toya engaged in mathematics in her everyday life, Nicole did not include Toya's experiences with these school-like activities. I did not inquire about this observation with Nicole and hence she was not offered an opportunity to clarify this with me.

Interpreting a Collage of Toya's Everyday Mathematical Events

An examination of the mathematical events coded from field notes and transcript data gathered from observation visits with Toya revealed the content of number as a prominent theme. Toya frequently and spontaneously engaged in multiple meanings of number across various contexts in her everyday life. As such, Toya's developing understandings of early number concepts, including the various meanings for which number can signify, naturally emerged from her everyday experiences, particularly at home.

*Conceptualizing a Wider View of
Number As Mathematical*

Toya interpreted, identified, and employed verbal (i.e., number words) and symbolic (i.e., numerals) representations of elements from the set of natural numbers (i.e., 0, 1, 2, 3, 4...) to signify nominal, ordinal, and/or numeric meanings.

Number words and numerals conveyed nominal meanings when they functioned as arbitrary names, labels, or identities for specific items or distinct entities, without reference or significance to an entity's position in a set (Sophian, 1996).

Number words, numerals, and ordinal number names signified ordinal meanings when they specifically designated an individual element by its position within an ordered set (Sophian, 1996).

Lastly, for the purposes of this study, numeric meanings and applications of number included activities such as: (a) expressing number words orally in a count sequence without reference to discrete objects or events (e.g., Baroody & Wilkins, 1999; Ginsburg et al., 2006; Ginsburg & Ertle, 2008; Sophian, 1996, 2007); (b) demonstrating concepts of enumeration (i.e., determining a quantity's numerosity), including developing understandings of one-to-one correspondence, cardinality, as well as enumeration strategies such as subitizing, using explicit number words to assign to each item in a quantity while simultaneously physically keeping track of each item in the quantity (e.g., pointing to elements and saying "This is one, this is two..."), and/or counting objects or events (e.g., Ginsburg et al., 1999; Ginsburg et al., 2006; Ginsburg & Ertle, 2008; Klein & Starkey, 1988; Sarama & Clements, 2008)

with or without one-to-one correspondence and/or cardinality; and (c) employing number words to describe or interpreting numerals to identify attributes of physical quantities or entities (e.g., Sophian, 1996, 2007) including relative magnitude of a given attribute across a particular classification of objects.

Toya routinely engaged in nominal meanings of number in her daily activities at home. While early number concepts pervade research dedicated to the study of young children's understandings of mathematics, these discussions primarily focus on numeric meanings of number, in particular counting concepts, with less attention given to ordinal meanings. Children's understandings of nominal meanings of number, however, are typically not addressed.

By excluding nominal meanings of number, the extant literature seems to suggest that children's activities involving nominal meanings of number are not mathematical. I contend that children's emerging understandings of numeric and ordinal meanings of number coexist and may be intertwined with their developing constructions of nominal meanings of number. As children's understandings of number mature so does their capacity to discriminate between the various meanings or significations of number. Given the contemporaneous, interrelated nature of young children's experiences with and their emergent understandings of nominal, ordinal, and numeric meanings of number, it therefore seems appropriate and productive to examine young children's activities and understandings that involve emergent concepts of numeric and ordinal meanings of number, which the extant literature deems as mathematical, alongside their activities and understandings in which they

engage in nominal meanings of number. As such, I considered Toya's engagement with nominal meanings of number as constituting evidence of mathematical thinking.

In the opening pages of her book *Children's Numbers*, Sophian (1996) presented a collection of statements that a young child might hear to illustrate the different uses or purposes of the number word "three" (e.g., "'On your birthday you will be *three* years old.'.... 'My phone number is *three*-two-*three*...' 'One, two, buckle my shoe, *three*, four close the door...'" (p. 3-4)). Sophian (1996) aimed "to appreciate the range of different activities and ideas that 'learning about numbers' encompasses" (p. 5) and thus she provided a rare exception among the extant research literature by introducing a more inclusive picture of the various meanings of number in which young children can engage.

Sophian (1996) identified and briefly discussed the following four kinds of number that she claimed children encounter: cardinal numbers, ordinal numbers, nominal numbers, and numbers as a symbol system. According to Sophian cardinal numbers are used to quantify a set of objects whereas numbers as a symbol system, while also quantitative, may not correspond to a set of objects. These two kinds of number reflect the notion of number that is typically emphasized in research and literature focused on young children's early mathematical understandings and young children's early number concepts.

Sophian's (1996) book did not pursue further children's understandings of nominal and ordinal numbers, but instead, consistent with the extant literature, focused on young children's development and understandings of cardinal numbers and numbers as a symbol system. However, Sophian's inclusion of nominal numbers,

in particular, as part of children's "learning about numbers" supports my assertion, and thus informed this study's conceptualization of the content of number and its various meanings articulated above.

Considering a wider view of number as mathematical, a perspective which encompasses nominal meanings along with numeric and ordinal meanings of number, provides an appropriate analytic tool with which to examine inclusively the contemporaneous various meanings for which Toya interpreted, identified, and employed number words and numerals in her everyday experiences within and across diverse contexts.

Composition

The analysis of Toya's collection of significant mathematical events is organized by the various content meanings of number identified above. I employ the remaining dimensions of purpose and setting, values and beliefs, and social relations to present and to examine a set of Toya's mathematical events that illustrate each content meaning.

I include field notes and transcription data of Toya's mathematical events mainly within the dimension of purpose and setting to highlight and then to discuss the specific contexts that most significantly seemed to give rise to the mathematical content observed. Within my discussions of purpose and setting, I also examine the mathematical understandings that Toya appeared to demonstrate either in a specific mathematical event or, more generally, in the purpose and setting presented. On a few occasions, I also incorporate Toya's mathematical events and/or other observation or transcription data in my discussions of values and beliefs and/or social relations to

exemplify my analysis of these dimensions in the given purpose and setting framing a particular mathematical event or set of events. In such occasions, where appropriate, I also examine the mathematical meanings that Toya seemed to demonstrate in these events.

I acknowledge that varying meanings of number, as instantiated by Toya, may be interpreted beyond the analysis presented. To the extent possible, I aimed to keep the discussion concerning Toya's understandings of number focused on what appeared to me as the most salient meanings (e.g., nominal, ordinal, or numerical) for which Toya interpreted, identified, and/or employed number words and/or numerals in a given mathematical event or set of events.

For several mathematical events, I provide lengthy transcriptions from the data to portray an essence of everydayness in the mathematical events observed, including capturing Toya's personality and the candid nature of the social interactions that transpired.

Content: Number as Label

Nominal meanings of number played a prominent role in Toya's daily experiences at home. Although nominal uses of number words and numerals readily appeared throughout Toya's everyday environment and experiences (e.g., numerals appearing on and number words identifying McDonald's value meals, a string of number words indicating a specific phone contact sung in commercial jingles, and numerals labeling racecar images displayed on Toya's racecar-shaped bed), Toya seemed to engage in the content of number as label most significantly when watching television and when talking to either of her grandmothers on the telephone.

1a. Purpose and setting. Toya interpreted, identified, and employed number words and numerals in her practice of watching television. Toya used her remote control to select a particular television program or channel she wanted to view. More specifically, Toya knew the distinct combinations of number words assigned to her preferred programs/channels and she identified the corresponding numerals on her remote control.

Using the remote control as an input device, Toya recognized and pressed the numeral buttons in sequence as each numeral appeared (from left to right) in the unique number label assigned to her favorite programs/channels. Toya verified her channel selection by confirming either the numeral label that appeared on her television screen or the reception of her choice of program/channel. Identifying the correct combinations of numerals and successfully inputting these using her remote control yielded desired outcomes for Toya's television use, such as viewing a program of her choice or, when she chose to play video games, enabling specific video-game-related functions.

The following pair of mathematical events occurred on a Friday evening home visit with Toya. As was typical of Toya's weekends with her mother Nicole, Toya's cousin Ricky was also present.

Channel Four Four (Part 1)

We arrived at Nicole's house around 6 p.m. Toya summoned Ricky and myself to follow her into her room, "Com'on y'all." Nicole gave each of the children half of a popsicle to eat as they played and watched television in Toya's room. Ricky was trying to talk to or to ask Toya about one of her toys. Seemingly annoyed with Ricky's talking, Toya responded, "No! Turn it [the television volume] up." The theme song of the television show got louder while Toya attempted to sing along. Out of curiosity, I asked,

Grace: What TV show is this?

Toya: Four four.

Grace: [I experienced difficulty deciphering Toya's words.]
 Foo-Foo?
 [Ricky and Toya (R & T) simultaneously responded.]
 Ricky/Toya: Forty-four./Four four.
 Grace: Forty-four?
 R & T: [They both affirmed.] Mm-hmm.
 Grace: Why do they call it 'forty-four'?
 Toya: Cause.
 Ricky: Cause she wanted that channel.
 Toya: Cause I watch this [indiscernible phrase, sounded like, 'every time'].

Toya ignored Ricky as he tried to show her he could jump rope. Toya instead focused her attention on the television show, *The Suite Life of Zack and Cody*. (June 2, 2006 – Home Visit with Toya)

Although I did not understand it at the time of this mathematical event, “four four” or “forty-four” was the number word label corresponding with Toya’s and Ricky’s selection of the Disney Channel. Toya’s response to my inquiry regarding what she was watching did not clearly indicate her interpretation of the question I posed. For example, it was not clear whether or not Toya thought I was asking, “What channel are you watching?” or “What is the name of this show?” or whether or not she even distinguished between the two. Her response to me, however, seemed meaningful as she identified what she was watching with the number word label “four four.”

While Toya’s verbal interpretation of the symbol “44” was “four four,” her cousin Ricky referred to the same symbol as “forty-four.” Both Ricky’s statement and my use of “forty-four” later in the event implied that “forty-four” and “four four” both referred to the numeral 44. In the context of watching television, Toya’s cousin Ricky modeled for Toya a more developmentally sophisticated verbal interpretation of the numeral “44” as “forty-four.” Toya’s everyday leisure routine of watching television, particularly in the company of family members, therefore seemed to

provide a supportive, non-threatening, meaningful context in which Toya's own understanding of numbers could continue to mature.

The following mathematical event, occurring later that evening, demonstrated Toya's emerging, more developmentally sophisticated, verbal representation of the numeral 44 from "four four" to "forty-four."

Channel Four Four (Part 2)

Just before 7 p.m., Toya, Ricky and I were in Toya's room. Toya decided she was done with the Dora the Explorer jigsaw puzzle she had been working on and was in search of a new activity. Before Ricky gladly stepped in to put the remaining pieces together on his own, he noted that the music from the *Kindercise with the Kinderman* CD, which was playing from Toya's television set, had stopped.

Toya: Wait a minute. Where's my remote?

Grace: Your remote?

[All three of us looked for it and eventually I found it.]

Grace: Toya, it's right here.

[She excitedly came to me uttering, "Aah, aah, aah."]

Toya: Four, four, four.

Ricky: Forty-four

Toya: Forty-four.

[The television powered on to the series *The Suite Life of Zack and Cody* on the Disney Channel. A character from the program stated, "...takes me to a parallel universe".]

Ricky: Or forty-three. Let's see what's on forty-three. Is *SpongeBob* on? [After a few moments of Toya not responding, Ricky repeated himself.]

Let me see. Is *SpongeBob SquarePants* on?

Toya: [Toya snapped back with a sharp tone.] No!

Ricky: Let's see.

Toya: [After using the arrow keys on her remote to change the channel two or three times to check for *SpongeBog*, she tentatively verified,]

I don't think it's on. [She uttered aloud to herself,]

Let me see this. [After another try with changing the channel, Toya confidently stated,]

Nope. You're gonna watch forty-four.

Ricky: Wait, Toya, I think that was, it was on.

[Toya ignored Ricky and did not change the channel. In frustration with his cousin, Ricky retreated and mumbled aloud,]

Fine, I hope you find someone to play with. I'm doing my own thing.

(June 2, 2006 – Home Visit with Toya)

Ricky promoted Toya's understanding of number words by stating "forty-four" in reaction to Toya's "four, four, four." Toya appeared to recognize Ricky's intent to clarify the number name of the channel Toya wished to view as evidenced in her repeating Ricky by stating aloud herself "forty-four." The meaning of Toya's initial utterance of "four, four, four" is unclear. It may be that Toya was following the pattern of her utterance moments before when she expressed, "Aah, aah, aah." Alternatively, saying four multiple times, although the actual number label of the desired channel only contained two numeral 4's, may have indicated a sense of excitement in Toya upon taking possession of the remote control and/or an enthusiastic urge to watch channel 44.

The above event also provided a glimpse into how the context of Toya watching television in her room with her cousin Ricky, with both children employing number words and interpreting numerals to identify channels or shows they each wished to view, could have also potentially afforded Toya an opportunity to explore further the sequential quality of number. For example, if Ricky had said, "Go down one," as an alternative directive to Toya to check for *SpongeBob SquarePants* on channel 43, Toya might have begun to make connections between her understandings of what it meant when she asked Ricky to "turn it [television volume] up" in the channel four-four (part 1) event and what it might mean if she were to "go down one," or press the arrow down channel button once in terms of channel selection. For example in the former, Toya seemed to understand that the more you press the arrow

facing “up” volume button on the remote control, the louder the television sound. In the latter scenario of going down one in channel by pressing the channel down arrow key once, Toya might have therefore begun to make meaning of the numeral 43 that would appear on the screen, and how this corresponded to both Ricky’s request for channel “forty-three” and the act of pressing the arrow down channel button once resulting in the change in numerals on the screen from “44” to “43”.

Ib. Values and beliefs. Watching television appeared to be a salient form of leisure in Toya’s family life with Nicole. Nicole’s apartment furnishings included three television sets, one in each bedroom and a third in the living room. Whenever I visited Toya at home in the evenings after day care, all three televisions were typically operating simultaneously.

On my visit with Toya and Nicole to Nicole’s sister’s (Pam) apartment one evening when Toya was to get her hair washed and braided, I observed a large television set in the living room that also remained on throughout my stay. Pam braided Toya’s hair in the living room so that they could watch television with others in the apartment. Toya’s cousin Ricky had a Disney television set similar to Toya’s, except his set had bold primary colors and reflected a Mickey Mouse motif. I learned that Nicole and Pam purchased these television sets from Target as Christmas presents for Toya and Ricky.

The practice of watching television, a valued activity and significant source of leisure in Toya’s family life with Nicole, also afforded Toya meaningful opportunities in which to develop her capacity to interpret, recognize, and employ symbolic and verbal representations of some numbers. The following mathematical event primarily

illustrated the value Toya attached to watching television, as well as her desire to control channel selection. To some extent, the event also demonstrated the ways in which use of the remote control seemed to support Toya's developing meanings of number.

"We got cable...!"

It was just shy of 4 p.m. Toya, Nicole and I were gathered in Toya's room. Toya spotted, eagerly approached, and then held up the white cable wire dangling behind her pink television set and exclaimed,

Toya: Oh, we got cable right here! Hey, mommy!

Nicole: Yes?

Toya: Look this is cable right here. Mom, can you put this, mommy, can you put this in? We got cable.

Nicole: It's really not cable, Toya. It's just that some of the channels are still showing.

[Nicole then connected the cable wire to the wall, turned the television power to on, and proceeded to press the controls on the set to try to find a clear channel. After only finding static reception, Nicole asked Toya,]

Nicole: Where's your remote?

[Toya searched for the television remote control... After expressing frustration of not being able to find the remote control initially, Toya eventually located the device... Nicole then proceeded to use the remote to try to locate a channel with clear reception.]

Toya: I'll show you which number I want [Toya said an indiscernible word that sounded like 'show'.] on. I can do it!

Nicole: I know what number you want, but PBS is usually on twenty-something, I think.

Toya: [After a short while of not observing any success come of her mother's efforts, Toya stated,] Okay mom, I'm watching a tape.

Nicole: Alright. Watch a tape then. [Then to herself, said something that sounded like, "Let me see. Let me check forty-five."] Or fifty-four. [Then Nicole resigned from her efforts.] Okay.

Toya: [Newly sparked by Nicole's attempt with these channels, Toya resumed back to her initial desire to watch television. She said something that sounded like, "That was four."]

Let me do it. I could do it. [Toya stated an indecipherable phrase that sounded like 'Let me show you'] how to do it. You push [indecipherable phrase that sounded like 'on the button'] like this and you [indecipherable phrase that sounded like 'push the other button'] like that.

[Having no successful response from her television, Toya scolded it saying,]

Uh-uh!! You TV, you better stop doing that before I pop you!
 [Nicole and I tried to suppress our giggles. Toya then gently said the following as she pressed the following numeral buttons on the remote control,] I put a four and a three. [Toya continued to direct the TV,] Alright. Put that, put that [indecipherable phrase that sounded like ‘thing out’.]
 Y’better put that [indecipherable phrase that sounded like ‘thing out’] if you wanna get on!

Nicole: [She then intervened as the television was still not responding to Toya’s efforts,]
 What do you wanna do? Play a tape?

Toya: Huh?

Nicole: What ch’ya rather do, play a tape?

Toya: Yeah.
 [As Nicole proceeded to insert a videotape into the connected VCR to play, Toya changed her mind and taking hold of the remote announced,]

Toya: I’m trying to find PBS.
 [After a valiant, but unsuccessful attempt at trying to find a channel with clear reception, Toya tried another tactic and cheered on the television,] Come on TV! You can do it! You can do it!
 [After a few moments, Toya stated,] Wait a minute. Push three. Push four. [With still no response from her television, Toya powered it off and declared,]
 Alright, the TV not working. We’re not doing this.

(June 20, 2006 – Home Visit with Toya)

Although Nicole had canceled her cable service and was in the process of finding another provider, Toya yearned to watch television. Nicole appeased Toya’s desire to watch television by reinserting the cable wire into the outlet and using the controls on the set and then the remote control to search for channels with clear reception. Toya eventually insisted that she handle the remote control so she could show Nicole which “number” she wanted, indicating that Toya understood that channel labels (both verbal and symbolic) consisted uniquely of numbers.

In Nicole’s attempt to find channels with clear reception, she expressed aloud number words assigned to the particular networks she thought would have reception including, “twenty-something,” “forty-five,” and “fifty-four.” Toya also employed

number words “four” and “three” and identified and pressed the corresponding numerals on her remote control buttons. Thus the activity of searching for channels inherently engaged Toya and her mother in employing, interpreting, and identifying certain number words and their corresponding numerals.

Toya’s actions of talking to her television, including scolding it and cheering it on, seemed to further indicate the significance that Toya placed in watching television.

Ic. Social relations. In the context of Toya watching television for leisure with family members in her own room, two interrelated types of social relations seemed to invoke the content of number as label.

One kind of interaction involved the activity of selecting which channel to watch. Given that Toya considered the television as belonging to her, she sought to maintain a significant amount of control in determining which “number” or channel to view. Her desire for control over her television was evidenced in Toya’s response to Ricky, with whom she negotiated very little, when he requested that she check “forty-three” for *SpongeBob SquarePants*. Toya also tried to assert her control of and to demonstrate her capacity to operate successfully her television in the cable event, insisting to her mother, “I’ll show you which number I want... I can do it!” While circumstances of not having cable service prohibited Toya’s practice of watching television, the social relations evident in Toya’s response to this situation and to her mother reflected Toya’s perseverance and intrinsic desire to participate in this valued activity and to exercise her authority to control which “numbers” (i.e., channels) to watch. Toya spontaneously employed meanings associated with number as labels and

utilized familiar tools (the television remote control) to problem solve in an attempt to attain her goal to watch television.

The second type of social interaction, related to selecting which channels to view, involved Toya's family members' modeling for Toya more sophisticated verbal representations of number when naming specific channels and Toya's interpretation and response to these actions. Having her family members verbalize the number words for certain double-digit numerals fostered Toya's understandings and use of these number words (e.g., forty-four) and their corresponding numerals (e.g., 44). These candid social interactions seemed to reflect aspects of guided participation (e.g., Rogoff, 2003) in the activity of watching television. More specifically, when Ricky, Nicole, and even I, employed more sophisticated verbal representations for numeral labels of particular channels (e.g., "forty-four," "forty-three," "fifty-four"), Toya responded accordingly to these number words in a meaningful manner. Toya thus demonstrated her developing capacity not only to interpret but also to begin to employ such verbal representations in this context.

2a. Purpose and setting: Phoning her grandmothers, from home or other everyday settings, with Nicole. Toya continued to develop her facility to interpret, identify, and employ number words and numerals in her practice of calling to speak with her grandmothers on the telephone. Toya relied on Nicole's assistance to recite, one number word at a time, the unique sequence of number words assigned to the telephone of each of Toya's grandmothers. For each given single-digit number word, Toya had to identify the corresponding numeral on the telephone keypad. At the time of data collection, particularly during my early visits with Toya, she was still learning

to identify the numeral 0 and those that were greater than 4. As such, Nicole would offer Toya hints on how to find these numerals on the keypad.

The following event took place during my first home visit with Toya and Nicole.

“Call your grandma...”

It was close to 7:30 p.m. and Nicole and I had been sitting in the living room talking when Toya emerged from her room... Toya picked up the telephone in the living room and pretended to call someone.

Nicole: What are you doing?

Toya: I'm calling my [indiscernible word, sounds like, 'princess'].

Nicole: Your who?

Toya: My friend.

Nicole: What friend? Come here... Okay, call your grandma [i.e., Toya's paternal grandmother].

Toya: Huh?

Nicole: Call your grandma and see if you're going over there.

Toya: Um, what's the number?

Nicole: Three. [Pause] You got the three? Zer-, [Nicole changed her mind on how she would say the next numeral '0', opting to say the letter name "Oh" instead of "zero."] O,

Toya: O.

Nicole: Look. See the 'O'?

Toya: Where is it?

Nicole: Come over here.

Toya: Like this?

Nicole: Yep. [Pause.] One. [Nicole realized that the buttons on the phone were not operating well.] That phone is so funny acting. Come here. Cause I gotta make sure it,

Grace: Cause you don't hear it [the sound indicating a numeral button had been pressed].

Nicole: Yeah,

Toya: I started over.

Nicole: You have to, here, hit the buttons kind of hard... Okay. Three. O. No, you didn't hit three two times. Three.

Toya: Oh this three,

Nicole: You gotta hit three hard. Let me see. Okay, here you go. Zero. Um-hmm, hit it again. One. Hit it again. Hit it again. Uh-oh. Okay, go get the other phone 'cause this phone doesn't act right... [Toya returned with another phone and proceeded to dial as Nicole dictated the phone number.]

Nicole: Zero ['beep'], one ['beep'], um, four. You know what the four is? Which one? Um-hmm ['beep'], eight ['pause'] eight, okay no listen. Remember eight is two circles. Right...

Toya: [Toya spoke into the receiver,] Huh? I can't hear you. [Then to Nicole,]
Gotta leave a message. Leave, gotta leave a message.

Nicole: Okay, you can leave a message.

Toya: Can I call mama [i.e., Toya's maternal grandmother/Nicole's mother]?

Nicole: Mama might be in the bed. You know mama don't stay up late. [Toya dismissed Nicole's apprehension and started to dial her mama's telephone number,]

Toya: Oh, three? ... [Awaited her mother's assistance,] Three, mom?
(March 22, 2006 – Home Visit with Toya)

Toya's grandma's (her paternal grandmother's) telephone number consisted primarily of the numerals 0, 1, 2, 3, and 4 and the digit 8 appeared twice. The frequent occurrence of the digits 1 thru 4 in her grandma's telephone number may have been particularly useful for Toya's meaning making of number as young children's number sense builds on their knowledge of small numbers (Baroody, 1987; Fuson, 1988). The context or practice of Toya regularly phoning each of her grandmothers provided Toya ongoing meaningful opportunities in which to develop her understanding of interpreting, identifying, and employing certain number words and numerals less than 10.

As demonstrated in the above event, this context also afforded natural opportunities for numeric meanings and other mathematical content to be drawn upon in meaningful ways, particularly through the support that Nicole provided in this practice. For example, in the above event, Nicole provided the hint, "eight is two circles" to assist Toya in locating the numeral 8 on the keypad. In order for Toya to successfully utilize Nicole's clue, not only did she have to conceptualize what "circle" meant (i.e., geometric thinking), but Toya also needed to understand the

numerosity two (i.e., enumeration). Nicole also employed numeric meanings when she said, “No, you didn't hit 3 two times.” For Toya to interpret this comment as Nicole intended, she would have to distinguish the meanings and use of the number word “three” as compared to the number word “two.” “Three” referred to the numeral 3 and its corresponding button, while “two” referred to the numerosity (i.e., enumeration) of times Toya pressed the number 3 button. Furthermore, the notion of numerosity as employed in Nicole’s hint demonstrated for Toya that enumeration not only includes determining the numerosity of quantities of concrete objects and visual attributes of objects or symbols (“eight is two circles”), but enumeration also includes determining the numerosity of actions (i.e., pressing a button two times).

Toya’s practice of phoning her grandmothers afforded Toya a context in which to support her developing understandings of number as label with respect to telephone numbers more generally.

Toya’s Version of Phone Numbers

At about quarter past 7 p.m., Nicole, Toya and I were sitting in the living room with the television on. I asked Nicole about her plans for the upcoming weekend and I confirmed if she had all my contact information. Nicole then took out a cell phone.

Nicole: Okay, Grace, what is it?

Grace: Three-oh-one,

Nicole: Um-hmm.

Toya: [As I recited my telephone number to Nicole, Toya chimed in,]
Four four four, two two two, [?], three, three, three, four, four,
four, [eh, eh, eh], one, one, one,

Nicole: Stop it. Go get tissues for your nose.

(March 22, 2006 – Home Visit with Toya)

Toya’s response to, or imitation of, the interaction between Nicole and myself seemed to reflect an understanding that telephone numbers consisted uniquely of a sequence of number words. In Toya’s version of a phone number, it also seemed as

though she perceived the structure of phone numbers to be comprised of a string of groups of three number words. Within the Washington, D.C. metropolitan area, callers must dial all ten digits of a local phone number in order to place a call. This typically results in people chunking phone numbers into a 3-3-4-digit patterned grouping of utterances when verbally stating a telephone number or dialing a number using a telephone keypad. As such, Toya's version of a phone number, which consisted of a repeated, 3-digit-patterned groupings of utterances, closely resembled this practice. Furthermore, Toya's version of a phone number demonstrated her knowledge and ease of use of the number words for the small numbers 1-4.

Nicole informed me that she began to let Toya tap the keypad to call her grandmothers around the time that Toya turned 4 years of age. Thus, by my first home visit, Toya had been engaged in the activity of identifying and locating numerals on a telephone keypad for approximately 1 month. On a few occasions, I observed Nicole engaging Toya in phoning her grandmothers outside of Nicole's home (e.g., in the afternoons when Nicole arrived at Ms. Franklin's to pick up Toya). In these events, Nicole used her cellular phone and encouraged Toya to enter all 10 digits of the phone number rather than using a speed dial function. By my final home visits with Toya, although I observed that Nicole still needed to recite the phone number to Toya, Toya needed less support from her mother to identify and to locate particular numerals.

Nicole also noticed Toya's increased independence in identifying and locating on a keypad the numerals assigned to her grandmother's telephone.

... now if I tell her the numbers to her grandmother's phone number, she can pretty much do that on the phone. But then if the same numbers, like the eight

and the six are just on a piece of paper, she has to be like, I have to coach her on it. But when she has the phone in her hand and I'm like, "Dial four, eight, four, zero," she does that with no problem.
(August 27, 2006 – Visit with Nicole)

According to Nicole's observations, Toya's capacity to identify larger single-digit numerals seemed context dependent. While Toya was able to identify and to locate independently the numerals 8 and 6 on a telephone keypad, she required her mother's assistance to identify these same numerals in other settings. In the context of phoning her grandmothers, it could be that Toya may have drawn on the mathematical content of spatial thinking to memorize the sequence and/or the locations of the buttons she needed to press in order to place the call.

2b. Values and beliefs. Toya's practice of regularly calling and talking to her grandmothers on the telephone seemed to reflect Nicole's value of maintaining close relationships with her family, especially with her mother and her sister Pam. The importance that Nicole placed on her family was instantiated through routine phone conversations with relatives, physical proximity of her residence to that of her mother's, and frequent visits and outings with her sister Pam and Pam's son Ricky. By engaging Toya in calling her grandmothers, Nicole demonstrated for Toya the value of regularly connecting with family. While Nicole's reasons for having Toya call her grandma (paternal grandmother) on the telephone may have also been motivated by logistical concerns such as coordinating Toya's transitions between her two domiciles, this activity still underscored the importance for Toya to interact regularly with family members on both her mother and her father's side.

Toya expressed an eagerness to engage in this practice, especially the activity of pressing the numeral buttons on the telephone keypad. Toya's interest in using the

keypad helped to further promote her practice of phoning her grandmothers as a meaningful context in which Toya could develop mathematical meanings involving number as label, numeric meanings (e.g., enumeration), and other mathematical content areas (e.g., geometry and spatial thinking).

2c. Social relations. Nicole fostered Toya's understandings about number as label and her capacity to interpret, identify and employ single-digit number words and numerals through guided participation (Rogoff, 2003) in the context of a familial cultural endeavor (i.e., phoning relatives) aimed at maintaining close communication and relationships with family members and kin. For the most part, Nicole managed this practice by directing Toya to call her grandmothers, reciting to Toya the numerals to input into the telephone, and providing hints when needed to Toya on how to identify particular numerals on the keypad. However, I also observed Toya occasionally initiate the practice of calling her grandmothers as well. When Nicole first began to let Toya use the telephone keypad to call her grandmothers, Toya relied heavily on Nicole's assistance to identify and to locate certain numerals on the keypad. By operating within Toya's zone of proximal development (Rogoff, 2003; Vygotsky, 1978) when providing hints as needed to help Toya successfully input the numbers on the keypad, Nicole supported Toya's increased independence in this activity.

Content: Number to Order

Toya engaged in emerging ordinal meanings of number when actively participating in familiar experiences that inherently involved identifying a particular location, action, or person according to its sequence in a set.

When Toya accompanied her mother at work on Saturday mornings, the experience of riding the elevator provided a meaningful context in which Toya could begin to relate number and order. They entered the office building on floor “L2,” located on a lower level, and then took the elevator up to floor “8,” or the top level, on which Nicole worked. Toya regularly demonstrated her interest in riding the elevator as she eagerly asked Nicole, “What button do I push?” Oftentimes, Nicole affirmed and responded to Toya’s eagerness by stating to Toya the number word signifying which floor Nicole wished to go and allowing Toya to find and to push the appropriate numeral button. Nicole provided Toya hints as needed to identify certain numerals. Through supporting Toya’s interest and active participation in operating the elevator, Nicole expressed or demonstrated meanings of number to order by using phrases like “we’re going up to” or “we’re going down to” along with stating the floor number of her destination.

Toya and Ricky, if he was also present, also joined Nicole as she delivered mail to various offices located on the top three floors of the building that were occupied by the law firm. This activity that entailed riding the elevator and taking the stairs to travel to different floor levels, afforded Toya additional opportunities to develop emerging ordinal concepts of number and to engage in interpreting and/or employing number words and identifying numerals for certain numbers.

Toya also showed evidence of beginning understandings of number to order in her meaningful use of the ordinal word “first.” Toya used the term “first” when she demonstrated and verbally described procedural tasks or directions, either to herself or to others, by saying, “First, you...” Toya also employed the word “first” when she

wanted to emphasize turn-taking (e.g., announcing, “I was here first!” when taking turns riding on a swing with her cousin Ricky).

Nicole’s ongoing acknowledgement of Toya’s birthday and age and Toya’s interest in her own birthday seemed significant in helping to foster Toya’s beginning understandings of number to order. Nicole’s attention to Toya’s birthday and her age also supported Toya’s capacity to interpret, identify, and employ number words and numerals for small numbers. In a conversation I had with Nicole after completing my home visits with her and Toya, I asked Nicole about when she thought Toya started to recognize numbers. Nicole reported,

Recognizing numbers, I think she may have been about, I think I always pointed them out as far as like her birthday and how old she is. So she always knew, could recognize the number of the age she was... and I think even now I don't think she can really recognize anything past five... Like she could say them, but to look at them, I don't think she could recognize anything past five without me reminding her or giving her a hint...
(August 27, 2006 – Visit with Nicole)

Like most youngsters her age, Toya proudly recited the date of her birthday (“February twenty-six”) whenever prompted or whenever the subject of birthdays came within ear-shot of her. As is evidenced by Toya’s statement of her birth date, she expressed the day of the month as a number word as compared to an ordinal number word. It was unclear whether or not Nicole expressed Toya’s birthday date to her in the same fashion or if Nicole instead employed the ordinal number word “twenty-sixth” and Toya, at the time of data collection, had not yet been able to discern the “th” following the “six.”

The following analysis demonstrates how Toya’s interest in her birthday, in particular when “reading” her collection of birthday cards, prompted Toya to engage

in beginning ordinal meanings of number.

Purpose and setting: “Reading” a collection of old birthday cards stored as keepsakes in an accessible location in Toya’s room. On a home visit with Toya, I observed Toya spontaneously “reading” through her collection of old birthday cards. These cards were kept in a plastic chest of drawers in her bedroom that also stored her socks, undergarments, and pajamas.

Birthday Cards

It was just before 7:30 p.m. and Toya, Ricky, and I were in Toya’s room. The television was on and set to the Disney Channel... Toya spontaneously acquired her birthday cards and began to talk to herself and ‘read’ these aloud as she flipped through her collection of cards:

Toya: Come get your card. From my mom...this one says, Happy birthday to, ... Happy birthday, I’m 4-years-old. Yup. I’m 3-years-old... Happy birthday... Happy [indcipherable word/number/ordinal number] birthday... Toya, I’m 3-years, yup, I’m 3-years-old... Happy birthday. 3-years-old... This one says, I love you. ... Okay, Happy birthday, [indcipherable word(s)] 3-years-old. Hi, I’m this birthday. I’m 3-years-old... Oh, that’s mine! That’s so [indcipherable word(s)]. I’m 4-years-old. Hey! Oh, this side. 4-years-old. Toya, me... All mine. These are mine. And nobody can touch them. Only me. Happy [indcipherable word/number/ordinal number, sounds like the word ‘this’] birthday! I’m 4-years-OLD! Let me see. I’m 8-years-old!? Uh-uhh! That, that say I’m 8-years-old? ... I’m still 4-years-old! I’m still 3-years-old... I’m 3-years-old! I’m 3-years-old. Aww, I got a Batman one! Happy [indcipherable word/number/ordinal number, sounds like the word ‘this’] birthday. 4-years-old.

Grace: You have a lot of cards.

Toya: Yeah, I got one more card though. Aww, I got one more card to go!... 3-years-old. You’re 3-years-old...

(June 2, 2006 – Home Visit with Toya)

In observing Toya “reading” aloud her birthday cards to herself, it seemed as though she understood the purpose of the cards and she recalled the nature of the sentiments that typically were expressed in both the pre-printed messages and images that appeared on the card as well as the hand-written notes from whom the card was

received. I noticed that, at least some of the time, the number word that Toya expressed did match the birthday age numeral I observed printed on the card that she was “reading” at a particular moment. For those occasions when Toya’s number word did not match the numeral on the card, it was not clear from my observations if Toya could not identify the numeral or if Toya’s interest in making up what she was “reading” in the course of play took precedence over any effort or desire she might have had to identify a given numeral correctly. Regardless of Toya’s intent, “reading” her birthday cards did provide Toya the opportunity to interpret, identify, and/or employ numerals and number words of small numbers while developing her understandings of number to order.

Toya seemed to be playing make-believe when she expressed aloud, “I’m 8-years-old!?” and then immediately responded to herself, “Uh-uhh! That, that say I’m 8-years-old? ... I’m still 4-years-old!” Toya’s statement seemed to reveal that, at least in the context of understanding her own age, Toya demonstrated emerging meanings of the ordinal or sequential quality of number. More specifically, Toya’s comments suggested that she understood that since she was “still 4-years-old” she had not yet turned 8-years-old; this further implied her understanding that the number 8 or her 8-years-old birthday (i.e., her eighth birthday) will come *after* the number 4 or her 4-years-old birthday (i.e., her fourth birthday).

Values and beliefs. Toya’s own interest in her birthday and in her age seemed, in part, to motivate her to read spontaneously past birthday cards. Nicole reported her own values and beliefs regarding greeting cards that she had received,

I don't like throwing away cards... I've just been like you're not supposed to throw away cards... They've written in it. Their own words are in there. I just don't feel like throwing them away. I have a boat-load of cards... And I'm like, "Are you supposed to throw these away?" I just don't feel like you're supposed to throw away cards.

(August 27, 2006 – Visit with Nicole)

Nicole also informed me that she had been doing the same with cards addressed to Toya and saving these as keepsakes as well. One of the statements 'read' aloud by Toya as she flipped through her collection of birthday cards included, "This one says, 'I love you'." It therefore seemed that people with whom Toya had a significant relationship sent her birthday cards both as a gesture to celebrate the occasion of Toya's birthday and to express their love to her. Nicole's value and practice of saving greeting cards, especially since these included personal written messages and sentiments of love, provided Toya meaningful artifacts with which Toya could engage or 'read' at her leisure, including employing meanings of number to order in the context of focusing on her birthday and understanding her age.

Social relations. In the birthday card event, Toya primarily engaged in the activity of "reading" aloud her birthday cards by herself, even expressing aloud, "These are mine. And nobody can touch them!" As such, Toya 'read' her birthday cards as she liked, sometimes making-up birthday ages and employing number words at her choosing and whim. Within this 'play reading' Toya spontaneously created for herself a scenario ("...that say I'm 8-years-old? ... I'm still 4-years-old!") in which she could reason mathematically, employing notions of number to order, regarding her age or "birthday."

Content: Expressing Number Words Orally

In a Descending Count Sequence

Purpose and setting: Verbally counting down to an anticipated action while playing. In the time I spent with Toya, I did not observe her engaging in rote counting. Instead, I witnessed Toya expressing number words orally in a count sequence primarily for the purpose of what I describe as *counting down to an anticipated action*. During these mathematical events, Toya employed number for numeric purposes as she stated a short sequence of small counting number words in descending order in the moments just before undertaking a particular action during unstructured play with/or alongside at least one other child. The following mathematical event took place on a weekday evening, shortly before 7 p.m., during a visit with Toya to her Auntie Pam's apartment.

"Blast off!"

(Note: During this visit, Toya played with her cousin, Ricky, and Pam's significant other's son, Michael. My recording device failed to operate during this visit so I could not obtain transcripts of the dialogue that occurred.)

Toya, Ricky, and Michael played outside on the apartment balcony, going back and forth between the balcony and the apartment's living/dining room area via a sliding glass door. The glass door, a brick wall to the left, and a metal L-shaped railing running along the front and right side defined the rectangular space of the balcony. A two-seater couch sat catty-corner opposite from the brick wall. Eventually, Toya and Ricky each transformed into their own imaginary astronaut/rocket. I watched as they engaged in a pattern of play – pretending to first "blast off" from the brick wall, then running a few steps across the balcony to *fly* to their destination, and finally taking a dramatic leap to *land* onto the couch. They completed repeated *missions*, each running back to their respective *base* on the brick wall and then ceremoniously counting down together, "5, 4, 3, 2, 1, blast off!"
(March 30, 2006 – Visit with Toya at her Auntie Pam's Apartment)

During their unstructured play, Toya and her cousin Ricky seemed to mimic real-life events of shuttles launching into outer space, including counting aloud in

descending order the remaining moments leading up to launch. Simultaneously counting down aloud enabled Toya and Ricky to coordinate their timing of when to bound from their respective places on the brick wall toward the sofa located on the opposite side of the balcony.

I also observed Toya verbally counting down to an anticipated action while playing in Ms. Franklin's backyard, in particular, when taking turns with her peers to go down a slide. Each time Toya took her turn at the top of the slide, she counted aloud "five, four, three, two, one" just before descending along the slide's ramp. Toya's verbal countdown seemed aimed at announcing the impending occurrence of her descent on the slide.

In both the blast off and the slide events, Toya's verbal countdown occurred while she played, and in particular, just before she engaged in a physical motion initiated from a still position. Toya's verbal countdowns further seemed to reflect her eagerness to anticipate the actions of running (or 'blasting off') and sliding, respectively. As such, Toya's activity of counting down aloud in these events seemed social in nature, as evidenced in Toya and Ricky coordinating their 'blast-offs' through their countdowns and Toya announcing her impending descent on the slide with her own countdown. During these events, Toya appeared to demonstrate understanding that when verbally expressing number words in a count sequence, even when starting with the number word "five" and ending with the number word "one", that these number words followed a specific order or sequence.

Additional evidence and analysis of Toya expressing number words orally in a descending count sequence. On one occasion, I observed Toya expressing number

words orally in a descending count sequence aside from the purpose of her counting down aloud to an anticipated action. I include this event in this discussion to augment my analysis of Toya's understandings expressing number words orally in a descending count sequence. The following event took place at day care just two days prior to the blast off event.

Placemats

Aaron and Toya finished their lunch and waited at the table for naptime as Ms. Franklin put down the sleeping cots. Aaron and Toya still had each of their placemats for eating in front of them... On Toya's placemat appeared a graphic of the numerals 1, 2, 3, 4, and 5. Toya pointed to each numeral in order, from left to right, starting with the numeral 1, while she counted down aloud, "Five, four, three, two, one."
(March 28, 2006 – Day Care Visit)

In the placemats event, Toya and Aaron entertained themselves with their placemats while Ms. Franklin prepared the space for naptime. During this time, Toya spontaneously expressed number words in a descending count sequence while referring to the numerals on her placemat. It was unclear if Toya incorrectly identified the numerals on her placemat as she pointed to each or if she instead expressed the number words five to one in reverse order in one-to-one correspondence with the images of numerals on her placemat without concern for identifying the number words she expressed with the numerals she observed on her placemat. It was also unclear whether Toya arbitrarily chose to count down from five to one and it just so happened to correspond with the number of images/numerals displayed on her placemat or if she noticed that there were five images and wanted to count these in reverse order.

While the mathematical meanings of Toya's actions in this placemats event remain uncertain, she appeared nonetheless to demonstrate knowledge of the

descending count sequence of small numbers and to apply this sequence in one-to-one correspondence to distinct images. As such, Toya seemed to recognize that the count sequence “five, four, three, two, one” represented 5 distinct number words or entities as opposed to thinking of the sequence as signifying a single, multi-syllabic number word, such as “four four.”

Earlier in the day when this placemats event occurred, I observed Toya counting aloud, with one-to-one correspondence and cardinality, a set of seven pom-poms that she observed in a photo image of a scarf. I discuss this pom-poms event later in the content section focusing on enumeration. As Toya demonstrated in the pom-poms and placemats events, she expressed number words in an ascending and descending count sequence, respectively, and assigned each number word in one-to-one correspondence with a distinct image. More specifically, Toya employed a consistent sequence for the number words one through five, maintaining the relative position of each number word with the others in either direction, across these two events.

In the placemats event, since Toya demonstrated one-to-one correspondence with distinct images while she expressed number words in a descending count sequence, Toya’s meanings of expressing “five, four, three, two, one” did not appear limited to a social activity in which she engaged while playing with and/or alongside others for the sole purpose to announce an impending, eagerly anticipated action. Rather, in the placemats event, Toya also appeared to exhibit developing meanings with respect to early enumeration concepts. In particular, Toya seemed to employ a count sequence more broadly, not just when expressing numbers words in ascending

order while enumerating a small set of objects as evidenced in the pom-poms event, but by exploring the activities of expressing number words in a descending count sequence in one-to-one correspondence with pointing to distinct images.

Values and beliefs. Nicole's value in nurturing family relationships and her practice of having Toya spend significant time with kin afforded Toya regular opportunities to play with Ricky, her other cousins, and Michael. Through play, Toya spontaneously engaged in meaningful experiences involving number. Likewise, Ms. Franklin provided regular time for unstructured play for the children to explore and to engage in activities of their choosing. During free play, Toya naturally employed numeric meanings of number words.

Social relations. When playing with Ricky, her other cousins, and Michael, Toya spontaneously engaged in activities in which she observed and participated in employing number words for both social and numeric purposes.

In the blast off event, while it was unclear who initiated the countdown, for the most part, Toya and Ricky engaged in this activity together, as a shared experience, primarily for the purpose of coordinating their timing of when to "blast off" from the brick wall. In addition, shortly following the blast off event, Toya, Ricky, and another cousin who was also present in the apartment, in unison, spontaneously did jumping jacks with the boys leading the oral count sequence from 1 to 20 in correspondence with each jumping jack repetition. This activity resembled a fitness practice of counting repetitions of a particular physical motion or exercise. Thus, under the guidance of her older cousins, Toya engaged in meanings of

enumeration, as she mimicked their activity and recitation of the count sequence in one-to-one correspondence to each jumping jack.

At day care, Toya initiated her verbal countdown in the slide event. Not only did Toya's expression of a descending count sequence seemed to indicate her excitement about going down the slide, but it also seemed, in part, aimed at seeking the attention of her peers during this activity. On the other hand, during the placemat event, Toya and Aaron primarily engaged in parallel play to entertain themselves while waiting for naptime, although some of their activities did seem to feed off of one another. In this event, Toya spontaneously expressed number words in a descending sequence in one-to-one correspondence with the images of numerals on her placemat, as an individual activity. As will be discussed later in Aaron's chapter, Aaron observed Toya's activity and attempted to approximate her behavior.

Content: Number to Enumerate

Toya also employed number for numeric means when she enumerated objects in her everyday settings and/or activities. Toya primarily engaged in enumeration for the following purposes: (1) to determine spontaneously the numerosity of objects she observed in her immediate environment with or without further motive, (2) to inquire candidly about the numerosity of a meaningful object; or (3) to respond to the question, "How many?" I also observed Toya meaningfully engaging in enumeration concepts while interacting with her mother in the context of her bedtime routine.

1a. Purpose and setting: Number to enumerate – Spontaneously enumerating objects observed in everyday settings and activities. I observed Toya, through her own initiative, naturally enumerate small sets of objects she observed in her

immediate environment. I discuss two of these mathematical events below. These events took place amidst Toya's engagement in meaningful everyday experiences at day care and at home, respectively.

(i) Apple Slices

Ms. Franklin prepared pre-sliced apples for the children's lunchtime dessert. I gave Toya her apples, serving her one apple slice at a time. As I placed the first slice on Toya's plate she spontaneously announced, "One,". When I put another slice on her plate, Toya said, "two."... After Toya finished her two apple slices, she asked for "two more" and she announced she ate all of her vegetables... I noticed that both Aaron and Toya each finished their apple slices. I had three more apple slices left to distribute so I explained to them that I would give them one slice each and that I would have one for myself so it was fair. (Note: my priority at this time was to avoid a contentious situation over how to share the remaining apple slices so I therefore decided it would be appropriate for me to express a mathematical idea here.) Moments after I gave each child their one apple slice, Toya looked over at me and nodded her head in agreement with my decision, saying something to the nature of, "It's fair." (March 28, 2006 – Day Care Visit)

(ii) Movies

While Nicole and I were chatting in her living room, I observed Toya noticing a set of DVD's on the end table nearby. She soon became interested in buying these and tried to interject herself into our conversation. As Nicole continued her story about Toya with me, Toya repeated aloud, "Mom, I wanna buy this movie!"

Nicole: [Nicole briefly turned her attention to Toya,] Hmm?

Toya: And um, this movie?

Nicole: What is that?

Toya: And um, a scary movie? And can I, and can I, can I buy this, and this, and um, this?

Nicole: I don't know.

Toya: Three of them.

Nicole: I don't know. [Nicole then returned her attention back to me and continued telling me her story.]

(June 20, 2006 – Home Visit with Toya)

In the apple slices event, Toya first spontaneously employed meanings of enumeration when she used explicit number words to count, with one-to-one correspondence, each apple slice I served to her. Although Toya did not demonstrate evidence regarding her understandings of cardinality with respect to these apple

slices, Toya appeared to determine the numerosity of the apple slices for no further reason or goal other than simply to count them. Toya also employed enumeration concepts later in this event when she stated the number word “two” to specify how many additional apple slices she wished to have after finishing her initial serving. Further, when Toya requested to have “two more” apple slices this supported the assertion that Toya’s activity when she received her initial serving of apple slices reflected meanings of number to enumerate rather than number as label, or assigning the first apple slice the name “one” and naming the second slice “two.” Finally, Toya appeared to employ meanings of enumeration when she voluntarily affirmed the fairness of me giving to each Toya, Aaron, and myself one slice of the three remaining apple slices. That is, Toya seemed to recognize that each of us had the same numerosity of apple slices and that there appeared to be no other alternative way to share the remaining apple slices fairly between us.

The movies event occurred during a period when Toya had left her room to explore other activity in the apartment. She noticed a set of DVD movies on a table in her living room and at once eagerly began to ask her mother if she could buy these. Toya spontaneously specified that she wanted to buy “Three of them.” It was not clear whether Toya subitized the set of DVD movies in which she was interested to determine its numerosity or if Toya implicitly engaged in some aspect of counting as she physically pointed to, held up, touched, and verbally referred to each DVD movie while she persisted in asking her mother to buy these for her. For instance, when pleading with Nicole, each of Toya’s following phrases appeared to suggest that Toya identified three movies she wished to have/buy: "Mom, I wanna buy this movie! ...

And um, this movie? ... And um, a scary movie?"; "... can I buy this, and this, and um, this?" While Toya's enumeration method was unclear, she nonetheless demonstrated emerging meanings of enumeration, as evidenced in Toya employing the number word three to inform her mother specifically how many movies she hoped Nicole would buy for her.

Ib. Values and beliefs. In the apple slices event, Toya demonstrated her tendencies to finish her food and to request for second servings during meal and snack times at day care. Toya's spontaneous enumeration of her initial serving of apple slices may have also indicated her eager anticipation to eat these for dessert. My response to Toya's request for "two more" apple slices communicated the importance of sharing fairly the remaining apple slices between Toya and Aaron since they both appeared to want more of these to eat. Although my method for achieving a fair outcome included giving one apple slice to myself, Toya conferred that the result of each person having one apple slice seemed fair. As such, in the name of fairness, Toya appeared to understandably accept that she could not have two more apple slices as she requested.

Nicole's family value and practice of watching television or DVD movies as a significant form of leisure provided a meaningful context for Toya to engage spontaneously in concepts of enumeration. I often observed small sets of DVD movie rentals lying on Nicole's kitchen counter or on the tables in her living room, thus allowing Toya easy access to these. Toya's persistence in asking her mother to buy the particular set of DVD movies in which she took interest in the movies event

further exemplified Toya's intrinsic desire to have these movies as well as the importance Toya placed in watching television and DVD movies for entertainment.

1c. Social relations. Toya's activity of counting each apple slice as I placed each one on her plate did not seem to reflect a regular mealtime practice within Toya's day care or home settings. Prior to this event, I did not observe Toya engaging in similar behavior at day care with Ms. Franklin or at home with her mother, nor did I observe Ms. Franklin or Nicole modeling this behavior for Toya during meal or snack times. Toya's spontaneous response of enumerating verbally the apple slices in this event therefore suggested that Toya's activity seemed to reflect more so an intrinsic desire to enumerate rather than a regular practice that either Ms. Franklin or Nicole fostered in their everyday experiences of feeding Toya. Further, Toya's enumeration of the apple slices also did not appear driven by a desire to please me as I did not affirm Toya's mathematical activity during this event. Toya's request to have "two more" apple slices indicated that she felt both welcomed to ask for second servings during meals and snack times at day care, and that to the extent possible and as deemed appropriate by Ms. Franklin or another adult assistant at the day care (e.g., her Auntie Pam or myself), her request would be granted.

Toya initiated the movies event as she tried to interject herself verbally in the conversation Nicole and I were having. However, Nicole seemed focused on telling me a story about Toya while Toya attempted to plead with Nicole to buy her three DVD movies. Nicole's eventual reply to Toya seemed aimed instead at diffusing Toya's insistent pleas and stopping Toya's fixation over the movies so that Nicole could resume her conversation with me. That is, Nicole did not appear interested in

seriously considering Toya's request. If she did entertain Toya's request, Nicole may have further engaged Toya in concepts of enumeration, such as negotiating with Toya how many movies Nicole would be willing to buy.

2a. Purpose and setting. Number to enumerate – Inquiring about the numerosity of a meaningful object at day care. In addition to enumerating spontaneously a small set of objects as demonstrated in the apple slices and movies events, Toya also engaged candidly in meanings of number to enumerate when she inquired about the numerosity of a set of handi-wipes refill packages with which she engaged with me at day care.

Handi-Wipes

I invited Toya to help me refill her empty handi-wipes container. Toya seemed eager to help me in this task. She acquired and gave me the unopened package of handi-wipes refills that she brought with her from home. I immediately became concerned that the refill package seemed to contain more wipes than what I estimated could fit in Toya's handi-wipes container. I asked aloud to Ms. Franklin, who was upstairs from the day care area, if Toya's container could accommodate the entire refill package. Ms. Franklin responded that normally there are two packs or bags of refill wipes within the larger refill package... I then asked Toya to get me a pair of scissors so that I could open the refill package. She found a pair of scissors and handed them to me handle first. Once I cut open the bag, Toya inquired, "Are there two in there," referring to whether there were two smaller packs of refill wipes inside the larger bag as Ms. Franklin stated. I posed the question back to Toya as I invited her to look into the bag and determine herself if there were indeed two smaller packs inside... Together, Toya and I took turns re-filling her handi-wipes container with the set of new handi-wipes.
(April 4, 2006 – Day Care Visit)

While helping me with the task of refilling her handi-wipes container with the new package of handi-wipes, Toya demonstrated her interest in the activity by asking me, once I opened the refill package, "Are there two in there?" Toya's question indicated that she attended to and made meaning from the verbal exchange between Ms. Franklin and me when Ms. Franklin informed me that the larger refill package

contained two smaller refill packs. In response to her candid inquiry, I prompted Toya to check for herself, and she confirmed that there were indeed two smaller packs in the larger bag. Toya seemed to subitize the small set of smaller packs to determine whether it had a numerosity of two.

2b. Values and beliefs. Ms. Franklin required that each child at the day care had their own individual particular supplies, including a container of handi-wipes. Toya's handi-wipes appeared to be an important and meaningful, everyday resource to her since she often utilized these to clean her congested/runny nose. As such, when Ms. Franklin tasked me to refill Toya's handi-wipes container, I thought Toya might like to help me in this activity. Toya appeared to take interest in this task, as evidenced in both her willingness to assist me and in her inquiry about the numerosity of the smaller refill bags.

2c. Social relations. The task of refilling Toya's handi-wipes container became a shared experience between Toya and me. Toya actively participated in the activity by bringing her bag of refills to me; attending to the verbal exchange between Ms. Franklin and me; obtaining a pair of scissors for me upon my request; inquiring about the numerosity of and then upon my prompting, enumerating the smaller refill packs; and helping me re-fill her container with the new handi-wipes. While Toya asserted herself as an active participant in the process involved in completing the task of refilling her handi-wipes container, she appeared to perceive me as the primary doer as evidenced in her deferring to me and asking me about the numerosity of the smaller refill bags. That is, Toya could have instead chosen to look directly inside the bag herself once I opened it to enumerate the smaller packs, but instead she appeared

to defer to my leadership regarding her curiosity about the numerosity of the smaller packs.

Because Toya initiated her candid inquiry to me regarding if there were indeed two smaller refill packs within the larger bag as noted by Ms. Franklin, I chose to take the methodological path of inviting Toya to determine the numerosity of the smaller packs instead of simply answering her question. Inviting Toya to enumerate and to confirm the numerosity of these smaller packs did not appear to impact the essence of everydayness of this event and it allowed Toya to answer her own mathematical inquiry of which she seemed intrinsically curious.

3a. Purpose and setting. Purpose: Number to enumerate – Responding to the question, “How many?” In the prior mathematical events in which Toya engaged in meanings of number to enumerate, Toya spontaneously enumerated or initiated an inquiry regarding the numerosity of everyday objects. In the two events that follow, Toya engaged in enumeration concepts when prompted by the question, “How many?” The first event occurred while Toya and Aaron interacted with me at day care. The second event took place during an observation visit with Toya, her mother, and her Auntie Pam to the grocery store. Given that I prompted Toya with the question of “how many” in the former, I limit my discussion of this event to the dimension of content, focusing my analysis on the mathematical understandings that Toya appeared to demonstrate in this event. On the other hand, the latter event exemplified a familiar practice in which Toya seemed to regularly participate with Nicole and her Auntie Pam. As such, I employ all four analytic dimensions to examine and discuss this event.

The following mathematical event took place at Ms. Franklin's day care.

Pom-Poms

Aaron and Toya came to stand on either side of me while I sat with my laptop opened in front of me. Aaron looked on quietly as Toya pointed to and recognized a few icons that represented different software applications (i.e., music, calendar, pictures) displayed on the screen. Toya then requested to see pictures so I opened iPhoto to share a few with her and Aaron. While I scrolled through my pictures Toya asked me to stop at certain images. One was a picture of a multi-colored scarf I knitted that included seven different colored pom-pom embellishments. I spontaneously asked Toya how many pom-poms she saw on the scarf. She pointed to each pom-pom and counted aloud in sequence from one to seven. I asked Toya again how much, and she told me seven. Aaron, without prompting directly from me, appeared to attempt to count the pom-poms as well. He pointed, without one-to-one correspondence, to the images of the pom-poms in the picture while he stated, "Three, four, five, six." Toya informed Aaron that he missed one when pointing to the pom-poms.

(March 28, 2006 – Day Care Visit)

Given that Toya appeared interested in looking at the digital photo image of the scarf, of which the pom-poms were a prominent feature, I chose at that moment to ask her how many pom-poms she saw on the scarf. I therefore engaged Toya in a school-like discourse by posing a known-answer question (Rogoff, 2003) to her.

Toya's response demonstrated the following understandings relevant to enumeration: a stable order count of at least 1 through 7, one-to-one correspondence, and cardinality. Toya's numeric knowledge as evidenced in this pom-poms event seemed particularly significant given that she counted static images of the pom-poms appearing on a digital photo and that the set of pom-poms had a numerosity of seven. On formal measures assessing young children's understandings of cardinality, a similar task of counting static images typically would include six such images. Toya's comment to Aaron that he "missed one" pom-pom not only further exhibited Toya's understanding of cardinality but it also seemed to indicate evidence of her developing

knowledge of concepts of counting on. That is, since Aaron's verbal count sequence ended at six, Toya recognized that the next number word that he should have stated was seven.

The following event took place during a grocery shopping trip observation visit with Toya.

Counting Apples

Toya opted to sit and ride in the shopping cart as Nicole and Pam shopped for items on and off their list. I loaned my cellular phone to Nicole when she asked if I had a calculator she could borrow to help her keep track of her expenses. Nicole and Pam first stopped in the produce (fruit) section. Nicole invited Toya to help her pick a few golden delicious apples and put these in a plastic bag. Once they collected three apples in the bag, Nicole asked Toya how many apples were in the bag. Toya looked at the bag of apples and responded, "Four." Nicole asked Toya to count again. This time, Toya pointed to the apples in the bag as she stated "One, two, three." Nicole praised Toya for her response and then proceeded to weigh the apples using a nearby scale. (June 11, 2006 – Grocery Store Visit with Toya)

In the counting apples event, Toya employed number to enumerate to respond to her mother's prompt regarding how many apples were in the bag. In Toya's initial response, it seemed as though she attempted to subitize the set of apples given her one word response of, "Four." Realizing that Toya had erred in her response, Nicole invited Toya to count the set of apples again. During this second attempt, Toya demonstrated understanding of one-to-one correspondence as she pointed to each apple and stated, "One, two, three." While Toya's response did not indicate cardinality, given her capacity to enumerate with cardinality a set of seven pom-poms in a digital photo image more than two months prior to this event, Toya more than likely understood that they collected three apples in the bag although she did not demonstrate this knowledge in her response.

Toya's initial error in attempting to subitize the set of apples may have been due to the fact that the apples were collected in a plastic bag and may not have been easily distinguishable in this arrangement when quickly viewing these. Toya's successful enumeration of the set of apples in her second attempt may have been supported through Toya's physical action of pointing to each apple in one-to-one correspondence with her verbal count sequence, thus being able to account for and keep track of all the apples in the bag.

3b. Values and beliefs. Nicole's effort to ask Toya how many apples they collected in the bag seemed to demonstrate Nicole's formal perspective of mathematics and her desire for Toya to "know her work," including developing Toya's facility to count objects, before she started formal schooling. Nicole's engagement with Toya in this event seemed to support Ms. Franklin's positive perception of Nicole and her efforts to try to "work with" Toya at home on school-like tasks. Nicole's interaction with Toya further seemed to indicate that Nicole appeared to place value in Toya's capacity to respond accurately to her school-like questions, as evidenced in Nicole's request for Toya to count the apples again after her initial error and in Nicole's praise of Toya after her second response.

3c. Social relations. Toya's regular experiences at the grocery store alongside her mother and her aunt afforded Toya meaningful opportunities to observe and to engage in the various tasks and activities that typically constituted Nicole's and Pam's shared practice of grocery shopping. In this sense, Nicole engaged Toya in guided participation (Rogoff, 2003) in this practice.

At times the tasks and activities involved in Nicole's practice of grocery shopping provided a natural context in which Toya could engage, either spontaneously or through prompting, in numeric meanings. As evidenced in the counting apples event, Nicole involved Toya in the activity of selecting produce for purchase by providing Toya a plastic bag in which to collect the apples that Nicole selected. Nicole further utilized this activity of selecting produce as a context in which to engage Toya in school-like discourse by posing a known-answer question (Rogoff, 2003) to Toya regarding how many apples they collected in the bag. Nicole's expectations for accuracy, her efforts to prompt Toya to respond as such, and her praise of Toya once she appeared to answer the question correctly, also reflected traditional school-like practices. Furthermore, Nicole did not proceed to weigh the apples until it appeared to her that Toya had successfully responded to her question. That is, Nicole did not move on to the next activity until she was satisfied with Toya's response.

Additional analysis regarding Toya's mathematical events involving number to enumerate when prompted with "How many?" Analysis of the following transcription data provides further insight regarding this discussion of Toya's mathematical events in which she engaged in meanings of enumeration when responding to her mother's prompt to count everyday objects.

During a conversation I had with Nicole after completing my home visits with Toya, I asked Nicole about the extent to which she engaged Toya in counting. Nicole responded,

Nicole: Yeah we count. A lot.

Grace: Now when did you start doing that?

Nicole: Probably when she was two. A little over two.
Grace: And in what context?
Nicole: Just everything. We count everything, everything that there's more one of we count. And that's just my,
Grace: For example? [Note: In reflecting on this conversation, I realized that a more productive course of action for me to have taken here would have been to just let Nicole continue her thought rather than redirecting her toward providing specific examples of what they counted.]
Nicole: Just like, I mean anything. Shoes.
Grace: Um-hmm.
Nicole: Food, if there is more than one different kind of food on her plate.
Grace: Um-hmm.
Nicole: Um, cars, like, 'We have two cars now.'
Grace: Um-hmm.
Nicole: Just anything.
Grace: Okay.
(August 27, 2006 – Visit with Nicole)

Although Nicole reported in the above post-observation-visits conversation with me that she counted “a lot” with Toya since she was about 2-years-old, Nicole did not mention counting with Toya as something that was “maybe math” on my first home visit with her and Toya. The closest example Nicole provided regarding Toya and counting involved Nicole specifying to Toya how many cookies she could have whenever Toya asked to have these as a snack.

During my first home visit with Toya and Nicole, I was not expecting Nicole to provide an exhaustive list of the range of activities in which she perceived Toya engaged in mathematics in the course of her everyday activities. However, as was posited earlier, Nicole’s perceptions regarding what activities might be mathematical, in her life as well as in Toya’s, reflected a preeminence of number and operations. As such, counting objects solely for the sake of counting would seem to fit within this characterization of what constituted an activity that was “maybe math.” Although

Nicole later reported that she counted “a lot” with Toya, it may be that Nicole did not share her practice of prompting Toya to enumerate objects for the sole purpose of counting as an activity that was “maybe math” simply because Nicole did not think of it at the time during my first home visit.

I did observe evidence of Nicole’s reported practice of prompting Toya to count in the counting apples event. However, this was the only occasion, aside from observing Nicole assist Toya in completing a school-like mathematics worksheet sent home from day care, in which I observed Nicole asking Toya to count or enumerate a set of objects (greater than 1) for the sole purpose of counting or enumerating. This is not to say that Nicole did not engage Toya in counting during those times when I was not present. Rather, during my observation visits, this particular type of event did not appear to occur as frequently as implied by Nicole.

While it seemed that Nicole and Toya did try to live and act as they normally would during my visits, it may be that Nicole recognized the activity of selecting and collecting apples at the grocery store as an appropriate context in which to have Toya count a small set of objects while in my presence. Despite what seemed to be an inconsistency between what Nicole reported and what I actually was able to observe, the inclusion and analysis of the counting apples event as part of the discussion of Toya’s everyday mathematical events does reflect the perceived salience of Nicole’s practice of prompting Toya to count as reported by Nicole in our post-observation-visits conversation.

4a. Purpose and setting. Setting: Meaningfully engaging in enumeration concepts during bedtime routines. In the following episode, two mathematical events

that engaged Toya in enumeration concepts emerged from Toya's interaction with her mother during her bedtime routine. The first event stemmed from Nicole's practice of reading a bedtime story to Toya, and the second event arose shortly afterwards when Toya requested to have some cookies before bedtime.

Bedtime: Rungs and Cookies

After Toya took her bath she sat, wrapped in her bath towel, in her Dora the Explorer saucer chair as Nicole, who sat on Toya's bed facing Toya, read Toya a Dora the Explorer story that Toya selected. Although Toya's attention occasionally shifted to the television set that remained on during this time, she did engage with the story as evidenced in the following,

Nicole: [Nicole read the story aloud to Toya,] "We made it to the rainforest. Look! There's a zip cord. We can ride the zip cord through the treetops and zip through the rainforest. Do you see a ladder we can climb up?"

Toya: Yes! It's missing pieces!

Nicole: "Oh no! The ladder is missing six rungs. Let's find the rungs. Do you see any of them?"

Toya: Yes. I find one, two, three, four, five, six! [Toya pointed to each rung she observed in the picture book in one-to-one correspondence with each number word she stated. She expressed the number word six with a slightly higher pitched tone of voice.]

Nicole: Yup. That's all six of them. [Nicole turned page and continued to read aloud,] "Thanks for helping find the six ladder [sic]. Now let's go save Baby Jaguar. Weee!" [Nicole continued and finished reading aloud the story.]

After finishing the story, Nicole began to look for Toya's "nighties" and "undies" to dress Toya for bed for the evening. As Nicole gathered Toya's bedtime garments, Toya asked,

Toya: Mom, can I have some cookies?

Nicole: What?

Toya: Can I have cookies? [Toya then said something that sounded like, "Ask cookies?"]

Nicole: [Nicole replied in a firm voice,] Two.

Toya: Okay.

Nicole: 'Cause it's bedtime.

[Toya briefly bickered with her mother over socks she wanted to wear for bed.]

Nicole: [Nicole refocused Toya's attention,] Go get your cookies! [Toya then headed to the kitchen to get her cookies.]

Nicole: Put 'em on a napkin. [Nicole emphasized again,] Two cookies.

Grace: [I asked Nicole,] Where are the cookies?

Nicole: On the counter right there.

Grace: Okay. Is that like a normal routine for her, to ask for cookies?
Nicole: Yeah, my mother usually bakes her some cookies.
Grace: That's what I heard from Pam. Yeah.
Nicole: Um-hmm. [Nicole then directed her attention to Toya who seemingly had difficulty locating the cookies on the kitchen countertop,] Look, look some more. Look again cause they're up there.

I casually followed Toya out to the kitchen and watched as she put a piece of a cookie in her mouth and another cookie on a paper towel. With her two cookies acquired, she then returned to her room.
(March 29, 2006 – Home Visit with Toya)

In the rungs-of-the-ladder event, the Dora the Explorer story that Nicole read to Toya appeared to prompt Toya to count the rungs pictured in the book. While the text did not explicitly state for Toya to count the rungs nor did the text include the question of “How many,” the book did invite Toya to help “find” the missing rungs, to which she responded by enumerating the rungs. Toya’s response demonstrated evidence of Toya’s stable order count, specifically the number words one to six, as well as one-to-one correspondence. Although Toya’s activity in the rungs event did not clearly indicate that she demonstrated cardinality, she did show evidence of cardinality the day before in the pom-poms event when she enumerated a set of seven pom-poms pictured in a digital image. Furthermore, Nicole’s affirmation of, “Yup. That's all six of them,” also seemed to reinforce for Toya the notion of cardinality with respect to the numerosity of the rungs.

In the cookies event, Toya requested to have “some cookies.” Nicole emphasized to Toya that she could only have “two” cookies since it was bedtime. As evidence in Toya’s response, she interpreted the number word two and constructed a set of two cookies. Toya’s activity of placing one cookie in her mouth and the other cookie in a paper towel may have indicated her emerging understandings of part-part-

total concepts. That is, one cookie in her mouth (part) and one cookie in the paper towel (part) make two cookies (total).

4b. Values and beliefs. Nicole reported that she read to Toya at bedtime, although not every night. Toya had her own collection of books in her room, and Nicole also occasionally borrowed books from the local library to read to Toya at night as well. Nicole therefore appeared to recognize the importance of books in Toya's life and to value the practice of reading to Toya. As such, exposing Toya to books and sometimes reading to her at bedtime seemed to support Nicole's desire for Toya to "know her work" prior to starting school that fall.

Although the television set remained on while Nicole read the story to Toya and Toya occasionally turned her attention toward the television, Toya demonstrated interest in the story as well, as evidenced in her active participation with noticing and counting the missing rungs.

Toya's mama regularly baked cookies for Toya, a gesture that also seemed to reflect her love for her grandchild. Toya enjoyed eating her mama's cookies for snacks, especially at bedtime. Nicole demonstrated her concern for monitoring Toya's intake of sweets in the cookies event by emphasizing to Toya that she could only have two cookies.

4c. Social relations. While Nicole managed Toya's bedtime routine activities from helping Toya finish her bath to preparing the television such that Toya had something to watch as she settled in her bed to go to sleep, Nicole also encouraged Toya's active participation in this process as well. For instance, in the above bedtime episode, Nicole invited Toya to select which story she preferred to read with Nicole.

Nicole's read aloud of the story, and Toya's own interest in the story, prompted Toya to actively participate in the read aloud and to engage with both her mother and the illustrations in the book. For example, when Nicole read, "Do you see a ladder we can climb up," Toya immediately responded, "Yes! It's missing pieces." It appeared that Toya and her mother had read this story together before as the next line that Nicole read aloud was, "Oh no! The ladder is missing six rungs. Let's find the rungs. Do you see any of them?" Toya eagerly responded, "Yes," and then she proceeded to count aloud the missing rungs she observed in the book's illustration. Consistent with Nicole's desire for Toya to count sets of objects correctly, Nicole affirmed Toya's activity by stating, "Yup. That's all six of them."

While Toya initiated her request to have "some cookies," Nicole managed Toya's consumption of these by limiting the number of cookies she could have. However, Nicole encouraged Toya's developing meanings of small numbers and enumeration concepts by permitting Toya to construct on her own the set of cookies with numerosity as enforced by Nicole.

Content: Number to Signify Discernable

Attributes of a Physical Quantity or Entity

Toya engaged in number for nominal and emerging numeric purposes when she identified numerals and employed and interpreted number words that signified discernable attributes of physical quantities or entities. While the event in which Toya "read" past birthday cards did refer to age, an attribute describing how old a person is, Toya did not conceive of age as a measure of time with a metric of years. Rather, Toya appeared to refer to age more in terms of order. For instance, Toya seemed to

understand that the number word eight comes after the number word four when expressing number words in an ascending count sequence, and accordingly, she recognized that she could not yet be 8-years-old as she was “still 4-years-old.”

The following set of spontaneous mathematical events illustrates two contexts in which Toya identified numerals and interpreted and employed number words that signified a discernable attribute of a physical quantity or entity. The first context involved Toya at the grocery store while her Auntie Pam weighed the set of apples that Nicole and Toya collected. The second context involved Toya engaging in meanings of clothing and shoe “size” or “number” during everyday activities and settings including initiating a conversation at home and shopping with her mother at a familiar store.

1a. Purpose and setting: Number to signify a characteristic of a collection of apples placed on a scale at a grocery store. The following mathematical event took place immediately following the counting apples event discussed previously.

Apples on a Scale

Nicole weighed the apples using a nearby single-pan, analog scale. Toya looked on from her perch in the shopping cart. Toya eagerly read aloud the numerals on the scale to which the scale’s needle pointed, specifically calling out “Two!” when the scale’s needle bounced close to the large, printed numeral “2” (indicating 2 pounds of weight), and similarly, “One!” when the needle bounced closer to the large, printed numeral “1” (indicating 1 pound of weight). Nicole wanted to purchase only 1 pound of apples so she and Pam worked together to swap out bigger apples for smaller ones until the weight of the set of apples came close to 1 pound.

(June 11, 2006 – Grocery Store Visit with Toya)

When Nicole placed or removed apples in the pan to weigh them, the pan lowered or raised and the needle moved along the curved number line indicating the measure of weight of the contents in the pan. The needle moved from an initial vertical position

pointing to the numeral 0, toward or past the numeral 1, and toward and back away from the numeral 2.

Toya spontaneously verbally identified the numerals “2” and “1” to which the needle pointed as Nicole weighed the apples. Because the numerals to which the needle pointed along the scale’s number line increased as Nicole placed the bag of apples into the pan, and subsequently decreased when she swapped out larger apples for smaller ones to manage the weight of apples, Toya’s observation of this activity exposed her to the notion of employing number, referenced by numerals and number words, to signify a discernable characteristic of the collection of apples beyond its numerosity. That is, while Toya enumerated the initial collection of apples as having numerosity three, this meaning differed from the numeric meaning of “2” indicated by the needle’s position. Likewise, when Nicole and Pam swapped out bigger apples for smaller apples, keeping the numerosity of apples at three, the meaning of “1” indicated by the needle’s position signified a numeric meaning about the set of apples distinct from its numerosity.

It was not clear to what extent Toya recognized or understood that the numerals she observed and verbally identified had something to do with the weight of the particular collection of apples in the pan. Nonetheless, Toya’s active participation in this activity appeared to contribute to her developing emerging numeric meanings beyond concepts of enumeration, as she noticed and engaged in the use of number to signify a discernable attribute of a physical quantity.

1b. Values and beliefs. Nicole’s purpose in using the scale stemmed from her need to monitor and therefore to limit the weight of the apples she would purchase.

This activity contributed to Nicole's and Pam's broader goal of managing their finances over the course of this shopping trip in order to ensure they had enough cash to pay for their groceries. The collective efforts of shopping for and paying for groceries with her sister reflected the significant value Nicole placed not only on maintaining family relationships but also in supporting kin. Given Nicole's and Pam's concerns regarding managing their grocery budget, the activity of weighing produce seemed to be a regular part of their grocery shopping practice. Furthermore, their efforts to limit the weight of produce to be purchased to a single pound or even less, afforded Toya the opportunity to participate in this activity as she could identify small numerals.

Toya seemed to be familiar with the activity of weighing produce as demonstrated in this apples-on-a-scale mathematical event. Although Nicole did not invite Toya to help her weigh the apples or encourage Toya to read aloud the numerals indicated by the scale, while Nicole and Pam worked together to obtain an acceptable weight of apples, Toya closely observed this activity from her seat in the grocery cart and enthusiastically read aloud the numerals indicated by the scale's needle position.

Toya's spontaneous enthusiastic response to her observations further seemed to demonstrate not only Toya's familiarity with the activity of weighing produce but also her interest in this activity, particularly observing the scale's needle move between different numerals. This appeared worthy of her attention and participation. Based on my observation of Toya at day care, Toya seemed to enjoy eating apples.

Her preference for apples may have further motivated her interest in the weighing activity.

1c. Social relations. Toya's experiences grocery shopping with her mother and aunt provided a meaningful context in which Toya could engage spontaneously in activities that fostered her developing meanings of number, including using number to signify a discernable attribute of a physical quantity. According to Rogoff (2003), "Whether or not parents focus on helping children learn, children may take initiative in observing and becoming involved in ongoing activities (Rogoff, 1990; Rogoff et al. 2003)" (p. 283). While Nicole and Pam did not directly involve Toya in their efforts to manage the weight of the apples they intended to purchase, Toya demonstrated her initiative and involved herself in their efforts by reading aloud the numerals indicated by the scale. Since neither Nicole nor Pam affirmed Toya's participation in this activity, it appeared that an intrinsic desire motivated Toya's initiative and involvement.

2a. Purpose and setting: Number to signify shirt and shoe "size" during spontaneous verbal interactions at home and other everyday contexts. The following event occurred on a weekday evening at approximately 5 p.m. when I visited Toya and her mother at home, soon after Nicole had picked up Toya from Ms. Franklin's day care. Toya noticed a stain on the t-shirt I was wearing. I explained to her that I had accidentally spilled some tea on my shirt earlier that day. Toya asked me, "Why you didn't clean it up [or] get another shirt?" I informed Toya that I did not have another shirt into which I could change. After Toya suggested that I "buy one," she

asked me, “What size do you wear?” This conversation eventually led to a lengthy debate between Toya and her mother Nicole over what size shoe Toya wore.

Shirt and Shoe Size

Toya: [Referring to my shirt size, Toya asked me,] What size do you wear?
Grace: What size?
Toya: Uh-huh.
Grace: Probably a small.
Toya: Huh?
Grace: Small.
Toya: What *number*?
Grace: Interesting. Well, you tell me what number do you wear. What size do y- [Toya immediately replied with,]
Toya: 4. A 4.
Grace: 4?
Toya: And my sister wear a 4 too. Her name is Alexis. She wear a 4. [Note: Alexis is Toya’s baby half-sister.]
Grace: Alexis wears a 4 too? Are you guys the same size?
Toya: Uh-hmm [affirmative]. We're the same size.
Grace: I thought Alexis was a baby.
Toya: She is a baby. She still a baby.
Grace: But then how do you all wear the same size if she's a baby and you're a little girl now?
Toya: I'm a big girl now.
Grace: Okay, so you're a big girl and Ale- [Toya immediately responded with,]
Toya: And she a little girl.
Grace: So how, how do you all wear the same size?
Toya: When we try [‘buy’?] our shoes. When, when we met, me and my grandma and her got her, got her first, g- some shoes for her birthday....
Grace: And what size shoes did you buy for Alexis?
Toya: A 4....
Grace: What size shoe do you wear?
Toya: Huh? A 4 too.
Nicole: [Nicole joined in the conversation,] No you don't.
Toya: Mm-hmm! [As if to say, “Yes, I do!”]
Nicole: You don't wear a 4 in shoes. Your *clothes* are a 4. Your *shoes* are not a 4. [Toya tried to interject, but Nicole insisted,] No listen to me. Your *shoes* are a 10. Your *clothes* are a 4.
Toya: Well, *I'm* 4.
Nicole: That doesn't mean that... [Toya continued, saying,]
Toya: My *shoes* 4. [Nicole chuckled.] Uh, my *shirt* is 4.

Nicole: No, your *shirt* is a 4. Your *clothes* are a 4. But your *shoes* are a 10
 [Toya cross-talked, indecipherable]....

Nicole: Am I your mother?

Toya: Yes.

Nicole: Okay. So, shouldn't I know what size you wear?

Grace: Yeah.

Toya: No.

Nicole: Why?

Toya: Cause.

Nicole: Cause what?

Toya: Cause you lie.

Nicole: [Nicole didn't quite understand what Toya said,] Cause I'm 9? I'm 9?!

Toya: Cause you're ly-ing.

Nicole: Because I'm lying? Toya, I'm not lying. No I'm not!

Toya: Yes, you are cause you're [indecipherable word(s)].

Nicole: Do you want me to show you your shoe? And you know how to, you know what the number 10 looks like?

Toya: 4....

Nicole: Do you know what the number 4 looks like?

Toya: Um-hmm [affirmative].

Nicole: Okay, get down and go get your shoe.
 [Toya got down from sitting on the kitchen counter and we all proceeded toward Toya's bedroom. Toya found a sandal she had been wearing recently and gave the shoe to her mother.]

Nicole: [Nicole referred to the label printed on the shoe, a numeral 9 with a line underneath it to distinguish it from the numeral 6.] Mm, what is that? That's a 9.

Toya: Let me see.

Nicole: You, okay, go -

Toya: I don't wear no 9! [Nicole chuckled.]

Nicole: Can you fit these?

Toya: No.

Nicole: [Nicole raised her voice, now in a stern tone,] Toya, can you fit these shoes?

Toya: My, my feet is so big. I cannot wear those.

Nicole: Okay, if your feet are too big for a 9, then how are you gonna wear a 4? A 4 is *smaller* than a 9.

Toya: No it's not.

Nicole: So where's your tennis shoes at?

Toya: Tennis shoes right there.

Nicole: The shoes you took off. Get 'em. [Toya started to look for her tennis shoes or sneakers]. The shoes you took of- the shoes you took off.
 [We waited until Toya located the shoes Nicole had asked her to find. After a few moments, Toya handed Nicole one of her tennis shoes.]

Nicole: Okay, [Nicole exposed the tongue of the shoe to show Toya the numeral 10 printed on it,] what size is that?
 Toya: A 4.
 Nicole: A 10! Y-, you know what a 4 is, right?
 Toya: Mm-hm [affirmative.] [Indecipherable phrase.]
 Nicole: [Nicole stated firmly,] No. You know what a 4 looks like?
 Toya: No.
 Nicole: You don't know what a 4 looks like? So you don't know how to call your grandmother then?
 Toya: Four. [Toya recited the numerals of her grandmother's telephone number, using "oh" for the numeral 0,] 3 - 0 - 1 - 4...
 Nicole: Okay, is that a 4?
 Toya: Where?
 Nicole: Right there. [Nicole pointed to each of numerals in the number symbol '10' printed on the shoe as she said,] The 1 and the 0.
 Toya: No.
 Nicole: You know what that 1 and 0 is?
 Toya: Um-mm [negative, as in 'no'].
 Nicole: Ten.
 Toya: Ten. Alright, you cheated.
 Nicole: So am I right?
 Toya: No.
 Nicole: Girl. Mm.
 Toya: Cheater, cheater, cheated.
 Nicole: You *are* your father's child.
 (June 20, 2006 – Home Visit with Toya)

In Toya's initial questioning of me regarding what "size" shirt I wore, it became evident that Toya expected my response to consist of a number word, as my descriptive signifier for size, "a small," did not appear to make sense to her. When Toya clarified her inquiry regarding my shirt size by asking, "What *number*," this seemed to demonstrate Toya's understanding that number signified the "size" of clothing and, as evidenced later in the episode, shoes as well. Rather than answering Toya's question with a number, I instead chose the methodological path of turning the question back at Toya. Given that Toya initiated the conversation with me about my shirt and that she eventually inquired about my shirt "size," my decision seemed to be

in keeping with the natural flow of the conversation that unfolded and thus seemed to maintain an essence of everydayness.

Toya stated that she and her baby sister Alexis were “the same size,” in particular “a 4.” Given Toya’s further explanation, it appeared that she meant that the numbers indicating their shoe, and not their shirt/clothing, “sizes” were the same. At the time of data collection, Alexis was approximately 14-18 months of age. Since infant and young toddler clothing sizes in the U.S. typically are identified using age in months, this also suggests that Toya’s observation that she and Alexis were “the same size” or number more than likely referred to their shoes and not their clothing.

In the U.S. system for indicating toddler and young children’s shoe sizes, although the numeric signifiers for size do not reflect a ratio or multiplicative relationship, the system does reflect a positive numeric relationship between the values of the signifiers for size and the corresponding measures of foot length to which each signifier is assigned (e.g., a US size 4 toddler/young children’s shoe fits a toddler with a 11.4 cm foot length while a US size 10 toddler/young children’s shoe fits a young child with a 16.5 cm foot length). Thus, within the few months prior to this observation visit, Toya may have very well gone on a shopping trip with her paternal grandmother and purchased a pair of size 4 shoes for Alexis. At that time Toya may have noticed that the number (“a 4”) that signified Alexis’ shoe “size” and the number (“a 4 too”) that signified her own shoe “size,” according to Toya’s understandings, were the same.

Toya’s comments, however, did not indicate nor suggest that she perceived that the global magnitude of her shoes was the same as the global magnitude of her

baby sister's shoes. For example, Toya's clarification that she was a "big girl" and that Alexis was "still a baby" and a "little girl," suggested that Toya understood the difference in global magnitude between herself and Alexis. This seemed to imply further that Toya recognized that her feet were bigger than that of her sister's and, consequently, that her shoes were bigger than Alexis' shoes. Thus, Toya appeared to understand the concept of global magnitude, with regard to shoes, as distinct from the meaning that she ascribed to the "size" of shoes.

The "number" signifying the "size" of Toya's and Alexis' shoes did not appear to indicate for Toya a numeric meaning that expressed the relative global magnitude of their respective shoes. Instead, in this specific context, Toya appeared to understand the "size" or the "number" of a shoe, and in particular "a 4," as having a nominal meaning, serving as an arbitrary label assigned to her and Alexis' shoes.

When Toya informed me that her shoe size was "a 4 too," Nicole immediately joined the conversation and attempted to correct Toya, "Your *shoes* are a 10. Your *clothes* are a 4." Toya argued in response, "Well *I'm* 4. ... My *shoes* 4. Uh, my *shirt* is four." Toya seemed to observe an association between the number that signified her shirt/clothing "size" and the number that indicated her age. In particular, Toya appeared to notice a similarity between her age and clothing "size." This observed association or similarity seemed salient to Toya as she appeared to extend it further to her shoe "size."

Toya seemed to construct a pattern in which she posited that not only was the "number" or "size" of her shirts/clothing the same as the number that indicated her age but that the "number" or "size" of her shoes must be the same as her age as well.

According to Ginsburg and Ertle (2008), “pattern is so important in mathematics ... it involves the search for abstractions that underlie the observed” (p. 52). Toya thus appeared to engage in abstract thinking or mathematical reasoning when connecting number to order with respect to her age and number as label with respect to signifying both her shirt/clothing “size” and her shoe “size.”

Toya’s earlier claim that she and her baby sister Alexis both wore a size 4 in shoes, however, did not reflect Toya’s reasoning regarding her own age, shirt/clothing “size,” and shoe “size.” At the time of this observation visit, Toya did not seem to recognize the distinction between Alexis’ reported shoe size (size 4) and Alexis’ age (1-years-old). As stated above, Toya’s observation regarding the sameness between her and Alexis’ shoe “size” instead seemed to reflect primarily nominal meanings of “size” and the number 4.

The prominence of the number 4 in Toya’s reasoning about her shoe “size” may have stemmed in part from Toya’s interest in her age as well as her developing understandings of and familiarity with numbers less than 5. More specifically, the observed everyday mathematical events in which Toya engaged in the content of number primarily consisted of contexts bounded by numbers less than 9. Within these contexts, as Nicole reported and as evidenced in many of the mathematical events previously discussed, Toya appeared to demonstrate a greater capacity to identify, interpret, and/or employ the numbers 1 through 4. Thus, when Nicole attempted to challenge Toya’s reasoning by showing Toya two of her current pairs of shoes and then pointing out and reading aloud the printed numerals that signified the sizes of each, the numbers 9 and 10 did not appear meaningful to Toya. Nonetheless, Nicole’s

efforts did draw Toya's attention to these larger numbers with which she seemed to have limited prior experiences and contexts in which to make sense of these.

Despite Nicole's attempts, Toya maintained a steadfast notion that her shoe "size" was "a 4." The combination of Toya's primarily nominal meanings of "size" regarding clothing and shoes and the numbers that signify "size," her reasoning and firm perception that her shirt/clothing "size" and shoe "size" must both reflect her age, her interest in and focus on her age, her prior experiences and understandings regarding number, along with Toya's persistence to maintain her stance in this debate appeared to lead her to reject any cognitive dissonance that might have arose from Nicole's efforts to prove to Toya that she wore a size 10 tennis shoe (as well as a size 9 sandal) and the fact that the sizes or numbers on these shoes did not match Toya's age of 4.

When Nicole showed Toya a pair of size 9 sandals and asked Toya if she could fit these, Toya confidently replied, "My feet is so big. I cannot wear those." Toya seemed to argue that she could no longer wear those particular sandals as her feet had outgrown these. Toya's observation of fit regarding the sandals seemed reasonable given Nicole's claim that Toya wore a size 10. Toya's response notably seemed to indicate her awareness of magnitude, regarding in particular the continuous attribute of length, which differentiated her feet from the sandals. According to Clements and Sarama (2007), "Preschool children know that continuous attributes such as mass, length, and weight exist, although they cannot quantify or measure them accurately" (p. 518). Toya specifically appeared to evaluate the length of her feet in comparison to that of the sandals. Toya's understandings reflected Seo and

Ginsburg's (2004) findings that preschool-aged children engage in the content of magnitude during their free play through activities such as "describing the global magnitude of the [sic.] objects, making direct or side-by-side comparison of objects, or making magnitude judgments with or without quantification" (p. 94).

Given, in particular, that Toya did not seem to have a meaningful understanding of the number 9 and that she appeared to perceive the "size" or "number" of clothing and shoes as having a nominal meaning, Toya did not seem to recognize the numeric contradiction between her comment that her feet were "so big" for the size 9 sandals and her understanding that her shoes were a size 4. However, Nicole did try to point this out to Toya by stating, "Okay, if your feet are too big for a 9, then how are you gonna wear a 4? A 4 is *smaller* than a 9." Nicole's argument expressed the positive numeric relationship between a shoe's global magnitude or overall size, as defined primarily by the continuous attribute of length, and the value of the number that signifies the size of a given shoe (i.e., the smaller the global magnitude of the shoe, the smaller the value of the number that signifies the size of the shoe, and further, the smaller one's feet must be to fit the given shoe).

Nicole's response also provided Toya some exposure and meaning to the number 9 as she related its value to the number 4 within the familiar context of shoe size and fit. By stating, "A four is *smaller* than a nine," Nicole attempted to convey to Toya the ideas that a size 4 shoe ("a 4") is smaller in global magnitude than a size 9 shoe ("a 9") and that this is consistent with the numeric relationship that the number 4 is less than the number 9.

Through Nicole's efforts to clarify Toya's understanding of her shoe size, this event provided a meaningful context in which (a) Toya attempted to discount her mother's arguments by employing concepts of magnitude and attribute comparison with respect to the length of her feet and her sandals, and (b) Nicole exposed Toya to the intended meaning of the "size" of a shoe as referring to the magnitude of the shoe and the "number" signifying "size" as having a numeric meaning indicating the relative magnitude of a shoe. While Toya may not have developed yet the capacity to make the connection between her observations of magnitude and the mathematical content of Nicole's arguments, this event nonetheless provided a meaningful experience and context on which Toya's understandings of shoe and clothing "size" and the "numbers" that signify an item's size could continue to mature and develop.

2b. Values and beliefs. Toya's attention to and awareness of number as a way of signifying the "size" of clothing and shoes seemed to stem from her mother's value and efforts in maintaining Toya's well-stocked wardrobe and shoe collection. Nicole enjoyed shopping for clothes and shoes for Toya. A number of days after the shirt-and-shoe-size event, I joined Nicole, Toya, Pam, and Ricky on a routine shopping trip to Target that included stops in both the girls' shoe and clothing departments. The following mathematical event occurred as Nicole and Toya browsed through the shoe department.

Boots

Nicole noticed a rack of girls' pink cowboy boots,

Nicole: Toya, aren't these soo cuuute?

Toya: YESSS!

Nicole: You should try them on.

Toya: Mom,

Nicole: Yes?

Toya: [Toya referred to pair of boots on the same rack,] What about these shoes?
Nicole: They're all the same. You have to get the right size.
Toya: [Indecipherable phrase.] Where's the 4?
Nicole: Huh?
Toya: Where's the 4?
Nicole: What ch'ya mean, where's the 4? [Nicole stated an indecipherable phrase that seemed aimed at quickly dismissing Toya's question.]
Nicole selected a size 11 pair of boots and directed Toya to try on one of the boots. After determining that the pair was "way too big" for Toya, Nicole uttered aloud "ten" as she searched for and then selected a size 10 pair for Toya to fit.
(June 30, 2006 – Shopping Trip Visit with Toya)

Toya's enthusiasm over the cuteness of the boots mirrored that of Nicole's. Toya also demonstrated interest in the activity of shopping for shoes as she actively participated by browsing the shoe rack and inquiring about the boots she noticed. Further, when Nicole casually commented about finding the right size of the boots, Toya spontaneously responded by asking, "Where's the 4" thus indicating that she maintained an understanding that her shoe "size" was "a 4" and that they should look for a pair of size 4 boots for Toya to try on for fit. While Nicole did not pursue the meaning of or address specifically Toya's question and instead she directly selected a size 11 and then a size 10 pair of boots for Toya to try on, Nicole did attempt to clarify Toya's shoe size previously at home. However, Nicole's efforts in the shirt-and-shoe-size event did not appear to influence Toya's subsequent thinking as evidenced in the boots event.

Although the tags on the boots indicated size 11 and 10, respectively, thus not a size 4, and given that Nicole uttered the word "ten" to signify the intended shoe size for the second pair of boots selected that were subsequently chosen for purchase, Toya again did not appear to experience any cognitive dissonance or explanatory

challenge to her understanding that she wore a size 4 in shoes. Later in the shopping trip, Toya willingly tried on a number of different articles of size 4 clothing from the outfits that Nicole selected. This activity seemed to reinforce both Toya's connection between clothing "size" and age and, as such, her persistent reasoning that shoe "size" must also match age.

Toya's earlier interest in my t-shirt, including her inquiry regarding what "size" of "number" I wore and her suggestions of how I could have handled the situation of having a stained shirt, seemed to indicate not only her appreciation for clothes and attention to clothing "size" but it also demonstrated Toya's capacity for relating her experiences with clothing to the experiences of others.

2c. Social relations. Toya initiated the shirt-and-shoe-size event through her interest and inquiry about my shirt. Our conversation evolved into a debate between Nicole and Toya over Toya's shoe size. Later that evening, Toya continued both to insist that her shoes were a size 4 and to dismiss Nicole's efforts to prove otherwise as evidenced in the following event that occurred while we drove back to Nicole's apartment after picking up Alan from the Metro station.

"Look at these shoes!"

Toya: [Toya addressed Alan, who sat next to Toya in the van while Nicole and I sat in the front,] Look at these shoes!

Alan: Look at 'em.

Toya: These are 4.

Alan: It says, "10."

Nicole: What does it say, Alan?

Alan: Ten.

Nicole: Tennen!

Toya: You lose! [Toya referred to her mother and me as I had echoed Nicole's sentiments,] Both of ya'll lose.

Nicole: No, *you* lose!

Toya: Both of ya'll [indecipherable phrase].

Nicole: [Nicole explained to Alan that Toya,] ...insists that she wears a 4 in shoes...Because her clothes are a 4 and she's 4. So she thinks...
[Nicole addressed Toya,] Toya, are my shoes a size 28 because I'm 28-years-old?
Toya: No.
Nicole: Oh, okay. Because my shoes are not even a 6. So does that mean I'm 6-years-old?
Toya: Yes.
[Nicole did not hear Toya's response as she now turned her attention back to driving.]
Toya: [Noticing that Nicole did not hear her initial response, Toya loudly repeated,] Yes!
[Nicole ignored Toya's comment.]
(June 20, 2006 – Home Visit with Toya)

Nicole's explanation of Toya's reasoning to Alan seemed to affirm to Toya that her mother understood her rationale. While Nicole acknowledged Toya's thinking, Nicole still sought to challenge Toya's rationale by applying it to her own shoe size and age, a case that Toya briefly appeared to concede. However, when Nicole furthered her argument by noting that her shoes were not even a size 6 and then asked Toya if her mother was 6-years-old, Toya boldly responded with, "Yes." While Toya's capacity to understand age beyond that of her own, including making sense of the age of adults, was unclear, given that Nicole had just clarified her own age with Toya, Toya's affirmative response seemed to reflect the social intent of not backing down from her mother in this debate, rather than meaning that Toya truly thought that her mother was 6 years of age.

Nicole's and Toya's interaction in both the shirt-and-shoe-size event and the "Look-at-these-shoes" event reflects patterns of behavior that Kochman (1981) found among African Americans as they participated in argument for persuasive purposes. According to Kochman (1981) the mode in which African American's tended to engage in debate is "animated, interpersonal, and confrontational... [it] is

characteristic of involvement; it is heated; loud, and generates affect” (p. 18).

Kochman asserted that these qualities of interaction reflect African American cultural values including individual self-assertion and self-expression, spontaneity and emotional expressiveness, individual distinctiveness, and forthrightness.

Kochman (1981) further claimed that African Americans “consider debate to be as much a contest between individuals as a test of opposing ideas... winning the contest requires that one outperform one’s opponents: outthink, outtalk, and outstyle them” (p. 24). According to Kochman, the outcome of this contest is decided once someone is unable to come back with an effective reply. In this sense, Toya’s persistent unwillingness to accept Nicole’s arguments and her unrelenting, verbal rejections to these demonstrated Toya’s desire not to lose this contest. In order to avoid surrendering to Nicole, Toya employed the tactic of outtalking her mother by responding to her points with dismissive one-liners. For example, when Nicole argued that because she is Toya’s mother, she should therefore know Toya’s shoe size, Toya responded, “You lie,” and when Nicole clarified that, “A 4 is *smaller* than a 9,” Toya forthrightly disagreed, “No, it’s not.”

Toya and Nicole further demonstrated a sense of engaging in a contest with the other in this debate in their word choice and/or tone of some of their comebacks. For example, in the shirt-and-shoe-size event, after Nicole established that Toya’s tennis shoes were a size 10, Toya dismissed Nicole and her argument by stating, “You cheated.” This is a comment of unfair methods normally applied in the context of playing a game or participating in a competition. Later that evening, after Toya attempted to reassert her self back into a debate with Nicole by spontaneously asking

Alan to look at her shoes, at which time he confirmed that her shoes were a size 10, Nicole gloated back to Toya, “Tennnn!” Unwilling to give in to Nicole (i.e., her opponent), even after a third-party (Alan) agreed with Nicole, Toya resorted to proclaiming, or rather implying, victory over her mother by announcing to Nicole, “You lose!”

The manner in which Toya and Nicole debated not only revealed Toya’s understandings of her shoe “size” or “number” but it demonstrated Toya’s tenacity to maintain her position when confronted with an opposing idea and supporting arguments. Toya did assert her rationale or attempt to outthink her mother initially by arguing that since she was age 4 that the size of her clothes and her shoes must also be 4. However, once Nicole countered with a series of arguments that proved otherwise, Toya turned to outtalking her mother as recourse for a defensive strategy. Toya’s use of outthink tactics before outtalk methods, seemed to suggest that Toya recognized and valued mathematical reasoning as a viable strategy to employ in this debate.

Characterizing Toya’s Everyday Mathematical Events

A subsequent examination of the collection of Toya’s everyday mathematical events revealed that these events tended to:

- Nurture Toya’s on-going development and understandings of number, in particular numbers less than 5, and its use for nominal, ordinal, and numeric purposes;
- Arise within practices valued by Toya’s mother, Nicole;

- Emerge spontaneously from Toya’s intrinsic motivation or goal-directed interests;
- Involve Toya in verbal interactions with family members and known others.

I discuss each of these characteristics of Toya’s everyday mathematical events below.

Nurturing Understandings and

Various Meanings of Small numbers

Number initially emerged as significant in the analysis of Toya’s everyday mathematical events not only because of the frequency of observed events involving number and the meaningful contexts in which Toya interpreted, identified, or employed number, but also because of the multiple meanings for which Toya seemed to discern number across and within these events.

Toya appeared to develop and engage in nominal, ordinal, and numeric meanings of number. Toya’s developing meanings or purposes of number constituted early principled understandings within the broader content of early number concepts. This does not mean that Toya consciously recognized or distinguished these meanings of number as differing or related “principled understandings” of number. Rather, Toya exhibited these principles throughout the collection of mathematical events discussed above as evidenced in her verbal and social interactions with others and her responses to and management of objects she observed and utilized in her everyday settings and activities.

Similar and different meanings of number across contexts. Toya demonstrated similar and different meanings of number depending on the context of a given event.

Toya employed number for nominal purposes across contexts involving television viewing, telephone use, and clothing and shoe size talk. Toya understood that “numbers” identified distinct television programs or channels, located particular telephone users, and indicated clothing and shoe sizes. For example, in the “We-got-cable event,” Toya insisted to her mother, “I’ll show you which *number* I want,” and when placing a telephone call to her grandmother, Toya asked her mother, “What’s the *number*?” Also, when it appeared to Toya that I did not understand her question regarding what size shirt I wore, she clarified, “What *number*?”

Although nominal meanings of number seemed prominent in Toya’s everyday experiences, she demonstrated understandings of different meanings of number across contexts as well. For instance, Toya’s nominal understanding of “a four and a three” in the context of selecting the television channel 43 in the “We-got-cable” event differed from the ordinal meaning she appeared to ascribe to these numbers in the context of “reading” her past birthday cards, “I’m this birthday. I’m 3-years-old. ... I’m 4-years-old.” Toya’s nominal and ordinal meanings, respectively, for these numbers in the prior contexts further differed from the numeric purpose that she indicated with stating “four” and then counting up to “three” in her two attempts to enumerate a collection of apples at the grocery store upon her mother’s prompting.

In addition to employing number for a numeric purpose when responding to an external stimulus to enumerate a quantity of objects or images (e.g., seven pom-poms, six rungs of a ladder), Toya also applied numeric meanings for number across settings (e.g., home, day care, grocery store) as her own intrinsic or goal-directed interests motivated Toya either to determine the numerosity of a quantity (e.g., two

apple slices, three movies), to inquire about the numerosity of a quantity (e.g., two handi-wipes refill packs), to construct a quantity of a given numerosity (e.g., two cookies), or to identify numerals that signified a discernable attribute of a physical quantity (e.g., the numerals 1 and 2 indicated by a scale weighing a collection of apples).

Various meanings of number within an event. In some cases, Toya managed and negotiated between various meanings of number within a particular practice, episode, or event.

While Toya primarily employed a nominal meaning for number in her practice of phoning her grandmothers, she discerned this meaning for number as distinct from the numeric meaning for number indicated in Nicole's hints to help Toya in locating particular numerals on the keypad (e.g., "Remember, 8 is two circles.") as well as Nicole's other comments to Toya that spontaneously arose through their interaction in this shared practice (e.g., "No, you didn't hit 3 two times.").

Toya appeared to differentiate numeric meanings of number in the related pair of apple events that occurred at the grocery store. The numeric meaning of number indicated in Toya's enumeration of the apples collected in the bag differed from the meaning of number represented by the numerals on the scale that Toya verbally identified. Although both of these uses of number indicated numeric meanings, the former signified the numerosity of the apples while the latter described a discernible attribute (weight not numerosity) associated with a quantity of apples.

In the shirt-and-shoe-size event, while Toya primarily conceived of clothing and shoe size in a nominal sense, she attempted to relate her clothing and shoe size to her age, thus demonstrating Toya's emerging capacity to reason mathematically by constructing an association that related her emergent ordinal meaning of the number 4 as age to the nominal meaning of 4 that, according to Toya's understandings, signified the "size" of her shoes as well as her clothing. Furthermore, Nicole's efforts to clarify Toya's shoe size also exposed Toya to yet another numeric meaning for number that associated shoe size with the global magnitude of shoes.

Numbers as distinct from words. Not only did Toya demonstrate multiple meanings for number in her everyday experiences, but she also recognized that numbers were distinct from letters, or as she referred to them, "words."

Numbers vs. Words

Toya and I were sitting at the kitchen table eating pizza for dinner while Nicole and Alan sat in the living room watching television. Shortly after Toya finished her dinner she walked to the refrigerator and spontaneously began to manipulate the magnetic colorful, wooden letters and numerals that appeared randomly arranged on its door. I watched from the kitchen table as Toya moved all the letters toward the right side of the refrigerator door and left the numbers toward the left hand side. I later asked her about what she had done. When I inquired specifically about the magnets she did not move from the left hand side of the refrigerator she simply stated, "Numbers." I then pointed to the magnets that she moved to the right side of the door and asked her about these, to which Toya succinctly responded, "Words."

(June 20, 2006 – Home Visit with Toya)

Toya understood number, regardless of its purpose, as its own distinct entity signified through a unique set of verbal and symbolic representations. Ginsburg and Ertle (2008) noted that when children can distinguish numbers from words, this reflects young children's capacity to be "splendid little mathematicians ... [who] deal spontaneously and joyfully with mathematical ideas" (p.55). Furthermore, Ginsburg

and Ertle (2008) claimed that young children's early mathematics is "deep" and that it extends beyond what is typically considered "basic" and the low-level meanings that are often associated with early mathematical learning (memorization, mechanical skills, and operating on a concrete level). I contend that Toya's growing facility to develop, manage, and negotiate meaningfully between various purposes of number – numerical, ordinal, *and* nominal – across and within contexts in her everyday experiences demonstrated her capacity to be a "splendid little mathematician."

Arising within Practices Valued

By Toya's mother, Nicole

Toya's everyday mathematical events involving number often arose within practices that Nicole valued and thus regularly maintained in her family life with Toya.

Nurturing family relationships and supporting kin. Nicole placed a substantial value on family, nurturing relationships with and supporting kin. Nicole engaged Toya in on-going practices that afforded her regular contact and quality time to spend and to bond with extended family members. These practices included routine phone calls with Toya's grandmothers, regular visits to her mama's (Nicole's mother) and to her Auntie Pam's homes, weekend stays with her cousin Ricky (at either Nicole's, Pam's, or mama's homes), and shopping trips and other outings with her mother, Pam and Ricky. Within these everyday practices, Toya developed and employed meanings of number for nominal, ordinal, and numeric purposes.

Toya employed nominal meanings of number when phoning her grandmothers. Nicole's hints to assist Toya in locating particular numerals on the

keypad also engaged Toya in numeric meanings of number, in particular enumeration concepts. Nicole's practice of having Toya regularly bond with her mama nurtured a reciprocal relationship between Toya and her mama. As an expression of love for her grandchild, Toya's mama often baked cookies for her and Toya enjoyed having these as a snack before bedtime. Nicole fostered Toya's understandings of enumerating small quantities by managing Toya's consumption of these treats through telling Toya how many cookies she could have and allowing Toya to construct a set of the indicated amount of cookies on her own.

Bonding and playing with her cousin Ricky provided Toya significant and meaningful opportunities to engage in nominal meanings of number and to explore ordinal meanings of number. Toya and Ricky regularly employed nominal meanings of number when negotiating channel selection while watching television together. Toya explored ordinal meanings of number when managing turn taking with Ricky, for example, as demonstrated when claiming a swing at the playground by saying, "I was here first!" Ricky also supported and encouraged Toya's interest in controlling the elevator at Nicole's work place. Riding the elevator provided Toya a meaningful context in which to develop emerging understandings of number to order. When Nicole pressed the elevator button for the destination floor instead of allowing Toya to find and to press the button as was typical, Ricky reacted immediately in disappointment, "Why did you do that?!" Ricky knew that Toya enjoyed pressing the elevator floor button and so he was dismayed when she lost that particular opportunity to do so.

Playing with Ricky also provided Toya meaningful experiences to develop

numeric understandings of number. For example, when pretending to be rocket ships, Toya and Ricky expressed number words in a reverse counting sequence to mark the time remaining and to coordinate their “blast-offs.” Also, Toya mimicked Ricky and her other cousin’s play and enumeration activity as they did 20 jumping jacks, counting each repetition.

Supporting kin seemed very important to Nicole. Nicole, Pam, and their mother shared responsibility for the child care and rearing of Toya and Ricky. Nicole and Pam also helped one another while shopping, including sharing the expenses of their grocery purchases. Accompanying her mother and aunt on grocery-shopping trips afforded Toya experiences to employ her current understandings and to develop further meanings of number. For example, after enumerating a set of apples that she and her mother selected, as Toya observed her aunt and mother work together to manage the weight of the set of apples to be purchased, Toya explored number as a way of signifying a discernable attribute of the set of apples as she spontaneously read aloud the numerals indicated by the scale’s needle.

Watching television. Watching television programs or DVD movies at home seemed to be a significant leisure activity for Nicole and her family. The presence of a television set in the living room as well as in each bedroom in each of Nicole’s and Pam’s apartments, the observed frequency and duration of multiple televisions being in use, the steady supply of newly rented DVD movies available in Nicole’s home, Toya’s persistence to find a working channel in the “We-got-cable” event, along with Toya’s insistent plea for Nicole to buy her three DVD movies each demonstrated the value that Nicole and her family, including Toya, accorded television as a substantial

form of entertainment and leisure. Family interaction revolving around television or DVD viewing provided Toya with ongoing opportunities to engage with nominal meanings of number as she participated in or managed channel selection or device operation. The routine and valued practice of watching television programs or DVD movies at home provided a meaningful context in which Toya developed her capacity for identifying, interpreting, and employing certain number words and numerals.

Toya's wardrobe, birthday cards, and preparation for school. Nicole took pride in Toya's appearance, and she took pleasure in shopping for Toya's clothing and shoes. Nicole's value in maintaining Toya's wardrobe and her inclusion of Toya in her shopping practices seemed to foster Toya's attention to and interest in clothing and shoes, in particular the "size" or "number" assigned to these articles. The pom-poms event also seemed to demonstrate Toya's interest in clothing, as she was drawn to pictures of knitted clothing and accessories.

Nicole deeply valued the thoughtfulness and personal written sentiments others expressed to her in greeting cards and she made a practice of saving these cards as keepsakes. Nicole extended this practice to Toya's birthday cards, saving and storing these in Toya's room. Having easy access to her cards, Toya spontaneously "read" these at her leisure, thus supporting her emergent understandings of ordinal meanings of number.

Finally, Nicole expressed concern for Toya to "know her work," or to be prepared to start formal preschool that following fall. In addition to purchasing workbooks for Toya and assisting her in completing these, helping her finish worksheets sent home from day care, and reading books to her at bedtime as

evidenced in the rungs-of-a-ladder event, Nicole also embedded efforts to help Toya “know her work” in daily routines, such as when she asked Toya how many apples they selected at the grocery store. Nicole’s practices aimed at helping Toya “know her work” tended to support Toya’s understandings of numeric purposes of number, in particular enumeration concepts.

Emerging from Toya’s Intrinsic Motivation

Or Goal-Directed Interests

Most of Toya’s everyday mathematical events involving number emerged spontaneously from Toya’s intrinsic motivation or goal-directed interests. Toya either initiated these events on her own or through her interactions with a familiar other as together they progressed toward a common goal.

Initiated by Toya. Toya initiated mathematical events in which she spontaneously employed nominal, ordinal, and numeric meanings for number for intrinsic and/or goal-directed purposes. Toya’s intrinsic desire to watch television prompted the “We-got-cable” event while her goal to locate a preferred channel with good reception led her to employ nominal meanings of number. Toya’s spontaneous observation and inquiry of interest prompted the “shirt-and-shoe-size” event while her intent persistence and reasoning regarding her shoe size fueled the ensuing mother-daughter debate over this issue and sparked the “Look at these shoes” event. Both intrinsic and goal-directed interests seemed to motivate Toya in these events regarding clothing and shoe size.

Toya engaged with beginning ordinal meanings of number for intrinsic purposes as she entertained herself by “reading” her saved birthday cards. An

intrinsic urge also seemed to trigger Toya's spontaneous utterance of her version of a phone number while she listened to Nicole exchange phone contact information with me. Similarly, an intrinsic or natural response to her activities appeared to prompt Toya to employ spontaneously numeric meanings for number as she (a) expressed number words in a reverse counting sequence before descending on a slide while engaging in free play, (b) enumerated the apple slices she received for dessert during lunch at day care, and (c) explored number as a way to signify a discernable attribute of a quantity of apples as she verbally identified the numerals indicated on a scale at the grocery store.

On the other hand, goal-directed interests seemed to drive Toya to enumerate a quantity within a set of DVD movies, as she pleaded with Nicole to purchase these, and to request for a particular number of additional apple slices to eat at day care. In other numeric mathematical events that Toya initiated, while she had a goal in mind that involved an unspecified quantity, such as securing *some* of her mama's cookies before bedtime, Nicole provided the numerical referent and prompted Toya to construct a set of cookies having that numerosity.

In each of these events initiated by Toya, she meaningfully identified, interpreted, or employed small numbers up to 4 (except in the birthday card event in which she also employed and interpreted the number 8) as she pursued either intrinsically motivated or goal-directed interests.

Initiated by Toya +1. Toya's interaction with a significant other when moving toward a shared goal or common interest also prompted mathematical events in which

Toya developed and employed meanings of number for nominal and numeric purposes.

Most significantly, Toya's practice of watching television with Ricky and her practice of calling her grandmothers with Nicole's assistance afforded Toya ongoing meaningful opportunities to engage in number. As evidenced in the channel-four-four events, Toya and Ricky's unique individual program preferences often led to their negotiation of channel selection, an activity in which Toya employed number for a nominal purpose. While Nicole often prompted Toya to call her grandmothers, Toya expressed great interest in this practice as well, particularly the activity of using the keypad to input her grandmothers' phone numbers. As such, phoning her grandmothers appeared to be a shared endeavor between Toya and her mother. Drawing on her understandings of nominal meanings for number, Toya eagerly asked her mother, "What's the number" so she could begin to place a call to either of her grandmothers.

In addition to Toya's practices of watching television with Ricky and phoning her grandmothers with Nicole's assistance, Toya's play with Ricky and their intrinsic desire to engage in shared-play activities yielded mathematical events that engaged Toya with numeric meanings for number, as evidenced in the "Blast-off" event and Toya's attempt to complete and count 20 jumping jacks.

Toya's participation in these shared experiences seemed to foster her understandings and meanings of number through a pattern of guided participation that Rogoff (2003) described as "intent participation in community activities." Toya's interactions with significant others permitted "learning from observation and

increasing participation in mature activities of the community” (Rogoff, 2003, p. 302).

Since Ricky was older than Toya and had already started formal schooling, he had deeper understandings of number that he modeled in authentic ways when they watched television together and during their shared play. Noticing Ricky’s use of the number word “forty-four” to name channel 44, Toya demonstrated increased development regarding the number-word name she ascribed to this channel, initially identifying it as “four four” and then, on her own initiative, using the number word “forty-four” instead. While Toya may not have been able to keep up with her cousins’ count of 20 jumping jacks, she nonetheless experienced a meaningful use of counting through their shared-play while being exposed to counting numbers beyond 7.

In the practice of phoning her grandmothers, as implied by Nicole’s reports, Nicole initially inputted the numbers to place calls for Toya until about the time of data collection when Toya turned 4 years of age. Nicole then began to let Toya assume this task. As needed, Nicole provided hints to Toya to locate particular numerals. Over time, Toya became increasingly independent in identifying the numerals of her grandmothers’ phone numbers on the telephone keypad as Nicole dictated these to her.

Initiated by Toya through her active involvement in an activity prompted by others. In a few cases, the activities of others prompted Toya’s mathematical events. However, Toya did demonstrate initiative in employing number meanings through her purposeful involvement in these contexts.

In the placemats events, Toya initially observed Aaron pointing to and seemingly counting the images on his placemat. This prompted Toya to engage with her own placemat by pointing to the numerals 1 through 5 on her placemat, demonstrating one-to-one correspondence while expressing the number words “five” to “one” in a reverse counting sequence. In turn, Aaron observed Toya’s behavior and tried to approximate it as well. Toya and Aaron engaged with their placemats to entertain themselves while waiting for naptime.

In the handi-wipes event, while I asked Toya to help me in this task, she spontaneously inquired about the number of smaller handi-wipe refill packs, thus demonstrating her genuine interest in this activity, and her curiosity regarding the numerosity of the packs. In the pom-poms event, although I prompted Toya to count the pom-poms she observed on the scarf, she still maintained interest in the social interaction regarding the photo image as she spontaneously noted to Aaron that he “missed one” when he approximated Toya’s behavior of enumerating the pom-poms, thus demonstrating further her understanding of cardinality in this event.

In the rungs-of-a-ladder event, Toya actively participated with Nicole and the story as she read the book aloud. For example, when Nicole read, “Do you see a ladder we can climb up,” Toya enthusiastically replied, “Yes! It’s missing pieces!” Accordingly, when the story then prompted Toya to find the missing rungs, she spontaneously counted aloud each rung she located in the book’s illustration, demonstrating one-to-one correspondence. When shopping with her mother in the boots event, Toya asserted her nominal meanings of “size” by spontaneously inquiring, “Where’s the 4?”

While Toya did not initiate these activities that led to a mathematical event, her active involvement demonstrated her initiative to employ number for numeric or nominal meanings, for intrinsic reasons or goal-directed purposes. Toya seemed to count the numerals in reverse order on her placemat for intrinsic reasons while goal-directed purposes seemed to motivate her to confirm the existence of two handi-wipe refill packs, to inform Aaron that he missed counting one pom-pom, to locate the six missing rungs in the story, and to indicate to her mother that they should look for a size 4 pair of boots.

Involving Toya in Verbal Interactions

In her everyday mathematical events, Toya tended to engage in verbal interactions with others, oftentimes within those mathematical events noted previously as emerging from her intrinsic or goal-directed interests. As such, the prior analysis complements and provides the organizational structure of this discussion. Toya's verbal interactions involving meanings of number primarily occurred through her individual initiative, her intent participation with another as they pursued a shared interest or common goal, or her active verbal involvement in activities prompted by others. Furthermore, Toya's verbal interactions typically consisted of either minimal or brief comments/responses or lengthier discourse with others.

Initiated by Toya. Toya prompted some of her mathematical events through her verbal interactions with others in order to pursue an intrinsic or goal-driven interest. Longer verbal exchanges occurred between Toya and Nicole when Nicole (a) decided to help Toya find a working channel in the "We-got-cable" event; (b) permitted and informed Toya to construct a set of two cookies to have as a snack

before bedtime; and (c) attempted to clarify Toya's shoe size in the mother-daughter debate over this issue. However, in the movies event, Nicole did not appear interested in entertaining Toya's pleas for her mother to purchase three DVD movies. Despite Toya's persistence, Nicole responded to her with only a brief verbal comment primarily aimed at squashing Toya's fixation over the DVDs.

While Toya initiated these events through her verbal requests of or comments to her mother, the duration of the verbal exchanges in these events seemed dependent on both the context of the event, especially the purpose of Toya's initial verbal interaction, as well as Nicole's willingness to engage with Toya in the given context. Likewise, when Toya requested two additional apple slices at day care, the resulting exchange occurred because only three apple slices remained and I sought to distribute these fairly to both Toya and Aaron. After explaining my reasoning for distributing the remaining slices, Toya verbally confirmed that it was fair.

Initiated by Toya +1. Toya's intent participation in shared practices with significant others afforded her meaningful verbal interactions through which she continued to develop nominal and numeric meanings for number. For example, the shared common goal of watching television encouraged Toya and Ricky to interact verbally and to employ nominal meanings of number, in particular when managing channel selection. Toya's participation in phoning her grandmothers with Nicole's assistance prompted verbal exchanges that supported Toya's understandings of nominal meanings of number as well as her developing capacity to identify numerals on the keypad per Nicole's hints. Nicole's verbal prompts, such as "Remember, 8 is two circles," also fostered Toya's numeric meanings of number as well as her

emerging understandings of shape. Finally, Toya and Ricky's intrinsic interest to engage in shared-play in the "Blast off" event prompted them to bond through their spontaneous play and shared chatter as they repeatedly counted down from 5 together to mark the time remaining and coordinate their activity.

Initiated by Toya through her active verbal involvement in an activity prompted by others. In those activities initiated by others, Toya demonstrated her active participation in these contexts through her spontaneous verbal interactions indicating meanings of number. Although I invited Toya to help me in the handi-wipes event, Toya's interest in the handi-wipes refill package and its contents, including the numerosity of the smaller refill packs, fostered our verbal exchange. While Nicole managed Toya's bedtime routine, including reading a story to Toya, Toya actively engaged in the rungs-of-the-ladder event as she anticipated and noticed the missing rungs of the ladder and subsequently verbally enumerated these. The nature of Nicole's practice of shared reading with Toya encouraged meaningful discourse around the story. In both of these events, Toya's shared interest and desire to actively participate in the activity or practice initiated by a known adult prompted Toya to engage in meaningful verbal interactions with others in which she employed number for numeric purposes.

In the boots event, while an initial verbal interaction between Nicole and Toya commenced once Nicole noticed the pink boots, this exchange was cut short when Nicole dismissed Toya's inquiry of, "Where's the 4?" As asserted previously, the context of an event and Nicole's willingness to engage with Toya seemed to influence the duration of the ensuing verbal exchange between them. Nicole's decision not to

verbally pursue Toya's comment in the boots event seemed informed, in part, by Nicole's unwillingness to indulge Toya's persistence regarding her shoe size. The fact that the boots event occurred in a public setting may have also factored into Nicole's dismissive response as the prior debates concerning Toya's shoe size took place in the privacy of their home and Nicole's van.

Prompted by others to enumerate. In the prior mathematical events discussed, Toya demonstrated initiative and interest in employing understandings of number through her active participation and verbal interactions with others. In contrast, Toya demonstrated more passive verbal involvement in the counting apples event at the grocery store, in which Nicole prompted Toya to count the set of apples they collected.

Toya's passive verbal responses in the counting apples event also seemed distinct from her verbal engagement in the pom-poms event. Although I too posed a known-answer question to Toya, thus prompting her to enumerate the pom-poms, she appeared to participate more actively in this event as demonstrated in her initial interest to look at the digital photo image of the scarf and later in her response to Aaron that he missed counting one pom-pom. The difference between Toya's participation (passive versus active) in these two events may in part be due to the nature of her initial engagement or attention to the objects to be counted. In the apples event, Nicole prompted Toya first to help her select apples and then to count these, while in the pom-poms event I asked "how many" of Toya only after she took the initiative to ask to look at the picture of the scarf and was already engaged in observing it.

Final Remarks

The first characteristic of Toya's everyday mathematical events indicates that Toya's significant mathematical events collectively nurtured her understandings of various meanings of number. The second characteristic of Toya's everyday mathematical events, that they tended to arise from practices valued by Nicole, reflected all but those events that took place at day care. Regarding the third characteristic, the few events that were not initiated by Toya in some way were those events in which someone or something prompted Toya to enumerate a quantity of objects. Finally, the fourth characteristic of Toya's everyday mathematical events, addressing Toya's verbal interaction with others, encompassed the majority of Toya's mathematical events. In those events not characterized as such, Toya instead seemed to be engaged in talking aloud to herself. This set of characteristics seems to capture the salient qualities portrayed across Toya's everyday mathematical events.

Chapter 5: Anika

Impressions of Anika and Capturing Glimpses of Her Everyday Life

At Day Care

Notes from Ms. Franklin. Anika started attending Ms. Franklin's day care in mid-February. It appeared to Ms. Franklin that Anika had not spent much time around other young children, as she noted that Anika often chose to sit back and watch the other children as they played. These behaviors, along with other unreported mannerisms, led Ms. Franklin to presume that Anika was probably more accustomed to being around adults, and as such, she described Anika as being like a "little old lady."

When living at her mother's home, Anika was scheduled to be at day care for 4 days each week, from about 12 noon to 8:30 at night. She did not attend Ms. Franklin's day care during the month of May.

Meeting Anika. I observed Anika at Ms. Franklin's day care from early-March through June, beginning just 2 days prior to her fourth birthday. Anika had dark-brown skin, was slim, and stood noticeably taller than her same-aged peers. Anika dressed in neat, well-matched outfits including coordinating hair accessories and name-brand sneakers. When I first met Anika, she appeared reserved and observant, and my presence seemed to pique her interest. Anika initiated a conversation with me about her birthday. In her soft voice, she informed me that she had a party "with Dora, Boots ... and Cousin Daisy," referring to her Dora the Explorer-themed birthday party, and that she blew out her birthday candles. When I asked Anika, "how many" or to "show me" with her fingers how many candles she blew out, she

displayed her right hand, palm facing me, with all five fingers extended but gave no verbal response. When I prompted her again with “how many,” Anika stated, “Four.”

In the moments that followed, Anika continued to interact with me as her attention shifted between other objects or activities and me. First, Anika noticed that I had been writing in my journal. She eventually requested to “see” (i.e., hold) my four-color ink pen and then proceeded to press down on the different colored buttons corresponding to the various choices of ink color. Next, she briefly played on the combination climbing-and-slide structure, occasionally pausing in her activity and calling for my attention, saying, “Look at me, I can do this!” Anika then returned by my side and extended one of her hands to me, palm down, and asked, “You wanna see my nails?” I observed that three of her fingernails were painted and verbally responded that they were pretty. As evidenced in our first meeting, Anika appeared comfortable with me and actively engaged with her environment, using her activity or observations as a means for initiating verbal interactions with me.

A Reluctant, Picky Eater with a Love for Sweets

Whether she was having a meal or a snack, at home or at day care, Anika usually took her time to eat and often did not finish her food. While Anika was a slow, often reluctant eater, she did have an enthusiastic preference for sweets. According to Anika’s mother, Anika loved candy, cake, and ice cream. Anika’s mother also informed me that, with the exception of sweets, Anika was a picky eater and that, “she will eat what she wants when she wants.”

Displaying a Range of Temperaments

As I came to know Anika, I noticed that she displayed a range of dispositions both at the day care and at home. Upon arriving at day care in the midst of lunch time each day, Anika usually seemed reserved and did not engage much with the other children as they ate lunch. Instead, she frequently asked to see what I was writing in my field journal. During the late-afternoon snack time, Anika oftentimes sat quietly at the gathering table while she picked away at her snack, occasionally shifting her attention between observing the other children as they engaged in free play and watching a program on the television set mounted in an upper corner of the day care space. When Anika engaged in free play, she often chose to play either by herself or in parallel play with the younger children at the day care, rather than interacting with her 4-year-old peers. Later in the day, when the after-school children arrived at Ms. Franklin's day care, Anika's energy level seemed to increase, and her behavior became less reserved. She frequently chose to play with the girls from after-school care who were a few years her senior. They sometimes engaged in role-play, for example, pretending to be a family and going through and ordering food from an imaginary McDonald's drive-thru. Although Anika was younger than her school-aged playmates, she also participated in the role of "mom" in her play family.

Anika also exhibited a range of temperaments at home with her mother, Christina, and Christina's significant other, Leroy. Similar to her behavior at day care, Anika demonstrated her reserved side as she quietly ate her dinner alone at the dining room table while watching a DVD on the television in the living room. On one late-evening home visit, Anika displayed an intense, lively version of herself as she

continuously giggled and chatted while bouncing on top of the queen-size bed that she and her mother shared. Christina reported that Anika typically behaved this way when she was tired.

Noticing and Participating in Adults' Everyday

Verbal Interactions, Cultural Interests, and Tasks

At times, Anika seemed to take interest in mundane verbal interactions among adults in her presence, and she often echoed the comments she heard in an effort to participate in these conversations.

On one evening, during a home observation visit, Christina prompted Anika to play an alphabet game on one of her VTech electronic toys. As Christina guided Anika through the activity, she and Leroy occasionally commented in a spirited manner on what they perceived were poor choices of certain word clues and images that the toy utilized to help youngsters learn to identify the image associated with the word, the initial sound of the word and its corresponding letter of the alphabet. When the toy presented the question, "Where is the yacht," to prompt the user to find the picture of the yacht as well as the letter "y," Christina responded in frustration, "Where's the yacht? How in the world is she supposed to know what a yacht is? Where's the boat 'Nika? The boat?" In reaction to a few of the word/image clues given, Leroy quipped, "That was crazy!" As Anika continued to play the game, she attempted to join the adults in their side commentary and banter. For example, when Christina wondered aloud, "Where's the letter W," Anika responded, "W? Ain't no W!" Shortly after her mother put away the toy, Anika expressed her sentiments regarding the game, "That was crazy! Crazy!"

Earlier that evening, Anika also demonstrated interest and desire to participate in adult verbal interactions. She talked about having her own iPod after she observed Leroy inquiring about my iPod recording device and our subsequent discourse about this item. Similarly during a morning home-observation visit, while Christina talked to me about her tattoo, Anika chimed in and claimed that she too had a tattoo.

Anika also seemed to have a keen interest in expressions shaped by Hip-Hop culture and the music that played in her everyday environment. Within casual conversations Anika employed current jargon appropriately and, at her whim, Anika spontaneously uttered or sang portions of lyrics from popular songs. For example, Anika talked about the “bling” or “diamonds” she noticed on her belt while her mother dressed for work and she uttered the lyrics “shake it like a salt shaker” while taking her bath. At one point during the VTech alphabet activity episode described above, Christina asked Anika if she was tired, to which Anika responded by saying, “Nope,” and followed with a spontaneous rap, “What ch’ya gonna do with all that junk, all that junk inside that trunk?” It appeared that Anika wanted to put off her bedtime routine and singing these lyrics seemed to help boost her energy so she could stay awake a bit longer.

Anika’s awareness of and interest in adult tasks appeared in her play. While playing in Ms. Franklin’s backyard, Anika sat on one of the toy cars and, as she pretended to drive, she placed a call on her imaginary cellular phone and informed her school-aged playmate, “Hello. You need to pick up your daughter. I’m driving. I’m going to work.”

Anika's interest in the activities of adults, as evidenced in her play, use of current jargon and lyrics from popular songs in her everyday activities, and intentional observation of and participation in verbal interactions among adults, seemed consistent with both Anika's demonstrated preference to engage with older companions at day care and Ms. Franklin's perception of Anika as being like a "little old lady."

Christina, Anika's Mother

At the time of this study Christina was a 27-year-old, single mother of two children. She lived in a 1-bedroom unit in a garden-style apartment complex located off a major thoroughfare in the Southeast region of Washington, D.C., just a few minutes drive from Ms. Franklin's day care. She had a 10-year-old son who lived primarily with his father. Anika lived mostly with Christina; however, according to Christina, Anika spent significant time with her father as well. For this study, I observed Anika during those times when she was living with Christina. Leroy also stayed with Christina and Anika in their apartment and was present during two home observation visits. Because Christina did not yet have her driver's license, Leroy provided transportation for her and Anika whenever he could.

Christina was tall, slender, and had dark-brown skin. The striking resemblance between Christina and Anika appeared in their facial features as well. Christina had an easy going temperament, laughed a lot, seemed at ease with me and, like the other parents in this study, she made me feel welcomed during my home visits, each time offering me something to drink and to eat. In an early chance meeting with Christina when we ran into each other at a coffee shop nearby Ms. Franklin's day care, I

learned that both she and Leroy attended the same high school as I did. This high school connection along with the fact that I grew up in the nearby area appeared to give Christina the sense that I had experiences with Black folk, and in particular the local African-American community. Having these experiences along with a familiarity of the local community seemed to afford me some initial credibility and trust with Christina.

Whenever I saw Christina going to or coming from work, she routinely appeared well groomed and fashionably dressed. Christina had been working at a pawnshop for 3 years. She described her job as "not stable" and shared a similar sentiment about the people with whom she worked, noting, "All of them have problems." According to Christina, "The only reason why I go back there [to her job at the pawnshop] is that I have no where else to go and because I have nobody to help me." In the time prior to her employment at the pawnshop Christina worked at McDonald's for about 3 years and, for at least one period in her life with Anika, Christina "wasn't working." Because of her dissatisfaction with her job and co-workers, Christina actively sought alternative work opportunities and career paths during the course of this study (e.g., researching online degree programs for accounting, attending a job fair, pursuing work as a fire fighter).

In addition to juggling the activities of a single, working mother who was taking steps to improve or change her employment situation, Christina was also dealing with the very recent passing of her father. Given her responsibilities and life circumstances, Christina often stated that she was tired. She also worried regularly

about money and whether she had adequate financial resources available to pay for her daily expenses and monthly bills.

Christina reported that she was raised predominately by her mother, who had Christina when she was very young, and, to a lesser extent, by her maternal great-grandmother. Christina was the oldest of four sisters, one of whom was a half-sister, with whom she was close. When Christina's mother was pregnant with Christina's next sibling, they were "homeless for a while" needing to stay in "shelters and stuff like that." When Christina was about 9 years old, she and her sisters were placed in a foster home by child protective services for approximately a month because of the long hours they were left alone at home while her mother worked. She reported that staying in that foster home was "the absolute worst experience in my life." Christina's willingness to share candidly these personal details about her life history seemed to further demonstrate Christina's level of comfort with and trust in me.

Christina's Home

Christina rented a relatively spacious apartment that she kept tidy. The living room and dining area combined to form an L-shaped, open-spaced configuration with a small kitchen located by the dining area. Large windows spanned the length of the rectangular apartment and overlooked the busy city street. According to Christina, when she first moved into this apartment, she started off with no furniture except a single television set. In the 2 years since she moved into the apartment she acquired additional furnishings and electronics including a sofa and dining room set along with an older-model desktop computer. Christina and Anika shared the single bedroom, sleeping together in a queen-sized bed. Their bedroom was equipped with a stereo, a

second television set, and a collection of DVDs. Christina stored most of Anika's toys in the bedroom closet while she placed the larger of these along the walls in the living/dining areas.

Obstacles to Observation Visits

While Christina expressed a willingness to have me visit Anika at home or join them on a shopping trip, challenging circumstances oftentimes hindered my opportunities to observe Anika in any setting. Anika unexpectedly stayed under the care of her father or her father's relatives for much of the time during data collection. Christina informed me that she was dealing with a child support case with Anika's father and that she did not have his address or a phone number with which to contact him. According to Christina, Anika's father instead called "from someone else's number," and she worried, "I don't know where they're going." Such conditions resulted in Anika staying with her father or his relatives for indefinite periods of time.

During those times when Anika was under Christina's care, other issues arose that also made it difficult for me to complete observation visits with her. On a few occasions, Christina reported that Anika was ill. While Christina occasionally mentioned how much she and Anika loved shopping at Walmart and she seemed eager at the prospect of going there for one of my observation visits, she consistently determined that her financial situation during a given week did not afford her the economic means to go shopping at that time.

Glimpses of Anika's Everyday Life at Home

My sense of Anika's everyday life at home stems primarily from her mother's verbal accounts of Anika's activities and the observational data collected during the

limited visits I spent with Anika at home.

Weekdays. On a weekday morning when Christina was scheduled to work, Christina helped groom and dress Anika for the day. Depending on what time she woke up, Anika typically ate breakfast and/or lunch before taking a taxi with her mother to Ms. Franklin's day care. After Anika ate dinner at day care, Leroy and Christina arrived to pick her up. If Christina had no other errands to run in the evening (e.g., going to the laundry mat), they would go home.

On one weekday evening, I joined Christina, Anika and Leroy back at their home. Anika's nighttime activities included preparing for and taking a bath, completing an elaborate post-bath, skin-care routine that included a full-body moisturizing rub, getting dressed for bed, brushing her teeth, and interacting and playing with the adults while they ate at the dining room table and watched a DVD.

Christina assisted and engaged with Anika during each of these activities. For example, during Anika's bath time, Christina negotiated with Anika regarding which toys she could have in the tub. Anika selected several toys with which she played, including a Dora doll and a set of 3 plastic ponies of increasing size. While Christina, Leroy and I sat and conversed at the dining room table, Anika asked to write in my journal and began to scribble in it. This sparked Anika to inquire, "How you write my name?" She then asked Christina, "Mommy, can you write my name?" After Christina helped Anika to write her name in my journal, she prompted Anika to play games on her VTech electronic toy. I was not present that evening to observe Anika's bedtime routine. However, Christina reported that she and Anika usually watched DVDs before Anika went to sleep.

Wednesdays. Christina and Leroy did not work on Wednesdays. According to Christina, they normally slept in, did some cleaning, and typically ran errands. During a Wednesday afternoon home visit in June, Anika entertained herself while Christina and Leroy lounged around and relaxed for much of the time, including talking to one another and to me and watching either local television programs or DVD videos that Christina purchased from her pawnshop. Anika engaged in independent, physical activity (e.g., dancing to music videos) or play (e.g., climbing on the sofa), and every now and then she engaged briefly with Christina. Occasionally, on her own will, Anika also interacted with me by showing me her belongings from around her home.

On this visit, Christina also showed me two placemats that Anika's godmother gave to Anika about 3 months prior on her fourth birthday. The placemats displayed bare and pictorial representations of addition and subtraction problems.

She pays no attention to these (referred to bare computation problems printed on the placemat). I don't know if they're too complicated right now, or, but um, I try to tell her the answers. You know. That's about it. But um she understands these (referred to subtraction and addition equations, 1 through 10) before she drew all over them (referred to pen marks scribbled over the placemat). But um, you know "one plus one" these are addition, subtraction, and um, these here just basically count all of them that are in here and they'll tell you to add or subtract or how many are left. I try to go over these with her. (June 14, 2006 – Home Visit with Anika)

Later that afternoon, Anika retrieved from her toy bin another VTech electronic game that had the following academic categories of play: math, vocabulary, and spelling. According to Christina, Anika's grandmother gave this toy to Anika when she was 3-years-old. Christina proceeded to guide Anika through the toy's activities.

After a while of helping Anika play the spelling activity, Christina asked Anika, “You wanna go to math? (Paused while setting up game.) We wanna go with addition.” Once set up, the game prompted, “Addition. What is the correct answer? Forty-four plus three.” Christina proceeded to model for Anika the two-column algorithm with her fingers, encouraging Anika to try to follow along. After about 5-minutes, with Anika’s attention quickly waning and given her difficulty following her mother’s prompts, Anika finally pleaded, “Can I cut it off?”

Anika’s reported and observed lack of interest, respectively, with the computation problems on the placemat and the VTech math game, demonstrated the difficult and abstract nature of the content of these activities relative to Anika’s mathematical understandings and interests. Despite Christina’s well-intentioned efforts to engage Anika in mathematical experiences at home, these activities did not appear to support or draw upon Anika’s developing meanings of number. It was not clear to me whether the observed interactions with the VTech math game transpired for my benefit or were representative of Christina’s periodic efforts to engage Anika with the device.

Weekends. Anika usually stayed with her father over the weekends, from Friday evening through Sunday morning. Christina typically went grocery shopping before Anika returned from being with her father. However, if Christina planned to shop at Walmart, she had to wait and take Anika with her, otherwise, Anika would "catch an attitude." On Sundays, Anika attended church with Christina and in the evening Christina typically washed and “did” (i.e., combed, plaited, or styled in

ponytails, cornrows, or individual braids) Anika's hair. Christina prepared dinner and afterwards they watched DVDs before going to bed.

Candid Reflections of Mathematics:

Perspectives from Anika's Mother, Christina

When I asked Christina about her experiences in school and in particular her experiences in school mathematics, much of the content of her responses tended to focus on particular social memories from her school-aged years rather than on her school mathematics or even broader academic experiences. The following discussion begins with the few reflections that Christina did share regarding her mathematical experiences in elementary school.

"I was always doing it wrong."

While Christina exclaimed, "I love math" when we met by chance at the coffee shop, she conveyed a different relationship toward the subject when discussing her early experiences in school mathematics.

Christina: When I first started, I would say like elementary, I used to hate math.

Grace: Why?

Christina: I could never get it right. Uh, cause I had my own way and my own method of adding my numbers together or subtracting my numbers, and dividing my numbers... I had my own method of trying to come up with an answer for, you know, the mathematical problem. But I could never get it right. I was always doing it wrong. And somebody else would try to show me. I didn't want to listen to them at all, so.

Grace: ...what would be your own way of doing math problems when you were little?

Christina: The wrong way. If it, says subtract, I'll add. If it says add, I'll divide... basically, especially when we had homework...I would sit there and try to just write anything to hurry up and go outside... And that was it.

(June 12, 2006 – Home Visit with Anika)

Christina seemed to view school mathematics as not only something to get “right” but that it also required using a single right method to solve problems (e.g., computation tasks). Her negative feelings toward mathematics appeared to stem from her frustration and belief that she “could never get it right” and from her resistance to others who tried to help her. Believing that her own way of doing mathematics was “wrong,” Christina resorted to writing “anything” as answers to problems so she could quickly complete assignments and move on to more enjoyable activities like playing.

Christina’s inferior perception regarding her own methods to solve mathematical problems also seemed to extend to the computation strategies she employed at work, a setting in which she believed she used mathematics in her daily life.

Christina: Especially, um, the discounts that we give customers, like on the jewelry and stuff. Like, um, it used to be fifty percent off, so that wasn't so hard to do. But now it's back to thirty percent, so. Like, um, Saturday, this man, it was a chain that was four ninety-nine [\$499], so he was like "Well, how much would this be with thirty percent off?" And we didn't have a calculator in sight! Four ninety-nine. All I have to do is just round it off to the nearest dollar, which is five hundred [\$500]. And then, I took the long way around, so I said "Okay multiply it times seventy, okay it'll be three fifty [\$350.00]." I was so happy!

Grace: Wait, how did you get that?...

Christina: Okay, I, first of all, it was four ninety-nine so I rounded it off to five hundred. And then, um, there's a hundred percent, in a dollar there's a hundred [indiscernible word, sounds like 'coins'], I mean, there's a hundred percent in one dollar. Start from a hundred percent. If something is thirty percent off, you take away thirty you have seventy percent left. So, you multiply, the five, well seven times five is thirty-five, so, that's how I got three fifty. Other people have, you know, different ways of doing it, that's what I meant about the different methods of doing it. I took the long way around. [Indiscernible phrase follows.]

Grace: But, as long as it works and makes sense to you, really, it doesn't matter which way you go.
Christina: Yeah.
(June 12, 2006 – Home Visit with Anika)

Christina determined the estimated sale price of the chain (\$350) by subtracting 30% from \$500, then taking the result of \$325 and multiplying it by five (to find 70% of \$500). Although Christina employed a strategy that made sense to her, was mathematically sound, and resulted in the correct discounted price, she considered this approach as taking the “long way around,” suggesting that her method was less efficient or sophisticated compared to other, possibly more formal, methods or the “right” way for computing the sale price.

Consistent with Christina’s beliefs that there is a “right way” to solve problems, particularly in school mathematics, she used her fingers to model for Anika the two-column traditional algorithm for adding to solve the problem $44+3$ posed by the VTech toy as opposed to presenting an alternative strategy such as counting on. “...we count...add and subtract...”

Christina's perception of mathematics seemed to encompass primarily a formal perspective of mathematics, characterized by a preeminence of number (e.g., rote counting, enumerating physical quantities, and recognizing numerals) and numeric operations (e.g., addition, subtraction, multiplication, and division).

Christina referred explicitly to numeric operations when recalling her elementary mathematics experiences and explaining her use of mathematics at work. She also focused on number and operations when reflecting on when she thought Anika engaged in mathematics in her everyday life, referring to the placemats discussed previously.

Like when we count, we got like a, a board...it's like a table mat. And Anika will try to sit there and add and subtract... I try to get a chance to help her with it, or Leroy...

(June 12, 2006 – Home Visit with Anika)

Christina expressed concern for or value in developing Anika's computation skills when she observed Anika's difficulty in responding to the VTech toy's math activity.

Christina and Leroy determined that they could help Anika by purchasing from Walmart some "flashcards" that displayed addition and subtraction facts and reviewing these with Anika.

Christina also reported counting with Anika as another example of when she thought Anika engaged in mathematics in her everyday life. Christina stated that she had Anika rote count to 20 and helped her along whenever she stopped. According to Christina, Anika spontaneously counted everyday objects in her home (e.g., bathroom toys, crayons).

Money

Not only did Christina reference money when she helped customers to determine the price of items at the pawnshop, but she also recalled how she learned to count with money from her maternal great-grandmother (whom she referred to as her "grandmother").

... when like people in our family would give us money, I learned from like money. Whenever they would give us money and stuff...my grandmother would say, "I'm not going to give you this until you tell me what it is," and she'll take it back. Or if I get change, say we're at a store, she'll make us stand there and count out the change back...like when the man would give us change, she'll ask, "How much is that in your hand?" And if we couldn't tell her she'll take it from us...

(June 12, 2006 – Home Visit with Anika)

Through everyday activities with money, Christina claimed, “I learned from experiences like that. That’s how Anika learns.” Christina reported that she began to help Anika learn to identify the monetary values of different coins and bills. During a trip to the laundry mat, she observed that Anika recognized the different buttons on the washing machines that corresponded to the particular value of coin(s) to be inserted.

Christina discussed how her mother emphasized the financial importance of knowing mathematics,

... my mother used to get on us all the time, "You know you're going to need math in the real world. You're gonna get tired of people cheatin' you out of your money. You're going to need to know. Especially when you get jobs." And she used to go on and on and on and on...
(June 12, 2006 – Home Visit with Anika)

Money therefore provided a meaningful context in which Christina and, reportedly, Anika developed particular mathematical competencies involving number and numeric operations.

Interpreting a Collage of Anika’s Everyday Mathematical Events

An examination of the data of Anika’s everyday mathematical events revealed concepts of difference and similarity as a salient content theme. Before I present and analyze below a collection of Anika's everyday mathematical events involving meanings of difference and similarity, I explain my conceptualization of these concepts as being mathematical.

Conceptualizing Meanings of Difference and

Similarity as Mathematical Content

The NCTM (2000) identified data analysis as one of five major content

strands in which children should develop understandings in school mathematics. In addition to counting, young children's capacities to classify objects is a fundamental mathematical process that contributes to their evolving knowledge of data analysis concepts (Clements, 2004; Clements & Sarama, 2007; Sarama & Clements, 2008). Classification involves the process of grouping, categorizing, or sorting objects according to an established criteria or attribute (Seo & Ginsburg, 2004). Toddlers and preschool children informally classify objects as they spontaneously select or group identical or similar objects together in their everyday settings and activities (Clements & Sarama, 2007; Seo & Ginsburg, 2004).

Research on preschool-aged children's learning of data analysis concepts and related processes is limited (Clements, 2004; Clements & Sarama, 2007). In particular, more specific analyses of the developmental trajectory of young children's growing capacities related to data analysis concepts and processes, namely classification and its preceding or relevant competencies, are lacking in comparison to, for example, the vast literature available that addresses young children's growing understandings of number and operations. Nevertheless, recent characterizations of preschool children's informal mathematical activities include classification (e.g., sorting) as well as number concepts (e.g., enumeration) (Clements & Sarama, 2007; Seo & Ginsburg, 2004).

I drew inferences from the limited scope of literature within the field of mathematics education research that addresses young children's development of classification processes to inform my interpretation of Anika's collection of significant everyday mathematical events in which she spontaneously observed and

identified differences and/or similarities between various entities. I posited that a mathematical quality, involving Anika's meanings of difference and similarity, characterized these events. My assumptions initiated from Clements and Sarama's (2007) claim that *oddity* (i.e., "which one is not like the others") is a "basic competency" in classification (p. 527). I contend that, conversely, the capacity to identify entities or attributes of entities that are the same could also be considered a basic competency in classification.

A distinct set of attributes defines an entity. Various types of attributes can be discerned when examining an entity, including: physical, material, functional, and/or categorical qualities. In this study, physical qualities refer to attributes (measurable or non-measurable) that can be perceived or approximated by sight (e.g., color, shape, figure, design, size). Material qualities refer to attributes that can be perceived or approximated through senses other than sight (e.g., taste). Functional qualities can include the perceived purpose, use, or operation of an entity. Categorical qualities can include the perceived classification or grouping criteria ascribed to an entity.

For the purposes of this study, *difference* refers to the perception of a set of entities, or a set of corresponding attributes that define these entities, as being distinguishable or not the same. Meanings of *similarity* include concepts of sameness and resemblance. *Sameness* refers to the perception of a set of entities, or a set of corresponding attributes that define these entities, as being indistinguishable or identical. *Resemblance* refers to the perception of a set of non-identical entities as having at least one identical or nearly identical attribute in common. Resemblance can also refer to the perception of a set of corresponding attributes, attributes which

define a set of non-identical entities, as identical or nearly identical. The perceived degree of resemblance among a set of entities, or the corresponding attributes that define these entities, can vary depending on the attributes, qualities, and/or other meanings that are considered when evaluating a set of entities or their corresponding attributes.

Composition

The following analysis presents Anika's collection of significant everyday mathematical events in which she demonstrated meanings of difference and similarity. The discussion is organized into five sections.

The first three sections encompass a set of mathematical events in which Anika observed and identified differences and/or similarities within pairs of entities. Each of these sections focuses on one of three types of entity pairings: *two like objects*, *an object and an image*, and *two unlike objects*. Each type of pairing is further explained in its respective section below. The spontaneous nature underlying these events involving paired entities demonstrated Anika's significant inclination toward and interest in observing and identifying differences and similarities within various pairs of entities she encountered in her everyday settings and activities.

The fourth section presents an analysis of a mathematical event in which Anika exhibited beginning understandings of classification as she spontaneously sorted a collection of fruit snacks.

In the final section, I discuss a mathematical event in which Ms. Franklin directed Anika to complete a worksheet on identifying two same images.

For each mathematical event presented, I discuss the adapted dimensions of content, purpose and setting, values and beliefs, and social relations that together framed the event.

Across this collection of mathematical events, Anika employed, constructed, and expressed meanings of difference and similarity. The following discussion of mathematical events illustrates Anika's developing mathematical understandings and competencies relevant to classification and, more broadly, data analysis concepts.

Content: Observing and Identifying Differences

And/or Similarities - Between Two Like Objects

Anika meaningfully noticed, engaged with, and evaluated familiar items in her everyday environment while she actively participated in routine activities at home and at day care. It was not uncommon for Anika to observe and identify spontaneously differences and/or similarities within pairs of entities she found interesting. In most of these mathematical events, Anika focused on *two like objects*. In this study, two like objects refers to either a pair of identical items or a pair of items that had considerable resemblance, having only minor distinctions in terms of their physical, material, functional, and/or categorical qualities. In the following mathematical events involving two like objects, the extent of similarity, including their degree of sameness or resemblance, varied among each pair. Also, the extent to which Anika identified differences and/or similarities within each pair of like objects varied as well.

These pairs of like objects included familiar or commonplace items found in either Anika's home with Christina or in Ms. Franklin's day care. At home, Anika

observed and/or identified differences and/or similarities within the following pairs of like objects: two barrettes, two pairs of slippers, and two juice drinks. Anika noticed and/or engaged with these like objects while getting her hair done, taking her bath, and participating in dinnertime activities, respectively. During free-play time at day care, Anika observed and identified a difference between two baby dolls and she also recognized that two plastic teacup saucers were the same. In these mathematical events at home and at day care, Anika observed particular physical (e.g., color) or material (e.g., taste) qualities that she identified as different and/or similar between the two like objects with which she noticed and/or engaged. Each of these mathematical events in which Anika demonstrated meanings of difference and similarity in her observations of two like objects is discussed in greater detail below.

1a. Content: Observing and identifying a similarity and a difference within a pair of barrettes. The following mathematical event occurred during my first home visit with Anika as Christina did Anika's hair in their bedroom. Throughout this visit I focused my attention mostly on establishing a rapport with Christina through casual conversation. The field notes excerpt below describes an event that took place as Christina and I chatted while she did Anika's hair.

Barrettes

Anika entertained herself while she was getting her hair done by playing with her stash of hair accessories (a variety of plastic barrettes, or "bows" as Christina called them, and ball ponytail holders, contained in a clear, plastic shoe bin). Anika pretended to cook with the barrettes and occasionally offered me an imaginary taste of her cooking. As Christina and I continued our conversation, I noticed Anika pausing as she held a pair of yellow plastic barrettes, that were not only the same color, but appeared to be the same relative size and shape. She was also talking to herself and I thought I heard her say the words "same" and "color." As Anika continued examining the barrettes, she touched or pointed to the unique designs in the center of each barrette (I believe one was a flower and the other was a bird). Anika's body

language and verbal cues suggested to me that she recognized the difference between these two, otherwise very similar, barrettes.
(March 31, 2006 – Home visit with Anika)

Because I was engaged in a conversation with Christina it was difficult for me to fully attend to what Anika was doing or saying during this event. However, based on what I was able to observe with respect to Anika's body language, coupled with a few key words that I thought I heard Anika utter aloud to herself ("same" and "color"), Anika appeared to identify both a similarity and a difference between the two, nearly identical barrettes. More specifically, Anika noticed that each barrette was yellow and thus determined that the barrettes were the same color (physical quality). She also recognized that the barrettes' center designs (physical quality) were different from one another. Given that Anika paused in her pretend cooking to examine closely these two barrettes in particular, suggested that Anika may have taken interest in the pair of like objects because she noticed, upon closer inspection, that the barrettes, although they looked nearly identical, were actually not the same since their center designs were in fact different. Anika seemed to observe and identify the single physical feature that distinguished the two yellow barrettes from one another.

1b. Purpose and setting. The barrettes mathematical event took place at Anika's home while Christina did her hair, a routine practice that normally took Anika's mother several minutes to complete each day. While waiting for her mother to finish doing her hair, Anika chose to entertain herself and to play with her barrettes and ponytail holders. Anika's collection of hair accessories provided her with a variety of colorful objects with which she could play as well as examine. An intrinsic interest seemed to motivate Anika to notice and evaluate the physical attributes of the

two yellow barrettes.

1c. Values and beliefs. Getting her hair done was a significant, routine practice in Anika's family life with Christina. Christina made an effort to try to do Anika's hair every morning, including reserving a few hours on Sunday evenings to wash and to "really" do Anika's hair. When Anika's hair was styled in individual, thin braids, Christina reported that it took several hours not only to plait Anika's hair initially but it took just as long (over a couple of days) to undo the braids. In addition to the various tools and hair products Christina used to do Anika's hair, she also had a growing collection of colorful barrettes and ponytail holders with which to embellish Anika's hair styles. Each day, Christina carefully selected those accessories to place in Anika's hair that would complement her outfit. The time, effort, and care that Christina devoted to the practice of maintaining and doing Anika's hair suggested that Christina took pride in her child's appearance, not only with respect to the shoes and clothing that Anika wore, but also in how Anika's hair was kept.

1d. Social relations. Anika's mother, Christina, primarily shaped the context of the barrettes event as she managed the practice of doing Anika's hair.

Christina assumed responsibility for grooming Anika's hair each day, creating and maintaining Anika's hair style, purchasing and selecting "bows" with which to accessorize Anika's hair, and washing and "really" doing Anika's hair at least once a week.

In addition to the above activities, the practice of doing Anika's hair also called for Christina to provide and manage activities or objects with which Anika could engage or be entertained while waiting for Christina to finish doing her hair. In

the barrettes mathematical event, Christina provided a collection of various hair accessories and placed these in a container that she allowed Anika to freely access as Christina did Anika's hair. On another occasion when I was present as Christina did Anika's hair, Christina permitted Anika to select a DVD movie and to operate the DVD player so she could watch the movie while getting her hair done. Managing and negotiating the appropriate use of for-entertainment-use objects and activities also afforded Anika and Christina a meaningful context in which to engage each other in dialogue as Christina did Anika's hair.

While Christina primarily managed the practice of doing Anika's hair, Anika freely chose to play with her hair accessories and she initiated her observations of similarity and difference regarding the pair of yellow barrettes.

Anika's observations, as expressed through her physical engagement with the barrettes and her utterances, did not appear to be directed at either Christina or myself. While I observed a number of occasions during my home visits with Anika in which she either attempted to interject her voice into adult conversation or to initiate a verbal interaction with an adult, Anika's desire to gain adult attention did not seem important to her during this particular event. Instead, Anika appeared to be thinking aloud to herself as she uttered the thoughts and meanings she constructed when examining the pair of barrettes. Anika's spontaneous pause in her play to inspect and to evaluate the two yellow barrettes also suggested an intrinsic motivation might have prompted Anika to engage in meanings of difference and similarity.

2a. Content: Observing and identifying a similarity and a difference between two pairs of slippers. In the following mathematical event, Anika observed and

identified a similarity and a difference between a pair of slippers that were physically in view during the episode and a pair of slippers that were not physically present. The event took place on a weekday evening home visit during Anika's bath time. I sat in the bathroom while Anika played in the tub. Christina had just left the bathroom to put away Anika's excess bath toys when the following event occurred.

Slippers

Anika: Miss Grace.

Grace: Yes, Anika.

Anika: [She observed the pair of slippers I was wearing. The slippers were ballet-style, made of black satin, with a bow on the front.] My mommy got slippers [indecipherable word/s].

Grace: Your mom has slippers, too?

Anika: Yeah.

Grace: Yeah?

Anika: She has 'em.

Grace: [I recalled the slippers I observed Christina wearing on my first home visit,] I think she has, I think blue fuzzy ones.

Anika: No.

Grace: She has slippers like this? [I pointed to my slippers.]

Anika: No.

Grace: I thought her slippers were blue. They're slip on. They may have like a rainbow or a heart, something with art, some kind of little picture on her slippers.

Anika: She got the same,

Grace: She has the same kind like me? Yeah? [Crosstalk with Anika below.]

Anika: But it's, it's, [Crosstalk with Grace below.]

Grace: I haven't seen those yet.

Anika: But it's not the same color. [Crosstalk, Indecipherable word(s).]

Grace: Oh really? What color are hers?

Anika: It's like the same color like, um, of, of, um, it was like this color. [She pointed to the off-white, beige-like color of the tub.]

Grace: What do you mean, like th-, that white? Or yellow?

Anika: White.

Grace: It's white? Okay.

(April 10, 2006 – Home Visit with Anika)

While soaking and playing in the tub, Anika spontaneously observed my slippers and seemed to recognize a similarity between my slippers and a pair of

slippers she stated belonged to her mother. Anika referenced a pair of Christina's slippers that I had not seen and were not in view during this event. The only slippers I observed Christina wear were a light-blue, fuzzy, slip-on pair. This is not to say that the slippers Anika appeared to picture in her mind and tried to describe to me did not exist. Anika seemed adamant that the particular pair of her mother's slippers she envisioned were not the "blue fuzzy ones" I described.

When I first asked Anika if the pair of slippers she envisioned were "like" my slippers, she responded "no." However, as our conversation continued, Anika stated that her mother "got the same" slippers, presumably, as me. When I clarified her comment by asking Anika if her mother had "the same kind (of slippers) like me," Anika confirmed in the affirmative. Thus, when I initially asked Anika if her mother's slippers were "like" my pair, Anika might have interpreted my question or my use of "like" to mean whether the pairs of slippers were indistinguishable or identical (e.g., same fabric, color and style), which, based on her subsequent descriptions, they were not. Upon noticing my slippers, Anika appeared to observe a resemblance between these and her mother's slippers. Anika's meaning of "same kind" seemed to indicate that she perceived that the style (physical quality) of my slippers, ballet-style, was the same as the style of the slippers she appeared to have in mind.

All three pairs of footwear (my black satin, ballet-style pair; Christina's blue fuzzy, slip-on-style pair; and the pair of slippers that Anika envisioned) could be perceived as similar, or more specifically, as having resemblance in that they all can be categorized as slippers. However, Anika's discernment between the footwear

seemed either to go beyond or to disregard their categorization as slippers. Anika instead appeared most concerned with the physical attributes of the pairs of slippers rather than their classification. In our discourse, when I mentioned the “blue fuzzy ones,” Anika not only rejected these as the pair of slippers she pictured in her mind, but she also appeared to dismiss the “blue fuzzy ones” as a potential resource from which she could characterize the slippers that she envisioned. Rather, Anika maintained her focus on the style of my slippers as the primary attribute underlying her evaluation of sameness.

Anika also identified a difference between the two pairs of slippers, reporting that her mother’s slippers were “not the same color” (physical quality) as my pair. Anika did not have a word to describe or to name the color of her mother’s slippers. Instead, she utilized the bathtub/shower surface as a resource to help her portray this physical quality of color to me as she pointed to the nearby, off-white, beige-like-colored bathtub/shower wall and informed me, “... it was like this color.” She recognized a resemblance between the color of the slippers she mentally pictured and the color of the bathtub/shower surface. Thus, not only did Anika observe and identify a sameness in the style (physical quality) as well as a difference in the color (physical quality) between the two like slippers, but she also observed and identified a resemblance in color (physical quality) between two unlike objects, the slippers she envisioned and the nearby bathtub/shower wall.

2b. Purpose and setting. Anika’s daily personal hygiene regimen typically included taking a bath in the evenings. In the slippers event, I sat in the bathroom during Anika’s bath time, which lasted approximately 20- to 25-minutes. As part of

Anika's bathing practice, Christina allowed Anika time to soak and play in the bathtub as she wished, within established behavior expectations. During this unstructured soaking time, Anika freely played and chattered with herself, her mother, and me while she frolicked in the bathtub.

The slippers event occurred while Anika leisurely soaked in the bathtub. Because I sat in the bathroom while Anika bathed, not only did Anika have the opportunity to interact with me but my presence along with my belongings and attire added to this bath time setting as well. At one point when Christina left the bathroom, Anika immediately initiated discourse with me regarding the pair of slippers she observed me wearing. Anika's spontaneous observation and desire to share her thoughts with me seemed to indicate that Anika deemed what she noticed as interesting and worth my attention.

2c. Values and beliefs. A significant portion of Anika's bath time routine consisted of an unstructured soaking period. Thus, in addition to maintaining Anika's personal cleanliness, providing Anika some time and freedom to conduct her own amusement activities while she soaked in the bathtub seemed to be another important aspect of Anika's bath time practice. Christina supported Anika's leisure interests during bath time by providing Anika an opportunity to freely play or amuse herself while soaking, supplying Anika with a selection of various bath toys, negotiating with Anika which of these toys she could play with during her bath, and welcoming and responding to Anika's talk and song as she soaked in the bathtub.

The slippers event illustrates Anika's desire to interact with known adults during everyday activities such as bath time. Immediately after Christina informed

Anika that she was going to leave the bathroom to put away the remaining bath toys in the closet, Anika invited me into conversation with her by saying, “Miss Grace.” This gesture seemed to suggest a genuine intention on Anika’s part to engage with me verbally during her bath time. Anika could have chosen instead to play on her own without seeking or gaining my attention. Anika’s interest to talk to me seemed consistent with other occasions during home visits in which I observed Anika actively seeking to converse with known adults or to involve herself in discussions among adults.

2d. Social relations. For the most part, Christina managed Anika’s bath time routine. Activities such as negotiating what toys Anika could have in the tub, establishing a comfortable water temperature, and assisting Anika in scrubbing her body afforded Anika and her mother natural opportunities to interact verbally with one another. In addition to these bath activities, an unstructured soaking period afforded Anika the occasion and freedom to engage in various activities of her choice.

In the slippers event, Anika initiated a conversation with me regarding my footwear. Anika’s particular attention to my slippers and her desire to share her observations with me appeared to reflect either a social or observation-oriented interest. In terms of a social aim, Anika may have taken notice of my slippers initially as a means to interact with me. On the other hand, Anika’s observations of similarity and difference may have itself sparked her intent to engage with me.

3a. Content: Observing a difference between two juice drinks. The following mathematical event took place around 9:15 p.m., after Anika’s bath time referenced in the slippers event discussed above. Christina, Leroy, Anika, and myself were

gathered in the dining room and kitchen area of the apartment.

Juice

Christina: [To Anika] Not hungry?

Anika: I'm, I'm. I'm gonna get that other juice cause I like it [she left the table to return the juice she had and to retrieve another juice from the refrigerator; meanwhile, Christina and I chatted between ourselves].

Anika: [She returned to the table and said during a pause in my conversation with Christina,] This not the same juice that I had. [Paused. Then after no response, she repeated in the same tone as before,] This not the same juice that I had.

Grace: Is that another juice?

Anika: It's another juice.

(April 10, 2006 – Home Visit with Anika)

In this event, Anika decided to exchange her initial juice drink for one that she liked. Once Anika returned to the table with her new juice, she announced that the juice was “not the same” as her initial juice drink. Anika observed a difference between two like objects when she determined that the two juice drinks were not the same. Although Anika did not identify a specific quality that differed between the two juices, the difference she noticed did not appear to reflect simply that the drinks were exchanged. Rather, Anika's initial comments that she wanted to get a juice that she liked suggested that Anika preferred the taste of the latter drink. Thus, Anika appeared to observe a difference in taste (material quality) between the two juice drinks.

3b. Purpose and setting. The juice event occurred on a weekday evening in Christina's dining room as she prepared dinner. We were gathered in the dining room and kitchen area of the apartment where we could also view the DVD movie playing on the living room television set. Although Anika did not wish to eat at that time, she did accept something to drink. Anika exchanged the initial juice drink she received for

one that she seemed to prefer in taste. Anika's mother, Christina, purchased various drinks and food products and made a few of these items accessible in her kitchen for Anika to select and acquire independently. When Anika spontaneously announced, during a pause in my chatter with Christina, "This is not the same juice that I had," neither of us acknowledged Anika's comment. Anika therefore repeated her statement and I responded accordingly. Anika's persistence suggested that she wished to direct her observation to either or both Christina and me.

3c. Values and beliefs. The juice episode demonstrated Anika's growing self-reliance to obtain nourishment, in particular, with respect to helping herself to certain refrigerator items. Anika's ease in obtaining a new drink on her own, with no concern demonstrated from her mother, suggested that Christina supported and possibly encouraged Anika's independence in this context.

Although Christina seemed to encourage Anika's independence to select and acquire her own beverages, Christina also expressed concern regarding Anika's nighttime energy level as well as her dental health. Christina regulated Anika's evening sugar intake, including the amount of juice she consumed. Later that evening when Anika indicated that she wanted to drink more of this juice, Christina immediately denied this request because of her apprehension with the juice's high sugar content. The juice event also appeared to illustrate Anika's dispositions with respect to food and eating. Anika's dissatisfaction with her initial juice drink seemed to reflect both her preference for sweets as well as her tendency to be a picky eater.

Anika stated and repeated the comment, "This is not the same juice that I had." Anika's actions suggested that she wished to gain either or both Christina's and

my attention and to share her observation with us. Thus, this episode also seemed to reflect Anika's desire to interact verbally with known adults in her daily settings and activities.

3d. Social relations. The juice event occurred, more broadly, within the context of Anika's dinnertime activities at home. Christina primarily managed the foundational aspects of dinnertime, including: purchasing and storing away grocery items; cooking meals; assessing Anika's hunger; and setting aside time for dinner. On the other hand, Anika appeared to have a significant say in determining whether she actually ate dinner and what, if anything, she consumed during this activity. For example, if Anika indicated she was not hungry, Christina did not seem to insist that she eat. In the juice event, Anika opted only to have something to drink. Anika's lack of hunger during this episode could have been due to, as Christina reported, Anika's tendency to "eat what she wants when she wants."

Anika's composed persistence in sharing her observation with us about her drink suggested that Anika perceived that her mother and Leroy welcomed her talk and that Anika's vocal participation during dinnertime activities appeared to be a routine aspect of this practice.

4a. Content: Observing and identifying a difference and a similarity within pairs of like toys. Anika also spontaneously observed and identified differences and similarities between like objects at day care. The following two mathematical events occurred during free-play time.

Baby dolls

Anika played with a baby doll she named Keekee. Anika and Nevin (a toddler at the day care) each placed their baby doll in a toy, child safety car-seat. Later, Anika and Nevin came over to me with their baby dolls. Anika pointed

to the torso area, near the neck of each doll, and informed me that Nevin's doll's outfit had writing (that to me looked like an abstract, cursive script, continuous pattern design) on it and her doll did not. The two children then continued to play with their dolls.
(March 21, 2006 – Day Care Visit)

The two dolls with which Anika and Nevin played had considerable resemblance. Both toys were Black, baby-girl dolls and had non-identical, corresponding body parts (functional qualities) in common. Several non-identical, physical qualities distinguished each doll. Anika observed in particular a difference between the two dolls' outfits, one had writing or a scribble pattern on it while the other did not. Anika's attention to the dolls' outfits suggested that Anika viewed this physical quality as salient in distinguishing these. Anika physically and verbally referenced the distinct physical quality of each baby doll's outfit when she shared her observations of difference with me.

Teacup saucers

As Anika played by herself, she rummaged through a toy bin selecting various smaller toys with which to play. Eventually, she found one plastic teacup saucer and then another similar one. Upon inspecting both saucers with her eyes, she immediately held up the pair and exclaimed to me, "These are the same!"
(April 3, 2006 – Day Care Visit)

Anika spontaneously noticed the sameness of the two, toy teacup saucers she found.

4b. Purpose and setting. The baby dolls and teacup saucers events took place at Ms. Franklin's day care during free-play time. During this activity, children had access to various toys, climbing structures, puzzles, drawing supplies, and books with which they could engage. This wide selection and variety of choices afforded Anika with a range of objects and materials with which she could freely explore and engage

in meanings of differences and similarities. In the baby dolls event, Anika and Nevin engaged in parallel play with their dolls before they approached me. In the teacup saucers event, Anika played independently as she typically did during free-play time. In both events, Anika deemed her observation of difference and similarity, respectively, as worthy to share with me.

4c. Values and beliefs. Ms. Franklin believed that children at the day care benefitted from having opportunities to engage in unstructured play. As such, Ms. Franklin included free-play time as part of the children's daily schedule and she provided a variety of stimuli (e.g., toys, climbing structures, books) with which children could play and explore during this period.

In the baby dolls and teacup saucers events, Anika demonstrated her temperament to interact with known adults. Although other children were present during these events, Anika did not share her observations with them.

4d. Social relations. Ms. Franklin primarily managed free-play time by establishing this activity as a routine practice in the children's day care schedule, providing a variety of child-friendly stimuli with which day care attendees could play and explore, and setting and reinforcing appropriate behavior expectations for the children to follow. During free-play time, Ms. Franklin allowed the children to play with little interaction from her, except when she needed to correct inappropriate behavior.

Anika appeared to prefer to play alone, especially when the school-aged children were not present. While Anika occasionally interacted, to some degree, with the day care toddlers, she rarely played with her 4-year-old peers.

As illustrated in the baby dolls and teacup saucers events, Anika willingly paused in her play to convey her observations of difference and similarity to me, and immediately afterwards she resumed her self-directed play. This suggested that Anika did not intend to engage in discourse with me. Instead, it appeared that Anika simply aimed to announce what she noticed, deeming her observations of difference and similarity worthy of comment.

Content: Observing and Identifying a Similarity

And Difference - Between an Object and an Image

In this section, I discuss two mathematical events in which Anika demonstrated meanings of similarity and difference while observing an object and an image. In the first mathematical event, Anika identified differences between the actual appearance of children and a drawing of those children. In the second event, Anika identified a difference between a toy xylophone and a pictorial representation of another xylophone.

In these mathematical events, despite the structural (e.g., medium, dimension, and scale) distinctions within each object-image pair, Anika appeared to recognize a conceptual similarity within each pair. The subsequent differences that Anika observed within each object-image pair seemed to focus on a particular physical quality that distinguished the appearance of an attribute represented by one entity from the other rather than on the entities' disparate structures. In each of these mathematical events, Anika seemed to understand that the image represented the actual object or an object that closely resembled the actual object.

1a. Content: Identifying differences between the actual appearance of children and a drawing of those children. The following mathematical event occurred one late afternoon at day care. Anika stood by me as I sat in the lounge chair.

Drawing

Anika asked me to draw her and the others (Toya and Nevin) in my journal. She specifically asked me to draw her with 3 plaits (she actually had about 15 plaits in her hair). After Anika inspected my drawings of Toya and Nevin, she communicated non-verbally to me (by motioning with her hands and body) that Toya should have a zipper on the top part of her shirt and that Nevin should have a little bow on the collar of his shirt (he was wearing his sister Kalia's top as he stained the clothes he had on earlier).
(March 21, 2006 – Day Care Visit)

In this drawing event, Anika requested that I draw her and two other children who were present at the day care. Once I sketched the children, Anika spontaneously examined my drawing compared to her observations of the actual subjects' appearance. Using hand and body gestures, Anika identified two shirt details (i.e., Toya's zipper and Nevin's bow) that she wanted me to add to the drawing. As such, Anika appeared to notice a difference between my sketch of the shirts and the corresponding actual physical appearance of Toya's and Nevin's shirt collars. Anika's attention to the shirt collar of each child seemed consistent with the attribute of outfit appearance that she focused on in the baby dolls event reported above. This suggested that clothing, if present, may have represented a significant physical quality that Anika noticed when she observed and evaluated entities for differences and similarities.

When Anika requested that I draw her with three plaits, it was not clear if Anika intended to have the drawing of her depict fewer plaits than what she actually had, or if Anika understood that she did have a multiple number of plaits and that she

tried to enumerate these by using the word "three."

1b. Purpose and setting. The drawing event occurred at Ms. Franklin's day care, following afternoon snacks and a period of free play, during an unstructured time for Anika. As Ms. Franklin began to work on writing skills with Toya, Anika remained by me as I sat in a chair from which I could observe day care activities. By examining my picture and identifying two missing details that she observed in my drawing, Anika sought to help me make the images of Toya and Nevin more similar to Toya's and Nevin's actual appearance.

1c. Values and beliefs. The drawing event illustrated not only Anika's inclination to engage with known adults but it also demonstrated her interest to write or to draw in my field journal. Over the course of my visits with her, Anika consistently expressed interest in my practice of writing in my field journal, and she often asked if she could write or draw in it herself. As the situation permitted, I allowed Anika to use my pencil and to write or draw in my journal. This activity also supported Anika's interest to interact with a known adult.

Before the drawing event, Anika chose to spend much of her free-play time by my side, at one point asking me to draw a dress in my journal. Anika augmented the drawing by adding a body, head, and limbs to what she identified as being a picture of herself. Once free play was over and the other children transitioned into other activities, Anika remained by my side and continued to focus her attention on me and on drawing in my journal.

1d. Social relations. My presence at the day care together with my practice of recording field notes in my observation journal appeared to become sources of

interest and engagement for Anika. Although I occasionally helped Ms. Franklin when she needed an extra hand, I typically sat in the lounge chair located near the center of the room that gave me a good vantage point of the day care space. I welcomed the children to approach me and to inquire about my journal. Anika's desire to interact with me in the drawing event may have stemmed not only from her inclination to engage with known adults but also from the novelty of my presence and her interest in writing or drawing in my journal.

While I primarily managed my journal and its written content, to the extent possible, I allowed the children, including Anika, to initiate and shape our interactions involving my journal. As such, this mathematical event evolved into a joint endeavor between Anika and me in which we aimed to represent Toya's and Nevin's actual appearance, as determined by Anika, in a drawing. Anika and I also engaged in a shared drawing experience using my journal in the incident preceding this when Anika asked me to draw a dress.

2a. Content: Identifying a difference between a toy xylophone and a pictorial representation of another xylophone. The following event occurred during free-play time as Anika played independently.

Xylophones

Anika paused to examine a drawing of a xylophone in a picture book she found and an actual toy xylophone instrument that rested on the floor nearby. After a few seconds, she announced, "It's not the same as this. This doesn't have a bar like this one."

(June 5, 2006 – Day Care Visit)

While the image of the xylophone in the picture book was not a direct representation of the toy xylophone, Anika seemed to recognize that the image of the xylophone was conceptually similar to the actual toy xylophone that she noticed on the floor nearby.

Although a number of differences distinguished the xylophone pictured in the children's book from the toy xylophone (e.g., colors, number of pitch bars, types of fasteners), Anika seemed to notice one particular physical quality, a support bar, that appeared on one xylophone but not the other. As such, Anika concluded that the two xylophones were "not the same."

2b. Purpose and setting. The xylophones event took place during free-play time at day care. As Anika played independently, she spontaneously noticed an image of a xylophone in a picture book and compared its appearance to a nearby toy xylophone. Anika not only commented to herself that the two xylophones were not the same but she also verbally identified to herself the particular difference she observed between the two.

2c. Values and beliefs. The values and beliefs instantiated within the practice of free play have been discussed in the dolls and teacup saucers events. While Anika demonstrated her inclination to interact with adults, albeit briefly, in those events, she did not engage with adults or her peers in the xylophone event.

2d. Social relations. The social relations that shaped the practice of free play have been discussed in the dolls and teacup saucers event. However, unlike her activity in those events, Anika did not announce her xylophone observations with anyone. Instead, Anika behaved similarly to her actions in the barrettes event. She seemed to think aloud to herself and appeared absorbed in the moment with her observations, showing no concern for affirmation from others. As such, Anika's behavior and utterances seemed intrinsically focused. After she identified to herself

the difference she noticed between the image of a xylophone and the toy xylophone, Anika resumed her independent play.

Content: Observing and Identifying a Similarity Between Two Unlike Objects

Anika observed and identified a similarity between two unlike objects that had little to no resemblance with respect to their physical, material, or functional qualities.

Content. The following mathematical event took place during a Wednesday afternoon home observation visit with Anika when Christina and Leroy were off from work. I brought a box of Hello Kitty-themed fruit snacks for Anika as a small hospitality gift for Christina.

Hello Kitty

Anika: I got Hello Kitty like you (she referred to my Hello Kitty pen that I was holding at the time along with my journal).

Grace: You do? Where?

Anika: My candy that you bought me.

Grace: Oh, it's supposed to be fruit snacks. It's supposed to be semi-healthy.

(June 14, 2006 – Home Visit with Anika)

Although the “candy” (i.e., fruit snacks) and pen appeared to have little to no physical resemblance, Anika identified a similarity between these two unlike objects. Anika recognized that both items displayed the same Hello Kitty iconic face. In this sense, Anika classified my pen and her “candy” as sharing the same attribute that she identified as “Hello Kitty.” Anika’s conclusion that she had Hello Kitty “like” me, seemed to reflect an understanding that two structurally-distinct objects can have a “like”-ness or similarity through a shared membership in a group or category even if those objects differ substantially with respect to their physical, material, or functional qualities.

Purpose and setting. While Christina, Leroy, Anika and I relaxed in the living room and chatted while watching television, Anika approached Christina requesting to have some, what sounded like, “fruity bears.” Christina recognized that Anika was referring to the fruit snacks that I brought for her. As Christina was in the kitchen getting the snack, Anika eagerly informed me that, with the “candy” that I gave to her, she now had “Hello Kitty” just like me. Anika seemed pleased with knowing that she had a Hello Kitty brand item as she noticed over time that I had several Hello Kitty items of my own.

Values and beliefs. According to Christina, she typically ran errands on Wednesday afternoons, but because of the rainy weather on this day, she opted to stay home and to relax instead. Watching television/DVDs seemed to be a significant leisure activity in Anika’s home life with Christina. During situations when only one television set was in use and managed by the adults in the household, Anika seemed content with entertaining herself by engaging in various activities around her home, including initiating brief exchanges with Christina or other adults present.

Anika’s interactions with her mother and me during the Hello Kitty event may have been prompted by a couple of Anika’s interests and dispositions beyond her inclined nature to engage with adults. First, Anika’s love for sweets appeared to motivate her request to have some “fruity bears.” Just prior to the Hello Kitty event, Anika asked to have some of her mother’s Altoids, a breath freshener that Anika considered and ate as if it were candy. Anika likewise regarded the Hello Kitty fruit snacks as “candy” and she eagerly requested to have these as well. Second, this event seemed to demonstrate Anika’s awareness of and tendency to approximate the

interests of known adults. Anika noticed that I owned a number of Hello Kitty items. As such, Anika seemed excited to have something “Hello Kitty” in common with me.

Social relations. While Christina, Leroy and I chatted while the television simultaneously played, Anika amused herself in various activities including climbing on the sofa, showing me some of her belongings, dancing along to a music video, and locating a DVD she wanted to watch. Not only did Anika seem comfortable approaching Christina at any point, but Christina readily responded to Anika’s requests (e.g., allowing Anika to have “just one” Altoid) or condition (e.g., checking on Anika after she fell from climbing on the couch) without hesitation or concern. It therefore appeared that Anika’s self-directed activities during this unstructured time of family relaxation were not only familiar but were also, for the most part, welcomed. Christina seemed to shift her attention with ease between watching television, engaging in a conversation, and tending to Anika. In an effort to maintain some degree of consistency or approximate an essence of everydayness in Anika’s home settings and activities, I tried to adopt a similar interaction style with Anika during this visit.

Content: Sorting Fruit Snacks into Same-Kind Groups

Anika demonstrated beginning understandings of classification when she employed meanings of difference and similarity to sort spontaneously.

Content. The following sorting event took place moments after the Hello Kitty event discussed above.

Sorting

Anika and I sat at the dining room table to enjoy our snacks. Anika had a pouch of Hello Kitty fruit snacks. She opened her pouch and dumped all her fruit snacks on top of the table. I noticed that a few snacks were clumped

together. Anika proceeded to separate her snacks into the following piles: a group of four red Hello Kitty heads, a group of three yellow flowers; and a single blue snack (I could not tell what it was supposed to be). She first ate the blue snack, then two yellow flowers. Anika then made a fruit snack sandwich or tower, squishing two red Hello Kitty heads, one on top of and the other below the remaining yellow flower. Before Anika ate all her fruit snacks, as casually as possible, I called Christina's attention to the piles that Anika created and inquired if she has ever noticed Anika "doing this" (i.e., sorting) before. Christina glanced over at the table and replied, "Oh, that's something new. Nope."

(June 14, 2006 – Home Visit with Anika)

Anika employed meanings of difference and similarity based on physical qualities when she sorted the fruit snacks according to their color/shape. As evidenced in the same-kind piles she created, Anika not only identified which snacks were the same, but she also identified the different types of fruit snacks, each physically defined by a distinct color and shape, that were contained in her pouch. Although Christina reported that she had never seen Anika sorting before, it is unclear whether Anika had sorted previously without her mother's knowledge.

Purpose and setting. This event took place during an unstructured family relaxation period at home. Anika sorted and ate her snacks at the dining room table while Christina painted her toenails and watched the local news nearby in the living room. Anika appeared to take delight in taking stock of her pouch contents, dumping all of the fruit snacks onto the table and spontaneously sorting them by their color/shape, as well as in eating the sorted snacks in a deliberate fashion, including consuming the triple-snack sandwich that she constructed.

Values and beliefs. In an effort to monitor Anika's consumption of sweets, Christina permitted Anika to have one pouch of the fruit snacks. Anika enjoyed eating sweets, especially candy, and she considered the fruit snacks as "candy."

Social relations. After Christina directed Anika to eat her snacks at the dining room table, Anika seemed independently engrossed and amused with her fruit snacks, managing, sorting, and eating these as she wished.

Content: Identifying Images that are the Same

In the mathematical event presented below, Ms. Franklin engaged Anika in a worksheet activity with which she prompted Anika to identify an image from a set of images that was the same as a target image.

Content.

Bears Worksheet

Ms. Franklin gave Anika a worksheet activity to complete. Each item consisted of a row of bears, beginning on the left with an image of a bear followed by a set of four distinct bear images. The orientation of the bears' arms and legs differentiated the images from one another. The images were simple, black-line drawings resulting in a coloring-book-like representation of bears. The task was to identify and color which bear among the set of four bears was the same as the initial or target bear. Ms. Franklin explained the task to Anika.

In her first try and without Ms. Franklin's assistance, Anika pointed to an incorrect bear. Ms. Franklin then intervened to help Anika, providing Anika specific cues to help her identify the correct bear that was the same as the target bear. For instance, Ms. Franklin prompted Anika to look at the orientation of the feet of the target bear (e.g., "this one has toe to toe") and then she pointed to each of the following bears in the row and asked Anika whether or not the given bear had "toe to toe." With Ms. Franklin's assistance, Anika eventually pointed to the correct bear that was the same as the target bear, however, she only colored the target bear. Ms. Franklin directed Anika's attention to the second row of bears, noting first that the target bear had "elbow to elbow." As with the initial example, Ms. Franklin pointed to each of the following bears, one by one, and asked Anika, "Does this one have elbow to elbow?" Ms. Franklin continued to systematically prompt Anika to compare each subsequent bear's limbs to the target bear's limbs, thus helping Anika to identify the correct bear that was the same as the target bear. Ms. Franklin guided Anika's through to the rest of the items in which Anika identified the correct bears by pointing to, rather than coloring, each.
(April 7, 2006 – Day Care Visit)

In Anika's initial attempt to respond to this worksheet task, she did not identify the correct bear that was the same or identical to the target bear. However, with Ms. Franklin's systematic prompting, Anika successfully identified the bears that were the same as the target bears presented for each item. Because I did not interview Anika regarding her initial response to this task, it was unclear to what extent Anika understood the task when Ms. Franklin initially explained it to her. It was also unclear to what extent, if any, Anika drew upon her prior understandings of difference and similarity in her process of completing this activity.

Responding to a worksheet task of identifying an image that was the same as a target image seemed unfamiliar to Anika. The formal nature of the task may have presented a cognitive obstacle for Anika as she may have been unsure how to respond in this new context.

Also, the type of entities and the appearance of the images to be examined in this task may have been interpreted by Anika in such a way that was not anticipated by the developers of the task or by Ms. Franklin. The same bear image appeared throughout each item with variations in the orientation of its limbs. Thus, Anika may have initially perceived all the bear images to be essentially the same, because she believed that the same bear appeared in each image except the positioning of its legs and arms differed. These distinctions among the bears may have been too subtle for Anika to recognize without Ms. Franklin's targeted scaffolding. Ms. Franklin's prompting focused Anika's attention to the specific orientation of each item's initial bear's limbs and guided Anika to systematically compare this feature of the target bear to each of the following bears. With Franklin's assistance, Anika was able to

determine which following bear's limbs looked the same and which bear's limbs looked different than the initial bear's limbs.

Purpose and setting. The bears worksheet event occurred at day care just before lunch time. On this day, Anika arrived earlier than her normal scheduled time and therefore had already spent a couple of hours at day care before the event took place. While the other children were occupied in other activities, Ms. Franklin opted to use this opportunity to work with Anika on the matching bears task. Anika sat at the gathering table as Ms. Franklin directly guided her through each item, calling Anika's attention to the specific features defining each bear and prompting Anika with questions to identify which bear was the same as the target bear.

Values and beliefs. Ms. Franklin considered her day care to be informal in the sense that she did not follow a prescribed academic curriculum or educational agenda and that her day-care setting and activities did not substantially reflect a formal school-like context. However, Ms. Franklin did occasionally engage the preschool-aged children in activities and tasks to develop their fine motor and cognitive skills that she believed could help prepare them for later schooling. By engaging Anika in the bears worksheet, Ms. Franklin exposed Anika to the formal practice of completing paper and pencil tasks, including the practice of writing her name on her paper, and also fostered Anika's capacity to identify difference and similarity between images in this context.

Social relations. Ms. Franklin managed the paper and pencil tasks she gave to the preschool-aged children by providing occasional opportunities for them to engage in this formal activity, selecting tasks for children, and guiding them as needed to

complete the activity. In the bears worksheet event, Ms. Franklin focused on Anika identifying the correct bear that was identical to each item's target bear. As such, when Anika identified an incorrect bear in her initial attempt at the task, Ms. Franklin responded by prompting Anika to observe the particular orientation of the target bear's limbs and to determine if the corresponding limbs of each remaining bear looked the same. By systematically guiding Anika through each item, Ms. Franklin significantly managed Anika's subsequent responses to the activity. Having only colored the initial bear on the worksheet, Anika appeared to show minimal interest in this task and continued to assume a passive role as Ms. Franklin guided her through each item.

Characterizing Anika's Everyday Mathematical Events

A subsequent examination of the collection of Anika's everyday mathematical events revealed that Anika's engagement in early classification processes tended to:

- Focus upon familiar entities with which Anika amused herself or took interest while she participated in daily routines at home or free play at day care;
- Be intrinsically motivated;
- Prompt Anika to initiate verbal interactions with me; and/or
- Demonstrate that Anika observed and noted differences and/or similarities between entities or their attributes for personal mathematical interest.

I discuss each of these characteristics of Anika's everyday mathematical events below.

*Focusing upon Familiar Entities In
Everyday Contexts*

This characteristic of Anika's mathematical events primarily reflects the objects with which and the contexts in which Anika tended to engage in early classification processes. At home, Anika primarily observed and identified differences and similarities between familiar items with which she entertained herself or found interesting while participating in routine activities. At day care, Anika tended to employ meanings of difference and similarity to pairs of objects that drew her attention during free play.

While Anika's daily routines at home of getting her hair done, taking a bath, dinnertime, or passively watching television with her family as well as her free play at day care were not themselves purposefully mathematical, Anika freely engaged in early classification processes as she amused herself with and/or noticed familiar items and their attributes during these everyday contexts. At home, Anika's daily routines and activities afforded her opportunities to observe and evaluate a range of meaningful objects at her leisure. At day care, Ms. Franklin provided Anika a variety of toys and materials with which Anika could play and observe during free play.

In the mathematical events examined, Anika's attention to entities that she found interesting during routine activities at home and free play at day care seemed to focus on the entities' physical traits, such as their figures, colors, or designs, as well as on material (i.e., taste) and categorical attributes. Anika noticed differences, for example, in the physical features of objects, as she discerned the decorative figures that distinguished an otherwise nearly identical pair of barrettes, noticed the unique

appearances of two baby dolls' outfits, and sorted a bag of fruit snacks by their color/figure. Anika also identified physical and categorical similarities between entities, respectively, when she noticed that two teacup saucers were identical and when she observed that both she and I had items branded with Hello Kitty.

As Anika participated in her daily routines at home and free play at day care, her attention naturally gravitated toward select items, and she seemed to take further interest in particular physical, material, or categorical qualities that defined these entities. While Anika appeared not to have yet developed a more descriptive vocabulary to identify verbally the particular attributes she noticed, she often referred to specific traits of items either by pointing to or touching them. Distinctions and/or commonalities in the physical appearance, material quality, and/or categorization of entities that Anika encountered in her everyday contexts provided her with concrete, familiar items and attributes with which to reference and to engage spontaneously in her understandings of the mathematical concepts of difference and similarity.

Intrinsically Motivated

Although Anika's mother, Leroy, and I were present when I observed Anika's daily routines at home and while Ms. Franklin and I were present during observed free-play time, we did not prompt Anika to take interest in any particular entity or to notice differences or similarities between entities during these everyday activities. Instead, Anika's engagement in early classification processes seemed intrinsically motivated. Anika, through her own accord, took interest in various objects, spontaneously noting differences and/or similarities between entities or their attributes. With the exception of the bears worksheet event, Anika initiated all the

previously discussed mathematical events.

Anika's motivation to recognize and to identify difference and/or similarity between entities appeared to stem in part either from a desire to seek experiences, pursue interactions, or acquire items of interest. As the discussion of the prior characteristic indicated, a majority of Anika's mathematical events involving early classification processes took place either as she entertained herself during daily routines at home or engaged in free play at day care. Within these contexts, Anika initiated some kind of play, leisure, or other pursuit that drew her attention toward particular objects, and then this activity eventually led her to notice spontaneously a difference and/or similarity between entities.

The barrettes, baby dolls, teacup saucers, and xylophones mathematical events all emerged from Anika's play, while the drawing event stemmed from Anika's occupation with my journal. In the slippers and Hello Kitty mathematical events, Anika's observations appeared to reflect her temperament to attend to adult interests and possessions by identifying, respectively, similar belongings between her mother and me and herself and me. The slippers event also seemed to serve, in part, to pass time as Anika soaked during her bath time. Anika's initial desire to have some "candy" (i.e., fruit snacks) to eat prior to the Hello Kitty episode may have also prompted this mathematical event. The subsequent fruit snacks sorting event not only appeared to arise from Anika's pleasure with having and eating sweets, but it also seemed to serve her playful interests as she appeared to enjoy her fruit snacks by taking stock of them, sorting the snacks by their color/figure, and by combining the snacks into towers before eating them.

Both Anika's quiet observations and verbal sharing of difference and similarity seemed to substantiate further the characterization of Anika's engagement in early classification processes as being intrinsically motivated. In the barrettes, xylophone, and sorting mathematical events, Anika quietly noted to herself a difference between these objects as she entertained herself with these. Anika's observations appeared to be a natural extension of her play and interest in the objects. In contrast to this self-directed response to her observations, in the baby dolls and teacup saucers mathematical events, Anika paused in her play to seek my attention and to inform me verbally of the difference and similarity, respectively, that she noticed within each pair of toys. Once Anika announced her observations to me, she quickly resumed her independent play. Anika's deliberate efforts to notify me of her observations, in these two events as well as her persistence to inform me of her observation in the juice event, suggested that Anika felt a sense of pride in her understandings of difference and similarity as she appeared to deem these significant and worthy to share with me.

Verbal Interactions

Anika oftentimes verbally shared her spontaneous observations of difference and similarity. These verbal interactions ranged from simple announcements to lengthier conversations involving her observations. While Anika was otherwise engaged in play or in acquiring something to drink or eat, in the baby dolls, teacup saucers, juice, and Hello Kitty mathematical events, she drew my attention to state briefly her observation of difference or similarity. In the drawing event, Anika first examined my sketch, then commented on a few differences she noticed between my

drawing and the children she observed in the day care, and finally proceeded to look on with approval as I incorporated these details into my drawing. Lastly, our conversation in the slippers event evolved from Anika's initial observation that her mother had slippers like the pair she noticed me wearing to a discourse encompassing Anika's attempts to describe her mother's slippers to me.

As indicated above, Anika primarily directed her unprompted verbal insights regarding her observations of difference and/or similarity, as either brief announcements or those that extended into longer verbal exchanges, to me. While Anika initiated these instances of stating what she noticed, with the exception of the juice event in which Anika appeared to address her observation of difference to both Christina and me, I did not observe Anika directing comments about difference or similarity to other adults or to her peers at day care.

Employing Content for Its Own Sake

Other than the drawing event, Anika appeared to observe and identify difference or similarity between the entities with which she engaged with no particular pragmatic purpose in mind. Anika seemed to employ early classification processes spontaneously for its own sake, with no prompting from others.

Anika's mathematical events often stemmed from her own interests and daily activities. For example, in the barrettes, xylophones, baby dolls, and teacup saucers events, Anika's observations of difference or similarity seemed to extend naturally from her play and curiosity. The sorting of her fruit snacks also reflected a form of leisure activity, allowing Anika to enjoy taking stock of her pouch of sweet treats. Anika's play therefore appeared to support her development of early classification

processes and her interest in noticing and comparing attributes within a set of entities.

Although I attempted to refrain from reacting positively to Anika when she verbally expressed her observations of similarity and difference to me, it could be that Anika's verbal interactions indicated a desire for approval or affirmation from an adult. In this sense, the barrettes, xylophones, and fruit snack events appeared significant particularly as Anika did not direct her observations of difference or similarity to anyone. Anika remained engrossed in her play and observations, and she appeared to regard the differences and similarities she noticed in these events as cognitively interesting. Anika thus seemed to engage in observing and identifying difference and similarity as a meaningful activity in itself that emerged naturally from her everyday experiences.

A Final Observation

The prior discussion of characteristics of Anika's everyday mathematical events involving early classification processes did not address Anika's experience with the bears worksheet. In this mathematical event, Anika did not seem to be familiar with, in general, the intent of paper-and-pencil tasks nor the bear images displayed on this particular worksheet. Anika did not appear intrinsically motivated to complete the worksheet, as she did not spontaneously engage in this activity. Furthermore, her initial responses were not successful. While verbal interaction occurred in this event, it differed from Anika's other mathematical events in which verbal interaction took place in that she did not initiate the interaction nor did she verbally express the difference and/or similarities she observed. Anika engaged in the bears worksheet passively, and her understandings of difference and similarity were

not drawn upon in her initial response to this task nor were they readily accessed in her initial reaction to Ms. Franklin's efforts to guide Anika through the activity.

Anika's experiences in and the nature of the bears worksheet mathematical event appeared in strong contrast to the qualities of familiarity of activities and objects, intrinsic motivation, verbal engagement, and mathematical curiosity that characterized the remaining collection of Anika's significant mathematical events. The limited nature of Anika's engagement and mathematical understandings of difference and similarity in response to the bears worksheet may in part reflect the unfamiliar symbolic and representational expectations or demands of the task.

Each of Anika's significant everyday mathematical events (excluding the bears worksheet event), in which Anika appeared to employ early classification processes meaningfully and with understanding, were characterized by at least the following two qualities: (a) Anika engaging with familiar objects of interest in her daily settings as a context that gave rise to her use of early classification processes and, and (b) Anika being intrinsically motivated to observe and to identify difference and similarity spontaneously in the course of her everyday activities. This pair of characteristics of Anika's everyday mathematical events thus seemed to provide conditions that promoted and nurtured Anika's capacity to develop, to make sense of, and to apply her mathematical understandings of difference and similarity. On the other hand, the absence of these characteristics, as discussed above with the bears worksheet event, did not seem to support Anika's facility to meaningfully and productively engage in early classification processes.

Chapter 6: Aaron

Impressions of Aaron and Capturing Glimpses of His Everyday Life

At Day Care

I first met Aaron at Ms. Franklin's day care in late March, about one week after he started attending. Toya and Anika were also present that day and, upon my arrival, all three children huddled around me and I greeted each one with a hug. Even though this was my first encounter with Aaron, he seemed to reciprocate my gesture with ease. Aaron regularly attended Ms. Franklin's day care every weekday from about 8:15 a.m. to 4:30 p.m. for a period of approximately 2 months. Aaron's fourth birthday occurred that following early July, making him the youngest participant in the study.

Aaron had a light complexion and, as did many of the boys at Ms. Franklin's day care, he wore his hair in braids. He often dressed in baggy cargo pants or jeans, a stylish shirt, and well-kept, name-brand sneakers. Because I had some difficulty understanding Aaron's speech, I often repeated or rephrased statements back to him to verify that I understood what he said.

In general, I found Aaron to be a polite little boy as he readily said, "Please," "Thank you," and "Excuse me" when appropriate. He complied willingly when adults made requests for him to share toys with his peers or to take turns with others. Aaron had lots of energy, and he enjoyed playing both on his own and with the younger boys at day care. Ms. Franklin frequently deemed his exuberance as inappropriate indoor behavior, resulting in timeout restrictions. Aaron rarely slept during naptime. His inability to lie still for an extended period of time inevitably led to disturbances

and subsequent timeouts. During a timeout, Aaron was required to sit quietly on the steps adjacent to the play area, with no interaction from others. When Aaron's mother came for him in the afternoons, she appeared disappointed whenever she saw her son relegated to the steps or when he received a negative "daily bear" report. When Aaron was not in a timeout, he seemed happy at day care.

I first noticed Aaron's ebullience during my second observation of him, when Ms. Franklin took the three 4-year-old children to the local library for story time. At the beginning of the session, Aaron, Toya, and Anika observed passively from the rear of the room while the other attending children joined in an opening song-and-dance activity as led by the librarian. However, once the librarian began to read aloud, Aaron displayed an enthusiastic interest in the stories as demonstrated in his verbal responses and physical behavior. Subsequently, he participated eagerly in the kinesthetic activities that the librarian facilitated between each book read. Out of all the stories read aloud, Aaron seemed to enjoy the book *Tall* (Alborough, 2005) in particular.

This illustrated book told the story of a monkey who sought to be taller than each of the animals it encountered. Previously, the librarian had instructed the children to sit quietly with their legs crossed as she read each book. As the story unfolded, Aaron would run excitedly to the front of the group while pretending to be the different animals that the monkey met. I decided to sit next to Aaron and to keep him within arms' reach for the remainder of the story. Aaron then told me what was happening in the story and he mimicked the sounds and actions that are characteristic of each animal he observed. Rather than using the term "taller" to describe the monkey's increasing height, Aaron used the word "big."
(March 23, 2006 – Day Care Visit, Field Trip to the Library)

As evidenced in this episode, Aaron seemed to take pleasure in listening to and engaging with the story, and he demonstrated beginning understandings of

magnitude. Aaron's deep interest in this particular book appeared to motivate his spontaneous, enthusiastic behavior. Although Aaron's active behavior could be deemed as inappropriate for story time at the library, it did demonstrate his full comprehension of and engagement with the story.

At Home

Aaron's family had recently moved to the Washington, D.C. metropolitan area, where Aaron's father had spent some of his formative years. His parents, Keisha and Aaron Sr., were married approximately 6 months prior to this period of data collection. Both Aaron Sr. and Keisha acknowledged that one of their reasons for agreeing to participate in this study was because I was an ethnic-minority student. As explained by Aaron Sr., "I feel if it wasn't for a student, and an ethnic student, I don't think I'd be involved [in the study]. The only thing that really makes me want to get involved is you getting through school" (April 8, 2006 – Home Visit with Aaron).

Aaron lived with his parents in a two-bedroom apartment located within a short walking distance from Ms. Franklin's home/day care. The living and dining areas formed one large, open rectangular space and a separate, small kitchen was located off the dining room. The Hamilton's welcomed me into their home and allowed me to observe in all spaces of their apartment except the master bedroom. They had a tidy home furnished modestly with older-modeled furniture and electronics, including some items that had been in their families for some time. The bedrooms and living room each included a television set. I observed a collection of DVD movies in the living room and Aaron's room as well. The television in the

living room remained in use the majority of the time during my visits, mostly playing DVDs.

Managing and shaping Aaron's behavior and character. In contrast to his active behavior at day care, Aaron seemed more subdued when I observed him at home. Aaron's behavior at home appeared to be significantly shaped by his father, who proudly claimed his position as "the man" in the household. Aaron appeared to regard Aaron Sr. as more authoritative than his mother Keisha. As such, Aaron acquiesced more readily to his father's direction and discipline. Keisha characterized Aaron Sr.'s nature and role in their family when she commented,

... he has a dominant, a very dominant presence about him... He's really determined to be a father because he didn't have a father figure in his life. He's really determined to do that. And that's why I guess he just wants to be the dominant one... he wants to be both parents.
(April 9, 2006 – Home Visit with Aaron)

The dynamics of Aaron's family life, in particular, the role his father played in shaping Aaron's activities and character, provides some context for interpreting Aaron's experiences at home and, to some an extent, his behavior at day care. Aaron Sr. spoke extensively about the importance of being a strong, aggressive male and his expectation that his son develop these qualities as well,

I do want him [Aaron] to play sports. It develops a certain type of male, I think... They grow up to be the kind of man I think women feel safe with, the kind of men that their mom's are proud of them. They walk into a room and there's a certain – it comes from certain things, from being around a group of guys, playing sports, being hurt, being – all of this for him to develop into a better Hamilton than me. I come from a line of Hamiltons, though. My uncles are all strong, aggressive males. He's [Aaron] naturally a strong, aggressive male, and every male ain't that way.
(April 8, 2006 – Home Visit with Aaron)

The décor of Aaron's room reflected his father's value in molding Aaron to become a strong, aggressive male. A poster of Al Pacino's gangster character from the movie *Scarface* (Bregman & De Palma, 1983) with the phrase "The world is yours," hung above Aaron's bed. Aaron Sr. reported that Aaron, "knows what *Scarface* is and all that stuff" (April 8, 2006 – Home Visit with Aaron). A theme of successful and strong Black men seemed to characterize the majority of the images displayed. Two posters of Bob Marley hung next to the *Scarface* poster and pictures of famous African-American men including rappers, hip-hop artists, and athletes, along with photos of male relatives, hung on the wall opposite of Aaron's bed near his television set. In addition to these images and cartoon graphics of the Incredible Hulk, a large framed collage of photos including Aaron Sr. and significant male family members, as well as framed baby photos of Aaron with and without his father, adorned the walls and dresser in Aaron's room. I observed just one image of a woman in Aaron's room, a caricature drawing of Keisha that hung next to a growth chart.

Aaron Sr. set clear boundaries for Aaron regarding non-acceptable behaviors. Aaron's behavior at day care, as reported on his daily bear reports, had consequences at home. According to Aaron Sr., Aaron's experience at home "adjusts off of really what that card says" (April 8, 2006 – Home Visit with Aaron). On my first weekend visit, Aaron Sr. reported that Aaron did not have a toy in his room because he repeatedly received negative daily bear reports from day care. Aaron's parents asked him to show me his large bin of toys that they secured in the living room closet until he earned it back.

Aaron seemed to understand that his behavior at day care had potential not only for consequences at day care, but also for consequences at home. In the following episode that occurred two days prior to my first visit to his home, Aaron conveyed the hope that his good behavior at day care would mean positive outcomes at home,

Ms. Franklin enthusiastically shared the good news with Keisha that Aaron had earned a sticker for his good behavior and that he stayed out of timeout. Upon hearing this, Aaron began to mention some of the privileges that had been taken away from him at home that he hoped he would soon get back. With animated gestures and words, Aaron described how his toys were put out of his room and that his TV set had been turned around. It seemed as if this was Aaron's way of asking his mother if he was going to get these privileges back given that he had been good today.
(April 6, 2006 – Day Care Visit)

Because Aaron frequently received negative daily bear reports, it appeared that Aaron spent a significant period of time at home, particularly on weekends, in punishment. This typically entailed being assigned to the bed in his room where he was to stay, whether he napped or not, for substantial periods of time with little to no access to his toys or his television/DVDs. Despite this, Aaron still struggled with his behavior at day care. His continued infractions during naptime may have occurred in part because Aaron may not have interpreted naptime as a time to rest, but rather as a form of punishment. This in turn may have led him to stay awake and unwittingly to disturb those around him.

Aaron's love for books. When told how much Aaron seemed to enjoy the day care trip to the library, Keisha responded that she was not surprised to hear this news, as she regularly took Aaron to the library and she frequently read to him at home. Thus Aaron had consistent, meaningful experiences not only with visiting libraries,

but also with books and reading at home. During my initial home visit with Aaron, his parents gave me a tour of his room, which included showing me Aaron's collection of books, and they described his typical activities in this space.

Aaron Sr.: He sits here and watches a lot of movies. [I observed a mini-lounge chair in Aaron's room as well as a television, a set of DVDs, and a DVD player that were all within easy reach for Aaron.]

Keisha: ... These are the stor-, the books that I read to him. They're up here. [Keisha pointed to a shelf in Aaron's closet.]

Aaron: Mommy, read me my books.

Keisha: Hold on, pumpkin.

Aaron: Mommy, read some books please?

Aaron Sr.: Aaron, calm down.

Keisha: And then he has all his other toys and, like, so,

Aaron: Mommy get some books to read.

Keisha: Hold on, pumpkin.

As Keisha continued to tour me around Aaron's room and to point out to me the sports-related balls that Aaron had in his collection of toys, Aaron continued to insist, "Mommy, I wanna read some books."

(April 8, 2006 – Home Visit with Aaron)

Despite the other potentially enjoyable toys or activities at his disposal, Aaron instead focused squarely on the books, demonstrating his preferred interest in reading as he pleaded with his mother to read to him.

During my final home visit with Aaron, Keisha reported that she had gone to the library earlier without Aaron to acquire some new books for him to read. She also informed me that she had been reading to Aaron at bedtime since "forever." On this visit, Keisha invited Aaron to choose from the pile of library books and to read one story to his parents and me. Aaron chose a book about transportation with a fire truck on the cover. He engaged thoroughly in this activity for over 30 minutes, as he carefully examined the cover of the book and each page, telling us what he observed and asking lots of questions. Occasionally, his parents reminded him that he was the one reading the story so he had to tell us what he thought was happening.

As Aaron continued to engage with the book although it appeared as though the pictures seemed to become increasingly abstract and more difficult to interpret, Aaron Sr. asked Aaron at three different points in time if he wanted to play a video game instead. Each time, Aaron declined and insisted that he wanted to continue to read to us. Eventually, Aaron Sr. convinced Aaron to stop reading and to play his video game in the living room. It was not entirely clear if Aaron resisted the invitation to play his videogame because he had difficulty playing it or because he found reading to be a more enjoyable activity. In either case, Aaron demonstrated great focus and interest in reading the book to us. Later that night, Aaron and his mother spent over 30 minutes reading books at bedtime, including re-reading the book he read to us earlier that day.

Weekdays. I did not have an opportunity to observe Aaron at home during a weekday. This discussion draws mostly on reports from Aaron's parents. While Aaron's parents described Aaron's typical weekday and weekend activities at home, Aaron Sr. noted that "... things aren't always that structured ... We never do the same things whenever ... I don't think no one does the same thing everyday ... every weekend, everyday after work" (April 8, 2006 – Home Visit with Aaron).

Generally in the mornings, Aaron woke up and ate his breakfast. Afterwards, Keisha groomed Aaron and then he watched cartoons in his room while his mother ironed his clothes and then dressed him. Aaron hugged and kissed his father before Aaron Sr. left the home for work, and then Keisha took Aaron to day care on her way to work.

In the afternoons, Keisha picked up Aaron from day care after work. According to Keisha, once home, Aaron typically asked for something to drink. The television set in the living room was normally in use while Aaron either played with his toys in his room or helped his mother “clean” while Keisha prepared dinner. Keisha vacuumed the apartment everyday. Aaron liked to help her clean by vacuuming after his mother and following her pattern of going from room to room. According to Aaron Sr., Aaron’s typical weekday afternoon/evening activities depended on what kind of day Aaron had at day care,

Aaron Sr.: ... I can tell from looking at Aaron, either he’s happy to see me or he’s scared to death. If he’s scared to death, that means he got a bad day. He goes to his room and lays down... He comes out around dinnertime and eats. We talk about what happened, then he goes back to his room. On a good day... he’s happy. Keisha’s smiling, and that’s a whole different mode... There’s always usually a,

Keisha: He gets a treat.

Aaron Sr.: Or candy. I go to the store and get a pack of M&M’s. Because you was good, this happened. We’re going to actually take a trip to [indecipherable word] or McDonalds. You know, something happens on a good day. We come back in, he eats and then he’s kind of in Cloud 9 mode.

(April 8, 2006- Home Visit with Aaron)

After dinner on a “good day” there was about an hour of “random ... not really structured” time when Aaron either played independently or colored in his room, played a videogame or shot his mini-basketball with his father, or watched television with his parents in their room. However, according to Aaron Sr., Aaron had to be “extra calm” to stay in their room, otherwise he was sent back to his room. Aaron also liked to “read” books during this time,

Keisha: Sometimes he picks up books and, you know, and tries to read it by himself.

Aaron Sr.: Right, he'll be saying words or, like, tell a story, but it don't be what you read. That's why we'll say, "You want us to read you a story." He'd rather make up a story. It's the process of him turning pages. [Aaron Sr. acted as if to read a story as Aaron would.] "And then he said," and then turn the page.

Grace: And that's reading for him.

Aaron Sr.: Right.

Keisha: Yeah, I take him to the library. I read him the story first, and then I let him take the book and then he tells me the story. What I read, he'll say, what I um, tell me back what I read...

(April 8, 2006 – Home Visit with Aaron)

Although Aaron's parents did not specifically mention bath time as part of Aaron's typical weeknight evening activities, on my final weekend visit I did observe Aaron's bath time and post-bath time routine. On that day, Aaron took his bath before eating dinner with his family. As Keisha bathed Aaron, she verbally engaged and played with him as he played with his toy fish, alligator and lizard. Aaron also had his own scrubby or rag that he used to wash himself once Keisha finished. Aaron seemed to enjoy his bath and repeatedly stated, "I wanna stay in the bathroom and play with my toys" (June 4, 2006 – Home Visit with Aaron). Keisha informed me that she normally let him stay in the tub for a little while longer so he could play independently. Keisha also managed Aaron's post-bath ritual including moisturizing his skin, dressing him, and dabbing his sleepwear with his father's cologne.

Keisha reported that Aaron's bedtime routine started around 8 p.m. with Keisha typically reading about five books to and with Aaron. Afterwards, she turned on Aaron's television set and DVD player to either a cartoon or a movie for him to view alone while tucked in bed. Aaron eventually fell asleep around 10 p.m.

Weekends. Keisha typically went to work on Saturdays. Aaron Sr. reported that he slept or relaxed in his room until about noon. During this time, Aaron

entertained himself in his bedroom, watching cartoons from his own stack of DVDs. Keisha explained that Aaron knew how to switch DVDs once he was finished watching one and ready to view another.

Aaron Sr. stated that Aaron typically ate lunch alone at the dining room table, facing away from the living room, with his father overseeing his eating. Aaron Sr., explained, "... I can control his eating; get it over with. He can't play at the table" (April 8, 2006 - Home Visit with Aaron). In the evening when Keisha was home from work, they ate together as a family. When I observed Aaron eating a meal at home, either alone or with his parents, he always blessed the food upon his father's prompting, praying, "God is great. God is good. And we thank Him for our food. Amen."

Aaron Sr., who took pride in his Mercedes Benz, would typically take Aaron outside with him on Saturdays while he washed his car. According to Keisha,

He'd [Aaron Sr.] go outside and clean his car, and Aaron would be outside running around with little kids and what not or go back and forth to his father's car dancing to the music and singing the songs...He always likes to try to help his father wash his car.... He loves his daddy's car. (April 8, 2006 – Home Visit with Aaron)

Aaron Sr. also noted that Aaron could distinguish his father's car from other Mercedes Benz models.

Keisha referred to Sundays as "family day." Aaron Sr. stated, "Keisha tries to get me to go to a restaurant or go out ... or something. You know, that type of thing. ... If I can go see my mother or something, usually, it's a Sunday" (April 8, 2006 – Home Visit with Aaron). Aaron Sr.'s mother lived about a 30-minute drive away from the Hamilton's apartment. Aaron Sr. and Keisha also mentioned that

occasionally on Sundays their family would go to Chuck E. Cheese's or to the local flea market where they could find bargains on household goods and fashion items. In general, however, Keisha stated that she did not like to go out and that she preferred to stay at home instead and watch movies, an activity that she emphasized their family did "a lot."

In addition to spending time together as a family on Sundays, Keisha typically groomed Aaron's hair on Sunday afternoons. On every other Sunday, Keisha washed Aaron's hair and then put it in a single, plaited ponytail. Later in the week, she styled his hair in multiple, usually six to ten, braids. On alternating Sundays, Aaron's braids were undone and his hair was re-braided.

I observed on a Sunday when Aaron Sr. undid Aaron's braided hair and Keisha re-braided it. To encourage Aaron to endure the discomfort of having his hair braided without crying, Aaron Sr. informed Aaron that if he was "good," did not cry or fidget much, then he would get to go to the park and play. As Keisha braided Aaron's hair and he began to whimper, Keisha repeatedly tried to console Aaron by reminding him of his father's promise of taking him to the park and by telling Aaron that she would give him some M&M's after she was done. Keisha also played a DVD of a children's animated movie to help distract Aaron's attention away from any pain he may have felt while his hair was being braided. On this visit, it took Keisha approximately 1 hour and 20 minutes to braid Aaron's hair. Aaron Sr. finished the process by using clippers to shape his son's hairline. When I asked Keisha if she considered this mommy-son bonding time, she responded that "in a way" it was.

Aaron the Observer

When Aaron “read” a book, he described the pictures he saw to help him tell the story, an activity encouraged by his parents. When Aaron read with his mother, he frequently asked “why” questions, or if he did not recognize an object in an illustration, he inquired, “What’s that?” I also observed moments when Aaron noticed an object, either in a book or in his everyday settings, which looked similar to another object he had seen before, he described or named it as the familiar object. Once during Aaron’s bath, he questioned why his fingers were shriveled up and Keisha responded that they looked like raisins because he had been in the water for a while. Afterwards during dinnertime, Aaron interrupted a conversation among the adults and held up his hands exclaiming, “Look! The raisins are away!”

In addition to Aaron’s tendency to observe and describe those images and objects as encountered in his everyday settings, Aaron also spontaneously determined and expressed cause-and-effect-type of relationships. For instance, at day care, Aaron took interest in my mechanical pencil and after examining it for a short while, he noted to me that when you push down the top of the pencil, the lead gets bigger. Aaron often used if-then statements, many of which had to do with avoiding injury at day care. For example, while playing on the slide, he commented that if he went down the slide head first, then he would “bust” his head.

Aaron’s parents’ also reported on his observant nature. For example, Aaron Sr. noted that Aaron could distinguish his father’s car from other Mercedes Benz models. Likewise, Keisha informed me that Aaron vacuumed the apartment the same

way she did, suggesting that Aaron noticed and followed the order of rooms by which Keisha vacuumed the apartment.

Keisha, Aaron's Mother

Keisha was light-skinned and had long, dark curly hair that she typically wore in a slicked-backed ponytail. She often dressed in athletic apparel. Whenever Keisha arrived at the day care to drop off or to pick up Aaron, she did not stay long to visit or to chat with Ms. Franklin or myself. She seemed polite, yet a bit shy. However, when I visited Aaron and his family at home, Keisha seemed more open and at ease, and she made me feel very comfortable in her home. At the time of the study, Keisha was 21-years-old and expecting her second child.

Keisha's family life. Keisha's family came to the U.S. from Jamaica as immigrants. Her father is Chinese-Jamaican, and her mother is Black-Jamaican. Keisha spent her childhood years in an area of the Bronx, New York, heavily populated with Jamaicans where she lived near and amongst numerous relatives. When she was approximately 11 years of age, her family moved to Georgia, and she lived there until moving to the D.C. metropolitan area with Aaron Sr. and Aaron.

Keisha described her family life growing up:

Well, me growing up is totally different from Aaron Sr. My household is more sheltered. We didn't really - weren't exposed to a lot of things. It was more family-oriented. It was just family, family, family. Basically, we had to go to school, we had to get good grades, we have to go to college.

(April 8, 2006 – Home Visit with Aaron)

By sheltered, Keisha meant that a majority of her social interactions involved her family and relatives and that she had minimal social experiences beyond this context. Throughout her conversations with me, Keisha gave numerous examples of how the

importance of family was impressed upon her since she was young. Even as an adult with a family of her own, Keisha continued to bond with her relatives, often with Aaron by her side, by attending family reunions and regularly returning to Georgia to visit her family as permitted by budget and time.

Keisha talked extensively about her parents whom she stated were very different from one another. Whereas Keisha's father encouraged her to "go with what your heart feels like doing," Keisha's mother encouraged her to seek a more financially secure career rather than pursuing her interest in interior design. Keisha struggled with her parents' two different points of views,

I'm actually more -- I am good at math, but I'm more artistic. I know how to draw and stuff like that, so it's kind of hard growing up in that kind of family cause you have your mother pulling you one way and you have your father pulling you another way.

(April 8, 2006 – Home Visit with Aaron)

Aaron's interest in books and reading appeared to stem from Keisha's mother's passion for reading. According to Keisha, "My mother's a bookworm. She doesn't watch TV at all. She sits in her room and reads, reads, reads" (April 8, 2006 – Home Visit with Aaron). As a child, Keisha was expected to read books instead of watching television. Keisha fondly reported, "My mom read to me all the time, and she still reads to me today" (June 4, 2006 – Home Visit with Aaron). Keisha not only read to Aaron since "forever" but she also read to her unborn child, noting, "They say that it's good to read books [to your unborn baby]" (June 4, 2006 – Home Visit with Aaron).

Keisha's school experiences. Keisha's parents emphasized the importance of education and expected their children to make school a priority. As soon as she came

home from school, her parents would say, “Do you have homework? You need to do your homework” (April 8, 2006 – Home Visit with Aaron). Keisha explained that the importance of school as stressed in her family reflected broader Jamaican cultural values regarding school success. She perceived that schooling was not valued as much in American culture.

When reflecting upon her school mathematics experiences, Keisha focused mainly on those that occurred while she was in high school. She reported that she “liked” mathematics and that it was her third best subject, behind English and art, with art being her favorite subject. Keisha stated that she took “a lot” of mathematics courses in high school including a money management course, Algebra I, Algebra II, geometry, and trigonometry. She attributed her success or difficulty in these classes primarily to the teachers who taught her, to the racial bias that she perceived the White teachers had toward the majority White student population, and to the exclusion she felt from her White peers when tasked to work in groups. Keisha’s positive high school mathematics experiences seemed to result from teaching approaches that resonated with Keisha’s own sense of creativity and classroom atmospheres that encouraged and supported learning amongst all racial groups.

After high school Keisha was admitted to a private art school where she intended to study interior design. Although Keisha received a scholarship from the state, her mother, in addition to expressing concern about the lack of job opportunities in interior design, discouraged Keisha from attending this school because the tuition was too expensive. Keisha thus enrolled in a local, public two-year college and pursued accounting because she had an uncle who was an accountant, and he was

“making a lot of money” in his career. Keisha viewed this as a practical choice given her financial situation and her mother’s advice. However, Keisha completed only about one year’s worth of coursework, as she met Aaron Sr., became pregnant, moved in with Aaron Sr., and began to work. Keisha explained that, after she had her second baby and established in-state residency, she hoped to be able to transfer her coursework and continue her studies in accounting in an evening degree program.

Keisha’s job. At the time of the study, Keisha worked at a dollar store.

Although Keisha once served as an assistant manager of the store, she was demoted to cashier after she became pregnant.

Aaron Sr., Aaron’s Father

Aaron Sr. was dark-skinned, muscular, tattooed, and shaved his head bald. At the time of this study, Aaron was 27-years-old and expecting his third child (his eldest child was a daughter from a previous relationship). Aaron Sr. looked forward to having more children and hoped that Keisha was having a boy. On my visits, although Aaron Sr. assumed a dominant posture, he also exhibited a sense of humor and light-heartedness, often joking and laughing with me amidst his serious responses to my inquiries and his unyielding management of Aaron’s behavior.

Aaron’s Sr.’s early life. Aaron Sr. reported that he was raised primarily by his mother and, to a lesser extent, by his grandmother who lived in Rhode Island. He moved from one location to another, spending part of his formative years living with his mother, who managed a home day care in a suburb near the D.C. metropolitan region. His birth father did not appear to be present.

When reflecting upon his experiences in school, Aaron Sr. stated, “I was a terror, like naturally bad behavior.” He further characterized himself as one of the “tough” or “funny” guys in school who was often sent to the office either for fighting or for cracking jokes in class. Aaron Sr. described himself as an average student who “never” did his homework but still managed to score “high” on tests.

By age 17, Aaron Sr. had passed the American Council on Education’s Test of General Educational Development, acquired a barber’s license, and moved into his own apartment in the city. Aaron Sr. clarified that his choice to move out on his own, “wasn’t because my people didn’t have ... I chose not to want to live out there [with his mother in the suburbs] ... I just wanted to be grown. If I could go back, I’d still be at home, but ... I did my own thing” (April 8, 2006 – Home Visit with Aaron). Aaron Sr.’s motivation to move out on his own at age 17 also reflected his “household’s” expectation of him to leave home after high school.

Aaron Sr.’s school mathematical experiences. When I inquired specifically about his experiences in school mathematics, Aaron Sr. briefly responded, “Math came easy to me. ... Like, I wouldn’t say I liked math ... it just clicked naturally to me. I was able to BS, do nothing ... and still get by” (April 8, 2006 – Home Visit with Aaron). Rather than elaborating further on his experiences in school mathematics, Aaron Sr. subsequently discussed social aspects of his life in school and described his aspirations for Aaron to attend college and become a professional athlete.

Aaron’s job. According to Keisha, their family moved to the area because of a job opportunity for Aaron Sr. At the time of the study, Aaron Sr. worked for a

construction company as a “supervisor, slash, foreman.” When I asked Aaron Sr. to clarify what that meant, he responded,

The carpenters, the plumbers and everybody, they – I’m like their boss. I tell them what time they come in, what time they leave. When things have to get done, I’m the one that says, “Now, go do this.” I don’t necessarily do it; I just make sure they get it done.

(April 8, 2006 – Home Visit with Aaron)

Aaron’s Parents’ Perspectives on Mathematics

In their Everyday Lives

At Work

Aaron Sr.’s and Keisha’s perceptions of the mathematics they typically used in their respective workplaces primarily involved number and numeric operations.

Aaron Sr. briefly commented, “Well, I’m a carpenter, so I use it daily. ... I keep time sheets. I’m always clocking time. There’s material lists. I think it’s really simple to see how numbers fall into carpentry” (April 8, 2006 – Home Visit with Aaron). Aaron Sr. perceived indicating time and determining elapsed time as involving “numbers,” but it was unclear if he viewed these activities as reflecting measurement concepts in particular. He did not clarify the mathematics he used with respect to “material lists.”

Aaron Sr. referred to number and numeric operations specifically when he described a “game” that he used to play at his job, in which he competed against his boss and other coworkers to see who could add numbers faster,

At my job, my boss and them will take the calculator... I’m used to counting, you feel what I’m saying? They take the numbers and they use their calculator, and I can add them up, but I add numbers funny. Like, I don’t add it like 25 and 37. I take the 2 off the seven and make it 35 in my mind. Take that 5 off that, add it to 25. That makes it 30 and 30. Then, I add the 2 back.

My mind will do that real fast. That's different than they do with the calculator. I can usually beat them with numbers... but, math came easy to me.

(April 8, 2006 – Home Visit with Aaron)

When adding numbers, Aaron Sr. employed his own invented strategy that involved decomposing and recomposing the addends to find the sum. Because Aaron Sr.'s approach did not reflect the traditional algorithm for adding multi-digit addends, he described his way of adding numbers as “funny.” Although Aaron Sr. perceived that his “counting” strategy was unconventional or “different,” he found it more efficient than using the traditional algorithm or a calculator to find the sum of two multi-digit addends.

Keisha believed that she did not use much mathematics as a cashier at the dollar store. She stated, “Everything's a dollar so there's not much [mathematics]. As cashier, I count the money and that's about it. I count my register down, and I count the register before I start ringing customers, and that's about it” (April 8, 2006 – Home Visit with Aaron). The mathematics that Keisha appeared to observe in her job involved enumeration, both enumerating money as well as enumerating items in one-to-one correspondence with each product's cost of one dollar to determine the pre-tax amount of each sale.

At Home

Aaron Sr. and Keisha described activities in which they perceived they used or experienced mathematics at home. Most of their reported activities focused on Aaron and involved numeric purposes for number.

Keisha and Aaron Sr. claimed that they observed Aaron “counting” since he was about 2 years of age as he began to participate with his father in working out and play boxing,

Keisha: Like, sometimes Aaron Sr. would – he works out a lot; he does push-ups and what-not, and Aaron will like to do push-ups with him. So he’d go right beside his father and he’d be like, “One, two,” and count for every push-up.

Aaron Sr.: And we’ve been doing that forever. And then we play boxing... When the guy falls to the ground, the ref’s going, “One, two,”... And when we be playing boxing, when I knock him out, he’s on the ground counting, [Keisha joined in, “He starts counting, “one, two, three.”] “One, two.” He’s doing what the ref does, not really realizing he’s counting... Or even when we’re doing punches and he hits my hand [Aaron Sr. demonstrated the action of throwing punches.] “One, two – one, two, three.” I think math, which you know this, math is actually involved in everything...

(April 8, 2006 – Home Visit with Aaron)

According to his parents’ report, Aaron seemed to observe and to mimic the practice of counting aloud in each of these activities: counting repetitions while exercising, counting elapsed time once a boxer falls on the ground, and counting in rhythmic fashion each punch a boxer throws in a combination. Counting while doing push-ups and counting while throwing punches, in particular, not only involved Aaron in expressing numbers orally in a count sequence, but these activities may have also exposed Aaron to concepts of enumeration as each number was expressed in one-to-one correspondence with an action.

Over the course of my first weekend visit with the Hamilton family, Keisha recognized activities that she and/or Aaron Sr. initiated in which they engaged Aaron in mathematics. Each activity involved enumeration concepts. Keisha noticed, “... Today we were playing basketball in his room. ...we were trying to teach him how to

shoot and, you know, counting how many shots each of us did” (April 9, 2006 – Home Visit with Aaron). Rather than assigning each completed shot with a point value greater than one, Keisha and Aaron Sr. modeled the activity of enumerating the set of successful shots completed per person when playing basketball in this context. Keisha also realized that when she braided Aaron’s hair, “I was just thinking I usually always do this with his hair, tell him how many [braids] he has” (April 9, 2006 – Home Visit with Aaron). Keisha demonstrated enumeration concepts for Aaron by reportedly informing him of the numerosity of braids she had completed. When I observed Keisha braiding Aaron’s hair, I also noticed that she frequently employed small number words to inform Aaron of the amount of braids she had left to do in an effort to ease Aaron through his discomfort by suggesting that she was almost done braiding his hair.

Meal times provided a meaningful context in which Aaron’s parents exposed him to the concept of “half.” Aaron Sr. reported,

...we break the sandwich in half..., “You [Aaron] can get a sip [of a drink] after eating half of his food.” So he knows after he can eat half his sandwich and half of his fries, he can get away with asking me for a sip of his drink... I give him a sip of a drink for eating half. That’s why his sandwich is always cut in half.

(April 8, 2006 – Home Visit with Aaron)

Aaron’s parents divided Aaron’s sandwiches into two smaller, equal-sized portions and described each portion as a “half” as a means to help manage Aaron’s eating and drinking habits during meals.

A Formal Perspective of Mathematics

When reflecting upon the everyday activities or practices in which they engaged in mathematics at home, apart from describing those number-focused

activities involving Aaron, Aaron's parents mainly discussed their use of mathematics when paying or calculating their monthly bills and budgeting their household expenses with respect to their weekly incomes. Thus, number (e.g., rote counting, identifying numerals, and enumerating physical quantities/actions) and numeric operations (e.g., addition, subtraction, multiplication, and division of whole and decimal numbers) appeared significant in Keisha's and Aaron Sr.'s perceptions regarding the mathematics they and/or Aaron used or experienced in their everyday activities at work and at home.

An emphasis on number and operations was further evidenced in Aaron Sr.'s comments below.

I think it all falls back to that same math I learned in school. But in ninth grade...I didn't do a lot of that math... It think it goes more back to the simple: the counting, multiplication, division, fractions. I think that right there is what really falls into everyday life... Anything you look at – a pizza, you'll be like, it's four people, you got division with your eyes and you're already counting and see your part of the pizza by the way it's cut. You feel what I'm saying? Or with paying rent, I'm always like, "Wow, okay, I make how much a week? Four weeks in a month. Okay, subtract how much rent? That leaves me this much. I gotta pay this. So that leaves me that so when I go to the bar I can spend \$40.00." So, even for the simplest things, you break it all [emphasis on "all"] the way down.
(April 8, 2006 – Home Visit with Aaron)

Aaron Sr. asserted that "the simple" mathematics that he learned prior to ninth grade involving concepts of number and numeric operations constituted the mathematics that he employed in his daily activities.

Earlier, when Aaron Sr. discussed the mathematics he used in his job as "a carpenter," Aaron Sr. did not mention any activity that involved measurement or geometry. It was unclear whether Aaron Sr. considered, for example, the activity of measuring length as mathematical. Similarly, Keisha nor Aaron Sr. perceived the

growth chart they displayed, and presumably used, in Aaron's room as being mathematical.

Aaron Sr.'s and Keisha's reflections and comments appeared to indicate that they held a formal perspective of mathematics. Through this perspective, Aaron's parents appeared to view number and operations as primarily composing the domain of mathematics. Both Keisha and Aaron Sr. expressed a sense of confidence in their capacities for this domain of formal mathematics.

Home Visits with Aaron Ended

During my first two home observation visits with Aaron and his family, I spent a significant portion of time in formal conversation with Keisha and Aaron Sr. discussing questions relevant to my study. By my third home visit, Keisha and Aaron Sr. appeared to be more comfortable with my presence and I had more occasions to observe Aaron engaging in his normal routines and initiating his own activities. At the end of this visit, I made plans with Keisha to observe Aaron the following weekend during a family outing to Chuck E. Cheese's. However, I was unable to complete this observation or any further observations as I lost contact with Aaron's family after my third visit with them. By this time, Aaron had also stopped attending Ms. Franklin's day care.

Interpreting a Collage of Aaron's Everyday Mathematical Events

In my analysis of the data of Aaron's everyday mathematical events, two content themes seemed to emerge as most salient: numeric meanings of number and geometric thinking and spatial reasoning.

Aaron engaged in number for numeric purposes, spontaneously and when prompted, at day care and at home. These events demonstrated Aaron's developing understandings regarding expressing number words orally in a count sequence, enumerating a quantity of two or more objects or images, employing the word "two" as a descriptive narrative device to indicate the numerosity of imagined subjects in his activities of story telling, and distinguishing number words and numerals from letter names and symbols.

I observed a small set of mathematical events at Ms. Franklin's day care in which Aaron engaged in early geometric thinking and spatial reasoning. In addition to demonstrating Aaron's observant tendencies, these events seemed to reveal a notable capacity and interest in Aaron to think geometrically and reason spatially. As such, I considered the events to be significant as these appeared to indicate a potential strength in Aaron's mathematical thinking.

The following discussion presents an analysis first of those mathematical events in which Aaron engaged in numeric meanings in number followed by the set of mathematical events in which Aaron engaged in geometric thinking and spatial reasoning.

Content: Expressing Number Words

Orally In a Count Sequence

According to Aaron's parents, Aaron started "counting" when he was approximately 2 years of age, as he participated with his father in sports-related activities (i.e., play boxing) and fitness practices. These reports suggested that Aaron had regular, meaningful experiences at home in which to express the number words,

one through possibly ten, orally in a count sequence. However, I did not observe these kinds of activities during my home visits with Aaron.

I observed two mathematical events, one at his home and one at day care, in which Aaron specifically engaged in expressing number words orally in a count sequence. Assertions of Aaron's content understandings as revealed in each event are discussed, respectively, within the dimensions of purpose and setting and content.

1a. Purpose and setting. Engaging in rote counting as prompted by his father's use of a wall activity. A colorful, fabric wall activity hung next to the door in Aaron's room. Entitled "Tell Me About Your Day," the school-like wall activity addressed calendar concepts, including days of the week, days of the month, months, years, yesterday/today/tomorrow; seasons; and weather. Hook-and-loop word, numeral, and icon markers for each concept could be moved and placed on the display according to each day's date, season, and weather conditions. Ten pockets, arranged in two rows, each held a numeral marker (1 through 9 and 0) and were labeled accordingly. The wall activity hung above Aaron's eye level thus suggesting that Aaron's engagement with this activity required the assistance of either of his parents.

When Aaron's parents first invited me inside his room, I noticed the wall activity and the following event transpired.

Wall Activity

Grace: Can you tell me about that?
Aaron Sr.: [He addressed Aaron.] This is your time to talk to her.
Aaron: This is my numbers.
Aaron Sr.: Your what?
Grace: Say that again?
Aaron: My numbers.
Aaron Sr.: Your numbers.

Grace: Your numbers?
 Aaron: Yes.
 Aaron Sr.: What, come hear you count. What are these numbers right here?
 [Aaron Sr. pointed to the pocket labeled with “1”.]
 Aaron: That, that one.
 Aaron Sr.: What’s that? [Aaron Sr. pointed to the pocket labeled with “2”.]
 Aaron: That, that’s three.
 Aaron Sr.: No, that’s a three [He pointed to the pocket labeled with “3.”].
 [Aaron Sr. then pointed to each successive numeral while he said the following.] That’s 1, 2, 3, 4 –
 Aaron: [He joined his father at “3, 4,” then said the rest by himself as Aaron Sr. proceeded to point to each numeral in sequence.] 5, 6, 7, 8, 9, 10!
 Aaron Sr.: Good boy. What’s your name?
 Aaron: Aaron Hamilton...
 Aaron Sr.: Aaron Hamilton? When’s your birthday?
 Aaron: My birthday July...
 Aaron Sr.: July?
 Aaron: Yes.
 Aaron Sr.: July what?
 Aaron: July for birthday.
 Aaron Sr.: That’s when your birthday, what day, though? July?
 Aaron: July 8th.
 Keisha: Yes.
 Aaron Sr.: Good. Where’s the 8 at?
 Aaron: The 8 right here.
 Aaron Sr.: That’s a 6. Which one – where’s the 8?
 Aaron: The 8 right there, daddy.
 Aaron Sr.: No, 6. [Aaron Sr. pointed to each numeral in sequence as he said the following.] That’s 6. Then it’s 7, 8.
 Aaron: 8.
 (April 8, 2006 – Home Visit with Aaron)

Aaron referred to the wall activity as his “numbers.” This appeared to indicate not only that Aaron had some degree of familiarity with the display but also that Aaron seemed to view the numerals on the pockets as the most salient aspect of this activity. In Aaron’s parents’ reports of him “counting” or engaging with number in his everyday activities, they did not mention this wall activity or other experiences that involved engaging Aaron in numeral recognition. In this wall activity event, it appeared that Aaron Sr. intended to engage Aaron in identifying the numerals. Aaron

Sr.'s actions may have been prompted by my inquiry about the wall activity as well as an interest to demonstrate Aaron's understandings of number to me.

Although Aaron appeared to recognize the numeral 1, when his father pointed to the numeral 2 and asked, "What's that," Aaron responded, "That's three." It was unclear to what extent Aaron was familiar with his father's questioning activities regarding the wall activity. Aaron's responses in the wall activity event seemed to indicate an emergent understanding of numeral recognition, as he identified the numeral 2 as "three" and pointed to the numeral 6 when asked to identify the numeral 8.

In the counting-related activities reported by Aaron's parents, it appeared that Aaron tried to join his father in expressing number words orally in a counting sequence as Aaron Sr. engaged in counting repetitions or elapsed time as a natural part of his everyday fitness or play activities with Aaron. In comparison, in the wall activity event Aaron Sr. appeared to emphasize numeral recognition along with expressing number words orally in a counting sequence.

At first, Aaron did not seem to interpret Aaron Sr.'s gestures of pointing to the first two numerals and asking "What's that," as a cue to engage in rote counting as these behaviors did not reflect other counting activities in which Aaron and his father reportedly engaged. Rather, Aaron Sr.'s initial facilitation of the wall activity with Aaron may have inadvertently disrupted the verbal flow between the number words "one, two" that Aaron otherwise may have been able to recognize as the beginning of a counting sequence. In other words, given the reported counting-related activities in which Aaron regularly engaged with his father, Aaron may have been accustomed to

joining in his father's verbal counting sequence beginning at the number word "three."

Once Aaron Sr. fluidly pointed to the numerals 1 then 2 while saying the number words "One, two," in a counting sequence, Aaron recognized his father's prompt to count. He readily stated "three, four" with his father, and then, on his own, Aaron proceeded to express the number words five through ten orally in a counting sequence as his father continued to point to the remaining numerals, with the final depicted numeral actually being 0 and not 10. Only after hearing his father say, "one, two" did Aaron recite the counting sequence of 3, 4, 5, 6, 7, 8, 9, 10.

Ib. Values and beliefs. This event demonstrated Aaron's parents' formal perspective of mathematics. Aaron's parents also appeared to find value in the wall activity as they displayed it prominently in Aaron's room and, as evidenced in this event, utilized aspects of this tool to encourage Aaron to recognize the numerals 1 through 9 and to rote count.

Ic. Social relations. The question-answer exchange characterizing the social interaction between Aaron Sr. and Aaron reflected a school-like mathematical practice in which the teacher determines and manages the discourse and the content in a particular mathematical event (Street et al., 2005) and indicates to the student when a correct and incorrect response has been given. Aaron Sr. directed Aaron's mathematical activity in this event and conveyed to Aaron the appropriateness of his responses, thereby suggesting that certain ways of engaging in this activity are more favorable than others as judged by the adult leading the activity.

Aaron Sr.'s questioning of Aaron regarding his name and birthday reflects typical verbal exchanges that occur between parents in the U.S. and their young children in which parents seek to develop their children's capacities to respond to such identifying questions.

2a. Content – A stable-order count revealed through a spontaneous expression of number words in a count sequence. At day care, I observed one event in which Aaron spontaneously expressed number words in a count sequence. This is not to say that this event was the only occurrence in this setting in which Aaron engaged in number for this purpose. I often split my attention and observations between the participant children when two or three of them were present at the day care. As such, I might have missed other similar events with respect to Aaron and this content. Whenever Ms. Franklin played the *Kindercise with the Kinderman* (Taylor, 1998) video, Aaron did sing along during the "Friends" song that began with, "Friends, friends, 1, 2, 3. All my friends are here with me."

While I did not observe any additional events, at home or at day care, in which Aaron spontaneously expressed number words in a count sequence, the event below does provide insight into Aaron's developing stable-order count.

Making a Pretend Hamburger

Aaron found a play, plastic mini-burger patty and came to me and asked me to taste it and to tell what else it needed on it. I pretended to bite the burger and said, "I think it needs more ketchup." Aaron responded by repeating the ingredient I named and then he ran up the slide of the climbing structure and pretended the little archway at the top of the slide was a food preparation station or kitchen where he pretended to add the ketchup to the hamburger patty. Then he ran back down the slide and back toward where I was sitting and offered me another taste of the burger. I pretended to taste the burger again and named another needed ingredient. Aaron subsequently named the

ingredient and again ran up the slide to add this to the burger and ran back down the slide to have me taste the burger again and tell him what else it needed. This pattern repeated several more times. On one of his trips up the slide, Aaron spontaneously expressed aloud the following number words sequence, “Three, four, five, six, seven.” As Aaron ran back down the slide, Anika (who was still sitting at the table eating her goldfish snack) said aloud, “eight”.

(March 23, 2006 – Day Care Visit)

Aaron spontaneously expressed the number words three through seven in a stable count sequence while running up the slide on his way to make another pretend hamburger. Aaron’s stable-order count seemed to start with the number word three. A few days following this event, Aaron also employed a similar stable-order count beginning with the number word three when he appeared to enumerate images in the pom-poms event discussed below.

During this period of time, given Aaron’s parents’ reports of his “counting” activities at home, it may be that Aaron’s stable-order count started with “three” as he may have interpreted his father’s fluid, verbal initiation of, “One, two,” for example while Aaron Sr. was counting repetitions when exercising, as a cue to rote count, to which Aaron responded by expressing a string of number words, beginning with “three,” in a counting sequence. The wall activity event, that took place approximately 2 weeks after the pretend hamburger event, provides further evidence to support this assertion as it demonstrated Aaron’s tendency to begin rote counting with the number word three once his father stated “one, two.”

Although at day care Aaron often sang along with the Kinderman “Friends” song, that included the count sequence “1, 2, 3,” Aaron may not have yet recognized or made the connection between this three-element count sequence and the count

sequences, either 1 through 10 or 3 through 10, that he reportedly expressed while engaging with his father in play boxing or exercising.

2b. Purpose and setting. The pretend hamburger event took place during an unstructured time late in the afternoon at day care when the children occupied themselves in various activities as they waited for their parents. Aaron's purpose for expressing the number words three to seven in a count sequence seemed unclear. It may be that Aaron spontaneously expressed number words in a count sequence in approximate correspondence with each step he took while running up the slide toward his pretend food preparation station. Or, it may be that Aaron expressed the number words in a count sequence with no specific purpose in mind.

Aaron's behavior simply may have been a verbal indication of his exuberance during his play, particularly while running up the slide. While the content of expressing number words orally in a count sequence did not appear to be "integral" to Aaron's "interactions and... interpretive processes," (Baker et al., 2003, p. 12) when making his pretend hamburgers, the physical activity of running up the slide during his play may have prompted Aaron's spontaneous oral expression of number words in a counting sequence.

2c. Values and beliefs. Ms. Franklin afforded children the opportunity to engage in various activities while they waited for their parents to pick them up in the afternoons. These activities included unstructured play, or more formal learning activities such as completing worksheets as directed by Ms. Franklin, perusing through books, playing learning games on the computer, or entertaining themselves with educational tools such as putting together board puzzles. This afternoon

unstructured time thus promoted children's independent play, self-directed exploration, and learning of particular skills through adult or computer-based support.

In the hamburger event Aaron engaged me in his play, demonstrating his level of comfort with me.

2d. Social relations. Ms. Franklin managed the children's activities as well as their behavior. As with the case of Aaron, this also meant that Ms. Franklin placed him in timeout whenever she deemed this consequence necessary. Ms. Franklin's assessment of Aaron's good behavior on this afternoon afforded him the opportunity to engage in free play.

Since Aaron initiated his play with me, it seemed appropriate to respond to him every time he invited me to taste his pretend hamburger and to suggest what other ingredient it needed. My choosing to play along seemed to encourage further Aaron to run several times up and down the slide to prepare a pretend hamburger.

Aaron's spontaneous oral expression of number words in a count sequence while running up the slide seemed to reflect in part Aaron's parents' reports of Aaron's verbal participation with his father while they engaged in physical activity together (e.g., play boxing, exercising). However, in this event, Aaron's expression of number words in a count sequence did not occur in parallel with the behavioral and verbal actions of another person. He ran and expressed number words in a counting sequence on his own accord. Rather than disrupting the spontaneity and rhythm of Aaron's play, I opted not to respond to Aaron's mathematical activity and allowed his play to continue on his own terms.

*Content: Number to Enumerate Objects/Images,
When Prompted with, “How Many?”*

In the set of four mathematical events that follows, I shaped the events by initiating and asking the question, “How many?” In doing so, I engaged Aaron (as well as Toya, who was also present in two of the events below) in school-like discourse by posing a known-answer question (Rogoff, 2003) to them. However, I posed the question of “how many” in the midst of interactions in which the focus children either appeared to initiate interest in observing, having, and/or enumerating a particular quantity of objects or images. As such, a verbal exchange explicitly concerning each quantity’s numerosity seemed to be a reasonable extension of the social interaction at the given moment.

While the following events did not reflect an essence of everydayness consistent with the purpose of this study, I include these events in this discussion as they reveal evidence regarding Aaron’s emerging understandings of enumeration and the related competencies involved in enumerating quantities. As such, my analysis of the events in this section focuses primarily on the dimension of content, particularly the mathematical understandings Aaron appeared to demonstrate in each event. In order to situate the events below, I also include brief statements regarding the dimension of purpose/setting. This discussion also provides evidence that supports the previous analysis of Aaron’s stable-order count, and it informs the analysis in the subsequent section regarding Aaron’s spontaneous moments of enumerating objects/images.

1. Content. On the day of the pom-poms event below, Aaron and Toya were present at day care and I was using my laptop computer to record field notes instead of using my field journal and pencil as I had done so on previous visits. I did not anticipate that both children would take interest in my laptop. I responded to Aaron and Toya's intrigue in my laptop by sharing it briefly with them. Exposing Aaron and Toya to my laptop served in part to satisfy their initial awe with it so that they could go back to their typical activities afterward. Their interest in my laptop, while a bit more intense, seemed consistent with the interest that all three participating children occasionally demonstrated throughout the study with respect to my field journal and pencil.

The following mathematical event took place in the morning while I shared with Toya and Aaron photos stored on my laptop computer.

Pom-Poms

Aaron and Toya came to stand on either side of me while I sat with my laptop opened in front of me. Aaron looked on quietly as Toya pointed to and recognized a few icons that represented different software applications (i.e., music, calendar, pictures) displayed on the screen. Toya then requested to see pictures so I opened iPhoto to share a few with her and Aaron. While I scrolled through my pictures Toya asked me to stop at certain images. One was a picture of a multi-colored scarf I knitted that included seven different colored pom-pom embellishments. I spontaneously asked Toya how many pom-poms she saw on the scarf. She pointed to each pom-pom and counted aloud in sequence from one to seven. I asked Toya again how much, and she told me seven. Aaron, without prompting directly from me, appeared to attempt to count the pom-poms as well. He pointed, without one-to-one correspondence, to the images of the pom-poms in the picture while he stated, "Three, four, five, six." Toya informed Aaron that he missed one when pointing to the pom-poms.

(March 28, 2006 – Day Care Visit)

The different-colored pom-poms were a prominent feature of the scarf. While Toya seemed to take interest in and observed the picture of the scarf, I spontaneously

asked her how many pom-poms she saw. After Toya determined the numerosity of the pom-poms, and without my direct prompting, Aaron actively participated in the social interaction around the picture of the scarf by pointing to the pom-poms in the image while expressing the number words three through six in a counting sequence. This string of number words reflected a portion of the same stable-order count beginning with the number word three that Aaron employed a few days prior in the pretend hamburger event.

Although Aaron did count to the number word seven in the pretend hamburger event, and despite the fact that Toya enumerated the pom-poms by counting aloud from 1 to 7 and by confirming the presence of “seven” pom-poms on the scarf, Aaron’s counting sequence ended with the number word six. It could be that the act of pointing to the images of the pom-poms while stating a sequence of number words might have impeded Aaron’s facility to express a counting sequence past six.

The extent to which Aaron understood his participation, or even Toya’s gestures and statements, in this event to be that of enumerating or determining the numerosity of the set of pom-poms was unclear. However, Aaron’s verbal and physical actions did demonstrate an emerging understanding of enumeration and early number concepts. This included Aaron pointing to the pom-poms in the image, albeit without one-to one correspondence, while expressing a stable-order counting sequence of “three, four, five, six.” To my knowledge, I did not indicate outwardly to Toya whether or not her responses were correct. Such acknowledgement from me therefore did not appear to motivate Aaron’s participation and mathematical activity.

While I did not prompt Aaron directly to count the pom-poms, my questions of “how many” to Toya appeared to pique his interest. Aaron’s participation in this event seemed to indicate a desire to engage actively in the social interaction concerning the scarf/pom-poms, and/or to attempt to respond accordingly to my questions to Toya, and/or to follow Toya’s lead (i.e., mimic Toya). Aaron’s tendency to observe and to notice images, particularly those in picture books, may have also prompted him to employ early enumeration concepts in observing the photo of the scarf with pom-poms.

2. *Content.* Later, on the same day as the pom-poms event, the following event took place during lunchtime. I typically helped Ms. Franklin distribute food to the children during their meal/snack times.

Apple Slices

Ms. Franklin prepared sliced apples for the children to eat for their dessert. I gave Toya her apples first. As I put one apple slice on her plate Toya spontaneously said “one.” When I put another apple slice on her plate, Toya said, “two.” I then asked Aaron how many I gave to Toya. He didn’t respond. I proceeded to put two apple slices, one at a time, on Aaron’s plate and then I asked him how many he had. Aaron pointed to one of the apple slices and said “four” and then he pointed to other apple slice and said “six.” Ms. Franklin overheard the verbal exchange and wanted to know what Aaron said. After I informed Ms. Franklin of Aaron’s response, she asked me if I had corrected him. When I told her I hadn’t corrected Aaron, Ms. Franklin came over to the table and pointed to the apple slice that Aaron had in his mouth and said, “That’s one,” and then she pointed to the other apple slice on his plate and said, “That’s two.”...

After Toya finished her two apple slices, she asked for “two more.” Then Aaron stated, “I want apples.” I noticed that when Aaron wants more of something, rather than saying that he wanted “more” of it or the numerosity of how many more he wanted, he typically said that he “wants ____” and then he named what he wanted more of. Ms. Franklin counted as she gave Aaron two apple slices.

(March 28, 2006 – Day Care Visit)

Given that Toya spontaneously counted her apple slices as I gave them to her (i.e., she initiated the use of number at that moment), and given the pom-poms event earlier that day in which Aaron spontaneously participated and appeared to engage in early enumeration concepts after he observed Toya doing so, I chose to ask Aaron how many slices I gave him, especially since the numerosity of the apple slices was just two. I sought to examine Aaron's understandings of enumeration with respect to a very small quantity. Aaron's response seemed to indicate that he interpreted my question of "how many" to mean that he was to respond in a similar fashion as Toya, by pointing to each object and assigning or naming each with a number word. As such, Aaron demonstrated one-to-one correspondence and an emerging understanding of using explicit number words to determine the numerosity of a quantity.

When comparing Aaron's response in the apple slices event compared to his response in the pom-poms event, a few interesting differences seemed worthy of noting. Aaron's success with one-to-one correspondence in the apples event, as opposed to the absence of one-to-one correspondence in the pom-poms event, appeared to stem from the fact that there were only two apple slices and the slices were tangible. In the pom-poms event, Aaron may have recognized a counting sequence in Toya's enumeration activities as she counted fluidly from 1 to 7, which therefore may have prompted Aaron to employ his own stable-order count. In the apples event, however, Toya stated the words "one" then "two" as I placed each apple slice on her plate. In this sense, Toya did not express these number words fluidly as if in a counting sequence that Aaron might have otherwise recognized. Since Aaron's

stable-order count did not appear to include the number words one and two, this too may have hindered his capacity to identify a counting sequence in Toya's utterances.

Aaron most likely did not interpret my question to mean for him to count on from where Toya left off since the first number word Aaron stated was "four" and not "three," and after "four" he stated "six" and not "five." In this sense, Aaron may have interpreted Toya's verbal behavior as that of assigning or labeling each apple slice with any number word and so he responded to my prompt accordingly.

It was unclear why Aaron did not respond to me when I asked him how many apples I had given to Toya, but he did respond when I asked him about the number of apples I had given to him. Given Aaron's polite manner, it may be that he did not want to point over Toya's food but he was fine with pointing over his own food when seemingly assigning a number name to each slice.

During my visits at the day care, I observed Ms. Franklin teaching the children about their five senses and occasionally giving them mathematics worksheets to complete. However, prior to the apple slices event I did not observe Ms. Franklin asking the children, "How many?" or explicitly counting familiar objects with the children during their daily activities. This is not to say that Ms. Franklin never engaged in these activities with the children. Toya's responses in the pom-poms and apple slices events suggested that she developed early number and enumeration concepts at some point, perhaps across her home and day care settings.

My interactions with Aaron and his response in the apple slices event prompted Ms. Franklin to model an enumeration strategy for Aaron, by pointing to each apple slice while saying "this is" and expressing the first then second counting

number. When Aaron stated, “I want apples,” Ms. Franklin modeled the same enumeration strategy again for Aaron as she gave him two more apple slices. Aaron’s statement of “I want apples,” seemed to also suggest that, at least in the context of requesting more of something, Aaron seemed unfamiliar with employing number words to specify the numerosity of objects that he wished to have.

3. *Content.* I observed the following mathematical event at day care just over a week after the apple slices event. It is worth noting that approximately 30 minutes before the event below occurred, I observed Aaron spontaneously subitizing two pieces of a broken cookie that he noticed while eating these from his bag of cookies during snack time. I discuss the cookies event in greater detail in the next content section involving mathematical events in which Aaron enumerated spontaneously.

Hands

Aaron played trucks with 2-year-old Nevin. While rolling their trucks along the floor, they chased each other’s trucks around and frequently collided their trucks into one another. At one point during their play, Ms. Franklin summoned Aaron to come to her. She asked Aaron how many hands he had. At first he just raised his right hand, but with encouragement from Ms. Franklin, he raised both hands and eventually said something like, “This one is four. This one is six,” referring to his right then his left hand. Ms. Franklin responded as she had done so before except she also demonstrated cardinality to Aaron by saying something to the effect of, “This is one. This is two. You have two hands.”

A few minutes afterwards, Aaron and Nevin became interested in my mini-container of hand sanitizer that was hanging from my backpack. They each pretended to squirt some sanitizer on their hands, and then they each rubbed their hands together. While Aaron and Nevin were pretending to use my hand sanitizer, Aaron asked me to take a picture of his hands as he held his hands together with his palms facing up. Given that Ms. Franklin had asked Aaron moments before how many hands he had, it seemed appropriate to ask Aaron the same question, especially since he was presenting his hands to me. He immediately responded, “I have two hands.” I then took a picture of Aaron and of Nevin with each of their hands open and then shared the images with them. They seemed to enjoy looking at their photos.

(April 6, 2006 – Day Care Visit)

When Ms. Franklin pulled Aaron aside from playing trucks with Nevin to ask him how many hands he had, Ms. Franklin seemed to be following up on what she observed the prior week in the apple slices event. That is, Ms. Franklin appeared to ask Aaron how many hands he had to assess his understanding of counting or enumerating a quantity of two objects. Ms. Franklin's approach to Aaron reflected a school-like practice in that it seemed "teacher focused" and "top down" with Ms. Franklin determining and managing the context and the interaction around the mathematics content (Street et al., 2005).

Despite having demonstrated the capacity to subitize two pieces of a broken cookie approximately 30 minutes earlier, Aaron initially appeared unsure of Ms. Franklin's question as he hesitantly raised a single hand and, with Ms. Franklin's guidance, eventually raised the other. Aaron's uncertainty may have stemmed from the nature of his interaction with Ms. Franklin at that moment. Aaron may have questioned why Ms. Franklin pulled him aside to ask him about his hands as it appeared disjointed from his play with the trucks. Aaron may have even wondered whether or not he was in trouble and therefore his thinking may have been focused instead on that.

Eventually, Aaron responded to Ms. Franklin in a manner similar to his response to me in the apple slices event as he said, "This is one is four. This is one is six." The meanings underlying Aaron's use of "four" and "six," in particular, in both the apple slices and hand events remains unclear. With Ms. Franklin's prompting, Aaron appeared to approximate the enumerating activities that she modeled for him previously, by referring to each hand and uttering the phrase "This is (number

word).” At this point in data collection, I had not observed Aaron spontaneously express number words in a count sequence beginning with, “One, two.”

Given my observations of the cookies event earlier that day and Ms. Franklin’s interaction with Aaron about his hands, when Aaron presented his hands to me after pretending to use my hand sanitizer and his focus remained on his hands including asking me to take a picture of them, this appeared to be an appropriate moment for me to ask “how many” to further understand Aaron’s thinking with respect to enumerating small quantities, the number word two, and the numerosity two. Aaron’s immediate response of, “I have two hands,” may have reflected his developing understandings of enumerating quantities with numerosity two via his interaction with Ms. Franklin moments before as well as their interaction about a week prior in the apple slices event, or it may be that Aaron simply repeated what Ms. Franklin said to him earlier.

The ease with which Aaron responded to me, compared to the uncertainty he displayed with Ms. Franklin, seemed to suggest that the context (i.e., purpose and setting) may have also shaped (a) Aaron’s interpretation of the question (“How many?”) that Ms. Franklin and I each posed, as well as (b) his response to each of us, including his emotional reaction to the context as well as the content he expressed when stating how many. For instance, Aaron may have simply known that he had two hands and the context of the event with me afforded him the opportunity to share this knowledge without hesitation. When I asked Aaron how many hands he had, he had already initiated showing his hands enthusiastically with me and asking me to take a picture of them. Furthermore, my inquiry of how many addressed the very objects to

which Aaron's attention was already focused and with which he was actively engaged.

In comparison, when Ms Franklin asked Aaron how many hands he had, Aaron's attention up until that point had been on playing trucks with Nevin. When compared to the apple slices event, while Aaron's interest was on the apples, Aaron's role seemed to be that of a passive recipient of the slices as I determined how many to give him. When I asked Aaron the question of "How many?" based on a context driven by Aaron (him playing with my hand sanitizer, then showing me his hands, and finally asking me to take a picture of his hands) this may have supported Aaron's capacity either to subitize his hands or simply to demonstrate to me that he automatically knew that he had two hands.

4. *Content.* The following event took place at Aaron's home, just 3 days after the hands event. I was sketching the layout of the apartment in my field journal when Aaron approached me.

Pillows

Aaron: Miss Grace, you watch the movie?

Grace: I'm listening to the movie. I'm actually drawing your house here so I can remember what it looks like.

Aaron: Let me see my house.

Grace: Well, it's like I'm above in the ceiling and looking down on it. That's your TV, that's your mag-, that's your table, coffee table right there. You see that?

Aaron: Yeah.

Grace: And that's the plant. And here are the two ashtrays. Here's the window... And then I'm gonna draw - I forgot that chair. That chair right there. Then I need to draw the pillows.

Aaron: [Aaron went over to point to the pillows.] The pillow? Is that a pillows?

Grace: How many pillows are on there?

Aaron: [Aaron touched the pillows as he responded to me.] That - this - that - this is one. This is two. This is _____. [Aaron Sr. cross-talked over Aaron as he appeared to be counting.]

Aaron Sr.: [Aaron Sr. addressed his concern that Aaron might get the pillows dirty.] Wipe your hands. Be careful. Your hands are dirty.
Grace: Yeah. How many pillows are on there?
Aaron: These two pillows right there.
Grace: Oh, okay. The two pillows right there? Okay. [I proceeded to sketch the pillows.]
Aaron: [Aaron agreed.] Uh-hmm.
(April 9, 2006 – Home Visit with Aaron)

In this pillows event, Aaron expressed a desire to see my sketch of his family's apartment. As Aaron observed the sketch, I described what I had drawn thus far and what else I needed to add to the picture. When I mentioned that I needed to draw the pillows, Aaron took particular interest in these as evidenced in his verbal and physical reactions to my statement. Given Aaron's interest in the pillows, I asked Aaron "How many pillows?" as if to request his assistance to help me document the appropriate amount of pillows in my sketch.

Aaron's response demonstrated the following understandings with respect to enumeration: using explicit number words to determine the numerosity of a quantity; assigning each number in the count sequence with each distinct object in the set (i.e., one-to-one correspondence); and identifying the numerosity, in this case correctly, of a quantity (i.e., cardinality). Aaron's enumeration behaviors reflected those that Ms. Franklin modeled from him and he employed the number words one and two to determine the numerosity of the pillows.

Aaron's experiences at day care in the several days prior to this event at home seemed to have informed his response to my inquiry of "How many?" pillows. At day care, Aaron had demonstrated a facility to subitize a quantity of two pieces of a broken cookie. Aaron's understanding of the need to use explicit number words, along with one-to-one correspondence, to determine the numerosity of a quantity

appeared to be emerging (as evidenced in the apple slices event and the first part of the hands event). The experiences of observing Toya enumerating quantities either spontaneously or when prompted, and observing Ms. Franklin's direct modeling of enumerating quantities of two objects seemed to contribute to Aaron's developing understandings of numeric meanings for number, and in particular, enumeration concepts.

Content: Number to Enumerate Objects/Images,

Spontaneously

After the pom-poms and apple slices events, I began to observe Aaron spontaneously enumerating objects, images, and people in different everyday contexts. He demonstrated various mathematical understandings to determine a quantity's numerosity including: (1) subitizing quantities with numerosity two, (2) verbally and physically keeping track of each item in a quantity of images having numerosity two, and (3) counting aloud while attempting to physically keep track of each item in a quantity of images having numerosity greater than two.

The set of mathematical events below complements the previous events in which Aaron was prompted to respond to the question of "How many?" Collectively, these mathematical events demonstrated Aaron's developing understandings of numeric meanings, especially enumerating quantities with numerosity two.

Content - Subitizing quantities with numerosity two. The following three mathematical events each demonstrated Aaron's capacity to subitize two objects. That is, without using explicit number words or counting, Aaron spontaneously recognized each set of quantities below as having numerosity two.

1a. Purpose and setting – Noticing two pieces of a broken cookie. The following event took place at day care during snack time, approximately 30 minutes before the hands event occurred.

Cookies

Ms. Franklin gave Aaron a bag of frosted animal cookies for snack time. I observed Aaron as he began to eat his cookies. He pulled out a pink frosted cookie from his bag and said something referring to the cookie. I casually tried to ask Aaron what he had said about the cookie, but I still could not understand his attempts to repeat himself. Darlene, an after-school-care child, who was sitting to Aaron's right eating her own snack, informed me that that Aaron said that there were circles on the cookies (referring to the different colored, round sprinkles that stuck to the pink frosting of the cookie). I watched Aaron closely examine each cookie he took out of the bag. He pulled another pink frosted cookie out of the bag, examined the cookie briefly, and set it down on the table. Aaron then took a broken piece of a white frosted cookie out from his bag and took a moment to examine it. Aaron looked back into the bag, taking a bit longer this time as it seemed as though he was looking for a particular cookie. He eventually pulled out another piece of a broken white frosted cookie. Aaron turned toward Darlene and me to show us the two broken pieces of a white frosted cookie he held in each hand and stated something to the effect of, "There are two pieces 'cause it broke." (April 6, 2006 – Day Care Visit)

Aaron appeared to take interest in visually examining the individual pieces or cookies he pulled from his bag. Once Aaron retrieved a part of a white frosted cookie from the bag he seemed intent on finding the other part of the cookie. Once he found the other piece, Aaron held both parts of the cookie and announced his observation that there were two pieces of the cookie because it broke.

1b. Values and beliefs. The cookies event seemed to instantiate Aaron's observant nature. Rather than simply taking out the cookies and placing each in his mouth, he took time to observe and examine each piece before taking a subsequent action with it. He noticed the sprinkles on the cookies and, according to Darlene's interpretation of what he said, Aaron described these sprinkles as circles. Aaron took

particular interest in the broken, white frosted cookie. It appeared that Aaron might have noticed the initial broken piece in the bag as he placed the pink frosted cookie that he took out just before it on the table. Once he had this first piece of a broken white frosted cookie, Aaron appeared to look for the other part of the cookie in the bag. Consistent with his temperament to observe, when Aaron held both pieces in his hands, not only did he announce their numerosity, but he also explained why there were two. As evidenced in his announcement, Aaron appeared to emote a sense of success and delight in finding the pair and noticing why there were two of them.

1c. Social relations. The particular observations that Aaron noticed about the cookies prompted him to express his thinking verbally to those nearby. Aaron's verbal and physical actions thus suggested that Aaron appeared to deem his observations worthy of sharing with others. Aaron transformed what would otherwise be an individual activity of eating and observing his snack of cookies into a social experience.

2a. Purpose and setting – Spontaneously noticing numerosities of twos in his environment. On my first home visit with Aaron, as Aaron's parents and I talked in the family room while Aaron ate his lunch in the dining room area, he spontaneously announced from the dining room, "That two girls," (referring to his mother and me) and, "That two boys," (referring to his father and himself) (April 8, 2006 – Home Visit with Aaron). Since I did not hear what Aaron said as I was in the midst of responding to a question from Aaron Sr., I continued speaking and did not acknowledge Aaron's statements. Aaron Sr. interjected and pointed out to me what Aaron said and then asserted, "Aaron's smart. He realizes that numbers are the

generalization of our conversation.” As his father said that, Aaron stated something to the effect of, “That one girl. That one girl. That one boy. That one boy,” (referring to each of us in the room).

While Aaron Sr. argued that Aaron’s statements reflected his awareness that the focus of his parents’ conversation with me were about “numbers,” I posit that Aaron’s use of number, in this case two then one, to subitize the people he observed in the space emerged from Aaron’s temperament to observe, coupled with his developing understandings of enumerating quantities with numerosity two. Aaron’s growing awareness of the numerosity two seemed to have stemmed, at least in part, from the mathematical events Aaron experienced at day care just prior to my initial home visit with him. In many of these events, I played an active role in introducing or shaping the content of enumeration, such as prompting Aaron with the question of “How many?” Additionally, most of these day care events involved quantities with numerosity two, including the cookies event in which Aaron demonstrated the capacity and intrinsic interest to subitize two pieces of a broken cookie.

Aaron’s statements of “two girls” and “two boys” and later when he pointed out the singleness of each us present, “That one girl...” suggested that Aaron’s capacity to enumerate at this point in data collection seemed developed for quantities of up to two familiar objects or images of the same type or category, in this case, gender of people (i.e., enumerating girls and then enumerating boys). In this sense, Aaron did not appear to demonstrate abstraction when enumerating people.

2b. Values and beliefs. This event illustrated Aaron’s observant nature. When I did not respond to Aaron’s spontaneous initial observation of two girls and two

boys, Aaron Sr. called my attention to what Aaron said. Aaron Sr.'s efforts seemed to reflect not only his awareness of my presence and his perceived understanding of the purpose of my visits/observations but also a desire for Aaron to appear favorably in this study. As such, Aaron Sr. wanted to make sure that I took note of Aaron's statements and that I recognized how "smart" Aaron was.

2c. Social relations. Aaron's growing awareness of identifying and/or enumerating quantities with numerosity two may have motivated him to announce what he noticed. In other words, given his recent day care experiences involving enumeration, when Aaron recognized that both the girls *and* the boys in the room had numerosity two, he may have found this coincidence interesting and thus worthy of our attention.

Aaron Sr. believed in the notion that children should be seen and not heard. As such, whenever Aaron tried to enter into a conversation among adults, Aaron Sr. typically would either remind Aaron to not interject, reprimand Aaron for interrupting, or simply ignore Aaron when he said something while the adults were talking. In contrast to these typical responses to Aaron, Aaron Sr. interrupted my response to his question in order to bring my attention to Aaron's spontaneous comments. In this event, Aaron Sr. appeared to view Aaron's comments from afar to be of value to me and hence worthy of my attention.

3a. Purpose and setting – Describing "Dog" to Miss Grace. On my third and final visit to Aaron's home, Aaron Sr. was not present when I arrived, and Keisha informed me that they had been house-sitting a puppy for a couple days to see if it would be a good match for their family. The puppy stayed in a crate in Aaron's room

near his bed. Upon my arrival, Aaron invited me into his room to show me the puppy which he had named Dog. Aaron discussed his experiences with Dog the day before and earlier that day. In addition, Aaron began to describe Dog to me, saying in particular, “He got two eyes,” and other descriptive statements about the puppy including, “He got a tail,” “He’s got a nose,” “He scratched me right there with his nails,” “He says ‘woof woof,’” and “He has a really big tongue” (June 4, 2006 – Home Visit with Aaron).

While most of the physical features that Aaron described to me about Dog had a numerosity of one, Aaron did recognize that Dog had two eyes. He also demonstrated understandings of global magnitude as he characterized the size of Dog’s tongue as being, “really big.”

3b. Values and beliefs. This event illustrated Aaron’s inclination to observe aspects about his environment or entities that he found of interest and to express to others what he noticed. In getting to know Aaron, I observed that he enjoyed reading and telling stories about animal characters, watching animated movies with animal characters, playing with animal toys, and making animal sounds and gestures. As such, Aaron seemed to take pleasure in having the puppy in his home as evidenced in the attention he gave to Dog and the time he spent talking about Dog, including its physical features, the “belt” (i.e., collar) it wore around its neck, the “cheerios” (i.e., dog food) it ate, and the experiences Dog had while in his home.

Aaron Sr. mentioned to me a few times that he had bred dogs in the past. Thus, bringing a male puppy into his household with Aaron, Keisha, and their unborn child seemed consistent with Aaron Sr.’s interests.

3c. Social relations. In my previous home visits with Aaron, Aaron Sr. was present and most of my conversations and interactions involved Aaron's parents. I noticed during those visits that Aaron did not freely engage spontaneously in activities or interactions of his choice as his father often checked or managed Aaron's behavior. However, during the dog event when his father not present, Aaron freely interacted with me as he immediately invited me into his room to show me the puppy and to talk about Dog as his mother was elsewhere in the apartment. Aaron seemed comfortable engaging with me as he had done so at day care. I participated in a conversation with Aaron by listening to Aaron, asking him questions where I felt it was appropriate, and occasionally remarking on his comments. Thus, the ease with which Aaron seemed to interact with me, along with his interest in the puppy, supported a context in which Aaron could observe and describe features about Dog, including enumerating one of its physical traits.

Content - Verbally and physically keeping track of each item in a quantity of images having numerosity two. In the following event, which took place during a home visit with Aaron, Aaron demonstrated a capacity to combine verbal and physical strategies to keep track of and then to express the numerosity of two bugs he attempted to draw.

Bugs

[Aaron had been writing/drawing in my field journal.]

Grace: That's a what?

Aaron: A bug in my bed.

[A subsequent verbal exchange occurred in which I tried to clarify what Aaron was saying since I had a difficult time understanding his speech. Then the following dialogue took place.]

Aaron: Okay. I draw, ... that a bug, and I draw, ... that a bug. I draw, ...that *two* [said with emphasis] bugs. [It was difficult for me to decipher what Aaron said. He either said, 'I draw a bug and I draw

a bug. I draw *two* bugs,' or 'That a bug and that a bug. That *two* bugs.']

Grace: That's a bug and that's a bug, so there's two bugs?

Aaron: Yeah.

Grace: Yeah?

Aaron: Two bugs - they gonna climb up my bed and eat all my face up.

(April 9, 2006 – Home Visit with Aaron)

In this episode, Aaron spontaneously enumerated the images of the bugs he drew. Aaron pointed to the markings he drew in my journal while saying something to the effect of, “That a bug and that a [another] bug. That two bugs.” This appeared similar to the enumeration strategy that Ms. Franklin directly modeled for Aaron at least twice at day care in the days prior to this home visit. However, Aaron did not use explicit number words, and instead verbally identified each marking of a bug as a singular marking that he drew, by using the determiner “a” to correspond with each marking of a “bug.” Afterwards, Aaron identified the numerosity of his images of bugs by stating, “That two bugs.” Aaron appeared to demonstrate developing understandings of both the meaning of enumeration as well as enumeration strategies.

Purpose and setting. Aaron had just come out from his room after being sent there for some time by Aaron Sr. for not being able to sit quietly on the couch while watching television. Aaron noticed that I had been writing in my field notes journal and asked me what I was doing. I told him that I was, “just writing” and he requested, “I wanna write.” So I handed him my journal and pencil and Aaron proceeded to draw different pictures and describe to me what he had written or drawn in my journal. Eventually he was inspired to draw bugs and began to tell a story about the bugs. Aaron provided details regarding his drawings and story including noting how many bugs were on his bed and were going to eat his face.

Values and beliefs. Aaron, like his peers in this study, often expressed interest in my field notes journal and seemed to take pleasure in writing or drawing in it. As he drew in my journal, Aaron commented to me about his markings. Aaron's description regarding the bugs seemed to reflect Aaron's interests in reading and telling stories as well as watching scary movies.

Social relations. Given our prior interactions at day care, Aaron knew he was welcomed to approach me and to inquire about my journal and even to write in it. Aaron chose to engage with me through drawing in my journal and describing to me what he was drawing, rather than sitting quietly watching television. During this event, Aaron's parents were in the kitchen preparing an early dinner and as such Aaron Sr., was not present to monitor Aaron's behavior. Aaron therefore seemed free to interact with me regarding the markings he drew in my field journal.

Content - Counting aloud while attempting to physically keep track of each item in a quantity of images having numerosity greater than two.

Images of Book Covers

Upon my arrival at the day care this morning, Ms. Franklin informed me that Aaron was to stay seated at the table and watch *Sesame Street* (Broder, Burgund, & Dallard, 2006). After some time had passed, Aaron was still restricted to the table. Eventually Aaron picked up a book that was resting nearby on the table. It was a simple picture book about weather that included words such as rain, snow, windy, hail, and fog. I noticed how Aaron proceeded to read the book out loud to himself, narrating a story based on the pictures he observed. Aaron eagerly asked if he could read the book to me. After reading the book to me, Aaron examined the cover of the book. On the rear cover of the book was of a row of three rectangular images, each representing a cover of another book written by the same author as the book that Aaron had in hand. Aaron spontaneously pointed to each image, and said, "One, two, three". I asked Aaron, "How many?" and he pointed again to the images, and this time said, "One, two, three, four".
(May 5, 2006 – Day Care Visit)

Aaron spontaneously attempted to enumerate the images he observed on the rear cover of the book. Aaron's stable-order count sequence had evolved from weeks earlier as his counting sequence now began with "One, two, three, four." In Aaron's initial attempt to enumerate, by counting aloud while keeping track of each item in the set through physically pointing to each image, he appeared to demonstrate one-to-one correspondence. However, when I asked Aaron, "How many?" his recount of the images while pointing to keep track, did not exemplify one-to-one correspondence. Neither of Aaron's enumeration attempts demonstrated that he understood cardinality, at least in this context that involved a quantity of *images* with numerosity three. Nonetheless, Aaron's understandings of enumeration appeared to develop as evidenced in his evolved stable-order count and his initiative to enumerate a set of similar images having numerosity greater than two.

Purpose and setting. After being restricted in his activities, Aaron sought relief in a nearby book. Aaron's attempt to enumerate the set of similar images appeared to be a spontaneous response to his interest in and observations of this book.

Values and beliefs. This mathematical event demonstrated not only Aaron's interest in books and reading, but also his disposition to observe and to take particular notice of things/images that he found of interest in his everyday environment or activities.

Social relations. While Ms. Franklin required Aaron to stay seated at the table and watch television as a means of managing his behavior, she did not appear to mind when Aaron began entertaining himself by reading a book, as she seemed to deem this as appropriate behavior that would not disrupt the other children or their

activities. Through Aaron's experiences at home, he was accustomed in engaging in reading as both an individual and shared experience. As such, he asked if he could read the book to me after he read it aloud to himself.

Aaron's spontaneous attempt to enumerate the images he noticed on the rear cover of the book appeared to reflect an intrinsic interest to do so and not an action that he intended to direct toward me. Since Aaron, without external prompting, already employed number to enumerate the images he observed, I felt it was appropriate to respond by asking him, "How many?" as a means to further examine Aaron's mathematical understandings with respect to enumeration concepts, including whether or not he demonstrated cardinality in this particular event.

Content: Employing the Number Word Two

As a Descriptive Narrative Device

Aaron's experiences of enumerating quantities of objects, images, and/or people with numerosity two, both when prompted and when doing so spontaneously, seemed to inspire Aaron to employ the number word two as a descriptive narrative device when "reading" or telling stories, responding to illustrations, and when participating in a pretend scenario. That is, Aaron appeared to use the number word two to provide more specific details regarding the numerosity of a character or object he observed or created in a story or make-believe situation.

1a. Purpose and setting – Describing how many animals/lions when telling Miss Grace a story. The following event took place just as Aaron was describing Dog to me. It seemed as if Aaron had transitioned into story-telling mode about a lion and other animals, once he claimed that Dog looked like a lion.

“Them got two lions.”

Aaron mentioned that, “He [Dog] looks like a lion.” Aaron proceeded to talk about how a truck took the lion(s) and other animals, including a “monkey” an “elephant,” “Them got two animals,” and “Them got two lions,” to the jungle. It seemed as if Aaron was telling me the plot from one or two recent animated movies with which I was familiar. I asked Keisha if her family had seen either movie. She informed me that they had not but that she and Aaron had recently read the book *The Lion King* together.

(June 4, 2006 – Home Visit with Aaron)

In my understanding of Aaron’s story, Aaron specifically mentioned that a truck got “two animals” and “two lions,” in addition to other animals, and then transported these animals to the jungle. The remaining animals that Aaron mentioned each appeared to be single in numerosity.

Ib. Values and beliefs. Aaron enjoyed reading books and telling stories or excerpts from books or movies he had read or seen. In this event, Aaron spontaneously transitioned into telling me a story once he noticed that Dog looked like a lion to him. As Keisha reported, she and Aaron had recently read a book about *The Lion King* story together. As such, the experiences Aaron had when reading that story may have prompted Aaron to tell me a similar story or his version of the story. Aaron’s interest in animals in general may have also sparked him to tell me an impromptu story.

Ic. Social relations. Aaron typically engaged in reading books and story-telling as a shared experience, most often with his mother Keisha. Aaron appeared to be at ease with me and as such, he felt comfortable sharing and talking about Dog with me and eventually telling me a story about lions and other animals. My interactions with Aaron as he told his story to me were mostly characterized by

comments or questions of clarification to help me understand the story Aaron was attempting to convey.

2a. Purpose and setting – Verbally reacting to a picture in a book. The following event occurred just before 10 p.m. as Aaron began reading his third and final book for bedtime.

“I want two...”

As Aaron inspected a picture in the book, he said, “I want two firefires [fire trucks?], two toys, two houses.”

Aaron’s observation of the picture elicited a verbal response from him that included a repetitive use of the number word two to describe a set of objects that Aaron viewed in the picture or envisioned in his mind. While there were images of trucks and buildings, more than two of each in the picture, no toys were visible. Aaron’s mention of toys in his reaction might have stemmed from the fact that earlier that evening, including moments before Aaron selected this third book to read for bedtime, Aaron occasionally commented that his toys were still left in the tub from his bath time. That is, Aaron seemed to be mindful of his toys in the tub and had repeatedly inquired about them. Given that this event occurred relatively late in the evening, it could be that Aaron’s verbal reaction to the picture he observed showed evidence of fatigue as Aaron mentioned an object in his response that was not present in the story or picture.

2b. Values and beliefs. Keisha valued reading and sought to instill this in Aaron by regularly exposing him to books and reading to and with him, especially as part of Aaron’s bedtime routine. On this evening, Keisha mostly encouraged Aaron to read his books to her while she occasionally read to him some excerpts from the books he selected.

2c. Social relations. Aaron engaged in bedtime stories as a shared experience with his mother Keisha. At times, Aaron inquired about an object or animal (e.g., a dog) that was not present in the story to which Keisha responded by explaining why it was not visible in the picture (e.g., regarding the dog, she said, “It was in the backyard.”). Because it was late in the evening, Keisha also seemed tired and as such she appeared to not interject much and instead encouraged Aaron simply to move on and continue to “read” the story.

3a. Purpose and setting – Responding to ‘how many’ when prompted by his father in a pretend scenario. After Keisha finished braiding Aaron’s hair, Aaron Sr. used clippers to shape Aaron’s hairline and then he paused to check out the work he had done. Seemingly satisfied with his son’s appearance, Aaron Sr. engaged with Aaron in the following mathematical event:

Date Night

Aaron Sr.: Oh you look nice. Oh yeah. You gotta date tonight? How many girls coming to get you?

Aaron: Two.

Aaron Sr.: Two coming? Whoaaa! You’re gonna be back by 10 o’clock?

Aaron: Yeah. [Aaron ran to his mother then to the front door, and said the following,] I’ll be right back.

Grace: You’ll be right back? Where you going?

Aaron: Out. Two girls are gonna get me.

(April 9, 2010 – Home Visit with Aaron)

Aaron Sr. initiated this mathematical event with Aaron by presenting a pretend “date” scenario, including asking Aaron how many girls were coming to get him for a date. Prompted by his father’s question of, “How many girls?” Aaron correctly interpreted the meaning of the question by providing a numeric response. It may be that Aaron’s response demonstrated his understanding that his father’s question implied that more than one girl was coming to get him for a date. Or it may

be that Aaron responded with “two” as this had been a number with which Aaron had frequent experiences over the few days preceding this event. Aaron proceeded to play along with the scenario by acting as if he was leaving the apartment to go out on a date with the two girls who were coming to get him.

3b. Values and beliefs. In my formal conversations with Aaron Sr., he spoke about his son’s attractiveness and the strong likelihood that many girls and then women will pursue Aaron. After making the final touches on his son’s freshly done hair, Aaron Sr.’s make-believe scenario reflected his belief regarding Aaron’s future dating prospects. Aaron Sr.’s question of “How many?” as well as his mention of what time Aaron would come back from his date also appeared to demonstrate Aaron Sr.’s heightened awareness of my presence and a possible desire to have his family and son appear favorably in this study.

3c. Social relations. In this event, Aaron Sr. engaged Aaron in light-hearted pretend play by acting as if Aaron was going out on a date. Aaron seemed to respond positively to his father’s prompting and, as such, willingly responded, both verbally and physically, in playing along with the make-believe scenario.

Content: Number Words and Symbols as

Distinct from Letter Names and Symbols

The mathematical events discussed above also demonstrated that Aaron understood that number words and symbols (numerals) are distinct from letter names and symbols. For example, in those events in which a caregiver or I asked Aaron, “How many?” Aaron’s responses consisted of number words. Also, while Aaron’s wall activity included words, pictures, and numerals, he referred to this activity as his

“numbers” as his parents most likely focused Aaron’s attention on recognizing the numerals appearing on this display.

The following mathematical event provided further evidence that Aaron understood, in particular, that number words constitute a distinct category. This episode occurred at day care about 20-minutes after the apple slices event took place.

Placemats

Aaron and Toya finished their lunch and waited at the table for naptime as Ms. Franklin put down the sleeping mats. Aaron and Toya still had each of their placemats for eating in front of them. Aaron pointed to Count Dracula and another character on his placemat. He said something like, “This is one and this is two,” while pointing to Count Dracula and something else on his mat. Then Toya came over to Aaron’s mat and started pointing to different character images while saying, “This is mine and this is mine.”

Aaron and Toya continued to wait at the table with their placemats in front of them. On Toya’s placemat appeared a graphic of the numerals 1, 2, 3, 4, and 5. Toya pointed to each numeral in order, from left to right, starting with the numeral 1, while she said the following, “Five, four, three, two, one.”

Both children continued to entertain themselves with their placemats. At one moment, Aaron pointed to various images/locations on his mat and said, “Eight, nine, four, five,” and then, “nine, ___, seven, seven, nine.” Ms. Franklin then came over to the table where both children sat and commented, “I’m glad that you are counting.” The children then prepared for naptime. (March 28, 2006 – Day Care Visit)

At the beginning of this placemats event, when Aaron spontaneously pointed to two distinct characters on his placemat while stating something to the effect of, “This is one and this is two,” he appeared either to be mimicking or applying what Ms. Franklin had directly modeled for him shortly before in the apple slices event to the images he pointed to on his placemat. While Aaron demonstrated one-to-one correspondence, it was not clear whether or not he was enumerating the images or assigning a number word label to each one. That is, it was unclear at this point what meanings Aaron developed with respect to enumeration and enumeration practices. As for Toya, she appeared to mimic what she observed Aaron doing, but she put her

own non-numeric, Toya-like twist to it by pointing to two distinct images and stating, “This is mine and this is mine.”

In the second half of the episode, Aaron appeared to mimic what he observed Toya spontaneously doing when she pointed to the numerals on her placemat and, with one-to-one correspondence, counted aloud in reverse order from 5 down to 1. On his own, Aaron attempted to approximate Toya’s behavior as he pointed to random images/locations on his placemat and announced what appeared to be an arbitrary collection of number words, without one-to-one correspondence to with his gestures.

Aaron seemed to interpret and/or to recognize Toya’s utterance of counting numbers in reverse order as consisting of a random string of number words. This is evidenced in Aaron’s own arbitrary set of number words that he stated. That is, Aaron appeared to determine that Toya did express solely a collection of number words but that this particular set of number words, to Aaron, did not seem to reflect a count sequence. As such, Aaron did not express his own stable-order count sequence, beginning with the number 3, when mimicking Toya in this event. This is in contrast to Aaron’s response in the pom-poms event discussed previously when he counted aloud from 3 through 6 while pointing to the pom-pom images after he observed Toya expressing the numbers 1 through 7 orally in a count sequence while pointing to the pom-pom images.

Purpose and setting. Aaron and Toya appeared to entertain themselves with their placemats while they waited for Ms. Franklin to prepare the day care space for naptime. Given the apple slices event that took place just prior to this event, Aaron seemed interested in continuing what he had experienced or observed in the apple

slices event by seemingly mimicking Ms. Franklin and pointing to images on his placemat and saying in turn, “This is one and this is two.” Just as Toya had appeared to copy somewhat Aaron’s spontaneous behavior at the beginning of this event, Aaron later copied or attempted to approximate Toya’s spontaneous behavior when she counted in reverse order while pointing to numerals on her placemat.

Values and beliefs. Aaron’s initial behavior in this mathematical event seemed to extend from his experiences in the prior apple slices event. Aaron might have observed that the activity of pointing to two entities while saying, respectively, “This is one and this is two,” seemed to be of value to Ms. Franklin as she made it a point to correct Aaron’s initial response in the apple slices event of, “This is four. This is six,” and to model for Aaron the response she desired.

Aaron also appeared to take notice of and interest in Toya’s activities when she spontaneously used number words when occupying herself with her placemat. This was evidenced in Aaron’s attempt to copy Toya’s actions when he seemed to state an arbitrary string of numbers while pointing to various items or locations on his own placemat.

Ms. Franklin explicitly expressed her praise of the children, in particular Aaron, for “counting” when she heard them state a collection of number words in the later portion of this event. As such, this affirmation may have encouraged Aaron to perceive that Ms. Franklin valued his actions concerning number words.

Social relations. My initial prompting of “How many?” in the apple slices event just prior to this seemed to serve as a catalyst for Aaron’s efforts to enumerate quantities with numerosity two. When I posed this question and Aaron’s initial

response was in error, Ms. Franklin sought to correct Aaron and to model for him how to count a set of two objects. Subsequently, Ms. Franklin's response to Aaron in the apple slices event appeared to shape, in part, the ways in which Aaron chose to entertain himself while waiting for naptime in the placemats event. Furthermore, while Aaron and Toya did not directly engage with one another in the placemats event, their individual actions when they employed number words seemed to feed off of one another. At one point, they each continued to amuse themselves independently by following or approximating what the other one was doing. When Ms. Franklin praised the children for "counting," this suggested that their use of number words was significant or of value to her.

Content: Geometric Thinking/Spatial Reasoning

On a few occasions at day care, I noticed Aaron engaging in geometric thinking and spatial reasoning. In addition to the two events that I discuss in greater detail below, I observed two other mathematical events in which Aaron exhibited beginning understandings with respect to shape in particular. During the broken cookie event presented above, Aaron noticed and seemed to say that there were "circles" on the cookies, referring to the sprinkles he observed on his animal cookie snacks. The other event occurred during structured playtime with wooden blocks. While playing with his blocks, Aaron paused when he came across a triangular prism. He examined the piece in his hands briefly and then asked me, "What's this?" Aaron demonstrated interest in and attention to my response. For example, while I recognize that it was not developmentally appropriate for me to eventually describe the piece to Aaron as a "triangular prism," Aaron, on his own accord, took several attempts to

pronounce the word “prism” with my assistance. Shortly afterwards, when Ms. Franklin asked the children to clean up and put their blocks and other toys away, I noticed that Aaron paused when he noticed and picked up a plastic, square prism (which was not included with the set of wooden solids with which he engaged earlier), and said aloud to himself, “This is a block,” before putting the toy away.

These events not only reflected Aaron’s observant nature but they also seemed to indicate Aaron’s interest in and emerging mathematical understandings about 2-dimensional and 3-dimensional shapes. According to the *EYCM* (Clements et al., 2004) flexible, developmental guidelines for young children’s learning of geometry and spatial sense mathematical content, and in particular the subtopic/content of shape, Aaron appeared to demonstrate the big idea that “analyzing, comparing, and classifying shapes helps create new knowledge of shapes, and their relationships” (p. 44). More specifically, Aaron used a shape-class name informally when he identified the sprinkles on the cookies as “circles.” In the blocks event, Aaron appeared to engage in naming and comparing 3-dimensional shapes through his informal play and building with wooden solids. Aaron’s inquiry of “What’s this?” with respect to the triangular prism (i.e., naming) suggested that he noticed that this piece seemed qualitatively different than the other wooden solids with which he played. Aaron’s informal description of the plastic cube he found during clean-up time as a “block,” indicated that his conceptual referent for cube, informally termed “block,” was determined by its shape and not by the material composing it. Aaron identified the plastic cube as a “block” even though it was not made of wood.

The two mathematical events (labeled “Puzzle” and “Baby Seat”) that follow demonstrated not only Aaron’s emerging understandings with respect to analyzing and comparing 3-dimensional solids in his everyday activities, but they also exhibited Aaron’s early capacity to engage in notions of orientation, motion and perceptual analysis with respect to everyday objects.

The extant literature on young children’s geometric and spatial thinking primarily addresses their understandings with respect to images or models of geometric 2-dimensional and 3-dimensional shapes that accurately depict the mathematical properties of the shape (e.g., standard drawing of an equilateral triangle, pattern blocks, wooden solids, geometric tiles). As such, in the fields of mathematics education and early childhood research, literature regarding young children’s mathematical understandings involving geometric thinking and spatial reasoning with everyday 3-dimensional objects, pictures, or sketches is sparse.

For example, puzzles (e.g., jigsaw, board, table puzzles) are popular toys in home and in early-childhood care and school settings as it is widely assumed that puzzles afford children a wide range of educational and developmental benefits (Maldonado, 1996; Montford & Readdick, 2008). With respect to mathematics learning in particular, Hansen (2005) stated, “Jigsaw puzzles are superior for helping preschool and kindergarten children develop spatial ability. They must look at the sizes and shapes of puzzle pieces and how the pieces must be oriented to complete the puzzle” (p. 211). However, according to Montford and Readdick (2008), “no research exists that specifically investigates the skills children employ when they are working with puzzles” (p. 548). That is, no studies have examined the understandings and

competencies, including those that are mathematical, that children utilize when they engage with puzzles. The lack of studies in mathematics education research may suggest that the field has not clarified the potential of puzzles or other everyday objects as mechanisms for fostering or supporting young children's mathematical understandings of geometric and spatial relationships.

Two research studies seem relevant to my analysis. First, Montford and Readdick (2008) found preliminary evidence that young children's puzzle making is related to part-whole perception, which involves the capacity "to distinguish visually an object in its entirety and the features that comprise that whole as distinct" (p. 548). With regard to puzzles, this means being able to recognize that the parts or pieces of a puzzle form a whole or an entire picture (Maldonado, 1996). Second, Owens (1999) proposed a theoretical framework of young students' "early spatio-mathematical development" (p. 222) to examine the range of imagery strategies that children use to respond to a variety of tasks involving 2-dimensional geometric shapes. Owens (1999) argued that children's imagery strategies reflected their evolving understandings regarding the following aspects of spatial knowledge and shape: orientation and motion, part-whole recognition, and classification and language.

In my analysis of the following mathematical events, I extend Owens' (1999) proposed framework, and in particular the "emergent" and "perceptual" imagery strategies that children employ with respect to their understandings of orientation and motion and part-whole relationships, to include everyday, concrete, 3-dimensional objects. The relationship between part-whole perception and puzzle making found in Montford and Readdick's (2008) research and the inclusion of part-whole

relationships as an aspect of spatial knowledge and shape in Owens' (1999) framework both reflect the significant idea in geometry that shapes can be composed and decomposed into other shapes (Clements, 2004; NRC, 2009).

Content – Analyzing and transforming parts of a 3-dimensional object to compose a whole object. In each of the following two mathematical events, Aaron assembled parts of an everyday, 3-dimensional object to form the whole entity.

1. Puzzle

Ms. Franklin called Aaron and the two younger boys to the table so they could work on the new puzzles she purchased. There were three puzzles (Trucks, Fire Engine, Tools) that the boys were supposed to rotate among them. The Fire Engine puzzle was the most difficult of the three. Initially, Ms. Franklin gave it to one of the younger boys to complete, but shortly after he started, she switched his puzzle with Aaron's puzzle. I observed Aaron while Ms. Franklin worked with the younger boys. With seemingly relative ease, Aaron placed the front of the truck, the two wheels, hydrant, the engine logo, the dog and person 1 and person 2 onto the puzzle board. For each of the eight puzzle piece he placed, Aaron seemed to examine carefully the board and the piece in his hand to figure out where each piece was to be placed. After a few minutes of working on his puzzle and reaching a point where the remaining four pieces seemed more challenging for Aaron to place independently, Aaron announced that he was done, placing the remaining pieces within the open space on the board, without interlocking them.

After a number of minutes passed and Aaron announced for the third time that he was done, Ms. Franklin sat beside Aaron to help him place the remaining pieces in the puzzle. She asked him, "What looks like ___ (pointing to an area on the board that has not yet been completed) over here (pointing to the remaining pieces to be placed)?" On a single occasion, Ms. Franklin reduced the number of puzzle pieces from which Aaron could compare and choose when responding to her prompt. Ms. Franklin also tried to have Aaron focus on the different arrangement of colors appearing on the pieces already placed in the puzzle compared to the colors appearing on the remaining pieces. In addition, she cued Aaron to turn or move pieces accordingly that were in or near the correct area of the puzzle but not positioned correctly. (March 29, 2006 – Day Care Visit)

Puzzles (e.g., jigsaw, board, table) are considered as 3-dimensional objects.

Aaron had no prior experience with this 12-piece puzzle and therefore his strategy to

assemble the pieces did not include him drawing from memory knowledge of where the various pieces were to be placed. It was unclear to what extent Aaron had prior experience with puzzles in his home or other everyday settings outside of day care.

Aaron's success with placing the initial eight pieces may have reflected a combination of puzzle factors and puzzle-making strategies. With respect to the visual and physical aspects of the puzzle, each of the initial eight pieces that Aaron placed correctly constituted a complete or whole image (e.g., a wheel, a dog) (Maldonado, 1996) and many of these pieces belonged along the perimeter of the cut-out of the puzzle board. These features of the puzzle may have supported Aaron to employ "image" and, perhaps to a lesser extent, "form" puzzle-making strategies (Montford & Readdick, 2008) when assembling these initial whole-image puzzle pieces on the board. That is, for each whole-image puzzle piece or figure, Aaron seemed to consider not only the image depicted on the piece, but he also appeared to analyze the shape of the piece and to compare it to the contours of the visible spaces on the board in order to determine the piece's placement. According to Montford and Readdick (2008), with respect to puzzle making, image strategies are considered the most rudimentary approaches after trial and error strategies. Aaron may have also drawn upon his prior knowledge of images of fire engines in picture books to inform his assembly.

When attempting to place the remaining four pieces of the puzzle, Aaron appeared to use, without success, a trial-and-error strategy. Aaron's difficulty may have been due in part because these pieces reflected parts of the remaining image of the chassis of the fire engine. Placing these pieces required Aaron to also employ a

color-matching strategy (Barron, 1999; Maldonado, 1996; Montford & Readdick, 2008) along with moving and turning these pieces as necessary. Ms. Franklin eventually guided Aaron through these strategies when she intervened to help him complete the puzzle.

In terms of part-whole perception as it relates to puzzle making (Montford & Readdick, 2008), Aaron appeared to recognize that the individual, whole-image puzzle pieces and figures (e.g., dog, hydrant, each wheel) together formed a broader fire engine scene. Aaron's successful placement of these particular pieces seemed to demonstrate part-whole perception. With the availability of whole-image puzzle pieces and figures, Aaron seemed to demonstrate a beginning capacity to engage in *perceptual* imagery strategies with respect to the aspects of spatial knowledge and shape that concern part-whole relationships and orientation and motion (Owens, 1999). According to Owens (1999), "Students using perceptual strategies are attending to spatial features and beginning to make comparisons, relying on what they can see or do" (p. 224).

While Aaron primarily seemed to rely on image puzzle-making strategies, focusing on the picture or images that he observed, he also seemed to attend simultaneously to the spatial features of each whole-image piece (i.e., the shape or figure of each piece) and the board (i.e., the contours of the visible spaces on the board) prior to manipulating each piece physically to fit on the board. That is, Aaron appeared to demonstrate early perceptual imagery strategies (Owens, 1999) by seemingly recognizing (a) each whole-image piece or figure would be used to build the picture of the fire engine scene (part-whole relationships), and (b) the figure of

each whole-image piece, despite its initial random position nearby on the table, to determine its approximate location on the board and then physically manipulating each one to fit on the board (orientation and motion).

Because each of the remaining four pieces constituted a part of the chassis of the fire engine, as opposed to the whole image of the chassis, Aaron was not successful when relying on his image puzzle-making strategies (Montford & Readdick, 2008) and his perceptual imagery strategies (Owens, 1999) to place these final pieces. As demonstrated in Aaron's trial and error approach to assemble these final pieces of the puzzle, he seemed to employ emergent imagery strategies with respect to his understandings of part-whole relationships as an aspect of spatial knowledge and shape (Owens, 1999).

The following event involved Aaron putting together parts of an everyday, non-educational-focused object.

2. Baby Seat

Baby D's tray had fallen off of his floor-level baby seat. Aaron noticed the detached tray lying on the ground and proceeded to place it back on Baby D's baby seat. Instead of placing the tray on top of the seat and trying to force it to fit, Aaron purposefully inspected or examined the bottom of the tray while looking at D's seat. Aaron repositioned the tray in his hands and lifted it up to place on D's seat. In repositioning the tray, Aaron seemed to be looking for the oval hole on either side of the tray so that he could match these up with or fit them onto the oval notches on D's baby seat.

(March 23, 2006 – Day Care Visit)

In this event, Aaron seemed to engage in perceptual imagery strategies with respect to the aspects of spatial knowledge and shape that concern part-whole relationships and orientation and motion (Owens, 1999). Aaron's actions indicated his understandings of a part-whole relationship in that the tray and seat were parts that together formed the baby seat, the whole. Rather than using trial and error to place the

tray back onto the seat, Aaron appeared to consider not only which side was the right-side-up regarding the tray, but he also seemed to examine the physical features/shapes of each part, namely the holes on the tray and the corresponding notches on the seat. Per his visual analysis of each part of the whole baby seat, Aaron demonstrated a perceptual capacity to engage in orientation and motion when he re-orientated the tray in his hands so that the notches on the seat would fit into the holes on the tray as he successfully placed the tray back onto the seat.

In addition to the puzzle and baby seat events, the broken cookie event also seemed to demonstrate Aaron's emerging understandings of part-whole relationships with regard to spatial knowledge and shape as he identified the two unique pieces or parts that formed a particular whole cookie.

The puzzle, baby seat, and broken cookie events revealed Aaron's developing mathematical understandings with respect to early geometric thinking and spatial reasoning. Aaron's mathematical understandings reflected three fundamental ideas in young children's learning of geometry (Clements, 2004; NRC, 2009). First, these events demonstrated Aaron's beginning capacity to analyze and to compare parts and features of 3-dimensional objects. That is, in order to compose the whole object from its parts, Aaron engaged in perceptual analysis of the features of the parts as well as the features of the whole. Second, by composing or putting together 3-dimensional objects to make the whole, Aaron demonstrated understandings that shapes or objects can be composed into other shapes or objects and that these forms can be decomposed into their component parts. Third, by physically composing whole 3-dimensional

objects, Aaron informally engaged in mathematical transformations, such as slides, turns, and flips, as he moved and repositioned these 3-dimensional objects precisely.

Purpose and setting. Both of these events took place at Ms. Franklin's day care. The puzzle event occurred as a structured activity prompted by Ms. Franklin to engage Aaron and his younger peers in solving puzzles. On the other hand, the baby seat event emerged spontaneously during an unstructured period as Aaron expressed, through his physical actions, a desire to fix Baby D's seat once Aaron noticed the tray had fallen off the seat.

Values and beliefs. Ms. Franklin provided several puzzles of varying difficulty at her day care. On the day of the puzzle event, she introduced Aaron to three new puzzles that she purchased. Ms. Franklin's supply and use of puzzles at her day care reflected what Maldonado (1996) reported:

In early childhood practice, puzzles have been seen as structured learning material that enhances cognitive, perceptual, and motor development ... Children work toward cognitive closure as they engage in and achieve completion of a puzzle. This requires attention and concentration on their part.
(p. 5)

Ms. Franklin's use of puzzles at the day care demonstrated her value in preparing the children for formal schooling by providing them various activities aimed at fostering their development in particular areas typically drawn upon in formal settings. The novel fire engine puzzle appeared to hold Aaron's attention, even several minutes after he announced the he was finished and that he could not assemble the final pieces. As mentioned above, Aaron's sustained attention toward

the puzzle may have also reflected his interest in and prior knowledge of fire engines. The challenge of completing the puzzle afforded Aaron and Ms. Franklin the opportunity to engage in a formal learning experience as Ms. Franklin scaffolded the task and guided Aaron through a color-matching strategy (Maldonado, 1996).

The baby seat event seemed to be prompted, in part, by Aaron's polite disposition, including a willingness to help others. This event also appeared to reflect Aaron's tendency to observe objects in his everyday environment, particularly things that seemed broken and thus needed to be completed or fixed, as was also evidenced in the broken cookie event discussed previously.

Social relations. The puzzle provided Aaron an activity in which he could focus and engage in independent problem solving for a relatively extended period of time. Because Ms. Franklin introduced the puzzles as a structured activity, rather than spontaneously occupying himself with another activity once Aaron determined he was "finished" with the puzzle, as would probably be the case had this occurred during free play, Aaron instead waited until Ms. Franklin could attend to him. Beginning as an independent, structured activity for Aaron, the puzzle task evolved into a social activity with Ms. Franklin once she noticed that Aaron had not successfully placed the last few remaining pieces and sought to help him complete the puzzle.

While Aaron, on his own accord, noticed and independently fixed Baby D's baby seat, he appeared to take satisfaction in knowing that his actions were in the service of helping others in the day care.

Characterizing Aaron's Everyday Mathematical Events

A subsequent examination of the collection of Aaron's everyday mathematical events discussed above revealed that Aaron's engagement with mathematical content tended to:

- Arise spontaneously either from Aaron's observant nature and/or from his interest in reading books/telling stories;
- Involve Aaron in verbal interactions with known adults or peers;
- Focus predominantly on number although emerging geometric and spatial awareness was evident; and/or
- Demonstrate Aaron's seemingly peaked interest in noticing and describing quantities of two.

I discuss each of these characteristics of Aaron's everyday mathematical events below.

Arising Spontaneously from Aaron's Observant Nature or Aaron's Interest in Reading Books/Telling Stories

The majority of Aaron's engagement with early number concepts and geometric and spatial thinking seemed to emerge spontaneously from his observant nature and/or his interest in reading books and telling stories.

Aaron's observant nature helped to support his growing understandings of early enumeration concepts. Aaron spontaneously observed and verbally subitized the two parts of a cookie which he noticed formed a whole cookie as he ate these treats for snack time, the two "girls" and two "boys" he saw in his living room as I talked to his parents on my first home visit, and the number of Dog's eyes as Aaron described

the puppy to me. In addition to observing the number of objects and people he noticed in his everyday settings, Aaron also observed the mathematical activity of Toya at day care and attempted to approximate her enumerating behavior and utterances as demonstrated in the pom-poms and placemats events.

Aaron's observant tendencies also seemed to support and to develop Aaron's emerging geometric and spatial thinking. In the broken cookie event, Aaron noticed the circles, or sprinkles, on the cookies he ate. While cleaning up after a structured playtime with wooden solids or blocks, Aaron paused to observe and to identify a plastic, square prism, as "a block." Aaron also appeared to attend closely to the physical form of parts of everyday objects with respect to how they fit together to make a whole. In addition to noticing the two pieces that formed the broken cookie, Aaron's initiative to fix Baby D's seat with his purposeful reorientation of the fallen tray before correctly placing it back onto the baby seat and Aaron's facility to put together successfully the majority of pieces of a new board puzzle helped to demonstrate his observant quality.

Aaron's interest in reading books and telling stories often provided a context within which Aaron naturally engaged in mathematical content. For example, he employed enumeration concepts, albeit counting without one-to-one correspondence, to the images he noticed on the rear cover of the weather book he read spontaneously at day care. Aaron also enumerated the bugs, animals, and images he noticed or included in his storytelling, either when reading books aloud (or telling his own version of a given book) or when voluntarily sharing stories he improvised while interacting with known adults. Aaron's pleasure in reading books and storytelling

provided a potentially rich, meaningful context in which he could engage not only in enumeration concepts but also in other mathematical content as well. For example, on his trip to participate in story time at the library with Ms. Franklin and his 4-year-old peers, Aaron employed the concept of magnitude as he actively engaged in the read-aloud of the picture book *Tall* (Alborough, 2005).

Verbal Interactions

Aaron's spontaneous mathematical events often involved him verbally interacting with familiar others. In some mathematical events, Aaron's observant nature seemed to prompt him to interact verbally with known adults as he spontaneously expressed the numerosity of everyday objects and images he noticed at home and at day care. Other mathematical events demonstrated an inherent social quality and a natural use dialogue, such as Aaron's everyday home practice of reading books with his mother as well as his spontaneous storytelling with others.

Some of Aaron's mathematical events appeared to stem both from an interest to observe the social activity of others and an intrinsic desire to participate accordingly, including interacting verbally with others. For example, Aaron observed my interaction with Toya in the pom-poms event and without others directly prompting him, he proceeded to approximate Toya's enumeration of the pom-poms, including expressing number words in a count sequence. Toya verbally affirmed Aaron's active participation in our social interaction focused on the image of the scarf by noting to him that he missed (counting) one. In the pillows event, Aaron verbally expressed interest in my activity of sketching his living room and this evolved into

me pretending to require Aaron's help by asking him "How many?" pillows he saw on a nearby sitting area.

Aaron also verbally interacted with known adults when prompted to respond to questions posed to him. Aaron's father, Ms. Franklin, and I directly prompted Aaron to engage in enumeration concepts by asking him "How many?" in the date night, apple slices, hands, and pillows events, respectively. Furthermore, in the hands and pillows events, Aaron seemed to respond with meaningful, correct numerosities during those events in which enumerating the target quantity appeared to be a natural extension of the social and verbal interactions in which Aaron actively engaged. In contrast, when Ms. Franklin pulled Aaron aside and, seemingly at random, asked him "How many?" hands he had, Aaron initially appeared slightly confused and did not initially respond verbally or physically. Aaron's father engaged him in verbal interaction around concepts of number in the wall activity event. Finally, the challenge of placing the final few pieces in the puzzle event afforded Aaron the opportunity to engage in scaffolded puzzle-making strategies with Ms. Franklin as she verbally and physically guided him to complete the puzzle.

While not discussed in the prior analysis of Aaron's mathematical events, the practice of grooming Aaron's hair also provided him a meaningful, social context in which to interpret his parents' verbal use of number and enumeration concepts. For example, to help ease Aaron's discomfort, Keisha frequently reminded Aaron how many braids she had left to complete, thus suggesting to him that it would not be too much longer until she finished braiding his hair.

*Focus on Number with Notable Evidence of
Emerging Geometric and Spatial Awareness*

The majority of Aaron's significant mathematical events involved number, in particular numeric meanings. Among these events, Aaron primarily engaged in enumeration concepts.

In Aaron's spontaneous attempts to enumerate objects and images in his everyday settings, he employed a variety of strategies including subitizing, using explicit number words while physically keeping track of entities, and counting aloud while attempting to keep track of a quantity of images (albeit without cardinality and one-to-one correspondence in the pom-poms event and the event involving images on the rear cover of a book about weather).

Known adults also engaged Aaron in enumeration concepts by prompting Aaron with the question "How many?" It was not clear, however, to what extent my prompting of Aaron with the question of "How many?" in the apple slices event, and later Ms. Franklin's attention to this content with Aaron, facilitated, informed or motivated Aaron's subsequent spontaneous events concerning enumeration.

Nonetheless, these later events did appear intrinsically driven by Aaron's observant nature and/or his desire to read books or tell stories.

Aside from enumeration concepts, Aaron also engaged in numeric meanings when he spontaneously expressed number words in a count sequence as evidenced in the pretend hamburger event and when prompted to rote count in the wall activity event.

Aaron's use of the number word two as a descriptive narrative device suggested that his use of number for this purpose served to describe the numerosity of the subject/objects of his stories. As such, these events also seemed to demonstrate Aaron's developing understandings of enumeration concepts.

Aaron's engagement with numeric meanings also indicated an understanding that number words and numerals are distinct from letter names and symbols. In addition to the events referred to previously, Aaron also demonstrated this understanding during the placemats event when he attempted to approximate the behaviors and utterances he observed Ms. Franklin and Toya employ when they enumerated objects/images.

Although the majority of the observed significant everyday mathematical events in which Aaron engaged involved number, and in particular numeric meanings, the remaining few mathematical events are just as significant as these revealed Aaron's interest in and inclination toward early geometric thinking and spatial reasoning. The raw, unprompted quality of these events, including no observed prompting from known adults concerning this content in previous contexts, suggested that Aaron seemed to have both an intrinsic desire to notice the form and shape of everyday objects and a natural, emerging capacity to perceive part-whole spatial relationships.

Two

Several of Aaron's mathematical events involving numeric meanings seemed to capture his developing understandings of enumeration of small quantities, and in particular, Aaron's awareness of quantities with numerosity two. Not only did Aaron

spontaneously observe the numerosity two in the familiar objects, people, and images he noticed in his various everyday contexts, but Aaron also seemed to employ the number word two as a descriptive narrative device to signify numerosity in his story reading and story telling.

Aaron's recognition of the numerosity two and his use of the number word two as a descriptive narrative device appeared significant not only due to the observed prominence of these activities across his mathematical events, but also given my early observations of (a) Aaron's stable-order count that began with the number word three as evidenced in the pretend hamburger and pom-poms events, and (b) Aaron's initial responses of, "This is four. This is six," when prompted with "how many" in the apple slices and hands events. These initial observations of Aaron suggested that at that time neither the number word two nor the numerosity two appeared as salient concepts in Aaron's emerging numeric understandings. Aaron's subsequent peaked activity of noticing the numerosity two and using the number word two as a descriptive narrative device therefore seemed significant with respect to Aaron's evolving sense of number, and enumeration in particular.

As stated earlier, it was unclear to what extent Ms. Franklin's and my prompting Aaron to respond to "How many?" with respect to a set of objects with numerosity two encouraged Aaron's later spontaneous mathematical activity and tendency to notice and to employ the numerosity two.

Final Remarks

The identified characteristic of Aaron's mathematical events concerning content reflects the collection of Aaron's significant mathematical events in that

together these events indicated that Aaron engaged primarily in numeric meanings of number and demonstrated emerging early geometric and spatial awareness. In other words, each of Aaron's significant mathematical events reflected this characteristic as Aaron engaged in either numeric meanings of number or geometric thinking/spatial reasoning in each of these events.

In addition to portraying the content-focused characteristic, the majority of Aaron's significant everyday mathematical events reflected at least two of the remaining three identified characteristics of Aaron's everyday mathematical events. The verbal interactions and thus social nature characterizing Aaron's engagement with mathematical content seemed particularly salient for both spontaneous and prompted events. When considering the set of spontaneous mathematical events initiated by Aaron that seemed to reflect an essence of everydayness, all but one of these events reflected the first identified characteristic and thus emerged from Aaron's intrinsic desire to observe aspects of his everyday settings and/or to read or tell stories. That is, Aaron's own interests fostered his engagement with and developing understandings regarding numeric meanings and early geometric thinking and spatial reasoning.

Aaron's tendency to notice and to employ the numerosity two seemed the most conditional among the identified characteristics of Aaron's everyday mathematical events in that it indicated a quality of Aaron's mathematical events most particular to the snapshot of time during which I observed Aaron.

Chapter 7: Analysis of Mathematical Events and Characteristics of these Events Across All Three Children

This study examined the everyday mathematical experiences and understandings of three, 4-year-old, African-American children from working-class family backgrounds. This study drew upon Street et al.'s (2005) perspective of mathematics as social and the analytic concept of mathematical events and identified the characteristics of those mathematical events. In particular, the four interrelated dimensions of content, purpose and setting, values and beliefs, and social relations, that were adapted for this study and constituted the mathematics in and the social aspects situating mathematical activity, provided the conceptual framework with which this study identified, analyzed and discussed the children's collections of significant mathematical events and the characteristics of these events.

Each of the prior three chapters presented a case of one of the children, beginning with a description of a child and the child's typical everyday activities, followed with a discussion of a collection of everyday mathematical events that emerged as significant for the child and the characteristics of these events.

This chapter presents an analysis across the collections of children's everyday mathematical events and across the corresponding collective characteristics of these events. The analysis reveals both distinctions and commonalities across the children's significant everyday mathematical events and identifies commonalities across the characteristics of these events.

Mathematical Content and Understandings

Distinctions Across the Children

All three children engaged in mathematical events, either spontaneously or when prompted, in the course of their daily activities at home, day care, and other familiar settings. The analysis across the children's mathematical events revealed distinctions in the mathematical content that emerged as most salient for each child as well as the mathematical understandings that each child demonstrated through her/his events.

Number. Although number emerged as a significant mathematical content in Toya's as well as in Aaron's everyday experiences, and both children recognized numerals and number words as distinct from letters and letter names, their experiences with and understandings of number and its meanings varied.

Toya regularly interpreted, identified, and employed multiple meanings of number within and across various everyday contexts. For example, Toya routinely engaged in nominal meanings of number in her daily practices of watching television and phoning her grandmothers and also occasionally in her activity of chatting with others about clothing or shoe size. In addition when phoning her grandmothers, the hints that Nicole provided to Toya to help her identify certain numerals on the telephone keypad also engaged Toya in numeric meanings of number. Toya also employed number for numeric purposes when she expressed number words in a count sequence as she played with her cousin Ricky, enumerated her refill packs of hand-wipes that she helped to replenish at day care, and responded to her mother's prompt to count a set of apples while they shopped with her Aunt Pam at the grocery store.

Toya also appeared to develop emerging ordinal meanings of number as she “read” her collection of birthday cards. Through her mathematical events Toya demonstrated growing understandings of small numbers and an increased capacity to employ and interpret number for various purposes in her everyday life. For instance, she identified with increasing independence the numerals 1 through 4, 6, and 8 when phoning her grandmothers, constructed a set of two cookies that she ate as a bedtime snack, and enumerated a quantity of seven pom-poms in a photo.

On the other hand, Aaron’s mathematical events centered on numeric meanings of number and demonstrated more limited emergent understandings of this content emphasizing, in particular, Aaron’s awareness of and interest in the number 2 and of quantities having this numerosity. Although at day care, Ms. Franklin and I both prompted Aaron to enumerate a quantity of everyday objects having numerosity two, on his own accord, Aaron spontaneously enumerated quantities of two objects and images that he observed at day care and at home, such as two pieces of a broken cookie, two eyes of a puppy, two “girls” and two “boys,” and two bugs that he drew. Aaron employed two strategies to determine the numerosities of these pairs, namely, subitizing and verbally and physically keeping track of each item in a quantity (i.e. while pointing to each image he drew, Aaron noted, “That a bug. That a bug. That’s two bugs!”). The latter of these two strategies appeared to reflect in part the counting strategy that Ms. Franklin modeled for Aaron. Aaron also demonstrated his attention to quantities of two in his reading and storytelling activities as he employed the number word “two” as a descriptive narrative device.

Thus, Aaron's and Toya's everyday lives evidenced their engagement in differing experiences involving number and its meanings, experiences that reflected differing levels of cognitive demand, and, as such, they demonstrated different understandings of this content.

Geometric thinking and spatial reasoning. In addition to indicating the significance of number, Aaron's mathematical events at day care also demonstrated geometric thinking and spatial reasoning as a salient mathematical content in which Aaron meaningfully engaged in this setting. Aaron spontaneously observed and examined the form and shape of 3-dimensional objects, and he identified these solids, using a shape-class name informally when he identified the sprinkles on his animal cookie snacks as "circles" and naming a plastic cube as a "block." Aaron also demonstrated an emergent capacity to analyze and to compare parts and features of everyday 3-dimensional objects, perceptual awareness of part-whole spatial relationships with beginning understandings of the ideas that shapes can be composed from and decomposed into smaller parts, and early knowledge of the transformation concepts of orientation and motion as he physically assembled parts of objects into their whole (i.e., a broken cookie, a baby seat with tray, and pieces of a board puzzle) and, in the broken cookie event, as he stated the part-whole spatial relationship that he observed.

Difference and similarity. For Anika, early concepts of classification, in particular meanings of difference and similarity, emerged as salient in her everyday mathematical events at home and at day care. Anika employed, constructed, and expressed meanings of difference and similarity, including concepts of sameness and

resemblance, as she spontaneously observed pairs of entities and identified differences and/or similarities between these or their corresponding attributes. For example at home, as Anika played with her stash of hair accessories while her mother Christina groomed Anika's hair, she noticed that a pair of yellow barrettes differed in their designs. At day care, Anika paused during her independent play when she came across two identical teacup saucers and announced to me, "These are the same!" Anika also demonstrated understandings of classification as she spontaneously sorted a collection of fruit snacks by shape/color before she ate these as a snack at home. While Anika primarily focused on physical attributes when identifying differences and/or similarities between entities, she also noticed and compared functional, categorical, and material attributes of entities as well.

*Broadening Conceptualizations of Evidence of
Mathematical Thinking in Young Children: A
Commonality Across the Children*

While the mathematical content and understandings that emerged as most salient for each child were distinct, each child's case required that I extend existing conceptualizations regarding what constitutes evidence of mathematical thinking in young children and, consequently, what mathematical content and understandings could be discerned in such activity, beyond that which is currently identified in the extant research literature encompassing young children's mathematical understandings.

In each of the children's chapters, I discussed the literature gaps I encountered when searching for prior characterizations of young children's mathematical activities and understandings.

As evidenced by Toya's collection of salient mathematical events, I assert that young children's developing understandings of and their everyday experiences engaging in early number concepts involves the contemporaneous construction of nominal, ordinal, and numeric meanings of number. Given that the extant literature on young children's development of early number concepts typically excludes children's nominal meanings for number, and drawing on Sophian's (1996) discussion of the different kinds of number she claimed children encounter, I argued for, presented, and applied in Toya's case a wider conceptualization of young children's thinking about number as mathematical, a conceptualization that included nominal, ordinal, and numeric meanings.

For Anika, I argued that meanings of difference and similarity constituted early classification concepts. Literature on young children's mathematics frequently identifies classification as an important mathematical process in early childhood (Clements, 2004; Clements & Sarama, 2007; Sarama & Clements, 2008).

With regard to Aaron's case, the research on children's early geometric thinking and spatial reasoning focuses on children's understandings of formal representations of geometric 2-dimensional and 3-dimensional shapes and therefore is somewhat inadequate for characterizing the mathematical content and for examining the mathematical understandings instantiated in Aaron's activities of manipulating and composing the parts of more complex, everyday 3-dimensional objects into their

wholes. Findings from two research projects, one that examined children's puzzle-making strategies (Montford & Readdick, 2008) and another that studied children's imagery strategies when responding to formal tasks involving 2-dimensional geometric shapes (Owens, 1999), provided useful conceptual tools with which to examine and discuss Aaron's mathematical activity and understandings.

Purposes and Settings, Values and Beliefs

Distinctions Among Contexts and Values

That Prompted Mathematical Events

The purposes and settings (i.e., contexts) that tended to give rise to the children's mathematical events at home and/or the values and the beliefs, including the observed tendencies or nature of a child, that either seemed to prompt or appeared to be instantiated in their mathematical events at home or at day care differed across the children.

Watching television. All three participating families seemed to place a significant value on television and/or video viewing as an everyday form of leisure and entertainment at home. Each family demonstrated this significant value by furnishing their living room and each bedroom of their residence with a television set and video player, obtaining a steady supply of new video titles to view, and regularly keeping their television(s) in continual use during home observation visits. However, the practice of watching television and videos emerged as a significant context that fostered mathematical activity only in the case of Toya.

Neither Aaron's parents nor Anika's mother had cable service and instead their families primarily viewed movies or other programs on video. Therefore they

and their children appeared to have little need to change channels on their television sets. On the other hand, Toya had her own television set in her room with access to both cable television and videos. Because Nicole permitted Toya control, more or less, over her television set, Toya regularly used the numeral buttons on her remote control to select different channels or programs when viewing television.

Furthermore, Toya's cousin Ricky was often present while she watched television and this prompted Toya to manage and negotiate channel selection with him which in turn, through guided participation with Ricky, afforded Toya a meaningful context in which to develop her capacity to identify and interpret particular two-digit numbers (e.g., 43, 44) as either the named number or separate specification of digits. Even when Nicole temporarily suspended her cable service, Toya demonstrated the significant value she placed on watching television and her developing understandings to interpret and employ number names and to interpret and identify numerals as she persisted to find channels with reception in the "We got cable" event.

Toya's mathematical events nurtured by her mother's values. All of Toya's mathematical events at home, at the grocery store, and when shopping for clothing emerged from contexts that supported or instantiated important values that her mother fostered in Toya's everyday life.

Nicole placed a significant value on family, including maintaining relationships with and supporting kin. The importance of family support in Toya's life with Nicole, as evidenced in the frequent shared responsibility between Nicole, her sister Pam, and their mother in caring for Toya and her cousin Ricky, reflects research findings regarding the significance of extended families and kin in the care,

socialization, and developmental outcomes of African American children (e.g., Allen, 1995; García-Coll et al., 1996; Harrison et al., 1990). Nicole nurtured Toya's sense of family by having Toya phone her grandmothers on a daily basis and providing Toya a regular time to spend and bond with her mama (Toya's maternal grandmother), Auntie Pam, and cousin Ricky.

Nicole's practice of having Toya call her grandmothers and assisting her in this activity supported Toya's capacity to identify and interpret numerals and number words for nominal and numeric purposes. The cookies that Toya's mama regularly baked especially for her naturally led to Toya's frequent requests to have some of these cookies as a special snack before bedtime, a request that Nicole assented to while encouraging Toya to construct small quantities of cookies.

Toya's play and interactions with Ricky afforded her ongoing opportunities to engage in meaningful activities that supported the development of her understandings of nominal, numerical, and emergent ordinal meanings of number. In addition to identifying, interpreting, and employing number words, number names, and numerals when managing channel selection while watching television with Ricky, Toya joined or mimicked Ricky in expressing number words orally in a count sequence while they played together, and Ricky attempted to encourage Toya's facility to identify the numeral-labeled buttons with which they engaged on the elevator when they accompanied Nicole at work on Saturdays.

Nicole's values of saving greeting cards, shopping for Toya's wardrobe, and preparing Toya for later schooling also supported Toya's mathematical understandings regarding number. Nicole's value and practice of saving greeting

cards as meaningful keepsakes allowed Toya to "read" her past birthday cards at her leisure and to develop emergent ordinal meanings of number during independent play with these. Nicole's pleasure in shopping for Toya's clothing and shoes, and Toya's experiences shopping for these items with her mother, naturally gave rise to and helped foster social interactions with others in which Toya noticed, employed, or interpreted number as a way to identify the "size" of these articles. Nicole's concerns for Toya to "know her work" and to prepare Toya for later schooling informed her efforts to prompt Toya to enumerate a set of apples at the grocery store and to read a picture book to Toya at night, which included prompting Toya to count a set of images in an illustration.

Aaron's mathematical events prompted by his interest in reading/storytelling and inclination to observe. Aaron's spontaneous mathematical events primarily tended to arise from either his joy of reading and storytelling, as encouraged and valued by his mother, or his inclination to observe, including noticing shapes and part-whole spatial relationships and recognizing quantities having numerosity of two.

Aaron's own interest (i.e., value) and engagement in reading books and storytelling (i.e., purposes) reflected the significant value his mother Keisha expressed toward reading and indicated through her reported regular practices of taking Aaron to the library, obtaining books for him on her own, and reading books to and with him. As Keisha reported, her own mother was a "bookworm," and she read to Keisha "all the time." An emphasis on reading in Keisha's family life as a child also supported the value placed on schooling and education, which Keisha attributed to her Jamaican culture. Thus, Keisha's practice of reading to and with Aaron seemed

aimed at possibly two outcomes, to inculcate in him a personal interest in reading and books and to provide foundational experiences on which later success in school could be built.

Not only did Keisha emphasize reading for pleasure, but she also engaged Aaron in activities that developed his emergent literacy skills, especially with respect to comprehension. For example, she frequently encouraged Aaron to discuss what he thought was happening in the story by talking about what he observed in the pictures and she regularly invited Aaron to “read” a book back to her after she read it once to him.

Keisha did not appear to use the pictures or the story of a book to engage Aaron in mathematical meanings related to the story. However, on his own initiative and given his interest in reading/storytelling and tendency to observe, Aaron used the number word two as a descriptive narrative device when he “read” books aloud or when he engaged in impromptu storytelling. Aaron’s interest in books combined with his inclination to observe also seemed to lead Aaron to notice and then to attempt to enumerate a set of four small images of book covers that appeared on the rear cover of a book that he “read” at day care.

Aaron’s tendency to observe seemed to prompt and support him in noticing and enumerating quantities of two both at home (e.g., two boys and two girls) and at day care (e.g., two pieces of a broken cookie), noticing and identifying shapes (e.g., “circles” on a cookie), and noticing part-whole spatial relationships in everyday objects and physically manipulating the object’s parts to compose these into their whole (e.g., baby seat and tray).

It was unclear to what extent Aaron's tendency to observe and to note subsequently, for example, the numerosity of objects or images in his environment indicated an innate quality and/or if Keisha nurtured Aaron's intent observant behavior through her reading practices with him. Given that Keisha did not appear to engage Aaron specifically in enumerating images in picture books or in identifying or describing the shape of objects illustrated in books as part of their regular reading practices, Aaron's spontaneous specification of numerosities of quantities he observed in picture books, imagined when storytelling, or noticed in his immediate environment, and his unprompted observations and inquiry of form and shape of everyday and novel objects suggested not only that Aaron engaged intrinsically in these mathematical activities but that these events further demonstrated a natural inclination in Aaron to observe.

Anika's mathematical events emerged from her play or casual interests.

Anika's meanings of difference and/or similarity tended to emerge naturally from her play, as she observed items with which she amused herself during routine activities at home (e.g., barrettes and sorting fruit snacks events) or during free play (e.g., teacup saucers event) or unstructured times (e.g., baby dolls event) at day care. Anika's mathematical events also demonstrated her desire to interact socially with adults as she initiated verbal exchanges with me through her observations of similarity and/or difference (e.g., slippers and drawing events) and paused in her independent play to share her observations with me (e.g., teacup saucers and baby dolls events).

Contexts for Mathematical Events at Day Care:

Commonalities and Distinctions Across Children

The children's spontaneous mathematical events at day care emerged from the various contexts that Ms. Franklin incorporated and managed in this setting including free play and unstructured periods, structured play and activities, and meal and snack times. Commonalities and distinctions occurred across the children with regard to the extent to which each child engaged in her or his respective, salient mathematical content in these various contexts.

Free play. Ms. Franklin aimed to nurture the children's sense of curiosity, imagination, and exploration and to encourage them to pursue activities and materials of their choice by providing them daily opportunities for free play. As such, the context of free play tended to foster the children's mathematical events, particularly for Anika and Aaron. For example, Anika noticed two identical teacup saucers, and she observed a difference between a toy xylophone and one pictured in a nearby book; Aaron candidly uttered the number words three through seven in a counting sequence while pretending to make hamburgers. Anika's mathematical engagement stemmed from her exploration with the materials she encountered during free play along with her intrinsic interest in meanings of difference and similarity, while Aaron's verbal expression of a count sequence naturally occurred in his play as he ran up the slide on his way to pretend to make another hamburger.

Unstructured periods. Like free play, unstructured times, such as the late afternoon period when the children waited for their parents to pick them up from day care, afforded the children the opportunity to also engage in activities of their choice

as permitted by Ms. Franklin. While Ms. Franklin did not express particular cognitive goals for the children with respect to these unstructured periods, the nature of this context, particularly the relative flexibility that Ms. Franklin granted the children regarding their choice of activity, nonetheless seemed to provide a supportive context in which Anika and Aaron spontaneously engaged in mathematical events. For example, Anika noticed a difference in the appearance of two baby dolls' outfits as she played alongside one of the toddlers while waiting for the other children's parents to come pick them up, and Aaron, during another down-time period, employed part-whole perceptual strategies and demonstrated concepts of orientation and motion as he replaced the fallen tray back on Baby D's baby seat.

Structured play or activities. Through structured play or activities, Ms. Franklin aimed to foster the children's cognitive, social, and motor-skill development to help prepare them for formal schooling. During these activities, Ms. Franklin directed children to play freely with particular materials (e.g., puzzles, blocks, computer games) or to complete specific tasks (e.g., art projects, crafts that involve fine motor skills, worksheets).

The children's interactions with Ms. Franklin during structured play or activities varied depending on the activity, the children's capacity to independently engage in the task, and Ms. Franklin's availability to work individually with the children. For example, during the puzzle event, after Aaron placed the initial pieces on his own, Ms. Franklin worked with Aaron on particular strategies to help him complete the puzzle.

During structured play with blocks, the children typically seemed to play freely on their own with little to no interaction with Ms. Franklin or their peers. In this sense, opportunities to discuss mathematical ideas such as magnitude, patterns, and shape, if shared publically, appeared to depend on or to emerge primarily from the children themselves. For instance, during his structured play with blocks, Aaron approached me with a triangular prism in hand and expressed a deep interest in identifying and naming this object. Structured play and activities with blocks and puzzles seemed most significant and meaningful for Aaron as these supported his interest in and fostered his propensity to engage in geometric thinking and spatial reasoning.

Paper-and-pencil tasks. The bears worksheet event with Anika demonstrated Ms. Franklin's occasional practice of engaging the children in worksheet tasks or other paper-pencil tasks, as she viewed these activities as valuable in preparing young children for formal schooling. Although Anika spontaneously expressed meanings of difference and similarity with everyday objects or images she encountered in her play, the formal nature of the worksheet along with the potential difficulty in discerning an image of a bear presented in varying orientations, seemed to inhibit Anika's capacity to draw upon these mathematical understandings. Upon observing Anika's initial difficulty in responding to the task, Ms. Franklin guided Anika through each item, posing questions and prompting Anika to examine systematically each image to assist her in identifying the same image as the focus bear.

In general, paper-and-pencil tasks did not arise as a meaningful or salient context in which children's significant everyday mathematical events emerged.

Meals and snack times. Meal and snack times provided a meaningful context in which the children, namely Toya and Aaron, spontaneously employed mathematical content. Toya candidly enumerated the two apple slices that she eagerly received for dessert and, later during that same lunch, she asked for two more slices to eat. At snack time, Aaron employed enumeration concepts and geometric thinking/spatial reasoning, respectively, when he identified the two pieces of a broken cookie in his bag of animal cookies and when he described the sprinkles on the cookies as “circles.” For both Toya and Aaron, their mathematical thinking focused on the food they were about to eat. While meals and snack times did not have a cognitive focus, as intended by Ms. Franklin, Toya’s and Aaron’s mathematical events involving numeric meanings of number and geometric thinking/spatial reasoning seemed to emerge naturally from their activities in these everyday contexts.

As evidenced above, the values underlying particular contexts (i.e., free play, structured play and activities) as intended by Ms. Franklin seemed to support and foster the children’s mathematical events, while in other contexts (i.e., unstructured periods, meal and snack times), for which Ms. Franklin did not have intentional goals for developing children’s cognitive growth, children’s mathematical events appeared to extended naturally from their self-directed activities.

Social Relations and Characterizing Children’s

Mathematical Thinking

Commonalities and distinctions emerged across the three children’s collections of significant mathematical events with respect to the following aspects relevant to the dimension of social relations: who initiated a mathematical event and

for what motive or purpose; who managed the identified mathematical content; and what was the nature of social interaction, if any, that occurred around the given mathematical content. Analysis across the children's mathematical events revealed that the majority of these events seemed to fall into one of three initiation-management types: child-driven, child-and-other-driven, and adult-driven mathematical events. Furthermore, distinct characterizations of children's mathematical thinking evidenced across these three categories also emerged.

Child-Driven Mathematical Events

All three children engaged in mathematical meanings through their own initiative. The children's primary motives for engaging in mathematical content appeared either goal-related or purely spontaneous.

In goal-related events, children seemed to initiate or to apply mathematical content for a specific purpose including to indicate the numerosity or relative amount of objects desired, to clarify physical attributes of objects, to complete a desired task, and to provide numeric descriptive information regarding events when engaged in narrative activities.

In purely spontaneous events, the children appeared to initiate mathematical content for its own sake, with no apparent or specific purpose in mind, not including or beyond a desire to indulge or to satisfy a personal trait or value (e.g., Aaron's inclination to observe or to tell stories, Anika's preference to interact with and to connect socially with adults). As such, child-driven, spontaneous events included those in which the children appeared to examine describe, enumerate, or inquire about objects or occurrences they encountered in their day-to-day activities and immediate

environment with no further intention beyond simply to note, to themselves or to others, what they observed.

Child-driven, goal-directed events. All three children initiated and engaged in child-driven, goal-directed mathematical events.

For Toya, most of her child-driven, goal-directed mathematical events involved her employing a number word or a relational term to indicate the amount of desired objects she wished to acquire (i.e., “three” DVD movies, “two more” apple slices, and “some” of her mama’s cookies). Once Toya initiated the mathematical content in these events by indicating the amount that she wished to have, her mother and I also participated in managing the mathematical content in response to Toya’s request. With respect to Toya’s DVD request, Nicole simply dismissed it. Regarding the cookies, as Nicole reportedly practiced, she specified to Toya the amount of cookies (e.g., two) that she could have and expected Toya to construct a set of two cookies on her own, thus fostering Toya’s capacity to construct small quantities of objects. In the apple-slices event, only three apple slices remained, thus in an effort to be fair, I opted to give one slice each to Toya and Aaron and to take one for myself. I explained my rationale for distributing the remaining slices with Toya, and she agreed that it was fair, appearing content with her one apple slice. In this sense, Toya and I shared in managing the content of enumeration.

Anika engaged in two child-driven, goal-directed mathematical events in which she initiated meanings of difference and similarity while interacting socially with me. In the latter portion of the slippers event, given that I continued to experience difficulty in interpreting Anika’s description of her mother’s slippers,

Anika aimed to clarify the appearance, including the color, of her mother's slippers to me. My need for clarification encouraged Anika to employ meanings of difference and similarity further in order to convey more clearly her observation that Christina had a pair of slippers "like" mine. In the drawing event, Anika sought to improve the accuracy of my drawing by using hand and body gestures to indicate two shirt details that she noticed missing from my sketch, detailing that she observed on the actual subjects. I revised my drawing accordingly, thus affirming Anika's initial observations of difference. In these events, through our social and verbal interactions, Anika managed the content of difference and similarity in response to my reactions to her descriptions.

Aaron engaged in child-centered, goal-directed events when he reassembled Baby D's baby seat and when he employed the number word "two" as a descriptive narrative device.

In the baby seat event, Aaron used his geometric thinking and spatial reasoning to fix or reassemble the baby seat without verbal or social interactions involving others. Ms. Franklin's lack of comment suggested that Aaron's activities were appropriate and non-disruptive to the day care setting.

Aaron employed the number word "two" as a descriptive narrative device in his goals of "reading" books and engaging in impromptu storytelling. When Aaron employed the number word two as he "read" books back to his mother, Keisha did not appear to respond to his comments. It was unclear whether Aaron's use of the number word "two" as a descriptive narrative device reflected actual images in the books he "read" or if Keisha sought to encourage Aaron's passion for reading and

chose not to interrupt Aaron's interpretations, whether or not his descriptions matched the story or its illustrations. When Aaron engaged in impromptu storytelling with me, I simply listened when he described events and enumerated objects that he imagined. I asked questions only when I needed clarification regarding, for example, the sequence of events. Furthermore, when Aaron employed the number word two to describe objects he envisioned, I did not respond particularly to these numeric details, and I let him continue on with his story. Aaron therefore managed, with seemingly no input or response from his audience, his use of the number word "two" as a descriptive narrative device when "reading" books or telling stories based on his imagination.

Child-driven, purely spontaneous events. All three children initiated and engaged in child-driven, purely spontaneous mathematical events. These types of events seemed most significant in Anika's case.

As the discussion of the characteristics of Anika's everyday mathematical events indicated, she appeared to engage in meanings of difference and/or similarity for its own sake. Anika appeared fully consumed in her meanings of difference and similarity instantiated in the barrettes, xylophones, and sorting fruit snacks events, indicating her intrinsic desire to engage independently in this content for its own sake with no concern to share her observations with adults or to seek affirmation for her understandings.

In other events, Anika did share her observations of difference and similarity with adults. A slight nuance appeared to distinguish Anika's social interactions in the baby dolls and teacup saucers events from those in the juice and Hello Kitty events.

In the former, Anika's attention primarily focused on her meanings of difference and similarity, with her social interactions taking a secondary role or demonstrating a natural reaction to her observations as indicated by Anika immediately resuming her play after noting her observation to me. In the latter, Anika's desire to interact with and to connect socially with familiar adults seemed as important to her as determining difference and similarity as she appeared to seek adult affirmation of her observations. As evidenced by these social interactions, Anika managed the mathematical content of difference and similarity in these events just as she did when she addressed this content on her own without social interactions with others.

Aaron's child-driven, purely spontaneous mathematical events tended to emerge from his inclination to observe his environment and/or from his propensity to engage in geometric thinking and spatial reasoning. For example, Aaron engaged in identifying a plastic cube as a "block" on his own with no evidence of seeking social interaction and thus managed his emergent understandings of shape. Aaron also managed the mathematical content he employed when he noticed the "circles" on his cookies and identified the two pieces that formed a broken cookie. In these cookie events, Aaron's activity of sharing his observations seemed similar to the social interactions characterizing Anika's baby dolls and teacup saucers events and thus demonstrated his interest to engage in enumeration, geometric thinking, and part-whole spatial relationships for its own sake.

In the triangular prism event, Aaron found interest in this object and sought my help to identify this solid. Thus, Aaron initiated the mathematical content to which I contributed by informing him of the name of this solid. Despite the formal

name I provided, Aaron continued to demonstrate intrigue in this content as he persisted in efforts to pronounce the words “triangular prism.”

One of Aaron’s child-driven, purely spontaneous events occurred while I chatted with Aaron’s parents in their family room. Aaron demonstrated his seemingly peaked interest in quantities of two as he spontaneously noticed “two girls” (i.e., Keisha and me) and “two boys” (Aaron Sr. and Aaron). As Aaron Sr. indirectly affirmed his son’s management of enumeration by stating to me how smart Aaron was, Aaron proceeded to explain his thinking by stating something to the effect of, “That one girl. That one girl. That one boy. That one boy.” It was unclear to what extent, if any, Aaron’s father’s indirect comments shaped Aaron’s subsequent observation. Nonetheless, Aaron demonstrated an intrinsic desire to enumerate the “girls” and “boys” he noticed in his space.

Finally, Toya appeared to engage in two child-directed, purely spontaneous mathematical events that coincidentally both involved apples. At day care, Toya spontaneously enumerated apple slices as I placed these on her plate during lunch. In an effort to avoid influencing Toya’s management of number in this context, I did not respond to her comment. Toya’s enumeration activity appeared purely spontaneous and intrinsically driven as neither Ms. Franklin, nor Toya’s mother, appeared to encourage Toya to count food items during meals or snack times. Toya’s enumeration of the apple slices may have also indicated her preference for apples, as later demonstrated when she asked for “two more” slices to eat. In the apples on a scale event, Toya spontaneously verbally identified the numerals indicated by the scale’s

needle as her mother weighed sets of apples. Toya did not seem to notice or to be bothered by the fact that her mother, her Aunt Pam, nor I responded to her activity involving number. This further demonstrated Toya's intrinsic desire to identify numerals she recognized in this context simply for its own sake.

Child-and-Other-Driven Mathematical Events

This type of mathematical event emerged as significant in Toya's case. Two qualities characterize this type of mathematical event. First, a child and a significant other participated together in an everyday, goal-directed practice in which both employed mathematical meanings for a specific purpose. Second, through interactions with significant others, both the child's capacities to engage in the goal-directed practice and understandings of mathematical content appeared to develop through a pattern of learning through guided participation described as "intent participation in community activities" (Rogoff, 2003).

Toya's everyday home practices of phoning her grandmothers with her mother's assistance, watching television with family members, and engaging in shared-play with her cousin Ricky supported Toya's developing understandings of nominal and numeric meanings of number and fostered her capacity to participate meaningfully and independently in these everyday practices. A detailed analysis of these mathematical events, including the shared management of the content of nominal and numeric meanings of number and the nature of social interactions involving this content appears in Toya's chapter in the discussion of the characteristics of her everyday mathematical events.

Adult-Driven Mathematical Events

Both Toya's enumeration of a collection of apples at the grocery store as prompted by her mother Nicole and Aaron's identification of numerals and stating of number words in a count sequence in the wall activity event as prompted by his father exemplify adult-driven mathematical events. In addition to a parent prompting or initiating the event, the following three qualities also characterize these adult-driven mathematical events.

First, the content of number instantiated in the event reflected the parent's formal perspective of mathematics and, as such, the parent viewed the activity as mathematical.

Second, the motive to prompt the child to engage in numeric meanings of number seemed to stem in part from the parent's concern for or value of engaging the child in activities perceived to be helpful in preparing the child for school mathematics. That is, the parent aimed to develop, support, or encourage the child's mathematical thinking.

Finally, the nature of social interaction between the parent and the child in the event seemed to reflect traditional, school-like practices as evidenced in the parent selecting the mathematical content and activity, directing the child's engagement in the mathematical content, posing known-answer questions to the child (e.g., Nicole asking Toya "how many" apples; Aaron Sr. pointing to the numeral 2 and asking Aaron, "What's that?"), and emphasizing and affirming correct responses from the child (e.g., Nicole prompting Toya to count the apples again after her initial incorrect response and praising Toya on her successful second attempt; Aaron Sr. locating and

naming the numeral 3 after Aaron identified the numeral 2 as “three,” and later saying “Good boy” to Aaron after he expressed the number words five through ten in a counting sequence). The adult-driven emergence, selection, and management of the mathematical content and activity and the nature of socialization involving the content of number evidenced in these events reflect some of the school domain mathematical practices found by Street et al. (2005).

While Anika’s collection of significant mathematical events did not include the content of number, her mother’s attempt to engage Anika in solving a 2-digit computation problem displayed on the VTech electronic game demonstrates the identified qualities that characterize an adult-driven mathematical event. Because of the cognitive demand and abstract nature of the task, despite Christina’s encouragement, Anika soon experienced frustration and difficulty, even in following her mother’s prompts as Christina modeled the algorithm for addition with her fingers. This was evidenced in Anika’s inattentive behavior and plea of, “Can I cut it [the game] off?” Christina’s use and modeling of the traditional algorithm over informal or alternative strategies, such as the one she described she used at work when determining the sale cost of an item of jewelry, suggested that she perceived the traditional or more formal approach as more valuable or appropriate to employ in this context. Recognizing Anika’s difficulty with this activity, Christina informed me that she intended to purchase flash cards to help Anika in math. Christina’s intent to use flash cards with Anika further demonstrated Christina’s formal perspective of mathematics as flash cards typically display bare computation problems for children to read and solve quickly.

Aaron's wall activity, the computation problems presented on Anika's VTech electronic game, and Christina's modeling of the traditional algorithm for two-column addition, reflected formal, abstract, school-like mathematics materials and activities involving number and operations.

Toya's adult-driven mathematical event of enumerating apples at the grocery store seemed qualitatively different than Aaron's and Anika's adult-driven mathematical events in that Nicole incorporated a known-answer question within Toya's familiar experience of grocery shopping with her mother and aunt. Given her understandings of Toya's capacity to count small quantities, Nicole viewed that moment of selecting apples as a meaningful context for Toya to determine "how many," albeit for the sole purpose of responding to her mother's prompt, and Nicole considered the numerosity of the apples collected as appropriate for Toya to count.

It was unclear to what extent, if any, Toya's initial response of "four" to her mother's prompt to enumerate the quantity of three apples indicated a simple counting error or if the question came unexpectedly to Toya, suggesting that Nicole's activity of prompting Toya to enumerate a quantity for no other purpose may have been unfamiliar to Toya and thus possibly occurred for my benefit.

The children's experiences engaging in numeric meanings of number when completing worksheets provided by Ms. Franklin at day care, and at home with her mother's help in Toya's case, also demonstrated adult-driven mathematical events. Regarding the bears worksheet that Ms. Franklin tasked and guided Anika to complete, it was unclear whether Ms. Franklin viewed the concepts necessary to

complete this task as mathematical, although she did perceive the activity, in general, as promoting Anika's development and preparation for later schooling.

Distinctions in Children's Mathematical Thinking

As demonstrated in the above discussion, the characterizations of children's mathematical thinking across each initiation-management group of events appeared distinct.

In child-driven mathematical events, a child's mathematical thinking seemed intrinsically motivated as the child aimed either to pursue a goal-related task or to discern, identify, or arrange objects, people, or symbols with greater specificity without further motive. A child's spontaneous mathematical thinking indicated the child's in-the-moment understandings as she/he applied mathematical content appropriately or demonstrated interest in mathematical meanings for its own sake in the course of the child's everyday activities. The development of mathematical thinking did not appear as a salient quality among child-driven mathematical events. However, mathematical learning seemed to occur in the event where Aaron inquired about a triangular prism as the feedback requested contributed to his knowledge about this shape.

Child-and-other-driven mathematical events shared some similar characteristics with child-driven events in that the child's engagement in mathematical thinking appeared intrinsically motivated and the child's use of mathematical content served to increase the child's capacity to perform and manage everyday, goal-directed activities. However, in child-and-other-driven mathematical events, the development of the child's mathematical thinking or learning of

mathematical content through “intent participation” (Rogoff, 2003) in family activities also seemed significant.

In child-and-other-driven mathematical events, the child’s development or learning of mathematical content occurred either intentionally (e.g., as Nicole provided hints to help Toya identify certain numerals on the telephone keypad in order to place a call to her grandmother) or spontaneously (e.g., when Toya initially identified the channel numeral label 44 as “four four” and moments later employed the number word “forty-four” after she heard Ricky use it as they were watching television). The child’s mathematical thinking therefore seemed to emerge with ease and to develop naturally from the child’s interest and participation in valued family activities.

In adult-driven mathematical events, a parent provided the extrinsic motivation for the child to engage in mathematical content with an aim to reinforce or to develop the child’s mathematical thinking. A child’s mathematical thinking appeared labored in varying degrees as the child attempted to respond to the parent’s prompts and use of typically formal materials or symbolic representations.

The reinforcement or meaningful development of a child’s mathematical thinking and the degree to which the child’s thinking appeared labored in adult-driven mathematical events also seemed to reflect the extent to which a given task was familiar to the child and was developmentally appropriate and, accordingly, the extent to which a child required parental or adult guidance with a task. For example, after Toya’s initial error in enumerating the set of apples at the grocery store in response to Nicole’s prompt to count them, Nicole only needed to direct Toya to count the set of

apples again in order to reinforce Toya's capacity to enumerate small quantities. In order to help Aaron continue to develop his facility to rote count and to identify numerals, Aaron Sr. provided his son with supportive prompts and gestures in the wall activity event, such as counting aloud the first few numerals in the count sequence with Aaron while pointing to each numeral displayed. In Anika's case, the nature of the Vtech computation task fell well beyond that of Anika's mathematical understandings. As such, this task, along with the resulting frustration that Anika developed in response to it, did not provide appropriate conditions in which to draw upon, much less to develop, Anika's mathematical understandings regarding number.

Common Characterizations of Children's

Everyday Mathematical Events

The set of characteristics of the children's everyday mathematical events identified below are portrayed across and within each of the children's collections of significant mathematical events. I determined these common characteristics of the children's significant everyday mathematical events by drawing on both an analysis across the three sets of characteristics of everyday mathematical events (one set for each child) identified previously as well as the discussion of social relations inherent in the child-driven mathematical events presented above.

The children's everyday mathematical events tended to:

- Emerge and evolve spontaneously from the children's intrinsic motivation,
- Demonstrate children's meaningful application of mathematical content or active engagement in mathematical thinking as they pursued goal-directed

activities or engaged in mathematical meanings for its own sake in the context of their everyday settings and activities, and

- Promote purpose-oriented verbal interactions (dialogue, negotiation, description) involving mathematical content between the children and their significant others.

These common characteristics portrayed across the children's collective significant, everyday mathematical events reflect many of the emergent qualities that Street et al. (2005) identified as characterizing the mathematical activities of young children in their home or out-of school settings.

However, one aspect that most distinguishes the common set of characteristics of children's everyday mathematical events revealed in this study from those (i.e., home mathematical practices) identified by Street et al. (2005) is that, in addition to applying mathematical content for the purpose of pursuing goal-directed activities, Anika, Aaron, and Toya also appeared to engage spontaneously in mathematical meanings simply for its own sake. In these events, the children verbally expressed or physically demonstrated their mathematical understandings and interests as they naturally examined, described, and enumerated everyday entities (e.g., barrettes, dolls, people, cookies, plastic block, and apples); sorted items (fruit snacks); inquired about novel objects (e.g., a triangular prism); and identified numerals (numerals on a produce weight scale) they encountered in their everyday settings and activities. In this way, identifying differences and similarities between objects, enumerating quantities, examining and identifying shapes, and recognizing numerals emerged as

natural activities, in and of themselves, in which the children engaged in their everyday experiences.

Some Final Thoughts

An analysis across the three sets of characteristics of the children's significant everyday mathematical events did not reveal commonalities or themes addressing the dimension of values and beliefs, in particular with respect to those values or beliefs that parents fostered in their children's everyday lives. This does not suggest that parents' values and beliefs were not salient in shaping their children's significant mathematical events. For example, as previously discussed, Nicole's value of family and kinship support and Keisha's strong emphasis on reading seemed significant in creating contexts for Toya and Aaron, respectively, that promoted in some way their understandings of number.

Alternatively, an analysis across the children's mathematical events and across the corresponding sets of characteristics of these events coupled with an examination of both the children's reported everyday activities and evidence from guided conversations with parents revealed some common values and beliefs shared across the parents. However, the extent to which these shared values and beliefs helped to support and foster their children's mathematical understandings varied. The following illustrations provide evidence of this finding.

As discussed in Aaron's chapter, Keisha, similar to Nicole, also deeply valued her family of origin. Keisha reported that as a child she not only grew up living in the same building as her extended family but that her social interactions primarily centered around her relatives. However with a family of her own, due to geographic

and financial constraints, Keisha and Aaron could only visit her family of origin a few times a year during special occasions, family reunions, or holidays. As such, Aaron had limited opportunities to socialize and learn from extended family members in the way that Toya, and perhaps his own mother, experienced.

All of the parents took great pride and care in their children's appearance, both in terms of dressing them and in maintaining and grooming their hair. However, activities involving clothing, shoes, and shopping for these items appeared mathematically meaningful only in Toya's case as these experiences seemed to encourage and promote her understandings regarding number. While each of the parents committed time during the week, some periods lasting an hour or even several hours, to maintain and style their child's hair, Christina's practice of letting Anika play with her stash of hair accessories while she did Anika's hair seemed particularly supportive in developing Anika's mathematical understandings as it encouraged her propensity to notice differences and similarities across items.

As discussed previously, all three sets of parents valued watching television or DVDs as a routine form of leisure in their family life, and all three children spent significant amounts of time watching television. However, the practice of watching television appeared significant, in terms of fostering mathematical thinking, only for Toya as the conditions shaping her experiences of watching television afforded her ongoing opportunities to notice, identify, and interpret particular numeral labels.

As these illustrations indicate, although the parents expressed or demonstrated some common values and beliefs evidenced in their routine family activities or practices, the unique ways in which these were operationalized or instantiated in each

of their children's everyday experiences, combined with each child's distinct dispositions and understandings, shaped the extent to which these values and beliefs supported contexts or practices that fostered the children's mathematical thinking. These findings not only further demonstrate the interrelated nature of the four adapted dimensions that frame the concepts of mathematical events and the characteristics of these events, but these findings also underscore the complexity in examining children's everyday mathematical experiences and understandings from a broad social perspective.

Finally, although not a feature or quality found salient across the children's collections of significant everyday mathematical events, the notion that number and numeric operations constitute the domain of mathematics emerged as a salient belief in an analysis across the data of guided conversations with the parents. The parents' formal perspective of mathematics informed their perceptions of when they thought they used or encountered mathematics in their daily lives at home and at work and when they thought their children engaged in mathematics in their day-to-day activities.

In addition to holding a formal perspective of mathematics, all of the parents also appeared to value their children's success in future schooling as they occasionally engaged their children in activities that they believed would help prepare their children for school mathematics. Parents utilized formal-like visual displays (e.g., placemats and wall calendar activity), electronic toys (VTech game), or everyday objects (apples) to prompt or to guide their children in rote counting, recognizing numerals, enumerating small quantities, or computing sums.

The parents' formal perspective of mathematics appeared to stem from their own experiences in school mathematics. These experiences also seemed to inform how parents perceived their own informal or alternative computation strategies in comparison to traditional or formal methods that they employed in school and out-of-school settings, the formal-like mathematical activities in which they engaged their children (i.e. adult-driven mathematical events), and the nature of their social and verbal interactions with their children during adult-driven mathematical events.

While adult-driven mathematical events were not often observed during data collection and therefore did not emerge as salient in the analysis of children's everyday mathematical events, this does not deny the fact that all of the parents in this study recognized in some way the importance of engaging their children in mathematical activities before they started school and that they all made efforts, as informed in part by their formal perspective of mathematics and their own experiences learning mathematics in school, to prepare their children for school mathematics.

Chapter 8: Discussion and Implications

As attention to early childhood mathematics education continues to rise, mainstream discourses concerning equity continue to characterize young children from underserved groups as deficient. This study aims to contribute to the work of researchers and practitioners who are committed to creating more socially just (Gutiérrez, 2010; Gutiérrez & Dixon-Román, 2011; Martin, 2009a, 2009b) mathematics education practices, and who seek to understand rather than to position the mathematical experiences and understandings of members of underserved groups. These efforts are particularly critical for African-American children and their families as mainstream education policy, and educational practices based on those policies, continue to position African Americans at the bottom of what Martin (2009a, 2009b) refers to as the racial hierarchy of mathematics ability.

Under the discursive frame of the achievement gap (Martin, 2009a), or what Gutiérrez (2008) terms as gap gazing, research and policy narratives (e.g., Lee & Burkham, 2002) portray young African-American children from lower-income backgrounds, before they even start formal schooling, as being inferior or at a disadvantage mathematically compared to their White, middle-class peers because it is believed that their home environments do not provide them with adequate or appropriate experiences to prepare them for school mathematics.

In order to gather factual information rather than presuming the veracity of assumed beliefs, this study investigated the everyday mathematical experiences and understandings of three, 4-year-old, African-American children from working-class backgrounds. The discussion and implications of findings from this research,

including those that address themes of inquiry that have emerged from a sociopolitical stance within mathematics education, are presented below. Limitations of the current study and additional recommendations for future research are also presented.

*Addressing Sociopolitical Themes of Inquiry in
Mathematics Education*

The discussion and implications of findings from this study address the following sociopolitical themes of inquiry in mathematics education: studying subordinate populations for their own sake (Gutiérrez & Dixon-Román, 2011) or, more specifically, understanding the mathematical experiences of African-American children without comparison to other racial groups (Martin, 2009a); broadening what counts as evidence of mathematical thinking (Gutiérrez & Dixon-Román, 2011; Martin, 2009a); and recognizing that examining the mathematical experiences and understandings of children from underserved groups can contribute to more general theories about education and learning (Gutiérrez & Dixon-Román, 2011). Findings from this study may inform efforts that seek to re-conceptualize prekindergarten and kindergarten mathematics teaching and learning practices and to examine assumptions characterizing young children's prior mathematical knowledge and experiences presumed by school mathematics practice and policy.

*Understanding the Everyday Mathematical
Experiences of Young African-American Children*

Much of the extant literature that investigates the mathematical experiences of African-Americans from a sociopolitical perspective concerns students from upper-

elementary school through college and employs identity as a construct with which to examine how students make sense of and negotiate between their racial socialization and mathematics socialization (Martin, 2009a) experiences and understandings. Findings from this study add to this body of research by portraying the everyday mathematical experiences and understandings of three African-American children before they began formal schooling.

Rather than addressing identity, this study examined the children's everyday mathematical experiences and understandings with particular attention to the social aspects in which these experiences occurred, drawing upon Street et al.'s (2005) ideological model of mathematics as "social," employing their analytic concept of mathematical events, and including characteristics of mathematical events. Conceptualizing and examining children's mathematical experiences in this way recognized ecological (e.g., García-Coll et al., 1996), ecocultural (e.g., Gallimore et al., 1993; Weisner, 2002), and sociocultural (e.g., Rogoff, 2003, Vygotsky, 1978) perspectives on child and human development, as well as a proposed person-centered cultural psychology (Gjerde, 2004) perspective.

This study identified for each child a collection of the child's significant everyday mathematical events and determined characteristics portrayed across and within these events. Analysis focused on the mathematical content in which the children engaged in their everyday settings and their mathematical understandings of this content as well as examined the adapted interrelated dimensions of purposes and settings, values and beliefs, and social relations (Street et al., 2005) that situated the children's everyday mathematical activities.

Toya, Anika, and Aaron each engaged in spontaneous mathematical events in the course of their daily activities. Each child demonstrated distinct mathematical understandings that reflected the child's unique family life and individual predispositions and knowledge development.

For Toya, her mother's values (nurturing family relationships and support, watching television for leisure, saving greeting cards, maintaining Toya's wardrobe) tended to give rise to many of the contexts that fostered Toya's mathematical thinking regarding nominal, ordinal, and numeric meanings for number.

Anika spontaneously engaged in meanings of difference and similarity, oftentimes in the context of her play with everyday objects (e.g., barrettes, toys at day care, fruit snacks) at home and at day care, and she tended to share her understandings with others, demonstrating her desire to interact socially and verbally with adults.

Aaron's emergent understandings of numeric meanings for small number, geometric thinking, and spatial sense tended to emerge either from his personal interest in reading, a practice valued and fostered by his mother, or his inclination to observe.

Although the children's mathematical events were distinct, in terms of the children's mathematical content and thinking and the social aspects situating these, the following three common characteristics emerged as salient from an analysis across the children's mathematical events. The children's mathematical events tended to: (a) emerge and evolve spontaneously from the children's intrinsic motivation, (b) demonstrate the children's meaningful application of mathematical content or active

engagement in mathematical thinking as they pursued everyday goal-directed activities or engaged in mathematical meanings for its own sake, and (c) promote purpose-oriented verbal interactions (dialogue, negotiation, description) involving mathematical content between the children and significant others.

Although these findings indicate that Toya, Anika, and Aaron meaningfully and actively engaged in mathematical thinking in their everyday lives, their experiences and understandings would most likely be positioned as inadequate or underdeveloped within current early childhood mathematics education discourse practices and narratives regarding the kinds of mathematical experiences and understandings young children “are expected” to develop prior to school. In this sense, the institutional mechanisms that maintain the racial hierarchy of mathematical ability (Martin, 2009a) seem to lock African-American children from working-class families, such as Toya, Anika, and Aaron, in place even before they enter school by dismissing, or relegating as deficient, the kinds of mathematical knowledge and experiences these African-American children *did* develop and engage in prior to formal schooling.

The characteristics of Toya, Anika, and Aaron’s collective everyday mathematical events are distinct from characteristics of children’s school mathematical events. For example, children’s school mathematical events tend to be teacher-directed, curriculum-driven, and traditionally didactic (Street et al., 2005). As Street et al. contend, this disjunction between characterizations of children’s everyday or home mathematical events and characterizations of children’s school mathematical events often results in student disengagement in school mathematics.

In order to move toward a more liberatory mathematics education for young African-American children (Martin, 2009a), I propose that mathematics education research, policy, and practice broaden what counts as evidence of young children's mathematical thinking, re-conceptualize what early school mathematics teaching and learning might look like, and examine critically what role parents may have in their young children's mathematical development.

*Broadening What Counts As Evidence of
Young Children's Mathematical Thinking*

In analyzing the set of mathematical events that emerged as most salient for each child, I encountered evidence of the child's thinking that I believed were mathematical in nature but were not addressed or identified in the extant literature as such. In order to examine these events, I extended existing conceptualizations regarding what constitutes young children's mathematical thinking and, consequently, what mathematical content and understandings could be discerned in such activity.

Given that events in all three children's cases required me to broaden the notion of what counts as evidence of young children's mathematical thinking, this suggests that the current knowledge base regarding young children's mathematical understandings and content for early childhood mathematics programs needs further examination and re-conceptualization if we are to recognize, learn from, and build upon the kinds of everyday mathematical experiences and understandings in which young children engage and develop in their everyday lives.

In Toya's case, nominal meanings of number emerged as significant, yet existing research instead privileges numeric and ordinal meanings for number when

addressing children's early number concepts. Young children encounter and must discern and navigate between nominal, ordinal, and numeric meanings of number in the course of their everyday activities, a task that seems nothing short of complex. Future research studies can help tease apart the ways in which young children make meaning of these multiple purposes of number and may provide insights into how prekindergarten and kindergarten mathematics programs can better support children's learning of number in its broader, not just numeric, sense.

In characterizing Anika's thinking regarding identifying differences and similarities between everyday entities as mathematical, I found that these processes were not addressed in the extant mathematics education literature focusing on young children's understandings in the domain of geometry, even though Anika was engaged in examining particular attributes that defined the everyday objects she observed. I argued that Anika's understandings seemed to fall within the content of data analysis, as it is currently defined in the mathematics education literature, under the concept of classification.

Anika, Aaron, and Toya, like many other young children, spontaneously engaged in observing their environment and made sense of or described what they noticed. There seems to be a lack of research in the extant mathematics education literature that examines young children's understandings as they discern qualities between or across various *everyday* objects. Research of this kind may have potential implications for mathematics education as analyzing data, examining number patterns, identifying visual patterns constructed from formal geometric shapes, and

noticing differences and similarities within these activities seem to be processes that are drawn upon regularly in existing school mathematics practices.

Aaron appeared to indicate a proclivity to engage in geometric thinking and spatial reasoning. Yet, I found limited research that could support my assertion that his activities and thinking were mathematical as much of the existing literature addressing young children's geometric thinking and spatial reasoning focuses on children's thinking with formal geometric shapes in formal contexts or school settings.

As demonstrated in Aaron's case, children's everyday experiences are filled with everyday objects. Some objects are intended to promote children's cognitive development (e.g., jigsaw puzzles), while other objects do not have an educational purpose. Everyday objects often have complex shapes and therefore do not reflect the seemingly sterile, plain, formal 2-dimensional and 3-dimensional shapes that are oftentimes the objects of young children's learning of geometry in mathematics education research and school contexts. Investigating young children's mathematical understandings as they spontaneously examine, manipulate, and engage with everyday objects can both further our knowledge regarding how young children develop geometric thinking and spatial sense in their everyday experiences and inform early childhood mathematics teaching practice.

Broadening what counts as evidence of young children's mathematical thinking to include young children's everyday mathematical understandings that are not currently privileged or recognized in mainstream early childhood mathematics education discourse also has implications for teacher education. In particular, I

propose that pre-service and in-service teacher education programs include instruction on developing teachers' capacity to recognize when children engage in mathematical meanings in their everyday activities at home and in informal activity when they are at school, and to examine what mathematical understandings children demonstrate in a particular moment or event.

Strengthening teachers' skills in identifying and analyzing what mathematical understandings young children demonstrate informally through their everyday activities and settings can inform teachers' instructional decision making and develop their capacity to support and foster more effectively student mathematics learning. Furthermore, vignettes from cases in the study can be developed and used for teacher professional development as a means for teachers to engage in instructional decision-making based on the data presented.

Re-conceptualizing Early Childhood Mathematics

Teaching and Learning Practices

Evidence from this study of Toya, Anika, and Aaron's everyday mathematical experiences and understandings offers insights into how prekindergarten and kindergarten mathematics teaching and learning might be changed to support more socially just practices. Although the following proposals have critical implications for African-American children and other populations who have been underserved historically in school mathematics, I contend that these proposals can be generalized to benefit all groups of children.

Broadening what counts as evidence of young children's mathematical thinking is one way we can begin to re-conceptualize prekindergarten and

kindergarten mathematics teaching and learning. It allows us to recognize the mathematical meanings developed by young children in their everyday lives and to understand how young children's mathematical thinking can naturally emerge from and be supported through their day-to-day activities. In this sense, it gives us a way to think about teaching and learning mathematics in early childhood contexts that is different from the taken-for-granted assumption and practice of developing children's mathematical understandings primarily through engaging in mathematics as a formal learning endeavor with little attention paid to how young children experience and develop mathematics in their everyday settings and activities. Broadening what counts as evidence of young children's mathematical thinking while critically examining what kinds of (mathematical) thinking are currently privileged in early childhood mathematics education discourse and settings, also supports sociopolitical calls to re-examine or interrogate normative assumptions regarding the mathematical development of (young) children.

The analysis of Toya, Anika, and Aaron's mathematical thinking in child-driven, child-and-other-driven, and adult-driven mathematical events presented in the prior chapters provides insights into the kinds of conditions that might prove more supportive in promoting the mathematical development of young children in schools.

Current practices in school mathematics seem to reflect adult-driven mathematical events in which *learning as an endeavor* was employed to develop a child's mathematical understandings and that a formal or non-familiar item was typically used as the object of a child's mathematics learning activity. However, as revealed in this study, a child's learning in an adult-driven mathematical event

appeared more passive and labored rather than as a natural, intrinsic outcome of the child's participation in a meaningful activity.

In child-and-other-driven mathematical events, a child's development of mathematical understandings appeared to emerge spontaneously as the child actively engaged in intent participation in a community (i.e., family) practice (Rogoff, 2003) with a significant other. In some child-and-other-driven mathematical events, the child expressed interest in participating in a particular practice and therefore teaching was intentional (e.g., when calling her grandmother, Toya interpreted Nicole's hints to find certain numerals on the telephone keypad). In other child-and-other-driven mathematical events, a child's mathematical development occurred when an opportunity to learn was seized once the child spontaneously revealed mathematical understandings or inquired about something while a significant, more knowledgeable other was present and therefore responded to the child in a way that promoted the child's mathematical thinking (e.g., while watching television, Toya identified the numeral 44 as "four four," then later as "forty-four," after she noticed Ricky gently correcting her). This characterization also reflects the mathematics learning that emerged in child-driven mathematical events (e.g., when Aaron inquired about a triangular prism).

Thus, child-and-other-driven and child-driven mathematical events, as examined in this study, both offer insights regarding conditions in formal contexts that might result in more meaningful, engaged mathematics learning for young children. These findings seem worthy of consideration when re-conceptualizing prekindergarten and kindergarten mathematics teaching and learning practices.

Emphasizing child-and-other-driven and child-driven mathematical events in school mathematics practices suggests that early childhood mathematics learning environments might better serve children if less time was spent on adult-driven, contrived learning tasks or activities of learning as an endeavor, that may hold little to no meaning for young children, and more time was spent eliciting children's active participation in meaningful, shared community practices from which mathematical concepts can emerge either naturally or through the intentional design of the teacher.

As Gutiérrez and Dixon-Román (2011) cautioned, this does not mean merely incorporating an activity or context in a task that is believed to be common among children's everyday lives as a way to "hook" them into learning mathematics. Not only can these learning activities appear fabricated, but, as this study revealed, even though children may share common experiences (e.g., watching television) this does not mean that the mathematical meanings that emerge for children in their unique experiences are similar across children nor does it guarantee that their understandings will relate to a given task.

By shared community practices, I am suggesting that an early childhood classroom engage children in activities that they experience or share together as a classroom community, where the focus is not on learning as an endeavor, but rather involves the care of and participation by both the students and teacher and that can have the potential to promote children's mathematical understandings through their intent participation in the shared practice. For example, the teacher and students can work together to negotiate and establish practices for organizing classroom materials and space. Organizational arrangements, routines, and talk centered around cleaning-

up or putting materials away can be designed to promote, for instance, children's understandings of sorting (e.g., identifying differences or similarities across attributes); early number concepts including nominal, ordinal, and numeric meanings for number; and geometric thinking and spatial sense as children actively participate in the daily practice of caring for, organizing, and maintaining their classroom materials and space.

As reported in this study, child-driven mathematical events did not tend to result in a child's learning of mathematical content, except, for example, in the case where Aaron specifically inquired about a formal 3-dimensional shape. Yet, these kinds of conditions could promote children's mathematical learning if the event could be transformed into an opportunity to learn through interaction with a knowledgeable other or an adult.

Child-driven mathematical events tended to be spontaneous, intrinsically motivated, and involve a child in applying mathematical content to pursue goal-directed activities or to engage in mathematics for its own sake. As teachers develop their capacity to recognize children's everyday mathematical experiences and understandings and to build upon these, they will be better prepared to notice a child-driven mathematical event and to use it as an opportunity to promote the child's mathematical thinking. This kind of learning would seem to support the mathematical thinking of individuals or small groups of learners during casual moments as child-driven events spontaneously occur in the classroom. On the other hand, learning through intent participation in shared community practices could support mathematics learning at the whole group, small group, and individual level.

These proposals for re-conceptualizing early childhood mathematics teaching and learning also reflect the common characteristics of the children's everyday mathematical events that emerged from an analysis across the three children in this study. As such, a similarity in the characteristics of children's mathematical events across their everyday (home) and school settings provides greater support for developing and building upon young children's mathematical experiences and understandings as they transition between these two contexts.

Examining the Role of Parents in Young

Children's Mathematical Development

Currently there are expectations that a young child comes to formal schooling with particular, (privileged) mathematical understandings and that (responsible/supportive) parents play an active role in developing their child's mathematical thinking before the child begins formal schooling. One concern with these assumptions is that parents' own mathematical experiences differ, especially with respect to their experiences, understandings, and perceptions regarding school mathematics. As a result, parents who are positioned among the top of the racial hierarchy in mathematics ability (Martin, 2009a, 2009b) have experiences and understandings that will be inherently privileged over others and therefore likely to support the favorable positioning of their young children in early childhood school mathematics settings. This in turn fosters the persistent, disparate sorting of children by racial groups in early childhood mathematics education policy and practice.

Walkerdine (1988) critically inquired, "Since when did mothers (in particular, not fathers) become pedagogues" (parenthesis in original, p. 213). Considering the

role, if any, parents might play in the mathematical development of their young children is a matter that deserves close examination and critique, particularly given the diversity and range of not just parents' mathematical experiences and understandings but also, more generally, the diversity in family contexts and experiences that exist not only across different racial, cultural, and economic groups but also within groups that share categorical memberships. As this study revealed, although Toya, Anika, and Aaron's families shared a number of categorical similarities, each child demonstrated distinct mathematical experiences and understandings. The examination of this matter is beyond the scope of this discussion, however, I offer below some insights about this issue based on findings from this study.

Nicole, Christina, and Aaron Sr. and Keisha each held a formal perspective of mathematics in which they perceived number and operations as constituting the domain of mathematics, and they each demonstrated concern about the future success of their child in school. As such, they each engaged their child in adult-driven mathematical events that they believed would help prepare their child for school mathematics. In this way, each of the parents *chose* to act as pedagogues in the mathematical development of their child. However, each parent engaged in different ways with their child in activities that they viewed as mathematical and appropriate for their child.

For Nicole, this included having Toya construct a set of cookies whenever she requested to have some, having her identify money, engaging her in completing workbook exercises, and asking Toya to enumerate a set of apples as they engaged in

the practice of grocery shopping. However, Nicole did not consider helping Toya identify television channels or numerals on the telephone keypad when calling her grandmothers as mathematical activities. Nicole also demonstrated awareness of Toya's developing understandings of number as she reported Toya's context-dependent capacity to recognize certain numerals on a telephone keypad but not in other contexts. In addition to engaging her child in activities that she believed would promote Toya's mathematical understandings, she also occasionally read to Toya at bedtime as she believed this too would help Toya "know her work" when she began school.

Aaron Sr. utilized a wall calendar activity, a formal resource that might be found in an early childhood care setting or classroom, to develop Aaron's capacity to rote count and to identify numerals. Keisha valued reading and frequently took Aaron to the library and engaged him in reading activities.

Christina sought to develop Anika's mathematical understandings by occasionally engaging her in bare computation tasks that were beyond Anika's developmental understanding.

These findings have potential implications both for parents, regardless of the extent to which they currently assume the role as pedagogues in the mathematical development of their young children, and for mathematics education to consider as a way to move toward a more liberatory (Martin, 2009a) school mathematics. The statements that follow are not meant to imply that if parents do not do what is suggested then their children are doomed to begin school with a deficit in

mathematics. Thus, the following suggestions are offered with caution and sensitivity to the unintended consequences that may result.

In the same way that there are national campaigns for parents to read to their young children as a way to promote literacy development, similar efforts might be made regarding mathematics. These efforts could (a) support parents in broadening their own perspectives of what is mathematics and recognizing how mathematics is embedded in their and their children's everyday lives, (b) encourage parents to engage their young children in mathematical meaning making within the context of their family routines and children's day-to-day activities and through culturally-relevant social and verbal interactions, and (c) inform parents about appropriate everyday activities and contexts in which they could foster their children's meaningful mathematics development.

I propose that policy and parental efforts focus on activities that support and encourage the kinds of children's everyday mathematical experiences and understandings characterized in this study and proposed in the section above. This includes developing and fostering young children's: understandings of multiple meanings of number; geometric thinking and spatial sense; and capacity to observe, describe, enumerate, and examine the objects or events they naturally encounter in their day-to-day activities and settings.

Parental efforts should not add a burden to family practices and instead be infused in established family routines and day-to-day activities (Gallimore et al., 1993) and reflect patterned ways of engaging in social and verbal interactions within the family setting. For example, as Nicole demonstrated, parents can be encouraged

to have their children use the telephone keypad to place calls to loved ones as a way to develop their capacity to identify numerals in a meaningful context. With today's technology, many calls are placed using either a speed-dial function or a direct-dial command from a phone number stored in a contact list on a given cellular phone or handset. Encouraging children instead to enter the full telephone number of the desired contact can support their developing understandings of number. Finally, efforts should promote children's mathematics learning primarily through child-and-other-driven as well as child-driven characterized mathematical events.

Limitations

One major limitation of this study concerned the amount of time I was able to visit and observe the different children. I spent considerably more hours observing Toya than the other children, including eight visits to Toya's home and other everyday settings outside of day care. Also, unlike in the cases of Anika and Aaron, the times I was able to visit and observe Toya represented a broad picture of her typical activities over a given week. For example, in addition to observing her on weekdays at day care and at home in the evenings, I also observed Toya when she and her cousin Ricky accompanied Nicole to work on a Saturday morning, when she visited her Auntie Pam's and Ricky's apartment, when she joined her mother and aunt to shop at a grocery store, and when she and Ricky went on a shopping trip with their mothers. The extensive observation hours and broad range in visits provided a richer set of data for Toya compared to the data I collected for Anika and Aaron.

The difference across the kinds and richness of mathematical experiences I was able to portray across the children may be in part because of the disparity of time

I had in observing each child at home or in other everyday settings outside of day care. While I had made plans with both Anika's mother and Aaron's parents for future home visits during my last home visit with each of them, and the parents seemed enthusiastic about these subsequent observation visits, I was not able to follow through with the parents on these plans. Although I did speak with and even met with Christina in the months following my last home visit with Anika, Christina reported that Anika had been staying with her father and his relatives and Christina did not know when Anika would return to her home. After my last visit with Aaron at home, I lost contact with his family as Aaron no longer attended day care and I was unable to reach his parents by phone. These experiences reflect challenges that researchers might face in trying to collect naturalistic observation data in children's homes and other everyday settings.

A second limitation of this research is that I was not able to share my ongoing interpretations and final analysis with the parents in an effort to get their input and feedback and to include their interpretive voice in this report. Although the study focused on the children, because of their age and level of development, their parents would have assumed a voice for their child in addition to their own. In Tillman's (2002, 2006) framework for conducting culturally sensitive research in African-American communities, she includes culturally sensitive data interpretations where, "Analysis and presentation that is appropriate to the research topic and individual group under study is co-constructed" (Tillman, 2006, p. 270). A co-constructed analysis of the data would have therefore reflected the parents' interpretations of the data in addition to that of my own.

A third limitation of this study concerns the selective nature of the interpretive process in focusing analysis on particular mathematical events and not others. The examination of the data of children's everyday mathematical events resulted in identifying content theme(s) that emerged as most salient for each child. As such, other mathematical content and understandings in which a child engaged was not presented. For example, Toya also seemed to engage significantly in spatial reasoning, or more specifically early navigation or way-finding activities. However, given the extent of the data that involved number meanings, I focused the case of Toya around these mathematical events and did not include navigation as an additional content theme.

Although the analysis employed allowed for a rich examination of a set of events within each identified mathematical content theme, the broad range of mathematical content observed was not portrayed. Additional analysis would be required to characterize this and would provide further context in which to interpret the findings reported in this study.

Finally, a fourth limitation of this study involves the confounded nature of race, culture, social class, and socioeconomic status in shaping the children's everyday mathematical experiences and understandings. Although the research questions and observation data collected were intended to capture the complex (social) nature of children's everyday mathematical experiences and understandings, distinguishing the nature and degree of influence each construct had on shaping each child's mathematical activities remained an elusive task.

However, in a few instances, the data provided evidence with which I identified connections between a child's family culture and the child's mathematical experiences and understandings. These occasions demonstrated how the data instantiated aspects of the child's family culture. However, this study's conceptual framework and the data collected did not appear to provide adequate analytic tools with which to determine explicit links between the other constructs, such as socioeconomic status, and the children's mathematical activities. As Hill, Murry, and Anderson (2005) reported, disentangling these constructs in terms of their influences on children's developmental outcomes continues to be a methodological issue for research on African-American and other ethnic minority families.

Additional Recommendations for Future Research

This study examined the everyday mathematical experiences and understandings of three, 4-year-old, African-American children from working-class backgrounds. Given the dearth of research that addresses young children's everyday mathematical experiences and understandings before they enter school, further research is needed to better understand this issue. I propose that future research focus on African-American and other children of color who have been underserved historically in schools, including children who come from a range of socioeconomic backgrounds and communities, as little is known regarding how mathematics is experienced, developed, and supported in the everyday lives of children from these populations. Efforts to disentangle the influences of race, ethnicity, culture, socioeconomic status, social class, and community context with regard to young children's everyday mathematical experiences and understandings can further

strengthen such research. Examination of findings across these studies can reveal commonalities and distinctions across and within these groups.

Regarding the learning and development of African-American children in particular, education researchers have called for examining race “as a social and political construction that affects the everyday experiences of people of color, particularly of African Americans” (Lee, Spencer, Harpalani, 2003, p. 11). Likewise within mathematics education research, Martin (2009a, 200b) has argued for perspectives that frame mathematics learning and participation as racialized forms of experience in order to examine race, as a social and political construct, when investigating the mathematical experiences and outcomes of African-American students.

In this study, Keisha reported racialized experiences in her high school mathematics classes as she felt her teachers favored and supported White classmates over her; and Aaron Sr. not only discussed his desire and efforts to develop Aaron into a strong Black male, but he also shared concerns for Aaron’s future social experiences of having to navigate life as a light-skinned Black male. What is unclear is how or the extent to which, if any, Keisha’s racialized experiences in high school mathematics and Aaron Sr.’s goals for Aaron’s racial (and gender) socialization, for example, impacted Aaron’s everyday mathematical experiences and understandings.

Although child development research identifies racial socialization as significant in African-American children’s development (Garcia et al., 1996; Harrison et al., 1990), and studies in mathematics education have begun to link students’ racial socialization with their experiences in school mathematics, what this means for young

African-American children's everyday mathematical experiences and understandings *prior* to schooling remains unclear. Furthermore, Van Ausdale and Feagin (2001) found that children in preschool demonstrated racial understandings and meanings, including racial-ethnic social status identities reflected in the larger society. How can future research addressing the everyday mathematical experiences and understandings of African-American children (or other children of color) before they enter school examine the children's racial understandings, and once these are identified, in what ways do these meanings inform, shape, or interact with the children's everyday mathematical experiences and understandings?

Although García-Coll et al.'s (1996) proposed integrative theoretical model for the study of developmental competencies in minority children highlights race (as one of four social position variables) as a primary construct that is mediated through the mechanism of racism, examining in what ways race and racism are instantiated in subsequent forces in the model, including adaptive culture and family, that are identified as proximal to or directly influencing a child's development (including a child's mathematical understandings), needs further investigation. Continued research is needed to theorize, examine, and explicate more robustly the ways in which race and racism are implicated in the everyday mathematical experiences and understandings of young African-American children, and other children of color, before they enter school.

Knowledge of African-American children's everyday language use and experiences, practices, and knowledge has demonstrated implications for culturally responsive school practice, for example, in early literacy settings (e.g., Heath, 1983)

and literary instruction in secondary contexts (e.g., Lee et al., 2003). However the links between reported or presumed “distinctive patterns of practice within the African-American community (Boykin, 1994)” (Lee, 2002, p. 285) or African-American cultural processes (Lee, 2002) and African-American children’s everyday mathematical experiences and understandings remain unclear. Although some aspects of African-American family life and culture reported in the research literature were evidenced in this study (e.g., emphasis on maintaining family relationships and supporting kin, racial socialization of children), findings did not appear to indicate patterns across the children’s mathematical experiences or to reflect practices of engaging in mathematics across the children that may be attributed to African-American cultural processes. Further research is needed to clarify the ways, if any, in which (African-American) culture is instantiated in the everyday mathematical experiences and understandings of African-American children.

The current study adapted and employed Street et al.’s (2005) analytic framework to examine the everyday mathematical experiences and understandings of young African-American children. As this discussion of proposed future research suggests, and as called for by other scholars (Lee, 2008; Lee et al., 2003), further research is needed to generate new theories and to develop or revise existing analytic frameworks (e.g., Street et. al, 2005; Garcia-Coll et al., 1996) that can examine the complex processes of learning and development within the proximal ecological and cultural settings and broader institutional and historical contexts in which these processes are situated.

Rogoff (2003) and Lee (2002, 2008) further asserted that the task of examining learning and development in its ecological, sociocultural, historical, and institutional context is by far not an individual undertaking, and instead requires a joint effort of researchers across disciplines (e.g., education, human development, psychology, anthropology). Future investigations addressing the everyday mathematical experiences and understandings of young African-American children would benefit from collaborative, cross-disciplinary research efforts while findings from such studies can suggest further re-conceptualizations of prekindergarten and kindergarten mathematics practice and promote a more socially just vision of early childhood mathematics education.

References

- Alborough, J. (2005). *Tall*. Somerville, MA: Candlewick Press.
- Allen, W. R. (1995). African American family life in societal context: Crisis and hope [Special issue on African Americans and sociology: A critical analysis]. *Sociological Forum*, 10(4), 569–592.
- Anderson, A., Anderson, J., & Thauberger, C. (2008). Mathematics learning and teaching in the early years. In O. N. Saracho & B. Spodek (Eds.), *Contemporary Perspectives on mathematics in early childhood education* (pp. 95–132). Charlotte, NC: Information Age Publishing.
- Anderson, D. D. & Gold, E. (2006). Home to school: Numeracy practices and mathematical identities. *Mathematical Thinking and Learning*, 8(3), 261–286.
- Angrosino, M. V. (2005). Recontextualizing observation: Ethnography, pedagogy, and the prospects for a progressive political agenda. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (3rd ed.) (pp. 729–746). Thousand Oaks, CA: Sage Publications.
- Anyon, J. (1997). *Ghetto schooling: A political economy of urban educational reform*. New York: Teachers College Press.
- Apple, M. W. (1996). Taking power seriously: New directions in equity in mathematics education and beyond. In W. G. Secada, E. Fennema & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 329–348). New York: Cambridge University Press.

- Aubrey, C., Bottle, G. & Godfrey, R. (2003). Early mathematics in the home and out-of-home contexts. *International Journal of Early Years Educaiton*, 11(2), 91–103.
- Baker, D., Street, B., & Tomlin, A. (2003). Mathematics as social: Understanding relationships between home and school numeracy practices. *For the Learning of Mathematics*, 23(3), 11–15.
- Baker, D., Street, B., & Tomlin, A. (2006). Navigating schooled numeracies: Explanations for low achievement, in mathematics of UK children from low SES background. *Mathematical Thinking and Learning*, 8(3), 287–307.
- Banks, J. (1994). Ethnicity, class, cognitive, and motivational styles: Research and teaching implications. In J. Kretovics & E. Nussel (Eds.), *Transforming urban education* (pp. 277– 290). Boston, MA: Allyn and Bacon. (Reprinted from *The Journal of Negro Education*, 57, 452–466.)
- Baroody, A. J. (1987). *Children's mathematical thinking: A developmental framework for preschool, primary, and special education teachers*. New York: Teachers College Press.
- Baroody, A. J. & Wilkins, J. L. M. (1999). The development of informal counting, number, and arithmetic skills and concepts. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 48–65). Reston, VA: National Council of Teachers of Mathematics.
- Barron, M. (1999). Three- and four-year-olds completing 150-piece puzzles? Impossible! *Young Children*, 54(5), 10–11.

- Bernstein, B. (1975). On the classification and framing of educational knowledge. In *Class, codes and control, vol. 3: Towards a theory of educational transmissions* (pp. 85–115). London: Routledge & Kegan Paul.
- Bishop, A. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics, 19*(2), 179–192.
- Bishop, A. (2002). Critical challenges in researching cultural issues in mathematics education. *Journal of Intercultural Studies, 23*(2), 119–131.
- Boaler, J. (2000a). Exploring situated insights into research and learning. *Journal for Research in Mathematics Education, 31*(1), 113–119.
- Boaler, J. (2000b). Introduction: Intricacies of knowledge, practice, and theory. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 243–259). Westport, CT: Ablex Publishing.
- Bregman, M. (Producer) & De Palma, B. (Director). (1983). *Scarface* [Motion picture]. United States: Universal Pictures.
- Bronfenbrenner, U. (1979). *The ecology of human development*. Cambridge, MA: Harvard University Press.
- Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32–42.
- Carraher, T. N., Carraher, D. W. & Schliemann, A. D. (2000). Mathematics in the streets and in schools. In T. P. Carpenter, J. A. Dossey & J. L. Koehler (Eds.), *Classics in mathematics education research* (pp. 187–193). Reston, VA: National Council of Teachers of Mathematics. (Reprinted from *British Journal of Developmental Psychology, 3* (1985): 21–29.)

- Caughy, M. O., O'Campo, P. J., Randolph, S. M., & Nickerson, K. (2002). The influence of racial socialization practices on the cognitive and behavioral competence of African American preschoolers. *Child Development, 73*(5), 1611–1625.
- Civil, M. (2002). Culture and mathematics: a community approach. *Journal of Intercultural Studies, 23*(2), 133–148.
- Clements, D. H. (2004). Major themes and recommendations. In D. Clements & J. Sarama (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 7–72). Mahwah, NJ: Erlbaum.
- Clements, D. H. & Sarama, J. (2007). Early childhood mathematics learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 461–556). Charlotte, North Carolina: Information Age Publishing.
- Clements, D. H., Sarama, J., & DiBiase, A. (Eds.). (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Erlbaum.
- Cobb, P. (2000). The importance of a situated view of learning to the design of research and instruction. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 45–82). Westport, CT: Ablex Publishing.
- Cobb, P., Jaworski, B. & Presmeg, N. (1996). Emergent and sociocultural views of mathematical activity. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin & B.

- Greer (Eds.), *Theories of mathematical learning* (pp. 3–20). Mahwah, NJ: Erlbaum.
- Cobb, P. & Yackel, E. (2004). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. In T. P. Carpenter, J. A. Dossey & J. L. Koehler (Eds.), *Classics in mathematics education research* (pp. 208–226).
- Cohen, D. H., Stern, V., & Balaban, N. (1997). *Observing and recording behavior of young children* (4th ed.). New York: Teachers College Press.
- Common Core State Standards Initiative. (2010). *The common core state standards for mathematics*. Retrieved from:
<http://www.corestandards.org/the-standards/mathematics>
- Crawford, K. (1996) Cultural processes and learning: Expectations, actions, and outcomes. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin & B. Greer (Eds.), *Theories of mathematical learning* (pp. 131–148). Mahwah, NJ: Erlbaum.
- Delpit, L. D. (1988). The silenced dialogue: Power and pedagogy in educating other people's children. *Harvard Educational Review*, 58(3), 280–298.
- Diversity in Mathematics Education (DiME) Center for Learning and Teaching. (2007). Culture, race, power, and mathematics education. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 405–433). Charlotte, North Carolina: Information Age Publishing.
- Forman, E. A. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform. In L. P. Steffe, P.

- Nesher, P. Cobb, G. A. Goldin & B. Greer (Eds.), *Theories of mathematical learning* (pp. 115–130). Mahwah, NJ: Erlbaum.
- Foreman, E. A. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, G. W. Martin & D. Schifter (Eds.), *A research companion to "Principles and Standards for School Mathematics"* (pp. 333–352). Reston, VA: National Council of Teachers of Mathematics.
- Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematical literacy curriculum. *Journal of Negro Education*, 59(3), 336–347.
- Freire, P. (2002). *Pedagogy of the oppressed* (30th anniv. ed.). New York: Continuum.
- Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.
- Gallimore, R., Goldenberg, C. N., & Weisner, T. S. (1993). The social construction and subjective reality of activity settings: Implications for community psychology. *American Journal of Community Psychology*, 21(4), 537–559.
- García-Coll, C., Lamberty, G., Jenkins, R., McAdoo, H. P., Crnic, K., Wasik, B. H., et al. (1996). An integrative model for the study of developmental competencies in minority children. *Child Development*, 67(5), 1891–1914.
- Gilbert, D. (1998). *The American Class Structure*. New York: Wadsworth Publishing.
- Ginsburg, H. P., Cannon, J., Eisenband, J. G., & Pappas, S. (2006). Mathematical thinking and learning. In K. McCartney & D. Phillips (Eds.), *Handbook of early child development* (pp. 208–229). Oxford: Blackwell.

- Ginsburg, H. P. & Ertle, B. (2008). Knowing the mathematics in early childhood mathematics. In O. N. Saracho & B. Spodek (Eds.), *Contemporary Perspectives on mathematics in early childhood education* (pp. 45–66). Charlotte, NC: Information Age Publishing.
- Ginsburg, H. P. & Golbeck, S. L. (2004). Thoughts on the future of research on mathematics and science learning and education [Special issue on early learning in math and science]. *Early Childhood Research Quarterly*, 19(1), 190–200.
- Ginsburg, H. P., Inoue, N. & Seo, K. H. (1999). Young children doing mathematics: Observations of everyday activities. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 88–99). Reston, VA: National Council of Teachers of Mathematics.
- Ginsburg, H. P., Klein, A., & Starkey, P. (1998). The development of children's mathematical thinking: Connecting research with practice. In I. E. Sigel & K. A. Renninger (Eds.), *Handbook of child psychology* (5th ed.) (pp. 401–476). New York: John Wiley & Sons.
- Ginsburg, H. P., Lin, C., Ness, D., & Seo, K. (2003). Young American and Chinese children's activity. *Mathematical Thinking and Learning*, 5(4), 235–258.
- Ginsburg, H. P. & Russell, R. L. (1981). Social class and racial influences on early mathematical thinking. *Monographs of the Society for Research in Child Development*, 46(6), serial no. 193.
- Ginsburg, H. P. & Seo, K.H. (1999). Mathematics in children's thinking. *Mathematical Thinking and Learning*, 1(2), 113–129.
- Gjerde, P. F. (2004). Culture, power, and experience: Toward a person-centered

- cultural psychology. *Human Development*, 47, 138–157.
- Gonzalez, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed At Risk*, 6(1&2), 115–132.
- Guberman, S. R. (1999). Cultural aspects of young children’s mathematics knowledge. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 30–36). Reston, VA: National Council of Teachers of Mathematics.
- Guberman, S. R. (2004). A comparative study of children’s out-of-school activities and arithmetical achievements. *Journal for Research in Mathematics Education*, 35(2), 117–150.
- Gutiérrez, R. (2008). A “gap gazing” fetish in mathematics education? Problematizing research on the achievement gap. *Journal for Research in Mathematics Education*, 39(4), 357–364.
- Gutiérrez, R. (2010). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*. Retrieved from:
[http://www.nctm.org/uploadedFiles/Journals_and_Books/JRME/articles/JRME_Special_Equity_Issue/jrme2010-08-5a\(1\).pdf](http://www.nctm.org/uploadedFiles/Journals_and_Books/JRME/articles/JRME_Special_Equity_Issue/jrme2010-08-5a(1).pdf)
- Gutiérrez, R. & Dixon-Román. (2011). Beyond gap-gazing: How can thinking about education comprehensively help us (re)envision mathematics education? In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (21–34). New York: Springer.
- Hansen, L. (2005). ABCs of early mathematics experiences. *Teaching Children Mathematics*, 12(4), 208–212.

- Harrison, A. O., Wilson, M. N., Pine, C. J., Chan, S. Q., & Buriel, R. (1990). Family ecologies of ethnic minority children. *Child Development*, 61, 347–362.
- Heath, S. B. (1983). *Ways with words: Language, life, and work in communities and classrooms*. Cambridge, MA: Cambridge University Press.
- Heckman, P. E. & Weissglass, J. (1994). Contextualized mathematics instruction: Moving beyond recent proposals. *For the Learning of Mathematics*, 14(1), 29–33.
- Hill, N. E., Murry, V. M., & Anderson, V. D. (2005). Sociocultural contexts of African American Families. In V. C. McLoyd, N. E. Hill, & K. A. Dodge (Eds.), *African American family life: Ecological and cultural diversity* (pp. 21–44). The Guilford Press: New York.
- hooks, b. (1994). *Teaching to transgress: Education as the practice of freedom*. New York: Routledge.
- Kirshner, D. & Whitson, J. (1997) Editors' introduction to 'Situated Cognition: Social, Semiotic, and Psychological Perspectives. In D. Kirshner & J. A. Whitson (Eds.) 1997. *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 1–16). Mahwah, NJ: Erlbaum.
- Klein, A. & Starkey, P. (1988). Universals in the development of early arithmetic cognition. In G. B. Saxe & M. Gearhart (Eds.), *Children's mathematics*. New Directions for Child Development, no. 41, (pp. 5–26). San Francisco: Jossey-Bass.
- Kochman, T. (1981). *Black and white styles in conflict*. Chicago: The University of Chicago Press.

- Ladson-Billings, G. (1995). But that's just good teaching! The case for culturally relevant pedagogy. *Theory into Practice*, 34(3), 159–165.
- Ladson-Billings, G. (1997). It doesn't add up: African-American students' mathematics achievement. *Journal for Research in Mathematics Education*, 28(6), 697–708.
- Ladson-Billings, G. & Tate, W. F. (1995). Toward a critical race theory of education. *Teachers College Record*, 97 (1), 47–68.
- Lareau, A. (2000). *Home advantage: Social class and parental intervention in elementary education* (Updated ed.). New York: Rowman & Littlefield.
- Lave, J. (1988). *Cognition in practice*. Cambridge, UK: Cambridge University Press.
- Lave, J. (1997). The culture of acquisition and the practice of understanding. In D. Kirshner & J. A. Whitson (Eds.) 1997. *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 17–36). Mahwah, NJ: Erlbaum.
- Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.
- Lee, C. D. (2002). Interrogating race and ethnicity as constructs in the examination of cultural processes in developmental research. *Human Development*, 45, 282–290.
- Lee, C. D. (2008). The centrality of culture to the scientific study of learning and development: How an ecological framework in education research facilitates civic responsibility. *Educational Research*, 37(5), 267–279.
- Lee, C. D., Spencer, M. B., & Harpalani, V. (2003). 'Every shut eye ain't sleep': Studying how people live culturally. *Educational Researcher*, 32(5), 6–13.

- Lee, V. & Burkam, D. (2002). *Inequality at the starting gate: Social background differences in achievement as children begin school*. Washington, D.C.: Economic Policy Institute.
- Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport, CT: Ablex Publishing.
- Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education*, 31(4), 454–482.
- Maldonado, N. (1996). Puzzles: A pathetically neglected, commonly available resource. *Young Children*, 51(4), 4–10
- Martin, D. B. (2000). *Mathematics success and failure among African American youth: The roles of sociohistorical context, community forces, school influence, and individual agency*. Mahwah, NJ: Lawrence Erlbaum.
- Martin, D. B. (2006). Mathematics learning and participation as racialized forms of experience: African American parents speak on the struggle for mathematics literacy. *Mathematical Thinking and Learning*, 8(3), 197–229.
- Martin, D. B. (2009a). Liberating the production of knowledge about African American children and mathematics. In D. B. Martin (Ed.), *Mathematics teaching, learning, and liberation, in the lives of Black children* (pp. 3–36). New York: Routledge.
- Martin, D. B. (2009b). Researching race in mathematics education. *Teachers College Record*, 111(2), 295–338.

- Moll, L. C. & Gonzalez, N. (2004). Engaging life: A funds-of-knowledge approach to multicultural education. In J. A. Banks & C. A. Banks (Eds.), *Handbook of research on multicultural education* (2nd ed.) (pp. 699–715). San Francisco: Jossey-Bass.
- Montford, E., & Readdick, C. (2008). Puzzlemaking and part-whole perception of two-year-old and four-year-old children. *Early Child Development and Care*, 178(5), 537–550.
- Moschkovich, J. N. (1999). Understanding the needs of Latino students in reform-oriented mathematics classrooms. In W. Secada (Ed.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 5–12). Reston, VA: National Council of Teachers of Mathematics.
- Murrell, P. C. (2009). Identity, agency, and culture: Black achievement and educational attainment. In L. Tillman (Ed.), *The Sage handbook of African American education* (pp. 89–105). Los Angeles: Sage.
- National Association for the Education of Young Children (NAEYC) & National Council of Teachers of Mathematics (NCTM). (2002/2010). *Early childhood mathematics: Promoting good beginnings*. A joint position statement. Retrieved from: <http://www.naeyc.org/files/naeyc/file/positions/psmath.pdf>
- National Center for Education Statistics (NCES). (2011). *The nation's report card: Mathematics 2011* (NCES 2012–458). Washington, D.C.: Institute of Education Sciences, U.S. Department of Education. Retrieved from: <http://nces.ed.gov/nationsreportcard/pdf/main2011/2012458.pdf>

- National Commission on Excellence in Education. (1983). *A Nation at Risk: The imperative for educational reform*. Washington, D.C.: U.S. Department of Education.
- National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Research Council (NRC). (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Committee on Early Childhood Mathematics, C.T. Cross, T. A. Woods, & H. Schweingruber (Eds.). Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.
- National Research Council (NRC), Mathematical Sciences Education Board. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.
- Oaks, J. & Lipton, M. (1996). Developing alternatives to tracking and grading. In L. I. Rendon & R. O. Hope (Eds.), *Educating a new majority: Transforming America's educational system for diversity* (pp. 168–200). San Francisco: Jossey-Bass.
- Owens, K. (1999). The role of visualization in young students' learning. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 220–234). Haifa, Israel: Technion.

- Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. New York: Oxford University Press.
- Rogoff, B. (2003). *The cultural nature of human development*. New York: Oxford University Press.
- Sarama, J. & Clements, D. H. (2008). Mathematics in early childhood. In O. N. Saracho & B. Spodek (Eds.), *Contemporary perspectives on mathematics in early childhood education* (pp. 67–94). Charlotte, NC: Information Age Publishing.
- Saxe, G. B. (1988). Candy selling and math learning. *Educational Researcher*, 17(6), 14–21.
- Saxe, G. B. (1991). *Culture and cognitive development: Studies in mathematical understanding*. Hillsdale, NJ: Erlbaum.
- Saxe, G. B. & Burmudez, T. (1996). Emergent mathematical environments in children's games. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin & B. Greer (Eds.), *Theories of mathematical learning* (pp. 51–68). Mahwah, NJ: Erlbaum.
- Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development*, 52(2), serial no. 216.
- Secada, W. G. (1989). Agenda setting, enlightened self-interest, and equity in mathematics education. *Peabody Journal of Education*, 66(2), 22–56.

- Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623–660). New York: Macmillan.
- Secada, W. G. (1995). Social and critical dimensions for equity in mathematics education. In W. G. Secada, E. Fennema & L. Byrd Adajian (Eds.), *New directions for equity in mathematics education* (pp. 146–164). New York: Cambridge University Press.
- Seo, K. & Ginsburg, H. P. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. Clements, J. Sarama, & A. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 91–104). Mahwah, NJ: Erlbaum.
- Slaughter, D. T. & Epps, E. G. (1994). The home environment and academic achievement of Black American children and youth: An overview. In J. Kretovics & E. Nussel (Eds.), *Transforming urban education* (pp. 79–95). Boston: Allyn and Bacon. (Reprinted from *The Journal of Negro Education*, 20 (1), 3–20.)
- Sophian, C. (1996). *Children's numbers*. Boulder, CO: Westview Press.
- Sophian, C. (2007). *The origins of mathematical knowledge in childhood*. New York: Erlbaum.
- Spencer, J. A. (2009). Identity at the crossroads: Understanding the practices and forces that shape African American success and struggle in mathematics. In

- D. B. Martin (Ed.), *Mathematics teaching, learning, and liberation, in the lives of Black children* (pp. 200–230). New York: Routledge.
- Stanic, G. M. (1989). Social inequality, cultural discontinuity, and equity in school mathematics. *Peabody Journal of Education*, 66(2), 57–71.
- Stinson, D. W. (2009). Negotiating sociocultural discourses: The counter-storytelling of academically and mathematically successful African American male students. In D. B. Martin (Ed.), *Mathematics teaching, learning, and liberation, in the lives of Black children* (pp. 265–288). New York: Routledge.
- Street, B., Baker, D., & Tomlin, A. (2005). *Navigating numeracies: Home/school numeracy practices*. Dordrecht, The Netherlands: Springer.
- Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory into Practice*, 34(3), 166–173.
- Tate, W. F. (1997). Race-ethnicity, SES, gender, and language proficiency trends in mathematics achievement: An update. *Journal for Research in Mathematics Education*, 28(6), 652–679.
- Taylor, J. (1998). *Kindercise with the Kinderman* [Videotape]. Columbia, MD: Author.
- Tillman, L. C. (2002). Culturally sensitive research approaches: An African American perspective. *Educational Researcher*, 31(9), 3–12.
- Tillman, L. C. (2006). Researching and writing from an African American perspective: reflective notes on three research studies. *International Journal of Qualitative Studies in Education*, 19(3), 265–287.

- Tudge, J. R. & Doucet, F. (2004). Early mathematical experiences: Observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19*(1), 21–39.
- U.S. Department of Education, National Center for Education Statistics. (2004). *ECLS-K, Base Year Public-Use Data File, Kindergarten Class of 1998-99: Data Files and Electronic Code Book (Child, Teacher, School Files), and User's Manual* (CD-ROM). (NCES 2001-029 rev). Washington, DC: Author.
- Valero, P. (2004). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 5–23). New York: Kluwer Academic Publishers.
- Van Ausdale, D. & Feagin, J. R. (2001). *The first R: How children learn race and racism*. Lanham, MD: Rowman & Littlefield Publishers.
- Villegas, A. M. (1994). School failure and cultural mismatch: Another view. In J. Kretovics & E. Nussel (Eds.), *Transforming urban education* (pp. 347–359). Boston: Allyn and Bacon. (Reprinted from *The Urban Review, 20*, 253–265.)
- Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. New York: Routledge.
- Walkerdine, V. (1997) Redefining the subject in situated cognition theory. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 57–70). Mahwah, NJ: Erlbaum.

- Weisner, T. S. (2002). Ecocultural understandings of children's developmental pathways. *Human Development, 45*, 275–281.
- Zevenbergen, R. (2001). Mathematics, social class, and linguistic capital: An analysis of mathematics classroom interactions. In B. Atweh, H. Forgasz & B. Nebres (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 201–215). Mahwah, NJ: Erlbaum.