

ABSTRACT

Title of dissertation: ESSAYS ON ISSUES IN NEW PRODUCT
 INTRODUCTION: PRODUCT
 ROLLOVERS, INFORMATION
 PROVISION, AND RETURN POLICIES

Eylem Koca, Doctor of Philosophy, 2011

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In this dissertation we study several key issues faced by firms while introducing new products to market. The first essay looks at product rollovers: introduction of a new product generation while phasing out the old one. We study the strategic decision of dual vs. single roll jointly with operational decisions of inventory and pricing during this transitional period. Our results confirm previous findings and uncover the role and interaction of several parameters that were not examined before.

In the second essay, we investigate the role of information provision and return policies in the consumer purchasing behavior and on the overall market outcome. We build a novel model of consumer learning, and we attain significant analytical findings without making any distributional assumptions. We then fully study the joint optimization problem analytically under uniform valuations.

In the third essay, we study competition in the framework described in the second essay and we identify the potential Nash equilibria and associated conditions. Our findings demonstrate the effect of competition on return policy and information provision decisions and provide insight on some real-life observations.

ESSAYS ON ISSUES IN NEW PRODUCT INTRODUCTION:
PRODUCT ROLLOVERS, INFORMATION PROVISION, AND
RETURN POLICIES

by

Eylem Koca

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2011

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Dedication

I dedicate this dissertation to the two women who define me: my guardian angel, my mother, Halime Hatun, who taught me dedication and perseverance among other things – she’s my light, my inspiration; and my dear love, my wife, Glaucia, without whom this dissertation would neither be possible nor mean anything – she’s my rock, my soulmate.

Acknowledgments

I owe my deepest gratitude to my advisor, Prof. Gilvan C. Souza, for his invaluable academic and otherwise counsel, and for not giving up on me even in the hardest times. He has been not only an incredible mentor, but also a true friend.

I would like to give my heartfelt thanks to my dissertation co-chair, Prof. Yi Xu, who was always there when I needed his guidance and help, and to Prof. Cheryl Druehl for her support over the years, and for her advice and contributions especially in the first part of this dissertation.

I would like to express my thanks Prof. Michael Ball, Prof. Peter Cramton, and Prof. Martin Dresner for agreeing to serve on my dissertation committee and for taking the time to review the manuscript. My thanks are also due to Prof. Itir Karaesmen-Aydin and Dr. Barney Corwin, for their continuous support and guidance.

Lastly, I am greatly indebted to all who helped me through my Ph.D. studies with their support, advice, and friendship.

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Chapter 1

Managing Product Rollovers

1.1 Introduction

Firms, particularly in high-tech markets, increasingly see new product introduction as a tool to gain or maintain market share, to sustain growth, and to create profits. Accordingly, firms are under constant pressure for faster time-to-market and shorter life cycles for many products, and face the challenges of managing these. In addition to traditional new product development issues such as cost, quality, and time-to-market trade-offs, more frequent product introductions result in more frequent product rollovers—the process of phasing out the old generation while introducing the new to the market. Therefore, successful product introduction requires successful management of product rollovers, which involves several interrelated decisions including those on timing, pricing, preannouncing, and controlling inventory.

There are two basic product rollover timing strategies available to a firm. In a dual product rollover (dual roll), the old generation remains in the market for some time after the launch of the new; in a single product rollover (single roll), the old generation is discontinued as soon as the new generation arrives (Billington et al. 1998). Both of these strategies have implications on the operational decisions that a firm must make. In a single roll, sharp price markdowns may be necessary to clear excess inventory of the old product. Under a dual roll, the old product retards

the diffusion of the new product into the market; this may be undesirable as new products typically command higher margins.

There are numerous real-life examples attesting to the interplay and consequences of inventory, pricing, and timing decisions. Intel had scheduled the launch of its X48 chipset for PC motherboards in January 2008, when the X38 chipset would be replaced in the high-end market. However, the launch was delayed for two months due to pressure from the world's largest motherboard manufacturer, ASUSTEK, on the grounds that it had too much inventory of X38-based parts that were marginally inferior to X48-based parts. Other manufacturers who had no inventory problems had to wait until March 2008 although they were ready for launch in January. Another motherboard manufacturer, ASRock, was first to launch its P43-based motherboards to the mainstream market in June 2008, while all other manufacturers were struggling with their inventory of older P35-based parts even with significant price cuts. In November 2007, AMD introduced deep price cuts for its older Athlon based processors and rushed its long-awaited, quad-core Phenom processors to the market before the holiday season, even though the processors had a fault which caused unexpected crashes. Further, AMD was unable to meet the demand at launch and prices remained higher than Intel's competing quad-core processors that already greatly dominated the market and performed better. In the end, although the fault was corrected by March 2008, the highly anticipated Phenom architecture failed to capture the market share expected (various online technology news sources).

Despite their importance, product rollovers are commonly mismanaged in

practice, while understudied in the academic literature. A study of 126 U.S. durable goods firms reports that 40% of new products failed after launch (Ettlie 1993), one possible reason being mismanaged rollovers such as the previous examples. Another study by Greenley and Bayus (1994) indicates that most U.S. and U.K. firms do not have a formal decision process for product rollovers. Not only is there just a handful of scholarly papers that discuss product rollover strategies, but there is little consensus among them on what rollover strategy to use under what condition. Saunders and Jobber (1994) identify 11 rollover strategies, which they call “phasing.” They survey U.S. and U.K. managers and find that some sort of dual roll was used in slightly more than half of them. Billington et al. (1998) and Erhun et al. (2007) present managerial papers that provide understanding and guidelines derived from intuition and hands-on experience, but no formal treatment of the problem. While Billington et al. (1998) associate single (dual) roll with low (high) supply and demand risk, Erhun et al. (2007) state that oftentimes the industry dictates this decision and that dual roll is an industry standard for high-tech markets even with low supply and demand risks. The only two papers to our knowledge that attempt a formal analysis of product rollovers are Levinthal and Purohit (1989) and Lim and Tang (2006), but neither model incorporates diffusion, a key attribute of high-tech markets. Although they use different terminology, Levinthal and Purohit (1989) consider three alternative strategies: single roll, dual roll, and dual roll with buy-back of the old generation. They find that single roll is always better than dual roll, and that single roll is better than dual roll with buy-backs for modest performance improvements of the new product over the old. Contrast this finding with the rec-

ommendation of Billington et al. (1998), who suggest that a large technological gap between generations (large product risk) favors dual roll. Lim and Tang (2006) find that dual roll is optimal when marginal costs across generations are similar, using a linear deterministic demand structure. A few other authors (Carrillo 2005, Li and Gao 2008, Druehl et al. 2009) simply assume a particular rollover strategy in their models, regardless of the environment.

1.1.1 Contribution of This Study

We are not aware of an academic study that provides an integrated, formal treatment of product rollovers that incorporates the dynamics discussed above; we address this gap using a comprehensive model of product rollovers that includes pricing, inventory, product diffusion, and new product preannouncement (before introduction). More specifically, our key contribution in this essay is to identify the conditions under which a particular rollover strategy (single vs. dual) is preferred, and which factors play the most significant role in this strategy decision. We describe our approach below.

We focus on successive improved generations of a single product by a firm such as ASUSTEK. The fact that high-tech products are often introduced on a relatively regular basis supports our model; this notion of (time) pacing of product updates may also improve a firm's product development capability (Eisenhardt and Brown 1998). For example, the pacemaker company Medtronic has successfully used a time-pacing strategy (Christensen 1997). We adapt the multi-generation diffusion

process by Norton and Bass (1987) to model the arrival process of potential customers through the life-cycle of a product. Here, however, an arriving customer buys the product if the price is lower than her reservation price. In addition, the firm preannounces the new product sometime before its launch and we study different levels of the market's responsiveness to preannouncements to account for the potential changes in consumer purchasing behavior due to the preannouncement. The firm first adopts a product rollover strategy, single or dual roll, then decides on the quantity for the final build of the old product and the price paths for both products.

We find that the decision between dual and single roll is not trivial and depends on a number of (exogenous) factors considered in our model. Specifically, dual roll is preferred to single roll if (i) the time between product introductions is short, (ii) the preannouncement occurs at the later stages of the life-cycle, (iii) the old product keeps more of its value at the end-of-life, (iv) the market is less responsive to preannouncements, (v) the new product is expected to have a slower market diffusion, and/or (vi) performance improvement between the new and old products is smaller ¹. We also find that the optimal price paths closely follow customer reservation prices over time.

In the next section, we show how our work relates to and differs from the existing literature. We then present our model and its analytical solutions in Section

¹Although some of these factors, such as timing of preannouncement, are in reality not exogenous but decided on by the seller, we treat them as exogenous for tractability and to focus on the two rollover strategies, and we perform a sensitivity analysis to study their impact on profit.

1.3, and Section 1.4, a comprehensive numerical analysis of the factors impacting the optimal rollover strategy. We conclude in Section 1.5.

1.2 Related Literature

We bring together elements from a diverse literature, incorporating rollover strategies, diffusion of innovations (Norton and Bass 1987, Bass and Bass 2004), preannouncements (Farrell and Saloner 1986, Manceau et al. 2002, Su and Rao 2008), dynamic pricing (Bitran and Mondschein 1997, Smith and Achabal 1998, Elmaghraby and Keskinocak 2003), and inventory management at the end of life (Cattani and Souza 2003).

A stream of research has considered the interaction of diffusion and new product generations. Savin and Terwiesch (2005) model the diffusion effects in a duopoly and find the optimal launch time. Our model differs from theirs in that we study a multi-generation scenario and the implications of single versus dual roll strategies. Earlier, Wilson and Norton (1989) determined the optimal time to introduce a product line extension; thus, the rollover strategy is not relevant. They found that the second product should generally be introduced immediately or not at all, but ignored price and inventory considerations. Mahajan and Muller (1996) extended this result in a multi-generational scenario where they found that a monopolist should introduce the next generation either early in the first product's life cycle, or wait until it has reached maturity (i.e., sales have peaked).

Pricing of a product over its life-cycle has been addressed by a large number of

researchers. Several have focused on finding an optimal pricing pattern, assuming the sales follow the Bass (1969) model (e.g., Robinson and Lakhani 1975, Bass 1980, Dolan and Jeuland 1981, Kalish 1983, Horsky 1990). However, these studies found a pricing pattern that follows the sales growth curve, which is not supported by empirical data (Krishnan et al. 1999). In more recent work, Krishnan et al. (1999) present a model extending the Generalized Bass Model (Bass et al. 1994), to find an optimal price path. None of these papers study pricing considering the next product generation.

The sequence and timing of new product introductions for two or more products with differing quality levels has been considered as a way to alleviate cannibalism (Moorthy and Png 1992, Chen and Yu 2002, Battacharya et al. 2003, Krishnan and Zhu 2006). Dhebar (1994) examines the pricing and quality level decisions for a monopolist introducing two generations of products at fixed times. He finds that the firm may limit the quality (or features) offered in each generation to minimize consumer regret. In our setting, the effects of cannibalization are modeled, but sequence is not considered, and higher quality is always valued more by the customers.

A successful rollover requires inventory management for a product (generation 1) at the end of its life. A related stream of literature focuses on determining the optimal size of a “final buy” (or “final build”) for a product nearing the end of its life when there is uncertain demand (Teunter and Fortuin 1998, Cattani and Souza 2003). Like this stream of research, we also determine the optimal size of the final build for generation 1, which in our model is being phased out for introduction of generation 2. Unlike this stream of research, our model considers the demand

interactions – cannibalization – between old and new generations.

In summary, although there is a significant body of research analyzing product introduction management, modeling life-cycle demand, and considering pricing implications, no single work demonstrates the role and interaction of these in the product rollover process. Our contribution to the literature is to investigate all these aspects of the problem and present an analytical framework for a unified understanding.

1.3 Model

1.3.1 Planning Horizon

Consider an infinite horizon where, every τ periods, a firm introduces successive new generations of a certain product, in order to replace the existing old generation. In such a setting, a transition takes place between two consecutive product generations every τ periods; our model focuses on *one* product rollover that is representative of this repetitive process. The notation used in this essay is explained in Table 1.1.

Let $t = 0$ be the time when generation 1 is introduced (made available) to the market; accordingly, generation 2 is introduced at $t = \tau$. Throughout the essay, the following terms are used interchangeably: generation 1 (2), product 1 (2), and old (new) product. The planning horizon starts at $t = \alpha\tau$, $\alpha \in (0, 1)$, which marks the time when i) generation 2 is *preannounced*, and ii) the firm produces a *final build* of generation 1 and starts concentrating her production capabilities into assuring that generation 2 is ready to launch at $t = \tau$. An immediate extension of separating these

Table 1.1: Notation.

Symbol	Explanation
i	Index for product (generation); $i = 1, 2$
j	Index for rollover strategy; $j = S$ (single); $j = D$ (dual)
τ	Time between product introductions
$\alpha\tau$	Time of final build after launch of a product; $0 < \alpha < 1$
T^j	Time that product 1 is taken out of the market for rollover strategy j
Ω	The performance of the old generation
γ	Performance improvement per generation
$h(\gamma)$	Logit probability that an old product customer instead buys the new
$\lambda_i^j(t)$	Arrival rate of customers for product i under rollover strategy j at time t
M_i	Cumulative market potential with generation i
m_i	Incremental market potential for generation i
$F^j(t)$	Fraction of market potential achieved at time t under rollover strategy j
p	Coefficient of innovation for diffusion process
q	Coefficient of imitation for diffusion process
ϕ	Customer responsiveness to preannouncements; $\phi \in (0, \infty)$
$p_i(t)$	Price of product i at time t (control variable)
$G_{it}(\cdot)$	Cumulative probability function of reservation price for product i at time t
\bar{G}_{it}	$1 - G_{it}$
$g_{it}(\cdot)$	Probability density function of reservation price for product i at time t
$I(t)$	Inventory of product 1 at time t
I_0	Size of final build for product 1 at start of planning horizon ($I_0 = I(\alpha\tau)$)
c_p	Unit production cost
c_h	Unit holding cost per period
c_s	Unit salvage value (for leftover inventory)
δ	Continuous time discount rate
$N(\cdot)$	Non-stationary Poisson process

two event times is possible; however, this can be effectively achieved by appropriately modifying the market responsiveness parameter, as seen later. A final build at $t = \alpha\tau$ means that there will be no further inventory replenishments for generation 1. This may occur because, for instance, the same production facility needs to be reconfigured to produce the new product. The end of the planning horizon is $t = (1 + \alpha)\tau$ when generation 3 is announced and a final build for generation 2 is due. The time when generation 1 is taken out of the market is T^j , $j \in \{S, D\}$, where $T^S = \tau$ for the single roll, and $T^D = (1 + \alpha)\tau$ for the dual roll (that is, in a dual roll, generation i is taken out of the market at the time generation $i + 2$ is announced). Figure 1.1 depicts the sequence of events in our model.

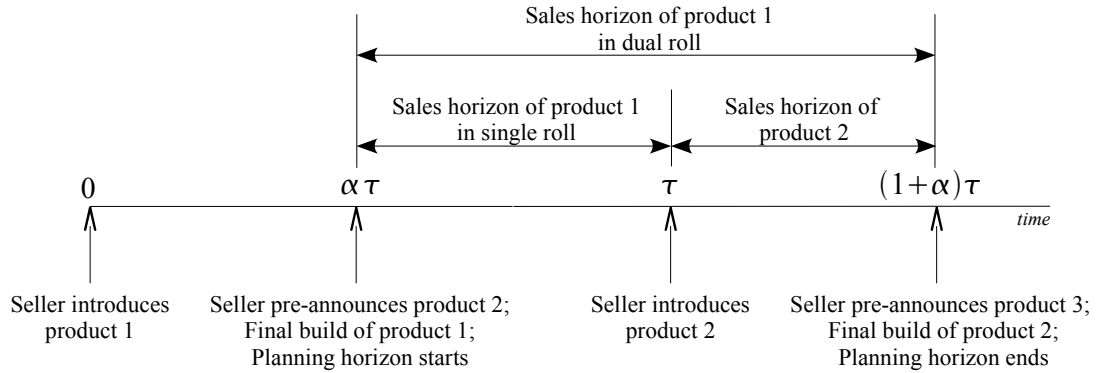


Figure 1.1: Planning horizon and sequence of events.

We assume that each new generation brings performance improvements over the old one, some of which are observable by the end users. Examples include new and/or enhanced features, better compatibility, and better environmental properties. Let Ω denote the performance (a sum of performance attributes) of the old generation. Then, the performance of the new generation is $(1 + \gamma)\Omega$, where $\gamma \geq 0$

is the performance improvement parameter.

1.3.2 Demand Process

The preannouncement of generation 2 at $t = \alpha\tau$, $\alpha \in (0, 1)$ —some time after the introduction of generation 1—starts a diffusion of “awareness” of the new product. People first become aware of the new generation even before it is launched, due to the preannouncement. They then decide to “adopt” (to become a “potential customer” for) the new generation; this decision is made based on the performance of generation 2 vis-a-vis that of generation 1. If a customer is a potential customer of generation 2, then she becomes an *actual* customer—buys product 2—when product 2 is available (and that occurs only after product 2 is launched at $t = \tau$) and if the product’s price is below her reservation price (maximum willingness-to-pay for the product at that time). We elaborate on this process below.

After generation 2 is preannounced, potential generation 1 customers *eventually* become aware of generation 2. Speed of this pre-launch information diffusion process depends on the “responsiveness” of the customer population to preannouncements (Farrell and Saloner 1986, Manceau et al. 2002, Su and Rao 2008). That is, if customers are fully responsive, then the diffusion of awareness (again, not of sales, as the product is not yet available) starts as if the product had been released; the diffusion of awareness is slower if customers are less responsive to preannouncements; finally, the diffusion of awareness does not start until the product is released if customers are not responsive. Once customers become aware of generation 2, they

decide whether to adopt the new generation by comparing the new generation’s performance relative to that of the old one. The higher the performance of the new generation relative to the old one, the higher the likelihood of a person to adopt the new product. This choice between generation 1 and 2 is modeled through the Logit model of discrete choice with a logarithmic utility function. Under this model, when a potential customer of the old generation becomes aware of the new generation, the probability that she adopts the new generation (i.e., becomes a potential customer for the new generation) is $h(\gamma) = (1 + \gamma)/(2 + \gamma)$. See Appendix A for a derivation of this formula.

We model the diffusion of awareness of the new generation through the Norton and Bass model (Norton and Bass 1987), which we refer to as N&B, as follows. In N&B, a new generation of an existing technology replaces the old generation through a process of adoption and substitution; this process continues for subsequent generations. In our model, the *arrival process of potential customers* to online and/or physical stores is described by a process similar to N&B, appropriately modified to take into account the impact of preannouncement on the diffusion of awareness of the new generation, and the likelihood of adoption $h(\gamma)$ by potential customers of the old generation who become aware of the new generation before its launch date. This is done as follows. Denoting the cumulative and incremental market potentials for generation i with M_i and m_i , respectively, the potential customer arrival rate for generation 1 is

$$\lambda_1^j(t) = \begin{cases} (M_0 + m_1) F^j(t) [1 - h(\gamma)F^j(t - \tau)], & \text{for } t \in [0, T^j] \\ 0, & \text{otherwise,} \end{cases} \quad (1.1)$$

with the *fraction* of potential customers for a generation at time t and rollover strategy j , $F^j(t)$, given by

$$F^j(t) = \begin{cases} F^A(t), & \text{if } t \leq 0 \\ \frac{1 - p \frac{1-F^A(0)}{p+qF^A(0)} e^{-(p+q)t}}{1 + q \frac{1-F^A(0)}{p+qF^A(0)} e^{-(p+q)t}}, & \text{if } 0 < t < T^j \\ 1, & \text{if } t \geq T^j, \end{cases} \quad (1.2)$$

where $F^A(\cdot)$ (Manceau et al. 2002) is given by

$$F^A(t) = \begin{cases} 0, & \text{if } t \leq -(1 - \alpha)\tau \\ \frac{1 - e^{-\phi(p+q) \left[e^{\frac{t}{\phi}} - e^{-\frac{(1-\alpha)\tau}{\phi}} \right]}}{1 + \frac{q}{p} e^{-\phi(p+q) \left[e^{\frac{t}{\phi}} - e^{-\frac{(1-\alpha)\tau}{\phi}} \right]}}, & \text{if } -(1 - \alpha)\tau < t \leq 0. \end{cases} \quad (1.3)$$

Equation (1.1) is similar to N&B except for the multiplier $h(\gamma)$, such that $h(\gamma)F^j(t - \tau)$ is the fraction of potential customers of generation 1 who switch to generation 2 due to its performance improvement. The fraction of potential customers for a generation at time t , $F^j(t)$, is higher than or equal to the corresponding $F(t)$ in N&B due to the preannouncement effect; p and q are N&B's coefficients of innovation and imitation, respectively. In N&B, $F(t) = 0 \forall t \leq 0$, but here, there is adoption of the new generation after preannouncement (but before introduction time, i.e., for $t \leq 0$), which is denoted by $F^A(t)$. The parameter $\phi \in (0, \infty)$ represents the responsiveness of customers to preannouncements. If

$\phi \rightarrow 0$, then customers are not responsive to preannouncements, and the diffusion process approaches N&B starting at $t = 0$. If, however, $\phi \rightarrow \infty$, then customers are fully responsive to preannouncements, and (1.2) is equivalent to N&B starting at $t = -(1 - \alpha)\tau$, the announcement of generation 1. Note that the time argument in Equation (1.3) is negative as $F^A(\cdot)$ represents the diffusion process due to preannouncement *before* a generation is introduced; $t \leq 0$.

For generation 2, the arrival rate of potential customers is

$$\lambda_2^j(t) = \begin{cases} [(M_0 + m_1) h(\gamma) F^j(t) + m_2] F^j(t - \tau), & \text{for } t \in (\tau, (1 + \alpha)\tau] \\ 0, & \text{otherwise.} \end{cases} \quad (1.4)$$

We further assume that market potentials follow a growth pattern according to the performance improvement: $m_i = \gamma M_{i-1}$ and $M_i = M_{i-1} + m_i$, where m_i (M_i) is the incremental (cumulative) market potential for generation i . Figure 1.2 demonstrates potential customer arrival intensities during the planning horizon for $p + q = 0.3$, $q/p = 25$, $M_0 = 100$, $\gamma = 0.5$, $\tau = 20$, $\alpha = 0.5$, and $\phi = 6.275$. Note that the arrival rate for generation 1 is independent of the rollover strategy used for $t \leq \tau$, but for $t \geq \tau$, $\lambda_1^S(t) = 0$. Because the market is somewhat responsive to preannouncements ($\phi > 0$), the arrival rate for generation 2 at the time of its introduction at τ is larger than 0 (zero would be the traditional diffusion pattern of N&B). We also have $\lambda_2^S(t) > \lambda_2^D(t)$ because there is some cannibalization of generation 2 by generation 1 in a dual roll. In Figure 1.3, the effect of ϕ is illustrated using $p + q = 0.3$, $q/p = 25$, $M_0 = 100$, $\gamma = 0.5$, $\tau = 20$, and $\alpha = 0.5$. Note that for $\phi \rightarrow 0$, the market is unresponsive to preannouncements, and thus the diffusion of generation 2 only starts when it is actually introduced at $t = \tau$. For $\phi \rightarrow \infty$, the

market is fully responsive to preannouncements, and diffusion starts immediately after preannouncement at $\alpha\tau$, as if generation 2 had been introduced at that time.

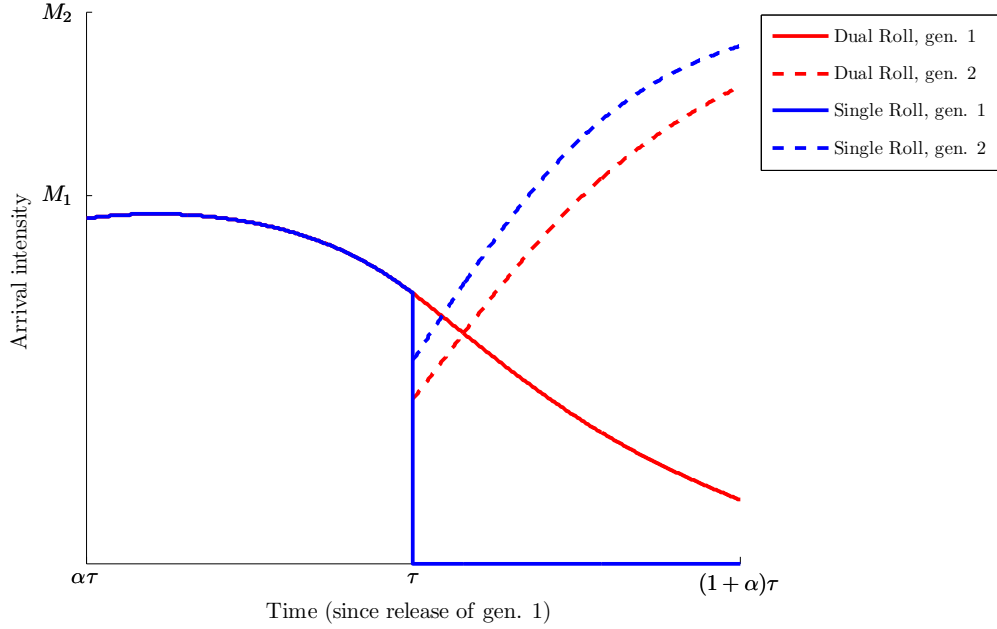


Figure 1.2: Customer arrival intensities for each rollover strategy.

As stated before, the *actual sales* rate of generation i at time t depends on the arrival rate of potential customers $\lambda_i^j(t)$, price $p_i(t)$ of product i , and the distribution of customer reservation prices. Price will be discussed in the next section. Customers of product i at time t have reservation prices distributed according to the cumulative distribution function (cdf) $G_{it}(\cdot)$, and probability density function (pdf) $g_{it}(\cdot)$. We assume that $G_{it}(\cdot)$ has the shape of a Weibull distribution, as this distribution is able to capture a variety of consumer behavior and has been used previously in the literature (Bitran and Mondschein 1997). The Weibull distribution has two parameters; the mean is mainly determined by the scale parameter β , and the variance by both β and the shape parameter k . For illustration, Figure 1.4 plots the

reservation price distribution $g_{it}(\cdot)$ for different shape and scale parameters. The firm knows the distributions for both products at any time; this knowledge feature is common in most marketing and operations models of consumer behavior.

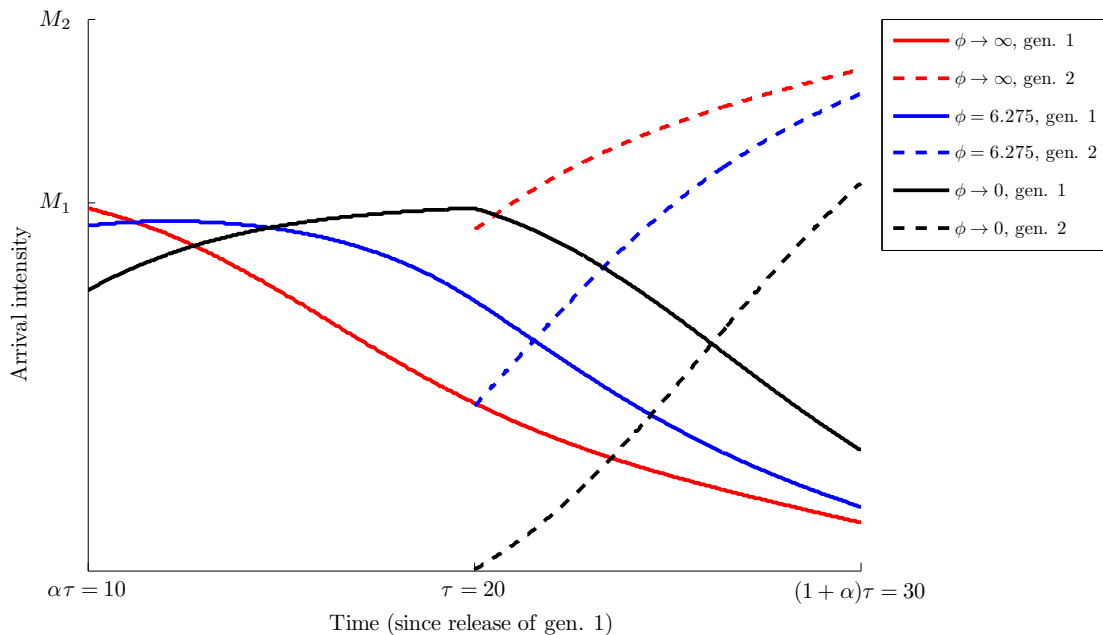


Figure 1.3: Customer arrival intensities for different responsiveness parameters.

We state the assumptions underlying the demand process in this essay as follows; we comment on these assumptions later in Section 1.5:

- (i) There are no explicit competing firms or products or expectation of any.
- (ii) Product generations interact only through the arrival process described above.

Once a customer makes a decision to adopt the new generation, her actual purchase decision is based on the price of the new generation; she does not re-evaluate her decision (i.e., consider the old generation) if the price of the new generation is higher than her reservation price.

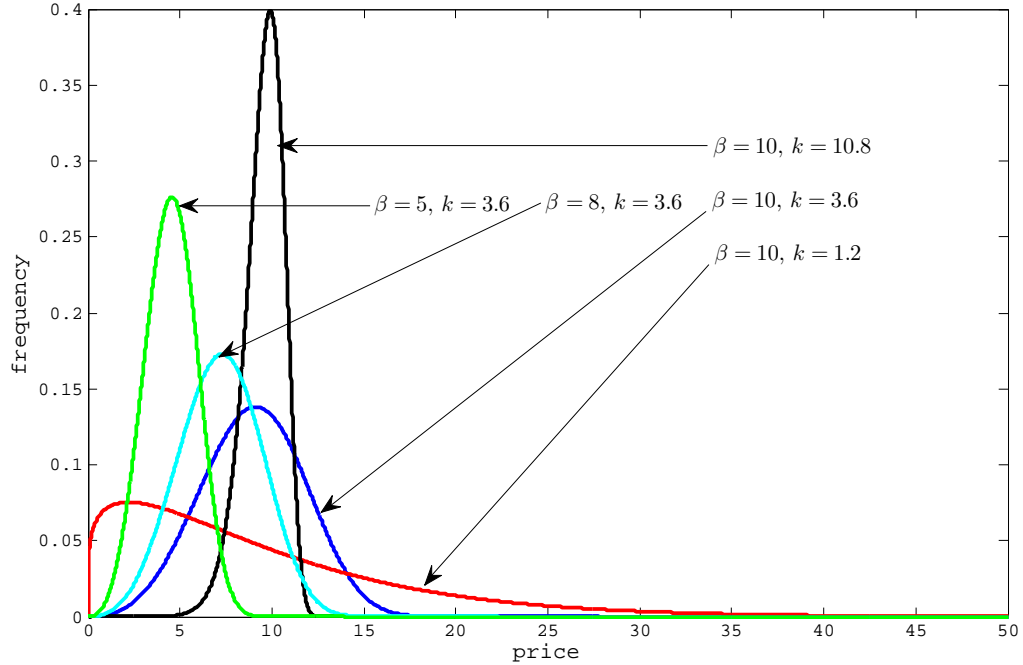


Figure 1.4: Weibull densities with different shape and scale parameters.

- (iii) At any time, there are at most two product generations in the market.
- (iv) Prices have no influence on the customer arrival processes, although they impact actual sales, because an arriving customer only buys if the price is below her reservation price. Thus, increasing prices decreases sales monotonically.
- (v) Customers are neither price strategic nor do they expect a new generation to be introduced before its announcement.

1.3.3 Optimization Problem

At the start of the planning horizon, the firm decides on the inventory level for product 1, denoted by $I_0 = I(\alpha\tau)$, and the price paths for both products, $p_i(t)$, $i = 1, 2$, throughout the horizon such that expected discounted profits are maximized. We

assume a unit production cost, c_p , that is constant over time and across generations. For product 1, there is a constant holding cost of c_h per unit per time, and a constant unit salvage value $c_s < c_p$ for any remaining inventory at time T^j . We assume that the firm uses a continuous review, instantaneous replenishment inventory control policy for product 2; therefore, there are no holding costs and no lost sales. We find that the inventory control policy for product 2 does not significantly affect the comparison of rollover strategies, enabling us to make this simplifying assumption. We have also analyzed periodic review order-up-to policies and found that the single vs. dual roll comparison was not significantly affected by the number of inventory reviews. This result is primarily driven by our assumption that the firm faces no supply constraints for the new generation. Although some firms face capacity constraints for new products, particularly immediately after introduction if the product is popular, our model does not capture this effect, and we leave investigation of supply constraints for future research.

The discount rate is δ and a non-stationary Poisson process, with time-dependent arrival intensity $\lambda_i^j(t)$ as its argument, is denoted by $N(\cdot)$. The profit maximization problem depends on the rollover strategy and is solved separately for each strategy. Given the underlying diffusion dynamics, the arrival processes for the two generations are independent from each other; there are no price or inventory interactions between the two arrival processes. Thus, for each strategy, we can partition the optimization problem into separate problems for each product.

By selecting a rollover strategy $j \in \{S, D\}$, the firm faces the following continuous-time stochastic optimization problem for product 1, where $\bar{G}_{it} = 1 - G_{it}$

denotes the tail distribution:

$$\begin{aligned} \max_{I_0, p_1(t)} \Pi_1^j = \mathbb{E} & \left[- \int_{\alpha\tau}^{T^j} e^{-\delta(t-\alpha\tau)} p_1(t) dI(t) + e^{-\delta(T^j-\alpha\tau)} c_s I(T^j) \right. \\ & \left. - c_h \int_{\alpha\tau}^{T^j} e^{-\delta(t-\alpha\tau)} I(t) dt - c_p I_0 \right] \end{aligned} \quad (1.5)$$

s.t.

$$I(T^j) \geq 0$$

$$I(t) = I_0 - N \left(\int_{\alpha\tau}^t \lambda_1^j(u) \bar{G}_{1u}(p_1) du \right), \quad t \in [\alpha\tau, T^j].$$

Note that $I(t)$ describes the inventory remaining at time t and we require that inventory is nonnegative when the product is pulled from the market. The first term of the expectation is price times sales rate, the second and third terms account for salvage and holding costs, respectively, and the last is the cost of final build.

The firm's problem for product 2 is:

$$\max_{p_2(t)} \Pi_2^j = \mathbb{E} \left[\int_{\tau}^{(1+\alpha)\tau} e^{-\delta(t-\alpha\tau)} (p_2(t) - c_p) N(\lambda_2^j(t) \bar{G}_{2t}(p_2)) dt \right] \quad (1.6)$$

1.3.4 Solution

The optimal price path for product 2 can be determined in a straightforward manner, as shown in Proposition 1 below.

Proposition 1 *The optimal price path for product 2 satisfies:*

$$p_2(t) - \frac{\bar{G}_{2t}(p_2(t))}{g_{2t}(p_2(t))} = c_p, \quad \forall t \quad (1.7)$$

This price path is unique if and only if $\frac{(\bar{G}_{2t})^2}{g_{2t}}$ is a decreasing function of $p_2(t)$, $\forall t$.

Proof See Appendix A. ■

Proposition 1 shows that the price for generation 2 at any point in time depends simply on the production cost and the consumer's reservation price distribution at that time. Due to Assumption 2 in the previous section, there is no interaction with generation 1 customers. Inventory availability also does not affect price due to the assumptions on reservation prices and inventory replenishment. If we assume that $G_{2t}(\cdot) = G_2(\cdot) \forall t$, then $p_2(t)$ will be constant.

The optimization problem (1.5) for product 1 is not tractable due to its stochastic nature and the existence of multiple decision variables. Proposition 2 below shows that the deterministic version of this problem is asymptotically optimal as arrival intensities grow large.

Proposition 2 *Solution to the following deterministic optimal control problem is asymptotically optimal to (1.5) as M_0 grows large.*

$$\begin{aligned} \max_{I_0, p_1(t)} \Pi_1^j &= \int_{\alpha\tau}^{T^j} e^{-\delta(t-\alpha\tau)} (p_1 \lambda_1^j(t) \bar{G}_{1t}(p_1) - c_h I(t)) dt + e^{-\delta(T^j-\alpha\tau)} c_s I(T^j) - c_p I_0 \\ \text{s.t.} \\ I(T^j) &\geq 0 \\ \frac{dI(t)}{dt} &= -\lambda_1^j(t) \bar{G}_{1t}(p_1) \text{ for } t \in [\alpha\tau, T^j] \end{aligned} \tag{1.8}$$

The optimal price path for product 1 from Equation (1.8) satisfies

$$p_1(t) - \frac{\bar{G}_{1t}(p_1(t))}{g_{1t}(p_1(t))} = e^{\delta(t-\alpha\tau)} \left(c_p + \frac{c_h}{\delta} \right) - \frac{c_h}{\delta}, \forall t \tag{1.9}$$

and is unique if and only if $\frac{(\bar{G}_{1t})^2}{g_{1t}}$ is a decreasing function of $p_1(t)$, $\forall t$.

The associated optimal initial inventory is

$$I_0 = \int_{\alpha\tau}^{T^j} \lambda_1^j(t) \bar{G}_{1t}(p_1) dt \tag{1.10}$$

Proof See Appendix A. ■

Note that the optimal price path for product 1 closely follows the reservation price curve, very similarly to the price path for product 2. The only difference is that the price for product 1 at any time t also accounts for the holding cost of inventory incurred between $\alpha\tau$ and t . If the reservation price curve for product 1 is decreasing in t , which is a reasonable scenario considering that the product is ending its life, then the optimal price will also decrease in t accordingly. Because we find the solution to the asymptotically optimal deterministic problem, the final build inventory level I_0 will be exactly sufficient to satisfy all demand between $[\alpha\tau, T^j]$ and there will be no leftover inventory.

To find a price path, one needs to solve equations (1.7) and (1.9) for each time point t . Given the Weibull distributions for G_{it} , there are no closed form solutions for $p_i(t)$. Numerically, however, this is straightforward: discretize the planning horizon, and solve (1.7) and (1.9), through any line-search algorithm, for each discrete t . We study the problem numerically in the next section.

1.4 Comparison of Rollover Strategies: Numerical Analysis

To develop further insight into the choice of rollover strategy, we turn to numerical analysis and run a full-factorial experimental design with eight model parameters at three levels each (low, medium and high). This allows us to better understand under which conditions of parameter values a particular rollover strategy is preferred, based on maximal profits resulting from the optimization procedure described in

Section 1.3. We now describe our experiment.

1.4.1 Parameters Describing the Planning Horizon: τ, α

The planning horizon is $[\alpha\tau, (1 + \alpha)\tau]$; however, sales horizons for the two products differ as shown in Figure 1.1. For the old product, length of the sales horizon (during the planning horizon) for single roll is $(1 - \alpha)\tau$, while that for dual roll is τ . For the new product, length of the sales horizon does not depend on the rollover strategy and is always equal to $\alpha\tau$. Therefore, given τ , a small (large) α indicates a long (short) sales horizon for the old product under single roll. Consequently, we expect dual roll to result in higher average profits compared to single roll as α increases. The effect of τ , however, is not as straightforward. A longer time horizon means higher total sales; however, price may decrease more and there may be downward substitution, negatively affecting the profit rate. In the following numerical studies, the time unit is months, and we use (10, 20, 30) months for τ ; these are typical times between product introductions in the high tech industry (Druehl et al. 2009). Given that $0 < \alpha < 1$, we use $\alpha \in \{0.3, 0.5, 0.7\}$ in order to capture scenarios where the final build decision occurs very early in the life-cycle ($\alpha = 0.3$) to late in the life-cycle ($\alpha = 0.7$), reflecting, for example, different manufacturing lead times.

1.4.2 Parameters Describing the Arrival Process: $p, q, \theta, \gamma, M_0$

Recall the arrival process consists of a non-stationary Poisson process whose mean depends on the diffusion process. First, noting that the initial market size is just

a scale parameter, we set $M_0 = 100$. For the Bass coefficients p and q , rather than using individual parameters, we use $p + q$ and q/p , as done by Krishnan et al. (1999) and Druehl et al. (2009). We experiment with $(0.2, 0.3, 0.4)$ for $p + q$ and with $(1, 5, 25)$ for q/p . These values, which remain constant over time and across generations (Norton and Bass 1987), capture diffusion processes with different characteristics, as shown in Figure 1.5. The diffusion rate is faster with increasing $p + q$, and/or with decreasing q/p . We expect slower diffusion rates to favor dual roll; as the new product garners adoptions, the old product continues selling at a higher rate.

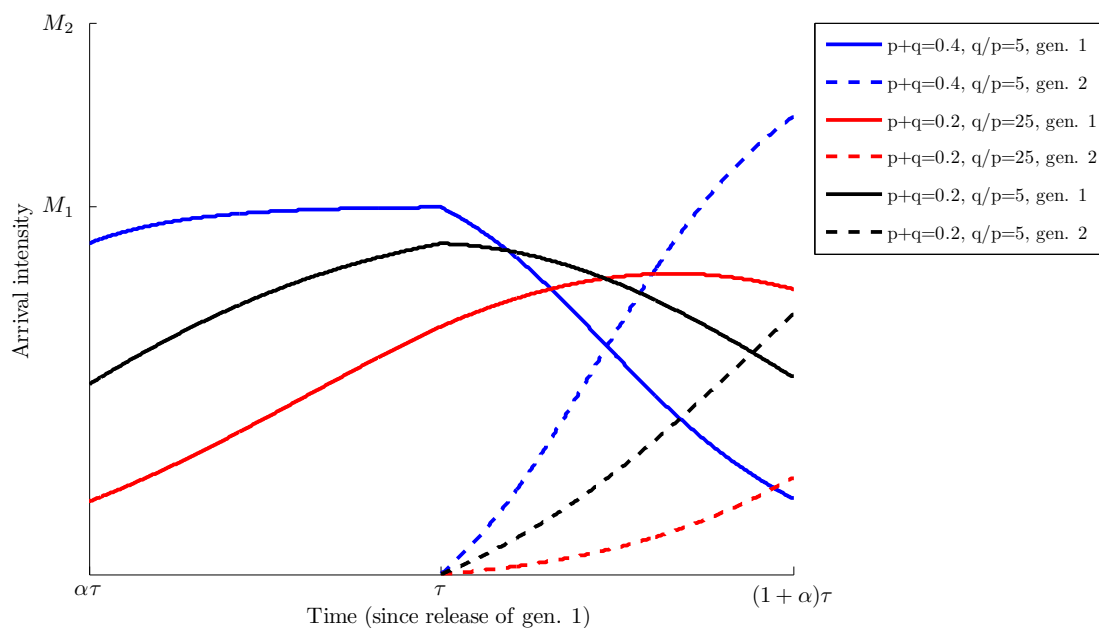


Figure 1.5: Arrival rates for different diffusion parameters.

Recall that the market responsiveness to preannouncements parameter ϕ takes values from 0 (no responsiveness) to ∞ (full responsiveness). As a result, we consider in our study three levels of ϕ , as shown in Figure 1.3: 0 (low), 6.275 (medium), and

∞ (high). Higher ϕ indicates higher diffusion rates for the new product and therefore lower market risk; thus, we expect higher ϕ to favor the single product roll.

The levels for the performance improvement per generation, γ , are $(0, 0.5, 1)$, corresponding to no improvement, 50% improvement, or 100% improvement. Recall that γ plays a role in the arrival process of the new generation in two ways. First, performance improvement expands the potential customer base according to $M_i = (1 + \gamma)M_{i-1}$. Thus the levels of γ correspond to no growth, 50% growth, and the market doubling in size. Second, performance improvement increases the probability of customers choosing the new product over the old according to $h(\gamma)$. The levels correspond to probabilities of choosing new over old of $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{2}{3}$. In other words, higher γ refers to lower market risk for the new product and should favor the single roll. Finally, γ affects reservation prices as described next.

1.4.3 Parameters Describing the Reservation Prices: μ, k

Recall that customer reservation prices follow Weibull distributions. In order to reflect diminishing customer valuations over time, we let the scale parameter β of the old generation decrease linearly with a slope per τ equal to $\mu \leq 0$ ². Let $\beta_i(t)$ be the Weibull scale parameter for product i at time t and define

$$\beta_1(t) = \beta_1(\alpha\tau) \left[1 + \mu \left(\frac{t}{\tau} - \alpha \right) \right], \quad t \in [\alpha\tau, (1 + \alpha)\tau].$$

²Other decreasing patterns, such as convex, concave, or piecewise linear, can be used; however, preliminary tests showed that this choice does not significantly impact our results.

We set $\beta_1(\alpha\tau) = 10$, without loss of generality, and experiment with μ using $(-0.2, -0.5, -0.8)$. By definition, higher μ means that customer reservation prices decline slowly and the old product can thus sell at higher prices longer. Therefore, higher μ should favor dual roll, where the old product remains in the market longer.

For product 2, we find that an increasing or a decreasing pattern for $\beta_2(t)$ does not significantly affect the results but does complicate the model. Thus we use a constant scale parameter where $\beta_2(t) = \beta_1(\alpha\tau)(1 + \gamma) = 10(1 + \gamma)$, $\forall t$. We base this assumption on the fact that typically customer willingness to pay increases with performance, and on the definition of the reservation price R_{it} (as discussed in Appendix A). Using this definition requires that $\beta_2((1 + \alpha)\tau) = \beta_1((1 + \alpha)\tau) = \beta_1(\alpha\tau)(1 + \gamma)$.

We assume that the Weibull shape parameter k_i is the same for both generations and is time invariant. We run experiments with $k_1 = k_2 = k = (1.2, 3.6, 10.8)$, which reflect different variability levels in consumer valuations (see Figure 1.4). Note also that because we assume $G_{2t}(\cdot) = G_2(\cdot) \forall t$, according to Proposition 1, the price for the new product will be constant during the planning horizon.

1.4.4 Auxiliary Parameters and Summary of Runs

The unit production cost is a scale parameter and we set $c_p = 1$. Choice of unit salvage value is arbitrary as long as it satisfies the condition that marginal cost is positive; in other words, c_s makes the right-hand side of Equation (A.9) in Appendix A greater than zero. We set the unit holding cost per period to $c_h = 0.025$,

corresponding to approximately 34.5% of unit production cost annually. Furthermore, we set the continuous time discount rate to $\delta = 0.005$ per period, $\approx 6.2\%$ per year. Preliminary tests showed that c_h and δ were not significantly influential on the comparison between rollover strategies and are held fixed. We summarize our experimental design in Table 1.2.

Table 1.2: Experimental design.

Parameter	Levels
τ	(10, 20, 30)
α	(0.3, 0.5, 0.7)
$p + q$	(0.2, 0.3, 0.4)
q/p	(1, 5, 25)
ϕ	(0, 6.275, ∞)
γ	(0, 0.5, 1)
μ	(-0.2, -0.5, -0.8)
k	(1.2, 3.6, 10.8)

In total, there are $3^8 = 6,561$ experiments. For each experiment (run), we use $dt = 0.01$ and use Propositions 1 and 2 to compute the prices, sales, inventory levels and resulting profits in time, for both the dual and single roll strategies. Then, we calculate the difference in profits (reported as dual minus single roll) for each of the 6,561 cases. Finally, we conduct statistical analyses to understand the role of experimental (input) parameters on the profit difference. Specifically, we use regression with the experimental parameters as independent variables and analyze the impact of each parameter on profit difference, the dependent variable, through the t-statistic for its respective regression coefficient; this *global* sensitivity analysis

is suggested by Wagner (1995), and it has been used in previous research as we discuss below.

Table 1.3 gives the summary statistics for the difference in profits between the rollover strategies. The average profit difference between strategies (profit in a dual roll minus profit in a single roll) is -24.7, with a minimum, median, and maximum values of -9944.7, 154.8, and 11086.2. This means that the choice between a single and dual roll strategy is not trivial; values of the input parameters determine the best strategy. These results confirm the earlier research (Billington et al. 1998, Erhun et al. 2007) on rollover strategy that argues that the choice is situation dependent. Similar to those papers, our purpose is not to decide which rollover strategy is better, but rather to investigate which one is better under what setting in a more concrete analytical setting. Therefore, we run a series of regression analyses in order to understand the effect of input parameters in this comparison of the two strategies.

Table 1.3: Statistics for difference in profits (dual minus single).

Statistic	Value
Minimum	-9,944.7
25 th percentile	-780.7
Median	154.8
Mean	-24.7
75 th percentile	877.3
Maximum	11,086.2

1.4.5 Statistical Analysis

We first pre-process the input parameters so that the regression results yield more meaningful insights. For k and q/p , we employ a log transformation; we then normalize ϕ such that $\phi \rightarrow \infty$ corresponds to 1, $\phi \rightarrow 0$ corresponds to 0, and accordingly, $\phi = 6.275$ to 0.5 (see Appendix A for the normalizing relationship); finally, we center all variables around their respective means. We perform multiple linear regression with the profit difference as the dependent variable and the processed input parameters as the independent variables. The summary of this regression is given in Table 1.4. We present the t-values and the statistical significance levels to summarize the level and direction of the particular variable's influence (Druehl et al. 2009, Souza et al. 2004). All variables are significant at $p < 0.0001$ (except for $\log(k)$, which is significant at $p < 0.05$), and we note that the signs on the estimates confirm our initial expectations.

We also performed two more multiple linear regressions with the profits from dual and single roll, respectively, as the dependent variables to examine the effect of the input parameters on these profits. All variables are significant at $p < 0.0001$ and again the signs on the estimates confirm expectations.

We make the following observations:

- (i) Increasing τ , the time between introductions, increases the sales horizon for both the old and new product under either single or dual roll, and therefore results in higher profits under either strategy. Although the theoretical implications of τ are unclear, in our setting we find that its effect on single roll is

greater and therefore that larger τ favors single roll.

Table 1.4: Statistics of multiple linear regression: Main effects.

Dependent variable:	Dual - Single profit difference	
Factor	t-value	
(intercept)	-1.6	.
τ	-36.4	***
α	5.4	***
$\log(k)$	-2.3	*
μ	21.1	***
γ	-21.3	***
$p + q$	-26.1	***
$\log(q/p)$	40.8	***
ϕ	-56.5	***
Adj. R-sq.	0.543	

Statistical significance codes: ‘***’: $p \approx 0$; ‘*’: $0.01 < p < 0.05$; ‘.’: $p > 0.1$

- (ii) As α increases, the dual roll strategy tends to have higher profits compared to single roll. This is because a higher value of α shortens the selling horizon for the first generation in single roll and as a result decreases sales.
- (iii) As expected, higher diffusion speed—higher $p + q$, lower q/p , higher ϕ —favors single product rollovers, while increasing the profits for both strategies on average.
- (iv) Similarly, a higher performance improvement amount, γ , results in faster diffusion and a larger and more favorable market for the new product. Consequently, a higher γ also favors single roll as well as increases profits on average under either strategy.

- (v) Increasing μ or k increases profits under either strategy. A larger μ , affecting the reservation price scale parameter, indicates a more favorable market for the old generation, and therefore a dual roll tends to result in higher profits. A larger k (shape parameter) favors a single roll, but it has the least statistical significance in the comparison. We conclude that lower variance in reservation prices increases profits under dual and single roll in almost the same way, and thus loses its significance when they are compared.
- (vi) Given by their t -values, parameters are ordered as $\phi > q/p > \tau > p+q > \gamma > \mu > \alpha > k$ with respect to their influence on the profit difference. With respect to profit levels, the ordering is $\gamma > \tau > k > \alpha > \phi > q/p > p+q > \mu$. This indicates that in the decision of whether to use a single or dual roll strategy, the diffusion rates, time between introductions, and performance improvement are the most important. Recall that ϕ , q/p and $p+q$ are all measures of the speed of diffusion. Given a particular rollover strategy, the strongest influencers of profit are performance improvement, time between introductions, and the shape parameter.

An interesting observation from the experimental runs is when the maximum and minimum differences between dual and single profits occur. Dual (single) roll achieves the greatest profit advantage when q/p is at the highest (lowest) level, $p+q$ and ϕ are at their lowest (highest) levels, while k , α , μ , τ , and γ are at their highest levels. This seems to indicate that market risk, and in particular, speed of diffusion, is a key factor in the decision of rollover strategy, again confirming

managerial prescriptions (Billington et al. 1998, Erhun et al. 2007).

We also consider the effect of interactions between the eight parameters because the adjusted R -squared value of 0.543, while it does not take away from our first-order insights, indicates that the main effects-only model is not sufficient to fully explain the profit difference between dual and single roll. Therefore, in order to obtain deeper insights, we conduct additional regression analyses incorporating two-way interaction effects between all input parameters. Here we summarize our results which are given in Table A.1 in Appendix A. All two-way interactions are highly significant ($p < 0.0001$), with the exception of τ and μ , α and $\log(k)$. The adjusted R -squared value increases to 0.837, showing that the optimal rollover strategy decision is driven by the interplay between the input parameters. We find that the shape parameter k , whose main effect on profit differences is not highly significant, is highly significant in interaction with other independent variables. Moreover, we observe that for all variables, the interaction terms with k have the same sign as their main effects in Table 1.4. In other words, higher k amplifies the main effects of all other variables. This means that if the variability of customer reservation prices is (or, is predicted to be) low (i.e., high k), then any errors in estimating (or, deciding on, when applicable) the other parameters will have more serious consequences and may easily result in an erroneous choice between single and dual roll strategies.

1.5 Conclusion

We study the decision process a firm undertakes when introducing a new generation of a product while phasing out the old. We address many aspects of this problem, including product rollover strategy (single vs. dual), dynamic pricing and inventory control, preannouncements, and technology advancement. Erhun et al. (2007) and Billington et al. (1998) indicate that the demand situation influences the choice of rollover strategy. Accordingly, we use a demand model that incorporates an information diffusion process based on preannouncements and the Bass model, as well as a heterogeneous customer base with reservation prices that may change in time and exhibit various levels of responsiveness to preannouncements. To our knowledge, we are the first to provide a formal analysis of this problem in a unified analytical framework with a more complete demand model; previous literature looked at only a subset of these aspects at a time. Our novel model enables us to incorporate several decision parameters jointly for an overall understanding and sensitivity analysis.

The results of our study confirm some managerial prescriptions and uncover interactions of several parameters that were not examined before. The optimal price path closely follows changes in reservation price curves for the two products over time. We find that lower market risk characterized by faster diffusion speed favors single roll compared to dual roll, confirming the managerial prescriptions by Billington et al. (1998); however, we also show that the rate of performance improvement and the market responsiveness to preannouncements play an important role in the diffusion process and thus in the comparison of rollover strategies. Furthermore, we

find that timing decisions (introduction of new product, and preannouncement/final build of the old) are highly related to the rollover strategy decision. Ultimately, we show that the interaction of all these parameters/decisions drives the choice between dual and single roll.

We acknowledge the limitations posed by the assumptions of our model. We do not explicitly study competition and its influence on the decision process; however, parameters regarding the diffusion process and reservation price distribution may be modified to reflect the existence of competition. We reduce the interaction of new and old product to an (exogenous) diffusion process since doing so helps simplify the dynamic pricing mechanism. We leave the investigation of joint pricing of old and new generations to future research. Finally, one can investigate the possible supply related issues especially for the new product; this is one of the common reasons for new product failure and should be incorporated in an ideal decision framework.

Chapter 2

Return Policies and Seller-Provided Information in Experience Good Markets

2.1 Introduction

Consumers are typically uncertain about what to expect from a product that they have no experience with, even though they do not lack information regarding the feature set, performance specs or the quality of the good. The source of uncertainty lies in their individual preferences and/or utility regarding that good; they are unsure if they will like an upcoming book, a new dress, if they prefer a particular furniture design alternative over another, or if they can comfortably use a new electric razor. For many—if not the vast majority of—goods, consumers can have but a vague idea about the value of using/owning it before having personal experience with it; there is always a possibility that it is not a perfect fit. This type of good is commonly referred to as “experience good” (Nelson 1970), while this phenomenon is particularly prevalent for new-to-market goods.

Consumers learn more of their true valuation as they gather more information regarding the good (Akerberg 2003), and they can make a better purchase decision given more information. Direct contact—through trial versions or periods, free samples, previews, fit-rooms, test-drives, and so forth—with the exact or a similar

product, interaction with sales personnel, peer or expert reviews, and other forms of word-of-mouth, which we collectively refer to as *informational tools*, serve this purpose. We observe that many sellers put effort to make informational tools available to consumers. For example, fit-rooms are standard practice in department stores such as Macy's; Adobe Systems, Inc., like many software companies, offers a 30-day trial of its products such as Acrobat Professional; Costco.com offers free samples of select items; in a recent campaign, Mitsubishi let people over the Internet to remotely test-drive an actual vehicle on a closed course; Best Buy spends as much as 5% of its total sales on employee training (Kump 2005). Some of these informational tools serve as a means for advertising, however we focus only on the information regarding the product that is conveyed to the consumer.

While seller-provided informational tools help the consumers make a better purchase decision, they do not fully resolve the uncertainty regarding product fit. According to a recent survey by the National Retail Federation, more than \$185 billion worth of retail merchandise, amounting to 8% of total retail sales, was returned to sellers in 2009 in the US (Davis 2010). More strikingly, recent studies show that a large percentage of these returns have no verifiable defect; for example, they account for up to 80% of HP printer returns (Ferguson et al. 2006), and 95% of all electronics purchases (Lawton 2008). These returns are made possible through the customized return policies offered by the sellers. For example, Amazon.com has 31 different product-specific return policies with restocking fees of up to 50%, Best Buy has a customized return policy with restocking fees up to 25%, Nordstrom offers full refunds for any return to their stores. In effect, return policies enable

consumers to defer their ownership decisions until after they gain some experience with the product (for a fee, in the case of partial refunds). In other words, they act as a contract that allocates the product fit-risk between the consumer and the seller; with full refunds, the seller takes on all the risk.

The interaction between the informational state of a consumer and returns is natural: The more informed the consumer, the less the likelihood of a return. However, it is neither practically possible to fully inform each and every consumer in order to stop any returns due to product misfit, nor it is clear that the seller should stop all such returns. It is further unclear how providing partial information effects the consumer return behavior, and therefore how the seller should design her return policy given the consumers' imperfect information level. While lenient return policies are highly valued by all consumers, they are open to abuse by some. The National Retail Federation reports that more than \$9.5 billion worth of retail returns in 2009 was deemed to be fraudulent returns (Davis 2010). Lenient return policies may also leave the seller with too many returned items, resulting in high processing and supply chain management costs, and possible negative brand implications, due to the product being perceived as a high return rate brand; for example, Lawton (2008) reports that 25% of people who return an item refrain from buying that same brand again, while 14% of such people are unlikely to buy from the same seller again. Stringent return policies—high restocking fees or non-refundable charges, limited eligibility or time period, etc.—may result in negative perception of brand, worsened consumer satisfaction, and eventually lost sales, due to some dissatisfied buyers being unable to return.

Our observations of the interaction between informational tools and return policies in experience good markets lead us to important research questions: Given that successful reverse supply chain management practices can maximize the value of returns (Guide et al. 2006), is there an optimal return rate for the seller? Considering the interaction between product information and returns, how much information should the seller provide and how should she design her return policies in order to maximize her profits while managing consumer dissatisfaction?

In order to answer these research questions, we develop a novel model of a consumer's learning process through informational tools provided by the seller. Our two-period model captures the interaction between information provision, return policies, prices and consumer dissatisfaction, and we analytically show that this interaction governs a consumer's pre- and post-purchase decisions, and therefore the market outcome overall. In this study, we focus on the implications of return policies regarding brand/product perception (pre-purchase effect), consumer satisfaction (post-purchase effect), and word-of-mouth (delayed future effect).

We contribute to the existing literature in several ways. To our knowledge, this is the first analytical study that addresses the role of partial information in consumer purchasing behavior, and its implications on product return decisions. We treat the information state of a consumer as a continuous variable and build a tractable yet comprehensive model around it. Furthermore, we consider the full spectrum of return policies from zero to 100% refunds, and incorporate pricing into the seller's decision process. This is the first study to tackle joint optimization of information, return policies and prices in a continuous decision space. We identify conditions

under which a seller chooses not to provide any information even if it is costless. Furthermore, we find that even when it is optimal to provide full information (so that all consumers know their true valuation of the product before purchase), the seller can instead provide only partial information and offer a specific partial refund return policy such that the same outcome is achieved. We further find that it is never strictly optimal to offer full refunds. We note that these conclusions are robust to distributional assumptions.

The rest of the essay is organized as follows. Section 2.2 gives a review of the relevant literature. In Section 2.3, we develop our basic model, and we provide a full analysis of the monopoly case in Section 2.4. In Section 2.5 we solve the seller's joint optimization problem analytically and discuss the findings. Section 2.6 summarizes our findings, and provides directions for extensions.

2.2 Literature Review

The paper that most closely relates to ours is by Shulman et al. (2009), who investigate a single seller with two products on a single-period setting, and they consider product returns and exchanges given that the consumers are uncertain of their valuations as well as the products' fit, and their valuations are uniformly distributed. They assume that the consumers are either fully uninformed or fully informed. In contrast, we capture penalty for consumer dissatisfaction due to product misfit through market growth in the second period. We model the purchasing behavior of a *partially* informed consumer, address both sources of uncertainty without using

exogenous parameters, and we obtain several results without explicit distributional assumptions. The main findings of Shulman et al. (2009) are that a seller's optimal refund amount can be higher or lower than the salvage value and it depends on her cost structure as well as consumer preferences, and that eliminating returns by providing full information is not always optimal. We find that the optimal refund amount depends on consumer dissatisfaction, and that a refund amount more than the salvage value is never exercised. Like Shulman et al. (2009), we find that eliminating returns is not always optimal. We further find, however, that the seller can devise a particular partial refund return policy that eliminates returns. In other words, our result suggests using return policies as an alternative to providing information in order to eliminate returns.

Matthews and Persico (2007) consider a single-period model where consumers are uninformed and may seek information to learn their valuation of the product. In their model, consumers are either fully uninformed or fully informed, and the seller decides on the refund amount and price. As a result of the single-period setting, the seller is motivated to use return policies to induce consumers to not seek information and to make an uninformed purchase decision. In our model, all consumers are information seekers but it is the seller who decides on the *amount of information* to provide, and we find conditions under which it is optimal to withhold information.

A crucial aspect of our study is that we investigate the future consequences of dissatisfied consumers by incorporating market growth and the effect of consumer dissatisfaction and returns on sales through word-of-mouth. This is the first study

to address this aspect of consumers returns; all other works in relevant literature assume a single-period, myopic setting (Shulman et al. 2009, Su 2009, Ketzenberg and Zuidwijk 2009, Matthews and Persico 2007, Yalabik et al. 2005, Davis et al. 1995). This assumption forces these studies to further assume the salvage value of a returned item to be less than its production/acquisition cost, which may not be realistic when the return occurs not due to product failure but product misfit (Ferguson et al. 2006). While Shulman et al. (2009) provide insights that relate a seller's forward channel cost structure to optimal refunds and information provision, incorporating market growth enables us to extend their work in this respect and show that a seller's cost structure as well as market diffusion capabilities in her forward channel are important in her decision process.

Regarding product fit, the common practice in the literature is to assume an exogenous fit probability (Shulman et al. 2009, Yalabik et al. 2005, Chu et al. 1998, Hess et al. 1996, Davis et al. 1995). In contrast, we model the fact that a consumer makes her purchase and return decisions considering both the value of the product and its price. For example, a consumer who purchased, from an online store, a pair of shoes that turned out to be too tight is dissatisfied not because the shoe has absolutely no value to her (she may gift it to a friend, etc.), but because the realized value is not a match to the price she paid for it (whereas she thought, pre-purchase, that it was). Another common practice in the literature is to use a parameter representing the consumer's cost of returning in order to explain the phenomenon where a dissatisfied consumer is unwilling to return (Shulman et al. 2009, Matthews and Persico 2007, Yalabik et al. 2005, Chu et al. 1998, Davis et al. 1998, Hess et al.

1996). Our model endogenously captures this phenomenon without using a hassle cost parameter.

Che (1996) and Davis et al. (1995) compare a full-refund return policy to a no-returns policy. Su (2009) considers partial refunds, however he assumes that consumers have no ex-ante information regarding the product, and he does not consider acquisition or provision of information. Ketzenberg and Zuidwijk (2009) also considers partial refunds but they assume an exogenous return probability in a deterministic model of product returns and remanufacturing. Heiman et al. (2001) also assume an exogenous return probability and value of information, and they compare the alternatives of demonstrating the product and/or offering a full refund. Akerberg (2003) empirically finds that, in experience goods markets, the primary effect of advertising is that of informing the consumers of their valuations. We build our research on this fact and we model the learning process of a consumer and study its endogenous consequences on her pre- and post-purchase behavior, given seller provided information tools and a full spectrum of return policies from zero to 100% refunds. To our knowledge, this is the first work that investigates optimal refunds, optimal provision of information and optimal prices jointly in a continuous decision space.

To summarize, the existing literature on consumer returns does not study the impact of seller-provided (partial) information in consumer pre- and post-purchase behavior, or the interaction of return policies and knowledge state of consumers. It also does not consider the negative impact of consumer dissatisfaction and product returns on the firm's brand and consequent potential market size in the future. Our

research addresses these shortcomings by endogenizing different market outcomes that were treated exogenously in the existing literature.

2.3 Model

We build our model based on the real-life observations pointed out in Section 2.1. In the following, we first describe how we model consumer valuations given partial information on the product. Then, we detail the setting and the planning horizon over which the seller maximizes profits.

2.3.1 Consumer Uncertainty

We model a consumer's valuation of a product as a learning process. Consider a new product being introduced to the market. With no private information regarding the product, a consumer can only have a naïve expected valuation. Therefore, at this zero-state, the consumer pool is homogeneous in terms of their ex-ante valuations; we denote the zero-state ex-ante consumer valuation as v_0 .

Seller-provided informational tools help a consumer draw inference (prior to purchase) about his true ex-post valuation, denoted by v . Without loss of generality, we assume $v \in [0, 1]$. We consider a mapping from the amount of seller's effort to the amount of consumer learning, $\alpha \in [0, 1]$, and we assume that consumers learn homogeneously. We do not model how the seller's efforts, which may be quantified in monetary terms, map to consumer learning; we merely postulate that given some (if any) effort to inform the consumers, a consumer's knowledge state is $\alpha \in [0, 1]$,

where $\alpha = 1$ means a fully informed consumer. We assume that the seller knows this mapping; that is, while deciding how much effort to put on informational tools, the seller effectively decides on α . As a result, we hereinafter refer to α as the “amount of information provided by the seller.” In order to isolate and investigate the underlying motivations for the seller to provide information or not, we assume that providing information has no cost to the seller; if there are conditions where the seller chooses not to provide information when it’s free, then it would mean there is a wider range of such conditions with costly information provision. We later comment on the impact of costly information provision on our results.

We model the valuation learning process such that, at an α -state, a consumer of type v has ex-ante valuation of

$$v_\alpha = (1 - \alpha)v_0 + \alpha v. \tag{2.1}$$

In other words, consumers approach their idiosyncratic ex-post valuations as they learn more about the product. With full information ($\alpha = 1$), there is no valuation uncertainty since each consumer realizes his ex-post valuation even before the purchase. In Figure 2.1, we give an illustration of consumer learning and resulting heterogeneity, as modeled in this study.

We assume that there is a single pool of consumers with individual valuations, V , drawn identically and independently from a publicly known distribution, F , defined over $[0, 1]$. Then, in the absence of any return policy, $v_0 = E[V]$. However, given a return policy with a refund factor of $\beta \in (0, 1]$, consumers have the opportunity to re-consider their initial purchase decision as follows. Denoting the sales price

by p , a consumer will either keep the product (if $v \geq p\beta$) or return it for refund (if $v < p\beta$). Thus, given β , $v_0 = E[\max\{V, p\beta\}]$. Substituting in (2.1), the ex-ante valuation of a consumer with ex-post valuation v is given by

$$v_\alpha = (1 - \alpha)E[\max\{V, p\beta\}] + \alpha v. \quad (2.2)$$

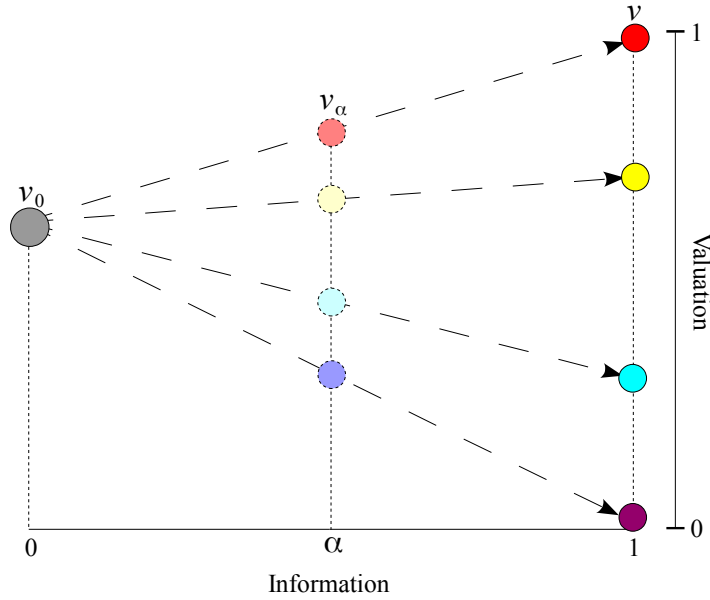


Figure 2.1: Illustration of consumer learning and heterogeneity with information.

At an α -state, by definition, the consumer purchases the product if $v_\alpha \geq p$.

From (2.2), one can show that this condition is equivalent to $v \geq v_\theta$, where

$$v_\theta(\alpha, \beta) \triangleq p + \frac{(1 - \alpha)}{\alpha} (p - E[\max\{V, p\beta\}]), \quad \alpha \in (0, 1], \beta \in [0, 1] \quad (2.3)$$

is the threshold ex-post valuation for purchase. Note that the definition of v_θ above presumes $\alpha > 0$, since we have homogeneity when $\alpha = 0$; either all consumers purchase if $v_0 \geq p$, or none of them purchase if $v_0 < p$.

2.3.2 Market Demand

We conceptualize a two-period model to capture the long-run consequences of seller’s decisions. At the *beginning* of the first period, the seller announces a new product to be introduced to the market, and sets the first-period price, p_1 and the refund factor, β . At the same time, the seller also decides on α and accordingly provides tools to help the consumers make a more informed purchase decision. The seller announces the second-period price, p_2 , after the first-period arrivals and their purchase decision. The product is made available for purchase only at the *end* of each period; there are thus two purchase points. The chronology of events is summarized in Figure 2.2. We assume that this product is sufficiently distinguished from existing products in the market to induce an uncertainty in consumer valuations as studied in this essay.

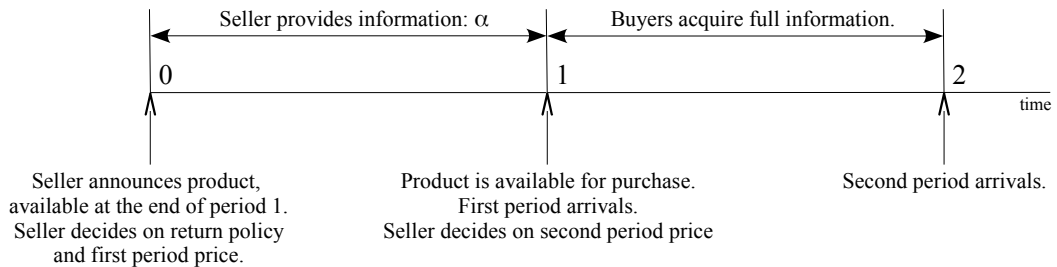


Figure 2.2: Chronology of events in the two-period setting.

We normalize the *initial size* of the *potential* consumer population to 1. These first-period arrivals are analogous to the “innovators” as described in Bass (1969), and are uncertain in their valuations as described above. During the second period, the innovators “spread the word” such that each first-period arrival who is *not dissatisfied* creates a second-period consumer base of $g \geq 0$; we assume this process

to be deterministic for parsimony ³. Therefore, the more consumers are dissatisfied in the first period, the less potential buyers in the second period; this dynamic effectively captures the future consequences of consumer dissatisfaction. We assume that even with full refunds ($\beta = 1$), the consumers who return their items do not contribute to market growth in the second period; this effectively incorporates the negative impact of returns to brand and seller image (Lawton 2008). As a result, in our model, the larger the parameter g , the greater the negative consequences of causing consumer dissatisfaction and/or returns. Therefore, hereinafter, we aptly refer to g as “misfit penalty.”

We further assume that the second-period arrivals have full information regarding their valuations (through owner experiences and reviews as well as seller provided informational tools); there is no valuation uncertainty in the second period. Finally, we assume that any returns occur at the end of the second period and any returning consumers leave the market.

This setting enables us to study interesting aspects of seller’s decisions:

- Through v_θ , a consumer’s purchasing decision in the first period is determined not only by p_1 , but also by α and β .
- Although providing information has a possible cost to the seller, it enables

³With this definition, we assume that any first-period arrival contributes to the second-period market even if he leaves without purchasing. The underlying reasoning is that the diffusion (of information) is triggered by being aware of the product, not by purchasing it. While we take it as given, the value of g is an indicator of the product’s market diffusion speed which is affected by market and product characteristics.

consumers to make more informed decisions in the first period and therefore decreases returns.

- Offering a generous return policy (high β) increases sales in the first period. However, as we show in the following section, a sufficiently high β results in consumers purchasing and being dissatisfied post-purchase, which in turn decreases sales in the second period.

In the next section, we show how the interplay of these three decisions determine the overall outcome both for the seller and the consumers. We analyze the seller's optimal decision strategy in detail and solve her profit maximization problem under a more specific setting.

2.4 Analysis of the Model

2.4.1 Structural Properties

We start our analysis by showing some structural properties of the seller's (α, β) decision space for a given p_1 . First recall that under our setting, the condition for consumer purchasing in the first period, $v_\alpha \geq p_1$, is equivalent to $v \geq v_\theta$; however, each consumer realizes her own ex-post valuation, v , only after she purchases the product. If it turns out that $v \geq p_1$, then the consumer is satisfied. If $v < p_1$ she is dissatisfied; in this case, if her valuation is as low as to be below the refund amount ($v < p_1\beta$), then she returns the item, otherwise she keeps it although she is dissatisfied. As a result, there are three possible market outcomes depending on

the value of v_θ . Figure 2.3 illustrates these cases, which we explain below:

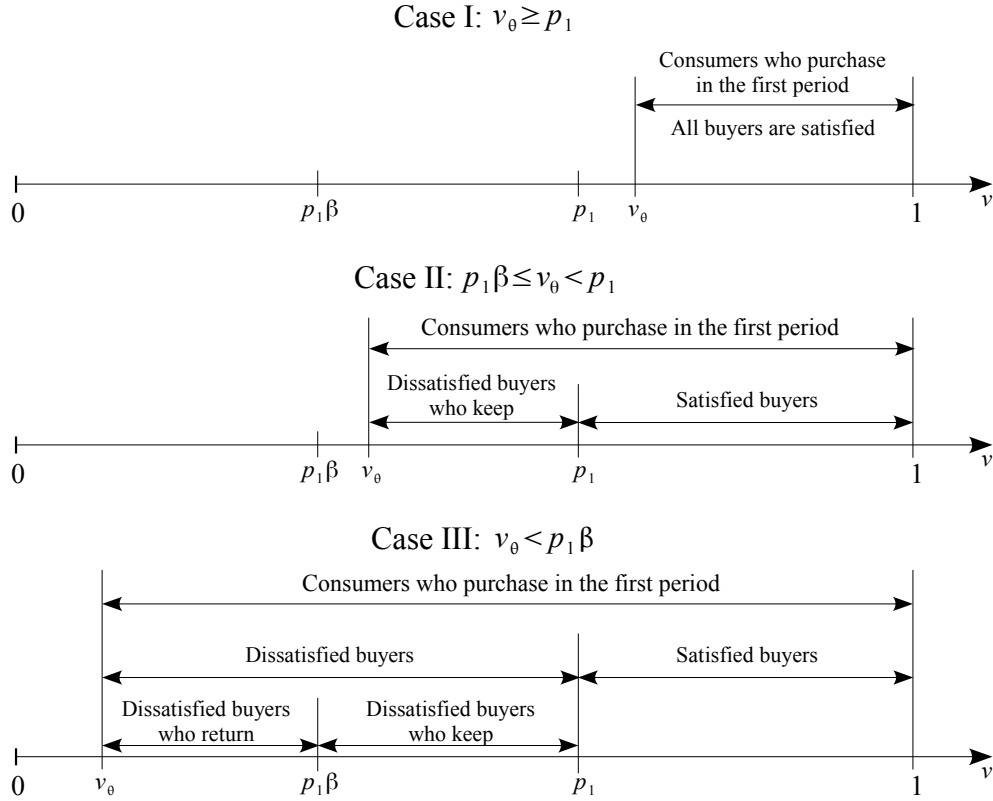


Figure 2.3: Possible cases for v_θ and corresponding market outcomes.

Case I ($v_\theta \geq p_1$): If $v_\theta \geq p_1$, all consumers who purchase the product have non-negative surplus ($v \geq p_1$), since they purchase only if $v \geq v_\theta$. All buyers are satisfied, and there are no returns.

Case II ($p_1 > v_\theta \geq p_1\beta$): In this case, there are some dissatisfied buyers ($v < p_1$), but all buyers have $v \geq p_1\beta$: There are some consumers who are dissatisfied with their purchase but none of them return their item as the refund amount is not sufficiently attractive.

Case III ($v_\theta < p_1\beta$): When $v_\theta < p_1\beta$, there are some dissatisfied buyers with $p_1 >$

$v \geq p_1\beta$ who keep their items but also some with $v < p_1\beta$ who return for a refund; there are some buyers who are dissatisfied but not all of them return their items.

The boundary $v_\theta = p_1$ is of particular importance since it marks the condition for efficient allocation of the product: When $v_\theta = p_1$, a consumer's purchasing condition becomes $v \geq p_1$ and therefore, 1) all consumers who value the product at least as much as its price purchase the product, and 2) all consumers who purchase the product value it at least as much as its price. Note that, when $v_\theta = p_1$, this efficient allocation is achieved ex-ante, as opposed to ex-post (which is possible through a full refund return policy, $\beta = 1$). From (2.3), $v_\theta = p_1$ is satisfied for $\alpha = 1$ (full information) regardless of the value of β . We show that it can also be satisfied with partial information ($0 < \alpha < 1$) and if $p_1 \geq E[V]$, and we state this result in Proposition 3 below.

Proposition 3 *With partial information, i.e. $0 < \alpha < 1$, and for $p_1 \geq E[V]$, there exists a $\beta_p \in [0, 1]$ such that ex-ante efficient allocation is achieved, i.e., $v_\theta(\alpha, \beta_p) = p_1$. Furthermore, $v_\theta(\alpha, \beta) > p_1$ if $\beta < \beta_p$ and $v_\theta(\alpha, \beta) < p_1$ if $\beta > \beta_p$.*

Proof See Appendix B. ■

Proposition 3 is a significant result as it means that, even without providing full information, the seller can achieve ex-ante efficient allocation of the product through a partial-refund return policy, i.e. by setting $\beta = \beta_p$. In other words, return policies can be used to substitute for full information in order to minimize consumer dissatisfaction and returns, even when providing information is costless.

This means that under costly information provision, the seller has a clear incentive to design such a return policy in order to minimize consumer dissatisfaction and returns.

We see from (2.3) that for a given p_1 , any fixed v_θ value results in a relationship between α and β . Therefore, the three cases above translate into three regions on the (α, β) plane, as shown in Figure 2.4. In essence, v_θ is on the z -axis in Figure 2.4 and each boundary seen on the (α, β) plane represents the curve satisfying the specified relationship. Although the graph is plotted for uniform valuations and a particular p_1 value, we show in Appendix B that it is representative of the general case (in terms of the signs of first and second order derivatives). Note that the cases $p_1 \geq E[V]$ and $p_1 < E[V]$ result in different graphs since in the latter, $v_\theta > p_1$ for all $\beta \in [0, 1]$ without full information ($0 < \alpha < 1$), and therefore Region I does not exist (see proof of Proposition 3 in Appendix B).

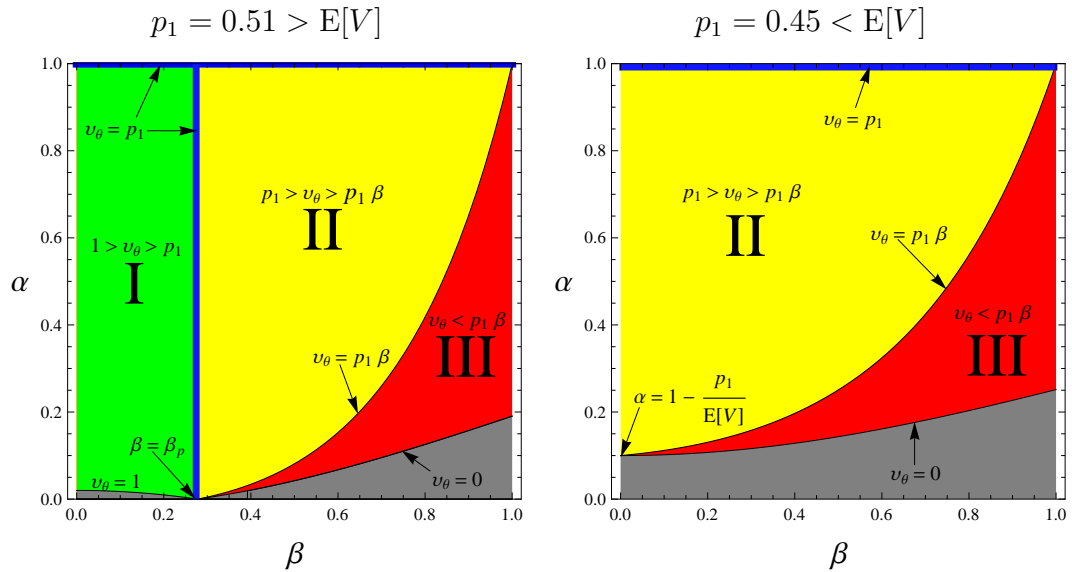


Figure 2.4: Seller's (α, β) decision space for $V \sim U(0, 1)$ at different price points.

We now turn our attention to the seller's general profit maximization problem.

2.4.2 Seller's Optimization Problem

Facing the market dynamics described in Section 2.3, the seller maximizes two-period profits by determining the optimal set of decisions, $(\alpha^*, \beta^*, p_1^*, p_2^*)$. Demand in the first period is $(1 - F(v_\theta))$, since we have unit market size. Demand in the second period is $(g(1 - F(p_2))(1 - L))$, where $L \triangleq \max\{0, F(p_1) - F(v_\theta)\}$ is the rate of dissatisfaction in the first-period market. Fraction of returns is given by $M \triangleq \max\{0, F(p_1\beta) - F(v_\theta)\}$ and an amount of $p_1\beta$ is refunded for each return. Production cost per unit is c and each returned product (if any) has a net salvage value of s . We assume reasonably that $s \in (0, 1)$. Note that we allow for $s > c$, in which case there is a profitable market for returned items. We write the seller's optimization problem in general form as follows and characterize the optimal second-period price in Proposition 4.

$$\begin{aligned} \max_{\alpha, \beta, p_i} R &= (p_1 - c)(1 - F(v_\theta)) + (-p_1\beta + s)M + g(p_2 - c)(1 - F(p_2))(1 - L) \\ \text{s.t.} \quad \alpha, \beta &\in [0, 1] \end{aligned} \tag{2.4}$$

Proposition 4 *The optimal price in the second period is given by*

$$p_2^* = \arg \max_p (p - c)(1 - F(p)).$$

Proof The solution for p_2 follows since p_2 appears only in the final term in the objective function and since $g(1 - L) \geq 0$ under any circumstances. ■

Proposition 4 shows that the optimization of p_2 is decoupled from the rest of the decision process. Therefore in the rest of the essay, we continue studying the joint optimization of α, β and p_1 . In the next section, we determine strictly dominated regions and identify conditions for optimality of others.

2.4.3 Characterization of Optimal Information and Refunds

As discussed in Appendix B, boundaries in the (α, β) decision space, which are critical for consumer dissatisfaction and existence of returns, are functions of p_1 . Although this complicates the seller's problem of jointly optimizing α, β and p_1 , we find that a general characterization of the optimal α and β is possible for a given p_1 . We summarize our findings in Proposition 5. Note that we do not yet make any assumptions as to how the consumer valuations are distributed. In the rest of the essay, we use superscripts for association to the indicated region in the (α, β) decision space, and we use the subscript θ to indicate a threshold value.

Proposition 5 *For a given p_1 , the optimal (α, β) corresponds to one of the two candidate solutions below, depending on the values of p_1, g, c and s , and on the distribution F . We depict these solutions in Figure 2.5.*

$$i) \text{ Solution (D): } (\alpha^*, \beta^*) = \left\{ (\alpha, \beta) \mid \alpha \in \left[0, 1 - \frac{p_1}{E[\max\{V, p_1\beta\}]} \right], p_1\beta = s - \frac{F(p_1\beta)}{F'(p_1\beta)} \right\}$$

This solution lies on the region where $v_\theta = 0$ and $p_1\beta \leq s$. All consumers purchase but the optimal refund amount is less than the salvage value.

$$ii) \text{ Solution (E): } (\alpha^*, \beta^*) = \{(\alpha, \beta) \mid v_\theta = p_1\}$$

This solution implies ex-ante efficient allocation, i.e. $v_\theta = p_1$, which is satisfied when $\alpha = 1$ for any β , or if $p_1 \geq \mathbb{E}[V]$, when $\beta = \beta_p$ for any $\alpha > 0$.

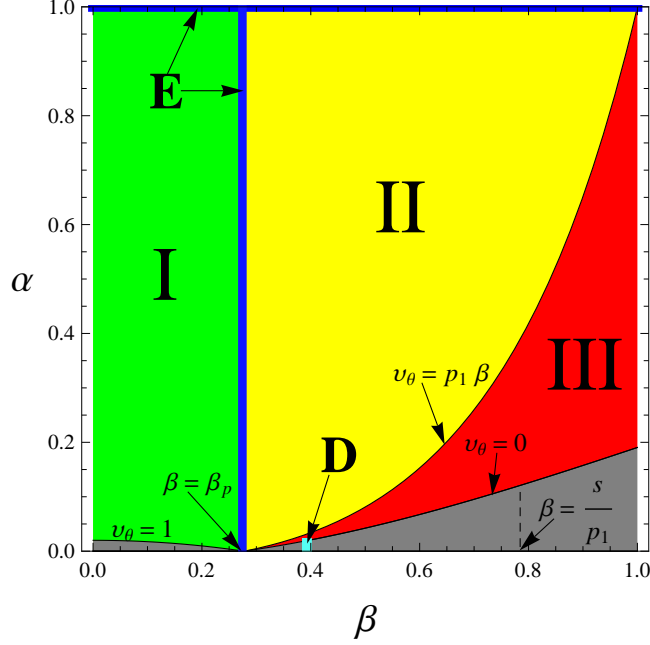


Figure 2.5: Candidate solutions in the seller's (α, β) decision space for $V \sim U(0, 1)$.

Specifically, if $g > \bar{g}_\theta^E$, then Solution (E) is optimal; if $g < \bar{g}_\theta^E$, then Solution (D) is optimal, where

$$\bar{g}_\theta^E = \frac{\left((p_1 - c) + (s - p_1 \beta^*) \frac{F(p_1 \beta^*)}{F(p_1)} \right)}{(p_2 - c)(1 - F(p_2))}, \quad (2.5)$$

and β^* satisfies

$$p_1 \beta^* = s - \frac{F(p_1 \beta^*)}{F'(p_1 \beta^*)}.$$

Proof See Appendix B. ■

Proposition 5 finds that for sufficiently small misfit penalty, Solution (D), which sells to all consumers in the first period, is optimal; equivalently, for suffi-

ciently large misfit penalty, Solution (E), which suggests no returns through ex-ante efficient allocation, is optimal. This result has three immediate corollaries. The first is that there are conditions that makes the ex-ante efficient allocation undesirable for the seller. Specifically, if the net profit of selling a product to a consumer in the first period exceeds the expected loss in the second period due to dissatisfying that consumer, then the seller chooses (α, β) such that every consumer in the first period purchases a product, regardless of his valuation. Since the expected loss in the second period increases with misfit penalty, we conclude that if the misfit penalty is sufficiently small, then it is optimal to sell to all consumers in the first period. Furthermore, observe that \bar{g}_θ^E , which is the threshold for absolute dominance of Solution (E), increases in s ; the larger the salvage value, the narrower the dominance region of Solution (E).

Second, offering a refund amount of more than the salvage value of returned items is not optimal unless it is optimal to provide full information. On the other hand, in case of full information, the return policy is redundant (since there are no returns) and the seller is indifferent in choosing a refund amount. Therefore, we say that it is weakly suboptimal to offer a refund amount more than the salvage value. Note that as long as the price is larger than the salvage value, this also means that it is weakly suboptimal to offer a full (100%) refund.

Finally, note that in the case of costly information provision, if $p_1 \geq E[V]$, providing full information is never optimal since the seller can instead design a return policy, by setting $\beta = \beta_p$, to achieve the same effect. If, however, $p_1 < E[V]$, then the seller would have more incentive to choose Solution (D); that is, Solution

(D) would be optimal for a wider range of misfit penalty.

2.5 Jointly Optimal Information, Refund and Price Strategy

In the previous section, we identified the two candidate solutions for optimal (α, β) without making any distributional assumptions, but under the assumption that p_1 is given. We observe that there is no clear dominance relationship between these solutions if p_1 is a decision variable as well. In this section, we assume uniform valuations, $F(p) = p$, and identify the candidate solutions for jointly optimizing information, refund and *price*, and determine the conditions that lead to the optimality of each solution. Uniform valuations is the most common assumption in the operations management, marketing and economics literatures (Shulman et al. 2009, Chesnokova 2007, Villas-Boas 2006, Davis et al. 1998, Chu et al. 1998). We employ this assumption in this essay in order to facilitate closed-form optimal solutions for better interpretation. This essentially constitutes the optimal $(\alpha, \beta, p_1, p_2)$ strategy for the seller, contingent on the values of g, c , and s as given in Proposition 6. We find that there is a boundary, critical for shaping the optimal strategy, in the (s, c) plane, which we plot in Figure 2.6.

Proposition 6 *With uniform valuations, the joint optimal $(\alpha, \beta, p_1, p_2)$ strategy is such that;*

For (s, c) such that $c \geq c_\theta(s)$ – Region (1) in Figure 2.6 – we have $g_\theta^{E,D} \leq g_\theta^{E,C} \leq g_\theta^{D,C}$, and

(a) If $g \geq g_\theta^{E,C}$, the optimal solution is Solution (E),

(b) If $g < g_{\theta}^{E,C}$, the optimal solution is Solution (C);

For (s, c) such that $c < c_{\theta}(s)$ – Region (2) in Figure 2.6 – we have $g_{\theta}^{D,C} < g_{\theta}^{E,C} < g_{\theta}^{E,D}$, and

(a) If $g \geq g_{\theta}^{E,D}$, the optimal solution is Solution (E),

(b) If $g_{\theta}^{D,C} < g < g_{\theta}^{E,D}$, the optimal solution is Solution (D),

(c) If $g < g_{\theta}^{D,C}$, the optimal solution is Solution (C),

where

$$c_{\theta}(s) = 2\sqrt{\frac{1+s^2}{3}} - 1,$$

$$g_{\theta}^{E,C} = \frac{4(1+s^2) - 2(1+c)^2}{(1+s^2)(1-c)^2},$$

$$g_{\theta}^{E,D} = \frac{4 + 12(1+s^2) - 8(1+c)^2}{(4+s^2)(1-c)^2},$$

$$g_{\theta}^{D,C} = \frac{4}{3(1-c)^2},$$

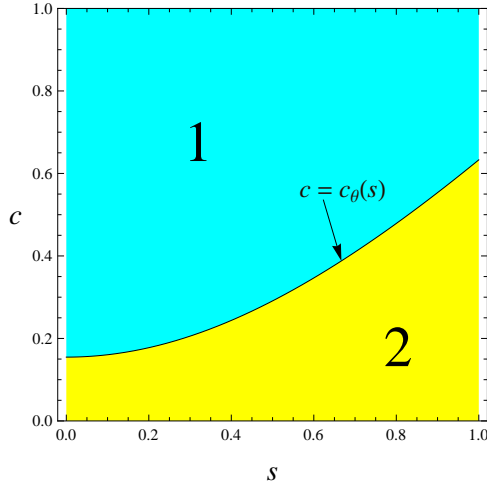


Figure 2.6: Critical regions in the (s, c) plane.

and

$$\text{Solution (E): } (\alpha^*, \beta^*) = \{(\alpha, \beta) \mid v_\theta = p_1^*\}, p_1^* = p_2^* = \frac{1+c}{2},$$

$$\text{Solution (D): } (\alpha^*, \beta^*, p_1^*, p_2^*) = \left(0, \frac{4s}{4+s^2}, \frac{4+s^2}{8}, \frac{1+c}{2}\right),$$

$$\text{Solution (C): } (\alpha^*, \beta^*, p_1^*, p_2^*) = \left(0, \frac{2s}{1+s^2}, \frac{1+s^2}{2}, \frac{1+c}{2}\right).$$

Proof See Appendix B. ■

We show in Proposition 6 that, for the joint optimization of α , β , p_1 and p_2 , there are three solutions among which the seller chooses, depending on the relationship between market parameters (c , s and g). Solution (E) corresponds to any (α, β) pair that satisfies $v_\theta = p_1$. We show in Section 2.4.1 that this implies efficient allocation of the product at the time of purchase even under valuation uncertainty. In other words, exactly those consumers who would purchase the product with full information do purchase the product. We also show that this can be achieved either by providing full information or, without full information, by setting the refund amount to a certain level $\beta = \beta_p$; when providing information is costless, the seller is indifferent between these two options. However, under costly information provision, Solution (E) reduces to $\alpha^* \gtrsim 0$ and $\beta^* = \beta_p = \frac{2\sqrt{c}}{1+c}$. In either case, since the consumer purchasing behavior is identical to full information case, the seller sets the classical monopoly prices: $p_1^* = p_2^* = \frac{1+c}{2}$.

Solutions (C) and (D) are both characterized by $v_\theta = 0$ but at different prices and refund factors; Solution (C) suggests higher values in both: $\beta^*(C) > \beta^*(D)$ and $p_1^*(C) > p_1^*(D)$. Therefore, Solution (C) offers a larger refund amount than Solution (D); the former offers a refund amount equal to the salvage value ($p_1^*\beta^*(C) = s$), while the latter offers half the salvage value ($p_1^*\beta^*(D) = \frac{1}{2}s$). In both Solutions (C)

and (D), the seller provides no information and ensures that all consumers purchase in the first period. Solution (C) offers a larger refund amount than Solution (D) and attains higher ex-ante consumer valuations in the first period; as a result, the seller charges more: $p_1^*(C) = \frac{1+s^2}{2} > p_1^*(D) = \frac{4+s^2}{8}$. Due to higher price and higher refund amount, Solution (C) results in both larger dissatisfaction rate (which is equal to the price), and larger return rate (which is equal to the refund amount).

The optimal strategy states that Solution (E) should be preferred only when the misfit penalty (g) is sufficiently large. In other words, if the misfit penalty is sufficiently small, the seller chooses to maximize her profits at the expense of consumer satisfaction. We further see that if the cost of production is sufficiently large, Solution (D) is never optimal; higher production costs require higher prices to compensate for them and Solution (C) is preferred. With low production costs, there is a range of misfit penalty for which Solution (D) is optimal; for sufficiently low misfit penalty, Solution (C), which is greedier, is optimal.

We attain an intuitive corollary out of Proposition 6, by noting that the optimality threshold for Solution (E) in Region (1) is $g_\theta^{E,C}$; in Region (2), the optimality threshold for Solution (E) is $g_\theta^{E,D}$. Next, we observe that $\frac{\partial g_\theta^{E,C}}{\partial s} = \frac{4s(1+c)^2}{(1+s^2)^2(1-c)^2} > 0$, and $\frac{\partial g_\theta^{E,D}}{\partial s} = \frac{16s(5+2c+c^2)}{(4+s^2)^2(1-c)^2} > 0$. Therefore, for a constant c , larger s results in larger thresholds, which in turn limits the optimality of range for Solution (E). We conclude that when salvage value is higher, it is easier for the seller to reject ex-ante efficient allocation, thus to allow returns.

2.6 Conclusion

In this essay, we study the profit maximization problem for a seller who optimizes information provision, return policy and prices for a new experience good, over a two-period horizon. With no information, consumers are fully uncertain of their valuations of the product. However, given more information, they learn more, approaching to individual valuations; information creates ex-ante heterogeneity among consumers. On the other hand, being aware of a return option, consumers update their valuations of the purchase decision.

We make several important contributions with this study. We devise a novel approach in understanding and modeling the process of a consumer's learning of own valuation, taking into consideration both partial information and partial-refund return policies. Our model incorporates two key parameters; 1) market growth rate, g , which represents the seller's forward channel capability; 2) s , which determines the value of returned items and therefore points to the seller's reverse channel capabilities. Building on the dynamics of interaction between information and return policy in consumer valuations, we treat the seller's optimization problem analytically, show structural properties of her decision space, and characterize the optimal solution for the general case. These characterizations lead to three major findings that are robust to distributional assumptions. First, if the market growth rate—and hence the future penalty due to consumer dissatisfaction and/or returns—is sufficiently low, then the seller may choose to provide no information to consumers even if it is costless to do so. We also show that as the salvage value increases, it

becomes easier for the seller to withhold information from consumers. This shows how the seller’s reverse channel capabilities interact with her forward channel decisions. Second, we show that even when it is optimal to ensure ex-ante efficient allocation, Solution (E), it is not necessary to provide full information as this can be achieved by devising the return policy appropriately (by setting $\beta = \beta_p$) and providing only partial information. This is a significant result as it showcases a situation where the return policy can be used to substitute for informational tools. Third, we find that offering a refund amount that is more than the salvage value is never exercised; the seller can advertise such a refund amount when Solution (E) is optimal, in which case there are no returns. Lastly, assuming uniform valuations, we determine the optimal decision strategy for the seller, which dictates the optimal values for information provision, refund factor and prices given model parameters.

Future work could investigate opportunistic consumer behavior where a consumer “purchases without intention to keep.” A seller can be exposed to such behavior if she offers lenient returns; tightening return policies for some sellers is attributed to their losses due to this type of consumer behavior (Davis 2010). Davis et al. (1995) and Hess et al. (1996) examine consumers who purchase without intention to keep; however, they assume that consumers have no pre-purchase information, and they do not consider information provision.

Chapter 3

Managing Return Policies and Information Provision under Competition

3.1 Introduction

More than 8% of total retail sales, \$185 billion worth of retail merchandise, was returned to sellers in 2009 in the US, and predictions for the near future indicate similar outcomes (Davis 2010). While the substantial implications, in terms of direct and overhead costs, of these returns for the whole supply chain makes the study of consumer returns valuable, it is further interesting to observe that a significant amount of these returns have no verifiable defect. For example, they account for up to 80% of HP printer returns (Ferguson et al. 2006), and 95% of all electronics purchases (Lawton 2008).

The primary reason for these “false-failure” returns is that the consumers learn—only after the purchase—that the good is not a perfect fit to their tastes, preferences, usage norms, established settings, etc. Take for example a consumer purchasing a new electric razor only to realize that its grip is not as comfortable as the old one he had, or a curtain set to be brought home only to notice it does not match the color of furniture at home. When lacking experience with the good before the purchase, the consumers cannot be certain of the true value of the good

for them, and thus, there is a possibility for a misfit. This type of good is commonly referred to as “experience good” (Nelson 1970). As a result, lack of information—regarding the value of the good—is the underlying driver of false-failure returns of experience goods.

Before purchasing an experience good, consumers mainly rely on the seller to gain access to, and—however limited—experience with the good; this is especially true in case of a new-to-market good. Consider for example trial versions of commercial software, test-drive events organized by auto manufacturers, fit-rooms that are a standard in all department stores, electronic stores with items displayed openly with trained sales personnel present, free samples of cosmetic products made available through online or physical channels, product samples sent to expert reviewers, etc. While the level of these efforts by firms vary greatly, their main purpose is to reduce false-failure returns by providing the consumers information regarding the true value of the goods. On the other hand, the sellers also offer customized return policies that facilitate product returns. For example, Amazon.com has 31 different product-specific return policies with restocking fees of up to 50%, Best Buy has a customized return policy with restocking fees up to 25%, Nordstrom offers full refunds for any return to their stores. In effect, return policies enable consumers to defer their ownership decisions until after they gain some experience with the product (for a fee, in the case of partial refunds).

If a seller’s objective is to maximize consumer satisfaction, the initial intuition is that this goal can be achieved either by providing full information to all consumers, or by offering a full-refund return policy. While providing full information

to each and every consumer would cut all false-failure returns, it is also practically impossible to achieve. Offering full refunds would enable any misfit alleviated, however at the cost of the seller—on top of immediate financial costs, negative brand implications can be substantial; Lawton (2008) reports that 25% of people who return an item refrain from buying that same brand again, while 14% of such people are unlikely to buy from the same seller again. On the other hand, we show in Essay 2 that a monopoly seller can design a partial-refund return policy to get rid of false-failure returns, while providing only partial information. In Essay 2, we also identify conditions where it is in fact optimal for the monopoly seller to minimize false-failure returns. Then, the question is ‘what happens when there is competition?’ Specifically, we pursue the following research questions in this essay:

- 1) Given competition, is it still possible to design a return policy to effectively minimize false-failure returns without having to provide full information? If it is, is such an outcome ever desirable for the sellers?
- 2) Are there any equilibrium return policy and information provision decisions?
 - (a) How do they differ from the decisions of a monopoly?
 - (b) Under what conditions do they exist?

To address these questions, we build on the basic two-period model described in Essay 2, with the exception of assuming uniform valuations for tractability. In order to isolate the effect of competition, we conceptualize a perfectly symmetric duopoly setting, and examine equilibrium information provision and return policy

decisions. To our knowledge of the literature, this is the first scholarly work that analytically studies the effects of competition on joint information provision and return policy decisions. We identify all potential Nash equilibria and their respective conditions of existence. Contrasting the results to the monopoly case, we find that, while competition can induce sellers to withhold information from the consumers under certain conditions, it forces them to offer full refunds.

The rest of the essay is organized as follows. Section 3.2 reviews the relevant literature. In Section 3.3, we describe and analyze the competition model, and we examine and discuss the equilibrium in Section 3.4. We conclude in Section 3.5.

3.2 Literature Review

Among the very few studies that investigate competition in a similar context as ours, the paper by Shulman et al. (2011) is the most relevant. In Shulman et al. (2011), they examine equilibrium prices and return policies in a competitive market where consumers are not informed of their tastes or valuations. On a single-period horizon, the sellers offer horizontally-differentiated products but provide no information to consumers, and they extract no value out of returned items. Our competition setting is significantly different from theirs in that we look at equilibrium information provision and return policies incorporating consumer dissatisfaction in the second period. We show that consumer dissatisfaction and salvage value are *critical* in determining the market equilibrium. In direct contrast to our findings, Shulman et al. (2011) conclude that competition may induce higher restocking fees, whereas

we find that sellers typically offer full refunds in a competitive setting. Their result can be explained by noting that they do not consider the impact of high restocking fees on consumer dissatisfaction (given their single period setting), and therefore the return policy effectively becomes a tool to discourage consumers from returning, in order to maximize short-term profits. Our findings help explain why full refunds are observed in competitive retail markets.

Aside from Shulman et al. (2011), Chesnokova (2007) considers a duopoly where the firms engage in a product reliability and price competition, and returns are in the form of repairs, not refunds; i.e., the source of returns in her model is product reliability, and not consumer tastes and preferences as in our model. In the context of experience goods, Doganoglu (2010), Villas-Boas (2006) and Villas-Boas (2004) study the price competition of two sellers over an infinite horizon; however, neither paper considers return policies or pre-purchase information provision. We study a duopoly case where two identical sellers engage in return policy and information competition over two periods; this is the first scholarly work to our knowledge to study the effects of competition on seller decisions on return policies and provision of information.

3.3 Competition Model

We build the competition setting on the same framework as described in Section 2.3 in Essay 2. Specifically, we conceptualize a consumer's valuation of a product as a learning process, given information of amount $\alpha \in [0, 1]$ by a seller. Further given a

return policy with a refund factor of $\beta \in (0, 1]$, such that the refund amount is $p\beta$ where p is the purchase price, each consumer has the opportunity to re-consider her initial purchase decision. As in Essay (2), we assume costless information provision in our analysis, and we later comment on the impact of costly information provision.

We consider a duopoly case with identical sellers, Y and Z ; sellers have identical unit costs c , net salvage values s , and market growth rates g , and they introduce new products at the same time. We assume that consumers equally value the products from both sellers; that is, the products are perfect substitutes of each other. In other words, there is a single, seller-independent distribution F , of consumer valuations V , which we assume to be uniformly distributed: $F(p) = p$. We further assume identical period prices, p_1 and p_2 , for both sellers. While we do not assume identical products, we assume that information provided by a seller on her product does not contribute to information on the other seller's product; while this assumption does not hold in general, it is valid for many—if not all—experience goods⁴. These assumptions help us focus on information and return policy (α and β) competition, as well as allowing a tractable solution.

We employ the same two-period setting as in the monopoly case, with the chronology of events shown in Figure 2.2 in Essay 2. As in the monopoly case, we assume that consumers attain full information on the products in the second period, regardless of the information provided in the first period. At the start of the first

⁴Consider for example two electric razors of different brands. Even after having used one of them, a consumer would have no understanding regarding how well the other product will perform, how comfortable it will feel in his hand, how comfortable a shave it will provide, etc.

period, the sellers simultaneously decide on their respective α and β . In the first period, we assume a unit market size, which is shared between the sellers according to consumer valuations given the sellers' α and β decisions. Specifically, a consumer with ex-post valuation v perceives ex-ante valuations of $v_{\alpha,Y}$ and $v_{\alpha,Z}$ for the sellers' products, and since prices are equal, chooses the seller with the larger $v_{\alpha,j}$, where

$$\begin{aligned} v_{\alpha,j} &= \alpha_j v + (1 - \alpha_j) \mathbb{E}[\max\{V, p_1 \beta_j\}] \\ &= \alpha_j v + (1 - \alpha_j) \frac{1 + (p_1 \beta_j)^2}{2} \end{aligned}$$

for uniform valuations. In the second period, the market size for each seller grows with rate g in the same manner as in the monopoly case: Consumers who are dissatisfied or who return their purchases in the first period do not contribute to market growth. Therefore, similarly, we refer to g as “misfit penalty.”

3.3.1 Market Share Dynamics

In preparation for the equilibrium analysis, we here analyze the market share dynamics *given* the sellers' α and β decisions. Suppose first that $\alpha_Y = \alpha_Z$. If $\alpha_Y = \alpha_Z < 1$ and if, without loss of generality, $\beta_Z > \beta_Y$, then $v_{\alpha,Z} > v_{\alpha,Y}$, $\forall v$. That is, in the case of symmetric, partial information, the seller offering a more lenient return policy captures the whole market. If $\alpha_Y = \alpha_Z = 1$, then $v_{\alpha,Y} = v_{\alpha,Z}$, $\forall v$; if both sellers provide full information, then the consumers are indifferent between the sellers regardless of the return policies. Suppose without loss of generality that $\alpha_Y < \alpha_Z$. Then, there is a threshold valuation, v_θ^{YZ} , such that all consumers with $v > v_\theta^{YZ}$ prefer seller Z to seller Y , while those with $v < v_\theta^{YZ}$ prefer seller Y to seller Z ,

where

$$v_{\theta}^{YZ} \triangleq \frac{(1 - \alpha_Y)(1 + (p_1\beta_Y)^2) - (1 - \alpha_Z)(1 + (p_1\beta_Z)^2)}{2(\alpha_Z - \alpha_Y)}. \quad (3.1)$$

Therefore, the first-period market share for seller Y is v_{θ}^{YZ} ; for seller Z , it is $(1 - v_{\theta}^{YZ})$. If $\beta_Y = \beta_Z = \beta$, we have $v_{\theta}^{YZ} = E[\max\{V, p_1\beta\}]$; that is, in case of identical return policies, the threshold valuation is independent of the level of information provided as long as $\alpha_Y \neq \alpha_Z$.

Further analysis of the market share dynamics reveals that the seller with a more lenient return policy can set an appropriate level of information to achieve a desired market share. Consequently, she can set an appropriate level of information to achieve 100% market share, that is, drive the other seller out of the market. This is formalized in Proposition 7 below.

Proposition 7 *Suppose, without loss of generality, that $\alpha_Y < 1$ and let Z be the seller offering a more lenient return policy, i.e., $\beta_Z > \beta_Y$. Then, seller Z can achieve a desired market share, \hat{v} , by setting $\alpha_Z = \alpha_{\hat{v}}$, where*

$$\alpha_{\hat{v}} \triangleq \alpha_Y + p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{2\hat{v} + (p_1\beta_Z)^2 - 1}.$$

In addition, seller Z can drive seller Y out of the market by setting $\alpha_Z \in [\underline{\alpha}, \bar{\alpha}]$, where

$$\underline{\alpha} \triangleq \max \left\{ 0, \alpha_Y - p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{1 - (p_1\beta_Z)^2} \right\},$$

and

$$\bar{\alpha} \triangleq \alpha_Y + p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{1 + (p_1\beta_Z)^2}.$$

Proof See Appendix C. ■

Regardless of how the market is shared, the dynamics between the consumer and the seller is identical to the monopoly case: Given that a consumer prefers seller Z ($v > v_{\theta}^{YZ}$), he purchases only if $v_{\alpha,Z} \geq p_1$, or equivalently if $v \geq v_{\theta,Z}$. If $p_1\beta_Z \leq v < p_1$, he is unhappy with the purchase but is not willing to return; if $v < p_1\beta_Z$, he is unhappy and would like to return. Therefore, all the findings regarding the structure of the (α, β) decision space carries on from the monopoly case. Furthermore, since we assume equal first-period prices, unit costs and net salvage values, the sellers have identical (α, β) decision spaces.

3.4 (α, β) Equilibrium

In this section, we analyze the sellers' (α, β) decision space in the light of the results for the monopoly case and the analysis of market shares above, in order to identify the possible Nash equilibria. In other words, we investigate whether and when there exists a pair of decisions (α_Y, β_Y) and (α_Z, β_Z) such that the former is seller Y 's best response to the latter, which in turn is seller Z 's best response to the former. When a pure-strategy Nash equilibrium does not exist, we identify the mixed-strategy equilibrium. The intuitive corollary of Proposition 7 suggests the non-existence of a Nash equilibrium where both sellers set $(\alpha_j < 1, \beta_j < 1)$, since given, without loss of generality, $(\alpha_Y < 1, \beta_Y < 1)$, seller Z has a potential best response where she sets a more lenient return policy and an appropriate level of information to capture the whole market. In fact, we find that capturing the whole market is the best response to $(\alpha_Y < 1, \beta_Y < 1)$, and we summarize our findings in Proposition 8 below.

Proposition 8 *In the duopoly where both sellers have identical p_1, p_2, c, s and g , and consumer valuations are uniformly distributed, we identify four thresholds on g (as functions of other variables) that are critical for the existence and the form of (α, β) Nash equilibria: $\bar{g}_\theta^{III}, g_\theta^{IV}, g_\theta^V$ and g_θ^{VI} . Furthermore, we find that the ordering of these functions is determined by the value of s compared to a threshold function s_θ . Specifically, the potential (α, β) Nash equilibria and the associated conditions for their existence are as follows.*

For $s > s_\theta$, we have $g_\theta^V > \bar{g}_\theta^{III}$, and

- (i) If $g < \bar{g}_\theta^{III}$, then there is a symmetric pure-strategy Nash equilibrium where both sellers provide no information but offer full refund return policy: $(\alpha_j = 0, \beta_j = 1)$ for both sellers,*
- (ii) If $\bar{g}_\theta^{III} < g < g_\theta^V$, there is no pure-strategy Nash equilibrium. There is a mixed-strategy Nash equilibrium where both sellers set $\beta_j = 1$ and pick $\alpha \in [0, 1]$ randomly,*
- (iii) If $g > g_\theta^V$, there is no pure-strategy Nash equilibrium. There is a mixed-strategy Nash equilibrium where both sellers set $\beta_j = 1$ and pick $\alpha \in [\hat{\alpha}, 1]$ randomly;*

For $s < s_\theta$, we have $g_\theta^V < g_\theta^{VI} < \bar{g}_\theta^{III} < g_\theta^{IV}$, and

- (i) If $g < g_\theta^V$, then there is a symmetric pure-strategy Nash equilibrium where both sellers provide no information but offer full refund return policy: $(\alpha_j = 0, \beta_j = 1)$ for both sellers,*

- (ii) If $g_\theta^V < g < g_\theta^{VI}$, then there is an asymmetric pure-strategy Nash equilibrium where one seller provides full information and offers an arbitrary return policy, while the other seller provides no information but offers a full refund return policy: without loss of generality, $(\alpha_Y = 1, \beta_Y \in [0, 1])$ and $(\alpha_Z = 0, \beta_Z = 1)$,
- (iii) If $g_\theta^{VI} < g < g_\theta^{IV}$, then there is a symmetric pure-strategy Nash equilibrium where both sellers provide full information and offer arbitrary return policies: $(\alpha_j = 1, \beta_j \in [0, 1])$ for both sellers,
- (iv) If $g > g_\theta^{IV}$, there is no pure-strategy Nash equilibrium. There is a mixed-strategy Nash equilibrium where both sellers set $\beta_j = 1$ and pick $\alpha \in [\hat{\alpha}, 1]$ randomly,

where

$$\begin{aligned}
s_\theta &= 1 - p_1 + c \left(2 - \frac{1}{p_1} \right), \\
\bar{g}_\theta^{III} &= \frac{s - c}{(p_2 - c)(1 - p_2)}, \\
g_\theta^{IV} &= \frac{(p_1 - c)(1 - p_1)}{p_1(p_2 - c)(1 - p_2)}, \\
g_\theta^V &= \frac{s - p_1(1 - p_1 + c)}{(1 - p_1)(p_2 - c)(1 - p_2)}, \\
g_\theta^{VI} &= \frac{2s - c - p_1(1 - p_1 + c)}{(2 - p_1)(p_2 - c)(1 - p_1)},
\end{aligned}$$

and

$$\hat{\alpha} = \frac{(1 - p_1)^2(g(1 - p_2)(p_2 - c) + c - s)}{(c - s)(1 - p_1)^2 - 2p_1(1 - p_1)(p_1 - c) + g(1 - p_2)(p_2 - c)(1 - 2p_1 + 3p_1^2)}.$$

Proof See Appendix C. ■

We see from Proposition 8 that for sufficiently small misfit penalty, g , both sellers find it the best decision to provide no information to sell to all consumers and share the market equally, although there are some dissatisfied consumers as well as some returns. Moreover, we observe that as the salvage value increases, the range of g , where providing no information is the best decision, grows. Both of these results are consistent with the monopoly case (Proposition 5).

We observe that for sufficiently large misfit penalty ($g > \bar{g}_\theta^{III}$ for $s > s_\theta$, and $g > g_\theta^{IV}$ for $s < s_\theta$), there is no pure-strategy Nash equilibrium since both sellers always find a best response where they capture the whole market alone. The mixed-strategy equilibrium we identify for $s > s_\theta$ and $\bar{g}_\theta^{III} < g < g_\theta^V$ suggests that both sellers offer full refunds and pick α randomly in $[0, 1]$. The second mixed-strategy Nash equilibrium suggests that for sufficiently large g , both sellers' best decision is to offer full refunds and provide at least partial information; they randomly set an information level between $\alpha = \hat{\alpha}$ and $\alpha = 1$. This means that for sufficiently large g , it is not a best decision to provide little or no information; a result that is consistent with the monopoly case. Both mixed-strategy equilibria suggest that the market is not necessarily covered in the first period (i.e., there are some consumers who leave without purchasing), since full market coverage requires $\alpha_j \leq \alpha(v_\theta = 0)$ for at least one of the sellers and this is not necessarily the case. Furthermore, recalling that for a monopoly, the optimal decision given $\beta = 1$ and $g > \bar{g}_\theta^{III}$ is to provide full information, so that there are no dissatisfied consumers, we note that both mixed-strategy equilibria imply partial information, resulting in sub-optimal outcomes where there are some dissatisfied consumers and some returns.

For sufficiently small net salvage value, $s < s_\theta$, intermediary misfit penalty values result in two different pure-strategy Nash equilibria. In the lower g range ($g_\theta^V < g < g_\theta^{VI}$), one seller provides full information and offers an arbitrary return policy, while the other provides no information but offers a full refund return policy. This implies full market coverage in the first period (i.e., all consumers purchase), and a market allocation such that one seller serves the consumers with higher valuations and sees no dissatisfied consumers and no returns, while the other seller serves the remainders and sees some dissatisfied consumers and some returns. This is an interesting result given that the sellers are identical. In the higher g range ($g_\theta^{VI} < g < g_\theta^{IV}$), both sellers provide full information and offer arbitrary return policies. In this case, while consumers are indifferent between the sellers, only the consumers with ex-post valuations at least as high p_1 purchase in the first period and there is ex-ante efficient allocation of the goods.

We note that costly information provision may significantly alter the Nash equilibria, even if the sellers would have the identical cost structure. For example, it is conceivable that in the mixed-strategy equilibria described above, the range for α would be capped from above since neither seller would have an incentive to incur high information provision costs. Furthermore, the pure-strategy equilibria where one or both sellers provide full information, $\alpha = 1$, may not exist; in that case, a mixed-strategy equilibrium where both sellers offer full refunds, $\beta = 1$, with α randomized over a range may prevail.

Regarding the refund factor, β , recall that in the monopoly solution described in Proposition 5, full refunds are never exercised; Solution (D) suggests partial

refunds, and when Solution (E) is optimal, amount of refund is arbitrary. However, we see from Proposition 8 that competition changes the picture abruptly. In the duopoly case, all but one Nash equilibria suggest that at least one seller offers full refunds; for $s < s_\theta$ and $g_\theta^{VI} < g < g_\theta^{IV}$, both sellers provide full information and the refund amount becomes arbitrary.

Consequently, we conclude, contrasting with the monopoly case, that while competition results in one or both of the sellers withholding information from consumers in certain cases, it typically forces them to offer full refunds. That helps explain why we observe full refunds in practice (e.g., Nordstrom.)

3.5 Conclusion

In this essay, we study competition in the context of information provision and return policies in experience good markets. In order to isolate the effects of competition on our results in the monopoly model given in Essay 2, we consider a duopoly case where two identical sellers engage in information provision and return policy competition. We identify the possible pure-strategy Nash equilibria, or if none exists, the mixed-strategy Nash equilibria, and the associated conditions where they take place. We find, in contrast to the monopoly case, that while competition can cause the sellers to withhold information under certain conditions, it typically forces them to offer full-refund return policies. This finding can shed light on some real-life phenomena where sellers offer full refunds and/or they do not put much effort to provide informational tools to consumers.

Appendix A

Appendix for Essay 1

A.1 Derivation of $h(\gamma)$

Let R_{it} denote the reservation price for product i at time t , a random variable. We write $R_{1t} = u(\Omega)\varepsilon_{1t}$, where $u(\cdot)$ is a (deterministic) *linear* mapping function and ε_{1t} is a random variable with a Weibull distribution (so that R_{it} has a Weibull distribution); similarly $R_{2t} = u((1+\gamma)\Omega)\varepsilon_{2t}$. We define customer utility \mathcal{V}_{it} as a log function of the customer's reservation price $\mathcal{V}_{it} = \ln(R_{it}) = \ln(u(\Omega)) + \ln(\varepsilon_{it})$. Because ε_{it} has a Weibull distribution, \mathcal{V}_{it} has a Gumbel distribution; this is consistent with the Logit model for choice. As a result, the probability that a customer adopts the new generation is

$$h(\gamma) = \frac{e^{\ln(u((1+\gamma)\Omega))}}{e^{\ln(u((1+\gamma)\Omega))} + e^{\ln(u(\Omega))}} = \frac{u((1+\gamma)\Omega)}{u(\Omega) + u((1+\gamma)\Omega)} = \frac{1+\gamma}{2+\gamma}. \quad (\text{A.1})$$

A.2 Proof of Proposition 1

For a non-stationary Poisson process with intensity $\Lambda(t)$, $\text{E}[N(\Lambda(t))] = \Lambda(t)$, and thus (1.6) becomes

$$\begin{aligned} \max_{p_2(t)} \Pi_2^j &= \int_{\tau}^{(1+\alpha)\tau} e^{-\delta(t-\alpha\tau)} (p_2(t) - c_p) \text{E} [N(\lambda_2^j(t)\bar{G}_{2t}(p_2))] dt \\ &= \int_{\tau}^{(1+\alpha)\tau} e^{-\delta(t-\alpha\tau)} (p_2(t) - c_p) \lambda_2^j(t)\bar{G}_{2t}(p_2) dt \end{aligned} \quad (\text{A.2})$$

This is a simple optimal control problem, with the first-order necessary condition given in (1.7). The solution is similar to that found in Bitran and Mondschein (1997). For uniqueness of the solution to (1.7), we need

$$K_{2,t} = p_2(t) - \frac{\bar{G}_{2t}}{G_{2t}}$$

to be an increasing function of $p_2(t)$ since $\lim_{p_2(t) \rightarrow \infty} K_{2,t} = \infty$. Therefore, we need

$$0 < \frac{dK_{2,t}}{dp_2} = 1 - \frac{d}{dp_2} \frac{\bar{G}_{2t}}{G_{2t}} = 1 - \frac{-G_{2t}^2 - \bar{G}_{2t}g'_{2,t}}{G_{2t}^2}$$

which becomes

$$0 > \frac{-2G_{2t}^2\bar{G}_{2t} - \bar{G}_{2t}^2g'_{2,t}}{G_{2t}^2} = \frac{d}{dp_2} \frac{\bar{G}_{2t}}{G_{2t}}. \blacksquare$$

A.3 Proof of Proposition 2

We show through fluid approximations (Mandelbaum and Pats 1998) that the solutions to the deterministic version of (1.5) is asymptotically optimal as initial maximum arrival rate, M_0 , and I_0 grow proportionally large. However, since I_0 is a decision variable, we first show that it is optimal to select I_0 proportionally large as M_0 .

Consider a sequence of instances of problem (1.5) indexed by $n \in \mathbb{Z}_+$. Let M_0^n denote the initial maximum arrival rate and λ_1^{jn} be the resulting arrival rate intensity function for the n^{th} instance. Let

$$\lim_{n \rightarrow \infty} \frac{M_0^n}{n} = M_0.$$

Thus, we have

$$\lim_{n \rightarrow \infty} \frac{\lambda_1^{jn}}{n} = \lambda_1^j.$$

Let I_0^n be the decision parameter for the final build and $I^n(t)$ denote the corresponding inventory trajectory for the n^{th} instance, and let all other parameters be held constant, independent of n . For the n^{th} instance, (1.5) becomes

$$\begin{aligned} \max_{I_0^n, p_1(t)} \mathbb{E} & \left[- \int_{\alpha\tau}^{T^j} e^{-\delta(t-\alpha\tau)} p_1(t) dI^n(t) + e^{-\delta(1-\alpha)\tau} c_s \left(I_0^n + \int_{\alpha\tau}^{T^j} dI^n(t) \right) \right. \\ & \left. - c_h \int_{\alpha\tau}^{T^j} e^{-\delta(t-\alpha\tau)} I^n(t) dt - c_p I_0^n \right] \\ \text{s.t.} & \end{aligned} \tag{A.3}$$

$$- \int_{\alpha\tau}^{T^j} dI^n(t) \leq I_0^n$$

$$I^n(t) = I_0^n - N \left(\int_{\alpha\tau}^t \lambda_1^{jn}(u) \bar{G}_{1u}(p_1) du \right) \text{ for } t \in [\alpha\tau, T^j],$$

where we wrote $I(T^j) \geq 0$ as $-\int_{\alpha\tau}^{T^j} dI(t) \leq I_0$. After dividing the second constraint by n , taking limits on both sides, and applying Lebesgue's monotone convergence theorem, we get

$$\lim_{n \rightarrow \infty} \frac{1}{n} I^n(t) = \lim_{n \rightarrow \infty} \frac{1}{n} I_0^n - N \left(\int_{\alpha\tau}^t \lambda_1^j(u) \bar{G}_{1u}(p_1) du \right).$$

Similarly, from the first constraint in (A.3), we have

$$\lim_{n \rightarrow \infty} -\frac{1}{n} \int_{\alpha\tau}^{T^j} dI^n(t) \leq \lim_{n \rightarrow \infty} \frac{1}{n} I_0^n.$$

Therefore, applying the same transformation to the objective function, we can rewrite (A.3) as

$$\lim_{n \rightarrow \infty} \frac{1}{n} \max_{I_0^n, p_1(t)} \mathbb{E} \left[- \int_{\alpha\tau}^{T^j} e^{-\delta(t-\alpha\tau)} p_1(t) dI^n(t) + e^{-\delta(1-\alpha)\tau} c_s \left(I_0^n + \int_{\alpha\tau}^{T^j} dI^n(t) \right) - c_h \int_{\alpha\tau}^{T^j} e^{-\delta(t-\alpha\tau)} I^n(t) dt - c_p I_0^n \right]$$

s.t.

$$\begin{aligned} \lim_{n \rightarrow \infty} -\frac{1}{n} \int_{\alpha\tau}^{T^j} dI^n(t) &\leq \lim_{n \rightarrow \infty} \frac{1}{n} I_0^n \\ \lim_{n \rightarrow \infty} \frac{1}{n} I^n(t) &= \lim_{n \rightarrow \infty} \frac{1}{n} I_0^n - N \left(\int_{\alpha\tau}^t \lambda_1^j(u) \bar{G}_{1u}(p_1) du \right) \text{ for } t \in [\alpha\tau, T^j]. \end{aligned} \tag{A.4}$$

Suppose (I_0^*, p_1^*) is an optimal solution to (1.5), with the optimal objective function value π_1^* . Then, (I_0^{n*}, p_1^*) is an optimal solution to (A.4) with the objective function value π_1^{n*} , such that I_0^{n*} and π_1^{n*} satisfy $\lim_{n \rightarrow \infty} I_0^{n*}/n = I_0^*$ and $\pi_1^{n*} = \pi_1^*$, respectively. This follows by observing that (A.4) is equivalent to problem (1.5) divided by n and taking limits as $n \rightarrow \infty$. As a result, we have shown that it is optimal to let the final build, I_0 , grow proportionally large as M_0 in the asymptotic regime.

Noting that the demand intensity process

$$\int_{\alpha\tau}^t \lambda_1^j(u) \bar{G}_{1u}(p_1) du$$

is continuous and uniformly bounded in $[\alpha\tau, T^j]$, and we find that in the limit as $n \rightarrow \infty$, $I^n(t)/n$ converges (almost surely and uniformly over a compact set) to $I(t)$, given by

$$I(t) = I_0 - \int_{\alpha\tau}^t \lambda_1^j(u) \bar{G}_{1u}(p_1) du.$$

Further details regarding the proof of this convergence result can be found

in Mandelbaum and Pats (1998). In this asymptotic regime, the stochastic optimization problem in (1.5) reduces to the optimal control problem in (1.8), where $I_0 + \int_{\alpha\tau}^{T^j} dI(t)$ is replaced with $I(T^j)$, and the second constraint is substituted into the first term in the objective function.

The solution to (1.8) can be found as follows. Treating $I(t)$ as the state variable and $p_1(t)$ as the control variable, and letting ν and $\omega(t)$ be the multipliers for the first and second constraints in (1.8), the Hamiltonian is $H = e^{-\delta(t-\alpha\tau)}(\lambda_1^j \bar{G}_{1t} p_1 - c_h I) - \omega \lambda_1^j \bar{G}_{1t}$, where arguments have been suppressed for simplicity. The optimality conditions are:

$$\frac{\partial H}{\partial p_1} = 0 \quad \therefore \quad \lambda_1^j [e^{-\delta(t-\alpha\tau)} (-p_1 G_{1t} + \bar{G}_{1t}) + \omega(t) G_{1t}] = 0, \quad (\text{A.5})$$

$$\frac{\partial H}{\partial I} = -\frac{\partial \omega}{\partial t} \quad \therefore \quad c_h e^{-\delta(t-\alpha\tau)} = \frac{\partial \omega}{\partial t}, \quad (\text{A.6})$$

$$\omega(T^j) = \nu + e^{-\delta(T^j-\alpha\tau)} c_s \quad \text{and} \quad \nu I(T^j) = 0. \quad (\text{A.7})$$

A first-order condition for I_0 is obtained by considering that the marginal revenue from the last unit must equal to its marginal cost (including the procurement cost and cumulative holding costs in time). That is,

$$\omega(T^j) = c_p + c_h \int_{\alpha\tau}^{T^j} e^{-\delta(u-\alpha\tau)} du. \quad (\text{A.8})$$

Combining (A.7) and (A.8), we get

$$\nu = c_p + c_h \int_{\alpha\tau}^{T^j} e^{-\delta(u-\alpha\tau)} du - e^{-\delta(T^j-\alpha\tau)} c_s. \quad (\text{A.9})$$

However, we must have $\nu > 0$, otherwise $I_0 \rightarrow \infty$ is optimal and the problem in (1.8) is unbounded. Therefore, from (A.7), $I(T^j) = 0$. In other words, the entire

initial inventory is depleted during the sales horizon. To find $p_1(t)$, we proceed as follows. From (A.5),

$$\omega(t) = e^{-\delta(t-\alpha\tau)} \left(p_1 - \frac{\bar{G}_{1t}}{G_{1t}} \right). \quad (\text{A.10})$$

On the other hand, (A.7) and (A.8) yield

$$\omega(t) = c_p + c_h \int_{\alpha\tau}^t e^{-\delta(u-\alpha\tau)} du. \quad (\text{A.11})$$

We combine (A.10) and (A.11) to obtain the necessary condition for the optimal price pattern for product 1, given in (1.9). The proof of uniqueness follows the same steps as in the proof for Proposition 1.

Once the optimal price path is determined using (1.9), and given that $I(T^j) = 0$, the optimal initial inventory is equal to the total sales through the planning horizon. ■

A.4 Normalization of ϕ for the Regression

We normalize the parameter ϕ , for the purposes of running the regression, so that it takes values between 0 and 1, instead of between 0 and ∞ . We do this by mapping ϕ to a new parameter θ , according to the normalizing relationship:

$$\phi = (1 - \alpha)\tau / \left(\frac{1}{\theta} + W \left(-\frac{1}{\theta} \times e^{-\frac{1}{\theta}} \right) \right), \quad (\text{A.12})$$

where $W(\cdot)$ is the Lambert W function. The Lambert W function is the inverse of $f(w) = we^w$ and we use the zeroth branch which is single valued and real for the range of θ considered. It is easily verified that $\lim_{\phi \rightarrow \infty} \theta = 1$, and $\lim_{\phi \rightarrow 0} \theta = 0$.

Table A.1: Statistics of multiple linear regression: Two-way interaction effects.

Factor	t-value	
(intercept)	-2.7	**
τ	-60.9	***
α	9.0	***
$\log(k)$	-3.8	***
μ	35.3	***
γ	-35.7	***
$p + q$	-43.7	***
$\log(q/p)$	68.3	***
ϕ	-94.5	***
$\tau : \alpha$	-20.9	***
$\tau : \log(k)$	-17.7	***
$\tau : \mu$	-1.8	o
$\tau : \gamma$	-35.0	***
$\tau : p + q$	-30.4	***
$\tau : \log(q/p)$	25.1	***
$\tau : \phi$	-48.1	***
$\alpha : \log(k)$	1.9	o
$\alpha : \mu$	6.3	***
$\alpha : \gamma$	-12.0	***
$\alpha : p + q$	-20.0	***
$\alpha : \log(q/p)$	30.7	***
$\alpha : \phi$	-9.5	***
$\log(k) : \mu$	10.0	***
$\log(k) : \gamma$	-10.3	***
$\log(k) : p + q$	-11.3	***
$\log(k) : \log(q/p)$	17.7	***
$\log(k) : \phi$	-24.4	***
$\mu : \gamma$	-4.0	***
$\mu : p + q$	-11.2	***
$\mu : \log(q/p)$	15.7	***
$\mu : \phi$	-21.9	***
$\gamma : p + q$	-16.3	***
$\gamma : \log(q/p)$	25.8	***
$\gamma : \phi$	-35.2	***
$p + q : \log(q/p)$	-6.3	***
$p + q : \phi$	-4.2	***
$\log(q/p) : \phi$	-6.0	***
Adj. R-sq.	0.837	

Statistical significance codes: '***': $p \approx 0$; '**': $0.001 < p < 0.01$; 'o': $0.05 < p < 0.1$

Appendix B

Appendix for Essay 2

B.1 Proof of Proposition 3

Under partial information, using the definition of v_θ , the condition $v_\theta = p_1$ translates to $p_1 = E[\max\{V, p_1\beta\}]$. Suppose that $p_1 \geq E[V]$. Then, since $p_1 \leq E[\max\{V, p_1\}]$ and through the intermediate value theorem, there exists $\beta_p \in [0, 1]$ such that $p_1 = E[\max\{V, p_1\beta_p\}]$. Note that if $p_1 < E[V]$, no such $\beta_p \in [0, 1]$ exists and there are no first period buyers with positive surplus for $\alpha < 1$. Thus, if $p_1 \geq E[V]$, setting $\beta = \beta_p$ is equivalent to providing full information as it completely nullifies the consequences of valuation uncertainty regardless of the value of α . ■

B.2 Structural Properties of the (α, β) Decision Space

B.2.1 Boundary for Returns: $v_\theta(\alpha, \beta) = p_1\beta$

The condition $v_\theta(\alpha, \beta) = p_1\beta$ is critical for the existence of returns. Assuming $\alpha > 0$, this condition is equivalent to $p_1 + \frac{(1-\alpha)}{\alpha} (p_1 - E[\max\{V, p_1\beta\}]) = p_1\beta$, which reduces to

$$\alpha = \alpha_r(\beta) \triangleq \frac{E[\max\{V, p_1\beta\}] - p_1}{E[\max\{V, p_1\beta\}] - p_1\beta}.$$

Note that, given p_1 , this equation represents a curve in the (α, β) space. Suppose first that $p_1 \geq E[V]$. Then, by Proposition 3, $\beta_p \geq 0$ exists and therefore $\frac{\partial v_\theta}{\partial \alpha} = -\frac{1}{\alpha^2} (p_1 - E[\max\{V, p_1\beta\}]) > 0$. Furthermore, $\alpha_r(\beta) > 0$ if and only if $\beta > \beta_p$, since the denominator in $\alpha_r(\beta)$ is always positive. Thus, $v_\theta(\alpha, \beta) < p_1\beta$ if and only if $\alpha < \alpha_r(\beta)$. Finally, $\alpha_r(\beta)$ is strictly convex and increasing, since for a continuously differentiable F , $\frac{d\alpha_r}{d\beta} = \frac{p_1(E[\max\{V, p_1\beta\}] - p_1(1 - (1-\beta)F(p_1\beta)))}{(E[\max\{V, p_1\beta\}] - p_1\beta)^2} > 0$, and $\frac{d^2\alpha_r}{d\beta^2} = p_1^2 \frac{(2(1-F(p_1\beta))(E[\max\{V, p_1\beta\}] - p_1) + p_1(1-\beta)F(p_1\beta)) + p_1(1-\beta)(E[\max\{V, p_1\beta\}] - p_1)F'(p_1\beta)}{(E[\max\{V, p_1\beta\}] - p_1\beta)^3} > 0$ for all $\beta > \beta_p$. Suppose $p_1 < E[V]$. Then, we have $\frac{\partial v_\theta}{\partial \alpha} > 0$, $\alpha_r(\beta) > 0$, $\frac{d\alpha_r}{d\beta} > 0$, and

$\frac{d^2\alpha_r}{d\beta^2} > 0$ satisfied for all $\beta \in [0, 1]$.

B.2.2 Boundary for 100% Sales: $v_\theta(\alpha, \beta) = 0$

The condition $v_\theta(\alpha, \beta) = 0$ reduces to

$$\alpha = \alpha_0(\beta) \triangleq 1 - \frac{p_1}{\mathbb{E}[\max\{V, p_1\beta\}]}.$$

Similar to above, when $p_1 \geq \mathbb{E}[V]$, $\beta_p \geq 0$ exists and for $\beta > \beta_p$, $\alpha_0(\beta) > 0$ is well defined and we have $\frac{\partial v_\theta}{\partial \alpha} > 0$. Finally, the function $\alpha_0(\beta)$ is strictly increasing if F is continuously differentiable, since $\frac{d\alpha_0}{d\beta} = \frac{p_1^2 F(p_1\beta)}{(\mathbb{E}[\max\{V, p_1\beta\}])^2} > 0$. If $p_1 < \mathbb{E}[V]$, then we have $\frac{\partial v_\theta}{\partial \alpha} > 0$, $\alpha_0(\beta) > 0$ and $\frac{d\alpha_0}{d\beta} > 0$ for all $\beta \in [0, 1]$.

B.2.3 Boundary for No Sales: $v_\theta(\alpha, \beta) = 1$

Note that $v_\theta(\alpha, \beta) = 1$ is possible only if $p_1 > \mathbb{E}[V]$, and it reduces to

$$\alpha = \alpha_1(\beta) \triangleq \frac{p_1 - \mathbb{E}[\max\{V, p_1\beta\}]}{1 - \mathbb{E}[\max\{V, p_1\beta\}]}.$$

Then, it is seen that $\alpha_1(\beta) > 0$ only if $0 \leq \beta < \beta_p$. On the other hand, we have $\frac{\partial v_\theta}{\partial \alpha} < 0$ for all $\beta < \beta_p$; hence, $v_\theta(\alpha, \beta) > 1$ for all $\beta \in (0, \beta_p)$ and $\alpha < \alpha_1(\beta)$. Finally, for a continuously differentiable F , $\frac{d\alpha_1}{d\beta} = -\frac{p_1(1-p_1)F(p_1\beta)}{(1-\mathbb{E}[\max\{V, p_1\beta\}])^2} < 0$, and $\frac{d^2\alpha_1}{d\beta^2} = -\frac{p_1^2(1-p_1)(2(F(p_1\beta))^2 + (1-\mathbb{E}[\max\{V, p_1\beta\}])F'(p_1\beta))}{(1-\mathbb{E}[\max\{V, p_1\beta\}])^3} < 0$; therefore, $\alpha_1(\beta)$ is strictly concave and decreasing in β .

B.2.4 Redundant Regions

We observe that two regions are rendered redundant in our model with the assumption of costless information. First, note the region where $\alpha < \alpha_0(\beta)$. By definition, any (α, β) pair in this region results in $v_\theta < 0$; all consumers purchase since $v \in (0, 1)$. Furthermore, this result is also achieved with no information ($\alpha = 0$), as long as $\beta > \beta_p$; when $\beta > \beta_p$, we have $v_0 = \mathbb{E}[\max\{V, p_1\beta\}] > p_1$ by definition. Therefore, for a given $\beta > \beta_p$, the seller is indifferent in choosing an $\alpha \in [0, \alpha_0(\beta)]$

when information is costless. Consequently, in our analyses in this study, we treat the conditions $v_\theta = 0$ and $\alpha = 0$ as equal at a given $\beta > \beta_p$.

Similarly, the second region is where $\alpha < \alpha_1(\beta)$, for which $v_\theta > 1$; there are no sales since consumer valuations are in $(0, 1)$. We see that “no sales” is achieved for any $\alpha \in [0, \alpha_1(\beta)]$, which is possible for $\beta < \beta_p$.

B.3 Analysis of the α, β Decision Space

B.3.1 Ex-Ante Efficient Market: $v_\theta = p_1$

When $v_\theta = p_1$, there are no dissatisfied consumers and there are no returns; i.e., $L = M = 0$. Then, the seller’s optimization problem takes the following form, where we name the region where $v_\theta = p_1$ as Region (E):

$$\begin{aligned} \max_{\alpha, \beta, p_i} R^E &= (p_1 - c)(1 - F(p_1)) + (p_2 - c)g(1 - F(p_2)) \\ \text{s.t.} \quad v_\theta &= p_1 \end{aligned} \tag{B.1}$$

The optimal prices are determined as $p_1^* = p_2^* = \arg \max_p (p - c)(1 - F(p))$. Since the objective function does not include α or β , the seller is indifferent in choosing them as long as they satisfy $v_\theta = p_1$. We name this as Solution (E).

B.3.2 Positive Consumer Surplus: $v_\theta > p_1$

Regardless of the value of $\alpha \in (0, 1)$, all buyers have strictly positive surplus when $\beta < \beta_p$ since $v_\theta^I > p_1$; we name this region as Region (I). In other words, under partial information, setting a stringent returns policy scares away some consumers with $v_\theta > v \geq p_1$ due to high risk of fit. Seller’s optimization problem in this region is

$$\begin{aligned} \max_{\alpha, \beta, p_i} R^I &= (p_1 - c)(1 - F(v_\theta)) + (p_2 - c)g(1 - F(p_2)) \\ \text{s.t.} \quad v_\theta &> p_1 \end{aligned} \tag{B.2}$$

We observe that $\frac{\partial R^I}{\partial v_\theta} = -F'(v_\theta)(p_1 - c) < 0$; it is optimal to decrease the purchasing threshold at any price point. We conclude that any (α, β) decision in

Region (I) is strictly dominated by Region (E) since $v_\theta^E = p_1 < v_\theta^I$; the seller never lets the consumers have positive surplus while there is an option to have efficient allocation.

B.3.3 Some Dissatisfied Consumers, No Returns: $p_1 > v_\theta \geq p_1\beta$

In the region where $p_1 > v_\theta \geq p_1\beta$, Region (II), there are some dissatisfied buyers but they are not willing to return their items as the refund amount is not high enough. We have $L = F(p_1) - F(v_\theta)$, and since there are no returns, $M = 0$. Seller's profit maximization problem in this region is

$$\begin{aligned} \max_{\alpha, \beta, p_i} R^{II} &= (p_1 - c)(1 - F(v_\theta)) + (p_2 - c)g(1 - F(p_2))(1 - F(p_1) + F(v_\theta)) \\ \text{s.t.} \quad p_1 > v_\theta &\geq p_1\beta \end{aligned} \tag{B.3}$$

The partial derivative with respect to v_θ is

$$\frac{\partial R^{II}}{\partial v_\theta} = F'(v_\theta)(-p_1 + c + g(p_2 - c)(1 - F(p_2)))$$

which is positive for

$$g > g_\theta^{II} \triangleq \frac{(p_1 - c)}{(p_2 - c)(1 - F(p_2))}, \tag{B.4}$$

and negative for $g < g_\theta^{II}$. Then, at any given price point (p_1, p_2) , it is optimal to increase v_θ if $g > g_\theta^{II}$ and it is optimal to decrease v_θ if $g < g_\theta^{II}$. Therefore, if $g > g_\theta^{II}$, Region (II) is dominated by Region (E) since $v_\theta^E = p_1 > v_\theta^{II}$; if $g < g_\theta^{II}$, Region (II) is dominated by the boundary where $v_\theta = p_1\beta$. As a result, no internal solution is optimal in Region (II).

B.3.4 Some Dissatisfied Consumers, Some Returns: $v_\theta < p_1\beta$

Region (III) is characterized by $v_\theta^{III} < p_1\beta$, which means there are some dissatisfied consumers ($L = F(p_1) - F(v_\theta)$) and a portion ($M = F(p_1\beta) - F(v_\theta)$) of these consumers are willing to return their purchases. Thus, the seller's optimization

problem in this region is as follows:

$$\begin{aligned} \max_{\alpha, \beta, p_i} R^{III} &= (p_1 - c)(1 - F(v_\theta)) + (-p_1\beta + s)(F(p_1\beta) - F(v_\theta)) \\ &\quad + g(p_2 - c)(1 - F(p_2))(1 - F(p_1) + F(v_\theta)) \end{aligned} \quad (\text{B.5})$$

$$\text{s.t.} \quad v_\theta < p_1\beta$$

In order to solve this problem, we look at the partial derivative of the objective function with respect to v_θ : $\frac{\partial R^{III}}{\partial v_\theta} = F'(v_\theta)(-p_1 + c + p_1\beta - s + g(p_2 - c)(1 - F(p_2)))$ which is positive if

$$g > g_\theta^{III} \triangleq \frac{((p_1 - c) + (s - p_1\beta))}{(p_2 - c)(1 - F(p_2))} \quad (\text{B.6})$$

and negative if $g < g_\theta^{III}$. Then, since $v_\theta^{II} > v_\theta^{III} > 0$, if $g > g_\theta^{III}$, Region (II) dominates Region (III), and if $g < g_\theta^{III}$, the boundary where $v_\theta = 0$ dominates Region (III). We name the part of this boundary region where $\beta > \frac{s}{p_1}$ as Region (B) and the part where $\beta \leq \frac{s}{p_1}$ as Region (D).

Note that if $\beta > \frac{s}{p_1}$ then $g_\theta^{III} < g_\theta^{II}$, and if $\beta < \frac{s}{p_1}$ then $g_\theta^{III} > g_\theta^{II}$. Therefore, combining our results so far for Regions (II) and (III), for $\beta < \frac{s}{p_1}$, there is a range $g_\theta^{II} < g < g_\theta^{III}$ where Region (E) dominates Region (II) and Region (D) dominates Region (III) and however it is not obvious which one of the two dominates the other. In order to identify the threshold g value, we write the profit functions for the two regions equal, $R^E = R^D$, and solve for g ;

$$g_\theta^E \triangleq \frac{\left((p_1 - c) + (s - p_1\beta) \frac{F(p_1\beta)}{F(p_1)} \right)}{(p_2 - c)(1 - F(p_2))}. \quad (\text{B.7})$$

Observing that $\frac{\partial R^E}{\partial g} > \frac{\partial R^D}{\partial g}$, we conclude that Region (E) dominates Region (D) if $g > g_\theta^E$. Note that for $\beta < \frac{s}{p_1}$, $g_\theta^{II} < g_\theta^E < g_\theta^{III}$; wherever Region (E) dominates Region (D), it dominates all other regions, and wherever Region (D) dominates Region (E), it dominates all other regions.

The following summarizes our findings in determining the optimal region in the (α, β) decision space given β , p_1 , p_2 , c , s , and g .

1. For $\beta > \frac{s}{p_1}$, we have $g_\theta^{III} < g_\theta^{II}$, and

- (i) If $g > g_{\theta}^{II}$, then Region (E) dominates all regions; it is optimal to set $\alpha = 1$.
 - (ii) If $g_{\theta}^{III} < g < g_{\theta}^{II}$, then the boundary region between Regions (II) and (III) where $v_{\theta} = p_1\beta$, Region (A), dominates all regions.
 - (iii) If $g < g_{\theta}^{III}$, then Region (B) dominates all regions.
2. For $\beta \leq \frac{s}{p_1}$, we have $g_{\theta}^{III} \geq g_{\theta}^E \geq g_{\theta}^{II}$, and
- (i) If $g > g_{\theta}^E$, then Region (E) dominates all regions.
 - (ii) If $g < g_{\theta}^E$, then Region (D) dominates all regions.

We see that there are four candidate regions (A, B, D and E) for optimality. Figure B.1 gives an illustration of these regions for a specific case. Note again that the findings above are for given β , p_1 , p_2 , c , s , and g values and point to the best α decision depending on the relationships between these “parameters”. For example, consider a price taking seller for whom the restocking fee, hence β , is also dictated by either the industry or some trade regulations. Then, the above rules apply directly to find the optimum amount of information to be provided, assuming it is costless.

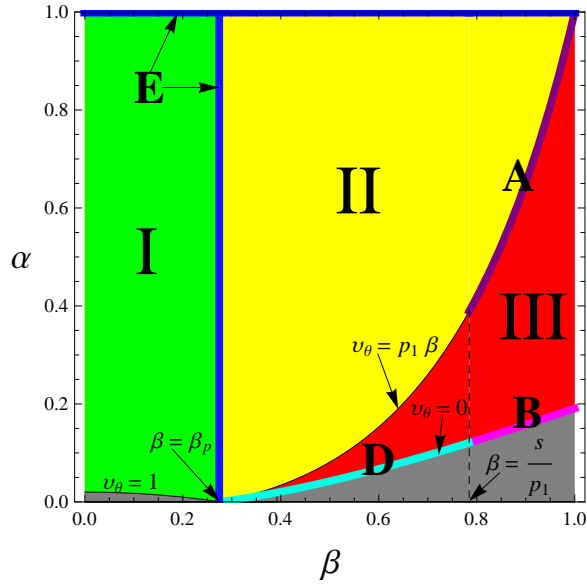


Figure B.1: Candidate regions for optimality for $V \sim U(0, 1)$, $p_1 = 0.51$ and $s = 0.4$.

We continue our analysis considering a seller who can set all α , β and p_1 freely, and use the above rules as a guideline to find the optimum strategy.

B.4 Analysis of Candidate Regions and Proof of Proposition 5

B.4.1 Region (A)

We start with looking at Region (A), which is optimal if $g_\theta^{III} < g < g_\theta^{II}$, which is in turn possible if $\beta > \frac{s}{p_1}$. Plugging in the defining constraint, $v_\theta = p_1\beta$, to the objective function and taking partial derivative with respect to β , we get

$$\frac{\partial R^A}{\partial \beta} = p_1 F'(p_1\beta) (-p_1 + c + g(p_2 - c)(1 - F(p_2))),$$

which is negative for $g < g_\theta^{II}$. Therefore, it is optimal to decrease β in Region (A), where $\sup(\beta) = \frac{s}{p_1}$. As a result, setting $\beta = \frac{s}{p_1}$ and α such that $v_\theta = p_1\beta$ dominates Region (A). However, from the discussion above, when $\beta = \frac{s}{p_1}$, Region (D), where $v_\theta = 0$, is optimal. This therefore establishes that Region (D) dominates Region (A).

B.4.2 Regions (B) and (D)

We collectively represent Regions (B) and (D) as $v_\theta = 0$. Thus, the objective function for these regions is

$$R^{B,D} = (p_1 - c) + (-p_1\beta + s) F(p_1\beta) + g(p_2 - c)(1 - F(p_2))(1 - F(p_1)).$$

The first order condition (FOC) for β is

$$\frac{\partial R^{B,D}}{\partial \beta} = p_1 (-F(p_1\beta) + F'(p_1\beta) (-p_1\beta + s)) = 0,$$

or equivalently,

$$\beta^* = \frac{s}{p_1} - \frac{1}{p_1} \frac{F(p_1\beta^*)}{F'(p_1\beta^*)}. \quad (\text{B.8})$$

Note that since the second term on the right is positive, at optimality, we have $\beta^* < \frac{s}{p_1}$. This means that Region (D) dominates Region (B) since the latter is defined for $\beta > \frac{s}{p_1}$.

Combining our results so far, we see that when $g < g_{\theta}^{II}$, it is never optimal to have $\beta > \frac{s}{p_1}$; Region (D) dominates both Regions (A) and (B). Therefore, we have two candidate optimal solutions left: Solution (D), which is defined by $v_{\theta} = 0$ (or equivalently, $\alpha \in [0, \alpha_0(\beta)]$, as shown above) and the FOC given in Equation B.8, and Solution (E) which is defined by $v_{\theta} = p_1$. This constitutes the proof of Proposition 5. ■

B.5 Proof of Proposition 6

From the analysis of the structural properties of seller's (α, β) decision space, we know that changing p_1 changes the (α, β) decision space as all the critical boundaries is a function of p_1 . Therefore, when p_1 is a decision variable as well, the seller has the tool to change the (α, β) decision space in order to maximize her profits. Considering this, we observe that the point $\beta = \frac{s}{p_1}$ on Region (D) structurally changes the profit function when solving for the optimal p_1 ; when $\beta = \frac{s}{p_1}$, refund is equal to salvage value and each return has zero net after-sales revenue. Therefore, we take this point explicitly and define Region (C): $\{\alpha = 0, \beta = \frac{s}{p_1}\}$. Profits for Region (C) is given by $R^C = (p_1 - c) + g(p_2 - c)(1 - F(p_2))(1 - F(p_1))$. Since Region (C) is a single point on the (α, β) decision space, the seller's profits here is a function of only the prices.

In the following, we solve for the optimal α , β , p_1 and p_2 for each of the regions (C), (D), (E), and we identify the optimal strategy for uniformly distributed consumer valuations; $F(p) = p$. Using Proposition 4, the optimal second period price is $p_2^* = (1 + c)/2$ for all regions.

B.5.1 Region (C)

In order to solve for the optimal profit function, we take the partial derivative with respect to p_1 , $\frac{\partial R^C}{\partial p_1} = 1 - g(p_2 - c)(1 - p_2)$, which is positive for $g < g_{\theta}^E$. Recall that Region (C) is optimal only if $g < g_{\theta}^E$; thus, it is optimal to increase p_1 as much as possible in the feasible range for Region (C). Constrained by the equality

$\beta = \frac{s}{p_1}$ for Region (C), the largest value that p_1 can attain is determined by the smallest value that β can take, which is equal to β_p . With uniform valuations, we have $E[\max\{V, p_1\beta\}] = \frac{1+(p_1\beta)^2}{2}$, and we find that

$$\beta_p = \{\beta \mid p_1 = E[\max\{V, p_1\beta\}]\} = \frac{\sqrt{2p_1 - 1}}{p_1}$$

from Proposition 3. Therefore, the optimal price should satisfy $\sqrt{2p_1 - 1} = s$, and it yields $p_1^* = \frac{1+s^2}{2}$. Then, we find the optimal refund factor as $\beta^* = \frac{s}{p_1} = \frac{2s}{1+s^2}$. As a result, recalling that $\alpha = 0$ in Region (C) by definition, Solution (C) is given by

$$(\alpha^*, \beta^*, p_1^*, p_2^*)_C = \left(0, \frac{2s}{1+s^2}, \frac{1+s^2}{2}, \frac{1+c}{2}\right)$$

and it yields the net profits of

$$R^{C*} = \frac{1}{8}g(1-c)^2(1-s^2) + \frac{s^2}{2} - c + \frac{1}{2}.$$

Note that $\beta^* = \frac{2s}{1+s^2} < 1$ for all $s < 1$; it is not optimal to offer full refunds when Solution (C) is optimal.

B.5.2 Region (D)

With uniform valuations, the FOC for β yields $\beta = \frac{s}{2p_1}$ in Region (D). Plugging this equality in the objective function, we get $\frac{\partial R^D}{\partial p_1} = \frac{\partial\left((p_1-c) + \frac{1}{4}(s)^2 + g(p_2-c)(1-p_2)(1-p_1)\right)}{\partial p_1} = 1 - g(p_2 - c)(1 - p_2)$. Therefore, profits are increasing in p_1 for $g < g_\theta^E$, for which Region (D) is optimal; i.e., it is optimal to increase p_1 as high as possible in Region (D). Given the FOC for β , the highest value for p_1 is determined by lowest value of β , which is equal to $\beta_p = \frac{\sqrt{2p_1-1}}{p_1}$. Therefore, the optimal price should satisfy $p_1\beta_p = \sqrt{2p_1-1} = \frac{s}{2}$, which yields $p_1^* = \frac{4+s^2}{8}$. The corresponding β is $\beta^* = \frac{s}{2p_1} = \frac{4s}{4+s^2}$. Then, these values constitute Solution (D),

$$(\alpha^*, \beta^*, p_1^*, p_2^*)_D = \left(0, \frac{4s}{4+s^2}, \frac{4+s^2}{8}, \frac{1+c}{2}\right),$$

which results in the profits of

$$R^{D*} = \frac{1}{32}g(1-c)^2(4-s^2) + \frac{3s^2}{8} - c + \frac{1}{2}.$$

Note that $\beta^* = \frac{4s}{4+s^2} < 1$ for all $s < 1$; full refunds are not optimal when Solution (D) is optimal.

B.5.3 Region (E)

Given that $V \sim U(0, 1)$, the optimal pricing decision in Region (E) is $p_1^* = p_2^* = \frac{(1+c)}{2}$.

In this case, the seller's net profit is equal to

$$R^{E*} = \frac{1}{4}(1-c)^2(1+g).$$

As we pointed out above, the seller is indifferent in deciding on an (α, β) point in Region (E), or more formally, in choosing between $(\alpha^* = 1, \beta^* \in [0, 1])$, and given $p_1 > E[V]$, $(\alpha^* \in (0, 1], \beta^* = \frac{\sqrt{2p_1-1}}{p_1})$.

B.5.4 Deriving the Optimal Strategy

To summarize our analysis above, there are three solutions that the seller can choose among, depending on the parameters incorporated in this study:

Solution (C): $(\alpha^*, \beta^*, p_1^*, p_2^*) = \left(0, \frac{2s}{1+s^2}, \frac{1+s^2}{2}, \frac{1+c}{2}\right)$, with profits R^{C*} .

Solution (D): $(\alpha^*, \beta^*, p_1^*, p_2^*) = \left(0, \frac{4s}{4+s^2}, \frac{4+s^2}{8}, \frac{1+c}{2}\right)$, with profits R^{D*} .

Solution (E): $(\alpha^*, \beta^*) = \{(\alpha, \beta) \mid v_\theta = p_1^*\}$, $p_1^* = p_2^* = \frac{(1+c)}{2}$, with profits R^{E*} .

In order to determine the ultimate optimal strategy, we conduct a three-way comparison of the net profits offered by these optimal solutions. First, we find the thresholds on g by conducting three pairwise comparisons between the above net profits. That is, we determine $g_\theta^{i,j}$ as a function of c and s by setting $R^{i*} = R^{j*}$ and solving for g ; $g_\theta^{i,j} \triangleq \{g \mid R^{i*} = R^{j*}\}$. Through algebraic manipulations, we get

$$\begin{aligned} g_\theta^{D,C} &= \frac{4}{3(1-c)^2}, \\ g_\theta^{E,C} &= \frac{4(1+s^2) - 2(1+c)^2}{(1+s^2)(1-c)^2}, \\ g_\theta^{E,D} &= \frac{4 + 12(1+s^2) - 8(1+c)^2}{(4+s^2)(1-c)^2}. \end{aligned}$$

Then, we verify that

$$\begin{aligned}\frac{\partial(R^{D^*} - R^{C^*})}{\partial g} &= \frac{3}{32}(1-c)^2 s^2 > 0, \\ \frac{\partial(R^{E^*} - R^{C^*})}{\partial g} &= \frac{1}{8}(1-c)^2(1+s^2) > 0, \\ \frac{\partial(R^{E^*} - R^{D^*})}{\partial g} &= \frac{1}{32}(1-c)^2(4+s^2) > 0,\end{aligned}$$

establishing that Solution (i) is preferred to Solution (j) if $g > g_\theta^{i,j}$, and Solution (j) is preferred to Solution (i) if $g < g_\theta^{i,j}$. However, this gives only a partial ordering; in order to develop a full ordering, we seek the ordering of these thresholds on g . Pairwise comparisons of these thresholds show that there is a critical c value that renders $g_\theta^{D,C} = g_\theta^{E,C} = g_\theta^{E,D}$: $c_\theta(s) = \{c \mid g_\theta^{D,C} = g_\theta^{E,C} = g_\theta^{E,D}\}$. We determine

$$c_\theta(s) = 2\sqrt{\frac{1+s^2}{3}} - 1,$$

and observe that

$$\begin{aligned}\frac{\partial\left(\frac{g_\theta^{D,C}}{g_\theta^{E,C}}\right)}{\partial c} &= \frac{4(1+s^2)(1+c)}{3(2+2s^2-(1+c)^2)^2} > 0, \\ \frac{\partial\left(\frac{g_\theta^{D,C}}{g_\theta^{E,D}}\right)}{\partial c} &= \frac{4(4+s^2)(1+c)}{3(4+3s^2-2(1+c)^2)^2} > 0, \\ \frac{\partial\left(\frac{g_\theta^{E,C}}{g_\theta^{E,D}}\right)}{\partial c} &= \frac{s^2(4+s^2)(1+c)}{(1+s^2)(4+3s^2-2(1+c)^2)^2} > 0.\end{aligned}$$

Therefore, we infer that if $c > c_\theta(s)$, then $g_\theta^{E,D} < g_\theta^{E,C} < g_\theta^{D,C}$, and if $c < c_\theta(s)$, then $g_\theta^{D,C} < g_\theta^{E,C} < g_\theta^{E,D}$. Proposition 6 follows by observing the ordering of solutions (C), (D) and (E) with respect to g , which is depicted in Figure B.2. ■

B.6 On the Value of Optimal Refund Amount

It is interesting to investigate if the “refund amount” exceeds the salvage value or not: does the seller allow returns even when they have negative net revenues? By definition, Solutions (C) and (D) do not offer a refund amount more than the salvage value; they offer exactly equal to, and exactly half of the salvage value, respectively.

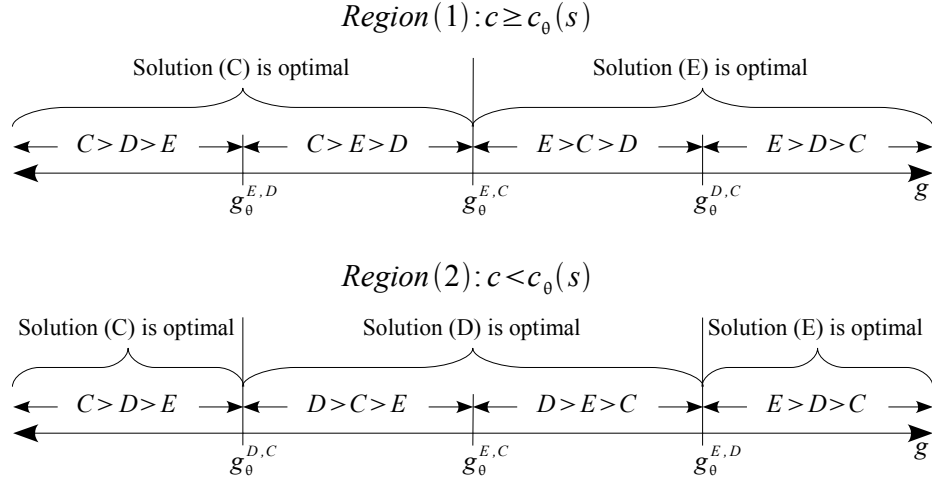


Figure B.2: Ordering of solutions according to the the regions in Figure 2.6.

In case of Solution (E), if the seller provides full information, there are no returns and the seller can “advertise” any return policy. On the other hand, if she provides only partial information and sets $\beta^* = \beta_p = \frac{\sqrt{2p_1-1}}{p_1}$, the refund amount is equal to $p_1^*\beta^* = \sqrt{2p_1^*-1} = \sqrt{c}$. Therefore, the seller can advertise a refund amount of more than the salvage value if $\sqrt{c} > s$, or $c > s^2$. However, since Solution (E) does not actually *exercise* returns, we conclude that a refund amount of more than the salvage value is never exercised.

Appendix C

Appendix for Essay 3

C.1 Proof of Proposition 7

Suppose, without loss of generality, that $\alpha_Z > \alpha_Y$. Then, the condition for seller Z capturing the whole market is $v_\theta^{YZ} = 0$. Solving from (3.1) for α_Z that satisfies this, we get

$$\alpha_Z = \bar{\alpha} \triangleq \alpha(v_\theta^{YZ} = 0) = \alpha_Y + p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{1 + (p_1\beta_Z)^2}.$$

Note that $\bar{\alpha} > \alpha_Y$ only if $\beta_Z > \beta_Y$ and $\alpha_Y < 1$. Furthermore, we observe that

$$\frac{\partial v_\theta^{YZ}}{\partial \alpha_Z} = p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{2(\alpha_Z - \alpha_Y)^2} > 0$$

if $\beta_Z > \beta_Y$ and $\alpha_Y < 1$. Then, any $\alpha_Z < \bar{\alpha}$ results in $v_\theta^{YZ} < 0$, and thus seller Z captures the whole market. Therefore, given $\alpha_Y < 1$ and $\beta_Z > \beta_Y$, $\alpha_Z \in [\alpha_Y, \bar{\alpha}]$ results in seller Z capturing the whole market.

Suppose now that $\alpha_Z < \alpha_Y$, which is possible only if $\alpha_Y > 0$. In this case, v_θ^{ZY} is defined analogous to v_θ^{YZ} in (3.1); all consumers with $v > v_\theta^{ZY}$ prefer seller Y to seller Z , while those with $v < v_\theta^{ZY}$ prefer seller Z to seller Y . Then, seller Z captures the whole market if $v_\theta^{ZY} = 1$. Solving for α_Z from (3.1), we find

$$\alpha_Z = \alpha(v_\theta^{ZY} = 1) \triangleq \alpha_Y - p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{1 - (p_1\beta_Z)^2}.$$

Observe that $\alpha(v_\theta^{ZY} = 1) < \alpha_Y$ as long as $\beta_Z > \beta_Y$ and $\alpha_Y < 1$, and that $\alpha(v_\theta^{ZY} = 1) > 0$ as long as $\alpha_Y > 0$. Moreover, we have

$$\frac{\partial v_\theta^{ZY}}{\partial \alpha_Z} = p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{2(\alpha_Y - \alpha_Z)^2} > 0$$

if $\beta_Z > \beta_Y$ and $\alpha_Y < 1$. Then, any $\alpha_Z > \alpha(v_\theta^{ZY} = 1)$ results in $v_\theta^{ZY} > 1$, and thus seller Z captures the whole market. This means, given that $\alpha_Y \in (0, 1)$ and $\beta_Z > \beta_Y$, $\alpha_Z \in [\alpha(v_\theta^{ZY} = 1), \alpha_Y]$ results in seller Z capturing the whole market, or

given that $\alpha_Y = 0$ and $\beta_Z > \beta_Y$, $\alpha_Z = 0$ results in seller Z capturing the whole market.

Combining the two results above, we conclude that given $\alpha_Y < 1$, seller Z captures the whole market if she sets $\beta_Z > \beta_Y$ and α_Z such that $\alpha_Z \in [\underline{\alpha}, \bar{\alpha}]$, where $\underline{\alpha} \triangleq \max\{0, \alpha(v_\theta^{ZY} = 1)\}$.

Suppose seller Z would like to have only the consumers with valuation greater than $(1 - \hat{v})$ prefer seller Z over seller Y ; that is she would like to have $v_\theta^{YZ} = 1 - \hat{v}$. From (3.1), we see that this is possible only if $\alpha_Z > \alpha_Y$ and solving for α_Z , we find

$$\alpha_Z = \alpha_{\hat{v}} \triangleq \alpha(v_\theta^{YZ} = 1 - \hat{v}) = \alpha_Y + p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{2\hat{v} + (p_1\beta_Z)^2 - 1}.$$

Then, seller Z can set $\beta_Z > \beta_Y$ and $\alpha_Z = \alpha_{\hat{v}} > \alpha_Y$ to attain $v_\theta^{YZ} = 1 - \hat{v}$, as long as $\alpha_Y < 1$ and $\hat{v} > \frac{1 - (p_1\beta_Z)^2}{2}$. Note that $v_\theta^{YZ} = 1 - \hat{v}$ means seller Z has a market share of $1 - v_\theta^{YZ} = \hat{v}$. ■

C.2 Proof of Proposition 8

Before we proceed with the proof, we note that the crucial aspect of the duopoly case in this essay is that the market is being divided among the sellers, and that there is no value creation as a result of competition. In other words, there are no win-win scenarios and the game is rather close to a constant-sum game. In the light of this observation and the market share dynamics described in Proposition 7, our first intuition is that setting (α_Z, β_Z) to capture the whole market is a potential best response of seller Z to seller Y 's ($\alpha_Y < 1, \beta_Y < 1$). Note that once seller Z captures the whole market, she is effectively a monopoly and the results for the monopoly case directly apply. Being a monopoly, seller Z clearly prefers to be at the monopoly optimal solution described in Proposition 5 (recall that in the duopoly case, we assume p_1 and p_2 are given, and therefore Proposition 5 applies). However, we see from Proposition 7 that conditions for seller Z to become a monopoly is not arbitrary, and that she is not necessarily able to attain the monopoly optimal solution while becoming a monopoly. In the proof below, we first identify the cases where seller Z can capture the whole market at the monopoly optimal solution. Then, we look

at the remainder cases step-by-step and investigate whether capturing the whole market is profitable given that monopoly optimal solution is not attainable. We ultimately find that under any condition, seller Z 's best response to $(\alpha_Y < 1, \beta_Y < 1)$ is to appropriately set (α_Z, β_Z) to capture the whole market. Since the sellers are identical in terms of p_1, p_2, c, s and g , we conclude that there is no Nash equilibrium where a seller sets $(\alpha_j < 1, \beta_j < 1)$. Given this result, we analyze best responses in the form of full refund $(\alpha_j < 1, \beta_j = 1)$ and full information $(\alpha_Y = 1, \beta_Y \in [0, 1])$, and identify the potential Nash equilibria and the associated conditions as given in Proposition 8.

We start by summarizing Proposition 5 for uniformly distributed valuations: The optimal (α, β) for a monopolistic seller when p_1, p_2, c, s and g are given is that the seller chooses either Solution (D) if $g < \bar{g}_\theta^E$, or Solution (E) if $g > \bar{g}_\theta^E$, where, for uniformly distributed valuations,

$$\text{Solution(D)} : (\alpha^*, \beta^*) = \left\{ (\alpha, \beta) \mid \alpha \in \left[0, 1 - \frac{2p_1}{1 + (p_1\beta)^2} \right], \beta = \frac{s}{2p_1} \right\},$$

$$\text{Solution(E)} : (\alpha^*, \beta^*) = \{(\alpha, \beta) \mid v_\theta = p_1\}, \text{ and}$$

$$\bar{g}_\theta^E = \frac{p_1 - c + \frac{s^2}{4p_1}}{(p_2 - c)(1 - p_2)}.$$

Suppose first that $g > \bar{g}_\theta^E$. Then, Solution (E) is optimal for a monopoly seller and thus, her optimal decision is to set α and β such that v_θ is as close to p_1 as possible. Therefore in the duopoly case, following Propositions 3 and 7, if $g > \bar{g}_\theta^E$, then the best response of seller Z to $(\alpha_Y < 1, \beta_Y < \beta_p)$ is to set $(\alpha_Z \in [\max\{0, \alpha(v_\theta^{ZY} = 1)\}, \alpha(v_\theta^{YZ} = 0)], \beta_Z = \beta_p)$ achieving $v_{\theta,Z} = p_1$ while capturing the whole market. If $\beta_Y \geq \beta_p$, however, seller Z cannot capture the whole market and set $v_{\theta,Z} = p_1$ at the same time. However, from Proposition 7, if $\alpha_Y < 1$, she can achieve $v_\theta^{YZ} = p_1$ by setting $\beta_Z > \beta_Y$ and

$$\alpha_Z = \alpha(v_\theta^{YZ} = p_1) = \alpha_Y + p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{1 + (p_1\beta_Z)^2 - 2p_1}.$$

Note that, since $\beta_Z > \beta_Y \geq \beta_p$, we have $1 + (p_1\beta_Z)^2 - 2p_1 > 0$ and $v_{\theta,Z} \leq p_1$. By setting $v_\theta^{YZ} = p_1$, seller Z ensures that only those consumers with valuation greater

than the price prefer seller Z over seller Y , and since $v_{\theta,Z} \leq p_1$, all such consumers purchase from seller Z . In other words, seller Z achieves monopoly optimal profits. We conclude, due to symmetry, that if $g > \bar{g}_\theta^E$, there is no equilibrium where a seller sets $(\alpha_j < 1, \beta_j < 1)$, since the other seller can always capture the whole market profitably.

Suppose that $g < \bar{g}_\theta^E$. In this case, we know that Solution (D) is optimal for a monopoly seller, and that she would set α as low as possible so that she can sell to as many consumers as possible. Thus, if seller Y chooses $(\alpha_Y < 1, \beta_Y < \frac{s}{2p_1})$, seller Z can set $\beta_Z = \frac{s}{2p_1}$ and $\alpha_Z = \max\{0, \alpha(v_\theta^{ZY} = 1)\}$, capturing the whole market profitably. Therefore due to symmetry, if $g < \bar{g}_\theta^E$, there is no equilibrium where a seller sets $(\alpha_j < 1, \beta_j < \frac{s}{2p_1})$.

Recall by definition in (2.5) that $\bar{g}_\theta^E = g_\theta^E(\beta = \frac{s}{2p_1})$ for uniformly distributed valuations. Then, compare (B.4) and (B.7) in Appendix B to observe that

$$g_\theta^{II} = g_\theta^E(\beta = \frac{s}{p_1}) = \frac{p_1 - c}{(p_2 - c)(1 - p_2)}$$

for uniformly distributed valuations. Given that

$$\frac{\partial g_\theta^E}{\partial \beta} = \frac{s - 2p_1\beta}{(p_2 - c)(1 - p_2)} < 0$$

for all $\beta > \frac{s}{2p_1}$, we conclude that g_θ^E strictly decreases from \bar{g}_θ^E to g_θ^{II} as β goes from $\frac{s}{2p_1}$ to $\frac{s}{p_1}$. As a corollary, if $g_\theta^{II} < g < \bar{g}_\theta^E$, then there exists a critical return factor, $\frac{s}{2p_1} < \hat{\beta} < \frac{s}{p_1}$, for which $g = g_\theta^E(\hat{\beta})$; through algebraic operations, we determine $\hat{\beta} = \left(s + \sqrt{s^2 + 4p_1(p_1 - c - g(p_2 - c)(1 - p_2))} \right) / (2p_1)$. Suppose $g_\theta^{II} < g < \bar{g}_\theta^E$. Then, as long as $\frac{s}{2p_1} < \beta \leq \hat{\beta}$, we have $g < g_\theta^E(\beta)$, and following the results in Appendix B, it is profitable for a monopoly seller to sell to all consumers. In other words, in the duopoly case, a seller would like to decrease her α as much as possible while capturing the whole market. Suppose $(\alpha_Y < 1, \beta_Y < \hat{\beta})$, then seller Z can set $(\alpha_Z = \max\{0, \alpha(v_\theta^{ZY} = 1)\}, \beta_Z = \hat{\beta})$, and thus capture the whole market and sell to as many consumers as possible. If $(\alpha_Y < 1, \beta_Y \geq \hat{\beta})$, seller Z must offer $\beta_Z > \beta_Y \geq \hat{\beta}$ in order to capture the whole market; however, for $g > g_\theta^{II}$ and $\beta > \hat{\beta}$, it is no longer optimal to have any dissatisfied buyers for a monopoly. Therefore, while capturing

the whole market, seller Z sets $\alpha_Z = \alpha(v_\theta^{YZ} = p_1)$ and $\beta_Z > \beta_Y$, achieving the monopoly optimal profits; $v_\theta^{YZ} = p_1$ means all consumers with $v > p_1$ prefer seller Z , and since $v_{\theta,Z} < p_1$, they all purchase. Note that since $\frac{\partial \alpha(v_\theta^{YZ} = p_1)}{\partial \beta} > 0$, there exists $\beta_Z > \beta_Y$ such that $1 > \alpha(v_\theta^{YZ} = p_1) > \alpha_Y$, given that $\alpha_Y < 1$ and $\beta_Y < 1$. As a result, if $g_\theta^{II} < g < \bar{g}_\theta^E$, and given that $\alpha_Y < 1$, seller Z has a best response that enables him to capture the whole market profitably for any $\beta_Y < 1$. We conclude due to symmetry that if $g_\theta^{II} < g < \bar{g}_\theta^E$, there is no equilibrium with $(\alpha_j < 1, \beta_j < 1)$ for any seller.

So far we showed that if $g > g_\theta^{II}$, there is no Nash equilibrium where any seller sets $(\alpha_j < 1, \beta_j < 1)$. Then, suppose that $g < g_\theta^{II}$. Given this sufficiently small g , it is optimal for a monopoly seller to sell to all consumers as long as $\beta \leq \frac{s}{p_1}$. Therefore in the duopoly case, as response to $(\alpha_Y < 1, \beta_Y < \frac{s}{p_1})$, seller Z can set $(\alpha_Z = \max\{0, \alpha(v_\theta^{ZY} = 1)\}, \beta_Z = \frac{s}{p_1})$, and thus capture the whole market selling to as many consumers as possible. Due to symmetry, we conclude that if $g < g_\theta^{II}$, neither seller sets $(\alpha_j < 1, \beta_j < \frac{s}{p_1})$ in an equilibrium.

Now recall from (B.6) and the subsequent analysis in Appendix B that if $\beta > \frac{s}{p_1}$, it is optimal for a monopoly seller to sell to all consumers as long as $g_\theta^{III} > 0$ and if $g < g_\theta^{III}$. Since g_θ^{III} is decreasing in β , if $g_\theta^{III}(\beta = 1) > 0$, then $g_\theta^{III} > 0$ for all $\beta < 1$; otherwise, since $g_\theta^{III}(\beta = \frac{s}{p_1}) = g_\theta^{II} > 0$, there is a $\frac{s}{p_1} < \bar{\beta} < 1$ such that $g_\theta^{III}(\beta = \bar{\beta}) = 0$. Therefore, as long as $\max\{0, g_\theta^{III}(\beta = 1)\} < g < g_\theta^{II}$, there is a critical return factor, $\frac{s}{p_1} < \tilde{\beta} < \min\{1, \bar{\beta}\}$, such that $g = g_\theta^{III}(\beta = \tilde{\beta})$. In other words, given $\max\{0, g_\theta^{III}(\beta = 1)\} < g < g_\theta^{II}$, it is profitable for a monopoly seller to sell to all consumers as long as $\beta \leq \tilde{\beta}$. Thus in the duopoly case, if $\max\{0, g_\theta^{III}(\beta = 1)\} < g < g_\theta^{II}$, seller Z 's best response to $(\alpha_Y < 1, \beta_Y < \tilde{\beta})$ is to set $(\alpha_Z = \max\{0, \alpha(v_\theta^{ZY} = 1)\}, \beta_Z = \tilde{\beta})$, capturing the whole market profitably. Due to symmetry, we conclude that if $\max\{0, g_\theta^{III}(\beta = 1)\} < g < g_\theta^{II}$, there is no equilibrium where a seller sets $(\alpha_j < 1, \beta_j < \tilde{\beta})$.

Consider then $(\alpha_Y < 1, \beta_Y \geq \tilde{\beta})$ in case of $\max\{0, g_\theta^{III}(\beta = 1)\} < g < g_\theta^{II}$. From Appendix B, for any given $\beta > \tilde{\beta}$, if $\max\{0, g_\theta^{III}(\beta = 1)\} < g < g_\theta^{II}$, a monopoly seller would like to set $v_\theta = p_1\beta$ so that no buyer returns even though

they may be dissatisfied. Thus, seller Z can set $\beta_Z > \beta_Y$ and

$$\alpha_Z = \alpha(v_\theta^{YZ} = p_1\beta_Z) = \alpha_Y + p_1^2(1 - \alpha_Y) \frac{\beta_Z^2 - \beta_Y^2}{(1 - p_1\beta_Z)^2},$$

ensuring that none of her buyers return while all (if any) buyers of seller Y return their purchases. Note that given $\alpha_Y < 1$ and $\beta_Y < 1$, there exists $1 > \beta_Z > \beta_Y$ such that $1 > \alpha(v_\theta^{YZ} = p_1\beta_Z) > \alpha_Y$. Therefore, due to symmetry, if $\max\{0, g_\theta^{III}(\beta = 1)\} < g < g_\theta^{II}$, there is no equilibrium where a seller sets $(\alpha_j < 1, \beta_j \geq \tilde{\beta})$. Combining with the above result, we conclude that if $\max\{0, g_\theta^{III}(\beta = 1)\} < g < g_\theta^{II}$, neither seller sets $(\alpha_j < 1, \beta_j < 1)$ in an equilibrium.

Suppose $\bar{g}_\theta^{III} \triangleq g_\theta^{III}(\beta = 1) > 0$, which translates to $s > c$. Then, if $g < \bar{g}_\theta^{III}$, it is profitable for a monopoly seller to sell to all consumers for any $\beta \in [0, 1]$. Thus in the duopoly case, in response to $(\alpha_Y < 1, \beta_Y < 1)$, seller Z can set $\beta_Z > \beta_Y$ and $\alpha_Z = \max\{0, \alpha(v_\theta^{ZY} = 1)\}$, capturing the whole market and sell to as many consumers as possible. Due to symmetry, we conclude that there is no equilibrium where a seller sets $(\alpha_j < 1, \beta_j < 1)$ if $0 < g < \bar{g}_\theta^{III}$, which is possible only if $s > c$.

Combining our analysis so far, we established that for any value of $g > 0$ there is no Nash equilibrium where $(\alpha_j < 1, \beta_j < 1)$ for any seller. Then, consider $(\alpha_Y < 1, \beta_Y = 1)$. In this case, seller Z 's best response is either $(\alpha_Z < 1, \beta_Z = 1)$, or $\alpha_Z = 1, \beta_Z \in [0, 1]$. Note that once $\alpha_Z = 1$, the value of β_Z is irrelevant for both sellers. Furthermore, as long as $\alpha_Z > \alpha_Y$ and $\beta_Y = \beta_Z = 1$, the value of α_Z is irrelevant for both sellers and the value of α_Y is irrelevant for seller Z . Therefore, essentially, seller Z has three potential best responses: 1) $(\alpha_Z < \alpha_Y, \beta_Z = 1)$, 2) $(\alpha_Z = \alpha_Y, \beta_Z = 1)$, or 3) $(\alpha_Z = 1, \beta_Z \in [0, 1])$. In the first, seller Z provides less information than seller Y and we have $v_\theta^{ZY} = \frac{1+p_1^2}{2} > p_1$; in the second, seller Y and seller Z are identical and they equally share the profits; in the third, seller Z provides full information and we have $v_\theta^{YZ} = \frac{1+p_1^2}{2} > p_1$. We write seller Z 's net

profits under each decision as follows:

$$\begin{aligned}
R_Z(\alpha_Z < \alpha_Y, \beta_Z = 1) &= (p_1 - c) \left(\frac{1 + p_1^2}{2} - v_{\theta, Z} \right) + (s - p_1)(p_1 - v_{\theta, Z}) \\
&\quad + g(p_2 - c)(1 - p_2) \left(\frac{1 + p_1^2}{2} - p_1 + v_{\theta, Y} \right), \\
R_Z(\alpha_Z = \alpha_Y, \beta_j = 1) &= \frac{1}{2} ((p_1 - c)(1 - v_{\theta, Y}) + (s - p_1)(p_1 - v_{\theta, Y}) \\
&\quad + g(p_2 - c)(1 - p_2)(1 - p_1 + v_{\theta, Y})), \\
R_Z(\alpha_Z = 1, \alpha_Y < 1, \beta_Y = 1) &= (p_1 - c) \left(1 - \frac{1 + p_1^2}{2} \right) \\
&\quad + g(p_2 - c)(1 - p_2) \left(1 - \frac{1 + p_1^2}{2} \right).
\end{aligned}$$

We observe from the profit function in the first case that seller Z would like to increase $v_{\theta, Z}$ as much as possible if $g > \bar{g}_\theta^{III}$ and would like to decrease it if $g < \bar{g}_\theta^{III}$. Suppose $g > \bar{g}_\theta^{III}$; then seller Z 's optimal decision given $\alpha_Z < \alpha_Y$ is to set α_Z as close to α_Y as possible; we denote this by $\alpha_Z \lesssim \alpha_Y$, and thus $v_{\theta, Z} \lesssim v_{\theta, Y}$ since $\beta_Y = \beta_Z = 1$. Comparing the profits for the three decisions, we find that $R_Z(\alpha_Z \lesssim \alpha_Y, \beta_Z = 1) > R_Z(\alpha_Z = \alpha_Y, \beta_j = 1) > R_Z(\alpha_Z = 1, \alpha_Y < 1, \beta_Y = 1)$ if $g > g_\theta^{IV}$ and $v_{\theta, Y} > \hat{v}_\theta$, otherwise if $g > g_\theta^{IV}$ and $v_{\theta, Y} < \hat{v}_\theta$, or if $g < g_\theta^{IV}$, then $R_Z(\alpha_Z \lesssim \alpha_Y, \beta_Z = 1) < R_Z(\alpha_Z = \alpha_Y, \beta_j = 1) < R_Z(\alpha_Z = 1, \alpha_Y < 1, \beta_Y = 1)$, where

$$\hat{v}_\theta = \frac{p_1(g(1 - p_1)(p_2 - c)(1 - p_2) + p_1(1 - p_1 + c) - s)}{g(p_2 - c)(1 - p_2) + c - s}, \quad (\text{C.1})$$

and

$$g_\theta^{IV} \triangleq \frac{(p_1 - c)(1 - p_1)}{p_1(p_2 - c)(1 - p_2)}.$$

As a result, depending on the value of g and $v_{\theta, Y}$, and hence on α_Y , seller Z 's best decision is either to set either $\alpha_Z \lesssim \alpha_Y$ or $\alpha_Z > \alpha_Y$; it is never the best decision to provide the same level of information given that $\alpha_Y < 1$. We observe from above that $\hat{v}_\theta = 0$ only if $g = g_\theta^V$, where

$$g_\theta^V \triangleq \frac{s - p_1(1 - p_1 + c)}{(1 - p_1)(p_2 - c)(1 - p_2)}.$$

Furthermore, we find that $\frac{\partial \hat{v}_\theta}{\partial g} > 0$ only if $s > s_\theta$, where

$$s_\theta \triangleq 1 - p_1 + c \left(2 - \frac{1}{p_1} \right).$$

Finally, we observe that if $s > s_\theta$, then $g_\theta^V > \bar{g}_\theta^{III} > g_\theta^{IV}$, and if $s < s_\theta$, then $g_\theta^V < \bar{g}_\theta^{III} < g_\theta^{IV}$. To sum, if $s > s_\theta$ and $\bar{g}_\theta^{III} < g < g_\theta^V$, then seller Z 's best response is $\alpha_Z \lesssim \alpha_Y$ for all $\alpha_Y < 1$; if $s > s_\theta$ and $g > g_\theta^V$, then $\alpha_Z \gtrsim \alpha_Y$ is the best response if $v_{\theta,Y} > \hat{v}_\theta$, and $\alpha_Z > \alpha_Y$ is the best response if $v_{\theta,Y} < \hat{v}_\theta$. Suppose that we have $s > s_\theta$ and $\bar{g}_\theta^{III} < g < g_\theta^V$; then, due to symmetry, there is no pure-strategy Nash equilibrium where a seller sets $\alpha_j < 1$ because providing a marginally less information than competition is always the best response for both sellers and there is a continuum of such best responses.

Suppose that $s > s_\theta$ and $g > g_\theta^V$. Then, both sellers' best response is to provide marginally less information than the competition until, without loss of generality, $v_{\theta,Z} = \hat{v}_{\theta,Z}$, at which point seller Y 's best response is $\alpha_Y = 1$. Seller B 's best response to $\alpha_Y = 1$ is either $(\alpha_Z \lesssim 1, \beta_Z = 1)$ with profits $R_Z(\alpha_Z \lesssim \alpha_Y = 1, \beta_Z = 1)$, or $\alpha_Z = 1$ with profits

$$R_Z(\alpha_j = 1) = \frac{1}{2} ((p_1 - c)(1 - p_1) + g(p_2 - c)(1 - p_2)).$$

We find that $R_Z(\alpha_Z \lesssim \alpha_Y = 1, \beta_Z = 1) > R_Z(\alpha_j = 1)$ only if $g > g_\theta^{IV}$; however, given $s > s_\theta$, we have $g_\theta^V > \bar{g}_\theta^{III} > g_\theta^{IV}$. Therefore, if $s > s_\theta$ and $g > g_\theta^V$, seller Z 's best response to $\alpha_Y = 1$ is $(\alpha_Z \lesssim 1, \beta_Z = 1)$. Given $(\alpha_Z \lesssim 1, \beta_Z = 1)$, seller Y 's best response is $(\alpha_Y \lesssim \alpha_Z, \beta_Z = 1)$ and the sellers are back in the loop of a continuous series of best responses where they unilaterally deviate from an equilibrium. As a result, there is no pure-strategy Nash equilibrium if $s > s_\theta$ and $g \geq g_\theta^V$. Combined with the above result, we conclude that if $s > s_\theta$ and $g > \bar{g}_\theta^{III}$, there is no pure-strategy Nash equilibrium in the duopoly.

On the other hand, consider, for $s > s_\theta$ and $\bar{g}_\theta^{III} < g < g_\theta^V$, the case where seller Y sets $\beta_Y = 1$ and picks $\alpha_Y \in [0, 1]$ arbitrarily. Without knowing where seller Y is located in terms of α , seller Z is forced to randomize his decision as well and her best response is similarly to set $\beta_Z = 1$ and choose $\alpha_Z \in [0, 1]$ randomly. Therefore, given $s > s_\theta$ and $\bar{g}_\theta^{III} < g < g_\theta^V$, there is a mixed-strategy Nash equilibrium where both sellers set $\beta_j = 1$, and pick $\alpha_j \in [0, 1]$ randomly. Next, consider for $s > s_\theta$ and $g > g_\theta^V$, the case where seller Y sets $\beta_Y = 1$ and picks $\alpha_Y \in [\hat{\alpha}, 1]$ randomly, where

$\hat{\alpha} \triangleq \alpha(v_\theta = \hat{v}_\theta)$ can be found by substituting (C.1) and $\beta = 1$ into (2.3). From the above analysis, we see that seller Z 's best response is to set $\beta_Z = 1$ and randomize $\alpha_Z \in [\hat{\alpha}, 1]$. As a result, given $s > s_\theta$ and $g > g_\theta^V$, there is a mixed-strategy Nash equilibrium where both sellers set $\beta_j = 1$ and pick $\alpha_j \in [\hat{\alpha}, 1]$ randomly.

Consider now $s > s_\theta$ and $g < \bar{g}_\theta^{III}$. In this case, seller Z 's best response to $(\alpha_Y \in (0, 1), \beta_Y = 1)$ given $\alpha_Z < \alpha_Y$ is to set $\alpha_Z = 0$, resulting in

$$R_Z(\alpha_Z = 0, \alpha_Y > 0, \beta_Z = 1) = (p_1 - c) \left(\frac{1 + p_1^2}{2} \right) + (s - p_1)p_1 + g(p_2 - c)(1 - p_2) \left(\frac{1 + p_1^2}{2} - p_1 \right).$$

We determine that if $g < g_\theta^V$, then

$$R_Z(\alpha_Z = 0, \alpha_Y > 0, \beta_Z = 1) > R_Z(\alpha_Z = \alpha_Y, \beta_j = 1) > R_Z(\alpha_Z = 1, \alpha_Y < 1, \beta_Y = 1)$$

and if $g > g_\theta^V$, then

$$R_Z(\alpha_Z = 0, \alpha_Y > 0, \beta_Z = 1) < R_Z(\alpha_Z = \alpha_Y, \beta_j = 1) < R_Z(\alpha_Z = 1, \alpha_Y < 1, \beta_Y = 1)$$

for any α_Y . Since for $s > s_\theta$ we have $g_\theta^V > \bar{g}_\theta^{III}$, we conclude that if $s > s_\theta$ and given $g < \bar{g}_\theta^{III}$, seller Z 's best response to $(\alpha_Y \in (0, 1), \beta_Y = 1)$ is $(\alpha_Z = 0, \beta_Z = 1)$. Consider then $(\alpha_Y = 0, \beta_Y = 1)$; seller Z 's best response is either $(\alpha_Z > 0, \beta_Z = 1)$ with profits equal to $R_Z(\alpha_Z = 1, \alpha_Y < 1, \beta_Y = 1)$, or $\alpha_Z = 0$ resulting in

$$R_Z(\alpha_j = 0, \beta_j = 1) = \frac{1}{2} ((p_1 - c) + (s - p_1)p_1 + g(p_2 - c)(1 - p_2)(1 - p_1)).$$

We see from above that $R_Z(\alpha_j = 0, \beta_j = 1) > R_Z(\alpha_Z = 1, \alpha_Y < 1, \beta_Y = 1)$ if $s > s_\theta$ and $g < \bar{g}_\theta^{III}$; in other words, both sellers' best response to the competition providing no information and offering full refund is to provide no information and offer full refund. This result leads us to a Nash equilibrium where both sellers provide no information and offer a full refund return policy, $(\alpha_j = 0, \beta_j = 1)$, in case of $s > s_\theta$ and $g < \bar{g}_\theta^{III}$.

Suppose now $s < s_\theta$, in which case we have $g_\theta^V < \bar{g}_\theta^{III} < g_\theta^{IV}$. Therefore, if $g < g_\theta^V$, then we have $g < \bar{g}_\theta^{III}$ readily satisfied, and following the analysis above,

seller Z 's best response to $(\alpha_Y \in [0, 1], \beta_Y = 1)$ is $(\alpha_Z = 0, \beta_Z = 1)$. As a result, $(\alpha_j = 0, \beta_j = 1)$ is the only Nash equilibrium if $s < s_\theta$ and $g < g_\theta^V$.

Given $s < s_\theta$, suppose $g > g_\theta^V$; then, seller Z 's best response to $(\alpha_Y = 0, \beta_Y = 1)$ is $(\alpha_Z > 0, \beta_Z = 1)$. In other words, both sellers' best response to the competition setting $(\alpha_j < 1, \beta_j = 1)$ is to provide more information than the competition, and therefore due to symmetry, there is no equilibrium where both sellers have $(\alpha_j < 1, \beta_j = 1)$. Consider then $(\alpha_Y = 1, \beta_Y \in [0, 1])$; seller Z 's best response is either $(\alpha_Z = 0, \beta_Z = 1)$ with $R_Z(\alpha_Z = 0, \alpha_Y > 0, \beta_Z = 1)$, or $(\alpha_Z = 1, \beta_Z \in [0, 1])$ with $R_Z(\alpha_j = 1)$. Comparing the profits, we find that if $g < g_\theta^{VI}$, seller Z 's best response to $(\alpha_Y = 1, \beta_Y \in [0, 1])$ is $(\alpha_Z = 0, \beta_Z = 1)$, and if $g > g_\theta^{VI}$, then it is $(\alpha_Z = 1, \beta_Z \in [0, 1])$, where

$$g_\theta^{VI} \triangleq \frac{2s - c - p_1(1 - p_1 + c)}{(2 - p_1)(p_2 - c)(1 - p_1)}.$$

We further find that given $s < s_\theta$, we have $g_\theta^V < g_\theta^{VI} < \bar{g}_\theta^{III}$. Therefore, if $s < s_\theta$ and $g_\theta^V < g < g_\theta^{VI}$, seller Z 's best response to $(\alpha_Y = 1, \beta_Y \in [0, 1])$ is $(\alpha_Z = 0, \beta_Z = 1)$, to which seller Y 's best response is $(\alpha_Y = 1, \beta_Y \in [0, 1])$. We conclude that, if $s < s_\theta$ and $g_\theta^V < g < g_\theta^{VI}$, there is a Nash equilibrium where one seller provides full information and offers an arbitrary return policy, while the other seller provides zero information but offers a full refund return policy.

Consider now the case where $s < s_\theta$ (for which $g_\theta^{VI} < \bar{g}_\theta^{III} < g_\theta^{IV}$) and $g_\theta^{VI} < g < g_\theta^{IV}$. We know from above that if $g < \bar{g}_\theta^{III}$, then seller Z 's best response to $(\alpha_Y = 1, \beta_Y \in [0, 1])$ given $\alpha_Z < \alpha_Y$ is $(\alpha_Z = 0, \beta_Z = 1)$, and therefore her overall best response for $g_\theta^{VI} < g < \bar{g}_\theta^{III}$ is $(\alpha_Z = 1, \beta_Z \in [0, 1])$. Furthermore, we know that if $g > \bar{g}_\theta^{III}$, then seller Z 's best response given $\alpha_Z < \alpha_Y$ is $(\alpha_Z \lesssim \alpha_Y, \beta_Z = 1)$, and therefore for $\bar{g}_\theta^{III} < g < g_\theta^{IV}$, her overall best response to $(\alpha_Y = 1, \beta_Y \in [0, 1])$ is $(\alpha_Z = 1, \beta_Z \in [0, 1])$. To sum, if $s < s_\theta$ and $g_\theta^{VI} < g < g_\theta^{IV}$, seller Z 's best response to $(\alpha_Y = 1, \beta_Y \in [0, 1])$ is $(\alpha_Z = 1, \beta_Z \in [0, 1])$. In other words, if $s < s_\theta$ and $g_\theta^{VI} < g < g_\theta^{IV}$, there is a Nash equilibrium where both sellers provide full information and offer arbitrary return policies.

Finally, if $s < s_\theta$ and $g > g_\theta^{IV}$, we have $g > \bar{g}_\theta^{III}$ and $g > g_\theta^V$ readily satisfied,

and as shown above, there is a continuous series of best responses where both sellers unilaterally deviate from an equilibrium. As a result, if $s < s_\theta$ and $g > g_\theta^{IV}$, there is no pure-strategy Nash equilibrium in the duopoly. On the other hand, there is a mixed-strategy Nash equilibrium where both sellers set $\beta_j = 1$ and pick $\alpha_j \in [\alpha(v_\theta = \hat{v}_\theta), 1]$ randomly. ■

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