ABSTRACT

Title of Document: ESTIMATING THE FRICTION PERFORMANCE OF HOT MIX ASPHALT PAVEMENTS BASED ON AGGREGATE PROPERTIES AND ROUTE CHARACTERISTICS: ANALYSIS, MODELING AND VALIDATION

Girum S. Awoke, Ph.D Civil Engineering, 2011

Directed By: Associate Professor, Dr. Dimitrios Goulias, Department of Civil and Environmental Engineering

Traffic accidents are one of the major causes of death in the United States. In 2008 alone, more than 37,000 fatalities occurred, accounting for one fatality every thirteen minutes. More than one tenth of fatal accidents occur when pavements are wet and slippery. In wet conditions, a water film is created between the pavement surface and the tire, thereby reducing the amount of available friction.

There are several factors that affect the level and type of friction between tires and a wet pavement surface. Some of these factors are microtexture and macrotexture, age of pavement, seasonal and environmental factors, traffic level and composition, individual and blend aggregate properties, binder used in mix, and road location/geometry. The research presented in this dissertation explores the impact of aggregate and mixture properties as well as the role of route characteristics, such as traffic intensity and composition, on the friction performance of Hot Mix Asphalt (HMA) pavements.

In the research, various databases for construction, material, pavement management and traffic condition were examined. The data included 5 years of pavement friction readings, construction and material data, and traffic monitoring data. The research included reviewing aggregate quality requirements and friction measurements, and compiling, categorizing and examining the various databases to develop a working dataset/s**.** In addition, a methodology was developed to isolate and analyze data specific to a given roadway constructed using a known type of aggregate and mix material. The results were then used to estimate pavement friction service life in terms of cumulative traffic loading. Multivariate Regression methods were employed to establish the relationship between Friction Number (FN) and cumulative AADT, for specific aggregates.

The research also included establishing relationships between material properties/route characteristics and pavement friction, and investigating/developing a model that can be used to predict the friction performance of pavements based on these factors. Partial Least Squares (PLS) Regression, a type of Structural Equation Modeling (SEM) method, was used to extract factors from datasets in order to formulate, test and validate several models out of which the most significant model was selected.

Keywords: Aggregate Properties, Cumulative AADT, ESAL, Friction Number, PLS Regression, Pavement Friction, Service Life, Skid Resistance, Structural Equation Models

Estimating The Friction Performance of Hot Mix Asphalt Pavementse Based on Aggregate Properties and Route Characteristics: Analysis, Modeling and Validation

By

Girum S. Awoke.

Dissertation submitted to the Faculty of the Graduate School of the
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Advisory Committee:
Dr. Dimitrios Goulias, Chair
Dr. M. Sherrif Aggour
Dr. Amde M. Amde
Dr. Ahmet Aydilek
Dr. Sung Lee, Dean's Representative

Dedication

This dissertation is dedicated to my late aunt, Tsehay Alemayehu, who instilled in me the love of learning.

Acknowledgment

I would like to thank my advisor, Professor Dimitrios Goulias, for his patience, feedback, and relentless help throughout the research and dissertation process. I would also like to thank members of my dissertation committee for taking the time to review and comment on my work. I am also indebted to Dr. Sushant Upadhyaya and Dr. Haejin Kim for their moral support and guidance. I appreciate the engineers and project managers at SHA's Office of Materials Technology and Office of Policy and Research for providing technical assistance and for their input on submissions.

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Above all, I thank God for allowing me to experience the challenges as well as the joy associated with such a remarkable journey.

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Abbreviations

AADT – Annual Average Daily Traffic

AASG - Allegany Aggregates Short gap

AASHTO – American Association of State Highway and Transportation Officials

AC – Asphalt Content

AESAL – Average Daily Equivalent Standard Axle Load

AIR – Acid Insoluble Residue

AIR - Aggregate Industries Rockville

ANOVA – Analysis of Variance

ASTM – American Society for Testing and Materials

BG – Binder Grade

BPN – British Pendulum Number

BPT - British Pendulum Test

CumAADT – Cumulative Average Annual Daily Traffic

CumESAL – Cumulative Equivalent Standard Axle Load

CV – Coefficient of Variation

Df – Direction Factor

DumTrk = Dummy Variable used for Equipment Type in Regression

ESAL – Equivalent Standard Axle Load

FHWA – Federal Highway Administration

FN – Friction Number

ICM – Independent Construction Materials

KLC - Keystone Lime Company

LAA – Los Angeles Abrasion

LCH - Lafarge Churchville

LEF – Load Equivalency Factor

LF - Lafarge Frederick

Lf – Lane Factor

LS – Latent Structures

LV – Latent Variables

LW – Lafarge Warfordsburg

MER - Multiple Exponential Regression

MER - Multiple Exponential Regression

MLR - Multiple Linear Regression

MLR - Multiple Linear Regression

MMI - Maryland Materials Incorporated

MMW - Martin Marietta Woodsboro

MP – Mile Point

MSHA/SHA – Maryland State Highway Administration

NMAS – Nominal Maximum Aggregate Size

NYSDOT – New York State Department of Transportation

OGFC – Open Graded Friction Course

OLS – Ordinary Least Squares

PG - Performance Grade

PIARC – Permanent International Association of Road Congresses (World Road Association)

PLS - Partial Least Squares

PV – Polish Value

SA - Sensitivity Analysis

SER - Simple Exponential Regression

SEM - Structural Equation modeling

SLR - Simple Linear Regression

SUPERPAVE - SUperior PERforming Asphalt PAVEments

TESAL – Terminal Equivalent Standard Axle Load

VMH - Vulcan Materials Hanover

VMHDG - Vulcan Materials Havre De Grace

VMW - Vulcan Materials Warrenton

YBPBv- York Building Products Belvedere

Chapter 1. Introduction

1.1. Background

Traffic accidents are one of the major causes of death in the United States. In 2008 alone, more than 37,000 fatalities occurred out of more than 10 million motor vehicle accidents (Census, 2010). The National Highway Traffic Safety Administration (NHTSA) estimates the rate of fatality at 1(one) fatality every 13 minutes; In addition, the cost of traffic crashes is estimated at more than $200 Billion every year (Noyce et al., 2005; NHTSA, 2004; NHTSA, 2007). Based on national estimates, approximately 13.5 percent of fatal accidents occur when pavements are wet and slippery. In addition, a report by the Maryland State Highway Administration indicated that approximately 18% of fatal accidents and 24.3% of all accidents occur when pavements are wet (Chelliah et al, 2003).

Many studies have indicated that there is a significant relationship between wet pavements and traffic crashes. In wet conditions, a water film is created between the pavement surface and the tire, which acts as a lubricant, thereby reducing the amount of contact between the grooving on the tire and the aggregates that make up the pavement surface (Flintsch et al., 2005). The water film results in hydroplaning, a condition in which there will be no or minimal friction between the tires of a vehicle and the pavement surface. In such a case, the driver of the vehicle would be unable to stop or steer the vehicle in the desired direction, especially at high speeds.

There are several key factors that affect the level and type of friction between vehicle tires and a wet pavement surface. Some of these factors are microtexture and macrotexture of pavement surface, age of pavement surface, seasonal and environmental factors, traffic level and composition, individual and blend aggregate properties, type and grade of binder used in mix, and road location/geometry. It is important to ensure that the design, construction and maintenance of pavements take into consideration these factors in order to maximize the friction performance of pavements. This research attempts to estimate the friction performance of pavements based on five years of pavement friction readings in Maryland and material data related to mix and aggregate properties, as well as route related information such as pavement age, traffic count and composition.

1.2 Problem statement

There is a significant amount of research conducted to increase the life span and performance of pavement materials. However there are currently no direct specifications available for the selection and use of aggregate and mixture design to assure satisfactory frictional performance. Moreover, there is not enough research on the interdependency of factors that affect pavement friction and how they can be used in combination to estimate the overall performance and friction-related lifespan of pavements. Over the years Maryland State Highway Administration (SHA) has encountered issues related to aggregate quality in regards to pavement friction. Furthermore, increased variability in aggregate friction test results has prompted a review of the existing approach to aggregate friction evaluation. To address this issue, SHA has established on-going partnering and quarry inspections with aggregate suppliers, and has previously conducted a research project (Phase I Aggregate Data Study) that had an objective to evaluate existing aggregate data including laboratory test results and petrographic analysis with a particular focus on the frictional properties of aggregates. The objective of this research project was to i) estimate pavement friction service life for mixtures with aggregates from a variety of quarries that supply material for Maryland SHA's roadway projects, and ii) relate pavement friction performance to aggregate and mix material properties as well as route related factors.

1.3. Objectives of research

The overall goal of this research project was to develop a methodology for predicting pavement friction life (friction performance) for mixtures with aggregates from a variety of quarries, and eventually relate pavement friction to aggregate properties. The specific objectives were:

1. Identify the major factors affecting field pavement friction;

2. Using the SHA pavement friction records, examine which parameters affect pavement friction for specific mixtures and aggregates;

3. Develop a methodology for predicting pavement friction life;

4. Combine SHA pavement friction and mixture data to identify any relationships between aggregate material properties and field pavement friction.

5. Develop a model that can be used to predict the friction performance of pavements.

1.4. Research Approach and Methodology

To achieve these objectives, the work under this research included:

i) reviewing the current state of practice in aggregate quality requirements and pavement friction measurements;

ii) compiling, categorizing and examining SHA's database for the following datasets:

   a. Pavement friction data
   b. Field pavement friction testing equipment variability data
   c. Aggregate quality database along with the aggregate quality requirements identified in the Phase I research study;
   d. Traffic and Construction data

iii) identifying the need for any additional field and lab testing data needed to complement the existing aggregate material and friction databases;

iv) developing a methodology for predicting pavement friction performance of selected mixture types and aggregates; and,

v) establishing the relationships between material properties/route characteristics and pavement friction.

1.5 Organization of document

This dissertation is organized into 8 chapters as described briefly in the following paragraphs:

Chapter 1: Introduction

Chapter 2: Literature Review – This chapter provides an overview and background information on pavement-tire friction (skid resistance), as well as the physical mechanism of pavement friction. This chapter also discusses the primary factors involved in tire-pavement friction interaction by categorizing them into Material Related, Loading/Age Related, Environmental/Site Related and Testing/Vehicle Related.

Chapter 3: Database and records used in research - This chapter outlines the types and extent of data used to conduct the research. The data sources include pavement friction records, materials / mix design data, aggregate lab test information and equipment repeatability test data.

Chapter 4: Equipment Variability Study – This chapter discusses the statistical analysis conducted to investigate equipment repeatability and variability among the pavement friction testing equipment.

Chapter 5: Initial Analysis on Evaluation Factors Affecting Pavement Friction – This chapter discusses the preliminary data investigation and analysis to identify the various variables that are related to pavement friction. The analysis was conducted on major data sets to assess quality and validity of the data.

Chapter 6 : Methodology for Predicting Pavement Friction Life – This chapter discusses the '10- step' methodology that was followed in identifying, categorizing, simplifying and analyzing the bulk friction and material data into a usable form. This chapter also provides a description on the approach that was followed to identify specific aggregates sources and the various analysis techniques. The analysis investigated the use of Cumulative Annual Average Traffic (Cum AADT) and Equivalent Standard Axle Load (ESAL) based analysis to describe the friction performance of pavements together with aggregate and route characteristics.

Chapter 7: Detailed Analysis and Modeling –This chapter builds on the steps, assumptions and analysis on the dataset and conclusions that resulted from the work in preceding chapters. This chapter outlines the approach and various attempts considered to arrive at valid mathematical models including Ordinary Least Squares and Structural Equation Model techniques. These models can be used to estimate friction performance of pavements based on the material, traffic and construction information of a given pavement.

Chapter 8: Summary and Conclusions – Summary of the results and outcomes of the research as well as conclusions based on the data analysis and modeling are presented in this chapter.

Chapter 2. Literature Review

2.1. Background on Pavement-Tire Friction (Skid Resistance)

Friction is generally defined as the resisting force created between a surface (or two surfaces) and an object, acting in the opposite direction of the intended motion. Pavement Friction (Skid resistance) can be defined as the resistance force developed at the interface of a pavement surface and the tire of a vehicle traveling on the road. The interaction between the rubber and the pavement surface can be in the form of sliding or rolling (AASHTO Guide, 2008). When any two materials come into contact, energy dissipation occurs as rubber from the tire interacts with surface material from the pavement. The two types of energy dissipation are hysteresis and adhesion (AASHTO Guide, 2008; FHWA, 2006). During contact, the tire (which is made up of a visco-elastic material) undergoes deformation while the pavement, being relatively rigid, suffers minimal or small deformation. Energy is dissipated during the interaction between the tire and the pavement surface. This phenomenon is known as hysteresis (Li, Noureldin, and Zhu 2003). The greater the energy dissipation of the tire in contact with the pavement, the better the skid resistance of the subject pavement. On the other hand, when the tire is pressed against the pavement material, molecular bonds are formed between the tire and surface particles. The larger the number of bonds formed in such manner, the greater the energy required to break the bonds and therefore better skid resistance is achieved. The shearing of these bonds is called adhesion (Li, Noureldin, and Zhu 2003).

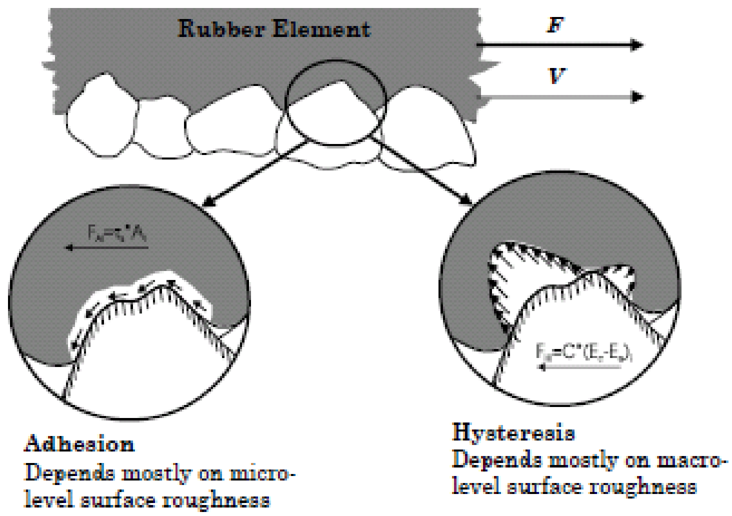

Figure 2-1.Key mechanisms of pavement–tire friction (AASHTO Guide, 2008)

In terms of skid resistance, there are two kinds of friction: static and kinetic friction. Static friction is the result of the interlocking of the irregularities of two surfaces (tire and pavement) to prevent any relative motion until and up to some motion occurs. Just after the motion occurs, the two surfaces start moving against one another and static friction will give way to kinetic friction. The purpose of the kinetic friction is to keep the object in motion. Usually the kinetic friction is less in magnitude than the static friction.

Friction is often represented by a coefficient that is unitless (designated as μ). The coefficient of friction is a function of the normal (reaction) force in a direction perpendicular to the surface (and the resisting force which is parallel to the surface and acting in the opposite direction to the motion). The coefficient of friction is given as follows per the Law of Coulomb/Amonton:

$\mu = F/N$ (Eq.2.1)

Where μ = coefficient of friction;

F = tractive/friction force

N = normal force on tire (Equal to Weight on wheel, Fw)

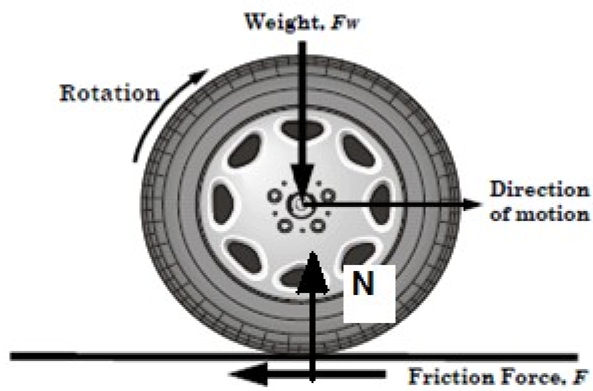

Figure 2-2. Simplified Diagram of Forces Acting on a Rotating Wheel (Adopted from AASHTO Guide, 2008)

2.2. Mechanism of Pavement Friction

For a vehicle traveling on given pavement, there are two forms of friction acting on the tire of the vehicle – longitudinal and side force friction. In longitudinal friction, there are two modes of operation between the pneumatic tire and road surface; rolling and constant-braked. In the free rolling mode (no braking), the relative speed between the tire circumference and the pavement, also known as the slip speed, is zero. In the constant-braked mode, the slip speed increases from zero to a potential maximum of the speed of the vehicle. A locked-wheel state is often referred to as a 100 percent slip ratio and the free-rolling state is a zero percent slip ratio. This relationship is depicted as follows (Meyer, 1982):

$$S = V - V_p; \text{ where } V_p = (0.68\ \omega\ r) \qquad \text{(Eq. 2.2)}$$

Where: S = Slip speed, mi/hr.

V = Vehicle speed, mi/hr.

VP = Average peripheral speed of the tire, mi/hr.

ω = Angular velocity of the tire, radians/sec.

r = Average radius of the tire, ft.

2.3. Primary Factors Involved in Tire-Pavement Friction Interaction

The factors that determine the friction outcome of a given pavement can be summarized in four major categories namely Material-Related, Loading/age-related, environmental/site- related, and Testing/Vehicle Operation- related.

2.3.1. Material Related

Materials involved in the tire-pavement interaction are the rubber that makes up the tire, and materials that make up the pavement surface structure (aggregates and asphalt binder in the case of Flexible Pavements; aggregates and Portland cement in the case of Rigid Pavements). The tire, being a viscoelastic material, is susceptible to significant temperature and moisture changes. Pavement wetness especially has an impact on the dissipation of energy at the contact surface between the tire and the pavement. In addition, the condition and type of tire plays a significant role on how water film trapped between the rubber and the pavement can drain out, leading to an increase in the adhesion between the tire and the pavement. Draining of water out of the tire-pavement interlock is a function of the tire tread design and the level of smoothness of the tire. Macrotexture (the series of larger irregularities formed by the spaces between individual aggregate particles) provides channels through which water can be expelled out of the tire-pavement interface. At high speeds, tread depth is particularly important for vehicles driving over thick films of water. Therefore smooth tires have a significantly lower wet friction resistance compared to well-treaded tires (Henry, 1983). Moreover, deflated tires exhibit lower friction resistance on wet pavements, especially at higher speeds, because of the longer residence time of the water film between the rubber and the pavement interface (Henry, 1983; Kulakowski, 1990).

There are two basic components that make up a pavement surface: aggregates (coarse and fine aggregates graded and blended as required) and a binding agent (Asphalt or Portland Cement) that are mixed together to form a durable matrix. Depending on the type, size and proportion of aggregates used in the pavement mixture, the pavement surface will have certain texture characteristics that determine the pavement's skid resistance. Pavement texture influences both parameters of friction – hysteresis and adhesion. Pavement surface texture refers to the irregularities on the pavement as well as the various irregularities on each aggregate particle used on the pavement surface. The surface irregularity of individual particles is referred to as "Microtexture". Microtexture ranges in size from 0.0004 in. to 0.02 in. The larger irregularities formed by the spaces between individual particles on the pavement surface are called "Macrotexture". Macrotexture can range in size from 0.02 in. to 2 in. Microtexture and adhesion are the prevailing factors influencing skid resistance at speeds less than 30 mph (AASHTO Guide, 2008). Other surface irregularities that are larger in size than 2 inches and less than 20 inches are called Megatexture. (PIARC, 1987). Irregularities that are larger than 20 inches are considered as roughness and have minimum bearing in pavement skid resistance (Henry 2000).

Table 2-1. Factors affecting pavement friction

Material Related	**Loading/Age Related**	**Environmental/Site Related**	**Testing/Vehicle Related**

Tire Rubber
 	
Pavement Surface Materials
• Micro-texture
• Macro-texture
• Megatexture/Unevenness
• Binder Type and Content
• Mix Properties
  o Mix Type
  o Mix Characteristics
• Aggregate Properties
  o Gradation/Parti
	Traffic Volume (AADT)
	
	Traffic Composition/truck percentage
	
	Pavement Construction year/Pavement Age
		Urban/Rural
		
		Road Geometry
		• Vertical Alignment
		• Horizontal Alignment
		• Cross Slope
		
		Temperature (Pavement and Air)
		
		Rainfall
		
		Pavement Surface cleanliness
			Vehicle Speed/Slip Speed
			
			Tire Tread (Design, smooth vs ribbed)

cle Size ○ Angularity/Asperity ○ Toughness ○ Carbonate/non-carbonate ○ Silica content			

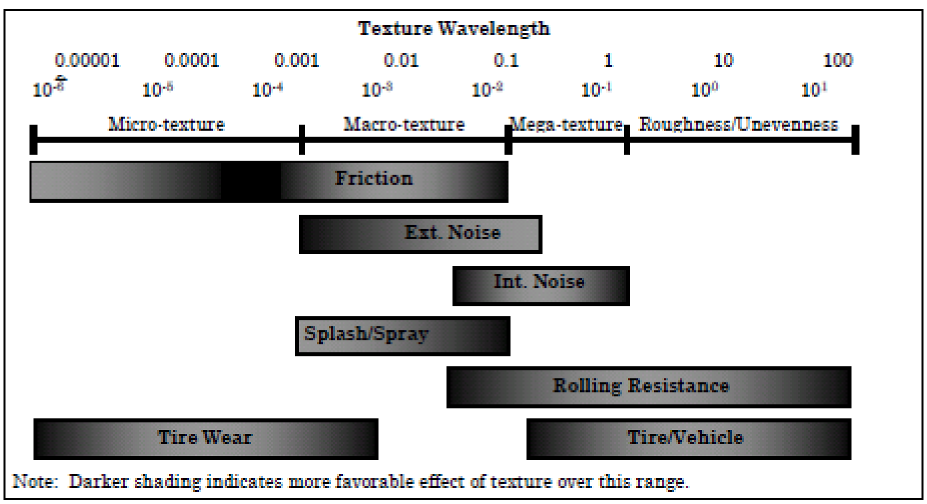

Figure 2-3. Texture wavelength influence on pavement–tire interactions (AASHTO Guide, 2008)

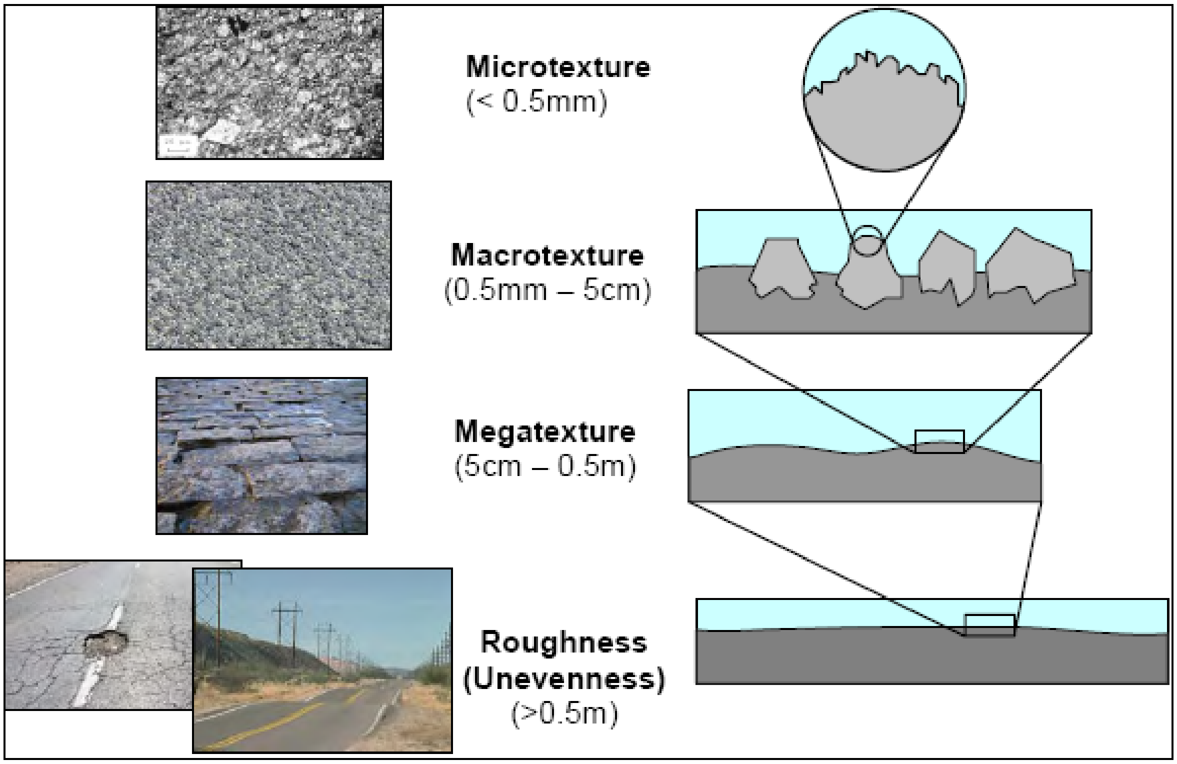

Figure 2-4. Representation and examples of surface textures (FHWA, 2006)

Several literature indicate that microtexture and macrotexture ultimately determine wet-pavement friction. This is because the adhesion force component depends on the microtexture and the hysteresis force component on the macrotexture (Henry, 2000). Also, surface drainage depends on the separation between individual particles which is represented by the macrotexture. A pavement with high roughness does not necessarily have large surface friction. On the other hand, an attempt to enhance pavement friction by making surface too coarse or too smooth may result in high noise, splash, or spray problems. The design of surface texture therefore requires a balance and compromise among skid resistance, internal/external noise, tire wear, and splash/spray.

Aggregate and mix characteristics, surface treatments such as tinning and other surface finishes influence both microtexture and macrotexture. Individual and grouped aggregate resistance to polishing and abrasion has direct contribution to friction resistance of the pavement surface while the type and amount of binder used in a particular mixture determines the coating on each aggregate, thereby affecting both macrotexture and microtexture . The type and composition of pavement mixture (type and grade of binder and gradation of aggregate blend) has been found to be significantly correlated to of the effect of polishing using the British Pendulum Number (BPN) (Bazlamit, 2005). The following aggregate properties have correlations with the friction performance of a pavement:

- Presence of Carbonates: Skeritt discussed the various impacts of the three different types of aggregates – homogenous, sandy and blend - classified based on their polishing characteristics; one significant element of aggregates that has been found to have an impact on polishing resistance is the presence of carbonates in the mineralogical composition of the aggregates. The lower the percentage of carbonates available in an aggregate blend, the higher the resistance of the aggregate blend to polishing. (Skeritt, 1993)
- Presence of Silica: Skeritt found out that, generally sandy rocks have a higher resistance to polishing irrespective of the traffic level. One way of quantifying this quality is by using the Acid Insoluble Residue (AIR) test. This test measures the percentage of acid insoluble residue that withstood degradation from a chemical action. NYSDOT specifies that good polishing resistant aggregates should have an AIR of 15% or more. (Skeritt, 1993).

- Toughness: Toughness, as measured by the Los Angeles Abrasion or the Micro-Deval Test, is another method of quantifying how a bulk of aggregates is able to resist abrasion and degradation from mechanical and physical impacts. It is important to note that, though toughness might not directly relate to polishing resistance of aggregates on the actual pavement, it can be correlated to other more directly applicable tests such as the British Wheel (reported as the British Pendulum Number, BPN). (SHA Phase I study 2008; Massad, 2008)
- Gradation and Angularity – Luce et al (2007) have investigated the impact of aggregate gradation using samples obtained from various sources and used in three different mixes. In this study, it was observed that aggregate gradation, represented using certain model coefficients, can be used to predict skid resistance of pavements.
- Chemical Reactivity /Inertness – Good aggregates are those that are inert, i.e. do not chemically interact with other compounds unless needed. One test that measures durability of aggregates against chemical action is called "Magnesium Sulphate Soundness Test", which measures the percent loss of aggregates due to chemical weathering. Since the pavement surface is exposed to various pollutants and chemicals, it is important that aggregates used on pavement surfaces be highly resistant to weathering by chemical action.
- Clay Content/Friable Particles – It has been found that excessive clay lumps and friable particles in aggregate intended for use on pavements may interfere with the bonding between the aggregate and the binding material. This will result in spalling, raveling, or stripping and create weak points and pop-outs out of the

pavement structure hence compromising its skid resistance and other qualities. (Kandhal, 1998). One standard test to measure this phenomenon is AASHTO T 112 (Clay Lumps and Friable Particles in Aggregates).

- Resistance to Polishing as measured by the Accelerated Polishing Test (Using British Wheel) and the British Pendulum Test (BPT) – The resistance to polishing and abrasion is not dependent on one particular aggregate property. As a result, it is important to measure the actual performance of the resistance of an aggregate blend or mixture to continued physical and mechanical abrasion using the above tests (FHWA, 2006).

In addition to individual and group aggregate properties, the type and composition of the pavement surface mixture also plays a significant role in determining the friction performance. Studies have shown the impact of texture and aggregate surface characteristics on the outcome of pavement friction for various Hot Mix Asphalt mixtures that were made up of aggregates obtained from various sources and with varying mineralogical compositions (Masad 2007, 2008; Luce et al 2007; Li et al 2007). Li et al investigated friction performance of various mixes in Open Graded Friction Courses (OGFC), Stone Matrix Asphalt (SMA) and Superpave mixes that were made of steel slag, crushed gravel or naturally obtained aggregates. Luce et al also investigated quartzite, sandstone, and siliceous gravel, combined in three different mix types referred to as Superpave, CMHB-C, and Type C (Texas Specific Mixes). The type and performance of the binder used in mixes also plays a role in the friction performance as investigated by Luce et al. In addition, aggregate spacing together with gradation determines the type and size of Macrotexture of an aggregate blend. Fwa et al (2003) have shown that aggregate spacing (within a blend) and mineralogy have an impact on skid resistance of pavements. Cafiso et al demonstrated using aggregate imaging and photographic techniques that the British Polish Number has a significant correlation with surface smoothness/roughness of aggregate particles, by using various descriptors of the aggregate surface (Cafiso et al 2006). Moreover, petrography and rock composition of aggregates used in the preparation of a pavement mixture play a significant role in the friction performance of the pavement (Masad, 2008; SHA Phase I study, 2008).

2.3.2. Loading/Age Related

Pavement friction performance can be attributed to factors pertaining to the age of the pavement surface and the amount and type of traffic applications on the particular pavement section. The rate of polishing of a given pavement surface is a direct result of the number and type of traffic applications on the pavement. Studies have shown that friction performance increases gradually for the first year or two after construction – attributed to binder flushing – and decreases thereafter with an increasing traffic loading (Li et al, 2007). It has also been shown that pavements constructed with different aggregate types exhibit varying rate of decline in friction performance (Skerritt, 1993; Crouch et al, 1998). Rate of friction performance as a result of repeated traffic loading is also dependent on the homogeneity of the aggregate blend.

Pavement Construction year/Pavement Age – The number of years a pavement surface has been in service determines how the surface would perform in terms of skid resistance.

Studies have shown that the skid resistance of pavements decreases from an initially higher value to a somewhat constant value in a matter of a few years (Masad 2008; Li et al 2007).

Traffic Volume (AADT) and Traffic Composition (ESAL) – Pavement aging can be enhanced by the amount and type of traffic using the road on a continuous basis. It has been found out that the decline of skid resistance can be attributed to the Average Annual Daily Traffic (AADT) (Skeritt, 1993) and the traffic mix as expressed in terms of Equivalent Standard Axle Loading (ESAL) (Li et al, 2007).

2.3.3. Environmental/Site Related

The main environmental or site related factors that have an impact on pavement friction are road geometry as represented by general location of route (urban versus rural) horizontal and vertical geometry (grade, curvature, cross slope), pavement and air temperature, rainfall (frequency and severity), pavement wetness, presence of snow/ice and general pavement surface cleanliness.

Temperature - Because tires are made up of rubber which is a visco-elastic material, their characteristics are affected by higher temperature (caused by repeated and sudden braking) which causes hydroplaning as a result of melting of the rubber material. This condition causes a reduction in the hysteresis component of the friction resistance. The hysteresis component is found to comprise a larger portion of the total friction force than the adhesion component as measured with the British pendulum tester. The hysteresis component of friction decreases with increased temperature regardless of surface texture state. The adhesion component of friction decreases with increased temperature for a polished pavement surface. (Bazalmit et al, 2005)

Smith, Chen, Song and Hedfi have found by studying the climate and friction records of the pavement network in Maryland that One degree (^{0}F) increase in temperature leads to one unit decrease in FN (Chen et al, 2005). It has been found that skid resistance decreases with increased temperature, and an approximately linear relationship exists between skid resistance and temperature with resulting models relating British Pendulum Number (BPN) and skid number obtained at any arbitrary temperature to a reference temperature of 293.15 K ~68°F (Bazlamit et al, 2005)

Pavement wetness - The two mechanisms by which energy is dissipated and friction force is developed through transfer of energy are hysteresis (loss of heat from the rubber) and adhesion (transfer of energy by contact). Generally adhesion is related to Microtexture while hysteresis is related to Macrotexture. When the pavement is wet, a water film is created between the two materials causing a drop in the adhesion component. The presence of water film between tire and pavement creates a condition called hydroplaning, which results in an almost zero friction resistance of the pavement. It has been discovered that the effect of water film is not significant at speeds less than 25 mph while it has been found that it has a negative impact on the friction performance of the pavement at speeds higher than 40mph (AASHTO Guide, 2008).

Snow/Ice- Related to pavement wetness, snow and ice also create a film between the tire and the pavement which reduces the skid resistance of the pavement.

2.3.4. Testing/Vehicle Related

Many states use tractor-trailer assembly to measure the skid resistance of a pavement surface as prescribed in the ASTM E 274 testing procedures. In this test a tractor trailer combination consisting of a mid-size truck and a two-wheel trailer are driven over the pavement to record the skid resistance of the pavement surface by using a two-axis force transducer(s) mounted on the axle assembly. As a result the quality of the friction readings recorded using this equipment are dependent on the following factors:

Slip Speed - the speed at which the vehicle is traveling has a direct relationship with the slip speed. It has also been discovered that the coefficient of friction between a tire and the pavement changes with varying slip (Henry, 2000). Skid Resistance increases sharply with increasing slip to a peak value that usually occurs between 10 to 20 percent slip. The friction then decreases to a value known as the coefficient of sliding which stabilizes to a 100 % slip and a constant value of coefficient of friction which occurs at 100 percent slip. Speed also impacts the side friction resistance. The following figure shows the relationship between percentage of tire slip and coefficient of friction.

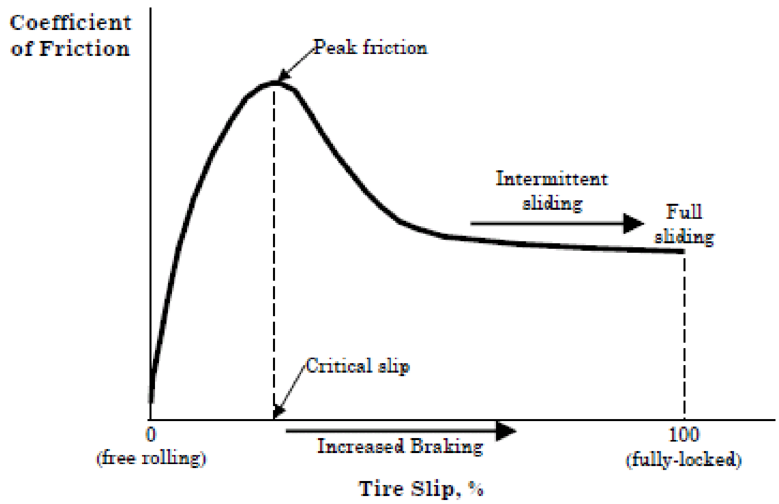

Figure 2-5. Pavement Longitudinal Friction versus Tire Slip (Henry, 2000)

Research has also shown that Friction Number (FN) varies with changes in test speed, and that there is a strong linear correlation between readings done using the ribbed tire for the ASTM E-274 test at 25 mph and 40 mph with the best relationship between these two parameters found to be to be polynomial (Jackson, 2008; Li et al, 2007). It has also been shown using actual friction readings in Maryland that FN values decrease at a rate of approximately 9 FN units per an increase of 5 mph in test speeds (Goulias et al, 2007). At speeds less than 40 mph, the microtextue (adhesion) component contributes greatly to the skid resistance, while macrotexture governs at higher speeds the (Dewey et al, 2001).

Chapter 3. SHA Materials database and Pavement Friction Records

In order to study the relationships between pavement friction and aggregate properties the following databases and records were used.

3.1. Pavement Friction Records

The pavement friction records considered in this study included 5 years friction data, from 2004 to 2008 with approximately 160,000 records. Overall the data are organized by Year and Route. The fields included in the database are shown in Table 3-1, while a screen shot of the Friction Database in Microsoft Access is shown in Figure 3-1. Most of the data were collected from early spring to late Fall. However there was variability in the timing of surveys at the same location from year to year. For example the data collected in 2004 were collected from March to September, while for 2006, the friction surveys were run from April to November, and so on. About 72% of the friction surveys represent sections with FN (Friction Number) values between 36 and 55. The data include sections that have been surveyed for the 5 consecutive years, and thus include the historical change of FN over time. Any missing values and/or values outside the expected range of FN were identified and flagged in the database. About 50% of the sections were surveyed at the specified slip speed of 40mph, and about 84% of the sections surveyed between 2004 and 2008 were evaluated at speeds between 38 and 42 mph. The reported AADT (Annual Average Daily Traffic) values reflect the local conditions (Rural vs Urban). About 95,000 of the surveyed sections were collected on rural conditions. Inconsistencies in AADT counts between consecutive years for the same sections were examined. In some cases there were missing AADT entries and/or very low values. These data were further examined. The contract numbers were cross referenced from the friction database to the construction database in order to include information regarding the year of rehabilitation/maintenance (“ACTION_YEAR”) related to the specific sections that the friction surveys were conducted, and for identifying the type of material used (“Material_Type”). For a certain number of sections, no rehabilitation information was available, and as a result the age of the existing roadway surface is unknown. In the data, there are also sections that have not received any rehabilitation in the last 40 to 50 years. These data were further examined. In terms of materials, the majority of the roadway surfaces in the database represent HMA (Hot Mix Asphalt) mixtures. Therefore, HMA was primarily targeted for the analysis of this study.

Table 2-2. Name, Type and description of fields in the "Friction" Database

Field Name	Data Type	Description
YEAR	Number	
CODE	Number	County Code
MUN	Number	
ROUTE	Text	
RNUM	Number	RouteNumber
RSUFF	Text	
Mile	Number	
DIRECTION	Text	
SPEED	Number	
FN	Number	
DATE	Date/Time	Date of Survey
AADT	Number	
UorR	Text	Urban vs Rural
ACTION_YEAR	Number	from construction history
CONTRACT	Text	This is from Construction History
Material_Type	Text	This is type of material used
DayCompleted	Date/Time	This is maintenance date
MonthCOmpleted	Text	This is maintenance month
YearCOmpleted	Number	This is maintenance year
MaintenanceContract	Text	This is contract number in maintenance history
ProjectType	Text	Maintenance Type
Truck	Number	5: International Cybernetics; 6: Dynatest

	YEA	CC	ML	RO	RNU	RS	Mile	DI	SPEED	FN	DATE	AADT	U	ACTI	CONTRACT	Material_Typ	Day	MonthCo	Year	Maintena	ProjectT	Truck
	2004	11		MD	42		1.28	N	38	57	8/25/2004	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2004	11		MD	42		1.33	S	43	56	8/25/2004	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2004	11		MD	42		1.58	N	39	55	8/25/2004	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2004	11		MD	42		1.63	S	40	59	8/25/2004	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2004	11		MD	42		1.88	N	38	61	8/25/2004	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2004	11		MD	42		1.93	S	39	57	8/25/2004	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2005	11		MD	42		1.33	N	38	57	9/1/2005	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2005	11		MD	42		1.41	S	39	57	9/1/2005	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2005	11		MD	42		1.63	N	40	54	9/1/2005	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2005	11		MD	42		1.71	S	40	52	9/1/2005	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2005	11		MD	42		1.93	N	39	56	9/1/2005	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2006	11		MD	42		1.18	S	40	55	8/29/2006	3440	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2006	11		MD	42		1.18	N	40	58	8/29/2006	3440	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2006	11		MD	42		1.48	N	38	56	8/29/2006	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2006	11		MD	42		1.48	S	40	58	8/29/2006	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2006	11		MD	42		1.78	N	40	61	8/29/2006	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2006	11		MD	42		1.78	S	40	60	8/29/2006	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	5
	2007	11		MD	42		1.19	N	39	63	6/6/2007	3440	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	6
	2007	11		MD	42		1.36	S	36	61	6/6/2007	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	6
	2007	11		MD	42		1.52	N	36	62	6/6/2007	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	6
	2007	11		MD	42		1.64	S	39	60	6/6/2007	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	6
	2007	11		MD	42		1.87	N	40	67	6/6/2007	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	6
	2008	11		MD	42		1.28	N	41	33	10/5/2008	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	6
	2008	11		MD	42		1.37	S	34	28	10/5/2008	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	6
	2008	11		MD	42		1.58	N	40	25	10/5/2008	3241	R	2008	XX9115177	HMA 12.5mm, 6	19	November	2007	XX8115177	Patching	6

Figure 3-1. Screen Shot of Pavement friction Data in Access

3.2. Materials / Mix Design Database.

The SHA Mix design database provides infrormation regarding the materials and mixtures used in pavement construction, including aggregate and binder information, source of materials and proportioning (Table 3-3). Specifically for the aggregate source, the aggregate gradation is often composed of a blend of aggregates from different sources, providing a blend of different aggregate types for each mixture (Figure 3-3). This has been a limitation in the research study in terms of identifying the effects of a single aggregate type/source on pavement friction performnace.

3.3. Merged Material and Friction Database.

In order to relate pavement friction to aggregates, the pavement friction records and the mixture databases were merged by using the "Contract" Column form the Friction Database with the "Project ID" column from the Mix Design Database. This resulted in about 52,000 records. The merged database (master database) was used to extract material and pavement friction related information for detailed data analysis.

3.4. Aggregate Bulletin Database

The Aggregate Bulletin database contains a list of tests that are performed annually by the Maryland State Highway Administration (except Polish Value, Soundness and Alkali-Silica Reactivity tests which are done every three years), on samples obtained from producers. Figure 3.3 provides an example of the data in the Aggregate Bulletin. In addition to the Aggregate Bulletin data, any information related to petrographic/ texture aggregate characteristics in the SHA records were used, when found appropriate. These included among others:

- General Information
    - Supplier (Source Location), Date, sample information, SHA Track Series
- General Classification (Carbonate or Non-Carbonate)
- Insoluble Residue Analysis
- Textural Description
- General Aggregate Testing Results:
    - Specific Gravity
    - Absorption
    - Los Angeles Abrasion
    - Sodium Sulphate Soundness
    - Polish Value
    - British Pendulum Number (BPN)

Table 3-1. Name, type and description of Fields in the "Mix Design" database

Column	Format
ID	Number
Mix Design	Text
Mix Size	Text
Date Approved	Text
Date Verified	Text
Current	Yes/No
Final	Yes/No
Date rescinded	Text
Traffic Level	Text
Plant	Text
Gmm	Number
Gmb	Number
Binder%	Number
Binder Source	Text
Binder Grade	Text
Gb	Number
MixTemp	Text
MoldTemp	Text
Gsb	Number
D/B	Number
50	Number
37.5	Number
25	Number
19	Number
12.5	Number
9.5	Number
4.75	Number
2.36	Number
1.18	Number
0.6	Number
0.3	Number
0.15	Number
0.075	Number

Table 3-1: Name, type and description of Fields in the “Mix Design” database (Continued)

AS1	Text
AS1 Old	Text
AS1%	Number
PV1	Text
AS2	Text
AS2Old	Text
AS2%	Number
PV2	Text
AS3	Text
As3Old	Text
AS3%	Number
PV3	Text
AS4	Text
AS4Old	Text
AS4%	Number
PV4	Text
AS5	Text
AS5Old	Text
AS5%	Number
PV5	Text
AS6	Text
AS6Old	Text
AS6%	Number
PV6	Text
AS7	Text
AS7Old	Text
AS7%	Number
PV7	Text
RAPCA%	Number
RAP Binder%	Number
MixPV	Number
Mineral Filler Source	Text
TSR	Number
Log Number	Number
Comments	Text

Mix Design All data

ID	Mix Design #	Mix Size	Date Approved	Date Verified	Current	Final	Date Recinded	Traffic Level	Plant	Gmm	Gmb	Binder %	Binder Source
494	S16525R2C51F	25	6/3/2008		Yes	No		2 - 0.3 to < 3.0	165	2.553	2.413	0	Citgo
498	S16512R1C03F	12	6/3/2008		Yes	No		1 - < 0.3	165	2.525	2.424	5	Citgo
499	S16512R2C03F	12	6/3/2008		Yes	No		2 - 0.3 to < 3.0	165	2.533	2.432	5	Citgo
502	S16525R3C51F	25	6/3/2008		Yes	No		3 - 3.0 to < 10.0	165	2.561	2.423	2.57	Citgo
544	S01225R4C50F	25	6/12/2008		Yes	No		4 - 10 to < 30	012	2.576	2.473	4.2	Chevron
550	S01225R3C5F	25	6/12/2008		Yes	No		3 - 3.0 to < 10.0	012	2.59	2.486	4.4	Chevron
622	S15119R4C50F	19	6/16/2008		Yes	No		4 - 10 to < 30	151	2.54	2.438	4.5	Chevron
623	S15109R1C04F	09	6/16/2008		Yes	No		1 - < 0.3	151	2.492	2.39	5.7	Chevron
624	S15109R2C03F	09	6/16/2008		Yes	No		2 - 0.3 to < 3.0	151	2.49	2.39	5.7	Citgo
625	S15109R2C04F	09	6/16/2008		Yes	No		2 - 0.3 to < 3.0	151	2.5	2.4	5.5	Chevron
626	S15109R3C04F	09	6/16/2008		Yes	No		3 - 3.0 to < 10.0	151	2.51	2.41	5.3	Chevron
627	S15109R4C03F	09	6/16/2008		Yes	No		4 - 10 to < 30	151	2.5	2.4	5.4	Chevron
628	S15109V4F04F	09	6/16/2008		Yes	No		4 - 10 to < 30	151	2.482	2.383	5.2	Citgo
631	S15119R2C50F	19	6/16/2008		Yes	No		2 - 0.3 to < 3.0	151	2.614	2.509	2.72	Citgo
700	S15704V2C02F	04	6/17/2008		Yes	No		2 - 0.3 to < 3.0	157	2.469	2.37	6.4	Valero
702	S15709R2E02F	09	6/17/2008		Yes	No		2 - 0.3 to < 3.0	157	2.504	2.404	5.2	Valero

Binder Gra	Gb	MixTemp	MoldTer	Gsb	D/B	50_0	37_5	25_(	19_0	12_50	9_50	4_75	2_36	1_18	0_60	0_30	0_15
64-22	1.03			2.695	0.00	100	100	99	88.00	64	55	35	24.00	17	13	10	
64-22	1.03			2.709	0.00	100	100	100	100.00	96	85	49	33.00	23	17	12	
64-22	1.03			2.709	0.00	100	100	100	100.00	96	85	49	33.00	23	17	12	
64-22	1.03			2.695	0.00	100	100	99	88.00	64	55	35	24.00	17	13	10	
64-22	1.03			2.74	0.00	100	100	100	84.00	71	67	33	24.00	19	13	9	
64-22	1.03			2.74	0.00	100	100	97	84.00	71	67	33	24.00	19	13	9	
64-22	1.03			2.704	0.00	100	100	100	97.00	86	77	38	26.00	18	15	10	
64-22	1.03			2.683	0.00	100	100	100	100.00	100	98	59	33.00	22	18	12	
64-22	1.03			2.693	0.00	100	100	100	100.00	100	98	53	29.00	20	15	12	
64-22	1.03			2.683	0.00	100	100	100	100.00	100	98	59	33.00	22	18	12	
64-22	1.03			2.683	0.00	100	100	100	100.00	100	98	59	33.00	22	18	12	
64-22	1.03			2.683	0.00	100	100	100	100.00	100	98	59	33.00	22	18	12	
76-22	1.03			2.671	0.00	100	100	100	100.00	100	98	58	32.00	22	16	11	
64-22	1.03			2.774	0.00	100	100	99	82.00	61	53	34	17.00	10	8	6	
64-22	1.03			2.663	0.00	100	100	100	100.00	100	100	96	61.00	33	20	13	
64-22	1.03			2.665	0.00	100	100	100	100.00	100	95	56	36.00	25	18	11	

Figure 3-2. Screen Shot of Mix Design database

0_075	AS1	AS1Old	AS1%	PV1	AS2	AS2Old	AS2%	PV2	AS3	AS3Old	AS3%	PV3
3.7	farge Frederick	LaFarge - Frede	40	0	Lafarge Frederi	LaFarge - Frede	20.00	0.00	:ville (Travilah)	LaFarge - Frede	15	0.00
3.7	farge Frederick	LaFarge - Frede	38	0	Lafarge Frederi	LaFarge - Frede	30.00	0.00	:ville (Travilah)	LaFarge - Texas	12	0.00
3.7	farge Frederick	LaFarge - Frede	38	0	Lafarge Frederi	LaFarge - Frede	30.00	0.00	:ville (Travilah)	LaFarge - Texas	12	0.00
3.7	farge Frederick	LaFarge - Frede	40	0	Lafarge Frederi	LaFarge - Frede	20.00	0.00	farge Frederick	LaFarge - Frede	15	0.00
3.9	farge Frederick	LaFarge - Frede	40	0	Lafarge Frederi	LaFarge - Frede	30.00	0.00	:erials Hanover	Vulcan - Hanov	5	0.00
3.9	farge Frederick	LaFarge - Frede	40	0	Lafarge Frederi	LaFarge - Frede	30.00	0.00	:erials Hanover	Vulcan - Hanov	25	0.00
4.7	farge Frederick	LaFarge - Frede	35	0	Lafarge Frederi	LaFarge - Frede	24.00	0.00		LaFarge - Frede	8	0.00
4.5	farge Frederick	LaFarge - Frede	35	0		Maryland Ston	10.00	0.00	farge Frederick	BBS PIT	15	0.00
5.5	farge Frederick	LaFarge - Frede	35	0		Maryland Ston	10.00	0.00	farge Frederick	BBT PIT	15	0.00
4.5	farge Frederick	LaFarge - Frede	35	0		Maryland Ston	10.00	0.00	farge Frederick	BBT PIT	15	0.00
4.5	farge Frederick	LaFarge - Frede	35	0		Maryland Ston	10.00	0.00	farge Frederick	BBS PIT	15	0.00
4.5	farge Frederick	LaFarge - Frede	25	0		Maryland Ston	10.00	0.00		LaFarge - Frede	35	0.00
3.9	farge Frederick	LaFarge - Frede	30	0	Lafarge Frederi	LaFarge - Frede	36.00	0.00		Maryland Ston	15	0.00
2.8	farge Frederick	LaFarge - Frede	24	0	Lafarge Frederi	LaFarge - Frede	35.00	0.00		LaFarge - Frede	8	0.00
9	farge Frederick	LaFarge - Frede	100	0			0.00	0.00			0	0.00
6	farge Frederick	LaFarge - Frede	50	0	Lafarge Frederi	LaFarge - Frede	20.00	0.00	lucts Perryville	Pareville MD	15	0.00
6	farge Frederick	LaFarge - Frede	45	0	Lafarge Frederi	LaFarge - Frede	45.00	0.00	lucts Perryville	Parville MD	10	0.00

AS4	AS4Old	AS4%	PV4	AS5	AS5Old	AS5%	PV5	AS6	AS6Old	AS6%	PV6	AS7	AS7Old
		0.00	0.00			0	0			0	0	Aggregate Indu	Aggregate Indu
		0.00	0.00			0	0			0	0	Aggregate Indu	Aggregate Indu
		0.00	0.00			0	0			0	0	Aggregate Indu	Aggregate Indu
		0.00	0.00			0	0			0	0	Aggregate Indu	Aggregate Indu
		0.00	0.00			0	0			0	0	RAP	RAP
		0.00	0.00			0	0			0	0	RAP	RAP
BBS Pit	BBS Pit	8.00	0.00			0	0			0	0	RAP	RAP
LaFarge - Frede	LaFarge - Frede	25.00	0.00			0	0			0	0	RAP	RAP
LaFarge - Frede	LaFarge - Frede	25.00	0.00			0	0			0	0	RAP	RAP
LaFarge - Frede	LaFarge - Frede	15.00	0.00			0	0			0	0	RAP	RAP
LaFarge - Frede	LaFarge - Frede	25.00	0.00			0	0			0	0	RAP	RAP
BBT PIT	BBT PIT	15.00	0.00			0	0			0	0	RAP	RAP
BBS PIT	BBS PIT	19.00	0.00			0	0			0	0		
BBS PIT	BBS PIT	8.00	0.00			0	0			0	0	RAP	RAP
		0.00	0.00			0	0			0	0		
		0.00	0.00			0	0			0	0	RAP	RAP
		0.00	0.00			0	0			0	0		

Figure 3-2: Screen Shot of Mix Design database (continued)

AS7%	PV7	RAPCA%	RAP Binder %	MixPV	Mineral Fille	Mineral Fille	TSR	Log Numbe	Comments
25	0	0	0	0		0	0	0	
20	0	0	0	0		0	0	0	
20	0	0	0	0		0	0	0	
25	0	0	0	0		0	0	0	
25	0	0	0	0		0	0	0	
5	0	0	0	0		0	0	0	
25	0	0	0	0		0	0	0	
15	0	0	0	0		0	0	0	
15	0	0	0	0		0	0	0	
25	0	0	0	0		0	0	0	
15	0	0	0	0		0	0	0	
15	0	0	0	0		0	0	0	
0	0	0	0	0		0	0	0	
25	0	0	0	0		0	0	0	
0	0	0	0	0		0	0	0	
15	0	0	0	0		0	0	0	
0	0	0	0	0		0	0	0	

Figure 3-2: Screen Shot of Mixture database (continued)

Maryland State Highway Administration Coarse Aggregate Properties for 2005 Test Data

Producer	SPGR	ABS	LA	UWLSE	UWROD	SOUND	BPN	PV	ASR
La Farge Churchville, MD	2.96	0.5	22	94.9	104.8	0.4	22	*	0.03
La Farge Frederick, MD	2.70	0.4	22	87.4	93.4	0.2	24	6	*
La Farge Marriotsville, MD	2.72	0.4	41	89.0	96.4	1.5	26	7	0.02
La Farge Medford, MD Nr. Westminster, MD	2.73	0.5	22	94.7	100.5	0.1	21	3	0.01
La Farge Texas, MD	2.88	0.4	21	98.2	103.2	0.3	24	6	0.02
Laurel S&G Woodsboro, MD Barrick Quarry	2.70	0.3	24	87.5	92.5	0.1	28	5	0.11
Luck Stone Corp Leesburg, VA (Leesburg Pit)	2.95	0.8	13	99.3	109.0	0.2	27	*	0.02

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Figure 3-3. Example of data in the 2005 SHA Aggregate Bulletin (Coarse Aggregates)

3.5. Equipment Repeatability Data

SHA is conducts repeatability pavement Friction tests annually on test sections along the I-795 corridor. The records shown in Table 3-4 were provided for the analysis of this research study. These included equipment repeatability runs on both flexible and rigid pavements in 2006 and 2007 using SHA's Locked Wheel pavement friction trucks, Truck #5 - International Cybernetics Corporation, and Truck #6 - Dynatest.

Table 3-2. Summary of Equipment Variability Tests

Test Date	**Equipment**	**Flexible/Rigid**
01/27/2006	Truck 5	Both
09/06/2006	Truck 5	Both
03/20/2007	Truck 5	Both
03/20/2007	Truck 6	Both
06/21/2006	Truck 5 & 6	Flexible

Chapter 4. Equipment Variability Study

The Maryland State Highway Administration uses Locked-Wheel Skid Testers (LWST), for its annual pavement friction surveys. The friction database used in this research project contains Friction readings collected using equipment designated as Truck 5 and Truck 6 in the database. The friction surveys are conducted per ASTM-E274. Equipment Assembly (Truck) 5 is an older, standard Dynatest 1295 model vehicle with the following specifications:

- Computer controlled Pavement friction tester developed to operate between 20 and 70 mph while computing the dynamic skid number (Dynatest Operating Manual, 2003)
- The tractor trailer combination consists of a mid-size truck (e.g. GMC Sierra) and a two-wheel trailer which uses one or two model 1270 two-axis force transducer(s) mounted on the axle assembly.
- The truck is equipped with portable computer and printer with Windows Operating System

Equipment Assembly 6 uses an identical trailer but with a larger truck (a "Custom International Truck Chassis with utility body" – Dynatest Website).

In both equipment the on-board computer calculates the dynamic Skid Number FN (t) from the two-axis force transducer(s) in real time, and displays the friction and speed traces on the portable test screen. The test headers, skid numbers and other information can be stored to a hard disk or sent directly to printer.

This research involved reviewing friction readings collected using both equipment along the I-795 corridor in Baltimore County and investigating the repeatability of data within and between the two equipment.

As mentioned above, SHA (State Highway Administration) uses two locked wheel friction devices, Truck #5 - International Cybernetics Corporation model, and Truck #6 the Dynatest model, to conduct annual pavement friction surveys. As listed in Table 3-4, the friction devices were used to collect repeatability and side by side comparison data on both flexible and rigid pavement sections of I-795 at different times. These readings were analyzed as part of the research study and the results are presented next.

4.1. Individual Equipment Repeatability.

4.1.1. Truck #5 - International Cybernetics Corporation model

A series of repeatability testing records collected on the same mile post and same day were examined. This included repeated testing conducted on both flexible and rigid pavement sections on the following dates:

01/27/06 (at 9:43am, and 10:07 am);

09/06/06 (at 11:41am, 11:59am, 12:54pm, and 1:14pm); and,

03/20/07 (8:47am, 9:11am, 9:43am, and 12:03pm).

The milepost numbers of the surveyed sections were matched so as to compare the FN (Friction Number) values for the same sites. An example of such data is shown in Tables 4-1 and 4-2 along with the summary statistics, and based on the four repeatability records of 09/06/06. As it can be seen in the tables and figures, the average value of the coefficient of variation (CV) for this devise ranges from 2% to 3% with individual values all the way up to 7%. Figures 4-1 and 4-2 show the FN measurements in the flexible and rigid sections of I-795 in relation to the milepost. Examining the repeatability data collected on other testing dates for this device, it was concluded that for flexible pavements, the average CV ranged from 1% to 2%, while for rigid pavements, the average CV ranged from 1% to 3%. Considering the level of FN values (average FN of 60 for the flexible and FN of 50 for the rigid pavement sections) the equipment repeatability introduce into the friction measurements, on the average, a variability of +/- 1.2FN and 1.5FN units for flexible and rigid pavements respectively.

In addition to the variability analysis, ANOVA (Analysis of Variance) was conducted on the repeated runs for assessing whether the measurements collected from the repeated runs can be statistically considered from the same population. The analysis are presented in Tables 4-3 and 4-4 for the FN measurements in the flexible and rigid sections of I-795 that were collected on 09/06/2006. As it can be concluded from the statistical analysis, the null hypothesis (i.e., there is not significant variability among the means of the four different runs) is accepted since the F-calculated/Observed < F critical at an alpha value of 0.05. The same conclusions were obtained with the data collected on other dates.

4.1.2. Truck #6 - Dynatest model

For this device the repeated runs collected on 03/20/07 (11:49am, 12:11am, and 12:32pm for flexible pavements, and 11:49am, and 12:32pm for rigid pavements) were used. The milepost numbers of the surveyed sections were matched and compared so as to evaluate the corresponding FN values. This data is shown in Tables 4-5 and 4-6 along with the summary statistics. As it can be seen the average value of the coefficient of variation (CV) for this device is ranging from 5% to 6%, with individual values all the way up to 20%. Figures 4-3 and 4-4 show the FN measurements in the flexible and rigid sections of I-795 in relation to the milepost. Considering this magnitude of variability along with the level of FN values (average FN of 60 for the flexible and FN of 55 for the rigid pavement sections) the equipment repeatability introduce into the friction measurements, on the average, a variability of +/- 3.0 FN and 3.3FN units for flexible and rigid pavements respectively.

In addition to the variability analysis, t-test and ANOVA was conducted on the repeated runs for assessing whether the measurements collected from the repeated runs can be considered – statistically- to be from the same population. While the ANOVA showed that the null hypothesis was rejected (i.e., there is significant variability among the means of the different runs), the t-test showed that the records collected from the repeated runs, when compared two at a time, can be considered to be from the same population. These results are further examined from the research group along with the individual values reported.

4.2. Equipment Side by Side Comparison.

For the comparison of the friction measurements between these two devices, the data collected on 06/21/06 were used. The milepost numbers of the surveyed sections were cross linked, specifically for the flexible test sections, so as to compare and analyze the FN values representing the same sections. For the rigid pavement sections, the reported mileposts between the two devices did not match, thus the analysis where not included. The comparison for the flexible sections is shown in Table 4-7 along with the summary statistics. As it can be seen, the average difference (CV) between the values produced by these two devices is of the order of 7%, and with individual values all the way up to 13%. Truck #6 always provided higher values than Truck #5. Figure 4-5 shows the FN measurements reported for the two friction trucks in relation to mileposts. Considering the level of FN values where these measurements were taken (average FN of 55), it is expected to observe a higher FN value of about + 6.5 FN units when truck # 6 is used in relation to #5. This is often reflected in the friction database when different devices are used, year after year, for surveying the same sections. In addition to the variability analysis, t-test and ANOVA was conducted on the data collected from the two trucks. As expected, both the t-test and ANOVA showed that neither the set of individual values (t-test) nor their averages (F-test) can be considered - statistically speaking - to represent samples from the same population. These results are shown in Table. 4-8.

Table 4-1. Repeatability of Truck #5 International Cybernetics Corporation on Flexible Pavement Sections of I-795 (09/05/06)

11:41:29 AM		11:59:21 AM		11:54:22 AM		1:14:11 PM	
MP	FN Reading	MP	FN Reading	MP	FN Reading	MP	FN Reading
0.183	62.4	0.189	64.2	0.191	63.9	0.192	62.8
0.281	60.6	0.286	59.6	0.29	61.4	0.292	58.9
0.381	60.5	0.388	62	0.39	61.7	0.391	62.6
0.482	58.7	0.486	58.7	0.49	59.4	0.491	61.3
0.581	53.2	0.587	55.6	0.589	54.2	0.591	54.6
0.682	60.3	0.686	59.3	0.69	61.3	0.69	59.6
0.782	63.5	0.787	59.6	0.791	64.2	0.791	62.3
0.882	63.6	0.887	62	0.889	63.2	0.891	61.2
0.983	63.5	0.986	62.4	0.99	64.8	0.991	61.8

Average	SD	Variance	COV
63.3	0.9	0.7	1%
60.1	1.1	1.2	2%
61.7	0.9	0.8	1%
59.5	1.2	1.5	2%
54.4	1.0	1.0	2%
60.1	0.9	0.8	1%
62.4	2.0	4.1	3%
62.5	1.1	1.2	2%
63.1	1.3	1.7	2%
	Average CV		2%

Table 4-2. Repeatability of Truck #5 International Cybernetics Corporation on

Rigid Pavement Sections of I-795 (09/05/06)

11:41:29 AM		11:59:21 AM		11:54:22 AM		1:14:11 PM	
MP	FN Reading	MP	FN Reading	MP	FN Reading	MP	FN Reading
4.381	49.8	4.382	51.9	4.386	46.2	4.384	50.2
4.481	50.9	4.481	49	4.486	47	4.484	47.9
4.581	50.6	4.581	50.8	4.586	46.8	4.583	48.4
4.681	48.8	4.681	50.6	4.686	49.1	4.683	56
4.781	52.1	4.782	51.9	4.786	52.2	4.784	51
4.881	49.5	4.881	50.3	4.885	49.2	4.883	50.7
4.98	47.3	4.981	51.4	4.986	46.3	4.983	48.9
5.081	49.4	5.082	50.5	5.086	53.2	5.084	47.1
5.25	47.9	5.243	49.9	5.24	47.8	5.237	46.5
5.372	50.2	5.343	49.3	5.34	48.8	5.338	49.6

Average	SD	Variance	COV
49.5	2.4	5.7	5%
48.7	1.7	2.8	3%
49.2	1.9	3.6	4%
51.1	3.3	11.2	7%
51.8	0.5	0.3	1%
49.9	0.7	0.5	1%
48.5	2.2	4.9	5%
50.1	2.5	6.4	5%
48.0	1.4	2.0	3%
49.5	0.6	0.3	1%
	Average CV		3%

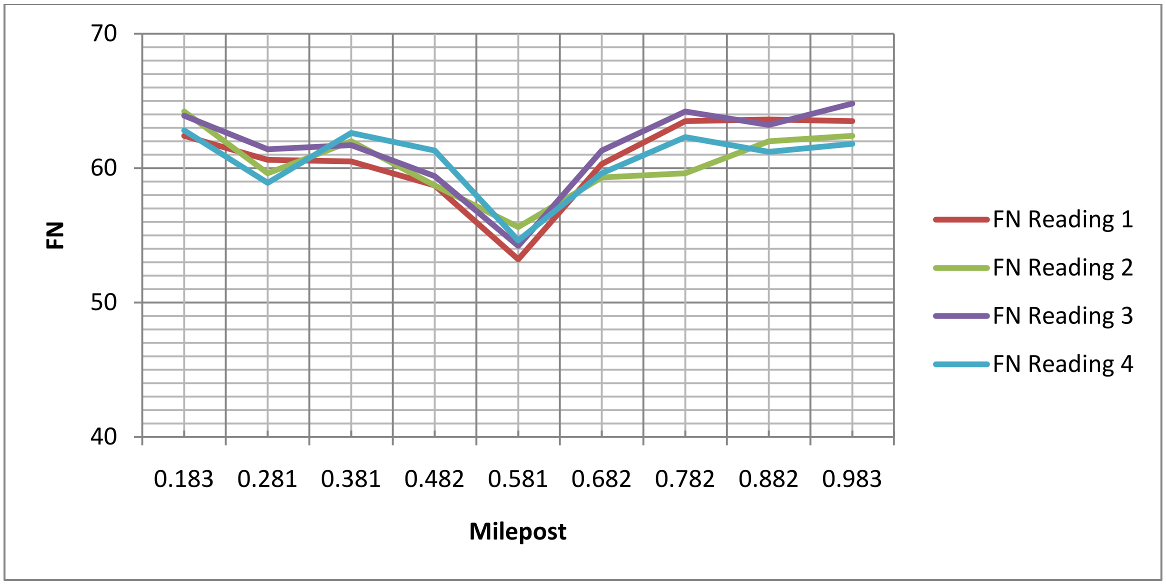

Figure 4-1. Repeatability of Truck #5 International Cybernetics Corporation on Flexible Pavement Sections of I-795 (09/05/06)

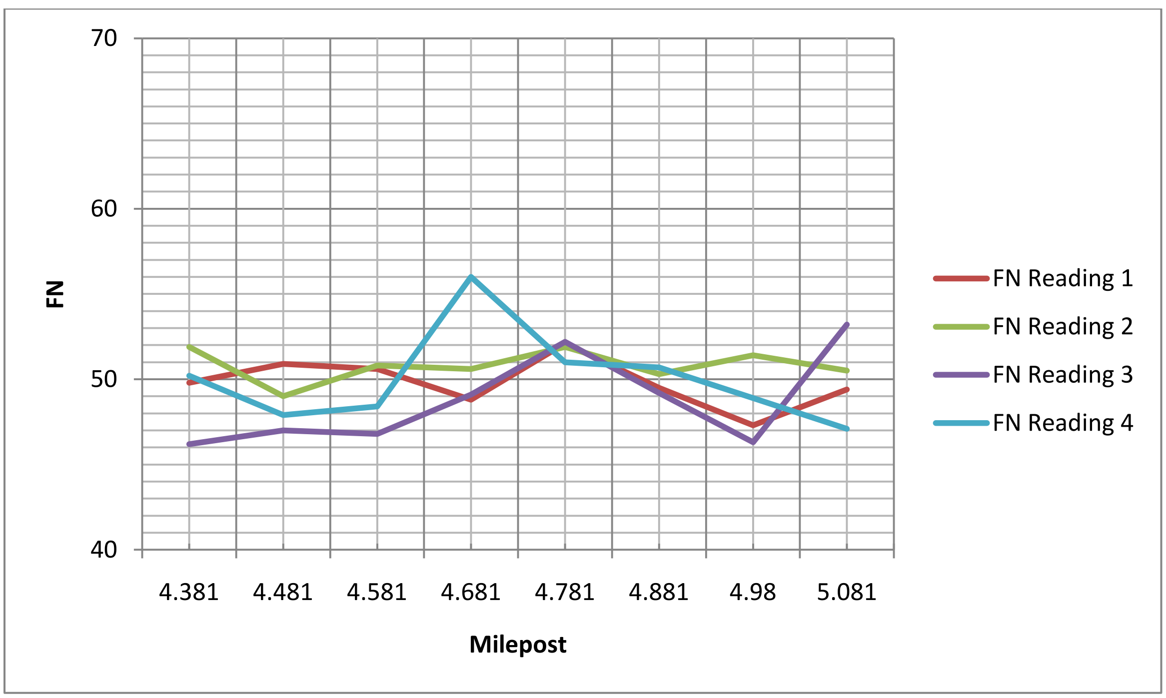

Figure 4-2. Repeatability of Truck #5 International Cybernetics Corporation on

Rigid Pavement Sections of I-795 (09/05/06)

Table 4-3. ANOVA for Repeatability of Truck #5 International Cybernetics Corporation on Flexible Pavement Sections of I-795 (09/05/06)

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Column 1	9	546.3	60.7	10.93
Column 2	9	543.4	60.38	6.49
Column 3	9	554.1	61.56	10.5725
Column 4	9	545.1	60.56	6.7375

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	7.4741	3	2.491	0.286	0.834	2.90
Within Groups	277.895	32	8.684			
Total	285.369	35				

Table 4-4. ANOVA for Repeatability of Truck #5 International Cybernetics Corporation on Rigid Pavement Sections of I-795 (09/05/06)

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Column 1	8	398.4	49.8	2.091
Column 2	8	406.4	50.8	0.91
Column 3	8	390	48.75	7.34
Column 4	8	400.2	50.025	7.730

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	17.16	3	5.72	1.26	0.30	2.94
Within Groups	126.555	28	4.51			
Total	143.718	31				

Table 4-5. Repeatability of Truck #6 Dynatest on Flexible Pavement Sections of I-795 (03/20/07)

11:49:00 AM		12:11:00 PM		12:32:00 PM	
MP	FN Reading	MP	FN Reading	MP	FN Reading
1.11	62.1	1.152	59.2	1.109	56.8
1.21	58.5	1.252	65.6	1.205	61.6
1.31	52.1	1.352	59.2	1.307	61.3
1.409	50.9	1.452	61	1.407	63.6
1.507	56.4	1.552	58.4	1.507	58
1.602	57.9	1.652	59.4	1.611	61.8
1.71	57.8	1.752	59.9	1.71	63
1.811	60.3	1.852	62.1	1.804	61.8
1.912	49	1.952	58.2	1.907	61.2
2.009	57.8	2.052	53.4	2.008	62.9

Average	SD	Variance	COV
59.4	2.7	7.0	4%
61.9	3.6	12.7	6%
57.5	4.8	23.2	8%
58.5	6.7	45.0	11%
57.6	1.1	1.1	2%
59.7	2.0	3.9	3%
60.2	2.6	6.8	4%
61.4	1.0	0.9	2%
56.1	6.4	40.4	11%
58.0	4.8	22.6	8%
	Average CV		6%

Table 4-6. Repeatability of Truck #6 Dynatest on Rigid Pavement Sections of I-795 (03/20/07)

11:49:00 AM		12:32:00 PM	
MP	FN Reading	MP	FN Reading
5.082	54.2	5.077	51.7
5.18	54.1	5.185	54.7
5.27	56.1	5.275	56.7
5.38	56.5	5.378	58.7
5.475	54.6	5.479	45.9
5.579	54.5	5.581	57.3
5.682	53.3	5.677	55.6
5.781	57.1	5.778	43.1
5.901	55.9	5.9	56.4
5.999	54.6	6.003	53.2

Average	SD	Variance	COV
53.0	1.8	3.1	3%
54.4	0.4	0.2	1%
56.4	0.4	0.2	1%
57.6	1.6	2.4	3%
50.3	6.2	37.8	12%
55.9	2.0	3.9	4%
54.5	1.6	2.6	3%
50.1	9.9	98.0	20%
56.2	0.4	0.1	1%
53.9	1.0	1.0	2%
	Average CV		5%

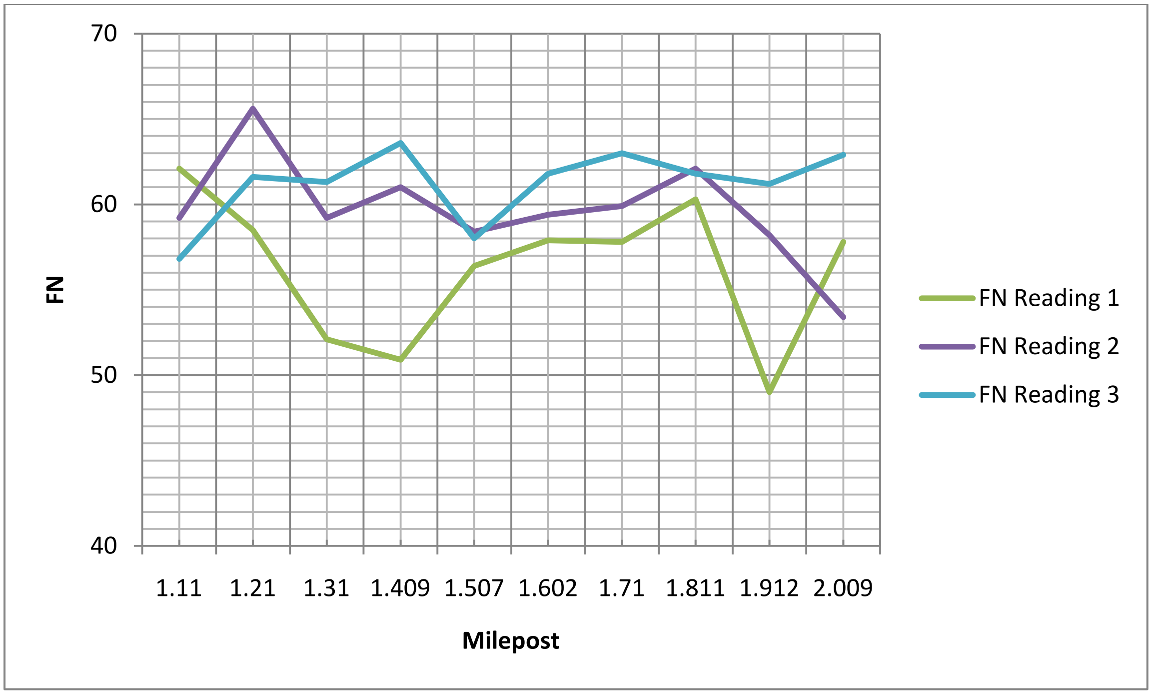

Figure 4-3. Repeatability of Truck #6 Dynatest on Flexible Pavement Sections of I-795 (03/20/07)

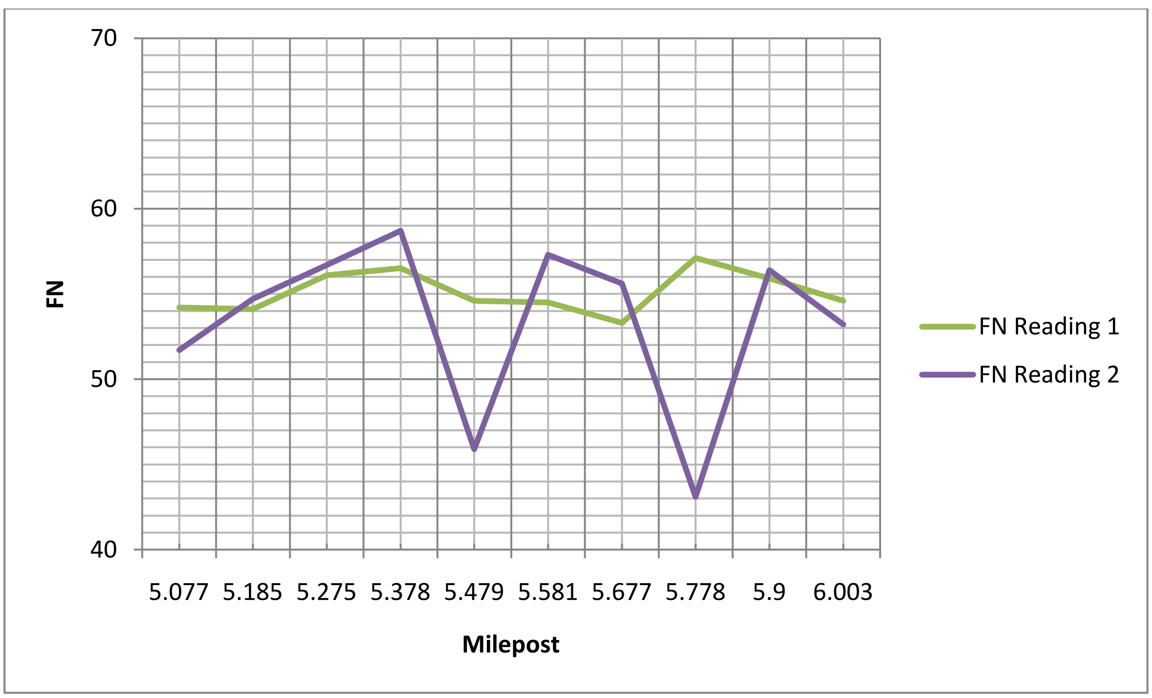

Figure 4-4. Repeatability of Truck #6 Dynatest on Rigid Pavement Sections of I-795 (03/20/07)

Table 4-7. Side by Side Comparison for Truck #5 and #6 on Flexible Pavement Sections of I-795 (06/21/06)

Truck #5		Truck #6	
MP	FN Reading	MP	FN Reading
1.144	52.9	1.155	60.9
1.243	51.9	1.255	62.2
1.342	52.1	1.355	54.5
1.443	52	1.455	60.9
1.542	49.4	1.555	53.9
1.643	52.8	1.655	56.4
1.743	53.5	1.755	60.1
1.842	53.4	1.855	57.6
1.942	54.6	1.955	57.2
2.042	53.1	2.055	58.4

Average	SD	Variance	COV
56.9	5.7	32.0	10%
57.1	7.3	53.0	13%
53.3	1.7	2.9	3%
56.5	6.3	39.6	11%
51.7	3.2	10.1	6%
54.6	2.5	6.5	5%
56.8	4.7	21.8	8%
55.5	3.0	8.8	5%
55.9	1.8	3.4	3%
55.8	3.7	14.0	7%
	Average CV		7%

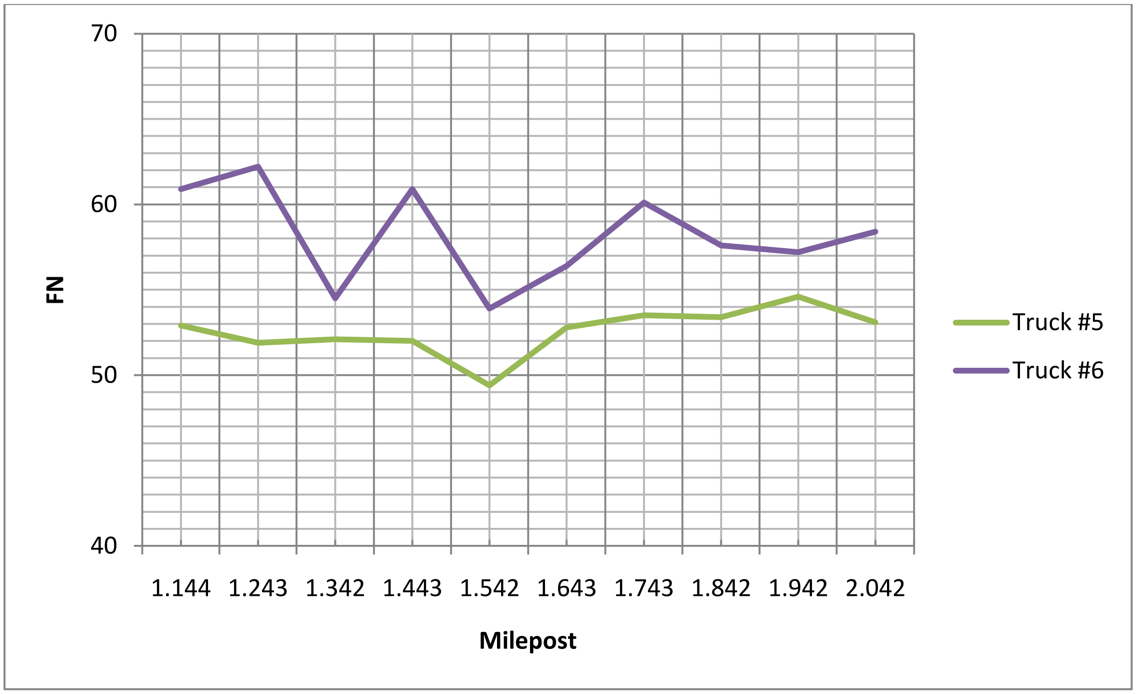

Figure 4-5 . Side by Side Comparison for Truck #5 and #6 on Flexible Pavement Sections of I-795 (06/21/06)

Table 4-8. Statistical Analysis for Side by Side Comparison of Truck #5 and #6 on

Flexible Pavement Sections of I-795 (06/21/06)

T-test

	Variable 1	*Variable 2*
Mean	52.57	58.21
Variance	1.906	7.889
Observations	10	10
Hypothesized Mean Difference	0	
df	13	
t Stat	-5.698	
P(T<=t) one-tail	3.6573E-05	
t Critical one-tail	1.770	
P(T<=t) two-tail	7.31E-05	
t Critical two-tail	2.160	

Analysis of Variance

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Column 1	10	525.7	52.57	1.90
Column 2	10	582.1	58.21	7.88

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	159.048	1	159.048	32.46	2.1E-05	4.413
Within Groups	88.17	18	4.898			
Total	247.218	19				

Chapter 5. Initial Analysis on Evaluation of Factors Affecting Pavement Friction

An initial set of analyses were conducted using all the friction records between 2004 and 2008. The friction records were analyzed by grouping either by individual or by group of counties, rural versus urban, or by specific route. At the onset, it was decided to eliminate any potential friction records related to potential data entry errors (i.e., FN<15 and FN>70), and analyze the records from Interstates separately from local (US and MD) roads. Some of the results are shown in Figures 5-1 through 5-12 (all figures and tables are located at the end of the chapter). As it can be seen from the analysis, the scatter/ variability in relating Friction Number (FN) to Annual Average Daily Traffic (AADT) and/or years-since-last-rehabilitation is considerable thus providing insignificant relationships (poor R^2). This is true whether the data are analyzed by group of counties, by county or by roadway type (Interstates, US and MD roads). Even in the case of analyzing the data by specific roadway and using the actual AADT values, such relationships are still insignificant. (Figures 5-11 and 5-12 provide the analysis for I-68 as an example). The reasons for such effects are related to the impact of several additional variables on friction performance including:

- Equipment and repeatability
- Seasonal effects on friction testing
- Local conditions
- Surface characteristics during testing
- Aggregate type and abrasion resistance quality
- Surveying speed
- other

The effect of survey speed on FN has been extensively studied in the past with SHA data (Goulias et. al. 2007). Those analyses conducted with approximately 1000 records per county, have reinforced the hypothesis that there is an inverse relationship between test speed and friction values. Furthermore, the analyses have shown that an increase in testing speed of 5 mph reduces friction number by about 9.1 FN units. An example of such a relationship, with data from Charles County, is shown in Figure 5- 13. The data selected for those analyses included friction readings taken in Charles County (CH), on the same day, on sections that have similar AADT and have received the same level of maintenance for the analysis period.

5.1. Systematic Evaluation of Variables Affecting FN

Since the objective of this research study was to identify the effects of aggregate on pavement friction, there was a need to systematically examine the contribution of various other parameters on FN. It is expected that different aggregates will have different effects on FN, and their role might be related to the type of mixtures in which they are used. At the same time, traffic level and pavement age will affect the degree of FN change. Since all remaining parameters (such as survey speed, equipment repeatability, seasonal effects, and so on) affect FN measurements, their impact has to be considered as well. Thus, it was the objective of this research project to isolate the effects of some of these variables. Exploratory analysis were conducted by considering subgroups of the data such as similar mixture type, a specific AADT level, constant survey speed and so on. According to the SHA friction data records, the HMA 12.5mm mixture represents the most popular material used in Maryland. As a result, mixture specific data were used for the analysis.

5.1.1. Friction Analysis for HMA 12.5 mm PG 70-22 – all types

Similar to the previous analysis, the HMA 12.5mm PG 70-22 friction data were used in examining the effects of survey speed, CumAADT (Cumulative AADT) and years-since-last-rehabilitation (pavement age) on FN. As it can be seen from Figures 5-14 through 5-16, dealing with friction surveys on MD and US routes, no acceptable relationships can be established due to the effects of the remaining parameters on FN which causes significant level of variance. The same is observed when the data from Interstate highways are examined as shown in Figures 5-17 through 5-19.

5.1.2. Friction Analysis for HMA 12.5 mm PG 70-22 & Uniform AADT~ 10,571

In the next step, sections with the same contract number and same AADT level were included in the analysis. The AADT in the friction surveys for this and the following analysis was replaced with the actual AADT values reported in the Traffic Monitoring System web site of SHA. The selected sections are shown in Table 5-1. The effects of speed (using data from 2004), years-since-last-rehabilitation and CumAADT are shown in Figures 5-20 to5-22. Overall, the relationships between these variables and FN has improved, however there is still a significant variability in the data due to the additional parameters affecting FN. Multiple linear regression analysis was also performed on these data. The results are shown in Table 5-2. Based on the analysis, the model below was obtained (F theoretical << F observed) relating FN with CumAADT, survey speed and age. However these parameters have t-observed close to the t-theoretical value at 95% confidence level (significant when t-observed - absolute value - is larger than t theoretical).

$$FN = 1.18\ Speed + 21.85\ Age - 0.0023\ CumAADT + 109.62 \quad (Eq.\ 5.1)$$

As expected pavement age (years-since-last-rehabilitation) was also insignificant since this variable is correlated to the CumAADT (Cumulative AADT = Age *AADT).

5.1.3. Friction Analysis for HMA 12.5 mm PG 70-22 with Uniform AADT= 9000 & Survey Speed of 40mph

In the next step, sections with the same contract number and AADT level were included along with a constant survey speed of 40 mph. The selected sections are shown in Table 5-3. The effects of year since last rehabilitation and CumAADT are shown in Figures 5-14 to 5-15. The relationships between these variables and FN are relatively poor due to a significant variability in the data introduced from additional parameters affecting FN which will be discussed in detail in the next chapters.

5.2. Tables and Figures

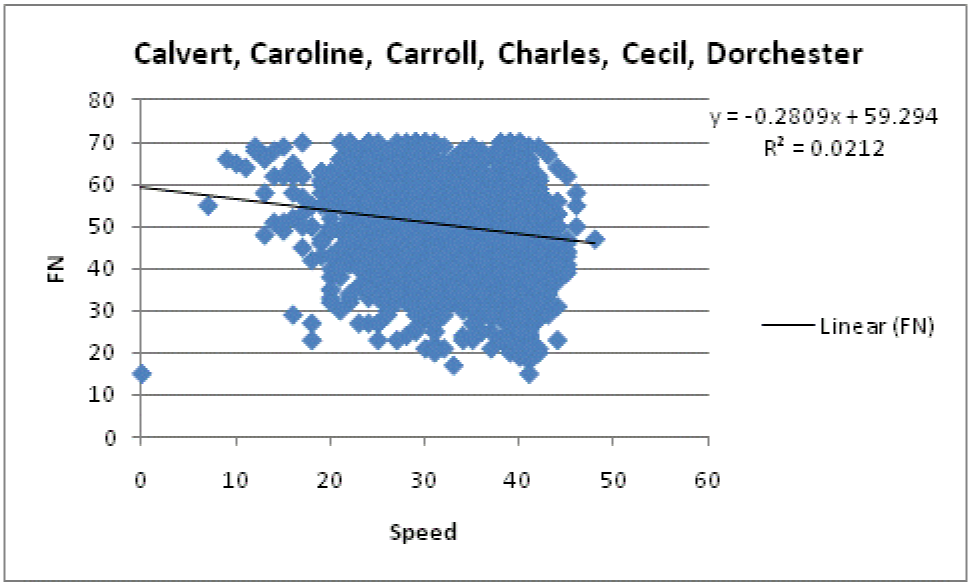

Figure 5-1. Speed vs FN for Selected Counties (MD and US Roads, n=28,216)

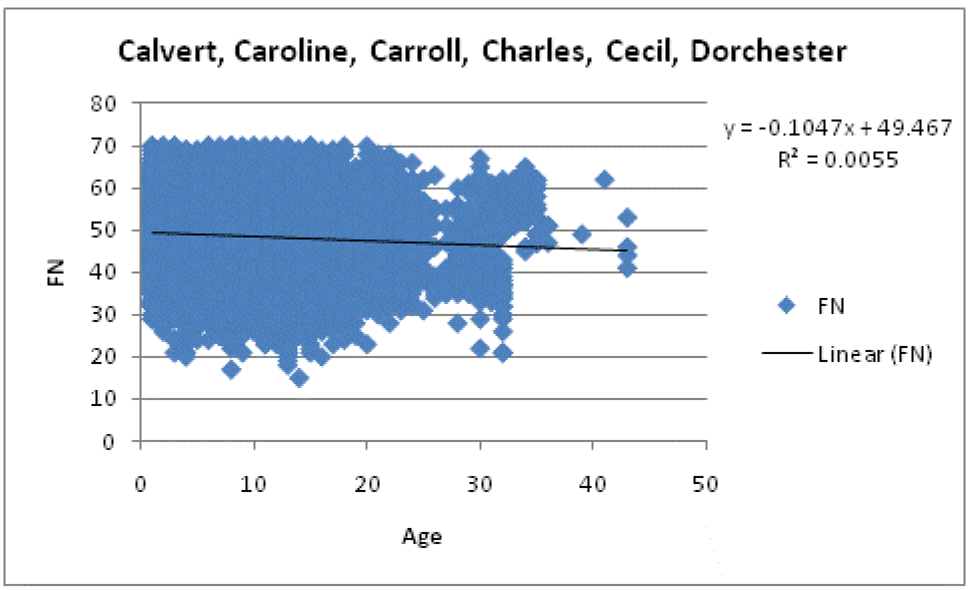

Figure 5-2. Age vs FN for Selected Counties (MD and US Roads, n=28,216)

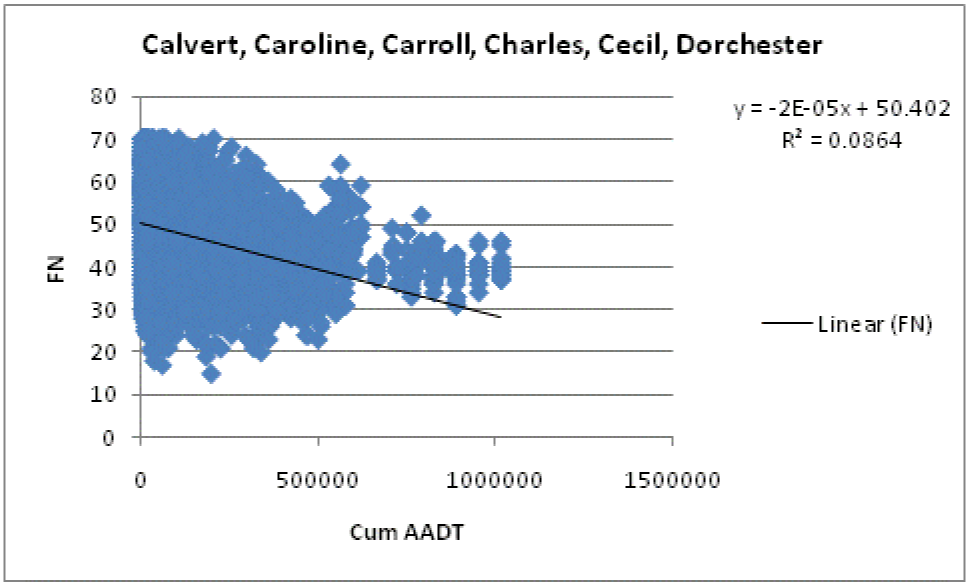

Figure 5-3. CumAADT vs FN for Selected Counties (MD and US Roads, n=28,216)

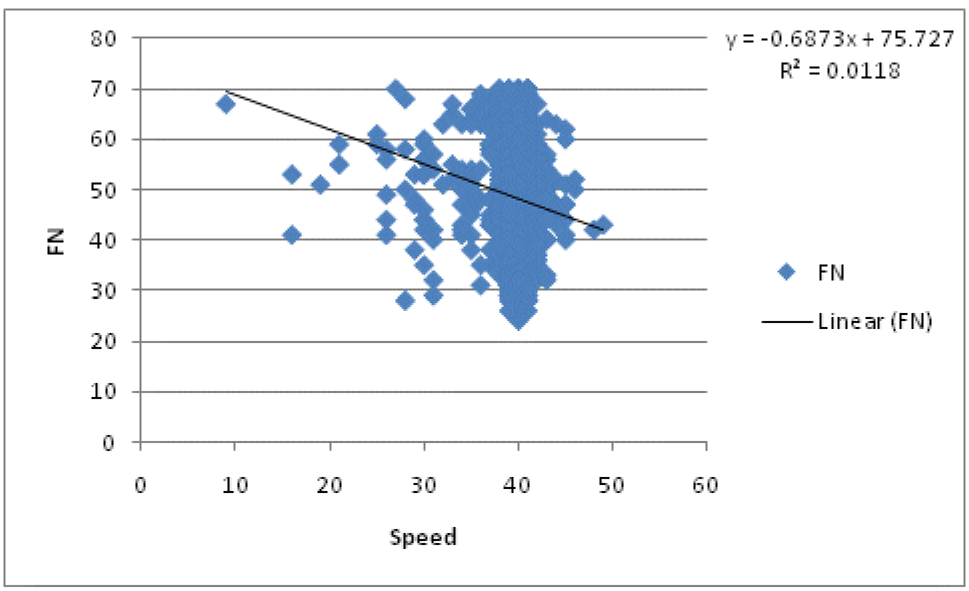

Figure 5-4. Speed vs FN for all Interstates – Statewide (n=10,828)

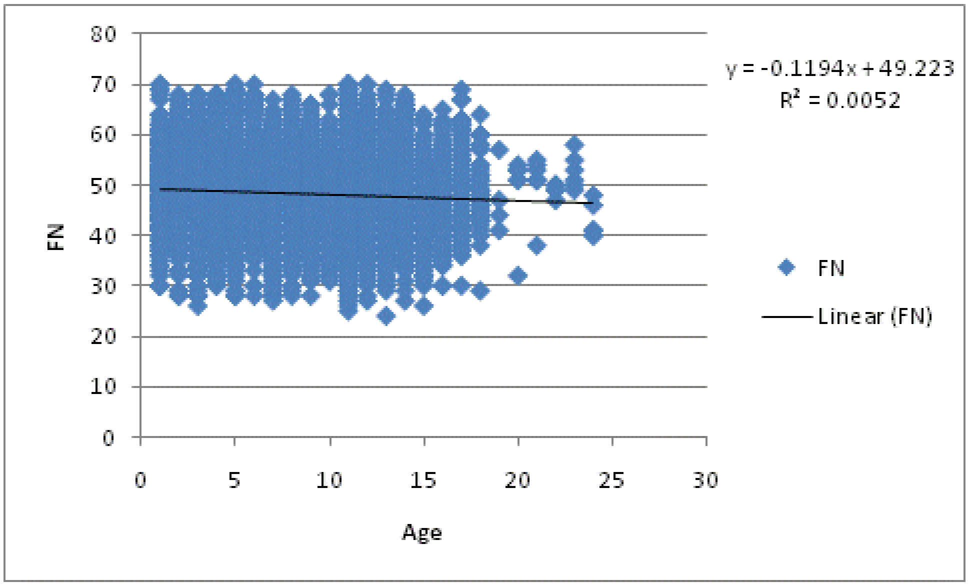

Figure 5-5. Age vs FN for all Interstates – Statewide (n=10,828)

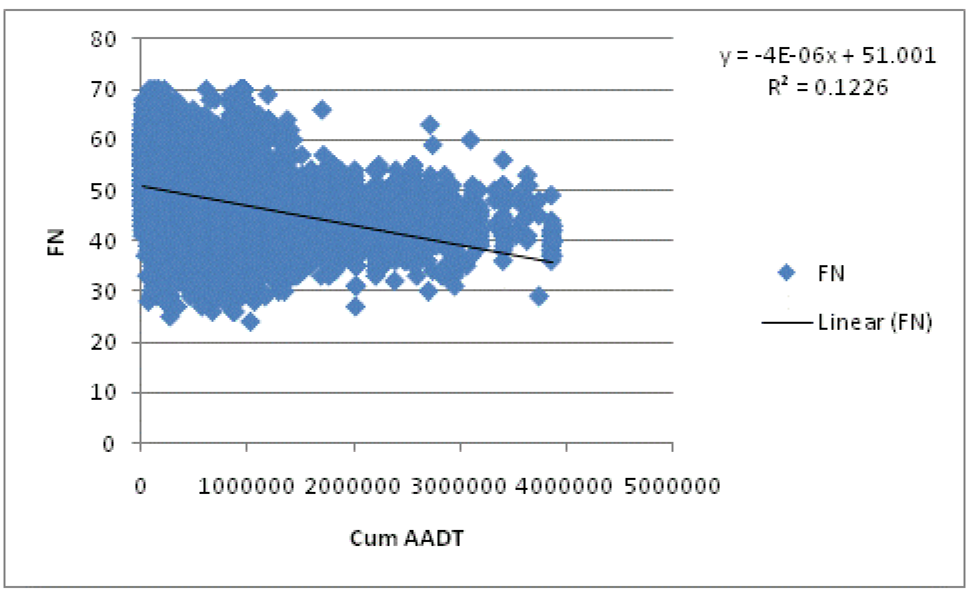

Figure 5-6. CumAADT vs FN for all Interstates – Statewide (n=10,828)

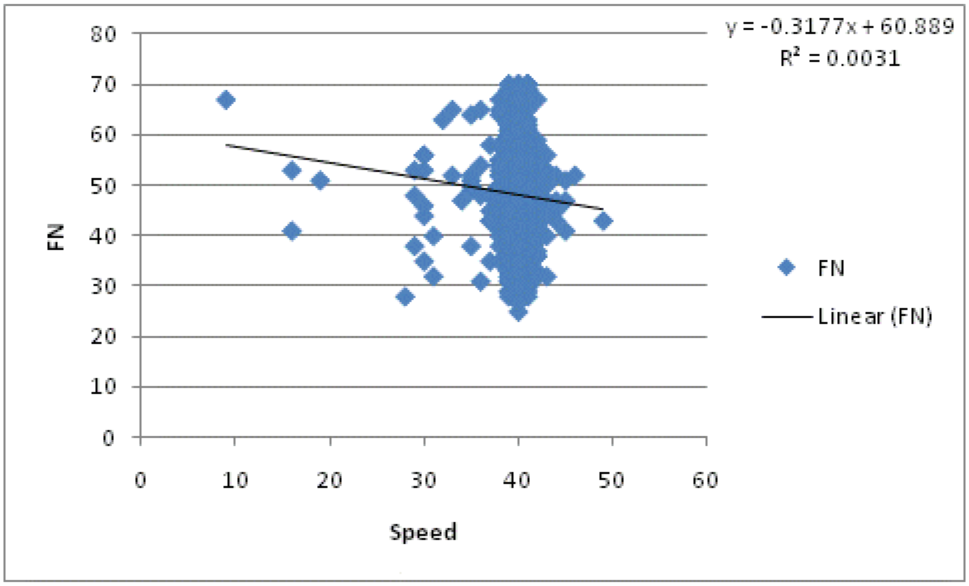

Figure 5-7. Speed vs FN for Interstates in Allegany, Anne Arundel, Baltimore, Calvert, and Charles Counties (n=3,602)

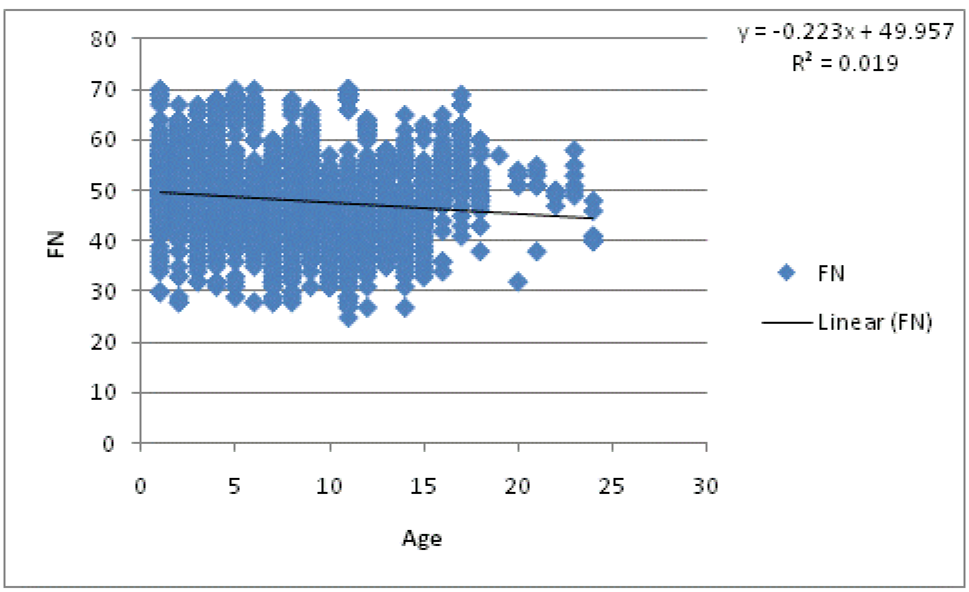

Figure 5-8. Age vs FN for Interstates in Allegany, Anne Arundel, Baltimore, Calvert, and Charles Counties (n=3,602)

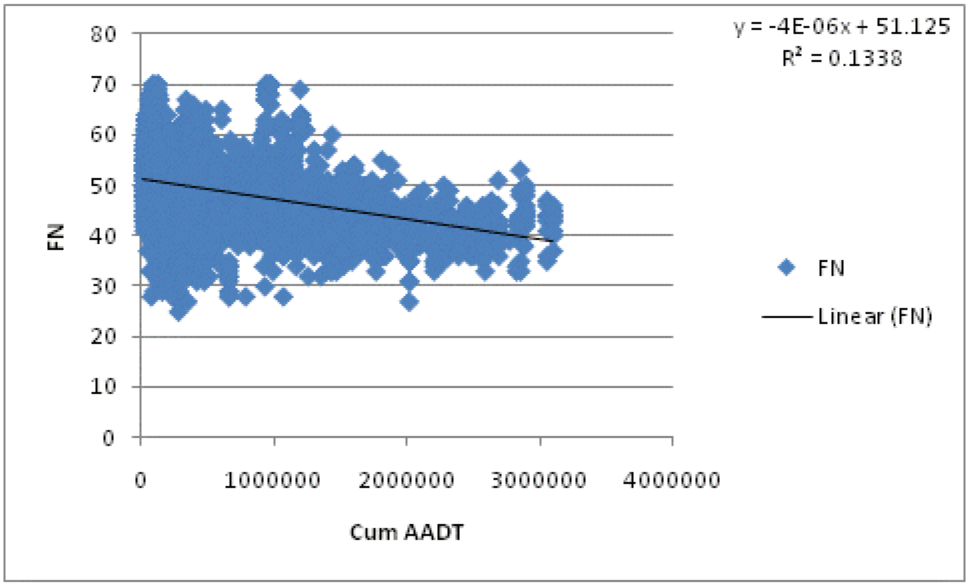

Figure 5-9. CumAADT vs FN for Interstates in Allegany, Anne Arundel, Baltimore, Calvert, and Charles Counties (n=3,602)

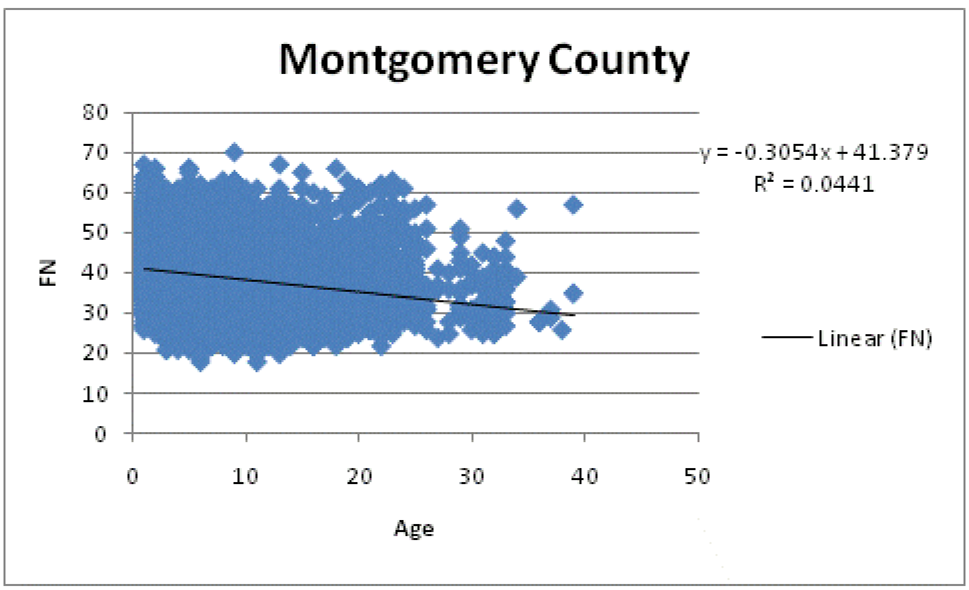

Figure 5-10. Years since Last Rehab vs. FN for MD and US routes in Montgomery County (n=7,904)

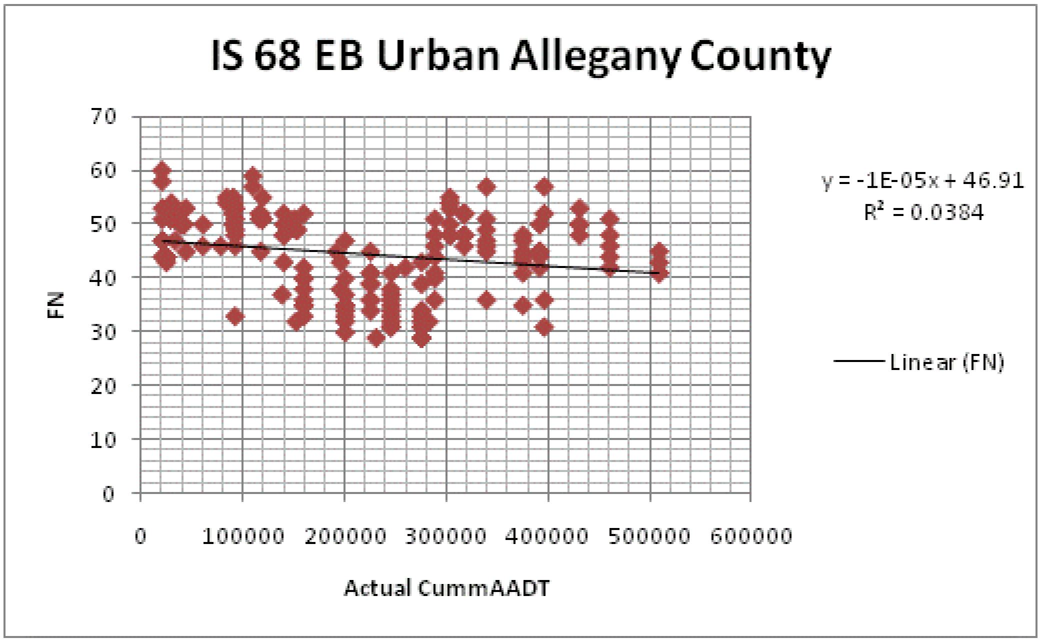

Figure 5-11. Actual CumAADT vs FN for IS 68 Eastbound (n=170)

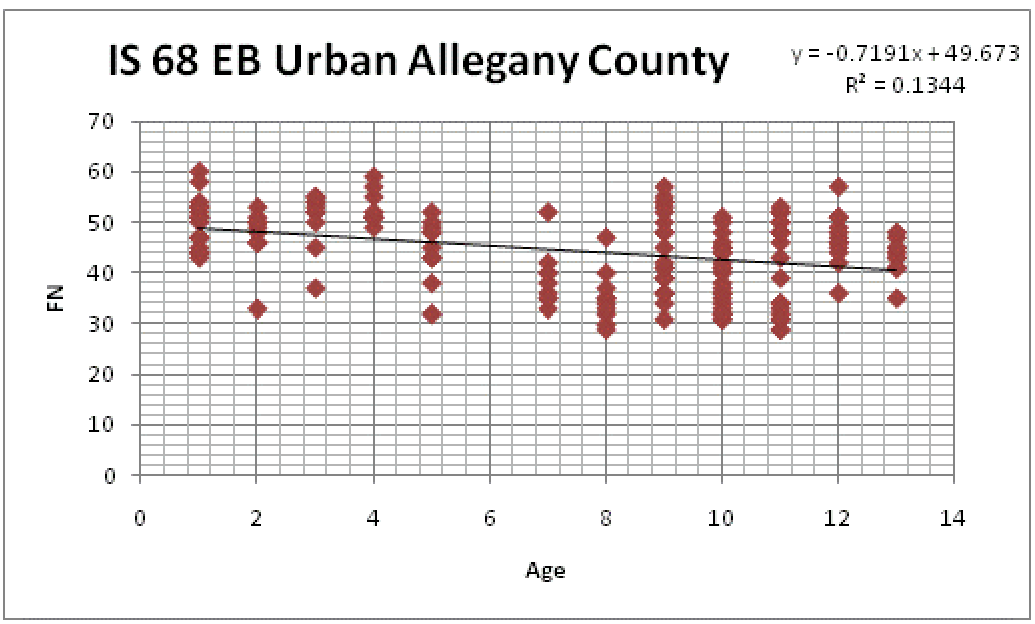

Figure 5-12. Years since Last Rehab vs FN for IS 68 Eastbound (n=170).

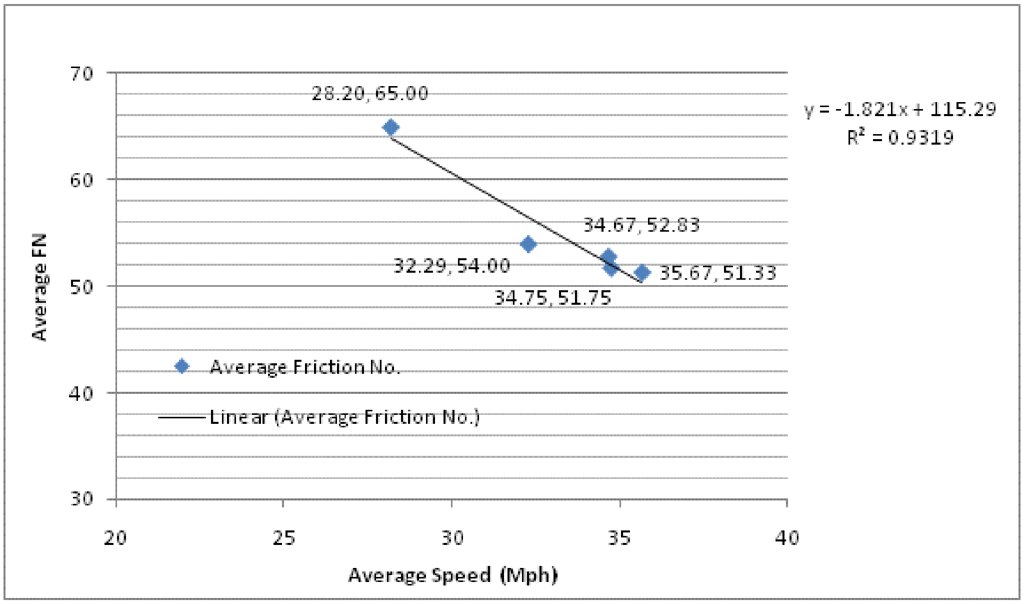

Figure 5-13. Survey Speed versus FN based on Average Values (Charles County)

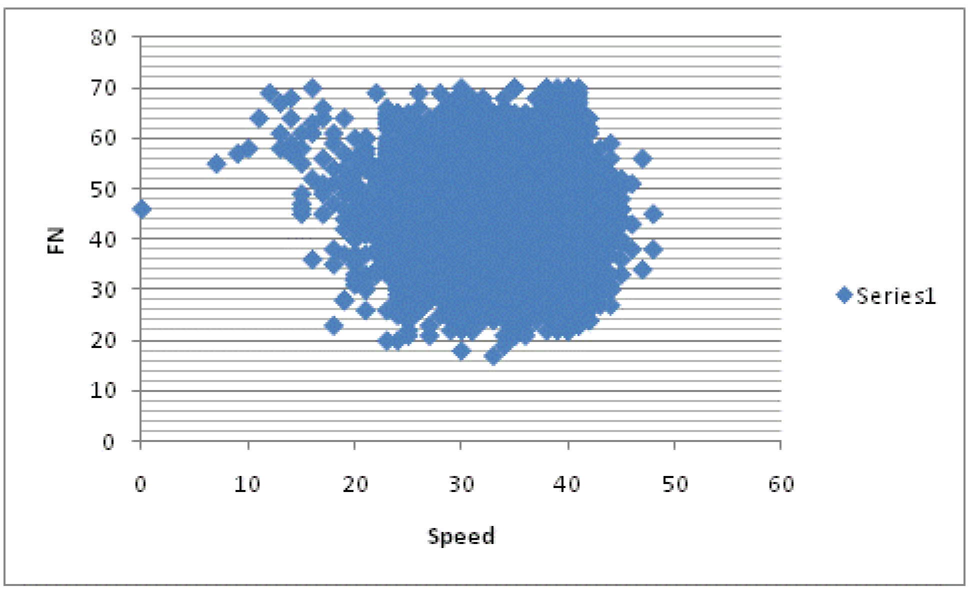

Figure 5-14. Survey Speed vs FN for HMA 12.5mm PG 70-22 in MD & US Routes

(n=22,338)

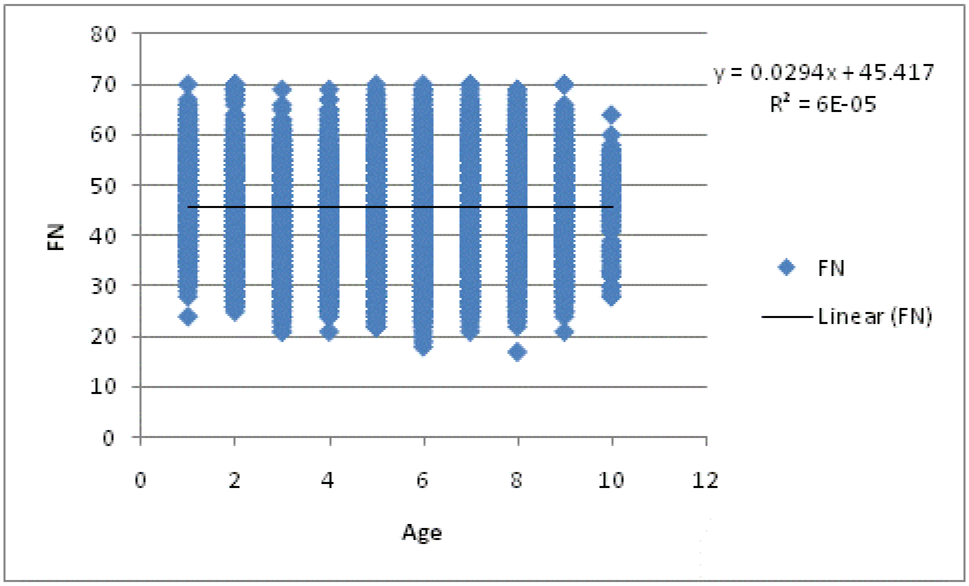

Figure 5-15. Age vs FN for HMA 12.5mm PG 70-22 in MD & US Routes (n=22,338)

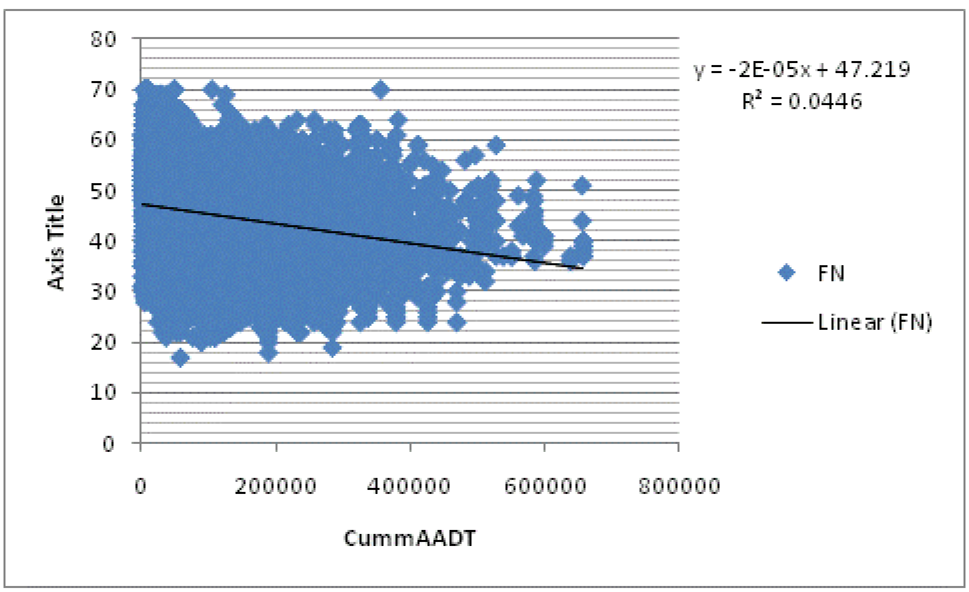

Figure 5-16. CumAADT vs FN for HMA 12.5mm PG 70-22 in MD & US Routes (n=22,338)

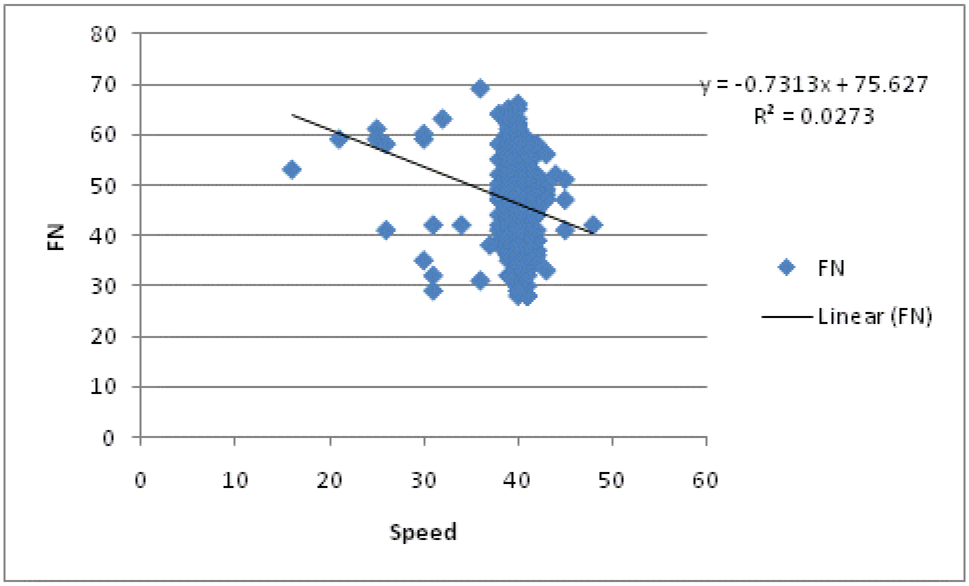

Figure 5-17. Survey Speed vs FN for HMA 12.5mm PG 70-22 in Interstates (n=2,031)

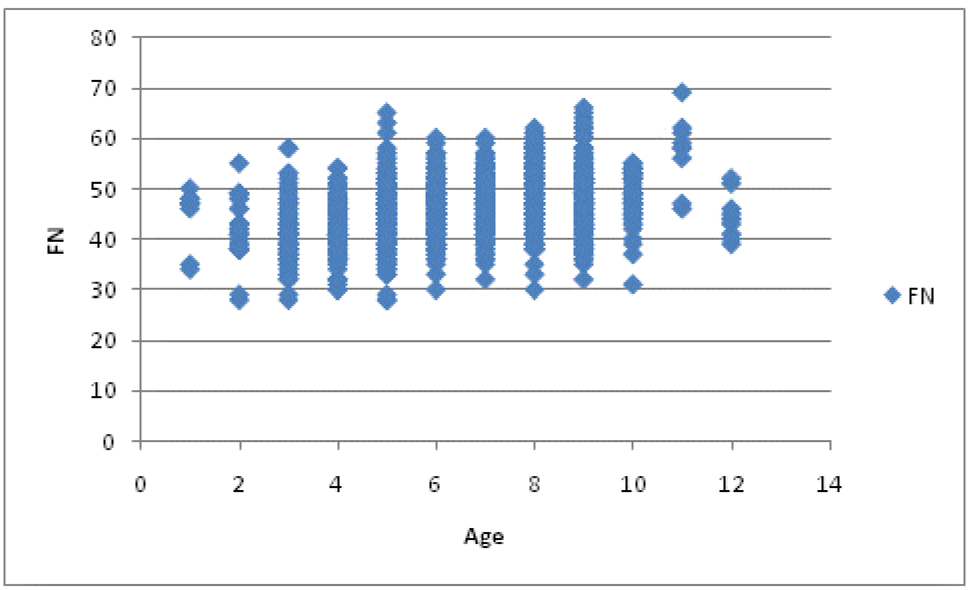

Figure 5-18. Age vs FN for HMA 12.5mm PG 70-22 in Interstates (n=2,031)

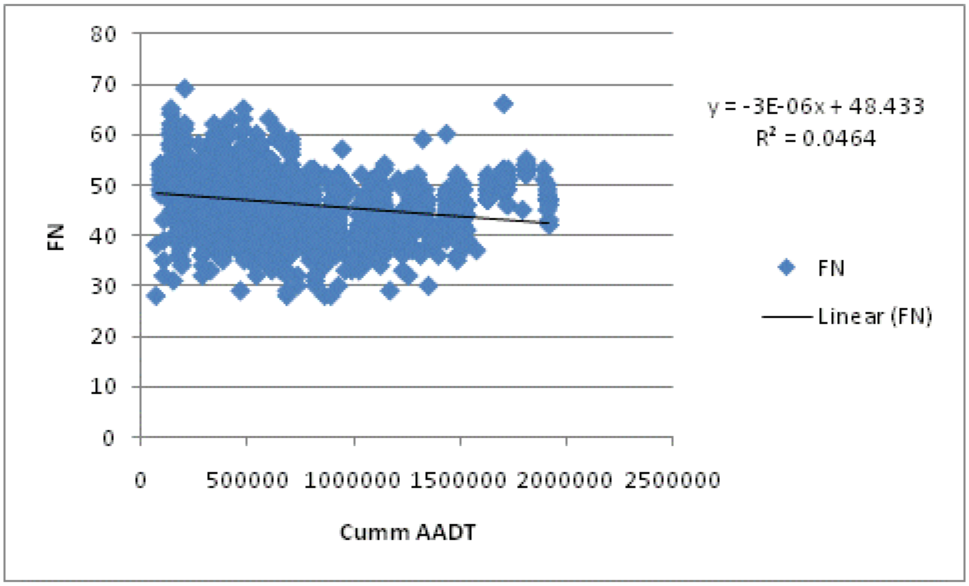

Figure 5-19. CumAADT vs FN for HMA 12.5mm PG 70-22 in Interstates (n=2,031)

Table 5-1. Selected Sections with Same AADT level and Contract Number/Mixture

(12.5mm PG70-22), and Variable Speed

YEAR	ROUTE	RNUM	Mile	DIR	SPEED	Pav. Age	Cum AADT	FN	AADT	Actual AADT	ACTION_ YEAR	CONTRACT
2004	MD	4	0.34	S	41	3	33675	50	10571	11225	2001	SM793B5D
2004	MD	4	0.65	S	41	3	33675	52	10571	11225	2001	SM793B5D
2004	MD	4	0.95	S	38	3	33675	54	10571	11225	2001	SM793B5D
2004	MD	4	1.24	S	42	3	33675	51	10571	11225	2001	SM793B5D
2004	MD	4	1.54	S	41	3	33675	52	10571	11225	2001	SM793B5D
2004	MD	4	1.84	S	40	3	33675	51	10571	11225	2001	SM793B5D
2004	MD	4	2.15	S	39	3	33675	52	10571	11225	2001	SM793B5D
2005	MD	4	0.19	S	40	4	42700	49	10571	10675	2001	SM793B5D

Table 5-1. Selected Sections with Same AADT level and Contract Number/Mixture (12.5mm PG70-22), and Variable Speed (Continued)

YEAR	ROUTE	RNUM	Mile	DIR	SPEED	Pav. Age	Cum AADT	FN	AADT	Actual AADT	ACTION_ YEAR	CONTRACT
2005	MD	4	0.57	S	40	4	42700	51	10571	10675	2001	SM793B5D
2005	MD	4	0.87	S	40	4	42700	49	10571	10675	2001	SM793B5D
2005	MD	4	1.17	S	41	4	42700	49	10571	10675	2001	SM793B5D
2005	MD	4	1.47	S	39	4	42700	50	10571	10675	2001	SM793B5D
2005	MD	4	1.77	S	40	4	42700	51	10571	10675	2001	SM793B5D
2005	MD	4	2.07	S	40	4	42700	51	10571	10675	2001	SM793B5D
2006	MD	4	1.16	S	39	5	52855	57	10571	10571	2001	SM793B5D
2006	MD	4	1.46	S	38	5	52855	58	10571	10571	2001	SM793B5D
2006	MD	4	1.76	S	39	5	52855	56	10571	10571	2001	SM793B5D
2006	MD	4	2.06	S	38	5	52855	55	10571	10571	2001	SM793B5D
2007	MD	4	0.20	S	43	6	62832	49	10571	10472	2001	SM793B5D
2007	MD	4	0.50	S	39	6	62832	52	10571	10472	2001	SM793B5D
2007	MD	4	0.80	S	41	6	62832	51	10571	10472	2001	SM793B5D
2007	MD	4	1.10	S	38	6	62832	56	10571	10472	2001	SM793B5D
2007	MD	4	1.40	S	41	6	62832	49	10571	10472	2001	SM793B5D
2007	MD	4	1.70	S	39	6	62832	48	10571	10472	2001	SM793B5D
2007	MD	4	2.00	S	39	6	62832	51	10571	10472	2001	SM793B5D
2007	MD	4	2.30	S	39	6	62832	49	10571	10472	2001	SM793B5D
2008	MD	4	0.30	S	40	7	75110	46	10571	10730	2001	SM793B5D
2008	MD	4	0.60	S	40	7	75110	43	10571	10730	2001	SM793B5D
2008	MD	4	0.90	S	41	7	75110	43	10571	10730	2001	SM793B5D
2008	MD	4	1.20	S	40	7	75110	42	10571	10730	2001	SM793B5D
2008	MD	4	1.50	S	40	7	75110	40	10571	10730	2001	SM793B5D
2008	MD	4	1.80	S	41	7	75110	46	10571	10730	2001	SM793B5D
2008	MD	4	2.10	S	40	7	75110	44	10571	10730	2001	SM793B5D

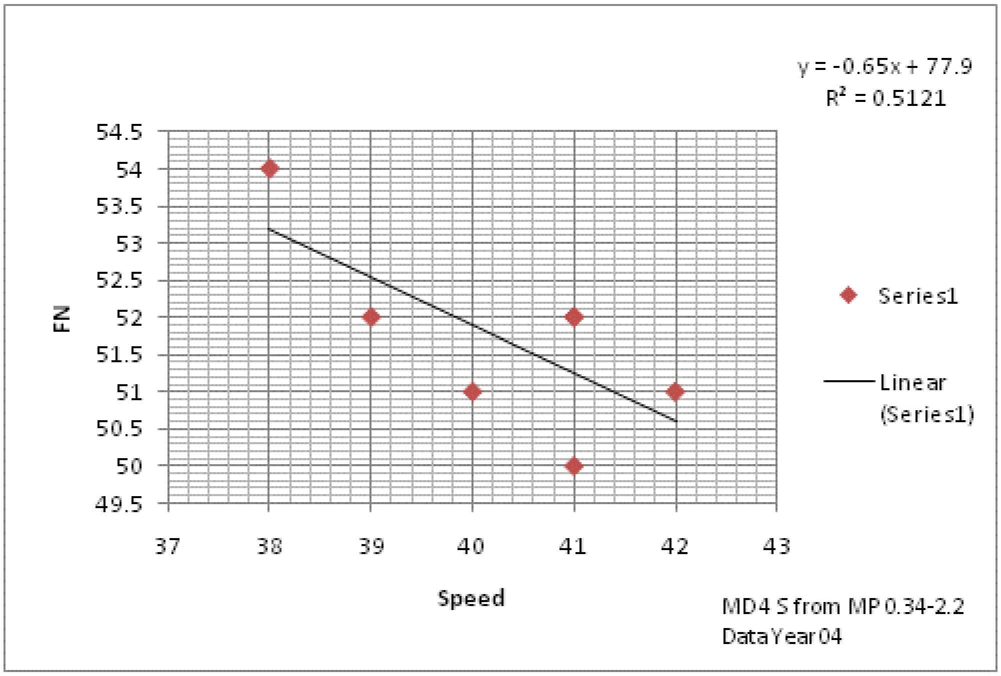

Figure 5-20. Speed vs FN for Sections with Same AADT level (~10,571) and Contract Number/ Mixture (12.5mm PG 70-22), at Variable Speed

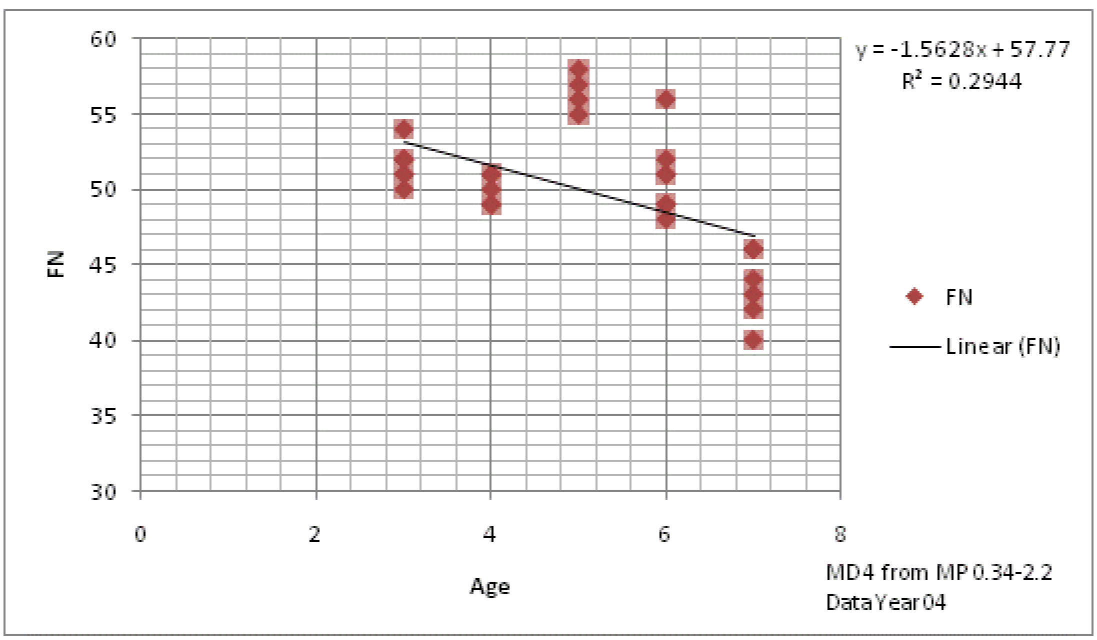

Figure 5-21. Age vs FN for Sections with Same AADT level (~10,571) and Contract Number/ Mixture (12.5mm PG 70-22), at Variable Speed

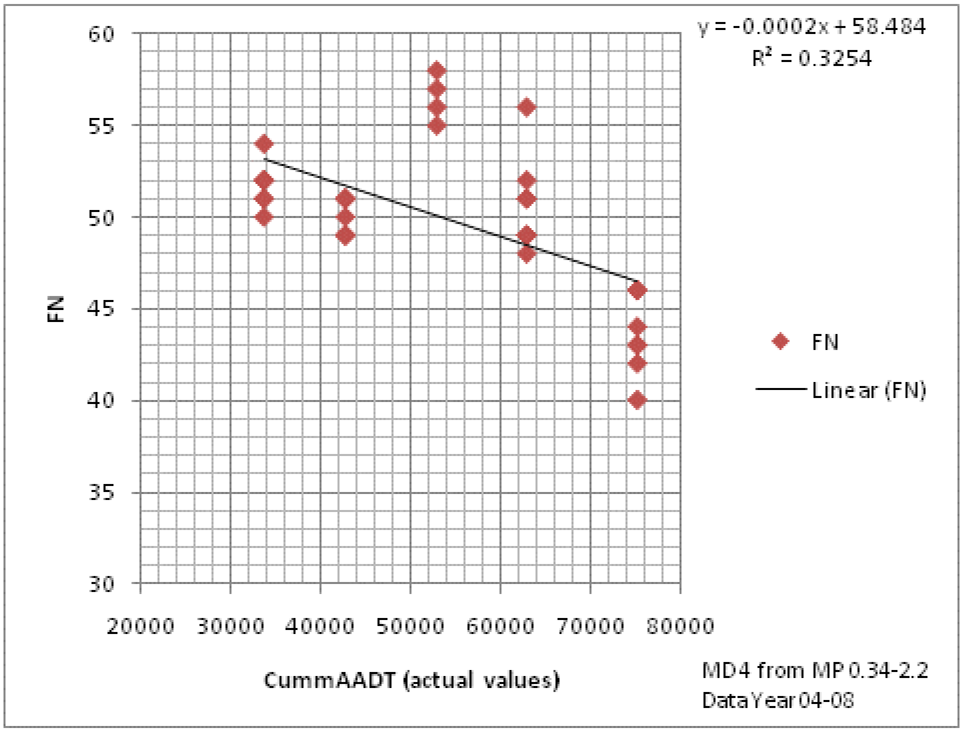

Figure 5-22. CumAADT vs FN for Sections with Same AADT level (~10,571) and Contract Number/ Mixture (12.5mm PG 70-22), at Variable Speed

Table 5-2. Multiple Linear Regression Analysis for Sections with Same AADT level and Contract Number/Mixture (12.5mm PG 70-22), at Variable Speed.

Coefficients

	AADT	**Age**	**Speed**	**b**
Coeff.	-0.00228	21.85137	-1.18182	109.6226
Std Errors	0.000568	5.85265	0.40014	15.81381
	0.671672	2.581737		
	19.77544	29		
	395.4316	193.2956		

Table 5-3. Multiple Linear Regression Analysis for Sections with Same AADT level and Contract Number/Mixture (12.5mm PG 70-22), at Variable Speed (Continued)

Model F Test Results

R^2	0.67
df	29
n	33
v1	1
v2	29
Fdist.	0.42
Fobs.	19.78

T- test Results

Variable	t-observed Value	Abs Value of t
Speed	-2.95	2.95
Age	3.734	3.73
AADT	-4.010	4.01
T-critical (at 95% confidence)		2.045229611

Table 5-4. Selected Sections with Same AADT level and Contract Number/Mixture

(12.5mm PG70-22), at Constant Speed of 40 mph.

YEAR	ROUTE	R NUM	Mile	DIR	SPEED	Age	Cum ADT	FN	AADT	ACTUAL AADT	ACT _YR	CONTRCT
2004	MD	5	14.87	S	40	5	32250	48	9000	6450	1999	SM793B53
2004	MD	5	15.17	S	40	5	32250	46	9000	6450	1999	SM793B53
2004	MD	5	15.47	S	40	5	32250	46	9000	6450	1999	SM793B53
2004	MD	5	15.77	S	40	5	32250	46	9000	6450	1999	SM793B53
2004	MD	5	16.67	S	40	5	42750	43	9000	8550	1999	SM793B53
2004	MD	5	16.97	S	40	5	42750	45	9000	8550	1999	SM793B53
2005	MD	5	14.86	S	40	6	40350	42	9000	6725	1999	SM793B53
2005	MD	5	15.17	S	40	6	40350	48	9000	6725	1999	SM793B53
2005	MD	5	16.06	S	40	6	40350	45	9000	6725	1999	SM793B53
2005	MD	5	16.36	S	40	6	53550	48	9000	8925	1999	SM793B53
2005	MD	5	16.67	S	40	6	53550	48	9000	8925	1999	SM793B53
2006	MD	5	15.69	S	40	7	49420	44	9000	7060	1999	SM793B53
2006	MD	5	16.59	S	40	7	63000	44	9000	9000	1999	SM793B53
2006	MD	5	16.89	S	40	7	63000	47	9000	9000	1999	SM793B53
2007	MD	5	14.81	S	40	8	55928	42	9000	6991	1999	SM793B53
2007	MD	5	15.11	S	40	8	55928	43	9000	6991	1999	SM793B53
2007	MD	5	16.31	S	40	8	71288	45	9000	8911	1999	SM793B53
2008	MD	5	15.48	S	40	9	59778	42	9000	6642	1999	SM793B53
2008	MD	5	15.78	S	40	9	59778	43	9000	6642	1999	SM793B53
2008	MD	5	16.38	S	40	9	76248	43	9000	8472	1999	SM793B53
2008	MD	5	16.68	S	40	9	76248	39	9000	8472	1999	SM793B53

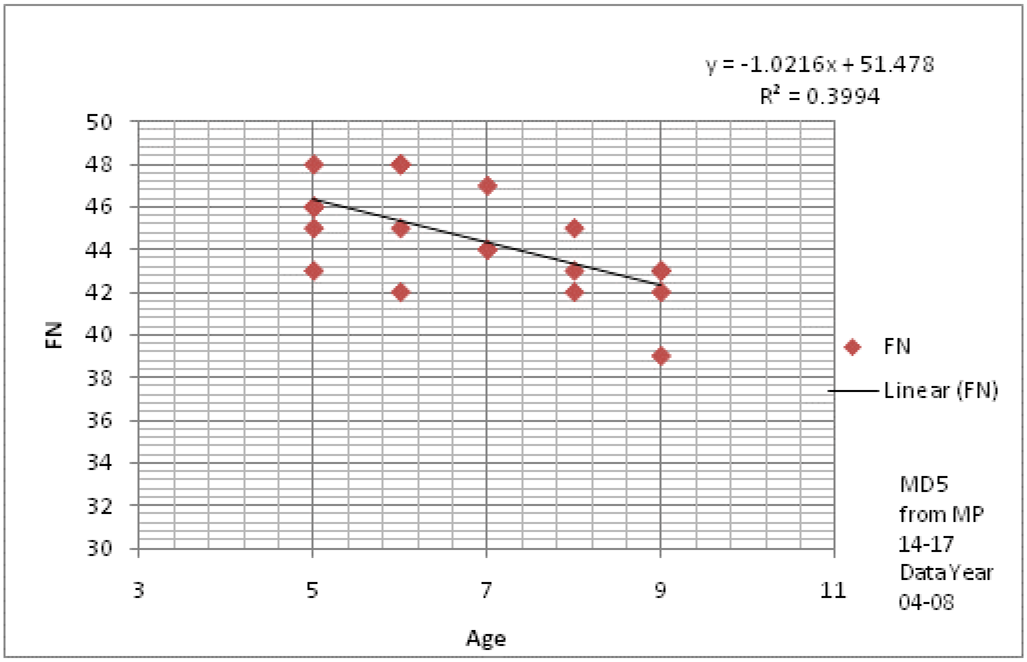

Figure 5-23. Pavement Age (Years Since last Rehab) vs FN for Sections with Same AADT level (~9000) & Contract Number/Mixture (12.5mm PG 70-22), at Constant Speed of 40mph.

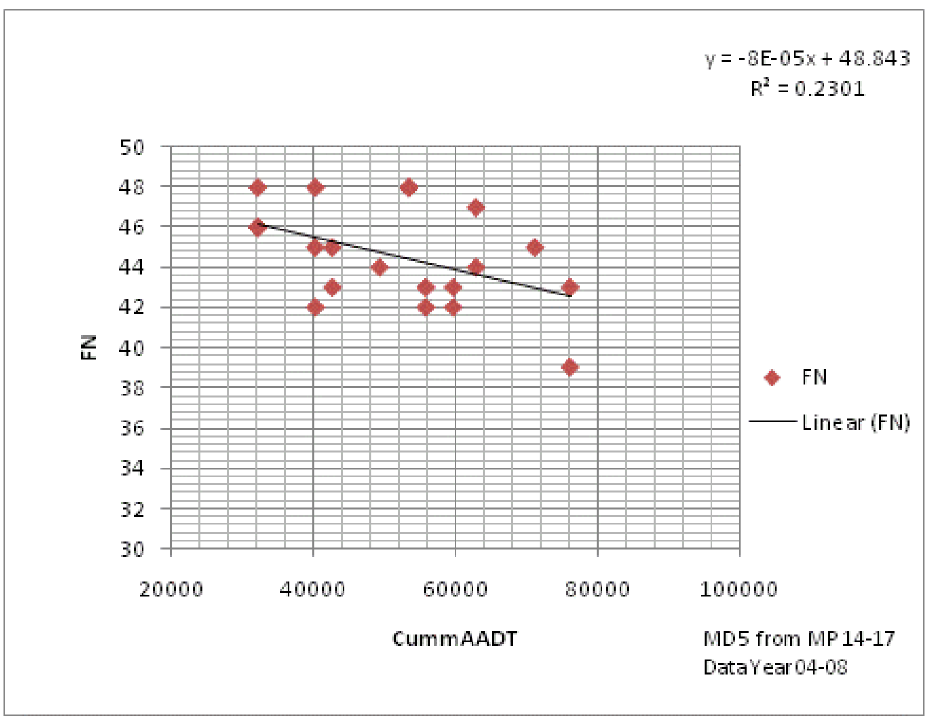

Figure 5-24. Cumulative AADT vs FN for Sections with Same AADT level (~9000) and Contract Number/Mixture (12.5mm PG 70-22), at Constant Speed of 40mph.

Chapter 6. Methodology for Predicting Pavement Friction Life & Relating Aggregates to Pavement Friction

Once the mixture/aggregate data and friction survey records from 2004-2008 were related using the contract/project IDs, the analysis were directed towards identifying a methodology for predicting pavement friction life using these five years of friction records for each pavement section, and then relate such friction life to the aggregates used in each mixture. The merging of the friction data and the mixture design database provided about 51,000 records consisting of friction and material data for the years 2004 through 2008. Projects constructed in 2004 represent cases where 4 to 5 years of historical friction data are available. Therefore, the records of these projects have been targeted as the first group to examine. As mentioned previously, the direction to follow for the analysis was to consider mixture specific data and with a significant number of records. Thus, the analysis focused first on the 12.5 mm, PG 64-22, HMA mixture that has a total of 11,131 friction records. Table 6-1 shows the contract numbers for the projects constructed with this mixture in 2004 and the aggregate sources (AS1, AS2 etc) used in the mixture. Similarly Table 6-2 and 6-3 show the records for the projects constructed in 2005 and 2006. As it can be seen from these tables, there are 385, 760 and 1,243 records in each one of these years where the project ID between friction data and mixture data matched. Furthermore it can be observed that, in many cases, different aggregate stockpiles/sources (AS1, AS2, etc…) were used for producing the desired aggregate gradation for the mix.

Table 6-1. Paving Projects Constructed with HMA 12.5mm, PG 64-22 in 2004 with Friction Records and Aggregate Sources.

Construction Year 2004 (385 records).

Contracts	Count
BA440B5B	77
CL821B5T	59
CL821B5Y	23
FR349B5V	74
HA250B55	22
HA250B57	111
HA250B5A	19
Grand Total	**385**

AS1	Count
Lafarge Frederick	74
Martin Marietta Woodsboro	159
Vulcan Materials Havre De Grace	152
Grand Total	**385**

As2	Count
Lafarge Frederick	74
Martin Marietta Woodsboro	159
York Building Products Belvedere Plant	152
Grand Total	**385**

AS3	Count
--------	74
Lafarge Medford	82
Martin Marietta Woodsboro	77
Vulcan Materials Havre De Grace	152
Grand Total	**385**

AS4	Count
-------	74
Arundel - Havre De Grace	152
Barricks - Woodsboro	77
LaFarge - Medford - Limestone	82
Grand Total	**385**

AS5	Count
------	385
Grand Total	**385**

AS6	Count
-------	385
Grand Total	**385**

AS7	Count
-------	156
Finksburg	77
ICM	152
Grand Total	**385**

Table 6-2. Paving Projects Constructed with HMA 12.5mm, PG 64-22 in 2005 with Friction Records and Aggregate Sources.

Construction Year 2005 (760 records).

Contracts	Count
AL877B5Q	19
AL877B5S	190
BA440B5K	150
CL821A5W	86
CL821B5Z	25
WA992B5Y	38
XX6215177	252
Grand Total	**760**

AS1	Count
------	440
Allegany Aggregates Short Gap	209
Martin Marietta Woodsboro	111
Grand Total	**760**

AS2	Count
--------	440
Allegany Aggregates Short Gap	209
Martin Marietta Woodsboro	111
Grand Total	**760**

AS3	Count
------	440
Keystone Lime Company, Inc. Springs	209
Lafarge Medford	111
Grand Total	**760**

AS4	Count
-----	649
LaFarge - Medford - Limestone	111
Grand Total	**760**

AS5	Count
------	760
Grand Total	**760**

AS6	Count
------	735
Miller	25
Grand Total	**760**

AS7	Count
------	760
Grand Total	**760**

Table 4. Paving Projects Constructed with HMA 12.5mm, PG 64-22 in 2006 with Friction Records and Aggregate Sources.

Construction Year 2006 (1243 records)

Contracts	Count
AL3195130	1
AL6155177	168
BA508A5X	118
BA508A5Z	24
BA508B5J	182
FR3735176	11
HA250B5S	69
HA250B5T	21
HA250B5W	95
HA250B5X	93
HA250B5Y	64
HA309B51	11
XX6015177	50
XX8015177	243
XX8135177	93
Grand Total	**1243**

AS1	Count
-------	196
Allegany Aggregates Short Gap	297
Lafarge Churchville	272
Lafarge Frederick	73
Lafarge Texas	181
Martin Marietta Woodsboro	118
Vulcan Materials Havre De Grace	106
Grand Total	**1243**

AS2	Count
-------	196
Allegany Aggregates Short Gap	297
Lafarge Churchville	272
Lafarge Frederick	254
Martin Marietta Woodsboro	118
York Building Products Belvedere Plant	106
Grand Total	**1243**

AS3	Count
---------	207
Allegany Aggregates Short Gap	81
Keystone Lime Company, Inc. Springs	216
Lafarge Frederick	62
Lafarge Texas	299
Vulcan Materials Havre De Grace	106
York Building Products Belvedere Plant	272
Grand Total	**1243**

Table 6-4. Paving Projects Constructed with HMA 12.5mm, PG 64-22 in 2006 with Friction Records and Aggregate Sources (continued).

Construction Year 2006 (1243 records)

As4	Count
------	685
Arundel - Havre De Grace	106
Barricks - Woodsboro	118
Kline	31
LaFarge - Frederick	31
York Build Prods - Belvedere	272
Grand Total	**1243**

AS5	Count
------	1125
LaFarge - Texas	118
Grand Total	**1243**

AS6	Count
------	1243
Grand Total	**1243**

AS7	Count
------	678
finksburg	118
ICM	106
LaFarge - Texas	181
MD Paving	160
Grand Total	**1243**

The 4-5 year friction records for each project were then examined to generate the data needed to study changes in FN (Friction Number) for a specific aggregate (or aggregate blend). In order to compare the friction number of a section year after year – taking into account increase in traffic - the milepost values were used. This was necessary since Annual Average Daily Traffic (AADT) may change at different mileposts. The friction readings were compared and contrasted by milepoint for the 4-5 year friction surveys which were collected on the pavement section under consideration.

Another consideration on grouping the data was related to the use of different friction equipment. Maryland SHA owns and operates two pavement friction survey equipment (designated as “Truck 5” and “Truck 6”) for collecting friction readings once a year throughout the state. In some cases Truck 5 was used for readings in some years, while in the remaining years Truck 6 was used and vice versa. The side by side repeatability analysis included in this dissertation (Chapter 4) indicated that, for flexible sections, Truck 5 shows on the average a lower value of FN, by 6.5 FN units. Thus, the FN data recorded using different equipment on the same section of roadway needed to be adjusted in order to account for equipment variability.

Furthermore, studies have shown that friction survey speed affects FN readings, i.e. survey speed is indirectly proportional to friction readings (Henry 2000; Goulias et. al. 2007). As a result, during the grouping of the data, Friction Number (FN) records that were collected at a speed of 38-41 mph were used so as to minimize the effect of variability due to survey speed. Finally, the grouped FN values were examined for potential outlier values. In this case the Chauvenet's criterion was used. In statistical terms, this requires to first calculate the mean and standard deviation of the observed data, then use the normal distribution function to determine the probability that a given data point is an outlier, and then multiply such probability by the number of data points considered. If that value is below 0.5, then the value may be flagged as an outlier (i.e., a data point may be rejected if the probability of obtaining the particular deviation from the mean is less than 1/(2n) where 'n' is the number of data points).

In summary, the procedure/methodology followed in the analyses includes the following steps:

STEP 1: Identify mixtures with the higher number of friction records and available aggregate information;

STEP 2: Merge friction records with mixture and aggregate data using Contract IDs;

STEP 3: Identify the construction year and group friction data for the following years using milepost information;

STEP 4: Update AADT for each milepost with the actual records from the Traffic Monitoring System web site;

STEP 5: Include the truck type (truck # 5 or #6) used in the friction surveys;

STEP 6: Run outlier analysis for subgroups of data representing uniform conditions;

STEP 7: Calculate the average FN values for subgroups of data representing uniform conditions;

STEP 8: Adjust FN values for considering the use of different friction equipment;

STEP 9: Use average FN values and AADT records to obtain the relationship between FN and traffic for a specific aggregate/ aggregate blend;

STEP 10: Use an interpolation/extrapolation function to calculate: i) the "friction drop rate" (FN drop/ 10k AADT) for each aggregate/aggregate blend, and ii) estimate "useful aggregate friction design life" (i.e., at what cumulative AADT a terminal FN of 32 is reached).

The terminal friction value (FN=32) chosen corresponds to the threshold value for the coefficient of friction (μ or f=0.32) as set by the Stopping Sight Distance criteria for a design speed of 40 mph in the AASHTO Geometric Design Guidelines (also known as the Green Book). Based on the results of a number of studies that measured the locked-wheel skid resistance on poor wet pavements, the 1994 Green Book calls for an f=0.32 for V=40 mph . This design value also corresponds to a comfortable deceleration rate of 6 to 8 mph/second, depending on initial speed. The 2001 AASHTO Green Book uses a ratio of the average deceleration rate to the acceleration due to gravity, g (32.2 ft/sec^2) to determine the coefficient of friction which yields comparable values.

In addition to the 10-step methodology for relating aggregate properties to pavement friction, the research was expanded to include the following:

1) Simple Regression models using

Raw data – all data, both directions combined

Combined data - Filtered for speed, adjusted for equipment;

Directional Data - grouped by year of survey;

2) Multiple Regression models

With adjusted data for friction equipment and considering the following Variables: CumAADT, Speed and FN;

With no adjustment for equipment and considering the following variables: CumAADT, Speed, Equipment and FN;

3) Considering data with friction survey speed of 40 mph and models relating CumAADT and FN;

4) Using data from combined contracts (all directions and speed ranges): Simple and Multiple Regression analysis as indicated above.

(The concept and methods of multivariate regression analysis are presented in detail in the next chapter.)

6.1. Example Analysis for a Specific Aggregate Source (Lafarge Frederick Quarry)

This section provides in summary an example of the analysis used for each pavement section/contract in the database using aggregate from a specific source, and for which sufficient friction data were available. The results of the analysis from this specific supplier were selected to be included herein since: i) the aggregate gradation was designed primarily with material from a single quarry (see table 6-4), and ii) there were sufficient number of friction records on which the analysis could be developed. The database provided records from two different contracts, FR349B5T on route MD 31, and MO4335177 on route MD 121, that met the above listed criteria. The following four approaches were used for analyzing the data and the outcome of the analysis are shown in table 6-5:

1. Analysis on UNFILTERED data (any speed, equipment, etc) combining N/S or E/W RAW DATA (Modeling and Graphs included)
2. Analysis on UNFILTERED data for each direction (Modeling and Graphs included)
3. Analysis on filtered (for speed) and adjusted (for equipment) data for both directions combined (Modeling and Graphs included)
4. Analysis on filtered (for speed) and adjusted (for equipment) data for each directions (Modeling and Graphs included)

As it can be seen from the models and analysis of Table 6.5 the friction records from a single contract and in the direction of MD 31E provided the model with the higher R^2. Overall it was observed that combining friction records from different route directions or different contracts increased the data variability, and thus reduced the coefficient of correlation for the model. Furthermore, the multiple regression models often provided lower R^2, and / or the model was reduced down to a simple linear regression form since most of the variables like survey speed, cumAADT and/or survey track equipment were statistically insignificant.

Table 6-5. Aggregate Supplier Data

FR349B5T on MD 31

Route	RNUM	Action Yr	AS1	AS1%	AS2	AS2%	Data Count
MD	31	2004	Lafarge Frederick	50	Lafarge Frederick	50	84

Supplier Source	Year Sampled	Carbonate?	Rock Analysis	Textural Description	Rock Category	BPN	PV	SG	LAA (% Loss)	Soundness (% Loss)
Lafarge Frederick	2004	Yes	No	Very-fine grained	Limestone	28	6	2.72	23	0.2

MO4335177 on MD 121

Route	RNUM	Action Yr	AS1	AS1%	AS2	AS2%	Data Count
MD	121	2006	Lafarge Frederick	50	Lafarge Frederick	50	52

Supplier Source	Year Sampled	Carbonate?	Rock Analysis	Textural Description	Rock Category	BPN	PV	SG	LAA (% Loss)	Soundness (% Loss)
Lafarge Frederick	2005	Yes	No	Medium Gray fine to Medium grained	Carbonate-Limestone	24	6	2.70	22	0.4

Table 6-6. Summary of Analysis for Friction Records related to a Specific Aggregate Source

(Lafarge Frederick Quarry)

Contract	Route	Analysis Type/ Data used for analysis	Model Type (Viable)	Equation	R^2	N	Terminal CumAADT at FN=32
FR349B5T	MD 31 (E+W)	Combined Directional Data (Filtered for Speed and Adjusted for Equipment)	SLR	FN = -0.0006 (CumAADT) + 52.66	0.41	78	34,000
		All Combined Directional Data (Un-Filtered and Un-adjusted)	MER (CumAADT, Speed and Equipment)	FN= 22.03* ((0.9999^CumAADT)*(1.001^Speed)*(1.186^DumTrk))	0.5	44	6,000(Using Truck 5) 8,000 (Using Truck 6)
	MD 31E	Directional Data (Filtered for Speed and Adjusted for Equipment)-Averages	SLR	FN = -0.0006 (CumAADT) + 52.473	0.76	40	34,200
		All Directional Data (Un-Filtered and Un-adjusted)	MLR (CumAADT and Equipment)	FN= -0.00089 (CumAADT) + 9.237 (DumTrk) + 7.75	0.49	44	24,000(Using Truck 5) 36,000 (Using Truck 6)
			MER CumAADT, Speed &Equipment)	FN= 26.102 ((0.9999^CumAADT)*(0.996^Speed)*(1.195^DumTrk))	0.52	44	6,000 (Truck 5) 8,000 (Truck 6)

Table 6-6. Summary of Analysis for Friction Records related to a Specific Aggregate Source (Lafarge Frederick Quarry) (Continued)

Contract	Route	Analysis Type/ Data used for analysis	Model Type (Viable)	Equation	R^2	N	Terminal CumAADT at FN=32
FR349B5T	MD 31W	Directional Data (Filtered for Speed and Adjusted for Equipment)-All data	SLR	FN = -0.0003 (CumAADT) + 52.907	0.08	38	81,000
		Directional Data (Filtered for Speed and Adjusted for Equipment)-Averages	SLR	FN = -0.0006 (CumAADT) + 52.966	0.73	38	35,000
		All Directional Data (Adjusted for Equipment)	MLR (CumAADT and Speed)	FN= -0.000568 (CumAADT) + 0.622 (Speed) + 27.61	0.52	40	36,000
		All Directional Data	MER (CumAADT, Speed and Equipment)	FN= 14.5*(0.9999^CumAADT)*(1.0126^Speed)*(1.1742^DumTrk))	0.45	40	5,000 (Truck. 5) 7,000 (Truck. 6)
MO4335177	MD 121 (N+S)	All Combined Directional Data (Un-Filtered and Un-adjusted)	MER(CumAADT, Speed and Equipment)	FN= 231.75*(0.9999^CumAADT) * (0.9962^Speed)* (0.8222^Dum-Trk)	0.29	52	6,500 (Truck 5)
	MD 121 N	All Directional Data (Un-Filtered and Un-adjusted)	SLR	FN= -0.0002 (CumAADT) + 55.23	0.006	26	13,000
		All Directional Data (Un-Filtered and Un-adjusted)	MER (CumAADT, Speed and Equipment)	FN= 1052.64*(0.9999^CumAADT)*(0.9920^Speed)*(0.6823^DumTrk)	0.56	26	12,000(Truck 5) 16,000(Truck 6)

Table 6-6. Summary of Analysis for Friction Records related to a Specific Aggregate Source (Lafarge Frederick Quarry) (Continued)

Contract	Route	Analysis Type/ Data used for analysis	Model Type (Viable)	Equation	R^2	N	Terminal CumAADT at FN=32
FR349B5T and MO4335177	MD 31 (E+W) and MD 121 (N+S)	All Combined/merged Data (Un-Filtered and Un-adjusted)	SLR	FN= -1E-04 (CumAADT) + 53.871	0.004	136	184,000
		Combined/merged Data (Filtered and adjusted)	SLR	FN = -0.0004 (CumAADT) + 53.328	0.06	128	48,000
		All Combined/merged Data (Adjusted for Equipment)	MER (CumAADT and Speed)	FN= 67.559* (0.9999^CumAADT)* (0.9940^Speed)	0.11	136	5,000
		All Combined/merged Data	MER (CumAADT, Speed and Equipment)	FN= 69.560* (0.9999^CumAADT)* (0.9921^Speed)*(1.0094^DumTrk)	0.08	136	5,000 (Truck 5 and 6)

Note:

SLR= Simple Linear Regression

MER = Multiple Exponential Regression

SER= Simple Exponential Regression

CumAADT= Cumulative Annual Average Daily Traffic

MLR= Multiple Linear Regression

DumTrk = Dummy Variable used for Equipment (Dumtrk=5 for truck 5; Dumtrk=6 for truck 6)

6.2. Analysis for Relating Friction to Pavement Traffic in terms of Cumulative AADT and ESAL.

The analyses outlined in the previous sections were conducted on all projects with valid and sufficient mixture and aggregate source data. Table 6-6 identifies the list of quarries/ suppliers considered in the study.

Table 6-7. Aggregate Quarries Considered in the Analysis

AIR	Aggregate Industries Rockville
AASG	Allegany Aggregates Short gap
KLC	Keystone Lime Company Inc. Springs
LCH	Lafarge Churchville
LF	Lafarge Frederick
LW	Lafarge Warfordsburg
MMI	Maryland Materials Incorporated
MMW	Martin Marietta Woodsboro
VMH	Vulcan Materials Hanover
VMHDG	Vulcan Materials Havre De Grace
VMW	Vulcan Materials Warrenton
YBPBv	York Building Products Belvedere

While the merged SHA friction records and mixture material database provided records for the 12 quarries shown in Table 6-6, aggregate petrographic/polishing properties for only a subset of these were available. Furthermore, for one quarry only limited FN/ milepost records were available (AASG), while for another (KLC) the aggregate properties were significantly different than the rest of the aggregates.

The result of the regression models between friction life and traffic are shown in Table 6-7. As can be seen from this table, the simple linear regression analysis provided the best relationships between Friction Number (FN) and CumAADT (Cumulative Annual Average Daily Traffic). For the multiple regression analysis relating FN to CumAADT, speed and equipment type, either the models had a lower R^2 or the variables turn out to be insignificant. The details of the best models are shown in Table 6-7. Based on these models and scatter plots, the CumAADT corresponding to a terminal FN of 32 (μ or f=0.32) were calculated and reported. Furthermore the CumAADT over the average AADT throughout the years was used to calculate the expected friction life in years. The Friction Number drop per 10,000 CumAADT value (FN drop/10kCumAADT) is also reported in this table. Examples of the relationships between FN and CumAADT are shown in Figures 6-1 to 6-4. As shown in Table 6-7 and in Figures 6-1 to 6-4, the models obtained from the 2004 to 2008 friction data were used to estimate the friction pavement life for each case, in terms of years (i.e., CumAADT over the average AADT throughout the years) and terminal cumulative AADT at a final value of FN 32. This FN value represents the minimum acceptable design value used by SHA and many other states. Furthermore the drop in FN for every 10,000 CumAADT is also reported.

From the comparison of the CumAADT at a terminal FN value of 32, it can be observed that there is a big difference in the order of magnitude of these values. This reflects the different traffic mix characteristics that each roadway experiences during its service life. Since the AADT does not reflect the diverse truck loading conditions on each roadway, there was a need to convert AADT to Equivalent Standard Axle Load (ESAL), considering the truck distribution factors on the projects and the mileposts considered in the analysis.

The AADT conversion into Equivalent Standard Axle Loading (ESAL) can be achieved by either: i) directly converting the Cumulative AADT obtained at the FN 32 value for each case, or ii) by converting AADT data to ESAL at each milepoint. In either case, the AADT data and truck percentage factor obtained from the traffic monitoring web site of SHA were used to calculate ESAL using the equivalency load factors analysis. These two methods were used in a couple of projects for assessing whether there is a difference in the approach used. Table 6-8 and 6-9 and Figures 6-5 to 6-8- present the results from these analysis for a couple of cases (AIR and AASG). As it can be seen in these results, whether the AADT to ESAL conversion is performed at the milepost level or on the CumAADT values, the calculated values are similar. Thus the latter method was used for converting AADT to ESAL for all cases. The details of these calculations and analysis are also included in Table 6-7.

Table 6-8. Regression Analysis Relating Friction Life to CumAADT and Aggregate Properties

(AIR, AASG, KLC, LCH, LF)

Material Source	AIR	AASG	KLC	LCH	LF
Aggregate type (If known)	Serpentine	Carbonate Dolomitic Limestone (2005 Petrography)	Carbonate-Siliceous limestone(2005 Petrography)	Hornlende Gnesiss (2005 Petrography)	Limestone
Supplier	Aggregate industries	Allegany Aggregates	Keystone Lime Company, Inc.	Lafarge	Lafarge
Quarry	Aggregate Industries Rockville	Allegany Aggregates Short Gap	Keystone Lime Company, Inc. Springs	Lafarge Churchville	Lafarge Frederick
Contract No	MO3285177	AL6165177	GA6455177	BA508B5J	FR349B5T
BPN/PV	22/5 (2004)	26/5 (2005)	34/10 (2005)	22/6 (2005)	24/6 (2005)
LAA/ Soundness	18% /4.5% (2004 tests)	15% / 2.8% (2005 tests)	18% / 1% (2005 tests)	22/0.4 (2005 tests)	22% / 0.2% (2005 tests)
Carbonate (yes/No/N/A)	N/A	Yes	N/A	No	Yes
Mix Type	HMA 12.5 70-22 8 PV	HMA 12.5mm, 64-22, Surface, L 4	HMA 12.5mm, 70-22, Surface, L 3	HMA 12.5mm, 64-22, Surface, L 2	HMA 12.5mm, 64-22, Surface, L 2
Supplier 1/% Composition	AIR/75%	AASG/100%	KLC/100%	LCH/65%	LF/100%
Supplier 2/% Composition	Plant 128 Stockpile/25%	N/A	N/A	YBPBv/25%; MD Pavng/10%	N/A
County	Montgomery	Allegany	Garrett	Baltimore	Frederick
Route	MD 190 (E+W)	US 220 (N+S)	US 219 (N+S)	MD 43 (E+W)	MD31(E+W)
MP	0-6.5	3.3-6.6	33.2-37.2	0-3.5	0-3.2
Action Year	2004	2006	2005	2006	2005
No of Lanes	2	2	3	4	2
Direction used in Analysis (Resulted in better models)	MD 190E	US 220S	US219S	MD 43E	MD 31E

Table 6-8. Regression Analysis Relating Friction Life to CumAADT and Aggregate Properties (Continued)

(AIR, AASG, KLC, LCH, LF)

Material Source	AIR	AASG	KLC	LCH	LF
AADT (Averaged over Mile points and over survey years)	3,573	7,201	4,762	36,320	3,435
Truck Percentage(2005-7 Data)					
Single	11.2	7.7	8.2	3.4	8.5
Combination	2.6	4	4.6	0.8	3
Passenger/Other	86.2	88.3	87.2	95.8	88.5
Truck Percentage(2008 Data)					
Single	10.6	7.6	8.2	3.4	9.6
Combination	2.1	2.2	4.6	0.8	3
Passenger/Other	87.3	90.2	87.2	95.8	87.4
Average Percentages					
Single	10.9	7.65	8.2	3.4	9.05
Combination	2.35	3.1	4.6	0.8	3
Passenger/Other	86.75	89.25	87.2	95.8	87.95
Load Equivalency Factors, LEF (SN=5, Pt=2.5)					
Single	**1.857**	**1.857**	**1.857**	**1.857**	**1.857**
Combination	**2.714**	**2.714**	**2.714**	**2.714**	**2.714**
Passenger/Other	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**
Directional Distribution Factor	0.5	0.5	0.5	0.5	0.5
Lane Distribution Factor	1	1	0.7	0.7	1
Terminal CumAADT [CumAADT where FN=32]	**66,000**	**57,200**	**30,000**	**195,000**	**40,000**

Table 6-8. Regression Analysis Relating Friction Life to CumAADT and Aggregate Properties (Continued)

(AIR, AASG, KLC, LCH, LF)

Material Source	AIR	AASG	KLC	LCH	LF
ESAL= Terminal CumAADT*T*Df*Lf*LEF*365					
Single	**2,438,065**	**1,482,970**	**583,590**	**1,572,847**	**1,048,937**
Combination	**768,218**	**878,275**	**478,465**	**540,873**	**508,183**
Passenger/Other	**2,090**	**1,863**	**668**	**4,773**	**1,098**
Terminal ESAL	**3,208,372**	**2,363,108**	**1,062,723**	**2,118,493**	**1,558,218**
Most Significant model/[Equation]	SLR/[FN = -0.0002* CumAADT + 47.75]	SLR/[FN = -0.0003*CumAADT + 49.157]	SLR/[FN= -0.0013*CumAADT + 71.011]	SLR/[FN= -1E-04*CumAADT + 51.64]	SLR/[FN= -0.0005* CumAADT+ + 52.165]
R2/n	0.17/85	0.12/20	0.65/39	0.72/18	0.75/40
FN Drop/10k AADT (in FN units)	-2	-3	-13	-1	-5
Other Models/ [R2/n/Terminal CumAADT based on SA]	MLR/[0.11/174/ 5,500]	MER/[0.78/20/ 20,000]	MER/[0.88/42 /14,000]	MER/[0.42/21/ 170,000]	MER/[0.52/44/ 14,000]
Expected Life in Years (Based on Terminal CumAADT)= CumAADT/Average AADT	*18.47*	*7.94*	*6.30*	*5.37*	*9.96*

Note:
SLR= Simple Linear Regression
SER = Simple Exponential Regression
MLR= Multiple Linear Regression
MER= Multiple Exponential Regression
SA= Sensitivity Analysis

Table 6-8. Regression Analysis Relating Friction Life to CumAADT and Aggregate Properties (continued)

(LW, MMI, MMW, VMH, VMHDG, VMW, YBPBv)

Material Source	LW	MMI	MMW	VMH	VMHDG	VMW	YBPBv
Aggregate type (If known)	Limestone			Limestone		Diabase (2004)	
Supplier	Lafarge	Maryland Material Inc	Martin Marietta	Vulcan Materials	Vulcan Materials	Vulcan Materials	York Building Products
Quarry	Lafarge Warfordsburg	Maryland Material Inc NE	Martin Marietta Woodsboro	Vulcan Materials Hanover	Vulcan Materials Havre De Grace	Vulcan Materials Warrenton	York Building Products Belvedere Plant
Contract No	WA1005177	CE785A5N	BA440B5B	AA3285177	WO750B5O	MO9005171	CE785B5H
BPN/PV	35/6(2008)	32/HPV(2009)		21/4 (2005)	31/HPV (2008)	26/- (2005)	
LAA/ Soundness	20%/0.6% (2008 tests)	21%/0.9%(2009 tests)		25% / 0/7% (2005 tests)	14%/0.1% (2008 tests)	11/0.3 (2005 tests)	
Carbonate (yes/No/N/A)	N/A	N/A	N/A	Yes	N/A	N/A	N/A
Mix Type	HMA 12.5mm, 64-22, Surface, L 4	HMA 9.5mm, 64-22, Surface, L 2	HMA 12.5mm, 64-22, Surface, L 2	HMA 9.5mm, 70-22, Surface, L 3	HMA 9.5mm, 70-22, Surface, L 3	HMA 12.5mm, 76-22, Surface, 8 PV, L 4	HMA 9.5mm, 70-22, Surface, L 2
Supplier 1/% Composition	LW/100%	MMI/78%	MMW/75%	VMH/85%	VMHDG/68%	VMW/75%	YBPBv=72%
Supplier 2/% Composition	N/A	YBPBv/7&; Edgemoor/15%	BW/15% Finksburg/10%	Flanigan/15%	JML GT/32%	AGI-S/10%; ½ RAP=15%	SDM&S EM=18%; ICM=10%
County	Washington	Cecil	Baltimore	Anne Arundel	Worcester	Montgomery	Cecil
Route	US 40 (E+W)	MD342 (N+S)	MD 30 (N+S)	MD100W	US 113 (E+W)	MD 650 (N+S)	MD 276 (N+W)

Table 6-8. Regression Analysis Relating Friction Life to CumAADT and Aggregate Properties (continued)

(LW, MMI, MMW, VMH, VMHDG, VMW, YBPBv)

Material Source	**LW**	**MMI**	**MMW**	**VMH**	**VMHDG**	**VMW**	**YBPBv**
MP	28-32	0-2.5	5-7.5	11.0-15.0	26 - 30	3.6 -- 5.3	3.5-6.5
Action Year	2005	2006	2004	2005	2005	2005	2004
No of Lanes	2	2	2	**4 (in one direction-WB)**	4	6	
Direction used in Analysis (Resulted in better models)	US 40W	MD 342N	MD 30S	MD 100W	US 113S	MD 650 N	MD 276 N
AADT (Averaged over Mile points and over survey years)	4,390	498	8,780	60,260	11,490	51,320	8,520
Truck Percentage(2005-7 Data)							
Single	6.4	0	7.5	2.2	9.8	2.9	8.3
Combination	1.9	0	4.1	0.5	8	1.5	5.7
Passenger/Other	91.7	100	88.4	97.3	82.2	95.6	86
Truck Percentage(2008 Data)							
Single	6.4	0	7.5	2.2	9.8	2.9	8.3
Combination	1.9	0	4.1	0.5	8	1.5	5.7
Passenger/Other	91.7	100	88.4	97.3	82.2	95.6	86
Average Percentages							
Single	6.4	0	7.5	2.2	9.8	2.9	8.3
Combination	1.9	0	4.1	0.5	8	1.5	5.7
Passenger/Other	91.7	100	88.4	97.3	82.2	95.6	86
Load Equivalency Factors, LEF (SN=5, Pt=2.5)							

Table 6-8. Regression Analysis Relating Friction Life to CumAADT and Aggregate Properties (continued)

(LW, MMI, MMW, VMH, VMHDG, VMW, YBPBv)

Material Source	LW	MMI	MMW	VMH	VMHDG	VMW	YBPBv
Single	**1.857**	**1.857**	**1.857**	**1.857**	**1.857**	**1.857**	**1.857**
Combination	**2.714**	**2.714**	**2.714**	**2.714**	**2.714**	**2.714**	**2.714**
Passenger/Other	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**	**0.0002**
Directional Distribution Factor	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Lane Distribution Factor	**1**	**1**	**1**	**0.7**	**0.7**	**0.7**	**1**
Terminal CumAADT [CumAADT where FN=32]	**14,100**	**4,500**	**76,000**	**510,000**	**72,000**	**480,000**	**54,000**
ESAL= Terminal CumAADT*T*Df*Lf*LEF*365							
Single	**305,826**	**0**	**1,931,744**	**2,661,740**	**1,673,907**	**3,302,266**	**1,518,961**
Combination	**132,692**	**0**	**1,543,370**	**884,119**	**1,997,070**	**2,496,337**	**1,524,549**
Passenger/Other	**472**	**164**	**2,452**	**12,679**	**1,512**	**11,724**	**1,695**
Terminal ESAL	**438,990**	**164**	**3,477,567**	**3,558,538**	**3,672,489**	**5,810,328**	**3,045,205**
Most Significant model/[Equation]	SLR/[FN = -0.0018* CumAADT + 57.363]	SLR/[FN = -0.0085* CumAADT + 66.25]	SLR/[FN = -0.0085*CumAADT + 66.25]	SLR/[FN= -2E-05*CumAADT + 40.042]	SLR/[FN=-0.0003* CumAADT+ 53.192]	SLR/[FN= -3E-05* CumAADT + 47.497]	SLR/[FN=-0.0004* CumAADT + 56.283]

Table 6-8. Regression Analysis Relating Friction Life to CumAADT and Aggregate Properties (continued)

(LW, MMI, MMW, VMH, VMHDG, VMW, YBPBv)

Material Source	LW	MMI	MMW	VMH	VMHDG	VMW	YBPBv
R2/n	**0.97/28**	**0.58/11**	**0.56/27**	**0.14/34**	**0.38/23**	**0.29/12**	**0.92/36**
FN Drop/10k AADT (in FN units)	**-18**	**-8.5**	**-2**	**-0.5**	**-0.3**	**-0.3**	**-5**
Other Models/ [R2/n/Terminal CumAADT based on SA]	**MER/[0.54/30/ 12,000]**	**MER/[0.69/21/ 4000]**	**MER/[0.09/31/ 14000]**	**MER/[0.2/39/ 28,000]**	**MER/[0.27/27/ 6000]**	**MER/[0.25/20/ 4000]**	**MLR/[0.8 1/37/ 56000]**
Expected Life in Years (Based on Terminal CumAADT)=CumAADT/Average AADT	*3.21*	*9.04*	*8.66*	*8.46*	*6.27*	*9.35*	*6.34*
Remark	Note: Material from both LW and LF was used in this contract			Note: This contract was used to construct MD 100 WB only			

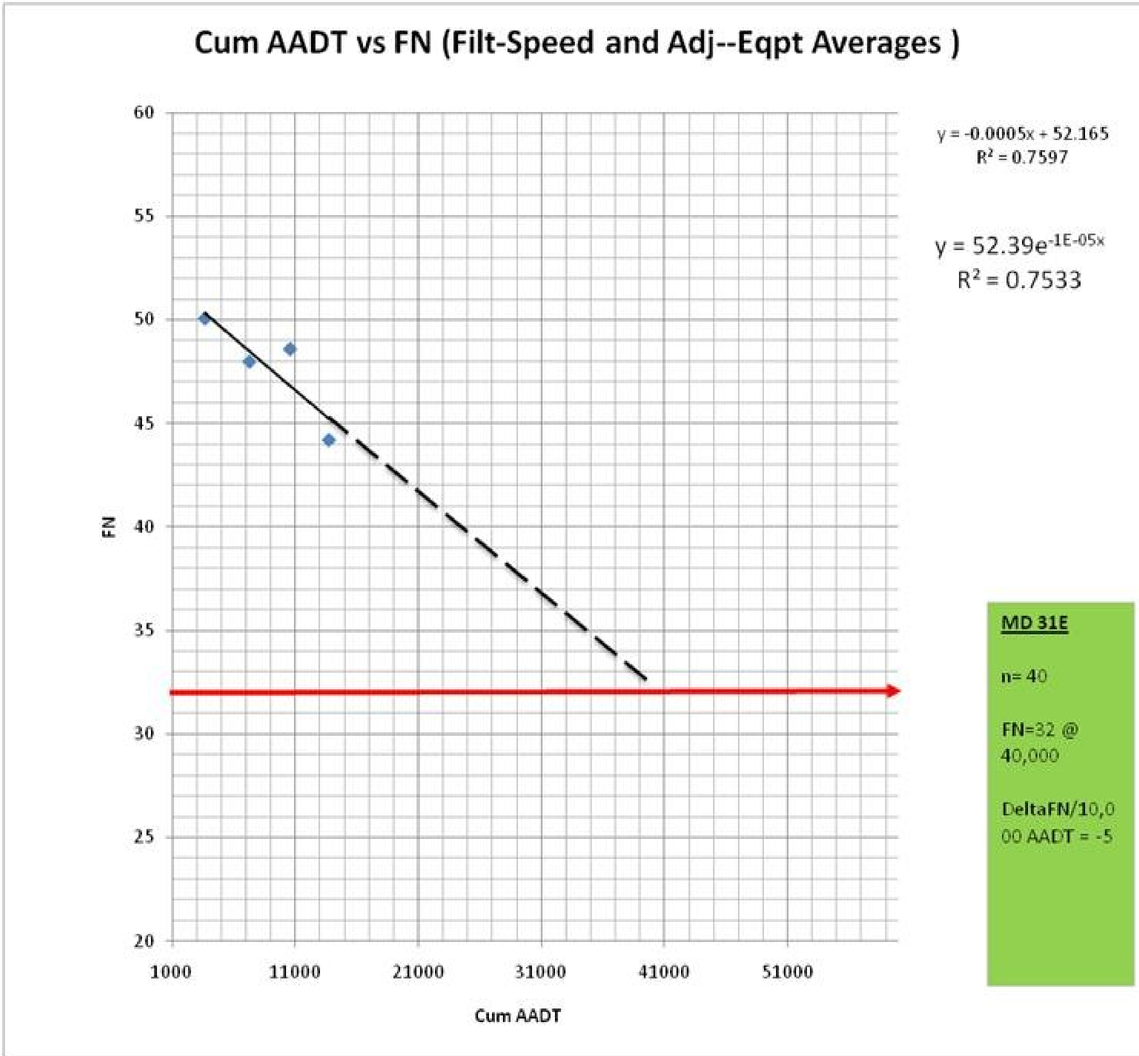

Figure 6-1. Cum AADT vs FN (filtered and adjusted Data – Averages)

N= 40, R^2= 0.76

Lafarge Frederick

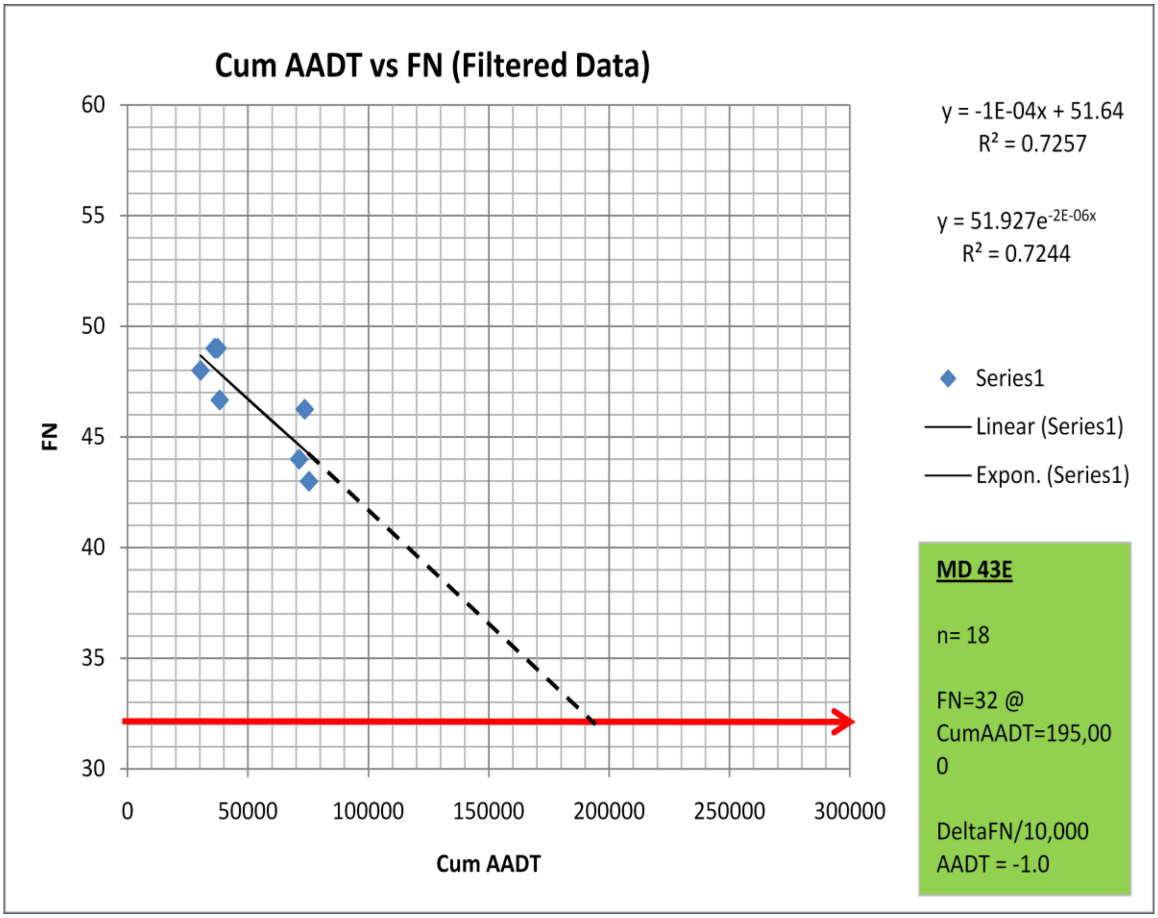

Figure 6-2. Cum AADT vs FN (filtered and adjusted Data – Averages)

N= 18, R^2= 0.73

Lafarge Churchville (Aggregate Blend)

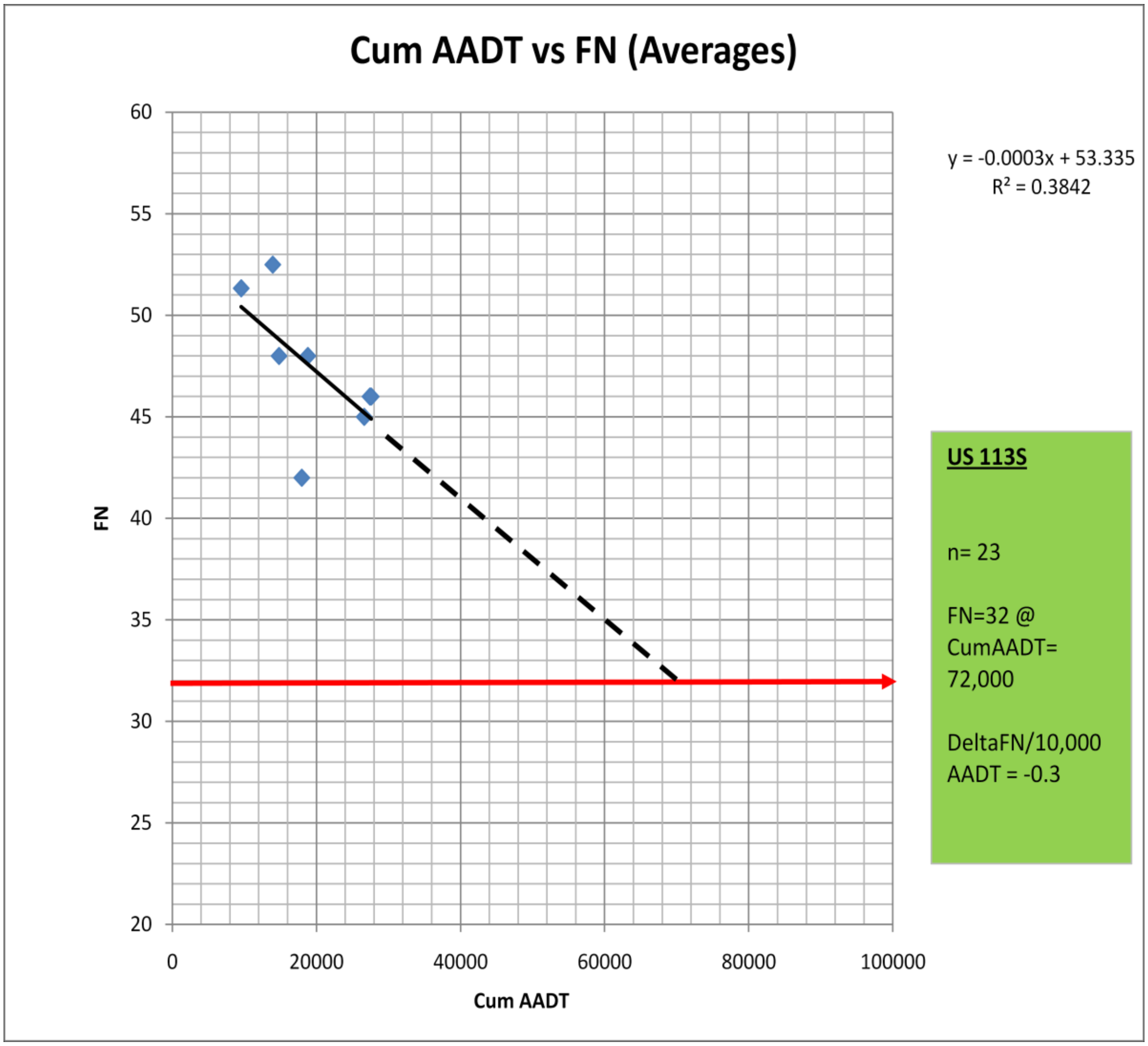

Figure 6-3. Cum AADT vs FN (filtered and adjusted Data – Averages)

N= 23,R^2= 0.38

Vulcan Materials Havre De Grace (Aggregate Blend)

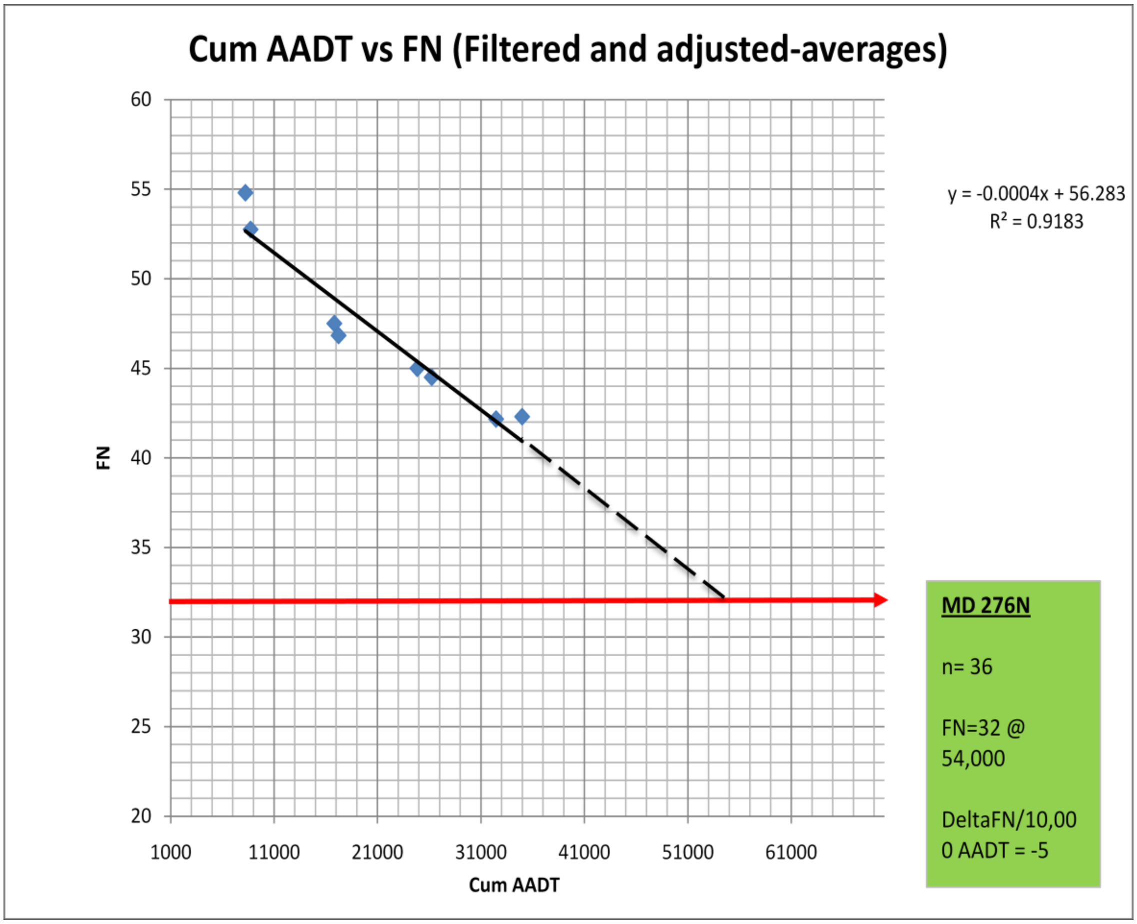

Figure 6-4. Cum AADT vs FN (filtered and adjusted Data – Averages)

N=36,R^2= 0.92

York Building Products Belvedere Plant (Aggregate Blend)

Table 6-9. Comparison of CumAADT converted to CumESAL and ESAL computed at Milepoint level

Supplier = AIR, Route= MD 190 E

a. Conversion of CumAADT to CumESAL

Material Source	**AIR**
Contract No	MO3285177
Mix Type	HMA 12.5 70-22 8 PV
County	Montgomery
Route	MD 190 (E+W)
MP	0-6.5
No of Lanes	2
AADT (Averaged over Milepoints and over survey years)	3,573
Direction used in Analysis	MD 190E
Truck Percentage(2005-7 Data)	
Single	11.2
Combination	2.6
Passenger/Other	86.2
Truck Percentage(2008 Data)	
Single	10.6
Combination	2.1
Passenger/Other	87.3
Average Percentages	
Single	10.9
Combination	2.35
Passenger/Other	86.75

b. Computation of ESAL at milepoint to obtain CUMESAL

Material Source	AIR
Contract No	MO3285177
Mix Type	HMA 12.5 70-22 8 PV
County	Montgomery
Route	MD 190 (E+W)
MP	0-6.5
No of Lanes	2
AADT (Averaged over Milepoints and over years)	3375
Direction used in Analysis	MD 190E
Truck Percentage(2008 Data)	
Single	10.6
Combination	2.1
Load Equivalency Factors, LEF (SN=5, Pt=2.5)	
Single	1.857
Combination	2.714
Directional Distribution Factor	0.5
Lane Distribution Factor	1
Terminal CumESAL **[CumESAL where FN=32] – From Graph (Fig 6-6)**	**3,150,000**

Table 6-9. Comparison of CumAADT converted to CumESAL and ESAL computed at Milepoint level (Continued)

Supplier = AIR, Route= MD 190 E

a. Conversion of CumAADT to CumESAL (Continued from above)

Load Equivalency Factors, LEF (SN=5, Pt=2.5)	
Single	1.857
Combination	2.714
Passenger/Other	0.0002
Directional Distribution Factor	0.5
Lane Distribution Factor	1
Terminal CumAADT	**66,000**
[CumAADT where FN=32]	
ESAL=Terminal CumAADT*T*Df*Lf*LEF*365	
Single	2,438,065
Combination	768,218
Passenger/Other	2,090
Total	**3,208,372**

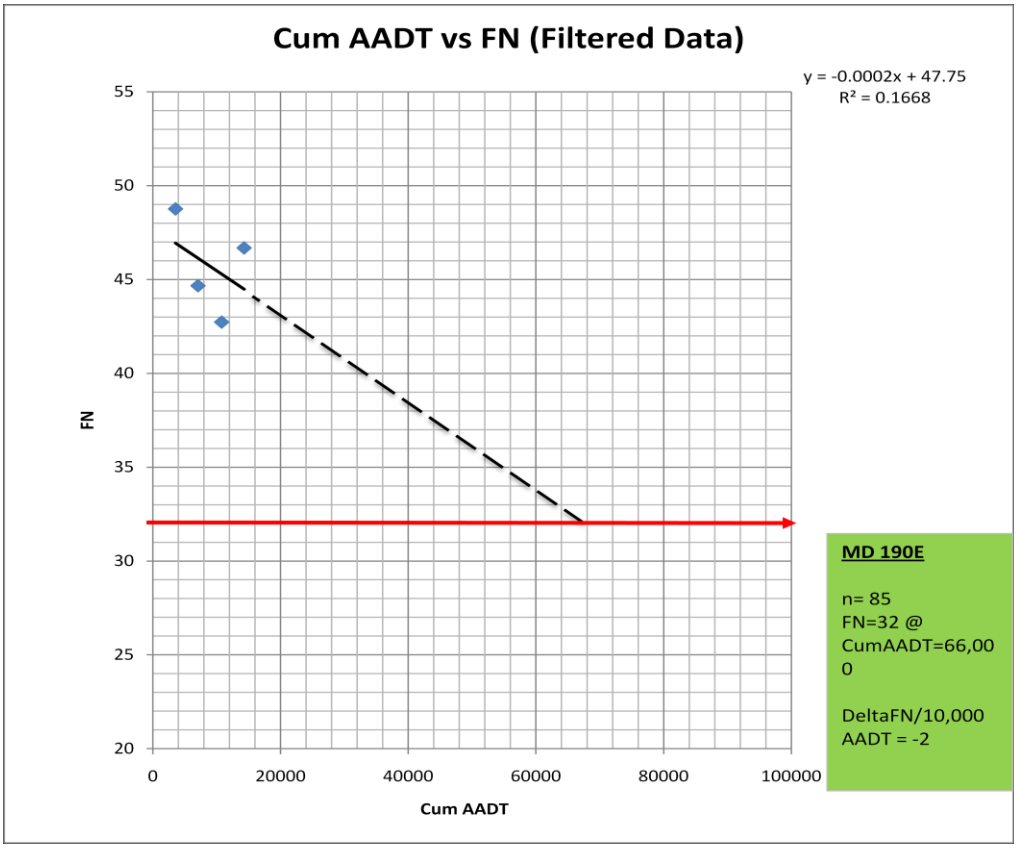

Figure 6-5. CumAADT vs FN (MD 190E

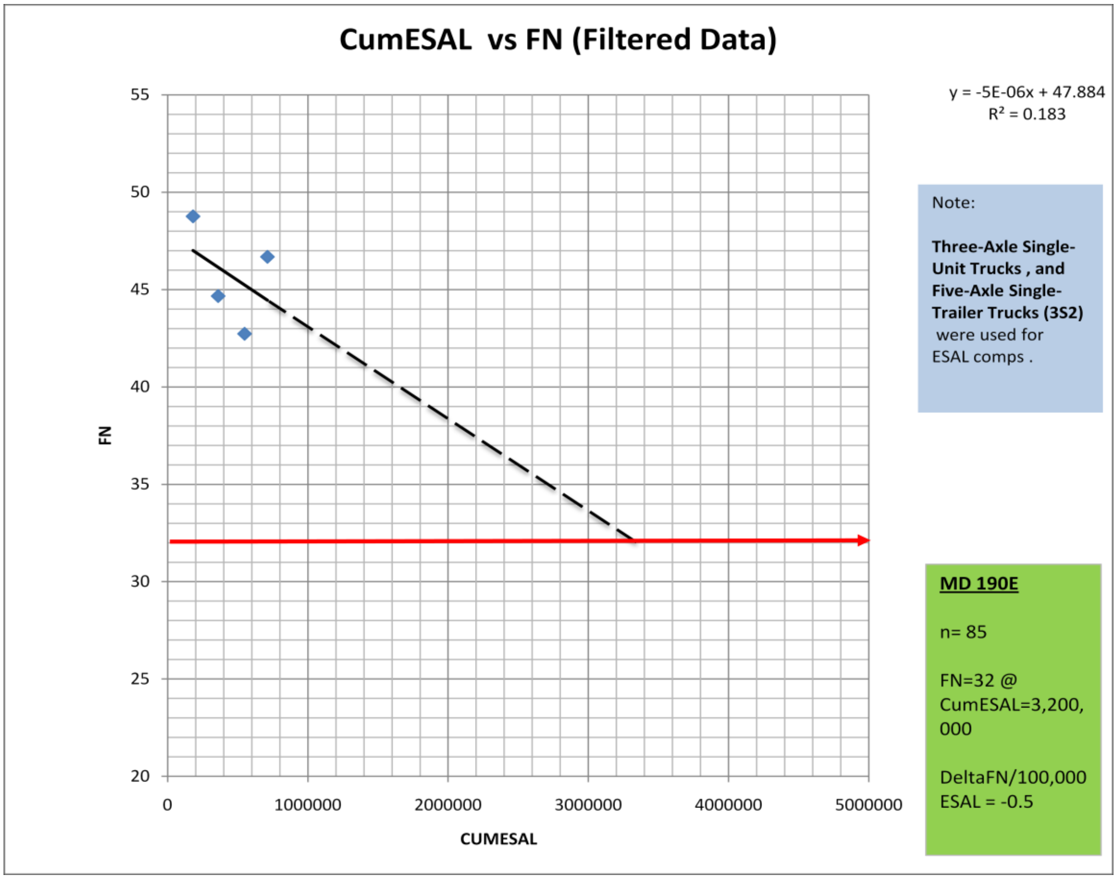

Figure 6-6. CumESAL vs FN (MD 190E)

Table 6-10. Comparison of CumAADT converted to CumESAL and ESAL computed at Milepoint level

Supplier = AASG, Route= US 220 S

a. Conversion of CumAADT to CumESAL

Material Source	AASG
Contract No	AL6165177
Mix Type	HMA 12.5mm, 64-22, Surface, L 4
County	Allegany
Route	US 220 (N+S)
MP	3.3-6.6
No of Lanes	2
AADT (Averaged over Milepoints and over survey years)	7,201
Direction used in Analysis	US 220S
Truck Percentage(2005-7 Data)	
Single	7.7
Combination	4
Passenger/Other	88.3
Truck Percentage(2008 Data)	
Single	7.6
Combination	2.2
Passenger/Other	90.2

b. Computation of ESAL at milepoint to obtain CUMESAL

Material Source	AASG
Contract No	AL6165177
Mix Type	HMA 12.5mm, 64-22, Surface, L 4
County	Allegany
Route	US 220 (N+S)
MP	3.3-6.6
No of Lanes	2
AADT (Averaged over Milepoints and over years)	7201
Direction used in Analysis	US 220N
Truck Percentage(2008 Data)	
Single	7.7
Combination	4
Load Equivalency Factors, LEF (SN=5, Pt=2.5)	
Single	1.857
Combination	1.857
Directional Distribution Factor	0.5
Lane Distribution Factor	1
Terminal CumESAL [CumESAL where FN=32] – From Fig 6-8	**2,500,000**

Table 6-10. Comparison of CumAADT converted to CumESAL and ESAL computed at Milepoint level (Continued)

Supplier = AASG, Route= US 220 S

a. Conversion of CumAADT to CumESAL (Continued from above)

Average Percentages	
Single	7.65
Combination	3.1
Passenger/Other	89.25
Load Equivalency Factors, LEF (SN=5, Pt=2.5)	
Single	1.857
Combination	2.714
Passenger/Other	0.0002
Directional Distribution Factor	0.5
Lane Distribution Factor	1
Terminal CumAADT	**57,200**
[CumAADT where FN=32]	
ESAL= Terminal CumAADT*T*Df*Lf*LEF*365	
Single	1,482,970
Combination	878,275
Passenger/Other	1,863
Total	**2,363,108**

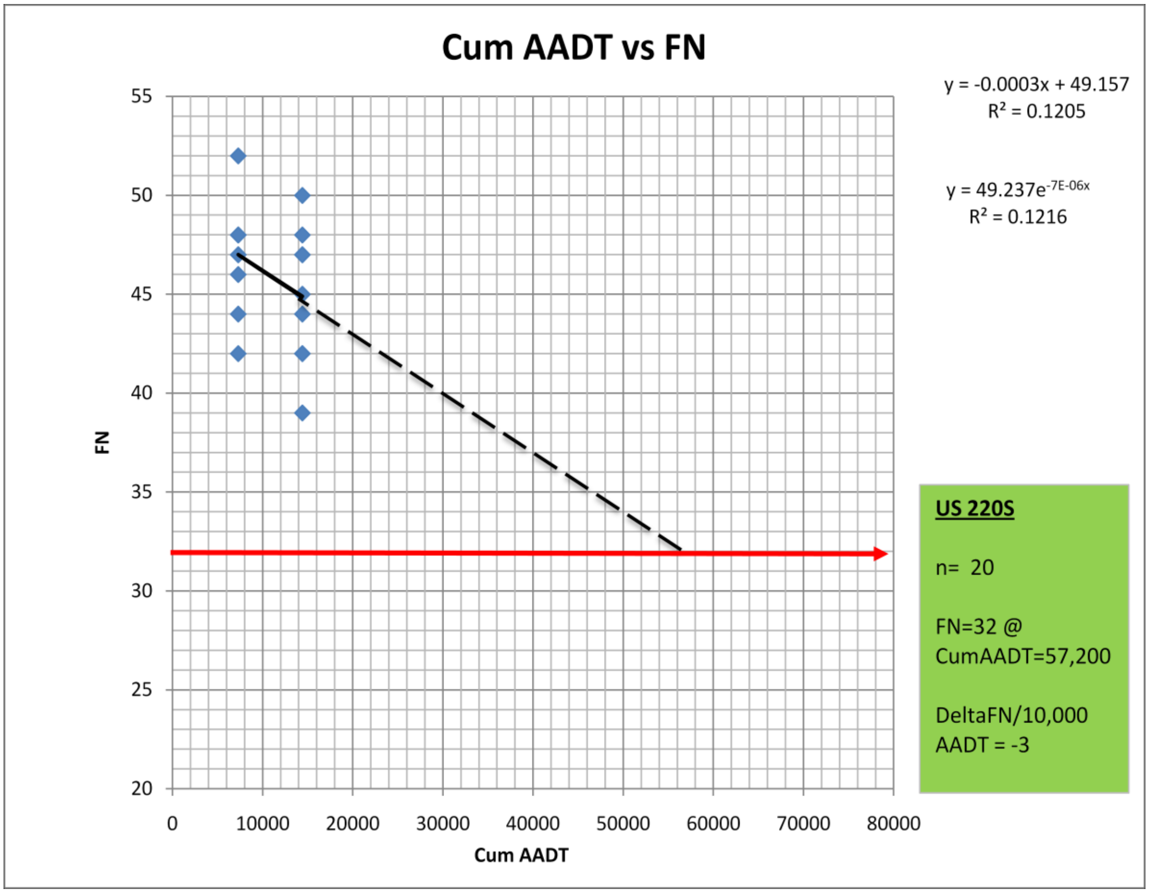

Figure 6-7. CumAADT vs FN (US 200S)

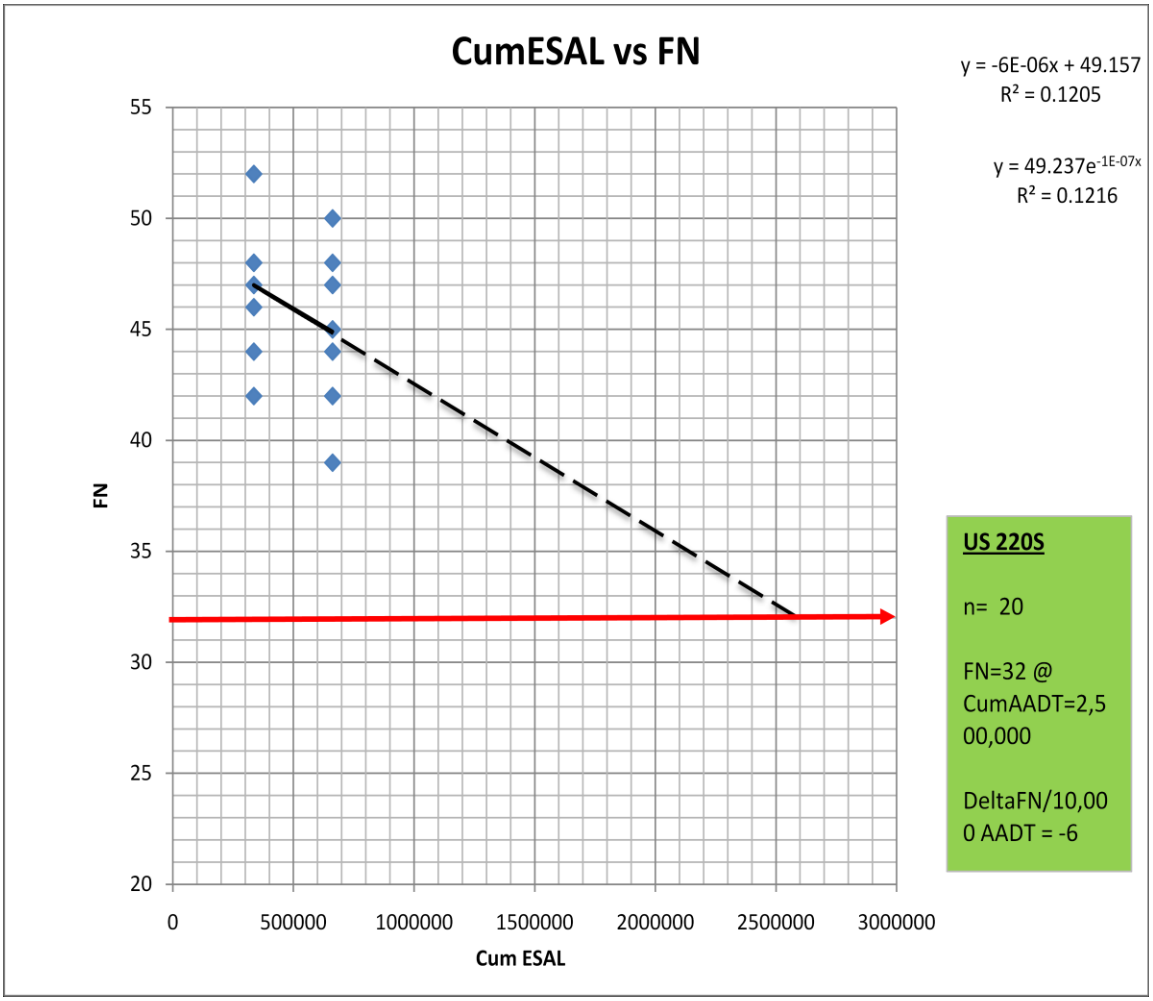

Figure 6-8. CumESAL vs FN (US 220S)

6.3 Aggregate Properties and Pavement Friction

The relationships between aggregate properties, such as Los Angeles Abrasion (LAA), British Pendulum Number (BPN), Polish Value (PV) and Magnesium Sulphate Soundness, and the expected pavement friction life (in terms of total cumulative AADT, expected pavement friction life in years, FN Drop/10k AADT) were then examined even though a limited number of aggregate quality data were available as reported in section 6.2 and Table 6-6. Table 6-10 summarizes these values and Figures 6-9 and 6-10 present example plots for BPN and PV. As it can be seen from these plots these relationships were not meaningful. Similar effects were observed for the FN drop/10k AADT, Table 6-11 and Figure 6-11, recognizing once more, the limited aggregate quality data available for these analyses, and the fact that AADT does not reflect the diverse truck loading conditions on each roadway. Similarly, the relationships between aggregate properties and total cumulative ESAL were examined. Table 6-12 summarizes these values and Figures 6-12 and 6-13 present example plots for BPN and LAA. As it can be seen from these plots while the excepted trends may be present for some of these aggregate properties, the BPN versus the total ESAL relationship is not meaningful, while the relationship between LAA and total ESAL has an R^2 of 0.36.

Table 6-11. Expected Life versus Aggregate Properties

Supplier	Exp Life (Years)	BPN	PV	LAA (%)	Soundness (%)
AASG	*7.94*	26	5	15	2.8
LCH	*5.37*	22	6	22	0.4
LF	*11.64*	24	6	22	0.2
AIR-70-22	*18.47*	22	5	18	4.5
VMH	*8.46*	21	4	25	0.7
VMW	*9.35*	26		11	0.3

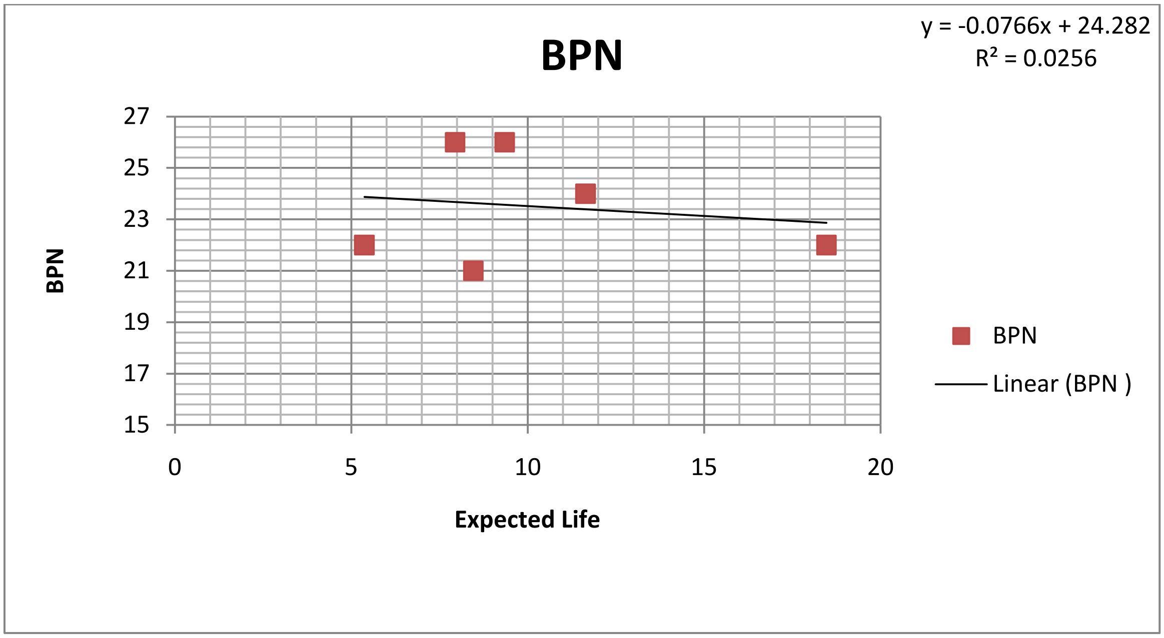

Figure 6-9. Expected FN Life vs BPN

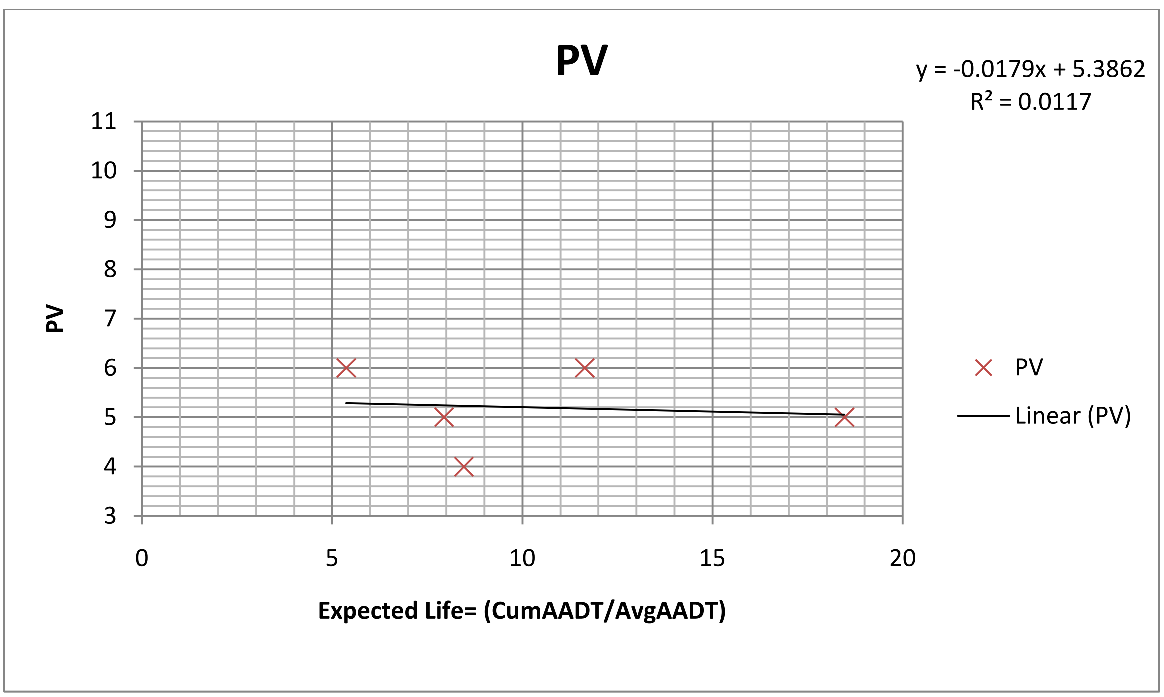

Figure 6-10. Expected FN Life vs PV

Table 6-12. FN Drop/ 10k AADT versus Aggregate Properties

Supplier	FN Drop/10k AADT (in FN units)	BPN	PV	LAA (%)	Soundness(%)
AASG	2	26	5	15	2.8
LCH	1	22	6	22	0.4
LF	5	24	6	22	0.2
AIR-70-22	2	22	5	18	4.5
VMH	1	21	4	25	0.7
VMW	0.30	26		11	0.3

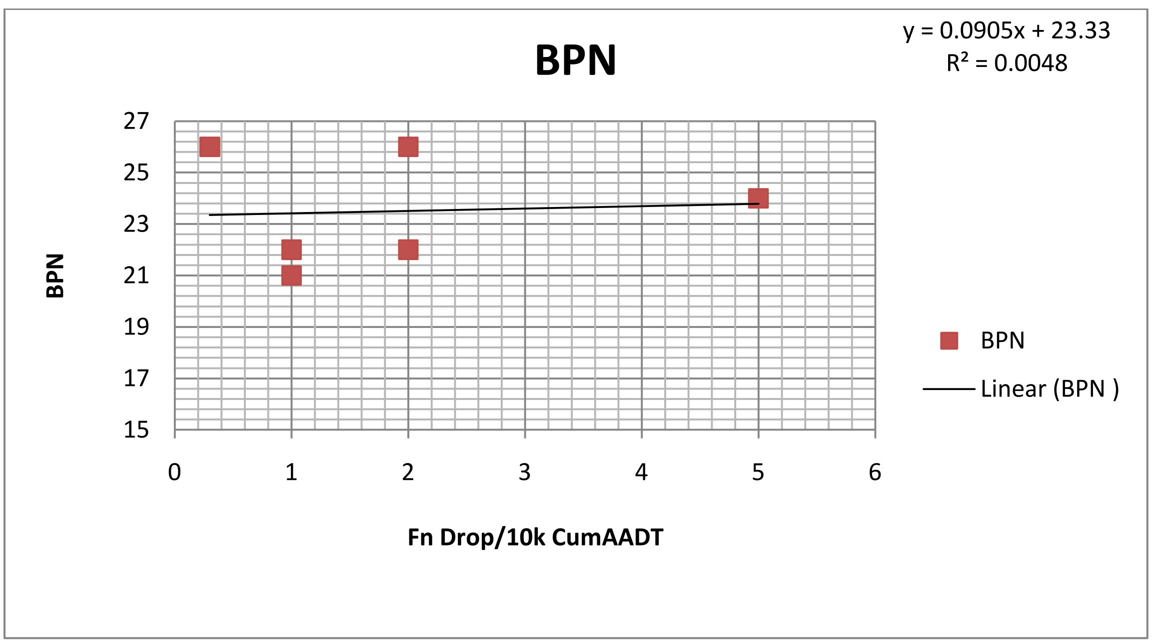

Figure 6-11. FN Drop/ 10k AADT versus BPN

Table 6-13. Terminal ESAL versus Aggregate Properties

Supplier	Terminal ESAL	BPN	PV	LAA (%)	Soundness (%)
AASG	2,363,108	26	5	15	2.8
LCH	2,118,493	22	6	22	0.4
LF	1,822,477	24	6	22	0.2
AIR-70-22	3,208,372	22	5	18	4.5
VMH	3,558,538	21	4	25	0.7
VMW	5,810,328	26		11	0.3

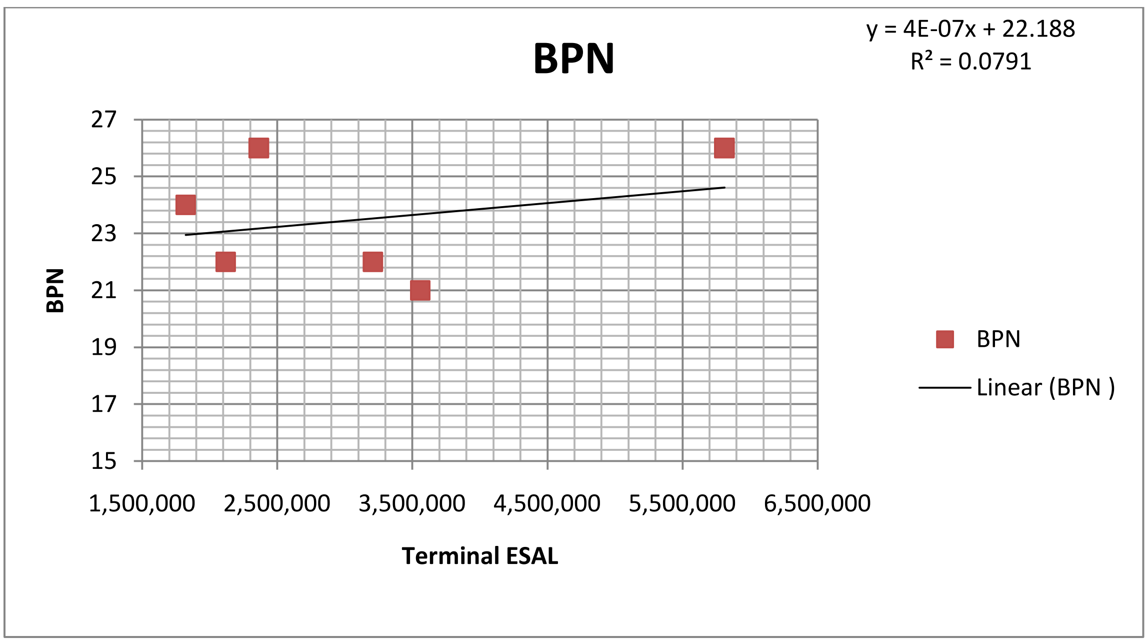

Figure 6-12. Terminal ESAL versus BPN

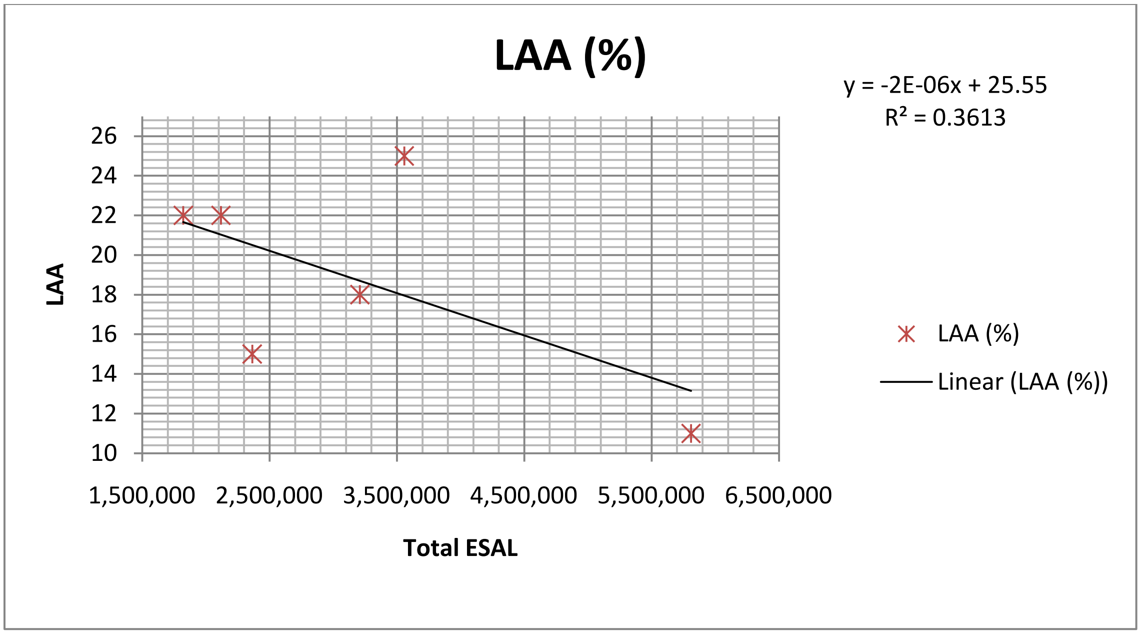

Figure 6-13. Terminal ESAL versus LAA

Chapter 7. Detailed Analysis and Research Modeling

7.1. Introduction

The purpose of the detailed analysis and modeling step in this research is to find meaningful and significant relationships between the predictor (independent) variables and the response (dependent) variable using the dataset obtained from work in the preceding chapters. The variables used in the modeling process are derived from three major data sources categorized as follows:

1. Pavement friction performance indicators: These variables are not directly measured/observed in the database. However, they are indirectly computed from statistical analysis using actual recorded values as discussed in the preceding chapters. The variables that fall in this category are "Terminal CumAADT" and "FN Drop/10,000 CumAADT "(computed from the Cumulative Annual Average Daily Traffic (CumAADT) versus Friction Number (FN) plots for various routes and suppliers; the terminal Cumulative AADT is read from this curve for an FN value of 32 (μ=0.32)), "Expected Pavement Life" (calculated by dividing "CumAADT" with the AADT of the route averaged over the years of survey and milepoint), and "Terminal ESAL" (terminal Equivalent Standard Axle Load Computed from CumAADT by using factors specific to the type and class of roadway).

2. Route descriptors: These are actual characteristics of the roadway that are specific to the pavement under consideration; they include pavement age (obtained from construction history database), Annual Average Daily Traffic

(obtained from SHA's traffic database) as well as Average Daily Equivalent Standard Axle Loading, AESAL, (computed from AADT by applying factors specific to the roadway such as truck percentage, lane and directional distribution factors, and load equivalency factors.)

3. Aggregate/Mix property descriptors: These are physically recorded, measured or observed values for specific materials/performance indicators and they are primarily obtained from the material and mix database, lab and field test results and supplier/contractor submittals. The variables in this category include material source information, source blend proportion information, aggregate gradation values (aggregate material pass sieve numbers), aggregate quality test results (British Pendulum Number –BPN, Polish Value –PV, Los Angeles Abrasion- LAA, Magnesium Sulphate Soundness, Binder Grade, Asphalt Content etc.)

Terminal Equivalent Standard Axle Loading (Terminal ESAL also referred to in this dissertation as TESAL) was selected to represent the friction performance of a pavement given that it is a more commonly used measure of pavement performance, and that this variable is well correlated to the other three descriptors in its group (see table 2). The models developed using the selected response variable and all (or a combination of) significant predictor variables can be used to estimate pavement friction performance in terms of Terminal (expected) ESAL. The model can also be used to estimate values of predictor variables that will yield higher friction performance. The various analysis techniques considered and the research modeling process followed are discussed in the next sections.

Table 7-1. Pavement Friction Performance Indicators by Supplier

Supplier	Terminal ESAL	CumAADT	FN Drop/10,000 CumAADT	Expected Pavement Life in Years
AASG	2,363,108	57,200	3	7.9
AIR	3,208,372	66,000	2	18.5
KLC	1,062,723	30,000	13	6.3
LCH	2,118,493	195,000	1	5.4
LF	1,558,218	40,000	5	11.6
LW	438,990	14,100	18	3.2
MMW	3,477,567	76,000	2	8.7
VMH	3,558,538	510,000	0.5	8.5
VMHDG	3,672,489	72,000	0.3	6.3
VMW	5,810,328	480,000	0.3	9.4
YBPBV	3,045,205	54,000	5	6.3

Table 7-2. Correlation Coefficients between Terminal ESAL and other Pavement Friction Performance indicators

	Terminal ESAL
Terminal ESAL	1
Terminal CumAADT	0.665266
FN Drop/10,000 CumAADT	-0.77491
Expected Life	0.318088

7.2. Dataset for Preliminary Investigation

As discussed above, several variables were obtained from the preliminary and detailed data analysis and investigation in chapters 5 and 6. The final output from the data analysis in the preceding chapters can be summarized as shown in the following tables:

Table 7-3. List of Variables and their descriptions

	Variable	Description
Response, (Dependent) Variable	Terminal ESAL	Equivalent Standard Axle Load (ESAL) at FN=32, for specific material and route
Predictor (Explanatory, Independent) Variables	Blend %	Proportion of major aggregate source
	BPN	British Pendulum Number
	PV	Polish Value
	LAA	Los Angeles Abrasion
	Soundness	Magnesium Sulphate Soundness
	Binder Grade	Binder Grade used in HMA mix
	Binder % (AC)	Asphalt Content in HMA Mix
	AESAL	Average Daily Equivalent Standard Axle Load (Computed from AADT)
	NMAS (Mix Size)	Nominal Maximum Aggregate Size (12.5mm etc.)
	12.5	Sieve Size = 12.5 mm (1/2 Inch)
	9.5	Sieve Size = 9.5 mm (3/8 Inch)
	4.75	Sieve Size = 4.75.5 mm (No. 4)
	2.36	Sieve Size = 2.36 mm (No. 8)
	1.18	Sieve Size = 1.18 mm (No. 16)
	0.6	Sieve Size = 0.6 mm (No. 30)
	0.3	Sieve Size = 0.3 mm (No. 50)
	0.15	Sieve Size = 0.15 mm (No. 100)
	0.075	Sieve Size = 0.075 mm (No. 200)
	Pan	Sieve Size = 0 mm

Table 7-4. Summary of dataset from detailed analysis in previous chapters

Source	Rock Type	Carbonate*	Surface Material	Action Year	Mix Size (mm)	Blend %	BPN	PV	LAA	Soundness	Binder Grade	Binder % (AC)	AADT
AASG	Dolomitic Limestone	1	HMA 12.5mm, 64-22, Surface, L 4	2006	12.5	100	26	5	15	2.8	64-22	5.7	7201
AIR	Serpentine	0	HMA 12.5mm, 70-22, 8 PV, L 3	2004	12.5	75	22	5	18	4.5	70-22	4.8	3573
KLC	Siliceous limestone	0	HMA 12.5mm, 70-22, Surface, L 3	2005	12.5	100	34	10	18	1	70-22	5.30	4762
LCH	Gnesiss	2	HMA 12.5mm, 64-22, Surface, L 2	2006	12.5	65	22	6	22	0.4	64-22	4.3	36320
LF	Limestone	1	HMA 12.5mm, 64-22, Surface, L 2	2005	12.5	100	24	6	22	0.2	64-22	5.30	3435
LW	Limestone	0	HMA 12.5mm, 64-22, Surface, L 4	2005	12.5	100	35	6	20	0.6	64-22	5.3	4390
MMW	Limestone()	0	HMA 12.5mm, 64-22, Surface, L 2	2004	12.5	75	27	6	18	1.2	64-22	4.8	8780
VMH	Limestone	1	HMA 9.5mm, 70-22, Surface, L 3	2005	9.5	85	21	4	25	0.7	70-22	5.4	60260
VMHDG	Gabbro ()	0	HMA 9.5mm, 64-22, 8 PV, L 2	2005	9.5	68	31	8	14	0.1	64-22	5.4	11490
VMW	Diabase	0	HMA 12.5mm, 76-22, Surface, 8 PV, L 4	2005	12.5	75	26	8	11	0.3	76-22	4.8	51320
YBPBV	Limestone()	0	HMA 9.5mm, 70-22, Surface, L 2	2004	9.5	72	27	6	18	1.2	70-22	5.3	8520

Note:

* Carbonate Information: 0= No information available on carbonates, 1= Carbonate Rock, 2= Non-Carbonate Rock
+ AADT values were converted to Daily Average Equivalent Standard Axle Load (AESAL) to account for variability in traffic
() Table was completed using data from additional sources such as supplier website, Maryland Geological Survey maps and SHA aggregate bulletin from other years

Table 7-4. Summary of dataset from detailed analysis in previous chapters (continued)

Source	AESAL+	Terminal Cum AADT	Exp Life (yrs)	Terminal ESAL	FN Drop/10K CumAADT	Number of records (n)
AASG	815	57200	7.9	2,363,108	-3	85
AIR	362	66000	18.5	3,208,372	-2	20
KLC	462	30000	6.3	1,062,723	-13	39
LCH	1081	195000	5.4	2,118,493	-1	18
LF	429	40000	11.6	1,558,218	-5	40
LW	980	14100	3.2	438,990	-18	28
MMW	1101	76000	8.7	3,477,567	-2	27
VMH	862	510000	8.5	3,558,538	-0.5	34
VMHDG	732	72000	6.3	3,672,489	-0.3	23
VMW	967	480000	9.4	5,810,328	-0.3	12
YBPBV	657	54000	6.3	3,045,205	-5	36

Note:

[*] Carbonate Information: 0= No information available on carbonates, 1= Carbonate Rock, 2= Non-Carbonate Rock
[+] AADT values were converted to Daily Average Equivalent Standard Axle Load (AESAL) to account for variability in traffic
[0] Table was completed using data from additional sources such as supplier website, Maryland Geological Survey maps and SHA aggregate bulletin from other years

Table 7-5. Aggregate Gradation (Percent Passing Sieve) by Supplier

Supplier	Mix Material	Mix Size	Percent Passing Sieve Size (mm)												
			50	37.5	25	19	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075
AASG	HMA 12.5mm, 64-22, Surface, L 4	12.5	100	100	100	100	97	86	53	35	22	14	10	7	5.8
AIR	HMA 12.5mm, 70-22, 8 PV, L 3	12.5	100	100	100	100	97	83	52	36	23	17	10	6	4.7
KLC	HMA 12.5mm, 70-22, Surface, L 3	12.5	100	100	100	100	91	77	52	33	21	14	10	8	6.1
LCH	HMA 12.5mm, 64-22, Surface, L 2	12.5	100	100	100	100	98	83	44	30	23	17	11	7	4.1
LF	HMA 12.5mm, 64-22, Surface, L 2	12.5	100	100	100	100	95	87	66	40	25	15	9	7	6
LW	HMA 12.5mm, 64-22, Surface, L 4	12.5	100	100	100	100	95	85	54	35	22	15	10	8	6.5
MMW	HMA 12.5mm, 64-22, Surface, L 2	12.5	100	100	100	100	97	83	44	26	21	17	12	8	5
VMH	HMA 9.5mm, 70-22, Surface, L 3	9.5	100	100	100	100	100	97	70	48	30	22	16	11	6.3
VMHDG	HMA 9.5mm, 64-22, 8 PV, L 2	9.5	100	100	100	100	100	99	66	41	29	20	12	9	6.9
VMW	HMA 12.5mm, 76-22, Surface, 8 PV, L 4	12.5	100	100	100	100	99	90	52	37	26	16	10	6	4.9
YBPBV	HMA 9.5mm, 70-22, Surface, L 2	9.5	100	100	100	100	100	95	58	32	24	18	12	8	5.6

7.3. Selection of Analysis and Modeling Dataset

The final dataset for detailed analysis and modeling was selected based on the significance of the variables identified in the previous sections. In addition, some of the variables had to be converted into a differnet form to bee able to be included in the statistical analysis. . For example, "Binder Grade" which is a categorical variable had to be converted into dummy variables (numerical values) such that the contribution of this parameter can be accounted for in the resulting model. Moreover, the Annual Average Daily Traffic (AADT) values were converted to Daily Average Equivalent Standard Axle Load (AESAL) based on the actual traffic and roadway characteristics of the route under consideration. Furthermore, it was possible to combine the data from two separate mix sizes (12.5 mm and 9.5 mm Nominal Maximum Aggregate sizes) given that all surface mixes essentially follow the same mix design methodology (Superpave). Tables 7-6 and 7-7 contain the "master" dataset that was used in the model development process. .

The model development process first starts by attempting to fit a multivariate linear regression model to the data, assessing the outcome, and then moving into alternative curve/model fitting methods that may yield better results, depending on the data structure as well as the significance and correlation of the variables in the dataset. The following flowchart demonstrates the research modeling methodology:

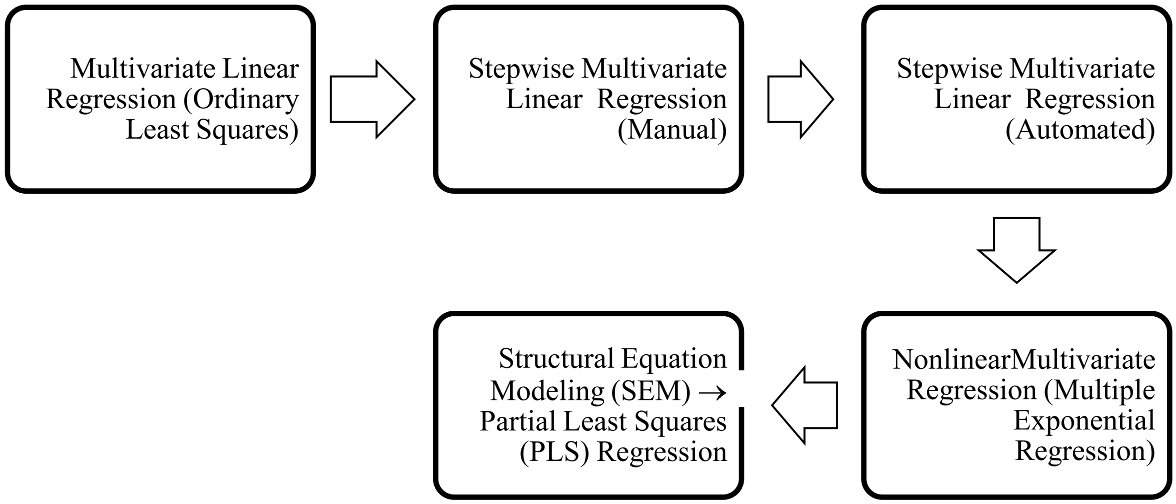

Figure 7-1. Flowchart of the model development process

Table 7-6. Final dataset used for subsequent model development (Percent Passing Sieve)

Supplier	Total ESAL	Blend %	BPN	PV	LAA	Sound ness	Binder Grade*	Binder %	AESAL
AASG	2,363,108	100	26	5	15	2.8	1	5.7	815
AIR	3,208,372	75	22	5	18	4.5	2	4.8	362
KLC	1,062,723	100	34	10	18	1	2	5.30	462
LCH	2,118,493	65	22	6	22	0.4	1	4.3	1081
LF	1,558,218	100	24	6	22	0.2	1	5.30	429
LW	438,990	100	35	6	20	0.6	1	5.3	980
MMW	3,477,567	75	27	6	18	1.2	1	4.8	1101
VMH	3,558,538	85	21	4	25	0.7	2	5.4	862
VMHDG	3,672,489	68	31	8	14	0.1	1	5.4	732
VMW	5,810,328	75	26	8	11	0.3	3	4.8	967
YBPBV	3,045,205	72	27	6	18	1.2	1	5.3	657

* 1= PG 64-22, 2= PG 70-22, 3= PG 76-22

Table 7-6. Final dataset used for subsequent model development (Percent Passing Sieve) (Continued)

Supplier	Percent Pass Sieve Size (mm)												
	50	**37.5**	**25**	**19**	**12.5**	**9.5**	**4.75**	**2.36**	**1.18**	**0.6**	**0.3**	**0.15**	**0.075**
AASG	100	100	100	100	97	86	53	35	22	14	10	7	5.8
AIR	100	100	100	100	97	83	52	36	23	17	10	6	4.7
KLC	100	100	100	100	91	77	52	33	21	14	10	8	6.1
LCH	100	100	100	100	98	83	44	30	23	17	11	7	4.1
LF	100	100	100	100	95	87	66	40	25	15	9	7	6
LW	100	100	100	100	95	85	54	35	22	15	10	8	6.5
MMW	100	100	100	100	97	83	44	26	21	17	12	8	5
VMH	100	100	100	100	100	97	70	48	30	22	16	11	6.3
VMHDG	100	100	100	100	100	99	66	41	29	20	12	9	6.9
VMW	100	100	100	100	99	90	52	37	26	16	10	6	4.9
YBPBV	100	100	100	100	100	95	58	32	24	18	12	8	5.6

Table 7-7. Final dataset used for subsequent model development (Percent Retained Sieve)

Supplier	Total ESAL	Blend %	BPN	PV	LAA	Sound ness	Binder Grade*	Binder %
AASG	2,363,108	100	26	5	15	2.8	1	5.7
AIR	3,208,372	75	22	5	18	4.5	2	4.8
KLC	1,062,723	100	34	10	18	1	2	5.30
LCH	2,118,493	65	22	6	22	0.4	1	4.3
LF	1,558,218	100	24	6	22	0.2	1	5.30
LW	438,990	100	35	6	20	0.6	1	5.3
MMW	3,477,567	75	27	6	18	1.2	1	4.8
VMH	3,558,538	85	21	4	25	0.7	2	5.4
VMHDG	3,672,489	68	31	8	14	0.1	1	5.4
VMW	5,810,328	75	26	8	11	0.3	3	4.8
YBPBV	3,045,205	72	27	6	18	1.2	1	5.3

Table 7-7. Final dataset used for subsequent model development (Percent Retained Sieve)(Continued)

Supplier	AESAL	Percent Retained Sieve Size (mm)++													
		50	37.5	25	19	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075	Pan (0)
AASG	815	0	0	0	0	3	11	33	18	13	8	4	3	1.2	5.8
AIR	362	0	0	0	0	3	14	31	16	13	6	7	4	1.3	4.7
KLC	462	0	0	0	0	9	14	25	19	12	7	4	2	1.9	6.1
LCH	1081	0	0	0	0	2	15	39	14	7	6	6	4	2.9	4.1
LF	429	0	0	0	0	5	8	21	26	15	10	6	2	1	6
LW	980	0	0	0	0	5	10	31	19	13	7	5	2	1.5	6.5
MMW	1101	0	0	0	0	3	14	39	18	5	4	5	4	3	5
VMH	862	0	0	0	0	0	3	27	22	18	8	6	5	4.7	6.3
VMHDG	732	0	0	0	0	0	1	33	25	12	9	8	3	2.1	6.9
VMW	967	0	0	0	0	1	9	38	15	11	10	6	4	1.1	4.9
YBPBV	657	0	0	0	0	0	5	37	26	8	6	6	4	2.4	5.6

++ Percent retained values computed from actual percent pass values

7.4. Multivariate Linear Regression Analysis

The number of predictor variables is between seventeen to eighteen, as can be seen from tables 7-6 and 7-7 respectively. On the other hand, there are several observations within each one of the eleven merged friction and mixture combinations shown in these tables The reduced amount of observations in relation to the variables considered in the modeling, as well as the correlation between these variables limit the validity of any traditional regression analysis. It was therefore important to find alternative methods of analysis. One of the alternatives in this regard was to reduce the number of predictor variables, for example by reducing the number of sieves in the dataset. This was possible by eventually considering variables that can represent the aggregate gradation as follows:

1. Use a measure of particle size distribution such as Coefficient of Uniformity (Cu) and Coefficient of Curvature (Cc) for the percent passing (cumulative) aggregate fractions;
2. Find alternative gradation curvature parameters that represent the best fit equation for the percent retained gradation (cumulative) and use these parameters in multivariate regression;
3. Select specific sieve sizes that represent the breakdown between coarse and fine aggregates (example the #4 sieve percent passing) and thus exmaine their contribution to friction;Select specific sieves (percent retained) to examine dominant sizes within the gradation in regards to friction.

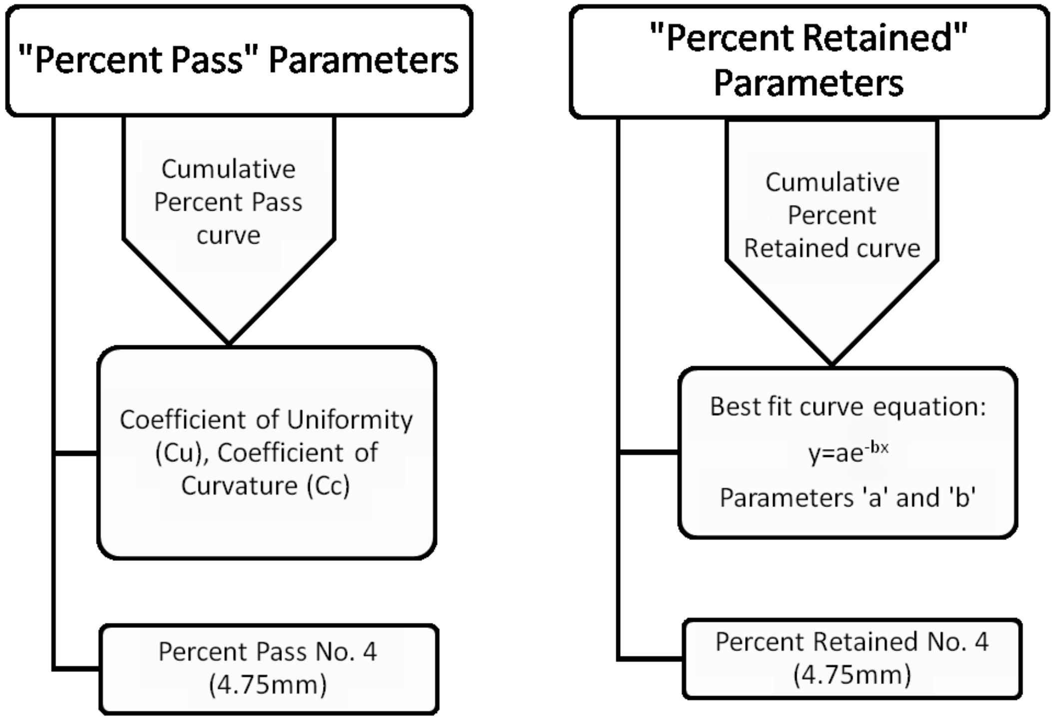

Figure 7-2. Flowchart of variable reduction options

The following tables summarize as an example the relationship between the response variable "Terminal ESAL" and selected sieve size (Sieve No. 4) distribution, percent pass gradation curve parameters (Cu and Cc), and percent retained curve fit parameters ('a' and 'b'):

Table 7-8. Relations between Terminal ESAL and gradation parameters

Supplier	Terminal ESAL	% pass #4	% retained #4	Cu	Cc	a	b
AASG	2363108	53	33	19.19	2.104	107.22	0.252
AIR	3208372	52	31	19.92	1.838	106.45	0.245
KLC	1062723	52	25	20.90	2.267	98.296	0.175
LCH	2118493	44	39	25.52	3.167	112.32	0.266
LF	1558218	66	21	9.05	1.946	98.645	0.229
LW	438990	54	31	18.90	2.136	102.23	0.222
MMW	3477567	44	39	29.77	7.243	109.5	0.244
VMH	3558538	70	27	24.92	3.880	110.32	0.227
VMHDG	3672489	66	33	19.43	2.102	126.18	0.285
VMW	5810328	52	38	19.17	1.501	116	0.323
YBPBV	3045205	58	37	22.25	3.785	112.1	0.188

Table 7-9. Correlation matrix for gradation parameters

	Terminal ESAL	%pass #4	% retained t#4	Cu	Cc	a	b
Terminal ESAL	1.000	0.051	0.495	0.253	0.116	0.700	0.680
%pass #4	0.051	1.000	-0.619	-0.481	-0.289	0.131	-0.109
%retained #4	0.495	-0.619	1.000	0.645	0.393	0.607	0.465
Cu	0.253	-0.481	0.645	1.000	0.746	0.320	-0.016
Cc	0.116	-0.289	0.393	0.746	1.000	0.073	-0.199
a	0.700	0.131	0.607	0.320	0.073	1.000	0.648
b	0.680	-0.109	0.465	-0.016	-0.199	0.648	1.000

The approach and computations used to investigate alternative means of representing gradation, so as to reduce the number of predictor variables in the dataset, are presented in detail in Appendix B.

The attempt to identify alternative gradation representation parameters yielded several parameters which can be used in multivariate regression analysis with reduced number of variables. As a result, the number of independent (predictor) variables was reduced from 17 to 10 for the percent passing gradation cases, and from 18 to 10 for the percent retained gradation alternatives. The results of such analyses, shown in Table 7-10, did not produce a significant model.

Table 7-10. Output from Multivariate Regression Analysis with 10 predictors

SUMMARY OUTPUT

Regression Statistics	
Multiple R	1
R Square	1
Adjusted R Square	65535
Standard Error	0
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	10	2.19E+13	2.19E+12	#NUM!	#NUM!
Residual	0	0	65535		
Total	10	2.19E+13			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-949926	0	65535	#NUM!	-949926	-949926	-949926	-949926
Cu	-364941	0	65535	#NUM!	-364941	-364941	-364941	-364941
Cc	675752.7	0	65535	#NUM!	675752.7	675752.7	675752.7	675752.7
Blend %	-125949	0	65535	#NUM!	-125949	-125949	-125949	-125949
BPN	-120817	0	65535	#NUM!	-120817	-120817	-120817	-120817
PV	670036.2	0	65535	#NUM!	670036.2	670036.2	670036.2	670036.2
LAA	114585.1	0	65535	#NUM!	114585.1	114585.1	114585.1	114585.1
Soundness	890493.1	0	65535	#NUM!	890493.1	890493.1	890493.1	890493.1
Binder Grade	1671818	0	65535	#NUM!	1671818	1671818	1671818	1671818
Binder %	3280790	0	65535	#NUM!	3280790	3280790	3280790	3280790
AESAL	6436.782	0	65535	#NUM!	6436.782	6436.782	6436.782	6436.782

The next step of the analysis was to reduce the number of predictor varaibles to nine recognizing the small size of the observations. Consequently, Sieve No. 4 (4.75 mm) was selected to represent gradation for the following reasons:

- It is the sieve that, on average, nearly 50% of the aggregate material passes for all suppliers;
- This sieve exhibits high variability as demonstrated in Table 7-11 (in the case of percent passing gradation);
- This sieve represents the peak value in the sieve size versus percent retained graph as shown in Figure 7-4 (in the case of percent retained gradation);
- This sieve represents the 'fraction between coarse and fine aggregate particles in the material.

Table 7-11. Descriptive Statistics for aggregate passing gradation parameters

Parameter	**12.5 mm**	**9.5 mm**	**4.75 mm**	**2.36 mm**	**1.18 mm**	**0.6 mm**	**0.3 mm**	**0.15 mm**	**0.075 mm**
Mean	97.18	87.73	55.55	35.73	24.18	16.82	11.09	7.73	5.63
Standard Error	0.83	2.05	2.61	1.78	0.92	0.75	0.58	0.43	0.26
Median	97.00	86.00	53.00	35.00	23.00	17.00	10.00	8.00	5.80
Mode	97.00	83.00	52.00	35.00	22.00	17.00	10.00	8.00	#N/A
Standard Deviation	2.75	6.81	8.64	5.90	3.06	2.48	1.92	1.42	0.86
Sample Variance	7.56	46.42	74.67	34.82	9.36	6.16	3.69	2.02	0.73
Kurtosis	1.30	-0.66	-0.76	0.99	-0.09	0.53	4.03	1.96	-0.73
Skewness	-1.08	0.41	0.40	0.53	0.97	0.92	1.81	1.10	-0.35
Range	9.00	22.00	26.00	22.00	9.00	8.00	7.00	5.00	2.80
Minimum	91.00	77.00	44.00	26.00	21.00	14.00	9.00	6.00	4.10
Maximum	100.00	99.00	70.00	48.00	30.00	22.00	16.00	11.00	6.90
Sum	1069.0	965.0	611.0	393.0	266.0	185.0	122.0	85.0	61.9
Count	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00
COV (Coeff. Of variation) =SD/Mean	0.028	0.078	0.156	0.165	0.127	0.148	0.173	0.184	0.152

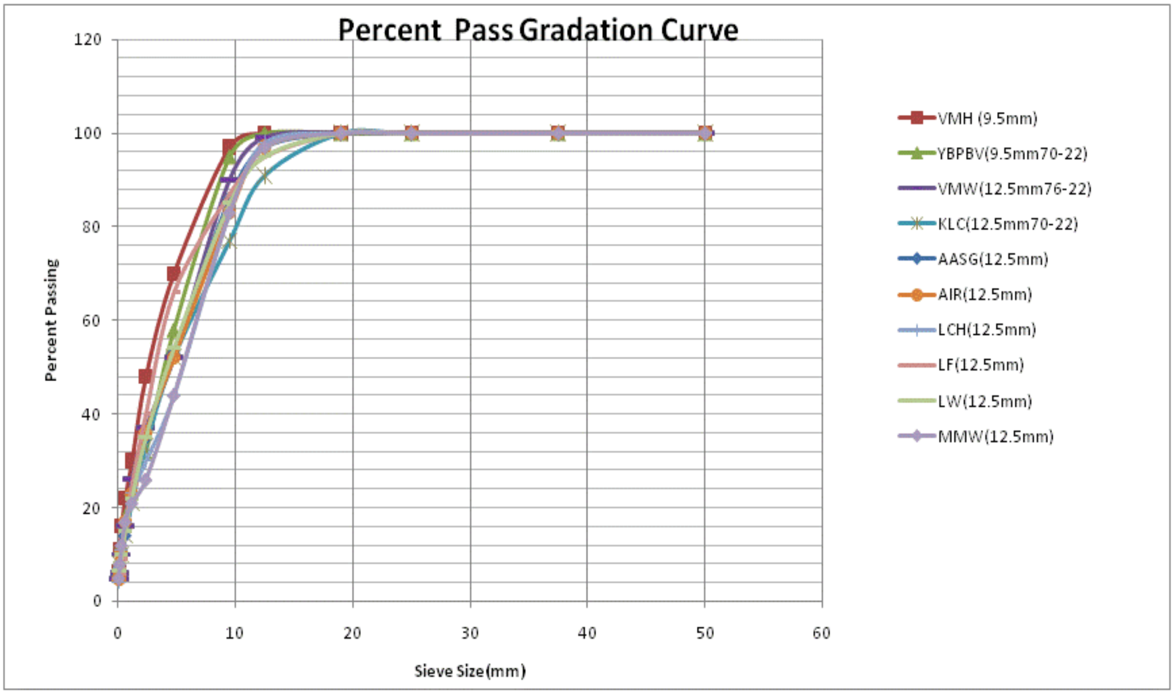

Figure 7-3. Aggregate Percent passing versus sieve size

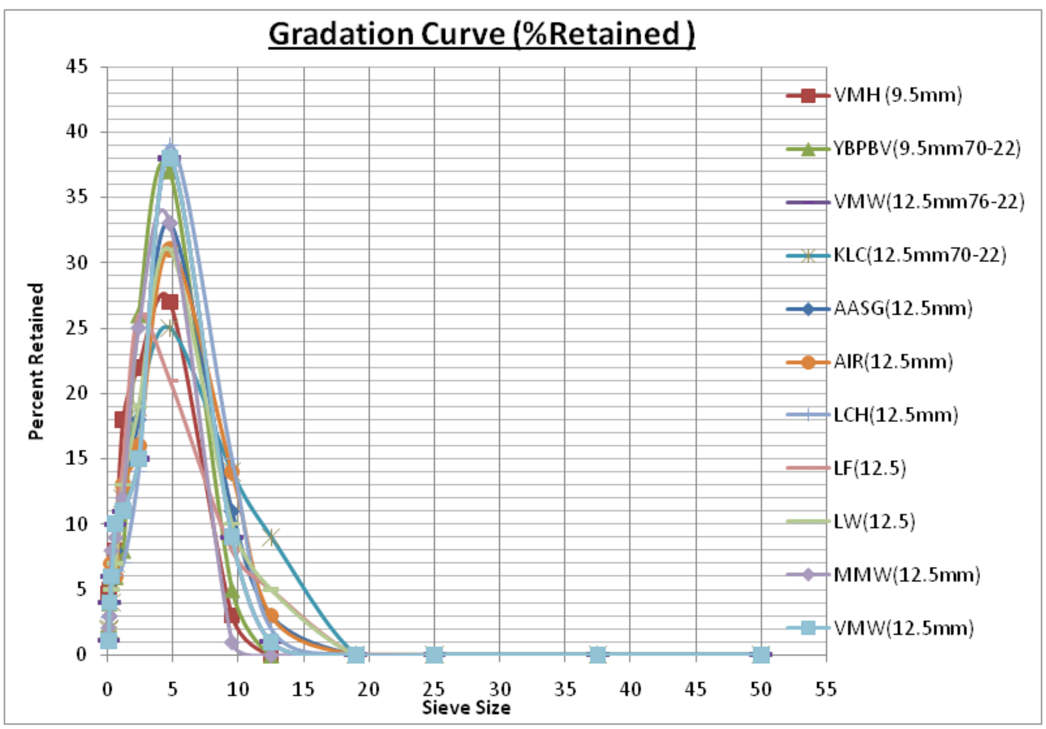

Figure 7-4. Aggregate Percent Retained versus sieve size

Based on the above selection, the dataset was revised to include nine independent (predictor) variables, one response variable, and inlcuding eleven distcinct cases of observations with multiple, datapoints n, as shown below:

Table 7-12. Reduced dataset for multivariate regression analysis (Percent Passing on Sieve No. 4)

Source	Terminal ESAL	%pass #4 (4.75mm)	Blend %	B P N	P V	L A A	Sound ness	BG*	Binder % (AC)	AESAL
AASG	2,363,108	53	100	26	5	15	2.8	1	5.7	815
AIR	3,208,372	52	75	22	5	18	4.5	2	4.8	362
KLC	1,062,723	52	100	34	10	18	1	2	5.30	462
LCH	2,118,493	44	65	22	6	22	0.4	1	4.3	1081
LF	1,558,218	66	100	24	6	22	0.2	1	5.30	429
LW	438,990	54	100	35	6	20	0.6	1	5.3	980
MMW	3,477,567	44	75	27	6	18	1.2	1	4.8	1101
VMH	3,558,538	70	85	21	4	25	0.7	2	5.4	862
VMHDG	3,672,489	66	68	31	8	14	0.1	1	5.4	732
VMW	5,810,328	52	75	26	8	11	0.3	3	4.8	967
YBPBV	3,045,205	58	72	27	6	18	1.2	1	5.3	657

Table 7-13. Reduced dataset for multivariate regression analysis (Percent Retained on Sieve No. 4)

Source	Terminal ESAL	%Retained #4 (4.75mm)	Blend %	B P N	P V	L A A	Sound ness	BG*	Binder %	AESAL
AASG	2,363,108	33	100	26	5	15	2.8	1	5.7	815
AIR	3,208,372	31	75	22	5	18	4.5	2	4.8	362
KLC	1,062,723	25	100	34	10	18	1	2	5.30	462
LCH	2,118,493	39	65	22	6	22	0.4	1	4.3	1081
LF	1,558,218	21	100	24	6	22	0.2	1	5.30	429
LW	438,990	31	100	35	6	20	0.6	1	5.3	980
MMW	3,477,567	39	75	27	6	18	1.2	1	4.8	1101
VMH	3,558,538	27	85	21	4	25	0.7	2	5.4	862
VMHDG	3,672,489	33	68	31	8	14	0.1	1	5.4	732
VMW	5,810,328	38	75	26	8	11	0.3	3	4.8	967
YBPBV	3,045,205	37	72	27	6	18	1.2	1	5.3	657

* BG= Binder Grade: 1= PG 64-22, 2= PG 70-22, 3= PG 76-22

7.4.1. Multivariate Linear (Ordinary Least Squares) Regression

A multiple (multivariate) linear regression model was assessed to fit all nine predictor variables listed in table 7-12. The multiple linear regression model has the following form:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + \varepsilon \qquad \text{(Eq. 7.1)}$$

Where:

y = the response variable

β_i = regression coefficients, and

x_i = predictor variables

ε = error term

The multiple linear regression method stipulates a linear relationship between a dependent (response) variable and a set of independent (predictor) variables. The algorithm for a multivariate linear regression has the objective of finding a vector of regression coefficients that will result in the least sum of squares (errors). This approach to multivariate linear regression is also referred as the Ordinary Least Squares (OLS) method. In matrix notation, the multivariate linear regression can be expressed as:

$$y = X\beta + \varepsilon \qquad \text{(Eq. 7.2)}$$

Where:

y = a vector (column matrix) of responses (an Nx1 matrix)
X = a matrix of exploratory variables (an NxK matrix where N is the number of rows –observations- and K is the number of columns -predictor variables)
B = a vector of regression coefficients (a Kx1 matrix)
ε = error term (an Nx1 matrix)

In its basic form, OLS is a data fitting mechanism based on minimizing the sum of squared residuals or residual sum of squares (RSS). The error (residual) from a regression equation can be defined as:

$$e = y - \hat{y} \qquad \text{(Eq. 7.3)}$$

Where:

y = actual response values

ŷ = predicted response values

The regression coefficient vector (column matrix), β, can be computed using the following matrix operation (Johnson et al 2007):

$$\beta = (X'X)^{-1}X'y \qquad \text{(Eq. 7.4)}$$

In addition to finding a set of regression coefficients that yield the least sum of squared residuals, the linear regression method also tests for the validity of the components of the resulting model. This is done using the following tests (Allen, 1997; Johnson et al 2007):

- Significance of independent coefficients: This is a test used to evaluate whether an exploratory variable contributes significantly to the model. This test investigates the null hypothesis which states that the regression coefficient for a certain variable is equal to zero (meaning that the particular predictor variable has little or no effect on the response). This corresponds to a t-test statistic which can be computed as the ratio of the estimated coefficient over its standard error. This test statistic follows a Student's 't' distribution with N-K degrees of freedom. If, according to this reference distribution, the probability that a value equal to or larger than the t-value - for a one-sided test - occurs (also referred to as 'P' value) is very small, the null hypothesis will be rejected and the coefficient may be taken to be significant, which also means that the contribution of the corresponding variable to the model is significant. Typically a probability test value ('alpha') of 0.05 is used as a measure of significance.

- Significance of the model: The second model validity test involves proving/disproving the null hypothesis that the regression model as whole is insignificant (that is all regression coefficients are equal to zero). This test, known as the Fisher's Statistic test (F-test), is constructed by comparing the residual sum of squares (RSS) obtained from the model with the sum of squares (RSSc) computed from the actual response values (computed as the average of the sum of the differences between each value of 'y' –i.e. y_i - and the mean of 'y' –i.e. $\bar{y}$). The 'F' statistic is given as:

$$F = \left[\frac{[(RSSC - RSS)/(K-1)]}{[\frac{RSS}{N-K}]} \right]$$

(Eq. 7.5)

The 'F' value obtained from above is compared to a published 'F' value for the given degrees of freedom of the model and the selected maximum probability ('alpha') value. For this research, an alpha value of 0.05 will be used as a measure of significance for the whole model as well as for the individual regression coefficients. The results of the multivariate regression performed on the dataset in table 7-12 are presented in the following table:

Table 7-14. Output of multivariate regression analysis (for dataset in table 7-12)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.973988
R Square	0.948652
Adjusted R Square	0.486522
Standard Error	1059295
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*tcritical*
Regression	9	2.07E+13	2.3E+12	2.052783	0.497164	2.262157
Residual	1	1.12E+12	1.12E+12			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	8073682	12721874	0.63463	0.639995	-1.5E+08	1.7E+08	-1.5E+08	1.7E+08
%pass #4	-11865.9	173701	-0.06831	0.956579	-2218947	2195215	-2218947	2195215
Blend %	-56788.4	56329.46	-1.00815	0.497417	-772522	658945.3	-772522	658945.3
BPN	-110772	156288.5	-0.70877	0.607469	-2096606	1875062	-2096606	1875062
PV	-225631	837387.1	-0.26945	0.832444	-1.1E+07	10414381	-1.1E+07	10414381
LAA	-187633	147845.1	-1.26912	0.424848	-2066183	1690917	-2066183	1690917
Soundness	-311281	1010998	-0.30789	0.809852	-1.3E+07	12534668	-1.3E+07	12534668
Binder Grade	1044310	943807.2	1.106486	0.467845	-1.1E+07	13036517	-1.1E+07	13036517
Binder %	1284143	2986348	0.430004	0.741468	-3.7E+07	39229286	-3.7E+07	39229286
AESAL	214.6552	4572.041	0.04695	0.970133	-57878.6	58307.94	-57878.6	58307.94

Table 7-15. Output of multivariate regression analysis (for dataset in table 7-13)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.974871
R Square	0.950373
Adjusted R Square	0.503733
Standard Error	1041391
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	9	2.08E+13	2.31E+12	2.127827	0.489725	2.262157
Residual	1	1.08E+12	1.08E+12			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	9878230	15689649	0.629602	0.642282	-1.9E+08	2.09E+08	-1.9E+08	2.09E+08
%Retained#4	-41936.7	210983.8	-0.19877	0.875089	-2722740	2638866	-2722740	2638866
Blend %	-61603	52878.42	-1.16499	0.451577	-733487	610281	-733487	610281
BPN	-110826	149588.3	-0.74087	0.59407	-2011525	1789874	-2011525	1789874
PV	-180240	557919.3	-0.32306	0.801073	-7269276	6908797	-7269276	6908797
LAA	-208245	175723.2	-1.18508	0.446206	-2441020	2024529	-2441020	2024529
Soundness	-195766	529134	-0.36997	0.774409	-6919051	6527519	-6919051	6527519
Binder Grade	943088.6	666620	1.414732	0.391717	-7527122	9413299	-7527122	9413299
Binder %	1038018	1574304	0.659351	0.628901	-1.9E+07	21041446	-1.9E+07	21041446
AESAL	1058.955	3705.411	0.285786	0.822787	-46022.8	48140.67	-46022.8	48140.67

As can be seen in the above tables, the model as a whole is valid as demonstrated by the higher ‘F’ statistic values compared to the corresponding “significance F” values, per the chosen ‘alpha’ value of 0.05. However, none of the regression coefficients yielded t-test values greater than the critical value of 2.262 (for an alpha value of 0.05) which makes all coefficients, including the constant term (intercept) insignificant. This is further confirmed by the high ‘P’ values (a ‘P’ value of less than 0.05 is needed for significance).

7.4.2. Stepwise Multivariate Linear Regression

The next approach that was employed to improve the outcome of the multivariate linear regression analysis was stepwise regression method. Stepwise regression analysis is a systematic method of adding and/or removing variables from a multivariate model based on their statistical significance (Matlab Handbook). The method starts with an initial model and then incrementally compares the explanatory power of larger or smaller models by adding or removing variables. At each step, the model significance is assessed as a whole using the 'p' value of an 'F'-statistic with and without a potential term.

In stepwise regression, if a variable is currently not in the model, the null hypothesis is that that particular predictor variable would have a zero coefficient if added to the model, and therefore will be considered insignificant to the model. However, if there is sufficient evidence (per the 'F' statistic results) to reject the null hypothesis, the variable is added into the model. On the other hand, if a predictor variable is currently in the model, the null hypothesis is that the variable has a zero coefficient. If there is insufficient evidence to reject the null hypothesis, the term will be removed from the model.

In this research, both manual and automated stepwise regression methods were considered. In the manual method, variables are removed from the model one (or more than one if they have relatively high 'p' values) at a time and multiple linear regression analysis is performed using the remaining variables until the model is found to be significant, and enough predictor variables prove to be significant for the formation of the model. In the automated method, a built-in computer program is used to automatically add/remove variables interchangeably until enough predictor variables are found to produce a significant model as well as significant regression coefficients. The detailed procedure and outputs for the manual stepwise regression are included in Appendix C.

The following tables show the final outputs from both manual and automated stepwise regression analysis on the dataset contained in tables 7-12 and 7-13 above:

Table 7-16. Output of "manual" stepwise multivariate regression analysis (for Table 7-12 dataset)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.81473
R Square	0.66378
Adjusted R Square	0.57973
Standard Error	958341
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	2	1.45E+13	7.25E+12	7.897120262	0.012778	2.306
Residual	8	7.35E+12	9.18E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.3E+07	2715162	4.927417	0.001153185	7117559	2E+07	7E+06	19639908
BPN	-208838	67734.22	-3.0832	0.015043718	-365034	- 52643	- 4E+05	-52642.9
LAA	-274849	80897.37	-3.39751	0.009395025	-461399	- 88300	- 5E+05	-88299.8

Table 7-17. Output of "automated" stepwise multivariate regression analysis (for Table 7-12 dataset)

Regression Statistics	
Multiple R	0.819068
R Square	0.670872
Adjusted R Square	0.58859
Standard Error	948186.5
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	2	1.47E+13	7.33E+12	8.153315	0.011734	2.306004
Residual	8	7.19E+12	8.99E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	6319581	1925870	3.281416	0.011164	1878516	10760646
Blend %	-62455.8	21126.96	-2.95621	0.018252	-111175	-13736.9
Binder Grade	1121604	436901.2	2.567179	0.033273	114107.8	2129100

Table 7-18. Output of "manual" stepwise multivariate regression analysis (for Table 7-13 dataset)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.81907
R Square	0.67087
Adjusted R Square	0.58859
Standard Error	948187
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	2	1.47E+13	7.33E+12	8.153314627	0.011734	2.306
Residual	8	7.19E+12	8.99E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	6319581	1925870	3.281416	0.011164182	1878516	1E+07	1878516	1.1E+07
Blend %	-62456	21126.96	-2.95621	0.018251582	-111175	-13737	-111174.6	-13736.9
Binder Grade	1121604	436901.2	2.567179	0.033273239	114107.8	2E+06	114107.8	2129100

Table 7-19. Output of "automated" stepwise multivariate regression analysis (for Table 7-13 dataset)

Regression Statistics	
Multiple R	0.819068
R Square	0.670872
Adjusted R Square	0.58859
Standard Error	948186.5
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	2	1.47E+13	7.33E+12	8.153315	0.011734	2.306004
Residual	8	7.19E+12	8.99E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	6319581	1925870	3.281416	0.011164	1878516	10760646
Blend %	-62455.8	21126.96	-2.95621	0.018252	-111175	-13736.9
Binder Grade	1121604	436901.2	2.567179	0.033273	114107.8	2129100

7.4.3. Multivariate Linear Regression "by-parts"

As can be inferred from the outputs of both the standard and stepwise multivariate linear regression analysis presented above, there has been limited success in finding a model that is significant, when evaluating for all variables as a whole or for independent variables individually. The next approach considered for improving the regression model was to break the dataset into two groups of variables that share at least one common variable, and then perform regression analysis separately. This approach will result in less number of variables than there are observations, which is likely to improve the significance of the contribution of the predictor variables. Consequently, the dataset from table 7-12 was divided into the following groups:

Table 7-20. Divided dataset for "by-parts" Multivariate Linear Regression (Group 1)

Terminal ESAL	%pass #4	Blend %	BPN	LAA	AESAL
2,363,108	53	100	26	15	815
3,208,372	52	75	22	18	362
1,062,723	52	100	34	18	462
2,118,493	44	65	22	22	1081
1,558,218	66	100	24	22	429
438,990	54	100	35	20	980
3,477,567	44	75	27	18	1101
3,558,538	70	85	21	25	862
3,672,489	66	68	31	14	732
5,810,328	52	75	26	11	967
3,045,205	58	72	27	18	657

Table 7-21. Divided dataset for "by-parts" Multivariate Linear Regression (Group 2)

Terminal ESAL	%pass #4	PV	Soundness	Binder Grade	Binder %	AESAL
2,363,108	53	5	2.8	1	5.7	815
3,208,372	52	5	4.5	2	4.8	362
1,062,723	52	10	1	2	5.30	462
2,118,493	44	6	0.4	1	4.3	1081
1,558,218	66	6	0.2	1	5.30	429
438,990	54	6	0.6	1	5.3	980
3,477,567	44	6	1.2	1	4.8	1101
3,558,538	70	4	0.7	2	5.4	862
3,672,489	66	8	0.1	1	5.4	732
5,810,328	52	8	0.3	3	4.8	967
3,045,205	58	6	1.2	1	5.3	657

Table 7-22. Output for "by-parts" Multivariate Linear Regression (First group)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.92021
R Square	0.84679
Adjusted R Square	0.69358
Standard Error	818300
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	5	1.85E+13	3.7E+12	5.527054934	0.041968	2.5706
Residual	5	3.35E+12	6.7E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1E+07	3158493	3.216147	0.023566634	2039012	2E+07	2039012	1.8E+07
%pass #4	58574.4	33690.93	1.738581	0.142605596	-28030.9	145180	-28030.89	145180
Blend %	-30518	22419.68	-1.36121	0.231581814	-88149.4	27114	-88149.44	27113.8
BPN	-163584	66638.89	-2.45478	0.057596893	-334885	7716.9	-334884.6	7716.9
LAA	-263306	76016.12	-3.46382	0.017968414	-458711	-67900	-458711.5	-67900
AESAL	1407.7	1120.684	1.256109	0.264571922	-1473.11	4288.5	-1473.108	4288.51

Table 7-23. Output for "by-parts" Multivariate Linear Regression (Second group)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.7764
R Square	0.60279
Adjusted R Square	0.00698
Standard Error	1473111
Observations	11

ANOVA

	df	SS	MS	F	Significance F	t-critical
Regression	6	1.32E+13	2.2E+12	1.01171316	0.519971	2.7764
Residual	4	8.68E+12	2.17E+12			
Total	10	2.19E+13				

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	-7E+06	10478679	-0.63903	0.5575636	-3.6E+07	2E+07	-3.6E+07	22397318
%pass #4	199551	147739.7	1.350691	0.2481483	-210640	609742	-210640	609741.7
PV	505985	583694.9	0.866866	0.4349209	-1114612	2E+06	-1114612	2126582
Soundness	1023825	888355.5	1.152494	0.3133070	-1442646	3E+06	-1442646	3490295
Binder Grade	663765	892298.1	0.743883	0.4982696	-1813651	3E+06	-1813651	3141182
Binder %	-2E+06	1999752	-1.10441	0.3313796	-7760748	3E+06	-7760748	3343657
AESAL	5594.92	3806.193	1.469951	0.2155194	-4972.77	16163	-4972.77	16162.6

It can be seen from the above two tables that this approach resulted in some improvement of the 'p' values for some variables, especially from the first group. However, this improvement is not any more significant than the results obtained by the stepwise regression analysis that yielded only a couple of variables as significant. Over the years, past studies have concluded that specific polishing related variables – such as British Pendulum Number and Los Angeles Abrasion - are related to friction performance of pavements, Similar results were obtained for the dataset in table 7-13 (for the case where percent retained on sieve No. 4 was used instead of percent passing)

7.5. Non-Linear Multivariate Regression (Exponential model)

Another method of regression analysis considered to establish a relationship between the response variable and the predictor variables is non-linear multivariate regression analysis, in this case exponential multiple regression. Exponential multiple variable regression has the form:

$$y = b * (m_1{}^{x1}) * (m_2{}^{x2}) * \ldots * (m_n{}^{xn}) \qquad \text{(Eq. 7.6)}$$

Where:

y: response variable

b: regression constant

$x_1 \ldots x_n$: predictor variables

$m_1 \ldots m_n$: exponential regression coefficients

It is important to note that the exponential regression can be converted to a linear form by simply taking the logarithm (natural logarithm) of both sides of the equation. In addition, the model and variable significance is measured in the same way as the multivariate linear regression. The built-in Microsoft excel function, LOGEST, was used to compute the multivariate exponential regression function for the dataset discussed in the previous sections. The results of the analysis for the dataset in table 7-12 are shown in the next tables.

Table 7-24. Output for Multivariate Exponential Regression for dataset in table 7-12 (Coefficients)

	AESAL	Binder %	Binder Grade	Sound ness	LAA	PV	BPN	Blend %	%pass #4	b
Coefficients	1.00033	3.306	1.216	0.985	0.954	1.134	0.8758	0.96791	1	3E+06
Standard Errors	0.00241	1.577	0.498	0.534	0.078	0.4422	0.0825	0.02975	0.1	6.719
R2 and Sev	0.93894	0.559								
F and df	1.7087	1								
Ssreg and Ssresid	4.81388	0.313								

Table 7-24: Output for Multivariate Exponential Regression for dataset in table 7-12 (Model Parameters) (Continued)

Model Parameter	Value	
r^2	0.938944	
df	1	
n	11	
v1 (degree of freedom 1)	9	
v2(degree of freedom 2)	1	
Fdist (F-critical)	0.536147	
Fobs	1.708702	**Model Acceptable**

Table 7-24: Output for Multivariate Exponential Regression for dataset in table 7-12 (Coefficient significance) (Continued)

Variable	t-observed Value (Absolute)	Variable significance Check T-critical value (alpha=0.05) = 12.706
AESAL	414.244	**Significant**
Binder %	2.09621	**Not Significant**
Binder Grade	2.43988	**Not Significant**
Soundness	1.84545	**Not Significant**
LAA	12.2186	**Not Significant**
PV	2.56474	**Not Significant**
BPN	10.6104	**Not Significant**
Blend %	32.5329	**Significant**
%pass #4	10.7732	**Not Significant**
b	472005	**Significant**

7.5.1. Multivariate Exponential regression with Select variables

In addition of initially using all variables in the multivariate exponential regression, further attempts considered reducing the number of variables to five based on their level of significance (variables with t-value of less than 10 were removed). . The removed variables were “Blend percentage”, “Polish Value”, Binder Grade” and “Soundness”. The resulting dataset and the output of the regression analysis is presented in the Table 7-25.

Table 7-25. Output for Multivariate Exponential Regression for reduced dataset (5-variables)

	AESAL	LAA	BPN	Blend %	%pass #4	b
Coefficients	1.0002	0.900267448	0.907484	0.982370327	1.022665	229870545
Standard Errors	0.0005	0.034014764	0.029819	0.010032083	0.015076	1.4133239
R2 and Sev	0.86924	0.366162831				
F and df	6.64781	5				
Ssreg and Ssresid	4.45653	0.670376095				

Model Parameter	Value	
r2	0.869244	
df	5	
n	11	
v1	5	
v2	5	
Fdist	0.029002	
Fobs	6.647807	**Model Acceptable**

Variable	**t-observed Value (Absolute)**	**Variable significance Check** T-critical value (alpha=0.05) = 2.57
AESAL	1994.53	**Significant**
LAA	26.467	**Significant**
BPN	30.4333	**Significant**
Blend %	97.9229	**Significant**
%pass #4	67.8358	**Significant**
b	1.6E+08	**Significant**

The above output was tested for validity by removing one observation from the dataset, running the multivariate regression again and plugging the omitted values back into the model, and thus comparing the predicted value to the actual observed value. To accomplish this, the median observation corresponding to Terminal ESAL value of 3,045,205 was removed from the dataset and the following results were obtained, see Table 7-26.

Table 7-26. Output for Multivariate Exponential Regression for reduced dataset (5-variables, 10-observations)

	AESAL	**LAA**	**BPN**	**Blend %**	**%pass #4**	**b**
Coefficients	1.00021	0.899871649	0.906801	0.982861421	1.022514	225133204
Standard Errors	0.00057	0.038183807	0.033989	0.012091315	0.016883	1.5891624
R2 and Sev	0.86737	0.408768571				
F and df	5.23194	4				
Ssreg and Ssresid	4.37107	0.668366978				

Model Parameter	**Value**		
r2	0.867373		
df	4		
n	10		
v1	5		
v2	4		
Fdist	0.067008		
Fobs	5.231936	**Model Acceptable**	

Variable	**t-observed Value (Absolute)**	**Variable significance Check** T-critical value (alpha=0.05) = 2.77
AESAL	1740	**Significant**
LAA	23.5668	**Significant**
BPN	26.6796	**Significant**
Blend %	81.2866	**Significant**
%pass #4	60.5634	**Significant**
PV	1.4E+08	**Significant**

The resulting model was:

$$TESAL = 225133204.25 * (X_1^{1.022}) * (X_2^{0.983}) * (X_3^{0.907}) * (X_4^{0.899}) * (X_5^{1.000}) \quad \text{(Eq. 7.7)}$$

Where:

TESAL = Terminal ESAL

X_1 = %Pass#4

X_2 = Blend %

X_3 = BPN

X_4 = LAA

X_5 = AESAL

Plugging in the values of the removed observation:

$$TESAL = 225133204.25 * (58^{1.022}) * (72^{0.983}) * (27^{0.907}) * (18^{0.899}) * (657^{1.000})$$

= **1.68*10^17 ESALs**

It can be seen from the output of Table 7-26 that the prediction is signifcantly large, which indicates that the model 'over fits' the data to the actual observations.

7.6. Variable Transformation

It is evident from the results of the multivariate linear and exponential regression analyses that the reduced dataset (nine predictor variables) did not produce a model that is significant both in temrs overall model validity as well as significance of predictor variables. As a result, a variable transformation/modification technique was considered, which can potentially help improve the correlation of the response variable (Terminal ESAL) with the individual predictor variables. To accomplish this, the correlation coefficient between each variable and the response variable was assessed and several options were investigated. The coefficient of correlation (r) measures the linear dependence between two variables, with values ranging between -1 and +1 inclusive. The correlation matrix for the variables is shown in Table 7-27.

Table 7-27. Correlation matrix for analysis dataset

(original variables - before transformations)

	Term. ESAL	**%pass #4**	**%Ret'd #4**	**Blend %**	**BPN**	**PV**	**LAA**	**Sound ness**	**Binder Grade**	**Binder %**	**AE SAL**
Terminal ESAL	1.000	0.051	0.495	-0.632	-0.423	-0.020	-0.514	0.007	0.558	-0.247	0.229
%pass #4	0.051	1.000	-0.619	0.203	-0.078	-0.159	0.245	-0.268	0.005	0.646	-0.393
%Retained #4	0.495	-0.619	1.000	-0.715	-0.094	-0.047	-0.431	0.022	-0.046	-0.529	0.685
Blend %	-0.632	0.203	-0.715	1.000	0.375	0.034	0.192	0.039	-0.060	0.630	-0.301
BPN	-0.423	-0.078	-0.094	0.375	1.000	0.657	-0.336	-0.282	-0.156	0.371	0.007
PV	-0.020	-0.159	-0.047	0.034	0.657	1.000	-0.507	-0.383	0.274	-0.031	-0.163
LAA	-0.514	0.245	-0.431	0.192	-0.336	-0.507	1.000	-0.112	-0.306	-0.055	-0.040
Soundness	0.007	-0.268	0.022	0.039	-0.282	-0.383	-0.112	1.000	0.097	0.009	-0.408
Binder Grade	0.558	0.005	-0.046	-0.060	-0.156	0.274	-0.306	0.097	1.000	-0.195	-0.121
Binder %	-0.247	0.646	-0.529	0.630	0.371	-0.031	-0.055	0.009	-0.195	1.000	-0.334
AESAL	0.229	-0.393	0.685	-0.301	0.007	-0.163	-0.040	-0.408	-0.121	-0.334	1.000

<u>Table 7-28. Correlation matrix for analysis dataset</u>

<u>(Modified variables - after transformations)</u>

	Term. ESAL	**%Ret #4**	**Blend %**	**-EXP (0.6* BPN)**	**- Exp (PV)**	**LAA**	**Ln (Soundness)**	**Binder Grade**	**Exp (Binder %)**	**(AESAL) ^-0.4**
Terminal ESAL	1.000	0.495	-0.632	**0.638**	**0.278**	-0.514	**-0.116**	0.558	**-0.270**	**-0.239**
%Ret #4	0.495	1.000	-0.715	0.205	0.352	-0.431	0.097	-0.046	-0.466	-0.655
Blend %	-0.632	-0.715	1.000	-0.516	-0.326	0.192	0.168	-0.060	0.618	0.267
-EXP (0.6*BPN)	0.638	0.205	-0.516	1.000	0.236	-0.126	0.014	0.123	-0.144	0.118
-Exp(PV)	0.278	0.352	-0.326	0.236	1.000	0.144	0.002	-0.323	-0.088	-0.334
LAA	-0.514	-0.431	0.192	-0.126	0.144	1.000	0.045	-0.306	-0.050	0.080
ln(Soundness)	-0.116	0.097	0.168	0.014	0.002	0.045	1.000	0.073	0.060	0.244
Binder Grade	0.558	-0.046	-0.060	0.123	-0.323	-0.306	0.073	1.000	-0.260	0.145
Exp (Binder %)	-0.270	-0.466	0.618	-0.144	-0.088	-0.050	0.060	-0.260	1.000	0.106
(AESAL)^-0.4	-0.239	-0.655	0.267	0.118	-0.334	0.080	0.244	0.145	0.106	1.000

It can be seen from the above analysis that the variable transformation approach resulted in better correlation between the response variable (Terminal ESAL) and some of the predictor variables. The variable "percent passing sieve no.4" (%pass#4) was not trasnformed since as it can be observed from the correlation matrix in table 7-27 this variable showed a high correlation with the response variable.

7.6.1. Multivariate Regression with transformed variables

Multivariate linear regression, stepwise regression and non-linear (exponential) regression were performed on the dataset using the transformed variables. The results are shown in Tables 7-29 through 7-33.

Table 7-29. Output of Multivariate linear regression (for transformed variables)

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.98157
R Square	0.96347
Adjusted R Square	0.63474
Standard Error	893426
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	9	2.11E+13	2.34E+12	2.93085	0.426514	12.706205
Residual	1	7.98E+11	7.98E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.9E+07	28240819	0.681061	0.619364	-3.4E+08	378067336	-3.4E+08	378067336
%Ret#4	-147803	331512.7	-0.44584	0.733007	-4360071	4064465.9	-4360071	4064465.9
Blend %	-62788	67984.49	-0.92356	0.525285	-926613	801037.08	-926612.7	801037.08
-EXP(0.6*BPN)	0.0012	0.000915	1.308096	0.415520	-0.01043	0.0128229	-0.010429	0.0128229
-Exp(PV)	49.2743	56.44498	0.872962	0.543114	-667.927	766.47579	-667.9272	766.47579
LAA	-179586	215829.3	-0.83208	0.558189	-2921958	2562785.4	-2921958	2562785.4
ln(Soundness)	179748	667860.7	0.26914	0.832626	-8306227	8665723	-8306227	8665723
Binder Grade	977788	764720.7	1.278621	0.422541	-8738910	10694486	-8738910	10694486
Exp(Binder %)	915.587	10025.23	0.091328	0.942019	-126467	128298.23	-126467.1	128298.23
(AESAL)^-0.4	-6E+07	93964450	-0.63833	0.638320	-1.3E+09	1.134E+09	-1.25E+09	1.134E+09

Table 7-30. Output of Stepwise Multivariate linear regression (for transformed variables)

Regression Statistics	
Multiple R	0.824384
R Square	0.679609
Adjusted R Square	0.599511
Standard Error	935516.2
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	2	1.49E+13	7.43E+12	8.484743	0.010537	2.306004
Residual	8	7E+12	8.75E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	1600842	713520.4	2.243583	0.055116	-44538.9	3246223
-EXP(0.6*BPN)	0.002101	0.000693	3.032437	0.016249	0.000503	0.003698
Binder Grade	1082783	431998.1	2.506454	0.036572	86593.85	2078973

Table 7-31. Output of Multivariate Exponential regression (using transformed variables)

	(AESAL)^-0.4	Exp (Binder %)	Binder Grade	Ln (Sound)	LAA	-Exp (PV)	-EXP (0.6* BPN)	Blend %	%Ret #4	b
Coefficients	2.45151E-13	1.001	1.221	1.185	0.929	1.000	1.000	0.966	0.918	1.87 E+10
Standard Errors	31.1116	0.0033	0.2532	0.2211	0.0715	0.0000	0.0000	0.0225	0.109	9.3505
R2 and Sev	0.982932204	0.295								
F and df	6.398874684	1								
Ssreg and Ssresid	5.039401834	0.087								

Model Parameter	**Value**	
r2	0.982932	
df	1	
n	11	
v1	9	
v2	1	
Fdist	0.298186	
Fobs	6.398875	**Model Acceptable**

Variable	**t-observed Value (Absolute)**	**Variable significance Check** T-critical value (alpha=0.05) = 12.77
(AESAL)^-0.4	7.87973E-15	**Not Significant**
Exp (Binder %)	301.4404839	**Significant**
Binder Grade	4.823893536	**Not Significant**
ln(Soundness)	5.359856035	**Not Significant**
LAA	13.00636834	**Significant**
-Exp(PV)	53507.88861	**Significant**
-EXP(0.6*BPN)	3300852808	**Significant**
Blend %	42.90268635	**Significant**
%Ret#4	8.360999024	**Not Significant**
b	1996852565	**Significant**

Table 7-32. Output of Multivariate Exponential regression (using reduced number of transformed variables)

	Exp (Binder %)	LAA	-Exp(PV)	-EXP(0.6*BPN)	Blend %	b
Coefficients	1.0009	0.958371	0.999994159	1.000000001	0.984299	19820980
Standard Errors	0.00227	0.028914	1.86996E-05	3.10383E-10	0.012224	0.8096871
R2 and Sev	0.89004	0.335784				
F and df	8.09419	5				
Ssreg and Ssresid	4.56315	0.563756				

Model Parameter	Value	
r2	0.867373	
df	4	
n	11	
v1	5	
v2	4	
Fdist	0.067008	
Fobs	5.231936	**Model Acceptable**

Variable	t-observed Value (Absolute)	Variable significance Check T-critical value (alpha=0.05) = 2.57
Exp (Binder %)	1740	**Significant**
LAA	23.5668	**Significant**
-Exp(PV)	26.6796	**Significant**
-EXP(0.6*BPN)	81.2866	**Significant**
Blend %	60.5634	**Significant**
b	1.4E+08	**Significant**

Table 7-33. Output of Multivariate Exponential regression (using reduced number of transformed variables with only 10 observations)

	Exp (Binder %)	LAA	-Exp(PV)	-EXP(0.6*BPN)	Blend %	b
Coefficients	1.00167	0.956334	0.999992801	1.000000001	0.979804	26394751
Standard Errors	0.00326	0.032309	2.08864E-05	3.62985E-10	0.01829	1.1811124
R2 and Sev	0.88839	0.369187				
F and df	6.36752	4				
Ssreg and Ssresid	4.33945	0.545197				

Model Parameter	Value	
r2	0.888385435	
df	4	
n	10	
v1	5	
v2	4	
Fdist	0.048548878	
Fobs	6.367523315	**Model Acceptable**

Variable	t-observed Value (Absolute)	Variable significance Check T-critical value (alpha=0.05) = 2.77
Exp (Binder %)	307.202	Significant
LAA	29.5996	Significant
-Exp(PV)	47877.8	Significant
-EXP(0.6*BPN)	2.8E+09	Significant
Blend %	53.5691	Significant
b	2.2E+07	Significant

The resulting model from such analysis using the reduced-variables, reduced-observations multivariatethe transformed variables was:

$$TESAL = 26394750.7 * (X_1^{0.979}) * (-\exp(0.6 * X_2)^{1.000}) * (-\exp(X_3)^{1.000}) * (X_4^{0.956}) * (\exp(X_5)^{1.002}) \quad \text{(Eq. 7.8)}$$

Where:

TESAL = Terminal ESAL

X_1 = Blend %

X_2 = BPN

X_3 = PV

X_4 = LAA

X_5 = Binder Grade

Plugging in the values from the removed median observation corresponding to Terminal ESAL value of 3,045,205 yields:

Terminal ESAL = **1.63*10^24 ESALs**

So far, numerous variable reduction and transformations were considered in order to use multivariate linear, stepwise and non-linear regression modeling and come up with a valid and significant relationship between the response variable (Terminal ESAL) and the various predictor variables. As can be concluded from the outputs of these analyses, none of the modeling approaches was able to produce acceptable models. . This could be attributed to many factors, the most important of which is that there is correlation/collinearity (multi-collinaearity) among one or more independent variables. In other words, there is a high interdependency between the independent variables themselves which makes the model fitting inadequate. In addition, the reduced number of observations in relation to the high number of predictor variables lead to the need of Structural Equation modeling (SEM).

7.7. Structural Equation Modeling

As it was conlcuded previously, the Ordinary Least Square (OLS) analysis as well as non-linear multivariate regression analyses did not provide acceptable models even if the predictor variables were transformed. In addition to the multicollinearity and data over-fitting problem observed in the preceding modeling, the level of correlation among the predictor variables also signifies that there is more than one 'layer' of causation between predictor and dependent variables that needs to be investigated. Structural Equation modeling (SEM) is one of the primary methods often employed to deal with complicated data structure such as the one obtained from this research.

Structural Equation Modeling is a general method of analysis for testing and estimating causal relations among variables (dependent and independent), using a combination of statistical data and qualitative causal assumptions (Kline, 2005; Silva et al, 2008). Through SEM, relationships among predictor/ exploratory and response variables can be established and/or confirmed, and these relationships can be modeled to predict possible outcomes. SEM allows for complicated variable relationships to be expressed through hierarchical or non-hierarchical, recursive or non-recursive structural equations, to present a more complete picture of the entire model (Gefen et al, 2000; Garson, 2010).

One of the strength of SEM is the ability to construct latent variables - variables which are not directly measured, but are estimated in the model from several measured variables. Latent Variables/constructs can be used to represent 'unobservable' variables in a structural model. Unobservable variables are generally categorized into three groups:

(a) variables that are unobservable in principle (e.g., theoretical terms); (b) variables that are unobservable in principle but either imply empirical concepts or can be inferred from observations; and (c) unobservable variables that are defined in terms of observables (Haenlein et al 2004). The representation of unobserved/unobservable variables in the model through latent variables allows for capturing any unreliability of measured values as well as not-readily known/observed causalities among the various potentially contributory variables within the model. (Kline, 2005; Garson, 2010; Gefen et al, 2000). The SEM model generally contains two inter-related models - the measurement model and the structural model. The measurement model defines the constructs (latent variables) that the model will use, and assigns observed variables to each. The structural model defines the causal relationship among these latent variables (Gefen et al, 2000).

The following figure shows the concept and formulation of the Structural Equation Modeling approach:

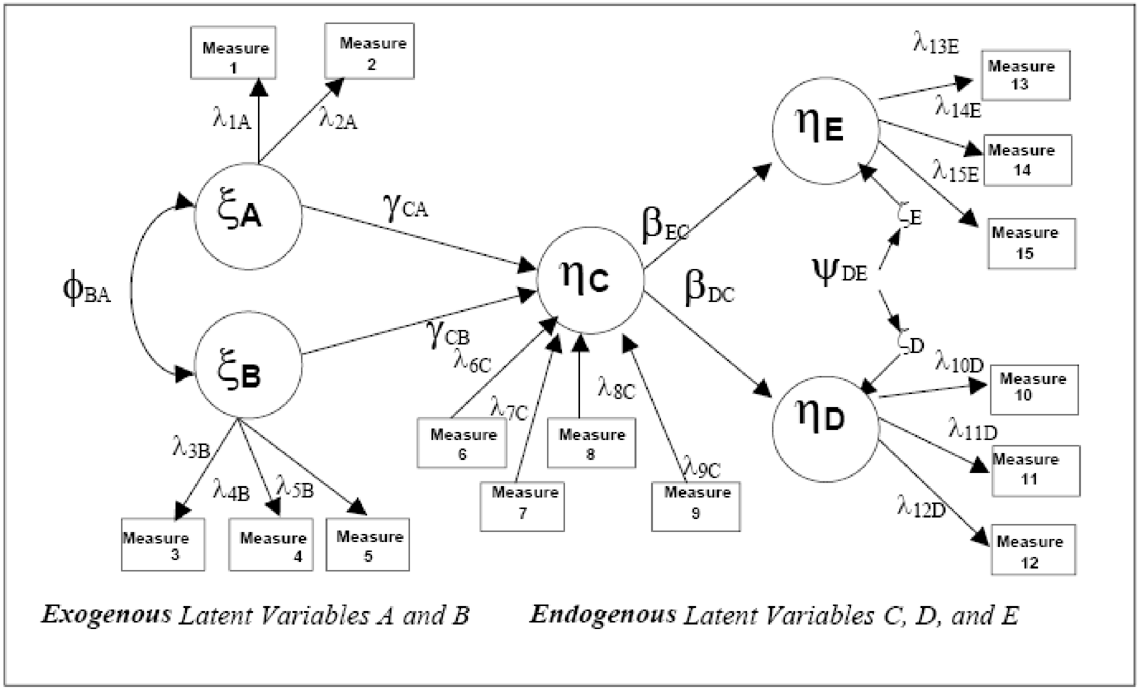

Figure 7-5. Illustration of the SEM approach (Gefen et al, 2000)

The variables, arrows and relationships shown in the above figure help define the overall structure of the model. The structural part of the model consists of the following (Gefen et al, 2000; Haenlein et al, 2004):

- Latent Variables – constructs (variables) that are not measured directly, but are measured indirectly through observable variables that reflect or form the construct:
    - Exogenous latent variables (ξ) – variables that act only as a predictor or "cause" other constructs in the model. They have only causal arrows leading out of them and are not predicted by any other variables in the model.

- Endogenous latent variables (η) - variables that depend on or are caused by at least one causal relationship. There are one or more arrows leading into an endogenous variable.

- Several Paths connecting the various variables considered in the model as follows:
    - Paths connecting exogenous and endogenous variables with coefficients indicating the strength of the relationship (γ)
    - Paths connecting endogenous variables with coefficients indicating the strength of the relationship (β).
- Shared correlation matrix among exogenous variables (ϕ).
- Error terms ("errors in assumed equations/path relationships") (ζ)
- Shared correlation matrix among the error terms of the endogenous variables (ψ).

The measurement model contains the following:

- Measured Observations or actual data collected, designated as X and Y. X is a measure of exogenous constructs while Y represents endogenous constructs.
- The path between an observed variable X and its exogenous counterpart is designated as λ_X while the path between an observed Y variable and its endogenous counterpart is designated as λ_Y. The term λ (lambda) represents the loading of a given observed item on the latent variable formed from it or reflected by it (Gefen et al, 2000.)

As in any analysis that is based on linear equations, Structural Equation methods utilize matrix operations that step by step formulate the model structure. Using observed and latent variables, an SEM system can be expressed as follows (Haenlein et al, 2004; Silva et al, 2008):

$$\eta = B\eta + \Gamma x + \zeta \qquad \text{(Eq. 7.9)}$$

Where:

- η is the vector of *p* endogenous variables;
- X is a vector of *q* exogenous variables;
- ζ is a vector of *p* disturbances (errors)
- B is (*p*x*p*) matrix containing the coefficients for the equations relating the endogenous variables;
- Γ is a (*p*x*q*) matrix containing the regression coefficients for the equations relating endogenous and exogenous variables.

The measured (observed) variables x and y can be decomposed into the latent variables as follows (Haenlein et al, 2004; Johnson et al, 2007):

$$x = \wedge_x \zeta + \delta, \qquad \text{(Eq. 7.10)}$$

$$y = \wedge_y \eta + \varepsilon \qquad \text{(Eq. 7.11)}$$

Where:

- $\wedge_x$ and $\wedge_y$ are the scores of x and y respectively (to be discussed in depth later)
- δ and ε are error terms

- ζ and η are latent variables (constructs)

7.7.1. Methods of SEM Analysis

There are two distinct techniques by which the rigorous SEM analysis can be performed (W. Chin 1998; Kline, 2005; Haenlein et al, 2004):

1. Covariance based analysis and
2. variance based (component based, also known as Partial Least Squares) analysis

These techniques differ in the statistical assumptions they are based on and on the nature of the goodness-of-fit statistics they produce with which the model validity is assessed.

In **covariance-based modeling**, the relationship between exogenous and endogenous variables is assessed so as to fit the covariance structure of the proposed model to a best possible fit covariance structure. This means, the covariance based SEM tests a previously assessed (a priori) relationship/model against population estimates derived from the sample. The modeling process examines whether the data is statistically congruous with an assumed multivariate distribution. This requires the proposed model to have a sound theoretical base. The objective of covariance based SEM is to show that the complete set of paths as specified in the proposed model being analyzed is valid (Gefen et al, 2000; Kline, 2005).

On the other hand, the objective of **Partial Least Squares (PLS) based (variance based)** SEM - as in other linear regression methods - is to reject the null hypothesis which states that the coefficients of the independent variables for a proposed model are invalid (Allen 1997; Gefen et al, 2000). To achieve this, PLS based SEM computes statistics such as R^2 and t-values to measure the goodness of fit of the model. The PLS algorithm was first introduced by H. Wold in 1975. The algorithm focuses on maximizing the variance of the dependent variables - explained by the independent variables – as opposed to reproducing the empirical covariance matrix as in the covariance-based approach. Like any SEM, a PLS model consists of a structural part, which reflects the relationships between the latent variables, and a measurement component, which shows how the latent variables and their indicators are related; PLS also has a third component known as weight relations, which are used to estimate case values for the latent variables (Haenlein et al, 2004; Gefen et al, 2000; Maitra et al 2008). PLS is designed to explain variance using Ordinary Least Squares (OLS) as an estimation technique which allows performing an iterative set of factor and path analyses until the difference in the average R^2 of the constructs (components) becomes insignificant (Gefen et al, 2000).

Through the Ordinary Least Squares iteration, PLS investigates components that will minimize the residual variance of all the dependent variables in the model, analyzing one construct at a time. Because of the iterative nature of the process, PLS algorithm is not impacted by deviations of variables from multivariate normal distribution which also makes it less susceptible to smaller sample sizes.

In Partial Least Squares (PLS) - also known as Projection to Latent Structures - the X variables (the predictors) are reduced to principal components, as are the Y variables (the dependents). The components of X are used to predict the scores on the Y components, and the predicted Y component scores are used to predict the actual values of the Y variables. In constructing the principal components of X, the PLS algorithm iteratively maximizes the strength of the relation of successive pairs of X and Y component scores by maximizing the covariance of each X-score with the Y variables. This strategy means that while the original X variables may be multicollinear, the 'new' X components used to predict Y will be orthogonal (uncorrelated.) (Garson, 2010; Haenlein et al, 2004)

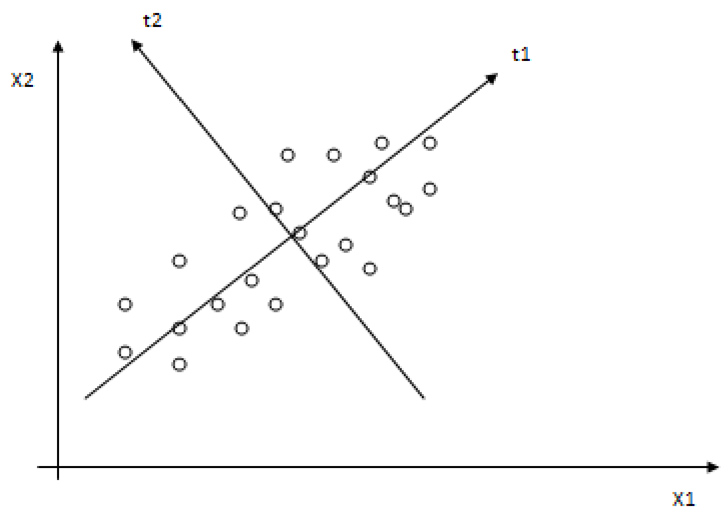

Figure 7-6. Illustration of the projection to components (X-data)

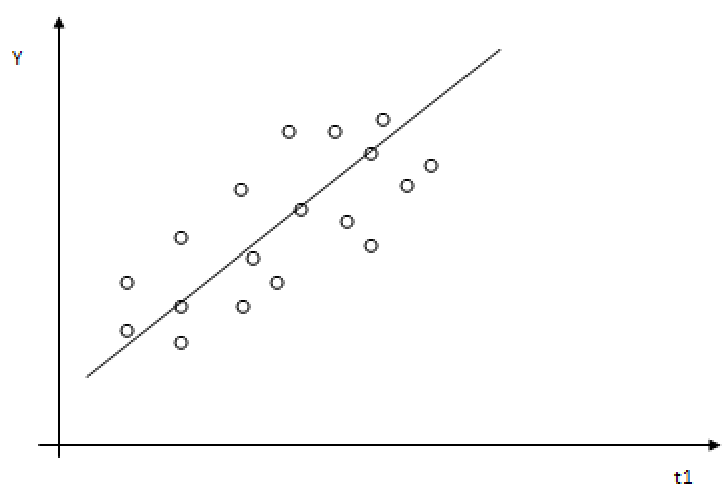

Figure 7-7. Illustration of the projection to components (Y-data)

In PLS, the main purpose of the model development process is to find uncorrelated components of the predictor set of variables so as to regress these components against the dependent variables and come up with a valid model. "Consider a data set with response variables Y (in matrix form) and a large number of predictor variables X (in matrix form), some of which are highly correlated (multicollinearity). A PLS algorithm computes the factor score matrix T=XW for an appropriate weight matrix W, and then considers the linear regression model Y=TQ+E, where Q is a matrix of regression coefficients (loadings) for T, and E is an error (noise) term. Once the loadings Q are computed, the above regression model is equivalent to Y=XB+E, where B=WQ, which can then be used as a linear predictive model." (Statsoft Handbook; SAS/STAT User's Guide)

In the context of this research project, the Partial Least Squares (PLS) based SEM approach will be used to assess the relationship between the predictor (building blocks of the exogenous constructs) variables and the response (building blocks of the endogenous constructs) variables. The reasons for choosing this method for modeling are:

1. Prediction rather than confirmation – The research project presented in this dissertation is based on prior determination of the relationship among the response variable (Pavement friction) and select independent variables (e.g. British Polish Number). There is currently no empirical model that can be used to simultaneously formulate and evaluate the impact of a combination of the various variables identified in this dissertation to the expected friction performance of a

pavement. Consequently PLS - being well suited for prediction modeling - was chosen as analysis tool to build and verify an empirical model.

2. Sample size – Among the SEM methods available, Partial Least Squares method works well with small sample sizes (Gefen et al, 2000). Given the limited number of observations that resulted from work in the preceding chapters, this method was found to be more appropriate.

3. Variance in response variables – Being a variance based SEM method, PLS is better suited to explain the variance in Y (the response variable matrix) and make use of this variance in component analysis.

4. Easier to validate – The process of model validation is more straightforward in PLS since it is possible to do jackknifing or bootstrapping for cross-validation of the resulting model.

5. Availability of application – There are readily available applications that perform PLS regression with cross validation including SYSTAT®, The Unscrambler®, Matlab®, and Excel Add-in programs such as XLSTAT.

7.7.2. Component Extraction

To demonstrate how PLS works, assume X (a set of predictor variables) is a $n \times p$ matrix and Y (a set of response variables) is a $n \times q$ matrix. The PLS technique works by successively extracting factors from both X and Y such that covariance between the extracted factors is maximized (Abdi, 2003; Maitra et al 2008; Statsoft Handbook). Even though PLS method can work with multivariate response variables (i.e., when Y is an $n \times q$ vector with $q>1$), it will be assumed in this case that there is a single response (target) variable (Y is $n \times 1$ and X is $n \times p$ matrix.)

The purpose of the PLS algorithm will then be to find a linear decomposition of *X* and *Y* such that (Maitra et al 2008):

$$X = TP^T + E \text{ and} \qquad \text{(Eq. 7.12)}$$

$$Y = UQ^T + F, \qquad \text{(Eq. 7.13)}$$

Where:

$T_{n \times r}$ = X-scores	;	$U_{n \times r}$ = Y-scores	
$P_{p \times r}$ = X-loadings	;	P^T = Denotes the Transpose of P	
$Q_{1 \times r}$ = Y-loadings	;	Q^T = Denotes the Transpose of Q	
$E_{n \times p}$ = X-residuals	;	$F_{n \times 1}$ = Y-residuals	

The decomposition of the X and Y variables into their respective scores is progressed so as to eventually maximize covariance between T (the X-scores) and U (the Y-scores). In this manner, there will be no (or very insignificant) correlation among the resulting scores, thereby eliminating the problem of multicollinearity in the model. Though there are many ways (algorithms) available to solve the PLS problem, all algorithms follow an iterative process to extract the X-scores and Y-scores.

In PLS Regression, the factors/scores for X and Y are extracted successively. The number of factors extracted (r) depends on the rank of X and Y. Rank of a matrix is defined as the maximum number of linearly independent rows or columns of a matrix. In this case, Y is a vector (an nx1 matrix), so only all possible X factors will be extracted. Depending on the number of variables (independent) that are correlated with one another, the rank of $X n\times p$, will be less than or equal to p.

Each extracted set of x-scores is some linear combination of X. For example, the first extracted x-score t of X is of the form t = Xw, where w is the eigen vector corresponding to the first eigen value of X^TYY^TX. Similarly the first y-score is u=Yc, where c is the eigen vector corresponding to the first eigen value of Y^TXX^TY (Maitra et al 2008).

> [Eigen Vector is defined as a non-zero vector which, after being multiplied by a matrix, remains proportional to the original vector – it changes only in magnitude and not direction. The factor by which each eigenvector is multiplied is called Eigen value (Johnson et al, 2007.) For instance, if *A* is a square matrix, a non-zero vector **v** is an eigenvector of *A* if there is a scalar λ (lambda) such that: Av = λ v]

Where:

X^T = the transpose of Matrix X

X^TY = the covariance of Matrices X and Y.

Once the first factors have been extracted, the original values of X and Y will be diminished/deflated to:

$X_1 = X - tt^T X$, and (Eq. 7.14)

$Y_1 = Y - tt^T Y$ (Eq. 7.15)

The above process is now repeated to extract the second PLS factors. This process continues until all possible latent factors t and u are extracted, i.e., until X is reduced to a null matrix. The rank of the matrix X determines the number of latent factors extracted. In most cases, a good fit is obtained after the first two or three components are extracted (Matlab Handbook.)

Comparable to the Ordinary Least Squares model significance tests, the validity and significance of a PLS Regression model is assessed using the following measures:

- Proportion of variance explained by scores: This value measures the proporiton of variance explained (both for the predictor and repsonse varibles) by the kth factor and is computed as follows:

$$\mathrm{VarProp}\ k\ (Y) = \frac{SS_k(Y)}{\mathrm{trace}(Y'Y)}, \text{ and } \mathrm{VarProp}\ k\ (X) = \frac{SS_k(X)}{\mathrm{trace}(X'X)} \quad \text{(Eq. 7.16)}$$

Where:

$$SS\ k\ (Y) = \left(t'(k)t(k)\right) \cdot \left(c'(k)c(k)\right) \quad \text{(Eq. 7.18)}$$

$$SS\ k\ (X) = \left(t'(k)t(k)\right) \cdot \left(p'(k)p(k)\right) \quad \text{(Eq. 7.19)}$$

The "cumulative X variance" is the percent of variance in the X variable(s) accounted for by the latent factors. The "cumulative Y variance" is the percent of variance in the Y variable(s) accounted for by the latent factors. These measures, calcualted as cumulative R-square ($CumR^2X$ and $CumR^2Y$) in regression, are computed as follows:

$$\text{CumVarProp}_k(Y) = \sum_{i=1}^{k} Var_i(Y), \text{ and,}$$

$$\text{CumVarProp}_k(X) = \sum_{i=1}^{k} Var_i(X) \qquad \text{(Eq. 7.20)}$$

- Variable Importance in Projection (VIP) (for individual predictor variables).The Variable Importance in Projection (VIP) coefficients measure the relative importance of each predictor (X_i) variable for each X factor (t_i) in the prediction model. As a result, VIP coefficients represent the importance of each X variable in fitting both the X- and Y-scores as the Y-scores are predicted from the X-scores. The rule rule of thumb for the threshold value of VIP is 0.8 (Wold, 1994). Any independent variable with a VIP value of less than 0.8 and/or very small regression coefficient may be considered insignifcnat and can therefiore be removed from the model.

- Distance to the Model: This measure evaluates the 'distance' of each variable to the model and is computed as follows:

$$DModX\ i = \sqrt{e'_i e_i} \quad \text{and} \quad DModY\ i = \sqrt{f_i f_i} \qquad \text{(Eq. 7.21)}$$

Where e_i and f_i are errors of prediction.

- The PRESS (Predictive Error Sum of Squares) statistic: This measure, computed as the sums of squares of the prediction residuals for observations not used in model development, is used as another measure of model validity as a whole.

- The Q^2cum index : This index measures the global contribution of the h first components to the predictive quality of the model. This index is computed as :

$$Q^2cum(h) = 1 - \prod_{j=1}^{h} \frac{\sum_{k=1}^{q} PRESS_{kj}}{\sum_{k=1}^{q} SSE_{K(j-1)}} \qquad (Eq. 7.22)$$

Where:

PRESS = the Predictive Sum of Squares

SSE = the Sum of Squares of Error

- The coefficient of determination (R^2) between the actual and predicted variables is also a useful measure of goodness of fit.

The complete PLS algorithm and discussion of model significance measures is included in Appendix D.

7.8. Partial Least Squares (PLS) Regression modeling

7.8.1. Preliminary PLS Modeling

In the preliminary modeling analysis, 12 various modeling approaches were considered using datasets created as follows:

- All **original (untransformed)** predictor variables with all **nine** sieves representing "**Percent Passing**" (Dataset used in Model number M1)
- All **original (untransformed)** predictor variables with all **nine** sieves representing " **Percent Retained"** (Dataset used in Model number M2)
- All predictor variables (including some that are transformed) with all **ten** sieves representing "**Percent Passing**" (Dataset used in Model number M3)
- All predictor variables (including some that are transformed) with all **ten** sieves representing " **Percent Retained**" (Dataset used in Model number M4)

The models numbered 5 through 12 are mere variations of the datasets discussed above. The following table shows the types and description of the various variables used in the model development:

Table 7-34. List of Variables and their descriptions

	Original	Modified (Transformed)	Description
Response, (Dependent) Variable	Terminal ESAL	Terminal ESAL	Equivalent Standard Axle Load (ESAL) at FN=32, for specific material and route
Predictor (Explanatory, Independent) Variables	Blend %	Blend %	Proportion of major aggregate source
	BPN	-EXP(0.6*BPN)	British Pendulum Number
	PV	-Exp(PV)	Polish Value
	LAA	LAA	Los Angeles Abrasion
	Soundness	Ln (Soundness)	Magnesium Sulphate Soundness
	Binder Grade	Binder Grade	Binder Grade used in HMA mix
	Binder % (AC)	Exp (Binder %)	Asphalt Content in HMA Mix
	AESAL	(AESAL)^-0.4	Average Daily Equivalent Standard Axle Load (Computed from AADT)
	12.5	12.5	Sieve Size = 12.5 mm (1/2 Inch)
	9.5	9.5	Sieve Size = 9.5 mm (3/8 Inch)
	4.75	4.75	Sieve Size = 4.75.5 mm (No. 4)
	2.36	2.36	Sieve Size = 2.36 mm (No. 8)
	1.18	1.18	Sieve Size = 1.18 mm (No. 16)
	0.6	0.6	Sieve Size = 0.6 mm (No. 30)
	0.3	0.3	Sieve Size = 0.3 mm (No. 50)
	0.15	0.15	Sieve Size = 0.15 mm (No. 100)
	0.075	0.075	Sieve Size = 0.075 mm (No. 200)
	Pan	Pan	Sieve Size = 0 mm

Description of the 12 preliminary PLS modeling approaches is presented below:

M1 : In this first approach, all **eight** independent variables (Blend Percentage, BPN, PV, LAA, Soundness, Binder Grade, Asphalt content (Binder %) , Average Daily Equivalent Standard Axle Load (AESAL)) in their original form as well as all **nine** sieves that represent Percent **Pass** gradation – 12.5mm, 9.5mm, 4.75mm , 2.36mm, 1.18mm, 0.6m, 0.3mm, 0.15mm, 0.075mm - were used in the Partial Least Square Regression Modeling .

M2: In this approach, all **eight** independent variables (Aggregate Blend Percentage, BPN, PV, LAA, Soundness, Binder Grade, Asphalt Content, Average Daily Equivalent

Standard Axle Load) in their original form as well as all **ten** sieves that represent Percent **Retained** gradation – 12.5mm, 9.5mm, 4.75mm, 2.36mm, 1.18mm, 0.6m, 0.3mm, 0.15mm, 0.075mm, 0 mm (Pan) - were used in the Partial Least Square Regression Modeling.

M3: In this approach, all **eight** independent variables, some of which are transformed for better correlation with the dependent variable, were used. The **transformed** variables are (-Exp (0.6*BPN), -Exp (PV), Ln (Soundness), Exp (Binder %), AESAL^-0.4). The **untransformed** variables that were used in their original form are (Blend Percentage, LAA and Binder Grade). Also, all **nine** sieves that represent Percent **Pass** gradation – 12.5mm, 9.5mm, 4.75mm, 2.36mm, 1.18mm, 0.6m, 0.3mm, 0.15mm, 0.075mm - were used in the Partial Least Square Regression Modeling.

M4: In this approach, all **eight** independent variables, some of which are transformed for better correlation, were used. The **transformed** variables are (-Exp (0.6*BPN), -Exp (PV), Ln (Soundness), Exp (Binder %), AESAL^-0.4). The **untransformed** variables are (Blend Percentage, LAA and Binder Grade). Also, all **ten** sieves that represent Percent **Retained** gradation – 12.5mm, 9.5mm, 4.75mm, 2.36mm, 1.18mm, 0.6m, 0.3mm, 0.15mm, 0.075mm, 0 mm (Pan) - were used in the Partial Least Square Regression Modeling .

M5: In this approach, all **eight** independent variables (Blend Percentage, BPN, PV, LAA, Soundness, Binder Grade, Asphalt content (Binder %), Average Daily Equivalent Standard Axle Load (AESAL)) in their original form as well as **five** sieves that represent Percent **Pass** gradation and that yielded a **VIP** (Variable Importance in the Projection) value of greater than **0.8** were used (Russolillo, 2009; Wold 1994). The selected sieves based on VIP values are – 12.5mm, 9.5mm, 1.18mm, 0.6m, 0.3mm.

M6: In this approach, all **eight** independent variables (Blend Percentage, BPN, PV, LAA, Soundness, Binder Grade, Asphalt content (Binder %) , Average Daily Equivalent Standard Axle Load(AESAL)) in their original form as well as **four** sieves that represent Percent **Retained** gradation and that yielded a **VIP** (Variable Importance in the Projection) value of greater than **0.8** were used. The selected sieves based on VIP values are – 12.5mm, 4.75mm, 0.3mm and 0.15mm.

M7: In this approach, all **eight** independent variables, some of which are transformed for better correlation, were used. The **transformed** variables are (-Exp (0.6*BPN), -Exp (PV), Ln (Soundness), Exp (Binder %), AESAL^-0.4). The **untransformed** variables are (Blend Percentage, LAA and Binder Grade). Also, **five** sieves that represent Percent **Pass** gradation and that yielded a **VIP** (Variable Importance in the Projection) value of greater than **0.8** were used. The selected sieves based on VIP values are – 12.5mm, 9.5mm, 1.18mm, 0.6m, 0.3mm.

M8: In this approach, all **eight** independent variables, some of which are transformed for better correlation, were used. The **transformed** variables are (-Exp(0.6*BPN), -Exp(PV), Ln(Soundness), Exp (Binder%), AESAL^-0.4). The **untransformed** variables are (Blend Percentage, LAA and Binder Grade).Also **four** sieves that represent Percent **Retained** gradation and that yielded a **VIP** (Variable Importance in the Projection) value of greater than **0.8** were used. The selected sieves based on VIP values are – 12.5mm, 4.75mm, 0.3mm and 0.15mm.

M9: In this approach, all **eight** independent variables (Blend Percentage, BPN, PV, LAA, Soundness, Binder Grade, Asphalt content (Binder %), Average Daily Equivalent Standard Axle Load(AESAL)) in their original form as well as **one** sieve (4.75mm) that represents Percent **Pass** gradation was used. This sieve was selected because it is the sieve that, on average, passes nearly 50% of the material and it exhibits significant variability amongst the various suppliers as shown table 10 above.

M10: In this approach, all **eight** independent variables (Blend Percentage, BPN, PV, LAA, Soundness, Binder Grade, Asphalt content (Binder %), Average Daily Equivalent Standard Axle Load(AESAL)) in their original form as well as **one** sieve (4.75mm) that represents Percent **Retained** gradation was used. This sieve was selected because it is the sieve that represents the peak values of the gradation distribution curve as shown in figure 5 above.

M11: In this approach, all **eight** independent variables, some of which are transformed for better correlation, were used. The **transformed** variables are (-Exp (0.6*BPN), -Exp (PV), Ln (Soundness), Exp (Binder %), AESAL^-0.4). The **untransformed** variables are (Blend Percentage, LAA and Binder Grade).Also, **one** sieve (4.75mm) that represents Percent **Pass** gradation was used.

M12: In this approach, all **eight** independent variables, some of which transformed for better correlation, were used. The **transformed** variables are (-Exp(0.6*BPN), -Exp(PV), Ln(Soundness), Exp (Binder%), AESAL^-0.4). Also, **one** sieve (4.75mm) that represents Percent **Retained** gradation was used.

Table 7-35. Complete Dataset of "original" predictor variables with all "Percent Pass" sieves (dataset for M1)

Dependent Variable	Predictor Variables (other than sieves)								Sieve Size (mm): Percent Pass								
Terminal ESAL	Blend %	BPN	PV	LAA	Soundness	Binder Grade	Binder % (AC)	AESAL	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075
438,990	100	35	6	20	0.6	1	5.3	980	95	85	54	35	22	15	10	8	6.5
1,062,723	100	34	10	18	1	2	5.3	462	91	77	52	33	21	14	10	8	6.1
1,558,218	100	24	6	22	0.2	1	5.3	429	95	87	66	40	25	15	9	7	6
2,118,493	65	22	6	22	0.4	1	4.3	1081	98	83	44	30	23	17	11	7	4.1
2,363,108	100	26	5	15	2.8	1	5.7	815	97	86	53	35	22	14	10	7	5.8
3,045,205	72	27	6	18	1.2	1	5.3	657	100	95	58	32	24	18	12	8	5.6
3,208,372	75	22	5	18	4.5	2	4.8	362	97	83	52	36	23	17	10	6	4.7
3,477,567	75	27	6	18	1.2	1	4.8	1101	97	83	44	26	21	17	12	8	5
3,558,538	85	21	4	25	0.7	2	5.4	862	100	97	70	48	30	22	16	11	6.3
3,672,489	68	31	8	14	0.1	1	5.4	732	100	99	66	41	29	20	12	9	6.9
5,810,328	75	26	8	11	0.3	3	4.8	967	99	90	52	37	26	16	10	6	4.9

Table 7-36. Complete Dataset of "original" predictor variables with all "Percent Retained" sieves (dataset for M2)

Dependent Variable	Predictor Variables (other than sieves)								Sieve size (mm) :Percent Retained									
Terminal ESAL	Blend %	BPN	PV	LAA	Soundness	Binder Grade	Binder %	AESAL	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075	Pan
438,990	100	35	6	20	0.6	1	5.3	980	5	10	31	19	13	7	5	2	1.5	6.5
1,062,723	100	34	10	18	1	2	5.3	462	9	14	25	19	12	7	4	2	1.9	6.1
1,558,218	100	24	6	22	0.2	1	5.3	429	5	8	21	26	15	10	6	2	1	6
2,118,493	65	22	6	22	0.4	1	4.3	1081	2	15	39	14	7	6	6	4	2.9	4.1
2,363,108	100	26	5	15	2.8	1	5.7	815	3	11	33	18	13	8	4	3	1.2	5.8
3,045,205	72	27	6	18	1.2	1	5.3	657	0	5	37	26	8	6	6	4	2.4	5.6
3,208,372	75	22	5	18	4.5	2	4.8	362	3	14	31	16	13	6	7	4	1.3	4.7
3,477,567	75	27	6	18	1.2	1	4.8	1101	3	14	39	18	5	4	5	4	3	5
3,558,538	85	21	4	25	0.7	2	5.4	862	0	3	27	22	18	8	6	5	4.7	6.3
3,672,489	68	31	8	14	0.1	1	5.4	732	0	1	33	25	12	9	8	3	2.1	6.9
5,810,328	75	26	8	11	0.3	3	4.8	967	1	9	38	15	11	10	6	4	1.1	4.9

Table 7-37. Complete dataset of "modified (transformed)" predictor variables with all "Percent Pass" sieves (dataset for M3)

Dependent Variable	Predictor Variables (other than sieves): transformed as needed								Sieve size (mm) :Percent Pass								
Terminal ESAL	Blend %	-EXP (0.6*BPN)	-Exp (PV)	LAA	Ln (Soundness)	Binder Grade	Exp (Binder %)	(AESAL)^-0.4	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075
438990	100	-1.30E+09	-403.429	20	-0.51083	1	200.3368	0.063606	95	85	54	35	22	15	10	8	6.5
1062723	100	-7.20E+08	-22026.5	18	0	2	200.3368	0.085918	91	77	52	33	21	14	10	8	6.1
1558218	100	-1.80E+06	-403.429	22	-1.60944	1	200.3368	0.088533	95	87	66	40	25	15	9	7	6
2118493	65	-5.40E+05	-403.429	22	-0.91629	1	73.69979	0.061159	98	83	44	30	23	17	11	7	4.1
2363108	100	-6.00E+06	-148.413	15	1.029619	1	298.8674	0.068474	97	86	53	35	22	14	10	7	5.8
3045205	72	-1.10E+07	-403.429	18	0.182322	1	200.3368	0.074659	100	95	58	32	24	18	12	8	5.6
3208372	75	-5.40E+05	-148.413	18	1.504077	2	121.5104	0.094777	97	83	52	36	23	17	10	6	4.7
3477567	75	-1.10E+07	-403.429	18	0.182322	1	121.5104	0.06072	97	83	44	26	21	17	12	8	5
3558538	85	-3.00E+05	-54.5982	25	-0.35667	2	221.4064	0.066968	100	97	70	48	30	22	16	11	6.3
3672489	68	-1.20E+08	-2980.96	14	-2.30259	1	221.4064	0.071487	100	99	66	41	29	20	12	9	6.9
5810328	75	-6.00E+06	-2980.96	11	-1.20397	3	121.5104	0.06394	99	90	52	37	26	16	10	6	4.9

Table 7-38. Complete dataset of "modified (transformed)" predictor variables with all "Percent Retained" sieves (dataset for M4)

Dependent Variable	Predictor Variables (other than sieves): transformed as needed								Sieve size (mm) :Percent Retained									
Terminal ESAL	Blend %	-EXP (0.6*BPN)	-Exp (PV)	LAA	Ln (Sound.)	BG	Exp (Binder %)	(AESAL)^-0.4	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075	Pan
438990	100	-1.30E+09	-403.429	20	-0.51083	1	200.3368	0.063606	5	10	31	19	13	7	5	2	1.5	6.5
1062723	100	-7.20E+08	-22026.5	18	0	2	200.3368	0.085918	9	14	25	19	12	7	4	2	1.9	6.1
1558218	100	-1.80E+06	-403.429	22	-1.60944	1	200.3368	0.088533	5	8	21	26	15	10	6	2	1	6
2118493	65	-5.40E+05	-403.429	22	-0.91629	1	73.69979	0.061159	2	15	39	14	7	6	6	4	2.9	4.1
2363108	100	-6.00E+06	-148.413	15	1.029619	1	298.8674	0.068474	3	11	33	18	13	8	4	3	1.2	5.8
3045205	72	-1.10E+07	-403.429	18	0.182322	1	200.3368	0.074659	0	5	37	26	8	6	6	4	2.4	5.6
3208372	75	-5.40E+05	-148.413	18	1.504077	2	121.5104	0.094777	3	14	31	16	13	6	7	4	1.3	4.7
3477567	75	-1.10E+07	-403.429	18	0.182322	1	121.5104	0.06072	3	14	39	18	5	4	5	4	3	5
3558538	85	-3.00E+05	-54.5982	25	-0.35667	2	221.4064	0.066968	0	3	27	22	18	8	6	5	4.7	6.3
3672489	68	-1.20E+08	-2980.96	14	-2.30259	1	221.4064	0.071487	0	1	33	25	12	9	8	3	2.1	6.9
5810328	75	-6.00E+06	-2980.96	11	-1.20397	3	121.5104	0.06394	1	9	38	15	11	10	6	4	1.1	4.9

The following table summarizes the output from the 12 modeling approaches:

Table 7-39. Summary of outputs from the Preliminary PLS Model Development Process*

Method No/ Model No	Descrip.	Total Indep-endent Variables	No of Comp-onents	No of sieves	Cum Q^2 Indx	R^2 Cum Y	R^2 Cum X	No of var. with VIP>0.8	No Sieves With VIP>0.8	R^2	Q^2/R^2	VIP >0.8/Total	Sieves VIP> 0.8/Total	Abs (1-R^2Pass /R^2Ret'd)
M1	Original Variables - all sieves (Pass)	17	2	9	0.474	0.868	0.511	9(t1); 9(t2)	5	0.87	0.546	0.529	0.556	0.176
M2	Original Variables - all sieves (Retained)	18	1	10	0.369	0.742	0.256	8(t1)	4	0.74	0.498	0.471	0.400	
M3	Modified Variables - all sieves (Pass)	17	2	9	0.580	0.891	0.532	8(t1); 9(t2)	4(t1); 5(t2)	0.89	0.651	0.529	0.556	0.161
M4	Modified Variables - all sieves (Retained)	18	1	10	0.451	0.768	0.269	8(t1)	4	0.77	0.587	0.471	0.400	
M5	Original Variables - selected sieves (Pass)	13	2	5	0.597	0.898	0.507	7(t1); 9(t2)	4(t1); 5(t2)	0.90	0.665	0.692	1.000	0.007
M6	Original Variables - selected sieves (Retained)	12	2	4	0.538	0.904	0.486	8(t1); 8(t2)	4(t1); 4(t2)	0.90	0.595	0.667	1.000	

*See Appendix F for detailed PLS regression outputs.

Table 7-39: Summary of outputs from the preliminary PLS model development process* (continued)

Method No/ Model No	Descripti	Total Indep-endent Variables	No of Comp-onents	No of sieves	Cum Q^2 Indx	R^2 Cum Y	R^2 Cum X	No of var. with VIP>0.8	No Sieves With VIP>0.8	R^2	Q^2/R^2	No.(VIP>0.8)/Total #	No. Sieves (VIP>0.8) / Total	Abs (1-R2Pass /R2Ret'd)
M7	Modified Variables - selected sieves (Pass)	13	2	5	0.744	0.941	0.520	8(t1); 8(t2)	4(t1);4(t2)	0.94	0.790	0.615	0.800	0.000
M8	Modified Variables - selected sieves (Retained)	12	2	4	0.721	0.941	0.493	8(t1); 8(t2)	4(t1);4(t2)	0.94	0.766	0.667	1.000	
M9	Original Variables - Sieve#4 (Pass)	9	1	1	0.173	0.766	0.216	4(t1)	0.000	0.77	0.226	0.444	0.000	0.126
M10	Original Variables - Sieve#4 (Retained)	9	1	1	0.182	0.680	0.280	5(t1)	1 (#4)	0.68	0.268	0.556	1.000	
M11	Modified Variables - sieve #4 (Pass)	9	2	1	0.523	0.953	0.395	4(t1); 5(t2)	0.000	0.95	0.549	0.556	0.000	0.036
M12	Modified Variables - sieve #4 (Retained)	9	2	1	0.540	0.920	0.459	5(t1); 6(t2)	1(t1);1(t2) -- #4	0.92	0.587	0.667	1.000	

*See Appendix F for detailed PLS regression outputs.

Definition of terms and expressions used in the above table are presented below:

$CumQ^2Indx$ = A model quality index that measures the cumulated contribution of the **components**; It measures the **global contribution** of the **h** first components to the predictive quality of the **model** (and of the sub-models if there are several dependent variables).

R^2CumY = A model quality index which is the sum of the coefficients of determination between the **dependent** variables and the h first **components**. It is therefore a measure of the **explanatory power** of the **h** first components for the **dependent variables** of the model. Since there is only one dependent variable in this case, this value is the same as the **coefficient of determination** (R^2) for the model.

R^2CumX = another model quality index which is the sum of the coefficients of determination between the **explanatory** variables and the h first **components**. It is therefore a measure of the **explanatory power** of the **h** first components for the **explanatory variables** of the model.

R^2 = Overall Predictive measure of the model (the **coefficient of determination** between the **predicted** and **actual** dependent variable values).

Based on the output shown in the above table, six models were selected for further analysis using the following criteria:

i. Coefficient of determination (R^2) value: this index is one of the most important indicators of the goodness of fit for the research model.
ii. Model quality index (Q^2) value: this index is also important because it measures how well the extracted factors (components) 'replace' the observed variables.
iii. The ratio between the coefficient of determination (R^2) and the model quality index (Q^2) =(Q^2/R^2) :
iv. Ratio between the coefficients of determination (R^2) for the case where Percent Pass gradation was used versus where the Percent Retained gradation was used (R^2Pass/R^2Retained): The closest this ratio is to a value of one, the better the model validity as it indicates that the model is not sensitive to the type of gradation representation considered.
v. Total number of variables with Variable Importance in the Projection (VIP) value greater than 0.8: Commonly used threshold value for VIP=0.8 (Wold, 1994; Russolillo, 2009).
vi. Total Number of sieves with VIP >0.8

The following models satisfied all or most of the above criteria and were selected for further analysis:

- M5, M6, M7, M8, M11, M12

7.8.2. PLS Model Validation/Verification

7.8.2.1. Model Validation Phase 1

In the preliminary modeling process, 12 different methods/approaches were considered to assess the relationship between the predictor variables (independent) and the response (dependent) variable (Terminal ESAL). Based on the preliminary model selection criteria, six models were identified for further analysis. These models were evaluated for model verification by varying the number/type of observations and/or predictor variables and assessing the resulting models for validity. The first phase of model verification involved removing one observation from the dataset at a time and running Partial Least Square (PLS) regression on the remaining 10 observations. The predictor variables from the omitted observation are then plugged into the produced model and the resulting output (predicted Terminal ESAL) is compared with the actual (observed) Terminal ESAL. This process is repeated until enough observations have been analyzed. This model verification phase resulted in 36 sub models that were compared with one another for predictive qualities. Table 8 shows the models and sub models resulting from the model identification and preliminary verification/validation process.

Table 7-40. Summary of outputs from the first PLS model validation/verification phase

Model No	Description	Total Ind. Vars.	No of comp.	No of sieves	Cum Q^2 Indx	R^2 CumY	R^2 CumX	No.Var. with VIP>0.8	No. Sieves VIP>0.8	R^2	Predicted	Actual	Residual (Error)	% Error	Remark
M5	Modified Variables - sieve #4 (Pass)	13	2	5	0.597	0.898	0.507	7(t1); 9(t2)	4(t1); 5(t2)	0.898	N/A	N/A	N/A	N/A	All Observations Used
M5.1	M5Data with 10 observations; Removed Obs 1: TESAL=438990)	13	2	5	0.420	0.894	0.504	8(t1); 9(t2)	4(t1); 5(t2)	0.894	1597433	438990	- 1158444	263.9	10 out of 11 observations used
M5.2	M5Data with 10 observations; Removed Obs 5: TESAL=2363108)	13	2	5	0.587	0.896	0.527	8(t1); 9(t2)	4(t1); 5(t2)	0.896	1819405	2363108	543702.2	23.0	10 out of 11 observations used
M5.3	M5Data with 10 observations; Removed Obs 8: TESAL= 3477567)	13	2	5	0.747	0.959	0.521	8(t1); 9(t2)	4(t1); 5(t2)	0.959	2151264.	3477567	1326302	38.1	10 out of 11 observations used
M5.4	M5Data with 10 observations; Removed Obs 9: TESAL= 3558538)	13	2	5	0.538	0.912	0.492	8(t1); 8(t2)	4(t1); 4(t2)	0.912	3353054	3558538	205483.7	5.8	10 out of 11 observations used
M5.5	M5Data with 10 observations; Removed Obs 10: TESAL= 3672489)	13	2	5	0.617	0.904	0.548	8(t1); 8(t2)	4(t1); 4(t2)	0.904	4217799	3672489	-545310	14.8	10 out of 11 observations used
M5.6	M5Data with 10 observations; Removed Obs 11: TESAL= 5810328)	13	1	5	0.558	0.709	0.372	7(t1)	5(t1)	0.709	2785924	5810328	3024403	52.1	10 out of 11 observations used

Table 7-40: Summary of outputs from the first PLS model validation/verification phase (Continued)

Model No	Description	Total Ind. Vars.	No comp.	No of sieves	Cum Q^2 Indx	R^2 CumY	R^2 CumX	No. var. with VIP>0.8	No Sieves VIP>0.8	R^2	Predicted	Actual	Residual (Error)	% Error	Remark
M6	Modified Variables - sieve #4 (Retained)	12	2	4	0.538	0.904	0.486	8(t1); 8(t2)	4(t1); 4(t2)	0.904	N/A	N/A	N/A	N/A	All Observations Used
M6.1	M6Data with 10 observations; Removed Obs 1: TESAL=438990)	12	2	4	0.399	0.895	0.494	8(t1); 8(t2)	4(t1); 4(t2)	0.895	1603209	438990	-1164219	265.2	10 out of 11 observations used
M6.2	M6Data with 10 observations; Removed Obs 5: TESAL=2363108)	12	2	4	0.524	0.903	0.511	8(t1); 8(t2)	4(t1); 4(t2)	0.903	2013423	2363108	349684.8	14.8	10 out of 11 observations used
M6.3	M6Data with 10 observations; Removed Obs 8: TESAL= 3477567)	12	3	4	0.688	0.985	0.616	8(t1); 8(t2); 9(t3)	4(t1); 4(t2); 4(t3)	0.985	2039165	3477567	1438402	41.4	10 out of 11 observations used
M6.4	M6Data with 10 observations; Removed Obs 9: TESAL= 3558538)	12	2	4	0.601	0.908	0.547	7(t1); 7(t2)	4(t1); 4(t2)	0.908	2512964	3558538	1045574	29.4	10 out of 11 observations used
M6.5	M6Data with 10 observations; Removed Obs 10: TESAL= 3672489)	12	2	4	0.493	0.901	0.532	8(t1); 8(t2)	4(t1); 4(t2)	0.901	3502956	3672489	169532.9	4.6	10 out of 11 observations used
M6.6	M6Data with 10 observations; Removed Obs 11: TESAL= 5810328)	12	1	4	0.462	0.696	0.347	6(t1);	4(t1)	0.696	3188598	5810328	2621729	45.1	10 out of 11 observations used

Table 7-40: Summary of outputs from the first PLS model validation/verification phase (Continued)

Model No	Description	Total Ind. Vars.	No comp.	No of sieves	Cum Q^2 Indx	R^2 CumY	R^2 CumX	No. var. with VIP>0.8	No Sieves VIP>0.8	R^2	Predicted	Actual	Residual (Error)	% Error	Remark
M7	Modified Variables - selected sieves (Pass)	13	2	5	0.744	0.941	0.520	8(t1); 8(t2)	4(t1); 4(t2)	0.941	N/A	N/A	N/A	N/A	All Observations Used
M7.1	M7Data with 10 observations; Removed Obs 1: TESAL=438990)	13	2	5	0.680	0.931	0.526	10(t1); 10(t2)	4(t1); 4(t2)	0.931	923265	438990	-484276	110.3	10 out of 11 observations used
M7.2	M7Data with 10 observations; Removed Obs 5: TESAL=2363108)	13	2	5	0.713	0.933	0.547	8(t1); 8(t2)	4(t1); 4(t2)	0.933	1872520	2363108	490587.2	20.8	10 out of 11 observations used
M7.3	M7Data with 10 observations; Removed Obs 8: TESAL= 3477567)	13	3	5	0.884	0.995	0.637	8(t1); 8(t2); 8(t3)	4(t1); 4(t2); 4(t3)	0.995	2233785	3477567	1243782	35.8	10 out of 11 observations used
M7.4	M7Data with 10 observations; Removed Obs 9: TESAL= 3558538)	13	2	5	0.749	0.952	0.535	8(t1); 8(t2)	4(t1); 4(t2)	0.952	3070205	3558538	488332.7	13.7	10 out of 11 observations used
M7.5	M7Data with 10 observations; Removed Obs 10: TESAL= 3672489)	13	2	5	0.733	0.944	0.525	8(t1); 8(t2)	4(t1); 4(t2)	0.944	4110491	3672489	-438002	11.9	10 out of 11 observations used
M7.6	M7Data with 10 observations; Removed Obs 11: TESAL= 5810328)	13	2	5	0.708	0.924	0.515	7(t1); 8(t2)	5(t1); 5(t2)	0.924	3658971	5810328	2151356	37.0	10 out of 11 observations used

Table 7-40: Summary of outputs from the first PLS model validation/verification phase (Continued)

Model No	Description	Total Ind. Vars.	No comp.	No of sieves	Cum Q^2 Indx	R^2 CumY	R^2 CumX	No. var. with VIP>0.8	No Sieves VIP>0.8	R^2	Predicted	Actual	Residual (Error)	% Error	Remark
M8	Modified Variables - selected sieves (Retained)	12	2	4	0.721	0.941	0.493	8(t1); 8(t2)	4(t1); 4(t2)	0.941	N/A	N/A	N/A		All Observations Used
M8.1	M8Data with 10 observations; Removed Obs 1: TESAL=438990)	12	2	4	0.653	0.923	0.522	10(t1); 9(t2)	4(t1); 3(t2)	0.923	1089919	438990	-650929	148.3	10 out of 11 observations used
M8.2	M8Data with 10 observations; Removed Obs 5: TESAL=2363108)	12	2	4	0.690	0.939	0.518	8(t1); 8(t2)	4(t1); 4(t2)	0.939	2291640	2363108	71467.09	3.0	10 out of 11 observations used
M8.3	M8Data with 10 observations; Removed Obs 8: TESAL= 3477567)	12	3	4	0.868	0.996	0.598	8(t1); 8(t2); 8(t3)	4(t1); 4(t2); 4(t3)	0.996	2303290	3477567	1174276	33.8	10 out of 11 observations used
M8.4	M8Data with 10 observations; Removed Obs 9: TESAL= 3558538)	12	2	4	0.770	0.949	0.546	8(t1); 7(t2)	4(t1); 3(t2)	0.949	2604860	3558538	953677.5	26.8	10 out of 11 observations used
M8.5	M8Data with 10 observations; Removed Obs 10: TESAL= 3672489)	12	2	4	0.670	0.939	0.518	8(t1); 8(t2)	4(t1); 4(t2)	0.939	3525228	3672489	147260.9	4.0	10 out of 11 observations used
M8.6	M8Data with 10 observations; Removed Obs 11: TESAL= 5810328)	12	1	4	0.522	0.752	0.355	5(t1)	3(t1)	0.752	3381123	5810328	2429204	41.8	10 out of 11 observations used

Table 7-40: Summary of outputs from the first PLS model validation/verification phase (Continued)

Model No	Description	Total Ind. Vars.	No comp.	No of sieves	Cum Q^2 Indx	R^2 CumY	R^2 CumX	No. var. with VIP>0.8	No Sieves VIP>0.8	R^2	Predicted	Actual	Residual (Error)	% Error	Remark
M11	Modified Variables - sieve #4 (Pass)	9	2	1	0.523	0.953	0.395	4(t1); 5(t2)	0	0.953	N/A	N/A	N/A	N/A	All Observations Used
M11.1	M11Data with 10 observations; Removed Obs 1: TESAL=438990)	9	2	1	0.306	0.937	0.429	6(t1); 6(t2)	0	0.937	485388	438990	-46399	10.6	10 out of 11 observations used
M11.2	M11Data with 10 observations; Removed Obs 5: TESAL=2363108)	9	3	1	0.596	0.977	0.532	4(t1); 5(t2); 5(t3);	0	0.977	3120503	2363108	-757396	32.1	10 out of 11 observations used
M11.3	M11Data with 10 observations; Removed Obs 8: TESAL= 3477567)	9	2	1	0.675	0.988	0.372	4(t1); 4(t2)	0	0.988	2393993	3477567	1083574	31.2	10 out of 11 observations used
M11.4	M11Data with 10 observations; Removed Obs 9: TESAL= 3558538)	9	1	1	0.601	0.924	0.242	4t1)	0	0.924	1914169	3558538	1644368	46.2	10 out of 11 observations used
M11.5	M11Data with 10 observations; Removed Obs 10: TESAL= 3672489)	9	2	1	0.440	0.955	0.415	4(t1); 5(t2)	0	0.955	3645051	3672489	27437.76	0.7	10 out of 11 observations used
M11.6	M11Data with 10 observations; Removed Obs 11: TESAL= 5810328)	9	1	1	0.227	0.768	0.216	3(t1)	0	0.768	3461747	5810328	2348580	40.4	10 out of 11 observations used

Table 7-40: Summary of outputs from the first PLS model validation/verification phase (Continued)

Model No	Description	Total Ind. Vars.	No comp.	No of sieves	Cum Q^2 Indx	R^2 CumY	R^2 CumX	No. var. with VIP>0.8	No Sieves VIP>0.8	R^2	Predicted	Actual	Residual (Error)	% Error	Remark
M12	Modified Variables - sieve #4 (Retained)	9	2	1	0.540	0.920	0.459	5(t1); 6(t2)	1(t1); 1(t2) – Sieve #4	0.920	N/A	N/A	N/A		All Observations Used
M12.1	M12Data with 10 observations; Removed Obs 1: TESAL=438990)	9	2	1	0.328	0.890	0.502	7(t1); 7(t2)	1(t1); 1(t2)	0.890	709974	438990	-270985	61.7	10 out of 11 observations used
M12.2	M12Data with 10 observations; Removed Obs 5: TESAL=2363108)	9	4	1	0.747	0.983	0.718	5(t1); 6(t2); 6(t3); 6(t4)	1(t1); 1(t2); 1(t3); 1(t4)	0.983	3909674	2363108	-1546567	65.4	10 out of 11 observations used
M12.3	M12Data with 10 observations; Removed Obs 8: TESAL= 3477567)	9	2	1	0.541	0.944	0.437	5(t1); 6(t2)	1(t1); 1(t2)	0.944	2682958	3477567	794608.1	22.8	10 out of 11 observations used
M12.4	M12Data with 10 observations; Removed Obs 9: TESAL= 3558538)	9	2	1	0.704	0.959	0.506	5(t1); 5(t2)	1(t1); 1(t2)	0.959	1656488	3558538	1902050	53.5	10 out of 11 observations used
M12.5	M12Data with 10 observations; Removed Obs 10: TESAL= 3672489)	9	2	1	0.445	0.921	0.472	5(t1); 6(t2)	1(t1); 1(t2)	0.921	3180434	3672489	492054.8	13.4	10 out of 11 observations used
M12.6	M12Data with 10 observations; Removed Obs 11: TESAL= 5810328)	9	1	1	0.171	0.653	0.288	4t1)	1(t1);	0.653	3502881	5810328	2307446	39.7	10 out of 11 observations used

7.8.2.2. Model Validation Phase 2

The second phase of model verification involved using reduced number of predictor variables (other than the sieves) and running a Partial Least Squares (PLS) regression using these variables as predictors. Following the first model verification phase, two **sub models** (M8.2 and M8.5) that had very good predictive qualities - as measured by the predicted values that were obtained by plugging in the omitted values into the model, as well as the coefficient of determination (R^2) and the model quality index (Q^2) – were identified for the second phase of model verification. The dataset for these models were obtained by eliminating variables that did not meet the VIP (Variable Importance in the Projection) threshold of 0.8 in Model M8 (Wold, 1994; Russolillo, 2009). It can be inferred from the above tables that the outputs from Model M8 and its sub models had the highest number of predictor variables meeting the VIP threshold requirement of 0.8. Consequently, the following variables were selected to build the dataset:

Table 7-41. List of Variables and their descriptions

		Variable	**Description**
Response Variable		Terminal ESAL	Equivalent Standard Axle Load (ESAL) at FN=32, for specific material and route
Predictor Variables		Blend %	Proportion of major aggregate source
		-EXP(0.6*BPN)	British Pendulum Number
		LAA	Los Angeles Abrasion
		Binder Grade	Binder Grade used in HMA mix
	Percent **Retained** on Sieve	12.5	Sieve Size = 12.5 mm (1/2 Inch)
		4.75	Sieve Size = 4.75.5 mm (No. 4)
		0.3	Sieve Size = 0.3 mm (No. 50)
		0.15	Sieve Size = 0.15 mm (No. 100)

This dataset was used together with all eleven observations in PLS regression. The model from this analysis is designated as M13.0. The analysis was further expanded by eliminating the fifth and tenth observations interchangeably and running additional PLS regression analysis to ensure validity of this model. The models created by this analysis are designated as M13.1 and M13.2. The results of this model verification process are shown in Table 7-42.

Table 7-42. Summary of outputs from the second PLS model validation/verification phase

Model No	Description	Total Ind. Vars.	No comp.	No of sieves	Cum Q^2 Indx	R^2 CumY	R^2 CumX	No. var. with VIP>0.8	No Sieves VIP>0.8	R^2	Predicted	Actual	Residual (Error)	% Error	Remark
M13.0	Selected Modified Variables - selected sieves (Reduced M8 data)	8	2	4	0.783	0.932	0.628	7(t1); 7(t2)	3(t1); 3(t2)	0.932	N/A	N/A	N/A		All Observations Used
M13.1	M13Data with 10 observations; Removed Obs 5: TESAL=2363108)	8	2	4	0.770	0.931	0.660	7(t1); 7(t2)	3(t1); 3(t2)	0.931	2467299	2363108	-104192	4.4	10 out of 11 observations used
M13.2	M13Data with 10 observations; Removed Obs 10: TESAL= 3672489)	8	2	4	0.770	0.930	0.661	7(t1); 7(t2)	3(t1); 3(t2)	0.930	3450591	3672489	221897	6.0	10 out of 11 observations used

Note: Detailed output of the PLS regression for the final model dataset is included in Appendix E.

7.8.3. Final Model Selection

As can be seen in the output from the second model verification phase, all three modeling approaches yielded very close model quality indices. It is also evident that in this phase, all but one variable met the VIP threshold value of 0.8. The one variable that did not meet this threshold – in all three cases – was the sieve size 0.3mm; In fact, the VIP values for this variable are just short of 0.8, specifically 0.76, 0.77 and 0.72 on average for models 13.0, 13.1 and 13.2 respectively. Even though Model 13.1(model created using M13.0 data with only 10 observations, by removing the fifth observation from the dataset) yielded the lowest error value in terms of prediction of omitted observations, M13.2 (model created using M13.0 data with only 10 observations, by removing the tenth observation from the dataset) was selected to represent the relationship between the significant predictor variables and the response variable. The reasons this model was adopted as the ultimate model are the following:

- This model has practically the same R^2 (0.93) as Model 13.1
- This model resulted in comparably low prediction error (4.4% for M13.1 versus 6% for M13.2)
- In four out of the six models retained for further analysis (Models M6,M7, M11 and M12), the sub models that were created by removing the tenth observation yielded the least prediction error while in the remaining two they yielded the second least prediction error (see table 7-40)

Consequently, the final model relating the response variable (Terminal ESAL) to the various predictor variables identified in the previous sections is listed below:

$$Y = -8497.057 * X_1 + 7.551E-04 * -EXP\,(0.6 * X_2) - 123371.616 * X_3 + 831645.296 * X_4 - 99337.952 * X_5 + 21920.225 * X_6 + 80937.506 * X_7 + 285585.366 * X_8 + 2786940.049 \quad \text{(Eq. 7.23)}$$

Y: Terminal ESAL at FN=32

X_1: Percentage of Material from Primary Source [Blend Percentage]

X_2: British Pendulum Number [BPN]

X_3: Los Angeles Abrasion Value [LAA]

X_4: Binder Grade Code [1= PG 64-22, 2=PG 70-22, 3=PG 76-22]

X_5: Percent of Aggregate Retained on 12.5mm Sieve

X_6: Percent of Aggregate Retained on 4.75mm Sieve

X_7: Percent of Aggregate Retained on 0.3mm Sieve

X_8: Percent of Aggregate Retained on 0.15mm Sieve

Note: The PLS regression was carried out using the Microsoft Excel Add-in program XLSTAT® (Addinsoft). The outputs from XLSTAT were analyzed with outputs from other statistical and Mathematical applications, namely SYSTAT and Matlab, and they proved to be comparable.

7.8.4. Discussion of Modeling Output/Results

As can be seen in the resultant model, out of 17 (in the case of percent aggregate **pass** sieve sizes) or 18 (in the case of percent aggregate **retained** sieve sizes) possible predictor variables, 8 were identified to have good projection onto latent structures (factors, components) and proved to have sound prediction capabilities, using the Partial Least Squares Regression method. It is evident from the final model that the amount of Equivalent Standard Axle Load (Terminal ESAL) that a given pavement can sustain before reaching a predetermined pavement skid resistance value at a Friction Number (FN) of 32 (μ=0.32) is largely dictated by the proportion of blend, gradation and physical characteristics of the aggregates used as well as the binder grade used in the Hot Mix Asphalt (HMA) mixture.

It is also evident from the regression coefficients that the model is most sensitive to the Binder Grade and Los Angeles Abrasion (LAA) values and less affected by the British Pendulum Number (BPN) value and the percentage of material passing the 12.5mm sieve. The percentage of material from the primary source (Blend %) shows a negative contribution to the response variable. In cases where there is more than one aggregate supplier, information was not readily available regarding the material from the second or third supplier. As a result, not much can be inferred about this variable from this model alone. This problem can be easily mitigated in the future when enough data can be assembled for observations in which a single supplier provided the aggregate material, or when/if enough information about all suppliers becomes available.

In addition, computing the average values of the model forming variables, for all observations, and plugging these values into the model, it is possible to see that the contribution of the British Pendulum Number (BPN) is only about 0.2% of the predicted Terminal ESAL value. The linear product of the regression coefficient for and the transformed value for the average BPN value of 27 yields only about 7,400 ESALs, which is really not significant in terms of the typically large Terminal ESAL values. This is also made clear by the very small coefficient for this variable.

Moreover, gradation plays a major role as demonstrated by the types of sieves that are found to be significant in the model; the 12.5mm and 4.75mm sieves represent the coarser aggregate material which is mainly associated with the macrotexture component of the pavement surface. The 0.3mm and 0.15mm sieve sizes, which represent the fine aggregate and typically associated with microtexture, are also well represented in the model. The 0.15mm (No. 50) sieve proved to have a higher contribution among the four sieves followed by the 4.75mm (No. 4) sieve, which supports the hypothesis that both microtexture and macrotexture have significant contribution to the skid resistance of pavements. Generally, based on the final model, the material retained on the 4.75mm, 0.3mm and 0.15mm sieve contribute positively to the friction performance of the pavement while material retained on the 12.5mm showed a negative contribution. Still, the negative contribution of the 12.5mm aggregate material is half of that of the 4.75mm and one-third of the 0.15mm material. In addition, it is important to note that there are some “0” entries for the 12.5mm variable in the dataset for observations that contain a 9.5mm nominal mix size aggregate. This problem can be avoided by separating the

12.5mm and 9.5mm nominal mix size materials and performing Partial Least Square Regression independently. This could not be done in this research due to the limited number of observations available and the fact that observations from the two different mix sizes were combined. However, the model can be further qualified in the future by using a controlled dataset, in terms of the nominal mix aggregate size.

It is also interesting to note that, out of the variables that were transformed to alternative forms, in order to achieve better correlation with the response variable, only one, namely British Pendulum Number (BPN), was found to be significant all the way into the final model. In addition, the predictor variables that were not found to be significant for the formation of this particular model can be incorporated into the model since most of them are correlated to one or more of the other predictor variables, in a two-stepped procedure. For example, it has been observed that Polish Value (PV) is highly correlated with British Pendulum Number (BPN). Therefore, this model can still be used to predict the friction performance of a pavement – in terms of terminal ESAL - using PV values even when/if BPN data is unavailable. Finally, this model can be amended/updated as more material and friction data become available in the future.

Chapter 8. Research Summary,Conclusions & Recommendations

8.1. Research Summary

This dissertation presented the methodology, data analysis and modeling, and the results of a research project that investigated the major factors affecting pavement friction, including material, traffic, age and environment related factors, and a systematic approach that can be used to estimate the friction performance of Hot Mix Asphalt (HMA) Pavements. The literature review provided an overview and background information on the mechanism of pavement-tire friction and discussed in-depth the primary factors that affect/contribute to pavement friction.

For the purpose of this research, more than 160,000 material and pavement friction records were assembled, categorized, filtered and analyzed to produce sufficient and dependable datasets that were eventually used in detailed analysis and modeling. The data sources included pavement friction records, material/mix design data, aggregate lab test information, equipment repeatability test data, construction history, and route AADT /truck percentage data. As part of the study, statistical analyses were performed on equipment repeatability/variability test data to account and correct for any discrepancies arising from the use of different equipment in the annual pavement friction surveys. In addition, preliminary scatter plots and simple and is it multiple linear regression or multivariate? multiple linear regression analysis were performed on combined data that were grouped under major categories, such as region (counties), route characteristics (Interstate vs Local), traffic level and testing speed, to identify the main variables that are related to pavement friction and to also assess the quality and validity of the data as a whole.

Following the preliminary analysis, the large database was broken down into various components so as to identify and categorize the pavement friction and material data into a more readily usable form. In this case, a 10-step methodology was adopted to organize and analyze the pavement friction and material data based on material supplier and route information. Route direction, survey location by milepoint, test speed, action year, actual AADT and survey equipment were the most important variables from the friction data, while the aggregate supplier and aggregate/mix properties were the main sources of data from the material database that were employed for detailed analysis. The main outputs from the detailed analysis were supplier and route specific pavement friction performance indicators such as cumulative AADT, cumulative ESAL or expected pavement life in years of service.

The last chapter of the dissertation discussed the methodology followed in assembling the output from the detailed analysis phase for further analysis and modeling of the pavement friction performance indicators. The detailed analysis and research modeling phase investigated numerous alternatives to relate the selected response variable (Terminal ESAL) to the various predictor variables identified in the preceding chapters using multivariate regression methods. In addition, the analysis introduced the concept of Structural Equation Modeling (SEM) and the method of Partial Least Squares (PLS) Regression that was used to investigate, develop and validate several models for relating the friction performance of pavements to aggregate properties and route characteristics. The conclusions from this research are presented below.

8.2. Research Conclusions

This research investigated the primary factors affecting pavement friction with emphasis on the effect of aggregate properties and route characteristics. The following conclusions were obtained from this research:

1) A step-by-step methodology was developed for isolating and analyzing data to predict pavement friction life for any mixture and aggregate. The various analyses produced pavement friction performance indicators, namely Cumulative AADT and Cumulative ESAL at terminal FN value, FN drop/10,000 Cumulative AADT or Cumulative ESAL, or expected/useful friction life in years, all of which can be used to compare and select pavement materials based on these performance measures.

2) Since different pavement sections are exposed to different traffic loading, the analysis considered converting Annual Average Daily Traffic (AADT) values to Equivalent Standard Axle Load (ESAL) values, either at the milepoint level in the actual database or after the terminal cumulative AADT has been computed from the regression analysis. Conversting AADT to ESAL at the milepoint level and evaluating the terminal ESALproved to have similar results to converting the terminal cumulative AADT into terminal ESAL after the regression analysis has been completed.

3) In the detailed analysis phase in which pavement friction data were analyzed for specific routes and suppliers, it was discovered that the simple linear regression analysis between Cumulative AADT (CumAADT) and Friction Number (FN) yielded the best possible relationships, while multivariate regression among cumulative AADT, Speed and equipment produced lower R^2 values, statistically insignificant regression coefficients or comparatively low terminal cumulative AADT values when evaluated using sensitivity analysis.

4) The results from the detailed data analysis helped develop a dataset that consisted of supplier and material information, pavement friction performance indicators, and traffic and route characteristics for various construction contracts that were in turn used to produce and test various research models.

5) In the research model development phase, several modeling methods were considered to formulate and test the relationship between the selected pavement friction performance indicator variable (Terminal ESAL) and the predictor variables that were obtained from detailed analysis in preceding chapters. Several multivariate regression models as well as Partial Least Square regression were tried and tested for model development. After numerous reductions, variations and iterations, a final model was obtained that proved to be valid in terms of model significance and predictive ability as measured by cross validation tests . The final model derived identified the most ctitical factors affecting pavement friction based on the datasets used in this study. Based on this model it is possible to estimate the friction performance of pavements in terms of terminal

ESAL (the maximum number of Equivalent Standard Axle Loads the pavement will 'sustain' before reaching a friction value of FN=32) given certain aggregate and mix properties such as gradation, binder grade used in the HMA mixture, Los Angeles Abrasion (LAA) and British Pendulum Number (BPN) values.

6) Based on the correlation analysis on the various aggregate properties, some of the predictor variables included in the model can estimated from other highly correlated aggregate properties that eventually were not part of the model. For example, it has been observed that Polish Value (PV) is highly correlated with British Pendulum Number (BPN). Therefore, this model can still be used to predict the friction performance of a pavement – in terms of terminal ESAL - using PV values even when/if BPN data is unavailable This can be significantly helpfull since SHA agencies do not always run all aggregate tests from every single aggregate quarry on a regular basis.

8.3 Recomendations

From the analysis and modeling of this study, it was evident that there is a need to control and reduce variability in friction measurements due to the various parameters affecting pavement friction. Thus, it is suggested to eventually complement the analysis and modeling by developing controlled experiments with the following considerations:

- identify projects that use a single aggregate source in the gradation of the mixtures (i.e., AS1, AS2, AS3 etc) for each pavement section;

- consider pavement sections that use aggregates *from different sources and one type of asphalt mixture*, at a time, so as to eliminate asphalt mixture design effects (i.e., effects of binder content and other mix design volumetric parameters that may affect binder film thickness around the aggregate, and thus pavement friction values);

- conduct repeated FN friction measurements at the same test sections and at specific times of the year to measure and isolate *seasonal effects*;

- collect FN measurements for at least 5-7 years on the same sections in order to better capture potential *microtexture renewal* effects;

- consider more *accurate traffic measurements* (AADT, truck distribution factors) and traffic lane distribution.

- use *a single friction equipment*, or side by side measurements of track #5 and#6, on a wider variety of pavement friction levels;

- control *survey speed* at 40 mph during the above testing.

- The detailed analysis included adjusting the data for equipment and survey speed, for reducing testing variability in the model development stage. A follow up analysis and modeling using many years of continuous data (5-7 years) for pavement sections surveyed at the specified testing speed, and using the same equipment, or well calibrated equipment, can increase the quality of the final model. Any follow up analysis should also include material (mix and aggregate) data that are up to date and accurate in terms of the construction contract they are used in so as to increase the quality of the input data.

- Finally, the selected model can further be amended/updated as more material and friction data become available in the future.

Appendix A

Table A- 1. Mineral Composition by Supplier:

Supplier	Year Sampled	Mineralogical Composition (%) Per Whole Rock Analysis				Remark
AIR	N/A					
AASG	2004	Calcite (95%)	Quartz (5%)			
ICM	2004	Dolomite (90%)	Siliceous Silt (10%)			
KLC	2006	Calcite (48%)	Quartz (50.4%)	Pyrite (+-1%)		
LCH	2005	Feldspar (35-40%)	Pyroboles (55-60%)	Opaques (<=5%)		
LF	2006	Calcite (78%)	Quartz (22%)			
LM	2006	Calcite (98%)	Pyrite (<1%)			Lafarge Medford (North Westminster MD)
LW	N/A					
LT	2005	Quartz (70-90%)	Muscovite (10-30%)	Pyrite (1%)		Lafarge Texas (Texas, MD)
MMI	N/A					
MMW	2004	Calcite (85-90%)	Quartz (3-5%)	Clays (5-8%)	Pyrite (<1%)	
VMH	2004	Carbonates (99%)	Clay (1%)			
VMHDG	2005	Quartz (+-25%)	Feldspar (30-35%)	Pyroboles (35-40%)	Opaques (<=5%)	Based on Arundel Corp Havre De Grace Quarry results
VMW	N/A					
YBPBv	N/A					
YBPRv	2004	Calcite, Dolomite (95%)	Silts, Clay (5%)			York Building Products (Roosevelt Avenue #1M)

Appendix B

Representation of Gradation using alternate parameters

The gradation of the aggregate material for the various suppliers (in terms of percent passing and percent retained on sieves) is presented below:

Table B- 1. Aggregate Gradation (Percent Passing Sieve) by Supplier

Supplier	Mix Size	Percent Passing Sieve Size (mm)												
		50	37.5	25	19	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075
AASG	12.5	100	100	100	100	97	86	53	35	22	14	10	7	5.8
AIR	12.5	100	100	100	100	97	83	52	36	23	17	10	6	4.7
KLC	12.5	100	100	100	100	91	77	52	33	21	14	10	8	6.1
LCH	12.5	100	100	100	100	98	83	44	30	23	17	11	7	4.1
LF	12.5	100	100	100	100	95	87	66	40	25	15	9	7	6
LW	12.5	100	100	100	100	95	85	54	35	22	15	10	8	6.5
MMW	12.5	100	100	100	100	97	83	44	26	21	17	12	8	5
VMH	9.5	100	100	100	100	100	97	70	48	30	22	16	11	6.3
VMHDG	9.5	100	100	100	100	100	99	66	41	29	20	12	9	6.9
VMW	12.5	100	100	100	100	99	90	52	37	26	16	10	6	4.9
YBPBV	9.5	100	100	100	100	100	95	58	32	24	18	12	8	5.6

Table B- 2. Aggregate Gradation (Percent Retained on Sieve) by Supplier

Supplier	Mix Size	Percent Retained Sieve Size (mm)													
		50	37.5	25	19	12.5	9.5	4.75	2.36	1.18	0.6	0.3	0.15	0.075	Pan (0)
AASG	12.5	0	0	0	0	3	11	33	18	13	8	4	3	1.2	5.8
AIR	12.5	0	0	0	0	3	14	31	16	13	6	7	4	1.3	4.7
KLC	12.5	0	0	0	0	9	14	25	19	12	7	4	2	1.9	6.1
LCH	12.5	0	0	0	0	2	15	39	14	7	6	6	4	2.9	4.1
LF	12.5	0	0	0	0	5	8	21	26	15	10	6	2	1	6
LW	12.5	0	0	0	0	5	10	31	19	13	7	5	2	1.5	6.5
MMW	12.5	0	0	0	0	3	14	39	18	5	4	5	4	3	5
VMH	9.5	0	0	0	0	0	3	27	22	18	8	6	5	4.7	6.3
VMHDG	9.5	0	0	0	0	0	1	33	25	12	9	8	3	2.1	6.9
VMW	12.5	0	0	0	0	1	9	38	15	11	10	6	4	1.1	4.9
YBPBV	9.5	0	0	0	0	0	5	37	26	8	6	6	4	2.4	5.6

B.1. Coefficient of Uniformity (Cu) and Coefficient of Curvature (Cc) computations

Coefficient of Uniformity, designated as Cu, is computed as a ratio of the grain size (diameter) corresponding to the material at 60% passing (D_{60}) and grain size at 10% passing (D_{10}), and is calculated as follows:

$$Cu = \frac{D_{60}}{D_{10}} \qquad (Eq. B.1)$$

Coefficient of Curvature, designated as Cc, is computed as a ratio of the square of the grain size (diameter) corresponding to the material at 30% passing (D_{30}) and the product of the grain size for the material at 60% passing (D_{60}) and grain size at 10% passing (D_{10}), and is calculated as follows:

$$Cc = \frac{(D_{30})^2}{D_{10} * D_{60}} \qquad (Eq. B.2)$$

The resuls for each supplier are as shown below:

Table B- 3. Aggregate Gradation Parameters by Supplier (Percent Passing)

Supplier	D_{10}	D_{30}	D_{60}	Cu	Cc
AASG	0.300	1.906	5.758	19.19	2.104
AIR	0.300	1.815	5.976	19.92	1.838
KLC	0.300	2.065	6.270	20.90	2.267
LCH	0.263	2.360	6.699	25.52	3.167
LF	0.375	1.573	3.393	9.05	1.946
LW	0.300	1.906	5.669	18.90	2.136
MMW	0.225	3.304	6.699	29.77	7.243
VMH	0.120	1.180	2.991	24.92	3.880
VMHDG	0.200	1.278	3.886	19.43	2.102
VMW	0.300	1.609	5.750	19.17	1.501
YBPBV	0.225	2.065	5.007	22.25	3.785

B.2. "Percent Retained" gradation parameters ('a' and 'b') computations

The percent retained and cumulative percent retained relationships were plotted as shown in the figures below. It can be seen from the graphs that relationships between the sieve sizes and cumulative percentage retained can be approximated using an exponential relationship and the best fit curve can be described using the following equation:

$$y = ae^{-bx} \qquad \text{(Eq. B-3)}$$

Where:

y = Cumulative percent retained

a,b = equation (model) parameters

x = Sieve Size

Table B- 4. Percent Retained and Cumulative Percent retained (4 suppliers)

Sieve Sizes (mm)	**12.5NMAS(AASG)**		**9.5NMAS(YBPBV)**		**12.5NMAS(LCH)**		**9.5NMAS(VMH)**	
	%Ret	CumRet	%Ret	CumRet	%Ret	CumRet	%Ret	CumRet
12.5	3	3	0	NA	2	2	0	NA
9.5	11	14	5	5	15	17	3	3
4.75	33	47	37	42	39	56	27	30
2.36	18	65	26	68	14	70	22	52
1.18	13	78	8	76	7	77	18	70
0.6	8	86	6	82	6	83	8	78
0.3	4	90	6	88	6	89	6	84
0.15	3	93	4	92	4	93	5	89
0.075	1.2	94.2	2.4	94.4	2.9	95.9	4.7	93.7
Equations	$y = 107.22e^{-0.252x}$		$y = 112.1e^{-0.188x}$		$y = 112.32e^{-0.266x}$		$y = 110.32e^{-0.227x}$	
R2	0.96		0.7903		0.898		0.84	

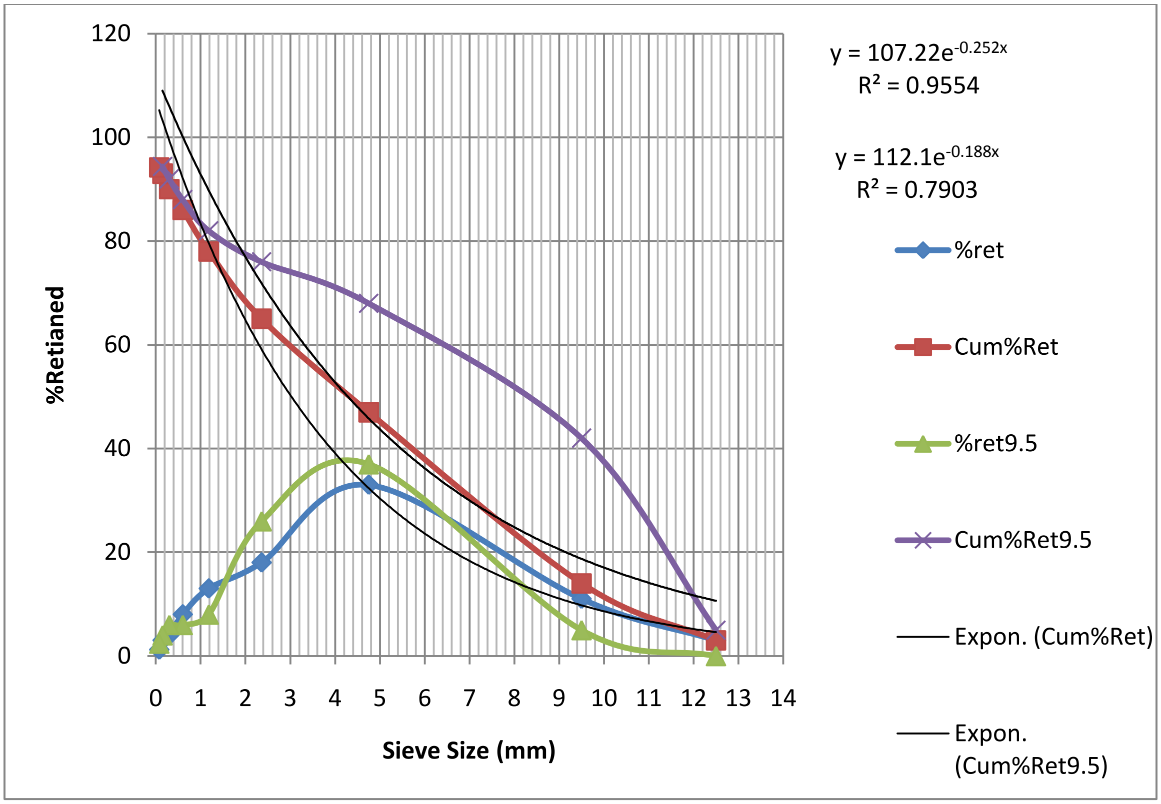

Figure B- 1. Percent Retained and Cumulative Percent retained (AASG and YBPBV)

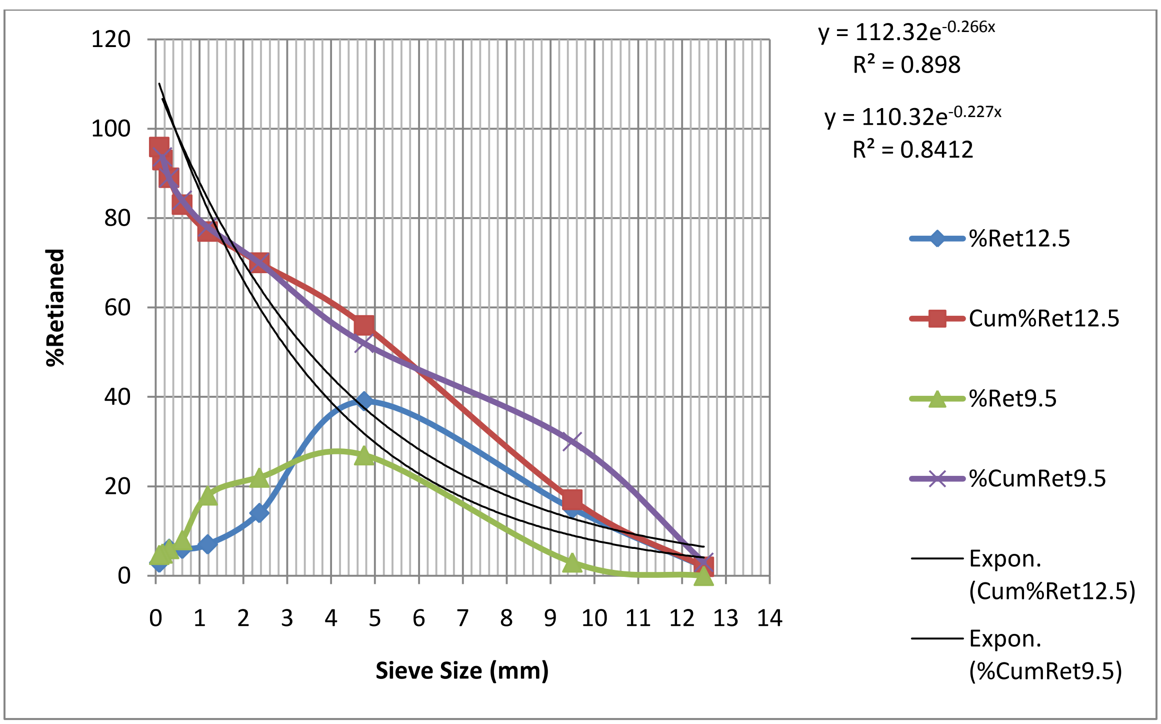

Figure B- 2. Percent Retained and Cumulative Percent retained (LCH and VMH)

Table B- 5. Percent Retained and Cumulative Percent retained (4 suppliers)

Sieve Sizes (mm)	12.5NMAS(AIR)		12.5NMAS(KLC)		12.5NMAS(LF)		12.5NMAS(LW)	
	%Ret	CumRet	%Ret	CumRet	%Ret	CumRet	%Ret	CumRet
12.5	3	3	9	9	5	5	5	5
9.5	14	17	14	23	8	13	10	15
4.75	31	48	25	48	21	34	31	46
2.36	16	64	19	67	26	60	19	65
1.18	13	77	12	79	15	75	13	78
0.6	6	83	7	86	10	85	7	85
0.3	7	90	4	90	6	91	5	90
0.15	4	94	2	92	2	93	2	92
0.075	1.3	95.3	1.9	93.9	1	94	1.5	93.5
Equations	$y = 106.45e^{-0.245x}$		$y = 98.296e^{-0.175x}$		$y = 98.645e^{-0.229x}$		$y = 102.23e^{-0.222x}$	
R2	0.94		0.98		0.99		0.98	

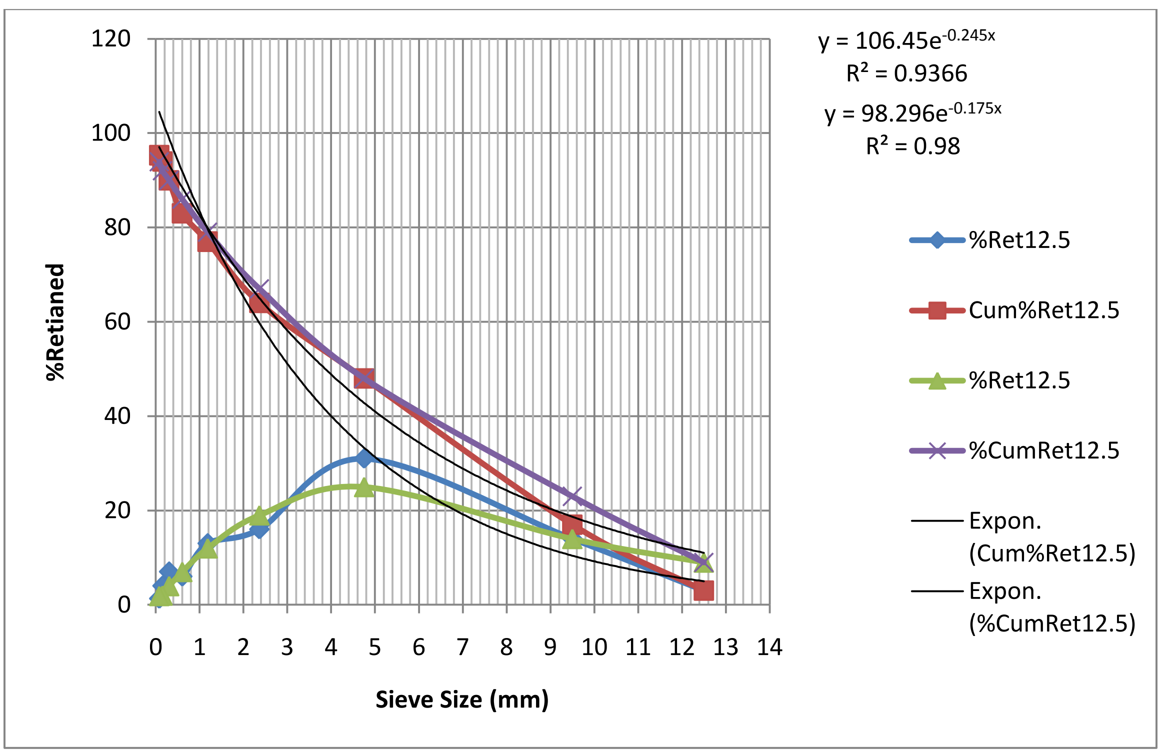

Figure B- 3. Percent Retained and Cumulative Percent retained (AIR and KLC)

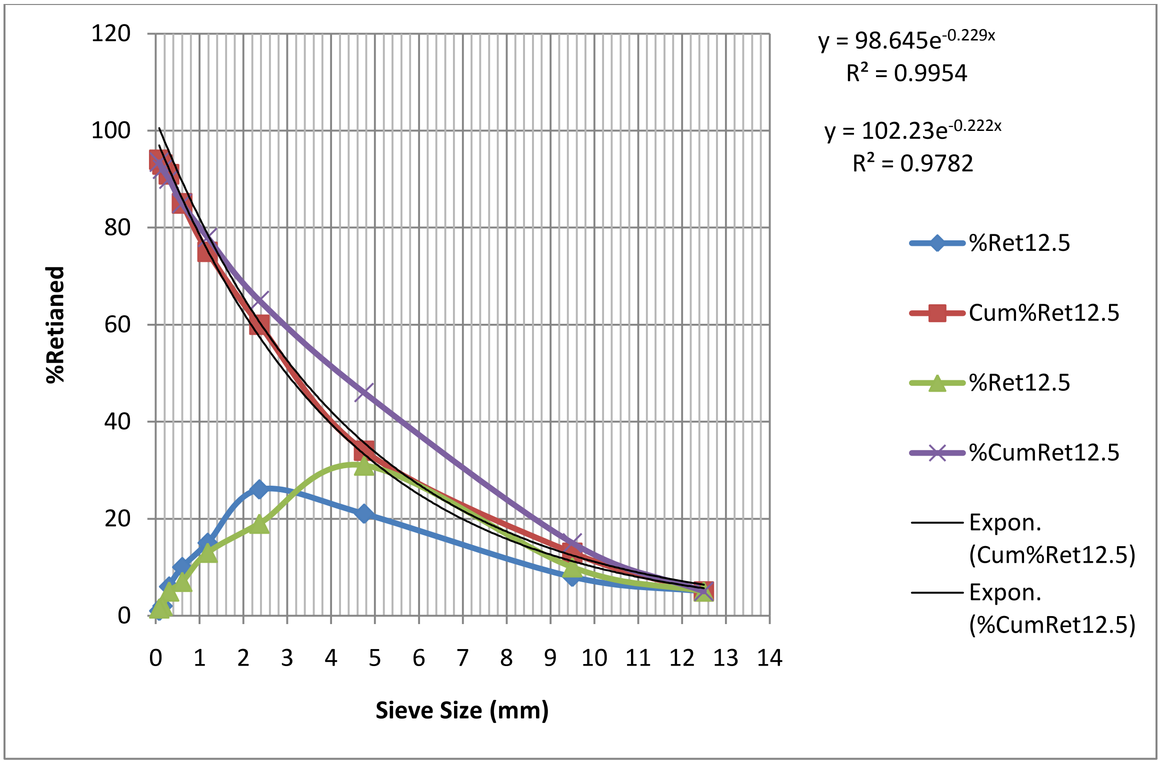

Figure B- 4. Percent Retained and Cumulative Percent retained (LF and LW)

Table B- 6. Percent Retained and Cumulative Percent retained (3 suppliers)

Sieve Sizes (mm)	**12.5NMAS(MMW)**		**9.5NMAS(VMHDG)**		**12.5NMAS(VMW)**	
	%Ret	CumRet	%Ret	CumRet	%Ret	CumRet
12.5	3	3	0	NA	1	1
9.5	14	17	1	1	9	10
4.75	39	56	33	34	38	48
2.36	18	74	25	59	15	63
1.18	5	79	12	71	11	74
0.6	4	83	9	80	10	84
0.3	5	88	8	88	6	90
0.15	4	92	3	91	4	94
0.075	3	95	2.1	93.1	1.1	95.1
Equations	$y = 109.5e^{-0.244x}$		$y = 126.18e^{-0.285x}$		$y = 116e^{-0.323x}$	
R2	0.92		0.75		0.92	

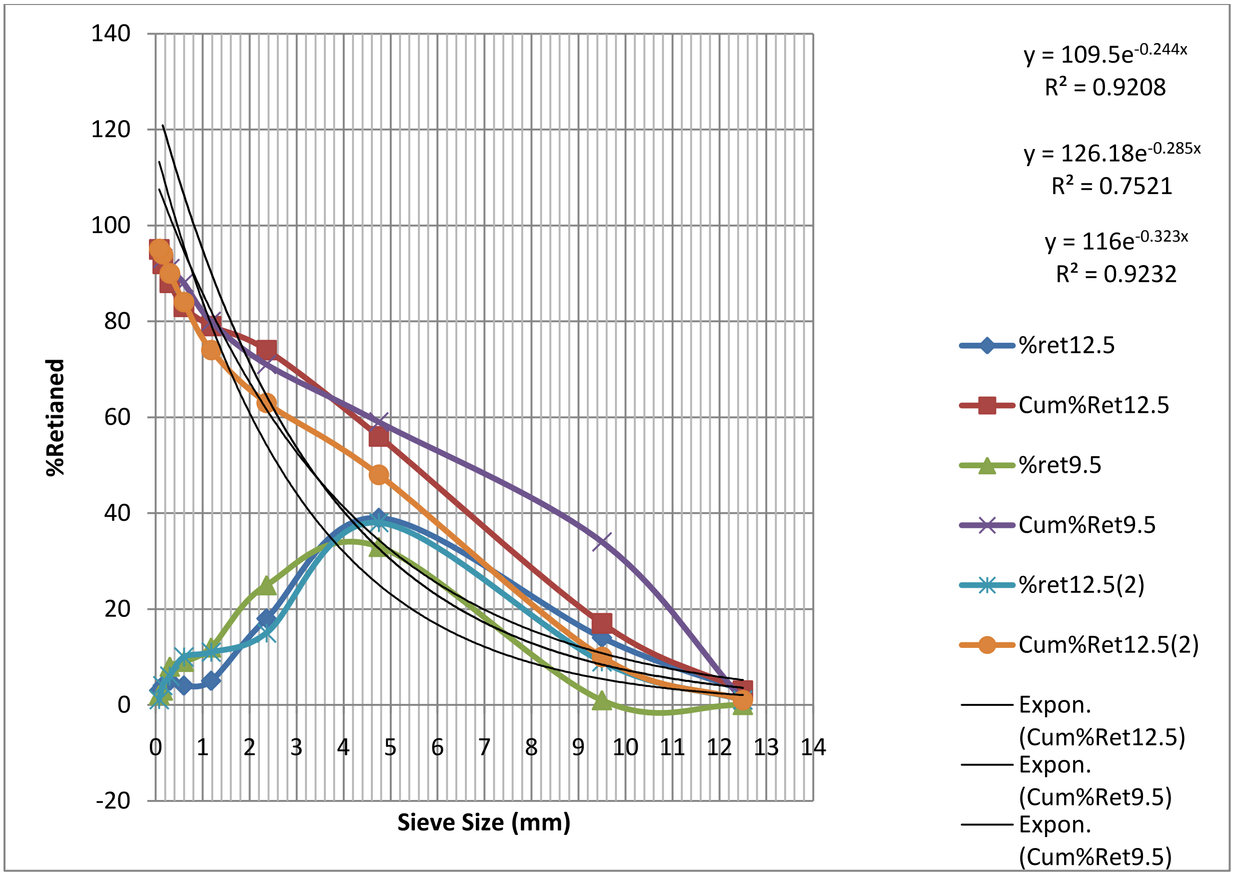

Figure B- 5. Percent Retained and Cumulative Percent retained (MMW,VMHDG and VMW)

Table B- 7. Summary of curve fit parameters

Chapter 9. Source	NMAS	%Retained On Sieve No.4	Curve Equation Parameters		Coefficient of determination
			a	b	R^2
AASG	12.5	33	107.22	0.252	0.96
AIR	12.5	31	106.45	0.245	0.94
KLC	12.5	25	98.296	0.175	0.98
LCH	12.5	39	112.32	0.266	0.90
LF	12.5	21	98.645	0.229	0.99
LW	12.5	31	102.23	0.222	0.98
MMW	12.5	39	109.5	0.244	0.92
VMH	9.5	27	110.32	0.227	0.84
VMHDG	9.5	33	126.18	0.285	0.75
VMW	12.5	38	116	0.323	0.92
YBPBV	9.5	37	112.1	0.188	0.79

Appendix C

Outputs from the Stepwise Multivariate Linear Regression

C.1. Using Original Variables and Percent Pass #4 Sieve data

Table C- 1. Multivariate Regression output using all original variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.97399
R Square	0.94865
Adjusted R Square	0.48652
Standard Error	1059295
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*T-critical*
Regression	9	2.07E+13	2.3E+12	2.05278325	0.497164	2.2622
Residual	1	1.12E+12	1.12E+12			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*ttest*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	8073682	12721874	0.63463	4.9885E-08	0.639995	-2E+08	1.7E+08	-1.5E+08	169720414
%pass #4	-11866	173701	-0.06831	-3.9327E-07	0.956579	-2E+06	2195215	-2218947	2195214.9
Blend%age	-56788	56329.46	-1.00815	-1.7897E-05	0.497417	-772522	658945.3	-772522	658945.26
BPN	-110772	156288.5	-0.70877	-4.535E-06	0.607469	-2E+06	1875062	-2096606	1875061.9
PV	-225631	837387.1	-0.26945	-3.2177E-07	0.832444	-1E+07	10414381	-1.1E+07	10414381
LAA	-187633	147845.1	-1.26912	-8.5841E-06	0.424848	-2E+06	1690917	-2066183	1690917.2
Soundness	-311281	1010998	-0.30789	-3.0455E-07	0.809852	-1E+07	12534668	-1.3E+07	12534668
Binder Grade	1044310	943807.2	1.106486	1.1724E-06	0.467845	-1E+07	13036517	-1.1E+07	13036517
Binder %	1284143	2986348	0.430004	1.4399E-07	0.741468	-4E+07	39229286	-3.7E+07	39229286
AESAL	214.655	4572.041	0.04695	1.0269E-05	0.970133	-57879	58307.94	-57878.6	58307.944

Table C- 2. Output of Stepwise (Manual) Multivariate Regression using 7 original variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.97275
R Square	0.94624
Adjusted R Square	0.8208
Standard Error	625778
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	7	2.07E+13	2.95E+12	7.543537787	0.062409
Residual	3	1.17E+12	3.92E+11		
Total	10	2.19E+13			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	9511776	4051943	2.347461	0.100537428	-3383314	22406866	-3E+06	22406866
Blend%age	-53525	21419.4	-2.49892	0.08779052	-121691	14640.81	-1E+05	14640.81
BPN	-96765	75805.75	-1.27649	0.291632277	-338013	144482.6	-3E+05	144482.6
PV	-270973	221875.5	-1.22129	0.309200139	-977080	435133.3	-1E+06	435133.3
LAA	-205059	72834.66	-2.8154	0.066992652	-436851	26733.65	-4E+05	26733.65
Soundness	-320480	182194.4	-1.759	0.176813208	-900304	259343.4	-9E+05	259343.4
Binder Grade	1011883	364578	2.77549	0.069250535	-148367	2172133	-1E+05	2172133
Binder %	910761	720741.4	1.263644	0.295625239	-1382960	3204482	-1E+06	3204482

Table C- 3. Output of Stepwise (Manual) Multivariate Regression using 5 original variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.93049
R Square	0.86582
Adjusted R Square	0.73163
Standard Error	765812
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	1.89E+13	3.78E+12	6.452430943	0.030816
Residual	5	2.93E+12	5.86E+11		
Total	10	2.19E+13			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.3E+07	2853370	4.449628	0.006704646	5361614	20031256	5E+06	2E+07
Blend%age	-32247	20822	-1.54871	0.182133987	-85771.9	21277.41	-85772	21277
BPN	-156746	71903.14	-2.17996	0.08111719	-341579	28087.01	-3E+05	28087
LAA	-205259	80376.37	-2.55372	0.051034446	-411873	1355.025	-4E+05	1355
Soundness	-238566	202842	-1.17612	0.292490516	-759988	282855.7	-8E+05	3E+05
Binder Grade	672371	392798.6	1.711745	0.147624538	-337350	1682092	-3E+05	2E+06

Table C- 4. Output of Stepwise (Manual) Multivariate Regression using 4 original variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.91033
R Square	0.82869
Adjusted R Square	0.71449
Standard Error	789892
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	1.81E+13	4.53E+12	7.256228462	0.017525
Residual	6	3.74E+12	6.24E+11		
Total	10	2.19E+13			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.1E+07	2720776	4.1962	0.005709827	4759421	18074420	5E+06	2E+07
Blend%age	-39116	20614.67	-1.89747	0.106542216	-89558.1	11326.47	-89558	11326
BPN	-120841	67147.65	-1.79963	0.122014179	-285146	43463.16	-3E+05	43463
LAA	-175150	78586.12	-2.22876	0.067380379	-367443	17143.38	-4E+05	17143
Binder Grade	710761	403748.2	1.760407	0.128822582	-277175	1698698	-3E+05	2E+06

Table C- 5. Output of Stepwise (Manual) Multivariate Regression using 3 original variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.86036
R Square	0.74021
Adjusted R Square	0.62888
Standard Error	900567
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	1.62E+13	5.39E+12	6.648366976	0.018578
Residual	7	5.68E+12	8.11E+11		
Total	10	2.19E+13			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.4E+07	2593366	5.415714	0.00099165	7912593	20177266	8E+06	2E+07
Blend%age	-33291	23198.35	-1.43505	0.194405249	-88146.2	21564.59	-88146	21565
BPN	-159468	72353.39	-2.20401	0.063354516	-330556	11620.69	-3E+05	11621
LAA	-232219	81618.49	-2.84518	0.02485943	-425216	-39222.26	-4E+05	-39222.262

Table C- 6. Output of Stepwise (Manual) Multivariate Regression using 2 original variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.81473
R Square	0.66378
Adjusted R Square	0.57973
Standard Error	958341
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	2	1.45E+13	7.25E+12	7.897120262	0.012778	2.306
Residual	8	7.35E+12	9.18E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.3E+07	2715162	4.927417	0.001153185	7117559	2E+07	7E+06	19639908
BPN	-208838	67734.22	-3.0832	0.015043718	-365034	-52643	-4E+05	-52642.9
LAA	-274849	80897.37	-3.39751	0.009395025	-461399	-88300	-5E+05	-88299.8

C.2. Using Transformed Variables and Percent Retained on #4 Sieve data

Table C- 7. Multivariate Regression output using all transformed variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.97487
R Square	0.95037
Adjusted R Square	0.50373
Standard Error	1041391
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*T-critical*
Regression	9	2.08E+13	2.31E+12	2.127827446	0.489725	2.2622
Residual	1	1.08E+12	1.08E+12			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	9878230	15689649	0.629602	0.642282341	-1.9E+08	2E+08	-1.89E+08	2.1E+08
%Ret#4	-41937	210983.8	-0.19877	0.875088688	-2722740	3E+06	-2722740	2638866
Blend%age	-61603	52878.42	-1.16499	0.451576891	-733487	610281	-733487	610281
BPN	-110826	149588.3	-0.74087	0.594070494	-2011525	2E+06	-2011525	1789874
PV	-180240	557919.3	-0.32306	0.801073283	-7269276	7E+06	-7269276	6908797
LAA	-208245	175723.2	-1.18508	0.446206467	-2441020	2E+06	-2441020	2024529
Soundness	-195766	529134	-0.36997	0.774409316	-6919051	7E+06	-6919051	6527519
Binder Grade	943089	666620	1.414732	0.391716572	-7527122	9E+06	-7527122	9413299
Binder %	1038018	1574304	0.659351	0.628901263	-1.9E+07	2E+07	-18965409	2.1E+07
AESAL	1058.96	3705.411	0.285786	0.822786646	-46022.8	48141	-46022.76	48140.7

Table C- 8. Output of Stepwise (Manual) Multivariate Regression using 7 transformed variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.97275
R Square	0.94624
Adjusted R Square	0.8208
Standard Error	625778
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	7	2.07E+13	2.95E+12	7.543537787	0.062409	3.1824
Residual	3	1.17E+12	3.92E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	9511776	4051943	2.347461	0.100537428	-3383314	2E+07	-3383314	2.2E+07
Blend%age	-53525	21419.4	-2.49892	0.08779052	-121691	14641	-121691.4	14640.8
BPN	-96765	75805.75	-1.27649	0.291632277	-338013	144483	-338012.8	144483
PV	-270973	221875.5	-1.22129	0.309200139	-977080	435133	-977080.2	435133
LAA	-205059	72834.66	-2.8154	0.066992652	-436851	26734	-436851.1	26733.6
Soundness	-320480	182194.4	-1.759	0.176813208	-900304	259343	-900304.3	259343
Binder Grade	1011883	364578	2.77549	0.069250535	-148367	2E+06	-148367.3	2172133
Binder %	910761	720741.4	1.263644	0.295625239	-1382960	3E+06	-1382960	3204482

Table C- 9. Output of Stepwise (Manual) Multivariate Regression using 5 transformed variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.8913
R Square	0.79442
Adjusted R Square	0.58884
Standard Error	947896
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	5	1.74E+13	3.47E+12	3.864310559	0.082114	2.5706
Residual	5	4.49E+12	8.99E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	2931020	4994700	0.586826	0.582815051	-9908267	2E+07	-9908267	1.6E+07
Blend%age	-79940	29004.79	-2.75608	0.040020363	-154499	-5380.3	-154498.7	-5380.32
LAA	-71992	85201.46	-0.84496	0.43668439	-291010	147025	-291009.6	147025
Soundness	-42240	227436.6	-0.18572	0.859961248	-626885	542404	-626884.8	542404
Binder Grade	1117844	479618.5	2.330693	0.067153294	-115055	2E+06	-115055	2350742
Binder %	1211907	1037138	1.168511	0.29527804	-1454141	4E+06	-1454141	3877955

Table C- 10. Output of Stepwise (Manual) Multivariate Regression using 4 transformed variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.89051
R Square	0.793
Adjusted R Square	0.655
Standard Error	868286
Observations	11

ANOVA

	df	SS	MS	F	Significance F	t-critical
Regression	4	1.73E+13	4.33E+12	5.74647518	0.02997	2.4469
Residual	6	4.52E+12	7.54E+11			
Total	10	2.19E+13				

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	2857046	4560647	0.626456	0.554086285	-8302454	1E+07	-8302454	1.4E+07
Blend%age	-80304	26507.92	-3.02943	0.023114741	-145167	-15441	-145166.5	-15441.5
LAA	-70377	77637.85	-0.90647	0.39962443	-260349	119596	-260349.5	119596
Binder Grade	1112992	438685.3	2.537108	0.044253189	39567.82	2E+06	39567.82	2186416
Binder %	1218129	949537.3	1.282866	0.246863799	-1105305	4E+06	-1105305	3541563

Table C- 11. Output of Stepwise (Manual) Multivariate Regression using 3 transformed variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.87445
R Square	0.76465
Adjusted R Square	0.66379
Standard Error	857156
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	3	1.67E+13	5.57E+12	7.581182803	0.013303	2.3646
Residual	7	5.14E+12	7.35E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	656663	3811486	0.172285	0.868088572	-8356071	1E+07	-8356071	9669396
Blend%age	-88426	24628.04	-3.59045	0.008851866	-146662	-30190	-146661.7	-30189.6
Binder Grade	1257874	403296	3.118984	0.016867681	304230.2	2E+06	304230.2	2211517
Binder %	1487132	890411.4	1.670163	0.138814115	-618356	4E+06	-618356.3	3592620

Table C- 12. Output of Stepwise (Manual) Multivariate Regression using 2 transformed variables

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.81907
R Square	0.67087
Adjusted R Square	0.58859
Standard Error	948187
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*	*t-critical*
Regression	2	1.47E+13	7.33E+12	8.153314627	0.011734	2.306
Residual	8	7.19E+12	8.99E+11			
Total	10	2.19E+13				

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	6319581	1925870	3.281416	0.011164182	1878516	1E+07	1878516	1.1E+07
Blend%age	-62456	21126.96	-2.95621	0.018251582	-111175	-13737	-111174.6	-13736.9
Binder Grade	1121604	436901.2	2.567179	0.033273239	114107.8	2E+06	114107.8	2129100

Appendix D

Partial Least Squares (PLS) Regression Algorithm

D.1. Description of method

The Partial Least Squares (PLS) regression is a method of fitting a model for one or more dependent (response) variables based on one or more independent (predictor) variables. It is typically very useful when the following conditions exist:

i. There is more number of predictors compared to the number of observations

ii. There is multicollinearity among the predictor variables (i.e. two or more predictors are linearly correlated)

The main purpose of the PLS regression methodology is to find few components/factors that can replace the predictor variables and at the same time can explain the variance in the response variable, thereby fitting a valid model to the data by eliminating the multicollinearity problem.

The goal of the PLS regression is to find components and loadings to the predictor matrix (X) and response matrix (Y) as follows:

$$\mathbf{X = TP' + E} \qquad \textbf{(Eq. D.1)}$$

$$\mathbf{Y = UQ' + F} \qquad \textbf{(Eq. D.2)}$$

And eventually obtain the relationship: $\mathbf{Y = XB + F}$ **(Eq. D.3)**

The PLS regression algorithm and methodology can be summarized as shown in the following figure:

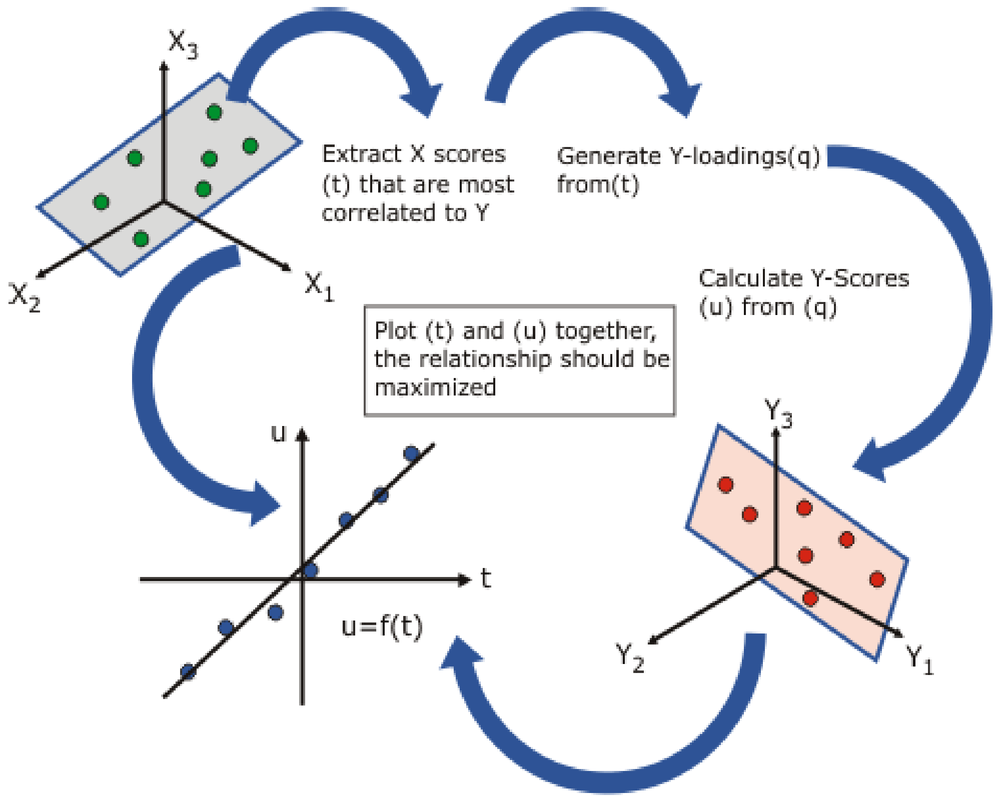

Figure D- 1. PLS regression Methodology (The Unscrambler® Handbook – PLS Theory)

The approach, notations and step by step methodology/algorithm will be presented in the following sections.

Table D- 1. Notations used in the PLS algorithm (adopted from SPSS Handbook):

Notation	Description
X	$N \times n$ design matrix of independent variables, centered and perhaps standardized. Note that there is no intercept term.
Y	$N \times m$ matrix of dependent variables, centered and perhaps standardized
c	$m \times 1$ column vector of weights
u	$N \times 1$ column vector of Y scores
w	$n \times 1$ column vector of weights
t	$N \times 1$ column vector of X scores
d (or h)	number of PLS factors to extract
p	$n \times 1$ loading vector
q	$m \times 1$ loading vector
P	$n \times d$ loading matrix
Q	$m \times d$ loading matrix
T	$N \times d$ score matrix, T= XW*
U	$N \times d$ score matrix
W	$n \times d$ matrix of X-weights
W*	$n \times d$ matrix of X-weights in original coordinates; these weights can be directly applied to **X**, $W^* = W(P'W)^{-1}$
C	$m \times d$ matrix of Y-weights; these weights can be directly applied to **Y**.
B	$n \times m$ matrix of regression parameters, $B = W^* C'$
E	$N \times n$ matrix of residuals, $\mathbf{E} = \mathbf{X} - \mathbf{TP}'$
F	$N \times m$ matrix of residuals, $\mathbf{F} = \mathbf{Y} - \mathbf{UQ}' = \mathbf{Y} - \mathbf{XB}$
DModX	$N \times 1$ vector of distances of **X** variables to the model
DModY	$N \times 1$ vector of distances of **Y** variables to the model
VIP	$n \times d$ matrix of Variable Importance in the Projection

D.2. the step-by-step algorithm (Adopted from SPSS Handbook)

1. Given a matrix of predictor variables '**X**' and response variables '**Y**', compute the mean and standard deviation of each variable (i.e. $\bar{X}_i$ and Sxi) and replace **X** with the centered and standardized variables. Do the same for the Y matrix.

$X := (X - \mu_X)\, \mathbf{\Sigma_X}^{-1}$

Where "$\mathbf{\Sigma}_X$" is a diagonal matrix of standard deviations and "μ_X" is the vector of means;

$Y := (Y - \mu_Y)\, \mathbf{\Sigma_Y}^{-1}$

2. There are various algorithms that can be used based on the number of dependent variables. For the case of one dependent (response) variable, the NIPALS (Non-linear Iterative Partial Least Squares) algorithm is used. NIPALS algorithm is useful when there are missing values in the data and when only the first few factors of a data set – often between 2 to 5 factors yield good results - need to be calculated.

NIPALS (Non-linear Iterative Partial Least Squares) Algorithm:

The NIPALS algorithm explicitly takes '**c**' and '**w**' to have unit norm. If there is only one response variable, then '**c**' will be a 1×1 unit vector (c = 1), and this will be the start of the analysis: initialize u = Y; when c = 1, the NIPALS converges in only one iteration.

Begin the loop with m =1, **c** = 1, begin at step 1 with **u** = **Y**:

Repeat until convergence:

1. $\mathbf{w} = \mathbf{X'u}/(\mathbf{u'u}) = \mathbf{X}'u(u'u)^{-1}$ compute the first weight vectors (for X)
2. $\mathbf{w} := \mathbf{w}/||\mathbf{w}||$ ……………… reduce '**w**' using determinant (absolute value)
3. $\mathbf{t} = \mathbf{Xw}$ … ………………….. compute the first '**t**' component (score)
4. $\mathbf{c} = \mathbf{Y't}/(\mathbf{t't}) = \mathbf{Y't}(\mathbf{t't})^{-1}$ ……… compute the next '**c**' weight (score) (for Y)

5. $\mathbf{c} := \mathbf{c}/||\mathbf{c}||$ …………….. deflate/reduce '**c**' using determinant

6. $\mathbf{u} = \mathbf{Yc}$ … ……………… compute the first '**u**' component (score)

Repeat the above until convergence and until all factors/components are extracted.

3. Computation of loadings to be used in the deflation of the **X** and **Y** matrices

3.1. Regress **X** on **t** and **Y** on **u**:

1. $\mathbf{p} = \mathbf{X't}/(\mathbf{t't}) = \mathbf{X't}\,(\mathbf{t't})^{-1}$ ….. Compare with OLS's β coefficients
2. $\mathbf{q} = \mathbf{Y'u}/(\mathbf{u'u}) = \mathbf{Y'u}\,(\mathbf{u'u})^{-1}$

3.2. Deflate **X** and **Y** matrices:

1. $\mathbf{X} := \mathbf{X} - \mathbf{tp'}$
2. $\mathbf{Y} := \mathbf{Y} - \mathbf{tc'}$ (use **c** from step 4, not step 5, above)

At this stage, the deflated matrices are the errors **E**, **F** at that stage (see Equations D.1, D.2 and D.3).

The vector of regression coefficients, **B,** to be used for predicting **Y** from **X** is given by:

$$B = W^{*}C'$$
$$B = W(P'W)^{-1}C'$$
$$B = X'U(T'XX'U)^{-1}T'Y$$

W and **C** are obtained by assembling the **w** and **c** vectors into $n\times d$ and $m\times d$ matrices to solve the PLS Regression equation (Eq. D.3):

$$\mathbf{Y} = \mathbf{XB} + \mathbf{F}$$

Up to this point, the **X** and **Y** matrices have been assumed to be centered (reduced by subtracting the mean for each X vector), and (optionally) standardized (by dividing into the standard deviation).

In order to express the relationship between the response variable (Y) and the predictor variables (X), the parameters **B** and residuals **E** and **F** must be restored to the original coordinates as follows:

$$\mathbf{B^* = (\Sigma_X^{-1})\ B\ (\Sigma_Y)\ ,}$$

$$\mathbf{E^* = E\ (\Sigma_X)\ ,}$$

$$\mathbf{\hat{y} = XB^* + (\mu_Y - \mu_X B^*)}$$

$$\mathbf{F^* = F\ (\Sigma_Y)}$$

Where :

$$\mathbf{F = Y - XB}$$

D.3. Output Statistics

The following are the main outputs from the PLS algorithm (discussed in Chapter 7):

1. Proportion of variance explained by scores: This value measures the proporiton of variance explained (both for the predictor and repsonse varibles) by the kth factor and is computed as follows:

$$\text{VarProp } k\,(Y) = \frac{SS_k(Y)}{\text{trace}(Y'Y)} \quad \text{and} \quad \text{VarProp } k\,(X) = \frac{SS_k(X)}{\text{trace}(X'X)}$$

Where:

$$SS\,k\,(Y) = \left(t'(k)t(k)\right)\cdot\left(c'(k)c(k)\right)$$

$$SS\,k\,(X) = \left(t'(k)t(k)\right)\cdot\left(p'(k)p(k)\right)$$

2. Variable Importance in Projection (VIP) (for individual predictor variables).The Variable Importance in Projection (VIP) coefficients measure the relative importance of each predictor (X_i) variable for each X factor (t_i) in the prediction model:

$$\text{VIPjk} = \sqrt{\frac{n\sum_{l=1}^{k} {w_{jl}^{*}}^{2}\cdot SS_l(Y)}{\sum_{l=1}^{k} SS_l(Y)}}$$

$1 \leq j \leq n$ and $1 \leq k \leq d$; w^*_{jk} is the *j*th element of $w^*_{(k)}$, where $w^*_{(k)}$ is the *k*th column of **W**.

3. Distance to the Model: This measure evaluates the 'distance' of each variable to the model and is computed as follows:

$$DModX\ i = \sqrt{e'_i e_i} \quad \text{and} \quad DModY\ i = \sqrt{f_i f_i}$$

Where e_i and f_i are errors of prediction.

For each row e_i of E and f_i of F, the distance to the model may be normalized as:

$$DModX_i = \sqrt{\frac{N}{N-d-1} \mathbf{e}'_i \mathbf{e}_i}$$

$$DModY_i = \sqrt{\frac{N}{N-d-1} \mathbf{f}_i \mathbf{f}_i}$$

4. The PRESS (Predictive Error Sum of Squares) statistic: This measure, computed as the sums of squares of the prediction residuals for observations not used in model development, is used as another measure of model validity as a whole. This index is computed as follows:

$$\text{PRESS} = \sum_{i=1}^{N} DModY_i^2$$

or,

$$PRESS = \sum_{i=1}^{N} \mathbf{f}'_{(i)} \mathbf{f}_{(i)}$$

5. The Q^2cum index : This index measures the global contribution of the h first components to the predictive quality of the model. This index is computed as :

$$Q^2cum(h) = 1 - \prod_{j=1}^{h} \frac{\sum_{k=1}^{q} PRESS_{kj}}{\sum_{k=1}^{q} SSE_{K(j-1)}}$$

Where:

PRESS = the Predictive Sum of Squares

SSE = the Sum of Squares of Error

6. The coefficient of determination (R^2): computed as a simple linear regression output between the actual response variables (as independent) and the predicted response variables (as dependent).

Appendix E

Partial Least Squares (PLS) Regression Modeling Detailed Outputs (Using XLSTAT® Application)

E.1. Preliminary Modeling Results

In the preliminary modeling analysis, 12 various modeling approaches were considered using datasets developed in the detailed data analysis phase. Out of the 12 modeling approaches considered, the model for Model 8 was found to produce more sound and valid models. In this modeling approach, all **eight** independent variables, some of which are transformed for better correlation, were used. The **transformed** variables were (-Exp (0.6*BPN), -Exp(PV), Ln(Soundness), Exp(Binder %), AESAL^-0.4. The **untransformed** variables are (Blend Percentage, LAA and Binder Grade). Also **four** sieves that represent Percent **Retained** gradation and that yielded a **VIP** (Variable Importance in the Projection) value of greater than **0.8** were used. The selected sieves based on VIP values are – 12.5mm, 4.75mm, 0.3mm and 0.15mm. The dataset is shown below:

Table E- 1. Dataset of modified (transformed) independent variables with selected "Percent Retained" sieves (dataset for M13)

No	Terminal ESAL	Blend %	-EXP (0.6*BPN)	LAA	Binder Grade	12.5	4.75	0.3	0.15
1	438990	100	-1.30E+09	20	1	5	31	5	2
2	1062723	100	-7.20E+08	18	2	9	25	4	2
3	1558218	100	-1.80E+06	22	1	5	21	6	2
4	2118493	65	-5.40E+05	22	1	2	39	6	4
5	2363108	100	-6.00E+06	15	1	3	33	4	3
6	3045205	72	-1.10E+07	18	1	0	37	6	4
7	3208372	75	-5.40E+05	18	2	3	31	7	4
8	3477567	75	-1.10E+07	18	1	3	39	5	4
9	3558538	85	-3.00E+05	25	2	0	27	6	5
10	3672489	68	-1.20E+08	14	1	0	33	8	3
11	5810328	75	-6.00E+06	11	3	1	38	6	4

Table E- 2. Dataset of modified (transformed) independent variables with selected "Percent Retained" sieves (dataset for M13.1)

No	Total ESAL	Blend %	-EXP(0.6*BPN)	LAA	Binder Grade	12.5	4.75	0.3	0.15
1	438990	100	-1.30E+09	20	1	5	31	5	2
2	1062723	100	-7.20E+08	18	2	9	25	4	2
3	1558218	100	-1.80E+06	22	1	5	21	6	2
4	2118493	65	-5.40E+05	22	1	2	39	6	4
6	3045205	72	-1.10E+07	18	1	0	37	6	4
7	3208372	75	-5.40E+05	18	2	3	31	7	4
8	3477567	75	-1.10E+07	18	1	3	39	5	4
9	3558538	85	-3.00E+05	25	2	0	27	6	5
10	3672489	68	-1.20E+08	14	1	0	33	8	3
11	5810328	75	-6.00E+06	11	3	1	38	6	4

Table E- 3. Dataset of modified (transformed) independent variables with selected "Percent Retained" sieves (dataset for M13.2)

No	Total ESAL	Blend %	-EXP(0.6*BPN)	LAA	Binder Grade	12.5	4.75	0.3	0.15
1	438990	100	-1.30E+09	20	1	5	31	5	2
2	1062723	100	-7.20E+08	18	2	9	25	4	2
3	1558218	100	-1.80E+06	22	1	5	21	6	2
4	2118493	65	-5.40E+05	22	1	2	39	6	4
5	2363108	100	-6.00E+06	15	1	3	33	4	3
6	3045205	72	-1.10E+07	18	1	0	37	6	4
7	3208372	75	-5.40E+05	18	2	3	31	7	4
8	3477567	75	-1.10E+07	18	1	3	39	5	4
9	3558538	85	-3.00E+05	25	2	0	27	6	5
11	5810328	75	-6.00E+06	11	3	1	38	6	4

Table E- 4. Summary of outputs from the second PLS model validation/verification phase

Model No	Description	Total Ind. Vars.	No comp.	No of sieves	Cum Q^2 Indx	R^2 CumY	R^2 CumX	No. var. with VIP>0.8	No Sieves VIP>0.8	R^2	Predicted	Actual	Residual (Error)	% Error	Remark
M13.0	Selected Modified Variables - selected sieves (Reduced M8 data)	8	2	4	0.783	0.932	0.628	7(t1); 7(t2)	3(t1); 3(t2)	0.932	N/A	N/A	N/A		All Observations Used
M13.1	M13Data with 10 observations; Removed Obs 5: TESAL=2363108)	8	2	4	0.770	0.931	0.660	7(t1); 7(t2)	3(t1); 3(t2)	0.931	2467299	2363108	-104192	4.4	10 out of 11 observations used
M13.2	M13Data with 10 observations; Removed Obs 10: TESAL= 3672489)	8	2	4	0.770	0.930	0.661	7(t1); 7(t2)	3(t1); 3(t2)	0.930	3450591	3672489	221897	6.0	10 out of 11 observations used

E.2. Model Validation Results

The outputs from the model development and validations steps (for the final model selected) are presented in the next sections.

Table E- 5. Summary of outputs from the first PLS model validation/verification phase

Model quality Indexes		
Index	**Comp1**	**Comp2**
Q^2 cum	0.608	0.770
R^2Y cum	0.806	0.930
R^2X cum	0.478	0.661

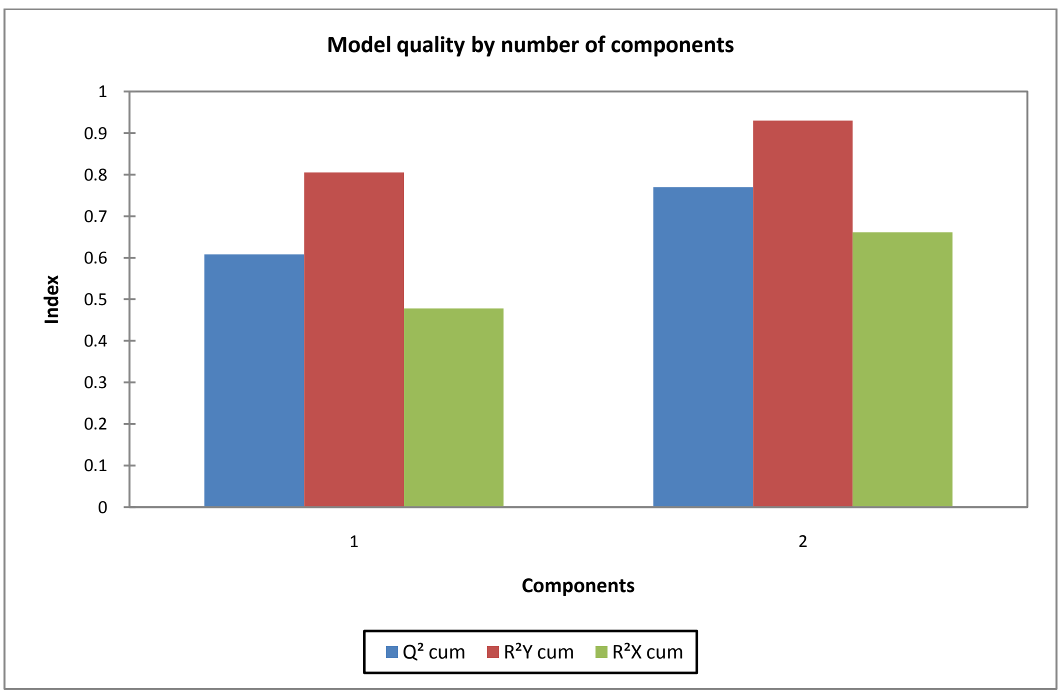

Figure E-1: Model quality by number of components

Table E- 6.Components (T matrix) for X variables (predictors)

Variable	t1	t2
Blend %	-0.862	0.329
-EXP(0.6*BPN)	0.716	-0.088
LAA	-0.286	-0.744
Binder Grade	0.420	0.739
12.5	-0.836	0.289
4.75	0.659	-0.141
0.3	0.615	-0.319
0.15	0.894	-0.212

Table E- 7.Weights (W) matrix for X variables (predictors)

Variable	w1	w2
Blend %	-0.358	0.392
-EXP(0.6*BPN)	0.385	0.035
LAA	-0.283	-0.544
Binder Grade	0.370	0.614
12.5	-0.405	0.144
4.75	0.291	-0.228
0.3	0.253	-0.289
0.15	0.440	-0.128

Table E- 9.Weights (W*) matrix for original coordinates for X variables (predictors)

Variable	w*1	w*2
Blend %	-0.358	0.305
-EXP(0.6*BPN)	0.385	0.129
LAA	-0.283	-0.613
Binder Grade	0.370	0.704
12.5	-0.405	0.046
4.75	0.291	-0.157
0.3	0.253	-0.227
0.15	0.440	-0.021

Table E- 8.Loadings (P) matrix for X variables (predictors)

Variable	p1	p2
Blend %	-0.454	0.280
-EXP(0.6*BPN)	0.377	-0.074
LAA	-0.150	-0.632
Binder Grade	0.221	0.628
12.5	-0.440	0.245
4.75	0.347	-0.119
0.3	0.324	-0.271
0.15	0.471	-0.180

Table E- 10. Weights (C matrix) for Y variable (response)

Variable	c1	c2
Terminal ESAL	0.472	0.299

Table E- 12.Components (T) matrix for, for Y Vector (response)

Observation	t1	t2
438990	-2.729	-0.482
1062723	-2.692	1.445
1558218	-1.953	-0.403
2118493	1.039	-1.702
2363108	-0.711	0.806
3045205	1.344	-0.911
3208372	1.338	0.119
3477567	0.654	-0.611
3558538	0.986	-0.494
5810328	2.722	2.234

Table E- 11.Components (U) matrix for, for Y Vector (response)

Observation	u1	u2
438990	-3.089	-0.568
1062723	-2.223	0.739
1558218	-1.535	0.658
2118493	-0.758	-2.835
2363108	-0.418	0.463
3045205	0.529	-1.287
3208372	0.756	-0.919
3477567	1.129	0.750
3558538	1.242	0.403
5810328	4.368	2.597

Table E- 13. Q^2 values

Q² quality index:		
Component	Total ESAL	Total
Comp1	0.608	0.608
Comp2	0.412	0.412
Comp3	-0.495	-0.495
Cumulative Q² quality index:		
Component	Total ESAL	Total
Comp1	0.608	0.608
Comp2	0.770	0.770
Comp3	0.656	0.656

Table E- 14.Variable Importance in projection (VIP) values

Variable	**VIP (t1)**	**VIP (t2)**
0.15	1.243	1.165
Binder Grade	1.047	1.163
12.5	1.145	1.076
Blend %	1.014	1.027
-EXP(0.6*BPN)	1.090	1.015
LAA	0.799	0.933
4.75	0.824	0.802
0.3	0.716	0.731

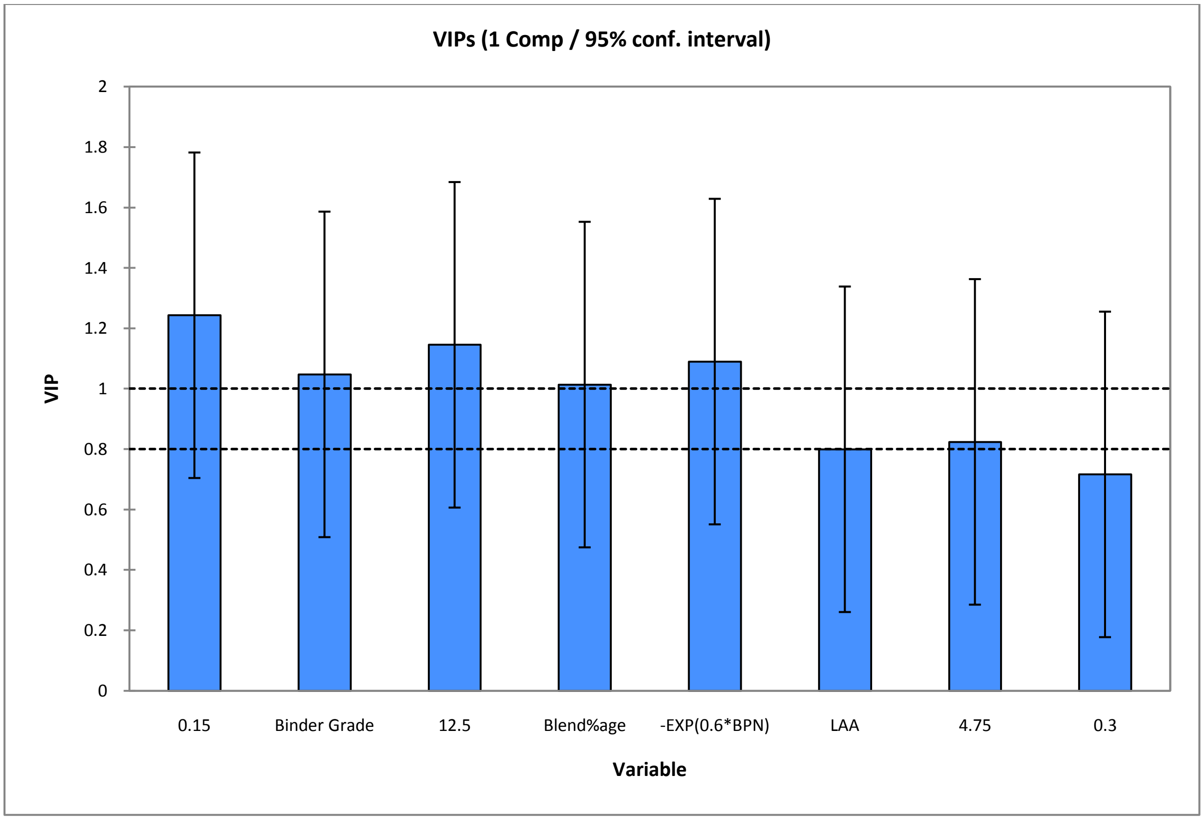

Figure E- 1.Variable Importance in projection (VIP) plots by variable (t1)

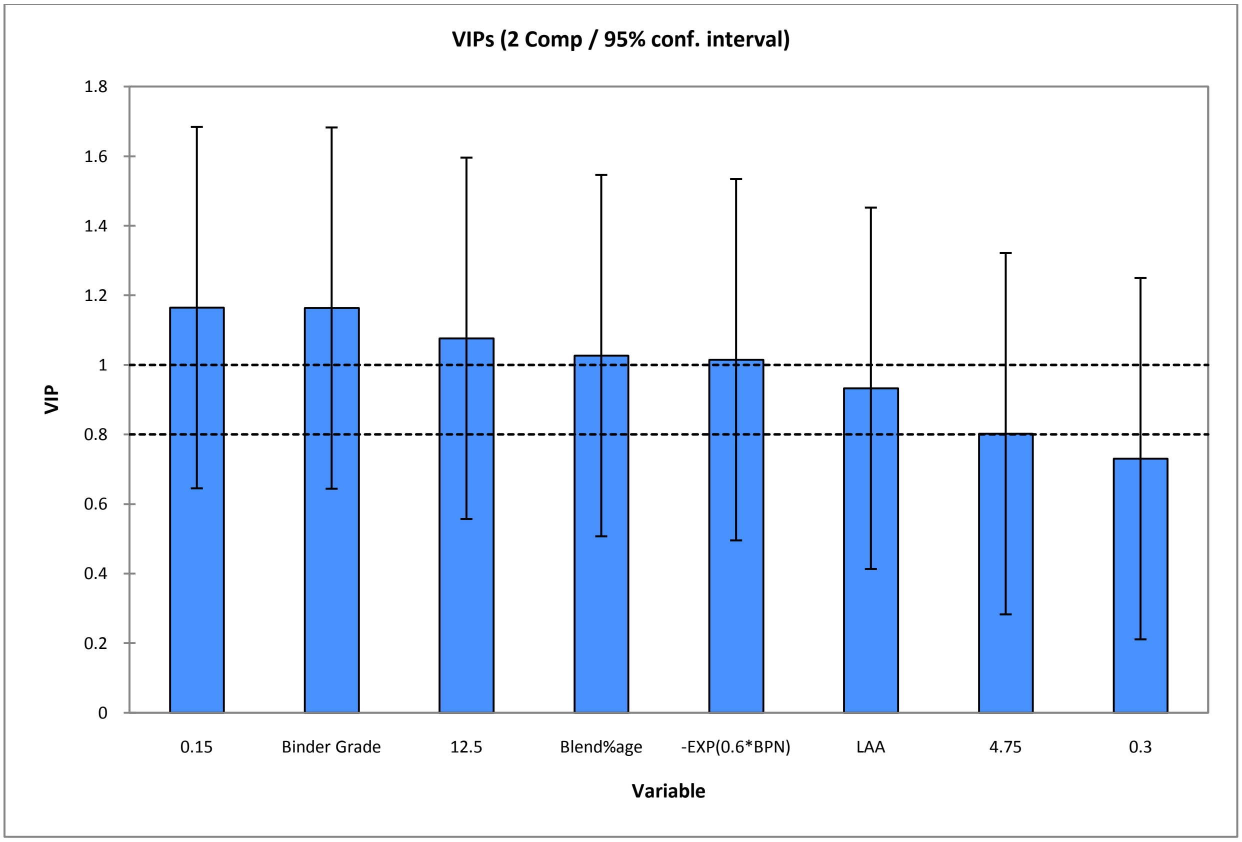

Figure E- 2.Variable Importance in projection (VIP) plots by variable (t2)

Table E- 16. Model Parameters (Coefficients)

Variable	Terminal ESAL
Intercept	2786940.049
Blend %	-8497.057
-EXP(0.6*BPN)	0.001
LAA	-123371.616
Binder Grade	831645.296
12.5	-99337.952
4.75	21920.225
0.3	80937.506
0.15	285585.366

Table E- 15.Goodness of fit statistics

Observations	10.000
Sum of weights	10.000
DF	7.000
R^2	0.930
Std. deviation	457903.548
MSE	146772961462.001
RMSE	383109.595

Table E- 17.Distance to the Model Indexes

Observation	DModX	DModY
438990	0.853	0.031
1062723	0.505	0.253
1558218	0.990	0.380
2118493	0.457	0.406
2363108	0.982	0.123
3045205	0.421	0.135
3208372	0.709	0.371
3477567	0.710	0.487
3558538	1.173	0.321
5810328	0.328	0.130

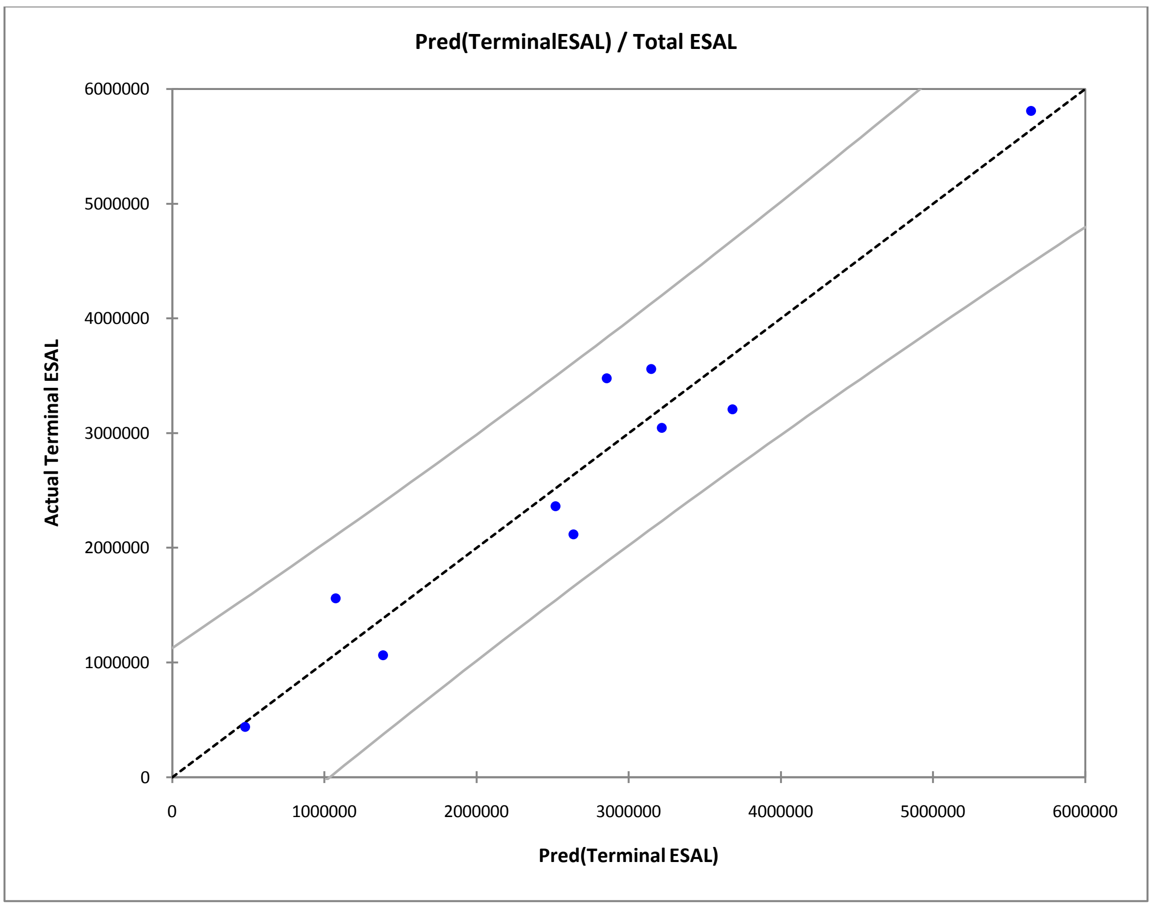

Figure E- 3.Predicted vs. Actual Terminal ESAL values for model (R^2=0.93)

Appendix F

Partial Least Squares (PLS) Regression Modeling Detailed Outputs (Using Alternative Applications)

F.1. PLS Modeling Results using SYSTAT 13®

Table F- 1. Dataset used in SYSTAT

Y	X1	X2	X3	X4	X5	X6	X7	X8
438,990.00	100	-1.30E+09	20	1	5	31	5	2
1,062,723.00	100	-720,000,000.00	18	2	9	25	4	2
1,558,218.00	100	-1,800,000.00	22	1	5	21	6	2
2,118,493.00	65	-540,000.00	22	1	2	39	6	4
2,363,108.00	100	-6,000,000.00	15	1	3	33	4	3
3,045,205.00	72	-11,000,000.00	18	1	0	37	6	4
3,208,372.00	75	-540,000.00	18	2	3	31	7	4
3,477,567.00	75	-11,000,000.00	18	1	3	39	5	4
3,558,538.00	85	-300,000.00	25	2	0	27	6	5
5,810,328.00	75	-6,000,000.00	11	3	1	38	6	4

Y: Terminal ESAL at FN=32

X_1: Percentage of Material from Primary Source [Blend %]

X_2: British Pendulum Number [-exp(0.6*BPN)]

X_3: Los Angeles Abrasion Value [LAA]

X_4: Binder Grade Code [1= PG 64-22, 2=PG 70-22, 3=PG 76-22]

X_5: Percent of Aggregate Retained on 12.5mm Sieve

X_6: Percent of Aggregate Retained on 4.75mm Sieve

X_7: Percent of Aggregate Retained on 0.3mm Sieve

X_8: Percent of Aggregate Retained on 0.15mm Sieve

Summary of Output

Dependent Variable(s): Y

Independent Variable(s): X1 X2 X3 X4 X5 X6 X7 X8

Number of Observations	:	10
Number of Factors Extracted	:	2

The NIPALS algorithm has been used to estimate the model.

Estimates of Regression Coefficients		
	ESTIMATE	**Standard Error**
Constant	2,786,940.049	1,178,157.533
X1	-8,497.057	11,237.231
X2	0.001	0.000
X3	-123,371.616	70,512.070
X4	831,645.296	307,966.084
X5	-99,337.952	46,225.745
X6	21,920.225	26,150.820
X7	80,937.506	133,035.845
X8	285,585.366	104,071.875

Analysis of Variance for Y					
Source	**SS**	**df**	**Mean Squares**	**F-Ratio**	**p-Value**
Regression	1.946E+013	2	9.731E+012	46.407	0.000
Error	1.468E+012	7	2.097E+011		

Percent Variation Explained by Factors for Predictors and Responses				
Factors	**Variation Explained for Predictor(s)**		**Variation Explained for Response(s)**	
	Percentage	**Cum. Percentage**	**Percentage**	**Cum. Percentage**
1	47.814	47.814	80.561	80.561
2	18.331	66.145	12.426	92.987

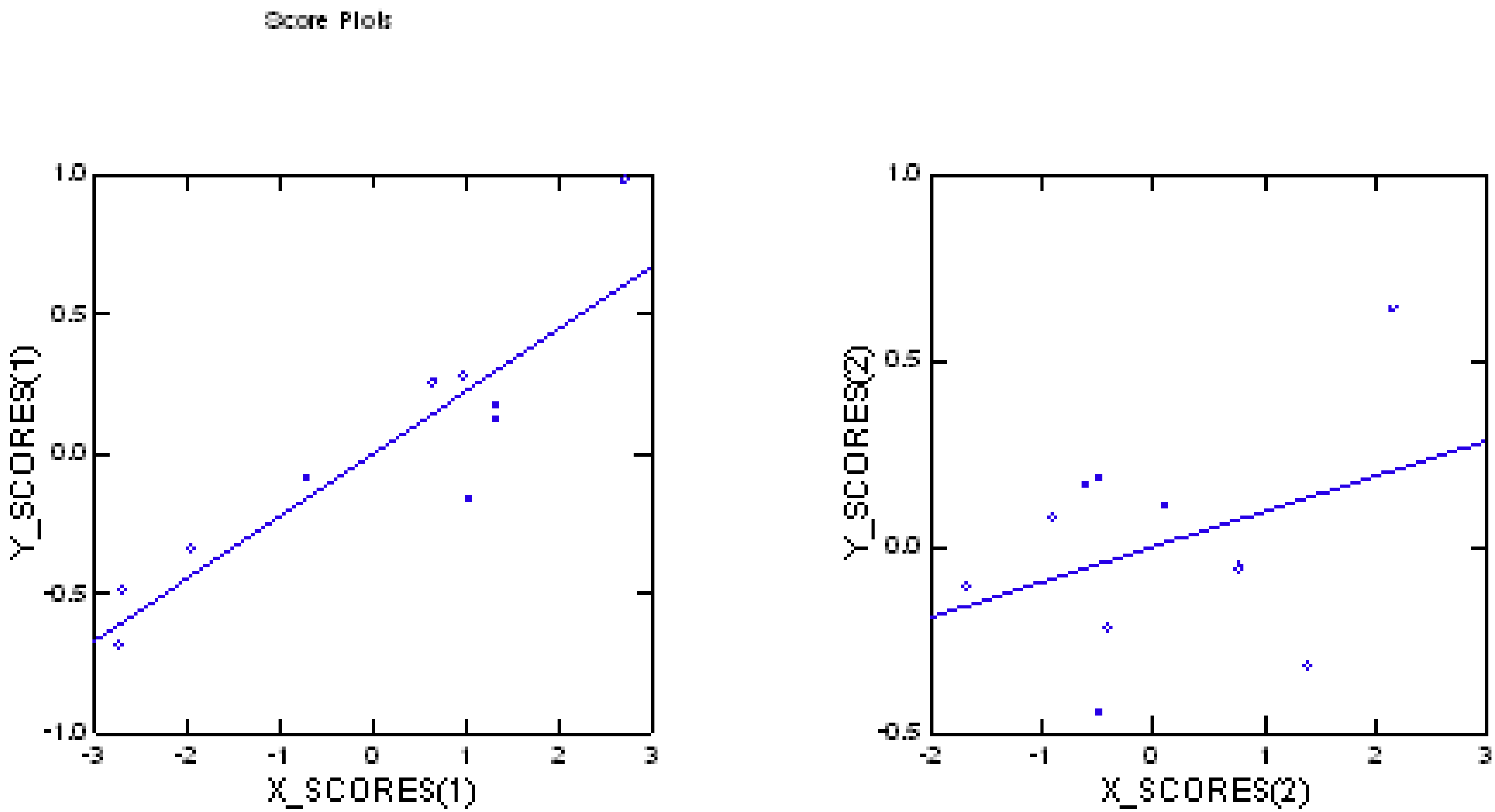

Figure F- 1.Plot of scores (Components for X and Y matrices)

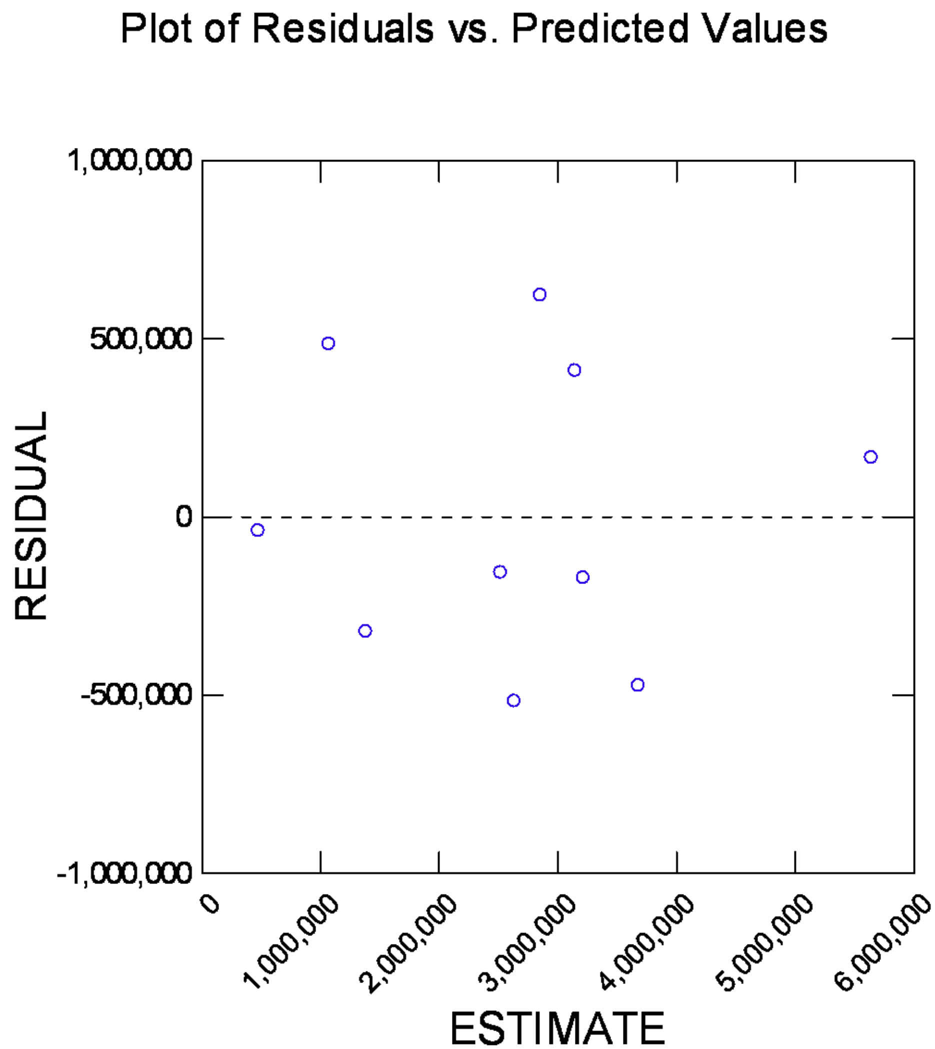

Figure F- 2.Plot of Predicted values versus residuals (errors of estimate)

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