ABSTRACT

Title of Document: RESOURCE ALLOCATION IN AIR TRAFFIC FLOW-CONSTRAINED AREAS WITH STOCHASTIC TERMINATION TIMES.

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In this dissertation we address a stochastic air traffic flow management problem. This problem arises when airspace congestion is predicted, usually because of a weather disturbance, so that the number of flights passing through a volume of airspace (flow constrained area – FCA) must be reduced. We formulate an optimization model for the assignment of dispositions to flights whose preferred flight plans passed through the FCA. For each flight, the disposition can be either to depart as scheduled but via a secondary route thereby avoiding the FCA, or to use the originally intended route but to depart with a controlled (adjusted) departure time and accompanying ground delay. We model the possibility that the capacity of the FCA may increase at some future time once the weather activity clears. The model is a two-stage stochastic program that represents the time of this capacity windfall as a random variable, and determines expected costs given a second-stage decision, conditioning on that time. We also allow the initial reroutes to vary from a conservative or pessimistic approach where all reroutes avoid the weather entirely to an optimistic or hedging strategy where some or all reroute trajectories can presume that the weather will clear by the time the FCA is reached, understanding that a drastic contingency may be necessary later if this turns out not to be true. We conduct experiments allowing a range of such trajectories and draw conclusions regarding appropriate strategies.

RESOURCE ALLOCATION IN AIR TRAFFIC FLOW-CONSTRAINED AREAS WITH STOCHASTIC TERMINATION TIMES.

By

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CHAPTER 1: INTRODUCTION

Air traffic management in the United States has two main components. The first component is Air Traffic Control (ATC), whose primary purpose is to maintain the safety of flights mainly by applying separation rules in the airspace among all flights nationwide. The second component is Air Traffic Flow Management (ATFM), which basically controls any demand-capacity imbalances. This is done by a set of operational decisions that makes sure the demand does not exceed the NAS resources capacities.

In the air traffic management system, the demand can be defined as a time-dependent need for the NAS resources by the air carriers or particularly aircraft flying through the system. Capacity, on the other hand, can be defined as a time-dependent supply for those needs. The capacity of the NAS resources can be variable due to several factors such as the operational level of physical infrastructure, staffing, and weather. For example, the number of runways and their configurations, the number of staff available at each period of time, and weather condition will determine the capacity of an airport and its upstream airports as well. Adverse weather such as convective weather, fog, snow, low ceilings and poor visibility can substantially reduce the capacity of NAS resources. The volume of airspace that experiences convective weather can simply be non-flyable, or its capacity can be reduced significantly, leading to a greater separation needed for safety issues. This reduction of capacity can cause congestion and delays. By some estimates, 70% of the total delay in the NAS is attributable to weather (Robinson et al., 2004).

Under adverse weather conditions, the decisions of ATFM become important in order to reduce possible airborne holdings of flights, as well as to control the workload of the ATC facilities. Some of the ATFM remedies include ground delay programs, rerouting flights to avoid weather-affected areas, and imposing airborne delays through miles-in-trail restrictions. Among these actions, the ground delay program, under which flights are assigned to a later departure time, is the favored recourse, because it imposes all delay costs on the ground and avoids additional pecuniary costs associated with reroutes such as extra fuel burned. Airborne delay is deemed costlier, both because of the extra resources consumed, and because it is thought less safe than holding flights on the ground before they have departed.

The uncertainty about the future conditions is what makes the stochastic ATFM problem most difficult and interesting. It is impossible to reliably forecast weather with the lead time and levels of temporal and spatial resolution necessary to perform ATFM deterministically (Evans, 2001). An ATFM planning tool must therefore take into account the uncertainty about the future capacities of NAS resources such as airports, sectors, fixes, and airways. The uncertainty about the demand for NAS resources is also a challenge to decision-makers, and this issue has received some specific attention in recent studies (Ball et al., 2003; Vossen, 2004). However, in practice, ATFM decisions are made by assuming deterministic time-varying capacities and demands, but with certain ad hoc procedures such as exemptions of long-haul flights, the goal of which is to ameliorate that uncertainty, perhaps at the expense of system efficiency.

Many optimization models for ATFM have been developed, and almost all of them share the same objective, which is to minimize the system-wide delay cost. The delay cost has two attributes – the airborne delay cost and the ground delay cost. Most of the optimization models found in the literature are formulated using either integer or non-integer linear programming. Both deterministic and stochastic optimization models have been used to address and solve the ATFM problems.

Many of the ATFM research efforts have focused on the problem that can be modeled as a single bottleneck (i.e. the destination airport). Other studies that investigate multiple capacity constraint areas include en route sectors and origin airports in their models. These models are mostly deterministic, so it is assumed there is perfect knowledge about the weather condition and any other resources that are included in their modeling. This is a significant limitation that has been addressed and studied in some later works.

Stochastic optimization models have been developed mainly for the single airport ground holding problem (SAGHP) (Richetta and Odoni, 1993), where flights are assigned specified amounts of ground delay at their origins in order to prevent/reduce the arrival demand-capacity imbalance at the destination. SAGHP studies are motivated by the need for decision support models for planning and implementing a ground delay program (GDP) at an airport. The early stochastic optimization models for SAGHP were mostly *static*, meaning that the delays are assigned at the beginning of a planning period, and are not revised later. More research has since been conducted on dynamic stochastic optimization models for the SAGHP. These studies have tried to better utilize the new information available about the capacity of an airport, as the time of day progresses, to revise ground delay decisions of flights. While the focus of most GDP problems is stationary airports as a source of the capacity limitation, bad weather frequently has impacts at locations other than airports. Thus, a more recent avenue of research has adopted methods of GDP planning where appropriate to resource problems in the en route airspace.

A flow-constrained area (FCA) is a region of the national airspace system (NAS) where a capacity-demand imbalance is expected, due to some unexpected condition such as adverse weather, security concerns, special-use airspace, or others. FCAs might be drawn as polygons in a two-dimensional space, although in practice they are usually represented by a single straight line, functioning as a cordon.

When an FCA has been defined, it is then often the case that an airspace flow program (AFP) is invoked by the Federal Aviation Administration (FAA). An AFP is a traffic management initiative (TMI) issued by the FAA to resolve the anticipated capacity-demand imbalance associated with the FCA. While the concept of the FCA has been confronted by FAA in practice through AFP and applying the remedies such as re-routes and airborne delays however a lack of comprehensive analysis and optimization models was the motivation for this dissertation. It is the goal of this dissertation to develop a method by which, given the aggregate data described here, specific orders for individual flights can be developed for a single FCA that a) maximize the utilization of the constrained airspace, b) prevent the capacity of the FCA from being exceeded, and c) achieve a system-wide delay minimization objective. It should be recognized that this model cannot be directly applied to AFP planning as it does not address issues related to the manner in which the FAA and the flight operators collaborate in reaching a final decision regarding each flight. The goal here is to develop relevant stochastic optimization models. We will address some of the issues related to collaborative decision making (CDM) in this dissertation.

CHAPTER 2: BACKGROUND AND RELATED RESEARCH

2.1 ATFM in the US

The Air Traffic Control System Command Center (ATCSCC) located in Herndon, Virginia and, known as the "Command Center", performs ATFM on a national basis. ATCSCC monitors the traffic situation in the NAS, and controls the level of congestion in case of any demand-capacity imbalance. (See FAA, 2004, for a detailed description of ATFM in the United States.). All the data and information on demands and capacities of different NAS components are stored in a computerized system known as Traffic Flow Management System (TFMS). TFMS is utilized with several tools to monitor the current traffic status as well as its possible future projection. Whenever demand is going to exceed capacity (for 15 minutes or more), ATCSCC uses some control measures to mitigate the problem. The main control measures are:

· *Ground Delay Program (GDP):* A GDP is implemented when the arrival capacity of an airport becomes less than the demand for some period of time. The capacity reduction is most likely the effect of bad weather impact on arrival capacity. In a GDP, some of the flights whose scheduled arrival times are within the congested time period are delayed at their origin airport and are rescheduled for a later arrival time. The base intuition for GDP is that it is less expensive and safer to hold and delay a flight on the ground rather than up in the air (airborne delay). GDPs are also issued in regard with the AFPs in the case of congestion in the en route airspace in the vicinity of an airport.

Ground Stops are closely related to GDPs, which are implemented in case of unexpected and severe congestion on airspace. In this case, all flights that have not yet departed will be held on the ground until the congestion is resolved.

· *Rerouting:* whenever some part of a flight's primary path is affected by the presence of bad weather conditions or some other restrictions, the flight can be rerouted via alternative routes. Rerouting not only forces a flight to undergo a longer path toward its destination but also can cause congestion in some condition-free parts of the airspace possibly near the conditioned area as a result of increased traffic, and may require miles-in-trail restriction.

· *Metering Flow at En route Fixes (Miles-in-Trail Restriction):* Miles-in-Trail (MIT) restriction is a distance-based metering technique used in different airspace fixes in order to ensure that the flow of traffic in the en route sectors or congested regions in the NAS does not exceed the capacities. MIT restrictions control traffic flow by imposing a minimum separation between two consecutive aircraft flying across the same fix. MIT restrictions can cause airborne delay but it is less expensive and less disruptive than airborne holding.

2.2 Current GDP practice

The time dependent demands for NAS airports are derived from the flight schedule that is published by Official Airlines Guide (OAG). Airlines report their flights schedules and their daily updates including delays and cancelations to the ATCSCC. The ATCSCC uses a decision support tool called Flight Schedule Monitor (FSM), to track the arrival demand-capacity status of an airport and implements a GDP whenever demand exceeds capacity.

The GDP is usually planned several hours prior to that time period. Demand-capacity imbalances most likely happen due to bad weather conditions and that makes it difficult to anticipate it accurately even for a few hours in advance. Although the ATCSCC usually faces uncertainty about the airport arrival/departure capacity regarding a GDP plan, it assumes a deterministic capacity profile and plans the GDP accordingly.

As a primary technique to manage uncertainty in forecasts, the ATCSCC excludes some of the flights from the GDP and allows them to depart on time. This exemption applies to those flights whose origin airports are beyond a certain distance from their destination airports. The distance is chosen based on the predicted severity of the demand-capacity imbalance.

The logic behind the exemptions is that delays assigned to long haul flights cannot be recovered if airport capacity increases later, and this can result in under-utilization. On the other hand, ground delays assigned to short haul flights can be adjusted tactically according to changes of the airport capacity throughout the day. Another exemption is obviously applied to flights already airborne at the time of GDP implementation as they cannot be assigned ground delays.

The equity issue is the disadvantage of the GDP exemption rule. The airlines with more long haul flights will benefit more and an airline that operates mostly short haul flights might face significant ground delays. Another issue with this rule is that it is only advantageous if the probability of weather clearance (i.e. capacity increase) is high. If it is low, then the exemption strategy may not only be unfair but also may be inefficient.

When the exempt flights are assigned to their scheduled arrival slots, the remaining arrival slots are assigned to the non-exempt flights based on Ration-by-Schedule (RBS) mechanism. The non-exempt flights are sorted according to their scheduled arrival times, and receive the arrival slots according to their position in the list. Hoffman (1997) provides a detailed discussion of the actual RBS algorithm.

2.3 Optimization Models for ATFM

Much research has been conducted on ATFM optimization for more than a decade. We can categorize this research into two main areas: 1) those optimization models that only consider the limited capacity of the airports as origins and/or destinations while ignoring the en route capacity. 2) Optimization models that take into account the en route capacity as well as the airport capacity.

The first set of models address what is known as the ground holding problem (GHP), and the seconds set refers to the multi-airport air traffic management problem.

In the following section we present a brief review on the several key optimization models for ATFM. Many of these models and research are dedicated to the ground holding problem (GHP). The objective of the GHP is to minimize the total delay costs whenever a capacity-demand imbalance is anticipated at the destination airports. This is achieved by assigning ground delays to the flight on the ground waiting to depart. The GDP itself can be broken into two sub-problems: the single airport ground holding problem (SAGHP) and the multi-airport ground holding problem (MAGHP).

In the SAGHP only one airport is considered while in the MAGHP a network of airports is taken into account for optimization modeling. In the MAGDP the delay assigned to a flight can propagate to its downstream destinations. Some versions of MAGDP consider crew and passenger connectivity effects.

The GHP was mathematically modeled first by Odoni (1987). Other deterministic models were subsequently, developed, where airport capacities assumed to be known in advance, regardless of their dynamic nature. Also a deterministic model was developed by Terrab and Odoni (1993) for SAGHP. Their model minimizes the total ground delay cost for a set of flights with linear delay cost function for input parameters. The concept of banking constraints was used in a deterministic model for the SAGHP by Hoffman (1997) and Hoffman and Ball (2000). Their model requires group(s) of flights to arrive within pre-specified time windows. Adding such constraints is useful to model hub-spoke operations at major airports; see Hoffman (1997) for a discussion on hub-spoke operations. Vossen (2002) proposed an optimization model for mitigating bias from exempting flights from a GDP. Vossen and Ball (2003) described a general framework for equitable allocation procedures within the context of ATM, and illustrated its use in reducing certain systematic biases that exist under current procedures.

Vranas et al. (1994) used the IP formulation to model a Deterministic optimization of MAGHP for the first time. However their model did not performed well for the large scale problems computationally. A stronger formulation was developed by Bertsimas and Stock (1998). Navazio and Romanin-Jacur (1998) proposed a deterministic optimization model for MAGHP with banking constraints.

Uncertainty in airport capacities has been addressed mainly in the context of the SAGHP, although Vranas et al. (1994) provides some treatment of a stochastic version of the MAGHP. Richetta and Odoni (1993) proposed a static stochastic IP formulation for solving the SAGHP under uncertainty in airport arrival capacities. Hoffman (1997) and Ball et al. (2003) proposed a modified version of the static stochastic optimization for SAGHP, which solves for the optimal number of planned arrivals of aircraft during different time intervals. In the static models, decisions related to departure delays of flights are taken once at the beginning of the planning horizon, and are not revised later. This limitation was addressed by Richetta and Odoni (1994), who formulated a multistage stochastic IP with recourse for the SAGHP. In their model, the ground delays of flights are not decided once at the beginning; rather they are decided at the scheduled departure time of the flights. However, ground delays once assigned cannot be revised later in their model.

Deterministic optimization models addressing en route capacity constraints were formulated as multi-commodity network flow problem by Helme et al. (1992), and more recently by Bertsimas and Stock (2000). Unlike single-commodity flow network formulations, these models are computationally harder and do not guarantee integer solutions from LP relaxations. One of the assumptions made by Helme et al. (1992) was that each aircraft route is pre-determined before its departure. The Bertsimas and Stock (2000) model addresses routing as well as scheduling decisions, but it produces non-integer solutions for even small scale problems. Therefore the authors suggested heuristics to achieve integer solutions.

Disaggregate deterministic integer programming models for deciding departure time and route of individual flights were formulated by Lindsay et al. (1993), and more recently by Bertsimas and Stock (1998). Although both formulations produce non-integer solutions from LP relaxation, the latter model achieves integrality in many more instances compared to the former, by virtue of its formulation. Goodhart (2000) formulated disaggregate deterministic models for ATFM, in which airlines' priorities for various flights were accommodated. Such formulations are useful for applying optimization techniques in ATFM under the paradigm of CDM.

Weather-related uncertainty in the en route airspace congestion was addressed by Nilim et al. (2002). Their work focuses on dynamically rerouting an aircraft across a weather impacted region.

2.4 Weather Activities effect on ATM

Weather is a major limiting factor in the National Airspace System (NAS) today, accounting for roughly 70% of all traffic delays .Because we cannot control the weather and because safety must be maintained in the presence of weather related hazards, our ability to predict the weather and its influence on air traffic is a critical element of new developments for future ATM. Hazardous weather conditions such as convective weather (e.g., lightning, tornados, turbulence, icing, and hail), extreme weather events (hurricanes, blizzards, and large scale weather systems), low visibility (due to fog, haze, or clouds), clear air turbulence, snow (including surface snow removal and aircraft de-icing), and wind shifts (that affect safe takeoff and landing of aircraft) pose challenges to the NAS on nearly a daily basis. Something as simple as wet runways can cause a major airport (e.g., Chicago O'Hare International Airport (ORD)) to lower its arrival rate due to the reduced ability of aircraft to brake during landing. (Krozel et al. 2003)

These problems are not related only to weather affected areas. Because delays propagate through the NAS, weather-related problems in one region (especially in the northeast) often propagate through a greater portion of the NAS, both in time and spatial dimensions. The key to greater capacity in the NAS lies in our ability to accurately predict and adjust the future state and resources of the NAS on a timescale consistent with critical NAS response times. Historically, prediction and adjustment of the NAS has been limited by the uncertainties of weather, a lack of adequate reasoning in relation to these uncertainties within Decision Support Tools (DSTs), and a lack of tools to support distributed and coordinated decision making and shared situation awareness among the users.

Additionally, a limited set of capabilities for setting long-term Traffic Flow Management (TFM) initiatives required to sufficiently deal with the effects of weather influences capacity prediction and adjustment. These and other weather-related limitations must be overcome in the future in order to increase NAS capacity.

Probabilistic forecast methods may be used to estimate the capacity of a particular airspace resource influenced by the presence of weather activities. Capacity estimation involves the analysis of both the demand on a resource and the weather hazard.

2.5 Collaborative Decision Making (CDM)

Collaborative decision making concept was first introduced in the mid 1990s. Prior to that ATFM was performed based on a centralized setting and ATCSCC had the authority to impose restrictions on flights routes and departure times. The developments of CDM have allowed participation of airlines in ATFM decision making (FAA, 1999). The collaboration and information sharing between the FAA and airlines has increased their ability to resolve day-to-day NAS NAS congestion problems more efficiently.

Creating a common situational awareness among the FAA and airlines about NAS capacity and demand is the main objective of CDM. Possible information exchange between the FAA and airlines provides airlines with greater flexibility to make their own decisions.

Under the CDM concept, ATFM is performed in a de-centralized setting. FAA (or ATCSCC) monitors the status of the NAS demands and resources capacity and allocates constrained resources among the users more equitable and efficient. Airlines are supposed to make their operational plans and strategies available to the FAA and in return, receive flexibility to make use of their share of resource capacities based on their individual business goals.

The Collaborative Decision Making (CDM) program has been a very successful paradigm for allocation of airport arrival slots. Aside from the detail operational considerations, the essence of the allocation principle is "first-scheduled, first-served", meaning that the earlier arrival slots are assigned to the flights that are scheduled to arrive earlier. Experiments have shown that this approach is an equitable treatment and provides an efficient use of resources as well. Prior to CDM, effective GDP initiatives were based on dated flight data that did not reflect the airline's schedule changes and adjustments on the day of operation. This had negative effect on flow control and led to inefficiencies.
The airline believed that they were not treated equitably and the information that they provide might be used in the favor of their companies. This caused they hesitate to provide up-to-date information. By introducing the new resource allocation mechanism that were base on first scheduled, first served standard yet allowed them to use their share of resources based on their own objectives they were eager to provide up-to-date information used for CDM purposes. The cooperation among the users and controllers has ever since improved the system efficiencies.

Initially the concept of the CDM was used to increase the efficiency of the GDP (Ball et al. 2000). Based on the observed benefits of applications of CDM on GDP, the Collaborative routing concept has been motivated. The idea of CR is to mitigate the airspace congestion by increased collaboration among airlines and the FAA (Burke, 2002)

Collaborative Trajectory Options Program (C-TOP) previously named System Enhancements for Versatile Electronic Negotiation (SEVEN) is a new concept for managing en route congestion that allows NAS customers to submit cost weighted sets of alternative trajectory options for their flights. C-TOP provides traffic managers with a tool that algorithmically takes these customer costs into consideration as it assigns reroutes and delays to flights subject to traffic flow constraints. This concept has the potential to reduce the workload of traffic managers while allowing them finer control over traffic in uncertain weather situations and gives NAS customers greater flexibility to operate their flights according to their business priorities. One of the most significant benefits is the ability to recapture system capacity that is currently lost when severe weather (or other capacity limiting factors) does not materialize as predicted. Presently, the CDM Future Concepts Team (FCT) is evaluating this concept through a series of storyboards, simulations and human-in-the-loop exercises with operational personnel from the airlines, general aviation, and traffic management communities (FAA, 2008).

There are two key enabling ideas at the core of C-TOP. The first is that NAS customers are able to submit cost weighted sets of trajectory options to the traffic management system (TMS), which they are able to update as often as needed. The second is that traffic managers manage demand on resources (such as Flow Constrained Areas (FCAs) by setting capacities on those resources then running allocation algorithms that adjust demand to meet those capacities while attempting to place each flight on the lowest cost option available.

An algorithmic approach was developed by Krozel et al. (2006), for airspace flow programs dealing with the FCAs. Their algorithm includes ground delay, route selection, and airborne holding as decision variables for departing and en route flights. A dynamic FCA capacity-estimation algorithm uses weather forecast information to produce time-varying entry and exit points as well as maximum flow rates through FCAs.

2.6 Related research

This proposal builds on stochastic ground holding models. Several stochastic integer programming models have been developed (Ball et al. (2003), Mukherjee et al. (2007), Mukherjee et al.(2009), Richetta et al.(1993), as mentioned above. While my model of FCA capacity is conceptually similar to airport arrival capacity models, I also explicitly represent the possibility of reroutes, including their dynamic adjustment under stochastic changes in FCA capacity.

While there is a growing body of literature on airspace flow management problems, this work also builds on earlier work by Nilim and his co-authors on the use of “hybrid” routes that hedge against airspace capacity changes. In Nilim et al. (2001), the rerouting of a single aircraft to avoid multiple storms and minimize the expected delay was examined. In their model, the weather uncertainty was treated as a two-state Markov chain, with the weather being stationary in location and either existing or not existing at each phase in time. A dynamic programming approach was used to solve the routing of the aircraft through a gridded airspace, and the aircraft was allowed to hedge by taking a path towards a storm with the possibility that the storm may resolve by the time the aircraft arrived. The focus of the work was on finding the optimal geometrical flight path of the aircraft, and not on allocation of time slots through the weather area as in the case of my model. Follow-on work expanded to modeling multiple aircraft with multiple states of weather and attempted to consider capacity and separation constraints at the storms, Nilim et al. (2003).

Initial steps at a concept of operations that describes the terminology, process, and technologies required to increase the effectiveness of uncertain weather information and the use of a probabilistic decision tree to model the state space of the weather scenarios was provided in Ball et al. (2003). Making use of this framework is a model recently proposed that uses a decision-tree approach with two-stage stochastic linear programming with recourse to apportion flows of aircraft over multiple routing options in the presence of uncertain weather, by Hoffman et al. (2007). In the model, an initial decision is made to assign flights to various paths to hedge against imperfect knowledge of weather conditions, and the decision is later revised using deterministic weather information at staging nodes on these network paths that are close enough to the weather that the upcoming weather activity is assumed known with perfect knowledge. Since this is a linear programming model, only continuous proportions of traffic flow can be obtained at an aggregate level, and not decisions on which individual flights should be sent and when they should arrive at the weather. In Mukherjee et al.(2009), a stochastic integer programming model is developed based on the use of scenario trees to address combined ground delay-rerouting strategies in response to en route weather events. While this model is conceptually more general than mine, by developing a more structured approach we hope to develop a more scalable model.

Recently, a Ration-by-Distance (RBD) method was proposed as an alternative to the Ration-by-Schedule (RBS) method currently used for Ground Delay Programs (GDPs). The RBD method maximizes expected throughput into an airport and minimizes total delay if the GDP cancels earlier than anticipated. This approach considers probabilities of scenarios of GDP cancellation times and assigns a greater proportion of delays to shorter-haul flights such that when the GDP clears and all flights are allowed to depart unrestricted, the aircraft are in such a position that the expected total delay can be minimized. While this problem was applied to GDPs, the principles of a probabilistic clearing time where there is a sudden increase in capacity and making initial decisions such that the aircraft are positioned to take the most advantage of the clearing are similar to my problem.

CHAPTER 3: THE MODEL

3.1 Introduction

In this chapter we present an optimization model for the assignment of dispositions to flights whose preferred flight plans pass through an FCA. For each flight, the disposition can be either to depart as scheduled but via a secondary route, or to use the originally intended route but to depart with a controlled (adjusted) departure time and accompanying ground delay. We model the possibility that the capacity of the FCA may increase at some future time once the weather activity clears. The model is a two-stage stochastic program that represents the time of this capacity windfall as a random variable, and determines expected costs given a second-stage decision, conditioning on that time. The model is extended by allowing the initial reroutes to vary from a conservative approach, where initial trajectories avoid the weather entirely, to an optimistic approach, where initial trajectories can be assigned as if the weather were not present.

3.2 Model Inputs

Our base model inputs consist of information about the FCA, which is consistent with the information used in AFP planning:

- Location of the FCA
- Nominal (good weather) capacity of the FCA
- Reduced (bad weather) capacity of the FCA
- Start time of the AFP
- Planned end time of the AFP

The FAA Web site, http://www.fly.faa.gov/, provides near real-time status information about the NAS. We may also receive information through the local air traffic facility (including flight service stations), airline, flight department, or other professional organizations (e.g., National Business Aviation Association, Air Transport Association, Aircraft Owners and Pilots Association).

Operational Information System (OIS) is a Web page managed by the ATCSCC that provides current information about the status of the NAS (Find it at http://www.fly.faa.gov/ois).

An advisory is a message that is disseminated electronically by the ATCSCC or ARTCC. It contains information pertaining to the NAS. Advisories are normally used for but not limited to:

- Ground stops
- GDP
- AFP
- Route information
- FCA

Any time there is information that may be beneficial to a large number of fliers, an advisory may be sent. There may be times when an advisory is not sent due to workload or the short duration of the activity. We can access U.S. and Canadian advisories for the current day and the previous 14 days at (http://www.fly.faa.gov/adv/advADB.jsp).

Route information is published in various sources for different purposes, different users and different time frames. Route information is contained in the:

- Airport Facility Directory
- Preferential route information in the host computer
- Route Management Tool (RMT)
- North American route advisory circular
- OIS
- Federal Air Regulations (FAR)
- NOTAM

From a list of scheduled flights and their flight plans, we determine the set of flights whose paths cross the FCA and which therefore would be subject to departure time and/or route controls under an AFP. We also require a set of alternate routes for each flight. The alternate route for each flight should be dependent on the geometry of the FCA and the origin-destination pair it serves. These most likely would be submitted by carriers in response to an AFP; for the purposes of this study it is assumed they are submitted exogenously, although for testing purposes it was necessary to synthesize some alternate routes.

3.3 Controls

In order not to exceed the (reduced) FCA capacity, each flight will be assigned one of two dispositions in the initial plan reacting to the FCA:

1. *The flight is assigned to its primary route, with a controlled departure time that is no earlier than its scheduled departure time.* Given an estimate of en route time, this is

tantamount to an appointment (i.e., a slot) at the FCA boundary. Some flights might be important enough that they depart on time, the AFP notwithstanding. Other flights might be assigned some ground delay.

2. *The flight is assigned to its secondary route, and is assumed to depart at its scheduled departure time.* As an extension to our basic model, we allow a much more flexible and general definition of secondary route. A “conservative” secondary route would employ a trajectory directed around the periphery of the FCA (this was the case considered in the earlier papers). An “optimistic” secondary route would fly directly at the FCA (even though the flight did not have an appropriate slot). If the weather did not clear, an aircraft following such an optimistic route would have to turn away from the FCA as late as possible and then fly around the FCA periphery. Inspired by the work by Nilim et al. (2001) [(24)], we also consider intermediate routes, which hedge between the optimistic and conservative ones.

Several assumptions underlie our model:

- We do not consider airborne holding as a metering mechanism to synchronize a flight on its primary route with its slot time at the FCA.
- We assume that any necessary number of flights can be assigned to their secondary routes without exceeding any capacity constraints in other parts of the airspace.
- We assume that, when the weather clears, the FCA capacity increases immediately back to the nominal capacity.
- The random variable is the time at which the FCA capacity increases back to its nominal value. We assume that perfect knowledge of the realization of this random variable is not gained until the scenario actually occurs, and so no recourse can be taken until the scenario is realized.

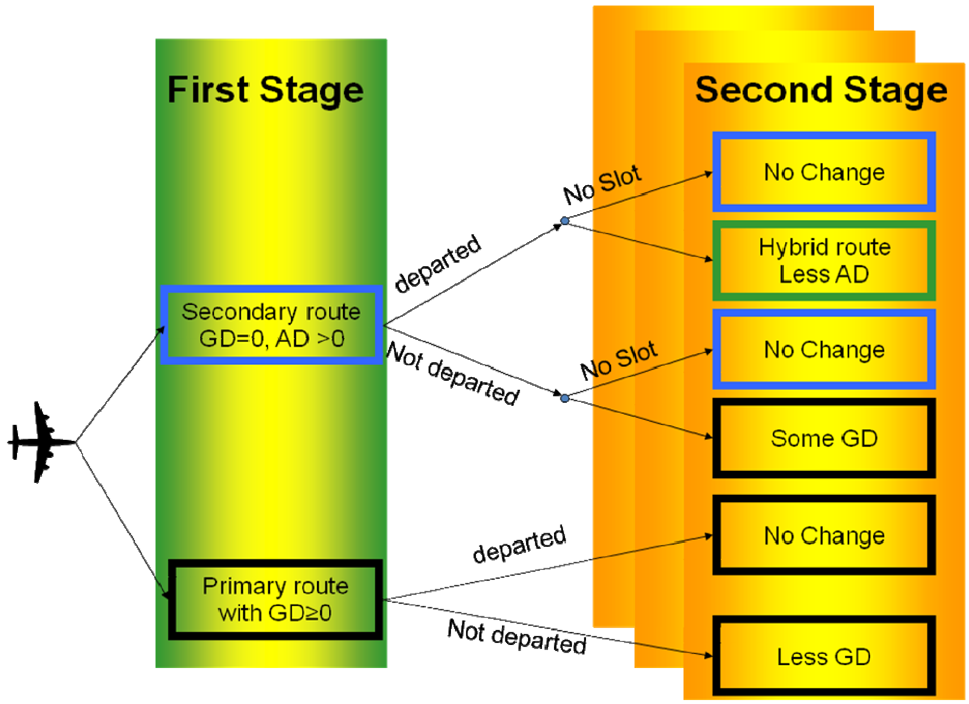


Figure 3-1: Model Control Structure

3.4 Scenarios and future responses

The outputs of this model are:

1. An initial plan that designates whether a flight is assigned to its primary route or secondary route; for those assigned to their primary route an amount of ground delay (possibly zero) is assigned. For those assigned to their secondary route a specific directional angle (possibly zero) is assigned.
2. A recourse action for each flight under each possible early clearance time.

We model the time at which the weather clears (i.e. FCA capacity increases) as a discrete random variable, with some exogenous distribution. For any realization of the capacity increase time, the flights in question will be in some particular configuration as specified in the initial plan. Some will have departed, either on their primary or secondary routes, some will already have completed their journeys, and some will still be at their departure airports.

Flights that were originally assigned to their primary route and that have already taken off will be assumed to continue with that plan. For any such flight, the primary route is assumed to be best, so no recourse action is necessary.

We now consider flights originally assigned to their primary route that have not yet taken off. We need not consider transferring them to their secondary routes, because if that were a good idea in the improved capacity situation, it would also have been a good idea in the initial plan. Thus, the only possible change in disposition for these flights involves potentially changing their controlled departure time, i.e. reducing their assigned ground delay.

All other flights not yet considered were originally assigned to their secondary routes, with departure times as originally scheduled. These secondary routes avoid the FCA somehow. Under the FCA capacity windfall, some of those flights may now have an opportunity to use the FCA. If a flight has not yet taken off, and it is decided that it can use the FCA, the lowest cost way to do this is to re-assign it back to their primary route, with some controlled departure time no earlier than their scheduled departure time. If, on the other hand, the flight has already taken off, then the only mechanism to allow it the use of the FCA is a hybrid route that includes that portion (and perhaps more) of the secondary route already flown, plus a deviation that traverses the FCA and presumably rejoins the primary route at some point after the FCA (see Figure 3-1). A flight that is already en route via its secondary route may or may not prefer such a hybrid path, depending on the difference in cost (time, fuel, etc.) between doing that and continuing on its secondary route. There may be many possible hybrid routes, and perhaps only a limited set of those would be reasonable.

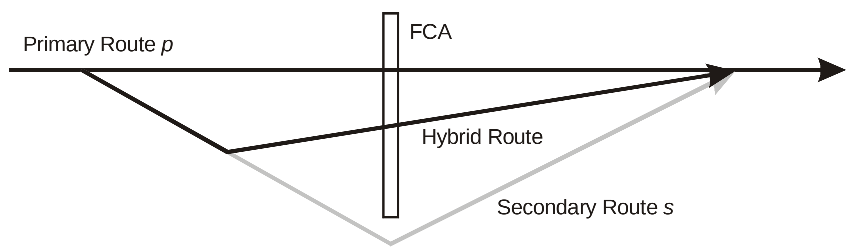


Figure 3-2: Reverting from secondary route back to primary route through FCA.

For each possible value of the capacity windfall time, we determine the expected locations of all affected flights at that time, and also what would be the best change in disposition, if any, for each of those flights according to a system performance metric. With this information, we can compute the conditional cost associated with these flights adjusted based on the realization of the stochastic event.

Ultimately, then, the goal of the optimization problem is to minimize the expected total cost, given these conditional costs and their probabilities.

3.5 Model Developments

We start by defining the discrete lattice on which time will be represented. We assume there is an index set $\{1,\ldots,T\}$ of size T that demarcates equally spaced time slots, each of duration Δt. Each of these represents a possible appointment time window at the FCA. The nominal capacity of the FCA should be specified in terms of the maximum number of flights permissible during one of these time windows. The number of time slots T then depends directly on Δt and the total duration of an AFP, perhaps inflated to allow for ending times later than the original estimate. The reference time $t=1$ can be chosen as the earliest scheduled departure time of all of the affected flights. The actual time indicated by the index t is then $(t-1/2)\Delta t$.

The flights affected by the FCA can be determined from the filed flight plans for that day, minus known cancellations and re-routes at the time the AFP is invoked. These flights are indexed according to the set $\{1,\ldots,F\}$. In the rest of the paper, any specific reference to a time period t and flight f assumes that $t \in \{1,2,\ldots,T\}$ and $f \in \{1,\ldots,F\}$.

3.5.1 Initial Plan

There are two sets of assignment variables that are related to decisions about the dispositions of flights. One set represents the initial plan, which is the set of decisions provided by the model that will be enacted immediately once the model is run and the AFP is declared. The second set represents conditional decisions (recourse actions) based on the random variable representing the time at which the capacity windfall takes place, which we do not know at the time of the execution of this optimization problem, but that we condition for when determining the best initial plan.

For the initial plan, we define the following set of binary decision variables:

$$x^p_{f,t} = \begin{cases} 1, & \text{if flight } f \text{ uses its primary route and} \\ & \text{has an appointment time } t \text{ at the FCA} \\ 0, & \text{otherwise} \end{cases}$$

$$x^s_{f,r} = \begin{cases} 1, & \text{if flight } f \text{ is assigned to its secondary route} \\ & \text{that has a directional angle r} \\ 0, & \text{otherwise} \end{cases}$$

Every flight f needs to have an assigned disposition under the initial plan thus:

$$\sum_t x^p_{f,t} + \sum_r x^s_{f,r} = 1 \quad \forall f \tag{3.1}$$

We require that any flight that is assigned to its primary route cannot be given an appointment slot at the FCA that is earlier than its scheduled departure time plus the expected en route time required to arrive at the FCA. If $E_f \Delta t$ represents the en route time (from its origin to the FCA) for flight *f*, and $D_f \Delta t$ is the scheduled departure time for flight *f*, then:

$$\sum_{t=1}^{D_f + E_f} x^p_{f,t} = 0 \quad \forall f \tag{3.2}$$

No similar constraint is applied to flights assigned to their secondary routes under the initial plan, because they are not metered at any point and hence are expected to depart at their originally scheduled departure time. There is no provision in the model for a flight to depart early, despite the fact that the secondary route takes more time than the primary route (since, subject to minor variations, airlines do not allow flights to take off before their scheduled departure times).

It might be the case that for a particular flight *f*, there is a latest slot time l_f at the FCA that the carrier who owns that flight would be willing to accept. Slots later than l_f can be prevented via the following constraint:

$$\sum_{t=l_f+1}^{T} x^p_{f,t} = 0 \tag{3.3}$$

For any flight for which l_f is not explicitly provided, l_f is the time beyond which the secondary route will be chosen.

The initial constrained capacity (maximum number of flights) for time window *t* can now be defined as C^0_t and the constraint to enforce it is:

$$\sum_f x^p_{f,t} \le C^0_t \quad \forall t \tag{3.4}$$

3.5.2 Second Stage

The variables and constraints defined so far represent the first stage of the stochastic program. It is assumed that these decisions will be enacted deterministically immediately after the FCA is declared. Next, we describe the second stage of the stochastic program – those variables that represent the conditional decisions we expect would be made if any of a number of possible capacity windfall times happens to come true in the future. We model the time slot at which this occurs as a discrete random variable with domain Ω and probability mass function

$$f_U(u) = \Pr\{U = u\} \quad \forall u \in \Omega$$

Under a capacity windfall, a flight that was originally assigned to its primary route with a controlled departure time might still be given the same general disposition, although its departure time could be moved earlier if that were beneficial to the system goal. We let

$$y_{f,t}^{p} \mid u = \begin{cases} 1, \text{ if at the time } U = u \text{ of the capacity windfall,} \\ \text{ flight } f \text{ is assigned to its primary route with} \\ \text{ appointment slot } t \text{ at the FCA} \\ 0, \text{ otherwise} \end{cases}$$

We will (shortly) introduce other variables for the other possible second stage flight dispositions, and we will require that all flights be assigned a disposition under every possible realization of the stochastic event *U*. For now, we proceed by obviating values of $y_{f,t}^{p} \mid u$ that would either be physically infeasible or politically imprudent. Later, structural constraints plus pressure from the objective function will lead to the best possible selection of second stage dispositions for all flights.

First, it is impossible to assign a flight to a slot that would require it to depart before its scheduled departure time:

$$y_{f,t}^{p} \mid u = x_{f,t}^{p} \quad \forall f, u, \quad \forall t \in \{1, ..., D_f + E_f\} \tag{3.5}$$

This constraint works with constraint (3.2) to achieve the required result.

Given the timing U of the capacity windfall, some flights may already have taken off. If they did so via their primary route (with a controlled departure time), then their second stage disposition should match that of the first stage:

$$y_{f,t}^{p} \mid u = x_{f,t}^{p} \quad \forall f, u, \quad \forall t \in \left\{1, \ldots, u + E_f\right\} \tag{3.6}$$

A closer look at constraint (3.6) reveals that it also satisfies an important requirement for flights that have not yet taken off. For any particular flight f and given the capacity windfall time u, the collection of primary stage variables $\left\{x_{f,t}^{p}\right\}_{t=1}^{t=u+E_f}$ will either contain one at exactly one position or it will consist entirely of zeros. In the former case, this means that the flight has already taken off, and that situation has been dealt with. In the latter case, this is indicative of the fact that these slot times are infeasible. Thus, even for flights that have not yet taken off, constraints (3.2) and (3.6) insure that they will not be assigned, in the second stage, to their primary routes with slot times that they cannot achieve.

Looking at constraints (3.5) and (3.6), it is clear that they can be combined:

$$y_{f,t}^{p} \mid u = x_{f,t}^{p} \quad \forall f, u, \quad \forall t \in \left\{1, \ldots, \max\left(u, D_f\right) + E_f\right\} \tag{3.7}$$

On the other hand, for flights that already took off via their secondary routes (and therefore at their scheduled departure times), the only possible second stage dispositions are secondary or hybrid routes, so assignments to primary routes for these flights must be prevented:

$$\sum_{t} y_{f,t}^{p} \mid u \leq 1 - \sum_{r} x_{f,r}^{s} \quad \forall u, \forall f \ni D_f < u \tag{3.8}$$

In addition, we will not allow a flight whose controlled departure time is being moved in the face of a capacity windfall to be worse off than it was before this event materialized:

$$y_{f,t}^{p} \mid u \leq \sum_{q \geq t} x_{f,q}^{p} + \sum_{r} x_{f,r}^{s} \quad \forall u, f, t \tag{3.9}$$

Notice that we want to allow for the possibility that flights originally assigned to their secondary routes can revert under appropriate circumstances, and if the optimization decides this is best, to their primary route if they have not already taken off, which is why the variable $x^{s}_{f,r}$ appears in constraint (3.9).

For flights that were originally assigned to the secondary route, the increased capacity at the FCA might allow some of these flights to pass through the FCA and thus improve their flight path by returning to the primary route at some point after the FCA or continuing directly to the destination. For a flight that has not yet departed, the same structure can apply, but the portions of the total flight path spent on the secondary and reverting routes have length zero. We define the second-stage decision variables for this choice as follows:

$$y^{h}_{f,t,r} \mid u = \begin{cases} 1, \text{ if flight } f \text{ was originally assigned to its} \\ \text{secondary route with directional angle } r, \\ \text{but under capacity clearing time } u \text{ has} \\ \text{been assigned an FCA appointment slot } t \\ 0, \text{ otherwise} \end{cases}$$

This decision can only be reached for flights that were originally assigned to their secondary routes:

$$y^{h}_{f,t} \mid u \le x^{s}_{f} \quad \forall u, f, t \qquad (3.10)$$

However, we note that the objective function will enforce this behavior implicitly. Such a flight will be on its secondary route, which may be altered to become a hybrid route that passes through the FCA. We need to impose constraints that insure that these flights are only assigned to FCA time slots they can feasibly reach. If a flight diverts from its secondary route to its hybrid route at time t^{d} there will be an earliest time it can reach the FCA. Figure 3-2 illustrates the geometry used to compute the parameter used by our model:

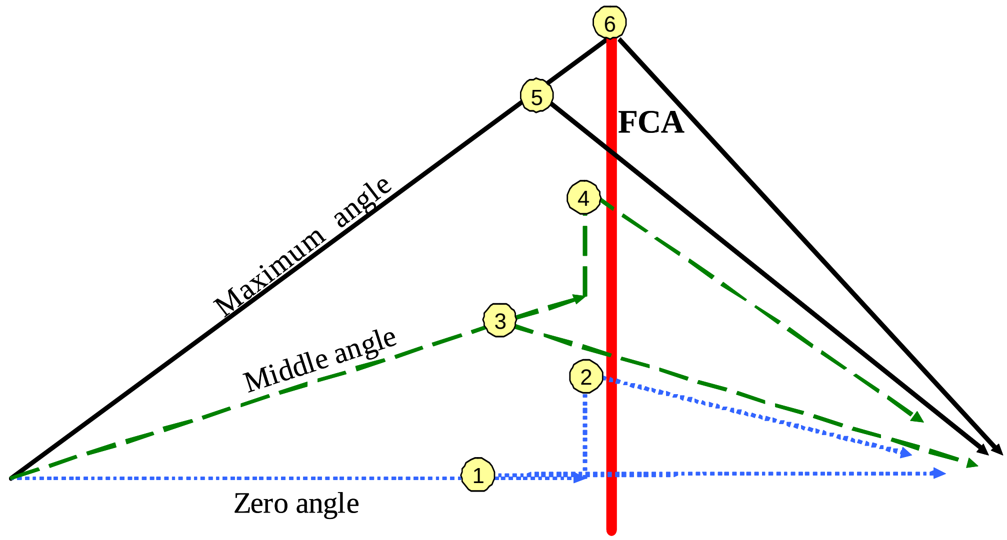


Figure 3-3. Possible conversions of a secondary route into hybrid route.

$t^d_{f,t,r}$ is the time at which flight f must alter its secondary route to become a hybrid route that arrives at the FCA at time t. Figure 3-2 illustrates six different $t^d_{f,t,r}$ values, which depend on the initial secondary route, the clearance time and the associated geometry.

The following constraint prevents a flight from diverting to its hybrid route before the weather is actually cleared.

$$y^h_{f,t,r} \big| u = 0 \quad \forall f, r, u \quad \forall t \big| t^d_{f,t} \leq u$$

In addition, the time slot assignment cannot be later than the latest time for which it would be reasonable to accept an assignment at the FCA considering the geometry of its secondary route:

$$y^h_{f,t} \,|\, u = 0 \quad \forall f, u, \quad \forall t > l_f$$

The final option possible is that a flight carries out its originally planned secondary route:

$$y^s_{f,r} \,|\, u = \begin{cases} \text{1, if flight } f \text{ was originally assigned to its} \\ \text{ secondary route with directional angle } r, \\ \text{and if, under AFP stop time } u\text{, that} \\ \text{decision remains unchanged} \\ \text{0 otherwise} \end{cases}$$

Practically speaking, it would never make sense to assign a flight to its secondary route under the recourse if it had not also been given the same assignment in the initial plan. It might seem, therefore, that the following constraint is necessary:

$$y_f^s \,|\, u \le x_f^s \quad \forall u, f \tag{3.11}$$

However, it can be seen that the objective function enforces this behavior implicitly. If it were cost-effective to assign a flight to its secondary route under the recourse, it would also be cost-effective to do so under the initial plan.

Constraints (3.10) and (3.11) can be combined into a single constraint:

$$\sum_t y_{f,t,r}^h \,|\, u + y_{f,r}^s \,|\, u \le x_{f,r}^s \quad \forall u, f, r$$

It would be possible, given the constraints developed so far, to assign a flight to a hybrid route that essentially reverts to the primary route immediately. In other words, this would be an assignment that is tantamount to taking off on the primary route at the scheduled departure time, which is a more logical way to interpret this outcome. Therefore we introduce the following constraint to enforce this behavior:

$$y_{f,D_f+E_f,r}^h \,|\, u = 0 \quad \forall f, u, r$$

For each time scenario *u*, every flight *f* must be assigned to one of these dispositions. Furthermore, if the disposition involves being scheduled into a slot appointment at the FCA, no more than one slot can be assigned to a given flight. Given that the decision variables are required to be binary, the following constraint addresses both of these concerns

$$\sum_t y_{f,t}^p \,|\, u + \sum_r \sum_t y_{f,t}^h \,|\, u + \sum_r y_f^s \,|\, u = 1 \quad \forall u, f \tag{3.12}$$

For any value $U = u$, there will be a new capacity profile $C^u(t)$ that agrees with $C^0(t)$ up to time $t = u$, but represents an increase in capacity beyond that point. For example, if $C^0(t)$ had been a constant vector, then $C^u(t)$ could be a step function that makes a jump at time $t = u$. On the other hand, if $C^0(t)$ had been a periodic 0-1 function, then $C^u(t)$ might just have an increased duty cycle after time $t = u$. A wide variety of profiles for $C^u(t)$ are possible; the only real requirements are that it agree with $C^0(t)$ prior to time $t = u$, and that after that time, it supports a higher rate of flow than was possible under the initial plan. The capacity constraint under the scenario $U = u$ can now be written as:

$$\sum_f y^p_{f,t} \mid u + \sum_r \sum_f y^h_{f,t,r} \mid u \le C^u_t \quad \forall u,t \tag{3.13}$$

3.5.3 Objective Function

Since our model involves the specification of decisions that are conditioned random events, the objective function will be an expected value. To emphasize the paradigm of creating a plan (our initial plan) together with contingency plans (our recourse actions), we represent the objective function as the sum of the deterministic cost of the initial plan minus the expected savings from recourse actions.

Therefore the objective function can thus be represented as:

$$Min\left[C(X) - \sum_u P_u S(Y_u)\right] \tag{3.14}$$

or more precisely:

$$Min \qquad z^1 + z^2 - \sum_u P_u \left(z^3_u + z^4_u\right) \tag{3.15}$$

where,

$$z^1 = \sum_f \sum_t c^p_{f,t} x^p_{f,t} \tag{3.16}$$

$$z^2 = \sum_r \sum_f c^s_{f,r} x^s_{f,r} \tag{3.17}$$

$$z^3_u = z^1 - \sum_f \sum_t c^p_{f,t} y^p_{f,t} | u + \sum_r \sum_f \sum_t c^s_{f,r} s^p_{f,t,r} | u \tag{3.18}$$

$$z^4_u = \sum_r \sum_f \sum_t sv^h_{f,t,r} y^h_{f,t,r} | u \tag{3.19}$$

where

$c^p_{f,t}$ is the cost of assigning flight *f* to its primary route so that it arrives at the FCA at time *t*.

$c^s_{f,r}$ is the cost of assigning flight *f* to its secondary route with directional angle *r*.

$sv^h_{f,t,r}$ is the savings incurred if flight *f* starts out on its secondary route with directional angle *r* but reverts to a hybrid route that arrives at the FCA at time *t*.

$s^p_{f,r}$ is a dummy binary variable that works as an indicator. It takes value of one when a flight initially assigned to its secondary route is assigned back to its primary route under revised plan.

So;

$$s^{p}_{f,r} = Min\left(x^{s}_{f,r}, \sum_{t} y^{p}_{f,t,r} \right) \tag{3.20}$$

Constraint 20 can be presented with two inequality constraints. However presenting its functionality with the following equality constraint will improve the performance of the model by providing stronger LP relaxation optimal solution.

$$s^{p}_{f,r} = 0.5\left(x^{s}_{f,r} - \sum_{t} x^{p}_{f,t} + \sum_{t} y^{p}_{f,t} \mid u - \sum_{t} y^{h}_{f,t,r} \right) \quad \forall u, f, r \tag{3.21}$$

In addition to the capacity constraints for the first and the second stage we still need to make sure that the number of flights that are rerouted around the FCA will not exceed an acceptable level of throughput, C^{s}_{t} . The following constraint will limit the maximum number of flights passing through the two corridors adjacent to the end points of the FCA for the time window *t*.

$$\sum_{f} \sum_{r} \mu^{s}_{f,t,r} x^{s}_{f,r} \le C^{s}_{t} \quad \forall t \tag{3.22}$$

Parameter $\mu^{s}_{f,t,r}$ is defined as:

$$\mu^{s}_{f,t,r} = \begin{cases} 1, \text{ if flight } f \text{ originally assigned to its} \\ \text{ secondary route with directional angle } r, \\ \text{would be at the FCA at time } t \\ 0 \text{ otherwise} \end{cases}$$

CHAPTER 4: THE PARAMETERS

4.1 Introduction

This chapter is devoted to explain the details of the model parameters. Our model results strongly depend on the geometry of the FCA, which in turn implies the significance of the parameters; consequently that geometry must be carefully studied. The chapter starts with the simplistic flight path geometry and then moves on to more general flight path geometry.

The unit for all distances throughout this dissertation is deliberately chosen to be equivalent to one minute of flying with a constant speed of a typical passenger aircraft (e.g. 500 mile/hour) unless mentioned otherwise. For example $a = 5$ means it will take 5 minutes for an aircraft flying 500 mile/hr to traverse "*a*".

4.2 Simplistic flight path geometry

In this flight path geometry we represent the FCA as a straight line perpendicular to the flight's primary path. The portion of the FCA that the flight has to circumvent, if it is not allowed to fly through the FCA, along with the flight primary path, forms a right triangle that will be used as the framework of our flight path geometry (see Figure 4-1).

4.2.1 Pessimistic Approach Geometry

In this approach, for each flight, the angle α is called the directional angle, and it measures the angular difference between the secondary path trajectory that circumvents the FCA and the primary path trajectory. In other words, if the flight starts its journey with deviational angle α from its primary path and continues in a straight line, and the FCA does not move, the flight will eventually reach the edge of the FCA and at that point can change its direction straight toward its destination.

As shown in Figure 4-1, a is the distance from the origin to the FCA and b is the distance from the FCA to the destination. Both a and b are shown on the horizontal line representing the primary flight path. The total length of the primary path is $a + b$. Now assume the flight was scheduled to depart at time t^0 but instead will depart with some ground delay, which leads to an arrival at the FCA at time t. The cost of ground delay then can be calculated as follows:

$$c_t^p = t - a - t^0 \qquad \forall t \tag{4.1}$$

If the flight is assigned to its secondary route with a directional angle of α, the amount of airborne delay incurred by that longer path is:

$$c^s = \sqrt{a^2+c^2} + \sqrt{b^2+c^2} - a - b \tag{4.2}$$

Or equivalently:

$$c^s = \frac{a}{\cos\alpha} + \sqrt{b^2+\left(a\tan\alpha\right)^2} - a - b \tag{4.3}$$

Next we represent the hybrid routes as alternative routes to the longest secondary route. If the flight reverts from its secondary route after d_1 minutes from its departure then it has to continue flying $x_1 + x_2$ minutes to get to its destination (Figure 4-1). First we calculate $x = x_1 + x_2$ as a function of d_1.

$$x = \sqrt{{d_1}^2 + \left(a+b\right)^2 - 2d_1\left(a+b\right)\cos\alpha} \tag{4.4}$$

Now we need to know when our flight would arrive at the FCA should it revert from its secondary path after d_1 minutes of its departure. To answer this question we need to calculate x_1.

$$x_1 = \frac{a - d_1\cos\alpha}{a+b-d_1\cos\alpha}x \tag{4.5}$$

So if the flight departs at time t_0, provided it will revert to the hybrid path after d_1 minutes, it will arrive at the FCA at time t:

$$t = t_0 + d_1 + x_1 \tag{4.6}$$

The savings incurred by reverting from the secondary path to a hybrid path as a more direct path toward the destination is:

$$sv_t^h = c^s - \left(t + x_2\right) = c^s - t - x + x_1 \tag{4.7}$$

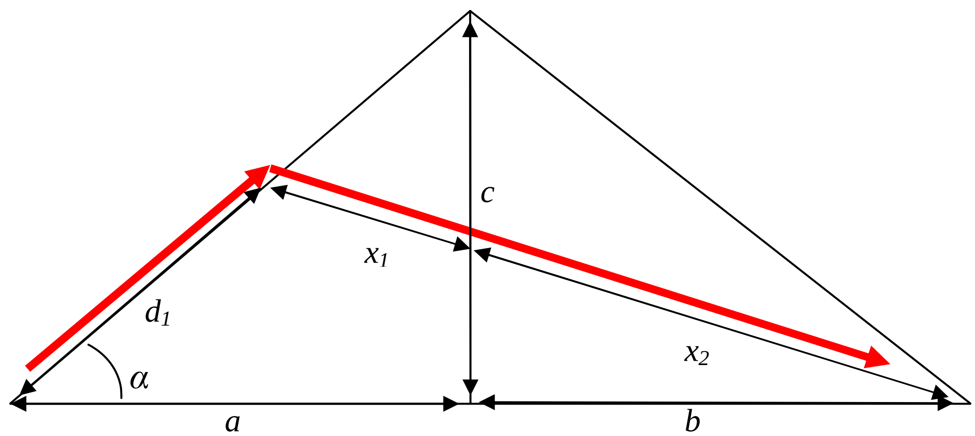


Figure 4-1 Flight path geometry for the pessimistic approach

4.2.2 Optimistic approach geometry

As an extension to the initial model (conservative approach), we considered multiple reroutes option presented in Figure 4-2.

In this approach, we consider a departure angle $\beta \le \alpha$ that attempts some diversion from the primary path, but not so severe as the angle α, which is designed to skirt the FCA entirely, assuming the FCA does not move. The idea is that, given the stochastic nature of the termination time of the capacity disruption, it may make sense to hedge against the two extreme possibilities:

1. The weather clears before the flight reaches the FCA, even by the primary path. In this case, departing immediately on the primary path *would have been* the least expensive decision.
2. The weather will still be in place when the flight reaches the FCA, even if it takes the widest sensible detour route. In this case, the secondary route with a departure angle α would have been the best decision.

Depending on the distribution of possible termination times for the weather disruption, the truth might often fall in between these two extremes. The idea here is to present an "optimistic" trajectory that hedges against the extremes. This plan must, of course, be coupled with recourse actions that describe the best thing to do when the true weather situation manifests itself, for better or worse. If the weather clears before the flight reaches the FCA, it can turn immediately towards its destination. If the weather still exists when the flight reaches the FCA, the flight must travel parallel to the FCA, in the direction of the tip of the storm. It can turn directly towards its destination either when the weather clears or when it reaches the tip of the FCA, whichever comes first.

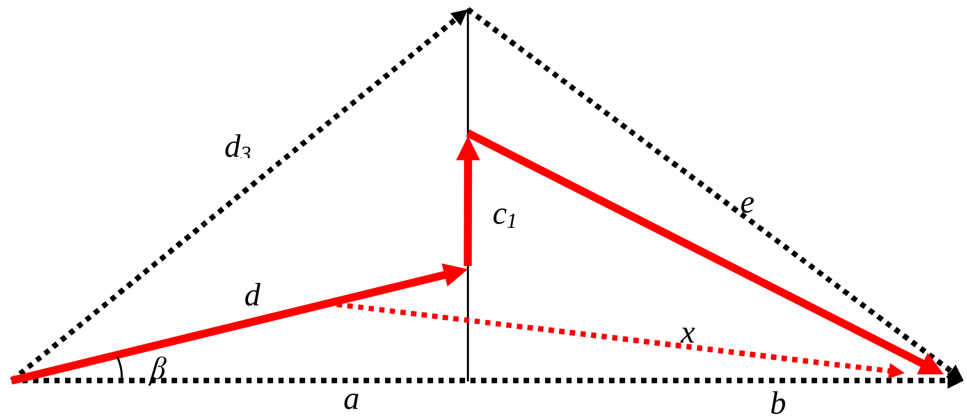


Figure 4-2: Flight path geometry for the optimistic approach

The details of the cost functions calculations for the optimistic approach are presented in section 5.2.3.

4.3 General flight path geometry

In the previous section, we laid out a simple path geometry in which the FCA was represented with a line perpendicular to the flight primary path. Although this simplifies the calculation of the input parameters and also might be a good approximation for many cases, there are other cases that it will not fit well. One such case is shown in Figure 4-3.

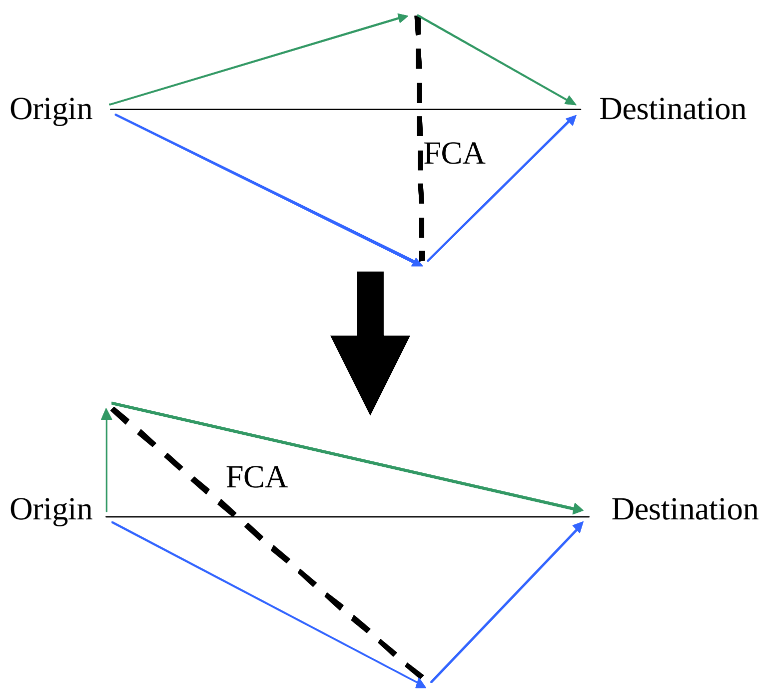


Figure 4-3: Special case of flight path geometry

The problem arises when the line of the FCA is not perpendicular to the flight primary path. In many cases the angle might be quite obtuse. This small generalization of the previously analyzed geometry can be overcome by introducing the FCA angle γ, which is measured relative to the primary flight path. To overcome the problem with such cases, I introduce a more general flight path geometry illustrated in Figure 4-4.

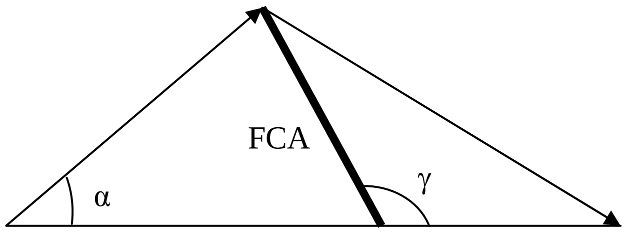
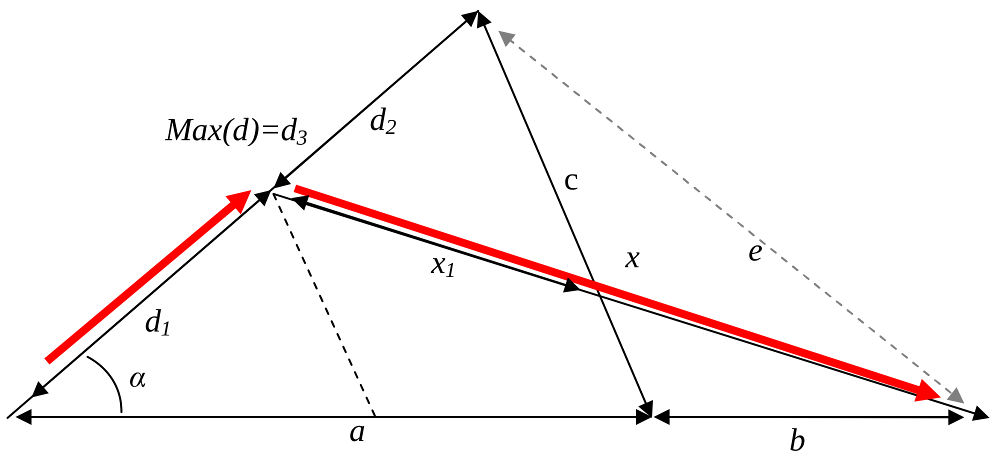


Figure 4-4: General flight path geometry

Based on the different angles chosen for the secondary route and the random time of the weather clearance, we may face three different layouts of the general flight path geometry shown in Figure 4-5(a, b, c):



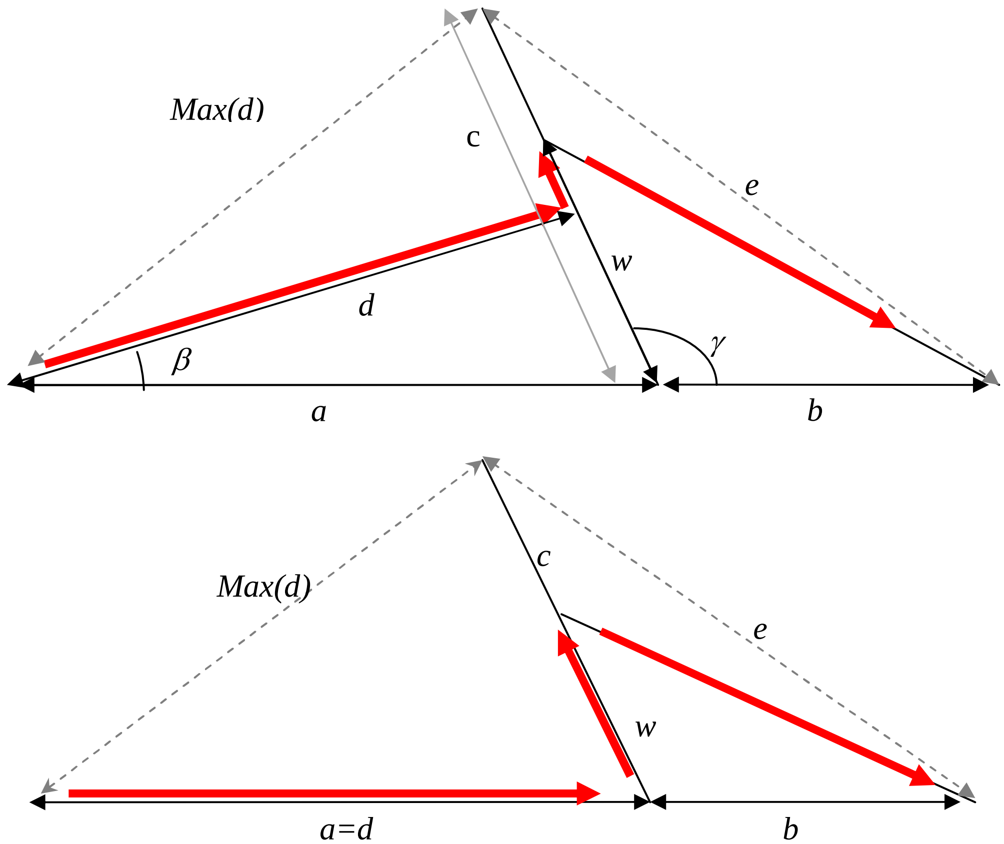


Figure 4-5 (a, b, c): Different layouts of the general flight path geometry.

Here we present the equations used to calculate the model parameters for the general flight path geometry. A MATLAB code was developed that uses the primary data and information of each individual flight such as schedule departure time, the distance from the origin to the FCA, the distance from the FCA to the destination and the part of the FCA that the flight has to circumvent, to generate the necessary input parameters for the model in a readable format for XPRESS (solver). We have a set of flights, that for each flight *f* we need to calculate three main cost functions; the primary path cost function, the secondary cost function and the hybrid route cost saving function. As our model is a time dependant model so are its cost functions. Here we present the calculations for the mentioned main cost functions.

In the following equations:

a: Distance from origin to the FCA on the primary path.

b: Distance from the FCA to the destination on the primary path.

c: Distance from the intersection of the primary path and the FCA to the closest edge of the FCA.

d: Distance from the origin to the FCA on the secondary path.

d_3: Maximum d, associated with the maximum directional angle.

e: Distance from the closest edge of the FCA to the destination.

x: Distance from the turning point (from secondary path to a hybrid path) to the destination.

x_1: Distance from the turning point (from secondary path to a hybrid path) to the FCA.

w: Distance from the intersection of the primary path and the FCA to the turning point.

t: Appointment time at the FCA.

t_0: Scheduled departure time.

β: Directional angle of the secondary path (i.e. the angular difference between the primary and the secondary path trajectories.

γ: Angle between the FCA representative line and the primary path.

n_t: Number of time slots used for the model.

c_β^s : Airborne delay associated with the secondary path with a directional angle β.

c_t^p : Ground delay associated with the primary path with an FCA appointment time t.

$sv_{t,\beta}^h$: Savings incurred if the aircraft starts out on its secondary route with directional angle β but reverts to a hybrid route that arrives at the FCA at time t.

$t_{t,\beta}^d$: Time at which the aircraft must alter its secondary route to become a hybrid route that arrives at the FCA at time t

$$\alpha = \cos^{-1}\left(\frac{a^2 + {d_3}^2 - c^2}{2ad_3}\right) \tag{4.8}$$

$$\gamma = \cos^{-1}\left(\frac{c^2 + b^2 - e^2}{2bc}\right) \tag{4.9}$$

$$c_\beta^s = d + c - \sqrt{a^2 + d^2 - 2ad\cos(\beta)} + e - a - b \tag{4.10}$$

$$c_t^p = t - a - t_0 \tag{4.11}$$

$$x = \sqrt{{d_1}^2 + (a+b)^2 - 2d_1(a+b)\cos(\alpha)} \tag{4.12}$$

$$x_1 = \frac{xad_2}{ad_2 + db} \tag{4.13}$$

$$d = \frac{a\sin\gamma}{\sin(\gamma - \beta)} \tag{4.14}$$

The following pseudo codes represent the process of generating the input parameters for the general flight path geometry. Note that both pessimistic and optimistic approaches are applicable through this process.

$\forall t$

$c_t^p = t - a - t_0$

if $\quad t > d + t_0$

$\quad w = t - t_0 - d + \sqrt{a^2 + d^2 - 2ad\cos(\beta)}$

$\quad$ *if* $\; w <= c \;$ *then*

$\quad\quad sv_{t,\beta}^h = c_\beta^s - \; (\; t - t_0 + \sqrt{w^2 + b^2 - 2wb\cos(\gamma)} \; - a - b)$

$\quad\quad t^d{}_{t,\beta} = t$

$\quad$ *else*

$\quad\quad sv_{t,\beta}^h = -n_t;$

$\quad\quad t^d{}_{t,\beta} = -n_t;$

$\quad$ *end*

else if $\quad t > a + t_0 \;\; \& \;\; t <= d + t_0$

$\quad$ *Find* $\; d_1 \; \left| d_1 + t_0 + x_1 \cong t \right. \qquad ***$

$\quad t^d{}_{t,\beta} = d_1 + t_0;$

$\quad sv_{t,\beta}^h = a + b + c_\beta^s - (d_1 + x)$

else if $\quad t = a + t_0$

$\quad$ *if* $\; \beta = 0$

$\quad\quad t^d{}_{t,\beta} = a + t_0$

$\quad$ *else*

$\quad\quad t^d{}_{t,\beta} = t_0$

$\quad$ *end*

$\quad sv_{t,\beta}^h = c_\beta^s$

else if $\quad t < a + t_0$

$\quad t^d{}_{t,\beta} = 0$

end

*** t as a function of d_1 is given here.

$$t = t_0 + d_1 + \frac{a - d_1\cos\alpha}{a + b - d_1\cos\alpha}\sqrt{d_1{}^2 + (a+b)^2 - 2d_1(a+b)\cos\alpha} \tag{4.15}$$

We used a MATLAB function to find numerically, d_1 as a function of t which is what we need in order to find the best turning point of each flight on its secondary route so that it can be at the FCA at a given time t .

4.4 Real data and the Great Circle effect on the input file

So far, we have assumed that our flight path trajectories are laid out on a flat plane. However, to be able to use the real data involving the actual locations of the airports we need to consider the curvature of the earth's surface by using the great circle calculus. In this section I briefly explain how we can consider the effect of the earth's curvature in our flight path geometry calculations.

We can find the Great Circle distance between two points (origin-destination) on the Earth using the following equation:

$$\begin{aligned} &\varphi = \cos^{-1}\left[\sin(lat_1)\ \sin(lat_2)\ +\ \cos(lat_1)\ \cos(lat_2)\ \cos(lon_2\ -\ lon_1)\right] \\ &\quad if\ \ (\varphi < 0) \quad then \qquad \varphi = \varphi + \pi \\ &dist = 3959\varphi \qquad \text{miles} \end{aligned} \tag{4.16}$$

where lat is the latitude and lon is the Longitude of a point on Earth. 3959 miles is the radius of the best spherical approximation to the shape of the Earth.

Our general flight path geometry introduced in the previous section, enables us to better approximate the real flight path trajectories on the great circle especially interacting with a more realistic FCA configuration in a sense that they may have any intersectional angle other than orthogonal. Four key points will form the framework of the general flight path geometry on the great circle. These points are: 1-Origin, 2-Destination, 3-Intersection of the FCA and the primary path, 4-Closest edge of the FCA to the primary path. Once we have the latitude and the longitude of these key points we can use Equation 4.15 to measure the distances between them. For simplicity we do not use the Great Circle calculus into the full extend. This means once we obtain the critical distances needed, we assume they are the same on a hypothetical flat plane and we continue our geometric calculations using the Euclidian Geometry.

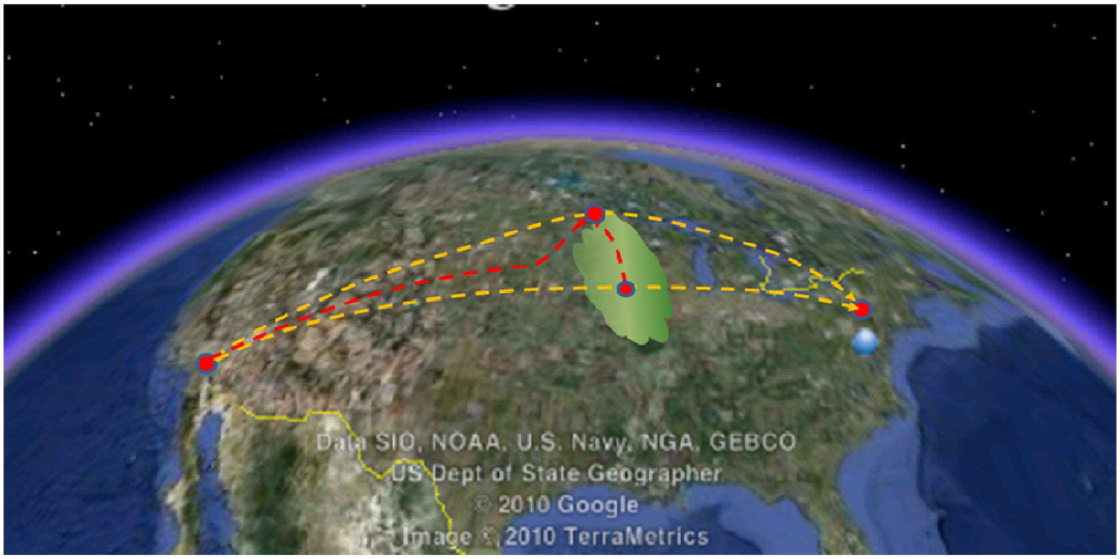


Figure 4-6: Real data and the Great Circle effect

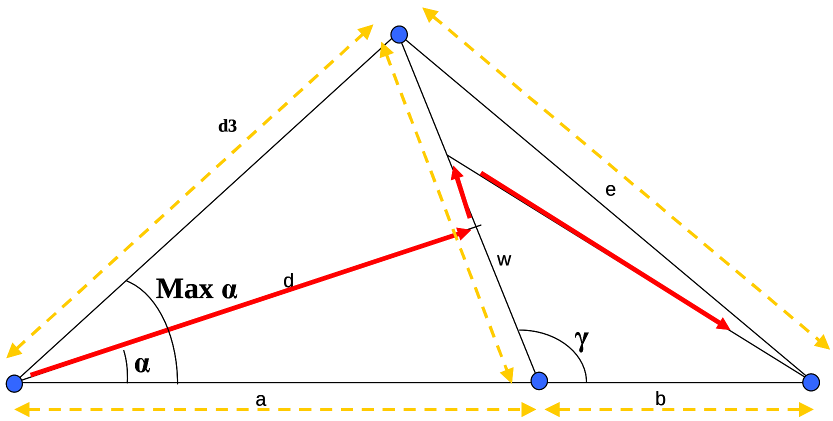


Figure 4-7: General flight path geometry to contain Real data and the Great Circle effect

A MATLAB code was developed to:

1-Read the data file of one full day of the schedule of U.S. domestic flights provided by the FAA and available to the public.

2- Locate all the airports by their latitude and longitude on a U.S. map and project the Great Circle flight paths between their origin and destination airports.

3-Create a virtual FCA on the U.S. map and then find all the flights whose primary paths pass through the FCA.

4-Generate the primary data and information that we need to generate the input file of the model.

The following figures represent a plot of the above process.

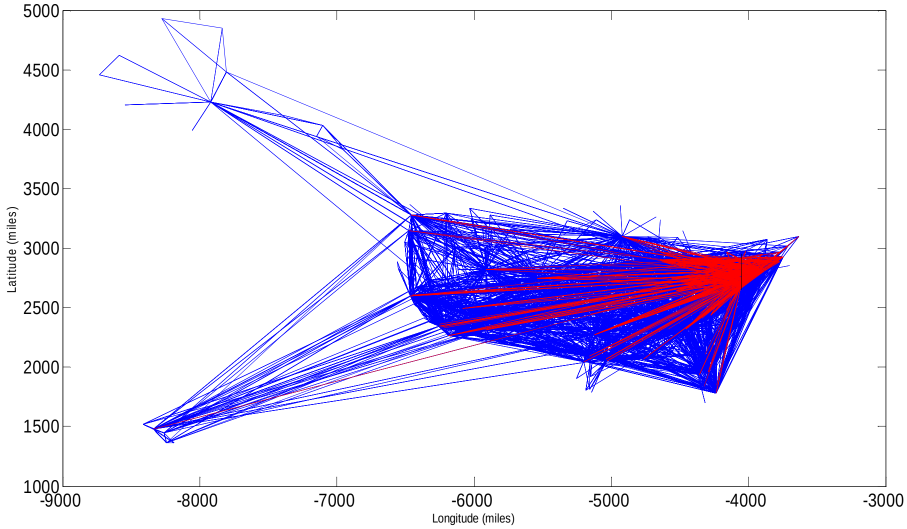


Figure 4-8 All the US domestic flight paths for one day (blue) /
All the US domestic flight paths passing through a virtual FCA on a 6-hour period of the same day (red)

Next we present a case study to evaluate the performance of the model with real data. As illustrated in figure 4-8 a virtual 300 miles long FCA was put in place where it intersects with 678 (out of 17141) flights primary paths that their scheduled departure times were between 12:00 PM and 18:00 PM. We have only considered pessimistic reroutes for affected flights. Table 4-1and Table 4-2 present the results for cases where we have 3 scenarios and 7 scenarios for early weather clearance times respectively. An FCA slot was equivalent of 2 minutes flying time.

Table 4-1: Case Study for Real Data (3 Scenarios)

q	p[q]	c(xp=1)	c(xs=1)	sv(yp=1)	sv(yh=1)	c(q)	n(xp=1)	n(yp=1)	n(xs=1)	n(yh=1)	n(ys=1)
15	0.3	87.2	947.2	0.4	270.7	1439.9	48	129	630	22	527
45	0.3			12.1	147.4	1674.8		91		15	572
75	0.3			32.0	7.8	1934.0		48		13	617

Table 4-2: Case Study for Real Data (7 Scenarios)

q	p[q]	c(xp=1)	c(xs=1)	sv(yp=1)	sv(yh=1)	c(q)	n(xp=1)	n(yp=1)	n(xs=1)	n(yh=1)	n(ys=1)
15	0.13	89.5	946.3	1.1	270.3	1440.4	47	131	631	21	526
25	0.13			1.2	239.9	1501.1		122		14	542
35	0.13			3.7	199.2	1580.2		109		12	557
45	0.13			13.8	146.9	1674.6		91		15	572
55	0.13			21.3	92.8	1775.4		78		13	587
65	0.13			40.0	41.8	1858.5		53		24	601
75	0.13			29.0	9.2	1934.8		47		15	616

The first and the second columns are the clearance time and its probability. The third and the fourth columns are the total costs for ground delays and airborne delays of the first stage. The fifth and the sixth columns are the savings occurred to the first stage costs (recourses) due to increased capacity on the second stage. The seventh column is the total cost of the system for each realization of the random variable (Clearance time). The next columns are the number of flights respectively. The objective function value is 1712.7 units of time for the first case and 1707.8 units of time for the second case which are the minimum expected total costs. The units of all costs are “number of time slots” that can be transferred to minutes or dollars.

CHAPTER 5: MODEL CHARACTRISTICS (DISCUSSION)

5.1 Introduction

In this chapter I will discuss some of the main characteristics of our main model. The idea is to familiarize the reader with principles, intuitions and assumptions that we used along with limitations that we faced in order to develop our main optimization model and to strengthen its performance and practicality afterward. Another advantage of this section of my proposal would be to provide useful insights and motivations to generate more constructive experiments and perhaps guidelines to develop heuristics.

First, I will define some of the deriving principles of our model and will demonstrate them by a simple and tractable numerical example. Then I present two analytical sub models that mainly analyze the problem for the single flight assignment. At the end I present three conceptual experiments to explore the behavior of our model for the special cases.

5.2 Analytical Discussion

5.2.1 The Model Concepts and Principles

In this section we will discuss some of the main ‘principles’ that guide the model in selection of an optimal solution and that demonstrate the power of the model. Conclusions guided me to what type of experiments to construct.

To facilitate discussion, I define these categories of flights that are output by the model:

F = set of all flights

P = set of flights assigned to their primary route

S = set of flights assigned to their secondary route

P can be decomposed into these two sets:

P_k = set of k flights that are assigned to their primary route and assigned to one of the FCA slots in the worst-case scenario. The assignment to those k slots is implicit by the model; it can be determined through post-processing of the solution by adding the ground delay of the flight to its original FCA arrival time.

P_0 = set of flights assigned to their primary route that did not receive one of the above slots, i.e. they are assigned to the last slot (with infinite capacity). Again, assignment to the last slot is implicit and can be determined through post-processing.

Let p be the probability of the FCA clearance at some time prior to the end of the program. Then $1 - p$ is the probability of clearance at the end of the program[1].

Deterministic Case

In the deterministic case, where the weather clearance time is assumed to be known exactly, the model is forced to plan (and execute) the worst-case scenario, namely reduced FCA capacity all the way to the end of the planning horizon. The model will fill all the FCA arrival slots (i.e. under the reduced capacity) with flights on their primary routes. To do this, many of the flights will need to absorb ground delay. The remaining flights will be launched on their secondary routes. That is, the last slot (i.e. with infinite capacity) is empty. The total ground delay is fixed, and deterministic. There is still a cost-related issue of *which* flights to include in P_k versus S, and within P_k , there is an issue of flight ordering.

> *Pessimistic Rerouting Principle: Under the deterministic case,* P_k *will contain exactly* k *flights (assuming adequate demand), and* P_0 *will be empty (all other flights will be rerouted).*

The only decision for the model to make is how to split flights between P_k and S. We expect flights with high secondary routing costs to be placed in P_k. Moreover, it makes no difference to the objective function how flights are ordered within P_k (assuming uniform ground holding costs).

Stochastic Case

[1] By assumption, the probability of clearance *by* end time is 1.0. But the probability of clearance *at* this time could be small.

We expect that the driving principle at work within P_k will be ration-by-distance (RBD):

> *RBD Principle: RBD will prevail in set P_k, when secondary routing costs are extremely high (as compared to primary ground delay), meaning that for any given slot s in the k-slot set, s will be given to the longest-haul flight that can feasibly arrive on time to make that slot.*

(Taking this one step further, it may be that RBD prevails within P_k even when secondary routing costs are not high.) The intuition is that if secondary routes are completely impractical, then every flight will be put on its primary route, and the model will horde short-haul flights in the ground-based inventory. This means whenever there is competition for a slot, it should be awarded to the longest-haul flight that can physically make that slot. In other words, the model tends to give more delay to short-hauls.

> *Primary Optimism Principle: As p increases, the model will tend to increase the total amount of planned ground delay.*

Furthermore, for $p > 0$, the minimum amount of ground delay is fixed, and P_k has a fixed size. This means that we can restate this RBD principle as "as p increases, P_0 will grow in size". Intuitively, this is because for large values of p, the likelihood of having to serve all that planned ground delay is small (expected ground delay is quite low). Flights are kept on their primary route because the model is optimistic that things will clear up.

The exception to this principle is flights with a very "shallow" rerouting angle, meaning that their secondary route is a minor deviation from their primary route. These flights, which otherwise would have been in P_0, might be launched on their secondary route because it follows their primary route so closely that they might as well start making progress toward the FCA (rather than serving time on the ground). We may be able to predict and compute this angle a priori, to force certain decision variables, thereby reducing solver time.

A *recovery route* is a route through the FCA dynamically taken by a flight from its secondary route.

Capacity Recovery Principle: As recovery routes become more viable (meaning they are explicitly allowed by the model and are deemed cost effective), the model will shift flights from the P_0 category to the S category.

The intuition for this is that some flights will be launched on their secondary route in anticipation of being able to capitalize on suddenly available capacity in the FCA from the air (via their recovery routes).

Experiments

We ran experiments to validate and demonstrate these three principles. The number of flights was kept small enough that we could examine the solutions on a flight-by-flight basis (or even solve the problem by hand). 12 flights sufficed. There are long-haul, short-haul, and medium-haul (4 of each). This was also broken into shallow angle and non-shallow angle flights.
The data used for the following experiments are presented in Table 5-1. We have 200 time slots each usable by only one flight except the last slot which is un-capacitated.

Table 5-2: Data for conceptual experiments

Flight	Departure Time	Enroute time: from origin to FCA	Enroute time: from FCA to destination	Angle
1	0	30	30	59
2	0	90	90	29
3	0	150	150	18
4	20	30	30	59
5	20	150	150	18
6	20	90	90	29
7	40	30	30	59
8	40	90	90	29
9	60	150	150	18
10	100	150	150	18
11	100	90	90	29
12	200	30	30	59

k=8 slots are available with reduced capacity

P_8={ 30,60,90,120,150,180,210,240}

Weather definitely clears after 4.5 hours so we have an un-capacitated slot at 270

P_0={270}

After the weather clears the capacity increases from "1 flight every 30 minutes" to "1 flight every 5 minutes"

Experiment 0: Confirms pessimistic rerouting by solving the deterministic case with one (pessimistic) scenario or by using multiple scenarios with (p = 0). We should see 8 slots assigned (randomly) and all other flights placed on their secondary route.

In the following the airborne delay cost is 2 times the ground delay cost per unit of time.**Table 5-3: Demonstrative experiment for pessimistic rerouting principle**

Flight	Departure	Arrive at FCA	xp	t	xs
1	0	30	1	30	0
2	0	90	0	0	1
3	0	150	1	150	0
4	20	50	1	60	0
5	20	170	1	180	0
6	20	110	1	120	0
7	40	70	1	90	0
8	40	130	0	0	1
9	60	210	1	210	0
10	100	250	0	0	1
11	100	190	0	0	1
12	200	230	1	240	0

Table 5-2 shows that exactly 8 flights were assigned to their primary route (i.e. In 4^{th} column xp:Decision variable for primary route=1). The rest of flights are assigned to their secondary routes (i.e. In 6^{th} column xs:Decision variable for secondary route=1).

Experiment 1: Confirms the RBD principle by making secondary route costs very high. The model should assign all flights to their primary route (set *P*), and the earlier slots will go to the longer-haul flights. There is only one slot available every 30 minutes (i.e @ 30,60,90,…,270).

Table 5-4: Demonstrative experiment for RBD principle

Flight	Departure	Arrive at FCA	Initial Assignment	Revised Assignment
1	0	30	30	30
2	0	90	90	90
3	0	150	150	150
4	20	50	60	60
5	20	170	180	180
6	20	110	120	120
7	40	70	270	95
8	40	130	270	155
9	60	210	240	210
10	100	250	270	250
11	100	190	210	190
12	200	230	270	230

a	u	Total Delay	Probability	Expected delay
5000	270	480	0.50	280
5000	60	80	0.50	

They are assigned based on RBD principle

One should note that the difference between the second and the third column of Table 5-3 is the flight's distance from the FCA. Between any two flights that are both candidates for the same earliest slot appointment at the FCA, the one with the longer distance from the FCA is granted the slot. For example if we compare flight 3 and 8, both flights are candidates for the earliest slot available to them which is at 150th minute. That slot is given to flight 3 (150 minutes away from the FCA) as it has a longer distance from the FCA than flight 8 (90 minutes away from the FCA). The mathematical proof of RBD principle is provided by Hoffman et al. (2007)[12]

Experiment2: Confirms the primary optimism principle. For a reasonable data set, we resolve the problem by gradually decreasing the value of *p*, with all other parameters being fixed, to show that total <u>planned</u> ground delay goes down but expected ground delay (Objective value) goes up.

Table 5-5: Demonstrative experiment for primary optimism principle.

Objective: 157 p0= 0.05 p= 0.95

	Dep	@FCA	xp	t	yp	t	xs	yh	t	td	ys
1	0	30	1	30	1	30	0	0	0	1000	0
2	0	90	1	90	1	90	0	0	0	1000	0
3	0	150	1	150	1	150	0	0	0	1000	0
4	20	50	1	60	1	60	0	0	0	1000	0
5	20	170	1	180	1	180	0	0	0	1000	0
6	20	110	1	120	1	120	0	0	0	1000	0
7	40	70	1	270	1	125	0	0	0	1000	0
8	40	130	1	270	1	185	0	0	0	1000	0
9	60	210	1	210	1	210	0	0	0	1000	0
10	100	250	1	270	1	250	0	0	0	1000	0
11	100	190	1	240	1	190	0	0	0	1000	0
12	200	230	1	270	1	230	0	0	0	1000	0
a	q	c(xp)	c(xs)	sv(yp)	sv(yh)	c(q)	nxp	nyp	nxs	nyh	nys
5	90	480	0	340	0	140	12	12	0	0	0

Objective: 308.575 p0= 0.5 p= 0.5

	Dep	@FCA	xp	t	yp	t	xs	yh	t	td	ys
1	0	30	1	30	1	30	0	0	0	1000	0
2	0	90	1	90	1	90	0	0	0	1000	0
3	0	150	1	180	1	180	0	0	0	1000	0
4	20	50	1	60	1	60	0	0	0	1000	0
5	20	170	0	0	0	0	1	1	175	107	0
6	20	110	1	120	1	120	0	0	0	1000	0
7	40	70	1	270	1	125	0	0	0	1000	0
8	40	130	1	150	1	150	0	0	0	1000	0
9	60	210	1	210	1	210	0	0	0	1000	0
10	100	250	1	270	1	250	0	0	0	1000	0
11	100	190	1	270	1	190	0	0	0	1000	0
12	200	230	1	240	1	230	0	0	0	1000	0
a	q	c(xp)	c(xs)	sv(yp)	sv(yh)	c(q)	nxp	nyp	nxs	nyh	nys
5	90	380	16.23	255	10.03	156	11	11	1	1	0

Objective: 385.2 p0= 0.95 p= 0.05

	Dep	@FCA	xp	t	yp	t	xs	yh	t	td	ys
1	0	30	1	30	1	30	0	0	0	1000	0
2	0	90	0	0	0	0	1	0	0	1000	1
3	0	150	0	0	0	0	1	0	0	1000	1
4	20	50	1	60	1	60	0	0	0	1000	0
5	20	170	1	180	1	180	0	0	0	1000	0
6	20	110	1	120	1	120	0	0	0	1000	0
7	40	70	1	90	1	90	0	0	0	1000	0
8	40	130	1	150	1	150	0	0	0	1000	0
9	60	210	1	210	1	210	0	0	0	1000	0
10	100	250	1	270	1	250	0	0	0	1000	0
11	100	190	1	240	1	190	0	0	0	1000	0
12	200	230	1	270	1	230	0	0	0	1000	0
a	q	c(xp)	c(xs)	sv(yp)	sv(yh)	c(q)	nxp	nyp	nxs	nyh	nys
5	90	180	42.14	110	0	280.7	10	10	2	0	2

The numbers in the red squares represent the expected total delay and those in blue squares represent the total planned ground delay cost.

Experiment 3: Confirms capacity recovery principles. We fix the parameters and data set so that primary optimism dominates the solution, i.e., the set *S* is empty. We do this by initially disallowing recovery routes or by making them prohibitively expensive (Table 5-5-a). We then introduce recovery routes (or decrease their cost to something reasonable) (Table 5-5-b). We should see at least one flight leave category P_0 and join category *S*.

As it is shown in Table 5-5-a, all 12 flights are assigned to their primary route (i.e. in fourth column; all x_p=1 and set S is empty) since the secondary routes deliberately were set to be expensive *enough* and furthermore no recovery route was allowed. In the second set of results (Table 5-5-b) however the recovery routes were introduced and as a result flight number 8 got assigned to its secondary routes (i.e. for f=8; x_p=0, x_s=1 and y_h=1). The objective function value is reduced from 280 to 272.225.

Table 5-6: Demonstrative experiment for capacity recovery principles

Objective: 280 P0=50% P=50% No recourse (Yh=0 for all)

	Dep	@FCA	xp	t	yp	t	xs	yh	t	td	ys
1	0	30	1	30	1	30	0	0	0	1000	0
2	0	90	1	90	1	90	0	0	0	1000	0
3	0	150	1	150	1	150	0	0	0	1000	0
4	20	50	1	60	1	60	0	0	0	1000	0
5	20	170	1	180	1	180	0	0	0	1000	0
6	20	110	1	120	1	120	0	0	0	1000	0
7	40	70	1	270	1	95	0	0	0	1000	0
8	40	130	1	270	1	155	0	0	0	1000	0
9	60	210	1	240	1	210	0	0	0	1000	0
10	100	250	1	270	1	250	0	0	0	1000	0
11	100	190	1	210	1	190	0	0	0	1000	0
12	200	230	1	270	1	230	0	0	0	1000	0
a	q	c(xp)	c(xs)	sv(yp)	sv(yh)	c(q)	nxp	nyp	nxs	nyh	nys
5	60	480	0	400	0	80	12	12	0	0	0

Objective: 272.225 P0=50% P=50%

	Dep	@FCA	xp	t	yp	t	xs	yh	t	td	ys
1	0	30	1	30	1	30	0	0	0	1000	0
2	0	90	1	90	1	90	0	0	0	1000	0
3	0	150	0	0	0	0	1	1	155	87	0
4	20	50	1	60	1	60	0	0	0	1000	0
5	20	170	1	180	1	180	0	0	0	1000	0
6	20	110	1	120	1	120	0	0	0	1000	0
7	40	70	1	150	1	95	0	0	0	1000	0
8	40	130	0	0	0	0	1	1	135	76	0
9	60	210	1	270	1	210	0	0	0	1000	0
10	100	250	1	270	1	250	0	0	0	1000	0
11	100	190	1	210	1	190	0	0	0	1000	0
12	200	230	1	240	1	230	0	0	0	1000	0
a	q	c(xp)	c(xs)	sv(yp)	sv(yh)	c(q)	nxp	nyp	nxs	nyh	nys
5	60	220	42.14	165	30.39	113.75	10	10	2	2	0

5.2.2 Analytic Model for Comparing Reroute and Ground Holding Strategies

In this section, we consider the case of a single flight and compare the cost of a reroute strategy (RR) and a ground holding strategy (GD). We develop a stochastic model that considers recourse options in both cases. Specifically, for the reroute case, we consider the possibility of returning to the most direct route if the weather clears and for the ground delay case, we consider the possibility of canceling any remaining ground delay and immediately departing if the weather clears.

For the reroute option, we use a relatively simple "stylized" model as depicted in Figure 5.1. Here the horizontal line represents the direct line of travel that is the preferred path for the flight. The vertical line represents an impediment (weather/FCA) that must be circumvented by the reroute.

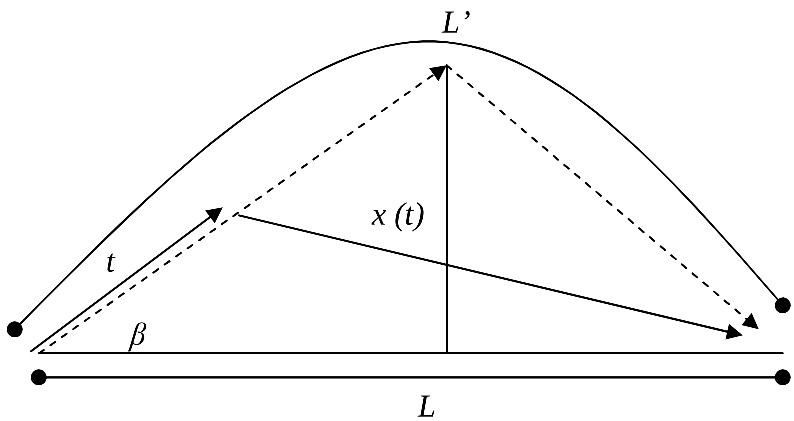


Figure 5-1: Simple "stylized" model of the flight path

The relevant parameters are:

L = length of preferred route (in time)

L' = length of alternate route around FCA

β = angle of reroute

t = time index { measures time elapsed since scheduled departure time

$x(t)$ = length of 2nd portion of route if weather clears at time t.

We will now define generic cost models and then later show how the model given above can be used to estimate these.

$G(t)$ = excess ground delay under GD if weather clears at time t
$A(t)$ = excess air delay under RR if weather clears at time t
$f(t)$ = probability density function for weather clearance time
T = length of problem time horizon
c_g = cost per unit time of ground delay
c_a = cost per unit time of air delay
c_a^0 = fixed cost for planning a reroute
C_{GD} = expected cost of GD strategy
C_{RR} = expected cost of RR strategy

The goal is to compute/estimate C_{GD} and C_{RR} and then to draw some conclusions regarding when each strategy should be used. Based on the definitions given so far, we can write down the basic definitions:

$$\begin{aligned} C_{GD} &= \int_0^T c_g G(t) f(t)\, dt \\ C_{RR} &= c_a^0 + \int_0^T c_a A(t) f(t)\, dt \end{aligned} \qquad (5.1)$$

We will now apply the reroute model given earlier and illustrated in Figure 5.1 to define the functions $G(t)$ and $A(t)$. We assume that time starts at the scheduled departure time of the flight. Under our model, if the weather clears before the scheduled departure time, then the best strategy would be to allow the flight to depart at its scheduled time and use its preferred route (independent of whether RR or GD has originally been planned). Thus, we need not perform any analysis related to weather clearance before the scheduled departure time.

We associate with the GD strategy a parameter, g, which is the amount of assigned ground delay. This is the amount of delay the flight incurs if the weather does not clear. Thus, we have:

$$\begin{aligned} G(t) &= t \quad \text{for } 0 \le t \le g \\ G(t) &= g \quad \text{for } t > g \end{aligned} \qquad (5.2)$$

Now referring to the Figure 5.1, under RR, we note that if the weather clears any time after the flight reaches the top of the vertical line, then the flight time is L' and the maximum airborne penalty is incurred. On the other hand if the weather clears before the flight reaches this point, then the flight time is reduced to $t + r(t)$. Note that the time required to reach the top of the vertical line is 0.5L'. Thus, we have:

$$\begin{aligned} A(t) &= t + x(t) - L \quad \text{for } 0 \le t \le 0.5L' \\ A(t) &= L' - L \quad \text{for } t > 0.5L' \end{aligned} \qquad (5.3)$$

Further, we can apply trigonometry to relate L' and $r(t)$ to basic problem parameters:

$$L' = \frac{L}{\cos\beta}$$
$$x(t) = \sqrt{t^2 + L^2 - 2Lt\cos\beta} \qquad (5.4)$$

The final undefined element is $f(t)$. Of course, many distribution functions could be used, but probably the easiest to start with would be a uniform distribution, i.e. $f(t) = 1/T$.

At this point we now have the capability to compute and compare C_{GD} and C_{RR}. These can be computed as a function of the input parameters: c_g , c_a , c^0_a , β , T , L , g. As is normal we probably would express costs in time so that c^0_a would be a time penalty, e.g. 15 minutes. Also, c_a would be expressed as a multiplier over c_g and c_g would be set to one. The key parameter we would vary to see when one option becomes better than the other is θ. Also, the relationship among g, T and 0.5L' would be very important.

For the first experiment we set the input parameters as follow:

T=120, g=40, L=180, C_a*=2,* C^0_a*=15.*

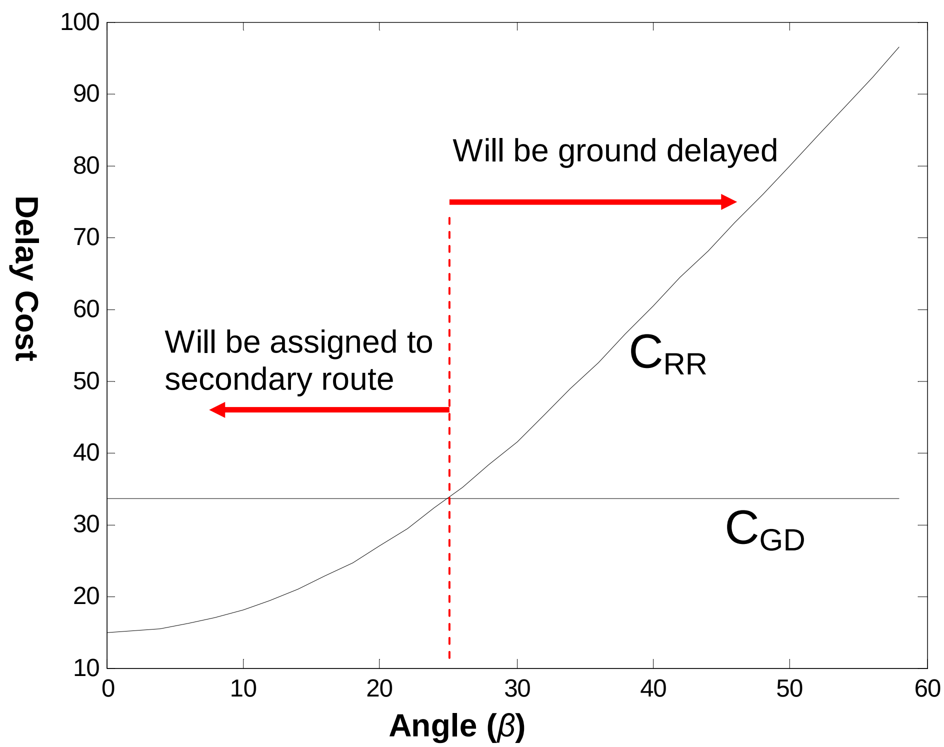


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Figure 5-2: Comparing Reroute and Ground Holding Strategies

In figure 5.2 we platted the results of the above analytical model. We varied the angle β from 0 to 60 degrees and calculated the associated delay cost for both GD and RR plan. First thing to notice is that the C_{GD} is independent of the reroute angle and is equal to 100/3 for this example.

However C_{RR} non-linearly increases as β increases. The 25° is the break-even point between the GD and RR plan for these set of input parameters. This means if the deviation of the reroute from the primary path is more than 25°, the GD plan should be chosen.

To further explore the behavior of our model and get better insight we also varied other important input parameters. The following four experiments represent the effect of different T, L, g and c_a on C_{GD} and C_{RR} values respectively.
The input parameters for each experiment are set to the following:
Exp1: T={50,100,150,200}, g=50, L=180, Ca=2, C0a=15
Exp2: T=200, g=50, L={50,100,300,600}, Ca=2, C0a=15
Exp3: T=200, g={10,20,50,100), L=200, Ca=2, C0a=15
Exp4: T=200, g=50, L=200, C0a=15, Ca={2,4}

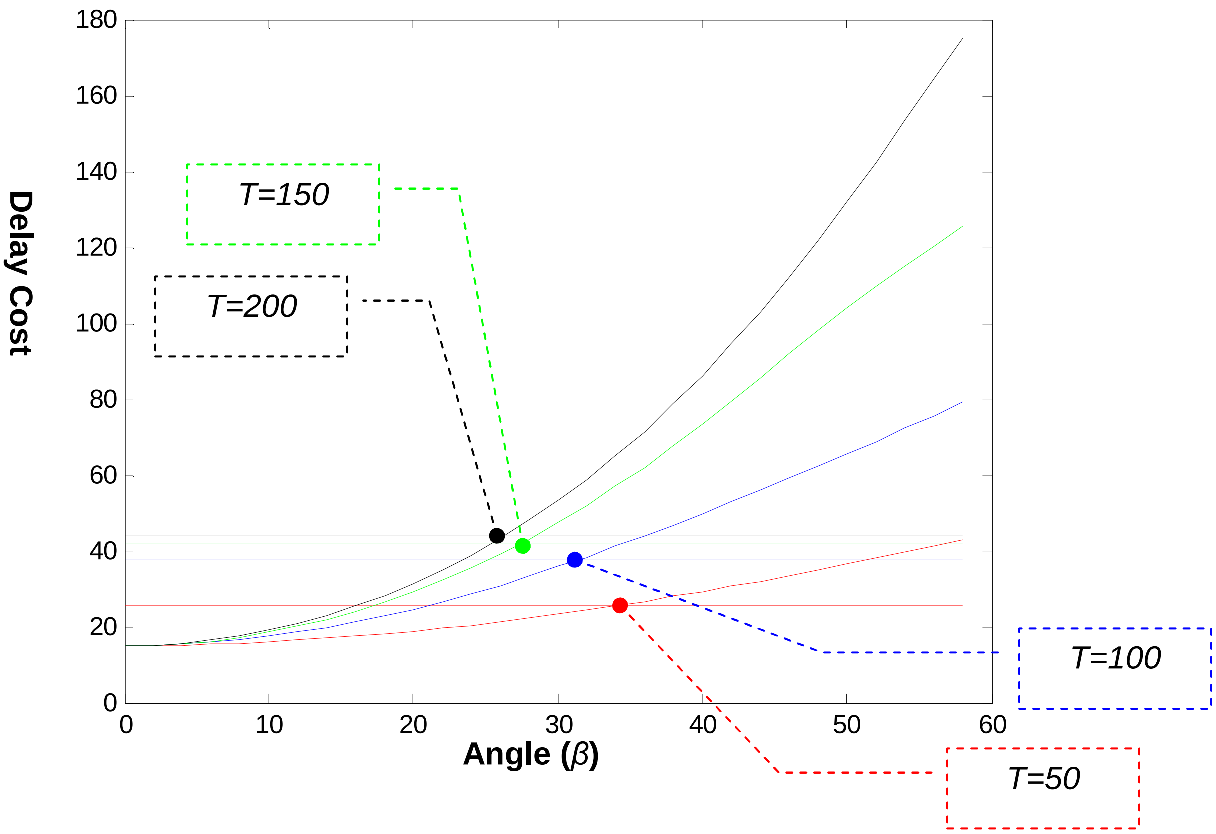

Figure 5-3: the results of Exp1

The result of the first graph suggests that the as the duration of the FCA (i.e. *T*) decreases, even longer reroutes now would be preferred over the ground delay option. With this we may conclude, for example, that if we were dealing with only brief degraded weather conditions, then we expect that our model would assign more flights to their secondary routes and fewer flights would be held on the ground. The intuition for this is the fact that flights assigned to their secondary routes will start progressing toward their destinations immediately, and probably soon after their departure, they get to recover from that path as the FCA capacity increases. Their ultimate trajectories, therefore, do not differ significantly from their primary paths.

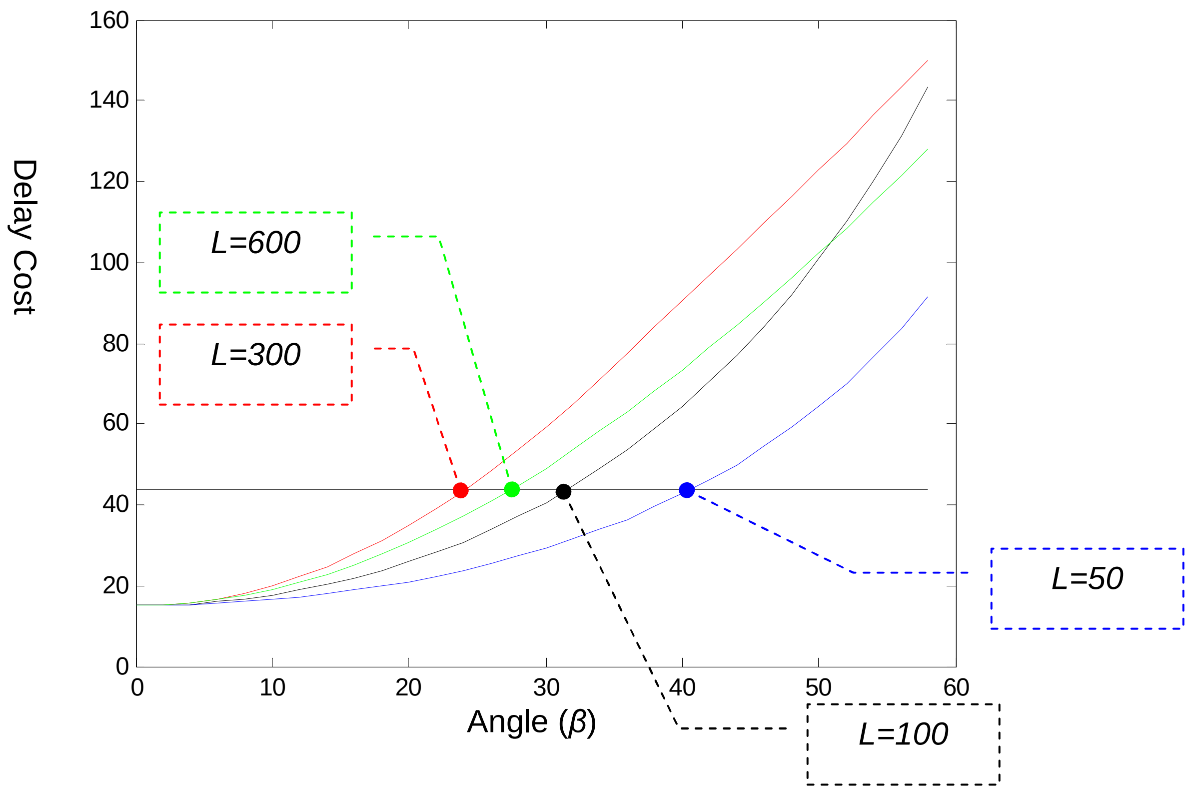

Figure 5-4: Results of Exp2

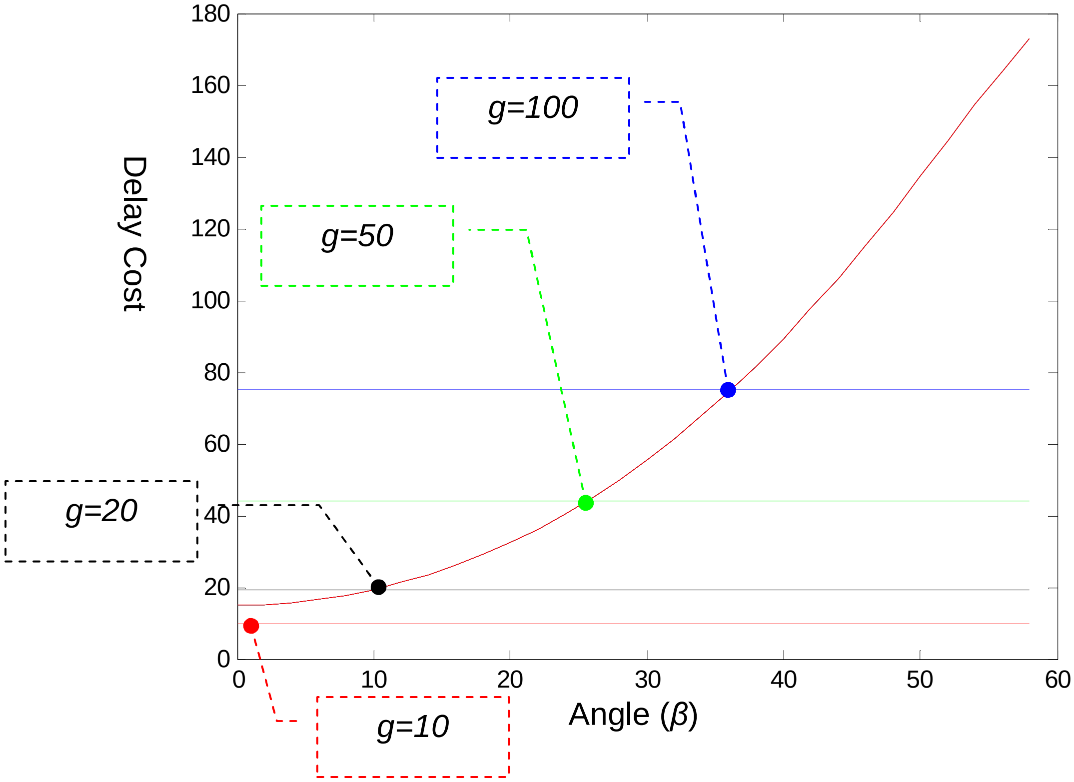

Figure 5-5: Results of Exp3

The results shown in Figure 5-5 indicate that when the maximum penalty for the ground delay is reduced, the reroute options become less attractive.

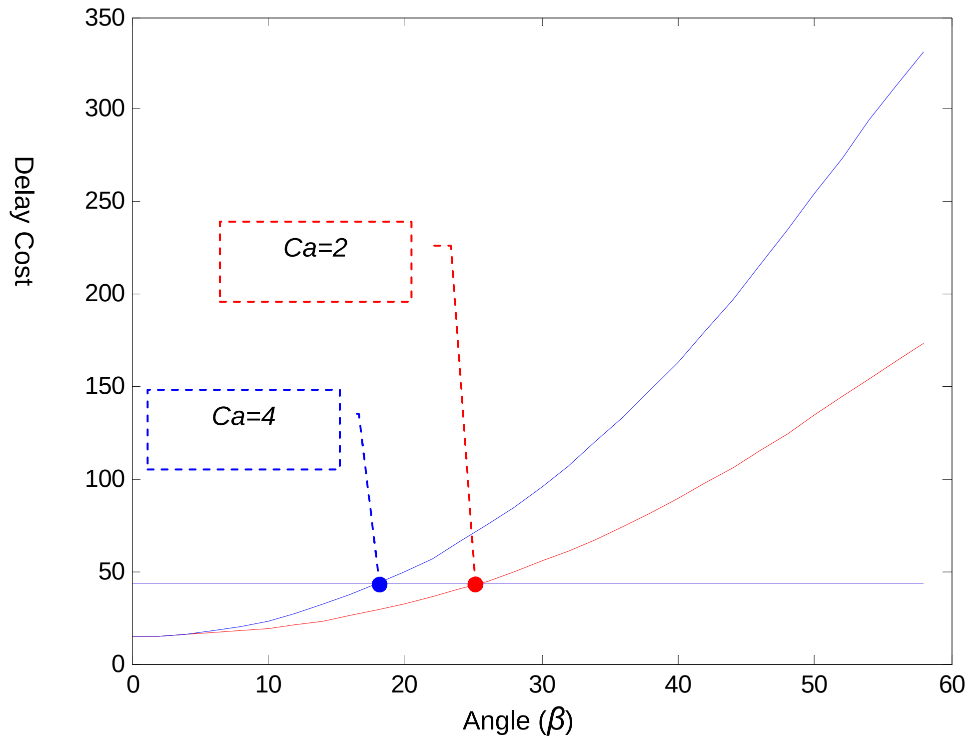

Figure 5-6: Results of Exp4

The results in Figure 5-6 also confirm that more expensive reroutes will have lower possibility of being the preferred option.

Finally in the last experiment (Exp 5) we will explore the effect of different weather clearance times on the expected delays of GD and RR plans. We have conducted four different reroute functions versus a single ground delay function. The ground delay function is shown by the black line. The maximum ground delay is set to be 90 minutes. As is shown, the GD function increases linearly from 0 to 90 minutes delay and then remains at its maximum, as expected. On the other hand four different reroute functions follow a similar pattern of non-linear increase followed by a maximum plateau once the aircraft reaches the edge of the FCA.

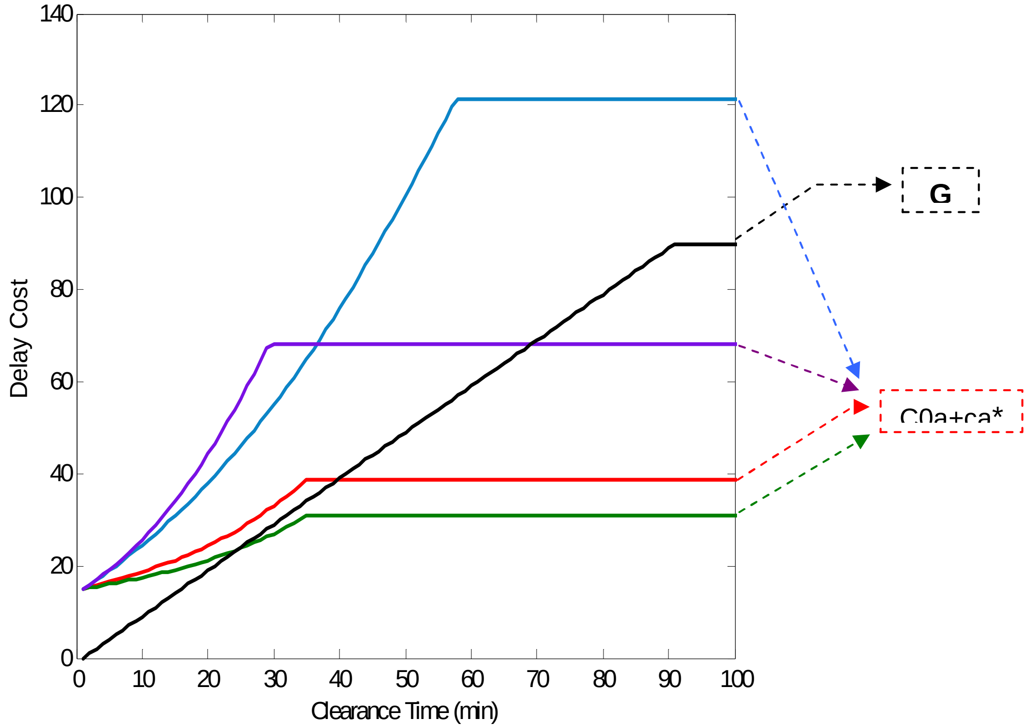

Figure 5-7: Total Delay incurred for a specific weather clearance time

The input parameters used for each reroute functions are given in the following table.

Table 5-6: Exp 5 parameters

	Red	Green	Blue	Purple
C0a	15	15	15	15
g	90	90	90	90
L	60	60	60	40
T	120	120	120	120
ca	3	2	2	2
Angle	30	30	60	60

5.2.3 Analytical Model on Optimistic Reroute

This section extends our earlier work on the problem by allowing the initial reroutes to vary from pessimistic (initial trajectory avoids weather entirely) to optimistic (initial trajectory assumes weather not present). Yoon et al. (2010) proposed a geometric model to generate optimal route choice to hedge against weather risk. While they proceeded with an analytical approach here we used a numerical approach just to explore the properties of different reroutes and their effect on our model decision making. We conduct experiments allowing a range of such trajectories and draw conclusions regarding appropriate strategies.

Once again we consider the case of a single flight and compare the cost of different reroutes strategy (RR) and a ground holding strategy (GD). We develop a stochastic model that considers recourse options in both cases. Specifically, for the reroute case, we consider the possibility of having a set of different directional angles (i.e. as an initiative trajectory of a reroute) with their corresponding recovery plan for returning to the most direct route as soon as the weather clears. As before, for the ground delay case, we consider the possibility of canceling any remaining ground delay and departing immediately if the weather clears. The FCA has limited capacity. Thus, a fight whose preferred flight plan goes through the FCA has the option of being ground delayed or to leave on time via one of its optimistic reroutes. Underlying these options is a stochastic model. Under this model, it is possible that, at any (or a discrete set of) time(s), the weather will clear. We assume that if the weather clears there is unlimited capacity. Thus, in the ground delay case, the flight may immediately depart and in the air delay case, it may immediately alter its route to a route that goes directly to the destination airport.

For the reroute option, we use a relatively simple "stylized" model as depicted in the Figure 5-8. Here the horizontal line represents the direct line of travel that is the preferred path for the flight. The vertical line represents an impediment (weather/FCA) that must be circumvented by the reroute. The red line represents an optimistic reroute that goes toward the FCA until it actually reaches it and then continues along the FCA until the very end of it.

Here are the parameters

a = Distance from origin to the FCA

b = Distance from FCA to the destination

c = Portion of FCA that the flight need to circumvent to not to go through

U = weather clearance time since actual departure

AT = Airborne Time

Now we can calculate the AT for different possible case that a flight might face considering the different weather clearance time. If the weather clears before the flight reaches the FCA boundary;

$$AT = U + \sqrt{(U \sin \beta)^2 + (a + b - U \cos \beta)^2} \qquad \text{for} \qquad U < \frac{a}{\cos \beta} \qquad (5.5)$$

On the other hand, if the flight reaches the FCA before the weather clears then it has to fly along the FCA until either the FCA opens up or ends. For this case, we introduce the x variable to measure how far from the intersection of the FCA and the primary path, the flight has to fly along the FCA until it gets to pass through the FCA.

$$x = T - \frac{a}{\cos \beta} + a \tan \beta \qquad \text{for} \qquad U \geq \frac{a}{\cos \beta} \qquad (5.6)$$

In this case if the value of x is less than c, the airborne delay will be:

$$AT = U + \sqrt{x^2 + b^2} \qquad \text{for} \qquad x \leq c \qquad (5.7)$$

But if the x is greater than c, this means the flight will reach the end of the FCA before it opens up, therefore:

$$AT = \frac{a}{\cos \beta} + c - a.\tan \beta + \sqrt{c^2 + b^2} \qquad \text{for} \qquad x > c \qquad (5.8)$$

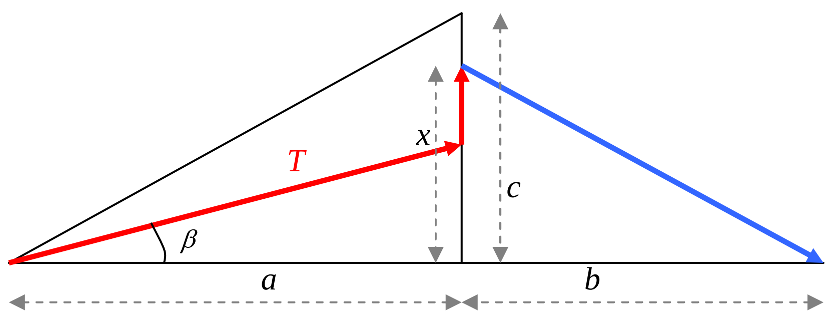

Figure 5-8: Optimistic flight path trajectory

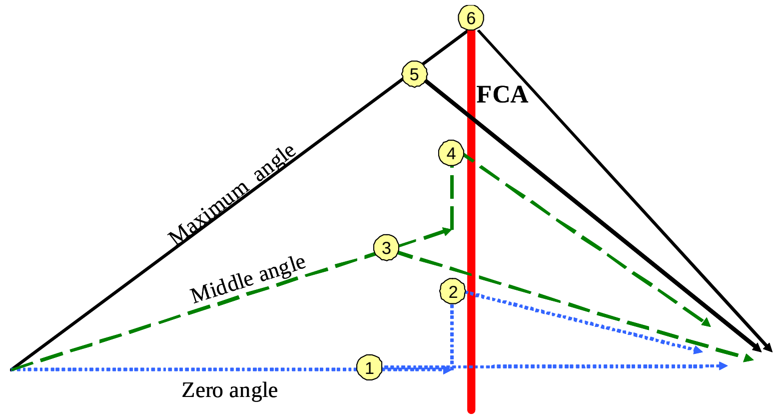

Figure 5-9: Different turning points for a flight on its reroute

With the following numerical example we will explore the results of the explained analytical model to get an insight about the possible outcomes of the different optimistic reroute that will be used by the model for the assignment of the dispositions.

For this example let:

a = 60, b = 45, c = 30, Max β = 26.56^{o} β= [0, 10, 20, 26]

As is shown in Figure 5-10, should the weather clear within 74 minutes of the departure time, the zero-angle path would be the best path. But the result for this specific example suggest that if the weather is not going to clear within 74 minutes then the max-angle path would be the best one to choose. One should notice that any point on this graph is the absolute value of the airborne time under exactly one realization of the random parameter of the weather clearance time. As we will not know the exact time of the weather clearance time we will present the expected value of the airborne delay in Figure 5-11.

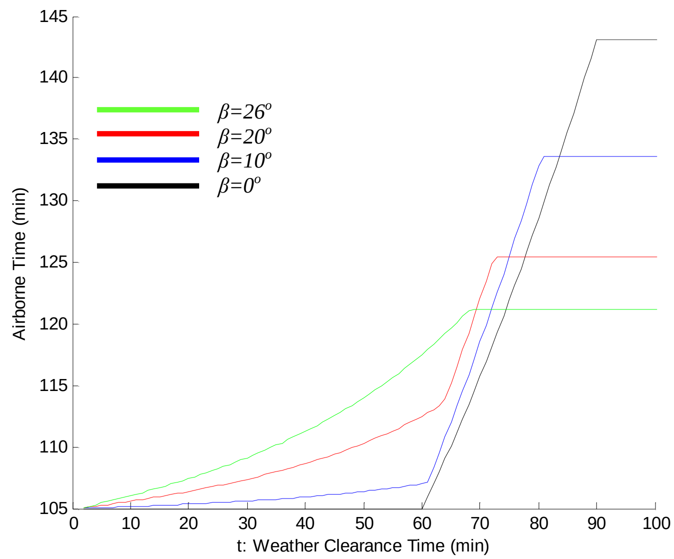

Figure 5-10: Airborne delay as a function of weather clearance time.

Assuming a uniform distribution for the weather clearance time, ranging from 0 to T, the probability density function would be p(t)=1/T.

$$E[AD] = \int_0^T AT(t).p(t)dt - a - b \tag{5.9}$$

Figure 5-11 shows the expected airborne delay values for different termination times.

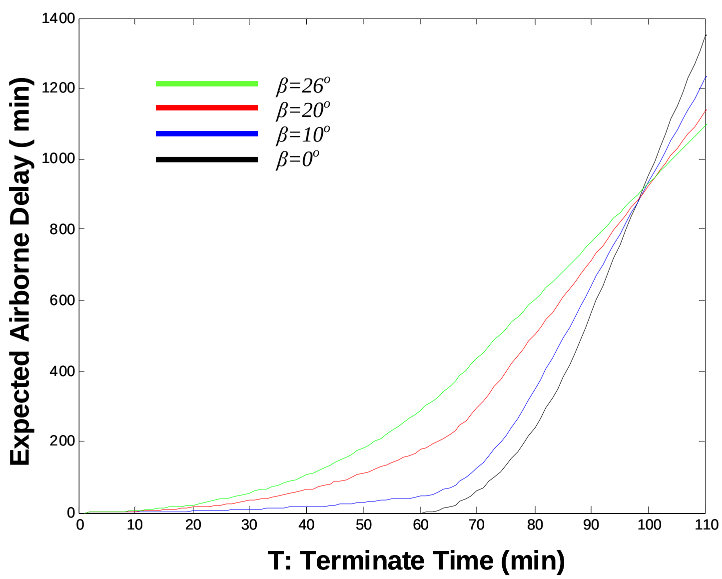

Figure 5-11: Expected Airborne delay as a function of weather termination time

We also extended our analytical model of optimistic reroute to achieve a higher level of precision and efficiency by adding one more degree of freedom on designing an optimistic reroute. We developed and algorithm that numerically search for optimal turning point (e.g. point

X in Figure 5-12. An optimal turning point is the outcome of a joint optimization of a directional angle (*β*) and a traversed portion of the primary path (*x*). At the turning point a flight will change its directional angle from the initial one (*β*) to what will aim the end point of the FCA line (e.g. point C) if the weather has not cleared by that time.

With this new approach we provide a new parameter that defines, for a flight currently heading toward the FCA without an appointment, where/when is the most beneficial location/time to divert. Because of one degree of freedom that we added to our reroute trajectory design, we can guarantee that the new trajectory always results the minimum expected airborne delay for a given PDF of weather clearance time. The proof lies in the fact the former forms of optimistic reroutes are all special cases of this new trajectory design. In the other word the new trajectory simply was set free to be at least as good as any other trajectory without an optimal turning point.

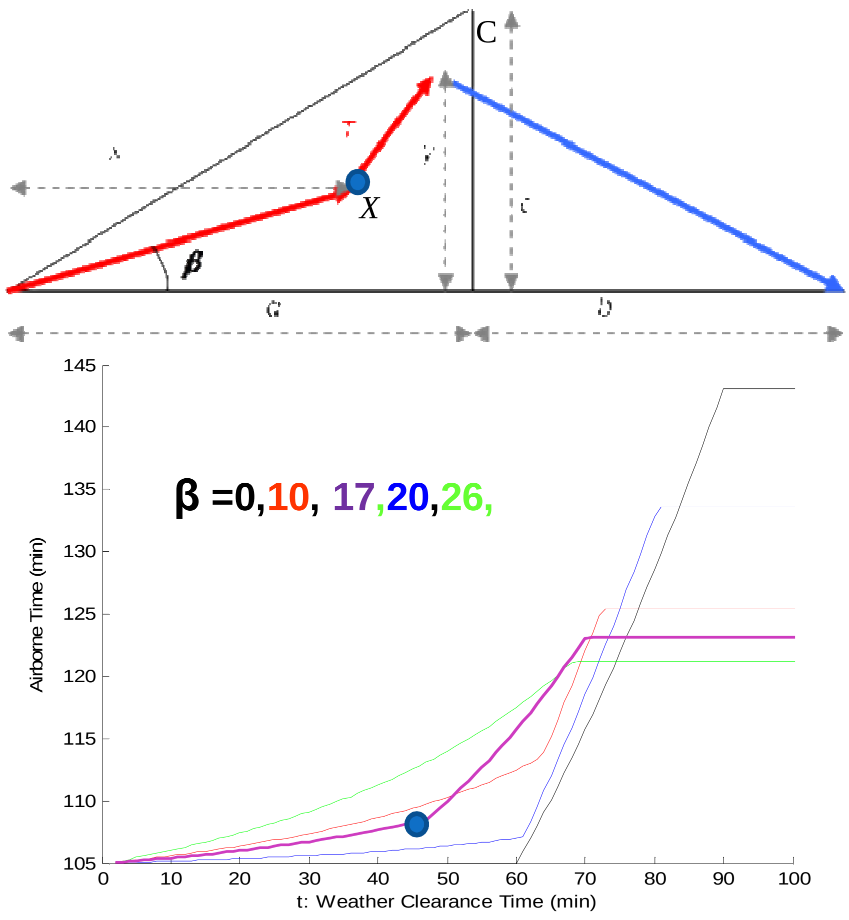

Figure 5-12: Optimistic flight path trajectory with individual optimal turning point

Figure 5-13 shows the expected airborne delay incurred to a flight for different FCA duration (i.e termination time). This graph compares the behavior of the expected airborne delay function for a flight following its most optimistic reroute (i.e. primary path-black line), the most conservative reroute (green line) or the path that optimally provides a turning point (purple line).

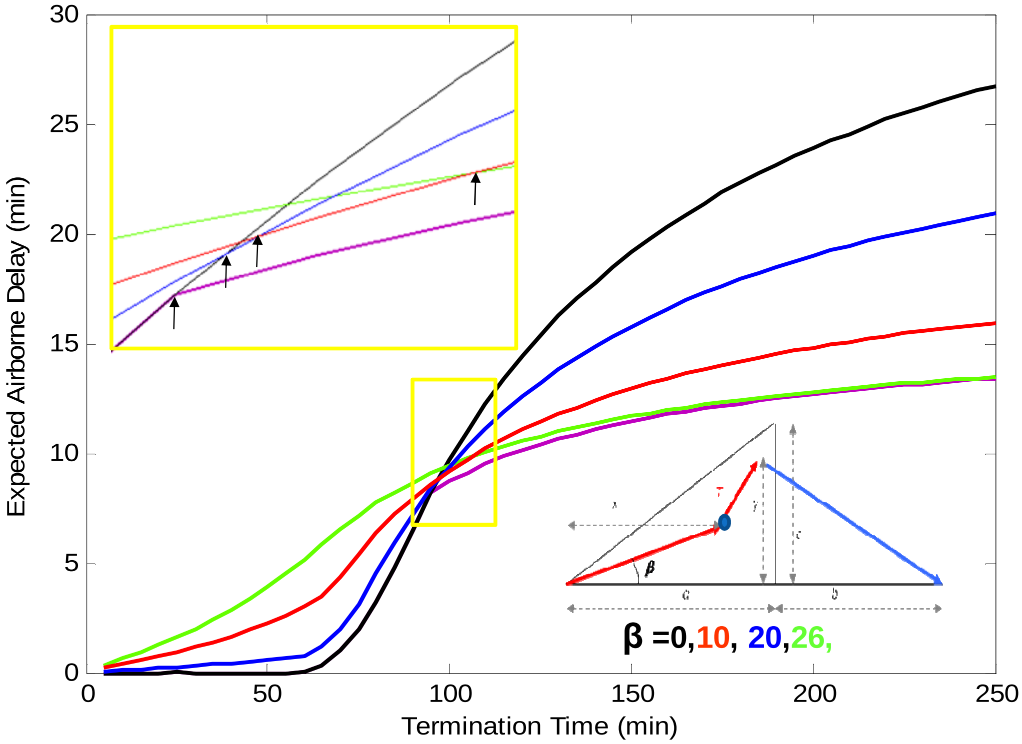

Figure 5-13: Expected Airborne delay as a function of weather termination time (enhanced)

It should be noticed that the optimal path initial angle and distance are different for each termination time. Table 5-7 provides the angle and the distance of each optimal turning point associated with every termination time considered for the above experiment (Figure 5-13).

Table 5-7: Optimal turning point associated angle and distance

Terminate Time	β	x	Terminate Time	β	x	Terminate Time	β	x	Terminate Time	β	x
5	0	5	65	0	60	130	21	40	195	23	36
10	0	10	70	0	60	135	21	39	200	23	35
15	0	15	75	0	60	140	21	38	205	23	35
20	0	20	80	0	60	145	22	40	210	24	39
25	0	25	85	0	60	150	22	39	215	24	38
30	0	30	90	0	60	155	22	38	220	24	38
35	0	35	95	0	60	160	22	37	225	24	38
40	0	40	100	18	45	165	22	36	230	24	37
45	0	45	105	18	43	170	23	39	235	24	37
50	0	50	110	19	42	175	23	38	240	24	36
55	0	55	115	20	42	180	23	38	245	24	36
60	0	60	120	20	41	185	23	37	250	24	35
			125	21	41	190	23	37			

Figure 5-14 shows the similar results to those presented in Figure 5-13 but for three different Normal probability distribution functions of weather clearance time rather than a uniform one.

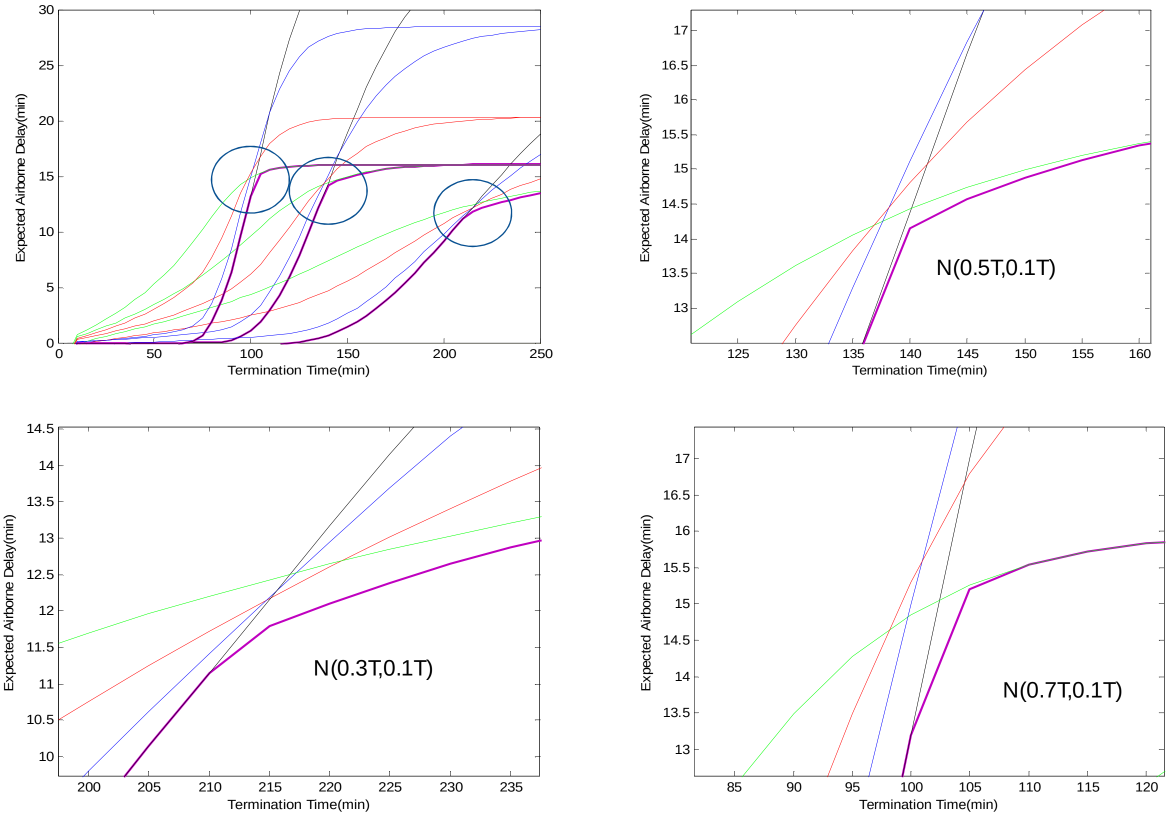

Figure 5-14: Expected Airborne delay as a function of weather termination time for different PDF of weather clearance time

pdf('Normal',1:T,0.5T,0.1T);
pdf('Normal',1:T,0.3T,0.1T);
pdf('Normal',1:T,0.7T,0.1T);

The optimality of the new trajectory design is only guaranteed as long as we eliminate the second stage capacity constraint. For the case of capacitated second stage, we may want to let the model decide for the most appropriate angle based on the results of the optimization. For this case we would have our algorithm to numerically calculate the optimal turning points locations for a given set of directional angles, for each flight. This means a flight assigned to one of its possible optimistic path would have a pre defined turning point location from which will deviate from its current trajectory to one of the end points of the FCA (which ever result in shortest path to its destination) given that the weather has not cleared by the time flight reaches that point.

5.3 *Experimental Discussion*

5.3.1 The Effect of Storm Duration

First we create an experiment where the FCA is impenetrable. This is a plausible scenario, as there are on occasion, for example, thunderstorms that it is not safe to fly through. Once such a storm is created, we would want to test the response of the model to an increase in the max duration, e.g.:

C1: Max duration 60 min with early clearance times of 15, 30 and 45 min.

C2: Max duration 90 min with early clearance times of 15, 30, 45, 60 and 75 min.

C3: Max duration 120 min with early clearance times of 15, 30, 45, 60, 75, 90 and 105 min.

C4: Max duration 150 min with early clearance times of 15, 30, 45, 60, 75, 90, 105, 120, and 135min.

This experiment should demonstrate two effects. First we would expect the amount of rerouting to increase. Of course, this is fairly obvious, but what should be more interesting is the percentage of increase in the total number of flights being rerouted. The other interesting thing will be to look at which flights are rerouted and which are ground-delayed. The ones with the shallower angles, which also correlate with the flights with the better reroute options, should be the ones that tend to be rerouted.

Table 5.8 shows the breakdown of the results in order to observe the above concepts.

Note: slot length= 2 Minutes, Total number of flights=200

Table 5-8: Results for experiment on the Effect of Storm Duration

	Number of Flights%			
	C1	C2	C3	C4
Low RR cost	7.5%	21.5%	45.5%	67.5%
Avg RR cost	2.5%	7.0%	12.5%	16.5%
High RR cost	0.0%	1.0%	5.0%	7.0%
Total	10.0%	29.5%	63.0%	91.0%
	Delay (minute)			
Initial GD	103	223	110	86
Initial RR	172	526	1152	1593
Expected	277	806	1515	2188

The total number of flights being rerouted for case C1 is only 10% of all flights in the program where the degraded conditions only last for an hour. That percentage increases to 29.5% for case C2 with the duration of 1.5 hours, 63% for case C3 with the duration of 2 hours and 91% in case

C4 which the degraded conditions last for 2.5 hours. In all cases, more than 70% of the rerouted flights are those with low RR cost and those with average RR cost are around 20%.

The second part of Table 5-8 provide the amount of delays expected for each case. For example in case C1, 90% of the flights assigned to their primary route received 103 minutes of delay in total. The remaining 10% of flights were assigned to their secondary routes and the total amount of airborne delay that they received was 172 minutes. The total expected delay for case C1 was 277 minutes.

5.3.2 Distributional Effect

Based on our model structure the penalty for both rerouting and GD are linear in the clearance time with maximum values after a certain time. Generally, rerouting will reach the maximum value earlier than GD. This is especially the case if the capacity of the FCA under the degraded conditions is fairly low. Suppose that there are three early clearance times, 30, 60 and, 90 minutes and consider five cases with the following probabilities;

C1: 60%, 20%, 10%, 10%;
C2: 40%, 30%, 20%, 10%;
C3: 30%, 30%, 30%, 10%;
C4: 20%, 30%, 40%, 10%;
C5: 10%, 20%, 60%, 10%;

(Note in all cases, there is 10% probability that storm goes to its maximum length).

Thus, there should be the following distributional effect: as we move from C1 to C5, we should see GD decrease and rerouting increase. The reason is that for the later clearance times, the rerouting alternatives will tend to have hit their maximum penalties and the GD alternatives will still be increasing in cost. (We have also investigated this effect in our Geometric model) Table 5-9 shows the results of this experiment. As we expected, the GD for case C1 is 1125 units of time and it decreases through the next cases as the higher probability densities of weather clearance time shift to the later times. The RR costs behave in a reverse order where is equivalent of 165 unit of time for case C1 and goes up for the next cases. The objective function which is the minimum expected total delay also increases from 577 units of time for case C1 to 1155 units of time for case C5.

Table 5-9: Results for the experiments on Distributional Effect

Case	C1	C2	C3	C4	C5
# flights GD	177	172	170	169	166
Initial GD plan cost	1125	938	880	789	643
# flights RR	23	28	30	31	34
Initial RR plan cost	165	197	222	234	271
Obj Function	577	759	880	994	1155

5.3.3 The effect of slot time length on model output and performance

A smaller number of slot times reduces the running time and saves memory usage. We conducted an experiment to find out how, on the other hand, this might affect the accuracy of the final results. In this experiment we had 100 flights of three classes; short, medium and long haul flights, scheduled to departure within a 100-minute time horizon. The reduced capacity was one flight per 4 minutes, which then increases to one flight per minute after the weather clears.

Three cases were generated for this experiment. These three cases are identical in all ways except for the number of slot times, which has been cut in half from one case to the next. This means that for the first case we had 300 time slots, while we had 150 time slots for the second case, and only 75 time slots for the last case. For each halving of the number of slots, we doubled the length of the slot time respectively.

The detailed results are shown in table 5-10;

Table 5-10: The effect of slot time length on model output and performance

Slot time length	1 min	2 min	4 min
RR/GD cost ratio	3	3	3
# flights GD	58	60	60
Initial GD plan cost	237	253	361
# flights RR	42	40	40
Initial RR plan cost	547	552	575
Obj Function	574	568	583
LP relaxation running time	52	18	9
Total running time	486	55	19

The objective function value remained approximately the same for all three cases; however the initial plans are not necessarily the same. With a simple data manipulation we found that 69% of the flights have been assigned similarly in all three cases, and the remaining 31% of the flights have been assigned differently from one case to another. To explore this further we also compared them in pairs. As a result of that we found the initial plans for case 1 and case 2 are similar for 88% of the flights for which the standard deviation of their appointment time difference at the FCA is 4.13 minutes. The complete results are shown below.

Table 5-11: The effect of slot time length on model initial plan similarity

	initial plan match	SD	Avg
1 & 2 & 3	69%		
1 & 2	88%	4.14	0.05
1 & 3	74%	3.66	-1.46
2 & 3	76%	3.81	-1.53

CHAPTER 6: COMPUTATIONAL EXPERIMENTS

Finally we have conducted a set of comprehensive computational experiments to investigate various properties of our main model. The first set of experiments was aimed at evaluating the computational efficiency and scalability of the model. The second set sought to evaluate the ability of the model to improve decision making and the quality of air traffic operations. For both sets of experiments, flights, their routes, and alternate routes were generated artificially.

6.1 Experiment1-Computational Efficiency

In the first set of experiments, several cases were considered with the lowest number of flights being 100 and the highest 500. Flight departure times were deterministically spaced evenly in time starting at 0 and ending at 100 minutes. We alternated among the three flight types. There were T=200 time slots; each slot had a width of Δt =2 minutes. Initially, the FCA had restricted capacity of 1 flight every two time slots (15 flights per hour). In all cases, the FCA cleared after 6 hours and 40 minutes (400 minutes), at which time the capacity rose to infinity. There were 3, 5, 7, or 9 possible early clearance times, each occurring with some positive probability; the sum of early clearance time probabilities equaled 0.9 so that the probability of no clearance was 0.1. In the event of early clearance, slot capacity rose from 1/2 to 2 flights for each time slot. For each number of scenarios (3, 5, 7 or 9) the model was run for a different number of flights starting from 100 flights and increasing by 100 flights up to 500 flights for each successive run. A 2.8 GHz Intel® Pentium® based computer was used with 3.99 GB of RAM. The IP solver used was XPress MP® vers 2007B.

The following figures provide a presentation of the model's computational performance. They show the manner in which various problem parameters can ultimately limit the size of problem that can be solved. Note that the running time increases nonlinearly as a function of the number of flights with a greater rate of increase for larger numbers of scenarios. All cases were solved to optimality. The limiting factor would not be the running time required to find an optimal solution but rather the memory required to initiate the solver.

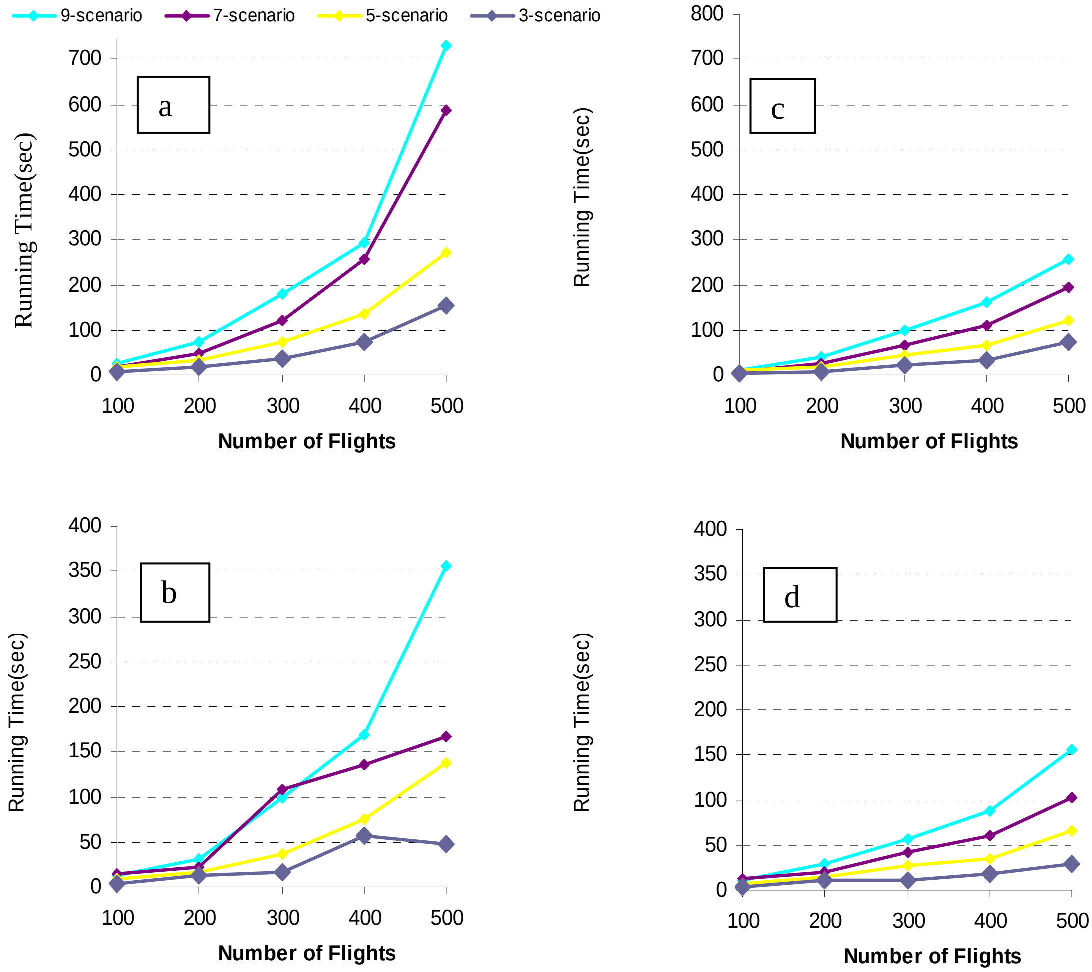

Figure 6-1 (a and b) Total running time (sec) versus number of flights
(c and d) LP relaxation running time (sec) versus number of flights
a and c:Optimistic Model b and d:Conservative Model

The following tables show the percentage gap between the LP relaxation solution and the final optimal integer solution regarding the above running times. An LP relaxation solution provides the initial bound therefore the closer it is to the optimal solution the faster the model may find an optimal integer solution. As an example if the objective value for the LP relaxation solution was 50 and the objective value for the optimal integer solution was 100, the percentage gap presented in these tables would be (100-50)/100=50% and the model most likely converged faster than if the objective value of the LP relaxation solution was -50 when the gap would have been (100-(-5))/100=150%.

The first table shows the results for the pessimistic approach in which we only considered the most conservative reroutes for flights. For the pessimistic approach the LP relaxation solutions are near their optimal integer solutions and the model converged quickly.

Table 6-1: Percentage gap between Initial bound and final integer solution (conservative model)

Number of Scenarios

(IP-LP)/IP	3	5	7	9
100	0.00%	0.00%	0.00%	0.00%
200	0.01%	0.00%	0.00%	0.00%
300	0.03%	0.07%	0.07%	0.04%
400	0.14%	0.07%	0.10%	0.04%
500	0.05%	0.05%	0.05%	0.03%

Number of Flights

The second table shows the results for the optimistic approach in which we considered both extreme reroute options (i.e. the most conservative and the most optimistic).

Table 6-2: Percentage gap between Initial bound and final integer solution (optimistic model)

Number of Scenarios

(IP-LP)/IP	3	5	7	9
100	177.64%	159.67%	184.45%	181.18%
200	110.56%	104.04%	115.96%	111.69%
300	79.93%	75.74%	83.20%	79.94%
400	67.18%	66.42%	71.70%	70.25%
500	44.99%	42.69%	45.73%	44.37%

Number of Flights

6.2 Experiment 2- Decision Impacts

To evaluate the decision impact of our model, we ran a set of experiments, where we varied the scope of the decision space of the model. In this way we were able to mimic alternate decision support environments where less powerful models or operational options were available. The table below lays out the possible options in the planning and execution of the traffic flow management initiative. The cases vary relative to the extent to which recourse actions are allowed and planned for. A recourse action is taken if the weather clears earlier than expected. In the ground delay case, this means a flight is released at a time earlier than its planned departure time. In the reroute case, this means a flight adjusts its original planned reroute to a more direct route. The key novel contribution of our model is its ability to take into account recourse actions when generating its initial plan. In the table of options below note that the manner in which recourse is handled can vary from the planning to execution steps. In fact, while in many operational contexts recourse actions are taken during execution, it is rarely the case that the initial plan is made anticipating the possibility of recourse actions.

	Reroute (RR)	**Ground Delay (GD)**
Plan	*none/static/recourse*	*none/static/recourse*

Execute	*none/static/recourse*	*none/static/recourse*

The following alternate cases were considered.

Case 1:	**Plan**	**RR:** *none*	**GD:** *recourse*
	Execute	**RR:** *none*	**GD:** *recourse*

This case eliminates totally the reroute option but uses the full power of the model in planning and executing the ground delay plan. This is perhaps not a realistic case, but by comparing it to Case 2, we can isolate the value of recourse in ground delay planning.

Case 2:	**Plan**	**RR:** *static*	**GD:** *static*
	Execute	**RR:** *static*	**GD:** *static*

This case chooses the best single static plan and then sticks with that plan during execution even if the weather clears early. In terms of practice this is probably an overly pessimistic scenario since usually there is some recourse in the execution step.

Case 3:	**Plan**	**RR:** *static*	**GD:** *static*
	Execute	**RR:** *static*	**GD:** *recourse*

This case finds the best static plan but only allows recourse in the ground delay execution. This is a plausible representation of reality.

Case 4:	**Plan**	**RR:** *static*	**GD:** *recourse*
	Execute	**RR:** *static*	**GD:** *recourse*

This case allows full recourse ground delay planning and execution but static reroute planning and execution. This is another realistic scenario under which TFM execution systems are not responsive enough to provide dynamic rerouting for en route flights.

Case 5:	**Plan**	**RR:** *static*	**GD:** *recourse*
	Execute	**RR:** *recourse*	**GD:** *recourse*

This case allows full recourse ground delay planning and execution but static reroute planning with recourse reroute execution.

Case 6:	**Plan**	**RR:** *static*	**GD:** *static*
	Execute	**RR:** *recourse*	**GD:** *recourse*

This case chooses the best single static plan but then allows recourse actions when the plan is executed. This is another plausible representation of reality, although a fairly optimistic one, in that the static plan is optimized and it is assumed that each flight is able to take the best recourse action during execution.

Case 7:	**Plan**	**RR:** *recourse*	**GD:** *recourse*
	Execute	**RR:** *recourse*	**GD:** *recourse*

This case applies the full power of the model.

Case 8-13:	**Plan**	**RR:** *recourse*	**GD:** *recourse*
	Execute	**RR:** *recourse*	**GD:** *recourse*

Case 8-13 applies the full power of the new model, structured to consider and evaluate alternate secondary route options sets.

The Table 6-3 and the Figure 6-2 below provide the results of an experiment under which all 13 cases were executed. Cases 1 through 7 use only the pessimistic reroute option as defined earlier. Among these, Case 7 applies the most powerful combination of planning and execution and thus generates the lowest cost solution. Perhaps not surprisingly, there is a very substantial cost difference in all experiments between Cases 2 and 7, which compare a totally static system with a totally dynamic one. What is perhaps more surprising is that substantial savings are still obtained when one compares Case 7 to Cases 3, 4, 5 and 6. Of particular note is that substantial savings still remain even when comparing Cases 7 and 6. Case 6 is a very optimistic representation of a TFM system that uses optimized static planning but then has the full (optimized) power of recourse during execution. Even in this case, the model can achieve savings in the range of 10 to 20%. It is also interesting to compare Cases 3 and 4. The only difference between these cases is that Case 4 takes recourse into account in making its ground delay decisions. It is noteworthy that this produces a very substantial impact – in some cases, savings of over 25%. Of course, we have already noted that planning with recourse in the context of ground delay programs is already practiced through the application of exemption policies, e.g. using distance-based GDP's (2). GDP recourse planning is fully analyzed in (5). The importance of the model developed in this paper is that it can simultaneously carry out recourse-based ground delay and reroute planning.

In case 8, we allow the directional angle of the secondary route for each flight to be chosen from two options. a) zero-angle (this is the "optimistic" case -- the secondary route has no angular deviation from its primary route); b) Max-angle (this is the "pessimistic" case – the trajectory for the secondary route totally avoids the FCA). It is interesting to note that substantial savings are achieved by case 8 relative to case 7. This shows that the strategy of flying towards a weather impacted area anticipating future clearance can produce the lowest expected cost. What is more important to note is that our model can determine when this represent an optimal strategy.

In case 9, we add a third option with a directional angle equal to the average of the optimistic and pessimistic cases (the average of the zero-angle and the max angle). Our results show that this produces very little additional benefit.

For case 10, we pre-compute the optimal directional angle of each flight independent of capacity constraints (here optimal is relative to the weather clearance time probability distribution and the geometry of the flight path). Note that the costs here are worse than in case 8, indicating the importance of considering the capacity constraints.

Cases 11, 12 and 13 are similar to cases 8, 9 and 10 respectively except that the capacity of the second stage was set to infinity (un-capacitated). In the absence of second stage capacity constraints, as one can expect, the approach that employs the optimal unconstrained directional angle (case 13) provides the lowest cost. *However, a very significant result is that the* *differences between case 13 and cases 11 and 12 are negligible. Thus, this experiment suggests that it is not necessary to consider reroutes that hedge against the extremes. Rather, one can achieve nearly all the benefits by choosing one extreme or the other.*

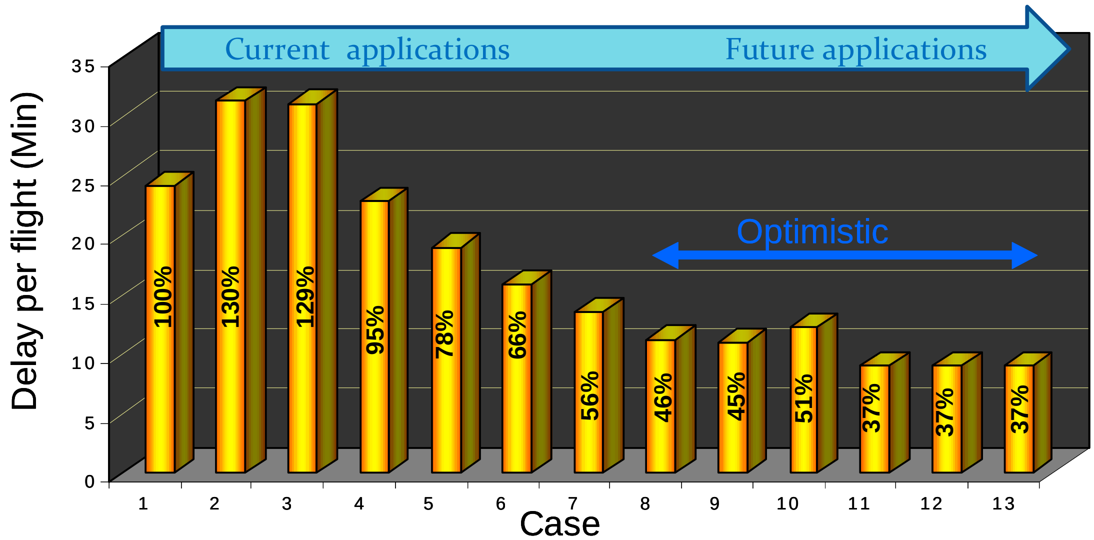

Figure 6-2: Normalized results of Decision Impacts Experiment

The second and the third columns are the clearance time and its probability. The fourth and the fifth columns are the total costs for ground delays and airborne delays of the first stage. The sixth and the seventh columns are the savings occurred to the first stage costs (recourses) due to increased capacity on the second stage. The eighth column is the total cost of the system for each realization of the random variable (Clearance time). The last column is the objective function value which is the minimum expected total cost. The units of all costs are "number of time slots" that can be transferred to minutes or dollars.

Table 6-3: Experiment 2 (Decision Impacts)

Case	q	p[q]	c(xp=1)	c(xs=1)	sv(yp=1)	sv(yh=1)	c(q)	Objective
1	15	0.5	12821	0	11079	0	1742	3619
	30	0.3			9511	0	3310	
	45	0.1			8096	0	4725	
2	15	0.5	355	1444	0	0	4686	4686
	30	0.3			0	0	4686	
	45	0.1			0	0	4686	
3	15	0.5	355	1444	56	0	4630	4658
	30	0.3			0	0	4686	
	45	0.1			0	0	4686	
4	15	0.5	8805	357	7764	0	2113	3423
	30	0.3			6675	0	3202	
	45	0.1			5692	0	4184	
5	15	0.5	8805	357	7652	298	1330	2837 ☆
	30	0.3			6570	216	2659	
	45	0.1			5537	158	3867	
6	15	0.5	355	1444	56	1035	1526	2373 ☆
	30	0.3			0	690	2615	
	45	0.1			0	371	3573	
7	15	0.5	1551	1318	1169	1104	1025	2021
	30	0.3			914	829	2105	
	45	0.1			728	504	3264	
8	15	0.5	927	2433	690	2340	519	1676
	30	0.3			522	2207	1084	
	45	0.1			398	1712	2692	
9	15	0.5	788	2380	566	2273	543	1638
	30	0.3			425	2162	1017	
	45	0.1			300	1647	2686	
10	15	0.5	1162	3286	839	3230	490	1834
	30	0.3			594	3174	903	
	45	0.1			424	2812	2159	
11	15	0.5	455	2655	360	2602	253	1342
	30	0.3			300	2484	669	
	45	0.1			240	2152	1723	
12	15	0.5	455	2618	360	2562	265	1341
	30	0.3			300	2443	681	
	45	0.1			240	2111	1735	
13	15	0.5	1075	3570	906	3561	195	1340
	30	0.3			751	3551	381	
	45	0.1			601	3404	971	

Experiment 1 note: reduced capacity = 1 flight every 4 minutes; increased capacity = 1 flights every 1 minutes; airborne delay cost/ground delay cost: a = 3; number of flights=160.

CHAPTER 7: THE EFFECT OF A MOVING FCA (MFCA)

In this chapter we will discuss the effect of possible movement of the FCA due to wind on our model results and will present a methodology to take into account such an effect assuming that the FCA will have constant velocity and fixed direction in two dimensional airspace. Base on historical data of weather forecasts, the displacement of weather activities such as thunder storm can be as fast as 40 miles per hour. Some slow moving weather activities can move 5 to 10 miles per hour. Relatively speaking a typical passenger aircrafts cruising speed is about 500 miles per hour. Therefore even the slow movement of an FCA can have a significant effect on flights reroute trajectories. This assumption was the base for our motivation to conduct a research to investigate the effect of a moving FCA on our model results. A deterministic displacement of the FCA assumption allowed us to prevent additional model complications and we were able to apply the necessary calculations and adjustment through our input parameters.

In the following section we will show how the movement of the FCA will affect our previous flight path geometries and we will provide the related calculations for each case. One can obtain the same functions for the case of a stationary FCA simply by setting v_a and v_c to zero.

First of all for each flight we need to find the relative location of the FCA at the time of its scheduled departure. For example if at time t=0 the FCA is 30 minutes away from the flight origin and flight departure time is at t=25 then we need to know where would be the new location of the FCA at time t= 25. This requires our flight path parameters to be time dependent as appose to those for the stationary FCA. For example if for the stationary FCA the distance between origin and FCA boundary for flight f is represented by a constant number, say *a*, then for the moving FCA we need to make *a* as a function of t_d, so we will have the up dated location of the FCA for the moment that each flight is actually going to depart. Once we have the adjusted flight path geometry parameters for the moment of flight departure time then we still need to find the possible changes that will occur in between the departure time and the time that flight reaches the boundary of the FCA. In the following sections we will show how the second part is calculated for different situations.

7.1 Primary route delay cost function

The first set of equations show how the primary route cost functions are recalculated. We assume that the intersection of the FCA and a flight primary path (i.e. the point that the flight will enter the FCA) will move with a constant speed either toward or away from the flight. In these equations, v_f is the speed of the aircraft along its path, v_a is the projection of the FCA speed vector on the flight's primary path, t is the time of arriving at the FCA, and t_d and t_s are the actual and scheduled departure times.

$$
\begin{aligned}
& v_f\left(t-t_d\right)+v_a\left(t-t_d\right)=a-v_a\left(t_d-t_s\right) \qquad \forall t \\
& t_d = t+\frac{v_a}{v_f}\left(t-t_s\right)-\frac{a}{v_f} \\
& c_{f,t}^{p} = t_d - t_s = \left(1+\frac{v_a}{v_f}\right)\left(t-t_s\right)-\frac{a}{v_f} \quad \text{and } c_{f,t}^{p} \geq 0
\end{aligned}
\tag{7.1}
$$

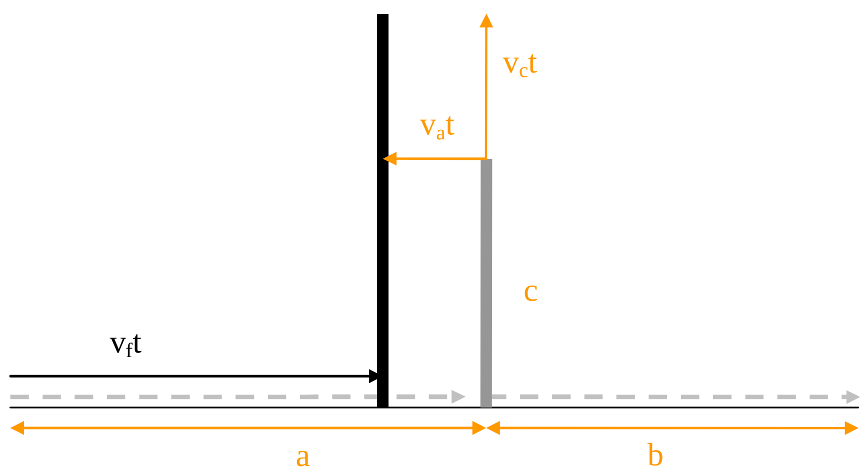

Figure7-1: The effect of MFCA on Primary route delay cost function.

The ground delay cost function of flight f, assigned to its primary path and scheduled to arrive at FCA at time t; $c_{f,t}^{p}$ is a function of a, distance from the FCA boundary at scheduled departure time, v_f :flight velocity, v_a: FCA velocity projected on flight primary path and t: FCA slot appointment time.

7.2 Secondary Route delay cost function

Next we show how the secondary cost functions are recalculated. Let α and β be the required directional angles of the reroute to avoid a moving and stationary FCA, respectively. The gray dashed lines represent the flight path in the case of a stationary FCA and the black lines represent those of a moving FCA. v_c is the orthogonal component of MFCA velocity.

$$\left.\begin{aligned} v_f\left(t-t_f^d\right)\cos\alpha &= a - v_a\left(t-t_f^d\right) \\ v_f\left(t-t_f^d\right)\sin\alpha &= c + v_c\left(t-t_f^d\right) \end{aligned}\right\} \Rightarrow$$

$$t-t_f^d = \frac{a}{v_f\cos\alpha + v_a} \Rightarrow \frac{a v_f \sin\alpha}{v_f\cos\alpha + v_a} = c + \frac{a v_c}{v_f\cos\alpha + v_a}$$

$$\Rightarrow a\sin\alpha - c\cos\alpha = \frac{c v_a + a v_c}{v_f}$$

$$d = \sqrt{c^2 + a^2},\quad \cos\beta = \frac{a}{d},\quad \sin\beta = \frac{c}{d}$$

$$\Rightarrow \frac{a}{d}\sin\alpha - \frac{c}{d}\cos\alpha = \frac{c v_a + a v_c}{d v_f} = \cos\beta\sin\alpha - \sin\beta\cos\alpha \Rightarrow$$

$$\sin\left(\alpha-\beta\right) = \frac{c v_a + a v_c}{d v_f} \tag{7.2}$$

With the new directional angle α, we can calculate the cost of airborne delay of the reroute;

$$\begin{aligned} d' &= \frac{a}{v_f\cos(\alpha) + v_a} \\ c_f^s &= d' + \sqrt{d'^2 + (a+b)^2 - 2\cos(\alpha)d'(a+b)} - a - b \end{aligned} \tag{7.3}$$

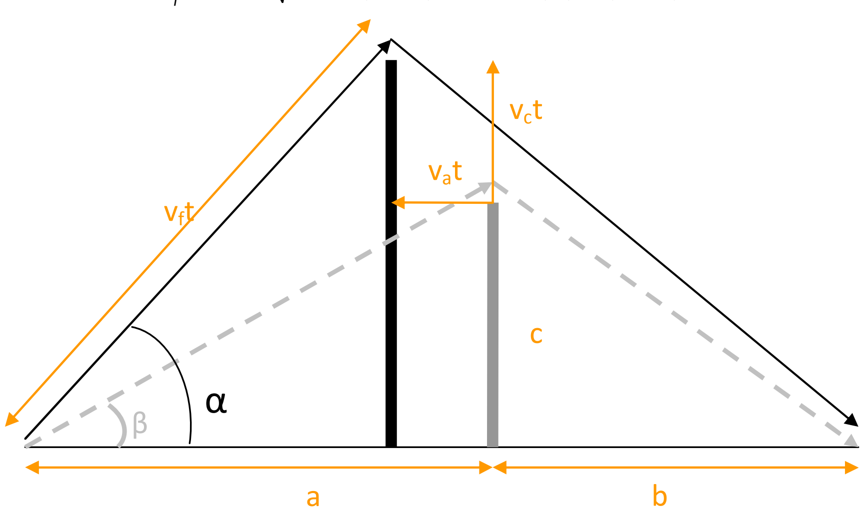

Figure 7-2: The effect of MFCA on secondary route delay cost function.

7.3 Hybrid Route delay cost saving function

Finally, once we have found the adjusting angle for the secondary route compromising the FCA movement, we need to calculate the interrelated changes to our hybrid route cost saving function as well. To do so we assume that the weather clears after *i* minutes of the flight's departure. With the speed of v_f, our flight would traverse a distance $\overrightarrow{AD} = i$ times v_f along its secondary route. As shown in Fig. 4, by knowing $\overrightarrow{AD}$ we can compute its counterpart angle μ.

$$\cos\mu = \frac{a + b - iv_f \cos\alpha}{\sqrt{\left(iv_f\right)^2 + \left(a+b\right)^2 - 2iv_f\left(a+b\right)\cos\alpha}} \tag{7.4}$$

Now that we have μ, we can build in our governing equation to calculate the time *t*, at which the flight arrives at FCA if it reverts from its secondary route after *i* minutes of its departure.

$$\begin{aligned} &v_f i\cos\alpha + v_f\left(t-i\right)\cos\mu = a - v_a t \Rightarrow \\ &t = \frac{a + iv_f\left(\cos\mu - \cos\alpha\right)}{v_f\cos\mu + v_a} \end{aligned} \tag{7.5}$$

This is only true if the flight has not yet reached the end of the FCA (i.e. point C). Therefore the following constraints should apply to maintain the feasibility of the above equations.

$$v_f i\sin\alpha - v_f\left(t-i\right)\sin\mu <= c + v_c t \tag{7.6}$$

and finally the saving incurred on the hybrid route:

$$sv_{f,t}^{h} = c_f^s - v_f i_t - \sqrt{\left(v_f i_t\right)^2 + \left(a+b\right)^2 - 2v_f i_t\left(a+b\right)\cos\alpha} \tag{7.7}$$

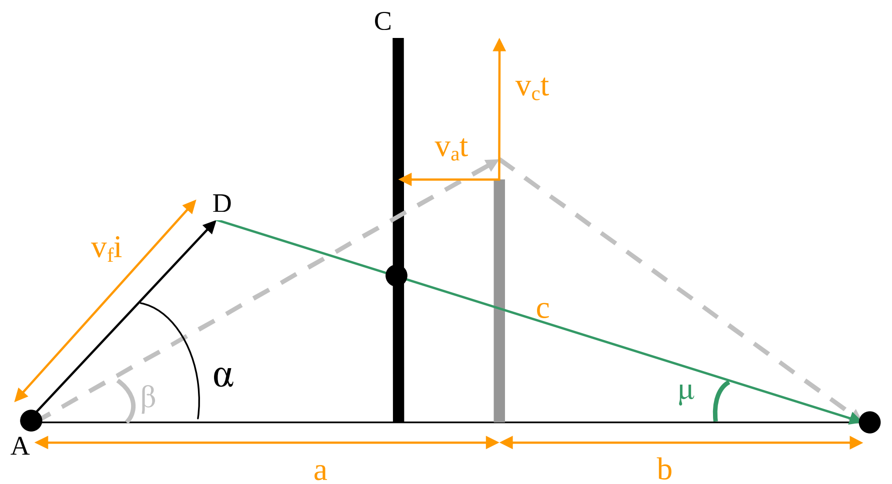

Figure 7-3: The effect of MFCA on Hybrid route delay cost saving function.

7.4 Optimistic Reroute delay cost function

With the similar approach, next we calculate the cost of the optimistic secondary routes:

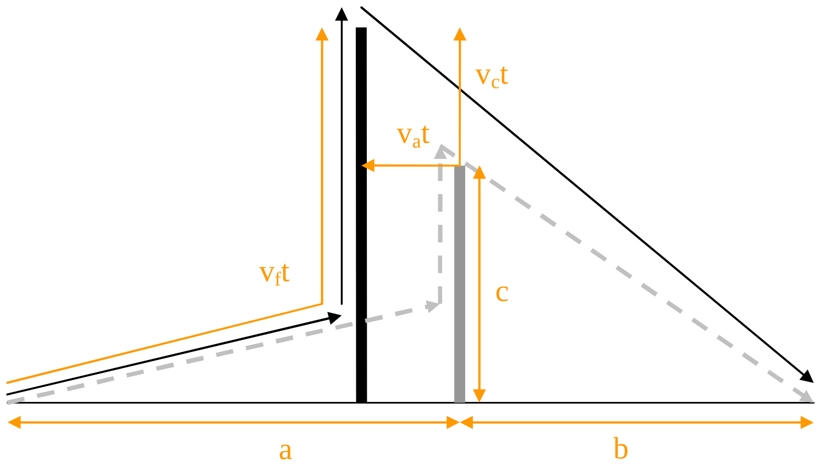

Figure 7-4: The effect of MFCA on Optimistic Reroute delay cost function.

Previously with the stationary FCA we had:

$$cs_{f,\alpha} = \frac{a}{\cos(\alpha)} + c - a\tan(\alpha) + \sqrt{b^2 + c^2} - ab \tag{7.8}$$

Considering a movement of the FCA, let's assume the flight will reach the end of the FCA after t minutes from its departure time. As illustrated in Figure 7.4 we can derive:

$$cs_{f,\alpha} = \frac{a - v_a t}{\cos(\alpha)} + (c + v_c t) - (a - v_a t)\tan(\alpha) + \sqrt{(b + v_a t)^2 + (c + v_a t)^2} - ab \tag{7.9}$$

Now that we have our cost function we need to eliminate t from it, by the following substitution. where,

$$\begin{aligned} &\frac{a - v_a t}{\cos(\alpha)} + (c + v_c t) - (a - v_a t)\tan(\alpha) = v_f t \\ &\Rightarrow t = \frac{c\cos(\alpha) - a\sin(\alpha) + a}{v_a(1 - \sin(\alpha)) + (v_f - v_c)\cos(\alpha)} \end{aligned} \tag{7.10}$$

7.5 General flight path geometry

Next we will discuss how we can apply the above methodology on our general flight path geometry where we had introduced the γ as an arbitrary angle as opposed to fixed right angle which were used for simplicity.

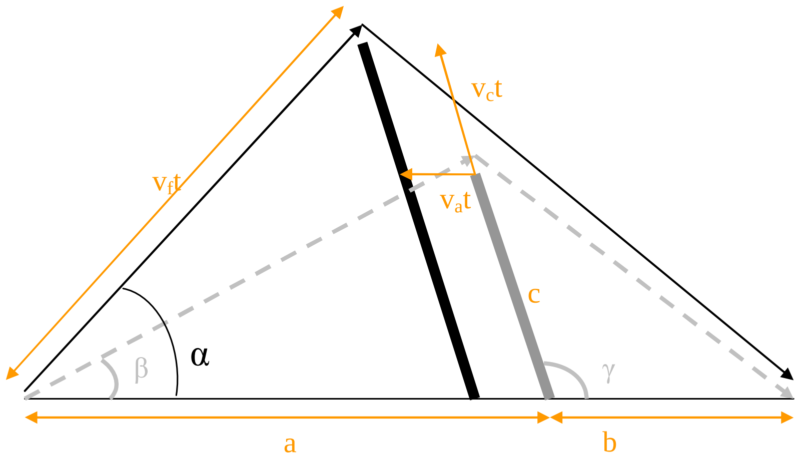

Figure 7-5: The effect of MFCA on secondary route delay cost function (General flight path geometry).

Once again we set up our governing equations. The first one maintain distance equalities on the flight primary path direction and the second one maintain distance equalities on a direction parallel to the FCA representative line :

$$v_f t\cos\alpha + (c + v_c t)\cos\gamma = a - v_a t$$
$$v_f t\sin\alpha = (c + v_c t)\sin\gamma \Rightarrow t = \frac{c\sin\gamma}{v_f\sin\alpha - v_c\sin\gamma} \tag{7.11}$$

If we substitute the t from the second equation into the first equation after some simple equation manipulation we can derive the following equalities:

$$cv_f\cos\alpha\sin\gamma + cv_c\cos\gamma\sin\gamma + cv_a\sin\gamma = (a - c\cos\gamma)(v_f\sin\alpha - v_c\sin\gamma)$$

$$\Rightarrow (a - c\cos\gamma)\sin\alpha - c\sin\gamma\cos\alpha = \frac{cv_a + av_c}{v_f}\sin\gamma$$

$$\Rightarrow d\cos\beta\sin\alpha - d\sin\beta\cos\alpha = \frac{cv_a + av_c}{v_f}\sin\gamma$$

$$\sin(\alpha - \beta) = \frac{cv_a + av_c}{dv_f}\sin\gamma \tag{7.12}$$

From equation (7.12) we can calculate the adjusting directional angle of the flight secondary path. Having that in hand we can find the new secondary route cost function and other necessary cost functions as mentioned in previous sections.

Finally we need to calculate our essential cost functions for the cases that we deal with both general and optimistic flight path geometry. As shown in figure(7-6) a flight is chosen to depart as scheduled via a secondary route with a directional angle α. This angle is chosen by optimization process so it is given. All we need to find is the time t at which the flight reaches one of the end points of FCA. Although the set up of the governing equations is conceptually similar to the previous cases, however it should be noted that the FCA speed vector components are chosen deliberately parallel to the specific directions as shown in Figure (7-6) to simplify the calculation.

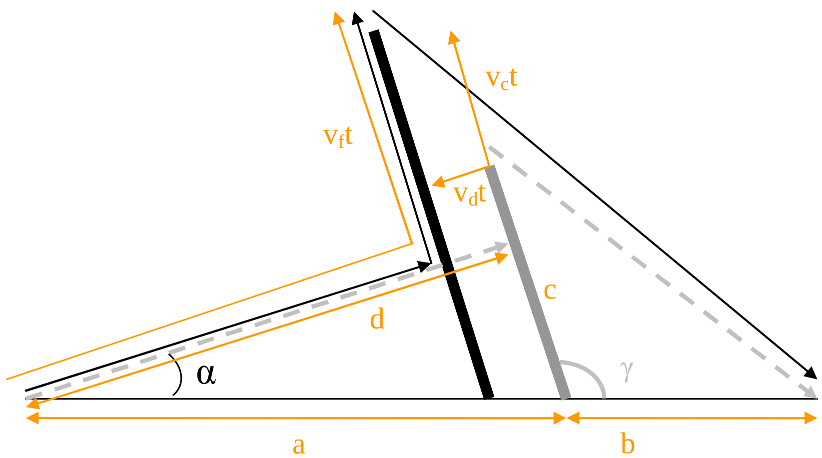

Figure 7-6: The effect of MFCA on optimistic reroute delay cost function (General flight path geometry).

$$\begin{aligned}&(d - v_d t) + (c + v_c t) - \frac{(d - v_d t)}{d}\sqrt{a^2 + d^2 - 2ad\cos\alpha} = v_f t \\ &\Rightarrow t = \frac{d + c - \sqrt{a^2 + d^2 - 2ad\cos\alpha}}{v_f + v_d - v_c - \frac{v_d}{d}\sqrt{a^2 + d^2 - 2ad\cos\alpha}}\end{aligned} \tag{7.13}$$

So the secondary route cost function is:

$$\begin{aligned}cs_{f,\alpha} &= d - v_d t + (c + v_c t) - \frac{(d - v_d t)}{d}\sqrt{a^2 + d^2 - 2ad\cos\alpha} \\ &+ \sqrt{(b + v_a t)^2 + (c + v_a t)^2 - 2(b + v_a t)(c + v_a t)\cos\gamma} - ab\end{aligned} \tag{7.14}$$

7.6 Decision Impacts

To evaluate the impact of the FCA movement on our model we ran a set of experiments, where we varied the direction of the FCA movement. In this way we were able to mimic more realistic environments where the flights' paths (primary and reroute) are affected by the movement of the FCA as well as its presence. The cases vary relative to the speed, direction and severity of the weather activity and recourse actions are allowed and planned respectively. A recourse action is taken if the weather clears earlier than expected. In the ground delay case, this means a flight is released at a time earlier than its planned departure time. In the reroute case, this means a flight adjusts its original planned reroute to a more direct route. The key novel contribution of our model is its ability to take into account recourse actions when generating its initial plan.

We now describe the problem data. Flights, their routes, and alternate routes were generated artificially based on the airspace geometry given in Fig. 1. There were F=200 flights with random departure times (t_s=0,…,60). There were T=200 time slots; each slot had a width of Δt =2 minutes. There were three possible early clearance times: $U \in \{30, 50, 70\}$ each occurring with probability 0.3 and 0.1 is the probability that the FCA does not clear until the end time of the AFP. The following alternate cases were considered. The ratio of airborne delay cost to ground delay cost was assumed to be 2.0.

Case 1: This case considers a stationary FCA and runs the model to find the best initial plan which will serve as a base for the purpose of evaluation of the other cases.

Case 2-9: in these cases the FCA has eight different directions with the same velocity approximately equal to 5% of the average flight speed. The reduced throughput of the FCA is one flight every 4 minutes and increased throughput is 2 flights per minute.

Case 10-17: these cases are similar to cases 2-9 except that the FCA velocity is approximately equal to 10% of the average flight speed.

Case 18: this case is similar to case 1 except that the reduced throughput of the FCA is one flight every 8 minutes and increased throughput is one flight per minute.

Case 19-26: these cases are similar to cases 10-17 except that the reduced throughput of the FCA is one flight every 8 minutes and increased throughput is one flight per minute.

The table below provides the results of an experiment under which all 26 cases were executed. First of all it should be clarified that for simplicity all the 200 flights are assumed to fly in the same direction but with different origin-destination distances, different scheduled departure times and different directional angles for their reroutes. Valuable insights should be obtainable even with this simplification, and more realistic scenarios can always be studied with the exact same formulation.

The first thing to notice is that the movement of the FCA can significantly change the total cost as well as the assignment of the dispositions to all flights affected by the presence of the FCA. The second interesting result is the consistent pattern with which the objective function value increases. In the result table we have sorted the similar cases (similar in terms of the FCA velocity and throughput) in an increasing order of the objective function value. In all three sections of the results table, perhaps not surprisingly, the maximum cost saving occurs when the FCA moves laterally (downward in Fig. 2 and Fig.3), in which case it either gets out of the way of the primary paths of the affected flights quickly or lowers the maximum length of the reroutes. The total cost is reduced by 38% and 64% with the low and high speed FCA, respectively.

One can observe that the effect of the longitudinal movement of the FCA, where it moves either toward or away from the oncoming traffic, is less significant than the lateral motion. The total cost is increased by 3% (8% for the high speed FCA) when the FCA is moving away from the traffic. When it is moving toward the traffic the total cost is increased by 19% (23% for the high speed FCA).

The second and the third columns are the total costs for ground delays and airborne delays of the first stage. The fourth and the fifth columns are the numbers of flights assigned to primary and secondary paths. The sixth column is the objective function value, which is the minimum expected total cost. The seventh and the eighth columns are the horizontal and the vertical component of the FCA velocity vector. The last column visualizes the FCA direction. The units of all costs are “numbers of time slots,” which can be converted readily to minutes, and presumably to dollars if the analyst has data on economic time values.\

Table 7-1: Experimental results on the effects of a moving FCA

Case	c(xp=1)	c(xs=1)	n(xp=1)	n(xs=1)	Obj	Va	Vc	Wind
1	80	459	67	133	379	0.00	0.00	●
2	66	225	60	140	236	0.00	-0.05	↓
3	97	285	65	135	310	-0.035	-0.035	↘
4	103	300	60	140	336	0.035	-0.035	↙
5	124	494	71	129	391	-0.05	0.00	→
6	108	437	61	139	452	0.05	0.00	←
7	130	689	70	130	543	-0.035	0.035	↗
8	117	670	67	133	572	0.035	0.035	↖
9	105	783	70	130	585	0.00	0.05	↑
10	65	93	58	142	135	0.00	-0.10	↓
11	98	161	63	137	185	-0.07	-0.07	↘
12	84	173	56	144	234	0.07	-0.07	↙
13	123	528	74	126	409	-0.10	0.00	→
14	102	451	61	139	466	0.10	0.00	←
15	150	1020	75	125	655	-0.07	0.07	↗
16	130	899	66	134	752	0.07	0.07	↖
17	135	1192	71	129	799	0.00	0.10	↑
18	65	717	36	164	605	0.00	0.00	●
19	55	190	32	168	247	0.00	-0.10	↓
20	85	331	36	164	327	-0.07	-0.07	↘
21	94	305	32	168	389	0.07	-0.07	↙
22	115	892	39	161	635	-0.10	0.00	→
23	90	712	34	166	711	0.10	0.00	←
24	133	1554	41	159	972	-0.07	0.07	↗
25	143	1333	34	166	1082	0.07	0.07	↖
26	107	1812	38	162	1178	0.00	0.10	↑

In this chapter we represented a methodology to derive our main input parameters of the cases for which we need to deal with a moving FCA rather than a stationary one. This is another step toward generalizing our model to better embrace the reality. It should be mentioned that neither the magnitude nor the direction of the FCA velocity vector is necessarily deterministic. However we assume it is a good approximation for the purpose of this study.

CHAPTER 8: SUMMARY AND CONCLUSIONS

In this dissertation I have defined the basics of a stochastic optimization model for simultaneously making ground delay and reroute decisions in response to en route airspace congestion. The model constraints and decision variables definition are presented in chapter 3.

In Chapter 4, the details of the model parameters were explained. Our model results strongly depend on the geometry of the problem, which in turn implies the significance of the parameters. The chapter starts with the simplistic flight path geometry and then moves on to more general flight path geometry. The simplistic flight path geometry is a rough estimation of a real flight path but it is the easiest way to generate the necessary input data. The more general flight path geometry covers the special cases more precisely and is more complex.

In Chapter 5, we discussed the main principles at work inside our optimization model. These principles are explained and a set of simple examples clarifies the intuitions behind those principles. In this chapter two sub-models are also presented. The goal of the first sub-model is to compute/estimate ground delay and reroute cost for a single flight and then to draw some conclusions regarding when each strategy should be used. The second sub model numerically generates optimal route choice for a single flight to hedge against weather risk. The goal is to explore the properties of different reroutes and their effect on our model decision making. We conduct experiments allowing a range of such trajectories and draw conclusions regarding appropriate strategies.

In Chapter 6, I have given the results of computational experiments that both test the computational efficiency and decision impact of the model. These results show that the model is tractable and can serve as a basis for solving practical TFM problems using commercial IP solvers. Further, the results show that the models have the potential to substantially improve TFM decision making.

In chapter 7, we represented a methodology to deal with a moving FCA (MFCA) rather than a stationary one. This was another step toward generalizing our model to better embrace the reality. We provided a mathematical framework to recalculate the parameters of our model. The new cost functions take into account the displacement of the FCA between present and the planed time of arriving at the FCA by assuming a constant speed and predefined direction. It should be mentioned that both the magnitude and the direction of the FCA velocity vector are stochastic variables. However we assumed a deterministic approximation suffices for the purpose of this study.

8.1 Operational issues and Considerations.

In order to take advantage of our model capabilities for the daily air traffic management and operations we need to address several operational considerations without which our model will not be widely acceptable for real world practices.

The air traffic management consists of several key players and any changes to the system need to be fully consistent with all elements of the system constraints and concerns.

While our model is able to produce better dynamic, stochastic TFM plans, it will only be useful if TFM systems can dynamically adjust to changing conditions. Today, TFM systems in the U.S. dynamically adjust ground delay decisions, e.g. by allowing flights given ground delays to leave early if the weather clears; however, there is less ability to dynamically reroute airborne flights to take advantage of newly availability capacity. It is also worth noting that such dynamically adaptive systems have less predictability than more static ones. In that sense, the use of models such as ours requires the user to make certain tradeoffs between expected delay and predictability.

Our model can be re-run if, and as often as, real-time information suggest that the data supporting a previous execution of the model have changed significantly, for example, if carriers cancel some additional flights, or if the probabilistic weather forecast changes. The model can be forced to preserve earlier decisions by additional constraints fixing those decisions for flights currently in the air.

In addition, Each pilot-in-command has the option to refuse a clearance for safety reasons. If a flight cannot comply with the clearance, it is required to advise ATC. At that time, different options may be presented to the flight, including the option of taking a delay on the ground until the situation in the airspace is resolved.

8.1.1 FCA representative shape and flight path geometry

The shape and dimensions of the weather activities play a major role in shaping the trend of air traffic managements. In this work we have modeled the FCA with a straight line which reduces the throughput of the intersecting traffic flow corridors. While this is a fairly close approximation of how a real weather activity affects the airspace, it is not the best of what might be achieved for the real case practices. Normally an irregular polygon with holes and gaps or a set of small polygons adjacent to each other would be a more appropriate demonstration of the real weather activities. By considering a more realistic shape for the FCA while the main structure of our model remains the same we need to refine our input generator algorithm to provide us with more realistic trajectories both in regard to nominal and off nominal jet routes. We anticipate that by introducing a set of fix points that are chosen based on standard real jet routes one can produce more acceptable and precise flight path geometry rather than using straight lines as an estimate for a flight flying path. Simulation and numerical computations can be used as alternative approaches to provide the necessary cost functions for our model if we need to replace the straight lines with a path that goes through typical route fix points.

8.1.2 FCA Capacity estimation

One of the key factors that directly control the reliability of our model results is the estimation of the FCA capacity. Although the estimation of the FCA capacity was not one of the goals of this dissertation, however the results of our model strongly depend on it. Basically the capacity of the FCA can be defined by the number of flights that controllers in charge of the impacted area are able to navigate safely through the FCA.

During convective weather normally the throughput significantly decreases due to safety considerations that require a larger separation standard and fewer and/or smaller passable gaps along the line of the FCA. This can be quantified by measuring the ratio of the total passable area (length) to the whole area (length) of the FCA.

The required separation (in time or distance) between the successive flights is a driving factor of the FCA throughput and depend on both severity of the weather and the portion of the FCA that is passable. On the other hand, a Required Navigation Performance (RNP) capability (that is, the ability to fly within *x* nmi of a given route, described by RNP-*x*), can affect our estimation of the

FCA capacity. For instance, a high performance aircraft (or pilot) may have an ability to fly RNP-1 routes while a General Aviation (GA) aircraft may only be able to fly RNP-10 routes. For a given RNP requirement, the estimation of capacity (expressed for instance in terms of the total number of lanes of traffic that can safely pass through the airspace) varies with the weather hazard constraint.

There are several factors that limit our ability to have a robust estimation of the FCA capacity. The main one is the uncertainty with the weather forecast specially when we need to have an estimate few hours in advance. Our prediction for both severity and the formation of the weather activity can be significantly different from the actual ones. While our model does not consider the stochasticity of the FCA capacity however the negative effect of this issue will be diluted through our ability to rerun our model anytime new updates are available.

8.1.3 Uncertainty with scheduled departure time.

Another element of system is facilities and surface domain services. Aside from weather related uncertainties, not exactly every process on the ground will prevail as we may expect. The scheduled departure time is one of the factors that our model assumes it is accurate and starts from there to provide an optimized decision for each flight regarding a new departure time or specific reroute. We understand that it is very difficult to maintain a departure time as scheduled. Even if there is no specific limiting factor, simply human performance can alter a departure time from its original plan by few minutes. We anticipate a need to replace a pin point assignment with a time window that allows a flexible departure time. Of course we can simply choose our time slot for our model to be longer (e.g. 5 minutes) that will take care of minor deviation of real departure times. However, once again our flight path geometry calculations won't be as meaningful as if we choose a tighter time slot (e.g. 1 or 2 minutes).

8.1.4 Objective function

The objective function of our optimization model takes into account the total cost of all possible ground and airborne delays over the next several hours of operation. The objective function while includes a specific amount of delay assigned to each and every individual flight affected by the FCA, it does not recognize any specific priority or classification among them. In other words, the airlines objectives or preferences are not reflected on our global objective function. The airline preferences can be accommodated through some post processing. For example each airline can utilize their share of resources (all the slots assigned to their flights) differently as long as they do not violate the main constraints of the model. Theoretically any changes to the original assignment of our model would have either negative or no effect on the global objective function value. However airlines simply might have different interpretations of delay costs based on their internal economic objectives. In general the cost of one minute of delay (ground or airborne) could be different at different time of the day and/or for different flights and/or for different airlines.

8.1.5 Collaborative decision making

The current version of our model acts as a centralized supporting management tool that assumes all the users will accept its results for the sake of system wide optimality and efficiencies. We understand that such a system is not acceptable in US as one of the key players of the system; airlines have their own rights and economics objective to follow. However our model is capable of providing few important features that take into account its users' preferences. One is a set of constrains to place a maximum cap on the amount of the ground delay that the model might assign to a flight. An airline that owns a flight may request for a maximum ground delay that they are willing to accept before they prefer a secondary route. The other option comes from the fact that our model is capable of accepting multiple secondary routes as an alternative reroutes. Airlines can have the option of submitting a set of secondary routes, and rank them based on their preferences. The airlines in return can expect that our model will assign the most efficient reroutes that is available based on their preference list. This concept is what C-TOP (or SEVEN) is pursuing via set of integrated algorithms, rules and procedures.

As an example the customer might choose to: a) File a route around the FCA (the secondary route). b) Submit a second trajectory that passes through the FCA, with (implicit) instructions that the flight is approved to use that route if the traffic manager, controller and pilot concur that the storm activity would be less severe than expected and that it is safe to fly through the FCA (an optimistic path).

c) Submit one or more contingency plans to deal with the situation if the weather develops more severely than expected. Note that such plans could involve the use of different altitudes (capping or topping), departure times or routes.

Minor tactical adjustments such as vectoring around a storm cell will always remain a part of the operation of the NAS.

8.1.6 Vertical controlling techniques

Our model currently does not take into account the possible vertical routing adjustments as an option while planning. However they are part of the air traffic control mechanisms. Traffic managers use different altitudes to segregate different flows of traffic or to distribute the number of aircraft requesting access to a specified geographic area. Some of the control mechanisms are Low Altitude Alternate Departure Route (LAADR), Capping, Tunneling and, Tower En Route descriptions. LAADR is a procedure whereby flight altitudes may be limited to flight level (FL) 230 and below.

LAADR procedures are primarily used in the departure phase of flight, but can be extended for an entire flight when operational benefits are achieved. “Capping” is a colloquialism for planning to hold aircraft at altitudes lower than their requested altitude until they are clear of a particular area. It may be in response to weather or other situations that have impacted air traffic controllers’ ability to provide service and it may be applied to the entire route of flight. It is used during constrained situations in the NAS and enables aircraft to continue to depart while remaining underneath a constrained airspace.

“Tunneling” is a colloquialism for descending traffic prior to the normal descent point at an arrival airport to keep aircraft clear of an airspace situation on the route of flight. It is used to avoid conflicting flows of traffic and holding patterns.

Tower-en route is a situation where the aircraft never reaches the en route stratum, but stays in the lower terminal altitudes being handed-off from one terminal facility (tower or TRACON) to another vs. center to center. This sometimes reduces delays, especially if the higher en route stratum is congested (FAA booklet).

8.1.7 Special use airspace

Our model can be used to provide set of decisions and constraints that are consistent with designated special use airspace constraints. “Special use airspace consists of airspace of defined dimensions identified by an area on the surface of the earth wherein activities must be confined because of their nature, or wherein limitations are imposed upon aircraft operations that are not a part of those activities, or both.”, (FAA, 2010).

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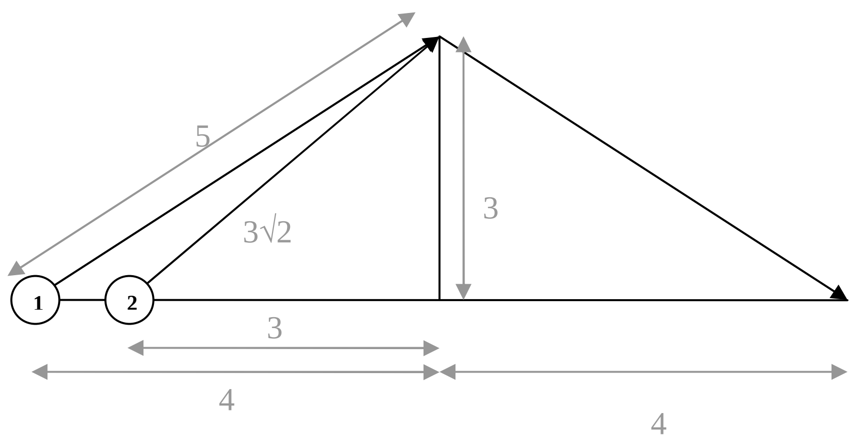
APPENDIX 1: PROOF REGARDING TABLE 6.3

While it may seem counter intuitive here is an example that proves:“ *it is possible that case 5 performs worse than case 6*”

Airborne/ground delay cost=2
Weather: 50% never clears, 50% clears after 0.5 hour
There are only two flights. One is 4 hours away from FCA the other one is 3 hours away from FCA. Under reduced capacity the first available slot is after 5 hours from now and the second is after 9 hours from now.
FCA is un-capacitated if it’s clear.
Flight 1 secondary route delay=5+5-8=2
Flight 2 secondary route delay=3√2+5-7=2.24
Flight 1 hybrid route delay| u=0.5 =0.5+7.6-8=0.1



assignment	initial plan		revised plan			execution		
		total cost		total cost	Obj		total cost	Obj
flight1-s	2	5	2	4	4.5	0.2	0.2	2.6
flight2-p	1		0			0		
flight1-s	2	8.48	2	8.48	8.48	0.2	0.2	4.34
flight2-s	2.24		2.24			0		
flight1-p	1	5.48	0.5	4.98	5.23	0.5	0.5	2.99
flight2-s	2.24		2.24			0		
flight1-p	1	6	0.5	0.5	3.25	0.5	0.5	3.25
flight2-p	5		0			0		

For case 6, the model chooses the first assignment based on yellow box and after execution we get the green box.
For case 5, the model chooses the fourth assignment based on blue box and after execution we get the red box which is worse than case 6.

APPENDIX 2: LIST OF ABBREVIATIONS

ARTCC: Air Route Traffic Control Center

ASPM: Aviation Systems Performance Metrics

ATC: Air Traffic Control

ATCSCC: Air Traffic Control System Command Center

ATFM: Air Traffic Flow Management

ATM: Air Traffic Management

CDM: Collaborative Decision Making

TFMS: Traffic Flow Management System

FAA: Federal Aviation Administration

FCA: Flow Constraint Areas

FSM: Flight Schedule Monitor

GDP: Ground Delay Program

GHP: Ground Holding Problem

IP: Integer Programming

LP Linear Programming

MAGHP: Multi-Airport Ground Holding Problem

MDP: Markov Decision Process

MIT: Miles-in-Trail

NAS: National Airspace System

RBS: Ration-by-Schedule

SAGHP: Single Airport Ground Holding Problem

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