ABSTRACT

Title of document: Multi-Period Natural Gas Market Modeling Applications, Stochastic Extensions and Solution Approaches

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This dissertation develops deterministic and stochastic multi-period mixed complementarity problems (MCP) for the global natural gas market, as well as solution approaches for large-scale stochastic MCP.

The deterministic model is unique in the combination of the level of detail of the actors in the natural gas markets and the transport options, the detailed regional and global coverage, the multi-period approach with endogenous capacity expansions for transportation and storage infrastructure, the seasonal variation in demand and the representation of market power according to Nash-Cournot theory. The model is applied to several scenarios for the natural gas market that cover the formation of a cartel by the members of the Gas Exporting Countries Forum, a low availability of unconventional gas in the United States, and cost reductions in long-distance gas transportation.[1] The results provide insights in how different regions are affected by various developments, in terms of production, consumption, traded volumes, prices and profits of market participants.

The stochastic MCP is developed and applied to a global natural gas market problem with four scenarios for a time horizon until 2050 with nineteen regions and containing 78,768 variables. The scenarios vary in the possibility of a gas market cartel formation and varying depletion rates of gas reserves in the major gas importing regions. Outcomes for hedging decisions of market participants show some significant shifts in the timing and location of infrastructure investments, thereby affecting local market situations.

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[1] www.gecforum.org

A first application of Benders decomposition (BD) is presented to solve a large-scale stochastic MCP for the global gas market with many hundreds of first-stage capacity expansion variables and market players exerting various levels of market power. The largest problem solved successfully using BD contained 47,373 variables of which 763 first-stage variables, however using BD did not result in shorter solution times relative to solving the extensive-forms. Larger problems, up to 117,481 variables, were solved in extensive-form, but not when applying BD due to numerical issues. It is discussed how BD could significantly reduce the solution time of large-scale stochastic models, but various challenges remain and more research is needed to assess the potential of Benders decomposition for solving large-scale stochastic MCP.

Multi-Period Natural Gas Market Modeling

Applications, Stochastic Extensions and Solution Approaches

By

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List of Abbreviations

BBL	Balgzand Bacton Line
bcf	billion cubic feet
bcm	billion cubic meters
bcm/y	billion cubic meter per year
BD	Benders decomposition
BP	British Petroleum
CHP	Combined Heat and Power
CO_2	Carbon dioxide
CQ	Constraint Qualification
CVaR	Conditional Value at Risk
DET	Deterministic
DOE	Department of Energy
DW	Dantzig-Wolfe decomposition
EC	European Commission
EEV	Expected value of the EVP solution
EIA	Energy Information Administration
EPEC	Equilibrium problems with equilibrium constraints
Eq.	Equation
ETS	Emissions Trading System
EU	European Union
EVP	Expected value problem
EVPI	Expected value of perfect information
GECF	Gas Exporting Countries Forum
IEA	International Energy Agency
k$	one thousand dollar
KKT	Karush-Kuhn-Tucker
LB	Lower bound
LCP	Linear complementarity problem
LNG	Liquefied Natural Gas
LP	Linear Program
LR	Lagrangian Relaxation
mboe	million barrels of oil equivalent
mcf	thousand cubic feet
mcm	million cubic meters
MCP	Mixed complementarity problem
MIP	Mixed Integer Program
MP	Master problem
MPEC	Mathematical problems with equilibrium constraints
NEMS	National Energy Modeling System
NG	Natural gas
NGL	Natural gas liquids
NLP	Nonlinear Program
OPEC	Organization of Petroleum Exporting Countries
QP	Quadratic Program
RES	Renewable Energy Sources
RGGI	Regional Greenhouse Gas Initiative
RP	Recourse problem

RPS	Renewable Portfolio Standards
SP	Subproblem
STO	Stochastic
Tcf	Trillion cubic feet
TOP	Take or pay
TPA	Third party access
TSO	Transmission system operator
UB	Upper bound
USD	U.S. dollar
VaR	Value at Risk
VI	Variational inequality
VVS	Value of the stochastic solution
WGM	World Gas Model

1 Background and Motivation

1.1 The increasing role of natural gas in the energy supply

Contemporary human societies depend heavily on the use of energy in any part of their daily activities. We need fuel for our cars to drive to the office in the morning, electricity to power lighting and our computers, and gas to heat our work spaces. Electricity is produced from sources such as coal and nuclear energy; renewable sources, such as solar, wind or hydropower and natural gas.

According to (International Energy Agency, 2008) the world-wide daily energy consumption in 2006 amounted to an equivalent of 250 million barrels of oil (mboe).[2] Energy consumption is expected to continue to increase, induced by a growing world population and economic growth. The International Energy Agency (IEA) projects a growth in energy use of 45% between 2006 and 2030 to 363 mboe (Figure 1, left).

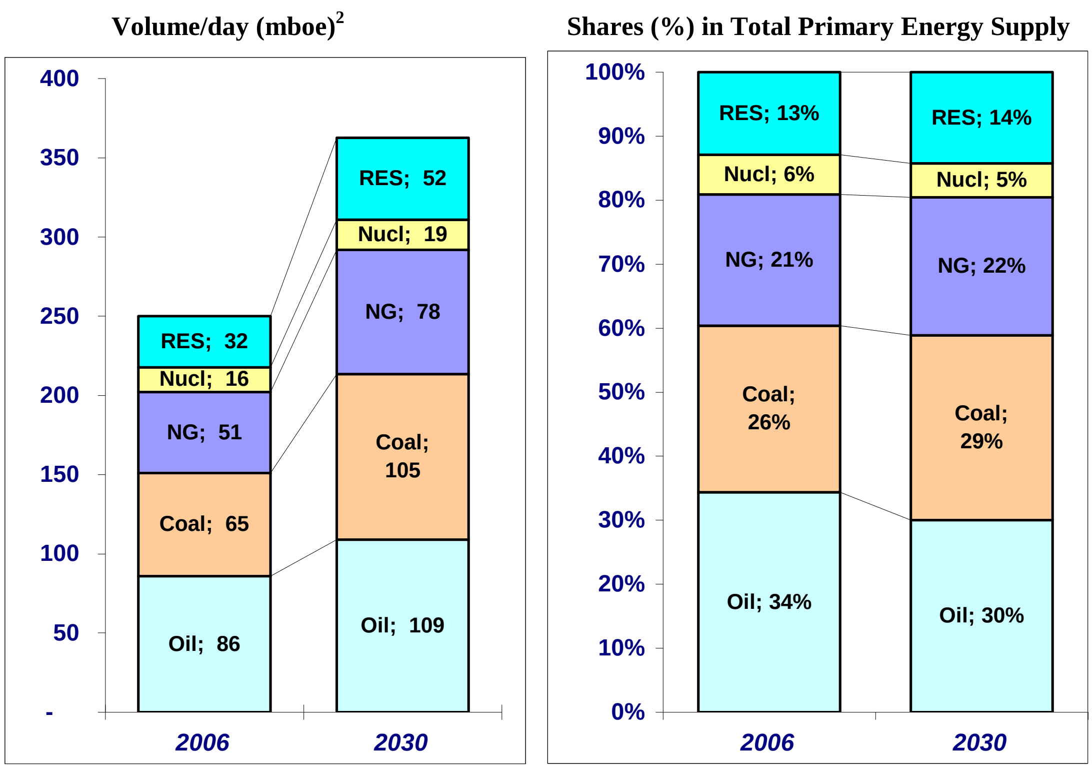

Figure 1: Development of total primary energy demand (IEA, 2008)

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[2] mboe = million barrels of oil equivalent

Fossil fuels are projected to grow 44%, renewable forms of energy (RES) 60% and nuclear energy 24%. Among the fossil fuels, growth rates vary considerably: coal (+61%) and natural gas (NG, +52%) grow more than total energy use, and oil (+27%) grows less. Thus, coal, gas and RES increase their market shares and the shares of oil and nuclear energy decrease (Figure 1, right).

1.2 Globalizing natural gas markets

Until recently, a global natural gas market was virtually non-existent. Several regional markets could be distinguished, based on geographical proximity of suppliers/exporters and consumers/importers. Most natural gas is transported through high-pressure pipelines onshore, a relatively small part via offshore pipelines or in ships in the form of liquefied natural gas (LNG).[3] LNG has been shipped and traded for over fifty years, however due to its high costs large-scale LNG imports were limited to some rich countries with few alternative supply options, notably Japan and South Korea. For several reasons, such as locally depleting reserves and supply security considerations, long-distance international gas trade has increased rapidly over the last years. A larger volume of LNG spot trade is the cause for regional natural gas markets to gradually merge into one global market.

The growth in international trade is illustrated by Figure 2 and 3. Figure 2 shows that between 2000 and 2009 global international pipeline and LNG trade increased rapidly, with 77% (from 389 to 634 bcm/y) and 63% (from 137 to 243 bcm/y) respectively. Both growth percentages are much larger than the 21% increase in worldwide gas consumption of (from 2435 to 2940 bcm/y)[4]

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[3] When natural gas is cooled to -260 degrees Fahrenheit it liquefies and becomes over 600 times denser (www.lngfacts.org, undated web references are dated early 2010.) The capital investments for a liquefaction facility are significant: the estimated investment costs are $900 million for a typical small plant with an output capacity of 4.8 bcm/y (Cayrade, 2004). On top of the high costs, there is a loss of about 12% of the natural gas used to power the liquefaction process. However, for transport over long distance, and/or when pipelines just cannot be built, LNG is a viable and competitive means to transport gas.

[4] bcm/y = billion cubic meters per year.

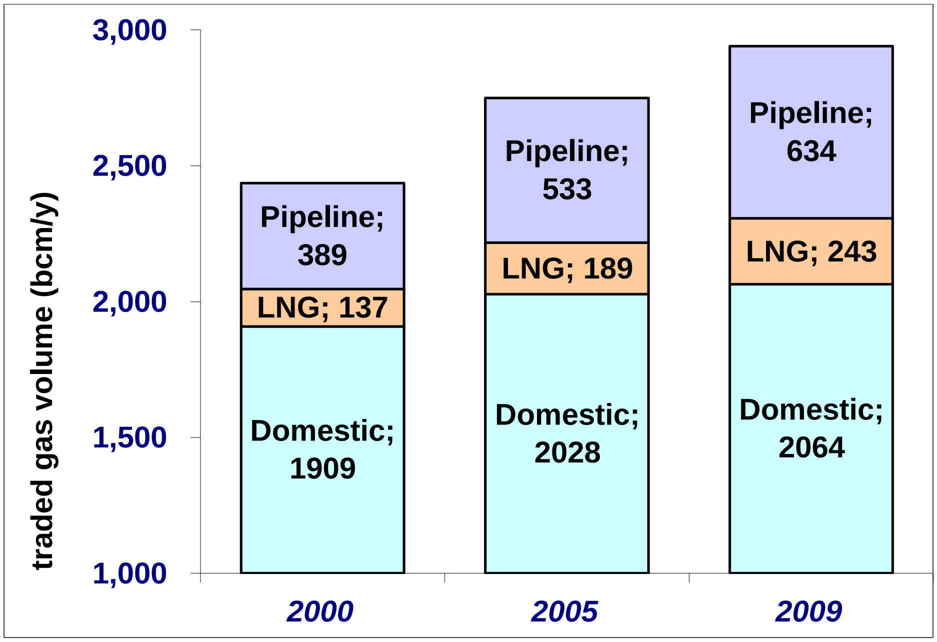

Figure 2: Global natural gas consumption and trade (BP various years)

As Figure 3 below illustrates, international natural gas trade is projected to outpace consumption growth in the coming decades (International Energy Agency, 2008). The expected increase in trade between 2006 and 2030 is about +150% in 2030, relative to a total global gas consumption increase of +52% (Figure 1).

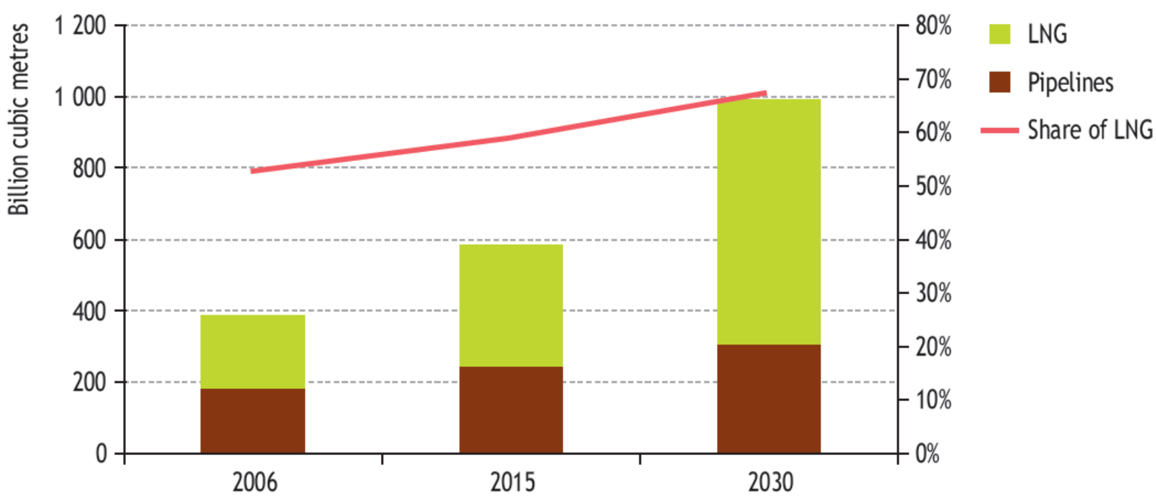

Figure 3: projected global natural gas trade. Source IEA 2008

To better capture the recent changes in global market dynamics caused by the rapid increase in international gas trade, advances in modeling approaches are helpful to allow policy makers and businesses to adequately address the growing interdependency among world regions and higher complexity of the structure and trade relations in the natural gas market.

1.3 Supply security and market power

Similar to the oil market, where a limited number of countries have the majority of proved reserves, gas resources are also unevenly spread among nations. The three countries with the biggest resources together have over half of all world reserves: Russia has 24%, Iran 16% and Qatar 14% (BP, 2010). And although the world's reserve-to-production ratio is about 60 years, the regional figures vary dramatically: from under ten years in Mexico and some European countries to over two centuries in the Middle East (BP, 2010). [5]

Many countries in North America, Europe and Asia have limited domestic reserves of natural gas, and are therefore dependent on imports from - sometimes nearby but often remote - production regions. For example, in recent years the United States imported over ten percent of its natural gas consumption from Canada (BP, 2010), and many European countries import gas from Russia, Norway and Algeria. The aggregate import shares of France, Germany and Italy from these three countries add up to 76%, 79% and 69% respectively (BP, 2010).

The two main ways to transport gas from production regions to consumption regions are through pipelines and as LNG. In most regions there is only a limited number of nearby suppliers with abundant reserves; lead times and capital costs for transport capacity expansions are significant for both transport options. The situation is often best described by the term 'natural monopoly' because the limitations in transportation infrastructure greatly hinder market access and thereby create limits for competition. In several regional markets there are signs of strategic behavior by producers. The exertion of market power can result in higher prices for gas consumers downstream. In the literature, Haurie et al. (1987) and Mathiesen et al. (1987) were among the first to address and analyze this issue for the European market.

[5] The reserve-to-production ratio indicates how many years of producing at current levels it would take to deplete current proved reserves. Section 3.2.2 in Chapter 3 discusses reserves and resources.

Several incidents in the past have shown the dependency of many European countries on Russia as a supplier. The consequences of one incident were severely felt in January of 2006 when GazExport, the trading arm of the large Russian gas and oil company GazProm, shut down gas flowing to Ukraine as leverage in a contractual dispute (Stern, 2006). Recently in January 2009 there was a very similar dispute with several countries - especially in Eastern Europe - experiencing serious problems in their gas supply situation.[6]

The incidents described above could arguably be seen as bilateral issues between Russia and the Ukraine with unfortunate collateral consequences for some countries in Western and Central Europe. However, there are developments that could have a much larger scope. In 2001 the Gas Exporting Countries Forum (GECF) was established and has raised concerns about the possible forming of a cartel in the world natural gas markets. Since 2001 the GECF has developed into a formal organization with broadening membership.[7] In December 2008, Russian prime-minister Putin was very explicit when he said that '*the time of cheap energy resources, and cheap gas, is surely coming to an end'*. [8] Other GECF participants stated objectives for the increased collaboration among GECF participants such as to coordinate investment plans, study ways to set global prices and represent the interests of producers and exporters on the international market. Although the recent downturn in the global economy and the consequently lower energy demand has undercut the market position for energy suppliers, the coordination among GECF members will likely increase in the long term.

In general, the exertion of market power will result in tighter supply and higher prices for gas, which can have adverse effects on economies highly dependent on gas imports. To analyze the impact of enhanced collaboration among gas suppliers on the economies of importing countries in quantitative terms, there is a need for natural gas market models

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[6] NY Times 2009; http://topics.nytimes.com/top/news/business/companies/gazprom/index.html
[7] In 2008, the GECF comprised Algeria, Bolivia, Brunei, Egypt, Equatorial Guinea (observer), Indonesia, Islamic Republic of Iran, Libya, Malaysia, Nigeria, Norway (observer), Qatar, Russian Federation, Trinidad & Tobago, United Arab Emirates and Venezuela, www.gecforum.com.qa, (July 4, 2008).
[8] NY Times 2008; www.nytimes.com/2008/12/24/business/worldbusiness/24gas.html

with a global coverage and modeling approaches that take market power aspects into account.

1.4 Liberalization and privatization

In many countries the supply and distribution of natural gas to their end-users has been a state-organized effort. Some exceptions include the United States, Canada and the United Kingdom. In the United States both public and private companies have been part of the supply chain for many years and deregulation in the 1970s enhanced the possibilities to compete for customers.[9] In the United Kingdom in the 1980s the administration of Prime Minister Thatcher included various market liberalizations which affected the energy sectors. A notable effect in the United Kingdom has been significantly lower gas prices than on the European mainland and the fast exploration of natural gas reserves in the years following the liberalization.[10] In other European countries the natural gas market remained state-owned until the mid-nineties, when several legislative and infrastructural measures were taken by the European Commission (EC).[11] These measures lead to legal unbundling - splitting of gas traders and network operators - and for mandatory Third Party Access (TPA) to transmission, distribution, storage and LNG regasification capacity.[12] To enhance market access opportunities the EC created lists of infrastructure priority projects (EC, 2000b) containing a variety of projects to introduce natural gas into new regions, interconnect regional gas networks and increase transport capacities.[13] Many of these infrastructure priority projects have actually been implemented in the last few years, or are currently under construction, favoring a continuing growth of natural gas use in Europe.

Since the European resources are limited, and the availability of natural gas to end-users must be secured, the dependence on external suppliers must be carefully managed. Supply diversification, buying the gas from several suppliers to reduce the dependence on a single supplier has always been one of the main means to mitigate risks. Supply

[9] http://www.ferc.gov/students/whatisferc/history.htm
[10] J.R. Branston (2000), A counterfactual price analysis of British electricity privatisation, Utilities Policy 9
[11] The first European gas directive 98/30/EC. For an overview of energy-related directives, see: www.energy.eu/#directives, http://europa.eu/legislation_summaries/energy/index_en.htm and http://europa.eu/legislation_summaries/energy/internal_energy_market/l27077_en.htm
[12] European directive 2003/55/EC. http://www.energy.eu/directives/l_17620030715en00570078.pdf
[13] European Commission decision No 761/2000/EC

diversification, technological progress and cost reductions (Cayrade, 2004) lead to increasingly cheaper long-distance LNG transports and continuing high growth in LNG trade for decades to come (cf. Figure 3).[14] This rise in LNG trade is creating one worldwide natural gas market in which the U.S. East Coast and Europe may be competing for LNG in the Atlantic Basin; the U.S. West Coast and India, China, South Korea and Japan may compete for South East Asian supplies; and the Middle East can act as a swing supplier between Europe and South East Asian LNG importers.

1.5 Environmental considerations

Besides security of supply and issues related to market liberalization there is another reason that natural gas has gained much attention in recent years. Regarding carbon, sulfur and nitrogen content it is the cleanest among fossil fuels. When burning natural gas, the emissions of carbon dioxide (CO_2), sulfur dioxide and nitrogen oxides are relatively low, and therefore it is often a preferred alternative over coal and oil in the electric power generation sector.[15] It is generally accepted that fossil fuels are necessary to meet a large part of the energy demand in the next couple of decades. However, shifts from coal and oil to gas can provide an intermediate step to reduce CO_2 emissions. President Obama's push for a cap-and-trade system in the United States and the yearly climate summits under the United Nations Framework Convention on Climate Change to negotiate follow-up agreements for the Kyoto Protocol are only two of the many major factors influencing the outlook for natural gas use from an environmental policy perspective.[16]

1.6 Making decisions in an unpredictable world

Although the upward trend in gas consumption has been very pronounced in the recent past, the future for the direction and magnitude of market developments are not clear at all. Globally, natural gas prices are much higher than they were in the 1990s. Figure 4 shows how the average prices for imported LNG in Japan almost doubled, spot prices in the European Union (EU) more than doubled, and spot prices in the United States almost

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[14] Report:DOE/EIA-0637, December 2003, The Global Liquefied Natural Gas Market: Status and Outlook http://www.eia.doe.gov/oiaf/analysispaper/global/lngindustry.html

[15] www.naturalgas.org/environment/naturalgas.asp

[16] www.nytimes.com/2009/02/28/science/earth/28capntrade.html ; http://unfccc.int/2860.php and http://unfccc.int/kyoto_protocol/items/2830.php

tripled. As previously discussed (see Figure 2), gas demand has continued to rise globally. However the price and demand trends vary among regions.

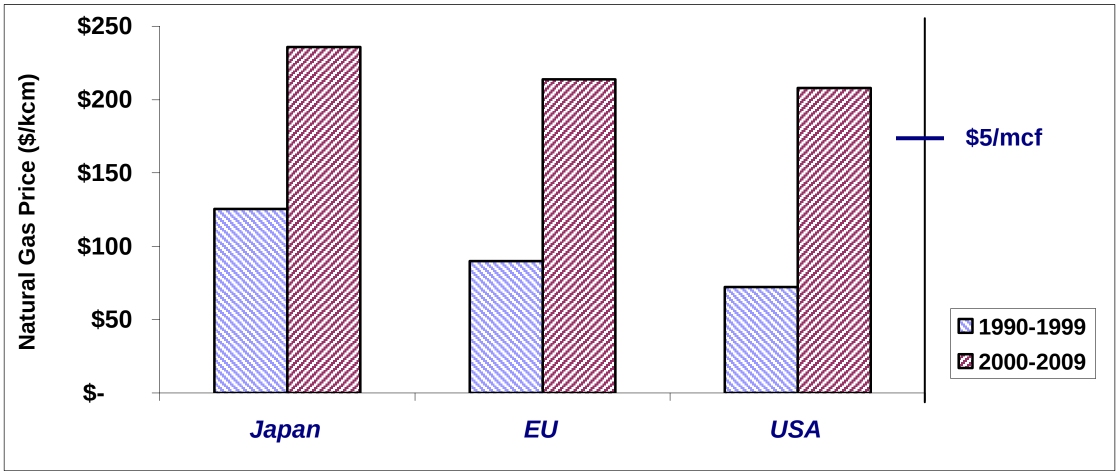

Figure 4: Average Gas Prices in the Last Two Decades ($/kcm). (BP, 2010)

In Japan the gas prices were relatively high but rather stable from 2000 to 2004 (not deviating more than 10% from an average $168/kcm) and then spiked in 2008 to peak prices of $450/kcm (details not in figure).[17] The price trend in the EU between 2000 and 2004 was gradually upward, more rapidly upward starting 2005 to peak at $412/kcm in 2008. In the U.S.A. prices decreased in the first two years of this century, to rapidly increase to a peak of $313/kcm in 2005. In 2008 there was a second peak, at $316/kcm (BP, 2010). Due to the global economic downturn, prices in 2009 were considerably lower in all the regions.

Another factor playing an important role is the need to reduce carbon dioxide emissions to address global climate change, resulting from the greenhouse effect. Although much cleaner than oil or coal, natural gas is a significant source of carbon dioxide and may not be a sustainable alternative fuel for power generation in the long run. For that reason gas consumption in several European countries is projected to start declining in only a few years from now (European Commission, 2008).

Whereas the previously described uncertainties concern the demand side of the market, the supply side also exhibits some uncertainties, including the amount of recoverable gas

[17] kcm = kilo cubic meter = 1000 m^3. One m^3 amounts to 35.31 cubic feet.

from the resource bases. Estimates for proved and probable reserves vary considerably. For instance, there are claims that under the Arctic resources as much as twenty years of total world demand are present, which could drastically increase global gas supply for a long period. Alternatively, some countries are notoriously unreliable and overstate when reporting on their domestic reserves, for example to gain leverage in negotiations about contracts or investments. On top of these long-run uncertainties that greatly affect the future trading volumes and prices, other factors (e.g., the weather, political disputes and activities of war) have large impacts on the daily market situation.

1.7 Motivation for the current research

Governments and companies alike have to deal with a rapidly changing uncertain environment. The use of quantitative tools can help them to make valid market assessments and support policy and business decisions.

The EC continues its efforts to make the European markets, including energy markets, more competitive.[18] Sometimes sub objectives to support the overarching aim of competitiveness are conflicting. The EC wants expansion of the gas transport network. It also pursues a market with more flexibility and fewer long term contracts. The EC also promotes TPA to available transport and storage capacities, so that no market player can limit competition by preventing other market players from market entry. All individual measures aim at enhancing competition, however side by side they may not lead to the desired outcome. For example, market participants who are willing to invest in additional capacity usually want guarantees that the capacity will be used at a high enough utilization rate to be profitable. For that reason the investors would prefer to either be able to claim the capacity for themselves, or sign contracts with other market agents that guarantee them a certain amount of revenues for an extended period of time. So to be willing to invest, they would either want to limit the TPA clauses, or have long-term contracts, thereby making the EC goal of network expansion conflicting with the goals for TPA and fewer long-term contracts.[19]

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[18] http://ec.europa.eu/growthandjobs/key/index_en.htm
[19] For a discussion on the instruments that can be used to provide for sufficient investments in electric power generation capacity in a competitive market see, e.g., (Oren, 2003)

Recognizing these conflicting sub objectives, the EC has developed legislation that allows for temporary exemption from TPA.[20] Some concrete examples of projects that might not have come on-line are the new pipeline from the Netherlands to the United Kingdom and a new regasification terminal in the Netherlands.[21]

Thus, exemptions can be made. But how should a regulator (e.g., FERC or the EC) decide whether to allow them? Or taking it from the other perspective: how should an investor balance and decide on the acceptable terms for him to invest in a project? Should he prefer a shorter TPA exemption period for 100% of capacity, or rather have a longer exemption period for a smaller fraction? Both the commission and the investor face difficult decision problems. In an uncertain and unpredictable world, they make decisions affecting millions of consumers and involving billions of dollars. Long-term demand and supply are uncertain, and will probably be affected by the capacity investment project under consideration. What both parties need to do is to make a good decision addressing the uncertainty, and hedge their positions so that a desirable outcome is reached whatever the future may bring.

Given the complexity of the market, the many factors that come into play and the interdependency of these factors, a quantitative model representing the market will be very helpful in making decisions. An example of the insights that can be gained can be found in (Egging and Gabriel, 2006). They show how additional pipeline capacity from the continent into the United Kingdom reduces the ability of market players to exert market power in the U.K. market, with consequently lower market prices, higher consumption volumes and consumer surplus.

Other factors and uncertainties that are highly relevant for decision makers in natural gas markets include the impact of price developments for carbon dioxide emission certificates in the European Emission Trading Scheme on gas demand. Another example is how the actual finalization date of the new huge Russia-Germany pipeline affects gas prices and desired gas storage levels in 2015, or how the ambitious expansion plans for

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[20] http://ec.europa.eu/energy/gas/infrastructure/exemptions_en.htm
[21] www.bblcompany.nl and www.gateterminal.com/en/

regasification and storage in the UK would influence the currently downward pointing EC demand projections (European Commission, 2008). [22]

In the past the modeling approach of preference has often been linear programming (LP), mainly due to its simplicity of application. Since the mid-nineties a shift can be observed to more advanced modeling approaches such as mixed complementarity problems (MCP).[23] One reason for that shift is the possibility to explicitly include market power characteristics in an MCP framework that are in line with game theory and Nash Cournot equilibria; something that cannot be done in LP models.[24] Therefore, the main models developed in this research are cast as MCP.

The above has illustrated several important features of the natural gas markets: globalization, market power aspects and uncertainty. In the research that is described in this dissertation, these and other aspects are addressed.

To illustrate the contribution of this research we will compare the characteristics of the developed World Gas Model with a number of state-of-the-art models.[25]

1.7.1 The state of the art in natural gas models

The National Energy Modeling System (NEMS) is a large-scale energy systems model developed by the Energy Information Administration of the U.S. Department of Energy.[26] NEMS provides a detailed bottom-up approach for supply and demand of energy in the U.S.A. for a time period of about 25 years. NEMS does not cover other regions of the world and has no provisions for the incorporation of market power à la Cournot for the

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[22] http://ec.europa.eu/environment/climat/emission/index_en.htm ; www.nord-stream.com/en/ and www.nationalgrid.com/uk/Gas/TYS/

[23] The EC sponsors energy market modeling efforts (see e.g. http://ec.europa.eu/research/fp7/index_en.cfm. GDF SUEZ, one of the European majors in the energy market is developing an MCP model for the natural gas market after discussing the results from a preliminary study (Gabriel et al., 2008), and the Danish network operator Energinet.dk has started developing efforts to improve their market models.

[24] In fact, linear programming models cannot model the demand responses to price changes directly. Quadratic programming models can have this feature. However both model types can only mimic market power behavior by assuming some price mark-ups for the marginal cost supply curve, which is, at best, a rough approximation of market power behavior.

[25] The World Gas Model will be described extensively in Chapter 3.

[26] www.eia.doe.gov/oiaf/aeo/overview/ (Accessed March 2010)

natural gas market as WGM provides. [27,28] FRISBEE is a partial equilibrium (PE) model developed by Statistics Norway.[29] It covers the global natural gas market on a rather aggregate level. The Rice World Gas Trade Model (RWGTM) is a computational general equilibrium (CGE) model developed by RICE University.[30]

Table 1: Overview of natural gas model characteristics

Model	Type[a]	Region(s)	Market power	Number of nodes	Time Scale	Density	Seasons	Sectors	Capacity expansions
NEMS	LP	USA+CAN	No	15[b]	2030	Yearly	2	5[c]	Endogenous
WGM	MCP	World	Yes	41	2030	Five years	2	3	Endogenous
FRISBEE	PE	World	No	13	2030	Yearly	1	3	Endogenous
RWGTM	CGE	World	No	460	2050	Five years	1	1	Endogenous
GASMOD[d]	MCP	Europe+LNG	Yes	6	2025	Ten years	1	1	Endogenous
GASTALE	MCP	Europe+LNG	Yes	19	2030	Five years	3	3	Endogenous
GRIDNET	LP	USA	No	18000	operational	Monthly	12	N/A	Exogenous
ICF GMM	NLP	USA	No	114	several years	Monthly	12	4	Exogenous

[a] LP: linear program; MCP: mixed complementarity problem; PE: partial equilibrium; CGE: computable general equilibrium.
[b] United States twelve, Canada two and Mexico one.
[c] Includes power generation, which is not considered as an end-use sector in NEMS.
[d] The dynamic version of GASMOD.

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[27] Actually, for the oil market NEMS allows market power exertion à la Cournot.

[28] A recently developed optimization model, the International Natural Gas Model l(INGM), provides projections for the global natural gas market (including LNG trade), with relatively much detail for North America and emerging countries (such as Russia, China and India) but not so much for Europe. It provides the global context for the NGTDM in the NEMS. The model is a Linear Program and market power aspects are addressed by setting tighter limits for future capacity expansions. Sources: EIA 2010, Models Used To Generate the IEO2010 Projections, www.eia.doe.gov/oiaf/ieo/pdf/appl.pdf (Accessed Nov 11, 2010), Personal communication Dr. A. Kydes: INGM Basics.pptx.

[29] See (Aune et al., 2009) and (Rosendahl and Sagen, 2009)

[30] Hartley, Peter, Kenneth B. Medlock, III and Jill Nesbitt. 2004a. Rice University World Gas Trade Model. James A. Baker, III Institute of Public Policy, Rice University, Houston Texas (March). http://www.rice.edu/energy/publications/docs/GSP_WorldGasTradeModel_Part1_05_26_04.pdf (Accessed March 2010). ; Hartley, Peter, Kenneth B. Medlock, III and Jill Nesbitt. 2004b. Rice University World Gas Trade Model. James A. Baker, III Institute of Public Policy, Rice University, Houston Texas (December). http://www.forum.rice.edu/presentations/Forum04/Peter%20Hartley%20-%20Presentation%20-%20An%20Economic%20Model%20of%20the%20Gas%20Industry.pdf (Accessed March 2010). ; Hartley, Peter, Kenneth B. Medlock, III. 2005. The Baker Institute World Gas Trade Model. James A. Baker, III Institute of Public Policy, Rice University, Houston Texas. and www.rice.edu/energy/publications/docs/GAS_BIWGTM_March2005.pdf (Accessed March 2010).

Among the models in the table, RWGTM represents the world with the most geographical detail, however it does not distinguish demand sectors within countries and also it has no capabilities of representing market power à la Cournot, both of which are features of the WGM. Relative to FRISBEE, the WGM offers three times the geographical detail and includes market power aspects. GASMOD, developed by DIW Berlin and GASTALE, developed by Energy Research Center of the Netherlands do implement MCP, thereby allowing an adequate representation of market power.[31] Both models' coverage is limited to the European natural gas market. In contrast, the WGM has global coverage and includes Europe in similar (GASTALE) or more detail (GASMOD) compared to these two models. The last two models in the table, GRIDNET and GMM provide much detail for the U.S. gas market.[32] They are designed for decision support by natural gas businesses with a short to medium-term time-horizon. This type of short-term operational model cannot provide the type of market analysis for which the WGM was designed and do also not provide the global coverage desired. Table 1 provides an overview with more information for all these models.

1.7.2 Contributions of this research

The models presented in this dissertation are large-scale game theoretic models that address both the increasing complexity and the increasing uncertainties in the natural gas market. The resulting model sizes potentially induce large calculation times, an issue that needs to be addressed.

- The first contribution of this research is the development of a representative global natural gas market model that can satisfactorily address relevant policy issues. This model, the World Gas Model, is unique in the combination of:
    - The level of detail wherein market agents are incorporated.
    - The level of detail wherein the transport options are included.
    - The global coverage and depth of the regional coverage.
    - The multi-period approach with endogenous capacity expansions.

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[31] GASMOD: (Holz et al., 2008) and (Holz, 2009). GASTALE: (Lise and Hobbs, 2009)

[32] www.rbac.com, Brooks, Robert E. and C.P. Neill.2010. GRIDNET: Natural Gas Operations Optimizing System. http://rbac.com/Articles/GRIDNETNaturalGasOperationsOptimizingSystem/tabid/67/Default.aspx (Accessed March 2010). ICF International. 2009. GMM, Model Overview. ICF International.2010. Gas Market Model http://www.icfi.com/markets/energy/doc_files/nangasweb.pdf (Accessed March 2010).

    - The inclusion of multiple seasons and storage facilities.
    - The representation of market power.
- A second major contribution of this research is the development of a large-scale stochastic natural gas market model that can adequately address input parameter uncertainty and allow market agents to hedge their decisions. The stochastic model is applied to a problem with four scenarios for the global natural gas market for a time horizon until 2050. The problem contains nineteen geographical regions and includes 78,768 variables. The largest stochastic MCP solved, contains eight scenarios for the period up to 2040, having 117,481 variables and solving in just under 5¼ hours on a dual core 2x1.2 GHz, 2GB computer.
- A third major contribution is the application (i.e., the adjustment, extension and implementation) of a Benders decomposition approach for large-scale stochastic mixed complementarity models, thereby addressing the so-called *curse of dimensionality*. Computational issues prevented the successful solution of the largest problems tried.

Chapters 2 through 4 discuss the first contribution, Chapters 5 through 7 the second and third contribution. Chapter 2 provides an overview of the literature relevant for natural gas market modeling. In Chapter 3 the various actors in the global natural gas market are introduced and discussed and the mathematical formulation of the WGM is presented. Chapter 4 provides the results of some numerical case studies with the WGM. Chapter 5 provides another literature overview, addressing stochastic modeling approaches and solution approaches to large-scale problems. In Chapter 6 a stochastic natural gas market model is presented and applied to a stochastic problem and Chapter 7 presents a Benders decomposition approach for stochastic mixed complementarity problems.[33]

The following chapter will provide an overview of existing literature for game theory and natural gas market modeling.

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2 Literature Review

In this section literature relevant for natural gas market modeling is presented. Some mathematical concepts and notation are introduced as well as concepts from game theory. In later chapters more literature relevant for specific sections is presented, and Chapter 5 provides a literature overview specific to stochastic modeling. This chapter provides a background for the research proposed, and should provide a stepping stone to the description of a full-scale deterministic natural gas market model in the next chapter.

2.1 Some mathematical concepts

In an economic model with multiple goods $\{q_1, q_2, .., q_m\}$ it is convenient to have a short notation. Vector notation provides this. A vector $\bar{q} = \begin{pmatrix} q_1 \\ q_2 \\ ... \\ q_m \end{pmatrix}$ represents all the *m* goods. In text it is generally more convenient to use the transpose of a vector: $\bar{q}^T = \{q_1, q_2, .., q_m\}$. Then, for example to write the total revenues $\sum_{i=1}^{m} p_i q_i$ of a company selling *m* products q_i at prices p_i, the following vector multiplication provides the succinct expression: $\bar{p}^T \bar{q}$, which is often just written as $p^T q$ when it is clear that p and q are vectors.

A matrix is an array of numbers. For example, to denote the prices for two goods in a three-period model, the following matrix with two rows and three columns can be used: $P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{pmatrix}$.

Function $f: \mathbb{R}^n \to \mathbb{R}$ is linear, if $\forall x, y \in \mathbb{R}^n, \forall \lambda, \rho \in \mathbb{R}: f(\lambda x + \rho y) = \lambda f(x) + \rho f(y)$. A function $g: \mathbb{R}^n \to \mathbb{R}$ is affine, if there are a linear function $f: \mathbb{R}^n \to \mathbb{R}$ and a vector $b \in \mathbb{R}$ such that: $g(x) = f(x) + b$. A function $f: \mathbb{R}^n \to \mathbb{R}$ is convex if

$\forall x, y \in \mathbb{R}^n, \lambda \in [0,1]: \; f(\lambda x + (1-\lambda) y) \le \lambda f(x) + (1-\lambda) f(y)$.

If $\forall x, y \in \mathbb{R}^n, \lambda \in (0,1): \; f(\lambda x + (1-\lambda) y) < \lambda f(x) + (1-\lambda) f(y)$, f is strictly convex (Nash and Sofer, 1996). A function $f(x)$ is concave, if and only if $-f(x)$ is convex. A function that is both convex and concave is affine.

A region or set *S* is convex if: for any two points *x* and *y* in *S*, any convex combination $\lambda x + (1-\lambda) y, \lambda \in [0,1]$ is also in the set: $\forall x \in S, y \in S, \lambda \in [0,1] \Rightarrow (\lambda x + (1-\lambda) y) \in S$ (Nash and Sofer, 1996). A problem with a convex objective and a convex feasible region is a convex programming problem.

An ε-neighborhood around a point $x \in E$ is the set $N_\varepsilon(x) = \{y : \|x - y\| < \varepsilon\}$. A point $x \in S \subset E$ is in the closure of S, $x \in cl\, S$, if $S \cap N_\varepsilon(x) \ne \varnothing, \forall \varepsilon > 0$. If $S = cl\, S$, then S is closed. If there is a ball with a large enough radius that can contain S, then S is bounded. A set *S* that is closed and bounded is compact (Bazaraa et al., 1993). A set defined by a finite number of linear constraints is a polyhedral set, or a polyhedron (Nash and Sofer, 1996). A (square) matrix $M \in \mathbb{R}^{n \times n}$ is positive semi-definite if $x^T M x \ge 0, \forall x \in \mathbb{R}^n$. If $x^T M x > 0, \forall x \in \mathbb{R}^n \setminus \{0\}$, then M is positive definite (Cottle et al., 1992).

The gradient (vector) of a real-valued function $h : \mathbb{R}^n \to \mathbb{R}$ is the vector of first order (partial) derivatives $\nabla h(x) = \begin{pmatrix} \frac{\partial h(x)}{\partial x_1} \\ \frac{\partial h(x)}{\partial x_2} \\ \vdots \\ \frac{\partial h(x)}{\partial x_m} \end{pmatrix}$. A stationary point of a function is a point $(x_1, x_2, ,.., x_m)^T$ for which the gradient is 0: $\nabla h(x) = 0$ (Nash and Sofer, 1996).

A collection of real-valued functions $h_1, h_2, ..., h_m$ can succinctly be written as a vector function h, whose k^{th} component is h_k. The Jacobian of h is: $\nabla h(x) = \begin{bmatrix} \nabla h_1(x) \\ \nabla h_2(x) \\ \\ \nabla h_m(x) \end{bmatrix}$, a matrix whose rows contain the gradients of $h_1, h_2, ..., h_m$ (Bazaraa et al., 1993). The Hessian $\nabla^2 h(x)$ is the matrix of second order partial derivatives. Entries of this matrix can be written as: $\left[\nabla^2 h(x)\right]_{kl} = \frac{\partial^2 h(x)}{\partial x_k \partial x_l}$ (Bazaraa et al., 1993).

Optimization is solving problems to find an optimum (minimum or maximum) of a function (the objective) for a specified set of allowed values, the feasible region. If there is just one point in the feasible region for which the objective function takes on its optimum, the problem is said to have a unique solution. It is also possible to have multiple optima or no optimum at al. There are several reasons for problems to not have a solution. The feasible region may be empty or not compact and some functional forms of the objective (e.g., $\frac{1}{x}$ or $\ln(x)$) are not bounded on regions containing the value *0*. Generally every optimization and many economic problems can be formulated as follows: $\min_{x \geq 0} h(x), \forall x \in S \subset \mathbb{R}^n$, $h: \mathbb{R}^n \rightarrow \mathbb{R}$ convex and S compact.[34] If $S \neq \varnothing$ (non-empty) there is at least one optimal solution. If $S \neq \varnothing$ and $h(x)$ is strictly convex, the problem has a unique solution. (Cf., necessary and sufficient optimality conditions in (Bazaraa et al., 1993)). If the objective function and equations (constraints) specifying the feasible region are all affine, the optimization problem is a linear program (LP). A problem with an objective of the form $c^T x + x^T Mx$ is a quadratic program (QP).

Every LP can be written in matrix notation as: $\min_{x \geq 0} c^T x$. s.t. $Bx - d \geq 0$ ($x \in \mathbb{R}^n, c \in \mathbb{R}^n$, $B \in \mathbb{R}^{nxm}$ and $d \in \mathbb{R}^m$). When an LP is optimally solved, beside the optimal values for *x*, and $c^T x$, also a set of values related to the constraints is determined. These values are the

[34] Maximization problem $\max_{x \geq 0} h(x)$ can be written equivalently as minimization problem $\min_{x \geq 0} -h(x)$.

dual variables also known as dual prices, shadow prices or Lagrange multipliers. Every LP has an associated problem, the dual problem, with an objective function and feasible region stated in terms of the dual variables. The dual of the above LP can be written as: $\max_{y\geq 0} d^T y$ s.t. $B^T y - c \leq 0$. To distinguish the original LP and the dual LP, the original LP is referred to as the primal.

The solutions to the primal and the dual are very much related through the Weak and Strong Duality Theorems (Nash and Sofer, 1996) and the Complementarity Slackness Conditions (Bertsimas and Tsitsiklis, 1997). Weak duality is the characteristic that for any two feasible points *x* and *y* for the above primal and dual problems, $c^T x \leq d^T y$. Strong duality is the characteristic that if one of the two of primal and dual problems has an optimal solution, so does the other, and $c^T \hat{x} = d^T \hat{y}$ for the optimal solution vectors $\hat{x}$ and $\hat{y}$. The Complementarity Slackness Conditions can be seen as a special case of the mixed complementarity problem (MCP), the approach implemented in this dissertation. The Complementarity Slackness Conditions state that if for some specific vectors *x* and *y* the following conditions are true: $x^T\left(c - B^T y\right) = 0$ and $y^T\left(Bx - d\right) = 0$ for x, y *feasible* ($x \geq 0$, $y \geq 0$, $Bx - d \geq 0$ and $B^T y - c \leq 0$) then *x* and *y* are optimal solutions to the primal and dual problems.

In economic problems the dual variables of constraints often have a very intuitive interpretation. For example, for resource constraints the shadow prices are the marginal values of the resources. If in an optimal solution the resource is not fully used (there is slack), the dual price is zero. But if it is fully used, and having more of the particular resource would allow for a better solution, the dual price indicates how much it would be worth to obtain more of that resource.

For a problem $\min_{x\in X}$ f(x) s.t. $k\left(x\right) \leq 0$ and $l\left(x\right) = 0$ ($X \subset \mathbb{R}^n$ and $k, l : \mathbb{R}^n \rightarrow \mathbb{R}$) with *v* and *w* as dual variables, the Karush-Kuhn-Tucker (KKT) conditions are:

stationarity:[35] $\nabla f(x)+\sum_{i=1}^{n} v_i \nabla k_i(x)+\sum_{j=1}^{m} w_j \nabla l_j(x)=0$

primal feasibility: $k(x)\leq 0$ and $l(x)=0$

nonnegativity of multipliers: $v \geq 0$

complementarity slackness: $v^T k(x)=0$.

The combination of $v \geq 0$, $k(x)\leq 0$ and $v^T k(x)=0$ is usually abbreviated to $0 \leq v \perp k(x) \leq 0$.[36]

For some types of problems, KKT conditions are necessary or sufficient for an optimal solution. Constraint qualifications (CQ) are mathematical properties to problems that guarantee that every KKT point provides an optimal solution. In (Bazaraa et al., 1993) several CQ are discussed that are useful for the type of models developed in this dissertation. The objective functions to be minimized by model agents are convex and twice differentiable. All feasible regions are polyhedral, specified by affine inequalities and linear equality conditions. For such problems, with a feasible region defined by linear constraints KKT are necessary for optimal solutions, independent of the functional form of the objective. For minimization problems with convex objectives and a feasible region defined by convex inequalities and affine equality conditions KKT points are sufficient for global optimality. Thus, KKT conditions are necessary and sufficient for optimal solutions for the models in this dissertation.

A linear complementarity problem (LCP) for a vector *b*, matrix *A* and variables x is to find x such that: $0 \leq x \perp Ax+b \geq 0$ (Cottle et al., 1992). In nonlinear complementarity problems (NCP) the expression $Ax+b$ can be replaced by nonlinear functions. MCP are a generalization of NCP, allowing for other than zero lower bounds ($\bar{l} : l_i \in \mathbb{R} \cup -\infty$) as well as upper bounds ($\bar{u} : u_i \in \mathbb{R} \cup +\infty$) to the decision variables. For an MCP with

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[35] Instead of stationarity, also *dual feasibility* is used.

[36] In this dissertation $0 \leq x \perp -k(x) \geq 0$ will be used since the modeling tool GAMS does not allow the $\leq$ variant and we prefer to have the code and the mathematical formulation consistent with each other. In some cases this affects how the sign of the free dual variable values should be interpreted.

(nonlinear) function $F:\mathbb{R}^n \rightarrow \mathbb{R}^n$, a vector $x \in \mathbb{R}^n$ must be found for which for each

element x_i: $\begin{array}{lll} a. & l_i = x_i & \Rightarrow F_i(x) \geq 0 \\ b. & l_i < x_i < u_i & \Rightarrow F_i(x) = 0 \\ c. & x_i = u_i & \Rightarrow F_i(x) \leq 0 \end{array}$

Another generalization of the NCP is the variational inequality (VI). For a function $h:\mathbb{R}^n \rightarrow \mathbb{R}^n$ find $x \in P$ such that: $h(x)^T(p-x) \geq 0, \forall p \in P$. When $P = \mathbb{R}_+^n$, the nonnegative orthant of a Euclidean space, a solution to the VI is equivalent to the solution of the NCP $0 \leq x \perp h(x) \geq 0$ (Cottle et al., 1992). There are more combinations of functions and feasible regions for which the VI problem is identical to an NCP, for instance when h is affine and P is polyhedral (Cottle et al., 1992). The model presented in Chapter 3 is an MCP, since the market-clearing conditions cannot be included in an NCP without loss of generality.

2.2 Game theory

A game has three elements: players $p \in P$, strategies of all players: $s_p \in S_p$, and payoff functions $u_p(s_p, s_p^-) : (S_p, S_p^-) \rightarrow \mathbb{R}$ that depend on the own strategy s_p and the strategies executed by the other players s_p^- (Fudenberg and Tirole, 1991). Players are the entities in the game that make decisions. They decide upon their optimal course of action, i.e., what strategy to execute. The payoff functions state for each player the benefit resulting from their strategy choice given the chosen strategies of all other players. In an economic context the players can include producers and consumers; and strategies the possible production and consumption levels. For instance, the payoff function for a producer is his profit level, the payoff function for the consumers is their consumer surplus. If the producers would decide to collaborate and maximize their aggregate profits, the resulting game would be a cooperative game; the game as previously described where all agents maximize their own payoff is non-cooperative. A game containing both cooperative and non-cooperative aspects is a hybrid game.

In an economic market model equilibria are points where supply and demand are equal. The game concept does not apply to (micro)economics only, and there are various equilibrium concepts for games. A concept often used in microeconomics is the Nash equilibrium. In a Nash equilibrium all players choose a strategy $\hat{s}_p$ that maximizes their payoff given the anticipated responses of all other players s_p^-. No player would benefit from changing his strategy unilaterally: $\forall p \in P: u_p\left(\hat{s}_p, s_p^-\right) \geq u_p\left(s_p, s_p^-\right), \forall s_p$ (Nash, 1951). Often, perfect information is assumed in game-theoretic models, as well as rationality, two rather intuitive concepts, which allow for accurate forecasts for other players' behavior. When all players communicate their (irreversible) decision at the same moment, the game is said to be a simultaneous game; otherwise it is a sequential game (e.g., the Stackelberg game, see the next section). A sequential game necessarily has more than one stage; however a simultaneous game can consist of multiple stages too. Games with several stages are dynamic games; one-period games are static games. There can be different information structures in dynamic games. In multi-stage games with a closed-loop information structure, or feedback strategies, at every stage, players consider former strategy decisions and outcomes when choosing a course of action. In contrast, in open-loop equilibria all decisions for all stages are set at the start of the game. Although the assumptions underlying the open-loop games are more restrictive than for closed-loop games, the resulting models are generally mathematically tractable. Therefore the open-loop analysis is used very often in analyses of long-term equilibria and trends.

In games, there may be no solutions, just one, or more than one (see (Nash, 1951), (Debreu, 1952), (Arrow and Debreu, 1954) and (Rosen, 1965)). Two well-known theorems that provide a basis for the existence of equilibria (e.g., for the World Gas Model) are the following. The Frank-Wolfe theorem: if $h \in C^0$ (continuous), $h: \mathbb{R}^n \rightarrow \mathbb{R}$, quadratic and bounded below on the polyhedral feasible region $P \neq \varnothing$ ($P \in \mathbb{R}^n$), then h attains its minimum on P (Cottle et al., 1992). As is illustrated in the next section, an equilibrium in a perfectly competitive market with quadratic costs can be calculated by maximizing social welfare. As long as all constraints, such as production capacity and pipeline flow limitations are linear, we have a polyhedral feasible region and the Frank-Wolfe theorem warrants that there is at least one feasible solution point that is optimal. As long as there are no lower bounds other than zero included in the model, the feasible region will always contain the zero vector for all primal variables, and therefore never be empty. Strict convexity of the (quadratic) social welfare function guarantees that the solution point is unique.

Since in some of our models we apply a non-quadratic functional form for production costs, the Frank-Wolfe theorem cannot always be applied. The Weierstrass theorem applies to a broader class of functions. If $h \in C^0$, $h: \mathbb{R}^n \to \mathbb{R}$, and $P \neq \emptyset$ and compact, then the problem [$\min h(x)$ s.t. $x \in P$] has a solution (Bazaraa et al., 1993). Note that this theorem guarantees the existence of a solution, but does not say anything about its uniqueness.

2.3 Economic market modeling

When designing economic market models various choices must be made regarding the market structure. The following picture is an adjusted version from (Shy, 1995).

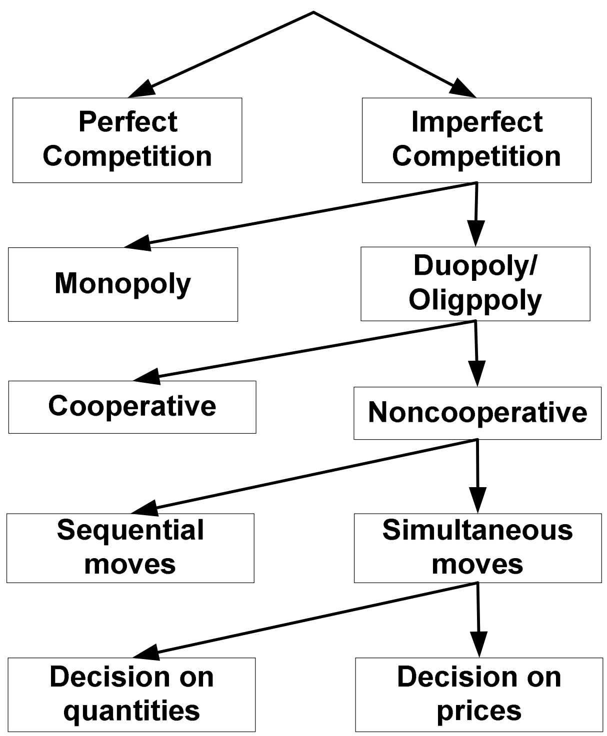

Figure 5: Market structures (Shy, 1995)

It distinguishes features according to the type of interaction among players, the number of players, the order of decision making, and whether quantities or prices are set by the suppliers. For example, an imperfectly competitive market, with multiple non-cooperative suppliers, who decide simultaneously on output quantities, is an oligopoly à la Cournot. The potential monopoly profit is often larger than the sum of the profits of suppliers in a Cournot oligopoly (e.g., the following example, where the monopoly profit of $42\frac{1}{4}$ is more than the aggregate duopoly profit of $2\times18\frac{7}{9}=37\frac{5}{9}$). Thus, there is an incentive for suppliers to collaborate and form a cartel.

Another market structure that is often assumed in market models is perfect competition, a concept first described by Walras in the late 1800s, wherein all market agents are price takers and cannot manipulate the market prices (Walras, 1977). In a perfectly competitive market the market equilibrium can be found by maximizing social welfare: the sum of profits of all players plus the consumer surplus (Bergson 1938).

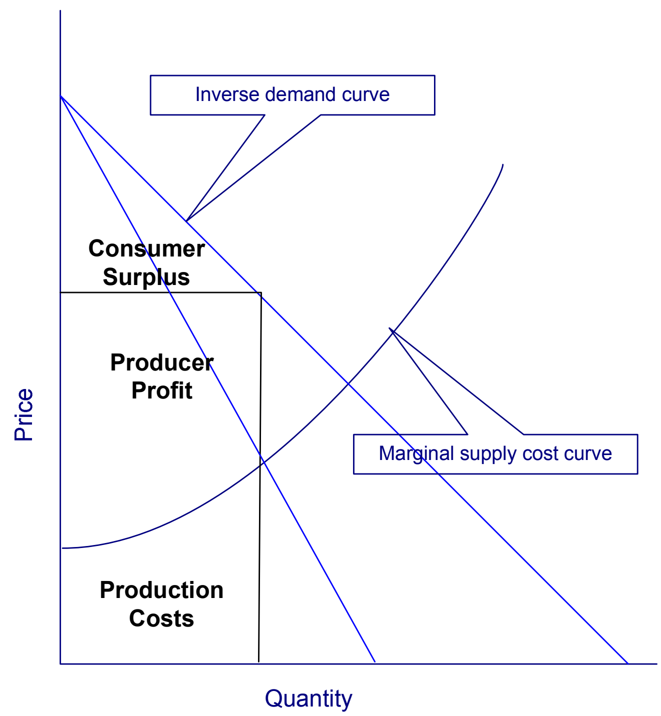

Figure 6: Social welfare components in a market equilibrium

As an example of the concept of some of the relevant economic concepts, assume a producer who is selling some commodity q to a market with a demand curve $q=15-p$. Commonly, when plotting demand in a graph, the inverse demand curve is used: $p=15-q$. Production costs are \$2 per unit. The consumer surplus can be expressed as $\frac{1}{2}(15-p)q$ =

$\frac{1}{2}q^2$. The social welfare is the sum of producer profit $pq-2q = (13-q)q = 13q-q^2$ and consumer surplus $\frac{1}{2}q^2$. To obtain a perfectly competitive market equilibrium the expression to be maximized is $13q-q^2+\frac{1}{2}q^2=13q-\frac{1}{2}q^2$. Setting the derivative equal to zero $13-q=0$ gives the optimal quantity: $q=13$.

Now assume that the producer realizes he is the only supplier to the market, and that his supply affects the market price. Hence, he is a monopolist, and ignores the consumer surplus when determining his optimal production level. The producer's optimization problem is to choose a nonnegative value for q that maximizes the quadratic expression $13q-q^2$. Setting the derivative equal to zero, reveals that the optimal quantity is $q=6\frac{1}{2}$, exactly half of the perfectly competitive supply, and a profit of $\left(15-2-6\frac{1}{2}\right)\times 6\frac{1}{2}=42\frac{1}{4}$.

For perfectly competitive and monopolistic market models with convex quadratic objective functions and affine, downward-sloping inverse demand curves the equilibrium can always be found through optimization. However for other types of markets, such as oligopolies, optimization cannot be used to adequately model them. The following is an example for modeling an oligopoly. In this case, the solution can be derived analytically, using symmetry of the market players.

Assume that there are two producers, identical to the one in the previous example, who are competing à la Cournot. Each producer i chooses a quantity q_i that maximizes the quadratic expression $pq_i-2q_i = (13-q_1-q_2)q_i$. To solve this for producer 1: $\max\left[(13-q_2)q_1-q_1^{\,2}\right]$, set $\frac{\partial\left[(13-q_2)q_1-q_1^{\,2}\right]}{\partial q_1}=0 \;\Rightarrow\; 13-q_2-2q_1=0 \;\Rightarrow\; q_1=\frac{13-q_2}{2}$.[37] Using symmetry of the producers to determine the solution: $q_1=q_2=4\frac{1}{3}$, and each producer

[37] We need to assert convexity of the minimization objective. Maximization of pq_i-2q_i is equivalent to minimization of $2q_i-pq_i$. Substituting in the inverse demand curve for p and taking the first partial derivative with respect to q_1 results in $2q_1+q_2-13$. The Hessian of $\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$ is positive definite, hence the objective function is strictly convex.

makes a profit of $\left(15-2-2\times4\frac{1}{3}\right)\times4\frac{1}{3}=18\frac{7}{9}$. The expression $q_1=\frac{13-q_2}{2}$ is known in the literature as the <u>optimal response curve</u> or <u>(Cournot) reaction curve</u>. It shows for each supply level of the competitor how much a firm should supply. For a duopoly of suppliers the optimal response curves can be drawn in a two-dimensional picture and used to derive the market equilibrium point, as is shown in Figure 7.

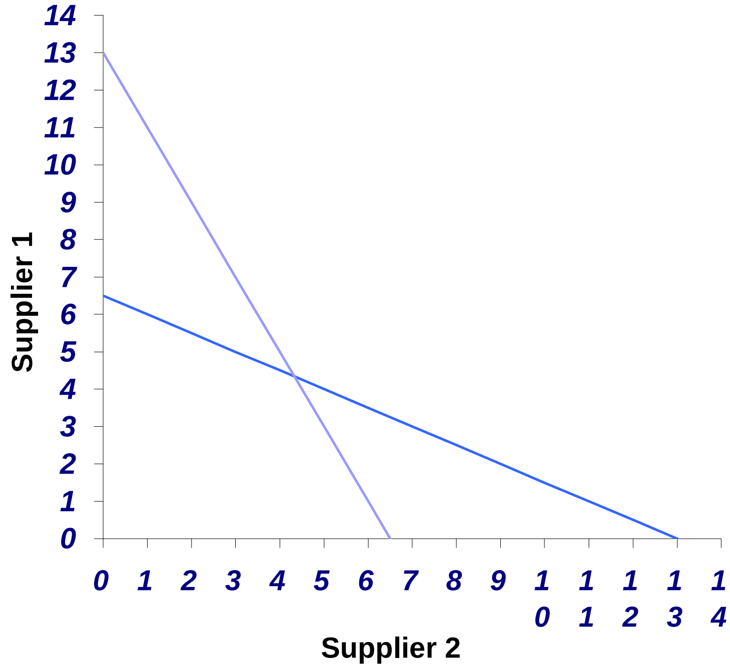

Figure 7: Duopoly market equilibrium using optimal response curves

Adding more producers to the problem with identical supply costs leads to a quadratic optimization objective for every producer of the form: $pq_i-2q_i=\left(13-\sum_j q_j\right)q_i$. Setting the partial derivative equal to zero: $13-\sum_j q_j-q_i=13-\sum_{j\neq i} q_j-2q_i=0$ results in the following expression for the optimal supply quantities in an n-firm oligopoly market: $q_i=\frac{13}{n+1},\forall i$.

We see that for this stylized example the market equilibrium can be determined analytically, and the outcomes are closed-form expressions. In a more general setting, such as the natural gas market model that we develop, with asymmetric costs, multiple supply and demand nodes, pipeline capacity restrictions and other complications, the complete system of equations can generally not be solved analytically. Unlike in the perfectly competitive market structure that maximize social welfare minimize (supply) costs, the Cournot oligopoly solution generally does not minimize the supply costs. All competitors each maximize their own objective function, which cannot be aggregated and represented as one optimization objective; therefore a convex programming approach or any other optimization approach cannot be used to find the market equilibrium. For such markets where market power plays a role, complementarity problems provide a viable modeling approach. For example, the above duopoly can be cast as the following LCP, consisting of KKT conditions for the profit of the suppliers:[38]

$$\begin{matrix} 0 \le q_1 \perp \frac{\partial\left[(13-q_2)q_1-q_1^2\right]}{\partial q_1} \ge 0 \\ 0 \le q_2 \perp \frac{\partial\left[(13-q_1)q_2-q_2^2\right]}{\partial q_2} \ge 0 \end{matrix} \Leftrightarrow \begin{matrix} 0 \le q_1 \perp 13 - q_2 - 2q_1 \ge 0 \\ 0 \le q_2 \perp 13 - q_1 - 2q_2 \ge 0 \end{matrix}$$

or in matrix notation LCP(b,M)

$$0 \le q \perp q^T \left(b + Mq\right) \ge 0 \text{, with } b^T = \begin{pmatrix} 13 & 13 \end{pmatrix} \text{ and } M = \begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}$$

An equivalent variational inequality for this problem is to find $\left(\tilde{q}_1, \tilde{q}_2\right) \in \mathbb{R}_+^2$ s.t. $\left(13 - q_2 - 2q_1 \quad 13 - q_1 - 2q_2 \ge 0\right) \begin{pmatrix} \tilde{q}_1 - q_1 \\ \tilde{q}_2 - q_2 \end{pmatrix} \ge 0$ for all $\left(q_1, q_2\right) \in \mathbb{R}_+^2$

Some markets are characterized by a dominant player and a fringe of followers. For example, some references argue that the oil market operates this way, with OPEC as the dominant player (e.g., (Al-Qahtani et al., 2008)). This leader-follower market structure is known as a Stackelberg game. A characteristic of this market structure is the sequential nature. In the first stage the leader decides on his output level; whereas in the second stage the followers decide on theirs. The leader is assumed to have full insight in the followers' willingness to supply and uses this information when setting his optimal output level. In the previous two-producer example, this information can be summarized in the optimal response curve. The following example shows the Stackelberg equilibrium where both leader and follower exert market power à la Cournot. Note that typically the

[38] See footnote 37 for a short proof that the Hessian of *–M* is positive definite, asserting concavity of the individual players maximization problems.

leader would have another cost structure than the follower, but in this example the same data as before are used.

In the first stage the leader maximizes the following expression: $(13 - q_1 - q_2)q_1$, anticipating the second stage response: $q_2 = \frac{13-q_1}{2}$. Substituting in the anticipated response in the objective function gives the following: $\left(13 - q_1 - \frac{13-q_1}{2}\right)q_1 \Leftrightarrow \left(\frac{13}{2} - \frac{3}{2}q_1\right)q_1$. Setting the first derivative equal to zero gives as the optimal supply quantity: $q_1 = \frac{13}{6} = 2\frac{1}{6}$, and for the follower: $q_2 = \frac{13-q_1}{2} = \frac{13}{2} - \frac{q_1}{2} = \frac{78}{12} - \frac{13}{12} = \frac{65}{12} = 5\frac{5}{12}$. The total supply to the market is: $\frac{65}{12} + \frac{26}{12} = \frac{91}{12} = 7\frac{7}{12}$, which is lower than the $8\frac{2}{3}$ in the previously shown duopoly results.

In general this type of multi-stage models cannot be formulated as complementarity problems, and a whole class of more general problems has been developed to model them; the mathematical problems (or equilibrium problems) with equilibrium constraints (MPEC/EPEC). See, e.g., (Luo et al., 1996) or the next Section 2.4.

2.4 Natural gas market modeling

One of the main purposes for developing natural gas market models has been to analyze the impact of policy and infrastructure developments on markets and consumers. Stoner (1969) may have been the first to present work on modeling a natural gas system. The early 1980s energy market liberalization efforts in the United States and the United Kingdom required politicians and regulators to gather information and boosted the development of quantitative models. A second boost came when in the late 1980s the European Commission started privatization and liberalization policy of the electricity market, and in the 1990s the natural gas markets (EC, various years).

The first market modeling efforts seem to have been executed by American researchers, for the North American natural gas market. Early work on natural gas market modeling with the direct objective to support policy development can be found in (O'Neill et al., 1979). That work had as main objective to reassign available gas supply to consumers in case of emergencies. Their model was solved approximately, linearizing non-linear equations and applying an iterative (modified Newton) method. Another model, the National Energy Modeling System (NEMS), and previous, related models from the U.S. Department of Energy contain a separate sub-system for modeling the U.S. natural gas market, the Natural Gas Transmission and Distribution Module (NGTDM) (Gabriel et al., 2001), (International Energy Agency, 1994). The NGTDM consists of various modules, including some for demand sectors, the supply side and conversion/transmission. An iterative approach (nonlinear Gauss Seidel) is used to solve the NEMS.

New developments in mathematical formulations and computer software have allowed for the representation of the specifics of actual markets in a single equilibrium framework. Developments and applications on the North American natural gas market include optimization-based equilibrium models (Gabriel et al., 2000, 2003) and mixed complementarity problems (Gabriel et al., 2005a, 2005b). A big advantage of the latter model types is assessing the impact of market power in a Nash–Cournot setting.

Another market, for which several models have been developed over time, is the European market. Haurie et al. (1987) developed a stochastic Nash–Cournot model. Mathiesen et al. (1987) investigated market power on the selling side of the European natural gas market. Another modeling approach was taken in (De Wolf and Smeers, 1997). They developed a two-stage stochastic Stackelberg game for the European gas market with one producer (Norway) as the leader and the others (Russia, Algeria, Netherlands and the U.K.) as followers. A stochastic approach was also developed by (Gürkan et al., 1999). They developed a Monte Carlo simulation based method to solve stochastic variational inequalities. Boots et al. (2004) constructed Gas mArket System for Trade Analysis in a Liberalizing Europe (GASTALE). That model, based in part on the work by (Golombek et al., 1995, 1998) used a successive oligopoly perspective. In further developments of GASTALE, Egging and Gabriel (2006) let go of the successive oligopoly approach. They added features like demand seasonality, a storage sector and transmission pipeline capacities. These extensions removed the possibility to have market power at two levels in the market, since the closed-form expressions needed to solve the model with double marginalization could not be derived anymore. The model agents in (Egging and Gabriel, 2006) included producers, a transmission system operator as well as storage operators and allowed for market power exertion in the interaction between producers and demand sectors. Various cases were analyzed with a focus on market power exertion in the European gas market. Egging et al. (2008) presented a new MCP for the European gas market with more detail than GASTALE. The trader was separated from the producer and the LNG supply chain was represented as liquefiers, shipping and regasifiers. The case studies included two disruption cases to illustrate the dependencies of Europe on Russian and Algerian supplies. Egging et al. (2010) further developed the model to the first version of the World Gas Model (WGM-2008). This model covered the whole world, multiple periods, a detailed representation of the LNG supply chain and allowed endogenous infrastructure expansions. Egging et al. (2009) implemented the model to study the impact of the coming into existence of a global gas market cartel. Two cases were studied relative to a reference scenario. The first case mimicked a cartel according to GECF membership.[39] In a second case, production capacities of cartel members were kept at 2005 levels throughout the time horizon, resulting in an about 50% decreased output by 2030 relative to the first cartel case. When implementing the cases some model limitations showed with regard to the representation of a cartel.[40] These limitations have been addressed when developing the model in this research. Egging et al. (2009) was a contribution to a Special Issue of The Energy Journal Vol. 30 as an outcome of the Energy Modeling Forum (EMF) 23.[41] Although with some limitations, among the participating models in EMF 23 the model version WGM-2008 in (Egging et al., 2009) and (Egging et al., 2010) was best suited to implement a global gas market cartel in a hybrid market setting with a fringe of Cournot and competitive players.

Continued GASTALE development in (Lise et al., 2008) and (Lise and Hobbs, 2008) addressed capacity expansions in a multi-period model. Other recent models for the European market include NATGAS (Mulder and Zwart, 2006) and GASMOD (Holz et al., 2008) and (Holz, 2009). Gabriel and Smeers (2006) provided a broad overview of natural gas market equilibrium models and provide suggestions for further mathematical approaches to address relevant issues in the natural gas markets. Although complementarity problems have clear advantages to simulate market power for certain

[39] Gas Exporting Countries Forum, gecforum.org.
[40] Due to the separate coordination of pipeline and LNG export.
[41] http://emf.stanford.edu/research/emf23/; http://emf.stanford.edu/files/pubs/22377/EMF23ReportWeb.pdf

types of analysis, optimization models may still be the method of choice. Tomasgard et al. (2007) in (Hasle et al., 2007) described optimization approach for various steps in the gas supply chain, and provide a stochastic modeling approach to address uncertainty.

An important consideration in natural gas markets is the finiteness of the resource. Not only daily production capacities are limited, but also the total production over time. Hotelling (1931) discussed the optimal depletion paths of exhaustible resources. His key result was that an optimal depletion path induces prices that, corrected for discount rates, are constant over time. The rationale is that if prices would move differently, it would be worthwhile to shift production between periods. This result is known as the Hotelling rule. Several recent publications that address finiteness of resources include (De Joode, 2003), who addressed depletion of gas reserves in the interaction between Russia and Europe. Benchekroun et al. (2006) analyzed different scenarios for how the threat of a forced break-up of a cartel impacts the extraction rate of the resource by the cartel in the cartel period. In honor of the 75th birthday of Hotelling's paper, Gaudet (2007) discussed many implications of the Hotelling rule in the present world. Zwart (2008) elaborated on the interplay between market power exertion and natural gas depletion in the European natural gas market.

The previous subsection (2.3) introduced the Stackelberg equilibrium in a two-stage game with market power. De Wolf and Smeers (1997) developed and applied a stochastic two-stage game for the European Gas Market with Norway as the leader and other suppliers as followers. Hobbs et al. (2000) developed an MPEC for an electricity market with a first stage wherein one or more individual firms decide on their supply *bid curves*; and in the second stage an integrated systems operator clears the electricity market through the profit maximization resulting from purchases from suppliers, deliveries to consumers, and prices.[42] Gabriel and Leuthold (2010) presented a two-stage model with discrete first state variables and developed a general solution method that transforms the model into a larger, but more easily solved, mixed integer program. They presented computational results for a fifteen node network covering the Netherlands, Belgium, and the French and German border regions.

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[42] A bid curve is an upward sloping curve indicating the willingness to supply for each price level.

Several papers developed solution procedures for MPEC without specific applications. Some approaches have used penalization and relaxation strategies. Penalization approaches remove complementarity conditions $xy = 0$ from the model restrictions, and add a penalty term to the objective that accounts for how much xy deviates from zero. DeMiguel et al. (2006) developed a two-sided relaxation scheme. The starting point was to reformulate the MPEC as a standard nonlinear program (NLP) by replacing the complementarity conditions by a set of smooth constraints. Doing so, the feasible region of the resulting NLP has no strictly interior points and as a consequence constraint qualifications are violated, and thereby the means are lost to check if found stationary points are optimal. The authors referred to work of Scheel and Scholtes (2000) and gave optimality conditions for MPEC. To maintain the applicability of constraint qualifications, DeMiguel et al. developed a sequential NLP approximation approach to the MPEC, in such a way that the relaxed NLP have a strictly feasible interior, even in the limit. To solve the NLP they proposed the use of an interior point method, which is described in their paper. Gabriel et al. (2009) developed a Benders decomposition approach to solve two-stage problems with discrete constraints. The approach combines a Benders algorithm with a procedure to decompose the domain of the upper-level discrete variable to ensure that the otherwise possibly concave subproblems are convex.

The above illustrates that there are many modeling approaches that allow the representation of market power à la Cournot. Still, many researchers use optimization approaches for their market models. Although optimization approaches are valid to analyze perfectly competitive markets, markets where market power can be expected are not well represented by them. MPEC are a more general class of models than what is needed in this research; for our purposes it is not necessary to implement these technically and computationally challenging modeling approaches that need considerably long calculation times to solve representative data sets. The main natural gas market models in this dissertation are market equilibrium approaches, in the form of mixed complementarity models.

Smeers (2008) discussed several major challenges that should be addressed by modelers to support the European Commission in the development of a regulatory framework for the European natural gas market. The three core objectives for the internal gas market are: i. to increase competition, ii. security and iii. sustainability of the energy supply. Smeers analyzed the contributions that existing models can make, consistencies and inconsistencies between model results and proposed legislation, and the potential for new models to provide insights. Smeers discussed various markets that can be distinguished related to natural gas: production, transportation, storage, trade ('supply' in (Smeers 2008)) and the retail market. Some major shortcomings addressed by Smeers are: the simplicity of the demand representation, models that do not allow for fuel substitution, oil price linkage, vertical integration, not representing the entry-exit system for domestic gas networks, how market-power exertion in gas supply is represented, that market-power exertion in capacity markets is not represented at all, the use of congestion pricing for the use of infrastructure capacities and not addressing environmental policies and sustainability issues.

Smeers (2008) posed many challenges and potentially defined a path of future research for many years to come. The models and methods presented in this dissertation do address representativeness of natural gas market models by scaling up the geographical region covered by a single model. Also, uncertainty in future developments is addressed, however most issues posed by Smeers remain untouched.

The World Gas Model (WGM-2009) that is introduced next in Chapter 3 is an improved version of the one that was presented and applied in (Egging et al., 2009, 2010) and (Huppman et al., 2010). WGM-2009 is presented in a recently submitted paper (Gabriel et al., 2010) and used in studies for Resources For the Future and Statistics Norway as well as several conference papers.

3 A Multi-Period Natural Gas Market Model: The World Gas Model

In this section the World Gas Model (WGM), a deterministic multi-period MCP for the global natural gas market is introduced. In the model various economic roles are distinguished by countries and geographical regions. Each such region can have a producer, trader, storage operator and a transmission system operator, which is responsible for managing all transport options. To provide a framework for the following discussion, Figure 8 below illustrates the interactions between the market participants that are represented in the model.

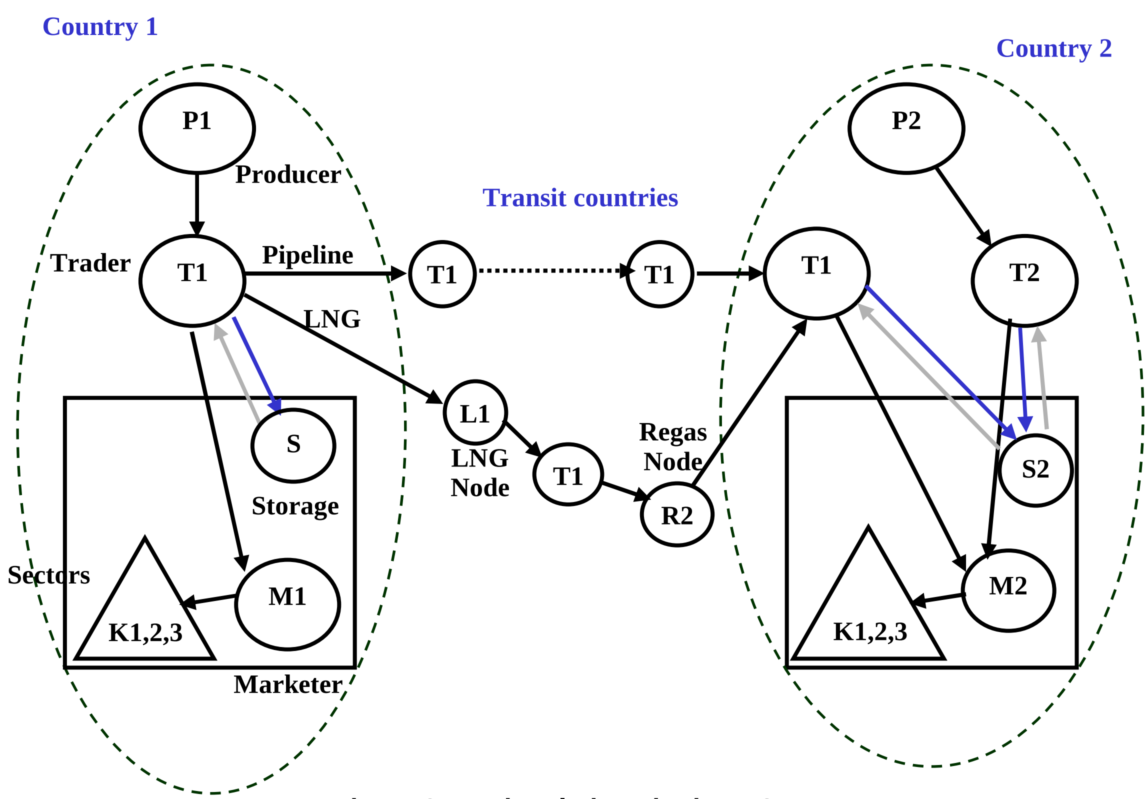

Figure 8: Trade relations in the WGM

Modeled market players are producers (P), traders (T), liquefiers (L), regasifiers (R), storage operators (S), marketeers (M) and several consumption sectors (K1, K2, K3). Producers sell gas to traders. Traders ship gas to consumer markets, domestically via distribution networks, or internationally via high pressure pipeline networks or LNG terminals, ships and regasifiers in other countries. Traders can make use of storage services to balance their flows among seasons.

The WGM is unique in the combination of the level of detail wherein market agents and transport options are incorporated, the global coverage and depth of the regional coverage, the multi-period approach with endogenous capacity expansions for transportation and storage infrastructure, addressing seasonality in the demand sector and how market power is represented.

We describe the players and some technical characteristics and economical roles in the natural gas market and how the characteristics and roles of the players are represented. The objective functions and constraints for the feasible regions are presented, as well as the Karush-Kuhn-Tucker (KKT) conditions, and the market-clearing constraints (mcc): the equations that tie the separate players' problems together into one MCP.

3.1 Introduction

Natural gas consumption and production can be found in most world regions. There are big differences between the regions though. For example, North America and Europe have well-developed gas pipeline systems to transport the gas from suppliers to consumers, possibly crossing several country borders on the way. In other parts of the world pipeline transmission systems are much less developed, and domestic distribution networks may only cover parts of the countries. About 70% of natural gas is used in the same country as where it is produced (BP, 2010). Of the remaining 30% about 50% is shipped internationally over relatively short pipelines, about 25% over long international pipelines and about 25% by LNG tankers (International Energy Agency, 2008).

The different aspects of the individual regions must be addressed when setting up a model. Infrastructure and market characteristics must be represented at an adequately detailed level to be able to draw useful conclusions. However much of the desired data is not (publicly) available, what puts limits to the level of detail that can be implemented.

Energy markets have many different types of agents and many possible interactions may occur among them. When formulating a model for an energy market many modeling decisions must be made regarding the representation of the actors and the technical and economical detail that can be represented. The focus in the modeling exercise presented in this chapter is on the market power aspects in the upstream market and the impact on production, consumption, traded volumes and prices. The emphasis in the model development is on the economic interactions prevalent in the natural gas market. Many technical aspects relevant for the natural gas market are addressed and discussed, however for tractability reasons many of them will not be incorporated in the actual model.

Before introducing the economic roles of all the players we will first start with an introduction of the fossil fuel that is the subject of production, trade and consumption in this dissertation.

3.2 Natural gas

Natural gas is a hydrocarbon consisting mostly of methane (CH_4), ethane (C_2H_6), some larger alkanes (C_xH_{2x+2}) and some components that are described in later sections. The existence and production of natural gas are linked to another hydrocarbon: oil, so describing the origins of natural gas means discussing hydrocarbons more generally. Most of the technical details are based on (Craft et al., 1991), however some web resources have been used as well.[43]

3.2.1 Hydrocarbons

Many million years ago dead organic material piled up on the bottom of the sea. Over time huge layers of sediments buried the organic material. Bacteria, pressure and heat degraded and decayed the organic material into fluid mixtures of hydrocarbons (C_xH_y). These mixtures of crude oil, natural gas and natural gas liquids (NGL) are nowadays denoted as petroleum.[44]

[43] www.metu.edu.tr/~kok/pete443.html; www.eia.doe.gov/neic/infosheets/natgassupply.html; www.naturalgas.org; http://fossil.energy.gov/education/energylessons/coal/gen_howformed.html; www.energy4me.org and www.most.gov.mm/techuni/media/PE_04025_13.pdf

[44] The terms petroleum and hydrocarbons can be used interchangeably. We will use the term hydrocarbons.

Natural gas is the part of the mixture that is gaseous at ambient temperature and under atmospheric pressure. In natural circumstances, when separated from crude oil after flowing out of a reservoir, natural gas contains water vapor, hydrogen sulfide, carbon dioxide, helium, nitrogen, and dissolved NGL such as: propane and butane.

Reservoirs with accumulated mixtures of hydrocarbons exist underground as subsurface porous sedimentary rocks, in and under the same sediments that buried them when the hydrocarbons were still organic material. These underground reservoirs of oil and gas are often connected to aquifers: porous rock systems containing water.

Reservoirs can contain hydrocarbons that are liquids, gases or both. The terms gases and liquids refer to the state of the hydrocarbons under atmospheric pressure and ambient temperature. Due to high reservoir pressures gases may have a liquid state; in contrast, liquids can have a gaseous state when temperatures are high. Dependent on the pressure and temperature in a reservoir, the mixture of hydrocarbons can be in a single-phase (either gaseous or liquid) or the two-phase state. Thus, if the single phase is a liquid phase, there may be gases present, dissolved in the oil. Alternatively, if the single phase is gaseous, any oil and NGL in the reservoir are vaporized. Typically, if the state of the reservoir is two-phase, there is a gas cap on top, and there are liquids in the lower part of the reservoir: the oil zone. Due to these various phase and substance combinations there can exist up to four types of hydrocarbon reserves in a reservoir: free gas, dissolved gas, crude oil and NGL (Craft et al., 1991).

The total content of a reservoir, the resource, is a fixed quantity. Generally not all contents can be recovered. How much can be, depends on the production methods used, the economic circumstances, and environmental and other governmental regulations.

3.2.2 Reserves

Due to the physical characteristics of hydrocarbon reservoirs, it is not easy to estimate the total volume of hydrocarbon contents in them. The volume-estimating activities to gauge reserves in an area where no production is taking place yet are called *exploration*. Over time, petroleum engineers have developed an advanced toolkit, including seismographic data collection and computer simulations, to gauge the total of reserves in reservoirs. Seismology studies how seismic wave energy moves differently through various types of terrestrial surface and underground formations. Seismic waves are created artificially by machinery, and the behavior of these waves is measured using sensitive tools, called geophones. Other data gathering activities include measuring magnetic properties and the gravitational field of the Earth.

Since there is a huge variation in the reliability of the assessments, and exploration activities may lead to drilling dry wells, but also to huge finds, various classifications of reserves estimates have been developed. The verbal indications of proved, probable and possible reserves are conceptually self-explanatory, however have varying meanings dependent on the institute that performed or reported the assessment, the assessment method used, and whether the assessment was deterministic or stochastic.

The *Society of Petroleum Engineers* has made huge efforts to compare and standardize reserves likelihood methodologies (Society of Petroleum Engineers, 2005a, 2005b). Naming conventions for reserves include: 1P for proved reserves, or *Low Estimate*; 2P, for proved plus probable reserves, or *Best Estimate* and 3P, for proved plus probable plus possible reserves, or *High Estimate*.

When stochastic assessments are performed, in terms of proved, probable and possible reserves, 1P is often taken equivalent to an at least 90% chance that eventually the recovered quantity will be the estimated amount; 2P to at least 50%, and 3P to at least a 10% probability of eventual recovery.

When other characteristics are taken into account, such as economic and technical recoverability, the (Society of Petroleum Engineers, 2005b) address that the reporting standards of many international agencies are rooted in the reporting methodology proposed by McKelvey (1974).

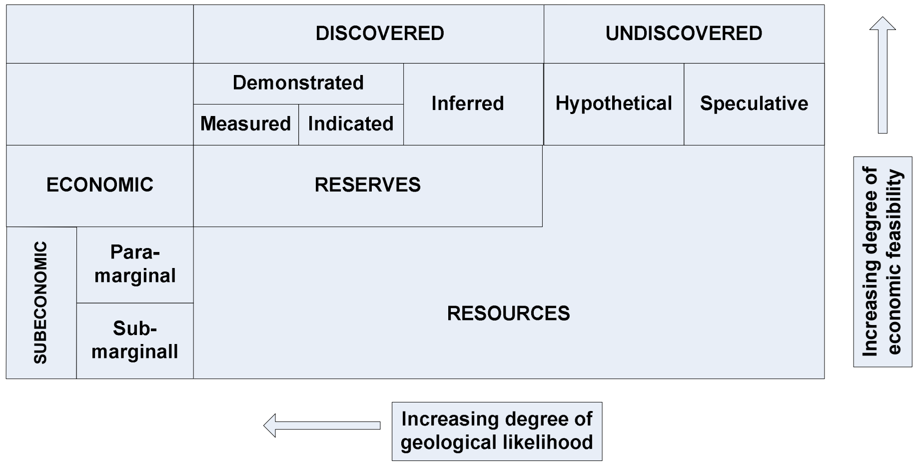

Figure 9: McKelvey Box in (Society of Petroleum Engineers, 2005b)

As stated above, the actual volume of hydrocarbons in a reservoir is hard to assess. Whenever an exploration team decides that a site has good prospects to find hydrocarbons, the next step will be to drill an extraction well. Permits need to be arranged, leases and rights of land use as well as arrangements with local or federal authorities about royalty and tax regimes. If a newly drilled well hits a significant hydrocarbon deposit with development potential, it is further enhanced to become a production well.

Dependent on how many cubic feet of gas are dissolved in the crude oil various types of *reservoirs*, or *wells*, are distinguished. *Oil wells* can have a *dissolved* gas content of up to a few 1000 cubic feet per barrel of crude oil. *Gas-condensate* reservoirs may have between five thousand and one hundred thousand cubic feet of gas per barrel of oil. Natural gas wells contain per one hundred thousand cubic feet of gas at most one barrel of *condensate*: mixtures of hydrocarbon liquids that are less dense than crude oil. Gas from condensate reservoirs is called *wet* gas, from gas reservoirs *lean* or *dry* gas. The various forms wherein gas can be present in different reservoirs have been given different names. Gas from oil wells is *associated* gas, which can be *associated-free* gas if it comes from the gas cap or *associated-dissolved* gas, or *solution* gas, if it was dissolved in the crude oil. *Non-associated* gas is gas from a reservoir that hardly contains any crude oil or condensates, with gas to oil ratios over one hundred thousand cubic feet of gas per barrel (Craft et al., 1991).

3.3 Production

Once a well is drilled in a reservoir, pressure differences will cause oil and gas to flow through the pores in the sedimentary rock to the well. Water from connected aquifers may further push out some of the oil and gas. The outflow of hydrocarbons causes the pressure and temperature to decrease, changing the physical properties of the mixture of hydrocarbons in the reservoir. For example when the pressure gets lower in a two-phase reservoir, the gas saturation in the oil zone will increase. When it reaches the critical gas saturation point, gas will flow out of the oil, changing the oil/gas ratio and this free gas may start flowing to the production wells.

The Schilthuis material balance equation (Craft et al., 1991) is helpful when analyzing shifts in those properties, pressure and temperature, when deciding on measures needed to prevent undesired changes. The material balance equation denotes a conservation of matter by accounting for volumes and quantities of fluids that were initially present, have been produced to date, have been injected into, and are still remaining in a reservoir. Essentially the equation presents a volumetric balance. Since the volume of a reservoir is constant, the sum of the volume changes of oil, free gas, water and rock must be zero.

Dependent on the physical properties of the hydrocarbons mixture, various production methods can be used. In general the initial energy contained in the reservoir will be enough to just let the gas and oil flow for some time. The methods that use the reservoir energy are the *primary production* methods. The pressure of ground water and dissolved gas will push out gas and oil. A second primary production method is fluid displacement of oil and gas by water inflow from aquifers. A third primary method is capillary expulsion, the process of water creeping up narrow pores in the rocks while pushing out the oil. A fourth method, gravity drainage, happens when oil moves to wells in lower parts of the reservoir.

Fluid displacement under the impact of *injected* water or gas is considered a *secondary recovery* method, as are other methods aiming at repressurizing the reservoir. Pumps can also be used to repressurize the reservoir. The injection of water is called water flooding, whereas the injection of natural gas is referred to as gas cycling. Injected gases can usually be recovered at the end of the oil production phase. However when the eventual recovery is in the far future, the lost revenues will be significant and other gases can be injected instead. In particular, the injection of carbon dioxide is rather commonly applied. A third set of methods is called *tertiary recovery*, also referred to as *improved* or *enhanced* recovery. These methods aim at lower *viscosity* (stickiness) of the oil, and include the injection of chemicals or chemically treated water. Also steam can be injected to increase the temperature of the oil and thereby its viscosity. [45]

Since conventional methods on average only produce about one third of the initial hydrocarbons in place, enhanced recovery methods have a large potential regarding the total output of production wells.

Besides production from natural gas from wells, there are also other sources of gas that add to the total natural gas supply. For example the Energy Information Administration (EIA) lists as supplemental gas supplies: blast furnace gas, refinery gas, propane-air mixtures, and synthetic natural gas manufactured from hydrocarbons or from coal. Although locally these supplies can be significant, they do not have huge impact on the global gas supply yet. In the future, biogas (e.g., from organic waste or manure) and gasification of coal possibly in combination with carbon sequestration and storage, could potentially play significant roles in the supply of natural gas. Biogas arises from the degeneration of organic material in the absence of oxygen.[46] The potential for biogas is huge, for instance, any wastewater treatment facility, landfill or dairy farm can be equipped to harvest the outflow of biogas.[47] The biogas can be used to produce heat and power in a Combined Heat and Power (CHP) plant, or distributed for use similar to natural gas. Due to the generally large shares of carbon dioxide, the calorific value is often too low for direct injection into the natural gas distribution grid. To do so the quality of the biogas must be upgraded. Mid 2008, biogas has become eligible in most U.S. states to fulfill the renewable energy targets laid out in their Renewable Portfolio

[45] fossil.energy.gov/programs/oilgas/eor/
[46] http://www1.agric.gov.ab.ca/$department/deptdocs.nsf/all/eng4447 (Accessed, Nov 8, 2010) ; http://www.iea-biogas.net/ (Accessed, Nov 8, 2010)
[47] http://www.epa.gov/chp/markets/wastewater.html (Accessed, Nov 8, 2010)

Standards (RPS). State's renewable energy targets are up to 30% for electricity supply, and the share of biogas in the total energy supply will likely increase.[48]

Similarly, in the EU directive 2009/28/EC has been adopted by the European Parliament to promote the use of renewable energy.[49] By 2020, 20% of energy should come from renewable sources, including biomass and biogas. In the European Union, in 2007 the total amount of biogas used was about 36 mboe, most of which was produced in Germany and the United Kingdom.[50] The huge potential for biogas and its status as a renewable energy source may affect the reserve base and R/P-ratios of natural gas, which is not accounted for in the case studies in this dissertation.

Lastly, in recent years more deep hydrocarbon wells have been drilled. The not-so-deep reservoirs, the 'low-hanging fruit' among the oil and gas wells, have been harvested and there is a trend towards new wells being deeper and deeper. When drilling deeper wells more gas and condensates are found, as well as more solution gas in the oil due to higher pressures on greater depths.

3.3.1 Processing

Produced natural gas is referred to as raw natural gas. It can be wet: containing large amounts of condensate; and sour: containing sulfur dioxide and/or carbon dioxide. Sour gas can cause corrosion in the pipeline system; and generally furnaces and other gas using appliances can only operate safely or efficiently using gas within a specific range of burning characteristics. Therefore, gas usually needs processing before it can be transported to and used by the final consumers. [51]

Raw natural gas may contain all kinds of materials such as: water vapor, hydrogen sulfide, carbon dioxide, helium, nitrogen, and dissolved NGL: ethane, propane, butane, isobutane, pentane and natural gasoline. Further processing is needed to separate all

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[48] http://www.epa.gov/chp/state-policy/renewable_fs.html (Accessed, Nov 8, 2010); http://www.epa.gov/agstar/tools/funding/renewable.html (Accessed, Nov 8, 2010)
[49] http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:32009L0028:EN:NOT (Accessed, Nov 8, 2010)
[50] http://www.eurobserv-er.org/pdf/baro186_a.pdf (Accessed, Nov 8, 2010)
[51] www.naturalgas.org/naturalgas/processing_ng.asp

hydrocarbons and fluids from the pure natural gas to get dry natural gas: mostly methane and a small fraction of ethane.

There are several steps in processing wet gas to dry gas. Some technically uncomplicated steps are performed close to the wellhead, whereas several more advanced steps are done in larger-scale facilities. The not-so-complicated steps include:

- Scrubbing, to remove sand and other large particles.
- Heating, to prevent too low a gas temperature which could induce gas hydrates (a solid ice-like substance) to form, accumulations of which could impede the flow through the pipeline network.
- Removing oil and gas condensates. This can be as simple as having a closed tank through which the gas-oil mixture is lead and wherein gravity separates the gases from the liquids. Dependent on the wet gas characteristics more specialized equipment uses pressure and temperature differences to separate oil, condensates, gas and sometimes water.
- Removing water. This can be done by separation (if the water is free and liquid), absorption or adsorption. For absorption a chemical agent with an affinity for water is used to absorb the water. Adsorption is cooling down the mixture and collecting the water vapor.

Consecutive processing steps are more advanced and usually done in a centralized fashion:

- Separating of NGL, including the extraction of NGL and fractionation of the various components. One method is absorption, similar to water removal, but with a different absorbing agent. Cryogenic expansion is cooling down the gas by rapidly expanding it. The drop in temperature makes all gases condense, except the methane. For the fractionation the different boiling points of different NGL are used to consecutively separate them from each other
- Removing sulfur and carbon dioxide. Taking the sulfur out is called sweetening, a process similar to the absorption processes for removal of water and NGL.

Due to limitations in model capabilities, but also data availability, not all steps and characteristics presented in the previous sections are included in our model. Before presenting the mathematical formulation in the next section, we introduce the notation used.

3.3.2 Nomenclature

The notational conventions used in the model formulation are mostly self-explanatory. Generally, sets and market-player indices are the first letter of their full name. For example, the variable $SALES^X$ are the total sales of a market agent of type X (cf., $SALES^P_{pdm}$ in Eq. (3.3.4)). Also, $PURCH^Y$ are the purchases of an agent of type Y. The set N denotes model nodes; and for subsets of nodes where a player x is present, we use $N(x)$. [52] To refer to individual nodes in this set, we write $n(x)$. Similarly, to denote the subset of agents X present at node n, we use: $X(n)$, (e.g. $T(n)$ are the traders with access to node n); and to refer to individual set elements of this set, we write $x(n)$. All market prices, i.e., dual variables to market-clearing conditions, are represented by the Greek letter π with appropriate subscripts and superscripts; and shadow prices of constraints are denoted as lower-case Greek symbols (e.g., α, β, γ for capacity constraints; ϕ for mass balance constraints and ρ for capacity expansion limitations). For completeness a full list of symbols used is shown below before the model formulation. Most costs and price-related data are in thousands of dollars per million cubic meters (k$/mcm); the unit of measurement for volume and flow data is million cubic meters per day (mcm/d).

3.3.2.1 Sets

$a \in A$	Gas transportation arcs, e.g., {NNED_GER, LNOR_FRA, RGER_GER}[53]
$d \in D$	Demand seasons, e.g., {low, high}
$p \in P$	Producers, e.g., {P_NOR, P_RUW, P_RUE }
$m \in M$	Years, e.g., {2005, 2010, 2015, 2020}

[52] Model nodes represent geographical regions in the world. They can be defined flexibly in the model data set. Due to the limited relevance and impact of countries that only produce and consume small amounts, several countries have been grouped with neighboring ones and are represented in the model data set on an aggregate level. For some countries the opposite is true: their consumption or production is so high, and the geographical distances so large, that a division of the countries in several regions is warranted. The regions used in the numerical analyses are introduced at the start of Chapter 4.

[53] The first letter indicates the type of arc, combinations of three letters denote the region of country name. NNED_GER represtents a pipeline from the Netherlands to Germany; LNOR_FRA an LNG shipping arc from the Norwegian liquefaction node to the regasification node of France and RGER_GER the arc from the German regasification node to the German country node. NNIG_LNG would denote the arc from the country node Nigeria to the Nigeriaion liquefaction node.

$n \in N$ Region nodes, e.g., {N_NOR, N_RUW}

$s \in S$ Storage facilities, e.g., {S_NED, S_GER}

$t \in T$ Traders, e.g., {T_NOR, T_RUS}

$a^{+}(n)$ Inward arcs into node *n*

$a^{-}(n)$ Outward arcs from node *n*

3.3.2.2 Constants/Input parameters

b_{am}^{A} Arc capacity expansion costs (k$/mcm/d)

b_{sm}^{SI} Storage injection capacity expansion costs (k$/mcm)

b_{sm}^{SX} Storage extraction capacity expansion costs (k$/mcm)

b_{sm}^{SW} Storage working gas capacity expansion costs (k$/mcm)

$c_{pm}^{P}(.)$ Production costs (k$/mcm)

$\overline{CAP}_{am}^{A}$ Arc capacity (mcm/d)[54]

$\overline{CAP}_{sm}^{SI}$ Storage injection capacity (mcm/d)[54]

$\overline{CAP}_{sm}^{SX}$ Storage extraction capacity (mcm/d)[54]

CON_{tadm}^{T} Contractual supply obligation (mcm/d)

δ_{tn}^{C} Level of market power exerted by trader in a market, $\delta_{tn}^{C} \in [0,1]$

0 is perfectly competitive, 1 is fully Cournot.

$days_{d}$ Number of days in season

$\overline{\Delta}_{am}^{A}$ Upper bound of arc capacity expansion (mcm/d)

$\overline{\Delta}_{sm}^{SI}$ Upper bound of storage injection capacity expansion (mcm/d)

$\overline{\Delta}_{sm}^{SX}$ Upper bound of storage extraction capacity expansion (mcm/d)

$\overline{\Delta}_{sm}^{SW}$ Upper bound of storage working gas capacity expansion (mcm)

γ_{m} Discount rate in year, $\gamma_{m} \in (0,1]$

INT_{ndm}^{W} Intercept of inverse demand curve (mcm/d)

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[54] Sub-script m is to account for expansions approved or under construction.

$loss_a$	Loss rate of gas in transport arc, $l_a \in [0,1)$
$loss_s$	Loss rate of gas storage injection, $l_s \in [0,1)$
$\overline{PR}^{P}_{pm}$	Production capacity (mcm/d)
$\overline{PH}^{P}_{p}$	Total accessible reserves in time horizon (mcm)
SLP^{W}_{ndm}	Slope of inverse demand curve (mcm/d/k$)
$\tau^{A,reg}_{adm}$	Regulated fee for arc usage (k$/mcm)
$\tau^{SI,reg}_{sdm}$	Regulated fee for storage injection (k$/mcm)
$\overline{WG}^{S}_{sm}$	Storage working gas capacity (mcm)[55]

3.3.2.3 Variables

Δ^{A}_{am}	Arc capacity expansion (mcm/d)
Δ^{SI}_{snm}	Storage injection capacity expansion (mcm/d)
Δ^{SX}_{snm}	Storage extraction capacity expansion (mcm/d)
Δ^{SW}_{snm}	Storage working gas capacity expansion (mcm/d)
$FLOW^{T}_{tadm}$	Arc flow by trader (mcm/d)
INJ^{T}_{tndm}	Quantity injected to storage by trader (mcm/d)
$PURCH^{T}_{tndm}$	Quantity bought from producer by trader (mcm/d)
$SALES^{A}_{adm}$	Pipeline capacity assigned to trader (mcm/d)
$SALES^{P}_{pdm}$	Quantity sold by producer to traders (mcm/d)
$SALES^{SI}_{sdm}$	Storage injection capacity assigned for use to traders (mcm/d)
$SALES^{SX}_{sdm}$	Storage extraction capacity assigned for use to traders (mcm/d)
$SALES^{T}_{tndm}$	Quantity sold to end-user markets by trader (mcm/d)
XTR^{T}_{tndm}	Quantity extracted from storage by trader (mcm/d)

[55] Sub-script m is to account for expansions approved or under construction.

When presenting restrictions in the formulations below, Greek symbols in parentheses represent the dual variables used in the KKT derivation.

$\alpha, \beta \geq 0$	dual variables to capacity restrictions
φ *free*	dual variables to mass balance constraints
$\rho \geq 0$	dual variables to capacity expansion limitations
π *free*	duals to market-clearing conditions for sold and bought quantities
τ *free*	duals to market-clearing conditions for capacity assignment and usage.

In what follows, we describe the representation of the producer and other players.

3.3.3 Producers

Some companies involved in natural gas production are Exxon-Mobil, British Petroleum, Shell, Statoil, Gazprom and Sonatrach. A main difference between the first three and the second three companies is that the first three operate globally and the second three mostly regionally. In many countries the production of gas is nationalized, especially in the countries that would potentially participate in a global gas cartel. Much of the company data for global firms is either not available, or would take more time to collect than can be justified for an academic study. For the type of studies performed with the model as part of this dissertation, focusing on upstream market power, a country-based representation is adequate to provide insight into the market developments.

In reality, natural gas production rates and costs will vary by reservoir and over time, and the costs for and success rates of drilling new reservoirs must be accounted for. However, data for costs and reserves of individual reservoirs are not publicly available or very hard to obtain. Also, the level of detail might have a potentially unmanageable impact on the model size of the resulting MCP. Therefore, in our model we limit the representation of the production side of the natural gas market to the economically most relevant aspects. We assume that all technical characteristics can be summarized into a supply cost curve, a limit to daily production and a bound to the total over the time horizon economically recoverable reserves. In the model there is one production quantity for every producer in every season and every year, which must be interpreted as the seasonal average production level. Summarizing all (marginal) supply costs into one curve is a simplification, but very common in the literature, and representative enough in the perspective of a long-term equilibrium model.

3.3.3.1 Assumptions

The model type used, MCP, requires that feasible regions and minimization objective functions are convex. Additionally, there are limits to data covered in publicly available sources. Also, the inclusion of many characteristics and much detail will lead to large models and possibly a large computational effort to solve the model. To develop a representative MCP that is computationally tractable, the assumptions must be made for the producer and other players. Hence, producers are assumed to:

- maximize their discounted profits over all periods
- have perfect information
- have exogenously given production cost functions, daily capacities and reserves
- have strictly increasing convex production costs, which include other relevant costs such as processing of the gas
- only sell gas to one given trader (which could be considered to be their trading arm)
- be price-takers with respect to their selling price and not exert market power.

3.3.3.2 Producer problem

A producer *p* is modeled as maximizing his discounted profits, which are the result of revenues from sales $SALES^P_{pdm}$ minus production costs. Cash flows in year *m* are discounted with a factor γ_m. Since sales rates are per day and may differ by season, the sales rates are multiplied by the number of days in the season *d*: $days_d$.

$$\max_{SALES^P_{pdm}} \sum_{m\in M} \gamma_m \sum_{d\in D} days_d \left[\pi^P_{n(p)dm} SALES^P_{pdm} - c^P_{pm}(SALES^P_{pdm}) \right] \quad \textbf{(3.3.1)}$$

The sales rate is restricted by a production capacity $\overline{PR}^P_{pm}$ (that can vary by year):

$$s.t. \quad SALES^P_{pdm} \le \overline{PR}^P_{pm} \quad \forall d,m \quad \left(\alpha^P_{pdm}\right) \quad \textbf{(3.3.2)}$$

Due to reserve limitations or governmental restrictions the aggregate production over all years in a time period is restricted by a production ceiling $\overline{PH}^P_p$.

$$\sum_{m \in M} \sum_{d \in D} days_d SALES^P_{pdm} \leq \overline{PH}^P_p \quad \forall m \quad \left(\beta^P_p\right) \tag{3.3.3}$$

Lastly, the sales-rate must be nonnegative:

$$SALES^P_{pdm} \geq 0 \quad \forall d, m \tag{3.3.4}$$

Note that all market-clearing conditions are shown at the end of this chapter. Also, the KKT conditions for the producer and all other agents can be found in Appendix 3.13. The market player that we describe in the following section is the trader.

3.4 Trade

Most natural gas trading companies also perform other activities than buying and selling of gas. Often gas trading companies have some vertical or horizontal integration, and the integrated companies can be active in all aspects of the natural gas market, from production and trade, to liquefaction, regasification, LNG transport, pipeline operation and storage. Examples of traders in today's natural gas marketplace include Gazexport, the trading arm for Gazprom (Russia) and GasTerra BV, the trading arm for NAM (Nederlandse Aardolie Maatschappij). A company that is active in Washington D.C. and Virginia is Washington Gas, and companies that are active in various southern U.S. states include San Diego Gas & Electric and Southern California Gas Co. Before discussing the role of the trader in the WGM, we elaborate on some aspects relevant for gas trade.

3.4.1 Market power

The Organization of Economic Co-operation and Development (OECD)[56] defines market power as: '*the ability of a firm (or group of firms) to raise and maintain price above the level that would prevail under competition is referred to as market or monopoly power. The exercise of market power leads to reduced output and loss of economic welfare.*'

Generally, the exertion of market power by particular market agents is hard to prove. Sometimes internal company memos are made public by angry ex-employees, but usually the information stays hidden. Typically one would want to show that prevailing market prices are higher than would be the case in a perfectly competitive market. To that end, we would need an adequately representative model and data set and show that price and

[56] http://stats.oecd.org/glossary/detail.asp?ID=3256

output values in a perfectly competitive market are lower than prevailing market prices. We would need to show that finite resource considerations (e.g., (Hotelling, 1931) and (De Joode, 2003)) and restrictions in production and transport capacity are not responsible for the difference between marginal supply costs and prevailing market prices. Proving the point this way is nearly impossible, however some past events have provided empirical evidence for market power exertion. In 1980, Algeria cut off supplies to American and European customers trying to force the acceptance of unilateral changes in contractual terms (International Energy Agency, 2004). More recently, Russia has disrupted supplies to the Ukraine and other CIS countries[57] as leverage in contract negotiations (Stern, 2006). The recent announcements around and developments of the Gas Exporting Countries Forum (GECF) are also an indication that market power exertion exists, or at the very least is contemplated by some gas exporting countries (see Chapter 1.)

Market power has been ignored for a long time in modeling energy markets. One reason is that a typical Cournot oligopoly cannot be represented as a linear or quadratic program which makes it a computationally challenging endeavor. Smeers (2008) provides an extensive discussion on alternative ways to model market power. Smeers argues that linear and quadratic programs can sufficiently capture market power aspects. He states that adding a mark-up to the pure competition price suffices, arguing that an exogenous mark-up is equally arbitrary as a conjectural variation or conjectured response approach, for instance as implemented in (Mulder and Zwart, 2006), (Boots et al., 2004) or (Egging and Gabriel, 2006). According to (Smeers, 2008), mark-ups can be easily interpreted and compared to market observations. However a mark-up approach fails to capture one important aspect of market power: the incentive for market power players to geographically diversify their supplies. This effect was illustrated and explained for the European gas market in (Egging and Gabriel, 2006). The explanation is along the following lines. In a perfectly market, wherein transport costs are minimized, most supplies are shipped to domestic and neighboring markets. When exerting market power, suppliers are inclined to supply lower amounts to the domestic and neighboring markets to drive up prices. However, the higher prices create opportunities for other suppliers,

[57] Commonwealth of Independent States: the countries within the former Sovjet Union

which can result in gas being shipped over longer distances in an imperfect market, relative to a perfectly competitive market. To illustrate this effect we have included an example at the end of this chapter.

3.4.2 Levels of market power[58]

There are many papers that develop, use or test for (concepts of) oligopolistic market power somewhere between perfectly competitive and Nash-Cournot oligopoly in markets as varying as agriculture, supermarkets and the airline industry. Some terms that have been used to indicate market power levels: the market power parameter (β^{mp} in (Raper et al., 1998); variable λ in (Steen and Salvanes, 1999), the conduct parameter, and conjectural elasticity (Taylor and Kilmer, 1988), (Weerahewa, 2003), conjectural variation (Garcia-Alcalde et al., 2002), and the numbers of equivalent Cournot firms (cf., the value of *n* in $q_i = \frac{13}{n+1}$ in Section 3 of Chapter 2), the Lerner index (Lerner, 1934) and the Hirschman - Herfindahl index (HHI). In our work we apply a mixed conjectural variation approach. We define a market power parameter for the traders $\delta^C_{t,n} \in [0,1]$, which is implicitly defined as follows: $\delta^C_{t,n} = \frac{\partial \sum_{t \in T(n)} SALES^{T \to M}_{tndm}}{\partial SALES^{T \to M}_{tndm}}$, the partial derivative of the total supply with respect to a firms own supply. A value of $\delta^C_{t,n} = 0$ means no market power: the conjecture is that a change in own supply will not induce a change in total supply (and thus in market price); a value of $\delta^C_{t,n} = 1$ means that the trader is a full Cournot player. As a full Cournot player the conjectured variation in supply by the other players is 0 (i.e., other players will not change their output volumes in response to a change in the market price induced by this player's changing output volume). Positive values lower than one indicate that we assume that some market power is exerted by the trader, but diluted relative to Cournot competition. This implies that the model as implemented is not a strict Cournot model, but rather a heuristic way to deal with varying degrees of market power and to calibrate the model. Long-term market share considerations and government policies are two of the factors that may not completely prevent, but may to some extend limit the exertion of market power.

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[58] This discussion on market-power parameters is a slightly adjusted version of the discussion in (Egging et al., 2008). Special thanks to *Energy Policy* for the permission to reuse some of the work in this dissertation.

In the model presented in this chapter, the market power is with the traders, both representing the pipeline and the LNG deliveries. This differs from (Egging et al., 2010) wherein both traders and regasifiers could exert market power. The latter formulation was not able to adequately represent cartel types of collusion. This is corrected by the current formulation wherein traders coordinate both pipeline and LNG flows originating from the same country.

3.4.3 Contracts

The subject of long-term contracts was briefly addressed in Chapter 1 as a means for market participants to secure a return on investments on expensive transportation infrastructure and a mechanism to allocate risk along the natural gas supply chain. Although decreasing in relative volume, long-term contracts are still a very important factor in natural gas markets. In the year 2007, 82.2% of global LNG (GIIGNL, 2008) was traded via long-term contracts, and a large part of the remaining 17.8% via short-term contracts.[59]

Most long-term contracts in the LNG market contain the *Take-Or-Pay* (TOP) clause: the buyer agrees to pay for a specific quantity of delivered LNG, even if he would not actually take it. A second option, the *flexibility clause*, allows the buyer to purchase an extra volume of LNG (often up to 50% of the minimum supplied amount) at the contractually agreed price. A third common contract term is the *destination clause*, which entails that the buyer cannot resell the gas to another party without permission of the supplier. The destination clause effectively segments the market, thereby hindering competition and market liquidity. It is one of the issues that has been heavily debated by European regulatory authorities, for example in the context of Russia's dominant supply situation (Finon and Locatelli, 2008).

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[59] For pipeline gas it was not possible to find any references that indicate the share of (long-term) contracts. For instance, (International Energy Agency, 2008) quotes GIINGL for the LNG contract shares, but gives no pipeline contracts share. To give some indication: in the United Kingdom, the earliest and most open market in Europe, about 70% of the produced gas was sold under long-term contracts, about 15% under short-term contracts, and about 15% on the spot market (International Energy Agency, 2004). In Belgium, which is completely import-dependent, long-term contracts for LNG and pipeline gas fulfill about 95% of domestic demand.

Recently there have been some developments in the structure and pricing of long-term gas supply contracts (International Energy Agency, 2004, 2006a). These developments include decreasing contract periods, smaller contract volumes and more flexibility in TOP obligations and diversion of the destination. Another development has been the decreasing share of long-term contracts relative to the total traded natural gas volumes, as discussed in (International Energy Agency, 2004) and (Neumann and Von Hirschhausen, 2004).

In the EU, a main reason for the decreasing share of contracts are the market liberalization efforts. The interdependence between market liberalization in the EU and decrease in the market share of contracts goes two ways. The regulatory framework has discouraged long-term contracts with little flexibility as they hinder competition. Alternatively, the liberalization has brought more players to the market, providing more alternative options and reducing the need to secure supplies from a specific source.

(Neuhoff and Von Hirschhausen, 2005) discussed the impact of long-term contracts on producer profits and consumer surplus in the context of the liberalizing European gas market. They argue that producers benefit from the risk-hedging aspects, while consumers benefit from lower prices resulting from spot market behavior of the producers given that the contractually delivered amounts and prices have been set. When long-term demand elasticities are significantly larger than short-term elasticity, the lower prices due to the spot deliveries induce higher gas consumption in the long run and higher profits for the producers than when no spot deliveries would be made. [60]

Allaz and Vila (1993) discussed the impact of contracts on market efficiency in a multi-stage multi-period duopoly and showed that contracts reduce the ability of market players to exert market power. The duopoly setting allows for an analytic approach using closed-form expressions based on optimal response curves (Figure 7 in Section 2.3). Calcagno and Sadrieh (2004) extended the results to storable products and risk-averse traders under demand uncertainty and Su (2007) addressed asymmetry in cost functions at the supply

[60] (Neuhoff and Von Hirschhausen, 2005) show for a stylized symmetric duopoly that an order of magnitude difference in short and long-term demand elasticities of five would be sufficiently large to induce the described effects.

side. Zhuang (2005) included contracts in an extensive-form two-stage stochastic duopoly cast as an MCP.

The important role of contracts in natural gas markets must be considered for the World Gas Model. However, including contracts poses several challenges. Some aspects have to do with the mathematical formulation, which can be overcome; other aspects are of a more practical nature. One major practical consideration is that it is difficult to obtain information on contract volumes and terms. This is more so for pipeline trade than for LNG. For LNG contracts most information in terms of prices and quantities is publicly available (GIIGNL, 2009), but this is not the case for pipeline contracts.

To adequately model the contracting process, we would need a multi-stage model, with contracting and spot-market phases (Neuhoff and Von Hirschhausen, 2005), (Zhuang, 2005). However, in a deterministic one-stage MCP with risk-neutral players, players are indifferent between contracted volumes and spot market sales. Since there is no uncertainty, there is no need to hedge decisions and forward and spot market prices in a deterministic model, the two sets of prices will be the same. Zhuang (2005) showed that in a stochastic duopoly setting, when supplied quantities are positive, the forward prices equal the expected spot market prices.

Another consideration when including contracts is that they typically contain a price and volume component, and that delivered prices are often indexed to crude oil prices. In an MCP we cannot set both the delivered prices and the delivered volumes. Setting both delivered prices and volumes to fixed values will likely cause infeasibility of the model due to inconsistency of these values with any feasible solution of the whole model. Instead we will only consider the contract as a lower bound to supplied amounts, and allow the delivered prices to be determined endogenously by the model.

Many actual gas deliveries that are done to meet contractual obligations would probably also have occurred if there had been no contracts in place, but only a spot market would have existed (e.g., contractually supplied volumes from Russia, Norway and Algeria to European countries). Therefore, many of these contractual trade flows will be captured by a long-run equilibrium model (c.f., (Holz, 2009) p. 73). What would not be captured by excluding contracts is explicit diversification of supplies by importing countries, and restrictions to redirecting of gas flows in disruption scenarios. The diversification aspect is more of an issue in the LNG market than in the pipeline market, since pipelines allow much fewer alternative destinations than LNG ships. The limitations caused by the destination clause are becoming less important, given the trend in loosening terms for reselling gas (c.f., IP/07/1074, the agreement between the EC and Algeria to drop territorial restrictions from all contracts). [61] It is not uncommon these days to redirect LNG supplies from their contractual destinations to some other market with higher spot prices, and the supplier and buyer sharing the benefits. Contractual terms are increasingly flexible relative to the destination of the gas. Also unilateral agreements among groups of countries (such as the European Union) to support each other in crisis situations reduce the impact of fixed contract volumes. Flexibility from one source can compensate for the rigidity of another, and even a relatively small fraction of non-fixed-destination gas and swing supply suffices to reallocate flows. A last consideration is that most models in the literature lack the inclusion of contracts (see Chapter 2). In conclusion, we forgo including pipeline contracts, but include LNG contracts when the appropriate data are present.

The traders in the WGM have a simplified role. They buy gas from one or more producers, and sell gas to one or more final consumption markets. This modeling approach can both represent a vertically integrated production and trading company (separate parts of the same overall organization with perfectly-competitive internal accounting prices) as well as an independent trader that purchases gas from one or several producers and who markets the gas to various consumer markets. In the WGM we distinguish two types of traders:

- Traders operating only at the domestic node of the producer, in case it is a small producer that does not export any gas. This applies to countries such as Germany and Italy that only produce about 16% and 10% respectively of domestic consumption.
- Traders that can operate at any consumption node that can be reached via the LNG supply chain or via pipelines through transit nodes from their own producer's node.

[61] http://europa.eu/rapid/pressReleasesAction.do?reference=IP/07/1074

An example is the Netherlands, both a gas producing and exporting country. The trader associated with the Dutch producer is present in European consuming countries such as the United Kingdom, Belgium, France, Germany, Poland, Austria, Italy, etc., but not in countries like:

- Algeria, because Algeria does not have incoming pipelines from the European Continent;
- South Korea, because South Korea cannot be reached by pipeline from the Netherlands and the Netherlands do not have any liquefaction facilities.

Modeling traders as separate participants is also consistent with legal requirements forcing unbundling of production and trade operations that have been pushed by the U.S. Federal Energy Regulatory Commission (FERC) and the European Commission (European Commission, 2003).[62]

3.4.3.1 Assumptions

The role of the trader in the WGM is completely disintegrated from other activities and does not surpass the trade of natural gas. In the WGM, traders only buy and sell gas and vertical nor horizontal integration is addressed and infrastructure services needed for transportation and storage of natural gas are purchased from other model players. Hence, traders are assumed to:

- maximize their discounted profits over all periods
- have perfect information
- only purchase gas from a specific set of producers
- only sell gas to markets in countries that are accessible via pipelines or the LNG supply chain
- not own any infrastructure (neither pipelines, LNG export or import terminals, vessels, storage facilities)
- be price-takers with regard to purchases from producers, as well as the usage of services from infrastructure operators
- exert market power to the consumer markets by strategically withholding part of the supplies according to a pre-specified market power parameter value

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[62] www.eia.doe.gov, www.ferc.gov/industries/gas.asp (e.g., FERC order 636).

3.4.3.2 Trader problem

The trader is modeled as maximizing the profits resulting from selling gas to marketers ($SALES^T_{tndm}$), net of the gas purchasing costs and other costs: a regulated fee $\tau^{\text{A,reg}}_{adm}$ plus a congestion fee τ^A_{adm}, to transport the gas ($FLOW^T_{tadm}$) over high pressure pipelines *a*. The parameter $\delta^C_{tn} \in [0,1]$ indicates the level of market power exerted by a trader at a consumption node, with 0 representing perfect competitive behavior and 1 representing perfect Nash-Cournot oligopolistic behavior. The expression $\left(\delta^C_{tn}\Pi^W_{ndm}(\cdot) + (1-\delta^C_{tn})\pi^W_{ndm}\right)$ can be viewed as a weighted average of market prices resulting from the inverse demand function $\Pi^W_{ndm}(\cdot)$ (Eq. (3.10.1)) and a perfectly competitive market-clearing wholesale price π^W_{ndm}. The trader also decides how much gas to inject in and extract from storage. Costs for injection are a regulated fee and a congestion rate; costs for extraction are a congestion rate only. Thus, trader *t* is modeled as solving the following optimization problem:

$$\max_{\substack{SALES^T_{tndm}\\ PURCH^T_{tndm}\\ FLOW^T_{tadm}\\ INJ^T_{tndm}\\ XTR^T_{tndm}}} \sum_{m\in M}\gamma_m \sum_{d\in D} days_d \left\{ \begin{array}{l} \displaystyle\sum_{n\in N(t)} \left[\begin{array}{l} \left(\delta^C_{tn}\Pi^W_{ndm}(\cdot) + (1-\delta^C_{tn})\pi^W_{ndm}\right) SALES^T_{tndm} \\ -\pi^P_{ndm} PURCH^T_{tndm} \\ \displaystyle -\sum_{s\in S(t)} \left(\begin{array}{l} \left(\tau^{SI,reg}_{sndm} + \tau^{SI}_{sndm}\right) INJ^T_{tndm} \\ + \qquad \tau^{SX}_{sndm} XTR^T_{tndm} \end{array} \right) \end{array} \right] \\ \displaystyle -\left(\sum_{a\in A(t)} \left(\tau^{\text{A,reg}}_{adm} + \tau^A_{adm}\right) FLOW^T_{tadm} \right) \end{array} \right\} \quad \textbf{(3.4.1)}$$

Mass balance at every node *n* in every season *d* of every year *m*:[63]

s.t.

$$PURCH^T_{tndm} + \sum_{a\in a^+(n)} \left(1 - loss_a\right) FLOW^T_{tadm} + XTR^T_{tndm} = \quad \textbf{(3.4.2)}$$

$$SALES^T_{tndm} + \sum_{a\in a^-(n)} FLOW^T_{tadm} + INJ^T_{tndm} \quad \forall n,d,m \ \left(\varphi^T_{tndm}\right)$$

There is no carry over of gas stored to following years. Thus, over the storage cycle in each year the total extracted volumes must equal the loss-corrected injection volumes.

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[63] Pipeline losses are accounted for in this mass-balance equation; in contrast, the storage loss rate is accounted for in the storage cycle constraint, equation (3.4.3).

$$(1-loss_s)\sum_{d\in D} days_d INJ^T_{tsdm} = \sum_{d\in D} days_d XTR^T_{tsdm} \quad \forall n, s\in S(N(t)), d, m \quad (\varphi^S_{tsdm}) \qquad \textbf{(3.4.3)}$$

Contractual arrangements to supply a minimum amount to a market *n* in a season *d*, and year *m* via a transport connection (arc) *a* provide lower bounds to some flows:

$$FLOW^T_{tadm} \geq CON^T_{tadm} \quad \forall a, d, m \quad (\varepsilon^T_{tadm}) \qquad \textbf{(3.4.4)}$$

All other constraints are nonnegativity of variables:

$$SALES^T_{tndm} \geq 0 \quad \forall n, d, m \qquad \textbf{(3.4.5)}$$

$$PURCH^T_{tndm} \geq 0 \quad \forall n, d, m \qquad \textbf{(3.4.6)}$$

$$FLOW^T_{tadm} \geq 0 \quad \forall a, d, m \qquad \textbf{(3.4.7)}$$

$$INJ^T_{tndm} \geq 0 \quad \forall n, d, m \qquad \textbf{(3.4.8)}$$

$$XTR^T_{tndm} \geq 0 \quad \forall n, d, m \qquad \textbf{(3.4.9)}$$

The inverse demand curve $\Pi^W_{ndm}(\cdot)$ is presented in the marketer section, 3.10. The following section presents the aspects relevant for natural gas liquefaction.

3.5 Liquefaction

Some of the major players in liquefied natural gas are Shell, GDF Suez and British Gas.[64] All of them are present in several LNG exporting countries and involved in other activities as well, such as production and marketing. The following picture characterizes the main elements of the LNG supply chain.

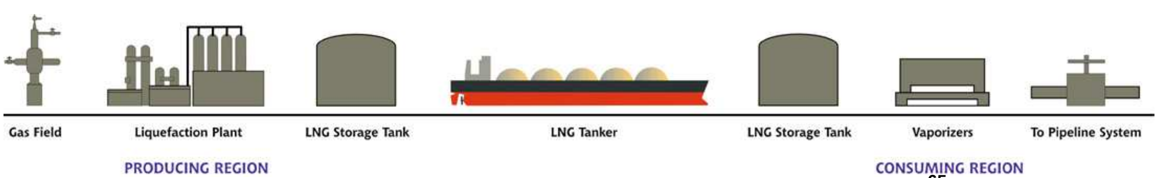

Figure 10: LNG supply chain. Source: Panhandle Energy[65]

On the supply side there is gas production and liquefaction, then the liquefied gas is shipped overseas. In the consuming region an importing facility re-gasifies the gas and pumps it into the local pipeline system.

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[64] www.shell.com/lng; www.gdfsuez.com/en/activities/our-energies/natural-gas-and-lng/natural-gas-and-lng/ and www.bg-group.com/OurBusiness/BusinessSegments/Documents/BG_LNGfactsheets2008.pdf

[65] www.panhandleenergy.com/images/content/lng_term_chain.jpg

3.5.1 Engineering considerations

The liquefaction of natural gas is a capital-intensive and technologically-advanced process. Differences in the constituents of gas in various production fields and the requirements in the importing markets need to be overcome. Local circumstances can drastically impact the construction of facilities as well as operational characteristics. For instance, arctic conditions in the Norwegian northern North Sea, the Russian Barents Sea and the Sakhalin island may impede the development of LNG infrastructure and the accessibility of the facilities by ships. However the much colder sea water as compared to the Middle Eastern countries, (e.g., Qatar), allow for significantly lower natural gas losses in the actual liquefaction process in the arctic regions.

Impurities and non-combustible components (such as carbon dioxide) do not add any value to the end-users and are removed from the gas to transport more energy content in the available shipping capacity. Other necessary steps in the LNG supply chain are:

- Sweetening: processing the gas to take out the acidic fractions such as water and carbon dioxide to prevent rusting of pipelines and containers.
- Separation: removing most of the heavier hydrocarbons (such as NGL).
- Compression and heat exchange: cooling and pressurizing the natural gas.
- Expansion: bringing the liquefied gas to atmospheric pressure.
- Storage: storing the gas between the liquefaction and the loading of the ship.
- Loading: bringing the liquefied gas on the ship.

Many technical aspects of the liquefaction process potentially cause non-linearities and non-convexities when representing them in a mathematical model. This can be problematic, since non-convex feasible regions undermine the assumptions underlying the mathematical approaches to find equilibrium solutions (c.f., Chapter 2). Capacity expansions are typically integer-valued, and carry economies of scale; integers cannot easily be incorporated in an MCP and representing economies of scale – decreasing marginal costs – induces concavity of minimization objectives. Linear approximations of non-linear technical characteristics might preserve the convexity required for solving the WGM, but are beyond the scope of this dissertation. The liquefaction of gas is modeled as capacity bounds, a loss rate, a regulated fee, and possibly a congestion charge.

For more details on the design of liquefaction plants, see for example the technical briefing (Denton and Barclay, 2005) that discusses some of the considerations and issues for the design of – relatively small scale – offshore LNG plants. For more pictures and extensive descriptions of LNG liquefaction technology (e.g., (Spilsbury et al., 2005). Note that there is no separate mathematical formulation for natural gas liquefaction. The relevant aspects are represented in the trader and the transmission system operator problems.

3.6 Regasification

When an LNG shipment arrives at its destination, the vessel must be unloaded and the gas must be brought back into a gaseous state to distribute it into the gas grid. Some players that are active in regasification are: GDF Suez, British Gas, Cheniere and Fluxys.[66] For an overview of all regasification terminals, see (GIIGNL, 2008). Similar to other players, regasifiers are represented in a simplified way in the WGM. For instance one of the aspects that is not captured is the following. Recently, some LNG receiving terminals have installed loading facilities. Liquefied gas that was unloaded and stored but not yet vaporized, can be loaded back onto ships and shipped to other destinations.[67] This loading equipment allows market players to benefit from short-term arbitrage opportunities, but could potentially provide swing supply in some markets. Due to the likely near-term excess of regasification capacity on the U.S. Gulf Coast, some analysts consider it likely that U.S. terminals may be equipped with loading facilities and possibly more storage tanks as well.[68] Terminals equipped as such could provide seasonal swing flexibility in the Atlantic Basin and enhance the security of supply situation on both sides of the ocean. The WGM will not incorporate these LNG storage and re-exportation capabilities of regasification terminals are not incorporated. The setting of a long-term equilibrium model with just a few demand seasons in a year does not capture short-term price-hikes that could provide the incentives for reloading LNG onto ships. Rather, the long-term seasonal average prices let LNG exporting traders decide where to direct their

[66] www.gdfsuez.com/en/activities/our-energies/natural-gas-and-lng/natural-gas-and-lng/ ; www.bg-group.com/OurBusiness/BusinessSegments/Documents/BG_LNGfactsheets2008.pdf ; www.cheniere.com/default.shtml and www.fluxys.com/en/Services/LNGTerminalling/LNGTerminalling.aspx

[67] www.fluxyslng.net/media/pdf/2008/PB_Fluxys_080731_EN.pdf

[68] www.chenierenergypartners.com/liquefaction_project/liquefaction_project.shtml (Accessed Nov 15, 2010)

LNG vessels dependent on seasonal demand patterns. Dependent on other variables not capturing this LNG reloading option in the model may result in lower future regasification capacities on the East Coast of the U.S. and higher regasification capacities and storage working gas in (Western) Europe in the model outcomes. However, we expect that the model results, especially the trends in long-term developments, will not be affected much by this modeling choice.

Contrary to liquefiers, we allow for more than one regasifier in a country. This choice allows countries like Spain, France and Mexico to have and expand import capacity on their respective east and west coasts. That choice is made as it could provide interesting insights in developments in the various basins.

3.6.1 Engineering considerations

The infrastructure for the regasification of gas is less capital-intensive than for liquefaction. Naturally when LNG is exposed to ambient temperature and atmospheric pressure it will return to a gaseous state. To speed up this process, heat exchangers can be used. Often sea water is used as the heat source. Sometimes the sea water is heated by boilers, using some of the gas that is regasified. The main steps in the regasification process are the following:[69]

- Unloading of the LNG shipping vessel.
- Vaporization of the liquefied gas to bring it back to a gaseous state.
- Storing the gas to allow for quick unloading of the vessel while not immediately bringing the gas into the pipeline system.
- Processing: sometimes some components must be added to decrease the calorific value to meet local requirements.
- Bringing gas into the pipeline system, the final step to distribute the gas from the terminal to the end-users.

The regasification of gas is modeled as capacity bounds, a loss rate, a regulated fee and an endogenously determined congestion charge (identical to liquefaction and pipeline

[69] www.cheniere.com/lng_terminals/terminals.shtml; www.canaportlng.com/faqs.php; www.fluxyslng.net and www.gate.nl/index.php?fotoalbum_id=&taal_id=2

infrastructure). There is no separate mathematical formulation for natural gas regasification. Instead, the relevant aspects are represented in the trader and the transmission system operator problems.

3.7 LNG shipment

The bulk of LNG is transported in dedicated LNG shipping vessels. In a few countries, such as Belgium, LNG is transported by trucks on a relatively small scale. [70] In the WGM the local truck distribution of LNG is ignored, as is done with all domestic distribution of natural gas. Also, the possible capacity restrictions resulting from a limited number of LNG shipping vessels is not included. Note that LNG shipment is not included as an independent player. Rather, it is represented by a shipping cost and gas loss rate in the trader problem.

The following section describes the storage operations.

3.8 Storage

Natural gas storage is used for a variety of reasons, including daily balancing and speculation, seasonal balancing and as a strategic backup supply to overcome temporary supply disruptions or to meet peak demand on extremely cold winter days. Examples of two companies involved in gas storage are Royal Dutch Shell and E.ON.[71]

3.8.1 Engineering considerations

The Energy Information Administration provides information regarding the storage of natural gas on their website.[72] There are various types of gas storage: depleted reservoirs in oil and gas fields, aquifers, and salt caverns. Each of them has different characteristics relative to the amount of gas that can be stored and the speed with which the gas can be injected or extracted. A specific amount of gas in the storage, called cushion gas, is never extracted to maintain a high enough pressure level. This cushion gas can take up to 80% of the available space in the storage. The amount of gas available for operation is the

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[70] www.fluxys.com/en/Services/LNGTerminalling/TruckLoading/TruckLoading.aspx
[71] www.shell.com/static/nam-en/downloads/Brochure_Underground_Gas_Storage.pdf and www.eon-uk.com/generation/gas_storage.aspx
[72] www.eia.doe.gov/pub/oil_gas/natural_gas/analysis_publications/storagebasics/storagebasics.pdf

working gas. Typically gas installations have minimum and maximum injection and extraction rates. To be able to inject the gas into storage compressors are used to increase pressure. These compressors use some of the gas as their energy source, therefore there is a loss rate. Many storage facilities are owned by other players in the gas market, such as gas producers, traders, and pipeline network owners. Other storage facilities provide a service. They inject, store and extract gas for a third party, and the storage operators never own the (non-cushion) gas stored in their facilities. In the WGM the storage operators are modeled as regulated players. This is different from (Egging et al., 2008) and (Egging et al., 2010)) where storage operators provided seasonal swing services by executing seasonal arbitrage. Having regulated players appears to be a better representation for most of the storage operators in the market. Also in former versions of the WGM we observed price-undercutting behavior in the high and peak demand seasons by storage operators. The price undercutting was due to storage operators buying at perfectly competitive prices in the low demand season, and selling at non-strategic, perfectly competitive prices to the marketer in the high and peak demand seasons, thereby undermining the position of traders exerting market power relative to these marketers. In the version of WGM that we present in this chapter, the traders coordinate the injection and extraction volumes, and the undercutting of prices in the high and peak demand seasons cannot occur.

3.8.2 Assumptions

There are several roles in the natural gas market executed by storage operators. In the WGM, storage is assumed to be a price-taking service provider and storage operators are assumed to:

- maximize discounted profits
- be regulated players
- allocate scarce capacities for injection, extraction and working gas, implemented through the maximization of congestion charges for capacity usage
- have perfect information
- not withhold existing capacities from the market to manipulate congestion charges
- be able to inject and extract gas in any season
- be able to inject and extract gas at any rate lower than the respective capacities
- not discriminate among traders (c.f., TPA).

- have linear injection and extraction restrictions, that do not depend on stored volumes

3.8.3 Storage operator problem

The storage operator provides an economic mechanism to efficiently allocate storage capacity to traders (cf., (Hobbs, 2001), (Egging and Gabriel, 2006) and (Egging et al. 2008, 2010)). The storage and transmission operators are both modeled using congestion-charge approaches. Smeers (2008) addressed some shortcomings of congestion-pricing approaches for allocating infrastructure capacities. For the USA, assuming a profit incentive for infrastructure operators does in itself not represent legislation as implemented by FERC. In the EU, member states can implement regulated or negotiated access regimes.[73] Representing the actual regulatory frameworks and allocation mechanisms for infrastructure capacity would pose large challenges with regard to modeling activities as well as data collection, which is not done yet for the WGM. Instead, the storage operator maximizes the discounted profit resulting from selling injection capacity $SALES_{sdm}^{SI}$ and extraction capacity to traders $SALES_{sdm}^{SX}$. In equilibrium the capacity sales rates $SALES_{sdm}^{SI}$ and $SALES_{sdm}^{SX}$ are equal to the aggregate injection and extraction rates. Generally, regulators set maximum infrastructure usage charges based on the long-term marginal costs, i.e. the operating and maintenance costs plus a margin to earn a return on investment.[74] Our simplified assumption is that the regulated fees collected from the traders equal the costs and therefore in the model the profit margin is equal to the congestion fees for injection τ_{sdm}^{SI} and extraction τ_{sdm}^{SX}. Note that these congestion fees are not paid in actuality, cf. the pipeline congestion fees. Besides the regulated tariffs for injection and extraction, costs may be accrued to expand capacities for injection, extraction and total working gas: $b_{sm}^{SI}\Delta_{sm}^{SI} + b_{sm}^{SX}\Delta_{sm}^{SX} + b_{sm}^{SW}\Delta_{sm}^{SW}$.

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73 http://ec.europa.eu/energy/gas_electricity/interpretative_notes/doc/implementation_notes/2010_01_21_third-party_access_to_storage_facilities.pdf (Accessed Nov 8, 2010) ; http://ec.europa.eu/energy/electricity/legislation/doc/notes_for_implementation_2004/exemptions_tpa_en.pdf (Accessed Nov 8, 2010)

74 For instance, refer to FERC www.ferc.gov/industries/gas.asp , EC directive 2009/73/EC or www.naturalgas.org/regulation/regulation.asp

$$\max_{\substack{SALES^{SI}_{sdm},\\ SALES^{SX}_{sdm},\\ \Delta^{SI}_{sm},\Delta^{SX}_{sm},\Delta^{SW}_{sm}}} \quad \sum_{m\in M}\gamma_m \sum_{d\in D} days_d \begin{Bmatrix} \tau^{SI}_{sdm}SALES^{SI}_{sdm} + \tau^{SX}_{sdm}SALES^{SX}_{sdm} \\ -b^{SI}_{sm}\Delta^{SI}_{sm} - b^{SX}_{sm}\Delta^{SX}_{sm} - b^{SW}_{sm}\Delta^{SW}_{sm} \end{Bmatrix} \tag{3.8.1}$$

As discussed in the introduction of this subsection the compressors that are used to inject the gas have a limited capacity. The aggregate injection rate in any season is restricted by the injection capacity. Since capacities can be expanded, the total capacity in a year is the sum of the initial capacity $\overline{INJ}^{S}_{s}$ and the aggregate previous yearly expansions $\sum_{m'<m}\Delta^{SI}_{sm'}$.

The modeling of storage is limited in both the number of seasons represented in the model, as well as not addressing the dependence of maximum injection and extraction rates on the amount of natural gas actually in storage. This will impact model results, and likely underestimate the use of storage and capacity additions. However, relative to a Base Case, results of other cases are still illustrative for long-term developments. Eq. (3.8.2) provides the limits to extraction from storage and Eq. (3.8.3) represents the working gas limitations.

s.t.

$$SALES^{SI}_{sdm} \le \overline{CAP}^{SI}_{s} + \sum_{m'<m}\Delta^{SI}_{sm'} \quad \forall m,d \quad \left(\alpha^{SI}_{sdm}\right) \tag{3.8.2}$$

$$SALES^{SX}_{sdm} \le \overline{CAP}^{SX}_{s} + \sum_{m'<m}\Delta^{SX}_{sm'} \quad \forall m,d \quad \left(\alpha^{SX}_{sdm}\right) \tag{3.8.3}$$

$$\sum_{d\in D} days_d SALES^{SX}_{sdm} \le \overline{WG}^{S}_{s} + \sum_{m'<m}\Delta^{SW}_{sm'} \quad \forall m \quad \left(\alpha^{SW}_{sm}\right) \tag{3.8.4}$$

Limitations to allowable capacity expansions:

$$\Delta^{SI}_{sm} \le \overline{\Delta}^{SI}_{sm} \quad \forall m \quad \left(\rho^{SI}_{sm}\right) \tag{3.8.5}$$

$$\Delta^{SX}_{sm} \le \overline{\Delta}^{SX}_{sm} \quad \forall m \quad \left(\rho^{SX}_{sm}\right) \tag{3.8.6}$$

$$\Delta^{SW}_{sm} \le \overline{\Delta}^{SW}_{sm} \quad \forall m \quad \left(\rho^{SW}_{sm}\right) \tag{3.8.7}$$

Also, all variables are nonnegative:

$$SALES^{SI}_{sdm} \ge 0 \quad \forall m,d \tag{3.8.8}$$

$$SALES^{SX}_{sdm} \ge 0 \quad \forall m,d \tag{3.8.9}$$

$$\Delta^{SI}_{sm} \ge 0 \quad \forall m \tag{3.8.10}$$

$$\Delta^{SX}_{sm} \ge 0 \quad \forall m \tag{3.8.11}$$

$$\Delta_{sm}^{SW} \geq 0 \quad \forall m \qquad \textbf{(3.8.12)}$$

Note that mass balance for each storage facility (the *storage cycle* constraint), including accounting for losses, is dealt with for each separate trader, in Eq. (3.4.3).

The next section describes the Transmission System Operator, who is responsible for assigning available capacities of international transportation infrastructure to the traders; as well as for expansions of the transportation infrastructure.

3.9 The pipeline network

Ownership, management and operation of the pipeline network is done differently in various countries. In the past it was very common that pipeline owners also traded the gas. However regulatory authorities have recognized that the ownership of a pipeline provides the owner with a monopoly position in the transport of the gas, and too much leverage in negotiations. In the 1980s and 1990s, the U.S. energy regulator FERC took measures to enhance the access to transport infrastructure for third parties (Third Party Access, TPA) by forcing an unbundling of pipeline ownership and gas trade. [75] In Europe, the EC started to take similar actions in the late 1990s; however the process is not complete yet (International Energy Agency, 2006b).[76] To clarify some of the data assumptions regarding pipelines, see Figure 11 that shows part of the international high pressure pipelines in Western Europe.

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[75] www.ferc.gov and www.eia.doe.gov/pub/oil_gas/natural_gas/analysis_publications/ngpipeline/regulatory.html

[76] http://ec.europa.eu/competition/sectors/energy/inquiry/index.html; http://ec.europa.eu/news/energy/081010_1_en.htm and http://europa.eu/legislation_summaries/energy/internal_energy_market/l27077_en.htm

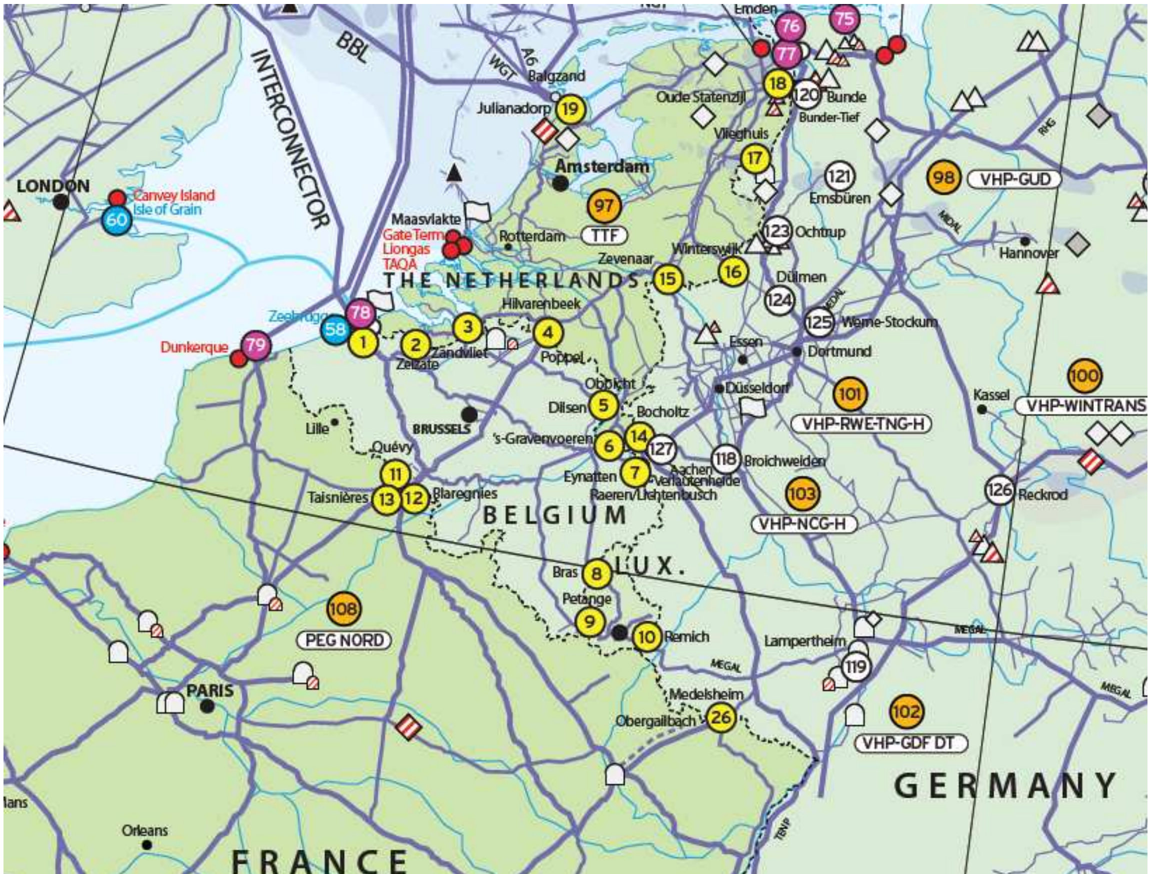

Figure 11: High-pressure pipelines in Western Europe – detail. Source: GTE[77]

In Figure 11 we can see a pipeline (BBL, 19) connecting the Netherlands with the United Kingdom, a pipeline connecting Belgium and the United Kingdom (Interconnector, 1) several pipelines (2-6) connecting the Netherlands and Belgium, and a number of pipelines (14-18) connecting the Netherlands and Germany.

What cannot be seen from the picture is that several of these numbers represent aggregated pipelines in reality, and also that there are pipeline networks for two different calorific values.[78] The WGM ignores the distinction between low and high calorific value gas as the relevance is limited in the context of a global natural gas market. Also, there is at most one pipeline going from one country to another, representing the aggregate pipeline capacities in one direction. This means for example that in the model there are two pipelines between the Netherlands and Germany, one in each direction, representing the five (groups of) pipelines in Figure 11 above.

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[77] http://gie.waxinteractive3.com/maps_data/capacity.html (Accessed June 9, 2009)
[78] http://gie.waxinteractive3.com/maps_data/downloads/GTE_CAP_DATA_April2009.xls

3.9.1 Engineering considerations

It is necessary to address pipeline capacities in an economic natural gas market model, since they limit the supplied volumes from producers to end-users. A big issue is that pipeline capacities depend on flows and pressure differences in neighboring pipelines and that these dependencies are non-linear.

There are various equations that relate pipeline diameter, length and pressure, such as *Weymouth* (Midthun, 2007) and *Colebrook and White* (More, 2006), and provide theoretically obtainable pipe flow rates in a pipeline network. The equations provide non-linear relations that cannot be directly included in an MCP. It would be possible to include linearized approximations, however at the expense of a significant increase in the number of equations and variables, and possibly very long solution times. Two recent dissertations have shed some light on how to address pipeline capacities in a modeling framework, both by using linearization techniques. Van Der Hoeven (2004) takes a simulation perspective and Midthun (2007) an optimization one.

Van Der Hoeven (2004) provided extensive details on gas quality, gas flow and pipeline network properties as well as various linearization approaches. His models included gas supply points, pipelines, compressor and pressure reduction stations, gas quality conversion stations and offtake points. He also addressed a distinguishing feature of the Dutch high pressure network of two somewhat linked networks, one for high and one for low calorific value natural gas.[79] This is due to the main production field in the Netherlands, the Groningen field, having much lower calorific value gas than most other fields in the Netherlands (and the rest of the world).

Midthun (2007) designed an optimization model for natural gas transportation, applied to the Norwegian continental shelf. His model includes the production fields, processing

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[79] Gross (Net) Calorific Value (GCV, NCV) = Upper (Lower) Wobbe index. The reason for having two different values is that not all heat from burning gas becomes available. Natural gas contains hydrogen atoms. When burning the gas the hydrogen atoms react with oxygen and form water vapor. The difference between GCV and NCV is the amount of energy needed to vaporize the water: the NCV excludes this energy. For natural gas the GCV is about 10% higher than the NCV. GCVs typically range from about 38 to 42 MJ/m^3, dependent on the local circumstances. Sources: www.iea.org, IEA 2006 Key World Energy Statistics p. 59, http://chp.defra.gov.uk/cms/fuel-calorific-value/

plants and delivery points to consumer markets. He addressed differences in calorific value and gas constituents from gas produced in different fields, as well as how pressure differences in one part of the pipeline system affect the transport capacity of other parts of the network. His objectives included minimizing deviations from producer's production schedules, minimizing the power usage in compressors and maximizing profits and consumer surplus while needing to meet contractual requirements for pressure, quality and volumes. The model was also equipped to manage disruptions (e.g., in pipelines) by deciding which production fields should provide swing in order to meet demand requirements.

Midthun refrained from the time-state, transitioning transient analysis and limited himself to steady-state analysis of pipeline flows, arguing that for planning purposes this limitation is minor.[80] However even the mathematical models for steady-state pipeline flows are non-linear and non-convex. He used linearization techniques to convexivy the feasible region and the objective functions. Midthun gave examples, such as how increasing a flow at some pipeline reduces the overall network capacity and how adding a pipeline at some place in the network requires additional compression at another part of the network to maintain flow capacities. In his last chapter he presented a two-stage stochastic MCP to model the booking process of pipeline capacity. From the perspective of the network operator three maximization objectives were compared: the aggregate flow, the aggregate value of the flows, and a social surplus measure with regard to the producers booking the transport capacities. He illustrated the approach with some small numerical examples.

Midthun et al. (2009)[81] discussed that ignoring the physical characteristics gas flows in a pipeline network will often lead to wrong conclusions regarding the maximum flows that can be transported through the network. The WGM does not explicitly contain the pipeline networks from production fields to the on-land off-take points. Only the onshore network is explicitly represented, although at an aggregate level. This differs from (Midthun, 2007) where the offshore pipelines are the main part of the network. Offshore

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[80] Some publications that have addressed transient analysis are: (More, 2006) and (Schroeder, Jr., 2001).
[81] Chapter 3 in Midthun (2007)

the production fields provide the pressure since no compressor stations can be put under water. However onshore, compressor stations can be used to increase pressure levels (although at the expense of some gas loss). It would be possible to use the approach developed by Midthun (2007) to represent physical pipeline characteristics in the model. This would significantly increase the number of equations and variables and require more computational effort to solve. Typically pipelines are designed to bear significantly higher pressures than the nominal ones, implying that maximum capacities are actually higher as well. Linepack, temporarily increasing the amount of gas and hence the pressure in part of the network is also a means of operational flexibility. As a result, actual capacities are not overstated by the nominal capacities in the dataset.

The demand variation in the model is aggregated to two seasons each year, and the operational consequences of daily and hourly changing flows are averaged out over these seasons. Given these considerations and the aggregation level of the pipelines in the data sets, the nominal values used for pipeline capacities are assumed to be representative for the actual pipeline capacities.

Some natural gas pipelines are bidirectional. In the WMG these pipelines are included as two separate capacities. Similar to (Mulder and Zwart, 2006), (Lise and Hobbs, 2009) and (Holz, 2009) there is no *netting* of flows. This will have no impact in a perfect competition setting, since in an optimal solution (assuming strictly positive costs or losses) at most one pipeline of the pair representing the bidirectional pipeline will be used. However traders exerting market power have an incentive to supply to other markets, what can result in congested pipelines in both directions. See Section 3.14.

3.9.2 Capacity expansions

There are limitations to how capacity investments and expansions can be modeled in an MCP. For example, at some point when designing an LNG plant, a decision about the gas turbines that power the compressors needs to be made. The used LNG processing equipment does not come in an unlimited variety of sizes (Spilsbury et al., 2005). Putting in a second turbine, or a bigger one, will generally results in a significant increase in both costs and capacity. This type of expansion is *integer-valued*, a characteristic that cannot be accommodated easily in an MCP. To preserve convexity, in our model we assume that any size of expansion can be made, i.e., all expansion variables are continuous.

If the integrality restrictions would be taken into account at the same time as the MCP, the resulting problem is difficult to solve and in some cases there may not be a solution. Several examples that do combine these two aspects of market equilibria include the work of (García-Bertrand et al., 2005, 2006), (Gabriel et al., 2006) and (Gabriel and Leuthold, 2010) in which electric power markets are modeled with the ability to turn certain power generating facilities on or off based on market conditions.

In practice, researchers often either fixed the level of investments exogenously or take a continuous relaxation of the integer restrictions but mostly in the context of solving an optimization problem and not an MCP. In the WGM we will adopt the relaxation approach. Zwart (2008) adopted a similar approach. Lise and Hobbs (2009) used a dynamic approach with a separate capacity expansion routine that was executed between periods in a forward-rolling single-period framework.

The WGM does not consider depreciation of existing infrastructure capacities, but assumes that the regulated fees or operational costs cover the needed investments for maintaining capacity levels. To see how capacity depreciation can be addressed in an MCP, see (Mulder and Zwart, 2006).

Since the capacity expansions are decided upon for all periods at once, the suggested modeling approach for capacity expansions entails an open-loop approach.

The number of gas transmission companies varies by country. In some countries, there is one organization responsible for the entire gas network, for instance in the Netherlands this is Gas Transport Services. In other countries, such as Denmark and the United Kingdom, there is one organization responsible for both the electricity and the natural gas grid.[82] In other countries there are several companies responsible for different parts of the gas transport network: for example, in Germany, there are sixteen companies and in the

[82] In Denmark this is Energinet.dk and in the United Kingdom: National Grid.

United States about two hundred. The model actor that manages the transportation network is the transmission system operators (TSO). Since the WGM is a sector model and in anticipation of a further unbundling of network ownership and trade, the TSO is a regulated player responsible for the gas transportation network only.

3.9.3 Assumptions

In reality, there is a large variation in gas transportation options. In the WGM, all transportation options are represented through a combination of capacity constraints, loss rates and transportation fees. The network is assumed to be managed by one operator, the TSO, which is assumed to:

- maximize discounted profits
- be a regulated player
- allocate scarce capacities for injection, extraction and working gas, implemented through the maximization of congestion charges for capacity usage
- have perfect information
- not withhold existing capacities from the market to manipulate congestion charges
- not discriminate among traders (c.f., TPA).

3.9.4 Transmission system operator problem

The TSO provides an economic mechanism to efficiently allocate transport capacity to traders.[83] As discussed in previous Subsection 3.8.3, Smeers (2008) addressed some shortcomings of congestion pricing approaches for allocating infrastructure capacities. A somewhat more enhanced approach would split the pipeline fees into a reservation and a usage charge. For instance, the reservation charge would have to be paid for the whole year over the maximum flow among the seasons, and usage charges for the seasonal flows. This approach would induce that in the model the traders would balance their flows more among seasons, and there would be an additional incentive to make use of storage. However, implementing such an approach would require significant additional effort in terms of modeling, data collection and computational power which is not done yet for the WGM. Instead, the TSO maximizes the discounted profit resulting from

[83] Note that the three parts in the LNG supply chain – liquefaction, shipment and regasification – are also represented as arcs, with appropriate costs, losses and capacities; and its management is also performed by the TSO. The underlying assumption is that all transportation infrastructure agents are regulated players.

selling arc capacity to traders $SALES_{adm}^{A}$ minus investment costs for capacity expansions Δ_{am}^{A}. Similarly as for the storage operator, we take as a starting point that the regulator sets a maximum capacity usage fee based on the long-term marginal costs. In the WGM the assumption is made that the regulated fees collected from the traders equal the costs; therefore the profit margin is equal to the congestion fee τ_{adm}^{A}.[84] Note that these congestion fees are not paid in actuality, but merely facilitate the efficient allocation of scarce capacity in the model. However, if the capacities would be auctioned, the congestion rates could be interpreted as the market-equilibrating bid price for capacity. This approach is not in conflict with the TSO (and storage operator) being regulated players.

$$\max_{\substack{SALES_{adm}^{A} \\ \Delta_{am}^{A}}} \quad \sum_{m \in M} \gamma_m \left\{ \sum_{d \in D} days_d \sum_a \tau_{adm}^{A} SALES_{adm}^{A} - \sum_a b_{am}^{A} \Delta_{am}^{A} \right\} \qquad \textbf{(3.9.1)}$$

The assigned capacity can be at most the available capacity. Available capacity at an arc *a* is the sum of the initial capacity $\overline{CAP}_{am}^{A}$ and expansions in the previous years $\sum_{m'<m} \Delta_{am'}^{A}$:

$$SALES_{adm}^{A} \leq \overline{CAP}_{am}^{A} + \sum_{m'<m} \Delta_{am'}^{A} \quad \forall a,d,m \quad \left(\alpha_{adm}^{A}\right) \qquad \textbf{(3.9.2)}$$

There may be budgetary or other limits to possible yearly capacity expansions:

$$\Delta_{am}^{A} \leq \overline{\Delta}_{am}^{A} \quad \forall a,m \qquad \left(\rho_{am}^{A}\right) \qquad \textbf{(3.9.3)}$$

Lastly, all variables are nonnegative:

$$SALES_{adm}^{A} \geq 0 \qquad \textbf{(3.9.4)}$$

$$\Delta_{am}^{A} \geq 0 \qquad \textbf{(3.9.5)}$$

The above sections have presented the optimization problems for all market agents that are incorporated in the WGM. Some other market agents are only incorporated implicitly. The main one being the final consumption sectors that are represented via an aggregation of the sector-level inverse demand functions, which in turn represents the marketer.

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[84] For instance, refer to FERC www.ferc.gov/industries/gas.asp, EC directive 2009/73/EC or www.naturalgas.org/regulation/regulation.asp

3.10 Marketer, distribution and consumption sectors

The last step in the natural gas supply chain is the distribution to final consumers. In Europe, marketing and distribution companies include RWE and GDF-Suez which are active in several countries. In the United States examples include San Diego Gas & Electric and the Southern California Gas Co which are active in some of the Southern U.S. States; and Baltimore Gas and Electric Company and Washington Gas who market their gas in Maryland and Washington DC. Often there is some vertical integration with producers, and horizontal integration, e.g., when utilities also sell electricity.

There are various sectors using natural gas. Mostly natural gas is used as a source of energy; however there are also non-energy uses, such as the production of fertilizers. The International Energy Agency provides detailed information for the natural gas use in most countries. Table 2 shows the sources and uses for natural gas in the USA in 2006 in Trillion cubic feet (Tcf) and billion cubic meters (bcm).[85]

Table 2: Natural gas supply and use in the USA in 2006. Source IEA.

Category	**Tcf**	**Bcm**
Production	**18.5**	**523.6**
From Other Sources	0.1	1.4
Imports	4.2	117.9
Exports	-0.7	-20.1
Stock Changes	-0.4	-12.3
Domestic Supply	21.6	610.5
Statistical Differences	0.1	2.7
Total Transformation	**7.0**	**198.0**
Electricity Plants	5.1	144.7
Combined Heat & Power Plants	1.9	53.3
Energy Sector	**1.8**	**51.6**
Total Final Consumption	**12.8**	**363.5**
Industry	4.5	126.6
Transport	0.6	17.3
Residential	4.4	123.7
Commercial and Public Services	2.8	78.9
Non-Energy Use	0.6	17.0
- of which Petrochemical Feedstocks	0.6	15.9

[85] www.iea.org/Textbase/stats/gasdata.asp?COUNTRY_CODE=US, gross calorific values, conversion of Terajoules to Tcf using factor 1.085, and Tcf to bcm using factor (1000/35.31). Definitions of categeries can be found at www.iea.org/Textbase/stats/defs/defs.htm

For the level and type of analysis of the world gas market for which the WGM is used, it would not help much to include all different sectors and/or different marketers in each country. Having fewer variables in the model will likely reduce the solution time needed . Therefore only the main sectors are selected and aggregated to form a representative and manageable data set. The equations representing the consumption are the inverse demand curves:

$$\left(\Pi_{ndm}^{M}\left(\cdot\right)=\right)\pi_{ndm}^{M}=INT_{ndm}^{M}+SLP_{ndm}^{M}\cdot\sum_{t}SALES_{tndm}^{T}\quad\forall n,d,m\quad\left(\pi_{ndm}^{M}\right)\qquad\textbf{(3.10.1)}$$

As such, the marketer is not incorporated as a profit-maximizing agent in the WGM. Although the inverse demand curves are on a country-level, the model has been calibrated by sector level, and after solving the model all sector consumptions can be inferred from a solution.[86]

3.11 Market-clearing conditions

There are four types of market-clearing conditions (mcc) tying the various optimization problems together into one market-equilibrium problem. Market clearing of produced volumes between producers and traders, and the market-clearing price $\pi_{n(p)dm}^{P}$:

$$SALES_{pdm}^{P}=\sum_{t(p)}PURCH_{tn(p)dm}^{T}\quad\forall p,d,m\quad\left(\pi_{n(p)dm}^{P}\right)\qquad\textbf{(3.11.1)}$$

Market clearing for injection capacities and volumes:

$$SALES_{sdm}^{SI}=\sum_{t\in T\left(N(s)\right)}INJ_{tsdm}^{T}\quad\forall s,d,m\quad\left(\tau_{sdm}^{SI}\right)\qquad\textbf{(3.11.2)}$$

Market clearing for extraction capacities and volumes:

$$SALES_{sdm}^{SX}=\sum_{t\in T\left(N(s)\right)}XTR_{tsdm}^{T}\quad\forall s,d,m\quad\left(\tau_{sdm}^{SX}\right)\qquad\textbf{(3.11.3)}$$

Market clearing between the TSO and the traders for arc capacities and flows:

$$SALES_{adm}^{A}=\sum_{t}FLOW_{tadm}^{T}\quad\forall a,d,m\quad\left(\tau_{adm}^{A}\right)\qquad\textbf{(3.11.4)}$$

[86] As long as market prices are higher than the slopes of the inverse demand curves of individual sectors, the results when using aggregate or non-aggregate inverse demand curves will be the same. This will is illustrated with an example in section 2.6 of Chapter 4.

3.12 Summary

This chapter has provided a discussion and considerations for the development of the World Gas Model. The following agents are represented in the WGM.

Table 3: Represented market participants in WGM

Agent	Role	Comment
Producer	Produces and sells natural gas.	See Section 3.3
Trader	Buys gas from producers as well as, transportation and storage services from the TSO and storage operator and sells gas to marketers.	See Section 3.4
Liquefier	Provides liquefaction services to traders.	See Section 3.5
LNG vessels	Provide LNG shipment services to traders.	Represented by distance-dependent costs and losses.
Regasifier	Provides regasification services to traders.	See Section 3.6
Storage Operator	Provides injection, storage and extraction services for the trader.	See Section 3.8
Transmission System Operator	Assigns arc capacities to traders who need to transport gas from one country to another; and is responsible for transportation network expansions.	See Section 3.9
Marketer	Buys natural gas from traders and distributes it to end-users	See Section 3.10
End-users	The three consumption sectors: power generation, industry and residential/commercial	See Section 3.10

For all players in the WGM, we have provided an explanation of their economic role; discussed engineering aspects related to hydrocarbon reservoirs, transportation and storage infrastructure; and presented a mathematical formulation for the optimization problems and market-clearing conditions.

The actual MCP consists of the KKT derived from the optimization problems and the market-clearing conditions described next.

3.13 Karush-Kuhn-Tucker conditions

We have introduced the symbols used below in Section 3.3.2. In the KKT the left-hand sides (relative to the $\perp$-sign) are the equations, the right-hand sides the variables. Primal variables are English words in capitals, and dual variables are written as Greek symbols.

3.13.1 KKT conditions for the producer's problem

$$0 \le \gamma_m days_d \left(-\pi^P_{n(p)dm} + \frac{\partial c^P_{pm}(SALES^P_{pdm})}{\partial SALES^P_{pdm}} \right) + \alpha^P_{pdm} + days_d \beta^P_p \perp SALES^P_{pdm} \ge 0, \forall d, m \qquad \textbf{(3.13.1)}$$

$$0 \le \overline{PR}^P_{pm} - SALES^P_{pdm} \quad \perp \quad \alpha^P_{pdm} \ge 0, \quad \forall d, m \qquad \textbf{(3.13.2)}$$

$$0 \le \overline{PH}_p - \sum_{m \in M} \sum_{d \in D} days_d SALES^P_{pdm} \quad \perp \quad \beta^P_p \ge 0 \qquad \textbf{(3.13.3)}$$

3.13.2 KKT conditions for the trader's problem

$$0 \le days_d \gamma_m \begin{pmatrix} \delta^C_{t,n} SLP^W_{ndm} SALES^T_{tndm} \\ -\left(\delta^C_{t,n} \Pi^W_{ndm} + (1-\delta^C_{t,n}) \pi^W_{ndm} \right) \end{pmatrix} + \phi^T_{tndm} \perp SALES^T_{tndm} \ge 0, \quad \forall t, n, d, m \qquad \textbf{(3.13.4)}$$

$$0 \le days_d \gamma_m \pi^P_{ndm} - \phi^T_{tndm} \quad \perp \quad PURCH^T_{tndm} \ge 0, \quad \forall n \in N(p(t)), d, m \qquad \textbf{(3.13.5)}$$

$$0 \le days_d \gamma_m \left(\tau^{SI,reg}_{ndm} + \tau^{SI}_{ndm} \right) + \phi^T_{tndm} - \left(1 - loss_n\right) days_d \phi^S_{tnm} \perp INJ^T_{tdnm} \ge 0, \quad \forall n, m \qquad \textbf{(3.13.6)}$$

$$0 \le days_d \gamma_m \tau^{SX}_{ndm} - \phi^T_{tndm} + days_d \phi^S_{tnm} \perp XTR^T_{tndm} \ge 0, \quad \forall n, m \qquad \textbf{(3.13.7)}$$

$$0 \le \begin{matrix} days_d \left[\gamma_m \left(\tau^A_{adm} + \tau^{\text{A,reg}}_{adm} \right) \right] \\ +\phi^T_{tn_{a-}dm} - (1-loss_a) \phi^T_{tn_{a+}dm} - \varepsilon^T_{tadm} \end{matrix} \quad \perp \quad FLOW^T_{tadm} \ge 0, \quad \forall a = \left(n_{a-}, n_{a+}\right), d, m \qquad \textbf{(3.13.8)}$$

$$0 = \begin{pmatrix} PURCH^T_{tndm} + \sum_{a \in a^+(n)} \left(1 - loss_a\right) FLOW^T_{tadm} + XTR^T_{tndm} \\ -SALES^T_{tndm} - \sum_{a \in a^-(n)} FLOW^T_{tadm} - INJ^T_{tndm} \end{pmatrix}, \quad \varphi^T_{tndm} \, free, \quad \forall n, d, m \qquad \textbf{(3.13.9)}$$

$$0 = \begin{matrix} \left(1 - loss_s\right) \sum_d days_d INJ^T_{tsdm} \\ -\sum_d days_d XTR^T_{tsdm} \end{matrix}, \quad \varphi^S_{tsm} \;\; free, \forall n, s \in S\left(N\left(t\right)\right), d, m \qquad \textbf{(3.13.10)}$$

$$0 \le FLOW^T_{tadm} - CON^T_{tadm} \quad \perp \varepsilon^T_{tadm} \ge 0, \forall a, d, m \qquad \textbf{(3.13.11)}$$

3.13.3 KKT conditions for storage operator's problem

$$0 \le -days_d \gamma_m \tau^{SI}_{sdm} + \alpha^{SI}_{sdm} \perp SALES^{SI}_{sdm} \ge 0, \quad \forall d, m \qquad \textbf{(3.13.12)}$$

$$0 \le -days_d \gamma_m \tau^{SX}_{sdm} + \alpha^{SX}_{sdm} + days_d \alpha^{SW}_{sm} \perp SALES^{SX}_{sdm} \ge 0, \quad \forall d, m \qquad \textbf{(3.13.13)}$$

$$0 \le \gamma_m b^{SI}_{sm} - \sum_{d \in D} \sum_{m' > m} \alpha^{SI}_{sdm'} + \rho^{SI}_{sm} \quad \perp \quad \Delta^{SI}_{sm} \ge 0, \quad \forall m \qquad \textbf{(3.13.14)}$$

$$0 \leq \gamma_m b_{sm}^{SX} - \sum_{d \in D} \sum_{m'>m} \alpha_{sdm'}^{SX} + \rho_{sm}^{SX} \quad \perp \quad \Delta_{sm}^{SX} \geq 0, \quad \forall m \tag{3.13.15}$$

$$0 \leq \gamma_m b_{sm}^{SW} - \sum_{d \in D} \sum_{m'>m} \alpha_{sm'}^{SW} + \rho_{sm}^{SW} \quad \perp \quad \Delta_{sm}^{SW} \geq 0, \quad \forall m \tag{3.13.16}$$

$$0 \leq \overline{CAP}_{sm}^{SI} + \sum_{m'<m} \Delta_{sm'}^{SI} - SALES_{sdm}^{SI} \ \perp \ \alpha_{sdm}^{SI} \geq 0 \quad \forall m, d \tag{3.13.17}$$

$$0 \leq \overline{CAP}_{sm}^{SX} + \sum_{m'<m} \Delta_{sm'}^{SX} - SALES_{sdm}^{SX} \ \perp \ \alpha_{sdm}^{SX} \geq 0 \quad \forall m, d \tag{3.13.18}$$

$$0 \leq \overline{WG}_s^{S} + \sum_{m'<m} \Delta_{sm'}^{SW} - \sum_{d \in D} days_d SALES_{sdm}^{SX} \ \perp \ \alpha_{sm}^{SW} \geq 0 \quad \forall m \tag{3.13.19}$$

$$0 \leq \overline{\Delta}_{sm}^{SI} - \Delta_{sm}^{SI} \quad \perp \quad \rho_{sm}^{SI} \geq 0 \quad \forall m \tag{3.13.20}$$

$$0 \leq \overline{\Delta}_{sm}^{SX} - \Delta_{sm}^{SX} \quad \perp \quad \rho_{sm}^{SX} \geq 0 \quad \forall m \tag{3.13.21}$$

$$0 \leq \overline{\Delta}_{sm}^{SW} - \Delta_{sm}^{SW} \quad \perp \quad \rho_{sm}^{SW} \geq 0 \quad \forall m \tag{3.13.22}$$

3.13.4 KKT conditions for the transmission system operator's problem

$$0 \leq -days_d \gamma_m \tau_{adm}^{A} + \alpha_{adm}^{A} \quad \perp \quad SALES_{adm}^{A} \geq 0 \quad \forall a, d, m \tag{3.13.23}$$

$$0 \leq \overline{CAP}_{am}^{A} + \sum_{m'<m} \Delta_{am'}^{A} \quad - SALES_{adm}^{A} \perp \quad \alpha_{adm}^{A} \geq 0 \quad \forall a, d, m \tag{3.13.24}$$

$$0 \leq \gamma_m b_{am}^{A} - \sum_{d \in D} \sum_{m'>m} \alpha_{adm'}^{A} + \rho_{am}^{A} \quad \perp \quad \Delta_{am}^{A} \geq 0 \quad \forall a, m \tag{3.13.25}$$

$$0 \leq \overline{\Delta}_{am}^{A} - \Delta_{am}^{A} \quad \perp \quad \rho_{am}^{A} \geq 0 \quad \forall a, m \tag{3.13.26}$$

3.13.5 Market clearing between producer and trader

$$0 = SALES_{pdm}^{P} - \sum_{t(n) \in T(p)} PURCH_{t(n)n(p)dm}^{T} \quad , \quad \pi_{pdm}^{P} \ free \quad \forall p, d, m \tag{3.13.27}$$

3.13.6 Market clearing for pipeline flows

$$0 = SALES_{adm}^{A} - \sum_{t} FLOW_{tadm}^{T} \quad , \ \tau_{adm}^{A} \ free \quad \forall a, d, m \tag{3.13.28}$$

3.13.7 Market clearing for injection and extraction volumes

$$0 = SALES_{sdm}^{SI} - \sum_{t \in T(N(s))} INJ_{tsdm}^{T} \quad , \quad \tau_{sdm}^{SI} \ free \quad \forall s, d, m \tag{3.13.29}$$

$$0 = SALES_{sdm}^{SX} - \sum_{t \in T(N(s))} XTR_{tsdm}^{T} \quad , \quad \tau_{sdm}^{SX} \ free \quad \forall s, d, m \qquad \textbf{(3.13.30)}$$

3.13.8 The inverse demand curve

$$0 = \pi_{ndm}^{W} - \left(INT_{ndm}^{M} - SLP_{ndm}^{M} \cdot \sum_{t \in T(n)} SALES_{tndm}^{T} \right) \ , \pi_{ndm}^{W} \ free \ \ \forall n, d, m \qquad \textbf{(3.13.31)}$$

The combination of all the Karush-Kuhn-Tucker conditions, the market-clearing conditions and inverse demand curves form the MCP. All profit functions are concave and differentiable, all cost functions are convex and differentiable and all feasible regions are polyhedral, thus, the KKT points for this system are necessary and sufficient for optimal solutions.[87]

Some market-clearing conditions can be specified and implemented as inequalities. Personal experience indicates that using inequality conditions results in somewhat shorter solution times relative to equality conditions.

3.14 Example: market-power players diversify supplies

Assume two suppliers at neighboring nodes *A* and *B*, each with a supply cost function: $c_X\left(s^X\right) = c_X\left(s^{XA} + s^{XB}\right) = s^{XA} + s^{XB}$, for $S = A, B$. There is consumption at both nodes with demand functions for q^A, and q^B depending on the price p^X: $q^x\left(p^X\right) = 10 - p^X$, for $X = A, B\ p^X$. Figure 12 below shows the market structure for this example.

[87] Since we are minimizing the negative of a concave profit function, we are effectively minimizing a convex function. See (Bazaraa et al., 1993) or Chapter 2 for more details on necessary and sufficient conditions.

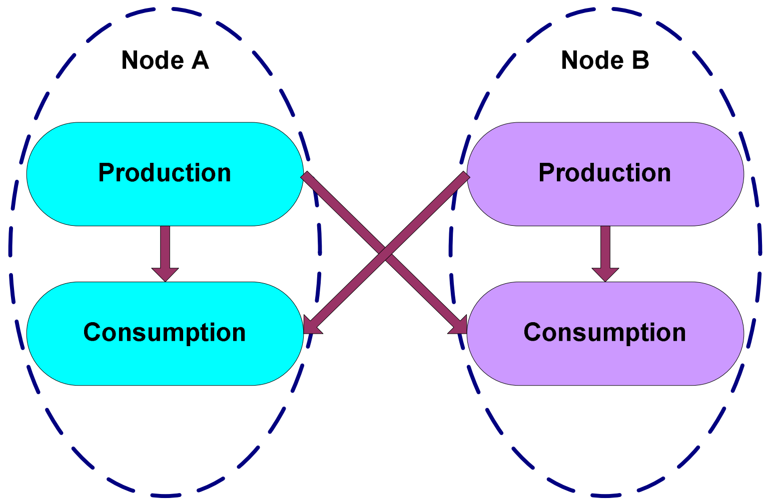

Figure 12: Market structure for example

The transport costs between the nodes is *1* per unit. There are no limitations to transported amounts. Let q^{XY} denote the supply from supplier at node X to the consumer at node Y. Also, $q^{XX} + q^{XY} = s^X$. First assume that the suppliers operate perfectly competitive. Then the profit for S^A:

$\Pi^A\left(q^{AA}, q^{AB}\right) = p^{AA}q^{AA} + p^{AB}q^{AB} - 1\left(q^{AA} + q^{AB}\right) - 1\left(q^{AB}\right)$ (due to symmetry, the profit function for S^B is identical). Because the marginal supply costs at both nodes are constant, we can completely separate the profit functions by node:

$\Pi^A\left(q^{AA}, q^{AB}\right) = \Pi^{AA}\left(q^{AA}\right) + \Pi^{AB}\left(q^{AB}\right)$, wherein:

$\Pi^{AA}\left(q^{AA}\right) = p^A q^{AA} - 1q^{AA}$ and $\Pi^{AB}\left(q^{AB}\right) = p^B q^{AB} - 2q^{AB}$. Taking the partial derivatives and setting them to zero: $\frac{\partial \Pi^{AA}(q_{AA})}{\partial q_{AA}} = p^A - 1 = 0$ and $\frac{\partial \Pi^{BA}(q_{BA})}{\partial q_{BA}} = p^B - 2 = 0$ gives us the prices p^A=1 and p^B=2. At these prices, S^A will only supply to node A since his costs to supply to node B are 2 per unit. Similarly, S^B will only to node B. Thus $q^A = q^{AA} = q^B = q^{BB} = 9$, $q^{AB} = q^{BA} = 0$.

Now assume that the suppliers compete à la Cournot. We first derive the optimal response curves. (Again, due to symmetry, they are identical). The profits for S^A contain the revenues from both nodes, and the production and transport costs:

$\Pi^{A}\left(q^{AA},q^{AB}\right)=$ $p^{AA}q^{AA}+p^{AB}q^{AB}-1\left(q^{AA}+q^{AB}\right)-1\left(q^{AB}\right)$ $=$

$\left(10-q^{AA}-q^{BA}\right)q^{AA}+\left(10-q^{AB}-q^{BB}\right)q^{AB}-1\left(q^{AA}+q^{AB}\right)-1\left(q^{AB}\right)$.

As before, we can completely separate the profit functions and optimal response curves for the players by node. For the profit functions we get:

$\Pi^{AA}\left(q^{AA}\right)=\left(10-q^{AA}-q^{BA}\right)q^{AA}-1q^{AA}$ and $\Pi^{AB}\left(q^{AB}\right)=\left(10-q^{AB}-q^{BB}\right)q^{AB}-2q^{AB}$.

To find the market equilibrium at node *A* we need to solve the first order conditions (cf., Section 2.3 in Chapter 2).

$\frac{\partial\Pi^{AA}\left(q^{AA}\right)}{\partial q^{AA}}=9-q^{BA}-2q^{AA}=0$, which gives an optimal response curve of $q^{AA}=\frac{9-q^{BA}}{2}$; and $\frac{\partial\Pi^{AB}\left(q^{AB}\right)}{\partial q^{AB}}=8-q^{BB}-2q^{AB}=0$, which gives an optimal response curve of $q^{BA}=\frac{8-q^{BB}}{2}$.

Some algebra reveals: $q^{AA}=q^{BB}=\frac{10}{3}$, $q^{AB}=q^{BA}=\frac{7}{3}$ and a market price at both nodes of $\frac{13}{3}$. We see that the resulting market price is $\frac{10}{3}$ higher than the perfectly competitive equilibrium price.

When we want to use the mark-up approach suggested by (Smeers, 2008), we would use a mark-up to the marginal supply costs of $\frac{10}{3}$, the difference between the perfectly competitive price of 1 and the observed price of $\frac{13}{3}$. The marginal supply function would look like: $c(q)=\left(1+\frac{10}{3}\right)q=\frac{13}{3}q$. Calculating the perfectly competitive equilibrium using the adjusted supply cost functions results in $q^{AA}=q^{BB}=\frac{17}{3}$, $q^{AB}=q^{BA}=0$. Indeed, the consumed quantities and prices are identical to the previous duopoly example, however there is no trade, since due to the assumption of perfect competition overall transport costs are minimized.

Generally using the mark-up approach will result in fewer and lower trader flows and no counter flows. To better represent these characteristics of the natural gas market, we think that it is important to model market power based on Cournot oligopoly theory, and not merely use mark-ups on supply cost functions.

4 World Gas Model Scenario Results

The World Gas Model (WGM) is a multi-period, mixed complementarity problem for the global natural gas market. The model represents the main economic agents in the natural gas markets and their interactions, addressing seasonal variation in consumption and allowing for endogenous capacity expansions in transportation and storage.

The consumers are represented via their aggregate inverse demand function. All other market players are modeled via their respective profit maximization problems under some player-specific operational and technical constraints. The implementation of the model is done in GAMS (Brook et al., 1988), and following good software development practices, the data files and model formulation are completely separate, to have maximum flexibility when developing scenarios or needing the use of alternative data sets.

The first part of this chapter will discuss the compilation of the data set and the development of a reference baseline scenario. The second part of the chapter will illustrate the insights that can be obtained from running the model on different scenarios. Beside the calibrated Base Case, three other cases are discussed. The investigated issues concern the formation of a global gas market cartel, the potential impact of much lower production of unconventional natural gas in the United States and the impact of lower transport costs (relative to the Base Case). The model outputs give insight in the impact of various developments on wholesale prices, profits of traders and consumer surplus in different countries and regions. Some more specific analyses will support that there is an economic rationale for LNG import terminals in the Netherlands, but that the construction of pipelines from the Caspian region to Europe via Turkey needs other than economic motivations. The analysis illustrates the strengths of the model: the combination of a representation of upstream market power and the level of detail and dependencies within and among continental regions. The results illustrate the added value of the modeling approach and the relevance for policy makers and business analysts.

4.1 Introduction

Figure 13 shows an overview of the forty countries and regions that are incorporated as nodes in the WGM.[88]

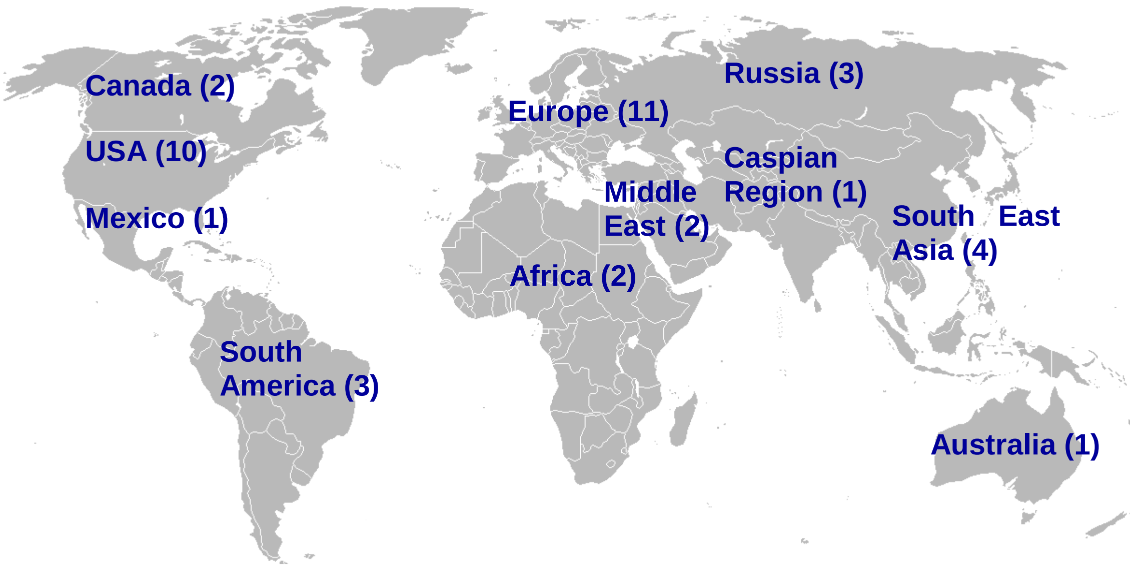

Figure 13: Model nodes in the WGM

The numbers in parentheses indicate the number nodes in that region in the model. Due to the limited relevance and impact of countries that produce and/or consume only small amounts of natural gas, several have been grouped with neighboring countries and are represented in the model on an aggregate level. For instance, the neighboring countries Belgium, Luxembourg and France are aggregated into one node since they all have negligible production and depend on imports to fulfill domestic consumption. For other countries the opposite is true: their consumption or production is so high, and the geographical distances so large, that a division of the countries into several regions is warranted. For example, as can be seen in Figure 14, the United States consists of ten model nodes. Having two nodes for Canada and one for Mexico, the average consumption of North American nodes in 2005 is 53 bcm/y and of European nodes 55 bcm/y.[89] This illustrates why the United States is split and in Europe most countries are aggregated into model nodes.

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[88] Sources for the blank maps in this chapter: http://en.wikipedia.org/wiki/Wikipedia:Blank_maps

[89] bcm/y = billion cubic meter per year

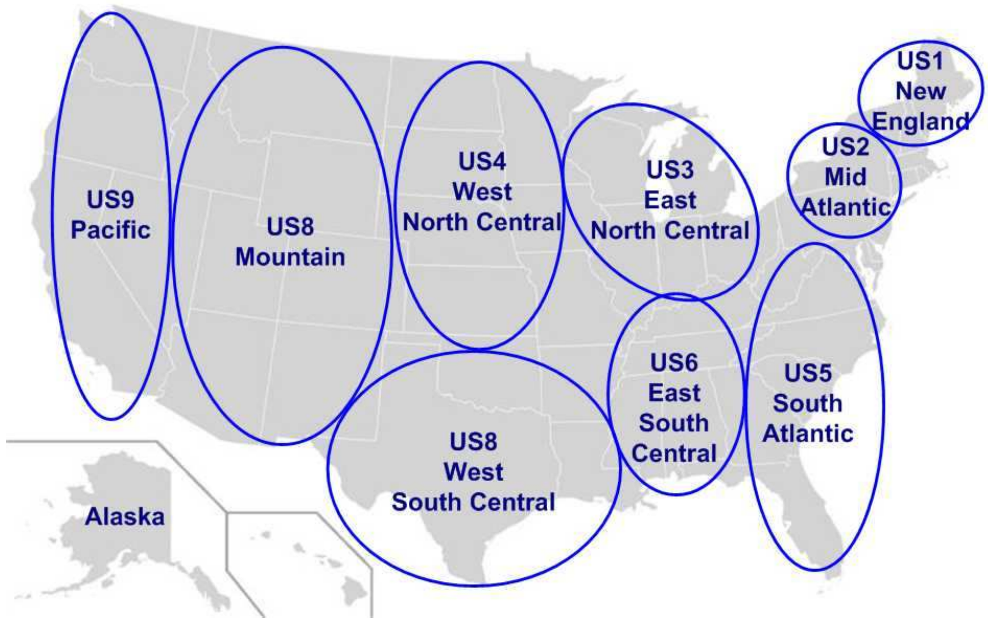

Figure 14: Ten model nodes for USA (incl. Alaska)

See Figure 15 for the aggregation level of the model nodes in Europe.

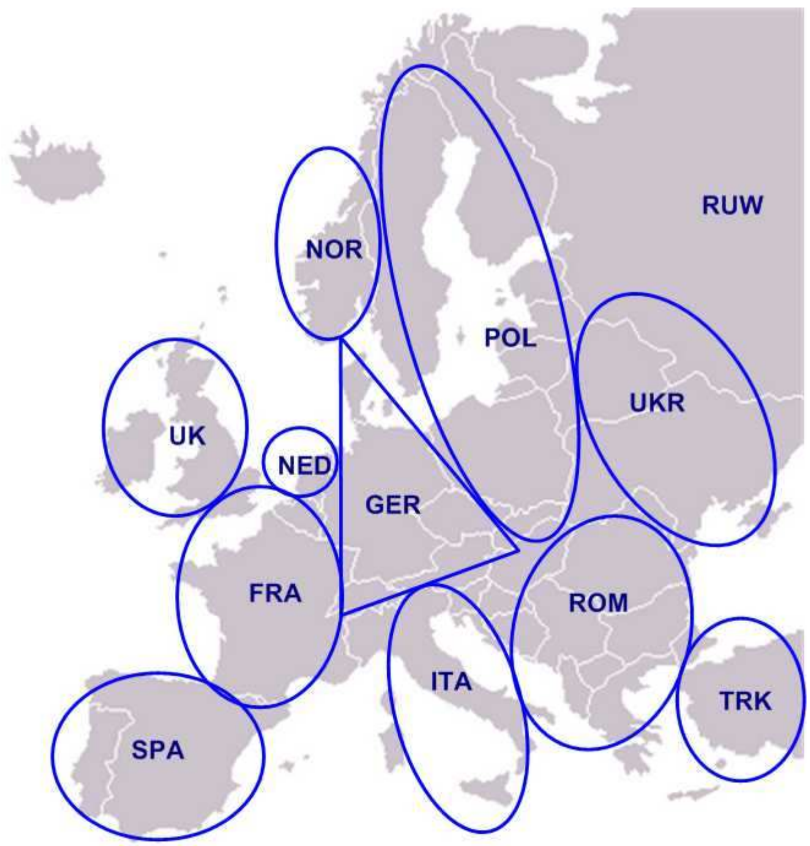

Figure 15: Ten model nodes in Europe plus Western Russia

A full list of model nodes countries can be found in Section 4.6 at the end of this chapter.

When compiling the model data set, the goal was to be thorough, representative and to cover virtually all production, consumption and trade in the present and near-future global natural gas market. The annual issues of Natural Gas Information (e.g., (International Energy Agency, 2009)) and Statistical Review of World Energy (e.g., (BP, 2009)) have been very helpful in the data collection process. The following will address the data sources used and some of the challenges when compiling the data sets for the various market agents.[90]

4.2 Data collection

In the data set usually one player is included of every type at every model node. For instance, at the node Netherlands, there is one producer, one trader, one marketer and one storage facility. However to facilitate the implementation of scenarios some other levels of detail were sometimes more appropriate. For example, in some case studies the impact of availability of unconventional natural gas was investigated (Gabriel et al., 2010) and therefore in the United States model nodes have two production entities: one for conventional and one for unconventional gas. In most other countries – except for Canada – unconventional gas either hardly contributes to total gas supply, or the data are simply not available to be able to distinguish conventional and unconventional gas production in the data set.[91] Consequently, most model nodes have just one production entity. In contrast to having multiple producers for U.S. nodes, there is only one trader in the United States that represents all producers. Having just one trader will limit the number of model variables drastically and will not affect the outcomes due to the assumed perfectly competitive nature of the upstream gas market in North America.[92] In contrast, all three Russian producers will operate through one shared trader allowing GazProm to coordinate all exports and potentially exert market power.

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[90] Dr. Franziska Holz contributed greatly to the data descriptions in (Egging et al., 2010).
[91] See, e.g., the website of the Canadian National Energy Board, www.neb-one.gc.ca
[92] Reducing the number of traders results in fewer variables for flows, storage injection, storage extraction, sales to end-users and various dual prices. The reduction in flow variables is the most significant. Having one instead of ten traders in the United States for eight periods, two seasons and 38 arcs results in $8 \times 2 \times (10-1) \times 38 = 5472$ fewer variables. Including reductions in other variables the total reduction is more than 12,000. Eventually the model as ran for the Base Case included 43,560 variables.

The WGM is a multi-period long term equilibrium model. A yearly representation of the market would be computationally challenging. In that regard it was necessary to define periods rather than years. The model data set includes every fifth year in the time horizon 2005 through 2030, with two added periods 2035 and 2040 to minimize distortions of endogenous capacity expansions in the last reporting year. The initial data set is built up for the year 2005; however known capacity expansions between 2005 and 2008 (time of data base construction) of pipelines, liquefiers, regasifiers and storage were exogenously included in the model year 2010. In future years the model allows for completely new pipelines and LNG capacities based on profit maximizing decisions of players in the model.

A 10% discount rate is used in the multi-period optimization decisions.[93] In general, the data are per day, distinguished by season (low and high demand) where applicable.[94] On the supply side, capacity constraints limit the amounts that can be produced and transported. Starting in 2010, there can be endogenous investments in transport and storage infrastructure. In order to maintain an MCP, capacity expansions need to be continuous. The expansions are limited in each period; where available project information is used to determine these limits, otherwise own assessments have been made.

4.2.1 Production data

The model formulation presented in Chapter 3 assumes convex production costs in order to make the Karush-Kuhn-Tucker conditions sufficient for optimality (e.g., (Bazaraa et al., 1993), or Chapter 2 of this thesis). Similar to (Boots et al., 2004), (Egging and Gabriel, 2006) and (Egging et al., 2008, 2010) to model the production costs the functional form proposed by (Golombek et al., 1995) is used. The production costs can be

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[93] The discount rate of 10% is a real discount rate including a risk-adder. All $-values in the model are in $ of 2005. The value of 10% is chosen in the range of values used by other models. For instance, depending on the analysis, the EIA uses varying discount rates to evaluate the cost-effectiveness of different investments. Low discount rates (3-4 percent) are generally used to capture a "societal" cost or benefit of a particular investment, while high rates (10-15 percent) are used to discount purchases by the typical consumer, paid for on credit. (http://www.eia.doe.gov/oiaf/servicerpt/eff/aircond.html, Accessed Nov 10, 2010) Commercial Another paper comparing various papers, presents values in the range of 5-12.5 percent (Table 1 in http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.163.9206 (Accessed Nov 10, 2010).

[94] The low-demand season is defined as the period from April through October. The high-demand season is the other half of the year.

expressed as $C(q)=(\alpha-\gamma)q+\frac{1}{2}\beta q^2+\gamma(Q-q)\ln\left(\frac{Q-q}{Q}\right)\ \forall q:0\leq q<Q$. Q is the production capacity, $\alpha>0$ is the minimum marginal unit cost term, $\beta\geq 0$ is the per unit linearly-increasing cost term, and $\gamma\leq 0$ is a term that induces high marginal costs when production is close to full capacity. The marginal supply cost curve for this expression is: $C'(q)=\alpha+\beta q+\gamma\ln\left(\frac{Q-q}{Q}\right)$. Figure 16 illustrates the shape of this curve for parameter values $\alpha=10$, $\beta=\frac{mmQ-\alpha}{Q}=\frac{40-10}{Q}=\frac{30}{Q}$ and $\gamma=\frac{mmQ-mmG}{6.90776}=\frac{-80}{6.90776}=-11.58$.[95]

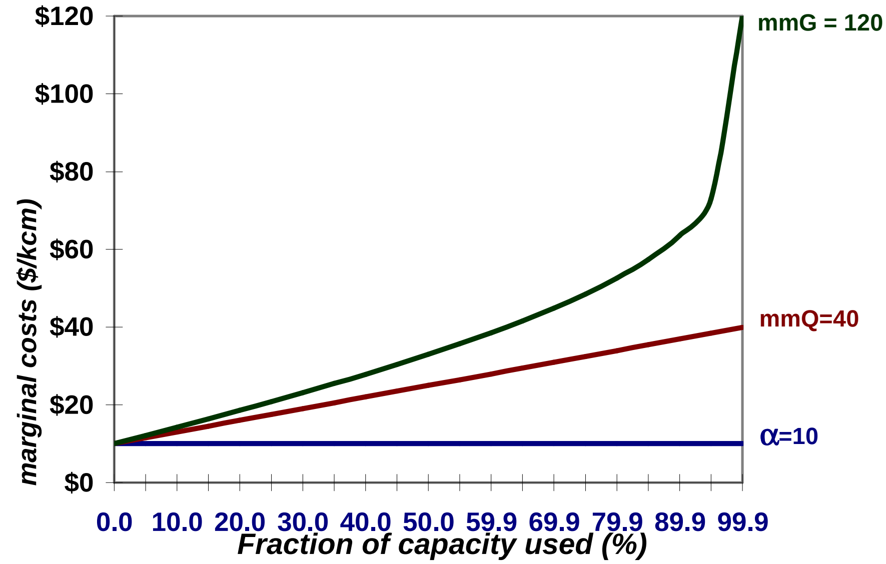

Figure 16: Example of a marginal production cost curve

Short-run production costs are similar to (Egging et al., 2010). The parameters for the cost functions were originally derived from (Observatoire Méditerranéen de l'Energie, 2003) and have been updated. Production capacity data originate from forecasts and information in the technical literature (e.g., (International Energy Agency, 2008), Oil and Gas Journal). Production capacity is determined exogenously for all model periods, thus there is no endogenous investment in production capacity expansions. As explained in Chapter 3, the data needed for modeling this part of the supply side of the natural gas market is not publicly available. Therefore, future production capacities are based on projections of the PRIMES model for Europe (European Commission, 2006, 2008), EIA

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[95] When developing the data set we defined parameters mmQ and mmG (see righthand side in Figure 16). β and γ are calculated based on α, mmQ and mmG.

projections for North America (Energy Information Administration, 2009) and the World Energy Outlook (International Energy Agency, 2008) for the rest of the world.

EIA, BP, IEA and other data sources generally report *gross* production, trade and consumption volumes. Not all information accounts for the fact that in the supply chain that brings gas from the production wells to the end-users, there are several steps that induce losses. There are also usage categories, such as gas injection for enhanced oil recovery, that are not represented in the WGM. The production capacities and volumes in the WGM are *net* production volumes, i.e., the volumes destined to a number of consumption sectors (see Section 4.2.6). To deduce the net from the gross production values, values are used from the IEA website. [96]

4.2.2 Trade

The traders in the model execute all midstream activities to bring the gas from the upstream producers to the downstream consumers. The WGM provides much detail in the representation of the trade and transport options and allows for modeling of Cournot market power. Additionally, this market power is especially relevant in light of the Gas Exporting Countries Forum (GECF) formed in May 2001 and when representing a potential gas cartel.[97] When Russia hosted the GECF annual ministerial meeting December 2008 the foundation of an organization was announced that according to Premier Vladimir Putin of Russia: ‘will study ways to set global prices and represent interests of producers and exporters on the international market’.[98] He also announced that ‘the time of cheap energy resources and cheap gas is surely coming to an end.’

Since in the WGM the trader does not own infrastructure but purchases services from infrastructure owners, most parameters relevant for the trader are collected in other player’s data sets. Contract data is described in Section 4.2.3. Determining the values for the market power parameter was a challenging exercise. (Section 3.4.1 and 3.4.2) As discussed in Chapter 3 it is very hard to prove market power exertion. If one would have an objective procedure to assess the level of market power, the information needed would

[96] www.iea.org/Textbase/stats/gasdata.asp and www.iea.org/Textbase/stats/defs/defs.asp
[97] www.gecforum.org and www.eia.doe.gov
[98] www.nytimes.com/2008/12/24/business/worldbusiness/24gas.html

not be publicly available. The values were set as a result of discussions between co-authors of (Egging et al., 2009), based on market shares, potential dominant positions and market expertise. To streamline the discussion the choices were limited to values in $\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right\}$.

4.2.3 Liquefaction, regasification and LNG shipment

The LNG transport value chain contains liquefaction, shipment and regasification. Liquefaction and regasification capacity data for 2005 and 2008 are from (International Energy Agency, 2006, 2009). For capacity expansion limits, including for new terminals, technical literature such as the Oil and Gas Journal, has been used. Anticipating the expiration of the Alaskan LNG export license in 2011 the capacity is left out completely starting for the model period 2015.[99] The WGM takes into account LNG contracts known as of 2008.[100] The contracting process is not included in the model and – somewhat arbitrarily – the choice could be made to extend contract periods to the end of the model time horizon. However, in the perspective of a shift towards shorter contract periods and more spot trades (see Section 3.4.3) contracts are phased out based on their currently known end dates and almost all LNG trade flows in later periods result from spot market trade. For the downstream actor in the LNG chain, the regasifier, (International Energy Agency, 2006, 2009) and the website of Gas Infrastructure Europe have provided the capacities.[101]

LNG shipment is optimized by the trader, given the distance-based transport costs. Distances (in sea miles) between each pair of liquefier node and regasifier node are obtained for the approximate location of the terminals.[102] There is no restriction on the trading pairs, so countries currently not maintaining diplomatic relations with each other can trade LNG. There are no limits on the LNG shipment capacity.

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[99] www.eia.doe.gov/oiaf/aeo/assumption/nat_gas.html (Accessed June 5, 2009)
[100] We thank Sophia Rüster and Anne Neumann from TU Dresden for sharing the contract information from their data base. See also (GIIGNL, 2010)
[101] www.gie.eu.com/maps_data/lng.html
[102] www.distances.com

The three steps in the LNG supply chain are modeled as arcs in the WGM (see Section 4.2.5). In the absence of detailed data for liquefaction and regasification costs, the same values are used for all countries. Shipment costs and losses are distance-based, $8 per kcm and 0.3% of the volume transported, both per one thousand sea miles.[103]

4.2.4 Storage

Storage is modeled as a regulated service provider, providing injection, extraction and storage services. In the data set there is at most one storage operator at a model node, for which a working gas capacity and a per unit injection cost value are needed as input. Storage capacities are obtained from (International Energy Agency, 2006, 2009). Storage costs were based on (International Energy Agency, 2006). Based on company information and dependent on local characteristics, storage losses are between 1% and 1.5%.[104] The storage operator is responsible for investment decisions, for which input parameter details are discussed in Section 4.2.7.

4.2.5 Pipeline and arc network

The minimum transport costs for pipelines are set by a regulated tariff. Additionally, an endogenously determined congestion fee ensures that the scarce pipeline capacities are allocated optimally. International pipeline transport is limited at the cross-border points. When there are several cross-border points between two adjacent model nodes, the capacities are aggregated to provide a single pipeline bound. Capacity data are obtained from Gas Infrastructure Europe for intra-European transport.[105] Data on pipeline capacities between the North American nodes were obtained from the Energy Information Administration.[106] For all other pipelines, company reports, an Excel file provided by the Energy Information Administration in July 2007, various websites as

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[103] See www.bg-group.com/OurBusiness/BusinessSegments/Documents/BG_LNGfactsheets2008.pdf
[104] See DONG, www.dongenergy.dk, dongstoragetariffspermay_tcm5-11450.pdf; Fluxys http://www.fluxys.com/en/Services/Storage/Storage.aspx, 20060101Tarieven_Stock_2006_E.pdf and Alkmaar Gas Storage: www.alkmaargasstorage.nl/Gas_Storage_Services_%20Agreement_2008-2010.pdf . Note that not all files are available online anymore, however newer versions are, e.g., www.dongenergy.dk/SiteCollectionDocuments/Doc_distribution/Storage/Tariffer/Storage%20Tariffs%202009-2010.pdf
[105] Formerly www.gie.eu.com/maps_data/capacity.html , currently www.entsog.eu/mapsdata.html
[106] 'Interstate Pipeline Capacity on a State-to-State level' Release date 9/1/2008, downloaded from www.eia.doe.gov/pub/oil_gas/natural_gas/analysis_publications/ngpipeline/usage.html,

well as technical literature were used.[107] For new Greenfield pipeline projects that are planned but do not exist yet, e.g., the Nabucco pipeline (see Section 4.4.4), the model includes a zero capacity in the base year and allows for positive capacity expansions in later periods (with the exact periods depending on the project specifics).

Short-term transport costs per pipeline (regulated fees) and pipeline losses are linear functions of the transported amounts, related to the length of the pipeline and whether the pipeline is onshore or offshore. Royalties are not incorporated as such, but implicitly assumed to be included in the fees. By default a lowest regulated fee of $10/kcm was chosen, cf., (Egging and Gabriel, 2006). As in (Egging et al., 2008, 2010), dependent on the pipeline characteristics (length, onshore/offshore), most regulated fees are between ten and thirty $/kcm, but some are higher (e.g., for the possible pipeline from Nigeria to Algeria). Most loss rates are between one percent and four percent, with higher values for extremely long pipelines.[108]

4.2.6 Consumption

The demand for natural gas is obtained from aggregating sector-specific consumption levels for each country. In its Monthly Natural Gas Survey, the International Energy Agency reports monthly consumption levels for the power generation, industrial and residential sectors as well as several other categories.[109] These data are aggregated by season (low and high demand)to determine a parameter value reflecting the intensity of seasonal variation of demand. For each sector-specific demand, a different price elasticity is assumed (between -0.25 and -0.75). For the construction of demand functions, reference prices are needed. With respect to price projections the outlooks varied dramatically. In (Energy Information Administration, 2009) the gas prices vary somewhat over time, but stay relatively stable. In (International Energy Agency, 2008) prices roughly quadruple in 4½ decades, a yearly average increase of 3.1%. An inflationary

[107] See, e.g., www.eia.doe.gov/cabs/

[108] Estimates are based on the value 0.22% per 100 km mentioned on page 78 of GTE 2003 European TPA Transmission Tariff Comparison 2003,www.gie.eu.com/adminmod/show.asp?wat=Tariff_Comp_V3.pdf

[109] www.iea.org/stats/surveys/archives.asp

trend is assumed in costs and consumer's willingness to pay of 1.5% so that prices gradually increase over the next decades.

To limit the number of variables in the model, the total demand for each model node is an aggregated function of the linear functions for each sector. Figure 17 below illustrates the aggregation of two inverse demand curves as a representative example of what is done in the WGM (and, e.g., in (Boots et al. 2004)). The line *aggreg* shows the actual aggregate demand curve consisting of two linear pieces (i.e., the curve is piecewise linear). However in the model single linear demand curves are used. Curve *model* is the curve how it would appear in the WGM. This curve is completely identical to curve *aggreg* for prices lower than six, however underestimates demand for prices higher than six.

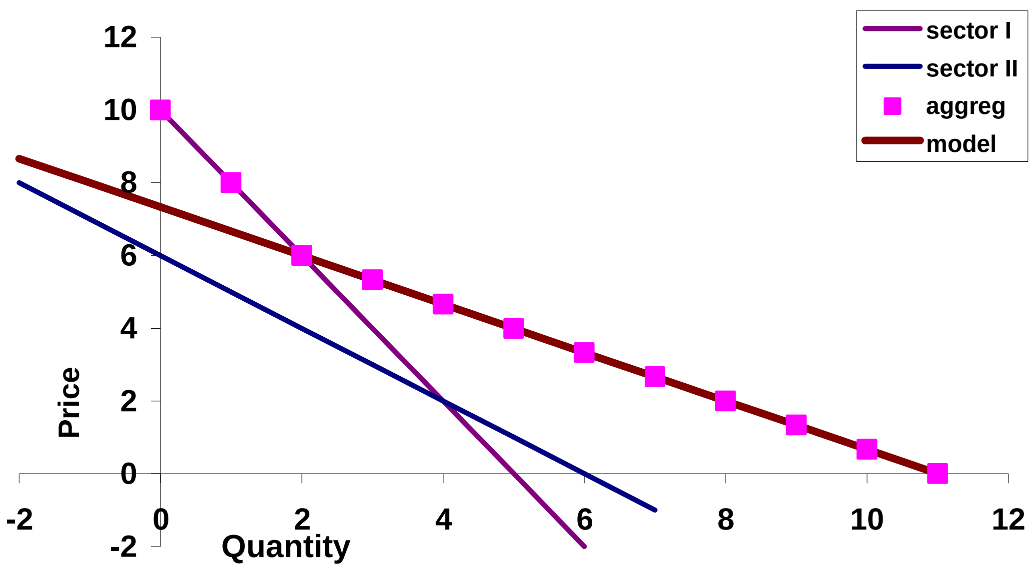

Figure 17: Inverse demand curve aggregation

More generally the linear aggregate curve represents the actual aggregate curve at price levels at which all sectors have nonnegative consumption. By calculating the consumption values of all sectors after finding a solution, using the disaggregate inverse demand curves, and checking that all sector consumptions are nonnegative, it is guaranteed that the single curve *model* results in the same consumption and price levels as separate curves would have given.

4.2.7 Capacity expansions

The WGM uses linear cost functions for the construction of incremental capacity of transport and storage: expansion levels are multiplied by a constant per unit cost. For pipelines, an investment cost of $50 million is used per mcm/d of new capacity.[110] If a pipeline is completely new the costs are doubled, if a pipeline is an offshore pipeline, the costs are also doubled, and for long pipelines the costs are doubled or tripled. These values are estimates based on reported project costs in technical literature such as the Oil and Gas Journal and company information. In the LNG value chain, the parameter values reflect that infrastructure for regasification is less capital intensive than for liquefaction.[111] Regasification expansions are as costly as the cheapest pipeline expansions and liquefaction expansions cost three times as much per unit of capacity: $150/mcm/d.

Storage expansions comprise expansion of injection, extraction and working gas capacity. Extra injection capacity is costlier ($3 million/mcm/d) than extraction capacity ($500,000/mcm/d). For working gas the investment costs are $150 million/bcm.

4.2.8 Model calibration

It is not possible to verify the outcomes of the type of natural gas market models presented in Chapter 3, that make projections about the future. Rather, model outcomes are compared to results of outside sources (e.g., the Energy Information Agency, or the Energy Information Administration) that are generally believed to present a reasonable, defendable outlook on the future natural gas market. The data inputs and modeling assumptions used, but also the transparency of the modeling approach support that model outcomes are representative and provide an insight. Typically, when a model is run on the data set initially compiled, the model outcomes do not compare very closely to these outside sources. Then, assuming that the model has been tested and corrected previously to eliminate mathematical and programming errors, some adjustments of the input data values are necessary so that the model outcomes do represent an outlook on the future that is believed to be reasonable. Typically, one should adjust the most unreliable input

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[110] mcm/d = million cubic meters/day; 1 cubic meter (m^3) = 35.31 cubic feet.
[111] See www.bg-group.com/OurBusiness/BusinessSegments/Documents/BG_LNGfactsheets2008.pdf

data first, and other input parameters should be kept as close as possible to collected data values. This process of input parameter adjustment is called 'model calibration'. Since needed changes in the model outcomes can usually by obtained by various kinds of adjustments (e.g., lower consumption levels can be obtained by higher production costs, higher transportation costs, shifting down the inverse demand curve, etc.) the calibrated model is not a unique outcome of a scientific formalized procedure. There are some judgment calls involved in this process, which require a thorough understanding of the model as well as natural gas market expertise.

The following section presents the case studies and discusses the results.

4.3 Description of cases

The cases presented in this section have been motivated by recent and actual developments in the natural gas market. Beside the Base Case, a business-as-usual scenario that provides a reference for comparison, another three case studies were developed. Two cases investigate a tighter supply of gas and one a less tight supply. The tighter supply is either induced by cartelization of the gas market (Cartel Case) or much lower availability of unconventional gas in the United States (Unconv Case). The less tight supply situation would be induced by a decrease in future transport costs (Transp Case).[112]

Table 4: Cases

Case name	**Abbreviation**	**Description**
Base Case	Base Case	Reference case
Cartel Case	Cartel Case	Cartel along the lines of GECF membership
Unconventional Gas Case	Unconv Case	Lower availability of unconventional gas in USA
Low Transport Cost Case	Transp Case	Lower transport costs

How countries are affected by different scenarios depends largely on their trade balance (in a business-as-usual situation). In the discussion of the case results there will be emphasis on an exporting country: the Netherlands, a transit country: Turkey and an

[112] The cases presented focus on structural changes in the future natural gas market relative to the Base Case. Another type of case can focus on short-term developments, e.g., sudden disruptions in supplies, for example the recent interruptions from Russian supplies (see Sections 1.3 and 3.4.1). Egging and Gabriel (2006) and Egging et al. (2008) discuss various disruption scenarios relevant for the European gas market.

importing region: United Kingdom and Ireland. To assess the profit potential for GECF members the profits and supplies by the (potential) cartel members is discussed.

4.3.1 Base Case

The Base Case is the reference for comparison. The model outcomes have been calibrated to closely match the state of the natural gas market in 2005 and the projections for the coming decades provided by the Annual Energy Outlook (Energy Information Administration, 2009), Natural Gas Information (International Energy Agency, 2009), European Energy and Transport: Trends to 2030 (European Commission, 2006, 2008) and the World Energy Outlook (International Energy Agency, 2008). Since none of these sources could provide us with the desired level of detail, multiple sources had to be used. Due to different modeling starting points, and some variations in the projections, the Base Case results differ slightly from each of the aforementioned projections. However, the results have a similar trend in terms of production and consumption growth. A notable point affecting the outcomes is the upwards revision of unconventional gas availability in the United States in the Annual Energy Outlook of 2009 (Energy Information Administration, 2009), resulting in much higher U.S. gas production in the longer term that were not accounted for yet in other projections. Naturally the higher U.S. gas production and lower imports affect LNG trade, regional trade balances, production and consumption globally.

4.3.2 Cartel Case

Market power is a significant issue in the global natural gas market. A major concern of gas importing countries is the potential for a cartelization of the gas market, comparable to the position of OPEC in the oil market. In the Cartel Case (Cartel) the member countries of the GECF will collude as a cartel.[113] In this case the GECF countries will collaborate and enforce market power by operating through a single trading entity. The model does not consider agreements about production quota or profit or revenue sharing agreements among the members (e.g., (Ikonnikova, 2007)). Instead, the cartel trader will

[113] See www.gecforum.com.qa/gecf/web.nsf/web/members. Member countries are taken as of mid 2009. The representation of this cartel in the WGM includes the following model nodes: North Africa, West Africa, Indonesia, North South America, Qatar and Russia. See Section 4.6 for the countries included in these model nodes. Note that there have been some shifts in the membership of the GECF after the WGM results were generated.

obtain the amounts of gas from each cartel producer so as to maximize aggregate profits for the cartel.

Figure 18 below shows a simplified representation of the traders in the Base Case: each trader buys from one producer.

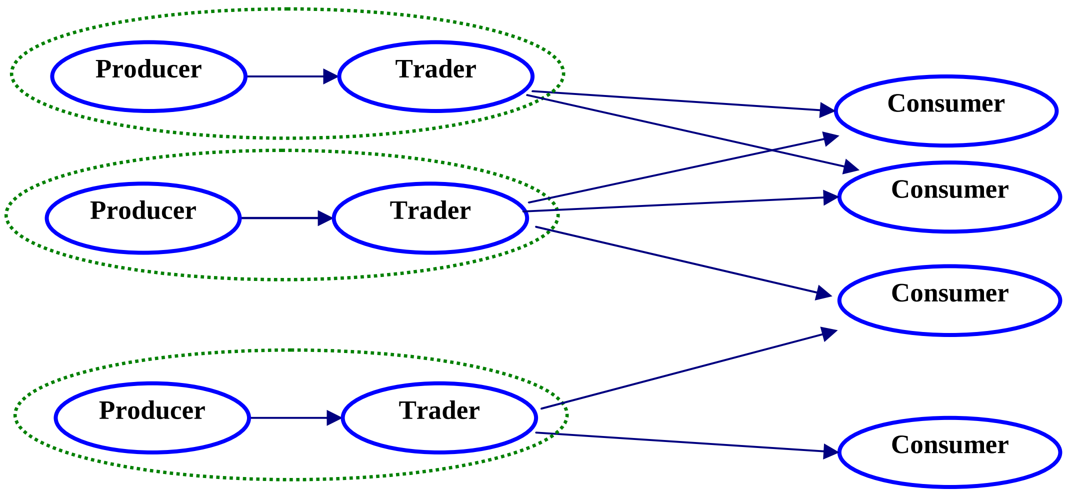

Figure 18 Standard trader representation: non-cooperative competition

In contrast, Figure 19 shows how in the Cartel Case one trader coordinates the supplies from various producers.

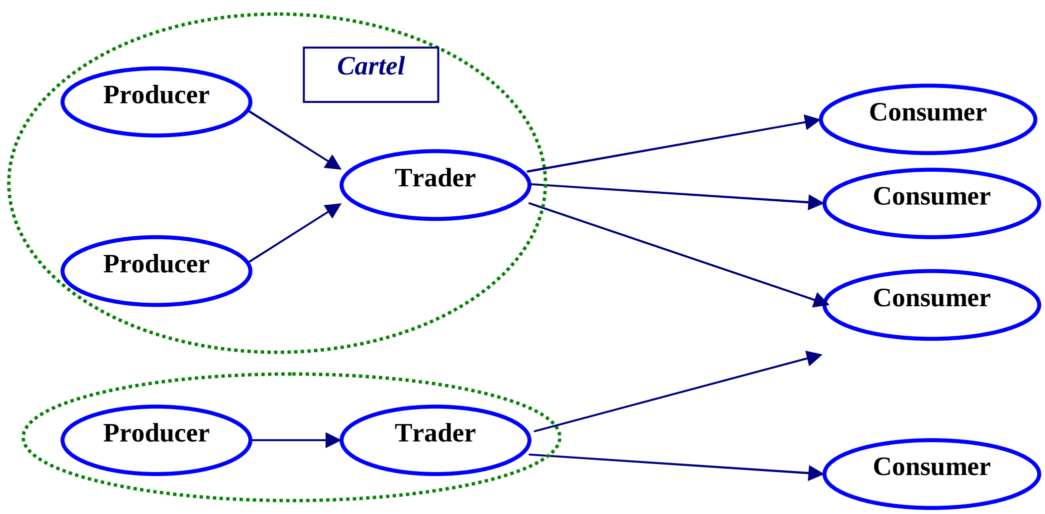

Figure 19 Cartel representation: cooperative competition in hybrid market setting

The anticipated results for the Cartel Case are that countries participating in the cartel will produce lower amounts of natural gas (relative to the Base Case) to drive up market prices. Countries highly dependent on gas imports to fulfill domestic consumption will see severely lower supplies and consequently much higher prices. It is likely that high-cost producers within the cartel give up more market share than low-cost producers. Exporting countries that do not participate in the cartel will reap the benefits from higher market prices by increasing their output and export levels.

4.3.3 Limited U.S. Unconventional Production: Unconv Case

Between 2008 and 2009 the Energy Information Administration significantly increased the U.S. production projections for unconventional gas, especially for shale gas. Although the resources are in place and can be produced economically in expected market circumstances, there is a potential problem in the form of the negative impact on the environment due to chemicals and water that are used in unconventional gas production.[114] To limit environmental damage, the government could develop policy to limit the production of unconventional gas.

This case addresses lower availability of unconventional gas in the United States. The production capacities of unconventional gas are reduced by 75% for all unconventional gas production entities in the United States. Because this reduction is applied to all unconventional gas, not only to shale, this case presents a very harsh scenario which could be seen as a worst case for the supply situation of the United States.

Lower U.S. domestic unconventional production rates will result in higher market prices, higher production in Alaska, and higher imports from Canadian pipeline gas and LNG from overseas. It is interesting to see how this "pull" of gas by the United States affects the world market in terms of trade and market prices.

4.3.4 Lower long-distance transportation costs: Transp Case

Gas transport costs have decreased due to technological progress and economies of scale and there is more potential for cost improvements (Cayrade, 2004), (Van Oostvoorn et

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[114] See, e.g., http://www.huntergasactiongroup.com.au/hgfracc.html ; www.energyindepth.org/frac-fluid.pdf; Federation of American Scientists: www.fas.org/sgp/crs/misc/R40894.pdf ; http://yosemite.epa.gov/opa/admpress.nsf/0/5ab7fab1665a0d698525774600606afe?OpenDocument ; www.propublica.org/article/frack-fluid-spill-in-dimock-contaminates-stream-killing-fish-921 ; www.propublica.org/article/gas-drilling-vs-drinking-water-new-york-city-fight-with-albany ; www.shalegaswiki.com ;

al., 2003). This fourth case will provide a sensitivity analysis on transport costs. Investments in new pipelines and liquefaction and regasification capacity are assumed to be 20% cheaper than in the Base Case, and operational costs and regulated fees for all transport options stay at present levels instead of increasing with an inflationary trend of 1.5% per year.

Since lower transportation costs make longer distance transports more attractive this scenario will result in a comparative advantage for suppliers further away from the importing markets. LNG exports will likely increase, as should long-distance pipeline exports from various regions.

The following sections discuss results illustrating some global and local effects on the natural gas market of the case assumptions.

4.4 Numerical results

The first subsection discusses and compares various aggregate results for the three cases and the Base Case, in terms of global and regional prices and production levels. Prices and production levels are discussed for the most mature gas markets: North America, Europe and Japan & South Korea. Subsequent sections present detailed results for an exporting country, a transit country, an importing country and the (potential) cartel members. Highlighted are: i. the Dutch trade balance, ii. pipeline transits through Turkey, iii. the breakdown of supplies to the United Kingdom & Ireland in the high-demand season, iv: production and profit levels of the cartel members and v. changes in consumer surplus in importing regions. Detailed results, also for other countries and regions, can be found in Section 4.7 and in (Gabriel et al., 2010).

4.4.1 Development of wholesale prices

In the Base Case, the worldwide volume-weighted average wholesale prices in 2015 in 2005$/kcm will be $194 ($5.50/mcf) and $240 ($6.81/mcf) in 2030.[115] In North America the prices in 2015 and 2030 will be $230 ($6.52/mcf) and $291 ($8.24/mcf) respectively, in Europe $283 and $368 and in Japan & South Korea $327/kcm and $398/kcm.

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[115] kcm=kilo cubic meter = 1000 m^3 ; mcf: thousand cubic feet = 1000 ft^3 ; 1 kcm = 35.31 mcf

Due to the nature of markets and market models, changes in assumptions affecting the supply will affect the demand situation, and will also impact the end-user prices. The regions that are likely to be most affected are regions that are – to some extent – dependent on imports to meet their domestic consumption. In the Base Case, North America will stay nearly self-sufficient throughout the model horizon, Europe will import 58% and 68% of total consumption in 2015 and 2030 respectively and Japan & South Korean will import about 98% throughout the time horizon. Figure 20 shows for all three cases the differences relative to the Base Case in volume-weighted average wholesale prices for the whole world, North America, Europe and Japan & South Korea.

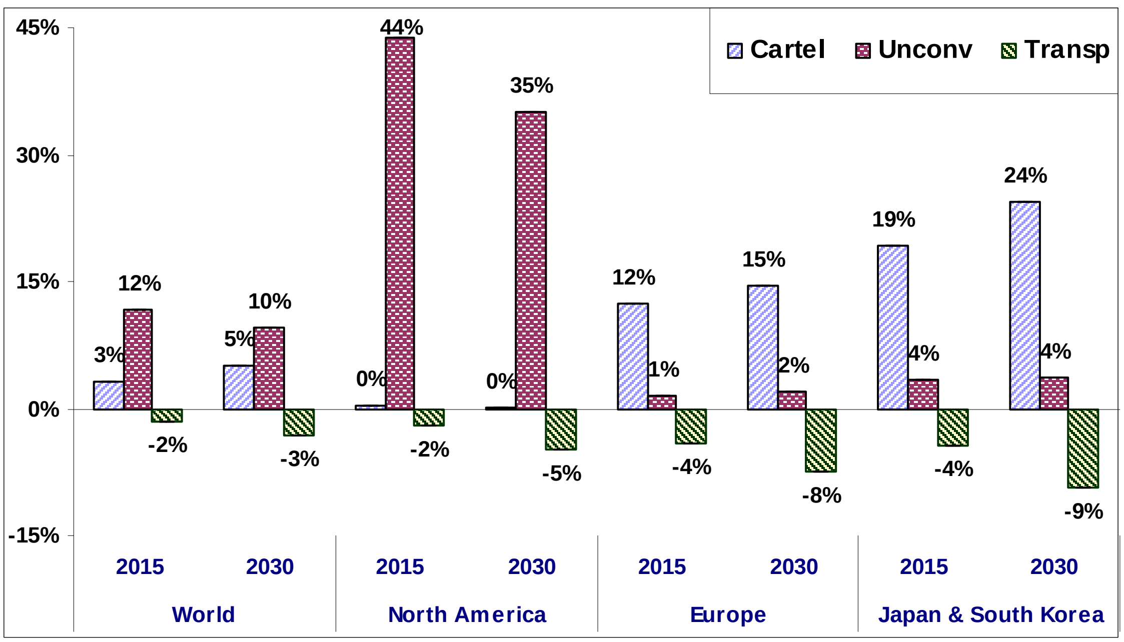

Figure 20: Volume-weighted average wholesale prices - Differences relative to Base Case

In the Cartel Case, Europe and Japan are affected quite harshly, with average prices between 12% and 24% higher than in the Base Case. North America, however, would hardly be affected. The lack of dependency on overseas external supplies shields North America from the impact of the cartelization. The global average impact of a cartel in terms of prices is quite modest, smoothed out by the negligible impact on the large North American market and lower prices in all countries participating in the cartel (not shown in the Figure 20).

Much lower unconventional production in the United States would have the largest impact on prices globally. In North America prices would be 44% higher than in the Base Case in 2015 and 35% higher in 2030. Canadian and Alaskan supplies could not make up completely for the lower unconventional resources and LNG imports from overseas would be significantly higher to cover domestic demand in the United States; actually drawing some LNG that in the Base Case would have been supplied to other LNG importing regions. Alaskan supplies would be higher, and the pipeline would eventually be expanded to 60.7 bcm/y of capacity in Unconv, about 7 bcm/y higher than in the Base Case. (Details not shown in Figure 20.) The pipeline from the Canadian Mackenzie region into Western Canada would be 46.8 bcm/y by 2030 vs. 41.2 in the Base Case and Eastern Canada would expand its regasification more, to 4.6 bcm/y by 2030 vs. 0.9 in the Base Case. The total regasification capacity in the United States in 2010 of 121.1 bcm/y (exogenously included) will not be expanded more in any of the cases. The much higher prices shown in Figure 20 for North America result from the tight external supply situation and higher marginal supply costs. Also, the overseas suppliers are assumed to exert market power in the North American market, which causes prices to increase relatively sharply when domestic supplies are tight. In Europe and Japan & South Korea the impact would be relatively small and prices would be at most 4% higher. Note that Case Unconv is the only case where market prices in European would be lower than in North America.

In the Transp Case the supply costs are lower for all regions importing gas. Consequently, all importing regions would benefit from lower prices. In North America the transports from Alaska and Canadian exports to the U.S. Lower 48 states would be somewhat cheaper than in the Base Case and imports from overseas would be more attractive too. Europe and Japan & South Korea, which depend on larger import amounts, would benefit more, with prices up to 9% lower in 2030 relative to the Base Case.

The next results to be discussed are the production levels.

4.4.2 Development of production

In the Base Case, global production in 2015 will amount to 2987 bcm, in 2030 global output will be 3846. Figure 21 shows production levels in 2015 and 2030; for the whole world, Europe and North America. Since domestic production in Japan & South Korea fulfills less than three percent of domestic consumption it will be neglected in this discussion.

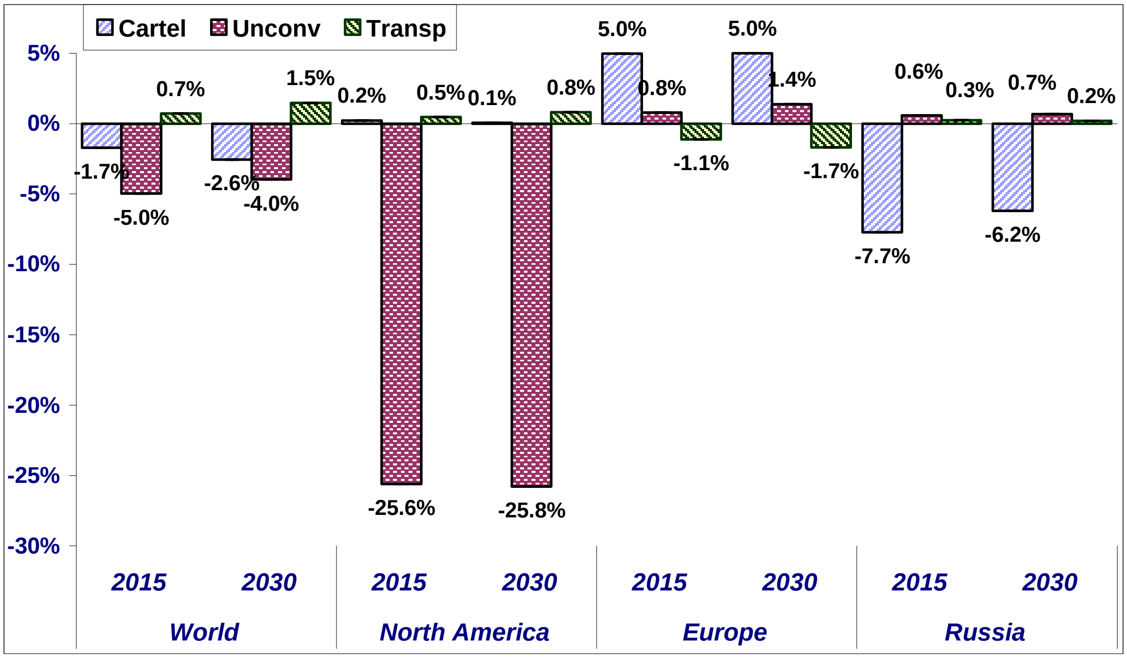

Figure 21: Production levels relative to the Base Case

Differences in global production levels among cases are relatively modest, varying at most 5% relative to the Base Case. However, the regional effects vary. Russian and European production are most affected by the Cartel Case assumptions. As one of the cartel members, Russia would produce significantly less than in the Base Case. In response to lower external supplies European countries would produce more domestically.

The lower availability of unconventional gas in the Unconv Case would result in about 25% lower production in North America compared to the Base Case in both periods. In absolute terms North American production would be 175 bcm less than the 684 of the Base Case in 2015, and 212 bcm less than the Base Case value of 821 in 2030. The much lower output of unconventional gas would have various consequences. North American market prices would significantly increase, which would reduce consumption levels and also trigger higher LNG imports and higher production in Canada and Alaska. Production in the Canadian Mackenzie Delta would be much higher earlier in the time horizon (32 vs. 18 bcm in 2015), and exports though the Alaskan pipeline would be larger than in the Base Case (65 vs. 54 bcm in 2030, not in the Figure).

The higher LNG imports to North America would reduce availability of LNG for other importing regions. Notably Europe would face higher LNG import prices, and consequently European production would be a few bcm per year higher. Since European countries are producing already close to capacity in the Base Case, these higher output levels can be considered to come from marginal fields, which in the business-as-usual situation would not be profitable. Russian would export slightly more than in the Base Case, both as LNG and via pipelines.

4.4.3 Focus on the Netherlands: supply, consumption and trade

From the early 1960s until 1994 the Netherlands was the biggest gas producer and exporter among the (current 27) members of the European Union, accounting for between 25% and 40% of total gas production. From 1995 until 2009 the United Kingdom has been the largest producer in the EU, however in 2009, due to declining reserves in the United Kingdom, the Netherlands was again the biggest gas producer among the EU countries (BP, 2010).

Figure 22: The Netherlands in Western Europe

The Dutch Groningen field is the largest onshore gas field on the European mainland and among the ten largest gas fields in the world. It has supplied more than half of all gas ever produced in the Netherlands. The rest of the Dutch gas production has originated from several smaller fields onshore and offshore. Due to its high pressure, the Groningen field has been used as a source of swing supply, allowing to meet short-term as well as seasonal fluctuations in the demand. Swing supply is an asset and since reducing production rates would maintain higher pressure levels, Dutch governments have provided tax incentives for the exploration and production from smaller gas fields, the so-called small-field policy.[116] Another measure has been to set production ceilings limiting the production from the Groningen field, e.g., for the ten-year period from 2006-2015 the production ceiling is 425 bcm, the ten-year period allowing for some variation in the yearly production rates. In spite of the measures after half a century the pressure in the Groningen gas field has dropped significantly, up to the point that compressors have been installed to produce gas from the field.

It is not clear how long the Netherlands will be able to export significant amounts of gas and the companies involved in the Dutch gas market are considering their options for the future, given the infrastructure in place and the expertise that has been gained in fifty years involvement in the gas business There are many pipelines from the Netherlands into the rest of Europe (see Figure 4 in Section 3.9), an LNG import terminal is being built, several storage facilities are under construction, and a Dutch company is shareholder in Nord Stream, the big new pipeline from Russia to Germany.[117]

Figure 23 shows the Dutch supply, demand and trade (LNG imports as well as net pipeline exports) in 2015 and 2030 for all four cases. The most-left bar presents the categories included in the graph: domestic production and LNG imports in the upper part and consumption and net pipeline exports in the bottom part (with negative values due to the structure of the graph). The figure shows that the Dutch production levels are hardly affected by the various case assumptions.

[116] See e.g., www.rijksoverheid.nl/onderwerpen/gas/gasexploratie-en-productie/groningenveld (Dutch)
[117] www.alkmaargasstorage.nl ; www.gate.nl and www.nord-stream.com/our-company/shareholders

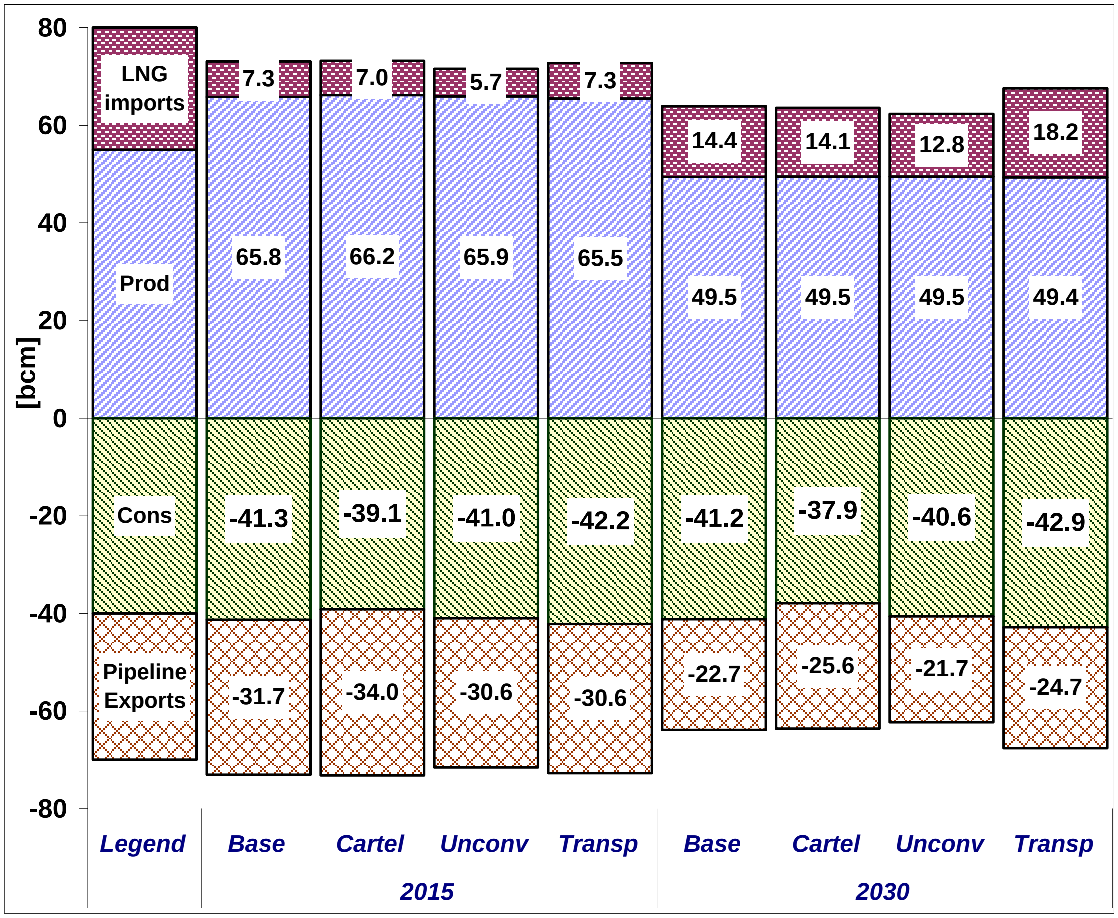

Figure 23: Supply and trade breakdown for the Netherlands (bcm/y)

In 2015 the Base Case production would amount to about 66 bcm, and in 2030 to just under 50 bcm. Consumption and trade are much more affected by the case assumptions. In both years in the Cartel Case, the supply of LNG would be lower due to the cartel members withholding supplies and the Dutch pipeline exports would increase due to higher market prices in surrounding countries. As a consequence, the consumption would be more than 5% lower in 2015 (39.1 instead of 41.3 bcm), and almost 8% lower in 2030 (37.9 instead of 41.2 bcm). Note that the 8% lower consumption in 2030 is an aggregate over all sectors and the various demand sectors have different price elasticities. For instance, the power sector is much more price-sensitive than the residential sector, and the higher prices of gas might significantly affect the fuel mix in the power generation sector.

Lower unconventional gas production in the United States (the Unconv Case) would be felt through lower LNG imports. They would be 1.6 bcm lower in both years. Dutch production would hardly change. Pipeline exports would be about 1 bcm lower in both years, and consumption 0.3 and 0.6 bcm lower, respectively.

A general decline in long-distance transport costs (Case Transp) would harm the competitive position of the Netherlands as an exporter to its neighboring countries in Europe. In 2015 the pipeline exports would be about 1.1 bcm lower than the 31.7 bcm in the Base Case. The lower pipeline exports would allow for higher consumption (0.9 bcm) and some imports would push out domestic production (0.3 bcm). In contrast, in 2030 there is an increase in pipeline exports of 2 bcm relative to the 22.7 in the Base Case. LNG imports would be so much cheaper, that they would be 3.8 bcm higher (18.2 vs. 14.4 bcm). This would allow a 1.7 bcm higher consumption as well as the observed higher pipeline exports.

These results show how a self-sufficient and gas exporting country such as the Netherlands would be affected by global developments as a consequence of the integrated nature of the global natural gas market. Domestic production levels would not vary much among the cases (which could be a consequence of the low production costs of the Groningen field in combination with the production ceiling). However different developments in the global market significantly affect the ability to draw LNG imports and the competitiveness of the Dutch pipeline exports.

The following subsection discusses the development of the role of Turkey in transiting exports from the Caspian region and the Middle East to Europe.

4.4.4 Focus on Turkey: Pipeline transits to Europe

In the past decades Russia, Algeria and Libya have exported large amounts of natural gas to Europe via pipelines. Other countries that potentially could export gas to Europe via pipelines are located in the Middle East and the Caspian region. This would allow these countries to monetize their reserves and could be interesting from a European perspective wanting to diversify supply sources. For various reasons pipelines from the Caspian regions would preferably avoid Russian territory and for Middle Eastern countries the shortest route to Europe is via Turkey. Therefore, Turkey is likely to become a major country for transiting gas from these regions to Europe.

The European Commission is supporting the construction of Nabucco, a major pipeline that should bring gas from Caspian countries to Europe. Mid 2009 the Nabucco Intergovernmental Agreement was signed between four Central and Eastern European EU members and Turkey.[118] Another pipeline, the Persian (Pars) pipeline would also pass through Turkey, but would have Middle Eastern countries as its supply source and target another part of the EU market, including Western Europe.[119]

Figure 24 shows the location of Turkey, between Asia and Europe, as well as several pipeline routes.

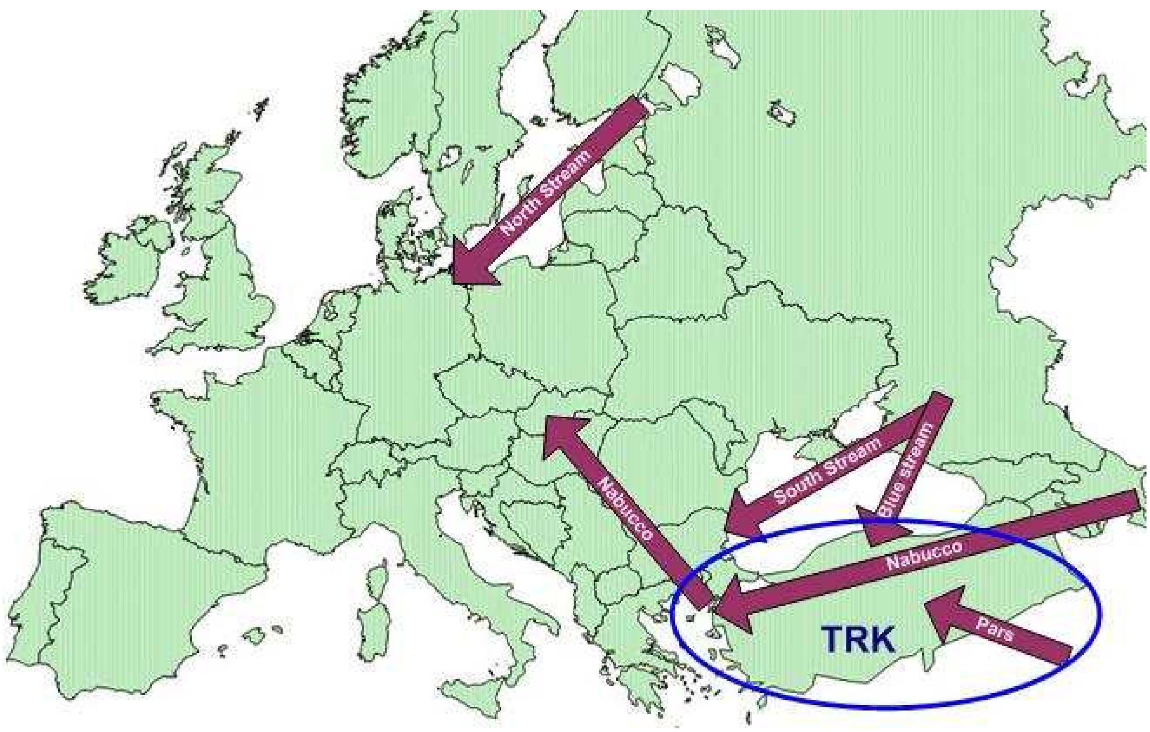

Figure 24: Transit country Turkey and various (proposed) pipeline routes

The pipeline routes depicted are at various stages of their development. Pars and South Stream are being discussed, Nabucco has political support, Nord Stream is being built

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[118] EU members: Bulgaria, Romania, Hungary and Austria, (ISEC/09/85: http://ec.europa.eu/unitedkingdom/press/press_releases/2009/pr0985_en.htm, 13 July 2009)
[119] EU members: Italy, Switzerland, Austria, Germany, (http://english.farsnews.com/newstext.php?nn=8707051084, 26 September 2008)

and Blue Stream has been in place since 2003. In 2011 27.5 bcm of Nord Stream capacity should become available and in 2012 its size should be doubled.[120] Political considerations often seem to play an important role in the decisions about new pipelines, leading to investments that are not necessarily the economically most viable ones. Our results show that the pipelines being built and expanded in the model, purely based on profit-maximization decisions, differ from the ones that are built in reality. This supports the idea that other – political – factors play a role in the decisions about major gas pipeline routes.

Although in today's reality the pipelines through Turkey are not being constructed yet, the gas market outlooks project such a huge supply gap for Europe by 2030 that much extra LNG and pipeline import capacity will be needed. In the Base Case the net imports to Europe will be 390 bcm in 2015, i.e., 100 bcm more than in 2005. In 2030, another 126 bcm will be added to the yearly imported amount, to bring the total to 516 bcm. In the Base Case the total pipeline capacity and exports into Europe from Russia, the Caspian region and the Middle East will increase significantly over time. It is interesting to see how the case assumptions would affect the expansions of the various pipeline capacities.

Figure 25 shows results for gas supply to and through Turkey. Each bar contains three categories: sales from (potential) cartel members to Turkey (Sales GECF), sales from the Caspian Region to Turkey and transits from (potential) cartel members via Turkey to the rest of Europe (Transits GECF). Total pipeline flows into Turkey would be significant, between 38 and 45 bcm in 2015 and between 67 and 86 bcm in 2030. but most of these supplies would stay in the country. Somewhat surprisingly the amount of gas transiting through Turkey would be very modest in all cases. None of these transits would originate from the Caspian region.

In the Cartel Case all transits are zero, in other cases the transit volumes (from Iran) would be at most 6 bcm. Naturally, in the Cartel Case the supplies to Turkey by cartel members would be lower than in the Base Case (e.g., -11 in 2015). Consequently, the supplies by the Caspian region would be higher (e.g., +7 in 2015. As Figure 26 below

[120] www.nord-stream.com/en/the-pipeline.html ; bcm = billion cubic meter.

shows, the total pipeline exports to Europe from Russia and the Caspian region would add up to large amounts.

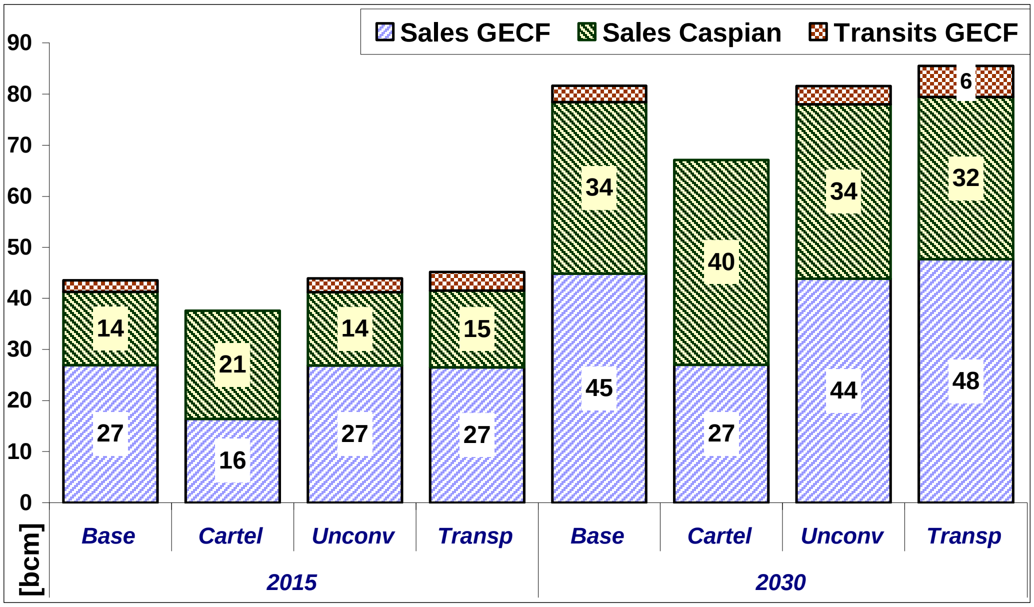

Figure 25: Pipeline exports to and transits through Turkey (bcm/y)

In the Cartel Case, total pipeline supplies to Europe by GECF countries in 2015 would be 61 bcm (36%) lower than in the Base Case and 72 bcm (33%) lower in 2030. Supplies by the Caspian region would be 23 resp. 25 bcm higher in these two years. In the other two cases pipeline supplies to Europe would be higher than in the Base Case.

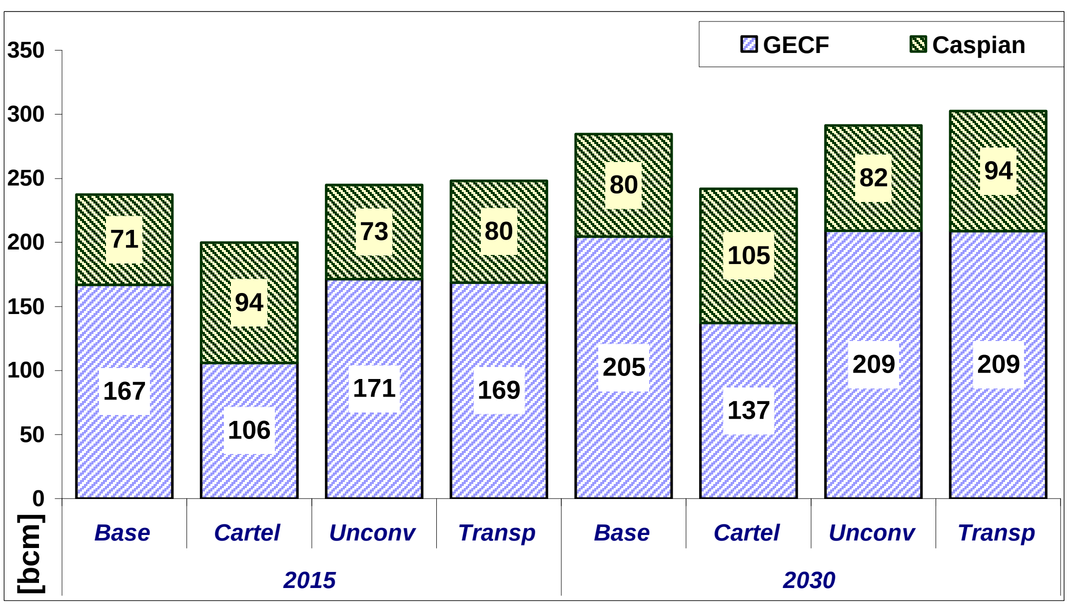

Figure 26: Pipeline exports to Europe from and via Western Russia (bcm/y)

In the Unconv Case the supplies would compensate for LNG drawn to the North America that in the Base Case is destined for Europe. In the Transp Case the higher pipeline supplies are due to the improved competitiveness of long-distance gas transports. In all cases in both years add up to at least 200 bcm, but as seen previously in Figure 25, the supplies to Europe would not be routed via Turkey. The Caspian region exports to Europe are sent through Western Russia and the Ukraine. Russia does not need to export to Europe via Turkey, but routes gas through the Ukraine. The Blue Stream pipeline would be used to supply Turkey and the Nord Stream pipeline would not be built. These results seem to contradict actual current developments. Various multi-billion dollar projects are under consideration or under construction to meet the increasing supply gap, but the model does not have them built.

Here some limitations of the model show up and a consequence of not having pipeline contract data incorporated into the model as well as only addressing economic factors in the investment decisions. Political considerations could only be addressed implicitly. That the Russian government wants to divert gas flows from the Ukraine and Belarus is not addressed in the model. It would have been possible to reduce the investment costs for the Nord Stream pipeline, or include part of its capacity exogenously, however that was not done.[121] Similarly, in the model all Caspian exports to Europe flow through Russia. In the Cartel Case the Caspian gas transiting through Russia would even be higher, when the Caspian region would fill up some of the pipeline capacity from Russia and the Ukraine to Europe that would be available due to the cartel withholding supplies. In contrast, the Nabucco pipeline will be built to be less dependent on the Russian transit route and especially in a cartel situation it would not be very likely that Russia would accommodate high amounts of gas transiting to Europe through their territory undercutting their own market position. Thus, interpretation of the model outcomes should be done carefully. Although the model does not have the Nabucco pipeline built, Caspian supplies to Europe in 2030 of 80 bcm in the Base Case and almost 105 bcm in the Cartel Case actually support the need for this pipeline, and in the long run for possibly much higher capacities than currently being discussed.

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[121] Note that in 2008, the time of data base construction, Nord Stream was in a planning stage only.

In conclusion, the transits through Turkey are surprisingly modest in all cases, which does not seem to support the case for major pipeline projects such as Nabucco. But when considering the model outcomes in a political context and looking at export volumes between 80 and 105 bcm from the Caspian region to Europe in 2030, supply security considerations do provide a rationale for having the Nabucco pipeline built.

The next subsection presents and discusses results for the supply situation of the United Kingdom and Ireland in the high demand season.

4.4.5 Focus on the U.K. & Ireland: supply in the high demand season

From 1994 until 2008 the United Kingdom was the largest gas producer in the European Union.[122] After peaking at 108 bcm in 2000, production has declined to slightly below 60 bcm in 2009.

Figure 27: United Kingdom and Ireland

Since 2005 the United Kingdom has had to import gas to meet domestic demand. In 2009 the net import share of the United Kingdom was about 30%. Gross imports amounted to

[122] Ireland is part of the node UK. Irish production is negligible and consumption just under 5 bcm in 2009.

about 41 bcm and the (re-)exports added up to 12 bcm. About 60% of the imports originated from Norway, 25% was imported as LNG and 15% came from the Netherlands. Almost half of the exports of the United Kingdom went to Ireland (BP, 2010).

Domestic production in the United Kingdom was booming in the 1990s, rapidly depleting the reserves. Companies involved in the U.K. gas market realized that the situation would change radically in the next decade and several new import pipelines and regasification terminals were developed. As a result, the added LNG regasification capacity at the Isle of Grain and Milford Haven since 2005 will total 47 bcm by 2011. In that same period two new pipelines (Langeled from Norway and the BBL from the Netherlands) and allowing flow reversal on a third one (the Interconnector from Belgium) add 70 bcm of import capacity and bring yearly U.K. import capacity to over 140 bcm/y by 2010.[123]

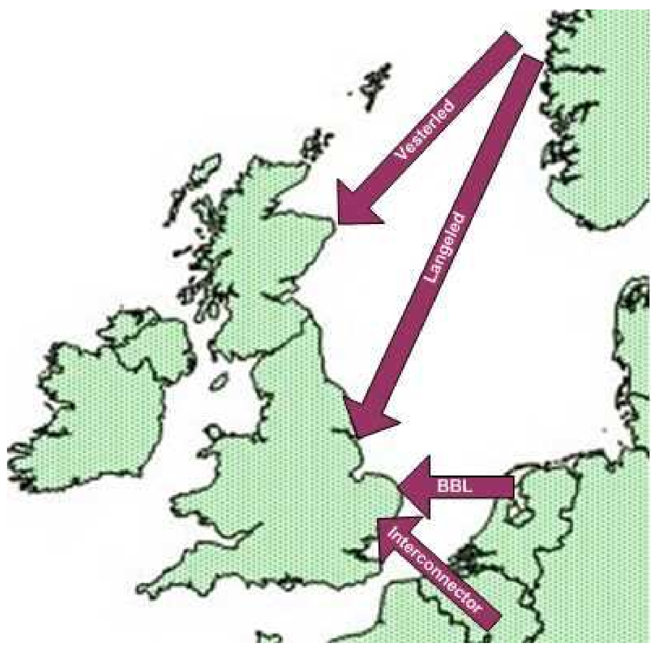

Figure 28: Pipelines into the United Kingdom

The U.K. network operator National Grid projects that by 2018 domestic production will have declined to something between 20 and 40 bcm per year. Combined with a demand

[123] Tampen Link, Vesterled, Langeled www.gie.eu.com and www.gassco.no/wps/wcm/connect/gassco-en/gassco/home/norsk-gass/gas-transport-system

projection of about 100 bcm this would imply net imports of 60 to 80 bcm.[124] These projections would imply that not all import capacity will be used at full capacity all the time. However, gas usage is not the same every day and slack capacity and storage are needed to provide daily and seasonal swing and to deal with supply interruptions. According to National Grid the typical peak winter day demand in the United Kingdom in the last decade has been about four hundred mcm/d: an annualized 146 bcm.[125]

In the short run there seems to be overcapacity for importing gas in the U.K. market. Given the various options that the United Kingdom have to fulfill the domestic demand, how will case assumptions affect what options will be used to which extend?
Seasonality representation in the WGM is limited to two seasons and the high demand season includes the period October through March. Thus, the very cold winter peak demand period is smoothed out somewhat by the demand characteristics of the late autumn and early spring months. In the WGM daily consumption in the cold half of the year is slightly more than 50% higher than daily demand in the warmer half.

In the Base Case, production of the United Kingdom and Ireland in 2015 would be a bit more than 32 bcm, and slightly below 17 bcm in 2030. Consumption in both years is projected to be just under 90 bcm. Domestic production levels do not depend much on the season or the case assumptions, however the consumption volumes do. Consumption in the high demand period would be around 55 bcm, or an annualized 110 bcm.

Since the seasonal swing of domestic production is very limited and building pipelines is expensive, storage and LNG imports are generally the cheaper options to meet variations in seasonal demand. However it is possible that when the supply of LNG is rather tight (as in the Cartel Case and the Unconv Case), or pipeline investment costs relatively low (as in the Transp Case), that different sources supply more or less than in a business as usual situation. Figure 29 below shows in the four cases how much domestic production, pipeline imports, LNG imports and withdrawals from storage are needed to cover winter demand in the United Kingdom and Ireland in the years 2015 and 2030.

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[124] http://www.nationalgrid.com/uk/Gas/TYS/ ;
[125] 400 mcm/day times 365 days → 0.4 bcm/d *365 days =146 bcm/y

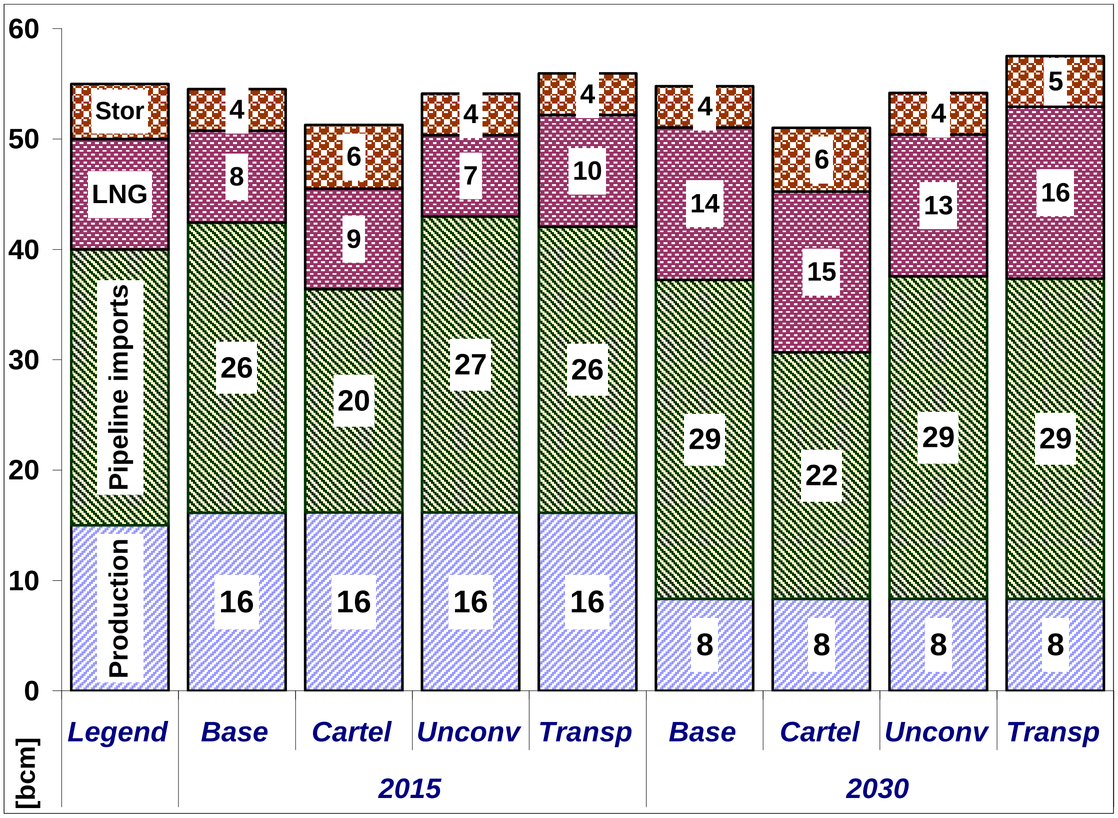

Figure 29: Supply in the high demand season United Kingdom and Ireland (bcm)

In all cases the variation in seasonal demand is very similar, with 39% of yearly demand falling in the low demand period, and 61% of demand in the high demand period (not shown in figure). The United Kingdom and Ireland would be affected more by a cartelization of the gas market than to much lower unconventional gas availability in the United States. In a cartel situation, the yearly consumption would be about 5 bcm lower in 2015, and 7 bcm lower in 2030. In contrast, in the Unconv Case, consumption would be just about 1 bcm lower in both years. Lower transport costs (Transp Case) would allow consumption to be about 2½ and 5 bcm higher in 2015 and 2030 respectively.

In the Cartel Case the pipeline supplies are significantly lower than in the Base Case. Supplies from Norway and the Netherlands that in the Base Case were destined for the United Kingdom are drawn to Central, Eastern and Southern Europe to fill the supply gap resulting from cartel members withholding supplies. The resulting much higher prices in the U.K. market would make it more attractive for LNG suppliers. In the Cartel Case, the LNG imports would even be a little bit higher than in the Base Case.

When the United States would have lower domestic production (Unconv Case) some of the LNG that otherwise could have been directed to the United Kingdom would go to North America. As a consequence, the U.K. LNG imports would be a little lower and pipeline supplies would increase slightly to compensate part of the redirected LNG flows.

Lower long-distance transport costs (Transp Case) would benefit the competitive position of LNG supplies, allowing for somewhat higher LNG imports. In 2015 LNG would even push out some pipeline supplies (-0.3 bcm, not visible in the figure), but by 2030 also the pipeline supplies would be slightly higher than in the Base Case (+0.1 bcm).

In 2015 the use of storage would be just under 4 bcm in all cases except the Cartel Case. In the Cartel Case it would be 2 bcm higher, due to a larger seasonal price difference resulting from withheld supplies by cartel countries. For 2030 the results are very similar, only in the Transp Case there would be a higher use of storage of almost 1 bcm relative to the Base Case. The use of storage in the model seems very low relative to the capacity expected to be available. Current working gas capacity in the United Kingdom & Ireland is about 4½ bcm, and there are plans for an additional 20 bcm.[126] Recall that in the WGM just two seasons are distinguished. In the model the annualized daily consumption in the high demand season is about 110 bcm/y. Compared to an annualized peak winter day demand of 146 bcm/y (see start of this section) and adding the need to deal with supply interruptions, more storage will be needed than what the model outcome indicates.

Table 5 below shows that in 2015 the seasonal price differential in the United Kingdom and Ireland is larger in the Cartel Case than in the other cases, even with the higher storage capacity used in the Cartel Case relative to the other cases. In 2030 the price differences among the cases are negligible, however with lower working gas available the price difference in the Cartel Case would have been larger.

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[126] http://www.gie.eu.com/maps_data/GSE/database/index.html

Table 5: Seasonal price differences in the United Kingdom and Ireland ($2005/kcm)

Year	Base	Cartel	Unconv	Transp
2015	$ 41	$ 47	$ 37	$ 39
2030	$ 58	$ 59	$ 58	$ 59

In this section a breakdown of the supply to the United Kingdom and Ireland in the high demand season was discussed. Due to depleting domestic reserves the countries will have to rely on imports to fulfill domestic demand throughout the year, and additional imports as well as storage to provide swing supply. Our results show that all supply options will be used to meet high season demand and that the supply mix is relatively independent of the case assumptions, except for significantly lower pipeline supplies in a cartel situation. A cartel would induce larger use of storage, due to a larger higher seasonal price difference. With the anticipated slack in pipeline and regasification capacities as well as the available storage working gas the United Kingdom seems well-prepared to deal with daily and seasonal variations in the demand.

Previous sections have discussed several effects of a cartelization of the gas market. In the following section the profitability of a cartelization of the gas market for the potential cartel members is discussed.

4.4.6 Focus on cartel members: cartelization profits

The idea behind a cartel exerting market power is that withholding supplies will drive up market prices and profits of the cartel members. However, the higher market prices will trigger higher supplies from non-cartel members, partly undercutting the intentions of the cartel members. Figure 30 shows the total net exports to Europe (incl. Turkey) by the main suppliers potentially participating in the cartel and the Caspian region in all cases.

The total net supplies to Europe in 2015 by a cartel would be 101 bcm (33%) lower than the aggregate supplies of the individual members in the Base Case, and 143 bcm (36%) lower in 2030. Higher Caspian supplies would fill part of the gap, with supplies to Europe being 26 bcm higher in 2015 and 27 bcm in 2030, relative to the Base Case. In the Unconv Case the LNG exporters among the GECF members would direct more supplies to North America, hence the lower supplies to Europe, which would induce slightly higher supplies by the Caspian region. In the Trans Case all long-distance supplies to Europe would increase.

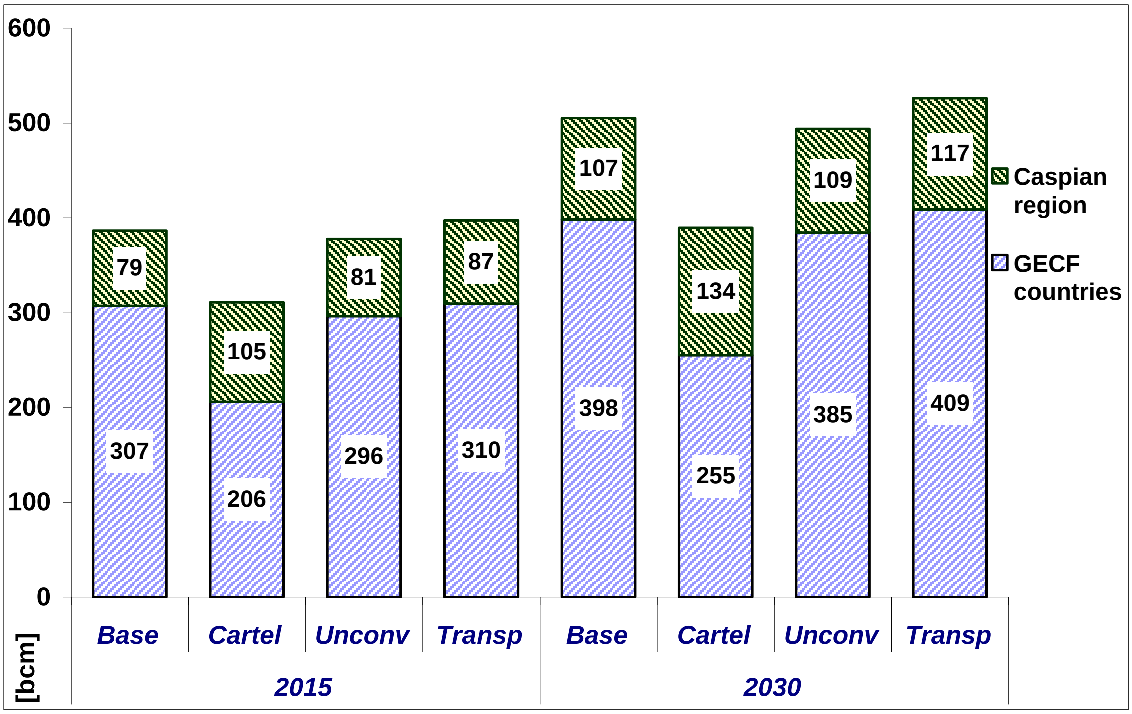

Figure 30: Net exports to Europe by cartel members and the Caspian Region (bcm/y)

In a cartel, the market prices in importing regions would be up to 24% higher in 2030 (Figure 20). How would the lower supplies to Europe and other regions in combination with the higher prices affect the profits of the potential cartel members? Figure 31 presents a breakdown of yearly trader profits in three groups: potential cartel members (GECF), the Caspian Region, and all other traders.

The trader profits are highest in the Cartel Case as would be expected. In 2015, the profits of the cartel members would be about 8% higher than the aggregate profits of the members in the Base Case. A result that is described more often in the literature (e.g., Farrell and Shapiro (1990, 1991), Salant et al. (1983)) is that the cartel profits in later periods are lower than without collaboration: in 2030 the aggregate profits of the cartel members would be almost $3 billion lower than in the Base Case.

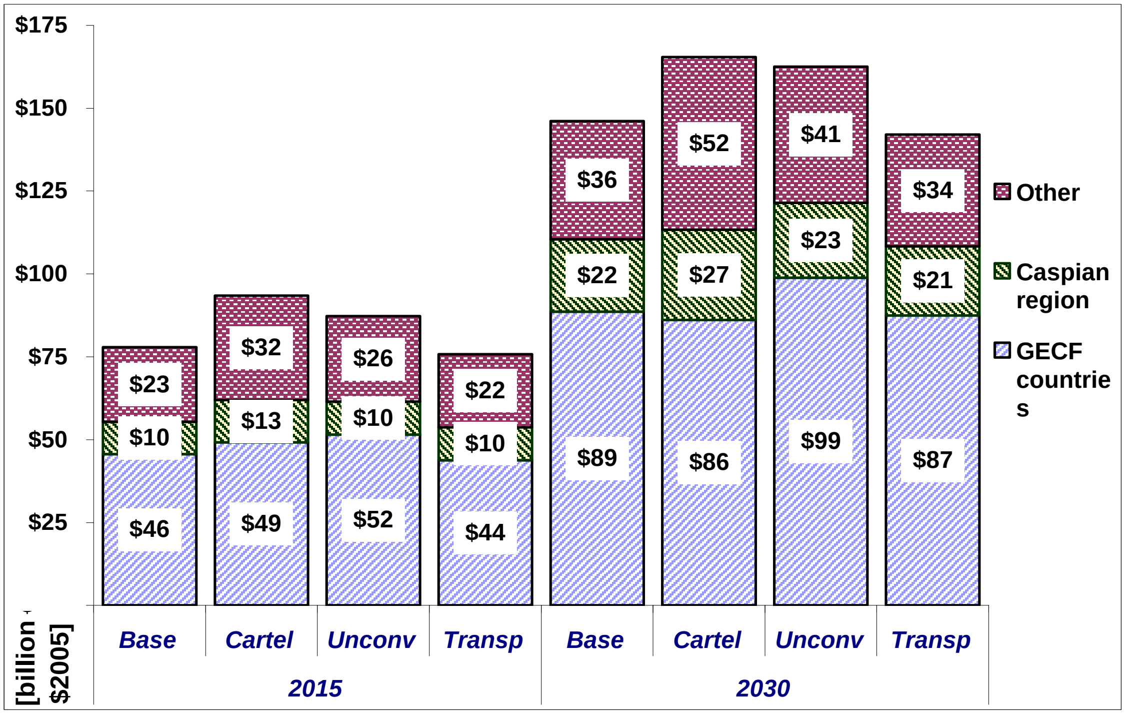

Figure 31: Aggregate profits of traders (billion $2005)

The explanation is that the cartel members maximize their discounted profits over the whole time horizon. The further into the future cash flows occur, the more they will be discounted and thus less influential in model decisions. Table 6 shows the discounted yearly profits of the cartel members. As it turns out, the yearly profits of the cartel members would be higher in the Cartel Case than in the Base Case until 2025, but lower in the rest of the time horizon.[127]

Table 6: Aggregate discounted yearly profits of cartel members (billion $2005)

Year	Base Case	Cartel Case	Relative Difference
2010	$ 20.9	$ 24.0	15.0%
2015	$ 17.6	$ 19.0	7.9%
2020	$ 13.5	$ 14.2	4.9%
2025	$ 10.3	$ 10.4	1.5%
2030	$ 8.2	$ 7.9	-2.9%

A back-of-the-envelope estimate for the aggregate discounted cartel profits would be $35-40 billion over the model period, about half of which would fall in the first five

[127] For completeness, the discounted aggregate profits in the Base and Cartel cases are in 2005: $24.8 vs. $29.5, in 2035 $5.4 vs. $5.3 and in 2040 $3.6 vs. $3.5.

years.[128] In the short run, a cartel along the lines of the GECF would withhold supplies and drive market prices up to significantly increase trader's profits. In the longer run, other suppliers such as the Caspian Region, Australia and Middle Eastern countries not taking part in the GECF would have time to expand supply capacities and export much higher volumes to importing markets, eventually at the expense of the cartel profits. Some caution when interpreting these results is recommended, since they do not account for the depletion of reserves and the possibility to change strategies over time. Alternatively, cartels with a broader membership could be considered. Gabriel et al. (2010) looks into such alternative cartels and also combines case assumptions to see how lower transport cost or lower North American unconventional gas production interacts with a cartelization of the gas market.

4.4.7 Focus on consumer surplus

Table 7 presents changes in consumer surplus based on differences in prices and consumption levels in the various cases. In the Cartel Case and the Unconv Case, the lower consumption levels and higher prices induce losses in consumer surplus of dozens of billion dollars. In contrast, consumer surplus in the Transp Case is much higher, due to higher consumption and lower prices.

Table 7: Changes in consumer surplus (billion $2005)

Region	Case / Year	Change in consumer surplus		
		Cartel	Unconv	Transp
World	2015	-27.6	-71.4	12.3
	2030	-62.6	-92.8	36.6
North America	2015	-0.6	-63.7	3.4
	2030	-0.2	-76.8	11.9
Europe	2015	-22.6	-2.7	7.9
	2030	-38.9	-5.8	21.6
Japan & South Korea	2015	-6.8	-1.3	1.6
	2030	-11.7	-1.9	5.1

In North America the lost consumer surplus is largest in the Unconv Case, totaling almost $64 billion in 2015 and close to $77 billion in 2030. In Europe and Japan & South Korea the Cartel Case has the most impact on consumer surplus, in 2030 Europe would gain

[128] The estimate is based on an interpolation for years that are not included in the model.

almost $39 billion and Japan & South Korea would gain $11.7. The higher the impact of case assumptions on the consumption, the larger the loss in consumer surplus.

4.5 Summary and conclusions

This chapter starts with a discussion of the input data collection and the development of a reference scenario for the global natural gas market in the WGM. In the second part of this chapter the results of three case studies are presented and discussed. The first case studies the impact of a gas cartel, a second case a lower availability of unconventional gas in the United States and a third case the impact of lower future transport costs.

A cartelization of the global gas market would severely impact the import dependent countries. In Europe the average prices would be between 12% and 15% higher, and in and Japan & South Korea between 19% and 24% higher than in the Base Case, and consumption levels would drop 6% to 8% in Europe, and 12% to 15% in Japan & South Korea. North America would hardly be affected, due to self-sufficiency of the region. In terms of consumer surplus, Europe and Japan would account for the majority of the losses, with among the two regions almost $30 billion lost in 2015 and $50 billion in 2030.

In contrast, much lower availability of unconventional gas in the United States would have a large impact on North America. North American prices would be up to 35% to 44% higher than in the Base Case, but the impact on other regions would be modest, with European, Japanese & South Korean prices not more than 4% higher. In this case the United States would be responsible for most of the loss in consumer surplus, losing almost $64 in 2015 and $77 billion in 2030.

Subsequent sections discuss detailed results for an exporting country (Netherlands), a transit country (Turkey) and an importing region (United Kingdom and Ireland), and the profit potential for GECF members were they to form a cartel. Highlighted are the impacts of the various case assumptions on the Dutch trade balance, the pipeline transits through Turkey destined for Europe, the breakdown of supply the to the United Kingdom and Ireland in the high demand season and the profits and supplies by the (potential) cartel members.

In the Netherlands, the domestic production levels do not vary much among the cases. However different developments in the global market significantly affect the ability to draw LNG imports and the competitiveness of the Dutch pipeline exports. The results support the development of LNG regasification capacity in the Netherlands as LNG imports are significant in all cases and periods.

The amounts of gas transiting through Turkey are surprisingly small in all cases, which does not seem to support the development of major pipeline projects such as Nabucco. Flows into Turkey would be significant in all cases, but with Turkey as their final destination. Exporters use other routes for shipping their gas into Europe. However, when considering the model outcomes for the various cases in a political context, and looking at a value of 80 to 105 bcm of Caspian exports to Europe in 2030 transited through Russia, clearly more pipeline capacity is needed. Supply security considerations could provide the rationale to not transit these exports through Russia, but indeed use the route through Turkey considered for Nabucco.

Next, the supply situation for the United Kingdom and Ireland in the high demand season is discussed. Anticipating the depletion of domestic reserves, the capacities of import pipelines, regasification capacity and storage working gas has been, and is being, expanded with large amounts. All supply options are used in all the cases considered, although there seems to be quite some slack in the import capacity that would only be used in the peak winter periods and to cope with disruptions. A cartel would induce larger use of storage, possibly due to a larger higher seasonal price difference.

Lastly, the production and profit developments of (potential) cartel members showed that non-cartel members would benefit most from the formation of a cartel. Non-cartel members benefit from the higher market prices induced by lower supplies from cartel countries, and raise their output levels. The cartel would benefit in the early part of the time horizon, but would give up so much market share in later years that in the long run yearly profits would be lower than in the Base Case. Over a thirty-year time horizon the total discounted profits for the cartel members would be about $35-40 billion higher than in the Base Case.

The case results illustrate that the increasing role of LNG trade in the global gas market has consequences for how local developments affect the global market. Changing domestic supply in one region causes shifts in LNG flows, which changes trade balances, prices and pipeline flows all over the globe. This ripple effect, or smoothing-out effect, is also discussed in (Nesbitt and Scotcher, 2009) and (Hartley and Medlock III, 2009).

This section concludes the discussion of the deterministic WGM. The following Chapters will introduce and discuss the stochastic version of the model and decomposition approaches to solve a large-scale stochastic version of the model.

4.6 Appendix

Region	Node	Countries
Africa	ALG	Algeria, Egypt, Libya, Morocco, Tunisia
	NIG	Nigeria, Angola, Equatorial Guinea, Mozambique, South Africa
Asia Pacific	AUS	Australia, New Zealand
	CHN	Burma, China, Singapore, Taiwan, Thailand
	IDA	India, Pakistan
	IDO	Indonesia, Brunei, Malaysia
	JAP	Japan, South Korea
Caspian	KZK	Kazakhstan, Azerbaijan, Turkmenistan, Uzbekistan, Armenia, Georgia
Europe	FRA	France, Belgium & Luxembourg
	GER	Austria, Czech Republic, Denmark, Germany, Switzerland
	ITA	Italy, Slovenia
	NED	Netherlands
	NOR	Norway
	POL	Poland, Sweden, Baltic Region, Finland, Slovak Republic
	ROM	Romania, Bulgaria, Greece, Hungary
	SPA	Spain, Portugal
	TRK	Turkey
	UK	United Kingdom, Ireland
	UKR	Ukraine, Belarus
Middle East	QAT	Qatar, Iran
	YMN	Kuwait, Oman, Saudi Arabia, UAE, Yemen
North America	CAE	Canada-East
	CAW	Canada-West, Mackenzie Delta
	MEX	Mexico
	US1	USA Census Region 1: New England[129]
	US2	USA Census Region 2:Middle Atlantic
	US3	USA Census Region 3:East North Central
	US4	USA Census Region 4:West North Central
	US5	USA Census Region 5:South Atlantic
	US6	USA Census Region 6:East South Central
	US7	USA Census Region 7:West South Central
	US8	USA Census Region 8:Mountain
	US9	USA Census Region 9: Pacific, except Alaska
	USL	USA Alaska
Russia	RUE	Russia-East
	RUL	Russia-Sakhalin
	RUW	Russia-West, Russia-Volga-Uralsk
South America	BRA	Brazil, Argentina
	CHL	Chile, Ecuador, Peru
	TRI	Trinidad & Tobago, Bolivia, Venezuela

[129] http://www.census.gov/geo/www/us_regdiv.pdf

4.7 Detailed case-study results

Table 8: Volume-weighted average wholesale prices ($2005/kcm)

Year	Case	North America	Europe	Asia Pacific	South America	Middle East	Africa	Caspian Region	Russia	World
2010	**Base**	$ 206	$ 252	$ 191	$ 233	$ 94	$ 84	$ 72	$ 79	$ 178
	Cartel	$ 206	$ 286	$ 198	$ 224	$ 97	$ 70	$ 85	$ 73	$ 184
	Unconv	$ 302	$ 257	$ 197	$ 237	$ 96	$ 89	$ 72	$ 80	$ 201
	Transp	$ 204	$ 247	$ 190	$ 230	$ 95	$ 85	$ 73	$ 80	$ 177
2015	**Base**	$ 230	$ 283	$ 200	$ 273	$ 102	$ 86	$ 80	$ 86	$ 194
	Cartel	$ 231	$ 318	$ 208	$ 263	$ 109	$ 72	$ 94	$ 79	$ 201
	Unconv	$ 331	$ 287	$ 204	$ 281	$ 105	$ 91	$ 81	$ 87	$ 217
	Transp	$ 226	$ 271	$ 196	$ 268	$ 105	$ 91	$ 85	$ 86	$ 191
2020	**Base**	$ 245	$ 305	$ 205	$ 269	$ 105	$ 89	$ 86	$ 92	$ 203
	Cartel	$ 247	$ 343	$ 216	$ 268	$ 114	$ 74	$ 103	$ 84	$ 211
	Unconv	$ 340	$ 309	$ 209	$ 272	$ 108	$ 104	$ 87	$ 93	$ 224
	Transp	$ 238	$ 288	$ 198	$ 262	$ 108	$ 97	$ 92	$ 93	$ 198
2025	**Base**	$ 261	$ 332	$ 229	$ 272	$ 111	$ 90	$ 101	$ 108	$ 218
	Cartel	$ 262	$ 376	$ 247	$ 279	$ 120	$ 77	$ 126	$ 97	$ 228
	Unconv	$ 351	$ 337	$ 233	$ 276	$ 114	$ 103	$ 103	$ 109	$ 238
	Transp	$ 250	$ 310	$ 220	$ 265	$ 115	$ 97	$ 108	$ 109	$ 212
2030	**Base**	$ 291	$ 368	$ 261	$ 282	$ 125	$ 91	$ 112	$ 121	$ 240
	Cartel	$ 291	$ 421	$ 281	$ 309	$ 134	$ 80	$ 143	$ 107	$ 253
	Unconv	$ 393	$ 375	$ 267	$ 293	$ 129	$ 102	$ 115	$ 122	$ 264
	Transp	$ 277	$ 340	$ 250	$ 279	$ 130	$ 98	$ 124	$ 121	$ 233

Table 9: Production, consumption and net trade – Europe (bcm)

Year	Data	Case	NOR	NED	UKD	FRA	GER	ITA	ROM	SPA	POL	TRK	UKR	EUR
2010	**Production**	**Base**	92	68	67	0	26	12	13	0	3	1	19	302
		Cartel	98	69	68	0	26	12	13	0	3	1	19	310
		Unconv	93	69	67	0	26	12	13	0	3	1	19	304
		Transp	91	68	67	0	26	12	13	0	3	1	19	301
	LNG imports	**Base**	-7	0	10	34	0	16	3	34	0	5	0	96
		Cartel	-7	0	16	27	0	17	2	36	0	5	0	95
		Unconv	-7	0	6	29	0	12	1	33	0	5	0	79
		Transp	-7	0	10	35	0	17	3	35	0	5	0	98
	Pipeline imports	**Base**	-84	-29	14	31	90	62	20	8	31	29	73	246
		Cartel	-90	-32	4	35	84	54	19	2	30	25	63	194
		Unconv	-86	-30	17	36	89	64	21	9	31	29	72	253
		Transp	-83	-28	15	31	91	62	20	8	32	30	73	249
	Consumption	**Base**	1	39	91	65	116	90	36	43	35	36	91	643
		Cartel	1	37	87	62	110	83	34	38	33	32	82	599
		Unconv	1	39	90	64	115	89	36	42	34	35	91	636
		Transp	1	40	92	66	117	91	37	43	35	36	92	648

Table 9: Production, consumption and net trade – Europe (bcm) continued

Year	Data	Case	NOR	NED	UKD	FRA	GER	ITA	ROM	SPA	POL	TRK	UKR	EUR
2015	Production	Base	111	66	32	0	21	11	14	0	3	1	19	279
		Cartel	124	66	32	0	21	11	14	0	3	1	19	293
		Unconv	113	66	32	0	21	11	14	0	3	1	19	281
		Transp	108	65	32	0	21	11	14	0	3	1	19	276
	LNG imports	Base	-7	7	14	28	4	20	4	38	2	5	0	114
		Cartel	-7	7	18	23	2	17	2	37	1	5	0	106
		Unconv	-7	6	11	25	4	17	2	37	2	5	0	100
		Transp	-7	7	17	28	4	22	4	40	2	5	0	122
	Pipeline imports	Base	-103	-32	43	37	93	62	21	12	34	37	72	277
		Cartel	-116	-34	33	38	90	58	21	5	33	33	67	227
		Unconv	-105	-31	45	40	93	64	23	12	34	37	71	284
		Transp	-100	-31	43	38	96	62	22	10	35	38	73	286
	Consumption	Base	1	41	89	65	118	93	39	49	39	43	90	669
		Cartel	1	39	84	62	113	86	36	42	38	39	86	625
		Unconv	1	41	88	65	117	92	38	48	39	43	90	664
		Transp	1	42	92	67	120	95	39	50	40	44	92	683

Year	Data	Case	NOR	NED	UKD	FRA	GER	ITA	ROM	SPA	POL	TRK	UKR	EUR
2020	Production	Base	122	64	26	0	18	9	14	0	3	1	19	276
		Cartel	139	64	26	0	18	9	14	0	3	1	19	293
		Unconv	125	64	26	0	18	9	14	0	3	1	19	279
		Transp	119	64	26	0	18	9	14	0	3	1	19	271
	LNG imports	Base	-7	10	20	31	5	23	4	35	4	3	0	129
		Cartel	-7	11	21	25	4	18	2	32	2	1	0	110
		Unconv	-7	9	16	26	5	22	3	34	4	2	0	115
		Transp	-7	11	22	34	5	27	5	38	4	4	0	143
	Pipeline imports	Base	-114	-32	45	37	100	70	24	12	37	50	75	303
		Cartel	-132	-36	38	39	96	67	24	8	37	46	71	258
		Unconv	-117	-31	48	41	99	70	25	12	37	50	75	309
		Transp	-111	-31	46	36	103	69	25	10	38	50	78	313
	Consumption	Base	1	42	91	68	123	101	42	47	44	54	94	707
		Cartel	1	39	86	64	117	93	40	40	42	49	89	661
		Unconv	1	41	90	68	122	100	42	46	43	54	94	702
		Transp	1	43	94	70	127	105	44	48	45	55	96	727

Table 9: Production, consumption and net trade – Europe (bcm) continued

Year	Data	Case	NOR	NED	UKD	FRA	GER	ITA	ROM	SPA	POL	TRK	UKR	EUR
2025	Production	Base	125	59	20	0	17	9	13	0	3	1	19	267
		Cartel	141	59	20	0	18	9	13	0	3	1	19	283
		Unconv	128	59	20	0	17	9	13	0	3	1	19	270
		Transp	120	59	20	0	17	9	13	0	3	1	19	262
	LNG imports	Base	-5	14	24	36	7	30	5	35	5	4	0	156
		Cartel	-7	14	26	26	6	20	3	28	3	1	0	120
		Unconv	-7	12	22	32	7	27	4	34	5	2	0	139
		Transp	-7	15	28	39	7	34	5	39	5	7	0	172
	Pipeline imports	Base	-119	-30	46	35	103	68	27	13	38	59	79	320
		Cartel	-133	-33	40	41	97	68	27	13	38	56	74	287
		Unconv	-120	-29	48	39	101	70	29	13	38	60	79	328
		Transp	-113	-29	47	35	106	68	29	11	39	58	82	334
	Consumption	Base	1	43	90	72	127	107	45	48	46	65	98	742
		Cartel	1	40	85	67	120	97	43	41	44	58	92	689
		Unconv	1	43	90	71	126	105	45	47	46	64	97	736
		Transp	1	45	94	74	131	111	47	50	48	66	101	768

Year	Data	Case	NOR	NED	UKD	FRA	GER	ITA	ROM	SPA	POL	TRK	UKR	EUR
2030	Production	Base	120	49	17	0	15	9	12	0	3	1	19	246
		Cartel	132	50	17	0	15	9	12	0	3	1	19	258
		Unconv	123	50	17	0	15	9	12	0	3	1	19	249
		Transp	116	49	17	0	15	9	12	0	3	1	19	241
	LNG imports	Base	-4	14	28	38	9	32	6	30	7	6	0	167
		Cartel	-4	14	29	28	7	21	3	23	4	3	0	129
		Unconv	-7	13	26	35	9	29	4	29	7	4	0	149
		Transp	-4	18	31	44	9	37	6	34	7	8	0	191
	Pipeline imports	Base	-115	-23	45	33	105	69	30	13	38	73	82	350
		Cartel	-127	-26	37	38	99	69	29	14	38	67	75	314
		Unconv	-115	-22	46	36	104	70	31	13	37	75	81	357
		Transp	-110	-25	47	31	110	69	32	12	39	73	85	362
	Consumption	Base	1	41	90	72	129	110	47	43	48	80	100	762
		Cartel	1	38	83	66	121	99	44	37	45	72	94	700
		Unconv	1	41	89	71	128	108	47	42	48	80	100	753
		Transp	1	43	95	75	134	115	50	46	50	83	103	793

Table 10: Production, consumption and net trade – Americas (bcm)

Year	Data	Case	USA	Of which Alaska	CAN	Of which Mackenzie	MEX	North America	BRA	TRI	CHL	South America
2010	Production	Base	514	4	144	0	38	696	46	79	6	131
		Cartel	515	4	145	0	38	697	46	72	6	124
		Unconv	340	4	147	0	38	526	46	79	6	132
		Transp	515	4	145	0	38	697	46	79	6	131
	LNG imports	Base	28	-2	0	0	5	33	5	-22	0	-17
		Cartel	25	-2	2	0	5	32	4	-13	0	-9
		Unconv	51	-2	9	0	7	67	4	-23	0	-19
		Transp	29	-2	0	0	5	34	5	-21	0	-17
	Pipeline imports	Base	35	0	-56	0	3	-18	9	-13	3	-1
		Cartel	37	0	-58	0	3	-18	9	-13	3	-1
		Unconv	78	0	-85	0	-8	-14	9	-13	3	-1
		Transp	35	0	-56	0	3	-18	9	-14	4	-1
	Consumption	Base	576	2	89	0	46	710	60	44	9	113
		Cartel	575	2	89	0	46	710	59	46	9	114
		Unconv	469	2	72	0	38	579	59	44	9	112
		Transp	577	2	89	0	46	712	60	44	10	114

Year	Data	Case	USA	Of which Alaska	CAN	Of which Mackenzie	MEX	North America	BRA	TRI	CHL	South America
2015	Production	Base	508	2	129	18	47	684	50	79	9	137
		Cartel	509	2	129	18	47	685	50	76	9	135
		Unconv	317	2	144	32	47	509	50	79	9	138
		Transp	507	2	133	22	47	687	50	79	9	137
	LNG imports	Base	35	0	0	0	5	40	9	-19	0	-10
		Cartel	31	0	2	0	5	38	7	-13	0	-6
		Unconv	63	0	10	0	8	81	8	-20	0	-12
		Transp	37	0	1	0	5	43	9	-19	0	-9
	Pipeline imports	Base	14	0	-36	-18	5	-17	9	-12	3	-1
		Cartel	16	0	-38	-18	5	-17	10	-13	3	-1
		Unconv	73	0	-79	-32	-8	-13	9	-12	3	-1
		Transp	18	0	-40	-22	5	-17	9	-13	3	-1
	Consumption	Base	557	2	93	0	57	706	68	47	11	126
		Cartel	556	2	93	0	56	705	67	50	11	128
		Unconv	453	2	75	0	48	576	67	46	11	125
		Transp	561	2	93	0	57	712	69	47	12	127

Table 10: Production, consumption and net trade – Americas (bcm) continued

Year	Data	Case	USA	Of which Alaska	CAN	Of which Mackenzie	MEX	North America	BRA	TRI	CHL	South America
2020	Production	Base	545	46	123	40	58	726	54	98	11	163
		Cartel	547	47	123	41	58	728	54	85	11	151
		Unconv	360	53	133	51	59	552	54	99	11	164
		Transp	548	52	126	44	58	732	54	98	11	163
	LNG imports	Base	33	0	0	0	5	38	12	-23	0	-11
		Cartel	27	0	3	0	5	35	10	-9	0	2
		Unconv	69	0	11	0	9	89	11	-23	0	-12
		Transp	36	0	1	0	5	42	13	-22	0	-8
	Pipeline imports	Base	-5	-44	-22	-40	8	-20	15	-19	3	-1
		Cartel	-3	-44	-24	-41	7	-20	14	-18	3	-1
		Unconv	48	-50	-57	-51	-8	-16	15	-19	3	-1
		Transp	-4	-49	-24	-44	9	-20	15	-20	3	-1
	Consumption	Base	572	2	102	0	71	744	81	56	14	151
		Cartel	571	2	101	0	70	743	78	59	14	151
		Unconv	477	2	88	0	60	625	81	56	14	151
		Transp	579	2	103	0	72	753	82	56	14	153

Year	Data	Case	USA	Of which Alaska	CAN	Of which Mackenzie	MEX	North America	BRA	TRI	CHL	South America
2025	Production	Base	588	56	128	41	70	786	59	105	17	182
		Cartel	591	56	128	41	70	789	60	87	17	163
		Unconv	398	67	138	51	71	607	59	106	17	183
		Transp	593	63	130	44	69	793	59	105	17	182
	LNG imports	Base	25	0	1	0	5	31	16	-23	0	-7
		Cartel	17	0	4	0	6	26	13	-4	0	9
		Unconv	73	0	11	0	9	93	15	-24	0	-9
		Transp	29	0	3	0	5	37	17	-23	0	-6
	Pipeline imports	Base	-6	-54	-20	-41	8	-18	19	-21	1	-1
		Cartel	-2	-54	-23	-41	6	-19	16	-18	1	-1
		Unconv	46	-65	-53	-51	-8	-15	19	-21	1	-1
		Transp	-6	-61	-23	-44	10	-19	19	-21	1	-1
	Consumption	Base	607	2	108	0	83	799	94	61	18	173
		Cartel	606	2	108	0	82	796	89	64	18	171
		Unconv	516	2	96	0	72	685	93	61	18	173
		Transp	616	2	111	0	84	811	95	61	18	175

Table 10: Production, consumption and net trade – Americas (bcm) continued

Year	Data	Case	USA	Of which Alaska	CAN	Of which Mackenzie	MEX	North America	BRA	TRI	CHL	South America
2030	**Production**	**Base**	619	56	127	41	75	821	65	112	22	199
		Cartel	620	56	127	41	75	822	65	102	23	190
		Unconv	396	67	137	52	76	609	65	118	22	205
		Transp	623	63	130	45	75	828	65	114	22	201
	LNG imports	**Base**	23	0	1	0	0	24	19	-23	0	-4
		Cartel	13	0	4	0	7	24	14	-17	0	-3
		Unconv	83	0	16	0	7	106	18	-31	0	-13
		Transp	29	0	4	0	0	34	21	-26	0	-5
	Pipeline imports	**Base**	-28	-54	-13	-41	23	-19	25	-24	-2	-2
		Cartel	-19	-54	-16	-41	16	-19	22	-19	-4	-1
		Unconv	38	-65	-53	-52	0	-15	24	-23	-3	-2
		Transp	-26	-61	-18	-45	25	-19	24	-23	-2	-2
	Consumption	**Base**	614	2	114	0	98	826	109	65	20	194
		Cartel	614	2	114	0	97	825	102	66	19	186
		Unconv	517	2	100	0	83	700	108	64	19	191
		Transp	625	2	117	0	100	842	110	65	20	194

5 Stochastic Market Modeling and Solution Approaches

There are many uncertain factors affecting the developments in the global natural gas market. Ignoring this uncertainty in modeling approaches leads to sub-optimal solutions. To address the uncertainty different perspectives can be taken and over the years many concepts have been developed. Before presenting an overview of various stochastic modeling methods, some terminology and definitions will be introduced that will prove helpful later in the chapter. As discussed in Chapter 2 some market-equilibrium models have equivalent optimization problems; therefore stochastic optimization approaches are not just a stepping stone, but also an option for methods to be implemented. After introducing stochastic optimization, several modeling and solution approaches for stochastic market-equilibrium problems will be introduced and illustrated. Along the way various methods will be presented addressing the computational challenges arising when solving large-scale stochastic models, including decomposition, relaxation, scenario reduction and sampling methods.

5.1 Introduction

Many factors in the demand side and the supply side of the natural gas market are inherently uncertain. The nature and the underlying factors driving the uncertainty vary. Uncertainty can be induced by human behavior or natural circumstances, and differ in characteristics such as the time-scale and the magnitude of the impact. For example, a factor with a large impact that changes on a daily basis is the weather. When temperatures are low, houses and offices need to be heated, directly increasing demand for gas, but also indirectly when electricity is used for space heating. In contrast, when temperatures are high, work and living spaces need to be cooled, as must perishable products. Air conditioning and refrigerators need power to run, and a higher demand for electricity will result in higher natural gas use in power generation.

Factors with a large impact on future natural gas demand are for instance the measures taken to mitigate global warming. When natural gas is burned CO_2 is emitted into the atmosphere, albeit in lower amounts per produced kilowatt hour than when burning oil or coal. Still, the CO_2 emissions are significant and natural gas may not be a sustainable fuel in the long run. How quickly will countries adopt policies to reduce fossil fuel usage?

Will natural gas be banned relatively quickly, or be used as a bridging fuel in the process of increasing the use of renewable energy sources? Will coal-fired power plants that capture and store emitted CO_2 be competitive with natural gas? The answers to these and other questions will have great impact on the future demand for natural gas.

Another uncertain factor is the actual natural gas reserve base. How much gas is still remaining in the production fields, and how much gas fields are yet undiscovered? Will there be as much unconventional gas in other world regions as in the United States? Will the Arctic and the North Pole be opened up for exploration and production of fossil fuels? Factors such as these influence matters like the profitability of investments, and the need for exploring alternative supply sources to be able to meet energy demand in the long run.

Policy makers and managers in companies have to make decisions facing many uncertain factors, often assigning multi-billion dollar budgets for years to come. As will be illustrated later, not addressing the uncertainty in quantitative modeling tools can lead to sub-optimal decisions. Although the underlying phenomena driving the uncertainty of the weather, the political playing field and the gas reserves estimates vary a lot, all can be addressed using stochastic modeling. Most energy market models developed in the past have not addressed uncertainty. Generally, input parameters are assumed to be known in advance and a deterministic model is solved. Sometimes low and high demand scenarios are analyzed to get some insight into the sensitivity of the model results regarding changing input assumptions.[130] The actual behavior and decisions of market players are not well addressed in such a scenario-analysis approach. In reality, market players hedge their decisions, taking into account the risks induced by possible variations in future developments. In contrast, in a single scenario specific circumstances prevail. Not having to address the different possible outcomes at once, induces that model agents make myopic decisions in the separate scenarios, only well-suited for that specific scenario but possibly bad in others. Averaging the profits over all scenarios will generally not be an adequate estimate of the expected profits, not to mention that using the average values of decision variables may not be a feasible solution. As for example shown in (Birge and

[130] Scenario: a combination of outcomes for random events that outlines one of the possible futures.

Louveaux, 1997) stochastic optimization is a necessity to address the behavior of market agents under uncertainty and to represent typical hedging behavior.

The first stochastic energy market models appeared in the literature more than twenty years ago (e.g., (Haurie et al., 1987, 1990)). Still, many researchers and policy makers use deterministic modeling approaches. Presumably, the need and benefits of stochastic modeling are not recognized by everybody, or there may be a – not completely unwarranted – fear for the mathematical and computational complexity, which restricts the size of models that can be solved within calculation time restrictions. Hopefully, this dissertation can contribute to a wider application of stochastic energy market models.

In the remainder of this chapter several stochastic modeling approaches will be presented and illustrated using a stochastic version of the problem introduced in Chapter 2.

5.2 Uncertainty, risk and stochastic models – some terminology

5.2.1 Optimization under uncertainty, risk attitudes and hedging

Optimization under uncertainty implies that the actual outcome may be better or worse than the expected outcome. There may be upward and downward potential (risk) for revenues, costs or profits. A risk-neutral decision maker doesn't care about possible asymmetric consequences of upward and downward variance in the outcomes and will take expected value maximization as the objective. However, often the consequences of high losses (e.g., bankruptcy) are unacceptable or at least undesirable, which makes it important to limit the consequences of lower than expected outcomes. A risk-averse decision maker is willing to give up some of the potential (future) benefits to limit the consequences of unfavorable outcomes. Thus, the sensitivity of a decision maker to the upward and downward outcomes affects what objective should be optimized and modeling approaches do not necessarily optimize expected value, but may have an alternative focus such as minimizing the maximum loss. Generally there are not only downward or upward potentials to be considered, but a combination of both. Balancing the upward and downward potential in such a way that an on average desirable position is achieved is called hedging. Hedging does not imply robustness against the actual outcomes of random events *per se*, but if the concept is broadened to include, for example, diversification as a means for hedging financial investments, robustness against the actual movements in the markets is a result of the hedging decisions.

Value-at-risk (VaR, e.g., (Duffie and Pan, 1997)) and conditional value-at-risk (CVaR) (Rockafellar and Uryasev, 2000) are metrics developed to measure risk exposure. VaR_γ is the maximum loss over a time period at a confidence (probability) level γ and $CVaR_\gamma$ is the expected value of the losses exceeding the VaR_γ:[131]

$$VaR_\gamma: \quad \min\left\{x \in \mathbb{R} : \mathrm{P}\left(Loss \geq x\right) \geq \gamma\right\} \tag{5.2.1}$$

$$CVaR_\gamma: \quad \tfrac{1}{1-\gamma}\mathrm{E}_{Loss}\left\{Loss : Loss \geq VaR_\gamma\right\} = \tfrac{1}{1-\gamma}\sum_{l \geq VaR_\gamma} l \times \mathrm{P}\left(Loss = l\right) \tag{5.2.2}$$

Confidence levels and expectations involve probability distributions for earnings and since earnings on investment show interdependencies, covariances are needed when calculating VaR and CVaR. Since investment portfolios of financial institutions typically contain tens of thousands of different investments, potentially millions of covariances must be calculated, posing a heavy computational burden. When the VaR or CVaR of a portfolio of financial assets not just need to be calculated but actually optimized for investment portfolios, approximations are needed. Although for instance Pang and Leyffer (2004) developed an approach for minimizing VaR and Künzi-Bay and Mayer (2006) developed a two-stage recourse problem for minimizing CVaR, all numerical examples in their work are of limited sizes. Kannan et al. (2009) developed a CVaR approach for bidding in forward and spot electricity markets while addressing the risks due to uncertainty in intermittent renewable energy sources. Besides providing a novel, risk-addressing framework, they developed several mathematical results, proposed and implemented a decomposition approach to address scalability issues. Cabero et al. (2010) developed a large-scale stochastic electricity market model, using CVaR for acceptable risk-levels in an oligopolistic setting among the producers. The model is solved using Benders decomposition and the paper is discussed in further detail in Section 5.4.3.

[131] The equations assume discrete probability distributions. Typically, γ is value close to but lower than 1. E.g., 0.95 or 0.99.

Most approaches to address uncertainty have their limitations. However, ignoring uncertainty and risk may lead to myopic and sub-optimal behavior. Addressing uncertainty is a necessity, but there is a cost involved: the additional investment in time and money to develop a more complicated model. To show the benefits of the additional modeling effort it is helpful to assess the gain of addressing uncertainty. The value of the stochastic solution provides a means to quantify this gain.

5.2.2 The value of the stochastic solution

Birge (1982) introduced the concept of the value of the stochastic solution (VSS) in stochastic optimization programs. The VSS is a measure for the added value of explicitly considering the stochasticity of uncertain aspects in a model instead of using expected values. Consider a situation where part of the decisions have to be taken immediately (here-and-now) and some (wait-and-see) decisions only after the uncertain outcomes are known. Such a problem is called a two-stage recourse problem, where the second-stage variables are the recourse variables. For instance, here-and-now decision could be how much should be invested in production capacity and the wait-and-see decision (to be taken after the uncertain demand has become known), how much to produce and sell. The condition that emphasizes the lack of knowledge about the future when deciding about the first-stage variables is called non-anticipativity (Wets, 1974).

Define a stochastic program with random outcomes χ, first-stage (here-and-now) decision variables x, second-stage decision variables y and objective function $a^T x + z\left(y\left(x,\chi\right)\right)$. The recourse problem (RP) is defined as (5.2.3) and its solution is the stochastic solution (SS). The problem with expected values for uncertain outcomes $\overline{\chi} = \mathrm{E}_\chi \chi$ is the expected value problem (EVP):

RP:

$$ESS = \min_{x,y} \mathrm{E}_\chi \left(a^T x + z\left(y\left(x,\chi\right)\right)\right) \tag{5.2.3}$$

EVP:

$$EV = \min_{x,y} \left(a^T x + z\left(y\left(x,\overline{\chi}\right)\right)\right) \tag{5.2.4}$$

Typically, the objective value EV does not represent what the myopic EVP solution variables would achieve in the stochastic setting. Calculating the objective value of the EVP in the stochastic setting gives the expected value of the EVP solution (EEV).

Intuitively, since the RP solution addresses the uncertainty which the EVP ignores, the RP should be better than the EV. For optimization problems this intuition is formalized and proved in (Birge and Louveaux, 1997):[132]

$$VSS = ESS - EEV = \min_{x,y} \mathrm{E}_{\chi}\left(a^T x + z\left(y\left(x,\chi\right)\right)\right) - \mathrm{E}_{\chi} \min_{x,y}\left(a^T x + z\left(y\left(x,\bar{\chi}\right)\right)\right) \geq 0 \qquad (5.2.5)$$

In contrast to optimization problems, there have been instances of complementarity problems for which some of the players had: $VSS < 0$ (E.g., (Zhuang, 2005) and (Genc et al., 2007)). Since convex problems for perfectly competitive markets and monopoly markets can be cast as optimization problems, negative VSS can only be observed for models with several players exerting market power.

Another useful concept is the value of perfect information. It represents how much one should be willing to pay a clairvoyant to get insight in future events relevant to the optimization problem. It can be calculated beforehand how much the information is expected to be worth. For all possible outcomes for random events χ, it can be determined what the best course of action would be and what the profits would be for each outcome. Weighting the profits for each scenario with the probability of the scenario p_{χ} gives the expected profits when having perfect information (EPI). The Value of Perfect Information (VPI, (Birge and Louveaux, 1997)) can then be calculated as the difference between EPI and ESS (Eq. **(5.2.3)** above). Eq. (5.2.6) provides the EVPI for discrete probability distributions.

$$EVPI = EPI - ESS = \sum_{\chi} p_{\chi} \min_{x,y}\left(a^T x + z\left(y\left(x,\chi\right)\right)\right) - \min_{x,y} \mathrm{E}_{\chi}\left(a^T x + z\left(y\left(x,\chi\right)\right)\right) \qquad (5.2.6)$$

A similar approach could be used to determine the value of (imperfect) information obtainable from industry experts, using conditional probabilities to assess the confidence in their expertise.

Sürücü (2005) provided an accessible introduction to modeling approaches for addressing risk management in energy markets. The work included a literature overview and a mathematical description of methods, but no numerical examples were given. Hu (2009) analyzed the impact of uncertainty in several energy markets. In a first case the value of

[132] Section 5.3.3.1 will present an example of a stochastic (two-stage recourse) cost minimization problem.

information for the United States was determined for demand load, natural gas prices and greenhouse gas policies (see also (Hu and Hobbs, 2010)). A second case addressed consistent over-estimation of consumer surplus due to uncertainty in technology costs for North West Europe. A third case analyzed the impact of different assumptions for the response of demand to price changes on the capacity-market outcomes for the Pennsylvania-New Jersey-Maryland (PJM) Electricity Market.

In the following sections more background will be provided on stochastic formulations and solution approaches for stochastic models. Along the way it will become clear what approach could be a good choice for modeling and solving the stochastic WGM.

5.3 Stochastic optimization

In Section 5.2 the case was made that the EV solution is generally sub-optimal. To illustrate this, a stochastic variant of the example introduced in Chapter 2 will be presented. Further in the chapter the same example will be used to illustrate various other stochastic modeling approaches.

5.3.1 Optimization with expected values of random outcomes

In the following example the small stochastic investment problem from Chapter 2 is extended to contain two scenarios: a low-demand scenario and a high-demand scenario. The investment decision will be the here-and-now decision and the sold quantities the recourse decisions that can vary by scenario.

5.3.1.1 Simple stochastic investment problem

Assume a producer is selling some commodity q in a two-period model. His production costs are negligible. Due to restrictions in his supply chain he can only sell five units of q in a time period; if he wants to sell more, he should invest in more capacity, $I \geq 0$, at a cost of \$2 per unit. The investment decision is to be made in the first period, and the sales will occur in the second period. Demand for q is stochastic, with two possible outcomes, q_1 and q_2. Prices are denoted by p_i and there is 50% chance that the inverse demand function is $p_1=10-q_1$ and 50% chance that it is $p_2=20-q_2$. How much should the producer invest to maximize his expected profits? To illustrate the (myopic) EV approach, the expected values of all random outcomes are taken, and the resulting deterministic model is solved.

The expected value for the random outcomes would here be the average of the low and high scenario inverse demand curves.

5.3.1.2 Solving with expected values for random outcomes

The average of both inverse demand curves is given by: $p=15-q$. The optimization problem then becomes:

$$\max_{q,I} (15-q)q - 2I \tag{5.3.1}$$
$$\textbf{s.t. } q \le I + 5$$
$$q \ge 0$$
$$I \ge 0$$

There are two possibilities. Either $q \le 5$ and $I = 0$ or $q > 5$ and since investment costs are positive, the capacity must be binding in an optimal solution: $I = q - 5$. $I = 0$ implies that the objective would be $\max_q (15-q)q$. Taking the derivative and setting it equal to zero: $15 - 2q = 0$, or $q = \frac{15}{2} = 7\frac{1}{2} > 5$, what contradicts with $I = 0$. Thus, $q > 5$ and $I = q - 5$. Substituting this into the objective (Eq. (5.3.1)), gives $\max_q (15-q)q - 2(q-5)$ = $\max_q (13-q)q + 10$. Setting the first-order derivative equal to zero $(13-2q) = 0$ leads to an optimal quantity of $\overline{q} = 6\frac{1}{2}$, an optimal investment of $\overline{I} = 1\frac{1}{2}$ an objective value of \$52¼.[133] This objective value is the EV, and at the end of the next subsection the EEV is determined.

An alternative approach is to solve all scenarios independently and average the outcomes: the scenario approach.

5.3.1.3 Solving scenarios and taking averages

In the low demand scenario: $p_1=10-q_1$, and the optimization problem for the producer is:

$$\max_{q,I} (10-q)q - 2I \tag{5.3.2}$$

[133] Note that for the quadratic maximization objective function in this and all other examples in this chapter, the second-order derivative is negative (e.g., here it is -2), and therefore the stationary point found by setting the first-order derivative equal to zero is indeed a maximum.

$$\text{s.t. } q \leq I + 5$$
$$q \geq 0$$
$$I \geq 0$$

This can be pictured as follows:

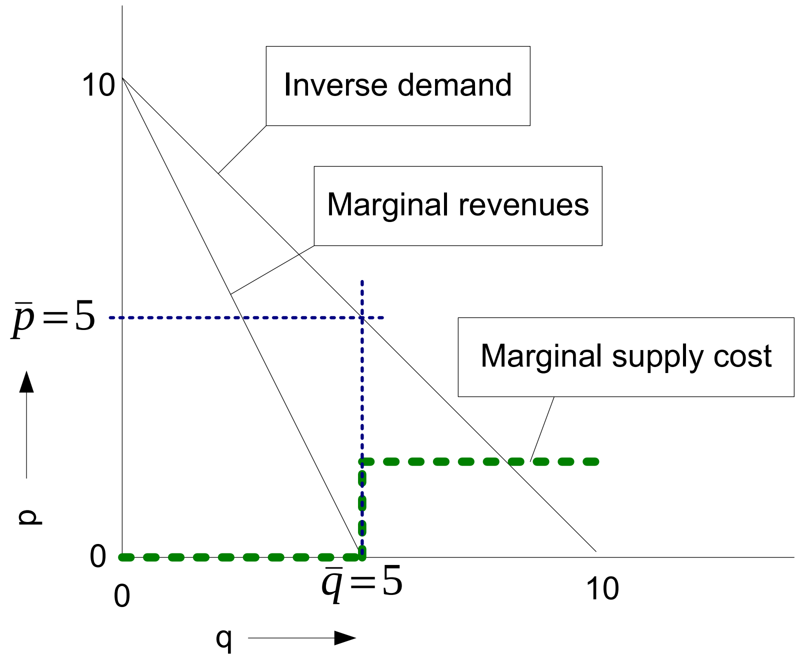

Figure 32: Low-demand scenario equilibrium

Following the same approach as in the former example, either $q \leq 5$ and $I = 0$ or $q > 5$ and $I = q - 5$. $I = 0$ implies that the objective would be problem would be: $\max_{q_1}\{(10 - q_1)q_1\}$. Setting the derivate equal to zero $10 - 2q_1 = 0$ gives the optimal quantity $\bar{q}_1 = 5$. Hence, $\bar{q}_1 = 5$ is the optimal quantity to the unconstrained problem, no investment is necessary and the profit would be \$25.

In the high demand scenario the inverse-demand curve is $p_2=20-q_2$ and the optimization problem for the producer is:

$$\max_{q,I}(20 - q)q - 2I \tag{5.3.3}$$

$$\text{s.t. } q \leq I + 5$$

$$q \geq 0$$

$$I \geq 0$$

This can be pictured as:

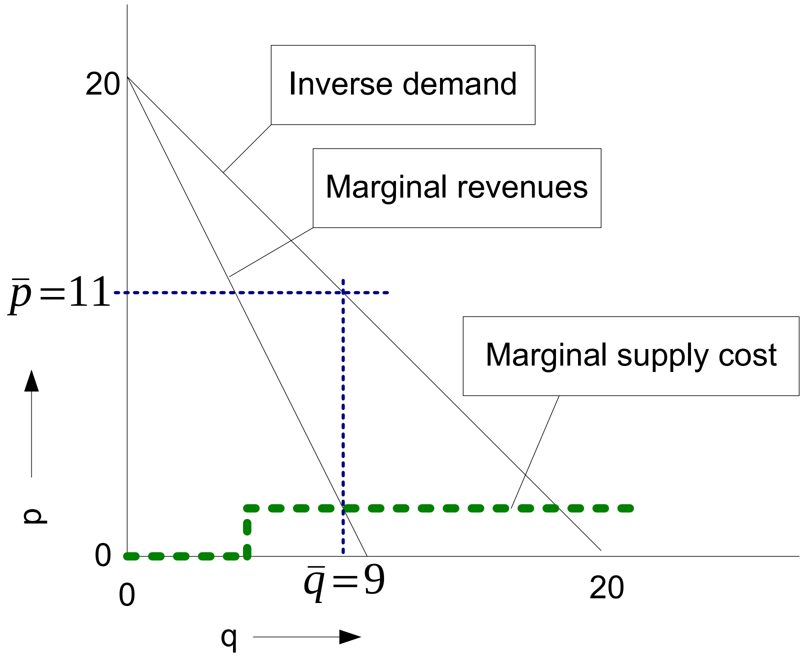

Figure 33: High-demand scenario equilibrium

Using the same approach as before, the optimal quantity for the unconstrained problem $\max_{q_2}(20-q_2)q_2$ is found as $\bar{q}_2 = 10$. This implies that for positive investment costs the optimal quantity will be at most ten and the optimal investment at most five. Assuming positive investments, the objective function becomes $10+18q-q_2^{\ 2}$, for which the optimal quantity is $\bar{q}_2 = 9$, $\bar{I} = 4$ with a profit of \$91. Combining both scenarios, the average investment would be two and the average profit $\frac{25+91}{2} = \$58$. This profit level compares favorably to the \$52¼ of the EVP solution. Unfortunately the \$58 outcome misrepresents the actual expected profits. To see that, the actual expected profit for an investment of two units must be calculated. In case of low demand the optimal quantity would be five and the total profit after subtracting investment costs for two units *25-4=* \$21. In case of high demand and maximizing $\max_{q}(20-q)q$ s.t. $q \leq 7$, the optimal quantity is seven and the total profit: $13\times 7-4=$ \$87. The expected profit of the scenario

approach solution is therefore: $\frac{21+87}{2} = \$54$ which is lower than the previously calculated average of \$58. This shows that the average outcome of scenarios will generally not provide us with a reliable solution. (Actually, \$58 is the EPI objective value.)
Thus, the scenario approach does not provide a valid outcome. What about the EV solution calculated in Section 5.3.1.2? The optimal decision in the EV was to invest in $\bar{I} = 1\frac{1}{2}$ units of additional capacity, at a cost of \$3. In the low-demand scenario the profit would be $(10-5)\times 5 - \left(2\times 1\frac{1}{2}\right) = \$22$ and in the high demand scenario the profit would be $\left(20-6\frac{1}{2}\right)\times 6\frac{1}{2} - \left(2\times 1\frac{1}{2}\right) = 13\frac{1}{2}\times 6\frac{1}{2} - 3 = \$84\frac{3}{4}$. Thus the expected profit (EEV) is: $\frac{22+84\frac{3}{4}}{2} = \$\,53\frac{3}{8}$, actually higher than the EV of \$52¼, but lower than the \$54 of the scenario approach. Although one might hope that scenario analysis would provide a better outcome, it depends on the situation whether scenario analysis or the EV approach gives a better outcome. However, generally both are not good approaches to model decisions under uncertainty, as is illustrated in a later example in Section 5.3.3.1.

Before continuing with general stochastic modeling approaches, some models will be discussed that have been developed for capacity expansion problems addressing uncertainty and gaming aspects.

5.3.2 Pipelines and other investment games under uncertainty

Murto and Keppo (2002) developed a game-theoretic investment decision model using real option theory. They analyzed how actions of competing investors affect each other's investment opportunities under different assumptions on the information that firms have about each other's project valuations. The existence of Nash equilibria was proved under different informational assumptions and it was shown that optimal strategies depend very much on the information availability. An illustration was given for setting up a telecommunications network given uncertainty in spot market prices for capacity usage.

Klaassen et al. (2004) developed a gas pipeline gaming model and applied it to a case study for the Caspian region. They developed a very detailed dynamical game model, cast as a dynamic nonzero-sum game with investment scenarios aiming to optimize the commercialization times of pipelines while addressing regulation of gas supply and formation of gas prices. Model players balanced between having the first-mover advantage (pushing out the other suppliers for a while) on the one hand and the possibility to wait and allow gas demand to grow as well as benefiting from potential technical innovations that could decrease investment costs, on the other hand. Their case study addressed the competition between the planned and proposed gas pipelines from Russia, Turkmenistan and Iran through the Caspian region. The net present values of the pipeline projects over time were projected for different scenarios. Interestingly they found that the Russian-built BlueStream, that was put in place several years ago, would never operate on full capacity, thereby supporting stubborn rumors that that pipeline was not built for commercial but rather political reasons.

Krey and Minullin (2005) extended the work of (Klaassen et al., 2004) using a mixed complementarity problem (MCP) setting. They developed two separate models, one for the natural gas supply game and another for the pipeline timing game. The supply game was modeled as an equilibrium problem and the timing game – using discrete time-steps – as a finite n-person game in normal form. To limit calculation times they restricted themselves to five market participants at most. They presented an application for different natural gas pipeline projects from various CIS countries to China.[134]

Tomasgard et al. (in Hasle et al. (2007)) presented an approach to manage and optimize the various parts of the natural gas supply chain from production to sales from the perspective of a Norwegian natural gas producer. Their paper took an integrated operational and financial perspective, taking into account uncertainty in both demand and prices in a two-stage recourse approach.

Kalashnikov et al. (2010) developed a bi-level approach for capacity booking in a pipeline network addressing the policy-induced separation of network ownership (network operator) and network usage (gas trader). The upper-level problem was formed by the gas trading company deciding on the capacities to be booked to maximize expected profits given uncertain demand. In the lower-level problem the network

[134] CIS: Commonwealth of Independent States

operator balanced capacity usage by charging penalties or providing bonuses with the aim to minimize the absolute value of the net penalties or bonuses.

The detailed game-theoretic approach developed in (Klaassen et al., 2004) and (Krey and Minullin, 2005) is very suitable for analyzing a limited number of investment options in a specific geographical region under uncertainty of prices and competitors' timing, but would be to computationally challenging for a model covering the global gas market. The same is true for the models in (Tomasgard et al., 2007). The added insight provided by the operational detail in (Kalashnikov et al. 2010) would not outweigh the extra time needed to solve a stochastic gas market model with global coverage. The WGM is an MCP with many players and periods and continuous capacity expansions. Developing a stochastic version with discrete variables is not an easy option, since KKT cannot accommodate discrete variables. Alternatively, a completely other stochastic modeling approach allowing for discrete variables could be considered. However, such approaches have limitations relative to the model size and computational tractability. The game-theoretic approaches in the former section are more suitable for games with fewer players and more detailed considerations for individual market agents. The modeling approach for the stochastic WGM should be scalable to deal with a large number of players. The approach must be able to address uncertainty in a multi-period setting, where the stochastic aspects are fully incorporated in the model. The first-stage decisions will balance the exposure to upward and downward risk while later-stage (recourse) decisions mitigate the consequences of varying outcomes of the random parameters. An approach providing those characteristics is the extensive-form stochastic model.

5.3.3 Extensive–form stochastic models with recourse

Extensive-form stochastic problem formulations include all considered futures (scenarios) explicitly and assign probabilities to all uncertain outcomes (Birge and Louveaux (1997)). A scenario tree can be drawn to represent the information structure (see Figure 34 below, showing the scenario tree for the two-stage extensive-form stochastic program with two scenarios used in the examples in this chapter.) Players with recourse options, i.e., players that are able to make other future decisions based on different outcomes, will have different decision variables for all future scenario-tree nodes. This ‘extensive’ way to define variables is the rationale for the name of this approach.

In the following risk neutrality is assumed and therefore expected values can be optimized. Continuing the small investment example: what would be the best course of actions for the producer when incorporating both low and high-demand scenarios in one framework and allowing him to hedge his decisions?

5.3.3.1 Solving the extensive-form stochastic problem

The scenario tree for the producer’s problem would have three scenario nodes. Let m_0 be the node in the first stage that represents the decision moment of the investment. Nodes m_1 and m_2 represent the low and the high demand scenarios. Then the tree can be depicted as follows:

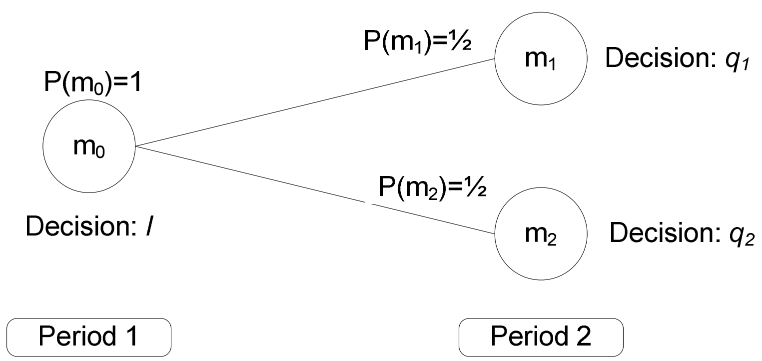

Figure 34: Scenario tree for small two-stage investment problem

The extensive-form formulation for the problem that needs to be solved is the following:

$$\max_{q_1, q_2, I} \left\{ \tfrac{1}{2}(10 - q_1) q_1 + \tfrac{1}{2}(20 - q_2) q_2 \right\} - 2I \tag{5.3.4}$$

$$\text{s.t. } q_1 \le 5 + I$$

$$q_2 \le 5 + I$$

$$q_1 \ge 0$$

$$q_2 \ge 0$$

$$I \ge 0$$

The calculations in Section 5.3.1.3 provided that regardless of the investment level, $\overline{q}_1 = 5$ and $\overline{q}_2 = 5 + \overline{I}$. Using this information, the objective function in (5.3.4) can be rewritten as $22\tfrac{1}{2} + 8q_2 - \tfrac{1}{2}q_2{}^2$, for which the optimal quantity is $\overline{q}_2 = 8$ and the optimal investment is $\overline{I} = 3$. The expected profits are $\tfrac{1}{2}(10-5)5 + \tfrac{1}{2}(20-8)8 - 2\times 3$ = \$54½. This outcome is called the stochastic solution. Naturally the expected profit of this outcome is lower than \$58, the – myopic and not attainable – average of the separate low and high demand scenario outcomes. However, it is higher than either of the other – feasible – expected profits that were calculated before in Section 5.3.1.3. The VSS turns out to be 54½ - 52¼ = 2¼, or almost 5%.

Explicitly enumerating all uncertain futures in one framework allows the market agents to hedge their decisions. A disadvantage is scalability: the model can grow quickly beyond sizes that can be solved in an acceptable amount of time. Large-scale models can often not be solved as a whole in their original form. Relaxation and decomposition are two of the approaches developed to solve large-scale problems. By iteratively adjusting the problem and improving the approximate solutions found, good eventual solutions can be obtained. The issue is how quickly the methods will converge to a true optimal solution.

5.3.4 Relaxation approaches to stochastic optimization

Relaxation approaches (e.g., (Nash and Sofer, 1996) or (Wolsey, 1998)) leave out some part of the problem, usually difficult constraints, to obtain an easier problem that can be solved quickly. Carøe and Schultz (1999) proposed a dual decomposition scheme to solve stochastic multi-stage integer programs with recourse. They implemented a decomposition scheme using branch and bound and Lagrangian relaxation (LR) with respect to the non-anticipativity condition (see Section 5.2.2). The original problem was decomposed in subproblems (SP) by scenario and each subproblem includes all first-stage variables. In the proposed LR procedure the differences among the first-stage variable values (that given the non-anticipativity condition have to be zero) are penalized using Lagrangian multipliers.

The Lagrangian dual of the problem was still a mixed integer program (MIP), however much smaller since it did not include the recourse variables and therefore the dual solved much quicker than the original problem. As the original problem was an MIP, the dual solution value could show a difference with the primal solution, the so-called duality gap. However, any optimal solution to LR provides an upper bound for the original (maximization) problem and for any relaxed solution feasible to the original problem, the duality gap is zero and the solution is optimal (Wolsey, 1998). The authors provided a small example showing that sometimes the duality gap was strictly positive. To find good feasible solutions they proposed a branch-and-bound procedure, wherein upper bounds were obtained from LR and lower bounds from feasible solutions. The authors reported great improvements in calculation times of their approach versus CPLEX MIP 4.0.[135]

Another LR approach was developed by (Nowak and Römisch, 2000) for the optimal scheduling of power-generation units under uncertainty. Their aim was to solve a huge large-scale mixed-integer problem with up to several hundred thousands of binary variables. Decisions included the generation loads and pumping cycles in terms of which units to turn on or off and the output and pumping levels. The problem was decomposed and solved using a relaxation scheme somewhat similar to (Carøe and Schultz, 1999) discussed above. To deal with the sheer size of the problem, different SP and algorithms for the different subproblems were applied, such as:

1. A linear-time descent algorithm for the stochastic hydro storage problems.
2. A stochastic dynamic program to select the output levels of the generation units.
3. A proximal bundle method to solve the decomposed dual problems.[136]
4. A Lagrangian heuristic to create a feasible schedule based on load and reserve expectations. The heuristic used the information from all former steps. Step 3 provided lower bounds on the production costs, since typically the coupling constraints for output and storage levels were violated.
5. Another descent algorithm adjusting the output levels to minimize costs for the schedule determined in step 4.

Their results showed a high computational efficiency relative to other methods and commercial solvers as well as duality gaps smaller than 1%.

[135] www.ibm.com/software/integration/optimization/cplex-optimizer/
[136] The bundle method is a variant of sub gradient methods. See (Nowak and Römisch, 2000).

5.3.5 Decomposition approaches

Dantzig-Wolfe (DW) and Benders decomposition (BD) are two approaches developed in the early 1960s for solving linear programs (LP) with specific challenges. DW handles complexities due to complicating constraints and BD can be applied to solve problems with complicating variables.

5.3.5.1 Dantzig-Wolfe decomposition

Dantzig and Wolfe (1960) developed a decomposition scheme for linear programs. Dantzig-Wolfe decomposition (DW) is a delayed column-generation approach, also referred to as inner linearization. It uses the fact that every solution of an LP can be expressed as a convex combination of the extreme points and extreme rays of the feasible region. DW can be a good decomposition approach when the constraints can be divided in a complicating and a non-complicating set. For instance, the complicating constraints can be coupling constraints, through which decision variables in multiple periods are connected. The algorithm starts by restating the objective function in terms of some of the extreme points and extreme rays, resulting in the so-called restricted master problem. That problem is solved, and then a set of subproblems is checked to see whether the solution is optimal or to identify an extreme point or extreme ray that should be added to form a new restricted master problem. This identification process is often called the pricing problem.

5.3.5.2 Benders decomposition

Benders (1962) proposed two iterative cutting-plane procedures for solving mixed integer programs.[137] He called the procedures partitioning approaches, but later literature referred to the methods as Benders decomposition (BD). It is a delayed constraint-generation approach. BD uses the fact that in an optimal solution to an LP typically only a small subset of the constraints is binding. It can be applied when there is a subset of complicating variables that prevents a solution of the problem by blocks.[138] When applied to (stochastic) multi-stage problems it is often called the L-shaped method (Birge and Louveaux, 1997).

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[137] The two partitioning procedures differ in whether the SP itself is solved or the dual of the SP.
[138] For example, when the problem matrix is block angular.

Define a minimization problem wherein x represent the complicating variables:

Original problem $$\min_{y\in\mathbb{R}_+^n, x\in K}\left\{c^T y + f(x) \middle| Ay + B(x) \geq b\right\}$$ **(5.3.5)**

Problem **(5.3.5)** can be transformed into **(5.3.6)** to facilitate the partitioning procedure:

Equivalent problem $$\min_{x\in K}\left\{f(x) + \min_{y\in\mathbb{R}_+^n}\left\{c^T y \middle| Ay \geq b - B(x)\right\}\right\}$$ **(5.3.6)**

BD starts with solving a simplified version of the original LP, the master problem (MP), in which the objective only contains the complicating variables and a new variable α that replaces all other terms and approximates the optimal value function $\alpha(x)$ = $\min_{y\in\mathbb{R}_+^n}\left\{c^T y \middle| Ay \geq b - B(x)\right\}$. The value of α is adjusted iteratively using outer linearization: an approximation from below (when the objective is to be minimized) using hyperplanes. The MP is a general programming problem that may be non-linear or discrete.

Initial master problem $$\min_{x\in K}\left\{f(x) + \alpha\right\}$$ **(5.3.7)**

Generally, there are three groups of constraints in the MP: the constraints in which only complicating variables appear (set K), the feasibility cuts and the optimality cuts. Feasibility cuts are bounds on the complicating variables that prevent infeasible solutions. Optimality cuts are linear approximations for the optimal value function $\alpha(x)$ at a particular point. The sets of feasibility and optimality are extended iteratively during the procedure.

The SP are parameterized in the decision variables of the MP and must be linear.

Primal subproblem: $$\min_{y\in\mathbb{R}_+^n}\left\{c^T y \middle| Ay \geq b - B(\hat{x})\right\}$$ **(5.3.8)**

The procedure repetitively solves MP and SP, with $\hat{y}$ (the MP solution) updated in the SP in every iteration. Typically the combined solution for SP and MP is not optimal to the original problem. In each iteration, when the SP is solved an optimality cut is generated and added to the MP to cut off parts of the feasible region. Some MP solutions may result in an infeasible SP, in which case feasibility cuts are generated and added to the MP. MP solutions provide lower bounds and SP solutions provide upper bounds for the optimal solution of the original minimization problem. The procedure continues until the lower and upper bound are close enough (or an iteration limit is reached).

Benders (1962) showed that the procedure converges to the optimal solution of the original problem in a finite number of steps. However, in later years complexity theory (e.g., Goldreich (2010)) showed that a finite number of steps does not necessarily means within practical time limits.

Geoffrion (1972) generalized BD to apply to mixed integer non-linear programs. In the following section Geoffrion's generalized BD will be applied to the small investment problem.[139]

5.3.5.3 Generalized Benders for small investment problem

When fixing the value for investment level I in the small investment problem, the problem decomposes in two quadratic SP, one for each scenario.[140] Naturally, when applying BD the first-stage decision variable would be I. Below, variable α is used as the optimal value function and $\Pi(q) = \Pi^1(q_1) + \Pi^2(q_2) = \frac{1}{2}(10 - q_1)q_1 + \frac{1}{2}(20 - q_2)q_2$ represents the revenues. The original quadratic programming problem, Eq. (5.3.4), can be written succinctly as:

Original problem:
$$\max_{q \in \mathbb{R}^2_+, I \geq 0} \left\{ -2I + \Pi(q) \middle| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} I \leq \begin{bmatrix} 5 \\ 5 \end{bmatrix} \right\} \qquad \textbf{(5.3.9)}$$

In terms of the equivalent problem, Eq. **(5.3.6)**, this would transform into:

Equivalent problem:
$$\max_{I \geq 0} \left\{ -2I + \max_{q \in \mathbb{R}^2_+} \left\{ \Pi(q) \middle| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \leq \begin{bmatrix} 5 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} I \right\} \right\} \qquad \textbf{(5.3.10)}$$

Decomposing this problem would result in a linear MP and two quadratic SP.

MP1:
$$\max_{I \geq 0} \{-2I + \alpha\} \qquad \textbf{(5.3.11)}$$

SP$_i$:
$$\max_{q_i \geq 0} \left\{ \Pi^i(q_i) \middle| q_i \leq 5 + I \right\} \quad i \in \{1, 2\} \qquad \textbf{(5.3.12)}$$

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[139] Generalized Benders decomposition will also be abbreviated with BD.
[140] Benders introduced the procedure with one MP and one SP. However, later on it was recognized that the SP can be split in parts, when constraints and variables form completely disjunctive problems.

The Benders procedure iteratively solves the MP and both SP. For numerical convenience constraint $I \leq 10$ is added.[141] Solving MP1 results in $I_1 = 0$ and $\alpha_1 = +\infty$. Solving both subproblems (5.3.12) with $I = 0$ results in the optimal quantities $q_1 = q_2 = 5$, with objective value $\frac{25+75}{2}$=\$50 and shadow prices $\lambda_1^1 = 0$ and $\lambda_2^1 = 5$ for the subproblem constraints $q_i \leq 5 + I$. For $I = 0$, the investment costs are zero, and a first feasible solution is obtained with objective value $Z_1 =$ \$50. Next, a cut is added to the MP. The cut is a linear approximation of how the SP objective values change when changing the first stage decision variable I. Using the SP shadow prices $\lambda_1^1 = 0$ and $\lambda_2^1 = 5$ and investment level $I_1 = 0$ the first cut is defined as an upper bound on the optimal value function: $\alpha \leq Z_1 + \lambda_1^1 (I - I_1) + \lambda_2^1 (I - I_1)$ = $50 + 5I$. Thus, the second master problem (MP2) is defined as follows:

MP2: **(5.3.13)**

$$\max_{I \geq 0} \{-2I + \alpha\}$$

$$\textbf{s.t. } I \leq 10$$

$$\alpha \leq 50 + 5I$$

Solving *MP2* results in $I_2 = 10$, and $\alpha_2 = 100$, providing an upper bound of $-2I_2 + \alpha_2$ = $-20 + 100$ = 80. Next, solving *SP₁* and *SP₂* with $I = 10$ results in optimal quantities $q_1 = 5$ and $q_2 = 10$ with objective values \$25 and \$100, an aggregate SP objective $Z_2 = \frac{25+100}{2}$=\$62½ and shadow prices $\lambda_1^2 = \lambda_2^2 = 0$. The objective value of the second feasible solution is $Z_2 - 2I_2$= \$42½. The difference between the last value for $\alpha : \alpha_2 = 100$ and the best feasible objective value Z_1=\$50 is the convergence gap: \$100 - \$50 = \$50. The second cut $\alpha \leq Z_2 + \lambda_1^2 (I - I_2) + \lambda_2^1 (I - I_2) = 62\frac{1}{2} + 0(I - 10) + 0(I - 10)$ is added to the MP, to form *MP3*:

MP3: **(5.3.14)**

$$\max_{I' \geq 0} \{-2I' + \alpha\}$$

$$\textbf{s.t. } I \leq 10$$

$$\alpha \leq 50 + 5I$$

$$\alpha \leq 62\frac{1}{2}$$

---

[141] This will prevent an unbounded MP solution $I = +\infty$ in the second iteration.

The solution to *MP3* is $I_3 = 2\frac{1}{2}$, $\alpha_3 = 62\frac{1}{2}$ and the new upper bound is \$57½. The results for consecutive iterations are presented in Table 11. Convergence (up to three digits) is reached in the ninth iteration.

Table 11: Convergence results for small investment problem

k	I_k	Z_k	q_1^k	q_2^k	λ_1^k	λ_1^k	***UB***	***LB***
1	0.000	50.000	5	5.000	0	5.000	∞	50.000
2	10.000	62.500	5	10.000	0	0.000	80.000	42.500
3	2.500	59.375	5	7.5.00	0	5.000	57.500	54.375
4	3.125	60.742	5	8.125	0	1.875	56.250	54.492
5	2.563	59.531	5	7.563	0	2.437	54.563	54.405
6	2.844	60.176	5	7.844	0	2.156	54.527	54.488
7	2.985	60.470	5	7.985	0	2.015	54.510	54.500
8	3.055	60.609	5	8.055	0	1.945	54.501	54.498
9	3.020	60.540	5	8.020	0	1.980	54.500	54.500

After this illustration of generalized Benders decomposition, the next subsection will present an alternative for decomposition approaches.

5.3.6 Scenario reduction

Scenario reduction methods reduce the number of scenarios in a model by aggregating scenarios with very similar characteristics. This aggregation process can be computationally challenging in itself and heuristic procedures have been developed (e.g., (Dupačová et al., 2003)). The resulting model sizes should be small enough to apply the usual solution algorithms and find solutions in acceptably short calculation times. Morales et al. (2009) applied scenario reduction to multi-period electricity markets and Gabriel et al. (2009) to natural gas market models (see Section 5.4.2).

A stochastic optimization variant of the World Gas Model (without market power) can be viewed as a multistage convex program with recourse. The first stage would encompass the minimization of capacity expansion costs and the second stage the maximization of social welfare. Clearly, for such an optimization model the techniques introduced in the previous sections could be applied. The following sections will elaborate on approaches for stochastic market-equilibrium problems that cannot be cast as optimization problems.

5.4 *Stochastic market models and algorithms*

Market-equilibrium models can be classified in various ways. One distinction among them is whether all decisions are taken at the same time, or that some decisions are taken before others, as in leader-follower games. Some games have optimization equivalents, others cannot even be represented as a complementarity problem (see Section 2.3). In the following sections stochastic variants of both game types are presented.

5.4.1 Gaming problems under uncertainty in energy markets

Haurie et al. (1987) were among the first to develop a stochastic market model incorporating market power aspects. The uncertainty was associated with the oil prices. They described the European gas market as an oligopoly and developed a stochastic nonlinear complementarity problem which was solved using a sequence of quadratic programs. They analyzed the main characteristics of long-term natural gas contracts and the market power aspects of producers vs. transmission companies, while addressing price escalation, oil price linkage as well as take-or-pay obligations. Contracts had recourse aspects, allowing different price-quantity combinations dependent on the prevailing oil prices.

Gürkan et al. (1999) set up a stochastic variational inequality and showed an application to the European gas market. They used an approximative sampling method to gain information about the second-stage objective values. Rather than solving the actual stochastic problem, the solution approach was deterministic and approximated the expected value solution (see Section 5.2.2).

Chen and Fukushima (2005) extended the expected residual minimization concept in (Lin and Fukushima, 2003) focusing on solving stochastic linear complementarity problems. An iterative solution method using a Monte Carlo sampling method was used to solve expected-value types of model approximations. By minimizing an error measure they provided better approximations than the expected value solution.

Genc et al. (2007) researched investment decisions in multi-period oligopoly problems with uncertainty. One of their main results was the occurrence of negative values of information (VI) for all players involved. The authors claimed that the result was due to the multi-player setting. However, the multi-player setting in itself cannot be the explanation, the gaming aspects are important as well (c.f., (Zhuang, 2005)). To clarify that the multi-player setting cannot be the full explanation, observe the following. Take a number of producers with identical characteristics (such as production costs) in a perfectly-competitive market. This problem can be modeled as an optimization problem, hence the value of the stochastic solution must be nonnegative (see Section 5.2.2). Since all producers are identical, all must have a nonnegative VSS. Genc et al. (2007) linked their result for the negative value of information to the prisoners' dilemma: individually dominant strategies resulting in a worse overall equilibrium than would be possible with coordination. Another interesting result was that higher price volatility induced higher expected profits. An alternative, hybrid game was proposed wherein each individual player solved a stochastic model assuming that all others use an expected value approach. In that setting the expected profits were higher. They concluded with an application of their model to the electricity market in Ontario.

(Zhuang, 2005) and (Zhuang and Gabriel, 2008) developed an extensive-form stochastic complementarity problem and provided a small-scale natural gas market implementation and various existence and uniqueness results. The authors showed that the value of information can be negative for market players not having market power.

In the following section methods are described for solving stochastic problems that cannot be cast as optimization problems: stochastic MCP.

5.4.2 Scenario reduction for stochastic MCP

Gabriel et al. (2009) applied the scenario reduction method of (Dupačová et al., 2003) to solve extensive-form stochastic natural gas market models. They applied the method using the GAMS scenario reduction package to a duopoly problem based on the North American gas market and a hybrid market problem based on the European gas market.[142] A main contribution of this paper was that theoretical convergence bounds were developed and proved. Some benchmarking was done to investigate how many scenarios

[142] www.gams.com/docs/contributed/financial/ngk_scenred.pdf

should be kept in a reduced scenario tree to still have appropriate representation of the original stochastic model. Monitoring functions were introduced to facilitate the benchmarking. These functions provided metrics showing how close the solution of a reduced scenario tree was to an optimal solution for the whole tree. In this experimental setting, the solutions could be determined for the models using the complete scenario trees; however generally for very large models this information would not be available. The results were encouraging, showing that rather large reductions in the number of scenarios can still provide good approximate solutions to the original problems.

5.4.3 Benders decomposition for stochastic MCP

Cabero et al. (2010) developed a Benders approach for linear complementarity problems (LCP). Their work took a risk-management perspective for companies operating in the Spanish electricity market. The risk measure used was conditional value at risk (see Section 5.2.1). The uncertainties considered include data for demand, fuel prices and the water inflow in reservoirs. Cabero et al. (2010) provided a large-scale implementation of Benders for LCP using realistic data and large first-stage problems. The MP were LCP that determined output quantities and acceptable risk-levels (CVaR) in an oligopolistic setting among the producers. The SP minimized cost for each producer to produce the output levels set by the MP. An initial set of 1,000 scenarios was clustered into a set of sixteen scenarios. The model contained close to 78,000 variables and the authors reported that direct solution with PATH (Dirkse and Ferris, 1995) was not possible. BD was applied for three hundred iterations, which took about twenty hours.[143] A first feasible solution was found after approximately one hundred iterations. The MP solution times grew from less than a minute for the initial iterations to about twelve minutes at the end. Consequently, the progress became very slow. The solution after three hundred iterations was given as a starting point for PATH. After almost another half hour the final solution was found.[144]

[143] On a Pentium IV, 3 GHz, 1 GB RAM

[144] Optimality is not clear. 'With this starting point PATH provides a better solution in 1,786 seconds' Given this remark, it is not clear whether the final solution was optimal, or why the extensive-form problem would not solve in PATH.

Gabriel and Fuller (2010) developed a decomposition method for general stochastic MCP and applied it to an electricity market model with stochastic demand. In contrast to (Cabero et al., 2010) the mathematical approach developed has a general – not problem specific – nature and the market power aspects were addressed in the subproblems. Therefore, the SP were complementarity problems that could not be cast as optimization problems, preventing the straight-forward application of BD. Gabriel and Fuller (2010) extended previous work by Fuller and Chung (2005), who developed a column-generation approach to solve variational inequalities and Fuller and Chung (2008) who developed a Benders decomposition approach for variational inequalities. The stochastic electricity model solved by (Gabriel and Fuller, 2010) was based on a deterministic model presented in (Hobbs, 2001). Model agents included power generators and electricity grid owners. The model consisted of two stages, with demand uncertainty in the second stage. In the first-stage the power generators decided on how much (low-cost) slow-ramping generation capacity should be brought online, while in the second stage decisions were made about (expensive) rapid-ramping capacity. Gabriel and Fuller (2010) developed theory for applying the Benders decomposition approach to stochastic MCP and established several convergence results. The method showed great reductions in calculation times and solved most problems in less than ten iterations. The most encouraging result was finding a solution within twenty minutes for a problem that in extensive form had several hundred thousands of variables and would not solve in four days of run time.

Mathematical programs with equilibrium constraints (MPEC) are a more general class of problems than MCP. For completeness, some solution approaches to stochastic MPEC are discussed next.

5.4.4 Stochastic mathematical programs with equilibrium constraints

De Wolf and Smeers (1997) presented a stochastic Stackelberg model. In their set-up the leader set his output level when future demand was still uncertain. After the demand value became known, the followers competed à la Cournot given the residual demand curve. De Wolf and Smeers (1997) showed that under reasonable assumptions there exists a unique equilibrium.[145] They addressed the possible non-convexity of the leader problem by using a piecewise-linear approximation of the aggregate response curve and solving the leader problem for all piecewise parts.

Lin et al. (2007) developed a not-exact approach for solving Stochastic MPEC by extending the expected residual minimization concept (see (Lin and Fukushima, 2003) and (Chen and Fukushima, 2005)). Essentially an expected value problem was solved, but by minimizing an error measure a better approximation was provided for the stochastic problem than the expected value solution. Birbil et al. (2004) developed a sampling approach to solve stochastic mathematical programs with complementarity constraints (SMPCC). Similar to the aforementioned paper by (Lin et al., 2007) an expected value problem is solved to approximate the stochastic solution. In consecutive iterations a deterministic problem is solved, in which the expected values are approximated using the average of simulated outcomes.

Patriksson and Wynter (1999) establish several convexity, differentiability and existence results for solutions of stochastic MPEC. The authors outline various parallel iterative solution methods, including sub-gradient methods and penalty approaches. Shapiro and Xu (2008) presented a less general SMPEC formulation than presented in (Patriksson and Wynter, 1999). In (Shapiro and Xu, 2008) the random outcomes affected the objective function values in both stages, but the first-stage decision did not affect the feasible region for the second-stage variables, as was the case in (Patriksson and Wynter, 1999). Shapiro and Xu (2008) developed a heuristic approach, incorporating a sample of the scenarios when considering the second-stage problems. Convergence properties for the approximations are shown and some benchmarking calculation time results are presented.

DeMiguel et al. (2006) developed an iterative relaxation scheme for MPEC that could possibly be adjusted to solve SMPEC as well (see Chapter 2). Lastly, Gabriel et al. (2009) developed a Benders decomposition approach for MPEC that could possible be extended for S-MPEC (see Chapter 2).

[145] For instance, their results hold when production costs are convex and twice differentiable, inverse-demand curves are concave, decreasing and twice differentiable and the aggregate marginal supply costs of the followers intersect with any of the possible future inverse-demand curves.

5.5 Summary and conclusions

In this chapter some motivation was provided for the need of stochastic modeling to adequately address input parameter uncertainties. As a stepping stone to stochastic market-equilibrium models an overview was given of stochastic optimization approaches. Next, more general stochastic equilibrium modeling approaches were discussed. Along the chapter illustrations were provided based on the small two-stage investment problem that was introduced in chapter 2.

Extensive-form stochastic MCP provide a means to address various types of uncertainty in the natural gas market. Possibly, the resulting problems will become very large, inducing long calculation times to solve them. A way to address calculation time restrictions is provided by the VI-based Benders decomposition approach developed in (Gabriel and Fuller, 2010) .

6 A Stochastic Multi-period Global Gas Market Model

The previous chapter provided a rationale why stochastic modeling is needed to address uncertainty in the natural gas market and provided an overview of stochastic modeling approaches. Stochastic modeling allows market agents to hedge their decisions anticipating uncertain future developments. In this chapter an extensive-form stochastic version of the World Gas Model (WGM) is presented and applied to a problem with four scenarios. These scenarios contain two uncertain events, the first in 2010 and the second in 2025. In 2010, a gas market cartel may come into existence and in 2025 production capacities in some importing countries may decrease significantly faster than in a business-as-usual situation. The combinations of the two events form four scenarios that are represented in a scenario tree, which is used as input for the model. Next, the input data is described and the model regions aggregation level used in the application is clarified. In the results section, various outcomes are presented, which at a first glance seem to show that on aggregate the impact of stochasticity is modest. However, when looking into detailed results various hedged decisions show that stochasticity affects both the timing as well as the sizes of capacity expansions as well as the development of production, consumption, trade volumes and prices over time.

This chapter provides the second major contribution of this dissertation in the development of a large-scale stochastic natural gas market model that can adequately address input parameter uncertainty and allow market agents to hedge their decisions. The stochastic model is applied to a problem with four scenarios for the global natural gas market for a time horizon until 2050. The problem contains nineteen geographical regions and includes 78,768 variables. Relative to Zhuang (2005) this application has more periods, uses a realistic data set for the global market and is about twelve times as large in terms of number of variables.

6.1 Introduction

This chapter analyzes the impact on model results due to the uncertainty of input parameters. Analyzing various scenarios separately (scenario analysis) can provide some insight into the consequences of different possible developments. However, to fully address the unpredictability of the future developments in the global natural gas market, the uncertainty should be addressed explicitly in the modeling approach. Thus, a stochastic version of the WGM has been developed. Aspects in the natural gas market that are uncertain include the development of demand, prices, reserves, production capacities and a possible cartelization. In the stochastic case, uncertainty is considered in two of these aspects. The first event, that may occur in 2010, is the establishment of a gas market cartel and the second event, that may occur in 2025, is a faster depletion of natural gas reserves and hence lower production in some major gas importing regions.[146] The combination of these events leads to four (in earlier periods overlapping) scenarios, that each are assumed to have an equal probability to realize. The following section briefly elaborates on the concept of a scenario tree that visualizes the extensive-form modeling approach.

6.1.1 Stochastic Scenario Tree

Figure 35 shows the scenario tree that is implemented for the stochastic case run. The scenario tree contains 31 nodes.

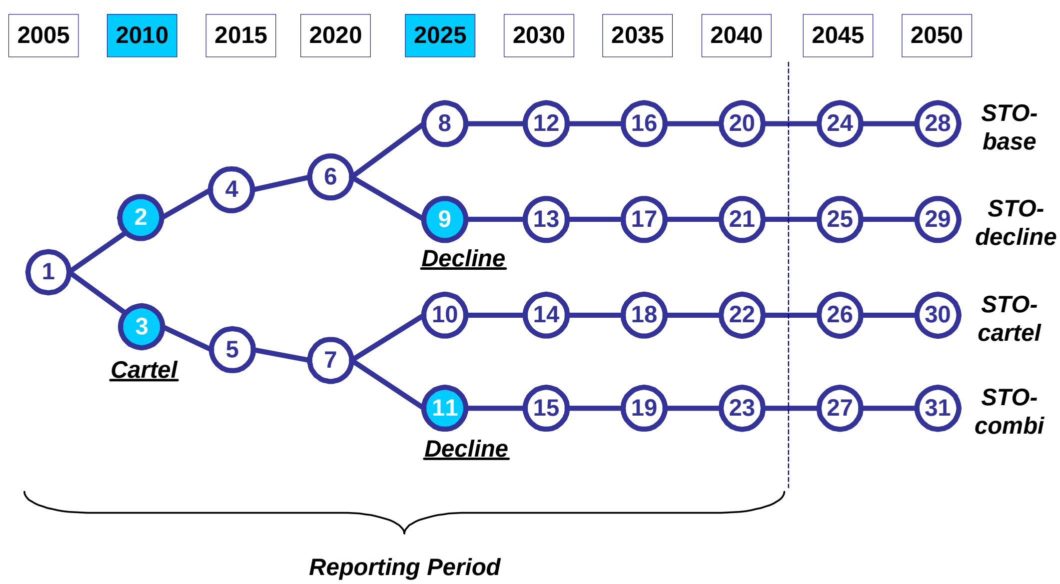

Figure 35: Scenario tree with four scenarios

The first node, node 1, represents the first year and is the common starting point for all scenarios. In years 2010 through 2020 there are two scenario nodes for each year. Each

[146] The cartel is formed by members of the Gas Exporting Countries Forum in 2009. (www.gecforum.org)

year after 2020 is represented by four different scenario nodes. All market players are assumed to be risk-neutral, maximizing their expected profits while having perfect information about all scenarios. Decisions in any scenario node *m*, notably investment levels, will be optimal 'on average' among the different scenarios of which the specific scenario node *m* is a part. Thus, in periods before 2025, the optimal decisions hedge against the outcomes of different futures. Any investment will result in additional capacity becoming available in the next period. In stochastic modeling terms, the investment in 2005 is a here-and-now decision. In any period all produced, consumed, traded and stored volumes are wait-and-see (recourse) decisions. Capacity expansions in later years have a mixed role. Relative to the capacity expansions in earlier years, capacity expansions are recourse decisions. However, relative to later years they are here-and-now decisions.

Decisions taken in 2005 have consequences for all future periods. In contrast, a decision taken in any scenario node in 2025 will only have consequences for its successor nodes in the remaining part of scenario STO-base, e.g., for node 8: nodes: 12, 16, 20, 24 and 28.

In the extensive-form approach, each scenario node has its own set of input parameters. In the case study, these scenario-specific input parameters differ in market power assumptions and production capacities. Each scenario node has its own set of decision variables and the outcomes for a variable in a certain year depend on the specific scenario that is playing out. For example, the investment level in a pipeline in 2035 depends on the scenario and may assume different levels among nodes 16, 17, 18 and 19. Similarly as for the deterministic model (see Chapter 4) two periods are added beyond the reporting period, to limit the distortion of the capacity expansion outcomes due to the end of the time horizon.

In a deterministic scenario tree, all probabilities equal one and the following model formulation would reduce to the one presented in Chapter 3, except for some differences in functionality.[147] In the next section the extensive-form stochastic market-equilibrium model is presented.

6.2 Formulating the stochastic global gas market model

The players involved in the natural gas market as well as the underlying assumptions and simplifications have been extensively described in Chapter 3. A general additional assumption for the stochastic model described next is risk-neutrality, so that players can be assumed to maximize expected profits. In the model, all volumes and capacities are in millions of cubic meters per day (mcm/d) except for storage working gas capacity which is in mcm. All operational costs and prices are in USD of 2005 per thousand cubic meters ($/kcm) and capital expansion costs are in USD per thousand cubic meter per day ($/mcm/d). All cost functions are convex. An overview of symbol names can be found in Appendix 6.6. The first player for which the stochastic optimization problem is given, is the producer.

6.2.1 Producer

Rather than optimizing over all years as in the deterministic problem, a producer p optimizes over all scenario tree nodes, weighting the individual node results with their respective probabilities. A producer p maximizes his expected profits (6.2.1) subject to a capacity constraint (6.2.2). The objective function (6.2.1) is a discounted, probability-weighted sum of revenues minus production costs, for all model nodes n, demand seasons d and random outcomes m. Production capacities are scenario-dependent, hence the values for CAP_{pnm}^{P} in Eq. (6.2.2) can vary by scenario node.[148]

$$\max_{q_{pndm}^{P\to T}} \sum_{n,d,m} p_m \gamma_m d_d \left(\pi_{ndm}^{p} q_{pndm}^{P\to T} - c_{pndm}^{P}\left(q_{pndm}^{P\to T} \right) \right) \tag{6.2.1}$$

s.t. $$q_{pndm}^{P\to T} \leq CAP_{pnm}^{P} \qquad \left(\alpha_{pndm}^{p}\right) \qquad \forall n,d,m \tag{6.2.2}$$

All primal variables are nonnegative. Greek symbols in parentheses (such as α_{pndm}^{p} in Eq. (6.2.2)) are the dual variables in the Karush-Kuhn-Tucker (KKT) conditions. Appendix

[147] The stochastic model is developed based on a previous version of the WGM where the storage operator was an arbitrageur and not a regulated service provider.

[148] Note that the production reserves constraint is not included in this formulation. Because no reliable data could be found for reserves, and using proved reserves would not be meaningful for most countries, the reserves constriction was not used.

6.7 presents the KKT conditions for the producer and all other players. Next, the trader problem is presented.

6.2.2 Trader

Traders face inverse demand curves $\Pi(\cdot) = INT^{W}_{ndm} - SLP^{W}_{ndm}\left(\sum_{t} q^{T\rightarrow W}_{tndm} + \sum_{s} q^{S\rightarrow W}_{sndm}\right)$ that may vary by scenario. Both the intercept and the slope are scenario dependent. The trader optimizes over all scenario tree nodes, weighting the individual node results with their respective probabilities. The level of market power δ^{MP}_{tnm} exerted by a trader at the nodes is scenario dependent. Thus, a trader t maximizes expected profits Eq. (6.2.3), resulting from sales to marketers and storage operators, minus purchase costs from producers and fees for using arcs. Arcs can be regular pipelines or represent parts of the LNG supply chain. Contractual supply obligations are incorporated through Eq. (6.2.4). Mass-balance Eq. (6.2.5) states that the sum of gas purchased and imported must equal the sum of gas sold to marketers, exported and sold to storage operators.

$$\max \sum_{n,d,m} p_m \gamma_m d_d \begin{pmatrix} \left(\delta^{MP}_{tnm}\Pi(\cdot) + \left(1-\delta^{MP}_{tnm}\right)\pi^{W}_{ndm}\right) q^{T\rightarrow W}_{tndm} \\ -\pi^{P}_{ndm} q^{T\leftarrow P}_{tndm} \\ -\sum_{a\in a^{+}(n)} \left(\tau^{reg}_{adm} + \tau^{A}_{ndm}\right) f^{A}_{tadm} + \pi^{TS}_{nd_1 m} q^{T\rightarrow S}_{tndm} \end{pmatrix} \quad \textbf{(6.2.3)}$$

s.t.

$$f^{A}_{tadm} \geq CON^{A}_{tam} \qquad \left(\varepsilon^{T}_{tadm}\right) \qquad \forall a,d,m \quad \textbf{(6.2.4)}$$

$$q^{T\leftarrow P}_{tndm} + \sum_{a\in a^{+}(n)} \left(1-l^{A}_{a}\right) f^{A}_{tadm} = q^{T\rightarrow W}_{tndm} + \sum_{a\in a^{-}(n)} f^{A}_{tadm} + q^{T\rightarrow S}_{tndm} \qquad \left(\varphi^{T}_{tndm}\right) \qquad \forall n,d,m \quad \textbf{(6.2.5)}$$

Next, the transmission system operator, who manages the transport network, is described.

6.2.3 Transmission system operator

The transmission system operator (TSO) is one of the two players making decisions directly affecting the future market, through making investment decisions in capacity additions and expansions. (The other player that makes investment decisions is the storage operator.) Being modeled as a regulated player and a price-taker, the TSO balances the investments in such a way that in expectation the revenues collected from capacity-congestion charges on additional capacities cover the investment costs. Thus, the TSO maximizes expected profits (6.2.6) resulting from congestion revenues minus arc expansion costs. Arc flows are subject to capacity constraints (6.2.7), which include capacity expansions in predecessor nodes: $\sum_{m' \in pred(m)} \Delta_{am'}^{A}$. Arc capacity expansions, i.e., pipeline, liquefaction and regasification expansions, are subject to limitations (6.2.8).

$$\max \sum_{am} p_m \gamma_m \left(\sum_d \left(d_d \tau_{adm}^{A} q_{adm}^{A \to T} \right) - c_a^{\Delta A} \Delta_{am}^{A} \right) \qquad \textbf{(6.2.6)}$$

s.t.

$$q_{adm}^{A \to T} \leq CAP_{am}^{A} + \sum_{m' \in pred(m)} \Delta_{am'}^{A} \qquad \left(\alpha_{adm}^{A} \right) \qquad \forall a,d,m \qquad \textbf{(6.2.7)}$$

$$\Delta_{am}^{A} \leq \bar{\Delta}_{am}^{A} \qquad \left(\rho_{am}^{A} \right) \qquad \forall a,m \qquad \textbf{(6.2.8)}$$

The next section presents the storage operator.

6.2.4 Storage operator

Storage operators execute seasonal arbitrage and make capacity expansion decisions. They maximize expected profit, Eq. (6.2.9) resulting from buying and selling gas and investment costs. They buy gas in the low-demand, low-price season and sell gas in the higher-priced high and peak-demand seasons.[149] Loss-corrected injections must equal the extractions in each year: Eq. (6.2.10). There are limitations on the injection rate, Eq. (6.2.11), extraction rate, Eq. (6.2.12) and availability of working gas: Eq. (6.2.13). The right-hand sides in the capacity constraints include the capacity expansions in predecessor nodes, e.g., $\sum_{m' \in pred(m)} \Delta_{nm'}^{I}$ for the injection capacities. Lastly, expansions are limited: Eq. (6.2.14)-(6.2.16).

$$\max \sum_{n,m} p_m \gamma_m \left(\begin{array}{l} \left(\sum_{d=2,3} d_d \pi_{ndm}^{W} q_{sndm}^{S \to W} \right) - d_1 c_{snm}^{SI} \left(q_{snd_1m}^{S \leftarrow T} \right) \\ -d_1 \pi_{nd_1m}^{TS} q_{snd_1m}^{S \leftarrow T} - c_{nm}^{\Delta I} \Delta_{nm}^{SI} - c_{nm}^{\Delta X} \Delta_{nm}^{SX} - c_{nm}^{\Delta W} \Delta_{nm}^{SW} \end{array} \right) \qquad \textbf{(6.2.9)}$$

s.t.

$$\left(1 - l_{sn}^{S} \right) d_1 q_{snd_1m}^{S \leftarrow T} = \sum_{d=2,3} d_d q_{sndm}^{S \to W} \qquad \left(\varphi_{snm}^{S} \right) \qquad \forall n,m \qquad \textbf{(6.2.10)}$$

$$q_{sndm}^{S \leftarrow T} \leq CAP_{snm}^{SI} + \sum_{m' \in pred(m)} \Delta_{snm'}^{SI} \qquad \left(\alpha_{sndm}^{SI} \right) \quad \forall n,d,m \qquad \textbf{(6.2.11)}$$

[149] There are one injection and two extraction seasons.

$$q_{sndm}^{S \to W} \leq CAP_{snm}^{SX} + \sum_{m' \in pred(m)} \Delta_{snm'}^{SX} \qquad (\alpha_{sndm}^{SX}) \quad \forall n,d,m \qquad \textbf{(6.2.12)}$$

$$\sum_{d=2,3} d_d q_{sndm}^{S \to W} \leq CAP_{snm}^{SW} + \sum_{m' \in pred(m)} \Delta_{snm'}^{SW} \qquad (\alpha_{snm}^{SW}) \quad \forall n,m \qquad \textbf{(6.2.13)}$$

$$\Delta_{snm}^{SI} \leq \overline{\Delta}_{snm}^{SI} \qquad (\rho_{snm}^{SI}) \quad \forall n,m \qquad \textbf{(6.2.14)}$$

$$\Delta_{snm}^{SX} \leq \overline{\Delta}_{snm}^{SX} \qquad (\rho_{snm}^{SX}) \quad \forall n,m \qquad \textbf{(6.2.15)}$$

$$\Delta_{snm}^{SW} \leq \overline{\Delta}_{snm}^{SW} \qquad (\rho_{snm}^{SW}) \quad \forall n,m \qquad \textbf{(6.2.16)}$$

Lastly, the downstream part of the natural gas market is represented in the inverse-demand curve.

6.2.5 Consumption

The inverse demand curve, Eq. (6.2.17) clears the market between the gas-selling traders and storage operators and the end-users.

$$\pi_{ndm}^{W} = INT_{ndm}^{W} - SLP_{ndm}^{W} \left(\sum_t q_{tndm}^{T \to W} + \sum_s q_{sndm}^{S \to W} \right) \qquad (\pi_{ndm}^{W}) \quad \forall n,d,m \qquad \textbf{(6.2.17)}$$

The conditions that tie the problems of the various players together to form one market-equilibrium problem are the market-clearing conditions.

6.2.6 Market-clearing conditions

Eq. (6.2.18) represents market clearing between producers and traders at every node.

$$\sum_p q_{pndm}^{P \to T} = \sum_t q_{tndm}^{T \leftarrow P} \qquad (\pi_{ndm}^{P}) \quad \forall n,d,m \qquad \textbf{(6.2.18)}$$

Market clearing condition (6.2.19) enforces equality of the total assigned arc capacity by the TSO and the aggregate arc flows by traders.

$$q_{adm}^{A \to T} = \sum_t f_{tadm}^{A} \qquad (\tau_{adm}^{A}) \quad \forall a,d,m \qquad \textbf{(6.2.19)}$$

Since the storage operator is not a service provider but rather a profit optimizing arbitrageur there is no market clearing for injection or extraction volumes. Instead, there is market clearing between trader and storage operators in the low demand season:

$$\sum_t q_{tnd_1m}^{T \to S} = \sum_s q_{snd_1m}^{S \leftarrow T} \qquad (\pi_{nd_1m}^{TS}) \quad \forall n,m \qquad \textbf{(6.2.20)}$$

All maximization objectives in the problem specifications above are concave. All restrictions are linear, which implies that all feasible regions are polyhedral. Hence, the

KKT conditions are necessary and sufficient for optimal solutions. The KKT conditions can be found in Section 6.7.

6.3 Input data

The stochastic model has been applied on a data set with fewer model regions than in Chapter 4. This was done because otherwise it would have taken a very long time for the model to solve. Running the original eighty-node data set with the eight-period determinist WGM takes about 3½ hours of run time.[150] Empirically we have seen that a doubling of the model-size induces a five to tenfold increase in run time. The ten-period model with the tree with four scenarios in Figure 35 has 31 nodes which implies an about fourfold increase in model size relative to the eight-period deterministic model. An estimate for the run time can be calculated as 25 to 100 times 3½ hours, or roughly between 3½ days and two weeks. Since this chapter aims at discussing consequences of stochasticity rather than solving large-scale models, the model size has been reduced by aggregating the data set to contain nineteen geographical regions only. This way, the number of variables is about four times smaller, and the resulting model size roughly the same as for the eighty-node deterministic model. Figure 36 shows the model regions included in the stochastic case. The countries included in all regions can be found in Appendix 6.8.

In the figure, the blue or darker shaded boxes are regions that can export LNG and the yellow or lighter shaded boxes are regions that can import LNG. Only the Caspian region is not involved in LNG trade. Arrows represent existing or optional pipelines. Regions that have their name underlined would take part in the cartel if it comes into existence.

The higher aggregation level has consequences for the results and some detailed insights may be lost. For instance, pipelines between regions that are grouped in the same model node will cease to exist in the model data set, so for the same projections for future demand and supply, an aggregated data set will show lower aggregate pipeline capacity expansions. Also, the characteristics of pipelines remaining in the model need to be

[150] GAMS (Brook et al., 1988) and solver PATH ((Dirkse and Ferris, 1995), (Ferris and Munson, 2000)). GAMS version 22.7.2, Computer specifications: 32-bit, 2GB dual core 2x 1.2 GHz

adjusted, since they run over longer distances and the losses and operating and expansions costs should be higher in a more aggregated data set. When demand and supply are for larger regions, seasonal price differentials will be smoothed out, and possibly storage will be used and expanded less.

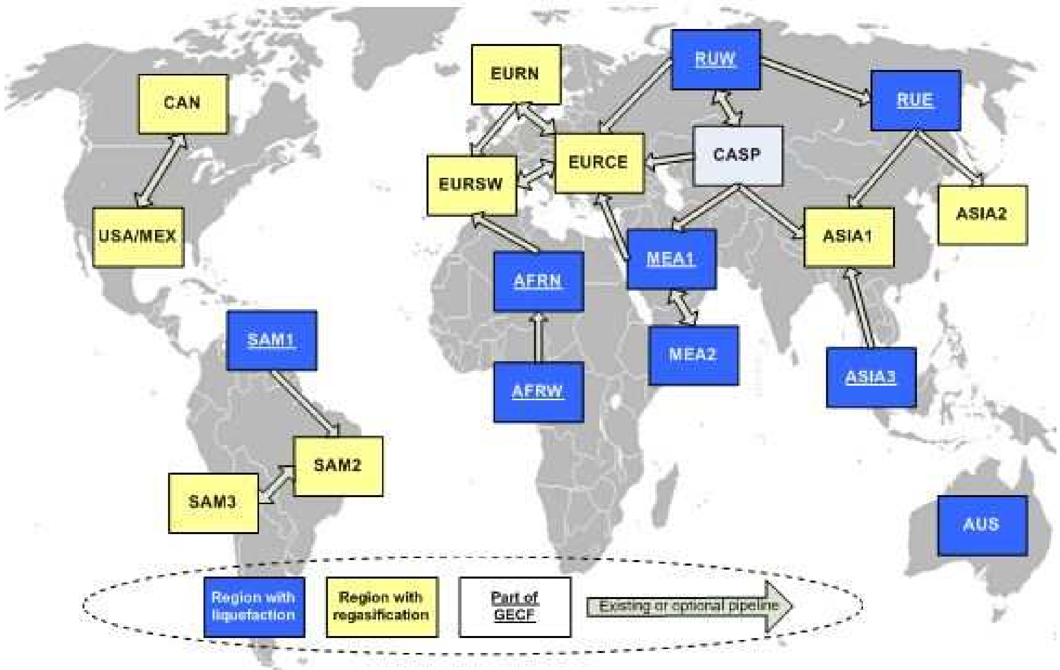

Figure 36: Regions represented in the stochastic gas market model

The input parameters used are an aggregation of the data used in Chapter 4. The reference scenario of the model has been calibrated to projections of the future natural gas demand and supply, namely PRIMES forecasts for Europe (European Commission, 2006a, 2008) and POLES forecasts for the rest of the world (European Commission, 2006b) The latter was published before the U.S. unconventional gas resources were revised and as a consequence in this model large imports of LNG by the United States are foreseen by 2030 and beyond.

The POLES projections reflect a worldwide increase in natural gas production and consumption of 70% in 2030 relative to 2005. In accordance with POLES projections, an average yearly price increase of 3% is used. The model output for global consumption in 2005 is 2362 bcm and for global production 2422 bcm, at an average wholesale price of

\$165/kcm.[151] For regions for which the data sources did not provide values beyond the year 2030, the same production capacities and reference demand levels have been assumed as for the year 2030. For infrastructure capacities, project and company information from various sources (e.g., Oil and Gas Journal, Gas Transmission Europe, and the Energy Information Administration) has been employed. See Chapter 4 for an extensive description of the data sources used.

6.4 Results and discussion

A stochastic problem with four scenarios will be analyzed in this section. The scenarios vary in that in 2010 a global gas cartel may come into existence and in 2025 production capacities may start to decrease significantly faster than in the base situation in a number of importing countries. Table 12 summarizes the main assumptions.

Table 12: Main case and scenario assumptions

	Base	**Decline**	**Cartel**	**Combi**
Market Power	In all periods: North America: 0, all other regions 0.25	Same as base	Starting 2010 cartel trader full market power	Same as Cartel
Production Capacities	Aggregates based on WGM	Starting 2025 lower for major importers	Same as Base	Same as Decline

To evaluate the stochastic model outcomes, the results are compared with four deterministic cases. The deterministic cases are:

- DET-base, the Base Case, is the calibrated reference case.
- DET-decline, the Decline Case that assumes faster depletion of gas reserves in North America, Europe and some Asian import countries from 2025 onwards.[152]
- DET-cartel, the Cartel Case that assumes a gas market cartelization in 2010.
- DET-combi, a case combining the assumptions for the Cartel and Decline Cases.

The terms deterministic counterpart and stochastic counterpart are used to refer to those cases/scenarios with identical input parameter values (compare Figure 35 in the introduction of this chapter and Figure 37 below).

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[151] kcm: 1000 m³; bcm: billion cubic meter; bcm/y: bcm per year. Note that the difference between production and consumption is due to losses in liquefaction, regasification, storage and pipelines.

[152] Implemented by assuming a linear decrease from the original 2020 values in 2020 to zero in 2070 for regions (see Figure 36): Asia1, Asia2, EURN, EURCE, EURSW, CAN and USA/MEX. The original production capacity is taken when that value is lower than the result of the aforementioned calculation.

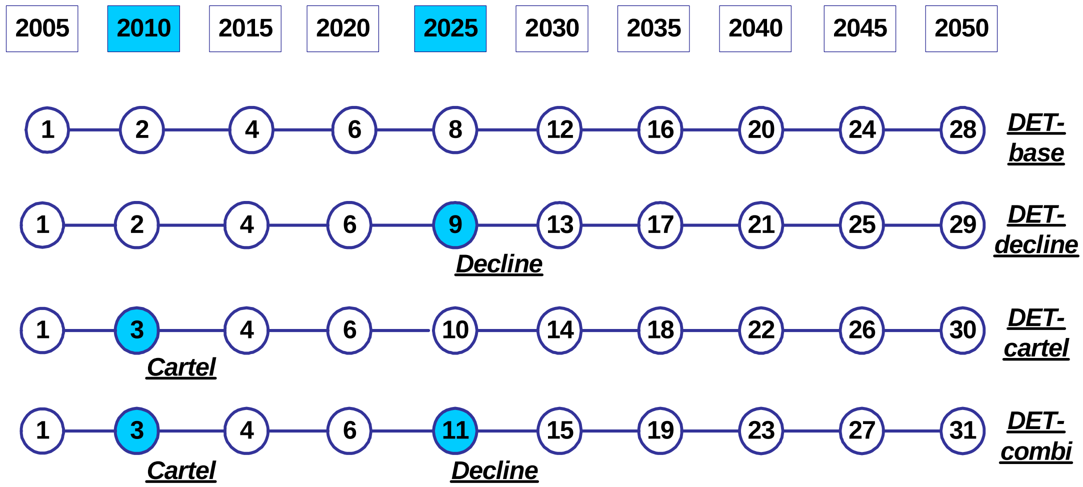

Figure 37: Deterministic counterparts of the stochastic scenarios

Each stochastic scenario has exactly the same input parameter values as one of the deterministic cases. For example, the deterministic counterpart of the uppermost scenario STO-base in Figure 35 is DET-base in Figure 37, the Base Case.

The regions that would take part in the cartel are: Russia, North and West Africa, the part of the Middle East with Iran and Qatar, the LNG exporters in South Asia (such as Indonesia) and the LNG exporters in South America (e.g., Trinidad and Tobago).

The following sections discuss several stochastic model outcomes and compare them with the deterministic case results. Since the impact of a possible cartelization is one of our main interests, the discussion will often contrast results for the cartel members and the importing regions.

Relative to the business-as-usual situation all other cases imply a tighter gas supply to the importing regions, however induced by different assumptions. Generally, the main means of a cartel to influence prices is by withholding supplies from the importing markets. Therefore, an anticipated effect of a cartelization would be lower production and export levels by members of the cartel. In contrast, suppliers that do not take part in the cartel may respond to higher market prices by bringing more gas to the markets, and reaping high benefits. A cartel would only harm regions that depend to some extent on imports to meet domestic demand, but lower domestic production capacities can harm countries that in the business-as-usual situation are self-sufficient. As such, lower domestic production is more likely to be compensated in a somewhat competitive market than in a cartel situation. The results presented include regional trade balances, production levels, liquefaction, regasification and pipeline capacity expansions, LNG trade and market prices. Tables in Section 6.9 present detailed results.

The first section discusses how production is affected by the various case assumptions. Production developments are compared for the group of cartel members and the other countries.

6.4.1 Production

The aggregate global production in 2005 is 2422 bcm in the (deterministic) Base Case DET-base and will steadily increase over time to reach 3828 bcm in 2040. Initially, production in non-GECF countries will grow quite fast, from 1389 bcm in 2005 to 1725 bcm in 2010. However the increase in later periods is modest and production will plateau at around 1800 bcm/y for the remainder of the time horizon. In 2005 the group of GECF countries produces 1033 bcm, a share of 43% of global production. After a slight dip in 2010 the share of GECF countries will grow to 54% by 2040 (just over 2000 bcm/y).

Figure 38 shows the differences in production levels among the different deterministic cases relative to the Base Case, aggregated by GECF and non-GECF countries. The production levels in the different cases vary considerably. Generally, a cartelization would induce lower production in GECF countries and higher production in others. In contrast, declining production rates in the major importing regions would induce higher production in all other countries, with the largest impact in both cartel cases.

In the cartel cases (DET-cartel and DET-combi), the output of GECF countries would be significantly lower than in DET-base, ranging from 69 bcm (5%) lower to 219 bcm (11%) lower in 2040.

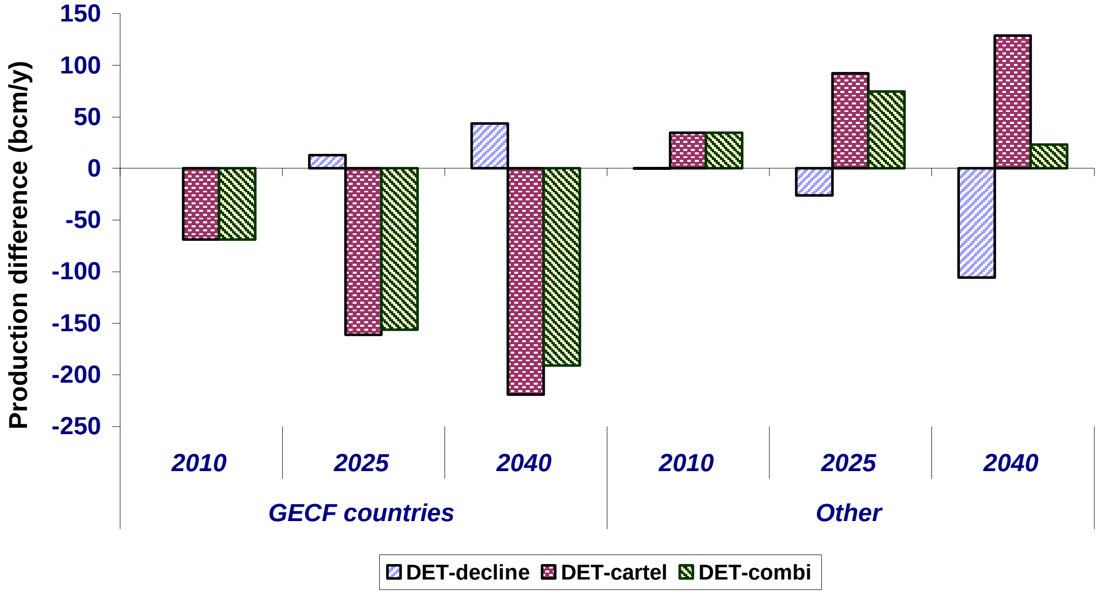

Figure 38: Differences in Production Levels relative to the Base Case

Figure 39 shows production differences in all scenarios and cases relative to the DET-base. In every period, the first, third, fifth and seventh bars present stochastic results.

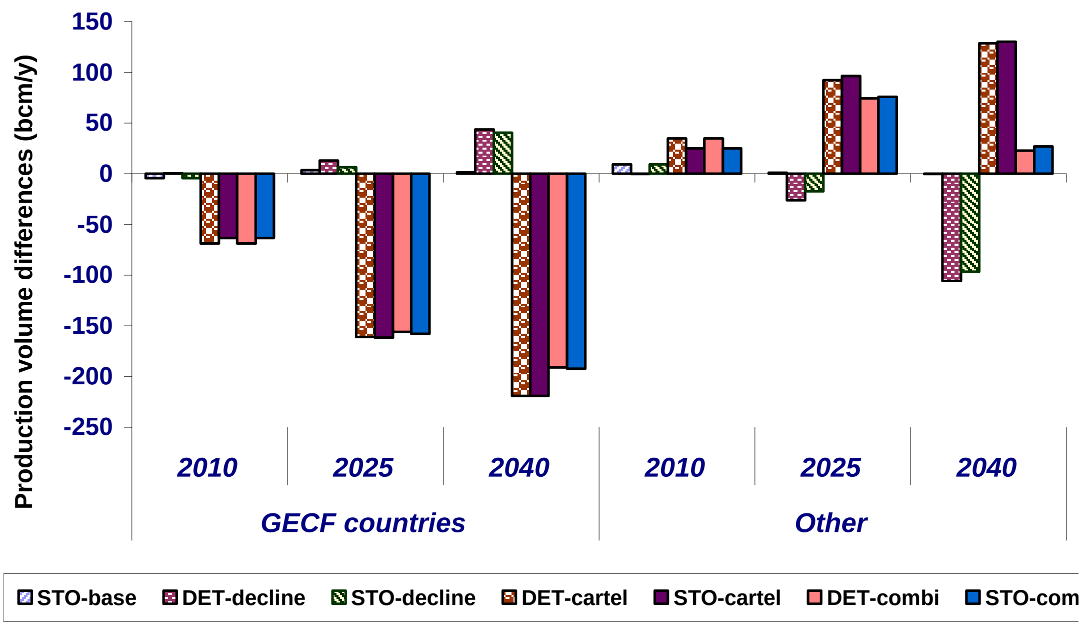

Figure 39: Production levels in all cases and scenarios

Generally, in the scenarios the aggregate production volumes are close to the volumes in their deterministic counterparts. In 2010, when the non-cartel scenarios STO-base and STO-decline still overlap completely (node 2 in Figure 35), the GECF countries produce slightly less than in the deterministic counterparts (-4 bcm, 1252 vs. 1256), whereas the other countries produce a bit more (+9 bcm, 1734 vs. 1725). In the cartel scenarios it is the other way around (GEFC countries 1193 vs. 1187 bcm, other 1750 vs. 1760 bcm). These modest shifts are a consequence of the hedging decisions in the first period. The hedged capacities expansions in the stochastic scenarios in 2005 generally lie between the minimum and maximum expansions among the four deterministic cases. The hedged capacity expansions in 2005 by cartel members are higher than in the deterministic counterpart DET-cartel and DET-combi, but lower than in DET-base and DET-decline. Clearly, capacities not added cannot be used, however once capacities are in place, they will likely be used. Hence, GECF members capacities will be more restrictive in 2010 in the STO-base (and STO-decline) than in the DET-base (and DET-decline) and less restrictive in the cartel scenarios relative to the deterministic counterpart cases.

Cartelization and depletion of domestic reserves in importing regions are two aspects affecting the developments in international gas trade. LNG trade is and will be responsible for an increasing share of long-distance international gas trade (see Chapter 1). The following section discusses the liquefaction capacity expansions.

6.4.2 Liquefaction capacities

International Energy Agency (2008) projected total LNG trade to be around 700 bcm/y by 2030, an increase by a factor of 3.5 relative to 2006. The model projects LNG exports of 437 bcm by 2030 in DET-base. Part of the difference between the projections is due to the aggregation level. For instance, some LNG flows occurring in reality do show up as pipeline flows in the model, e.g., from Norway to Belgium and France, or from Trinidad and Tobago to Brazil and Argentina. Another example is that the model allows for a pipeline from Russia-East (Sakhalin) towards Japan, which pushes out some of the LNG trade in later periods. Lastly, the model does not consider supply diversification motives, which favor LNG imports over pipeline supplies in some situations. However, the model does project a large increase in LNG trade, varying in the deterministic cases from 391 to 467 bcm. How do model assumptions and stochasticity affect the model outcomes?

There are nine LNG-exporting regions in the model, seven of which are potential cartel members. In the DET-base, the aggregate global LNG capacity expansions in 2040 add up to 291 bcm. In the non-cartel cases the expansions are equally divided between GECF and other countries, but in the cartel cases the non GECF countries add about double the amount of the cartel members. MEA2 and AUS are the only LNG exporters that would not participate in a cartel (see Figure 36). Figure 40 shows LNG capacity expansions by GECF countries in the deterministic cases. The total added capacity by GECF members in 2040 will range from 89 bcm in DET-cartel to 187 bcm in DET-decline. In all cases, most capacity is added in the three periods from 2015 through 2025. In DET-decline higher additions to liquefaction capacities occur to allow countries with lower domestic production to import more natural gas. As a consequence, in DET-decline the LNG capacity additions would be 32 bcm higher than in DET-base. In contrast, in DET-cartel when cartel members withhold supplies, the expansions by GECF members would total 89 bcm only, 40% less than in the DET-base. In DET-combi, both effects would occur and the aggregate LNG capacity expansions would be 115 bcm.

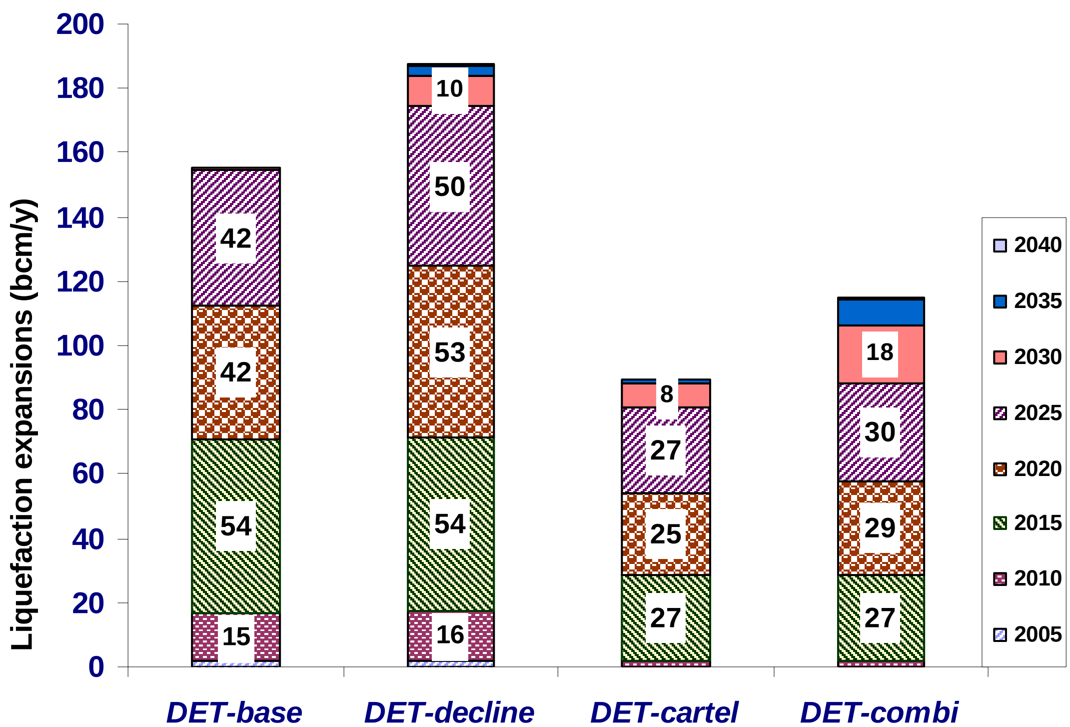

Figure 40: LNG Expansions in GECF countries in the Deterministic Scenarios

Figure 41 shows the LNG capacity expansions in the non-GECF countries: Australia and part of the Middle East.

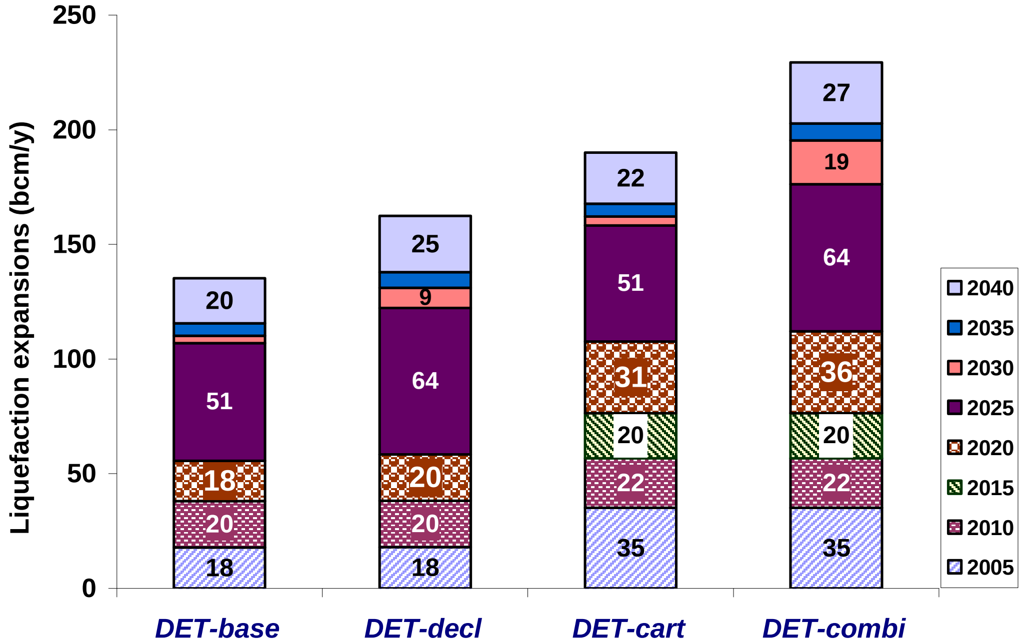

Figure 41: LNG Expansions in Non-GECF countries in the Deterministic Scenarios

The aggregate added capacities would range from 135 bcm/y in DET-base to 229 bcm/y in DET-combi. The non-cartel countries would add more export capacity to allow higher LNG exports so that importing regions could compensate lower domestic production as well as lower supplies by the cartel countries.

Australia started construction of its first LNG exporting facilities in 1985.[153] Four years later the first LNG cargos were shipped to Japan. At present, Australia has over 25 bcm/y of LNG export capacity, divided over two projects: the North West Shelf and Darwin.[154] August 2009 plans were announced for a floating liquefaction facility.[155] More developments are expected in the coming years.[156] Next, Australian expansions in the stochastic results are discussed in more detail. Figure 42 shows the expansions in the first two periods in all cases and stochastic scenarios.

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153 www.nwsalng.com
154 www.darwinlng.com, GIIGNL (2009)
155 www.gdfsuez.com
156 www.ret.gov.au

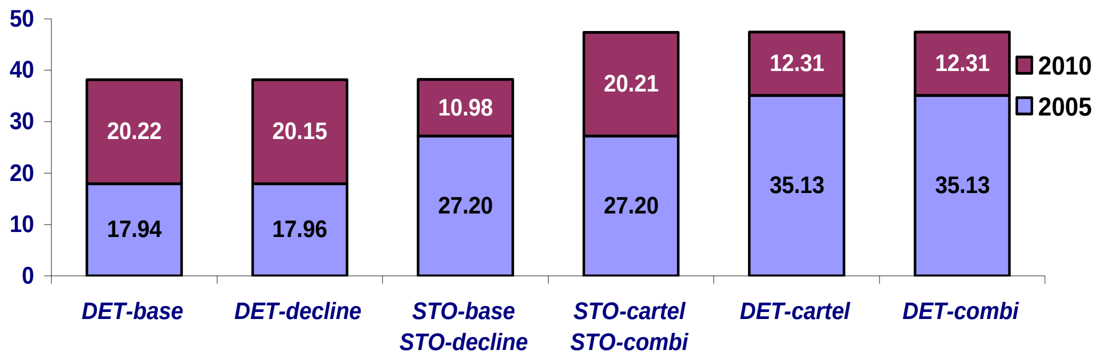

Figure 42: LNG Expansions in Australia in 2005 and 2010 (bcm/y)

In the stochastic model the first-period expansion is 27.20 bcm. Hedging for the various futures, the added capacity is just above the average of the added capacities in the deterministic counterparts. In the second period, when it has become clear whether or not a cartel will exist for the remainder of the time horizon, the added capacity in the STO-base and STO-decline is much lower than in the STO-cartel and STO-combi. As it turns out, once the uncertainty has disappeared, the additional capacity is such that the aggregate added capacity in the first two periods is very close to what would have been added in the deterministic counterparts.

Supposedly, the Australia liquefier has some characteristics that lead to a 'close to averages' hedging decision. It is a relatively small player, assumed to exert no market power, and therefore it has a modest impact on market prices. Also, in 2010 the next uncertain event will only materialize happen after 15 years, hence much of the uncertainty is discounted away.

Figure 43 shows that after 2020, when the second stochastic event about a decline in production capacities has become known, and there is no uncertainty about the future anymore, all aggregate expansions converge to their deterministic counterparts.

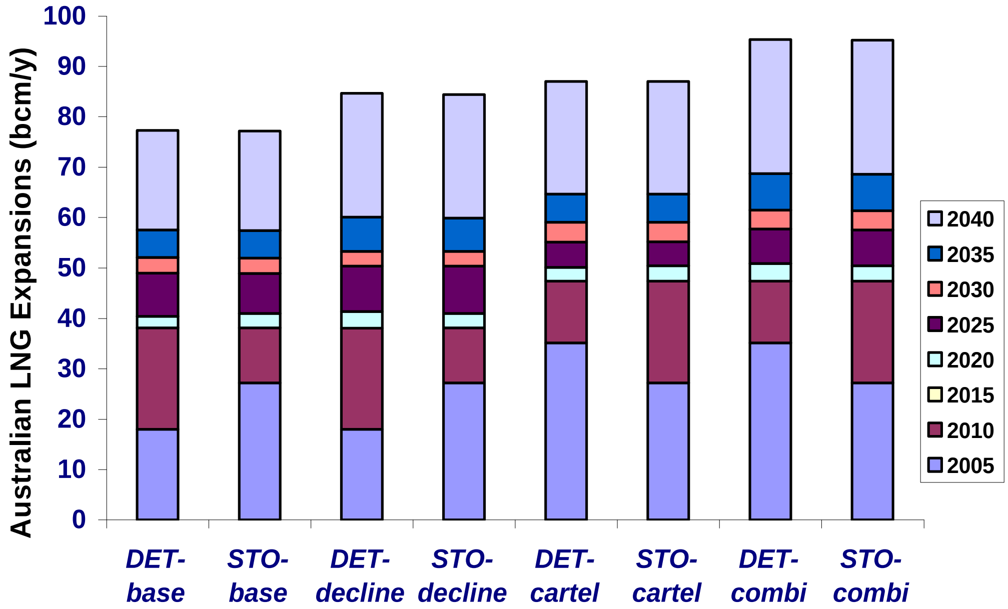

Figure 43: LNG Expansions in Australia

Some general observations should be noted. First of all, only at two moments an uncertain event may happen. Since future cash flows are discounted at 10%, events that are more than fifteen years in the future have a small impact on decisions. Secondly, risk-neutrality is assumed for all players. This can lead to hedging behavior similar to taking averages of the deterministic counterparts. In reality, such decisions may not be realistic, since overcapacity in liquefaction is very expensive and risk-conscious agents may be more conservative in their decisions to avoid bankruptcy if a pessimistic scenario occurred. Thirdly, contracting aspects are not captured that could ensure usage of constructed LNG capacity. LNG importers could - for the sake of supply diversification – prefer to contract LNG from supply sources such as Australia. Thus, the LNG capacity constructed in anticipation of a possible cartel would be used regardless whether a cartel would come into existence.

Additional liquefaction capacity will only be constructed when there would be enough capacity in place to regasify the LNG at the receiving end of the supply chain.

The ‘close to average’ hedging decisions seem to bear some similarities with the result in (Zhuang, 2005) that when supplied quantities are positive, the forward prices equal the expected spot market prices. However, those results were obtained within one model, whereas the outcomes discussed here result from comparing stochastic with deterministic models. Likely, the assumption of risk neutrality that the models share, is an explanation.

Next the developments in regasification capacities will be discussed.

6.4.3 Regasification capacities

Isolated countries lacking domestic reserves, such as Japan and South Korea, have imported large amounts of fossil fuels, including LNG, to fulfill their energy consumption. In contrast, a country such as the United Kingdom has imported significant volumes of LNG in the past, but due to boosting domestic production the regasification terminals could be mothballed. Due to depleting domestic reserves, in recent years the United Kingdom has been importing LNG again and depleting reserves and growing energy demand in other countries have boosted the interest in importing LNG all over the world. As a result, many countries have started the planning and construction of LNG import and regasification terminals (see Chapters 1 and 4, and (GIIGNL, 2009)).

Figure 44 shows the aggregate global expansions in regasification capacities. The aggregate added capacities in the deterministic cases vary from 252 bcm in DET-base to 333 in the DET-Combi. In the stochastic scenarios the range of aggregate values is similar. In earlier periods, until 2020, the expansions in the non-cartel scenarios and cases (base and decline) are largest, however in later periods, after 2025, the expansions are largest when there is no cartel. As discussed in the previous section, cartel members would hardly add LNG capacity. Therefore, in a cartelized market there would be much less supply of LNG and hence less reason to build additional regasification capacity. In DET-decline and DET-combi more capacity would be added in later periods than in DET-base, to compensate lower domestic production with higher LNG imports. In 2005 regasification expansions are low among all deterministic cases and in the stochastic problem. Developments in the rest of the time horizon are somewhat similar as what was observed for LNG capacities.

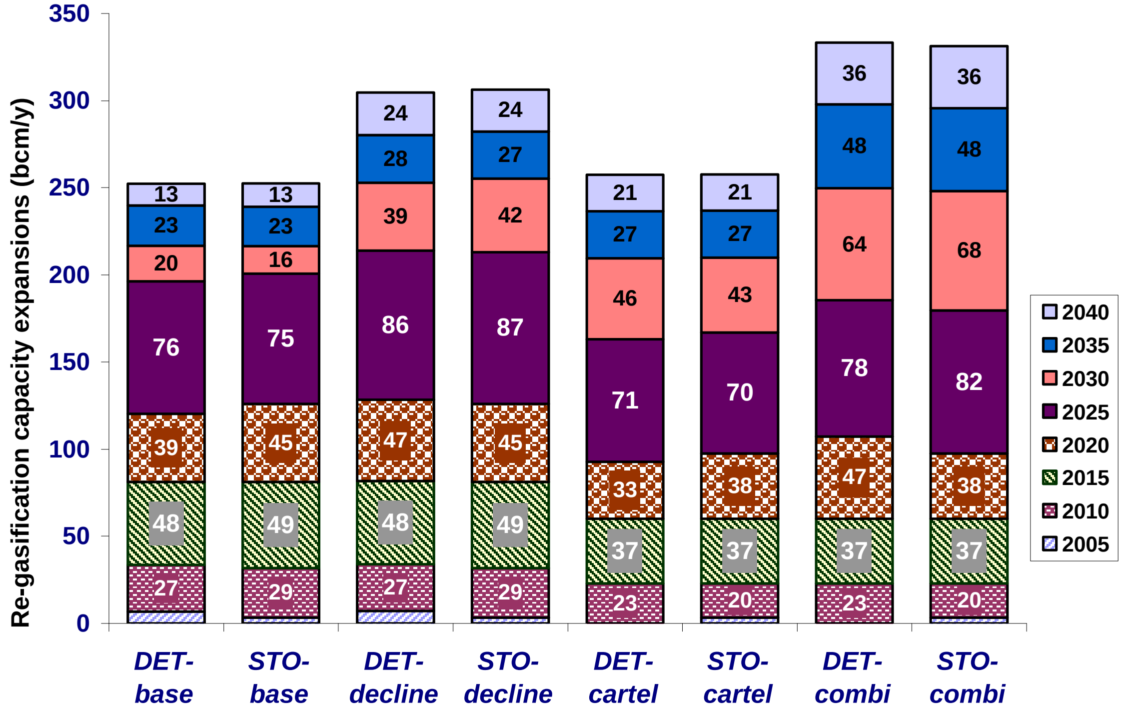

Figure 44: Global Regasification Expansions

The hedging decisions are quite close to the averages of the deterministic counterparts, and once an uncertain aspect has become known, the next (recourse) expansion decisions are such that the aggregate added capacity over the past periods is about the same as in the deterministic counterpart.

In a previous section major differences were found among the cases and scenarios regarding the countries that would expand LNG export capacities. This implies that the LNG trade patterns must also vary significantly among the cases and scenarios. The next section discusses LNG trade for region ASIA2, consisting of Japan and South Korea.

6.4.4 LNG imports Japan and South Korea

In 2000 Japan and South Korea were the first and second largest importers of LNG. Japan imported 72 bcm, which at the time was more than half of the global total LNG amount. Together with South Korea, that imported 20 bcm, they accounted for slightly over 2/3 of the global LNG imports (BP, 2001). In 2009, twenty-two countries imported LNG, twelve more than in 2000. Japan and South Korea were still the two largest LNG importers in the world. Japan imported 86 bcm of LNG and South Korea 34 bcm (BP, 2010). Together they imported half of the global LNG amount. Figure 45 shows the regions and supplied LNG volumes to Japan and South Korea in the stochastic model results.

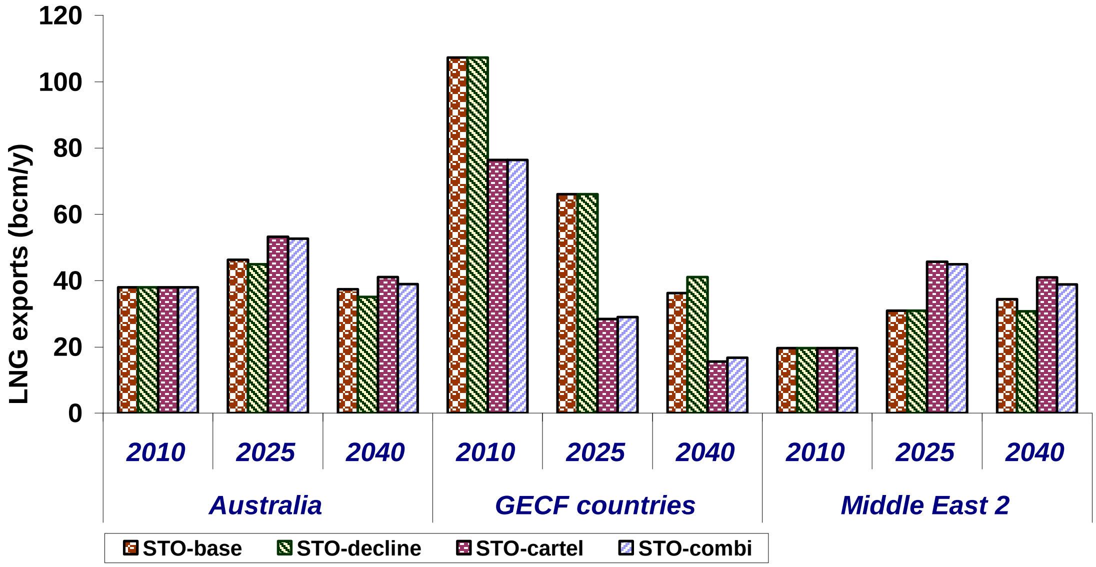

Figure 45: LNG Exports to Japan and South Korea

Two general observations are that the LNG imports by Japan and South Korea decrease over time in all of the scenarios and that the decrease is more pronounced in the cartel scenarios STO-cartel and STO-combi. Supplies by Australia are projected to grow from around 40 bcm in 2010 to around 50 bcm in the middle of the time horizon, but level off to around 40 bcm by 2040. Supplies by the non-GECF part of the Middle East are projected to grow from 20 bcm in 2010 to between 30 to 40 bcm in 2025 and then stay around those levels. The LNG supply that varies most among the scenarios, is that originating from the GECF countries. LNG supplies are projected to decrease over time, less in case of declining domestic production in ASIA2, but more harshly when a cartelization would happen. What is not shown in the figure is that in 2020 Russian pipeline exports from Sakhalin would start, growing from 18 bcm to 55 to 57 bcm/y by 2035. In the STO-cartel and STO-combi the Sakhalin pipeline would supply up to about 80% of the total GECF supplies. Due to the pipeline, the total supply to the ASIA2 region is impacted less by the varying case assumptions than other importing regions. Another explanation may be the already high prices in the ASIA2 region in the DET-base, making it a more beneficial export market than others.

The result tables in Section 6.9 show that there is a large variation in the net import and export values over time and among the cases and scenarios. The North American and European imports will grow significantly. By 2040, North American imports vary from 164 to 237 bcm/y and European imports would add up to an amount ranging from 430 to 528 bcm/y among the cases. The variation in the imports by the Asia Pacific region will vary less, with values ranging from 400 to 437 bcm/y. The impact for that region is reduced by the presence of cartel members as well as Japan and South Korea, as discussed in the previous section.

The following sections will provide more detailed insight on the pipeline trade from the Caspian region.

6.4.4.1 Caspian exports

In 2009 the Caspian region export 46 bcm. Of the exports 32 bcm had Russia as its destination, about 6 bcm went to Iran, about 5 bcm to Turkey and a few bcm to Eastern European countries (BP, 2010). The Caspian production and exports are projected to increase much over the next years, which means that additional transport capacities will be needed. In Chapter 4 the Nabucco pipeline (See Figure 24 in Section 4.4.4) was discussed from the Caspian region to Europe. The planned capacity for Nabucco is 31 bcm/y.[157] According to the Energy Information Administration, Kazakhstan, Turkmenistan and Russia have agreed to expand the pipeline capacities from the Caspian Region into and through Russia with about 20 bcm/y.[158] Between the Caspian countries and China there is agreement about a pipeline with a capacity of 40 bcm/y.[158] The three projects sum up to 91 bcm/y of additional Caspian export capacity compared to the current situation. One conclusion of Chapter 4 was that some of the pipelines that Russia has built recently and is building, seem motivated by political rather than economic reasons. A surprising result in Chapter 4 was that the model would not have Nabucco built, however in the analysis it was discussed that for reasons of supply diversification it could still be preferred over other options. As it turns out, in the stochastic model Nabucco would be built. This is possibly due to the different aggregation level of the

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[157] www.nabucco-pipeline.com
[158] www.eia.doe.gov/cabs/Kazakhstan/NaturalGas.html

model regions and clearly gives a somewhat different perspective on the natural gas market developments in the coming decades than previously obtained from the deterministic WGM.

Figure 46 shows the pipeline exports from the Caspian region to Europe and Asia, including the exports via Russia and the Middle East in the DET-base.[159]

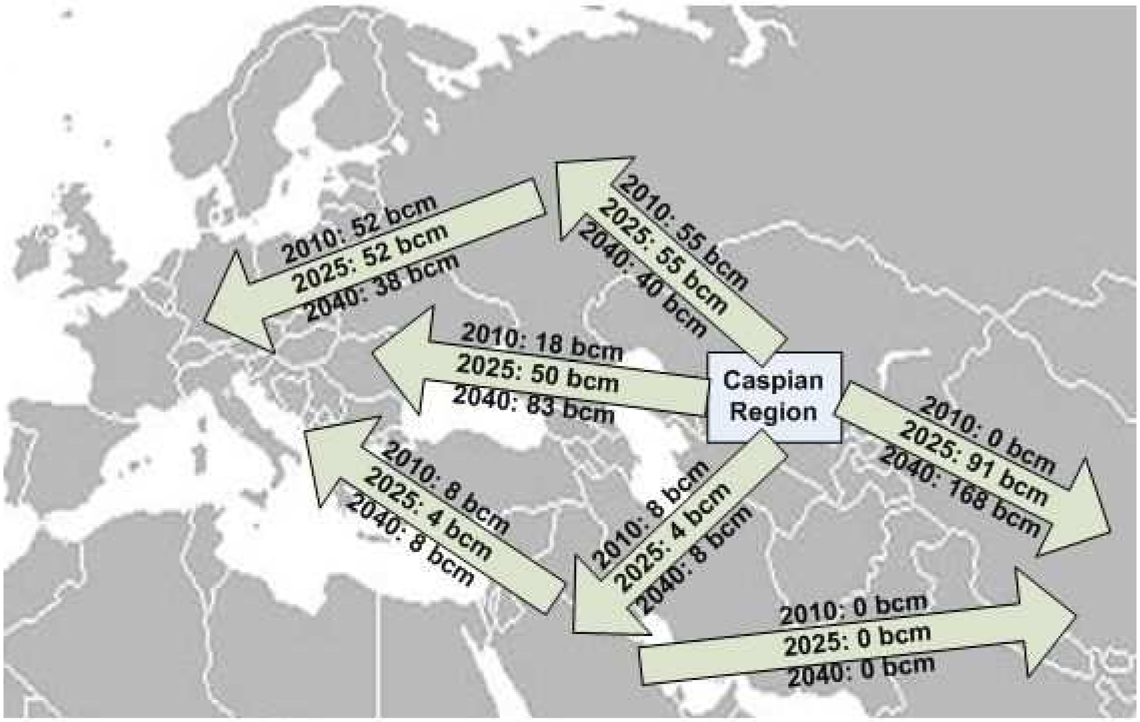

Figure 46: Pipeline exports from the Caspian region bcm/y DET-base

For 2010 the model projected 18 bcm of flows directly to Europe, 55 bcm via Russia and 8 bcm through the Middle East.[160] The Caspian pipeline exports add up to 81 bcm. This model result, based on projections of a few years ago, seems quite high in comparison with the actual exports in 2009 of 46 bcm, unless the exports would jump significantly between 2009 and 2010. A possible explanation is that due to the global economic downturn the demand for natural gas in 2009 has fallen instead of grown and in the last few years some pipeline construction projects have been postponed.

[159] Differences between inflows and outflows in Russia and the Middle East are due to pipeline losses.
[160] Note that Belarus, Turkey and Ukraine are part of EURSW, see Figure 36.

Figure 47 shows for the deterministic cases the additional export flows relative to DET-base in 2040.

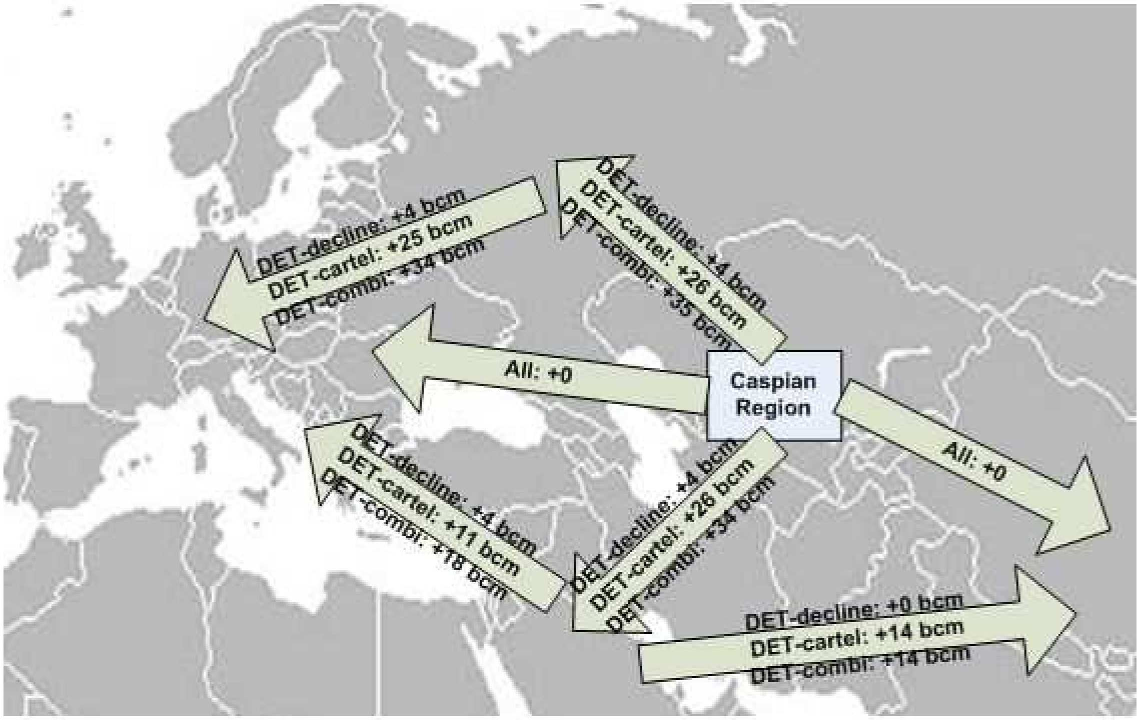

Figure 47: Pipeline exports from the Caspian region in 2040 bcm/y relative to DET-base

The direct exports to Asia and Europe are not affected by the case assumptions. Already in the DET-base in all periods the added capacities to the (lower-cost) direct pipelines are restricted by the expansion limits set (see Eq. (6.2.8)) and only pipelines in the other routes can be expanded more. That makes sense, since only when a cheaper option is fully used, a more expensive alternative should be considered. Additional exports vary from eight bcm (four via Russia and four via the Middle East) in DET-decline, to an additional 69 bcm (35 resp. 34) in the DET-combi case. In the latter case, total Caspian exports would add up to 369 bcm in 2040. The production levels needed to maintain such large export volumes would be more than double the actual 2005 levels and deplete the proved reserves of 13,000 bcm (BP, 2010) in 45 to 50 years. However, large areas in the region are still unexplored and more reserves will likely be found.[161] Except for reserves considerations, it is questionable whether the economic and political environment in the region can provide a sound basis for the enormous investments needed to construct so

[161] fpc.state.gov/documents/organization/9652.pdf

much additional pipeline capacity. Note that these high projected pipeline exports are part of the explanation for the low business-as-usual projections for global LNG trade.

In the following section the developments in the North American imports are discussed.

6.4.4.2 North American imports

The combined net natural gas imports by United States and Mexico in 2005 were 110 bcm, of which 94 bcm were pipeline imports from Canada (BP, 2006). In the DET-base in 2005, the combined region USA/MEX imported 109 bcm: 82 bcm from Canada and another 27 bcm as LNG. Only a few years ago the United States were expected to become major importers of LNG in the near future. Based on these projections by 2025 the imports from Canada will have dropped to 50 bcm and LNG imports will have risen to 164 bcm adding up to 214 bcm of total imports, almost twice the imported amount in 2005. Over time, the import dependency as a percentage of domestic consumption would grow from 19% in 2005 to 41% in 2040.

Figure 48 presents a breakdown by origin of the supply to the United States and Mexico in the stochastic scenarios.

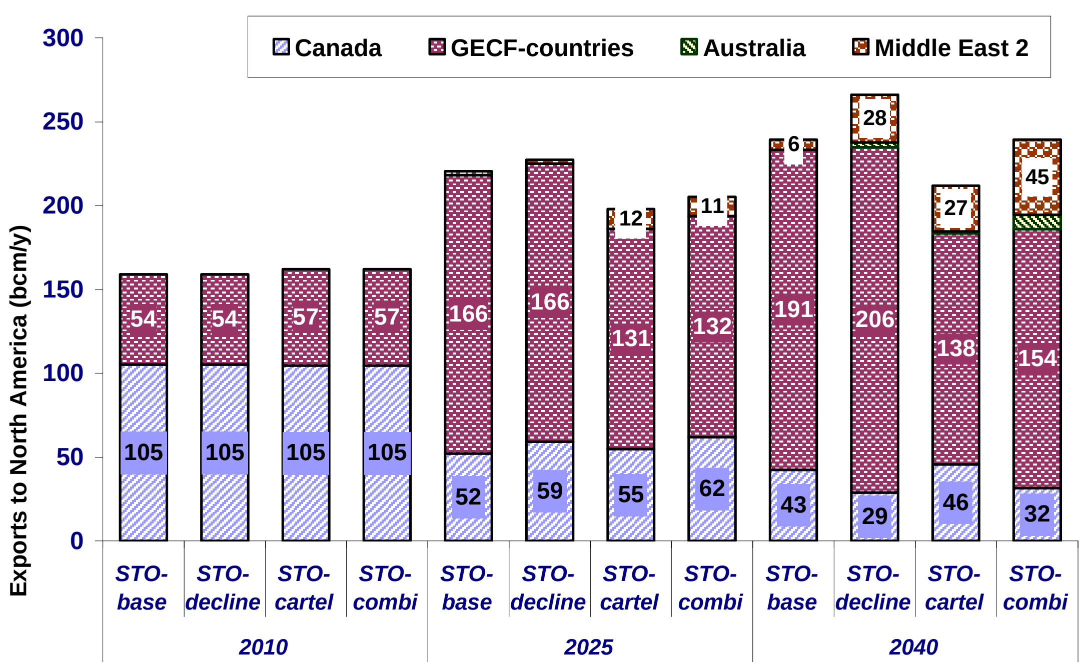

Figure 48: Exports to United States and Mexico

In 2010 the differences among scenario results are small. In the longer term, the development of the imports for the USA/MEX region are mostly determined by the cartel members. In cartel scenarios the GECF members would withhold LNG supplies (relative to the STO-base), which would be compensated partly by other LNG exporters and some pipeline supplies from Canada. In 2010 Canada will supply about 2/3 of total imports. Over time, the Canadian supplies decrease (in all scenarios), more harshly in the decline cases STO-decline and STO-combi when Canada also suffers from lower faster depleting reserves and production rates. LNG supplies from non-cartel regions would only come on stream in later periods. Only in the STO-combi case in 2040 the non-cartel LNG supplies would be larger than Canadian pipeline supplies. The GECF countries would provide between 25% and 35% of consumption and 64% to 80% of the imports by 2040.

The results show that the United States and Mexico could become very dependent on LNG imports from potential cartel member countries. Unlike Europe that is surrounded by several countries with huge gas reserves, USA/MEX only have Canada as a nearby source of pipeline supplies. However, the results are based on not up-to-date projections and the technological advances in unconventional natural gas production in recent years have changed the picture dramatically. Recent projections (EIA, 2009) foresee that the United States will be nearly self-sufficient for several decades and that North American LNG imports will be modest.

Next, the price developments in three major importing regions will be analyzed.

6.4.5 Prices

In 2005 Japanese LNG import prices were on average about $6/mcf, or slightly over $200/kcm (BP, 2010).[162] Prices in the EU were a few percent lower. Average spot-market prices in North America were much higher: $256/kcm in Canada and $310/kcm in the United States. In 2009, prices in Japan and the EU were over $300/kcm, much higher than in 2005. In contrast, in the United States and in Canada prices were lower in 2010 at $137/kcm resp. $119/kcm. Natural gas prices do not vary just as a result of demand and supply, but also due to factors such as price developments of substitute fuels (notably oil)

[162] mcf=1000 cubic feet; 1 kcm = 35.31 mcf

and speculation. Sector models, such as the ones developed in this dissertation, cannot capture all these aspects. Instead, the price paths will be smoother and not account for short-term price hikes. The model has been calibrated to have a gradual increase in prices over the time horizon. In the DET-base, volume-weighted global average prices rise from $165/kcm in 2005 to $479 in 2040. How the case assumptions and stochasticity affect the price developments is analyzed for three of the main importing regions: Central Europe, Unites States & Mexico and Japan & South Korea. Section 6.9 provides tables with the volume-weighted average prices for the whole world and the three regions.

Figure 49 shows the developments of the average wholesale prices in Central Europe relative to the DET-base.

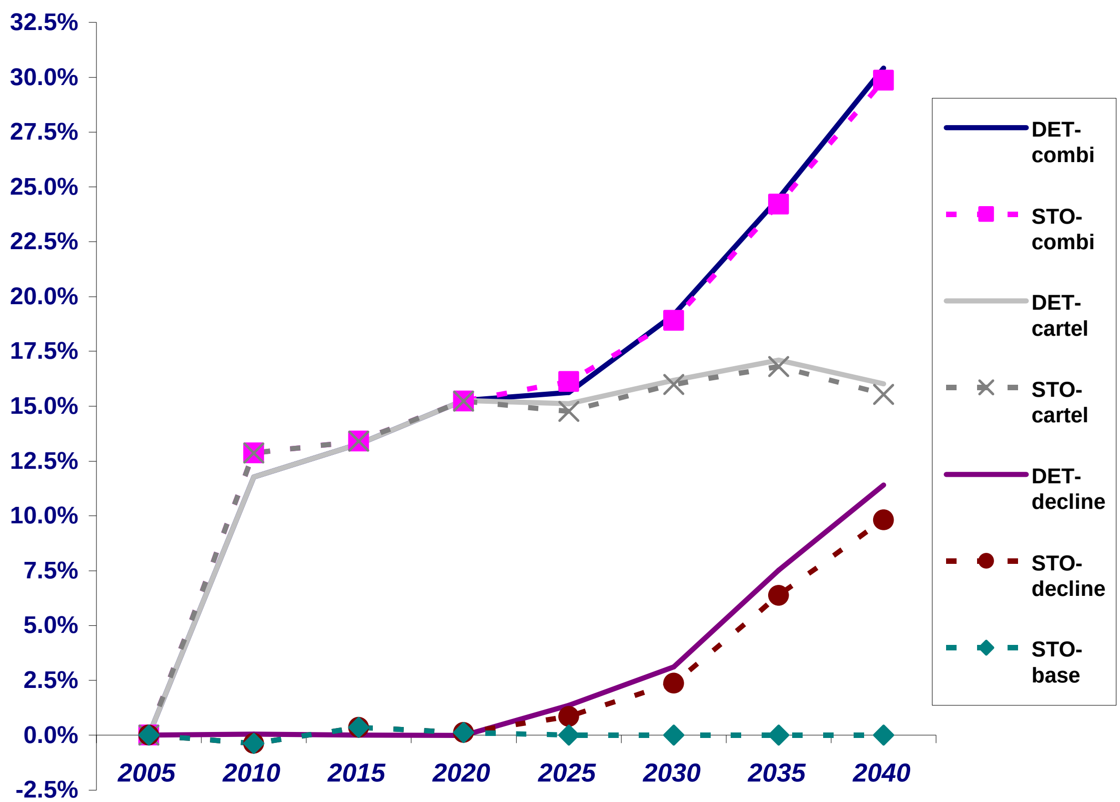

Figure 49: Development of prices in Central Europe, relative to DET-base

In the DET-base, prices in Central Europe in 2005 are $189/kcm, steadily rising to $520/kcm by 2040 (Table 23). The average prices in Central Europe in the scenarios match the prices in their deterministic counterparts rather closely. In 2010, when the cartelization may take place, the stochastic price results would deviate more noticeably from their deterministic counterparts, but in 2015 the deviations would all but disappear. In 2025, when the second uncertain event becomes known, price deviations would be more noticeable again, except in the STO-base scenario. From 2030 the STO-combi prices would be virtually identical to the DET-combi prices too. In the other two scenarios, STO-decline and STO-cartel, prices would stay somewhat lower than in their respective deterministic counterparts. When looking at the other regions in the next two graphs, the relatively large impact in Central Europe of the case assumptions can possibly be explained by the modest prices in the DET-base. Even in the harshest case and scenario DET-combi and STO-combi, the Central European prices would still be lower than the price levels in DET-base in USA/MEX and ASIA2.

Figure 50 shows the development of average prices in the United States and Mexico.

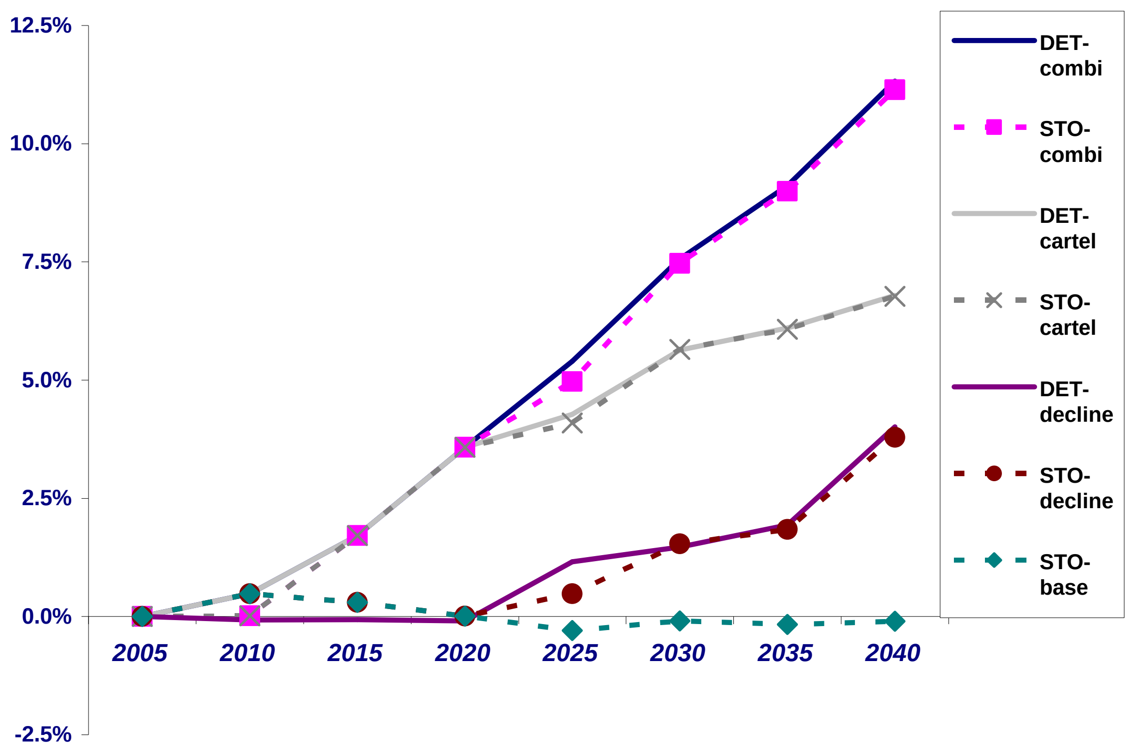

Figure 50: Development of Prices in United States and Mexico, relative to the DET-Base

In DET-base, prices in 2005 are \$206/kcm, increasing to \$746/kcm in 2040 (Table 24). The impact on prices in North America due to the cartel establishment and the declining production rates is modest relative to other importing regions, although it increases somewhat over time. The modest impact may be somewhat surprising, given the increasing dependency on cartel members for LNG supplies. However, already in the DET-base price levels in USA/MEX in 2040 will be much higher than they would be in the harshest case (DET-combi) in Central Europe, providing the financial incentive to exporters to supply to the USA/MEX market.

Figure 51 shows the price developments in Japan and South Korea.

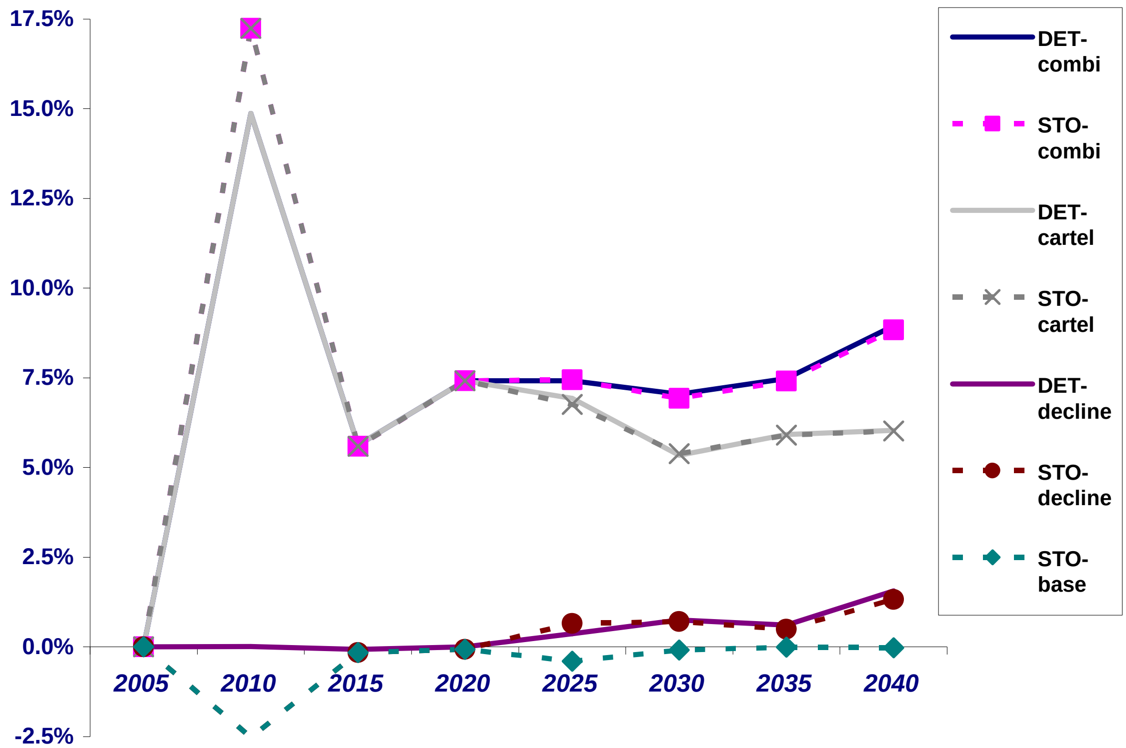

Figure 51: Development of Prices in Japan and South Korea, relative to the Base Case

In 2005 the average prices were \$282/kcm, growing to \$763/kcm in 2040 (Table 25). In the cartel cases there is a major price spike in the second period. In the long run the introduction of the pipeline from Sakhalin and additional supplies from Australia and the non-cartel part of the Middle East relieve the pressure on the gas market. In 2010 the prices in the STO-base and STO-decline drop below the DET-base levels. As discussed in Section 6.4.2 the non-cartel countries add more capacity in 2005 in the stochastic scenarios than in the non-cartel deterministic scenarios. When in the stochastic model the cartel is not formed, the potential cartel members have more capacity in place than in the deterministic counterpart cases and once in place, capacity would be used to export. From

2015 – except for 2025, when the other uncertain event becomes known – the stochastic price paths are virtually identical to the prices in the deterministic counterparts.

6.4.6 Expected profits and the value of the stochastic solution

Table 13 below presents the expected profits in the stochastic problem for the traders that make a positive profit in at least one of the periods.[163] The cartel countries account for most of the profits, especially in early periods (e.g., in 2005 the $19.23 billion is about 75% of the total of $25.72 billion. The share of the cartel countries in the yearly aggregate profits decreases gradually to 56% in 2040. In contrast, the share in the total profits of the Caspian region increases significantly from just over 9% in 2005 to 35% in 2040 ($111 billion of $317 billion).

Table 13: Expected profits for stochastic problem (undiscounted billion $ 2005)

	2005	2010	2015	2020	2025	2030	2035	2040
Cartel countries	19.23	31.75	51.80	78.65	107.05	143.58	161.75	178.95
Australia	0.09	3.03	4.78	4.81	5.45	6.12	6.68	7.59
South America 2	0.03	0.01	0.01					
Canada	0.76							
Caspian region	2.38	8.06	17.85	27.11	44.22	69.00	88.85	111.43
Netherlands & Norway	3.07	4.48	6.19	7.55	7.32	7.41	7.36	7.26
Middle East 2	0.17	0.81	1.66	2.62	4.65	9.35	10.75	12.02
Total	25.72	48.15	82.28	120.74	168.69	235.46	275.39	317.25

To determine the value of the stochastic solution (VSS, see Section 5.2.2) an additional deterministic case was run. In this additional case, the cartel exerts a market power level of 0.625 (the average of 0.25 in the Base Case and 1 in the Cartel Case) and the importing countries face only half of the additional decline of production capacities relative to the Base Case as in the Decline Case. The capacity results of this myopic deterministic case (EVP) were used to fix the capacities in a new run with the stochastic model, to determine the profit levels of the various traders (the EEV) and calculate the VSS.

This section focuses on VSS results for traders only, since the traders play a key role in the model and are the agents that exert market power. The VSS for other players are not discussed.

[163] Traders are assumed to not exert market power on their respective domestic nodes and therefore traders in the major importing regions do not make profits.

Table 14 below shows the differences in discounted expected yearly profits, and the yearly average difference in discounted profit between the EEV. It turns out that the trader 'Middle East 2', that would not take part in the cartel, has a negative VSS, while all other traders have a positive VSS.[164] This contrasts with results in Zhuang (2005) and (Genc et al., 2007) where negative VSS were found for all players of a specific type.

Table 14: VSS results for stochastic problem (discounted billion $2005)

	2010	2015	2020	2025	2030	2035	2040	Average
Cartel countries	-0.28	0.13	0.28	0.19	0.09	-0.09	-0.06	0.033
Australia	0.05	-0.09	0.00	0.02	0.01	0.02	0.00	0.001
Caspian	0.01	0.15	0.05	0.02	0.04	0.07	-0.02	0.041
Netherlands & Norway	0.00	0.01	0.02	0.01	0.02	0.00	0.00	0.007
Middle East other	0.01	0.05	0.00	-0.04	-0.03	-0.03	-0.03	-0.008
Total	-0.21	0.26	0.35	0.20	0.13	-0.03	-0.10	0.074

A possible explanation is that in those two papers the players exerting market power were more similar to each other – in terms of cost and capacity parameters – and in the numerical case in this chapter the characteristics of the players are very diverse. It must be noted that the magnitudes of the relative VSS are quite small, between -1.5% of the expected profit values for the Middle East other and +0.8% for the Caspian region.

6.5 Summary and conclusions

In this chapter a stochastic mixed complementarity problem for the global natural gas market has been presented. The model was applied to solve a stochastic problem with four scenarios. In the scenarios different assumptions applied for the market power exertion by members of the Gas Exporting Countries Forum (GECF), as well as for the domestic production capacities of some regions that rely on imports to cover significant parts of their domestic natural gas demand. Results of the stochastic problem were compared to the outcomes of four deterministic (counterpart) cases that each had exactly the same input parameter values as one of the stochastic scenarios. Results were discussed for production levels, regional trade balances, capacity expansions, LNG trade and wholesale prices. Special attention was given to the development of pipeline exports

[164] Note that all traders with an expected profit of zero in the stochastic problem have a VSS=0.

from the Caspian region to Europe, the development of Australian liquefaction capacity over time and supplies to and price developments in various importing regions.

On an aggregate level the consequences of the stochastic modeling approach seemed rather modest. However, when looking into the details several interesting results were found. Hedging behavior affected the timing and magnitude of capacity expansions, significantly affecting local market situations and prices. For example, for the Australian liquefier the shifts over time in capacity expansions were discussed; for North America the sources of imported supplies and the wholesale prices in Japan and South Korea. Probably due to assumed risk-neutral behavior of all players, the hedging lead often to capacity expansion decisions close to – but not equal to – the average decisions in the deterministic counterparts.

Only two uncertain events were included in the stochastic problem and after the first uncertain event would become known, the second would only happen after 15 years. As such, much of the future uncertainty was 'discounted away' when the recourse expansion decisions were taken. Also, once all uncertain events were known, the scenario results tended to converge in later years to the results of the deterministic counterparts. Hedging behavior was most pronounced in the period immediately before an uncertain event would occur, and recourse behavior most pronounced in the periods directly after. The effects were more clearly visible in the capacity additions, and less so in market prices. For the traders the values of the stochastic solutions were calculated and found to be relatively small, again likely due to the limited number of uncertain events (just two, leading to four scenarios) in the stochastic model.

Rather than concluding that stochastic modeling has limited impact only, the power of stochastic modeling would be more pronounced when more uncertain aspects are included in the problems, in all periods instead of just two. Also, other assumptions relative to risk-attitudes would likely have large impact, what will be briefly addressed in the future work section at the end of Chapter 7.

6.6 Nomenclature

6.6.1 Sets

A	Gas transportation arcs a
D	Demand seasons d
M	Scenario nodes m, m'
N	Country nodes n, n'
P	Producers p
S	Storage operators s
T	Traders t , t'
W	Wholesale markets w
$a^{+}(n)$	Inward arcs into node n
$a^{-}(n)$	Outward arcs from node n
$n^{+}(a)$	End node of arc a
$n^{-}(a)$	Start node of arc a
$pred(m)$	Predecessor nodes of scenario node m
$succ(m)$	Successor nodes of scenario node m

6.6.2 Constants/Input Parameters

p_m	Probability for scenario node
γ_m	Discount rate for scenario node
d_d	Number of days in season
δ_{tnm}^{MP}	Market power indicator
INT_{ndm}^{W}	Intercept of inverse demand curve
SLP_{ndm}^{W}	Slope of inverse demand curve
$c_{pnm}^{P}(\ )$	Production costs of producer
$c_{snm}^{SI}(\ )$	Storage injection costs of storage

$c_{am}^{\Delta A}$ Expansions costs for arc

$c_{snm}^{\Delta SI}$ Expansions costs for storage injection

$c_{snm}^{\Delta SX}$ Expansions costs for storage extraction

$c_{snm}^{\Delta SW}$ Expansions costs for storage working gas

CAP_{am}^{A} Initial capacity for arc

CAP_{pnm}^{P} Initial capacity for producer

CAP_{snm}^{SI} Initial storage injection capacity

CAP_{snm}^{SX} Initial storage extraction capacity

CAP_{snm}^{SW} Initial storage working gas capacity

$\overline{\Delta}_{am}^{A}$ Limit to arc expansion

$\overline{\Delta}_{snm}^{SI}$ Limit to storage injection capacity expansion

$\overline{\Delta}_{snm}^{SX}$ Limit to storage extraction gas capacity expansion

$\overline{\Delta}_{snm}^{SW}$ Limit to storage working gas capacity expansion

l_{a}^{A} Loss rate for shipments over arc

l_{sn}^{S} Loss rate for storage injection

τ_{adm}^{reg} Regulated tariff for shipments over arc

6.6.3 Primal variables

f_{tadm}^{A} Pipeline capacity purchased, i.e. flow, by trader *t* over arc *a*

q_{adm}^{A} Total pipeline capacity assigned for arc *a*

$q_{pndm}^{P \to T}$ Quantity sold by producer *p* at node *n* in season *d* of scenario node *m*

$q_{sndm}^{S \leftarrow T}$ Gas purchased, i.e., injection rate, by storage operator *s*

$q_{sndm}^{S \to W}$ Storage extraction rate at node *n* in season *d* of scenario node *m*

$q_{tndm}^{T \leftarrow P}$ Total amount purchased by trader *t* from producers at node *n*

$q_{tndm}^{T \to W}$ Total sold amount by trader *t* to consumers at node *n*

Δ_{am}^{A} Capacity expansion of arc *a*, in scenario node *m*

Δ_{snm}^{SI} Capacity expansion of storage injection

Δ_{snm}^{SX} Capacity expansion of storage extraction

Δ_{snm}^{SW} Capacity expansion of storage working gas

6.6.4 Dual variables

α_{adm}^{A} Dual to arc capacity limitation

α_{pndm}^{p} Dual to production capacity limitation for producer *p*

α_{sndm}^{SI} Dual to injection capacity limitation

α_{sndm}^{SX} Dual to extraction capacity limitation

α_{sndm}^{SW} Dual to working gas capacity limitation

ε_{tadm}^{T} Dual to supply contract obligation

φ_{tndm}^{T} Dual to mass balance for trader

π_{ndm}^{P} Market-clearing price between producers and traders

π_{ndm}^{TS} Market-clearing price for sales to storage

π_{ndm}^{W} Wholesale market price

$\rho_{am}^{\Delta A}$ Dual to expansion limitation for arc

ρ_{sndm}^{SI} Dual to expansion limitation for injection capacity

ρ_{sndm}^{SX} Dual to expansion limitation for extraction capacity

ρ_{sndm}^{SW} Dual to expansion limitation for working gas capacity

τ_{adm}^{A} Congestion rate for arc

6.7 Karush Kuhn-Tucker conditions

6.7.1 KKT conditions producer

Eq. (6.7.1) is stationarity with regard to the production volumes. Eq. (6.7.2) represents the production capacity constraint.

$$\forall n,d,m: \qquad 0 \le q_{pndm}^{P\to T} \perp \quad p_m \gamma_m d_d \frac{\partial c_{pndm}^{P}(\cdot)}{\partial q_{pndm}^{P\to T}} + \alpha_{pndm}^{p} - p_m \gamma_m d_d \pi_{ndm}^{p} \ge 0 \qquad \textbf{(6.7.1)}$$

$\forall n,d,m: \qquad 0 \le \alpha^{P}_{pndm} \perp \quad CAP^{P}_{pnm} - q^{P\to T}_{pndm} \ge 0$ **(6.7.2)**

6.7.2 KKT conditions trader

Eq. (6.7.3) and (6.7.4) are the stationarity conditions for the trader with respect to sales to the end-user and storage market. Eq. (6.7.5) is the stationarity condition with respect to purchases from producers. Eq. (6.7.6) gives the mass flow balance, (6.7.7) is the stationarity condition with respect to arc flows for arc $a = \left(n_{a-}, n_{a+}\right)$ and (6.7.8) represents the lower bound to supplies resulting from contractual obligations.

$\forall n,d,m: \qquad 0 \le q^{T\to W}_{tndm} \perp \quad p_m \gamma_m d_d \delta^{MP}_{tnm} SLP^{W}_{ndm} q^{T\to W}_{tndm} + \varphi^{T}_{tndm} - p_m \gamma_m d_d \pi^{W}_{ndm} \ge 0$ **(6.7.3)**

$\forall n,d,m: \qquad 0 \le q^{T\to S}_{tndm} \perp \quad \varphi^{T}_{tndm} - p_m \gamma_m d_d \pi^{TS}_{ndm} \ge 0$ **(6.7.4)**

$\forall n,d,m: \qquad 0 \le q^{T\leftarrow P}_{tndm} \perp \quad p_m \gamma_m d_d \pi^{p}_{ndm} - \varphi^{T}_{tndm} \ge 0$ **(6.7.5)**

$\forall a,d,m: \qquad 0 \le f^{A}_{tadm} \perp \quad \begin{pmatrix} p_m \gamma_m d_d \left(\tau^{reg}_{adm} + \tau^{A}_{adm}\right) \\ +\varphi^{T}_{tn^{-}(a)dm} \end{pmatrix} - \begin{pmatrix} \left(1 - l^{A}_{a}\right) \varphi^{T}_{n^{+}(a)dm} \\ +\varepsilon^{A}_{tadm} \end{pmatrix} \ge 0$ **(6.7.6)**

$\forall n,d,m: \qquad \varphi^{T}_{tndm} \ free, \quad \begin{pmatrix} q^{T\leftarrow P}_{tndm} + \\ \sum_{a\in a^{+}(n)} \left(1 - l^{A}_{a}\right) f^{A}_{tadm} \end{pmatrix} - \begin{pmatrix} q^{T\to W}_{tndm} + q^{T\leftarrow S}_{tndm} \\ + \sum_{a\in a^{-}(n)} f^{A}_{tadm} \end{pmatrix} = 0$ **(6.7.7)**

$\forall a,d,m: \qquad 0 \le \varepsilon^{T}_{tadm} \perp \quad f^{A}_{tadm} - CON^{A}_{tam} \ge 0$ **(6.7.8)**

6.7.3 KKT conditions TSO

Eq. (6.7.9) is stationarity for sales of pipeline capacity, and (6.7.10) for pipeline expansions. Eq. (6.7.11) provides the arc capacity limitation to aggregate flows; and (6.7.12) provides the limitation to capacity expansions.

$\forall a,d,m: \qquad 0 \le q^{A\to T}_{adm} \perp \quad \alpha^{A}_{adm} - p_m \gamma_m d_d \tau^{A}_{adm} \ge 0$ **(6.7.9)**

$\forall a,m: \qquad 0 \le \Delta^{A}_{am} \perp \quad p_m \gamma_m c^{\Delta A}_{am} + \rho^{A}_{am} - \sum_{d,m'\in succ(m)} \alpha^{A}_{adm'} \ge 0$ **(6.7.10)**

$\forall a,d,m: \qquad 0 \le \alpha^{A}_{adm} \perp \quad CAP^{A}_{am} + \sum_{m'\in pred(m)} \Delta^{A}_{am'} - q^{A\to T}_{adm} \ge 0$ **(6.7.11)**

$\forall a,m: \qquad 0 \le \rho^{A}_{am} \perp \quad \bar{\Delta}^{A}_{am} - \Delta^{A}_{am} \ge 0$ **(6.7.12)**

6.7.4 KKT conditions storage operator

Eq. (6.7.13) is the stationarity condition for gas injection, Eq. (6.7.14) for gas extraction. For every year, Eq. (6.7.15) is the gas cycle condition. The capacity restrictions for injection is provided by Eq. (6.7.16) and for extraction by Eq. (6.7.17). Eq. (6.7.18) is the working gas restriction. The stationarity conditions for and limitations to capacity expansions are very similar. For injection, extraction or working gas: stationarity of expansions Eq. (6.7.19) - (6.7.21) and the expansion limitations Eq. (6.7.22) - (6.7.24).

$$\forall n, d=1, m: \quad 0 \le q_{sndm}^{S\leftarrow T} \perp \quad p_m \gamma_m d_d \left(\pi_{ndm}^{TS} + \frac{\partial c_{sndm}^{SI}(\ )}{\partial q_{sndm}^{S\leftarrow T}} \right) + \alpha_{sndm}^{SI} - \left(1 - l_{sn}^S\right) d_d \varphi_{ndm}^S \ge 0 \qquad \textbf{(6.7.13)}$$

$$\forall n, d=2,3, m: \quad 0 \le q_{sndm}^{S\rightarrow W} \perp \quad \alpha_{sndm}^{SX} + d_d \alpha_{sndm}^{SW} - p_m \gamma_m d_d \pi_{ndm}^{W} \ge 0 \qquad \textbf{(6.7.14)}$$

$$\forall n, m: \quad \varphi_{snm}^S \ \ free, \quad \left(1 - l_{sn}^S\right) d_1 q_{snd_1m}^{S\leftarrow T} - \sum_{d=2,3} d_d q_{sndm}^{S\rightarrow W} = 0 \qquad \textbf{(6.7.15)}$$

$$\forall n, d=1, m: \quad 0 \le \alpha_{sndm}^{SI} \perp \quad CAP_{snm}^{SI} + \sum_{m' \in pred(m)} \Delta_{sndm'}^{SI} - q_{sndm}^{S\leftarrow T} \ge 0 \qquad \textbf{(6.7.16)}$$

$$\forall n, d=2,3, m: \quad 0 \le \alpha_{sndm}^{SX} \perp \quad CAP_{snm}^{SX} + \sum_{m' \in pred(m)} \Delta_{sndm'}^{SX} - q_{sndm}^{S\rightarrow W} \ge 0 \qquad \textbf{(6.7.17)}$$

$$\forall n, m: \quad 0 \le \alpha_{snm}^{SW} \perp \quad CAP_{snm}^{SW} + \sum_{m' \in pred(m)} \Delta_{ndm'}^{SW} - \sum_{d=2,3} d_d q_{sndm}^{S\rightarrow W} \ge 0 \qquad \textbf{(6.7.18)}$$

$$\forall n, m: \quad 0 \le \Delta_{snm}^{SI} \perp \quad p_m \gamma_m c_{snm}^{\Delta SI} + \rho_{snm}^{SI} - \sum_{m' \in succ(m)} \alpha_{snd_1m'}^{SI} \ge 0 \qquad \textbf{(6.7.19)}$$

$$\forall n, m: \quad 0 \le \Delta_{snm}^{SX} \perp \quad p_m \gamma_m c_{snm}^{\Delta SX} + \rho_{snm}^{SX} - \sum_{\substack{m' \in succ(m) \\ d=2,3}} \alpha_{sndm'}^{SX} \ge 0 \qquad \textbf{(6.7.20)}$$

$$\forall n, m: \quad 0 \le \Delta_{snm}^{SW} \perp \quad p_m \gamma_m c_{snm}^{\Delta SW} + \rho_{snm}^{SW} - \sum_{m' \in succ(m)} \alpha_{sndm'}^{SW} \ge 0 \qquad \textbf{(6.7.21)}$$

$$\forall n, m: \quad 0 \le \rho_{snm}^{SI} \perp \quad \overline{\Delta}_{snm}^{SI} - \Delta_{snm}^{SI} \ge 0 \qquad \textbf{(6.7.22)}$$

$$\forall n, m: \quad 0 \le \rho_{snm}^{SX} \perp \quad \overline{\Delta}_{snm}^{SX} - \Delta_{snm}^{SX} \ge 0 \qquad \textbf{(6.7.23)}$$

$$\forall n, m: \quad 0 \le \rho_{snm}^{SW} \perp \quad \overline{\Delta}_{snm}^{SW} - \Delta_{snm}^{SW} \ge 0 \qquad \textbf{(6.7.24)}$$

6.7.5 Market-clearing conditions

There are four market-clearing conditions: for produced volumes, Eq. (6.7.25), for assigned arc capacities, Eq. (6.7.26), for storage injection volumes Eq. (6.7.27) and the inverse demand curve Eq. (6.7.28):

$$\forall n,d,m: \quad \pi^{p}_{ndm} \ free, \quad \sum_{p} q^{P\rightarrow T}_{pndm} - \sum_{t} q^{T\leftarrow P}_{tndm} = 0 \tag{6.7.25}$$

$$\forall a,d,m: \quad \tau^{A}_{adm} \ free, \quad q^{A\rightarrow T}_{adm} - \sum_{t} f^{A}_{tadm} = 0 \tag{6.7.26}$$

$$\forall n,d,m: \quad \pi^{TS}_{ndm} \ free, \quad \sum_{t} q^{T\rightarrow S}_{tnd_1m} - \sum_{s} q^{S\leftarrow T}_{snd_1m} = 0 \tag{6.7.27}$$

$$\forall n,d,m: \quad \pi^{W}_{ndm} \ free, \quad \pi^{W}_{ndm} - \left(INT^{W}_{ndm} - SLP^{W}_{ndm} \left(\sum_{t} q^{T\rightarrow W}_{tndm} + \sum_{s} q^{S\rightarrow W}_{sndm} \right) \right) = 0 \tag{6.7.28}$$

6.8 Model regions

Region	**Node**	**Countries**
North America	CAN	Canada
	USA/MEX	Mexico, United States
South America	SAM1	Bolivia, Trinidad & Tobago, Venezuela
	SAM2	Argentina, Brazil
	SAM3	Chile, Ecuador, Peru
Africa	AFRN	Algeria, Egypt, Libya, Morocco, Tunisia
	AFRW	Angola, Equatorial Guinea, Nigeria
Europe	EURCE	Austria, Belarus, Belgium, Bulgaria, Czech Republic, Denmark, Estonia, Finland, Germany, Greece, Hungary, Latvia, Lithuania, Poland, Romania, Slovak Republic, Slovenia, Sweden, Switzerland, Turkey, Ukraine
	EURN	Netherlands, Norway
	EURSW	France, Ireland, Italy, Portugal, Spain, United Kingdom
Middle East	MEA1	Iran, Qatar
	MEA2	Kuwait, Oman, Saudi Arabia, UAE ,Yemen
Caspian	CAS	Armenia, Azerbaijan, Georgia, Kazakhstan, Turkmenistan, Uzbekistan
Asia Pacific	ASIA1	Burma, China, India, Pakistan, Singapore, Taiwan, Thailand
	ASIA2	Japan, South Korea
	ASIA3	Brunei, Indonesia, Malaysia
	AUS	Australia
	RUE	Russia-East, Russia-Sakhalin
	RUW	Russia-Volga-Uralsk, Russia-West

6.9 Detailed results

Table 15: Production, consumption and net trade 2010 (bcm/y)

Data	Case	Africa	Asia Pacific	Caspian Region	Europe	Middle East	North America	Russia	South America	Total
Production	DET-base	238	419	166	314	355	748	591	149	2980
	STO-base	238	427	166	313	355	750	588	148	2985
	DET-decline	238	419	166	314	355	748	591	149	2980
	STO-decline	238	427	166	313	355	750	588	148	2985
	DET-cartel	228	431	166	329	336	750	558	148	2946
	STO-cartel	230	425	166	329	338	748	560	148	2944
	DET-combi	228	431	166	329	336	750	558	148	2946
	STO-combi	230	425	166	329	338	748	560	148	2944
Net LNG imports	DET-base	-67	50	0	2	-58	57	-2	-20	
	STO-base	-67	46	0	6	-57	53	0	-20	
	DET-decline	-67	50	0	2	-58	57	-2	-20	
	STO-decline	-67	46	0	6	-57	53	0	-20	
	DET-cartel	-56	13	0	11	-35	53	0	-20	
	STO-cartel	-58	17	0	8	-36	57	0	-20	
	DET-combi	-56	13	0	11	-35	53	0	-20	
	STO-combi	-58	17	0	8	-36	57	0	-20	
net pipeline imports	DET-base	-76	0	-81	317	-3	-5	-187	-1	
	STO-base	-76	0	-81	315	-3	-5	-185	-1	
	DET-decline	-76	0	-81	317	-3	-5	-188	-1	
	STO-decline	-76	0	-81	315	-3	-5	-185	-1	
	DET-cartel	-76	0	-81	265	-3	-5	-132	-1	
	STO-cartel	-76	0	-81	267	-3	-5	-134	-1	
	DET-combi	-76	0	-81	265	-3	-5	-132	-1	
	STO-combi	-76	0	-81	267	-3	-5	-134	-1	
Consumption	DET-base	95	469	86	632	295	799	401	128	2905
	STO-base	95	473	86	633	295	798	403	127	2910
	DET-decline	95	469	86	632	295	800	401	128	2906
	STO-decline	95	473	86	633	295	798	403	127	2910
	DET-cartel	95	444	86	604	299	798	426	127	2879
	STO-cartel	95	441	86	602	299	799	425	127	2874
	DET-combi	95	444	86	604	299	798	426	127	2879
	STO-combi	95	441	86	602	299	799	425	127	2874

Table 16: Production, consumption and net trade 2015 (bcm/y)

Data	Case	Africa	Asia Pacific	Caspian Region	Europe	Middle East	North America	Russia	South America	Total
Production	DET-base	303	484	210	293	420	691	597	178	3176
	STO-base	302	484	210	293	420	692	596	178	3175
	DET-decline	303	484	210	293	420	691	597	178	3176
	STO-decline	302	484	210	293	420	692	596	178	3175
	DET-cartel	290	483	225	310	421	695	554	173	3151
	STO-cartel	290	483	225	310	421	695	554	173	3151
	DET-combi	290	483	225	310	421	695	554	173	3151
	STO-combi	290	483	225	310	421	695	554	173	3151
Net LNG imports	DET-base	-82	32	0	0	-58	86	-2	-20	
	STO-base	-81	34	0	0	-58	85	-1	-20	
	DET-decline	-82	33	0	0	-58	87	-2	-20	
	STO-decline	-81	34	0	0	-58	85	-1	-20	
	DET-cartel	-69	31	0	2	-61	76	0	-20	
	STO-cartel	-69	32	0	1	-61	76	0	-20	
	DET-combi	-69	31	0	2	-61	76	0	-20	
	STO-combi	-69	32	0	1	-61	76	0	-20	
net pipeline imports	DET-base	-115	31	-124	369	-14	-5	-183	-1	
	STO-base	-115	31	-124	368	-14	-5	-182	-1	
	DET-decline	-115	31	-124	369	-14	-5	-183	-1	
	STO-decline	-115	31	-124	368	-14	-5	-182	-1	
	DET-cartel	-115	31	-139	315	-14	-5	-113	-1	
	STO-cartel	-115	31	-140	316	-14	-5	-113	-1	
	DET-combi	-115	31	-139	315	-14	-5	-113	-1	
	STO-combi	-115	31	-140	316	-14	-5	-113	-1	
Consumption	DET-base	106	547	86	661	348	773	412	156	3089
	STO-base	106	548	86	661	348	772	412	156	3089
	DET-decline	106	547	86	661	348	773	412	156	3089
	STO-decline	106	548	86	661	348	772	412	156	3089
	DET-cartel	106	545	86	625	347	766	441	152	3068
	STO-cartel	106	545	86	625	347	766	441	152	3068
	DET-combi	106	545	86	625	347	766	441	152	3068
	STO-combi	106	545	86	625	347	766	441	152	3068

Table 17: Production, consumption and net trade 2020 (bcm/y)

Data	Case	Africa	Asia Pacific	Caspian Region	Europe	Middle East	North America	Russia	South America	Total
Production	DET-base	383	504	252	282	503	628	668	228	3448
	STO-base	383	504	252	282	502	628	667	228	3446
	DET-decline	383	504	252	282	503	628	668	228	3448
	STO-decline	383	504	252	282	502	628	667	228	3446
	DET-cartel	353	493	270	302	516	633	610	209	3386
	STO-cartel	353	493	271	302	516	633	609	209	3386
	DET-combi	353	493	270	302	516	633	610	209	3386
	STO-combi	353	493	271	302	516	633	609	209	3386
Net LNG imports	DET-base	-118	33	0	0	-58	133	-2	-38	
	STO-base	-117	32	0	0	-58	133	-1	-38	
	DET-decline	-118	33	0	0	-58	134	-2	-38	
	STO-decline	-117	32	0	0	-58	133	-1	-38	
	DET-cartel	-93	37	0	2	-79	113	0	-23	
	STO-cartel	-93	37	0	2	-79	113	0	-23	
	DET-combi	-93	37	0	2	-79	113	0	-23	
	STO-combi	-93	37	0	2	-79	113	0	-23	
net pipeline imports	DET-base	-148	129	-164	419	-47	-4	-232	-1	
	STO-base	-148	129	-165	418	-46	-4	-232	-1	
	DET-decline	-148	129	-164	419	-47	-4	-232	-1	
	STO-decline	-148	129	-165	418	-46	-4	-232	-1	
	DET-cartel	-143	120	-183	356	-43	-4	-150	-1	
	STO-cartel	-143	120	-184	356	-43	-4	-149	-1	
	DET-combi	-143	120	-183	356	-43	-4	-150	-1	
	STO-combi	-143	120	-184	356	-43	-4	-149	-1	
Consumption	DET-base	117	665	87	699	398	757	434	189	3346
	STO-base	117	666	87	699	399	757	434	189	3348
	DET-decline	117	665	87	699	398	758	434	189	3347
	STO-decline	117	666	87	699	399	757	434	189	3348
	DET-cartel	118	650	87	659	394	742	460	185	3295
	STO-cartel	118	650	87	659	394	742	460	185	3295
	DET-combi	118	650	87	659	394	742	460	185	3295
	STO-combi	118	650	87	659	394	742	460	185	3295

Table 18: Production, consumption and net trade 2025 (bcm/y)

Data	Case	Africa	Asia Pacific	Caspian Region	Europe	Middle East	North America	Russia	South America	Total
Production	DET-base	444	499	288	263	594	569	686	243	3586
	STO-base	444	498	288	263	597	569	685	245	3589
	DET-decline	444	488	288	256	600	559	688	248	3571
	STO-decline	444	489	288	259	597	566	687	245	3575
	DET-cartel	397	489	308	282	614	575	630	220	3515
	STO-cartel	397	489	312	282	615	575	629	220	3519
	DET-combi	398	479	319	272	618	565	630	222	3503
	STO-combi	398	479	312	276	616	572	630	220	3503
Net LNG imports	DET-base	-148	53	0	0	-81	164	-1	-41	
	STO-base	-147	56	0	0	-85	166	-1	-43	
	DET-decline	-148	60	0	0	-89	169	-1	-48	
	STO-decline	-147	56	0	0	-85	166	-1	-43	
	DET-cartel	-115	51	0	11	-113	140	0	-24	
	STO-cartel	-116	52	0	11	-114	141	0	-24	
	DET-combi	-117	53	0	11	-118	146	0	-26	
	STO-combi	-116	53	0	12	-115	141	0	-24	
net pipeline imports	DET-base	-170	188	-201	459	-76	-3	-248	-2	
	STO-base	-170	189	-201	459	-76	-3	-249	-2	
	DET-decline	-170	189	-201	462	-76	-3	-252	-2	
	STO-decline	-170	189	-201	461	-76	-3	-251	-2	
	DET-cartel	-153	183	-222	386	-72	-3	-168	-1	
	STO-cartel	-153	184	-226	388	-72	-3	-168	-1	
	DET-combi	-153	184	-233	395	-72	-3	-168	-1	
	STO-combi	-154	184	-226	390	-72	-3	-169	-1	
Consumption	DET-base	127	740	87	721	436	730	436	200	3477
	STO-base	127	744	87	721	436	731	436	200	3482
	DET-decline	127	737	87	717	435	726	435	199	3463
	STO-decline	127	734	87	719	436	728	435	200	3466
	DET-cartel	128	724	86	678	429	712	461	195	3413
	STO-cartel	128	725	86	679	429	713	462	195	3417
	DET-combi	128	716	86	676	428	707	461	195	3397
	STO-combi	128	715	86	675	428	709	461	195	3397

Table 19: Production, consumption and net trade 2030 (bcm/y)

Data	Case	Africa	Asia Pacific	Caspian Region	Europe	Middle East	North America	Russia	South America	Total
Production	DET-base	489	497	324	249	708	524	699	254	3744
	STO-base	488	497	324	249	709	523	698	256	3744
	DET-decline	489	476	326	233	729	503	704	259	3719
	STO-decline	489	477	326	236	727	505	702	258	3720
	DET-cartel	429	493	346	267	721	530	641	230	3657
	STO-cartel	429	493	349	267	722	530	640	230	3660
	DET-combi	433	473	357	247	733	509	641	233	3626
	STO-combi	433	474	355	250	731	511	642	233	3629
Net LNG imports	DET-base	-178	104	0	0	-136	189	-1	-41	
	STO-base	-177	104	0	0	-137	189	-1	-43	
	DET-decline	-178	118	0	0	-163	203	-1	-48	
	STO-decline	-177	117	0	0	-161	201	-1	-46	
	DET-cartel	-133	89	0	12	-161	159	0	-24	
	STO-cartel	-133	90	0	12	-161	159	0	-24	
	DET-combi	-137	95	0	13	-176	172	0	-27	
	STO-combi	-137	95	0	13	-175	172	0	-27	
net pipeline imports	DET-base	-173	242	-238	487	-104	-3	-264	-2	
	STO-base	-173	243	-238	487	-104	-3	-265	-2	
	DET-decline	-173	242	-241	494	-102	-2	-272	-2	
	STO-decline	-173	242	-240	493	-102	-2	-270	-2	
	DET-cartel	-155	239	-260	410	-100	-3	-183	-1	
	STO-cartel	-155	239	-264	411	-101	-3	-181	-1	
	DET-combi	-156	240	-272	421	-100	-3	-183	-1	
	STO-combi	-156	240	-270	419	-99	-3	-184	-1	
Consumption	DET-base	138	843	86	734	468	709	433	211	3622
	STO-base	138	844	86	734	468	710	433	210	3623
	DET-decline	138	837	86	725	464	704	430	209	3593
	STO-decline	138	836	86	728	464	703	431	210	3596
	DET-cartel	140	821	85	687	460	686	458	205	3542
	STO-cartel	140	822	85	688	460	686	458	205	3544
	DET-combi	140	808	85	679	456	679	458	204	3509
	STO-combi	140	808	85	679	457	679	458	204	3510

Table 20: Production, consumption and net trade 2035 (bcm/y)

Data	Case	Africa	Asia Pacific	Caspian Region	Europe	Middle East	North America	Russia	South America	Total
Production	DET-base	510	471	355	240	788	475	698	254	3791
	STO-base	509	471	355	240	788	475	698	256	3792
	DET-decline	519	454	357	205	814	439	706	259	3753
	STO-decline	519	454	357	210	812	442	705	258	3757
	DET-cartel	443	466	389	264	788	482	638	230	3700
	STO-cartel	443	466	391	263	788	482	637	230	3700
	DET-combi	452	450	402	217	809	446	643	234	3653
	STO-combi	452	450	400	221	808	448	644	234	3657
Net LNG imports	DET-base	-178	100	0	0	-136	195	-1	-41	
	STO-base	-178	100	0	0	-137	196	-1	-43	
	DET-decline	-188	112	0	0	-169	224	-1	-48	
	STO-decline	-187	111	0	0	-166	221	-1	-46	
	DET-cartel	-134	82	0	6	-153	166	0	-24	
	STO-cartel	-134	82	0	6	-152	166	0	-24	
	DET-combi	-141	85	0	15	-184	191	0	-29	
	STO-combi	-141	85	0	15	-183	189	0	-29	
net pipeline imports	DET-base	-177	294	-273	496	-131	-2	-265	-2	
	STO-base	-177	294	-273	496	-131	-2	-265	-2	
	DET-decline	-177	294	-274	511	-131	-2	-277	-2	
	STO-decline	-177	294	-274	509	-131	-2	-275	-2	
	DET-cartel	-151	291	-307	418	-119	-2	-185	-2	
	STO-cartel	-151	290	-308	419	-121	-2	-184	-2	
	DET-combi	-153	291	-320	437	-118	-2	-191	-1	
	STO-combi	-153	291	-318	435	-117	-2	-192	-1	
Consumption	DET-base	154	865	83	735	521	668	432	211	3669
	STO-base	155	865	83	735	521	668	432	210	3669
	DET-decline	154	859	83	714	514	661	428	209	3622
	STO-decline	154	859	83	718	515	661	428	210	3628
	DET-cartel	158	839	82	686	515	645	453	204	3582
	STO-cartel	158	839	82	686	515	645	453	204	3582
	DET-combi	158	826	82	668	507	634	452	204	3531
	STO-combi	158	827	82	668	507	634	452	204	3532

Table 21: Production, consumption and net trade 2040 (bcm/y)

Data	Case	Africa	Asia Pacific	Caspian Region	Europe	Middle East	North America	Russia	South America	Total
Production	DET-base	529	448	379	231	871	434	683	254	3829
	STO-base	528	447	379	231	871	434	683	255	3828
	DET-decline	542	436	387	174	892	377	698	260	3766
	STO-decline	542	436	385	182	892	381	695	259	3772
	DET-cartel	455	444	431	257	853	442	628	229	3739
	STO-cartel	454	444	433	256	854	442	627	229	3739
	DET-combi	465	434	447	186	870	382	637	239	3660
	STO-combi	464	434	445	190	868	385	638	239	3663
Net LNG imports	DET-base	-178	101	0	0	-136	193	-1	-41	
	STO-base	-178	102	0	0	-137	194	-1	-43	
	DET-decline	-189	101	0	0	-169	237	-1	-50	
	STO-decline	-188	100	0	0	-166	234	-1	-49	
	DET-cartel	-130	79	0	1	-146	164	0	-24	
	STO-cartel	-130	79	0	0	-145	164	0	-24	
	DET-combi	-135	66	0	13	-179	207	0	-36	
	STO-combi	-135	67	0	12	-178	205	0	-35	
net pipeline imports	DET-base	-177	335	-299	499	-160	-2	-255	-2	
	STO-base	-177	336	-299	499	-160	-2	-255	-2	
	DET-decline	-180	336	-308	528	-156	-1	-276	-2	
	STO-decline	-180	336	-306	524	-158	-1	-272	-2	
	DET-cartel	-146	332	-352	429	-135	-2	-186	-2	
	STO-cartel	-146	332	-354	431	-136	-2	-185	-2	
	DET-combi	-151	334	-369	454	-128	-2	-197	-1	
	STO-combi	-151	334	-367	452	-126	-2	-198	-1	
Consumption	DET-base	174	884	79	729	575	625	427	210	3703
	STO-base	174	885	79	729	575	626	427	209	3704
	DET-decline	173	872	79	701	567	612	421	208	3633
	STO-decline	173	872	79	705	568	613	422	208	3640
	DET-cartel	179	855	78	685	573	602	442	204	3618
	STO-cartel	179	855	78	686	573	602	442	204	3619
	DET-combi	179	833	78	651	564	587	440	202	3534
	STO-combi	179	834	78	653	564	587	440	202	3537

Table 22: Volume-weighted average wholesale prices. World ($2005/kcm)

Case	2005	2010	2015	2020	2025	2030	2035	2040
DET-base	$ 165	$ 195	$ 234	$ 270	$ 317	$ 374	$ 421	$ 479
DET-combi	$ 165	$ 198	$ 239	$ 280	$ 331	$ 395	$ 453	$ 525
DET-decline	$ 165	$ 195	$ 234	$ 270	$ 320	$ 380	$ 434	$ 502
DET-cartel	$ 165	$ 198	$ 239	$ 280	$ 328	$ 389	$ 440	$ 501
STO-combi	$ 165	$ 198	$ 239	$ 280	$ 331	$ 395	$ 453	$ 524
STO-cartel	$ 165	$ 198	$ 239	$ 280	$ 328	$ 389	$ 440	$ 500
STO-decline	$ 165	$ 194	$ 234	$ 270	$ 319	$ 379	$ 433	$ 500
STO-base	$ 165	$ 194	$ 234	$ 270	$ 316	$ 373	$ 421	$ 479

Table 23: Volume-weighted average wholesale prices in Central Europe ($2005/kcm)

Case	2005	2010	2015	2020	2025	2030	2035	2040
DET-base	$ 189	$ 228	$ 272	$ 311	$ 364	$ 429	$ 473	$ 520
DET-combi	$ 189	$ 255	$ 308	$ 359	$ 421	$ 511	$ 589	$ 678
DET-decline	$ 189	$ 228	$ 272	$ 311	$ 369	$ 443	$ 508	$ 579
DET-cartel	$ 189	$ 255	$ 308	$ 359	$ 419	$ 499	$ 554	$ 603
STO-combi	$ 189	$ 258	$ 308	$ 359	$ 423	$ 510	$ 587	$ 675
STO-cartel	$ 189	$ 258	$ 308	$ 359	$ 418	$ 498	$ 552	$ 600
STO-decline	$ 189	$ 227	$ 273	$ 312	$ 367	$ 439	$ 503	$ 571
STO-base	$ 189	$ 227	$ 273	$ 312	$ 364	$ 429	$ 473	$ 520

Table 24: Volume-weighted average wholesale prices in United States & Mexico ($2005/kcm)

Case	2005	2010	2015	2020	2025	2030	2035	2040
DET-base	$ 206	$ 257	$ 334	$ 416	$ 484	$ 562	$ 647	$ 746
DET-combi	$ 206	$ 258	$ 339	$ 431	$ 510	$ 604	$ 706	$ 830
DET-decline	$ 206	$ 257	$ 333	$ 416	$ 489	$ 570	$ 659	$ 775
DET-cartel	$ 206	$ 258	$ 339	$ 431	$ 505	$ 593	$ 686	$ 796
STO-combi	$ 206	$ 257	$ 339	$ 431	$ 508	$ 604	$ 705	$ 829
STO-cartel	$ 206	$ 257	$ 339	$ 431	$ 504	$ 593	$ 686	$ 796
STO-decline	$ 206	$ 258	$ 335	$ 416	$ 486	$ 570	$ 658	$ 774
STO-base	$ 206	$ 258	$ 335	$ 416	$ 482	$ 561	$ 645	$ 745

Table 25: Volume-weighted average wholesale prices in Japan & South Korea ($2005/kcm)

Case	2005	2010	2015	2020	2025	2030	2035	2040
DET-base	$ 282	$ 319	$ 375	$ 415	$ 481	$ 572	$ 658	$ 763
DET-combi	$ 282	$ 366	$ 397	$ 446	$ 517	$ 613	$ 707	$ 831
DET-decline	$ 282	$ 319	$ 375	$ 415	$ 483	$ 577	$ 662	$ 775
DET-cartel	$ 282	$ 366	$ 397	$ 446	$ 514	$ 603	$ 697	$ 809
STO-combi	$ 282	$ 374	$ 396	$ 446	$ 517	$ 612	$ 707	$ 830
STO-cartel	$ 282	$ 374	$ 396	$ 446	$ 513	$ 603	$ 697	$ 809
STO-decline	$ 282	$ 311	$ 375	$ 415	$ 484	$ 576	$ 661	$ 773
STO-base	$ 282	$ 311	$ 375	$ 415	$ 479	$ 572	$ 658	$ 763

7 Benders Decomposition for Large-Scale Stochastic Mixed Complementarity Problems

In this chapter a Benders decomposition (BD) approach for large-scale stochastic mixed complementarity problems (MCP) is presented. Convergence characteristics for the specialized method are compared with solution times of full-scale extensive-form MCP as well as convex optimization problems. Small-scale implementations are analyzed to show that small numerical deviations occur. These numerical deviations are the likely explanation for the problems encountered when trying to solve some of the large-scale problems.

In Chapter 6, a stochastic complementarity model was developed and applied to a natural gas market case with four stochastic scenarios. The calculation time to solve this four-scenario problem was a few hours and therefore short enough for most realistic applications such as policy or market analysis. However, analysts or regulators may be interested in including more scenarios or a less aggregated data set, which would likely result in much higher calculation times. To address the potential calculation time problems, in this chapter a Benders decomposition approach for large-scale stochastic mixed complementarity problems (MCP) is developed and applied. It will be clarified that there are two alternative routes to formulate the master and subproblems. The more difficult route follows the VI-based approach developed in (Gabriel and Fuller, 2010) and the easier route takes the generalized Benders approach in (Geoffrion, 1972) as its starting point. As a result, a number of master and subproblem variants are developed and applied, and are compared on their merits regarding calculation times and number of iterations needed to obtain a solution. Along the way it will become clear why in the implemented approach no feasibility cuts are needed, and that only optimality cuts are added in each iteration of the decomposition approaches. Some of the main findings in this chapter include that the approach developed in (Gabriel and Fuller, 2010) performs better on optimization problems in terms of the time needed to converge and solution accuracy relative to optimization-based approaches, based on (Geoffrion, 1972).

This chapter presents the third major contribution of this dissertation in the form of the application (i.e., the adjustment, extension and implementation) of a BD approach for large-scale stochastic MCP. No models are actually solved quicker using the decomposition method relative to solving the full-scale extensive-form models, but it is argued that implementing a parallel processing approach and using other software would make the decomposition method faster. Numerical computational problems prevented the successful solution of the largest problems tried. The largest stochastic MCP (problem B) solved with the BD approach contained 47,373 and solved in 13,684 seconds (wall-clock time) or 2,036 seconds of CPU time. Solving this problem in extensive-form took 16 minutes and 45 seconds. The largest stochastic MCP (problem E) solved in extensive-form contained 117,481 variables and solved in 18,679 seconds.

Due to the numerical difficulties the scalability of the approach was not proved, however the results support that the decomposition method has a good potential to significantly reduce solution times of large-scale stochastic MCP.

7.1 Introduction

A main objective of the research presented in this dissertation was to develop a method to solve large-scale stochastic mixed complementarity problems (MCP) that would address the memory and calculation time issues arising when setting up and solving full-scale extensive-form stochastic models. Chapter 5 discussed various methods to do so. The starting point for the approach are the methods developed by Benders (1962), which he called partitioning procedures, but which have later been referred to as Benders Decomposition.[165] Benders developed the decomposition method for linear optimization problems. Geoffrion (1972) extended the method to convex mixed-integer non-linear programming problems (NLP). Perfectly competitive and monopolistic markets can be modeled as convex NLP (see Chapter 2) and Geoffrion's approach is applied to a stochastic multi-period optimization model. However, markets with imperfect competition where more than one player exerts market power, such as the global natural gas market, cannot be modeled as optimization problems (Chapter 2). To represent the competition characteristics in these imperfect markets other model variants, such as MCP or variational inequalities (VI) are needed. Among the MP and SP variants developed in

[165] The article was reprinted as (Benders, 2005).

this chapter, only the MP-LCP, SP-MCP using the VI-based cuts can be applied to the stochastic MCP under consideration.

Fuller and Chung (2005) developed a Dantzig-Wolfe (DW, Dantzig and Wolfe (1960)) decomposition approach for VI, which in a later paper was used as the foundation for a Benders decomposition (BD) approach (Fuller and Chung, 2008). Gabriel and Fuller (2010) extended the Benders decomposition approach to be applicable to stochastic MCP. They provided mathematical details, proofs and several numerical examples. The numerical examples contained eight first-stage variables and up to 20,000 scenarios. Model agents include power generators and electricity grid owners. The model has two stages in which power generators face stochastic inverse demand curves in the second stage. The first-stage decision for the power generators is to decide on how much (low-cost) slow-ramping generation capacity to bring online, while in the second stage a decision is made about (expensive) rapid-ramping capacity.

In this chapter the Benders decomposition approach developed by Gabriel and Fuller is extended and applied to multi-period natural gas market problems. The problems will include more and different types of model agents, multiple future periods and a separation of first-stage capacity and second-stage quantity decisions. Generalized Benders decomposition for convex programming (Geoffrion 1972) is applied to profit and welfare maximization problems, for which under the assumptions of linear capacity investment costs, the master problems (MP) are linear programs (LP) and the subproblems (SP) convex non-linear optimization programs (NLP).[166] The mathematical formulations are derived and applied to numerical examples containing the aspects relevant for the natural gas market as modeled in the WGM (see Chapter 3). Next, the Karush-Kuhn-Tucker (KKT) conditions (e.g., (Cottle et al., 1992)) are derived for the linear MP and the convex SP. The KKT conditions of the MP and SP are used as alternative model formulations in a BD approach to investigate the impact on calculation times of using different model types and solvers.

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[166] SP is used to abbreviate the singular and plural words. Similarly for other abbreviations such as MCP, NLP and VI.

The decomposition method developed in (Gabriel and Fuller, 2010) is summarized. In an appendix to this chapter the groups of variables and equations in the natural gas market model are identified and matched with the variables and equations in the former papers (see Table 42 in Section 7.9 at the end of this chapter). Interestingly, except for the KKT conditions for the Benders cuts, the KKT conditions derived using the VI-based approach (Gabriel and Fuller, 2010) are identical to the KKT conditions derived from MP and SP following (Geoffrion, 1972). This is not a surprise, since in the generalized Benders decomposition approach, the economic considerations by the individual market agents are identical and represented accurately, however the optimality cuts in that approach misrepresent the obtainable SP solution values in hybrid and Cournot market structures and this is corrected by the optimality cut developed in (Gabriel and Fuller, 2010). [167]

Figure 52 below illustrates the alternative routes to derive the MP and SP developed in (Gabriel and Fuller, 2010). The starting point, in the left-upper corner are nonlinear programs (NLP) for which Geoffrion (1972) generalized Benders decomposition. The NLP formulation is presented in Appendix 7.8, and the MP and SP in Section 7.2. The KKT conditions derived from these MP and SP are presented in Section 7.3. The approach developed in (Gabriel and Fuller, 2010) is the second derivation route, clarified in Section 7.4.[168] As illustrated in the right part of Figure 52 both approaches result in the same KKT conditions systems (MP-LCP and SP-MCP), except for the optimality cuts.

[167] A hybrid market involves players exert varying levels of market power.
[168] The foundation for the decomposition approach developed in this chapter comes from (Fuller and Chung, 2005), (Fuller and Chung, 2008) and (Gabriel and Fuller, 2010). Fuller and Chung (2005) develop a Dantzig-Wolfe (DW) approach for VI. Using duality theory (see Chapter 2) (Fuller and Chung, 2008) developed a Benders decomposition (BD) approach for VI. Gabriel and Fuller (2010) extended the latter to be applicable to stochastic MCP.

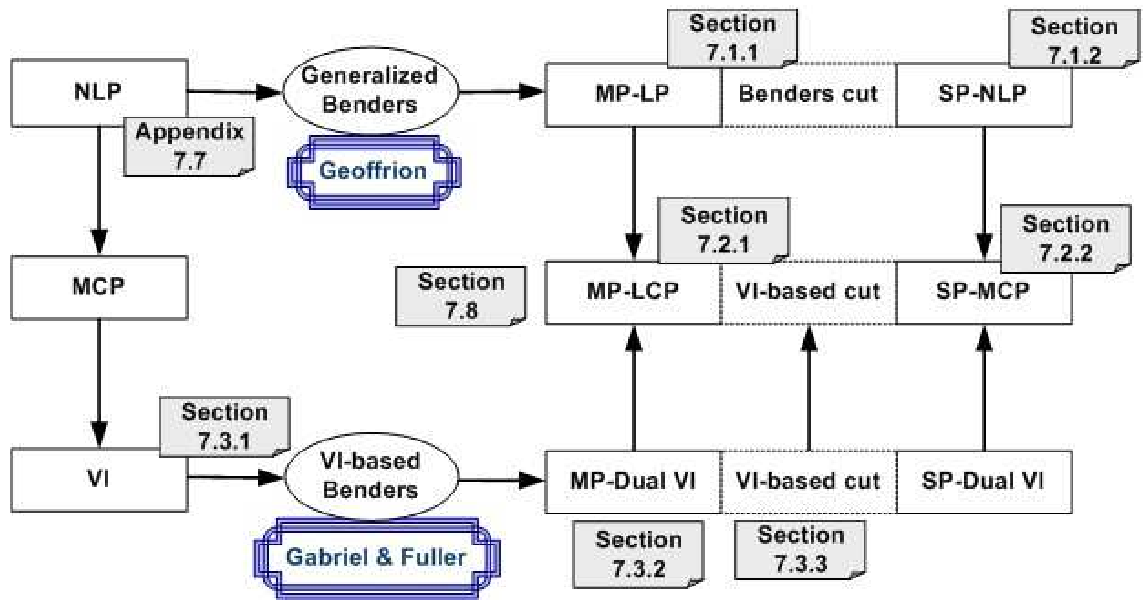

Figure 52: Alternative routes for deriving the VI-based master and subproblems

In the numerical experiments section the optimization problems for perfectly competitive and monopolistic markets are solved using all variants of MP and SP. Using various combinations the needed calculation times, numbers of iterations needed and convergence speed are compared. An interesting result is that the VI-based approaches needed fewer iterations and less calculation time than the optimization-based decomposition approaches, including the KKT variants. Next the computational results for a number of large-scale stochastic problems are presented. The chapter will conclude with a discussion of various implementation issues.

While performing this research several issues arose with the used software: GAMS (Brook et al., 1988) and solver PATH ((Dirkse and Ferris, 1995), (Ferris and Munson, 2000)). In the appendix at the end of this chapter limitations and workarounds necessary to implement the Benders approach in GAMS are discussed. Also, an extension to the approach is discussed that guarantees the feasibility of subproblems by setting minimum limits for aggregate capacity expansions in case of existing supply contracts with future start dates for which the current capacities do not suffice.[169]

[169] For problems without contracts, the zero vector is always feasible to the SP.

In the numerical sections of this chapter various alternatives for solving stochastic MCP are compared. There are two full-scale extensive stochastic model versions, one based on a stochastic social welfare maximization approach and one based on an MCP.

There are three alternative master problems:

1. a linear program (LP)
2. a linear complementarity problem based on generalized BD (Geoffrion, 1972) (LCP)
3. an LCP based on the VI approach of (Gabriel and Fuller, 2010).

Also, there are two alternative subproblem formulations:

1. a non-linear program (NLP)
2. a mixed complementarity problem (MCP).

The final goal of this chapter is to solve large-scale stochastic MCP. However, many interesting problems can be cast as optimization problems, and much insight can be obtained from analyzing and comparing related implementations. To solve stochastic optimization problems all MP and SP variants can be combined, resulting in six alternative ways to apply the decomposition approach for solving perfectly competitive and monopolistic markets. Tables 1 and 2 summarize the various solution approaches.

Table 26: Full-scale extensive-form solution approaches

Market forms	Approach	Abbreviation
Perfectly competitive and monopolistic markets	Surplus maximization	FullOPT
All markets, including Cournot and hybrid	Complementarity	FullMCP

Table 27: Decomposition approaches

Market forms	Sub / Master	SP-NLP	SP-MCP
Perfectly competitive and monopolistic markets	MP-LP	LP-NLP	LP-MCP
	MP-LCP	LCP-NLP	LCP-MCP
	MP-VI	VI-NLP	
All markets, including Cournot and hybrid	MP-VI		VI-MCP

The results of the various implementations show that the decomposition approaches need relatively much time to solve small-scale problems due to the overhead added (such as file-IO), but when problem sizes grow the calculation times increase much less compared to solving the full-scale extensive-form models. Interestingly, the (more general) decomposition approach based on (Gabriel and Fuller, 2010) outperforms other methods, needing fewer iterations and less calculation time to reach convergence. As a prelude to numerical challenges, some convergence results are discussed in detail, showing that even very small problems can have noticeable deviations in converged results. When solving bigger problems various numerical issues turned up (see Section 7.11). For some issues good work-around were found, however for implementing the approach on very large-scale problems it is recommended to use an alternative software platform and implement parallel processing (see Section 7.7).[170]

The first section will start with the optimization-based decomposition approaches.

7.2 Benders decomposition for convex non-linear programming

Benders decomposition was discussed in Chapter 5, including some numerical illustrations. Benders decomposition separates the problem at hand into various parts. One part is called the master problem (MP), the other part (or parts) the subproblem(s) (SP). The MP contains the 'complicating variables' that make the original problem difficult to solve, whereas the SP is relatively easy to solve. The MP and SP are solved iteratively. In each iteration, the MP determines a solution for the complicating variables and the SP is solved to determine the best solution possible, given the MP results.

The MP contains a variable α which approximates the optimal value function of the SP (Conejo et al., 2006). Information from the SP solutions in the form of dual prices, is used in the MP to improve the value of α. Every iteration provides a lower bound and an upper bound to the objective value of the original problem. Iteratively the MP and SP are solved, until the lower bound and upper bound are equal (or very close to each other) in which case the solution has converged.

To implement Benders decomposition the complicating variables must be identified. In multi-period, stochastic models the complicating variables are usually the variables that in some way link periods to each other, thereby preventing the solution of the original

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[170] The decomposition approach was implemented using GAMS (Brook et al., 1988).

problem by blocks (see Chapter 5, L-shaped method). Once the complicating variables are set to fixed values, the original problem decomposes into blocks: subproblems for all scenario tree nodes. See Figure 53 below for an example of a scenario tree.

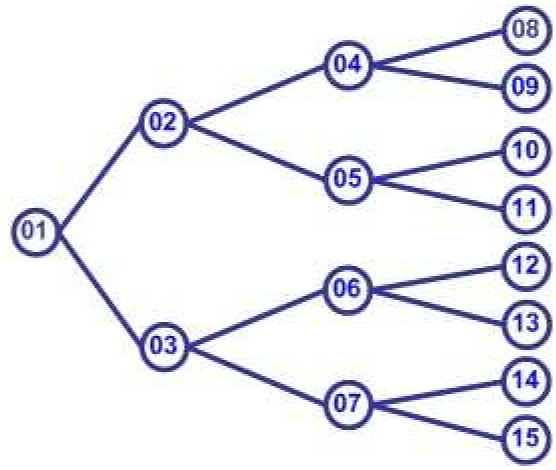

Figure 53: Scenario tree with four scenarios

In the WGM presented in Chapter 3 the complicating variables are the capacity expansions. The capacity expansions are complicating, since once capacity is in place it is available in all future periods.[171] Figure 54 shows four capacity constraints for a stylized problem. Variables Q_i are capacity-constrained quantities, and $\overline{CAP}$ is the initial capacity. The left-most large gray part are the capacity expansion variables (with a one-period time lag for expansions to become available).

			$-Q_1$				$\geq -\overline{CAP}$
Δ_1				$-Q_2$			$\geq -\overline{CAP}$
Δ_1	$+\Delta_2$				$-Q_3$		$\geq -\overline{CAP}$
Δ_1	$+\Delta_2$	$+\Delta_3$				$-Q_4$	$\geq -\overline{CAP}$

Figure 54: Block structure for capacity expansions

[171] The model introduced in Chapter 3 also includes complicating constraints for the gas reserves. To deal with this complicating aspect a DW approach could be considered. However, the gas reserves constraints are not used in the implementations due to unavailability of input data (see Chapter 4) and in this chapter the focus is on the capacity expansions as the complicating variables. If the gas reserves constraint would be used in implementations, the approach discussed in this chapter would have to be adjusted to accommodate incorporation. DW could be used, or some form of Lagrangian relaxation. Alternatively, the quantities produced could be included in the MP. This would, however, result in other challenges and possibly induce the need for feasibility cuts as well. See (Cabero et al., 2010).

Once the capacity expansions are taken out of the problem (or fixed to a value) the problem decomposes into blocks (Figure 55 below). Each of these blocks is a single-period market-equilibrium problem that can be solved separately.

$-Q_1$				$\geq -\overline{CAP}$
	$-Q_2$			$\geq -\overline{CAP} - \Delta_1$
		$-Q_3$		$\geq -\overline{CAP} - \Delta_1 - \Delta_2$
			$-Q_4$	$\geq -\overline{CAP} - \Delta_1 - \Delta_2 - \Delta_3$

Figure 55: Problem separation by block

Beside computational measures to reduce the calculation time, there are also options to reduce the model size and thereby the calculation time. For example, the storage operator problem can be modeled with fewer equations in a two-season data set, since not all three of the injection, extraction and working gas capacity restriction for each storage facility are needed. In a two-season data set, only injection, or extraction, or working gas can be limiting.[172] The working gas constraint can be used to represent all limitations to storage capacities. To do so, the working gas capacity data must be adjusted using the following equation: $\overline{WG}_s^S = \min\left\{\overline{WG}_s^S, (1-loss_s)\, days_L\, \overline{INJ}_s^S, days_H\, \overline{XTR}_s^S\right\}$. Also the expansion costs for working gas need to be adjusted to represent the costs for injection and extraction capacity expansions as well.

The market equilibrium in perfectly competitive markets can be found by maximizing social welfare (Walras, 1977). The market equilibrium in monopolistic markets can be found by maximizing the aggregate profit of the monopoly supplier and all other price-taking model agents. In Section 7.8 the mathematical formulation of a model to maximize expected social welfare is presented. In Section 7.8.1 the notation used throughout this chapter is introduced.

[172] Theoretically, there can be more than one binding constraint at the same time, but that does not affect the possibility to reduce the number of equations as discussed. Generally, there will be one restriction that is the most limiting, which makes the others redundant.

The generalized Benders decomposition developed in (Geoffrion, 1972) can be applied to optimization problems. Given the linear capacity expansion costs, the MP are LP and the SP are convex NLP. In the following, the master problem formulation for the decomposition method is presented based on the model formulation in Section 7.8.

7.2.1 MP for the multi-period optimization problem: MP-LP

The capacity expansions are considered the complicating variables, so they are included in the master problem, as well as the upper limits to capacity expansions. All other variables and equations are part of the subproblems. Following the Benders approach, a variable α is included in the MP as an approximation of the SP objective function values (Chapter 5, (Conejo et al., 2006)). To facilitate the derivation of the KKT conditions and the comparability of results, the MP and SP are written as minimization problems. Hence, the MP can be written as the minimization of the probability-weighted and discounted investment costs for capacity expansions plus α:

$$\min_{\Delta_{am}^{A},\Delta_{sm}^{S},\alpha} \sum_{m} p_m \gamma_m \left\{ \sum_{a} c_{am}^{\Delta A} \Delta_{am}^{A} + \sum_{s} c_{sm}^{\Delta S} \Delta_{sm}^{S} \right\} + \alpha \qquad \textbf{(7.2.1)}$$

To address limitations to capacity expansions Eq. (7.8.6) and (7.8.8) are included:

$$\Delta_{am}^{A} \leq \overline{\Delta}_{am}^{A} \quad \forall a,m \qquad \left(\rho_{am}^{A}\right) \qquad \textbf{(7.2.2)}$$

$$\Delta_{sm}^{S} \leq \overline{\Delta}_{sm}^{S} \quad \forall s,m \qquad \left(\rho_{sm}^{S}\right) \qquad \textbf{(7.2.3)}$$

The Benders optimality cuts approximate how varying the capacity expansions relative to former MP solutions will change the aggregate objective function value of the SP. In every iteration an optimality cut is added to the MP, containing the values of capacity expansions of the previous MP, the aggregate objective value and the dual prices to capacities from the previous SP and the decision variables of the current MP.

The following symbols are used in the cuts, for iteration *it*:

$\Delta_{am}^{A,it}$ MP solution value for arc capacity expansions in (mcm/d)

$\Delta_{sm}^{S,it}$ MP solution value for storage working gas capacity expansions (mcm)

$\lambda_{adm}^{A,it}$ SP solution value for dual price of arc capacity constraint (7.8.5) (k\$/mcm/d)

$\lambda_{sm}^{S,it}$ SP solution value for dual price of storage capacity constraint (7.8.7) (k\$/mcm/d)

Z_{it}^{SP} The probability-weighted discounted sum of SP objective values (k$)

In iteration *it+1* the *it*th cut is added to the MP. The set of cuts is written as: [173]

$$\begin{aligned}&\alpha \;+\sum_m p_m\gamma_m\left(\sum_{a,d}\lambda_{adm}^{A,it}\sum_{m'\in succ(m)}\Delta_{am}^{A}\;+\sum_s\lambda_{sm}^{S,it}\sum_{m'\in succ(m)}\Delta_{sm}^{S}\right)\\ &\geq Z_{it}^{SP}+\sum_m p_m\gamma_m\left(\sum_{a,d}\lambda_{adm}^{A,it}\sum_{m'\in succ(m)}\Delta_{am}^{A,it}+\sum_s\lambda_{sm}^{S,it}\sum_{m'\in succ(m)}\Delta_{sm}^{S,it}\right)\end{aligned}\quad \forall it \qquad \left(\theta^{it}\right) \qquad \textbf{(7.2.4)}$$

Lastly, a brief note on feasibility cuts, the other type of cuts commonly found in Benders decomposition approaches (see Chapter 5). When looking at the model presented in Section 7.8 and more specifically the subproblem definition, it is immediately clear that the zero vector, the vector where all (primal) decision variables are zero, is a feasible solution, regardless of the values for the capacity expansions. Hence, no infeasible SP will be encountered and no feasibility cuts are necessary in the implementation of the decomposition approach.

Next, the subproblem formulation is presented.

7.2.2 SP for the multi-period optimization problem: SP-NLP

The capacity expansions are determined in the MP, and enter the SP as fixed variables and the investment costs are not included in the SP objective. If all scenario nodes are combined into one large SP, the expected social welfare function to be maximized can be written as follows:

$$\max\sum_{m\in M}p_m\gamma_m\sum_{n\in N}\sum_{d\in D}d_d\left\{\begin{array}{l}\frac{1}{2}SLP_{ndm}^{W}\left(\sum_{t\in T(n)}Q_{tndm}^{T\to W}\right)^2\\ +\sum_{t\in T(n)}Q_{tndm}^{T\to W}\left(INT_{ndm}^{W}-SLP_{ndm}^{W}\sum_{t\in T(n)}Q_{tndm}^{T\to W}\right)\\ -\sum_{p\in P(n)}c_{pm}^{P}(Q_{pdm}^{P\to T})-\sum_{a\in a^+(n)}c_{adm}^{A}Q_{adm}^{A\to T}-\sum_{s\in S(n)}c_{sdm}^{I}Q_{sdm}^{S\to T}\end{array}\right\} \qquad \textbf{(7.2.5)}$$

[173] The weighting of the dual prices λ_{adm}^{A} and λ_{sm}^{S} with $p_m\gamma_m$ are addressed in the following paragraph.

Besides calculation time problems, another issue to be addressed by using decomposition techniques is size reduction of potentially large models that otherwise may not fit into a computer memory. Decomposition of the SP in smaller parts, for instance by separate scenario tree node, can facilitate separate or parallel processing of the SP with potentially large reductions in run times.[174] Therefore, the SP are decomposed by separate scenario node. To write the models and notation more succinctly, the probabilities and discount rates are left out of the subproblems. Doing so will not alter the obtained market equilibria in terms of produced, traded, and consumed volumes; however it will affect the scaling of the dual prices.[175] The SP for a scenario node *m* can be written as:

$$\max \sum_{n\in N}\sum_{d\in D} d_d \left\{ \begin{array}{l} \frac{1}{2} SLP^W_{ndm}\left(\sum_{t\in T(n)} Q^{T\to W}_{tndm}\right)^2 + \sum_{t\in T(n)} Q^{T\to W}_{tnd}\left(INT^W_{ndm} - SLP^W_{ndm}\sum_{t\in T(n)} Q^{T\to W}_{tnd}\right) \\ -\sum_{p\in P(n)} c^P_{pm}(Q^{P\to T}_{pd}) - \sum_{a\in a^+(n)} c^A_{ad} Q^{A\to T}_{ad} - \sum_{s\in S(n)} c^I_{sd} Q^{S\to T}_{sd} \end{array} \right\} \tag{7.2.6}$$

For simplicity and as a prelude to the implementation of the decomposition approach, in the parameters the index *m* is maintained, but not in the variables. The same is true for the restrictions presented below. This indicates that the model size of each SP according to Eq. (7.2.6) is an order of magnitude *m* smaller than the aggregate SP according to Eq. (7.2.5).

The SP for each scenario tree node *m* includes the production capacity restrictions (7.2.7), the nodal mass-balance Eq. (7.2.8) and the storage-cycle constraints (7.2.9):

$$Q^{P\to T}_{pd} \le \overline{CAP}^P_{pm} \quad \forall p,d \quad \left(\alpha^P_{pd}\right) \tag{7.2.7}$$

$$Q^{T\leftarrow P}_{tnd} + \sum_{a\in a^+(n)} \left(1-l_a\right) F^T_{tad} + X^T_{tnd} = Q^{T\to W}_{tnd} + \sum_{a\in a^-(n)} F^T_{tad} + I^T_{tnd} \quad \forall t,n,d \quad \left(\varphi^T_{tnd}\right) \tag{7.2.8}$$

$$\left(1-l_{sn}\right)\sum_{d\in D} d_d I^T_{tnd} = \sum_{d\in D} d_d X^T_{tnd} \quad \forall t,n \quad \left(\varphi^S_{tn}\right) \tag{7.2.9}$$

In the arc capacity restriction, the arc capacities added, are fixed to the last MP solution values $\Delta^{A,it}_{am'}$:

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[174] Run time: time elapsed, or total wall clock time needed to solve a problem. Run time includes all time needed for reading and processing data, generating models, file-I/O, communication with the solvers, and the net calculation time. Net calculation time: net CPU time consumed by the solvers. Gross calculation time: net calculation time plus communication time with the solvers.
[175] The scaling is done in the cuts, see equation (7.2.4).

$$Q_{ad}^{A\to T} \le \overline{CAP}_{am}^{A} + \sum_{m' \in pred(m)} \Delta_{am'}^{A,it} \quad \forall a,d \quad \left(\lambda_{ad}^{A}\right) \tag{7.2.10}$$

Similarly, in the storage capacity restriction the added working gas capacities are fixed to the MP solutions $\Delta_{sm'}^{S,it}$:

$$\left(1-l_{sn}\right)\sum_{d\in D} d_d Q_{sd}^{S\to T} \le \overline{CAP}_{sm}^{S} + \sum_{m' \in pred(m)} \Delta_{sm'}^{S,it} \quad \forall s \quad \left(\lambda_{s}^{S}\right) \tag{7.2.11}$$

Lastly, the three market-clearing conditions: between the producer and traders (7.2.12), for arc capacities between the TSO and the traders (7.2.13) and for storage (7.2.14):

$$Q_{pd}^{P\to T} = \sum_{t\in T(p)} Q_{tn(p)d}^{T\leftarrow P} \quad \forall p,d \quad \left(\pi_{n(p)d}^{P}\right) \tag{7.2.12}$$

$$Q_{ad}^{A\to T} = \sum_{t} F_{tad}^{T} \quad \forall a,d \quad \left(\tau_{ad}^{A}\right) \tag{7.2.13}$$

$$Q_{sd}^{S\to T} = \sum_{t\in T(n(s))} I_{tnd}^{T} \quad \forall n,d \quad \left(\tau_{sd}^{S}\right) \tag{7.2.14}$$

In each SP the objective function value for a single scenario tree node is determined. To determine the aggregate value for Z_{it}^{SP} that is used in the Benders cuts (Eq. (7.2.4)) the probability-weighted discounted sum of all SP objective function values is calculated. The weighting of the dual prices λ_{ad}^{A} and λ_{s}^{S} obtained from the SP is done in the cuts (Eq. (7.2.4)).

7.2.3 Bounds

Every iteration of the Benders method provides bounds to the objective value. In the minimization problem, expression Eq. (7.2.1) provides lower bounds, the theoretically best obtainable values:

$$LB^{it} = \sum_{m} p_m \gamma_m \left\{ \sum_{a} c_{am}^{\Delta A} \Delta_{am}^{A,it} + \sum_{s} c_{sm}^{\Delta S} \Delta_{sm}^{S,it} \right\} + \alpha^{it} \tag{7.2.15}$$

These lower bounds monotonically increase, $LB^{it} \ge LB^{it-1}$, hence the last one provides the tightest ((Benders, 1962), (Conejo et al., 2006)). Upper bounds are obtained from feasible solutions. They are not monotonically increasing and are determined through:

$$UB^{it} = \min\left(UB^{it-1}, \sum_{m} p_m \gamma_m \left\{ \sum_{a} c_{am}^{\Delta A} \Delta_{am}^{A,it} + \sum_{s} c_{sm}^{\Delta S} \Delta_{sm}^{S,it} \right\} + Z_{it}^{SP} \right) \tag{7.2.16}$$

and $UB^0 = +\infty$. UB^{it} is also referred to as the best feasible solution. The convergence gap is defined as $UB^{it} - LB^{it}$ and convergence is assumed when this value is lower than a predefined threshold value.

The decomposition approach consisting of the MP (Eq. (7.2.1)-(7.2.4)), SP (Eq. (7.2.6)-(7.2.14)) and bounds (7.2.15)-(7.2.16) are applied to several numerical examples for the natural gas market. Before presenting numerical results, the KKT conditions for the MP and SP are derived. These KKT conditions are used in a complementarity model alternative for the optimization-based approach in this section. Using alternative formulations allows for trying different solvers and investigating the differences in calculation times and other convergence characteristics.

7.3 Complementarity variant of Benders for optimization

Optimization problems can be cast as a complementarity problem (e.g., (Nash and Sofer, 1996) or Chapter 2). Under some common assumptions, such as convexity of objective functions and of the feasible region, the equivalence of the solutions often can be proved (see Chapter 2). For solving optimization problems and complementarity problems, different methods are needed, with different solution times dependent on the model structure and data characteristics. Since reducing calculation times is one of the research objectives, it is worthwhile to investigate solution times of various modeling and solution approaches.

7.3.1 KKT conditions for the MP of the optimization problem: MP-LCP

Deriving the KKT conditions for the linear optimization problem MP-LP results in six complementarity conditions that form the MP-LCP.[176] Eq. (7.3.1) represents the stationarity condition for arc capacity expansions; Eq. (7.3.2) is the limit to arc capacity expansions. Similarly Eq. (7.3.3) is stationarity for storage capacity expansions and (7.3.4) restricts the storage expansions. Eq. (7.3.5) provides the Benders cuts. The last condition, Eq. (7.3.6), results from the variable α.

$$0 \le \Delta_{am}^{A} \perp \quad p_m \gamma_m c_{am}^{\Delta A} + \rho_{am}^{A} - \sum_{it} \theta^{it} \sum_{m' \in succ(m)} p_{m'} \gamma_{m'} \sum_{d} \lambda_{adm'}^{A,it} \ge 0 \qquad \forall a, m \quad \textbf{(7.3.1)}$$

[176] For the notation used see Section 7.8.1

$$0 \le \rho_{am}^{A} \perp \quad \overline{\Delta}_{am}^{A} - \Delta_{am}^{A} \ge 0 \qquad \forall a,m \quad \textbf{(7.3.2)}$$

$$0 \le \Delta_{sm}^{S} \perp \quad p_m \gamma_m c_{sm}^{\Delta S} + \rho_{sm}^{S} - \sum_{it} \theta^{it} \sum_{m' \in succ(m)} p_{m'} \gamma_{m'} \lambda_{sm'}^{S,it} \ge 0 \qquad \forall s,m \quad \textbf{(7.3.3)}$$

$$0 \le \rho_{sm}^{S} \perp \quad \overline{\Delta}_{sm}^{S} - \Delta_{sm}^{S} \ge 0 \qquad \forall s,m \quad \textbf{(7.3.4)}$$

$$0 \le \theta^{it} \perp \quad \begin{array}{l} \alpha \; + \sum_{m} p_m \gamma_m \left(\sum_{a,d} \lambda_{adm}^{A,it} \sum_{m' \in succ(m)} \Delta_{am}^{A} + \sum_{s} \lambda_{sm}^{S,it} \sum_{m' \in succ(m)} \Delta_{sm}^{S} \right) - \\ Z_{it}^{SP} - \sum_{m} p_m \gamma_m \left(\sum_{a,d} \lambda_{adm}^{A,it} \sum_{m' \in succ(m)} \Delta_{am}^{A,it} + \sum_{s} \lambda_{sm}^{S,it} \sum_{m' \in succ(m)} \Delta_{sm}^{S,it} \right) \\ \ge 0 \end{array} \qquad \forall it \quad \textbf{(7.3.5)}$$

$$\alpha \; free, \quad \sum_{it} \theta^{it} - 1 = 0 \qquad \textbf{(7.3.6)}$$

The next sub section presents the KKT conditions for the subproblems.

7.3.2 KKT conditions for the SP of the optimization problem: SP-MCP

For clarity and consistency with previous chapters, the KKT conditions of the SP are presented grouped by player.[177]

7.3.2.1 KKT conditions for the producer problem

The SP-MCP contains two complementarity conditions for each producer. Eq. (7.3.7) is the stationarity condition for production by producer *p* and Eq. (7.3.8) provides the upper bound to the daily production rate.[178]

$$0 \le Q_{pd}^{P \to T} \perp \quad d_d \left(-\pi_{n(p)d}^{P} + \frac{\partial c_{pm}^{P}(Q_{pd}^{P \to T})}{\partial Q_{pd}^{P \to T}} \right) + \lambda_{pd}^{P} \ge 0 \qquad \forall d \quad \textbf{(7.3.7)}$$

$$0 \le \lambda_{pd}^{P} \perp \quad \overline{CAP}_{pm}^{P} - Q_{pd}^{P} \ge 0 \qquad \forall d \quad \textbf{(7.3.8)}$$

7.3.2.2 KKT conditions for the trader problem

For each trader *t* there are five stationarity conditions and two mass-balance conditions. Eq. (7.3.9) shows the stationarity condition for purchases from producers. Eq. (7.3.10)

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[177] One interesting fact to point out is that the KKT conditions based on the social welfare maximization approach are identical to the KKT conditions derived from a equilibrium modeling approach where different players are modeled as separate profit-maximizing entities.

[178] Note that the value for λ_{pd}^{P} in (7.3.7) is implicitly scaled with the number of days in the season d_d, but not with the probability or discount rate of the scenario.

and (7.3.11) are stationarity for storage injections and extractions respectively. Eq. (7.3.12) is stationarity of arc flows, where arc a starts at node n^{-} and ends in node n^{+}. Eq. (7.3.13) is stationarity of sales to end-users. In here, in optimization problems all Cournot coefficients $\delta_{tnm}^{T}=0$, except when a monopoly is modeled. In that case, there is only one trader and the value for all $\delta_{tn}^{T}=1$. Lastly, Eq. (7.3.14) provides mass balance by node and Eq. (7.3.15) is the storage cycle constraint.

$$0 \le Q_{tnd}^{T\leftarrow P} \perp \quad d_d \pi_{nd}^{P} - \phi_{tnd}^{T} \ge 0 \qquad \forall n,d \qquad \textbf{(7.3.9)}$$

$$0 \le I_{tnd}^{T} \perp \quad d_d\left(c_{nd}^{S} + \tau_{nd}^{S}\right) + \phi_{tnd}^{T} - \left(1 - l_{sn}\right) d_d \phi_{tn}^{S} \ge 0 \qquad \forall n,d \qquad \textbf{(7.3.10)}$$

$$0 \le X_{tnd}^{T} \perp \quad d_d \phi_{tn}^{S} - \phi_{tnd}^{T} \ge 0 \qquad \forall n,d \qquad \textbf{(7.3.11)}$$

$$0 \le F_{ad}^{T} \perp \quad d_d\left(c_{ad}^{A} + \tau_{ad}^{A}\right) + \phi_{tn^{-}(a)d}^{T} - (1 - l_a)\phi_{tn^{+}(a)d}^{T} \ge 0 \qquad \forall a,d \qquad \textbf{(7.3.12)}$$

$$0 \le Q_{tnd}^{T\rightarrow W} \perp \quad \phi_{tnd}^{T} - d_d\left(INT_{ndm}^{W} - SLP_{ndm}^{W}\left(\sum_{t'\in T} Q_{t'nd}^{T\rightarrow W} + \delta_{tn}^{T} Q_{tnd}^{T\rightarrow W}\right)\right) \ge 0 \qquad \forall n,d \qquad \textbf{(7.3.13)}$$

$$\varphi_{tnd}^{T} \;\; free \quad \begin{pmatrix} Q_{tnd}^{T\leftarrow P} + \sum_{a\in a^{+}(n)} \left(1 - l_a\right) F_{tad}^{T} + X_{tnd}^{T} \\ -Q_{tnd}^{T\rightarrow W} - \sum_{a\in a^{-}(n)} F_{tad}^{T} - I_{tnd}^{T} \end{pmatrix} \ge 0 \qquad \forall n,d \qquad \textbf{(7.3.14)}$$

$$\varphi_{ts}^{S} \;\; free \quad \left(1 - l_{sn}\right) \sum_{d} d_d I_{tsd}^{T} - \sum_{d} d_d X_{tsd}^{T} = 0 \qquad \forall s \qquad \textbf{(7.3.15)}$$

7.3.2.3 KKT conditions for the TSO

For the TSO Eq. (7.3.16) denotes the stationarity of arc capacities assigned and (7.3.17) the capacity limitation resulting from the initial capacity and the last MP solution:

$$0 \le Q_{ad}^{A} \perp \quad \lambda_{ad}^{A} - d_d \tau_{ad}^{A} \ge 0 \qquad \forall d \qquad \textbf{(7.3.16)}$$

$$0 \le \lambda_{ad}^{A} \perp \quad \overline{CAP}_{am}^{A} + \sum_{m'\in pred(m)} \Delta_{am'}^{A,it} - Q_{ad}^{A} \ge 0 \qquad \forall d \qquad \textbf{(7.3.17)}$$

7.3.2.4 KKT conditions for the storage operator

For the storage operator Eq. (7.3.18) denotes the stationarity of sold injection capacity, and Eq. (7.3.19) the restriction on total injections due to limited working gas capacity, resulting from the initial capacity and the last MP solution:

$$0 \le Q_{sd}^{S\to T} \perp \quad (1-l_{sn}) d_d \lambda_s^S - d_d \tau_{sd}^S \ge 0 \qquad \forall d \qquad \textbf{(7.3.18)}$$

$$0 \le \lambda_{sn}^S \perp \quad \overline{CAP}_s^S + \sum_{m' \in pred(m)} \Delta_{sm'}^{S,it} - (1-l_{sn}) \sum_{d \in D} d_d Q_{sd}^{S\to T} \ge 0 \qquad \textbf{(7.3.19)}$$

7.3.2.5 Market clearing

The market clearing of produced volumes between the producers and the traders is given by Eq. (7.3.20). The market clearing of arc capacities assigned and used between the TSO and the traders is given by (7.3.21). The market clearing for storage injection capacities is given by (7.3.22).[179]

$$\pi_{nd}^P \ \ free \quad Q_{p(n)d}^{P\to T} - \sum_{t \in T(n)} Q_{tnd}^{T\leftarrow P} = 0 \qquad \forall n,d \qquad \textbf{(7.3.20)}$$

$$\tau_{ad}^A \ \ free \quad Q_{adm}^{A\to T} - \sum_t F_{tadm}^T = 0 \qquad \forall a,d \qquad \textbf{(7.3.21)}$$

$$\tau_{sd}^S \ \ free \quad Q_{s(n)dm}^{S\to T} - \sum_{t \in T(n)} I_{tndm}^T = 0 \qquad \forall n,d \qquad \textbf{(7.3.22)}$$

This concludes the presentation of the KKT conditions derived from the master and subproblems for the optimization models. When the MP-LCP (with Benders cuts, Eq. (7.3.5)) and SP-MCP are used instead of the original optimization problems, this will be referred to as the complementarity variant. Note that the complementarity variant is only applied to optimization problems and that the same objective values and bounds (Section 7.2.3) can be calculated as for the original optimization-based problems: MP-LP and SP-NLP.

The following section provides more details about the VI-based decomposition approach.

7.4 VI-based decomposition for stochastic MCP

The foundation for the decomposition approach developed in this chapter comes from (Fuller and Chung, 2005), (Fuller and Chung, 2008) and (Gabriel and Fuller, 2010). Fuller and Chung (2005) develop a Dantzig-Wolfe (DW) approach for VI. Using duality theory (see Chapter 2) (Fuller and Chung, 2008) developed a Benders decomposition

[179] Note: the inclusion of the inverse demand curve as a separate market clearing condition is not necessary. Wherever the wholesale market-clearing price would show up after deriving the KKT conditions, the inverse demand curve can be substituted into its place. (After deriving the KKT conditions, since doing the substitution before deriving the KKT conditions would result in a Cournot Oligopoly formulation.)

(BD) approach for VI. Gabriel and Fuller (2010) extended the latter to be applicable to stochastic MCP. First, some key results of the BD approach in (Gabriel and Fuller, 2010) are summarized.

7.4.1 Benders decomposition for stochastic MCP – summary

To facilitate transparency in the development of the BD for stochastic MCP method, (Gabriel and Fuller, 2010) distinguish several categories of equations and variables. There are four types of variables, and three types of equations, presented in the following two lists. The symbols in parentheses are the notation used in the following paragraphs.

The four types of variables include:

(a.) the complicating, first-stage variables (Δ),

(b.) second-stage variables with a non-constant gradient (q),

(c.) second-stage variables that are free in sign, with a constant gradient (y),

(d.) second-stage variables that are nonnegative, with a constant gradient (f).

The three types of equations include:

(i.) equations that apply to first-stage variables only,

(ii.) inequality conditions applying to second and possibly first-stage variables,

(iii.) equality conditions applying to second and possibly first-stage variables.

Eq. (7.4.1)-(7.4.3) show the equations and variables, as well as the coefficient matrices and the dual variable vectors that are used in the following sections.

$$A\Delta \qquad\qquad\qquad\qquad \geq b \qquad (\rho \geq 0) \qquad \textbf{(7.4.1)}$$

$$\bar{A}\Delta \quad +\bar{B}q \quad +\bar{C}y \quad +\bar{D}f \quad \geq \bar{b} \qquad (\lambda \geq 0) \qquad \textbf{(7.4.2)}$$

$$\hat{A}\Delta \quad +\hat{B}q \quad +\hat{C}y \quad +\hat{D}f \quad = \hat{b} \qquad (\varphi \ free) \qquad \textbf{(7.4.3)}$$

In DW, the SP solutions are stored in a matrix: X_{SP}^{it}. To enhance transparency Gabriel and Fuller (2010) split X_{SP}^{it} in three parts to distinguish the solution values for different variable types: $X_{SP}^{it} = \begin{bmatrix} \lambda_{SP}^{1} & \lambda_{SP}^{2} & \dots & \lambda_{SP}^{it} \\ \varphi_{SP}^{1} & \varphi_{SP}^{2} & \dots & \varphi_{SP}^{it} \\ k_{SP}^{1} & k_{SP}^{2} & \dots & k_{SP}^{it} \end{bmatrix} = \begin{bmatrix} X_{\lambda,SP}^{it} \\ X_{\varphi,SP}^{it} \\ X_{k,SP}^{it} \end{bmatrix}$.

Here, the vectors k_{SP}^{it} contain the values of a new vector introduced in (Fuller and Chung, 2005). This vector is introduced to represent $F(q)$, the gradient for variables where it is non-constant. It facilitates the derivation of the MP and SP for the VI variant, but is eventually substituted out of the formulation before implementing the method. By using the results from the previous papers, introduction of the vector can be skipped, and below $F(q)$ is used. Write the SP solutions matrix: $X_{SP}^{it} = \begin{bmatrix} X_{\lambda}^{it} \\ X_{\varphi}^{it} \\ X_{F(q)}^{it} \end{bmatrix}$.

Introduce θ as the multiplier to the Benders cuts (7.4.6) and α as the dual to the convexity constraint (7.4.7).[180] Then the KKT conditions of the MP derived in (Gabriel and Fuller, 2010) form the following system. Eq. (7.4.4) is the stationarity condition for the first-stage variables. Eq. (7.4.5) provides the bounds to the first-stage variables. The Benders cuts are included by Eq. (7.4.6). Lastly (7.4.7) provides the convexity constraint. Here, e is the unit vector with length *it*.

$$0 \le \Delta \perp \quad d - A^T \rho - \bar{A}^T X_{\lambda}^{it} \theta - \hat{A}^T X_{\varphi}^{it} \theta \ge 0 \tag{7.4.4}$$

$$0 \le \rho \perp \quad A\Delta - b \ge 0 \tag{7.4.5}$$

$$0 \le \theta \perp \quad \left(X_{\lambda}^{it}\right)^T \left(\bar{A}\Delta - \bar{b}\right) + \left(X_{\varphi}^{it}\right)^T \left(\hat{A}\Delta - \hat{b}\right) + \left(X_{F(s)}^{it}\right)^T F^{-1}\left(X_{F(s)}^{it}\theta\right) + \alpha \ge 0 \tag{7.4.6}$$

$$0 \le \alpha \perp \quad \theta^T e - 1 = 0 \tag{7.4.7}$$

Details for the coefficient matrices used above and a mapping of the equations and variables in this chapter on the ones used in (Gabriel and Fuller, 2010) are presented in Appendix 7.9.

[180] Initially it may seem confusing to use variable α here again, however it turns out to have the same role as in the previous formulations when applying the method for solving optimization models cast as complementarity problems.

7.4.2 Benders decomposition for stochastic MCP – formulation

The former section showed the MP formulation. Gabriel and Fuller did provide the dual VI formulation for the SP, but did not present the KKT conditions. For the model presented in this chapter the KKT conditions for the SP-VI are identical to the system SP-MCP derived in Section 7.3.2.

Next, the derivations are made following Gabriel and Fuller to obtain the MP-VI. Conditions (7.4.4) result in the stationarity conditions (7.3.1) and (7.3.3) that were derived previously for the MP-LCP (see Section 7.3.1). Conditions (7.4.5) give the capacity restrictions Eq. (7.3.2) and (7.3.4) in the MP-LCP. The third condition (7.4.7) is the convexity constraint Eq. (7.3.6). Only when deriving the Benders cuts (7.4.6) there is a distinction with the cuts in MP-LCP given by Eq. (7.3.5), see the following section.

7.4.3 VI-based cuts

The cuts given by Eq. (7.3.5) in the MP-LCP (Section 7.3.1) contain variable: α, the aggregate SP objective: Z_{it}^{SP} and a with the capacity expansions weighted aggregate of the capacity shadow prices. The MP-VI cuts given by Eq. (7.4.6) (Section 7.4.1) contain similar elements, but differ in some major details. Variable α is present, and so are the shadow prices for added capacities. However in the MP-VI cuts, the aggregate SP objective is replaced by terms representing the objective function values of model agents that have an objective with a non-linear gradient: in this model the trader and the producer only (see group (b.) in Section 7.4.1), and shadow prices for capacities are also multiplied by the already existing capacities, not only by the additions.

Concretely, the VI-based cuts are as follows: $\forall it$

$$
0 \le \theta^{it} \perp \quad \alpha + \sum_m p_m \gamma_m \left(\begin{array}{l} \sum_{p,d} \lambda_{pdm}^{P,it} \left(\overline{CAP}_{pm}^{P} \right) \\ + \sum_{a,d} \lambda_{adm}^{A,it} \left(\overline{CAP}_{am}^{A} + \sum_{m' \in pred(m)} \Delta_{am}^{A} \right) \\ + \sum_{s} \lambda_{sm}^{S,it} \left(\overline{CAP}_{nm}^{S} + \sum_{m' \in pred(m)} \Delta_{sm}^{S} \right) \\ + \sum_{t,n,d,it'} d_d \left(INT_{ndm}^{W} - SLP_{ndm}^{W} \left(\begin{array}{l} \sum_{t' \in T} Q_{t'nd}^{T \to W,it} \\ + \delta_{tnm}^{T} Q_{tnd}^{T \to W,it} \end{array} \right) \right) \theta^{it'} Q_{t,n,d,m}^{T \to W,it'} \\ + \sum_{p,d,it'} d_d \left(\frac{\partial c_{pm}^{P}(Q_{pdm}^{P \to T,it})}{Q_{pdm}^{P \to T,it}} \right) \theta^{it'} Q_{pdm}^{P \to T,it'} \end{array} \right) \ge 0 \qquad \textbf{(7.4.8)}
$$

The former sections have discussed that the SP-VI and SP-MCP are identical, and between MP-VI and MP-LCP only the optimality cuts differ. Table 28 illustrates how the MP-LCP and SP-MCP (derived following the generalized Benders approach and taking KKT conditions) and the MP-VI and SP-VI (derived following (Gabriel and Fuller, 2010)) compare.

Table 28: Comparing problem parts for Generalized Benders vs. (Gabriel and Fuller, 2010)

Problem part	Geoffrion and KKT	Gabriel and Fuller
MP stationarity and feasibility	MP-LCP (Section 7.3.1)	MP-VI (Section 7.3.1)
Master problem optimality cuts	Eq. (7.3.5)	Eq. (7.4.8)
Subproblem	SP-MCP (Section 7.3.2)	SP-VI (Section 7.3.2)

In the above sections the master and subproblems of the VI-based decomposition method have been given. Since the decomposition method is an iterative method, a stopping criterion is needed. The stopping criterion developed in (Fuller and Chung, 2005) is a convergence gap, which is presented in the following subsection.

7.4.4 Convergence gap

Fuller and Chung (2005) defined the following convergence gap:

$$
CG^{it} = \left[H_2\left(x_{2,MP}^{it}\right) - \nabla q_2^{\ T}\left(x_{2,MP}^{it}\right)\gamma^{it} \right]^T \left(x_{2,SP}^{it+1} - x_{2,MP}^{it} \right).
$$

They showed that $CG^{it} \le 0$ and that when values $CG^{it} < 0$ (strictly), the addition of the last SP solution to the SP-solution matrix X_{SP}^{it} still enlarges the feasible region of the MP. And as long as the feasible region for the MP is enlarged by adding the last SP-solution, the MP may find another solution compared to the former iteration, and consequently the next SP-solution may also alter. Fuller and Chung (2005) proved that under mild assumptions for $CG^{it} = 0$ a solution to the problem is found.

For the model presented in this chapter, the following describes the convergence gap:

$$CG^{it} = \sum_m p_m \gamma_m \left(\begin{array}{l} \sum_{n,d} \left(\begin{array}{l} \left(INT_{ndm}^{W} - SLP_{ndm}^{W} \left(\sum_{t' \in T} Q_{t'ndm,SP}^{T \to W,it} + \delta_{tnm}^{T} Q_{tndm,SP}^{T \to W,it} \right) \right) \\ - \left(INT_{ndm}^{W} - SLP_{ndm}^{W} \left(\sum_{t' \in T} Q_{t'ndm,MP}^{T \to W,it} + \delta_{tnm}^{T} Q_{tndm,MP}^{T \to W,it} \right) \right) \end{array} \right) Q_{tndm,MP}^{T \to W,it} \\ + \sum_{p,d} \left(\frac{\partial c_{pm}^{P}(Q_{pdm,SP}^{P \to T,it})}{\partial Q_{pdm,SP}^{P \to T,it}} - \frac{\partial c_{pm}^{P}(Q_{pdm,MP}^{P \to T,it})}{\partial Q_{pdm,MP}^{P \to T,it}} \right) Q_{pdm,MP}^{P \to T,it} \\ + \sum_{p,d} \left(\lambda_{ndm,SP}^{P,it} - \lambda_{pdm,MP}^{P,it} \right) \left(\overline{CAP}_{pm}^{P} \right) \\ + \sum_{a,d} \left(\lambda_{adm,SP}^{A,it} - \lambda_{ndm,MP}^{A,it} \right) \left(\overline{CAP}_{nm}^{A} + \sum_{m' \in pred(m)} \Delta_{nm'}^{A,it} \right) \\ + \sum_{n} \left(\lambda_{nm,SP}^{S,it} - \lambda_{nm,MP}^{S,it} \right) \left(\overline{CAP}_{nm}^{S} + \sum_{m' \in pred(m)} \Delta_{nm'}^{S,it} \right) \end{array} \right)$$

Here, the production cost curve proposed by (Golombek et al., 1995) is used: $c_{pm}^{P}\left(Q_{pdm}^{P \to T}\right) = \left(c_{p,d,m}^{P,l} - c_{p,d,m}^{P,g}\right) Q_{pdm}^{P \to T} + c_{p,d,m}^{P,q}\left(Q_{pdm}^{P \to T}\right)^2 + c_{p,d,m}^{P,g}\left(\overline{CAP}_{p,m}^{P} - Q_{pdm}^{P \to T}\right) \ln\left(1 - \frac{Q_{pdm}^{P \to T}}{\overline{CAP}_{p,m}^{P}}\right)$, with $c_{p,d,m}^{P,l} > 0$ the minimum per unit costs, $c_{p,d,m}^{P,q} \ge 0$ the per unit linearly increasing cost term and $c_{p,d,m}^{P,g} \le 0$ a term that induces high marginal costs when production is close to full capacity $\overline{CAP}_{p,m}^{P}$. The marginal supply cost curve for this expression is:

$$c_{pm}^{P}{}'\left(Q_{pdm}^{P \to T}\right) = c_{p,d,m}^{P,l} + c_{p,d,m}^{P,q} Q_{pdm}^{P \to T} + c_{p,d,m}^{P,g} \ln\left(1 - \frac{Q_{pdm}^{P \to T}}{\overline{CAP}_{p,m}^{P}}\right)$$

Since several terms cancel out, the convergence gap can be simplified to the following:

$$CG^{it} = \sum_m p_m \gamma_m \left(\begin{array}{l} -\sum_{n,d} S_{ndm} \cdot \left(\begin{array}{l} \left(\sum_{t' \in T} Q^{T \to W,it}_{t'ndm,SP} + \delta^T_{tnm} Q^{T \to W,it}_{tndm,SP} \right) \\ -\left(\sum_{t' \in T} Q^{T \to W,it}_{t'ndm,MP} + \delta^T_{tnm} Q^{T \to W,it}_{tndm,MP} \right) \end{array} \right) Q^{T \to W,it}_{ndm,MP} \\ +\sum_{p,d} \left(\begin{array}{l} 2.c^{P,q}_{pdm} \cdot \left(Q^{P \to T,it}_{pdm,SP} - Q^{P \to T,it}_{pdm,MP} \right) \\ +c^{P,g}_{pdm} . \left(\ln\left(1\text{-}\frac{Q^{P \to T,it}_{pdm,SP}}{\overline{CAP}^P_{pm}} \right) \text{-} \ln\left(1\text{-}\frac{Q^{P \to T,it}_{pdm,MP}}{\overline{CAP}^P_{pm}} \right) \right) \end{array} \right) Q^{P \to T,it}_{pdm,MP} \\ +\sum_{p,d} \left(\lambda^{P,it}_{ndm,SP} - \lambda^{P,it}_{pdm,MP} \right) \left(\overline{CAP}^P_{pm} \right) \\ +\sum_{a,d} \left(\lambda^{A,it}_{adm,SP} - \lambda^{A,it}_{ndm,MP} \right) \left(\overline{CAP}^A_{nm} + \sum_{m' \in pred(m)} \Delta^{A,it}_{nm'} \right) \\ +\sum_{n} \left(\lambda^{S,it}_{nmSP} - \lambda^{S,it}_{nm,MP} \right) \left(\overline{CAP}^S_{nm} + \sum_{m' \in pred(m)} \Delta^{S,it}_{nm'} \right) \end{array} \right) \quad \textbf{(7.4.9)}$$

For this convergence gap to provide reliable information, the function F in the problem *VI(K,F)* must be strictly monotone (Fuller and Chung, 2005). For optimization problems this function F consists of the gradients of the subproblem objective functions. Unfortunately, in the numerical experiments the function *F* was not always strictly monotonic and this turned out to be a problem. Hence, alternative convergence criteria had to be used.

7.4.5 Convergence criteria

Three convergence criteria have been implemented. The first two criteria applied to optimization problems only. The first criterion was a threshold (10^{-4}) for the absolute difference between the lower bound and best upper bound lower than a specific threshold (AbsTol), see Eq. (7.2.15) and (7.2.16). The second criterion was a threshold (10^{-9}) for the relative difference between these lower and best upper bounds (RelTol). The third criterion, used for both optimization and hybrid market structure problems, was that the largest capacity expansion weighted with the square root of probability and discount rate

$\sqrt{p_m \gamma_m} \left| \Delta^{it} - \Delta^{it-1} \right|$ was lower than a threshold (10^{-2}) (Expans).[181] Once any of the criteria applicable for a model type was met, the decomposition run would be aborted.[182]

In the following section the various combinations of master and subproblems are implemented and compared to investigate what combination is the most promising regarding convergence speed and solution time.

The analysis shows, that the among the decomposition approaches, the MP-VI approaches need significantly fewer iterations and less time than the MP-LP and MP-LCP approaches. For the numerical cases solved the extensive-form solutions methods need less calculation than decomposition approaches.[183] However, the calculation times needed by decomposition approaches grow much less when increasing the size of the problems than calculation times of the full-scale extensive-form solution approaches. This is an indication that the decomposition approaches perform better on larger problems in terms of calculation time needed to solve the problems.

7.5 Decomposition approaches – numerical results

As previously discussed there are six ways of combining master and subproblems that can be applied to solve stochastic optimization models and MCP. Table 29 shows all combinations and the abbreviations that will be used to refer to them.

Table 29: Decomposition solution approaches

SP / **MP**	**SP-NLP**	**SP-MCP**
Optimization: MP-LP	LP-NLP	LP-MCP
Optimization: MP-LCP	LCP-NLP	LCP-MCP
Complementarity: MP-VI	VI-NLP	VI-MCP

[181] The reason for taking the square root is that when there are many scenarios or periods in the far future, the weight $p_m \gamma_m$ would become very small, potentially allowing for large deviations in future expansions relative to optimal solutions.
[182] The threshold values have been tuned during the numerical experiments to provide useful information regarding the convergence of the algorithms.
[183] One exception for which VI-MCP decomposition was faster but that is not presented in the subsection, is for a stochastic model with four identical scenarios.

All combinations are applied to several data sets and for varying market structures, but only VI-MCP can be used to solve problems where more than one player exercises market power.

The steps in the decomposition algorithms can be summarized as follows:

7.5.1 Algorithm steps

```
Read data set
Read scenario tree definition and probabilities
Define MP model
Define SP model
Iteration counter = 1
Loop:
    If (Iteration > 1)
        Solve MP
    Else
        Set all MP expansion variables equal to zero.
    Loop (scenario tree nodes m):
        Set parameters for SP(m)
        Solve SP(m)
    End Loop
    If (Iteration > 1)
        Calculate convergence metrics
    If (a convergence criterion is met)
        STOP
    Else
        Iteration←Iteration+1
End Loop
```

Next, some remarks relevant for the implementation in GAMS are given.

7.5.2 Implementation remarks

The solvers used are XPRESS for LP problems, PATH for MCP problems and CONOPT and MINOS for NLP.[184] In some numerical cases infeasibilities were encountered for theoretically feasible problems. Also, some runs resulted in deviating converged solutions for decomposed problems and even for full-scale optimization, accompanied by GAMS warning messages. Thus, algorithm termination did not always indicate that an actual solution was found to the problem and further checks had to be made. Appendix 7.11 discusses tricks and workarounds that have been used to solve some of these problems.

7.5.3 Stochastic non-linear optimization problems – results

The problems solved in this section are all small enough to be solved in extensive-form in a relatively short amount of time. Applying decomposition approaches to such modestly-sized problems will generally increase the needed calculation times. However, analyzing the convergence characteristics should help to provide insights into what decomposition approach is best suited for solving large-scale problems, in terms of convergence speed and solution accuracy.

For perfectly competitive and monopolistic markets, all variants of master problem and subproblem approaches have been applied. Calculation times, numbers of iterations needed and convergence speed using various combinations of master problems and subproblems are compared.

7.5.3.1 Small-scale problems

The convergence analysis presented in this subsection illustrates what many numerical experiments in this research have shown regarding varying solution paths leading to different convergence characteristics and small differences in converged results. For small-scale optimization problems the decomposition approaches need more time to solve them then solving the original larger problems in one piece. Among the decomposition

[184] CONOPT: CONOPT3. MINOS was only available in the final stage of the dissertation. The obtained performance improvement with MINOS for the NLP problems is large. According to GAMS documentation, MINOS is more suitable for NLP with linear constraints and CONOPT for NLP with non-linear constraints. Indeed, the problems in this chapter only have linear constraints.

approaches, the VI-based approaches need fewer iterations and less time than the LP and LCP approaches.

7.5.3.1.1 Convergence analysis for a seven-node problem

The first numerical comparisons are for a small data set, see Figure 56. This data set contains seven nodes, of which five are country nodes; one is a liquefaction node and one a regasification node. Three countries have production (NO, NL and FR); three have consumption (BE, DE and FR) and two have storage facilities (FR and DE). There are eight arcs, of which five represent pipelines and three a liquefaction shipping route.

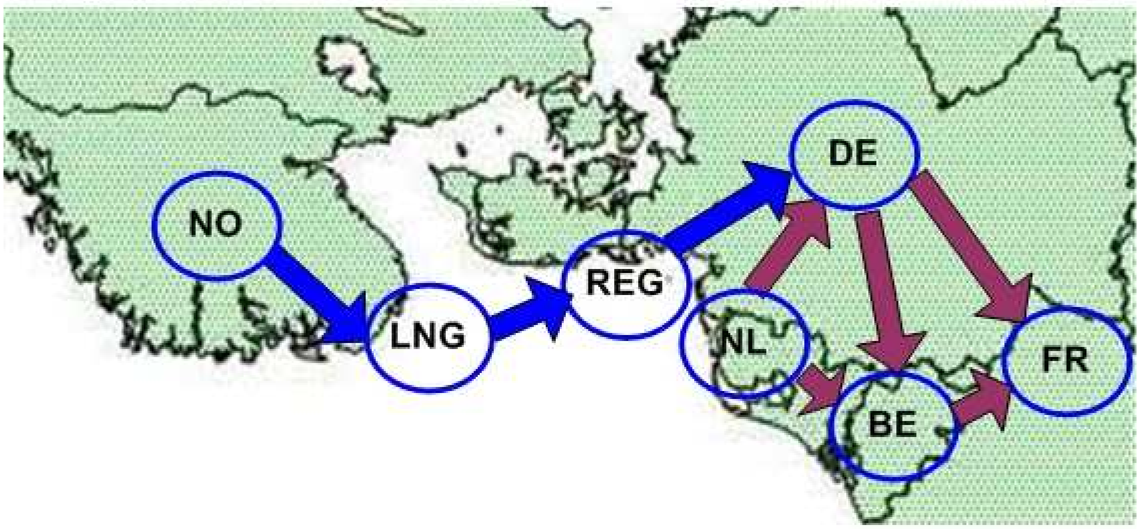

Figure 56: Nodes and arcs in small-scale problem

There are two or four model periods and two demand seasons in each period. The stochastic scenario tree splits after the first and after the second model period, resulting in 1+2=3 scenario tree nodes for the two period problems and in 1+2+4+4=11 for the four-period problems.[185] Table 30 presents the sizes of these problems.

[185] Compare with Figure 53 in the beginning of this chapter, the only difference is that in the numerical examples there are four scenarios instead of eight in the tree.

Table 30: Problem sizes in number of variables

Problem	Method	Total number of variables	Total number of expansion variables
Two-period monopoly	MCP	390	9
Two-period monopoly	OPT	190	9
Two-period perfect competition	MCP	493	9
Two-period perfect competition	OPT	260	9
Four-period monopoly	MCP	1506	63
Four-period monopoly	OPT	740	63
Four-period perfect competition	MCP	1905	63
Four-period perfect competition	OPT	1018	63

Note that the monopoly problems contain fewer variables than the perfectly competitive variants as all separate traders are replaced by one monopoly trader.

The following four tables present metrics for evaluating algorithm convergence: number of iterations needed (Num Iter), and seconds of calculation time used (Calc Time). Also output values are presented. From left to right: probability-weighted aggregate capacity expansions (mcm/d) for pipelines, liquefiers, regasifiers and storage working gas, investment costs, trader and producer profits and consumer surplus (k$).

Table 31 shows that the VI-based approaches need fewer iterations and less time to solve the small-scale two-period monopolistic problem than the other decomposition approaches. Even for this small problem some of the aggregate converged results deviate in the second digit. The VI-based decomposition methods give more precise results.[186]

[186] There are limits to the precision with which algorithms (solvers) can calculate solutions. Many solution methods go through repetitive calculation steps (iterations) and termination criteria for these steps usually allow for some minor deviations. When these deviations are smaller than a preset tolerance, they are acceptable. Usually, these tolerances are very small, e.g. 10^{-6} or 10^{-8}, and the deviations should be small enough so that for practical purposes the solution can be considered optimal. However, sometimes these small deviations do have an impact, as the results in this subsection clearly show.

Table 31: Convergence analysis for small two-period monopoly

Method	Convergence	Num Iter	Calc Time	Pipe-lines	Liquefier	Regasifier	Storage	Inv Cost	Trad Profit	Prod Profit	Cons Surp
Full MCP			<0.1	0.0	2.18	0.66	2.00	3.84	121.53	32.65	49.05
Full OPT^			<0.1	0.0	2.18	0.66	2.00	3.84	121.53	32.65	49.05
VI-MCP	Expans	10	2.9	0.0	2.18	0.66	2.00	3.84	121.53	32.65	49.05
VI-NLP^	Expans	10	1.6	0.0	2.18	0.66	2.00	3.84	121.53	32.65	49.05
LCP-MCP	AbsTol	18	5.8	0.0	2.18	0.65	2.00	3.84	121.53	32.65	49.04
LCP-NLP^	AbsTol	18	4.0	0.0	2.18	0.65	2.00	3.84	121.53	32.65	49.04
LP-NLP^	AbsTol	18	3.7	0.0	2.18	0.65	2.00	3.84	121.53	32.65	49.04
LP-MCP	AbsTol	18	6.0	0.0	2.18	0.65	2.00	3.84	121.53	32.65	49.04

^Calculation times for NLP in this and the following three tables are obtained using CONOPT3

Table 32: Convergence analysis for small four-period monopoly

Method	Convergence	Num Iter	Calc Time	Pipe-lines	Liquefier	Regasifier	Storage	Inv Cost	Trad Profit	Prod Profit	Cons Surp
Full MCP			0.3	1.80	3.82	2.29	3.12	9.45	268.00	74.45	118.46
Full OPT^			1.2	1.80	3.82	2.29	3.12	9.45	268.00	74.45	118.46
VI-MCP	Expans	36	28.2	1.81	3.82	2.29	3.13	9.44	268.00	74.45	118.46
VI-NLP^	Expans	36	32.9	1.81	3.82	2.29	3.13	9.44	268.00	74.45	118.46
LCP-MCP	Expans	205	194.8	1.81	3.82	2.28	3.12	9.45	268.00	74.46	118.47
LCP-NLP^	AbsTol	214	187.4	1.80	3.83	2.29	3.12	9.45	268.01	74.45	118.47
LP-NLP^	Expans	212	194.1	1.80	3.82	2.28	3.13	9.44	268.00	74.45	118.45
LP-MCP	Expans	213	206.9	1.81	3.83	2.29	3.11	9.45	268.00	74.46	118.46

Table 33: Convergence analysis for small two-period perfectly competitive problem

Method	Convergence	Num Iter	Calc Time	Pipe-lines	Liquefier	Regasifier	Storage	Inv Cost	Trad Profit	Prod Profit	Cons Surp
Full MCP			0.2	4.53	3.00	1.47	3.00	15.02	63.73	75.27	97.75
Full OPT			<0.1	4.53	3.00	1.47	3.00	15.02	63.73	75.27	97.75
VI-NLP^	Expans	8	1.7	4.53	3.00	1.47	3.00	15.02	63.73	75.27	97.75
VI-MCP	Expans	8	2.5	4.53	3.00	1.47	3.00	15.02	63.73	75.27	97.75
LCP-MCP	AbsTol	15	4.5	4.52	3.00	1.47	3.00	15.01	63.75	75.26	97.73
LCP-NLP^	AbsTol	15	3.9	4.52	3.00	1.47	3.00	15.01	63.74	75.26	97.73
LP-MCP	AbsTol	15	5.2	4.52	3.00	1.47	3.00	15.01	63.75	75.26	97.73
LP-NLP^	AbsTol	15	4.3	4.52	3.00	1.47	3.00	15.01	63.74	75.26	97.73

Table 34: Convergence analysis for small four-period perfectly competitive problem

Method	Convergence	Num Iter	Calc Time	Pipe-lines	Liquefier	Regasifier	Storage	Inv Cost	Trad Profit	Prod Profit	Cons Surp
Full MCP			0.4	8.76	7.76	6.20	7.02	-26.51	108.65	174.25	250.54
Full OPT^			0.2	8.76	7.76	6.20	7.02	-26.51	108.65	174.25	250.54
VI-MCP	Expans	55	41.7	8.76	7.77	6.20	7.02	-26.51	108.66	174.24	250.52
VI-NLP^	Expans	55	58.9	8.76	7.77	6.20	7.02	-26.51	108.66	174.25	250.53
LCP-NLP^	Expans	350	336.2	8.76	7.77	6.20	7.03	-26.51	108.63	174.25	250.55
LCP-MCP	Expans	349	308.3	8.77	7.76	6.20	7.03	-26.52	108.62	174.26	250.56
LP-NLP^	Expans	323	247.1	8.76	7.76	6.20	7.02	-26.51	108.62	174.26	250.56
LP-MCP	Expans	340	303.4	8.77	7.77	6.20	7.03	-26.52	108.61	174.26	250.57

Note that the solution time for a monopoly problem is generally shorter than for the perfectly competitive variants since it contains many fewer variables as all separate traders are replaced by one monopoly trader. Column ‘Calc Time’ shows that the decomposition approaches need significantly more time to solve these small problems than solving the original full-scale models in extensive-form.[187] Compared to solving the whole problem at once, which takes up to 1.2 seconds only, calculation times are sometimes several hundred times as long. The added overhead (e.g., file management) and processing steps of the decomposition approaches need relatively much time for these smaller problems.

Among the decomposition approaches, the VI-based approaches need significantly fewer iterations and less time than the LP and LCP approaches. For instance to solve the four-period monopoly the optimization-based approaches need about six times as many iterations (205-214 vs. 36) and also about six times as much time (187-207 seconds vs. 28-33). The LP and LCP approaches do not differ much among each other in number of iterations and calculation time needed to solve the problems. Deviations in the final solutions, in terms of aggregate expansions and welfare measures are small, only visible in the second digit. One might expect that the same sequence of master and subproblems is solved, what would result in the same solution path: the same optimal expansions, the

[187] Calculation times are the GAMS model attribute ‘resusd’. This is the CPU time used by solvers. Another time measure is the TimeElapsed, which represents the total duration of the GAMS run. TimeElapsed and Resusd are affected by the number of processes running on the computer. It is possible to run several models at the same time. In such cases the processes are competing for potentially scarce processing time, and the time measures may vary up to 20%.

same optimal quantities and prices, and the same shadow prices in all iterations. However this is not true, as the data in columns 'Num Iter' show. For the two VI approaches it is true for these four problems, but not in general as other numerical experiments have shown. Among LP and LCP approaches the number of iterations all differ somewhat for both four-period problems, deviations depend on both the master problem and the subproblem types. The explanation is that due to the possibility of multiple optimal solutions in some master problems (see the upcoming discussion and Table 35 and Table 36 below) and small numerical deviations due to solver tolerances, small differences in intermediate solutions occur that result in varying solution paths and different numbers of iterations needed to converge and deviations in the final solutions.

To illustrate these effects, some numerical data are presented for the coefficient values in the Benders cuts resulting from the SP and optimal capacity expansions as calculated by the MP.

7.5.3.1.2 Non-unique dual prices and solution paths in a five-node problem

To limit the amount of data presented, the illustration will use data from a smaller problem than before, which relative the former problems leaves out the two nodes BE and FR (see Figure 56). Table 35 below presents the discounted probability-weighted shadow prices of the infrastructure expansions: $p_m \gamma_m \lambda_{adm}^{A,it}$ and $p_m \gamma_m \lambda_{sm}^{S,it}$, the coefficients in the Benders cuts, Eq. (7.2.4).

In this deterministic two-period problem there is a pipeline from NED to GER, and an LNG shipping route from NOR to GER (see Figure 57), consisting of a liquefaction arc (NNOR_LNG), a shipping arc (assumed to have infinite capacity), and a regasification arc (RGER_GER). The expansion decisions are made in period 01, and the results are discussed for period 02, after the expansions are put in place.

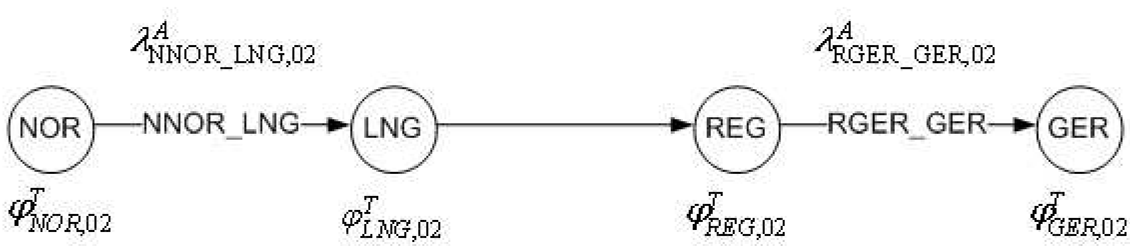

Figure 57: Three arcs in the LNG supply chain

The initial capacities are chosen the same for all three arcs. Eq. (7.3.17) for the flow restrictions $\overline{CAP}_{am}^{A} + \sum_{m' \in pred(m)} \Delta_{am'}^{A,it} - Q_{ad}^{A} \geq 0$ reduces to $2\frac{1}{2} - Q_{ad,02}^{A} \geq 0$ for all three and in the first iteration of the Benders method, when all expansions are zero, all arcs are fully used (bottlenecks): $\forall a \in A: Q_{ad}^{A} = \overline{CAP}_{am}^{A} + \sum_{m' \in pred(m)} \Delta_{am'}^{A,it}$, more specifically: $Q_{ad,02}^{A} = 2\frac{1}{2}$.

Table 35 shows that the dual prices for $\lambda_{\text{NNOR\_LNG,02}}^{A}$ and $\lambda_{\text{RGER\_GER,02}}^{A}$ differ between the SP types. In the MCP subproblem $\lambda_{\text{NNOR\_LNG,02}}^{A}$ = 8.1291, whereas in the NLP subproblem $\lambda_{\text{NNOR\_LNG,02}}^{A}$ = 0.2876 and $\lambda_{\text{RGER\_GER,02}}^{A}$ = 7.8415. That these values differ may seem strange, however 0.2876 + 7.8415 = 8.1291, see column 'LNG Chain'.

Table 35: Coefficients of capacity expansions and right-hand sides in Benders cuts

MP	SP	Iter	NNOR_LNG (k$/mcm/d)	RGER_GER (k$/mcm/d)	LNG Chain (sum left)	NNED_GER (k$/mcm/d)	STOR_GER (k$/mcm/d)	RHS (k$)
LCP	MCP	1	0.2876	7.8415	8.1291	8.1291	5.8764	-83.0048
LP	MCP	1	0.2876	7.8415	8.1291	8.1291	5.8764	-83.0048
LCP	NLP	1	8.1291		8.1291	8.1291	5.8764	-83.0048
LP	NLP	1	8.1291		8.1291	8.1291	5.8764	-83.0048
LCP	MCP	2	0.9032		0.9032			-104.5994
LP	MCP	2	0.9032		0.9032			-104.5994
LCP	NLP	2		0.9032	0.9032			-104.5994
LP	NLP	2		0.9032	0.9032			-104.5994
LCP	MCP	3	2.6651	5.4640	8.1291	8.1291		-85.7594
LP	MCP	3	2.6651	5.4640	8.1291	8.1291		-85.7594
LCP	NLP	3	8.1291		8.1291	8.1291		-85.7594
LP	NLP	3	8.1291		8.1291	8.1291		-85.7594
LCP	MCP	4	8.1291		8.1291	8.1291	5.8764	-83.0048
LP	MCP	4	8.1291		8.1291	8.1291	5.8764	-83.0048
LCP	NLP	4		8.1291	8.1291	8.1291	4.4073	-82.1440
LP	NLP	4		8.1291	8.1291	8.1291	4.4073	-82.1440

The explanation is that since both arcs in the LNG shipping route are bottlenecks, there are infinitely many solutions for the dual prices of the two arcs, as long as the sum of the dual prices equals 8.1291. The following illustrates that dual prices in LNG shipping routes are not always uniquely determined by the model.

Define the set of arcs in the LNG shipping route, consisting of the liquefaction arc, the LNG shipping arc, and the regasification arc: $A^{LR} = \{NNOR\_LNG, LNG\_REG, RGER\_GER\}$. Observe that the dual price for the trader's mass-balance constraint Eq. (7.3.14) for the production node NOR at the supply side of the LNG supply chain is determined by the production costs. With Eq. (7.3.7) and (7.3.9), $Q_{tnd}^{T \leftarrow P} > 0$ and production capacity not restrictive ($\lambda_{pd}^{P} = 0$):

$$\phi_{tndm}^{T} = d_d \pi_{n(p)dm}^{P} = d_d \frac{\partial c_{pm}^{P}(Q_{pd}^{P \to T})}{\partial Q_{pd}^{P \to T}} + \lambda_{pd}^{P} = d_d \frac{\partial c_{pm}^{P}(Q_{pd}^{P \to T})}{\partial Q_{pd}^{P \to T}} \quad \textbf{(7.5.1)}$$

The dual price for the mass-balance constraint Eq. (7.3.14) for the demand node GER at the receiving end of the LNG supply chain is determined by the wholesale market price and the exerted market power level. With Eq. (7.3.13) and $Q_{tnd}^{T \to W} > 0$:

$$\phi_{tnd}^{T} = d_d \left(INT_{ndm}^{W} - SLP_{ndm}^{W} \left(\sum_{t' \in T} Q_{t'nd}^{T \to W} + \delta_{tn}^{T} Q_{tnd}^{T \to W} \right) \right) \quad \textbf{(7.5.2)}$$

As follows from Eq. (7.3.12) the differences between the dual prices of mass-balance constraints for neighboring nodes define the shadow prices of the arcs. With $F_{ad}^{T} > 0$ $\forall a \in A^{LR}$, Eq. (7.3.16) and for convenience $d_d = 1$ (this only affects the scaling):

$\varphi_{GER,02}^{T} = \varphi_{REG,02}^{T} + d_d \left(c_{RGER\_GER,02}^{A} + \tau_{RGER\_GER,02}^{\mathrm{A}} \right) = \varphi_{REG,02}^{T} + c_{RGER\_GER,02}^{A} + \lambda_{RGER\_GER,02}^{\mathrm{A}}$,

$\varphi_{REG,02}^{T} = \varphi_{LNG,02}^{T} + c_{LNG\_REG,02}^{A} + \lambda_{LNG\_REG,02}^{\mathrm{A}}$ and

$\varphi_{LNG,02}^{T} = \varphi_{NOR,02}^{T} + c_{NNOR\_LNG,02}^{A} + \lambda_{NNOR\_LNG,02}^{\mathrm{A}}$.

By combining the three expressions the value of $\varphi_{GER,02}^{T}$ is linked to the value of $\varphi_{NOR,02}^{T}$ as follows: $\varphi_{GER,02}^{T} =$

$$\varphi_{NOR,02}^{T} + \left(c_{NNOR\_LNG,02}^{A} + c_{LNG\_REG,02}^{A} + c_{RGER\_GER,02}^{A} \right) + \left(\lambda_{LNOR\_LNG,02}^{\mathrm{A}} + \lambda_{LNG\_REG,02}^{\mathrm{A}} + \lambda_{RGER\_GER,02}^{\mathrm{A}} \right)$$

$$= \varphi^T_{NOR,02} + \sum_{a\in A^{LR}} \left(c^A_{a,02} + \lambda^{\mathrm{A}}_{a,02}\right).$$

Rearranging terms, and substituting in (7.5.1) and (7.5.2) gives:

$$\sum_{a\in A^{LR}} \lambda^{\mathrm{A}}_{a,02} = \varphi^T_{GER,02} - \varphi^T_{NOR,02} - \sum_{a\in A^{LR}} c^A_{a,02}$$

$$= INT^W_{GER,02} - SLP^W_{GER,02}\left(1+\delta^T_{GER,02}\right)Q^{T\to W}_{GER,02} - \frac{\partial c^P_{pm}(Q^{P\to T}_{pd})}{\partial Q^{P\to T}_{pd}} - \sum_{a\in A^{LR}} c^A_{a,02}$$

Hence: $\sum_{a\in A^{LR}} \lambda^{\mathrm{A}}_{a,02} = INT^W_{GER,02} - SLP^W_{GER,02}\left(1+\delta^T_{GER,02}\right)Q^{T\to W}_{GER,02} - \frac{\partial c^P_{pm}(Q^{P\to T}_{pd})}{\partial Q^{P\to T}_{pd}} - \sum_{a\in A^{LR}} c^A_{a,02}$, where all the terms on the right-hand side are determined individually, but the value for the left-hand-side term is determined on aggregate only.

Obviously, when in SP solutions the dual prices of the liquefaction and the regasification arc have other values (see Table 35), the Benders cuts, Eq. (7.2.4), will be different and this affects the solution for the capacity expansions determined by MP in the next iteration (see Table 36 below). In the second iteration the MP with the MCP subproblems expand the regasification arc, whereas the MP with the NLP subproblems expand the liquefaction arc to their maximum values (see Eq. (7.3.2)).

Table 36: Optimal expansions (mcm/d) for first five iterations of small-scale problem

Infrastructure	MP	SP	Iter 1	Iter 2	Iter 3	Iter 4	Iter 5
Arc NNOR_LNG	LCP	MCP					2.61
		NLP		**10.00**		2.32	2.63
	LP	**MCP**					2.61
		NLP		**10.00**		2.32	2.32
Arc RGER_GER	LCP	**MCP**		**10.00**		3.45	2.61
		NLP					2.82
	LP	**MCP**		**10.00**		3.45	2.61
		NLP					
Arc NNED_GER	LCP	MCP		10.00			
		NLP		10.00			
	LP	MCP		10.00			
		NLP		10.00			
Storage SGER	LCP	MCP		10.00	3.68		0.47
		NLP		10.00	3.68	0.47	0.47
	LP	MCP		10.00	3.68		0.47
		NLP		10.00	3.68	0.47	5.10

When solving the SP in the second iteration the arc not expanded is the (only) bottleneck and has a positive dual price of 0.9032, see Table 35 on page 232.

In the third iteration only the storage capacity is expanded by all approaches and in the fourth iteration the optimal capacity expansions by the master problems start to deviate, and consequently so do the solution paths and the bounds for optimal solutions. The varying solution paths result in a different number of iterations needed to converge as well as slightly different converged solutions. Table 37 summarizes the convergence results for the four different MP SP combinations. The columns present the total number of iterations until convergence ('#iter'), the iteration in which the best feasible solution was achieved ('Best'), total investment costs and the best objective value.[188]

Table 37: Summarized convergence results

MP	SP	#iter	Best	Investment Costs	Final objective value[189]
LCP	MCP	28	26	10.052	-92.555
	NLP	28	27	10.194	-92.548
LP	MCP	41	41	10.092	-92.564
	NLP	39	31	10.058	-92.562

The LCP-NLP and LCP-MCP converged after 28 iterations. After the same number of iterations the best feasible solutions of both other methods are very close to optimal (not shown in table), however the convergence gaps are still too large and several more iterations are needed to reach convergence.

An interesting observation is that the investment costs in the converged solutions vary noticeably. The highest and lowest values in column 'Investment Costs' differ 10.194 -10.052 = 0.142 (1.4%), but the corresponding objective values differ 0.007 only (-92.548 -92.555 = 0.007). Naturally, higher expansions are more expensive, but also allow for larger trade volumes, higher consumption surplus and possibly higher profits, that largely offset the higher costs in the overall objective value. Note that this does not mean that the solution is not unique, it merely shows that there are many feasible solutions with an objective value very close to optimal objective value. This

[188] Convergence criterion used for these runs is a gap smaller than 10^{-3}.
[189] The complementarity variants MP-LCP and SP-MCP are only applied to optimization problems. Hence, the same objective values and bounds can be calculated as for the original optimization problems.

characteristic could also be observed in the small example with one producer and one consumer in the previous chapter (Section 6.3.5.2.1) of generalized Benders decomposition. In that example, convergence was reached for an expansion of 3.020 units, 0.02 higher than the optimal solution of exactly 3 and the objective value for the best feasible solution 54.500 was identical to the optimal objective value up to three digits.

In the next subsection calculation times are compared for various problem sizes.

7.5.3.2 Medium-scale optimization problems – calculation times

For the experiments in this section a data set with forty model nodes is used. There are twenty country nodes. All countries have production and consumption. Nine countries have liquefaction, the other eleven have regasification, adding twenty model nodes to the data set. Nine countries have storage facilities. There are in total 144 arcs, of which twenty five represent pipelines, ninety-nine arcs represent a liquefaction shipping route and the remaining twenty arcs represent nine liquefaction and eleven regasification arcs. There are two demand seasons in each year. Three deterministic four-period cases are run: one for a perfectly competitive market, one for a monopolistic market and one for a hybrid market.

Compared to the previously presented results for small-scale data sets, the decomposition approaches do relatively better on these larger problems in terms of calculation time needed to solve the problems. This is an indication that the calculation times needed by decomposition approaches grow much less when increasing the size of the problems than calculation times of the full-scale solution approaches. Table 38 summarizes the ranking in the order of calculation times for the problems in the current and the previous section. The problems are from left to right increasing in problem size. The last problem cannot be cast as an optimization problem and is solved only in extensive form and using VI-MCP.

Table 38: Ranking in solution times for optimization problems

Data set		Former section: 6 nodes				This section: 40 nodes			
Market structure*		**mono**	**perf**	**mono**	**perf**	**perf**	**perf**&	**mono**	**hybrid**
Number variables		390	493	1506	1905	9924	9924	11,192	20,000
Solution Method	**Full MCP**	**1**	2	**1**	2	**1**	2	**1**	**1**
	Full OPT^	**1**	**1**	2	**1**	3	1	3	N/A
	VI-MCP	4	4	3	3	2	4	2	2
	VI-NLP^	3	3	4	4	6	3	5	N/A
	LCP-MCP	7	7	7	7	4	7	4	N/A
	LCP-NLP^	6	5	5	8	8	5	8	N/A
	LP-MCP	8	8	8	6	5	8	6	N/A
	LP-NLP^	5	6	6	5	7	6	7	N/A

* mono: monopolistic market, perf: perfectly competitive market
^ NLP results are obtained with CONOPT, except for &
& The second perfectly competitive run for the 40 nodes set used MINOS instead of CONOPT

The ranking based on calculation times and numbers of iterations needed by the decomposition approaches to solve the medium-sized problems is similar to the ranking for the small problems. Full-scale MCP is the quickest (unless the MINOS solver is available, more details in a later subsection) and the VI-based decomposition approaches are generally quicker than the optimization-based decomposition approaches. The only difference between the VI-MCP and the LCP-MCP approaches, is the definition of the Benders optimality cuts and it was not expected in advance that the different cut definition would have such a large impact on the number of iterations and the calculation times needed. Between the MCP and the optimization problems, generally PATH solves the MCP much faster than CONOPT solves the equivalent optimization problems. However, the availability of MINOS reduces the calculation times needed for NLP subproblems so much that using SP-NLP is faster than approaches using SP-MCP. However, the MP-VI needs fewer iterations and less calculation time than the MP-LP and MP-LCP approaches.

Next, Figure 58 shows the calculation times (in seconds, on the left vertical axis) and the ratio between the calculation times needed (on the right axis) for the four cases discussed in the previous section and the three new runs.

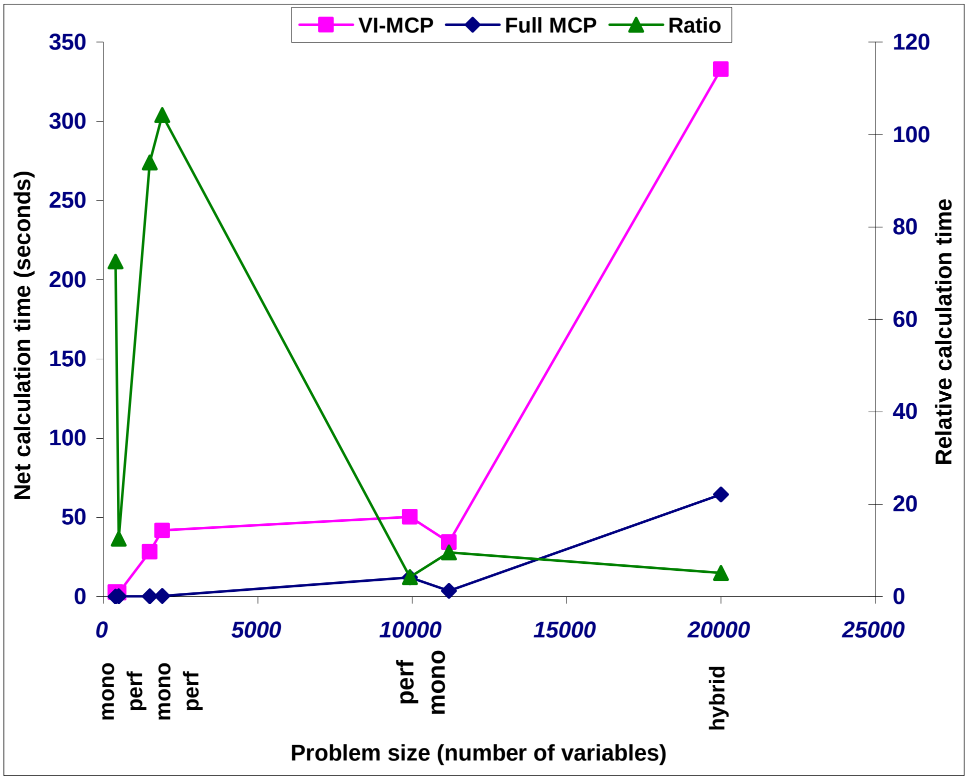

Figure 58: Calculation times ('resusd') for increasing problem sizes

The figure shows that for all these problems the calculation times for the extensive-form model solutions (Full MCP) are lower than the calculation times using decomposition approach VI-MCP. However, the ratio between the two is much smaller for the larger problems. This is another indication that the decomposition approaches perform relatively better on larger problems.

Table 39 shows for the medium-sized deterministic eight-period monopoly with 11,192 variables the CPU times and total run times needed. The MCP solver PATH is faster than MINOS for this full-scale problem, but among the decomposition approaches the availability of MINOS for solving SP-NLP would make VI-NLP the fastest solution approach.[190]

[190] Under the license used GAMS allows up to five runs to be executed at the same time on one computer. Calculation times (resusd) of larger models are affected by running them in parallel, hence calculation times vary somewhat when solving models multiple times.

Table 39: Impact of solvers on calculation time for deterministic eight-period problem

Method	Solver		CPU (s)	Total time (s)
Full MCP	Path		3.1	
Full OPT	MINOS		9.2	
Full OPT	CONOPT		72.2	
VI-NLP	Path	MINOS	13.6	91.2
VI-MCP	Path	Path	42.0	132.4
VI-NLP	Path	CONOPT	104.1	234.5
LP-NLP	XPRESS	MINOS	43.1	262.9
LP-MCP	XPRESS	Path	144.7	455
LP-NLP	XPRESS	CONOPT	293.3	517.2
LCP-NLP	Path	MINOS	46.1	302.2
LCP-MCP	Path	Path	100.3	348.5
LCP-NLP	Path	CONOPT	339.5	674.5

Some other comparisons of calculation times for large-scale deterministic problems indicated that MINOS was up to twenty-five times faster than CONOPT. Another advantage encountered in the numerical experiments was that when using MINOS for solving the NLP subproblems many fewer infeasible subproblems were encountered.[191]

In conclusion, for the medium-sized convex optimization problems, with a polyhedral feasible regions introduced in this chapter to find the equilibrium for perfectly completive or monopolistic markets, the VI-NLP decomposition outperforms the other decomposition approaches.

The next sub section will show results for some large-scale implementations.

7.5.4 Large-scale stochastic hybrid market-equilibrium problem – results

The data set used is the same as the one used in previous Section 7.5.3.2. There are forty model nodes in total, half of which are consumption and production nodes. The other half consists of liquefaction and regasification nodes. There are in total 144 arcs, of which forty-five represent arcs with capacities that may be expanded; the other ninety-nine are shipping routes. There are two demand seasons in each year. The market structure is

[191] That a particular solver cannot solve a problem does not mean that the problem is infeasible. According to the GAMS documentation both CONOPT and MINOS cannot solve 14 out of a specific set of 169 test problems. However, only four of the fourteen cannot be solved by both. See also Footnote 184.

hybrid with traders exerting various levels of market power on different markets. The model runs contain four, six or eight periods and for or eight scenarios. Table 40 presents convergence results for five runs, of which only the first two succeeded.

Table 40: Convergence results large-scale hybrid problems (all times in seconds)

	A	B	C (*)	D (*)	E (*)
Model periods	**4**	**6**	**6**	**8**	**8**
Scenarios	**4**	**4**	**8**	**4**	**8**
Scenario nodes	**11**	**19**	**31**	**27**	**47**
Num capacity expansion variables	339	763	1,187	1,187	2,035
Total num variables	27,221	47,373	77,177	67,525	117,481
Full MCP calc time	263	1,005	13,853	3,005	18,679
VI-MCP Net calc time	267	2,036	5,572	5,222	5,013
Num iterations	46	188	316	325	179
VI-MCP Gross calc time	521	13,684	52,272	51,207	32,502
feasible MP calc time&	4	129	502	550	333
infeasible MP calc time%	4	60	122	96	301
feasible SP calc time&	259	1,847	4,934	4,576	4,373
infeasible SP calc time%	0	0	7	0	6
VI-MCP calc time^	267	2036	5,572	5,222	5,013
Num infeasible MP	7	18	14	8	7
Num infeasible SP	0	0	1	0	1
Convergence criterion	Expans	Expans	MP infeas	MP infeas	MP infeas

(*) Run terminated due to the MP in the last iteration being infeasible for seven times[192]
& Time needed (resusd) for solving feasible problems
% Time needed (resusd) for solving infeasible problems
^ Time needed to solve all MP and SP until convergence was reached or the run terminated.

The results of the first two runs, A and B, are promising with regard to net calculation times needed when taking into account the possibility for parallel processing. In contrast, when solving larger problems in the other three runs (C, D and E) the decomposition approaches did do not succeed in due to repetitively infeasible MP.[192] Run B shows that encountering a few infeasible MP does not necessarily prevent the solution to converge. Often an infeasible MP solves after slightly relaxing the optimality tolerance and the iterative procedure could continue (see Appendix 7.11.4 at the end of this chapter). Before eventually converging in the 188th iteration, between the 176th and 185th iteration

[192] The GAMS code was adjusted to not immediately terminate when encountering an infeasible MP, but to change the solver option for the optimality tolerance and try again, for at most six times. If a later attempt to solve an MP succeeded, the optimality tolerance was set back to the initial (default) value.

a total of eighteen additional attempts were needed to solve MP with a relaxed tolerance.[193]

Why do the larger problems not solve? In the implementation phase of the decomposition approach many experiments have been done. Various tricks and workarounds have been implemented (see Appendix 7.11) with various degrees of success and, as run B shows, temporarily relaxing the solver optimality tolerance sometimes helps. Possible explanations for the infeasibility of larger problems are the impact of deviations caused by the solver and feasibility tolerances and the binary representation of real numbers in computer memory. The Benders cuts given by Eq. (7.4.8) contain many terms, the number of non-zeros in the MP-VI of problems C, D and E grows into the several hundred thousands. Complementarity induces that in various binding cuts (Eq. (7.4.8)) differently weighted summations of the same terms (capacity expansions and quantities produced and traded) have to add up to zero. Maybe the aggregate impact of all the small deviations is too large.

Problems C, D and E contain the exact same mathematical formulas and are implemented using the exact same GAMS code as problems A and B. Problems C, D and E only contain more first-stage variables and more subproblems. Extensive testing and the fact that A and B converge to the solutions of the full-scale extensive stochastic solutions is a strong reason to believe that the approach works and is implemented correctly.

Lastly, due to the long run times, time is an issue when trying alternative workarounds to resolve the infeasible MP. Parallel processing could potentially speed up the run times (although it would not resolve the MP infeasibility issues). This is addressed in Future Work Section 7.7.4.

7.6 Summary

This chapter discussed decomposition approaches to solve large-scale stochastic mixed complementarity problems (MCP). Various generalized Benders methods have been presented that can be used for solving large-scale stochastic optimization models,

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[193] Iteration: number of restarts, 176: 1, 179: 1, 181:4, 182:4, 183:3 and 185: 5

including social welfare maximization and monopoly market models. For the master problems, linear programs and linear complementarity problems were implemented and for the subproblems, non-linear programs and mixed complementarity problems. The convergence characteristics in terms of number of iterations and solution times of several numerical experiments have been discussed.

The Benders decomposition for stochastic optimization approaches provided a stepping stone for the approach to solve stochastic MCP. This approach implements the variational-inequality (VI) based decomposition approach developed in (Gabriel & Fuller 2010). This implementation is the first application of a decomposition approach to solve a large-scale stochastic multi-period natural gas market model with many hundreds of first-stage capacity expansion variables and traders exerting various levels of market power. Beside these characteristics, another difference compared to implementations in (Gabriel and Fuller, 2010) is that the first-stage decisions are not the quantities supplied, but capacity expansions setting upper limits to later period quantities. The complexity of the problems solved is illustrated by the numbers of iterations needed to solve them.

The results indicate that for solving the large-scale stochastic convex optimization problems with polyhedral feasible regions a VI-NLP decomposition should be implemented, with the master problems cast as MCP derived according to (Gabriel & Fuller 2010) and the subproblems cast as nonlinear programs.

For stochastic MCP the VI-MCP decomposition approach has the potential to greatly reduce the solution time of large-scale stochastic MCP. Due to numerical complications the size of the models in the numerical experiments was relatively modest. New implementations should use software that allows the minimization of times needed for file processing and model generation.

7.7 Future research

7.7.1 Functional extensions

The production reserves constraint was ignored in the implementation, and addressing this in a decomposition approach would be a valuable extension (see Footnote 171).

A start was made with the implementation of minimum expansions needed to guarantee feasibility of the SP in case of future contracts. See Appendix 7.10.

The WGM and stochastic models presented in this dissertation ignore most of the engineering aspects, for example relative to pipeline pressures and capacities and natural gas well production characteristics. Some of these aspects have been addressed in the literature (e.g., (Midthun, 2007), (Van Der Hoeven ,2004)) and could be included in the modeling framework. Other engineering aspects, involving non-convex characteristics, or that cannot be described mathematically in closed form, would need other, possibly heuristic approaches. For market equilibrium models that provide more detail in the periods, e.g., with a daily or weekly granularity, the engineering aspects and operational consequences of more and greater fluctuations in quantities and flows, will possibly have more impact and cannot be ignored.

As was discussed in Section 3.6 LNG regasification terminals may be equipped with loading facilities. The current WGM does neither explicitly include the availability of storage at LNG facilities nor the possibility to re-export LNG. Such a model extension might be relevant in light of the recent large upward revisions of unconventional natural gas reserves in the United States.

Risk neutrality is an assumption that does not well represent risk attitudes observed in reality. Cabero et al. (2010) addressed this, by developing and solving a stochastic electricity market model with players using CVAR as a risk metric instead. Other alternative assumptions regarding risk attitudes would address asymmetric information and differences in future beliefs. In our stochastic models it would be relatively easy to let probabilities vary by model agent and hence it seems possible to address differences in future beliefs in the modeling approach. Fan et al. (2010) address risk aversion when making investments in the power sector relative to the uncertainty in future CO_2 regulation. Ralph and Smeers (2010) considered a perfectly-competitive two-stage equilibrium game. The first stage encompassed investments in electricity generation by risk-averse producers and allowed for trade in financial instruments to hedge against rising future fuel costs.

In Section 3.9.4 two-part tariffs were discussed, distinguishing a reservation and usage charge for pipeline flows by traders. For an industry project I have implemented such a two-part tariff, which resulted in an increase in the number of equations and longer calculation times. As mentioned before, for models with a more operational orientation such an extension could be warranted.

An alternative approach for modeling investments uses a rolling horizon rather than perfect foresight, e.g., GASTALE (Lise and Hobbs, 2008) or NEMS. Such an approach offers another representation of investment decisions that will impact the timing and magnitude of expansions.

Smeers (2008) discussed that environmental issues are not well-addressed in models so far. Cap-and-trade systems (such as the ETS in the EU, RGGI for the U.S. East Coast and as discussed by the Obama administration) affect absolute and relative prices of fuels and hence induce substitution among fuels.[194] A multi-sector/multi-fuel model including restrictions on emissions and CO_2 pricing could provide a meaningful extension to the current state-of-the-art natural gas market models, including the WGM.

7.7.2 Using previous solutions as starting points

Solution times for an MP are generally less than two seconds, for each separate SP less than a second. Still, since so many thousands of them are solved, it could be worthwhile to use solutions from former iterations to provide good starting points for the solvers.

7.7.3 Other methods

In many operations research areas Lagrangian relaxation is applied successfully to solve difficult large-scale problems, possibly it could also be applied to stochastic MCP.

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[194] ETS = (greenhouse gas) emission trading system, http://ec.europa.eu/clima/policies/ets/index_en.htm (Accessed Nov 12, 2010.) RGGI = Regional Greenhouse Gas Initiative http://www.rggi.org/ (Accessed Nov 12, 2010.)

7.7.4 Potential gain of parallel processing and using other software

GAMS may not be the most viable tool to implement large-scale decomposition approaches. Compiled languages, such as C++ or FORTAN, might be better suited.

An analysis was made to see what could potentially be the gain of parallel processing. Table 41 below presents the shares in run time used for the main processing steps in the model run to solve problem A with a hybrid market structure, four periods, four-scenarios and eleven scenario nodes (see Table 40 above). The problem took forty-six iterations to solve, and the run time was 521 seconds of which 267 seconds were used by the solvers. An MP-VI was solved fifty three times, including seven restarts due to intermediate infeasibility. A total of 506 SP were solved (11 SP * 46 iter).

Table 41: Shares in processing times – Own calculations based on GAMS log files

	Total Run Time	Data Processing	Model Generation	Model Solution
MP	35%	17%	16%	2%
SP	65%	8%	6%	51%
	100%	25%	22%	53%

As the data in the first column show, although almost ten times fewer master than subproblems were solved, the MP run times make up 1/3 of the total time. Also, just over half of the total run time is used for solving models. A quarter of the run time is used for data processing and a little less than a quart for generating the models.

It would be possible to solve several subproblems in parallel. When applying parallel, processing the number of processors available would determine how many subproblems could be solved at the same time. To make a ballpark estimate for the gain in run time, assume that there are eight processors available. Then eight SP can be solved in parallel. Just dividing the total SP run time by eight would be the maximum potential gain, and the total run time would be: MP time + SP time/Number processors = 35% + [65%/8] = 43%, a reduction of 57%.

This is an overly optimistic value. Only eight processes can be started in parallel, which means that three processors have to solve two SP after another. Taking the average of SP run times, the total needed time would be: 35% + [65% * 2/11] = 47%, a reduction of 53%.

Lastly, a conservative estimate is calculated, based on maximum instead of average SP run times. For this model the maximum SP solution time in every iteration was on average 40% higher than the average SP solution time, or 1.4 times as long:
35% + [(8%+6%) * 2/11]+ [51% * 2/11*1.4] = 51%, or a reduction of 49%.

Based on these three values, parallel processing could cut the needed total run times roughly in half for this problem. This is a significant but not great improvement. The potential improvement becomes even smaller when considering that the MP grows with every iteration. For example, in problem E (see Table 40 above) after 180 iterations the data processing and model generation of the MP consume about 90% of the run time.[195] More specifically, every time an MP is generated it takes more than 2½ minutes and the other steps combined less than a ½ minute.[196] For every infeasible MP this time is added again to regenerate the model.

GAMS provides a utility for grid computing. When using this utility on some of the smaller problems the added overhead due to file-I/O took so much time, that total run times were not shorter. GAMS has a lot to support research and modeling when it comes to the user interface, availability of solvers and technical support. GAMS is an interpreted language, which means that when the program starts, the program processes the code line by line and executes the instructions. As such, it is not optimized for speed of data processing or model generation. An issue with the decomposition approach is that many times data need to be processed and models need to be generated. For the largest stochastic problem (E), in every iteration for an MP and forty-seven SP models need to be generated and data need to be processed. When there are several hundreds of iterations needed to reach convergence, many thousands of data processing and model generation steps are done. These overhead parts of the decomposition procedure should be done as efficiently as possible. Other programming languages, such as C++ and FORTRAN, are

[195] Due to a cut-clearing procedure after 180 iterations the MP size stays roughly constant. See Section 7.11.5

[196] This limits the number of iterations per hour to about twenty, less when some MP are infeasible.

compiled languages, and are able to process repetitive steps much quicker. Potentially the data processing and model generation time would vanish. A ballpark estimate for the run time of problem A would be 2% for the MP and [51% * 2/11*1.4]=11% for the SP. Total time would be 13%, a reduction of 87% or about eight times as quick. This could be promising, however trying other software is beyond the scope of this dissertation.

7.8 A stochastic multi-period energy market optimization model

7.8.1 Nomenclature

7.8.1.1 Sets

$a \in A$	Gas transportation arcs, e.g., {NNED_GER, LNOR_FRA, RGER_GER}
$d \in D$	Demand seasons, e.g., {low, high}
$p \in P$	Producers, e.g., {P_NOR, P_RUW, P_RUE }
$m \in M$	Scenario tree nodes, e.g., {01, 02, 03, 04, 05}
$n \in N$	Country nodes, e.g., {N_NOR, N_RUW}
$s \in S$	Storage facilities, e.g., {S_NED, S_GER}
$t \in T$	Traders, e.g., {T_NOR, T_RUS}
$a^-(n)$	Outward arcs from node *n*
$a^+(n)$	Inward arcs into node *n*
$n^+(a)$	End node of arc *a*
$n^-(a)$	Start node of arc *a*
$pred(m)$	Predecessor nodes in the scenario tree, e.g., pred(08) = {01, 02, 04}[197]
$succ(m)$	Successor nodes in the scenario tree, e.g., succ(04) = {08, 09, 16, 17}[198]
$n(s)$	Node where storage *s* is located
$P(n)$	Producers present at node *n*
$S(n)$	Storage facilities at node *n*

[197] See Figure 35 in Chapter 6 or Figure 53 in this chapter.
[198] See Figure 35 in Chapter 6.

$T(n)$ Traders present at node *n*

$T(p)$ Traders that can buy from producer *p*

7.8.1.2 Constants/Input parameters

c_{adm}^{A} Regulated fee for arc usage (k$/mcm)[199]

c_{sdm}^{I} Regulated fee for storage injection usage (k$/mcm)

$c_{pm}^{P}(.)$ Production costs (k$/mcm)

$c_{am}^{\Delta A}$ Arc capacity expansion costs (k$/mcm/d)

$c_{sm}^{\Delta S}$ Storage working gas capacity expansion costs (k$/mcm)

$\overline{CAP}_{pm}^{P}$ Production capacity (mcm/d)

$\overline{CAP}_{am}^{A}$ Arc capacity (mcm/d)[200]

$\overline{CAP}_{sm}^{S}$ Storage working gas capacity (mcm/d)[200]

d_d Number of days in season. $d_{low} = 183$ and $d_{high} = 182$

δ_{tnm}^{T} Market-power indicator, $\delta_{tnm}^{T} \in [0,1]$

$\overline{\Delta}_{am}^{A}$ Upper bound of arc capacity expansion (mcm/d)

$\overline{\Delta}_{sm}^{S}$ Upper bound of storage working gas capacity expansion (mcm)

γ_m Discount rate for scenario tree node, $\gamma_m \in (0,1]$

INT_{ndm}^{W} Intercept of inverse demand curve (mcm/d)

l_a Loss rate of gas in transport arc, $l_a \in [0,1)$

l_{sn} Loss rate of gas storage injection, $l_{sn} \in [0,1)$

p_m Probability of scenario tree node *m*, $p_m \in [0,1]$

SLP_{ndm}^{W} Slope of inverse demand curve (mcm/d/k$)

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[199] Units of measurement: k$: 1000 USD; mcm: million cubic meter; mcm/d: mcm per day. In applications costs are in the range of 10-100 k$/mcm; typical market prices are in the range of 100-800 k$/mcm. Typical quantities and flows are up to a few hundred mcm/d.

[200] The subscript m is to account for expansions under construction that are exogenously included.

7.8.1.3 Variables

All primal variables are nonnegative.

Δ_{am}^{A}	Arc capacity expansion (mcm/d)
Δ_{sm}^{S}	Storage working gas capacity expansion (mcm/d)
F_{tadm}^{T}	Arc flow by trader (mcm/d)[201]
I_{tndm}^{T}	Injection rate into storage by trader (mcm/d)
$Q_{adm}^{A\to T}$	Arc capacity assigned by TSO to trader (mcm/d)[202]
$Q_{pdm}^{P\to T}$	Quantity sold by producer to traders (mcm/d)
$Q_{sdm}^{S\to T}$	Storage injection capacity assigned to trader (mcm/d)
$Q_{tndm}^{T\leftarrow P}$	Quantity bought by trader from producer (mcm/d)
$Q_{tndm}^{T\to W}$	Quantity sold by trader to consumers (mcm/d)
X_{tndm}^{T}	Extraction rate from storage by trader (mcm/d)

Greek symbols in parentheses with appropriate sub and superscripts refer to dual variables to associated constraints in the KKT conditions.

$\lambda_{pdm}^{P} \geq 0$	dual variables to production capacity restrictions
$\lambda_{adm}^{A} \geq 0$	dual variables to arc capacity restrictions
$\lambda_{sm}^{S} \geq 0$	dual variables to storage capacity restrictions
φ_{tndm}^{T} *free*	dual variables to the trader's nodal mass balance constraint
φ_{tnm}^{S} *free*	dual variables to the trader's storage cycle constraint
$\rho_{am}^{A} \geq 0$	dual variables to arc capacity expansion limitations
$\rho_{sm}^{S} \geq 0$	dual variables to storage capacity expansion limitations
π_{ndm}^{P} *free*	dual variables to market clearing conditions for produced quantities
τ_{adm}^{A} *free*	dual variables to market clearing conditions for arc capacity

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[201] Arc flow is identical to arc capacity used.
[202] TSO: Transmission system operator. Arc capacity assigned must equal the arc capacities used by traders, see market-clearing conditions in Section 7.3.2.5.

τ_{sdm}^{S} *free* dual variables to market clearing conditions for storage working gas.

7.8.2 Model formulation

The social welfare ((Bergson, 1938), (Walras, 1977)) is the sum of consumer surplus and trader profits minus production costs and regulated fees and expansion costs for storage and transportation infrastructure.[203] Hence, the expression for the expected social welfare that is to be maximized is the following:

$$\max \sum_{m\in M} p_m \gamma_m \sum_{n\in N} \left[\sum_{d\in D} d_d \left\{ \begin{array}{l} \frac{1}{2} SLP_{ndm}^{W} \left(\sum_{t\in T(n)} Q_{tndm}^{T\to W} \right)^2 \\ + \sum_{t\in T(n)} Q_{tndm}^{T\to W} \left(INT_{ndm}^{W} - SLP_{ndm}^{W} \sum_{t\in T(n)} Q_{tndm}^{T\to W} \right) \\ - \sum_{p\in P(n)} c_{pm}^{P} (Q_{pdm}^{P\to T}) \\ - \sum_{a\in a^+(n)} c_{adm}^{A} Q_{adm}^{A\to T} - \sum_{s\in S(n)} c_{sdm}^{I} Q_{sdm}^{S\to T} \end{array} \right\} \begin{array}{l} - \sum_{a\in a^+(n)} c_{am}^{\Delta A} \Delta_{am}^{A} \\ - \sum_{s\in S(n)} c_{sm}^{\Delta S} \Delta_{sm}^{S} \end{array} \right] \tag{7.8.1}$$

The decision variables are limited by several restrictions. The production capacity limits the daily production:

$$Q_{pdm}^{P\to T} \le \overline{CAP}_{pm}^{P} \quad \forall p,d,m \quad \left(\lambda_{pdm}^{P}\right) \tag{7.8.2}$$

The trader needs to preserve mass balance at every node *n* in every season *d* of every year *m*. Thus, the total quantity bought from producers plus the net import and the extraction from storage must equal the total quantity sold to consumers, the exports and the injection into storage:[204]

$$Q_{tndm}^{T\leftarrow P} + \sum_{a\in a^+(n)} \left(1-l_a\right) F_{tadm}^{T} + X_{tndm}^{T} = Q_{tndm}^{T\to W} + \sum_{a\in a^-(n)} F_{tadm}^{T} + I_{tndm}^{T} \quad \forall t,n,d,m \quad \left(\varphi_{tndm}^{T}\right) \tag{7.8.3}$$

In each year the total extracted volumes must equal the loss-corrected injected volumes:

$$\left(1-l_{sn}\right) \sum_{d\in D} d_d I_{tndm}^{T} = \sum_{d\in D} d_d X_{tndm}^{T} \quad \forall t,n,m \quad \left(\varphi_{tnm}^{S}\right) \tag{7.8.4}$$

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[203] Several terms cancel out since revenues of many players are costs for another. For instance, congestion charges for infrastructure are profits for the system operators, but costs for the traders.

[204] Note that the arc losses are included in this mass-balance equation, but that storage losses are accounted for in the storage cycle constraint: (7.8.4).

The assigned arc capacity $Q_{adm}^{A\to T}$ is limited by the available capacity, which is the sum of the initial capacity $\overline{CAP}_{am}^{A}$ and the expansions in previous years $\sum_{m'\in pred(m)} \Delta_{am'}^{A}$:

$$Q_{adm}^{A\to T} \le \overline{CAP}_{am}^{A} + \sum_{m'\in pred(m)} \Delta_{am'}^{A} \quad \forall a,d,m \quad \left(\lambda_{adm}^{A}\right) \tag{7.8.5}$$

There may be budgetary or other limits restricting the arc capacity expansions.

$$\Delta_{am}^{A} \le \overline{\Delta}_{am}^{A} \quad \forall a,m \qquad \left(\rho_{am}^{A}\right) \tag{7.8.6}$$

The assigned storage injection capacity $Q_{sdm}^{S\to T}$ is restricted by the available working gas, which is the sum of the initial working gas and the expansions in former years:

$$\left(1-l_{sn}\right)\sum_{d\in D} d_d Q_{sdm}^{S\to T} \le \overline{CAP}_{sm}^{S} + \sum_{m'\in pred(m)} \Delta_{sm'}^{S} \quad \forall s,m \quad \left(\lambda_{sm}^{S}\right) \tag{7.8.7}$$

There may be budgetary or other limits restricting the storage working gas expansions:

$$\Delta_{sm}^{S} \le \overline{\Delta}_{sm}^{S} \quad \forall s,m \qquad \left(\rho_{sm}^{S}\right) \tag{7.8.8}$$

The market-clearing condition between the producers' sales and the traders' purchases is as follows:

$$Q_{pdm}^{P\to T} = \sum_{t\in T(p)} Q_{tn(p)dm}^{T\leftarrow P} \quad \forall p,d,m \quad \left(\pi_{n(p)dm}^{P}\right) \tag{7.8.9}$$

The market clearing of assigned arc capacities between the TSO and the traders:

$$Q_{adm}^{A\to T} = \sum_{t} F_{tadm}^{T} \quad \forall a,d,m \quad \left(\tau_{adm}^{A}\right) \tag{7.8.10}$$

Market clearing for assigned storage injection capacities:

$$Q_{sdm}^{S\to T} = \sum_{t\in T(n(s))} I_{tndm}^{T} \quad \forall s,d,m \quad \left(\tau_{sdm}^{S}\right) \tag{7.8.11}$$

The market-clearing conditions could be used to substitute out some decision variables. Instead, they are included explicitly to provide a stepping stone to the MCP formulation presented in the chapter.

Together, objective function (7.8.1), restrictions (7.8.2) - (7.8.8) and market-clearing conditions (7.8.9) - (7.8.11) provide the mathematical formulation of the optimization problem for maximizing expected social welfare for some general commodity that can be stored and transported, and for which capacity limitations apply. The applications in this dissertation are limited to natural gas markets; however the same model can be applied to other markets for energy carriers or commodities that can be produced, traded, transported and stored.

7.9 Matching equations with Gabriel and Fuller (2010)

In this section the groups of variables and equations in the model introduced in this chapter are matched with the variables and equations in (Gabriel and Fuller, 2010); Table 42 presents the mapping. The first column describes the equation group, the second indicates the equation numbers in (Gabriel and Fuller, 2010), the third column gives the equations from Sections 7.3.1 and 7.3.2 and the fourth provides the match with the variables and equations in Section 7.4.1.

Table 42: Matching equations groups

Group of equations	**Gabriel & Fuller***	**Sections 7.3.1 & 7.3.2**	**Section 7.4.1**
Stationarity of expansions	(8a)	(7.3.1)^ (7.3.3)^	(a.)
Stationarity of production and trader sales	(8b)	(7.3.7) (7.3.13)	(b.)
Stationarity of other decision variables	(8d)	(7.3.9) (7.3.10) (7.3.11) (7.3.12) (7.3.16)	(d.)
Expansion limits	(8e)	(7.3.2) (7.3.4)	(i.)
Capacity restrictions	(8f)	(7.3.8) (7.3.17) (7.3.18)	(ii.)
Mass balances	(8g)	(7.3.14) (7.3.15)	(iii.)
Market clearing		(7.3.20) (7.3.21) (7.3.22)	

*Note that model in this chapter does not contain any decision variables that are free in sign and therefore no equation group matching (8c) in (Gabriel and Fuller, 2010) exists.

^Eq. (7.3.1) is stationarity of arc expansions as part of the MP, which differs from the condition in the original problem: $p_m \gamma_m c_{am}^{\Delta A} + \rho_{am}^{A} - \sum_{m' \in succ(m)} \sum_{d} \lambda_{adm'}^{A} \geq 0$. Similarly, regarding (7.3.3) the reference should be to: $p_m \gamma_m c_{sm}^{\Delta S} + \rho_{sm}^{S} - \sum_{m' \in succ(m)} \lambda_{sm'}^{S} \geq 0$.

Next, the details are provided for the coefficient matrices in Eq. (7.4.1)-(7.4.3). Since there are no decision variables that are free in sign, matrices $\overline{C}$ and $\hat{C}$ are empty. Also, capacity expansions are not part of market-clearing or mass-balance Eq., and therefore the matrix $\hat{A}$ is empty too.[205] Thirdly, all equality conditions have zero right hand sides, implying that $\hat{b}$ is the zero vector.

Thus, in Eq. (7.4.1) $A\Delta \geq b$, the limits to capacity expansions, matrix $A = [-I]$ and vector $b = [-\overline{\Delta}]$, the negative of the capacity expansion limitation values.

The second set of Eq. (7.4.2) $\overline{A}\Delta + \overline{B}q + \overline{D}f \geq \overline{b}$, provide the capacity restrictions to production, pipeline flows and storage injection volume variables in the SP. Here, matrix $\overline{A}$ provides the coefficients for the additions to capacities. Matrices $\overline{B}$ and $\overline{D}$ provide the coefficients of the capacity-restricted variables. Vector $\overline{b}$ contains the negatives of the initial capacities. For instance, if there is an arc with capacity cap^A, that can be expanded $+\Delta^A$ restricting flow $q^{A\to T}$ and a production capacity cap^P restricting production q^P:

$$\begin{aligned} +1\cdot\Delta^A \qquad\qquad -1\cdot q^{A\to T} &\geq -cap^A \\ -1\cdot q^P \qquad\qquad\qquad &\geq -cap^P \end{aligned}$$

the matrices are: $\overline{A} = \begin{bmatrix} +1 \\ 0 \end{bmatrix}, \overline{B} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, \overline{D} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ and $\overline{b} = \begin{bmatrix} -cap^A \\ -cap^P \end{bmatrix}$

The third group of Eq. (7.4.3), $\hat{B}q + \hat{D}f = \overline{0}$ provides the mass balances and market-clearing conditions. Herein, matrices $\hat{B}$ and $\hat{D}$ provide the coefficients of the variables that are balanced; as clarified previously their right-hand sides are zero. For instance, for the following Eq., including mass balance between producer and trader: $q^{P\to T} = q^{T\leftarrow P}$, nodal mass balance for the trader: $q^{T\to W} = q^{T\leftarrow P} + f^{T\leftarrow A}$ and equality of flow with purchased arc capacity: $f^{T\leftarrow A} = q^{A\to T}$:

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[205] In (Gabriel and Fuller, 2010) the first-stage variables are not upper limits, but the actual values for the sold quantities. Hence, in their formulation first-stage variables appear in the market-clearing conditions.

$$
\begin{aligned}
+1\cdot q^{P\to T} \quad -1\cdot q^{T\leftarrow P} \quad &= 0\\
-1\cdot q^{T\to W} + 1\cdot q^{T\leftarrow P} + 1\cdot f^{T} \quad &= 0\\
-1\cdot f^{T} + 1\cdot q^{T\leftarrow A} &= 0
\end{aligned}
$$

the matrices would look as follows: $\hat{B} = \begin{bmatrix} +1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$, $\hat{D} = \begin{bmatrix} -1 & 0 & 0 \\ +1 & +1 & 0 \\ 0 & -1 & +1 \end{bmatrix}$ and $\hat{b} = \overline{0}$.

7.10 Extension to guarantee solutions in case of future contracts

Incorporating supply contracts could potentially cause the feasible region to the equilibrium problem to be empty. Without contracts, the feasible region always contains at least one point corresponding to the zero vector for all primal variables. Obviously, just setting all quantities and flows equal to zero provides a feasible solution. In a full-scale extensive-form stochastic model, providing high enough upper bounds (arc expansions limits) to capacity expansions will automatically let the model put in high enough capacity expansions to accommodate all future contracts. However, in the decomposition approaches discussed in this chapter contracts could be part of the SP.[206] The information about contracts would not automatically be available in the MP, so that must be provided for.

In this section an adjustment to the approach is presented that should generally guarantee the feasibility of the SP by setting minima for capacity expansions in case of supply contracts with future start dates for which the current capacities do not suffice.

Due to losses in preceding arcs that need to be accounted for, accommodating LNG contracts is more complicated than pipeline contracts. Hence, the LNG contracts are discussed. These contracts provide a minimum bound to trader's flows from liquefaction to regasification nodes. Thus, the liquefaction capacities must be large enough to satisfy the contractual amounts, as must production capacities on the nodes feeding into the liquefaction node and at the receiving end of the contractual flow: the regasification

[206] Contracts could also be included as part of the MP, but a disadvantage is that the number of first-stage variables is a major determinant in the number of iterations and run time needed.

capacities. Production capacities are exogenous to the model, and need to be set exogenously to high enough values.

Observe the following market structure (Figure 59). Initial liquefaction capacity at node C is twelve (mcm/d), at node D eight and regasification capacity at node E is fifteen. Let there be two traders who each have to meet a contract in the next model period. Both contracts amount to nine (18 in total). Further, assume that liquefaction losses amount to 10% and that LNG shipment losses are 2% from C to E, and 3% from D to E.

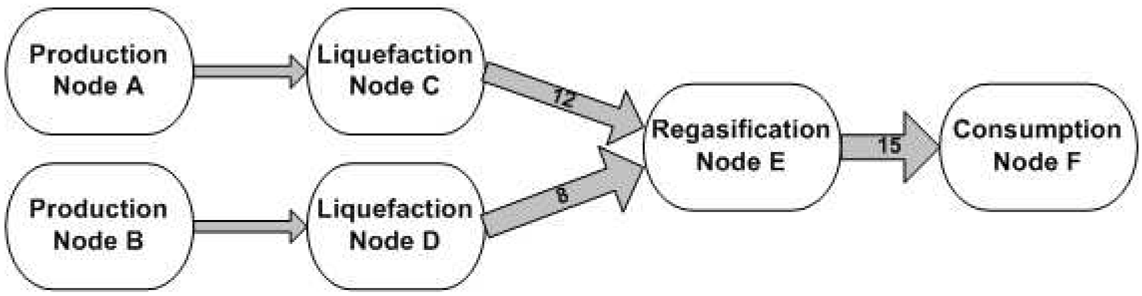

Figure 59: Two LNG supply routes

What expansions are needed to meet the contractual obligations?

First, new notation is introduced to facilitate writing up some equations.

Parameter and sets for contracts

CON^{T}_{tadm}	Contractual supply obligation (mcm/d)
$n^{-}(a)$	Start node of arc *a*
$n^{+}(a)$	End node of arc *a*
$a^{-}(n)$	Outward arcs from node *n*
$a^{+}(n)$	Inward arcs from node *n*
ε^{T}_{tadm}	Dual to contractual obligation (k$/mcm/d)
η^{A}_{am}	Dual to minimum expansion (k$/mcm/d)

The contractual minimum supply obligation for a trader to the marketer:

$$FLOW_{tadm}^{T} \geq CON_{tadm}^{T} \tag{7.10.1}$$

LNG contracts provide a lower bound to the flows from liquefiers to regasifiers, i.e., from node C to E and from node D to E. These flows are from the perspective of the liquefiers. The output of both liquefiers must be at least nine (mcm/d). The arc capacities in the model restrict the input flows. Since the losses are 10%, the input capacities must be at least: $\frac{9}{1-10\%} = 10$ mcm/d. Liquefier C has a capacity of 12, large enough, but the capacity of liquefier D of 8 must be expanded with at least 2 mcm/d.

The two contracts add up to 18 mcm/d. However, due to the shipment losses, the contractual amount arriving in the regasification node is somewhat lower: $9(1-2\%)+9(1-3\%)=17.55$ mcm/d. Hence, given the current capacity of 15, the minimum expansion for the regasifier is 2.55 mcm/d.

More generally, the minimum expansions for liquefiers are defined through:

$$\overline{CAP}_{am}^{A} + \sum_{m'<m} \Delta_{am'}^{A} \geq \frac{1}{1-l_a} \sum_{a':n^-(a')=n^+(a)} \sum_{t} CON_{ta'dm}^{T} \tag{7.10.2}$$

And for regasifiers the minimum expansions are defined through:

$$\overline{CAP}_{am}^{A} + \sum_{m'<m} \Delta_{am'}^{A} \geq \sum_{a':n^+(a')=n^-(a)} \left(1-l_{a'}\right) \sum_{t} CON_{ta'dm}^{T} \tag{7.10.3}$$

In general, the inclusion in the model of supply contracts will affect the feasible region. Specifically, contractual lower bounds that turn out to be binding in an optimal solution, will have made the feasible region smaller. The minimum expansion constraints defined through (7.10.2) and (7.10.3) are redundant to the full extensive-form stochastic model and will not affect the feasible region or the optimal solution. However, adding them is the stepping stone for deriving the new MP that addresses the minimum capacity expansions ensuring feasibility of the SP.

To extend MP-LP (as defined in Section 7.2.1) so that expansions are large enough to accommodate future contracts, Eq. (7.10.1) and (7.10.2) must be added. The SP-NLP (as defined in Section 7.2.2) must be extended with Eq. (7.10.3).

Deriving the MP-LCP and SP-MCP is quite straightforward. To allow for a succinct formulation an additional parameter is defined and calculated:

Parameter for minimum expansion constraints

$$\underline{\Delta}_{am}^{A} = \begin{cases} \max\left(0, \frac{1}{1-l_a} \sum_{a':n^-(a')=n^+(a)} \sum_t CON_{ta'dm}^{T} - \overline{CAP}_{am}^{A}\right), & \text{for liquefaction arcs} \\ \max\left(0, \sum_{a':n^+(a')=n^-(a)} (1-l_a) \sum_t CON_{ta'dm}^{T} - \overline{CAP}_{am}^{A}\right), & \text{for regasification arcs} \end{cases}$$

The MP-LCP in Section 7.3.1 needs to be extended with the following equation to ensure that the capacity expansions are large enough to accommodate the contracts:

$$0 \le \eta_{am}^{A} \perp \sum_{m'<m} \Delta_{am'}^{A} - \underline{\Delta}_{am}^{A} \ge 0 \qquad \forall a, m > 1 \qquad \textbf{(7.10.4)}$$

In addition, Eq. (7.3.1) (the stationarity condition for arc expansions) changes with a term representing the dual variables of future minimum expansion limits: $\sum_{m' \in succ(m)} \eta_{am'}^{A}$, to:

$$0 \le \Delta_{am}^{A} \perp \quad p_m \gamma_m c_{am}^{\Delta A} + \rho_{am}^{A} - \sum_{m' \in succ(m)} \left(p_{m'} \gamma_{m'} \sum_{it} \theta^{it} \sum_{d} \lambda_{adm'}^{A,it} + \eta_{am'}^{A} \right) \ge 0 \qquad \forall a, m \qquad \textbf{(7.10.5)}$$

In the SP-MCP as defined in Section 7.3.2.2 the condition for the contractual supply obligations must be included:

$$0 \le \varepsilon_{tadm}^{T} \perp \quad FLOW_{tadm}^{T} - CON_{tadm}^{T} \ge 0 \qquad \textbf{(7.10.6)}$$

Also, the stationarity condition for flows (7.3.12) would change to include the dual price for the contracts ε_{tadm}^{T}:

$$0 \le F_{and}^{T} \perp \quad d_d \left(c_{ad}^{A} + \tau_{ad}^{A} \right) + \phi_{tn^-d}^{T} - (1-l_a)\phi_{tn^+d}^{T} - \varepsilon_{tadm}^{T} \ge 0 \qquad \forall a, d \qquad \textbf{(7.10.7)}$$

Numerical evidence shows that for the optimization-based approaches binding contracts make the model find the solution quicker, i.e., needing fewer iterations.

7.11 Implementation issues

In this appendix several issues are discussed that may not have strong academic merits, but can be very useful and save a lot of time when coding up and implementing decomposition approaches similar to the ones described in this chapter.

7.11.1 Zero capacities

In some cases when initial arc capacities are zero their dual prices in optimization problems are not uniquely defined (this issue is somewhat comparable with the issue in Section 7.5.3.1). In MCP the duals are explicitly incorporated into the stationarity conditions and therefore much more often than in optimization problems uniquely defined. This can be confusing when testing the GAMS implementations and especially when comparing the outcomes of the SP-NLP and the SP-MCP decomposition approaches. Different dual prices induce different choices for what capacities to expand (see Section 7.5.3.1.) To improve the testing process the choice was made to include a small positive value for all arcs and storages with zero initial capacity. Minimum arc capacities were 10^{-5} and minimum storage capacities equal to (10^{-3} times the number of days) in an injection season. With the unit of measurement in mcm/day and most results only being reported up to two digits, the reported results in terms of capacity expansions and volumes and market prices were not affected. However the small positive initial capacities forced dual prices in optimization and complementarity subproblems to be equal and for several iterations the master problems would provide the same answers for expansions. This greatly facilitated the testing process of the implemented decomposition approaches.

7.11.2 The SP loop and initial starting points

In the decomposition approaches the SP are solved in a for-loop. At the start of the program the SP problem is defined in GAMS. Every time an SP is solved, the same problem structure is used, however with input data representing that particular SP. In GAMS, when the same problem is used, the starting point for the solver are the values for the last found solution. That is sometimes problematic, for example when some formerly optimal flows are larger than capacities of arcs in the current problem or when formerly optimal production quantities are larger than production capacities in the current problem. Therefore, after solving each SP and storing the output data, some of the variable values are initialized to zero.

7.11.3 Solver tolerances

The convergence gap for the VI-based approach contains terms: $Q_{pdm,MP}^{P\to T,it} = \sum_{it'<it} \theta_{it'} Q_{pdm,SP}^{P\to T,it'}$ (see Eq. (7.4.9) in Section 7.4.4). The produced quantities for the MP solution are calculated as the with θ^{it} weighted quantities in the SP solutions. Theoretically: with $Q_{pdm,SP}^{P\to T,it'} \le \overline{CAP}_{pm}^{P}$ and $\sum_{it} \theta^{it} = 1 \;\Rightarrow\; Q_{pdm,MP}^{P\to T,it} = \sum_{it'<it} \theta_{it'} Q_{pdm,SP}^{P\to T,it'} \le \overline{CAP}_{pm}^{P}$ (see Section 7.4.4).[207] However, in the GAMS implementation there were several instances where $Q_{pdm,MP}^{P\to T,it} \ge \overline{CAP}_{pm}^{P}$, causing infeasibilities due to log zero ($\ln(0)$) or even log of negative values in the production cost function. The workaround was to test $Q_{pdm,MP}^{P\to T,it} \ge \overline{CAP}_{pm}^{P} - 10^{-8}$ and if so, to set $Q_{pdm,MP}^{P\to T,it} := \overline{CAP}_{pm}^{P} - 10^{-8}$. To illustrate the adjustments made below a sample from a log file of the GAMS implementation. For instance, the first line in the report shows that in iteration 16 the quantity 121.36986393 was adjusted downward to 121.36986311, implying that before the adjustment the value was 8.1.10^{-7} larger than the capacity.

```
Iter 16 Adjust qps_MP outcome to prevent LOG error N_BRA L 01 121.36986393 121.36986311
 Iter 16 Adjust qps_MP outcome to prevent LOG error N_CHL L 02 27.39726048 27.39726037
Iter 18 Adjust qps_MP outcome to prevent LOG error N_BRA L 01 121.36986338 121.36986311
Iter 19 Adjust qps_MP outcome to prevent LOG error N_BRA L 01 121.36986407 121.36986311
 Iter 19 Adjust qps_MP outcome to prevent LOG error N_CHL L 02 27.39726051 27.39726037
Iter 21 Adjust qps_MP outcome to prevent LOG error N_BRA L 01 121.36986328 121.36986311
Iter 22 Adjust qps_MP outcome to prevent LOG error N_BRA L 01 121.36986336 121.36986311
```

Figure 60: Log file sample

7.11.4 Optimality tolerances

When doing the research many numerical data instances have been solved with the various full-scale and decomposition methods developed in this chapter. Somewhat surprisingly, sometimes the results varied. Sometimes the solutions were just not unique. More often there were many solutions with objective function values (for optimization problems) that were so close to the optimal value that they were within the optimality

[207] Actually, the functional form of the production costs would induce $\ln(0)$ for $Q_{pdm,SP}^{P\to T,it'} = \overline{CAP}_{pm}^{P}$ so theoretically it is even strictly smaller than.

tolerance. For example, in large-scale models when in an optimal solution two arcs are expanded with a large capacity, expanding one of them a little bit more and the other a little bit less or moving small part of a capacity expansion one period later, has very little impact on the total profits. Surprisingly, also for smaller test problems deviations were noticeable (e.g., Table 37 in Section 7.5.3.1). The solution would seem simple: just set the solution tolerances tighter. However, tighter solution tolerances sometimes caused infeasibilities. Thus, either some feasible problems do not solve because of too tight optimality tolerances, or some solutions will not be optimal. A workaround for the decomposition approaches was to not immediately terminate the whole program when encountering an infeasibility, but to stepwise temporarily increase the optimality tolerance and try to solve again. Increasing the tolerance was usually done in sequences like 10^{-6}, 3.10^{-6}, 10^{-5}, etc. This approach was quite successful in that intermediate infeasibilities were often overcome and the algorithm would converge, sometimes many iterations later (see Section 7.5.4 and Table 40). Unfortunately, in other occasions the solutions found deviated too much from optimal solutions. For the optimization-based MP this could pose a problem when the value found for α would be too low, affecting the calculated convergence gap and inducing the algorithm to terminate prematurely. Deviating dual prices in found SP solutions would induce cuts being specified wrongly, which can induce early program terminations due to cuts being too restrictive.

7.11.5 Increasing MP solution times

Generally, in the first iterations the time to generate and solve the MP is short, even for MP with thousands of variables this takes just a few seconds. However, the size of the MP grows with every cut added and when several hundred iterations have been executed, the number of variables has grown significantly and more time is needed to generate and solve the model, growing into several minutes per iteration. After making investigations, it turned out that many of the added cuts are only binding for some of the iterations, and at some point become redundant. To reduce the model sizes a procedure was created to remove all cuts that had not been binding for fifty iterations. The result of this was that MP solution times more or less stabilized and overall calculation times for large runs were cut dramatically.

Note: this cut removing procedure might accidentally remove a cut that would have been binding in a later iteration. However, that would not be a problem. In such a situation, the MP would find a solution violating this removed cut. When that solution would be suggested to the SP, the SP results would result in a new cut providing the necessary bounds to the MP.

7.11.6 Production costs and dual prices at full capacity

When solving an MCP with the Golombek functional form for production costs, some optimal production quantities may equal capacities. Then when calculating the production costs for the producers *ex-post*, GAMS will give an error message due to $\ln(0)$. Storing the solution and using it as a starting point in a new run, will cause GAMS terminate due to a function domain error. The work-around for this has been to slightly increase the capacity value that is used in the production cost calculations, but maintain the original value for the production capacity in the capacity restriction. This implies that the calculated (marginal) production costs are a bit too small, but the impact is negligible for a small enough adjustment. The value used is 10^{-5}.

7.11.7 Convex combination of binding constraints

Eq. (7.3.6) provides an equality condition: $\sum_{it} \theta^{it} = 1$. It is written as: $\sum_{it} \theta^{it} - 1 = 0$, however when implementing it in GAMS, it does not solve. What does solve is: $1 - \sum_{it} \theta^{it} = 0$. The explanation is that GAMS requires consistency for how stationarity conditions are specified. The value 1 comes from the coefficient of α in the objective function of the MP-LP: (7.2.1), and the -1 for all θ^{it} comes from the coefficient of α in the Benders cuts, Eq. (7.2.4), when specified as a less than or equal to zero constraint: $0 \geq -\alpha + (\text{other terms})$. Therefore: $1 - \sum_{it} \theta^{it} = 0$ is correct and should be used.

8 Summary

This dissertation develops deterministic and stochastic multi-period mixed complementarity problems (MCP) for the global natural gas market, as well as solution approaches for large-scale stochastic MCP. The contributions include the development of a detailed representative model for the global natural gas market, the development of a representative stochastic model for the global natural gas market and implementing and solving stochastic MCP with up to 117 thousand variables, and the application of a Benders decomposition approach for stochastic MCP.

Contemporary societies depend heavily on the use of energy. Currently, natural gas provides slightly over one-fifth of energy used worldwide. Projections show a growth in gas demand of 52% between 2006 and 2030, inducing a slight increase of the share of natural gas in the global primary energy supply (International Energy Agency, 2008).

Chapter 1 provides an introduction into the significant role of natural gas in the energy supply. Long-distance transport of liquefied natural gas has grown significantly in recent decades, and regional markets are gradually integrating into one global gas market. The importance of natural gas in the energy supply of many countries and the dependencies resulting from major gas imports have leaded to supply security considerations. Russia, Qatar and Iran hold over 50% of proved global natural gas reserves and their membership of the Gas Exporting Countries Forum gives rise to worries about market power exertion (BP, 2010).

Governments and companies have realized a need for good quantitative models to support policy development and businesses strategies. Other market developments that stimulated the development of quantitative models include policy for the liberalization and privatization of national energy markets in the United States and in the European Union and increasing concerns about the impact of greenhouse gases such as carbon dioxide on nature and the environment.

The three major contributions of this dissertation are

- The development of a multi-period global gas market model that can adequately represent market power and other main issues arising in policy development.
- The development of an extensive-form stochastic natural gas market model that can adequately address market uncertainties by allowing players to hedge decisions.
- The extension and application of a decomposition approach to solve large-scale stochastic natural gas market models.

Chapter 2 presents literature relevant for natural gas market modeling. Some mathematical and game theoretical concepts and notation are introduced. The advantages and disadvantages of various approaches are discussed, as well as the considerations to choose for mixed complementarity problems for modeling.

Chapter 3 gives an extensive overview of the various parts of the natural gas supply chain. Several simplifications and assumptions are necessary to develop a computationally tractable model representative for the global natural gas market. The resulting model, the World Gas Model (WGM), is described in detail in terms of the optimization problems, operational constraints and market-clearing conditions. The Karush-Kuhn-Tucker conditions are derived and the WGM is cast as a mixed complementarity problem.

Chapter 4 presents and discusses a number of cases analyzed with the World Gas Model. The first case is the Base Case which represents a business-as-usual scenario. The Base Case is calibrated so that the model outcomes closely match the state of the world and the projections provided by the institutions such as the International Energy Agency and the Energy Information Administration. Three alternative cases provide insight in how various regions are affected by different market developments, due to characteristics such as geographical location and the availability of domestic gas resources.

In the Cartel Case (the second case), the member countries of the Gas Exporting Countries Forum collude as a cartel.[208] The cartel enforces maximum market power by operating through a single trading entity. The lower supplies to importing regions such as

[208] www.gecforum.org

Europe and Japan and South Korea induce much higher market prices and higher profits for all gas traders. Non-cartel members enjoy the largest profit increases and by the end of the time horizon annual profits of cartel members are lower than in the Base Case. The impact of a gas market cartelization on self-sufficient regions such as North America is negligible.

The third case, the Unconv Case, addresses lower availability of unconventional gas in the United States. This case is inspired by the significant environmental concerns related to unconventional gas production and the possible consequences of strict environmental legislation. This case investigates the impact of a large reduction of unconventional gas production capacities in the United States. Relative to the Base Case North American prices would be dramatically higher, inducing large liquefied natural gas imports. Consequently there would be less liquefied natural gas available for other regions, resulting in slightly higher prices and pipeline trade in those other regions. Generally the impact in terms of market prices and consumed volumes would be relatively modest in most parts of the world except for North America.

The fourth case, the Transp Case, provides a sensitivity analysis on lower future transport costs. Investment costs in new infrastructure and the costs for infrastructure usage are reduced, thereby increasing the competitiveness of supply regions farther away from the importing markets. As a result global production and consumption would be higher than in the Base Case. The local effects vary. Supplies by most exporters increase, however some supplies from high-cost producers that trade most or all of their gas regionally, such as Norway and the Netherlands, are pushed out by the cheaper long-distance supplies.

Chapter 5 discusses various modeling approaches developed in the literature to address the effects of input parameter uncertainty on the optimal decisions of model agents. Stochastic modeling approaches are presented for several types of optimization and equilibrium models. Stochastic models can contain large numbers of variables and for many model types such large models can have very long calculation times. Decomposition methods can provide (approximate) solutions for such stochastic models in times short enough for practical applications. Several decomposition methods are discussed and some arguments are provided for the application of a Benders decomposition approach to solve a large-scale version of the World Gas Model.

Chapter 6 illustrates the consequences of implementing a stochastic approach for the natural gas market. A model with four scenarios is analyzed. The scenarios vary in the possible coming into existence of a natural gas market cartel in the second model period and a faster depletion of natural gas reserves in the major gas importing regions starting in the fifth model period.

The stochastic results show that on an aggregate level the effects of the stochastic modeling approach seem rather modest. The timing of investments in capacity expansions is affected, but once a random market characteristic has played out, model results seem to converge to the results of deterministic models. However, when looking into the details several interesting results can be found and significant shifts in the actual location of infrastructure investments are present, affecting local market situations.

Chapter 7 discusses two types of decomposition approaches. Various generalized Benders methods are presented that can be used for solving large-scale stochastic optimization models, including perfectly-competitive welfare-maximization market models. For several numerical experiments the convergence characteristics in terms of number of iterations and solution times are discussed. Next, the variational-inequality based decomposition approach developed in (Gabriel & Fuller 2010) is implemented but specialized to the setting of a stochastic multi-period natural gas market and applied to problems with many first-stage (complicating) decision variables.

This implementation is the first application of a decomposition approach to solve a large-scale stochastic natural gas market model with several hundreds of first-stage capacity expansion variables and market players exerting various levels of market power.[209] The results show that the decomposition approach has the potential to greatly reduce the

[209] An implementation of Benders decomposition for large-scale electricity market models is Cabero et al. (2010). They handle the complexities induced by market power exertion in the master problems. Their approach needs a hundred iterations before a first feasible solution is determined. A major advantage of the approach developed by Gabriel and Fuller (2010) is that all solutions are feasible.

solution time of large-scale stochastic models. An unanticipated research outcome is that the decomposition approach based on (Gabriel & Fuller 2010) greatly reduced calculation times for optimization models cast as MCP, compared to Benders decomposition approaches.

Numerical issues and some characteristics of the software and solvers used pose challenges and have limited the size of models solved successfully in the numerical experiments. Several numerical challenges have been addressed and more research is needed to assess the potential of Benders decomposition for solving large-scale stochastic MCP.

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