ABSTRACT

Title of dissertation: DELAY MINIMIZATION IN ENERGY CONSTRAINED WIRELESS COMMUNICATIONS

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In wireless communications and networks, especially for many real-time applications, the average delay packets experience is an important quality of service criterion. Therefore, it is imperative to design advanced transmission schemes to jointly address the goals of reliability, high rates and low delay. Achieving these objectives often requires careful allocation of given resources, such as energy, power, rate, among users. It also requires a close collaboration between physical layer, medium access control layer, and upper layers, and yields cross-layer solutions.

We first investigate the problem of minimizing the overall transmission delay of packets in a multiple access wireless communication system, where the transmitters have average power constraints. We formulate the problem as a constrained optimization problem, and then transform it into a linear programming problem. We show that the optimal policy has a threshold structure: when the sum of the queue lengths is larger than a threshold, both users should transmit a packet during the current slot; when the sum of the queue lengths is smaller than a threshold, only one of the users, the one with the longer queue, should transmit a packet during the

current slot.

Then, we study the delay-optimal rate allocation in a multiple access wireless communication system. Our goal is to allocate rates to users, from the multiple access capacity region, based on their current queue lengths, in order to minimize the average delay of the system. We formulate the problem as a Markov decision problem (MDP) with an average cost criterion. We first show that the value function is increasing, symmetric and convex in the queue length vector. Taking advantage of these properties, we show that the optimal rate allocation policy is one which tries to equalize the queue lengths as much as possible in each slot, while working on the dominant face of the capacity region.

Next, we extend the delay-optimal rate allocation problem to a communication channel with two transmitters and one receiver, where the underlying rate region is approximated as a general pentagon. We show that the delay-optimal policy has a switch curve structure. For the discounted-cost problem, we prove that the switch curve has a limit along one of the dimensions. The existence of a limit in the switch curve along one of the dimensions implies that, once the queue state is beyond the limit, the system always operates at one of the corner points, implying that the queues can be operated partially distributedly.

Next, we shift our focus from the average delay minimization problem to transmission completion time minimization problem in energy harvesting communication systems. We first consider the optimal packet scheduling problem in a single-user energy harvesting wireless communication system. In this system, both the data packets and the harvested energy are modeled to arrive at the source node randomly. Our goal is to adaptively change the transmission rate according to the traffic load and available energy, such that the time by which all packets are delivered is minimized. Under a deterministic system setting, we develop an optimal off-line scheduling policy which minimizes the transmission completion time, under causality constraints on both data and energy arrivals.

Then, we investigate the transmission completion time minimization problem in a two-user additive white Gaussian noise (AWGN) broadcast channel, where the transmitter is able to harvest energy from the nature. We first analyze the structural properties of the optimal transmission policy. We prove that the optimal total transmit power has the same structure as the optimal single-user transmit power. We also prove that there exists a *cut-off* power level for the stronger user. If the optimal total transmit power is lower than this level, all transmit power is allocated to the stronger user, and when the optimal total transmit power is larger than this level, all transmit power above this level is allocated to the weaker user. Based on these structural properties of the optimal policy, we propose an algorithm that yields the globally optimal off-line scheduling policy.

Next, we investigate the transmission completion time minimization problem in a two-user AWGN multiple access channel. We first develop a *generalized iterative backward waterfilling* algorithm to characterize the maximum departure region of the transmitters for any given deadline. Then, based on the sequence of maximum departure regions at energy arrival epochs, we decompose the transmission completion time minimization problem into a convex optimization problem and solve it efficiently.

Finally, we investigate the average delay minimization problem in a single-user communication channel with an energy harvesting transmitter. We consider three different cases. In the first case, both the data packets and the energy to be used to transmit them are assumed to be available at the transmitter at the beginning. In the second case, while the energy is available at the transmitter at the beginning, packets arrive during the transmissions. In the third case, the packets are available at the transmitter at the beginning and the energy arrives during the transmissions, as a result of energy harvesting. In each scenario, we find the structural properties of the optimal solution, and develop iterative algorithms to obtain the solution.

DELAY MINIMIZATION IN ENERGY CONSTRAINED WIRELESS COMMUNICATIONS

by

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DEDICATION

To my parents.

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Chapter 1

Introduction

In modern wireless communication systems, especially for many real-time applications, the delay packets experience is an important quality of service criterion. Therefore, in such systems, allocating the given resources, such as average power, energy, etc., to minimize the average delay, becomes an important issue. Since power and energy are physical layer resources, and the delay is a medium access control layer issue, such resource allocation problems require close collaboration of physical and medium access control layers, and yield cross-layer solutions. In addition, in many circumstances, such problems require treatments that combine information theory and queueing theory to obtain optimal solutions [1].

References [2–7] analyze the trade-off relationship between power and delay for a single-user communication system. Random arrivals queue at the transmitter to wait to be transmitted. In each slot, the transmitter adapts its service rate, i.e., transmission rate, according to the queue length and the channel state, as well as the average power constraint, to minimize the average delay. Reference [2] (see also [3]) formulates the problem as a dynamic programming problem and develops a delay-power tradeoff curve. References [4] and [5] determine some structural properties of the optimal power/rate allocation policy. Reference [6] derives bounds on the average delay in a system with a single queue concatenated with a multi-layer encoder. Reference [7] proposes a dynamic programming formulation to find optimal power, channel coding and source coding policies with a delay constraint. Reference [8] considers the delay-optimal transmission policy for the secondary user in a cognitive multiple access system. It formulates the problem as a one-dimensional Markov chain and derives an analytical result to minimize the average delay of the secondary user under an average power constraint.

The delay-optimal rate allocation in multiple access channels has been investigated in the literature as well. Reference [9] considers a symmetric Gaussian multiple access channel (MAC), whose capacity region for two-users is a symmetric pentagon. Reference [9] proves that in order to minimize the *packet* delay, the system should operate at an extreme point of the MAC capacity region, i.e., at one of the two corner points of the symmetric pentagon. In particular, [9] determines explicitly the corner point the system should operate at as a function of the queue sizes, by proving that the larger rate should be given to the user with the larger queue size, hence the name of the proposed policy: longer-queue-higher-rate (LQHR). Reference [10] generalizes [9] to a potentially asymmetric setting, and proves that the system should again operate at one of the two corner points of the capacity region, which in this case is a potentially asymmetric pentagon. This proves that the delay-optimal policy has a switch structure, i.e., that the queue state space should be divided into two, and in each region, the system should operate at one of the two corner points. However, unlike the symmetric case in [9], the explicit form of the switch curve is unknown. Reference [11] develops a policy named "modified LQHR" which works at a corner point of the pentagon when the queue lengths are different, and switches to the mid-point of the dominant face of the pentagon when the queue lengths become equal. The “modified LQHR” algorithm is shown to minimize the average *bit* delay in a symmetric system. The third chapter of [12] extends “modified LQHR” to a symmetric M-user scenario.

The trade-off relationship between delay and energy has been well studied in traditional battery powered (unrechargeable) systems. References [13–18] investigate energy minimization problems with various deadline constraints. Reference [13] considers the problem of minimizing the energy in delivering all packets to the destination by a deadline. It develops a *lazy scheduling algorithm*, where the transmission times of all packets are equalized as much as possible, subject to the deadline and causality constraints, i.e., all packets must be delivered by the deadline and no packet may be transmitted before it has arrived. This algorithm also elongates the transmission time of each packet as much as possible, hence the name, *lazy* scheduling. Under a similar system setting, [14] proposes an interesting novel calculus approach to solve the energy minimization problem with individual deadlines for each packet. Reference [15] develops dynamic programming formulations and determines optimality conditions for a situation where channel gain varies stochastically over time. Reference [16] considers energy-efficient packet transmission with individual packet delay constraints over a fading channel, and develops a recursive algorithm to find an optimal off-line schedule. This optimal off-line scheduler equalizes the energy-rate derivative function as much as possible subject to the deadline and causality constraints. References [17] and [18] extend the single-user problem to multi-user scenarios.

In Chapter 2, we generalize [8] to a two-user multiple access system, where both users have equal priority. Our goal is to minimize the average delay of the packets in the system under an average power constraint for each user. Unlike [2, 4, 5], where the rate per slot is a continuous variable, we restrict the transmission rate for each user in a slot to be either zero or one packet per slot. Our objective is to find a set of transmission probabilities according to the queue lengths of both users, so that the average delay in the system is minimized. Compared to [2, 4, 5], our model has a more restricted policy space at each stage, however, this model enables us to construct a two-dimensional discrete-time Markov chain and eventually gives us a closed-form optimal solution. We show that the optimal transmission policy has a threshold structure, i.e., if the sum of the queue lengths exceeds a threshold, both users transmit a packet from their queues, and if the sum of the queue lengths is smaller than a threshold, only one user, which has the larger queue length, transmits a packet from its queue, while the other user remains silent (equal queue length case is resolved by flip of a potentially biased coin).

In Chapter 3, we aim to minimize the average delay through rate allocation in a symmetric MAC. We consider a time-slotted system. In each slot, bits arrive at the transmitters randomly according to some general distribution. At the beginning of each slot, we allocate transmission rates from within the MAC capacity region to the users, based on their current queue lengths, to minimize the average delay. We formulate the problem as an average cost Markov decision problem (MDP). We first analyze the corresponding discounted cost MDP, and obtain some properties of the value function. Based on these properties, we prove that the delay optimal rate allocation policy for this discounted MDP is to equalize the queue lengths in each slot as much as possible. We then prove that this *queue balancing policy* is optimal for the average cost MDP as well.

In Chapter 4, we extend the delay-optimal rate allocation problem into a communication channel with a general pentagon rate region. Different from the Gaussian MAC capacity region, the pentagon we assume does not have a 45° dominant face. Through value iteration, we prove that a switch curve structure exists in the queue state space. Next, we prove that for the discounted-cost MDP, the switch curve has a limit on one of the queue lengths, i.e., when one of the queue lengths exceeds a threshold, the transmitters always operate at the corner point which has the larger sum-rate. That is, the delay-optimal policy favors throughput-optimality (i.e., larger sum-rate) unless the first queue gets close to empty, in which case, the policy favors balancing queue lengths. Our result has two practical implications: First, it gives a *partial analytical characterization* for the delay-optimal switch curve. Secondly, it implies that we can operate the queues *partially distributedly*, in that, if the current queue length of the first user is larger than the limit, then this user does not need to know the current queue length of the other user in order to decide about the rate point at which it should operate on the rate region. The optimal policy also indicates that always operating at the maximum sum-rate point is not optimal. The optimal rate allocation policy trades some of the instantaneously achievable sum-rate in favor of balancing the queue lengths, with the goal of minimizing the overall delay.

In Chapters 5, 6, 7, 8, we consider wireless communication networks where nodes are able to harvest energy from nature. In our work, we do not focus on how energy is harvested, instead, we focus on developing transmission methods that take into account the *randomness* both in the *arrivals of the data packets* as well as in the *arrivals of the harvested energy*. Since devising *on-line* algorithms that update the instantaneous transmission rate and power in *real-time* as functions of the current data and energy queue lengths is an intractable mathematical problem for now, here, we aim to develop optimal *off-line* algorithms instead.

In Chapter 5, we consider a single-user communication channel with an energy harvesting transmitter. We assume that an initial amount of energy is available at $t = 0$. As time progresses, certain amounts of energies will be harvested. For the data arrivals, we consider two different scenarios. In the first scenario, we assume that packets have already arrived and are ready to be transmitted at the transmitter before the transmission starts. In the second scenario, we assume that packets arrive during the transmissions. However, as in the case of energy arrivals, we assume that we know exactly when and in what amounts data will arrive. Subject to the energy and data arrival constraints, our purpose is to minimize the time by which all packets are delivered to the destination through controlling the transmission rate and power. Since we do not know the exact amount of energy to be used in the transmissions, we develop an algorithm, which first obtains a good lower bound for the final total transmission duration at the beginning, and performs rate and power allocation based on this lower bound. The procedure works iteratively until all of the transmission rates and powers are determined. We prove that the transmission policy obtained through this algorithm is globally optimum.

In Chapter 6, we consider a multi-user extension of the problem studied in Chapter 5. In particular, we consider a wireless broadcast channel with an energy harvesting transmitter and two receivers. References [19, 20] investigate two related problems. The first problem is to maximize the throughput (number of bits transmitted) with a given deadline constraint, and the second problem is to minimize the transmission completion time with a given number of bits to transmit. These two problems are "dual" to each other in the sense that, with a given energy arrival profile, if the maximum number of bits that can be sent by a deadline is B^* in the first problem, then the minimum time to transmit B^* bits in the second problem must be the deadline in the first problem, and the optimal transmission policies for these two problems must be identical. In Chapter 6, we will follow this "dual problems" approach. We will first attack and solve the first problem to determine the structural properties of the optimal solution. We will then utilize these structural properties to develop an iterative algorithm for the second problem. Our iterative approach has the goal of reducing the two-user broadcast problem into a single-user problem as much as possible, and utilizing the single-user solution in Chapter 5.

In Chapter 7, we consider the transmission completion time minimization problem in a two-user rechargeable multiple access channel. As in Chapter 6, we first aim to characterize the maximum number of bits both users can transmit for any given time T. We propose a *generalized iterative backward waterfilling* algorithm to achieve the boundary points of the maximum departure region for any given deadline T. Then, based on the solution of this "dual" problem, we go back to the transmission completion time minimization problem, simplify it into standard convex optimization problems, and solve it efficiently. In particular, we first characterize the maximum departure region for every energy arrival epoch, and based on the location of the given (B_1, B_2) on the maximum departure region, we narrow down the range of the minimum transmission completion time to be between two consecutive epochs. Based on this information, we propose to solve the problem in two steps. In the first step, we solve for the optimal power policy sequences to achieve the minimum T, so that (B_1, B_2) is on the maximum departure region for this T. This step can be formulated as a convex optimization problem. Then, with the optimal power policy obtained in the first step, we search for the optimal rate policy sequences from the capacity regions defined by the power sequences to finish B_1, B_2 bits. The second step is formulated as a linear programming problem. In addition, we further simplify the problem by exploiting the optimal structural properties for two special scenarios.

In Chapter 8, we revisit the average delay minimization problem in an energy harvesting single-user system. Under a deterministic setting, our aim is to adaptively allocate the energy over all packets according to the available amount of energy and number of packet at the transmitter, in a way to minimize the overall delay of the system. The most general version of the problem is complicated. In Chapter 8, we will consider three scenarios, starting with the simplest setting and proceeding with progressively more complicated settings. In the first scenario, we assume that the transmitter has a fixed number of packets to transmit, and a fixed amount energy to use in its transmissions. In the second scenario, we assume that the transmitter has a fixed amount of energy, but the packets arrive during the transmissions. In the third scenario, we assume that the transmitter has a fixed number of packets available at the beginning, but the energy arrives during the transmissions. This last setting models an energy harvesting transmitter which harvests energy from the nature by using a rechargeable battery. For each scenario, we develop iterative algorithms and dynamic programming formulation to obtain the optimal solution.

Chapter 2

Average Delay Minimization for Average Power Constrained Multiple Access Communications

2.1 Introduction

In many applications, the average delay packets experience is an important quality of service criterion. Therefore, it is important to allocate the given resources, e.g., average power, energy, etc., in a way to minimize the average delay packets experience. Since power and energy are physical layer resources, and the delay is a medium access control layer issue, such resource allocation problems require close collaboration of physical and medium access control layers, and yield cross-layer solutions. Our goal in this chapter is to combine information theory and queueing theory to devise a transmission protocol which minimizes the average delay experienced by packets, subject to an average power constraint at each transmitter.

Similar goals have been undertaken by various authors in recent years. Reference [21] considers a time-slotted system with N queues and one server. The length of the slot is equal to the transmission time of a packet in the queue. In each slot, the controller allocates the server to one of the connected queues, such that the average delay in the system is minimized. The authors develop an algorithm named longest connected queue (LCQ), where the server is allocated to the longest of all connected queues at any given slot. The authors prove that in a symmetric system, LCQ algorithm minimizes the average delay. Reference [21] does not consider the issue of power consumption during transmissions.

Reference [22] combines information theory and queueing theory in a multi-access communication over an additive Gaussian noise channel. Authors consider a continuous time system, where the arrival of packets is a Poisson process, and the service time required for each packet is random. Once a packet arrives, it is transmitted immediately with a fixed power, i.e., there are no queues at the transmitters. Each transmitter-receiver pair treats the other active pairs as noise. Because of the interference from the other transmitters, the service rate for each packet is a function of the number of active users in the system. Reference [22] derives a relationship between the average delay and a fixed probability of error requirement.

References [2–7, 9, 11, 23] consider the data transmission problem from both information theory and queueing theory perspectives. Reference [9] (see also [23], [11]) aims to minimize the average delay through rate allocation in a multiple access scenario in additive Gaussian noise. Unlike [22], in the setting of [9], packets arrive randomly into the buffers of the transmitters. When the queue at a transmitter is not empty, it transmits a packet with a fixed power. Simultaneously achievable rates are characterized by the capacity region of a multiple access channel (MAC), which, for the non-fading Gaussian case, is a pentagon. The purpose of [9] is to find an operating point on the capacity region of the corresponding MAC such that the average delay is minimized. The author develops the longer-queue-higher-rate

(LQHR) allocation strategy in the symmetric MAC case, which is shown to minimize the average delay of the packets. The LQHR allocation scheme selects an extreme point (i.e., a corner point) in the MAC capacity region.

Reference [2] (see also [3]) considers the problem of rate/power control in a single-user communication over a fading channel. It considers a discrete-time model, and investigates adapting rate/power in each slot according to the queue length, source state and channel state. The objective is to minimize the average power with delay constraints. It discusses two transmission models. In the first model, the transmission time of a codeword is fixed, while the rate varies from block to block. In the second model, the transmission time for each codeword varies. It formulates the problem into a dynamic programming problem and develops a delay-power tradeoff curve.

References [4–7] have similar formulations. Reference [4] uses dynamic programming to numerically calculate the optimal power/rate control policies that minimize the average delay in a single-user system under an average power constraint. Reference [6] derives bounds on the average delay in a system with a single queue concatenated with a multi-layer encoder. Reference [5] formulates the power constrained average delay minimization problem into a Markov decision problem (MDP) and analyzes the structure of the optimal solution for a single-user fading channel. Reference [7] proposes a dynamic programming formulation to find optimal power, channel coding and source coding policies with a delay constraint. As in [2], in these papers as well, because of the large number of possible rate/power choices at each stage, it is almost impossible to get analytical optimal solutions.

Reference [8] considers a cognitive multiple access system. In the model of [8], the primary user (PU) always transmits a packet during a slot whenever its queue is not empty. The secondary user (SU) always transmits when the PU is idle, and it transmits with some probability (which is a function of its own queue length) when the PU is active. The receiver operates at the corner point of the MAC capacity region where the SU is decoded first and the PU is decoded next, so that even though the SU experiences interference from the PU, the PU is always decoded interference-free. Reference [8] aims to minimize the average delay through controlling the transmission probability of the SU. It formulates the problem as a one-dimensional Markov chain and derives an analytical result to minimize the average delay of the SU under an average power constraint.

In this chapter, we generalize [8] to a two-user multi-access system, where both users have equal priority. Our goal is to minimize the average delay of the packets in the system under an average power constraint for each user. As in [2, 4, 5, 8], we consider a discrete-time model. We divide the transmission time into time slots. Packets arriving at the transmitters are stored in the queues at each transmitter. In each slot, each user decides on a transmission rate based on the current lengths of both queues. Unlike [2, 4, 5], where the rate per slot is a continuous variable, we restrict the transmission rate for each user in a slot to be either zero or one packet per slot. We define the probabilities of choosing each transmission rate pair, which can be $(0,0)$, $(0,1)$, $(1,0)$ or $(1,1)$, for each given pair of queue lengths.

Our objective is to find a set of transmission probabilities that minimizes the average delay while satisfying the average power constraints for both users. As in [8], there are two main reasons that we introduce transmission probabilities: First, a randomized policy is more general than a deterministic policy; probability selections of 0 and 1 correspond to a deterministic policy, which is a special case of the randomized policy. Secondly, since we cannot choose arbitrary departure rates in each slot, the use of transmission probabilities enables us to control the average rate per slot at a finer scale. Compared to [2, 4, 5], our model has a more restricted policy space at each stage, however, this model enables us to construct a two-dimensional discrete-time Markov chain and eventually gives us a closed-form optimal solution.

In the rest of this chapter, we first express the average delay and the average power consumed for each user as functions of the transmission probabilities and steady state distribution of the queue lengths. We then transform our problem into a linear programming problem, and derive the optimal transmission scheme analytically. We show that the optimal transmission policy has a threshold structure, i.e., if the sum of the queue lengths exceeds a threshold, both users transmit a packet from their queues, and if the sum of the queue lengths is smaller than a threshold, only one user, which has the larger queue length, transmits a packet from its queue, while the other user remains silent (equal queue length case is resolved by flip of a potentially biased coin). We provide a rigorous mathematical proof for the optimality of the solution. We also provide numerical examples for both symmetric and asymmetric settings.

2.2 System Model

2.2.1 Physical Layer Model

We consider a discrete-time additive Gaussian noise multiple access system with two transmitters and one receiver. The received signal is

$$Y = \sqrt{h_1}X_1 + \sqrt{h_2}X_2 + Z \tag{2.1}$$

where X_i is the signal of user i, $\sqrt{h_i}$ is the channel gain of user i, and Z is a Gaussian noise with zero-mean and variance σ^2. Here, h_1 and h_2 are real constants, with $h_1 \neq h_2$ in general.

In this two-user system, since the MAC capacity region is given as [24]

$$R_1 \leq \frac{1}{2}\log\left(1 + \frac{h_1 P_1}{\sigma^2}\right) \tag{2.2}$$

$$R_2 \leq \frac{1}{2}\log\left(1 + \frac{h_2 P_2}{\sigma^2}\right) \tag{2.3}$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + \frac{h_1 P_1 + h_2 P_2}{\sigma^2}\right) \tag{2.4}$$

the region of feasible received powers is given by [18]

$$h_1 P_1 \geq \sigma^2(2^{2R_1} - 1) \tag{2.5}$$

$$h_2 P_2 \geq \sigma^2(2^{2R_2} - 1) \tag{2.6}$$

$$h_1 P_1 + h_2 P_2 \geq \sigma^2(2^{2(R_1+R_2)} - 1) \tag{2.7}$$

In each slot, the transmitters adjust their transmitted powers to achieve the desired transmission rates. We assume that for each user, the average transmitted power over all of the slots must satisfy a constraint. We denote the average power constraints for the first and second user as P_{1avg} and P_{2avg}, respectively.

2.2.2 Medium Access Control Layer Model

In the medium access control layer, we assume that packets arrive at the transmitters at a uniform size of B bits per packet. We partition the time into small slots such that we have at most one packet arrive and/or depart during each slot. Let $a_1[n]$ and $a_2[n]$ denote the number of packets arriving at the first and second transmitters, respectively, during time slot n; see Figure 2.1. We assume that the packet arrivals are i.i.d. from slot to slot, and the probabilities of arrivals are

$$Pr\{a_i[n] = 1\} = \theta_i \tag{2.8}$$

$$Pr\{a_i[n] = 0\} = 1 - \theta_i \tag{2.9}$$

where θ_i is the arrival rate for user i, $i = 1, 2$.

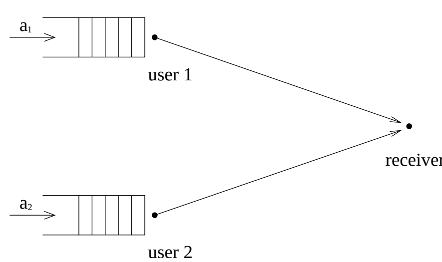


Figure 2.1: System model.

There is a buffer with capacity N at each transmitter to store the packets, where N is a finite positive integer. Once the buffer is not empty, the transmitters decide to transmit one packet in the slot with some probability based on the current lengths of both queues. Let $d_1[n]$ and $d_2[n]$ denote the number of packets transmitted in time slot n. The queue length in each buffer evolves according to

$$q_1[n+1] = (q_1[n] - d_1[n])^+ + a_1[n] \tag{2.10}$$

$$q_2[n+1] = (q_2[n] - d_2[n])^+ + a_2[n] \tag{2.11}$$

where $(x)^+$ denotes $\max(0, x)$.

The departure rate for each queue in each slot is either zero or one packet per slot, and the decision whether it should be zero or one packet per slot depends on the current queue lengths. When both queues are empty, the departure rates for both queues should be zero packet per slot. In all other situations where there is at least one packet in at least one of the queues, the departure rates for both queues should not be zero packet per slot simultaneously. This is because, keeping both transmitters idle does not save any power, but causes unnecessary delay. Therefore, in these situations, there are three possible departure rate pairs: $(d_1, d_2) = (1, 0), (0, 1)$ or $(1, 1)$, i.e., one packet is transmitted from queue 1 and no packet is transmitted from queue 2; no packet is transmitted from queue 1 and one packet is transmitted from queue 2; or, one packet is transmitted from each queue. We enumerate them as d^1, d^2, d^3. When the first queue length is i and the second queue length is j, we define the probabilities of choosing each pair of these departure rates as g_{ij}^1,

g_{ij}^2, g_{ij}^3, respectively. Note that $g_{ij}^1 + g_{ij}^2 + g_{ij}^3 = 1$. We also note that g_{ij}^1, g_{ij}^2, g_{ij}^3, for $i = 0, 1, \ldots, N$ and $j = 0, 1, \ldots, N$ are the main parameters we aim to choose optimally in this chapter.

The state space of the Markov chain consists of all possible pairs of queue lengths. We denote the state as $\mathbf{q} \triangleq (q_1, q_2)$. When both of the queues are empty, i.e., $\mathbf{q}[n] = (0, 0)$, transmitters have no packet to send, and from (2.10)-(2.11), $\mathbf{q}[n+1] = \mathbf{a}[n]$. The corresponding transition probabilities in this case are:

$$
\begin{aligned}
Pr\{\mathbf{q}[n+1] = (0,0)|\mathbf{q}[n] = (0,0)\} &= (1-\theta_1)(1-\theta_2) \\
Pr\{\mathbf{q}[n+1] = (1,0)|\mathbf{q}[n] = (0,0)\} &= \theta_1(1-\theta_2) \\
Pr\{\mathbf{q}[n+1] = (0,1)|\mathbf{q}[n] = (0,0)\} &= \theta_2(1-\theta_1) \\
Pr\{\mathbf{q}[n+1] = (1,1)|\mathbf{q}[n] = (0,0)\} &= \theta_1\theta_2
\end{aligned}
\tag{2.12}
$$

When one of the queues is empty, there is only one possible departure rate pair, which is either $(0, 1)$ or $(1, 0)$, depending on which queue is empty. Therefore, from our argument above, the departure probabilities should not be free parameters, but must be chosen as $g_{i0}^1 = g_{0j}^2 = 1$. The corresponding transition probabilities are:

$$
\begin{aligned}
Pr\{\mathbf{q}[n+1] = (i-1,0)|\mathbf{q}[n] = (i,0)\} &= (1-\theta_1)(1-\theta_2) \\
Pr\{\mathbf{q}[n+1] = (i-1,1)|\mathbf{q}[n] = (i,0)\} &= \theta_2(1-\theta_1) \\
Pr\{\mathbf{q}[n+1] = (i,0)|\mathbf{q}[n] = (i,0)\} &= \theta_1(1-\theta_2) \\
Pr\{\mathbf{q}[n+1] = (i,1)|\mathbf{q}[n] = (i,0)\} &= \theta_1\theta_2
\end{aligned}
\tag{2.13}
$$

A similar argument is valid when the first queue is empty, i.e., $\mathbf{q}[n] = (0, j)$. Transition probabilities in this case can be written similar to (2.13).

When neither of the queues is empty, i.e., for $\mathbf{q}[n] = (i, j)$, where $1 \leq i, j \leq N$, the transition probabilities are:

$$\begin{aligned}
Pr\{(i-1,j-1)|(i,j)\} &= g_{ij}^3(1-\theta_1)(1-\theta_2)\\
Pr\{(i-1,j+1)|(i,j)\} &= g_{ij}^1\theta_2(1-\theta_1)\\
Pr\{(i+1,j-1)|(i,j)\} &= g_{ij}^2\theta_1(1-\theta_2)\\
Pr\{(i,j+1)|(i,j)\} &= g_{ij}^1\theta_1\theta_2\\
Pr\{(i+1,j)|(i,j)\} &= g_{ij}^2\theta_1\theta_2\\
Pr\{(i-1,j)|(i,j)\} &= g_{ij}^3\theta_2(1-\theta_1)+g_{ij}^1(1-\theta_1)(1-\theta_2)\\
Pr\{(i,j-1)|(i,j)\} &= g_{ij}^3\theta_1(1-\theta_2)+g_{ij}^2(1-\theta_1)(1-\theta_2)\\
Pr\{(i,j)|(i,j)\} &= g_{ij}^1\theta_1(1-\theta_2)+g_{ij}^2\theta_2(1-\theta_1)+g_{ij}^3\theta_1\theta_2
\end{aligned} \tag{2.14}$$

For example, the first equation in (2.14) is obtained by noting that, for the next queue state to be $(i-1, j-1)$, we need to transmit one packet from each queue and we should have no arrivals to either of the queues. The probability of this event is g_{ij}^3, probability of transmitting one packet from each queue, multiplied by $(1-\theta_1)$, probability of having no arrivals to queue 1, and $(1-\theta_2)$, probability of having no arrivals to queue 2.

In this chapter, we assume that the average power constraints are large enough to prevent any packet losses. In order to prevent overflows, we always choose to transmit one packet from a queue whenever its length reaches N. Therefore, we have $g^1_{iN} = g^2_{Nj} = g^3_{NN} = 1$. The two-dimensional Markov chain is shown in Figure 2.2.

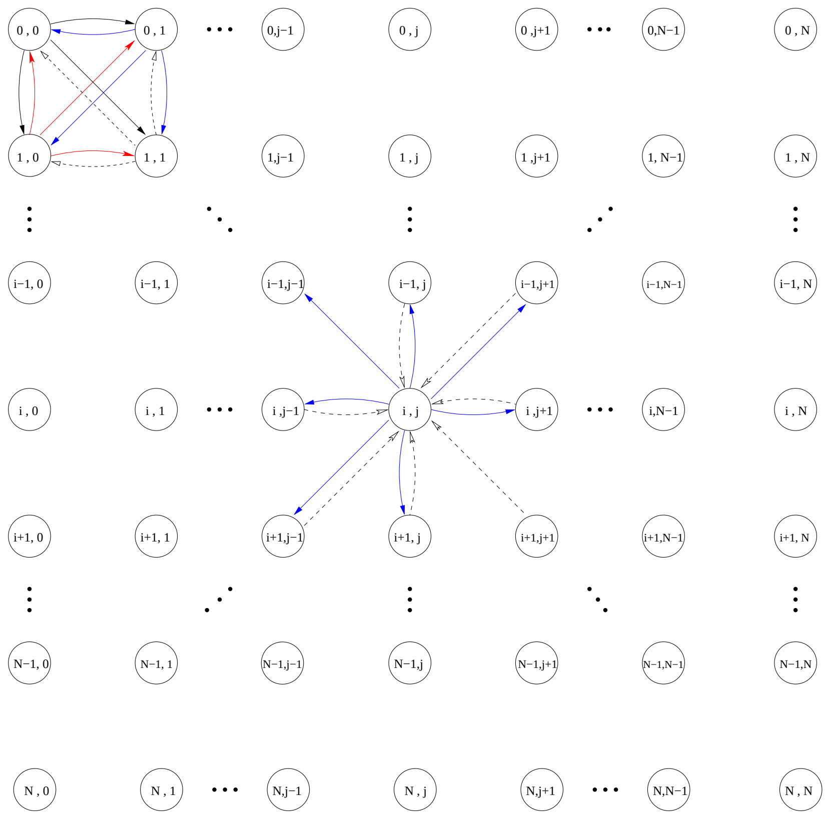


Figure 2.2: Two-dimensional Markov chain.

In [25], it is proven that, for all irreducible, positive recurrent discrete-time Markov chains with state space S, there exists a stationary distribution $\{\pi_s, s \in S\}$, which is given by the unique solution to

$$\sum_{s\in S} \pi_s p_{sr} = \pi_r, \qquad \sum_{s \in S} \pi_s = 1 \tag{2.15}$$

It is also stated that for a reducible Markov chain with a single closed positive recurrent aperiodic class and a nonempty set T, where for any $i \in T$, the probability of getting absorbed in the closed class starting from state i is 1, and the steady state distribution exists. In our problem, we first assume that the stationary distribution exists for the optimal solution. Once we determine the solution, we verify that the corresponding Markov chain has a unique stationary distribution.

Let us define the steady state distribution of this Markov chain as $\boldsymbol{\pi} = [\pi_{00}, \pi_{01}, \cdots, \pi_{0N}, \pi_{10}, \cdots, \pi_{NN}]$. Then, the steady state distribution must satisfy

$$\boldsymbol{\pi}\mathbb{P} = \boldsymbol{\pi}, \qquad \boldsymbol{\pi}\mathbf{1} = 1 \tag{2.16}$$

where $\mathbb{P}$ is the transition matrix defined by the transition probabilities (2.12)-(2.14). We can express the average number of packets in the system as $\sum_{i,j} \pi_{ij}(i + j)$. According to Little's law [25], for a fixed sample path in a queueing system, if the limits of average waiting time W and average arrival rate λ exist as time goes to infinity, then the limit of average queue length L exists and they are related as $L = \lambda W$. For our problem, in a system without overflow, these limits exist and the average delay D is equal to

$$D = \frac{1}{\theta_1 + \theta_2} \sum_{i,j} \pi_{ij}(i + j) \tag{2.17}$$

where $\theta_1 + \theta_2$ is the average arrival rate for the system.

2.3 Problem Formulation

The transmission rate for both transmitters during a slot is either one packet per slot or zero packet per slot. Equivalently, the transmission rate is either B/τ bits/channel use or 0 bits/channel use, where τ is the number of channel uses in each slot. We assume that in each slot we can use codewords with finite block length to get arbitrarily close to the boundary of feasible powers and achieve a satisfactory level of reliability.

Next, let us consider the power consumptions during each slot. When only one user transmits, since there is no interference from the other transmitter, the transmitted power for the active user needs to satisfy

$$h_i P_i \geq \sigma^2(2^{2R} - 1) \triangleq \alpha \tag{2.18}$$

where $R = B/\tau$. In order to minimize the power, the transmitted power for the active user should be α/h_i, depending on which user is transmitting. When both users transmit simultaneously, the received powers should additionally satisfy

$$h_1 P_1 + h_2 P_2 \geq \sigma^2(2^{4R} - 1) \triangleq \beta \tag{2.19}$$

The feasible transmitted power region is shown in Figure 2.3. Let us denote the received power pair as (β_1, β_2). In order to minimize the transmit power, this pair should be on the dominant face of the feasible power region, i.e., $\beta_1 + \beta_2 = \beta$. Then, the corresponding transmit power pair is $(\beta_1/h_1, \beta_2/h_2)$. Note that different operating points need different sum of transmit powers.

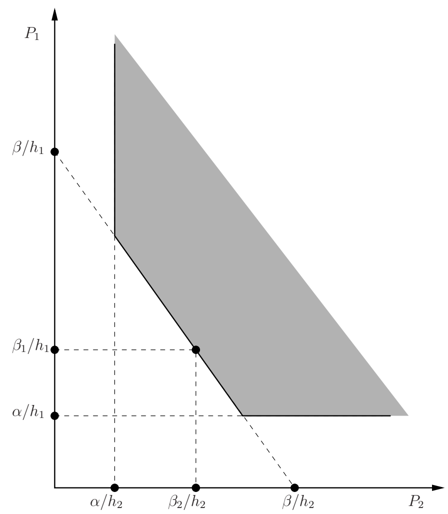


Figure 2.3: Feasible power region.

Thus, for any state $(i,j) \neq (0,0)$, the average power consumed for the first queue is $\frac{1}{h_1}(g_{ij}^1\alpha + g_{ij}^3\beta_1)$, while the average power consumed for the second queue is $\frac{1}{h_2}(g_{ij}^2\alpha + g_{ij}^3\beta_2)$. Our goal is to find the transmission policy, i.e., the probabilities g_{ij}^k, $k = 1,2,3$, $i = 0,1,\ldots,N$, $j = 0,1,\ldots,N$ along with the operating point (β_1, β_2), such that the average delay is minimized, subject to an average power constraint for each user. Therefore, our problem can be expressed as:

$$\min_{\mathbf{g},\beta_1,\beta_2} \quad \frac{1}{\theta_1+\theta_2}\sum_{i,j}\pi_{ij}(i+j) \tag{2.20}$$

$$\text{s.t.} \quad \frac{1}{h_1}\sum_{i,j}\pi_{ij}(g_{ij}^1\alpha+g_{ij}^3\beta_1)\leq P_{1avg} \tag{2.21}$$

$$\frac{1}{h_2}\sum_{i,j}\pi_{ij}(g_{ij}^2\alpha+g_{ij}^3\beta_2)\leq P_{2avg} \tag{2.22}$$

$$\boldsymbol{\pi}\mathbb{P}=\boldsymbol{\pi}, \quad \boldsymbol{\pi}\mathbf{1}=1 \tag{2.23}$$

$$g_{ij}^1+g_{ij}^2+g_{ij}^3=1, \quad i,j=0,1,\ldots,N \tag{2.24}$$

$$g_{ij}^k\geq 0, \quad i,j=0,1,\ldots,N, \quad k=1,2,3 \tag{2.25}$$

We note that the state transition matrix $\mathbb{P}$ is filled with variables in (2.12)-(2.14) which depend on g_{ij}^ks. Also, through (2.23), π_{ij}s depend on g_{ij}^ks, as well. Unlike [8], we have a two-dimensional Markov chain, and it does not admit closed-form expressions for the steady state distribution π_{ij}s in terms of g_{ij}^ks. Therefore, solving the above optimization problem becomes rather difficult. Our methodology will be to transform our optimization problem into a linear programming problem, and exploit its special structure to obtain the globally optimal solution analytically.

2.4 Analysis of the Problem

Note that $g_{ij}^1+g_{ij}^2+g_{ij}^3=1$ for any $(i,j)\neq(0,0)$, therefore $\pi_{ij}=\pi_{ij}(g_{ij}^1+g_{ij}^2+g_{ij}^3)$. Define $x_{00}=\pi_{00}$, $x_{ij}^k=\pi_{ij}g_{ij}^k$, $k=1,2,3$, $i=0,1,\ldots,N$, $j=0,1,\ldots,N$. Since g_{ij}^k is the conditional probability of choosing policy k when the system is in state (i,j), x_{ij}^k can be interpreted as the unconditional probability of staying in state (i,j) and choosing policy k. Our aim is to find optimal g_{ij}^ks. However, as we will see, our analysis will be more tractable with variables x_{ij}^ks. Once we find optimal x_{ij}^ks, we can obtain optimal g_{ij}^ks through

$$g_{ij}^k = \frac{x_{ij}^k}{\sum_{k=1}^3 x_{ij}^k} \tag{2.26}$$

Let us construct a vector of all of our unknowns $\mathbf{x} = [x_{00}, x_{01}^1, x_{01}^2, x_{01}^3, \ldots, x_{NN}^3]^T$.

First, we consider the average power consumption when average power constraints for both users are large enough such that each user is able to transmit a packet during a slot whenever its queue is not empty. In this scenario, the corresponding Markov chain has four non-transient states, (0,0), (0,1), (1,0), (1,1), and the stationary distribution is

$$\begin{aligned}
&\pi_{01} = \theta_2(1-\theta_1), \quad \pi_{00} = (1-\theta_1)(1-\theta_2), \\
&\pi_{10} = \theta_1(1-\theta_2), \quad \pi_{11} = \theta_1\theta_2
\end{aligned} \tag{2.27}$$

The average power consumption for each queue is

$$P_{1csmp} = \frac{1}{h_1}(\pi_{10}\alpha + \pi_{11}\beta_1) = \frac{1}{h_1}\left(\theta_1(1-\theta_2)\alpha + \theta_1\theta_2\beta_1\right) \tag{2.28}$$

$$P_{2csmp} = \frac{1}{h_2}(\pi_{01}\alpha + \pi_{11}\beta_2) = \frac{1}{h_2}\left(\theta_2(1-\theta_1)\alpha + \theta_1\theta_2\beta_2\right) \tag{2.29}$$

We note that

$$P_{1csmp}h_1 + P_{2csmp}h_2 = (\theta_1 + \theta_2 - 2\theta_1\theta_2)\alpha + \theta_1\theta_2\beta \tag{2.30}$$

From Figure 2.3, we note that $\beta_1, \beta_2 \geq \alpha$, therefore, each individual term in (2.30) must additionally satisfy

$$P_{1csmp} \geq \frac{1}{h_1}\theta_1\alpha \tag{2.31}$$

$$P_{2csmp} \geq \frac{1}{h_2}\theta_2\alpha \tag{2.32}$$

Therefore, if the average power constraints P_{1avg} and P_{2avg} satisfy the following inequalities

$$P_{1avg}h_1 + P_{2avg}h_2 \geq (\theta_1 + \theta_2 - 2\theta_1\theta_2)\alpha + \theta_1\theta_2\beta \tag{2.33}$$

$$P_{1avg} \geq \frac{1}{h_1}\theta_1\alpha \tag{2.34}$$

$$P_{2avg} \geq \frac{1}{h_2}\theta_2\alpha \tag{2.35}$$

then we can always find an operating point (β_1, β_2) such that $P_{1csmp} \leq P_{1avg}$ and $P_{2csmp} \leq P_{2avg}$, and we achieve the minimal possible delay in the system, which is one slot. The available power in this case is so large that the solution is trivial. If

$$P_{1avg}h_1 + P_{2avg}h_2 < (\theta_1 + \theta_2 - 2\theta_1\theta_2)\alpha + \theta_1\theta_2\beta \tag{2.36}$$

and P_{1avg} and P_{2avg} are large enough to prevent any overflows, both power constraints should be tight. Therefore, from (2.21)-(2.22), we have two equality power constraints,

$$\frac{1}{h_1}\sum_{i,j}(x_{ij}^1\alpha + x_{ij}^3\beta_1) = P_{1avg} \tag{2.37}$$

$$\frac{1}{h_2}\sum_{i,j}(x_{ij}^2\alpha + x_{ij}^3\beta_2) = P_{2avg} \tag{2.38}$$

Because the average arrival rate must be equal to the average departure rate when there is no overflow, we also have

$$\sum_{i,j}(x_{ij}^1 + x_{ij}^3) = \theta_1 \tag{2.39}$$

$$\sum_{i,j}(x_{ij}^2 + x_{ij}^3) = \theta_2 \tag{2.40}$$

Solving (2.37)-(2.40), we obtain

$$\beta_1 = \alpha + \frac{(\beta - 2\alpha)(P_{1avg}h_1 - \theta_1\alpha)}{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha} \tag{2.41}$$

$$\beta_2 = \alpha + \frac{(\beta - 2\alpha)(P_{2avg}h_2 - \theta_2\alpha)}{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha} \tag{2.42}$$

$$\sum_{i,j} x_{ij}^1 = \theta_1 - \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \tag{2.43}$$

$$\sum_{i,j} x_{ij}^2 = \theta_2 - \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \tag{2.44}$$

$$\sum_{i,j} x_{ij}^3 = \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \tag{2.45}$$

By jointly considering the normalization equation in (2.23), we also have

$$x_{00} = 1 - \frac{(\theta_1 + \theta_2)(\beta - \alpha) - (P_{1avg}h_1 + P_{2avg}h_2)}{\beta - 2\alpha} \tag{2.46}$$

Thus, we transform our optimization problem in (2.20)-(2.24) into

$$\min_{\mathbf{x}} \quad \sum_{i,j} \left(\sum_{k=1}^{3} x_{ij}^k (i+j) \right) \tag{2.47}$$

$$\text{s.t.} \quad x_{00} = 1 - \frac{(\theta_1 + \theta_2)(\beta - \alpha) - (P_{1avg}h_1 + P_{2avg}h_2)}{\beta - 2\alpha} \tag{2.48}$$

$$\sum_{i,j} x_{ij}^1 = \theta_1 - \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \tag{2.49}$$

$$\sum_{i,j} x_{ij}^2 = \theta_2 - \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \tag{2.50}$$

$$\sum_{i,j} x_{ij}^3 = \frac{P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1 + \theta_2)\alpha}{\beta - 2\alpha} \tag{2.51}$$

$$\mathbb{Q}\mathbf{x} = \mathbf{0}, \; x_{ij}^k \geq 0, \; i,j = 0, 1, \ldots, N, \; k = 1, 2, 3 \tag{2.52}$$

which is in terms of x_{ij}^ks. Here, $\mathbb{Q}$ is a $(N+1)^2 \times (4(N+1)^2 - 3)$ matrix defined by matrix $\mathbb{P}$. We get the equations in (2.52) from (2.23) by substituting $\pi_{ij} g_{ij}^k$ for x_{ij}^k.

The optimization problem in (2.47)-(2.52) is a linear programming problem. In addition, we observe that, in the objective function, all of the x_{ij}^ks with the same sum of indices share the same weight $i + j$. If we look into the two-dimensional Markov chain, this corresponds to the states on the diagonals running from the upper right corner to the lower left corner. This motivates us to group the x_{ij}^ks along the diagonals of the two-dimensional Markov chain in Figure 2.2 and define their sum, for the nth diagonal, as

$$y_n = \sum_{i=0}^{n} (x^1_{i,n-i} + x^2_{i,n-i}) \tag{2.53}$$

$$t_n = \sum_{i=0}^{n} x^3_{i,n-i} \tag{2.54}$$

Then, $y_n \geq 0$, $t_n \geq 0$, and the objective function in (2.47) is equivalent to

$$\sum_{n=1}^{2N} (y_n + t_n) n \tag{2.55}$$

We also get $2N$ flow-in-flow-out equations between the diagonal groups. For $n = 0, 1$, we have

$$x_{00} (\theta_1 + \theta_2 - \theta_1\theta_2) = (y_1 + t_2)(1 - \theta_1)(1 - \theta_2) \tag{2.56}$$

$$(x_{00} + y_1)\theta_1\theta_2 = (y_2 + t_3)(1 - \theta_1)(1 - \theta_2) + t_2 (1 - \theta_1\theta_2) \tag{2.57}$$

and for $n = 2, 3, \ldots, 2N - 2$, we have

$$y_n\theta_1\theta_2 = (y_{n+1} + t_{n+2})(1 - \theta_1)(1 - \theta_2) + t_{n+1} (1 - \theta_1\theta_2) \tag{2.58}$$

$$y_{2N-1}\theta_1\theta_2 = t_{2N} (1 - \theta_1\theta_2) \tag{2.59}$$

Figure 2.4 shows the transitions between diagonal groups for a system with $N = 3$; we use different colors to distinguish the transitions caused by different departure rate pairs.

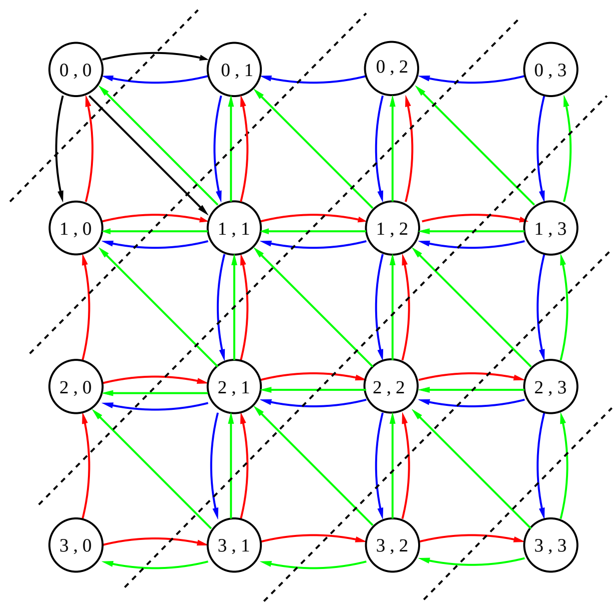


Figure 2.4: The transitions between diagonal groups when $N = 3$.

We multiply both sides of the nth equation in (2.56)-(2.59) with z^n and sum with respect to n to obtain

$$x_{00}(\theta_1+\theta_2-\theta_1\theta_2+\theta_1\theta_2 z)+\big(\theta_1\theta_2-(1-\theta_1)(1-\theta_2)z^{-1}\big)\sum_{n=1}^{2N} y_n z^n$$
$$-\big((1-\theta_1\theta_2)z^{-1}+(1-\theta_1)(1-\theta_2)z^{-2}\big)\sum_{n=1}^{2N} t_n z^n = 0 \tag{2.60}$$

Taking the derivative of (2.60) with respect to z and letting $z = 1$, we have

$$\sum_{n=1}^{2N} t_n n = \frac{1}{2-\theta_1-\theta_2}\Bigg((\theta_1+\theta_2-1)\left(\sum_{n=1}^{2N} y_n n\right)$$
$$+(1-\theta_1)(1-\theta_2)\left(\sum_{n=1}^{2N} y_n\right) \tag{2.61}$$
$$+\big(1-\theta_1\theta_2+2(1-\theta_1)(1-\theta_2)\big)\left(\sum_{n=1}^{2N} t_n\right)+x_{00}\theta_1\theta_2\Bigg)$$

From the definition of y_n and t_n in (2.53) and (2.54), we note

$$\sum_{n=1}^{2N} y_n = \sum_{n=1}^{2N}\sum_{i=0}^{n}(x^1_{i,n-i} + x^2_{i,n-i}) = \sum_{i,j}(x^1_{ij} + x^2_{ij}) \tag{2.62}$$

$$\sum_{n=1}^{2N} t_n = \sum_{n=1}^{2N}\sum_{i=0}^{n} x^3_{i,n-i} = \sum_{i,j} x^3_{ij} \tag{2.63}$$

From (2.62) and (2.63), and using (2.49)-(2.51), we conclude that $\sum_{n=1}^{2N} y_n$ and $\sum_{n=1}^{2N} t_n$ are constants that depend on system parameters α, β, θ_1, θ_2 and P_{1avg}, P_{2avg}. Using (2.62) and (2.49)-(2.50), for future reference, let us define

$$\begin{aligned}\sum_{n=1}^{2N} y_n &= \sum_{i,j} x^1_{ij} + \sum_{i,j} x^2_{ij}\\ &= \theta_1 + \theta_2 - \frac{2(P_{1avg}h_1 + P_{2avg}h_2 - (\theta_1+\theta_2)\alpha)}{\beta - 2\alpha} \triangleq \Psi\end{aligned} \tag{2.64}$$

Using the definition of y_n, t_n and (2.61), the objective function in (2.47) becomes

$$\sum_{n=1}^{2N}(y_n + t_n)n = \frac{1}{2-\theta_1-\theta_2}\left(\sum_{n=1}^{2N} y_n n\right) + C \tag{2.65}$$

where C is a constant, and $\frac{1}{2-\theta_1-\theta_2}$ is positive. Therefore, minimizing the original objective function in (2.47) is equivalent to minimizing $\sum_{n=1}^{2N} y_n n$. Since from (2.64) the sum of y_ns is fixed, and y_ns are positive, intuitively, the optimization problem requires us to assign larger values to y_ns with smaller indices n, without conflicting with the transition equation constraints.

2.5 The Modified Optimization Problem and a Two-Step Solution

In this section, we will prove the following main result of this chapter: If the average power constraints P_{1avg}, P_{2avg} are large enough to prevent any packet losses, the delay-optimal policy has a threshold structure. When the sum of the queue lengths is larger than the threshold, both users should transmit; when the sum of the queue lengths is smaller than the threshold, only the user with the longer queue should transmit; the equal queue length case can be resolved through flip of a potentially biased coin.

We propose to solve our original optimization problem in two steps. In the first step, we will consider the optimization problem in terms of y_ns and t_ns, where the objective function is $\sum_{n=1}^{2N} y_n n$, and the constraints are (2.64), (2.48), (2.56)-(2.59), and positivity constraints on y_ns and t_ns. The objective function of this optimization problem is exactly the same as that of our original optimization problem in (2.47)-(2.52), however, our constraints are more lenient than those of (2.47)-(2.52). First, (2.64) is weaker than (2.49)-(2.51), as it imposes a constraint on the sum while (2.49)-(2.51) impose constraints on individual terms. Secondly, the transition equations in (2.52) are between all of the states in the two-dimensional Markov chain, while the transition equations in (2.56)-(2.59) are only between the diagonal groups in the Markov chain. Finally, we do not explicitly impose the sum constraint on t_n on the new problem. These imply that, the result we obtain in the first step, in principle, may not be feasible for the original problem.

Therefore, in the second step we allocate y_ns and t_ns we obtain from the first step to x_{ij}^ks in such a way that the remaining independent transition equations in (2.52) are satisfied. We note that (2.39) and (2.40) can be derived from (2.52), therefore, once (2.52) is satisfied, (2.39) and (2.40) will be satisfied. Together with (2.64), we can make sure that (2.49)-(2.51) are all satisfied. Therefore, if we can find a valid allocation in the second step, we will conclude that the solution found in the first step is a feasible solution to our original problem. Since the problem we solve in the first step has the cost function of our problem, but is subject to more lenient constraints, its solution, in principle, may be better than the solution of our original problem. However, when we prove that the solution we obtain in our first step is within the feasible set of our original problem, we will have solved our original problem. In addition, once we prove the optimality of the solution in the first step, it will be globally optimal for the original problem.

First, we will minimize $\sum_{n=1}^{2N} y_n n$ subject to (2.64), (2.48), (2.56)-(2.59), and $y_n, t_n \geq 0$. This means that we will allocate Ψ to y_ns in a way to minimize $\sum_{n=1}^{2N} y_n n$. This will require us to allocate larger values to y_ns with smaller n, while making sure that (2.64), (2.48), and (2.56)-(2.59) are satisfied. We state the result of our first step in the following theorem.

Theorem 2.1 *The optimal solution of the problem*

$$\begin{aligned} \min \quad & \sum_{n=1}^{2N} y_n n \\ s.\ t. \quad & (2.64), (2.48), (2.56) - (2.59), \textit{ and } y_n \geq 0, t_n \geq 0, \forall n \end{aligned} \tag{2.66}$$

has a threshold structure. In particular, there exists a threshold $\bar{n}$ such that for

$n < \bar{n}$, $t_n = 0$ *and for* $n > \bar{n}$, $y_n = 0$.

The proof of this theorem is given in Appendix 2.8.1.

In the following, we consider the transition equations within groups for each state. Since adding more constraints cannot improve the optimization result, if we can find a way to allocate y_ns and t_ns to x_{ij}^ks, such that all of the remaining transition equations are satisfied, then we will conclude that the assignments we obtained in the first step are actually feasible for the original problem. Therefore, next, in our second step, we focus on the assignment of the y_ns and t_ns found in the first step to x_{ij}^ks.

First, we use a simple example to illustrate the procedure of allocation within each group, then, we generalize the procedure to arbitrary cases. In this simple example, we assume that $N = 4$.

Assume that after the group allocation, we obtained $y_1, \ldots, y_5$ and $t_5, t_6 \neq 0$, and the rest of the y_ns and t_ns are equal to zero. In order to keep the allocation simple, when we assign y_3, y_5, t_5 in each group, we assign them only to two states: $(1,2), (2,1)$ and $(2,3), (3,2)$, respectively; while we assign y_4 to three states: $(1,3), (2,2), (3,1)$, and we assign t_6 to a single state $(3,3)$. Figure 2.5 illustrates the allocation pattern within groups. We do not assign any values to the states with dotted circles. The dotted states will be transient states after the allocation. We need to guarantee that the nonzero-valued states only transit to other nonzero-valued states. This requires us to set $x_{12}^1 = x_{21}^2 = x_{23}^1 = x_{32}^2 = 0$, and $x_{13}^1 = x_{13}^3 = x_{31}^2 = x_{31}^3 = 0$. The valid transitions are represented as arrows in Figure 2.5. We can see that the transitions are within the positive recurrent class.

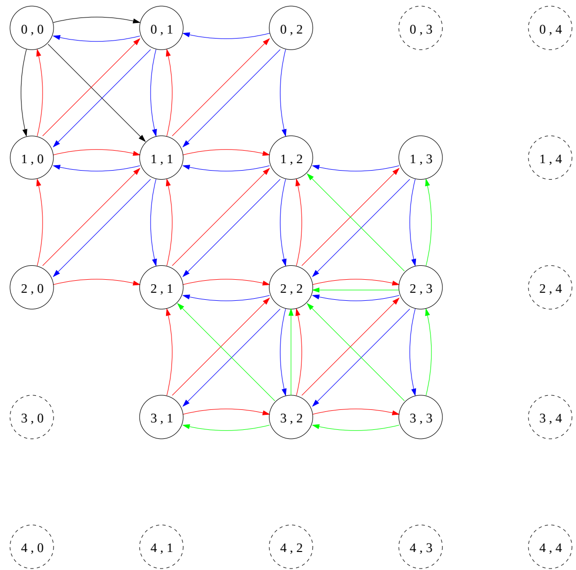


Figure 2.5: Example: allocation within groups when $N = 4$.

Then, let us examine each group and find transition equations to be satisfied for each state. For states $(0, 1), (0, 2), (1, 2), (1, 3), (2, 3)$, the transition equations to be satisfied are

$$\begin{aligned} x_{01}^2(1-\theta_2(1-\theta_1)) =& (x_{00}+x_{10}^1)\theta_2(1-\theta_1) \\ &+ (x_{02}^2+x_{11}^1)(1-\theta_1)(1-\theta_2) \\ x_{02}^2(1-\theta_2(1-\theta_1)) =& x_{11}^1\theta_2(1-\theta_1) \\ x_{12}^2(1-\theta_2(1-\theta_1)) =& (x_{02}^2+x_{11}^1)\theta_1\theta_2 + x_{21}^1\theta_2(1-\theta_1) \\ &+ (x_{13}^2+x_{22}^1+x_{23}^3)(1-\theta_1)(1-\theta_2) \\ x_{13}^2(1-\theta_2(1-\theta_1)) =& (x_{11}^1+x_{23}^3)\theta_2(1-\theta_1) \\ x_{23}^2(1-\theta_2(1-\theta_1)) + x_{23}^3(1-\theta_1\theta_2) =& (x_{13}^2+x_{22}^1)\theta_1\theta_2 + (x_{32}^1+x_{33}^3)\theta_2(1-\theta_1) \end{aligned} \quad (2.67)$$

We have five more similar transition equations for states $(0,1),(0,2),(1,2),(1,3),$ $(2,3)$. All the unknown variables are interacting with each other through these equations. How to find an allocation satisfying all of these equations simultaneously becomes rather difficult. After simple manipulations, equations in (2.67) become equivalent to

$$
\begin{aligned}
x^2_{01} =& (x_{00}+x^1_{10}+x^2_{01})\theta_2(1-\theta_1)+(x^2_{02}+x^1_{11})(1-\theta_1)(1-\theta_2)\\
x^2_{02} =& (x^1_{11}+x^2_{02})\theta_2(1-\theta_1)\\
x^2_{12} =& (x^2_{02}+x^1_{11})\theta_1\theta_2+(x^2_{12}+x^1_{21})\theta_2(1-\theta_1)+(x^2_{13}+x^1_{22}+x^3_{23})(1-\theta_1)(1-\theta_2)\\
x^2_{13} =& (x^1_{22}+x^2_{13}+x^3_{23})\theta_2(1-\theta_1)\\
x^2_{23} =& (x^2_{13}+x^1_{22})\theta_1\theta_2+(x^1_{32}+x^2_{23}+x^3_{33})\theta_2(1-\theta_1)-x^3_{23}(1-\theta_1\theta_2)
\end{aligned}
\tag{2.68}
$$

Observing the right hand sides of (2.68), we note that, x_{00}, $x^1_{10}+x^2_{10}$, $x^2_{12}+x^1_{21}$, $x^1_{32}+x^2_{23}$, x^3_{33} are known, therefore, the allocation for states $(0,1),(0,2),(1,2),(1,3),$ $(2,3)$ depends only on the values of $x^2_{02}+x^1_{11}$, $x^1_{22}+x^2_{13}$, and x^3_{23}. Similarly, the allocation for states $(1,0),(2,0),(2,1),(3,1),(3,2)$ also depends on the values of $x^1_{20}+x^2_{11}$, $x^2_{22}+x^1_{31}$, and x^3_{32} only. Since

$$y_2 = (x^2_{02}+x^1_{11})+(x^1_{20}+x^2_{11}) \tag{2.69}$$

$$y_4 = (x^1_{22}+x^2_{13})+(x^2_{22}+x^1_{31}) \tag{2.70}$$

$$t_5 = x^3_{23}+x^3_{32} \tag{2.71}$$

the allocation actually depends on how we split y_2, y_4 and t_5 between $(x^2_{02}+x^1_{11})$

and $(x^1_{20} + x^2_{11})$, $(x^1_{22} + x^2_{13})$ and $(x^2_{22} + x^1_{31})$, x^3_{23} and x^3_{32}, respectively. Once we fix the values of $x^2_{02} + x^1_{11}$, $x^1_{22} + x^2_{13}$, and x^3_{23}, we obtain the values of all of the states, completing the allocation. We note that there is more than one feasible allocation within groups, and for each feasible allocation, all of the transition equations are satisfied, and the power constraints are satisfied as well. In order to keep the solution simple, we let

$$x^2_{02} + x^1_{11} = y_2/2 \tag{2.72}$$

$$x^1_{22} + x^2_{13} = y_4/2 \tag{2.73}$$

$$x^3_{23} = t_5/2 \tag{2.74}$$

Plugging these into (2.68), we get

$$\begin{aligned}
x^2_{01} =& (x_{00} + y_1)\theta_2(1 - \theta_1) + \frac{1}{2} y_2(1 - \theta_1)(1 - \theta_2) \\
x^2_{02} =& \frac{1}{2} y_2\theta_2(1 - \theta_1) \\
x^2_{12} =& \frac{1}{2} y_2\theta_1\theta_2 + y_3\theta_2(1 - \theta_1) + \frac{1}{2}(y_4 + t_5)(1 - \theta_1)(1 - \theta_2) \\
x^2_{13} =& \frac{1}{2}(y_4 + t_5)\theta_2(1 - \theta_1) \\
x^2_{23} =& \frac{1}{2} y_4\theta_1\theta_2 + (y_5 + t_6)\theta_2(1 - \theta_1) - \frac{1}{2} t_5(1 - \theta_1\theta_2)
\end{aligned} \tag{2.75}$$

Going back to (2.72)-(2.73), we obtain

$$\begin{aligned} x_{11}^1 =& \frac{1}{2} y_2 (1 - \theta_2 (1 - \theta_1)) \\ x_{22}^1 =& \frac{1}{2} y_4 - \frac{1}{2} (y_4 + t_5) \theta_2 (1 - \theta_1) \end{aligned} \tag{2.76}$$

Since $y_n \geq t_{n+1}\rho/\delta$, we can easily verify that $x_{23}^2 \geq 0$, $x_{22}^1 \geq 0$. The allocation for the remaining half of the states has a similar structure. Thus, each state has a positive value, and the allocation is feasible.

Once we obtain the values of x_{ij}^ks, we can compute the transmission probabilities using $g_{ij}^k = \frac{x_{ij}^k}{\sum_{k=1}^3 x_{ij}^k}$. Here, we have

$$g_{11}^1 = \frac{1 - \theta_2(1 - \theta_1)}{2 - \theta_2(1 - \theta_1) - \theta_1(1 - \theta_2)} \tag{2.77}$$

$$g_{11}^2 = \frac{1 - \theta_1(1 - \theta_2)}{2 - \theta_2(1 - \theta_1) - \theta_1(1 - \theta_2)} \tag{2.78}$$

$$g_{22}^1 = \frac{y_4 - (y_4 + t_5)\theta_2(1 - \theta_1)}{2y_4 - (y_4 + t_5)(\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2))} \tag{2.79}$$

$$g_{22}^2 = \frac{y_4 - (y_4 + t_5)\theta_1(1 - \theta_2)}{2y_4 - (y_4 + t_5)(\theta_2(1 - \theta_1) + \theta_1(1 - \theta_2))} \tag{2.80}$$

We observe that a threshold structure exists. In this example, the threshold is 5. When the sum of the two queue lengths is greater than 5, both users transmit during a slot. When the sum of the two queue lengths is less than 5, only one user with longer queue transmits during a slot; in this case, if both queue lengths are the same, users transmit according to probabilities in (2.77)-(2.80).

Following steps similar to those in the example above, we can assign y_ns and t_ns to x_{ij}^ks and obtain a feasible allocation for general settings. The following theorem states this fact formally.

Theorem 2.2 *For the y_ns and t_ns obtained in the first step, there always exists a feasible x_{ij}^k assignment, such that x_{ij}^ks are positive and satisfy all of the transition equations.*

The proof of this theorem is given in Appendix 2.8.2. Since this is a constructive proof, it also gives the exact method by which y_ns and t_ns are assigned to x_{ij}^ks.

Therefore, in order to prove the optimality of the x_{ij}^k assignment, it suffices to prove the optimality of the solution of the first step. The following theorem proves the optimality of the first step.

Theorem 2.3 *The allocation scheme in Theorem 2.1 minimizes the average delay in the system.*

The proof of this theorem is given in Appendix 2.8.3.

In summary, the two-step allocation scheme is feasible and optimal for our original problem. The transition probabilities can be computed once we determine the allocation for each sate. From our allocation, we note that there exists a threshold $\bar{n}$, where $\bar{n}$ is the largest group index n such that $y_n \neq 0$. We have $t_n > 0$ only when $n \geq \bar{n}$. Since $g_{ij}^k = \frac{x_{ij}^k}{\sum_{k=1}^3 x_{ij}^k}$, we have $g_{ij}^3 = 1$ when $n > \bar{n}$. When $n < \bar{n}$, we have $g^1_{ij} = 1$ if $i > j$ and $g^2_{ij} = 1$ if $i < j$. Then, for $n \leq \bar{n}$, and n is even, we have

$$g^1_{n/2,n/2} = \frac{y_n - (y_n + t_{n+1})\theta_2(1-\theta_1)}{2y_n - (y_n + t_{n+1})(\theta_2(1-\theta_1) + \theta_1(1-\theta_2)) + t_n} \tag{2.81}$$

$$g^2_{n/2,n/2} = \frac{y_n - (y_n + t_{n+1})\theta_1(1-\theta_2)}{2y_n - (y_n + t_{n+1})(\theta_2(1-\theta_1) + \theta_1(1-\theta_2)) + t_n} \tag{2.82}$$

$$g^3_{n/2,n/2} = \frac{t_n}{2y_n - (y_n + t_{n+1})(\theta_2(1-\theta_1) + \theta_1(1-\theta_2)) + t_n} \tag{2.83}$$

If $t_n, t_{n+1} = 0$, which happens when $n < \bar{n} - 1$, (2.81)-(2.83) reduce to

$$g^1_{n/2,n/2} = \frac{1 - \theta_2(1-\theta_1)}{2 - \theta_2(1-\theta_1) - \theta_1(1-\theta_2)} \tag{2.84}$$

$$g^2_{n/2,n/2} = \frac{1 - \theta_1(1-\theta_2)}{2 - \theta_2(1-\theta_1) - \theta_1(1-\theta_2)} \tag{2.85}$$

Therefore, if the sum of the two queue lengths is greater than $\bar{n}$, both users should transmit one packet during the slot. If the sum of the two queue lengths is less than $\bar{n}$, only the user with the longer queue transmits one packet in the slot and the other user remains silent; if in this case both queues have the same length, then the probability that the first user transmits one packet while the second user keeps silent is $\frac{1-\theta_2(1-\theta_1)}{2-\theta_2(1-\theta_1)-\theta_1(1-\theta_2)}$, and the probability that the second user transmits one packet while the first user keeps silent is $\frac{1-\theta_1(1-\theta_2)}{2-\theta_2(1-\theta_1)-\theta_1(1-\theta_2)}$. When the system is symmetric, i.e., $\theta_1 = \theta_2$, these probabilities become 1/2 and 1/2.

2.6 Numerical Results

Here we give simple examples to show how our allocation scheme works. We choose $N = 10$, i.e., each queue has a buffer of size 10 packets. Therefore, the joint queue sates is represented by an 11×11 Markov chain.

First, we consider the symmetric scenario, where $\theta_1 = \theta_2 = \theta$, $h_1 = h_2 = h$ and $P_{1avg} = P_{2avg} = P_{avg}$. We assume the arrival rate $\theta = 1/2$, and the power levels $\alpha = 1$, $\beta = 3$. Therefore, we have $\eta = 3$, $\delta = 1$, $\rho = 3$. From the analysis, we know that if $P_{avg} \geq 5/8$, the average delay is one slot, which is the minimal possible delay in the system.

If $P_{avg} = 9/16$, we have $x_{00} = 1/8$, $\sum_{i,j} x_{ij}^1 = \sum_{i,j} x_{ij}^2 = 3/8$, $\sum_{i,j} x_{ij}^3 = 1/8$. Therefore, $\Psi = 3/4$. Following our allocation scheme, we have $y_1 = 3/8$, $y_2 = 3/8$, $t_3 = 1/8$. Then, we need to allocate these within groups.

We start with y_1. Because of the symmetry of the setting, we simply let $x_{10}^1 = x_{01}^2 = y_1/2 = 3/16$, $x_{12}^3 = x_{21}^3 = t_3/2 = 1/16$. Then, we consider y_2. We also let $x_{20}^1 = x_{02}^2$, $x_{11}^1 = x_{11}^2$. This symmetric allocation guarantees that the flow equations for states $(0, 1)$ and $(1, 0)$ are satisfied. The values of x_{20}^1 and x_{11}^1 also depend on the allocation of t_3. The state $(2, 0)$ must satisfy the transition equation

$$x_{20}^1 \left(\theta(1-\theta) + \theta^2 + (1-\theta)^2\right) = (x_{11}^2 + x_{21}^3)\theta(1-\theta)$$

Together with the symmetric allocation, we have

$$x_{20}^1 + x_{11}^2 = y_2/2 = 3/16$$

Solving these equations, we get the allocation for the second group as

$$x_{20}^1 = x_{02}^2 = 1/16, \qquad x_{11}^2 = x_{11}^1 = 1/8$$

We see that the two values are positive, thus feasible. Then, the transmission probabilities are $g_{11}^1 = g_{11}^2 = 1/2$, $g_{12}^3 = g_{21}^3 = 1$. The threshold of the sum of the queue lengths is 2 in this case. If the sum of the queue lengths is greater than 2, both users transmit, if the sum of the queue lengths is less than or equal to 2, only the user with the longer queue transmits and the other user remains silent; if both queues have one packet in their queues, each queue transmits with probability 1/2 while the other queue remains silent.

If $P_{avg} = 17/32$, we have $x_{00} = 1/16$, $\sum_{i,j} x_{ij}^1 = \sum_{i,j} x_{ij}^2 = 7/16$, $\sum_{i,j} x_{ij}^3 = 1/16$. Therefore, $\Psi = 7/8$. Following our allocation scheme, we have $y_1 = 3/16$, $y_2 = y_3 = 1/4$, $y_4 = 3/16$, $t_5 = 1/16$. Then, we assign these within groups. For y_1, we simply let $x_{10}^1 = x_{01}^2 = y_1/2 = 1/32$. Then, considering to allocate y_2, we have $x_{20}^1 = x_{02}^2 = 1/32$, $x_{11}^2 = x_{11}^1 = 3/32$. After completing the allocation, we have $x_{21}^1 = x_{12}^2 = 1/8$, $x_{31}^1 = x_{13}^2 = 1/32$, $x_{22}^2 = x_{22}^1 = 1/16$, $x_{23}^3 = x_{32}^3 = 1/32$. The transmission probabilities are $g_{11}^1 = g_{11}^2 = g_{22}^1 = g_{22}^2 = 1/2$, $g_{10}^1 = g_{01}^2 = g_{20}^1 = g_{02}^2 = g_{21}^1 = g_{12}^2 = g_{13}^1 = g_{31}^2 = g_{32}^3 = g_{23}^3 = 1$. The threshold of the sum of the queue lengths is 4 in this case. If the sum of the queue lengths is greater than 4, both users transmit, if the sum of the queue lengths is less than or equal to 4, only the user with the longer queue transmits and the other user remains silent; if both queues have equal length, which is either 1 or 2 in this case, each queue transmits with probability 1/2 while the other queue remains silent.

We compute the average delay as a function of average power for $\theta = 0.5$, $\theta = 0.48$ and $\theta = 0.46$, and plot them in Figure 2.6. We observe that it is a piecewise linear function, and each linear segment corresponds to the same threshold value. This is because based on our optimal allocation scheme, for a fixed threshold value, the objective function is a linear function in x_{00}, thus it is linear in P_{avg}. If P_{avg} increases, D_{avg} decreases, and the threshold decreases as well. The minimum value of P_{avg} on each curve corresponds to the maximum threshold, which is 19 in this example. This is also the minimum amount of average power required to prevent any overflows. We also observe that the delay-power tradeoff curve is convex, which is consistent with the result in [2]. We note that although these three values of θ are close to each other, the average delay varies significantly. This is because the average delay is not a linear function of θ.

For the asymmetric scenario, we assume $\theta_1 = 1/2$, $\theta_2 = 1/3$, then $\eta = 2$, $\delta = 1/2$, $\rho = 5/2$. We assume $h_1 = 1$, $h_2 = 2$. From (2.33), we know that if $P_{1avg}h_1 + P_{2avg}h_2 \geq 1$, $P_{1avg} \geq 1/2$, $P_{2avg} \geq 2/3$, then each user can always transmit a packet whenever its queue is not empty, and the average delay is one slot.

If $P_{1avg} = 19/36$, $P_{2avg} = 13/18$, then $P_{1avg}h_1 + P_{2avg}h_2 = 8/9$. Plugging these into (2.41)-(2.48), we have $\beta_1 = 1/2$, $\beta_2 = 1/2$, $\sum_{i,j}^{1} x_{ij}^1 = 4/9$, $\sum_{i,j}^{2} x_{ij}^1 = 5/18$,

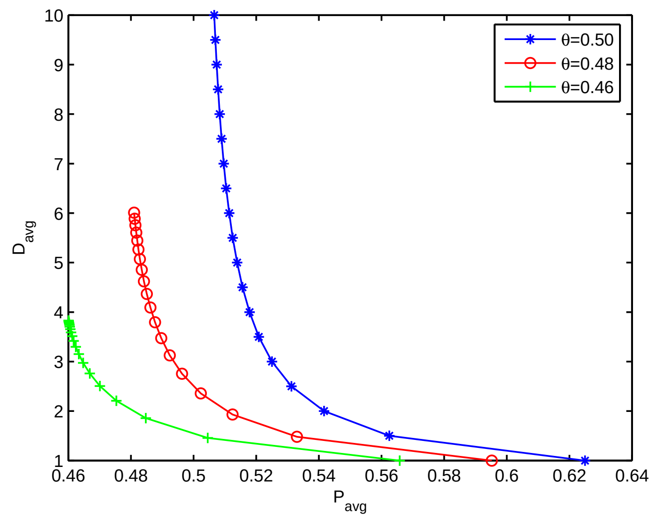


Figure 2.6: The average delay versus average power in the symmetric scenario.

$\sum_{i,j}^{3} x_{ij}^1 = 1/18$, $x_{00} = 2/9$. Then, $\Psi = 13/18$. Following the group allocation scheme, we have $y_1 = 4/9$, $y_2 = 5/18$, $t_3 = 1/18$. Then, we need to assign them within groups. From (2.118)-(2.124), we get $x_{01}^2 = 1/6$, $x_{10}^1 = 5/18$, $x_{02}^2 = 1/36$, $x_{11}^1 = 4/36$, $x_{11}^2 = 3/36$, $x_{20}^1 = 2/36$, and $x_{12}^3 = x_{21}^3 = 1/18$. The transmission probabilities are $g_{11}^1 = 4/7$, $g_{11}^2 = 3/7$, $g_{10}^1 = g_{01}^2 = g_{20}^1 = g_{02}^2 = g_{12}^3 = g_{21}^3 = 1$. The threshold is 2. If the sum of the queue lengths is greater than 2, both users transmit, if the sum of the queue lengths is less than or equal to 2, only the user with the longer queue transmits and the other user remains silent; if both queues have one packet in their queues, the first queue transmits with probability 4/7, and the second queue transmits with probability 3/7.

2.7 Conclusions

We investigated the average delay minimization problem for a two-user multiple access system with average power constraints for the general asymmetric scenario, where users have arbitrary powers, channel gains, and arrival rates. We considered a discrete-time model. In each slot, the arrivals at each queue follow a Bernoulli distribution, and we transmit at most one packet from each queue with some probability. Our objective is to find the optimal set of departure probabilities. We modeled the problem as a two-dimensional Markov chain, and minimized the average delay through controlling the departure probabilities in each time slot. We transformed the problem into a linear programming problem and found the optimal solution analytically. The optimal policy has a threshold structure. Whenever the sum of the queue lengths exceeds a threshold, both queues transmit one packet during the slot, otherwise, only one of the queues, which is longer, transmits one packet during the slot and the other queue remains silent; if both queues have the same length, only one of the queues transmits with a probability which depends on the arrival rates to both queues while the other queue remains silent.

2.8 Appendix

2.8.1 The Proof of Theorem 2.1

Let us define

$$\eta = \frac{\theta_1 + \theta_2 - \theta_1\theta_2}{(1-\theta_1)(1-\theta_2)} \tag{2.86}$$

$$\delta = \frac{\theta_1\theta_2}{(1-\theta_1)(1-\theta_2)} \tag{2.87}$$

$$\rho = \frac{1-\theta_1\theta_2}{(1-\theta_1)(1-\theta_2)} \tag{2.88}$$

Then, (2.56)-(2.59) are equivalent to

$$x_{00}\eta = y_1 + t_2 \tag{2.89}$$

$$(x_{00} + y_1)\delta = (y_2 + t_3) + t_2\rho \tag{2.90}$$

and for $n = 2, 3, \ldots, 2N-2$,

$$y_n\delta = (y_{n+1} + t_{n+2}) + t_{n+1}\rho \tag{2.91}$$

$$y_{2N-1}\delta = t_{2N}\rho \tag{2.92}$$

The optimization requires us to assign larger values to y_ns with smaller indices n as much as possible. Examining (2.89)-(2.92), we note that for fixed x_{00}, maximizing y_1, y_2, ... requires us to set t_2, t_3, ... to zero. Therefore, we choose

$$y_1 = x_{00}\eta \tag{2.93}$$

$$y_2 = (x_{00} + y_1)\delta \tag{2.94}$$

$$y_n = y_{n-1}\delta, \quad t_n = 0, \quad n = 1, 2, \ldots, n^* \tag{2.95}$$

where n^* is the largest integer satisfying $\sum_{n=1}^{n^*} y_n < \Psi$.

Let $\Delta = \Psi - \sum_{n=1}^{n^*} y_n$. We need to check that all of the group transition equations are satisfied.

Assume that $n^* > 2$. If $\Delta = y_{n^*}\delta\rho/(\delta + \rho)$, then let

$$y_{n^*+1} = \Delta, \quad \text{and} \quad y_n = 0, \quad n = n^* + 2, \ldots, N - 1 \tag{2.96}$$

$$t_{n^*+2} = y_{n^*+1}\delta/\rho, \quad \text{and} \quad t_n = 0, \quad n \neq n^* + 2 \tag{2.97}$$

We can verify that after this allocation, group transition equations (2.56)-(2.59) are satisfied. We also note that Ψ is allocated to $\{y_n\}_{n=1}^{n^*+1}$, among which, $\{y_n\}_{n=1}^{n^*}$ attain their maximum possible values. Therefore, the objective function achieves its minimal possible value for the first step.

If $\Delta \neq y_{n^*}\delta\rho/(\delta + \rho)$, if we assign it to y_{n^*+1} directly, the group transition equations are not satisfied automatically. In order to satisfy the group transition equations, we need to do some adjustments.

If $\Delta > y_{n^*}\delta\rho/(\delta + \rho)$, we assign Δ to y_{n^*+1} and y_{n^*+2} proportionally. Specifically, we let

$$y_{n^*+1} = \frac{\Delta(\rho+\delta) + y_{n^*}\delta\rho^2}{\rho^2+\delta\rho+\delta+\rho} \tag{2.98}$$

$$y_{n^*+2} = \frac{\Delta(\rho+\delta)\rho - y_{n^*}\delta\rho^2}{\rho^2+\delta\rho+\delta+\rho} \tag{2.99}$$

$$t_{n^*+2} = \frac{y_{n^*}\delta(\delta\rho+\delta+\rho) - \Delta(\rho+\delta)}{\rho^2+\delta\rho+\delta+\rho} \tag{2.100}$$

$$t_{n^*+3} = \frac{\Delta(\rho+\delta)\delta - y_{n^*}\delta^2\rho}{\rho^2+\delta\rho+\delta+\rho} \tag{2.101}$$

Since $y_{n^*}\delta > \Delta > y_{n^*}\delta\rho/(\delta+\rho)$, we can verify that each value above is positive, and the sum constraint and the group transition equations are satisfied. Among the non-zero $\{y_n\}_{n=1}^{n^*+2}$, although $\{y_n\}_{n=1}^{n^*}$ attain their maximum, y_{n^*+1} does not. Therefore, different from the first scenario, in this case, we cannot immediately claim that the result is optimal. We will give the mathematical proof for the optimality of this assignment later.

If $\Delta < y_{n^*}\delta\rho/(\delta+\rho)$, we need to remove some value from y_{n^*} and assign it to y_{n^*+1} to satisfy the equations. Define $\Delta' = \Delta + y_{n^*}$ and assign Δ' to y_{n^*} and y_{n^*+1} as follows

$$y_{n^*} = \frac{\Delta'(\rho+\delta) + y_{n^*-1}\delta\rho^2}{\rho^2+\delta\rho+\delta+\rho} \tag{2.102}$$

$$y_{n^*+1} = \frac{\Delta'(\rho+\delta)\rho - y_{n^*-1}\delta\rho^2}{\rho^2+\delta\rho+\delta+\rho} \tag{2.103}$$

$$t_{n^*+1} = \frac{y_{n^*-1}\delta(\delta\rho+\delta+\rho) - \Delta'(\rho+\delta)}{\rho^2+\delta\rho+\delta+\rho} \tag{2.104}$$

$$t_{n^*+2} = \frac{\Delta'(\rho+\delta)\delta - y_{n^*-1}\delta^2\rho}{\rho^2+\delta\rho+\delta+\rho} \tag{2.105}$$

Since $y_{n^*-1}\delta < \Delta' < y_{n^*-1}\delta(\delta\rho/(\delta+\rho)+1)$, we can also verify that each value above is positive, and the sum constraint and the group transition equations are satisfied. Similar to the second case, we cannot immediately claim that this result is optimal because after the adjustment, y_{n^*} does not achieve its maximum value. We will give the proof of optimality later.

When $n^* = 1$, the allocation will be in a different form. If $\Delta \geq (x_{00} + y_1)\delta\rho/(\delta + \rho)$, then we need to use $(x_{00} + y_1)$ instead of y_{n^*} in (2.96)-(2.101). If $\Delta < (x_{00} + y_1)\delta\rho/(\delta + \rho)$, then

$$y_{n^*} = \frac{\Psi(\rho+\delta) + x_{00}(\eta-\delta)\rho}{\rho^2+\delta\rho+\delta+\rho} \tag{2.106}$$

$$y_{n^*+1} = \frac{\Psi(\rho+\delta)\rho - x_{00}(\eta-\delta)\rho}{\rho^2+\delta\rho+\delta+\rho} \tag{2.107}$$

$$t_{n^*+1} = \frac{x_{00}(\eta\delta\rho+\eta\delta+\eta\rho^2+\delta\rho) - \Psi(\rho+\delta)}{\rho^2+\delta\rho+\delta+\rho} \tag{2.108}$$

$$t_{n^*+2} = \frac{\Psi(\rho+\delta)\delta - x_{00}(\eta-\delta)\delta}{\rho^2+\delta\rho+\delta+\rho} \tag{2.109}$$

When $n^* = 2$, if $\Delta \geq y_{n^*}\delta\rho/(\delta+\rho)$, the allocation of Ψ has the same form as in (2.96)-(2.101). If $\Delta < y_{n^*}\delta\rho/(\delta+\rho)$, then we need to use $(x_{00} + y_1)$ instead of y_{n^*-1} in (2.102)-(2.105).

2.8.2 The Proof of Theorem 2.2

While we generalize the simple example to an arbitrary setting, we follow the same basic allocation pattern. If n is odd, we assign y_n and t_n only to two states $\left(\frac{n+1}{2}, \frac{n-1}{2}\right)$ and $\left(\frac{n-1}{2}, \frac{n+1}{2}\right)$; if n is even, we assign y_n to three states: $\left(\frac{n}{2}+1, \frac{n}{2}-1\right)$, $\left(\frac{n}{2}, \frac{n}{2}\right)$,

$\left(\frac{n}{2}-1,\frac{n}{2}+1\right)$, and we assign t_n to a single state $\left(\frac{n}{2},\frac{n}{2}\right)$. We illustrate the allocation pattern in Figure 2.7.

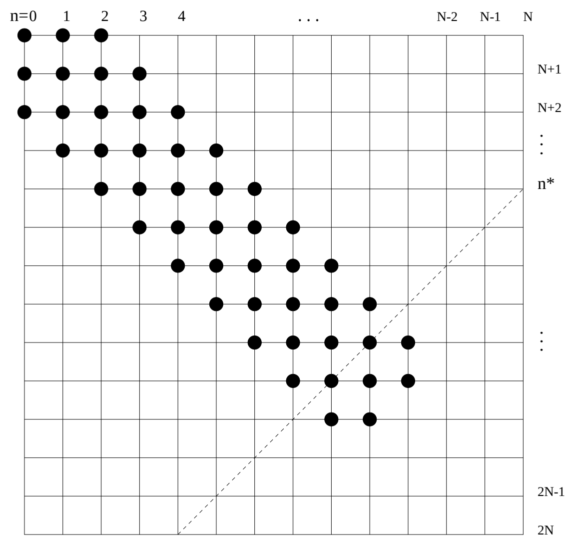


Figure 2.7: Allocation pattern within groups.

We need to make sure that the transitions only happen within the positive recurrent class. Therefore, when n is odd, we let $x^1_{\frac{n-1}{2},\frac{n+1}{2}} = x^2_{\frac{n+1}{2},\frac{n-1}{2}} = 0$; when n is even, we let $x^1_{\frac{n}{2}-1,\frac{n}{2}+1} = x^2_{\frac{n}{2}+1,\frac{n}{2}-1} = 0$. Then, let us examine the transition equations for the states. For $n = 1$, we have

$$x^2_{01}(1-\theta_2(1-\theta_1)) =(x_{00}+x^1_{10}+x^3_{11})\theta_2(1-\theta_1) + (x^2_{02}+x^1_{11}+x^3_{12})(1-\theta_1)(1-\theta_2) \quad (2.110)$$

For $n = 2, 3, \ldots$, if n is even, the transitions between states are illustrated in Figure 2.8. The transition equation for state $\left(\frac{n}{2}-1,\frac{n}{2}+1\right)$ is

$$x^2_{\frac{n}{2}-1,\frac{n}{2}+1} - \theta_2(1-\theta_1)) =(x^1_{\frac{n}{2},\frac{n}{2}} + x^3_{\frac{n}{2},\frac{n}{2}+1})\theta_2(1-\theta_1) \quad (2.111)$$

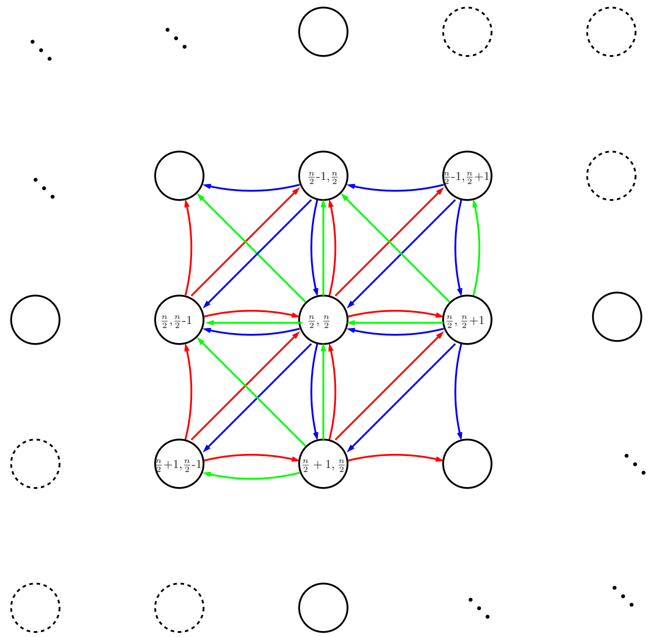


Figure 2.8: The transitions between states when n is even.

If n is odd, the transitions between states are illustrated in Figure 2.9. The transition equation for state $\left(\frac{n-1}{2}, \frac{n+1}{2}\right)$ is

$$\begin{aligned} & x^2_{\frac{n-1}{2},\frac{n+1}{2}}(1-\theta_2(1-\theta_1)) + x^3_{\frac{n-1}{2},\frac{n+1}{2}}(1-\theta_1\theta_2) \\ & \quad =(x^2_{\frac{n-3}{2},\frac{n+1}{2}} + x^1_{\frac{n-1}{2},\frac{n-1}{2}})\theta_1\theta_2 + (x^1_{\frac{n+1}{2},\frac{n-1}{2}} + x^3_{\frac{n+1}{2},\frac{n+1}{2}})\theta_2(1-\theta_1) \\ & \quad + (x^1_{\frac{n+1}{2},\frac{n+1}{2}} + x^2_{\frac{n-1}{2},\frac{n+3}{2}} + x^3_{\frac{n+1}{2},\frac{n+3}{2}})(1-\theta_1)(1-\theta_2) \end{aligned} \quad (2.112)$$

After a transformation, (2.110) is equivalent to

$$x^2_{01} =(x_{00} + x^1_{10} + x^2_{01} + x^3_{11})\theta_2(1-\theta_1) + (x^2_{02} + x^1_{11} + x^3_{12})(1-\theta_1)(1-\theta_2) \quad (2.113)$$

where x_{00} is known, $x^1_{10} + x^2_{01} = y_1$, $x^3_{11} = t_2$.

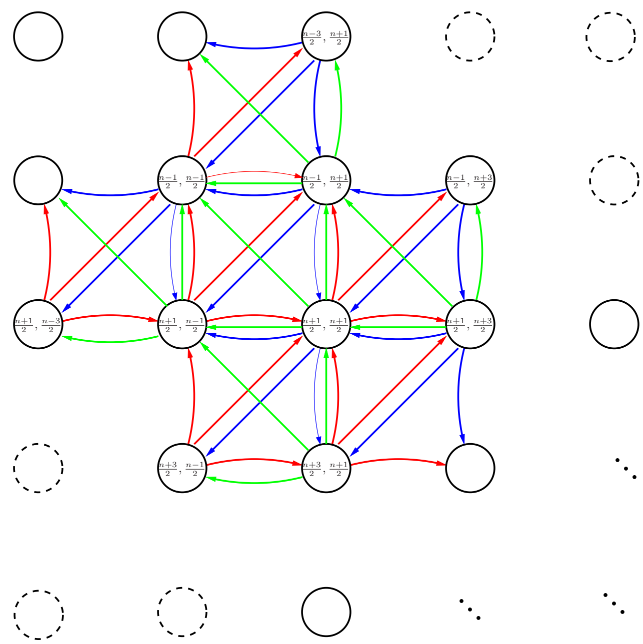


Figure 2.9: The transitions between states when n is odd.

For $n = 2, 3, \ldots$, when n is even, (2.111) is equivalent to

$$x^2_{\frac{n}{2}-1,\frac{n}{2}+1} = (x^1_{\frac{n}{2},\frac{n}{2}} + x^2_{\frac{n}{2}-1,\frac{n}{2}+1} + x^3_{\frac{n}{2},\frac{n}{2}+1})\theta_2(1-\theta_1) \tag{2.114}$$

and when n is odd, (2.112) is equivalent to

$$\begin{aligned} x^2_{\frac{n-1}{2},\frac{n+1}{2}} =& (x^2_{\frac{n-3}{2},\frac{n+1}{2}} + x^1_{\frac{n-1}{2},\frac{n-1}{2}})\theta_1\theta_2 - x^3_{\frac{n-1}{2},\frac{n+1}{2}}(1-\theta_1\theta_2) \\ &+ (x^1_{\frac{n+1}{2},\frac{n+1}{2}} + x^2_{\frac{n-1}{2},\frac{n+3}{2}} + x^3_{\frac{n+1}{2},\frac{n+3}{2}})(1-\theta_1)(1-\theta_2) \\ &+ (x^1_{\frac{n+1}{2},\frac{n-1}{2}} + x^2_{\frac{n-1}{2},\frac{n+1}{2}} + x^3_{\frac{n+1}{2},\frac{n+1}{2}})\theta_2(1-\theta_1) \end{aligned} \tag{2.115}$$

where $x^1_{\frac{n+1}{2},\frac{n-1}{2}} + x^2_{\frac{n-1}{2},\frac{n+1}{2}} = y_n$, $x^3_{\frac{n+1}{2},\frac{n+1}{2}} = t_{n+1}$.

The transition equations for the remaining half of the recurrent states can be expressed in a similar form. Therefore, the values of x^k_{ij}s are determined only by the allocation of y_n between $x^1_{\frac{n}{2}+1,\frac{n}{2}-1}+x^2_{\frac{n}{2},\frac{n}{2}}$ and $x^2_{\frac{n}{2}-1,\frac{n}{2}+1}+x^1_{\frac{n}{2},\frac{n}{2}}$ when n is even, and the allocation of t_n to $x^3_{\frac{n+1}{2},\frac{n-1}{2}}$ and $x^3_{\frac{n-1}{2},\frac{n+1}{2}}$ when n is odd. If we let

$$x^1_{\frac{n}{2},\frac{n}{2}}+x^2_{\frac{n}{2}-1,\frac{n}{2}+1}=y_n/2, \qquad \text{when } n \text{ is even} \tag{2.116}$$

$$x^3_{\frac{n-1}{2},\frac{n+1}{2}}=t_n/2, \qquad \text{when } n \text{ is odd} \tag{2.117}$$

and solve equations (2.113)-(2.115), then, for $n=1$, we obtain

$$\begin{aligned} x^2_{01} =&(x_{00}+y_1+t_2)\theta_2(1-\theta_1)+\frac{1}{2}(y_2+t_3)(1-\theta_1)(1-\theta_2) \\ x^1_{10} =&(x_{00}+y_1+t_2)\theta_1(1-\theta_2)+\frac{1}{2}(y_2+t_3)(1-\theta_1)(1-\theta_2) \end{aligned} \tag{2.118}$$

For $n=2,3,\ldots$, if n is even, we get

$$x^2_{\frac{n}{2}-1,\frac{n}{2}+1}=\frac{1}{2}(y_n+t_{n+1})\theta_2(1-\theta_1) \tag{2.119}$$

$$x^1_{\frac{n}{2}+1,\frac{n}{2}-1}=\frac{1}{2}(y_n+t_{n+1})\theta_1(1-\theta_2) \tag{2.120}$$

$$x^1_{\frac{n}{2},\frac{n}{2}}=\frac{1}{2}y_n-\frac{1}{2}(y_n+t_{n+1})\theta_2(1-\theta_1) \tag{2.121}$$

$$x^2_{\frac{n}{2},\frac{n}{2}}=\frac{1}{2}y_n-\frac{1}{2}(y_n+t_{n+1})\theta_1(1-\theta_2) \tag{2.122}$$

and if n is odd, we have

$$x^2_{\frac{n-1}{2},\frac{n+1}{2}} = \frac{1}{2}y_{n-1}\theta_1\theta_2 + (y_n + t_{n+1})\theta_2(1-\theta_1) + \frac{1}{2}(y_{n+1} + t_{n+2})(1-\theta_1)(1-\theta_2) - \frac{1}{2}t_n(1-\theta_1\theta_2) \tag{2.123}$$

$$x^1_{\frac{n+1}{2},\frac{n-1}{2}} = \frac{1}{2}y_{n-1}\theta_1\theta_2 + (y_n + t_{n+1})\theta_1(1-\theta_2) + \frac{1}{2}(y_{n+1} + t_{n+2})(1-\theta_1)(1-\theta_2) - \frac{1}{2}t_n(1-\theta_1\theta_2) \tag{2.124}$$

This completes the allocation. Note that $t_n \neq 0$ only when n is equal to $n^* + 1$, $n^* + 2$, and/or $n^* + 3$, depending on the value of Δ. When $t_{n+1} = 0$, it automatically disappears from the right hand sides of (2.118)-(2.124). From the group transition equations, we have $y_n \geq t_{n+1}\rho'/\delta'$, and it is easy to verify that all states have nonnegative assignments and the transition equations are also satisfied in this case. Therefore, there always exists a feasible allocation to satisfy all of the transition equations with y_ns defined through this allocation scheme.

2.8.3 The Proof of Theorem 2.3

In a convex optimization problem, where the inequality constraints are convex and the equality constraints are affine, if x^* is such that there exists a set of Lagrange multipliers which together with x^* satisfy the KKT conditions, then x^* is a global minimizer for the problem [26][27]. In the first step, we have a linear objective function and linear constraints. Therefore, if we prove that the point achieved by the assignment satisfies the KKT conditions, then we can say that it is the global minimizer for our problem.

In the allocation scheme, if $\Delta = y_{n^*}\delta\rho/(\delta+\rho)$, then it is easy to prove that the resulting allocation is optimal, since every $y_n, n < n^*$ achieves its maximum possible value. However, this is not the case when $\Delta \neq y_{n^*}\delta\rho/(\delta+\rho)$, because the second to last nonzero y_n does not achieve its maximum. In the following, we prove that our allocation is optimal for this case as well. Define $\mathbf{y} = [y_1, y_2, \ldots, y_{2N-1}, t_2, \ldots, t_{N-1}, t_{2N}]$. Then, the linear equality constraints, including the $2N$ group transition equations and the sum constraint can be written as a $(2N+1) \times 2(2N-1)$ matrix form as follows

$$\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ -\delta & 1 & 0 & \cdots & 0 & \rho & 1 & 0 & \cdots & 0 \\ 0 & -\delta & 1 & \cdots & 0 & 0 & \rho & 1 & \cdots & 0 \\ & \vdots & & \ddots & \vdots & & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & -\delta & 0 & 0 & 0 & \cdots & \rho \\ 1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \mathbf{y}^T = \begin{pmatrix} x_{00}\eta \\ x_{00}\delta \\ 0 \\ \vdots \\ 0 \\ 0 \\ \Psi \end{pmatrix}$$

which we write equivalently as,

$$\mathbb{A}\mathbf{y}^T = \mathbf{b} \tag{2.125}$$

by defining

$$\mathbf{b}^T = \begin{pmatrix} x_{00}\eta & x_{00}\delta(1+\eta) & x_{00}\delta^2(1+\eta) & \cdots & x_{00}\delta^{2N-1}(1+\eta) & \Psi \end{pmatrix}^T \tag{2.126}$$

and

$$
\mathbb{A} = \begin{pmatrix}
1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 & \rho+\delta & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 & (\rho+\delta)\delta & \rho+\delta & 1 & \cdots & 0 \\
 & \vdots & & \ddots & \vdots & & & & \ddots & \\
0 & 0 & 0 & \cdots & 1 & (\rho+\delta)\delta^{2N-3} & (\rho+\delta)\delta^{2N-4} & (\rho+\delta)\delta^{2N-5} & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0 & (\rho+\delta)\delta^{2N-2} & (\rho+\delta)\delta^{2N-3} & (\rho+\delta)\delta^{2N-4} & \cdots & \rho+\delta \\
1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0
\end{pmatrix} \tag{2.127}
$$

The Lagrangian is expressed as

$$
L(\mathbf{y}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \mathbf{c}^T\mathbf{y} - \boldsymbol{\lambda}^T(\mathbb{A}\mathbf{y} - \mathbf{b}) - \boldsymbol{\mu}^T\mathbf{y} \tag{2.128}
$$

where $\mathbf{c} = [1, 2, \cdots, 2N-1, 0, 0, \cdots, 0]$, $\boldsymbol{\lambda} \in \mathbf{R}^{2N+1}$ and $\boldsymbol{\mu} \in \mathbf{R}^{4N-2}$.

We need to prove that there exists a set of $\boldsymbol{\lambda}^*$, $\boldsymbol{\mu}^*$ associated with our allocation $\mathbf{y}^*$, such that they satisfy

$$
\boldsymbol{\mu}^* \geq \mathbf{0}, \quad \boldsymbol{\mu}^{*T}\mathbf{y}^* = 0 \tag{2.129}
$$

$$
\mathbf{y}^* \geq \mathbf{0}, \quad \mathbb{A}\mathbf{y}^{*T} = \mathbf{b} \tag{2.130}
$$

$$
\mathbf{c} = \mathbb{A}^T\boldsymbol{\lambda}^* + \boldsymbol{\mu}^* \tag{2.131}
$$

Consider the $\mathbf{y}$ we obtained with the algorithm. Let us consider the case when $\Delta < y_{n^*}\delta\rho/(\delta+\rho)$ first. The allocation indicates that $y_n > 0$ only when

$n = 1, 2, \ldots, n^* + 1$, and $t_n > 0$ only when $n = n^* + 1, n^* + 2$. Because of the complementary slackness in (2.129), we obtain

$$\mu_n = 0, \quad n = 1, 2, \ldots, n^* + 1, n^* + 2N - 1, n^* + 2N \tag{2.132}$$

Plugging this into (2.131), and solving the equations, we have

$$\begin{aligned}
\lambda_n &= \frac{1}{\rho + 1} + n - n^* - 1, \quad n = 1, 2, \ldots, n^* + 1 \\
\lambda_{2N+1} &= \frac{\rho}{\rho + 1} + n^* \\
\mu_{n+2N-2} &= -\left(\lambda_{n-1} + (\rho + \delta) \sum_{i=n}^{n^*-1} \lambda_i \delta^{i-n} + \rho \delta^{n^*-n} \lambda_{n^*} \right), \\
&\quad n = 2, 3, \ldots, n^*
\end{aligned} \tag{2.133}$$

Thus, we have $\lambda_n < 0$ when $n \leq n^*$, which guarantees the positiveness of $\{\mu_n\}_{n=2N}^{n^*+2N-2}$. We also have

$$\sum_{i=n^*+2}^{2N} \lambda_i \delta^{i-n^*-2} = -\frac{1}{(\rho + \delta)(\rho + 1)} \tag{2.134}$$

and

$$\begin{aligned}
\mu_n &= \frac{1}{\rho + 1} + n - n^* - 1 - \lambda_n, \quad n = n^* + 2, \ldots, 2N - 1 \\
\mu_n &= -\left(\lambda_{n-1} + (\rho + \delta) \sum_{i=n}^{2N} \lambda_i \delta^{i-n} \right), \quad n = n^* + 2N + 1, \ldots, 4N - 2
\end{aligned} \tag{2.135}$$

We can always find a set of negative $\{\lambda_i\}_{i=n^*+2}^{2N}$ to satisfy (2.134). Since they are all negative, this guarantees that $\{\mu_n\}_{n=n^*+2}^{2N-1}$ and $\{\mu_n\}_{n=n^*+2N+1}^{4N-2}$ are positive. Therefore, at the point $\mathbf{y}^*$, we can always find a set of multipliers satisfying all of the KKT constraints. This proves that the allocation our algorithm gives is a global minimizer.

Chapter 3

Delay Minimization in a Symmetric Multiple Access Channel

3.1 Introduction

Traditional information theory investigates transmission problems from a physical layer perspective. In the simplified source-channel-destination model, information-theoretic approaches assume the availability of an infinite number of bits at the transmitter before the transmission starts. The burstiness of the arrivals and the associated issue of delay are mostly ignored. In contrast, network theory gives sophisticated analysis of network layer issues, such as random arrivals and network delay. However, in network-theoretic approaches, the underlying physical layer model is usually very simplified, e.g., in most approaches simultaneous transmissions are not allowed, and even when they are allowed, a collision channel model is used, which is too simplistic to capture what can be achieved in the physical layer from an information-theoretic perspective.

In recent years, many authors have taken efforts to bridge the gap between information theory and network theory [1]. Reference [22] addresses the delay issue for an additive Gaussian noise multiple access channel (MAC). Packets with random sizes arrive according to a Poisson process, and are transmitted out immediately with a fixed power. At the physical layer, the receiver decodes a packet while treating other transmissions as noise. Consequently, the service rate becomes a function of the number of active users in the system. Reference [22] derives the relationship between the average delay and a fixed probability of error requirement. References [2], [4] and [5] consider a discrete-time model for a power-constrained single-user communication channel. Random arrivals queue at the transmitter to wait to be transmitted. In each slot, the transmitter adapts its service rate, i.e., transmission rate, according to the queue length and the channel state, as well as the average power constraint, to minimize the average delay. Reference [2] formulates the problem as a dynamic programming problem and develops a delay-power tradeoff curve. References [4] and [5] determine some structural properties of the optimal power/rate allocation policy.

Reference [9] uses a continuous-time queueing model to model the network layer behavior of a multiple access system. The packets arrive at the transmitters according to independent Poisson processes, and the packet lengths are exponentially distributed. The physical layer is modeled as an additive Gaussian noise channel, whose capacity region is a pentagon for the two-user case. The goal of [9] is to select an operating rate point inside the MAC capacity region, as a function of the current queue lengths, in order to minimize the average packet delay. The transmission rates selected from the capacity region serve as the current service rates of the queues. Reference [9] develops the longer-queue-higher-rate (LQHR) allocation strategy, which selects an extreme point in the capacity region of the MAC (i.e., a corner point of the pentagon). Reference [9] shows that LQHR minimizes the average delay of a symmetric system. Reference [10] extends [9] to a potentially asymmetric setting, and proves that the delay-optimal policy has a threshold (switch) structure. Reference [11] develops a policy named “modified LQHR” which works at a corner point of the pentagon when the queue lengths are different, and switches to the mid-point of the dominant face of the pentagon when the queue lengths become equal. The “modified LQHR” algorithm is shown to minimize the average bit delay in the system. The third chapter of [12] extends “modified LQHR” to an M-user scenario.

In this chapter, we consider a similar delay minimization problem. In order to track the relationship between the average delay and the transmission rates more accurately and also to consider more general arrivals, we adopt a discrete-time queueing model and consider the problem from a bit perspective rather than a packet perspective. We partition the time into small slots. In each slot, bits arrive at the transmitters randomly according to some general distribution. At the beginning of each slot, we allocate transmission rates from within the MAC capacity region to the users, based on their current queue lengths, to minimize the average delay. In our model, the number of bits transmitted in each slot is equal to the product of the transmission rate and the number of channel uses in each slot. We formulate the problem as an average cost Markov decision problem (MDP). We first analyze the corresponding discounted cost MDP, and obtain some properties of the value function. Based on these properties, we prove that the delay optimal rate allocation policy for this discounted MDP is to equalize the queue lengths in each slot as much as possible. We then prove that this *queue balancing policy* is optimal for the average cost MDP as well.

Essentially, both the “modified LQHR” and our policy aim to balance the queue lengths as well as to maximize the throughput at any time. However, the continuous model in [11, 12] allows the rates to be changed at any time, while our model allows us to make decisions only at the beginning of each slot. Consequently, the resulting optimal policies are different: The operating point of the "modified LQHR" algorithm is either one of the corner points or the mid-point of the dominant face of the pentagon, while the *queue balancing policy* here may operate at any point on the dominant face of the pentagon.

3.2 System Model and Problem Formulation

3.2.1 Physical Layer Model

We consider a two-user AWGN multiple access system

$$Y = X_1 + X_2 + Z \tag{3.1}$$

where X_i is the signal of user i, and Z is a Gaussian noise with zero-mean and variance σ^2. In this multiple access system, the capacity region is given by [24]

$$R_1 \leq \frac{1}{2}\log\left(1 + \frac{P_1}{\sigma^2}\right) \triangleq C_1 \tag{3.2}$$

$$R_2 \leq \frac{1}{2}\log\left(1 + \frac{P_2}{\sigma^2}\right) \triangleq C_2 \tag{3.3}$$

$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + \frac{P_1 + P_2}{\sigma^2}\right) \triangleq C_s \tag{3.4}$$

The capacity region is a pentagon, as shown in Figure 3.1. In this chapter we consider a symmetric two-user system, where $P_1 = P_2 = P$. Our results can be generalized to the symmetric K-user case.

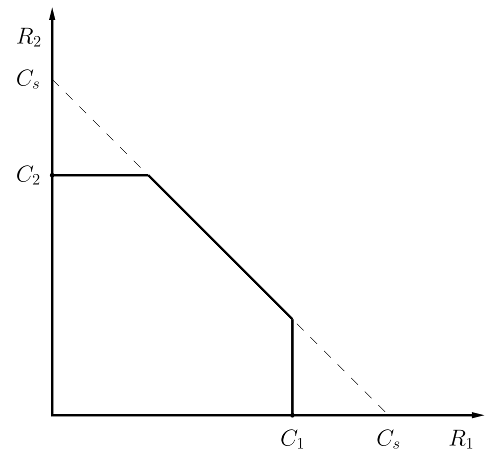


Figure 3.1: The capacity region for a two-user multiple access system.

3.2.2 Medium Access Control Layer Model

In the medium access control layer, we assume that the bits arrive at the transmitters in random numbers in each slot, see Figure 3.2. Let $a_1[n]$ and $a_2[n]$ denote the number of bits arriving at the first and the second transmitter, respectively, during time slot n. Here, $a_1[n]$ and $a_2[n]$ are two independent random variables with a common distribution F_a. We assume that the arrivals are i.i.d. in n.

There is an infinite capacity buffer at each transmitter to store the bits. Let $q_1[n]$ and $q_2[n]$ denote the number of bits in the first and the second buffer, respectively, at the beginning of the nth slot. At the beginning of each slot, the transmitters decide on how many bits to transmit in this slot based on the current lengths of the two queues. Let $d_1[n]$ and $d_2[n]$ denote the number of bits to be

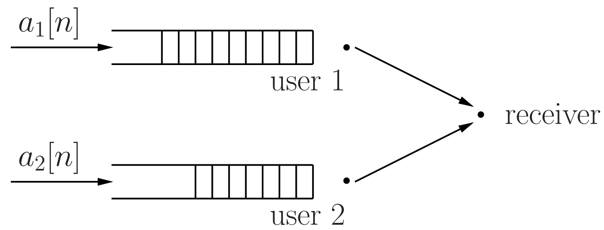


Figure 3.2: System model.

transmitted from the first and the second queue, respectively, in the nth time slot. Let us define $\mathbf{q}[n] \triangleq (q_1[n], q_2[n])$, $\mathbf{d}[n] \triangleq (d_1[n], d_2[n])$, and $\mathbf{a}[n] \triangleq (a_1[n], a_2[n])$. Then, the queue lengths evolve according to

$$\mathbf{q}[n+1] = (\mathbf{q}[n] - \mathbf{d}[n])^+ + \mathbf{a}[n] \tag{3.5}$$

where $(x)^+$ denotes $\max(0, x)$.

If the number of channel uses in a slot is τ, the transmission rate of user i becomes $R_i[n] = d_i[n]/\tau$. Consequently, the actual rates of the users that need to be selected from the capacity region described by (3.2)-(3.4), are proportional to $d_1[n]$ and $d_2[n]$, and therefore, $(d_1[n], d_2[n])$ can be viewed as (scaled) rates. In order to simplify the notation, we will call $d_i[n] = R_i[n]\tau$ as the *rate* of user i for slot n. The corresponding scaled capacity region that (d_1, d_2) should reside in is described by (3.2)-(3.4) by multiplying right hand sides by τ.

3.2.3 Formulation as an MDP

According to Little's law [25], minimizing the average delay in the system is equivalent to minimizing the average number of bits in the system, which is the average sum of queue lengths. If the system starts from state $\mathbf{q}[1]$, the delay minimization problem is to obtain optimal policy $\mathbf{d}[n]$, $n = 1, 2, \ldots$ to minimize

$$\limsup_{N\to\infty} \frac{1}{N} E\left[\sum_{n=1}^{N}(q_1[n] + q_2[n])\right] \tag{3.6}$$

Therefore, this problem can be formulated as a standard average cost MDP. The state space consists of all possible queue length vectors, while the policy space is the set of operating points within the multiple access capacity region. In principle, the values of $q_i[n]$, $d_i[n]$ can only be integers, however, for practical applications, one bit is a fine enough precision that we can use a fluid model to reasonably approximate the original discrete-state system.

3.3 The Discounted Cost Problem

Instead of considering the minimization problem with the average cost criterion in (3.6) directly, we first consider the following minimization problem with a total discounted cost criterion

$$E\left[\sum_{n=1}^{\infty} \beta^n (q_1[n] + q_2[n])\right] \tag{3.7}$$

where $0 < \beta < 1$ is the discount factor. We will return to the average cost criterion in (3.6) by letting β go to 1.

Let us define $V^\beta(\mathbf{q})$ to be the total discounted cost starting from an initial state $\mathbf{q}$. Then, for the optimization problem with criterion (3.7), $V^\beta(\mathbf{q})$ must satisfy the following optimality condition [28]

$$V^{\beta}(\mathbf{q}) = \min_{\mathbf{d}\in C} \left\{q_1 + q_2 + \beta E\left[V^{\beta}((\mathbf{q}-\mathbf{d})^{+} + \mathbf{a})\right]\right\} \tag{3.8}$$

We will first start with a discounted cost problem over finite horizon N. For this problem with an initial state $\mathbf{q}$, the dynamic programming formulation is

$$V_N^{\beta}(\mathbf{q}) = \min_{\mathbf{d}\in C} \left\{q_1 + q_2 + \beta E\left[V_{N-1}^{\beta}((\mathbf{q}-\mathbf{d})^{+} + \mathbf{a})\right]\right\} \tag{3.9}$$

with $V_0^{\beta}(\cdot) = 0$. Since the instantaneous cost $q_1[n] + q_2[n]$ is positive, and the policy space is finite [28]

$$V_N^{\beta}(\mathbf{q}) \to V^{\beta}(\mathbf{q}) \quad \text{as} \quad N \to \infty \tag{3.10}$$

where $V^{\beta}(\cdot)$ is the unique bounded solution of (3.8).

In the following, we will analyze the discounted cost problem and obtain structural properties of the value function $V^{\beta}(\mathbf{q})$. We will find these structural properties of $V^{\beta}(\mathbf{q})$ by examining the structural properties of the finite-horizon discounted cost problem $V_N^{\beta}(\mathbf{q})$.

Lemma 3.1 *$V^{\beta}(\mathbf{q})$ is increasing in q_1 and q_2.*

Proof: From (3.10), we know that proving $V^{\beta}(\mathbf{q})$ is increasing in q_1 and q_2 is equivalent to proving $V_N^{\beta}(\mathbf{q})$ is increasing in q_1 and q_2 for every N. We prove this through induction. First, when $N = 0, 1$, this is trivially true. Next, we assume that it is true for $N-1$. We will prove that $V_N^\beta(q_1+1,q_2) > V_N^\beta(q_1,q_2)$ for any positive (q_1,q_2).

$$
\begin{aligned}
& V_N^\beta(q_1+1,q_2) \\
& = q_1+q_2+1+\beta E\Big[V_{N-1}^\beta((q_1+1-d_1^*)^+ + a_1, (q_2-d_2^*)^+ + a_2)\Big] && (3.11) \\
& \geq q_1+q_2+1+\beta E\Big[V_{N-1}^\beta((q_1-d_1^*)^+ + a_1, (q_2-d_2^*)^+ + a_2)\Big] && (3.12) \\
& > \min_{\mathbf{d}\in C}\Big\{(q_1+q_2)+\beta E\Big[V_{N-1}^\beta\big((q_1-d_1)^+ + a_1, (q_2-d_2)^+ + a_2\big)\Big]\Big\} && (3.13) \\
& = V_N^\beta(q_1,q_2) && (3.14)
\end{aligned}
$$

where (d_1^*, d_2^*) in (3.11) is the point within the capacity region that minimizes $V_N^\beta(q_1+1,q_2)$, and (3.12) follows from the assumption that $V_{N-1}^\beta(q_1,q_2)$ is increasing for every q_1. Therefore, $V_N^\beta(\mathbf{q})$ is increasing in q_1 for every N. Using (3.10), this implies that $V^\beta(\mathbf{q})$ is increasing in q_1. Now, following the same procedure for q_2, we can prove that $V^\beta(\mathbf{q})$ is increasing in q_2 as well. □

Lemma 3.2 *In (3.8), the optimal operating point* $\mathbf{d}$ *must be on the boundary of the capacity region* C.

Proof: For an initial state $\mathbf{q}$, if the optimal operating point $\mathbf{d}=(d_1,d_2)$ is not on the boundary of the capacity region but on the interior of the capacity region, then, we can always find points $\bar{\mathbf{d}}=(d_1',d_2)$, $\tilde{\mathbf{d}}=(d_1,d_2')$ that are on the boundary of the capacity region with $d_1'>d_1$, $d_2'>d_2$. Note that $\bar{\mathbf{d}}\geq\mathbf{d}$ and $\tilde{\mathbf{d}}\geq\mathbf{d}$. Then, by Lemma 3.1, we have

$$E\left[V^{\beta}((\mathbf{q}-\bar{\mathbf{d}})^{+}+\mathbf{a})\right] \leq E\left[V^{\beta}((\mathbf{q}-\mathbf{d})^{+}+\mathbf{a})\right] \tag{3.15}$$

and

$$E\left[V^{\beta}((\mathbf{q}-\tilde{\mathbf{d}})^{+}+\mathbf{a})\right] \leq E\left[V^{\beta}((\mathbf{q}-\mathbf{d})^{+}+\mathbf{a})\right] \tag{3.16}$$

This contradicts the optimality of $\mathbf{d}$. Thus, $\mathbf{d}$ must be on the boundary of the capacity region. □

Lemma 3.3 *$V^{\beta}(\mathbf{q})$ is symmetric and jointly convex in $\mathbf{q}$.*

Proof: The symmetry property can be proved by induction. Note that $V_N^{\beta}(\mathbf{q})$ is symmetric for $N = 0, 1$. Assuming that $V_{N-1}^{\beta}(\mathbf{q})$ is symmetric, it is easy to see that $V_N^{\beta}(\mathbf{q})$ would be symmetric. Now, taking the limit $N \to \infty$, it follows that $V^{\beta}(\mathbf{q})$ is symmetric.

We prove the convexity of $V^{\beta}(\mathbf{q})$ through induction as well. When $N = 0, 1$, it is trivial to see that $V_N^{\beta}(\mathbf{q})$ is convex in $\mathbf{q}$. Next, we assume that $V_{N-1}^{\beta}(\mathbf{q})$ is convex in $\mathbf{q}$. Given two different queue length vectors $\mathbf{x} \triangleq (x_1, x_2)$ and $\mathbf{y} \triangleq (y_1, y_2)$, we have

$$
\begin{aligned}
&\lambda V_N^\beta(\mathbf{x}) + (1-\lambda)V_N^\beta(\mathbf{y}) \\
&\qquad = \lambda(x_1+x_2) + (1-\lambda)(y_1+y_2) + \lambda\beta E\left[V_{N-1}^\beta((\mathbf{x}-\mathbf{b}^*)^+ + \mathbf{a})\right] \\
&\qquad\quad + (1-\lambda)\beta E\left[V_{N-1}^\beta((\mathbf{y}-\mathbf{d}^*)^+ + \mathbf{a})\right] && (3.17) \\
&\qquad \geq \lambda(x_1+x_2) + (1-\lambda)(y_1+y_2) + \\
&\qquad\quad \beta E\left[V_{N-1}^\beta\big(\lambda(\mathbf{x}-\mathbf{b}^*)^+ + (1-\lambda)(\mathbf{y}-\mathbf{d}^*)^+ + \mathbf{a}\big)\right] && (3.18) \\
&\qquad \geq \lambda(x_1+x_2) + (1-\lambda)(y_1+y_2) + \\
&\qquad\quad \beta E\left[V_{N-1}^\beta\Big(\big(\lambda(\mathbf{x}-\mathbf{b}^*) + (1-\lambda)(\mathbf{y}-\mathbf{d}^*)\big)^+ + \mathbf{a}\Big)\right] && (3.19) \\
&\qquad \geq \min_{\mathbf{d}\in C}\Big\{\lambda(x_1+x_2) + (1-\lambda)(y_1+y_2) + \\
&\qquad\quad \beta E\left[V_{N-1}^\beta\Big(\big(\lambda\mathbf{x} + (1-\lambda)\mathbf{y} - \mathbf{d}\big)^+ + \mathbf{a}\Big)\right]\Big\} && (3.20) \\
&\qquad = V_N^\beta(\lambda\mathbf{x} + (1-\lambda)\mathbf{y}) && (3.21)
\end{aligned}
$$

where $\mathbf{b}^*$ and $\mathbf{d}^*$ are the minimizers for $V_N^\beta(\mathbf{x})$ and $V_N^\beta(\mathbf{y})$, respectively. Here, (3.18) follows from the assumption of the convexity of $V_{N-1}^\beta(\cdot)$, (3.19) follows from the convexity of the function $(\cdot)^+$, and (3.20) is valid because $\mathbf{b}^*, \mathbf{d}^* \in C$, and C is a convex set, implying $\lambda\mathbf{b}^* + (1-\lambda)\mathbf{d}^* \in C$. □

Before we move on to the next structural property of the function $V^\beta(\mathbf{q})$, we need to introduce the concepts of majorization and Schur-convexity.

Definition 3.1 ([29]) *Given* $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$*, we say that* $\mathbf{x}$ *majorizes* $\mathbf{y}$*, and we write*

$\mathbf{x} \succeq \mathbf{y}$, *if*

$$\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i, \quad k \in \{1, \ldots, d-1\} \tag{3.22}$$

$$\sum_{i=1}^{d} x_i = \sum_{i=1}^{d} y_i \tag{3.23}$$

where x_i *and* y_i *are the* i*th largest elements of* $\mathbf{x}$ *and* $\mathbf{y}$*, respectively.*

Definition 3.2 ([29]) *A function* $f : \mathbb{R}^d \to \mathbb{R}$ *is said to be Schur-convex when* $\mathbf{x} \succeq \mathbf{y}$ *implies* $f(\mathbf{x}) \geq f(\mathbf{y})$*.*

A function is Schur-convex if it is symmetric and convex [29]. Using Lemma 3.3, we conclude that $V^\beta(\mathbf{q})$ is Schur-convex. However, given that $\mathbf{x} \succeq \mathbf{y}$, we cannot directly claim that $V^\beta(\mathbf{x}+\mathbf{a}) \geq V^\beta(\mathbf{y}+\mathbf{a})$ for every $\mathbf{a}$. This is because the randomness of $\mathbf{a}$ may reverse the majorization relationship between $\mathbf{x} + \mathbf{a}$ and $\mathbf{y} + \mathbf{a}$. However, provided that $V^\beta(\mathbf{q})$ is symmetric and convex, and $\mathbf{a}$ has i.i.d. components, we can prove that $E[V^\beta(\mathbf{x} + \mathbf{a})] \geq E[V^\beta(\mathbf{y} + \mathbf{a})]$ if $\mathbf{x} \succeq \mathbf{y}$.

Lemma 3.4 *For i.i.d.* a_i*s* $\mathbf{x} \succeq \mathbf{y}$ *implies* $E[V^\beta(\mathbf{x} + \mathbf{a})] \geq E[V^\beta(\mathbf{y} + \mathbf{a})]$*.*

Proof: When $a_1 = a_2$, clearly, $\mathbf{x} + \mathbf{a} \succeq \mathbf{y} + \mathbf{a}$, and $V^\beta(\mathbf{x} + \mathbf{a}) \geq V^\beta(\mathbf{y} + \mathbf{a})$. When $a_1 \neq a_2$, we evaluate the functions $V^\beta(\mathbf{x}+\mathbf{a})$ and $V^\beta(\mathbf{y}+\mathbf{a})$ at two symmetric points (c_1, c_2) and (c_2, c_1). In order to simply the notation, for any vector $\mathbf{v} = (v_1, v_2)$, we define $\check{\mathbf{v}} = (v_2, v_1)$. Because a_is are i.i.d., the two points $\mathbf{c}$, $\check{\mathbf{c}}$ have the same probability mass. Without loss of generality, we assume $c_1 > c_2$, $x_1 \geq x_2$, $y_1 \geq y_2$. Since $\mathbf{x} \succeq \mathbf{y}$, we have $x_1 \geq y_1 \geq y_2 \geq x_2$.

Consider four vectors $(\mathbf{x}+\mathbf{c})$, $(\check{\mathbf{x}}+\mathbf{c})$, $(\mathbf{y}+\mathbf{c})$, $(\check{\mathbf{y}}+\mathbf{c})$. We see that they are four points on the line $q_1+q_2 = x_1+x_2+c_1+c_2$. Moreover, since $x_1 \geq y_1 \geq y_2 \geq x_2$, $(\mathbf{x}+\mathbf{c})$ and $(\check{\mathbf{x}}+\mathbf{c})$ are the two outer points, and the mid-point of these two points is the same as the mid-point of the other two points. Since $V^\beta(\mathbf{q})$ is convex, we have

$$V^\beta(\mathbf{x}+\mathbf{c}) + V^\beta(\check{\mathbf{x}}+\mathbf{c}) \geq V^\beta(\mathbf{y}+\mathbf{c}) + V^\beta(\check{\mathbf{y}}+\mathbf{c}) \tag{3.24}$$

We also note that because of the symmetry property of $V^\beta(\mathbf{q})$ we have $V^\beta(\check{\mathbf{x}}+\mathbf{c}) = V^\beta(\mathbf{x}+\check{\mathbf{c}})$. Similarly, we have $V^\beta(\check{\mathbf{y}}+\mathbf{c}) = V^\beta(\mathbf{y}+\check{\mathbf{c}})$. Therefore, (3.24) is equivalent to

$$V^\beta(\mathbf{x}+\mathbf{c}) + V^\beta(\mathbf{x}+\check{\mathbf{c}}) \geq V^\beta(\mathbf{y}+\mathbf{c}) + V^\beta(\mathbf{y}+\check{\mathbf{c}}) \tag{3.25}$$

Integrating over a_1, a_2, we get

$$\begin{aligned}
E[V^\beta(\mathbf{x}+\mathbf{a})] &= \int_{a_1>a_2} V^\beta(\mathbf{x}+\mathbf{a}) + \int_{a_1<a_2} V^\beta(\mathbf{x}+\mathbf{a}) + \int_{a_1=a_2} V^\beta(\mathbf{x}+\mathbf{a}) && (3.26)\\
&= \int_{a_1<a_2} (V^\beta(\mathbf{x}+\mathbf{a}) + V^\beta(\mathbf{x}+\check{\mathbf{a}})) + \int_{a_1=a_2} V^\beta(\mathbf{x}+\mathbf{a}) && (3.27)\\
&\geq \int_{a_1<a_2} (V^\beta(\mathbf{y}+\mathbf{a}) + V^\beta(\mathbf{y}+\check{\mathbf{a}})) + \int_{a_1=a_2} V^\beta(\mathbf{y}+\mathbf{a}) && (3.28)\\
&= E\left[V^\beta(\mathbf{y}+\mathbf{a})\right] && (3.29)
\end{aligned}$$

where the inequality follows from (3.25). □

We now combine Lemmas 3.1 through 3.4 to obtain the main result of this chapter which is given in Theorem 3.1.

Theorem 3.1 *To minimize the average delay, in each slot, the transmitters should choose an operating point on the dominant face of the capacity region that equalizes the queue lengths. If no such operating point exists, the transmitters should operate at a corner point which minimizes the queue length difference.*

Proof: We know from Lemma 3.2 that, in each slot, the transmitters must operate on the dominant face (sum-rate constrained face) of the multiple access capacity region.

First, we prove that if there exists a point on the dominant face that equalizes the queue lengths, then this point must be the optimal operating point. Given queue lengths $\mathbf{q} = (q_1, q_2)$, let $\mathbf{d} = (d_1, d_2)$ be such a point, i.e., $(q_1 - d_1)^+ = (q_2 - d_2)^+$. If $(q_1 - d_1)^+ = (q_2 - d_2)^+ = 0$, then, clearly, $\mathbf{d}$ is the optimal operating point. We consider the case when $q_1 - d_1 = q_2 - d_2 > 0$. To prove the claim by contradiction, let us assume that $\mathbf{d}$ is not optimal, but $\mathbf{b} = (b_1, b_2)$ is the optimal point on the dominant face. Since both $\mathbf{d}$ and $\mathbf{b}$ are on the dominant face of the capacity region: $d_1 + d_2 = b_1 + b_2$. Since with a fixed sum, the vector with identical components is majorized by any other vector [29], we have $(q_1 - b_1, q_2 - b_2) \succeq (q_1 - d_1, q_2 - d_2)$. Without loss of generality, we assume $q_1 - b_1 > q_2 - b_2$, i.e., $q_1 - b_1 > q_1 - d_1 = q_2 - d_2 > q_2 - b_2$. If $q_2 - b_2 \geq 0$, we have $((q_1-b_1)^+, (q_2-b_2)^+) \succeq ((q_1-d_1)^+, (q_2-d_2)^+)$, and using Lemma 3.4, this implies

$$E[V^\beta((\mathbf{q} - \mathbf{b})^+ + \mathbf{a})] \geq E[V^\beta((\mathbf{q} - \mathbf{d})^+ + \mathbf{a})] \tag{3.30}$$

On the other hand, if $q_2 - b_2 < 0$, we have

$$E[V^\beta((\mathbf{q}-\mathbf{b})^+ + \mathbf{a})] = E[V^\beta((q_1-b_1)+a_1, a_2)] \tag{3.31}$$

$$\geq E[V^\beta(q_1-d_1+a_1, d_1-b_1+a_2)] \tag{3.32}$$

$$= E[V^\beta(q_1-d_1+a_1, b_2-d_2+a_2)] \tag{3.33}$$

$$> E[V^\beta(q_1-d_1+a_1, q_2-d_2+a_2)] \tag{3.34}$$

$$= E[V^\beta((\mathbf{q}-\mathbf{d})^+ + \mathbf{a})] \tag{3.35}$$

where (3.32) follows from $(q_1 - b_1, 0) \succeq (q_1 - d_1, d_1 - b_1)$ and Lemma 3.4, (3.33) follows from the fact that $d_1 + d_2 = b_1 + b_2$, and (3.34) is valid because we assumed that $q_2 - b_2 < 0$, thus $q_2 - d_2 > b_2 - d_2$, and we apply Lemma 3.1. The results in (3.30) and (3.35) contradict the optimality of $\mathbf{b}$, and therefore, $\mathbf{d}$ must be the optimal operating point.

Next, we prove that if there does not exist a point on the dominant face of the capacity region which equalizes the queue lengths, then the optimal operating point must be one of the corner points. Let us assume that the optimal operating point $\mathbf{d} = (d_1, d_2)$ is not a corner point, and without loss of generality, let us assume that $(q_1 - d_1)^+ > (q_2 - d_2)^+$. If $q_1 - d_1 > q_2 - d_2 \geq 0$, we can always find a small enough $\delta > 0$, such that the operating point $(d_1 + \delta, d_2 - \delta)$ is also on the dominant face, and $q_1 - (d_1 + \delta) > q_2 - (d_2 - \delta) > 0$. Since $(q_1 - d_1, q_2 - d_2) \succeq (q_1 - (d_1 + \delta), q_2 - (d_2 - \delta))$, based on Lemma 3.4, we have $E[V^\beta((\mathbf{q}-\mathbf{d})^+ + \mathbf{a})] \geq E[V^\beta(q_1 - (d_1 + \delta) + a_1, q_2 - (d_2 - \delta) + a_2)]$, and this contradicts the optimality of $\mathbf{d}$. On the other hand, if

$q_1 - d_1 > 0 > q_2 - d_2$, we can also find a small enough $\delta > 0$, such that $q_1 - (d_1 + \delta) > 0 \geq q_2 - (d_2 - \delta)$, and $(d_1 + \delta, d_2 - \delta)$ is on the dominant face as well. Therefore, we have $0 < q_1 - (d_1 + \delta) < q_1 - d_1$, and $(q_2 - d_2)^+ = (q_2 - d_2 + \delta)^+ = 0$. According to Lemma 3.1, we have $V^\beta(q_1 - d_1 + a_1, a_2) > V^\beta(q_1 - (d_1 + \delta) + a_1, a_2)$ for any value of a_1 and a_2. Therefore, $E[V^\beta(q_1 - d_1 + a_1, a_2)] > E[V^\beta(q_1 - (d_1 + \delta) + a_1, a_2)]$, and this contradicts the optimality of $\mathbf{d}$. Hence, the optimal operating point, in this case, must be one of the corner points. □

Using Theorem 3.1, we express the optimal operating point $\mathbf{d}^* = (d_1^*, d_2^*)$ as a function of the queue lengths $\mathbf{q} = (q_1, q_2)$

$$\mathbf{d}^* = \begin{cases} \left(\frac{q_1 - q_2 + C_s}{2}, \frac{q_2 - q_1 + C_s}{2}\right), & |q_1 - q_2| < 2C_1 - C_s \\ (C_1, C_s - C_1), & q_1 - q_2 > 2C_1 - C_s \\ (C_s - C_2, C_2), & q_1 - q_2 < C_s - 2C_1 \end{cases}$$

This optimal rate allocation scheme works on the dominant face of the capacity region and therefore maximizes the number of bits transmitted in each slot; and, at the same time, it tries to balance the queue lengths as much as possible, which, in turn, minimizes the probability that any one of the queues becomes empty in the upcoming slots. When a queue becomes empty, the system resources cannot be utilized most efficiently, as even though the user with an empty queue has power to transmit, it does not have any bits to transmit.

Finally, while we developed Theorem 3.1 for the discounted cost criterion, we can find a sub-sequence of discount factors β_n such that $\beta_n \to 1$ as $n \to \infty$.

Therefore, the policy we developed is optimal for the average cost problem as well.

3.4 Numerical Results

We consider a two-user AWGN multiple access channel, with $C_1 = C_2 = 20$ bits/slot and $C_s = 30$ bits/slot. The number of bits arriving at the transmitters in each slot follows a Poisson distribution with parameter λ. We compare two policies: the optimal policy developed in this chapter which tries to balance the queue lengths in each slot and the LQHR algorithm developed in [9] which chooses a corner point of the capacity region and allocates the larger rate to the longer queue. We plot the average delay versus λ in Figure 3.3.

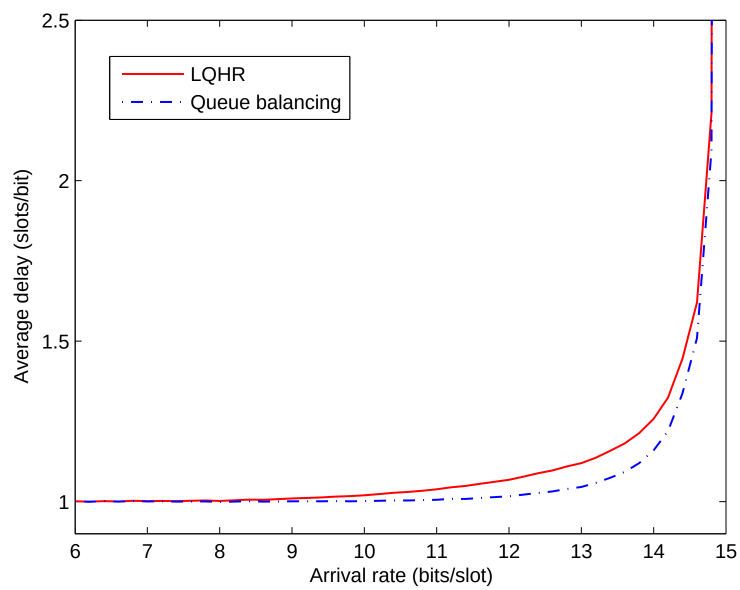


Figure 3.3: Average delay versus arrival arrival rate.

We observe that when λ is small, both the LQHR policy and the queue balancing policy yield delay close to one slot, and the difference between these two policies is insignificant. This is because, the system has a light traffic, and both policies empty both queues in almost all slots. When λ becomes very close to the boundary of the capacity region, the average delay grows rapidly under both policies, and again the difference between the two policies becomes insignificant. This is because, the system has a heavy traffic, and the probability that the queues become empty is very small under both policies, and the actual number of departures in each slot is almost the same for both policies. When λ is neither very small nor very large, the queue balancing policy outperforms the LQHR policy significantly. This is because, equalizing the queue lengths minimizes the probability that one queue is large while the other queue is empty or close to empty, and consequently utilizes the system resources more efficiently.

3.5 Conclusions

In this chapter, we investigated the delay-optimal rate allocation problem in a symmetric MAC. We formulated the problem as a discrete-time MDP, and analyzed the properties of the value function for the corresponding discounted cost MDP. Based on these properties, we proved that the delay optimal rate allocation policy is to equalize the queue lengths in each slot as much as possible.

Chapter 4

Delay Minimization with a General Pentagon Rate Region

4.1 Introduction

In Chapter 3, we investigate the delay-optimal rate allocation in a symmetric AWGN multiple access channel (MAC), where the underlying capacity region is a symmetric pentagon. We prove that the *queue length balancing* policy, which minimizes the queue length difference while working on the dominant face of the capacity region in each slot, minimizes the average bit delay in the system. The goal of this chapter is to use a general pentagon shaped underlying rate region (hence, non-time-divided transmissions) and determine the optimal rate allocation policy from this available rate region, as a function of the current queue sizes of the users, to minimize the delay.

Delay minimization for a single-user communication channel has been investigated in [2, 4, 5], where the structural properties of the optimum power/rate allocation policies, and relationships between average power and delay have been determined for fading channels, using dynamic programming and Markov decision process (MDP) formulations. In these works, due to the large number of possible rate/power choices at each channel state, it has been almost impossible to get analytical closed-form optimal solutions. For multi-user systems, even the properties of the optimum rate allocation have been impossible to obtain, except for special rate regions.

Reference [9] considers a symmetric Gaussian MAC, and proves that in order to minimize the *packet* delay, the system should operate at an extreme point of the MAC capacity region, and the larger rate should be given to the user with the larger queue size, hence the name of the proposed policy: longer-queue-higher-rate (LQHR). Reference [10] generalizes [9] to a potentially asymmetric setting, and proves that the delay-optimal policy has a switch structure, i.e., the queue state space should be divided into two, and in each region, the system should operate at one of the two corner points. However, unlike the symmetric case in [9], the explicit form of the switch curve is unknown. Reference [11] develops a policy named "modified LQHR" which works at a corner point of the pentagon when the queue lengths are different, and switches to the mid-point of the dominant face of the pentagon when the queue lengths become equal. The "modified LQHR" algorithm is shown to minimize the average *bit* delay in a symmetric system. The third chapter of [12] extends "modified LQHR" to a symmetric M-user scenario.

From the literature above, we observe that the explicit solution of the queue-length based delay-minimization problem is known only for the symmetric Gaussian MAC, where the underlying rate region is a symmetric pentagon. Even for the asymmetric pentagon, the delay-minimizing policy is not known. The reason for this is that delay-minimization requires maximizing the throughput at the current time as well as maximizing the throughput in the future. These are often conflicting objectives. The first objective requires maximizing the sum-rate while the second objective requires balancing the queue lengths. Unbalanced queue lengths increases the likelihood of one of the queues becoming empty, which results in inefficiency of transmission, as it decreases the future achievable sum-rates. Thanks to the special properties of the capacity region of the symmetric Gaussian MAC, these two objectives can be achieved simultaneously.

However, having a symmetric pentagon as a capacity region is a peculiarity of the symmetric Gaussian MAC. The capacity region of a general (non-Gaussian) MAC is not a pentagon, it is a union of pentagons [24]. Likewise, the capacity regions of the fading Gaussian MAC [30], the Gaussian MAC with multiple antennas [31], or the Gaussian MAC with user cooperation [32, 33] are not pentagons. In this chapter, we will consider a two-user communication channel with a general pentagon rate region. Different from the Gaussian MAC capacity region, the pentagon we assume does not have a 45° dominant face. The motivation to study such a rate region is two-fold: First, it is the simplest extension of the rate regions studied so far, that changes a characteristic of the rate region in a fundamental way. This characteristic is that the two corner points on the dominant face do not have equal sum-rates. Therefore, in this example rate region, we are able to observe the tension between throughput optimality, i.e., the desire to work at the point that yields the largest sum-rate, and balancing the queue lengths, i.e., the desire to favor the longer queue over the shorter one, more explicitly. Secondly, this asymmetric pentagon with a non-45° dominant face can be seen as a crude approximation of a general rate region, as shown in Figure 4.1. That is, we can imagine this asymmetric pentagon to be the largest such shape fitting in a general rate region, which may belong to a MAC with fading, multiple antennas, or cooperation.

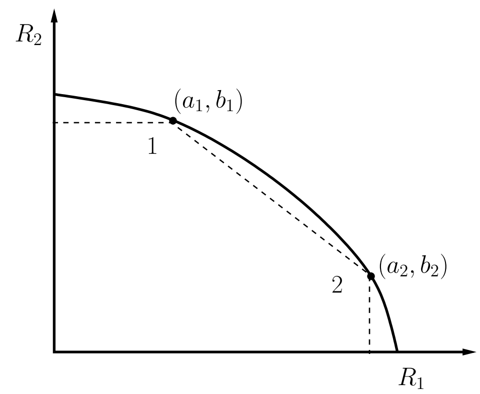


Figure 4.1: The asymmetric pentagon rate region with a non-45° dominant face. Corner point 2 has larger sum-rate, i.e., $a_2 + b_2 > a_1 + b_1$.

Our goal in this chapter is to assign rate pairs to users from the underlying rate region based on their current queue lengths in order to minimize the average delay in the system. We formulate the problem as an MDP and prove that the delay-optimal policy should operate at one of the two corner points of the rate region. Through value iteration, we prove that a switch curve structure exists in the queue state space. Next, we prove that for the discounted-cost MDP, the switch curve has a limit on one of the queue lengths, i.e., when one of the queue lengths exceeds a threshold, the transmitters always operate at the corner point which has the larger sum-rate (see Figure 4.5). That is, the delay-optimal policy favors throughput-optimality (i.e., larger sum-rate) unless the first queue gets close to empty, in which case, the policy favors balancing queue lengths. Our result has two practical implications: First, it gives a *partial analytical characterization* for the delay-optimal switch curve. Secondly, it implies that we can operate the queues *partially distributedly*, in that, if the current queue length of the first user is larger than the limit, then this user does not need to know the current queue length of the other user in order to decide about the rate point at which it should operate on the rate region.

Finally, we note that, according to the optimal policy, always operating at the maximum sum-rate point is not optimal. With the goal of maximizing the current sum-rate as well as the sum-rate in the future, depending on the current queue lengths, the optimal policy may switch from the maximum sum-rate point to the rate point that favors balancing the queue lengths. This action minimizes the probability that any one of the queues becomes empty in the future, hence maximizes the overall transmission rates, and consequently, minimizes the overall delay. Therefore, we observe that, the optimal rate allocation policy trades some of the instantaneously achievable sum-rate in favor of balancing the queue lengths, with the goal of minimizing the overall delay.

4.2 System Model and Problem Formulation

We consider a communication system with two transmitters, and one receiver, as in Figure 4.2. The underlying rate region is a general pentagon as shown in Figure 4.1. We denote the two corner points of the rate region as points 1 and 2, with rate pairs (a_1, b_1) and (a_2, b_2), respectively. Without loss of generality, we assume that $a_2 + b_2 > a_1 + b_1$, i.e., that point 2 has a larger sum-rate. We denote the difference between the two sum-rates by $\delta = a_2 + b_2 - (a_1 + b_1)$.

In the medium access control layer, we assume that packets arrive at the source nodes according to independent Poisson processes with parameters λ_1 and λ_2, see Figure 4.2. We also assume that the packet lengths are independent and identically distributed exponential random variables with unit mean. Therefore, for a given transmission rate r, the transmission time for a packet is an exponential random variable with parameter r. There is a buffer with infinite capacity at each transmitter, storing the packets until they are transmitted. Let $q_1(t)$, $q_2(t)$ denote the number of packets in the two buffers at time t. The transmitters determine their transmission rates, which are the components of the rate vector $\mathbf{r}$, where $\mathbf{r}$ is in the rate region, based on the current queue length vector $\mathbf{q}(t) = (q_1(t), q_2(t))$. Therefore, on the medium access control layer, the queue lengths evolve according to a continuous-time Markov chain, whose transition rates are determined by the arrival and transmission rates.

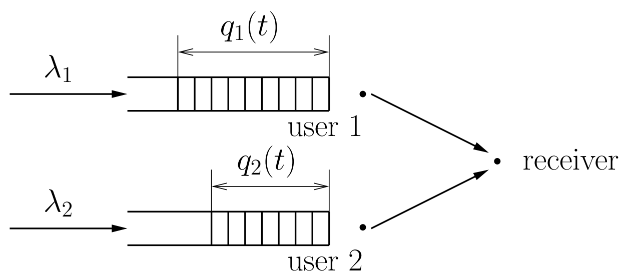


Figure 4.2: The system model.

According to Little's law [25], minimizing the average delay in the system is equivalent to minimizing the average number of packets in the system. Assuming that the system starts from state $\mathbf{q}(0)$, the delay minimization problem is to obtain an optimal policy $\mathbf{u}$, to minimize the long-term average cost:

$$\limsup_{t\to\infty} \frac{1}{t} E\left[\int_0^t \mathbf{q}(s)^T \mathbf{e} ds | q(0)\right] \tag{4.1}$$

where $\mathbf{e}$ is the vector of all ones.

Sampling the system at certain epoches, we can convert the original continuous-time problem into a discrete-time problem [28]. Intuitively, we intend to sample the system at any epoch when an arrival or departure occurs. However, because the transition rates are different at different operating points, the sampling frequency may be different for different states. In order to sample the system at a uniform frequency, we adopt the normalization method in [34]. Since $a_2 + b_2$ is the maximum sum of transmission rates, the maximum total transition rate of the system is $\lambda_1 + \lambda_2 + a_2 + b_2$, which we define as γ. Let us denote the transmission rates of the users as r_1 and r_2. If $r_1 + r_2 < a_2 + b_2$, we assume that there is a third transmitter transmitting a dummy packet with rate $a_2 + b_2 - (r_1 + r_2)$. Then, we sample at the epoches when either a packet arrives, or a packet (dummy or real) departs. Therefore, the sampling frequency for all of the states will be the same, and the corresponding discrete-time Markov chain will precisely represent the original system.

After sampling and discretizing the continuous-time system, our goal will be to choose $\mathbf{r}$ at every transition epoch to minimize the average delay. Let us denote the indices of the transition epoches as n, $n = 1, 2, \ldots$. Given the initial queue lengths $\mathbf{q}_0$, the delay minimization problem is to determine the optimal policy that minimizes:

$$\limsup_{N\to\infty} \frac{1}{N} E\left[\sum_{n=0}^{N-1} \mathbf{q}[n]^T \mathbf{e} | \mathbf{q}[0] = \mathbf{q}_0\right] \tag{4.2}$$

Let us define A_i and D_i to be an arrival or (potential) departure at the ith queue, $i = 1, 2$. For example, $A_1\mathbf{q} = (q_1+1, q_2)$, $D_1\mathbf{q} = ((q_1-1)^+, q_2)$. We first define the corresponding discounted-cost problem with a discount factor β, and obtain the dynamic programming formulation:

$$V_N^\beta(\mathbf{q}) = \mathbf{q}^T\mathbf{e}+\beta\gamma^{-1}\Big[\lambda_1 V_{N-1}^\beta(A_1\mathbf{q}) + \lambda_2 V_{N-1}^\beta(A_2\mathbf{q}) + \min_{\mathbf{r}\in C}\Big\{r_1 V_{N-1}^\beta(D_1\mathbf{q}) + r_2 V_{N-1}^\beta(D_2\mathbf{q}) + (a_2+b_2-r_1-r_2)V_{N-1}^\beta(\mathbf{q})\Big\}\Big] \tag{4.3}$$

where C is the rate region from which rates r_1 and r_2 are chosen. As $N \to +\infty$, $V_N^\beta(\mathbf{q}) \to V^\beta(\mathbf{q})$, which is the unique solution of the optimality equation:

$$V^\beta(\mathbf{q}) = \mathbf{q}^T\mathbf{e} + \beta\gamma^{-1}\Big[\lambda_1 V^\beta(A_1\mathbf{q}) + \lambda_2 V^\beta(A_2\mathbf{q}) + \min_{\mathbf{r}\in C}\Big\{r_1 V^\beta(D_1\mathbf{q}) + r_2 V^\beta(D_2\mathbf{q}) + (a_2+b_2-r_1-r_2)V^\beta(\mathbf{q})\Big\}\Big] \tag{4.4}$$

This is a two-dimensional MDP, which is difficult to solve in general. We first determine some structural properties of the optimal policy.

Lemma 4.1 *$V^\beta(\mathbf{q})$ is monotonically increasing in q_i, $i = 1, 2$.*

Proof: We prove this lemma using induction. First, since $V_0^\beta(\mathbf{q}) = 0$, $V_N^\beta(\mathbf{q})$ increases monotonically in q_1 and q_2 for $N = 0$. Then, we assume that this lemma holds for $V_N^\beta(\mathbf{q}), N > 0$, and prove it for $N+1$. Since

$$V_{N+1}^\beta(\mathbf{q}) = \mathbf{q}^T\mathbf{e} + \beta\gamma^{-1}\Big[\lambda_1 V_N^\beta(A_1\mathbf{q}) + \lambda_2 V_N^\beta(A_2\mathbf{q}) + \min_{\mathbf{r}\in C}\Big\{r_1 V_N^\beta(D_1\mathbf{q}) + r_2 V_N^\beta(D_2\mathbf{q}) + (a_2+b_2-r_1-r_2)V_N^\beta(\mathbf{q})\Big\}\Big] \tag{4.5}$$

Using the assumption that $V_N^\beta(\mathbf{q})$ is monotonically increasing in q_1 and q_2 and the fact that $\mathbf{q}^T\mathbf{e}$ is also monotonically increasing in q_1 and q_2, in order to prove the monotonicity of $V_{N+1}^\beta(\mathbf{q})$ in q_1 and q_2, we only need to show that

$$\min_{\mathbf{r}\in C}\Big\{r_1 V_N^\beta(D_1\mathbf{q}) + r_2 V_N^\beta(D_2\mathbf{q}) + (a_2+b_2-r_1-r_2)V_N^\beta(\mathbf{q})\Big\} \tag{4.6}$$

is monotonically increasing in q_1 and q_2. We compare the values of this expression at two states $A_1\mathbf{q}$ and $\mathbf{q}$ as follows

$$\min_{\mathbf{r}\in C}\Big\{r_1 V_N^\beta(D_1A_1\mathbf{q}) + r_2 V_N^\beta(D_2A_1\mathbf{q}) + (a_2+b_2-r_1-r_2)V_N^\beta(A_1\mathbf{q})\Big\} \tag{4.7}$$

$$= r_1^* V_N^\beta(D_1A_1\mathbf{q}) + r_2^* V_N^\beta(D_2A_1\mathbf{q}) + (a_2+b_2-r_1^*-r_2^*)V_N^\beta(A_1\mathbf{q}) \tag{4.8}$$

$$\geq r_1^* V_N^\beta(D_1\mathbf{q}) + r_2^* V_N^\beta(D_2\mathbf{q}) + (a_2+b_2-r_1^*-r_2^*)V_N^\beta(\mathbf{q}) \tag{4.9}$$

$$\geq \min_{\mathbf{r}\in C}\Big\{r_1 V_N^\beta(D_1\mathbf{q}) + r_2 V_N^\beta(D_2\mathbf{q}) + (a_2+b_2-r_1-r_2)V_N^\beta(\mathbf{q})\Big\} \tag{4.10}$$

where (r_1^*, r_2^*) minimizes the value of (4.6) at state $A_1\mathbf{q}$. Here the first inequality follows from the assumption that $V_N^\beta(\mathbf{q})$ is monotonically increasing in q_1 and q_2, and the second inequality follows from the fact that (r_1^*, r_2^*) may not be the minimizer of the function in (4.10).

Comparing (4.7) and (4.10), we conclude that the function in (4.6) is monotonically increasing in q_1 and q_2 for N. Then, since this is true for any N, by taking the limit $V^\beta(\mathbf{q}) = \lim_{N\to\infty} V_N^\beta(\mathbf{q})$ is monotonically increasing in q_1 and q_2. $\square$

Lemma 4.2 *The optimal operating point must lie on the boundary of the rate region. In addition, it must be one of the two corner points.*

Proof: The first half of Lemma 4.2 can be proved using Lemma 4.1. If the optimal operating point (r_1, r_2) is not on the boundary but is in the interior of the rate region, then, we can always find another operating point (r_1', r_2') on the boundary, where $r_1' \geq r_1$, and $r_2' \geq r_2$. Then, based on Lemma 4.1, the resulting value of (4.6) will be strictly smaller when operating at (r_1', r_2') compared to the value when operating at (r_1, r_2). This contradicts with the optimality of (r_1, r_2). Thus, the optimal operating point must lie on the boundary of the rate region. Therefore, we only need to focus on the dominant face of the capacity region. Any point (r_1, r_2) on the dominant face can be expressed as a linear combination of the two corner points. Thus, we have

$$\begin{aligned}
&\min_{\mathbf{r}\in C} \Big\{ r_1 V_{N-1}^\beta(D_1\mathbf{q}) + r_2 V_{N-1}^\beta(D_2\mathbf{q}) + (a_2 + b_2 - r_1 - r_2) V_{N-1}^\beta(\mathbf{q}) \Big\} \\
&\quad = \min_{\rho\in(0,1)} \Big\{ \rho \Big(a_1 V_{N-1}^\beta(D_1\mathbf{q}) + b_1 V_{N-1}^\beta(D_2\mathbf{q}) + \delta V_{N-1}^\beta(\mathbf{q}) \Big) \\
&\qquad\qquad + (1-\rho) \Big(a_2 V_{N-1}^\beta(D_1\mathbf{q}) + b_2 V_{N-1}^\beta(D_2\mathbf{q}) \Big) \Big\} \qquad (4.11) \\
&\quad = a_2 V_{N-1}^\beta(D_1\mathbf{q}) + b_2 V_{N-1}^\beta(D_2\mathbf{q}) \\
&\qquad + \min \Big\{ (a_1 - a_2) V_{N-1}^\beta(D_1\mathbf{q}) + (b_1 - b_2) V_{N-1}^\beta(D_2\mathbf{q}) + \delta V_{N-1}^\beta(\mathbf{q}), 0 \Big\} \qquad (4.12)
\end{aligned}$$

where the last equality follows from the fact that the minimizer for a linear function must be one of the end points. □

Let T be an operator defined on real-valued functions as:

$$Tf(\mathbf{q}) = \mathbf{q}^T\mathbf{e}+\beta\gamma^{-1}\Big[\lambda_1 f(A_1\mathbf{q}) + \lambda_2 f(A_2\mathbf{q}) + a_2 V^{\beta}_{N-1}(D_1\mathbf{q}) + b_2 V^{\beta}_{N-1}(D_2\mathbf{q}) \\ + \min\Big\{(a_1 - a_2)f(D_1\mathbf{q}) + (b_1 - b_2)f(D_2\mathbf{q}) + \delta f(\mathbf{q}), 0\Big\}\Big] \quad (4.13)$$

Therefore, the dynamic programming optimality equation can be written as

$$V^{\beta}_{N+1}(\mathbf{q}) = TV^{\beta}_{N}(\mathbf{q}) \quad (4.14)$$

4.3 An Inductive Proof of the Switch Structure

In this section, we prove that the delay-optimal policy has a switch structure. In order to prove that, we first define a set of functions with properties which are sufficient to have a switch structure. We show that these properties are preserved under the operator T. Since $V^{\beta}_0 = 0$ is within this set, using induction, we will show that V^{β} will be within this set.

Let us define $\mathcal{F}$ to be the set of real-valued functions such that:

1. $f(\mathbf{q})$ is increasing in q_1 and q_2.

2. $f(\mathbf{q} + \mathbf{x}) - f(\mathbf{q})$ is increasing in q_1 and q_2 for any fixed $\mathbf{x}$.

3. $(a_1 - a_2)f(D_1\mathbf{q}) + (b_1 - b_2)f(D_2\mathbf{q}) + \delta f(\mathbf{q})$ is increasing in q_1.

4. $(a_1 - a_2)f(D_1\mathbf{q}) + (b_1 - b_2)f(D_2\mathbf{q}) + \delta f(\mathbf{q})$ is decreasing in q_2.

Then, we have the following lemma.

Lemma 4.3 *If $f \in \mathcal{F}$, then $Tf \in \mathcal{F}$.*

The proof of Lemma 4.3, when $\delta = 0$, can be found in [35]. When $\delta \neq 0$, the proof is different, and is provided in Appendix 4.7.

Lemma 4.4 *$V_n^\beta(\mathbf{q}) \in \mathcal{F}$ for all n.*

This lemma can be verified as follows. Since $V_0^\beta = 0$, V_0^β is in $\mathcal{F}$. Using Lemma 4.3 recursively, we have $V_n^\beta(\mathbf{q}) \in \mathcal{F}$ for $n = 0, 1, 2, \ldots$.

We now define the switch function:

$$s_n(q_1) = \min \left\{ q_2 : (a_1 - a_2)f(D_1\mathbf{q}) + (b_1 - b_2)f(D_2\mathbf{q}) + \delta f(\mathbf{q}) \leq 0 \right\} \tag{4.15}$$

A generic switch function is shown in Figure 4.3. As we state in the following theorem, the optimal rate assignment problem has a switch structure.

Theorem 4.1 *The optimal policy for the discounted-cost MDP has a switch structure, i.e., $s_n(q_1)$ is increasing for every n.*

This theorem can be proved using properties 3) and 4) of $V_n^\beta(\mathbf{q})$. The switch curve partitions the queue state space into two parts, each corresponding to one of the two operating points (corner points of the pentagon). Following the arguments in [10, 35], we can prove that the switch structure still exists when $\beta \to 1$, i.e., for the average cost problem.

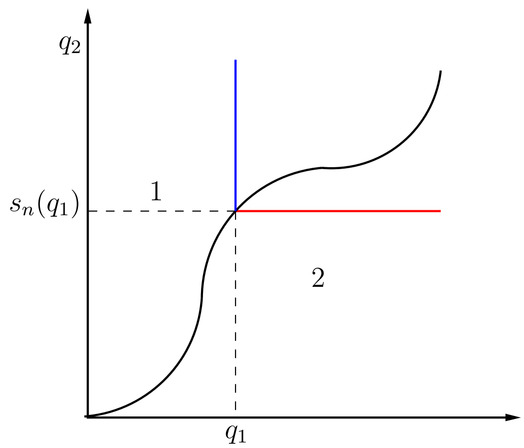


Figure 4.3: The switch structure of the optimal policy.

While we have proved that the optimal policy has a switch structure, i.e., that the queue state space is divided into two, where in each region the optimal policy operates the system at one of the two corner points, a closed form solution for this switch curve is not known in general. The switch curve is explicitly known only for one special case, which is the symmetric Gaussian MAC case, where the rate region is a symmetric pentagon with a 45° dominant face. In that case the switch curve is a 45° straight line emanating from the origin, i.e., $s_n(q_1) = q_1$, as shown in Figure 4.4. This implies that the system operates at one of the corner points when $q_1 > q_2$, and at the other corner point when $q_1 < q_2$. This results in the LQHR policy in [9]. In the asymmetric Gaussian MAC case, where the rate region is an asymmetric pentagon, but with still a 45° dominant face, even though it is known that a switch curve structure exists, the explicit form of the switch curve is not known [10]. In the next section, we will show that, in this more general case where we have an asymmetric pentagon rate region with a non-45° dominant face, even though we do not have an explicit formula for the switch curve, we show that we have a limit on the switch curve along one of the dimensions.

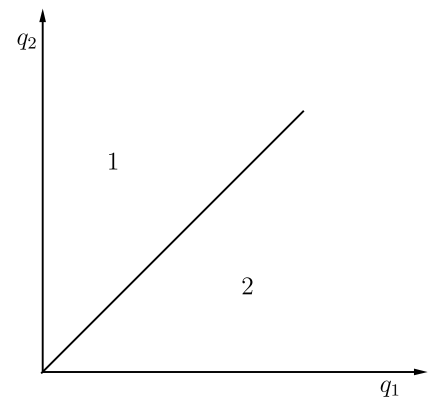


Figure 4.4: The switch structure for a symmetric Gaussian MAC.

4.4 The Limit on the Switch Curve

Although we have shown that the delay optimal policy has a switch structure, it is difficult to obtain the exact switch curve analytically. In this section, we will show that the switch curve is bounded in the q_1-dimension. In other words, we can find a threshold N, such that, for all q_1 greater than this threshold, the optimal operating point is the second corner point of the pentagon. In order to prove this, we start from an initial function f_0, which is linear in $q_1 + q_2$. We will use f_0 to approximate V^β over a large portion of the state space. Specifically, this region includes states $\mathbf{q}$ with $q_1, q_2 > N$, where N is a large enough number. Let us define:

$$f_0(\mathbf{q}) = \frac{1}{1-\beta}(q_1 + q_2) + \frac{\beta}{(1-\beta)^2} \frac{\lambda_1 + \lambda_2 - a_2 - b_2}{\lambda_1 + \lambda_2 + a_2 + b_2} \tag{4.16}$$

Clearly, $f_0 \in \mathcal{F}$. It is easy to verify that

$$T f_0(\mathbf{q}) - f_0(\mathbf{q}) = \begin{cases} 0 & q_1, q_2 \neq 0 \\ \frac{\beta(a_2+b_2)}{\gamma(1-\beta)} & \mathbf{q} = \mathbf{0} \\ \frac{\beta(a_1+\delta)}{\gamma(1-\beta)} & q_1 = 0 \\ \frac{\beta b_2}{\gamma(1-\beta)} & q_2 = 0 \end{cases} \tag{4.17}$$

that is, Tf_0 and f_0 differ only on the boundary, and for all states away from the boundary, these two functions have the same value. This is a key property that will be essential in this section. Note that under the operator T, the difference caused by the boundary only propagates into the interior region of the state space by one layer in each iteration; rest of the states are not affected by the operator.

Let us define:

$$|f|_k = \max\{f(\mathbf{q}) : q_1, q_2 \geq 0, q_1 + q_2 \leq k\} \tag{4.18}$$

which is the maximum value of the function f in the region where the sum of the queue lengths is less than k. Similarly, let us define

$$|f|_\infty = \sup\{f(\mathbf{q}) : q_1, q_2 \geq 0\} \tag{4.19}$$

which is allowed to be infinity. Then, we have the following property.

Lemma 4.5 *For* $\forall f, g \in \mathcal{F}$, $|Tf - Tg|_k \leq \beta |f - g|_{k+1}$.

Proof:

$$
\begin{aligned}
&Tf(\mathbf{q}) - Tg(\mathbf{q}) \\
&\quad = \beta\gamma^{-1}\Big[\lambda_1 f(A_1\mathbf{q}) + \lambda_2 f(A_2\mathbf{q}) - \lambda_1 g(A_1\mathbf{q}) - \lambda_2 g(A_2\mathbf{q}) \\
&\qquad + \min\Big\{a_1 f(D_1\mathbf{q}) + b_1 f(D_2\mathbf{q}) + \delta f(\mathbf{q}), a_2 f(D_1\mathbf{q}) + b_2 f(D_2\mathbf{q})\Big\} \\
&\qquad - \min\Big\{a_1 g(D_1\mathbf{q}) + b_1 g(D_2\mathbf{q}) + \delta f(\mathbf{q}), a_2 g(D_1\mathbf{q}) + b_2 g(D_2\mathbf{q})\Big\}\Big] \qquad (4.20)
\end{aligned}
$$

Since $|\min\{a,b\} - \min\{c,d\}| \leq \max\{|a-c|, |b-d|\}$, we have

$$
\begin{aligned}
&|Tf - Tg|_k \\
&\quad \leq \beta\gamma^{-1}\Big[\lambda_1|f-g|_{k+1} + \lambda_2|f-g|_{k+1} \qquad (4.21) \\
&\qquad + \max\Big\{a_1|f-g|_{k-1} + b_1|f-g|_{k-1} + \delta|f-g|_k, a_2|f-g|_{k-1} + b_2|f-g|_{k-1}\Big\}\Big] \\
&\quad \leq \beta\gamma^{-1}(\lambda_1 + \lambda_2 + a_2 + b_2)|f-g|_{k+1} \qquad (4.22) \\
&\quad = \beta|f-g|_{k+1} \qquad (4.23)
\end{aligned}
$$

completing the proof. □

Lemma 4.6 *$T^n f_0$ converges to a function f as $n \to +\infty$, and $Tf = f$.*

Proof: Since $f_0 \in \mathcal{F}$, $T^n f_0 \in \mathcal{F}$ for any $n > 0$.

$$|T^{n+1} f_0 - T^n f_0|_k \leq \beta |T^n f_0 - T^{n-1} f_0|_{k+1} \tag{4.24}$$

$$\leq \beta^n |T f_0 - f_0|_{k+n} \tag{4.25}$$

$$\leq \frac{\beta^{n+1}(a_2 + b_2)}{\gamma(1-\beta)} \tag{4.26}$$

where (4.26) follows from (4.17). We observe that (4.26) does not depend on k, thus, $|T^{n+1} f_0 - T^n f_0|_\infty$ is uniformly bounded by (4.26). Since $\beta < 1$, the right hand side of (4.26) forms a Cauchy sequence, therefore, $T^n f_0$ converges to a function f pointwise. In other words, for any ϵ, we can find an $N_1(\epsilon)$ such that when $n \geq N_1(\epsilon)$, we have $|f - T^{n-1} f_0|_\infty \leq \epsilon$. Thus, for such n, we have

$$|Tf - f|_\infty \leq |Tf - T^n f_0|_\infty + |T^n f_0 - f|_\infty \tag{4.27}$$

$$\leq \beta |f - T^{n-1} f_0|_\infty + |T^n f_0 - f|_\infty \tag{4.28}$$

$$\leq (\beta + 1)\epsilon = \epsilon' \tag{4.29}$$

Therefore, for any ϵ', we can find a $n > N_1(\frac{\epsilon'}{\beta+1})$, such that $|Tf - f|_\infty \leq \epsilon'$. In other words, Tf and f are arbitrarily close. Thus, $Tf = f$. □

Lemma 4.7 *Let $V_0^\beta(\mathbf{q}) = 0$, then, $V_n^\beta(\mathbf{q}) = T^n V_0^\beta(\mathbf{q})$ converges to $V^\beta(\mathbf{q})$, and $f(\mathbf{q}) = V^\beta(\mathbf{q})$.*

Proof: In order to prove that $f(\mathbf{q}) = V^\beta(\mathbf{q})$ pointwise, we start from the following:

$$|f - V^\beta|_k \le |f - T^n f_0|_k + |T^n f_0 - V_n^\beta|_k + |V_n^\beta - V^\beta|_k \tag{4.30}$$

$$\le |f - T^n f_0|_k + \beta|T^{n-1} f_0 - V_{n-1}^\beta|_{k+1} + |V_n^\beta - V^\beta|_k \tag{4.31}$$

$$\le |f - T^n f_0|_k + |V_n^\beta - V^\beta|_k + \beta^n|f_0 - V_0^\beta|_{k+n} \tag{4.32}$$

$$= |f - T^n f_0|_k + |V_n^\beta - V^\beta|_k + \beta^n\left(\frac{n+k}{1-\beta} + \frac{\beta}{(1-\beta)^2}\frac{\lambda_1 + \lambda_2 - a_2 - b_2}{\lambda_1 + \lambda_2 + a_2 + b_2}\right) \tag{4.33}$$

$$\le \epsilon_1 + \epsilon_2 + \epsilon_3 \tag{4.34}$$

where (4.31) follows from Lemma 4.5, (4.33) follows from the definition of f_0, and (4.34) follows from the fact that $T^n f_0$ converges to f_0, V_n^β converges to V^β, and $\beta^n n \to 0$. Therefore, when n is large enough, we have the difference bounded by (4.34). We note that (4.34) does not depend on k, thus $f(\mathbf{q}) = V^\beta(\mathbf{q})$ for any point $\mathbf{q}$. $\square$

Lemma 4.5 means that starting from f_0 and performing the iterations, V^β converges to the same function if we started from $V_0^\beta = 0$. The convergence point is the unique solution of the optimality equation (4.4). Next, we will prove that $f(\mathbf{q})$ gets arbitrarily close to $f_0(\mathbf{q})$ when $q_1, q_2 \to +\infty$.

Lemma 4.8 $|f - T^n f_0|_\infty \le \frac{\beta^{n+1}(a_2+b_2)}{\gamma(1-\beta)^2}$.

Proof:

$$
\begin{aligned}
|T^{n+p}f_0 - T^n f_0|_k \leq |T^{n+p}f_0 - T^{n+p-1}f_0|_k + |T^{n+p-1}f_0 - T^{n+p-2}f_0|_k + \cdots \\
+ |T^{n+1}f_0 - T^n f_0|_k \qquad (4.35) \\
\leq \left(\beta^{n+p-1} + \beta^{n+p-2} + \cdots + \beta^n\right) |Tf_0 - f_0|_{k+n+p} \qquad (4.36) \\
\leq \frac{\beta^n(1-\beta^p)}{1-\beta} \frac{\beta(a_2+b_2)}{\gamma(1-\beta)} \qquad (4.37)
\end{aligned}
$$

Note that (4.37) does not depend on k, therefore, $|T^{n+p}f_0 - T^n f_0|_\infty$ is uniformly bounded, and we have

$$
\begin{aligned}
|f - T^n f_0|_\infty = \lim_{p\to\infty} |T^{n+p}f_0 - T^n f_0|_\infty \qquad (4.38) \\
= \frac{\beta^{n+1}(a_2+b_2)}{\gamma(1-\beta)^2} \qquad (4.39)
\end{aligned}
$$

□

Theorem 4.2 *$f(\mathbf{q})$ gets arbitrarily close to $f_0(\mathbf{q})$ when $q_1, q_2 \to +\infty$. Therefore, the switch curve has a limit on q_1.*

Proof: For any fixed state $\mathbf{q}$, we have

$$
|f(\mathbf{q}) - f_0(\mathbf{q})| \leq |f(\mathbf{q}) - T^n f_0(\mathbf{q})| + |T^n f_0(\mathbf{q}) - f_0(\mathbf{q})| \qquad (4.40)
$$

Based on Lemma 4.8, we can see that for $\forall\epsilon$, there exists $N(\epsilon)$, such that $|f -$

$T^{N(\epsilon)} f_0|_\infty \leq \epsilon$. From the definition in (4.19),

$$|f(\mathbf{q}) - T^{N(\epsilon)} f_0(\mathbf{q})| \leq |f - T^{N(\epsilon)} f_0|_\infty \leq \epsilon \tag{4.41}$$

At the same time, from (4.17), we know that $T^{N(\epsilon)} f_0(\mathbf{q})$ only differs from $f_0(\mathbf{q})$ over the states which are within $N(\epsilon)$ layers away from the boundary. Thus, for all $q_1 > N(\epsilon), q_2 > N(\epsilon)$,

$$T^{N(\epsilon)} f_0(\mathbf{q}) - f_0(\mathbf{q}) = 0 \tag{4.42}$$

Therefore, combining (4.40)-(4.42), for any $\mathbf{q}$, $q_1 > N(\epsilon), q_2 > N(\epsilon)$, (4.40) is bounded by

$$|f(\mathbf{q}) - f_0(\mathbf{q})| \leq |f - f_0|_\infty + 0 = \epsilon \tag{4.43}$$

i.e., $-\epsilon \leq f(\mathbf{q}) - f_0(\mathbf{q}) \leq \epsilon$. Thus, in this region, as shown in Figure 4.5, we have

$$\begin{aligned}
&a_1 f(D_1\mathbf{q}) + b_1 f(D_2\mathbf{q}) + \delta f(\mathbf{q}) - a_2 f(D_1\mathbf{q}) - b_2 f(D_2\mathbf{q}) \\
&\quad = (b_1 - b_2) f(D_2\mathbf{q}) + \delta f(\mathbf{q}) - (a_2 - a_1) f(D_1\mathbf{q}) && (4.44) \\
&\quad \geq (b_1 - b_2)\left(f_0(D_2\mathbf{q}) - \epsilon\right) + \delta(f_0(\mathbf{q}) - \epsilon) - (a_2 - a_1)\left(f_0(D_1\mathbf{q}) + \epsilon\right) && (4.45) \\
&\quad = \frac{\delta}{1-\beta} - 2(a_2 - a_1)\epsilon && (4.46)
\end{aligned}$$

where the inequality follows from (4.43). Therefore, when

$$\epsilon \leq \frac{\delta}{2(a_2 - a_1)(1 - \beta)} \tag{4.47}$$

(4.46) is always greater than zero, thus point 2 is always better than point 1. From Lemma 4.8, let

$$\epsilon = \frac{\beta^{n+1}(a_2 + b_2)}{\gamma(1 - \beta)^2} = \frac{\delta}{2(a_2 - a_1)(1 - \beta)} \tag{4.48}$$

from which, we have

$$N(\epsilon) = \left\lceil \log_\beta \frac{\delta\gamma(1 - \beta)}{2(a_2 + b_2)(a_2 - a_1)} \right\rceil - 1 \tag{4.49}$$

Since we have proved in the previous section that the optimal policy must have a switch curve structure, for any $\mathbf{q}$, such that $q_1 \geq N(\epsilon)$, the optimal policy is always to operate the system at point 2. Thus, the switch curve has a limit. □

The result implies that when both q_1, q_2 are large, the objective of maximizing the sum-rate is more important than balancing the queue lengths in order to minimize the average delay. Thus, in this scenario, operating at point 2 is optimal. When one queue (q_1 in this chapter) becomes close to empty, the objective of balancing the queue lengths becomes more important, and the operating point must be switched from point 2 to point 1.

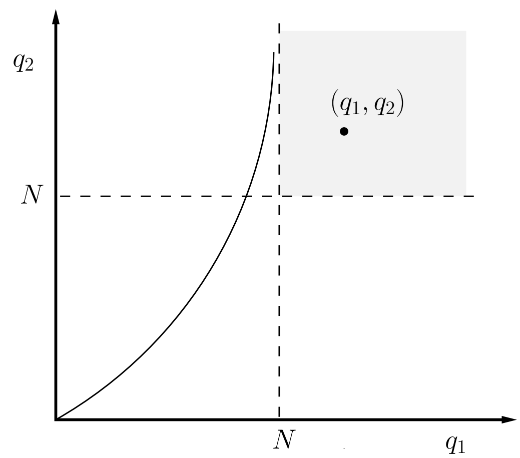


Figure 4.5: The switch curve of the discounted-cost MDP.

4.5 Numerical Results

We consider a system where the arrival rates for the first and second user are $\lambda_1 = 0.4$ packets/unit time, $\lambda_2 = 0.3$ packets/unit time, respectively. We assume that the packet sizes are exponentially distributed i.i.d. random variables with unit mean. We assume that the underlying rate region is a general pentagon, where the normalized coordinates of the first corner point is $(0.3, 0.5)$, and the normalized coordinates of the second corner point is $(0.7, 0.3)$. We first obtain the optimal policy with $\beta = 1$, which corresponds to the average delay minimization policy. We observe that the optimal policy has a switch structure. Then, we vary the value of β, and obtain the optimal policy for the corresponding discounted-cost problem. These curves are shown in Figure 4.6. We observe that for each curve, there is a limit on the dimension of q_1, and all of these curves are lower bounded by the curve with $\beta = 1$. This can be explained in this way: as β increases, the weight of future cost increases. Thus, balancing the queue lengths becomes progressively more important, and for some states, it overweighs maximizing the sum-rate at the current stage. Therefore, in this case, the set of states which operate at the first corner point enlarges.

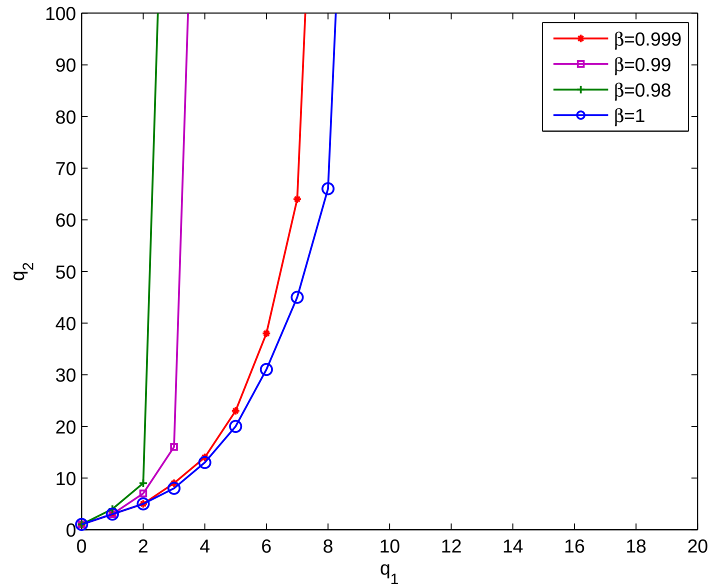


Figure 4.6: The switch curves for the discounted-cost MDP.

4.6 Conclusions

In this chapter, we investigated the delay minimization problem in a two-user communication channel, where the underlying rate region is approximated as a general pentagon. We assumed that the corner points of this pentagon have different sum-rates. We formulated the problem as an MDP, and proved that the delay-optimal policy always operates at one of the two corner points, and has a switch structure. This implies that for some states, the optimal policy requires trading a portion of the sum-rate for balancing the queue lengths in order to minimize the average delay.

We also proved that for the discounted-cost problem, the switch curve is bounded in one of the dimensions. This implies that the queues can be operated partially distributedly.

4.7 Appendix

We prove the properties 1) through 4) of Tf by induction. If $f \in \mathcal{F}$, then obviously, $\mathbf{q}^T\mathbf{e}$, $f(A_1\mathbf{q})$, $f(A_2\mathbf{q})$, $f(D_1\mathbf{q})$, $f(D_2\mathbf{q})$ are in $\mathcal{F}$. Then, it suffices to show that $\min\{(b_1-b_2)f(D_2\mathbf{q})+\delta f(\mathbf{q}),(a_2-a_1)f(D_1\mathbf{q})\}$ is also in $\mathcal{F}$. In order to simply the notation, we define

$$g(\mathbf{q}) = \min\{(b_1-b_2)f(D_2\mathbf{q})+\delta f(\mathbf{q}),(a_2-a_1)f(D_1\mathbf{q})\} \tag{4.50}$$

If $(b_1-b_2)f(D_2\mathbf{q})+\delta f(\mathbf{q}) < (a_2-a_1)f(D_1\mathbf{q})$, then, the optimal operating point for state $\mathbf{q}$ is corner point 1; otherwise, the optimal operating point is corner point 2. We will show that $g(\mathbf{q})$ also possesses the properties 1) through 4) of $f(\mathbf{q})$.

4.7.1 $g(\mathbf{q})$ is increasing in q_1 and q_2.

It is straight forward to prove this property. Hence, we omit its proof.

4.7.2 $g(\mathbf{q}+\mathbf{x}) - g(\mathbf{q})$ is increasing in q_1 and q_2 for any fixed $\mathbf{x}$.

For this property, we will prove that

$$g(A_1^2\mathbf{q}) - g(A_1\mathbf{q}) \geq g(A_1\mathbf{q}) - g(\mathbf{q}) \tag{4.51}$$

$$g(A_2^2\mathbf{q}) - g(A_2\mathbf{q}) \geq g(A_2\mathbf{q}) - g(\mathbf{q}) \tag{4.52}$$

$$g(A_1A_2\mathbf{q}) - g(A_2\mathbf{q}) \geq g(A_1\mathbf{q}) - g(\mathbf{q}) \tag{4.53}$$

First, we evaluate function g at points $\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2\mathbf{q}$, as shown in Figure 4.7.

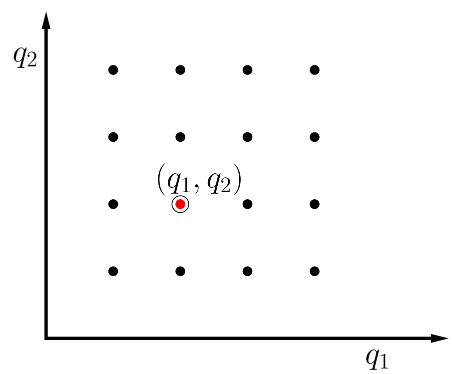


Figure 4.7: We compare the values of $g(\mathbf{q})$ at different states.

If the optimal operating point for state $\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2\mathbf{q}$ is corner point 1, then, we have

$$g(\mathbf{q}) = (b_1 - b_2)f(D_2\mathbf{q}) + \delta f(\mathbf{q}) \tag{4.54}$$

$$g(A_1\mathbf{q}) = (b_1 - b_2)f(D_2A_1\mathbf{q}) + \delta f(A_1\mathbf{q}) \tag{4.55}$$

$$g(A_1^2\mathbf{q}) = (b_1 - b_2)f(D_2A_1^2\mathbf{q}) + \delta f(A_1^2\mathbf{q}) \tag{4.56}$$

Comparing the difference of values between two adjacent states, we have

$$
\begin{aligned}
&g(A_1^2\mathbf{q}) - g(A_1\mathbf{q}) \\
&\quad = (b_1 - b_2)\left(f(D_2A_1^2\mathbf{q}) - f(D_2A_1\mathbf{q})\right) + \delta\left(f(A_1^2\mathbf{q}) - f(A_1\mathbf{q})\right) \qquad (4.57) \\
&\quad \geq (b_1 - b_2)\left(f(D_2A_1\mathbf{q}) - f(D_2\mathbf{q})\right) + \delta\left(f(A_1\mathbf{q}) - f(A_1\mathbf{q})\right) \qquad (4.58) \\
&\quad = g(A_1\mathbf{q}) - g(\mathbf{q}) \qquad (4.59)
\end{aligned}
$$

where the inequality follows from the assumption that $f(\mathbf{q})$ is in $\mathcal{F}$. Similarly, if the optimal operating point for state $\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2\mathbf{q}$ is corner point 2, i.e.,

$$g(\mathbf{q}) = (a_2 - a_1)f(D_1\mathbf{q}) \qquad (4.60)$$

$$g(A_1\mathbf{q}) = (a_2 - a_1)f(D_1A_1\mathbf{q}) \qquad (4.61)$$

$$g(A_1^2\mathbf{q}) = (a_2 - a_1)f(D_1A_1^2\mathbf{q}) \qquad (4.62)$$

we still have $g(A_1^2\mathbf{q}) - g(A_1\mathbf{q}) \geq g(A_1\mathbf{q}) - g(\mathbf{q})$.

If the optimal operating points for state $\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2\mathbf{q}$ are corner points 1, 2, 2, respectively, then, we have

$$g(\mathbf{q}) = (b_1 - b_2)f(D_2\mathbf{q}) + \delta f(\mathbf{q}) \qquad (4.63)$$

$$g(A_1\mathbf{q}) = (a_2 - a_1)f(D_1A_1\mathbf{q}) \qquad (4.64)$$

$$g(A_1^2\mathbf{q}) = (a_2 - a_1)f(D_1A_1^2\mathbf{q}) \qquad (4.65)$$

and,

$$g(A_1\mathbf{q}) - g(\mathbf{q}) = (a_2 - a_1)f(D_1A_1\mathbf{q}) - (b_1 - b_2)f(D_2\mathbf{q}) - \delta f(\mathbf{q}) \tag{4.66}$$

$$= (b_1 - b_2)\left(f(\mathbf{q}) - f(D_2\mathbf{q})\right) \tag{4.67}$$

$$g(A_1^2\mathbf{q}) - g(A_1\mathbf{q}) \geq (a_2 - a_1)f(A_1\mathbf{q}) - (b_1 - b_2)f(D_2A_1\mathbf{q}) - \delta f(A_1\mathbf{q}) \tag{4.68}$$

$$= (b_1 - b_2)\left(f(A_1\mathbf{q}) - f(D_2A_1\mathbf{q})\right) \tag{4.69}$$

Therefore, based on the second property of function f, $g(A_1^2\mathbf{q}) - g(A_1\mathbf{q}) \geq g(A_1\mathbf{q}) - g(\mathbf{q})$ still holds.

Similarly, if the optimal operating points for state $\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2\mathbf{q}$ are corner points 1, 1, 2, respectively,

$$g(\mathbf{q}) = (b_1 - b_2)f(D_2\mathbf{q}) + \delta f(\mathbf{q}) \tag{4.70}$$

$$g(A_1\mathbf{q}) = (b_1 - b_2)f(D_2A_1\mathbf{q}) + \delta f(A_1\mathbf{q}) \tag{4.71}$$

$$g(A_1^2\mathbf{q}) = (a_2 - a_1)f(D_1A_1^2\mathbf{q}) \tag{4.72}$$

Since the operating point at state $A_1\mathbf{q}$ is corner point 1, it implies that we have

$$g(A_1\mathbf{q}) - g(\mathbf{q}) \leq (a_2 - a_1)f(D_1A_1\mathbf{q}) - ((b_1 - b_2)f(D_2\mathbf{q}) + \delta f(\mathbf{q})) \tag{4.73}$$

$$= (b_1 - b_2)\left(f(D_1A_1\mathbf{q}) - f(D_2\mathbf{q})\right) \tag{4.74}$$

On the other hand, we have

$$g(A_1^2\mathbf{q}) - g(A_1\mathbf{q}) = (b_1 - b_2)\left(f(A_1\mathbf{q}) - f(D_2A_1\mathbf{q})\right) \tag{4.75}$$

$$\geq (b_1 - b_2)\left(f(\mathbf{q}) - f(D_2\mathbf{q})\right) \tag{4.76}$$

$$\geq g(A_1\mathbf{q}) - g(\mathbf{q}) \tag{4.77}$$

where the first inequality follows from the second property of function f.

Based on the assumption that $f \in \mathcal{F}$, if the optimal policy for any state $\mathbf{q}$ is to operate at corner point 2, then, because of the third property of f, all the states on its right should operate on point 2 also. In the analysis above, we discuss every possible policy at states $\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2\mathbf{q}$. For all possible cases, we have shown that $g(A_1^2\mathbf{q}) - g(A_1\mathbf{q}) \geq g(A_1\mathbf{q}) - g(\mathbf{q})$. Following similar procedure, we can prove that $g(A_2^2\mathbf{q}) - g(A_2\mathbf{q}) \geq g(A_2\mathbf{q}) - g(\mathbf{q})$, and $g(A_1A_2\mathbf{q}) - g(A_2\mathbf{q}) \geq g(A_1\mathbf{q}) - g(\mathbf{q})$. In summary, we conclude that the property 2) holds for $g(\mathbf{q})$.

4.7.3 $(a_1 - a_2)g(D_1\mathbf{q}) + (b_1 - b_2)g(D_2\mathbf{q}) + \delta g(\mathbf{q})$ is increasing in q_1.

We need to show that

$$\begin{aligned}&(a_1 - a_2)g(A_1A_2\mathbf{q}) + (b_1 - b_2)g(A_1^2\mathbf{q}) + \delta g(A_1^2A_2\mathbf{q})\\ &\qquad \geq (a_1 - a_2)g(A_2\mathbf{q}) + (b_1 - b_2)g(A_1\mathbf{q}) + \delta g(A_1A_2\mathbf{q})\end{aligned} \tag{4.78}$$

We evaluate function g at points $A_1A_2\mathbf{q}$, $A_1^2\mathbf{q}$, $A_1^2A_2\mathbf{q}$, $A_2\mathbf{q}$, $A_1\mathbf{q}$.

First, we note that if the optimal operating points for states $A_1A_2\mathbf{q}$, $A_1^2\mathbf{q}$,

$A_1^2A_2\mathbf{q}$ are corner points 1, 2, 2, respectively, as shown in Figure 4.8(a),

$$g(A_1A_2\mathbf{q}) = (b_1 - b_2)f(D_2A_1A_2\mathbf{q}) + \delta f(A_1A_2\mathbf{q}) \tag{4.79}$$

$$g(A_1^2\mathbf{q}) = (a_2 - a_1)f(D_1A_1^2\mathbf{q}) \tag{4.80}$$

$$g(A_1^2A_2\mathbf{q}) = (a_2 - a_1)f(D_1A_1^2A_2\mathbf{q}) \tag{4.81}$$

we have

$$\begin{aligned}
&(a_1 - a_2)g(A_1A_2\mathbf{q}) + (b_1 - b_2)g(A_1^2\mathbf{q}) + \delta g(A_1^2A_2\mathbf{q}) \\
&\quad = (a_1 - a_2)\big((b_1 - b_2)f(D_2A_1A_2\mathbf{q}) + \delta f(A_1A_2\mathbf{q})\big) \\
&\qquad + (b_1 - b_2)(a_2 - a_1)f(D_1A_1^2\mathbf{q}) + \delta(a_2 - a_1)f(D_1A_1^2A_2\mathbf{q}) \qquad (4.82) \\
&\quad = 0 \qquad (4.83)
\end{aligned}$$

This is an important policy pattern, and we will use it often in the proof afterwards.

Another important policy patten is to operate at corner point 1, 2, 1, for state $A_1A_2\mathbf{q}$, $A_1^2\mathbf{q}$, $A_1^2A_2\mathbf{q}$, respectively, as shown in Figure 4.8(b). In this scenario, we observe that

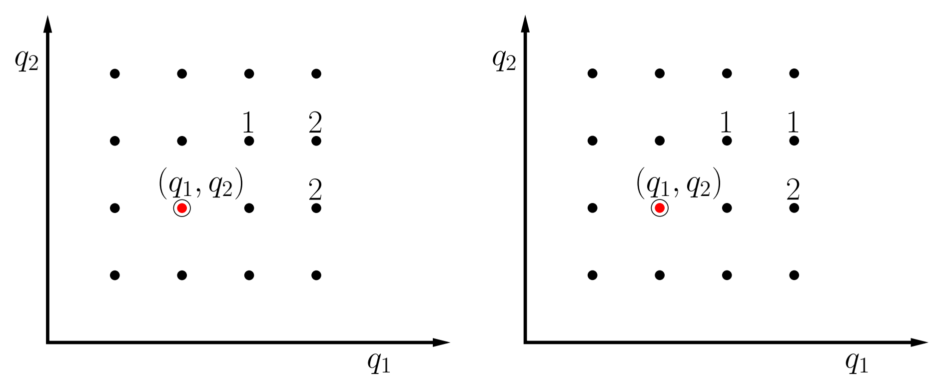
$$(a_1 - a_2)g(A_1A_2\mathbf{q}) + (b_1 - b_2)g(A_1^2\mathbf{q}) + \delta g(A_1^2A_2\mathbf{q}) \tag{4.84}$$

$$= (a_1 - a_2)\big((b_1 - b_2)f(D_2A_1A_2\mathbf{q}) + \delta f(A_1A_2\mathbf{q})\big)$$

$$+ (b_1 - b_2)(a_2 - a_1)f(D_1A_1^2\mathbf{q})$$

$$+ \delta\left((b_1 - b_2)f(D_2A_1^2A_2\mathbf{q}) + \delta f(A_1^2A_2\mathbf{q})\right) \tag{4.85}$$

$$= \delta\left((a_1 - a_2)f(A_1A_2\mathbf{q}) + (b_1 - b_2)f(A_1^2\mathbf{q}) + \delta f(A_1^2A_2\mathbf{q})\right) \tag{4.86}$$



(a) Pattern 1 (b) Pattern 2

Figure 4.8: Two special policy patterns.

If the optimal operating points at $A_1^2\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2A_2\mathbf{q}$, $A_1A_2\mathbf{q}$, $A_2\mathbf{q}$ are $2, 1, 2, 1, 1$, respectively, as shown in Figure 4.9. Then , if we switch the operating point at state $A_1A_2\mathbf{q}$ from corner point 1 to 2, the policy at point $A_2\mathbf{q}$, $A_1\mathbf{q}$, and $A_1A_2\mathbf{q}$ becomes the policy pattern discussed above, and we have

$$
\begin{aligned}
&(a_1 - a_2)g(A_2\mathbf{q}) + (b_1 - b_2)g(A_1\mathbf{q}) + \delta g(A_1A_2\mathbf{q}) \\
&\quad \le (a_1 - a_2)\big((b_1 - b_2)f(D_2A_2\mathbf{q}) + \delta(a_2 - a_1)f(D_1A_1A_2\mathbf{q}) \\
&\qquad + \delta f(A_2\mathbf{q})\big) + (b_1 - b_2)(a_2 - a_1)f(D_1A_1\mathbf{q}) && (4.87) \\
&\quad = 0 && (4.88) \\
&\quad = (a_1 - a_2)g(A_1A_2\mathbf{q}) + (b_1 - b_2)g(A_1^2\mathbf{q}) + \delta g(A_1^2A_2\mathbf{q}) && (4.89)
\end{aligned}
$$

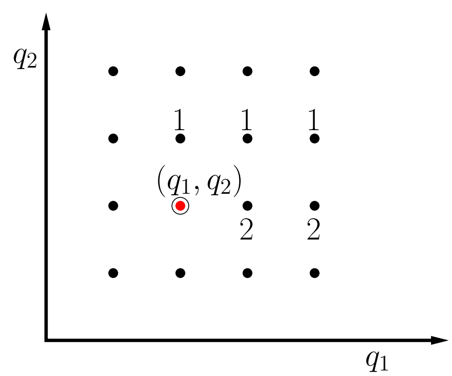


Figure 4.9: The optimal operating points at $A_1^2\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2A_2\mathbf{q}$, $A_1A_2\mathbf{q}$, $A_2\mathbf{q}$ are $2, 1, 2, 1, 1$, respectively.

Similarly, if the optimal operating points at $A_1^2\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2A_2\mathbf{q}$, $A_1A_2\mathbf{q}$, $A_2\mathbf{q}$ are $2, 2, 2, 1, 1$, or $2, 2, 2, 2, 1$, respectively, we can show that property 3) still holds.

If if the optimal operating points at $A_1^2\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2A_2\mathbf{q}$, $A_1A_2\mathbf{q}$, $A_2\mathbf{q}$ are

$2, 2, 1, 1, 1$, as shown in Figure 4.10, we have

$$
\begin{aligned}
&(a_1 - a_2)g(A_1A_2\mathbf{q}) + (b_1 - b_2)g(A_1^2\mathbf{q}) + \delta g(A_1^2A_2\mathbf{q}) \\
&\qquad = \delta\left((a_1 - a_2)f(A_1A_2\mathbf{q}) + (b_1 - b_2)f(A_1^2\mathbf{q}) + \delta f(A_1^2A_2\mathbf{q})\right) && (4.90)\\
&\qquad \geq \delta\left((a_1 - a_2)f(A_2\mathbf{q}) + (b_1 - b_2)f(A_1\mathbf{q}) + \delta f(A_1A_2\mathbf{q})\right) && (4.91)\\
&\qquad = (a_1 - a_2)g(A_2\mathbf{q}) + (b_1 - b_2)g(A_1\mathbf{q}) + \delta g(A_1A_2\mathbf{q}) && (4.92)
\end{aligned}
$$

where the inequality follows from the property 3) of function f, and the last inequality follows from the assumption that the policy at state $A_2\mathbf{q}$, $A_1\mathbf{q}$, $A_1A_2\mathbf{q}$ falls into the second policy pattern discussed above.

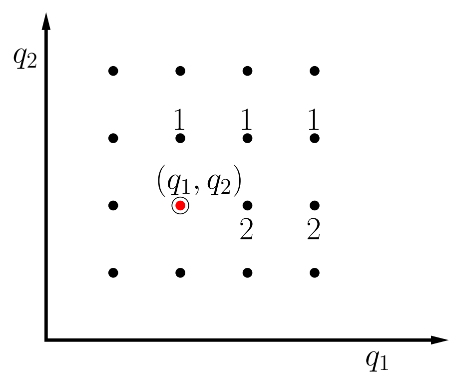


Figure 4.10: The optimal operating points at $A_1^2\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2A_2\mathbf{q}$, $A_1A_2\mathbf{q}$, $A_2\mathbf{q}$ are $2, 2, 1, 1, 1$, respectively.

Similarly, if the optimal operating points at $A_1^2\mathbf{q}$, $A_1\mathbf{q}$, $A_1^2A_2\mathbf{q}$, $A_1A_2\mathbf{q}$, $A_2\mathbf{q}$ are $2, 1, 1, 1, 1$, we have

$$
\begin{aligned}
&(a_1 - a_2)g(A_2\mathbf{q}) + (b_1 - b_2)g(A_1\mathbf{q}) + \delta g(A_1A_2\mathbf{q}) \\
&\qquad \leq \delta\left((a_1 - a_2)f(A_2\mathbf{q}) + (b_1 - b_2)f(A_1\mathbf{q}) + \delta f(A_1A_2\mathbf{q})\right) && (4.93)
\end{aligned}
$$

This is because $g(A_1\mathbf{q}) = (b_1 - b_2)f(D_2A_1\mathbf{q}) + \delta f(A_1\mathbf{q}) \leq (a_2 - a_1)f(\mathbf{q})$, and if we switch the policy from corner point 2 to corner point 1, it forms the second special policy pattern. Thus, the inequality still holds. In summary, for all possible cases, the function g preserves the property 3) of function f.

4.7.4 $(a_1 - a_2)g(D_1\mathbf{q}) + (b_1 - b_2)g(D_2\mathbf{q}) + \delta g(\mathbf{q})$ is decreasing in q_2.

We will evaluate g at points $A_1\mathbf{q}$, $A_1A_2\mathbf{q}$, $A_1A_2^2\mathbf{q}$, $A_2\mathbf{q}$, $A_2^2\mathbf{q}$. If the optimal operating points are $2, 2, 2, 2, 2$, or $1, 1, 1, 1, 1$, respectively, it is straightforward to show that the property still holds. If the optimal operating points are $2, 2, 2, 1, 1$, respectively, as shown in Figure 4.11, we note that the policy at these points is the first special policy patten discussed before, and

$$
\begin{aligned}
&(a_1 - a_2)g(A_2\mathbf{q}) + (b_1 - b_2)g(A_1\mathbf{q}) + \delta g(A_1A_2\mathbf{q}) \\
&\qquad = (a_1 - a_2)g(A_2^2\mathbf{q}) + (b_1 - b_2)g(A_1A_2\mathbf{q}) + \delta g(A_1A_2^2\mathbf{q}) && (4.94) \\
&\qquad = 0 && (4.95)
\end{aligned}
$$

If the optimal operating points are $2, 2, 1, 1, 1$, we have

$$
\begin{aligned}
&(a_1 - a_2)g(A_2^2\mathbf{q}) + (b_1 - b_2)g(A_1A_2\mathbf{q}) + \delta g(A_1A_2^2\mathbf{q}) \\
&\qquad = \delta\left((a_1 - a_2)f(A_2^2\mathbf{q}) + (b_1 - b_2)f(A_1A_2\mathbf{q}) + \delta f(A_1A_2^2\mathbf{q})\right) && (4.96) \\
&\qquad \leq 0 && (4.97) \\
&\qquad = (a_1 - a_2)g(A_2\mathbf{q}) + (b_1 - b_2)g(A_1\mathbf{q}) + \delta g(A_1A_2\mathbf{q}) && (4.98)
\end{aligned}
$$

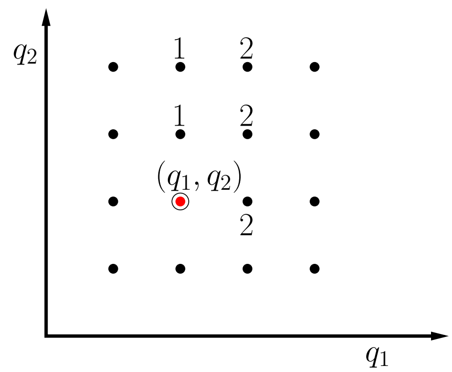


Figure 4.11: The optimal operating points at $A_1\mathbf{q}$, $A_1A_2\mathbf{q}$, $A_1A_2^2\mathbf{q}$, $A_2\mathbf{q}$, $A_2^2\mathbf{q}$ are $2, 2, 2, 1, 1$, respectively.

where the inequality follows from the assumption that at point $A_1A_2^2\mathbf{q}$, the optimal policy is to operate at corner point 1.

Similarly, for cases where the optimal operating points are $2, 2, 2, 2, 1$, or $2, 2, 1, 2, 1$, the property 4) still holds for g, this is because

$$(a_1 - a_2)g(A_2\mathbf{q}) + (b_1 - b_2)g(A_1\mathbf{q}) + \delta g(A_1A_2\mathbf{q}) \geq 0 \tag{4.99}$$

$$(a_1 - a_2)g(A_2^2\mathbf{q}) + (b_1 - b_2)g(A_1A_2\mathbf{q}) + \delta g(A_1A_2^2\mathbf{q}) \leq 0 \tag{4.100}$$

If the optimal operating points are $2, 2, 1, 1, 1$, we have

$$(a_1 - a_2)g(A_2^2\mathbf{q}) + (b_1 - b_2)g(A_1A_2\mathbf{q}) + \delta g(A_1A_2^2\mathbf{q}) \tag{4.101}$$

$$\leq \delta\left((a_1 - a_2)f(A_2^2\mathbf{q}) + (b_1 - b_2)f(A_1A_2\mathbf{q}) + \delta f(A_1A_2^2\mathbf{q})\right) \tag{4.102}$$

$$\leq \delta\left((a_1 - a_2)f(A_2\mathbf{q}) + (b_1 - b_2)f(A_1\mathbf{q}) + \delta f(A_1A_2\mathbf{q})\right) \tag{4.103}$$

$$\leq (a_1 - a_2)g(A_2\mathbf{q}) + (b_1 - b_2)g(A_1\mathbf{q}) + \delta g(A_1A_2\mathbf{q}) \tag{4.104}$$

where the first inequality follows from the assumption that the optimal policy for point $A_1 A_2 \mathbf{q}$ is corner point 1, thus the sum is smaller than operating at corner point 2. The second inequality follows from the property 4) of function f, and the last inequality follows from the assumption that the optimal policy for point $A_2 \mathbf{q}$ is the corner point 2.

In summary, for all possible cases, we have proven that properties 1) through 4) hold for g, thus, if f is in $\mathcal{F}$, then Tf is in $\mathcal{F}$.

Chapter 5

Optimal Packet Scheduling in a Single-User Energy Harvesting System

5.1 Introduction

We consider wireless communication networks where nodes are able to harvest energy from the nature. The nodes may harvest energy through solar cells, vibration absorption devices, water mills, thermoelectric generators, microbial fuel cells, etc. In this work, we do not focus on how energy is harvested, instead, we focus on developing transmission methods that take into account the *randomness* both in the *arrivals of the data packets* as well as in the *arrivals of the harvested energy.* As shown in Figure 5.1, the transmitter node has two queues. The data queue stores the data arrivals, while the energy queue stores the energy harvested from the environment. In general, the data arrivals and the harvested energy can be represented as two independent random processes. Then, the optimal scheduling policy becomes that of adaptively changing the transmission rate and power according to the instantaneous data and energy queue lengths.

While one ideally should study the case where both data packets and energy arrive randomly in time as two stochastic processes, and devise an *on-line* algorithm that updates the instantaneous transmission rate and power in *real-time* as functions

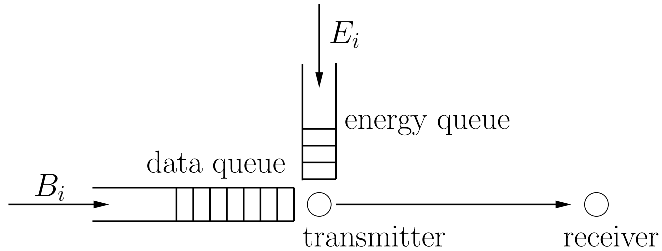


Figure 5.1: An energy harvesting communication system model.

of the current data and energy queue lengths, this, for now, is an intractable mathematical problem. Instead, in order to have progress in this difficult problem, we consider an idealized version of the problem, where we assume that we know exactly when and in what amounts the data packets and energy will arrive, and develop an optimal *off-line* algorithm. We leave the development of the corresponding *on-line* algorithm for future work.

Specifically, we consider a single node shown in Figure 5.2. We assume that packets arrive at times marked with $\times$ and energy arrives (is harvested) at points in time marked with $\circ$. In Figure 5.2, B_i denotes the number of bits in the ith arriving data packet, and E_i denotes the amount of energy in the ith energy arrival (energy harvesting). Our goal then is to develop methods of transmission to minimize the time, T, by which all of the data packets are delivered to the destination. The most challenging aspect of our optimization problem is the *causality* constraints introduced by the packet and energy arrival times, i.e., a packet may not be delivered before it has arrived and energy may not be used before it is harvested.

The trade-off relationship between delay and energy has been well investigated in traditional battery powered (unrechargeable) systems. References [13–18] inves-

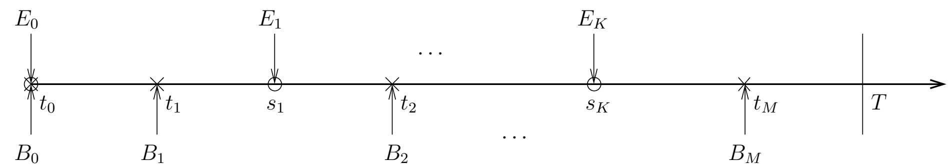


Figure 5.2: System model with random packet and energy arrivals. Data packets arrive at points denoted by $\times$ and energies arrive (are harvested) at points denoted by $\circ$.

tigate energy minimization problems with various deadline constraints. References [2, 4, 5, 9–12] investigate delay optimal resource allocation problems under various different settings. References [2, 4, 5] consider average power constrained delay minimization problem for a single-user system, while [9–12] minimize the average delay through rate allocation in a multiple access channel.

In this chapter, we consider a single-user communication channel with an energy harvesting transmitter. We assume that an initial amount of energy is available at $t = 0$. As time progresses, certain amounts of energies will be harvested. While energy arrivals should be modeled as a random process, for the mathematical tractability of the problem, in this chapter, we assume that the energy harvesting procedure can be precisely predicted, i.e., that, at the beginning, we know exactly when and how much energy will be harvested. For the data arrivals, we consider two different scenarios. In the first scenario, we assume that packets have already arrived and are ready to be transmitted at the transmitter before the transmission starts. In the second scenario, we assume that packets arrive during the transmissions. However, as in the case of energy arrivals, we assume that we know exactly when and in what amounts data will arrive. Subject to the energy and data arrival constraints, our purpose is to minimize the time by which all packets are delivered to the destination through controlling the transmission rate and power.

This is similar to the energy minimization problem in [13], where the objective is to minimize the energy consumption with a given *deadline* constraint. In this chapter, minimizing the transmission completion time is akin to minimizing the deadline in [13]. However, the problems are different, because, we do not know the exact amount of energy to be used in the transmissions, even though we know the times and amounts of harvested energy. This is because, intuitively, using more energy reduces the transmission time, however, using more energy entails waiting for energy arrivals, which increases the total transmission time. Therefore, minimizing the transmission completion time in the system requires a sophisticated utilization of the harvested energy. To that end, we develop an algorithm, which first obtains a good lower bound for the final total transmission duration at the beginning, and performs rate and power allocation based on this lower bound. The procedure works progressively until all of the transmission rates and powers are determined. We prove that the transmission policy obtained through this algorithm is globally optimum.

5.2 Scenario I: Packets Ready Before Transmission Starts

We assume that there are a total of B_0 bits available at the transmitter at time $t = 0$. We also assume that there is E_0 amount of energy available at time $t = 0$, and at times s_1, s_2, …, s_K, we have energies harvested with amounts E_1, E_2, …, E_K, respectively. This system model is shown in Figure 5.3. Our objective is to minimize the transmission completion time, T.

Figure 5.3: System model with all bits available at the beginning. Energies arrive at points denoted by $\circ$.

We assume that the transmitter can adaptively change its transmission power and rate according to the available energy and the remaining number of bits. We assume that the transmission rate and transmit power are related through a function, $f(p)$, i.e., $r = f(p)$. We assume that $f(p)$ satisfies the following properties: i) $f(0) = 0$ and $f(p) \to \infty$ as $p \to \infty$, ii) $f(p)$ increases monotonically in p, iii) $f(p)$ is strictly concave in p, iv) $f(p)$ is continuously differentiable, and v) $f(p)/p$ decreases monotonically in p. Properties i)-iii) guarantee that $f^{-1}(r)$ exists and is strictly convex. Property v) implies that for a fixed amount of energy, the number of bits that can be transmitted increases as the transmission duration increases. It can be verified that these properties are satisfied in many systems with realistic encoding/decoding schemes, such as optimal random coding in single-user additive white Gaussian noise channel, where $f(p) = \frac{1}{2}\log(1+p)$.

Assuming the transmitter changes its transmission power N times before it finishes the transmission, let us denote the sequence of transmission powers as p_1, p_2, ..., p_N, and the corresponding transmission durations of each rate as l_1, l_2, ..., l_N, respectively; see Figure 5.4. Then, the energy consumed up to time t, denoted as $E(t)$, and the total number of bits departed up to time t, denoted as $B(t)$, can be related through the function g as follows:

$$E(t) = \sum_{i=1}^{\bar{i}} p_i l_i + p_{\bar{i}+1} \left(t - \sum_{i=1}^{\bar{i}} l_i \right) \tag{5.1}$$

$$B(t) = \sum_{i=1}^{\bar{i}} f(p_i) l_i + f(p_{\bar{i}+1}) \left(t - \sum_{i=1}^{\bar{i}} l_i \right) \tag{5.2}$$

where $\bar{i} = \max\{i : \sum_{j=1}^{i} l_j \leq t\}$.

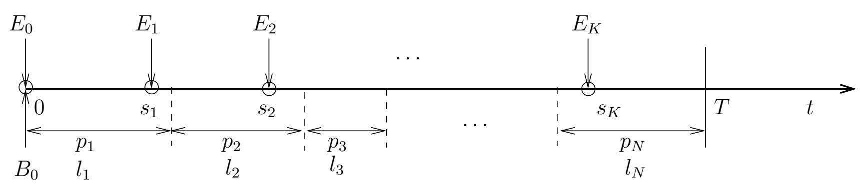


Figure 5.4: The sequence of transmission powers and durations.

Then, the transmission completion time minimization problem can be formulated as:

$$\begin{aligned} \min_{\mathbf{p},\mathbf{l}} \quad & T \\ \text{s.t.} \quad & E(t) \leq \sum_{i:s_i<t} E_i, \qquad 0 \leq t \leq T \\ & B(T) = B_0 \end{aligned} \tag{5.3}$$

First, we determine the properties of the optimum solution in the following three lemmas.

Lemma 5.1 *Under the optimal solution, the transmit powers increase monotonically, i.e.,* $p_1 \leq p_2 \leq \cdots \leq p_N$.

Proof: Assume that the powers do not increase monotonically, i.e., that we can find two powers such that $p_i > p_{i+1}$. The total energy consumed over this duration is $p_i l_i + p_{i+1} l_{i+1}$. Let

$$p_i' = p_{i+1}' = \frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}} \tag{5.4}$$

$$r_i' = r_{i+1}' = f\left(\frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}\right) \tag{5.5}$$

Then, we have $p_i' \leq p_i$, $p_{i+1}' \geq p_{i+1}$. Since $p_i' l_i \leq p_i l_i$, the energy constraint is still satisfied, and thus, the new energy allocation is feasible. We use r_i', r_{i+1}' to replace r_i, r_{i+1} in the transmission policy, and keep the rest of the rates the same. Then, the total number of bits transmitted over the duration $l_i + l_{i+1}$ becomes

$$\begin{aligned} r_i' l_i + r_{i+1}' l_{i+1} &= f\left(\frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}\right)(l_i + l_{i+1}) \\ &\geq f(p_i)\frac{l_i}{l_i + l_{i+1}}(l_i + l_{i+1}) + f(p_{i+1})\frac{l_{i+1}}{l_i + l_{i+1}}(l_i + l_{i+1}) \\ &= r_i l_i + r_{i+1} l_{i+1} \end{aligned} \tag{5.6}$$

where the inequality follows from the fact that $f(p)$ is concave in p. Therefore, the new policy departs more bits by time $\sum_{j=1}^{i+1} l_j$. Keeping the remaining transmission rates the same, the new policy will finish the entire transmission over a shorter duration. Thus, the original policy could not be optimal. Therefore, the optimal policy must have monotonically increasing powers (and rates). □

Lemma 5.2 *The transmission power/rate remains constant between energy harvests, i.e., the power/rate only potentially changes when new energy arrives.*

Proof: Assume that the transmitter changes its transmission rate between two energy harvesting instances s_i, s_{i+1}. Denote the rates as r_n, r_{n+1}, and the instant when the rate changes as s_i', as shown in Figure 5.5. Now, consider the duration $[s_i, s_{i+1})$. The total energy consumed during the duration is $p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')$. Let

$$p' = \frac{p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i} \tag{5.7}$$

$$r' = f\left(\frac{p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i}\right) \tag{5.8}$$

Now let us use r' as the new transmission rate over $[s_i, s_{i+1})$, and keep the rest of the rates the same. It is easy to check that the energy constraints are satisfied under this new policy, thus this new policy is feasible. On the other hand, the total number of bits departed over this duration under this new policy is

$$\begin{aligned} r'(s_{i+1} - s_i) &= f\left(\frac{p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i}\right)(s_{i+1} - s_i) \\ &\geq \left(f(p_n)\frac{s_i' - s_i}{s_{i+1} - s_i} + f(p_{n+1})\frac{s_{i+1} - s_i'}{s_{i+1} - s_i}\right)(s_{i+1} - s_i) \\ &= r_n(s_i' - s_i) + r_{n+1}(s_{i+1} - s_i') \end{aligned} \tag{5.9}$$

where the inequality follows from the fact that $f(p)$ is concave in p. Therefore, the total number of bits departed under the new policy is larger than that under the original policy. If we keep all of the remaining rates the same, the transmission will be completed at an earlier time. This conflicts with the optimality of the original policy. □

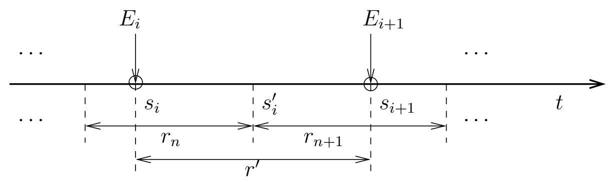


Figure 5.5: The rate must remain constant between energy harvests.

Lemma 5.3 *Under the optimal policy, whenever the transmission rate changes, the energy consumed up to that instant equals the energy harvested up to that instant.*

Proof: From Lemma 5.2, we know that the transmission rate can change only at certain energy harvesting instances. Assume that the transmission rate changes at s_i, however, the energy consumed by s_i, which is denoted by $E(s_i)$, is less than $\sum_{j=0}^{i-1} E_j$. We denote the energy gap by Δ. Let us denote the rates before and after s_i by r_n, r_{n+1}. Now, we can always have two small amounts of perturbations δ_n, δ_{n+1} on the corresponding transmit powers, such that

$$p_n' = p_n + \delta_n \tag{5.10}$$

$$p_{n+1}' = p_{n+1} - \delta_{n+1} \tag{5.11}$$

$$\delta_n l_n = \delta_{n+1} l_{n+1} \tag{5.12}$$

We also make sure that δ_n and δ_{n+1} are small enough such that $\delta_n l_n < \Delta$, and $p_n' \leq p_{n+1}'$. If we keep the transmission rates over the rest of the duration the same, under the new transmission policy, the energy allocation will still be feasible. The total number of bits departed over the duration $(\sum_{i=1}^{n-1} l_i, \sum_{i=1}^{n+1} l_i)$ is

$$f(p'_n)l_n + f(p'_{n+1})l_{n+1} \geq f(p_n)l_n + f(p_{n+1})l_{n+1} \tag{5.13}$$

where the inequality follows from the concavity of $f(p)$ in p, and the fact that $p_n l_n + p_{n+1}l_{n+1} = p'_n l_n + p'_{n+1}l_{n+1}$, $p_n \leq p'_n \leq p'_{n+1} \leq p_{n+1}$, as shown in Figure 5.6. This conflicts with the optimality of the original allocation. □

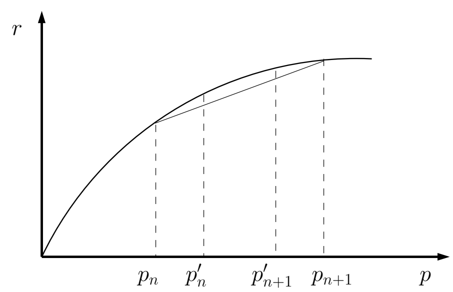


Figure 5.6: $f(p)$ is concave in p.

We are now ready to characterize the optimum transmission policy. In order to simplify the expressions, we let $i_0 = 0$, and let $s_{m+1} = T$ if the transmission completion time T lies between s_m and s_{m+1}.

Based on Lemmas 5.1, 5.2 and 5.3, we can characterize the optimal policy in the following way. For given energy arrivals, we plot the total amount of harvested energy as a function of t, which is a staircase curve as shown in Figure 5.7. The total energy consumed up to time t can also be represented as a continuous curve, as shown in Figure 5.7. In order to satisfy the feasibility constraints on the energy, energy consumption curve must lie below the energy harvesting curve at all times. Based on Lemma 5.2, we know that the optimal energy consumption curve must be linear between any two consecutive energy harvesting instants, and the slope of the segment corresponds to the transmit power level during that segment. Lemma 5.3 implies that whenever the slope changes, the energy consumption curve must touch the energy harvesting curve at that energy harvesting instant. Therefore, the first linear segment of the energy consumption curve must be one of the lines connecting the origin and any corner point on the energy harvesting curve before $t = T$. Because of the monotonicity property of the power given in Lemma 5.1, among those lines, we should always pick the one with the minimal slope, as shown in Figure 5.7.

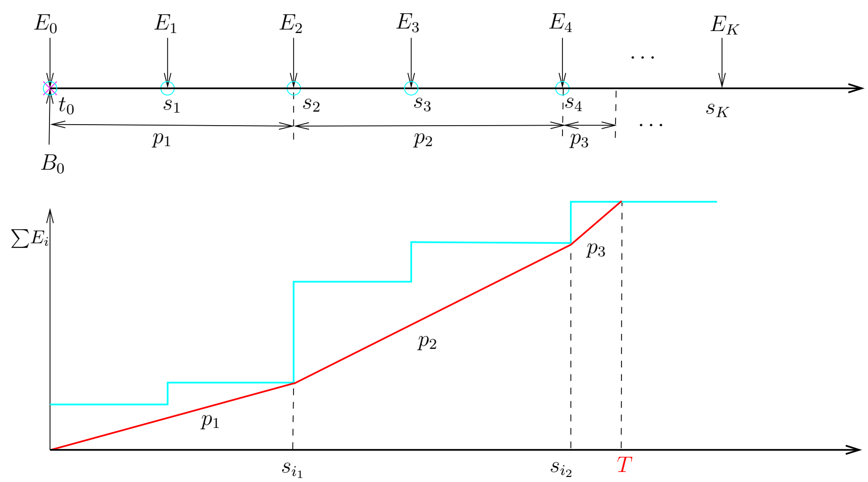


Figure 5.7: An interpretation of transmission policies satisfying Lemmas 5.1, 5.2, 5.3.

Otherwise, either the feasibility constraints on the energy will not be satisfied, or the monotonicity property given in Lemma 5.1 will be violated. For example, if we choose the line ending at the corner point at s_3, this will violate the feasibility constraint, as the energy consumption curve will surpass the energy arrival curve. On the other hand, if we choose the line ending at the corner point at s_1, then the monotonicity property in Lemma 5.1 will be violated, because in that case, the slope of the following segment would be smaller. These properties must hold similarly for p_2, p_3, ..., p_N. We also observe that, for given T, the optimal transmission policy is the tightest string below the energy harvesting curve connecting the origin and the total harvested energy by time T. This is similar to the structure in [14].

We state the structure of the optimal policy formally in the following theorem.

Theorem 5.1 *For a given B_0, consider a transmission policy with power vector $\mathbf{p} = [p_1, p_2, \ldots, p_N]$ and corresponding duration vector $\mathbf{l} = [l_1, l_2, \ldots, l_N]$. This policy is optimal if and only if it has the following structure:*

$$\sum_{n=1}^{N} f(p_n) l_n = B_0 \tag{5.14}$$

and for $n = 1, 2, \ldots, N$,

$$i_n = \arg \min_{\substack{i: s_i \leq T \\ s_i > s_{i_{n-1}}}} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \tag{5.15}$$

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} \tag{5.16}$$

$$l_n = s_{i_n} - s_{i_{n-1}} \tag{5.17}$$

where i_n is the index of the energy arrival epoch when the power p_n switches to p_{n+1}, i.e., at $t = s_{i_n}$, p_n switches to p_{n+1}.

The proof of this theorem is given in Appendix 5.6.1.

Therefore, we conclude that if the overall transmission duration T is known, then the optimal transmission policy is known via Theorem 5.1. In particular, optimal transmission policy is the one that yields the tightest piecewise linear energy consumption curve that lies under the energy harvesting curve at all times and touches the energy harvesting curve at $t = T$. On the other hand, the overall transmission time T is what we want to minimize, and we do not know its optimal value up front. Consequently, we do not know up front which energy harvests will be utilized. For example, if the number of bits is small, and E_0 is large, then, we can empty the data queue before the arrival of E_1, thus, the rest of the energy arrivals are not necessary. Therefore, as a first step, we first obtain a good lower bound on the optimal transmission duration.

We first illustrate our algorithm through an example in Figure 5.8. We first compute the minimal energy required to finish the transmission before s_1. We denote it as A_1, and it equals

$$A_1 = f^{-1}\left(\frac{B_0}{s_1}\right) s_1 \tag{5.18}$$

Then, we compare it with E_0. If $A_1 < E_0$, it implies that we can complete the transmission before the arrival of the first energy harvest, thus E_1 is not necessary for the transmission. We allocate E_0 evenly to B_0 bits, and the duration A_1 is the minimum transmission duration. On the other hand, if $A_1 > E_0$, which is the case in the example, the final transmission completion time should be longer than s_1. Thus, we move on and compute A_2, A_3, A_4, and find that $A_2 > \sum_{i=0}^{1} E_i$, $A_3 > \sum_{i=0}^{2} E_i$ and $A_4 < \sum_{i=0}^{3} E_i$. This means that the total transmission completion time will be larger than s_3 and energies $E_0, \ldots, E_3$ will surely be utilized. Then, we allocate

$\sum_{i=0}^{3} E_i$ evenly to B_0 bits and obtain a constant transmission power $\tilde{p}_1$, which is the dotted line in the figure. The corresponding transmission duration is T_1. Based on our allocation, we know that the final optimal transmission duration T must be greater than T_1. This is because, this allocation assumes that all E_0, ..., E_3 are available at the beginning, i.e., at time $t = 0$, which, in fact, are not. Therefore, the actual transmission time will only be larger. Thus, T_1 is a lower bound for T.

Next, we need to check the feasibility of $\tilde{p}_1$. Observing the figure, we find that $\tilde{p}_1$ is not feasible since it is above the staircase energy harvesting curve for some duration. Therefore, we connect all the corner points on the staircase curve before $t = T_1$ with the origin, and find the line with the minimum slope among those lines. This corresponds to the red solid line in the figure. Then, we update $\tilde{p}_1$ with the slope p_1, and the duration for p_1 is $l_1 = s_{i_1}$. We repeat this procedure at $t = s_{i_1}$ and obtain p_2, and continue the procedure until all of the bits are finished.

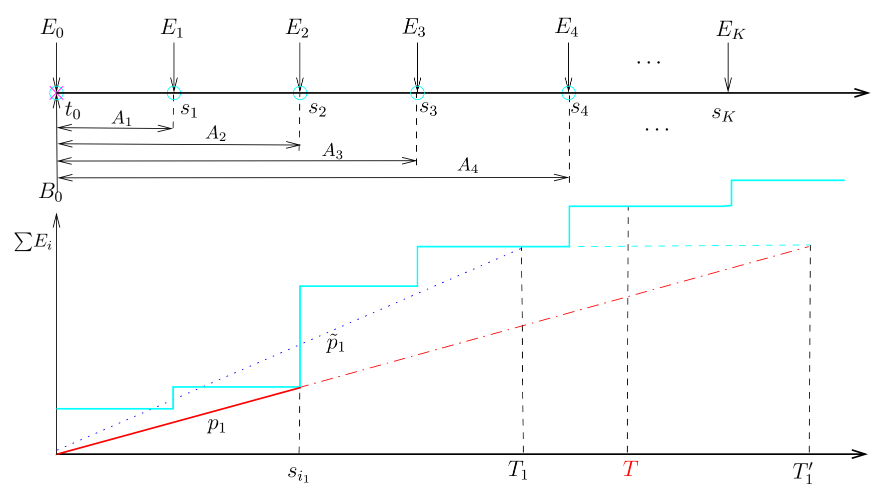


Figure 5.8: An illustration of the algorithm.

We state our algorithm for the general scenario as follows: First, we compute the amounts of energy required to finish the entire transmission before s_1, s_2, …, s_K, respectively, at a constant rate. We denote these as A_i:

$$A_i = f^{-1}\left(\frac{B_0}{s_i}\right) s_i, \quad i = 1, 2, \ldots, K \tag{5.19}$$

Then, we compare A_i with $\sum_{j=0}^{i-1} E_j$, and find the smallest i such that $A_i \leq \sum_{j=0}^{i-1} E_j$. We denote this i as $\tilde{i}_1$. If no such $\tilde{i}_1$ exists, we let $\tilde{i}_1 = K + 1$.

Now, we assume that we can use $\sum_{j=0}^{\tilde{i}_1-1} E_j$ to transmit all B_0 bits at a constant rate. We allocate the energy evenly to these bits, and the overall transmission time T_1 is the solution of

$$f\left(\frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1}\right) T_1 = B_0 \tag{5.20}$$

and the corresponding constant transmit power is

$$p_1 = \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1} \tag{5.21}$$

Next, we compare p_1 with $\frac{\sum_{j=0}^{i-1} E_j}{s_i}$ for every $i < \tilde{i}_1$. If p_1 is smaller than every term, then, maintaining p_1 is feasible, and the optimal policy is to transmit at a constant transmission rate $f(p_1)$ with duration T_1, which gives the smallest possible transmission completion time, $s_{i_1} = s_{\tilde{i}_1}$. Otherwise, maintaining p_1 is infeasible under the given energy arrival realization. Thus, we update

$$i_1 = \arg\min_{i<\tilde{i}_1} \left\{ \frac{\sum_{j=0}^{i-1} E_j}{s_i} \right\} \tag{5.22}$$

$$p_1 = \frac{\sum_{j=0}^{i_1-1} E_j}{s_{i_1}} \tag{5.23}$$

i.e., over the duration $[0, s_{i_1})$, we choose to transmit with power p_1 to make sure that the energy consumption is feasible. Then, at time $t = s_{i_1}$, the total number of bits departed is $f(p_1)s_{i_1}$, and the remaining number of bits is $B_0 - f(p_1)s_{i_1}$. Subsequently, with initial number of bits $B_0 - f(p_1)s_{i_1}$, we start from s_{i_1}, and get another lower bound on the overall transmission duration T_2, and repeat the procedure above. Through this procedure, we obtain $p_2, p_3, \ldots, p_N$, and the corresponding $i_2, i_3, \ldots, i_N$, until we finish transmitting all of the bits.

Based on our allocation algorithm, we know that p_1 is optimum up to time T_1, since it corresponds to the minimal slope line passing through the origin and any corner point before $t = T_1$. However, the algorithm also implies that the final transmission duration T will be larger than T_1. The question then is, whether p_1 is still the minimum slope line up to time T. If we can prove that p_1 is lower than the slopes of the lines passing through the origin and any corner point in $[T_1, T]$, then, using Theorem 5.1, we will claim that p_1 is the optimal transmission policy, not only between $[0, T_1]$, but also between $[0, T]$.

The fact that this will be the case can be illustrated through the example in Figure 5.8. We note that, clearly, T_1 is a lower bound on the eventual T. If we keep transmitting at power p_1, if no additional energy arrives, the energy harvested up until $s_{\tilde{i}_1}$, i.e., $\sum_{i=0}^{\tilde{i}_1-1} E_i$, will be depleted by time T_1'. We will next prove that T_1' is an upper bound on T. Because of the assumption that $f(p)/p$ is decreasing in p, we can prove that under this policy, the number of bits departed up to time T_1' is greater than B_0. Therefore, since potentially additional energy will arrive, T_1' provides an upper bound. Thus, we know that the optimal T lies between T_1 and T_1'. We next note that if we connect the origin with any corner point of the staircase curve between T_1 and T_1', the slope of the resulting line will be larger than p_1, thus, p_1 will be the smallest slope not only up to time T_1, which is a lower bound, but also up to time T_1', which is an upper bound. This proves that while we do not know the optimal T, if we run the algorithm with respect to the lower bound on T, i.e., T_1, it will still yield an optimal policy, in that the resulting policy will satisfy Theorem 5.1.

We prove the optimality of the algorithm formally in the following theorem.

Theorem 5.2 *The allocation procedure described above gives the optimal transmission policy.*

The proof of this theorem is given in Appendix 5.6.2.

5.3 Scenario II: Packets Arrive During Transmissions

In this section, we consider the situation where packets arrive during transmissions. We assume that there is an E_0 amount of energy available at time $t = 0$, and at times $s_1, s_2, \ldots, s_K$, energy is harvested in amounts $E_1, E_2, \ldots, E_K$, respectively, as in the previous section. We also assume that at $t = 0$, we have B_0 bits available, and at times $t_1, t_2, \ldots, t_M$, bits arrive in amounts $B_1, B_2, \ldots, B_M$, respectively. This system model is shown in Figure 5.2. Our objective is again to minimize the transmission completion time, T, which again is the time by which the last bit is delivered to the destination.

Let us denote the sequence of transmission powers by $p_1, p_2, \ldots, p_N$, and the corresponding transmission durations by $l_1, l_2, \ldots, l_N$. Then, the optimization problem becomes:

$$
\begin{aligned}
\min_{\mathbf{p},\mathbf{l}} \quad & T \\
\text{s.t.} \quad & E(t) \leq \sum_{i:s_i<t} E_i, \quad 0 \leq t \leq T \\
& B(t) \leq \sum_{i:t_i<t} B_i, \quad 0 \leq t \leq T \\
& B(T) = \sum_{i=0}^{M} B_i
\end{aligned} \tag{5.24}
$$

where $E(t)$, $B(t)$ are defined in (5.1) and (5.2). We again determine the properties of the optimal transmission policy in the following three lemmas.

Lemma 5.4 *Under the optimal solution, the transmission rates increase in time, i.e.,* $r_1 \leq r_2 \leq \cdots \leq r_N$.

Proof: First, note that since the relationship between power and rate, $r = f(p)$, is monotone, stating that the rates increase monotonically is equivalent to stating that the powers increase monotonically. We follow steps similar to those in the proof of Lemma 5.1 to prove this lemma. Assume that the rates do not increase monotonically, i.e., that we can find two rates such that $r_i > r_{i+1}$, with duration l_i, l_{i+1}, respectively. If $i+1 \neq N$, then, let

$$r_i' = r_{i+1}' = \frac{r_i l_i + r_{i+1} l_{i+1}}{l_i + l_{i+1}} \tag{5.25}$$

$$p_i' = p_{i+1}' = f^{-1}\left(\frac{r_i l_i + r_{i+1} l_{i+1}}{l_i + l_{i+1}}\right) \tag{5.26}$$

Since $r_i > r_i' = r_{i+1}' > r_{i+1}$, $p_i > p_i' = p_{i+1}' > p_{i+1}$, it is easy to verify that the new policy is feasible up to the end of l_{i+1}, from both the data and energy arrival points of view. On the other hand, based on the convexity of f^{-1}, the energy spent over the duration $l_i + l_{i+1}$ is smaller than $p_i l_i + p_{i+1} l_{i+1}$. If we allocate the saved energy over to the last transmission duration, without conflicting any energy or data constraints, the transmission will be completed in a shorter duration. If $i+1 = N$, then, we let

$$p_i' = p_{i+1}' = \frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}} \tag{5.27}$$

$$r_i' = r_{i+1}' = f\left(\frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}\right) \tag{5.28}$$

Then, from (5.6), under the new policy, the last bit will depart before the end of l_{i+1}. The energy and data arrival constraints are satisfied over the whole transmission duration. Consequently, the original policy could not be optimal. Therefore, the optimal policy must have monotonically increasing rates (and powers). □

Lemma 5.5 *Under the optimal policy, the transmission power/rate remains constant between two event epoches, i.e., the rate only potentially changes when new* *energy is harvested or a new packet arrives.*

Proof: This lemma can be proved through a procedure similar to that in Lemma 5.2. If power/rate is not constant between two event epoches, then, by equalizing the rate over the duration while keeping the total departures fixed, we can save energy. Allocating this saved energy to the last transmission duration, we can shorten the whole transmission duration. Thus, if power/rate is not constant between two event epoches, the policy cannot be optimal. □

Lemma 5.6 *If the transmission rate changes at an energy harvesting epoch, then the energy consumed up to that epoch equals the energy harvested up to that epoch; if the transmission rate changes at a packet arrival epoch, then, the number of packets departed up to that epoch equals the number of packets arrived up to that epoch; if the event epoch has both energy and data arrivals at the same time, then, one of the causality constraints must be met with equality.*

Proof: This lemma can be proved through contradiction using techniques similar to those used in the proof of Lemma 5.3. When the transmission rate changes at an energy harvesting epoch, if the energy consumed up to that time is not equal to the total amount harvested, then, without conflicting the energy causality constraint, we can always increase the rate before that epoch a little and decrease the rate after that epoch a little while keeping the total departures fixed. This policy would save some energy which can be used to shorten the transmission durations afterwards. Thus, the energy constraint at that epoch must be satisfied as an equality. The remaining situations can be proved similarly. □

Based on Lemmas 5.4, 5.5 and 5.6, we can identify the structure of the unique optimal transmission policy as stated in the following theorem. In order to simplify the notation, we define u_i to be the time epoch when the ith arrival (energy or data) happens, i.e.,

$$u_1 = \min\{s_1, t_1\} \tag{5.29}$$

$$u_2 = \min\{s_i, t_j : s_i > u_1, t_j > u_1\} \tag{5.30}$$

and so on, until the last arrival epoch.

Theorem 5.3 *For a given energy harvesting and packet arrival profile, the transmission policy with a transmission rate vector* $\mathbf{r} = [r_1, r_2, \ldots, r_N]$ *and the corresponding duration vector* $\mathbf{l} = [l_1, l_2, \ldots, l_N]$ *is optimal, if and only if it has the following structure:*

$$\sum_{i=1}^{N} r_i l_i = \sum_{i=0}^{M} B_i \tag{5.31}$$

$$r_1 = \min_{i:u_i \leq T} \left\{ f\left(\frac{\sum_{j:s_j<u_i} E_j}{u_i} \right), \frac{\sum_{j:t_j<u_i} B_j}{u_i} \right\} \tag{5.32}$$

Let i_1 *be the index of* u *associated with* r_1*. Then, with updated amount of bits and energy remaining in the system at* $t = u_{i_1}$*,* r_2 *is the smallest feasible rate starting from* u_{i_1}*, and so on.*

The proof of this theorem is given in Appendix 5.6.3.

For a given optimal transmission duration, T, the optimal policy which has the structure in Theorem 5.3 is unique. However, since we do not know the exact transmission duration up front, we obtain a lower bound on T first, as in the previous section. In this case also, we develop a similar procedure to find the optimal transmission policy. The basic idea is to keep the transmit power/rate as constant as possible throughout the entire transmission duration. Because of the additional casuality constraints due to data arrivals, we need to consider both the average data arrival rate as well as the average power the system can support for feasibility.

If $s_K \leq t_M$, i.e., bits have arrived after the last energy harvest, then, all of the harvested energy will be used. First, we assume that we can use these energies to maintain a constant rate, and the transmission duration will be the solution of

$$f\left(\frac{\sum_{j=0}^{K} E_j}{T}\right) T = \sum_{j=0}^{M} B_j \tag{5.33}$$

Then, we check whether this constant power/rate is feasible. We check the availability of the energy, as well as the available number of bits. Let

$$i_{1e} = \arg\min_{u_i < T} \left\{\frac{\sum_{j=0}^{i-1} E_j}{u_i}\right\}, \quad p_1 = \frac{\sum_{j=0}^{i_{1e}-1} E_j}{u_i} \tag{5.34}$$

$$i_{1b} = \arg\min_{u_i < T} \left\{\frac{\sum_{j=0}^{i-1} B_j}{u_i}\right\}, \quad r_1 = \frac{\sum_{j=0}^{i_{1b}-1} B_j}{u_i} \tag{5.35}$$

We compare $\min(p_1, f^{-1}(r_1))$ with $\frac{\sum_{j=0}^{K} E_j}{T}$. If the former is greater than the latter, then the constant transmit power $\frac{\sum_{j=0}^{K} E_j}{T}$ is feasible. Thus, we achieve the minimum possible transmission completion time T. Otherwise, constant-power transmission is not feasible. We choose the transmit power to be the smaller of p_1 and $f^{-1}(r_1)$, and the duration to be the one associated with the smaller transmit power. We repeat this procedure until all of the bits are transmitted.

If $s_K > t_M$, then, as in the first scenario where packets have arrived and are ready before the transmission starts, some of the harvested energy may not be utilized to transmit the bits. In this case also, we need to get a lower bound for the final transmission completion time. Let u_n be the energy harvesting epoch right after t_M. Then, starting from u_n, we compute the energy required to transmit $\sum_{j=0}^{M} B_j$ bits at a constant rate by u_i, $u_n \leq u_i \leq u_{K+M}$, and compare them with the total energy harvested up to that epoch, i.e., $\sum_{j:s_j<u_i} E_j$. We identify the smallest i such that the required energy is smaller than the total harvested energy, and denote it by $\tilde{i}_1$. If no such $\tilde{i}_1$ exists, we let $\tilde{i}_1 = M + K + 1$.

Now, we assume that we can use $\sum_{j:s_j<u_{\tilde{i}_1}} E_j$ to transmit $\sum_{j=0}^{M} B_j$ bits at a constant rate. We allocate the energy evenly to these bits, and the overall transmission time T_1 is the solution of

$$f\left(\frac{\sum_{j:s_j<u_{\tilde{i}_1}} E_j}{T_1}\right) T_1 = \sum_{j=0}^{M} B_j \tag{5.36}$$

and the corresponding constant transmit power is

$$p_1 = \frac{\sum_{j:s_j<u_{\tilde{i}_1}} E_j}{T_1} \tag{5.37}$$

Next, we compare p_1 with $\frac{\sum_{j:s_j<u_i} E_j}{u_i}$ and $f^{-1}\left(\frac{\sum_{j:t_j<u_i} B_j}{u_i}\right)$ for every $i < \tilde{i}_1$. If p_1 is smaller than all of these terms, then, maintaining p_1 is feasible from both energy and data arrival points of view. The optimal policy is to keep a constant transmission rate at $f(p_1)$ with duration T_1, which yields the smallest possible transmission completion time, $i_1 = \tilde{i}_1$. Otherwise, maintaining p_1 is not feasible under the given energy and data arrival realizations. This infeasibility is due to the causality constraints on either the energy or the data arrival, or both. Next, we identify the tightest constraint, and update the transmit power to be the power associated with that constraint. We repeat this procedure until all of the bits are delivered.

Theorem 5.4 *The transmission policy obtained through the algorithm described above is optimal.*

The proof of this theorem is given in Appendix 5.6.4.

5.4 Numerical Results

We consider a band-limited additive white Gaussian noise channel, with bandwidth $W = 1$ MHz and the noise power spectral density $N_0 = 10^{-19}$ W/Hz. We assume that the distance between the transmitter and the receiver is 1 Km, and the path loss is about 110 dB. Then, we have $f(p) = W \log_2\left(1 + \frac{ph}{N_0 W}\right) = \log_2\left(1 + \frac{p}{10^{-2}}\right)$ Mbps. It is easy to verify that this function has the properties assumed at the beginning of Section 5.2. For the energy harvesting process, we assume that at times $\mathbf{t} = [0, 2, 5, 6, 8, 9, 11]$ s, we have energy harvested with amounts $\mathbf{E} = [10, 5, 10, 5, 10, 10, 10]$ mJ, as shown in Figure 5.9. We assume that at $t = 0$, we have 5.44 Mbits to transmit. We choose the numbers in such a way that the solution is expressable in simple numbers, and can be potted conveniently. Then, using our algorithm, we obtain the optimal transmission policy, which is shown in Figure 5.9. We note that the powers change only potentially at instances when energy arrives (Lemma 5.2); when the power changes, energy consumed up to that point equals energy harvested (Lemma 5.3); and power sequence is monotonically increasing (Lemma 5.1). We also note that, for this case, the active transmission is completed by time $T = 9.5$ s, and the last energy harvest at time $t = 11$ s is not used.

Next, we consider the scenario where data packets arrive during the transmissions. We consider a smaller time scale, where each unit consists of 10 ms. We assume that at times $\mathbf{t} = [0, 5, 6, 8, 9]$, energies arrive with amounts $\mathbf{E} = [5, 5, 5, 5, 5] \times 10^{-2}$ mJ, while at times $\mathbf{t} = [0, 4, 10]$, packets arrive with equal size 10 Kbits, as shown in Figure 5.10. We observe that the transmitter changes its transmission power during the transmissions. The first change happens at $t = 5$ when energy arrives, and the energy constraint at that instant is satisfied with equality, while the second change happens at $t = 10$ when new bits arrive, and the traffic constraint at that time is satisfied with equality.

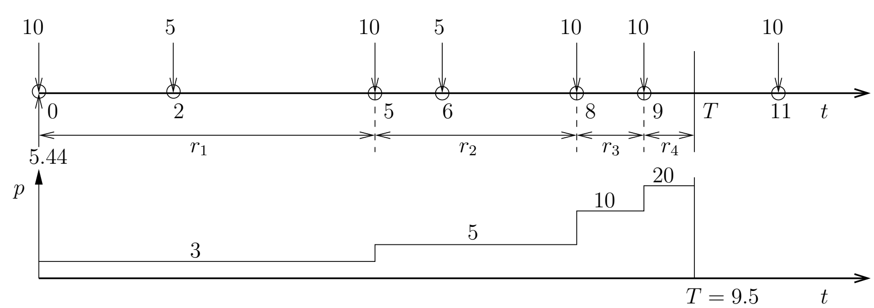


Figure 5.9: Optimal transmit powers $\mathbf{p} = [3, 5, 10, 20]$ mW, with durations $\mathbf{l} = [5, 3, 1, 0.5]$ s.

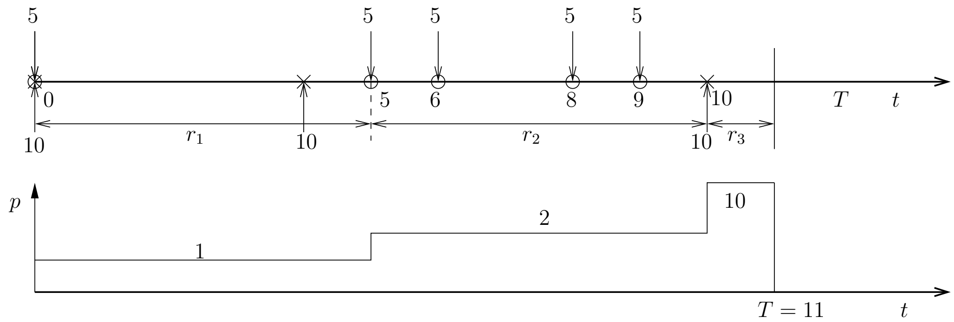


Figure 5.10: Optimal transmit powers $\mathbf{p} = [1, 2, 10]$ mW, with durations $\mathbf{l} = [5, 5, 1] \times 10^{-2}$ s.

5.5 Conclusions

In this chapter, we investigated the transmission completion time minimization problem in an energy harvesting communication system. We considered two different scenarios, where in the first scenario, we assume that packets have already arrived and are ready to be transmitted at the transmitter before the transmission starts, and in the second scenario, we assume that packets may arrive during the transmissions. We first analyzed the structural properties of the optimal transmission policy, and then developed an algorithm to obtain a globally optimal off-line scheduling policy, in each scenario.

5.6 Appendix

5.6.1 The Proof of Theorem 5.1

We will prove the necessariness and the sufficiency of the stated structure separately. First, we prove that the optimal policy must have the structure given above. We prove this through contradiction. Assume that the optimal policy, which satisfies

Lemmas 5.1, 5.2 and 5.3, does not have the structure given above. Specifically, assume that the optimal policy over the duration $[0, s_{i_{n-1}})$ is the same as the policy described in Theorem 1, however, the transmit power right after $s_{i_{n-1}}$, which is p_n, is not the smallest average power possible starting from $s_{i_{n-1}}$, i.e., we can find another $s_{i'} \leq s_{i_N}$, such that

$$p_n > \frac{\sum_{j=i_{n-1}}^{i'-1} E_j}{s_{i'} - s_{i_{n-1}}} \triangleq p' \tag{5.38}$$

Based on Lemma 5.3, the energy consumed up to $s_{i_{n-1}}$ is equal to $\sum_{j=0}^{i_{n-1}-1} E_j$, i.e., there is no energy remaining at $t = s_{i_{n-1}}^-$.

We consider two possible cases here. The first case is that $s_{i'} < s_{i_n}$, as shown in Figure 5.11(a). Under the optimal policy, the energy required to maintain a transmit power p_n over the duration $[s_{i_{n-1}}, s_{i'})$ is $p_n(s_{i'} - s_{i_{n-1}})$. Based on (5.38), this is greater than the total amount of energy harvested during $[s_{i_{n-1}}, s_{i'})$, which is $\sum_{j=i_{n-1}}^{i'-1} E_j$. Therefore, this energy allocation under this policy is infeasible.

On the other hand, if $s_{i'} > s_{i_n}$, as shown in Figure 5.11(b), then the total amount of energy harvested over $[s_{i_n}, s_{i'})$ is $\sum_{j=i_n}^{i'-1} E_j$. From (5.38), we know

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} > \frac{\sum_{j=i_{n-1}}^{i'-1} E_j}{s_{i'} - s_{i_{n-1}}} > \frac{\sum_{j=i_n}^{i'-1} E_j}{s_{i'} - s_{i_n}} \tag{5.39}$$

Thus, under any feasible policy, there must exist a duration $l \subseteq [s_{i_n}, s_{i'})$, such that the transmit power over this duration is less than p_n. This contradicts with Lemma 5.1. Therefore, this policy cannot be optimal.

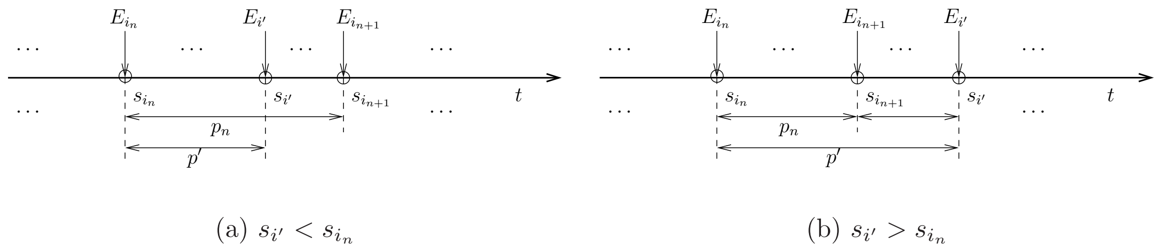


Figure 5.11: Two different cases in the proof of Theorem 5.1.

Next, we prove that if a policy with power vector $\mathbf{p}$ and duration vector $\mathbf{l}$ has the structure given above, then, it must be optimal. We prove this through contradiction. We assume that there exists another policy with power vector $\mathbf{p}'$ and duration vector $\mathbf{l}'$, and the transmission completion time T' under this policy is smaller.

We assume both of the policies are the same over the duration $[0, s_{i_{n-1}})$, however, the transmit policies right after $s_{i_{n-1}}$, which are p_n and p'_n, with durations l_n and l'_n, respectively, are different. Based on the assumption, we must have $p_n < p'_n$.

If $l_n < l'_n$, from Lemma 5.3, we know that the total energy available over $[s_{i_{n-1}}, s_{i_n})$ is equal to $p_n l_n$. Since $p_n < p'_n$, p'_n is infeasible over $[s_{i_{n-1}}, s_{i_n})$. Thus, policy $\mathbf{p}'$ cannot be optimal. Then, we consider the case when $l_n > l'_n$. If $T' \geq s_{i_n}$, then, the total energy spent over $[s_{i_{n-1}}, s_{i_n})$ under $\mathbf{p}'$ is greater than $p_n l_n$, since $p'_n > p_n$, and $p'_{n+1} > p'_n$ based on Lemma 5.1. If $T' < s_{i_n}$, since the power-rate function f is concave, the total number of bits departed over $[s_{i_{n-1}}, s_{i_n})$ under $\mathbf{p}$ is greater than that under $\mathbf{p}'$. Thus, policy $\mathbf{p}'$ cannot depart B_0 bits over T', and it cannot be optimal.

In summary, a policy is optimal if and only if it has the structure given above, completing the proof.

5.6.2 The Proof of Theorem 5.2

Let T be the final transmission duration given by the allocation procedure. Then, we have $B(T) = B_0$. In order to prove that the allocation is optimal, we need to show that the final transmission policy has the structure given in Theorem 5.1. We first prove that p_1 satisfies (5.16). Then, we can similarly prove that p_2, p_3, ... satisfy (5.16).

We know that if $T = T_1$, then it is the minimum possible transmission completion time. We know that this transmit policy will satisfy the structural properties in Theorem 5.1. Otherwise, the final optimal transmission time T is greater than T_1, and more harvested energy may need to be utilized to transmit the remaining bits. From the allocation procedure, we know that

$$p_1 \leq \frac{\sum_{j=0}^{i-1} E_j}{s_i}, \quad \forall i < \tilde{i}_1 \tag{5.40}$$

In order to prove that p_1 satisfies (5.16), we need to show that

$$p_1 \leq \frac{\sum_{j=0}^{i-1} E_j}{s_i}, \quad \forall i : s_{\tilde{i}_1} \leq s_i \leq T \tag{5.41}$$

If we keep transmitting with power p_1, then at $T_1' = \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{p_1}$, the total number of bits departed will be

$$f(p_1)T_1' \geq f\left(\frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1}\right) T_1 = B_0 \tag{5.42}$$

where the inequality follows from the assumption that $f(p)/p$ decreases in p. Then, (5.40) guarantees that this is a feasible policy. Thus, under the optimal policy, the transmission duration T will be upper bounded by T_1', i.e.,

$$T \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{p_1} \tag{5.43}$$

which implies

$$p_1 \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T} \tag{5.44}$$

If $T \leq s_{\tilde{i}_1}$, as shown in Figure 5.12(a), no future harvested energy is utilized for the transmissions. Then, (5.44) guarantees that (5.41) is satisfied.

If $T > s_{\tilde{i}_1}$, as shown in Figure 5.12(b), additional energy harvested after $s_{\tilde{i}_1}$ should be utilized to transmit the data. We next prove that (5.41) still holds through contradiction. Assume that there exists i' with $s_{\tilde{i}_1} \leq s_{i'} \leq T$, such that (5.41) is not satisfied, i.e.,

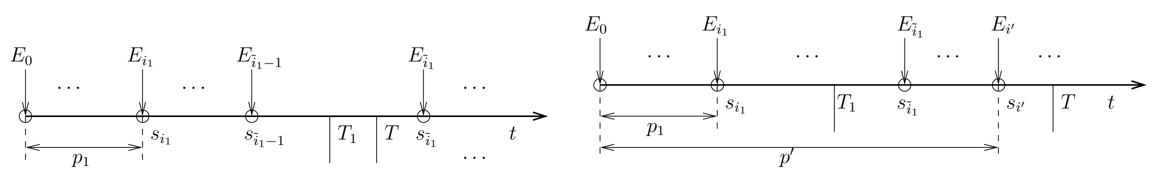
$$p_1 > \frac{\sum_{j=0}^{i'-1} E_j}{s_{i'}} \triangleq p' \tag{5.45}$$

Then,

$$\frac{\sum_{j=0}^{i'-1} E_j}{p_1} < s_{i'} \tag{5.46}$$

Combining this with (5.43), we have $T < s_{i'}$, which contradicts with the assumption that $s_{i'} \leq T$. Thus, (5.41) holds, p_1 satisfies the requirement of the optimal structure in (5.40).

We can then prove using similar arguments that p_2, p_3, ... also satisfy the properties of the optimal solution. Based on Lemma 5.1, this procedure gives us the unique optimal policy.



(a) $T \leq s_{\tilde{i}_1}$ (b) $T > s_{\tilde{i}_1}$

Figure 5.12: Two different cases in the proof of Theorem 5.2.

5.6.3 The Proof of Theorem 5.3

First, we prove that for the optimal transmission policy, r_1 must satisfy (5.32). We prove this through contradiction. If r_1 does not satisfy (5.32), then, we can always find another $u_{i'}$, such that

$$r_1 > \min\left\{ f\left(\frac{\sum_{j:s_j<u_{i'}} E_j}{u_{i'}}\right), \frac{\sum_{j:t_j<u_{i'}} B_j}{u_{i'}} \right\} \tag{5.47}$$

First, we assume that $f\left(\frac{\sum_{j:s_j<u_{i'}} E_j}{u_{i'}}\right) < \frac{\sum_{j:t_j<u_{i'}} B_j}{u_{i'}}$. Then, if $u_{i'} < u_{i_1}$, clearly r_1 is not feasible over the duration $[0, u_{i'})$, because of the energy constraint. If $u_{i'} > u_{i_1}$, then, the transmitter cannot maintain a transmission rate that is always greater than r_1 over $[u_i, u_{i'})$, from the energy point of view. This contradicts with Lemma 5.4. Similarly, if $f\left(\frac{\sum_{j:s_j<u_{i'}} E_j}{u_{i'}}\right) > \frac{\sum_{j:t_j<u_{i'}} B_j}{u_{i'}}$, the "bottleneck" is the data constraint. We can prove that r_1 is not feasible. Thus, r_1 must be the smallest feasible rate starting from $t = 0$, as in (5.32). We can also prove that r_2, r_3, ... must have the same structure, in the same way. Next, we can prove that any policy has the structure described above is optimal. We can prove this through contradiction. Assume that there exists another policy with a shorter transmission completion time. Based on Lemmas 5.4 and 5.6, we can prove that this policy could not be feasible.

5.6.4 The Proof of Theorem 5.4

First we prove that r_1 obtained through this procedure satisfies (5.32). If $T = T_1$, i.e., the constant rate is achievable throughout the transmission, then it is the shortest transmission duration we can get, thus, it is optimal. If $T \neq T_1$, from the procedure, we have

$$r_1 \leq \min_{1\leq i\leq \tilde{i}_1} \left\{ f\left(\frac{\sum_{j:s_j<u_i} E_j}{u_i}\right), \frac{\sum_{j:t_j<u_i} B_j}{u_i} \right\} \tag{5.48}$$

We need to prove that

$$r_1 \leq \min\left\{ f\left(\frac{\sum_{j:s_j<u_i} E_j}{u_i}\right), \frac{\sum_{j:t_j<u_i} B_j}{u_i} \right\} \quad \text{for } u_{\tilde{i}_1} < u_i \leq T. \tag{5.49}$$

Considering the policy with a constant power $p_1 = g^{-1}(r_1)$, then, at $T_1' = \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{p_1}$, the total number of bits departed will be

$$f(p_1)T_1' \geq f\left(\frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1}\right) T_1 = \sum_{j=0}^{M} B_j \tag{5.50}$$

while at $T_1'' = \frac{\sum_{j=0}^{\tilde{i}_1-1} B_j}{r_1}$, the total energy required will be

$$p_1 T_1'' \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1} \frac{\sum_{j=0}^{\tilde{i}_1-1} B_j}{f\left(\frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1}\right)} = \sum_{j=0}^{\tilde{i}_1-1} E_j \tag{5.51}$$

where the inequality follows from the assumption that $f(p)/p$ decreases in p. Therefore, maintaining a transmission rate r_1 until the last bit departs the system is feasible from both the energy and data arrival points of view. Thus, under the optimal policy, the transmission duration T will be upper bounded by T_1' and T_1'', i.e.,

$$T \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{p_1}, \quad T \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} B_j}{r_1} \tag{5.52}$$

which implies

$$p_1 \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T}, \quad r_1 \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} B_j}{T} \tag{5.53}$$

If no future harvested energy is utilized for the transmissions, (5.53) guarantees that (5.32) is satisfied.

If $T > u_{\tilde{i}_1}$, additional energy harvested after $u_{\tilde{i}_1}$ should be utilized to transmit the data. We next prove that (5.49) still holds through contradiction. Assume that there exists i' with $u_{\tilde{i}_1} \leq u_{i'} \leq T$, such that (5.49) is not satisfied, i.e.,

$$p_1 > \frac{\sum_{j=0}^{i'-1} E_j}{u_{i'}} \quad \text{or} \quad r_1 > \frac{\sum_{j=0}^{i'-1} B_j}{u_{i'}} \tag{5.54}$$

Then, we have

$$\frac{\sum_{j=0}^{i'-1} E_j}{p_1} < u_{i'} \quad \text{or} \quad \frac{\sum_{j=0}^{i'-1} B_j}{r_1} < u_{i'} \tag{5.55}$$

Combining this with (5.52), we have $T < u_{i'}$, which contradicts with the assumption that $u_{i'} \leq T$. Thus, (5.49) holds, r_1 satisfies the requirement of the optimal structure in (5.32). We can then prove using a similar argument that r_2, r_3, ... also satisfy the structure of the optimal solution. Based on Theorem 5.3, this procedure gives us the unique optimal transmission policy.

Chapter 6

Optimal Packet Scheduling in a Broadcast Channel with an Energy Harvesting Transmitter

6.1 Introduction

We consider a wireless communication network where users are able to harvest energy from the nature. Such energy harvesting capabilities make sustainable and environmentally friendly deployment of wireless communication networks possible. While energy-efficient scheduling policies have been well-investigated in traditional battery powered (un-rechargeable) systems [13–18], energy-efficient scheduling in energy harvesting networks with nodes that have rechargeable batteries has only recently been considered in Chapter 5. Chapter 5 considers a single-user communication system with an energy harvesting transmitter, and develop a packet scheduling scheme that minimizes the time by which all of the packets are delivered to the receiver.

In this chapter, we consider a multi-user extension of the work in Chapter 5. In particular, we consider a wireless broadcast channel with an energy harvesting transmitter. As shown in Figure 6.1, we consider a broadcast channel with one transmitter and two receivers, where the transmitter node has three queues. The data queues store the data arrivals intended for the individual receivers, while the energy queue stores the energy harvested from the environment. Our objective is to adaptively change the transmission rates that go to both users according to the instantaneous data and energy queue sizes, such that the total transmission completion time is minimized.

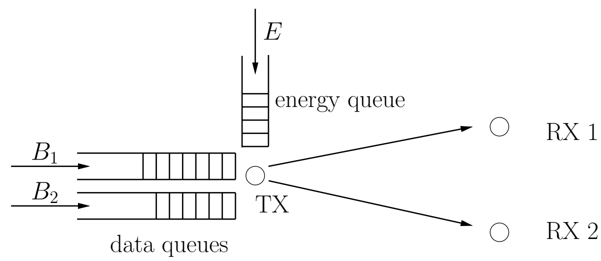


Figure 6.1: An energy harvesting two-user broadcast channel.

In this chapter, we focus on finding the optimum *off-line* schedule, by assuming that the energy arrival profile at the transmitter is known ahead of time in an off-line manner, i.e., the energy harvesting times and the corresponding harvested energy amounts are known at time $t = 0$. We assume that there are a total of B_1 bits that need to be delivered to receiver 1 and B_2 bits that need to be delivered to receiver 2, available at the transmitter at time $t = 0$. As shown in Figure 6.2, energy arrives (is harvested) at points in time marked with $\circ$; in particular, E_k denotes the amount of energy harvested at time s_k. Our goal is to develop a method of transmission to minimize the time, T, by which all of the data packets are delivered to their respective receivers.

The optimal packet scheduling problem in a single-user energy harvesting communication system is investigated in Chapter 5. In Chapter 5, we prove that the optimal scheduling policy has a "majorization" structure, in that, the transmit power

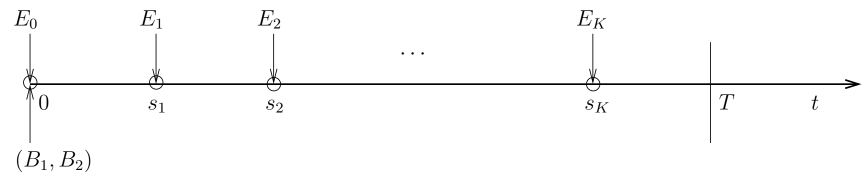


Figure 6.2: System model. (B_1, B_2) bits to be transmitted to users are available at the transmitter at the beginning. Energies arrive (are harvested) at points denoted by $\circ$. T denotes the transmission completion time by which all of the bits are delivered to their respective destinations.

is kept constant between energy harvests, the sequence of transmit powers increases monotonically, and only changes at some of the energy harvesting instances; when the transmit power changes, the energy constraint is tight, i.e., the total consumed energy equals the total harvested energy. In Chapter 5, we develop an algorithm to obtain the optimal off-line scheduling policy based on these properties. Reference [19] extends Chapter 5 to the case where rechargeable batteries have finite sizes. We extend Chapter 5 in [20] to a fading channel.

References [19, 20] investigate two related problems. The first problem is to maximize the throughput (number of bits transmitted) with a given deadline constraint, and the second problem is to minimize the transmission completion time with a given number of bits to transmit. These two problems are "dual" to each other in the sense that, with a given energy arrival profile, if the maximum number of bits that can be sent by a deadline is B^* in the first problem, then the minimum time to transmit B^* bits in the second problem must be the deadline in the first problem, and the optimal transmission policies for these two problems must be identical. In this chapter, we will follow this "dual problems" approach. We will first attack and solve the first problem to determine the structural properties of the optimal solution. We will then utilize these structural properties to develop an iterative algorithm for the second problem. Our iterative approach has the goal of reducing the two-user broadcast problem into a single-user problem as much as possible, and utilizing the single-user solution in Chapter 5. The second problem is also considered in the independent work [36] which uses convex optimization techniques to reduce the problem into local sub-problems that consider only two energy arrival epochs at a time.

We first analyze the structural properties of the optimal policy for the first problem where our goal is to maximize the number of bits delivered to both users under a given deadline constraint. To that end, we first determine the *maximum departure region* with a given deadline constraint T. The maximum departure region is defined as the set of all (B_1, B_2) that can be transmitted to users reliably with a given deadline. In order to do that, we consider the problem of maximizing $\mu_1 B_1 + \mu_2 B_2$ under the energy causality constraints for the transmitter, for all $\mu_1, \mu_2 \geq 0$. Varying μ_1, μ_2 traces the boundary of the maximum departure region. We prove that the optimal total transmit power policy is independent of the values of μ_1, μ_2, and it has the same "majorization" structure as the single-user non-fading solution. As for the way of splitting the total transmit power between the two users, we prove that there exists a *cut-off* power level for the stronger user, i.e., only the power above this *cut-off* power level is allocated to the weaker user.

We then consider the second problem, where our goal is to minimize the time, T, by which a given (B_1, B_2) number of bits are delivered to their intended receivers. As discussed, since the second problem is "dual" to the first problem, the optimal transmission policy in this problem has the same structural properties as in the first problem. Therefore, in the second problem as well, there exists a *cut-off* power level. The problem then becomes that of finding an optimal *cut-off* power such that the transmission times for both users become identical and minimized. With these optimal structural properties, we develop an iterative algorithm that finds the optimal schedule efficiently. In particular, we first use the fact that the optimum transmit power has the same structural properties as the single-user problem, to obtain the first optimal total power, P_1. Then, given the fact that there exists a *cut-off* power level, P_c, for the first user, the optimal transmit strategy depends on whether P_1 is smaller or larger than P_c, which, at this point, is unknown. Therefore, we have two cases to consider. If P_c is smaller than P_1, then the stronger user will always have a constant, P_c, portion of the total transmit power. This reduces the problem to a single-user problem for the second user, together with a fixed-point equation in a single variable (P_c) to be solved to ensure that the transmissions to both users end at the same time. On the other hand, if P_c is larger than P_1, this means that all of P_1 must be spent to transmit to the first user. In this case, the number of bits delivered to the first user in this time duration can be subtracted from the total number of bits to be delivered to the first user, and the problem can be started anew with the updated number of bits (B_1', B_2) after the first epoch. Therefore, in both cases, the broadcast channel problem is essentially reduced to single-user problems, and the approach in Chapter 5 is utilized recursively to solve the overall problem.

6.2 System Model and Problem Formulation

The system model is as shown in Figures 6.1 and 6.2. The transmitter has an energy queue and two data queues (Figure 6.1). The physical layer is modeled as an AWGN broadcast channel, where the received signals at the first and second receivers are

$$Y_1 = X + Z_1 \tag{6.1}$$

$$Y_2 = X + Z_2 \tag{6.2}$$

where X is the transmit signal, and Z_1 is a Gaussian noise with zero-mean and unit-variance, and Z_2 is a Gaussian noise with zero-mean and variance σ^2, where $\sigma^2 > 1$. Therefore, the second user is the *degraded* (weaker) user in our broadcast channel. Assuming that the transmitter transmits with power P, the capacity region for this two-user AWGN broadcast channel is [24]

$$r_1 \leq \frac{1}{2}\log_2\left(1 + \alpha P\right) \tag{6.3}$$

$$r_2 \leq \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha)P}{\alpha P + \sigma^2}\right) \tag{6.4}$$

where α is the fraction of power spent for the message transmitted to the first user. Let us denote $f(p) \triangleq \frac{1}{2}\log_2(1+p)$ for future use. Then, the capacity region is $r_1 \leq f(\alpha P)$, $r_2 \leq f\left(\frac{(1-\alpha)P}{\alpha P+\sigma^2}\right)$. This capacity region is shown in Figure 6.3.

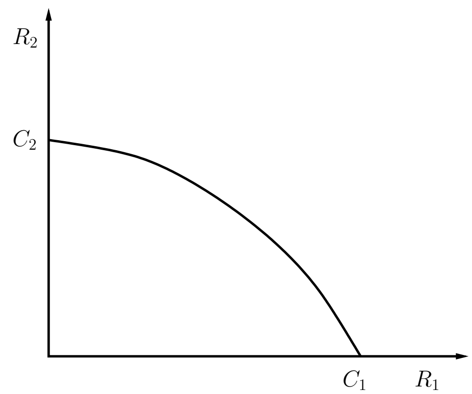


Figure 6.3: The capacity region of the two-user AWGN broadcast channel.

Working on the boundary of the capacity region, we have

$$P = 2^{2(r_1+r_2)} + (\sigma^2 - 1)2^{2r_2} - \sigma^2 \tag{6.5}$$

$$\triangleq g(r_1, r_2) \tag{6.6}$$

As shown in Figure 6.1, the transmitter has B_1 bits to transmit to the first user, and B_2 bits to transmit to the second user. Energy is harvested at times s_k with amounts E_k. Our goal is to select a transmission policy that minimizes the time, T, by which all of the bits are delivered to their intended receivers. The transmitter adapts its transmit power and the portions of the total transmit power used to transmit signals to the two users according to the available energy level and the remaining number of bits. The energy consumed must satisfy the causality constraints, i.e., at any given time, the total amount of energy consumed up to time t must be less than or equal to the total amount of energy harvested up to time t.

Before we proceed to give a formal definition of the optimization problem and propose the solution, we start with the "dual" problem of this transmission completion time minimization problem, i.e, instead of trying to find the minimal T, we aim to identify the maximum number of bits the transmitter can deliver to both users by any fixed time T. As we will observe in the next section, solving the "dual" problem enables us to identify the optimal structural properties for both problems, and these properties eventually help us reduce the original problem into simple scenarios, which can be solved efficiently.

6.3 Characterizing $\mathcal{D}(T)$: Largest (B_1, B_2) Region for a Given Deadline T

In this section, our goal is to characterize the maximum departure region for a given deadline T. We define it in the following way.

Definition 6.1 *For any fixed transmission duration T, the maximum departure region, denoted as $\mathcal{D}(T)$, is the union of (B_1, B_2) under any feasible rate allocation policy over duration $[0, T)$, i.e., $\mathcal{D}(T) = \bigcup_{r_1(t), r_2(t)} (B_1, B_2)(r_1(t), r_2(t))$, subject to the energy constraint $\int_0^t g(r_1, r_2)(\tau) d\tau \leq \sum_{i: s_i < t} E_i$, for $0 \leq t \leq T$.*

We call any policy which achieves the boundary of $\mathcal{D}(T)$ to be optimal. In the single-user scenario in Chapter 5, we first examined the structural properties of the optimal policy. Based on these properties, we developed an algorithm to find the optimal scheduling policy. In this broadcast scenario, we will first analyze the structural properties of the optimal policy, and then obtain the optimal solution based on these structural properties. The following lemma which was proved for a single-user problem in Chapter 5 was also proved for the broadcast problem in [36].

Lemma 6.1 *Under the optimal policy, the transmission rate remains constant between energy harvests, i.e., the rate only potentially changes at an energy harvesting epoch.*

Proof: We prove this using the strict convexity of $g(r_1, r_2)$. If the transmission rate for any user changes between two energy harvesting epochs, then, we can always equalize the transmission rate over that duration without contradicting with the energy constraints. Based on the convexity of $g(r_1, r_2)$, after equalization of rates, the energy consumed over that duration decreases, and the saved energy can be allocated to both users to increase the departures. Therefore, changing rates between energy harvests is sub-optimal. $\square$

Therefore, in the following, we only consider policies where the rates are constant between any two consecutive energy arrivals. We denote the rates that go to both users as (r_{1n}, r_{2n}) over the duration $[s_{n-1}, s_n)$. With this property, an illustration of the maximum departure region is shown in Figure 6.4.

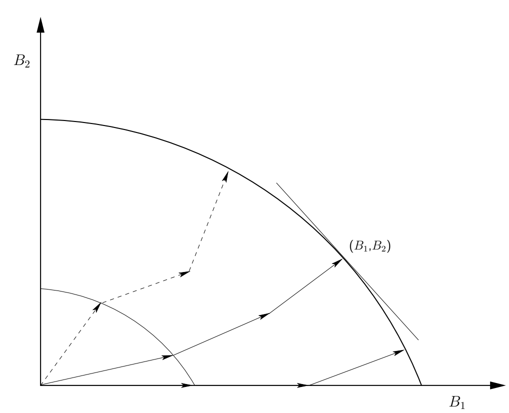


Figure 6.4: The maximum departure region and possible trajectories to reach the boundary.

Lemma 6.2 *$\mathcal{D}(T)$ is a convex region.*

Proof: Proving the convexity of $\mathcal{D}(T)$ is equivalent to proving that, given any two achievable points (B_1, B_2) and (B_1', B_2') in $\mathcal{D}(T)$, any point on the line between these two points is also achievable, i.e., in $\mathcal{D}(T)$. Assume that (B_1, B_2) and (B_1', B_2') can be achieved with rate allocation policies $(\mathbf{r}_1, \mathbf{r}_2)$ and $(\mathbf{r}_1', \mathbf{r}_2')$, respectively. Consider the policy $(\lambda\mathbf{r}_1 + \bar{\lambda}\mathbf{r}_1', \lambda\mathbf{r}_2 + \bar{\lambda}\mathbf{r}_2')$, where $\bar{\lambda} = 1 - \lambda$. Then, the energy consumed up to s_n is

$$\sum_{i=1}^{n} g(\lambda r_{1i} + \bar{\lambda} r_{1i}', \lambda r_{2i} + \bar{\lambda} r_{2i}') l_i \leq \lambda \sum_{i=1}^{n} g(r_{1i}, r_{2i}) l_i + \bar{\lambda} \sum_{i=1}^{n} g(r_{1i}', r_{2i}') l_i \tag{6.7}$$

$$\leq \lambda \sum_{i=0}^{n-1} E_i + \bar{\lambda} \sum_{i=0}^{n-1} E_i \tag{6.8}$$

$$= \sum_{i=0}^{n-1} E_i \tag{6.9}$$

Therefore, the energy causality constraint is satisfied for any $\lambda \in [0, 1]$, and the new policy is energy-feasible. Any point on the line between (B_1, B_2) and (B_1', B_2') can be achieved. When $\lambda \neq 0, 1$, the inequality in (6.7) is strict. Therefore, we save some amount of energy under the new policy, which can be used to increase the throughput for both users. This implies that $\mathcal{D}(T)$ is strictly convex. □

In order to simplify the notation, in this section, for any given T, we assume that there are $N - 1$ energy arrival epochs (excluding $t = 0$) over $(0, T)$. We denote the last energy arrival epoch before T as s_{N-1}, and $s_N = T$, with $l_N = T - s_{N-1}$, as shown in Figure 6.5.

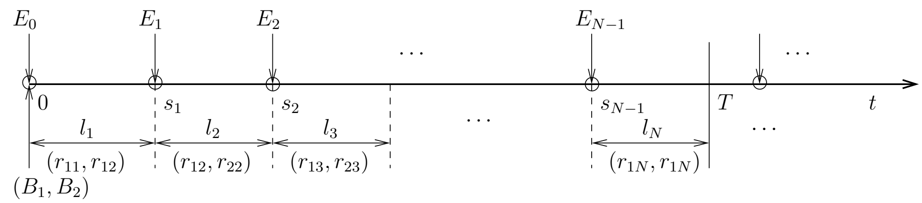


Figure 6.5: Rates (r_{1n}, r_{2n}) and corresponding durations l_n with a given deadline T.

Since $\mathcal{D}(T)$ is a strictly convex region, its boundary can be characterized by solving the following optimization problem for all $\mu_1, \mu_2 \geq 0$,

$$\begin{aligned} \max_{\mathbf{r}_1, \mathbf{r}_2} \quad & \mu_1 \sum_{n=1}^{N} r_{1n} l_n + \mu_2 \sum_{n=1}^{N} r_{2n} l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} g(r_{1n}, r_{2n}) l_n \leq \sum_{n=0}^{j-1} E_n, \quad \forall j : 0 < j \leq N \end{aligned} \tag{6.10}$$

where l_n is the length of the duration between two consecutive energy arrival instances s_n and s_{n-1}, i.e., $l_n = s_n - s_{n-1}$, and $\mathbf{r}_1$ and $\mathbf{r}_2$ denote the rate sequences r_{1n} and r_{2n} for users 1 and 2, respectively. The problem in (6.10) is a convex optimization problem with convex cost function and linear constraints, therefore, the unique global solution should satisfy the extended KKT conditions.

The Lagrangian is

$$\begin{aligned} \mathcal{L}(\mathbf{r}_1, \mathbf{r}_2, \boldsymbol{\lambda}, \boldsymbol{\gamma}) =& \mu_1 \sum_{n=1}^{N} r_{1n} l_n + \mu_2 \sum_{n=1}^{N} r_{2n} l_n - \sum_{j=1}^{N} \lambda_j \left(\sum_{n=1}^{j} g(r_{1n}, r_{2n}) l_n - \sum_{n=0}^{j-1} E_n \right) \\ &+ \sum_{n=1}^{N} \gamma_{1n} r_{1n} + \sum_{n=1}^{N} \gamma_{2n} r_{2n} \end{aligned} \tag{6.11}$$

Taking the derivatives with respective to r_{1n} and r_{2n}, and setting them to zero, we have

$$\mu_1 + \gamma_{1n} - \left(\sum_{j=n}^{N} \lambda_j\right) 2^{2(r_{1n}+r_{2n})} = 0, \quad n = 1, 2, \ldots, N \tag{6.12}$$

$$\mu_2 + \gamma_{2n} - \left(\sum_{j=n}^{N} \lambda_j\right) \left(2^{2(r_{1n}+r_{2n})} + (\sigma^2 - 1)2^{2r_{2n}}\right) = 0, \quad n = 1, 2, \ldots, N \tag{6.13}$$

where $\gamma_{1n} = 0$ if $r_{1n} > 0$, and $\gamma_{2n} = 0$ if $r_{2n} > 0$. Based on these KKT optimality conditions, we first prove an important property of the optimal policy.

Lemma 6.3 *The optimal total transmit power of the transmitter is independent of the value of μ_1, μ_2, and it is the same as the single-user optimal transmit power. Specifically,*

$$i_n = \arg \min_{i_{n-1} < i \leq N} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \tag{6.14}$$

$$P_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} \tag{6.15}$$

i.e., at $t = s_{i_n}$, P_n switches to P_{n+1}.

Proof: Based on the expression of $g(r_{1n}, r_{2n})$ in (6.6) and the KKT conditions in (6.12)-(6.13), we have

$$g(r_{1n}, r_{2n}) = \frac{\mu_2 + \gamma_{2n}}{\sum_{j=n}^{N} \lambda_j} - \sigma^2 \tag{6.16}$$

$$\geq 2^{2(r_{1n}+r_{2n})} - 1 \tag{6.17}$$

$$= \frac{\mu_1 + \gamma_{1n}}{\sum_{j=n}^{N} \lambda_j} - 1 \tag{6.18}$$

$$\geq \frac{\mu_1}{\sum_{j=n}^{N} \lambda_j} - 1 \tag{6.19}$$

where (6.17) becomes an equality when $r_{2n} = 0$. Therefore, when $r_{2n} > 0$, (6.16)-(6.19) imply

$$g(r_{1n}, r_{2n}) = \frac{\mu_2}{\sum_{j=n}^{N} \lambda_j} - \sigma^2 > \frac{\mu_1}{\sum_{j=n}^{N} \lambda_j} - 1 \tag{6.20}$$

When $r_{2n} = 0$, we must have $r_{1n} > 0$. Otherwise, if $r_{1n} = 0$, we can always let the weaker user transmit with some power over this duration without contradicting with any energy constraints. Since there is no interference from the stronger user, the departure from the weaker user can be improved, thus it contradicts with the optimality of the policy. Therefore, when $r_{2n} = 0$, $\gamma_{1n} = 0$, (6.16)-(6.19) imply

$$g(r_{1n}, r_{2n}) = \frac{\mu_1}{\sum_{j=n}^{N} \lambda_j} - 1 > \frac{\mu_2}{\sum_{j=n}^{N} \lambda_j} - \sigma^2 \tag{6.21}$$

Therefore, we can express $g(r_{1n}, r_{2n})$ in the following way:

$$g(r_{1n}, r_{2n}) = \max\left\{\frac{\mu_1}{\sum_{j=n}^{N}\lambda_j} - 1, \frac{\mu_2}{\sum_{j=n}^{N}\lambda_j} - \sigma^2\right\} \tag{6.22}$$

Plotting these two curves in Figure 6.6, we note that the optimal transmit power is always the curve on the top. If $\frac{\mu_2}{\sum_{j=n}^{N}\lambda_j} - \sigma^2 > \frac{\mu_1}{\sum_{j=n}^{N}\lambda_j} - 1$ for some $\bar{n}$, then, we have

$$\frac{\mu_2 - \mu_1}{\sum_{j=n}^{N}\lambda_j} \geq \frac{\mu_2 - \mu_1}{\sum_{j=\bar{n}}^{N}\lambda_j} > \sigma^2 - 1, \quad \forall n > \bar{n} \tag{6.23}$$

where the first inequality follows from the KKT condition that $\lambda_j \geq 0$ for $j = 1, 2, \ldots N$. Therefore, we conclude that there exists an integer $\bar{n}$, $0 \leq \bar{n} \leq N$, such that, when $n \leq \bar{n}$, $r_{2n} = 0$; and when $n > \bar{n}$, $r_{2n} > 0$.

Furthermore, (6.20)-(6.21) imply that, the energy constraint at $t = s_{\bar{n}}$ must be tight. Otherwise, $\lambda_{\bar{n}} = 0$, and (6.21) implies

$$g(r_{1\bar{n}}, r_{2\bar{n}}) = \frac{\mu_1}{\sum_{j=\bar{n}+1}^{N}\lambda_j} - 1 > \frac{\mu_2}{\sum_{j=\bar{n}+1}^{N}\lambda_j} - \sigma^2 = g(r_{1,\bar{n}+1}, r_{2,\bar{n}+1}) \tag{6.24}$$

which contradicts with (6.20). Therefore, in the following, when we consider the energy constraints, we only need to consider two segments $[0, s_{\bar{n}})$ and $[s_{\bar{n}+1}, s_N)$ separately.

When $n < s_{\bar{n}}$, based on (6.20), if $\lambda_n = 0$, we have $g(r_{1n}, r_{2n}) = g(r_{1,n+1}, r_{2,n+1})$. Starting from $n = 1$, $g(r_{1n}, r_{2n})$ remains a constant until an energy constraint becomes tight. Therefore, between any two consecutive epochs, when the energy constraints are tight, the power level remains constant. Similar arguments hold when $n \geq s_{\bar{n}}$. Therefore, the corresponding power level is

$$P_n = \frac{\sum_{j=i_{n-1}}^{i_n - 1} E_j}{s_{i_n} - s_{i_{n-1}}} \tag{6.25}$$

where $s_{i_{n-1}}$ and s_{i_n} are two consecutive epochs with tight energy constraint.

Finally, we need to determine the epochs when the energy constraint becomes tight. Another observation is that $g(r_{1\bar{n}}, r_{2\bar{n}})$ must monotonically increase in n, as shown in Figure 6.6. This is because both of these two curves monotonically increase, and the maximum value of these two curves should monotonically increase also. Therefore, based on the monotonicity of the transmit power, we conclude that

$$i_n = \arg \min_{i_{n-1} < i \leq N} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \tag{6.26}$$

This completes the proof. □

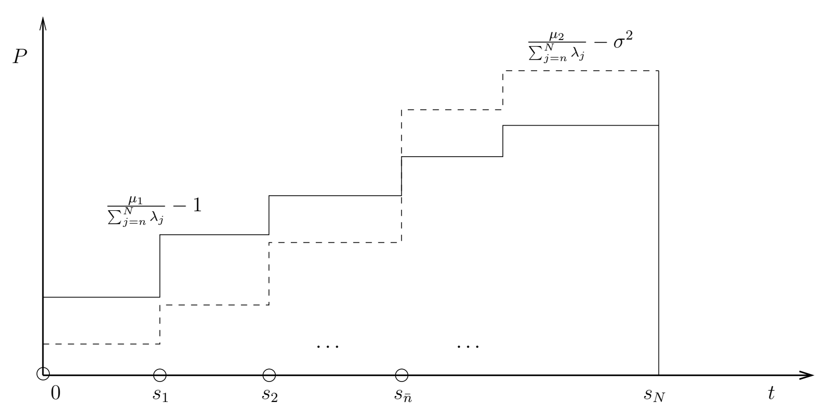


Figure 6.6: The value of the optimal transmit power is always equal to the curve on top.

Since the power can be obtained directly irrespective of the values of μ_1, μ_2, the optimization problem in (6.10) is separable over each duration $[s_{n-1}, s_n)$. Specifically, for $0 < n \leq N$, the local optimization becomes

$$\begin{aligned} \max_{r_{1n}, r_{2n}} \quad & \mu_1 r_{1n} + \mu_2 r_{2n} \\ \text{s.t.} \quad & g(r_{1n}, r_{2n}) \leq P_n \end{aligned} \tag{6.27}$$

We relax the power constraint to be an inequality to make the constraint set to be convex. Thus this becomes a convex optimization problem. This does not affect the solution since the objective function is always maximized on the boundary of its constraint set, i.e., the capacity region defined by the transmit power P_n.

When $\frac{\mu_2}{\mu_1} \leq \frac{P_n+1}{P_n+\sigma^2}$, the solution to (6.27) can be expressed as

$$r_{1n} = \frac{1}{2}\log_2(1 + P_n) \tag{6.28}$$

$$r_{2n} = 0 \tag{6.29}$$

In this scenario, all of the power P_n is allocated to the first user.

When $\frac{1+P_n}{\sigma^2+P_n} \leq \frac{\mu_2}{\mu_1} \leq \sigma^2$, we have

$$r_{1n} = \frac{1}{2}\log_2\left(\frac{\mu_1(\sigma^2 - 1)}{\mu_2 - \mu_1}\right) \tag{6.30}$$

$$r_{2n} = \frac{1}{2}\log_2\left(\frac{(\mu_2 - \mu_1)(P_n + \sigma^2)}{\mu_2(\sigma^2 - 1)}\right) \tag{6.31}$$

In this scenario, a constant amount of power, $\frac{\mu_1(\sigma^2-1)}{\mu_2-\mu_1} - 1$, is allocated to the first user, and the remaining power is allocated to the second user.

When $\frac{\mu_2}{\mu_1} > \sigma^2$, we have

$$r_{1n} = 0 \tag{6.32}$$

$$r_{2n} = \frac{1}{2}\log_2\left(1 + \frac{P_n}{\sigma^2}\right) \tag{6.33}$$

In this scenario, all of the P_n is allocated to the second user.

Let us define a constant power level as

$$P_c = \left(\frac{\mu_1(\sigma^2-1)}{\mu_2-\mu_1} - 1\right)^+ \tag{6.34}$$

Based on the solution of the local optimization problem (6.27), we establish another important property of the optimal policy as follows.

Lemma 6.4 *For fixed μ_1, μ_2, under the optimal power policy, there exists a constant cut-off power level, P_c, for the first user. If the total power level is below this cut-off power level, then, all the power is allocated to the first user; if the power level is higher than this level, then, all the power above this cut-off level is allocated to the second user.*

In the proof of Lemma 6.3, we note that the optimal power P_n monotonically increases in n. Combining Lemma 6.3 and Lemma 6.4, we illustrate the structure of the optimal policy in Figure 6.7. Moreover, the optimal way of splitting the power in each epoch is such that both user's share of the power monotonically increases.

In particular, the second user's share is monotonically increasing in time. Hence, the path followed in the (B_1, B_2) plane is such that it changes direction to get closer to the second user's departure axis as shown in Figure 6.4. The dotted trajectory cannot be optimal, since the path does not get closer to the second user's departure axis in the middle (second) power epoch.

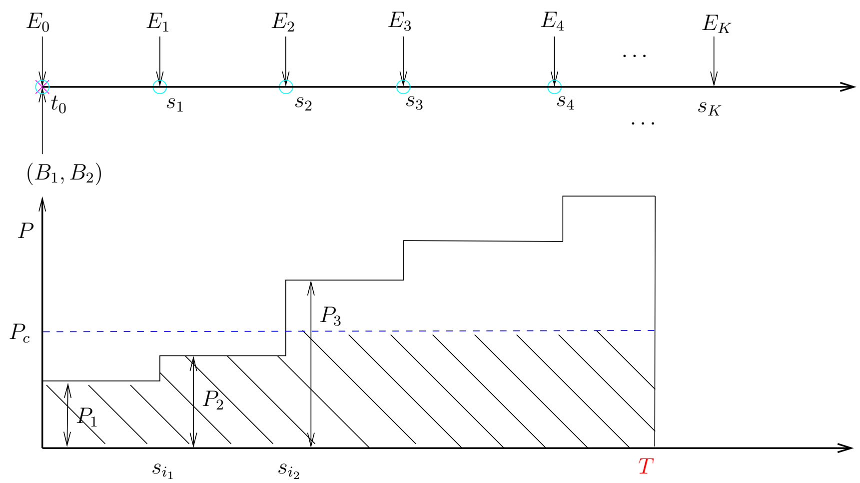


Figure 6.7: Optimally splitting total power between the signals that go to the two users.

Corollary 6.1 *Under the optimal policy, the transmission rate for the first user,* $\{r_{1n}\}_{n=1}^{N}$*, is either a constant sequence (zero or a positive constant), or an increasing sequence. Moreover, before* r_{1n} *achieves its final constant value,* $r_{2n} = 0$*; and when* r_{1n} *becomes a constant,* r_{2n} *monotonically increases in* n*.*

Based on Lemma 6.3, we observe that for fixed T, μ_1 and μ_2, the optimal power allocation is unique, i.e., does not depend on μ_1 and μ_2. However, the way the total power is split between the two users depends on μ_1, μ_2. In fact, the *cut-off* power level P_c varies depending on the value of μ_2/μ_1. Therefore, for different values of μ_2/μ_1, the optimal policy achieves different boundary points on the maximum departure region, and varying the value of μ_2/μ_1 traces the boundary of this region.

In this section, we characterized the maximum departure region for any given time T. We proved that the optimal total transmit power is the same as in the single-user case, and there exists a cut-off power for splitting the total transmit power to both users. In the next section, we will use these structural properties to solve the original transmission completion minimization problem.

6.4 Minimizing the Transmission Completion Time T for a Given (B_1, B_2)

In this section, our goal is to minimize the transmission completion time of both users for a given (B_1, B_2). The optimization problem can be formulated as

$$\begin{aligned} \min_{\mathbf{r}_1,\mathbf{r}_2} \quad & T \\ \text{s.t.} \quad & \sum_{n=1}^{j} g(r_{1n}, r_{2n}) l_n \leq \sum_{n=1}^{j-1} E_n, \quad \forall j : 0 < j \leq N(T) \\ & \sum_{n=1}^{N(T)} r_{1n} l_n = B_1, \quad \sum_{n=1}^{N(T)} r_{1n} l_n = B_2 \end{aligned} \tag{6.35}$$

where $N(T)-1$ is the number of energy arrival epochs (excluding $t=0$) over $(0,T)$, and $l_{N(T)} = T - s_{N(T)-1}$. Since $N(T)$ depends on T, the optimization problem in (6.35) is not a convex optimization problem in general. Therefore, we cannot solve it using standard convex optimization tools.

We first note that this is exactly the "dual" problem of maximizing the departure region for fixed T. They are "dual" in the sense that, if the minimal transmission completion time for (B_1, B_2) is T, then (B_1, B_2) must lie on the boundary of $\mathcal{D}(T)$, and the transmission policy should be exactly the same for some (μ_1, μ_2). This is based on the fact the $\mathcal{D}(T) \subset \mathcal{D}(T')$ for any $T < T'$. Assume (B_1, B_2) does not lie on the boundary of $\mathcal{D}(T)$. Then, either (B_1, B_2) cannot be achieved by T or (B_1, B_2) is strictly inside $\mathcal{D}(T)$ and hence (B_1, B_2) can be achieved by $T' < T$. Therefore, if (B_1, B_2) does not lie on the boundary of $\mathcal{D}(T)$, then T cannot be the minimum transmission completion time.

We have the following lemma.

Lemma 6.5 *When $B_1, B_2 \neq 0$, under the optimal policy, the transmissions to both users must be finished at the same time.*

Proof: This lemma can be proved based on Corollary 6.1. If the transmission completion time for both users is not the same, then over the last duration, we transmit only to one of the users, while the transmission rate to the other user is zero. This contradicts with the monotonicity of the transmission rates for both users. Therefore, under the optimal policy, the transmitter must finish transmitting to both users at the same time. □

This lemma is proved in [36] also, by using a different approach. The authors prove it in [36] mainly based on the convexity of the capacity region of the broadcast channel.

For fixed (B_1, B_2), let us denote the transmission completion time for the first and second user, by T_1, T_2, respectively. We note that T_1 and T_2 depend on the selection of the *cut-off* power level, P_c. In particular, T_1 is monotonically decreasing in P_c, and T_2 is monotonically increasing in P_c. Based on Lemma 6.5, the problem of optimal selection of P_c, can be viewed as solving a *fixed point* equation. In particular, P_c must be chosen such that, the resulting T_1 equals T_2. Therefore, we propose the following algorithm to solve the transmission completion time, T, minimization problem. Our basic idea is to try to identify the *cut-off* power level P_c in an efficient way.

Since the power allocation is similar to the single-user case (c.f. Lemma 6.3), our approach to find T will be similar to the method in Chapter 5. First, we aim to identify P_1, the first total transmit power starting from $t = 0$ in the system. This is exactly the same as identification of P_1 in the corresponding single-user problem. For this, as in Chapter 5, we treat the energy arrivals as if they have arrived at time $t = 0$, and obtain a lower bound for the transmission completion time as in Chapter 5. In order to do that, first, we compute the amount of energy required to finish (B_1, B_2) by s_1. This is equal to $g\left(\frac{B_1}{s_1}, \frac{B_2}{s_1}\right) s_1$, denoted as A_1. Then, we compare A_1 with E_0. If E_0 is greater than A_1, this implies that the transmitter can finish the transmission before s_1 with E_0, and future energy arrivals are not needed. In this case, the minimum transmission completion time is the solution of the following equation

$$g\left(\frac{B_1}{T}, \frac{B_2}{T}\right) T = E_0 \tag{6.36}$$

If A_1 is greater than E_0, this implies that the final transmission completion time is greater than s_1, and some of the future energy arrivals must be utilized to complete the transmission. We calculate the amount of energy required to finish (B_1, B_2) by s_2, s_3, ..., and denote them as A_2, A_3, ..., and compare these with $E_0 + E_1, \sum_{j=0}^{2} E_j, \sum_{j=0}^{3} E_j$, ..., until the first A_i that becomes smaller than $\sum_{j=0}^{i-1} E_j$. We denote the corresponding time index as $\tilde{i}_1$. Then, we assume that we can use $\sum_{i=0}^{\tilde{i}_1-1} E_i$ to transmit (B_1, B_2) at a constant rate. And, the corresponding transmission completion time is the solution of the following equation

$$g\left(\frac{B_1}{T}, \frac{B_2}{T}\right) T = \sum_{i=0}^{\tilde{i}_1-1} E_i \tag{6.37}$$

We denote the solution to this equation as $\tilde{T}$, and the corresponding power as $\tilde{P}_1$. From our analysis, we know that the solution to this equation is the minimum possible transmission completion time we can achieve. Then, we check whether this constant power $\tilde{P}_1$ is feasible, when the actual energy arrival times are imposed. If it is feasible, it gives us the minimal transmission completion time; otherwise, we get P_1 by selecting the minimal slope according to (6.15). That is to say, we draw all of the lines from $t = 0$ to the corner points of the energy arrival instances before $\widetilde{T}$, and choose the line with the smallest slope. We denote s_{i_1} as the corresponding duration associated with P_1. This is shown in Figure 6.8.

Once P_1 is selected, we know that it is the optimal total transmit power in our broadcast channel problem. Next, we need to divide this total power between the signals transmitted to the two users. Based on Lemma 6.4 and Corollary 6.1, if the *cut-off* power level P_c is higher than P_1, then, the transmitter spends all P_1 for

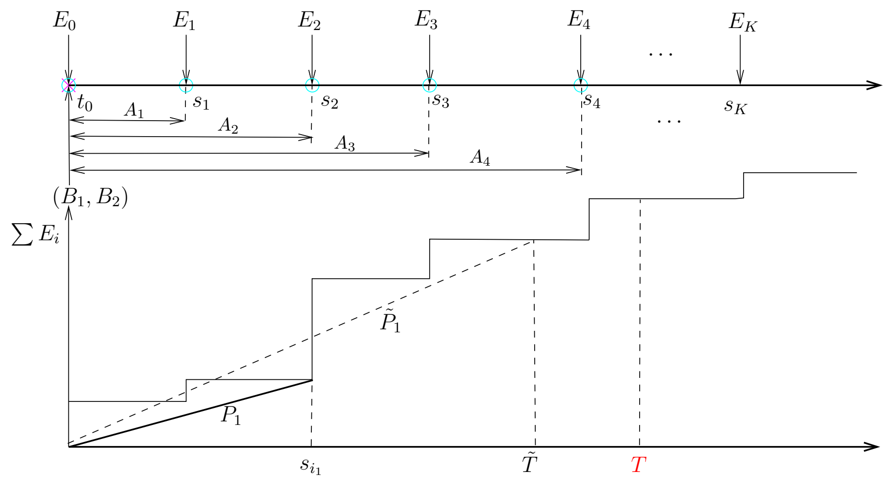


Figure 6.8: Determining the optimal total power level of the first epoch.

the stronger user; otherwise, the first user finishes its transmission with a constant power P_c.

We will first determine whether P_c lies in $[0, P_1]$ or it is higher than P_1. Assume $P_c = P_1$. Therefore, the transmission completion time for the first (stronger) user is

$$T_1 = \frac{B_1}{f(P_1)} \tag{6.38}$$

Once P_c is fixed, we can obtain the minimum transmission completion time for the second user, T_2, by subtracting the energy consumed by the first user, and treating P_1 as an interference for the second user. This reduces the problem to the single-user problem for the second user with fading, where the fading level is $P_1 + \sigma^2$ over $[0, T_1)$, and σ^2 afterwards. The single-user problem with fading is discussed in [20]. Since obtaining the minimal transmission completion time is not as straightforward for the fading channel, a more approachable way is to calculate the maximum number of bits departed from the second user by T_1, denoted as $D_2(T_1, P_c)$. In order to do that, we first identify the optimal power allocation policy with fixed deadline T_1. This can be done according to Lemma 6.3. Assume that the optimal power allocation gives us $P_1, P_2, \ldots, P_{N(T_1)}$. Then, we allocate P_1 to the first user over the whole duration, and allocate the remaining power to the second user. Based on (6.4), we calculate the transmission rate for the second user over each duration, and obtain $D_2(T_1, P_c)$ according to

$$D_2(T_1, P_c) = \sum_{i=1}^{N(T_1)} \frac{1}{2} \log\left(1 + \frac{P_n - P_c}{P_c + \sigma^2}\right)(s_{i_n} - s_{i_{n-1}}) \tag{6.39}$$

We observe that, given P_c, $D_2(T_1, P_c)$ is a monotonically increasing function of T_1. Moreover, given T_1, $D_2(T_1, P_c)$ is a monotonically decreasing function of P_c.

If $D_2(T_1, P_c)$ is smaller than B_2, it implies that $T_1 < T_2$, and we need to decrease the rate for the first user to make T_1 and T_2 equal. Based on Lemma 6.4, this also implies that the transmission power for the first user is a constant $P_c < P_1$. In particular, P_c is the unique solution of the following equation.

$$B_2 = D_2\left(\frac{B_1}{f(P_c)}, P_c\right) \tag{6.40}$$

Note that $D_2\left(\frac{B_1}{f(P_c)}, P_c\right)$ is a continuous, strictly monotonically decreasing function of P_c, hence the solution for P_c in (6.40) is unique. Since T_1 is a decreasing function of P_c and $D_2\left(\frac{B_1}{f(P_c)}, P_c\right)$ is a decreasing function of P_c, we can use the bisection method to solve (6.40). In this case, the minimum transmission completion time is $T = \frac{B_1}{f(P_c)}$.

If $D_2(T_1, P_c)$ is larger than B_2, that implies $T_2 < T_1$, and we need to increase the power allocated for the first user to make $T_1 = T_2$, i.e., $P_c > P_1$. Therefore, from Lemma 6.4, over the duration $[0, s_{i_1})$, the optimal policy is to allocate the entire P_1 to the first user only. We allocate P_1 to the first user, calculate the number of bits departed for the first user, and remove them from B_1. This simply reduces the problem to that of transmitting (B_1', B_2) bits starting at time $t = s_{i_1}$, where $B_1' = B_1 - f(P_1)s_{i_1}$. The process is illustrated in Figure 6.9. Then, the minimum transmission completion time is

$$T = s_{i_K} + \frac{B_1 - \sum_{i=1}^{K} f(P_k)(s_{i_k} - s_{i_{k-1}})}{f(P_c)} \tag{6.41}$$

where K is the number of recursions needed to get P_c.

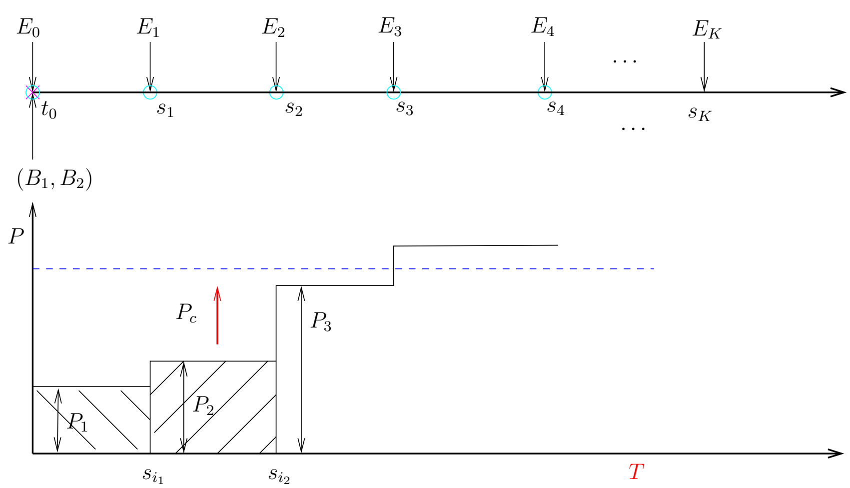


Figure 6.9: Search for the cutoff power level P_c iteratively.

In both scenarios, we reduce the problem into a simple form, and obtain the final optimal policy. Before we proceed to prove the optimality of the algorithm, we introduce the following lemma first, which is useful in the proof of the optimality of the algorithm.

Lemma 6.6 *$f(E/T)T$ monotonically increases in T; $f\left(\frac{\alpha E/T}{(1-\alpha E/t)+\sigma^2}\right)T$ monotonically increases in T also.*

Proof: The monotonicity of both functions can be verified by taking derivatives,

$$(f(E/T)T)' = f(E/T) - \frac{E}{(2\ln 2)(T+E)} \tag{6.42}$$

$$(f(E/T)T)'' = \frac{E}{2\ln 2}\left(\frac{1}{(T+E)^2} - \frac{1}{T(T+E)}\right) < 0 \tag{6.43}$$

where the last inequality follows since $E > 0$. Therefore, $f(E/T)T$ is a strictly concave function, and its first derivative monotonically decreases when T increases. Since when $\lim_{T\to\infty}(f(E/T)T)' = 0$, when $T < \infty$, we have $(f(E/T)T)' > 0$, therefore, the monotonicity follows.

Similarly, we have

$$\begin{aligned}\left(f\left(\frac{\alpha E/T}{(1-\alpha E/t)+\sigma^2}\right)T\right)' =& \frac{1}{2}\log_2\left(\sigma^2 + E/T\right) - \frac{1}{2}\log_2\left(\sigma^2 + (1-\alpha)E/T\right) \\ & - \frac{E}{2\ln 2}\frac{E}{E+\sigma^2 T} + \frac{E}{2\ln 2}\frac{(1-\alpha)E}{(1-\alpha)E+\sigma^2 T} \end{aligned} \tag{6.44}$$

$$\begin{aligned}\left(f\left(\frac{\alpha E/T}{(1-\alpha E/t)+\sigma^2}\right)T\right)'' =& \frac{E^2}{2T\ln 2}\left(\frac{1}{(\sigma^2 T/(1-\alpha)+E)^2} - \frac{1}{(\sigma^2 T+E)^2}\right) \\ <& 0\end{aligned} \tag{6.45}$$

Again, the concavity implies the first derivative is positive when $T < \infty$, and the monotonicity follows. □

Theorem 6.1 *The algorithm is feasible and optimal.*

Proof: We first prove the optimality. In order to prove that the algorithm is optimal, we need to prove that P_1 is optimal. Once we prove the optimality of P_1, the optimality of P_2, P_3, ... follows. Since the solution obtained using our algorithm always has the optimal structure described in Lemma 6.4, the optimality of the power allocation also implies the optimality of rate selection, thus, the optimality of the algorithm follows. Therefore, in the following, we prove that P_1 is optimal.

First, we note that P_1 is the minimal slope up to $\widetilde{T}$. We need to prove that P_1 is also the minimal slope up to the final transmission completion time, T. Let us define T' as follows

$$T' = \frac{\sum_{n=0}^{\tilde{i}_1} E_n}{P_1} \tag{6.46}$$

Assume that with $\tilde{P}_1$, we allocate $\alpha\tilde{P}_1$ to the first user, and finish (B_1, B_2) using constant rates. Then, we allocate αP_1 to the first user, and the rest to the second user. Based on Lemma 6.6, we have

$$f(\alpha P_1)T' \geq f(\alpha\tilde{P}_1)\tilde{T} = B_1 \tag{6.47}$$

$$f\left(\frac{\alpha P_1}{(1-\alpha)P_1 + \sigma^2}\right)T' \geq f\left(\frac{\alpha\tilde{P}_1}{(1-\alpha)\tilde{P}_1 + \sigma^2}\right)\hat{T} = B_2 \tag{6.48}$$

Therefore, T' is an upper bound for the optimal transmission completion time. Since P_1 is the minimal slope up to T', we conclude that P_1 is optimal throughout the transmission. Following similar arguments, we can prove the optimality of the rest of the power allocations. This completes the proof of optimality.

In order to prove that the allocation is feasible, we need to show that the power allocation for the first user is always feasible in each step. Therefore, in the following, we first prove that P_1 is feasible when we assume that $P_c = P_1$. The feasibility of P_1 also implies the feasibility of the rest of the power allocation. With the assumption that $P_c = P_1$, the final transmission time for the first user is

$$T_1 = \frac{B_1}{f(P_1)} \leq \frac{B_1}{f(\alpha P_1)} \tag{6.49}$$

Based on (6.47) and (6.48), we know that $T_1 < T'$. Since P_1 is feasible up to T', therefore, P_1 is feasible when we assume that $P_c = P_1$. The feasibility of the rest of the power allocations follows in a similar way. This completes the feasibility part of the proof. □

6.5 Numerical Results

We consider a band-limited AWGN broadcast channel, with bandwidth $W = 1$ MHz and the noise power spectral density $N_0 = 10^{-19}$ W/Hz. We assume that the path loss between the transmitter and the first receiver is about 100 dB, and the path loss between the transmitter and the second user is about 105 dB. Then, we have

$$r_1 = W \log_2\left(1 + \frac{\alpha P h_1}{N_0 W}\right) = \log_2\left(1 + \frac{\alpha P}{10^{-3}}\right) \text{Mbps} \tag{6.50}$$

$$r_2 = W \log_2\left(1 + \frac{(1-\alpha) P h_2}{\alpha P h_2 + N_0 W}\right) = \log_2\left(1 + \frac{(1-\alpha)P}{\alpha P + 10^{-2.5}}\right) \text{Mbps} \tag{6.51}$$

Therefore,

$$g(r_1, r_2) = 10^{-3}2^{r_1+r_2} + (10^{-2.5} - 10^{-3})2^{r_2} - 10^{-2.5} \quad \text{W} \tag{6.52}$$

For the energy harvesting process, we assume that at times $\mathbf{t} = [0, 2, 5, 6, 8, 9, 11]$ s, we have energy harvested with amounts $\mathbf{E} = [10, 5, 10, 5, 10, 10, 10]$ mJ. We find the maximum departure region $\mathcal{D}(T)$ for $T = 6, 8, 9, 10$ s, and plot them in Figure 6.10. We observe that the maximum departure region is convex for each value of T, and as T increases, the maximum departure region monotonically expands.

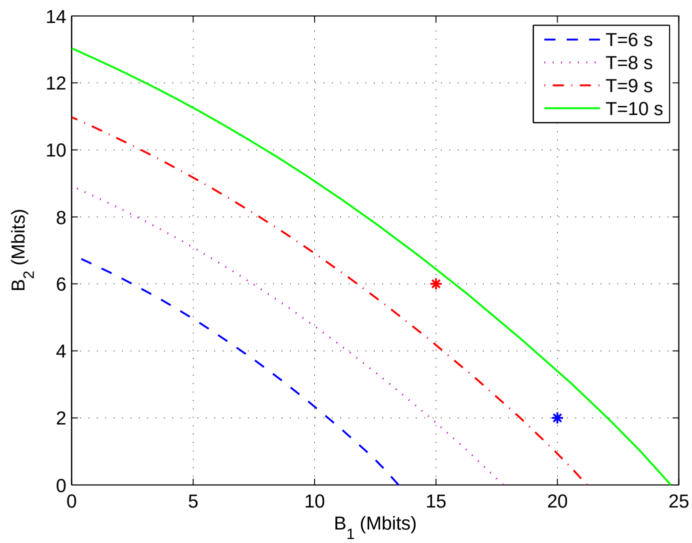


Figure 6.10: The maximum departure region of the broadcast channel for various T.

Then, we aim to minimize the transmission completion time with $(B_1, B_2) = (15, 6)$ Mbits. Following our algorithm, we obtain the optimal transmission policy, which is shown in Figure 6.11. We note that the powers change only potentially at instances when energy arrives (Lemma 6.1); power sequence is monotonically increasing and "majorized" over the whole transmission duration (Lemma 6.3). We also note that, for this case, the first user transmits at a constant rate, and the rate for the second user monotonically increases. The transmitter finishes its transmissions to both users by time $T = 9.66$ s, and the last energy harvest at time $t = 11$ s is not used.

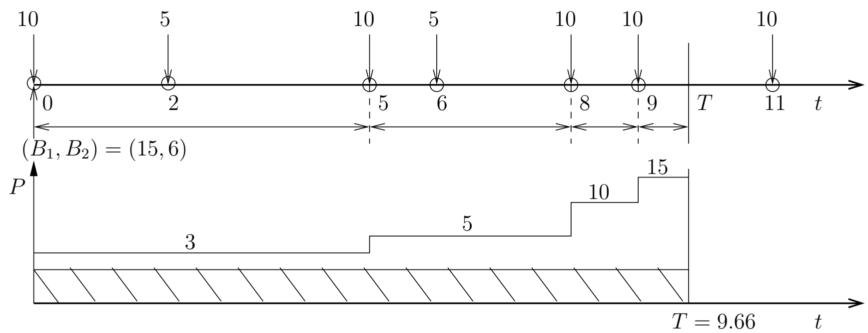


Figure 6.11: Cut-off power $P_c = 1.933$ mW. Optimal transmit rates are $r_1 = 1.552$ Mbps, $\mathbf{r}_2 = [0.274, 0.680, 1.369, 1.834]$ Mbps, with durations $\mathbf{l} = [5, 3, 1, 0.66]$ s.

Next, we consider the example when $(B_1, B_2) = (20, 2)$ Mbits, we have the optimal transmission policy, as shown in Figure 6.12. In this example, the cut-off power is greater than P_1, and therefore, P_1 is allocated to the first user only over $[0, 5)$ s, and after $t = 5$ s, the first user keeps transmitting at a constant rate until all bits are transmitted. In this case, the transmission rates for both users monotonically increase. The transmitter finishes its transmissions by time $T = 9.25$ s, and the last energy harvest is not used.

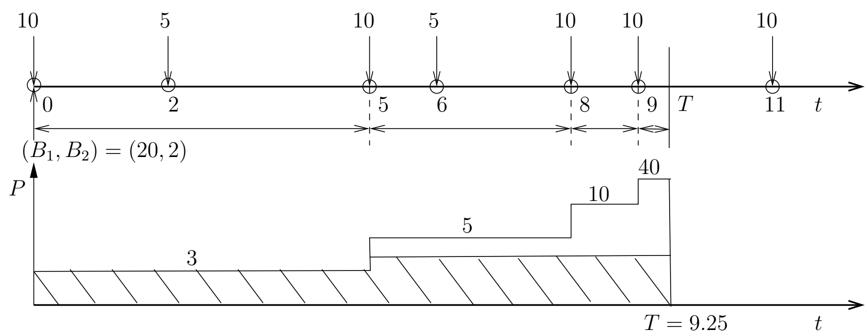


Figure 6.12: Cut-off power $P_c = 4.107$ mW. Optimal transmit rates $\mathbf{r}_1 = [2, 2.353, 2.353, 2.353]$ Mbps and $\mathbf{r}_2 = [0, 0.167, 0.856, 2.570]$ Mbps, with durations $\mathbf{l} = [5, 3, 1, 0.25]$ s.

6.6 Conclusions

In this chapter, we investigated the transmission completion time minimization problem in an energy harvesting broadcast channel. We assumed that there are certain number of packets at the transmitter, ready to be transmitted to both users before the transmission starts. We first analyzed the structural properties of the optimal transmission policy, and proved that the optimal total transmission power has the same structure as that in the single-user communication channel. We also proved that there exists a *cut-off* power for the stronger user. If the optimal total transmission power is lower than this cut-off level, all power is allocated to the stronger user, and when the optimal total transmission power is greater than this cut-off level, all power above this level is allocated to the weaker user. Based on these structural properties of the optimal policy, we developed an iterative algorithm to obtain the globally optimal off-line scheduling policy.

Chapter 7

Optimal Packet Scheduling in a Multiple Access Channel with Energy Harvesting Transmitters

7.1 Introduction

Efficient energy management is crucial for wireless communication systems, as it increases the throughput and improves the delay performance. Energy efficient scheduling policies have been well investigated in traditional battery powered (un-rechargeable) systems [13–18]. On the other hand, there exist systems where the transmitters are able to harvest energy from the nature. Such energy harvesting abilities make sustainable and environmentally friendly deployment of communication systems possible. This renewable energy supply feature also necessitates a completely different approach to energy management.

In this chapter, we consider a multi-user rechargeable wireless communication system, where data packets as well as the harvested energy arrive at the transmitters as random processes in time. As shown in Figure 7.1, we consider a two-user multiple access channel (MAC), where each transmitter node has two queues. The data queue stores the data arrivals, while the energy queue stores the energy harvested from the environment. Our objective is to adaptively change the transmission rate and power according to the instantaneous data and energy queue sizes, such that the transmission completion time is minimized.

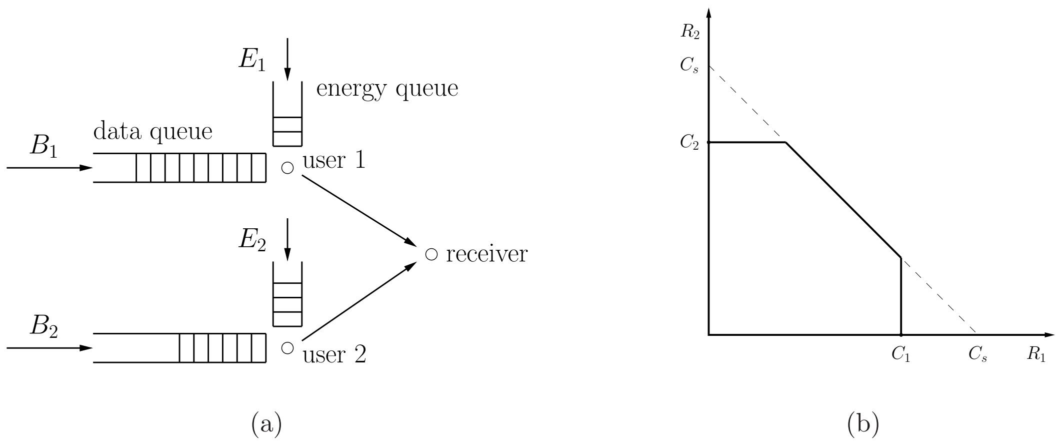


Figure 7.1: (a) An energy harvesting MAC model with energy and data queues, and (b) the capacity region of the additive white Gaussian noise MAC.

In general, the arrival processes for the data and the harvested energy can be formulated as stochastic processes, and the problem requires an *on-line* solution that adapts transmission power and rate in *real-time*. Since this seems to be an intractable problem for now, we simplify the problem by assuming that the data packets and energy will arrive in a deterministic fashion, and we aim to develop an *off-line* solution instead. In this chapter, we consider the scenario where packets have already arrived before the transmissions start. Specifically, we consider two nodes as shown in Figure 7.2. For the traffic load, we assume that there are a total of B_1 bits and B_2 bits available at the first and second transmitter, respectively, at time $t = 0$. We assume that energy arrives (is harvested) at points in time marked with $\circ$. In Figure 7.2, E_{1k} denotes the amount of energy harvested for the first user at time s_k. Similarly, E_{2k} denotes the amount of energy harvested for the second user at time s_k. If there is no energy arrival at one of the nodes, we simply let the corresponding amount be zero, which are denoted by the dotted arrows in Figure 7.2. Our goal then is to develop methods of transmission to minimize the time, T, by which all of the data packets from both of the nodes are delivered to the destination.

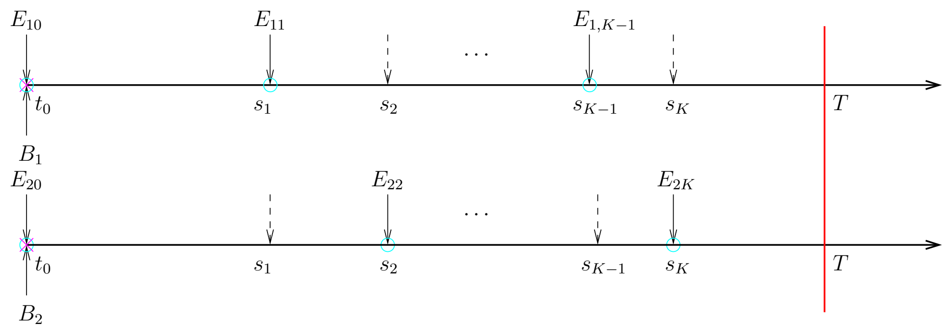


Figure 7.2: System model with all packets available at the beginning. Energies arrive at points denoted by $\circ$.

The optimal packet scheduling problem in a single-user energy harvesting communication system is investigated in Chapter 5. In Chapter 5, we prove that the optimal scheduling policy has a "majorization" structure, in that, the transmit power is kept constant between energy harvests, the sequence of transmit powers increases monotonically, and only changes at some of the energy harvesting instances; when the transmit power changes, the energy constraint is tight, i.e., the total consumed energy equals the total harvested energy. In Chapter 5, we develop an algorithm to obtain the optimal off-line scheduling policy based on these properties. Reference [19] extends Chapter 5 to the case where rechargeable batteries have finite sizes. We extend Chapter 5 in [20] to a fading channel. In the two-user MAC setting studied in this chapter, the scheduling problem is significantly more complicated. This is because the two users interfere with each other, and we need to select the transmission powers for both users as well as the rates from the resulting rate region, to solve the problem. In addition, because the traffic load and the harvested energy for both users may not be well-balanced, the final transmission durations for the two users may not be the same, further complicating the problem.

We first investigate a problem which is "dual" to the transmission completion time minimization problem. In this "dual" problem, we aim to characterize the maximum number of bits both users can transmit for any given time T. These two problems are "dual" to each in the sense that, if (B_1, B_2) lies on the boundary of the maximum departure region for time T^*, then, T^* must be the solution to the transmission completion time minimization problem with initial number of bits (B_1, B_2). We propose a *generalized iterative backward waterfilling* algorithm to achieve the boundary points of the maximum departure region for any given time T. Then, based on the solution of this "dual" problem, we go back to the transmission completion time minimization problem, simplify it into standard convex optimization problems, and solve it efficiently. In particular, we first characterize the maximum departure region for every energy arrival epoch, and based on the location of the given (B_1, B_2) on the maximum departure region, we narrow down the range of the minimum transmission completion time to be between two consecutive epochs. Based on this information, we propose to solve the problem in two steps. In the first step, we solve for the optimal power policy sequences to achieve the minimum T, so that (B_1, B_2) is on the maximum departure region for this T. This step can be formulated as a convex optimization problem. Then, with the optimal power policy obtained in the first step, we search for the optimal rate policy sequences from the capacity regions defined by the power sequences to finish B_1, B_2 bits. The second step is formulated as a linear programming problem. In addition, we further simplify the problem by exploiting the optimal structural properties for two special scenarios.

7.2 System Model and Problem Formulation

The system model is as shown in Figures 7.1 and 7.2. As shown in Figure 7.1, each user has a data queue and an energy queue. The physical layer is modeled as an additive white Gaussian noise channel, where the received signal is

$$Y = X_1 + X_2 + Z \tag{7.1}$$

where X_i is the signal of user i, and Z is a Gaussian noise with zero-mean and unit-variance. The capacity region for this two-user MAC is [24]

$$R_1 \leq f(P_1) \tag{7.2}$$

$$R_2 \leq f(P_2) \tag{7.3}$$

$$R_1 + R_2 \leq f(P_1 + P_2) \tag{7.4}$$

where $f(p) = \frac{1}{2}\log(1+p)$. We denote the region defined by these inequalities above as $\mathcal{C}(P_1, P_2)$. This region is shown on the right figure in Figure 7.1.

As shown in Figure 7.2, user i has B_i bits to transmit which are available at transmitter i at time $t = 0$. Energy is harvested at times s_k with amounts E_{ik}

at transmitter i. Our goal is to solve for the transmit power sequence, the rate sequence, and the corresponding duration sequence that minimize the time T by which all of the bits are delivered to the destination.

We assume that the transmitters can adapt their transmit powers and rates according to the available energy level and number of bits remaining. The energy consumed must satisfy the causality constraints, i.e., for each user, the total amount of energy consumed up to time t must be less than or equal to the total amount of energy harvested up to time t by that user.

Let us denote the transmit power for the first and second user at time t as $p_1(t)$ and $p_2(t)$, respectively. Then, the transmission rate pair $(r_1(t), r_2(t))$ must be within the capacity region defined by $p_1(t)$ and $p_2(t)$, i.e., $\mathcal{C}(p_1, p_2)(t)$. For user i, $i = 1, 2$, the energy consumed up to time t, denoted as $E_i(t)$, and the total number of bits departed up to time t, denoted as $B_i(t)$, can be written as:

$$E_i(t) = \int_0^t p_i(\tau) d\tau, \quad B_i(t) = \int_0^t r_i(\tau) d\tau, \quad i = 1, 2 \tag{7.5}$$

Here r_i and powers p_i are related through the f function as shown in (7.2)-(7.4). Then, the transmission completion time minimization problem can be formulated as:

$$
\begin{aligned}
\min_{p_1,p_2,r_1,r_2} \quad & T \\
\text{s.t.} \quad & E_1(t) \leq \sum_{n:s_n<t} E_{1n}, \quad 0 \leq t \leq T \\
& E_2(t) \leq \sum_{n:s_n<t} E_{2n}, \quad 0 \leq t \leq T \\
& B_1(T) \geq B_1, \quad B_2(T) \geq B_2 \\
& (r_1, r_2)(t) \in \mathcal{C}(p_1, p_2)(t),\ 0 \leq t \leq T
\end{aligned} \tag{7.6}
$$

The optimization problem in (7.6) is a difficult optimization problem in general. We first investigate a problem which is "dual" to this transmission completion time minimization problem. Specifically, we aim to characterize the maximum departure region, which is the region of (B_1, B_2) the transmitters can depart within a deadline T. Based on the solution for this "dual" problem, we will go back and decompose the original transmission completion time minimization problem into convex optimization problems, and solve it in an efficient way.

7.3 Characterizing $\mathcal{D}(T)$: Largest (B_1, B_2) Region for a Given Deadline T

In this section, our goal is to characterize the maximum departure region for a given deadline T. We define it in the following way.

Definition 7.1 *For any fixed transmission duration T, the maximum departure region, denoted as $\mathcal{D}(T)$, is the union of (B_1, B_2) under any feasible power and rate* *allocation policy over the duration* $[0, T)$.

We call any policy which achieves the boundary of $\mathcal{D}(T)$ to be optimal.

Lemma 7.1 *Under the optimal policy, the transmission power/rate remains constant between energy harvests, i.e., the power/rate only potentially changes at an energy harvesting epoch.*

Proof: Assume that the transmitter changes its transmission power between two energy harvesting instances s_i, s_{i+1}. Denote the transmit powers for the first and second user as p_{1n}, $p_{1,n+1}$, and p_{2n}, $p_{2,n+1}$, respectively. Denote the instant when the rate changes as s_i', as shown in Figure 7.3. Now, consider the duration $[s_i, s_{i+1})$. We equalize the transmit power of both users by letting

$$p_1' = \frac{p_{1n}(s_i' - s_i) + p_{1,n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i} \tag{7.7}$$

$$p_2' = \frac{p_{2n}(s_i' - s_i) + p_{2,n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i} \tag{7.8}$$

It is easy to check that the energy constraints are satisfied under this new power allocation policy, thus this new policy is feasible. On the other hand, the total number of bits departed over this duration under this new policy is a pentagon bounded by

$$f(p_1')(s_{i+1} - s_i) > f(p_{1n})(s_i' - s_i) + f(p_{1,n+1})(s_{i+1} - s_i')$$

$$f(p_2')(s_{i+1} - s_i) > f(p_{2n})(s_i' - s_i) + f(p_{2,n+1})(s_{i+1} - s_i')$$

$$f(p_1' + p_2')(s_{i+1} - s_i) > f(p_{1n} + p_{2n})(s_i' - s_i) + f(p_{1,n+1}, p_{2,n+1})(s_{i+1} - s_i') \tag{7.9}$$

where the inequality follows from the fact that $f(p)$ is strictly concave in p. We note that the right hand side of these inequalities characterizes the boundary of the departure region under the original policy over $[s_i, s_{i+1})$. Therefore, the departure region under the original policy is strictly inside the departure region under the new policy, which conflicts with the optimality of the original policy. □

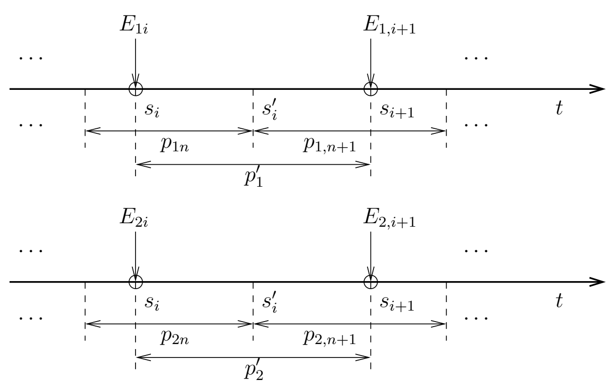


Figure 7.3: The power/rate must remain constant between energy harvests.

Therefore, in the following, we only consider policies where the rates are constant between any two consecutive energy arrivals. In order to simplify the notation, in this section, for any given T, we assume that there are $N-1$ energy arrival epochs (excluding $t=0$) over $(0,T)$. We denote the last energy arrival epoch before T as s_{N-1}, and $s_N = T$, with $l_n = T - s_{n-1}$. Let us define (p_{1n}, p_{2n}) to be the transmit power over $[s_{n-1}, s_n)$.

Lemma 7.2 *For any feasible transmit power sequences* $\mathbf{p}_1$, $\mathbf{p}_2$ *over over* $[0,T)$*, the total number of bits departed from both of the users, denoted as* B_1 *and* B_2*, is a* *pentagon defined as*

$$\left\{ (B_1, B_2) \;\middle|\; \begin{array}{l} B_1 \le \sum_{n=1}^{N} f(p_{1n}) l_n \\ B_2 \le \sum_{n=1}^{N} f(p_{2n}) l_n \\ B_1 + B_2 \le \sum_{n=1}^{N} f(p_{1n} + g_{2n}) l_n \end{array} \right\} \tag{7.10}$$

This lemma can be established based on the property of pentagon with 45° dominant face.

Lemma 7.3 $\mathcal{D}(T)$ *is a convex region.*

Proof: Consider two power policies $(\mathbf{p}_1, \mathbf{p}_2)$ and $(\bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2)$ over $[0, T)$. Without loss of generality, we assume that

$$\sum_{n=1}^{N} f(p_{2n}) l_n > \sum_{n=1}^{N} f(\bar{p}_{2n}) l_n \tag{7.11}$$

$$\sum_{n=1}^{N} f(p_{1n} + p_{2n}) l_n \le \sum_{n=1}^{N} f(\bar{p}_{1n} + \bar{p}_{2n}) l_n \tag{7.12}$$

Let us construct a new policy as a linear combination of these two policies over $[0, T)$, i.e., $\mathbf{p}'_i = \lambda \mathbf{p}_i + (1 - \lambda) \bar{\mathbf{p}}_i$, $i = 1, 2$, $0 < \lambda < 1$. It is straightforward to check that the energy constraints are still satisfied, thus the new policy is feasible. Consider the upper corner points of the departure region under the policies $(\mathbf{p}_1, \mathbf{p}_2)$

and $(\bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2)$. Because of the concave property of $f(p)$ in p, we have

$$\sum_{n=1}^{N} f(p'_{2n})l_n > \lambda \sum_{n=1}^{N} f(p_{2n})l_n + (1-\lambda)\sum_{n=1}^{N} f(\bar{p}_{2n})l_n \tag{7.13}$$

$$\sum_{n=1}^{N} f(p'_{1n}+p'_{2n})l_n > \lambda \sum_{n=1}^{N} f(p_{1n}+p_{2n})l_n + (1-\lambda)\sum_{n=1}^{N} f(\bar{p}_{1n}+\bar{p}_{2n})l_n \tag{7.14}$$

i.e., the upper corner point of the departure region under the new policy is always above the line connecting these two upper corner points under policies $(\mathbf{p}_1, \mathbf{p}_2)$ and $(\bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2)$. Therefore, the union of (B_1, B_2) over all feasible power allocation policies is a convex region. □

Lemma 7.4 *For any $T' > T$, $\mathcal{D}(T)$ is strictly inside $\mathcal{D}(T')$.*

Proof: For any policy achieving the boundary point of $\mathcal{D}(T)$, let us fix the power sequence for one user, and change the transmit power of the other user by removing part of its energy consumed before T and spend it over the duration $[T, T')$. Since there is no interference over $[T, T')$, the departures for the user can be potentially improved while the departures for the other user is unchanged. Therefore, $\mathcal{D}(T)$ must be strictly inside $\mathcal{D}(T)$. □

As a first step, we aim to explicitly characterize $\mathcal{D}(T)$ for any T. Similar to the capacity region of the fading Gaussian multiple access channel [30], where each boundary point is a solution to $\max_{\mathbf{R}\in\mathcal{C}} \boldsymbol{\mu}\cdot\mathbf{R}$, here, in our problem, the boundary points also maximize $\boldsymbol{\mu}\cdot\mathbf{B}$ for some $\boldsymbol{\mu}$. First, let us examine three different cases separately.

7.3.1 $\mu_1 = \mu_2$.

In this subsection, we consider the scenario where $\mu_1 = \mu_2$. Therefore, our problem becomes $\max_{\mathbf{p}_1,\mathbf{p}_2} B_1 + B_2$. In Chapter 5, we examined the optimal packet scheduling policy for the single-user scenario. We observe that for any fixed T, the optimal power allocation policy has the "majorization" property. Specifically, we have

$$i_n = \arg \min_{i_{n-1} < i \leq N} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \tag{7.15}$$

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n - 1} E_j}{s_{i_n} - s_{i_{n-1}}} \tag{7.16}$$

In this two-user multiple access channel, if we want to maximize the sum of departures, we conclude that the sum of powers must have the same "majorization" property as in the single-user scenario. Therefore, we merge the energy arrivals from both users, and obtain the sum of energy arrivals as a function of t. We can obtain the optimal sequence of sum of transmit powers, p_1, p_2, ..., p_n based on (7.15)-(7.16).

The sum of transmit powers and its corresponding duration defines $\sum_{n=1}^{N} f(p_n) l_n$. However, we can divide each p_n into p_{1n}, p_{2n} pair in infinitely many ways, such that their sums equal p_n for all n. Each feasible sequence of p_{1n} and p_{2n} gives a feasible region of (B_1, B_2), which is a pentagon. The dominant faces of all of these pentagons are on the same line. Therefore, the union of these pentagons is a larger pentagon. We need to identify the boundary of this larger pentagon, i.e., the end points of its dominant face.

With the sum of powers fixed, we want to find feasible power allocations which maximize B_1 and B_2, individually. As we proved for the single-user case, whenever the sum of powers changes, the total amount of energy consumed up to that instance must be equal to the total amount of energy harvested up to that instance. In other words, both users must deplete their energy completely at that moment. This adds additional energy constraints on both users besides the casuality constraints.

In order to maximize B_1, we plot the sum of E_{1n} as a function of t in Figure 7.4. Then, we equalize the transmit powers of the first user as much as possible with the casuality constraints on energy and the additional energy consumption constraints. This latter constraint requires us to empty the energy queue at given instances s_{i_1}, s_{i_2}, etc. The former constraint requires us to choose the minimum slope among the lines passing through the origin and any other corner point before the next energy emptying epoch, Chapter 5. This gives us the sequence of p_{1n}, as shown in Figure 7.4. Based on the concavity of the function $f(p)$, we can prove that this policy maximizes B_1 under the constraint that $B_1 + B_2$ is maximized at the same time.

Once p_{1n} is obtained, p_{2n} can be obtained by subtracting p_{1n} from p_n. Since p_n is always feasible in our allocation, the corresponding p_{2n} must be feasible as well. This power allocation defines a pentagon region for (B_1, B_2), where the lower corner point of this pentagon is also the lower point on the flat part of the dominant face of $\mathcal{D}(T)$, which is point 1 in Figure 7.5. Similarly, we can obtain the upper corner point on the flat part of the dominant face of $\mathcal{D}(T)$, which is point 2 in in Figure 7.5. Since any linear combinations of these two policies still achieves the sum rate, any

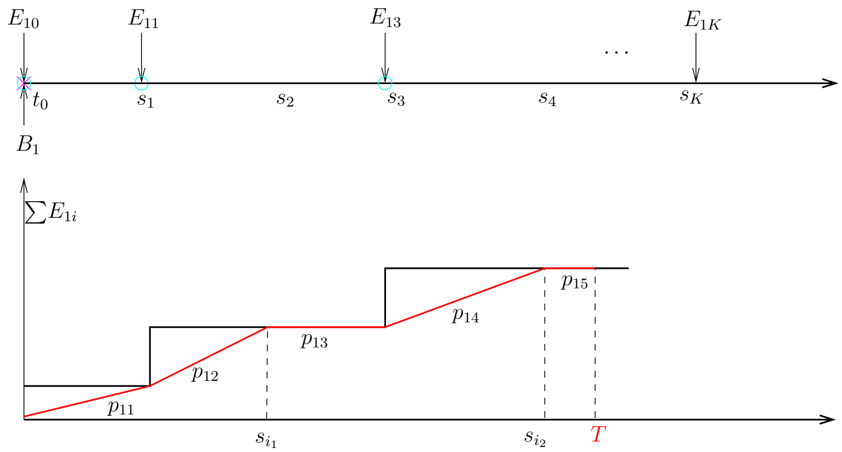


Figure 7.4: The transmit powers of individual user.

point on the flat part of the dominant face can be achieved. Therefore, the flat part of the dominant face of $\mathcal{D}(T)$ is bounded by these two corner points.

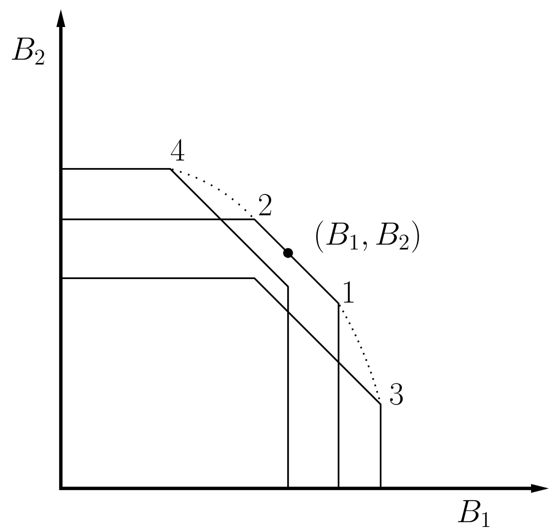


Figure 7.5: The departure region $\mathcal{D}(T)$.

7.3.2 $\mu_1 = 0$ or $\mu_2 = 0$.

In this subsection, we aim to maximize the departure from one user only. This procedure is exactly the same as the procedure in the single-user scenario. On top of that, we also want to maximize the departure from the other user. Without loss of generality, we aim to maximize B_1 first. This is a single-user scenario, and the optimal policy can be obtained according to (7.15)-(7.16). Given the allocation p_{1n}^*, in order to maximize the departure from the second user, we need to solve the following optimization problem

$$
\begin{aligned}
\max_{\mathbf{p}_2} \quad & \sum_{n=1}^{N} f(p_{1n}^* + p_{2n}) l_n \\
\text{s.t.} \quad & \sum_{n=1}^{j} p_{2n} l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad 1 \leq j \leq N
\end{aligned}
\tag{7.17}
$$

Theorem 7.1 *The optimal power allocation for (7.17) can be interpreted as a backward waterfilling process with base water level p_{1n}^* over $[s_{n-1}, s_n)$ for $1 \leq n \leq N$. Starting from $n = N$, we fill the energy $E_{2,N-1}$ over $[s_{N-1}, s_N)$, and get an updated water level as $p_{2N} + p_{1N}^*$; and then, we start to fill energy E_{N-2} over $[s_{N-2}, s_{N-1})$; once the water level exceeds $p_{2N} + p_{1N}^*$, we fill the remaining energy over $[s_{N-2}, s_N)$ until it is depleted. We continue this process until $n = 0$. The difference between the updated water level and base water level gives $\mathbf{p}_2$.*

Proof: We note that the constraint in (7.17) must be satisfied with an equality when $k = N$, otherwise, we can always increase some p_{2n} without conflicting with any other constraint, and the resulting number of departures is thus increased.

Based on this observation, (7.17) can be equivalently expressed as

$$\sum_{n=j}^{N} p_{2n} l_n \geq \sum_{n=j-1}^{N-1} E_{2n}, \quad 1 < j \leq N \tag{7.18}$$

$$\sum_{n=1}^{N} p_{2n} l_n = \sum_{n=0}^{N-1} E_{2n} \tag{7.19}$$

The Lagrangian becomes

$$\mathcal{L}(\mathbf{p}_2, \boldsymbol{\lambda}) = \sum_{n=1}^{N} f(p_{1n}^* + p_{2n}) l_n + \sum_{n=1}^{N} \lambda_n \left(\sum_{j=n}^{N} p_{2j} l_j - \sum_{j=n-1}^{N-1} E_{2j} \right) - \sum_{n=1}^{N} \gamma_n p_{2n} \tag{7.20}$$

where $\lambda_n \geq 0$ when $n > 1$, $\gamma_n \geq 0$ and $\gamma_n p_{2n} = 0$. The optimal solution must satisfy

$$p_{2n} = \left(\frac{1}{\lambda_1 - \sum_{j=1}^{n} \lambda_j} - p_{1n}^* - 1 \right)^+, \quad n = 1, 2, \ldots, N \tag{7.21}$$

$\frac{1}{\lambda_1 - \sum_{j=1}^{n} \lambda_j}$ can be interpreted as the "water" level over $[s_{n-1}, s_n)$, and $p_{1n}^* + 1$ is the base water level. If $\lambda_n > 0$, no energy flows across the epoch $t = s_{n-1}$, and we have,

$$\frac{1}{\lambda_1 - \sum_{j=1}^{n} \lambda_j} > \frac{1}{\lambda_1 - \sum_{j=1}^{n-1} \lambda_j} \tag{7.22}$$

i.e., the water level over $[s_{n-1}, s_n)$ must be higher than that over $[s_{n-2}, s_{n-1})$.

If $\lambda_n = 0$, energy harvested before flows across the epoch $t = s_{n-1}$, and we have,

$$\frac{1}{\lambda_1 - \sum_{j=1}^{n} \lambda_j} = \frac{1}{\lambda_1 - \sum_{j=1}^{n-1} \lambda_j} \tag{7.23}$$

i.e., the water level over $[s_{n-1}, s_n)$ is equal to that over $[s_{n-2}, s_{n-1})$. Therefore, energy flows across the epoch $t = s_{n-1}$ only when the water level $[s_{n-2}, s_{n-1})$ has the potential to surpass that over $[s_{n-2}, s_n)$, and the energy flow makes the water levels even. A backward waterfilling process naturally leads to the optimal power policy. □

The *backward waterfilling* procedure is shown in Figure 7.6. This power allocation defines another pentagon, and its lower corner point maximizes B_1, which is point 3 in Figure 7.5. Similarly, we can obtain another pentagon whose upper corner point maximizes B_2, which is point 4 in Figure 7.5. In general, points 3 and 4 do not coincide with the points 1 and 2, respectively, and consequently, there are curved parts connecting these corner points.

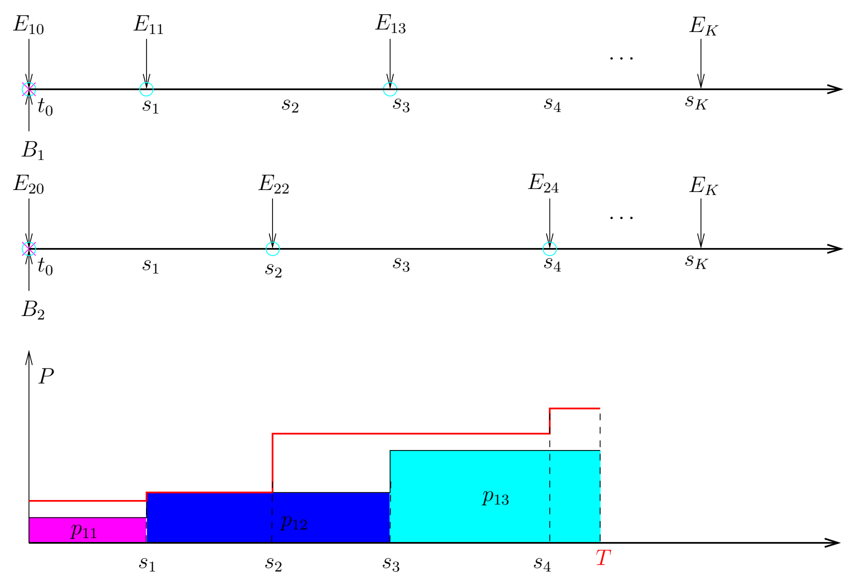


Figure 7.6: The optimal transmit power for the second user to maximize its departure.

7.3.3 General $\mu_1, \mu_2 > 0$.

The curved parts can be characterized through the solution of $\max_{\mathbf{B} \in \mathcal{D}(T)} \boldsymbol{\mu} \cdot \mathbf{B}$ for some $\boldsymbol{\mu} > \mathbf{0}$. Since each boundary point corresponds to a corner point on some pentagon, for $\mu_1 > \mu_2$, we need to solve the following problem:

$$\begin{aligned}
\max_{\mathbf{p}_1, \mathbf{p}_2} \quad & (\mu_1 - \mu_2) \sum_n f(p_{1n}) l_n + \mu_2 \sum_n f(p_{1n} + p_{2n}) l_n \\
\text{s.t.} \quad & \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j : 0 < j \leq N \\
& \sum_{n=1}^{j} p_{2n} l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad \forall j : 0 < j \leq N
\end{aligned} \tag{7.24}$$

The problem in (7.24) is a convex optimization problem with linear constraints, therefore, the unique global solution satisfies the extended KKT conditions as follows:

$$\frac{\mu_1 - \mu_2}{1 + p_{1n}} + \frac{\mu_2}{1 + p_{1n} + p_{2n}} \leq \sum_{j=n}^{N} \lambda_j, \quad 1 \leq n \leq N \tag{7.25}$$

$$\frac{\mu_2}{1 + p_{1n} + p_{2n}} \leq \sum_{j=n}^{N} \beta_j, \quad 1 \leq n \leq N \tag{7.26}$$

where the conditions in (7.25) and (7.26) are satisfied with equality if $p_{1n}, p_{1n} > 0$. When $\mu_1 \neq \mu_2$, it is difficult to obtain the optimal policy explicitly from the KKT conditions. Therefore, we adopt the idea of *generalized iterative waterfilling* in [37] to find the optimal policy.

Specifically, given the power allocation of the second user, denoted as $\mathbf{p}_2^*$, we optimize the power allocation of the first user, i.e., we aim to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{p}_1} \quad & (\mu_1 - \mu_2) \sum_{n=1}^{N} f(p_{1n}) l_n + \mu_2 \sum_{n=1}^{N} f(p_{1n} + p_{2n}^*) l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad 0 < j \leq N \end{aligned} \tag{7.27}$$

Once the power allocation of the first user is obtained, denoted as $\mathbf{p}_1^*$, we do a *backward waterfilling* for the second user to obtain its optimal power allocation. We perform the optimization for both users in an alternating way. Because of the concavity of the objective function and the Cartesian product form of the convex constraint set, it can be shown that the iterative algorithm converges to the global optimal solution, [38].

Because there is more than one term in the objective function of (7.27), the optimal policy for the first user does not have a backward waterfilling interpretation. However, using the method in [37], we can interpret the procedure for the first user as a *generalized backward waterfilling* operation. In order to see that, given $\mathbf{p}_2^*$, we define a generalized water level $b_n(p_{1n})$ as the inverse of the left hand side of (7.25), i.e.,

$$b_n(p_{1n}) = \left(\frac{\mu_1 - \mu_2}{1 + p_{1n}} + \frac{\mu_2}{1 + p_{1n} + p_{2n}^*} \right)^{-1} \tag{7.28}$$

and the base water level as $b_n(0)$, which can be seen as the modified interference plus noise level over the duration $[s_{n-1}, s_n)$. We generalize the form of the water level by taking the priority of users into account. Then, the KKT condition for this single-user problem is

$$\frac{1}{b_n(p_{1n})} \leq \sum_{j=n}^{N} \tilde{\lambda}_j, \quad n = 1, 2, \ldots, N \tag{7.29}$$

We note that $\tilde{\lambda}_j$ in general is different from the Lagrange multiplier λ_j in (7.25), since p_{2n}^* need not be the optimal $\mathbf{p}_2$. However, because of the convergence of the iterative algorithm, $\tilde{\lambda}_j$ converges to λ_j eventually as well.

Therefore, under the definition of the generalized water level $b_n(p_{1n})$, we can also interpret the optimal solution for the first user as a *generalized backward waterfilling* process. We first fill $E_{1,N-1}$ over the duration $[s_{N-1}, s_N)$, with the base water level $b_N(0)$. This step gives us an updated water level $b_N(E_{1,N-1}/l_N)$. Then, we move backward to the duration $[s_{N-2}, s_{N-1})$, and fill $E_{1,N-2}$ over that duration until it is depleted, or the water level becomes equal to $b_N(E_{1,N-1}/l_N)$. Once the latter happens, we fill the remaining energy over the durations $[s_{N-2}, s_{N-1})$ and $[s_{N-1}, s_N)$ in a way that the water level always becomes even. We repeat the steps until E_{10} is finished. This allocation gives the optimal $\mathbf{p}_1$ when the power of the second user is fixed. The optimality of this procedure can be proved in the same way as in the proof of Theorem 7.1.

Therefore, in this section, we determined the largest (B_1, B_2) region for any given T, i.e., $\mathcal{D}(T)$. We also determined the optimal power/rate allocation policy that achieves the points on the boundary of this (B_1, B_2) region. However, we recall that our goal is to find the minimum time, T, by which we can transmit given fixed number of bits (B_1, B_2). In the next section, we go back to our original problem, and provide a solution for it, using our findings in this section.

7.4 Minimizing the Transmission Completion Time T for a Given (B_1, B_2)

For a given pair (B_1, B_2), in order to minimize the transmission completion time of both users, we need to obtain T such that (B_1, B_2) lies on the boundary of the departure region $\mathcal{D}(T)$, as shown in Figure 7.5. However, $\mathcal{D}(T)$ depends on T, which is the objective we want to minimize, and is unknown upfront.

Therefore, in order to solve the problem, we first calculate $\mathcal{D}(t)$ for $t = s_1, s_2, \ldots, s_K$. Then, we locate (B_1, B_2) on the maximum departure region. If (B_1, B_2) is exactly on the boundary of $\mathcal{D}(t)$ for some $t = s_i$, then, based on the "duality" of these two problems, we know that this s_i is exactly the minimum transmission completion time the system can achieve, and the corresponding power and rate allocation policy achieving this point is the optimal policy.

If (B_1, B_2) is outside $\mathcal{D}(s_i)$ but inside $\mathcal{D}(s_{i+1})$ for some s_i, then, we conclude that the minimum transmission completion time, T, must lie between these two energy arriving epoches, i.e., $s_i < T < s_{i+1}$. Therefore, $T - s_i$, denoted as t here, is the duration we aim to minimize.

We propose to solve this optimization problem in two steps. In the first step, we aim to find a set of power allocation policy to ensure that (B_1, B_2) is on the boundary of the departure region defined by this power allocation policy. In the second step, with the power allocation obtained in the first step, we find a set of rate allocation within its corresponding capacity region, such that B_1, B_2 are finished by the minimal transmission duration obtained in the first step. The first step guarantees that such a rate allocation exists. Solving the problem through these two steps significantly reduces the complexity for each problem, since the number of unknown variables is about half in each problem. In addition, as we will observe, the first step can be formulated as a standard convex optimization problem, and the second step becomes a linear programming problem. Therefore, both steps can be solved through standard optimization tools in an efficient way.

Let us define the energy spent over $[s_{n-1}, s_n)$ by the first and second transmitter as e_{1n}, e_{2n}, respectively. Then, let $\mathbf{e}_1 = [e_{11}, e_{12}, \ldots, e_{1,i+1}]$, and $\mathbf{e}_2 = [e_{21}, e_{22}, \ldots, e_{2,i+1}]$, we formulate the optimization problem in the first step as follows

$$
\begin{aligned}
\min_{\mathbf{e}_1, \mathbf{e}_2, t} \quad & t \\
\text{s.t.} \quad & \sum_{n=1}^{j} e_{1n} \leq \sum_{n=0}^{j-1} E_{1n}, \quad 0 < j \leq i+1 \\
& \sum_{n=1}^{j} e_{2n} \leq \sum_{n=0}^{j-1} E_{2n}, \quad 0 < j \leq i+1 \\
& B_1 \leq \sum_{n=1}^{i} f\left(\frac{e_{1n}}{l_n}\right) l_n + f\left(\frac{e_{1,i+1}}{t}\right) t \\
& B_2 \leq \sum_{n=1}^{i} f\left(\frac{e_{2n}}{l_n} l_n\right) l_n + f\left(\frac{e_{2,i+1}}{t}\right) t \\
& B_1 + B_2 \leq \sum_{n=1}^{i} f\left(\frac{e_{1n} + e_{2n}}{l_n}\right) l_n + f\left(\frac{e_{1,i+1} + e_{2,i+1}}{t}\right) t
\end{aligned} \tag{7.30}
$$

where the last three inequality constraints simply mean that $(B_1, B_2) \in \mathcal{D}(s_i + t)$. We state the problem in this form, so that the constraint set becomes convex, and the problem is transformed into a standard convex optimization problem. The joint concavity of $f\left(\frac{e}{t}\right)t$ in (e,t) can be proved through taking second derivatives of the function with respect to e and t, and observing that the Hessian is always negative semidefinite. Therefore, the right hand side of these inequality constraints are all jointly concave, thus the constraint set is convex.

Once we obtain $\mathbf{e}_1, \mathbf{e}_2$ and t, we divide the energy by its corresponding duration, and get the optimal power policy sequences $\mathbf{p}_1$ and $\mathbf{p}_2$. Next, we perform the rate allocation in the second step. Therefore, the problem becomes that of searching for $\mathbf{r}_1$ and $\mathbf{r}_2$ from the sequence of capacity regions defined by the sequences $\mathbf{p}_1$ and $\mathbf{p}_2$ to depart B_1 and B_2. This solution may not be unique. Therefore, we formulate it as a linear programming problem as follows:

$$\begin{aligned}
\min_{\mathbf{r}_1,\mathbf{r}_2} \quad & r_{1,i+1} \\
\text{s.t.} \quad & \sum_{n=1}^{i} r_{1n} l_n + r_{1,i+1} t = B_1 \\
& \sum_{n=1}^{i} r_{2n} l_n + r_{2,i+1} t = B_2 \\
& (r_{1n}, r_{2n}) \in \mathcal{C}(p_{1n}, p_{2n}), \quad 0 < n \leq i+1
\end{aligned} \tag{7.31}$$

Here the objective function can be any arbitrary linear function in $\mathbf{r}_1$ and $\mathbf{r}_2$, since our purpose is only to obtain a feasible solution satisfying the constraints. We choose the objective function to be $r_{1,i+1}$ for simplicity. The solution of the optimization problem (7.30)-(7.31) gives us an optimal power and rate allocation policies, which minimize the transmission completion time for both users.

Obtaining $\mathcal{D}(s_i)$ for every s_i requires a large number of computations, and as we will see, it is not necessary. In order to reduce the computation complexity, we aim to explore two special cases of the problem, and use the algorithm in Chapter 5 to obtain a lower bound for T.

7.4.1 (B_1, B_2) lies on the flat part of the dominant face.

For a given pair of (B_1, B_2), the minimum possible transmission completion time can be achieved if it lies on the flat part of the dominant face of $\mathcal{D}(T)$ for some T. This corresponds to the scenario discussed in Section 7.3.1. Therefore, we can also treat these two users as a single-user system, and identify the value of T through the method discussed in Chapter 5.

Specifically, we calculate the minimum energy required to finish B_1+B_2 by s_1, this is equal to $2^{2\left(\frac{B_1+B_2}{s_1}\right)}-1$, denoted as A_1. Then, we compare A_1 with $E_{10}+E_{20}$. If A_1 is smaller than $E_{10}+E_{20}$, then, the minimum possible transmission completion time is the solution to the following equation

$$f\left(\frac{E_{10}+E_{20}}{T}\right) = \frac{B_1+B_2}{T} \tag{7.32}$$

In this case, the maximum departure region $\mathcal{D}(T)$ is a pentagon defined by $\mathcal{C}\left(\frac{E_{10}}{T}, \frac{E_{20}}{T}\right) T$. If $B_1 < f\left(\frac{E_{10}}{T}\right)T$ and $B_2 < f\left(\frac{E_{20}}{T}\right)T$, then, we always select a rate from $\mathcal{C}\left(\frac{E_{10}}{T}, \frac{E_{20}}{T}\right)$ to achieve the minimum transmission completion time.

If A_1 is greater than $E_{10}+E_{20}$, then, we continue to calculate the minimum energy required to finish B_1+B_2 by s_2, s_3, ..., denoted as A_2, A_3, ..., and compare these with $\sum_{j=0}^{1} E_{1j}+E_{2j}$, $\sum_{j=0}^{2} E_{1j}+E_{2j}, \ldots$, until the first A_i that becomes smaller than $\sum_{j=0}^{i-1} E_{1j} + E_{2j}$. Then, the minimum possible transmission completion time is the solution of

$$f\left(\frac{\sum_{j=0}^{i-1} E_{1j} + E_{2j}}{T}\right) = \frac{B_1 + B_2}{T} \tag{7.33}$$

Then, we need to determine whether this constant sum of transmit power is feasible when the energy arrival times are imposed. We merge the energy arrivals from both users and plot the sum of energy as a function of time. Then, we connect the corner points up to T with the origin, and the smallest slope among the lines gives us the first sum of the transmit power, p_1. We repeat this process, to obtain $p_2, p_3, \ldots$, until all of $B_1 + B_2$ bits are transmitted. This gives the shortest possible transmission completion time, T_1, for the system.

Next, we need to determine whether (B_1, B_2) lies on the flat part of the dominant face of $\mathcal{D}(T_1)$. We obtain the region $\mathcal{D}(T_1)$ and find the corner points of the flat part on its dominant face through the method described in Section 7.3.1, and compare them with (B_1, B_2). If (B_1, B_2) lies within the bound, as shown in Figure 7.5, this means that it is feasible to empty both queues by time T_1. The only remaining step is to identify a feasible power and rate allocation sequence to achieve this lower bound.

In order to obtain a feasible power allocation, we simplify the optimization problem in (7.30) into the following form

$$
\begin{aligned}
\min_{\mathbf{p}_1,\mathbf{p}_2} \quad & p_{11} \\
\text{s.t.} \quad & p_{1n} + p_{2n} = p_n, \quad 0 < n \leq i+1 \\
& B_1 \leq \sum_{n=1}^{i} f(p_{1n}) l_n + f(p_{1,i+1})(T_1 - s_i) \\
& B_2 \leq \sum_{n=1}^{i} f(p_{2n}) l_n + f(p_{2,i+1})(T_1 - s_i)
\end{aligned} \tag{7.34}
$$

Again, the objective function can be arbitrary since our purpose is only to obtain a feasible solution satisfying the constraints. We choose p_{11} for simplicity. Once the feasible power allocation is obtained, the optimal rate allocation can be obtained by solving (7.31).

7.4.2 (B_1, B_2) lies on the vertical or horizontal part.

If (B_1, B_2) does not lie on the flat part of the dominant face of $\mathcal{D}(T_1)$, then, it either lies on the vertical or horizontal parts of the boundary of $\mathcal{D}(T)$ for some T, or lies on the curved part of the boundary of $\mathcal{D}(T)$ for some T. Specifically, we assume that (B_1, B_2) is beyond the lower corner point of the flat part of the dominant face of $\mathcal{D}(T_1)$, as shown in Figure 7.7. This implies that if we keep transmitting with any policy corresponding to the point on the flat part of the boundary of $\mathcal{D}(T_1)$, by T_1, we have B_2 bits departed from the second user, however, there are still some more bits left in the queue of the first user. This situation motivates us to put more priority on the first user.

Therefore, as the second step, we consider the scenario that (B_1, B_2) lies on the vertical part of the boundary of $\mathcal{D}(T)$, for some duration T. We first ignore the second user, and treat the first user as the only user in the system. This is exactly the same situation as in the single-user scenario. We apply the algorithm in Chapter 5, and obtain the transmission duration for the first user, denoted as T_2. T_2 is the shortest possible transmission completion time for given B_1. If we can depart B_2 bits from the second user by T_2, then T_2 is the shortest transmission completion time for both users; otherwise, we cannot finish both data queues by T_2, and the final transmission time should be greater than T_2.

With T_2 fixed, we obtain the optimal energy allocation for the second user through the *backward waterfilling* procedure described in Section 7.3.2. Once p_{1n} and p_{2n} are determined, we can calculate the maximum number of bits departed from the second user under the assumption that the first user is the primary user. This gives us a number B_2'. If $B_2' \geq B_2$, as shown in Figure 7.7, it implies that our assumption is valid, and we can empty both queues by T_2, which is also the shortest possible transmission duration for the system.

If $B_2' < B_2$, this implies that we cannot depart B_2 bits from the second queue by T_2, therefore, the final transmission duration could not be T_2 either for the system. This leaves us with the last possibility that (B_1, B_2) must be on the curved part of some other region with some duration T, where $T > T_1, T_2$.

Therefore, up to this point, we obtained a lower bound for the transmission completion time T, which is $\max(T_1, T_2)$. In order to identify an upper bound for T, we only need to calculate the maximum departure region for the energy arriving

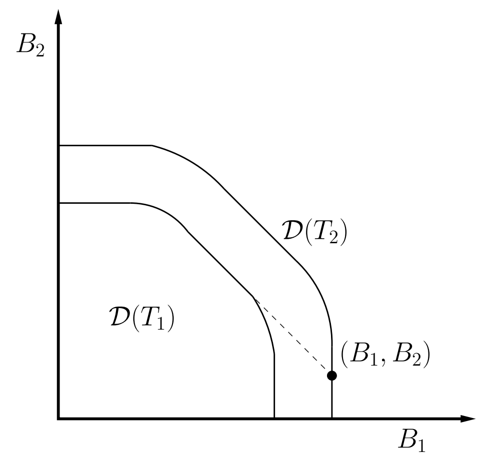


Figure 7.7: The minimum transmission completion time T to depart (B_1, B_2).

epochs right after $\max(T_1, T_2)$, until (B_1, B_2) is included for some $t = s_i$.

7.5 Numerical Results

We consider a band-limited additive white Gaussian noise channel, with bandwidth $W = 1$ MHz and noise power spectral density $N_0 = 10^{-19}$ W/Hz. We assume that the distance between the transmitters and the receiver is 1 Km, and the path loss is about 110 dB. Then, we have $f(p) = W \log_2\left(1 + \frac{ph}{N_0 W}\right) = \log_2\left(1 + \frac{p}{10^{-2}}\right)$ Mbps. For the energy harvesting process, we assume that at times $\mathbf{t} = [0, 2, 7, 11]$ s, we have energy harvested with amounts $\mathbf{E} = [5, 5, 10, 10]$ mJ for the first user; at times $\mathbf{t} = [0, 5, 8, 12]$ s, we have energy harvested with amounts $\mathbf{E} = [5, 10, 5, 10]$ mJ for the second user. We find the maximum departure region $\mathcal{D}(T)$ for $T = 7, 8, 11, 12$ s, and plot them in Figure 7.8. We observe that the maximum departure region is convex for each value of T, each boundary consists of three different parts, and as T increases, the maximum departure region monotonically expands.

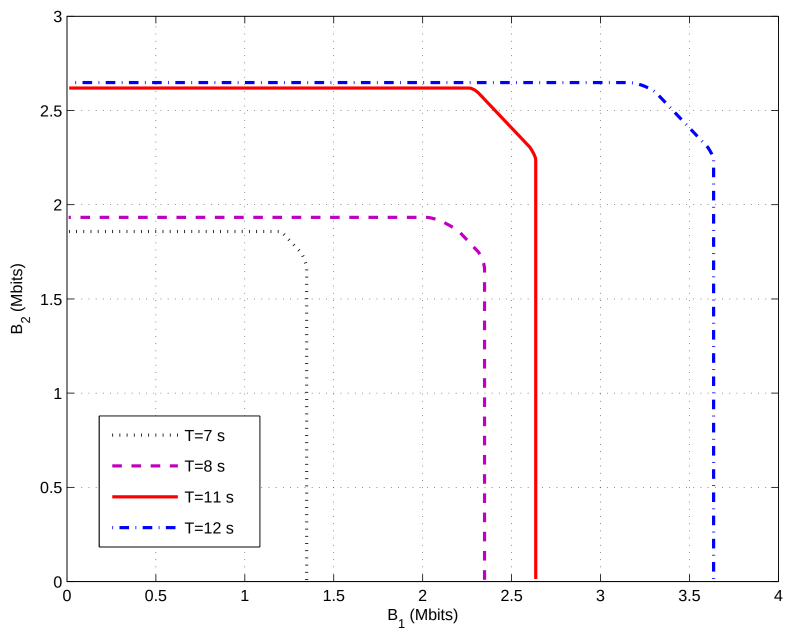


Figure 7.8: The maximum departure region of the multiple access channel for various T.

We assume that at $t = 0$, we have $B_1 = 2.5$ Mbits from the first user and $B_2 = 2.32$ Mbits from the second user to transmit. We choose the numbers in such a way that the solution is expressable in simple numbers, and can be plotted conveniently. Then, using the proposed algorithm, we obtain the optimal transmission policy, which is shown in Figure 7.9. We also determine the transmission rates as $\mathbf{r}_1 = [0.263, 0, 0.585, 0.3]$ Mbps and $\mathbf{r}_2 = [0.1155, 0.585, 0, 0.285]$ Mbps. We note that, for this case, the active transmission is completed by time $T = 10$ s, and the energy harvests at time $t = 11$ s and $t = 12$ s are not used. We also note that (B_1, B_2) lies on the flat part of the dominant face of $\mathcal{D}(10)$, therefore, we finish the transmission of both user simultaneously at $t = 10$ s. Since (B_1, B_2) is not at the corner point, the optimal policy is not unique. We may have different $\mathbf{p}_1$ and $\mathbf{p}_2$ and choose different rates accordingly to have the same departure time. However, the sequence of the sum of transmit powers is unique.

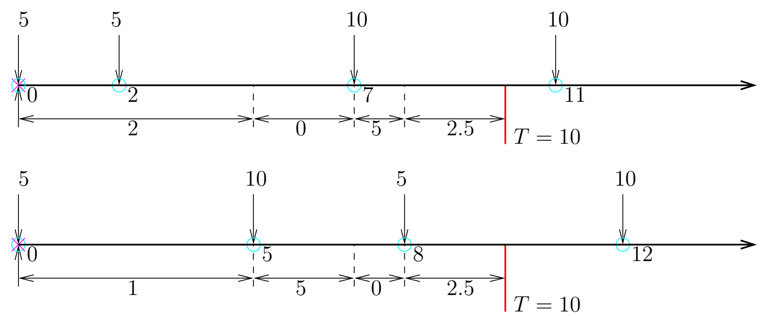


Figure 7.9: Optimal transmit powers $\mathbf{p}_1 = [2, 0, 5, 2.5]$ mW, $\mathbf{p}_2 = [1, 5, 0, 2.5]$ mW, with durations $\mathbf{l} = [5, 2, 1, 2]$ s.

If (B_1, B_2) is not well-balanced, then, it may not be on the dominant face of $\mathcal{D}(10)$, even though the sum $B_1 + B_2$ is the same. For example, if $B_1 = 2.63$ Mbits and $B_2 = 2.19$ Mbits, a simple calculation indicates that (B_1, B_2) lies beyond the range of the dominant face of $\mathcal{D}(10)$, and we cannot finish both queues at $t = 10$ s. Therefore, we take the first user as our primary user, and calculate the minimum possible transmission time for it. The optimal policy for the first user is $p_{11} = 1.43$ mW over $[0, 7)$ s, and $p_{12} = 2.67$ mW over $[7, 10.75)$ s. Based on this allocation, we perform the waterfilling procedure for the second user. The optimal allocation for the second user is shown in Figure 7.10, and the maximum number of bits departed from the second user is 2.22 Mbits, which is greater than B_2. This implies that the minimum transmission duration for both users is $T = 10.75$ s, and the data queue of the second user will be emptied earlier than the first user.

The value of (B_1, B_2) may be such that it is neither on the flat part of the dominant face nor on the vertical part of the boundary of any $\mathcal{D}(T)$. For example,

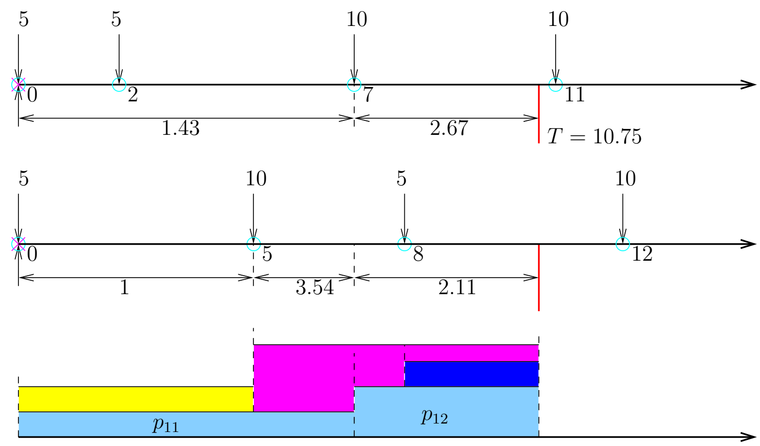


Figure 7.10: Optimal transmit powers $\mathbf{p}_1 = [1.43, 1.43, 2.67]$ mW, $\mathbf{p}_2 = [1, 3.54, 2.11]$ mW, with durations $\mathbf{l} = [5, 2, 3.75]$ s.

let $B_1 = 2.58$ Mbits and $B_2 = 2.24$ Mbits (note that the sum $B_1 + B_2$ is the same as in the previous two examples). From our first example, we know that it is beyond the dominant face of $\mathcal{D}(10)$. Then, we use the method for the second example to find the minimum transmission time for the first user by treating it as the primary user. Calculation indicates that the minimum transmission duration for the first user is $T = 9.7$ s, and the corresponding power allocation is $p_{11} = 1.43$ mW over $[0, 7)$ s, and $p_{12} = 3.7$ mW over $[7, 9.7)$ s. Then, since $T < 10$ s, and 10 s is the minimum possible transmission duration for the system, it implies that the total number of bits departed by $T = 9.7$ s is strictly less than $B_1 + B_2$. Therefore, we cannot finish the second queue by $T = 9.7$ s. Based on this analysis, we conclude that (B_1, B_2) must be on the curved part of $\mathcal{D}(T)$ for some T. Then, since it lies within $\mathcal{D}(11)$, together with the lower bound $\max(10, 9.7) = 10$ s, we solve the optimization problem described in (7.31). The optimal policy is shown in Figure 7.11. We observe that the sum of the transmit powers is always increasing, even though they are not monotonically increasing for each individual user. The power changes at $t = 2$ s and $t = 8$ s, where the energy constraints are satisfied with equality for the second user.

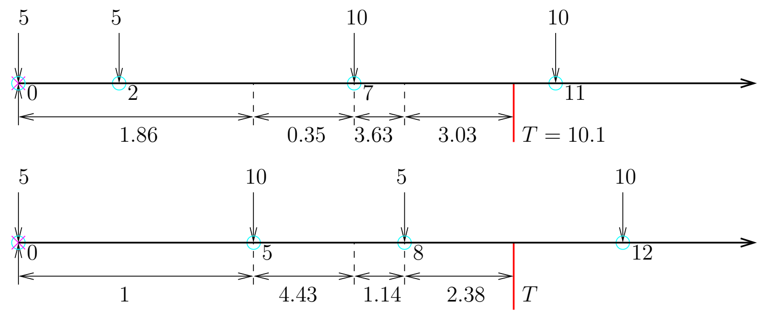


Figure 7.11: Optimal transmit powers $\mathbf{p}_1 = [1.86, 0.35, 3.63, 3.03]$ mW, $\mathbf{p}_2 = [1, 4.43, 1.14, 2.38]$ mW, with durations $\mathbf{l} = [5, 2, 1, 2.1]$ s.

These three pairs of (B_1, B_2) are plotted in Figure 7.12. Although the sum of B_1, B_2 is the same, they corresponds to different scenarios discussed before, and lies on different parts of the boundary of their corresponding maximum departure regions.

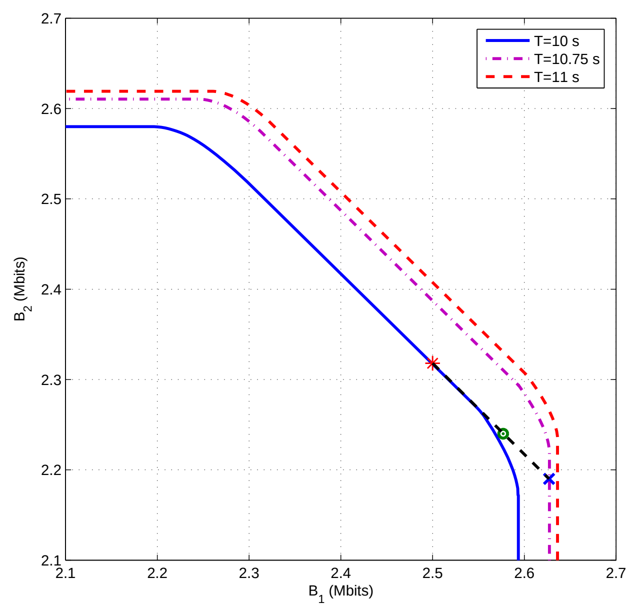


Figure 7.12: The maximum departure region of the multiple access channel for various T.

7.6 Conclusions

In this chapter, we investigated the transmission completion time minimization problem in an energy harvesting multiple access communication system. We assumed that the packets have already arrived and are ready to be transmitted at the transmitter before the transmission starts. We first proposed a *generalized iterative backward waterfilling* algorithm and characterized the maximum departure region for any given deadline constraint T. Then, based on these findings, we simplified the transmission completion time minimization problem into convex optimization problems, and solved it efficiently.

Chapter 8

Average Delay Minimization for an Energy Constrained Single-User Channel

8.1 Introduction

Our objective in this chapter is to minimize the packet delay in a general energy constrained system, where the transmitter may harvest energy from the nature. We aim to develop optimal transmission policies that take into account the *randomness* both in the *arrivals of the data packets* as well as in the *arrivals of harvested energy.* As shown in Figure 8.1, we will consider a single node, where packets arrive at random times marked with $\times$ and energy arrives (is harvested) at random points in time marked with $\circ$. In Figure 8.1, B_i denotes the number of bits in the ith arriving data packet, and E_i denotes the amount of energy in the ith energy arrival (energy harvesting). our objective is to minimize the overall delay of the packets subject to the energy constraints on the transmitter. The delay includes both the queuing time and the transmission time for the packet. Our aim is to adaptively allocate the energy over all packets according to the available amount of energy and number of packet at the transmitter, in a way to minimize the overall delay of the system.

The most general version of the problem is complicated. In this chapter, we will consider three scenarios, starting with the simplest setting and proceeding with

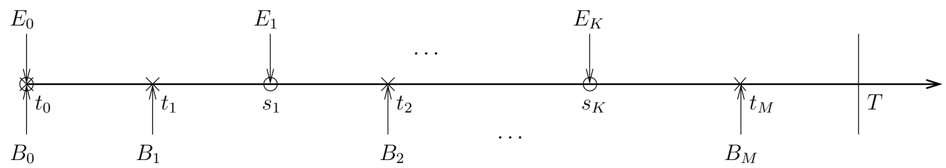


Figure 8.1: System model with random packet and energy arrivals. Data packets arrive at points denoted by × and energies arrive (are harvested) at points denoted by ◦.

progressively more complicated settings. In the first scenario, we assume that the transmitter has a fixed number of packets to transmit, and a fixed amount energy to use in its transmissions. We formulate the problem as a convex minimization problem. We use a Lagrangian based approach, and develop an iterative algorithm. The iterative algorithm is guaranteed to converge to the unique global optimum solution.

In the second scenario, we assume that the transmitter has a fixed amount of energy, but the packets arrive during the transmissions. We also formulate the problem as a convex minimization problem. However, even though the overall cost function is convex in the energies allocated to the packets, it is not differentiable. The reason for this is that the cost function takes different forms in different regions of allowable energy distributions. In other words, the energy allocated to a packet affects the form of the cost function for later packets. For this setting, unlike [13], the problem does not admit a closed-form solution. Therefore, we develop an iterative algorithm that is based on the principle of decreasing the overall delay at each iteration. We prove that the proposed algorithm decreases the overall delay monotonically. However, due to the non-differentiability of the overall delay function, the proposed algorithm may converge to a suboptimal fixed point. In order to overcome this problem, we use two modifications on our algorithm: increasing the dimensionality of the sub-problem solved at each iteration (i.e., considering more than two packets at any given iteration), and ϵ-perturbation of the sub-optimal fixed points. In addition, we develop a dynamic programming (DP) based formulation for the same problem.

In the third scenario, we assume that the transmitter has a fixed number of packets available at the beginning, but the energy arrives during the transmissions. This models an energy harvesting transmitter which harvests energy from the nature by using a rechargeable battery. In this scenario, a certain amount of energy from the battery is allocated to a packet for its transmission. In order to shorten the transmission time, a packet may hold its transmission until the battery gathers enough energy. This on the other hand increases its waiting time in the queue. Therefore, in this scenario, there is a trade-off between the waiting time and transmission time for the packets. This problem is not convex in general, and we develop a DP formation to obtain the optimal solution.

8.2 Scenario I: Packets and Energy Ready Before Transmission Starts

In many situations, such as multimedia communications, the source (video, music, etc.) may be available at the server waiting to be downloaded to their destinations. In sensor networks, a node may have gathered a number of packets before the transmission starts. In these scenarios, minimizing the overall transmission delay with a given amount of energy is an important problem.

We consider a non-fading single-user wireless channel. We assume that there are M packets available at the transmitter at $t = 0$; see Figure 8.2. The packets have a uniform size, which is B_0 bits per packet. The transmitter has a total energy constraint which is denoted by E_0. Let e_i denote the energy allocated for the transmission of packet i, then $\sum_{i=1}^{M} e_i \leq E_0$. We can express the relationship between the transmission duration of τ_i and the energy spent in its transmission e_i, for packet i, as a deterministic function $\tau_i = f(e_i)$. Without loss of generality, as in [13, 18], we assume that $f(e)$ satisfies the following properties: i) $f(e) \geq 0$, ii) $f(e)$ decreases monotonically in e, iii) $f(e)$ is strictly convex in e, iv) $f(e)$ is continuously differentiable, and v) $f(e) \to \infty$ as $e \to 0$. As shown in [13, 18], the first four conditions are satisfied in realistic channel coding schemes. The last condition is reasonable as a packet cannot be transmitted with zero energy.

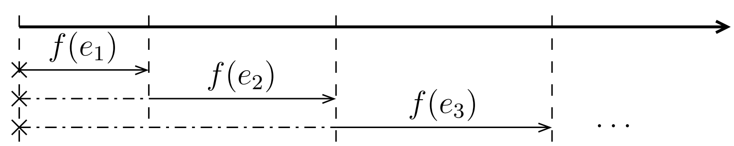


Figure 8.2: System model when all packets and energy are ready before the transmission starts.

Therefore, for the ith packet, the delay D_i can be expressed as

$$D_i = \sum_{k=1}^{i} \tau_k = \sum_{k=1}^{i} f(e_k) \tag{8.1}$$

Then, our optimization problem becomes

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} (M-i+1) f(e_i) \\ \text{s.t.} \quad & \sum_{i=1}^{M} e_i \leq E_0 \\ & e_i \geq 0, \quad i = 1, \ldots, M \end{aligned} \tag{8.2}$$

We note that, since all the packets have arrived before the transmission starts, the cost function has a fixed form. This makes the optimization problem tractable. The problem in (8.2) is a convex optimization problem, and there exists a unique global optimum solution that satisfies the KKT optimality conditions.

We note that because of property v) of $f(e)$, no e_i can be zero, as it would require the cost function to go to infinity. As a result, the KKTs can be expressed as

$$(M-i+1) f'(e_i) + \lambda = 0 \tag{8.3}$$

i.e., as

$$e_i = f'^{-1}\left(\frac{-\lambda}{M-i+1}\right), \quad i = 1, 2, \ldots, M \tag{8.4}$$

where λ is the non-negative Lagrange multiplier which is chosen such that $\sum_{k=1}^{i} e_k = E_0$.

In the following, we also devise an iterative algorithm to solve this problem. Initially, we allocate the total energy E_0 to the first packet. Then, we consider the first two packets, and optimize the distribution of the total energy E_0 over these two packets, in a way to minimize the overall delay, while we keep the energies allocated to the rest of the packets fixed. We continue this process until we reach the last packet, then we return to the first packet. The local optimization in the kth iteration becomes

$$\begin{aligned} \min \quad & (M-i+1)f(e_i^k) + (M-i)f(e_{i+1}^k) \\ \text{s.t.} \quad & e_i^k + e_{i+1}^k = e_i^{k-1} + e_{i+1}^{k-1}, \quad e_i^k, e_{i+1}^k \geq 0 \end{aligned} \tag{8.5}$$

It is easy to prove that this algorithm converges to a fixed point, since the algorithm monotonically decreases the cost function which is lower bounded by zero. Assume that $\mathbf{e}^k$ converges to a fixed point, $\bar{\mathbf{e}}$, we need to show that $\bar{\mathbf{e}}$ is the solution to (8.2). From the KKTs of the local optimization, we have

$$Mf'(\bar{e}_1) = (M-1)f'(\bar{e}_2) = \ldots = f'(\bar{e}_M) \tag{8.6}$$

We also have $\sum_{i=1}^{M} \bar{e}_i = E_0$. Therefore, $\bar{\mathbf{e}}$ satisfies the global KKT conditions in (8.3) and is the globally optimal point.

Based on the properties of $f(e)$, we know that $f'(e)$ is negative and monotonically increasing in e. From (8.6), we have $f'(\bar{e}_1) > f'(\bar{e}_2) > \ldots > f'(\bar{e}_M)$. Therefore, at the optimal point, the energy spent for each packet monotonically decreases in the order of transmission. Thus, earlier packets are assigned larger energies and therefore, are transmitted quicker than the later ones. Therefore, this model for the delay minimization problem yields a solution which is in contrast with the principle of *lazy scheduling* that the model in [13] resulted in.

8.3 Scenario II: Random Packet Arrivals

We assume that M packets arrive at the transmitter during the transmissions at times $t_1, t_2, \ldots, t_M$, where the inter-arrival times are denoted as $d_1, d_2, \ldots, d_{M-1}$; see Figure 8.3.

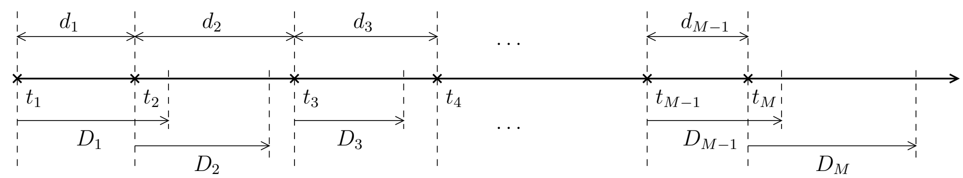


Figure 8.3: System model with random packet arrivals.

Let D_i denote the delay experienced by the ith packet, which includes the waiting time in the queue and the transmission time. Then, the delay experienced by each packet can be written recursively as,

$$\begin{aligned} D_1 &= f(e_1) \\ D_2 &= (D_1 - d_1)^+ + f(e_2) \\ D_3 &= (D_2 - d_2)^+ + f(e_3) \\ &\vdots \\ D_M &= (D_{M-1} - d_{M-1})^+ + f(e_M) \end{aligned} \tag{8.7}$$

where $(x)^+ = \max(0, x)$. Here, for the ith packet, $(D_{i-1} - d_{i-1})^+$ denotes the waiting time in the queue, and $f(e_i)$ denotes the actual transmission time. Then, we can express our optimization problem as

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} D_i \\ \text{s.t.} \quad & \sum_{i=1}^{M} e_i \leq E_0 \\ & e_i \geq 0, \qquad i = 1, 2, \ldots, M \end{aligned} \tag{8.8}$$

where the parameters of the optimization are the energies allocated to all packets, $\{e_i\}_{i=1}^{M}$, and the givens of the optimization problem are the total energy E_0 and the inter-arrival times of the packets $\{d_i\}_{i=1}^{M}$.

Intuitively, the optimization problem in (8.8) is a convex optimization problem since function $f(e_i)$ is convex and a linear combination of convex functions is convex. However, the existence of $(\cdot)^+$ function complicates matters, and the joint convexity of the cost function with respect to all e_i, i.e., with respect to $\mathbf{e} = [e_1 \ e_2 \ \ldots \ e_M]^\top$ needs to be proved.

Theorem 8.1 *The objective function in (8.8) is convex with respect to* $\mathbf{e}$*.*

Proof: We will prove the convexity recursively. First, we note that $D_1 = f(e_1)$ and $f(e_1)$ is convex in e_1. We also note that $D_2 = (f(e_1) - d_1)^+ + f(e_2)$ and the function $(f(e_1) - d_1)^+$ is convex in $\mathbf{e}$ because of the convexity of the function $f(e_1)$ in e_1. Thus, D_2 is convex in $\mathbf{e}$ also.

Then, we look at $D_3 = \left((f(e_1) - d_1)^+ + f(e_2) - d_2\right)^+ + f(e_3)$. We let $F(\mathbf{e}) = (f(e_1) - d_1)^+ + f(e_2) - d_2$. We note that $F(\mathbf{e})$ itself is convex in $\mathbf{e}$, and we need to prove that $(F(\mathbf{e}))^+$ is convex in $\mathbf{e}$ as well. Using the definition of $(\cdot)^+$, for any two vectors $\mathbf{e}$ and $\mathbf{e}'$ in the constraint set, we have

$$\begin{aligned}\lambda F(\mathbf{e})^+ + (1-\lambda)F(\mathbf{e}')^+ &\geq \lambda F(\mathbf{e}) + (1-\lambda)F(\mathbf{e}') \\ &\geq F(\lambda\mathbf{e} + (1-\lambda)\mathbf{e}') \end{aligned} \tag{8.9}$$

If $F(\lambda\mathbf{e} + (1-\lambda)\mathbf{e}')$ is positive, we have

$$\begin{aligned}\lambda F(\mathbf{e})^+ + (1-\lambda)F(\mathbf{e}')^+ &\geq F(\lambda\mathbf{e} + (1-\lambda)\mathbf{e}') \\ &= F(\lambda\mathbf{e} + (1-\lambda)\mathbf{e}')^+ \end{aligned} \tag{8.10}$$

If $F(\lambda\mathbf{e}+(1-\lambda)\mathbf{e}')$ is negative, then $F(\lambda\mathbf{e}+(1-\lambda)\mathbf{e}')^+ = 0$. Using the nonnegativity of the $(\cdot)^+$ function, we have

$$\begin{aligned}\lambda F(\mathbf{e})^+ + (1-\lambda)F(\mathbf{e}')^+ &\geq 0 \\ &= F(\lambda\mathbf{e} + (1-\lambda)\mathbf{e}')^+ \end{aligned} \tag{8.11}$$

Therefore, using (8.10) and (8.11), we conclude that

$$\lambda F(\mathbf{e})^+ + (1-\lambda)F(\mathbf{e}')^+ \geq F(\lambda\mathbf{e} + (1-\lambda)\mathbf{e}')^+ \tag{8.12}$$

which implies that $(F(\mathbf{e}))^+$ is convex in $\mathbf{e}$. Therefore, D_3 is convex in $\mathbf{e}$ as well.

The convexity of $(D_i - d_i)^+$ for $i = 4, \ldots, M-1$ can be proved in a similar manner. Since the objective function can be expressed as

$$\sum_{i=1}^{M} D_i = \sum_{i=1}^{M-1} (D_i - d_i)^+ + \sum_{i=1}^{M} f(e_i) \qquad (8.13)$$

and since each term in the cost function is convex in $\mathbf{e}$, the linear combination is convex in $\mathbf{e}$ as well. $\square$

Therefore, our problem is a convex minimization problem which has a convex objective function and linear constraints. However, there are two main difficulties in this optimization problem. First, since the overall delay includes both the queuing time and the transmission time of the packets, the transmission time for a packet affects the queuing time of all of the following packets. This causes the queuing time of earlier packets to be multiply counted in the objective function. This leads to the varying coefficients before $f(e_i)$'s in the cost function, which implies that the convexity of $f(\cdot)$ alone will not provide us a closed-form solution; we note that the convexity of the cost function alone provided a closed-form solution in [13] due to the symmetry in the cost function. Secondly, because of the existence of $(\cdot)^+$ function in the overall delay expression, the cost function has non-differentiable points. In addition, depending on whether the insides of $(\cdot)^+$ functions are negative or positive, we have 2^M possible cost functions. Since the number of different cost functions to consider grows exponentially with the number of packets, standard Lagrangian method is not tractable here. In the following, we will use a simple 3-packet problem to illustrate the difficulties involved in solving this convex optimization problem.

Using the definition of D_i in (8.7), the 3-packet problem is

$$\begin{aligned} \min \quad & f(e_1) + (f(e_1) - d_1)^+ + f(e_2) + \left((f(e_1) - d_1)^+ + f(e_2) - d_2\right)^+ + f(e_3) \\ \text{s.t.} \quad & e_1 + e_2 + e_3 \leq E_0, \qquad e_1, e_2, e_3 \geq 0 \end{aligned} \tag{8.14}$$

Opening the parentheses, we have four different possible cases:

Case 1: Both the transmission of the first and second packets end before the arrival of the next packet, i.e., insides of both $(\cdot)^+$ functions are negative. This case is illustrated in Figure 8.4(a). In this case, we have

$$\begin{aligned} \min \quad & f(e_1) + f(e_2) + f(e_3) \\ \text{s.t.} \quad & f(e_1) \leq d_1, \quad f(e_2) \leq d_2 \\ & e_1 + e_2 + e_3 \leq E_0, \qquad e_1, e_2, e_3 \geq 0 \end{aligned} \tag{8.15}$$

Case 2: The transmission of the first packet ends after the arrival of the second packet, while the transmission of the second packet ends before the arrival of the third packet. This case is illustrated in Figure 8.4(b). In this case, we have

$$\begin{aligned} \min \quad & 2f(e_1) + f(e_2) + f(e_3) - d_1 \\ \text{s.t.} \quad & f(e_1) > d_1, \quad f(e_1) + f(e_2) \leq d_1 + d_2 \\ & e_1 + e_2 + e_3 \leq E_0, \qquad e_1, e_2, e_3 \geq 0 \end{aligned} \tag{8.16}$$

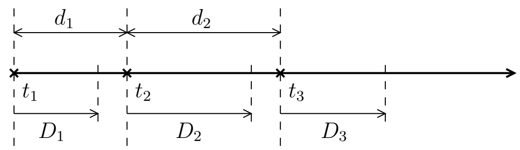
Case 3: The transmission of the first packet ends before the arrival of the second packet, while the transmission of the second packet ends after the arrival of the third packet. This case is illustrated in Figure 8.4(c). In this case, we have

$$\begin{aligned} \min \quad & f(e_1) + 2f(e_2) + f(e_3) - d_2 \\ \text{s.t.} \quad & f(e_1) \le d_1, \quad f(e_2) > d_2 \\ & e_1 + e_2 + e_3 \le E_0, \qquad e_1, e_2, e_3 \ge 0 \end{aligned} \tag{8.17}$$

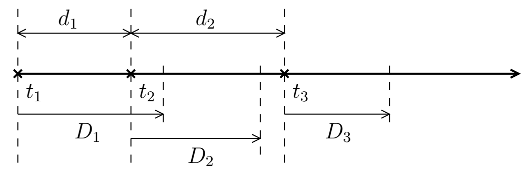
Case 4: The transmissions of both the first and the second packets end after the arrival of the next packet. This case is illustrated in Figure 8.4(d). In this case, we have

$$\begin{aligned} \min \quad & 3f(e_1) + 2f(e_2) + f(e_3) - 2d_1 - d_2 \\ \text{s.t.} \quad & f(e_1) > d_1, \quad f(e_1) + f(e_2) > d_1 + d_2 \\ & e_1 + e_2 + e_3 \le E_0, \qquad e_1, e_2, e_3 \ge 0 \end{aligned} \tag{8.18}$$

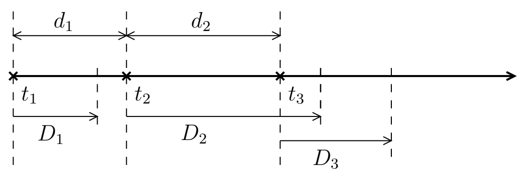
As we see, the sub-problems in (8.15), (8.16), (8.17) and (8.18) are similar in structure, except for different coefficients in front of the transmission delay times, $f(e_i)$, in the cost function. In addition, each problem has a different constraint set, which are all convex due to the monotonicity of $f(e_i)$ in e_i. In order to solve the optimization problem in (8.14), we need to solve the four optimization problems in (8.15), (8.16), (8.17) and (8.18), and take the solution that gives us the smallest cost function, i.e., overall delay. Even though each problem is differentiable and convex, the number of problems to be solved increases exponentially with the number of packets, making this approach intractable for practical scenarios with many packet arrivals.



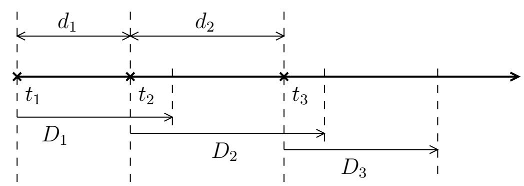
(a) Case 1.



(b) Case 2.



(c) Case 3.



(d) Case 4.

Figure 8.4: Four different cases.

8.3.1 An Iterative Approach

Because of the intractability of the global problem, in this section, we consider developing an iterative algorithm, which at any given iteration, considers a smaller local sub-problem. Similar to the *FlowRight* algorithm developed in [18], in this section, we consider optimizing two of the variables, the energies allocated to two consecutive packets, at any iteration, when the rest of the variables, the energies allocated to the rest of the packets, are fixed.

We follow the procedure of iterative algorithm described in the previous section. Initially, we allocate the total energy E_0 to the first packet. Then, we consider the first two packets, and optimize the distribution of the total energy E_0 over these two packets, in a way to minimize the overall delay, while we keep the energies allocated to the rest of the packets fixed. We continue this process until we reach the last packet, then we return to the first packet. We express the local optimization problem in terms of the energies of two consecutive packets, as follows

$$\begin{aligned} \min \quad & \sum_{j=i}^{M} D_j(e_i^k, e_{i+1}^k) \\ \text{s.t.} \quad & e_i^k + e_{i+1}^k = e_i^{k-1} + e_{i+1}^{k-1}, \quad e_i^k, e_{i+1}^k \geq 0 \end{aligned} \tag{8.19}$$

where e_i^{k-1} and e_{i+1}^{k-1} denote the energies of the packets in the previous iteration. This problem can be solved relatively easily as it essentially is a single-variable optimization problem.

Similarly, it is easy to prove that this algorithm converges to a fixed point, since the value of cost function monotonically decrease in each step, and it is lower bounded by zero. If the objective function had a fixed form and was twice-differentiable, as in the previous section, we could be sure that the algorithm converges to the globally optimum solution. Since our cost function is not differentiable at some points, the algorithm may converge to a strictly sub-optimal fixed point. Reference [38] proposes two approaches to solve the difficulty introduced by non-differentiability in network flow problems: "multiple node relaxation method", and "ϵ-relaxation method". We adopt these two methods here in order to escape sub-optimal fixed points. Following multiple node relaxation method, we consider sub-problems involving three or more packets, as opposed to two packets as we have done above. Similarly, following the ϵ-relaxation method, we move a small amount of energy from one packet to another to perturb a sub-optimal fixed point. Experimentally, we have observed that both methods improve the convergence of the algorithm.

8.3.2 A Dynamic Programming Approach

In this section, we develop a DP approach to our delay minimization problem. In particular, we partition the problem into M stages, and define the state space to be $\mathcal{E}\times\mathcal{A}$, where $\mathcal{E}$ includes the possible amounts of energy remaining at the current stage and $\mathcal{A}$ is the set of possible queuing times associated with the packet. Specifically, in stage n, we define $S_n(e,a)$ to be the minimal delay for the last $M-n+1$ packets, given the total energy remaining is e and the waiting time in the queue for the n-th packet is a, as shown in Figure 8.5. Then, we have the following recursive relationship

$$S_n(e,a) = \min_{0\le e_n\le e}\left\{a + f(e_n) + S_{n+1}\left(e-e_n, (a+f(e_n)-d_n)^+\right)\right\} \tag{8.20}$$

for $n=1,2,\ldots,M$, and $S_{M+1}(e,a)=0$.

During the process of solving the recursive equations backwards, we keep track of e_n that leads to the minimum value. Let us denote the minimizing values as $\hat{e}_n(e,a)$ for $n=1,2,\ldots,M$.

After computing the functions $\{S_n(e,a), 0\le e\le E_0\}$ in a backward recursion and obtaining the $\hat{e}_n(e,a)$, we get the optimal energy allocation strategy as $e_1 =$

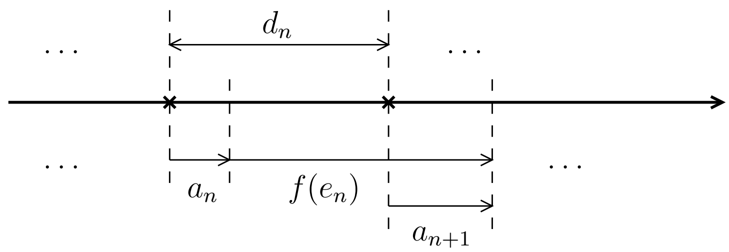


Figure 8.5: System model for the dynamic programming approach.

$\hat{e}_1(E_0, 0)$. For $n = 2, \ldots, M$,

$$\begin{aligned} a_n &= (a_{n-1} + f(e_{n-1}) - d_{n-1})^+ \\ e_n &= \hat{e}_n \left(E_0 - \sum_{i=1}^{n-1} e_i, a_i \right) \end{aligned} \tag{8.21}$$

Since getting a closed form solution for the recursive equations appears to be intractable, we perform numerical approximation instead. To this end, we quantize the state space into a finite number of discrete states. The step size of the quantization decides the size of the state space. Specifically, if there are N levels for the energy and J levels for the waiting time, for each packet we have $N \cdot J$ different states. The number of evaluations of $a + f(e_n) + S_{n+1}[e - e_n, (a + f(e_n) - d_n)^+]$ is once per quantized e_n for each quantized state for each stage. Thus, the number of basic evaluations is N^2JM, and the number of calculations grows linearly with the total number of packets M. We note that we can use the DP approach for more general cases where the packet arrivals are modeled as a random process, and the delays are calculated as expectations. In addition, we can incorporate the fading nature of the wireless channel, as well as develop *online* algorithms.

8.4 Scenario III: Random Energy Arrivals

In this section, we consider the situation where M packets are ready to transmit at $t = 0$. We assume that the packets have the same size, which is B_0 bits per packet. We also assume that there is E_0 amount of energy available at time $t = 0$, and at times s_1, s_2, ..., s_K, we have energies harvested with amounts E_1, E_2, ..., E_K, respectively. This system model is shown in Figure 8.6. Our goal is to adaptively choose the transmit rate according to the available energy and traffic level, in a way to minimize the average delay of the packets.

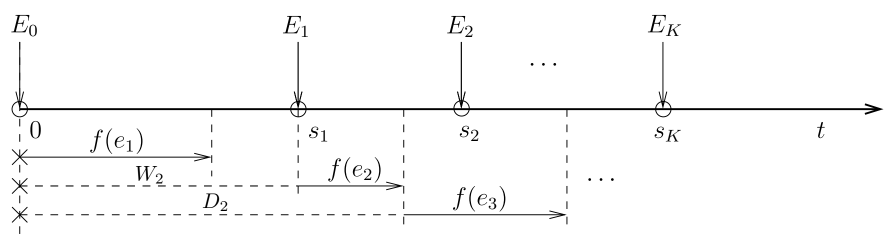


Figure 8.6: Average delay minimization with energy harvesting.

In order to make the system consistent with the model we have discussed in the previous sections, we assume that, the transmit rate of a packet is kept constant during its transmission. This assumption guarantees that the transmission time of a packet τ is a function of the energy allocated to it, i.e., $\tau_i = f(e_i)$. Moreover, we assume that at the time epoch right before packet i's transmission starts, we allocate a certain amount of energy e_i to it, and e_i cannot be greater than the total amount of energy available at that time epoch. This assumption is consistent with the causality constraint on energy, i.e., energy cannot be allocated before it has been harvested. This assumption also makes the stochastic optimal *online* algorithm possible, as we will see later in this section.

Again, we let D_i denote the delay experienced by the ith packet, which includes the waiting time in the queue and the transmission time. Different from previous sections, where the waiting time only includes the time waiting for all of the packets in front of it in the queue depart from the system, in this scenario, the waiting time may also include the time spent waiting for energy to become available.

Define W_i to be the earliest epoch when the energy allocated to the ith packet e_i becomes available. Then, given $e_1, e_2, \ldots, e_M$, we have

$$W_1 = \min_k \left\{ s_k : \sum_{j=0}^{k} E_j \geq e_1 \right\} \tag{8.22}$$

$$W_2 = \min_k \left\{ s_k : \sum_{j=0}^{k} E_j \geq e_1 + e_2 \right\} \tag{8.23}$$

$$\vdots$$

$$W_M = \min_k \left\{ s_k : \sum_{j=0}^{k} E_j \geq \sum_{i=1}^{M} e_i \right\} \tag{8.24}$$

From the definition, we note that if $e_1 \leq E_0$, then, $W_1 = 0$; otherwise, the first packet needs to wait for the arrivals of energy, until e_1 amount of energy becomes available. Then, the delay experienced by each packet can be expressed recursively as

$$D_1 = W_1 + f(e_1) \tag{8.25}$$

$$D_2 = \max(D_1, W_2) + f(e_2) \tag{8.26}$$

$$\vdots$$

$$D_M = \max(D_{M-1}, W_M) + f(e_M) \tag{8.27}$$

where $\max(D_{i-1}, W_i)$ denotes the waiting time in the queue, and $f(e_i)$ denotes the actual transmission time. Then, the average delay minimization problem becomes

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} D_i \\ \text{s.t.} \quad & e_i \geq 0, \qquad i = 1, 2, \dots, M \end{aligned} \tag{8.28}$$

where the parameters of the optimization are the energies allocated to all packets, $\{e_i\}_{i=1}^M$, and the givens of the optimization problem is the energy arrival profile.

Different from previous scenarios, where the optimization problems are convex, in this scenario, because of the existence of W_is in the cost function, the problem, in general, is not convex. From the definition of W_i and D_i in (8.22)-(8.24), we note that W_i and D_i are functions of $e_1, e_2, \dots, e_i$. Specifically, W_1 is a staircase function of e_1, and D_1 is a piecewise convex function, as shown in Figure 8.7. The expressions of W_is and D_is for $i > 1$ have more complex forms. As we can see from W_1 and D_1, in general, they are not convex in $\mathbf{e}$. However, for given W_is, the cost function is jointly convex in $\mathbf{e}$. We illustrate this fact through a simple two-packet case with two energy arrivals E_0, E_1 at times 0 and s_1, respectively.

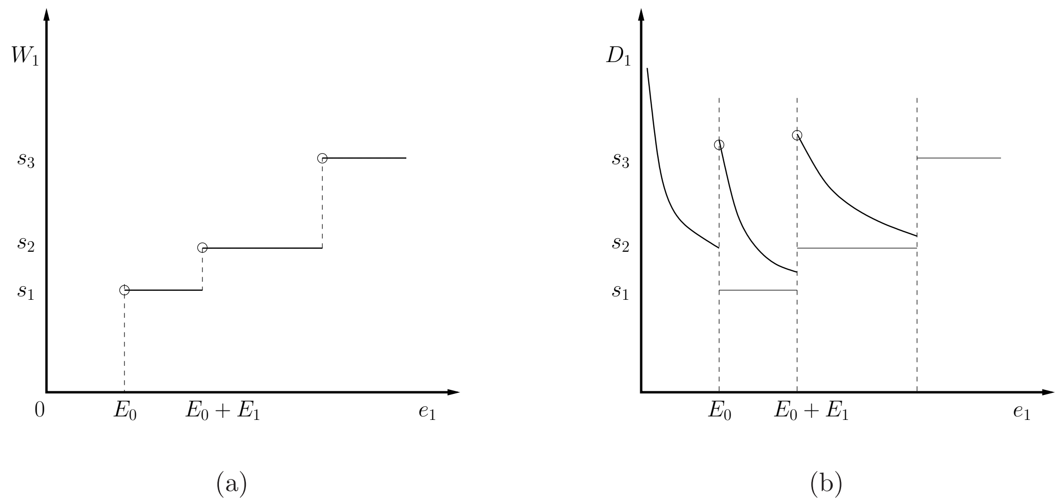


Figure 8.7: (a) The waiting time for the first packet, W_1 and (b) the delay for the first packet, D_1.

Using the definition of W_i and D_i, the optimization problem becomes

$$\min \quad W_1 + f(e_1) + \max(W_1 + f(e_1), W_2) + f(e_2) \tag{8.29}$$

$$\text{s.t.} \quad W_1 = \min_k \left\{ s_k : \sum_{j=0}^{k} E_j \geq e_1 \right\} \tag{8.30}$$

$$W_2 = \min_k \left\{ s_k : \sum_{j=0}^{k} E_j \geq e_1 + e_2 \right\} \tag{8.31}$$

$$e_1, e_2 \geq 0 \tag{8.32}$$

Opening the parentheses of the min function in the constraint, we have three different possible cases:

Case 1: $W_1 = W_2 = 0$, i.e., the second energy arrival is not utilized during the transmission. The optimization problem becomes

$$\begin{aligned} \min \quad & 2f(e_1) + f(e_2) \\ \text{s.t.} \quad & e_1 \le E_0, \quad e_1 + e_2 \le E_0, \quad e_1, e_2 \ge 0 \end{aligned} \tag{8.33}$$

Case 2: $W_1 = 0$, $W_2 = s_1$, i.e., the second packet is held until the second energy arrives, while the first packet's transmission is started at the beginning. The optimization problem becomes

$$\begin{aligned} \min \quad & f(e_1) + \max(f(e_1), s_1) + f(e_2) \\ \text{s.t.} \quad & e_1 \le E_0, \quad E_0 < e_1 + e_2 \le E_0 + E_1, \quad e_1, e_2 \ge 0 \end{aligned} \tag{8.34}$$

Depending on relative values $f(e_1)$ and s_1 in the cost function, we may have two different cases as follows:

Case 2a:

$$\begin{aligned} \min \quad & 2f(e_1) + f(e_2) \\ \text{s.t.} \quad & f^{-1}(s_1) < e_1 \le E_0, \quad E_0 < e_1 + e_2 \le E_0 + E_1, \quad e_1, e_2 \ge 0 \end{aligned} \tag{8.35}$$

Case 2b:

$$\begin{aligned} \min \quad & f(e_1) + f(e_2) + s_1 \\ \text{s.t.} \quad & e_1 \le \min(E_0, f^{-1}(s_1)), \quad E_0 < e_1 + e_2 \le E_0 + E_1, \quad e_1, e_2 \ge 0 \end{aligned} \tag{8.36}$$

Case 3: $W_1 = W_2 = s_1$, i.e., the first packet is held until the second energy is harvested, and the second packet's transmission is started after the first packet departs. The optimization problem becomes

$$\begin{aligned} \min \quad & 2f(e_1) + f(e_2) \\ \text{s.t.} \quad & e_1 > E_0, \quad e_1 + e_2 \leq E_0 + E_1, \quad e_1, e_2 \geq 0 \end{aligned} \tag{8.37}$$

As wee see, the sub-problems in (8.33), (8.35), (8.36) and (8.37) are similar in structure, and are all convex. In order to solve the average delay minimization problem in (8.32), we need to solve the optimization problem for each different case, and take the solution that gives us the smallest average delay. In the random packet arrivals scenario discussed in Section 8.3, the number of sub-problems to be solved increases exponentially with the number of packet arrivals. In this scenario, depending on the value selection of W_is, there are $\frac{K^M}{M!}$ different constraint sets, and each constraint set corresponds to multiple cost functions (*Cases 2a, 2b* in the example), which again increases the complexity of the problem. Therefore, solving the problem analytically becomes intractable with large number of packets and energy arrivals.

8.4.1 A Dynamic Programming Approach

In this section, we develop a DP approach to our delay minimization problem. In particular, we partition the problem into M stages, and define the state space to be $\mathcal{E} \times \mathcal{A}$, where $\mathcal{E}$ includes the possible amounts of energy remaining at the current stage and $\mathcal{A}$ is the set of possible epochs when the packet's transmission is started. Specifically, at stage n, we define $S_n(e,a)$ to be the minimal delay for the last $M-n$ packets, given the total energy remaining at $t=a$ is e.

Assuming there is no packet transmitting at $t=a$. Then, the transmitter may start to transmit the nth packet immediately, or it may postpone the transmission until more energy is harvested. Then, the start time is either $t=a$, or $t=s_i$ for some $s_i>a$. If we start to transmit nth packet at $t=a$ with energy e_n, where $e_n \leq e$, then, the transmission time for nth packet is $f(e_n)$. Since this transmission duration affects the queueing time of all these packets behind the nth packet, it should be counted $M-n+1$ times in the total delay. Once the transmission of the nth packet finishes, the system enters another stage $n+1$, with state $(e-e_n+\sum_{i:a<s_i\leq a+f(e_n)} E_i, a+f(e_n))$. If we hold the transmission of the nth packet until $t=s_i$ for $s_i>a$, then, the waiting time s_i-a should also be counted $M-n+1$ times in the total delay.

In order to simply the notation, we define $T_n(e,a)$ as the total minimal delay for the rest $M-n+1$ packet if the transmitter starts to transmit the n-th packet at $t=a$. Then, we have

$$T_n(e,a) = \min_{0<e_n<e} \left\{ (M-n+1)f(e_n) + S_{n+1}\Big(e-e_n+\sum_{a<s_j\leq a+f(e_n)} E_j, a+f(e_n)\Big) \right\} \tag{8.38}$$

$$S_n(e,a) = \min \left\{ T_n(e,a), (M-n+1)(s_i-a) + T_n\Big(e+\sum_{a<s_j\leq s_i} E_j, s_i\Big), \forall s_i>a \right\} \tag{8.39}$$

for $n = 1, 2, \ldots, M$, and $S_{M+1}(e, a) = 0$. The relationship between $T_n(e, a)$ and $S_n(e, a)$ is illustrated in Figure 8.8.

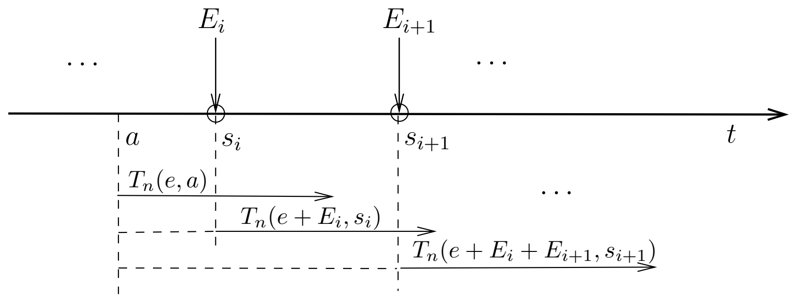


Figure 8.8: $T_n(e, a)$ in the dynamic programming formulation.

During the process of solving the recursive equations backwards, we keep track of e_n and the start point a_n that leads to the minimum value of $S_n(e, a)$. Let us denote the minimizing values as $\hat{e}_n(e, a)$ and $\hat{a}_n(e, a)$ for $n = 1, 2, \ldots, M$.

After computing the functions $T_n(e, a), S_n(e, a)$ in a backward recursion and obtaining the $\hat{e}_n(e, a)$, and $\hat{a}_n(e, a)$, we get the optimal energy allocation strategy as $e_1 = \hat{e}_1(E_0, 0)$, $a_1 = \hat{a}_1(E_0, 0)$. For $n = 2, \ldots, M$,

$$
\begin{aligned}
a_n &= \hat{a}_n \left(\sum_{s_j \le a_{n-1} + f(e_{n-1})} E_j - \sum_{i=1}^{n-1} e_i, a_{n-1} + f(e_{n-1}) \right) \\
e_n &= \hat{e}_n \left(\sum_{s_j \le a_{n-1} + f(e_{n-1})} E_j - \sum_{i=1}^{n-1} e_i, a_{n-1} + f(e_{n-1}) \right)
\end{aligned}
\tag{8.40}
$$

First, let us examine the example with two packets and two energy arrivals under the DP formulation. Based on (8.38)-(8.39), we have

$$T_2(e,a) = f(e) \tag{8.41}$$

$$S_2(e,a) = \min\left\{ f(e), (s_i - a) + f\Big(e + \sum_{a<s_j\le s_i} E_j\Big), \forall s_i > a \right\} \tag{8.42}$$

$$= \begin{cases} \min\{f(e), s_1 - a + f(e+E_1)\}, & a < s_1 \\ f(e), & a \ge s_1 \end{cases} \tag{8.43}$$

$$T_1(e,a) = \begin{cases} \min_{0\le e_1\le e} \Big\{ 2f(e_1) + \\ \quad \min\big(f(e-e_1), s_1 - a - f(e_1) + f(e-e_1+E_1)\big)\Big\}, & a + f(e_1) < s_1 \\ \min_{0\le e_1\le e}\{2f(e_1) + f(e-e_1+E_1)\}, & a + f(e_1) \ge s_1 \end{cases} \tag{8.44}$$

$$S_1(E_0,0) = \min\{T(E_0,0), 2s_i + T_1(E_0+E_1, s_1)\} \tag{8.45}$$

After taking derivatives of the functions on the right hand side of $T_1(e,a)$ and obtain the minimizers for each possible case, we can plug them in the expression of $S_1(e,a)$, and solve the problem explicitly. Although getting a closed form solution for the recursive equations becomes intractable when M becomes large, we can still perform numerical approximation to obtain the optimal energy allocation policy. The complexity is about KMN^2J, where there are N levels for the energy space and J levels for the time space. Based on the DP formulation, we can easily incorporate the random energy harvesting process to develop *online* algorithms.

8.5 Numerical Results

We consider a band-limited additive white Gaussian noise channel, with bandwidth $W = 1$ MHz and the noise power spectral density $N_0 = 10^{-19}$ W/Hz. We assume that the path loss between the transmitter and the first receiver is about 110 dB. We assume that the packets have a uniform size of 10 Kbits. Since the transmission rate with given power p is equal to

$$W \log_2 \left(1 + \frac{ph}{N_0 W}\right) = 10^6 \log_2 \left(1 + \frac{e}{10^{-2}\tau}\right) \tag{8.46}$$

the transmission time of a packet τ and the energy allocated to it e are related through the following equation

$$10^6 \log_2 \left(1 + \frac{e}{10^{-2}\tau}\right) = \frac{10^4}{\tau} \tag{8.47}$$

Although we cannot express τ as an explicit function of e, we can prove that the relationship between τ and e satisfies all of the stated properties for $f(e)$.

We assume that at time $\mathbf{t} = [0, 1.5, 2, 3.5, 5.25] \times 10^{-2}$ s, we have packets arrive at the transmitter. We use five algorithms, including our iterative algorithm, the versions of it with dimension relaxation, and ϵ-perturbation methods, DP based algorithm and built-in Matlab optimization functions.

Simulation results indicate that DP based algorithm always converges to the solution that the built-in Matlab function finds. In Figure 8.9, we observe that our iterative algorithm converges to the solution the built-in Matlab function finds.

However, in Figure 8.10, we observe that there is a gap between the convergence point of our iterative algorithm and the Matlab solution. We note that, at the point that our algorithm converges to, the departure time of the third packet coincides with the arrival time of the fourth packet. This means that our algorithm converges to a non-differentiable sub-optimal fixed point. When we apply dimension-3 relaxation and ϵ-perturbation methods, we observe that the modified version of our algorithm escapes the sub-optimal fixed point and converges to the optimal solution.

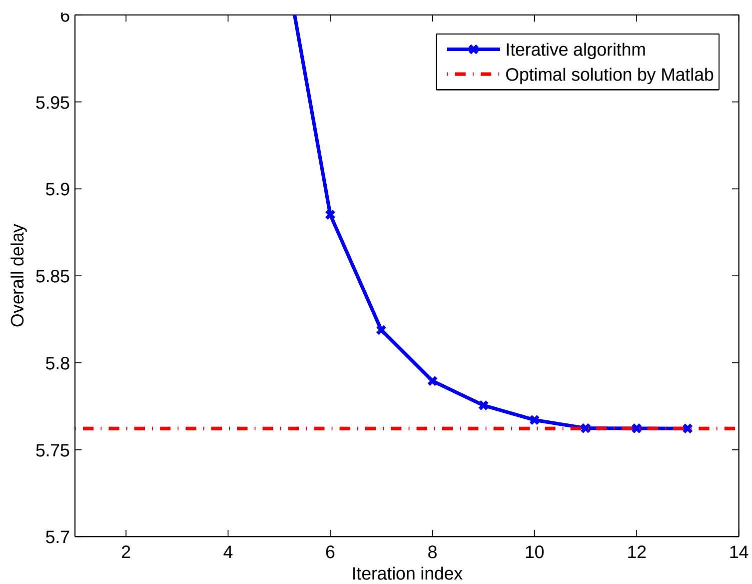


Figure 8.9: Overall delay as a function of the iteration index, when $E = 48 \times 10^{-2}$ mJ.

For the third scenario with energy arrivals, we assume that at $\mathbf{t} = [0, 2, 5, 6, 8, 9] \times 10^{-2}$ s, we have energy harvested with amount $\mathbf{E} = [10, 5, 10, 5, 10, 10] \times 10^{-2}$ mJ. We assume that at $t = 0$, we have four packets to transmit. We apply the DP algorithm, and obtain the policy as shown in Figure 8.11. We observe that since there is only a small amount of energy available at $t = 0$, in order to minimize the average delay, all of the packets except the first one have to wait for the arrivals of energy

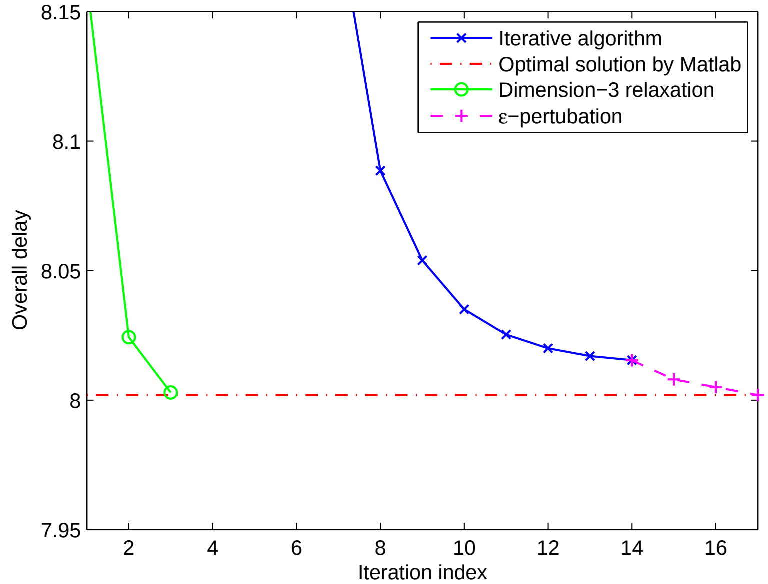


Figure 8.10: Overall delay as a function of the iteration index, when $E = 45 \times 10^{-2}$ mJ.

to transmit their packet, and the objective function in this case is equivalent to minimize $\sum_{i=1}^{4} f(e_i)$, i.e., the transmission time for each packet has the same weight in the cost function.Therefore they have the same value in the optimal solution.

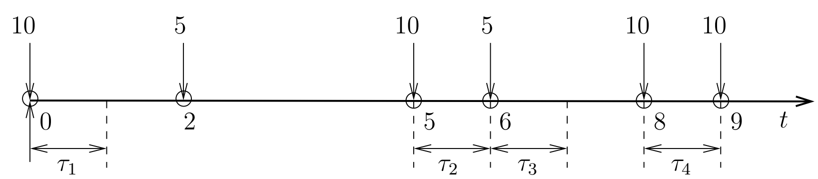


Figure 8.11: The optimal energy allocation $\mathbf{e} = [10, 10, 10, 10] \times 10^{-2}$ mJ, with duration $\boldsymbol{\tau} = [1, 1, 1, 1] \times 10^{-2}$ s, respectively.

When E_0 increases to 20×10^{-2} mJ and the rest E_is are the same, the optimal transmission policy is shown in Figure 8.12. We observe that in this scenario, the second packet starts its transmission right after the first packet, and last two packets finish their transmission with the same energy amount. Although the second packet has a longer transmission duration than the last two packet, the overall delay is minimized since the the waiting time for the second packet is avoided under this allocation.

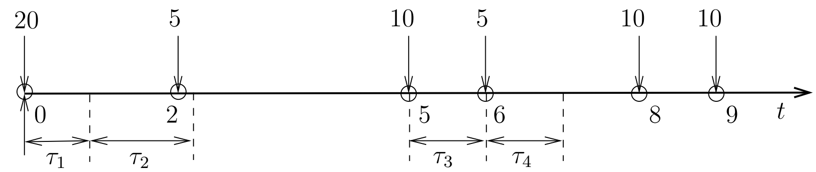


Figure 8.12: The optimal energy allocation $\mathbf{e} = [10.5, 9.5, 10, 10] \times 10^{-2}$ mJ, with duration $\boldsymbol{\tau} = [0.89, 1.15, 1, 1] \times 10^{-2}$ s, respectively.

When E_0 increases to 40×10^{-2} mJ, the optimal transmission policy is shown in Figure 8.13. This policy is identical to the policy in the first scenario when 45×10^{-2} mJ is available at $t = 0$. Because of the multiple counting of the transmission time for each packet in the cost function, the optimal policy has monotonically increasing transmission duration for the packets.

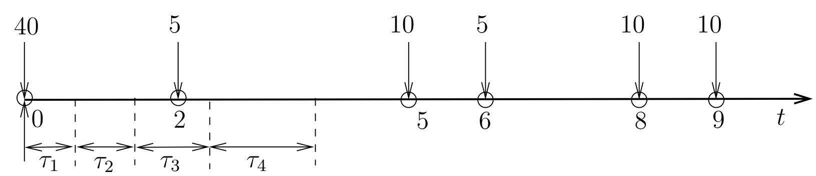


Figure 8.13: The optimal energy allocation $\mathbf{e} = [12.6, 11.8, 10.9, 9.7] \times 10^{-2}$ mJ, with duration $\boldsymbol{\tau} = [0.63, 0.70, 0.82, 1.07] \times 10^{-2}$ s, respectively.

8.6 Conclusions

In this chapter, we investigated the average delay minimization in a general energy harvesting system. Depending on the arrival profiles of the energy and data packets, the average delay minimization problem becomes different. We consider three different scenarios here. In the first scenario, we assume that both packets and energy are ready at the transmitter before the transmission starts. In the second scenario, we assume that packets may arrive during the transmission, while energy is ready at the transmitter before the transmission. These two scenarios correspond to traditional unrechargeable systems. We developed iterative approaches and DP formulation for both scenarios. For the third scenario, we assume that packets are ready before the transmission starts, and energy is harvested during the transmission. We first analyzed the structural properties of the optimal transmission policy, and developed an iterative algorithm and/or DP formulation to obtain the offline scheduling policy.

Chapter 9

Conclusions

In this dissertation, we investigated delay minimization problems in wireless communication channels with energy and power constraints at the transmitters. We combined queueing theory with information theory and designed queue-length based cross-layer transmission policies.

We first studied the average delay minimization problem in a two-user multiple access system, where each transmitter has an average power constraint. We analyzed the trade-off between the average power constraints and the average delay, and proved that the optimal transmission policy has a threshold structure, i.e., if the sum of the queue lengths exceeds a threshold, both users transmit a packet from their queues, and if the sum of the queue lengths is smaller than a threshold, the user with the larger queue length transmits a packet from its queue.

Delay-optimal rate allocation is another important research area in multi-user communications. We first studied the optimal rate allocation policies in a symmetric multiple access channel. We proved that the delay optimal rate allocation policy is to balance the queue lengths in each slot as much as possible. In order to observe the tension between maximizing the current throughput and balancing the queue lengths, we studied the optimal rate allocation policy in a system with a general pentagon rate region. We proved that a switch curve structure exists in the queue state space, and the switch curve has a limit on one of the queue lengths. The optimal policy implies that we can operate the queues *partially distributedly*. It also implies that the system may need to trade sum-rate for balancing queue lengths in order to achieve the optimal delay performance.

Next, we consider communication systems with rechargeable batteries, where the transmitters are able to harvest energy from the nature throughout the duration of the transmissions. We investigated the transmission completion time minimization problem in such systems. We first considered a single-user communication channel with an energy harvesting transmitter. We developed an iterative algorithm, and proved its global optimality. Then, we extended the single-user scenario to a broadcast channel and a multiple access channel. For these two scenarios, we first characterized the maximum departure region for a given deadline T, then, based on the "duality" between the departure region maximization and transmission completion time minimization problems, we simplified the transmission completion time minimization problem into simple single-user problems, and obtained the optimal scheduling policies efficiently.

Finally, we studied the average delay minimization problem in a single-user energy harvesting communication channel. We investigated three different scenarios. For each scenario, we first analyzed the structural properties of the optimal transmission policy, and developed an iterative algorithm and/or dynamic programming formulation to obtain the offline scheduling policy.

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