ABSTRACT

Title of dissertation: DESIGN OF DISCRETE AUCTION

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Doctor of Philosophy, 2010

Dissertation directed by: Professor Peter Cramton
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Chapter 1: Efficient Design of an Auction with Discrete Bid Levels

This paper studies one of auction design issues: the choice of bid levels. Full efficiency is generally unachievable with a discrete auction. Since there may be more than one bidder who submits the same bid, the auction cannot completely sort bidders by valuation. In effort to maximize efficiency, the social planner tries to choose the partition rule—a rule dictating how type space is partitioned to group bidders who submit the same bid together-to maximize efficiency. With the efficient partition rule, we implement bid levels with sealed-bid and clock auctions. We find that the efficient bid levels in the sealed-bid second-price auction may be assigned non-unique bid amounts and efficient bid increments in a clock auction with highest-rejected bid may be decreasing. We also show that revealing demand is efficiency-enhancing even in the independent private valuation setting where price discovery is not important.

Chapter 2: Pricing Rule in a Clock Auction

We analyze a discrete clock auction with lowest-accepted bid (LAB) pricing and provisional winners, as adopted by India for its 3G spectrum auction. In a perfect Bayesian equilibrium, the provisional winner shades her bid while provisional losers do not. Such differential shading leads to inefficiency. An auction with highest-rejected bid (HRB) pricing and exit bids is strategically simple, has no bid shading, and is fully efficient. In addition, it has higher revenues than the LAB auction, assuming profit maximizing bidders. The bid shading in the LAB auction exposes a bidder to the possibility of losing the auction at a price below the bidder's value. Thus, a fear of losing at profitable prices may cause bidders in the LAB auction to bid more aggressively than predicted assuming profit-maximizing bidders. We extend the model by adding an anticipated loser's regret to the payoff function. Revenue from the LAB auction yields higher expected revenue than the HRB auction when bidders' fear of losing at profitable prices is sufficiently strong. This would provide one explanation why India, with an expressed objective of revenue maximization, adopted the LAB auction for its upcoming 3G spectrum auction, rather than the seemingly superior HRB auction.

Chapter 3: Discrete Clock Auctions: An Experimental Study

We analyze the implications of different pricing rules in discrete clock auctions. The two most common pricing rules are highest-rejected bid (HRB) and lowest-accepted bid (LAB). Under HRB, the winners pay the lowest price that clears the market; under LAB, the winners pay the highest price that clears the market. Both the HRB and LAB auctions maximize revenues and are fully efficient in our setting.

Our experimental results indicate that the LAB auction achieves higher revenues. This also is the case in a version of the clock auction with provisional winners. This revenue result may explain the frequent use of LAB pricing. On the other hand, HRB is successful in eliciting true values of the bidders both theoretically and experimentally.

DESIGN OF DISCRETE AUCTION

by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2010

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Preface

Auctions have long been used to trade a variety of goods ranging from collectibles to agricultural products. In the past few decades, governments around the world have been sold publicly-owned assets such as spectrum licenses, carbon permits and offshore oil leases through auctions. Auctions have become more common among individuals since the emergence of internet auction websites where virtually any seller can put an item for sale and buyers can browse through a list of auctioned items. It is undeniable that auctions have become one of the important economic institutions. Undoubtedly, the design of auctions has gained attention from both academics and practitioners.

Bidding behavior depends on an auction format and so does the auction outcome. An auction designer primary role is to tailor auction rules so that the outcome aligns with the auction objective. Typical auction objectives are revenue and efficiency. On one hand, an auction house may focus on maximizing the auction revenue. The auctioneer may set a reserve price that might effectively hinder the efficient allocation (Riley & Samuelson, 1981; Myerson, 1981). The well-known revenue equivalence principle asserts that any standard auction format that allocates the good in similar manner generates the same expected revenue. On the other hand, a government who sells a publicly-owned asset to the private sector typically put an emphasis on allocative efficiency—the good is awarded to the one who values it most. Theoretically, any standard auction format—an auction that awards the good to the one who submits the highest bid—yields the efficient allocation. Since equilibrium bidding strategy is a one-to-one monotonic mapping from valuation, any standard auction will automatically award the good to the highest-valuation bidder.

The classic auction theory seemingly suggests that a choice of auction is irrelevant since any standard auction yields an efficient allocation and the same expected revenue. However, a relaxation of some assumption may make such desirable outcome deviate from the prediction. For example, Milgrom and Weber (1982) show that English and second-price auctions are no longer equivalent if valuations are interdependent as price discovery become relevant. Holt (1980) argues that the revenue of the first-price auction is greater than that of the second-price auction when bidders are risk-averse. Vickrey (1961) states that with asymmetric bidders, the revenue ranking between the first-price and second-price auctions are ambiguous and the allocation of the first-price auction may be inefficient. Che and Gale (1998) show that if bidders face budget constraints, the first-price auction is more efficient and yields higher expected revenue than the second-price auction.

While most auction literature assumes continuous bidding, most auctions in the real world are conducted in discrete manner. Conceptually, a discrete sealed-bid auction allows bidders to submit only bids chosen from a finite set of bid amounts. In the extreme case, bid amounts are restricted to the smallest currency unit. Discrete English auction requires bid amounts to meet a certain minimum which is a discrete jump from the previous round price. A clock price in a discrete clock auction is the previous clock price plus one discrete bid increment. Discrete bidding rounds in dynamic auctions are desirable in practice because of their robustness to communication failures and properties that mitigate tacit collusion (Ausubel & Cramton, 2004). Real-world examples of auctions with discrete bid levels are the spectrum auctions in US, Germany, Norway and so on. In these auctions, minimum allowable bid is implemented. In eBay auction, discrete bid increment varies depending on the current price. The relaxation of continuous bid space brings in an unexplored area of auction design. One of the implications of discrete auction is that the full efficiency is unattainable with a discrete bid levels. While the symmetric equilibrium bid function in the continuous case is one-to-one monotonic mapping from valuation, the symmetric equilibrium bid function in a discrete auction is many-to-one mapping since the number of possible valuations is greater than the number of bid levels. Particularly, there may be one or more bidder who submits the same bid and the auctioneer will treat them similarly. If another bidder submits the same bid as the highest-valuation bidder and wins the tie breaker, the allocation is inefficient.

For example, consider the first-price sealed-bid auction with continuous bid space and the one with ten bid levels. Suppose there are two bidders whose valuations are independently and uniformly distributed between the interval [0,1]. Figure 1 shows the equilibrium bid function in continuous (dashed) and discrete (solid) cases. When bidders are allowed to submit any real number, the bid function is defined B(x)=x/2 where x is valuation. If bidders are allowed to submit bid amounts chosen from a set of ten bid amounts, the equilibrium bid function is a step function. Bidders whose valuations are in the same domain of a step will submit the same bid amount. For instance, if a bidder with valuation of 0.71 and another with valuation of 0.77 will submit the same bid of 0.37. The auctioneer therefore randomly assigns the item to one of them. The auctioneer may misassign the item to the one with

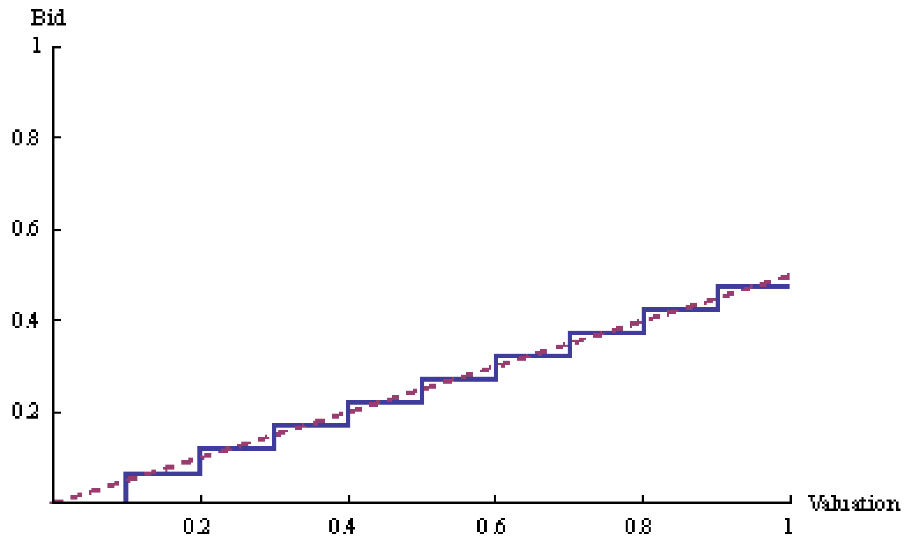

Figure 1: Bid functions in continuous (dashed) and discrete (solid) cases.

valuation 0.71 if she wins the tiebreaker.

In Chapter 1, we derive the maximum attainable efficiency of the standard auction mechanism given the number of bid levels and develop an approach to determine a set of bid levels that achieve such level of efficiency. The efficient discrete auction mechanism can be implemented in various auction formats.

In Chapter 2 and 3, we study a choice of pricing rule in a discrete clock auction. Clock auction is one of the most popular dynamic auctions and it has been used to sell commodities such as gas, electricity, spectrum right, carbon permit, and so on. With discrete bid levels, the pricing rule in an clock auction becomes relevant. While a continuous clock auction yields a unique market clearing price—the price where the marginal bidder drops out, there can be more than one market clearing prices in a discrete clock auction. For example, consider a clock auction where one item is being sold to two bidders (see Figure 2). At a price of 50, both bidders bid but at the price of 60, one bidder drops out and the other bidder bids. Obviously,

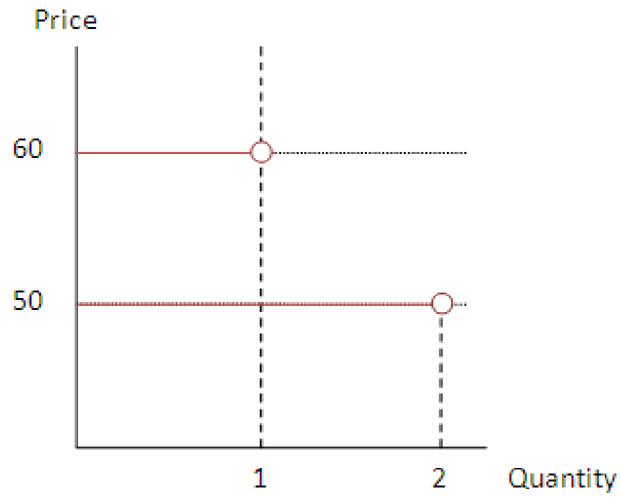

Figure 2: A clock auction with two bidders and one item for sale.

60 is one of the market clearing prices but any price between 50 and 60 can also clear the market. Each pricing rule elicits different bidding behavior and yields different outcome. Chapter 2 and 3 thus compare revenue and efficiency of various pricing rules in a discrete clock auction. The former provides a theoretical prediction whereas the latter shows an experiment evidence.

Dedication

To my parents, Pacharee and Suthirak Sujarittanonta,

and my wife, Pattraporn Tantayotin

Acknowledgments

This dissertation would not have been possible without the help of several people. It is a pleasure to convey my gratitude to them all in my humble acknowledgment.

I would like to thank my advisor, Peter Cramton, for his guidance and support since the early stage of my graduate study. He has introduced me to the fascinating world of auction and market design and taught me a countless number of lessons since then. I am greatly appreciated him giving me invaluable opportunities to work in several spectrum auction projects that have enhanced my understanding of auction design. I learned from his passionate and positive attitude how to enjoy academic life while balancing between research and practice. It is a great honor to work under supervision of such an extraordinary individual.

I gratefully thank Dan Vincent and Larry Ausubel for their theoretical and practical expertise. They have always made themselves available to help and provided constructive comments. This dissertation cannot be complete without them. I would also like to thank the rest of my dissertation committees, Rob Schwab and Subramanian Raghavan, for insightful comments and sparing their valuable time reviewing the manuscript.

A special thanks goes to Erkut Ozbay and Emel Filiz-Ozbay, who brought me to experimental and behavioral economics, for their advice and co-authorship. It has been my pleasure to collaborate with them.

I gratefully acknowledge Allan Ingraham who gave me an opportunity to work on several exciting auction consulting projects.

I would also like to acknowledge help and support from faculties and staffs at the Department of Economics, University of Maryland. Special thank to John Shea, Vickie Fletcher, Terry Davis, Lizzie Martinez and Heather Nalley for all administration matters. Thank to Qyunh Nguyen, Yeon Soo Kim, Jing Zhang and Lingsheng Meng who share up and down of our graduate life. Thank to colleagues at Inforum including Clopper Almon, Jeff Werling, Margaret McCarthy , Doug Meade, Doug Nyhus, Ron Horst, Troy Wittek, Matias Antonio and San Sampattavanija.

I would like to thank all folks at FED for their friendship over the years.

I owe my deepest gratitude to my family. Thank to my father, Suthirak Sujarittanonta, who has always been the one whom I look up to and my mother, Pacharee Sujarittanonta, for her love and care. Thank to my sister, Pachareerak Sujarittanonta, for taking care of our mom during my absence. I owe my thank to Bulpakdi family for their laughters every time we gathered and Sujarittanonta, Limtrakarn and Tantayotin families for their support and belief in me.

I was extremely fortunate to have my wife, Pattraporn Tantayotin, with me since the beginning of my study. I would like to thank her for always standing by me in every step I made.

Finally, I would like to thank everybody who has made this dissertation possible, as well as expressing my apology that I could not mention personally.

Contents

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List of Abbreviations

AWS	Advanced Wireless Services
FCC	Federal Communications Commission
HRB	Highest-rejected Bid
HRBD	Highest-rejected Bid with Demand Disclosure
LAB	Lowest-accepted Bid
LABD	Lowest-accepted Bid with Demand Disclosure
LABx	Lowest-accepted Bid with Exit Bids
PDF	Probability Density Function

Chapter 1

Efficient Design of an Auction with Discrete Bid Levels

1.1 Introduction

Auctions have been predominantly used by governments to sell a wide range of goods such as spectrum licenses, electricity, treasury bills and emission rights. Typically, a government would aim to allocate the good to those who value it most. Although auctions have been regarded as efficient selling mechanisms in literatures, designing efficient auctions is still an immense challenge. Some design issues have been cleared up but others remain unresolved (Cramton, 2009). One issue that has received little attention so far is the choice of discrete bid levels.

While most auction models have assumed that bidders can submit any bid chosen from a continuous interval, bid amounts are normally restricted to discrete bid levels in practice. For example, FCC's AWS and 700 MHz required that a valid bid amount must exceed a designated minimum acceptable bid-one bid increment plus the standing high bid. In the India 3G spectrum auction, a bid increment is a fraction of the previous clock price. Ausubel and Cramton (2004) stated that the motivation for discrete bid levels, as opposed to a continuous increment, is their robustness to communication failure and facilitation of the price discovery. Discrete bidding rounds give bidders time to fix communication issues and incorporate information revealed throughout the auction into their valuation models and bidding strategies.

Despite the practicality of discrete bidding, full efficiency is generally unattainable. Efficiency of the traditional auction model hinges on its ability to sort bidders by valuation. Since a bid function is a one-to-one monotonic mapping from valuation, any standard auction mechanism-an auction that awards the item to the bidder who submitted the highest bid-is fully efficient. On the other hand, a discrete auction can be perceived as a selling mechanism whose message space is a finite set. The type and message spaces are no longer isomorphic. A group of bidders with different valuations may submit the same message. The mechanism cannot rank these bidders and thereby assign them the same probability of trade and expected payment. At worst, the mechanism may misassign the item to another bidder who submits the same message as the highest-valuation bidder.

This paper develops a framework for determining bid levels that maximize efficiency in a single-item auction. In a symmetric equilibrium, the type space can be partitioned into segments such that bidders with valuations within the same segment follow the same bidding strategy. The social planner's problem is to choose a partition rule to maximize efficiency. We employ a mechanism with finite message space to determine the efficient partition rule for static auction. The framework is also applicable to a blind clock auction which is equivalent to a sealed-bid auction. We use a different approach for a dynamic auction with demand disclosure. We find that in a two-bidder case, the efficient partition is finer where a valuation is more likely to lie. With more than two bidders whose valuations are uniformly distributed, the higher segments become smaller.

The efficient partition rule can be implemented with bid levels in many discrete auction formats. In the sealed-bid first-price auction, efficient bid levels are strictly increasing whereas in the second-price auction, bid amounts may be non-unique. We demonstrate numerically that with two bidders whose valuations are uniformly distributed, the efficiencies of an efficient discrete auction and an auction with naively-chosen bid levels converge to the full efficiency level as the number of messages increases but the latter's rate of convergence is slower. The efficient bid levels in a clock auction with demand disclosure and highest-rejected bid can be either increasing or decreasing. One of our key results is that revealing demand improves efficiency even in the independent private valuation setting in which price discovery is irrelevant.

Several papers analyzed bidding behavior in discrete auctions. Chwe (1989) and Mathews and Sengupta (2008) examined the first-price and second-price sealed-bid discrete auctions, respectively. Cramton and Sujarittanonta (2010) compared performances of various pricing rules in ascending clock auctions with discrete bid increments. They also studied clock auctions in which intraround bidding is permitted. Another branch of literature that is closely related to our paper studied optimal discrete bid levels. Rothkopf and Harstad (1994) determined a set of optimal discrete bid levels in an English auction. David et al. (2007) extended the model of Rothkopf and Harstad and found that decreasing bid increments maximize revenue. In contrast to these works, our approach is not specific to any auction format. A few paper studied a mechanism design with a restriction on action space similar to our model. Blumrosen and Feldman (2006) characterized sufficient conditions for dominant-strategy implementability in a mechanism analogous to our model. Blumrosen, Nisan, and Segal (2007) studied an optimal mechanism with bounded action space.

The rest of this paper is organized as follows. In Section 1.2, we outline a mechanism with finite message space which will be used in designing the efficient static auction. We derive an efficient auction mechanism with finite message space in Section 1.3. Then, we implement the mechanism with sealed-bid and blind clock auctions in Section 1.4. In Section 1.5, we design an efficient clock auction with demand disclosure. Conclusion and discussion of future works are in Section 1.6. Missing proofs are presented in Appendix A.

1.2 Mechanism with Finite Message Space

There is one indivisible item for sale. A set of I bidders is denoted by $\mathcal{I} = \{1, 2, \ldots, I\}$. The seller values the item at $x_0 = 0$. Bidder i's private valuation is denoted by $x_i \in X_i$ where $X_i \equiv [\underline{x}_i, \overline{x}_i] \in \mathbb{R}^+$ is bidder i's type space and $\underline{x}_i \geq x_0$ for all $i \in \mathcal{I}$. Bidder i's valuation x_i is distributed according to a probability distribution F_i with associated density function f_i with a support X_i. Bidder i's expected payoff is $x_i q_i - t_i$ where q_i is the probability that bidder i obtains the item and t_i is the payment made to the seller by bidder i.

Let $\mathcal{M}_i \equiv \{m_{i0}, m_{i1}, \ldots, m_{i,M_i-1}\}$ be a finite set of messages available to bidder i where M_i is the number of messages. The vector of the numbers of messages $\mathbf{M} \equiv (M_1, M_2, \ldots, M_I)$ is exogenously given. Note that the messages space is not necessary orderable. For example, the message space can be $\{red, green, blue\}$ where each message corresponds with different probability of trade and expected payment. However, it is a common practice to name a message with an associated bid amount.

All bidders learn their types and submit reports chosen from their message spaces to the mechanism simultaneously. Let $\mathbf{r} \equiv (r_1, r_2, \ldots, r_I)$ be a vector of reports where $r_i \in M_i$ is bidder i's report. The mechanism with finite message space can be defined as follows.

Definition 1.1. *A mechanism with finite message space $\langle \mathbf{Q}, \mathbf{T} \rangle$ consists of a pair of functions: (1) allocation rule $\mathbf{Q} : \mathcal{M} \to [0,1]^I$ where $\sum_{i \in \mathcal{I}} Q_i r_i \leq 1$ and (2) payment rule $\mathbf{T} : \mathcal{M} \to \mathbb{R}^I$.*

In contrast to the traditional selling mechanism, allocation and payment rules are mappings from a finite set instead of a compact interval into outcomes. If bidder i reports r_i and others report $\mathbf{r}_{-i}$, bidder i's expected payoff is

$$U_i(r_i, \mathbf{r}_{-i}) = x_i Q_i(r_i, \mathbf{r}_{-i}) - T_i(r_i, \mathbf{r}_{-i})$$

We can define a direct mechanism with finite message space as follows.

Definition 1.2. *Define a partition rule as $\boldsymbol{\mu} : \mathbf{X} \to \mathcal{M}$. Direct mechanism with finite message space, $\langle \tilde{\mathbf{Q}}, \tilde{\mathbf{T}} \rangle \equiv \langle \mathbf{Q}, \mathbf{T}, \boldsymbol{\mu} \rangle$, consists of a pair of composite functions: (1) direct allocation rule $\tilde{\mathbf{Q}} = \mathbf{Q} \circ \boldsymbol{\mu}$ and (2) direct payment rule $\tilde{\mathbf{T}} = \mathbf{T} \circ \boldsymbol{\mu}$. There exists a direct mechanism with finite message space $\langle \tilde{\mathbf{Q}}, \tilde{\mathbf{T}} \rangle$ such that, in an equilibrium, each bidder reports her type truthfully and the outcome is identical to* *that of a mechanism with finite message space* $\langle \mathbf{Q}, \mathbf{T} \rangle$.

The direct mechanism with finite message space should not be confused with the traditional direct mechanism. Although bidders report their valuations truthfully, the valuations are first mapped into messages via the partition rule $\boldsymbol{\mu}$ and then these messages will be used to determine the outcome. The mechanism cannot handle the valuations directly.

If bidder i reports z_i and others report $\mathbf{x}_{-i}$, bidder i's expected payoff is

$$\begin{aligned} \tilde{U}_i(z_i, \mathbf{x}_{-i}) &= x_i Q_i(\mu_i(z_i), \boldsymbol{\mu}_{-i}(\mathbf{x}_{-i})) - T_i(\mu_i(z_i), \boldsymbol{\mu}_{-i}(\mathbf{x}_{-i})) \\ &= x_i \tilde{Q}_i(z_i, \mathbf{x}_{-i}) - \tilde{T}_i(z_i, \mathbf{x}_{-i}) \end{aligned}$$

If other bidders report truthfully, bidder i's expected payoff is

$$\tilde{q}_i(z_i)x_i - \tilde{t}_i(z_i)$$

where $\tilde{q}_i(z_i) = \int_{\mathbf{x}_{-i}} \tilde{Q}_i(z_i, \mathbf{x}_{-i})\mathbf{dF}_{-i}(\mathbf{x}_{-i})$ and $\tilde{t}_i(z_i) = \int_{\mathbf{x}_{-i}} \tilde{T}_i(z_i, \mathbf{x}_{-i})\mathbf{dF}_{-i}(\mathbf{x}_{-i})$. The incentive compatibility condition is characterized in Definition 1.3 and its implication is discussed in Proposition 1.1.

Definition 1.3. *The direct mechanism with finite message space is (Bayesian) incentive compatible if and only if* $\tilde{u}_i(x_i) \geq \tilde{q}_i(z_i)x_i - \tilde{t}_i(z_i)$ *for all* $z_i \in \mathcal{M}_i, x_i \in X_i$ *and* $i \in \mathcal{I}$ *where* $\tilde{u}_i(x_i) \equiv \tilde{q}_i(x_i)x_i - \tilde{t}_i(x_i)$ *is the equilibrium payoff function. Moreover, the incentive compatibility implies that* $\tilde{u}_i(x_i) = \max_{z_i \in X_i} \tilde{q}_i(z_i)x_i - \tilde{t}_i(z_i)$.

Proposition 1.1. *If a direct mechanism with finite message space* $\langle \mathbf{Q}, \mathbf{T}, \boldsymbol{\mu} \rangle$ *is incentive compatible, all* $i \in \mathcal{I}$, $\tilde{q}_i(x_i)$ *is non-decreasing in* x_i. *Specifically, for* $m_{is}, m_{it} \in \mathcal{M}_i$, $\alpha, \beta \in X_i$ *and* $\alpha > \beta$, $q_i(m_{is}) \geq q_i(m_{it})$ *where* $\mu_i(\alpha) = m_{is}$, $\mu_i(\beta) = m_{it}$ *and* $s \neq t$.

Proposition simply states that in an incentive compatible mechanism, a bidder with higher valuation will report a message with higher priority. The partition rule in a Bayesian incentive compatible mechanism is equivalent to a Bayesian equilibrium strategy profile.

Proposition 1.2. *If a direct mechanism* $\langle \mathbf{Q}, \mathbf{T}, \boldsymbol{\mu} \rangle$ *is incentive compatible, the expected payment is, for all* $x_i \in X_i$ *and* $i \in \mathcal{I}$,

$$\tilde{t}_i(x_i) = \tilde{q}_i(x_i)x_i - \int_{\underline{x}_i}^{x_i} \tilde{q}_i(z)\mathrm{d}z - \tilde{u}_i(\underline{x}_i).$$

(Revenue Equivalence Principle) *The expected payments in two incentive compatible mechanisms with finite message space are equivalent up to a constant if their allocation and partition rules are identical.*

In the traditional mechanism design, identical allocation rules imply revenue equivalence. In contrast, the expected payment in a mechanism with finite message space depends on both allocation and partition rules that together determine the actual allocation. Two distinct standard auction formats with the same set of bid levels does not imply revenue equivalence since bidding behavior, and thereby partition rule, may be different.

To simplify the analysis without loss of generality, we reorder messages such that a message with a higher index number yields a higher probability of trade.

Definition 1.4. *An ordered finite message space is a message space* $\mathcal{M}$ *such that* $q_i(m_{ij}) \geq q_i(m_{ik})$ *for all* $i \in \mathcal{I}$ *and* $j > k$*. Without loss of generality, any message space can be re-indexed into an ordered finite message space.*

Proposition 1.3. *Consider an incentive compatible direct mechanism with ordered finite message space* $\langle \mathbf{Q}, \mathbf{T}, \boldsymbol{\mu} \rangle$*. A partition rule* $\boldsymbol{\mu}$ *is characterized by a partition vector* $\hat{\mathbf{x}}_i \equiv (\hat{x}_{i0}, \hat{x}_{i1}, \ldots, \hat{x}_{iM_i})$ *for all* $i \in \mathcal{I}$ *such that if* $x_i \in [\hat{x}_{ij}, \hat{x}_{i,j+1})$*,* $\mu_i(x_i) = m_{ij}$ *where* $j = 0, 1, \ldots, M_i - 1$*,* $\hat{x}_{i0} = \underline{x}_i$*,* $x_{iM_i} = x_i$ *and* $\mu_i(x_i) = m_{i,M_i-1}$*. Therefore,* $\tilde{q}_i(x_i)$ *is a non-decreasing step function with* M_i *steps such that the k-th step lies upon the interval* $[\hat{x}_{i,k-1}, \hat{x}_{ik})$ *with the step height of* $q_i(m_{i,k-1})$ *and the height at* $\overline{x}_i$ *is* $q_i(m_{i,M_i-1})$*.*

The partition vector divides the type space into a number of convex segments. Bidders whose valuations lie within the same segment will submit the same message and thereby obtain the same probability of trade. Thus, the probability of trade in each segment is a constant function constituting a step with a size of the segment. Since the incentive compatibility implies that a bidder with higher valuation prefers a higher-priority message, the probability of trade can be represented as a non-decreasing step function.

To state Proposition 1.3 differently, a symmetric Bayesian equilibrium of an incentive compatible mechanism with ordered finite message space $\langle \mathbf{Q}, \mathbf{T} \rangle$ is fully characterized by the partition rule. If $x_i \in [\hat{x}_{ij}, \hat{x}_{i,j+1})$, bidder i submits a message

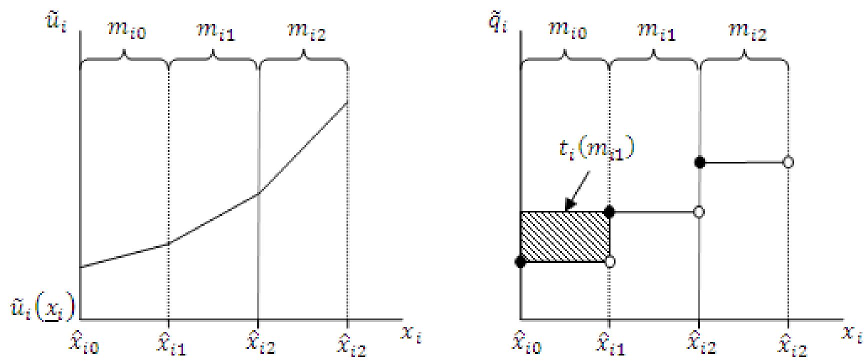

Figure 1.1: Expected utility, probability of trade and expected payment in a mechanism with ordered finite message space.

m_{ij}. A partition $\hat{x}_{ij}$ can be interpreted as a valuation of the bidder who is indifferent between reporting $m_{i,j-1}$ and m_{ij}. We however assume that she always submits the higher-priority message.

Given the allocation and partition rules, the payment rule can be rewritten as shown in Proposition 1.4.

Proposition 1.4. *The expected payment in an incentive compatible mechanism with ordered finite message space* $\langle \mathbf{Q}, \mathbf{T}, \boldsymbol{\mu} \rangle$ *is given by*

$$t_i(m_j) = \sum_{k=1}^{j} \big(q_i(m_{ik}) - q_i(m_{i,k-1})\big)\hat{x}_{ik} - \tilde{u}_i(\underline{x}_i)$$

Bidder i's expected utility, probability of trade and expected payment are shown in Figure 1.1. The expected utility is a convex function with a kink at each partition. The slope of expected utility (Figure 1.1 left) which is equal to the probability of trade (Figure 1.1 right) is constant within each segment. The expected payment when reporting m_j is the intersection between the areas above the probability of trade function and below the horizontal line with the Y-coordinate equal to $q_i(m_j)$. The intersection within the segment associated with the report is empty since the probability of trade function is also horizontal. Therefore, the expected payment is the area the intersection between the areas above the probability of trade function and below the horizontal line with the Y-coordinate equal to $q_i(m_{j-1})$ as shown in the shaded area in Figure 1.1.

1.3 Efficient Auction Mechanism with Finite Message Space

By the nondiscriminatory characteristics of an auction, all message spaces are identical. Specifically, $\mathcal{M}_i = \mathcal{M} \equiv \{m_0, m_1, \ldots, m_{M-1}\}$ for all $i \in \mathcal{I}$. In addition, the allocation rules as well as payment rules are the same for all bidders. That is, $Q_i(\mathbf{r}) = Q(r_i, \mathbf{r}_{-i})$, $T_i(r) = T(r_i, \mathbf{r}_{-i})$, $q_i(r_i) = q(r_i)$ and $t_i(r_i) = t(r_i)$ for all $i \in \mathcal{I}$ and $r_i \in \mathcal{M}$. To simplify the analysis, we assume that bidders are ex ante symmetric. That is, for all $i \in \mathcal{I}$, bidder i's valuation is independently distributed on the interval $[\underline{x}, \overline{x}]$ with associated distribution function F and density function f. The partition vector is redefined as $\hat{\mathbf{x}} = (\hat{x}_0, \hat{x}_1, \ldots, \hat{x}_M)$ where $\hat{x}_0 = \underline{x}$ and $\hat{x}_M = \overline{x}$. The partition vector is universal to all bidders. If $x_i \in [\hat{x}_j, \hat{x}_{j+1})$, $\mu(x_i) = m_j$.

In an ordered finite message space, a message with a higher index number implies a higher priority. Therefore, define a priority function $S : M \rightarrow \{0, 1, \ldots, M-1\}$ such that $S(m_j) = j$. The allocation rule of the symmetric auction mechanism with finite message space is defined as follows.

Definition 1.5. *A symmetric auction mechanism with ordered finite message space* *is a feasible and incentive compatible mechanism with ordered finite message space* $\langle \mathbf{Q}, \mathbf{T} \rangle$ *where* $\mathbf{Q} : \mathcal{M}^I \to [0,1]^I$ *and* $\mathbf{T} : \mathcal{M}^I \to \mathbb{R}^I$ *such that for every* $\mathbf{r} \in \mathcal{M}^I$*,*

$$Q_i(\mathbf{r}) \equiv Q(r_i, \mathbf{r}_{-i}) = \begin{cases} 0 & \textit{if } S(r_i) < \max_{j \neq i} S(r_j) \\ 1 & \textit{if } S(r_i) > \max_{j \neq i} S(r_j) \\ \frac{1}{k} & \textit{if } S(r_i) = \max_{j \neq i} S(r_j) \end{cases}$$

where the number of bidders who submits the message with the highest priority $k = \Big| \{ r_l | S(r_l) = \max_{j \in \mathcal{I}} S(r_j) \text{ and } l \in \mathcal{I} \} \Big|$

The mechanism allocates the item to the bidder who submits the highest-priority message. If there is a tie, the item is awarded randomly to one of the high bidders. With the predetermined allocation rule, we can define the probability of trade and expected payment as follows.

Proposition 1.5. *Consider an auction mechanism with ordered finite message space* $\langle \mathbf{Q}, \mathbf{T} \rangle$*. For each* $m_j \in \mathcal{M}$*, the probability of trade and expected payment*[1] *when reporting* m_j *are*

$$\begin{aligned} q(m_j) &= F(\hat{x}_{j-1})^{I-1} + \sum_{k=1}^{I-1} \frac{1}{k+1} \binom{I-1}{k} \big(F(\hat{x}_j) - F(\hat{x}_{j-1})\big)^k F(\hat{x}_{j-1})^{I-k-1} \\ &= F(\hat{x}_j)^{I-1} + \frac{1}{I\big(F(\hat{x}_{j+1}) - F(\hat{x}_j)\big)} \\ &\quad \cdot \Big[F(\hat{x}_{j+1})^I - I\big(F(\hat{x}_{j+1}) - F(\hat{x}_j)\big) F(\hat{x}_j)^{I-1} - F(\hat{x}_j)^I \Big] \end{aligned}$$

[1] Let $\mathcal{K}_k \equiv \big\{ \mathcal{K} | \mathcal{K} \subset \mathcal{I} \setminus \{i\} \text{ and } |\mathcal{K}| = k \big\}$ be a set of all possible set of k bidders. Without the assumption of ex ante symmetric bidders, bidder i's probability of trade and expected payment are, for each $m_j \in \mathcal{M}$, $q(m_j) = \prod_{j \in \mathcal{I} \setminus \{i\}} F(\hat{x}_{ij}) + \sum_{k=1}^{I-1} \sum_{\mathcal{L} \in \mathcal{K}_k} \frac{1}{k+1} \binom{I-1}{k} \Big(\prod_{j \in \mathcal{L}} \big(F(\hat{x}_{i,j+1}) - F(x_{ij})\big) \prod_{j \in \mathcal{I} \setminus \mathcal{L}} F(\hat{x}_j) \Big)$ and $t_i(m_j) = t(m_j) = \sum_{k=1}^{j} \big(q(m_k) - q(m_{k-1})\big) \hat{x}_{ik} - u_i(\underline{x}_i)$, respectively.

and

$$t(m_j) = \sum_{k=1}^{j} \big(q(m_k) - q(m_{k-1})\big)\hat{x}_k - u_i(\underline{x}).$$

The first term in the probability of trade is a probability that all bidders except bidder i submit messages with a priority lower than m_j. The second term is a summation of probability that k bidders submit m_j and $I-k-1$ bidders submit messages with a priority lower than m_j for $k = 1, 2, \ldots, I-1$. In this case, bidder i wins the tiebreaker with probability of $1/(k+1)$. The expected payment is similar to the one in Proposition 1.4 except that the messages are not bidder-specific.

The social planner's objective is to maximize efficiency. We consider an expected gain from trade as the social-choice function[2] defined as follows,

$$\Phi(\mathbf{x}, \mathbf{r}) = \mathbf{x} \cdot \mathbf{Q}(\mathbf{r})$$

The social-choice function is implementable in a dominant strategy since the sufficient conditions as characterized in Corollary 1 in Blumrosen and Feldman (2006) are satisfied. That is, the valuation function, $v_i(x_i, \mathbf{r}) \equiv x_i Q_i(\mathbf{r})$, is single crossing and linear in x_i for all $i \in \mathcal{I}$ and $\mathbf{r} \in \mathcal{M}$.

Since the allocation rule is predetermined, the ex ante expected gain from

[2] An alternative social-choice function is a probability that the allocation is inefficient. One criticism for this social-choice function is that it ignores the magnitude of the loss in gain from trade.

trade[3] is a function of a partition vector defined as follows.

$$\varphi(\hat{\mathbf{x}}) = \sum_{j=0}^{M-1} \sum_{k=1}^{I} \binom{I}{k} \big(F(x_{j+1}) - F(x_j)\big)^k F(\hat{x}_j)^{I-k} \frac{\int_{\hat{x}_j}^{\hat{x}_{j+1}} x \mathrm{d}F(x)}{F(\hat{x}_{j+1}) - F(\hat{x}_j)}$$

The equation is the sum of the probability that the highest valuation is in $[\hat{x}_j, \hat{x}_{j+1})$ multiplied by the expected highest valuation. The expected gain from trade depends only on the partition rule. Proposition 1.6 is immediately follows.

Proposition 1.6. (Efficiency Equivalence Principle) *Any two standard auctions with the same partition rule are equally efficient.*

The social planner problem is to choose the partition rule that maximize the social-choice function. Specifically, the social planner's problem is $\max_{\hat{x} \in \mathbf{X}} \varphi(\hat{\mathbf{x}})$ subject to $\underline{x} = \hat{x}_0 \leq \hat{x}_1 \leq \cdots \leq \hat{x}_M = \overline{x}$. A partition vector can be derived by solving $M+1$ equations, $\nabla \varphi = \mathbf{0}$, for $M+1$ unknowns $\hat{\mathbf{x}}$. Proposition states the relationship between efficiency and the number of bid levels.

Proposition 1.7. *Efficiency of an auction with discrete bid levels is weakly increasing in the number of messages.*

The social planner's problem is basically an optimization with a constrained choice set. The optimization problem with M messages is equivalent to the one with $M+1$ messages where one of constraints is $\hat{x}_{M-1} = \hat{x}_M$. If this constraint is relaxed

[3] In asymmetric case, the expected gain from trade can be written as

$$\varphi(\hat{\mathbf{X}}) = \sum_{j=0}^{M-1} \sum_{k=1}^{I} \sum_{\mathcal{L} \in \mathcal{K}_k} \left(\frac{1}{I}\right)^k \left(\prod_{j \in \mathcal{L}} \left(F(\hat{x}_{i,j+1}) - F(\hat{x}_{i,j})\right) \prod_{j \in \mathcal{I} \setminus \mathcal{L}} F(\hat{x}_j) \right) \mathrm{E}\big[x | x \in [\hat{x}_j, \hat{x}_{j+1})\big]$$

by permitting an additional message, the higher value of the social-choice function can be achieved. As the number of messages increases, the expected gain from trade becomes higher. If there are an infinite number of messages, the partition rule maps valuation into itself. Thus, the mechanism is able to sort bidders directly by their valuations and separate the bidder with highest valuation from others. The outcome is therefore fully efficient.

By differentiating the expected gain from trade with respect to each partition, we can derive the first-order conditions given in Proposition 1.8.

Proposition 1.8. *Efficient partition vector* $\hat{\mathbf{x}}$ *satisfies, for* $j = 0, 1, \ldots, M-1$,

$$IF(\hat{x}_j)^{I-1}\big(\mathrm{E}(x|x \in [\hat{x}_j, \hat{x}_{j+1})) - \mathrm{E}(x|x \in [\hat{x}_{j-1}, \hat{x}_j))\big) =$$

$$\lambda(\hat{x}_{j-1}, x_j)\big(\hat{x}_j - \mathrm{E}(x|x \in [\hat{x}_{j-1}, x_j))\big) + \lambda(\hat{x}_j, \hat{x}_{j+1})\big(\mathrm{E}(x|x \in [\hat{x}_j, x_{j+1})) - \hat{x}_j\big)$$

where $\lambda(a, b) = \frac{F(b)^I - F(a)^I}{F(b) - F(a)}$ *if* $b > a$ *and zero otherwise.*

Instead of analyzing the first-order condition directly, we make further assumptions in Corollary 1.1 and Corollary 1.2 to gain insights into characteristics of the efficient partitions.

Corollary 1.1. *If valuations are uniformly distributed, the efficient partition vector* $\hat{\mathbf{x}}$ *satisfies* $\hat{x}_j = \left(\frac{\hat{x}_{j+1} - \hat{x}_{j-1}}{I}\right)^{\frac{1}{I-1}}$ *for* $j = 1, 2, \ldots, M-1$. *When there are more than two bidders, the size of segment is strictly decreasing. That is,* $\hat{x}_j - \hat{x}_{j-1} > \hat{x}_{j+1} - \hat{x}_j$ *for* $j = 1, 2, \ldots, M-1$.

Figure 1.2 shows efficient partitions with ten messages when the number of

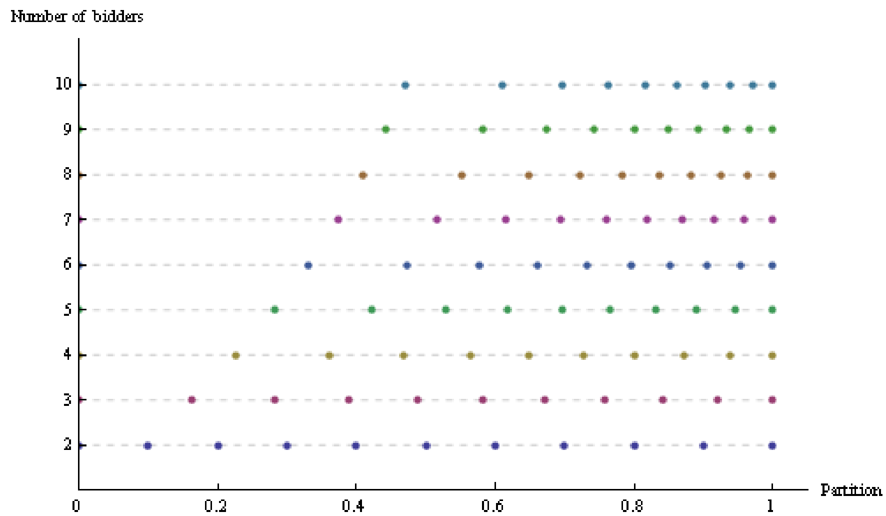

Figure 1.2: Efficient partition with 10 messages and uniform distribution when the number of bidders is two to ten.

bidders is two to ten. The efficient partition becomes finer in a higher segment as the number of bidders increases. Recall that the expected gain from trade is the sum over all segment j of the products between the probability that the highest valuation is in the segment j and the expected valuation of segment j. The probability that the highest valuation is in the segment j can be thought as a weight. The weights of the higher segments become larger as the number of bidders increases (similar to the negatively-skewed PDF of the first-order statistic). The social planner therefore squeezes the higher segments to the right to raise their expected valuations.

Corollary 1.2. *If there are two bidders, the efficient partition vector* $\hat{\mathbf{x}}$ *satisfies* $\hat{x}_j = \mathrm{E}[x | x \in [\hat{x}_{j-1}, \hat{x}_{j+1})]$ *for* $j = 1, 2, \ldots, M-1$.

In an interval $[\hat{x}_{j-1}, \hat{x}_{j+1})$, if the mass of bidders leans more to the right, the partition value $\hat{x}_j$ shifts to the right as well. As a result, a segment located in a denser portion of the type space is smaller. Figure 1.4 shows the case in which there

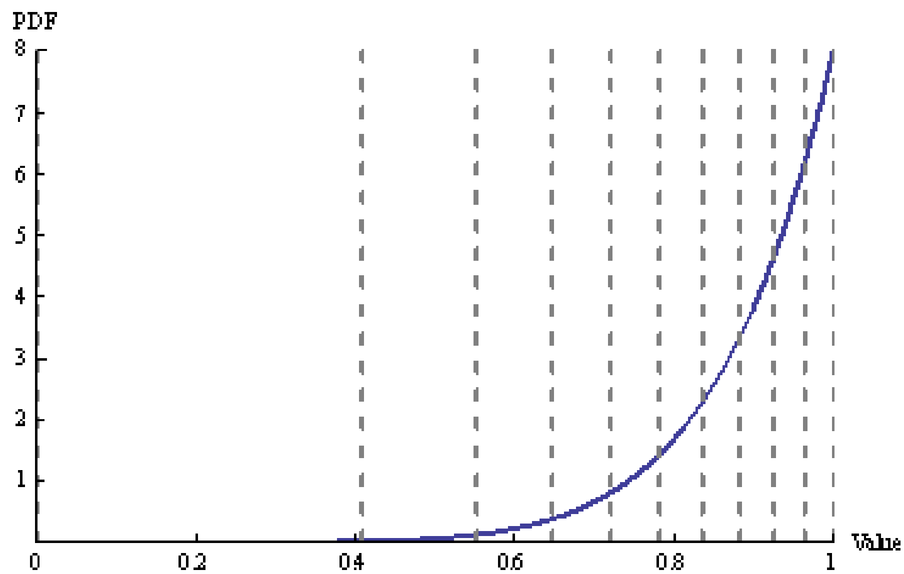

Figure 1.3: PDF of the first-order statistic (solid) and the efficient partition (dashed) when there are ten messages and eight bidders whose valuations are uniformly distributed on the interval $[0, 1]$.

are ten messages and two bidders whose valuations are distributed according to the Beta distribution with $\alpha = \beta = 2$. Since it is more likely that the highest valuation lies in the middle of the type space, the efficient partition is finer in the middle and becomes coarser further away.

1.4 Implementation

With the same number of messages, number of bidders and type space, the efficient partition rule is applicable to implementation of any auction format. Each auction format nonetheless requires a different set of bid levels to achieve the efficient partition rule. Ultimately, any efficient discrete auction format in an identical setting yields the same efficiency level.

In this section, we implement the efficient discrete auction mechanism with

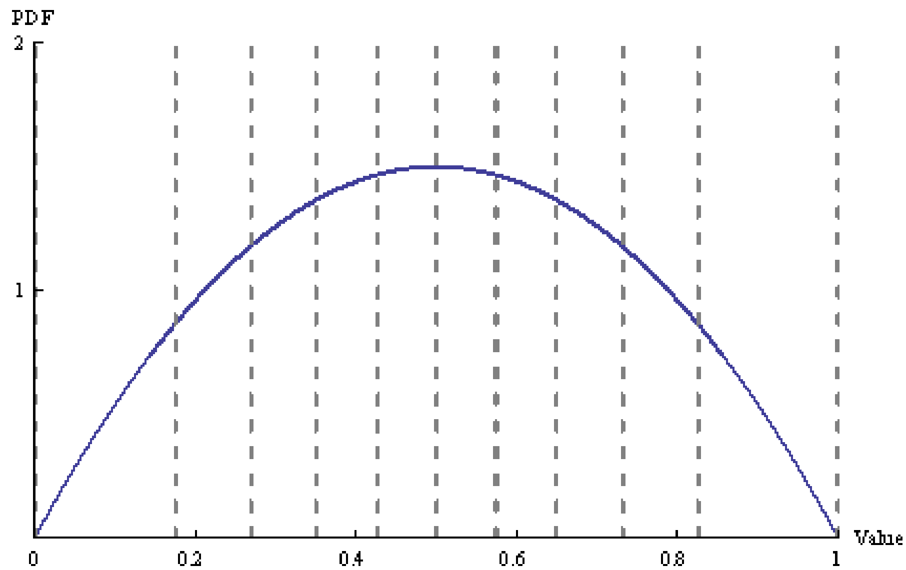

Figure 1.4: PDF of the first-order statistic (solid) and the efficient partition (dashed) when there are ten messages and two bidders whose valuation are distributed according to the Beta distribution with $\alpha = \beta = 2$.

various auction formats. Once the efficient partition rule is derived as demonstrated in the previous section, the probability of trade and expected payment are immediately identified. We can use them to determine the actual bid levels. To simplify the analysis, assume that valuations are independently distributed on $[0, 1]$. We first discuss about an implementation of sealed-bid auctions and then turn our attention to clock auctions.

1.4.1 Sealed-bid Auctions

In a sealed-bid auction, each bidder simultaneously selects a bid amount from available discrete bid levels. The item is awarded to the bidder who submits the highest bid. If there is a tie, the item is randomly assigned to one of the high bidders.

Let $\mathbf{p} \equiv (p_0, p_1, \dots, p_{P-1})$ be a vector of bid levels such that $p_0 \leq p_1 \leq \cdots \leq p_{P-1}$ where, for $i = 0, 1, \dots, P-1$, p_i is a bid amount and P is the number of bid levels. Clearly, the number of available messages M is equal to the number of bid levels P. Since higher bid amount implies higher priority, submitting a bid p_j corresponds to reporting a message m_j in an ordered message space. In an equilibrium, a bidder with valuation $x \in [\hat{x}_j, \hat{x}_{j+1})$ submits a sealed bid of p_j. We define the efficient bid levels in the first-price and second-price sealed-bid auctions as follows.

Definition 1.6. *The efficient bid levels of the first-price auction with P bid levels, $\mathbf{p}^I \equiv (p_0^I, p_1^I, \dots, p_{M-1}^I)$ are defined by, for $j = 0, 1, \dots, M-1$,*

$$p_j^I = \frac{t(m_j)}{q(m_j)}$$

The efficient bid levels of the second-price auction with P bid levels, $\mathbf{p}^{II} \equiv (p_0^{II}, p_1^{II}, \dots, p_{M-1}^{II})$, are defined by, for $j = 0, 1, \dots, M-1$,

$$\begin{aligned} t(m_j) &= p_j^{II} \sum_{k=1}^{I-1} \frac{1}{k+1} \binom{I-1}{k} \big(F(\hat{x}_{j+1}) - F(\hat{x}_j)\big)^k F(\hat{x}_j)^{I-1-k} \\ &+ \sum_{l=0}^{j-1} \sum_{k=1}^{I-1} \binom{I-1}{k} \big(F(\hat{x}_{l+1}) - F(\hat{x}_l)\big)^k F(\hat{x}_l)^{I-1-k} p_l^{II} \end{aligned}$$

In both auctions, the individual rationality condition implies that $p_0^I = p_0^{II} = 0$.

In the first-price auction, the winner pays her bid. Therefore, the expected payment is simply the product of probability of trade and bid amount. Since the lowest valuation is zero but the probability of trade of the lowest-priority message

m_0 is strictly positive. The individual rationality condition implies that $0 \cdot q(m_0) - t(m_0) \geq 0$. So, $t(m_0) = 0$ and thus $p_0^I = 0$. Each bid level can be calculated by solving the corresponding equation.

In the second-price auction, the payment when bidding p_j^{II} can be any price between p_0^{II} and p_j^{II} depending on the second-highest bidder. The first term is the expected payoff when one or more opponents submitted p_j^{II} where k is the number of opponents who submitted p_j^{II}. A high bidder is awarded the item with a probability $1/(k+1)$. The second term is the expected payment when the second-highest bid is less than p_j. The second-highest bid and the number of second-highest bidders are indexed by l and k, respectively. Individual rationality implies that $0 \cdot q(m_0) - t(m_0) = -p_0^{II} \cdot F(\hat{x}_1)^{I-1}/I \geq 0$. Therefore, $p_0^{II} = 0$. The efficient bid levels are characterized by M equations where the j-th equation has j unknowns, p_1^{II} to p_j^{II}. The efficient bid levels can be solved iteratively from zeroth to $(M-1)$-th equation.

Corollary 1.3. *The bid amounts in the efficient first-price auction are strictly increasing whereas those in the efficient second-price auction may be non-unique.*

Corollary suggested that bid amounts assigned to two different bid levels may be the same in the second-price auction. However, their priorities are different, and so are the probability of trade and expected payment. For instance, given a vector of bid levels $(0, 1/3, 1/3, 2/3, 2/3, 1)$, a bidder who submits the second $1/3$ will win over a bidder who submits the first $1/3$. When this feature is to be implemented in practice, two bid levels with the same bid amounts should be distinguishable.

In addition to bid amounts, we can designate each bid level with lexicographic indicators such as $1/3^a$ and $1/3^b$ where the bid level with higher lexicographic order has a higher priority. Another way to distinguish identical bid amounts is to use a time stamp to express priority. For example, if the auction is open for one hour, a bid submitted during the first thirty minutes has a higher priority than the same bid submitted during the last thirty minutes.

Intuitively, some bidders may bid above their valuations and some others may shade. Consider an auction with two uniform bidders and bid levels are $(0, 1/3^a, 1/3^b, 2/3^a, 2/3^b, 1)$. A bidder with a valuation sufficiently close to 1/3 from the left may find bidding 1/3 is more profitable in expectation than bidding zero since her opponent may bid zero so that she will receive a positive profit without having to face a tiebreaker. However, she risks winning the item at an unaffordable price if her opponent also bids 1/3. The same bidder may find $1/3^b$ less profitable than $1/3^a$ because the former's probability of winning at an unaffordable price is too high. Some other bidders with valuations close to 1/3 from the right may find that shading bid to $1/3^b$ is more profitable than $1/3^a$ and $2/3^a$. We show an example in Section 1.4.3.

1.4.2 Blind Ascending Clock Auctions

An ascending clock auction is one of the popular dynamic auction formats. In a clock auction with a single item for sale, the auctioneer announces a current round price and then asks bidders whether they want to bid or exit at the round price. If there is excess demand, the auctioneer increases the price by one increment. This process continues until there is no excess demand. The item is awarded to the active bidder. If there is excess supply, the item will be randomly awarded to the active bidders in the previous round. The auction starts at round 1. If the auction reaches the final round, the auction automatically ends and the item will be randomly awarded to one of active bidders. We also impose the monotonic activity rule—once a bidder exits, she cannot bid again in the subsequent rounds.

With discrete bid levels, there can be many market clearing prices. We consider two popular pricing rules: highest-rejected bid (HRB) and lowest-accepted bid (LAB). If there is only one active bidder in round t, HRB is the price in round $t-1$ and the LAB is the price in round t. When there is excess supply in round t, both HRB and LAB are equal to the price in round $t-1$.

With a blind clock feature, no information is revealed after each round. A blind clock auction is isomorphic to a sealed-bid auction. Specifically, exiting in round t is equivalent to submitting a proxy bid equal to the price in round $t-1$. Hence, a bidder is to choose an exit round. Similar to a sealed-bid auction, the numbers of messages and the number of bid levels are equal. HRB is equivalent to the second-price and thus their efficient bid levels are identical. We will not repeat the same analysis and characterize only a blind clock auction with LAB.

Definition 1.7. *The efficient bid levels of the blind clock auction with highest-rejected bid (HRB auction),* $\mathbf{p}^H \equiv (p_0^H, p_1^H, \ldots, p_{M-1}^H)$*, are the same as those of the second-price auction. The efficient bid levels of the blind clock auction with* *lowest-accepted bid (LAB auction),* $\mathbf{p}^L \equiv (p_0^L, p_1^L, \ldots, p_{M-1}^L)$*, are defined by, for* $j = 1, 2, \ldots, M-1$,

$$\begin{aligned} t(m_j) &= p_j^L \sum_{k=1}^{I-1} \frac{1}{k+1} \binom{I-1}{k} \big(F(\hat{x}_{j+1}) - F(\hat{x}_j)\big)^k F(\hat{x}_j)^{I-1-k} \\ &+ \sum_{l=0}^{j-1} \sum_{k=1}^{I-1} \binom{I-1}{k} \big(F(\hat{x}_{l+1}) - F(\hat{x}_l)\big)^k F(\hat{x}_l)^{I-1-k} p_{l+1}^L \end{aligned}$$

The individual rationality condition implies that the initial condition is $p_0^L = 0$*.*

The conditions are similar to those of the second-price auction except that the final price is the bid level above the second-highest bid instead of the one below. Thus, p_l^L in the second term is replaced by p_{l+1}^L. Individual rationality implies that $0 \cdot q(m_0) - t(m_0) = -p_0^L \cdot F(\hat{x}_1)^{I-1}/I \geq 0$. Therefore, $p_0^L = 0$. We can solve the system of equations iteratively in a manner similar to the second-price auction.

1.4.3 Analytical solution: two uniform bidders

In this section, assume that there are two bidders whose valuations are independently and uniformly distributed on $[0,1]$. According to Corollary 1, the efficient partitions are defined by

$$\hat{x}_j = \frac{\hat{x}_{j+1} + \hat{x}_{j-1}}{2}$$

for $j = 1, 2, \ldots, M-2$. It implies that segments are equally spaced. Therefore, $\hat{x}_j = j/M$ for $j = 0, 1, \ldots, M-1$. Substituting the efficient partition into the probability of trade and expected payment yields, for $j = 0, 1, \ldots, M-1$,

$$\begin{aligned} q(m_j) &= F(\hat{x}_j) + \frac{1}{2}\big(F(\hat{x}_{j+1}) - F(\hat{x}_j)\big) = \frac{2j+1}{2M} \\ t(m_j) &= \sum_{k=1}^{j} \big(q(m_k) - q(m_{k-1})\big)\hat{x}_k = \sum_{k=1}^{j} \frac{k}{M^2} = \frac{j(j+1)}{2M^2} \end{aligned}$$

In the first-price auction, $t(m_j) = p_j^I q(m_j)$ where p_j^I is j-th price level. Therefore,

$$p_j^I = \frac{j(j+1)}{(2j+1)M}$$

Since $\Delta_j = p_{j+1}^I - p_j^I = \frac{2(1+j)^2}{(4j^2+8j+3)M} > 0$ and $\Delta_{j+1} - \Delta_j = -\frac{2}{(8j^3+36j^2+46j+15)M} < 0$, the efficient bid increment is strictly decreasing in size.

In the second-price auction, the efficient bid levels are defined by, for $j = 1, 2, \ldots, M-1$,

$$t(m_j) = \frac{1}{2}(\hat{x}_{j+1} - \hat{x}_j)p_j^{II} + \sum_{k=0}^{j-1}(\hat{x}_{k+1} - x_k)p_k^{II}$$

Substituting $t(m_j)$ and $\hat{x}_j$ and rearranging yields

$$p_j^{II} = \frac{j(j+1)}{M} - 2\sum_{k=0}^{j-1} p_k^{II}$$

Therefore,

$$p_j^{II} - p_{j-1}^{II} = \frac{j(j+1)}{M} - \frac{j(j-1)}{M} - 2p_{j-1}^{II}$$

Solving the difference equation with the initial condition $p_1^{II} = 0$ yields

$$p_j^{II} = \frac{2j + 1 - (-1)^j}{2M}$$

Bid amounts of the $(2k)$-th and $(2k+1)$-th bid levels are the same where $k = 1, 2, \ldots, \lfloor \frac{M-1}{2} \rfloor$. The efficient bid levels in the second-price auctions are given by $\left(0, \frac{2}{M}, \frac{2}{M}, \frac{4}{M}, \frac{4}{M}, \ldots, 1\right)$ if M is an even number and $\left(0, \frac{2}{M}, \frac{2}{M}, \frac{4}{M}, \frac{4}{M}, \ldots, \frac{M-1}{M}\right)$ if M is an odd number. In an equilibrium, a bidder with valuation between $\frac{2k}{M}$ and $\frac{2k+1}{M}$, for $k = 0, 1, \ldots, \lfloor \frac{M-1}{2} \rfloor$, shades her bids and a bidder with valuation between between $\frac{2k-1}{M}$ and $\frac{2k}{M}$, for $k = 1, 2, \ldots, \lfloor \frac{M-1}{2} \rfloor$ overbids.

In an LAB auction, similar to the method used in the analysis of second-price auction, substituting $t(m_j)$ and $q(m_j)$ into the condition and rearranging yields

$$p_j^L = \frac{j(j+1)}{M} - 2\sum_{k=1}^{j-1} p_{k+1}^L$$

Therefore,

$$p_j^L - p_{j-1}^L = \frac{j(j+1)}{M} - \frac{j(j-1)}{M} - 2p_j^L$$

Solving the difference equation with the initial condition $p_1^L = 0$ yields

$$p_j^L = \frac{2j \cdot 3^j - 3^j + 1}{2M \cdot 3^j}$$

Figure 1.5 shows bid functions in the first-price, second-price and LAB auctions with two bidders and 10 bid levels. The dashed line is a 45 degree line. The efficient

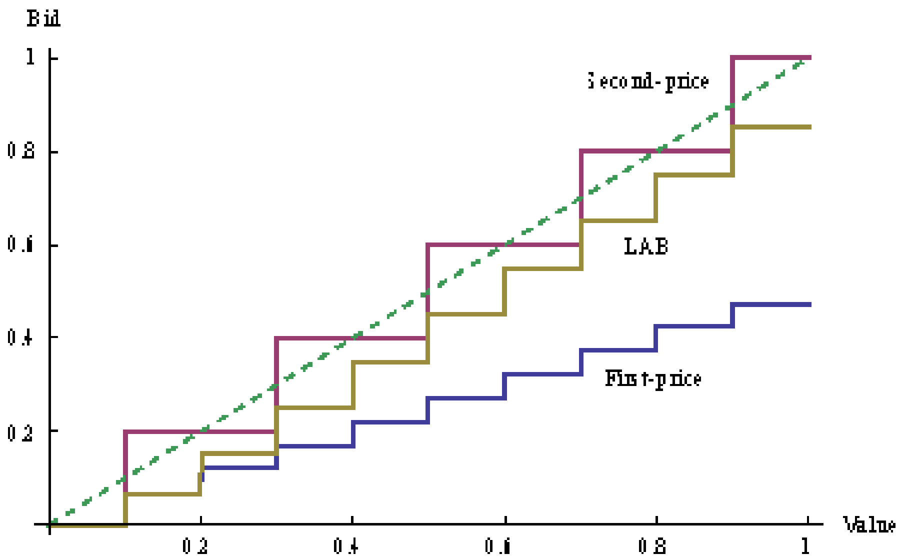

Figure 1.5: Bid functions in the first-price, second-price and clock LAB auctions with two bidders and 10 bid levels . Dashed line is a 45 degree line.

partition is $(0, 0.1, 0.2, \ldots, 1)$. The bid function in the first-price auction closely follows the bid function in a continuous case, $x/2$, and is slightly decreasing. The bid functions of the second-price and LAB auctions trace the truthful bidding function but the bid increments of the latter are slightly increasing. As discussed in Section 1.4.1, the bidding strategy in the second-price auction involves both overbidding and shading. Bidders with valuations lying on the left segment of the same bid amount overbid whereas ones in the right segment shade. The expected revenue is given as follows.

$$I \sum_{k=0}^{M-1} t(m_k) \cdot (\hat{x}_{k+1} - \hat{x}_k) = \frac{1}{3} - \frac{1}{3M^2}$$

The expected revenue in the continuous case, 1/3, is subtracted by the second term involving an inverse of the number of messages. Hence, the expected revenue is increasing in the number of messages. As the number of messages increases,

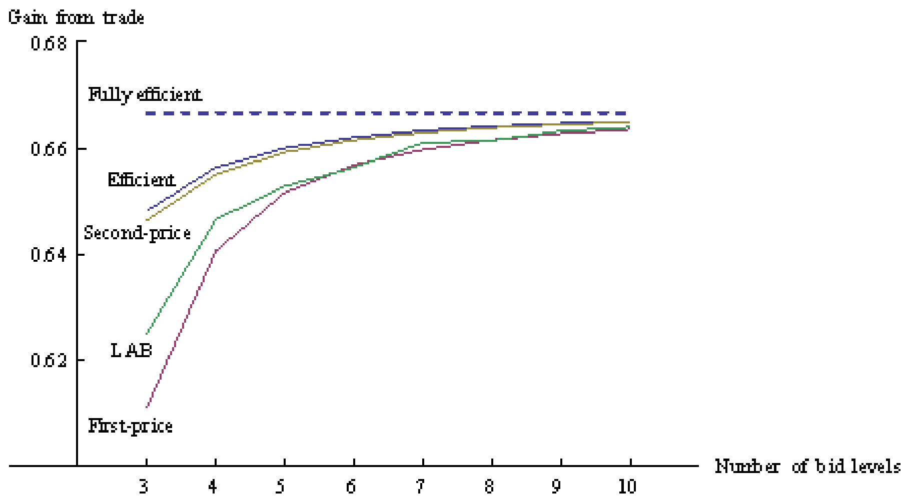

Figure 1.6: Gains from trade of the first-price, second-price and LAB auctions with three to ten equally spaced bid levels.

the second term gets smaller and the expected revenue converges to that of the continuous case.

It is interesting to see efficiency when bid levels are chosen naively. Without optimizing, it is natural to set the bid levels to be equally spaced and trace the equilibrium bid functions in the standard setting. Hence, the bid levels of the first-price, second-price and LAB auctions with M bid levels are defined as follows: $p_j^I = j/2(M-1)$, $p_j^{II} = j/(M-1)$ and $p_j^L = j/M$. The symmetric equilibria of these auctions can be solved numerically.

Figure 1.6 shows the comparison of the expected gains from trade of auctions with bid levels chosen naively and the optimal level. The efficiencies of all auction formats asymptotically go to the fully efficient level. Efficiency of the second-price auction lies closely to the efficient level while efficiencies of the first-price and LAB are below and converge to the efficient level later on. Naively-chosen bid levels can perform well with the large number of bid levels. With many bidders, an auction

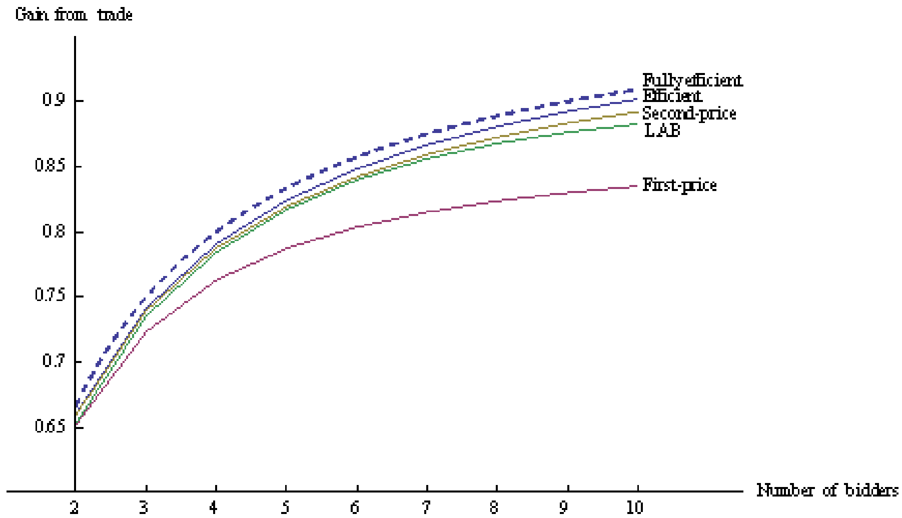

Figure 1.7: Gain from trade of the first-price, second-price and LAB auctions with five bid levels and two to ten bidders.

with naively chosen bid levels may yield considerably lower revenue than the efficient auction. Figure 1.7 shows gain from trade of various auctions with five bid levels and two to ten bidders. As the number of bidders increases, all gains from trade diverge from the efficient level. Remarkably, the gain from trade of the first-price auction departs from the efficient level at the highest rate.

1.5 Clock Auction with Demand Disclosure

In dynamic auctions, it is more common to reveal an aggregate demand after the end of each round. Bidders will bid more aggressively if they are aware of more competition. Moreover, they will make use of disclosed demand to make an inference about opponent's valuation. For instance, if an opponent bids when the demand is low, one can infer that the opponent's valuation is relatively high. On the contrary, if the opponent bids when the demand is high, the opponent's valuation might not be as high as in previous scenario as the competition could be another factor inducing her opponent to bid more aggressively. Thus, a bidder may bid in a round with a particular demand history but may drop out in the same round with different demand history. The history dependence complicates the use of the mechanism design approach. Given a probability of trade and expected transfer, the derivation of efficient bid levels requires a system of equations involving bid levels and probability of winning for all possible demand histories. We therefore approach the problem with indirect mechanism.

Suppose the number of rounds P is exogenously given. If the auction reaches round $P-1$, the auction automatically ends and the item is randomly awarded to one of the active bidders. Let $\mathbf{h}_t \equiv (h_1, h_2, \ldots, h_t)$ be a demand history in round t where $h_s \in \mathcal{I}$ is the number of active bidders at the beginning of round s, for $s = 1, 2, \ldots, t$. The monotonic activity rule implies that $I = h_1 \geq h_2 \geq \cdots \geq h_t \geq 2$. Let $\mathbf{H}_t \equiv \{\mathbf{h}_t | h_s \in \mathcal{I} \text{ for } s = 1, 2, \ldots, t \text{ and } I = h_1 \geq h_2 \geq \cdots \geq h_t \geq 2\}$ be a set of all possible demand histories in round t.

Let x_t^* be a critical valuation in round t. We consider a monotonic bidding strategy: in round t, a bidder with valuation x bids if $x \geq x_t^*$ and drops out if $x < x_t^*$. A bidder with valuation x_t^* is indifferent between bidding and dropping out. We assume that she always chooses to bid. Given this bidding strategy, the inferred lower bound of active opponent's valuation in round t is x_{t-1}^* because opponents with valuations less than x_{t-1}^* would have already dropped out in round $t-1$. The critical valuation x_t^* depends on the number of active bidders h_t and the inferred lower bound x_{t-1}^*. The recursive relationship of the critical valuation implies that the critical valuation x_t^* is a function of the entire demand history h_t. A symmetric equilibrium can be defined as follows.

Proposition 1.9. *Define a critical valuation function as* $x_t^* : \mathbf{H}_t \to [0,1]$. *A symmetric Bayesian equilibrium of a clock auction with demand disclosure is fully characterized by a set of critical valuation* $\mathbf{x}^* \equiv \{x_t^*(\mathbf{h}_t)\}_{t=1,2,\ldots,P-1,\mathbf{h}_t \in \mathbf{H}_t}$.

Similar to bidders, in round t with demand history h_t, the social planner learns about demand and the lower bound of bidders' valuations. Given that there are h_t bidders with valuations in the sub-type space $[x_{t-1}^*(\mathbf{h}_{t-1}), 1]$, the social planner chooses where to split the sub-type space to maximize the expected gain from trade. Thus, the partition rule is contingent on demand history as well. The critical valuation is equivalent to the partition rule by construction.

Define $\Phi_t : \mathbf{H}_t \to \mathbb{R}$ as expected gain from trade in round t. The expected gain from trade in round t can be defined recursively as follows.

$$
\begin{aligned}
\Phi_t(\mathbf{h}_t, \mathbf{x}^*) \;=\; & \left(\frac{F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))}{1 - F(x_{t-1}^*(\mathbf{h}_{t-1}))}\right)^{h_t} \cdot \frac{\int_{x_{t-1}^*(\mathbf{h}_{t-1})}^{x_t^*(\mathbf{h}_t)} x \mathrm{d}F(x)}{F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))} \\
+ \; & \frac{h_t\big(1 - F(x_t^*(\mathbf{h}_t))\big)\big(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\big)^{h_t - 1}}{\big(1 - F(x_{t-1}^*(\mathbf{h}_{t-1}))\big)^{h_t}} \cdot \frac{\int_{x_t^*(\mathbf{h}_t)}^{1} x \mathrm{d}F(x)}{1 - F(x_t^*(\mathbf{h}_t))} \\
+ \; & \sum_{s=2}^{h_t} \binom{h_t}{s} \cdot \frac{\big(1 - F(x_{t-1}^*(\mathbf{h}_{t-1}))\big)^k F(x_{t-1}^*(\mathbf{h}_{t-1}))^{h_t - k}}{\big(1 - F(x_{t-1}^*(\mathbf{h}_{t-1}))\big)^{h_t}} \cdot \pi_{t+1}(\mathbf{h}_t \cup (s))
\end{aligned}
$$

where $\Phi_P(\mathbf{h}_P, \mathbf{x}^*) = \frac{\int_{x_{P-1}^*(\mathbf{h}_{P-1})}^{1} x \mathrm{d}F(x)}{1 - F(x_{P-1}^*(\mathbf{h}_{P-1}))}$. The first two terms are the expected gain from trade if the auction ends in round t. The last term is a sum of expected gain from trade for all possible demand histories in round $t+1$. Note that $\Phi_P(\mathbf{h}_P, \mathbf{x}^*)$ is virtually a gain from trade if two or more bidders bid in round $P-1$. There is no actual bidding round P.

The social-choice function can be written as $\Phi(\mathbf{x}^*) \equiv \Phi_1(I, \mathbf{x}^*)$. The social planner's problem is to maximize the expected gain from trade in round 1 by choosing the critical valuations for all possible demand histories. The efficient partition rule in a clock auction with demand disclosure is characterized by $\mathbf{x}^* = \arg\max_{x^*} \Phi(\mathbf{x}^*)$ subject to (1) $x_t^*(\mathbf{h}_{t-1} \cup (s)) \geq x_{t-1}^*(\mathbf{h}_{t-1})$ for all $s = 2, 3, \ldots, h_{t-1}$ and $\mathbf{h}_{t-1} \in \mathbf{H}_{t-1}$ and (2) $x \in [0, 1]$ for $x \in x^*$. The first-order conditions are $\nabla\Phi = \mathbf{0}$. We can solve for $\mathbf{x}^*$ as the number of equations and the number of unknowns are both equal to the number of possible demand histories $\sum_{s=1}^{P-1} |\mathbf{H}_s|$. The social planner can commit to the partition rule since it is contingent on demand history.

Proposition 1.10. *A clock auction with demand disclosure is more efficient than the blind version with the same number of rounds, number of bidders and type space.*

It is known that revealing demand is efficiency enhancing in the affiliated valuation setting. Proposition 1.10 suggests that the demand disclosure can improve efficiency even in the independent private valuation setting where a price discovery is irrelevant. An implicit assumption however is that bid levels are chosen optimally.

One can implement an efficient partition rule in a blind clock auction with a clock auction with demand disclosure. The expected gain from trade in static auction $\varphi(\hat{\mathbf{x}})$ is equivalent to the one in dynamic auction $\Phi(\mathbf{x}^*)$ with constraints $x_t^*(\mathbf{h}_t) = \hat{x}_t$ for all $\mathbf{h}_t \in \mathbf{H}_t$ and $t = 1, 2, \ldots, P-1$. Note that a different set of bid levels is needed since bidders still respond to demand history. The social planner's problem in a blind clock auction is equivalently an optimization of a clock auction with demand disclosure with additional constraints. Since the optimization with fewer constraints yields higher value of objective function, a auction clock auction with demand disclosure is more efficient than the blind clock auction. When there are two bidders, demand history is irrelevant since a bidder can imply that her opponent is still active if the auction continues. In such case, both auctions are equally efficient.

Once we derive the efficient partition rule, we can implement them with bid levels. In each round with a particular demand history, we can choose a bid level such that the critical valuation coincides with the efficient partition rule. Hence, bid levels must be contingent on demand history as well. Bid levels in a clock auction with demand disclosure are given by $\{p_t(\mathbf{h}_t)\}_{t=1,2,\dots,P-1,\mathbf{h}_t\in\mathbf{H}_t}$. With HRB and LAB pricing rules, the efficient bid levels of a clock auction with demand disclosure are characterized in Definition 1.8.

Definition 1.8. *The efficient bid levels of the clock auction with highest-rejected bid and demand disclosure (HRBD),* $\mathbf{p}^{HD} \equiv \{p_t^{HD}(\mathbf{h}_t)\}_{t=1,2,\dots,P-1,\mathbf{h}_t\in\mathbf{H}_t}$*, are defined by, for* $t = 1, 2, \dots, P-1$ *and* $\mathbf{h}_t \in \mathbf{H}_t$*,*

$$\frac{1}{h_t}\left(x_t^*(\mathbf{h}_t) - p_{t-1}^{HD}(\mathbf{h}_{t-1})\right)\left(\frac{x_t^*(\mathbf{h}_t) - x_{t-1}^*(\mathbf{h}_{t-1})}{1 - x_{t-1}^*(\mathbf{h}_{t-1})}\right)^{h_t-1} =$$
$$\left(x_t^*(\mathbf{h}_t) - p_{t-1}^{HD}(\mathbf{h}_{t-1})\right)\left(\frac{x_t^*(\mathbf{h}_t) - x_{t-1}^*(\mathbf{h}_{t-1})}{1 - x_{t-1}^*(\mathbf{h}_{t-1})}\right)^{h_t-1} + \left(x_t^*(\mathbf{h}_t) - p_t^{HD}(\mathbf{h}_t)\right)\cdot$$
$$\sum_{j=1}^{h_t-1}\frac{1}{j+1}\binom{h_t-1}{j}\left(\frac{x_{t+1}^*(\mathbf{h}_t\cup(j)) - x_t^*(\mathbf{h}_t)}{1 - x_{t-1}^*(\mathbf{h}_{t-1})}\right)^j\left(\frac{x_t^*(\mathbf{h}_t) - x_{t-1}^*(\mathbf{h}_{t-1})}{1 - x_{t-1}^*(\mathbf{h}_{t-1})}\right)^{h_t-j-1}$$

The efficient bid levels of the clock auction with lowest-accepted bid and de-

mand disclosure (LABD), $\mathbf{p}^{LD} \equiv \{p_t^{LD}(\mathbf{h}_t)\}_{t=1,2,\ldots,P-1,\mathbf{h}_t\in\mathbf{H}_t}$*, are defined by, for* $t = 1, 2, \ldots, P-1$ *and* $\mathbf{h}_t \in \mathbf{H}_t$*,*

$$\frac{1}{h_t}\left(x_t^*(\mathbf{h}_t) - p_{t-1}^{LD}(\mathbf{h}_{t-1})\right)\left(\frac{x_t^*(\mathbf{h}_t) - x_{t-1}^*(\mathbf{h}_{t-1})}{1 - x_{t-1}^*(\mathbf{h}_{t-1})}\right)^{h_{t-1}} =$$

$$\left(x_t^*(\mathbf{h}_t) - p_t^{LD}(\mathbf{h}_{t-1})\right)\left(\frac{x_t^*(\mathbf{h}_t) - x_{t-1}^*(\mathbf{h}_{t-1})}{1 - x_{t-1}^*(\mathbf{h}_{t-1})}\right)^{h_{t-1}} + \left(x_t^*(\mathbf{h}_t) - p_t^{LD}(\mathbf{h}_t)\right) \cdot$$

$$\sum_{j=1}^{h_t-1}\frac{1}{j+1}\binom{h_t-1}{j}\left(\frac{x_{t+1}^*(\mathbf{h}_t \cup (j)) - x_t^*(\mathbf{h}_t)}{1 - x_{t-1}^*(\mathbf{h}_{t-1})}\right)^j\left(\frac{x_t^*(\mathbf{h}_t) - x_{t-1}^*(\mathbf{h}_{t-1})}{1 - x_{t-1}^*(\mathbf{h}_{t-1})}\right)^{h_t-j-1}$$

In both auctions, the individual rationality condition implies that $p_0^{HD}(\emptyset) = p_0^{LD}(\emptyset) = 0$*.*

The conditions in HRBD and LABD auctions are similar except that the final prices when all bidders drop out (the first term on the right-handed side) are $p_{t-1}^{HD}(\mathbf{h}_t)$ and $p_t^{LD}(\mathbf{h}_t)$, respectively. By construction, a bidder with a valuation $x_t^*(\mathbf{h}_t)$ is indifferent between bidding and dropping out in round t with a demand history $\mathbf{h}_t$. The expected payoff of dropping out is on the left-hand side. That is, the bidder wins only when all opponents also drop out and she wins the tiebreaker. The expected payoff of bidding is given on the right-hand side. The first term on the right-hand side is a scenario in which all opponents drop out. The second term on the right-hand side is the probability that the bidder wins the tiebreaker when one or more opponents bid. In this scenario, one or more opponents bid in round t and drop out in the next round and the bidder wins the tiebreaker. The number of opponents who bid ranges from one to $h_t - 1$ as indicated in the summation. A bidder with valuation of zero will drop out in the first round. Her expected payoff in HRBD or LABD auction is $x_1^*(I)^I \cdot (0 - p_0^{HD}(\emptyset))/I$ or $x_1^*(I)^I \cdot (0 - p_0^{LD}(\emptyset))/I$, respectively. Thus, individual rationality implies that $p_0^{HD}(\emptyset) = p_0^{LD}(\emptyset) = 0$.

1.5.1 Example: three bid levels and three bidders

Assume that bidders' valuations are independently and uniformly distributed on the interval $[0, 1]$. With three bid levels, there are two rounds and three possible active bidder histories: (3), $(3, 3)$ and $(3, 2)$. We can derive $x_1^*(3) = 0.42$, $x_2^*(3, 2) = 0.71$ and $x_2^*(3, 3) = 0.76$. As expected, the critical valuation is higher when there are more active bidders. In the first round, the social planner splits the type space at 0.42. If there are two active bidders in round 2, the efficient partition is equally spaced because of the uniform type space. The critical valuation in this case is $x_2^*(3, 2) = (1 + 0.42)/2 = 0.71$. Since the PDF of the first-order statistic is skewed more negatively when there are three active bidders, $x_2^*(3, 3) > x_2^*(3, 2)$.

The efficient bid levels in HRBD and LABD auctions can be derived by solving systems of equations defined in Definition 1.8. The efficient bid levels are shown in Table 1.1. In LABD auction, each critical valuation is higher than its associated bid level. All bidders shade due to the first-price incentive—an incentive to keep the final price low. In HRBD auction, some bidders may overbid when demand history is (3) or $(3, 3)$ as $p_1^{HD}(3)$ and $p_2^{HD}(3, 3)$ exceed their associated critical valuations. Overbidding in HRB pricing rule is profitable if all opponents drop out so that the final price is equal to the previous round price. Surprisingly, the bid level in round 1 is greater than that in round 2 when demand history is $(3, 2)$. The bid levels

Table 1.1: Efficient bid levels in HRBD and LABD auctions with three bid levels and three bidders

Demand history	Critical valuation	Bid level	
		HRBD	LABD
$(\emptyset)$	-	0	0
(3)	0.42	0.73	0.35
$(3,2)$	0.71	0.69	0.59
$(3,3)$	0.76	0.77	0.68

when the demand history is $(3,2)$ needs to be equal to 0.69 to make a bidder with valuation 0.71 indifferent between bidding and dropping out.

The peculiar characteristics of efficient bid levels HRBD auction may make it less appealing. A bidder who wins in round 1 may end up paying more than the one who wins in round 2. In practice, one would find the equally-efficient LABD auction more desirable as their bid levels are more intuitive.

1.6 Conclusion

This paper offers an insight into an important design aspect of the discrete auction: the choice of bid levels. The social planner's problem is to choose a partition rule to maximize efficiency. The efficient partition rule can be implemented with bid levels in various auction formats. Our approach has an advantage over the previous literature because it is not specific to a particular auction format. The efficient partition rule is applicable to many auction formats given that the type space, number of bidders and number of messages are the same. Any auction consistent with the efficient partition rule yields the maximum attainable efficiency. Intuitively, efficiency cannot be improved by altering the auction format as long as bid levels are chosen optimally.

One way to increase efficiency is to permit additional messages. In a sealed-bid auction, allowing bid levels to be as fine as the smallest currency unit is feasible and the cost is marginal, if not more. In addition, with such a large number of messages, the efficiency loss when bid levels are chosen naively is likely to be small. On the contrary, motivations for coarser bid levels in a dynamic auction are much stronger. Auction duration becomes a concern since the cost of conducting auction and bidders' participation costs are increasing as the auction goes on. While smaller bid increments promote efficiency, it may take longer to conclude and thus incurs more cost. The social planner therefore faces a tradeoff between efficiency loss and auction duration. We show that efficiency in a clock auction can be improved without prolonging the auction by revealing demand. To determine the optimal number of messages, we can setup the optimization problem where the objective function is the social gain-efficiency minus social cost.

Ausubel and Cramton (2004) suggested that the discreteness of bid levels can be overcome by the use of intra-round bidding. That is, a bidder can specify a price at which she wants to reduce demand. While it has been used in some auctions, it is not common. With this feature, the auction is fully efficient regardless of the choice of bid levels in the independent private value setting. However, in the affiliated value setting the choice of bid levels still affects efficiency through within-round price discovery because bidders as well as the social planner learn about demand only at the previous bid levels and prices below. The motivation for choosing efficient bid levels is even stronger when values are more correlated.

One natural extension is to design a revenue-maximizing discrete auction. The objective function takes the form of expected revenue instead of the expected gain from trade. It is also interesting to compare reserve prices between continuous and discrete auctions. Regardless, the concept remains the same: to choose the optimal partition rule and use it to determine the optimal bid levels as well as the reserve price.

Chapter 2

Pricing Rule in a Clock Auction

2.1 Introduction

In 2008, the government of India announced that radio spectrum for 3G mobile services would be auctioned in 2009. The 2.1 GHz spectrum is to be sold as paired spectrum (2x5 MHz blocks) in each of 22 regions covering India. There are one to four lots available in each region and each bidder can obtain at most one lot per region. The government's stated objective for the auction emphasizes revenue maximization rather than efficiency. In December 2008, the government announced the chosen auction design: a discrete clock auction with lowest-accepted bid (LAB) pricing and provisional winners (Telecommunications & Information Technology, 2008). Here we examine the equilibrium properties of such an auction in a simplified setting. As a comparison, we analyze a discrete clock auction with highest-rejected bid (HRB) pricing and exit bids. This format is often used in high-stake auctions in practice (Ausubel & Cramton, 2004).

Interestingly, with profit maximizing bidders, we find the HRB auction dominates the LAB auction in both efficiency and revenues. The HRB with exit bids is fully efficient, since it is a dominant strategy to bid up to one's valuation. In contrast, LAB with provisional winners has differential shading, since a provisional winner shades her bid, whereas a provisional loser does not. This differential shading creates an inefficiency and reduces revenues. Given this strong theoretical result, it may seem odd that India chose the LAB format.

One potential explanation comes from behavioral economics. If bidders anticipate the regret of losing at a profitable price, they may be reluctant to shade bids as a provisional winner. This fear of losing has been shown to explain overbidding in first-price sealed-bid auctions (Filiz-Ozbay & Ozbay, 2007; Engelbrecht-Wiggans & Katok, 2007, 2008). Risk aversion is an alternative explanation for overbidding, but has little empirical support (Engelbrecht-Wiggans & Katok, 2009). Delgado et al. (2008) provide a neurological foundation that fear of losing, not joy of winning, is the source of overbidding in first-price auctions. Loss aversion with a reference point of winning (Kőszegi & Rabin, 2006; Lange & Ratan, 2010) provides an analogous theory for fear of losing. When we extend the standard theory to include a fear of losing, we find that if bidders' fear of losing is sufficiently strong, then the LAB auction revenue dominates the HRB auction. This result provides an explanation for India's selection of a seemingly inferior auction format.

Most theoretical papers on clock auctions assume a continuous clock for convenience. In practice, clock auctions use a discrete price clock, since these auctions typically are conducted on the internet, and communication is not sufficiently reliable to bind bidders to higher prices with the continuous passage of time. With discrete bid levels, the two predominant pricing rules, lowest-accepted bid and highest-rejected bid, are distinct, and the auction designer must select a pricing rule as well as other elements of the design. Currently, there is little literature for the auction designer to turn to for help with this issue.

There are a limited number of papers investigating auctions with discrete bid levels. These papers focus on explaining bidding behavior. Chwe (1989) studied the first-price auction with discrete bid levels and showed that the expected revenue is less than its continuous counterpart. (Mathews & Sengupta, 2008) analyzed a sealed-bid second-price auction with discrete bids.

More closely related is the work that considers choices of bid levels in ascending auctions. Rothkopf and Harstad (1994) is an important early contribution, determining optimal bid levels that maximize expected revenue in an oral auction. The paper also introduces the trade-off between auction duration and bid increments. David et al. (2007) extend the model of Rothkopf and Harstad and find that decreasing bid increments are optimal. Although the pricing rule in our paper is the same as in Rothkopf and Harstad, the auction formats have important differences, which result in significantly different bidding behavior.

We consider a discrete clock auction with two pricing rules: highest-rejected bid and lowest-accepted bid. Bidders have independent private values and unit demands. We first analyze bidding behavior in an HRB auction. This is our benchmark for comparing performance with the LAB auction. The HRB auction is a useful benchmark because of its simplicity, its desirable properties (efficiency and truth dominance), and its use in practice. In contrast, the LAB auction forces bidders to engage in difficult tradeoffs. We are only able to solve for equilibrium bidding behavior in a simplified setting. Nonetheless, we show that an LAB auction is generally inefficient. Despite this inefficiency, the LAB auction can yield higher revenues if bidders anticipate the regret of losing at profitable prices, and therefore engage in less bid shading than a bidder focused solely on profit maximization.

2.2 Discrete clock auction with unit demand

There are K identical items for sale to N risk-neutral bidders indexed by $i = 1, \ldots, N$ where $N > K$. The seller values the items at zero. Bidder i demands at most one item. Her private valuation for the item is x_i which is independently drawn from the distribution F with associated density function f on the interval $[0, 1]$. (We use the independent private values model for simplicity. Our main results extend to a model with affiliated values.) Bidder i's payoff if she wins the item is $x_i - m_i$ where m_i is bidder i's payment and zero otherwise.

Before the auction starts, the seller announces a vector of bid levels, $\mathbf{P} = (P_0, P_1, \ldots, P_{T-1})$ where P_t is the clock price at round t for $t = 1, 2, \ldots, T-1$ and T is the number of bid levels. The clock price increases every round so that $P_0 < P_1 < \cdots < P_{T-1}$. Define the bid increment in round t as $\Delta_t = P_t - P_{t-1}$ for $t = 1, 2, \ldots, T-1$. Assume that $P_0 = 0$ and $P_{T-1} = 1$. The auction begins in round 1 at a price P_1. In round t, bidder i chooses either to bid at the current clock price, $q_{it} = 1$, or to exit, $q_{it} = 0$. Once a bidder exits she cannot bid again. Let $Q_t = \sum_{i=1}^{N} q_{it}$ be total demand in round t. If there is excess demand, $Q_t > K$, the auction proceeds to the next round. The auction ends in the round t such that $Q_t \leq K$.

2.2.1 Highest-rejected bid with exit bids

In the HRB format (highest-rejected bid with exit bids), if bidder i exits in round t, the bidder can submit an exit bid-a price between P_{t-1} and P_t at which she wants to exit. In round t such that $Q_t = K$, the items are awarded to the K active bidders and the final price is the highest exit bid among the inactive bidders . If $Q_t < K$, the items are awarded to the Q_t active bidders and to those bidders who submitted the $K - Q_t$ highest exit bids. The final price is the $K - Q_t + 1$-highest exit bid.

Proposition 2.1. *In the HRB auction, truthful bidding (bidding up to one's valuation) is a weakly-dominant strategy. Therefore, the HRB auction yields an efficient allocation. Each of the K-highest valuation bidders wins and pays the $K + 1$-highest valuation.*

This result follows immediately from the unit demand setting and the ability to submit exit bids at actual valuations. Unit demand guarantees that each winner pays the Vickrey price, thereby inducing truthful bidding. Since all bidders bid truthfully, the items are awarded to the bidders who value them the most.

2.2.2 Lowest-accepted bid with provisional winners

In the LAB format (lowest-accepted bid with provisional winners), if there is excess demand in any round, the seller randomly selects K provisional winners for the next round from active bidders and ranks them from K to one (highest rank is given priority). Other remaining active bidders are designated as provisional losers (rank of 0) for the next round. Each bidder is automatically assigned a rank in round 1.

The auction ends if there is no excess demand. The items are awarded to the active bidders and the remaining items are awarded to the inactive provisional winner in the current round with the highest rank and so on until all items are sold. Since bidders are not allowed to submit exit bids, the final price and bids are restricted to clock prices. The final price is uniform and determined by the K-highest winner's bid. If $Q_t = K$, the final price is P_t and if $Q_t < K$, the final price is P_{t-1}. Notice that the ranking is relevant in determining the allocation only when $Q_t < K$ since then there is excess supply at the final price P_{t-1}. Let $\theta \equiv \{0, 1, \ldots, K\}$ be the set of all possible ranks and $\mathbf{\Theta} \equiv \{\mathbf{X}_t | \mathbf{X}_t \in \theta^t\}$ be the set of all possible bidder's ranking histories up from round 1 to round t. Let $R_{it} \in \theta$ denote bidder i's rank in round t and $\mathbf{H}_{it} \equiv (R_{i1}, R_{i2}, \ldots, R_{it}) \in \mathbf{\Theta}_t$ be a vector of bidder i's ranking history from round 1 to round t. Bidder i's ranking history $\mathbf{H}_{it}$ is known only to bidder i.

One important difference between the LAB auction described here and an ascending bid auction with the same pricing rule is that in the LAB auction, a provisional winner must keep topping her own bid in order to be eligible to bid in subsequent rounds. In contrast, in an ascending bid auction with LAB, being a provisional winner is counted as being active. Therefore, a provisional winner does not need to bid in order to be eligible to bid in the subsequent rounds.

To better illustrate the LAB pricing rule with ranking, consider an auction with three bidders and two items for sale. Suppose all bidders bid in round $t-1$ and bidder 2 and 3 are selected as provisional winners with ranks of 1 and 2 respectively

Table 2.1: LAB auction with three bidders and two items in round t

Case	Bidder 1 ($R_{1t} = 0$)	Bidder 2 ($R_{2t} = 1$)	Bidder 3 ($R_{3t} = 2$)	Outcome
1	Bid	Bid	Bid	Proceed to round $t + 1$
2	Bid	Bid	Exit	Bidder 1 and 2 win and the final price is P_t
3	Bid	Exit	Bid	Bidder 1 and 3 win and the final price is P_t
4	Exit	Bid	Bid	Bidder 2 and 3 win and the final price is P_t
5	Bid	Exit	Exit	Bidder 1 and 3 win and the final price is P_{t-1}
6	Exit	Bid	Exit	Bidder 2 and 3 win and the final price is P_{t-1}
7	Exit	Exit	Bid	Bidder 2 and 3 win and the final price is P_{t-1}
8	Exit	Exit	Exit	Bidder 2 and 3 win and the final price is P_{t-1}

while Bidder 1 is a provisional loser. In round t, eight possible combinations of bids and corresponding allocations and final prices are shown in Table 2.1. If all bidders bid (case 1), the auction proceeds to round $t+1$ with a price of P_{t+1} and all bidders are assigned new ranks. Regardless of ranking, if there are exactly two active bidders (case 2 to 4), they win the items at the current clock price P_t. If there is only one active bidder which creates an excess supply at the current clock price (case 5 to 7), the item is awarded to the active bidder and an inactive provisional winner with highest rank at the previous clock price P_{t-1}. Finally, if there is no active bidder (case 8), bidders 2 and 3 who hold the highest ranks win at the previous clock price P_{t-1}.

Lemma 2.1. *For any K, in any round $t > s+1$ with any ranking history $\mathbf{H}_{it} \in \mathbf{\Theta}_t$, bidder i with valuation $x_i < P_{s+1}$ exits.*

This lemma simply states that a bidder with a valuation less than P_{s+1} never bids at a price P_{s+2} or above. In some situations a bidder may take a risk by bidding in round $s+1$ at a price P_{s+1} which is higher than her valuation in the hope that there will be excess supply and she will consequently win the item at the previous clock price P_s. When she find out that the auction actually continues to round $s+2$ it is a dominant strategy to exit immediately regardless of her ranking since the lowest possible final price is P_{s+1}, which is still higher than her valuation.

Lemma 2.2. *For any K, in any round t with any ranking history $\mathbf{H}_{it} \in \mathbf{\Theta}_t$, a provisional loser with valuation x_i bids if $x_i \geq P_t$. For any $K > 1$, in any round t with some ranking history $\mathbf{H}_{it} \in \mathbf{\Theta}_t$, a provisional loser with valuation $x_i < P_t$ bids in round t if x_i is sufficiently close to P_t.*

For a provisional loser, exiting yields a payoff of zero while bidding yields a positive expected payoff as long as her valuation is above the current clock price. Thus, a provisional loser never exits if her valuation exceeds the current clock price. For some ranking history, a provisional loser may bid at a price level above her valuation since there is a positive probability that less than $K-1$ bids are submitted and that by bidding, she can win the item at the previous clock price which is below her valuation. However, such a strategy entails a risk of winning the item at a price higher than her valuation. As a provisional loser's valuation is closer to P_t, the negative payoff she may receive if the auction continues is smaller while the positive payoff in case that she wins is larger. Therefore, a provisional loser who has a valuation closer to P_t may find that bidding is more attractive. If she does not win, she will exit in round $t+1$ according to Lemma 2.1.

Consider the example in Table 2.1. Suppose bidder 1 has a valuation between P_{t-1} and P_t. If she bids and bidder 2 and 3 exit as in Case 5 which has a positive probability of occurring, bidder 1 wins the item at P_{t-1} gaining a positive payoff. If bidder 1 bids and either bidder 2 or bidder 3 bid, as in Case 2 or 3, bidder 1 wins the item at P_t, receiving a negative payoff. If all bidders bid, bidder 1 will exit according to Lemma 2.1.

Lemma 2.3. *For any $K \geq 1$, there exists a round $t \leq s$ with some ranking history $\mathbf{H}_{it} \in \mathbf{\Theta}_t$ such that a provisional winner with a valuation $x_i \geq P_s$ exits if x_i is sufficiently close to P_s.*

Intuitively, a provisional winner faces the first-price incentive, an incentive to keep the price low, which results in exiting before her valuation is reached. A provisional winner can win the item at a lower price if she exits, but she risks losing at a profitable price as exiting is irrevocable. In contrast, if she stays in the auction, her probability of winning the item increases and so does the expected final price she pays. Hence, a provisional winner may find that exiting yields higher expected payoff than bidding.

In addition, a provisional winner with higher rank faces less risk of losing when exiting since a greater number of new bids are required to displace her provisional winning bid. Consequently, there is some ranking history such that a provisional winner with a particular valuation bids if her rank is less than r but one with the same valuation exits if her rank is equal to or greater than r.

In the scenario in Table 2.1, bidder 2 who is a provisional winner faces a tradeoff between bidding and exiting. Suppose bidder 1 will bid with certainty. By exiting, bidder 2 may win at the price P_{t-1} if bidder 3 exits (case 5) but she will lose if bidder 3 stays in. In contrast, by bidding, bidder 2 may win at the higher price P_t if bidder 3 exits (case 2).

Proposition 2.2. *The LAB auction is inefficient.*

There are two sources of inefficiency: discrete bid levels and asymmetric bidding behavior. First, discrete bid levels may prevent bidders from expressing a difference in their valuations. Any auction with discrete bid levels is generally inefficient. As the size of bid increments becomes small, so does the difference between the HRB and LAB pricing rules. The difference in efficiency is smaller as well. Second, although bidders and their bidding strategies are symmetric, asymmetric bidding behavior is introduced by naming and ranking provisional winners as discussed in Lemmas 2.2 and 2.3. In some situations, a bidder with a particular valuation bids if she is a provisional loser but in the same circumstances, she exits if she is a provisional winner. Differential bid shading is exhibited among provisional winners and bidding above valuation is found among provisional losers. These differences are a separate source of inefficiency.

Exit bids in the HRB auction overcome both sources of inefficiency of the LAB auction. In the HRB auction, there is no bid shading and all value differences are expressed.

2.3 Equilibrium characterization with one item and two bidders

In order to further characterize equilibrium behavior in the LAB auction, it is necessary to simplify the setting to one item and two bidders, 1 and 2. These assumptions are maintained through Section 2.8. Although this setting is limiting, it will provide intuition for cases with more items and more bidders.

With two bidders and a single item, there are two possible ranks: a provisional winner or a provisional loser. Once a bidder knows her rank, she can infer that her opponent's rank is the other. Specifically, for all t, $\mathbf{H}_{1t}$ is the complement of $\mathbf{H}_{2t}$ and vice versa. The auction ends when at least one bidder exits. If in round t only one bidder exits, the active bidder receives the item and pays P_t. If both bidders exit in round t, the provisional winner in round t gets the item at a price of P_{t-1}.

Definition 2.1. *Bidder i follows a straightforward bidding strategy in round t when*

$$q_{it}(x_i) = \begin{cases} 1 & ;\ \textit{if}\ x_i \geq P_t \\ 0 & ;\ \textit{if}\ x_i < P_t \end{cases}.$$

Lemma 2.4. *For a bidder with any rank, bidding above her valuation is a weakly dominated strategy. For a provisional loser, bidding straightforwardly is a weakly dominant strategy.*

As discussed in Lemma 2.2, a provisional loser bids if her valuation is higher than the current clock price. In the one-item case, the final price is at least the current clock price if one or more bids are submitted. In contrast to the case with two or more items, it is not profitable for a bidder with any rank to bid when the current clock price is above her valuation.

The fact that a provisional loser bids straightforwardly greatly simplifies the analysis. To construct an equilibrium, we need to solve for the bidding behavior of a provisional winner. A provisional winner's bidding strategy depends not only on her valuation, but also on the history of the opponent's ranks. For instance, a provisional winner may exit when a ranking history implies that a provisional loser's valuation has a lower bound less than the next clock price. In contrast, a provisional winner will continue to compete when an inferred provisional loser's valuation is higher than the current clock price. This dependence on the history of rankings implies that even with the same realized valuation different ranking histories may produce different allocations and final prices.

Lemma 2.5. *In any round t, a provisional winner's expected gain from exiting—expected payoff from exiting less expected payoff from bidding—is decreasing in her valuation.*

Simply put, a provisional winner's bidding strategy is monotonic in valuation. Intuitively, if an optimal strategy of a provisional winner with valuation x in any round t is bidding (exiting), it is an optimal strategy of a provisional winner with valuation $x' > x$ ($x' < x$) to bid (exit) in round t as well.

Let $\hat{x}_t : \mathbf{\Theta}_t \rightarrow [0, 1]$ be a critical valuation for a bidding strategy in round t. This function $\hat{x}_t(\mathbf{H}_{it})$ indicates that in round t, a bidder with a valuation at least $\hat{x}_t(\mathbf{H}_{it})$ will bid while one with valuation less than $\hat{x}_t(\mathbf{H}_{it})$ will exit. Moreover, if $P_{t-1} < \hat{x}_t(\mathbf{H}_{it}) < P_t$, $\hat{x}_t(\mathbf{H}_{it})$ is a valuation of a bidder who is indifferent between bidding and exiting in round t for a ranking history $\mathbf{H}_{it}$.

According to Lemma 2.4, the provisional loser bid straightforwardly regardless of the ranking history. Therefore, for any ranking history $\mathbf{H}_{it} \in \mathbf{\Theta}_t$ such that $R_{it} = 0$,

$\hat{x}_t(\mathbf{H}_{it}) = P_t$.

In each round, a bidder applies Bayesian updating to the lower bound of her opponent's valuation and uses this lower bound to determine an optimal strategy. For example, suppose in round t with bidder 1's ranking history of $\mathbf{H}_{1t}$, bidder 1 was a provisional loser and bidder 2 was a provisional winner. Suppose both of them have a valuation above P_{t+1} and bid at P_t. In round $t+1$, bidder 1 is selected as a provisional winner and she makes an inference that bidder 2's valuation must be in $[\hat{x}_t(\mathbf{H}_{2t}), 1]$. If for instance, $P_{t+1} < \hat{x}_t(\mathbf{H}_{2t}) < P_{t+2}$, bidder 1, aware that bidder 2 has a valuation higher than the current clock price and follows a straightforward bidding strategy, will bid in round $t+1$.

The lower bound is at least the previous clock price since no bidder bids above her valuation according to Lemma 2.4. It can be higher than the previous clock price when an equilibrium strategy suggests that the critical valuation of the provisional winner is higher than the previous clock price and she in turn bid. Importantly, being a provisional winner reveals more information on valuation to the opponent than being a provisional loser.

Let $l_t(\mathbf{H}_{jt}) \equiv \max\{P_{t-1}, \hat{x}_{t-1}(\mathbf{H}_{j,t-1})\}$ be an inferred lower bound of bidder j's valuation in round t as a function of a ranking history $\mathbf{H}_{jt}$. Consider a provisional winner i in round t with valuation is $x_i > P_t$. Let $\pi_{it}(q_{it}, x_i, \mathbf{H}_{it})$ be bidder i's expected payoffs in round t when submitting a decision $q_{it} \in \{0, 1\}$ given a ranking history $\mathbf{H}_{it}$ where 0 and 1 correspond to exiting and bidding respectively. If $l_t(\mathbf{H}_{jt}) \geq P_t$, a provisional winner's dominant strategy is bidding if $x_i \geq P_t$ and exiting otherwise. Next, consider the case where $l_t(\mathbf{H}_{jt}) \in [P_{t-1}, P_t)$. Let $\Pi_{i,t+1}$ be bidder i's expected payoff in round $t+1$ if the auction proceeds to round $t+1$. Bidder i's payoffs of bidding and exiting in round t are, respectively,

$$\begin{aligned}\pi_{it}(q_{it}=1, x_i, \mathbf{H}_{it}) &\equiv (x_i - P_t)\frac{F(P_t) - F(l_t(\mathbf{H}_{jt}))}{1 - F(l_t(\mathbf{H}_{jt}))} + \Pi_{i,t+1} \\ \pi_{it}(q_{it}=0, x_i, \mathbf{H}_{it}) &\equiv (x_i - P_{t-1})\frac{F(P_t) - F(l_t(\mathbf{H}_{jt}))}{1 - F(l_t(\mathbf{H}_{jt}))}.\end{aligned}$$

It is an optimal strategy to bid if $\pi_{it}(q_{it} = 1, x_i, \mathbf{H}_{it}) \geq \pi_{it}(q_{it} = 0, x_i, \mathbf{H}_{it})$ and exit if $\pi_{it}(q_{it} = 1, x_i, \mathbf{H}_{it}) < \pi_{it}(q_{it} = 0, x_i, \mathbf{H}_{it})$. $\hat{x}_t(\mathbf{H}_{it})$ can be either equal to a clock price or between two consecutive clock prices. It is equal to a clock price, say P_t for $t = 1, 2, \ldots, T-2$, if $\pi_{it}(q_{it} = 1, P_t, \mathbf{H}_{it}) \geq \pi_{it}(q_{it} = 0, P_t, \mathbf{H}_{it})$ and $\pi_{it}(q_{it} = 1, P_t - \epsilon, \mathbf{H}_{it}) < \pi_{it}(q_{it} = 0, P_t - \epsilon, \mathbf{H}_{it})$ where ϵ is a small positive number. $\hat{x}_t(\mathbf{H}_{it})$ will be between P_s and P_{s+1} if $\pi_{it}(q_{it} = 1, \hat{x}_t(\mathbf{H}_{it}), \mathbf{H}_{it}) = \pi_{it}(q_{it} = 0, \hat{x}_t(\mathbf{H}_{it}), \mathbf{H}_{it})$ and $\hat{x}_t(\mathbf{H}_{it}) \in (P_s, P_{s+1})$. We assign $\hat{x}_t(\mathbf{H}_{it}) = \infty$ if it is optimal for a provisional winner with any valuation to exit regardless of the ranking history. Obviously, $\hat{x}_{T-1}(\mathbf{H}_{i,t-1}) = \infty$ for any $\mathbf{H}_{i,t-1} \in \mathbf{\Theta}_{t-1}$.

Proposition 2.3. *An equilibrium of a discrete clock auction with LAB is characterized by $\hat{x}_t(\mathbf{H}_{it})$ for all $\mathbf{H}_{it} \in \mathbf{\Theta}_t$ and for all $t = 1, 2, \ldots, T-1$. In any round t with a ranking history $\mathbf{H}_{it} \in \mathbf{\Theta}_t$, a bidder i bids if her valuation is in $[\hat{x}_t(\mathbf{H}_{it}), 1]$ and exits if her valuation is in $[0, \hat{x}_t(\mathbf{H}_{it})]$.*

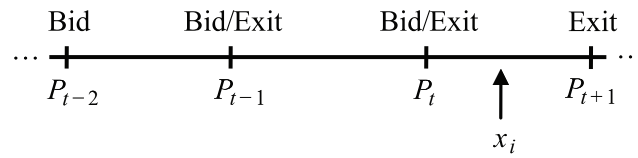

Figure 2.1: Strategy of a provisional winner with valuation $x \in [P_t, P_{t+1})$

2.4 Equilibrium characterization with a fixed bid increment and uniform valuations

To further characterize equilibrium behavior, it is helpful to assume a fixed bid increment and assume that valuations are uniformly distributed on $[0, 1]$. Thus, $\Delta_t = \Delta = \frac{1}{T-1}$ for all t and $F(x) = x$. We maintain these assumptions through Section 1.8.

Lemma 2.6. *In any round $t \leq s - 2$ with a ranking history $\mathbf{H}_{it} \in \mathbf{\Theta}_t$, a provisional winner in round t with a valuation $x \in [P_s, P_{s+1})$ always bids.*

Assuming fixed bid increments and uniform valuations greatly simplifies the equilibrium derivation, since we only need to solve for the provisional winner's bidding behaviors in the two rounds below her valuation. Figure 2.1 summarizes the strategy of a bidder with valuation $x \in [P_t, P_{t+1})$ when she is a provisional winner. According to Lemma 2.6, if she is a provisional winner in any round 1 to $t - 2$, she will bid. She may bid or exit, if she is a provisional winner in round $t - 1$ or t depending on her valuation and ranking history. By Lemma 2.4, she will exit in round $t + 1$ regardless.

Proposition 2.4. *Inefficiency in the LAB auction can be reduced by smaller bid*

Table 2.2: Valuations of a provisional winner whose optimal strategy is bidding in each round and ranking history

Round	Price	Ranking History	Value
1	1/3	(1)	$[4/5, 1]$
2	2/3	(1,1)	None
		(0,1)	$[2/3, 1]$
3	1	Any	None

increments.

In this setting, efficiency suffers from discrete bid levels and differential bid shading. However, according to Lemma 2.6, the amount of bid shading is limited to only two bid levels. Therefore, smaller bid increments reduce the absolute amount of bid shading as well as constraints on expressing value differences.

2.5 An example with four bid levels

To get a better sense of the equilibrium, consider an example with two bidders, one item and four bid levels: $\mathbf{P} = (0, \frac{1}{3}, \frac{2}{3}, 1)$ or $\Delta = 1/3$. Solving for an equilibrium yields a provisional winner's critical valuations, $\hat{x}_t(1) = 4/5$, $\hat{x}_t(0,1) = 2/3$, $\hat{x}_t(1,1) = \infty$ and $\hat{x}_t(\mathbf{H}_{i3}) = \infty$ for any $\mathbf{H}_{i3} \in \mathbf{\Theta}_3$. The bidding strategy of the provisional winner is summarized in Table 2.2.

A bidder never bids above her valuation. In round 1, if a provisional winner has a valuation less than 4/5, she will exit. To show how the ranking history affects inference of the lower bound of an opponent's valuation and bidding behavior, consider bidder 1 with valuation x_1 and bidder 2 with valuation x_2 where $x_1 > x_2 >$

4/5. Suppose she is a provisional winner in round 2. If the ranking history is $\mathbf{H}_{12} = (0, 1)$, the fact that bidder 2 bids in round 1 implies that her valuation is higher than 4/5 and thus she will bid in round 2. Since bidder 1 knows that bidder 2 will bid and her valuation is above the current clock price as well, it is a dominant strategy to bid. In contrast, if the ranking history is $\mathbf{H}_{12} = (1, 1)$, bidder 1 infers that bidder 2's valuation is higher than 1/3. Bidder 1 thus exits and loses. Bidder 1 would have done better if the ranking history had been $\mathbf{H}_{12} = (0, 1)$. The allocation in this case is inefficient.

This example shows that being a provisional loser provides a bidder a chance to learn about her opponent's valuation and that a ranking history affects a provisional winner's bidding strategy. It is this history dependence that makes the equilibrium calculation so difficult. With four bid levels, the expected revenue is 0.278 which is lower than that of the HRB auction, which is equal to 1/3.

2.6 Expected revenue with profit maximizing bidders

In this section, we continue to assume that there is a single item and two bidders whose valuations are uniformly distributed on $[0, 1]$. There are T bid levels, equally spaced. Since all bidders bid truthfully in the HRB auction, the expected revenue is $R_{HRB} = 1/3$.

Calculating an expected revenue of the LAB auction is tedious since all possible histories and associated outcomes have to be considered. Thus, the problem grows exponentially in the number of bid levels making derivation of an equilibrium with an arbitrary number of bid levels impossible. We however can calculate expected revenue for small T. We also can calculate revenue in the limit as the number of bid levels goes to infinity-bid increments become small. The limit result comes from the limit of upper and lower bounds on revenue. Since both limits converge to the same thing, the equilibrium expected revenue must converge as well.

Lemma 2.7. *Suppose all bidders follow the straightforward strategy. For any $T \geq 3$, the expected revenue is given by*

$$R_{LAB}^{SB}(T) = \frac{(T-2)(2T+3)}{6(T-1)^2}.$$

In addition, $\lim_{T\to\infty} R_{LAB}^{SB}(T) = R_{HRB}$.

A rational bidder infers a lower bound of her opponent's valuation from a ranking history and a previous clock price. The inferred opponent's valuation is at the previous clock price unless a ranking history implies that it is higher. The higher is the lower bound, the less is the bid shading. Consider a maximum shading strategy—in every round, a provisional winner infers that a lower bound of the provisional loser's valuation is a previous clock price. Therefore, the lowest inference of lower bound which leads to the highest amount of shading constitutes the maximum shading strategy as the name suggests. Explicitly, consider a bidder with valuation $x_t \in [P_t, P_{t+1})$ where $t \geq 2$. In any round $s \leq t-2$, she will bid regardless of her ranking according to Lemma 2.6. In round t, if she is a provisional winner, she will exit since $\pi_{it}(q_{it} = 1, x_i, \mathbf{H}_{it}) \equiv (x_i - P_t)\frac{\Delta}{1-F(P_{t-1})} + \frac{1}{2}(x_i - P_t)\frac{\Delta}{1-F(P_{t-1})} < (x_i - P_{t-1})\frac{\Delta}{1-F(P_{t-1})} \equiv \pi_{it}(q_{it} = 0, x_i, \mathbf{H}_{it})$. Given that she will exit in round t, in round $t-1$, we can calculate that $\hat{x}_{t-1}(\mathbf{H}_{it}) = P_t + \frac{2}{5}\Delta$.

Lemma 2.8. *Suppose a provisional winner follows the maximum shading strategy and a provisional loser follows a straightforward bidding strategy. For any $T \geq 5$, the expected revenue is given by*

$$R_{LAB}^{MS}(T) = \frac{T^3 - 195T^2 + 397T - 549}{150(T-1)^3}.$$

In addition, $\lim_{T\to\infty} R_{LAB}^{MS}(T) = R_{HRB}$.

Proposition 2.5. *The expected revenue of the LAB auction converges to that of an HRB auction as bid increments become small—as the number of bid levels goes to infinity.*

The expected revenue of the LAB auction when bidders follow a straightforward bidding strategy is an upper bound and the expected revenue of the LAB auction when a provisional winner follows a maximum shading strategy is a lower bound of the expected revenue of the LAB auction with profit maximizing bidders. Since the upper and lower bounds converge to 1/3, the expected revenue of the LAB auction converges to 1/3 as well.

With profit maximizing bidders, the expected revenues of the LAB auction with 4–7 bid levels are shown in Table 2.3. The expected revenue is increasing in the number of bid levels at a decreasing rate. We conjecture that the LAB auction always yields lower expected revenue than the HRB auction.

Table 2.3: Expected revenue of LAB auction with 4-7 bid levels

Number of bid levels	Expected revenue
4	0.278
5	0.301
6	0.311
7	0.317
∞	0.333

2.7 Extension to bidders with an anticipated loser's regret at profitable prices

In this section, we extend the model to include the possibility that bidders' anticipated regret of losing at profitable prices will cause them to bid more aggressively than under pure profit maximization. Again we consider the case with two bidders and a single item. Valuations are uniformly distributed on $[0, 1]$ with T equally spaced bid levels.

There is strong support for this view in first-price auctions (Engelbrecht-Wiggans & Katok, 2007, 2008; Filiz-Ozbay & Ozbay, 2007; Delgado et al., 2008). It seems plausible that the same behavioral bias-caring more about the negative emotion coming from losing at profitable prices than the positive emotion of extra profit from successful bid shading-may exist in the dynamic context. In a separate paper, we examine this possibility in the experimental laboratory (Cramton et al., 2009). Here we present the basic theory.

The behavioral theory posits that bidders systematically put too much weight on profits lost from losing at profitable prices and too little weight on profits gained from successful bid shading. In our context it is the bids of a provisional winner that are affected by this asymmetric treatment of profits, since only the provisional winner has an incentive to bid below her valuation.

Define $\tilde{\pi}_{it}(q_{it}, q_{jt}, R_{it})$ as bidder i's payoff if the auction ends given that bidder i submits q_{it}, bidder j submits q_{jt} and R_{it} is bidder i's rank in round t. Recognizing anticipated loser's regret, we define bidder i's payoff function as

$$
\begin{aligned}
\tilde{\pi}_{it}(q_{it}=1, q_{jt}=0, R_{it}=1) &= x_i - P_t \\
\tilde{\pi}_{it}(q_{it}=0, q_{jt}=0, R_{it}=1) &= -\alpha \max\{0, E_t[\tilde{\pi}_{i,t+1}(q_{it}=1, q_{jt}=1, R_{it}=1)]\} \\
\tilde{\pi}_{it}(q_{it}=0, q_{jt}=0, R_{it}=1) &= x_i - P_{t-1} \\
\tilde{\pi}_{it}(q_{it}=1, q_{jt}=0, R_{it}=0) &= x_i - P_t \\
\tilde{\pi}_{it}(q_{it}=0, q_{jt}=1, R_{it}=0) &= -\alpha \max\{0, E_t[\tilde{\pi}_{i,t+1}(q_{it}=1, q_{jt}=1, R_{it}=0)]\} \\
\tilde{\pi}_{it}(q_{it}=0, q_{jt}=0, R_{it}=0) &= -\alpha \max\{0, E_t[\tilde{\pi}_{i,t+1}(q_{it}=1, q_{jt}=0, R_{it}=0)]\}
\end{aligned}
$$

where α is the regret coefficient indicating the strength of anticipated regret. In this section, assume that $0 \leq \alpha < 1$.

Given this payoff structure, it is still a weakly dominant strategy for a provisional loser to bid straightforwardly. Consider a provisional loser with a valuation between P_t and P_{t+1}. In round $t+1$, she could win only at an unprofitable price by bidding. Hence, she does not have regret so she exits. Therefore, it is a weakly-dominated strategy to bid above her valuation. Moreover, exiting below her valuation is still a weakly-dominated strategy as doing so penalizes the bidder even more than in the no-regret case resulting in a negative expected payoff; whereas, bidding yields a positive expected payoff.

For a provisional winner who faces a tradeoff between exiting and bidding, exiting is now less attractive since she anticipates that she will regret if losing. A provisional winner is expected to bid more aggressively when anticipated regret is more intense.

Lemma 2.9. *Suppose a provisional winner follows the maximum shading strategy and a provisional loser follows the straightforward bidding strategy. For any* $T \geq 5$ *and* $0 \leq \alpha < 1$*, the expected revenue is given by*

$$\tilde{R}_{LAB}^{MS}(T) = \frac{1}{150(1+\alpha)^2(T-1)^3}\big(50T^3 - 195T^2 + 397 - 549 \\ +2\alpha(50T^3 - 150T^2 + 163T - 171) + \alpha^2(50T^3 - 105T^2 - 23T - 9)\big).$$

In addition, $\lim_{T\to\infty} \tilde{R}_{LAB}^{MS}(T, \alpha) = R_{HRB}$.

As in the no-regret case, $\tilde{R}_{LAB}^{MS}(T, \alpha)$ is a lower bound of the expected revenue of the LAB auction with fear of losing. The upper bound is the same as the no-regret case in Lemma 2.7. Therefore, the expected revenue of the LAB auction with fear of losing converges to $\frac{1}{3}$.

Nonetheless, in the interesting case of finite T as in any real auction, if the regret coefficient is sufficiently large, the LAB auction yields higher expected revenue than the HRB auction. The necessary condition is given in Proposition 2.6.

Proposition 2.6. *Suppose a provisional winner follows the maximum shading strat-*

egy and a provisional loser follows the straightforward bidding strategy. If $T \geq 7$ *and*

$$\frac{121 - 13T + 5\sqrt{3(27T^4 - 252T^3 + 896T^2 - 1328T + 468)}}{45T^2 - 173T + 41} \leq \alpha < 1,$$

then $\tilde{R}^{MS}_{LAB}(T, \alpha) \geq R_{HRB}$.

Define α^* as the critical regret coefficient such that $\tilde{R}^{MS}_{LAB}(T, \alpha^*) = R_{HRB}$. Figure 2.2 shows the relationship between the critical regret coefficient and the number of bid levels. With six bid levels a regret coefficient of one guarantees that the LAB auction yields higher revenue than the HRB auction. If $T \geq 7$ and α is sufficiently close to one, we can be certain that the expected revenue of the LAB auction is higher. In Section 8, we show that the actual regret coefficient (as opposed to the critical regret coefficient α^*) with six bid levels that makes the LAB and HRB auctions have equal revenues is 0.66, not 1.

2.8 An example with four and six bid levels and fear of losing

By using a similar solution technique as used in Section 5, we can explicitly calculate a perfect Bayesian equilibrium with fear of losing for a small number of bid levels. Consider the case of four bid levels. A provisional winner's bidding strategies when $0 \leq \alpha < 1$ and $\alpha \geq 1$ are as shown in Table 2.4 and 2.5, respectively.

The equilibrium strategy with $\alpha < 1$ is similar to the no-regret case but the critical valuation is instead a function of the regret coefficient. Moreover, in round

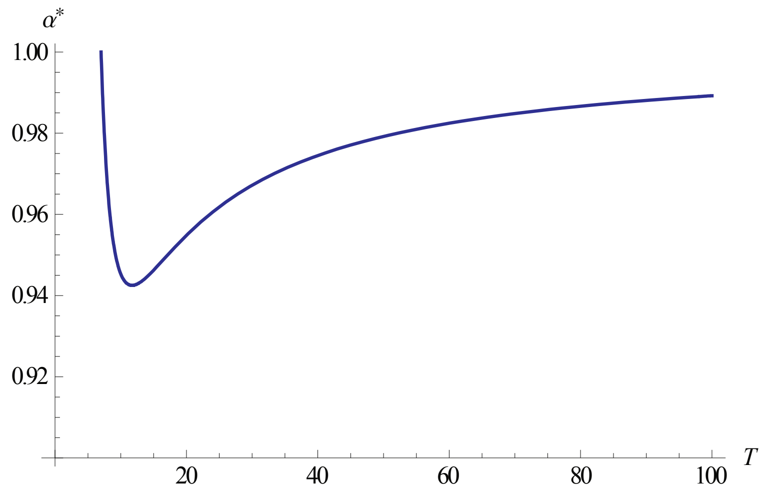

Figure 2.2: Relationship between the critical regret coefficient and the number of bid levels

Table 2.4: Valuation of a provisional winner whose optimal strategy is bidding in each round and ranking history when $0 \leq \alpha < 1$

Round	Price	Ranking History	Value
1	1/3	(1)	$[2/3 + \frac{2(1-\alpha)}{15(1+\alpha)}, 1]$
2	2/3	(1,1)	None
		(0,1)	$[2/3, 1]$
3	1	Any	None

Table 2.5: Valuation of a provisional winner whose optimal strategy is bidding in each round and ranking history when $\alpha \geq 1$

Round	Price	Ranking History	Value
1	1/3	(1)	$[1/3 + \frac{2}{3(1+\alpha)}, 1]$
2	2/3	(1,1)	None
		(0,1)	$[2/3, 1]$
3	1	Any	None

1 a provisional winner with any valuation below 2/3 exits if $\alpha < 1$ but if $\alpha \geq 1$, a provisional winner with valuation close to 2/3 may bid. In both cases, the larger is the regret coefficient, the lesser is the amount of shading and the higher is the expected revenue. Expected revenues when $\alpha < 1$ and $\alpha \geq 1$ are

$$\tilde{R}_{LAB}(4,\alpha) = \begin{cases} \frac{15+19\alpha}{54(1+\alpha)} & ; \text{if } 0 \leq \alpha < 1 \\ \frac{11\alpha^3+27\alpha^2+25\alpha+5}{27(1+\alpha)^3} & ; \text{if } \alpha \geq 1 \end{cases}.$$

Similar calculations can be done for the case of six bid levels. Figure 2.3 plots the expected revenue from the LAB auction with $T = 4$ (solid) and 6 (dashed) as a function of the regret coefficient. Revenue increases with the fear of losing, and exceeds the revenue of 1/3 from the HRB auction if the fear of losing is sufficiently strong. The x-axis is drawn at 1/3, so points above the axis are instances where the LAB auction yields higher revenues than the HRB auction. With four bid levels, the LAB auction yields higher expected revenue than the HRB auction if $\alpha > 1.52$; with six bid levels, the LAB auction yields higher expected revenue than the HBR auction if $\alpha > 0.66$. Since six or more bid levels is typical in practice, we conclude that the LAB auction may yield higher revenues than the HRB auction with plausible levels of fear of losing.

2.9 Lowest-accepted bid with exit bids

Our version of lowest-accepted bid is motivated from India's 3G auction. A variation which may be preferable is the lowest-accepted bid with exit bids (LABx).

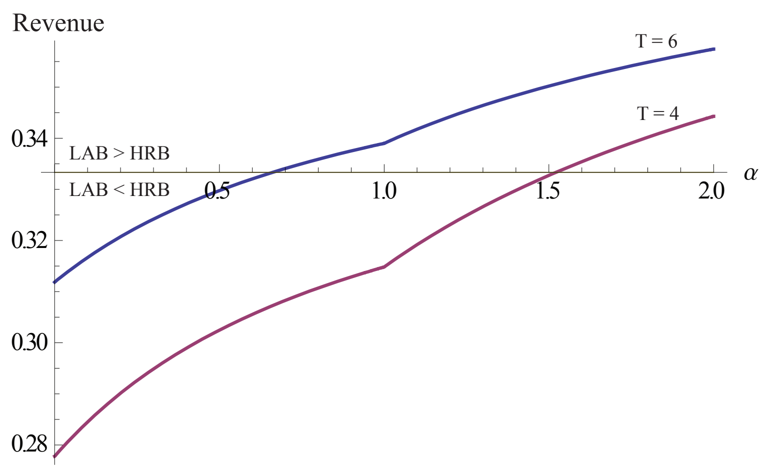

Figure 2.3: Expected revenue of the LAB auction with fear of losing, $T = 4$ **and** $T = 6$

This is identical to HRB in that exit bids are allowed, but the price is set by the lowest-accepted bid, so there is a first-price incentive to shade one's bid.[1]

Assume that there are N bidders whose valuations are distributed on the interval $[0, 1]$ with the distribution function F. If there is excess demand in any round, the auction proceeds to the next round. If a bidder exits in round t, the bidder can submit an exit bid—a price between P_{t-1} and P_t. The final price is determined by the lowest-accepted exit bid. That is, in round t such that $Q_t = K$, the auction ends and the final price is P_t and the items are awarded to active bidders. If $Q_t < K$, the $K - Q_t$-highest exit bid determines the final price and the items are awarded to the active bidders and exiting bidders with the highest $K - Q_t$ exit bids.

Let $\mathbf{M}_t = (M_1, M_2, \ldots, M_t)$ be a vector indicating the number of active bidders from round 1 to round t where $M_t \in \{2, \ldots, N\}$ is the number of active bidders in

[1]Another variation is the pricing rule analyzed in Rothkopf and Harstad (1994) and David et al. (2007). Moreover, such pricing rule is dominated in both revenue and efficiency by highest-rejected bid with exit bids, regardless of whether bidders fear losing.

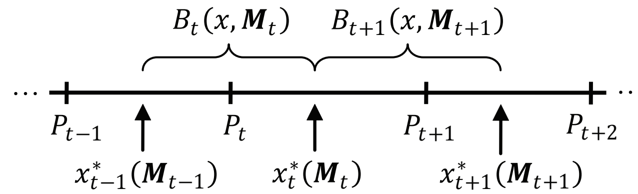

Figure 2.4: Equilibrium Bidding strategy in the LABx auction

round t and $M_1 \geq M_2 \geq \ldots \geq M_t$. Let $\mathbf{\Omega}_t = \{\mathbf{M}_t | M_1 \geq M_2 \geq \ldots \geq M_t\}$ be the set of all possible active bidder histories from round 1 to round t. Assume that the active bidder history $\mathbf{M}_t$ is common knowledge.

Given the history, the bidder forms a belief of a lower bound of active bidders' valuations and chooses whether to bid or submit an exit bid accordingly. Since it is a dominated strategy to bid above one's valuation, the lower bound is at least the previous clock price.

Let $x^*_t : [0, \infty) \times \mathbf{\Omega}_t \rightarrow [0, 1]$ be an intermediate valuation in round t given a regret coefficient and active bidder history. Let $B_t : [0, 1] \times [0, \infty) \times \mathbf{\Omega}_t \rightarrow [P_{t-1}, P_t)$ be an equilibrium exit bid function in round t given a valuation, regret coefficient and a history of the number of active bidders. An equilibrium bidding strategy is defined by these two functions. That is, a bidder with valuation $x \in [x^*_{t-1}(\alpha, \mathbf{M}_{t-1}, x^*_t(\alpha, \mathbf{M}_t)$ exits in round t and submits an exit bid $B_t(x, \alpha, \mathbf{M}_t)$. The equilibrium bidding strategy is illustrated in Figure 2.4.

Both $x^*_t(\alpha, \mathbf{M}_t)$ and $B_t(x, \alpha, \mathbf{M}_t)$ can be derived iteratively from round 1 to round T. In any round t and any $\mathbf{M}_t \in \mathbf{\Omega}_t$, bidders infer that their opponents' valuations are in $[x^*_{t-1}(\alpha, \mathbf{M}_{t-1}, 1]$. The exit bid function is

$$\tilde{B}_t(x, \alpha, z_t, M_t) = \frac{1}{G(s|z_t)^{1-\alpha}} \int_{z_t}^{x} s \mathrm{d}G(s|z_t)^{1-\alpha}$$

where $z_t = x^*_{t-1}(\alpha, \mathbf{M}_{t-1})$ and $G(s|z_t) = F(s|z_t)^{M_t - 1}$.

The critical valuation $x^*_t(\alpha, M_t) \equiv x^*$ can be obtained by solving the condition $\tilde{B}_t(x^*, \alpha, x^*_{t-1}(\alpha, \mathbf{M}_{t-1}), M_t) = P_t$. Note that the equilibrium exit bid function can be rewritten as $B_t(x, \alpha, \mathbf{M}_t) \equiv \tilde{B}_t(x, \alpha, x^*_{t-1}(\alpha, \mathbf{M}_{t-1}), M_t)$.

Proposition 2.7. *A symmetric equilibrium of LABx auction is characterized by critical valuation $x^*_t(\alpha, \mathbf{M}_t)$ and exit bid function $B_t(x, \alpha, \mathbf{M}_t)$ for $t = 1, 2, \ldots, T-1$ and for any $\mathbf{M}_t \in \mathbf{\Omega}_t$. Define $x^*_0(\alpha, \emptyset)$. In round t with a history $\mathbf{M}_t \in \mathbf{\Omega}_t$, a bidder with valuation $x \in [x^*_{t-1}(\alpha, \mathbf{M}_{t-1}), x^*_t(\alpha, \mathbf{M}_t))$ exits in round t and submits an exit bid equal to $B_t(x, \alpha, \mathbf{M}_t)$. For $t = 1, 2, \ldots, T-1$ and for any $\mathbf{M}_t \in \mathbf{\Omega}_t$, $x^*_t(\alpha, \mathbf{M}_t)$ is decreasing in α and $B_t(x, \alpha, \mathbf{M}_t)$ is increasing in x and α.*

Proposition 2.8. *The LABx auction is efficient.*

Because the bidding strategy is monotonic in valuation and symmetric, the allocation is efficient. The use of exit bids overcomes both the inefficiency arising from discrete bid levels and the asymmetry created by ranking.

Proposition 2.9. *Revenue equivalence between HRB and LABx auctions holds if $\alpha = 0$. If $\alpha > 0$, the LABx auction yields higher revenue than the HRB auction.*

Since the allocation rules and expected payoffs of the lowest-valuation bidder of the HRB and LABx auctions are the same, revenue equivalence immediately follows

Table 2.6: Bidding strategy of the LABx auction with four bid levels and fear of losing

Valuation	Exit round	Exit bid
$[0, \frac{2+\alpha}{3(1+\alpha)})$	1	$\frac{1+\alpha}{2+\alpha}x$
$[\frac{2+\alpha}{3(1+\alpha)}, \frac{(2+\alpha)(1+2\alpha)}{3(1+\alpha)^2})$	2	$\frac{1}{3(1+\alpha)} + \frac{1+\alpha}{2+\alpha}x$
$[\frac{(2+\alpha)(1+2\alpha)}{3(1+\alpha)^2}, 1]$	3	$\frac{1+2\alpha}{3(1+\alpha)} + \frac{1+\alpha}{2+\alpha}x$

when $\alpha = 0$. In contrast to the HRB auction, in the LABx auction, bidders submit exit bids below their valuations so that fear of losing at a profitable price impacts the bidding strategy in the LABx auction. Similar to the first-price auction, this fear of losing reduces the amount of shading resulting in higher exit bids relative to the $\alpha = 0$ case and thus implying higher expected revenue.

2.10 An example of the LABx auction with four bid levels and fear of losing

Assume that there are two bidders and each bidder's valuation is uniformly distributed over an interval $[0, 1]$. In the two-bidder case, an active bidder history is irrelevant since the auction ends when any bidder exits. The equilibrium bidding strategy is given in Table 2.6.

If $\alpha = 0$ and both bidders bid in round 1, it implies that both bidders have valuations in $[2/3, 1]$ and thus no bidder exits in round 2. The expected revenue

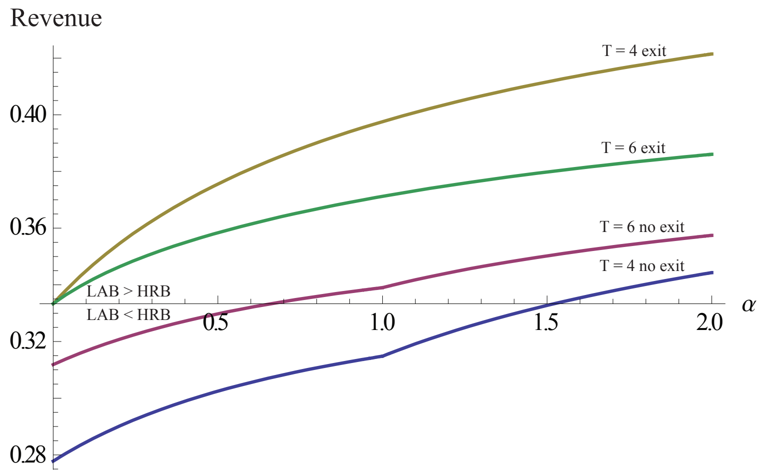

Figure 2.5: revenue of the LAB and LABx auctions with fear of losing, $T = 4$ and $T = 6$

when $\alpha > 0$ is given as follows.

$$\tilde{R}_{LABx}(4, \alpha) = \frac{13\alpha^7 + 98\alpha^6 + 314\alpha^5 + 553\alpha^4 + 580\alpha^3 + 361\alpha^2 + 124\alpha + 18}{27(2 + \alpha)(1 + \alpha)^6}$$

Expected revenues of the LAB and LABx auctions with fear of losing with four and six bid levels are shown in Figure 2.5. As the number of bid levels increases, the expected revenue when $\alpha > 0$ decreases. This is because LABx converges to HRB as the number of bid levels goes to infinity. The LABx auction achieves maximum revenues when the number of bid levels is set to two; that is, the first-price sealed-bid auction achieves the maximum revenues when bidders fear losing.

2.11 Conclusion

The pricing rule is of fundamental importance in practical auction design. It is now well understood that the pricing rule impacts both the efficiency and the revenues of the auction. Although there is an immense literature on the pricing rule in static (sealed-bid) auctions-first-price vs. second-price in single-unit auctions and pay-as-bid vs. uniform-price in multi-unit auctions-little is known about alternative pricing rules in dynamic auctions. This paper begins to fill that gap.

We find that the highest-rejected-bid auction with exit bids (HRB auction) is superior in both efficiency and revenues to the lowest-accepted-bid with provisional winners (LAB auction) when bidders seek to maximize profits. Given this, it may seem odd that India, with a stated objective of revenue maximization, chose the LAB auction.

Behavioral economics provides a plausible explanation for the choice. With the LAB auction, profit maximizing bidders engage in bid shading and therefore face the risk of losing at profitable prices. Bidders who fear losing at profitable prices reduce their bid shading to lessen this risk. Provided the fear of losing is sufficiently strong, the LAB auction revenues exceed those of the HRB auction. Thus, the LAB auction may achieve India's primary objective of maximizing revenues. However, the lowest-accepted bid with exit bids (LABx) is strictly superior to LAB in all cases in both revenue and efficiency. The use of exit bids eliminates both sources of inefficiency in the LAB auction—discrete bid levels and asymmetric bidding strategies caused by provisional winners.

In Cramton et al. (2009), we conducted laboratory experiments to test the theory. The experiments confirm bidding behavior consistent with a significant loser's regret coefficient. Both the LAB and LABx auction achieve significantly higher revenues than the HRB auction. Consistent with the theory, bidders in the

HRB auction tend to bid true values and the efficiency and revenue are as predicted by the theory. With LAB and LABx auctions, the bidders engage in much less bid shading than is seen in the standard theory without a fear of losing. This accounts for the significantly higher revenues under the lowest-accepted bid pricing rule.

Despite the possibility of higher revenues from the LAB format, we would recommend against its use in India or elsewhere. If there are revenue gains, the gains likely are modest (and tightly bounded as we have shown). Offsetting these potential revenue gains are inefficiencies. One source of inefficiency, which we have ignored so far, is bidder participation costs. As we have seen, bidding strategy in the LAB auction is incredibly complex even in the simplest cases. In sharp contrast, bidding strategy in the HRB auction is simple in simple settings: a bidder of modest size and with additive values across regions, which is often a good first approximation, can bid straightforwardly, raising the bid on each region until the bidder's value is reached. The great complexity of bidding strategy under the LAB format is an important reason to favor the strategically simpler and more efficient HRB format.

Our view is that India would be better off in the long-run if it focused on efficient auctions. Efficient auctions are much simpler for bidders and still raise substantial revenues. The long-run revenues of the state are apt to be highest from a policy that promotes the rapid and efficient development of wireless communications. Auction revenues are only one piece of the overall revenues. For a country like India, the much more important piece is the promise of long-term sustainable growth. The rapid and efficient development of wireless communications will play a big role in achieving this growth.

Chapter 3

Discrete Clock Auctions: An Experimental Study

3.1 Introduction

A common method to auction radio spectrum, electricity, gas, and other products is the discrete clock auction. The auctioneer names a price and each bidder responds with her desired quantity. If there is excess demand, the auctioneer then names a higher price. The process continues until there is no excess demand.

Discrete rounds are used in practice to simplify communication, make the process robust to communication failures, and mitigate tacit collusion (Ausubel & Cramton, 2004). An implication of discrete rounds is that the pricing rule matters. The two most common pricing rules are lowest-accepted bid and highest-rejected bid. Another issue is whether the bidder can specify an exit bid-a price less than the current price at which the bidder desires to reduce quantity. In the limit as the size of the bid increment goes to zero, the distinction between pricing rules is irrelevant and exit bids are unnecessary. However, in practical auctions where the number of rounds often ranges from 4 to 10, discreteness matters.

In this paper, we examine bidding behavior under three versions of a discrete clock auction. In each version, after each round the bidders learn the aggregate demand. To prevent bid-sniping, an activity rule requires that a bidder's quantity demanded cannot increase at higher prices. Bidders can only maintain or reduce quantity as the price rises. The three versions differ in the pricing rule and whether bidders make exit bids to express the price at which a quantity reduction is desired.

Highest-rejected bid (HRB). If the bidder reduces quantity in a round, the bidder names a price for each quantity reduction. The price of each reduction must be greater than the prior price and less than or equal to the current price. Each exit price is interpreted as the price at which the bidder is indifferent between the higher quantity and the lower quantity. If there is no excess demand at the current price, the supply is awarded to the highest bidders, and each winner pays the highest-rejected bid for the quantity won. The clearing price is the lowest price consistent with market clearing-the price at which supply equals demand.

Lowest-accepted bid (LAB). This is the same as HRB, except that the winners pay the lowest-accepted bid for the quantity won. The clearing price is the highest price consistent with market clearing.

Lowest-accepted bid with provisional winners (LABpw). This is the same as LAB, except there are no exit bids. Instead after each round, provisional winners are determined. Those with the highest price bid are selected first, and, in the event of a tie, the remaining provisional winners are selected at random.

The clock auction is best thought of as a dynamic version of a sealed-bid uniform-price auction. In the uniform-price auction, the auctioneer collects a demand curve from each bidder, forms the aggregate demand curve, and crosses it with the supply curve to determine the market clearing price and the quantity won by each bidder. The clock auction does the same thing, but gathers the demand curves from each bidder in a sequence of discrete rounds, and bidders receive information about excess demand at the end of each round. The uniform-price auction is just a single-round clock auction. In both clock auctions and uniform-price auctions, two pricing rules are commonly used: highest-rejected bid and lowest-accepted bid. This is the motivation for our HRB and LAB treatments.

The LAB rule is used in US Treasury auctions, as well as Treasury auctions in many other countries. These are sealed-bid uniform-price auctions. In contrast, clock auctions for electricity and gas products in Europe have used the HRB rule.

Our third treatment, lowest-accepted bid with provisional winners, is a version of the simultaneous ascending auction commonly used to auction radio spectrum. The government of India used this format for its 2010 3G spectrum auction. Similar approaches have been used elsewhere, such as in Italy's 3G spectrum auction.

At first glance, it would seem that selecting the highest clearing price (LAB) would result in greater revenue than selecting the lowest clearing price (HRB). The argument is incomplete, since the pricing rule influences behavior. LAB provides a stronger incentive for shading one's bid below valuation. In simple cases (assuming symmetry, independence, and risk neutrality), the greater bid shading under LAB exactly offsets the revenue gain from selecting the higher clearing price. Revenue equivalence obtains, and the two pricing rules result in the same expected revenue-at least in theory when bidders seek to maximize profits.

It is a robust and puzzling finding of the experimental literature that in second-price sealed-bid auctions subjects bid more than their value although they bid truthfully in its dynamic counterpart, the English auction (Cooper & Fang, 2008; Coppinger et al., 1980; Kagel et al., 1987; Kagel & Levin, 1993). The HRB auction is somewhere between the English and second-price auctions and these two are the limit cases of HRB. HRB converges to the English auction (continuous clock) when the number of rounds approaches to infinity. When there is only one round, HRB is the same as the second price auction. The exit bid decision in HRB requires similar strategic thinking as in a second-price sealed-bid auction. On the other hand, whether to stay in the auction is the same binary decision as in the English auction. Harstad (2000) finds that experience with the English auction leads to less overbidding in second-price auctions. Moreover, he argues that the binary choice aspect of English auctions explains an important part of this learning. The HRB format allows subjects to make that kind of binary choice for early price increments and when they come to the price interval when they want to exit, they have this experience. In this respect, studying discrete clock auctions helps us understand what makes subjects overbid in second-price auctions and bid truthfully in English auctions.

Since we find experimentally that subjects do not deviate from the straightforward bidding strategy in HRB, it is confirmed that dynamic formats make it easier for the bidders to recognize equilibrium strategies. Having a discrete clock makes HRB more practical than the English auction, and yet it still is successful in eliciting the true values in our experiment.

The LAB pricing rule is analogous to a first-price sealed-bid auction. In particular, if there is only one round and one good, the LAB and first price auctions are the same. Another robust finding of the experimental literature is that bidders of a first-price auction overbid compared to the risk-neutral Nash equilibrium prediction

(Cox et al., 1982, 1988; Kagel & Levin, 2008)[1]. Therefore, we expect to see some sort of overbidding in LAB auction.

The purpose of this paper is to examine the bidding behavior and outcomes, especially efficiency and revenue, under the three different formats experimentally. Our main hypothesis is that subjects will overbid under lowest-accepted bid, consistent with the first-price sealed-bid auction; whereas, under highest-rejected bid, bidders will bid truthfully. Thus, revenues under LAB will be higher than revenues under HRB. However, efficiency will be higher under HRB, as a result of the truthful bidding. Although the formats apply to the general case of auctioning many units of multiple products, for simplicity we restrict attention to the case of auctioning a single good.

Our results indicate that revenues under both LAB and LABpw are significantly higher than under HRB. Thus, in settings where revenue is the predominant objective, the seller may favor LAB, but in settings where both efficiency and simplicity are of greater concern, then the seller may favor HRB.

There are a number of experimental papers that use clock auctions. Some of these papers use a continuous clock (Kagel & Levin, 2001). Others use a discrete clock, and compare a sealed-bid auction with a particular discrete clock (Ausubel et al., 2009). This paper aims to understand the implications of different pricing rules in discrete clock auctions.

In Section 3.2, we begin with a presentation of the theory. Equilibrium bidding

[1] Risk aversion offers one explanation, but this has proven inadequate (Kagel, 1995). Several papers explain the overbidding phenomena with behavioral motives (Goeree et al., 2002; Crawford & Iriberri, 2007; Delgado et al., 2008; Filiz-Ozbay & Ozbay, 2007; Lange & Ratan, 2010)

strategies for the three versions of the discrete clock auction are characterized in Cramton and Sujarittanonta (2010). Here we summarize the results, and provide the equilibrium for bidding strategies for our experimental setting. The experimental design and the results are discussed in Sections 3.3 and 3.4. Section 3.5 concludes.

3.2 Theory

There is one indivisible good for sale to $N > 1$ risk neutral bidders. Bidder i's private value for the good is v_i where each v_i is independently drawn from a commonly know distribution F. Bidder i's payoff if she wins the good at a price p is $v_i - p$. Otherwise, it is equal to 0. The seller values the good at 0.

Before the auction starts, the seller announces a vector of bid levels, $P = (P_0, P_1, \ldots, P_{T-1})$ where P_t is the price at round t and T is the number of bid levels. The clock price increases every round so that $\underline{v} = P_0 < P_1 < \cdots < P_{T-1} = \bar{v}$. The auction begins in round one at a price P_1.

In each round t, each bidder chooses either to bid at the current clock price or to exit. Once a bidder exits she cannot bid again. If both bidders stay in, the auction proceeds to the next round. If one bidder exits, then the bidder who stayed in wins the good. The resolution of the other cases and the payments will depend on the auction format.

3.2.1 Highest-rejected-bid (HRB)

In the HRB format, the final price is determined by the highest-rejected bid. If a bidder exits in round t, the bidder submits an exit bid—a price between P_{t-1} and P_t at which she wants to exit. If more than one bidder remains, the auction continues to another round; if all but one bidder exits, the remaining bidder wins and pays the highest exit bid; if all bidders exit, the bidder with the highest exit bid wins and pays the second-highest exit bid.

Proposition 3.1. *In the HRB auction, truthful bidding (bidding up to one's value) is a weakly-dominant strategy. The HRB auction is efficient and maximizes seller revenue.*

The dominant strategy result is standard and holds regardless of the number of bidders, the number of goods, or how values are drawn. All that is required is that each bidder demands only a single good. Thus, highest-rejected bid is the Vickrey price, thereby inducing truthful bidding. Efficiency is an immediate implication of truthful bidding.

The HRB auction has extremely desirable properties in our setting. It is both efficient and maximizes seller revenues. Moreover, the bidding strategy is simple—just bid up to your true value—and is best regardless of what the other bidders are doing. Another important property of the HRB auction is that a bidder cannot lose at an affordable price as long as she bids her value. Hence, neither the winner nor the loser ever regrets having bid as they bid. The winner could not do better by exiting earlier; the loser could not do better by staying in longer.

3.2.2 Lowest-accepted-bid (LAB)

In the LAB format, the final price is determined by the lowest-accepted bid. If a bidder exits in round t, the bidder submits an exit bid-a price between P_{t-1} and P_t at which she wants to exit. If more than one bidder remains, the auction continues to another round; if all but one bidder exits, the remaining bidder wins and pays the current price; if all bidders exit, the bidder with the highest exit bid wins and pays her exit bid.

Proposition 3.2. *In the symmetric equilibrium of the LAB auction (see Table 3.1), a bidder, who exits bid at* P_t*, bids* $b(v_i, P_{t-1}) = E[Y|P_{t-1} < Y_k < v_i]$ *where* Y_k *is the highest of* $k-1$ *independently drawn values and* k *is the number of bidders present at* P_{t-1}*. The LAB auction is efficient and maximizes seller revenue.*

The LAB auction in our setting is efficient and maximizes seller revenues. Nonetheless, one might favor the HRB auction because of its simple bidding strategy without bid shading. In the LAB auction, bidders must do a difficult equilibrium calculation to determine the optimal level of bid shading.

3.2.3 Lowest-accepted-bid with provisional winners (LABpw)

Our third version of the discrete clock auction has provisional winners instead of exit bids. This approach is common in spectrum auctions, and was used in the 2010 India 3G spectrum auction. It is theoretically investigated in Cramton and Sujarittanonta (2010).

With two bidders, one of the bidders is selected at random as the provisional winner at the reservation price P_0. In each round, each bidder chooses either to bid at the current price or to exit. If both bidders stay in, one of the bidders is selected at random as the provisional winner and the auction proceeds to the next round. If one bidder exits, then the bidder who stayed in wins the good at the current price. If both bidders exit, then the provisional winner wins the good at the prior price.

An important difference between the auction described here and a standard ascending-bid auction is a provisional winner must keep topping her own bid in order to be eligible to bid in subsequent rounds. In contrast, in a standard ascending-bid auction, a provisional winner does not need to bid, since if her bid is topped, she can still bid in the next round.

Proposition 3.3. *In a perfect Bayesian equilibrium of the LABpw auction, the provisional loser stays in provided her value is not yet reached; the provisional winner exits at a level below her true value. The exit level depends on the provisional winning history. The outcome is inefficient and does not maximize seller revenue.*

The LABpw auction uses provisional winners, rather than exit bids, to determine who wins in the event both bidders exit in the same round. This creates two sources of inefficiency. First, without exit bids, there is no precise value information to determine who has the higher value. Second, the provisional winner designation creates differential bid shading (provisional winners shade bids, whereas provisional losers do not). Since in our setting there is no conflict between efficiency and revenue maximization, it is clear that the LABpw auction does not maximize seller revenue. The seller should favor HRB or LAB on both efficiency and revenue grounds. In

Table 3.1: Equilibrium Strategy for LAB auction

Value	Exit Round	Exit bid function
$[50, 70)$	1	$25 + \frac{1}{2}v_i$
$[70, 90)$	3	$35 + \frac{1}{2}v_i$
$[90, 100]$	5	$45 + \frac{1}{2}v_i$

addition, the equilibrium strategies in LABpw are greatly complicated by history dependence.

3.2.4 Theoretical predictions for the experiment

In our experiment, two bidders compete to buy a single good. Bidder i's private value for the good is v_i where each v_i is independently drawn from the uniform distribution on $[50, 100]$ and $\mathbf{p} = (50, 60, 70, 80, 90, 100)$. By Proposition 3.1, in HRB the bidder submits her value as an exit bid. By Proposition 3.2, the equilibrium strategy is given in Table 3.1.

Ex ante symmetry-the fact that the bidders' values are drawn from the same distribution-is critical for Proposition 3.2. This allows for a symmetric equilibrium. Since the exit bid functions are the same and strictly increasing, the bidder with the highest value wins and the outcome is efficient. Revenue maximization then follows from the revenue equivalence theorem. The assignment is the same as in the HRB auction, and both auctions give the bidder with a value of 50 a payoff of 0.

In LABpw, the provisional loser stays in provided her value is not yet reached; the exit strategy for the provisional winner is given in Table 3.2.

Table 3.2: Provisional Winner's Equilibrium Strategy for LABpw auction

Round	History[a]	Critical Value[b]
1	(1)	74
2	$(1, 1)$	84
	$(0, 1)$	70
3	$(1, 1, 1)$	94
	$(0, 1, 1)$	90.8
	$(0, 0, 1)$	80
4	$(1, 0, 0, 1)$, $(0, 1, 0, 1)$ and $(0, 0, 0, 1)$	90
	Otherwise	100
5	Any	100

[a] History vector denotes whether the bidder is a provisional winner (denoted by 1) or not (denoted by 0) in all the rounds up to the current round.
[b] Critical Value denotes the threshold such that the bidder stays in if her value is above the corresponding threshold.

3.3 Experimental method

The experiments were run at the Experimental Economics Lab at the University of Maryland. All participants were undergraduate students at the University of Maryland. The experiment involved six sessions. In each session one of the three treatments was administered. The numbers of participants in each treatment were 30 (HRB), 32 (LAB), and 30 (LABpw). No subject participated in more than one session. Participants were seated in isolated booths. Each session lasted about 80 minutes. Bidder instructions for each treatment are in the Appendix C. To test each subject's understanding of the instructions, the subject had to answer a sequence of multiple choice questions. The auctions did not begin until the subject answered all of the multiple choice questions correctly.

In each session, each subject participated in 21 auctions. The first auction was a practice auction. Each auction had two bidders, selected at random among the subjects. Bidders were randomly rematched after each auction. All bidding was anonymous. Bids were entered via computer. The experiment is programmed in z-Tree (Fischbacher, 2007). At the conclusion of each auction, the bidder learned whether she won and the price paid by the winning bidder.

Bidders had independent private values for a fictitious good. All values were uniformly distributed between 50 and 100, rounded to the nearest cent. Both bidders were IN at the starting price of 50. The price increased by 10 if both bidders stayed IN the prior round. Thus, the possible price levels in each auction were 50, 60, 70, 80, 90, and 100. There were a maximum of five rounds in the discrete clock auction. The auction concluded as soon as a round was reached in which one or both bidders stayed OUT.

Treatment HRB. In each round, the computer asks the bidder if she is IN or OUT at the current price. If the bidder stays OUT, the bidder must specify an exit bid between the current price and the prior price. If the bidder stays IN and the opponent stays OUT, then the bidder wins and pays the opponent's exit bid. If the bidder stays OUT and the opponent stays IN, then the opponent wins at the bidder's exit bid. If both stay OUT, then the bidder with the higher exit bid wins and pays the smaller of the exit bids. If both stay IN, then the price increases by 10 and the auction continues to the next round.

Treatment LAB. In each round, the computer asks the bidder if she is IN or OUT at the current price. If the bidder stays OUT, the bidder must specify an exit bid between the current price and the prior price. If the bidder stays IN and the opponent stays OUT, then the bidder wins and pays her exit bid. If the bidder stays OUT and the opponent stays IN, then the opponent wins and pays her exit bid. If both stay OUT, then the bidder with the higher exit bid wins and pays her exit bid. If both stay IN, then the price increases by 10 and the auction continues to the next round.

Treatment LABpw. One of the bidders is selected at random as the provisional winner at the price of 50. In each round, the computer asks the bidder if she is IN or OUT at the current price. If the bidder stays IN and the opponent stays OUT, then the bidder wins and pays the current price. If the bidder stays OUT and the opponent stays IN, then the opponent wins at the current price. If both stay OUT, then the provisional winner of the prior round wins and pays the prior round price. If both stay IN, then one of the bidders is selected at random as the provisional winner at the current price, price increases by 10 and the auction continues to the next round.

The winner in each auction earned her value minus the price paid in Experimental Currency Units (ECU). At the end of the experiment, total earnings were converted to US Dollars, at the conversion rate of 10 ECU = 1 US Dollar. Subjects also received a $5 show-up fee. Cash payments were made at the conclusion of the experiment. The average subject payment was $19.77.

Table 3.3: Outcomes of treatment HRB, LAB and LABpw

	HRB	LAB	LABpw
Frequency of efficient allocation	92.0%	90.3%	85.3%
	(1.57)	(1.65)	(2.04)
Revenue per auction	67.35	69.85	69.33
	(0.70)	(0.60)	(0.65)
Seller's share of gains from trade	80.8%	84.8%	85.3%
	(0.86)	(0.71)	(0.77)
Theoretical revenue per auction	67.06%	67.12%	65.27%
Number of auctions	300	320	300

Standard errors are shown in the parentheses.

3.4 Experimental results

As in any dynamic auction, bidding strategies of all bidders are not observable. For example, in the Dutch auction only the bid of the winner, or in the English auction only the bids of the losers are observable. Nonetheless, it is still possible to make revenue and efficiency comparisons, which are important in auction design. Later, we will study the bidding strategies based on the observable bids.

Table 3.3 shows the outcomes of each treatment. Treatment HRB, LAB and LABpw consist of 300, 320 and 300 auctions, respectively. Treatments HRB and LAB, which are theoretically efficient, yield the efficient allocation with a frequency of 92% and 90.3%, respectively. Treatment LABpw yields the efficient allocation 85.3% of the time. Using the Wilcoxon-Mann-Whitney test, LABpw is significantly less efficient than HRB ($z = 2.57, p = 0.01$), but there is no significant difference between LAB and HRB ($z = 0.74, p = 0.46$).

On the other hand, treatment LAB and LABpw yield higher revenues than

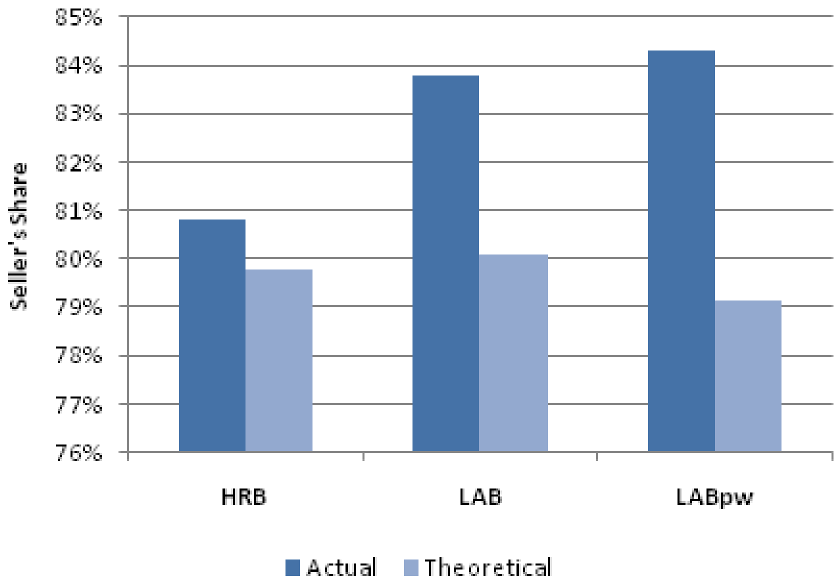

Figure 3.1: Actual and theoretical seller's share of gains from trade per auction

HRB (Table 3.3). The revenue results are robust even if we consider the seller's share of the gains from trade (the ratio of the price and winner's value) which can be thought of as the proxy for the revenues. Treatment HRB gives the seller a smaller share of the gains from trade than treatment LAB and LABpw. The Wilcoxon-Mann-Whitney test shows LAB and LABpw yield significantly higher seller's share than HRB ($z = -3.28, p < 0.01$ and $z = -3.25, p < 0.01$, respectively).

Figure 3.1 compares actual (what we observed in the experiment) and theoretical seller's share of the gains from trade per auction. Theoretical seller's share is the equilibrium seller's share given the realization of values. The actual seller's share of HRB is not significantly different from the theoretical prediction ($t = 0.41, p = 0.68$), but actual seller's share of LAB and LABpw are significantly higher than the theoretical share ($t = 4.51, p < 0.01$ and $t = 6.27, p < 0.01$). This evidence implies that

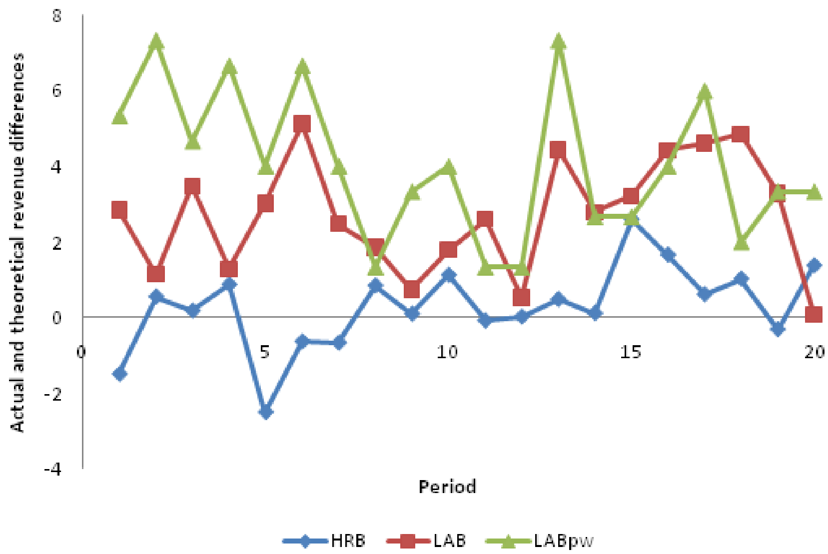

Figure 3.2: Average differences between actual and theoretical revenues per period

subjects bid more aggressively than the equilibrium prediction and as a result the seller receives higher revenue. These aggressive bidding behaviors are investigated in the next subsections.

Figure 3.2 plots the average differences between the actual and theoretical revenues period by period. The revenues we observed in the HRB are close to the prediction of the theory, however the actual revenues are higher than the theoretical ones both in LAB and in LABpw. These results suggest that the subjects follow a truthtelling strategy in HRB, but they overbid in LAB and LABpw. Next, we will study the bidding strategies in detail.

3.4.1 Bidding behavior in treatment HRB

Figure 3.3 plots exit bids and corresponding values in HRB. The dashed line is a truthful bid function. To further examine bidding behavior, we estimate regressions of exit bids on values with bidder fixed effects, clustered by bidder. Subjects submitted a total of 342 exit bids. The deviation from truthful bidding is on average close to zero but there are instances of bid shading and bidding above one's value. 83 percent of exit bids were submitted in the round that theory predicts. The vast majority of subjects were successful in waiting until the right round to exit. Since the binary decision of staying in or out is similar to the strategic thinking of the English auction, this finding is in line with the success of value bidding in English auction experiments. (Harstad, 2000) finds that subjects who gain experience in auctions with this kind of binary decision making perform better in discovering the value bidding strategy in second-price sealed-bid auctions. Staying in the HRB auction until the price reaches the correct level provides a similar experience as participating in an English auction. Hence, once a subject reaches that round her exit bid is close to the true value. In line with this argument, the deviations from truthful bidding diminish for higher valuations. The reason might be that a bidder with high value makes more binary decisions before the price reaches her value and this helps her discover equilibrium exit bids.

We regress exit bid on value with subject fixed effects, clustered by subject. The regression results are shown in Table 3.4. We cannot reject the null hypothesis that the coefficient of value is equal to one.

Table 3.5 shows the averages of the ratios of exit bids to values for each period. We observed 15 to 20 exit bids in each period. As the ratios in all periods are close to one, subjects on average bid truthfully throughout the experiment.

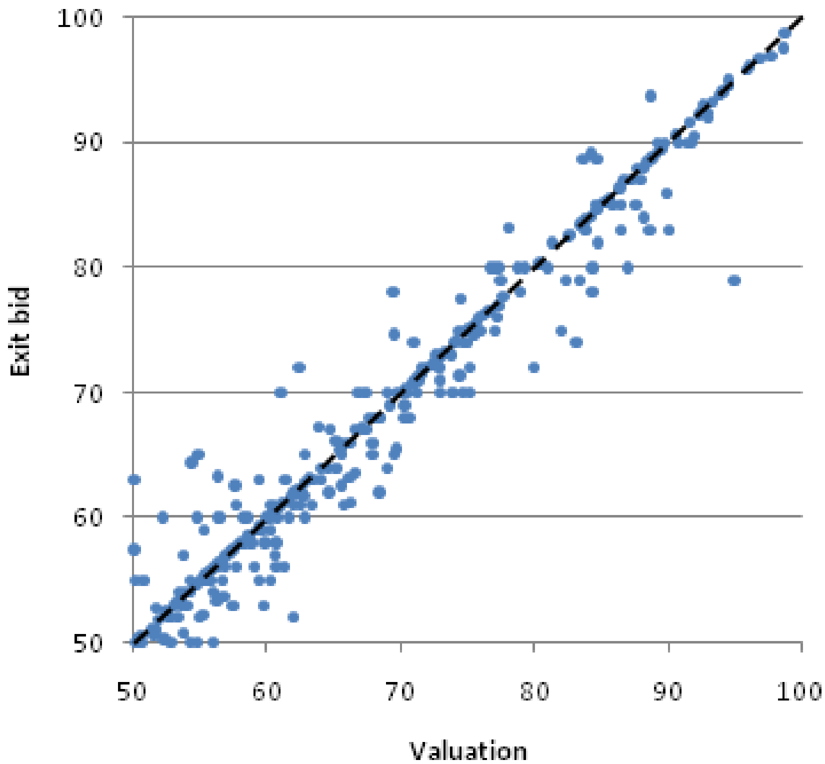

Figure 3.3: Plot between values and exit bids in HRB auction

Table 3.4: Regression of exit bids on values in HRB auction

Independent variables	(1)
Constant	1.93
	(1.20)
Value	(1)
	(0.02)
R-squared	0.949

* Significant at 95% confidence interval. Standard errors are shown in the parentheses. Sample size is 342.

Table 3.5: Average ratio of exit bid to value by period in HRB auction

Period	Exit bid/value	Period	Exit bid/value
1	0.98 (0.04)	11	1.00 (0.04)
2	0.98 (0.04)	12	1.00 (0.04)
3	0.99 (0.07)	13	1.00 (0.05)
4	1.00 (0.08)	14	1.00 (0.04)
5	0.97 (0.05)	15	1.01 (0.05)
6	0.98 (0.02)	16	1.00 (0.05)
7	0.99 (0.03)	17	1.01 (0.03)
8	0.99 (0.03)	18	1.01 (0.04)
9	1.00 (0.04)	19	1.00 (0.04)
10	1.00 (0.05)	20	0.99 (0.02)

Standard deviations are shown in the parentheses.

3.4.2 Bidding behavior in treatment LAB

A total of 386 exit bids were submitted in LAB. The actual seller's share of gains from trade per auction is significantly higher than the theoretical prediction. This is the case because subjects bid more aggressively than the theory predicted as shown in Figure 3.4. We consider an exit bid as equilibrium bidding if it is within 1 percent of the theoretical prediction, otherwise as overbidding or underbidding. Starting from the first period, overbidding is observed and it is persistent throughout the experiment. 66 percent of exit bids lie above the theoretical exit bid.

3.4.3 Bidding behavior in treatment LABpw

A total of 367 exit decisions are observed. As predicted by the theory (Propositions 3.3), provisional losers tended to stay in until value was reached (only 11

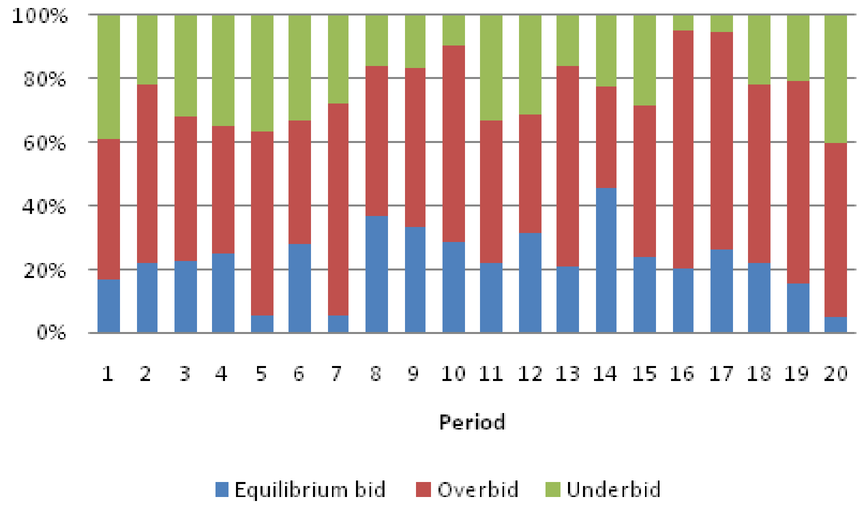

Figure 3.4: Percentage of over-, under-, equilibrium-bidding per period in LAB auction

violations out of 600 auctions). Additionally, similar to treatment LAB, aggressive bidding behavior in treatment LABpw is observed, as indicated by the higher revenue than the theoretical prediction and a mean exit price that is higher than the theoretical exit price. Figure 3.5 shows a histogram of differences between the actual and theoretical exit round. There are 91 instances of bidders staying in extra rounds versus 33 instances of bidders exiting early compared with the theory. In addition, among 233 active bidders at the end of the auction, 18 bidders bid more than value. It was mostly the provisional winners who failed to follow the equilibrium.

3.4.4 Learning

All auction formats are played 20 periods. In order to see if there is any difference between bidders' early and late plays, and whether learning is taking place, we compare the actual and theoretical exit bids throughout the experiment. We

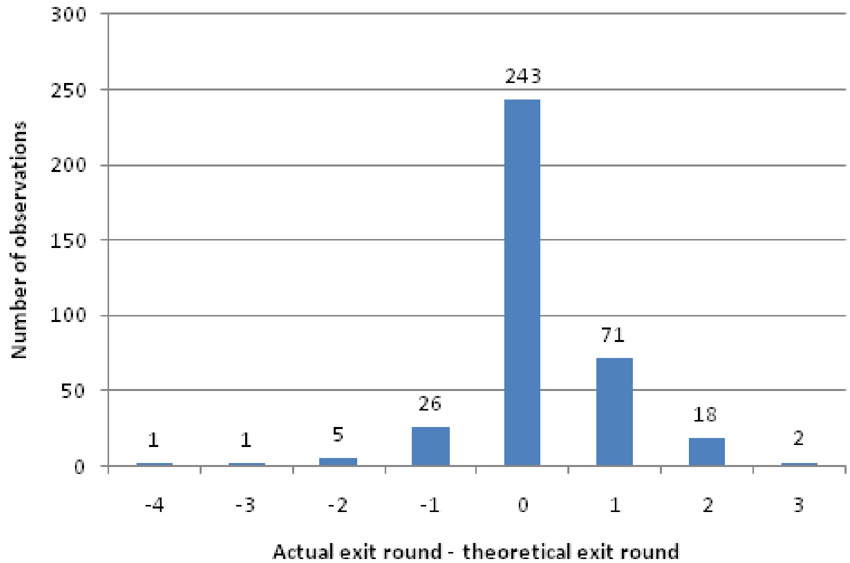

Figure 3.5: Histogram of differences between actual and theoretical exit rounds in LABpw

measure the deviations from equilibrium by the absolute percentage deviation: the absolute value of the difference between actual and theoretical exit bid divided by the theoretical exit bid for each auction. Figure 3.6 shows average absolute percentage deviations by period. Figure 3.6 suggests that bids in HRB and LAB auctions do not change much as bidders gain experience. HRB deviations are within 4 percent of the theoretical exit bids from the beginning of the session. LAB deviations do not vary much over time either. However, the deviations in LAB are consistently higher than in HRB. Absolute percentage deviations in LABpw are the highest, and show some tendency to decline over time, especially in the final five periods. Since provisional losers are successful in discovering equilibrium strategy, the variation in Figure 3.6 in LABpw is due to deviations of the provisional winners.

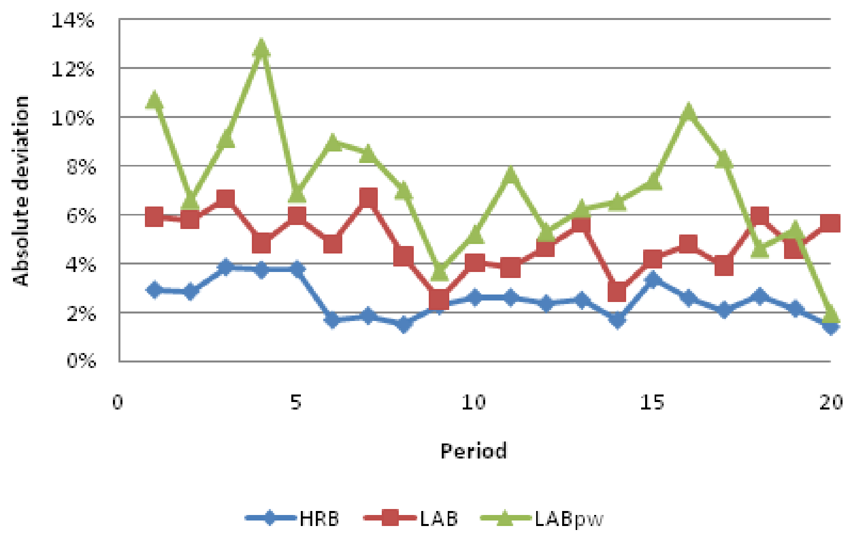

Figure 3.6: Average absolute deviation by period

3.5 Conclusion

The pricing rule is of fundamental importance in practical auction design. Pricing impacts both the efficiency and the revenues of the auction. Although there is an immense literature on the pricing rule in static (sealed-bid) auctions-first-price vs. second-price in single unit auctions and pay-as-bid vs. uniform-price in multi-unit auctions-little is known about alternative pricing rules in dynamic auctions commonly used in practice. We show how different pricing rules influence bidding behavior in discrete clock auctions in a simple setting.

Based on the standard theory in which bidders seek to maximize profits, the highest-rejected-bid (HRB) and the lowest accepted-bid (LAB) auctions seem equally good. They maximize revenues and are fully efficient in our unit-demand setting. Despite this theoretical result, LAB pricing is often used in practice. In our experiments, the LAB auction, both with and without exit bids, yielded higher revenues than the HRB auction. The HRB auction did better on efficiency grounds than LAB with provisional winner, but not significantly so when compared to LAB. Although there is a significant overbidding in LAB, this overbidding is not as severe as LAB's limit case, first-price sealed-bid auction. Perhaps, the dynamic nature of LAB enables bidders to handle the complex bidding strategies. It may be interesting to study the discrete clock auctions by varying the number of rounds and price increments. This may enable us to understand better the overbidding puzzle in sealed-bid auctions.

Developing strategies in HRB requires two things: when to exit as in an English auction, and what exit bid to submit as in a second-price sealed bid auction. Bidders submit exit bids close to their valuations. The tendency of bidding above value found in second-price auctions is not observed here. We conjecture that this is due to the experience bidders gain in deciding whether to exit each round. That thinking process is similar to decision making in an English auction. Harstad (2000) finds that practicing English auctions improves performance in second-price sealed-bid auctions. Similarly, our experiment shows that the multiple-round implementation of the second-price auction eliminates the tendency to overbid when faced with second-price incentives. The dynamic implementation also limits the spread between the highest bid and the price to at most one bid increment.

In situations like spectrum auctions where efficiency is the most important objective rather than revenue, we recommend the HRB auction since it achieves the efficient outcome with a simple value-bidding strategy. On the other hand, bidding strategy in the LAB auction is complex even in the simplest cases.

Appendix A

Missing proofs from Chapter 1

Proof of Proposition 1.1. According to Definition 1.3, incentive compatibility implies that $\tilde{u}_i(x_i) = \max_{z_i \in X_i} \tilde{q}_i(z_i)x_i - \tilde{t}_i(z_i)$. So, $\tilde{u}_i(x_i)$ is a convex function. With envelope theorem, $\tilde{u}_i'(x_i) = \tilde{q}_i(x_i)$. That is, the slope of $\tilde{u}_i(x_i)$ is $\tilde{q}_i(x_i)$. Convexity of $\tilde{u}_i(\cdot)$ implies that $\tilde{q}_i(\cdot)$ is non-decreasing. Suppose $\mu_i(\alpha) = m_{is}$ and $\mu_i(\beta) = m_{it}$ where $\alpha, \beta \in X_i$, $\alpha > \beta$, $m_{is}, m_{it} \in M_i$ and $s \neq t$. Since $\tilde{q}_i(\cdot)$ is non-decreasing, $\tilde{q}_i(\alpha) = q_i(\mu_i(\alpha)) = q_i(m_{is}) \geq q_i(m_{it}) = q_i(\mu_i(\beta)) = \tilde{q}_i(\beta)$. □

Proof of Proposition 1.2. Integrating both sides of $\tilde{u}_i'(x_i) = \tilde{q}_i(x_i)$ yields

$$\tilde{u}_i(x_i) = \tilde{u}_i(\underline{x}_i) - \int_{\underline{x}_i}^{x_i} \tilde{q}_i(z)\mathrm{d}z$$

Substituting $\tilde{u}_i(x_i) \equiv \tilde{q}_i(x_i)x_i - \tilde{t}_i(x_i)$ and rearranging yields the expected payment given in Proposition 1.2. □

Proof of Proposition 1.3. Since there are M_i elements in the message space $\mathcal{M}_i$, the range of $\mu_i(\cdot)$ also consists of M_i elements. Thus, bidder's type X_i can be partitioned into M_i segments defined by $\mathbf{Y}_i = (Y_{i0}, Y_{i1}, \ldots, Y_{i,M_i-1})$ such that $\mu_i(\alpha) = m_j$ where $\alpha \in Y_{ij}$ or, equivalently, $Y_{ij} = \mu_i^{-1}(\{m_j\})$. Let Y_{is} and Y_{it} be segments such that $\mu_i(\alpha) = m_{is}$ and $\mu_i(\beta) = m_{it}$ where $\alpha \in Y_{is}$ and $\beta \in Y_{it}$. The sufficient condition for incentive compatibility is that if $q_i(m_{is}) > q_i(m_{it})$, any $\alpha \in Y_{is}$ must be greater than all $\beta \in Y_{it}$. The condition holds when each segment is convex. For any $i \in \mathcal{I}$ and $j = 0, 1, \ldots, M_i - 1$, the partition is defined as $\hat{x}_{i,j-1} = \inf Y_{ij}$. We can redefine $\tilde{q}_i(x_i)$ as a step function $\{(\hat{x}_{ik}, \tilde{q}_i(x_{ik}))\}_{k=0}^{M_i-1}$. □

Proof of Proposition 1.4. Consider $x_i \in [\hat{x}_{ij}, \hat{x}_{i,j+1})$.

$$\begin{aligned} t_i(m_i) = \tilde{t}_i(x_i) &= q_i(m_{ij})x_i - \int_{\underline{x}_i}^{x_i} \tilde{q}_i(z)\mathrm{d}z - u_i(\underline{x}_i) \\ &= q_i(m_{ij})\hat{x}_{ij} + q_i(m_{ij})(x_i - \hat{x}_{ij}) - \int_{\underline{x}_i}^{\hat{x}_{ij}} \tilde{q}_i(z)\mathrm{d}z \\ &\quad - \int_{\hat{x}_{ij}}^{x_i} \tilde{q}_i(z)\mathrm{d}z - u_i(\underline{x}_i) \end{aligned}$$

Since $\tilde{q}_i(z)$ is constant for $z \in [\hat{x}_{ij}, x_i]$, $\int_{\hat{x}_{ij}}^{x_i} \tilde{q}_i(z)\mathrm{d}z = q_i(m_{ij})(x_i - \hat{x}_{ij})$. Hence,

$$\begin{aligned} t_i(m_i) &= q_i(m_{ij})\hat{x}_{ij} - \int_{\underline{x}_i}^{\hat{x}_{ij}} \tilde{q}_i(z)\mathrm{d}z - u_i(\underline{x}_i) \\ &= q_i(m_{ij})\hat{x}_{ij} - \sum_{k=1}^{j-1} \left(q_i(m_{ik}) - q_i(m_{i,k-1})\right)\left(\hat{x}_{ij} - \hat{x}_{ik}\right) - q_i(m_{i0})\hat{x}_{ij} - u_i(\underline{x}_i) \\ &= \sum_{k=1}^{j} \left(q_i(m_{ik}) - q_i(m_{i,k-1})\right)\hat{x}_{ij} \\ &\quad - \sum_{k=1}^{j-1} \left(q_i(m_{ik}) - q_i(m_{i,k-1})\right)\left(\hat{x}_{ij} - \hat{x}_{ik}\right) - u_i(\underline{x}_i) \\ &= \sum_{k=1}^{j} \left(q_i(m_{ik}) - q_i(m_{i,k-1})\right)\hat{x}_{ik} - u_i(\underline{x}_i) \end{aligned}$$

□

Proof of Proposition 1.5. The probability of trade can be derived from the allocation rule in Definition 1.5. Bidder i will win with message m_j when (1) all other bidders submit messages with lower priority or (2) some bidders submit the same message m_j, other bidders submit messages with lower priority and bidder i wins the tiebreaker. The probability that the former scenario happens is the first term. For the latter scenario, the number of bidders who submit the same message m_j ranges from one to $I-1$. Therefore, the second term is a sum of a probability that bidder i wins the tiebreaker with k-way tie for $k = 1, 2, \ldots, I-1$. Therefore,

$$\begin{aligned}
q(m_j) &= F(\hat{x}_{j-1})^{I-1} + \sum_{k=1}^{I-1} \frac{1}{k+1} \binom{I-1}{k} (F(\hat{x}_j) - F(\hat{x}_{j-1}))^k F(\hat{x}_{j-1})^{I-k-1} \\
&= F(\hat{x}_{j-1})^{I-1} + \sum_{k=2}^{I} \frac{1}{k} \binom{I-1}{k-1} (F(\hat{x}_j) - F(\hat{x}_{j-1}))^{k-1} F(\hat{x}_{j-1})^{I-k} \\
&= F(\hat{x}_{j-1})^{I-1} + \frac{1}{I(F(\hat{x}_j) - F(\hat{x}_{j-1}))} \\
&\quad \cdot \sum_{k=2}^{I} \binom{I}{k} (F(\hat{x}_j) - F(\hat{x}_{j-1}))^k F(\hat{x}_{j-1})^{I-k} \\
&= F(\hat{x}_{j-1})^{I-1} + \frac{1}{I(F(\hat{x}_j) - F(\hat{x}_{j-1}))} \cdot \\
&\quad \left[F(\hat{x}_j)^I - I(F(\hat{x}_j) - F(\hat{x}_{j-1}))F(\hat{x}_{j-1})^{I-1} - F(\hat{x}_{j-1})^I\right]
\end{aligned}$$

The expected payment is similar to the one in Proposition 1.4 except that the messages' subscript i's are dropped. □

Proof of Proposition 1.6. Since the expected gain from trade depends only on a partition rule $\hat{\mathbf{x}}$, the number of bidders I and type space F. Proposition 1.6 immediately follows. □

Proof of Proposition 1.7. In a mechanism with M messages, the social planner's problem is equivalent to the one in a mechanism with $M+1$ messages and an additional constraint, $\hat{x}_{iM} = \hat{x}_{i,M+1} = \overline{x}_i$ for all $i \in \mathcal{I}$. An optimization problem with fewer constraints yields weakly higher maximum than otherwise. Therefore, a mechanism with $M+1$ messages yields higher efficiency than the one with M messages. $\square$

Proof of Proposition 1.8. Differentiating the objective function with respect to $\hat{x}_j$ yields the first-order condition as follows.

$$\begin{aligned}\frac{\partial\varphi}{\partial\hat{x}_j} = & \frac{IF(\hat{x}_j)^{I-1}f(\hat{x})\int_{\hat{x}_{j-1}}^{\hat{x}_j}x\mathrm{d}F(x)}{F(\hat{x}_j)-F(\hat{x}_{j-1})} - \frac{IF(\hat{x}_j)^{I-1}f(\hat{x}_j)\int_{\hat{x}_j}^{\hat{x}_{j+1}}x\mathrm{d}F(x)}{F(\hat{x}_{j+1})-F(\hat{x}_j)} \\ & + \frac{\hat{x}_jf(\hat{x}_j)(F(\hat{x}_j)^I-F(\hat{x}_{j-1})^I)}{F(\hat{x}_j)-F(\hat{x}_{j-1})} - \frac{\hat{x}_jf(\hat{x}_j)(F(\hat{x}_{j+1})^I-F(\hat{x}_j)^I)}{F(\hat{x}_{j+1})-F(\hat{x}_j)} \\ & - \frac{(F(\hat{x}_j)^I-F(\hat{x}_{j-1})^I)f(\hat{x}_j)\int_{\hat{x}_{j-1}}^{\hat{x}_j}x\mathrm{d}F(x)}{(F(\hat{x}_j)-F(\hat{x}_{j-1}))^2} \\ & + \frac{(F(\hat{x}_{j+1})^I-F(\hat{x}_j)^I)f(\hat{x}_j)\int_{\hat{x}_j}^{\hat{x}_{j+1}}x\mathrm{d}F(x)}{(F(\hat{x}_{j+1})-F(\hat{x}_j))^2} = 0\end{aligned}$$

Substituting $\mathrm{E}(x|x\in[a,b]) = \frac{1}{F(b)-F(a)}\int_a^b x\mathrm{d}F(x)$ and $\lambda(a,b) = \frac{F(b)^I-F(a)^I}{F(b)-F(a)}$ yields the first-order condition. $\square$

Proof of Corollary 1.1. Substituting $F(x)=x$ into the first-order condition and rearranging yields $\hat{x}_j = \left(\frac{\hat{x}_{j+1}-\hat{x}_{j-1}}{I}\right)^{\frac{1}{I-1}}$. $\square$

Proof of Corollary 1.2. Substituting $I=2$ into the first-order condition and rearranging yields

$$\int_{\hat{x}_{j-1}}^{\hat{x}_j}x\mathrm{d}F(x)+\int_{\hat{x}_j}^{\hat{x}_{j+1}}x\mathrm{d}F(x)+\hat{x}_j(F(\hat{x}_j)+F(\hat{x}_{j-1}))-\hat{x}_j(F(\hat{x}_{j+1})+F(\hat{x}_j))=0$$

Therefore,

$$\hat{x}_j = \frac{1}{F(\hat{x}_{j+1}) - F(\hat{x}_{j-1})} \int_{\hat{x}_{j-1}}^{\hat{x}_{j+1}} x\mathrm{d}F(x) = E(x | x \in [\hat{x}_{j-1}, \hat{x}_{j+1}])$$

□

Proof of Corollary 1.3. That is, we will prove that $p_j^I - p_{j-1}^I > 0$. So,

$$\begin{aligned}
p_j^I - p_{j-1}^I &= \frac{t(m_j)}{q(m_j)} - \frac{t(m_{j-1})}{q(m_{j-1})} \\
&= \frac{1}{q(m_j)q(m_{j-1})} \left(q(m_{j-1}) \sum_{k=1}^{j} (q(m_k) - q(m_{k-1}))\hat{x}_k \right) \\
&\quad - \frac{1}{q(m_j)q(m_{j-1})} \left(q(m_j) \sum_{k=1}^{j-1} (q(m_k) - q(m_{k-1}))\hat{x}_k \right) \\
&= \frac{1}{q(m_j)q(m_{j-1})} \left((q(m_{j-1}) - q(m_j)) \sum_{k=1}^{j-1} (q(m_k) - q(m_{k-1}))\hat{x}_k \right) \\
&\quad + \frac{1}{q(m_j)q(m_{j-1})} \Big(\hat{x}_j q(m_{j-1})(q(m_j) - q(m_{j-1})) \Big) \\
&= \frac{q(m_j) - q(m_{j-1})}{q(m_j)q(m_{j-1})} \left(\hat{x}_j q(m_{j-1}) - \sum_{k=1}^{j-1} (q(m_k) - q(m_{k-1}))\hat{x}_k \right) \\
&= \frac{q(m_j) - q(m_{j-1})}{q(m_j)q(m_{j-1})} (\hat{x}_j q(m_{j-1}) - t(m_{j-1}))
\end{aligned}$$

Since $\frac{q(m_j)-q(m_{j-1})}{q(m_j)q(m_{j-1})}$ is strictly positive by construction and $\hat{x}_j q(m_{j-1}) - t(m_{j-1}) > \hat{x}_{j-1} q(m_{j-1}) - t(m_{j-1}) \geq 0$ by individual rationality condition, $p_j^I - p_{j-1}^I > 0$.

The example that the efficient bid levels of the second-price auction are non-unique is discussed in Section 1.4.3. □

Proof of Proposition 1.9. We will prove that the equilibrium bidding strategy is defined as follows: in round t with demand history $\mathbf{h}_t$, a bidder with valuation

$x \geq x_t^*(\mathbf{h}_t)$ bids whereas bidder with valuation $x < x_t^*(\mathbf{h}_t)$ drops out. Suppose all bidders except bidder i follows this bidding strategy. This bidding strategy is an equilibrium if it is optimal for bidder i to follow this bidding strategy as well. If bidder i bid and all other bidders drop out, bidder i wins and pays $p_t(bid)$. If all bidders drop out and bidder i wins the tiebreaker, bidder i pays $p_t(dropout)$. For most pricing rules, $p_t(bid) \geq p_t(dropout)$. Define $\Pi_t(x_i, \mathbf{h}_t)$ as bidder i's expected payoff if the auction continues to round t. For any incentive compatibility selling mechanism, $\Pi_t(x_i, \mathbf{h}_t)$ is increasing in x_i.

In round t with demand history $\mathbf{h}_t$, Bidder i's payoff of bidding can be written as

$$
\begin{aligned}
\pi_t(bid, x_i, \mathbf{h}_t) \quad = \quad & (x_i - p_t(bid))\big(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\big)^{h_t-1} \\
& + \sum_{k=1}^{h_t-1} \binom{h_t}{k} \big(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\big)^{h_t-1-k} \big(1 - F(x_t^*(\mathbf{h}_t))\big)^k \\
& \cdot \Pi_{t+1}(x_i, \mathbf{h}_t \cup (k))
\end{aligned}
$$

Bidder i's payoff of dropping out can be written as

$$
\pi_t(dropout, x_i, \mathbf{h}_t) = \frac{1}{h_t}(x_i - p_t(dropout))\big(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\big)^{h_t}
$$

Bidder i's bids if $\pi_t(bid, x_i, \mathbf{h}_t) \geq \pi_t(dropout, x_i, \mathbf{h}_t)$ and drops out otherwise. The gain from bidding can be defined as follows.

$$
\begin{aligned}
&\pi_t(bid, x_i, \mathbf{h}_t) - \pi_t(dropout, x_i, \mathbf{h}_t) \\
&= (x_i - p_t(bid)) \left(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\right)^{h_t-1} \\
&\quad + \sum_{k=0}^{h_t-2} \binom{h_t}{k} \left(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\right)^k F(x_t^*(\mathbf{h}_t))^{h_t-1-k} \\
&\quad - \frac{1}{h_t}(x_i - p_t(dropout)) \left(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\right)^{h_t} \\
&= \left(\frac{h_t-1}{h_t} x_i + p_t(dropout) - p_t(bid)\right) \left(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\right)^{h_t-1} \\
&\quad + \sum_{k=1}^{h_t-1} \binom{h_t}{k} \left(F(x_t^*(\mathbf{h}_t)) - F(x_{t-1}^*(\mathbf{h}_{t-1}))\right)^{h_t-1-k} \left(1 - F(x_t^*(\mathbf{h}_t))\right)^k \\
&\quad \cdot \Pi_{t+1}(x_i, \mathbf{h}_t \cup (k))
\end{aligned}
$$

Since $\pi_t(bid, x_i, \mathbf{h}_t) - \pi_t(dropout, x_i, \mathbf{h}_t)$ is strictly increasing in x_i, we can solve for a critical valuation $x_i \equiv x_t^*(\mathbf{h}_t)$ such that $\pi_t(bid, x_i, \mathbf{h}_t) - \pi_t(dropout, x_i, \mathbf{h}_t) = 0$. If $x_i < x_t^*(\mathbf{h}_t)$, $\pi_t(bid, x_i, \mathbf{h}_t) - \pi_t(dropout, x_i, \mathbf{h}_t) < 0$ and vice versa. Therefore, it is optimal for bidder i to follow the proposed equilibrium strategy. □

Proof of Proposition 1.10. We can implement an efficient partition rule in a blind clock auction with a clock auction with demand disclosure. The expected gain from trade in static auction $\varphi(\hat{\mathbf{x}})$ is equivalent to the one in dynamic auction $\Phi(\mathbf{x}^*)$ with additional constraints $x_t^*(\mathbf{h}_t) = \hat{x}_t$ for all $\mathbf{h}_t \in \mathbf{H}_t$ and $t = 1, 2, \ldots, P-1$. Relaxing these constraints yield higher value of the objective function. Thus, a clock auction with demand disclosure is more efficient than the blind clock auction. □

Appendix B

Missing proofs from Chapter 2

Proof of Proposition 2.1. A bidder's maximization problem is to choose an optimal exit bid. Consider bidder i with a valuation $x_i \in [P_t, P_{t+1})$. Suppose that $\tilde{x}$ is the K-highest competing bid among bidders other than bidder i. First, it is a weakly-dominated strategy to submit an exit bid $x' < x_i$. If $\tilde{x} \leq x' < x_i$, her payoff is equal to the one when she bid her valuation. If $x' \leq \tilde{x} < x_i$, she loses and is better off bidding her valuation. If $x' < x_i \leq \tilde{x}$, she loses regardless. Hence, she cannot be better off bidding $x' < x_i$. Second, it is also a weakly-dominated strategy to submit an exit bid $x' > x_i$. If $\tilde{x} \leq x_i < x'$, she receives the same payoff as submitting an exit bid of x_i. If $x_i < \tilde{x} \leq x'$, she wins the item at price above her valuation and receives a negative payoff. Finally, if $x_i < x' < \tilde{x}$, she loses regardless. Thus, she cannot gain by bidding $x' > x_i$. Therefore, it is a weakly-dominant strategy to submit an exit bid equal to the bidder's valuation. □

Proof of Lemma 2.1. In round $t > s + 1$, the final price is at least P_{t-1} which exceeds bidder i's valuation. Winning the item yields a strictly negative payoff regardless of her rank so that she is better off losing. Thus, it is a weakly-dominant strategy to exit immediately in round t to reduce a chance of winning the item if she is a provisional winner or to avoid winning at all if she is a provisional loser. □

Proof of Lemma 2.2. Consider a provisional loser who has a valuation less than the current clock price. Exiting immediately yields a payoff of zero while bidding yields a positive expected payoff since she may win the item in the subsequent round at a profitable price. Thus, a provisional loser will never exit before her valuation is reached.

Next, consider a provisional loser i who has a valuation $x_i \in [P_{t-1}, P_t)$ in round t. By remaining active, she may be able to win the item at P_{t-1} if fewer than $K-2$ bids in addition to her bid are submitted. However, bidding entails a risk of winning at a price of P_t resulting in a negative payoff. According to Lemma 1, if she bids in round t and the auction continues to round $t+1$, she will exit regardless of her rank. She will bid if the expected payoff of bidding—the expected payoff if she wins at P_{t-1} less the expected payoff if she wins at P_t—is positive.

She will win the item at P_t when (1) there are at least $K-1$ bids submitted in round t, (2) she is selected to be a provisional winner of rank L and (3) less than L bids from provisional losers are submitted in round $t+1$. Let $\lambda_{it}(m, r, \mathbf{H}_{it})$ be the probability that m bidders out of r remaining bidders excluding bidder i stay active in round t given a ranking history $\mathbf{H}_{it}$ and $\mu_{it}(m, r, \mathbf{H}_{it})$ be the probability that m provisional losers out of r provisional losers bid in round t given a ranking history $\mathbf{H}_{it}$.

Suppose there are R active bidders in round t. The provisional loser i's expected payoff is

$$(x_i - P_{t-1}) \sum_{j=0}^{K-2} \lambda_{it}(j, R, \mathbf{H}_{it}) + (x_i - P_t)\lambda_{it}(K-1, R, \mathbf{H}_{it})$$
$$+(x_i - P_t) \sum_{j=K}^{R} \lambda_{it}(j, R, \mathbf{H}_{it}) \sum_{L=1}^{K} \sum_{z=0}^{L-1} \frac{1}{j+1} \mu_{i,t+1}(z, j-K+1, \mathbf{H}_{it} \cup (L))$$

The first term is the expected payoff when less than $K-2$ bids are submitted. The second term is the expected payoff when exactly $K-1$ bids are submitted and the provisional loser i wins the item at P_t. The third term is the expected payoff when the auction continues to round $t+1$ and the provisional loser i wins the item. The last two summations aggregate probabilities that bidder i is selected as a provisional winner of rank $L = 1, \ldots, K$ and $z = 0, \ldots, L-1$ bids are submitted in round $t+1$. The first summation in the third term then aggregates over a chance that at least K bids are submitted in round $t+1$.

Since $P_{t-1} \leq x_i < P_t$ and $\lambda_{it}(m, r, \mathbf{H}_{it}) \geq 0$, the first term is weakly positive while the second and third terms are negative. For some ranking history, an expected payoff when less than $K-2$ bids are submitted is strictly positive. Because bidder i's valuation is closer to P_t, the first term is larger while the other two terms become smaller and the expected payoff of bidding increases. By using an intermediate valuation $x_i = P_t$, the expected payoff of bidding is positive. So, if bidder i's valuation is sufficiently close to P_t, the expected payoff is positive and as a result, bidding is profitable for bidder i. □

Proof of Lemma 2.3. Consider a provisional winner i in round t with valuation

$x_i \in [P_t, P_{t+1})$. It is optimal to exit in round t if the expected payoff of exiting exceeds the expected payoff of bidding. Using the same notations as in the proof of Lemma 2, suppose there are R bidders remaining in round t and the ranking history is $\mathbf{H}_{it}$. Let l be a provisional winner i's rank in round t. If her valuation is close to P_t, she will exit in round $t+1$ since a negative expected payoff arising from winning the item at an unprofitable price exceeds an expected payoff of exiting.

It is optimal to exit in round t if

$$
\begin{aligned}
(x_i - P_{t-1}) \sum_{j=0}^{l-1} \lambda_{it}(j, R-K, \mathbf{H}_{it}) > (x_i - P_t)\lambda_{it}(K-1, R, \mathbf{H}_{it}) \\
+(x_i - P_{t-1}) \sum_{j=0}^{K-2} \lambda_{it}(j, R, \mathbf{H}_{it}) \\
+(x_i - P_t) \sum_{j=K}^{R} \lambda_{it}(j, R, \mathbf{H}_{it}) \sum_{L=1}^{K} \sum_{z=0}^{L-1} \frac{1}{j+1} \mu_{i,t+1}(z, j-K+1, \mathbf{H}_{it} \cup (L))
\end{aligned}
$$

The left-hand side is the expected payoff if provisional winner i exits and less than l bids are submitted. The first term of the right-hand side is the expected payoff if provisional winner i bids and exactly $K-1$ bids are submitted so that provisional winner i gets the item at P_t. The second term of the right-hand side is the expected payoff if fewer than $K-1$ bids are submitted so that bidder i gets the item at P_{t-1}. The last term is the expected payoff if at least $K-1$ bids are submitted and the auction continues to round $t+1$. Provisional winner i is selected to be a provisional winner of rank L in round $t+1$. She then exits and wins the item at P_t. Assume for now that bidder i exits in round $t+1$. Then, we use an intermediate valuation P_t so that it is a dominant strategy for provisional winner i to exit in round $t+1$.

Also, let $l = K$ as the provisional winner with rank K is most susceptible to exit.

Rearranging yields

$$(x_i - P_{t-1})\lambda_{it}(K-1, R, \mathbf{H}_{it}) > (x_i - P_t)\lambda_{it}(K-1, R, \mathbf{H}_{it})$$
$$+(x_i - P_t)\sum_{j=K}^{R}\lambda_{it}(j, R, \mathbf{H}_{it})\sum_{L=1}^{K}\sum_{z=0}^{L-1}\frac{1}{j+1}\mu_{i,t+1}(z, j-K+1, \mathbf{H}_{it}\cup(L))$$

Because $x_i \in [P_t, P_{t+1})$ and $\lambda_{it}(m, r, \mathbf{H}_{it}) \geq 0$ for any m, r and $\mathbf{H}_{it}$, both sides of the inequality are positive. For some ranking history, the probability that $K-1$ bids are submitted in round t is strictly positive so that the left-hand side is strictly positive as well. As provisional winner i's valuation is closer to P_t, the left-hand side is larger while the right-hand side becomes smaller. By using an intermediate valuation, $x_i = P_t$, the inequality holds. That is, the expected payoff of exiting exceeds that of bidding. □

Proof of Proposition 2.2. According to Lemma 2.2 and 2.3, provisional losers may bid above their valuations and provisional winners may exit before their valuations are reached. For some ranking history and valuation, a bidder may bid if she is a provisional loser but she may exit if she is a provisional winner. Such an asymmetric bidding strategy leads to an inefficient allocation. □

Proof of Lemma 2.4. In the one-item case, the final price is equal to the highest bid. Regardless of her rank, a bidder with a valuation $x_i < P_t$ cannot profitably bid in round t since the final price will be at least P_t. Therefore, bidding above one's valuation is a weakly-dominated strategy.

If a bidder with valuation $x_i \geq P_t$ is a provisional loser in round t, it is a weakly-dominated strategy to exit since exiting yields a payoff of zero while bidding may give her a chance to win at a profitable price. Hence, the optimal strategy for a provisional loser is straightforward bidding. □

Proof of Lemma 2.5. To show this, consider a provisional winner with valuation x_i in round t. Let $\lambda_t(\mathbf{H}_{it})$ be a probability that the opponent bids in round t given a ranking history $\mathbf{H}_{it}$ and $\Pi_{i,t+1}(x_i, \mathbf{H}_{it})$ be an expected payoff in round $t+1$ given a valuation x_i and a ranking history $\mathbf{H}_{it}$. Note that $\Pi_{i,t+1}(x_i, \mathbf{H}_{it})$ is increasing in x_i. A provisional winner's expected gain from exiting in round t is given by

$$\begin{aligned} G_{it}(x_i, \mathbf{H}_{it}) &\equiv (x_i - P_{t-1})\lambda_t(\mathbf{H}_{it}) - (x_i - P_t)\lambda_t(\mathbf{H}_{it}) - \Pi_{i,t+1}(x_i, \mathbf{H}_{it}) \\ &= (P_t - P_{t-1})\lambda_t(\mathbf{H}_{it}) - \Pi_{i,t+1}(x_i, \mathbf{H}_{it}) \end{aligned}$$

Because the first term does not depend on x_i and the second term is decreasing in x_i, the gain from exiting is decreasing in x_i as well. □

Proof of Proposition 2.3. Weakly-dominant strategies are defined in Lemmas 2.4 and 2.5. □

Proof of Lemma 2.6. We will show that in any round $t \leq s-2$ a provisional winner will bid even in the scenario which is the most susceptible to bid shading. Suppose bidder i with a valuation $x_i \in [P_t, P_{t+1})$ is a provisional winner in round $t-2$. Consider a scenario that is most susceptible to bid shading—(1) bidder i has the lowest valuation, $x_i = P_t$, (2) it is profitable to exit in round $t-1$ given a ranking history and (3) the lower bound of her opponent's valuation is the lowest; that is, the ranking history implies that $\hat{x}_{t-2}(\mathbf{H}_{i,t-2}) = P_{t-3}$. Let $\Pi_{it}(x_i, \mathbf{H}_{it})$ be bidder i's expected payoff in round t. Her gain from exiting in round $t-2$ is

$$\begin{aligned}
G_{i,t-2}(x_i, \mathbf{H}_{i,t-2}) &= (x_i - P_{t-3})\frac{F(P_{t-2}) - F(P_{t-3})}{1 - F(P_{t-3})} \\
&- (x_i - P_{t-2})\frac{F(P_{t-2}) - F(P_{t-3})}{1 - F(P_{t-3})} \\
&- \frac{1}{2}(x_i - P_{t-2})\frac{F(P_{t-2}) - F(P_{t-3})}{1 - F(P_{t-3})} - \frac{1}{2}\Pi_{it}(x_i, \mathbf{H}_{it}) \\
&= \frac{3\Delta^2}{1 - F(P_{t-3})} - \frac{3\Delta^2}{1 - F(P_{t-3})} - \frac{1}{2}\Pi_{it}(x_i, \mathbf{H}_{it}) \\
&= -\frac{1}{2}\Pi_{it}(x_i, \mathbf{H}_{it}) < 0
\end{aligned}$$

The first term is the payoff of exiting when the opponent also exits in round $t-2$. The expected payoff of bidding consists of the last three terms. The second term is a payoff of bidding when the opponent exits in round $t-2$. The third term is a payoff when the opponent bids in round $t-2$, bidder i is selected as a provisional winner in round $t-1$ and the opponent exits in round $t-1$. Since $\Pi_{it}(x_i, \mathbf{H}_{it})$ is strictly positive, the gain from exiting in round $t-2$ is negative.

Since a provisional winner i with a valuation $x_i - P_t$ bids in round $t-2$, according to Lemma 5, any provisional winner with a valuation $x_i > P_t$ stays active. In other words, a provisional winner in round $t-2$ with a valuation $x \in [P_{t+j}, P_{t+j+1})$ for $j = 0, 1, \ldots, T-t-2$ or equivalently, a provisional winner in round $s = t-2-j$ with a valuation $x \in [P_t, P_{t+1})$ for $j = 0, 1, \ldots, t-3$, will bid as well. □

Proof of Proposition 2.4. Since the amount of bid shading is limited to only two bid levels according to Lemma 2.6, the absolute amount of bid shading shrinks as the bid increments become smaller. Moreover, the finer bid increments allow bidders to better express the value differences. Hence, the efficiency in the LAB auction is higher by reducing bid increments. □

Proof of Lemma 2.7. We have to calculate a probability that the final price is P_t for $t = 0, 1, \dots, T-1$. For $t > 0$, there are two possible cases: (1) both bidders have valuations in $[P_t, P_{t+1})$ and (2) one bidder has a valuation in $[P_t, P_{t+1})$ and the other bidder has a valuation in $[P_t, 1]$. The final price is P_0 only if both bidders have valuations in $[P_0, P_1)$. Thus, the expected revenue is

$$\begin{aligned} R^{SB}_{LAB} &= P_0\big(F(P_1) - F(P_0)\big)^2 + \sum_{t=1}^{T-1} P_t\Big[\big(F(P_{t+1}) - F(P_t)\big)^2 \\ &+ 2\big(1 - F(P_t)\big)\big(F(P_t) - F(P_{t-1})\big)\Big] \\ &= \frac{(T-2)(2T+3)}{6(T-1)^2} \end{aligned}$$

□

Proof of Lemma 2.8. The maximum shading strategies are given as follows. A provisional winner does not infer the opponent's valuation from the ranking history. That is, in any round t, she maximizes her payoff given that the opponent has a valuation in $[P_{t-1}, 1]$. Therefore, we can find that $\hat{x}_{t-1}(\mathbf{H}_{i,t-1}) = P_t + \frac{2}{5}\Delta$ for any $t \geq 2$ and for any $\mathbf{H}_{i,t-1} \in \mathbf{\Theta}_{t-1}$. If the final price is P_t , there are three possible combinations of bids determining the final price: (1) only a provisional winner exits in round t, (2) only a provisional loser exits in round t and (3) both of them exit

Table B.1: Possible scenarios in which the final price is P_t for $t = 3, 4, \ldots, T-4$

Ranking history	Valuation		Exiting bidder
	Provisional winner	Provisional loser	
$(R_{i,t-3}, R_{i,t-2}, R_{i,t-1}, R_{it}) = (1,1,1,1)$	$[P_t + 2\Delta/5, P_{t+1} + 2\Delta/5)$	$[P_t, 1]$	Proviosional winner
$(R_{i,t-3}, R_{i,t-2}, R_{i,t-1}, R_{it}) = (1,1,1,1)$	$[P_t + 2\Delta/5, 1]$	$[P_{t-1}, P_t)$	Proviosional loser
$(R_{i,t-3}, R_{i,t-2}, R_{i,t-1}, R_{it}) = (1,0,1,1)$	$[P_{t-1} + 2\Delta/5, P_{t+1} + 2\Delta/5)$	$[P_t + 2\Delta/5, 1]$	Proviosional winner
$(R_{i,t-3}, R_{i,t-2}, R_{i,t-1}, R_{it}) = (0,1,1,1)$	$[P_t + 2\Delta/5, P_{t+1} + 2\Delta/5)$	$[P_t, 1]$	Proviosional winner
$(R_{i,t-3}, R_{i,t-2}, R_{i,t-1}, R_{it}) = (0,1,1,1)$	$[P_{t+1} + 2\Delta/5, 1]$	$[P_{t-1}, P_{t+1} + 2\Delta/5)$	Proviosional loser
$(R_{i,t-3}, R_{i,t-2}, R_{i,t-1}, R_{it}) = (0,0,1,1)$	$[P_{t-1}, P_{t+1} + 2\Delta/5)$	$[P_t + 2\Delta/5, 1]$	Proviosional winner
$(R_{i,t-2}, R_{i,t-1}, R_{it}, R_{i,t+1}) = (1,1,1,1)$	$[P_{t+1} + 2\Delta/5, P_{t+2} + 2\Delta/5)$	$[P_t, P_{t+1})$	Both
$(R_{i,t-2}, R_{i,t-1}, R_{it}, R_{i,t+1}) = (0,0,1,1)$	$[P_{t+1} + 2\Delta/5, P_{t+2} + 2\Delta/5)$	$[P_t + 2\Delta/5, P_{t+1})$	Both

in round $t+1$. Since a provisional winner never shades more than two bid levels below her valuation, a ranking history of only three rounds before the final round is relevant. Suppose bidder i is a provisional winner at the final round. Hence, there are eight possible cases with a final price of P_t for $t = 3, 4, \ldots, T-4$ as shown in Table B.1.

We have to calculate a probability that the auction ends at P_0, P_1, P_2, P_{T-3} and P_{T-2} separately because their associated probabilities are different as shown in Table B.2.

Using Table B.1 and B.2, we can calculate the expected revenue of the LAB

Table B.2: Possible scenarios in which the final prices are P_0, P_1, P_2, P_{T-3} and P_{T-2}

Final price	Ranking history	Valuation		Exiting bidder
		Provisional winner	Provisional loser	
P_0	$(R_{i1}) = (1)$	$[P_0, P_2 + 2\Delta/5)$	$[P_0, 1]$	Both
P_1	$(R_{i1}) = (1)$	$[P_0, P_2 + 2\Delta/5)$	$[P_{t-1}, P_t)$	Proviosional winner
P_1	$(R_{i1}) = (1)$	$[P_2 + 2\Delta/5, 1]$	$[P_0, P_1]$	Proviosional loser
P_1	$(R_{i1}, R_{i2}) = (1, 1)$	$[P_2 + 2\Delta/5, P_3 + 2\Delta/5)$	$[P_1, P_2)$	Both
P_2	$(R_{i1}, R_{i2}) = (1, 1)$	$[P_2 + 2\Delta/5, P_3 + 2\Delta/5)$	$[P_2, 1]$	Proviosional winner
P_2	$(R_{i1}, R_{i2}) = (1, 1)$	$[P_3 + 2\Delta/5, 1]$	$[P_1, P_2)$	Proviosional loser
P_2	$(R_{i1}, R_{i2}) = (0, 1)$	$[P_1, P_3 + 2\Delta/5)$	$[P_2 + 2\Delta/5, 1]$	Proviosional winner
P_2	$(R_{i1}, R_{i2}, R_{i3}) = (1, 1, 1)$	$[P_3 + 2\Delta/5, P_4 + 2\Delta/5)$	$[P_2, P_3)$	Both
P_2	$(R_{i1}, R_{i2}, R_{i3}) = (0, 1, 1)$	$[P_3 + 2\Delta/5, P_4 + 2\Delta/5)$	$[P_2 + 2\Delta/5, P_3)$	Both
P_{T-2}	$(R_{i,T-6}, R_{i,T-5}, R_{i,T-4}, R_{i,T-3}) = (1, 1, 1, 1)$	$[P_{T-3} + 2\Delta/5, P_{T-2} + 2\Delta/5)$	$[P_{T-3}, 1]$	Provisional winner
P_{T-2}	$(R_{i,T-6}, R_{i,T-5}, R_{i,T-4}, R_{i,T-3}) = (1, 1, 1, 1)$	$[P_{T-2} + 2\Delta/5, 1]$	$[P_{T-4}, P_{T-3})$	Provisional loser
P_{T-2}	$(R_{i,T-6}, R_{i,T-5}, R_{i,T-4}, R_{i,T-3}) = (1, 0, 1, 1)$	$[P_{T-4} + 2\Delta/5, P_{T-2} + 2\Delta/5)$	$[P_{T-3} + 2\Delta/5, 1]$	Provisional winner
P_{T-2}	$(R_{i,T-6}, R_{i,T-5}, R_{i,T-4}, R_{i,T-3}) = (0, 1, 1, 1)$	$[P_{T-3} + 2\Delta/5, P_{T-2} + 2\Delta/5)$	$[P_{T-3}, 1]$	Provisional winner
P_{T-2}	$(R_{i,T-6}, R_{i,T-5}, R_{i,T-4}, R_{i,T-3}) = (0, 1, 1, 1)$	$[P_{T-2} + 2\Delta/5, 1]$	$[P_{T-4}, P_{T-2} + 2\Delta/5)$	Provisional loser
P_{T-2}	$(R_{i,T-6}, R_{i,T-5}, R_{i,T-4}, R_{i,T-3}) = (0, 0, 1, 1)$	$[P_{T-4} + 2\Delta/5, P_{T-2} + 2\Delta/5)$	$[P_{T-3} + 2\Delta/5, 1]$	Provisional winner
P_{T-2}	$(R_{i,T-5}, R_{i,T-4}, R_{i,T-3}, R_{i,T-2}) = (1, 1, 1, 1)$	$[P_{T-2} + 2\Delta/5, 1]$	$[P_{T-3}, P_{T-2})$	Both
P_{T-2}	$(R_{i,T-5}, R_{i,T-4}, R_{i,T-3}, R_{i,T-2}) = (0, 0, 1, 1)$	$[P_{T-2} + 2\Delta/5, 1]$	$[P_{T-3} + 2\Delta/5, P_{T-2})$	Both
P_{T-1}	$(R_{i,T-5}, R_{i,T-4}, R_{i,T-3}, R_{i,T-2}) = (1, 1, 1, 1)$	$[P_{T-2} + 2\Delta/5, 1]$	$[P_{T-2}, 1]$	Provisional winner
P_{T-1}	$(R_{i,T-5}, R_{i,T-4}, R_{i,T-3}, R_{i,T-2}) = (1, 0, 1, 1)$	$[P_{T-3} + 2\Delta/5, 1]$	$[P_{T-2} + 2\Delta/5, 1]$	Provisional winner
P_{T-1}	$(R_{i,T-5}, R_{i,T-4}, R_{i,T-3}, R_{i,T-2}) = (0, 1, 1, 1)$	$[P_{T-2} + 2\Delta/5, 1]$	$[P_{T-2}, 1]$	Provisional winner
P_{T-1}	$(R_{i,T-5}, R_{i,T-4}, R_{i,T-3}, R_{i,T-2}) = (0, 0, 1, 1)$	$[P_{T-3}, 1]$	$[P_{T-2} + 2\Delta/5, 1]$	Provisional winner

auction with maximum shading strategy as

$$
\begin{aligned}
R_{LAB}^{MS}(T) &= P_0\Big[\frac{1}{2}\Delta \cdot \frac{12}{5}\Delta\Big] + P_1\Big[\frac{1}{2}\Delta \cdot (1-\frac{12}{5}\Delta) + \frac{1}{2}(1-\Delta)\cdot\frac{12}{5}\Delta + \frac{1}{4}\Delta^2\Big] \\
&+ P_2\Big[\frac{1}{4}\cdot\frac{12}{5}\Delta\cdot(1-\frac{12}{5}\Delta) + \frac{1}{4}\Delta\cdot(1-\frac{17}{5}\Delta) + \frac{1}{4}\Delta(1-2\Delta) + \frac{1}{8}\Delta^2 \\
&+ \frac{1}{8}\Delta\cdot\frac{3}{5}\Delta\Big] + \sum_{t=3}^{T-4}\frac{1}{8}P_t\Big[\Delta(1-t\Delta) + \Delta(1-(t+1)\Delta - \frac{2}{5}\Delta) \\
&+ 2\Delta(1-t\Delta-\frac{2}{5}\Delta) + \Delta(1-t\Delta) + (1-(t+1)\Delta-\frac{2}{5}\Delta)\cdot\frac{3}{5}\Delta \\
&+ \frac{12}{5}\Delta\cdot(1-t\Delta-\frac{2}{5}\Delta) + \Delta^2 + \Delta\cdot\frac{3}{5}\Delta\Big] + \frac{1}{8}P_{T-3}\Big[\Delta(1-(T-3)\Delta) \\
&+ \Delta(1-(T-2)\Delta-\frac{2}{5}\Delta) + 2\Delta(1-(T-3)\Delta-\frac{2}{5}\Delta) \\
&+ \Delta(1-(T-3)\Delta) + (1-(T-2)\Delta-\frac{2}{5}\Delta)\cdot\frac{3}{5}\Delta \\
&+ \frac{12}{5}\Delta\cdot(1-(T-3)\Delta-\frac{2}{5}\Delta) + \Delta\cdot\frac{3}{5}\Delta + (\frac{3}{5}\Delta)^2\Big] \\
&+ \frac{1}{8}P_{T-2}\Big[(1-(T-2)\Delta)\cdot\frac{3}{5}\Delta + \frac{8}{5}\Delta\cdot(1-(T-2)\Delta-\frac{2}{5}\Delta) \\
&+ (1-(T-2)\Delta)\cdot\frac{3}{5}\Delta + 2\Delta\cdot(1-(T-2)\Delta-\frac{2}{5}\Delta)\Big]
\end{aligned}
$$

Replacing $\Delta = \frac{1}{T-1}$ and $P_t = \frac{t}{T-1}$ with some manipulation yields the expected revenue in Lemma 2.8. □

Proof of Proposition 2.5. According to Lemma 2.7 and 2.8, the lower bound and upper bound of the expected revenue converge to 1/3. The expected revenue of the LAB auction converges to 1/3 as well. □

Proof of Lemma 2.9. The calculation is similar to the proof of Lemma 8 with $\hat{x}_{t-1}(\mathbf{H}_{i,t-1}) = P_t + \frac{2(1-\alpha)}{5(1+\alpha)}$ for any $t \geq 2$ and for any $H_{i,t-1} \in \mathbf{\Theta}_{t-1}$ instead. □

Proof of Proposition 2.6. Using the lower bound defined in Lemma 2.9 we solve for $\alpha \in [0, 1)$ such that $\tilde{R}^{MS}_{LAB}(T, \alpha) \geq 1/3$. □

Proof of Proposition 2.7. The equilibrium bidding function is solved in the Section 2.9. The next step is to prove that if the optimal exit bid is higher than the current clock price, it is optimal to bid at the current clock price. Consider a bidder with a valuation x who bids as if he has a valuation x'. His payoff function is

$$\pi_t(x', x, \mathbf{M}_t) = xG(x'|z_t) - B_t(x', \alpha, \mathbf{M}_t)G(x'|z_t) - \alpha \int_{x'}^{B_t^{-1}(x')} x - B_t(x', \alpha, \mathbf{M}_t) dG(s|z_t)$$

Differentiating the payoff function with respect to x' yields

$$\begin{aligned} \frac{\delta}{\delta x'}\pi_t(x', x, \mathbf{M}_t) &= (1+\alpha)(x - B_t(x', \alpha, \mathbf{M}_t))g(x'|z_t) - B_t'(x', \alpha, \mathbf{M}_t)G(x'|z_t) \\ &= (1+\alpha)(x - x')g(x'|z_t) + (1+\alpha)(x' - B_t(x', \alpha, \mathbf{M}_t))g(x'|z_t) \\ &\quad - B_t'(x', \alpha, \mathbf{M}_t)G(x'|z_t) \\ &= (1+\alpha)(x - x')g(x'|z_t) \end{aligned}$$

Consider a bidder with valuation x such that $B_t(x, \alpha, \mathbf{M}_t) > P_t$. She is constrained to bid either at the current clock price or submit and an exit bid between P_{t-1} and P_t. Since $\pi_t(x', x, \mathbf{M}_t)$ is increasing for all $x' < x$, it is optimal to bid at the current clock price. □

Proof of Proposition 2.8. According to Proposition 2.7, the bidding strategy is symmetric and monotonic in valuation; thus, the LABx auction is efficient. □

Proof of Proposition 2.9. In the HRB and LABx auctions, the allocation rules and expected payoffs of the lowest-valuation bidder are the same. Furthermore, their payoff functions are the same when $\alpha = 0$. Thus, revenue equivalence holds.

Since bidders shade their bids in the LABx auction, fear of losing therefore impacts the bidding strategy. In contrast, bidders bid truthfully in the HRB auction so that the strategy is not affected by fear of losing. As the exit bid function is increasing in α, the expected revenue of the LABx auction is increasing in α as well. $\square$

Solving for the equilibrium with the uniform distribution, fixed increment and four bid levels

In this section, define $\pi_{it}(x_i, q_{it}, \mathbf{H}_{it})$ as bidder i's expected payoff if she submits $q_{it} \in \{0, 1\}$ given bidder i's ranking history $\mathbf{H}_{it}$. Note that we can omit the other bidder's ranking history because it is the complement of bidder i's ranking history. We will solve for the bidding strategy of the bidder with the lowest valuation first.

First, consider a bidder i with valuation $x_i \in [P_0, P_1)$. If she is a provisional winner in round 1, according to Lemma 2.3, she will exit in round 1.

Next, consider a bidder i with valuation $x_i \in [P_1, P_2)$. If she is a provisional winner in round 2, she will exit. When she is a provisional winner in round 1, she will exit as well since

$$\begin{aligned}
\pi_{i1}(x_i, 0, (1)) &= (x_i - P_0)\frac{F(P_1) - F(P_0)}{1 - F(P_0)} = (x_i - P_0)\Delta \\
&> \pi_{i1}(x_i, 1, (1)) = (x_i - P_1)\frac{F(P_1) - F(P_0)}{1 - F(P_0)} \\
&+ \frac{1}{2}(x_i - P_1)\frac{F(P_2) - F(P_1)}{1 - F(P_0)} \\
&= \frac{3}{2}(x_i - P_1)\Delta
\end{aligned}$$

Finally, consider a bidder i with valuation $x_i \in [P_2, P_3]$. If she is a provisional winner in round 3, she will exit. When she is a provisional winner in round 2 and $\mathbf{H}_{i2} = (1, 1)$ which implies that the opponent has a valuation in $[P_1, 1]$, she will exit since

$$\begin{aligned}
\pi_{i2}(x_i, 0, (1, 1)) &= (x_i - P_1)\frac{F(P_2) - F(P_1)}{1 - F(P_1)} = (x_i - P_1)\frac{\Delta}{1 - F(P_1)} \\
&> \pi_{i2}(x_i, 1, (1, 1)) = (x_i - P_2)\frac{F(P_2) - F(P_1)}{1 - F(P_1)} \\
&+ \frac{1}{2}(x_i - P_1)\frac{F(P_2) - F(P_1)}{1 - F(P_0)} \\
&= \frac{3}{2}(x_i - P_2)\frac{\Delta}{1 - F(P_1)}
\end{aligned}$$

If she is a provisional winner in round 2 and $\mathbf{H}_{i2} = (0, 1)$, then the opponent has a valuation greater than P_2 and thus the opponent will bid according to Lemma 2.4. Therefore, the provisional winner will also bid in round 2.

If she is a provisional winner in round 1, we can obtain $\hat{x}_1((1)) = P_2 + \frac{2}{5}\Delta$ by solving $\pi_{i1}(\hat{x}, 0, (1)) = \pi_{i1}(\hat{x}, 1, (1))$.

To calculate the expected revenue, we consider all possible realizations of val-

Table B.3: Possible outcomes for all realizations in round 1 and $\mathbf{H}_{11} = (1)$

Bidder 1's value	Bidder 2's value			
	$[P_0, P_1)$	$[P_1, P_2)$	$[P_2, P_2 + \frac{2}{5}\Delta)$	$[P_2 + \frac{2}{5}\Delta, P_3]$
$[P_0, P_1)$	P_0	P_1	P_1	P_1
$[P_1, P_2)$	P_0	P_1	P_1	P_1
$[P_2, P_2 + \frac{2}{5}\Delta)$	P_0	P_1	P_1	P_1
$[P_2 + \frac{2}{5}\Delta, P_3]$	P_1	Continue	Continue	Continue

Table B.4: Possible outcomes for all realizations in round 2 and $\mathbf{H}_{12} = (1, 1)$

Bidder 1's value	Bidder 2's value			
	$[P_0, P_1)$	$[P_1, P_2)$	$[P_2, P_2 + \frac{2}{5}\Delta)$	$[P_2 + \frac{2}{5}\Delta, P_3]$
$[P_0, P_1)$	-	-	-	-
$[P_1, P_2)$	-	-	-	-
$[P_2, P_2 + \frac{2}{5}\Delta)$	-	-	-	-
$[P_2 + \frac{2}{5}\Delta, P_3]$	-	P_1	P_2	P_2

uations and outcomes as shown in Tables B.3 to B.5. Outcomes when ranking histories are $\mathbf{H}_{11} = (0)$, $\mathbf{H}_{12} = (0, 0)$ and $\mathbf{H}_{12} = (0, 1)$ are similar to Tables B.3 to B.5, respectively, with bidder 1 and 2 swapped. Therefore,

$$\begin{aligned} R_{LAB}(4) &= 2\Big[\frac{1}{2}P_0\Delta \cdot \frac{12}{5}\Delta + \frac{1}{2}P_1(\Delta \cdot \frac{3}{5}\Delta + \frac{1}{2}(2\Delta) \cdot \frac{12}{5}\Delta + \frac{1}{4}\Delta \cdot \frac{3}{5}\Delta) \\ &+ P_2(\frac{1}{4}\Delta \cdot \frac{3}{5}\Delta + \frac{1}{4}(2\Delta) \cdot \frac{3}{5}\Delta)\Big] \end{aligned}$$

Substituting $P_t = t/3$ and $\Delta = 1/3$ yields an expected revenue of 0.2778.

Table B.5: Possible outcomes for all realizations in round 2 and $\mathbf{H}_{12} = (1, 0)$

Bidder 1's value	Bidder 2's value			
	$[P_0, P_1)$	$[P_1, P_2)$	$[P_2, P_2 + \frac{2}{5}\Delta)$	$[P_2 + \frac{2}{5}\Delta, P_3]$
$[P_0, P_1)$	-	-	-	-
$[P_1, P_2)$	-	-	-	-
$[P_2, P_2 + \frac{2}{5}\Delta)$	-	-	-	-
$[P_2 + \frac{2}{5}\Delta, P_3]$	-	P_2	P_2	P_2

Appendix C

Bidder Instructions

C.1 Treatment HRB

Experiment Instructions

Welcome to the auction experiment. In this experiment, you will participate in auctions as a bidder. The precise rules and procedures that govern the operation of these auctions will be explained to you below.

Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions you can finish the experiment with a considerable amount of money, which will be paid to you in cash at the end. The experiment will last about 80 minutes.

The type of currency used in this experiment is Experimental Currency Units (ECU). Participants completing the session do not risk losing any money. At the end of the experiment all your earnings will be converted to US Dollars. The conversion rate is 10 ECU = 1 US Dollar. You will be paid in cash when you finish the experiment. The more ECU you earn, the more US Dollars you earn. If you participate in this experiment until the session is over, then you will be paid an additional 5 US Dollars.

Auction Rules and Calculation of Earnings

You will be participating in 20 auctions. At the beginning of the first auction, you will be randomly matched with another participant in this room; every auction, you will be randomly re-matched with a different participant. In each auction you will be bidding against the participant with whom you are matched.

In a given auction, there is a fictitious good that is sold and you will be a bidder in this auction. When the first auction starts, you will observe your value of the fictitious good. Your value is a number between 50 and 100 and it is randomly selected from the [50,100] interval with equal probability and rounded to the nearest cent. The other bidder participating in this auction also receives his or her independent value for the fictitious good and his or her value is also randomly selected from [50,100] interval. Each bidder will know only his or her own value.

The price for the fictitious good will start at 50 and will gradually increase to 60, 70, 80, 90, and 100. At price level 50, both you and your opponent are in the auction. The computer will ask you if you would like to stay in the auction when the price increases to 60. You either stay IN which indicates that you are willing to pay 60 for the good or you stay OUT. If you stay out for at a price of 60, you need to enter an exit bid which must be an amount between 50 and 60. The exit bid is the maximum amount that you are willing to pay for the good. For example, if you stay out and indicate that your exit bid is 54, then it means you would be willing to pay at most 54 for the good. The exit bid has to be an amount between the previous price level and the current price at which you are staying out. There are four possible things that can happen:

- You stay IN, and your opponent stays OUT with an exit bid of, say, 55: Then you win the fictitious good and pay 55, your opponent's exit bid.
- You stay OUT with exit bid of, say 57, and your opponent stays IN: Then your opponent wins the fictitious good and pays 57, your exit bid.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays the smaller of the exit bids.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 70.

If both you and your opponent stay in the auction for price level 60, then the computer will ask if you would like to stay in the auction when the price increases to 70. Again there are four possible things that can happen:

- You stay IN, and your opponent stays OUT with exit bid of, say, 65: Then you win the fictitious good and pay 65, your opponent's exit bid.
- You stay OUT with exit bid of, say 67, and your opponent stays IN: Then your opponent wins the fictitious good and pays 67, your exit bid.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays the smaller of the exit bids.

- You stay IN, your opponent stays IN: Then the price moves to the next level which is 80.

If both you and your opponent stay in the auction for price level 70, then the computer will ask if you would like to stay in the auction at a price of 80.

The same procedure will repeat for price levels of 90 and 100. If both of you are still in the auction when the price level is 100, then the computer will randomly assign the fictitious good to one of the bidders with equal chance and that bidder will pay 100.

Please note that the price level increases only if both bidders stay in the auction. If only one bidder stays in and the other one stays out with an exit bid, the bidder who is in wins the good and pays the exit bid of the other bidder. Otherwise, if both bidders stay out then the one with the higher exit bid wins the good and pays the exit bid of the other bidder. For example, when the price level is 70, if you indicate to stay OUT with exit bid of 63, and your opponent indicates to stay OUT with exit bid of 67, then your opponent wins the good (he or she has the higher exit bid) and pays 63 (which is the smaller exit bid). If you both submit the same exit bid while staying out then, the computer randomly assigns the good to one of the bidders, and the winner pays the exit bid of the other.

When one bidder wins the good, the auction is over. If you are the winner at a certain price, then you earn the difference between your value and the price. For example, let us say you have a value of 82.55 for the fictitious good in the current auction and you win the good at a price of 61. Then your earning from this round is

Earning = 82.55 - 61 = 21.55 ECU

When the first auction is completed, the second auction will start. At the beginning of the second auction, you will be randomly matched with another participant in this room and play with that person in this round. First, the computer will show you your new value for the good for this auction. It is again a randomly selected number from [50,100] interval. Your opponent will also observe his or her own value for the good for this auction privately. The same auction rules as in the first auction will apply. There are 20 auctions in total. The computer will sum up your earnings in ECU in all auctions and convert this amount to the US Dollars by dividing by 10. We will pay you this amount in cash at the end of the experiment in person.

In order to make sure that you understand the rules of the auction, we have a test period before the real session starts. In this test period, you will see some multiple choice questions about the auction rules. Please answer those questions to the best of your knowledge. You may look at the hard copy of the instructions while answering them. Once you answer all the questions correctly, one practice auction will be conducted for which no payment will be made. Then the experiment will start with 20 real auctions.

Please ask if you have any questions.

Questions for the test period (asked to the subjects during the test period by the computer)

1) The price level just moved from 60 to 70 and the computer asks you if you would like to stay in at a price of 70. You said that you are IN and your opponent said that he or she is OUT with an exit bid of 65. Then what would be the outcome of the auction?

a) You would win the good and pay 60.

b) Your opponent would win the good and pay 70.

c) You would win the good and pay 65.

d) Nobody would win the good and the price would move to 80.

Answer: (c)

2) The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You decided to stay OUT for price level of 80. What are the possible exit bids you may enter?

a) Any amount between 65 and 70.

b) Any amount between 70 and 80.

c) Any amount between 60 and 70.

d) Any amount between 80 and 90.

Answer: (b)

3) Let us say, your value of the good is 91 for the current round. The price level just moved from 70 to 80 and the computer asks you if you would like to stay in

for price level 80. You said that you are OUT with an exit bid of 77 and your opponent said that he or she is OUT with an exit bid of 74. Then what would be your and your opponent's earnings for this round?

a) You would win the good and pay 77. You would earn 14 and your opponent would earn 0.

b) Your opponent would win the good and pay 74. You would earn 0 and you cannot know your opponent's earning without knowing his or her value.

c) You would win the good and pay 80. You would earn 11 and your opponent would earn 0.

d) You would win the good and pay 74. You would earn 17 and your opponent would earn 0.

Answer: (d)

C.2 Treatment LAB

Experiment Instructions

Welcome to the auction experiment. In this experiment, you will participate in auctions as a bidder. The precise rules and procedures that govern the operation of these auctions will be explained to you below.

Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions you can finish the experiment with a considerable amount of money, which will be paid to you in cash at the end. The experiment will last about 80 minutes.

The type of currency used in this experiment is Experimental Currency Units (ECU). Participants completing the session do not risk losing any money. At the end of the experiment all your earnings will be converted to US Dollars. The conversion rate is 10 ECU = 1 US Dollar. You will be paid in cash when you finish the experiment. The more ECU you earn, the more US Dollars you earn. If you participate in this experiment until the session is over, then you will be paid an additional 5 US Dollars.

Auction Rules and Calculation of Earnings

You will be participating in 20 auctions. At the beginning of the first auction, you will be randomly matched with another participant in this room; every auction, you will be randomly re-matched with a different participant. In each auction you will be bidding against the participant with whom you are matched.

In a given auction, there is a fictitious good that is sold and you will be a bidder in this auction.. When the first auction starts, you will observe your value of the fictitious good. Your value is a number between 50 and 100 and it is randomly selected from the [50,100] interval with equal probability and rounded to the nearest cent. The other bidder participating in this auction also receives his or her independent value for the fictitious good and his or her value is also randomly selected from [50,100] interval. Each bidder will know only his or her own value.

The price for the fictitious good will start at 50 and will gradually increase to 60, 70, 80, 90, and 100. At price level 50, both you and your opponent are in the auction. The computer will ask you if you would like to stay in the auction when the price increases to 60. You either stay IN which indicates that you are willing to pay 60 for the good or you stay OUT. If you stay out at a price of 60, you need to enter an exit bid which must be an amount between 50 and 60. The exit bid is the maximum amount that you are willing to pay for the good. For example, if you stay out and indicate that your exit bid is 54, then it means you would be willing to pay at most 54 for the good. The exit bid has to be an amount between the previous price level and the current price at which you are staying out. There are four possible things that can happen:

- You stay IN, and your opponent stays OUT: Then you win the fictitious good and pay 60.
- You stay OUT, and your opponent stays IN: Then your opponent wins the fictitious good and pays 60.
- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays his or her exit bid.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 70.

If both you and your opponent stay in the auction for price level 60, then the computer will ask if you would like to stay in the auction when the price increases to 70. Again there are four possible things that can happen:

- You stay IN, and your opponent stays OUT: Then you win the fictitious good and pay 70.

- You stay OUT, and your opponent stays IN: Then your opponent wins the fictitious good and pays 70.

- You stay OUT, your opponent stays OUT: Then the bidder who submitted the highest exit bid wins the fictitious good and pays his or her exit bid.

- You stay IN, your opponent stays IN: Then the price moves to the next level which is 80.

If both you and your opponent stay in the auction for price level 70, then the computer will ask if you would like to stay in the auction at a price of 80.

The same procedure will repeat for price levels of 90 and 100. If both of you are still in the auction when the price level is 100, then the computer will randomly assign the fictitious good to one of the bidders with equal chance and that bidder will pay 100.

Please note that the price level increases only if both bidders stay in the auction. If only one bidder stays in and the other one stays out, the bidder who is in wins the good and pays the price for which he or she indicated to stay in. Otherwise, if both bidders stay out then the one with the higher exit bid wins the good and pays his or her exit bid. For example, when the price level is 70, if you indicate to stay OUT with exit bid of 63, and your opponent indicates to stay OUT with exit bid of 67, then your opponent wins the good (he or she has the higher exit bid) and pays 67 (his or her exit bid). If you both submit the same exit bid while staying out then, the computer randomly assigns the good to one of the bidders, and the winner pays his or her exit bid.

When one bidder wins the good, the auction is over. If you are the winner at a certain price, then you earn the difference between your value and the price. For example, let us say you have a value of 82.55 for the fictitious good in the current auction and you win the good at a price of 61. Then your earning from this round is

Earning = 82.55 - 61 = 21.55 ECU

When the first auction is completed, the second auction will start. At the beginning of the second auction, you will be randomly matched with another participant in this room and play with that person in this round. First, the computer will show you your new value for the good for this auction. It is again a randomly selected number from [50,100] interval. Your opponent will also observe his or her own value for the good for this auction privately. The same auction rules as in the first auction will apply. There are 20 auctions in total. The computer will sum up your earnings in ECU in all auctions and convert this amount to the US Dollars by dividing by 10. We will pay you this amount in cash at the end of the experiment in person.

In order to make sure that you understand the rules of the auction, we have a test period before the real session starts. In this test period, you will see some multiple choice questions about the auction rules. Please answer those questions to the best of your knowledge. You may look at the hard copy of the instructions while answering them. Once you answer all the questions correctly, one practice auction will be conducted for which no payment will be made. Then the experiment will start with 20 real auctions.

Please ask if you have any questions.

Questions for the test period (asked to the subjects during the test period by the computer)

1) The price level just moved from 60 to 70 and the computer asks you if you would like to stay in at a price of 70. You said that you are IN and your opponent said that he or she is OUT with an exit bid of 65. Then what would be the outcome of the auction?

a) You would win the good and pay 70.

b) Your opponent would win the good and pay 70.

c) You would win the good and pay 65.

d) Nobody would win the good and the price would move to 80.

Answer: (a)

2) The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You decided to stay OUT for price level of 80. What are the possible exit bids you may enter?

a) Any amount between 65 and 70.

b) Any amount between 70 and 80.

c) Any amount between 60 and 70.

d) Any amount between 80 and 90.

Answer: (b)

3) Let us say, your value of the good is 91 for the current round. The price level just moved from 70 to 80 and the computer asks you if you would like to stay in for price level 80. You said that you are OUT with an exit bid of 77 and your opponent said that he or she is OUT with an exit bid of 74. Then what would be your and your opponent's earnings for this round?

a) You would win the good and pay 77. You would earn 14 and your opponent would earn 0.

b) Your opponent would win the good and pay 74. You would earn 0 and you cannot know your opponent's earning without knowing his or her value.

c) You would win the good and pay 80. You would earn 11 and your opponent would earn 0.

d) You would win the good and pay 74. You would earn 17 and your opponent would earn 0.

Answer: (a)

C.3 Treatment LABpw

Experiment Instructions

Welcome to the auction experiment. In this experiment, you will participate in auctions as a bidder. The precise rules and procedures that govern the operation of these auctions will be explained to you below.

Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions you can finish the experiment with a considerable amount of money, which will be paid to you in cash at the end. The experiment will last about 80 minutes.

The type of currency used in this experiment is Experimental Currency Units (ECU). Participants completing the session do not risk losing any money. At the end of the experiment all your earnings will be converted to US Dollars. The conversion rate is 10 ECU = 1 US Dollar. You will be paid in cash when you finish the experiment. The more ECU you earn, the more US Dollars you earn. If you participate in this experiment until the session is over, then you will be paid an additional 5 US Dollars.

Auction Rules and Calculation of Earnings

You will be participating in 20 auctions. At the beginning of the first auction, you will be randomly matched with another participant in this room; every auction, you will be randomly re-matched with a different participant. In each auction you will be bidding against the participant with whom you are matched.

In a given auction, there is a fictitious good that is sold and you will be a bidder in this auction.. When the first auction starts, you will observe your value of the fictitious good. Your value is a number between 50 and 100 and it is randomly selected from the [50,100] interval with equal probability and rounded to the nearest cent. The other bidder participating in this auction also receives his or her independent value for the fictitious good and his or her value is also randomly selected from [50,100] interval. Each bidder will know only his or her own value.

The price for the fictitious good will start at 50 and will gradually increase to 60, 70, 80, 90, and 100. At price level 50, both you and your opponent are in the auction. Then the computer will randomly determine one of you as a provisional winner. Each bidder has 50% chance of being a provisional winner. Next, the computer will ask you if you would like to stay in the auction when the price increases to 60. There are four possible things that can happen:

- You stay IN, your opponent stays OUT: Then you win the fictitious good and pay 60.
- You stay OUT, your opponent stays IN: Then your opponent wins the fictitious good and pays 60.
- You stay OUT, your opponent stays OUT: Then the provisional winner wins the fictitious good and pays 50.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 70.

If both you and your opponent stay in the auction for price level 60, then the computer will randomly determine a new provisional winner and ask if you would like to stay in the auction when the price increases to 70. Again there are four possible things that can happen:

- You stay IN, your opponent stays OUT: Then you win the fictitious good and pay 70.
- You stay OUT, your opponent stays IN: Then your opponent wins the fictitious good and pays 70.
- You stay OUT, your opponent stays OUT: Then the provisional winner wins the fictitious good and pays 60.
- You stay IN, your opponent stays IN: Then the price moves to the next level which is 80.

If both you and your opponent stay in the auction for price level 70, then the computer will randomly determine a new provisional winner and ask if you would like to stay in the auction when price increases to 80.

The same procedure will repeat for price levels 90 and 100. If both of you are still in the auction when the price level is 100, then the computer will randomly assign the fictitious good to one of the bidders with equal chance, and the winner will pay 100. Please note that the price level only increases if both bidders stay in the auction. Each time the price level increases a new provisional winner is randomly determined by the computer and each bidder has equal chance of being the provisional winner at that price level. The provisional winner wins the fictitious good at the previous price level if both bidders stay OUT at the current price level. Otherwise, if one bidder stays in and the other bidder stays out in the current round, then the bidder who is IN wins the good at the current price, regardless of the provisional winner designation.

When one bidder wins the good, the auction is over. If you are the winner at a given price, then you earn the difference between your value and the price. For example, let us say you have a value of 82.55 for the fictitious good in the current auction and you win the good at a price of 60. Then your earning from this round is

Earning = 82.55 - 60 = 22.55 ECU

When the first auction is completed, the second auction will start. At the beginning of the second auction, you will be randomly matched with another participant in this room and play with that person in this auction. First, the computer will show you your new value for the good for this auction. It is again a randomly selected number from [50,100] interval. Your opponent will also observe his or her own value of the good for this auction privately. The same auction rules as in the first auction will apply. There are 20 auctions in total. The computer will sum up your earnings in ECU in all auctions and convert this amount to US Dollars by dividing by 10. We will pay you this amount in cash at the end of the experiment in person.

In order to make sure that you understand the rules of the auction, we have a test period before the real session starts. In this test period, you will see some multiple choice questions about the auction rules. Please answer those questions to the best of your knowledge. You may look at the hard copy of the instructions while answering them. Once you answer all the questions correctly, one practice auction will be conducted for which no payment will be made. Then the experiment will start with 20 real auctions.

Please ask if you have any questions.

Questions for the test period (asked to the subjects during the test period by the computer)

1) The price level just moved from 60 to 70 and the computer determined you as the provisional winner for the current price. The computer asks you if you would like to stay in for price level 70. You said that you are IN and your opponent said that he or she is OUT. Then what would be the outcome of the auction?

a) Your opponent would win the good and pay 70.

b) You would win the good and pay 70.

c) You would win the good and pay 60.

d) Nobody would win the good and the price would move to 80.

Answer: (b)

2) The price level just moved from 60 to 70 and the computer determined your opponent as the provisional winner for the current price. The computer asks you if you would like to stay in for price level 70. You said that you are OUT and your opponent said that he or she is OUT. Then what would be the outcome of the auction?

a) You would win the good and pay 70.

b) Your opponent would win the good and pay 70.

c) You would win the good and pay 60.

d) Your opponent would win the good and pay 60.

Answer: (d)

3) Your value of the good is 91 for the current auction. The price level just moved from 70 to 80 and the computer determined you as the provisional winner for the current price. The computer asks you if you would like to stay in for price level 80. You said that you are OUT and your opponent said that he or she is OUT. Then what would be your and your opponent's earnings for this auction?

a) You would win the good and pay 70. You would earn 21 and your opponent would earn 0.

b) Your opponent would win the good and pay 70. You would earn 0 and you cannot know your opponent's earning without knowing his or her value.

c) You would win the good and pay 80. You would earn 11 and your opponent would earn 0.

d) Your opponent would win the good and pay 80. You would earn 0 and your opponent would earn 15.

Answer: (a)

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