ABSTRACT

Title of dissertation: PREFERENCE BASED FAIR ALLOCATION OF LIMITED RESOURCES

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The fair division of scarce resources among agents is a challenging issue across a range of applications, especially when there is competition among agents. One application of resource division is in Air Traffic Management (ATM). During severe weather, when there is a reduction in en-route capacity, a specialist using the TSD (Traffic Situation Display) identifies a problem area by creating a Flow Constrained Area (FCA). The air traffic flow management specialists at the Air Traffic Control System Command Center can enter the capacity of the FCA, expressed as the number of flights that can be managed per hour, and an Airspace Flow Program (AFP) will be run. Thus, affected flights will be delayed or rerouted.

Fair allocation of available resources among airlines is very challenging when there is a reduction in en-route resources. Each airline will typically place a different

relative weight on delays, rerouting and cancelation. Whereas some airlines would like to preserve the on-time performance for certain flights and cancel or reroute many other flights, other airlines prefer to have less rerouting and cancelations while tolerating higher total delay. Therefore, fairness concerns as well as the ability to respond to different user priorities have played an important role throughout the development of allocation procedures, and continue to be an essential factor. The notion of fairness in air traffic management is largely left implicit and there is no well-defined set of principles that defines what constitutes a fair distribution of resources.

This dissertation is motivated by the fairness issues that arise in the resource allocation procedures that have been introduced under Collaborative Decision Making (CDM). Fair rationing and allocation of available en-route time slots are two major challenges that we address in this research.

The first challenge, fair rationing, is about how to compute a fair share of available resources among agents, when the available resources fall below the total demand. Since the demand, (flights), are time dependent, we introduce a new rationing method that includes the time dependency of demand. The new procedure gives every flight that is disrupted by an AFP a share of available resources. This is in contrast to Ration-By-Schedule (RBS), the allocation method currently in use, where later scheduled flights do not receive any slots. We will discuss and prove the fairness properties of our novel rationing procedure.

The second challenge, allocation of en-route resources, is about how to allocate resources among competitive agents, (flight operators), when each agent has different preferences over resources, (time slots). We design four randomized procedures for allocating scarce resources when the airlines' preferences are included. These procedures use an exogenous fair share, which can be computed using the method described above, as a fairness standard for the allocation of slots among airlines.

The first two procedures, Preference Based Proportional Random Allocation (PBPRA) and Modified-PBPRA, implicity assume equal weight for each time slot. Compared to RBS, PBPRA and M-PBPRA reduce the total internal cost of airlines and also assign each airline a number of slots close (in expectation) to their fair share. The fairness, efficiency and incentive properties of PBPRA and M-PBPRA are evaluated.

The value (or cost of delay) an airline associates with a particular flight may vary substantially from flight to flight. Airlines who wish to receive priority for certain flights usually are willing to pay more for specific time slots. To address the need to express varying priorities, we propose two procedures, Dual Price Proportional Random Allocation (DP-PRA) and Modified-DP-PRA (MDP-PRA) , that assign dual prices to resources, i.e. time slots, in order to capture the airlines' preferences over delays, rerouting and cancelations. We explore the fairness, efficiency and incentive properties of DP-PRA and MDP-PRA.

Preference Based Fair Allocation of Limited Resources

by

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Dedication

To my parents

and

To my husband

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Chapter 1

Introduction

The air transportation system in the United States is one of the most complex logistical systems in the world. Each day, there are approximately 60,000 flights of commercial, military and general aviation aircraft, and as many as 10,000 aircraft may simultaneously occupy the airspace. Current projections expect air traffic to grow at annual rate of 4% to 5% over the next 15 years. Besides the sheer volume, the air transportation system gets more stressed by significant variation in airspace capacity, due to factors such as fluctuating weather conditions and equipment outages. Therefore, the coordination of air traffic requires a multitude of processes and involves large number of stake holders.

To manage airspace congestion better, the Federal Aviation Administration (FAA) has implemented a number of initiatives such as Miles-in-Trail(MIT), Ground Delay Program(GDP) and Metering . However, when there is an en-route capacity reduction due to severe weather, none of these initiatives are sufficient to address extended capacity reductions in the airspace, and the need for additional tool has been recognized. In the spring of 2006, the FAA proposed a new initiative called the Airspace Flow Program (AFP) to allow more efficient, effective, equitable, and predictable management of airborne traffic in congested airspace.

However, there are several potential weaknesses with the way traffic is handled today. Several Traffic Management Initiatives (TMI's) can create excessive delay. For example, Miles-In-Trail restrictions can propagate, resulting in longer restrictions; and rerouting is manually intensive, consuming controller time and attention while giving little consideration to NAS customer input. To address these problems, the Collaborative Decision Making (CDM) community has recognized a more collaborative and dynamic capability. The CDM Future Concepts Team (FCT) has focused on the development of two concepts over the past several years to address these needs; Integrated Collaborative Rerouting (ICR) and System Enhancements for Versatile Electronic Negotiation (SEVEN). ICR is an enhanced, more collaborative version of rerouting that involves customers early in the process and allows them to submit preferences for reroutes. SEVEN is a longer-term concept that allows much more collaboration between FAA traffic managers and NAS customers. SEVEN extends and makes more robust the current practice of AFP's while retaining the present capabilities.

The typical approach to managing airspace congestion today can be described as follows. When there is a reduction in enroute capacity due to sever weather, a traffic management specialist using the TSD (Traffic Situation Display), can identify a problem area by creating a Flow Constrained Area (FCA). The TFM specialists at the Air Route Traffic Control Center (ATCSCC) can enter the capacity of the FCA, expressed as the number of flights that can be managed per hour, and Flight Schedule Monitor (FSM) will then assign each flight a controlled departure time that will provide a smooth managed flow of traffic to the FCA. These departure times are sent to the customers for their planning and to the towers at the departure airports for enforcement. The AFP uses GDP like procedures such as Ration By Schedule(RBS), which is based on the first-scheduled-first-served principle, for resource rationing. However, GDP procedures implicitly assume all flights must be assigned an arrival slot. On the other hand, in the AFP or SEVEN setting, all flights on the demand list need not be granted access to the enroute resource.

There are important differences between resource allocation for GDPs and enroute resource allocation. In the GDP setting, demand is established based on the set of flights scheduled to arrive at the GDP airport. Since the authority to cancel a flight rests with the flight operator and not the FAA, GDP planning procedures must allocate a slot to all flights within the GDP demand list. Of course, in severe situations, the FAA will be forced to assign extreme flight delays, which may de facto necessitate the cancellation of certain flights. However, GDP procedures implicitly assume all flights must be assigned an arrival slot. On the other hand,in the AFP or SEVEN setting, all flights on the demand list need not be granted access to the enroute resource. The flight operator has the prerogative to cancel flights not given access or reroute such flights around the restricted airspace. Thus, enroute resource allocation decision models must both determine which flights gain access and assign an access time (slot) for those flights that do gain access.

1.1 Motivation

Fair allocation of available resources among airlines is very challenging when there is a reduction in en-route resources. Fairness concerns have played an important role throughout the development of allocation procedures, and continue to be an essential factor whenever extensions or modifications to these procedures are proposed.

Each airline will typically place a different relative weight on delays, rerouting and cancellation. Whereas some airlines would like to preserve the on-time performance for certain flights and cancel or reroute other flights, other airlines prefer to have less rerouting and cancellation while tolerating higher total delay.

Using fairness principles as a basis for allocating scarce resources provides our research with a novel focus. In fact, some proposals address the rationing of airport arrival capacity in the long run. Using methods ranging from auctions [52] and congestion pricing [50] to bargaining schemes [4]. The allocation of slots under CDM is different, in that slots must be assigned on a daily basis due to fluctuation in airport or en-route capacity. The dynamic nature of the allocation process makes it more complicated and fairness plays an important role in this environment.

Our research focuses on the development of new fair enroute resource rationing methods with specific emphasis on mechanisms for airspace flow programs. Our objective is to include airline preferences for trading-off delay and rerouting.

1.2 Research Contribution and Outline

Chapter 2 presents an overview of air traffic management, especially management of daily operations. We also explain the current air traffic flow management initiatives employed by the FAA. The new concept System Enhancement for Versatile Electronic Negotiation (SEVEN) will be explained. Both Airspace Flow Programs and SEVEN represent potential application domains for our research. We present an overview of slot allocation methods and discuss how fairness and user preferences play an important role in resource allocation.

Chapter 3 introduces a new way for fair rationing of resources, which in our context is the problem of how to compute a fair share of available resources among agents, when the available resources fall below the total demand. The new rationing is designed to consider time-based resources, time slots, and time-dependent demand, flights. The proposed rationing procedure is based on Proportional Random Assignment. We adapted fairness concepts from the economics literature as a basis of our rationing procedures. The key advantage of our proposed rationing algorithm is that it considers all flights disrupted by an AFP, which means all flights receive a positive share of available slots. However, it should be noted that the current policy used in practice, Ration-By-Schedule (RBS), provides a slot assignment only to the earliest disrupted flights so that later scheduled flights do not receive any slots in an AFP. We develop the new rationing algorithm, and also we present experimental results to compare it vs. RBS. Additionally, in order to investigate the fairness of the proposed rationing algorithm, we analytically prove fairness properties such as impartiality, consistency, equal treatment of equals, and demand monotonicity.

Chapter 4 introduces novel methods for allocation of limited resources, which in our context is the problem of how to allocate resources among competitive agents, (flight operators), when each agent, (airline), has a different preference over resources, (time slots). We design two randomized methods for allocating scarce resources when the airlines' preferences are included. They exploit an exogenous fair share, which can be computed using the method described in the chapter 3, as a fairness standard to drive the allocation. Our randomized procedures take into account airline preference information, and they implicity assign equal weight to each time slot. Key advantages of our algorithms are described: (1) they guarantee that the total number of slots any carrier receives is between the floor and ceiling of their fair share; (2) they assign each carrier a number of slots close (in expectation) to their fair share; (3) they reduce the total internal cost of each carrier compared to the existing resource allocation procedure, RBS. Our two proposed algorithms are compared experimentally vs. each other and their fairness, efficiency and incentive properties are analytically derived. We discuss the shortcomings of RBS, and we compare the performance of our algorithms with RBS based on data derived from a real application.

Chapter 5 defines and analysis two new alternative randomized allocation algorithms that employ richer agent preference information. The value (or cost of delay) an airline associates with a particular flight may vary substantially from flight to flight. Airlines who wish to receive priority for certain flights usually are willing to pay more for specific time slots. To accommodate richer carrier preferences so that airlines can express the relative importance of delays, rerouting and cancelations, new concepts of slot values and dual pricing are introduced. A key advantage of these methods is the sophistication that they provide to carriers for capturing their preferred slots. This provides flexibility to carriers to achieve their goals; and also allows carriers to receive "premium" slots for an extra "charge". The two designed algorithms are compared vs. each other and their fairness, efficiency and incentive properties are analytically derived. We analyze the performance of the new methods and compare them with RBS based on based on data derived from a real application.

Chapter 6 provides conclusions and discusses future research areas.

Chapter 2

Air Traffic Management

The air transportation system in the United States is one of the most complex logistical systems in the world. Each day, there are approximately 60,000 flights of commercial, military and general aviation aircraft, and as many as 10,000 aircraft may simultaneously occupy the airspace. Besides the sheer volume, the air transportation system gets more stressed by significant variation in airspace capacity, due to factors such as fluctuating weather conditions and equipment outages. Therefore, the coordination of air traffic requires a multitude of processes and involves large number of stake holders.

This chapter presents a general overview of Air Traffic Management with particular focus on Air Traffic Flow Management (ATFM) initiatives. We explain at a high level the characteristics of various initiatives. Next, we describe the importance of fair allocation of scarce resources in ATFM [8]. Finally, we explain the motivation behind this research.

2.1 Introduction

Air Traffic Management (ATM) consists of two major components. Air Traffic Flow Management (ATFM) and Air Traffic Control (ATC). ATC consists of processes that provide tactical separation services, that is, real-time separation procedures for collision detection and avoidance. Thus, ATC primarily addresses immediate safety concerns of airborne flights. On the other hand, Air Traffic Flow Management (ATFM) includes all activities related to the management of the flow of aircraft and related system resources from "block to block", including strategic system management of airport arrival and departure capacities, tactical enroute flow management, near-terminal area flow management, and ground traffic flow management. As such, ATC actions are of a more "microscopic" nature and have a very short time horizon. The aim of ATFM is to resolve a capacity imbalances by adjusting aggregate traffic flows to match scarce resources.

Odoni [49] classified the Air Traffic Flow Management into three different category: Long, Medium and Short-term:

- Long-Term approaches are mainly focused on increasing the capacity. This can be done by constructing a new airport or a runway at an existing airports. Such initiatives are usually very costly and subject to strict environmental regulations. Thus they may be difficult to implement.

- Medium-Term approaches are more administrative or economic and try to alleviate congestion by modifying spatial or temporal traffic patterns. For instance, at some airports flight schedules are coordinated bi-annually according to IATA guidelines [37]. Recent proposal suggests the use of slot auctions and congestion pricing [49].

- Short-Term approaches mainly try to make adjustments to air traffic flows to match demand with available capacity. Such short-term plans usually are performed a few hours in advance of predictable disruptions (usually caused by bad weather).

In the next section, we review the ATFM short-term initiatives. Through this dissertation, we use ATFM to refer to only short-term initiatives.

2.2 Air Traffic Flow Management Initiatives

The primary task of the Federal Aviation Administration (FAA), the U.S. Air Navigator Service produce, is to enforce the proper separation requirement in the controlled airspace. The United States air space has been divided to 22 areas. The Air Route Traffic Control Centers (ARTCCs) are responsible for aircraft separation within each area. Each ARTCC is divided to 20 to 80 smaller areas called sectors. Air Traffic Controllers (ATCs) guide aircrafts from one sector to another until they arrive within almost 200 miles from their destination airports. A controller is only responsible for the movement of aircraft within a specific sector and decisions are based on nearly real-time flight information when the flight enters the sector. There is coordination between controllers at adjacent sectors by transferring the responsibility for an aircraft when it passes sector boundaries. Finally, the control of aircraft is assumed by Terminal Radar Approach Control Facilities (TRACONs). The airport towers control aircraft while they taxi to and from runways and during takeoffs and landings.

The FAA uses the Enhanced Traffic Management System (ETMS) at the Air Traffic Control System Command Center (ATCSCC) and major Terminal Radar

Approach Control (TRACON) facilities to manage the flow of air traffic within the National Airspace System (NAS). Other organizations (e.g., the airlines, Department of Defense, NASA, and international sites) also have access to the ETMS software and/or data. The ETMS provides Traffic Management Specialists with tools such as Traffic Situation Display (TSD), and traffic counts for airspace sectors, airports, and fixes. The ATCSCC continuously monitors current and projected demand within the NAS. Whenever it is predicted that demand will exceed capacity limits for at least a 15 minutes duration, FAA regulation mandates a response. In that case, the ARTSCC generates and implements strategies to resolve the problem. The ATFM procedures that are used most often are ground delay programs or ground stops , flow constraint area/ flow evaluation area, metering, rerouting and recently air space flow programs(AFP's); there is also a new concept called System Enhancements for Versatile Electronic Negotiation (SEVEN) . We give a brief introduction to these initiatives in the following sections.

2.2.1 Ground Delay Program

Each airport is constrained by the rate at which they can land arriving aircrafts. Generally, when airports operate under normal circumstances, the scheduled aircraft flow does not exceed the arrival rate. But circumstances, most usually poor weather conditions, can lower the arrival rate so that the expected number of arriving aircrafts exceeds the capacity of the airport. In these circumstances, the ATCSCC reacts by issuing a Ground Delay Program (GDP). A GDP issues departure delay to aircraft expected to arrive at the constrained airport. These ground delays are less costly and safer than airborne delays that would result without such actions.

Flights destined for the affected airport are issued Controlled Departure Times (CDT) at their points of departure. Flights that have been issued CDTs are not permitted to depart until their Controlled Departure Times. These CDTs are calculated in such a way as to meter the rate that traffic arrives at the affected airport, ensuring that demand is equal to the acceptance rate. The length of delays that result from the implementation of a GDP is a function of two factors: how much greater than the acceptance rate the original demand was, and for what length of time the original demand was expected to exceed the acceptance rate.

A Ground Stop (GS) is closely related to a GDP. When there is an unexpected problem at an airport, e.g. a runway closure, the ATCSCC will stop all inbound traffic , i.e. indefinitely delay their departure, to reduce traffic flows. When ground stops become excessive or delay can be foreseen, a regular GDP often follows a ground stop.

2.2.2 Flow Constraint Area/ Flow Evaluation Area

The Traffic Situation Display (TSD) and the Common Constraint Situation Display (CCSD) provide traffic managers and flight dispatchers with the ability to define and display FEAs and/or FCAs. An FEA/FCA is a user-defined volume of airspace along with associated flight lists and filters. FEAs and FCAs are used to show areas where the traffic flow should be evaluated or where initiatives should be taken due to severe weather or volume constraints. Traffic managers or flight dispatchers define a geographic area of an FEA or an FCA by drawing a polygon or a line on the display and defining the ceiling and floor of the FEA/FCA using a dialog box. Alternatively, an FEA/FCA tool user can designate a NAS element as an FEA/FCA (e.g., a fix, an airport, a sector, or a TRACON). The tool user also defines criteria for filtering the flights that are predicted to intersect the drawn FEA/FCA (e.g., by airports, by center traversed, or by departure or arrival points). The FEA/FCA tool user also defines a time period for the FEA/FCA (maximum of 23 hours). It is useful here to distinguish FEAs from FCAs. A Flow Evaluation Area (FEA) is a two dimensional line or three-dimensional volume of airspace, along with filters and time boundaries, used to identify flights associated with a potential (or actual) constraint. FEAs can be built by Traffic Management Coordinators (TMCs) at Traffic Management Units (TMUs), Traffic Management Specialists (TMSs) at the Air Traffic Control System Command Center (ATCSCC), or by flight dispatchers at various flight operations centers (using CCSD). A Flow Constrained Area (FCA) is an FEA subject to an actual constraint. FCAs are built by the ATCSCC and require a traffic management initiative (TMI); for example, a reroute. Any FEA tool user can create "private" FEAs for viewing on their workstation to monitor traffic flows and evolving traffic flow situations. If it is determined that a developing constraint situation may impact any system stakeholder, an FEA/FCA tool user can create a "Shared FEA" in order to exchange information and facilitate collaboration with other system stakeholders. At this point, some voluntary action may be suggested and taken by stakeholders to help avoid a more drastic requirement or reroute initiative. The idea is to solicit “Operational Intent” information in order for Traffic Managers to assess whether more restrictive initiatives are warranted. Intent data can be submitted by NAS customers through the CCSD. [33]

2.2.3 Metering and Rerouting

Metering operations are mostly used in the context of en route Traffic Flow Management (TFM). There are two kinds of metering: the first, time-based metering, which controls the time at which an aircraft is to pass over the certain geographical point. Second, distance-based metering, which is better known as “Miles-in-Trail”(MIT) restriction. A MIT specifies a minimum separation (in miles) between aircraft moving across an airspace way point.

The primary use of time-based metering is to regulate flows into the terminal area of an airport. Here, time-based metering efficiently spaces aircraft for final approach. On the other hand, MIT restrictions are typically into a congested portion of enroute airspace (or terminal area) [35].

When bad weather is forecast to impact accessibility to a certain region of airspace, rerouting can be used as an option. Severe Weather Avoidance Procedures (SWAP) are applied to deal with such conditions. SWAP plans usually have a major impact on air traffic, including metering restrictions and/or GDP’s along with rerouting and/or AFP.

2.2.4 Airspace Flow Programs

When there is a capacity reduction due to the severe weather, rerouting flights is not sufficient to address extended capacity reductions in the airspace, and the need for additional tools has long been recognized. To meet that need the FAA introduced a new capability in the spring of 2006. The Airspace Flow Program (AFP) combines the power of GDP's and FCAs to allow more efficient, effective, equitable, and predictable management of airborne traffic in congested airspace.

When TFM specialists at the ATCSCC, in consultation with FAA field managers and customer representatives, decide that the weather conditions are appropriate they can plan and deploy an AFP. The first step is to use the Traffic Situation Display (TSD) to examine predicted weather and traffic patterns and identify the problem area by creating an FCA.

The Enhanced Traffic Management System (ETMS) takes the FCA description and produces a list of the flights that are expected to pass through the FCA and the time they are expected to enter. This list, updated with fresh information every five minutes, is sent to the Flight Schedule Monitor (FSM), which displays the projected demand in a number of formats designed to support effective planning. FSM creates a common situational awareness among all users and service providers in the National Airspace System. All parties need to be aware of NAS constraints in order to make collaborative air traffic decisions. It is designed to effectively interact with existing FAA systems, FSM displays the Aggregate Demand List (ADL) information for both airport and airspace data elements for its users, which means everyone is looking at the same picture.. The TFM specialists at the ATCSCC can enter the capacity of the FCA, expressed as the number of flights that can be managed per hour, and FSM will then assign each flight a controlled departure time so that the flow into the FCA does not exceed the declared capacity. These departure times are sent to the customers for flight planning and to the towers at the departure airports for enforcement.

The principal goal for the initial deployment of the AFP program is to better manage en route traffic during severe weather events. Compared to previous approaches, AFP's reduce unnecessary delays while providing better control of demand, more equity, and more flexibility for customers [43].

2.2.5 System Enhancements for Versatile Electronic Negotiation

As we mentioned in previous sections, the FAA has implemented a number of initiatives to manage airspace congestion better. However, a remaining shortcoming is the inability of carriers to express preferences based on their business needs. To manage en route congestion better and enable National Airspace System (NAS) customers to submit sets of alternative trajectory options for their flights, the CDM Future Concepts Team (FCT) has focused on the development of two concepts over the past several years to address these needs; Integrated Collaborative Rerouting (ICR) and System Enhancements for Versatile Electronic Negotiation (SEVEN).

ICR is an enhanced, more collaborative version of rerouting that involves customers early in the process and allows them to submit preferences for reroutes.

Initial ICR capabilities were deployed with Enhanced Traffic Management System (ETMS) version 8.3 (Fall 2006). SEVEN is a longer-term concept that allows much more collaboration between FAA traffic managers and NAS customers.

There are two main enabling concepts in SEVEN:

1. Customers can submit prioritized lists of route options
2. Traffic managers create Interactive Dynamic Flight Lists (IDFL's) that allow them to monitor key system resources

The first concept is extremely flexible, customers can include any option they like and can submit this list at any time. They can submit not only different physical routes, but different temporal routes as well. That is, they can adjust the times they are willing to fly certain routes or submit the same route flown at different times as options. SEVEN adds a default last choice option of ground delay. Thus there are always options in the system, and even without participation from the customers SEVEN falls back to an AFP-like Traffic Management Initiative.

The second main concept in SEVEN is its instrument control mechanism. Traffic managers create Interactive Dynamic Flight Lists (IDFL's) that allow them to monitor key system resources and adjust the demand on these resources quickly as conditions change. They do this by choosing flights to allow or disallow in the FCA, rather than manually rerouting flights. The IDFL provides an interface from which to do this, either by choosing flights manually or by automatically suggesting flights for the traffic manager. This functionality is combined with monitoring capability, condensing these traffic management tools into a single interface. This ability to monitor and alter demand without worrying about individual reroutes gives traffic managers finer and more efficient control over resource allocation [34].

The basic steps of SEVEN are explained as follows. These are not necessarily sequential steps, but rather elements of the concept which may occur at different times during operation.

1. **Customers submit prioritized lists of route options**. They submit this list to the Traffic Management System (TMS). If they so choose, they may edit the list at any time, because of changing weather conditions or changing priorities. Potentially, customers will develop their own software to automatically create the lists. Currently, customers can file a single route and they have no control over weather or not it is accepted. If a situation arises in which they cannot fly their current route, not only do they have little input into their reroutes, but the burden of finding and choosing reroute options is on the traffic manager.

2. **Traffic managers identify areas of interest by creating IDFL's**. When a region of airspace might become congested, traffic managers at the ARTCC Traffic Management Unit (TMU), in collaboration with the ATCSCC, establish an FEA or FCA. Traffic managers share the FEA/FCA with the customers, along with any additional constraints, remarks, and route guidance. Once the constrained area is defined, traffic managers generate an IDFL identifying the flights with current routes that take them through the constrained area. IDFL's are dynamic and updated as changes occur. The IDFL dis-

plays the pertinent data about these flights, including ACID (Aircraft ID), current state, origin, destination, entry and exit time, altitude, and any other data deemed necessary. The IDFL updates dynamically to reflect the changes made either by the traffic manager or by the customers. As route options are adjusted, flights will appear on or disappear from the list in real time. Once the IDFL is created, traffic managers monitor the demand and the next step is enacted.

3. **Customers receive notification for flights on the IDFL**. Once traffic managers generate or share an IDFL, dispatchers are notified that their flights are to be subject to the constraint associated with the IDFL. If a customer has not loaded any options, they are notified that they are at risk of taking ground delay if they are moved. This message also notifies dispatchers of the potential ground delay time.

4. **Traffic managers dynamically adjust demand up or down and choose flights to move on or off the IDFL**. Traffic managers have the ability to adjust the demand on a constrained area. The capacity is dynamic, and can change over time. Once the capacity is determined, the traffic managers meet the capacity by checking or unchecking flights to allow or disallow them from the constrained area, using their own operational knowledge to determine those flights which would have the least impact, be least affected by their moving, or other criteria determined to be best. Equity and efficiency concerns play a large role in this decision. Once a flight is removed from the

IDFL, it is moved to its highest priority option that does not intersect the constraint. In addition to manually selecting flights, IDFLs have a built in function which traffic managers can use to adjust demand. Traffic managers specify a reduction/expansion in demand to match capacity and rationing algorithms automatically choose flights to remove from the IDFL. Once the autosuggest function produces a recommended solution, traffic managers can preview the solution and accept, reject, or fine tune it.

5. **Automation assigns reroutes or ground delay for flights selected to move**. Once traffic managers select flights for removal, each moved flight is automatically rerouted to its highest priority option which takes it out of the constrained area (which may be ground delay). The system retains the route option list, and customers can still update the list.

6. **Traffic manages and ATCSCC monitor multiple constraints**. In some cases, such as a widespread weather pattern, traffic managers must create multiple IDFL's simultaneously. In such situations, there may be IDFL's interacting or covering multiple sectors. These would require coordination amongst the traffic managers responsible, as well as the ATCSCC. The ATCSCC specialists would monitor and control interactions amongst IDFL's, detect conflicts and redundant constraints, assign control in cases where an IDFL covered an area spanning multiple centers or sectors, and any other coordination needed amongst ARTCC's.

SEVEN extends and makes more robust the current practice of AFPs while retaining the present capabilities. In the absence of user submitted routing options, SEVEN emulates an AFP, but with options, SEVEN opens up a far more flexible set of capabilities that automatically work around constraints to maximize flow and minimize reaction time. In the IDFL, traffic managers are given a powerful tool to control the situation as much or as little as is necessary. This control is exerted without introducing excessive complexity into the controller environment. One of the most significant benefits of SEVEN is the ability to recapture system capacity that is currently lost when severe weather (or other capacity limiting factors) does not materialize as predicted. Traffic managers can handle uncertainty in both capacity and demand more easily as SEVEN makes it easy to quickly adapt to the situation as it unfolds [34].

2.2.6 Interaction

While the FAA is concerned about aggregate flows and capacity limits, the ultimate goal of airlines is to maintain their published flight schedule, which reflects its competitive strategy. Typically, Airlines coordinate their daily operation at centralized Airline Operational Control Centers (AOC's), which interact with airports, maintenance stations and pilots. Airline operations require a high degree of coordination, because of potential propagation effects of flight delays. This presents a challenge when airlines face irregular operations, usually caused by the need to respond to ATFM restrictions imposed by the FAA. The important functions that need to be performed by AOC's are schedule adjustment, flight planning and dispatch and flight monitoring ([1],[35]).

Any unpredicted events, such as delay or mechanical problems, may cause flight schedule disruption. To prevent the cascading of delays, the AOC's will adjust operations to return to more balanced conditions. This may be done by delaying a single flight, relocating the resources (aircraft, crew, airport arrival slots), canceling flights or creating flights to balance the schedule. Balancing the schedule may be interpreted differently by individual airlines: one airline's objective might be the ability to back to the normal schedule be the next day, while another might keep as many of its schedule flights as possible [35].

To minimize the cost, one important aspect of airline operations is to determine flight route and payload. Aircraft type, winds, complex trade-offs among speed, altitude, payload and fuel load, all will affect the choice of route.

The AOC's monitor all aspect of flights in progress, such as ensuring flights stay within safe and legal limits, assessing weather conditions (en route and arrival airport), and helping crews in solving problems that may arise.

It is necessary to have significant coordination between a number of stake holders on the side of the FAA and the airlines. On the FAA's side, the ATC-SCC predicts aggregate traffic flows and monitors current and projected capacity limits and demands. The ATCSCC usually initiates GDP's, SWAP's and AFP's and coordinates these ATFM initiatives with traffic management units at various Air Traffic Control Centers (ARTCCs), Terminal Radar Approach Control facilities (TRACON's) and towers. Also, when the ATCSCC predicts a sustained period of congestion, it responds to it with ATFM initiative, which is communicated to airlines

AOC's. Typically, these plans are formulated two to four hours in advance. The ARTCC's, TRANCON's and Towers also interact to coordinate air traffic between their regions. They delegate responsibilities to the individual air traffic controllers. The controllers at adjacent sectors interact to transfer the control of aircraft. On the airlines' side, the AOC's primary task is to coordinate the daily operations, such as gate assignment, maintenance, and flight dispatch. They employ constant communication with pilots to control and monitor the progress of individual flights.

2.3 Decentralized Air Traffic Management

As air traffic increases, significant change in ATM will be required. Airlines often believe that the restrictions implemented by the FAA are overly severe, so that unnecessary delays, congestion, and costs for the airlines result. The traditional approach largely followed is a central planning paradigm, in which users have to adhere to ATC decisions. The national air transportation system is moving toward an unprecedented, paradigm-shifting change. The next 10 years promise to be a pivotal time in the history of air transportation that will change the face of aviation. It is called the Next Generation Air Transportation System NextGen for short and it will forever redefine the management of national airspace system (NAS). To meet future demand, there must be a comprehensive system upgrade that will allow a fundamental change in the way that air traffic is managed. NextGen will enable critical transitions:

- From ground based to satellite based navigation and surveillance

- From voice communications to digital data exchange

- From a disparate and fragmented weather forecast delivery system to a system that uses a single, authoritative source

- From operations limited by visibility to sustaining the pace of operations even when impacted by adverse weather or difficult terrain.

Most significant, however, is the one transition that makes all the others possible moving from disconnected and incompatible information systems to a scalable, network centric architecture. This will ensure that everyone using the system has easy access to the same information at the same time, when needed. NextGen introduces new analytic tools that more pro actively detect adverse trends and identify precursors. These tools will allow to act on potential problems before they take shape. In addition, airports will benefit from increased safety, better use of existing capacity, greater design flexibility, and reduced environmental impacts. New technologies, standards, and procedures, in addition to new airside infrastructure, will allow airports to realize the benefits of NextGen [32].

2.3.1 Collaborative Decision Making

Collaborative Decision Making (CDM) was initially conceived in the mid 1990s within the FAA Airline Data Exchange (FADE) project. Under CDM, the AOC's have a significant decision making responsibilities about resource allocation and traffic flow management (for more information about CDM see [9], [11], [73]). The initial implementation of CDM, was focused on the development of new operational procedures and decision support tools for implementing and managing GDP's.

2.3.1.1 GDP's under CDM

The process of issuing GDP's had existed before the Collaborative Decision Making project got started. But the old system had many short comings that led to inefficiencies in the use of the valuable arrival resources.

Under the old model for running a GDP, called Grover Jack, flights were allocated slots by a priority based on their latest estimated time of arrival. This implied that if an airline reported a delay, then that airline would be a larger total ground delay than the airline did not share the information. This concept was referred to as the "double penalty". In addition, any canceled flights were not allocated arrival slots. It was in the airlines' best interests not to report a cancellation, wait until the GDP was issued, then cancel the flight and substitute another flight up to the vacant slot. To address this problem, a new algorithm was formulated called Ration By Schedule (RBS). Flights are now prioritized based on their original schedule times, even if they are canceled or delayed. If a delayed flight is given an arrival slot earlier that its delayed time, the airlines can use the substitution process to swap another of their flights into the earlier slot. Also, RBD is a new proposed algorithm [12] that prioritized flights based on their distance to the GDP airport.

Another problem was that very often valuable arrival slots were going unused occasionally during a GDP. An airline had to cancel a flight but was unable to substitute another of its flights into the vacant slot. There had been no mechanism to fill in these holes in the schedule. This has been addressed by the Compression algorithm. This is a rule-based algorithm that expands the idea of substitution across airlines. The algorithm processes each slot, which is open due to flight cancellation or delay. It first tries to find a flight operated by the same carrier to move up into the vacated slot. If one does not exist, the slot is then opened to the next available flight that can move up, regardless of which carrier operate the flight. The process continues, meanwhile always checking after each slot move to see whether the airline that owns the slot can now take the advantage of it. Compression results in reduction or no change to each flight's delay. This has proved to be a win-win concepts for both the FAA and airlines [10, 11, 70, 71].

The Ground Holding Problem (GHP) was first introduced in scientific literature by Odoni [49]. The GHP in its basic version [60] requires additional assumptions such as, discrete time horizon, deterministic demand and deterministic capacity. At the beginning of the planning horizon we need, a fixed and finite time period which has been discretized into contiguous time periods (slots), a complete list of flights bound to arrive at the congested airport and the airport arrival capacity in each time period. If $\mathcal{F}$, $\mathcal{S}$ are the set of flights and the set of available slots, $x_{fs} \in \{0, 1\}$ for $f \in \mathcal{F}$ and $s \in \mathcal{S}$ is the integer decision variable for assigning a slots to a flight. $C_f(d)$ is cost of assigning delay d to flight f, the capacity do each slot is considered to be one. Let t_s be the time of slot s and a_f the scheduled arrival time of flight f. The GHP can be formulated as an Integer Programming (IP) problem as:

$$Min \quad \sum_{f \in \mathcal{F}, s \in \mathcal{S}, t_s \geq a_f} C_f(t_s - a_f) x_{fs}$$

subject to:

$$\sum_{s \in \mathcal{S}, t_s \geq a_f} x_{fs} = 1 \qquad \forall f \in \mathcal{F}$$

$$\sum_{f \in \mathcal{F}, t_s \geq a_f} x_{fs} \leq 1 \qquad \forall s \in \mathcal{S}$$

This version of the GHP was studied in [7, 49]. More research on the GHP can be found in [6, 13, 35, 53, 54, 61, 70, 71]. The form of the delay cost in objective function is an important issue and most models employ a function in which marginal cost increases as a function of delay.

When airlines face a GDP, they respond to resulting schedule disruptions by trading-off flight cancellations and delays to minimize the cost of the disruption. Disruptions in flight schedules may have a cascading effect. To overcome this problem, airlines may cancel flights and substitute flight-slot assignments. The models that discuss resolving schedule disruption through slot swapping are proposed in [36, 40, 41, 48, 68]. Other models ([15, 24, 25, 58, 59, 62, 63]) attempt to find an operable, system-balanced flight schedule, that is they consider an airline's entire network of flights.

2.4 Slot Allocation

During severe weather, when there is a reduction in the en-route capacity, traffic management specialist using the TSD (Traffic Situation Display), can identify a problem area by creating a Flow Constrained Area (FCA). The TFM specialists at the ATCSCC can enter the capacity of the FCA, expressed as the number of flights that can be managed per hour, and FSM will then assign each flight a controlled departure time that will provide a smooth managed flow of traffic to the FCA. These departure times are sent to the customers for their planning and to the towers at the departure airports for enforcement.

Each airline will typically place a different relative weight on delays, rerouting and cancellation. Whereas some airlines would like to preserve the on-time performance for certain flights and cancel or reroute many other flights, other airlines prefer to have less rerouting and cancellations while tolerating higher total delay.

Since the en-route capacity is reduced the fair allocation of available resources among airlines arises. Fairness concerns have played an important role throughout the development of allocation procedures, and continue to be an essential factor whenever extensions or modifications to these procedures are proposed. The notion of fairness in ATM is largely left implicit in the procedures. Howevr, some recent research has developed fairness metrics and used these within TFM optimization models that tradeoff fairness and efficiency (see [12], [70], [71]).

The use of fairness as a basis for allocating scarce resources presents is a principal focus of our research. In fact, some proposals address the rationing of airport arrival capacity in the long run using methods ranging from auctions [52] and congestion pricing [50] to bargaining schemes [4]. The allocation of slots under CDM is different, in that slots have to be assigned on daily basis due to fluctuation in airport or en-route capacity. Fairness plays an important role in this environment.

Our research focuses on the development of a fair resource allocation mechanism for an airspace flow program. Our objective is to include airlines preferences for trading-off delay and rerouting. In our allocation procedures, each airline has been assigned a limited budget. We analyze our procedures using fairness principles.

The next chapter discusses the rationing of enroute resources. We propose a new resource rationing method that is designed specifically for our problem. We discuss the fairness of their new procedure.

Chapter 3

Computing a Fair Share of Limited Resources

3.1 Introduction

The FAA introduced a new capability in the spring of 2006 known as the Airspace Flow Program (AFP) that combines the power of Ground Delay Programs (GDP's) and Flow Constrained Areas (FCAs) to allow more efficient, effective, equitable, and predictable management of airborne traffic in congested airspace. The principal goal for the initial deployment was to provide enhanced en route traffic management during severe weather events.

In this research, we investigate a methodology to allocate available time slots among carriers according to their preferences during the AFP. For example, one carrier may want to increase its on time performance of certain flights, but reroute more of its other flights while another carrier may be less concerned with the flight delay, but prefer less rerouting, i.e. access to the FCA by more flights. Our model assumes an air traffic service provider (FAA) seeks to assign the available time slots among the carriers fairly while considering their preferences.

As discussed, the problem we address arises due to a capacity reduction in a section of airspace for a period of time. Based on flight plans, each flight has a scheduled arrival time at the boundary of the FCA. Since there is a reduction in the capacity of part of the airspace, it is not feasible for all flights whose scheduled arrival time at the boundary of the FCA during that period of time to continue in their preferred route and go through the FCA. Therefore, some of flights must be rerouted. The challenge is which flights get rerouted and which flight pass through the FCA. In this chapter, we propose a fair allocation method to determine a fair share for each flight operator from available time slots. These fair shares will then serve as allocation standards in the subsequent slot allocation processes developed in chapters 4 and 5.

In the section 3.2, we review the literature on fair division of resources. Our problem is described in section 3.3. In section 3.4 a method to determine the fair share of carriers from available slots will be explained. Section 3.5 describes the properties of our fair share methods. Experimental results based on our procedure is provided in section 3.6. Finally, conclusions are given in the section 3.7.

3.2 Background

The problem of sharing somehow "fairly" a given amount of resources is perhaps the oldest one faced by the economists. Brams' books ([19, 20]) are full of examples about divisions of goods. The fair division problem is simply stated in general terms: given a set Ω, given n individuals and given some fairness requirements, find an opportune division. The challenge of dividing indivisible goods has been studied in the literature [18, 22, 5].

When studying a simple model for the allocation of homogeneous indivisible units of a commodity, the problem can be posed either as a rationing or a scheduling "story". The various models of the rationing problem have been addressed in [51, 75, 76] for divisible goods and in [31, 27] for discrete items. Allocating resources in proportion to individual claims is the oldest formal rule of distributive justice. In case of indivisibility, the probabilistic rationing method gives an expected share to an agent proportional to his claim [44, 57]. The method meets the axiom of equity, consistency and equal treatment of equals. In the proportional random allocation method [44], the assumption is all items are homogeneous.

Two simple scheduling methods are discussed in the queuing literature. The *proportional* method seeks to treat equally each unit of claim [47]. In other words, the t-th unit goes to an agent with a probability proportional to unsatisfied demand. The *fair queuing* method solves this problem by allocating one unit per agent, irrespective of the size of individual demand, in a successive round-robin fashion. In each round the active agents (whose demands is not yet fully met) are randomly ordered (with uniform probability) and served one job in that order [46].

The problem of fair division when agents have heterogeneous preferences over the objects is studied as well. The division problem with single-peaked preferences is introduced in [55]. A considerable number of papers consider the ordinal extension of preferences e.g. [2, 17, 28, 30]. In those papers a probabilistic approach to the problem of assigning objects to the agents is suggested. The main normative requirement in mechanism design for dividing objects among agents with preference over the objects are efficiency and strategy-proofness; neither concept applies to the preference-free environment. In the full preference domain ([38, 16]), when the number of objects is equal to the number of agents, probabilistic rules are proposed based on a deterministic assignment.

In our problem, we face two difficulties. First, when the FAA as a coordinator must decide about each carrier's share, the underlying good, a time-slot, is not homogeneous in nature. The second difficulty is how to include carriers' preference in the assignment problem. In the following section we address the first difficulty while we try to meet fairness principles. The second difficulty is addressed in later chapters.

3.3 Problem Description

During severe weather events, reduction in en-route capacity can lead to a reduction in the number of flights that can pass through a portion of airspace. The traffic flow management (TFM) specialists at the air traffic control systems command center (ATCSCC) enter an FCA capacity, expressed as the number of flights that can be managed per hour, and then the decision support tool, FSM, assigns each flight a controlled departure time so that the flow into the FCA does not exceed the declared capacity.

In our research we assume that flights pass the boundary of FCA one at a time (this is consistent with current practice). Therefore we can express the capacity as the number of available time slots. We consider those flights that are "scheduled" to arrive at the boundary of FCA. Such a flight schedule can be derived based on each flights scheduled departure time and filed flight plan. Employing such a schedule can be problematic as it is not immune to gaming or strategic behavior on the part of flight operators.

For example, it could be worthwhile for a flight operator to file flight plans through the FCA, even though these are not the best routes for those flights. Such flights could improve that flight operators fair share. Later, these “extra” flights could be rerouted onto their most preferred trajectories. Our use of these schedules is consistent with current practice- we view finding an alternative standard as an open research question.

The fair share for each carrier can be found in many different ways. A principal goal we seek is to provide equity among carriers. The allocation of homogeneous demands, when the total demand exceeds total available resources is addressed in [51, 75, 76] and, in the case of scheduling problems, is treated in [47, 44, 46, 69](these models correspond to the situation in which all flights arrive at the beginning of the AFP). Vossen [69] uses a heterogeneous demand model to treat the different arrival times of flights. To allocate slots to flights, he uses “proportional random assignment” which randomly assigns slots to the carriers in proportion to the number of a carrier’s flights that can use a slot. In his method, slots sequentially are assigned to the carriers. The proportional random assignment method is a random allocation method. It gives one feasible solution of flights-slots assignment. As we can see, the procedure is time dependent. In the “proportional random allocation” method proposed by Moulin [44] there is no time dependency, which means that all agents can participate in the lottery at each time till their demand is met. In proportional random assignment, agents participate in the lottery if they can use the slot. We can use this method as a way to assign a fair share to each flight [66] (correspondingly each carrier). We will explain in the next section how to determine the fair share of each flight from available slots.

3.4 Determining Fair Share of Available Slots for Each Carrier

The goal of this section is to determine a fair share of available slots owed to each operator in expectation. We now define notation that we will use in this and later chapters.

Let $\mathcal{F} = \{f_1, f_2, \ldots, f_n\}$ be the set of flights and $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$ be the set of available slots ($m < n$) during the AFP. The capacity, c_j, of each slot s_j is considered one. Suppose there are K carriers $\mathcal{A} = \{A_1, A_2, ...A_K\}$, and $\mathcal{F}_i$ is the set of flights of carrier A_i such that

- $\mathcal{F}_i \subset \mathcal{F}$
- $\mathcal{F}_i \cap \mathcal{F}_j = \phi \qquad \forall \quad 1 \leq i, j \leq K, \qquad i \neq j$
- $\cup_{i=1}^{K} \mathcal{F}_i = \mathcal{F}$

a_f is the time flight f is scheduled to arrive at the boundary of FCA and t_j is the time of slot s_j. Flight f can be assigned to any slots s_j with $t_j \geq a_f$.

Our objective is to find each carrier's fair share from the available slots. It should be emphasized that the models and axioms we introduce here are based on the those proposed in the [44, 46, 69].

From the point of view of our allocation philosophy, two flights are equivalent if they have the same a_f. Thus, we refer to the a_f values as type designators (τ_f)

and say two flights are the same type if they have the same a_f value ($\tau_f = a_f$). Let $\tau \in \mathcal{N}_+^{\mathcal{F}}$ be the vector of all flight types; associated with each set of flights $\mathcal{F}$ there is a set of feasible allocations:

$$Q = \left\{ x \in \{0,1\}^{|\mathcal{F}| \times m} : \sum_{f \in \mathcal{F}} x_{f,j} = 1 \quad \forall \quad 0 \leq j < m, \quad \sum_{t_j \geq \tau_f} x_{f,j} \leq 1 \quad \forall f \in \mathcal{F} \right\}$$

Where the first constraint, $\sum_{f \in \mathcal{F}} x_{f,j} = 1 \quad \forall \quad 0 \leq j < m$, implies that all available slots are used, and the second constraint, $\sum_{t_j \geq \tau_f} x_{f,j} \leq 1 \quad \forall f \in \mathcal{F}$, assigns a flight to at most one slot.

For a given feasible set Q, any $f \in \mathcal{F}$, and any slot index $j : 0 \leq j \leq m$, we define the reduced feasible set $Q(f, j)$ as follow:

$$Q(f, j) = \{x \in Q : x_{f,j} = 1\}$$

$Q(f, j)$ represents the set of feasible allocations for the flights in $F - \{f\}$ while slot j is unavailable.

By considering $\sum_{f \in \mathcal{F}} x_{f,j} = 1 \quad \forall \quad 0 \leq j < m$, the constraint that assigns all slots, and $\sum_{t_j \geq \tau_f} x_{f,j} \leq 1 \quad \forall f \in \mathcal{F}$, the constraint that assigns at most one flight to any slot, it can easily be seen that, in order for all slots to be used, for any given slot time, the total number of slots up to that time must be less than or equal to the total number of flights that can use those slots. In other words, if $F_j = \{f : a_f \leq t_j\}$ then $|F_j| \geq j$ for all j. If this condition does not hold, then it is not possible to assign flights to all slots.

Therefore, we must "*decompose*" the set of flights-slots such that this condition holds. We define the decomposition procedure as follows:

Decomposition:

Step 0: Inputs: Set of flights $\mathcal{F}$, set of slots $\mathcal{S} = \{s_1, s_2, ..., s_m\}$, $j = 1$

Step 1: while $j \leq m$ Do:

Step 1a: $F_j = \{f : a_f \leq t_j\}$ if $|F_j| < j$ then delete s_j

Step 1b: $j = j + 1$

end while

This deletes "excess" slots. It should be noted that we call the resultant flight-slot non-decomposable but what the procedure really does is slot deletion.

We use a non-decomposable set of flights-slots to compute the fair share and also, as we will see in the later sections, to allocate slots to flights. From now on,we assume the set of flights-slots is not decomposable.

An *allocation problem* consists of a tuple (τ, Q), where τ represents the types for flights in $\mathcal{F}$. A *probabilistic allocation rule* $\mathbf{P}$ associates with each allocation problem (τ, Q), a *random* allocation in the feasible set Q. Thus, any allocation can be represented as a convex combination of the possible assignments, i.e.

$$\mathbf{P}(\tau, Q) = \sum_k p_k x^{(k)}, \quad p_k \geq 0, \quad \sum_k p_k = 1$$

where $x^{(k)}$ represents a possible assignment of flights to slots. In other words, the allocation rule $P(\tau, Q)$ selects each assignment $x^{(k)}$ with probability p_k. $P(\tau, Q)_{f,j}$ may be interpreted as the probability that f is assigned to slot j if $a_f \leq t_j$. A *uniform allocation rule* chooses each allocation $x^{(k)}$ with equal probability, $p_k = \frac{1}{|Q|}$.

To find the p_k, we need to know the total number of feasible flights to slots assignments, $|Q|$. In general, counting perfect matchings in bipartite graphs is an NP-hard [67] problem. As we show below, since there is time dependency between flights and slots, we can find the total number of flight-to-slot assignments in polynomial time.

To find the fair share of each carrier from the available slots, the original scheduled FCA arrival time of flights is considered, a_f. The flights whose original scheduled time is earlier than the latest available time slot, $a_f \leq t_m$, are considered and other flights are discarded. Here, we present a time-dependent method to find the share of each carrier from set of available slots. Our important equity principles states that:

- Each flight can use *at most* one slot.
- All flights have equal share of each slot that they can use in any feasible allocation.
- Each flight can be assigned to any slot later than its scheduled time of arrival.

3.4.1 Proportional Random Assignment

In a non-decomposable problem, there are many feasible flights-to-slots assignments. We use the allocation procedure called *proportional random assignment*, PRA [69].

PRA:

Step 1 : Set $F_1 = \{f \in \mathcal{F} : a_f \leq t_1)\}$ and $i = 1$

Step 2 : Choose an $f \in F_i$ with probability $\frac{1}{|F_i|}$ and assign f to s_i

Step 3 : Set $i = i + 1$

Step 4 : Set $F_i = \{f \in \mathcal{F} : a_f \leq t_i\} - \{f\}$

Step 5 : If $i \leq m$ Then go to Step 2.

End.

PRA uses three principles of equity and randomly assigns flights to slots. Note that PRA output is one assignment of flights-to-slots. However, we use PRA, viewed as a random process with associated probabilities, as a basis for computing flights fair share, FS_f, which is the goal of this section.

The probability $P(\tau, Q)_{f,j}$ can be interpreted as the (random) share of flight f in slot j. This probability can be computed as:

$$\mathbf{P}(\tau, Q)_{f,j} = \sum_{k:x^k_{f,j}=1} p_k$$

Therefore, we need to compute p_k in order to find the probability of assigning flight f to slot j. We can see in the next proposition that p_k can be found in a polynomial time.

Proposition 3.4.1 **PRA** *chooses a given flights-to-slots assignment with probability of:*

$$p_k = \frac{1}{\prod_{i=1}^{m}(n_i - (i-1))}$$

Also the probability that slot j is assigned to flight f with $a_f = t_k$ can be obtained as:

$$\mathbf{P}(\tau, Q)_{f,j} = \frac{\prod_{i=k}^{j-1}(n_i - i)}{\prod_{i=k}^{j}(n_i - (i-1))}$$

Proof Consider the following bipartite graph. There is an edge e_{ij} between any

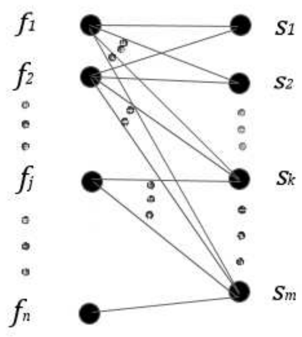

Figure 3.1: Bipartite graph

flight f_i and slot s_j if $a_{f_i} \leq t(s_j)$. n_j is the degree of node s_j which is equal to the number of edges connected to it.

Consider one particular assignment that assigns flights $F_i = \{f_{i_1}, f_{i_2}, \ldots, f_{i_m}\}$ to $\{s_1, \ldots, s_m\}$ (i.e. $f_{i_1} \rightarrow s_1, f_{i_2} \rightarrow s_2, \ldots, f_{i_m} \rightarrow s_m$).

$$Prob(Assigning\ F_i\ to\ S) = P(f_{i_1} \rightarrow s_1) \times \prod_{j=2}^{m} Prob(f_{i_j} \rightarrow s_j | \{f_{i_1}, \ldots, f_{i_j}\} \rightarrow \{s_1, \ldots, s_m\})$$

Start from s_1: n_1 flights can be assigned to slot s_1, based on algorithm the $Prob(f_{i_1} \rightarrow s_1) = \frac{1}{n_1}$. Remove f_{i_1} and s_1 and all of its connected edge from the graph. So, the degree of all nodes $s_2, ..., s_m$ is decreased by one. Now, there are $n_2 - 1$ flights that can be assigned to slot s_2. Therefore, $Prob(f_{i_2} \rightarrow s_2) = \frac{1}{n_2 - 1}$. Again remove f_{i_2} and its connected edges and s_2 from the graph. The degree of remaining slots reduced by 1. i.e the degree of s_j is $n_j - 2$. Continue this procedure till we get to the last slot, s_m, where we just have $n - (m - 1)$ flights available. So, $Prob(f_{i_m} \rightarrow s_m) = \frac{1}{n_m - (m-1)}$. Therefore the probability of choosing any perfect matching is:

$$p_k = \frac{1}{\prod_{i=1}^{m} (n_i - (i - 1))} \tag{3.1}$$

The probability that f can be assigned to s_j can be computed as:

$$P(\tau, Q)_{f,j} = \sum_{Number\ of\ assignemnt\ s.t.\ f\ is\ assigned\ to\ s_j} p_k = \sum_{k:x(f,j)=1} p_k \tag{3.2}$$

We need to compute the number of flight-to-slot assignments such that f is assigned to s_j, in other words the size of $Q(f, j)$. Suppose $a_f \geq t(s_k)$, so there is an edge between f and slots $s_k, ..., s_m$ and $k \leq j \leq m$. Remove f and s_j and all edges connected to f from the graph. Therefore, the degree of all nodes after s_k is reduced by one. Start from s_1, n_1 flights can use the slot. Since one flight is already assigned to s_1, n_2-1 flights can use the s_2 and so on till s_{k-1} where $n_{k-1}-(k-2)$ can use s_{k-1}. Since the degree of nodes after s_k is already reduced by one, so for $k \leq i \leq (j-1)$ the number of flights that can use slot s_i is $n_i - 1 - (i-1)$. Just f can use s_j. The number of flights that can use s_{j+1} is $n_{j+1} - 1 - (j-1)$ ($j-1$ flights is already assigned to slots s_1 to s_{j-1}). Thus, for $j+1 \leq i \leq m$ the number of flights that can use slot s_i is $n_i - 1 - (i-2)$. The size of set $Q(f, j)$ is:

$$\begin{aligned} |Q(f,j)| = n_1 \times (n_2 - 1) \times ... \times (n_{k-1} - (k-2)) \times (n_k - (k-1) - 1) \times ... \\ \times (n_{j-1} - (j-2) - 1) \times 1 \times (n_{j+1} - (j-1) - 1) \times ...(n_m - (m-2) - 1) \end{aligned} \tag{3.3}$$

Substitute 3.3 and 3.1 in 3.2, we will have:

$$P(\tau, Q)_{f,j} = \frac{\prod_{i=k}^{j-1}(n_i - i)}{\prod_{i=k}^{j}(n_i - (i-1))} \tag{3.4}$$

•

Corollary 3.4.1 *PRA chooses each flight-to-slot assignment with equal probability, and also the number of total flight-to-slot assignments is* $|Q| = \prod_{i=1}^{m}(n_i - (i-1))$.

Proof Proposition 3.4.1 the probability of one assignment can be computed. It is seen that computing the probability, p_k, is independent of k (i.e. it is independent of any assignment). Therefore PRA chooses each flight-to-slot assignment with equal probability. Therefore, PRA can choose any flight-to-slot assignment with equal probability then the number of feasible flight-to-slot assignment is $1/p$. •

The probability $P(\tau, Q)_{f,j}$ can be interpreted as the (random) share of flight f in slot j, $Share^{f}_{s_j}$. The Fair Share of carrier A_l from the available slots is:

$$FairShare^{A_l}_{\mathcal{S}} = \sum_{s_i \in \mathcal{S}} \sum_{f_j \in \mathcal{F}_l} Share^{f_j}_{s_i} \tag{3.5}$$

We denote $FairShare^{A_l}_{\mathcal{S}}$ as FS_l for simplicity. We call this method *Finding Fair Share Based on PRA*(FFS-PRA).

RBS considers only one possible flights-to-slots assignment, while in the new method, all flights-to-slots assignments are considered. PRA chooses each of these flights-to-slots assignment with equal probability. Therefore, it is possible for all flights included in an AFP to have a positive share of available slots.

Corollary 3.4.2 *In FFS-PRA, all flights have positive share from available slots.*

Proof Since PRA considers all possible flights-to-slots assignments, therefore if a flight,f, can use a slot, s_j, it is considered in some of flights-to-slots assignments. Since there is a probability associated to any feasible matching, and probability of assigning f to s_j is sum of probabilities of those flights-to-slots assignment in which f is assigned to j. Then f has a positive share of j. •

Flights	f_{1A}	f_{1B}	f_{2A}	f_{2B}	f_{1C}	f_{2C}
Airline	A	B	A	B	C	C
a_f	3:58	4:00	4:02	4:03	4:05	4:06

Table 3.1: Flight schedules of airline A, B and C

This is a very important point. Unlike RBS, that does not give any share to later flights, FFS-PRA gives a positive share to any flight included in an AFP. Also, it treats all flights with the same type equally. FFS-PRA chooses each flights-to-slots assignment with equal probability. We should note that flights that are scheduled earlier usually receive more share rather that later scheduled flights. Because the number of feasible matchings for earlier flights is more that the number of feasible matchings for the later flights. Therefore, FFS-PRA implicity gives higher share to earlier scheduled flights.

3.4.2 Example

Suppose we have three carriers and six flights. Assume only four flights can pass through the FCA. Table 3.1 shows the flights of three airlines, A, B and C, and their scheduled arrival times at the boundary of FCA. The available time slots are:

Slot:	s_1	s_2	s_3	s_4
Time:	4:00	4:02	4:04	4:06

Here we want to compare the result of FFS-PRA and RBS. Using RBS the flight slot assignment is:

$$f_{1A} \rightarrow s_1 \qquad f_{1B} \rightarrow s_2 \qquad f_{2A} \rightarrow s_3 \qquad f_{2B} \rightarrow s_4$$

So the share of each airline form available slots is:

$$Share_{\mathcal{S}}^{A} = 2 \qquad Share_{\mathcal{S}}^{B} = 2 \qquad Share_{\mathcal{S}}^{C} = 0$$

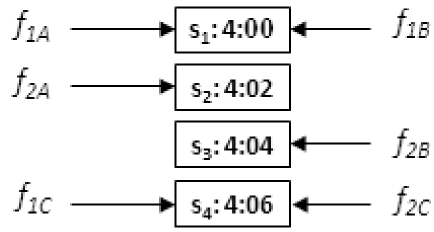

Figure 3.2: The earliest slot that each flight can use

Figure 3.2 shows the earliest slot that each flight can use.

The latest time slot is 4:06, hence the flights with scheduled time afterward are discarded. We are interested in finding the fair share of each carrier with respect to the four available slots, $\{s_1, ..., s_4\}$.

We would like to compute the (random) share of each flight from any slots. As we explained, Q is the set of feasible assignment of flights to slots. Figure 3.3 shows the graph of all possible combinations of slot assignments. Each level of the graph corresponds to one of the available four slots, and the numbers on the edges of the graph indicate the probability of assigning f_i to s_j for a given path.

For example, we would like to compute the share of flight f_{1A} from s_1, s_2, s_3 and s_4. The share of flight f_{1A} from s_1 is equal to the probability of assigning f_{1A} to

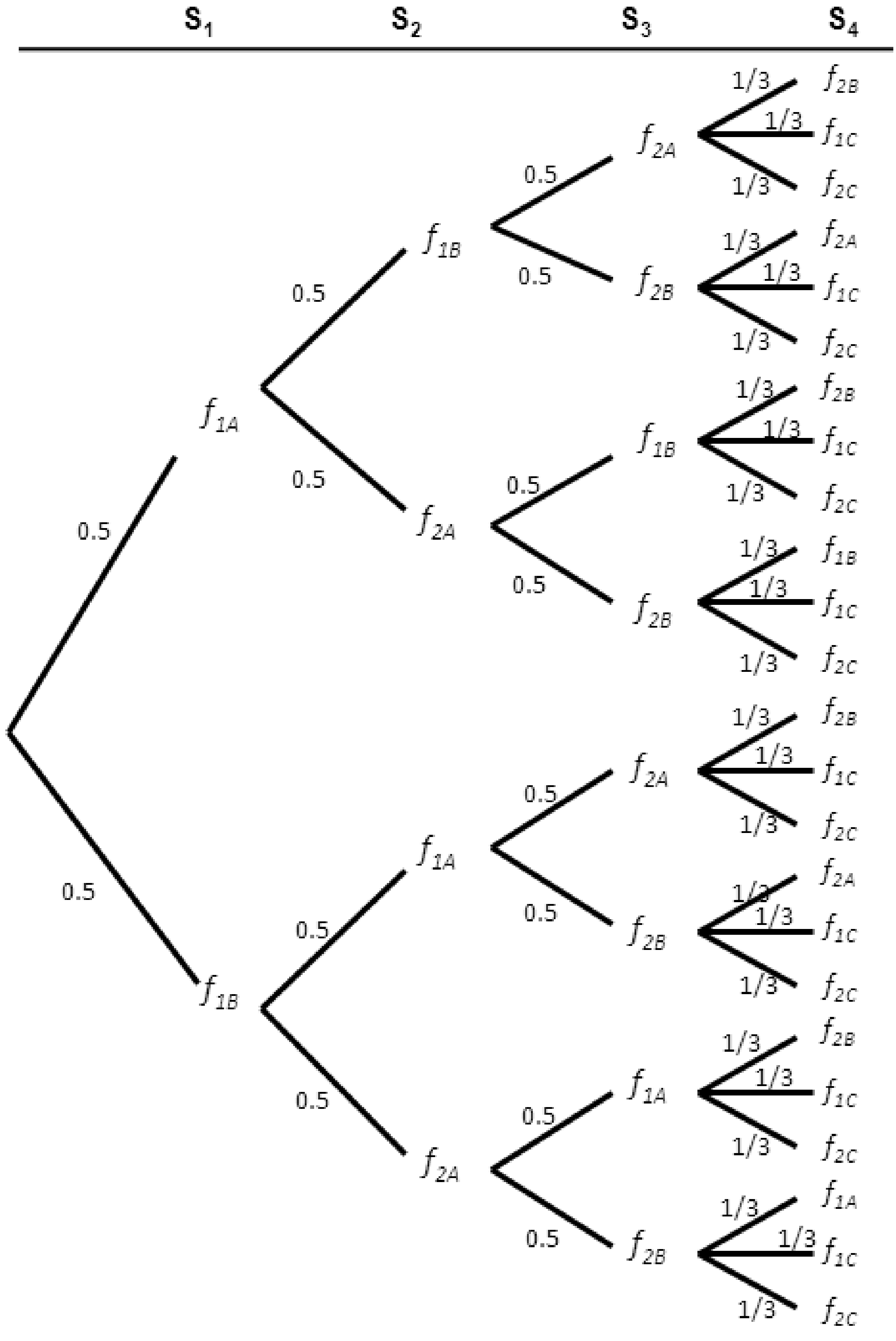

Figure 3.3: All possible allocations of slots to flights

s_1 that equals to $\frac{1}{2}$. We can see from figure 3.2 $n_1 = 2$, $n_2 = 3$, $n_3 = 4$ and $n_4 = 6$. Using the 3.5 gives us the probability (share) of flight f_1 from each slot:

$$Share_{s_1}^{f_1} = \frac{1}{2}$$

$$Share_{s_2}^{f_1} = \frac{1}{2\times(3-1)} = \frac{1}{2}$$

$$Share_{s_3}^{f_1} = \frac{1}{2\times(3-1)\times(4-2)} = \frac{1}{8}$$

$$Share_{s_4}^{f_1} = \frac{1}{2\times(3-1)\times(4-2)\times(6-3)} = \frac{1}{24}$$

	f_{1A}	f_{1B}	f_{2A}	f_{2B}	f_{1C}	f_{2C}
s_1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
s_2	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0	0	0
s_3	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	0	0
s_4	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$

Table 3.2: Share of flights for each slot

	A	B	C
s_1	$\frac{1}{2}$	$\frac{1}{2}$	0
s_2	$\frac{3}{4}$	$\frac{1}{4}$	0
s_3	$\frac{3}{8}$	$\frac{5}{8}$	0
s_4	$\frac{3}{24}$	$\frac{5}{24}$	$\frac{2}{3}$

Table 3.3: Fair share of airlines from each slot

Table 3.2 shows the share of each flight for any slot; the share of a carrier for a slot is equal to the summation of the share of its flights for that slot (Table 3.3). For example, the total fair share of airline A for s_4 is $\frac{3}{24}$ (i.e. $\frac{1}{24} + \frac{1}{12}$). As we can see, A, B and C have two flights, however, due to the scheduled time of these flights, their total share of slots are different. In addition, according to 3.5, the total share of each airline for all the available slots is:

$$FairShare_S^A = \frac{21}{12} \qquad FairShare_S^B = \frac{19}{12} \qquad FairShare_S^C = \frac{8}{12}$$

□

Thus, while under RBS the share of airline C was zero, airline C has a $\frac{2}{3}$ share under FFS-PRA. small carriers, which have fewer flights or carriers with flights scheduled late, still receive a share of available slots. We will see in the later chapters that we use the fair share of carriers as an input to our random allocation procedures. Therefore, carriers with positive share will have chance to receive slots.

In the next section, the equity properties of FFS-PRA will be discussed.

3.5 Equity of FFS-PRA

There are SEVERAL principles used to determine the fairness of an allocation. The fundamental principles of *fairness* are *impartially* and *consistency*, *Equal Treatment of Equals* and *Demand Monotonicity* [74]. The formal definition of each of these axioms is defined as:

Definition 3.5.1 *A probabilistic allocation rule P is impartial if for any allocation problem (τ_F, Q) and any permutation π of F,*

$$P(\tau_F \circ \pi, Q \circ \pi) = P(\tau_F, Q) \circ \pi$$

Impartiality states that allocation rule should not discriminate among the flights *except* insofar as they differ in type. In other words, if two flights are indifferent in type and in the feasible set, they will receive the same slot shares.

The origins of *consistency* principle is studied in [74], p.173 and [65]. Variations of the consistency were formulated independently on different allocation problems, such as the apportionment problem [14], the cost sharing and rationing problem [45], and bargaining problems [64]. The definition of consistency by Vossen [69] is based on the definition of consistency by Moulin[46], which recognizes the probabilistic nature of the underlying allocation problem. Define for a given feasible set Q, any $f \in F$, and any slot index $j : 0 \leq j \leq m$ the reduced feasible set $Q(f, j)$

$$Q(f, j) = \{x \in Q : x_{f,j} = 1\}$$

which represents the set of feasible allocation for the flights in $F - \{f\}$ with slot j unavailable. So, the consistency can be defined as:

Definition 3.5.2 *A probabilistic allocation rule P is consistent if for any allocation problem (τ, Q) and any $f, f' \in F$*

$$P(\tau, Q)_{f',j'} = \sum_{j=1}^{j'} P(\tau, Q)_{f,j} P(\tau_{\mathcal{F}-\{f\}}, Q(f, j))_{f',j'}$$

In other words, the consistency property states that the expected slot shares should be independent of the order in which flights are assigned to the slots.

The other important axiom of $fairness$ is called *Equal Treatment of Equals (ETE)*. In the *random allocation* method, ETE can take in two interestingly different forms[46]. Define Y_i as an integer valued random variable that gives the total number of slots assigned to carrier i.

Definition 3.5.3 *The random allocation rule P has Equal Treatment of Equals Ex*

Post(ETEP) property, if for any realization y of the random variable Y, we have

$$FairShare_{\mathcal{S}}^{A_i} = FairShare_{\mathcal{S}}^{A_j} \Rightarrow |Y_i - Y_j| \leq 1$$

and It has Equal Treatment of Equals Ex Ante (ETEA)property if:

$$FairShare_{\mathcal{S}}^{A_i} = FairShare_{\mathcal{S}}^{A_j} \Rightarrow Y_i \sim Y_j$$

(these two random variables have identical distribution)

Another important axiom is Demand Monotonicity(DM) that can be defined as:

Definition 3.5.4 *A probabilistic allocation rule P has Demand Monotonicity property if for any allocation problem* (τ, Q)*:*

$$FairShare_{\mathcal{S}}^{A_i} \leq FairShare_{\mathcal{S}}^{A_i}|_{F_i'=F_i+f*}$$

DM says that an increase in carrier i's demand F_i(extra flight $f*$), leaving number of available slots and other flight sets unchanged, can not deteriorate carrier i's (random) share.

Another strong property is that every carrier has a chance of receiving one slot, this property is called *Positive Share*. In other words, $FairShare_{\mathcal{S}}^{A} > 0$.

Theorem 3.5.1 *FFS-PRA meets impartiality, Equal Treatment of Equals(Ex-Post), consistency and Demand Monotonicity.*

<u>Proof</u> We skip the easy proof of impartiality, equal treatment of equals and consistency. Suppose airline A_i's demand increased by one flight f^*, where the

earliest slot that f^* can use is s_k. In other words, $a_{f^*} \leq t(s_i)$ for all $i \geq k$. Since share of f^* from all slots which have $a_{f^*} \leq t(s_i)$ is positive then there must be a reduction in the share of all flights from slots which have $a_{f^*} \leq t(s_i)$. Suppose δ_i is the change in the share of flight f_i. In other words, the new share of each flight is going to be $Share'^{f_i} = Share^{f_i} - \delta_i$. Thus, $Share^{f^*} = \sum_{f \in \mathcal{F}} \delta_i$. We can write the new share of airline i as:

$$\begin{aligned} FairShare'^{A_i}_{\mathcal{S}} &= \sum_{f \in \mathcal{F}_{A_i}} Share'^{f_i} + Share^{f^*} \\ &= \sum_{f \in \mathcal{F}_{A_i}} (Share^{f_i} - \delta_i) + \sum_{f \in \mathcal{F}} \delta_i \\ &= FairShare^{A_i}_{\mathcal{S}} + \sum_{f \notin \mathcal{F}_{A_i}} \delta_i \end{aligned}$$

As you can see $\sum_{f \notin \mathcal{F}_{A_i}} \delta_i > 0$ then the share of A_s is increased as its demand is increased. $\bullet$

3.6 Experimental Results

For our experiment, we used a test data set that had been employed by the CDM Future Concepts Team to perform human in-the-loop experiments related to SEVEN. It contained 386 flights with 38 flight operators. The data included scheduled arrival arrival times at an FCA boundary. The FCA duration was from 18:00 pm to 21:00 pm.

We compared the results of ration-by-schedule(RBS), which is currently used to allocate FCA access during airspace flow programs with the fair share that we computed.

	Number Of Slot Assigned for Different Capacity Reduction (RBS vs. Fair Share)									
Capacity Reduction	40%		50%		60%		70%		80%	
Airlines	RBS	FS	RBS	FS	RBS	FS	RBS	FS	RBS	FS
1	13	12.903	11	11.189	10	9.322	8	7.276	6	5.079
2	4	3.952	4	3.407	4	2.815	3	2.198	1	1.553
3	2	2.05	2	1.653	2	1.265	1	0.904	0	0.58
4	0	0.49	0	0.362	0	0.258	0	0.174	0	0.107
5	6	5.923	6	4.788	5	3.666	4	2.596	0	1.673
6	5	4.591	3	3.864	3	3.06	3	2.237	2	1.475
7	1	1.114	1	0.845	0	0.616	0	0.422	0	0.262
8	5	4.445	4	3.937	4	3.394	2	2.777	2	1.998
9	0	0.443	0	0.327	0	0.228	0	0.15	0	0.094
10	8	7.878	6	6.561	4	5.196	4	3.816	3	2.508
11	1	0.775	1	0.621	1	0.469	0	0.327	0	0.21
12	1	0.69	1	0.544	0	0.403	0	0.282	0	0.178
13	20	21.601	17	18.682	16	15.393	15	11.796	12	8.035
14	3	3.721	2	2.952	2	2.236	2	1.574	1	0.994
15	16	19.214	12	15.885	9	12.634	7	9.481	7	6.405
16	1	1.517	1	1.149	1	0.841	0	0.582	0	0.358
17	1	0.843	1	0.703	1	0.555	1	0.402	0	0.257
18	0	0.09	0	0.062	0	0.044	0	0.027	0	0.017
19	1	0.611	0	0.47	0	0.342	0	0.237	0	0.145
20	0	0.154	0	0.11	0	0.071	0	0.046	0	0.027
21	34	34.967	28	29.489	22	23.727	19	17.78	15	11.953
22	5	4.843	4	4.036	4	3.277	3	2.558	1	1.868
23	1	1.028	0	0.766	0	0.55	0	0.373	0	0.23
24	4	3.772	3	3.064	2	2.348	1	1.685	1	1.087
25	6	5.618	5	4.655	5	3.632	4	2.621	2	1.697
26	1	0.699	1	0.555	0	0.414	0	0.288	0	0.184
27	0	0.459	0	0.337	0	0.238	0	0.16	0	0.098
28	2	2.303	2	2.091	2	1.777	2	1.381	2	0.924
29	25	24.225	23	20.705	18	16.987	13	13.065	10	8.97
30	2	1.609	2	1.328	1	1.034	1	0.744	0	0.48
31	1	0.757	1	0.607	1	0.464	0	0.327	0	0.211
32	24	23.165	18	19.502	16	15.685	10	11.899	7	8.161
33	0	0.258	0	0.183	0	0.126	0	0.079	0	0.049
34	7	5.901	6	4.903	6	3.809	3	2.764	1	1.783
35	0	0.318	0	0.226	0	0.155	0	0.101	0	0.061
36	29	25.67	27	20.87	16	16.112	10	11.609	5	7.512
37	1	0.64	1	0.497	0	0.364	0	0.253	0	0.155
38	2	2.763	1	2.077	0	1.492	0	1.013	0	0.62

Figure 3.4: Share of each airline form avialable slots for diffeent capacity reduction. Comparing RBS with Fair Share

In our experiment, we considered 40%, 50%, 60%, 70% and 80% en-route capacity reduction for the FCA. Figure 3.4 shows the number of slots assigned to each airline for a specific capacity reduction for the RBS.

The cells, which are highlighted, show when the difference between the FFS-PRA allocation one and RBS exceed one

3.7 Discussion

So far, a method has been presented to determine the fair share of each carrier from the available slots. Our fair Share assignment meets equity principles such as impartiality, equal treatment of equals, consistency and demand monotonicity. The current flight assignment procedure used during AFP's uses ration-by-schedule (RBS). RBS works based on first scheduled first served. Thus, under RBS flights, which are scheduled late in the time horizon do not receive any share, while under FFS-PRA all flights get a positive share.

As we will see in other chapters of this dissertation, we will use this fair share as a parameter to assign flights to slots.

Chapter 4

Allocation of Limited Resources in a Full Preference Domain

4.1 Introduction

In the previous chapter a method has been proposed to determine a fair share of available slots for each carrier. Now, we would like to address how to assign slots to carriers in a way that includes carriers' preferences and maintains fairness. A carrier's $FairShare$, is interpreted as the number of slots the carrier should receive.

In section 4.2 we will review the literature of allocation of resources based on agents' preferences. Two algorithms will be proposed to assign slots to carriers based on their preferences in section 4.3 . The equity of the proposed algorithms will be discussed in section 4.4. Finally, we will provide some experimental results.

4.2 Background

The probabilistic allocation of indivisible objects has received significant research attention. This problem can be considered in two main scenarios. The first scenario, there are n objects and n agents and each agent receives exactly one object. Each agent has a strict preference over the set of objects, objects are distinct. In an application example agents can be workers and objects can be jobs in a company. In the second scenario there are k identical objects and n agents. Each agent receives a certain number of objects and each object is assigned to some agent. In an application example, there are identical jobs that have to assigned to workers.

The problem of n agents and n objects has been studied by Abdulkadiroglu and Sonmez [3], Bogomolnaia and Moulin [16], and Cres and Moulin [23]. Abdulkadiroglu and Sonmez showed that the only Pareto-Efficient matching mechanisms is a *serial dictatorship*, which are like random priority (RP), the priority ordering of agents choose randomly, except that initial ordering of the agents is chosen in a deterministic fashion. Svensson [56] showed that serial dictatorships are the only rule satisfying *strategyproofness, neutrality and nonbossiness.* RP may not be efficient if agents are endowed with utility functions consistent with their preferences. The main contribution of Bogomolnaia and Moulin [16] is the definition of *ordinal efficiency.* They showed that the *probabilistic serial* mechanism is weakly strategy proof, and achieves an envy free, ordinarily efficient solution. In a result parallel to Zhou's impossibility theorem [77], the showed that no strategy proofness mechanism can achieve both ordinal efficiency and fairness, even in the weak sense of equal treatment of equals. Katta and Sethuraman [38] addressed the problem in a full preference domain. Cres and Moulin [23] show that in their model Probabilistic Serial (PS) solution stochastically dominates the RP solution.

The second scenario has been addressed by Moulin [44] and Moulin and Stong [46] where each agent demands a certain number of objects and total demand is greater than the number of objects available. There is no preferences over the objects in their model discussed. Ehlers and Klaus [29], Kureishi [39] studied the case where each agents has a single peaked preference over the number of objects she may receive. A probabilistic rule chooses for each profile of preferences a probability distribution over the set of allocations. An agent compares two distributions over the set of allocations by evaluating the marginal distribution that are induced over her allotments. Bogomolnaia and Moulin [17] studied the case when agents have dichotomous preferences over the objects.

In our problem, we have set of slots, which can be considered as indivisible heterogenous goods, and we have agents, carriers, that have different preferences over the slots. We consider a full preference domain, where carriers can express their preferences over the slots [66]. As we will discuss later, the preference domain can be very rich and it is not as simple as ranking objects, slots.

4.3 Preference Based Proportional Random Allocation

As discussed, the problem of allocating of heterogeneous goods among agents with different preferences has been studied in many papers. In our problem, we have heterogeneous goods, slots, and agents, flight operators, with different preferences over the slots.

The cost per minute of delay can vary substantially from flight to flight. For example, the delay on a flight that has more passengers is more costly than a smaller flight; delay on a flight with connecting passengers is more costly than a flight with no connections. There are also significant cost implications of the status of the flight's crew. Thus, for each carrier the concept of preferences over slots is closely related with the concept of preferences over the flights.

Here we propose an algorithm that takes carriers' preferences and flight schedules as input, and allocates slots to carriers. Before explaining our algorithm we need to explain two concepts, one is decomposition and the other is carriers' preferences. The algorithm needs a priority list of flights-to-slots assignments from carriers as input.

4.3.1 Decomposition

As we explained in the previous chapter, when we compute the fair share for each carrier, we need to insure that if $F_j = \{f : a_f \leq t_j\}$ then $|F_j| \geq j$ for all j. We provided a decomposition procedure based on deleting certain slots.

In the allocation problem that we will introduce here, we need a stronger form of non-decomposability in order to prevent carriers from gaming the system. The stronger version is called *diverse non-decomposable.*

In order to achieve a diverse non-decomposable set from original set of flights-to-slots, we need to exclude two types of flights-to-slots subsets from the original set. The first type of subset includes those flights-to-slots sets such that all flights associated with the slots belong to the same carrier, i.e. there is no competition. In this case, these slots are assigned to that particular carrier, thus this part of the allocation problem is removed. The second type of subset is when $|F_j| = j$ which implies there are equal number of flights and slots. Allocation to each such subset can be solved separately. After excluding these two types of subsets by suitable decomposition, the remaining set of flights-to-slots satisfies the following

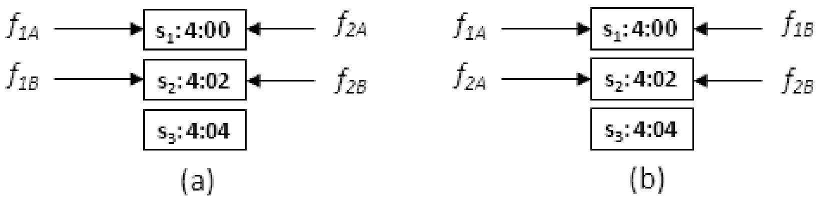

Figure 4.1: (a) non-decomposable (b) diverse non-decomposable problem

two properties:

1- If $F_j = \{f : a_f \leq t_j\}$ then $|F_j| > j$ for all $j < n$

2- There is always more than one carrier that can use each slot.

We develop a procedure that excludes these two types of subsets from the original set of flights-to-slots. Let us first clarify the concept of "non-decomposable" and "diverse non-decomposable" using following example, and then we formally define a procedure to achieve a diverse non-decomposable set from original set of flights-to-slots.

Figure 4.1 illustrates an example to clarify concept of "non-decomposable" and "diverse non-decomposable". Figure 4.1 shows two configurations. The first configuration, (a), shows a set of flights-slots. It is seen than the set is non-decomposable, but it is not diverse, because A is the only carrier that can use s_1. The second configuration, (b), is another set of flights-slots that is diverse non-decomposable.

The following procedure produces the appropriate decomposition.

Diverse Decomposition:

Step 0: Inputs: $\mathcal{S} = \{s_1, s_2, ..., s_m\}$ and $\mathcal{F} = \{f_1, f_2, ..., f_m\}$, $k = 1$,$j = 1$, $i = 1$

Step 1: while $j \leq m$, Do:

Step 1a: $F_j = \{f : a_f \leq t_j\}$

Step 1b: if $|F_j| = j$ then $\mathcal{S}_i = \{s_k, ..., s_j\}$, $\mathcal{F}_i = F_j$, $\mathcal{F} = \mathcal{F} - F_j$, $\mathcal{S} = \mathcal{S} - \mathcal{S}_i$, $k = j + 1$, $i = i + 1$

else if $|F_j| < j$ then delete s_j

else if $f \in F_j$ all belong to one carrier then $\mathcal{S} = \mathcal{S} - \{s_j\}$, $\mathcal{F} = \mathcal{F} - F_j$

Step 1c: $j = j + 1$;

end while

The output of this procedure are flight-slot sets $(\mathcal{F}_i, \mathcal{S}_i)$. Each of these should be analyzed separately. Further, the final line in step 1b deletes certain flight-slot pairs. In each case, the slot assigned to the associated carrier.

In the next section, we will explain how the decomposition procedure eliminates certain incentives for non-truthful preferences.

4.3.2 Priority List

We employ $FairShare$ as a standard that determines how many slots a carrier should receive in our slot allocation procedure. As part of our suggested slot allocation procedure, the FAA would inform each carrier of their fair share. We will prove later that our proposed slot allocation algorithms guarantee a carrier a will receive at least $\lfloor FairShare_a \rfloor$ slots and at most $\lceil FairShare_a \rceil$ slots. This means carriers know about the minimum and the maximum total number of slots they will receive before applying the slot allocation procedures. Carriers have precise knowledge of the number of slots they will receive, but they do not know which slots they will be receive.

As discussed earlier our slot allocation procedures require airline flight-slot preference information. There are two types of preference lists.

In first type, carriers submit to the FAA an ordered list of flight-to-slot assignments. For example, carriers submit an ordered list of (f_i, s_j) pairs. This type of list can be very long when the number of slots is large. The second type of list can be a compact version of the first type. Instead of submitting an ordered list of (f_i, s_j) pairs separately, carriers submit the pair of flights and bundle of slots. For example, if a carrier ordered preference list is (f_i, s_j), (f_i, s_{j+1}), (f_i, s_{j+2}) (f_l, s_k) then it can be expressed as $(f_i, s_j : s_{j+2})$, (f_l, s_k).

Under certain conditions, a carrier may prefer a later slot to an earlier one. A benefit of our slot allocation procedures is they provide flexibility for carriers to express such preferences. In the following example, we try to better clarify concept of priority list, and also to see how this priority list of flights could be used in some non-trivial cases (e.g. when a carrier can prefer a later slot to an earlier slot).

Suppose, carrier A has three flights $A101$, $A102$ and $A103$. And also assume there are six available slots, $s_1, \ldots, s_6$. The earliest slots, a_f, that each flight can be assigned could be:

Slot:	s_1	s_2	s_3	s_4	s_5	s_6
Flights:	$A101$	$A102$	$A103$			
a_f	s_1	s_4	s_6			

The following table illustrates a possible flight priority list that carrier A submits to FAA.

Preference List for A

Rank	(Flight,Slot)	Rank	(Flight,Slot)
1	$(A103, s_6)$	6	$(A101, s_3)$
2	$(A101, s_1)$	7	$(A101, s_4)$
3	$(A101, s_2)$	8	$(A101, s_5)$
4	$(A102, s_4)$	9	$(A101, s_6)$
5	$(A102, s_5)$	10	$(A102, s_6)$

For simplicity, the flight priority list can be shown as:

Rank	(Flight,Slot)	Rank	(Flight,Slot)
1	$(A103, s_6)$	4	$(A101, s_3 : s_4)$
2	$(A101, s_1 : s_2)$	5	$(A101, s_5 : s_6)$
3	$(A102, s_4 : s_5)$	6	$(A102, s_6)$

In this example, for carrier A, the highest preference is s_6, and it prefers later slots s_4 and s_5 to earlier slot s_3. This could happen if flight $A103$ had much higher delay and cancellation costs than $A101$ and $A102$. Further, it could be the case that $A101$ is delayed beyond slot s_2, its marginal delay cost become small so that saving delay in flight $A102$ become a higher priority.

It can be seen that by submitting a priority list of flights, carriers have the flexibility to express a range preferences based on their internal cost functions.

4.3.3 Preference-Based Proportional Random Allocation

In this section, we introduce a randomized allocation procedure, which we call *Preference-Based Proportional Random Allocation* (PBPRA). PBPRA, has the following objections/properties:

1− Slot allocation process that based on a randomized procedure.

2− Slot allocation process that includes carrier preferences.

3− Slot allocation process that assigns slots to carries in a way that each carrier receives a total number of slots as "close" to its fair share.

We first outline the steps of the PBPRA algorithm. Initially the diverse decomposition algorithm described in section 4.3.1 is applied to make sure any set of flights-to-slots is non-decomposable. The two primary inputs to PBPRA are: A fair share for each carrier and a flight priority list that is provided by each carrier.

The PBPRA execution involves two phases:

Phase 1: In this phase, the procedure starts by considering the fractional part of each carrier's fair share. In Phase 1, carriers are chosen randomly in proportion to these fractional parts. When a carrier is chosen, it is assigned the highest flight-to-slot assignment on its priority list. Each flight operator is assigned at most one slot during this phase. Small flight operators with $FairShare_i < 1$ are only considered in this phase.

Phase 2: The second phase also uses a randomized procedure where the remaining slots are considered from earliest to latest. Flight operators, who can use the slot in question, are chosen randomly in proportion to the integer part of the $FairShare_i$'s.

Our algorithm can be defined formally:

PBPRA:

Inputs: Carriers: $A_1, A_2, ..., A_K$,

Carrier Fair Shares: $FS_1, FS_2, ..., FS_K$,

Carrier Preference Lists: $PList_1, PList_2, ..., PList_K$

Step 0: Calculate $FS_1^F, FS_2^F, ..., FS_K^F$ and $FS_1^I, FS_2^I, ..., FS_K^I$, the fractional parts and integer parts of $FS_1, FS_2, ..., FS_K$; set $N_{fract} = \sum_i FS_i^F$;

Step 1: **PHASE 1** while $N_{fract} > 0$ Do:

Step 1a: from among all carriers A_i with $FS_i^F > 0$ choose A_{i^*} randomly in proportion to the value of $FS_{i^*}^F$.

Step 1b: Let (f', s') be the highest priority assignment on $PList_{i^*}$. Assign f' to s' and set $FS_{i^*}^F = 0$

Step 1c: Delete all assignments of the form $(f', *)$ from $PList_i$; delete all assignments of the form $(*, s')$ from all lists $PList_k$ for $k \neq i$,

Step 1d: Set $N_{fract} = N_{fract} - 1$;

end while

Step 2: **PHASE 2**

Step 2a: Let s' be the earliest unassigned slot. If no flights can be assigned to s', then delete s' and skip to Step 2d. Otherwise, from among all carriers A_i with $FS_i^I > 0$ that can use s', choose A_{i^*} randomly in proportion to the value of FS_i^I.

Step 2b: Let (f', s') be the highest priority assignment on $PList_{i^*}$. Assign f' to s'.

Step 2c: set $FS^I_{i*} = FS^I_{i*} - 1$; delete all assignments of the form $(f', *)$ from $PList_i$; delete all assignments of the form $(*, s')$ from all lists $PList_k$ for $k \neq i$,

Step 2d: If all slots have been assigned then stop; otherwise repeat Step 2.

It can be seen that PBPRA algorithm has has an initial stage and two execution phases. In the following, we are going to explain intuitively what PBPRA is trying to accomplish at initial stage, phase one and phase two.

At the initial stage we perform the diverse decomposition procedure on the set of flights-slots. This procedure reduces carrier gaming possibilities. Let us clarify this gaming prevention in more detail with an example.

Suppose the true preference of A in Figure 4.1 is (f_{1A}, s_1) and (f_{2A}, s_2). Also let $FS_a = 1.8$ and $FS_B = 1.2$. A may falsify its preferences, by indicating (f_{2A}, s_2) as its highest preference. If we consider a non-decomposable problem (Figure 4.1(a)), and if in the first phase A is chosen, then A receives s_2 and in the second phase it will receive s_1. But in a diverse non-decomposable problem (Figure 4.1(b)), if A lies, it receives s_2 and it may loose its chance to receive s_1. Thus, this is a penalty for non-truthfulness. In example Figure 4.1(a), the initial decomposition step would allocate s_1 to A and then apply PBPRA to the remaining flights/slots. We will prove later in this chapter, that PBPRA is resistent to gaming if we have a diverse non-decomposable set of flights-slots.

PBPRA requires as input a fair share for each carrier. We proposed a fair share algorithm in the previous chapter 3, the output of the FFS-PRA can be used as an input to PBPRA. There might be other methods to assign fair shares to carriers. In this dissertation, we always use the output of FFS-PRA as input to our procedures. The fair share can be used as a basis for allocating the slot to carriers. As mentioned, one of the goals for PBPRA is to allocate a total number of slots to carriers that is "close" to their fair shares. We will show in section 4.5, this goal will be archived.

The third input to PBPRA consist of the carriers' preferences lists. As explained in section 4.3.2 such flight priority lists provide flexibility for each carrier to express its preferenceamong slots. PBPRA takes into account the preferences of carriers when it allocates slots.

In the execution stage of PBPRA, there are two phases. The motivation behind doing allocation in two phases is that we implicity give priority to the small carriers (i.e. carriers with $FairShare < 1$). Also, carriers more explicitly influence the slots they receive.

In phase 1, we assign N_{fract} slots. In phase 2, every carrier a with $FairShare_a \geq 1$ receives $\lfloor FairShare_a \rfloor$ slot(s). Slots are assigned sequentially to carriers from earlier to later ones. Indeed, we assign slots to carriers and then based on their submitted priority list we assign the slot to the highest preferred flight according to the carriers' priority list.

4.3.4 Modified- PBPRA

We now propose an alternative to PBPRA that is less complex, and also is more immune to gaming. The modified PBPRA operates in just one phase, and we call it M-PBPRA. The M-PBPRA is presented as follows. It can bee seen that step 1 of PBPRA is deleted and the step 2a is modified as below:

Step 2a: Let s' be the earliest unassigned slot. If no flights can be assigned to s', then delete s' and skip to Step 2d. Otherwise, from among all carriers that can use s', among all A_i with $FS_i \geq 1$ choose A_{i^*} randomly in proportion to the value of FS_i else choose A_{i^*} randomly in proportion to the value of FS_i among all carriers A_i with $FS_i > 0$.

M-PBPRA successively assigns slots considering all carriers who can use that slot. M-PBPRA starts from the earliest slot and assigns slots one by one to each carrier. If a slot is assigned to a carrier, then its fair share is reduced by one. In each round, only carriers with a positive share remaining will be considered.

We will prove in section 4.4 , that in order to be immune to gaming, having a non-decomposable set is enough, while PBPRA needs a diverse non-decomposable set. It should be noted that in the decomposition procedure described in section 3.4 we only delete slots while in the diverse decomposition procedure we delete slots as well as assign and delete pairs of flight-to-slot assignemnt. This means PBPRA requires a stronger constraint (i.e. diverse non-decomposable) to be resistent to gaming. Let us clarify this in the following example.

Flights	f_{1A}	f_{1B}	f_{2A}	f_{2B}	f_{3A}	f_{3B}	f_{4A}	f_{1C}	f_{4B}	f_{2C}	f_{5A}	f_{3C}
Airline	A	B	A	B	A	B	A	C	B	C	A	C
a_f	3:58	4:00	4:01	4:02	4:03	4:03	4:05	4:06	4:07	4:08	4:09	4:09

Table 4.1: Flight schedules of airline A, B and C

As explained before, in Figure 4.1, suppose the true preference of A is (f_{1A}, s_1) and (f_{2A}, s_2). And also $FS_a = 1.8$ and $FS_B = 1.2$. A may falsify its preferences. This means A may declare (f_{2A}, s_2) its highest preference. In Figure 4.1, in both configurations, (a) and (b), since slots are assigned successively A can not improve its allocation through deception.

It can also be seen that M-PBPRA is performed in a single phase (i.e. easier for practical implementation), while PBPRA requires two phases. However, one limitation with M-PBPRA is that when a carrier is chosen its preference list only influences its flight-to-slot assignment, not the slot it receives. In PBPRA, carriers have the opportunity to choose the best proffered slot. We illustrate in the following example that PBPRA can assign the best proffered slot to carriers.

Table 4.1 shows the flight schedules of three carriers A, B and C. The available time slots are:

Slot:	s_1	s_2	s_3	s_4	s_5	s_6
Time:	4:00	4:02	4:04	4:06	4:08	4:10

Table 4.2 shows the flight priority of each airline.

As we explained in chapter 3, the fair share of each airline can be computed: $FS_A = 2.71$, $FS_B = 2.43$ and $FS_C = 0.86$. Therefore $N_{frac} = 2$.

A	B	C
$(f_{2A}, s_2 : s_3)$	$(f_{4B}, s_5 : s_6)$	$(f_{1C}, s_4 : s_5)$
$(f_{3A}, s_3 : s_4)$	$(f_{2B}, s_2 : s_4)$	$(f_{2C}, s_5 : s_6)$
$(f_{4A}, s_4 : s_6)$	$(f_{1B}, s_1 : s_3)$	(f_{3C}, s_6)
$(f_{1A}, s_1 : s_6)$	$(f_{3B}, s_3 : s_6)$	
(f_{5A}, s_6)		

Table 4.2: Preference list for airlines A, B and C

While executing PBPRA, if in the first phase A and B are chosen, then A and B receive their first preferences, i.e. A receives (f_{2A}, s_2) and B receives (f_{4B}, s_5). Instead if we run M-PBPRA and A and B are chosen, then A receives (f_{1A}, s_1) and B receives (f_{2B}, s_2) which are not their first preferences. This shows that in PBPRA, A and B have the opportunity to choose their best proffered slots while in M-PBPRA they have not received their first preferences .

The M-PBPRA procedure is very similar to PRA except that the demand (fair share of carriers) is not an integer value (In PRA, demand of each agent is an integer value). At each round of the procedure, we implicity give priority to the carriers with remaining fair share greater than 1. This helps to make sure every carrier receives at least $\lfloor FairShare \rfloor$. If in one round of the procedure, say the j^{th}, there is no carrier with $FS \geq 1$, then M-PBPRA considers all carriers that can use slot s_j with a positive remaining fair share.

In most of cases, PBPRA or M-PBPRA assigns all slots, however there are very rare instances that some slots could remain unassigned. For example, Figure 4.2 shows a non-decomposable problem, in which 6 airlines A, B, C, D, E and G

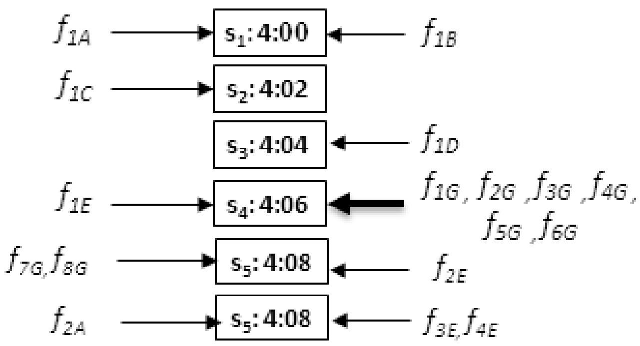

Figure 4.2: Example of unused slots in PBPRA or M-PBPRA

compete for 6 slots. Suppose the highest preference for airlines A, B, C, D, E and G are (f_{2A}, s_6), (f_{1B}, s_1), (f_{1C}, s_2), (f_{1D}, s_3), (f_{1E}, s_4) and (f_{2G}, s_5) respectively. As explained in chapter 3, the fair share of each airlines can be computed: $FS_A = 0.99$, $FS_B = 0.9$, $FS_C = 0.81$, $FS_D = 0.63$, $FS_E = 0.53$ and $FS_G = 2.14$. Therefore $N_{frac} = 4$. While executing PBPRA, if in the first phase, A, D, E and G are chosen, then s_6, s_3, s_4 and s_5 are assigned to A, D, E and G respectively. In the second phase PBPRA starts from first unassigned airline which is s_1. The only two airlines that can use s_1 are A and B. Since A and B's remaining fair share is zero then no airlines can use s_1. Therefore, s_1 remains unused.

We provide the following example to explain how these two procedures work.

4.3.5 Example

Consider example 3.4.2; there are three carriers each having two flights; each are competing for four available slots. We compute the fair share for each carrier:

$$FS_A = \frac{21}{12} \qquad FS_B = \frac{19}{12} \qquad FS_C = \frac{8}{12}$$

A	B	C
(f_{2A}, s_2)	(f_{2B}, s_3)	(f_{1C}, s_4)
(f_{2A}, s_3)	(f_{2B}, s_4)	(f_{2C}, s_4)
(f_{1A}, s_1)	(f_{1B}, s_1)	
(f_{1A}, s_2)	(f_{1B}, s_2)	
(f_{2A}, s_4)		

Table 4.3: Preference list for airlines A, B and C

Therefore the integer part and fraction part of each fair share is:

$$FS_A^I = 1 \qquad FS_B^I = 1 \qquad FS_C^I = 0$$

$$FS_A^F = \frac{9}{12} \qquad FS_B^F = \frac{8}{12} \qquad FS_C^F = \frac{8}{12}$$

Thus $N_{fract} = 2$. Suppose carriers submit their priority list of flights based on Table 5.2. In the first step, we compute the probability based on fractional part of each carrier, thus:

$$Prob_A = \frac{9}{25} \qquad Prob_B = \frac{8}{25} \qquad Prob_C = \frac{8}{25}$$

where $Prob_i$ is the probability that carrier i is chosen. Suppose, B is chosen, so B will receive its highest priority flight in its priority list, so B will receive s_3, $f_{2B} \to s_3$. Now, any $(*, s_3)$ is removed form preference tables. We reduce the $N_{frac} = 1$ and $FS_B^F = 0$. Next, we randomly choose A and C with probabilities proportion to their fraction part as follow:

$$Prob_A = \frac{9}{17} \qquad Prob_C = \frac{8}{17}$$

Suppose C is selected, so f_{1C} will be assigned to s_4. Update the preference table and remove any pair of $(*, s_4)$. Phase one of algorithm is done and we start with the first unassigned slot, s_1. A and B both can use s_1, so with probability of $\frac{1}{2}$ s_1 will be assigned to one of those, suppose s_1 assigned to A, based on A's priority list, f_{1A} goes to s_1. As you can see, f_{1A} is not its highest priority flight, but it is the only flight that can use s_1 therefore it must take s_1. Finally, B will receive s_2 and f_{1B} assigned to s_2.

M-PBPRA starts from s_1, A and B are the two carriers that can use s_1, both have a fair share greater than one. Therefore, we compute the probabilities:

$$Prob_A = \frac{21}{40} \qquad Prob_B = \frac{19}{40}$$

Suppose A is chosen then we reduce the fair share of A by one, $FS_A = FS_A - 1 = \frac{21}{12} - 1 = \frac{9}{12}$ and assign f_{1A} to s_1. We continue to the next slot, s_2; both A and B can use s_2 while B is the only airline with $FS_B \geq 1$. Thus, we assign s_2 to the highest priority flight in $PList_B$, so f_{1B} goes to s_2. Also, we reduce the B's fair share by one, $FS_B = FS_B - 1 = \frac{19}{12} - 1 = \frac{7}{12}$. In the next round, A and B compete for s_3; since both airlines can use s_3 and both have positive share (no one has fair share greater than one) . We compute the probabilities:

$$Prob_A = \frac{9}{16} \qquad Prob_B = \frac{7}{16}$$

Suppose B is chosen, since its fair share is less than one, the fair share of B will be set to zero and from $PList_B$, f_{2B} is assigned to s_3. Finally, in the last round of randomized procedure, A and C participate:

$$Prob_A = \frac{9}{17} \qquad Prob_B = \frac{8}{17}$$

Suppose C is chosen then we assign f_{1C} to s_4.

If we compare the result with RBS, we notice that under RBS, the flight-to-slot assignment will ba:

$$f_{1A} \rightarrow s_1 \qquad f_{1B} \rightarrow s_2 \qquad f_{2A} \rightarrow s_3 \qquad f_{2B} \rightarrow s_4$$

Note that C receives nothing. In both PBPRA and M-PBPRA, C has a chance to receive a slot. Of course, the second important point of allocation is that under PBPRA and M-PBPRA the assignment is based on carriers' preferences.

4.4 Equity of PBPRA and M-PBPRA

We assume that our problem is strictly non-decomposable. Let us define $\mathbf{Q}$ as set of all possible assignments of carriers to slots. It is convenient to think of a deterministic assignment as a 0-1 matrix, with rows indexed by carriers and columns indexed by slots. Each slot is assigned only to one carrier, i.e. there is exactly a single 1 in each column and the number of slots assigned to a carrier a does not exceed its $\lceil FairShare_a \rceil$. Formally, set of all possible assignments of carriers to slots, $\mathbf{Q}$, is:

$$\mathbf{Q} = \left\{ x_{i,j} \in \{0,1\}^{K \times m} : \sum_i x_{i,j} \leq 1 \quad \forall j, \lfloor FairShare_i \rfloor \leq \sum_j x_{i,j} \leq \lceil FairShare_i \rceil \quad \forall i \right\} \tag{4.1}$$

where i^{th} row of each assignment matrix shows the slots assigned to carrier i and K is the number of carriers.

We call a problem "*strictly non-decomposable*" if for all deterministic assignment in $\mathbf{Q}$, $\sum_i x_{ij} = 1 \quad \forall i$. It means that all slot has been assigned.

Corollary 4.4.1 *In a strictly non-decomposable set, PBPRA and M-PBPRA assign to any carrier a, at least $\lfloor FairShare_a \rfloor$ and at most $\lceil FairShare_a \rceil$ slots.*

Proof Since our problem is strictly non-decomposable which means all slots are assigned, all carriers are assigned $\lfloor FairShare_a \rfloor$ slots in the second phase of PBPRA. In the first phase, carriers with positive fractional part can participate once. Therefore, in the first phase no carrier can receive more than one slot. Thus, any carrier receives at most $\lceil FairShare_a \rceil$.

In M-PBPRA, at each step of the procedure, carriers with fair share greater or equal one are considered first. Since our problem is strictly non-decomposable and sum of all fair shares is equal to the number of available slots, each carrier, a, receives $\lfloor FairShare_a \rfloor$. During the those steps when carriers with remaining positive fair share are considered, if a carrier receives a slot then its fair share is reduced to zero. Thus, the total number of slots a carrier receives is at most $\lfloor FairShare_a \rfloor + 1$ or $\lceil FairShare_a \rceil$. •

We call the problem "*strictly diverse non-decomposable*" if all slots have been assigned and there is more than one carrier that can be assigned to any slot. In reality, the number of fights and airlines are large enough so that the set of flights-slots can always be considered strictly diverse non-decomposable. For example, based on some experimental results that we will analyze in following section, involving a set of 386 flights and 40 airlines, the problem always had the strictly diverse non-decomposable property after 40% or more capacity reduction. To make sure that we have a strictly non-decomposable set of flights-to-slots at each step of PBPRA, we may execute the diverse decomposition procedure after every round of slot allocation. This means the following step should be added before Step 2a of PBPRA:

Step 2a: Run diverse decomposition procedure

A *random feasible assignment*, P, can be represented as a probability distribution over all deterministic feasible assignments (Π_i); the corresponding convex combination of deterministic matrices is a matrix whose $(i,j)^{th}$ entry represents the probability with which carrier i receives slot j can be represented as p_{ij}. Let $\mathcal{P}$ be set of all random feasible assignments.

The set of all preference orderings is called a preference domain and denoted by $\mathcal{A}$. A *random assignment mechanism* is a mapping from $\mathcal{A}^K$ to $\mathcal{P}$. As we said, our objective is to show some desirable fairness properties for PBPRA and also M-PBPRA. To describe the fairness properties formally we need to extend the carriers' preferences over the slots to preferences over the random assignment.

Given two random assignments P and Q, we say carrier i prefers P to Q ($P \succ_i Q$) if the allocation P_i stochastically dominates the allocation Q_i (where P_i is the i^{th} row, which represents the allocation for carrier i in the random assignment P). Thus, formally we can define:

Definition 4.4.1 *Given two random assignments P and Q, P stochastically dominates Q with respect to carrier's i preference ordering if :*

$$P \succ_i Q \Leftrightarrow \sum_{k:k \succeq_i j} p_{ik} \geq \sum_{k:k \succeq_i j} q_{ik}, \qquad \forall j \in \mathcal{S} \tag{4.2}$$

Moreover, a random assignment P *dominates* random assignment Q if all carriers prefer P to Q, that is $P \succ_i Q$ for all $i \in A$. If one carrier prefers P to Q and another prefers Q to P then $P \not\succ Q$ and also $Q \not\succ P$. Now we are ready to define the efficiency and fairness properties.

There are three important properties : efficiency, fairness that includes *Equal treatment of equals* and *anonymity*, and *strategy proofness*. We describe each of these properties extensively as follows:

Efficiency- To understand the concept of efficiency we need to describe the concept of *Pareto Optimality*. The formal definition of *Pareto Optimality* is:

Definition 4.4.2 *A deterministic assignment* Π_i *is Pareto optimal if there is no other deterministic allocation,* Π'*, such that* $\forall\ A_i\ \Pi'_i \succsim \Pi_i$ *and at least for one* A_j*,* $\Pi'_j \succ \Pi_j$*.*

The *Pareto optimality* states that if one carrier can not do better unless another carrier is worse off. It should be noted for Pareto-Optimality property that we evaluate our procedures against it, slots are considered in an abstract form. This means total number of slots that a carrier receives does matter , and also earlier slots are better than later ones. This means utility of a carrier is not associated with a slot. This implies a weak notion of Pareto optimality. The stronger form of Pareto optimality is to look at the deterministic assignments that are aligned with carriers' preferences which does not hold for our procedures.

Proposition 4.4.1 *In a strictly non-decomposable set, all* Π *in* $\mathbf{Q}$ *are Pareto Optimal.*

<u>Proof</u> Case 1- Suppose in allocation Π carrier a receives its $\lfloor FairShare_a \rfloor$. Car-

rier a strictly prefers allocation Π' to Π if it receives its $\lceil FairShare_a \rceil$. In order for carrier a to receive one more slot, then there must be another carrier, b, that looses one slot. Since the total number of slots is constant, m. Thus, carrier b does not prefers Π' to Π. In other words, $\Pi'_b \nsucceq \Pi_b$. Case 2- Suppose in allocation Π carrier a receives its $\lceil FairShare_a \rceil$. Since this is the most slots that carrier a can receive, so it can not do better. Case 3- when the number of slots assigned to two carriers a and b do not change and they just exchange an slot. Suppose in assignment Π, s_j and s_i are assigned to a and b respectively, and s_i is earlier than s_j. Suppose a receives s_i and b receives s_j in assignment Π'. a prefers $\Pi' \succ \Pi_j$ since it receives an earlier slot while b does not prefers Π' to Π, $\Pi'_b \nsucceq \Pi_b$. •

We should have note that we look at the efficiency in an abstract way and we don't include the utility of the carriers. The random allocation is called efficient if:

Definition 4.4.3 *P is :*

(a) Ex post efficient iff it can be represented as a probability distribution over Pareto optimal deterministic assignments.

(b) Ex ante efficient iff for any profile of utility functions consistent with the preference profile of the agents, the resulting expected utility vector is Pareto efficient[1].

(c) Ordinally efficient iff it is not dominated by any other assignment Q.

[1]For any random assignment in which the expected utility of some agent is strictly greater, there must be another agent that whose expected utility is strictly lower.

It can be shown that ex post efficiency implies ordinally efficiency, which implies ex post efficiency. The relation between various notation of efficiency is explored in [2] and [42].

Fairness As we said in the previous chapter, two important axioms of $fairness$ is called *anonymity* and *Equal Treatment of Equals (ETE)*. You can refer to the previous chapter for the formal definition of these two properties. A random assignment mechanism is anonymous if the its out come is only depends on the preference profile and it is independent of the type of agents. Equal treatment of equals ex post states that carriers with the same schedule and same preference profile should have the same probability distribution over slots. On the other hand the stronger version of Equal treatment of equals is ETE-ex post: the outcome of allocation, the actual total number of slots received by a carrier, to two carriers with the same schedule flights should be different in at most one slot.

Incentive A random assignment mechanism is said to be *strategy-proof* if for each agent the true preference ordering is a dominant strategy. If B_i is the true valuation of slots for carrier i, then for any B_i compatible with true $\succ_i$, the expected return for agent i is higher than any other $\succ_i^*$ (false preference ordering). A weaker notation of strategy-proofness can be defined: a mechanism is weakly strategy proof if an agent by falsifying her preference list can not obtain an allocation that she strictly prefers to her true allocation.

Definition 4.4.4 *Given a mechanism $P(.)$, we define:*

strategy proofness: $P_i(\succ) sd(\succ_i) P_i(\succ |^i \succ_i^*) \qquad \forall i \in N, \succ^* \in (P), \succ \in \mathcal{P}^N$

Weak Strategy-proofness: $P_i(\succ |^i \succ_i^*) sd(\succ_i) P_i(\succ) \Rightarrow P_i(\succ |^i \succ_i^*) = P_i(\succ)$

With all this explanation we can have the following theorem:

Theorem 4.4.1 *(a) In a strictly non-decomposable problem, PBPRA and M-PBPRA meet anonymity, Equal treatment of equals (ex-post), ex post efficiency and M-PBPRA meets strategy proofness.*

(b) In a strictly diverse non-decomposable problem PBPRA meets strategy proofness.

Proof It is easy to see that both procedures are anonymous.

Our problem is strictly non-decomposable which means all slots are assigned. In PBPRA, we make sure that each carrier a receives its $\lfloor FairShare_a \rfloor$. The only difference would be in the fractional part; thus the difference between two carriers with same share will be at most one. In M-PBPRA, since in each round we give priority to the carriers with a fair share greater than one, then eac carrier a will receive at least $\lfloor FairShare_a \rfloor$. The rest of slots would be distributed based on a randomization among carriers with positive share; no carriers can receive more than one slot in this phase. Therefore, there will be at most one slot difference between carriers with the same fair share.

We can write P as a probability distribution over all Pareto Optimal deterministic assignments in $\mathbf{Q}$ so it is ex-post efficient.

To prove strategy proofness, we consider two procedures PBPRA and M–PBPRA separately. M-PBPRA acts like as random priority method (or a serial dictator). There is a known probability associated with each deterministic assignment. Computing the probabilities are independent from carriers' preferences. Thus, not telling the truth does not help a carrier to increase its chance of winning a better slot. In other words, carriers can not have an assignment that stochastically dominates another assignment by falsifying their preferences. Thus, there is no advantage for a carrier to express an untrue preference.

In PBPRA, suppose airline a does not say the truth about its preferences while other airlines are truthful. If $FS_a < 1$, then a can only be considered in the first phase, and if it is chosen it may receive an available slot it is proffered to the one it receives. Therefore, it can not do better. If $FS_a^F = 0$, then carrier a is only considered in the second phase. Since the first phase has already been run, then there is a known probability associated with each deterministic assignment. Also, the set of flights-slots is strictly diverse non-decomposable, which means that there is always more than one carrier compete for a slot. Therefore, a can not increase its chance of wining a proffered slot by falsifying its preferences. The last scenario is when $FS_a^F > 1$, i.e. a participates in both phases. Without loss of generality, suppose a prefers s_i to s_j, $s_i \succ_a s_j$, but it falsifies its preferences by $s_j \succ_a^* s_i$. If in the first phase a is chosen and the best available slot is s_i but it chooses s_j instead of s_i. Our problem is strictly diverse non-decomposable, which means s_i can be claimed by another carrier. Then by falsifying its preference, a increases the chance of another carrier wining slot s_i, i.e. it reduces its chance of wining s_i.

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4.5 Experimental Results

In our experiments, we use a test data set that had been employed by the CDM Future Concepts Team to perform human-in-the-loop experiments related to SEVEN. It contained 386 flights with 38 flight operators. The data included scheduled arrival arrival times at an FCA boundary. The FCA duration was from 18:00 pm to 21:00 pm. We generated a flight cost function that is described below. Given the cost function, we generated the priority list for each flight operator based on the following principle:

> Of all available flights that could use a slot, the flight operator preferred allocating the slot to the flight with the highest marginal cost of delay.

We consider the cost of each flight as a function of the number of passengers (actually number of aircraft seats) and the flight delay. The cost function (based on generic advice from airline dispatchers) was constructed based on the following general principles. First, the initial 15 minutes of delay is considered free. The Air Transportation Association (ATA) estimates that direct operating cost during block time is \$64 per minute. We assume ground cost is 1/2 as expensive as air cost (and this is an accepted practice in the literature), so \$64/2 = \$32. We can assume that there is also the possibility of rerouting the flight. This effectively caps the delay cost (once the delay cost curve exceeds the rerouting cost, the airline is better off rerouting the flight). As with the airline cost, we have assumed that passengers are willing to ignore the first 15 minutes of delay, that their time is worth \$0.60 for each minute thereafter, and that this linear function is capped after 15 hours [21]. The

\$0.60 figure again comes from the ATA web site, which cites \$34.88 per hour as an average cost of passenger time. This translates to \$34.88/60 = \$0.5813 per minute, per passenger, which we rounded to \$0.60 per minute. Our cost function should represent internal airline costs. Airlines are interested in providing good customer service but do not suffer the full brunt of passenger costs. We approximated this customer service perspective by multiplying the passenger cost function by 1/6 and adding the resultant cost to the flight delay costs as described above. Thus, we can write the flight delay cost function as:

$$C(x, P) = \begin{cases} 0 & x \leq 15 \\ (32 + 0.1P)(x - 15) & 15 < x \leq M_p \\ (32 + 0.1P)(M_p - 15) & x > M_p \end{cases}$$

Where M_p is a flight specific max delay. That is, it is assumed that after M_p minutes of delay, the flight operator would prefer to reroute the flight. Since the cost effectiveness of rerouting will vary with flight characteristics we chose M_p randomly with uniform likelihood between 30 to 90 minutes.

We compared the results of PBPRA and M-PBPRA against ration-by-schedule (RBS), which is currently used to allocate FCA access during airspace flow programs. Our version of RBS proceeded from the earliest to latest slot. At each step, it assigned the available flight with the earliest scheduled arrival time (ties were broken randomly with equal likelihood). Once we determined a flight-to-airline assignment, if multiple flights from the chosen airline could be assigned to the same slot, then we assigned the flight with the highest marginal cost of delay. In this way, at the end of the procedure, the airlines could not improve their cost function by doing an "internal" flight-to-slot reassignment.

In our experiments, we considered 40%, 50%, 60%, 70% and 80% en-route capacity reduction for the FCA. We performed 2000 repetitions of the procedure (note that since PBPRA and M-PBPRA use randomization their "expected" impacted can only be calculated by doing multiple repetitions). Table 4.4 shows the average number of slots assigned to each airline for a specific capacity reduction.

A noteworthy point to be made is that many airlines received 0 slots under RBS. Note that since RBS is a deterministic procedure if an airline receives 0 under one recitation it receives 0 under all repetitions. Such airlines had only a single flight demanding access to the FCA and that flight had a relatively late scheduled arrival time. On the other hand, the fractional values achieved by PBPRA indicate that on some repetitions PBPRA allocated such an airline a slot and on others it did not. Few would probably dispute that this is a more equitable outcome.

An important related issue is the degree to which PBPRA or M-PBPRA achieve (on the average) the flight operator fair shares (FS_i's). As stated earlier we cannot formally prove that this is the case. As the results in the table indicates experimentally both algorithms come very close achieving FS_i values. As would be expected, the RBS can diverge by fairly significant amounts. Table 4.5 shows the mean square error of PBPRA, M-PBPRA and RBS compare to the fair share for each capacity reduction.

Of course, a very fundamental implicit goal of our procedure is that flight operators should be able to improve their internal performance based on an allocation process that takes into account their preferences. The total cost saving over all

	Number of Slot Allocated to Each Flight Operator, RBS vs. PBPRA and M-PBPRA																			
	80% Capacity Reduction				70% Capacity Reduction				60% Capacity Reduction				50% Capacity Reduction				40% Capacity Reduction			
Airlines	PBPRA	M-PBPRA	RBS	Fair Share	PBPRA	M-PBPRA	RBS	Fair Share	PBPRA	M-PBPRA	RBS	Fair Share	PB-PRA	M-PBPRA	RBS	Fair Share	PBPRA	M-PBPRA	RBS	Fair Share
1	5.12	5.11	6	5.08	7.34	7.35	8	7.28	9.36	9.36	10	9.32	11.3	11.3	11	11.2	12.8	12.8	13	12.9
2	1.56	1.6	1	1.55	2.25	2.25	3	2.2	2.7	2.71	4	2.82	3.46	3.49	4	3.41	3.8	3.79	4	3.95
3	0.61	0.6	0	0.58	0.74	0.74	1	0.9	1.32	1.31	2	1.27	1.64	1.64	2	1.65	2.08	2.09	2	2.05
4	0.15	0.15	0	0.11	0.23	0.23	0	0.17	0.3	0.31	0	0.26	0.43	0.43	0	0.36	0.56	0.55	0	0.49
5	1.66	1.63	0	1.67	2.6	2.58	4	2.6	3.62	3.62	5	3.67	4.72	4.72	6	4.79	5.77	5.79	6	5.92
6	1.52	1.55	2	1.48	2.29	2.31	3	2.24	3.08	3.09	3	3.06	3.76	3.74	3	3.86	4.63	4.61	5	4.59
7	0.32	0.34	0	0.26	0.46	0.46	0	0.42	0.59	0.58	0	0.62	0.73	0.74	1	0.85	1.17	1.17	1	1.11
8	1.8	1.78	2	2	2.69	2.7	2	2.78	3.45	3.43	4	3.39	3.78	3.78	4	3.94	4.53	4.55	5	4.45
9	0.13	0.14	0	0.09	0.21	0.19	0	0.15	0.3	0.28	0	0.23	0.4	0.4	0	0.33	0.53	0.52	0	0.44
10	2.54	2.55	3	2.51	3.69	3.71	4	3.82	5.25	5.23	4	5.2	6.58	6.61	6	6.56	7.78	7.77	8	7.88
11	0.27	0.29	0	0.21	0.38	0.39	0	0.33	0.5	0.51	1	0.47	0.63	0.64	1	0.62	0.71	0.74	1	0.78
12	0.26	0.24	0	0.18	0.32	0.34	0	0.28	0.44	0.45	0	0.4	0.58	0.58	1	0.54	0.67	0.68	1	0.69
13	8.06	8.05	13	8.04	11.7	11.7	15	11.8	15.4	15.4	16	15.4	18.7	18.7	17	18.7	21.6	21.6	20	21.6
14	0.79	0.78	1	0.99	1.57	1.57	2	1.57	2.28	2.3	2	2.24	2.77	2.77	2	2.95	3.71	3.71	3	3.72
15	6.47	6.45	7	6.41	9.5	9.53	7	9.48	12.6	12.6	9	12.6	15.7	15.8	12	15.9	19.3	19.3	16	19.2
16	0.45	0.44	0	0.36	0.59	0.58	0	0.58	0.73	0.71	1	0.84	1.22	1.2	1	1.15	1.56	1.56	1	1.52
17	0.31	0.32	0	0.26	0.45	0.44	1	0.4	0.54	0.54	1	0.56	0.67	0.68	1	0.7	0.75	0.74	1	0.84
18	0.01	0.02	0	0.02	0.02	0.04	0	0.03	0.04	0.06	0	0.04	0.08	0.09	0	0.06	0.11	0.14	0	0.09
19	0.2	0.19	0	0.15	0.3	0.3	0	0.24	0.38	0.39	0	0.34	0.55	0.53	0	0.47	0.63	0.65	1	0.61
20	0.03	0.03	0	0.03	0.06	0.07	0	0.05	0.1	0.1	0	0.07	0.14	0.17	0	0.11	0.21	0.22	0	0.15
21	11.7	11.8	15	12	17.7	17.7	19	17.8	23.7	23.6	22	23.7	29.5	29.5	28	29.5	34.8	34.8	34	35
22	1.74	1.74	1	1.87	2.58	2.57	2	2.56	3.33	3.33	4	3.28	4.06	4.06	4	4.04	4.77	4.76	5	4.84
23	0.3	0.29	0	0.23	0.42	0.42	0	0.37	0.56	0.56	0	0.55	0.7	0.72	0	0.77	1.04	1.04	1	1.03
24	1.11	1.13	1	1.09	1.66	1.63	1	1.69	2.4	2.4	2	2.35	3.1	3.1	3	3.06	3.74	3.73	4	3.77
25	1.66	1.66	2	1.7	2.58	2.6	4	2.62	3.59	3.62	5	3.63	4.65	4.64	5	4.66	5.63	5.63	6	5.62
26	0.24	0.24	0	0.18	0.33	0.34	0	0.29	0.45	0.44	0	0.41	0.58	0.57	1	0.56	0.7	0.7	1	0.7
27	0.14	0.12	0	0.1	0.2	0.2	0	0.16	0.33	0.28	0	0.24	0.44	0.42	0	0.34	0.53	0.53	0	0.46
28	0.78	0.76	2	0.92	1.43	1.43	2	1.38	1.67	1.68	2	1.78	2.14	2.14	2	2.09	2.4	2.38	2	2.3
29	8.78	8.78	10	8.97	13.1	13.1	14	13.1	16.8	16.7	18	17	20.7	20.7	23	20.7	24.3	24.3	25	24.2
30	0.54	0.51	0	0.48	0.67	0.67	1	0.74	1.04	1.05	1	1.03	1.41	1.41	2	1.33	1.61	1.61	2	1.61
31	0.27	0.27	0	0.21	0.39	0.37	0	0.33	0.48	0.49	1	0.46	0.6	0.63	1	0.61	0.73	0.7	1	0.76
32	8.23	8.22	7	8.16	11.7	11.7	10	11.9	15.6	15.6	16	15.7	19.6	19.6	18	19.5	23.2	23.2	24	23.2
33	0.07	0.07	0	0.05	0.1	0.1	0	0.08	0.18	0.17	0	0.13	0.26	0.25	0	0.18	0.34	0.32	0	0.26
34	1.69	1.7	1	1.78	2.7	2.66	3	2.76	3.69	3.68	6	3.81	4.77	4.76	6	4.9	5.79	5.77	7	5.9
35	0.08	0.08	0	0.06	0.14	0.14	0	0.1	0.18	0.2	0	0.16	0.31	0.3	0	0.23	0.41	0.4	0	0.32
36	7.55	7.56	4	7.51	11.6	11.6	10	11.6	16.1	16.1	16	16.1	20.7	20.8	27	20.9	25.7	25.7	29	25.7
37	0.21	0.2	0	0.16	0.31	0.32	0	0.25	0.41	0.42	0	0.36	0.53	0.54	1	0.5	0.64	0.66	1	0.64
38	0.62	0.63	0	0.62	1.02	1.02	0	1.01	1.52	1.51	0	1.49	2.12	2.12	1	2.08	2.73	2.73	2	2.76

Table 4.4: Comparison of PBPRA, M-PBPRA and RBS allocations

% Capacity reduction	PBPRA	M-PBPRA	RBS
40	0.211	0.214	31.23
50	0.213	0.221	74.49
60	0.172	0.166	35.61
70	0.17	0.154	35.35
80	0.284	0.275	57.63

Table 4.5: Total mean square error from fair share, PBPRA and M-PBPRA vs. RBS

airlines of PBPRA compared to RBS is shown in Fig 4.3. We note that PBPRA consistently provides a significant savings. Table 4.5 provides the corresponding percentage savings.

There is a reduction in total delay of both algorithms vs. RBS, we can see the result in table 4.5. Note that PBPRA does a little bit better in terms of average cost saving and average total delay. However, the advantage of M-PBPRA is it has a smaller standard deviation than PBPRA. As can be seen in Figure 4.3 the average cost plus standard deviation of M-PBPRA falls below both PBPRA and RBS.

4.6 Discussion

In this chapter two randomized procedures have been proposed to assign flights to slots based on carriers' preferences. In RBS, flights-to-slots assignment is based on the flight schedule and later flights may not receive slots. We proposed randomized methods that use an exogenous fair share of carriers from available slots as a

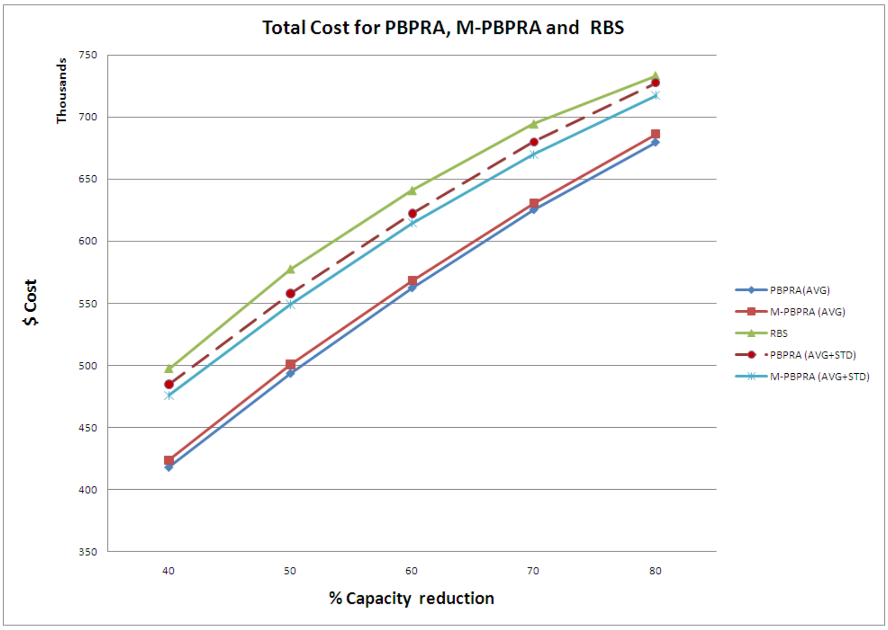

Figure 4.3: Total cost of all flight operators, RBS vs. PBPRA.

% Capacity reduction	PBPRA	M-PBPRA
40	15.96	14.76
50	14.53	13.3
60	12.22	11.33
70	9.97	9.198
80	7.36	6.42

Table 4.6: Total percent of cost savings PBPRA and M-PBPRA compared to RBS

% Capacity reduction	PBPRA	M-PBPRA
40	25.07	21.14
50	28.26	24.1
60	30.9	26.94
70	33.27	28.15
80	35.3	28.9

Table 4.7: Total percent of average delay reduction, PBPRA and M-PBPRA compared to RBS

parameter to assign flights to slots. Carriers are entitled to receive the number of slots based on their fair share. It is guaranteed that any carrier receives at least the floor of its fair share. These methods give carriers whose flights are scheduled late chance of receiving a slot. We also explored the principles of our allocation procedures. We showed that PBPRA and M-PBPRA meet equity principles and also have ex-post efficiency and strategy proofness properties. Also, we tested our algorithms on real data. Our algorithms showed improved performance compare to the RBS. In PBPRA and M-PBPRA, *expected total number of slots* that a carrier receives is very close to its fair share.

In PBPRA (or M-PBPRA), although slot preferences were employed we implicity assumed that all slots had equal values. In reality, some slots are worth more tan others. It is desirable for us that carriers can express their preference between delay and rerouting. As we mentioned, some carriers prefer to receive fewer slots but to preserve on-time performance for certain important fights while other ones prefer to receive more slots and can tolerate more delays. We will propose a new algorithm in the next chapter to address this problem.

Chapter 5

A New Randomized Allocation Using Dual Pricing of Resources

5.1 Introduction

In the case of capacity reduction in enroute resources we explained how to find a fair share for each carrier based on scheduled arrival times at a FCA. Also, we proposed probabilistic algorithms that assign slots to flights. In our algorithms we took into account the preference of carriers for each flight. The algorithms allocate slots successively to carriers while considering their fair shares. No carriers would receive more than the ceiling of its fair share.We demonstrated that both algorithms, meet fairness and efficiency properties.

In this chapter, we extend the allocation process to treat a new variant of the problem. We describe the problem of allocation of slots in section 5.2. In section 5.3 a new algorithm will be proposed to allocate slots. The equity and incentive of our algorithm will be discussed in section 5.4. Finally, we will explain the algorithm by a numerical example.

5.2 Problem Description

In PBPRA (or M-PBPRA), although slot preferences were employed we implicity assumed that all slots had equal values. Specifically, when measuring and allocation against a carrier's fair share, we only considered the total number of slots a carrier received. Clearly, given that a flight can use two slots, the earlier one is always preferred. Further, those carriers that would like to maintain their on time performance for key flights, may be willing to pay more than others for particular slots. We wish to allow carriers to "pay more" for earlier slots when they wish to do so.

Our objective here is somehow distinguish between those carriers who want to maintain the on-time performance for certain flights and in return receive fewer slots and those carriers who can tolerate more delay and would like to receive more slots.

The methods we propose to accomplish these objectives employ a new preference scheme. Our algorithm has two phases. We will explain each phase of algorithm in the following section.

5.3 Dual Price Proportional Random Assignment

In general, each carrier will have different cost for delay and rerouting (or cancellation). In PBPRA we include the carriers' preferences to allocate slots based on their exogenous fair share. We did not elicit preferences related to the trade off between delay and rerouting .

Dual-Price Proportional Random Assignment (DP-PRA) is a new algorithm that considers the carriers' tradeoff between delay and rerouting (or cancellation). The basic concept in the DP-PRA is : those airlines who want to receive fewer slots in order to get less delay should be able to do so in exchange for a reduction in the total number of slots they receive. That is, if one views the fair share as currency then they can pay more than one unit for highly desirable slots.

Consider the example of carrier A who prefers to receive priority for certain flights in exchange for receiving fewer slots in total. The algorithm employs a parameter which is the "value" of the higher priority slots distributed. If carrier A's fair share is 5.5 then it can receive two "high-priority" slots based on 2 ($\lfloor \frac{5.5}{2} \rfloor$). The remainder of its fair share is 1.5, which can then be used to receive later slots. It is very important to notice that only those carriers that can afford this trade off (have a fair share ≥ 2) are considered. If a small carrier with a small fair share prefers to receive good slots, if it does not have enough budget to give up a second flight, it can not be considered.

5.3.1 Slot Values

For illustration purposes, suppose we have two sets of airlines. Let A_1 be the set of airlines that prefer less delay and A_2 the set of airlines that prefer to receive more slots. In our allocation algorithm we initially give priority to the airlines in A_1. Therefore, they must pay more for each slot they initially receive because of the priority. Let us assume the price of each slot they receive is P_H. Since airlines in A_1 receive priority in the allocation process their exogenous fair share must be greater than P_H.

The FAA acts as an independent, fair moderator. The FAA announces the value of priority slots. This value must be greater than one. The process operates so that the total value of slots given away equals the number of slots available. Since the value of each slot for the airlines in set A_1 is P_H, we can compute the value of remaining slots. Thus, later (less preferred) slots will have a value less than one. Suppose there are m slots available, to compute the value of remaining slots, we need to find the number of slots that are assigned to airlines in A_1. Let us call this number m_1:

$$m_1 = (\sum_{a \in A_1} [FS_a - (FS_a\ mod^*\ P_H)])/P_H \tag{5.1}$$

Where FS_a is the fair share of carrier a. And $(FS_a\ mod^*\ P_H)$ is the reminder of FS_a from P_H[1]. Then the value of remaining slots can be computed as:

$$P_L = \frac{m - P_H \times m_1}{m - m_1} \tag{5.2}$$

As we can see the value of the remaining slots is less than one. Note that higher P_H values result in a smaller m_1. We will show the effect of varying P_H in our simulation results.

5.3.2 Flight Priority

Carries must submit a list of flight priorities. As we explained in the previous chapter, the priority list includes tuple of (f, s). To make the list somehow shorter, carriers can submit the list of flights and a range of slots. For example, $(f, \{s_i, s_{i+1}, ..., s_k\})$.

[1]Note: we use mod for integer values. For example $7\ mod\ 3 = 1$. Here, we use mod^* for positive real values. Then, $\forall a, b \in \mathbf{R}^+ a\ mod^*\ b = a - b\lfloor \frac{a}{b} \rfloor$. For example $2.5\ mod^*\ 0.2 = 2.5 - 0.2\lfloor \frac{2.5}{0.2} \rfloor = 0.1 and 9\ mod^*\ 2.5 = 9 - 2.5\lfloor \frac{9}{2.5} \rfloor = 1$.

5.3.3 DP-PRA

DP-PRA contains two phases: First phase allocates slots to the flights in the set A_1 and in the second phase all remaining slots are allocated from the earliest available to the latest available. The second phase can use one one the procedures in cahpter4, i.e. PBPRA or M-PBPRA.

As mentioned, we need to have a diverse non-decomposable set of flights-slots. Therefore, we run strong decomposition algorithm before applying DP-PRA. We define two policies. Policy P_1, where carriers prefer to receive priority that means these carriers prefer to receive fewer slots but less delay. Policy P_2, where carriers are not interested to receive priority instead they prefer to receive more slots. We develop the DP-PRA procedure in the formal way as below:

Step 2: PHASE 0

- **Step 0a:** Inputs: Set of flights $\mathcal{F}$, set of carriers $\mathcal{A}$, set of available slots $\mathcal{S}$, Carriers' preference lists: $PList_1, PList_2, ..., PList_K$ also P_H and carriers set $A_1 = \{a \in A : P_1 \succ_a P_2,\ FS_a \geq P_H\}$.
- **Step 0b:** Calculate the fair share of each airline FS_a based on PRA
- **Step 0c:** Calculate P_L based on 5.1 and 5.2

Step 1: PHASE 1 while $A_1 \neq \emptyset$ Do:

- **Step 1a:** $\forall a \in A_1$, Randomly choose an $a^* \in A_1$ in proportion to FS_{a^*}.
- **Step 1b:** From $PList_{a^*}$, assign the best slot available to the highest priority flight $(f*, s*)$

Step 1c: $FS_{a^*} = FS_{a^*} - P_H$, $PList_{a^*} = PList_{a^*} - \{f^*\}$ and $\mathcal{S} = \mathcal{S} - \{s*\}$ and $A_1 = \{a \in A_1 : FS_a \geq P_H\}$.

end while

Step 2: PHASE 2

Step 2a: $A = \{a \in A : FS_a > 0\}$.

Step 2b: for all a in A, $FS_a = FS_a/P_L$.

Step 2c: Run PBPRA.

The modified version of DP-PRA is MDP-PRA and in step 2-c we can run M-PBPRA instead of PBPRA.

In the first phase of algorithm we consider just carriers in A_1 who can afford a slot with value of P_H. A carrier will be chosen randomly based on its fair share, FS_{a^*}. Then from $PList_{a^*}$ we assign the best slot available to the highest priority flight, f^*. Assign f^* to s^* then remove f^* from $PList_{a^*}$ and s^* from $\mathcal{S}$. We reduce the fair share of a^* by P_H. We repeat this phase until A_1 becomes empty. Now, we move to the second phase.

In the second phase of the algorithm all airlines with positive fair share will be considered. The value of each slot in the second phase is P_L. We make the value of each slot one and increase the fair share of all airlines by $1/P_L$. Then, we execute PBPRA or M-PBPRA. A carrier will be chosen randomly in proportion to its fair share. From $PList_a$ the highest priority flight from carrier a will be chosen. Carrier a's fair share will be reduced by one.

Flights	f_{1A}	f_{1B}	f_{2A}	f_{2B}	f_{3A}	f_{3B}	f_{4A}	f_{1C}	f_{4B}	f_{2C}	f_{5A}	f_{3C}
Airline	A	B	A	B	A	B	A	C	B	C	A	C
a_f	3:58	4:00	4:01	4:02	4:03	4:03	4:05	4:06	4:07	4:08	4:09	4:09

Table 5.1: Flight schedules of airline A, B and C

A	B	C
$(f_{2A}, s_2 : s_3)$	$(f_{1B}, s_1 : s_3)$	$(f_{1C}, s_4 : s_5)$
$(f_{3A}, s_3 : s_4)$	$(f_{2B}, s_2 : s_4)$	$(f_{2C}, s_5 : s_6)$
$(f_{4A}, s_4 : s_6)$	$(f_{4B}, s_5 : s_6)$	(f_{3C}, s_6)
$(f_{1A}, s_1 : s_6)$	$(f_{3B}, s_3 : s_6)$	
(f_{5A}, s_6)		

Table 5.2: Preference list for airlines A, B and C

5.3.4 Example

Suppose we have 12 flights belonging to three airlines A, B and C. There are 6 slots available which means there is 50% capacity reduction. Table 5.1 shows the flights of three airlines and their scheduled arrival times at the boundary of the FCA. The available time slots are:

Slot:	s_1	s_2	s_3	s_4	s_5	s_6
Time:	4:00	4:02	4:04	4:06	4:08	4:10

Table 5.2 shows the flight priority of each airline. As we explained in chapter 3, the fair share of each airlines can be computed : $FS_A = 2.71$, $FS_B = 2.43$ and $FS_C = 0.86$.

Among these three airlines A and B choose policy P_1 while C selects policy P_2. This means A and B prefer to receive priority in order to receive fewer slots but less delay. In other words, C is not interested to receive priority instead it prefers to receive more slots. Assume the price of each slot in the first round is 2. Both airlines have fair shares greater than two, then $A_1 = \{A, B\}$. , so A and B each can afford one slot. There are four slots available which will be allocated in the second round. We can compute the value of remaining slots are$P_L = \frac{6-4}{6-2} = 0.5$

In first phase of the algorithm, A and B participate. In the first round we choose randomly A or B proportional to their fair share, so:

$$P(A) = \frac{2.71}{2.71 + 2.43} = 0.53 \qquad P(B) = \frac{2.43}{2.71 + 2.43} = 0.47$$

Suppose A is chosen, so $FS_A = 2.71 - 2 = 0.71$. From $PList_A$ the highest priority flight, f_{2A}, receives slot s_2. We update the table and remove any $(*, s_2)$. Now, B is the only airline that can afford a slot. Thus, we will assign s_1 to f_{1B}. The fair share of B is reduced by 2, $FS_A = 2.43 - 2 = 0.43$. We remove s_1 from any tuple $(*, s_1)$ in the preference table. Now we move to the second phase. We can run PBPRA and M-PBPRA. First, we adjust the fair shares by P_L:

$$FS_A = \frac{0.71}{0.5} = 1.42 \qquad FS_B = \frac{0.43}{0.5} = 0.86 \qquad FS_C = \frac{0.86}{0.5} = 1.72$$

If we run PBPRA: As explained, we only take into account the fractional part, $N_{frac} = 2$. Suppose B and C are chosen. We assign available slots to the highest priority flights of B and C. Thus, f_{2B} is assigned to s_3 and f_{1C} goes to s_4. The fair share of both airlines reduced. In the second part, the integer part of the fair shares are considered. Therefore, A and C are considered. We first assign s_5 and then s_6. Suppose A is chosen first and then C is chosen. Thus, f_{4A} and f_{2C} receive s_5 and s_6 respectively.

Slots	s_1	s_2	s_3	s_4	s_5	s_6
RBS	f_{1A}	f_{1B}	f_{2A}	f_{2B}	f_{3A}	f_{3B}
PBPRA	f_{1A}	f_{2A}	f_{1B}	f_{1C}	f_{4B}	f_{5A}
M-PBPRA	f_{1A}	f_{1B}	f_{3A}	f_{2B}	f_{1C}	f_{1A}
DP-PRA	f_{1B}	f_{2A}	f_{2B}	f_{1C}	f_{4A}	f_{2C}
MDP-PRA	f_{1B}	f_{2A}	f_{3A}	f_{1C}	f_{4B}	f_{2C}

Table 5.3: Flights-Slots assignments based on execution of different procedures

If instead of PBPRA, we run M-PBPRA. We start from s_3, A and B can use s_3 while A has fair share of greater than one. Thus,we assign f_{3A} to s_3 and remove any tuple $(*, s_3)$ form the table. The fair share of A is reduced by one so $FS_A - FS_A - 1 = 0.42$. The next slot is s_4, C is the only airline that can use and afford s_4 so, f_{1C} goes to s_4 and $FS_A - FS_A - 1 = 0.72$. For s_5, three airlines can use the slot (none of them has fair share ≥ 1 and we consider all of them).One of the airlines is chosen with probabilities:

$$P(A) = \frac{0.42}{0.42 + 0.86 + 0.72} = 0.21 \qquad P(B) = \frac{0.86}{2} = 0.43 \qquad P(B) = \frac{0.72}{2} = 0.36$$

Suppose B is chosen, therefore from $PList_B$ we assign f_{4B} to s_5. The fair share of B is reduced to zero. The last slot, s_6 is chosen by lottery between A and C. Suppose C receives the slot. Thus, f_{2C} goes to s_6.

Table 5.3 provides the flights to slots assignment for the five procedures DP-PRA, MDP-PRA, PBPRA, M-PBPRA and RBS. As you can see, in both DP-PRA and MDP-PRA A and B receive their first priority while, in the other three procedures, the airlines receive the second best preferences. In all proposed procedures C received one slot, while in RBS it does not receive any slots.

We will see in the experimental section of this chapter that DP-PRA and MDP-PRA assign fewer slots but less delays to airlines in A_1 who prefer to receive priority. In the other words, these two procedures assign more slots to airlines in $A - A_1$ who are interested to receive more slot instead of receiving priority.

5.4 Equity of DP-PRA

As explained in the previous chapter, we evaluate the equity of a procedure using three criteria : *fairness, efficiency* and *incentive*. One parameter to measure the *fairness* is called *Equal Treatment Of Equals*. Below, we discuss which equity principals are met by DP-PRA or MDP-PRA.

As we said, there are two sets of airlines. The first set of carriers, A_1, includes carriers who prefer to receive priority and receive fewer slots but less delay. The other set, $A - A_1$, includes carriers who are not interested to receive priority instead they prefer to receive more slots. Carriers must have sufficient budget ($FairShare \geq P_H$) to be considered in the first set. Without loss of generality, suppose $A_1 = \{a_1, ..., a_L\}$. The number of slots that will be assigned to each carrier in the first phase of algorithm is $m_{i,1} = (FS_i - (FS_i \ mod^* \ P_H))/P_H$. Therefore, for each $a \in A_1$ the first $m_{i,1}$ flight(s) in its priority list would be assigned slots in the first phase. Thus, the total number of slots assigned in the first phase is $m_1 = \sum_{i=1}^{L} m_{i,1}$. Formally, we can write $Q(F)$, the set of all deterministic allocations as:

$$Q(F) = \left\{ \begin{array}{l} x \in \{0,1\}^{K \times m} : \sum_{a \in A} x_{ij} \leq 1 \quad \forall j \in \mathcal{S} \\ \qquad\qquad\qquad\qquad \lfloor \frac{FS_i}{P_L} \rfloor \leq \sum_j x_{i,j} \leq \lceil \frac{FS_i}{P_L} \rceil \quad \forall i \notin \ A_1 \\ m_{i,1} + \lfloor \frac{FS_i \ mod^* \ P_H}{P_L} \rfloor \leq \sum_j x_{i,j} \leq m_{i,1} + \lceil \frac{FS_i \ mod^* \ P_H}{P_L} \rceil \quad \forall i \in A_1 \end{array} \right\}$$

Denote by $\Pi_i \in Q(F)$, one such allocation. Here, we assume that all slots can be assigned, or in the other words our problem is strictly non-decomposable, which means $\sum_{a \in A} x_{ij} = 1 \quad \forall j \in \mathcal{S}$.

The first constraint assigns one flight to one slot, the second constraint, assign flights to the carriers that prefer more slots (not belonging to set A_1). As explained, in the second phase of the algorithm the fair share of all carriers increased by factor of $1/P_L$. The last constraint is for carriers in A_1: they must receive $m_{i,1}$ slots during the first phase of algorithm, the remaining slots are assigned in the second phase.

Our procedure chooses each deterministic assignment with a probability. To see the efficiency of the algorithms, we have to show that every deterministic assignment is Pareto-Optimal.

Proposition 5.4.1 *In a strictly non-decomposable problem , all deterministic assignments in $Q(F)$ are Pareto-optimal.*

Proof Suppose carrier a prefers Π' to Π. Without loss of generality, suppose all carriers have the same allocation in Π and Π' except carrier b. We investigate three possible cases:

First case: $a, b \in A - A_1$. If $\Pi' \succ_a \Pi$ then two options are possible: 1- a has $\lfloor FS_a/P_L \rfloor$ in Π and it receives one more slot in Π'. Since our problem is strictly non-decomposable, therefore. there must be another carrier, b, to loose a slot. Then $\Pi' \not\succeq_b \Pi$. 2- If a exchanges slot s_j, in allocation Π, with s_i, where $t_i < t_j$. Therefore, $\Pi' \succ_a \Pi$. Since our problem is strictly non-decomposable, it means all slots have been used, then b has to loose earlier slot s_i and receive later slot s_j. Thus, b does not prefer Π' to Π.

Second case: $a \in A_1,\ b \in A - A_1$. In order for a to prefers allocation Π' to Π, three options are possible. 1- a receives one more slot in Π'. In that case, since the number of slots it receive in the priority is known, $m_{a,1}$, then a must receives one more slots in the second phase of algorithm. Since, our problem is strictly non-decomposable then b has to loose a slot. Therefore, $\Pi' \not\succeq_b \Pi$. 2- If a exchanges a slot with b. Say, slot s_j is exchanged with s_i, such that $\Pi' \succ_a \Pi$. In this case, since a is in high priority set, if s_j is obtained by a in the first phase, then it is not possible to exchange s_j with any slot of b, since b is not in the priority set, and obtain a better allocation. If s_j is obtained in the second phase, then in order to receive a better allocation, s_i has to be an earlier slot. Therefore, b has to loose s_i and receive s_j, since the problem is strictly non-decomposable. Thus, $\Pi' \not\succeq_b \Pi$.

Third case: $a, b \in A_1$. In this case two options are possible in order to $\Pi' \succ_a \Pi$: 1- a has $\lfloor FS_a/P_L \rfloor$ slots in Π and it receives one more slot in Π'. Because the number of slots a receives in the first phase is known, $m_{a,1}$, thus, a has to receive the extra slot in the second phase. Our problem in strictly non-decomposable therefore, b has to loose a slot. Thus $\Pi' \not\succeq_b \Pi$. 2- a receives s_i in Π' instead of s_j in Π. a prefers Π' to Π, therefore $s_i \succ_a s_j$. Thus, if s_i is a slot that a receives in the first phase in stead of s_j then, it means that b has to loose s_i. But b prefers s_i, because in allocation Π, it chooses s_i, therefore $\Pi' \not\succeq_b \Pi$. If s_i is in the set of slots allocate in the second phase, then s_i has to be an earlier slot in order to $\Pi' \succ_a \Pi$. We know that our problem is strictly non-decomposable, then b has to loose an earlier slot s_i and receive s_j. Therefore, $\Pi' \not\succeq_b \Pi$. •

Both DP-PRA and MDP-PRA are randomized procedures which means they choose each deterministic allocation with a probability. Therefore:

Corollary 5.4.1 *Both DP-PRA and MDP-PRA meet ex-post efficiency.*

It is clear that both DP-PRA and MDP-PRA are *anonymous* procedures, which means they assign flights to slots without taking into account the name of carriers. And also it is clear that the *ex-ante Equal Treatment of Equals* property is held by both of these procedures. However, behavior of DP-PRA and MDP-PRA on total slot value assigned to a carrier and also ex-post property which deals with the actual total number of slots assigned to the carrier are not clear or straight forward. In the following, we explain and investigate these two in more details.

It is well known that the stronger form of ETE is ex-post. Validity of the ex-post ETE property for a procedure implies if two carriers have equal fair share that belong to the same set, then there should not be more than one slot difference in the actual total number of slots the carriers will receive based on the procedure. Both DP-PRA and MDP-PRA procedures are maintaining the ex-post ETE property and it is proved in the next proposition.

We already defined concept of slot values, P_H and P_L, where P_H is assigned to a slot while P_L is calculated based on the procedure. Now, the actual total slot value for a carrier after using the procedure can be calculated based on wether slot value is P_H or P_L, and based on actual total number of slots received by that carrier. Both DP-PRA and MDP-PRA procedures are providing an interesting behavior on the actual total slot value for a carrier.

After applying any of DP-PRA and MDP-PRA, then for any two carriers with equal fair share the difference in actual total slot value for two carriers with the same fair share will be less than an upper bound of $2P_L$. To be more precise, if two carriers with equal fair share belong to the same set, then the difference in actual total slot value for each carrier is less than P_L. And if two carriers with equal fair share belong to two different sets, then the difference in actual total slot value for each carrier is less than $2P_L$. This means after applying any of DP-PRA and MDP-PRA procedures, then actual total slot value for airlines with equal fair share will be in reasonable bound. In the next proposition, we first formulate this behavior, and then we prove it.

If Y_i and V_i are the actual number of slots and actual total slot value a carrier receives then it follows :

Theorem 5.4.1 *In a strictly non-decomposable problem for two carriers a and b with the same fair share:*

(a) $a,\ b \in A_1\ \ |Y_a - Y_b| < 1$ *and also* $a,\ b \in A - A_1\ \ |Y_a - Y_b| < 1$

(b) $a,\ b \in A_1$ *or* $a,\ b \in A - A_1$ *then* $|V_a - V_b| \leq P_L$

(c) $a \in A$, $a \in A - A_1$ *then* $|V_a - V_b| \leq 2P_L$

where P_L *is the value of slots in the second phase of DP-PRA and MDP-PRA (note* $P_L < 1$*).*

<u>Proof</u>

Proof (a)- If we consider carriers a and b with the same share, $FS_a = FS_b$, in the set A_1 then the number of slots they receive in the first phase is equal to $m_{a,1} = (FS_a - (FS_a\ mod^*\ P_H))/P_H$ and the remainder of their share would be equal for the second phase. In the second phase, since the problem is strictly non decomposable, then all slots have been assigned. Consequently, based on Theorem 4.4.1, if Y_i is the number of slots that a carrier i receives in the second phase then:

$$|Y_a - Y_b| \leq 1$$

Proof (b) and (c)- In terms of the value of slots that an airline receives, if :

(i) $a \in A_1$ then $V_a = P_H \times m_{a,1} + Y_a \times P_L$

(ii) $a \in A - A_1$ then $V_a = Y_a \times P_L$

If two carriers belong to the same set, it can be concluded from *(a)* that $|V_a - V_b| < P_L$. But if a and b belong to different sets then : $V_a = P_H \times m_{a,1} + Y_a \times P_L$ and $V_b = Y_b \times P_L$. If we substitute the value of $m_{a,1}$ then we have $V_a = (FS_a - (FS_a\ mod^*\ P_H)) + Y_a \times P_L$. Define $R_a = (FS_a\ mod^*\ P_H)$.

In the second phase the number of slots that carrier a would definitely receive is: $m_{a,2} = (R_a - (R_a\ mod^*\ P_L))/P_L$ and for carrier b is $m_{b,2} = (FS_b - (FS_b\ mod^*\ P_L))/P_L$ if we substitute these two values we have:

$$V_a = (FS_a - R_a) + R_a - (R_a\ mod^*\ P_L) + Y'_a \times P_L$$
$$V_b = FS_b - (FS_b\ mod^*\ P_L) + Y'_b \times P_L$$

where Y' is a random variable with the value of 0 or 1. Therefore:

$$|V_a - V_b| = |Y'_a \times P_L - (R_a\ mod^*\ P_L) - Y'_b \times P_L + (FS_b\ mod^*\ P_L)|$$

If $Y'_a = Y'_b$ then $|V_a - V_b| = |(R_a\ mod^*\ P_L) - (FS_b\ mod^*\ P_L)|$. If $|Y'_a - Y'_b| = 1$ then:

$|V_a - V_b| = |P_L \pm ((R_a\ mod^*\ P_L) - (FS_b\ mod^*\ P_L))|$

$\leq P_L + |(R_a\ mod^*\ P_L) - (FS_b\ mod^*\ P_L)| \leq 2P_L$. •

As we can see, if two carriers have the same fair share and belong to the same set then *ex post equal treatment of equals* holds. However, there will be a difference in the value the they receive. This difference however is guaranteed to be small.

We now need to investigate the degree to which our algorithms encourage truthfulness relative to preference revelation. That is the degree to which carriers can manipulate preferences in order to receive a better allocation. We explained *strategy-proofness* in the previous chapter. Since we have a random allocation procedure, strategy proofness means that a carrier can not receive an allocation that stochastically dominates the current allocation by using non-truthful preferences.

Proposition 5.4.2 *In a strictly diverse non-decomposable problem both DP-PRA and MDP-PRA procedures meet strategy proofness.*

<u>Proof</u> Assume m_1 is total number of slots that are going to be assigned in the first phase. And suppose $a \in A_1$ wants to manipulate its preference while other airlines are truthful. Moreover, assume that a actually prefers s_i. There are there different cases that we discuss strategy proofness for them as follow.

Case 1: $a \in A$ and a falsifies its preference in the first phase. If a does not say its true preference in the first phase, for example $s_i \succ_a s_j$, but a falsifies its preference by $s_j \succ^*_a s_i$. Now in executing DP-PRA and MDP-PRA procedures, if

a is chosen in the first phase, then a receives s_j instead of s_i. So, a's fair share is reduced by value of P_H. Considering set of flight-to-slot is strictly diverse non-decomposable, so more than one airline competes for any slot. After receiving s_j by a at first phase, a has less faire share for next round if it is chosen by the procedures. Consequently, a's chance for wining s_i is decreased since a has to compete with some other airlines on slot s_i.

Case 2: $a \in A$ and a falsifies its preference in the second phase. The proof is similar to Theorem 4.4.1.

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5.5 Experimental Results

In our experiment, we use the same test data set as previous chapter. This data set that had been employed by the CDM Future Concepts Team to perform human-in-the-loop experiments related to SEVEN. It contained 386 flights with 38 flight operators. The data included scheduled arrival arrival times at an FCA boundary. The FCA duration was from 18:00 pm to 21:00 pm. As we explained in previous chapter a flight cost function can be generated as:

$$C(x,P) = \begin{cases} 0 & x \leq 15 \\ (32+0.1P)(x-15) & 15 < x \leq M_p \\ (32+0.1P)(M_p-15) & x > M_p \end{cases}$$

Where M_p is flight specific max delay. Given the cost function, we generated the priority list for each flight operator based on all available flights that could use a slot; and the assumption is that the flight operator preferred allocating the slot to the flight with the highest marginal cost of delay. The flight operators are randomly assigned to set A_1, the ones who prefer to receive better slots, or $A - A_1$, flight operators who prefer to receive more slots.

We compared the results of DP-PRA and MDP-PRA against ration-by-schedule (RBS), which is currently used to allocate FCA access during airspace flow programs. In our experiment, we considered 40%, 50%, 60%, 70% and 80% en-route capacity reduction for the FCA. We performed 2000 repetitions of the procedure since both procedures are random. In first part of all of our experiment we set $P_H = 2$. We

% Capacity reduction	List of Airlines	Number of slots
40	{1,3,5,6,21,25,28,29,34}	45
50	{1,5,6,21,25,28,29,34}	37
60	{1,5,6,21,25,29,34}	27
70	{1,5,6,21,25,29,34}	21
80	{1,21,29}	12

Table 5.4: List of airlines that can participate in the first phase and the number of slots are assigned

will show later the effect of changing P_H. For each capacity reduction, the number of carriers that can participate in the first phase of algorithm is different. It is clear that as capacity increases the fair share of each airline increases, consequently the number of airlines that can participate will increase as well. Airlines 1, 3, 5, 6, 7, 9, 17, 19, 20, 21, 25, 26, 28, 29, 30, 31, 34, 35 have the second policy. Table 5.5 shows

Number of Slot Allocated to Each Flight Operator, DPPRA and MDP-PRA																									
	80% Capacity Reduction					70% Capacity Reduction					60% Capacity Reduction					50% Capacity Reduction					40% Capacity Reduction				
	DPPRA		MDP-PRA			DPPRA		MDP-PRA			DPPRA		MDP-PRA			DPPRA		MDP-PRA			DPPRA		MDP-PRA		
Airlines	# Slots	Slot Value	# Slots	Slot Value	Fair Share	# Slots	Slot Value	# Slots	Slot Value	Fair Share	# Slots	Slot Value	# Slots	Slot Value	Fair Share	# Slots	Slot Value	# Slots	Slot Value	Fair Share	# Slots	Slot Value	# Slots	Slot Value	Fair Share
1	3.34	5.12	3.33	5.11	5.08	4.60	7.25	4.62	7.26	7.28	5.63	9.29	5.63	9.29	9.32	6.60	11.23	6.96	11.50	11.2	7.31	13.00	8.00	13.52	12.9
3	0.64	0.53	0.62	0.52	0.58	1.21	0.95	1.22	0.95	0.9	1.59	1.26	1.59	1.26	1.27	2.16	1.65	2.19	1.68	1.65	1.12	2.09	1.04	2.03	2.05
5	2.00	1.67	2.00	1.67	1.67	1.68	2.53	1.69	2.53	2.6	3.16	3.70	3.15	3.70	3.67	3.02	4.78	3.04	4.79	4.79	4.49	5.89	4.29	5.74	5.92
6	1.65	1.38	1.68	1.40	1.48	1.36	2.28	1.35	2.27	2.24	2.40	3.10	2.41	3.11	3.06	3.46	3.88	3.46	3.88	3.86	2.76	4.58	3.00	4.76	4.59
21	7.38	11.9	7.39	12.00	12	10.34	17.82	10.32	17.81	17.8	13.24	23.77	13.24	23.77	23.7	15.78	29.36	16.00	29.52	29.5	18.39	35.05	19.00	35.52	35
25	2.04	1.70	2.04	1.71	1.7	1.70	2.55	1.70	2.55	2.62	3.09	3.65	3.09	3.65	3.63	2.74	4.57	2.72	4.55	4.66	4.13	5.62	4.07	5.57	5.62
28	1.14	0.95	1.14	0.95	0.92	1.70	1.32	1.68	1.31	1.38	2.33	1.83	2.31	1.82	1.78	1.16	2.12	1.28	2.21	2.09	1.53	2.40	2.00	2.76	2.3
29	5.20	9.00	5.19	8.99	8.97	7.43	13.11	7.41	13.10	13.1	9.30	17.02	9.30	17.02	17	10.76	20.58	10.99	20.76	20.7	12.40	24.30	13.00	24.76	24.2
34	2.18	1.82	2.16	1.80	1.78	1.78	2.61	1.76	2.59	2.76	3.37	3.87	3.35	3.85	3.81	3.22	4.93	3.23	4.94	4.9	4.52	5.91	4.28	5.73	5.9
2	1.70	1.42	1.69	1.42	1.55	2.70	2.10	2.70	2.10	2.2	3.56	2.81	3.56	2.81	2.82	4.42	3.37	4.47	3.42	3.41	4.72	3.58	5.00	3.80	3.95
4	0.17	0.14	0.17	0.14	0.11	0.28	0.22	0.30	0.24	0.17	0.37	0.29	0.36	0.29	0.26	0.52	0.40	0.50	0.38	0.36	0.69	0.52	0.36	0.27	0.49
7	0.36	0.30	0.35	0.29	0.26	0.56	0.44	0.56	0.44	0.42	0.66	0.52	0.69	0.54	0.62	1.10	0.84	1.14	0.87	0.85	1.44	1.09	1.26	0.96	1.11
8	2.41	2.01	2.43	2.03	2	3.55	2.76	3.57	2.78	2.78	4.37	3.45	4.37	3.45	3.39	5.06	3.87	5.19	3.97	3.94	5.40	4.10	5.43	4.13	4.45
9	0.15	0.12	0.14	0.12	0.09	0.25	0.19	0.24	0.18	0.15	0.34	0.27	0.35	0.28	0.23	0.49	0.37	0.44	0.34	0.33	0.64	0.49	0.32	0.25	0.44
10	3.00	2.51	3.00	2.51	2.51	4.74	3.69	4.73	3.68	3.82	6.56	5.18	6.59	5.20	5.2	8.42	6.43	8.57	6.55	6.56	9.87	7.49	10.22	7.76	7.88
11	0.29	0.24	0.30	0.25	0.21	0.46	0.36	0.47	0.36	0.33	0.57	0.45	0.57	0.45	0.47	0.73	0.56	0.68	0.52	0.62	0.92	0.70	1.01	0.76	0.78
12	0.25	0.21	0.25	0.21	0.18	0.41	0.32	0.41	0.32	0.28	0.52	0.41	0.53	0.42	0.4	0.67	0.51	0.65	0.50	0.54	0.82	0.62	0.47	0.35	0.69
13	9.58	8.01	9.59	8.02	8.04	15.18	11.83	15.19	11.83	11.8	19.52	15.40	19.52	15.40	15.4	24.15	18.46	24.45	18.69	18.7	27.28	20.72	28.26	21.46	21.6
14	1.22	1.02	1.22	1.02	0.99	2.04	1.59	2.03	1.58	1.57	2.74	2.16	2.73	2.15	2.24	3.66	2.80	3.72	2.84	2.95	4.47	3.39	4.45	3.38	3.72
15	7.60	6.36	7.60	6.35	6.41	12.22	9.52	12.22	9.52	9.48	16.02	12.64	16.02	12.64	12.6	20.25	15.48	20.68	15.80	15.9	23.77	18.05	25.19	19.13	19.2
16	0.44	0.36	0.45	0.38	0.36	0.67	0.53	0.67	0.52	0.58	1.08	0.85	1.10	0.86	0.84	1.52	1.16	1.50	1.15	1.15	1.69	1.28	1.52	1.15	1.52
17	0.35	0.30	0.36	0.30	0.26	0.55	0.43	0.55	0.43	0.4	0.65	0.51	0.62	0.49	0.56	0.77	0.59	0.74	0.57	0.7	0.92	0.70	1.00	0.76	0.84
18	0.03	0.03	0.02	0.01	0.02	0.04	0.03	0.03	0.03	0.03	0.07	0.05	0.05	0.04	0.04	0.11	0.08	0.09	0.07	0.06	0.21	0.16	0.07	0.05	0.09
19	0.22	0.18	0.21	0.18	0.15	0.37	0.29	0.37	0.29	0.24	0.48	0.38	0.46	0.37	0.34	0.62	0.47	0.59	0.45	0.47	0.80	0.61	0.42	0.32	0.61
20	0.04	0.04	0.03	0.02	0.03	0.08	0.07	0.06	0.05	0.05	0.11	0.08	0.12	0.09	0.07	0.20	0.15	0.18	0.14	0.11	0.31	0.24	0.13	0.10	0.15
22	2.28	1.90	2.26	1.89	1.87	3.35	2.61	3.36	2.62	2.56	4.21	3.32	4.19	3.31	3.28	5.23	4.00	5.34	4.08	4.04	6.09	4.62	6.25	4.75	4.84
23	0.33	0.28	0.33	0.27	0.23	0.49	0.38	0.50	0.39	0.37	0.64	0.51	0.65	0.51	0.55	0.97	0.74	1.00	0.77	0.77	1.34	1.02	1.22	0.93	1.03
24	1.34	1.12	1.32	1.11	1.09	2.22	1.73	2.23	1.74	1.69	2.75	2.17	2.77	2.18	2.35	3.91	2.99	4.01	3.07	3.06	4.51	3.43	4.47	3.39	3.77
26	0.26	0.22	0.26	0.22	0.18	0.41	0.32	0.42	0.33	0.29	0.54	0.43	0.55	0.43	0.41	0.69	0.53	0.65	0.49	0.56	0.83	0.63	0.48	0.36	0.7
27	0.15	0.13	0.14	0.12	0.1	0.26	0.21	0.25	0.20	0.16	0.36	0.28	0.37	0.30	0.24	0.50	0.38	0.47	0.36	0.34	0.69	0.52	0.33	0.25	0.46
30	0.54	0.46	0.58	0.48	0.48	0.76	0.59	0.75	0.58	0.74	1.36	1.07	1.39	1.09	1.03	1.65	1.26	1.65	1.26	1.33	1.87	1.42	2.00	1.52	1.61
31	0.31	0.26	0.29	0.24	0.21	0.46	0.36	0.48	0.37	0.33	0.59	0.46	0.58	0.46	0.46	0.71	0.54	0.69	0.53	0.61	0.84	0.64	0.49	0.37	0.76
32	9.65	8.07	9.67	8.08	8.16	15.33	11.94	15.34	11.95	11.9	19.74	15.58	19.71	15.55	15.7	25.07	19.16	25.53	19.51	19.5	29.03	22.04	30.30	23.01	23.2
33	0.07	0.06	0.08	0.07	0.05	0.15	0.12	0.14	0.11	0.08	0.19	0.15	0.20	0.16	0.13	0.32	0.24	0.29	0.22	0.18	0.47	0.36	0.21	0.16	0.26
35	0.10	0.08	0.10	0.08	0.06	0.18	0.14	0.17	0.13	0.1	0.26	0.20	0.25	0.20	0.16	0.37	0.29	0.34	0.26	0.23	0.55	0.42	0.26	0.20	0.32
36	8.74	7.31	8.74	7.31	7.51	14.76	11.50	14.76	11.50	11.6	20.43	16.12	20.44	16.13	16.1	26.78	20.47	27.34	20.90	20.9	31.88	24.21	33.41	25.37	25.7
37	0.22	0.19	0.23	0.20	0.16	0.38	0.29	0.40	0.31	0.25	0.48	0.38	0.48	0.38	0.36	0.63	0.48	0.61	0.46	0.5	0.79	0.60	0.43	0.33	0.64
38	0.66	0.55	0.65	0.54	0.62	1.36	1.06	1.36	1.06	1.01	1.75	1.38	1.75	1.38	1.49	2.61	2.00	2.65	2.03	2.08	3.38	2.57	3.35	2.55	2.76

Table 5.5: Comparison of Average number of slots and average slot values received by each carrier

the airlines and number of flights (or slots) that are assigned in the first phase for each capacity reduction. The fair share of each airlines and the number of slots they received is shown in table 5.5. The first part of table (above the bold line) shows the airlines who like to receive less delay.

% Capacity reduction	DP-PRA	MDP-PRA
40	18.19	21.21
50	15.72	17.11
60	11.69	11.53
70	9.71	9.15
80	6.78	5.79

Table 5.6: Comparison of cost reduction for DP-PRA and MDP-PRA vs. RBS

We should note that for 40% capacity reduction our problem is not strictly non-decomposable. Table 5.5 shows the percentage of cost savings for the two algorithms when compared to RBS. If we compare this with the result of PBPRA or MPB-PRA from previous chapter, we notice that the cost saving is almost the same. But if we compare the total delay saving (Table 5.5) , DP-PRA and MDP-PRA are better than PBPRA and M-PBPRA.

The main advantage of DP-PRA or MDP-PRA compared to previous procedures is to meet carriers' preference better. Figure 5.1(a) shows the average number of slots carriers in A_1 receives compare to previous procedures for 60% capacity reduction. We can see the comparison of delay in figure 5.1(b). As we can see, airlines

% Capacity reduction	DP-PRA	MDP-PRA
40	50.36	51.85
50	47.67	45.81
60	37.96	36.70
70	36.44	35.27
80	33.55	31.8

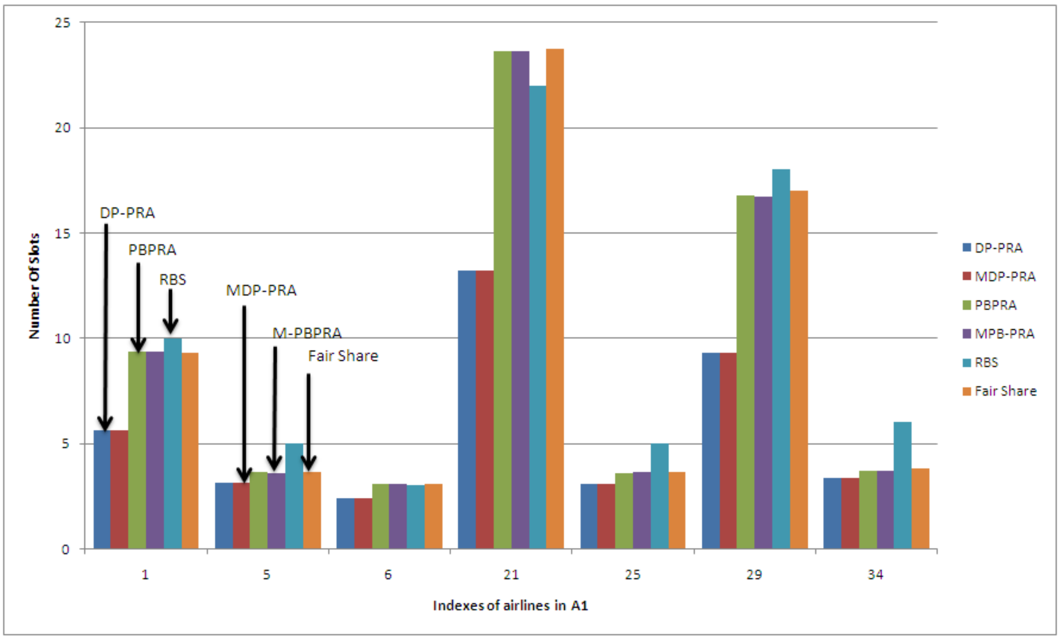
Table 5.7: Comparison of delay reduction for DP-PRA and MDP-PRA vs. RBS

in A_1 save more delay and in return they receive fewer slots.

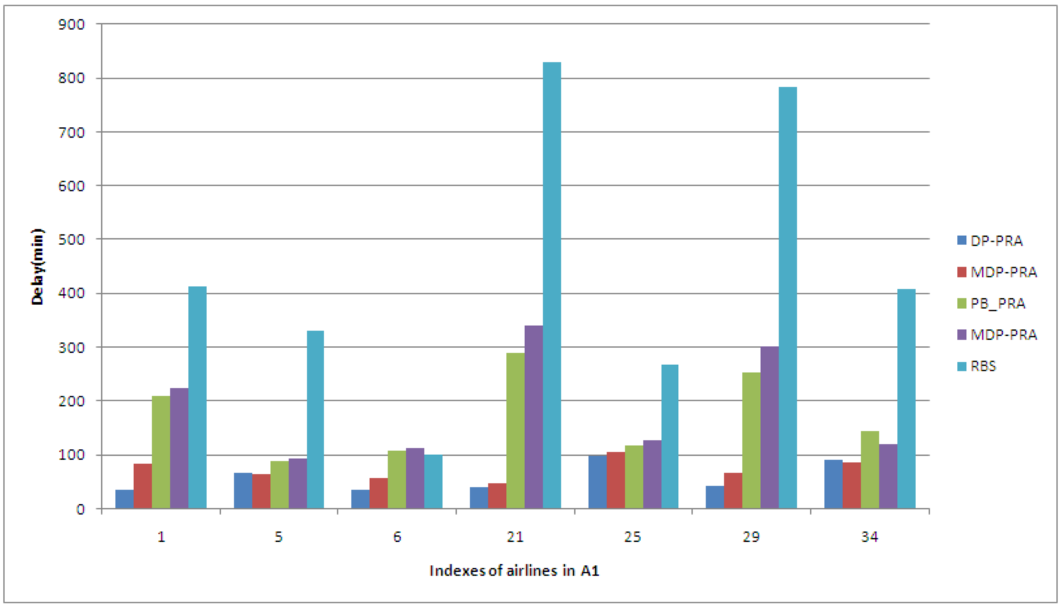
5.5.1 Effect of P_H

So far we have used $P_H = 2$ in all of our experiments. Here we want to investigate the effect of P_H in overall performance of DP-PRA and MDP-PRA. Choosing the right P_H is a challenge for the FAA. There can be many different performance criteria; for example, deviation from carriers' fair share, total internal cost, how many slots should be assigned in the first phase. Here we explain the effect of P_H on some of performance criteria. In all of our examples we consider 40% capacity reduction in enroute resources.

As we expect, when P_H increases the number of carriers in A_1 decreases. Also, the number of slots assigned in the first phase decreases as well (Figure 5.2). For example, if the FAA decides to assign 25% or 15% of available slots in the first phase then P_H must be chosen 1.5 or 2.5 respectively (the solid line in the figure). As can



(a)



(b)

Figure 5.1: (a) Comparison of number of slots received for airlines in A_1. (b) Comparison of delay for the airlines in A_1 for all procedures

be seen, the number of airlines is considered in the first phase is reduced from 8 to 7 airlines.

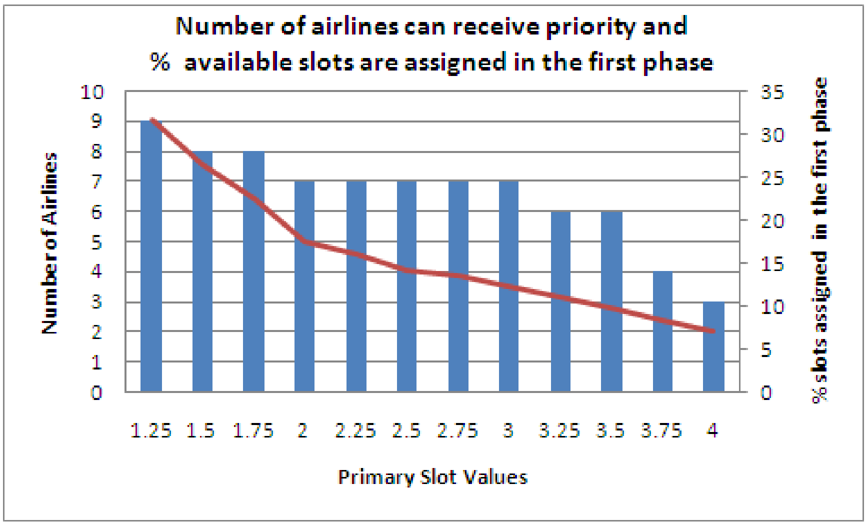


Figure 5.2: Effect of primary slot values on the number of slots assigned in the first phase, the bars show number of airlines that can participate in the first phase

The second performance criteria is to minimize the total internal cost. This is very hard for the FAA to measure because each carrier's cost information is private. Figure 5.3 shows the total internal cost of airlines for different primary slot values. As it can be seen we have two local minima: one occurs at at the value of 1.25 for DPPRA and 1.5 for MDP-PRA; the second local minimum occurs at $P_H = 3.25$ and $P_H = 3.0$ for DPPRA and MDP-PRA respectively.

The FAA can also consider the deviation from fair share as a one criteria. Figure 5.4(a) shows the total define Minimum Square Error (MSE) of slot values from carriers' fair share. As can be seen, a minimum occurs at $P_H = 2.75$ and $P_H = 3.5$ for both procedures.

The other criterion is that, those carriers who are not in A_1 they should receive more slots instead. The second graph (on the left vertical axis of Figure 5.4(b))

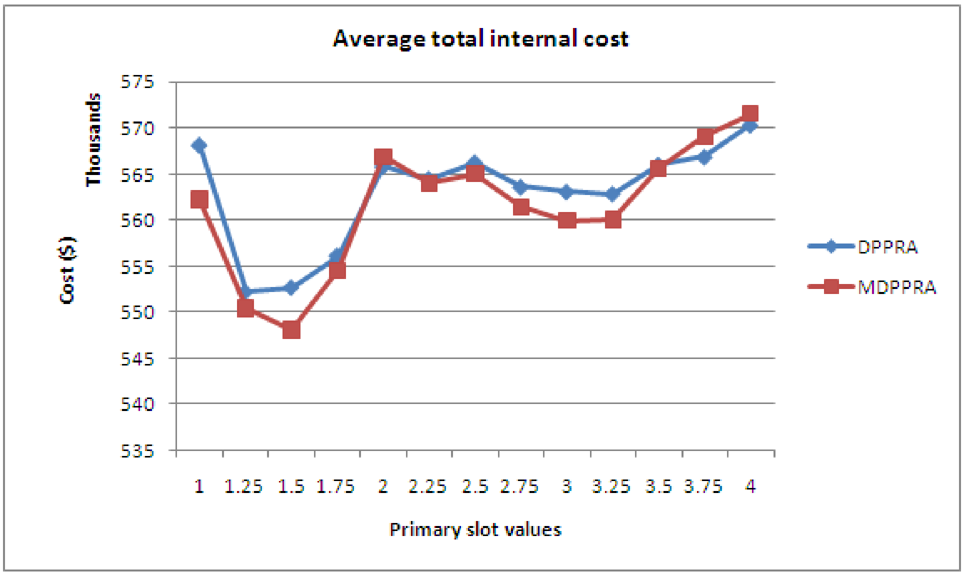


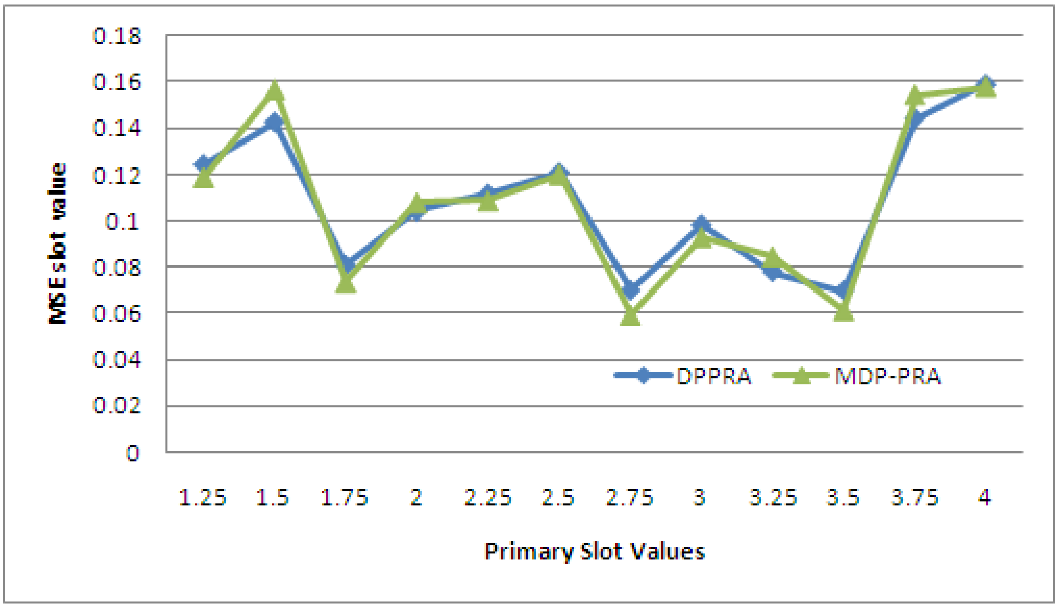
Figure 5.3: Effect of primary slot values on total internal cost

shows the total MSE of the difference between the number of slots and the carriers' fair share (carriers who are not in A_1 are considered). The maximum happens at $P_H = 3$ for both procedures.

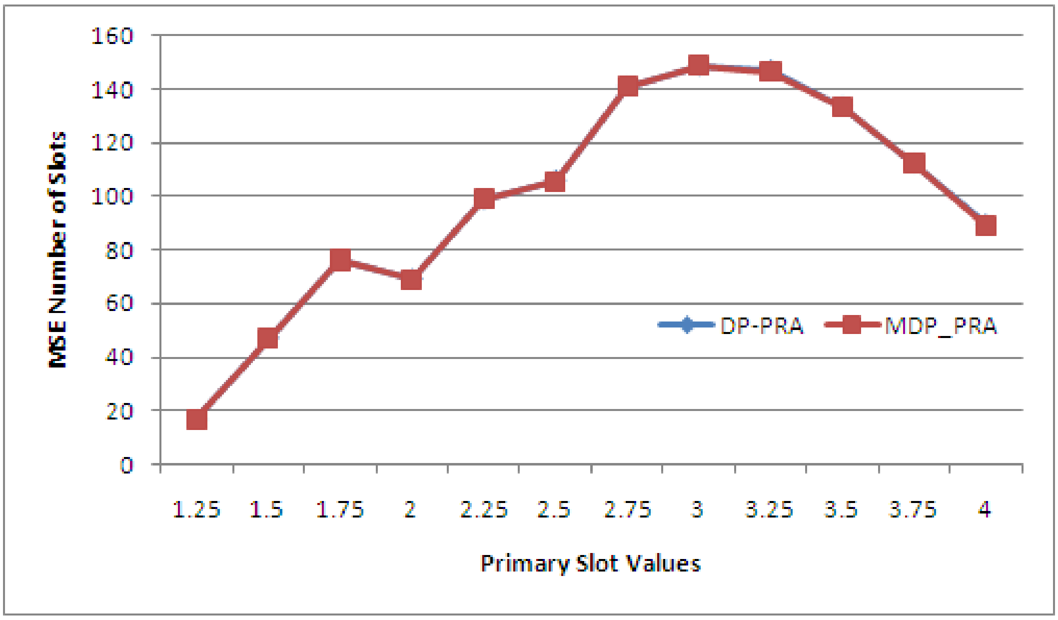
We can also consider the combination of all criteria together. In this case maybe choosing P_H between 3 to 3.5 is a good choice.

5.6 Discussion

In this section a new procedure for slot allocation has been proposed. Unlike PBPRA and M-PBPRA that assigns the same value to all slots. In DP-PRA and MDP-PRA, we consider two values for slots. The main goal is to address carriers' preferences better. As mentioned, carriers who wish to give priority to certain flights, may be willing to pay more for some particular slots. In our procedures, we use two prices for slots. Airlines, who wish to receive "premium" slots could do so but would



(a)



(b)

Figure 5.4: (a) Effect of primary slot values on MSE of slot values. (b) Effect of primary slot values on MSE of number of slots from fair share for airlines in $A - A_1$

be charged more for each premium slot they receive. Our procedures meets ETE, efficiency and strategy proofness. As it was shown in our experiment carriers can meet their preferences better than previous algorithms. A challenge here is to decide about the primary slot values. There can be different criteria that must be decides by the FAA.

Chapter 6

Conclusion

This research was motivated by fair allocation of scarce resources among flight operators, agents, especially when there is competition among agents. We specifically looked at problems that arise, when, due to bad weather, there is a capacity reduction in a part of the airspace for a period of time and it is not possible for all flights who are scheduled during that period of time to pass through constrained area. Therefore, some of flights must be rerouted or receive a departure delay.

Each airline will typically place a different relative weight on delays, rerouting and cancellation. Whereas some airlines would like to preserve on-time performance for certain flights and cancel or reroute many other flights, other airlines prefer to have less rerouting and cancellations while tolerating higher total delay. Therefore a key challenge is to determine how many slots each airline should receive and how we can include airlines' preferences while maintaining fairness.

We proposed a new rationing procedure that is based on Proportional Random Assignment. In contrast to RBS that gives access only to the earliest flights, the new rationing algorithm considers all flights that are disrupted by an AFP. Therefore, small carriers, which have fewer flights or carriers whose flights are scheduled late, can still have a share of available slots. The new method for computing a fair share of available resources implicity gives a larger share to the flights that are scheduled earlier. However, no flight can receive more than one slot as its fair share. We use the fair share of carriers as an input to our random allocation procedures. Therefore, carriers with positive share will have chance to receive slots. Our approach for computing a fair share achieves principles such as impartiality, equal treatment of equals, consistency and demand monotonicity.

In chapter 4 of this dissertation, we proposed randomized methods, Preference Based Proportional Random Allocation (PBPRA) and Modified PBPRA, that use an exogenous fair share as a parameter to assign flights to slots. Carriers are entitled to receive a number of slots based on their fair share. It is guaranteed that any carrier receives at least the floor of its fair share. These methods give carriers whose flights are scheduled late a chance of receiving a slot. Another main advantage of the new algorithms is to include carriers' preferences. Carriers' can express their preferences over slots and these preferences considered during the allocation process. We also explored the principles of our allocation procedures. We showed that PBPRA and M-PBPRA meet equity principles and also have ex-post efficiency and strategy proofness properties. Also, we tested our algorithms on real data. Our algorithms showed improved performance compare to the RBS. In PBPRA and M-PBPRA, *expected total number of slots* that a carrier receives is very close to its fair share.

In PBPRA (or M-PBPRA), although slot preferences were employed we implicity assumed that all slots had equal values. In reality, some slots are worth more tan others. Those carriers that would like to maintain on time performance for key flights may be willing to pay more than others for particular slots. It is typically the case that carriers have higher preference for earlier slots, while the later slots are less favored. Thus, it does make sense that earlier slots, which have more demand, have more weight than later slots, which are less preferred. In both algorithms, we did not elicit preferences related to the trade off between delay and rerouting. We addressed this problem with Dual Price Proportional Random Allocation(DP-PRA).

In DP-PRA and MDP-PRA, we consider two values for slots. The main goal is to meet carriers' preferences better. As mentioned, some carriers who like to maintain their on time performance for certain flights may be willing to pay more for some particular slots. Airlines who wish to receive better slots for select flights can do so, but in return are charged more for such slot they receive. Our procedures meets ETE, efficiency and strategy proofness. As it was shown in our experiments, carriers can meet their preferences better than previous algorithms. Also, carriers receive the *expected total value of slots* very close to their fair share.

In this dissertation, weather conditions are considered constant during the AFP period. A potential research is to investigate resource allocation considering dynamic weather. In reality, weather can change during the time horizon and so there is a change in available resources as well. To much more efficient use of available resources, it can be better to have a dynamic allocation that can adapts over time.

Bibliography

[1] Airline operational control overview, prepared by Airline Dispatchers Federation and Seagull Technology, Inc., Cooperative Research and Development Agreement 93-CRDA-0034, September 1995.

[2] A. Abdulkadiroglu, T. Sonmez. Ordinal efficeincy and dominated sets of assignment. *Journal of Economic Theory*, 112:157-172, 2003.

[3] A. Abdulkadiroglu, T. Sonmez. Random serial dictatorship and the core from random endowments in house allocation problems. *Econometrica*, 66: 689-701, 1998.

[4] M. Adams, S. Kolitz, J. Milner, and A. Odoni. Evolutionary concept for decentralized air traffic management. *Air Traffic Control Quarterly*, 4:281-306, 1997.

[5] A. Alkan, G. Demange, and D. Gale. Fair allocation of indivisible goods and criteria of justice. *Econometrica*, 59:1023-1039, 1991.

[6] G. Andreatta, A. Odoni, and O. Richetta. Models for the ground holding problem. In L. Bianco and A.R. Odoni, editors, *Large-Scale Computation and Information Processing in Air Traffic Control*, pages 125-168. Springer-Verlag, Berlin, 1993.

[7] G. Andreatta and G. Romanian-Jacur. Aircraft flow management under congestion. *Transportation Science*, 21:249-253, 1987.

[8] M. Ball, C. Barnhart, G. Nemhauser and A. Odoni, Air Transportation: Irregular Operations and Control, *Handbook of Operations Research and Management Science: Transportation*, C. Barnhart and G. Laporte, eds, Elsevier, Amsterdam, pp 1-67, 2007.

[9] M. Ball, C-Y. Chen, R. Hoffman, and T. Vossen. Collaborative decision making in air traffic management: Current and future research directions. In *Proceedings of ATM'99 Workshop on Advanced Technologies and Their Impact on Air Traffic Management in the 21st century*. 1999.

[10] M. Ball, R. Hoffman, W. Hall, and A. Muharremoglu. Collaborative decision making in air traffic management : a priliminary assessment. NEXTOR Research Report #RR-99-3, UC Berkeley. 1999.

[11] M. Ball, R. Hoffman, D. Knorr, J. Wetherly, and M. Wambsganss. Assessing the benefits of collaborative decision making in air traffic management. In *Proceedings of 3rd USA/Europe Air Traffic Management R&D Seminar*. 2000.

[12] M. Ball, R. Hoffman, A. Mukherjee. Ration-by-distance with equity guarantees: A new approach to ground delay program, em the 7th ATM RD Seminar,2007.

[13] M. Ball, R. Hoffman, A. Odoni, and R. Rifkin. A Stochastic Integer Program with Dual Network Structure and its Application to the Ground-Holding Problem, *Operations Research*, 51: 167 171, 2003.

[14] M.L. Balinski and H.P. Young. *Fair Representation: Meeting the Ideal of One Man, One Vote*. Yale University Press, 1982.

[15] J. Bard, G. Yu, and M.F. Arguello. A time-band optimization model for the irregular operations problem. *IIE Transportations on Operation Engineering*, 33:931-947, 2001.

[16] A. Bogomolnaia and H. Moulin. A new solution to the random assignment problem. *Journal of Economic Theory*, 100:295-328, 2001.

[17] A. Bogomolnaia and H. Moulin. Collecective choice under dichotomous preferences. *Journal of Economic Theory*, 122:165-184, 2005.

[18] S. J. Brams. Fair Division. Mimeo, New York University, Department of Political Science, 2005.

[19] S. J. Brams and A.D. Taylor. Fair Division: from Cake-Cutting to Dispute Resolution, Cambridge University Press, 1996.

[20] S. J. Brams and A.D. Taylor. The Win-Win Solution: Guaranteeing Fair Shares to Everybody. Norton & Co. 1999.

[21] S. Bratu and C. Barnhart, An analysis of passenger delays using flight operations and passenger booking data, em Air Traffic Control Quarterly, 13, 2005.

[22] I. Bezakova and V. Dani. Allocating indivisible goods. *ACM SIGecom Exchanges*, 5: 11-18, 2005.

[23] H. Cres and H. Moulin. Random priority: A probabilistic resolution of the tragedy of the commons. *Operations Research*, 49: 565-577, 2001.

[24] J.M. Cao and A. Kanafani. Real-time decision support for integration of airline flight cancellations and delays, part i: Mathematical formulation. *Transportation Planning and Technology*, 20: 183-199, 1997.

[25] J.M. Cao and A. Kanafani. Real-time decision support for integration of airline flight cancellations and delays, part ii: Algorithm and computational experiments. *Transportation Planning and Technology* , 20: 201-217, 1997.

[26] P. Cramton. Spectrum auctions. In M. Cave, S. Majumdar, I. Vogelsang, eds. *Handbook of Telecommunications Economics*. Elsevier Science B.V., Amesterdam, The Netherlands. 2002.

[27] A. Demers, S. Keshav, and S. Shenker. Analysis and simulation of fair queuing algorithm. *Internetworking: Res Experience*, 1: 3-26, 1990.

[28] L. Ehlers. Probablistic allocation rules and single-dipped preferences. *Social Choice and Welfare*, 19: 325-348, 2002.

[29] L. Ehlers and B. Klaus. Solidarity and probablistic target rules. *Journal of Public Economic Theory*, 3: 167-184, 2001.

[30] L. Ehlers, H. Peters, and T. Storcken. Strategy-proof proablistic decision schemes for one dimensional single-peaked preferences. *Journal of economic theory*, 105: 408-434, 2002.

[31] J. Elster. *Local Justice*. Russle Sage Foundation, New York, 1992.

[32] FAA's NextGen. http://www.faa.gov/.

[33] FCA/Rerout Workgroup. Flow Evaluation Area/Flow Constraint Area(FEA/FCA) Operational Concepts. http://cdm.fly.faa.gov/.

[34] N. Gaertner, M. Klopfenstein, G. Wilmouth. Updated Operational Concept for System Enhancements for Versatile Electronic Negotioation(SEVEN). Metron Aviation. September 2007.

[35] S. Grandeau, M. Clarke, and D. Mathaisel. The processes of airline system operations control. In G. Yu, editor, *Operations Research in the Airline Industry*, pages 312-369. Kluwer Academic Publisher. 1998.

[36] D. Hoitomt, D. Kraay, and B. Tang. Flight sequencing under the FAA's simplified substitution rules. Presentation. AGIFORS Operations, Joint Study Group Meeting, Control Study and Ground Resources. April 1999.

[37] Iata. Worldwide scheduling guidelines. International Air Trasport Association, 3rd Edition, December 2000.

[38] A. Katta and J. Sethurman. A solution to the random assignmnet problem on the full preference domain. *Journal of Economic Theory*, 131: 231-250, 2006.

[39] W. Kureishi and H. Mizukami. Equal probability for the best and the assignment of identical indivisible objects. *Economics Bulletin*, 4: 1-10, 2007.

[40] S. Luo and G. Yu. Airline schedule perturbation problem: Ground delay program with splittable resources. In G. Yu, editor, *Operations Research in the Airline Industry*, pages 404-432. Kluwer Academic Publishers, 1998.

[41] H. Luss. On equitable resourse allocation problem: A lexicographic minmax approach. *Operation Research*, 47: 361-378, 1999.

[42] A. McLennan. Ordinal efficiency and polyhedral separating hyperlane theorem. *Journal of Economic Theory*, 105: 435-449, 2002

[43] Metron Aviation. www.metronaviation.com/airspace-programs.php

[44] H. Moulin. The proportional random allocation of indivisible units. *Social Choice and Welfare*, 19: 381-413, 2002.

[45] H. Moulin. Axiomatic cost and surplus-sharing. Mimeo, Rice University, 2001.

[46] H. Moulin and R. Stong. Fair queueing and other probabilistic allocation methods. *Mathematics of Operations Research*, 27:1-30, 2002.

[47] H. Moulin and R. Stong. Filling a multicolor urn: an axiomatic analysis. *Game and Economic Behavior*, 45:242-269, 2003.

[48] T. Niznik. Optimization the airline response to ground delay program. Proceedings of AGIFORS Airline Operations Meeting, 2001.

[49] A. Odoni. The flow management problem in air traffic control. In A.R. Odoni, L.Bianco, and G. Szego, editors, *Flow Control of Congested Networks*, 269-288. Springer-Verlag, Berlin, 1987.

[50] A. Odoni and T.P. Fan. The potential of demand management as a short-term means of relieving airport congestion. In *ATM-2001 EUROCONTROL-FAA Seminar, Santa Fe, NM.* 2001.

[51] B. O'Neill. A prolem of rights arbitration from the talmud. *Mathematical Social Sciences*, 2:345-371, 1982.

[52] S. Rassenti, V. Smith, and R. Buffin. A combinatorial auction mechanism for airport time slot allocation. *Bell Journal of Economics*, 13: 402-417, 1982.

[53] O. Richetta. Optimal algorithms and a remarkably efficient heuristic for the ground-holding problem in air traffic control. *Operation Research*, 43: 758-770, 1995.

[54] O. Richetta and A. Odoni. Solving optimally the static ground holding policy problem in air traffic control. *Transportation Science*, 24: 228-238, 1993.

[55] Y. Sprumont. The division problem with single-peaked preferences: a characterization of uniform allocation rule. *Econometrica*, 59: 509-519, 1991.

[56] L. Svensson. Strategy-proof allocation of indivisible goods. *Social Choice Welfare*, 16: 557-567, 1999.

[57] A. Tasnadi. On probablistic rationing methods. *Mathematical Social Science*, 44: 211-221, 2002.

[58] D. Teodorovic and S. Guberinic. Optimal dispatching strategy on an airline network after a schedule perturbation. *Europe Journal of Operations Research*, 15: 178-182, 1984.

[59] D. Teodorovic and S. Guberinic. Model for operational daily airline scheduling. *Transportation Planning and Technology*, 14: 273-285, 1990.

[60] M. Terrab. *Ground Holding Strategies for Air Traffic Control*. PhD Thesis, Massachusetts Institute of Technology, Department of Civil Engineering, 1990.

[61] M. Terrab and A. Odoni. Strategic flow management for air traffic control. *Operation Research*, 41:138-152, 1993.

[62] B. Thengvall, J. Bard, and G. Yu. Multiple fleet aircraft schedule recovery following hub closures. *Transportation Research, Part A.*, 35: 289-308, 2001.

[63] B. Thengvall, J. Bard, and G. Yu. Balancing user preferences for aircraft recovery during airline irregular operations. *IIE Transactions on Operations Engineering*, 32: 181-193, 2000.

[64] W. Thomson. Monotonicity, stability, and egalitarianism. *Mathematical Social Sciences*, 8: 15-28, 1984.

[65] W. Thomson. The Consistency principle. In T. Ichiishi, A. Neyman, and Y. Tauman, editors, *Game Theory and Applications*, pages 187-215. Academic Press, New York, 1990.

[66] N. Vakili Pourtaklo, M. Ball. Equitable allocation of enroute airspace resources. *Eighth USA/Europe Air Traffic Management Research and Development Seminar*, 2009.

[67] L. G. Valiant. The complexity of enumeration and reliability problems. *SIAM Journal of Computing*, 8: 410-421, 1979.

[68] A. Vasquez-Marquez. American airlines arrival slot allocation system. *Interfaces*, 21:42-61, 1991.

[69] T.W.M. Vossen *Fair Allocation Methods in Air Traffic Management.* PhD Thesis, University of Maryland, R.H. Smith School of Business, 2002.

[70] T.W.M. Vossen, M. Ball. Slot trading opportunities in collaborative ground delay programs. *Transportation Science*, 40: 29-43, 2006.

[71] T.W.M. Vossen, M. Ball. Optimization and mediated bartering models for ground delay programs. *Naval Research Logistics*, 53: 75-90, 2005.

[72] T.W.M. Vossen, M. Ball, R. Hoffman, and M. Wambsganss, A General Approach to Equity in Traffic Flow Management and its Application to Mitigating Exemption Bias in Ground Delay Programs, *Air Traffic Control Quarterly* 11, 277-292, 2003.

[73] M. Wambsganns. Collaborative decision making through dynamic information transfer. *Air Traffic Control Quarterly,* 4: 107-123, 1996.

[74] H.P. Young. *Equity in Theory and Practice.* Princeton University Press, 1994.

[75] H.P. Young. Distruptive justice in taxation. *Journal of Economic Theory,* 48: 321-335, 1988.

[76] H.P. Young. Progressive taxation and equal sacrifice. *American Economic Review*, 80: 253-266, 1990.

[77] L. Zhou. On a conjecture by Gale about one-sided matching problems. Journal of Economic Theory, 52: 123-135, 1990.