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Research paper

Nonlinear analysis for proposing limit state criteria of reinforced concrete road bridge superstructures

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Abstract: In the research of long-time operating road bridge superstructures, it should note that when the internal forces in the beam elements reach specific values, the stiffness of the cross-section of these elements should decrease. Besides that, if there are damaged places in the beam-element of the road bridge superstructures, the element could not work normally, and the redistribution of internal forces between elements in the whole system would happen. This phenomenon was not taken into account in the initial design calculation. In practice, it shows that many road bridges are subjected to greater loads than the calculated loads in the design process, but they still normally operate. This article proposes the other limit state criteria in evaluating the load capacity reserve of road-bridge superstructures using nonlinear analysis based on nonlinear deformational models of modern construction codes. The proposed calculation procedure is established to explain the load capacity reserve of long-time operating road bridge superstructures in the case of the lack of experimental evaluation. From the obtained results, the suitable limit state criteria for road bridge superstructures are suggested, and the conclusions about the accuracy of the proposed approach of nonlinear structural analysis are recommended.

Keywords: limit state criteria, nonlinear analysis, road bridge superstructure, nonlinear deformational model, reinforced concrete element, pre-stressed reinforced element

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1. Introduction

Many operating road bridges were built from the 1950s onwards in Russia and Vietnam, these road bridges with superstructures designed according to the typical drawings with precast reinforced concrete as “Issue 1956”, “Issue 1956-D” and precast pre-stressed reinforced concrete with as “Issue 1947”, “Issue 1011”, “Issue 122-62” [1]. These typical drawings were published due to the requirements to speed up the construction progress in the historical moment in Russia and Vietnam. The operating time of these road bridges can now be approximately 50–70 years. According to statistics, if comparing the value of the used live load in a period that these designs came into practice with the current value of the live load, the current live load has increased more than two times [2, 3]. However, these above-mentioned road bridges are still in a normal working state. The elements of road bridges, such as their superstructures, are required to test the load-carrying capacity, but that has not been done simultaneously for all road bridges over the country in Russia and Vietnam. The article deals with the nonlinear analysis according to the nonlinear deformational models to propose the other evaluation criteria of limit states of road-bridge superstructures. From there, it explains the load-carrying capacity reserve of the superstructures, which is subjected to a higher load than the designed load in the case of a lack of experimental tests. The new evaluation criteria of limit states based on the nonlinear deformational analysis proposed in this research are especially effective in the case of that the superstructure has been damaged at the places of one or several its elements because it takes into account the stiffness decrease of elements of the superstructure. From the application of the nonlinear deformational analysis method in road bridge superstructure, the scope of application extended to the beam systems in the building structure [4, 5].

In addition, the nonlinear analysis of road-bridge structures has been studied by many researchers. In [6], the authors applied nonlinear models for T-beam to calculate the shear strength outcomes and in [7] the research determined the force-displacement relations for a reinforced concrete beam-column type element, the determination of this relationship needs to be computed internally at each step of the numerical integration process. In [7], the localized plasticity approach is used in which the nonlinear material properties are considered at the endpoints of the elements. In [8], application of programming software in calculation of bridge structures allows to static analysis without any difficulty. In [9, 10] the nonlinearity is considered in the analysis of bridge structures. From these researches, it is shown that with consideration of nonlinearity, the analysis results converge with the observed behavior of the bridge structures and make them more economical. In research [11–18], the authors have studied many types of road bridge superstructures considering the nonlinearity of material and used the nonlinear deformational models for compressive concrete and reinforcement, pre-stressed reinforcement in the analysis. In this article, besides the nonlinear deformational models in [19], the nonlinear stress-strain relationship of compressive concrete [20] is used to compare the obtained results. From there, it can be to verify the accuracy of the proposed nonlinear approach by authors in road bridge superstructures. Besides, in this research, the authors applied an iterative algorithm for applying the load process of structural analysis of road-bridge superstructures.

Furthermore, according to the common limit state, calculate the load-carrying capacity of any structure and road-bridge superstructures in particular; it is required to determine the calculated values of internal forces in their elements. After that, compare these values with the critical values of internal forces respectively in the most considered dangerous cross-sections of the elements. However, this approach is often used in the calculations of the design process. In the case of operating structures such as road-bridge superstructure that are subjected to a higher load than the design load and has elements with damaged places, this approach shows several disadvantages

- In the cross-section of an element, the bending moment of which achieved the critical value, the plastic hinge is formed in this section, this beam-element cannot continue to carry loading, but it does not mean that the whole system has reached the limit state;
- When the bending moment of the elements reaches specific values, the stiffness in the section gradually decreases. It will have happened the redistribution of the bending moment in the whole system of the structure. This leads to different results than those calculated in the design process's initial period. This article presents the evaluation criteria and calculation procedure based on the nonlinear deformational models of concrete and steel according to construction codes [19] and [20].

The calculation procedure of the proposed approach in this article consists of two steps:

- Establishment of relationships between parameters such as an algebraical sum of bending moments in considered sections, bending curvature, cross-section stiffness of elements, and strain of compressive concrete. In this step, the nonlinear deformational models according to [19] and [20] will be applied;
- Finite element analysis for road-bridge superstructure with gradually increasing reference lane of AK moving loads. This step considers the relationship between the algebraical sum of bending moments in the elements and the cross-section stiffness of elements, which was obtained in the previous step. From the calculation results in this step, the other evaluation criteria of limit state for road-bridge superstructures are proposed.

2. Nonlinear deformational models and nonlinear structural analysis

2.1. Nonlinear deformational models

Previously, the construction codes used models based on stress. When the internal forces and stresses in the elements of the structural system reach critical values, the elements cannot continue to carry loading. When Russian codes SP 52-101-2003 [21], SP 52-102-2004 [22], later version [19] and [20] were released, allowing nonlinear structural analysis based on stress-strain relations. According to these documents, the nonlinear deformational models describe the stress-strain state at any period of the loading process and consider the element stiffness decreasing when the load increases in the system. The stress-strain relations considered deformational models, which are presented in Fig. 1. In which the 3-stage line describes the stress-strain relation of concrete according to 6.1.19, 6.1.20, 6.2.14 of [19], while the curve describes the stress-strain relation of concrete according to 3.1.5 of [20].

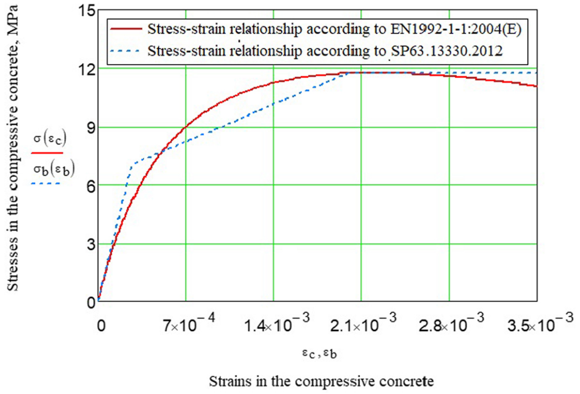


Fig. 1. The comparison diagram of stress-strain relations of materials for nonlinear analysis according to [19] and [20]

In Fig. 1, it presented the comparison between the stress-strain relations according to [19] and [20]. From the Fig. 1, it shows that the stress-strain relations of [19] and [20] are relatively similar with each other.

2.2. Approach of nonlinear structural analysis

In this research, the nonlinear structural analysis for beams of the road-bridge superstructures is problem of establishment of relationships between different parameters, which are strains of compressive concrete ε_b, bending curvature of cross-section ρ, bending moment M, cross-section stiffness $M/(1/\rho)\ldots$ The relation between section stiffness $M/(1/\rho)$ and bending moment M is $M/(1/\rho) = f(M)$ that obtained from this step is used into the second step of the analysis to describe the change of section stiffness when the load increases. In this research, it is studied the increase of loading from the value of internal forces that corresponds with static load to the value of internal forces that corresponds with the ultimate loading.

2.2.1. Formulation of nonlinear analysis for beam elements

a) Theoretical background

In order to establish the above-mentioned relations, it is necessary to determine the strains ε_b, ε_s, ε_{sc}, ε_p, of compressive concrete, tensile, compressive and pre-stressed reinforcement relatively and the corresponding stresses σ_b, σ_s, σ_{sc}, σ_p in accordance with the given variable strain of compressive concrete ε_{b0}. These strains and stresses distribute according to linear function by the height of the element as presented in Fig. 2.

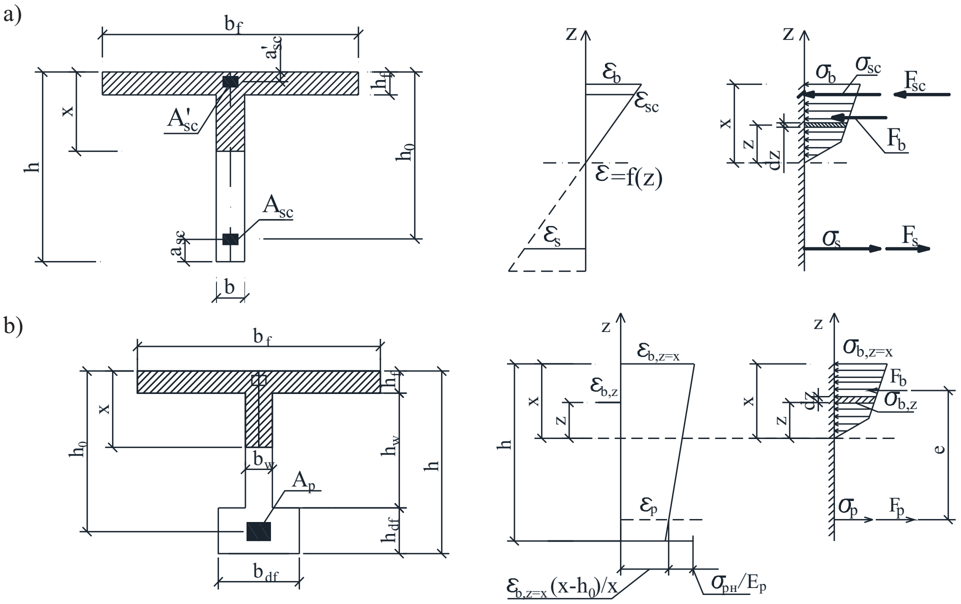


Fig. 2. Cross section ad schematic representation for nonlinear structural analysis: a) reinforced concrete; b) pre-stressed reinforced concrete

From the schematic representation in Fig. 2a, 2b, it is determined the forces raised in the compressive zone of concrete, in the tensile and compressive reinforcement, in the pre-stressed reinforcement by following formula [23]:

$$F_b = \int\limits_A \sigma_b \, \mathrm{d}A; \quad F_s = \int\limits_A \sigma_s A_s; \quad F_{sc} = \int\limits_A \sigma_{sc} A_{sc}; \quad F_p = \int\limits_A \sigma_p A_p \tag{2.1}$$

in which $\sigma_b, \sigma_s, \sigma_{sc}, \sigma_p$ – stresses in compressive concrete, tensile, compressive reinforcement, pre-stressed reinforcement; A, $\mathrm{d}A$ – areas of compressive concrete and infinitesimal zone in compressive concrete; A_s, A_{sc}, A_p – areas of tensile, compressive reinforcement, pre-stressed reinforcement.

It is necessary to note that in the pre-stressed reinforced concrete elements, the strain of the pre-stressed reinforcement determined by the following formula:

$$\varepsilon_p = \sigma_{pn}/E_p + \varepsilon_{b,z=x}(x - h_0)/x \tag{2.2}$$

in which σ_{pn} – stress of extensive period of reinforcement taking into account the stress loss; h_0 – effective depth of section; x – height of compressive concrete zone of section.

For each given value of variables $\varepsilon_{b,z=x}$, it is determined only one value of bending moment M and section height of compressive concrete x that satisfy the equilibrium equations for each case:

– for reinforced concrete element:

$$\sum F = F_b + F_s + F_{sc} = 0 \tag{2.3}$$

– for pre-stressed reinforced concrete element [23]:

$$\sum F = F_b + F_p = 0 \tag{2.4}$$

The bending curvature for section element is determined as $1/\rho = \varepsilon_{b,z=x}/x$ for reinforced concrete element and $1/\rho = (\varepsilon_p - \varepsilon_{b,z=x})/h_0$ for pre-stressed reinforced concrete element. Calculating the relationship between 3 parameters $\varepsilon_{b,z=x}$, M and x using Mathcad programming software to determine the strains, the stresses, bending curvature $1/\rho$ and bending stiffness $B = M/(1/\rho)$.

Here, it should note that, the way of calculation of the bending stiffness of cross section of element B is different in two period of load applying:

- For reinforced concrete elements, in the first period of load applying (when $\varepsilon_{b,z=x} \leq \varepsilon_{b1}$), the bending stiffness of cross section $B = M/(1/\rho)$ is constant and equals $E_b I_{\text{red}}$, in which I_{red} is equivalent moment of inertia of cross section and the tensile concrete zone is neglected. After crossing the transition point $|\varepsilon_{b,z=x}| = \varepsilon_{b1}$, $\sigma_{b1} = 0.6R_b$ as shown in the stress-strain relationship of compressive concrete of [19], this period is considered as second period, the bending stiffness of cross section is defined as ratio $M/(1/\rho) < E_b I_{\text{red}}$. The loss of load carrying capacity determined by conditions of which the strains of compressive concrete and reinforcement reached the values ε_{b2}, ε_{s2} as shown in stress-strain relationships of compressive concrete and reinforcement of [19] or the bending stiffness of cross section decreases to zero.
- For the pre-stressed reinforced concrete, the bending stiffness is also considered as $B = M/(1/\rho)$, which is changed in two period of load applying. In the first period, the cross section is compressive completely ($x = h$), the equivalent area A_{red} and equivalent moment of inertia I_{red} of cross section are calculated for the whole section and these parameters are unchangeable at this period. In the second period of load applying ($x < h$), the bending stiffness of cross section is considered as ratio $M/(1/\rho)$ (where $1/\rho = |\varepsilon_{b,z=x}| = (\varepsilon_p - \varepsilon_{b,z=x})/h_0$) which is calculated like the case of reinforced concrete elements.

b) Calculation procedure of establishment of relationships between different parameters for beam-elements:

- Listed input parameters of the analysis: elastic modulus and strength of concrete, reinforcement E_b, E_s, E_{sc}, E_p, R_b, R_s, R_p; geometrical dimensions of the element section b, b_f, h, h_f; parameters of reinforcement A_s, A_{sc}, A_p, a_p, a_s, a_{sc}, h_0; tensile stress of pre-stressed reinforcement σ_{pn}.
- Determine the geometrical characteristics of element section: equivalent area A_{red}, equivalent first moment of area S_{red}, equivalent moment of inertia I_{red}, equivalent section modulus $W_{\text{top\_red}}$, $W_{\text{bot\_red}}$, distances between centre of section and top or bottom edges of section y_{top}, y_{bot};

- Formulate the equation of forces to determine the height of compressive zone of concrete x. However, for pre-stressed reinforcement, it is necessary to calculated in two period as illustrated in [23].
- Write the distribution of stresses of compressive reinforcement, tensile reinforcement and pre-stressed reinforcement $\sigma_b(\varepsilon_{b,z}(\varepsilon_b,x)), \sigma_{sc}(\varepsilon_{sc}(\varepsilon_b,x)), \sigma_s(\varepsilon_s(\varepsilon_b,x)), \sigma_p(\varepsilon_p(\varepsilon_b,x))$, by the height of cross section according to formula in the sections 6.1.19, 6.1.20, 6.2.14 of [19] and 3.1.5 of [20];
- Determine the forces of compressive concrete, tensile, compressive and pre-stressed reinforcement as following:
  - for the reinforced concrete beam-element:

$$
\begin{aligned}
F_b(\varepsilon_{b,z},x) =& b_f \int\limits_0^x \sigma_b(\varepsilon_{b,z})\mathrm{d}z \quad \text{if } x \le h_f\\
F_b(\varepsilon_{b,z},x) =& b_w \int\limits_0^{x-h_f} \sigma_b(\varepsilon_{b,z})\mathrm{d}z + b_f \int\limits_{x-h_f}^{x} \sigma_b(\varepsilon_{b,z})\mathrm{d}z \quad \text{if } h_f < x:
\end{aligned}
\tag{2.5}
$$

$$F_{sc}(\varepsilon_{b,z},x) = \sigma_{sc}(\varepsilon_{b,z},x) \cdot A_{sc} \tag{2.6}$$

$$F_s(\varepsilon_{b,z},x) = \sigma_s(\varepsilon_{b,z},x) \cdot A_s \tag{2.7}$$

  - for the pre-stressed reinforced concrete beam- element like in [23]
- Determine the bending moment of the forces at the neutral axis of the section:
  - for the reinforced concrete beam-element:

$$
\begin{aligned}
M_b(\varepsilon_{b,z},x) =& b_f \int\limits_0^x \sigma_b(\varepsilon_{b,z})z\mathrm{d}z \quad \text{if } x \le h_f\\
M_b(\varepsilon_{b,z},x) =& b_w \int\limits_0^{x-h_f} \sigma_b(\varepsilon_{b,z})z\mathrm{d}z + b_f \int\limits_{x-h_f}^{x} \sigma_b(\varepsilon_{b,z})z\mathrm{d}z \quad \text{if } h_f < x:
\end{aligned}
\tag{2.8}
$$

$$M_{sc}(\varepsilon_{b,z},x) = F_{sc}(\varepsilon_{b,z},x) \cdot (x - a_{sc}) \tag{2.9}$$

$$M_s(\varepsilon_{b,z},x) = M_s(\varepsilon_{b,z},x) \cdot (x + a_s - h) \tag{2.10}$$

$$M_{ex} = \sum M(\varepsilon_{b,z},x) = M_b(\varepsilon_{b,z},x) + M_s(\varepsilon_{b,z},x) + M_{sc}(\varepsilon_{b,z},x) \tag{2.11}$$

  - for the pre-stressed reinforced concrete beam-element like in [23]

$$M_p(\varepsilon_{b,z},x) = F_p(\varepsilon_{b,z},x) \cdot (x - h_0) \tag{2.12}$$

$$M_{ex} = \sum M(\varepsilon_{b,z},x) = M_b(\varepsilon_{b,z},x) + M_p(\varepsilon_{b,z},x) \tag{2.13}$$

$$M_{\sum} = M_{ex} + \sigma_p \cdot A_p(y_{\text{bot}} - a_p) \tag{2.14}$$

- Determine the bending curvature $1/\rho$ and the stiffness $B = M/(1/\rho)$ of particle of beam-element with length $\mathrm{d}l$:

- for the reinforced concrete beam-element:

$$\frac{1}{\rho\left(\varepsilon_{b,z=x},x\right)}=\left|\frac{\varepsilon_{b,z=x}}{x}\right|;\quad B\left(\varepsilon_{b,z=x},x\right)=\frac{M_{ex}}{1/\rho\left(\varepsilon_{b,z=x},x\right)}\tag{2.15}$$

- for the pre-stressed reinforced concrete beam-element:

$$\frac{1}{\rho\left(\varepsilon_{b,z=x},x\right)}=\left|\frac{\varepsilon_{b,z=x}}{x}\right|=\frac{\varepsilon_p-\varepsilon_{b,z=x}}{h_0};\quad B\left(\varepsilon_{b,z=x},x\right)=\frac{M_{\Sigma}}{1/\rho\left(\varepsilon_{b,z=x},x\right)}\tag{2.16}$$

Finally, draw the schematic representation of relationship between above determined parameters (strain of compressive concrete, bending curvature, stiffness of cross section . . .): $M=f(\varepsilon_{b,z})$, $1/\rho=f(M)=f(\varepsilon_{b,z})$; $B=f(\varepsilon_{b,z})$, $B=f(M)$.

Using the above calculation procedure and the proposed mentioned approach of nonlinear structural analysis for beam-element, the authors established the algorithm, wrote the programming routine in Mathcad calculation software for reinforced concrete, and pre-stressed reinforced concrete beam-elements. The results of this routine is applied in the second period of the nonlinear structural analysis of road bridge superstructures.

2.2.2. Nonlinear structural analysis for road-bridge superstructures

The nonlinear structural analysis for road-bridge superstructures is implemented using structural analysis software SCAD Office 21.1. In the analysis, it is applied the frame-shell model as shown in Fig. 3a, 3b. In the analysis, it is necessary to take into account the difference in height of center of beam elements and central axis of road bridge desks. For this purpose, the hard connection is applied as shown in Fig. 3a, 3b [2, 3].

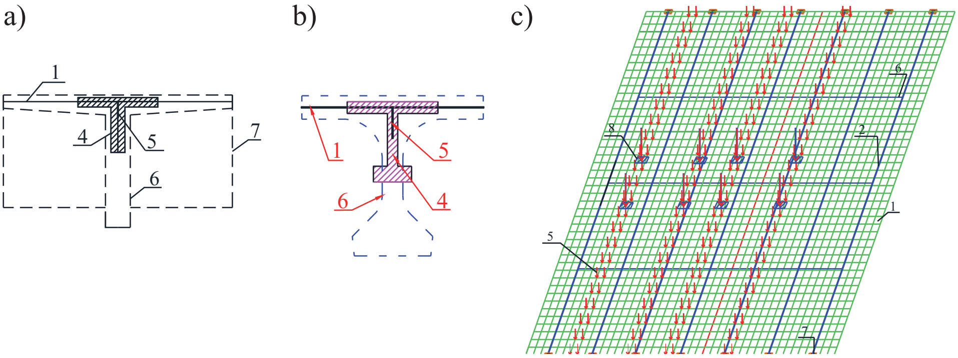


Fig. 3. Schematic representation for structural analysis of road-bridge: a) scheme of frame-shell models for road-bridge superstructures of “Issue 56”; b) scheme of frame-shell models for road-bridge superstructures of “Issue 122-62”; c) model of road-bridge superstructures

The Fig. 3c shows an example of model of road-bridge superstructures that subjected to two lanes of live road loading type AK; it presents the discretization of desks and beam-elements into finite elements, the distribution of AK load lane [2]. The selection of frame-shell model in nonlinear structural analysis of road-bridge superstructure allows presentation of the change in section stiffness $B = M/(1/\rho)$ in the beam-elements according to the relationship $B = f(M)$ obtained from previous step of analysis in the case of that the live load gradually increases.

The loading in the system followed the principle:

- In the first time of loading, in the road-bridge superstructures, it is applied only a part of the live load of AK loading type which corresponding to the compressive concrete stress raised in the middle section of the span of the highest bearing load beam-element $\sigma_{b,z=x} = \sigma_{b1}$;
- In the following times of loading, the unit live load A1 of the AK load added with the previous live load AK for each time, the displacements of the beam-elements recorded repeatedly. The load applied gradually until to the moment of reaching of the limit state. The diagram of relationship between these displacements and the gradually applied AK live load is established.

  In this research, the limit state can be recommended according to following criteria:
- The displacement of the maximum bearing load beam-element in the superstructure reaches to the critical value of displacement in the case of beam-element of road-bridge superstructure;
- The bending moment of maximum bearing load beam-element reaches to the critical value according to limit state of strength;
- The number of formed plastic hinges in road-bridge superstructures;
- The signal of displacement progression in the maximum bearing load beam-element;
- The condition of transformation of the boundary beam into the maximum bearing load beam.

3. Numerical investigation

3.1. Reinforced concrete road-bridge superstructure

In this research, it is implemented the example of nonlinear structural analysis for reinforced road-bridge superstructure of typical drawing Issue 56 with the length of beam 14.06 m (calculated span 13.7 m) and other parameters of dimensions and geometrical characteristics are given in Fig. 4 and Fig. 5.

Material characteristics: Elastic modulus of concrete (M300) $E_b = 25650$ MPa, reinforcement $E_s = 2.06 \cdot 10^5$ MPa; compression strength of concrete $R_b = 11.75$ MPa, reinforcement $R_s = 236$ MPa.

In the Table 1, it is presented the results of nonlinear analysis of reinforced concrete beam with length 14.06 m of Russian typical drawing Issue 56.

- Through Table 1, we could see that the results of distribution of stresses of compressive concrete calculated according to [19] and [20] are relatively converged.

In the Fig. 6, it is presented the results of nonlinear analysis of beam with 14.06m length of Russian typical drawing of Issue 56, which are calculated to nonlinear models according

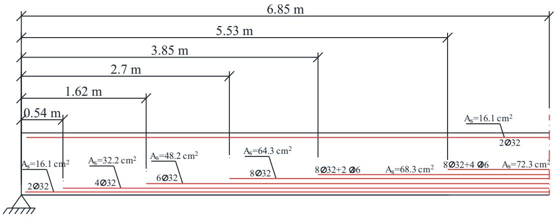


Fig. 4. Schematic presentation of reinforcement along length of 14.06 m beam of typical drawing Issue 56

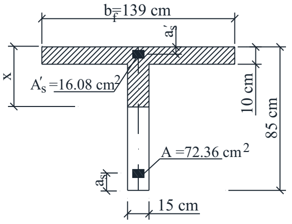
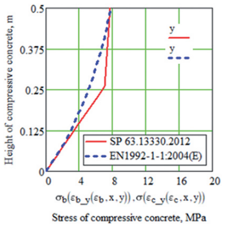
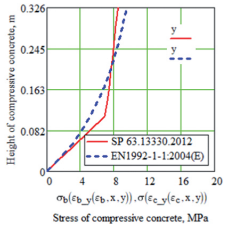
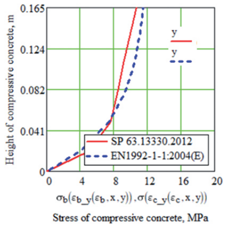
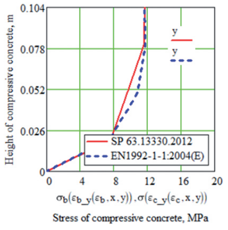


Fig. 5. Cross-section of 14.06 m beam

Table 1. The distribution of stress by height of cross-section of 14.06 m beam of Russian typical drawing Issue 56

$\varepsilon_b = 2.5 \cdot 10^{-4}$	$\varepsilon_b = 8 \cdot 10^{-4}$	$\varepsilon_b = 16 \cdot 10^{-4}$	$\varepsilon_b = 25 \cdot 10^{-4}$
			

to different construction codes. The Fig. 6 shows that the results of calculation according to Russian construction code [19] and [20] are not much different with each other. This can evince that the accuracy of proposed approach of nonlinear analysis of beam elements.

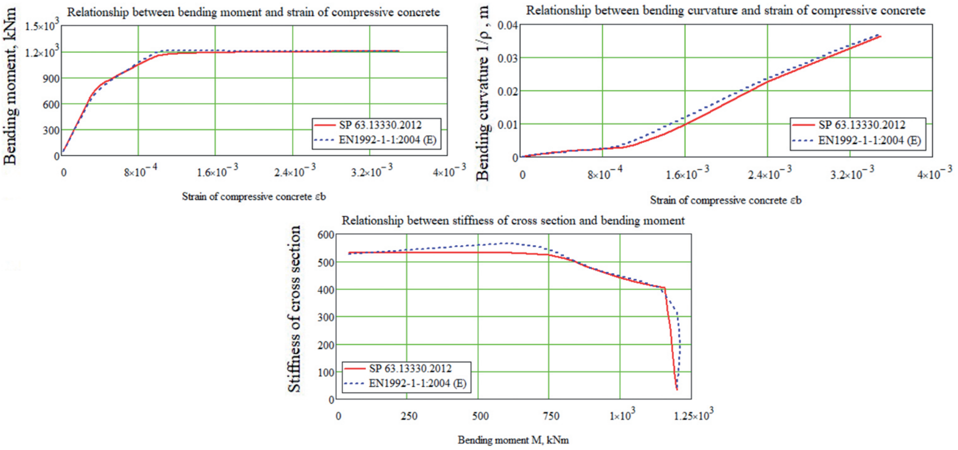


Fig. 6. Results of nonlinear analysis of beam with length 14.06 m of typical drawing of Issue 56

The second period of nonlinear structural analysis of road-bridge superstructures implemented using the results of nonlinear analysis of beam elements with 14.06 m length and dimensions of the superstructure given as shown in Fig. 7.

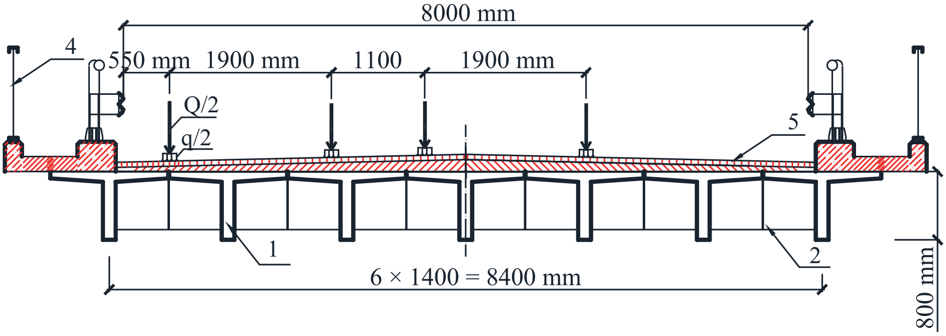


Fig. 7. Cross section of reinforced concrete road-bridge superstructure with length of beam 14.06 m of typical drawing Issue 56: 1 – beam element, 2- diaphragm, 3- metal barriers, 4 – reinforced concrete sides

The authors implemented the nonlinear structural analysis of road-bridge superstructure in the research using the structural analysis software SCAD [2, 3]. The models that used in the research are frame-shell models as above mentioned with gradually applying of AK type load. The Fig. 8 shows some results of analysis, in the Fig. 8a; it presents the changes of bending moment along the length of all beams of road-bridge superstructure. The red lines M_B_{const} of the Fig. 8a presents the values of bending moment corresponding with the constant stiffness of cross section of the beams. When the value of bending moment of beams reaches to the maximum value of bending moment, which corresponds with the value of cross section stiffness, is approximately zero, it is considered to form the plastic hinges in the beams. If in the road-bridge superstructure, it is formed 2 or 3 plastic hinges, the superstructure reaches to limit state by criterion of number of plastic hinges. Besides that, in the Fig. 8b, it is presented the diagram of relationship between displacements and AK load. From Fig. 8b, it can be seen that after load A17 the displacements will increases rapidly in almost beams. So, the load A17 of AK type load can be considered as the limit load for the criterion of displacement progression.

From the results of nonlinear structural analysis, the Table 2 presents the comparison of AK load corresponding with three proposed types of limit states:

- Achievement of critical value of bending moment according to the condition of reaching of ultimate strength in one (maximum bearing load) beam;
- Achievement of critical value of displacement, which equals 0.004 of calculated span length (l_c) in maximum bearing load beam;
- Formation of two plastic hinges in beams of superstructure;

The data in Table 2 shows that the application of proposed approach of nonlinear structural analysis in road-bridge superstructures and proposed criteria of limit states

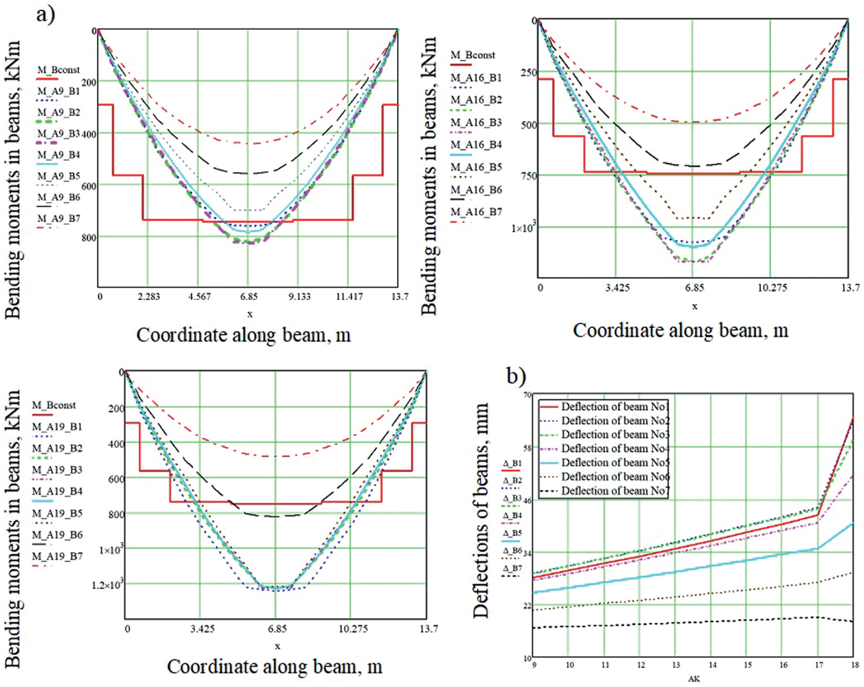


Fig. 8. Results of nonlinear structural analysis: a) the change of bending moment of beams under reference lane of AK loads; b) displacements of beams

allows increasing class of loading of type AK about 2–3 units: from A10–A11 to A12–A13 and from A14 to A16–A17.

Table 2. The comparison of AK load corresponding with different limit states

Beam length, m	Area of steel (in middle span), cm^2	Achievement of critical value of bending moment calculated by ULS method	Displacement of max. bearing load beam 0.004 l_c	Formation of 2 plastic hinges in superstructure	Achievement of max.disp. in boundary beam B1
14.06	72.4	A16	A19	A18	A17

3.2. Pre-stressed reinforced concrete road bridge superstructure

Similarly, in the research, it is also performed the example of nonlinear structural analysis for pre-stressed reinforced road-bridge superstructure of typical drawing Issue 122-62 with 22.16 m beam (calculated span 21.5 m) and other parameters of dimensions and geometrical characteristics are given in Table 3 and Fig. 9. Material characteristics:

Elastic modulus of concrete (class B30) $E_b = 29250$ MPa, pre-stressed reinforcement (type Bp-II) $E_s = 177000$ MPa; compressive strength of concrete $R_b = 15.5$ MPa, reinforcement $R_p = 995$ MPa, $\sigma_{pn} = 897$ MPa, in the middle section of beam $M_{\text{init}} = 2102$ kN·m, $\varepsilon_{b0} = -3.75 \cdot 10^{-4}$, $\sigma_{p0} = -9.5$ MPa.

Table 3. Dimensions and geometrical characteristics of cross-section of pre-stressed beam element

Sec. a, m	b_w, cm	h, cm	h_f, cm	h_w, cm	A_p, cm^2	a_p, cm	h_0, cm	A_{red}, cm^2	Y_{bot}, cm	I_{red}, 10^5cm^4	$W_{\text{bot\_red}}$ 10^5cm^3	$W_{\text{top\_red}}$ 10^5cm^3
0	16	120	10	35	28.3	33	87	4796	74.3	82.96	1.116	1.817
5	16	120	10	35	28.3	25	105	4796	74	84.37	1.141	1.832
10.75	16	120	10	35	28.3	13	107	4796	73.6	86.25	1.171	1.861

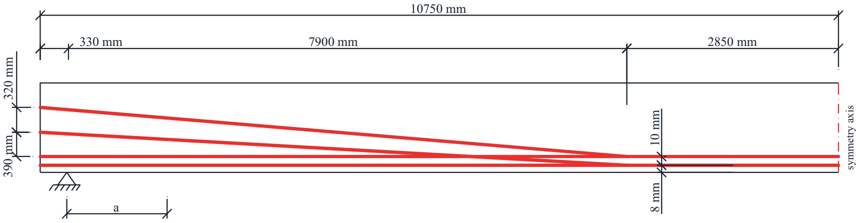
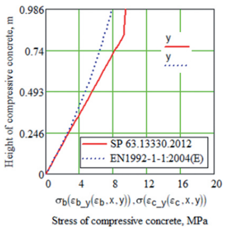
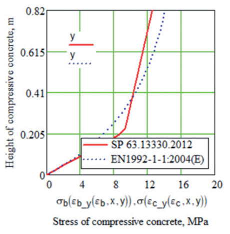
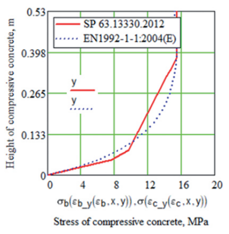
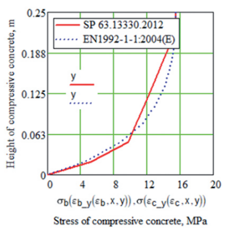


Fig. 9. Schema of reinforcement of pre-stressed beam with 22.16 m length of Issue 122–62

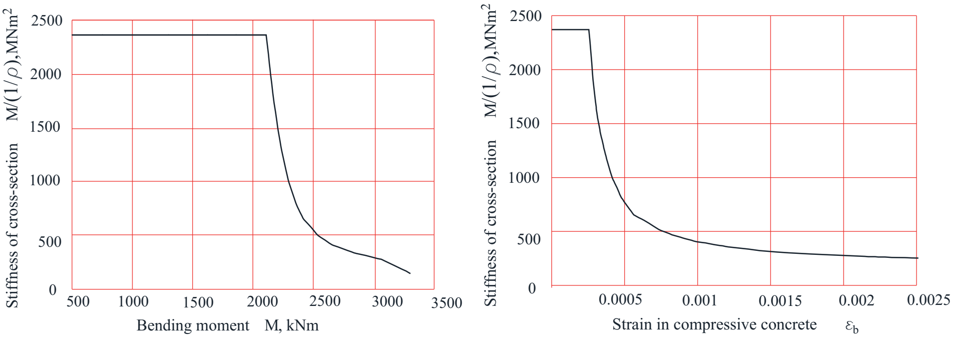
In the Table 4, it is presented the results of nonlinear analysis of pre-stressed reinforced concrete beam with length 22.16 m. From Table 4, it can be seen that the results of distribution of stresses of compressive concrete calculated according to [19] and [20] are relatively converged.

Table 4. The distribution of stresses σ_b by height for middle cross section $a = 10.75$ m (section 1–1) of 22.16 m beam of Russian typical drawing Issue 122–62

$\varepsilon_b = 3.75 \cdot 10^{-4}$	$\varepsilon_b = 8 \cdot 10^{-4}$	$\varepsilon_b = 16 \cdot 10^{-4}$	$\varepsilon_b = 20 \cdot 10^{-4}$
			

The Fig. 10 presents the results of nonlinear analysis of pre-stressed beam with 22.16 m of Typical drawing of Issue 122–62.

In the Fig. 11, it presents cross-section of road bridge superstructure with 22.16 m beam (calculated length of beam 21.5 m) and dimension G8 + 2 × 1.0. Similarly, in the second



(a) (b)

Fig. 10. Diagram of relationships of pre-stressed beam with length 14.06 m of typical drawing of Issue 122-62: a) $M_{\sum}/(1/\rho) = f(M_{ex})$, b) $M_{\sum}/(1/\rho) = f(\varepsilon_{b,z=x})$

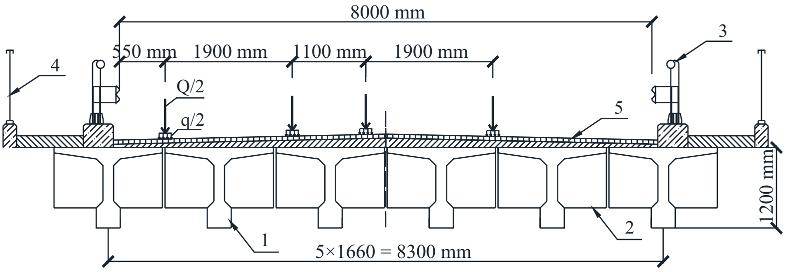


Fig. 11. Cross section of pre-stressed RC road-bridge superstructure with beam length 22.16 m of typical drawing Issue 122–62: 1 – beam element, 2 – diaphragm, 3 – metal barriers, 4 – reinforced concrete sides

period of nonlinear structural analysis of road-bridge superstructure also implemented using the results of nonlinear analysis of beam in the first period, which shown in the diagrams of Fig. 10 and the parameters of the superstructure in Fig. 11. The authors also implemented the nonlinear structural analysis of road-bridge superstructure in the research using the structural analysis software SCAD and the used models like in previous example. The reference lane of moving load also is A11 load with gradually applying unit load of AK type load. The Fig. 12 shows the results of analysis like in previous example: the changes of bending moment along the length of all beams.

In the structural analysis, it is assumed that all reference lanes of live load are static. On the other hand, by other words, in the research, the dynamic effect is neglected.

Like in previous example with reinforced concrete road bridge superstructure, in the example with pre-stressed reinforced concrete road bridge superstructure, it also observed and investigated the moment of formation of plastic hinges in beams of superstructures. The moment of forming of two or three plastic hinges considered as the moment of reaching of limit state of the superstructure by criterion of number of plastic hinges. In addition, in the example, it is also investigated the moment of displacement progression.

The Fig. 12b shows that after applying A18 load, the displacements in almost beams of the superstructure increases rapidly. So, the load A18 can be considered as the limit load for the criterion of displacement progression.

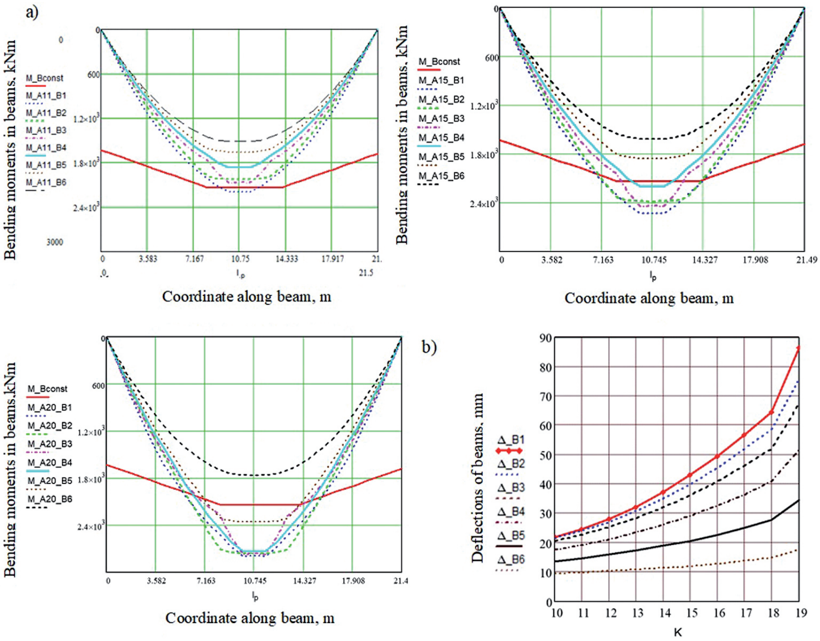


Fig. 12. Results of nonlinear structural analysis: a) The change of bending moment of beams under reference lane of moving load AK; b) Displacements of beams

- The obtained results of this example are presented in the Table 5 which referred the moving load AK corresponding with three proposed criteria of limit states.
- Reaching the point ε_{b0} corresponding with the value $M_{\text{init}} = M$ that raised in the maximum bearing load beam B1 under the reference lane of moving load A11;
- The first plastic hinge that formed in beam B1 under the reference lane of moving load A16;

Table 5. The comparison of AK load corresponding with different limit states

Beam length, m	Area of pre-stressed reinforcement of class BII (in middle span) cm^2	Achievement of critical value of bending moment calculated by ULS method	Displacement of max. bearing load beam $0.004l_c$	Formation of 2 plastic hinges in road-bridge superstructure	Achievement of max. displacement in boundary beam B1
22.16	28.26	A14.6	A17	A17	A18

Under the moving load A11, the displacements of beam reached the value: B1–64,94 mm, B2–58,5 mm, B3–51,9 mm; and the average displacement 43 mm, the number of plastic hinges in the superstructure is three (formed in beams B1, B2, B3).

The data in Table 5 shows that the application of proposed approach of nonlinear structural analysis in road-bridge superstructures and proposed criteria of limit states allows increasing class of loading of type AK about 2–3 units: from A14–A15 to A17–A18.

4. Conclusions

In the research, the authors proposed an approach of nonlinear structural analysis for reinforced concrete and pre-stressed reinforced concrete beams of road-bridge superstructures according to nonlinear deformational models of construction codes [19] and [20]. Based on the proposed approach, the algorithm and procedure of analysis are established, and the routine of calculation is written using Mathcad software. From the obtained results, it is built different relationships between internal forces, the strain of compressive concrete, bending curvature, and cross-section stiffness. The obtained results calculated according to various codes are relatively converged. This allows us to freely choose the deformational models of the Russian code or the Eurocode2 in structural analysis.

The results of nonlinear analysis applied for beam elements in this research help to perform a systematic description of the development of the stress-strain state of elements at any period of the loading process. The relationships between bending moment and cross-section stiffness used in the structural analysis of road-bridge superstructures allow unequal section stiffness change in relationship with raised bending moment. In addition, for this research, the most effective model of finite elements is the frame-shell model, which consists of a finite shell element with three degrees of freedom at the node of the element and the frame element with the entire section stiffness. The adopted models applied in nonlinear structural analysis for road-bridge superstructures allow for reflection of the unequal section stiffness of beams that changes along the length of beams.

The approach of nonlinear structural analysis proposed by authors applied in the road-bridge superstructures helps present the stress-strain state's development. Moreover, it helps describe the internal forces of any beam element of superstructures from the beginning of loading to the exhaustion of bearing capacity. The reference lanes of loading type AK = A1 that are taken as the step of live load at the final stages of loading allow taking into account the decrease of the section stiffness corresponding to raised bending moment of elements.

In this research, the recommended criteria for limit states of nonlinear structural analysis are the following:

- The formation of two plastic hinges in the superstructure (with at least six beams);
- The transformation (in the process of step loading) of the boundary beam into a maximum loaded one;
- The achievement of the critical value of displacement in the maximum loaded beam or the degree of progression of deflection in the most loaded beam.

According to these proposed criteria for limit states, that could reasonably explain the load capacity reserve of the road-bridge superstructures, which have a long-time operation process under greater load than the initially designed load. This is important in evaluating the load capacity of reinforced concrete and pre-stressed concrete road-bridge superstructures in the case of a lack of experiments.

References

[1] Road Project Union, “Standard designs, product and components of buildings and structures. Prefabricated reinforced concrete superstructures 12, 15, 18, 21, 24, 33m length from I beam with pre-stressed reinforcement for bridges and overpasses located on roads, streets in cities”, Russia. Series 3.503.1-81, 1981–1985.

[2] D.M. Shapiro, A.V. Agarkov, and T.T.T. Van, “Spatial non-linear deformational analysis of road bridge multi-beam superstructures”, *Scientific Journal of Voronezh State University of Architecture and Civil Engineering, Series of Construction and Architecture*, no. 2, pp. 29–37, 2008. [Online]. Available: http://vestnikvgasu.wmsite.ru/vypuski/vypusk-2-10-2008/prostranstvennyj-nelinejnyj-deformacionyj-raschet-balochnyh (in Russian).

[3] D.M. Shapiro and A.V. Agarkov, “Deformative non–linear calculation of ferroconcrete t-section beams”, *Scientific Journal of the Voronezh State Architecture and Civil Engineering, Series of Construction and Architecture*, no. 1, pp. 38–44, 2008. [Online]. Available: http://vestnikvgasu.wmsite.ru/vypuski/vypusk-1-9-2008/deformacionnyj-nelinejnyj-raschet-zhelezobetonnyh-balok (in Russian).

[4] E. Spacone and J.P. Conte, “Nonlinear analysis of frame structures”, in *Structural Engineering and Geomechanics*, S. K. Kunnath, Ed., vol. 1. EOLSS Publications, 2020. [Online]. Available: http://www.eolss.net/sample-chapters/c05/E6-139-03.pdf.

[5] T.T.T. Van and V.T.B. Quyen, “Non-linear deformational analysis of reinforced concrete frame”, in *Lecture Notes in Civil Engineering 54, CIGOS 2019, Innovation for Sustainable Infrastructure*. Springer Nature, 2020, pp. 317–323, DOI: 10.1007/978-981-15-0802-8.

[6] O.M.O. Ramadan, A.H.A. Kareem, H.R. Abusafa, and I.A. El-Azab, “Nonlinear Analysis of Reinforced Concrete T-Beam”, *Al-Azhar University Civil Engineering Research Magazine (CERM)*, vol. 43, no. 2, 2021. [Online]. Available: https://bu.edu.eg/portal/uploads/Engineering,%20Benha/Civil%20Engineering/2291/publications/Ibrahim%20Ali%20Ibrahim%20El%20Azab_Nonlinear%20Analysis%20of%20Reinforced%20Concrete%20T-Beam.pdf.

[7] A. Karakaplan, D.R. Petricone, and P.E.T. Barnard, “A Computer Model for Nonlinear Analysis of Bridge Structures”, Studies in Applied Mechanics, vol. 35, pp. 423–432, 1994, DOI: 10.1016/B978-0-444-89918-7.50027-7.

[8] P. Hawryszków and B. Czaplewski, “Application of Matlab software in static calculations of bridge structures”, *Archives of Civil Engineering*, vol. 68, no. 1, pp. 299–317, 2022, DOI: 10.24425/ace.2022.140169.

[9] M. Fawad, K. Koris, M. Salamak, M. Gerges, L. Bednarski, and R. Sienko, “Nonlinear modelling of a bridge: A case study-based damage evaluation and proposal of Structural Health Monitoring (SHM) system”, *Archives of Civil Engineering*, vol. 68, no. 3, pp. 569–584, 2022, DOI: 10.24425/ace.2022.141903.

[10] K. Kaptan, “Non-linear analysis of bridge structures”, *Trakya University Journal of Engineering Sciences*, vol. 18, no. 1, pp. 17–29, 2017. [Online]. Available: https://dergipark.org.tr/tr/download/article-file/388796.

[11] D.M. Shapiro and A.Y. Sukhareva, “Engineering method of nonlinear calculation of plate and beam systems used in bridge building”, *Scientific Journal of Voronezh State University of Architecture and Civil Engineering. Series of Construction and Architecture*, vol. 3, no. 22, pp. 52–62, 2019. [Online]. Available: https://cchgeu.ru/science/nauchnye-izdaniya/stroitelnaya-mekhanika-i-konstruktsii/spisok-vypuskov/22.pdf.

[12] D.M. Shapiro, A.V. Agarkov, N.N. Melnichuk and T.T.T. Van, “The non-linear methods of analysis in modern designing (by the example of geo-technics facilities and bridges)”, *Scientific Journal of the Voronezh State University of Architecture and Civil Engineering. Construction and Architecture*, vol. 3, no. 7, pp. 46–58, 2010. [Online]. Available: http://vestnikvgasu.wmsite.ru/ftpgetfile.php?id=228.

[13] D.M. Shapiro and A.P., Tyutin, "Reliability and durability of concrete bridge spans", *Scientific Journal of the Voronezh State University of Architecture and Civil Engineering. Construction and Architecture*, vol. 2, no. 13, pp. 89–99, 2010. [Online]. Available: https://cchgeu.ru/science/nauchnye-izdaniya/stroitelnaya-mekhanika-i-konstruktsii/spisok-vypuskov/13.pdf.

[14] D.M. Shapiro and A.P. Tyutin, "Experimental research of pre-stressed reinforced beam with length 28m", *Scientific Journal of the Voronezh State University of Architecture and Civil Engineering. Construction and Architecture*, vol. 2, pp. 99–104, 2013.

[15] D.M. Shapiro and A.P. Tyutin, "Theory of numerical calculation of plate-beam reinforced concrete road-bridge superstructures", *Scientific Journal of Construction and Architecture*, vol. 2, no. 54, 2019, DOI: 10.25987/VSTU.2019.54.2.012.

[16] O.V. Zyuzina, "Experimental studies of reinforced concrete structures of hydraulic structures strengthened with prestressed transverse reinforcement", *Structural Mechanics of Engineering Constructions and Buildings*, vol. 16, no. 6, pp. 504–512, 2020, DOI: 10.22363/1815-5235-2020-16-6-504-512.

[17] D. M. Shapiro and A. P. Tyutin, "Distribution of stresses in the supporting areas of pre-stressed reinforced concrete beams of road-bridge superstructures", *Scientific Journal of the Voronezh State University of Architecture and Civil Engineering. Construction and Architecture*, vol. 1, no. 10, pp. 88–96, 2015.

[18] A.A. Piskunov, T.A. Zinnurov, D.V. Berezhnoy, B.S. Umarov, and A.R. Voltaire, "On the results of experimental and numerical studies of the stress-strain state pre-stressed reinforced concrete structures with polymer composite rods", *Russian Journal of Transport Engineering*, vol. 5, no. 2, 2018, DOI: 10.15862/02SATS218.

[19] SP 63.1330.2012 Set of rules: Concrete and Reinforced concrete structures. Ministry of Regional Development of the Russian Federation, 2012. [Online]. Available: https://kupdf.net/download/sp-63-13330-2012-concrete-and-reinforced-concrete-structures-revised-ed_58ec3a45dc0d60ef23da97e2_pdf.

[20] EN 1992-1-1:2004 Eurocode 2: Design of concrete structures. Part 1-1 General rules and rules for buildings. European Committee for Standardization, 2004. [Online]. Available: https://www.phd.eng.br/wp-content/uploads/2015/12/en.1992.1.1.2004.pdf.

[21] SP 52-101-2003 Set of rules: Concrete and Reinforced concrete structures without pre-stressed reinforcement, Ministry of Regional Development of the Russian Federation, 2003. [Online]. Available: https://www.academia.edu/8077683/SP_52-101-2003_Concrete_and_reinforced_concrete_structures_made_without_reinforcement_prestressing_System_of_regulations_in_construction_activity_SET_OF_RULES_SP_FOR_DESIGN_AND_CONSTRUCTION_CONCRETE_AND_REINFORCED_CONCRETE_STRUCTURES_WITHOUT_PRESTRESSING

[22] SP 52-102-2004 Set of rules: pre-stressed reinforced concrete structures. Ministry of Regional Development of the Russian Federation, 2004. [Online]. Available: https://files.stroyinf.ru/Data2/1/4294813/4294813056.pdf.

[23] T.T.T. Van and D. Shapiro, "Nonlinear deformation analysis for precast pre-stressed concrete beam systems", in *E3S Web of Conferences*, vol. 97. 2019, DOI: 10.1051/e3sconf/20199703039.

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