

ABSTRACT

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ASSUMPTIONS: AN ACTION RESEARCH
STUDY OF TEACHER QUESTIONING
PATTERNS IN 7TH AND 8TH GRADE
MATHEMATICS CLASSROOMS

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This action research project explores the link between a teacher's questioning patterns and the modes of thinking, analyzing, evaluating and communicating that are developed in his 7th and 8th grade math students. The highly qualitative analysis focuses on three videotaped lessons from his 7th and 8th grade classrooms, and evaluates the lessons according to four categories or "lenses": cognitive demand, task completion, self-efficacy, and metacognitive activity. It then seeks to identify and codify the predominant questioning pattern used in each lesson, and connect this pattern to the levels of success exhibited in each of the four categories. Four principal patterns are observed and discussed in the lessons: Unilateral Inquiry Response Evaluation, Multilateral Inquiry Response Evaluation, Inquiry Response Collection, and Inquiry Response Revoicing Controversy. The fourth pattern is proposed as a tool for managing classroom discourse that involves a variety of (sometimes competing) student opinions.

SUBTLE CUES AND HIDDEN ASSUMPTIONS: AN ACTION RESEARCH
STUDY OF TEACHER QUESTIONING PATTERNS IN 7TH AND 8TH
GRADE MATHEMATICS CLASSROOMS

By

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Chapter 1: An Overview of the Research Project

Despite having a substantial amount of experience teaching math 7th and 8th graders, I have never been able to step back and observe how my own patterns of interaction with students shape the development of the very habits of mind I wish for them to acquire during their two years under my direction¹. Moreover, I have not been able to truly evaluate the extent to which my own teaching philosophy – how I feel students learn best, what skills I believe are most important for them to acquire, how I would like them to perceive my role as their teacher, etc -- actually informs and guides the patterns of interaction I establish consciously or unconsciously with them over the course of the year.

It is certainly possible for a teacher to enter each class with a certain philosophy in mind, but not actually demonstrate that philosophy in the way that he or she teaches. Stigler and Hiebert (1999) illustrate this phenomenon powerfully in their comparative study of German, Japanese, and U.S. classrooms. They show, for example, that the U.S. cultural norm that labels a student's frustration and confusion as a product of teaching "failure" can prompt teachers to lower the level of difficulty of the tasks they give students, even if the teacher strongly believes in the positive benefits of maintaining high levels of rigor.

¹ At my school, middle school math teachers teach most of their students for both 7th and 8th grades.

Teaching and learning are, in many ways, activities that are structured by habits and routines. As a teacher initiates his or her students into new subject matter, he or she models certain conventions of communication between him or her and the students he or she teaches. These conventions are sometimes explicit, as when a teachers instructs students to use certain phrases in presenting arguments (“I believe this is true because.....” or “We can’t assume this is always true because of this counterexample..”), but more often than not the modeling is implied. Many teachers, for example, will respond to correct student responses by either affirming that they are correct, or writing the students work on the board, and not commenting at all. Teachers often reserve questions such as “Can you prove why this is true?” or “Can you explain to us how you found that answer?” for situations when a student has made an error. As this pattern becomes a established, students quickly learn to pick up subtle cues from the teacher as to whether what they are saying is deemed valid or invalid by their teacher.

Whether this “arrangement” of subtle cuing from teacher to student is carried out consciously or unconsciously by the teacher, it can have a profound effect on how students perceive the teacher’s role and their own role in the process of classroom discourse. These patterns of interaction between teacher and student, then, are powerful indicators of what skills and habits of mind a teacher deems most important for students to acquire. Since the teacher, as a creature of habit, carries these patterns into many of the activities that he engages in with students, these patterns can essentially establish, through their persistence and omnipresence, paradigms of learning for students as they progress through the year’s curriculum.

The Research Project

Curious about the extent to which my teaching practice matched my teaching philosophy, I decided to embark on an action research project investigating patterns of student-teacher interaction in my 7th and 8th grade math classrooms. Since asking students questions is a central tool that I use to facilitate their learning, I focused my observations on sequences during lessons in which I posed queries to either individual students or the class as a whole. By videotaping and transcribing the student and teacher talk during these sequences, I was able to track closely how I responded to student responses to these questions. My goal was to perform a fine-grained analysis of what underlying patterns of interaction were structuring the learning environment for my students, so I could better understand what paradigms of learning and discourse I was consciously or unconsciously developing in my students. Ultimately, I hoped to better understand how various patterns of interaction – whether old ones I had internalized over my teaching career or new ones I had adopted into my pedagogical repertoire during the research study -- could bring about successful interactions between me and students. Perhaps, then, I could apply these patterns more judiciously and mindfully, knowing the benefits and dangers that each carried in bringing about success.

As I conducted this action research project with my 7th and 8th grade classes, I was guided by the following central questions:

-What are some typical patterns of questioning that I use with students during my classes?

-In what ways do these typical patterns of questioning develop students' cognitive growth, metacognitive skills, and confidence in their math abilities? In what ways do these typical patterns limit or undermine students' progress in these areas?

-Are there other patterns of questioning (besides my typical patterns) which are more effective in bringing about student cognitive growth, metacognitive ability and self-confidence? If so, what are they, and what would it be like to apply these patterns in the course of a lesson?

-Given the range of questioning patterns available, what the potential benefits and dangers of each pattern, what learning contexts would be best suited for the application of each pattern?

Research Methodology, Rationale and Process: An Overview

With the purpose and aim of my research project firmly established, I proceeded forward with the pre-data collection phases (**Research Stages 1 and 2** below) and data collection phases (**Research Stages 3 through 6** below) of the project.

Research Stage 1: Defining What Success “Looks Like”

(Constructing an Analytical Framework)

In order to judge the success of my interactions with students, and the role of specific questioning patterns in bringing about this success, I needed a clear definition of what success would look like in my teaching practice. Furthermore, I wanted a framework for success in place *before* I began videotaping my lessons. This posed a small dilemma: how could I generate an analytical framework before I had any real data to analyze?

The solution was to construct a series of fictional scenarios that represented typical student-teacher communication in my classrooms. Some of these scenarios felt “successful” to me in an intuitive sense, and others felt “unsuccessful”. I then probed the successful scenarios for the common threads or themes that could best articulate what it meant for student and teacher exchanges to be considered successful in my eyes. I also examined the unsuccessful scenarios to determine whether there were certain common aspects that many of them lacked.

Four principle characteristics of success emerged from as I explored the hypothetical scenarios. Not surprisingly, three of these four expressed fundamental commitments that I have in my teaching: presenting students with tasks that demand critical thinking skills, developing in my students a positive view of themselves as math students, and training students to monitor and assess their own thinking processes. The fourth characteristic -- task completion -- reflected the reality that as a teacher, there is a certain

body of knowledge that I am expected to convey to my students within the limited amount of time that I share with them during the school year.

In **Chapter 2**, I describe the four components of this framework for success: cognitive demand, self-efficacy, metacognitive skill, and task completion. Like a magnifying lens, each component focuses in on a specific characteristic of student-teacher interactions that is deemed desirable, and assesses the extent to which that characteristic is present in the interactions. In addition to describing the contours of the four “lenses”, I situate each in the context of current research that addresses communication patterns and social norms that shape student learning.

Research Stage 2: Previewing My Questioning Patterns

(Evaluating Success and Identifying Patterns in the Hypothetical Scenarios.)

With my success framework well established, I decided to return to the hypothetical scenarios that generated the framework itself, and inspect them with greater precision. I could now use the framework as a *tool* to articulate particular dynamics and processes present in the scenarios that helped make the interactions successful, as well as specific factors that undermined the success of the interactions. In other words, I could speak about the “success” of the interactions in much more specific and well-defined language.

But the success framework, while essential as an analytical tool, was not the central motivation for constructing the scenarios. In writing up these scenarios, I hoped to

identify any recurring questioning patterns – that is, the ways that I pose questions to students -- that I might expect to see in my videotaped lessons. In identifying these patterns *before* the data collection phase, I hoped to be more aware of them as I was teaching during my videotaped lessons. Perhaps, I thought, this exercise of identifying patterns would give me a deeper “felt” sense of the patterns as I was applying them in the classroom during the videotaping process. In bringing a consciousness of these patterns into my videotaped lessons, I hoped to gain a greater awareness of what motivated them in the first place.

In **Chapter 3**, I present the transcripts of four hypothetical scenarios, each of which illustrates a slightly different pattern of questioning that may occur in a typical lesson. In my discussion of each scenario, I accomplish two central tasks: I highlight and describe the main questioning pattern illustrated by the scenario, and I evaluate the scenario’s success using the four-part framework detailed in Chapter 2. Using these identified patterns, I preview what types of questioning I *might expect* to observe in my actual lessons, and connect these patterns to ones that have been documented in the literature. One documented pattern in particular, the Inquiry Reponse Evaluation (IRE) pattern, appears frequently in the scenarios, and seems to undermine levels of cognitive demand and metacognitive skill acquisition. Another pattern documented in the literature -- the technique of revoicing -- is introduced as a means of transcending some of the limitations posed by the IRE pattern.

Research Stage 3: Viewing the IRE Pattern in Action:

(Applying the Analytical Framework to Lesson Clip 1)

With a more intimate feel for the typical questioning patterns that seemed to pervade my teaching, I felt ready to engage in data collection using my 7th and 8th grade classes. I was eager to see whether the predictions I had made (using the hypothetical scenarios) about probable questioning patterns were actually confirmed by the videotapes. In **Chapter 4**, I take a close look at a video clip of an exchange I have with a student in an 8th grade algebra class who is working his way through the simplification of an algebraic quotient involving a binomial dividend and monomial divisor. As I dissect the lesson transcript, and evaluate its success using the four-part framework, I take a keen interest in how I respond to mistakes that the student makes in his reasoning.

Not surprisingly, I notice the preponderance of the IRE pattern that persisted in the imagined scenarios. More specifically, I observe a recurring pattern that can best be described as *Unilateral Inquiry Response Evaluation (UIRE)*, since the entire exchange takes place between myself and one student. The unilateral nature of the exchange poses serious limitations for the maintenance of high cognitive demand and metacognitive skill for both the student and the rest of his classmates. Consequently, I enter my next videotaped lesson – an 8th grade algebra class exploring polynomial factoring strategies - - with intention to include more student voices in my discussions around a specific question.

Research Stage 4: A First Attempt to Expand Student Participation:

(Applying the Analytical Framework to Lesson Clip 2)

In **Chapter 5**, I present the transcript this second lesson, and assess its success using the four part framework. In particular, I track the extent to which I am able to expand the scope of student participation. In some ways, this lesson breaks new ground, elevating the level of metacognitive training by involving several students in a discussion of each others' ideas. At the same time, the pattern -- categorized as *Multilateral Inquiry Response Evaluation (MIRE)* -- shows many of the same characteristics as the UIRE pattern evident in the first lesson clip.

I observed that I was constantly evaluating students' responses immediately after they were given through explicit or implicit cues. This meant that students were not being encouraged to evaluate their classmates' responses on their own, and thus lost valuable opportunities to judge the accuracy of their peers' statements. In assessing the validity of another student's thinking, a student can gain a deeper awareness of how mathematicians discern the accuracy of a given assumption or claim -- an essential metacognitive skill. Without any incentive to do this type of peer evaluation, students learn to accept that only their teacher has the authority to determine whether a statement is valid or invalid. For my third and final lesson, I clearly needed to find a radically different questioning pattern if I wanted to break out of the predictable and habitual IRE format.

Research Stage 5: A Second Attempt to Expand Student Participation

(Applying the Analytical Framework to Lesson Clip 3)

In response to this realization, I design and videotape a lesson comparing absolute change to percent change for my 7th grade class. In designing the lesson, I hoped to create a learning environment that was conducive to questioning patterns that encouraged greater peer-to-peer evaluation and discussion. In **Chapter 6**, I present the transcript of part of this third lesson and apply the success framework. In the process of analyzing the clip, I discover a new questioning form, the Inquiry Response Collection (IRC) pattern. While successful in promoting more student engagement and participation, this pattern also has the potential of lowering cognitive demand and metacognitive skill acquisition. Moreover, the absence of the appropriate sociomathematical norms for constructive peer-to-peer debates poses a problem for the stability of the discussion, which is threatened by tension between students with conflicting opinions.

From this vantage point, I propose another questioning pattern, which seeks to address these potential pitfalls, while at the same time maintaining student participation and engagement. Labeled the Inquiry Reponse Revoicing Controversy (IRRC) pattern, it is envisioned as a form that is more conducive to a successful implementation of revoicing techniques.

Research Stage 6: Imagining a New Pattern

(Applying the Analytical Framework to Future “Controversial” Scenarios)

In **Chapter 7**, I describe this IRRC pattern, and demonstrate how it addresses some of the limitations observed in the UIRE, MIRE and IRC patterns. In addition, I confront the dilemmas a teacher may face in trying to implement this pattern while at the same time preserving positive student morale and mathematical rigor are explored deeply. The importance of establishing sociomathematical norms that guide student behavior and attitudes during the discussion of thorny mathematical questions, or "controversies," is addressed as an essential consideration in pattern implementation. Finally, I describe a vision for my own teaching practice that both embraces multiple questioning patterns -- each with its own potential benefits and trade-offs -- and seeks to manage the dilemma of balancing the needs of the four lenses as best as possible.

Research Stage 7 - Implementing the New Pattern

(Applying the Analytical Framework to Lesson Clip 4)

In **Chapter 8**, I discuss my fourth and final videotaped lesson, in which I attempt to implement the IRRC pattern while engaging students with a “controversial” mathematical task. I also detail the process through which I prepare the students for this fourth lesson, through a sequence of activities exposing them to the art of constructing compelling mathematical arguments and proofs. After providing the transcript of the

lesson, I explore the extent to which I faithfully implemented the IRRC pattern, and apply the four-part framework to analyze the success of the lesson.

In **Chapter 9**, I address what subconscious assumptions could potentially derail my efforts to continue implementing the IRRC pattern in future discussions with my students. Finally, I analyze the extent to which my research findings correspond with some of the NCTM Principles and Standards, and what these links imply for the larger community of math educators.

Chapter 2: Constructing an Analytical Approach

Honoring My Commitments

Every teacher brings a set of values and philosophical aims into the classroom. These commitments establish priorities for a teacher as he or she navigates the typically messy and unpredictable terrain of engaging students in the learning process. If a teacher is truly committed to improving his or her practice, and is aiming to apply “best practice”—that is, an optimal way of engaging students in the learning process -- it is important for him or her to keep these commitments in mind as he or she makes choices in managing these complex scenarios.

For example, if a teacher is asked an insightful question by a student during a discussion that is related but not central to the topic at hand, the teacher must decide whether to dismiss the question as irrelevant, or engage with it as a tool for deepening student learning². If the teacher embraces the question, and helps to facilitate a meaningful discussion about an important topic, he or she may not complete some the tasks that he or she was planning to that day with the students. Without a larger perspective on the importance of student queries and their potential ability to promote critical thinking, a teacher may feel that this spontaneously triggered discussion was a waste of time. If however, having meaningful discussions like this one is a priority for that teacher, he or she will see it as an enriching experience for the students.

² Of course, the teacher could also respond to the student by acknowledging the question’s importance, but explaining that due to time constraints, the question is best addressed at a future point. All the same, the question is “dismissed” in the immediate sense.

There are many values that I carry with me into the classroom that shape the countless decisions I make on the spot in responding to students' questions, comments, and behavior. However, three of these values seem paramount, and hold the greatest sway in determining what I deem to be most important in any given interaction with a student. In essence, my ongoing assessment of how well I am doing as a teacher is guided principally by these three aims.

One commitment is to appropriately challenge my students with tasks that engage them in critical and independent thinking processes. I would like them to be able to grow more confident facing problems that require the novel application of a new concept or set of ideas, rather than the blind implementation of a memorized procedure. To this end, I would like them to build their understanding of new concepts through the experience of wrestling with problems that require them to reflect upon the essence of these concepts. In the literature, these types of problems are described as *cognitively demanding tasks* (Stein et al. 1996)

Another principal commitment is to help them see themselves as self-sufficient and competent mathematical thinkers who can approach these cognitively demanding tasks with calm and self-assurance. No matter what kind of experiences they have had in previous math classes, I would like them to leave my class feeling like that have the ability to be successful in math classes. This belief in their ability as math students, termed *self-efficacy* in the literature, can have a tremendous affect on their achievement

in math, particularly during their middle school years. On the positive side, a high self-efficacy could inspire them to pursue challenging math classes through high school and college, and could motivate them to enter into math-related professions. On the negative side, low self-efficacy could discourage them from investing time and effort into their math classes, since they assume that they don't have the ability to succeed anyway. Thus, maintaining and building high self-efficacy is vital for supporting student achievement.

Finally, I am committed to helping them developing the ability to reflect about how they approach problems, and how they can apply more effective strategies to new problem solving situations. Furthermore, I want them to gain a deeper sense of what it means to assess whether a mathematical statement is true or false (or sometimes true and sometimes false), and how one might go about making this assessment in the context of a class discussion. At their core, these *metacognitive skills* are addressing students' ability to learn and analyze new material effectively, and to determine if and how they need to make adjustments in their learning strategies as they explore novel conceptual territory.

While these principal commitments to cognitively demanding tasks, high self-efficacy and metacognitive skill development are not the only objectives that shape my overall teaching approach, they are ones that I felt would most interesting to investigate during this research project. In surveying the literature, I found a host of scholars who shared

my appreciation of these three pedagogical priorities. Included in this collection of like-minded educators were the writers of the NCTM Principles and Standards (2000).³

The Importance of Cognitively Demanding Tasks

Given the inevitable confusion and struggle that arises when students face cognitively demanding tasks, and the ensuing dilemmas that are posed for the teacher as a result, one might question whether these tasks are worth the challenges that they bring about for teachers. And yet, the extensive implementation of cognitively demanding tasks is vital, according to Stein and Henningsen (1997), in developing what Schoenfeld (1992) labels a “mathematical disposition” among students, which is “characterized by such activities as looking for and exploring patterns to understand mathematical structures and underlying relationships; using available resources effectively and appropriately to formulate and solve problems; making sense of mathematical ideas, thinking and reasoning in flexible ways: conjecturing, generalizing, justifying, and communicating one’s mathematical ideas; and deciding whether mathematical results are reasonable.”

However, as Stein et al (1996) report in a landmark study on cognitively demanding tasks, the implementation of cognitively demanding tasks does not guarantee that their level of cognitive demand will remain high. Since these tasks often involve certain levels of ambiguity and complexity, students may be initially uncomfortable facing them. This student discomfort, coupled with the time constraints of a 45-minute class period, may prod the teacher to “routinize certain problematic aspects of the tasks”, thus

³ See Chapter 8 for a discussion of how the NCTM Principles and Standards address these commitments

leading to a decline in the tasks' levels of cognitive demand. While Stein et al. admit to a dearth of research about what instruction fosters high-level thinking, they do cite certain techniques that have been shown to sustain student effort and motivation in the face of high cognitive demand – scaffolding, modeling of high-level problem solving by capable students, selecting tasks to build on prior student knowledge, encouragement of student self-monitoring, and a consistent press for student explanation and justification (Stein et al 462).

The Importance of Promoting Self-Efficacy

These techniques can do little for a student, however, if a student's self-efficacy is low. Given how influential a student's frame of mind is in shaping their success in facing these highly cognitively demanding tasks, it is essential to acknowledge how an interaction makes a student feel, both in the moment as he or she answers a question in front of the class, and more generally as he or she develops a sense of his or her ability as a math student. Eight years of teaching 7th and 8th graders mathematics has shown me that a student's self-efficacy plays a central role in motivating him or her to face these cognitively demanding tasks. In their study of student's mathematical self-efficacy, Lloyd et al. report that 7th grade students show lower levels than 4th graders, indicating middle school students are more likely to view themselves as "bad" math students than elementary school students (Lloyd et al. 2005). If a student carries this attitude into high school, he or she is much more likely to avoid math classes at the college level, when they are largely optional.

In fact, Pajares (1996) has shown that a negative mathematical self-image is responsible more than any other factor for students' avoidance of math-related careers. If a student feels that he or she is "not good at math", he or she will be more likely to respond to a challenging task with trepidation, fearing that it will once again confirm that he or she "doesn't have what it takes" to do complex math problems. This anxiety inevitably undermines a student's ability to think clearly and calmly about the many resources and tools that he or she has at his or her disposal, thereby hampering his or her ability to solve the problem successfully, and confirming his or her self-doubts.

In his study, Pajares found that this vicious cycle of self-doubt and consequent failure was more pronounced in girls, who tended to underestimate their math abilities and view any successful mathematical achievement as the product of external factors (luck, the teacher, their mood) rather than their own innate skill. Indeed, I have noticed that, on average, the girls that I teach are more likely to doubt themselves when facing an ambiguous task, and assume that they don't have the ability to face the task. Given this tendency among female students, which is propagated and established in the media by the lack of appearance of women who are mathematically gifted, I felt it was even more significant to acknowledge any social or psychological factors that may influence how my students approach their interaction with me during a class discussion.

The Importance of Developing Metacognitive Skills

Aside from establishing an environment favorable to the implementation of cognitively demanding tasks and high levels of student self-efficacy, I feel it is essential for students to develop an awareness of how they approach learning tasks. My experiences with scores of gifted math students as a teacher at an elite middle school have shown me that the most successful problem solvers know themselves well enough as students to determine what strategies will be most advantageous in given scenarios, and how to adapt or tweak these strategies if necessary. Since each student carries with him or her a slightly different tool kit of abilities and affinities, it is essential that each develops an awareness of how to approach challenging new ideas and problems in a strategic manner, given the resources that he or she has at his or her disposal. Schoenfeld beautifully distills the somewhat elusive concept of metacognition into three essential questions:

-How accurate are you in describing your own thinking?

-How well do you keep track of what you are doing when you are solving problems, and how well do you use the input from those observations to guide your problem solving actions?

-What ideas about mathematics do you bring to your work in mathematics, and how does that shape the way that you do mathematics?

(Schoenfeld 1987)

In each of these three questions, there is a built-in element of reflection. The student is being asked to observe his or her own thoughts and describe them accurately. For this reason, classroom discourse, whether it involves the entire class or a small group of problem solvers, is an indispensable component of this metacognitive development.

Vygotsky (1962, 1978) theorized that all higher order thinking skills originate in and are internalized by interaction with others. Only in articulating their ideas and arguments to others can students truly solidify the understanding they have of a particular idea and grow cognitively. In this sense, high cognitive demand and metacognitive skill development are inextricably linked. To the extent that a student can be more aware of what he or she knows about an idea, and expresses this knowledge to the teacher and his classmates, the more a student can gain higher levels of cognitive mastery.

Vygotsky's theory of the "zone of proximal development" (ZPD) holds that children can function at a somewhat higher level working with more capable peers (or the adult teacher) than they would working alone. I have seen this theory in action on countless occasions in my classroom. In responding to the comments and insights of more able peers during group problem solving sessions or class discussions, students are able to internalize more readily these insights into their own problem solving. In the process of responding to each others' comments, students are forced engage in the metacognitive processes of justifying, clarifying and sharpening their arguments for others.

In my research study, I looked carefully for instances in which student-to-student interaction helped foster these zones of proximal development for students, thereby spurring them to engage in metacognitive behavior. Moreover, I evaluated the extent to which my exchanges with students encouraged this type of peer-to-peer reflective discourse.

Competing Commitments: A Teacher's Dilemma

It is all well and good for a middle school math teacher to enter the classroom with a commitment to engage the students in cognitively demanding tasks, boost student levels of self-efficacy, and strengthen student's metacognitive skills. The reality is, however, that the developmental realities of the students he or she teaches, combined with the complexities of managing the distinct needs of 15 students simultaneously, make it very challenging to fully meet *all* of these commitments on a consistent basis. A teacher must always be aware of what students are capable of doing, what they are prepared to do (through previous training), and what they are willing to do.

The typical 13 or 14 year-old student, for example, is highly sensitive to the possibility of peer ridicule or disapproval. This self-consciousness can discourage students from sharing insights, commenting on an idea, or making a query to the teacher or class, if they feel that their contribution make trigger a negative peer reaction in some way. The teacher, then, must always make sure that the classroom environment that his or her students inhabit is safe enough for them to feel comfortable taking appropriate risks.

Perhaps more complex, however, is the challenge of responding to student errors in reasoning. Suppose a question is posed to the class addressing a particular concept, and a student responds with an invalid statement. The teacher then has to decide how to respond to this statement. Should the teacher try to communicate to the student – either explicitly or implicitly – that the comment is flawed or erroneous in some manner? Should he withhold any evaluation and ask the student’s fellow classmates to comment on the response? However the teacher decides to respond, how can he or she do so in a way that helps students maintain confidence in their math abilities?

If the teacher avoids any evaluation and leaves it completely up to the students to find the error, the error may go unnoticed. Or, other students may address the error, but make other errors of their own in reasoning as they respond to it. A discussion could quickly veer off course into a tangled web of flawed logic that is never corrected by anyone – student or teacher. Even worse, students may lapse into personal attacks on other students whose ideas they believe are incorrect. If the teacher does not involve himself or herself in the discussion to manage these potentially divisive confrontations, students’ levels of self-efficacy could be seriously threatened.

If, on the other hand, the teacher is *too* involved in correcting the student, the student may become dependent on the teacher to evaluate all his or her responses. Moreover, other students may avoid thinking critically about the student’s comment, since they are accustomed to having the teacher do this for them. In taking on the explicit role of

evaluator, the teacher has cast aside an opportunity for students to practice an important metacognitive skill – discerning the validity of another person’s statement.

Thus, a thorny dilemma confronts the teacher: How can he or she respond in a way that maintains the integrity of the material while promoting independent thinking skills in students? Faced with these seemingly dueling commitments – to metacognitive skill development and high cognitive demand on the one hand, to the maintenance of the material’s integrity and preservation of student self-efficacy on the other – is it possible (and worthwhile) for a teacher seek a path that does service to all his or her commitments?

This dilemma is common to most self-reflective teachers (Ball 1993), so it is no surprise that I face it constantly in my classroom as I navigate my way through the unpredictable currents of a class discussion. All too often, however, I leave these experiences of dilemma management feeling that I have completely abandoned one or more of my central teaching commitments in the process. Perhaps I significantly lower the cognitive demand of a task because I worry it may provoke self-doubt (and lowered self-efficacy) in the minds and hearts of my students. Or maybe I deprive a student of valuable metacognitive training by telling him exactly what is wrong with his answer (instead of letting another student do this) because I am afraid that a peer’s response to his mistaken answer will confuse him or discourage him. The more I have taught, the stronger my desire has become to manage these dilemma experiences in a way that stays as true as possible to my fundamental teaching commitments.

This quest for improved dilemma management became the central motive for conducting a fine-grained analysis of my videotaped lessons. Perhaps, I thought, by better understanding the process of delicately balancing dueling commitments, I could gain a clearer sense of what it would mean to manage these commitments successfully. In order to develop a more focused picture of what a successful approach to this dilemma might look like, however, I needed to construct a mechanism to analyze its success.

Building the Success Framework

As a first step in building this framework, I decided to conduct a thought experiment. I composed a host of teaching vignettes⁴ between a fictional teacher and his students, some of which I thought to be “successful” in some way, and others of which I deemed “unsuccessful.” While the scenarios were intentionally composed to illustrate typical dynamics that I encounter in my practice, I tried my best to make the exchanges as realistic as possible. While reading through the scenarios, I analyzed each to determine what aspects of the scenario seemed successful. The more I identified successful features, the more I realized that there were really *multiple* levels of success, and that I would need to investigate each of these types separately if I wanted to understand the success of the scenario as a whole. Not surprisingly, three of the four types of success coincided with the three commitments that frame my teaching style. I added a fourth – task completion – to acknowledge the reality that students need to cover a certain core

⁴ See Chapter 3 for transcriptions and analyses of each of the vignettes.

body of material with their teacher in order to be adequately prepared for success in their next math course.

Thus, the success framework that emerged from this thought experiment consisted of four components: **task completion, cognitive demand, self-efficacy**, and **metacognitive skill**. As I worked with all four components of the framework, I began to view each as a magnifying “lens” that helped me zoom in on a particular aspect of the interaction. Conceivably, an interaction could be successful through one lens, and unsuccessful for another. This multi-faceted approach to success was necessary, I felt, for generating a more detailed and nuanced picture of success. Managing the dilemma of teacher evaluation is an art -- not a science. Dismissing an episode as an overall failure simply because it was flawed according to one lens could ignore positive and successful aspects of the interaction that may exist from the perspective of another lens.

The Task Completions Lens

The first of the four, or task completion lens, evaluates the extent to which the interaction resulted in the student successfully completing the mathematical task in question. This task could be answering a teacher’s query, or solving a problem correctly. At first glance, the task completion lens seems too mundane to exist in the same framework with such lofty ideas as cognitive demand, self-efficacy of metacognition. And yet, it is essential for it be acknowledged and kept in mind as the others are discussed. After all, what teacher could feel successful about his

students' learning and growth if they are consistently solving problems incorrectly? If a teacher's students consistently engage in non-routine problem solving, but are rarely able to solve the problems correctly, a teacher may think twice about the value (or appropriateness) of these problem solving tasks, even if they are cognitively demanding. Furthermore, if the students and their teacher participated in enriching discussions about mathematics on a consistent basis, but covered little of the material in the curriculum, students could be severely disadvantaged in their subsequent math course the next year.

The Cognitive Growth Lens

The cognitive growth lens measures how well a task engages students in high-level thinking which promotes deep and integrated understanding. More often than not, the success of the interaction from this lens rests on the extent to which the task or question posed to a student exerts a high "cognitive demand" on the student. Stein et al. (1996) define cognitive demands as "the kind of thinking processes entailed in solving the task as announced by the teacher". This cognitive demand could be low, as in the case of simply applying well-established procedures without attention to understanding to a set of similar exercises, or high, such as making and justifying generalized mathematical claims.

The Psychological Lens

The psychological lens charts how a child's level of self-efficacy is affected by the interaction with the teacher and other classmates. The following questions convey the emotional significance of this lens:

Does the exchange help the student gain a stronger sense of his or her ability to tackle non-routine math problems independently?

Does the student leave the exchange feeling affirmed and supported by the teacher and/or his or her classmates?

Does the student leave the exchange feeling confident that he or she can successfully face problem solving challenges in the future?

Clearly, this lens is the most difficult to apply as a research observer. In this project, no surveys were given, nor were any students interviewed about how they felt during the lessons that were videotaped and analyzed. Admittedly, much of the analysis according to this lens is inferential, based on what I remember observing of students during the lessons I taped. And yet, its low level of exactness in depicting students' actual emotional states does not render the lens useless. In fact, by suggesting possible thoughts and feelings that students might have had during their exchanges with the teacher, I expanded my awareness of the spectrum of student emotional responses.

Since a classroom is always composed of an eclectic mix of student personalities, each with his or her own distinct emotional make-up, the teacher's awareness of this range of emotional possibilities is actually an asset rather than a liability.

The Metacognitive Lens

Finally, the metacognitive lens addresses whether or not the interaction helped the student(s) develop a keener awareness of what they know to be true, and how they know it is true. In order for students to handle increasingly complex problem solving, they must be able to ascertain what they can assume to be true in a given scenario, and how they can justify these assumptions.

In essence, all four of these lenses point to essential attributes that a student must possess if he or she is to develop a sound mathematical mind. While a given interaction may not lead to success on all four categories, it is certainly a desirable end for the teacher to seek ways to attain success on as many levels as possible for each interaction that he or she has with students.

Managing the Dilemma

While each category of "success" - high cognitive demand, metacognitive skill, task completion, and self-efficacy - is significant and vital in its own right in determining the ultimate success of an interaction, it is not always easy to shape interactions so that all

types of success are reached. For example, suppose a 4th grade teacher recognizes that students have a variety of opinions surrounding the statement that “dividing makes things smaller.” To engage the students with this idea, the teacher may initiate a debate about the statement, asking students to share their argument with the rest of the class. From a metacognitive and cognitive perspective, this activity has lots of potential. In hearing a variety of perspectives from their peers, students will be forced to examine their own beliefs about division – in essence, examine their own thinking processes. However, the debate could easily devolve into a contentious shouting match between rival camps, leading all the participants to come away from the interaction feeling frustrated and dispirited about the process of mathematical discourse. From a psychological perspective, students whose ideas were labeled “stupid” by others may come away from the discussion with lowered levels of self-efficacy.

Thus, in attempting to facilitate a successful interaction on all levels, the teacher may face a dilemma: seeking to address the success of one component may come at the expense of another. As Ball (1993) points out, it is often not clear how productive it is for a discussion to remain “stuck” on an idea that is mathematically incorrect for an extended period of time without the teacher providing an explanation or explicit correction. Ball addresses many dilemmas a teacher faces, but perhaps the one most relevant to this project is the role of the teacher as an authority figure. On the one hand, the teacher has a responsibility to help make students aware of statements that contradict each other, or of mathematical generalizations (like “dividing makes things smaller”) that don’t hold up under analysis. On the other hand, the teacher cannot control a

student's thought process. If the teacher is committed to not doing the thinking for a child, he or she must walk a fine line between making students aware of errors without making students depending on him or her to find the errors.

In the videotaped lessons I analyzed, these dilemmas consistently resurfaced and demanded to be addressed. While not always resolved, they at least clarified the choices I had as a teacher at particular junctures in the class discussion. In this sense, Ball's (1993) and Lampert's (1985) notion of these dilemmas as challenges to be managed rather than problems to be avoided proved to be freeing as I made difficult choices on the spot about how to respond to student errors or misconceptions. All the same, I was curious as to whether there ways in which I could have managed these dilemmas more successfully.

Questioning Patterns: An Entry Point for Structural Analysis

While the success framework provided me with a mechanism for assessing the effectiveness of my dilemma management in a given scenario, I also needed a way of describing my exchanges with students from a broader structural perspective. Were there overarching patterns in the way that I communicated with students during these experiences of dilemma management? Did I have a distinctive dilemma management approach that could be described and categorized, just as personality types have been codified by Myers and Briggs (Pittenger 1993)? Was this approach consistent, or did it change depending on the nature of the scenario?

My hypothesis at the start of the research project was that I *did* have a distinct dilemma management style. As a creature of habit, I theorized, I would surely develop conditioned responses to familiar dilemmas, given the fact that I faced them on a daily basis. Furthermore, these conditioned responses would no doubt vary in nature depending on the context, since there was no “one size fits all” approach that would work effectively in all situations.

In investigating my dilemma management style, it seemed most logical to focus on the questions that I pose to my students. More than any other teaching technique, the act of posing a carefully constructed query to students is the most common method that I use to assert my own influence in guiding a discussion or brief exchange with students. Indeed, the majority of the “teacher talk” in my classes comes in the form of questions, whether they are seeking clarification from a student whose logic is not clear, or challenging the class to consider a connection between seemingly unrelated topics. In analyzing the purpose, content, and timing of the questions I pose to students during these dilemma management experiences, I felt that I could gain an intimate sense of my approach to dilemma management.

A central research hypothesis of this study is that the nature of these questions – how they are constructed, whom they are posed to, and what they communicate to students – plays a key role in shaping how students engage with me during our exchanges. As I seek to honor the sometimes competing commitments of cognitive demand, self-efficacy

and metacognitive skill, do these patterns of questioning limit or enhance my ability to manage the dilemma effectively? The analyses of both hypothetical and real teaching scenarios in the ensuing chapters seek to form a clearer link between these questioning patterns and the mental and emotional processes that I engender in my students. I do not try to present one questioning pattern as the most effective pattern for bringing about success on all four levels in every situation. Instead, it is hoped that by analyzing the application of a number of different patterns in several classroom scenarios, the link between questioning patterns and “success” -- according to the four lenses of cognitive demand, self-efficacy, metacognitive skill, and task completion -- will be clearer.

Chapter 3 – Highlighting Questioning Patterns: A Thought Experiment

A Return to the Hypothetical Scenarios

With the four-part framework for success established and codified, I could now conduct a rigorous evaluation of the hypothetical scenarios using specific and well-defined language from the literature. While initially an outgrowth of the scenarios themselves, the framework could now function as a *tool* to help articulate the contours and attributes (both positive and negative) of teacher-student exchanges in specific and well-defined language. Moreover, the transcriptions of the scenarios provided fertile ground for the identification of recurring communication patterns between my students and me.

Since I knew that posing carefully constructed questions was a common technique that I used to direct and manage discussions with students, it seemed that a focus on questioning patterns could provide a concrete mechanism for understanding the manner in which I manage the dilemma of teacher evaluation in my practice. With a deeper awareness of the common questioning patterns that could infuse my teaching, I felt that I could approach my videotaped lessons with a keener awareness of their benefits and risks as I implemented the patterns. In essence, these fictional vignettes could serve as “previews” of the videotaped lessons to come, and give me an opportunity to highlight essential aspects of the transcribed teacher-student exchanges in order to determine their success.

Exploring the Scenarios: A Two-Pronged Approach

In the first portion of this chapter, I demonstrate how a student-teacher exchange around a given problem could be developed in four different ways⁵, each of which illustrates a slightly different questioning pattern. Following the transcription of each vignette, I conduct a two-pronged discussion of the scenario. The first component is the identification and description of the questioning pattern(s) most prominent in the exchange. The second component is the evaluation of the scenario's success from the perspective of the four lenses: task completion, cognitive demand, self-efficacy and metacognitive skill. A working hypothesis is that these two components are in fact related --- that the type of questioning pattern used may enhance or undermine the success of the scenario according to one or more of the four lenses. In analyzing four different scenarios from the dual perspectives of questioning pattern and success, I hope to discern whether any links may exist between the type of questioning pattern used, and the success of the scenario according to one or more of the lenses. These links, if they do exist, might help me develop a better awareness of the benefits and dangers of specific questioning patterns that I use in my actual lessons.

Before describing the hypothetical scenarios, however, an important distinction between the processes of questioning and hinting needs to be made.

⁵ Of course, these four methods are not meant to be exhaustive. Hopefully, they give the reader a sense of the variety of approaches available.

Questioning vs. Hinting: A Clarification

Questioning refers to an ongoing dialogue between the teacher and a student or entire class that is employed when a student answers a question incorrectly and displays a lack of understanding of a particular concept. The questions asked by the teacher normally address fundamental concepts that are essential for the comprehension of the misunderstood concept at hand. The responses given by the student provide the teacher with more information and clarity about the nature of the student's misunderstanding. At the same time, the questions point the student's attention to relevant mathematical principles in order to encourage him or her to view the concept at hand in the context of these principles. This back and forth process continues iteratively, with student responses greatly influencing subsequent questions, until ultimately the student displays improved understanding.

Hinting refers to a technique of questioning and suggesting employed by the teacher when a student, a small group of students, or an entire class appears unable to make progress with a particular problem or question posed to them. It may take the form of a question (Ex: Is it possible for a linear system to have exactly two solutions?") or a suggestion (Ex: "Try looking for a pattern in the ones digits of this sequence of powers of 2.") In either case, however, the intention is to point students in a direction that helps them to be cognizant of the tools they have at their disposal to make progress. These tools may be previously learned knowledge or techniques that they had neglected to consider, or simply the application of number sense or spatial visualization. It is

distinguished from questioning in that it tends to be more unilateral, from teacher to student, and does not draw upon student responses to craft subsequent questions or suggestions.

In short, questioning consists of a student-teacher dialogue meant to help a student perceive his or her misunderstanding and recognize a valid solution method. Hinting directs student attention to the necessary tools they need to solve a problem.

In this project, I focused on my questioning patterns rather than my hinting patterns. For this reason, the scenarios that following address the questioning patterns. However, an example of “hinting” scenario, along with the analysis of its success, is available in the Appendix for comparison.

A Questioning Scenario:

The following is the transcript of a hypothetical scenario involving the process of questioning in a 7th grade lesson on the Pythagorean Theorem. At a certain point early on the scenario, the teacher faces a typical dilemma – a student is confusing the square of the hypotenuse with the hypotenuse itself in a problem involving a 6-8-10 right triangle. I provide four different approaches with the student when confronted with this dilemma. The first approach is to direct the student’s attention immediately to their mistake with a pointed question. The second uses manipulatives to help the student perceive a logical contradiction in his argument. The third spends more time focusing on the concept of the square root, and its inverse relationship to the square function. The

fourth approach involves telegraphing the answer to the student through verbal cues, without addressing the student's conceptual misunderstanding.

In each case, I have represented the student, Justine, by "S" and the teacher (myself) by "T." The teacher and student comments are numbered for reference purposes.

- 1 T: OK, guys. Suppose I have a right triangle with legs of 6 cm and 8 cm. (*Draw triangle on SmartBoard.*) How would I find the length of the hypotenuse?
- 2 S: Well, you would square 6 and get 36, then square 8 and get 64, and add those two together to get 100. So the long side [hypotenuse] is 100.
- 3 T: 100? [*in case I heard her incorrectly*]
- 4 S: Uhh, yeah, $36+64$ is 100. 100.

At this point, I recognize immediately that the student has forgotten that the squares of the legs add up to the square of the hypotenuse, not the hypotenuse itself. I could take a number questioning routes at this point. Each route is characterized by a specific questioning approach. In the ensuing pages, I imagine four different approaches to this scenario with Justine, and assess their strengths and weaknesses. Each scenario begins with Comment 5, picking up where the above episode leaves off.

Questioning Pattern #1 : Directing a Student's Attention to the Mistake

The first option, which is the most typical one I use, could be represented by the exchange below:

- 5 T: OK, remind me what formula we discovered a few days ago that relates the three sides of a right triangle
- 6 S: $a^2 + b^2 = c^2$.
- 7 T: Do you remember the name of that formula?
- 8 S: Ahhhhh, umm, Pythagoran or something like that.
- 9 T: Very close – does anyone remember?
- 10 Other student: Pythagorean Theorem
- 11 T: Good memory – now let's get back to this triangle. Julia (student's name), what are "a", "b", and "c" in this case [*I point to the triangle drawn on the board*]
- 12 S: Well, a is 6, b is 8, and well we don't know c yet.
- 13 T: OK, great.

This is an important "choice point" in the conversation between the student and me. I could ask her if a could be 8 and b could be 6, but given the flow of our dialogue and the fact that she is on the spot, I decide not to. This is a question I would ask the entire class after we solved the problem, or if we were doing it together, I would pose that question to the class at the beginning. I don't want to overwhelm her with too many tangential

questions at once. If she were a very strong student with lots of verbal confidence, I would consider doing it for the benefit of the class.

- 14 T: Now take me through how you got 100 again, but show me how you used the formula as you explain.
- 15 S: OK, well, $a^2 + b^2 = c^2$, so $a^2 = 36$ and $b^2 = 64$ so $a^2 + b^2$ is 100 -- that's c^2
- 16 T: Is c^2 the length of hypotenuse?
- 17 S: [*pause, ...then a look of realization*] – ohhhhh, noo – you need to ah, ah, ah square [*searching for word...*] square root it.
- 18 T: Square root what?
- 19 S: 100
- 20 T: Why?
- 21 S: Because 100 is c^2 not c .
- 22 T: Oh, OK. So what is the length of the hypotenuse?
- 23 S: 10
- 24 T: 10 what? 10 jelly beans?

This last question is a common rhetorical device I use when a student doesn't include the units of measurement in their answer. While it does telegraph that they need units, it also reinforces why including units are important. I could spend more time trying to get the student to realize on his own that he left out the units, but I am more concerned that

he understands why the units are important in the first place. I sacrifice a little student autonomy to make a point.

25 S: 10 centimeters.

26 T: Excellent, Justine!

Analysis of Questioning Pattern #1: Highlighting the Pattern

Notice that in this pattern, the teacher call's the student's attention directly to the mistake made by asking, "Is c^2 the hypotenuse?" By forcing the student to address this issue, the teacher is encouraging him to reconsider his answer. Even though the teacher's comment is *phrased* as a question, it is actually not a question at all. The question actually functions an implicit cue to the student that his answer was somehow flawed, and needs to be corrected. Indeed, at each step in the process, the teacher either affirms the student's response, or asks the student to consider a particular idea more carefully. In both cases, the teacher's response is a form of evaluation. Thus, the series of teacher questions and responses follows a predictable pattern – teacher question, followed by a student response, followed by a teacher's evaluation (positive or negative) of the response. The exchange is largely confined to a unilateral discussion between the teacher and the student (S), so the student responses are limited to one individual.

Analysis of Questioning Pattern #1: Evaluating Its Success

From a task completion perspective, this approach yielded moderate success. The student was able to correctly find the hypotenuse with some prodding and questioning from the teacher. And yet, from a cognitive demand standpoint, the level of thinking asked of the student in this interaction was relatively routine. The teacher merely led the student through the formulaic application of the Pythagorean Theorem until the student noticed the mistake. From the perspective of self-efficacy, the student probably left feeling confident in his or her ability, given the success in task completion. However, the success was developed with a heavy dependence on the teacher's help, so this self-efficacy is somewhat superficial. Metacognitively, there was not much asked of the student. While the student was forced to re-assess his answer, he was not encouraged to reflect on his own about the response, nor were other students encouraged to do so. From multiple vantage points, then, the approach can be considered an unsuccessful one.

Questioning Pattern #2 : Using Manipulatives to Make Logical Judgments

A second approach, which involves more visual representation and concrete manipulatives, is demonstrated below in another version of the above scenario.

5 T: I am a visual learner. Would you mind helping me see this right

triangle with some Unifix cubes?

A few days before, we performed an exploration that looked at triangles that were right and ones that were not right triangles. We built the legs of the triangles with Unifix cubes and then built the third side of the triangle. If the third side fit, we decided the three side lengths could be a right triangle. If not, the triangle wasn't considered a right triangle. The kids discovered that in all the right triangle cases, the sum of the squares of the legs equaled the square of the third "long" side. They enjoyed working with the cubes, so using the cubes usually engages them well.

- 6 S: Sure
- 7 T: Great, folks at the table, could you pitch in so we can do this quickly? I'd like to see the two legs built.
- 8 S: [they build the two legs]
- 9 T: Now, what was our third side length? 100? Could you guys build that side?
- 10 S: [Immediately S has good-natured grin on her face and chuckles] OK-
- so 100 is clearly not right
- 11 T: Why not?
- 12 S: Well, clearly 100 won't fit on this triangle.
- 13 T: OK, let's go back to the triangle. Why don't you take me through how you found the third side?
- 14 S: well, $a^2 + b^2 = c^2$ so $a^2 = 36$ and $b^2 = 64$ so $a^2 + b^2$ is 100, and such c

is 10 --- oh yeah, of course – it's 10, not 100.

Discussion of Questioning Pattern# 2: Highlighting the Pattern

In this scenario, the questions the teacher poses come across more as requests or instructions. The teacher's main teaching tool is the activity of creating the triangle with the Unifix cubes, so the diagnostic quality of the questioning in the first scenario is noticeably absent. And yet, the teacher is no less involved in guiding the students' thought process. Instead of using questions to send kids subtle cues about the validity of their responses, he is using them to issue instructions to the students about how they should approach building the triangle, with the hope that they will realize their mistake. Indeed, instead of letting the students determine why 100 "won't fit on this triangle," he directs their attention to the process of finding the third side. In this process, a pattern develops that starts with a teacher command, is followed by a student observation, which is followed by another teacher command. Again, the exchange is limited to a one student, although a couple other students did participate in the construction of the triangle with Unifix cubes.

Discussion of Questioning Pattern #2: Evaluating Its Success

From a task completion perspective, the scenario was certainly a success. The student was able to realize her answer of 100 was incorrect once she constructed the triangle.

Upon being asked to recount her solution method, she approached it again with more careful thought, and derived the correct answer. And yet, the task of determining her error was by no means a cognitively demanding one. She was merely following the explicit guidance of her teacher, who told her and her peers at the table exactly what to do with the Unifix cubes. It seems that the teacher is in somewhat of a hurry to get the triangle built so that the student's error can be quickly exposed (see Comment 7). The student does not need to think about *how* Unifix Cubes could be used to assess her own answer, nor do her classmates need to think about how they would use the Unifix cubes to evaluate it. While the student does catch her answer upon recounting her process, she has been directed to do this *by the* teacher. In terms of self-efficacy, the student probably left the exchange feeling good about arriving at a correct answer. However, the student may not see any value in the use of the Unifix cubes and may not know how to approach a future problem that cannot realistically be solved with the use of manipulatives. Once again, the student's dependence on the teacher limits her self-efficacy to a more temporary feeling of accomplishment. Metacognitively, very little activity has occurred. Students did not need to generate any methods for testing the answer on their own – their method was provided to them by the teacher. On the whole, then this scenario could be termed more of a failure than a success, even though it involved a helpful visualization tool in the Unifix Cubes.

Questioning Pattern #3: An Emphasis on the Concept behind the Computation

A third approach, which focuses on the nature of the square root and what it means, is demonstrated below:

Let's pick up the dialogue at the line below....

- 5 T: A question for you. You said $a^2 + b^2 = c^2$. What does c squared stand for – the hypotenuse?
- 6 S: Ohh, nooo!! C is the hypotenuse...
- 7 T: OK, so if we know c squared is 100,, how can we find c?
- 8 S: [confusion, blank look...] uhhhh..., divide by 2?
- 9 T: You don't seem convinced.
- 10 S: I'm not.
- 11 T: What does it mean to square a number?
- 12 S; Multiply it by itself.
- 13 T: Good. Let's consider an actual square, since we're talking about "squares". What makes a square a square?
- 14 S: All four sides are the same.
- 15 T: Excellent.

Depending on the flow and the students comfort level,. I may ask, "Anything else necessary for it to be a square?" If she says no, I will draw a rhombus on the board and

label all sides the same length and ask, “Is this a square?” to which she will say no. And I’ll say, “What does it need to be a square?” and she’ll say, “the right angles. However, this could get us out of our rhythm, so I would probably hold off on that interlude.

- 15 T: If the side length of the square is 5 cm, what’s the area?
- 16 S: 25.
- 17 T: 25 pizzas?
- 18 S: 25 square centimeters. (smiling)
- 19 T: How did you find that?
- 20 S: I just multiplied 5 by 5.
- 21 T: What’s another way of writing a number times itself like 5 times 5, or 6 times 6?
- 22 S: Squared.
- 23 T: So another way of writing c times c would be...
- 24 S: c squared
- 25 T: [*I draw another square on the board but don’t label its side lengths.*] What if I told you that the area of this square was 100 square inches. How would you find the length of each side? Would you divide 100 by 2?
- 26 S: no... uhh. It would be ...10
- 27 T: Why?
- 28 S; Because 10 times 10 is 100

- 29 T: Wouldn't it be great if we could find what number was multiplied by itself to get any answer, not just 100, or 25 or 36? Is there any tool on the calculator that helps us do that?
- 30 S: Oh yeah, the little heartbeat symbol [class laughter]
- 31 T: Yeah it does sort of look like an EKG. So what do you call that symbol? [*This question I would pose to the class to take the student out of the spotlight for a second.*]
- 32 Other student: The square root
- 33 T: Right -- the square root. So, Justine what did you do to 100 to get 10?
- 34 S: I took the square root.
- 35 T: Excellent.

Analysis of Questioning Pattern #3: Highlighting the Pattern

In this scenario, the teacher periodically asks yes or no questions that are in reality statements or cues to the students. For example, when the student says the area is 25, the teacher asks "25 pizzas?" which is a question whose answer is clearly "no" (See Comment 14). By asking the student this outlandish question, the teacher is silently communicating to the student that he has not included the units of measurement with his area calculation. In Comment 26, the teacher uses a similar technique, but this time asks a leading question whose solution is implied to be "yes" – after all, why would the teacher ask about a tool on the calculator (which is rarely used in class) if it did not

exist? Thus, the teacher continues to use questions to communicate silent messages to the student by asking questions with a clear “yes” or “no” answer. As in previous scenarios, the exchange is largely unilateral, between the teacher and one student.

Analysis of Questioning Pattern #3: Evaluating its Success

The students completed the task successfully after being reminded of the existence and function of the square root. The level of cognitive demand was slightly higher than the first two scenarios, since the teacher ask the student to think how the geometry concept a square related to the idea of squaring a number. Moreover, by asking the student to find the side length of a square whose area was 100 (Comment 25), the teacher was forcing the student to engage with the notion of a square root as the inverse operation of squaring a number. From a self-efficacy standpoint, the student no doubt enjoyed being to draw on her knowledge of the square root button to solve the problem. Once again, however, the student’s feeling of success could very well be short-lived, however, since she was dependent on her teacher for the success. Finally, the teacher did not engage the class in any metacognitive analysis of the student’s conception of the square root. When the student offers “dividing by 2” as a possible method for finding the square root (Comment 7), the teacher could have facilitated a discussion about whether there was any difference between dividing by 2 and square root, posing queries such as the following:

Does dividing by 2 ever have the same effect as square rooting a number? If so, does dividing by 2 always have the same effect as square rooting a number? How could you prove your answers are correct, no matter what number you choose?

Thus, while the teacher broke new ground by engaging students in a study of what a square root means, he squandered some opportunities for metacognitive skill acquisition.

Questioning Pattern #4 : Telegraphing the Answer

A fourth approach, which involves “telegraphing the answer” to a student with explicit verbal and non-verbal cues, is demonstrated below in the following exchange, starting at line:

- 5 T: Are you sure the third side is 100? Isn't that a little big?
- 6 S: Well, yeah, I guess you're right
- 7 T: Take me through your process.
- 8 S: Well $a^2 + b^2 = c^2$ so 6 squared plus 8 squared is 36 plus 64, which is 100
- 9 T: But, Justine, does a squared plus b squared equal C? [In my tone of voice, I am implying to her that the answer to this question is NO!]
- 10 S: No.. Ohh, it's c squared
- 11 T: So if its c squared how would I “undo” the square to get c? [*Here I am telling her what to do*]
- 12 S: I'm not sure – divide by 2, by itself?

- 13 T: What button on our calculator did we use to undo squares yesterday?
- 14 S: Oh year, square root
- 15 T; OK so what's the square root of 100? [Notice I am telling her how to use the square root.)
- 16 S: 10
- 17 T: So the hypotenuse is 10
- 18 S: Yup.
- 19 T: Just 10? You mean 10?
- 20 S: Oh, 10 cm

Analysis of Questioning Pattern #4: Highlighting the Pattern

Out of the four questioning patterns presented, the one presented in this scenario depicts the highest level of teacher involvement in the evaluation of student responses. Instead of *implying* subtly that a student response is incorrect with a request for clarification, the teacher in this scenario sends *explicit* messages to the student in the way he phrases his questions. If a student's response is incorrect, he directly communicates his doubts about the student's reasoning by questioning *the response itself*. In Comment 5, he asks the student whether she's sure the hypotenuse is 100, given the fact that 100 seems "a little big"? There is absolutely no doubt in the student's mind that 100 is the incorrect answer, and that the correct answer is smaller than 100, since the teacher said it was too big. In Comment 9, the teacher makes it clear to Justine that her mistake lies in her mixing up c and c^2 . In Comment 11, the question that the teacher poses is actually a

command in disguise (i.e. that she should “undo” the squaring process to get c.) In essence, the teacher has a goal of “getting” Justine to a statement of the correct answer, in the fewest number of steps. If she is confused or unsure about how to proceed, the teacher will step in and give her explicit instructions about what to do next. This pattern -- a teacher inquiry, followed by a student response, then followed immediately by a teacher evaluation, closely parallels the processes we have seen in previous questioning scenarios. Moreover, like the other scenarios, the discussion is confined to one student who is “led” by the teacher to a correct answer through strategic and suggestive questioning. The only small difference in this vignette is that the teacher’s tacit cues to the student are significantly more explicit and transparent for all to see.

Analysis of Questioning Pattern #4: Evaluating its Success

While the student did ultimately find the correct answer to the problem posed to her, she gained very little from this interaction in terms of cognitive growth, heightened self-efficacy, and metacognitive skill acquisition. For one, she had to do very little thinking – the teacher virtually scripted her responses for her through his leading questions. Moreover, she never really had to grapple with the concepts that confused her, such as the inverse relationship between the square and square root functions. This was due to the fact that the teacher stepped in and told her how to proceed at every uncertain junction in her path of logical reasoning. He never asked her to justify her claims with arguments, or evaluate her correct statements. For example, in Comment 17, the teacher could have asked the student how she knew for sure that the square root of 100 was 10,

or how she knew that *any* number was in fact another's square root, for that matter. By simply affirming her answer of 10, the teacher lost a valuable opportunity for developing her metacognitive skills. Finally, it is unlikely that the exchange boosted her self-efficacy very much, if at all, given the fact that she was completely dependent on the teacher's guiding instructions throughout her problem solving process.

General Patterns: A Summary of the Hypothetical Scenarios

In dissecting both the hinting and questioning scenarios above, it is clear that the exchanges between teacher and student, or teacher and class, follow a familiar pattern. The teacher asks a question, the student responds, and the teacher indicates to the student whether that response is correct or not. An affirmation like "That's great!" or "OK, good." Indicates the response is correct, while a question that asks for further clarification like "could you explain how you got your answer?" subtly communicates to the student that he or she has made a mistake in computation or reasoning. Thus, the IRE paradigm is highly visible in all of the questioning and hinting scenarios, and establishes the role of the teacher as an authority who determines what is correct and what is incorrect for the benefit of the students.

From the perspective of the four lenses, the success of each scenario is mixed at best. While some involve kids that come away feeling happier or more confident, the levels of self-efficacy are compromised by the fact that the students' successes are closely intertwined with the constant guidance of the teacher. While most scenarios ultimately

ended with the student(s) successfully completing the given task (or correcting an error correctly), it is questionable how much the exchanges represented cognitively demanding tasks. With such extensive teaching involvement, it is difficult to ascertain what level of autonomous analysis was going on in the minds of students. There is little evidence of the development of metacognitive habits such as strategizing, logical argumentation, and self-monitoring among the students.

The Persistent Pattern: Inquiry Response Evaluation

In reading through the transcripts of each of the hypothetical scenarios described above, it was clear to me that my patterns of questioning fall into a predictable and well-established rhythm. They begin with an inquiry, where I ask a question, to which there is a student response. This response is followed by my evaluation of this student's response using various "cues" that tell the student whether I "approve" or "disapprove" of the response. For example, I may affirm a response's validity by stating "OK, that's great" or by simply writing the student's ideas up on the board. In contrast, I may indicate to a student that his or her response is flawed by questioning how they come up the response, posing a request such as, "Explain to me how you came up with that answer." Whether my evaluation is explicit or implicit, it is consistently immediate and directly aimed at the student who gave the response. Consequently, students in these scenarios (and those in my classes by extension) would naturally become conditioned to look for cues from me indicating whether or not the response they gave was valid.

The pattern closely resembles the Initiation-Response-Evaluation (IRE) sequence explored by Cazden (1988) and Mehan (1979). With the IRE pattern so firmly established in my teaching practice, I do not leave much space for multiple student responses to a particular question, nor do I provide incentives for students to evaluate each other's responses for themselves. After all, they know they can count on me to do the evaluating for them. This dependence on teacher evaluation poses a threat to success from three of the four lenses. From the self-efficacy perspective, the student loses the opportunity to develop confidence in his or her ability to assess the validity of a statement on his or her own. How can he or she truly possess confidence in his or her math ability if it is dependent on the teacher's constant guidance? From the cognitive demand level, the student could navigate his or her way through an exchange with me by reading my cues instead of drawing on his or her firm understanding of the concepts at hand. In this sense, the student could be lowering the cognitive demand of the interaction by following the teacher's directions instead of reasoning his or her way to a solution. In addition, student conditioned to follow the IRE pattern does not have the opportunity to judge the validity of other's comments, and argue for the validity of his or her own ideas. This denies the student an opportunity to think about someone else's thinking, a key metacognitive skill.

Limitations of the Unilateral Approach

In addition to reflecting the IRE paradigm, the questioning style was frequently characterized by a unilateral approach towards teacher-student exchanges. More often

than not, the teacher in the scenarios limited his interaction to one or two students, as he sought to correct their misconceptions through a string of targeted questions. This focus on one student probably sprung from the teacher's desire to ensure that a particular student had the opportunity to work his way through the entire reasoning process along with the teacher. However, by excluding the rest of his students from the exchange, the teacher lost an opportunity to engage *them* in the cognitively demanding task he was exploring with the single student. If other students aren't asked to engage in the exchange between the teacher and the student, what motivation will they have to listen to the teacher and student's comments? If they get the sense that the exchange is just between the responding student and her teacher, they will quickly lose interest in the comment of both teacher and student, making the discussion irrelevant to all but two individuals in the classroom. This limited range of interaction then, would appear to seriously hamper the teacher's ability to promote a student-to-student commentary and analysis that would significantly enhance the metacognitive abilities of participating students.

Revoicing: An Alternative Approach to Managing Student Responses

Given the limitations that the IRE pattern and unilateral approach appear to present, it would seem helpful to seek an alternative approach to managing student responses that expands student participation and facilitates success on all four levels. O'Connor and Michaels (1993) point to the technique of revoicing as a potential tool for managing the flow of student contributions while at the same time maintaining learning objectives. In

this technique, the teacher re-articulates student contributions in such a way that students are able to respond more pointedly to each other without becoming overly confrontational. This technique may allow a teacher to engage students in a discussion with high cognitive demand without immediately evaluating student comments as they are given. In order to be able to respond to other student's comments, students must first understand the essence of their peers' contributions. Using revoicing, a teacher can act as an effective discussion facilitator, enabling student comprehension of student comments while withholding immediate evaluation. In this way, the teacher can promote metacognitive skills among his students, both through the encouragement of student comments and through the careful facilitation of student-to-student communication. Finally, as O'Connor and Michaels point out, a student whose voice is heard is more likely to feel valued as a member of his or her classroom community, which could potentially lift his or her self-efficacy. At the same time, if the teacher mismanages a class debate and allows students to disrespect or ridicule each other's opinions, a student's self-efficacy may suffer as a result of the discussion.

It would appear, then, that the use of revoicing could potentially engender success on the cognitive growth, metacognitive and psychological levels. On the task completion level, the interaction may not be as successful, since the students might not reach the desired conclusion or valid claim as quickly. This dilemma of time management is one the teacher will have to weigh carefully. It may force a teacher to revise his or her learning objectives a particular lesson to accommodate time restraints.

Applying Revoicing: Another Imagined Scenario

Consider the following hypothetical scenario involving an 8th grade algebra student named Robert is studying systems of linear equations in two variables. In this scenario, the teacher uses a revoicing strategy to give him a more empowered voice. He is presented with the following word problem:

I am thinking of a two-digit number. When 54 is added to the number, the result is the number with its digits reversed. The ones digit is 6 more than the tens digit. Find the two-digit number.

Here is the teacher's attempt to engage Robert in explanation his solution process:

- 1 Teacher: So, Robert, how would you approach this problem?
- 2 Robert: Well, we can write the two-digit number as $10x + y$, so we could see $10x + y + 54 = 10y + x$. That's one equation.
- 3 Teacher: OK, good. What do x and y represent?
- 4 Robert: x represents the... tens digit and y represents the ones digit.
- 5 Teacher: Alright, are there any more equations you could set up using the given information?
- 6 Robert: Hmm – it talks about digits. $Y = 10x + 6$ is another equation.
- 7 Teacher: Tell me how you derived that equation.
- 8 Robert: Well, that's what it says, Mr. C – the

- 9 Ones digit is 6 more than the tens digit, so I added six.
- 10 Teacher: OK, but how did you get the 10x part?
- 11 Robert (*starts looking a little flustered and irritated*) - because it's the TENS digit, so I need to multiply it by 10.

The teacher realizes Robert is struggling to communicate his thoughts, so he decides to apply the revoicing strategy to help Robert make his voice heard in the discussion.

- 12 Teacher: So you're saying that the number in the tens place actually represents something that is ten times that number. Is that what you mean?
- 13 Robert: Yeah.
- 14 Teacher: I think I understand. It would help me to understand the statement about the digits if we took a sample two-digit number. Help – I need a two-digit number. Robert, what's your favorite two digit number?
- 15 Robert: 48!
- 16 Teacher: Alright, 48. So what would you like x to be and what would you like y to be, Robert?
- 17 Robert: Let's make x the tens, and y the ones.
- 18 Teacher: So what's the value of x and what's the value of y in this case?
- 19 Robert: x is 4 and y is 8.
- 20 Teacher (*writes blanks underneath each digit, and then x under the 4*

and y under the 8)

- 21 Teacher: Could anyone make a statement about the two digits in the number 48?
- 22 Christina: The ones digit is twice the tens digit.
- 23 Teacher: That's great. Could someone write Christina's statement in the form of an algebraic equation with x and y ?
- 24 Erica: $y = 2x$
- 25 Teacher: Any other equations that could relate x and y ? Philip?
- 26 Philip: $y = x + 4$
- 27 Teacher: Do you agree with Philip, Betsy?
- 28 Betsy: Yeah, sure.
- 29 Teacher: Huh, that's interesting, Betsy. Why wouldn't Philip want to write this..... [*Teacher writes $y = 10x + 4$ on the board*]
- 30 Betsy: Because that doesn't work. $10(4) + 4$ doesn't equal 8.
- 31 Teacher: Oh, OK. It seems like Betsy is saying we don't need to multiply by 10. What do you think, Sheila?
- 32 Sheila: No, we don't because we're just talking about the digits.
- 33 Teacher: So is it important for us to know whether we are talking about a digit all by itself or what the digit stands for, Chris?
- 34 Chris: Yeah, definitely.
- 35 Teacher: So let me read that statement from the last problem again. The ones digit is 6 more than the tens digit. Are we talking about the digits all on their own or what they stand for? What do you think, George?

- 36 George: All on their own.
- 37 Teacher: OK. Peter, could you give me an example of a number where the ones digit is 6 more than the tens digit?
- 38 Peter: Sure – 93.
- 39 Teacher: So if x is the tens digit and y is the ones digit (teacher write two blanks with x underneath the first and y underneath the second) then what equation could we write with x and y , Robert?
- 40 Robert: $y = x + 6$
- 41 Teacher: Folks, notice how Robert made a really important point. Since we are just talking about the digits on their own, and not what they stand for, he wrote $y = x + 6$ instead of $y = 10x + 6$. The first equation that Robert came up with: $10x + y + 54 = 10y + x$ has to do with the value of the two digit number, and the second has to do with the digits themselves. Let's try to go ahead and solve this system. Anyone have any suggestions?
- 42 Susan: Substitution would work since y is all alone.
- 43 Teacher: So how would you apply substitution here?
- 44 Susan: Just put $x+6$ where you see y in the first equation.
- 45 Teacher: Great. Any other suggestions?
- 46 David: Well, we might want to make the first equation simpler first and get all the x s and y s on one side.
- 47 Teacher: So you're suggesting that we can save ourselves some work in the substitution if we simplify the first equation?

- 48 David: Yeah
- 49 Teacher: OK, why don't a few people try Susan's method and a few others try David's method? (*Asks for volunteers.*) Kids solve the system using different methods.
- 50 Teacher: How many used Susan's? (3 students' hands goes) How many used David's? (*9 students' hands go up*). Those of you who chose substitution right off the bat, what made you prefer that?
- 51 Maria: I just like having an equation with one variable right away.
- 52 Teacher: And those who chose David's?
- 53 Peter: Well, there was less simplifying to do.
- 54 Teacher: What did people get as a two-digit number?
- 55 Maria: 28
- 56 Teacher: OK. You used the "substitution right away method". Anyone else get that answer who used a different method?
- 57 Robert: Yeah. I simplified first and then used substitution, and I still got 28.
- 58 Teacher: So it seems like there are different approaches to solving this system, based on your preferences. Is that right?
- 59 Class: Yeah.

Discussion of Scenario: Highlighting the Pattern

Throughout the revoicing scenario, the teacher applies a fairly consistent approach. He poses a question to the class, and asks for a response. After receiving a student's response, he calls for another method or response that might be different from the first one (and maybe even a third or fourth distinct method). Then he asks the class to comment on the responses that have been given by the various students. If he thinks that a response needs to be clarified, he revoices the response in language that he thinks will both convey the student's intended logic while at the same time highlighting an important mathematical idea. After revoicing a student's contribution, he continues to refer to that student comment when the idea encapsulated by the comment arises later in the discussion. In this way, he keeps the ideas presented by students alive and visible for all to draw upon as the discussion progresses.

Discussion of Scenario: Evaluating the Success of Revoicing Strategies

This interaction was at least mildly successful on all the levels. For one, Robert, was able to successfully solve the problem correctly, so from a task achievement level, he succeeded. From a cognitive perspective, Robert was able to recognize the difference between a digit "on its own" and what the digit "stands for". This was in part enabled by the teacher's formulation of the dichotomy between digits on their own and the value they represent. However, Robert still needed to be able to distinguish between these two possibilities, and he did so correctly in the interaction.

From a psychological perspective, the teacher's use of revoicing allowed Robert to see his realization (that multiplying by 10 wasn't necessary in the second equation) as a valued contribution to the class discussion. Consider this excerpt from the discussion:

Folks, notice how Robert made a really important point. Since we are just talking about the digits on their own, and not what they stand for, he wrote $y = x + 6$ instead of $y = 10x + 6$. The first equation that Robert came up with: $10x + y + 54 = 10y + x$ has to do with the value of the two digit number, and the second has to do with the digits themselves. (Comment 41)

By making the summary statement above, the teacher clarifies Robert's misunderstanding to him and the entire class while at the same time giving him credit for the clarification. This was a powerful way of allowing Robert to see his initial mistake as a slight misstep instead of a humiliating lack of understanding. Robert can come away from this interaction feeling positive not purely for getting the right answer, but also for making an important contribution to the class discussion. This will do much more for Robert's self-esteem as a math student than the first interaction, in which he got the correct answer by merely doing what the teacher told him to do.

Finally, from a metacognitive standpoint, Robert was building a number of habits of mind during the interaction. The first was to use sample numbers to help create algebraic representations of generalized scenarios. The "48" portion of the dialogue established for Robert the absurdity of writing $y = 10x + 4$ to describe the digits in 48, because he

could see that these numbers didn't fit that rule. The second habit of mind was to see the value of multiple solution methods, and to evaluate the benefits of each. Robert was able to simultaneously see why Maria's method was valid (it still resulted in the correct answer) and why he preferred his method. Developing the ability to view solution methods as options or tools to attain specified problem-solving goals rather than pre-ordained procedures dictated by the textbook or teacher is essential for students to become more autonomous as problem solvers. By evaluating the option before him and choosing one that he deemed most effective, Robert was exercising this metacognitive skill.

In conclusion, the revoicing strategies demonstrated by the teacher in this scenario helped to make the interaction successful on a number of levels. By reformulating the problem in terms of a sample number (48), the teacher helped situate Robert in a context where he was much more comfortable without dwelling on his confusion. This gave Robert the time and space he needed to make his realization without feeling as put "on the spot." Furthermore, by opening up the discussion to other students towards the end of the interaction, the teacher took Robert out of the spotlight momentarily, and shone it on other students and their approaches to solving the system. When Robert "returned" to the discussion by sharing his solution method, he could return as a contributing member of the group rather than as an individual being focused upon. The teacher was able to both honor Robert's contribution to the discussion in an empowering way, while at the same time giving attention to other students' ideas so that Robert didn't feel singled out.

While the revoicing strategy appears to be a powerful tool for enabling students' voices to be heard, it needs to be used judiciously by the teacher. A teacher must skillfully reformulate student responses in ways that illuminate concepts without telling students what to think, or doing the thinking for them. If a student feels that a teacher's question that is in the form of "So what you really are trying to say is" is basically a disguised way of telling the student what to say, then he or she will automatically assent without necessarily understanding the concept that the teacher is addressing. In order to avoid this pitfall, a teacher may need to use revoicing to reformulate false ideas and see if the class can discern that they are false. If this "false revoicing" is done, it needs to be done in a way that doesn't single out certain students in a bilateral teacher-student exchange, but rather gives the group space and time to mull it over carefully with the guidance of the teacher.

Looking Ahead: New Aims and Intentions for the Data Collection Process

As I moved into the month-long data collection portion of my research project, I carried a much clearer awareness of how the IRE paradigm infused my questioning pattern, along with the limitations for success that it presented. The host of hypothetical scenarios that I imagined had done their job --they had helped me identify recurring patterns in my teaching and determine how these patterns had undermined or enhanced my interactions from the perspective of task completion, cognitive demand, self-efficacy, and metacognitive skill. Furthermore, I left this pre-data collection stage inspired to use the technique of revoicing in my lessons to engage more students in

thoughtful and discerning discourse, while shifting the role of evaluator from teacher to students. With this four-part framework established and an intention to use new techniques to enhance my questioning patterns, I stepped back, rolled the camera, and observed my teaching in action.

Chapter 4 – The IRE Pattern in Action

Background: The Students

For my first classroom observation, I decided to film one of my 8th grade Algebra I classes. I teach at an elite private school in Washington, DC, where the students are highly motivated, the classes are very small (12 – 15 students), and where behavior problems are minimal. Starting in the 5th grade, the students are grouped according to their ability into three groupings . The first is an extra support group (10% of the students) for students with significant math skill deficiencies. The second is a "mainstream" group (about 55% of the students) that covers the standard grade level material with substantial teacher direction and guidance. The third is an accelerated group (35% of the students) that approaches the grade level material at a greater pace, with higher levels of abstraction, and with an expectation that students will work independently on complex problems with minimal teacher direction. I usually teach a mix of accelerated and mainstream classes across the 7th and 8th grades. This year, I am teaching two sections of accelerated Algebra I classes, and two sections of "mainstream" 7th grade math classes.

The typical student in my accelerated Algebra I classes is a 13 or 14 year-old 8th grader who scores in the top 2% nationally on the quantitative portion of standardized tests, maintains high levels of focus and engagement in class, and eagerly embraces new problem solving challenges. As a result of these students' considerably high math

aptitude and level of focus, I am able to delve into rich discussions about the course material without having spend lots of time on behavioral management. On most days, I will begin my lesson with a set of two or three warm up problems that prime the students for the day's activities. Sometimes this warm up serves as a review mechanism, reminding students of recently explored concepts or techniques that are relevant to the day's lesson. On other occasions, it presents problems to students that prod them to apply previous knowledge to new problem solving contexts, foreshadowing the novel territory they will cover during the lesson. Often, I will write the warm up on the SmartBoard in the front of my class. SmartBoards allow you to solve what you have written on them, so I was able to record my SmartBoard writing during this lesson. Some of this writing has been displayed in the discussion below.

Background: The Lesson

In the class transcribed for analysis, I began the class with a warm up that including the following problem:

Simplify $\frac{3x^5 + 6x^2}{3x^3}$

I constructed this problem intentionally to address an error that I had seen on a number students' assessment in recent days. Some were simplifying algebraic fractions involving polynomial numerators and monomial denominators as if the numerators were products instead of sums. For example, they might simplify $\frac{a + b}{a}$ into $1 + b$, having

“cancelled out” the two a’s and turning them into 1’s. In making this mistake, students are no doubt confusing the associativity of multiplication, which allows the expression $\frac{ba}{a}$ to be simplified to $1(b)$ or b [since $(b \cdot a) \cdot \frac{1}{a}$ is equivalent to $b \cdot (a \cdot \frac{1}{a})$], with the distributive nature of multiplication over addition. I was curious to see whether these same students would simplify the expression on the warm up in the former “associative” manner. Since we were in the midst of a unit that explored the properties of exponents, I also wanted to see how students managed the fraction $\frac{6x^2}{3x^3}$, whose denominator’s degree is larger than the numerator’s degree. The following is a transcript of a discussion I had with one of the students (whom we’ll call Brian) about this problem.

The First Lesson Clip:

Key: AC = The teacher (myself) B = Brian

Part 1 – Brian attempts to simplify a rational expression of two polynomials, and makes an error.

$$\frac{3x^5 + 6x^2}{3x^3}$$

(The above expression is written on the SmartBoard.)

- 1 AC: Can you simplify that, Brian?
- 2 B: Three x squared over....2x... wait..yeah 2x... *(While he is doing this I write down $3x^2$)*

- 3 AC: Three x squared over 2x?
- 4 B: That's not right...
- 5 Alan: Oh, I know
- 6 AC: Tell me about your process...
- 7 B: Well I did the two sides separately (*I cross out the original $3x^2$ that I wrote*)
- 8 AC: So what was the first thing you were trying to simplify?
- 9 B: $3x^5$ over $3x^3$
- 10 AC: OK Good -- $3x^5$ over $3x^3$. (*I write this down*) What else were you trying to simplify, Brian?
- 11 B: $3x^2$ over $3x^3$.

Here is the graphic of what I have on the SmartBoard so far in my exchange with Brian:

$$\begin{array}{ccc}
 \cancel{3x^2} & \frac{3x^5}{3x^3} & \frac{6x^2}{3x^3}
 \end{array}$$

- 13 AC: OK, so what would the first fraction become?
- 14 B: $3x^2$
- 15 AC: Cool. How did you get $3x^2$?

This question tips Brian off that there is something wrong with the response $3x^2$ (Otherwise I would have continued without asking him to clarify his answer.) Thus, it serves as a form of evaluation, following my initial inquiry and his accompanying response. The IRE pattern is clearly alive and well here.

Part 2 – Brian corrects his mistake, and I walk him through an explanation of the simplification process.

16 B: Well um, the x^2 and um, and no wait, it's just x^2 .

17 AC: Just x^2 ?

18 B: Yeah because I forgot to divide...

19 AC: Because the three become 1s? (*Cross out threes...*)

20 B: Yeah

21 AC: What about this one? (*Point to the second fraction*)

22 B: Well um, it would go..well the way I do it is the other way...So x^3 minus...ah it would be...2 over x?

23 AC: (*I write that down*) Very good, two over x, yeah!... So x^2 and do we have plus or minus here?

24 B: Plus

25 AC: So in other words sometimes we might get expressions where this exponent (point to numerator) is smaller than the other one. What would be another option here – someone else – instead of ... (point to $2/x$) -- Bill?

26 Bill: $2x^{-1}$

27 AC: Nice $x^2 + 2x^{-1}$

Predictions Confirmed: The UIRE Pattern at Work

As I read through the transcript, it became clear that the hypothetical scenarios had done an impressive job of foreshadowing the questioning patterns that were demonstrated throughout this lesson.⁶ The same IRE pattern that characterized the teacher's questioning style in the hypothetical scenarios appeared frequently during my exchanges with Brian, as I used subtle hints and cues to provide him with constant evaluation. Since virtually all the student contributions in this exchange come from Brian, we can more specifically label the enduring questioning pattern in this lesson as Unilateral Inquiry Response Evaluation (UIRE). Notice how I respond to Brian's initial mistake by asking him to explain how he got $3x^2$ from the fraction $\frac{3x^5}{3x^3}$ in Comment 15. Due to his conditioning in the IRE pattern, Brian has a sense that he may have made a mistake (which prompted me to ask him this question instead of affirming his answer.) Thus, he goes back and evaluates his own response, discovers his mistake, and makes the correction. At several instances in the exchange, I deprive Brian and other classmates to assess the validity of Brian's comments by jumping in and doing the evaluating for them. For example, at one point I confirm the validity of an answer he has given immediately (Comment 23), and at another point I provide a justification for one of his answers instead of asking the class to explain why his move was mathematically "legal"(Comment 19). In each of these cases, I am acting as the master evaluator, giving

⁶ See Chapter 3

Brian the authority to move on by confirming the validity of his responses. As a result, he has no incentive to evaluate his responses himself and assume this authority as an evaluator. Whether my evaluation comes in the explicit form of an affirmation like “Cool” or “Very Good” or the implicit form of a question which asks Brian to justify his response (read: find his mistake), it is consistently presented as the “final word” on the comment at hand.

The Participant Framework

Perhaps the most limiting aspect of this exchange, however, was the narrow “participant framework” I constructed. O’Connor and Michaels (1993) see this framework as the constellation of student voices that are heard and referred to by a teacher during a discussion. The teacher can establish this framework publicly by engaging as many students as possible in the discussion, and by referring to student’s comments explicitly as he or she moderates the exchanges between students. In so doing, the teacher makes it clear to the entire class that the discourse is not a private one between a student and the teacher, but a public forum for constructive argumentation and dialogue. By contrast, the only student engaged in the discussion about the warm up (until Comment 26, the second to last line of the exchange) was Brian. In keeping the exchange a unilateral one between me and Brian, I provided no portals for other students to enter our discourse and voice their ideas about the concepts we were exploring.

Alternatives to the UIRE Approach

There are a number of alternative approaches I could have taken to expand the participant framework. Instead of staying with Brian, I could have included other students in the discussion by asking them about moves Brian has made. I could have asked for more algebraic proof of some of the statements we were making. For instance, after Comment 7, when Brian describes simplifying a fraction with a binomial numerator and monomial denominator as a process involving two “separate sides” I could have asked the class to explain what they think Brian meant by that statement, and how we could prove that that statement was true for any value of x . I could have also simplified a number of expressions myself on the SmartBoard, deliberately made a mistake in one or two of the expressions, and asked the students to make sure all my work was correct. Finally, I could have also had Brian call on other people to make insights about his work, so that he felt more empowered during the exchange. This would have engaged other students in the discussion, and given them opportunities to analyze his thought process.

The Lesson Report Card: Applying the Four-Part Framework

From a task achievement perspective, this interaction was successful for Brian. Brian was able to correct his mistake, and correctly simplify the rational algebraic expression. However, it is unclear to what extent this interaction with Brian helped other peers in the class simplify algebraic fractions effectively.

For a cognitive demand perspective, this exchange seemed to be successful in some ways. Brian was able to flesh out his understanding of the distributive property as a simplifying tool when an expression is divided by a monomial. By articulating how he treats this rational expression, with its “two sides”, Brian has bolstered his understanding of algebraic fractions and how they can be simplified. On the other hand, the consistent evaluating presence of the teacher deprived Brian of potential cognitively demanding tasks, such as arguing why his “two sides” method of simplifying fractions works. His work was more procedural, following prescribed rules for how one is “supposed to” simplify fractions, rather than analytical, exploring the logic behind his procedures.

From a psychological perspective, the results were probably mixed. On the one hand, it was most likely a satisfying experience for Brian to ultimately correct his initial mistake and work his way towards a correct answer with the teacher’s guidance. However, he might have felt somewhat “on the spot” during the exchange since the teacher focused on him for the entire exchange. While he did not seem visibly distressed or uncomfortable, the experience may have been comfortable for him if the teacher had shifted the focus of conversation to other students and come back to him

From a metacognitive perspective, the dialogue was successful in some regards. For one, Brian was developing his ability to review his own work and determine its accuracy. He gave himself time to acknowledge that something “wasn’t right” about his

response, and that he needed to think through the problem more carefully. Moreover, in communicating his problem solving method, he developed a keener awareness of how he thinks, and what the form of his problem-solving approach is like. This self-reflectiveness and awareness of what is being done in the problem solving process is a vital metacognitive ability for a math student. The more that he or she can step back, assess the validity of a certain approach, and adjust if necessary, the more flexible he or she will be in facing difficult challenges without getting bogged down in an unproductive strategy that is not going anywhere.

At the same time, little was done by the teacher to engage other students in assessing what Brian's thought process was, and how he was conceptualizing the problem. The teacher could have easily asked other students to explain what Brian meant by a fraction having "two sides", for example. The teacher could have then had students generate examples of fractions with a sum in the numerator and number in the denominator and had them analyze different ways of simplifying this fraction. The teacher could have called upon different students to present various ways to simplify algebraic fractions that are in the same form as the one being simplified by Brian. This would have given other students an opportunity to generalize overarching principles governing the simplification of fractions in this form, whether they are numerical or algebraic.

New Intentions for the Next Lesson

After processing the successes and failures of this lesson from the perspective of the four lenses, it became clear to me that I needed to approach future lessons with a deeper commitment to expanding the participant framework. Many of the shortcomings of this lesson seem to be tied to the narrow, unilateral format of the dialogue between myself and Brian, which could be more accurately described as a tutorial than a class discussion. Indeed, it is noteworthy that I began the data collection phase with a commitment to use revoicing⁷ techniques to expand the participant framework and involve more students; unfortunately, these intentions were never realized during the actual lesson. Clearly, the orientation of the UIRE pattern – which emphasizes the teacher’s strict control of a focused, unilateral exchange between him or her and *one* student – clashes with the spirit of the revoicing technique, which seeks to *open up* a discussion to multiple voices.

Looking ahead to my next videotaped lesson, I resolved to constantly look for opportunities to involve as many students as possible. I hoped that this would allow me to break away from the confines of the IRE paradigm, and shift the responsibility of evaluation from my own shoulders to those of the students. Specifically, I aimed to use revoicing techniques to make this more inclusive and open discussion style visible to my students. To this end, I returned to the same 8th grade Algebra I class a week later, and observed how these intentions actually translated into my teaching practice.

⁷ See the end of Chapter 3, where I speak about the intention to use revoicing in my upcoming lessons

Chapter 5: Expanding the Participant Framework

Background on the Lesson

Excited to expand the participant framework, I entered my next videotaped accelerated Algebra I class (see Chapter 4 for a profile of this group of students) with the objective of improving students' comfort factoring polynomials whose leading coefficient was not 1. They were a couple weeks into a unit on polynomials, and had developed some methods for facilitating the factoring process. First, they learned about how to determine the greatest common factor of a set of monomials and "factor out" this term by applying the distributive property. They also developed some visual model based on rectangular areas to represent both the process of multiplying two polynomials and the process of factoring trinomials into two binomials. For example, the product $(x+3)(x+4)$ could be represented by the following diagram:

	x	4
x		
3		

If each of the four rectangular boxes' areas are computed, the diagram can be filled in to create the following:

	x	4

x	x^2	4x
3	3x	12

The total area of the four boxes could be simplified to $x^2+7x+ 12$, the product of the two binomials. With my guidance, the students developed a method for factoring trinomial in the form $ax^2 + bx + c$ (where a is nonzero, and a, b, and c are integers) into two binomials of the form $(px +q)(rx+ t)$, where p, q, r, and t are nonzero integers by reversing the process described above.

For example, if the trinomial is $x^2 - 6x + 8$, the students might begin by setting up model such as:

	x	
x	x^2	
		8

and quickly realize that -2 and -4 multiply to give 8 and add to give the desired coefficient of the -6x term, yielding the following:

	x	-4
x	x^2	-4x
-2	-2x	8

Notice that here the concrete notion of area breaks down for this model, since a rectangle can't have a negative area or side length. However, the model still helps

students to visualize the four partial products that compose the product of the two binomials, and can be used as a tool to try possible factoring configurations in an ordered fashion.

With these factoring techniques established, student had had practice the day before solving quadratic equations using factoring and the zero product property. They had also been given trinomials to factor whose leading coefficient was not 1. Predictably, these were more challenging for the students at first, although the area model described above proved to be a helpful resource for students as they search for the right combination of factors.

In this lesson, my objective was to have students articulate strategies for approaching these harder factoring problems (with non-unit leading coefficients) among themselves in small groups, and subsequently in a class discussion. I began the class with a warm up containing some introductory problems geared towards reviewing the use of factoring and ZPP to solve quadratics. Then the following problem was introduced on the SmartBoard:

Solve for x using factoring

$$15x^2 + 20x - 20 = 0$$

Below is a transcription of the discussion that followed:

The Second Lesson Clip: The Transcript

(Note: I have used pseudonyms for each student for privacy purposes.)

Key: AC = the teacher (myself)

Part I- Evelyn offers to present her factoring method in front of the class

- 1 AC: What would be your ideal leading coefficient?
- 2 Class (*various students simultaneously*): 1
- 3 AC: 1. Love it! The one you just did was one once you factor out that x.
What's the leading coefficient now?
- 4 Class (*various students simultaneously*): 15
- 5 AC: That's sort of gross. (*Evelyn raises her hand.*) Evelyn?
- 6 Evelyn: Can I come up to the board?

Here, I see a golden opportunity for a student to take on the teaching role, shifting the responsibility of evaluation from my shoulders to hers. I gladly welcome Evelyn's request, while uncertain about how well she will fare in communicating her method to the class.

Part 2 : Evelyn articulates her model for factoring the polynomial

- 7 AC: Yes you can.
- 8 Evelyn: OK, so what I did is I took out the 5. 5 times three x squared plus 4x – 4 = 0.

(Evelyn writes the following on the board : $5 \cdot 3x^2 + 4x - 4$)

Part 3 – I make an attempt to expand the participant framework by asking Timothy to evaluate what Evelyn has written. However, notice how I quickly revert back to a unilateral IRE exchange with Evelyn.

9 AC: *(Interrupts her)* OK, so Timothy, if you multiplied this out, what would multiply the 5 by?

10 Timothy: The three x squared, the four x and then....

11 AC: *(Interrupts Timothy)* So how would you know to multiply the 5 by all of those terms?

Evelyn fills in parentheses around the $3x^2 + 4x - 4$, giving $5(3x^2 + 4x - 4)$

12 AC: Does it matter whether she puts parentheses or not?

13 Timothy: Yes, because otherwise it's $3x^2$ times 5 and then you add the 4x and the -4.

14 AC: So if we just had a 5, and then a $3x^2$, would it be implied that the five is multiplied by the rest of this stuff?

15 Evelyn: No

16 AC: So do we need the parentheses there?

17 Evelyn: Yeah.

18 AC: OK

19 Evelyn: And then I made the box, so on one side, I usually just put the coefficient, if its not a perfect square, I put $3x$ and x . (*Draw the box*) and then here (pointing to upper right and lower left portions of box model) we don't know, and then you have $3x^2$, 4

Part 4 ; I invite Evelyn to call on students to interpret her work. Note the IRE exchange that ensues between her and Kristina. It closely parallels the style I have modeled for Evelyn during the lesson.

20 AC: Could you stop for a second? In fact, would you...I would like you to include the rest of the class...in other words – you've set up the box but see if you can include the rest of the class.

21 Evelyn: (*to the class*): What makes -4 ? (*points to the lower right hand box, where she has already written -4 as the product*). Kristina?

22 Kristina: negative 2 and 2

23 Evelyn: OK, so I put -2 and 2 , and then I multiplied...

Part 5: I reclaim my role as the leader of the discussion, and fall comfortably into IRE exchanges with Charlie and Evelyn, respectively.

24 AC: (*Interrupts her*) Why did Evelyn put the 2 with the $3x$ and the negative 2 with the x ?

25 Evelyn: Actually I did it the other way around.

- 26 AC: You did it the other way around.
- 27 Evelyn: Yeah I put the 2 with the 2 and the -2 with the $3x$...
- 28 AC: (*To class*) OK, so why do you think she put the 2 up in that position and the -2 down there. Charlie?
- 29 Charlie: So that she could get the 2 to make $4x$.
- 30 AC: OK, so first she just started off. (*To Evelyn*) Do you want to show us what you did originally?
- 31 Evelyn: This is what I did.
- 33 AC: This is what you did originally? OK Cool.
- 34 Evelyn: I just, because the coefficient (*points to $3x$*) is more and that (*points to -2*) is less.So then you get $6x$ and $-2x$ (write these in the respective sections of the area model) and then the negative 2 plus 6 is $4x$ and ...and THEN
- 35 AC: Hold up, Evelyn. Timothy's got a question?

Part 6 – Timothy poses a query about whether the factoring strategy can be generalized, leading to a peer-to-peer exchange between him and Charlie.

- 36 Timothy: Is there a general rule for putting um, factors there? Like -2 and 2...is there a general rule for figuring out actually where they go?

Here, I see an opportunity to expand the participant framework by posing Timothy's query to the entire class.

37 AC: That's a great question. I want to ask the class. What were some of the things.. because this is like a puzzle, right...

38 AC: OK, Timothy asked a really good question which is there a rule. What were some of the strategies you were thinking about in terms of the 2 and the -2. What did Charlie talk about it in terms of what factored into it. Charlie?

39 Charlie: We since the middle term is a possible number – it was $4x$ – you want to multiply by a positive number to yield a larger number x .

40 AC: Gotcha. (To class) Make sense? (AC notices Timothy is still confused.) OK, Charlie do you want to say that again?

41 Charlie: OK $3x$ and x are the two ...um variables

42 AC: Weights?

43 Charlie: (To Timothy) Yeah. You want a positive number so you want to multiply by a larger number that's positive and then you combine it with $-2x$ and then....

44 Timothy: OK, yeah.

45 AC: OK, great. (Noticing Evelyn has left out $= 0$) OK, so all of a sudden we went from an equation to an expression. (She quickly realizes her mistake and writes " $=0$ ").

A Variation on the UIRE Pattern: The MIRE Pattern

On one level, the form of questioning in the lesson took on a slightly different shape. Instead of confining the conversation to one student, as I did in the first lesson clip, I expanded the scope of student participation by asking students to address questions

posed by their peers. By placing Evelyn in the teacher's role, I literally and symbolically shifted my role from the director of the discussion to the facilitator of student-to-student exchanges.

While this lesson involved a slight expansion of the participant framework (from one student to 2 or three students), it did little to shift the burden of evaluation from the teacher to the students. Even as I facilitated a peer to peer exchange between Charlie and Timothy, I still evaluated the validity of their responses once they shared them, instead of calling upon other students to do this. It seemed that my desire to keep the discussion headed in a specific direction compelled me to retain the role of "master evaluator" as I responded to student comments. For example, when Evelyn omits parentheses in factoring out a 5 of the original trinomial, I respond by asking her if it was implied that the 5 was multiplied by each term in the expression (Comment 14). Under the firmly established IRE pattern that she has grown accustomed to, this question of mine is an implicit cue that she has made a mistake. By asking her if she needs parentheses, I make it clear that parentheses are necessary in this case. Thus, I initiate the evaluation process for her with my first question and assist her in making the proper evaluation with my follow-up question.

It would seem, then that opening up the discussion space to more students did not automatically give students more responsibility in evaluating their peer's work. The enduring presence of the IRE pattern (albeit with different students responding instead of one) maintained my established role as the ultimate evaluator of student comments.

This encouraged students to still look to me to approve their responses as they made contributions to the discussion, and did little to encourage them to assess the validity of their peers' responses (since I was still doing this evaluation for them.) As such, we can label the prominent questioning pattern of this lesson as **Multilateral Inquiry Response Evaluation (MIRE)**, a slight variation on the **Unilateral Inquiry Response Evaluation** pattern from the first lesson clip.

Lesson Report Card: Applying the Four-Part Framework

From a task completion level, the students were largely successful. With Evelyn leading the way, some members of the class were able to factor the trinomial and explain strategies for making the factoring process easier. At the same time, it is difficult to determine how many of the students accurately factored the trinomial on their own, since not every student's work was checked. Accuracy aside, the discussion certainly explored the issue I wanted the students to address – what are effective strategies for approaching the factoring of polynomials with non-unit leading coefficients?

From a cognitive demand standpoint, the results were mixed. On the one hand, students were exposed to Charlie's idea about balancing large coefficients with positive multipliers to yield a net positive sum for the middle term. For Charlie and Timothy, the recipient of Charlie's explanation, this dialogue was a helpful illumination of factoring "strategies" for trinomials with leading coefficients larger than one. Charlie was clearly engaged in the process of applying his own numerical logic to the procedure of

factoring, a clear sign that the cognitive demand of the task was relatively high for him. In fact, Timothy's question (Comment 36) about whether a rule could be established for the procedure exposed the task's level of cognitive demand. Charlie's response suggested that the polynomial could not be factored by simply applying a memorized procedure, but rather by strategically positioning factors so that large or small sums or difference would result between the coefficients of the two first degree terms in the product. Charlie was essentially saying to Timothy that this task of factoring polynomials had a higher level of cognitive demand than Timothy had initially thought – he could not simply apply a consistent rule to each new factoring situation.

However, it is unclear whether this exposition really helped the development of other students' mastery of factoring techniques, and maintained a high level of cognitive demand. It is entirely possible that some students may have waited until Evelyn's explanation of her process, and copied her process instead of thinking about factoring strategies themselves. Moreover, some students left the class without really understanding the logic that Charlie articulated for choosing and positioning factors. In these students minds, Charlie's comments may have represented simply another procedure that they had to memorize and apply blindly, instead of an example of logical reasoning that was specific to this factoring scenario.

From a self-efficacy standpoint, this appeared to be a successful experience for Evelyn, who was able to demonstrate confidence and eloquence in explaining her methodology. She was clearly enjoying herself as she explained her factoring method, and relished

taking on the role of the teacher. By taking on the teacher's role, she was empowered to evaluate the responses of other students, and focus on the fundamental precepts involved in the factoring process. The tacit public affirmation of her "performance" in front of the class no doubt spurred her confidence and self-esteem. Given Pajares' studies (1996) on the tendency of girls to doubt their math ability, Evelyn's confidence and self-assurance in leading provided her female peers with a powerful counterexample to this destructive social stereotype.⁸

For Charlie, being able to explain a nuanced idea to one of the strongest students in the class, Timothy, was no doubt a highly affirming experience. And yet, because the interaction was largely limited to 2 or 3 students, it is difficult to ascertain the level of psychological success among the remaining students. They probably were more engaged in following the discussion since one of their peers was leading it. At the same level, they could have been mere observers of the dialogue between Evelyn, Timothy, and Charlie, instead of active participants.

From a metacognitive standpoint, Evelyn, Charlie and Timothy all engaged in successful behavior. Evelyn was able to reflect about her approach to the factoring problem and communicated effectively in front of the class. She was also able to determine what questions would be helpful to ask in her role as a teacher to prod the class towards a solution. Charlie was able to detail a strategy that informed his application of the binomial factoring technique. He was able to articulate a deeper understanding for how

⁸ Of course, I can never absolutely be sure of how Evelyn was feeling about herself during her experience leading the class, and how it affected her view of herself as a math student. I can only infer from her cheerful and comfortable manner that this was a positive experience for her.

binomial multiplication works, and was apply to this understanding towards an effective strategy. Furthermore, Charlie was able to provide an alternate version of his explanation to Timothy when Timothy did not understand what his was saying at first. This shows an ability to communicating ideas flexibly in different forms, a metacognitive skill. Finally, Timothy was able to ask a question that attempted to generalize the strategy that Evelyn used. (“Is there a specific method that always works?”) This type of querying on Timothy’s part indicates the kind of reflection about his learning that is an essential metacognitive skill. In essence, he is asking himself the question, “are there implications of what I am learning right now that I can deduce?”

And yet, at the same time, many opportunities for metacognitive success were overlooked. For example, instead of having Charlie provide an explanation of his own strategy, I might have asked the class, “How could Timothy determine if there is a general formula or rule for assigning factors to given positions in binomials when you are trying to factor a trinomial?” In other words, instead of addressing Timothy’s query directly (albeit using a student to provide this response), I could have prodded the class to think more deeply about Timothy’s question could be addressed and studied. This would help the students develop their ability to conceive of a strategy for approaching a question that they have on their minds, instead of depending on their teacher or their peers to answer it for them. One of the goals in developing a student’s metacognitive skills is to train them to naturally pose questions such as the following when a query arises in their mind about a particular mathematical phenomenon or pattern:

What do I need to know to better understand what I don't know or don't understand?

What approaches would help me to make a definitive statement about this issue?

How would I be able to tell if my answer were true for all cases?

While asking the class these questions would have prolonged our discussion and perhaps precluded a discussion of our second topic for the lesson, the benefits of undergoing this type of thinking far outweigh the "loss" of time not spent on the second topic.

Moreover, the discussion would have given students an even deeper insight into strategies for factoring trinomials in more complex scenarios. That is to say, the extra time would not have been time wasted.

Designing a New Pattern: Looking Ahead to Lesson #3

Given the fact that the IRE paradigm persisted in this second lesson, despite my strong intention to avoid it, it became clear that I needed to make a fundamental change in my approach. I needed to create a lesson in which two main goals were soliciting as many student contributions as possible, and minimizing the amount of explicit or implicit teacher evaluation of student responses. Moreover, I had to find a way build the revoicing technique into the very structure of the lesson, so that I could easily apply it during my facilitation of student-to-student discourse.

In order to meet these objectives, I needed a task that was open-ended enough to encourage multiple student perspectives, and to student-centered enough to wean students from the habit of looking to me to give them (or confirm for them) “the right answer.” It was with these motives in mind that a lesson on progressive and absolute salary raises – the focus of my third clip – took shape.

Chapter 6 – An Experiment in Revoicing

Background: The Students

After taping two classes with one of my accelerated 8th grade Algebra classes, I decided to turn my attention to another age group and ability level. The students in my “mainstream” 7th grade math topics class range from the 50th percentile to 90th percentile on nationally administered tests. While generally attentive and conscientious, they need considerably more scaffolding from the teacher in grasping new concepts and techniques. Whereas students in the 8th grade accelerated class notice connections between concepts immediately, these 7th graders need to be pointed towards these connections. Mastery of new principles usually only comes with extensive practice and application of these principles. Few of the students see themselves as “smart” math students, and a handful see themselves as weak math students.

As a result, the “mainstream” 7th grade students I teach tend to be less confident in their ability to communicate mathematical ideas verbally. Not surprisingly, they are less likely than my accelerated 8th graders to make unsolicited contributions to class discussions⁹. Given my interest in encouraging extensive student participation in discussions, it seemed more appropriate to try out revoicing techniques with a group of students who were naturally more resistant to participating in classroom discourse. If class participation improved, the case for using revoicing techniques to open discussions to more students would be made more compelling. In order to engender more frequent

⁹

and full participation from the entire class, however, I first needed to find a task that would easily generate student responses.

Background: The Lesson

In searching for an appropriate task to facilitate student participation in a discussion, I looked for a problem-solving scenario which would generate a host of distinct and valid answers. I wanted a task whose solution depended on a student's point of view and value system, rather than on solely the correct application of mathematical techniques. In fact, I decided to make the actual computation portion of the task relatively straightforward, so that the main focus of student's attention could be the interpretation of the quantitative data they generated.

The students were presented with the following scenario:

You are in charge of determining the salaries of your company's employees for the coming year. You want to give all of your employees a raise, but you haven't decided how to determine what the raise will be for each employee. Two options come to mind:

Option A: *Give every employee a \$3000 raise, no matter what his or her salary is*

Option B: Give every employee, including yourself, the CEO, a 4% raise. (The average “cost of living” usually goes up by 3% every year, so you figure this will both cover the cost of living increase, and also give the employees an extra bonus.)

The students were given time to determine what the salaries would be of each employee under Option A and under Option B. They filled this information on the chart below:

Employee	Present Salary (in \$)	Option A Salary	Option B Salary
Bill	25,000		
Susan	40,000		
Philip	40,000		
Andrew	35,000		
Collette	70,000		
Mark	60,000		
David	80,000		
Phoebe	100,000		
Alice	250,000		
Nina	90,000		
Christina	80,000		

Then they were asked to decide which option was fairer, and then to write down a few major points backing up their claim. At the end of this reflection time, I initiated a discussion about the pros and cons of each plan.

Lesson Clip #3: The Transcript

Note: Pseudonyms have used in place of actual student names for privacy reasons.

Key: AC = The teacher (myself)

Part 1 – The class discusses the benefits of Option A

AC: Let's start with the pros of Option A. Could someone give me a pro for option A? You don't have to tell me what you voted for. You don't have to commit. I just want you to give me a pro for Option A

Colin: Everyone gets the same amount of money.

AC: So you're saying equality. You're saying equality. *(I write this on the board)*

OK. Great. Another plus for Option A?

Sylvia: For several of the lower paid employees, three thousand dollars would be more than they would get under Option B.

AC: OK. So helps out lower paid employees. *(I write this on the board.)*

AC: Any other pluses? Alright. Let's look at the minuses of option A. Susan? A minus for Option A.

Susan: Oh, I thought you were going to do pluses for Option B.

Part 2- The class discusses the benefits of Option B

AC: Alright. We can do pluses for Option B. Pluses for Option B.

Susan: It covers the average cost of living and gives everyone 1% more.

Garth: Well 3000 would have, too.

AC: OK. So covers cost of living, gives them a bonus? Garth?

Garth: Well for the people who make above 80,000...because the person who makes 80,000 gets 83,200, so the people who make over 80,000 get more than 3000.

AC: OK. So over 80,000 benefits than if it's more. (*I write this on the board.*)

Garrett: Yeah. Cuz they get more the other plan would.

AC: What else for pluses for Option B? Colin?

Kevin: The amount of money they get is based on their current salary so...it's not really the same amount of dollars but it's the same amount, according to their salary.

AC: (*I write on the board*) Equal according to salary. Any other pluses? Minuses? Let's go to minuses for Option A.

Part 3 – The class discusses the downsides of Option A

Garth: It wouldn't be good for people who make over 80,000. They would make less than they would under...[Option B]

AC: Writing - It's not fair for people more than 80,000.... Any other minuses?

OK. Minuses for Option B. OK we got some minuses here.

Jenna: The people who get paid more already have more, and then they get a bigger raise.

AC: OK so richer have....how do you want me to...richer people already have more...I don't want to put words in your mouth. Is that what you mean?

Jenna: Yeah.

AC: I am going to put this [richer] in quotes. They might be really really rich but have a smaller salary but (*writing*) "richer people have more of a raise but they already have more"

Shawn: If they're all equally valued and work equally as hard, why would they get different amount of money.

AC: OK, Shawn can you help me out? Are you talking about a minus for Option A or a minus for Option B?

Shawn: If they are equally valued, why would they [the higher paid employees] get more in a raise, than others.

AC: Any other minuses? OK, Garth?

Garth: I have another minus for Option B. I actually wrote this as one of my points. I said that there are 11 people on this list, and I calculated that out of the 11 people – that's more than half would get less than 3000.

AC: OK so you did a little study, and so...(writing) "6 out of 11 would make less under Option B.....

Garrett: They would make less than they would...

Here is the list of pros and cons that was written on the SmartBoard:

Option A	Option B
<p>Pluses: equality helps out lower paid employees</p>	<p>Pluses: extra bonus over 80,000 benefits more equal according to current salary</p>
<p>Minuses: - unfair for people more than 80,000</p>	<p>Minuses: - richer people make more of a raise when they already have more - if they're equally valued why are some raises higher - 6 out of 11 would make less</p>

Part 4 - After a brief period of reflection, the students cast their vote for Option A or Option B. All vote for Option A, with the sole exception of Kevin, who presents a case for Option B. I facilitate a discussion of various students' rationales for choosing their particular option.

AC: OK. What I would like you to do now is look at what we have on the board, meditate on it for a second.....and then I want you to decide what you think is more fair, so...we're going to hear from people here. Give yourself a minute to make your decision, and then we'll take a little bit of a vote. Option A or Option B. Assume

you're in the CEO chair and you need to decide what the salaries are for your employees. Your guiding principle is fairness.

Greg: Oh this is hard for me.

AC: Sometimes there are hard decisions in life, Greg.

AC: Who's ready to make your vote? Raise your hand if you're ready to make your vote. Raise your hand if you can't decide, and you just can't sleep at nightAlright, let's try. *[I take a vote, and the results are: 10 for Option A, 1 for Option B]*

AC: Now, I don't mean to pick on you, Kevin [the only voter for Option B], I actually have a lot of respect for you because voted for something that no one else did. Would you be willing to explain why you stuck with Option B?

Kevin: I think it's more fair that everyone...No, it's based on the salary instead of everyone being the same thing.

AC: That's good. So it's counts for the salary...(to Kevin) Talk to the people. Talk to the folks.

Kevin: Like, like, hummm. Well that's what I said. I think it's more fair because it's according to the salary.

AC: So there's a reason that someone has a higher salary. They should earn more

Shawn: I thought you said there was no reason.

Sylvia: Yeah I thought you said there was no reason.

AC: You're saying that someone getting a higher salary, should in general get a higher raise. OK, Susan, what was the most important point for you? If you can boil it down...

Susan: Well there wasn't really a specific point. For B, there were 3 pluses and 3 minuses, so it kinda cancelled each other out and in A there was 2 pluses and 1 minus. I thought if the good outweighs the bad, then that should be the better [option]

AC: There are more minus here [in Option B] than there are [in Option A]

Susan: Well there are the same number of pluses and minuses in Option B they just cancel each other out. Excellent Susan. Someone who had a different...

Sylvia: I think [Option A] practices a "bottom up" philosophy. I mean one could argue that the cost of living is more.. but for the people who are making more money, maybe they are living a better lifestyle. So the people who need help the most also need more money to spend...

[Bell Rings]

AC: Jenna, what did you stick with? Why did you stick with Option A?

Jenna: Because it's a fairer plan I think on the whole. The people who get paid more receive more [under Option B] , and that's not fair because people already have more. And that's not fair because people who have less don't get as much percent raise.

AC: So Jenna's point is that people who have more don't really need it as much. And the people who have less...

Jenna: It's also a pretty good raise.

Part 5 - Troubled by Option A's "communist" overtones, Greg and Ted decide to join Kevin in supporting Option B, triggering minor pandemonium among their peers.

Greg: Hey, I'm changing

AC: Why?

Kevin: I say I say, Option A is like communism.

Greg: I feel like if you don't work yourself to a higher place in society

Shawn: But you said that everybody who worked there was equal.

Ted: Oh my god -- that IS communism.

[Chaos and pandemonium. The kids are talking over one another. I clap for some order.]

AC: There is no right answer to this question. There is no right answer to this question.

What do you think the purpose of asking you a question with no right answer was? Why do you think I asked you this question?

Susan: So that we could think about it on our own.

AC: She just said so you could be individuals and think on your own. Why I did I ask you a question with no correct answer? Colin?

Colin: So you could test our moral state.

AC: OOOH. I like that! Sonya?

Sylvia: So you can see if our economic philosophy is "bottom up" or "trickle down".

Expanding the Participant Framework: The Inquiry Response Collection (IRC) Pattern

Given the open-ended nature of the task, coupled with the teacher's comment that no "wrong answers" existed, I was able to introduce a new pattern that could be described as **Inquiry Response Collection (or IRC)**. The main focus was not on evaluation, since I had stated that I had no real need to evaluate their answers. (The students performed some evaluation of their own beliefs about how money should be distributed, but they did not specifically evaluate or respond to each others' comments.) Instead, the focus was on the collection of student contributions. Since students were not afraid of being wrong, nor of competing with another student to be the winner in an argument, they were much more comfortable sharing their rationale for making their decision. Moreover, since I took the time to document multiple students' ideas on the SmartBoard, students had time to digest what they were hearing and formulate well-thought out responses. Garth, for example, was able to do a quick study of the employees during the reflection period to see how many benefited from each of the respective plans. Under a different instructional pattern that involved more immediate teacher evaluation, students would not have the same amount of time to reflect about ideas and construct thoughtful responses to the query. Instead, they would be more focused on responding to the teacher's evaluation of their responses on the spot, in front of the rest of the class. In this sense, this task had the effect of opening up the entire class to a much more reflective discourse.

While this IRC pattern certainly expanded the participant framework, as I had hoped, it avoided the central dilemma of teacher evaluation. Since I had made it clear to students that multiple opinions or answers were acceptable, I had little need to evaluate the validity of student responses, since they were largely based on value judgments. Furthermore, students had no need to evaluate each other's responses directly. They could merely state their opinion and let others state theirs, without engaging in direct dialogue with their peers around specific ideas. In this sense, the IRC pattern addresses the issue of student involvement, but does not deal with the dilemma of teacher evaluation. There seems to be the need for a fourth stage in the pattern (after the Collection phase) that positions students to be able to critically assess each other's comments in a non-threatening, reflective, and well-supported manner.

Lesson #3 Report Card: Applying the Success Framework

From a task achievement perspective, this task could be classified as moderately successful. The students all successfully computed the salary amounts under each plan, and most of them realized that the "justness" of the plan really depended on which employee was being considered. Also, most of them realized that the problem could be viewed with multiple perspectives, and that certain value judgments (such as the notion that lower salaried employees "need" a higher raise than their higher-salaried counterparts do) were at the basis of deciding which perspective should be the guiding one in making a decision.

From a cognitive perspective, a number of important insights emerged among the students during their process of computing the new salaries and then justifying their selection of the plan. Some students picked up on some shortcuts that could simplify their computation of salaries under plan B. Some realized that since 4% of 10,000 is 400, they simply take a salary, divide it by 10,000 (an easy task considering most salaries were multiples of 10,000) and multiply the result by 400 in order to yield that raise for each employee under Plan B. Furthermore, the discrepancies between the amounts in each of the two columns in the chart made it clear to most students that a equal percent increase does not imply an equal absolute increase. Both Colin and Kevin referred to Plan B demonstrating an equality “based on the salary”, as opposed to the same for everyone. However, for students to more fully appreciate the “fairness” of a progressive raise policy (Plan B), it might have been helpful to ask the class which plan would have been more fair if the “raises” were actually taxes charged to the employees by the federal government. They would have to delve more deeply into the question of whether lower-salaried employees should still be prioritized over higher-salaried employees in the realm of tax payments (versus raises).

This said, the task itself did not demand a very sophisticated level of analysis. The students were not asked to, for example apply the concept of a proportion raise to another scenario and determine its validity in this new context. In filling out the chart, it was pretty clear, who benefitted and who suffered under each plan. All students had to do was decide who should benefit more, on average. Students simply applied a

consistent procedure to find each new raise (either add 3000 or add 4% of the original number), filled in each of the two columns on their charts, and then compared the two columns to determine the winner. Given the low cognitive demand, one might ask whether students would have been forthcoming with insights if they had been asked to envision a new scenario with the same general principle (comparing absolute change and proportional change) but different circumstances (such as tax rates).

From a psychological perspective, the open-ended nature of the problem, coupled with the teacher's pronouncement that there was no "correct" answer to the problem, created a considerably comfortable and relaxed space for the students as they contributed their insights. Since they were asked to generate pros and cons for each plan before revealing their selection, there were no incentives for students to compete with each other for making the "right" decision. The only moments of tension, potential discomfort came after the class vote was taken, when Kevin had to defend himself as the sole proponent of Plan B. His justification for the decision – that higher-salaried employees deserved a higher raise more than the lower-salaried employees – prompted proponents of Plan A to immediately denounce his selection, and the class begin to coalesce into two clear factions pitted against each other. It took the teacher's redirection to get the students focused on why they were being asked to solve an open-ended problem with no clear solution (instead of being focused on which camp was "right") to calm the passions of each group's most vocal members. Nevertheless, students felt safe to share their opinions for the majority of the discussion, and the inclusion for virtually every student in the discussion affirmed each student's value as a contributor.

From a metacognitive standpoint, the lesson was successful on one level. In addressing which guiding principles should guide the answer to this open-ended dilemma students gained an awareness of how assumptions can frame problem-solving strategies, and how central these assumptions are in helping problem-solvers navigate their way through scenarios that do not present definitely “correct” solutions. Given the fact that most of the problems that students are faced are implied to have specific a correct solution or set of solutions, the realization some solutions can be deemed better than others based on certain principles or norms was a novel and enriching idea for several of the students. This training in viewing a problem from multiple perspectives not only helps student become more comfortable facing problems without easily visible answers, but it also encourages them to patiently consider multiple solution paths. With an awareness that multiple solution methods exist for most problems, students tend to be more willing to test one or two strategies that fail before finding one that is successful. These more flexible problem solvers are less bent on finding the correct strategy than they are at finding a strategy that turns out to be successful.

At the same time, the teacher’s pronouncement that no “correct answer” existed drew the students’ attention away analyzing the validity of their classmates specific rationales. Instead of addressing each other, students were really just sharing their ideas with the teacher, who was revoicing them and recording them on the SmartBoard on the Pros and Cons page. With students tacitly assuming that any answer would be deemed correct by the teacher, they had no incentive to justify their position versus another student’s

position. Of course, this type of back and forth between students could have generated competitiveness and confrontational dialogue. If this dialogue were not managed carefully by the teacher, its antagonistic nature could have undermined the psychological success of the lesson for the class as a whole. And yet, the teacher could have found a way to keep students attention focused on the substantive ideas based each rationale given in this debate between students, rather than on which group had the “winning” argument. In the process, students would have been developing the ability to interpret and judge the validity of their peers’ statements, which would in turn have helped them get a feel for the ways in which normative assumptions shape the way a person discerns the validity of a given idea.

Revoicing in the Lesson: Constrained by Norms

While the IRC pattern failed to address the challenge of teacher evaluation, it did provide many opportunities for me to use the technique of revoicing. The act of collecting student input and writing it on the SmartBoard in chart format helped to keep my focus on the students’ ideas, rather than on my evaluation of those ideas. This, in turn, helped me to resist the habitual pull towards an IRE pattern of dialogue with students, which had shaped my previous two videotaped lessons.

And yet, my use of revoicing was more implicit than the explicit style articulated by O’Connor and Michaels (1993). Instead of attaching students’ identities to the comments they had made and asking others to assess these particular students’ ideas, I

merely collected the ideas of all the students and “revoiced” them by writing them on the chart of pros and cons. This certainly helped students to become more aware of the constellation of opinions and issues at work, and aided them in weighing the benefits and costs of each plan. At the same time, I did little to articulate the mathematical meaning behind these student comments.

Part of this passive non-identifying aspect of the revoicing was intentional on my part. I was worried that if students’ ideas were too strongly linked to them, then disagreements between students over conflicting would escalate into personal attacks. If the environment became tense or contentious, it could potentially deter students from sharing their opinions, for fear that they might be ridiculed or “disagreed with”. Since students were used to an IRE model in which questions were answered quickly by students (whose answers were approved by the teacher) or the teacher, it could be possibly destabilizing for a class to address a question whose answer is either not definite or not addressed immediately by the teacher.

Indeed, the “chaos and pandemonium” that erupted near the end of the lesson foreshadowed the potential disruptiveness of a contentious debate like this one. In order for students to be comfortable with the presence of unresolved controversy, they need have normalized the experience of engaging in critical discussions with each other involving disagreement. Since this norm had yet to established, I avoided presenting a more controversial and cognitively demanding task, for fear that the revoicing process would be subsumed by the larger dilemma of managing students’ disagreements.

Aligning Sociomathematical Norms with the Demands of Revoicing Scenarios

As the instability towards the end of Lesson Clip #3 suggests, a shift from the IRE pattern also demands a shift in some of the norms that shape how students participate in class discussions. These “sociomathematical norms” are described by Yackel and Cobb (1996) as the “normative aspects of mathematical discussion that are specific to students’ mathematical activity.” As Lampert (1990) and Chazan and Schnepf (2002) demonstrate, students need to be trained in the importance of defending conjectures, evaluating assumptions made by their classmates, and critiquing the contributions of others with defensible logic. The teacher must consistently demand high levels of independent thinking from his or her students, and must often refrain from commenting directly on the validity of student responses in order to force the students to do the evaluating on their own. For students conditioned by the frequent use of IRE patterns of interaction, in which the teacher is assumed to be the sole person in the class with ultimate authority in determining “right” answers from “wrong” answers, this demand for student evaluation can be intimidating and bewildering. Used to leaning upon their teacher for cues to guide them, students may feel adrift without the teacher pointing them in the “right” direction.

Another pressing issue for teachers seeking to engage their students in peer-to-peer exchanges, then, is the necessity of training them in sociomathematical norms that support these student-driven discussions. As different patterns of questioning are applied

by teacher, careful attention needs to be paid to the kind of preparation students need to be able to realize the pattern's potential benefits.

Managing an Unresolved Controversy: The Need for a New Pattern

Is it possible, then, to use revoicing to engage students positively as they confront a controversial and cognitively demanding task? How does a teacher manage the inevitable disagreements or controversies that emerge in the discussion of an open-ended, cognitively demanding task so that self-efficacy is not significantly threatened? What sociomathematical norms need to be in place in order for the teacher to successfully promote metacognitive skill and self-efficacy while at the same time involving as many students in the discussion?

Clearly, this dilemma of maintaining self-efficacy and high cognitive demand while at the same time encouraging and promoting student-to-student evaluation demands a new kind of questioning pattern to be managed effectively. It must draw upon the inclusive power of the IRC pattern, which encourages participation by all students, but also provide a structure through which the teacher can manage the conflicting opinions of many students as they respond to a mathematical controversy.

And yet, merely formulating a new pattern for managing controversy would not be sufficient. The successful implementation of this new pattern would seem to depend greatly on the extent to which students were comfortable exploring an unresolved

controversy with their peers in a critical and thoughtful manner. Given the developmental realities of early adolescents, whose attentiveness and self-control can lapse in an instant, the establishment of sociomathematical norms that guide appropriate behavior during these discussions would no doubt be a gradual process, with frequent setbacks. Despite the challenging nature of this process, it would seem to be a vital element in promoting self-efficacy, metacognition, and student-to-student evaluation in a cognitive demanding learning environment.

Chapter 7: A New Vision for Controversy Management

Before formulating a new questioning pattern that addresses the complex challenges of controversy management, it would seem appropriate to cast a careful glance at the questioning patterns that dominated the three lesson clips that were described in Chapters 4, 5, and 6. Each of these patterns, while limiting in some way, had beneficial components that could be helpful in shaping the design of a new pattern. In the three lesson clips, three patterns of student-teacher dialogue emerged: Unilateral Inquiry Response Evaluation (UIRE), Multilateral Inquiry Response Evaluation (MIRE), Inquiry Response Collection (IRC). From the standpoint of task completion, cognitive demand, psychological well-being, and metacognitive development, each of the patterns had mixed results in its ability to engender success.

Unilateral Inquiry Response Evaluation (UIRE) Pattern

The first, Unilateral Inquiry Response Evaluation (UIRE), involves a teacher working through a concept with a particular student. Often, the student may make an error in his or her thinking, and the teacher asks questions in order to prod the student to diagnose and fix his or her error. This pattern dominated the first lesson clip, in which the teacher works with a student to simplify a rational expression involving a binomial and monomial (see Chapter 4 for more discussion of this lesson clip.) The pattern can be largely categorized under the IRE paradigm, with all the responses (R) coming from one student. The teacher provides evaluation through either tacit approval (affirming a

student's response by not questioning it) or tacit disapproval (hinting to a student that he or she has made a mistake by asking him or her to explain his or her reasoning). While the teacher asks the student to explain and justify correct responses, the majority of correct responses are met without a request for justification. Thus, in the student's mind, a question from the teacher indicates there is a strong possibility that an error was made. This pattern is perpetuated in large part because the teacher wants the student error addressed, and wants the student to understand the conceptual misunderstanding that is behind the error. The cycle of IRE continues until the teacher is satisfied that the student has recognized his or her mistake, made a cognitive adjustment, and provided a correct response. Lessons that have as one of their objectives the mastery of certain tasks or techniques may be more likely to produce this pattern. This is because the teacher may fear that opening up the dialogue he or she is having with the student to other students may distract the student from the process of recognizing the error. Also, opening up the discussion could introduce more flawed contributions from other students, which would then need to be addressed or corrected. If the teacher is feeling pressed to have a certain skill or idea mastered by the end of class, he or she will be less likely to deviate from this pattern, since its ultimate aim is the correct application of certain technique. The conceptual understanding that the teacher is hoping to engender through his diagnostic questioning is more a means to this successful task completion, than it is an end in and of itself.

Thus, the pattern benefits from a certain level of time efficiency from a task completion perspective. Furthermore, it is easy for the teacher to maintain control of the discussion

and keep it focused on a particular under this paradigm. At the same time, this method excludes almost the entire class from engaging in the discussion, since it is isolated to one student. The other students have no incentive to follow the exchange between the teacher and targeted student, and thus they lose an opportunity to follow the thought process of one of their peers and evaluate the validity of their peer's comments on their own. Furthermore, this approach tends to avoid addressing tangential ideas connected to the problem at hand, since the targeted student is being guided by the teacher towards reaching mastery of a specific concept or technique. While addressing tangential ideas may take time, doing this could provide other gateways for other students in the class to understand the topic being discussed. All too often, the teacher can subconsciously that his or her thought process or conceptualization of an idea is shared by all of the students. An alternate solution method or perspective contributed by a student other than the targeted one could be the idea that helps some in the class make the necessary neural connections to achieve deep understanding.

Multilateral Inquiry Response Evaluation (MIRE) Pattern

A second closely related pattern that uses a more inclusive approach involves the same process of Inquiry Response and Evaluation, but instead of limiting the responses to one student, the teacher calls on different students to respond to either one of his comments or to another student's comments. This pattern was prevalent in the second lesson clip, as the teacher engage a handful of students in a discussion surrounding factoring strategies (see Chapter 5 for more discussion). However, the teacher still uses cues to

evaluate a student's answer: asking a question for clarification if the student makes an error, and showing tacit acceptance when a student's work is correct. Again, the goal of bringing students to a certain level of technical mastery by the end of class may motivate the teacher to alert students to their mistakes instead of letting the class take the responsibility for evaluating student comments.

As mentioned above, the benefits of involving more students in the discussion are varied. From a psychological perspective, a student is more likely to feel engaged and important and interested in class activities if he or she feels that her attention and contributions are valued and needed by the teacher. While the extended discussion may take time away from the targeted student's task completion, the process of hearing students articulate their "takes" on the problem may actually expedite the learning process for many other students in the classroom. Other students who are intently following the conversation may be more likely to internalize an idea that is articulated intelligently and compellingly by one of their peers than they would by their teacher.

At the same time, the consistent presence of teacher cues, and the tacit understanding between the teacher and the class that these cues will always be given to guarantee that no errors are left unaddressed by the teacher, undermines the potential for metacognitive development under this paradigm. Students are still responding first and foremost to the teacher's evaluation of their contributions (whether the feedback is affirmative or critical) rather than to their own sense of what is valid, what is not valid, and what is a valid way to determine or justify an idea's "correctness". In this sense, the lowering of

metacognitive demand for the entire class may actually hamper the cognitive growth of the class, since students' efficacy could become overly dependent on the presence of a teacher who is constantly at their side evaluating each move they make.

Inquiry Reponse Collection (IRC) Pattern

A third pattern could best be described as Inquiry Reponse Collection (IRC). Under the IRC model, the teacher poses an open-ended question that does not have one fixed correct answer, and simply collects responses from many students without making any explicit or implicit judgment of the responses' validity. This pattern defined the third lesson clip, in which the teacher collects students' opinions about which option for salary raises (absolute or proportional) is more fair for the employees of a hypothetical company whose salaries are given (see Chapter 6 for an in-depth discussion of this pattern.) If students believe that any response will be accepted by the teacher (since he or she has stated that there is no one "right" answer), they may be less afraid of sharing an opinion or statement that contradicts a statement made previously by one of his or her peers. In this pattern, the stage of evaluation is relatively absent, since the students are essentially evaluating each other's responses, and the teacher is playing the role of scribe and discussion moderator. This pattern tends to emerge when a task's objective is not primarily to get students to master a specific problem-solving technique, but more to encourage students to reflect about a particular idea. Open-ended questions that could be answered in different ways based on one's implicit assumptions tend to lend themselves well to this type of pattern.

Since many student voices were heard and valued, this pattern tends to be engaging and successful from a psychological standpoint. (Indeed, one of the students' parents emailed the teacher saying how much she enjoyed this foray into a real world dilemma that connected to the curriculum but also integrated politics and sociology.) Also, the highly generalized and open-ended nature of the task makes task completion fairly straightforward. All the students could feel satisfied that they completed the task of deciding which plan was more just, based on their beliefs about the distribution of wealth in a society. From the metacognitive and cognitive standpoints, however, the demand that these lessons place on students is less great since they are being asked to merely investigate a certain idea rather than apply it successfully to new situations. In the third lesson clip, for example, students were able to determine with little thought (using the chart they filled in with each employee's salary under each plan) whether a plan was advantageous or disadvantageous to a given employee. Since the numbers in the chart explicitly indicate who wins and loses under each plan, little is required of students in terms of evaluating the validity of their work. (The students can debate which employees deserve to be given preference, but the actual salary figures are non-controversial from a mathematical perspective, since they are based on a simple calculation.)

The analysis and thought required of students was more focused on notions of what was equitable and morally defensible rather than the mathematical nature of progressive increases vs fixed increases. A more cognitively demanding extension may have been to

present more salary raise options to students (perhaps Option C and Option D) that were a little more complex, and have them predict (without calculating new salaries as they had done before) who would benefit more from each plan.

Drawing upon the directed and efficient quality of the UIRE and MIRE patterns, coupled with the expansive and inclusive quality of the IRC pattern, we may imagine a new pattern that addresses the need to draw students into a critical dialogue with each other around a mathematical controversy.

The Inquiry Response Revoicing Controversy (IRRC) Pattern

In the Inquiry Response Revoicing Controversy pattern, students are presented with a task which is geared to expose certain assumptions that they may have about a particular mathematical phenomenon. An example of this type of task for a 4th grade student might be the following:

Investigate the following statement, and decide whether you agree or disagree:

When you divide a number into another, the result is a number that is smaller than the divided (the number you divided into.)

Justify your answer with logic and numerical examples or counterexamples.

The teacher would solicit responses from a variety of students, and then discern contradictions that may exist between two or more student comments. Alternatively, the

teacher may observe contradictions in two parts of a student's argument. The teacher then artfully revoices the essence of each contradictory idea, and presents this controversy to the students, without hinting at the correct resolution to this contradiction. The teacher does not take a side in resolving one of these contradictions, but also does not avoid controversy by establishing that all responses are valid. Instead, the teacher forces students to face the contradictions created by their arguments. Moreover, he requires them to use mathematical logic to justify their resolution of the apparent contradiction. In this paradigm, the element of teacher evaluation, whether unconditionally affirming (as in the IRC paradigm) or implicitly telegraphed through verbal cues (as in the first two IRE paradigms), is noticeably hidden to the students. Of course, the teacher is continually evaluating the validity of student responses as they are shared; he or she simply doesn't communicate these evaluations implicitly or explicitly to the students. However, by modeling the process by which ideas are justified and examined from the basis of mathematical logic, the teacher is training students to be their own evaluators, instead of relying on the teacher to do the evaluating for them.

This IRRC paradigm has significant metacognitive and cognitive benefits. For one, students are being trained in mathematical habits of mind, such as the necessity of justifying arguments with proof and or sound logic. Moreover, students are developing tools to help them examine the validity of a mathematical idea intelligently and communicate their opinions clearly and respectfully to each other. Moreover, in bringing to light a host of student assumptions and arguments, the teacher helps the students become aware of misunderstandings that they may share with their peers. If

resolution is reached between students regarding one or more of the contradictions introduced, those who held erroneous beliefs or assumptions may be more likely to correct them and apply valid assumptions to future tasks. Furthermore, those students who were persuaded by peers to let go of flawed assumptions are more likely to internalize the valid ideas that won them over, and understand why these ideas are in fact valid.

At the same time, this paradigm is not free of dangers. For one, the element of conflict and controversy between various students' ideas could easily devolve into mean-spirited and antagonistic confrontation between students. Students holding flawed assumptions may be reluctant to let go of them even if they recognize that they are invalid if they fear being ridiculed as the "losers" of the argument. Pride may motivate them to stubbornly cling to their ideas even in the face of overwhelming proof that they are wrong. If a teacher senses that students are taking an overly personal stake in being on the "winning" side of an argument, he or she will need to act quickly to separate the ideas that students are contributing from the students themselves.

A possible strategy for mitigating potential divisiveness in the face controversy could be the normalization of the "devil's advocate" role among students. Student could either be assigned this role, or be free to take on this role at any time in the discussion. The more that the teacher can communicate the inherent value that contradictory ideas have – in generating fruitful discussion, in clarifying valid from invalid assumptions – the more comfortable students may feel sharing their conceptualization of an idea, even if it turns

out that their conception of the idea is flawed. A teacher who engages students in resolving these “controversies” needs to first lay the foundation for respectful and civil discourse by giving students opportunities to debate various sides of a controversial argument in highly structured contexts. Students and teacher need agreed upon norms in terms of how students address each other and listen to each other, and these norms can only take hold if they are modeled and practiced by the teacher repeatedly. Students need to feel that whatever the outcome of a controversy – stalemate, resolution, etc – they are guaranteed respect from their teacher and peers.

Clearly, the successful implementation of the IRRC pattern is no small feat in any classroom. A teacher must be highly sensitive to the trajectory of a discussion, and know how to gracefully and subtly steer it in the right direction when it veers too far off course. He or she must find a way to re-articulate students’ voices in authentic ways while at the same time managing the complex personalities of the students’ themselves.

Implementation of the IRRC Pattern: The Need for Training

Clearly, training the students to engage in respectful and rigorous mathematical argumentation before presenting them with controversies seems essential in order to avoid the divisiveness and high levels of student frustration described above. The palpable instability that emerged towards the end of the Salary Raise lesson described in Chapter 6 was a clear indicator of the danger of implementing the IRRC pattern too quickly. Without sociomathematical norms established in the classroom to guide them

through this new type of discussion centered around a math controversy, students would understandably feel unclear as to how to proceed. Students need to be comfortable with the notion of mathematical controversy as an acceptable and non-threatening classroom dynamic before I can apply a questioning pattern that so openly embraces, and even seeks out, controversy itself. As I looked ahead to my fourth and final videotaped lesson, I committed myself to investing sufficient class time in training students in both the art of constructing mathematical arguments, and the skill of respectfully but rigorously analyzing the validity of their peers' arguments. (See Chapter 8 for a discussion of my implementation of the IRRC pattern in my fourth videotaped lesson.)

The Toolkit of Questioning Patterns: A Strategic Approach

The presence of both benefits and dangers for each questioning pattern discussed in the preceding chapters suggests that teachers need to think carefully about what their objectives are before employing a given pattern. If he or she wants tight control of the evaluation of student comments, then he or she might favor the strict and consistent application of the UIRE or MIRE pattern, since it provides the teacher with the space and sanctioned responsibility to immediately evaluate every comment. (Indeed, students are expecting the teacher to fulfill this responsibility under these patterns.) If, on the other hand, the teacher is more concerned with first understanding how students are thinking about a problem, he or she may try to first collect student contributions and use revoicing techniques to help students conceptualize what mathematical ideas have been presented. This latter approach may produce patterns of questioning, such as

the IRC and IRRC patterns, in which teacher evaluation is much less present, and student debate or “controversy” is more prevalent. In these patterns, a teacher’s role would be understood more as the moderator/facilitator of a discussion than as the conductor of a series of exchanges.

The UIRE, MIRE, IRC and IRRC patterns are by no means mutually exclusive as pedagogical tools. It may be that a teacher uses a variety of patterns through his or her lesson in order to achieve specific aims for particular parts of the lesson. *All these patterns have limitations, and should be implemented wisely according to what is needed in any given teaching context.* More important than *which* pattern a teacher uses in a given situation is his or her awareness of what kind of orientation the pattern encourages in his or her students.

Whatever approach a teacher chooses for a given scenario, however, his or her questioning patterns can only be carried out if they are in accord with sociomathematical norms that have already been established in the classroom by the teacher. If, for example, the students are accustomed to having the teacher tell them whether an answer is right or wrong, it may be very difficult for them to respond to a teacher using the latter revoicing approach. The desire among students to know “who is right” may be so strong that students refuse to share their own insights and join the “controversy” until the teacher has made a pronouncement about which answer is valid and which is invalid.

This then presents the teacher with another dilemma – if presenting a certain questioning pattern will challenge the norms in a classroom, does a teacher forge ahead and face these conflicts or avoid them by abandoning the “norm-challenging” pattern? The dilemma is by no means simple to resolve, and may involve a teacher introducing students to new norms in smaller, more manageable doses until they are ready to fully embrace a new pattern which requires them to accept new norms of surrounding the evaluating of student ideas.

Chapter 8: Engaging in Controversy

Background: The Students

For my fourth and final videotaped lesson, I decided to try implementing the IRRC pattern while engaging one of my 7th grade accelerated classes in a task designed to generate mathematical controversy. These talented 7th graders, like the 8th graders in the first two lesson video tapings, are in the top 30% of their grade in terms of mathematical aptitude. (To put this in a broader perspective, this means that nearly all of them are in the 99th percentile nationwide according to their ERB standardized test scores.) They tend to be independent thinkers who are accustomed to grappling with highly demanding cognitive tasks. Their high mathematical aptitude is clearly an asset as they engage in mathematical argumentation, since they are able to apply their excellent number sense in constructing cogent and valid explanations of mathematical claims. Indeed, this ability to articulate mathematical ideas well was one of the principal reasons I decided to use this group of students for my experimentation with the IRRC pattern. With a limited amount of time left in my study, I knew that I needed a group of students that would be immediately receptive to the importance of constructing logically valid arguments.

At the same time, their very ability poses its own challenges. Since they generally have a high view of their own ability, they can sometimes get defensive when challenged by peers. Alternatively, some students in the class can be intimidated by the presence of exceptionally able students whose aptitude they feel is significantly above their own. As

a result, these less confident students might be less likely to voice an opinion that is contrary to the opinions voiced by those students seen as the most able in the class. It was thus essential for me to establish guidelines for these discussions that would guard against overly contentious debates and intimidating classroom environments.

Background: Preparation for the Lesson

Given the norm-challenging nature of the IRRC pattern, along with the intrinsic sensitivity of my students to criticism, it was clear to me that I needed to prepare my students carefully before engaging them in debates surrounding mathematical “controversies” -- mathematical tasks that generated multiple and conflicting student answers, without an immediate resolution from the teacher. (See Chapter 7 for a more detailed discussion of the dangers of engaging students in mathematical debates without adequate preparation.) To this end, I designed a sequence of training exercises to help students gain a better feel for what it meant to present a compelling argument during one of these mathematical debates.

In the first activity, I asked the students to develop skits that modeled civil, productive and respectful discourse, and then follow them with ones that modeled disrespectful and counterproductive discussion. As they developed these skits, I asked them to reflect on what words or phrases could trigger positive feelings of respect among discussion participants, and which words or phrases could antagonize students against one another.

In their skits, students used these words or phrases to reflect their power to influence the emotional climate in the classroom.

In the second preparatory activity, I asked students (in groups) to choose one of the following three statements:

An even number plus an even number is always an even number.

An even number plus an odd number is always an odd number.

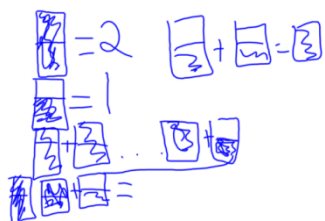
An odd number plus an odd number is always an even number.

They were then required to present three mathematical arguments of varying quality (poor, average, excellent) that proved the statement that they chose. After each argument presented, the rest of the class evaluated the quality of the argument, and voted in the end for which argument was the most elegant. Then, the class had a discussion about what characterized elegant and compelling mathematical arguments. Here are some tips students came up with for those presenting mathematical arguments, as transcribed by myself on the SmartBoard:

- present it confidently
- logical, examples
- present supporting examples (concrete)
- Simple
- prove general pattern

Two common themes emerged from this discussion regarding what makes an argument powerful. The first was that good arguments are usually simply stated, and clear to those who hear it. Some of the students who presented arguments remarked that it can be sometimes daunting to maintain your train of thought when expressing a mathematical claim in front of the class, and the simpler your argument is, the easier it is

to express to your peers. Secondly, many mentioned the power of specific and concrete examples in illustrating points. While students mentioned that examples don't necessarily prove an idea is true (since counterexamples may exist), demonstrating a "general pattern" that can be applied to all examples is powerful. The prize for "most elegant" argument went to two students who proved that an odd and an odd make an even using the metaphors of full and half-full glasses. Even numbers, they stated, could be seen as a group of full glasses, each of which has two halves, since even numbers are just sets of 2 halves. Odds, on the other hand, could be seen as a set of full glasses and then an extra half class, since they are one more "half" than an even number (of half glasses). When two odds are combined, the two leftover half full glasses are combined to make a full glass, and a set of full glasses, which by definition represents an even number, is created. Here is their diagram and work:



Students liked this argument for its simplicity, its clarity and its generality. (It was not restricted to a finite set of examples.) In fact, many students' "average" arguments included merely specific examples of odds and odds equaling evens, evens and evens equaling evens, etc, rather than general theories about what happens when odds and evens are combined with each other. In other words, students deliberately chose "examples only" arguments to demonstrate less than compelling arguments as a part of this exercise.

Finally, I decide to give the students a sheet [see Appendix B for the full text of the handout] in which they explored properties of exponents such as the multiplication property of exponents, and the power to a power property in a purely exploratory fashion using only numbers instead of algebraic generalizations. I did not explicitly teach them the power to a power property, but rather gave them numerical expressions involving powers raised to powers, asked them to write these expressions in expanded form, and then asked them to generalize any patterns they noticed. I purposely did not discuss the sheet with them afterwards – it was more to give me a sense of what kind of knowledge they were carrying regarding the behavior of exponential expressions. In particular, I was interested in their understanding of the power to a power property, which would play a role in the controversial task to be presented in the IRRC-based lesson. About two thirds of the kids seemed to have a solid understanding of the concept, while one third were still confused about its logic. From this data, I knew that the task I chose was likely to expose some of these misconceptions about exponents, and thus would serve as a helpful tool for addressing them. With guidelines for presenting compelling mathematical arguments in a friendly and respectful environment established, as well as an introduction to the behavior of exponents, I decided that the students were ready to engage in a controversial mathematical task dealing with exponents.

Background: The Lesson, the Task, and its Rationale

I decided to make the controversial task center around exponents. I gave the following task to students:

Without using a calculator, determine which number is the biggest: 2^{134} , 8^{46} , or 21^{34} ? Explain your reasoning.

In the next five minutes, make a choice (which number is biggest) and write a mathematical argument that supports your choice. Think about what makes a good mathematical argument, and how your argument can be presented in the clearest, most compelling manner. You may use diagrams, equations, metaphors, your knowledge of exponents – anything that will make your argument more compelling and logical. Use the space provided below.

I constructed the task with a number of pedagogical objectives in mind. First, I wanted to give the students a problem that related to a topic that they were familiar enough with to explore strategically, but not so familiar with that they had achieved full conceptual mastery. Their relative familiarity with the *purpose* of exponents – to denote repeated multiplication in an efficient manner – would give them enough knowledge to enter into the task and imagine what the expressions 2^{134} , 8^{46} , and 21^{34} would look like if they were written in expanded form. This “footing”, I felt, was essential for the kids to be able to confidently present arguments in front of their peers.

At the same time, I hoped that their lack of mastery regarding the properties of exponents would mean that some of the students would make some erroneous assumptions about how exponential behavior. This, in turn, would mean that at least some of the students would make arguments for the incorrect answer, even while

genuinely feeling that their argument was valid. I needed this diversity of valid and invalid arguments, confidently presented by different students, to create a “controversial” mathematical scenario in which it was not immediately clear which of the students was right. Moreover, I wanted students to be evaluating each other’s *arguments*, rather than evaluating each other. I hypothesized that their lack of mastery of properties of exponents meant that they might be more open to a variety of opinions, listening to a spectrum of arguments mindfully before settling on an answer that seemed to be the most compellingly justified. This openness, I felt, would help facilitate constructive and respectful discourse, and a willingness to listen to one another’s ideas.

I chose the numbers 2^{134} , 8^{46} , and 21^{34} to tease out students’ understanding of a handful of concepts surrounding exponents. The first was the notion that exponential expressions whose bases are composite can be rewritten as powers of some of their prime factors. In this case, I wanted them to see the value in writing 8^{46} as a power of 2. I also wanted them to use their knowledge of expanded form to imagine what 2^{134} and $(2^3)^{46}$ would look like if written out with all the powers of 2 listed. I hoped that they would draw on their discovery -- on the Exponent Exploration handout I had given them the previous week -- that powers have a multiplicative effect on other powers. Even if they hadn’t taken away a clear sense of this multiplicative effect from the exponent handout, I hoped that they might at least “rediscover” this multiplicative effect by imagining 8^{46} written out as 46 groups of $2*2*2$.

I added 21^{34} as a third number to increase the complexity of the task and add a level of ambiguity. An extremely astute student might have developed an understanding of the distributive property of exponentiation over multiplication from the exponent exploration, and used it in the following manner to prove that $21^{34} > 8^{46}$ (or 2^{138}) $> 2^{134}$:

$$21^{34} > 20^{34} = (2 \cdot 10)^{34} = 2^{34} \cdot 10^{34} > 2^{34} \cdot 10^{33} \approx 2^{34} \cdot (2^{10})^{11} = 2^{144}, \text{ where } 2^{10} \approx 10^3$$

(Note: The approximation error incurred by rounding 1024 down to 1000 can be neglected, since 1000 is $2^{9.97}$, such that $2^{34}(2^{9.97})^{11}$ is 2^{143} , which is still greater than 2^{138} .)

Since it was highly unlikely that any of the students would be able to articulate and present this argument for 21^{34} , I felt confident that we would probably leave the end of the lesson *without* absolute proof that 21^{34} was in fact the greatest number. This probable ambiguity, I theorized, would serve as a tool to help students distinguish between a compelling, logical proof based on their definition of exponents (like the one proving that 8^{46} was larger than 2^{134} by substituting 2^3 for 8), and a conjecture founded on student “hunches” about what *seemed* to make 21^{34} greater than 8^{46} . Undoubtedly, I thought, students would conjure up their own properties (without any justification for using them) to defend these hunches that 21^{34} or 8^{46} was the greater of the two.

Subsequently, this would provide an opportunity for students to investigate the logical basis for these hypothetical claims, perhaps exposing the flaws in their conjectures.

As they made arguments for one of the three numbers, I expected students to draw upon four major pieces of mathematical knowledge that I knew had been established and reinforced in previous math classes they had taken in 5th and 6th grade. The first was the

notion an exponent is just a way of expressing the repeated multiplication of a given number, or base. The second was that any exponential expression, a^b could be written in expanded form as the product of a with itself b times. The third idea was that if two expressions have the same (positive) base, then the greater expression is the one with the higher exponent. Finally, I expected them to know that substituting a different form of a quantity for another (such as 2^3 in place of 8) does not change the value of the expression.

At the same time, I was not necessarily expecting kids to refer explicitly to properties of exponents that they might have discovered in completing the Exponent Exploration handout, such as the multiplication property of exponents, power to a power property, and distributive property of exponentiation over multiplication. While these properties (particularly the latter two) would be very useful to apply to the task I presented them with, I was more interested in how they could use their fundamental knowledge of exponents as symbols of repeated multiplication to “rediscover” the multiplicative effect of raising powers to powers during the exploration of the task. I wanted them to develop a deeply ingrained sense of where this multiplicative effect comes from, rather than simply memorizing the power to a power property by rote and mechanically applying it to write 8^{46} as power of 2.

After the allotted five minutes, I asked students to present their arguments for 2^{134} , 8^{46} , and 21^{34} , respectively. Instead of having a “back and forth” dialogue between students about which number was bigger, I decided to begin the discussion by simply having a

number of students present conflicting opinions. I knew that one of the limiting factors in my implementation of the IRRC pattern would be my willingness to take the class time to let these discussions unfold without seizing control of the discussion with the verbal cues and teacher-directed evaluation exemplified in the UIRE and MIRE paradigms described earlier. By having students present their opinions one after the other with no discussion in between, I hoped to give myself the opportunity to step back and allow a variety of student opinions to be heard without my intervening guidance, thus creating the “controversy” without making any efforts to immediately resolve it.

During this presentation portion of the lesson, I asked six students to come up in present their arguments for a given number – two from those who chose 2^{134} , two from those who chose 8^{46} , and two from those who chose 21^{34} . As students presented their arguments, using the SmartBoard to note down important equations and work, I periodically sought to “revoice” student ideas with brief summaries of their arguments. Once all six arguments had been presented, I asked the students if any of them had changed their opinion of which number was the greatest simply based on the power of an argument presented by one of their peers for one of the other numbers. We also touched upon which arguments were the most elegant, and what made them so. Below is a transcript of the discussion that followed once the 5-minute reflection time that students had to generate their arguments had transpired.

Lesson Clip #4 - The Transcript

Key: AC = the teacher (myself)

Part 1: The teacher sets the stage for the discussion, and Michael presents his case for 2^{134}

- 1 AC: We are list what people's points are for each number. Could I have someone who thought that 2^{134} was the biggest?
- 2 AC: I want you to be honest and just share what you wrote in your argument. I don't want you to change midstream now. Read what you shared – you'll have some time to revise your argument later, but right now just share what you wrote. Yeah, Michael?

(Michael volunteers and offers to come to the SmartBoard.)

- 3 Michael: The reason that I think 2^{134} power is a bigger number than the others is that....in my previous experiments, ...experiences, on the sheet*, I found that the bigger the exponent, the greater the value of the number. A little smaller version what it's kinda like here....
- 4 Michael: 2 to the 12 equals,....just to see if it corresponded to what I thought...equals 4096, and if you did 21^2 , or 441, it's obviously a lot smaller.

Here is what Michael wrote on the SmartBoard:

$$2^{134}$$

$$2^{12} = 4,096$$

$$21^2 = 441$$

- 5 AC: (To class) OK. So, just to rearticulate, Michael is suggesting that exponents

kind of have more “power” using the same digits, than the base. OK?

6 Michael: Yeah

Despite realizing Michael’s misconception – that digits can be shifted from exponent to base to decrease a number, a vice versa, I withhold any comment about his theory. I want to allow this misconception to be expressed without critique so that we can address it later in the discussion.

7 AC: Do we have someone else who wants to share for 2^{134} ? [Alex volunteers.]

OK Adam. [Alex comes to the SmartBoard.]

Part 2: Alex presents his case for 2^{134}

8 Alex: My argument is basically proving this statement $2^{134} > 8^{46} > 21^{34}$. So what I did is I started with 8 [to the 46th] being bigger than that [21^{34}], and what I said is that since 8 times 8 = 64, if you just did 8^2 , so 8^{46} is 64^{44} , which is obviously bigger because both the base and the exponent are bigger than in 21^{34} .

9 Alex: You have 2^{134} , it’s basically....it’s 2 to the um 2^4 , which equals 16, which is 16^{130} , and both the base and the exponent are larger than in 8^{46} . So I said 2^{134} is greater.

Here is what Alex wrote on the SmartBoard:

$(2^{134}) > 8^{46} > 21^{34}$
 $2^4 = 16$
 $2^{134} = 16$
 $1708 \times 8 = 64$
 $8^{46} = 64$

10 AC: Thank you, Alex. So what Alex is saying, just to replay his idea here, is

that ah... since 2^4 equals 16, I can kind of “take off” those 4 twos and write that as the base that 16 and write that as the base, and then he can use the rest of the exponents [130] to raise it [16] to a power.

Again, I hold off here in commenting about Alex’s logic, which, like Michael’s, involves shifting powers of 2 from the exponent to the base and assuming equivalency incorrectly. I want students to be able to see how a flawed argument can be presently in a systematic and somewhat logical fashion. One can follow the logic that Alex applies in borrowing from the exponents and giving to the base, even though it is flawed mathematically.

- 11 AC: OK, now even if you have 2^{134} [as your greatest number] I want to go some of these other ones, to give them a chance, so anyone for 8^{46} ? Did we have any takers for 8^{46} ? I’ll take Tim, and then Nick. Tim?

Part 3 – Tim presents his argument for 8^{46}

- 12 Tim: OK, so this is not as mathematical, but...OK, so it has the biggest speed over 2^{134} because $8*8*8...$
- 13 AC: (writing on the SmartBoard): It advances more quickly because of the base
- 14 Tim: It also beats the 2^{134} because it has a bigger power, which means it will multiply more, so and there’s also...it [8^{46}] has a bigger starting number than the 2^{134} .
- 15 AC: (writing on the Smartboard): So bigger exponent than $2^{134}...$
- 16 Tim : So it [8^{46}] has both [the exponent and the base] pretty high up, so I think that [it is the highest number]

- 17 AC: So it's sort of combining the power of the exponent with the power of the base – the best of both worlds, is what you're suggesting (class laughs).

In Tim's case, I deliberately stay away from probing the mathematical details of his argument. Instead, I encapsulate his general theory with a "best of both worlds" description. Both he and the class have a sense here that the argument has not been mathematically justified, that it is more of a "hunch" than solid proof. I decide to let the rest of the class observe this fact for themselves.

Here is what I wrote on the SmartBoard for Tim's argument,

- it advances more quickly because of the base. ($8 \rightarrow 2$)
- has bigger exponent than 2^{134}
- "in the middle"

- 18 AC: Is that what you meant? I don't want to put words in your mouth.

- 19 Tim: Yeah.

- 20 AC: OK, let's take one more for 8^{46} and then we'll take 2^{134} . OK, Nick? You want to come up or do you want to [stay at your seat]...OK, come on up. Speak nice and loud.

Part 4 - Nick presents his argument for 8^{46}

- 21 Nick: Pretty much what I did is 8^{46} is bigger than 2^{134} . Two is, um... 2 times 2 times 2 equals 8, and so to get 8^{46} , you want to convert that...if you want to convert 8 into a smaller number, it would be 46, to the 92^{nd} , I think, so...I just

lost my train of thought. (Nick goes back to his seat to get his notes.)

22 AC: So, I think what Nick is trying to say, is that he's trying to convert 8^{46} into a power of 2.

Here, I see that Nick is floundering as he gives his explanation, and self-conscious about doing so in front of the class. I decide that this is an opportune time for me to revoice some of his ideas so that he will feel more supported. I can tell that Nick wants to transform 8^{46} into a power of 2 for comparison with 2^{134} , which is exactly what I am looking for students to do. As such, I think it is important for his idea to get communicated clearly, so I revoice more than I have with some of the other presenters.

23 Nick: Yeah, pretty much.

24 AC: So in other words, multiplying 8 is the same as multiplying 2 how many times. Is that the gist of your question?

25 Nick: Yeah, and so....*(pauses for five seconds and looks down, stuck in thought and flushed with embarrassment)*

26 AC: And how many twos [multiply to give] one 8?

Here, I see that he is having a tough time finishing his argument, but I can clearly see that he is trying convert 8^{46} into a power of 2. I ask this question to help him move forward with the explanation he has started.

27 Nick: It's 3 twos.

28 AC: 3 twos? So if you had 46 of those three twos, how many twos would there be?

29 Nick; Yeah there would be...46 times 3....*(thinking)*...

30 Tim: (*Calls out*) 138.

31 Nick: Wait, what did you say, Tim?

32 Tim: 138

33 Nick: Yeah, so it's just bigger [than 134], but not by that much.

34 AC: So (*to Nick*), could you write that out? So, do people follow his logic?

He seems to be saying that 8 as 2 to the third power,....so 46 8's is the same as 138 2's.

Again, I step in here to make sure that the class was following what he is saying, using some revoicing to clarify his main point.

35 Nick: So with [two repeated] three times [for each 8] that would be 2^{138} is greater than 2^{134} . *Here is what Nick wrote on the SmartBoard during his presentation..*

Handwritten notes in green ink:

$$2^{138} \rightarrow 2^{134}$$

$$2 * 2 * 2 = 8$$

Below the first '2' is a '4'. Below the second '2' is an '8'. A '+' sign is between the second and third '2'.

36 Nick: 21^{34} is definitely smaller because I found out how many times you multiply 2 to get 8, and 8 goes into 21 less than three times....that's it.

37 AC: OK, now let's take 21^{34} . I got a whole bunch here now. OK, Lisa

Part 5 – Lisa presents her argument for 21^{34}

38 Lisa: OK, kinda like Nick was saying...what I did, I knew that 8^{46} had to be bigger than 2^{134} because $2*4$ is 8 and approximately 46 times 2.5 is 134, and

then to get from 8 to 21, its [multiplied by] 2.65, but you only multiplied 46 by 2.6 to get 134, so I think that 21^{34} is the biggest.

Here is Lisa's work on the SmartBoard:

Handwritten work showing calculations related to 21^{34} and 2.65 . The work includes a division of 21^{34} by 2.6 resulting in 8 , and a multiplication of 8 by 2.65 resulting in 21.34 . There are checkmarks and arrows indicating the relationship between the two calculations.

39 AC: OK, so what Lisa is saying is that because we multiplied the base in 8^{46} by 2.65, whereas we only multiplied the exponent by 2.6 to get 134, 21^{34} is bigger. *At this point, I have to make a decision about whether to investigate the incorrect “scale factor” theory that Lisa was proposing. I could address her misconception by presenting another example with smaller exponents and bases to let students see if her theory holds up. However, my gut instinct tells me that Diana really doesn't have sturdy mathematical justification for her hunch, and I am not ready to expose this lack of proof just yet. As in Tim's case, I want the rest of the students to be able to judge her argument on their own without my guidance. Hoping to come back to Diana's claim later, I move on to the next student volunteer.*

40 AC: OK, Peter?

Part 6 – Peter presents his argument for 21^{34}

41 Peter: OK, mine is kind of the same. So basically, I thought 2 times 2 times 2 equals 8. It's 2 to the third power is 8, and then this [46] goes into 134 almost

the same, 46 [multiplied by 3] is almost 134 I think, so that 8^{46} is bigger.

42 Peter: And then, 21 to the 34th power, its like, the power is one fourth less than 46, so its not that much smaller, but it's almost three times the base, so for that reason I think in this example its 2 to the third power, 46 times 3 more, but it should match up, but this (*points to 21^{34}*) is way more because it's a third less than 46, but its three times more than the 8. And I also have an example....

43 AC: So could I just stop you for a second? What he is saying, is that we didn't lose a lot of exponents with the 34, but we [more than] doubled the base, two and a half as much. That doubling and a half outweighed the drop in the exponents.

I decide to intervene here and revoice because I get the sense from the blank stares around me that the class is not completely following Peter's argument for 21^{34} in Comment 42.

44 Peter: Yes.

45 AC: That's your idea, OK.

46 AC: Um, can we just stop right there?

Here, I feel tempted to investigate Peter's theory with some probing questions.

However, I worry again that, as with Lisa, Peter does not have sufficient mathematical justification for the conjecture he presents in Comment 42. I decide that, having done my best to articulate the gist of Peter's theory, it will be better to let students judge it for themselves, and come back to analyzing it during the discussion that follows.

47 Peter: Yes.

Part 7 – The class discusses whether any conclusions can be made, and whether anyone has been persuaded to change their answer.

48 AC: I want to wrap up, we've got just a few minutes left. Are there any of these [three numbers] that people would like to eliminate, that they're sure cannot be the greatest of those three? Who feels convinced that at least one of them can be thrown away? Kevin?

With time running out, I realize that we won't have time during this lesson to address all the claims made by the six presenters during this lesson. I decide to postpone this in-depth analysis of their claims to a future discussion, and focus instead in getting students' feedback on what argument(s) they thought were most compelling in the remaining few minutes. I decide that getting their reaction immediately after the presentations will give me a more accurate sense of how they interpreted their peers' ideas.

49 Kevin: I think 2^{134} .

50 AC: You're convinced? What convinced you the most? What argument convinced you the most that 2^{134} , in your opinion, was not the highest? Yolanda?

51 Yolanda: Nick's argument.

52 AC: What did you like about his argument?

53 Yolanda: It had a lot of good proof.

54 AC: Let me go back to this. (*I find the SmartBoard page that Nick wrote on.*) I think this one was Nick's, right? What did you like about Nick's argument

that convinced you that you could throw away 2^{134} ?

55 Yolanda: Well um it had a lot of proof that 8^{46} was more than 2^{134} .

56 AC: Alright, Michael?

I feel here that we need more specifics about what made Nick's argument "a lot of proof". Since I see Michael raising his hand in support of Nick's argument, and Michael has originally chosen 2^{134} , I view this as an opportunity to hear from someone who has changed his or her mind.

57 Michael: Well, it basically is saying that 2^{134}no, sorry, 8^{46} is 2 to the third, to the 46th.

58 AC: Stop right there, let me write that down. Matt is saying that Nick's argument, is that 8 to the 46th is...

59 Michael: 2 to the third to the 46th.

60 AC: *(writing on SmartBoard) 2 to the third...to the 46th, like that. (I write Michael's representation of 8^{46} to Nick's original work, as shown below on the upper left of the SmartBoard image.) In doing this, I am seeking to provide a visual representation of his comment that students can stare it and digest even after his comment is made. I also want students to see a more refined mathematical representation of his ideas next to his own. Note: The outer set of parentheses were unnecessary in my expression. I was so excited by Michael's connection that I neglected to erase them.*

$$8^{46} = ((2^3)^{46})$$

$2^{138} \rightarrow 2^{134}$
 $2 * 2 * 2 = 8$
 $4 * 8 = 32$

61 Michael: And in the sheet where we were exploring exponents [see Appendix] I think a problem like that where there's two exponents, that kind of equation I don't know what you call that....

62 AC: A power to a power

63 Michael: Yeah. Then you just multiply them together, multiply the exponents, and that's what the number is, cause 3 times 46 is 138, which is a greater number than 134. So numerically, 2^{138} is a larger number than 2^{134} .

64 AC: OK.

65 Michael: He [Nick] did a good job.

66 AC: That was a good job by Nick.

67 Michael: Yeah.

68 AC: Ahh, we didn't really have any resolution about 8^{46} and 21^{34} . But we were able to....Now, I want to come back to this. Whose opinion has been changed during the class period about which one was the greatest?

(Alex and Michael, both of whom presented arguments for 2^{134} , raise their hands eagerly)

69 AC: OK, alright, so your opinion was changed maybe because it was a good argument, it was a logical argument, it helped you out. It sounds like to me it's not quite clear yet which one [8^{46} or 21^{34}] is bigger, is that the general sense? Because I get the feeling that not everyone is ready to vote for 21 or [vote for] 8. Raise your hand if you would vote for 8.

I want to remind students here that, despite Nick's excellent proof that 8^{46} was

in fact larger than 2^{134} , we still have not seen any proofs that 21^{34} was bigger than 8^{46} . Lisa and Peter have both made conjectures that 21^{34} seems bigger, but they have not proved their conjectures in a systematic way.

(A few raise their hands.)

70 AC: What about 21?

(More raise their hands.)

71 AC: OK. I'll see you Thursday.

Lesson Analysis: How faithfully was the IRRC pattern applied?

In studying the transcript of Lesson #4, I was principally interested in the extent to which my questioning patterns reflected two essential qualities of the Inquiry Response Revoicing Controversy (IRRC) pattern imagined in Chapter 7. First, I wanted to see whether I used revoicing strategies to help empower students in making their arguments clear to their classmates. Secondly, I also wanted to study the extent to which I allowed students to make judgments about their peers' arguments without any guidance (explicit or implicit) from me. In other words, did I allow students to express conflicting opinions about a mathematical issue, and grapple with this "controversy" on their own, rather than taking on the responsibility of evaluating their peers' arguments for them?

In terms of revoicing, a close analysis of the transcript reveals a consistent rhythm of student responses (presentations of their arguments for a particular answer), followed by my brief revoicing summaries of their responses. I do this for Michael (Comment 5),

Alex (Comment 10), Tim (Comment 17), Nick (Comments 22, 24, 34), Lisa (Comment 39), and Peter (Comment 43) , almost always at the end of their presentations. In Nick's case, I intervene during his presentation, in addition to summarizing his argument. (This was largely because he seemed to be getting flustered by the task of explaining the argument to the class, and I thought revoicing and clarifying his comments would help put him at ease.) I also revoice Michael's ideas at the end of the lesson as he explains why Nick's argument that 8^{46} is larger than 2^{134} convinced him to adjust his belief that 2^{134} was the largest of the three (Comments 58, 60, and 62).

In terms of the second essential quality of the IRRC pattern – allowing and encouraging student controversy without immediate resolution by the teacher -- I hewed to the spirit of the pattern for most of the lesson. The structure of the presentation portion of the lesson – with two presentations for each of the three answers following each other in immediate succession – greatly facilitated my ability to refrain from evaluating student arguments. I literally didn't provide any space for discussion of the six arguments until after all of them had been presented. In fact, it is not until Part 7 of the lesson, at Comment 47, that I finally ask students to make a judgment about the arguments and eliminate any that they feel can certainly not be the greatest number of the three.

Some might claim that my question in Comment 47 was slyly directing students to eliminate 2^{134} , since a number of students had mentioned it in the presentations as being less than at least one of the two other numbers, due to the properties of exponents. And yet, I felt the question was necessary to jumpstart the discussion about which number

was the greatest. As noted in the transcript notes, it did not seem like any student had made a compelling case for 21^{34} being greater than 8^{46} , so focusing which number was *the* greatest was more likely to generate confusion among the students. In order to help foster some discussion about what made certain arguments, I thought it would be more helpful to focus students' attention and what they *did* feel was demonstrated in a compelling manner.

Indeed, Michael's reflections in Part 7 of the lesson about what made Nick's argument for 8^{46} convincing for him, and how Nick's argument related to what he had just recently explored in the handout on exponents that he had completed, suggest that Michael did in fact evaluate his own argument (for 2^{134}) a second time after having heard's Nick's argument without any explicit prodding from me. Michael was engaging in a deeply metacognitive process of assessing the arguments presented to him, comparing the ideas presented in the arguments to his own mathematical knowledge of exponents, and adjusting his argument accordingly when he realized a dissonance between what he knew about exponents and what his argument for 2^{134} was suggesting. Thus, it would be appear that the IRRC pattern, in addition to being implemented with relative fidelity, also encouraged some of the thinking and learning I seek to develop in my students.

At the same time, by avoiding discussion until after the arguments were all presented, I may have suppressed dialogue between the students that could have among one another about their individual arguments. If the lesson were viewed as a "debate", all that really took place were the "opening arguments" for each case. There was little "cross-

examination” between students, as they did not have the opportunity to immediately comment on what their peers were stating during their presentations. By the time that the lesson reached its end, most students had reached the conclusion that 8^{46} was in fact greater than 2^{134} , and that 21^{34} *seemed* to be the biggest of all, even though no one had proved that conclusively. However, there were many erroneous ideas presented during the presentations that were never questioned or examined, by myself or by students. Alex, for example, felt that by removing exponents from an exponential expression and translating them into a new base with the leftover exponents as the new power, one does not change the value of the expression. Lisa posited that one can compare sizes of two exponential expressions by comparing scale factors that relate the corresponding bases and exponents to see if they are multiplicative inverses. By not addressing these erroneous claims, nor giving students opportunities to challenge and investigate them immediately as they arose, was I depriving the students of a valuable opportunity to identify the flaws in their peers’ logic?

Perhaps by structuring the lesson into a presentation part, followed by a discussion part, I did lose out on the possibility for constructive student exchanges back and forth regarding what was presented. And yet, I also knew, from the diagnostic/exploratory handout I had given and collected from the students on exponents, that their knowledge of exponents was characterized more by exposure than mastery. I was worried that by addressing the erroneous claims about exponents head on during the presentations, I would feel pressure to teach the kids about fundamental properties of exponents so that they could articulate what made these claims erroneous. In the process, I worried, we

would get bogged down in discussions about exponents that the students as a whole weren't developmentally ready yet to digest, and lose the opportunity to hear multiple student arguments, which was my highest priority. More important, I thought, was that the student presenters felt comfortable making their presentations, rather than feel forced to defend "on the spot" concepts about exponents that they had limited exposure to.

Moreover, I felt that by having a greater variety of arguments presented, rather than just one argument whose discussion monopolized most of the class, students would develop a deeper appreciation for what made some arguments more elegant and compelling than others. After all, the principal goal of the task was not to teach students about exponents and their properties. The topic of exponents, rather, was strategically chosen as a portal through which students could engage with mathematical controversy.

As was hoped for in my construction of the task, the fact that the students were familiar with the exponents topic, without having mastered it fully, made it ideal as a source of mathematical controversy for this group of gifted students. Since no one in the class had a deep mastery of the topic, students may have felt more confident sharing opinions that conflicted with others in class, even if those opinions were coming from those viewed as most able in the class. As a result, the presence of contrasting opinions did not seem to foment conflict or competition or intimidation among the students. When I asked the class at the end whether we had resolved the question of which was the greatest, none of the students felt that we had done this convincingly. Since no one was completely sure of the correct answer, their minds may have been more open to listen to opinions that

might have been different from their own, and they may have felt more empowered to respectfully disagree with their peers as they presented their arguments.

In order to provide time and space for the students to study the claims presented in the six presented arguments in greater detail, I would follow this lesson with another lesson focused solely on critiquing the arguments. I would summarize each of the significant mathematical claims made during the presentations on a handout, distribute this summary handout to all the students, and have the students investigate the validity of each of the claims closely in small groups. I would instruct them to try to determine with greater confidence and mathematical support whether each of the claims was true or not. If they felt claim was invalid, I would prod them to provide a counterexample to support their opinion. If they felt a claim was valid, I would ask them to find some systematic way of proving the claim's validity so that its generality could not be questioned (e.g. its truth was not limited to a finite number of cases). If no students were able to prove that 21^{34} was greater than 8^{46} , I would show them the proof mentioned earlier in the chapter using the distributive property of exponentiation over multiplication. Hopefully, by addressing the presenters' claims a day later in small groups, rather than posing questions or critiques directly at them during their presentations, I would help those presenters who made invalid claims feel less self-conscious about their misconceptions.

Lesson Clip #4: Lesson Report Card

From a task completion perspective, the lesson was largely successful. While the class did not reach a consensus about which of the three numbers was the greatest, most completed the task of reflecting about the problem and generating an argument for one of the three numbers. Most of the class members (all but 2 or 3) volunteered at some point in the lesson to present their arguments in front of the class, and six of the members actually made presentations. (Were it not for the time limitations, more would have certainly presented.) Moreover, several students were able to articulate in the “post presentation” discussion what made the most compelling arguments elegant, a key goal of the exercise.

From a cognitive perspective, a number of students were able to make a connection between the exploratory exponent work they had done the previous week on the exponent handout (see Appendix B) , either during their presentations, or during their comments about valid arguments. In particular, their understanding of the power to a power property, which they were exposed to during their exponent exploration but were never explicitly taught, became a key linchpin in several students’ proofs that 8^{46} is larger than 2^{134} .

Beyond their deepened understanding of exponents, however, students showed a more refined appreciation for what constitutes a good argument. The preparatory exercise during the previous week, in which the students modeled excellent, average and poor

arguments, had established the importance of providing not only specific examples, but also irrefutable, logical proof in supporting mathematical claims. When Yolanda stated, for example, that Nick's argument had "a lot of good proof" (Comment 53), she was implicitly contrasting his approach, which clearly shows that 46 factors of 8 is equivalent to 138 factors of 2, and thus greater than 2^{134} , to other student approaches like Lisa, which used unsupported conjectures to justify claims of absolute proof. Thus, students were developing the ability to construct compelling arguments based on both their mathematical knowledge and their criteria for valid argumentation.

From a psychological perspective, the IRRC pattern seemed to promote a number of opportunities for improved self-efficacy among the students. For one, students with elegant arguments, such as Nick, received public praise for the quality of their work, an incalculable boost to their confidence in their ability to present mathematical arguments. In addition, students such as Michael, whose presented arguments were logically flawed, were given a space to adjust their positions without feeling defeated. The eagerness with which Michael and Alex (the two presenters who argued for 2^{134}) enthusiastically and shamelessly shot up their hands when the class was asked if anyone's opinion had been changed speaks to the constructive (rather than humiliating) effect that hearing other students' arguments had on them. Finally, the generally positive, non-competitive, and encouraging atmosphere that infused the room during the presentations and discussions modeled for the students that the classroom is a safe place to respectfully disagree with one's peers, and that anyone (not simply the most talkative participants) with a compelling argument has the power to change other's opinions. Nick, who is

generally one of the more timid and tacit students during our daily discussions, manifested this very phenomenon beautifully. The smile of satisfaction and pride that spread across his face when his peers praised his argument still gives me chills.

Metacognitively, a significant amount of evaluation and discerning judgment seemed to be characterizing students' thought processes during the lessons. Some students, such as Tim (Comment 12) even acknowledged when their arguments weren't as "mathematical" as others. Others, such as Peter, pointed out similarities between their arguments and others' arguments (Comment 41). In other words, at least some of the students were clearly engaged in listening to the arguments of others and juxtaposing them with their own. Furthermore, they were able to intuitively observe that Nick's idea -- that 8 could be replaced by 2^3 to make an easier comparison with 2^{134} -- was more "provable" and "mathematical" than Tim's idea that 8^{46} is the greatest because it has the "best of both worlds" (a larger base than 2^{134} , and larger exponent than 21^{34}). While these 7th graders might not have been able to articulate these metacognitive developments explicitly in their comments and presentations, they were certainly no less aware that certain standards needed to be met before an idea could be proved to be true.

The IRRC pattern, then, seemed to provide both the necessary ethos and structure for students to both reflect about the problem, and intently evaluate the validity of their fellow classmates' ideas. More importantly, it gave them space to adjust their opinions based on compelling proof that another opinion was more valid without feeling defeated or humiliated in front of their peers.

Clearly, managing this spirit during a back and forth dialogue between students (rather than a sequence of presentations with no intervening discussions) would require more frequent and purposeful questioning from the teacher to point student's attention towards misconceptions that have been raised by students so that students are able to confront their misconceptions head on. And yet, the very act of establishing the value and criteria for elegant argumentation seems to be a worthy accomplishment on its own for a lesson. Hopefully, as I continue to experiment with the implementation of the IRRC pattern in a variety of lesson formats (presentations, debates, back and forth dialogues), I gain a better feel for how I can engender the spirit of respectful but rigorous peer-to-peer evaluation during all teacher-student and student-student exchanges in my classes.

Chapter 9: A Vision for Normative Discourse

In the next five years, I would like to work towards training my students more intentionally in sociomathematical norms that support critical thinking and student-to-discourse. Since I teach many of my students for two years in a row (in 7th and 8th grade), I have an invaluable opportunity to lay the foundations for this normative training at the beginning of the 7th grade course, and build upon those foundations in the ensuing two years. I would like to feel more comfortable facilitating discussions about mathematical ideas that engage students in cognitively demanding tasks and metacognitive thinking on a consistent basis. Ideally, by the time my students leave me at the end of Algebra I in 8th grade, I want them to feel comfortable assessing the validity of a mathematical statement on their own, without any implicit or explicit cues from me.

Furthermore, I want them to possess a tolerance for the inherent uncertainty that infuses discourse surrounding mathematical controversies. If an idea is presented by a student, and I refuse to affirm or challenge that idea, I want other students to feel completely natural evaluating the idea for themselves and expressing their opinion about its validity without fearing threatening confrontation or ridicule from others. Moreover, I would like my students to openly receive critiques of their statements from peers, and to see these critiques as an essential, non-threatening aspect of typical classroom discussions. In order for me to train my students in these habits of discussing, sharing, critiquing and assessing, however, I will need to challenge several of my own tightly held assumptions

about my purpose as a teacher. While I disagree with these assumptions intellectually, they have long held sway in my practice, and will thus provide a source of resistance unless I address them head-on.

Assumption #1: The more material covered in a course, the better.

The first assumption is that I need to cover all the material prescribed for the course I am teaching for that year, and that covering more than the prescribed material is a desired bonus. I have realized that sometimes my own personal pride drives me to pack more into units than is absolutely necessary, which forces me to rush through discussions at times. A hurried attitude and the kind of metacognitive training I described above are simply incompatible. If I am worried about having enough time to cover all the points I want for a given lesson, I will be much less receptive to letting my students struggle with a thorny concept or problem. Instead, I will seize the role of director, evaluating the students at every turn and/or issuing instructions (disguised as questions) as to how they should proceed. I may think that by asking questions (instead of issuing explicit directives) I am giving the students the responsibility for the evaluation of their peers' contributions. However, if I am unwilling to let a discussion unfold in its own way, colored by the students' unique perspectives, then the questioning that I use will merely be a front for my own driven desire to get students to complete tasks quickly and accurately, as efficiently as possible.

Assumption #2: A teacher should always relieve students of intellectual discomfort

The second assumption is that students need to be comfortable in my class at all times.

As a teacher who is sensitive to the anxieties and insecurities of my students, I am all too often influenced by the fear that certain tasks will make students initially uncomfortable, and overwhelm them. Of course, it is important to gauge what level of abstraction students are ready for before handing them a task. If the level of abstraction or complexity is too high – that is, if the task demands skills or abilities that are far beyond the reach of the students, then it could trigger levels of anxiety, confusion, and frustration that threaten students' self-efficacy. A student who is consistently overwhelmed by the difficulty of the task he is given is more likely to consider himself incompetent as a math student, and avoid pursuing math in college or beyond.

And yet, it is certainly possible for a teacher to provide students with appropriately challenging tasks that, while initially producing uncertainty or confusion, can ultimately be completed by the students with the right teacher support. In my own practice, I have noticed that I sometimes lower my level of cognitive demand when I sense that students are becoming frustrated by a problem or idea. In these “rescue” scenarios, I provide hints and shortcuts for them that remove the burden of having to grapple with a particular tricky concept or idea. Indeed, the IRE pattern is well suited to “rescuing” students from the uncertainty that surfaces in problem solving when the solution method

is not apparent; the carefully phrased and inflected questions give the students specific instructions about how to proceed, thus removing the uncertainty.

As I move forward in my teaching practice, I would like to think more creatively about ways to support and encourage students to maintain their self-efficacy, without taking away opportunities for cognitive growth. A more patient and unhurried attitude could certainly help me in this delicate balancing act – students are much more likely to experience high levels of anxiety and frustration if they feel there is a time pressure bearing down on them as they face these cognitively demanding tasks. Moreover, the cultivation of a classroom environment in which being “stuck” is normal and accepted by all as a natural aspect of problem solving, could help students feel less embarrassed when they feel that are at a impasse with a particular problem.

Assumption #3: Mathematical Debates are Too Risky for Middle Schoolers

A final assumption that limits my ability to honor my commitments to cognitive demand, self-efficacy and metacognitive skills is that facilitating debates about mathematical concepts is too risky in a 7th or 8th grade classroom. The fear that students will quickly devolve into ridicule and aggressive critiquing if a debate is opened has kept me from collecting multiple student responses to a question, as well as student comments about their peers' responses. As was discussed in Chapter 7, this fear is a helpful caution – if not managed correctly, discussions surrounding a statement whose validity is not evident *can* easily turn into shouting matches, with students leaving the

discussion feeling either vindicated or demoralized. At the same time, it is possible for students to be trained in the protocols of respectful debate, so that the process of a discussion about a math controversy with many different opinions is not threatening or unsafe for any students.

In the upcoming years, I would like to invest more time and energy into this training process so that I can gradually introduce debates into the flow of discussion without destabilizing the classroom dynamic. This will require me to design activities that build student comfort with the debating dynamic at an appropriate pace, so that they are ultimately ready to receive and share comments in a peaceful and calm manner when a potentially divisive controversy arises in a discussion. Without feeling confident in their ability to logically assess someone else's claims and express that assessment out loud comfortably, students will never be able to develop the metacognitive skills they need to think independently.

The Questioning Patterns: A Tool for Self-Awareness

Given these two imminent goals for my practice -- to train students in sharing and receiving critiques of one another's claims, and to help them calmly and persistently work their way through problems whose solutions are not readily apparent -- it will be essential for me to maintain high levels of self-awareness regarding what questioning patterns I am using in my classroom. In this sense, the work I have done identifying and analyzing these questioning patterns in context has provided me with a powerful tool for

deepening this self-awareness. If I catch myself using the IRE pattern in order to hurry a child through problem, or to “rescue” him or her from initial confusion, I will be more likely to consider other approaches to the discussion that might promote higher levels of cognitive, metacognitive or psychological success for more of the students.

Furthermore, my exposure to the revoicing technique, along with the conception of the IRRC pattern for moderating math “debates”, have given me new tools for managing the dilemma of teacher evaluation as it arises in different teaching scenarios. It has also demonstrated to me the importance of preparing students for math “controversies” through training in the basic norms of respectful and constructive teacher-student and student-student exchange during these debates. Finally, in examining my teaching style, I have developed a conscious appreciation of many of the latent forces that have shaped how I approach my lessons, how I interact with my students, and, ultimately, how my students approach the learning mathematics. While it has been humbling to discover aspects of my current teaching style which undermine my commitment to cognitive demand, self-efficacy, and metacognitive skill, it has been gratifying to realize the benefits of applying new questioning patterns that are informed by my research in my classes. Indeed, I see this action research project as an ongoing one that will continue (albeit in a less structured form) in subsequent school years.

Implications for the Broader Public: A Comparison with the NCTM Standards

While my study was certainly a small scale one, it still has implications for teachers at all grade levels and school environments. The three principal commitments – high cognitive demand, metacognitive skill development, and high self-efficacy -- that framed my success framework are not specific to my teaching practice. In the NCTM Principles and Standards (2000 Version), all three commitments are highlighted in some shape or form.

The NCTM Perspective on Cognitively Demanding Tasks

In the Standards, there is a recurring focus on tasks that promote certain types of reasoning, visualizing, and analyzing. In fact, virtually all of the ten standards contain some mention of tasks in the Standard sub headings. In the Communication Standard, for example, the following statement is made:

Students need to work with mathematical tasks that are worthwhile topics of discussion. Procedural tasks for which students are expected to have well-developed algorithmic approaches are usually not good candidates for such discourse. Interesting problems that "go somewhere" mathematically can often be catalysts for rich conversations.

(NCTM Standards and Principles 2000)

This is a clear reference to the notion of cognitive demanding tasks that is so central to this project. In order to engage students in meaningful mathematical explorations, the tasks they are given need to demand independent thinking from students rather than the blind application of algorithms.

The NCTM Perspective on Self-Efficacy

Moreover, the notion of self-efficacy is touched upon in the discussion of the Equity Principle that guides the NCTM standards.

The vision of equity in mathematics education challenges a pervasive societal belief in North America that only some students are capable of learning mathematics. This belief, in contrast to the equally pervasive view that all students can and should learn to read and write in English, leads to low expectations for too many students. Low expectations are especially problematic because students who live in poverty, students who are not native speakers of English, students with disabilities, females, and many nonwhite students have traditionally been far more likely than their counterparts in other demographic groups to be the victims of low expectations.

(NCTM Standards and Principles 2000)

In essence, the above passage refers to the internalized belief that many students have that they are not able math students – in other words, a low self-efficacy. And yet, while the NCTM traces this self-efficacy to low expectations, I address self-efficacy from a

slightly different angle. In the elite and competitive independent school environment where I teach, teachers have *high* expectations for their students, which are shared by their parents. (At times, the parents' expectations can be unrealistically high, to the detriment of the student). In this setting, self-efficacy tends to stem from students' *own* feelings of shame or failure about poor academic performance, rather than from the negligence or low expectations of a teacher. For example, a student in my setting may equate a B or C on a math test with a failing grade, since he or she was aiming to get an A, and failed to reach that goal.

At the same time, students who have experienced consistent failure in previous math classes *do* tend to develop low expectations for *themselves*. This, in turn, makes them less confident in their ability to face cognitively demanding tasks successfully, and increases their levels of anxiety when they are taking assessments or struggling to understand a concept. Thus, it is imperative for teachers to present the experience of struggle or confusion during the problem solving process as a natural aspect of mathematical inquiry, rather than an indication of some deficiency in the student's math ability.

The NCTM Perspective on Metacognitive Skill

The NCTM Communication Standard also addresses the need for metacognitive skill development through students' frequent presentation and critique of logical claims. The

act of listening to a peer's comments and analyzing their validity is viewed as a tremendously valuable learning activity.

When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing. Listening to others' explanations gives students opportunities to develop their own understandings. Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. Students who are involved in discussions in which they justify solutions—especially in the face of disagreement—will gain better mathematical understanding as they work to convince their peers about differing points of view.

(Hatano & Inagaki 1991 in NCTM Principles and Standards 2000).

The phrase “especially in the face of disagreement” dovetails well with my concept of the math debate or controversy, which inevitably produces differences of opinion that need to be managed by the teacher.

The NCTM Perspective on Sociomathematical Norms

Moreover, in the description of the NCTM Communication Standard, it is acknowledged that “students do not necessarily talk about mathematics naturally; teachers need to help them learn how to do so” (NCTM Principles and Standards 2000). This resonates strongly with my finding that students need to be trained in the norms of mathematical discourse in order for exchanges to be constructive and successful. The revoicing technique described and demonstrated in Chapters 3, 6, and 7, also serves a mechanism for supporting students as they learn how to communicate their ideas effectively. The

Communication Standard goes on to suggest the following approach to training middle school students in these norms:

Students in the middle grades are often reluctant to stand out in any way during group interactions. Despite this fact, teachers can succeed in creating communication-rich environments in middle-grades mathematics classrooms. By the time students graduate from high school, they should have internalized standards of dialogue and argument so that they always aim to present clear and complete arguments and work to clarify and complete them when they fall short. Modeling and carefully posed questions can help clarify age-appropriate expectations for student work.

(NCTM Principles and Standards 2000)

The “carefully posed questions” mentioned in the description could just as well be queries issued by the teacher when applying the IRRC questioning pattern during a math controversy. By posing questions without providing immediate answers or forms of evaluation, the teacher can model critical and independent thinking patterns for his students.

Understandably, the NCTM Standards and Principles contain many more “commitments” that are deemed vital for effective mathematics instruction. The three commitments I have chosen are not meant to be exhaustive, nor are they meant to prescribe the three greatest priorities for any teacher. Rather, they shine a light on three areas of *my* teaching practice that I have been particularly interested in investigating. Perhaps, in future (probably more informal) action research projects, I will use the Principles and Standards as a resource for suggesting other points of focus.

Lessons Learned: A Personal Reflection

As I move forward in my own teaching practice, I know that I will carry with me an abundance of new insights into what my tendencies are as a teacher, and how my awareness of these tendencies can help me approach my classes with an intention to better promote cognitive growth, self-efficacy, and metacognitive skill among my students. This project has provided me with an invaluable opportunity to observe my teaching at close range, challenge hidden assumptions, and move forward with new tools to successfully manage the dilemma of honoring my teaching commitments. Indeed, I have already observed three new aspects to my teaching that have stemmed from my experience of conducting an action research project in the past year.

The first aspect is simply a heightened awareness of the IRE pattern when I use it. I am now consciously aware of when I am in a hurry to complete a task before the bell rings, and I clearly understand how this hurried attitude can motivate me to intervene quickly to evaluate student responses without allowing for discussion. With this new awareness, I have tried to be intentional about the way I react to incorrect and correct student answers during discussions so that students are forced to evaluate the answers on their own. For example, instead of simply accepting a correct answer when it is provided, I am now consistently asking the class whether they are comfortable accepting an answer as valid before moving forward. Since they can no longer count on me to “telegraph” my evaluation of a response with a response (or lack of response), they are thinking more intentionally about whether they personally agree with answers provided by their classmates. By slowing myself down, and focusing students’ attention on evaluating the

contributions of their peers, I have been rewarded with significant gains in overall student understanding and engagement. As a result, I have started to strengthen the habit of reacting to incorrect and correct answers more equally, and my IRE tendencies have softened somewhat.

The second new aspect of my teaching that has emerged is my commitment to training students in the art of constructing and evaluating mathematical arguments. The activities that I conducted with my 7th graders prior to the fourth and final videotaped lesson continue to yield major benefits as I engage them in argumentation from day to day. They already seem more comfortable stating their disagreements with other students' arguments during discussions, and they will frequently mention why they prefer one argument over another. I am now committed to making these activities a regular part of my beginning of the year curriculum for all my classes, and to periodically intersperse similar activities into my lessons from time to time.

Finally, I have found myself highlighting disagreements between students more frequently as they arise in course of discourse. Instead of responding only to the correct answer (and ignoring the incorrect ones) when a number of competing answers are provided simultaneously by different students, I capitalize on these scenarios to bring students' attention to the controversy at hand. I talk about the "controversy" with enthusiasm, eagerly invite the opinions from various students, and then ask for a "verdict" from the class. More often than not, these verdicts are arrived at relatively quickly as the correct answer's validity is revealed and confirmed by the entire class.

However, these discussions can occasionally become prolonged, bringing many opportunities for students to practice their skills in mathematical argumentation. Instead of intentionally avoiding debates as “risky” activities for middle school students, I am now intentionally embracing them. As I do this, I try my best to let the students’ arguments drive the discourse, while at the same time keeping them from losing focus and direction. This process is certainly art rather than a science. At the same time, the more I practice this role as “controversy facilitator”, the more faith I develop in my ability to effectively manage new controversial scenarios as they in my classroom.

Conclusion

Managing dilemmas in the classroom is a fundamental feature of the teaching practice. No matter what questioning pattern he or she is using, the teacher will always need to make decisions about how to help students evaluate the validity of the statements made by themselves and others without doing the evaluating for them. Furthermore, the teacher will have to balance a desire to hear as many voices in the classroom while at the same time keeping students’ attention focused on the key mathematical ideas at hand. The teacher may need to quite explicit in telling students which comments in particular they would like to be addressed, and which comments seem to be in conflict with each other. In addition, the teacher will need to decide to how to provide closure to a lesson if the class ends with the controversy unresolved. Moreover, the teacher will need to decide which patterns students are prepared to respond to, and which learning activities are best suited to each pattern. The implementation of these patterns may not always

yield success according to the four lenses of self-efficacy, high cognitive demand, task completion and metacognition. Nevertheless, the thoughtful application of these patterns, combined with a careful attention to students' readiness for the participation in these patterns, can only help deepen the teacher's awareness of how the very questions he or she uses shape the way his or her students learn.

Appendix A: Hinting Scenarios

Below, I present hypothetical scenarios that illustrate hinting patterns. As in the presentation of the scenarios, I evaluate the success of the scenarios based on the analytical framework.

Background:

In an 8th grade accelerated algebra class, we are beginning a unit on systems of equations. The first topic is solving systems of linear equations by graphing. At the beginning of class, before we discuss what a system is or what it means to solve a system, I give the students the following system:

$$\begin{cases} x + 27 = -3 \\ y = 3x + 5 \end{cases}$$

I tell them that I would like them to work in groups to find an ordered pair that is a solution to both equations at the same time. They can use any method they want. Then, I tell them to figure out whether the solution they find is the *ONLY* solution, or whether others exist. Most of them find the ordered pair in a few minutes using substitution (we haven't formally discussed the substitution method, but we've used substitution recently in composing two functions, and this is an accelerated class). None of them use graphing, which is exactly what I predicted.

However, none of them could say for sure whether it was the only ordered pair solution. Some thought that there were an infinite amount of ordered pair solutions, and others

thought there was one, but couldn't prove it. So they were stuck on this issue. I decided to engage in some hinting with them. As in the questioning scenarios presented in Chapter 3, I offer a couple different approaches, or hinting patterns, that the teacher could apply in addressing this dilemma of student confusion.

Key for all scenarios: T = teacher and G = the group of students.

Hinting Pattern #1: Reviewing Fundamental Concepts

In this pattern, the teacher identifies key related concepts that need to be understood well by the students in order for them to answer his question. Through his questions, he focuses their attention on the most important aspects of these ideas, hoping that the students will infer connections between them, and ultimately understand the problem from a more grounded conceptual standpoint.

1 T: Is there any other way you could represent these two equations with x and y ?

A variety of student responses may occur at this point:

Response 1: Students realize graphing is an option immediately

2 G: You could graph it!!

3 T: But how would you find an ordered pair that works for both?

4 G: Just look at where they intersect?

5 T: Does graphing help you make a statement about how many solutions exist?

That would be the end of this hint, since inevitably students who made the graphing realization also have the spatial sense to know that if two different lines cross once, they can't cross again.

Response 2: Students can't think of another way of representing the equations

Here is a possible approach to the situation, in which the teacher leads the students through a review of basic concepts involving linear graphs before coming back to the solution to the system. It picks up at Comment 2, after the initial query made by the teacher.

2 G: I don't know

3 T: Well, let's look at the 2nd equation. Can you give me an ordered pair solution to that equation?

4 G: (0,5)

5 T: Are there any more?

6 G: Yeah, (3,8), (4, 11), etc

7 T: How many are there?

8 G: Lots – an infinite number!

9 T: OK. What's a way to represent an ordered pair? What can each of the two

numbers stand for?

10 G: x and y

11 T: So where have we represented something with an x part and a y part this year?

12 G: Graphing!

13 T: OK, so how could you represent each equation in another way?

14 G: By graphing!

15 T: How could you use the graphs to find an ordered pair that works for both equations?

16 G: By looking at where intersect?

17 T: How many times could the lines intersect? What's the maximum number of times they could cross?

18 G: Once

19 G: Or more than once, if they're the same line

20 T; EXCELLENT INSIGHT! How do we know if these lines are the same line?

21 G: Put them in the same form

22 T: OK, do that. Are they the same line?

23 G: Nope.

24 T: So how many solutions are there?

25 G: Just one – the one we found.

Discussion of Hinting Pattern # 1: Highlighting the Pattern

In many ways, this hinting pattern exhibits an even higher level of teacher evaluation than any of the questioning patterns discussed in Chapter 3. Every comment made by the Group is followed by a question posed by the teacher that suggests at the next appropriate step. The hinting questions that the teacher asks focus students' attention on particular ideas in order to help them make useful connections between concepts. In accepting each student response without a request for justification, the teacher is subtly giving cues about the validity of each group answer. The questions are straightforward enough that no correcting maneuvers from the teacher are necessary. (Of course, this is not always the case.) Perhaps this pattern could be labeled **Inquiry Response Evaluation Suggestion** (IRES). The teacher *suggests* a way forward the content of his follow up questions. The evaluation phase of each IRES is principally a tacit affirmation of the validity of the group's response.

Discussion of Hinting Scenario 1: Evaluating its Success

From a task completion standpoint, the scenario is successful, since the students are able to correctly visualize the geometric implications of two lines graphed on a coordinate plane. Moreover, the task has a relatively high level of cognitive demand, since the students are being asked to integrate their knowledge of geometry and linear equations in a novel manner. Instead of simply completing an exercise in applying the substitution

method to solve the system, they are reflecting about the nature of a system's solution, and what it implies. Psychologically, the students' self-efficacy was supported, since it appeared as if they were able to reason their way to a solution. At the same time, their dependence on the teacher, who led them through the entire process, might be an obstacle as they seek to build confidence in their abilities to work independent of the teacher. Metacognitively, there was some level of skill acquisition here. The task was fundamentally asking: *how do we know it is true that there is only one solution?* The exercise of constructing an argument that defends the "exactly one solution" claim is certainly valuable for their metacognitive development. At the same time, the persistent presence of the teacher in the students' thinking process may have limited opportunities for the students to assess the validity of their responses before moving on. The teacher, for example, could have temporarily stopped giving the students hints after Comment 9, to see if they could make a connection between the system and the coordinate plane on their own. Thus, the success of this scenario is mixed – it certainly shows signs of high cognitive demand and metacognitive training, but would be improved by a reduced teacher presence.

Hinting Pattern # 2: Telegraphing the Answer

In this pattern, the teacher essentially tells the students how to approach by suggesting that they consider certain important ideas. In contrast to Hinting Pattern 1, the teacher makes the connections between important ideas for the students, and then asks leading questions whose answers are implied.

- 2 T: Hey, guys! What would happen if you graphed both lines? Would that help you at all?
- 3 G: I guess we could try that.
- 4 T: Try graphing it and see if that helps you figure out how many solutions exist.
- 5 G: [Students graph both lines] Oh, I see, the lines only cross once. So there's only one solution. Thanks, Mr. C!

Discussion of Hinting Pattern # 2: Highlighting the Pattern

The pattern in this exchange can be best described as the **Suggestion Application Interpretation (SAI) Pattern**. The teacher *suggests* a problem solving approach to the students, the students *apply* this approach, and then the teacher *interprets* what the students have done for them in order to help them see its significance. In essence, the Suggestion phase is really a Direction phase, since the students have been conditioned to believe that if the teacher suggests a solution method, it will probably be a valid one. So while the teacher phrases these hints as suggestions, the students really hear them as directions.

Discussion of Hinting Pattern # 2: Evaluating its Success

As with the first pattern, this scenario was successful from a task completion standpoint, because the students ended up discovering that there was exactly one solution to the system. Cognitively, however, this scenario demonstrated very low levels of cognitive demand. The students were basically being asked by the teacher to complete a straightforward graphing task, look at their graph, and determine by inspection how many times the lines crossed. The students were merely following a memorized procedure for graphing lines. Psychologically, the experience was probably neutral for the students. They may have felt relieved that they were able to solve the problem, but since not much thought or effort was invested in reaching this solution, I doubt that they would feel much satisfaction. In this sense, their self-efficacy was not threatened, nor was it boosted. Finally, the low level of cognitive demand of the teacher's approach precludes the possibility of metacognitive skill acquisition. The students are never asked to justify their claims, nor are they asked to assess the validity of any statements. They are merely blindly following their teacher's commands. Overall, then, this approach was certainly an unequivocal failure.

Appendix B: Exponents Handout

(This was handed out to students without any introduction to the properties of exponents. The only introduction given to students in class before receiving the handout was a clarification of what “expanded form” and “standard form” meant.)

Exponents Exploration

We say that an expression is in exponential form if it is written using bases and exponents to show repeated multiplication. We say an expression is in expanded form if it is written showing all the repeated multiplication without any exponents. For example, 2^5 would be in exponential form, whereas $2*2*2*2*2$ would be in expanded form.

1. A) Write the following expression in expanded form: $2^3 * 2^4$

B) Now take the above expression, and rewrite it in exponential form as a power of 2. You do not need to compute the actual number.
2. Write each of the following expressions in exponential form. (You do not need to compute the actual number.) It may be helpful to imagine what the expressions would look like in expanded form if you took the time to write them out in expanded form.
 - a) $3^4 * 3^{10}$
 - b) $7^5 * 7^{13}$
 - c) $2^{12} * 2^6 * 2^{20}$
3. Does $2^5 * 3^4 = 2^9$? Explain why or why not in complete sentences.
4. Suppose a, m and n are positive whole numbers. What is a simpler way to write $a^m * a^n$?
5. A) Write the expression $(2^4)^2$ in expanded form (no exponents in the expression) in the space below.

B) Now use this expression to find the value of n in the equation below
 $(2^4)^2 = 2^n$

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