ABSTRACT

Title of dissertation: A CROSS-LAYER STUDY OF THE SCHEDULING PROBLEM

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This thesis is inspired by the need to study and understand the interdependence between the transmission powers and rates in an interference network, and how these two relate to the outcome of scheduled transmissions. A commonly used criterion that relates these two parameters is the Signal to Interference plus Noise Ratio (SINR). Under this criterion a transmission is successful if the SINR exceeds a threshold. The fact that this threshold is an increasing function of the transmission rate gives rise to a fundamental trade-off regarding the amount of time-sharing that must be permitted for optimal performance in accessing the wireless channel. In particular, it is not immediate whether more concurrent activations at lower rates would yield a better performance than less concurrent activations at higher rates. Naturally, the balance depends on the performance objective under consideration. Analyzing this fundamental trade-off under a variety of performance objectives has been the main steering impetus of this thesis.

We start by considering single-hop, static networks comprising of a set of always-backlogged sources, each multicasting traffic to its corresponding destinations. We study the problem of joint scheduling and rate control under two performance objectives, namely

sum throughput maximization and proportional fairness. Under total throughput maximization, we observe that the optimal policy always activates the multicast source that sustains the highest rate. Under proportional fairness, we explicitly characterize the optimal policy under the assumption that the rate control and scheduling decisions are restricted to activating a single source at any given time or all of them simultaneously.

In the sequel, we extend our results in four ways, namely we (i) turn our focus on time-varying wireless networks, (ii) assume policies that have access to only a, perhaps inaccurate, estimate of the current channel state, (iii) consider a broader class of utility functions, and finally (iv) permit all possible rate control and scheduling actions. We introduce an online, gradient-based algorithm under a fading environment that selects the transmission rates at every decision instant by having access to only an estimate of the current channel state so that the total user utility is maximized. In the event that more than one rate allocation is optimal, the introduced algorithm selects the one that minimizes the transmission power sum. We show that this algorithm is optimal among all algorithms that do not have access to a better estimate of the current channel state.

Next, we turn our attention to the minimum-length scheduling problem, i.e., instead of a system with saturated sources, we assume that each network source has a finite amount of data traffic to deliver to its corresponding destination in minimum time. We consider both networks with time-invariant as well as time-varying channels under unicast traffic. In the time-invariant (or static) network case we map the problem of finding a schedule of minimum length to finding a shortest path on a Directed Acyclic Graph (DAG). In the time-varying network case, we map the corresponding problem to a stochastic shortest path and we provide an optimal solution through stochastic control methods.

Finally, instead of considering a system where sources are always backlogged or have a finite amount of data traffic, we focus on bursty traffic. Our objective is to characterize the stable throughput region of a multi-hop network with a set of commodities of anycast traffic. We introduce a joint scheduling and routing policy, having access to only an estimate of the channel state and further characterize the stable throughput region of the network. We also show that the introduced policy is optimal with respect to maximizing the stable throughput region of the network within a broad class of stationary, non-stationary, and anticipative policies.

A Cross-Layer Study of the Scheduling Problem

by

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Dissertation submitted to the Faculty of the Graduate School of the
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Dedication

To the memory of my father, Andreas,

who taught me to be tenacious and hard-working.

To my mother, Elma,

who taught me to always follow my conscience.

Acknowledgements

My Ph.D journey has been a unique learning and living experience for me. It is associated with many events, pleasant and not, that utterly changed my life and my views of looking into life. Thus, it means to me much more than just a Ph.D. in Electrical Engineering. In this road I was blessed to have the invaluable guidance and support of several people without whom this Ph.D would not have been materialized.

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Last, but not least, I would like to express my gratitude to my parents and brothers for their inexhaustible love from which I draw strength. I would like to thank especially my mother for her endless love and trust in me and for always being an impartial adviser. I would also like to thank my father for always believing in me and for fighting hard to be here with us today to see this Ph.D. come true. Dad, you are surely greatly missed! Finally, I would like to thank my brothers, Andreas and Manos, for their love and support.

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Chapter 1

Introduction

1.1 A Relationship between Transmission Powers and Rates

In a wireless environment where concurrent transmissions from multiple users interfere among each other, the exact coupling between the transmission powers and achievable rates of the various users remains unclear. The problem arises since the existing models to capture the interference are inadequate.

A commonly used criterion, borrowed from point-to-point communications, is the *Signal to Interference plus Noise Ratio* (SINR). Under the SINR model, a transmission is successful if the ratio of the signal power at a receiver to the noise and the total interference power exceeds a certain threshold. This model is approximate in general as it models the interference as Gaussian noise. However, it is intuitive and accounts for the fact that, depending on the channel conditions and the transmission powers, in a wireless environment *all* the concurrently transmitting nodes may interfere and cause a transmission to fail. Thus, in this thesis we will employ the SINR interference model to incorporate the physical layer in the scheduling decisions. The precise value of the SINR threshold depends on various communication related parameters, such as the transmission rate, the target probability of bit error, the modulation and coding techniques employed for the transmission, etc. It follows from the fundamental principles of wireless communications that the transmission rate is an increasing function of the SINR threshold (see Appendix

A). This is the core idea around which this thesis develops.

1.2 A Fundamental Trade-Off

A fundamental question in multiple-access is which nodes should access the channel at any given time and at which transmission powers and rates. Given the relationship between the transmission rate and the SINR threshold, the following trade-off arises.

By lowering the transmission rate, the corresponding value of the SINR threshold decreases, and thus more transmissions can jointly satisfy the SINR criterion. Alternatively, by increasing the transmission rate, the SINR threshold increases, and therefore, the number of transmitters that can be successful in accessing concurrently the wireless channel decreases. Thus, it is not immediate whether it is preferable to allow more nodes to concurrently transmit at lower rates or whether permitting fewer of them to transmit simultaneously at higher rates will yield a better performance.

In this thesis, we investigate under which cases “more time-sharing” (fewer concurrent transmissions at higher rates) is preferable compared to more concurrent transmissions at lower rates. In one extreme, a single transmitter can transmit at any given time at its highest achievable rate, as in a Time Division Multiple Access (TDMA) scheme. In another extreme, all network nodes can simultaneously access the wireless channel successfully, at perhaps arbitrarily low rates depending on the amount of interference that one causes to the other. All possible rate assignments between the two extremes are also possible. It is natural to expect that more time-sharing is preferable under high interference while on the other hand if the nodes do not interfere much among each other more concurrent transmissions should be preferred. Certainly, the optimal answer depends on the selected performance objective.

1.3 Scheduling Complexity

It is easy to observe that deciding the transmission rates and powers at which the network nodes can operate has a *combinatorial* flavor when the set of available power selections is discrete. This is natural since it involves the following two-stage procedure. First, all possible ways of assigning the transmission powers must be identified. Next, for each such possible power assignment the maximum rates that ensure the success of the scheduled transmissions according to the SINR criterion must be selected. Clearly, even for the most simplistic case of power control with binary decisions (either transmit at the maximum transmission power or remain silent), the number of potential transmitter activations increases exponentially in the number of nodes in the network. This renders the scheduling problem non-scalable. This issue of increased complexity together with the fact that scheduling needs to be solved repeatedly over time as the network conditions change, necessitates the introduction of alternative efficient solutions. Such alternatives can be heuristics that achieve efficiency by compromising optimality in performance.

One such approach is to simplify the scheduling problem by reducing the set of possible rate control and scheduling decisions that a policy can choose from. In a part of this thesis instead of considering all potential scheduling decisions (a set that grows exponentially in the number of nodes) we provide a simplification to the scheduling problem by allowing only decisions given by the aforementioned extreme types of communication, namely (i) *one at a time* and (ii) *all together*. Although the above two schemes represent a severe restriction of the action space, we expect to obtain useful insights regarding this trade-off which can facilitate the discovery of better heuristics.

1.4 Performance Measures

1.4.1 Stable Throughput

An important criterion to measure network performance is to maximize the rates at which data can be sent through the network while guaranteeing that the network queues remain finite. This is the stable throughout of the network. Under stability, these rates coincide with the exogenous arrival rates. The set of all such rates for which the network queues remain stable is called the stable throughput region of the network. In this thesis, we consider the problem of stable throughput maximization under anycast traffic.

1.4.2 Utility Maximization

Since the network resources are limited, they must be appropriately allocated to the network users. "Resource" can be the time that a network node has access to the wireless channel or the average rate that it receives. In this thesis we consider the latter case. We are interested in the scheduling problem for maximizing the user utility. In our framework, we consider arbitrary utility functions that are concave, continuously differentiable, and strictly increasing in the average rate.

The general problem of utility maximization inherently captures several commonly used performance criteria, such as the total throughput and fairness. As an example, an interesting utility function is that of α-fairness introduced by [1] where the corresponding utility function $U^\alpha(r)$ is given by

$$U^\alpha(r) = \begin{cases} \log(r) & \text{if } \alpha = 1 \\ (1-\alpha)^{-1} r^{1-\alpha} & \text{otherwise.} \end{cases} \tag{1.1}$$

The parameter α denotes the amount of "fairness" the utility function provides to the users. For instance, $\alpha = 0$ yields the criterion of total throughput under which the objective is to find the maximum throughput rate that the network can support. When $\alpha = 1$ this utility yields the objective of *proportional fairness* and it further leads to *max-min* fairness as α grows to infinity.

Clearly, maximizing the total throughput of the network leads to an efficient utilization of the network resources since the network sends traffic at the maximum rate that it can support. Nevertheless, it can lead to serious unfairness among the users since the optimal action set may totally exclude users with poor channel conditions, prohibiting them from accessing the channel. Thus, in this thesis we pay special attention to the criterion of proportional fairness [2], which has been widely used as a performance metric in wireless networks. Our focus on the criterion of proportional fairness stems from the fact that it provides a good compromise between efficiency and fairness [3].

1.4.3 Minimum-Length Scheduling

The performance metrics of stable throughput and utility maximization rely on the basic assumption that the corresponding average rate regions are well defined. Such an assumption requires that the wireless channel has a stationary and ergodic behavior. However, in practice the wireless channel evolution may neither be stationary nor ergodic such as in the cases of arbitrary mobility or networks with finite lifetime. This renders the above criteria inappropriate for such cases.

An alternative metric to stable throughput and utility maximization that can characterize the traffic-carrying capabilities of wireless networks with non-stationary and non-ergodic channel behavior is to construct schedules of minimum-length ([4], [5], [6], [7], [8]). This problem involves obtaining a sequence of activations of wireless nodes so that a finite, fixed amount of data traffic, residing at a set of source nodes, is routed to get delivered in minimum time to its intended destinations. In fact, a schedule of minimum-length is closely related to maximizing the network throughput since by minimizing the time to send a fixed amount of data, the effective rate at which data traverses the network is maximized.

1.5 Outline of the Thesis

In Chapter 2 we start our analysis by considering single-hop, static networks under multicast traffic. All traffic sources are assumed to be backlogged. We explicitly characterize the optimal joint scheduling and rate control policy under the objective of sum throughput maximization. Under the objective of proportional fairness we formulate the problem as a convex problem with a large number of variables. To explicitly characterize the optimal policy we consider a restricted set of scheduling actions given by activating a single transmitter at any given time or all of them simultaneously. Under this restricted framework, we explicitly characterize how the optimal proportionally fair scheduling and rate control decisions relate to the current channel conditions.

Next, in Chapter 3 we consider single-hop, *time-varying* wireless networks comprising of a set of backlogged multicast sources. We consider policies that take decisions only based on a possibly inaccurate estimate of the current channel state. We introduce an online gradient-based algorithm under a fading environment that selects the transmission rates at every decision time. We show optimality of this algorithm for a large class of utility functions by making use of the theory of stochastic approximation under a utility maximization framework. In the event that more than one rate allocation is optimal, the algorithm selects the one that minimizes the power sum.

In Chapter 4 we focus on the problem of obtaining schedules of minimum length for single-hop wireless networks under unicast traffic. We introduce an optimal joint scheduling and rate control policy that minimizes the required time for all network sources to deliver their data traffic to their respective destinations. We consider both static and time-varying networks. In the static network case, the optimality of the introduced policy is established using graph theory and methods from stochastic control theory are employed for the time-varying case.

In Chapter 5 we turn our focus on the objective of stable throughput maximization for a set of commodities of anycast traffic for multi-hop wireless networks. Each commodity is assigned a weight of preference. We introduce a joint scheduling and routing policy that has access to only an estimate of the channel state. We characterize the stable throughput region of the network under uncertainty in the channel state by using quadratic Lyapunov methods. We show that the introduced policy is optimal with respect to maximizing the stable throughput of the network within a broad class of stationary, non-stationary, and anticipative policies, irrespective of the weight assignment.

Finally, Chapter 6 summarizes our contributions and discusses a few potential future research directions.

Chapter 2

Sum Throughput Maximization and Proportional Fairness for Multicast Traffic in Static Networks

2.1 Background

The problem of scheduling in wireless networks has been studied extensively under various assumptions and performance criteria ([4], [5], [9], [10], [11]), and in particular in the context of joint scheduling and rate control (e.g., [12], [13]). In [12], scheduling of unicast transmissions in static networks is considered, where the wireless channel between any two nodes depends only on the path loss and attenuation due to shadow fading. The optimal solution for the problem of maximizing the sum throughput of the network with and without a minimum rate requirement for every transmitter is obtained. It is further shown that in the presence of minimum rate constraints and when the transmission powers are large, a pure Time Division Multiple Access (TDMA) scheme, that allows a single node to transmit at any given time, is optimal with respect to maximizing the sum throughput of the network. In addition, the problem of obtaining a max-min fair and a proportionally fair rate allocation is formulated in terms of a linear and a non-linear program respectively. However, these problems are not solved and the optimal solution is not characterized in either formulation.

In this chapter, we are interested in a cross-layer view of the scheduling problem by extending our earlier work [14] in which we obtained preliminary results. Since multicast traffic comprises a large volume of traffic in many network applications, we consider a single-hop network of multiple transmitters, each *multicasting* traffic destined for a set of receivers. The cases of unicast and broadcast traffic are naturally special cases in our formulation. Each transmitter is associated with a multicast session and the receivers of various sessions are allowed to overlap. We are interested in the problem of jointly scheduling the transmitters and controlling their rates under two different criteria, namely *sum throughput* and *proportional fairness*. We first obtain the optimal rate control and scheduling policy to maximize the sum throughput of the network. Since maximizing the sum throughput can be unfair to users with poor channel conditions, we also consider the objective of proportional fairness. We formulate the problem of obtaining the proportionally fair schedule as a convex problem. Next, by focusing on a restricted subset of the possible rate control and scheduling actions, similarly to [14], we are able to analytically solve the corresponding convex problem and obtain a proportionally fair solution over the reduced set of rate control and scheduling decisions. Our results generalize [14] in two respects: (i) we consider multicast, rather than unicast, traffic, and (ii) we employ a weaker set of assumptions. Our framework includes unicast and broadcast traffic as special cases. Unlike in [12], our objective is to *explicitly characterize* the optimal solution and how it relates to the current channel conditions. Similarly, this chapter is different from a body of work that studies the joint scheduling and rate control problem under time-varying channels for unicast ([13], [15], [16], and [17]) and multicast traffic [18]. The focus of the above works is to provide algorithmic solutions to maximizing the user utility. In contrast, our focus is, rather, to explicitly characterize the exact relation between the current channel conditions and the optimal scheduling decisions both for unicast, and multicast traffic.

2.2 Model Formulation

We consider a set of single-hop, wireless multicast links from T transmitters to D receivers as shown in Fig. 2.1, that operate in slotted time. Let $\mathcal{T}$ and $\mathcal{D}$ be the *sets* of transmitters and receivers in the network respectively. Each transmitter $k \in \mathcal{T}$ wishes to multicast at a *common* rate (single rate multicast) to a set of receivers $\mathcal{D}(k) \subseteq \mathcal{D}$. The pair $(k, \mathcal{D}(k))$ is called a *multicast session*. Note that this model is general enough to account for the special cases of unicast ($|\mathcal{D}(k)| = 1$) and broadcast ($|\mathcal{D}(k)| = D$) traffic, where $|\mathcal{D}(k)|$ denotes the cardinality of set $\mathcal{D}(k)$. We assume that a receiver $d \in \mathcal{D}$ can be a member of more than one multicast session, i.e., for multicast transmitters $j, k \in \mathcal{T}$, it is possible that $\mathcal{D}(k) \cap \mathcal{D}(j) \neq \emptyset$. In this work we assume that each transmitter has a saturated buffer with unlimited reservoir of data traffic; that is we do not consider the case of stable throughput, finite delays, and bursty traffic.

Let $P_n(k)$ represent the transmission power of transmitter k at time slot n. The variable $P_n(k)$ is assumed to take two possible values, namely $P_k^{\max}$ (when transmitter k is activated) and 0 (when it remains silent). We denote by $\mathbf{P}_n$ the T-dimensional vector of transmission powers at time slot n, i.e., $\mathbf{P}_n = (P_n(k),\ k \in \mathcal{T})$. We also denote with $N(d)$ the noise power level at receiver $d \in \mathcal{D}$. Although we restrict our attention to single-hop networks, our model can be used to address the scheduling and rate control problem in full-fledged multi-hop networks under fixed routing. However, we do not consider this

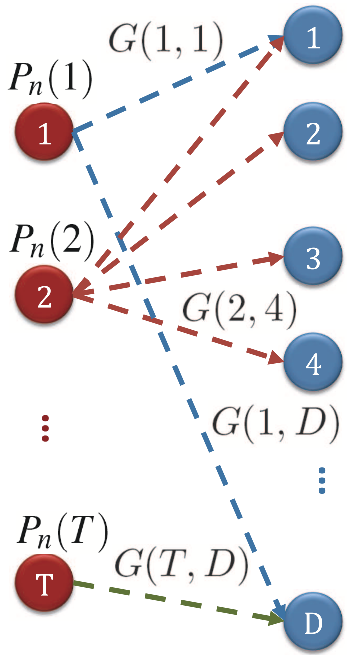

Figure 2.1: A single-hop network of T multicast transmitters and D receivers.

extension in this thesis.

We assume that the channel conditions between every transmitter and receiver in the network do not vary with time and are due to pure path loss. Hence, we denote by $G(i, j)$ the path loss between every transmitter $i \in \mathcal{T}$ and receiver $j \in \mathcal{D}$.

In our model we employ the *Signal to Interference plus Noise Ratio (SINR)* criterion to determine the outcome of a transmission. In the case of multicasting successfully the same message to a set of receivers the SINR criterion has to be satisfied at every receiver. Let $\gamma_{n,d}(r)$ be the threshold at time slot n at receiver $d \in \mathcal{D}$ that corresponds to transmission rate r. We will say that at time slot n a transmitter k successfully *multicasts* at a *common* rate r to all its intended receivers in the set $\mathcal{D}(k)$, if the SINR at each receiver $d \in \mathcal{D}(k)$ exceeds the corresponding threshold, i.e.,

$$\frac{P_n(k)G(k,d)}{N(d)+\sum_{j\in\mathcal{T},j\neq k}P_n(j)G(j,d)}\geq\gamma_{n,d}(r),\;\;\forall d\in\mathcal{D}(k). \tag{2.1}$$

In our model we consider receivers with multi-packet reception (MPR) capabilities. Under MPR a receiver may successfully receive concurrently from multiple transmitters as long as the SINR from each one of them exceeds the required threshold. Hence, two multicast transmitters with overlapping receiving nodes can concurrently transmit successfully. Each receiver is equipped with a detector that has multiple matched filters so that it can receive successfully from multiple transmitters at any given time as long as the corresponding SINR at each one of them exceeds the required threshold. If the SINR threshold is not exceeded at all intended receivers, we do *not* assume the transmission successful.

There exist 2^T-1 possible subsets of transmitters that can be activated at any given time, each corresponding to different threshold selections. These amount to all the possible ways of activating at least one out of the T transmitters. For a given activation, the transmission rates of active transmitters are set to the highest possible rates satisfying the condition that the SINR values at all respective receivers exceed the thresholds associated with that rate. Consequently, there exist 2^T-1 possible scheduling and rate control decisions that we will call *actions* for simplicity. Let us denote by $\mathcal{A}$ the set of all possible actions, i.e., $|\mathcal{A}|=2^T-1$. The optimal action selection depends on the adopted performance objective and on the link channel conditions.

We denote by r_k^j the *instantaneous* rate at which transmitter $k\in\mathcal{T}$ transmits to all of the receivers $\mathcal{D}(k)$ in its multicast session, under Action $j\in\mathcal{A}$. Since we consider single-rate multicast, the rate of transmitter k is equal to the rate of every receiver d in its multicast group, i.e., $d \in \mathcal{D}(k)$. Thus, we can characterize the rate of each receiver $d \in \mathcal{D}(k)$ through the transmission rate of its corresponding transmitter. Let $\boldsymbol{\pi} = (\pi_0, \ldots, \pi_{|\mathcal{A}|-1})$ denote a probability distribution over the set of all possible rate control and scheduling actions in $\mathcal{A}$. That is, we randomize the policy decision so that in every slot Action j is taken with probability π_j. This formulation by-passes one aspect of combinatorial complexity that arises when we associate each action in a deterministic way with each slot. We assume that such probability distribution exists, e.g., by requiring ergodicity on the action selection. Since a transmitter is not activated at the same rate in every slot, we define the *effective rate* $r_k(\boldsymbol{\pi})$ of transmitter $k \in \mathcal{T}$ to be the average rate over the action distribution $\boldsymbol{\pi}$, i.e.,

$$r_k(\boldsymbol{\pi}) = \sum_{j \in \mathcal{A}} r_k^j \pi_j.$$

Although in a unicast transmission there is no ambiguity regarding how to define throughput, this is not the case for multicasting where throughput can be measured both in terms of the transmission rate as well as with respect to the received rate. Defining throughput in terms of the transmission rate of a multicast transmitter would give two transmitters operating at the same rate equal weights, regardless of the number of receivers to which each of them transmits. In this chapter we define throughput as the *overall* traffic that reaches all the receivers of a multicast session. Thus, for any two multicast transmitters that operate at equal rates, the transmitter that has a higher number of receivers is assumed to contribute more in terms of throughput. In other words, our criterion is the *received throughput* which reflects the number of receivers in the multicast group.

2.3 Total Throughput Maximization

In this section we obtain a scheduling and rate control policy that maximizes the total (sum) throughput of the network. The maximization problem can be posed as follows:

$$\max_{\boldsymbol{\pi}} \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| r_k(\boldsymbol{\pi}) \tag{2.2}$$

s.t.

$$\pi_j \geq 0, \qquad j \in \mathcal{A}, \tag{2.3}$$

$$\sum_{j \in \mathcal{A}} \pi_j = 1. \tag{2.4}$$

We call the above problem described by (2.2)-(2.4) Problem I. Consider also the closely associated surrogate problem called Problem II defined as

$$\max_{j \in \mathcal{A}} \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| r_k^j. \tag{2.5}$$

The following theorem shows how these two problems relate.

Theorem 1 *Let $\mathcal{A}^\star \subseteq \mathcal{A}$ denote the set of actions solving Problem II defined in* (2.5)*. The optimal probability assignment solving Problem I defined in* (2.2)-(2.4) *satisfies* $\sum_{j \in \mathcal{A}^\star} \pi_j = 1$.

The proof of Theorem 1 is presented in Section 2.7. It is clear that optimizing the total throughput of the network leads to an efficient utilization of the network resources since the network sends traffic at the maximum rate that it can support. Nevertheless, it can lead to serious unfairness among the transmitters since the optimal action set may totally exclude transmitters with poor channel conditions, prohibiting them from accessing the channel. In the next section, we consider the utility of proportional fairness [2] which has been widely used as a performance metric in wireless networks, as it provides a good compromise between efficiency and fairness [3].

2.4 Proportional Fairness

In this section we focus on the objective of proportional fairness. As it was shown in [2] and also in [1] the objective of proportional fairness is equivalent to maximizing the sum of the logarithms of the user rates over the long-term average feasible rate region.

Recall that r_k^j is the instantaneous transmission rate of transmitter k under Action j. We are interested in obtaining an optimal probability distribution so that the *effective* rates of each receiver $d \in \mathcal{D}$ are assigned in a proportionally fair way. This can be expressed as a convex optimization problem as follows:

$$\max_{\boldsymbol{\pi}} \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| \log \left(r_k(\boldsymbol{\pi}) \right)$$

s.t.

$$\pi_j \geq 0, \quad \forall\, j \in \mathcal{A},$$

$$\sum_{j \in \mathcal{A}} \pi_j = 1,$$

where π_j is the probability that in a given slot Action j is chosen. Although this is a

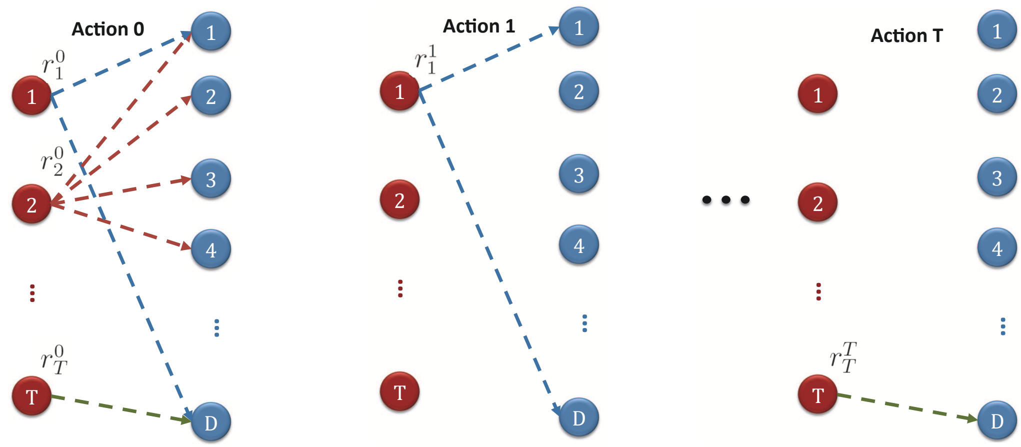

Figure 2.2: The $T+1$ possible actions obtained by either scheduling T transmitters one-by-one or by allowing all the transmitters to transmit simultaneously. The rate of transmitter k under Action j is denoted by r_k^j.

convex problem, the number of possible actions, and hence constraints, increases exponentially in the number of multicast transmitters. Therefore, although numerical solutions can be obtained (for example, through interior-point methods [19],) when the number of transmitters in the network is sufficiently small, computing the optimal solution analytically is infeasible.

Consequently, in what follows, we consider a suboptimal solution by restricting the set of feasible actions. These actions include (i) the simultaneous activation of *all* T multicast transmitters operating successfully and at instantaneous rates that ensure all SINR threshold inequalities are satisfied (we call this operation "all-at-once" or "Action 0") and (ii) the individual activation of each transmitter separately (we call this operation "one-at-a-time" or "Action k" when transmitter k is activated). Clearly, under Action k the instantaneous rate is the maximum possible that permits the SINR for the given transmission power to exceed the corresponding threshold at each receiver $d \in \mathcal{D}(k)$.

The above two modes of operation yield a total of $T+1$ actions, as shown in Fig. 2.2.

Restricting attention to these two modes of operation is somewhat natural since it permits comparison between two extreme cases, namely the cases of "all-at-once" and "one-at-a-time" operation. Note that since we don't consider power control, under Action 0 the individual rates are likely to be low due to the effects of interference. On the other hand, although under Action k the instantaneous rate of the kth transmitter will likely be much higher (than the corresponding rate under concurrent operation), the effective rate may be lower due to the effect of time sharing. Although this represents a severe restriction of the action space, it is expected to provide an insight into the trade-off between concurrent and individual activation.

Next, we find the optimal proportionally fair probability distribution over the aforementioned restricted set of actions by solving the following problem:

$$\max_{\boldsymbol{\pi}} \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| \log(\pi_0 r_k^0 + \pi_k r_k^k) \tag{2.6}$$

s.t.

$$\pi_j \geq 0, \quad \forall j \in \{0, 1, \ldots, T\}, \tag{2.7}$$

$$\sum_{j=0}^{T} \pi_j = 1. \tag{2.8}$$

Before we characterize the optimal policy solving (2.6)-(2.8), we provide some useful definitions. Let $\mathcal{J}$ be a subset of the set $\mathcal{T}$, such that for every $j \in \mathcal{J}$ it is true that $\pi_j > 0$. Also, let the complement $\mathcal{J}^{\mathrm{c}}$ of the set $\mathcal{J}$ be a set such that for every $i \in \mathcal{J}^{\mathrm{c}}$ it follows that $\pi_i = 0$, i.e., $\mathcal{J}^{\mathrm{c}} = \mathcal{T} \setminus \mathcal{J}$.

Theorem 2 *Let* $\boldsymbol{\pi}^\star = (\pi_0^\star, \ldots, \pi_T^\star)$ *denote the solution to* (2.6)-(2.8) *above. Then we have:*

1. *If*

$$\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k} \leq 1,$$

each multicast transmitter $k \in \mathcal{T}$ *is scheduled to transmit individually with probability*

$$\pi_k^\star = \frac{|\mathcal{D}(k)|}{\sum_{j \in \mathcal{T}} |\mathcal{D}(j)|}, \quad \forall k \in \mathcal{T},$$

and the probability of concurrent operation satisfies $\pi_0^\star = 0$*.*

2. *If*

$$\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k} > 1,$$

the optimal policy is of a threshold type with threshold $R(\mathcal{J})$ *given by*

$$R(\mathcal{J}) = \frac{1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j}{\sum_{m \in \mathcal{J}^c} |D(m)|}. \tag{2.9}$$

Specifically:

(a) *A multicast transmitter* $j \in \mathcal{T}$ *is scheduled to transmit individually with probability* $\pi_j^\star > 0$ *(i.e.,* j *is activated individually and belongs to* $\mathcal{J}$*) given by*

$$\pi_j^\star = \frac{|\mathcal{D}(j)| - \sum_{i \in \mathcal{J}^c} |\mathcal{D}(i)| \frac{r_j^0 / r_j^j}{1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j}}{\sum_{k \in \mathcal{T}} |\mathcal{D}(k)|}, \tag{2.10}$$

if and only if

$$\frac{r_j^0}{|\mathcal{D}(j)| r_j^j} < R(\mathcal{J}). \tag{2.11}$$

(b) All transmitters operate concurrently with probability $\pi_0^\star$ *given by*

$$\pi_0^\star = \frac{\sum_{m \in \mathcal{J}^c} |\mathcal{D}(m)|}{\left(\sum_{k \in \mathcal{T}} |\mathcal{D}(k)|\right) \left(1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j\right)}. \tag{2.12}$$

The proof of Theorem 2 is provided in Section 2.8. The quantity $\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k}$ determines, in a sense, the relative degree of interference in the network. Clearly, for any transmitter the instantaneous rate under concurrent operation is no greater than its corresponding rate under individual transmission. If it is also true that $\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k} \leq 1$, then the transmitters interfere among themselves sufficiently, so that their corresponding rates under concurrent operation are much lower than the corresponding rates under individual operation. Hence, when $\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k} \leq 1$, the optimal policy would never activate all transmitters concurrently ($\pi_0 = 0$); instead the optimal scheduling and rate control solution is to activate a single transmitter at a time as in a TDMA fashion. On the other hand, if $\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k} > 1$, the interference among the transmitters when they concurrently transmit is not so severe, and hence, the individual rates under concurrent operation result in levels that are "comparable" to those achieved under individual operation. Thus, the optimal policy assigns a positive probability to Action 0.

Theorem 2 characterizes the optimal solution based on the threshold function $R(\mathcal{J})$ which itself is a function of the set $\mathcal{J}$. Hence, in order to completely characterize the optimal policy we need to characterize the composition of $\mathcal{J}$. Note that since the optimal policy is of threshold type, the cardinality $|\mathcal{J}|$ of the "individually activated" set suffices to completely determine the set $\mathcal{J}$ itself, provided we label the transmitters appropriately. To simplify the notation in the sequel we will write $R(j)$ to denote $\{R(\mathcal{J}) : |\mathcal{J}| = j\}$.

Let us reorder the multicast sessions with respect to their corresponding values of the ratios $r_j^0 / \left(|\mathcal{D}(j)| r_j^j\right)$, $j \in \mathcal{T}$ in increasing order, i.e.,

$$\frac{\tilde{r}_1^0}{|\tilde{\mathcal{D}}(1)|\tilde{r}_1^1} \leq \frac{\tilde{r}_2^0}{|\tilde{\mathcal{D}}(2)|\tilde{r}_2^2} \leq \ldots \leq \frac{\tilde{r}_T^0}{|\tilde{\mathcal{D}}(T)|\tilde{r}_T^T}, \tag{2.13}$$

where the rates $\tilde{r}_j^0$, $\tilde{r}_j^j$, and the set of receivers $\tilde{\mathcal{D}}(j)$ denote the quantities r_j^0, r_j^j, and $\mathcal{D}(j)$ respectively of the jth transmitter under the new ordering. From now on, unless otherwise stated, the transmitter j is the jth transmitter under this new ordering. We will make use of the following property of the threshold function $R(j)$ to obtain the cardinality of the set $\mathcal{J}$.

Lemma 1 *Under the ordering of* (2.13)*, the threshold function* $R(j)$ *defined in Theorem 2 satisfies the following:*

$$R(j-1) \leq R(j), \quad \textit{if and only if} \;\; j \in \mathcal{J}.$$

The proof of Lemma 1 is proved in Section 2.9. From Lemma 1 it follows that $R(j)$ is increasing for all $j \in \mathcal{J}$ and decreasing for all $j \in \mathcal{J}^{\mathrm{c}}$. Using this fact, the following result follows directly using the definition of $R(\mathcal{J})$.

Theorem 3 *The cardinality of the set $\mathcal{J}$ under the optimal policy specified in Theorem 2 is given by*

$$|\mathcal{J}| = \underset{\ell \in \{0,1,\dots,T\}}{\arg\max} \frac{1 - \sum_{j=1}^{\ell} \tilde{r}_j^0 / \tilde{r}_j^j}{\sum_{m=\ell+1}^{T} |\tilde{\mathcal{D}}(m)|}. \tag{2.14}$$

From Theorems 2 and 3 it follows that the set $\mathcal{J}$ contains the $|\mathcal{J}|$ transmitters with the lowest values of the ratios $r_j^0 / \left(|\mathcal{D}(j)| r_j^j\right)$ for $j \in \mathcal{T}$. Hence, in the optimal solution the transmitters that are selected to be activated individually are the most "disadvantaged" multicast transmitters, i.e., those that either (i) can only achieve very low rates under concurrent operation compared to individual operation or (ii) those that multicast to a large number of receivers.

Consider a single-hop network of T transmitter and receiver pairs, where each transmitter sends unicast traffic to its corresponding receiver. Note that under the restricted set of actions the optimal proportionally fair probability distribution for the unicast case follows directly from our formulation by simply setting the cardinality of the set $\mathcal{D}(k)$ for every transmitter $k \in \mathcal{T}$ equal to one, i.e., $|\mathcal{D}(k)| = 1$. Thus, the solution of the unicast case is given next.

Corollary 1 *Let $\boldsymbol{\pi}^{\mathrm{u}\star} = (\pi_0^{\mathrm{u}\star}, \dots, \pi_T^{\mathrm{u}\star})$ be the optimal proportionally fair probability distribution for unicast traffic. Then we have:*

1. *If*

$$\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k} \leq 1,$$

each transmitter $k \in \mathcal{T}$ is scheduled to transmit individually with probability

$$\pi_k^{u\star} = \frac{1}{T}, \quad \forall k \in \mathcal{T},$$

and the probability of concurrent operation is zero , i.e., $\pi_0^{u\star} = 0$.

2. *If*

$$\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k} > 1,$$

the optimal policy is of a threshold type with threshold $R(\mathcal{J})$ *given by*

$$R(\mathcal{J}) = \frac{1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j}{T - |\mathcal{J}|}. \tag{2.15}$$

Specifically,

(a) *A transmitter* $j \in \mathcal{T}$ *is scheduled to transmit individually with probability* $\pi_j^{u\star} > 0$ *(i.e.,* j *is individually activated and belongs in* $\mathcal{J}$*) given by*

$$\pi_j^{u\star} = \frac{1}{T}\left(1 - \sum_{i \in \mathcal{J}^c} \frac{r_j^0 / r_j^j}{1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j}\right), \tag{2.16}$$

if and only if

$$\frac{r_j^0}{r_j^j} < R(\mathcal{J}). \tag{2.17}$$

(b) *All transmitters operate concurrently with probability* $\pi_0^{u\star}$ *given by*

$$\pi_0^{u\star} = \frac{T - |\mathcal{J}|}{T\left(1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j\right)}. \tag{2.18}$$

Corollary 2 *The cardinality of the set $\mathcal{J}$ under the optimal policy specified in Corollary 1 is given by the following:*

$$|\mathcal{J}| = \arg\max_{\ell \in \{0,1,\ldots,T\}} \frac{1 - \sum_{j=1}^{\ell} \tilde{r}_j^0 / \tilde{r}_j^j}{T - \ell}. \tag{2.19}$$

Corollaries 1 and 2 extend our prior work [14] where we had assumed that for every unicast transmitter $j \in \mathcal{T}$ the rates under individual operation r_j^j were all equal to each other.

2.5 Simulation Results

In this section, we analyze the performance of the proposed policies through a set of numerical experiments. First, we consider the special case of purely unicast traffic. Then, we proceed to a more general case that involves both unicast and multicast sessions. Throughout this section, we focus only on the criterion of proportional fairness. To illustrate our results we assume that the data rate $r(\cdot)$ is given by the single user Shannon formula under the assumption of unit bandwidth (See e.g., (A.2) in Appendix.). We could just as well use other expressions for different modulation schemes, e.g., (A.1) in the Appendix, corresponding to M-ary Phase Shift Keying (PSK) modulation with symbol rate control. Finally, we assume that the duration of a time slot is equal to one unit of time.

2.5.1 Unicast Case

The first wireless network we consider is shown in Fig. 2.3. It is a single-hop, static network of three transmitter/receiver pairs of unicast data traffic. The maximum transmission powers at the transmitters are $P_1^{\max} = P/2, P_2^{\max} = P, P_3^{\max} = 6 * P$, where $P = 6.0 * 10^{-5}$ Watts. Further, the power of the thermal noise is assumed to be common at all receivers and equal to $N = 3.34 * 10^{-6}$ Watts.

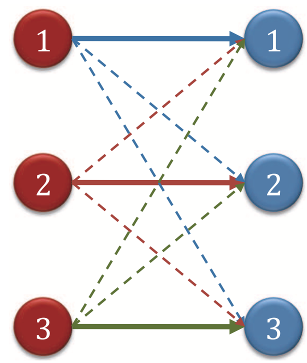

Figure 2.3: A static network of three transmitter/receiver pairs under unicast traffic.

We also parameterize the path loss matrix $\mathbf{G}$, defining the path losses between the 3 transmitters and the 3 receivers, as

$$\mathbf{G} = \begin{bmatrix} 0.9 & 0.9 * \beta & 0.9 * \beta \\ 0.9 * \beta & 0.9 & 0.9 * \beta \\ 0.9 * \beta & 0.9 * \beta & 0.9 \end{bmatrix},$$

where $\beta \in [0, 1]$ is a parameter, we dub as the *interference coefficient*, since it scales the degree of interference in the cross-channels. If $\beta = 0$, the three sessions can operate in parallel in an interference free manner. As β increases, the cross-channel qualities improve and the amount of interference between the sessions increases. When $\beta = 1$, the path losses between the direct and the cross channels become equal to each other at 0.9.

Under this channel model, we compare the performance of two proportionally fair policies. By proportionally fair we mean that the corresponding probabilities with which the different actions are chosen solve the unicast problem (2.6)-(2.8) obtained by replacing $|\mathcal{D}(k)| = 1$ for each k in (2.6). The first policy we consider is a proportionally fair TDMA scheme that activates a single transmitter at any given time (at its highest possible rate) with a probability optimized to ensure proportional fairness in the effective received rates when only TDMA actions are considered. The second policy is a restricted rate control policy that can choose to activate the transmitters one at a time or all together. Again, the probability with which each action is selected is optimized so that the effective rate at each receiver is proportionally fair under this restricted set of actions.

Fig. 2.4 shows the variation of the effective proportionally fair rates of the three transmitter/receiver pairs under the considered policies as the interference level in the system increases. First, we observe that the proportionally fair rate of the transmitter/receiver pair 3 is higher than the corresponding rates of the other two pairs and that the transmitter/receiver pair 1 has the lowest rate under all values of the interference coefficient. This is a natural outcome stemming from the specific selections on the maximum transmission powers of the respective transmitters. Our second observation confirms our intuitive explanation that the rate control policy performs strictly better than the pure TDMA scheme at low levels of interference, i.e., when the interference coefficient β is small. However, the performance gains of the proportionally fair rate control policy over the proportionally fair TDMA policy diminish rather quickly as β increases. For any interference coefficient $\beta > 0.2$, we observe that the rate control scheme converges to a TDMA scheme and thus both policies achieve the same performance. In other words, after a certain level of interference, a proportionally fair TDMA scheme becomes the optimum choice.

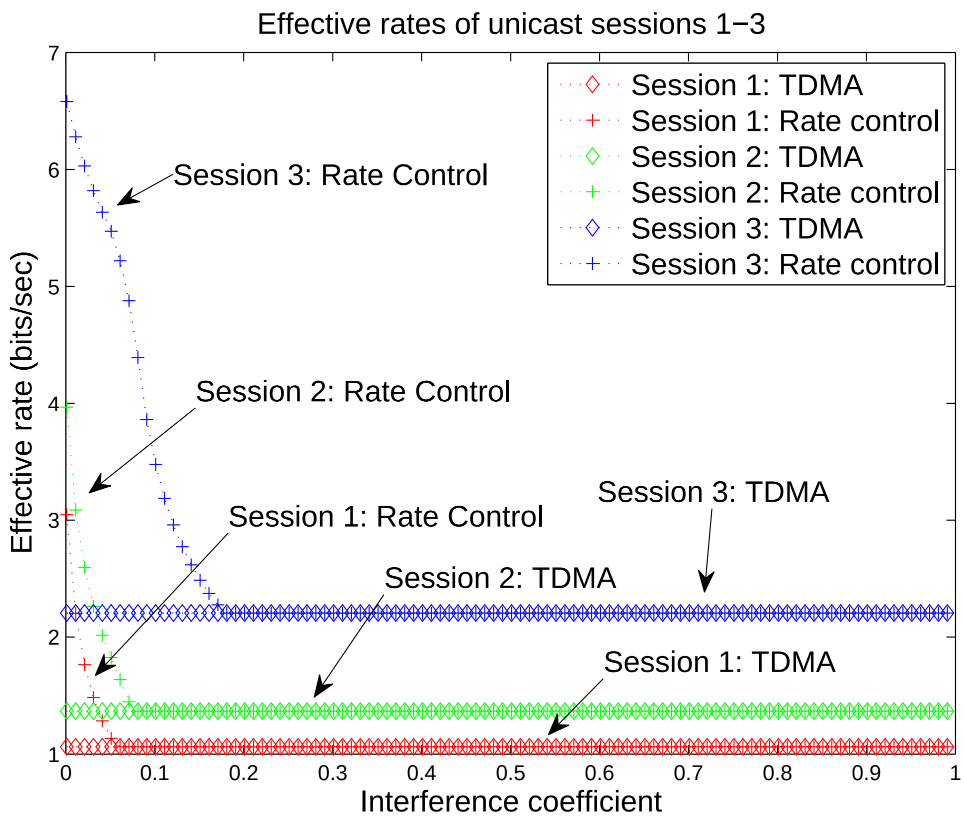

Figure 2.4: Effective rate of the three unicast sessions with respect to the interference coefficient β.

2.5.2 Multicast Case

In this subsection, we consider a static single-hop network with three transmitters and six receivers as shown in Fig. 2.5. The sets of the receivers for each transmitter

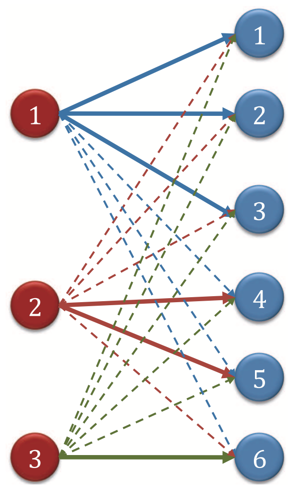

Figure 2.5: A single-hop static network of 3 multicast transmitters and 6 receivers.

are $\mathcal{D}(1) = \{1, 2, 3\}$, $\mathcal{D}(2) = \{4, 5\}$, and $\mathcal{D}(3) = \{6\}$, in other words transmitters 1 and 2 multicast to their respective receivers while transmitter 3 is a unicast source. We set the maximum transmission powers to be equal, i.e., $P_k^{\max} = P$, $k = 1, 2, 3$, where $P = 6.0 * 10^{-5}$ Watts. As in the previous section, the noise power is assumed to be common at all receivers and equal to $N = 3.34 * 10^{-6}$ Watts.

The path losses between the 3 transmitters and the 6 receivers are captured by the path loss matrix $\mathbf{G}$, given as

$$\mathbf{G} = \begin{bmatrix} 0.8 & 0.9 & 0.75 & \beta & \beta & \beta \\ \beta & \beta & \beta & 0.85 & 0.9 & \beta \\ \beta & \beta & \beta & \beta & \beta & 0.7 \end{bmatrix}.$$

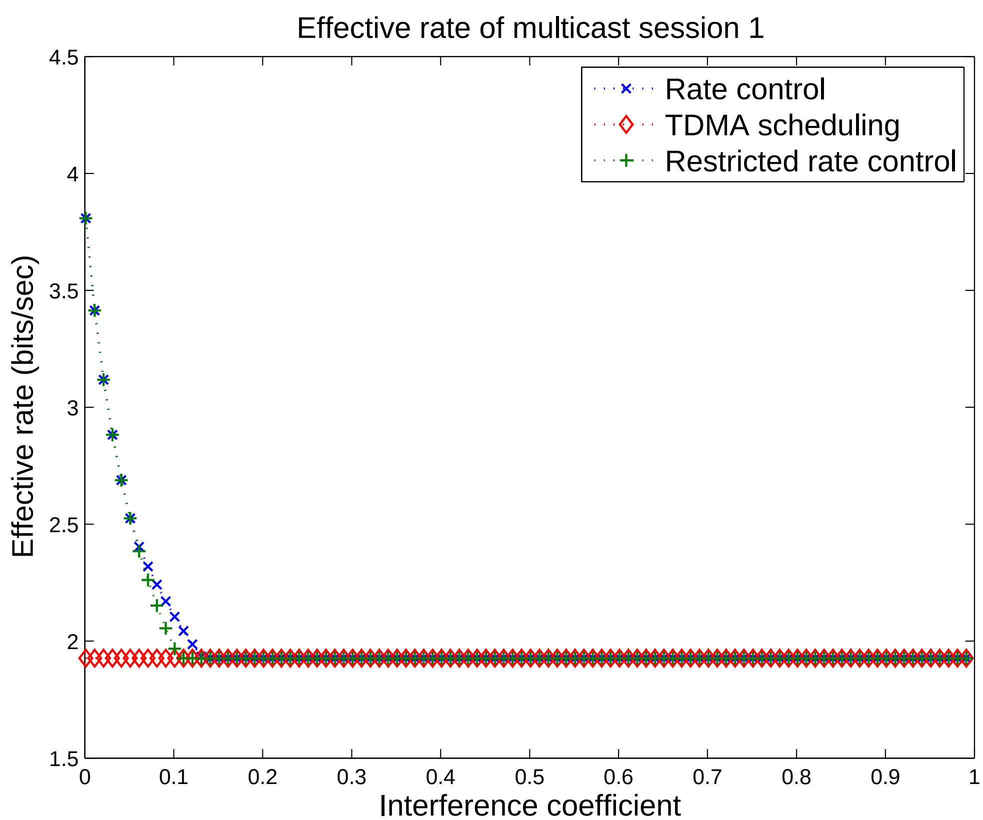

Figure 2.6: Effective rate of transmitter 1 with increasing β.

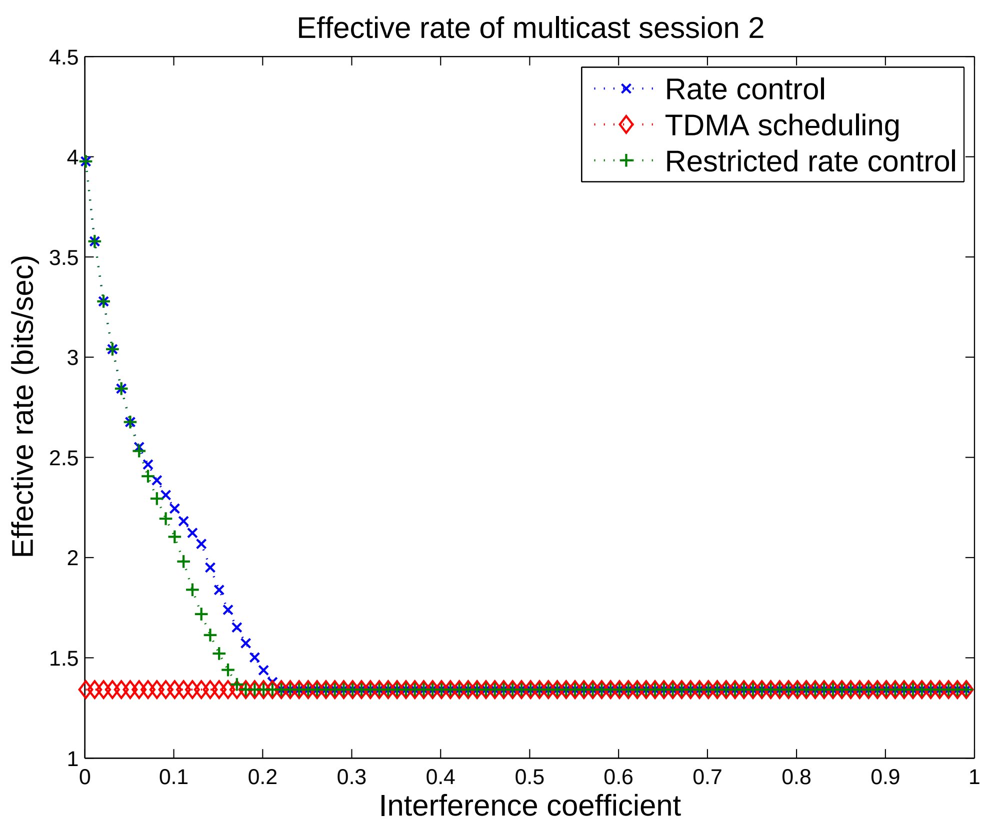

Figure 2.7: Effective rate of transmitter 2 with increasing β.

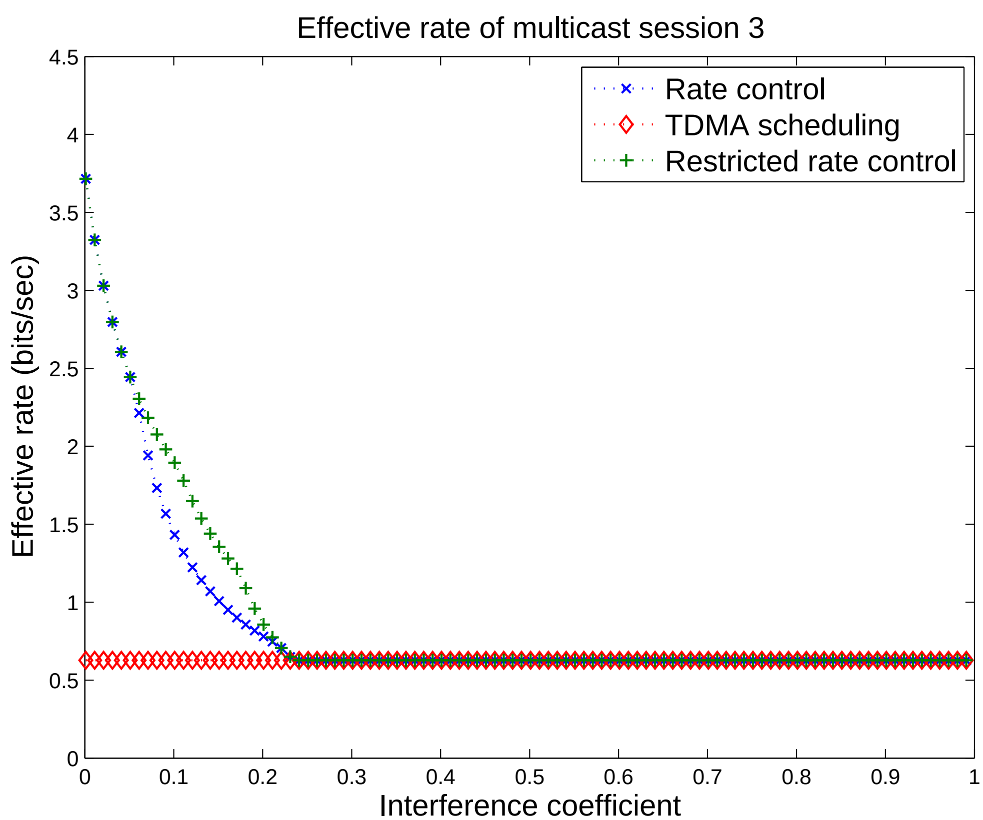

Figure 2.8: Effective rate of transmitter 3 with increasing β.

As before, $\beta \in [0,1]$ represents the *interference coefficient*. In Fig. 2.6, Fig. 2.7 and Fig. 2.8, the proportionally fair rates of each multicast session are plotted as a function of the interference coefficient β for three policies. Specifically, we consider (i) a proportionally fair scheme that allows all $2^3 - 1$ possible rate control and scheduling actions of activating the 3 transmitters, (ii) a proportionally fair TDMA scheme, where a single transmitter is activated at any given time, and (iii) the restricted scheme that considers either "all-at-once" operation or one at a time. As in the unicast experiment, the corresponding action probabilities are optimized so that the effective received rates are proportionally fair.

Similarly to the unicast case, when the levels of interference are low (i.e., β is close to 0), the two proportionally fair rate control schemes achieve much higher rates than the corresponding TDMA scheme. Furthermore, both rate control schemes converge fast, as expected, to the TDMA scheduling policy as the interference coefficient β increases.

2.6 Summary

In this chapter, we obtained a joint scheduling and rate control policy that assigns a probability distribution to the set of feasible rate control and scheduling actions under two performance objectives. We first considered sum throughput maximization and then proportional fairness. The identity of the transmitters that access the channel and their respective rates was selected according to this probability distribution.

In Section 2.2, we presented the network model under consideration. In Section 2.3 we focused on the criterion of total throughput maximization. We explicitly characterized an optimal scheduling and rate control policy. In Section 2.4 we focused our attention on the criterion of proportional fairness. Specifically, due to the complexity of the general problem we restricted the set of feasible actions to only actions given by concurrent operation of the transmitters all together or one at a time. For this restricted model we characterized the exact conditions under which a pure TDMA scheme should be employed instead of concurrent transmission for the "average rate" of each receiver to be proportionally fair. We showed that under this restricted framework the optimal proportionally fair solution is of a threshold type. We verified our analytical results through a set of numerical experiments in Section 2.5. Finally, the proofs of our main results appear in Sections 2.7, 2.8, and 2.9.

2.7 Proof of Theorem 1

We can write the Lagrangian of the problem defined in (2.2)-(2.4) as:

$$L(\boldsymbol{\pi}, \boldsymbol{\mu}, \lambda) = \sum_{k\in\mathcal{T}} \sum_{j\in\mathcal{A}} |\mathcal{D}(k)| r_k^j \pi_j - \lambda\big(\sum_{j\in\mathcal{A}} \pi_j - 1\big) + \sum_{j\in\mathcal{A}} \mu_j \pi_j, \qquad (2.20)$$

where $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_{|\mathcal{A}|})$ and λ are the Lagrange multipliers for the inequality and the equality constraints respectively. The Karush-Kuhn-Tucker (KKT) conditions yield:

$$\frac{\partial L(\boldsymbol{\pi}, \boldsymbol{\mu}, \lambda)}{\partial \pi_j} = \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| r_k^j - \lambda + \mu_j = 0 \quad \text{for all } j \in \mathcal{A}. \qquad (2.21)$$

$$\frac{\partial L(\boldsymbol{\pi}, \boldsymbol{\mu}, \lambda)}{\partial \lambda} = -\sum_{j\in\mathcal{A}} \pi_j + 1 = 0. \qquad (2.22)$$

$$\mu_j \pi_j = 0, \quad \mu_j \geq 0,\ \pi_j \geq 0, \qquad \text{for all } j \in \mathcal{A}. \qquad (2.23)$$

Let $\mathcal{J}$ denote a subset of the set $\mathcal{A}$, such that for every $j \in \mathcal{J}$ it is true that $\pi_j > 0$, and $\mathcal{J}^{\mathrm{c}} = \mathcal{A} \setminus \mathcal{J}$ denotes the complement set such that for every $i \in \mathcal{J}^{\mathrm{c}}$, $\pi_i = 0$. Then from (4.18) it follows that

$$\sum_{j \in \mathcal{J}} \pi_j = 1. \tag{2.24}$$

Also, from (4.19) we conclude that $\mu_j = 0$ for every $j \in \mathcal{J}$ and from (2.21) it follows that

$$\lambda = \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| r_k^j, \qquad \text{for every } j \in \mathcal{J}. \tag{2.25}$$

Moreover, from (4.19) we obtain that $\mu_i \geq 0$ for every $i \in \mathcal{J}^{\mathrm{c}}$, and from (2.21) it follows that

$$\lambda = \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| r_k^i + \mu_i, \qquad \text{for every } i \in \mathcal{J}^{\mathrm{c}}. \tag{2.26}$$

Then, from (2.25) and (2.26) and the fact that $\mu_i \geq 0$ we obtain that

$$\sum_{k \in \mathcal{T}} |\mathcal{D}(k)| r_k^j \geq \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| r_k^i, \quad \text{for every } j \in \mathcal{J}, \text{ and } i \in \mathcal{J}^{\mathrm{c}}. \tag{2.27}$$

Thus, from (2.25) and (2.27) it follows that any action $j \in \mathcal{J}$ has to be a solution of Problem II, i.e., $j \in \mathcal{A}^\star$. Therefore, we can conclude that $\mathcal{J}$ is a subset of $\mathcal{A}^\star$, i.e., $\mathcal{J} \subseteq \mathcal{A}^\star$. As a result, we can obtain the desired using (2.24):

$$\sum_{j \in \mathcal{A}^\star} \pi_j = 1.$$

■

2.8 Proof of Theorem 2

The Lagrangian function of the problem defined in (2.6)-(2.8) is given by

$$L(\boldsymbol{\pi}, \boldsymbol{\mu}, \lambda) = \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| \log(\pi_0 r_k^0 + \pi_k r_k^k) + \lambda(1 - \sum_{j=0}^{T} \pi_j) + \sum_{j=0}^{T} \mu_j \pi_j$$

where $\boldsymbol{\mu}$ and λ represent the Lagrange multipliers. The Karush-Kuhn-Tucker (KKT) conditions yield:

$$\frac{\partial L(\boldsymbol{\pi}, \boldsymbol{\mu}, \lambda)}{\partial \pi_0} = \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| \frac{r_k^0}{\pi_0 r_k^0 + \pi_k r_k^k} - \lambda + \mu_0 = 0. \tag{2.28}$$

$$\frac{\partial L(\boldsymbol{\pi}, \boldsymbol{\mu}, \lambda)}{\partial \pi_k} = |\mathcal{D}(k)| \frac{r_k^k}{\pi_0 r_k^0 + \pi_k r_k^k} - \lambda + \mu_k = 0 \quad \text{for all } k \in \mathcal{T}. \tag{2.29}$$

$$\frac{\partial L(\boldsymbol{\pi}, \boldsymbol{\mu}, \lambda)}{\partial \lambda} = 1 - \sum_{j=0}^{T} \pi_j = 0. \tag{2.30}$$

$$\mu_j \pi_j = 0, \quad \mu_j \geq 0, \pi_j \geq 0 \qquad \text{for all } j \in \{0, 1, \ldots, T\}. \tag{2.31}$$

Consider the following cases:

Case 1: Consider the case where Action 0 is never employed, i.e., $\pi_0 = 0$. It is easy to see that in this case, $\pi_k > 0$ for every $k \in \mathcal{T}$. Hence, from (2.31) it follows that $\mu_0 \geq 0$ and $\mu_k = 0$ for every $k \in \mathcal{T}$. Then, from (2.29) we obtain,

$$\pi_k = \frac{|\mathcal{D}(k)|}{\lambda}. \tag{2.32}$$

Using (2.30) and (2.32) and the fact that $\pi_0 = 0$, we can solve for λ as

$$\lambda = \sum_{k \in \mathcal{T}} |\mathcal{D}(k)|, \tag{2.33}$$

which implies that

$$\pi_k = \frac{|\mathcal{D}(k)|}{\sum_{j\in\mathcal{T}} |\mathcal{D}(j)|}, \quad \forall k \in \mathcal{T}. \tag{2.34}$$

Finally, from (2.28) it follows that

$$\sum_{k\in\mathcal{T}} \frac{r_k^0}{r_k^k} \leq 1. \tag{2.35}$$

Case 2: Now let us consider the alternative case that Action 0 is employed with strictly positive probability, i.e., there will be a fraction of the time that all transmitters operate concurrently. Also, assume that a subset $\mathcal{J}$ of the transmitters is further individually activated with positive probability, while the rest of the transmitters, $\mathcal{J}^{\mathrm{c}} = \mathcal{T} \setminus \mathcal{J}$ are not chosen for individual operation. This implies that $\pi_0 > 0$, $\pi_j > 0$ for every $j \in \mathcal{J}$, and $\pi_i = 0$ for every $i \in \mathcal{J}^{\mathrm{c}}$. Hence, (2.31) yields $\mu_0 = 0$, $\mu_j = 0$ for every $j \in \mathcal{J}$, and $\mu_i \geq 0$ for every $i \in \mathcal{J}^{\mathrm{c}}$. From (2.28) we have

$$\lambda = \sum_{j\in\mathcal{J}} \frac{|\mathcal{D}(j)| r_j^0}{\pi_0 r_j^0 + \pi_j r_j^j} + \sum_{i\in\mathcal{J}^{\mathrm{c}}} \frac{|\mathcal{D}(i)|}{\pi_0}. \tag{2.36}$$

Also from (2.29) it follows that for all $j \in \mathcal{J}$ we have

$$\lambda = \frac{|\mathcal{D}(j)| r_j^j}{\pi_0 r_j^0 + \pi_j r_j^j}, \tag{2.37}$$

or equivalently

$$\pi_j = \frac{|\mathcal{D}(j)|}{\lambda} - \pi_0 \frac{r_j^0}{r_j^j}. \tag{2.38}$$

Using the fact that $\pi_0 + \sum_{j \in \mathcal{J}} \pi_j = 1$, we obtain

$$\pi_0 = \frac{\lambda - \sum_{j \in \mathcal{J}} |\mathcal{D}(j)|}{\lambda \left(1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j\right)}. \tag{2.39}$$

Combining (2.36), (2.37), and (2.39) yields

$$\lambda = \sum_{k \in \mathcal{T}} |\mathcal{D}(k)|, \tag{2.40}$$

$$\pi_0 = \frac{\sum_{m \in \mathcal{J}^c} |\mathcal{D}(m)|}{\left(\sum_{k \in \mathcal{T}} |\mathcal{D}(k)|\right) \left(1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j\right)}, \tag{2.41}$$

$$\pi_j = \frac{|\mathcal{D}(j)| - \sum_{i \in \mathcal{J}^c} |\mathcal{D}(i)| \frac{r_j^0 / r_j^j}{1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j}}{\sum_{k \in \mathcal{T}} |\mathcal{D}(k)|}. \tag{2.42}$$

In addition, for all $i \in \mathcal{J}^c$, from (2.29) it is true that

$$\frac{r_i^0}{|\mathcal{D}(i)| r_i^i} \geq \frac{1}{\lambda \pi_0}. \tag{2.43}$$

Furthermore, using (2.40) and (2.41) we rewrite the RHS of (2.43) to obtain

$$\frac{r_i^0}{|\mathcal{D}(i)| r_i^i} \geq \frac{1 - \sum_{j \in \mathcal{J}} r_j^0 / r_j^j}{\sum_{m \in \mathcal{J}^c} |D(m)|}, \quad \forall i \in \mathcal{J}^c. \tag{2.44}$$

After some straightforward manipulation (2.44) yields

$$\sum_{k \in \mathcal{T}} \frac{r_k^0}{r_k^k} \geq 1, \tag{2.45}$$

providing the necessary and sufficient condition for $\pi_0 > 0$.

Note also that from (2.9) the right hand side in (2.44) is the threshold $R(\mathcal{J})$. Hence, $i \in \mathcal{J}^{\mathrm{c}}$ if and only if

$$\frac{r_i^0}{|\mathcal{D}(i)| r_i^i} \geq R(\mathcal{J}). \tag{2.46}$$

Also, (2.41) can be written in terms of (2.9) as

$$\pi_0 = \frac{1}{\left(\sum_{k \in \mathcal{T}} |\mathcal{D}(k)|\right) R(\mathcal{J})}. \tag{2.47}$$

Then, (2.37), (2.40) and (2.47) together yield

$$|\mathcal{D}(j)| r_j^j = \frac{r_j^0}{R(\mathcal{J})} + \pi_j r_j^j \sum_{k \in \mathcal{T}} |\mathcal{D}(k)|.$$

By dividing both sides of the equality by $|\mathcal{D}(j)| r_j^j$ and by using the fact that $\frac{\pi_j}{|\mathcal{D}(j)|} \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| > 0$ for $j \in \mathcal{J}$, it is easy to obtain that $j \in \mathcal{J}$ if and only if

$$\frac{r_j^0}{|\mathcal{D}(j)| r_j^j} < R(\mathcal{J}). \tag{2.48}$$

Therefore, from (2.46) and (2.48) it is clear that the optimal policy is of threshold type.

■

2.9 Proof of Lemma 1

For notational convenience, let us define the following quantities:

$$v_k := \frac{\tilde{r}_k^0}{\tilde{r}_k^k},$$
$$M := \sum_{k=1}^{T} |\tilde{\mathcal{D}}(k)|,$$
$$m_k := |\tilde{\mathcal{D}}(k)|.$$

Under the ordering given in (2.13), for every $k \in \mathcal{J}$ we have by definition that

$$\frac{v_k}{m_k} < R(|\mathcal{J}|) < \frac{1 - \sum_{j=1}^{k} v_j - \sum_{j=k+1}^{|\mathcal{J}|} v_j}{M - \sum_{j=1}^{k} m_j - \sum_{j=k+1}^{|\mathcal{J}|} m_j}.$$

The above can be rewritten as

$$\frac{v_k}{m_k}(M - \sum_{j=1}^{k} m_j) < 1 - \sum_{j=1}^{k} v_j - \sum_{j=k+1}^{|\mathcal{J}|} \left(v_j - \frac{v_k}{m_k} m_j \right).$$

Due to the fact that under the ordering of (2.13) we have $\frac{v_k}{m_k} \leq \frac{v_j}{m_j}$ for all $j \geq k$, we obtain

$$\frac{v_k}{m_k} < \frac{1 - \sum_{j=1}^{k} v_j}{(M - \sum_{j=1}^{k} m_j)}, \quad \text{for every } k \in \mathcal{J}$$
$$\frac{v_k}{m_k} < R(k), \quad \text{for every } k \in \mathcal{J}. \tag{2.49}$$

Furthermore, from (2.49) we obtain that for every $k \in \mathcal{J}$ the following is true

$$\frac{v_k}{m_k} < \frac{1 - \sum_{j=1}^{k-1} v_j - v_k}{M - \sum_{j=1}^{k-1} m_j - m_k},$$

which can also be written as

$$v_k(M - \sum_{j=1}^{k-1} m_j) < m_k(1 - \sum_{j=1}^{k-1} v_j)$$

or

$$\frac{v_k}{m_k} < R(k-1). \tag{2.50}$$

On the other hand, using the definition of $R(k)$, we obtain

$$\begin{aligned}
R(k) - R(k-1) &= \frac{1 - \sum_{j=1}^{k-1} v_j - v_k}{M - \sum_{j=1}^{k} m_j} - R(k-1) \\
&= \frac{R(k-1)(M - \sum_{j=1}^{k-1} m_j) - v_k}{M - \sum_{j=1}^{k} m_j} - R(k-1) \\
&= \frac{R(k-1)(M - \sum_{j=1}^{k-1} m_j) - R(k-1)(M - \sum_{j=1}^{k} m_j)}{M - \sum_{j=1}^{k} m_j} \\
&\quad - \frac{v_k}{M - \sum_{j=1}^{k} m_j} \\
&= \frac{m_k R(k-1) - v_k}{M - \sum_{j=1}^{k} m_j},
\end{aligned} \tag{2.51}$$

which implies that

$$R(k-1) \leq R(k) \iff \frac{v_k}{m_k} \leq R(k-1). \tag{2.52}$$

Hence, from (2.49) and (2.50) we obtain that

$$R(k-1) \leq R(k), \quad \text{for all } k \in \mathcal{J}. \tag{2.53}$$

Now, let us consider $k \in \mathcal{J}^c$. By definition we have

$$\frac{v_k}{m_k} \geq R(|\mathcal{J}|),$$

By simply letting $k = |\mathcal{J}| + 1$ we have

$$\frac{v_{|\mathcal{J}|+1}}{m_{|\mathcal{J}|+1}} \geq R(|\mathcal{J}|).$$

However, from (2.52) it follows that

$$R(|\mathcal{J}|) > R(|\mathcal{J}| + 1).$$

Combining the above two results, we have

$$\frac{v_{|\mathcal{J}|+1}}{m_{|\mathcal{J}|+1}} \geq R(|\mathcal{J}|) \geq R(|\mathcal{J}| + 1). \tag{2.54}$$

Furthermore, due to the aforementioned ordering, we also have

$$\frac{v_{|\mathcal{J}|+2}}{m_{|\mathcal{J}|+2}} \geq \frac{v_{|\mathcal{J}|+1}}{m_{|\mathcal{J}|+1}} \geq R(|\mathcal{J}| + 1), \tag{2.55}$$

which, from (2.52), implies that

$$R(|\mathcal{J}| + 1) \geq R(|\mathcal{J}| + 2).$$

Repeating the same pattern it is easy to see that

$$R(k) \geq R(k+1), \quad \text{for all } k \in \mathcal{J}^c. \tag{2.56}$$

Finally, combining (2.53) and (2.56) yields the desired result.

■

Chapter 3

Utility Maximization for Multicast Traffic in Time-Varying Networks

3.1 Background

The policy of Chapter 2 focused on characterizing the exact relation between the current channel conditions and the optimal rate control and scheduling decisions, where optimality was assumed with respect to sum throughput maximization and proportional fairness. Nevertheless, it has four limitations, namely it (i) assumes that the wireless channel does not change with time, (ii) assumes the network control policy has perfect channel state information at every decision instant, (iii) is limited to the objectives of proportional fairness and total throughput maximization, and (iv) considers a restricted set of rate control actions in the analysis of proportionally fair schedules.

In this chapter, we consider time-varying networks, with channel conditions that are potentially not perfectly known by the network control policy, under a general family of utility functions. We focus on finding a rate and power control algorithm for the problem, rather than an explicit characterization of the scheduling decisions with respect to the channel conditions. Specifically, we consider a system of multiple transmitters with *multicast* traffic destined for a set of receivers. Each transmitter is associated with a multicast session and the receivers of different sessions can be overlapping. We are interested in the problem of scheduling the transmitters through joint rate and power control decisions so that the overall system utility, measured in terms of the average rate of each receiver, is maximized. We obtain an optimal policy that jointly allocates the transmission rates and powers of each transmitter by having access to only a perhaps inaccurate estimate of the wireless channel state. We prove optimality of this policy through the theory of stochastic approximation for any utility function that is strictly concave, continuously differentiable, and increasing in the average rate.

The problem of joint scheduling and rate control has been studied extensively in the literature. A large body of work focuses on scheduling of the downlink channel of a base station transmitting *unicast* data traffic to a set of mobile terminals. The base station at any given time has to select a single terminal to transmit to according to a Time Division Multiple Access (TDMA) scheme. One particular example is the proportional fair sharing scheduler (PFS) introduced by Qualcomm. The PFS selects a single terminal for transmission at any given time, the one that maximizes the ratio of a user's instantaneous rate to the average rate it has received so far in order to achieve proportional fairness. Therefore, those terminals that received comparably lower average data rates until the current decision instant are more likely to be selected in the optimal solution ([17], [20], [21]).

However, as it is shown in a variety of settings ([14], [22], [23]), TDMA scheduling of a set of nodes one at a time need not be optimal. In fact, it is shown in [14] that in a static wireless network, depending on the channel conditions it may be beneficial to allow all the nodes to operate concurrently for a certain period of time under the objective of proportional fairness for unicast traffic. Similar results are obtained in [23] for the case of multicast traffic.

In a different work [16], the authors consider the problem of rate control for unicast traffic in time-varying wireless networks. Their formulation permits the scheduling of concurrent unicast transmissions. The authors introduce an optimal rate control policy under the objective of maximizing the sum of user utilities for utility functions that are strictly concave, increasing in the average rate of each receiver, and continuously differentiable. A subsequent work, [13], considers the problem of optimal rate allocation for a switch serving a set of queues under the objective of utility maximization. Although a broader class of utility functions is considered in [13], the switch is restricted to change states according to a finite-state, stationary and ergodic Markov Chain.

Although [13], [14], [16], [17], [20], [21], and [23] consider the problem of rate control for utility maximization under unicast traffic, a large amount of traffic in networks is comprised of multicast data. In [18] the authors consider a base station that multicasts traffic to various groups of receivers. It is assumed that only a single multicast group can be chosen for transmission at any given time and that all the terminals in the multicast group receive at the same rate (i.e., single-rate multicast). The multicast scheduler needs to decide which *unique* group to serve and at which *rate* under two objectives; when the objective is to be proportionally fair with respect to the (i) total rate of each multicast group and (ii) overall rate of each terminal when it is a member of various multicast groups. Further, in a recent work [15] we considered the problem of utility maximization for *multicast* traffic in time-varying wireless networks, through joint rate and power control decisions by permitting concurrent node activations.

However, a fundamental assumption in all prior work is the availability of *perfect* channel state information to the scheduling policies at each decision instant[1]. In practice,

[1]In [16], a limited discussion on the subject of channel estimation is presented only for the restricted case of TDMA scheduling.

the channel conditions can only be estimated, and hence exact knowledge of the current channel state is unlikely to be available. Depending on the quality of the channel estimates the performance degradation can be high. For example, in a fast fading environment the channel state at the time it is observed can be significantly different from the channel state at which the actual transmissions take place (see e.g., [24], [25]). The effect of this discrepancy in the channel state may be two-folded; first, certain scheduled transmissions may fail, and second, transmissions which would be successful may never be activated.

In this chapter we study the problem of utility maximization for time-varying wireless networks under channel estimation. We assume a set of multicast transmitters that are always backlogged. The set of receivers of different multicast sources may be overlapping. The objective is to schedule the transmitters by selecting their transmission rates and powers so that the sum of user utilities is maximized. We consider policies that take scheduling decisions based *only* on a possibly inaccurate estimate of the wireless channel state. We introduce an on-line, gradient-based policy and establish its optimality among all policies that have access only to the current estimate. We employ the theory of stochastic approximation to prove our results.

In this chapter, we further generalize prior works of [13], [14], [15], [16], [17], [18], [20], [21], and [23] by considering the problem of utility maximization for multicast traffic under channel estimation. We also extend the results of [18] in two aspects: we (i) consider a wireless network where multiple multicast transmissions can be scheduled concurrently, and (ii) assume a broad class of utility functions, that includes the utility of proportional fairness.

3.2 Model Formulation

3.2.1 Network Model

We consider a time-varying, single-hop, wireless network, consisting of T transmitters and D receivers. We denote by $\mathcal{T} = \{1, 2, \ldots, T\}$ and $\mathcal{D} = \{1, 2, \ldots, D\}$ the sets of transmitters and receivers in the network respectively. Each transmitter $k \in \mathcal{T}$ is associated with a *multicast* session and multicasts traffic to a set of receivers $\mathcal{D}(k) \subseteq \mathcal{D}$. We denote by $|\mathcal{D}(k)|$ the cardinality of the set $\mathcal{D}(k)$. Our model captures the special cases of *unicast* ($|\mathcal{D}(k)|$ = 1) and *broadcast* traffic ($|\mathcal{D}(k)|$ = D). We assume that different multicast sessions may have overlapping receiver sets, i.e., for any two transmitters $j, k \in \mathcal{T}$ it is possible that $\mathcal{D}(j) \cap \mathcal{D}(k) \neq \emptyset$. As an example, in Fig. 3.1 both transmitters 1 and 2 multicast to receiver 1. In this chapter, again, we assume that each multicast source is always backlogged and has enough data to send whenever it is activated. This traffic model is to be distinguished from other alternatives that assume burstiness in traffic.

We consider a slotted-time model. We denote by $P_n(k)$ the transmission power level of transmitter k at time slot n. We also denote by $\mathbf{P}_n$ the T-dimensional vector of transmission powers of every transmitter at time slot n, i.e., $\mathbf{P}_n = (P_n(k),\ k \in \mathcal{T})$. We further assume that for every slot n the power vectors $\mathbf{P}_n$ take values from a compact set $\mathcal{P}$ of allowable power allocations, i.e., $\mathbf{P}_n \in \mathcal{P}$. Finally, we denote the thermal noise power at receiver $d \in \mathcal{D}$ by $N(d)$.

We consider a channel process $\{\mathbf{G}_n\}_{n=0}^{\infty}$ with channel state $\mathbf{G}_n = \{\mathbf{G}_n(i,j),\ \ i \in \mathcal{T},\ j \in \mathcal{D}\}$ at every slot n representing the channel conditions between each transmitter i

and receiver j. We assume that $\{\mathbf{G}_n\}_{n=0}^{\infty}$ follows a block fading model, namely it changes at the beginning of every slot and stays constant therein. By assuming that slot durations can be sufficiently small this assumption becomes less restrictive. The channel process reflects the variations of the channel quality that can be due to node mobility, channel fading, path loss, shadowing, etc. We make the assumption that $\{\mathbf{G}_n\}_{n=0}^{\infty}$ is *stationary* and *ergodic*.

A fundamental aspect of our model that contrasts it from prior work [15] is the fact that at the beginning of each time slot n the network controller is assumed to have access to only an estimate of the true channel state. This is in fact the reality in wireless systems; the channel can only be estimated and this estimate can be highly misleading. The effects of the inaccuracy of the available channel state information at the network controller can be two-fold: (i) it can lead to the failure of certain scheduled transmissions and (ii) it can prohibit certain transmissions from being activated although they would have been successful. These effects get mitigated as the qualities of the estimates improve.

Let the estimate of the channel state $\mathbf{G}_n$ at time slot n be denoted by $\hat{\mathbf{G}}_n = \{\hat{G}_n(i,j), \quad \forall i \in \mathcal{T}, \; j \in \mathcal{D}\}$. This estimate represents the *estimated channel state* $\hat{G}_n(i,j)$ between each transmitter $i \in \mathcal{T}$ and receiver $j \in \mathcal{D}$ at slot n. Naturally, at any given time slot n the estimated channel state $\hat{\mathbf{G}}_n$ and the true channel state $\mathbf{G}_n$ are correlated. In fact, they can be identical under perfect estimation. The estimated channel process $\{\hat{\mathbf{G}}_n\}_{n=0}^{\infty}$ also follows a block fading model. It is also assumed to be stationary and ergodic with stationary distribution given by $f_{\hat{\mathbf{G}}}(\cdot)$. In this chapter, we restrict our attention to network control policies that at any given time slot n take scheduling decisions based only on the estimate $\hat{\mathbf{G}}_n$. To have a "fair" comparison, we only consider policies that have access to a common channel estimate $\hat{\mathbf{G}}_n$. We denote the class of these policies by $\hat{\mathbf{\Pi}}$. We further assume that both process $\{\mathbf{G}_n\}_{n=0}^{\infty}$ and $\{\hat{\mathbf{G}}_n\}_{n=0}^{\infty}$ take values from a common state space, that is a continuous set $\mathcal{G}$. The above are illustrated in Fig. 3.1.

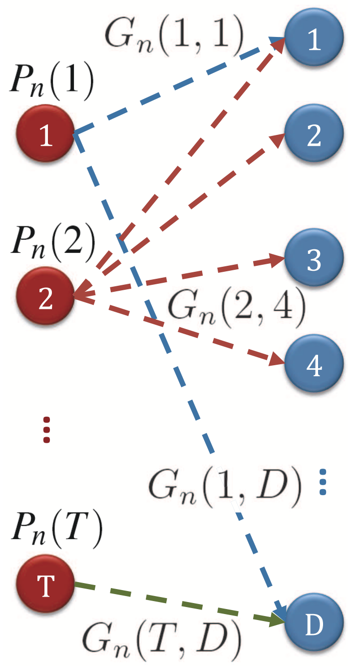

Figure 3.1: A network of T multicast transmitters and D receivers.

Again, we capture the effects of interference in the network through the SINR interference model under two types of receivers; (i) receivers that can only receive from a single transmitter at any given time (i.e., single packet reception, SPR), and (ii) receivers with multi-packet reception (MPR) capabilities. Under MPR a receiver may successfully receive concurrently from multiple transmitters as long as the SINR from each one of them exceeds the required threshold. Hence, two multicast transmitters with overlapping receiving nodes can concurrently transmit successfully, unlike the SPR case where only a single transmission can be received successfully at any given time.

Under unicast traffic, throughput is unambiguously defined as the rate at which data is delivered to a receiver successfully. However, this is not the case under multicast traffic. It is possible that a multicast transmission reaches successfully only a subset of the receivers. In such a case depending on the requirements of the application under consideration this transmission can be assumed to be successful, and hence count as throughput, or not. For example, if the application expects that at least *some* of the receivers obtain the message successfully then such a transmission is assumed to be successful. As an alternative, the requirements of the application may be more strict and require that *all* the receivers of the multicast group receive the message. In this chapter, we consider the latter case and will assume that a transmission from transmitter k is successful if it is received by all the receivers in the multicast group $\mathcal{D}(k)$. If any receiver fails to receive the message, then the transmission is assumed to fail and the message has to be retransmitted.

As in the previous chapter, we focus on the dependence of the threshold only on the transmission rate. Assume that the transmission rate and power of each transmitter $k \in \mathcal{T}$ at time slot n are $r_n(k)$ and $P_n(k)$ respectively. Let $\mathbf{r}_n = (r_n(k), k \in \mathcal{T})$ and $\mathbf{P}_n = (P_n(k), k \in \mathcal{T})$ be the respective transmission rate and power vectors. Then the SINR at each receiver $d \in \mathcal{D}(k)$ at time slot n is given as

$$\mathrm{SINR}_n^{\mathbf{P}_n}(k,d) := \frac{P_n(k)G_n(k,d)}{N(d) + \sum_{j\in\mathcal{T},\, j\neq k} P_n(j)G_n(j,d)}. \tag{3.1}$$

As discussed previously, we will assume that transmitter k *multicasts* successfully at rate $r_n(k)$ if the SINR at each receiver $d \in \mathcal{D}(k)$ exceeds the required threshold. We denote by $\gamma_{n,d}(r)$ the SINR threshold at time slot n that represents the minimum value of SINR that allows successful transmission at rate r at receiver d. Then, for a given pair of vectors $\mathbf{r}_n$ and $\mathbf{P}_n$ a multicast transmission from the kth transmitter is successful if

$$\text{SINR}_n^{\mathbf{P}_n}(k,d) \geq \gamma_{n,d}(r_n(k)), \quad \forall d \in \mathcal{D}(k). \tag{3.2}$$

In this chapter we assume policies that take decisions at every given time slot n based on the channel estimate $\hat{\mathbf{G}}_n$. Thus, the transmissions are scheduled based on the estimated SINR, namely

$$\widehat{\text{SINR}}_n^{\mathbf{P}_n}(k,d) := \frac{P_n(k)\hat{G}_n(k,d)}{N(d) + \sum_{j\in\mathcal{T},\, j\neq k} P_n(j)\hat{G}_n(j,d)}. \tag{3.3}$$

and a transmission is expected to be successful by the scheduling policy if

$$\widehat{\text{SINR}}_n^{\mathbf{P}_n}(k,d) \geq \gamma_{n,d}(r_n(k)), \quad \forall d \in \mathcal{D}(k). \tag{3.4}$$

Since the policies under consideration take decisions based on the estimated SINR criterion (3.4), it is possible that certain scheduled transmissions fail. To capture this effect we introduce the $T \times T$ diagonal matrix $\mathbf{Q}_n^{(\mathbf{r}_n,\mathbf{P}_n)}$, whose (k,k)th entry satisfies

$$Q_n^{(\mathbf{r}_n,\mathbf{P}_n)}(k,k) = \begin{cases} 1, & \text{if } \text{SINR}_n^{\mathbf{P}_n}(k,d) \geq \gamma_{n,d}(r_n(k)), \;\; \forall d \in \mathcal{D}(k) \\ 0, & \text{otherwise.} \end{cases} \tag{3.5}$$

In other words the matrix $\mathbf{Q}_n^{(\mathbf{r}_n,\mathbf{P}_n)}$ is an indicator diagonal matrix whose (k,k)th diagonal entry takes the value one if a scheduled transmission based on (3.4) is in fact also successful with respect to the true SINR criterion (3.2).

We also define by $\bar{\mathbf{Q}}_{\mathbf{g}}^{(\mathbf{r},\mathbf{P})}$ the $T \times T$ dimensional matrix whose (k,k)th entry gives the probability of success of a scheduled transmission from transmitter k to all receivers $d \in \mathcal{D}(k)$ at rate $r(k)$ and power $P(k)$ given that the estimated channel state is $\mathbf{g} \in \mathcal{G}$. Specifically, the matrix $\bar{\mathbf{Q}}_{\mathbf{g}}^{(\mathbf{r},\mathbf{P})}$ is defined as

$$\bar{\mathbf{Q}}_{\mathbf{g}}^{(\mathbf{r},\mathbf{P})} = \mathbb{E}[\mathbf{Q}_n^{(\mathbf{r},\mathbf{P})} | \hat{\mathbf{G}}_n = \mathbf{g}], \quad \mathbf{g} \in \mathcal{G}. \tag{3.6}$$

Note that the $\bar{\mathbf{Q}}_{\mathbf{g}}^{(\mathbf{r},\mathbf{P})}$ is stationary with respect to time since the true and estimated channel processes are both stationary.

We proceed to define the set of feasible *instantaneous* transmission rates that can be achieved through all possible rate and power control actions under the current estimated channel state conditions. Note that the set of feasible rates depends on the capabilities of the receivers as well, specifically on whether they have SPR or MPR capabilities. Let $\mathcal{R}^{\text{SPR}}(\hat{\mathbf{G}}_n)$ and $\mathcal{R}^{\text{MPR}}(\hat{\mathbf{G}}_n)$ be the feasible rate regions corresponding to channel state $\hat{\mathbf{G}}_n = \mathbf{g}$, $\mathbf{g} \in \mathcal{G}$ under SPR and MPR respectively. Since under SPR a transmission cannot be successful if more than one transmitter transmits to the same receiver, we first identify subsets of the transmitters with non-overlapping receivers. We define a *valid activation vector* $\mathbf{c}$ to be a binary T-element vector that takes values in $\{0,1\}^T$. All the non-zero entries of a valid activation vector correspond to transmitters with non-overlapping receiver sets that can be activated successfully under some rate and power allocation. In other words, for any two elements $c(j), c(k)$ of an activation vector $\mathbf{c}$ with $c(j) = c(k) = 1$ it must be true that $\mathcal{D}(j) \cap \mathcal{D}(k) = \emptyset$. We further define the *constraint set* $\mathcal{C}$ to be the set containing *all* such activation vectors. Based on the above, $\mathcal{R}^{\text{SPR}}(\hat{\mathbf{G}}_n)$ is defined as follows:

$$\mathcal{R}^{\mathrm{SPR}}(\hat{\mathbf{G}}_n) = \mathrm{co}\Bigg\{\mathbf{r} = (r(k),\ k \in \mathcal{T}):\ \exists\, \mathbf{P} \in \mathcal{P},\ \exists\, \mathbf{c} \in \mathcal{C},\ \text{such that } \forall\, d \in \mathcal{D}(k)$$
$$\frac{c(k)P_n(k)\hat{G}_n(k,d)}{N(d) + \sum_{j\in\mathcal{T}, j\neq k} c(j)P_n(j)\hat{G}_n(j,d)} \geq \gamma_{n,d}(r(k))\ \Bigg\}, \tag{3.7}$$

where $\mathrm{co}(\cdot)$ is the convex hull of the set. Hence, the set $\mathcal{R}^{\mathrm{SPR}}(\hat{\mathbf{G}}_n)$ is the set obtained by time-sharing of feasible rates, achieved by some power vector $\mathbf{P} \in \mathcal{P}$ such that concurrent transmission from two or more transmitters to a common receiver is prohibited. Similarly, $\mathcal{R}^{\mathrm{MPR}}(\hat{\mathbf{G}}_n)$ is given by

$$\mathcal{R}^{\mathrm{MPR}}(\hat{\mathbf{G}}_n) = \mathrm{co}\Bigg\{\mathbf{r} = (r(k),\ k \in \mathcal{T}):\ \exists\, \mathbf{P} \in \mathcal{P},\ \text{such that } \forall\, d \in \mathcal{D}(k),$$
$$\widehat{\mathrm{SINR}}_n^{\mathbf{P}}(k,d) \geq \gamma_{n,d}(r(k))\ \Bigg\}. \tag{3.8}$$

3.2.2 Problem Formulation

In this chapter, we are interested to maximize the sum of utilities of all receivers where the utility is defined in terms of the long-term average throughput. We assume utility functions $U(\cdot)$ that are strictly concave, increasing, and continuously differentiable with respect to the received user rate. As an example, a utility function that satisfies these properties is the utility of α-fairness presented in Chapter 1.

To distinguish from the region $\mathcal{R}^{\mathrm{SPR}}(\hat{\mathbf{G}}_n)$ of *instantaneous* transmission rates, we also define the average rate region, $\bar{\mathcal{R}}^{\mathrm{SPR}}$ when the receivers have only single-packet reception capabilities as

$$\bar{\mathcal{R}}^{\mathrm{SPR}} = \Big\{ \bar{\mathbf{r}} = (\bar{r}(k), k \in \mathcal{T}) : \exists\, \mathbf{r}^{\mathbf{g}} \in \mathcal{R}^{\mathrm{SPR}}(\mathbf{g}),\ \forall\, \mathbf{g} \in \mathcal{G}$$
$$\text{s.t. } \bar{r}(k) = \int_{\mathcal{G}} r^{\mathbf{g}}(k) \bar{Q}^{\boldsymbol{\pi}}_{\mathbf{g}}(k,k) f_{\hat{\mathbf{G}}}(\mathbf{g}) d\mathbf{g} \Big\}. \quad (3.9)$$

This region corresponds to the long term average rate region of throughput rates that are achievable when the scheduling decisions are based only on an estimate of the true channel state. It is easy to see that no rate outside the region $\bar{\mathcal{R}}^{\mathrm{SPR}}$ is achievable unless a policy $\boldsymbol{\pi} \notin \hat{\mathbf{\Pi}}$, i.e., has access to a better estimate of the channel state which would improve the probabilities of the matrix $\bar{\mathbf{Q}}^{\boldsymbol{\pi}}_{\mathbf{g}}$. The corresponding average rate region under MPR capabilities, $\bar{\mathcal{R}}^{\mathrm{MPR}}$, is defined similarly by replacing $\mathcal{R}^{\mathrm{SPR}}(\mathbf{g})$ with $\mathcal{R}^{\mathrm{MPR}}(\mathbf{g})$ wherever it appears in (3.9).

From now on, to simplify our notation we will write $\mathcal{R}(\hat{\mathbf{G}}_n)$ to refer to the instantaneous feasible rate region $\mathcal{R}^{\mathrm{SPR}}(\hat{\mathbf{G}}_n)$ or $\mathcal{R}^{\mathrm{MPR}}(\hat{\mathbf{G}}_n)$, depending on the receiver capabilities. Further, we denote by $\bar{\mathcal{R}}$ the corresponding average rate region. Finally, we denote $\bar{Q}^{(\mathbf{r},\mathbf{P})}_{\mathbf{g}}(k,k)$ and $Q^{(\mathbf{r}_n,\mathbf{P}_n)}_{n}(k,k)$ by $\bar{Q}^{\boldsymbol{\pi}}_{\mathbf{g}}(k,k)$ and $Q^{\boldsymbol{\pi}}_{n}(k,k)$ respectively where the superscript $\boldsymbol{\pi}$ is used to denote the pair of rate and power choices by a policy $\boldsymbol{\pi}$.

Given the above definitions, the utility maximization problem under consideration can be formulated as follows:

$$\max_{\bar{\mathbf{r}} \in \bar{\mathcal{R}}} \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| U(\bar{r}(k)), \quad (3.10)$$

where $U(\bar{r}(k))$ is the rate utility that any receiver $d \in \mathcal{D}(k)$ receives from the kth transmitter. By multiplying this utility with the number of receivers in the multicast group, we obtain the overall utility of all receivers.

Note that this is a convex optimization problem. One approach to solve this problem is to use interior point methods to obtain the optimal vector of average rates and in the sequel map these average rates to a sequence of instantaneous transmission rate selections (and corresponding powers) over the time. However, due to the large number of variables involved in the optimization, finding these instantaneous rates can be difficult. Further, the complexity of this reverse process is exacerbated by the fact that at any given slot there may be more than one such instantaneous rate selection. In this chapter, we follow a different approach by introducing an on-line, gradient-based solution that at every time slot selects the instantaneous rates so that the long-term average rate is the maximizer of (3.10).

3.3 Optimal Rate and Power Control Policy under Uncertainty

In this section we specify an optimal, centralized policy $\boldsymbol{\pi}^{\star}$, which takes rate and power control decisions at the beginning of each time slot and is claimed to solve (3.10).

Let $r_n^{\boldsymbol{\pi}}(k)$ denote the transmission rate of transmitter k under policy $\boldsymbol{\pi}$ at time slot n and let $\mathbf{r}_n^{\boldsymbol{\pi}}$ denote the T dimensional vector of rates $\mathbf{r}_n^{\boldsymbol{\pi}} = (r_n^{\boldsymbol{\pi}}(k), k \in \mathcal{T})$. Also let us define by $\theta_n^{\boldsymbol{\pi}}(k)$ the time-average rate of transmitter k under a policy $\boldsymbol{\pi}$ until time slot n. From now on we will refer to this time-average rate as the *effective rate* of transmitter k at time slot n. Note that this coincides with the rate that each receiver $d \in \mathcal{D}(k)$ receives up to time slot n. Specifically,

$$\theta_n^{\boldsymbol{\pi}}(k) = \frac{1}{n}\sum_{\nu=1}^{n} r_\nu^{\boldsymbol{\pi}}(k) Q_\nu^{\boldsymbol{\pi}}(k,k). \tag{3.11}$$

We also define the vector of effective rates of all transmitters up to time slot n by $\boldsymbol{\theta}_n^{\boldsymbol{\pi}} = (\theta_n^{\boldsymbol{\pi}}(k), k \in \mathcal{T})$. The vector $\boldsymbol{\theta}_n^{\boldsymbol{\pi}}$ can be written recursively according to

$$\boldsymbol{\theta}_{n+1}^{\boldsymbol{\pi}} = \boldsymbol{\theta}_n^{\boldsymbol{\pi}} + \epsilon_n \mathbf{Y}_n^{\boldsymbol{\pi}}, \tag{3.12}$$

where

$$\mathbf{Y}_n^{\boldsymbol{\pi}} = {\mathbf{r}_{n+1}^{\boldsymbol{\pi}}}^\top \mathbf{Q}_{n+1}^{\boldsymbol{\pi}} - \boldsymbol{\theta}_n^{\boldsymbol{\pi}}, \tag{3.13}$$

and

$$\epsilon_n = \frac{1}{n+1}. \tag{3.14}$$

Assume that at time slot n, the estimated channel state satisfies $\hat{\mathbf{G}}_n = \mathrm{g}$ for some $\mathbf{g} \in \mathcal{G}$ and the effective rates at the previous time slot $n-1$ are given as $\boldsymbol{\theta}_{n-1}^{\boldsymbol{\pi}^\star} = \boldsymbol{\theta}$. Then, the *rate and power control* policy $\boldsymbol{\pi}^\star$ under channel state uncertainty is defined as

$$[\mathbf{P}_n^{\boldsymbol{\pi}^\star,\boldsymbol{\theta},\mathbf{g}}, \mathbf{R}_n^{\boldsymbol{\pi}^\star,\boldsymbol{\theta},\mathbf{g}}] = \left\{ \underset{\mathbf{r}\in\mathcal{M}^{\boldsymbol{\theta},\mathbf{g}}, \mathbf{P}\in\mathcal{P}}{\arg\min} \sum_{k\in\mathcal{T}} P(k) : \forall k \in \mathcal{T}, \ \ \forall\, d \in \mathcal{D}(k) \right\}, \tag{3.15}$$

where $\mathcal{M}^{\boldsymbol{\theta},\mathbf{g}}$ is given by

$$\mathcal{M}^{\boldsymbol{\theta},\mathbf{g}} = \left\{ \tilde{\mathbf{r}} = \underset{\mathbf{r}\in\mathcal{R}(\mathbf{g})}{\arg\max} \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| r(k) \bar{Q}_{\mathbf{g}}^{\boldsymbol{\pi}}(k,k) \frac{\partial U(\theta(k))}{\partial \theta(k)} \right\}. \tag{3.16}$$

Note that the set $\mathcal{M}^{\boldsymbol{\theta},\mathbf{g}}$ may not be well defined when the gradient of the utility function is not finite. An example to this is the utility of proportional fairness where for values of $\theta(k)$ close to 0 the gradient is ill-defined. However his problem can be overcome by adding arbitrarily small initialization constants to the argument of the gradient of the utility, as was done in [15] and [21].

The optimality of $\boldsymbol{\pi}^\star$ defined in (3.15) and (3.16) with respect to the utility maximization problem of (3.10) is established next.

3.4 Asymptotic Analysis of the Optimal Policy

To show the optimality of the policy $\boldsymbol{\pi}^\star$ given by (3.15) and (3.16) we use the theory of stochastic approximation ([26], [27]). Note that the recursion (3.12) is in the standard stochastic approximation form with decreasing step size ϵ_n. Let $t_0 = 0$ and for $n = 1, \ldots$ let $t_n = \sum_{i=0}^{n-1} \epsilon_i$. We define the continuous time interpolation process $\boldsymbol{\theta}^{0,\boldsymbol{\pi}^\star}$ on $(-\infty, +\infty)$ as follows:

$$\boldsymbol{\theta}^{0,\boldsymbol{\pi}^\star}(t) = \begin{cases} \boldsymbol{\theta}_0^{\boldsymbol{\pi}^\star}, & \text{if } t < t_0, \\ \\ \boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star}, & \text{if } t_n \leq t < t_{n+1}. \end{cases}$$

We further define the *shifted* process $\boldsymbol{\theta}^{n,\boldsymbol{\pi}^\star}$ on $(-\infty, +\infty)$ as:

$$\boldsymbol{\theta}^{n,\boldsymbol{\pi}^\star}(t) = \boldsymbol{\theta}^{0,\boldsymbol{\pi}^\star}(t_n + t), \ \ \forall t \in (-\infty, +\infty).$$

The basic idea behind this method is to interpolate the discrete process of effective rates $\boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star}, n \in \{0, 1, 2, \ldots\}$ to a continuous process $\boldsymbol{\theta}^{0,\boldsymbol{\pi}^\star}$, with interpolating length equal to the decreasing step size ϵ_n of the algorithm. The shifted version $\boldsymbol{\theta}^{n,\boldsymbol{\pi}^\star}$ of the continuous process is created by shifting $\boldsymbol{\theta}^{0,\boldsymbol{\pi}^\star}$ to start at the nth interpolation interval. It is easy to see that the tail of the sequence $\boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star}$ follows that of the process $\boldsymbol{\theta}^{n,\boldsymbol{\pi}^\star}$. Showing that the latter converges to a set of limit points of an ordinary differential equation (ODE) proves that the sequence $\boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star}$ converges to the same set of limit points.

The convergence can be either with probability one or in distribution. Although weaker, convergence in distribution often yields the same information about the asymptotic behavior in practical applications as the probability one methods [26]. Hence, we only focus on convergence in distribution.

Due to the fact that the bandwidth of the communication is finite and the power vectors are chosen from a compact set $\mathcal{P}$, the rate region $\mathcal{R}(\mathbf{g})$ is compact. Let $\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}$ denote the transmission rates assigned by policy $\boldsymbol{\pi}^\star$ until time n, i.e., $\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star} = \{\mathbf{r}_\nu^{\boldsymbol{\pi}^\star}, \nu \leq n\}$. From the above it follows that $\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}$ also belongs in a compact set. We denote the later by Ξ.

To show optimality of the policy $\boldsymbol{\pi}^\star$ described by (3.15) and (3.16) we make the following assumptions.

Assumption 1 *For every time slot n and sequence $\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}$ the function $g^{\boldsymbol{\theta},\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}}$ defined as*

$$g_n^{\boldsymbol{\theta},\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}} := \mathbb{E}[\mathbf{r}_{n+1}^{\boldsymbol{\pi}^\star}{}^\top \mathbf{Q}_n^{\boldsymbol{\pi}^\star} | \boldsymbol{\theta}, \boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}] - \boldsymbol{\theta}, \tag{3.17}$$

is measurable with respect to the σ-algebra generated by $\{\boldsymbol{\theta}, \boldsymbol{\xi}_i^{\boldsymbol{\pi}^\star}, i = 1, \ldots, n\}$. Furthermore, for every compact set $\Delta \subset \Xi$, the function $g_n^{\boldsymbol{\theta},\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}}$ is continuous in $\boldsymbol{\theta}$ uniformly in n and in $\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star} \in \Delta$.

Assumption 1 guarantees that small changes in the current time average rate will not affect significantly the rate selection of the next decision instant.

Assumption 2 *The function $\bar{g}^{\boldsymbol{\theta}}$ defined as*

$$\bar{g}^{\boldsymbol{\theta}} := \mathbb{E}[\mathbf{r}_{n+1}^{\boldsymbol{\pi}^\star}{}^\top \mathbf{Q}_{n+1}^{\boldsymbol{\pi}^\star}] - \boldsymbol{\theta}, \tag{3.18}$$

is continuous in $\boldsymbol{\theta}$ and satisfies

$$\lim_{m,n\to\infty} \frac{1}{m} \sum_{\ell=n}^{n+m-1} \left(\mathbb{E}[\mathbf{r}_{\ell+1}^{\boldsymbol{\pi}^\star}{}^\top \mathbf{Q}_{\ell+1}^{\boldsymbol{\pi}^\star} | \boldsymbol{\theta}_0^{\boldsymbol{\pi}^\star}, \boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}] - \boldsymbol{\theta} \ - \bar{g}(\boldsymbol{\theta}) \right) 1\{\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star} \in \Xi\} = 0, \tag{3.19}$$

where the limit is in the mean and taken as $n \to \infty$ and $m \to \infty$ simultaneously in any way at all.

The second part of Assumption 2 resembles a weaker version of the law of large numbers, since we only require that the time average of a sequence of expected values must converge. When the channel process is ergodic, then (3.19) holds even without the expectation. The following two theorems establish the optimality of the proposed policy.

Theorem 4 *Consider the policy $\boldsymbol{\pi}^\star \in \hat{\boldsymbol{\Pi}}$ specified by* (3.15) *and* (3.16)*. Under Assumptions 1-2 and for any initial condition, $\boldsymbol{\theta}^{n,\boldsymbol{\pi}^\star}$ converges in distribution to the set of limit points of the ODE given by*

$$\dot{\boldsymbol{\theta}} = \mathbb{E}[\mathbf{r}_{n+1}^{\boldsymbol{\pi}^\star}{}^\top \mathbf{Q}_{n+1}^{\boldsymbol{\pi}^\star}] - \boldsymbol{\theta}. \tag{3.20}$$

The proof is given in Section 3.7.

Theorem 5 *The ODE given in* (3.20) *has a unique limit point $\boldsymbol{\theta}^\star \in \bar{\mathcal{R}}$ where $\boldsymbol{\theta}^\star$ is the solution to* (3.10)*, i.e.,*

$$\boldsymbol{\theta}^\star = \arg\max_{\bar{\mathbf{r}} \in \bar{\mathcal{R}}} \sum_{k \in \mathcal{T}} |\mathcal{D}(k)| U(\bar{r}(k)).$$

The proof of Theorem 5 is given in Section 3.8.

3.5 Simulation Results

In this section, we analyze the performance of our proposed policy through a set of simulations both for unicast and multicast traffic. For the sake of simplicity, we assume perfect channel estimation, i.e., $\mathbf{G}_n = \hat{\mathbf{G}}_n$ for every time slot n. Throughout this section, we consider the utility of proportional fairness. We observe the performance of our policy both in the presence and absence of channel fading. As we will see later in this section unicast traffic is significantly benefitted from the presence of fading. These benefits are mitigated in the case of multicast.

Throughout our simulation analysis we consider a single-hop, wireless network with three transmitters and three receivers. The duration of a time slot is assumed to be equal to one second. For simplicity in our simulations we only consider rate control, and assume that each transmitter k at every time slot n can either remain silent or transmit at a maximum power $P(k),\ k = 1, 2, 3$. Specifically, we assume that the transmission powers satisfy $P(1) = 6.0 * 10^{-5}$ Watts, $P(2) = 3.0 * 10^{-5}$ Watts, and $P(3) = 2.0 * 10^{-5}$ Watts. Further, the power of the Additive White Gaussian Noise is assumed to be $N(d) = 3.34 * 10^{-6}$ Watts at all receivers.

In our model we consider quasi-static, Rayleigh fading. Let the received signal power under path loss and shadowing between transmitter i and receiver j at time slot n be denoted by $P_n^r(i, j)$. Moreover, let the average received power be $\bar{P}^r(i, j)$. Then, under Rayleigh fading the received signal power $P_n^r(i, j)$ is exponentially distributed with mean $\bar{P}^r(i, j)$ (see e.g., Chapter 3 in [28]).

Let us define the average path loss matrix $\bar{\mathbf{G}} = (\bar{G}(i, j), i, j = 1, 2, 3)$, which is obtained by averaging the path losses over all different states of the channel (fading) process $\{\mathbf{G}_n\}_{n=0}^{\infty}$. With the common assumption that the shadowing process varies according to a zero mean Gaussian random variable it follows that the channel coefficients $G_n(i,j)$ satisfy $G_n(i,j) = P_n^r(i,j)/P(i)$ at time n, and furthermore the average path loss is equal to $\bar{G}(i,j) = \bar{P}^r(i,j)/P(i)$. We study the performance of our policy under two scenaria, namely under (i) pure path loss and (ii) Rayleigh fading. For the comparison to be meaningful, we assume that the matrix of path losses under scenario (i) is given by the matrix $\bar{\mathbf{G}}$, which is the mean of the Rayleigh fading of scenario (ii). The matrix $\bar{\mathbf{G}}$ is parameterized as follows:

$$\bar{\mathbf{G}} = \begin{bmatrix} 0.9 & \beta\, 0.9 & \beta\, 0.9 \\ \beta\, 0.9 & 0.9 & \beta\, 0.9 \\ \beta\, 0.9 & \beta\, 0.9 & 0.9 \end{bmatrix},$$

where $\beta \in [0,1]$ multiplies the cross channel path losses. For the case of unicast traffic, we dub the parameter β as the *interference coefficient* while we call it *cross-link coefficient* for the case of multicast. In the case of unicast traffic, this parameter reflects the level of interference in the network. For example, when $\beta = 0$, the channels between the three transmitter/receiver pairs can be seen as three parallel channels that can operate simultaneously without causing any interference to each other. On the other extreme, when $\beta = 1$, the path losses at the direct channels between every transmitter and receiver are equal to the path losses over the cross channels, and therefore the level of interference at every receiver is very high. In the case of multicast traffic the parameter β gives the quality of the cross links and has an effect not only on the interference, but also on the transmission rate. Throughout the section, we obtain the data rate $r(\cdot)$ at the receivers through the single-user Shannon formula, i.e., $r(\mathrm{SINR}) = \log_2(1 + \mathrm{SINR})$, by assuming unit bandwidth.

3.5.1 Case I - Unicast Sessions

In this subsection, we consider the special case of unicast sessions, i.e., $\mathcal{D}(1) = \{1\}$, $\mathcal{D}(2) = \{2\}$, and $\mathcal{D}(3) = \{3\}$.

In Fig. 3.2, we present the convergence effective rates of each receiver when we employ the optimal policy presented in Section 3.3 under a fading and a non-fading channel model. The interference coefficient is set to $\beta = 0.2$. From Fig. 3.2 it follows that the effective rate of each receiver quickly converges to its corresponding proportionally fair rate. We also observe that naturally the effective rates are proportional to the transmission powers, and thus receiver one has a higher rate than receiver two, and the latter has a higher rate than the third receiver.

From the figure we can also draw an important conclusion; the effective rate of each receiver is higher under fading than in the absence of fading. This demonstrates the opportunistic nature of our policy. If a transmitter sees a bad channel at the current time slot, the policy will not activate this transmitter in general, since with *high probability* in the future its channel conditions will improve. Moreover, at the current time slot with high probability some other transmitter having a better channel will be activated by the policy.

We proceed to compare the proposed optimal policy when (i) it can take all possible

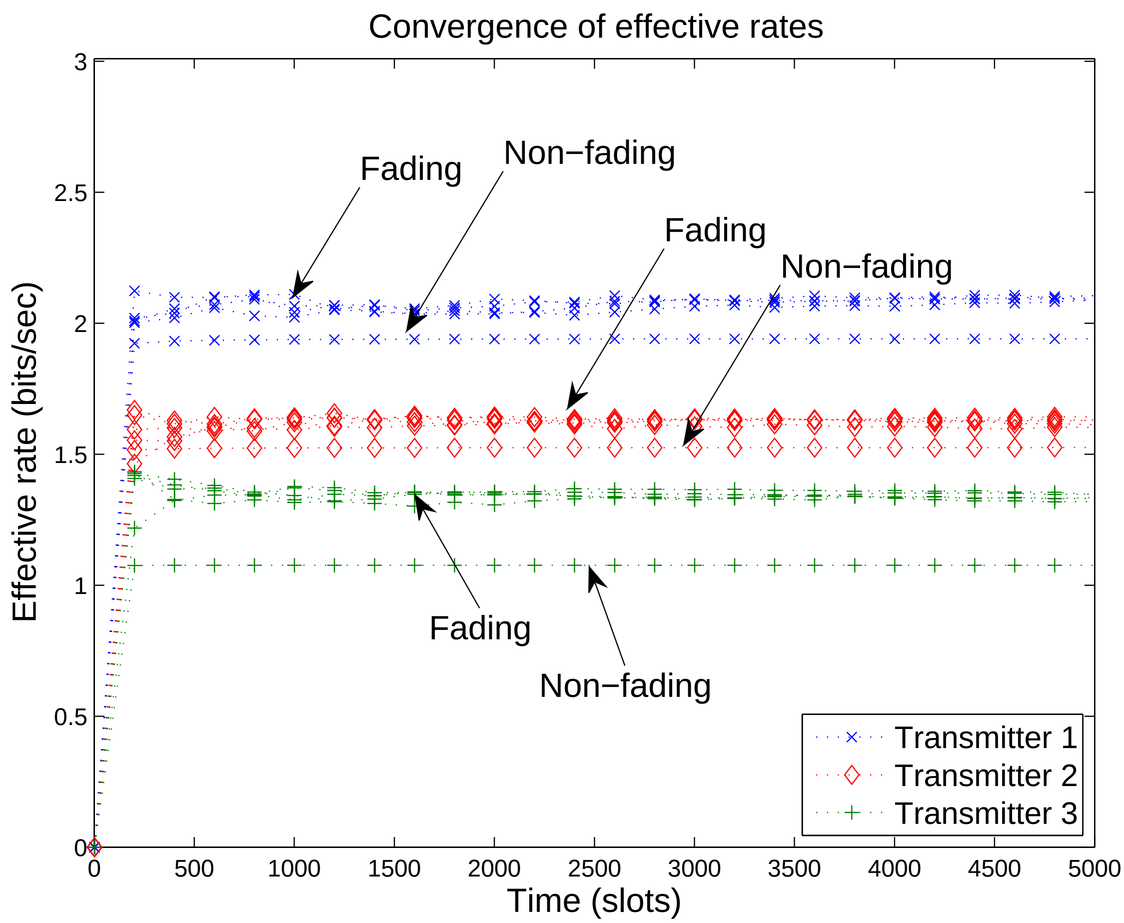

Figure 3.2: Convergence of the utility optimal policy $\pi^{\star}$ for unicast traffic.

rate control actions in the set $\mathcal{R}(\mathbf{G}_n)$, and (ii) it is restricted to take TDMA scheduling and rate control actions, i.e., it can only activate a single transmitter at any given time at its maximum achievable rate. Note that since the transmission powers take binary values from the set $\{0, P(k)\}$ for every transmitter k, the set $\mathcal{R}(\mathbf{G}_n)$ contains 7 rate vectors obtained by finding all possible subsets of transmitters and assigning them the maximum transmission rates such that the SINR criterion is jointly satisfied at all their receivers. We refer to the former as the "optimal rate control policy" and to the latter as the "TDMA scheduling policy". The comparison of the two policies is performed under various interference levels.

In Fig. 3.3 the proportionally fair effective rates at each receiver are plotted as a function of different values of the interference coefficient β for the optimal rate control policy and for the TDMA scheduling policy in the absence of fading. We observe that when the interference levels are relatively low, the optimal rate control policy achieves higher rates for every transmitter and receiver pair, as opposed to the TDMA scheduling policy. We also observe that the two policies have comparable performance under higher interference levels. This result is natural since the proposed policy exploits the potential benefits of concurrent transmissions when the interference is relatively low and it effectively operates as a proportionally fair TDMA scheduling when the interference is relatively high. A similar pattern is observed in Fig. 3.4, where fading is considered.

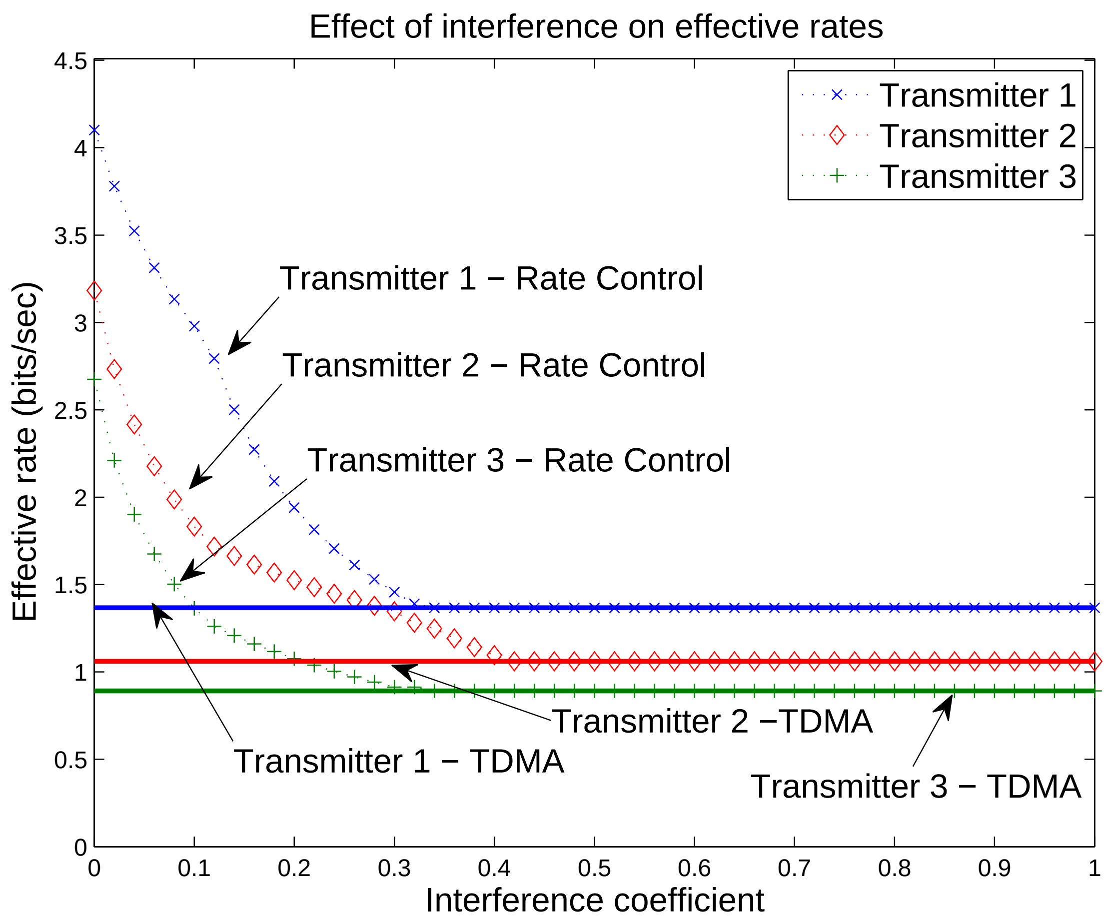

Figure 3.3: Proportionally fair rates with increasing β in the absence of fading.

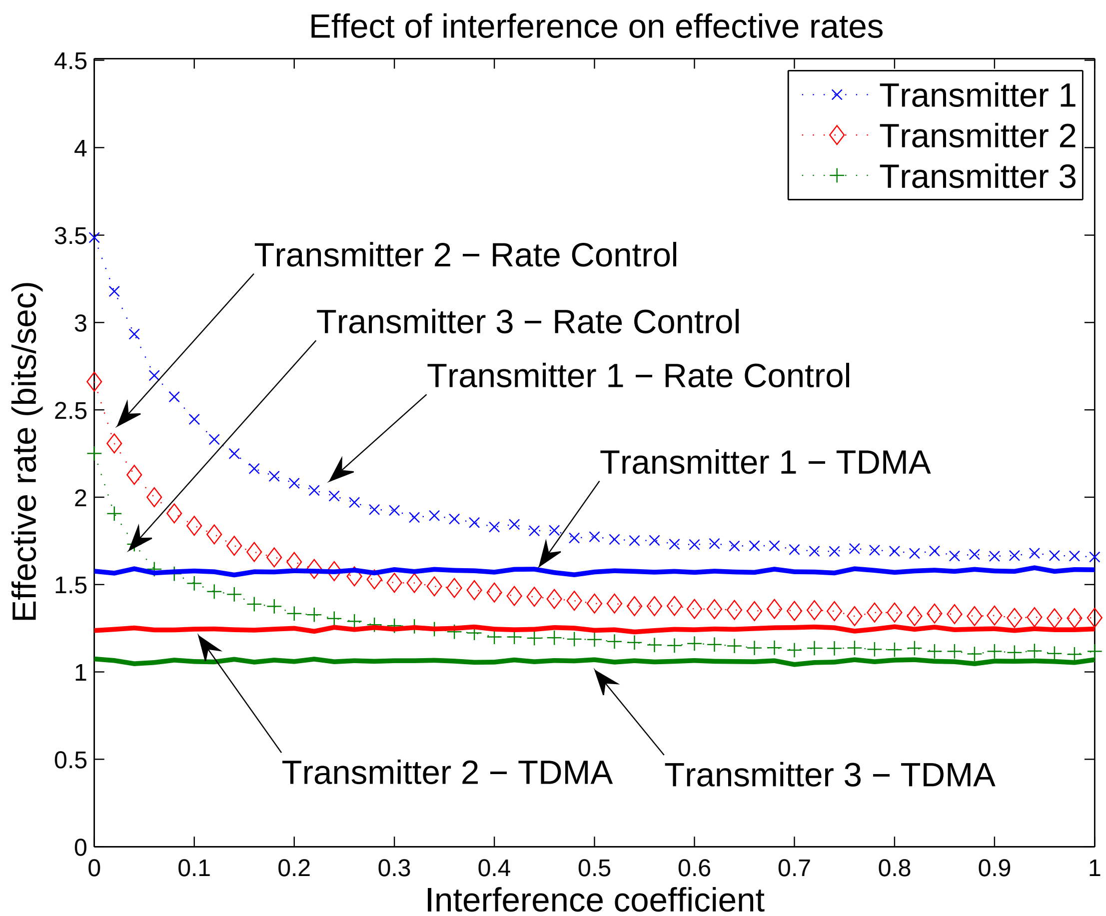

Figure 3.4: Proportional fair rates with increasing β under Rayleigh fading.

3.5.2 Case II - Multicast Sessions

In this subsection, we consider the case where there are three sessions, one broadcast, one multicast, and one unicast. Specifically, $\mathcal{D}(1) = \{1, 2, 3\}$, $\mathcal{D}(2) = \{1, 2\}$, and $\mathcal{D}(3) = \{3\}$. We assume multi-packet reception capabilities (MPR) at the receivers, e.g., both sessions 1 and 2 can be activated simultaneously as long as the SINRs for each received transmission at every receiver exceed the appropriate thresholds. In Fig. 3.5, we present the convergence of the effective rates of each receiver achieved under the proposed policy by setting the cross-link coefficient equal to $\beta = 0.2$.

Again in this figure we observe that the effective rate of each multicast session converge quickly to its respective proportionally fair rate. We further observe that unlike unicast, in the case of multicast traffic it is no longer true that the rates under fading are always better than the corresponding rates in the absence of fading. The reason behind this observation is the fact that now a transmission involves multiple links and the multicast rate is constrained by the link with the worst channel due to the single rate multicast assumption. Hence, for transmitter 1 to effectively observe a "good" channel, all the three channels to which it broadcasts have to be good simultaneously. Clearly, the probability of occurrence of this event decreases as the number of receivers of a multicast session increases. Therefore, the average received multicast rate of the broadcast session is naturally worse under fading. On the other hand, the average received rate under multicast session 2 and under the unicast from transmitter 3 is still better under fading due to the opportunistic nature of the optimal policy.

In Fig. 3.6 and 3.7 the proportionally fair rates of each multicast session are plot-

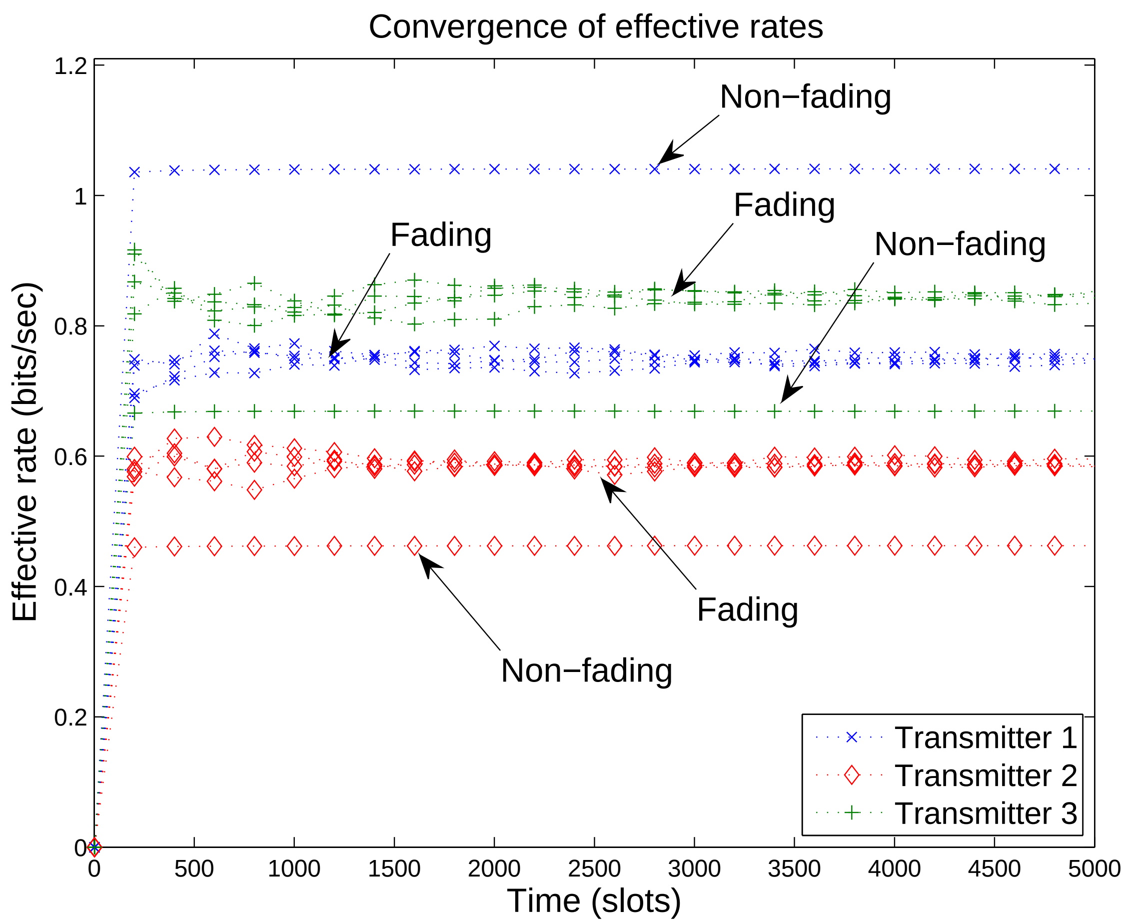

Figure 3.5: Convergence of the utility optimal policy $\pi^\star$ for multicast traffic.

ted as a function of the cross-link coefficient β under a non-fading and a fading channel model. As in the unicast case, we plot both the proposed optimal rate control policy, as well as the TDMA scheduling policy. From Fig. 3.6 we observe that the only session that enjoys a higher rate under the optimal rate control policy is the unicast session. For the broadcast transmission from transmitter 1 and the multicast transmission from transmitter 2, no benefits are observed under concurrent operation of the transmitters, even for small values of the parameter β. The reason behind this observation is the multi-packet reception capabilities of the receivers in adjunction with the fact that the quality of the direct links is fixed and equal to 0.9 in this numerical experiment. For example, in the case of broadcasting from transmitter 1, the high quality of the direct link will not only potentially increase the rate of the broadcast session using this link, but will also cause high interference to the multicast session using receiver 1. The same observation is true for the multicast session. Therefore, regardless of the value of β rate control does not provide any additional gains in terms of rate compared to TDMA for multicast and broadcast.

However, the above discussion is valid only under non-fading channels. As shown in Fig. 3.7 the statistical gains observed by allowing more multicast sessions to operate concurrently makes a TDMA based scheduling suboptimal in the presence of fading.

3.6 Summary

In this chapter, we obtained a joint rate and power control policy that allocates the transmission rates and powers to each multicast transmitter optimally so that the total utility of the average rate at each receiver is maximized. We considered policies that have

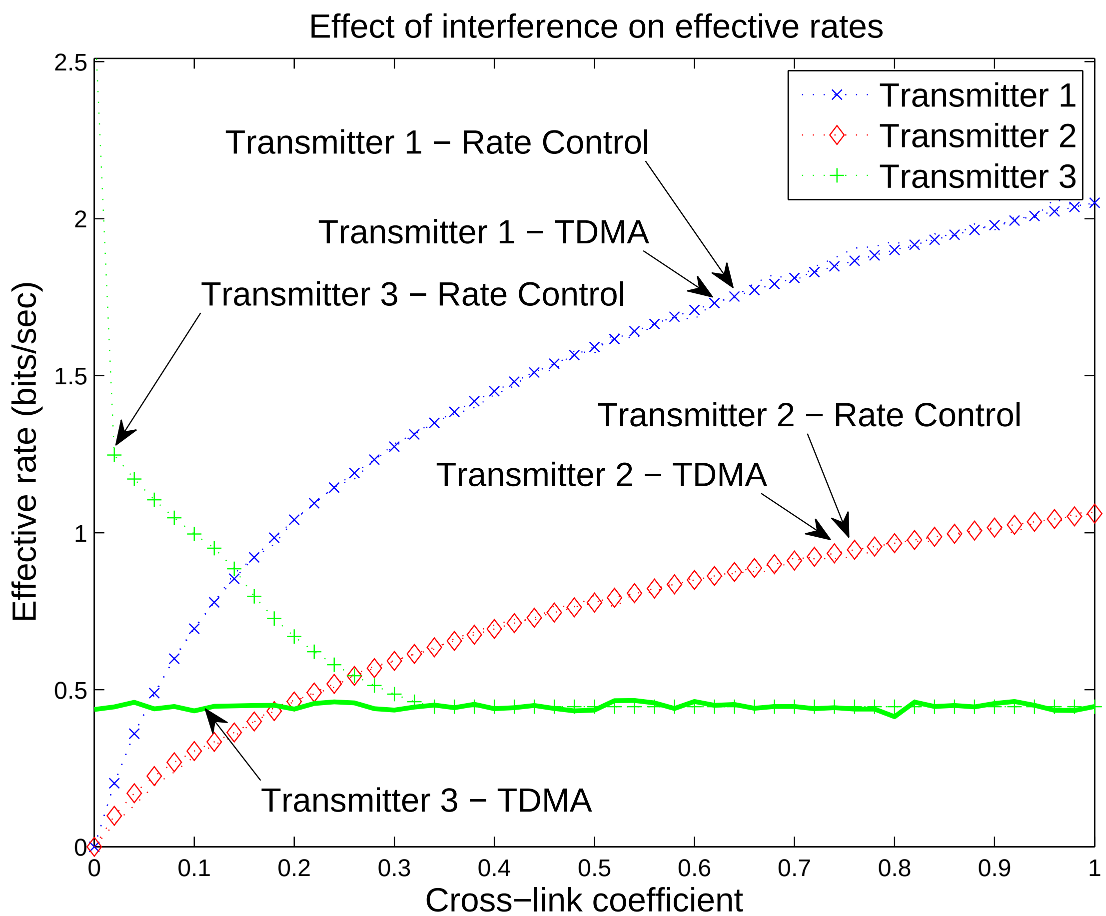

Figure 3.6: Proportional fair rates with increasing β in the absence of fading.

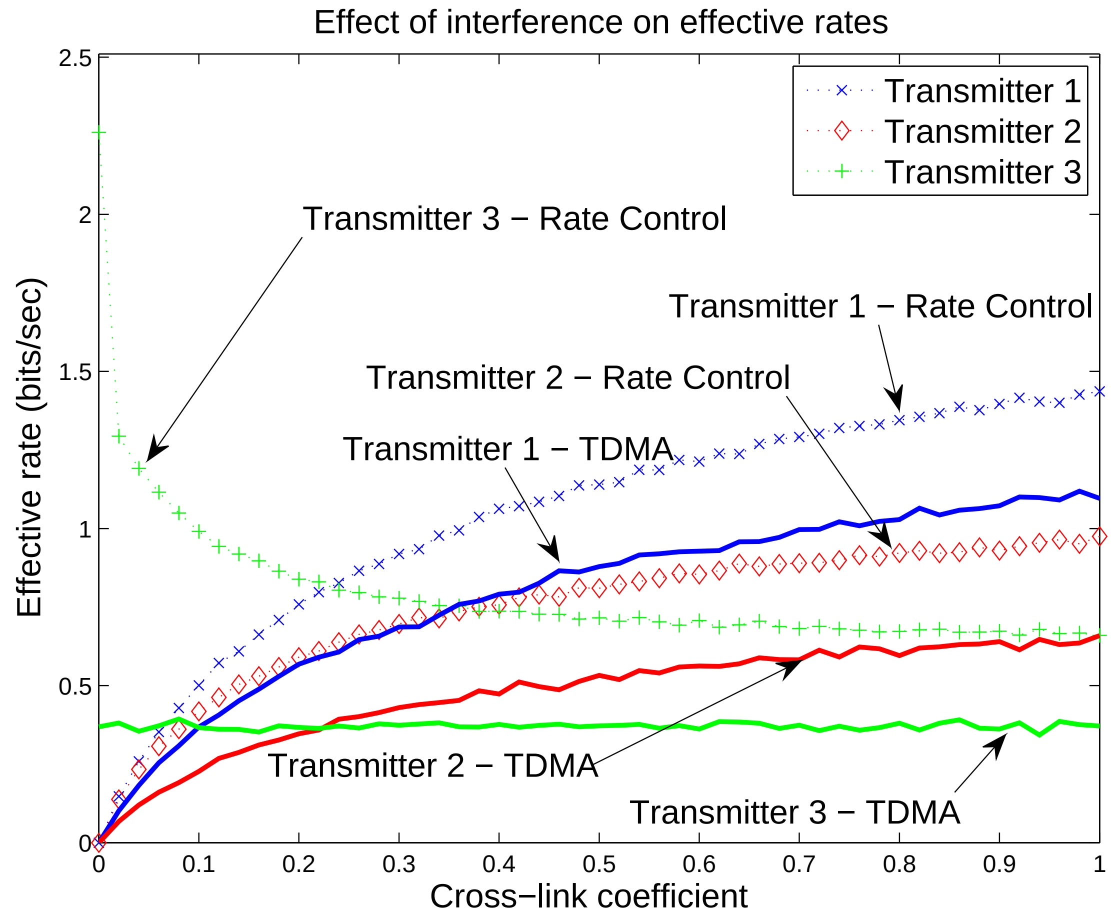

Figure 3.7: Proportional fair rates with increasing β under Rayleigh fading.

access to only an estimate of the channel state, that can be possibly inaccurate.

In Section 3.2, we presented the network model under consideration and we defined the utility maximization problem. In Section 3.3 we introduced an online gradient-based algorithm claimed to be optimal with respect to the utility maximization problem of Subsection 3.2.2. The main results of optimality of the proposed solution were presented in Section 3.4. In Section 3.5 we presented numerical examples that complement our analytical results by providing engineering insights on the optimal scheduling solution. In particular, we confirmed that the average received rate under unicast can be higher under a fading environment than in a non-fading one with path loss equal to the average path loss under fading. This improvement is decreasing for multicast traffic as the number of multicast receivers increases. Further, our numerical results showed that under unicast traffic the optimal solution achieves strictly higher rates than a pure TDMA scheduling policy in the low interference regime, and as interference increases our approach remains at least as good. The optimality of the algorithm was shown by employing the theory of stochastic approximation. The proofs of our results appear in Section 3.7, 3.8 and 3.9.

3.7 Proof of Theorem 4

We make use of Theorem 2.3, p.258 in [26] stated below:

Lemma 2 *Consider the algorithm given by* (3.12), (3.13) *and* (3.14) *and where the step sizes ϵ_n satisfy:*

$$\sum_{n=0}^{\infty} \epsilon_n = \infty, \quad \epsilon_n \geq 0, \text{ and } \epsilon_n \to 0 \text{ for } n \geq 0; \epsilon_n = 0, \text{ for } \ n < 0. \tag{3.21}$$

Further, assume that the following assumptions are true:

(A.1) The sequence $\{\mathbf{Y}_n^{\boldsymbol{\pi}}\}_{n=0}^{\infty}$ *is uniformly integrable.*

(A.2) There are measurable functions $g_n^{\boldsymbol{\theta}_n,\boldsymbol{\xi}_n}$ *and random variables* β_n *such that*

$$\mathbb{E}_n[\mathbf{Y}_n^{\boldsymbol{\pi}}] = g_n^{\boldsymbol{\theta}_n,\boldsymbol{\xi}_n} + \beta_n. \tag{3.22}$$

(A.3) For each compact set $\Delta \subset \Xi$*, the function* $g_n^{\boldsymbol{\theta},\boldsymbol{\xi}_n}$ *is continuous in* $\boldsymbol{\theta}$ *uniformly in* n *and in* $\boldsymbol{\xi}_n \in \Delta$.

(A.4) For each $\delta > 0$*, there exists a compact set* $A_\delta \subset \Xi$ *such that*

$$\inf_n P[\boldsymbol{\xi}_n \in A_\delta] \geq 1 - \delta. \tag{3.23}$$

(A.5) The following sets $\{g_n^{\boldsymbol{\theta}_n,\boldsymbol{\xi}_n}\}$*,* $\{g_n^{\boldsymbol{\theta},\boldsymbol{\xi}_n}\}$ *for each* $\boldsymbol{\theta}$ *are uniformly integrable.*

(A.6) The following is true:

$$\lim_{n,m\to\infty} \frac{1}{m} \sum_{i=n}^{n+m-1} \mathbb{E}_n[\beta_i] = 0, \tag{3.24}$$

where the limit is in the mean.

(A.7) There is a continuous function $\bar{g}^{\boldsymbol{\theta}}$ *such that for each* $\boldsymbol{\theta}$ *and compact set* Ξ *it is true that*

$$\lim_{n,m\to\infty} \frac{1}{m} \sum_{i=n}^{n+m-1} \mathbb{E}_n[g_i^{\boldsymbol{\theta},\boldsymbol{\xi}_i} - \bar{g}^{\boldsymbol{\theta}}]1\{\boldsymbol{\xi}_n \in \Xi\} = 0, \tag{3.25}$$

where $1\{\cdot\}$ *is the indicator function and where the limit is in the mean.*

(A.8) The decreasing sequence ϵ_n *changes slowly in the sense that there is a sequence of integers* $\lambda_n \to \infty$ *such that*

$$\limsup_{n\to\infty,\, 0\leq i\leq\lambda_n} \left|\frac{\epsilon_{n+i}}{\epsilon_n} - 1\right| = 0. \tag{3.26}$$

Then for every subsequence of $\boldsymbol{\theta}^n(\cdot)$ there is a further subsequence, which will be indexed by n_k and a process $\boldsymbol{\theta}(\cdot)$ such that $\boldsymbol{\theta}^{n_k}(\cdot) \Rightarrow \boldsymbol{\theta}(\cdot)$ (in distribution), where

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}(0) + \int_0^t \bar{g}^{\boldsymbol{\theta}(s)} ds.$$

For any $\delta > 0$, the fraction of time that $\boldsymbol{\theta}^n(\cdot)$ spends in a δ-neighborhood of L_H on $[0, \tau]$ goes to one (in probability) as $n \to \infty$ and $\tau \to \infty$, where L_H is the set of limit points of the ODE $\dot{\boldsymbol{\theta}} = \bar{g}^{\boldsymbol{\theta}}$.

The proof of Theorem 4 is readily obtained by verifying that the conditions of Lemma 2 are satisfied under Assumptions 1 - 2. First, note that the required conditions regarding the step size ϵ_n in (3.21) are satisfied by our choice of step size given in (3.14). As we mentioned previously since the bandwidth of the communication is finite and the power vectors are chosen from a compact set $\mathcal{P}$, the achievable rate region $\mathcal{R}(\mathbf{g})$ for every $\mathbf{g} \in \mathcal{G}$ is compact. Hence, for every slot n it follows that both the transmission rate allocation $\mathbf{R}_n^{\boldsymbol{\pi}^\star, \boldsymbol{\theta}_{n-1}^{\boldsymbol{\pi}^\star}, \hat{\mathbf{G}}_n}$ of policy $\boldsymbol{\pi}^\star$, as well as the effective rate $\boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star}$, are bounded almost surely. Moreover, from (3.13), the sequence $\{\mathbf{Y}_n^{\boldsymbol{\pi}^\star}\}_{n=0}^{\infty}$ is almost surely bounded and hence uniformly integrable, i.e.,

$$\sup_n \mathbb{E}[\mathbf{Y}_n^{\boldsymbol{\pi}^\star} 1\{\mathbf{Y}_n^{\boldsymbol{\pi}^\star} > c\}] \to 0, \quad \text{as } c \to \infty.$$

Thus condition $(A.1)$ of the Lemma 2 is satisfied. In addition, from (3.13) it follows that

$$\begin{aligned}\mathbb{E}[\mathbf{Y}_n^{\boldsymbol{\pi}^\star} | \boldsymbol{\theta}_0^{\boldsymbol{\pi}^\star}, \boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}] &= \mathbb{E}[{\mathbf{r}_{n+1}^{\boldsymbol{\pi}^\star}}^\top \mathbf{Q}_{n+1}^{\boldsymbol{\pi}^\star} - \boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star} | \boldsymbol{\theta}_0^{\boldsymbol{\pi}^\star}, \boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}] \\ &= \mathbb{E}[{\mathbf{r}_{n+1}^{\boldsymbol{\pi}^\star}}^\top \mathbf{Q}_{n+1}^{\boldsymbol{\pi}^\star} | \boldsymbol{\theta}_0^{\boldsymbol{\pi}^\star}, \boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}] - \boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star}.\end{aligned}$$

Hence, by choosing the function the $g_n^{\boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star},\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}}$ as $g_n^{\boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star},\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}} = \mathbb{E}[{\mathbf{r}_{n+1}^{\boldsymbol{\pi}^\star}}^\top \mathbf{Q}_{n+1}^{\boldsymbol{\pi}^\star} | \boldsymbol{\theta}_0^{\boldsymbol{\pi}^\star}, \boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}] - \boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star}$ and the random variables β_n as $\beta_n = 0$, for every n, $(A.2)$ is satisfied. From Assumption 1, $(A.3)$ follows. Condition $(A.7)$ follows from Assumption 2. Condition $(A.4)$ follows trivially from the fact that Ξ is a compact set and therefore every subset of Ξ is compact as well. Since $\{\mathbf{Y}_n^{\boldsymbol{\pi}^\star}\}_{n=0}^{\infty}$ is uniformly integrable, $(A2.5)$ is satisfied from the definition of $g_n^{\boldsymbol{\theta}_n^{\boldsymbol{\pi}^\star},\boldsymbol{\xi}_n^{\boldsymbol{\pi}^\star}}$. Also, since $\beta_n = 0$ for every n, $(A.6)$ trivially follows. Finally, for ϵ_n given by (3.14), it is easy to verify $(A.8)$.

Hence, since the conditions $(A.1)$-$(A.8)$ and (3.21) are all satisfied we conclude that $\boldsymbol{\theta}^{n,\boldsymbol{\pi}^\star}$ converges in distribution to the set of limit points of the ODE given in (3.20).

■

3.8 Proof of Theorem 5

We need to show that the ODE of (3.20) has a unique limit point $\boldsymbol{\theta}^\star$ irrespective of the initial conditions, where $\boldsymbol{\theta}^\star$ is the solution of (3.10) and hence the process $\boldsymbol{\theta}^n(t)$ converges to $\boldsymbol{\theta}^\star$ as $n \to \infty$.

From (3.16), it follows that $\mathbf{R}_{n+1}^{\boldsymbol{\pi}^\star,\boldsymbol{\theta},\mathbf{g}} \in \mathcal{M}^{\boldsymbol{\theta},\mathbf{g}} \subseteq \mathcal{R}(\mathbf{g})$, $\forall \boldsymbol{\theta}$. Let us define the set $\mathcal{M}^{\boldsymbol{\theta}}$ for some $\boldsymbol{\theta}$ according to

$$\mathcal{M}^{\boldsymbol{\theta}} = \left\{ \tilde{\mathbf{r}} : \tilde{\mathbf{r}} = \arg\max_{\bar{\mathbf{r}} \in \bar{\mathcal{R}}} \left\{ \sum_{k \in \mathcal{T}} \sum_{d \in \mathcal{D}(k)} \bar{r}(k) \frac{\partial U(\theta(k))}{\partial \theta(k)} \right\} \right\}, \tag{3.27}$$

where $\bar{\mathcal{R}}$ is given by (3.9) in the case of SPR capable receivers and is defined similary in the MPR case.

Lemma 3 *The following is true:*

$$\bar{\mathbf{R}}^{\boldsymbol{\pi}^\star} := \mathbb{E}[{\mathbf{r}_{n+1}^{\boldsymbol{\pi}^\star}}^\top \mathbf{Q}_{n+1}^{\boldsymbol{\pi}^\star}] \in \mathcal{M}^{\boldsymbol{\theta}}, \tag{3.28}$$

where $\mathcal{M}^{\boldsymbol{\theta}}$ is given by (3.27).

The proof is presented in Section 3.9. For $\boldsymbol{\theta}^\star$ to be an limit point we need to have that $\boldsymbol{\theta}^\star \in \bar{\mathcal{R}}$ and $\dot{\boldsymbol{\theta}} = \mathbf{0}$, i.e., $\mathbb{E}[{\mathbf{r}_n^{\boldsymbol{\pi}^\star}}^\top \mathbf{Q}_n^{\boldsymbol{\pi}^\star}] = \boldsymbol{\theta}^\star$. Since $\boldsymbol{\theta}^\star \in \mathcal{M}^{\boldsymbol{\theta}^\star}$ from (3.27) it follows that for every $\boldsymbol{\theta} \in \bar{\mathcal{R}}$

$$\sum_{k \in \mathcal{T}} |\mathcal{D}(k)| \frac{\partial U(\theta^\star(k))}{\partial \theta^\star(k)} \big[\theta(k) - \theta^\star(k)\big] \le 0,$$

which from Proposition 2.1.2 of [29] implies that $\boldsymbol{\theta}^\star$ maximizes the utility problem defined in (3.10).

Further, to show that $\boldsymbol{\theta}^\star$ is a stable equilibrium point to which the ODE converges we use Lyapunov stability criteria. We will use the utility function $U(\cdot)$ as a Lyapunov function. We then have

$$
\begin{aligned}
\frac{d}{dt}U(\boldsymbol{\theta}_t) &= \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \frac{\partial U(\theta_t(k))}{\partial \theta_t(k)} \dot{\theta}_t(k) \\
&= \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \frac{\partial U(\theta_t(k))}{\partial \theta_t(k)} \left[\bar{R}^{\boldsymbol{\pi}}(k) - \theta_t(k) \right] \\
&= \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \frac{\partial U(\theta_t(k))}{\partial \theta_t(k)} \times \arg\max_{\bar{\mathbf{r}}\in\bar{\mathcal{R}}} \left(\sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \bar{r}(k) \frac{\partial U(\theta_t(k))}{\partial \theta_t(k)} \right) \\
&\quad - \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \frac{\partial U(\theta_t(k))}{\partial \theta_t(k)} \theta_t(k) \\
&= \max_{\bar{\mathbf{r}}\in\bar{R}} \left(\sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \frac{\partial U(\theta_t(k))}{\partial \theta_t(k)} \bar{r}(k) \right) \\
&\quad - \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \frac{\partial U(\theta_t(k))}{\partial \theta_t(k)} \theta_t(k) \geq 0, \quad \forall \boldsymbol{\theta}_t \in \bar{\mathcal{R}}.
\end{aligned}
$$

Therefore the utility $U(\boldsymbol{\theta}_t)$ is a Lyapunov function for the ODE since it is strictly increasing with time t unless the equilibrium point $\boldsymbol{\theta}^\star$ is reached. In such a case, i.e., when $\boldsymbol{\theta}_t = \boldsymbol{\theta}^\star$, the above inequality holds with equality proving that the ODE defined in (3.20) converges to $\boldsymbol{\theta}^\star$. This completes the proof.

■

3.9 Proof of Lemma 3

We have the following:

$$
\begin{aligned}
\bar{\mathbf{R}}^{\boldsymbol{\pi}^\star}_{n+1} &:= \mathbb{E}[{\mathbf{r}^{\boldsymbol{\pi}^\star}_{n+1}}^\top \mathbf{Q}^{\boldsymbol{\pi}^\star}_{n+1}] \\
&= \mathbb{E}[\mathbb{E}[{\mathbf{r}^{\boldsymbol{\pi}^\star}_{n+1}}^\top \mathbf{Q}^{\boldsymbol{\pi}^\star}_{n+1} | \hat{\mathbf{G}}_{n+1} = \mathbf{g}]] \\
&= \mathbb{E}[{\mathbf{r}^{\boldsymbol{\pi}^\star}_{n+1}}^\top \bar{\mathbf{Q}}^{\boldsymbol{\pi}^\star}_{\mathbf{g}}] \\
&= \int_{\mathcal{G}} {\mathbf{r}^{\boldsymbol{\pi}^\star}_{n+1}}^\top \bar{\mathbf{Q}}^{\boldsymbol{\pi}^\star}_{\mathbf{g}} f_{\hat{\mathbf{G}}}(\mathbf{g}) d\mathbf{g}.
\end{aligned}
$$

Therefore, by the definition of $\bar{\mathcal{R}}$, it follows that $\bar{\mathbf{R}}^{\boldsymbol{\pi}^\star}_{n+1} \in \bar{\mathcal{R}}$. As a result we have

$$\sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \bar{R}^{\boldsymbol{\pi}^\star}_{n+1}(k) \frac{\partial U(\theta(k))}{\partial\theta(k)} \leq \max_{\bar{\mathbf{r}}\in\bar{\mathcal{R}}} \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \bar{r}(k) \frac{\partial U(\theta(k))}{\partial\theta(k)}.$$

Further, $\mathbf{R}^{\boldsymbol{\pi}^\star,\theta^\star,\mathbf{g}}_{n} \in \mathcal{M}^{\boldsymbol{\theta}^\star,\mathbf{g}}$, hence for every other policy $\tilde{\boldsymbol{\pi}}$ we have

$$\begin{aligned}
\sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \bar{R}^{\boldsymbol{\pi}^\star}_{n+1}(k) \frac{\partial U(\theta(k))}{\partial\theta(k)} &= \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \int_{\mathcal{G}} r^{\boldsymbol{\pi}^\star}_{n+1}(k) \bar{Q}^{\boldsymbol{\pi}^\star}_{\mathbf{g}}(k,k) f_{\hat{\mathbf{G}}}(\mathbf{g}) d\mathbf{g} \frac{\partial U(\theta(k))}{\partial\theta(k)} \\
&\geq \int_{\mathcal{G}} \sum_{k\in\mathcal{T}} r^{\tilde{\boldsymbol{\pi}}}_{n+1}(k) |\mathcal{D}(k)| \bar{Q}^{\tilde{\boldsymbol{\pi}}}_{\mathbf{g}}(k,k) f_{\hat{\mathbf{G}}}(\mathbf{g}) d\mathbf{g} \frac{\partial U(\theta(k))}{\partial\theta(k)} \\
&\geq \max_{\bar{\mathbf{r}}\in\bar{\mathcal{R}}} \left\{ \sum_{k\in\mathcal{T}} |\mathcal{D}(k)| \bar{r}(k) \frac{\partial U(\theta(k))}{\partial\theta(k)} \right\}.
\end{aligned}$$

This concludes that $\bar{\mathbf{R}}^{\boldsymbol{\pi}^\star} = \mathbb{E}[{\mathbf{r}^{\boldsymbol{\pi}^\star}_{n+1}}^\top \mathbf{Q}^{\boldsymbol{\pi}^\star}_{n+1}] \in \mathcal{M}^{\boldsymbol{\theta}}$.

■

Chapter 4

Minimum-Length Scheduling

4.1 Background

The performance metric of utility optimization studied in Chapters 2 - 3 relies on the fundamental assumption that the average rate is well defined and the average rate region can be characterized. This is also a common assumption when the objective is to maximize the network stability region or the information theoretic capacity region. However, for instance, the unpredictability of the wireless channel or the finite energy of the wireless nodes can lead to non-stationary and non-ergodic channel behavior. For this reason, alternative measures should be investigated to account for the cases of non-ergodic and non-stationary wireless channel processes.

In this chapter we consider an alternative approach, that of minimum-length scheduling. The problem of minimum-length scheduling involves obtaining a sequence of activations of wireless nodes so that a finite amount of data, residing at a subset of the nodes in the network reaches its intended destinations in minimum time. This topic has attracted a lot of attention recently ([4], [5], [6], [7], [8]). It is closely related to the problems of network throughput or stable throughput maximization, since minimizing the time to deliver a fixed amount of data, can be seen as maximizing the effective rate at which data traverses the network. Furthermore, it is a useful alternative metric that characterizes the traffic-carrying capabilities of wireless networks with non-stationary and non-ergodic channel variations, where the commonly used performance criteria of stable throughput and network capacity are not well defined. Although in this chapter we focus on networks with stationary and ergodic channel behavior, we expect our analysis to yield valuable insights regarding the more general case of non-ergodic and non-stationary wireless channels.

In [4], the authors obtain a centralized, polynomial-time algorithm for static networks that finds a schedule of minimum-length satisfying a set of link traffic requirements. However, in [4] modeling of the physical layer is overly simplified as it is assumed that any two links can be successfully activated simultaneously as long as they do not share any common vertices. This simplification relates the minimum-length scheduling problem to the problem of obtaining a maximal matching in a non-bipartite graph [30]. However, due to the broadcast nature of the wireless medium *all* concurrent transmissions can potentially contribute to the total amount of interference at each receiver and make its reception to fail.

In [5], the authors consider the problem of obtaining a schedule of minimum-length under the SINR interference model. They assume that the transmission rates are fixed and each transmitting node selects its transmission power optimally. In [5], the minimum-length scheduling problem is formulated as a linear program [31], that can possibly have a prohibitively large number of variables and thus is hard to solve. In [6] and [7] the authors consider the minimum-length scheduling problem under different sets of optimization parameters. Specifically, they consider the cases where (i) both the transmission powers and rates are fixed, (ii) the transmission powers can be optimized but the transmission rates are fixed, and (iii) the transmission powers are fixed and each transmitter is allowed to choose its rate from a predetermined, finite set of rates, that is common among transmitters. In [6] and [7] the minimum-length scheduling problem is also formulated as a complex linear program, with a relatively small number of constraints and a large number of variables. To address the high complexity, the authors employ the technique of column generation [31], whose running time is faster on the average than that of the original linear program. However, the worst case performance of column generation can be significantly worse than that of the original linear program.

Most of the prior work on the minimum-length scheduling problem focuses on selecting the transmission powers while keeping the transmission rates fixed. Due to the coupling between the physical layer and the medium access control in wireless systems, it is clear that a joint optimization of link activation and rate control will yield a better performance, which is the focus of this chapter. In the first part of this chapter, we consider static networks where the channel effect is due to pure path-loss. We first assume a slotted-time model, and formulate the minimum-length scheduling problem as a shortest path between a given source-destination pair on a Directed Acyclic Graph (DAG). We obtain an optimal joint scheduling and rate control solution that provides a shortest path on a DAG. Although finding a shortest path on a DAG has a polynomial complexity in the number of its vertices and edges, this number in our DAG construction grows exponentially as the size of the network and initial data traffic increase. For this reason, we make the following simplifications. We first map the discrete-time problem to a continuous-time equivalent, where slots are replaced with periods of time. We then reduce the possible scheduling and rate control decisions to include only "one at a time" or "all together" communication and explicitly characterize the optimal solution of this reduced problem. Understanding the behavior of the optimal policy, even for the reduced problem, is significant since it provides valuable intuition about which scheduling and rate control actions are expected to be in the optimal solution, i.e., in the minimum-length schedule. This intuition, for example, can improve the performance of the column generation technique in [6] and [7] by providing the algorithm with those scheduling and rate control actions that are expected to be employed by an optimal policy.

Further, all prior work (see e.g., [4], [5], [6], [7], [8]) studies the minimum-length scheduling problem *only* for wireless networks with time-invariant channel conditions, which is not the case in reality. Thus, in the second part of this chapter, we extend prior work by considering time-varying wireless networks. Our goal in the time-varying network case is to find an optimal policy that minimizes the *expected time* required to deliver all the traffic to its respective destinations. We solve the minimum-length scheduling problem by formulating it as a *stochastic shortest path*, which is a special case of a Markov Decision Process (MDP). We obtain an optimal scheduling and rate control policy through stochastic control methods. For time-invariant channel processes, this model reduces to finding a shortest path on a DAG and methods described in the first part of this chapter are applicable to compute the optimal solution.

The results presented in this chapter differ from [4] since we model the interference more accurately through the SINR interference model. We follow a different approach from [5], [6], and [7] since we formulate the minimum-length scheduling problem as finding a shortest path on a single-source DAG, and we give an optimal graph-theoretic algorithm. Furthermore, we provide an explicit characterization of an optimal policy for a simplified model that is obtained by reducing the set of feasible scheduling and rate control decisions to either communication “one at a time” or “all together”. Our results are different from [5] since we consider joint scheduling and rate control decisions. Finally, we generalize existing work in this subject to time-varying channels.

4.2 Model Formulation

We consider a slotted-time single-hop, wireless network comprising of K transmitter and receiver pairs. Without loss of generality, the slot duration is equal to one second. Each transmitter has a finite amount of data units, e.g., a file to deliver to its corresponding receiver. The objective is to activate the transmitters so that the time to deliver all the traffic to its intended receivers is minimized. The single-hop network assumption, albeit simplifying, is interesting since it captures the fundamental problems that arise due to the interference when multiple nodes attempt to obtain channel access. We denote by $\mathcal{K} = \{1, \ldots, K\}$ the set of all transmitter and receiver pairs in the network. At every time slot, each transmitter $k \in \mathcal{K}$ can either transmit at its maximum transmission power $P_k^{\max}$ or remain silent. We denote the transmission power of the kth transmitter at time slot t by $P_k(t)$, where $P_k(t) \in \{0, P_k^{\max}\}$.

It is assumed that each transmitter k has a *fixed* amount of d_k bits to deliver to its corresponding destination. We denote by $\mathbf{d} = (d_1, \ldots, d_K)$ the vector of initial data traffic at each transmitter. We also denote by $X_k(t)$ the queue size at transmitter k at time slot t and by $\mathbf{X}(t) = (X_1(t), \ldots, X_K(t))$ the corresponding vector of queue sizes at all transmitters in the network. The queue size of each transmitter at time slot 0 is equal to its initial data traffic, i.e., $\mathbf{X}(0) = \mathbf{d}$. The state space of the process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ is denoted by $\mathcal{X}$.

We also consider a channel process $\{\mathbf{G}(t)\}_{t=0}^{\infty}$ that takes values from a finite set $\mathcal{G}$. For every time slot t, the channel state $\mathbf{G}(t) = (G_{(k,j)}(t), \forall k, j \in \mathcal{K})$ gives the channel quality between every transmitter k and receiver j. This model captures the effects of channel variations due to e.g., node mobility, fading, or fixed path loss. It is assumed that the channel follows a block fading model with block length equal to the duration of a time slot. Hence, the channel conditions change *only* at the beginning of each time slot and remain constant throughout the slot duration. The above notions are summarized in Fig. 4.1.

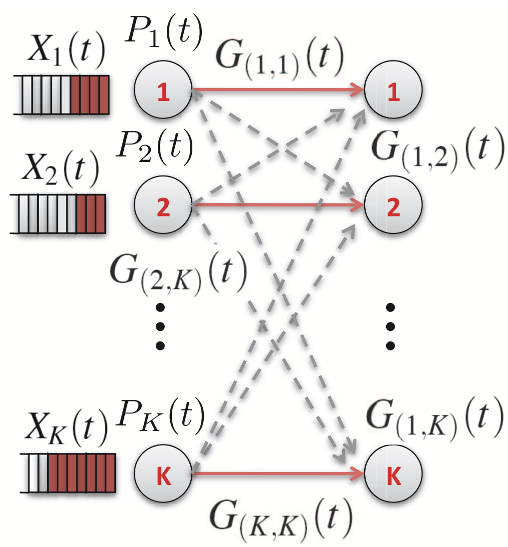

Figure 4.1: A network of K transmitter/receiver pairs.

We model the physical layer by adopting the Signal to Interference plus Noise Ratio (SINR) criterion. We denote by $\gamma_{t,k}(r_k(t))$ the SINR threshold value at receiver k that must be met or exceeded in order to receive successfully from transmitter k at rate $r_k(t)$ at time slot t. Consequently, we say that at slot t transmitter k transmits successfully to receiver k at rate $r_k(t)$ if

$$\text{SINR}_k(t) = \frac{P_k(t)G_{(k,k)}(t)}{N_k + \sum_{j=1,j\neq k}^{K} P_j(t)G_{(j,k)}(t)} \geq \gamma_{t,k}(r_k(t)), \tag{4.1}$$

where N_k is the thermal noise power at receiver k.

A joint scheduling and rate control policy at any given time needs to decide (a) which transmitters to activate and (b) their respective transmission rates. This information can be captured by the K-dimensional rate vector $\mathbf{r}(t) = (r_1(t), \ldots, r_K(t))$, where $r_k(t)$ is the rate of transmitter k at slot t. If a transmitter is assigned a zero rate then it is not activated by the policy. In other words, a transmission rate vector implicitly specifies the scheduling decisions. The set of all feasible rate vectors contains those that are obtained by the following two-step procedure. We first identify all possible subsets of activated transmitters (by assigning to each transmitter k either power 0 or $P_k^{\max}$) and then we assign them the maximum rates that allow all activated transmitters to jointly satisfy the SINR criterion. Thus, there exist $2^K - 1$ such K-dimensional transmission power vectors, each of which corresponds to an achievable rate vector. Clearly, the set of achievable rates depends on the current channel state $\mathbf{g} \in \mathcal{G}$. Hence, for every channel state $\mathbf{g}$, we denote by $\mathcal{R}(\mathbf{g})$ the finite, discrete set of K-dimensional rate vectors. Then, the cardinality of $\mathcal{R}(\mathbf{g})$, i.e., $|\mathcal{R}(\mathbf{g})|$, is equal to $2^K - 1$ for every channel state $\mathbf{g} \in \mathcal{G}$.

In this chapter we are interested to obtain optimal policies that take joint scheduling and rate control decisions under the objective of minimizing the (expected) time to deliver all data to its corresponding destinations. The policies we consider are aware of the network queue-sizes. Further, they are assumed to know the current channel conditions in order to make accurate scheduling decisions. For every slot t the pair of the channel state $\mathbf{G}(t)$ and queue sizes $\mathbf{X}(t)$ comprises the system state $\mathbf{S}(t)$. We denote by $\mathcal{S}$ the state space of the system state process $\{\mathbf{S}(t)\}_{t=0}^{\infty}$ which is given by

$$\mathcal{S} := \{(\mathbf{x}, \mathbf{g}) : \ \ \mathbf{x} \in \mathcal{X}, \ \mathbf{g} \in \mathcal{G}\}. \tag{4.2}$$

We restrict our attention to *stationary* policies that take decisions merely based on the *current system state information*. Let the system state at time slot t satisfy $\mathbf{S}(t) = i = (\mathbf{x}, \mathbf{g}) \in \mathcal{S}$. Then, we consider policies that are given by the mapping

$$\mathbf{r}(t) = \boldsymbol{\pi}(i), \quad \boldsymbol{\pi} : \mathcal{X} \times \mathcal{G} \rightarrow \mathcal{A}(i) \subseteq \mathcal{R}(\mathbf{g}). \tag{4.3}$$

The set $\mathcal{A}(i)$ is a subset of the overall feasible decisions. If it is a strict subset, scheduling will be suboptimal in general at the benefit of decreased complexity. Further, it is possible by "smartly" choosing the elements of the set $\mathcal{A}(i)$ to obtain performance close to optimal while achieving considerable reduction in computational complexity.

We assume that every admissible policy uses the channel state information rationally so that a scheduled transmission is always successful. Naturally, as reflected by the cardinality of the set $\mathcal{R}(\mathbf{g})$, the policies we consider are *non-idling*, i.e., they always activate at least one transmitter that has a non-empty queue until all the queues in the network are empty. Otherwise, an idling policy would potentially waste a slot by not activating any transmissions. We call the class of stationary, non-idling policies given by the mapping (4.3) as *admissible* and denote them by $\mathbf{\Pi}$.

Consider a scheduling and rate control policy $\boldsymbol{\pi}$ that at every slot selects the transmission rates of all the transmitters. Then, the queue size process evolves according to the following equation

$$\mathbf{X}(t+1) = \Big[\mathbf{X}(t) - \boldsymbol{\pi}(t)\Big]^+, \tag{4.4}$$

where $[z]^+ = \max\{z, 0\}$.

Clearly, the queue size at each transmitter k takes its maximum value at time slot 0, when it is equal to the initial demand d_k, and due to the absence of external arrivals it keeps decreasing over time until it reaches zero. Under the above model, we proceed to formulate the minimum length scheduling problem for static and time-varying networks.

4.3 Static Networks

In this section, we restrict our attention to static networks, where the channel qualities $G_{(k,j)}(t)$ are equal for every time slot t, i.e., we ignore effects of fading or user mobility. Thus, the cardinality of the set $\mathcal{G}$ is equal to one. To simplify notation, in this section we denote the channel quality $G_{(k,j)}(t)$ as $G(k,j)$. We will drop this assumption in Section 4.4 where we will consider time-varying channel processes. Further, to simplify notation we denote $\mathcal{R}(\mathbf{g})$ for $\mathbf{g} \in \mathcal{G}$ by $\mathcal{R}$ and $\mathcal{A}(i)$ for $i = (\mathbf{g}, \mathbf{x})$, $\mathbf{g} \in \mathcal{G}$ by $\mathcal{A}$. At every time slot t the scheduling and rate control policy identifies a rate vector $\mathbf{r}(t) = (r_1(t), \ldots, r_K(t)) \in \mathcal{A} \subseteq \mathcal{R}$ that specifies which transmitters are activated at time slot t and their respective rates.

We can formulate the minimum-length scheduling problem as follows:

$$\mathbf{minimize}: \quad T \tag{4.5}$$

$$\mathbf{subject\ to}: \quad \mathbf{X}(T) = 0, \ \mathbf{X}(0) = \mathbf{d}, \tag{4.6}$$

$$T \in \mathbb{N}. \tag{4.7}$$

In the specific case of pure Time Division Multiple Access (TDMA) scheduling, combined with rate control, where only a single transmitter can be active at any given time, the solution of the above problem becomes trivial. Specifically, each transmitter must be active for as many time slots as needed to empty its queue. The required number of such time slots for each transmitter k is equal to the ratio of its initial demand d_k divided by its corresponding rate when it accesses the channel individually. Then, the minimum total time that is needed until all the queues are empty is equal to the sum of the time slots required by each transmitter. The order in which the transmitters must be activated is immaterial; they can be chosen in a round-robbin or random fashion or a single transmitter may keep transmitting until its queue empties, after which time another transmitter with a non-empty queue is chosen.

However, the solution of the optimization problem given by (4.5)-(4.7) is in general a non-trivial discrete optimization problem. In the following subsections, we provide an optimal graph-theoretic algorithm by mapping it to a shortest path problem on a DAG and we also give an explicit characterization of the optimal policy for a reduced version of this problem.

4.3.1 The Equivalent DAG Representation

To solve the optimization problem defined in (4.5)-(4.7), we follow a graph-theoretic approach, and formulate it as a single source shortest path problem on an equally-weighted DAG.

We construct the weighted DAG $G = (V, E)$ as follows: We assume that every vertex $\mathbf{u} \in V$ of the DAG represents a queue-size vector that can be obtained through some scheduling and rate control action chosen from the set $\mathcal{A}$ starting from a vector of queues, $\mathbf{X}(t)$. Further, every directed edge $(\mathbf{u}, \mathbf{v}) \in E$ represents one such action in $\mathcal{A}$. We say that the edge $(\mathbf{u}, \mathbf{v})$ is *incident from* $\mathbf{u}$ and *incident to* $\mathbf{v}$. Hence, from every vertex $\mathbf{x}_i$ we can have $|\mathcal{A}|$ edges that are incident from $\mathbf{x}_i$, each corresponding to a different rate vector $\mathbf{r}^i, i = 1, \ldots, |\mathcal{R}|$. Each such edge is incident to a node $\mathbf{y}_i = [\mathbf{x}_i - \mathbf{r}^i]^+$. We disallow those edges that correspond to rate vectors, which activate transmitters with empty queues. Therefore, the actual number of edges that are incident from a vertex can be less than $|\mathcal{A}|$. The weight of each edge is equal to one. From now on, we will refer to action $\mathbf{r}^i$ through the edge $(\mathbf{x}_i, \mathbf{y}_i)$. The unique source node $\mathbf{s}$ of the DAG represents the vector of initial demands, $\mathbf{X}(0)$.

In Fig. 4.2 we give an example of such a graph for a network of two transmitters and two receivers. We assume that the initial demands are $d_1 = 4$ bits and $d_2 = 6$ bits and that we have three possible scheduling and rate control actions: (i) only transmitter 1 accesses the channel at a rate of 3 bits/sec, (ii) only transmitter 2 accesses the channel at a rate of 3 bits/sec, and (iii) both transmitters concurrently transmit at a rate of 2 bits/sec. Fig. 4.2 depicts the DAG that is obtained by these three actions. Note that from each vertex all the three rate control actions are allowed, as long as each action schedules transmitters with non-empty queues. For example, in Fig. 4.2 the only viable rate control action for the queue-size vector $[4, 0]$ is to activate transmitter 1 individually.

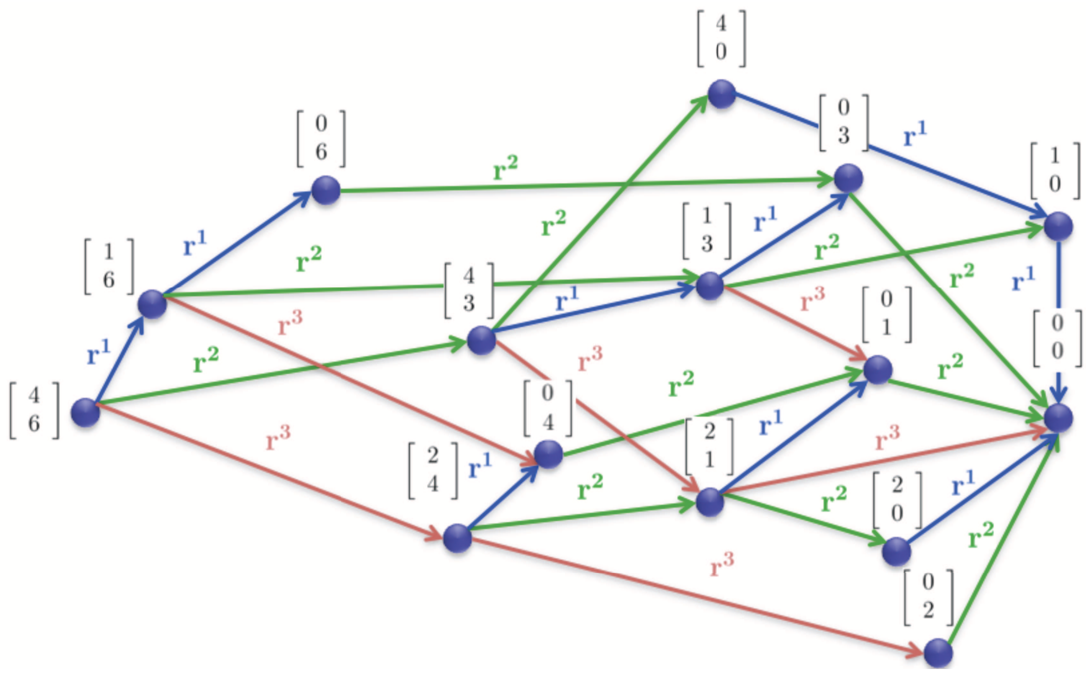

Figure 4.2: A DAG construction corresponding to initial demands $d_1 = 4$ bits and $d_2 = 6$ bits and three rate control actions $\mathbf{r}^1 = [3, 0]$, $\mathbf{r}^2 = [0, 3]$, and $\mathbf{r}^3 = [2, 2]$.

As we observe from Fig. 4.2 for any path of vertices $< \mathbf{s}, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m >$ the queue-size vector of each vertex in the path has to be component-wise larger or equal to the queue size of any other vertex that succeeds it in the path and the queue-size vectors of any two vertices on the graph cannot be the same. As a result, the overall graph representing the queue size dynamics is a DAG. Further, it is clear that every path starting at the source $\mathbf{s}$ ends at the $\mathbf{0}$-vector. Moreover, the weight of any sub-path $< \mathbf{s}, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m >$ is equal to its length m, which is effectively the number of time slots to go from $\mathbf{s}$ to $\mathbf{x}_m$ on the specified path, as each weight of the DAG represents the duration of one time slot.

Thus, the initial problem given by (4.5)-(4.7) is transformed into a single-source shortest path problem on a weighted DAG.

4.3.2 Finding A Shortest Path on a DAG

Shortest path problems on single-source DAGs can be solved optimally in polynomial time [30]. In [30] an optimal algorithm that finds a shortest path on a DAG is presented. Below, for the purpose of completeness, we provide this algorithm.

In order to compute a shortest path, we first need to sort the DAG in *topological order* and then use a sequence of edge *relaxations* until we obtain a shortest path from the source $\mathbf{s}$ to the vertex corresponding to the $\mathbf{0}$-vector. Topological order is a linear ordering of all the vertices of the DAG so that for every edge $(\mathbf{x}_i, \mathbf{x}_j)$, the vertex $\mathbf{x}_i$ appears before $\mathbf{x}_j$ in the ordering. The process of edge relaxation verifies whether the current best-known path from the source $\mathbf{s}$ to a vertex $\mathbf{y}$ can be improved by passing through a different vertex $\mathbf{x}$.

We proceed with a few definitions that will be useful in the rest of this subsection. We define the *distance* of a vertex $\mathbf{x}$ to be the minimum distance from the source in terms of edges that must be traversed to reach $\mathbf{x}$. We also denote by $\delta[\mathbf{x}]$ an *upper bound* on the distance of vertex $\mathbf{x}$. For every edge $(\mathbf{x}, \mathbf{y})$ we say that $\mathbf{x}$ is the predecessor of $\mathbf{y}$ and we write $\mathbf{x} = \pi[\mathbf{y}]$. We denote by Adj[$\mathbf{x}$] a list that contains all the vertices $\mathbf{y}$ that are adjacent to $\mathbf{x}$, i.e., such that there exists an edge $(\mathbf{x}, \mathbf{y}) \in E$. The pseudo-code of the algorithm is provided below.

DAG-SHORTEST-PATHS($G, \mathbf{s}$)

1 topologically sort the vertices of G

2 INITIALIZE-SINGLE-SOURCE($G, \mathbf{s}$)

3 **for** each vertex $\mathbf{x}$ taken in topologically sorted order

4   **do for** each vertex $\mathbf{y} \in \text{Adj}[\mathbf{x}]$

5   **do** RELAX($\mathbf{x}, \mathbf{y}$)

The topological sorting of the first line of the algorithm can be completed in $\Theta(|V|+|E|)$ time, by running a Depth-First Search (DFS) [30]. The second line of the algorithm involves the initialization of various variables as shown next:

INITIALIZE-SINGLE-SOURCE($G, \mathbf{s}$)

for each vertex $\mathbf{x} \in V$

    do $\delta[\mathbf{x}] \leftarrow \infty$

        $\pi[\mathbf{x}] \leftarrow NIL$

$\delta[\mathbf{s}] \leftarrow 0$

This process requires time of the order of $\Theta(|V|)$. Finally, in lines 3-5 of the DAG-SHORTEST-PATHS($G, \mathbf{s}$) algorithm, at each time step the next vertex in the topological order is selected and a sequence of relaxations over all edges that are incident from this vertex is performed. The procedure RELAX($\mathbf{x}, \mathbf{y}$), given next, verifies whether the current shortest path from $\mathbf{s}$ to $\mathbf{y}$ can be improved by passing through $\mathbf{x}$.

RELAX($\mathbf{x}, \mathbf{y}$)

if $\delta[\mathbf{y}] > \delta[\mathbf{x}] + 1$

    then $\delta[\mathbf{y}] \leftarrow \delta[\mathbf{x}] + 1$

$\pi[\mathbf{y}] \leftarrow \mathbf{x}$

For the example given in Fig. 4.2, the shortest path algorithm selects the sequence of actions $\mathbf{r}^3, \mathbf{r}^3, \mathbf{r}^2$. Note that the sequences of actions $\mathbf{r}^2, \mathbf{r}^3, \mathbf{r}^3$ and $\mathbf{r}^3, \mathbf{r}^2, \mathbf{r}^3$ are also optimal as the order in which the actions are taken is immaterial in terms of minimizing the time needed to empty the queues, under the assumption of static channels. Also, it is worth to mention that the length of the optimal schedule obtained through rate control is naturally shorter than that of TDMA which is, in this example, of length 4. Further, it is reasonable to expect that the difference between the two would become significant as the number of transmitter/receiver pairs in the network increases as well as for larger values of initial demands.

The optimality of DAG-SHORTEST-PATHS(G, s) can easily be verified (see e.g., [30], Theorem 24.5). Also, it is easy to see that its overall running time is $\Theta(|V| + |E|)$. Hence, the number of operations needed to compute a shortest path of a single-source DAG is of polynomial complexity on the number of vertices and edges. However, in our DAG construction this number grows exponentially (i) in the number of transmitters when $\mathcal{A} = \mathcal{R}$ since from every vertex there exist $2^K - 1$ potential edges that are incident from it and (ii) as the initial demands increase. The above render the overall complexity of the algorithm exponential.

4.3.3 Continuous Time Model

As it is clear from the above discussion, the DAG solution becomes infeasible quickly as the number of transmitters and initial demands increase. In this subsection, to decrease the complexity that stems from the discrete nature of this problem, we map the problem given by (4.5)-(4.7) to a continuous time one. Therefore, instead of seeking for the minimum number of time slots required to deliver all data traffic to its respective destinations, we are interested to obtain the minimum "duration" or "period of time" that has to elapse until all network queues empty. In this way, the minimum length scheduling problem becomes a linear program with a relatively small number of constraints and a large number of variables as in the formulations of [5], [6], and [7]. In order to solve this linear program, we follow a different approach than [5], [6], and [7]. In particular, we reduce the number of variables involved, i.e., the scheduling and rate control decisions that the policy employs, and then obtain an optimal solution for this reduced problem.

Specifically, we restrict the set $\mathcal{A}$ to contain feasible rate vectors obtained by two simple schemes, namely scheduling a *single* transmitter at a time or concurrently activating *all* the transmitters, as considered in [14], [23]. By doing so, we decrease the cardinality of $\mathcal{A}$ to $K+1$. Although such a reduction is expected to be suboptimal, we anticipate to gain valuable insights regarding the nature of optimal scheduling and rate control for the general problem.

We define Action k for $k \in \mathcal{K}$ to be the action of individually activating transmitter k and Action 0 to be the corresponding action when all K transmitters are activated simultaneously. Let the rate of transmitter k under individual operation be r_k^k and the corresponding rate under concurrent operation be r_k^0. Further, let us denote by τ_i for $i \in \{0, \ldots, K\}$ the period of time that Action i is utilized. Then, the continuous time equivalent of (4.5)-(4.7) under the reduced space of actions is:

$$\textbf{minimize}: \quad \sum_{i=0}^{K} \tau_i \tag{4.8}$$

$$\textbf{subject to}: \quad d_k \le \tau_k r_k^k + \tau_0 r_k^0, \qquad \forall k \in \mathcal{K} \tag{4.9}$$

$$\tau_i \ge 0, \qquad i \in \{0, \ldots, K\} \tag{4.10}$$

The following theorem characterizes an optimal scheduling and rate control policy that solves (4.8)-(4.10).

Theorem 6 *A minimum-length scheduling and rate control policy solving* (4.8)-(4.10) *takes actions according to the following:*

1. *If it is true that*

$$\sum_{k=1}^{K} \frac{r_k^0}{r_k^k} \le 1,$$

for every $k \in \mathcal{K}$ Action k is chosen for a duration of

$$\tau_k = \frac{d_k}{r_k^k},$$

and Action 0 is never employed, i.e.,

$$\tau_0 = 0.$$

2. *If it is true that*

$$\sum_{k=1}^{K} \frac{r_k^0}{r_k^k} \geq 1,$$

then a subset of transmitters $\mathcal{J}$ *is chosen such that for every* $k \in \mathcal{J}$ *Action* k *is chosen for a duration of*

$$\tau_k = \frac{d_k - \tau_0 r_k^0}{r_k^k},$$

and Action 0 *is selected for a period of*

$$\tau_0 = \max_{i \in \mathcal{K} \setminus \mathcal{J}} \frac{d_i}{r_i^0}.$$

The proof appears in Section 4.6. To completely characterize the policy we need to specify the set $\mathcal{J}$. The following result is true:

Lemma 4 *Consider an ordering of the transmitters in decreasing order of their values* d_k/r_k^0 *for every* $k \in \mathcal{K}$*. Let the corresponding indexing of the transmitters be* $\{\ell_k\}_{k=1}^K$ *such that* $d_{\ell_1}/r_{\ell_1}^0 \geq \ldots \geq d_{\ell_K}/r_{\ell_K}^0$*. Then, the set* $\mathcal{J}$ *contains those transmitters with the highest* d_k/r_k^0 *ratios and the cardinality* $|\mathcal{J}|$ *of the set* $\mathcal{J}$ *is given by*

$$|\mathcal{J}| = \operatorname*{arg\,min}_{k \in \{0,\ldots,K\}} \left\{ \frac{d_{\ell_{k+1}}}{r_{\ell^0_{k+1}}} + \sum_{j=\ell_1}^{\ell_k} \frac{d_j r^0_{\ell_{k+1}} - d_{\ell_{k+1}} r_j^0}{r_j^j r^0_{\ell_{k+1}}} \right\}.$$

The proof of the lemma appears in Section 4.7.

From the above we conclude that the set $\mathcal{J}$ contains the transmitters with the highest $|\mathcal{J}|$ values of d_k/r_k^0, where $|\mathcal{J}|$ is given by Lemma 4. Hence, an optimal scheduling and rate control policy individually activates the transmitters that either have a very high initial demand or whose rates under concurrent operation are very low, e.g., due to excessive amounts of interference caused by other concurrent transmissions. Those transmitters must be further assisted towards emptying their queues by being granted individual access to the channel.

4.4 Time-Varying Networks

In the previous section, we focused on time-invariant channels. However, the wireless channel is actually time-varying, due to fading, node mobility etc. In this section, we extend our model by considering time-varying channels. We make the following assumption on the wireless channel process $\{\mathbf{G}(t)\}_{t=0}^{\infty}$.

Assumption 3 *The channel process* $\{\mathbf{G}(t)\}_{t=0}^{\infty}$ *varies according to a stationary Markov Chain with transition probability to go from some channel state* $\mathbf{g} \in \mathcal{G}$ *to another channel state* $\mathbf{g}' \in \mathcal{G}$ *given by*

$$p_{\mathbf{G}}(\mathbf{g}, \mathbf{g}') := P[\mathbf{G}(t+1) = \mathbf{g}' \mid \mathbf{G}(t) = \mathbf{g}], \quad \forall \mathbf{g},\ \mathbf{g}' \in \mathcal{G}. \tag{4.11}$$

Due to the time variability of the channel process the length of the schedule T is a random variable and thus "minimum-length" is meant "in the expected sense". This can be formulated as follows:

$$\textbf{minimize}: \quad \mathbb{E}[T] \tag{4.12}$$

$$\textbf{subject to}: \quad \mathbf{X}(T) = 0,\ \mathbf{X}(0) = \mathbf{d}, \tag{4.13}$$

$$T \in \mathbb{N}. \tag{4.14}$$

We proceed to present a solution to the problem of (4.12)-(4.14) through stochastic control methods by considering admissible policies in the class $\mathbf{\Pi}$.

4.4.1 Stochastic Shortest Path Formulation

Since the wireless channel process $\{\mathbf{G}(t)\}_{t=0}^{\infty}$ is Markov and the queue size process evolves according to (4.4), for every admissible policy, it is easy to show that the system process $\{\mathbf{S}(t)\}_{t=0}^{\infty}$ is also a Markov Chain, with state space $\mathcal{S}$ given by (4.2). We further define a subset $\mathcal{S}_{\text{term}}$ of the state space $\mathcal{S}$ to be the set of *terminating states* that correspond to empty queues, i.e.,

$$\mathcal{S}_{\text{term}} := \{(\mathbf{x}, \mathbf{g}) : \mathbf{x} = \mathbf{0},\ \mathbf{g} \in \mathcal{G}\}. \tag{4.15}$$

Evidently, from (4.4) it follows that once the system reaches any state in $\mathcal{S}_{\text{term}}$ it remains there forever. The objective is then to reach a terminating state in minimum expected time by choosing the next state. This will yield the schedule of minimum expected-length. Note that, by construction this Markov Chain is absorbing and from every non-terminating state a terminating state is reached with probability one in finite time under all admissible policies. This is a *stochastic shortest path* problem, which is a special case of an MDP. In the case where we assume that there is no randomness in the channel state, i.e., the entire wireless channel state realization is known at priori at the very first time slot, our results in Section 4.3 follow from this model.

The set of feasible scheduling and rate control actions corresponding to each system state $i = (\mathbf{x}, \mathbf{g}) \in \mathcal{S}$ is the set $\mathcal{A}(i) \subseteq \mathcal{R}(\mathbf{g})$. Further, the system is driven by the time-varying channel process $\{\mathbf{G}(t)\}_{t=0}^{\infty}$. Taking an action leads to different states with different probabilities depending on the evolution of the channel process unless the system has already reached to a terminating state.

Let $p_{\mathbf{r}}(i, j)$ be the transition probability of going from system state $i = (\mathbf{x}, \mathbf{g})$ to state $j = (\mathbf{x}', \mathbf{g}')$ by taking action $\mathbf{r} = \boldsymbol{\pi}(\mathbf{x}, \mathbf{g}) \in \mathcal{A}(i)$. Then we have

$$p_{\mathbf{r}}(i, j) = P[\mathbf{X}(t+1) = \mathbf{x}',\ \mathbf{G}(t+1) = \mathbf{g}' \,\big|\, \mathbf{X}(t) = \mathbf{x},\ \mathbf{G}(t) = \mathbf{g},\ \boldsymbol{\pi}(\mathbf{x}, \mathbf{g}) = \mathbf{r}].$$

From (4.4) and Assumption 3 it is easy to see that $p_{\mathbf{r}}(i, j)$ can be written as

$$p_{\mathbf{r}}(i, j) = \begin{cases} p_{\mathbf{G}}(\mathbf{g}, \mathbf{g}'), & \text{if } (\mathbf{x} - \mathbf{r})^{+} = \mathbf{x}',\ i, j \in \mathcal{S} \\ 0, & \text{otherwise.} \end{cases} \tag{4.16}$$

Note that from the Markovianess of the channel process and the admissibility of the policy $\boldsymbol{\pi}$, the transition probability $p_{\mathbf{r}}(i, j)$ is time invariant and does not depend on the previous system states.

We define the cost of taking action $\mathbf{r}$ and going from state i to state j as $\tilde{c}_{\mathbf{r}}(i, j)$. For every system state i, action $\mathbf{r} \in \mathcal{A}(i)$, and system state j such that $p_{\mathbf{r}}(i, j) > 0$, we assume that $\tilde{c}_{\mathbf{r}}(i, j) = 1$. This represents the fact that in order to go from state i to state j by taking this action one needs to spend the duration of one time slot. Let us further define the *cost per stage* $c_{\mathbf{r}}(i)$ to be the expected cost when at state $i \in \mathcal{S} \setminus \mathcal{S}_{\text{term}}$ control

$\mathbf{r} \in \mathcal{A}(i)$ is chosen. It is clear that $c_{\mathbf{r}}(i) = \sum_{j \in \mathcal{S}} p_{\mathbf{r}}(i,j)\tilde{c}_{\mathbf{r}}(i,j) = 1$. Once a terminal state $i \in \mathcal{S}_{\text{term}}$ is reached no more cost is incurred and the system remains there forever, i.e., $c_{\mathbf{r}}(i) = 0, \quad \forall\, \mathbf{r} \in \mathcal{A}(i),\ i \in \mathcal{S}_{\text{term}}$.

4.4.2 An Optimal Policy

Let $\mathcal{T}^{\boldsymbol{\pi}}(i)$ be the expected time to empty the queues in the network starting from state i under a policy $\boldsymbol{\pi} \in \boldsymbol{\Pi}$. Then the minimum expected schedule length $\mathcal{T}^{\star}(i)$ is given by

$$\mathcal{T}^{\star}(i) = \min_{\boldsymbol{\pi} \in \boldsymbol{\Pi}} \mathcal{T}^{\boldsymbol{\pi}}(i), \quad \forall i \in \mathcal{S} \setminus \mathcal{S}_{\text{term}}.$$

A policy $\boldsymbol{\pi}^{\star}$ is optimal if it achieves the minimum $\mathcal{T}^{\star}(i)$ for every non-terminating state $i \in \mathcal{S} \setminus \mathcal{S}_{\text{term}}$, i.e.,

$$\mathcal{T}^{\boldsymbol{\pi}^{\star}}(i) = \mathcal{T}^{\star}(i), \quad \forall i \in \mathcal{S} \setminus \mathcal{S}_{\text{term}}.$$

To optimally solve the above shortest path problem two commonly used methods are policy iteration and value iteration [32]. Due to the large state space of the problem, value iteration is easier to compute and hence will be used here. Consider the value iteration algorithm and the corresponding "expected" time $\mathcal{T}_k(i)$ to empty the queues starting from state i at the kth iteration. Assume that $\mathcal{T}_0(i) = \infty$ for all states $i \in \mathcal{S}$. We borrow the following properties from [32].

Lemma 5 *The value iteration method converges to the optimal cost function, i.e.,*

$$\mathcal{T}^\star(i) = \lim_{k\to\infty} \mathcal{T}_k(i), \quad \forall i \in \mathcal{S} \setminus \mathcal{S}_{\text{term}},$$

where

$$\mathcal{T}_{k+1}(i) = 1 + \min_{\mathbf{r}\in\mathcal{A}(i)} \sum_{j\in\mathcal{S}} p_{\mathbf{r}}(i,j)\mathcal{T}_k(j), \quad i \in \mathcal{S} \setminus \mathcal{S}_{\text{term}}.$$

Lemma 6 *The optimal solution to a stochastic shortest path problem must satisfy Bellman's equation, i.e., for every non-terminating state $i \in \mathcal{S} \setminus \mathcal{S}_{\text{term}}$ it is true that*

$$\mathcal{T}^\star(i) = 1 + \min_{\mathbf{r}\in\mathcal{A}(i)} \left\{ \sum_{j\in\mathcal{S}} p_{\mathbf{r}}(i,j)\mathcal{T}^\star(j) \right\}$$

Hence, the optimal scheduling and rate control policy $\boldsymbol{\pi}^\star$ for every state $i \in \mathcal{S}\setminus\mathcal{S}_{\text{term}}$ is given by

$$\boldsymbol{\pi}^\star(i) = \arg\min_{\mathbf{r}\in\mathcal{A}(i)} \left\{ \sum_{j\in\mathcal{S}} p_{\mathbf{r}}(i,j)\mathcal{T}^\star(j) \right\}, \quad \forall i \in \mathcal{S} \setminus \mathcal{S}_{\text{term}}.$$

Although the value iteration method optimally solves the aforementioned stochastic shortest path problem, in general it may require an infinite number of iterations until it converges. However, if the Markov Chain of the system evolution is acyclic, then it was shown in [32] that the value iteration method for each state converges in a finite number of iterations (at most as many as the non-terminating states of the Markov Chain).

It is easy to see that the Markov Chain driving our system is acyclic. This is because starting from one of the states i whose queue size satisfies $\mathbf{X}(0) = \mathbf{d}$, the queue sizes in the network constantly decrease with time as given in (4.4) under any admissible policy. This ensures that, the Markov Chain is acyclic and terminates at some node $i \in \mathcal{S}_{\text{term}}$.

4.4.3 Numerical Results

In this subsection, we illustrate our analytical results through a few numerical experiments. We consider a network of two transmitter and two receiver pairs. The channel process $\{\mathbf{G}(t)\}_{t=0}^{\infty}$ is Markov, and switches between two states, namely a *good* state, $\mathbf{G}$, and a *bad* state, $\mathbf{B}$. When the channel is in good state, both transmitters have channels of good quality to their receivers otherwise both channels are bad. The transition probabilities of this Markov Chain are shown in Fig. 4.3.

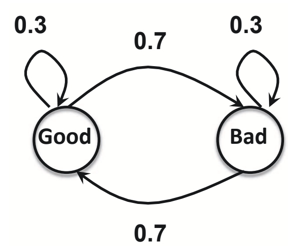

Figure 4.3: A two-state Markovian channel process.

Since we have 2 transmitter/receiver pairs, there exist 3 possible rate vectors corresponding to each channel state. Thus, we denote by $\mathbf{r}^i(\mathbf{g})$, $i = 1, 2$ the rate vector when only the ith transmitter is activated under channel state $\mathbf{g} \in \{\mathbf{B}, \mathbf{G}\}$. We also denote by $\mathbf{r}^3(\mathbf{g})$ the corresponding rate vector when both transmitters are activated.

We first consider that the initial demands are $d_1 = 4$ bits and $d_2 = 6$ bits which is the case discussed in Subsection 4.3.1. We consider 3 scenarios associated to different achievable rates corresponding to different channel states.

- **Scenario 1:** We consider the case that under both channel states, when the ith transmitter is activated alone its achievable rate is 3 bits/slot and when both transmitters are activated simultaneously, the corresponding rates are 2 bits/slot for each. In this case, the channel realization is immaterial and the minimum expected time to empty the queues is 3 slots, i.e., equal to the result of the static network case of Subsection 4.3.1.

- **Scenario 2:** We assume that under good channel state the achievable rates are equal to the case of Scenario 1, i.e., when the ith transmitter is activated alone its achievable rate is 3 bits/slot and when both transmitters are activated simultaneously, the corresponding rates are 2 bits/slot for each. However, under bad channel the achievable rates are strictly worse (2 bits/slot for individual transmission and 1 bit/slot for each transmitter under concurrent transmission). Naturally, we observe that the expected time required to empty the queues is more than 3 slots.

- **Scenario 3:** We assume that under bad channel state the achievable rates are equal to the ones in Scenario 1 but the good channel is better and thus allows higher rates (4 bits/slot when a transmitter is activated individually and 3 bit/slot when they are both activated simultaneously). Naturally, the expected time to empty the queues will decrease to a value less than 3.

The same pattern was observed for higher initial demands ($d_1 = 100$ bits and $d_2 = 100$ bits). The above are shown in Table 4.1 by assuming that the channel starts from a good channel state.

Further, in Fig. 4.4 we illustrate the performance comparison between the optimal

Demands	Good Channel			Bad Channel			$\mathbb{E}[T]$
	$\mathbf{r}^1(\mathbf{G})$	$\mathbf{r}^2(\mathbf{G})$	$\mathbf{r}^3(\mathbf{G})$	$\mathbf{r}^1(\mathbf{B})$	$\mathbf{r}^2(\mathbf{B})$	$\mathbf{r}^3(\mathbf{B})$	
$\begin{bmatrix} 4 \\ 6 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	3.00
	$\begin{bmatrix} 3 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	3.86
	$\begin{bmatrix} 4 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	2.91
$\begin{bmatrix} 100 \\ 100 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	50.00
	$\begin{bmatrix} 3 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	66.95
	$\begin{bmatrix} 4 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	40.37

Table 4.1: Expected time required to empty queues for different values of initial demands, under Scenarios 1-3, assuming that the channel starts from a good state.

policy and a pure TDMA scheme that activates only a single transmitter at any given time. Specifically, we consider the same single-hop network of two transmitter/receiver pairs discussed above under Scenario 2. Further, we vary the values of initial data traffic. For simplicity the initial queue sizes at each node are assumed to be equal. As expected, we observe from the figure that the difference between the expected time to empty the queues under the optimal policy and under the TDMA scheme grows as the initial queue sizes increase. This result illustrates the fact that employing concurrent transmissions can provide considerable gains.

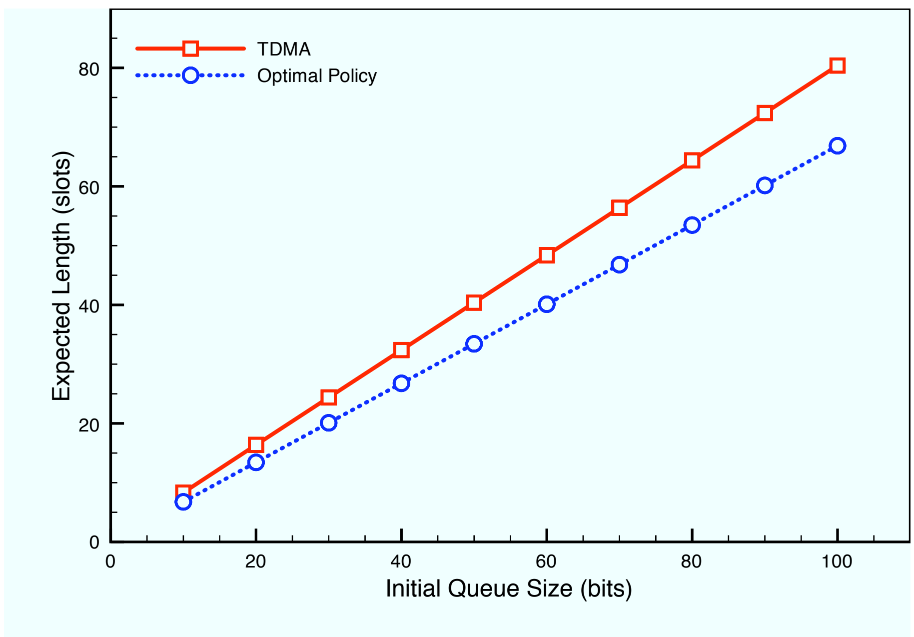

Figure 4.4: Performance comparison of the optimal policy with respect to TDMA scheduling.

4.5 Summary

In this chapter we focused on the problem of joint scheduling and rate control in single-hop wireless networks under the objective of minimizing the required time to deliver all data traffic to its respective destinations.

In Section 4.2 we presented the network model. In the first part of this chapter, i.e., in Section 4.3, we considered networks with time-invariant links. Under this assumption, in Subsection 4.3.1 we presented a graph-theoretic formulation for the minimum-length scheduling problem. An optimal algorithm was given in Subsection 4.3.2. Motivated by the combinatorial nature of this problem, in Subsection 4.3.3 we first mapped the problem to continuous time and then restricted the set of feasible scheduling and rate control actions that can be chosen. By doing so, we were able to explicitly characterize an optimal policy that finds a minimum-length schedule.

In the second part of this chapter, i.e., in Section 4.4, we considered time-varying wireless networks. In Subsection 4.4.1 we formulated the minimum-length scheduling problem as a stochastic shortest path and in Subsection 4.4.2 we introduced an optimal policy by employing the principles of stochastic control theory. Specifically, we employed the value iteration method to optimally solve the stochastic shortest path problem, which under our framework is guaranteed to converge in a finite number of iterations. A set of numerical experiments complementing our analytical results were presented in Subsection 4.4.3. The proofs of our results appear in Chapters 4.6 and 4.7.

4.6 Proof of Theorem 6

We can write the Lagrangian of the above problem as:

$$L(\boldsymbol{\tau}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = -\sum_{i=0}^{K} \tau_i + \sum_{k=1}^{K} \mu_k(\tau_k r_k^k + \tau_0 r_k^0 - d_k) + \sum_{i=0}^{K} \lambda_i \tau_i,$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ represent the Lagrange multipliers. The Karush-Kuhn-Tucker (KKT) conditions yield:

1. For every Action $k \in \mathcal{K}$ we have

$$\frac{\partial L(\boldsymbol{\tau}, \boldsymbol{\mu}, \boldsymbol{\lambda})}{\partial \tau_k} = -1 + \mu_k r_k^k + \lambda_k = 0. \tag{4.17}$$

2. For Action 0 we have

$$\frac{\partial L(\boldsymbol{\tau}, \boldsymbol{\mu}, \boldsymbol{\lambda})}{\partial \tau_0} = -1 + \sum_{k=1}^{K} \mu_k r_k^0 + \lambda_0 = 0. \tag{4.18}$$

3. For every Action $k \in \mathcal{K}$ it must be true that

$$\begin{aligned} \mu_k(\tau_k r_k^k + \tau_0 r_k^0 - d_k) = 0 \quad \Rightarrow \\ \mu_k \geq 0, \quad \tau_k r_k^k + \tau_0 r_k^0 \quad \geq \quad d_k. \end{aligned} \tag{4.19}$$

4. For all actions $i \in \{0, \ldots, K\}$ we have

$$\lambda_i \tau_i = 0 \Rightarrow \lambda_i \geq 0, \quad \tau_i \geq 0. \tag{4.20}$$

Consider the following cases:

Case 1: Assume that Action 0 is never employed, i.e., $\tau_0 = 0$. Since the traffic demands of every transmitter must be met we have that $\tau_k > 0$ for every $k \in \mathcal{K}$. Hence, from (4.20) it follows that $\lambda_0 \geq 0$ and $\lambda_k = 0$ for every $k \in \mathcal{K}$. From (4.17) we obtain,

$$\mu_k = \frac{1}{r_k^k}. \tag{4.21}$$

Further, since $\mu_k > 0$ and $\tau_0 = 0$, (4.19) yields

$$d_k = \tau_k r_k^k,$$

i.e., for every $k \in \mathcal{K}$ we get

$$\tau_k = \frac{d_k}{r_k^k}. \tag{4.22}$$

Finally, from (4.18) and (4.21) it follows that

$$\sum_{k=1}^{K} \frac{r_k^0}{r_k^k} \leq 1. \tag{4.23}$$

Case 2: Assume that Action 0 is employed and also a subset $\mathcal{J}$ of the transmitters are further selected to transmit individually. This implies that $\tau_0 > 0$, $\tau_j > 0$ for every $j \in \mathcal{J}$ and $\tau_i = 0$ for every $i \in \mathcal{K} \setminus \mathcal{J}$. Hence, (4.20) yields $\lambda_0 = 0$, $\lambda_j = 0$

for every $j \in \mathcal{J}$ and $\lambda_i \geq 0$ for every $i \in \mathcal{K} \setminus \mathcal{J}$. Also, for every $j \in \mathcal{J}$, (4.17) yields

$$\mu_j = \frac{1}{r_j^j}, \tag{4.24}$$

and for every $i \in \mathcal{K} \setminus \mathcal{J}$ it follows that

$$\mu_i \leq \frac{1}{r_i^i}. \tag{4.25}$$

Moreover, from (4.19) and (4.24) for every $j \in \mathcal{J}$ we get

$$\tau_j = \frac{d_j - \tau_0 r_j^0}{r_j^j}, \tag{4.26}$$

and from (4.19) and (4.25) for every $i \in \mathcal{K} \setminus \mathcal{J}$ we have

$$d_i \leq \tau_0 r_i^0, \tag{4.27}$$

or equivalently

$$\tau_0 \geq \max_{i \in \mathcal{K} \setminus \mathcal{J}} \frac{d_i}{r_i^0}. \tag{4.28}$$

Finally from (4.18), (4.24), and (4.25) it follows that

$$\sum_{k=1}^{K} \frac{r_k^0}{r_k^k} \geq 1. \tag{4.29}$$

■

4.7 Proof of Lemma 4

From (4.26), (4.27), and the fact that $\tau_j > 0$ for every $j \in \mathcal{J}$, for $i \in \mathcal{K} \setminus \mathcal{J}$ it follows that

$$0 < \tau_j \leq \frac{d_j r_i^0 - r_j^0 d_i}{r_j^j r_i^0},$$

which yields that

$$\frac{d_i}{r_i^0} < \frac{d_j}{r_j^0}. \tag{4.30}$$

Hence, from (4.30) it follows that there exists a threshold, i.e., a transmitter index, below which all the transmitters must belong in the set $\mathcal{J}$ and above which all of them must belong in the set $\mathcal{K} \setminus \mathcal{J}$. Since the objective is to minimize $\tau_0 + \sum_{j \in \mathcal{J}} \tau_j$, from (4.26) and (4.28) it follows that

$$\begin{aligned} |\mathcal{J}| &= \arg\min_{k \in \{0,\ldots,K\}} \left\{ \tau_0 + \sum_{j=\ell_1}^{\ell_k} \tau_j \right\} \\ &= \arg\min_{k \in \{0,\ldots,K\}} \left\{ \tau_0 + \sum_{j=\ell_1}^{\ell_k} \frac{d_j - \tau_0 r_j^0}{r_j^j} \right\} \\ &= \arg\min_{k \in \{0,\ldots,K\}} \left\{ \frac{d_{\ell_{k+1}}}{r_{\ell_{k+1}}^0} + \sum_{j=\ell_1}^{\ell_k} \frac{d_j r_{\ell_{k+1}}^0 - d_{\ell_{k+1}} r_j^0}{r_j^j r_{\ell_{k+1}}^0} \right\}. \end{aligned}$$

■

Chapter 5

Stable Throughput Maximization under Channel Uncertainty

5.1 Background

In this chapter, we turn our focus on obtaining joint scheduling and routing network control policies that maximize the stable throughput region of time-varying wireless networks.

There exists a rich literature on the subject of stable throughput maximization (see e.g., [9], [10], [33], [34]). Specifically in [33], a scheduling policy that maximizes the stable throughput in single-hop time-varying networks is identified. Moreover, in [9], the authors characterize the stable throughput region of static, multi-hop radio networks with multiple commodities, and propose a centralized, stationary, scheduling and routing rule, commonly referred as the "back-pressure", that maximizes the stable throughput. The "back-pressure" policy forwards the traffic through the network from queues with high loads to queues with lower loads and achieves stability by load-balancing the queues in the network. Furthermore, the authors in [9] show that their proposed policy is at least as good as any stationary policy. Under the assumption that a scheduled transmission is always successful, they prove that their policy performs at least as well as any non-stationary policy with respect to maximizing the stable throughput region of the network. In fact, the "back-pressure" algorithm of [9] has been shown to maximize the stable throughput region under a variety of contexts. In [34], we proved optimality of a policy inspired by the back-pressure algorithm of [9] within the set of all stationary policies in the more general setting of wireless networks with *time-varying topologies*. Further, [34] also differs from [9] in that our proposed policy gives priority to each commodity according to a preassigned commodity weight. In both [9] and [34], it is assumed that links are imperfect and that a scheduled transmission may fail, based on a link failure probability, which is independent of the identity, and the number of the simultaneously activated links. Finally, in another related study, [10], a joint scheduling, routing and power control policy, also inspired by the back-pressure algorithm, is proposed that maximizes the stable throughput region of time-varying wireless networks. The authors in [10] consider a time-varying process of perfect channels, i.e., a transmission through a link is always successful.

However, in practice the channel conditions can only be estimated, and hence exact knowledge of the current channel state is likely to be unavailable. The effect of this discrepancy in the channel state may be two fold; first, certain scheduled transmissions are going to fail, and second, transmissions through certain links which would be successful if scheduled, are not activated. Naturally, this situation will affect the set of stabilizable rates and will result in a smaller stable throughput region that is a subset of the stable throughput region under perfect links or under perfect channel estimation.

In this chapter, we are interested in capturing the effect of imperfect channel estimation and characterize the maximum achievable stable throughput region. We also obtain a policy that maximizes the stable throughput region under this setting. Towards this end, our results are different from [10], and generalize [9] and [34], in that we consider policies with knowledge of only an *estimate* of the true channel state. Specifically, we propose a stationary, joint scheduling, and routing policy for *multi-hop, time-varying* networks that maximizes the stable throughput region of the network by having access to only a, perhaps highly inaccurate, *estimate* of the current channel state. Our proposed policy, inspired by the “back-pressure” idea of [9], is shown to be optimal within a broad class of stationary, non-stationary, even anticipative policies. We improve on the results of [9] and [34] in two aspects. First, we show that our proposed policy performs at least as well in terms of stable throughput as a large class of policies that do not have more information on the current true channel state than our policy and where this information is limited to be given through an estimate of the channel state. In contrast with [9], this result holds even when scheduled transmissions are not guaranteed to succeed. Second, our model of uncertainty in the channel state is more sophisticated than the simplistic model used in [9] and [34] in two respects: (i) the existence of a link is explicitly modeled through the Signal to Interference plus Noise Ratio criterion imposed by the physical layer and (ii) our model accounts for the fact that the probability of success of a transmission is affected by the interference caused by other nearby concurrent transmissions.

5.2 Model Formulation

We consider slotted time and a wireless network consisting of N, possibly mobile, nodes each of which is equipped with a single transceiver. We denote by $\mathcal{N} = \{1, 2, \ldots, N\}$ the set of all nodes in the network. Each node $n \in \mathcal{N}$ transmits at a fixed power level P_n.

We also consider a set $\mathcal{J} = \{1, 2, \ldots, J\}$ of distinct commodities of traffic with packet lengths equal to one time slot. The number of exogenous packet arrivals of commodity j at node n during time slot t is denoted by $A_{nj}(t)$. We let $\mathbf{A}^j(t)$ denote the N-vector $(A_{nj}(t) : n = 1, 2, \ldots, N)$ of arrivals of the j^{th} commodity during time slot t at every node in the network and $\mathbf{A}(t)$ denote the $N \times J$ matrix $(A_{nj}(t), n = 1, 2, \ldots N, j = 1, 2, \ldots J)$ of arrivals in time slot t at every node n and for every commodity j. Traffic of commodity $j \in \mathcal{J}$ is routed in a multi-hop fashion through the network until it reaches *any* node in a set of *exit nodes* for that commodity, $V_j \subset \mathcal{N}$, where it exits the network. For any commodity $j' \neq j$, the sets $V_{j'}$ and V_j may overlap. We further assume that there are no exogenous arrivals of a particular commodity at the exit nodes of that commodity, i.e., $A_{nj}(t) = 0$ for all $n \in V_j,\ j \in \mathcal{J}$.

At each node n there exist J infinite capacity buffers, each holding separately the packets of a particular commodity $j \in \mathcal{J}$ that have reached node n. We denote the queue size for commodity j at node n *at the end* of time slot t by $X_{nj}(t)$. At time slot 0 the queue sizes at all nodes are arbitrary but finite, i.e., $X_{nj}(0) \geq 0$ for every node $n \in \mathcal{N}$ and commodity $j \in \mathcal{J}$. Moreover, the queue size at each exit node $n \in V_j$ of some commodity j and for all time slots $t \geq 0$ satisfies $X_{nj}(t) = 0$. Finally, for every commodity $j \in \mathcal{J}$ we denote by $\mathbf{X}^j(t)$ the N-vector $(X_{nj}(t), n = 1, 2, \ldots N)$ of queue sizes of the j^{th} commodity at every node in the network at the end of time slot t and by $\mathbf{X}(t)$ the $N \times J$ matrix $(X_{nj}(t), n = 1, 2, \ldots N, j = 1, 2, \ldots J)$ of queue sizes of every commodity at every node in the network at the end of time slot t. The set of possible values of $\mathbf{X}(t)$, i.e., the state space of the process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$, is denoted by $\mathcal{X}$.

The channel process $\{\mathbf{S}(t)\}_{t=1}^{\infty}$ defines the channel conditions between any pair of nodes in the network and is assumed to change only at the beginning of each time slot $t \in$

$\{1, 2, \ldots\}$. Specifically, at time slot t, the channel state $\mathbf{S}(t) = \{(G_{(n,m)}(t), N_{o(m)},\ \forall n, m \in \mathcal{N}\}$ is characterized by the path loss $G_{(n,m)}(t)$ between each pair of nodes n, m, as well as the noise power, $N_{o(m)}$, at each receiving node m. A fundamental aspect of our model that contrasts it from prior work of [33], [34], and [10] is that at the beginning of each time slot t the network controller has access only to an *estimate* $\hat{\mathbf{S}}(t) = \{(\hat{G}_{(n,m)}(t), \hat{N}_{o(m)}(t),\ \forall n, m \in \mathcal{N}\}$ of the current channel state $\mathbf{S}(t)$. The *estimated* channel state $\hat{\mathbf{S}}(t)$ during slot t is characterized by the *estimated* path loss $\hat{G}_{(n,m)}(t)$ between each pair of nodes n, m and the *estimated* noise power $\hat{N}_{o(m)}(t)$ at each receiving node m. Note that although the noise power $N_{o(m)}$ is time invariant, its estimate $\hat{N}_{o(m)}(t)$ depends on time, since as time progresses we may naturally get a monotonically improving estimate.

We further assume that the state space of the *true* and *estimated* channel processes is a finite set of cardinality K, which is naturally assumed to be common for both $\{\mathbf{S}(t)\}_{t=1}^{\infty}$ and $\{\hat{\mathbf{S}}(t)\}_{t=1}^{\infty}$. For example, that would be the case if we consider node mobility that is restricted to occur only among points of a finite grid. We denote this common set by $\mathcal{S} = \{\mathbf{S}^{(1)}, \mathbf{S}^{(2)}, \ldots, \mathbf{S}^{(K)}\}$. We will further denote by $\mathcal{K} = \{1, 2, \ldots, K\}$ the set of indices that label the elements of $\mathcal{S}$.

At every time slot t, a (unidirectional) link $\ell = (n, m)$ from node n to node m under the true channel state $\mathbf{S}(t) \in \mathcal{S}$ is defined to exist, if the Signal to Noise Ratio (SNR) at m exceeds a certain, non-negative, threshold γ_m, i.e.,

$$\mathrm{SNR}(\ell, t) := \frac{P_n\ G_{(n,m)}(t)}{N_{o(m)}} \geq \gamma_m. \tag{5.1}$$

We denote the source node n of link ℓ by $s(\ell)$ and its destination node m by $d(\ell)$. Given the time variability of the channel conditions, and the fact that nodes are mobile, the total number of links, L, can be as large as $N \times (N-1)$. We denote by $\mathcal{L} = \{1, 2, \ldots, L\}$ the set of indices of all links in the network.

The fact that the wireless medium is a shared resource poses limitations on the set of nodes that may successfully transmit simultaneously. Hence, not every subset of links in $\mathcal{L}$ can be concurrently activated. In order to take the physical layer access constraints into account, appropriate medium access control schemes need to be introduced. In this chapter, we focus on conflict free scheduling. Towards this end, we define an *activation vector* to be any L-element binary vector, each entry of which corresponds to a (unidirectional) link. At any time slot t, the entries of this vector are equal to one for those links that are concurrently activated at time slot t and zero for all other links. We also require that an activation vector complies with the single transceiver assumption. This assumption implies that simultaneous transmission and reception from the same node as well as receiving/transmitting simultaneously from/to multiple nodes are not allowed. We further define an activation vector $\mathbf{c}$ to be *valid* with respect to some channel state $\mathbf{S}(t)$ if for every link $\ell \in \mathcal{L}$ such that the ℓ^{th} entry c_ℓ of $\mathbf{c}$ satisfies $c_\ell = 1$, the SINR criterion as shown in (5.2)

$$\text{SINR}^{\,\mathbf{c}}(\ell, t) := \frac{P_{s(\ell)}\, G_{(s(\ell),d(\ell))}(t)}{N_{o(d(\ell))} + \sum\limits_{\substack{\ell' \in \mathcal{L} \setminus \{\ell\} \\ \text{s.t. } c_{\ell'} = 1}} P_{s(\ell')}\, G_{(s(\ell'),d(\ell))}(t)} \geq \gamma_{d(\ell)}, \qquad (5.2)$$

is satisfied with $c_{\ell'}$ being the ℓ'^{th} entry of $\mathbf{c}$. The criterion of (5.2) implies that the corresponding transmissions through all links $\ell \in \mathcal{L}$ with $c_\ell = 1$ will be successful under channel state $\mathbf{S}(t)$. Similarly, the estimated SINR criterion under $\hat{\mathbf{S}}(t)$ can be written as

$$\widehat{\text{SINR}}^{\mathbf{c}}(\ell,t) := \frac{P_{s(\ell)}\ \hat{G}_{(s(\ell),d(\ell))}(t)}{\hat{N}_{o(d(\ell))}(t) + \sum_{\substack{\ell' \in \mathcal{L} \setminus \{\ell\} \\ \text{s.t. } c_{\ell'} = 1}} P_{s(\ell')}\hat{G}_{(s(\ell'),d(\ell))}(t)} \geq \gamma_{d(\ell)}. \tag{5.3}$$

Note that due to the inaccuracy of the estimate, an activation vector selected at time slot t may be valid with respect to the estimated channel state $\hat{\mathbf{S}}(t)$ at slot t, but not valid with respect to the true channel state $\mathbf{S}(t)$ and vice versa.

For every possible channel state $\mathbf{S}^{(k)} \in \mathcal{S}$ where $k \in \mathcal{K}$, we denote by $\mathcal{T}_k$ the *constraint set* of $\mathbf{S}^{(k)}$, i.e., the set of all *valid* activation vectors with respect to $\mathbf{S}^{(k)}$. Note that for every activation vector $\mathbf{c}' \in \{0,1\}^L$ that is componentwise smaller than some vector $\mathbf{c} \in \mathcal{T}_k$, i.e., $\mathbf{c}' \leq \mathbf{c}$, it follows that $\mathbf{c}' \in \mathcal{T}_k$. This is natural because for any collection of links that jointly satisfy the SINR criteria of (5.2) - (5.3), these criteria will still be satisfied by switching off certain transmissions. From the above observation it follows trivially that for every $k \in \mathcal{K}$ the $\mathbf{0}$-vector is also a valid activation vector for each channel state $\mathbf{S}^{(k)} \in \mathcal{S}$.

For each commodity j, consider a process $\{\mathbf{E}^j(t)\}_{t=1}^{\infty}$ that for every time slot t gives the link activations for packets of commodity j. In other words for every time slot t the vector $\mathbf{E}^j(t)$ is an L-element binary vector, the entries of which are equal to one for those links that are simultaneously activated and packets of commodity j are transmitted through them, and are equal to zero otherwise. Further, for every time slot t we define $\mathbf{E}(t) := \sum_{j=1}^{J} \mathbf{E}^j(t)$. The process $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ corresponds to the overall link activations for every time slot t and it is such that whenever the at time slot t the estimated channel process is in state $\mathbf{S}^{(k)}$, the vector $\mathbf{E}(t)$ is a valid link activation vector with respect to $\mathbf{S}^{(k)}$. This means that $\mathbf{E}(t)$ is a vector from the constraint set $\mathcal{T}_k$, i.e, $\mathbf{E}(t) \in \mathcal{T}_k$. We call the process $\{\mathbf{E}^j(t)\}_{t=1}^{\infty}$ an *activation process*. Recall that the constraint set has the property that for any vector in the constraint set, any other vector that is smaller component-wise must be in the constraint set as well. Since $\mathbf{E}(t) \in \mathcal{T}_k$, the aforementioned property implies that for every commodity j the corresponding vector $\mathbf{E}^j(t)$ is also a valid activation vector with respect to $\mathbf{S}^{(k)}$, i.e., it satisfies $\mathbf{E}^j(t) \in \mathcal{T}_k$. Further, we require that for each commodity j, a vector $\mathbf{E}^j(t)$ must be such that its ℓ^{th} component, $(\mathbf{E}^j(t))_\ell$, takes the value zero for all those time slots t that the queue size at source node of the link, $s(\ell)$, for commodity j is equal to zero at the time of the link activation, i.e., $X_{s(\ell)j}(t-1) = 0$. We say that every such process $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ is an *admissible policy* and the process $\{\mathbf{E}^j(t),\ j \in \mathcal{J}\}_{t=1}^{\infty}$ is an *admissible policy corresponding to the* j^{th} *commodity*. Unless otherwise specified all the policies we consider are valid.

Further, for every time slot t where $\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}$ for some $k \in \mathcal{K}$ and for any activation vector $\mathbf{c} \in \mathcal{T}_k$, we construct the $L \times L$ diagonal indicator matrix $\mathbf{Q}^{\mathbf{c}}(t)$, whose ℓ^{th} diagonal entry, $(\mathbf{Q}^{\mathbf{c}}(t))_\ell$, satisfies

$$(\mathbf{Q}^{\mathbf{c}}(t))_\ell = \begin{cases} 1, \text{ if } \Big(\text{SINR}^{\mathbf{c}}(\ell,t) \geq \gamma_{d(\ell)}, \widehat{\text{SINR}^{\mathbf{c}}}(\ell,t) \geq \gamma_{d(\ell)}\Big) \text{ or} \\ \qquad \Big(\text{SINR}^{\mathbf{c}}(\ell,t) < \gamma_{d(\ell)}, \widehat{\text{SINR}^{\mathbf{c}}}(\ell,t) < \gamma_{d(\ell)}\Big), \\ 0, \text{ otherwise.} \end{cases} \tag{5.4}$$

Intuitively, for any given activation vector $\mathbf{c} \in \mathcal{T}_k$ and estimated channel state $\mathbf{S}^{(k)}$, the ℓ^{th} entry of the matrix $\mathbf{Q}^{\mathbf{c}}(t)$ takes the value one only when the estimator estimates the channel correctly in the sense that the values of the corresponding SINRs under both the *true* and *estimated* channel state lie on the same side of the inequality. Note that whether $(\mathbf{Q}^{\mathbf{c}}(t))_\ell$ is equal to one or zero depends on the overall link activations given by the vector $\mathbf{c}$. In the ideal case of perfect channel estimation, the matrix $\mathbf{Q}^{\mathbf{c}}(t)$ is the identity matrix, i.e., $\mathbf{Q}^{\mathbf{c}}(t) = \mathrm{I}$, for every time slot t where the estimated channel state is in state $\mathbf{S}^{(k)}$ for some $k \in \mathcal{K}$ and for any activation vector $\mathbf{c} \in \mathcal{T}_k$.

Also, for every commodity j we define the matrix $\mathbf{R}^j$ as an $N \times L$ matrix that denotes the changes in the queue sizes after a successful link activation. The (n, ℓ) entry, $R^j_{n\ell}$, of this matrix equals

$$R^j_{n\ell} = \begin{cases} 1, & \text{if } n = d(\ell) \notin V_j, \\ -1, & \text{if } n = s(\ell), \\ 0, & \text{otherwise.} \end{cases} \tag{5.5}$$

Note that $R^j_{n\ell} = 0$ when $n = d(\ell) \in V_j$, as packets of commodity j arriving at n exit the system. Overall, the above yields the following dynamic equation for the queue sizes

$$\mathbf{X}^j(t+1) = \mathbf{X}^j(t) + \mathbf{R}^j\ \mathbf{Q}^{\mathbf{E}(t+1)}(t+1)\ \mathbf{E}^j(t+1) + \mathbf{A}^j(t+1),\ \ t \geq 0. \tag{5.6}$$

Throughout this chapter we make use of the following assumption on the input processes.

Assumption 4 *(a) The triplet* $\{\mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t)\}_{t=1}^{\infty}$ *is i.i.d. over time and independent of* $\mathbf{X}(0)$. *(b) The arrival process has finite second moments, i.e.,* $\mathbb{E}[\mathbf{A}(t)^2] < \infty$.

Assumption 4 (a) guarantees that each of the processes $\{\mathbf{S}(t)\}_{t=1}^{\infty}$, $\{\hat{\mathbf{S}}(t)\}_{t=1}^{\infty}$, and

$\{\mathbf{A}(t)\}_{t=1}^{\infty}$ are individually i.i.d, and hence have a stationary distribution. In particular, the probability $p_{\hat{\mathbf{S}}}(k)$ of the occurrence of *estimated* channel state $\mathbf{S}^{(k)} \in \mathcal{S}$, given by

$$p_{\hat{\mathbf{S}}}(k) := P[\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}], \quad \forall k \in \mathcal{K}, \tag{5.7}$$

does not depend on t. Without loss of generality, we assume that

$$p_{\hat{\mathbf{S}}}(k) > 0, \quad \forall\, k \in \mathcal{K}. \tag{5.8}$$

Indeed, all our results are probabilistic in nature, and are not affected if we discard sample paths corresponding to a nullset of outcomes. Moreover, from Assumption 4(a) it follows that although the processes are i.i.d. in time, for any particular time slot t they can be correlated among themselves. For example, the true and estimated channel states $\mathbf{S}(t)$ and $\hat{\mathbf{S}}(t)$ are naturally correlated but not $\mathbf{S}(t)$ and $\hat{\mathbf{S}}(t-1)$.

From Assumption 4(b), it follows that the first moments of the arrival process $\{\mathbf{A}(t)\}_{t=1}^{\infty}$ are also finite, i.e., $\lambda_{nj} := \mathbb{E}[A_{nj}(t)]$, where the quantity λ_{nj} corresponds to the arrival rate of commodity j at node n. We also denote by $\boldsymbol{\lambda}$ the *arrival rate matrix* $(\lambda_{nj},\ n = 1, 2, \ldots N,\ j = 1, 2, \ldots J)$ of arrival rates at every node in the network and for every commodity. Finally, for each commodity $j \in \mathcal{J}$ we write $\boldsymbol{\lambda}^j$ for the N-vector $\boldsymbol{\lambda}^j = (\lambda_{nj},\ n = 1, 2, \ldots N)$ of arrivals of the j^{th} commodity at every node in the network. All arrival rates in our model are measured in terms of packets per time slot.

The nomenclature defined so far is summarized through an example in Fig. 5.1, where we consider a network of 3 nodes, i.e., $\mathcal{N} = \{1, 2, 3\}$. Nodes 1 and 2 transmit at a fixed powers P_1 and P_2 respectively. We consider that the channel conditions are such that we have two possible channel states, namely $\mathcal{S} = \{\mathbf{S}^{(1)}, \mathbf{S}^{(2)}\}$. On the left side of the figure, we give the possible links that can be established under channel state $\mathbf{S}^{(1)}$ and on the right side of the figure we give the set of possible links under channel state $\mathbf{S}^{(2)}$. Specifically, when the estimated channel state is $\mathbf{S}^{(1)}$, there exist two possible links, namely links 1 and 2, where a “link” satisfies the SNR criterion of (5.1) and when it is $\mathbf{S}^{(2)}$ no connectivity exists among the nodes. Hence, $\mathcal{L} = \{1, 2\}$. Further, although both links 1 and 2 are in $\mathcal{L}$, we assume that they cannot be activated simultaneously due to the fact that they do not jointly satisfy the physical layer constraints of SINR. Specifically, we assume that at most one of them can be activated at any given time. Since the constraint set $\mathcal{T}_k$ for channel state $\mathbf{S}^{(k)}$ contains all the valid activation vectors with respect to $\mathbf{S}^{(k)}$, we have that $\mathcal{T}_1 = \{[0, 0], [0, 1], [1, 0]\}$ and $\mathcal{T}_2 = \{[0, 0]\}$. There exist two commodities of traffic in the network, i.e., $\mathcal{J} = \{1, 2\}$. $A_{11}(t)$ and $A_{22}(t)$ denote the arrivals in packets per slot, during time slot t, of commodity 1 at node 1 and of commodity 2 at node 2 respectively. We assume that packets of each commodity exit the network at node 3, i.e., $V_j = \{3\}$, for $j = 1, 2$. At every node in the network, there exist two infinite capacity buffers, that hold separately the packets of each commodity. We indicate the queue size of commodity 1 at node 2 at the end of time slot t by $X_{21}(t)$ and the queue size of commodity 2 at the same node by $X_{22}(t)$. Note that, due to the estimation errors, the policy may schedule e.g., link 1 assuming that the current channel state is $\mathbf{S}^{(1)}$ when in fact the current state is $\mathbf{S}^{(2)}$ and hence the scheduled transmission through link 1 will fail.

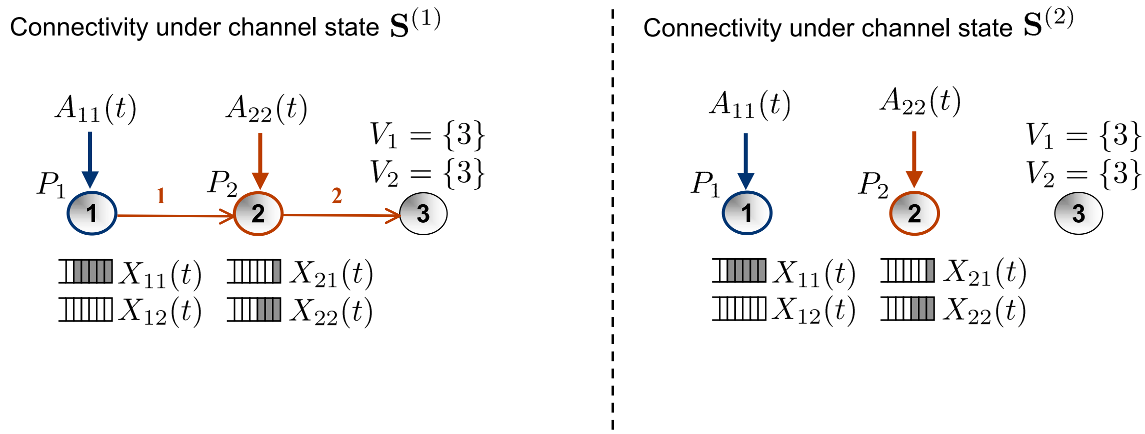

Figure 5.1: The possible connectivities of a 3 node network under 2 possible channel states, $\mathbf{S}^{(1)}$ and $\mathbf{S}^{(2)}$.

5.3 Stable Throughput Maximization under Channel State Uncertainty

In this section, we consider a policy that maximizes the stable throughput region of the network by making use of *only* an estimate of the true channel state. Our policy is built upon the "back-pressure" idea in [9]. As its name suggests, this policy attempts to maximize the stable throughput by spreading the traffic from the more congested to the less congested areas in the network. Accordingly, the policy we introduce activates the nodes of the network in a way that the weighted queue sizes for every commodity j will be kept as close to equal as possible, while at the same time the constraints imposed by the physical layer are being satisfied. Since the physical layer information available to our policy is limited due to the uncertainty in the channel state, our policy will try to maximize the stable throughput region of the network, within a broad class of policies, by having access to only an estimate of the channel conditions.

The routing component of the introduced policy resembles the so-called "hot-potato" routing approach in which nodes simply unload packets to neighboring nodes with smaller queue loads ([35]). In fact, in our model, the route any packet follows is determined by the link activation schedule that aims at maximizing the stable throughput region of the network. Hence, although an individual packet may follow a circuitous route towards one of its exit nodes, the overall characteristics of the routes are expected to be reasonable, albeit non-optimal. Since our objective is to achieve maximum stable throughput, this sort of routing is legitimate. No other routing will increase the stable throughput region, although it may decrease the delay that packets of the different commodities experience in the network.

The introduced policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ is parameterized by a weight assignment $\mathbf{w} = (w_j, j = 1, 2 \ldots, J)$, where w_j is a positive weight assigned to each commodity j. Packets corresponding to a commodity of a larger weight are given priority over the others, by being scheduled and routed through the network more frequently. For every given weight vector $\mathbf{w}$, the *stationary policy* $\mathbf{E}(t) := \boldsymbol{\pi}_0^{\mathbf{w}}(t)$ is a certain J-tuple of mappings $\boldsymbol{\pi}_0^{\mathbf{w}j} : \mathcal{X} \times \mathcal{S} \to \{0,1\}^L$, each corresponding to a commodity j and where $\mathbf{E}^j(t) := \boldsymbol{\pi}_0^{\mathbf{w}j}(t)$. So, we also have that $\boldsymbol{\pi}_0^{\mathbf{w}} = \sum_{j=1}^{J} \boldsymbol{\pi}_0^{\mathbf{w}j}$. For every time slot t, the quantity $\boldsymbol{\pi}_0^{\mathbf{w}j}(t)$ indicates the link activations for packets of commodity j and $\boldsymbol{\pi}_0^{\mathbf{w}}(t)$ gives the overall link activations in the network.

We proceed by specifying the stable throughput maximizing policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ in detail. Given the current queue size matrix $\mathbf{x} \in \mathcal{X}$, weight assignment $\mathbf{w}$ and activation vector $\mathbf{c} \in \mathcal{T}_k$, for every estimated channel state $\mathbf{S}^{(k)}$, let

$$\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x}) := -w_j \, \tilde{\mathbf{Q}}_k^{\mathbf{c}} \, \mathbf{R}^{j\top} \mathbf{x}^j, \quad k \in \mathcal{K}, \; j \in \mathcal{J}, \; \mathbf{c} \in \mathcal{T}_k, \tag{5.9}$$

where

$$\tilde{\mathbf{Q}}_k^{\mathbf{c}} := \mathbb{E}\left[\mathbf{Q}^{\mathbf{c}}(t) \mid \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}\right]. \tag{5.10}$$

From this definition it follows that the matrix $\tilde{\mathbf{Q}}_k^{\mathbf{c}}$ is an $L \times L$ diagonal matrix. Its ℓ^{th} diagonal entry $(\tilde{\mathbf{Q}}_k^{\mathbf{c}})_\ell$ gives the conditional probability that both the estimated and true SINR values corresponding to ℓ lie at the same side of the inequality, provided that the overall link activations in the network are determined through the activation vector $\mathbf{c}$ and the estimated channel state is $\mathbf{S}^{(k)}$. For any given link ℓ, our model allows this probability to be dependent on the concurrent transmissions. For example, this probability is expected to be higher when link ℓ is the only link activated than when link ℓ is activated along with other concurrent nearby transmissions. Also, Assumption 4(a) guarantees that the matrix $\tilde{\mathbf{Q}}_k^{\mathbf{c}}$ for every $k \in \mathcal{K}$ and $\mathbf{c} \in \mathcal{T}_k$, defined in (5.10), is time invariant.

Since the queue size x_{nj} is equal to zero whenever $n \in V_j$, it follows that the ℓ^{th} component $\left(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})\right)_\ell$ of $\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})$ is the weighted queue size difference

$$\left(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})\right)_\ell = w_j(\tilde{\mathbf{Q}}_k^{\mathbf{c}})_\ell(x_{s(\ell)j} - x_{d(\ell)j}). \tag{5.11}$$

For every link $\ell \in \mathcal{L}$, let

$$(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x}))_\ell := \max_{j \in \mathcal{J}} \left(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})\right)_\ell, \tag{5.12}$$

and

$$\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x}) := \left((\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x}))_\ell, \quad \ell = 1, \ldots, L \right). \tag{5.13}$$

Finally, define

$$(j_k^\star(\mathbf{x}))_\ell := \text{ arg } \max_{j \in \mathcal{J}} \left\{ \left(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x}) \right)_\ell \right\}, \tag{5.14}$$

to be the maximizer in (5.12) and also let

$$\mathbf{c}_k^\star(\mathbf{x}) := \text{ arg } \max_{\mathbf{c} \in \mathcal{T}_k} \left\{ \mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x})^\top \mathbf{c} \right\}. \tag{5.15}$$

Recall that the entries of every valid activation vector $\mathbf{c} \in \mathcal{T}_k$ are either 0 or 1, with 1 indicating activation of the corresponding link. Hence $\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x})^\top \mathbf{c}$ is a partial sum of weighted queue size differences over all the links, maximized over all the elements of the constraint set $\mathcal{T}_k$. If there exist more than one maximizer in (5.15) ties are resolved arbitrarily provided that a link ℓ will be left inactive whenever the corresponding maximum weighted difference associated with that link is 0. Furthermore, if there exist more than one maximizer in (5.14), ties are resolved arbitrarily. With the above in hand, and in the spirit of the optimal policy of [9], our proposed policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ is such that its ℓ^{th} entry $(\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}))_\ell$ is given by

$$(\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}))_\ell = \begin{cases} 1, & j = (j_k^\star(\mathbf{x}))_\ell, (\mathbf{c}_k^\star(\mathbf{x}))_\ell = 1, \text{ and } x_{s(\ell)j} > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{5.16}$$

where $(\mathbf{c}_k^\star(\mathbf{x}))_\ell$ is the ℓ^{th} entry of the vector $\mathbf{c}_k^\star(\mathbf{x})$. When a link ℓ is activated, i.e., $\left(\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x}, \mathbf{S}^{(k)})\right)_\ell = 1$, the policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ will select for transmission through that link a packet of one of the classes j that achieves the "max" in (5.14). Note that from (5.14), (5.15), and (5.16) the policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ also satisfies

$$\left(\mathbf{D}_k^{\mathbf{w}}(\mathbf{x})^\top - \mathbf{D}_k^{\mathbf{w}j}(\mathbf{x})^\top \right) \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}) = 0. \tag{5.17}$$

Note that the matrix $\tilde{\mathbf{Q}}_k^{\mathbf{c}}$ is all the information our policy has regarding the current channel conditions as shown through (5.11), (5.14), and (5.15). The policy employs this information by giving a higher preference to those links for which both the true and the estimated SINRs lie at the same side of the inequality. Specifically, the policy will have the tendency to activate links that have a higher chance of successful transmission.

Clearly, for every commodity j we have that $\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}) \in \mathcal{T}_k$. Note further that for every link ℓ that is activated, a packet of a single commodity j is transmitted, and hence there will exist a single $\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)})$ that satisfies $(\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}))_\ell = 1$. From this observation it follows that $\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x}, \mathbf{S}^{(k)}) \in \mathcal{T}_k$. The above, along with the fact that the policy leaves a link ℓ inactive whenever the maximum weighted difference over that link is 0, guarantees that $\boldsymbol{\pi}_0^{\mathbf{w}}$ satisfies the conditions for being an admissible policy. In Section 5.5, we will show the maximizing property of this policy under the following mild assumption.

Assumption 5 *Let $n' \in \mathcal{N}$ be a node such that for some $n \in \mathcal{N}$, $j \in \mathcal{J}$ with $\lambda_{nj} > 0$ there exists a sequence of links $\{\ell_i\}_{i=1}^m \in \mathcal{L}$, with $s(\ell_1) = n$, $d(\ell_i) = s(\ell_{i+1}),\ i = 1, \ldots, m-1$, and $d(\ell_m) = n'$ such that $\forall\ i = 1, \ldots, m$*

$$P[\mathrm{SNR}(\ell_i, t) \geq \gamma_{d(\ell_i)},\ \text{and}\ \widehat{\mathrm{SNR}}(\ell_i, t) \geq \gamma_{d(\ell_i)}] > 0, \tag{5.18}$$

where $\mathrm{SNR}(\ell, t)$ is obtained through (5.1) *and $\widehat{\mathrm{SNR}}(\ell, t)$ is defined similarly as*

$$\widehat{\mathrm{SNR}}(\ell, t) := \frac{P_{s(\ell)}\ \hat{G}_{(s(\ell),d(\ell))}(t)}{\hat{N}_{o(d(\ell))}(t)}. \tag{5.19}$$

Then, there exists a node $n'' \in V_j$ *and a sequence of links* $\{\ell'_i\}_{i=1}^{m'} \in \mathcal{L}$ *with* $s(\ell'_1) = n'$, $d(\ell'_i) = s(\ell'_{i+1}),\ i = 1, \ldots, m'-1$*, and* $d(\ell'_{m'}) \in V_j$ *such that* (5.18) *holds with* $\{\ell_i\}_{i=1}^{m}$ *replaced by* $\{\ell'_i\}_{i=1}^{m'}$.

Assumption 5 is an assumption on sufficient connectivity of the network. Specifically it requires that for any node that may receive traffic of a particular commodity, there should also exist a downstream path of links to some exit node for that commodity under both the true and estimated channel states.

5.3.1 System Stability

The state of our system is driven by the process of the queue sizes. In this section, we show that under Assumption 4(a) and policy $\boldsymbol{\pi}_0^{\mathbf{w}}$, the queue size process defined by (5.6), i.e., the state of our system, evolves according to a homogeneous Markov Chain. Our aim is to show that this Markov Chain is stable and thus derive network stability for as large a set of arrival rates as possible.

Proposition 1 *Under Assumption 4(a), the process* $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ *generated by* (5.6) *with* $\mathbf{E}^j(t) = \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{X}(t-1), \hat{\mathbf{S}}(t))$ *for every* $j \in \mathcal{J}$ *is a homogeneous Markov chain. Furthermore,* $\mathbf{X}(t)$ *is independent of* $(\mathbf{S}(t'), \hat{\mathbf{S}}(t'), \mathbf{A}(t'))$ *for all* $t' > t \geq 0$.

The result in the above proposition is a direct consequence of the fact that any process defined by a recurrence equation driven by white noise input, with initial value independent of the input, is Markov (See, e.g., [36, Theorem 2.1].).

A usual definition for stability of an irreducible Markov Chain is that the Markov Chain is positive recurrent. When the Markov Chain is not guaranteed to be irreducible, a more general definition for stability needs to be employed. Following [9], we adopt the following definition for stability of a (not necessarily irreducible) homogeneous Markov Chain.

Definition 1 *[9] Let* $\{Y(t)\}_{t=0}^{\infty}$ *be a Markov Chain with, possibly empty, transient class* $\mathcal{Y}$ *and recurrent communicating classes* $\mathcal{Z}_i$*,* $i = 1, 2, \ldots$*. Then* $\{Y(t)\}_{t=0}^{\infty}$ *is* stable *if*

$$P\left[\min\{\tau \geq 0 \,:\, Y(\tau) \notin \mathcal{Y}\} < \infty \,|\, Y(0) = y\right] = 1, \,\forall\, y \in \mathcal{Y},$$

and all states $z \in \cup_{i=1}^{\infty} \mathcal{Z}_i$ *are positive recurrent.*

We will say that the network is stable if the state process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ is stable, as defined in Definition 1.

5.4 A Broad Class of Policies under Channel State Uncertainty

In this section, we introduce a general class of policies, $\mathcal{E}$. Our objective will be to compare the performance of the members in $\mathcal{E}$ to $\boldsymbol{\pi}_0^{\mathbf{w}}$ with respect to maximizing the stable throughput region of the network. This comparison will be performed in Section 5.5.

In order to specify the class $\mathcal{E}$ we define $n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(t; k, \mathbf{c}, \mathbf{Q})$ to be the number of time slots in the interval $[0, t]$ that the estimated channel state is in state $\mathbf{S}^{(k)}$, the activation vector $\mathbf{E}(t)$ takes value $\mathbf{c} \in \mathcal{T}_k$ and the matrix $\mathbf{Q}^{\mathbf{E}(t)}(t)$ is equal to $\mathbf{Q} \in \mathcal{Q}$. Here $\mathcal{Q}$ is the set of all $L \times L$ diagonal matrices whose diagonal is in the set $\{0, 1\}^L$. Also, we define $n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c})$ to be the number of time slots in the interval $[0, t]$ that the estimated channel state is $\mathbf{S}^{(k)}$ and the activation vector $\mathbf{E}(t)$ takes value $\mathbf{c} \in \mathcal{T}_k$. We define the set $\mathcal{E}$ as follows. We say that a policy $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ belongs to $\mathcal{E}$ if for every $k,\ k' \in \mathcal{K}$ and time slot $t \in \{1, 2, \ldots\}$ the following is true

$$P[\mathbf{S}(t) = \mathbf{S}^{(k')}|\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}, \mathbf{E}(t) = \mathbf{c}] = P[\mathbf{S}(t) = \mathbf{S}^{(k')}|\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}], \tag{5.20}$$

and for every $k \in \mathcal{K}$, activation vector $\mathbf{c} \in \mathcal{T}_k$, and matrix $\mathbf{Q} \in \mathcal{Q}$ the following is true

$$\frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(t; k, \mathbf{c}, \mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c})} \to \frac{P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q}, \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}, \mathbf{E}(t) = \mathbf{c}]}{P[\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}, \mathbf{E}(t) = \mathbf{c}]},\ \text{almost surely as } t \to \infty, \tag{5.21}$$

when $n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c}) \neq 0$ as $t \to \infty$. Note that if $n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c}) = 0$ as $t \to \infty$, then the corresponding activation vector $\mathbf{c}$ is not used by the policy. In such a case, this activation vector can be eliminated from its constraint set. Recall that the constraint set is the set of all valid activation vectors with respect to the current channel state estimate.

The condition (5.20) is natural. It requires that at any time slot t, $\mathbf{E}(t)$ and the true channel state $\mathbf{S}(t)$ are conditionally independent given the estimate $\hat{\mathbf{S}}(t)$. In other words, all policies $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ we may consider have no more information on the true channel state $\mathbf{S}(t)$ than the stationary policy $\boldsymbol{\pi}_0^{\mathbf{w}}$. Naturally, a policy that has additional information regarding the true channel state at time slot t can potentially exploit this knowledge and for example avoid collisions by not scheduling the corresponding nodes. Also, (5.21) is natural and it is in spirit similar to regular ergodicity conditions. From (5.20) and (5.21) we may easily deduce that

$$\frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(t; k, \mathbf{c}, \mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c})} \to P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q}|\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}], \tag{5.22}$$

where from Assumption 4(a), $P[\mathbf{Q^c}(t) = \mathbf{Q}|\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}]$ is independent of time t. Note also that the set $\mathcal{E}$ includes all the stationary policies since for stationary policies both (5.20) and (5.21) are being satisfied. It may further include some *non-stationary*, as well as *anticipative* policies as long as they comply with the conditions for being in set $\mathcal{E}$. Finally, we remind the reader that anticipative network control policies are all those policies that have knowledge on the future values of the quantities that affect the evolution of the state process, driven by (5.6).

5.4.1 The Notion of Intermittent Boundedness

When the policy $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ belongs to the class $\mathcal{E}$, the resulting queue size process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ generated by (5.6) is not necessarily a Markov Chain. Therefore, the stability definition according to Definition 1 is not applicable anymore. Instead, we will make use of a weaker notion of stability, that of intermittent boundedness.

Definition 2 *The random process $\{Y(t)\}_{t=0}^{\infty}$ is almost surely intermittently bounded, if there exists a subset W of the sample space, with $P[W] = 1$, such that for every $\omega \in W$ there exists a sequence $\{t_i\}_{i=1}^{\infty}$ and a finite $Y_{\max}$ for which $|Y(\omega, t_i)| < Y_{\max},\ \forall\ i = 1, 2, \ldots$, where $Y(\omega, t)$ denotes the sample path of the process $\{Y(t)\}_{t=0}^{\infty}$ corresponding to outcome ω. Further, $\{Y(t)\}_{t=0}^{\infty}$ is said to be intermittently bounded with positive probability, if there exists a subset W of the sample space, with $P[W] > 0$, such that for every $\omega \in W$ there exists a sequence $\{t_i\}_{i=1}^{\infty}$ and a finite $Y_{\max}$ for which $|Y(\omega, t_i)| < Y_{\max},\ \forall\ i = 1, 2, \ldots$.*

5.5 Optimality of the Proposed Policy

In this section we will prove optimality of the policy introduced in Section 5.3 with respect to maximizing the stable throughput region of the network under uncertainty in the channel state. We will first define some sets of rates that are important in our proofs.

In a stable network, traffic at any given node $n \in \mathcal{N}$ cannot accumulate without bound. Hence, stability can be viewed through the concept of *flow conservation*, namely that for any commodity the sum of departing flows at any node, except for the exit nodes for this commodity, must be equal to the sum of arriving flows for this commodity. Therefore, we define the set of *feasible* arrival rates $\mathbf{\Lambda}$ as

$$\mathbf{\Lambda} = \left\{ \boldsymbol{\lambda} \in \mathbb{R}_+^{NJ} : \exists \mathbf{f}_k^j \in \mathbb{R}_+^L, \text{ such that } \quad \boldsymbol{\lambda}^j = -\mathbf{R}^j \sum_{k=1}^{K} p_{\hat{\mathbf{S}}}(k) \mathbf{f}_k^j, \text{ and } \sum_{j=1}^{J} \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k) \right\}, \tag{5.23}$$

where $\tilde{\mathcal{Q}}_k = \{\tilde{\mathbf{Q}}_k^{\mathbf{c}}\ \mathbf{c},\ \ \mathbf{c} \in \mathcal{T}_k\}$, $\mathbf{f}_k^j$ are flow vectors of the j^{th} commodity under estimated channel state $\mathbf{S}^{(k)}$ and $\mathrm{co}(\cdot)$ denotes the convex hull of a set. Further, let the stable throughput region $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$ under $\boldsymbol{\pi}_0^{\mathbf{w}}$ be defined as

$$\begin{aligned} \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}} \quad = \quad & \big\{ \text{The set of arrival rates } \boldsymbol{\lambda} \text{ such that for all processes } \left\{ \mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t) \right\}_{t=1}^{\infty}, \\ & \text{satisfying Assumptions 4 and 5, where } \boldsymbol{\lambda} = \mathbb{E}[\mathbf{A}(t)], \text{ the network is stable} \\ & \text{under } \boldsymbol{\pi}_0^{\mathbf{w}}. \big\} \end{aligned}$$

We also denote by $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$ the following set of rates

$$\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}} = \Big\{\text{The set of rates } \boldsymbol{\lambda} \text{ such that for all processes } \Big\{\mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t)\Big\}_{t=1}^{\infty}\text{, satisfying Assumptions 4 and 5, where } \boldsymbol{\lambda} = \mathbb{E}[\mathbf{A}(t)]\text{, the process of the queue sizes is almost surely intermittently bounded under } \boldsymbol{\pi}_0^{\mathbf{w}}.\Big\}$$

Finally, to compare with $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$ and $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, we introduce the set of arrival rates $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$ as

$$\widetilde{\mathbf{C}}^p_{\mathcal{E}} = \Big\{\text{ The set of rates } \boldsymbol{\lambda} \text{ such that for some processes } \Big\{\mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t)\Big\}_{t=1}^{\infty}\text{, satisfying Assumption 4 where } \boldsymbol{\lambda} = \mathbb{E}[\mathbf{A}(t)]\text{, the process of the queue sizes is intermittently bounded with positive probability under some policy } \{\mathbf{E}(t)\}_{t=1}^{\infty} \in \mathcal{E}.\Big\}$$

Note that although the requirement for an arrival rate being in $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$ is that the process of the queue sizes is stable under $\boldsymbol{\pi}_0^{\mathbf{w}}$, the set of arrival rates $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$ only requires that the queue size process satisfies the weak notion of intermittent boundedness with positive probability.

Let $\mathrm{ri}(\cdot)$ denote the relative interior of a set. The following theorem states our main result. The proof can be found in Section 5.7.

Theorem 7 *The set* $\boldsymbol{\Lambda}$ *is a convex polytope. Furthermore, for* all *weight assignments* $\mathbf{w} = (w_j, j = 1, 2, \ldots, J)$*, with* $w_j > 0$ *for every commodity* $j \in \mathcal{J}$*, the following relationships hold*

$$\mathrm{ri}(\boldsymbol{\Lambda}) \subseteq \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}} \subseteq \widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}} \subseteq \widetilde{\mathbf{C}}^p_{\mathcal{E}} \subseteq \boldsymbol{\Lambda}. \tag{5.24}$$

We proceed to give some more insight into the meaning of this theorem. From (5.24) it follows that for all weight assignments $\mathbf{w}$, the rate regions $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, and $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$ are all squeezed between the convex polytope $\boldsymbol{\Lambda}$, and its relative interior. Hence, the sets of rates $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, and $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$ can differ by at most points on the relative boundary of $\boldsymbol{\Lambda}$, and therefore they are almost identical sets. In fact, this implies that for any rate, except perhaps for a few rates in the relative boundary of $\boldsymbol{\Lambda}$, that cannot be stabilized by our introduced stationary policy $\boldsymbol{\pi}_0^{\mathbf{w}}$, there exists no policy in the large class $\mathcal{E}$ that can even make the process of the queue sizes intermittently bounded with some positive probability.

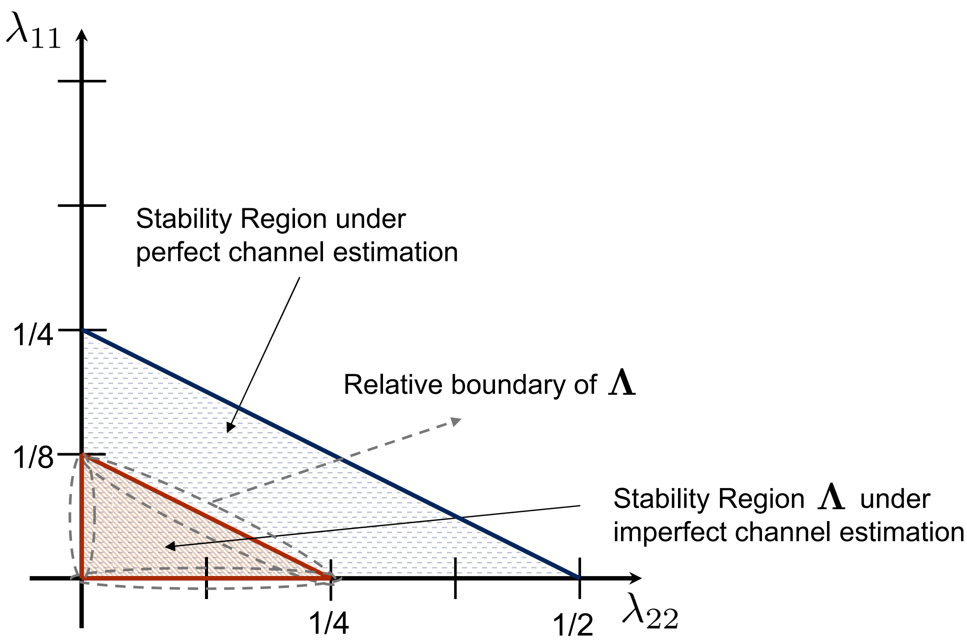

Figure 5.2: Stable throughput region of the network presented in Fig. 5.1 under perfect and imperfect channel estimation.

As an example, by utilizing (5.23), in Fig. 5.2 we depict the stable throughput region for the example network presented in Fig. 5.1. Here, it is assumed that the channel estimation is such that the matrices $\tilde{\mathbf{Q}}_1^{[0,0]^T}$, $\tilde{\mathbf{Q}}_1^{[0,1]^T}$, $\tilde{\mathbf{Q}}_1^{[1,0]^T}$ are all equal to a diagonal matrix with diagonal entries given by 0.5, while the values of $\tilde{\mathbf{Q}}_2^{[0,0]^T}$ are immaterial due to the fact that there are no links available under channel state $\mathbf{S}^{(2)}$. Further, we assumed that the stationary probabilities of the estimated channel states are both equal to 0.5, i.e., $p_{\hat{\mathbf{S}}}(1) = p_{\hat{\mathbf{S}}}(2) = 0.5$. As discussed above, the set of stable achievable rates may differ from $\mathbf{\Lambda}$ by only the relative interior of $\mathbf{\Lambda}$, which is the union of three line segments shown in Fig. 5.2. Further, in Fig. 5.2 we also provide the stable throughput region of the network under perfect channel estimation, obtained by replacing $\tilde{\mathbf{Q}}_1^{[0,0]^T}$, $\tilde{\mathbf{Q}}_1^{[0,1]^T}$, and $\tilde{\mathbf{Q}}_1^{[1,0]^T}$ with the identity matrix in (5.23). It is evident that the channel estimation errors have a significant impact on the stable throughput region.

5.6 Summary

In this chapter, we characterized the stable throughput region of a multi-hop network with multiple commodities in which the true channel state cannot be known by the network control policy.

In Section 5.2 we presented the network model. In Section 5.3 we discussed the problem of stable throughput maximization under channel state uncertainty. We defined the notion of stability considered in this work. Specifically, we assumed that the system is stable if the underlying Markov Chain of the network queue sizes is positive recurrent. We introduced a joint scheduling and routing policy that assigns weights of preference to each commodity and attempts to maximize the stable throughput region of time-varying wireless networks, independently of the weight assignment, while having access only to a possibly inaccurate estimate of the true channel state. In Section 5.4 we introduced a large class of stationary, non-stationary, perhaps anticipative policies. A restriction we posed on these policies was that they are not permitted to know more about the current true channel state than what the estimate reveals. Since under the broad class of policies the queue size process need not be a Markov Chain any more, in the sequel we gave an alternative, very weak definition for stability called as intermittent boundedness. In Section 5.5 we characterized the common set of stable arrival rates that our optimal policy supports and proved its optimality with respect to maximizing the stable throughput region of the network within a broad class of stationary, non-stationary, and possibly anticipative policies, under some mild conditions. We finally showed through an example that the network stable throughput region can be considerably smaller than the corresponding stable throughput region under perfect channel estimation. The proofs of our results appear in Section 5.7.

5.7 Proof of Theorem 7

In this section we prove each individual inclusion relationship of Theorem 7. The third inclusion, that is $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathrm{w}}} \subseteq \widetilde{\mathbf{C}}^p_{\mathcal{E}}$, follows trivially from the definitions of the sets $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathrm{w}}}$, and $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$. Next, we prove the three remaining inclusions, namely that (i) $\mathrm{ri}(\boldsymbol{\Lambda}) \subseteq \mathbf{C}_{\boldsymbol{\pi}_0^{\mathrm{w}}}$, (ii) $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathrm{w}}} \subseteq \widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathrm{w}}}$, and (iii) $\widetilde{\mathbf{C}}^p_{\mathcal{E}} \subseteq \boldsymbol{\Lambda}$.

5.7.1 Proof of $\mathrm{ri}(\mathbf{\Lambda}) \subseteq \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$

Consider a rate $\boldsymbol{\lambda} \in \mathrm{ri}(\mathbf{\Lambda})$. We show that $\boldsymbol{\lambda} \in \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, i.e., that this rate is stabilized by our proposed policy $\boldsymbol{\pi}_0^{\mathbf{w}}$. We make use of Extended Foster's Theorem ([9]), which provides a sufficient condition for stability.

Theorem 8 (Extended Foster Theorem) *Consider a Homogenous Markov Chain $\{Y(t)\}_{t=0}^{\infty}$ with state space $\mathcal{Y}$. Suppose there exists a real valued, function $V : \mathcal{Y} \to \mathbb{R}$, that is bounded from below, such that*

$$\mathbb{E}[V(Y(t+1)) \mid Y(t) = y] < \infty,\ \forall y \in \mathcal{Y}, \tag{5.25}$$

and such that for some $\epsilon > 0$, and some finite subset $\mathcal{Y}_0$ of $\mathcal{Y}$

$$\mathbb{E}[V(Y(t+1)) - V(Y(t)) \mid Y(t) = y] < -\epsilon, \quad \forall\, y \notin \mathcal{Y}_0 \tag{5.26}$$

Then, $\{Y(t)\}_{t=0}^{\infty}$ is stable in the sense of Definition 1.

We will show that the process of the queue sizes $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ satisfies the conditions of this theorem. For compactness of notation, we use t^+ to denote $t+1$. Given $\mathbf{w} > 0$, and $\mathbf{x} \in \mathcal{X}$, let $V(\mathbf{x}) := \sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \mathbf{x}^j$, be a candidate Lyapunov function. We show that, with $V(\cdot)$ thus defined under policy $\boldsymbol{\pi}_0^{\mathbf{w}}$, and given any process $\{\mathbf{A}(t)\}_{t=1}^{\infty}$, such that $\mathbb{E}[\mathbf{A}(t)] = \boldsymbol{\lambda}$, the process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ given by (5.6) with $\mathbf{E}^j(t) = \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{X}(t-1), \hat{\mathbf{S}}(t))$ for all $j \in \mathcal{J}$ satisfies the conditions of Theorem 8.

First, it is immediate that $\mathbb{E}[V(\mathbf{X}(t^+)) \mid \mathbf{X}(t) = \mathbf{x}] < \infty, \forall \mathbf{x} \in \mathcal{X}$. To see this, let $\mathbf{x} \in \mathcal{X}$, and let

$$\mathbf{G}^j(t) := \mathbf{x}^j + \mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t))}(t) \boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t)) + \mathbf{A}^j(t). \tag{5.27}$$

Note that for every t the matrix $\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t))}(t)$ is a function of $\mathbf{S}(t)$, and $\hat{\mathbf{S}}(t)$. Since by Proposition 1, the variables $\mathbf{S}(t^+)$, $\hat{\mathbf{S}}(t^+)$, $\mathbf{A}(t^+)$ are independent of $\mathbf{X}(t)$, (5.6) yields

$$\mathbb{E}[V(\mathbf{X}(t^+)) \mid \mathbf{X}(t) = \mathbf{x}] = \sum_{j=1}^{J} w_j \mathbb{E}\left[\mathbf{G}^j(t^+)^\top \mathbf{G}^j(t^+)\right], \tag{5.28}$$

which is finite for all $\mathbf{x}$ since from Assumption 4 (b) the process $\{\mathbf{A}(t)\}_{t=1}^{\infty}$ is assumed to have finite second moments, and further the policy $\boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t^+))$, as well as the process $\{\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t))}(t)\}_{t=1}^{\infty}$ take values in finite sets. This in fact holds independently of the choice of stationary policy $\boldsymbol{\pi}$, and of the arrival rate $\boldsymbol{\lambda}$. To complete the proof, we show that, when policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ is used, there exists a finite set $\mathcal{X}_0$ such that (5.26) holds. For compactness of notation, we define

$$\Delta V(\mathbf{x}) := \mathbb{E}\left[V(\mathbf{X}(t^+)) - V(\mathbf{X}(t)) \mid \mathbf{X}(t) = \mathbf{x}\right].$$

We first prove two lemmas that will be useful in proving the desired result.

Lemma 7 *Given any policy $\boldsymbol{\pi}$, arrival rate $\boldsymbol{\lambda}$, and queue size matrix $\mathbf{x} \in \mathcal{X}$, the Markov Chain $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ given by* (5.6) *satisfies*

$$\Delta V(\mathbf{x}) \leq 2 \left(\sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j - \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{j=1}^{J} \mathbf{D}^{\mathbf{w}j}_{k\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \, \boldsymbol{\pi}^j(\mathbf{x}, \mathbf{S}^{(k)}) \right) + B, \tag{5.29}$$

where B does not depend on $\mathbf{x}$.

Proof: From (5.28), and the definition of our candidate Lyapunov function we have

$$\begin{aligned}
\Delta V(\mathbf{x}) &= \sum_{j=1}^{J} w_j \mathbb{E}\left[\left(\mathbf{X}^j(t^+) - \mathbf{X}^j(t)\right)^\top \left(\mathbf{X}^j(t^+) + \mathbf{X}^j(t)\right) \mid \mathbf{X}(t) = \mathbf{x}\right] \\
&= \sum_{j=1}^{J} w_j \mathbb{E}\left[\left(\mathbf{X}^j(t^+) - \mathbf{X}^j(t)\right)^\top \left(2\mathbf{X}^j(t) + \mathbf{X}^j(t^+) - \mathbf{X}^j(t)\right) \mid \mathbf{X}(t) = \mathbf{x}\right] \\
&= 2\sum_{j=1}^{J} w_j \left(\mathbf{x}^{j\top} \mathbb{E}\left[\mathbf{X}^j(t^+) - \mathbf{X}^j(t) \mid \mathbf{X}(t) = \mathbf{x}\right]\right) \\
&+ \sum_{j=1}^{J} w_j \mathbb{E}\left[(\mathbf{X}^j(t^+) - \mathbf{X}^j(t))^\top (\mathbf{X}^j(t^+) - \mathbf{X}^j(t)) \mid \mathbf{X}(t) = \mathbf{x}\right].
\end{aligned}$$

By using (5.6) we obtain

$$\begin{aligned}
\Delta V(\mathbf{x}) = 2\sum_{j=1}^{J} &\left(w_j \mathbf{x}^{j\top} \mathbb{E}\left[\mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t^+))}(t^+) \boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t^+)) + \mathbf{A}^j(t^+) \mid \mathbf{X}(t) = \mathbf{x}\right]\right) \\
+ \sum_{j=1}^{J} w_j \mathbb{E} &\left[\left(\mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t^+))}(t^+) \boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t^+)) + \mathbf{A}^j(t^+)\right)^\top \right. \\
&\left. \left(\mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t^+))}(t^+) \boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t^+)) + \mathbf{A}^j(t^+)\right) |\mathbf{X}(t) = \mathbf{x}\right].
\end{aligned}$$

Since $\{\mathbf{A}(t)\}_{t=1}^{\infty}$ is stationary, and has finite first and second moments, and the policy $\boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t^+))$, as well as the process $\{\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t))}(t)\}_{t=1}^{\infty}$, where $\boldsymbol{\pi}(\mathbf{x}, \hat{\mathbf{S}}(t)) = \sum_{j=1}^{J} \boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t))$, take values in finite sets, the second term is finite and bounded for every $j \in \mathcal{J}$ by a quantity independent of the queue size matrix $\mathbf{x}$, and time slot t. Hence for every $\mathbf{x} \in \mathcal{X}$,

$$\Delta V(\mathbf{x}) \leq 2\sum_{j=1}^{J} \left(w_j \mathbf{x}^{j\top} \mathbb{E}\left[\mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t^+))}(t^+) \boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t^+)) + \mathbf{A}^j(t^+) \mid \mathbf{X}(t) = \mathbf{x}\right]\right) + B$$

for some B independent of $\mathbf{x}$, and t. Further by making use of Proposition 1, namely that $\mathbf{A}(t^+)$ is independent of $\mathbf{X}(t)$, and using conditional expectations it follows that

$$
\begin{aligned}
\Delta V(\mathbf{x}) \quad &\leq 2 \sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j + B \\
&+ \quad 2 \sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \mathbf{R}^j \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \mathbb{E}\left[\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}(t^+) | \mathbf{X}(t) = \mathbf{x}, \hat{\mathbf{S}}(t^+) = \mathbf{S}^{(k)} \right] \boldsymbol{\pi}^j(\mathbf{x}, \mathbf{S}^{(k)}).
\end{aligned}
$$

Using (5.10), and the fact that $\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}(t^+)$, and $\hat{\mathbf{S}}(t^+)$ are independent of $\mathbf{X}(t)$ we obtain

$$
\Delta V(\mathbf{x}) \quad \leq \quad 2 \sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j - 2 \sum_{j=1}^{J} \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \left(-w_j \tilde{\mathbf{Q}}_k^{\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})} \mathbf{R}^{j\top} \mathbf{x}^j \right)^{\top} \boldsymbol{\pi}^j(\mathbf{x}, \mathbf{S}^{(k)}) + B. \tag{5.30}
$$

Finally, by using (5.9), the above equation becomes

$$
\Delta V(\mathbf{x}) \leq 2 \left(\sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j - \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{j=1}^{J} \mathbf{D}_{k\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}^{\mathbf{w}j}(\mathbf{x})^{\top}\ \boldsymbol{\pi}^j(\mathbf{x}, \mathbf{S}^{(k)}) \right) + B,
$$

which completes the proof.

■

When an arrival rate $\boldsymbol{\lambda}$ belongs to $\mathrm{ri}(\boldsymbol{\Lambda})$, a useful upper bound can be obtained on the first term in the parenthesis of (5.29), by means of the following lemma.

Lemma 8 *Let $\boldsymbol{\lambda} \in \mathrm{ri}(\boldsymbol{\Lambda})$. Then there exist nonnegative scalars $\mu'^{\mathbf{c}}_k$, for all $\mathbf{c} \in \mathcal{T}_k$, $k \in \mathcal{K}$, with $\sum_{\mathbf{c} \in \mathcal{T}_k} \mu'^{\mathbf{c}}_k < 1$, such that, for all $\mathbf{x} \in \mathcal{X}$,*

$$
\sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j \leq \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{\mathbf{c} \in \mathcal{T}_k} \mu'^{\mathbf{c}}_k \mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x})^{\top} \mathbf{c}. \tag{5.31}
$$

Proof: Let rate $\boldsymbol{\lambda} \in \mathrm{ri}(\boldsymbol{\Lambda})$. Then $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}$, as $\mathrm{ri}(\boldsymbol{\Lambda}) \subseteq \boldsymbol{\Lambda}$. Hence, with reference to (5.23) there exists a scalar $\delta > 1$, and non-negative flow vectors $\mathbf{f}_k^j \in \mathbb{R}_+^L$ such that

$$\boldsymbol{\lambda}^j = -\mathbf{R}^j \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \mathbf{f}_k^j, \tag{5.32}$$

and where $\delta \sum_{j=1}^{J} \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k)$ i.e., for some $\mu_k^{\mathbf{c}} \geq 0$ such that $\sum_{\mathbf{c} \in \mathcal{T}_k} \mu_k^{\mathbf{c}} = 1$ we have

$$\delta \sum_{j=1}^{J} \mathbf{f}_k^j = \sum_{\mathbf{c} \in \mathcal{T}_k} \mu_k^{\mathbf{c}} \tilde{\mathbf{Q}}_k^{\mathbf{c}} \mathbf{c}. \tag{5.33}$$

Note that from (5.33) it follows that, for all $j \in \mathcal{J}$, and $k \in \mathcal{K}$, we have

$$(\mathbf{f}_k^j)_\ell = 0\,, \quad \forall \ell \notin \mathbf{S}^{(k)}. \tag{5.34}$$

Using (5.32), and the fact each of the vectors $\mathbf{f}_k^j$ are non-negative component-wise we can write

$$\begin{aligned} \sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j &\leq \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{j=1}^{J} \left(\max_{j \in \mathcal{J}} \left(-w_j \mathbf{x}^{j\top} \mathbf{R}^j \right) \mathbf{f}_k^j \right) \\ &= \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \max_{j \in \mathcal{J}} \left(-w_j \mathbf{x}^{j\top} \mathbf{R}^j \right) \sum_{\mathbf{c} \in \mathcal{T}_k} \frac{\mu_k^{\mathbf{c}}}{\delta} \tilde{\mathbf{Q}}_k^{\mathbf{c}} \mathbf{c}, \end{aligned} \tag{5.35}$$

where (5.35) follows by making use of (5.33). Let $\mu'^{\mathbf{c}}_k := \frac{\mu_k^{\mathbf{c}}}{\delta}$. By definition, $\mu'^{\mathbf{c}}_k \geq 0$. Also, since $\sum_{\mathbf{c} \in \mathcal{T}_k} \mu_k^{\mathbf{c}} = 1$, and $\delta > 1$, it follows that $\sum_{\mathbf{c} \in \mathcal{T}_k} \mu'^{\mathbf{c}}_k < 1$. Further, (5.35) can be written as

$$\begin{aligned} \sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j &\leq \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{\mathbf{c} \in \mathcal{T}_k} \mu'^{\mathbf{c}}_k \max_{j \in \mathcal{J}} \left(\left(-w_j \tilde{\mathbf{Q}}_k^{\mathbf{c}} \mathbf{R}^{j\top} \mathbf{x}^j \right)^\top \right) \mathbf{c} \\ &= \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{\mathbf{c} \in \mathcal{T}_k} \mu'^{\mathbf{c}}_k \mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x})^\top \mathbf{c}, \end{aligned} \tag{5.36}$$

where (5.36) follows by making use of (5.9), (5.12), and (5.13). This completes the proof of Lemma 8.

■

We proceed to finalize the proof of the claim that $\mathrm{ri}(\boldsymbol{\Lambda}) \subseteq \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$. From Lemmas 7 and 8 we conclude that, given $\boldsymbol{\lambda} \in \mathrm{ri}(\boldsymbol{\Lambda})$, there exist nonnegative scalars $\mu'^{\mathbf{c}}_k$, for all $\mathbf{c} \in \mathcal{T}_k$, and $k \in \mathcal{K}$, with $\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k < 1$, such that, for all $\mathbf{x} \in \mathcal{X}$, and all stationary policies $\boldsymbol{\pi}$,

$$\Delta V(\mathbf{x}) \leq 2\sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \left(\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k \mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top \mathbf{c} - \sum_{j=1}^{J} \mathbf{D}^{\mathbf{w}j}_{k\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \boldsymbol{\pi}^j(\mathbf{x},\mathbf{S}^{(k)}) \right) + B. \tag{5.37}$$

So far $\boldsymbol{\pi}$ was an arbitrary stationary policy. We now focus on the policy $\boldsymbol{\pi}_0^{\mathbf{w}}$. In view of the fact that $\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)}) = \sum_{j=1}^{J} \boldsymbol{\pi}^j(\mathbf{x},\mathbf{S}^{(k)}) \in \mathcal{T}_k$, from (5.17), and of the definition of $\boldsymbol{\pi}_0^{\mathbf{w}}$, we obtain

$$\begin{aligned} \sum_{j=1}^{J} \mathbf{D}^{\mathbf{w}j}_{k\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x},\mathbf{S}^{(k)}) &= \mathbf{D}^{\mathbf{w}}_{k\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \sum_{j=1}^{J} \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x},\mathbf{S}^{(k)}) \\ &= \mathbf{D}^{\mathbf{w}}_{k\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x},\mathbf{S}^{(k)}) \\ &= \max_{\mathbf{c}\in\mathcal{T}_k} \{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top \mathbf{c}\}. \end{aligned}$$

By substituting into (5.37), we get

$$\begin{aligned}\Delta V(\mathbf{x}) &\leq B + 2\sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k)\left(\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_{k}\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c} - \max_{\mathbf{c}\in\mathcal{T}_k}\{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c}\}\right) \\ &\leq B - 2\sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k)\max_{\mathbf{c}\in\mathcal{T}_k}\{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c}\}\left(1 - \sum_{\mathbf{c}\in\mathcal{T}_k}\mu'^{\mathbf{c}}_{k}\right) \\ &\leq B - \rho\max_{k\in\mathcal{K}}\max_{\mathbf{c}\in\mathcal{T}_k}\{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c}\},\end{aligned}$$

where from (5.7), and the fact that $\sum_{\mathbf{c}\in\mathcal{T}_k}\mu'^{\mathbf{c}}_{k} < 1$

$$\rho := 2\min_{k\in\mathcal{K}}\left(p_{\hat{\mathbf{S}}}(k)\left(1 - \sum_{\mathbf{c}\in\mathcal{T}_k}\mu'^{\mathbf{c}}_{k}\right)\right) > 0.$$

Now, let $\mathbf{x} \in \mathcal{X}$, with $\mathbf{x} \neq \mathbf{0}$, and suppose $\mathbf{X}(t) = \mathbf{x}$. Choose a node n, and a commodity j such that $x_{nj} > 0$. The Markov property of $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ implies that

$$\Delta V(\mathbf{x}) = \mathbb{E}\left[V(\mathbf{X}(t^+)) - V(\mathbf{X}(t)) \mid \mathbf{X}(t) = \mathbf{x}, \mathbf{X}(0) = \mathbf{0}\right].$$

Hence, without loss of generality, assume that the queue size process at time slot 0 satisfies $\mathbf{X}(0) = \mathbf{0}$. Since $X_{nj}(t) = x_{nj} > 0$, and $X_{nj}(0) = 0$, there must exist a sequence of links in $\mathcal{L}$ from some node n', with $\lambda_{n'j} > 0$, to node n that satisfy Assumption 5. Further, Assumption 5 then implies that there exist links $\ell_i \in \mathcal{L}$, $i = 1, \ldots, z$, for some z, satisfying $0 < z < N$, such that $n = s(\ell_1)$, and nodes $n_1, \ldots, n_z$, such that $d(\ell_1) = n_1$, $s(\ell_{i+1}) = n_i$, $d(\ell_{i+1}) = n_{i+1}$, $i = 1, \ldots, z-1$, and $n_z \in V_j$. For notational simplicity, also let $n_0 := n$. Since $x_{n_z j} = 0$, whenever $n_z \in V_j$, we can write

$$x_{nj} = \sum_{i=1}^{z}(x_{n_{i-1}j} - x_{n_i j}) \leq z\max_{i,j}(x_{n_{i-1}j} - x_{n_i j}). \tag{5.38}$$

It follows that there exists some link $\ell_{i^\star}$ for which the above queue size difference through it, is maximized for some commodity $j^\star \in \mathcal{J}$. Let $n_{i^\star-1} = s(\ell_{i^\star})$, and $n_{i^\star} = d(\ell_{i^\star})$. Then, from (5.38) we have

$$x_{n_{i^\star-1}j^\star} - x_{n_{i^\star}j^\star} \geq \frac{x_{nj}}{z} \geq \frac{x_{nj}}{N}. \tag{5.39}$$

Recall that $\ell_i \in \mathcal{L}$ for all $i = 1, \ldots, z$. Further, let $k^\star$ be such that $\ell_{i^\star}$ satisfies (5.1) under the estimated channel state $\hat{\mathbf{S}}(t) = \mathbf{S}^{(k^\star)}$. Let $\mathrm{e}_{\ell_{i^\star}} \in \mathbb{R}^L$ be a vector with its ${\ell_{i^\star}}^{\text{th}}$ component equal to 1, and with all other components equal to 0. Then, from the property of the constraint set it follows that $\mathrm{e}_{\ell_{i^\star}} \in \mathcal{T}_{k^\star}$. Also, it follows from (5.12) and (5.13) that

$$\begin{aligned}
\max_{k\in\mathcal{K}} \max_{\mathbf{c}\in\mathcal{T}_k} \{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top \mathbf{c}\} \;&\geq\; \max_{\mathbf{c}\in\mathcal{T}_{k^\star}} \{\mathbf{D}^{\mathbf{w}}_{k^\star\mathbf{c}}(\mathbf{x})^\top \mathbf{c}\} \\
&\geq\; \mathbf{D}^{\mathbf{w}}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})^\top \mathrm{e}_{\ell_{i^\star}} = \left(\mathbf{D}^{\mathbf{w}}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})\right)_{\ell_{i^\star}} \geq \left(\mathbf{D}^{\mathbf{w}j^\star}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})\right)_{\ell_{i^\star}},
\end{aligned}$$

where $\left(\mathbf{D}^{\mathbf{w}j^\star}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})\right)_{\ell_{i^\star}}$ is the ${\ell_{i^\star}}^{\text{th}}$ entry of the vector $\mathbf{D}^{\mathbf{w}j^\star}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})$. In view of (5.11), and (5.39), it follows that

$$\max_{k\in\mathcal{K}} \max_{\mathbf{c}\in\mathcal{T}_k} \{\mathbf{D}^{\mathbf{w}}_{k}(\mathbf{x})^\top \mathrm{e}_{\ell_{i^\star}}\} \geq w_{j^\star} (\tilde{\mathbf{Q}}^{\mathrm{e}_{\ell_{i^\star}}}_{k^\star})_{\ell_{i^\star}} (x_{n_{i^\star-1}j^\star} - x_{n_{i^\star}j^\star}) \geq \frac{w_{\min}\, \tilde{q}_{\min}\, x_{nj}}{N},$$

where $(\tilde{\mathbf{Q}}^{\mathrm{e}_{\ell_{i^\star}}}_{k^\star})_{\ell_{i^\star}}$ is the $\ell_{i^\star}^{\text{th}}$ diagonal entry of the matrix $\tilde{\mathbf{Q}}^{\mathrm{e}_{\ell_{i^\star}}}_{k^\star}$, while

$$w_{\min} := \min_{j\in\mathcal{J}} w_j > 0,$$

and, in view of Assumption 5, $\tilde{q}_{\min} > 0$. Note that the entries $w_{\min}$ and $\tilde{q}_{\min}$ do not depend on $\mathbf{x}$. Overall, we have

$$\Delta V(\mathbf{x}) \leq B - \frac{\rho\, w_{\min}\, \tilde{q}_{\min}\, x_{nj}}{N}$$

so that, given any $\epsilon > 0$,

$$\Delta V(\mathbf{x}) < -\epsilon, \quad \forall \mathbf{x} \notin \mathcal{X}_0 := \left\{ \mathbf{x} \in \mathcal{X} \ : \ x_{nj} \leq \frac{N(B+\epsilon)}{\rho\, w_{\min}\, \tilde{q}_{\min}} \right\}.$$

Since vectors in $\mathcal{X}$ have integer components, the set $\mathcal{X}_0$ is finite, and the proof is complete.

■

5.7.2 Proof of $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}} \subseteq \widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$

Consider an arrival rate $\boldsymbol{\lambda} \in \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$. In order to prove that $\boldsymbol{\lambda} \in \widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, we need to show that stability according to Definition 1 implies intermittent boundedness with probability 1. We proceed by giving a theorem that gives a sufficient condition for intermittent boundedness of a Markov Chain.

Theorem 9 *Let $\{Y(t)\}_{t=0}^{\infty}$ be a Markov Chain, with $\mathcal{Y}$ the, possibly empty, set of its transient states. If $\{Y(t)\}_{t=0}^{\infty}$ almost surely exits the set of transient states in finite time, i.e. if*

$$P\left[\min\{\tau \geq 0 \ : \ Y(\tau) \notin \mathcal{Y}\} < \infty \mid Y(0) = y\right] = 1, \quad \forall y \in \mathcal{Y} \tag{5.40}$$

(which holds vacuously when $\mathcal{Y}$ is empty), then $\{Y(t)\}_{t=0}^{\infty}$ is intermittently bounded with probability 1.

Proof: Consider the Markov Chain $\{Y(t)\}_{t=0}^{\infty}$ that satisfies (5.40). Then with probability 1, the Markov Chain $\{Y(t)\}_{t=0}^{\infty}$ will be eventually confined within a single recurrent class. It follows (e.g. from Theorem 7.3 in Chapter 2 of [36]) that, with probability 1, some (recurrent) state will be visited infinitely many times. Hence, there exists a set W, that is a subset of the sample space Ω, i.e. $W \subseteq \Omega$, with $P[W] = 1$ such that for every event $\omega \in W$, there exist a state y, and a sequence $\{t_i\}_{i=1}^{\infty}$, such that in the sample path ω the process satisfies

$$Y(\omega, t_i) = y, \ \forall i = 1, 2, \ldots .$$

Hence, by Definition 2 it follows that $\{Y(t)\}_{t=0}^{\infty}$ is intermittently bounded with probability 1.

■

A direct consequence of Theorem 9 is Corollary 3, that we state next.

Corollary 3 *Let $\{Y(t)\}_{t=0}^{\infty}$ be a stable Markov Chain. Then, $\{Y(t)\}_{t=0}^{\infty}$ is intermittently bounded with probability* 1.

From Corollary 3, the desired result follows.

5.7.3 Proof of $\widetilde{\mathbf{C}}_{\mathcal{E}}^{p} \subseteq \mathbf{\Lambda}$

We need to show that if $\boldsymbol{\lambda} \in \widetilde{\mathbf{C}}_{\mathcal{E}}^{p}$ then $\boldsymbol{\lambda} \in \mathbf{\Lambda}$. We start by introducing the notation required for our proof. We define the random variable $n_{\hat{\mathbf{S}}}(t;k)$ to be the number of time slots τ in the interval $[0,t]$ during which $\hat{\mathbf{S}}(\tau)$ takes the value $\mathbf{S}^{(k)}$. Moreover, we denote by $\{n_{\hat{\mathbf{S}}}(\omega,t;k)\}_{t=1}^{\infty}$, $\{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega,t;k,\mathbf{c})\}_{t=1}^{\infty}$, $\{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega,t;k,\mathbf{c},\mathbf{Q})\}_{t=1}^{\infty}$ the sample path ω of the corresponding processes (Recall that the processes $\{n_{\hat{\mathbf{S}}\mathbf{E}}(t;k,\mathbf{c})\}_{t=1}^{\infty}$, $\{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(t;k,\mathbf{c},\mathbf{Q})\}_{t=1}^{\infty}$ are defined in Section 5.4.). Finally by $\{\mathbf{A}(\omega,t)\}_{t=1}^{\infty}$, $\{\hat{\mathbf{S}}(\omega,t)\}_{t=1}^{\infty}$, $\{\mathbf{E}(\omega,t)\}_{t=1}^{\infty}$, $\{\mathbf{Q}^{\mathbf{c}}(\omega,t)\}_{t=1}^{\infty}$ and $\{\mathbf{X}(\omega,t)\}_{t=1}^{\infty}$ we denote each of the sample paths ω of the respective processes.

Since $\boldsymbol{\lambda} \in \widetilde{\mathbf{C}}_{\mathcal{E}}^{p}$, there exists a policy $\{\mathbf{E}(t)\}_{t=1}^{\infty} \in \mathcal{E}$ and an i.i.d. process $\{\mathbf{S}(t),\hat{\mathbf{S}}(t),\mathbf{A}(t)\}_{t=1}^{\infty}$ such that $\mathbb{E}[\mathbf{A}(t)] = \boldsymbol{\lambda}$. In particular

$$P\left[\omega : \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=1}^{t} \mathbf{A}^{j}(\omega,\tau) = \boldsymbol{\lambda}^{j}\right] = 1, \quad \forall j \in \mathcal{J}, \tag{5.41}$$

$$P\left[\omega : \lim_{t\to\infty} \frac{n_{\hat{\mathbf{S}}}(\omega,t;k)}{t} = p_{\hat{\mathbf{S}}}(k)\right] = 1, \quad \forall k \in \mathcal{K}. \tag{5.42}$$

Furthermore, from (5.22) we have that

$$P\left[\omega : \lim_{t\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega,t;k,\mathbf{c},\mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega,t;k,\mathbf{c})} = P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q}|\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}]\right] = 1. \tag{5.43}$$

Also, since the process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ is intermittently bounded with positive probability it follows that

$$P\left[\omega : \mathbf{X}(\omega,\tau_i) < \mathbf{X}_{\max}, \ \text{ for some finite } \mathbf{X}_{\max}, \text{ and for some sequence } \{\tau_i\}_{i=1}^{\infty}\right] > 0. \tag{5.44}$$

Since the events in (5.41), (5.42) and (5.43) have probability 1 and the event in (5.44) has a positive probability, their intersection will have a positive probability. Hence, it follows that the 4 events have a non-empty common intersection. We first fix an outcome ω' that belongs to this common intersection and once ω' is selected, we identify an $\mathbf{X}_{\max}$ and a sequence $\{t_i\}_{i=1}^{\infty}$ as specified by (5.44). We have

$$\lim_{i\to\infty}\frac{1}{t_i}\sum_{\tau=1}^{t_i}\mathbf{A}^j(\omega',\tau)=\boldsymbol{\lambda}^j \tag{5.45}$$

$$\lim_{i\to\infty}\frac{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}{t_i}=p_{\hat{\mathbf{S}}}(k) \tag{5.46}$$

$$\lim_{t\to\infty}\frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega',t;k,\mathbf{c},\mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega',t;k,\mathbf{c})}=P[\mathbf{Q}^{\mathbf{c}}(t)=\mathbf{Q}|\hat{\mathbf{S}}(t)=\mathbf{S}^{(k)}] \tag{5.47}$$

$$\mathbf{X}(\omega',t_i)<\mathbf{X}_{\max},\qquad \text{for some }\ \mathbf{X}_{\max},\ \ \forall i=1,2,\ldots. \tag{5.48}$$

We now proceed to first sum both sides of (5.6) from time slot 0 to t_i for some $i=1,2,\ldots$ and cancel the identical terms. Then, by dividing both sides of the resulting equation by t_i we obtain

$$\frac{1}{t_i}\mathbf{X}^j(\omega',t_i)=\frac{1}{t_i}\mathbf{X}^j(\omega',0)+\frac{1}{t_i}\sum_{\tau=1}^{t_i}\mathbf{R}^j\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)+\frac{1}{t_i}\sum_{\tau=1}^{t_i}\mathbf{A}^j(\omega',\tau). \tag{5.49}$$

From (5.48), we have

$$\lim_{i\to\infty}\frac{1}{t_i}\mathbf{X}^j(\omega',t_i)=0, \tag{5.50}$$

and

$$\lim_{i\to\infty}\frac{1}{t_i}\mathbf{X}^j(\omega',0)=0. \tag{5.51}$$

Taking the limit in (5.49) as $i\to\infty$, and by using (5.45), (5.50) and (5.51) we obtain

$$\begin{aligned}
\boldsymbol{\lambda}^j &= -\lim_{i\to\infty}\left\{\frac{1}{t_i}\sum_{\tau=1}^{t_i}\mathbf{R}^j\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}\\
&= -\lim_{i\to\infty}\left\{\mathbf{R}^j\sum_{k\in\mathcal{K}}\frac{1}{t_i}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}\\
&= -\lim_{i\to\infty}\left\{\mathbf{R}^j\sum_{k\in\tilde{\mathcal{K}}}\frac{1}{t_i}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}, \qquad (5.52)
\end{aligned}$$

where

$$\tilde{\mathcal{K}} = \left\{k\in\mathcal{K} \text{ s.t. } \hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)} \text{ for some } \tau\in\{1,\ldots,\infty\}\right\}.$$

Thus, for $k\in\tilde{\mathcal{K}}$, and for i large enough it follows that $n_{\hat{\mathbf{S}}}(\omega',t_i;k)>0$. Without loss of generality (by redefining the sequence $\{t_i\}_{i=1}^{\infty}$ if necessary), assume that $n_{\hat{\mathbf{S}}}(\omega',t_i;k)>0$ for all $k\in\tilde{\mathcal{K}}$ and $i=1,2,\ldots$. Then, (5.52) can be written as

$$\boldsymbol{\lambda}^j = -\lim_{i\to\infty}\left\{\mathbf{R}^j\sum_{k\in\tilde{\mathcal{K}}}\frac{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}{t_i}\frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}. \qquad (5.53)$$

Note that $\mathbf{E}^j(\omega',\tau)\in\mathcal{T}_k$ whenever $\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}$. Also, for every time slot τ, the matrix $\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)$ is a diagonal matrix, whose diagonal entries take values in the set $\{0,1\}$. Therefore, it is also true that the product $\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\,\mathbf{E}^j(\omega',\tau)\in\mathcal{T}_k$. Also, since

$$\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)} = \frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}1 = \frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}\,n_{\hat{\mathbf{S}}}(\omega',t_i;k)=1,$$

we have that for every $i \in \{1, \ldots\}$, $j \in \mathcal{J}$ and $k \in \tilde{\mathcal{K}}$,

$$\frac{1}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \sum_{\substack{\tau \in \{1, \ldots, t_i\} \\ \text{s.t. } \hat{\mathbf{S}}(\omega', \tau) = \mathbf{S}^{(k)}}} \mathbf{Q}^{\mathbf{E}(\omega', \tau)}(\omega', \tau)\mathbf{E}^j(\omega', \tau) \in \mathrm{co}(\mathcal{T}_k).$$

Since $\tilde{\mathcal{K}}$ is a finite set and since for every k, the set $\mathrm{co}(\mathcal{T}_k)$ is a compact set, there exists a subsequence $\{t_{i_\ell}\}_{\ell=1}^{\infty}$ and vectors $\mathbf{f}_k^j$ such that

$$\lim_{\ell \to \infty} \left\{ \frac{1}{n_{\hat{\mathbf{S}}}(\omega', t_{i_\ell}; k)} \sum_{\substack{\tau \in \{1, \ldots, t_{i_\ell}\} \\ \text{s.t. } \hat{\mathbf{S}}(\omega', \tau) = \mathbf{S}^{(k)}}} \mathbf{Q}^{\mathbf{E}(\omega', \tau)}(\omega', \tau)\mathbf{E}^j(\omega', \tau) \right\} = \mathbf{f}_k^j, \tag{5.54}$$

for all $j \in \mathcal{J}, k \in \tilde{\mathcal{K}}$. Hence from (5.46), (5.53) and (5.54) we obtain

$$\boldsymbol{\lambda}^j = -\mathbf{R}^j \sum_{k \in \tilde{\mathcal{K}}} p_{\hat{\mathbf{S}}}(k)\mathbf{f}_k^j, \quad \forall k \in \tilde{\mathcal{K}}. \tag{5.55}$$

Finally, by letting the corresponding $L \times 1$ vector $\mathbf{f}_k^j$ be the $\mathbf{0}$-vector, whenever $k \in K \setminus \tilde{\mathcal{K}}$ we conclude that

$$\boldsymbol{\lambda}^j = -\mathbf{R}^j \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k)\mathbf{f}_k^j, \quad \forall k \in \mathcal{K}. \tag{5.56}$$

Clearly, $\mathbf{f}_k^j \in \mathbb{R}_+^L$ for every $k \in \mathcal{K}$ and $j \in \mathcal{J}$. To complete the proof we need to show that $\sum_{j=1}^{J} \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k)$ for every $k \in \mathcal{K}$. We consider two cases.

1. $k \in \mathcal{K} \setminus \tilde{\mathcal{K}}$: For every $k \in \mathcal{K} \setminus \tilde{\mathcal{K}}$, we have that

$$\sum_{j=1}^{J} \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k), \tag{5.57}$$

since $\mathbf{0} \in \mathcal{T}_k$ for every $k \in \mathcal{K}$.

2. $k \in \tilde{\mathcal{K}}$: From (5.54), and since $\mathbf{E}(\omega', \tau) = \sum_{j=1}^{J} \mathbf{E}^j(\omega', \tau)$, for all $k \in \tilde{\mathcal{K}}$ we have

$$\begin{aligned}
\sum_{j=1}^{J} \mathbf{f}_k^j &= \lim_{i\to\infty} \left\{ \sum_{\substack{\tau \in \{1,\ldots,t_i\} \\ \text{s.t. } \hat{\mathbf{S}}(\omega',\tau) = \mathbf{S}^{(k)}}} \frac{1}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega', \tau) \mathbf{E}(\omega', \tau) \right\} \\
&= \lim_{i\to\infty} \left\{ \frac{1}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \sum_{\mathbf{c}\in\mathcal{T}_k} \sum_{\mathbf{Q}\in\mathcal{Q}} \sum_{\substack{\tau \in \{1,\ldots,t_i\} \\ \text{s.t. } \hat{\mathbf{S}}(\omega',\tau) = \mathbf{S}^{(k)}, \\ \mathbf{E}(\omega',\tau) = \mathbf{c}, \\ \mathbf{Q}^{\mathbf{c}}(\omega',\tau) = \mathbf{Q}}} \mathbf{Q}\ \mathbf{c} \right\} \\
&= \lim_{i\to\infty} \left\{ \sum_{\mathbf{c}\in\mathcal{T}_k} \sum_{\mathbf{Q}\in\mathcal{Q}} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \mathbf{Q}\ \mathbf{c} \right\} \\
&= \lim_{i\to\infty} \left\{ \sum_{\mathbf{c}\in\mathcal{T}_k} \sum_{\mathbf{Q}\in\mathcal{Q}} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \frac{t_i}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \mathbf{Q}\ \mathbf{c} \right\}. \qquad (5.58)
\end{aligned}$$

Since each of the terms involved in the sum are non-negative, and since the outer limit exists, it follows that each of the product terms in the limit are bounded. Further, since $\frac{n_{\hat{\mathbf{S}}}(\omega', t_i; k)}{t_i}$ converges to a non-zero value, we may extract a converging subsequence such that $\lim_{i\to\infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \right\}$ exists, and therefore

$$\sum_{j=1}^{J} \mathbf{f}_k^j = \sum_{\mathbf{c}\in\mathcal{T}_k} \sum_{\mathbf{Q}\in\mathcal{Q}} \lim_{i\to\infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \right\} \frac{1}{p_{\hat{\mathbf{S}}}(k)}\ \mathbf{Q}\ \mathbf{c}. \qquad (5.59)$$

Note also that $\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{t_i}$ exists and can be written as a finite sum of existing limits as

$$\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{t_i} = \lim_{i\to\infty} \sum_{\mathbf{Q}\in\mathcal{Q}} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} = \sum_{\mathbf{Q}\in\mathcal{Q}} \lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i}, \qquad (5.60)$$

where we made use of the fact that the limit $\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i}$ exists. As discussed in Section 5.4, for all $\mathbf{c} \in \mathcal{T}_k$, the quantity $n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c}) \neq 0$ as $t \to \infty$. Hence, we can write

$$\lim_{i\to\infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \right\} = \lim_{i\to\infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \frac{n_{\hat{\mathbf{S}}}(\omega', t_i; k)}{t_i} \right\}. \tag{5.61}$$

It follows from (5.46) and (5.60) that

$$\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} = \frac{\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{t_i}}{\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}}(\omega', t_i; k)}{t_i}}$$

exists. Let this limit be equal to

$$\gamma_k^{\mathbf{c}} := \lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)}. \tag{5.62}$$

From (5.46), (5.47) and (5.62) it follows that the individual limits in (5.61) exist. Hence, it can be written as

$$\lim_{i\to\infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \right\} = P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q} | \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}]\, \gamma_k^{\mathbf{c}}\, p_{\hat{\mathbf{S}}}(k). \tag{5.63}$$

By replacing (5.63) in (5.59) we get

$$\begin{aligned} \sum_{j=1}^{J} \mathbf{f}_k^j &= \sum_{\mathbf{c}\in\mathcal{T}_k} \sum_{\mathbf{Q}\in\mathcal{Q}} \gamma_k^{\mathbf{c}}\, P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q} | \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}]\, \mathbf{Q}\, \mathbf{c} \\ &= \sum_{\mathbf{c}\in\mathcal{T}_k} \gamma_k^{\mathbf{c}} \tilde{\mathbf{Q}}_k^{\mathbf{c}} \mathbf{c}, \end{aligned} \tag{5.64}$$

where (5.64) follows by employing (5.10). Consequently, it follows that

$$\sum_{j=1}^{J} \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k),$$

and the proof is complete.

■

Chapter 6

Concluding Remarks

6.1 Thesis Contributions

The main contribution of this thesis is to shed light in the scheduling problem by understanding whether it is preferable to allow more concurrent transmissions at lower rates or fewer concurrent transmissions at higher rates. We studied this trade-off under various performance objectives.

In Chapter 2 we considered static networks comprising of a set of, always back-logged, sources, each *multicasting* traffic to its corresponding destinations. First, we considered the problem of *joint scheduling and rate control* under the objective of *sum throughput* maximization and then *proportional fairness*. We introduced an optimal joint *scheduling and rate control* policy that assigns a *probability distribution* to the set of feasible rate control and scheduling decisions. In the case of proportional fairness, we *restricted* the set of feasible rate control and scheduling decisions to either activation of one transmitter at a time, in a pure Time Division Multiple Access (TDMA) manner or all together. Under this restricted framework we obtained the optimal probability distribution for the restricted set of actions so that the average rate of a receiver is proportionally fair. The corresponding optimal policy for the special cases of unicast and broadcast traffic follows from our analysis. These results were also published in [14] and [23].

Next, in Chapter 3 we considered *time-varying* wireless networks and a broader class of *utility functions* that are strictly increasing, continuously differentiable, and concave functions of the average rate. These utility functions include the utilities of total throughput maximization and proportional fairness studied in Chapter 2. We considered the problem of scheduling a set of multicast sources with the objective to maximize the total user utility. We assumed policies that do not accurately know the current channel conditions but rather base their decisions on an *estimate* of the channel state. We obtained an *online* algorithm that yields the *optimal* transmission rate among all policies with the same estimate of the current channel state. In the case where more than one rate allocations is optimal, the optimal algorithm selects the one that minimizes the power sum. We proved optimality of the proposed algorithm through the theory of stochastic approximation. A related work corresponding to the case of perfect channel estimation appeared in [15].

Unlike Chapters 2 - 3 where saturated networks were considered, in Chapter 4 we assumed that the network sources have a finite amount of data traffic to send to their corresponding destinations. We considered unicast traffic. We studied the problem of *joint scheduling and rate control* in wireless networks with the objective to minimize the required time for all network sources to deliver the traffic demands to their respective destinations. We considered both *static* and *time-varying* networks. In the static network case we mapped the minimum-length scheduling problem into finding a *shortest path* on a Directed Acyclic Graph (DAG). In the time-varying network case the corresponding problem was mapped to a *stochastic shortest path* and an optimal solution was provided through stochastic control methods. The case of time-invariant channels was published in [8].

Unlike the saturated queue assumption of previous chapters, traffic in reality is bursty and guaranteeing stability of the network is of paramount importance. Thus, in Chapter 5 we turned our focus on the objective of *stable throughput* maximization for a set of commodities of *anycast* traffic for *multi-hop* wireless networks. Each commodity is assigned a weight of preference. We introduced a *joint scheduling and routing* policy, having access to only an *estimate* of the channel state. We incorporated the physical layer into the scheduling and routing decisions through the SINR interference model. We assumed that the SINR thresholds that determine the outcome of a transmission are fixed, i.e., the transmission rate is *constant* and each packet is assumed to be comprised of a fixed number of bits. We characterized the stable throughput region of the network. Moreover, we showed that the introduced policy is optimal with respect to maximizing the stable throughput region of the network, irrespective of the weight assignment, within a broad class of stationary, non-stationary, and anticipative policies. These results appeared in [34], [37], [38], and [39].

6.2 Future Work

In this thesis, we studied the scheduling problem under various contexts and assumptions. However, there are still a lot of questions on this subject awaiting to be answered and thus, we conclude this thesis with a few potential future directions.

1. **Distributed Solutions**

   One of the basic assumptions in this thesis was the existence of a centralized scheduler. This assumption allowed us to obtain optimal results. However, in practice the existence of such a controller may be infeasible. By using our centralized results as benchmarks, it will be of great interest to investigate alternative solutions that are distributed.

2. **Modeling the Interference**

   In this thesis we employed the SINR model to account for the interference. This model albeit tractable and widely used, it is approximate and assumes that the interference behaves as Additive White Gaussian Noise. Given the strong coupling between the physical layer and the layers above it, it is natural that the network performance can be improved by modeling the physical layer in a more accurate fashion. It will be of great merit to obtain alternative models that describe the physical layer properties more appropriately.

3. **Dealing with Non-Stationary and Non-Ergodic Behaviors**

   Commonly employed performance measures in communication networks are those of utility maximization, stability, and delay. However, as we mentioned previously, these performance measures depend critically on the assumption that the wireless channel process is stationary and ergodic. In reality, fading effects are rather unpredictable, network nodes have finite energy reservoirs, and may move in arbitrary patterns. Thus, it is likely to observe a non-ergodic and non-stationary behavior. It will be of interest to study and explore new measures that can be meaningful in describing performance of wireless systems under conditions of *non-stationarity* and *non-ergodicity*.

Appendix A

Rate Formulas

Assume that a transmission is successful if the received SINR exceeds a threshold γ, i.e., SINR $\geq \gamma$. By successful transmission we mean that for a given modulation scheme the probability that a bit is received erroneously is below a target probability of bit error P_b. It follows from the principles of wireless communications [28] that the threshold value γ is a decreasing function of the probability of bit error for a given modulation. Moreover, the threshold γ depends on the *transmission rate*. In this section, we will exemplify this by relating the maximum transmission rate for successful communication to the SINR threshold γ for the specific case of M-ary Phase Shift Keying (PSK) modulation with symbol rate control where the target probability of bit error is fixed. However, rate expressions under different modulation schemes can be obtained in a similar fashion.

Let W be the available bandwidth of the communication. Let also $T_{\rm s}$ be the symbol duration, $R_{\rm s} = \frac{1}{T_{\rm s}}$ be the symbol rate and M be the number of distinct symbols in the alphabet. From [28], for general pulses the symbol rate must satisfy $R_{\rm s} = W/k$ for some constant k. Here we assume that $k = 1$, which results in a maximum symbol rate value $R_{\rm s}^{\max}$ equal to $R_{\rm s}^{\max} = W$. Under M-ary PSK modulation [28] the relation between the SINR threshold γ and the symbol rate R_s so that the target probability of bit error is P_b is given by

$$\gamma = \begin{cases} \frac{R_s}{2}\left[Q^{-1}(P_b)\right]^2, & M = 2 \\ 2R_s\left[Q^{-1}(P_b)\right]^2, & M = 4 \\ R_s\left(\frac{1}{2\sin^2(\pi/M)}\right)\left[Q^{-1}\left(\frac{P_b\log(M)}{2}\right)\right]^2, & M > 4, \end{cases}$$

where $Q(x)$ is defined to be the probability that a Gaussian random variable with zero mean and unit variance exceeds the value x. Hence, the maximum bit rate under M-ary PSK modulation for any fixed M is given by

$$R^{M,\,\mathrm{PSK}}(\gamma) = \begin{cases} \min\left\{\frac{2\,\gamma}{[Q^{-1}(P_b)]^2}, R_s^{\max}\right\}, & M = 2 \\ 2\min\left\{\frac{\gamma}{2[Q^{-1}(P_b)]^2}, R_s^{\max}\right\}, & M = 4 \\ \log(M)\min\left\{\frac{2\sin^2(\pi/M)\gamma}{\left[Q^{-1}\left(\frac{P_b\log(M)}{2}\right)\right]^2}, R_s^{\max}\right\}, & M > 4. \end{cases}$$

Moreover, by further optimizing the distinct number of symbols M the maximum bit rate is given by

$$R^{\mathrm{PSK}}(\gamma) = \max_{M=2,4,\ldots} R^{M,\,\mathrm{PSK}}(\gamma). \tag{A.1}$$

In Fig. A.1 we illustrate the maximum achievable rate under M-ary PSK modulation ($M = 2, 4, 8, 16, 32, 64$) as a function of the SINR threshold γ when the bandwidth equals 1 Hz (spectral efficiency) and the target probability of bit error is $P_b = 10^{-6}$. The corresponding rate when the symbol rate and the number of distinct symbols, M, are jointly controlled is also shown in the figure by the dashed line.

We observe that the rate function is a piecewise increasing function of the SINR threshold, where each increasing segment corresponds to a different value of M. Further,

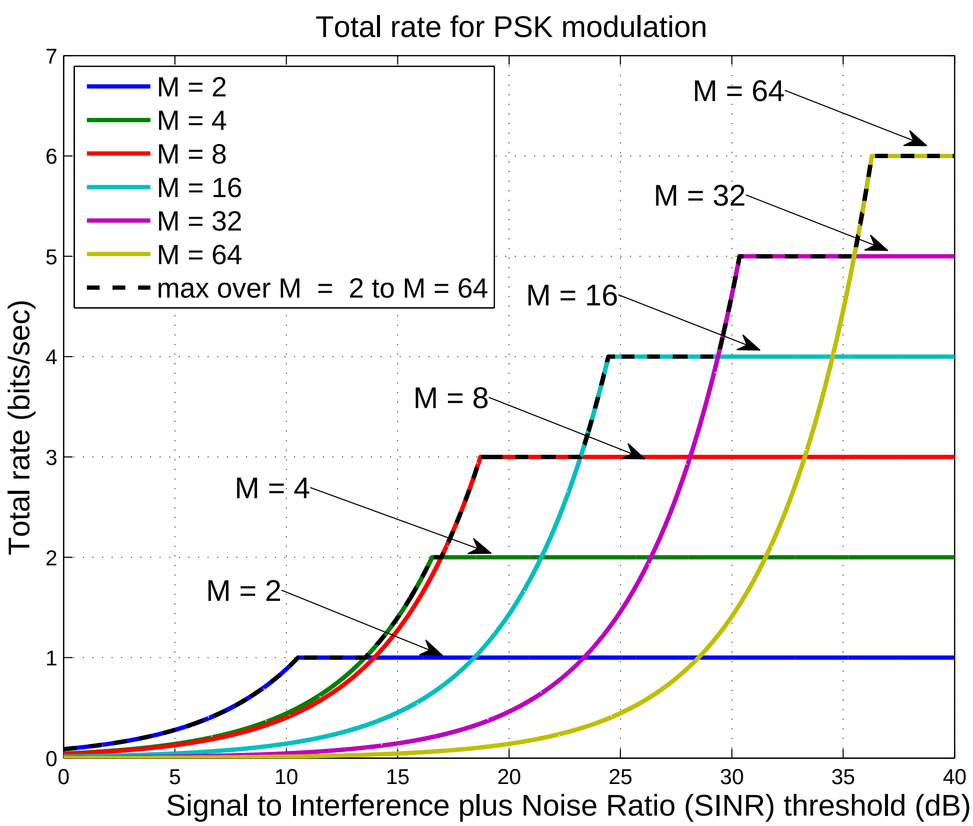

Figure A.1: The maximum achievable rate (bits/sec) as a function of the SINR threshold γ (dB) for M-ary PSK modulation, i.e., 2-PSK, 4-PSK, 8-PSK, 16-PSK, 32-PSK and 64-PSK. ($W = 1$ Hz, $P_b = 10^{-6}$)

from Fig. A.1 we observe that the maximum transmission rate over all M depicted by the dashed line is an *increasing* function of the SINR threshold.

In the literature, the single-user Shannon formula is commonly used to tie γ with the corresponding maximum achievable rate. The Shannon rate $R^{\mathrm{Sh}}(\gamma)$ that corresponds to a given threshold is given by the following expression

$$R^{\mathrm{Sh}}(\gamma) = W \log_2 (1 + \gamma) \, . \tag{A.2}$$

This formula is an upper bound on the achievable rate that can be achieved asymptotically through coding. It further assumes that the probability of bit error of the communication approaches zero. Although both expressions are approximate for multi-user systems, they provide useful insights on how the physical layer channel conditions relate to the maximum achievable rate.

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