Mathematical modelling in upper secondary school

A case study of Norwegian curriculum discourses

Ingeborg Katrin Lid Berget

Thesis for the degree of Philosophiae Doctor (PhD)
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Foreword

When I worked as a mathematics teacher in an upper secondary school, my students sometimes asked me, "What do we need this for? Why are we learning this?". Some of my students which were studying sports education did not find mathematics to be as relevant to their lives as sports-related school subjects. I strived to make mathematics relevant, as I found it important both in the short and long term for the students.

I was once asked on short notice to be a substitute teacher for another mathematics class, the second year of "Practical mathematics, 2P". I was surprised by the textbook I was handed because it did not include much mathematical content that had not already been covered in previous school years. Was it all repetition, or was there an overarching aim for the school subject that I did not grasp from the experience as a substitute teacher?

A year later, I worked as a lecturer in mathematics for student teachers. There I was involved in a course called *Mathematical Problem Solving and Modelling*. I did not have any experience with mathematical modelling in mathematics education from my own education. I dove into the literature on mathematical modelling in mathematics education and discovered an entire field focused on making mathematics teaching relevant. Later, when I reread the curriculum for the mathematics class in which I was a substitute teacher, I realised that modelling was one of the main content areas. How could I have missed this? My first-hand experience led to more questions. Was I the only mathematics teacher who did not have experience with mathematical modelling? How long had it been a part of the Norwegian curriculum? How is modelling taught? Is it presented in textbooks? Does it

appear on the national exam? I wanted to find out more, and this desire resulted in this work, which included more questions and development.

I thank my colleagues and students in the upper secondary school and Volda University College for their interesting discussions and for sharing educational challenges. Also, to the teachers and students who participated in this study for generously welcoming me into their classrooms: thank you for including me in your teaching and learning of mathematical modelling!

Special thanks to my supervisors Hilde Opsal at Volda University College and Mette Andresen at the University of Bergen for their questions and critical considerations over the years.

I also want to thank my crew at home; my husband Aleksander and our three boys – Haakon, Emil and Olai – who are reminding me of what is most important in life: solid ground.

Ingeborg Katrin Lid Berget
Volda, Norway
February 2023

Abstract

The research field of mathematical modelling within mathematics education has maturated, with the participation of researchers from all parts of the world. Different perspectives on mathematical modelling have emerged, and a gap has been identified between official regulations and the educational debate, on one hand, and the everyday teaching practice of mathematical modelling, on the other hand. In Norway, mathematical modelling has been an explicit part of the curriculum in upper secondary schools for the last 30 years. Through this case study of Practical mathematics 2P in the second year of the Norwegian upper secondary school, this main research question has been studied: Which perspectives on mathematical modelling can be identified in the different discourses of the mathematics curriculum, and which discursive and social practices can be identified within and between the discourses? These discourses are included in the study: The *ideological* (research literature and frameworks), the *intended* (the Norwegian curricula R94, L97, LK06 and LK20), the *instructional* (textbook tasks), the *perceived* (teacher interviews), the *enacted* (classroom observations) and the *assessed* (exam tasks) curriculum. The study is framed within case study theory, framework for curriculum assessment, Discourse Analysis and Positioning Theory. Moreover, a modelling cycle is central to the task analysis, and a theory of modelling perspectives is included in the meta-analysis. In this thesis, new approaches within mathematics education research are developed: To apply Positioning Theory to analyse the teaching of mathematical modelling, and a comprehensive curriculum assessment of mathematical modelling drawing on discourse theory. The different curriculum discourses are seen in relation to each other, and it illuminates how they influence each other and how discursive and social constructions allow certain understandings of mathematical modelling to develop.

In the *ideological* and the *intended* curriculum, I identified a diversity of modelling perspectives, which require students to make assumptions and analyse and develop mathematical models to solve everyday problems. I did not to a large extent find traces of *consumption* of these curriculum discourses within the others. In the *instructional* curriculum, the textbook tasks can be solved by given procedures. Earlier given exam tasks were included in the textbooks. In the *perceived* curriculum teachers expressed they were not familiar with mathematical modelling from their own education and did not recognise mathematical modelling as relevant for mastering real-life situations. Within the *enacted* curriculum, mathematical modelling was connected to one content area of mathematics, and the teachers referred to the textbook and the exam to justify their choices regarding the teaching of mathematical modelling. That is, the *assessed* curriculum was consumed in the production of the *instructional, perceived* and *enacted* curriculum. Since the practices of the *assessed* curriculum, a 5-hour written exam, do not provide for solving holistic modelling tasks, this might lead to the development of certain understandings of mathematical modelling. Even if teachers acknowledged open-ended modelling tasks as relevant to the students' lives, they did not experience that the curriculum discourses allowed or provided such tasks. At the closure of the thesis, I suggest how to reduce the gap between the educational debate and the everyday practices of teaching and learning mathematical modelling by pointing out the power relations revealed in this study.

Samandrag (Norwegian abstract)

Innanfor matematikkdidaktisk forsking har matematisk modellering utvikla seg som forskingsfelt, der forskarar frå alle verdsdelar er aktive. Ulike perspektiv på matematisk modellering har vakse fram, og det er identifisert eit gap mellom korleis matematisk modellering er framstilt i forsking og undervisingsdebattar på den eine sida, og korleis matematisk modellering blir arbeida med i klasseromma på den andre sida. I Noreg har matematisk modellering vore ein del av læreplanen i matematikk for vidaregåande skule dei siste 30 åra. I denne avhandlinga, ein case-studie av praktisk matematikk 2P i den norske vidaregåande skulen, blir dette forskingsspørsmålet svara på: Kva perspektiv på matematisk modellering kan identifiserast i ulike diskursar av læreplanen, og kva for diskursive og sosiale praksisar kan identifiserast i og mellom diskursane? Desse diskursane er inkluderte i studien: Den *ideologiske* (forskingslitteratur og rammeverk), *intenderte* (dei norske læreplanane R94, L97, LK06 og LK20), *instruktive* (lærebokoppgåver), *oppfatta* (lærarintervju), *utøvde* (klasseromsobservasjonar) og *vurderte* (eksamensoppgåver) læreplanen. Som teoretisk rammeverk har eg tatt utgangspunkt i *case study theory, framework for curriculum assessment*, *discourse analysis* og *positioning theory*. I tillegg er ein modelleringssyklus sentral i analysane av matematikkoppgåvene, og ein teori om ulike modelleringsperspektiv er inkludert i metaanalysen. I denne avhandlinga er det utvikla nye tilnærmingsmåtar innanfor matematikkdidaktisk forsking: Å nytte positioning theory til å analysere undervising av matematisk modellering, og gjennomgripande læreplanvurdering av matematisk modellering ved hjelp av diskursteori. I studien har dei ulike læreplandiskursane blitt sett i samanheng, og det er trekt fram korleis dei påverkar kvarandre, og korleis sosiale og diskursive strukturar kan føre til at visse forståingar av matematisk modellering utviklar seg.

I den *ideologiske* og den *intenderte* læreplanen har eg identifisert eit mangfald av modelleringsperspektiv som krev at elevane sjølv må ta avgjersler, analysere og utvikle matematiske modellar, og nytte desse til å løyse problem utanfor skulematematikken. Eg har ikkje funne særleg spor av *konsumering* av desse læreplandiskursane i dei andre. I den *instruktive* læreplanen kan lærebokoppgåvene bli løyste ved å hugse og å bruke framgangsmåtar. Tidlegare gitte eksamensoppgåver er inkluderte i lærebøkene. I den *oppfatta* læreplanen uttrykte lærarar at dei ikkje kjente til matematisk modellering frå eiga utdanning innanfor undervising av matematikk, og såg ikkje på matematisk modellering som viktig for at elevane skal meistre kvardagssituasjonar. I den *utøvde* læreplanen blei matematisk modellering knytt til eitt matematisk kunnskapsområde, og lærarane viste til læreboka og eksamen for å argumentere for vala knytt til undervising av matematisk modellering. Med andre ord blei den *vurderte* læreplanen konsumert i produksjonen av den *instruktive, oppfatta* og *utøvde* læreplanen. Sidan praksisane i den *vurderte* læreplanen, ein 5-timers skriftleg eksamen, ikkje legg til rette for å løyse holistiske modelleringsoppgåver, kan dette føre til at visse forståingar av matematisk modellering oppstår. Sjølv om lærarar vedkjente at opne modelleringsoppgåver hjelper elevane til å sjå matematikk som relevant for livet utanfor skulen, opplevde dei ikkje at læreplandiskursane tillet eller la til rette for arbeid med slike oppgåver. I slutten av denne avhandlinga kjem eg med nokre forslag til korleis ein kan minske gapet mellom forskings- og utdanningsdebatten og korleis matematisk modellering kjem til uttrykk i klasserommet, ved å peike på maktrelasjonar som er avdekka i denne studien.

Contents

1 Introduction

1.1 Background

Twenty years ago, Blum (2002) noted that there was “a substantial gap between the ideals of educational debate about modelling on the one hand, and everyday teaching practice on the other hand” (p.150). Years have gone by, but although there are good reasons for teaching modelling and “although an increasing number of countries include modelling explicitly in their standards and curricula, there is still a considerable gap between official regulations and the educational debate, on one hand, and everyday teaching practice, on the other hand” (Blum & Pollak, 2018, p. viii).

Galbraith (2012) argued that the lack of common use of terms among researchers in the field of mathematical modelling in education is one possible reason for the gap between how modelling is discussed in the educational debate and how it is enacted in classrooms: “Confusing voices stand to compromise progress at any level, for if those working in a field give mixed messages, why should others listen to their advocacy” (Galbraith, 2012, p. 13). Therefore, Galbraith (2012) tried to clarify some concepts by categorising using the two different perspectives on mathematical modelling presented by Julie and Mudaly (2007) as *modelling-as-vehicle* and *modelling-as-content*. These terms are presented in Chapter 2.1 of this thesis.

In a study of the Norwegian *intended* curriculum, Berget and Bolstad (2019) compared the *intended* curriculum that was to expire with the proposal for a new *intended* curriculum in terms of the two perspectives on modelling which Galbraith (2012) built on. The understanding of mathematical modelling was not clearly expressed in either of the *intended* curricula, but by analysing every use of the term “model*”, we found that both perspectives were included. In the recent *intended* curriculum, which was implemented in 2020, mathematical modelling was expressed as a *core competence* in school mathematics at all levels (grades 1–13) and seemed to form a large part of the *intended* curriculum (Berget & Bolstad, 2019). Indeed, modelling has been an explicit part of the *intended* curriculum in upper secondary schools in Norway since 1994 (The Norwegian Royal Ministry of Church and Education and Research, 1994); however, to date, no further research on the implementation of mathematical modelling has been conducted.

In other countries, the implementation of mathematical modelling has been studied through textbooks and/or national exam tasks, for example Sweden (Frejd, 2011), Denmark (Jessen & Kjeldsen, 2021), Turkey (Urhan & Dost, 2018), and Australia and Iran (Gatabi et al., 2012). Teachers' conceptions of mathematical modelling have commonly been explored through teacher interviews, for example in Sweden (Frejd, 2012; Ärlebäck, 2010), China (Xu et al., 2022), South Africa (Jacobs & Durandt, 2017) and Turkey (Özdemir et al., 2017). Research on the implementation of mathematical modelling in German-speaking countries has been conducted, among others, by Vorhölter et al. (2019). Extant literature involving teacher interviews, questionnaires, textbooks and student tests provides knowledge about the implementation of mathematical modelling is presented in Chapter 4. The findings indicate the gap noted by Blum and Pollak (2018). However, how mathematical modelling is communicated in the daily teaching of mathematical modelling in the classroom discourse is rarely explored. This topic is addressed in this dissertation and is understood through classroom observations, alongside teacher interviews, questionnaires, textbooks and exam tasks.

1.2 Aims and research questions

The aims and research questions for this study are presented in Table 1, which includes the overarching research question for the entire study and the more specific research questions included in the three articles that are part of this dissertation.

Study purpose	To gain a deeper understanding of how mathematical modelling is expressed in the teaching and learning of mathematics.		
Main research question	Which perspectives on mathematical modelling can be identified in the different discourses of the mathematics curriculum, and which discursive and social practices can be identified within and between the discourses? • the *ideological* curriculum discourse (e.g., frameworks: PISA and KOM) • the *intended* curriculum discourse (curriculum document) • the *instructional* curriculum discourse (textbook tasks) • the *perceived* curriculum discourse (teacher interviews) • the *enacted* curriculum discourse (classroom observations) • the *assessed* curriculum discourse (exam tasks)		
	Article 1	Article 2	Article 3
Part of curriculum	*Instructional* and *assessed*	*Ideological* and *perceived*	*Enacted*
Title	Mathematical modelling in textbook tasks and national examination in Norwegian upper secondary schools	Mathematical modelling in the discourses of the KOM and PISA frameworks and teacher interviews	Identifying positioning and storylines about mathematical modelling in teacher–student dialogues in episodes from two upper secondary classrooms
Research questions	Which steps in the modelling cycle are needed to solve textbook modelling tasks and tasks from national examinations?	What possible tensions in the approaches to mathematical modelling can be identified from a discourse analysis of relevant framework documents and interviews with four teachers?	What are the storylines of the teacher–student dialogues in the teaching of mathematical modelling in which students take the initiative to positioning? How do teachers position the students to give them agency when working with mathematical modelling?

Sample	514 tasks from textbooks by the three largest textbook publishers in Norway and 112 tasks from 10 exams	Interviews with 4 teachers and PISA and KOM frameworks	Episodes from two classrooms when teaching mathematical modelling.
Data	Mathematics tasks	Transcription of interviews and policy documents	Transcription of classroom audio recordings and observation schemas.
Analysis	Document analysis tool developed from a modelling cycle	Discourse analysis	Discourse analysis, Positioning Theory

Table 1: Overview of the study

The identification of the perspectives on mathematical modelling expressed in the main research question is based on Blum's (2015) presentation of six different perspectives on mathematical modelling that are grounded in aims, activities and modelling cycles. These three indicators operate as guides to identify perspectives on mathematical modelling in a meta-analysis of the results from the three articles. The *intended* curriculum is not analysed in any of the three articles but is included in the discussion in article 1. The *intended* curriculum is also presented in Chapter 2.5 in this extended summary and included in the meta-analysis as described in Chapter 5.4.4.

The interactions between the curriculum discourses mentioned in the main research question will be explored in the discussion in Chapter 7. The discussion is based on the findings in the three articles, the meta-analyses and the researcher's interpretation of the diverse experiences from the case study, and the theoretical and empirical background presented in this extended summary.

1.3 Overview of the dissertation

In Chapter 2, I first present the different aims for including mathematical modelling in mathematics education, modelling cycles and examples of modelling activities; next, I outline Blum's (2015) six different perspectives on mathematical modelling connected to these three indicators. The six perspectives (Section 2.4) are included in the presentation of empirical background (Chapter 4) and formed the basis of the meta-analysis (presented in 5.4.4, results in 6.4).

Chapter 3 outlines the further theoretical perspectives of each of the three articles, in addition to what is presented in Chapter 2: Curriculum assessment, discourse theory and Positioning Theory. In this extended summary, I discuss the combination of the perspectives (Section 3.4).

Chapter 4 presents the empirical background of the dissertation. Previous research is presented and discussed concerning the perspectives on mathematical modelling presented in Section 2.4 within each of the curriculum discourses outlined in Section 3.1.

In Chapter 5, the methodological issues are addressed. This chapter presents the methodological approach, research methods and data generation, analysis and meta-analysis. Ethical considerations are also included.

Chapter 6 describes the results of the three articles related to the different curriculum discourses presented in Section 3.1. Article 1 explores the *instructional* curriculum (textbook tasks) and the *assessed* curriculum (exam tasks), which is seen in light of the *intended* curriculum. The *ideological* curriculum (PISA and KOM frameworks) and *perceived* curriculum (teacher interviews) are investigated in article 2. In article 3, the *enacted* curriculum (classroom observation) is analysed. Further, the result from the meta-analysis is presented as a response to the first part of the main research question, identifying modelling perspectives in the different curriculum discourses.

In Chapter 7, the second part of the main research question is discussed in light of the results from the three articles and meta-analysis presented in Chapter 6, the perspectives on mathematical modelling presented in Chapter 2 and the empirical background given in Chapter 4 using the theoretical lens presented in Chapter 3. Conclusions are drawn from this discussion. Retrospective reflections are made, and implications for practice and future research are outlined.

2 Perspectives on mathematical modelling

Mathematical modelling can be expressed as the process of describing a real-world problem in mathematical terms, and by working mathematically to solve the problem. Within mathematics education, mathematical modelling has since the 1960s gained research interest and has reached maturation as a research discipline (Blum et al., 2007); it "is presently a strong and internationally well-recognized research field in mathematics education" (Borromeo Ferri, 2021, p. 103). The teaching of mathematical modelling has expanded in mathematics classrooms worldwide (Burkhardt, 2006, p. 178). There is not an unambiguous perception of mathematical modelling in mathematics education, and it comprises many different practices (Jablonka & Gellert, 2010; Kaiser & Sriraman, 2006). Several aims for teaching and learning mathematical modelling have been expressed in the literature; accordingly, different perspectives on mathematical modelling have been developed.

Kaiser and Sriraman (2006) presented five different perspectives based on previous research and discussions at conferences. The perspectives are not seen as mutually exclusive (Kaiser et al., 2007); nonetheless, they effectively demonstrate the differences in the perspectives on mathematical modelling in research in an organised way (Abassian et al., 2020). These normative theoretical approaches (Kaiser et al., 2007) are useful for communicating across the research field of mathematical modelling in education (Kaiser, 2006) and were developed to distinguish between different modelling perspectives in research papers. Inspired by this, Blum (2015) conceptualised perspectives on mathematical modelling based on its aims, the types of activities, and modelling cycles. I will now first present different aims for including mathematical modelling in mathematics education, followed by a presentation of different modelling activities, and cycles of the modelling process. Finally, I present Blum's (2015) six perspectives on mathematical modelling and discuss the Norwegian intended curriculum concerning these modelling perspectives. Blum's (2015) perspectives are chosen for this dissertation because the aim here is to identify the conceptions of mathematical modelling in the different curriculum discourses, and not only in research papers as Kaiser and Sriraman's (2006) perspectives.

2.1 Different aims for mathematical modelling in mathematics education

Blum (2015) presented four justifications, or aims, for including mathematical modelling in mathematics education: 1) pragmatic, 2) formative, 3) cultural and 4) psychological. Within the pragmatic justification, students should be engaged with concrete, authentic examples from society because this teaches them to put mathematics into play in argumentation in everyday life. The formative justification includes cognitively rich examples accompanied by meta-cognitive activities. The aim is for students to become active and reflective learners. The cultural justification emphasises the role of mathematics, particularly how strongly it shapes the world while often remaining hidden and invisible. "The role of mathematics and its relation to the real world must be made more conscious" (Blum, 2015, p. 82), and the ability to de-mathematise situations and models, and to identify mathematical structures presented by others is seen as important. The psychological justification concerns how modelling can motivate the learning of mathematics. Within this justification, authentic problems are not very significant; however, it is important to be honest, and not present constructed, non-realistic situations as authentic (Blum, 2015).

Blum (2015) argued that the pragmatic justification deals with mathematics as an aid for the real world, whereas within the three others, the real-world acts as an aid for mathematics in a broad sense. Later, Niss and Blum (2020) formulated, in simplistic terms, the two distinct overarching reasons for including mathematical modelling as a significant part of mathematics: 1) *mathematics for the sake of modelling* and 2) *modelling for the sake of mathematics*. They underlined that these two reasons were not contradictory but rather that each reason prioritised different activities. Within the first, the aim is to learn to perform all parts of the modelling process, and the process of mathematising is emphasised to engage in a real-world situation and express the model in mathematical terms. If learning mathematics is the main goal of the activity, working mathematically using the mathematical model might be the focus. Julie and Mudaly's (2007) two perspectives correspond with these two reasons; they are expressed as 1) *modelling-as-content*, in which the aim is the modelling itself, and 2) *modelling-as-vehicle*, in which modelling is a way of working to achieve other aims, such as developing an understanding of a mathematical concept. These two perspectives were, as mentioned in the introduction of this dissertation, built on by Galbraith (2012) as an overarching guide to classify other perspectives. A third perspective, presented by Barbosa (2006), is *modelling-as-critic*, in which one learns about modelling to be able to critically assess models presented in society. This is related to Blum's (2015) cultural justification, of the role of mathematics in society. Barbosa (2006) emphasised the importance of being able to critically assess models in society, to better master life, which also points to the pragmatic justification.

Blomhøj and Ärlebäck (2018) divided modelling into two overarching aims, expressed as a means for 1) developing modelling competence and 2) learning mathematics. Modelling competency includes both being able to actively build models and critically assess one's own and others' modelling work. Accordingly, this includes both the pragmatic justification, to build and use models to master real-life situations, and the cultural justification, to de-mathematise models used in society. Modelling as a means for learning mathematics expresses the same aim as *modelling for the sake of mathematics*: to learn mathematics, which also includes Blum's (2015) formative, cultural and psychological justifications.

I have now argued that, although expressed in different terms, the other presented aims for mathematical modelling in mathematics education are included in Blum's (2015) four justifications. Further, I will present activities connected to these justifications, and how they indicate the six perspectives on mathematical modelling.

2.2 Different types of modelling tasks

Literature uses different terms to express what provokes modelling activity – a problem, task, exercise, question, or example. This thesis does not focus on comparing the different terms; rather, the terms expressed in the different studies referred to are used.

The four justifications presented by Blum (2015) require specific types of examples; within a justification, two types of examples are possible, which indicates two perspectives on mathematical modelling grounded in the same justification. As presented in Table 2, both the cultural and psychological justifications include two types of examples. The different types of examples are presented by Blum (2015). The formulations presented in Table 2 are also inspired by the description of Blum's (2015) examples given by Xu et al. (2022).

Justification	Examples, modelling activities	Perspective
Pragmatic	Concrete, authentic examples (Pragmatic | authentic)	Applied modelling
Formative	Cognitively rich examples, accompanied by meta-cognitive activities (Formative | cognitively rich)	Educational modelling
Cultural	Authentic examples that show students how strongly mathematics shapes the world (Cultural, with an emancipatory intention | authentic)	Socio-critical modelling
	Epistemologically rich examples that shed light on mathematics as a science (Cultural, concerning mathematics | epistemologically rich)	Epistemological modelling
Psychological	Interesting examples for motivational or illustration purposes to make mathematics more marketable for students (Psychological, with marketing intention | motivating)	Pedagogical modelling

	Mathematically rich examples that make certain mathematical topics more comprehensible (Psychological | mathematically rich)	Conceptual modelling

Table 2: Blum's (2015) justification including aims and types of required examples and modelling perspectives. The given descriptions are inspired by Xu et al. (2022).

Blum (2015) noted that "examples are not good or bad per se; it depends on their purpose" (p. 82). While Borromeo Ferri (2018) characterises authenticity, in contrast to pseudo-reality, as one criterion of a modelling problem. She claims that mathematical modelling does not mean "having a *pseudo-realistic problem*, in which all data are given, or you only have to exercise algorithms" (Borromeo Ferri, 2018, p. 13), such as the second task presented in the interview in Appendix A in this thesis. Borromeo Ferri's (2018) understanding of a modelling problem points to both a pragmatic and a cultural justification because the connection to an authentic, real-life situation is seen as crucial. Drawing on Blum (2015), a modelling problem based on a non-realistic fantasy context, or a pseudo-reality, could be useful within the psychological justification but might not be a good modelling problem within the pragmatic justification. However, as pointed out by Vos (2018), students might find mathematics less relevant if unrealistic real-world contexts are used. If fantasy contexts make the students less motivated, they might not be a good choice within the psychological justification either.

Further, I will present different modelling cycles, models of the modelling process, which is the third indicator Blum's (2015) perspectives are based on, in addition to aims and activities.

2.3 Models of the process of modelling

Several models have been developed to show the modelling process (see e.g., Borromeo Ferri (2018, pp. 20-27)), and different models show emphasis on different parts of the modelling process and different views and perspectives on mathematical modelling. The PISA framework is central in article 2 of this thesis, and I will therefore include a reflection on the modelling cycles presented in the PISA frameworks (2003, 2012 and 2021) and relate it to Blum's (2015) perspectives. In the 2003 PISA framework, the cycle is referred to as the *mathematisation cycle*; however, in later frameworks, the same cycle is referred to as a *modelling cycle*, both within the later PISA framework (OECD, 2018) and by others (e.g. Stacey (2015)).

In modelling cycles within the pragmatic justification, the real world is often separated from mathematics to emphasise modelling as a way of connecting mathematics to the rest of the world or to use mathematics to answer real-world problems – such as the cycle in the 2003 PISA framework (see Figure 1) referred to in article 2.

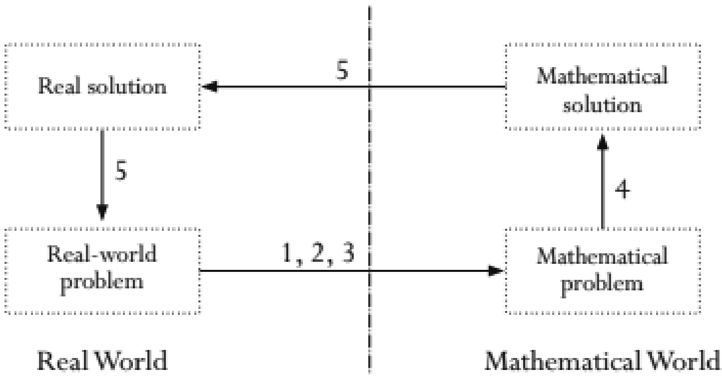

Figure 1: The mathematisation cycle in the 2003 PISA framework (OECD, 2003, p. 38)

The five steps are expressed (OECD, 2003):

1. Starting with a problem situated in reality;
2. Organising it according to mathematical concepts and identifying the relevant mathematics;
3. Gradually trimming away reality through processes such as making assumptions, generalising and formalising, which promote the mathematical features of the situation and transform the real-world problem into a mathematical problem that faithfully represents the situation;
4. Solving the mathematical problem; and

5. Making sense of the mathematical solution in terms of the real situation, including identifying the limitations of the solution (p. 38).

Here, the first part of the modelling process – to go from the real world to the mathematical world – is emphasised and divided into three operations. This communicates a focus on the mathematisation of a real-world situation.

In comparison, in the 2012 PISA framework, the modelling cycle was expressed as four separate processes directly linked to the reporting of student proficiency, represented by arrows in the model of mathematical literacy in practice (see Figure 2).

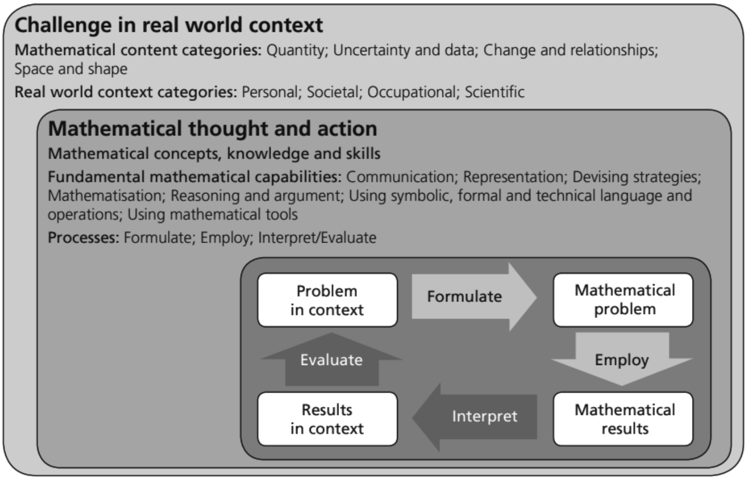

Figure 2: The modelling cycle in the 2012 PISA framework (OECD, 2013, p. 26)

Stacey (2015) argued that the two modelling cycles – from 2003 (Figure 1) and 2012 (Figure 2) – were the same, except for the spatial orientation of the figure and the fact that the five processes from the 2003 modelling cycle are expressed by four arrows in the 2012 modelling cycle. I will point out three other differences. In the modelling cycle in the 2012 PISA framework, "Real-world problem" and "Real solution" from the 2003 cycle are reformulated into "Problem in context" and "Results in context". In the modelling cycle from 2012, no distinction was indicated between the real world and the mathematical world. Moreover, there is an emphasis on the first process in the 2003 cycle, indicating the move from the real-world problem to the mathematical problem in three steps. In PISA 2012, formulation, employment and interpretation/evaluation were assessed as three separate processes. The PISA 2012 framework included perspectives on mathematical modelling in addition to those expressed in the 2003 PISA framework. For example, in the PISA 2003 framework, *a dressed-up task* (Vos, 2020) and other mathematical tasks expressed in a context would not be considered a modelling task because of the descriptions of the real world. In the 2012 PISA framework, decision-making is not explicitly expressed in the modelling cycle as it is in the 2003 framework, and the problem in context is not divided from the mathematical world. This indicates a shift from the pragmatic justification, where involvement in a real-world situation is important, to awareness of solving problems in context as within the pedagogical modelling perspective.

In the latest framework, the 2021 PISA framework (OECD, 2018), mathematical modelling is referred to as a cornerstone of the earlier frameworks. The 2021 cycle (see Figure 3) is presented as a *problem-solving (modelling) cycle* instead of a

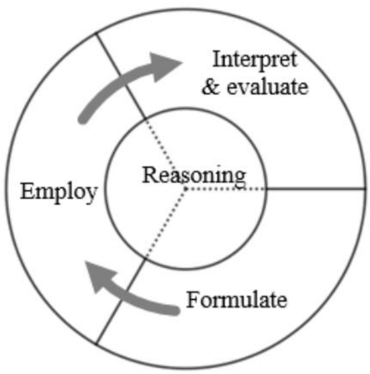

Figure 3: The problem-solving (modelling) cycle in the 2021 PISA framework (OECD, 2018, p. 8)

modelling cycle, as in the earlier framework. It is more general; it includes other parts of the PISA framework (reasoning, mathematical contents, context and selected 21st-century skills) instead of being a tool for presenting the work of the modelling process. It does not highlight the specific processes that include engagement in a real-world situation; instead, the term "mathematical reasoning" is emphasised. This may indicate a change in the perspective on mathematical modelling in the PISA framework, at least the cycle shows less emphasis on the mathematisation process and the connection between a real-world context and mathematics. It rather includes general mathematical processes; mathematical modelling as expressed within the epistemological modelling perspective presented in the next chapter.

2.4 Six perspectives on mathematical modelling

Based on different aims, examples and cycles of mathematical modelling, Blum (2015) presented six perspectives on modelling. These perspectives are used for distinguishing between how mathematical modelling is implemented in different countries (Ferri, 2013). And also to identify mathematicians', mathematics educators' and mathematics teachers' conceptions of mathematical modelling in school (Xu et al., 2022) which is drawn on in this study. Blum's (2015) perspectives were developed for identifying the conceptions of mathematical modelling within the school context by aim, examples of activity and modelling cycles. In the following paragraphs, I will present each of the perspectives.

Applied modelling (understanding and mastering real-world situations)
Within this perspective, a strong focus is on using realistic, authentic, real-world examples (Ferri, 2013). Moreover, this perspective stresses the goal of understanding the real world and finding solutions to real-world problems (Xu et al., 2022). For

example, the modelling cycle by Blum and Leiß (2007) (see article 1) expresses this focus – to find solutions to real-world problems and present "mathematics" apart from "the real world" – to better show the processes of mathematising and interpreting. Within this perspective, the modelling process as a whole is emphasised (Xu et al., 2022), and holistic tasks, which include all the steps of a modelling cycle, are preferable. The first steps of mathematising are seen as important within this perspective (Xu et al., 2022), where the focus is mainly on solving a real-world problem rather than on developing mathematical knowledge, as in the epistemological modelling perspective (Ferri, 2013).

Educational modelling (realising one's own growth in competency)
From the educational modelling perspective, the development of modelling competency is the focus. Here, the goal is to develop the awareness and ability to solve real-world problems by applying mathematics and acquiring a study perspective to analyse and solve real-world problems (Xu et al., 2022). Modelling competency is divided by Blomhøj and Jensen (2003) into several sub-competencies (such as the steps of the modelling cycle in article 1). They emphasised the importance of both working with a full-scale mathematical modelling process, where all steps of the modelling cycle are needed to solve the task, holistic task, and working on atomistic tasks, where the focus is on a specific step or some of the steps in the modelling process. Kaiser and Sriraman (2006) also included the perspective of educational modelling: on the one hand, learning processes and structures are promoted, and on the other hand, the understanding of mathematical concepts is promoted. As such, holistic modelling is emphasised, and moving the activity to the meta-level by discussing the steps of the modelling process is encouraged within this perspective (Ferri, 2013). A modelling cycle is seen as helpful for the students' meta-reflections, and they can better perform the modelling processes when they are familiar with the modelling cycle (Grigoraş et al., 2011).

Socio-critical modelling (understanding the role of mathematics)
From this perspective, “the role of mathematics and its relationship to the real world must be more conscious” (Blum, 2015, p. 82). It has pedagogical goals but is concerned in particular with critically seeing and understanding the surrounding world (Ferri, 2013). This perspective emphasises the role of mathematics in society and asserts the need to support critical thinking about this role as well as about the role and nature of mathematical models and the function of mathematical modelling in society (Kaiser & Sriraman, 2006). Barbosa (2006) labelled this perspective as *modelling-as-critic*, to use mathematical concepts to make decisions in society. It shows both the power of mathematics in decision-making and how mathematics can be used as a tool to make decisions (Abassian et al., 2020). Within this perspective, the step of validating and assessing the model and answers given by the model is emphasised.

Epistemological modelling (comprehending mathematics as a science)
Modelling is, from this perspective, seen as making use of the learned mathematical knowledge to find the best mathematical means to solve real-world problems. Modelling can be integrated into one’s entire mathematical learning and study process (Xu et al., 2022). Within this perspective, again, “the role of mathematics and its relationship to the real world must be made more conscious” (Blum, 2015). The focus in the modelling process lies not specifically in the transition from reality to mathematics and vice versa, as in the applied modelling perspective, but in the inner mathematical structures of the problem (Ferri, 2013). The aim of modelling is to form a mathematical scheme, and modelling is seen as a natural cognitive path through which one can understand how mathematics is gradually generated from real-world problems (Xu et al., 2022).

Pedagogical modelling (enjoying the activity of mathematics)

The aim of pedagogical modelling is to make mathematics more marketable to students using interesting examples for either motivation or illustration purposes (Blum, 2015). The role of modelling in inspiring students' interest, exploration and talent is emphasised, and modelling is seen as an approach to cultivating one's creativity (Xu et al., 2022). From this perspective, authentic examples are not required; they might quite well be *dressed-up tasks* and imaginary contexts. If so, however, it is important, to be honest, and not present them as real-world examples (Blum, 2015). A representation of the real world might not be included in the modelling cycle within this perspective, but rather as a problem in a context as presented in the PISA 2012 cycle in Figure 2.

Conceptual modelling (understanding mathematical concepts)

From this perspective, modelling is a way to understand and learn mathematics and to develop students' mathematical competencies rather than just their ability to solve real-world problems (Xu et al., 2022). The modelling tasks within this perspective should be mathematically rich examples that make certain mathematical topics more comprehensible (Blum, 2015). This perspective also includes modelling within Realistic Mathematics Education (Blum, 2015). Here, mathematical modelling is expressed as emergent modelling, where models are seen as representations to visualise a concrete situation and generalise to provide an understanding of other situations. For example, to express an algebraic problem as a geometric model (van den Heuvel-Panhuizen, 2003). A mathematical model, then, is seen as a concrete representation of an abstract mathematical problem (see Gravemeijer, 1999), rather than an abstract mathematical model of a concrete real-world situation.

These perspectives are drawn on in the meta-analysis of this extended summary. The aim of the meta-analysis is not only to find out which of these perspectives could be identified but by trying to identify the perspectives also reveal the discursive practices of the different curriculum discourses.

2.5 Mathematical modelling in the Norwegian *intended* curriculum

In this section, I will present how mathematical modelling has been expressed in the *intended* curricula in Norway over the last 30 years and point to the six modelling perspectives by Blum (2015).

In Norway, the *intended* curriculum is established as regulations by the Ministry of Education and Research, and all schools and teachers are obliged to follow these regulations. During the last 30 years, a new curriculum has been established three times. Before 2006 there were separated *intended* curricula for grades 1-10 (L97) and for upper secondary school, grades 11-13 (R94). A new curriculum was established in 2006 (LK06), and again in 2020 (LK20).

In L97, the *intended* curriculum for grades 1-10, mathematical modelling was not explicitly included, but *mathematics in everyday life* was expressed as one of five main areas of school mathematics. For grades 5-7 the aim within this area was that "the students should gain experience with mathematics as a relevant tool also in other school subjects, and in daily life be able to use mathematics regarding situations at home and in society" (The Norwegian Directorate for Education and Training, 1997, p. 162). And for grades 8-10 this was for the students

> to learn to use their mathematical knowledge as a useful tool in problems from their everyday life and society. The students should from a relevant theme or problem area systematise and formulate it in mathematical language, develop results by using known methods and tools, and evaluate the use related to the given context (The Norwegian Directorate for Education and Training, 1997, p. 166).

Here we can recognise the aim from the applied modelling perspective, to understand the real world and solve real-world problems. Mathematics is seen as a tool, which refers to *mathematics for the sake of modelling* which includes the applied modelling perspective. When it comes to the process of modelling, the starting point is formulated as “everyday life and society”, and further the process is described as “systematise and formulate in mathematical language”, which relates to the first three steps in the PISA 2003 cycle in Figure 1. The next step is in L97 formulated as using known methods and tools (working mathematically), and to evaluate. Here, the two last steps in the PISA 2003 cycle are formulated, to solve the mathematical problem and to make sense of it related to the original problem. Even if the word modelling is not included in L97, I have now argued that modelling was a part of the aims expressed.

In the *intended* curriculum for upper secondary school, R94, mathematical modelling is explicitly expressed.

> When we study problems from everyday life, the students should be involved as much as possible in the whole modelling process – they should be given the opportunity to formulate the original problem mathematically, choose appropriate methods, solve the problem, and finally interpret and evaluate the answer in the original situation (The Norwegian Royal Ministry of Church and Education and Research, 1994, p. 3).

This may point to the perspective of educational modelling because all parts of the modelling process are emphasised, and the students’ development regarding the sub-processes. Further, it is expressed that “students should also be given challenging tasks from the world of mathematics – tasks where they have to identify relations, look for patterns, make examples, experiments and simulations” (The Norwegian Royal Ministry of Church and Education and Research, 1994, p. 3). Both when working with problems from reality and when exploring purely mathematical questions, “students should experience that mathematics is not only a collection of formulas and algorithms for solving routine problems but also a toolbox with tools for solving problems that require imagination and insight” (The Norwegian Royal Ministry of Church and Education and Research, 1994, p. 3). According to this, R94 also includes the perspective of conceptual modelling – aiming to understand mathematical concepts.

The epistemological modelling perspective could be recognised in the next *intended* curriculum, LK06. Mathematics is used and developed “to systematise experiences, to describe relationships in nature and society and to explore the universe” (Norwegian Ministry of Education and Research, 2013, p. 2). Mathematics is comprehended as science, and mathematics is used as a tool to solve problems in the world and to explore, as expressed within the epistemological modelling perspective.

Further in LK06, we can recognise several modelling perspectives: “Active democracy requires citizens who are able to study, understand and critically assess quantitative information, statistical analyses and economic prognoses. Hence mathematical competence is required to understand and influence processes in society”(Norwegian Ministry of Education and Research, 2013, p. 2). Within the applied modelling perspective, the aim is to understand and master the real world, as expressed here, to understand processes in society. And within the socio-critical perspective, activities are expressed as to analyse and assess models from society, as expressed here, be able to “study, understand and critically assess”. In LK06 it is further emphasised using “modelling to analyse and transform a problem into mathematic form, solve the problem and evaluate the validity of the solution. This also has linguistic aspects, such as communicating, discussing and reasoning” (Norwegian Ministry of Education and Research, 2013, p. 2). This points to the learning of mathematics within the conceptual modelling perspective, and also educational modelling with a focus on all parts of the modelling process, and also reflecting upon it. Within LK06 also emotions are mentioned: "Another source of inspiration for the development of [mathematics] has been the joy people have felt when simply working with mathematics" (Norwegian Ministry of Education and Research, 2013, p. 2), as within the pedagogical modelling perspective, where student motivation is emphasised.

Within Practical mathematics 2P of the second year of upper secondary school, in which this study is conducted, modelling was included as one of the four main subject areas:

> The main subject area modelling provides an overarching perspective on the subject of mathematics. Modelling is a fundamental process in the subject where the starting point is something that actually exists. This is described in mathematical terms through a formulated model, and the results are discussed in light of the original situation. (The Norwegian Ministry of Education, 2013, p. 3)

Modelling as an overarching process points to the epistemological perspective of modelling (Xu et al., 2022). The starting point of the modelling process is expressed as "something that actually exists" (Norwegian Ministry of Education and Research, 2013, p. 3), which is emphasised within the applied modelling perspective where the activities are realistic and authentic real-world examples.

Modelling is also included in the *intended* curriculum through the *basic skill* of mathematical literacy, one of five *basic skills* included in LK06 and continued in LK20, to be integrated and developed in all subjects across the curriculum. This also includes the epistemological and socio-critical perspectives of using mathematics as a path towards formulating arguments and making decisions in other school subjects.

In the latest *intended* curriculum in Norway, LK20, *modelling and applications* is included as one of the six *core competences* in school mathematics on all levels, from primary to upper secondary school:

> A model in mathematics is a description of reality using mathematical language. The pupils shall gain insight into how mathematical models are used to describe everyday life, working life and society in general. Modelling in mathematics means creating such models. It also means to critically evaluate whether the models are valid and what limitations the models have, evaluate the models in view of the original situations, and evaluate whether they can be used in other situations. (The Norwegian ministry of Education, 2019, pp. 2-3)

LK20 was first implemented for the second year of upper secondary school in the school year 2021–22, after the completion of data collection for this study. The description of mathematical modelling in this *intended* curriculum could include several perspectives on mathematical modelling. "To gain insight into how mathematical models are used" could point to the epistemological modelling perspective – to see the usefulness of mathematics in society. However, "creating such models" points to the pedagogical perspective, including modelling competency, where building models is seen as important. Furthermore, different parts of a modelling cycle are described, pointing to the holistic process of mathematical modelling emphasised in the applied modelling perspective. Critical assessment models are also emphasised within the socio-critical perspective. To "evaluate whether they can be used in other situations" points to the conceptual perspective, which aims to develop a model for understanding other situations, or to the epistemological modelling perspective, which focuses on inner mathematical structures instead of real-world problem-solving.

I have now argued that the six perspectives on mathematical modelling could be identified in the *intended* curriculum over the last 30 years. But to identify known perspectives is different than for a teacher to grasp the aims for mathematical modelling expressed through the *intended* curriculum, recognise the different parts of the modelling process, and decide which activities to present for the students. To assess how mathematical modelling is implemented in the teaching and learning of mathematics, other discourses of the curriculum should also be considered. In the next chapter, further theoretical frameworks for curriculum assessment are presented.

3 Further theoretical background

3.1 Curriculum assessment

The theoretical framework presented here builds on two existing conceptual frameworks relevant to research on the implementation of curricula: Goodlad (1979) and Porter (2006). Goodlad (1979) presented a framework for studying curriculum practices in which the curriculum was divided into five categories: *ideological* (system of ideas), *formal* (documents), *perceived* (in the teacher's mind), *operational* (in the classroom) and *experienced* (students' individual experience). The *ideological* curriculum could be seen as the idea behind the *formal* curriculum. It is based on educational research and comprises subject policy documents, such as the KOM and PISA frameworks: "The *ideal* reflects beliefs, opinions and values of disciplinary and educational scholars, but likeliest does not exist in reality" (Remillard & Heck, 2014). The *formal* curriculum is the approved curriculum in a country. This can be seen as a contract between the school and the society/student and contains descriptions of what students are expected to learn. Goodlad (1979) also included instructional materials as part of the *formal* curriculum because they are written statements of intent. The *perceived* curriculum is, according to Goodlad (1979), the one with the greatest influence on the practical use and realisation of the curriculum. It is the result of individual teachers' implementation of the *formal* curriculum. Individual implementation is performed based on teachers' personal experiences and what they consider possible for teaching. The *operational* curriculum is the *perceived* curriculum as it is acted out in the classroom. It can be affected by both external factors and teachers' beliefs regarding mathematics and teaching. The *experienced* curriculum refers to the personal experiences of students.

Porter (2006) assessed mathematics curricula and divided them into the *intended*, *enacted*, *assessed* and *learned* curricula. The *intended* curriculum is the overt curriculum in a country, which is acknowledged in policy documents regarding what schools are aiming to accomplish. This corresponds with the *formal* curriculum, as expressed by Goodlad (1979). The *enacted* curriculum refers to instruction, or what happens in the classroom, which Goodlad (1979) called the *operational* curriculum. The *assessed* curriculum refers to student achievement tests, such as national examinations. This is not expressed as its own category by Goodlad (1979), but it is included in the operational curriculum. The last, the *learned* curriculum, is what the students have actually learned in school, and, as Porter (2006) pointed out, this can be both more or less than what is assessed. It is also individual for each student, and Goodlad (1979) called this the individual student experience. The perceived curriculum included by Goodlad (1979) emphasised the role teachers play in mediating between what is authorised in policy and what occurs in the classrooms (Remillard & Heck, 2014), which is not as prominently emphasised by Porter (2006). However, Porter (2006) did acknowledge this when elaborating on the *enacted* curriculum: "teachers are emphasising different things, and there are various reasons for their choices. They might teach what they believe is most important, what they think the students are ready to learn, or what is most enjoyable and easy to teach" (Porter, 2006, p. 142). Goodlad (1979) and Porter (2006) included curriculum implementation at all phases but emphasised different parts.

The textbooks in Norway are not government approved but are published by independent publishers. I, therefore, do not see them as part of the *intended* curriculum. In Nordic countries, textbooks have been extensively used in mathematics education (Grevholm, 2017). Textbook tasks can therefore reflect what is taught in the classroom and could be included in the *enacted* or *operational* curriculum. However, textbook tasks do not reflect different teachers' choices, so textbook analysis might not be considered a full-fledged analysis of the *enacted*/*operational* curriculum. Textbook authors aim to cover all parts of the mathematical content of the curriculum, and according to Grevholm (2017) and the experience of this research, teachers follow the structure of the textbook and assign the suggested tasks. Therefore, textbooks are seen as a distinct part of the curriculum assessment, which I present as the *instructional* curriculum. In this research, I include the following parts in the curriculum assessment: *ideological*, *intended*, *instructional*, *perceived*, *enacted* and *assessed* curricula. To delimit this study, the *learned* curriculum is not included in the data or the analysis.

In the development of this thesis, the cultural historical anthropological theory (CHAT) by Chevallard (2006) was considered as an overarching theory by including the different spheres, the didactic transpositions between scholarly knowledge, knowledge to be taught, taught knowledge and learned/available knowledge (Chevallard & Bosch, 2020). This was not straightforward because the connection of school mathematics to everyday life through mathematical modelling affects the transpositions. And where the question is posed, of which mathematical modelling is the answer, could both be where mathematics is used in everyday life, the knowledge of professional modellers, and the research area of mathematical modelling in mathematics education. Even if CHAT was not included, it has inspired the way the different curriculum discourses are seen in this thesis, and also the awareness of monuments; activities in the classrooms which are proceeded without knowing the reason for it (Chevallard, 2006). Discourse theory combined with curriculum assessment is rather built on as a theoretical lens in this study, as presented in the following sections.

3.2 Discourse

Discourse can be seen as an "interrelated set of text, and the practices of their production, dissemination and reception, that brings an object into being […]. In other words, social reality is produced and made real through discourses" (Phillips &

Hardy, 2002, p. 3). Or as expressed by Bryman (2016), discourse is seen as a process through which meaning is created. That is, the meaning of mathematical modelling is created through texts and the practices of their production in the curriculum discourses.

Kim et al. (2017) examined the historical development of the concept of discourse and understood mathematical discourse through Vygotsky's (1962) view that language serves as the instrument to develop thought. They also referred to Wittgenstein's argument from 1953 (republished in Wittgenstein and Anscombe (2001)) that people use language not only to reflect the world in words but also to create meanings through language with logical structures. In this light, learning mathematics is facilitated by language and the meanings that people create in discourse. Sfard (2008) considered the learning of mathematics as involving discursive activities, cognition and communication. Indeed, "[i]n that sense, the aforesaid language shall be the objectified mathematics as well as the language of individuals participating in mathematical discourse" (Kim et al., 2017, p. 448). Individuals' understanding of mathematics, in this view, can be seen as that created through discourse. As Bryman (2016) noted, there has been a shift from treating language as a resource, as in traditional views in social research, to treating it as a topic. Language becomes a focus of attention in its own right within Discourse Analysis.

According to Morgan and Sfard (2016), mathematical discourse comprises the four aspects presented by Sfard (2008): *word use*, *visual mediators*, *routines* and *endorsed narratives*. *Word use* includes how participants use mathematical vocabulary (and in this study, vocabulary concerning mathematical modelling). *Visual mediators* refer to objects, including symbols and diagrams, used for mathematical communication. *Routines* refer to various meta-rules that regulate participants' actions in discourse and patterned ways of performing mathematical tasks. *Endorsed narratives* refer to any story considered by a mathematical community as a useful and reliable description of the mathematical universe (Morgan & Tang, 2016). Morgan and Sfard's (2016) Discourse Analysis includes both the textual dimension and discursive practice. In article 2 of this thesis, the *ideological* curriculum (frameworks) and the *perceived* curriculum (teacher interviews) are analysed based on Morgan and Sfard's (2016) framework developed to analyse mathematical discourse. The *endorsed narratives* about mathematical modelling developed within the *ideological* and *enacted* curricula were explored, which was at the centre of my interest: how mathematical modelling is expressed within these discourses.

Fairclough (1992) presented a three-dimensional framework for discourse analysis. The first is the textual dimension, which draws on Halliday (1978) and considers discourse as exclusively linguistic elements. The next dimension is discursive practice – an examination of the form of discursive interaction used to communicate meaning and beliefs. The third dimension is the social context in which the discursive event is taking place – the social practice dimension. A constraint of this framework is the lack of distinction between the discursive and the non-discursive, and the boundaries between discourse analysis and the analysis of social practice are not clarified (Jørgensen & Phillips, 1999). Similar to Fairclough (1992), Morgan and Sfard (2016) did not distinguish between discursive and non-discursive phenomena but included both in the four aspects of communication. The frameworks referred to here, use different terms, but both Fairclough (1992) and Morgan and Sfard (2016) emphasise the context of the discourse. The term "intertextuality" was introduced by Fairclough (1992). As Bryman (2016) noted, "This notion draws attention to connections between texts so that any text that is being subjected to scrutiny is considered in relation to other related texts" (Bryman, 2016, p. 540). In this thesis, Fairclough's (1992) framework is not included as an analysis tool in either of the three articles but is used as a guide for the overall discussion of this thesis – a discussion that involves the curriculum discourses analysed in the three different articles and the interactions between them, expressed by the discursive and social practices.

Within Critical Discourse Analysis, discourse is seen as an important social practice that both constitutes the social world and is constituted by other social practices. That is, discourse is dialectically related to other social practices; it contributes to the shaping and reshaping of social structures, but it also reflects them (Jørgensen & Phillips, 1999). For example, the social structure in the classroom affects the discourse in which the teacher and students are involved. It is socially constructed; at the same time, it is an existing construction containing practices, relations and identities in the teacher and student roles. Within Critical Discourse Analysis, one aim is to explore why some meanings are taken for granted and others become marginalised (Bryman, 2016). Critical Discourse Analysis aims to reveal the role of discursive practice in the maintenance of the social world and the power in social relations (Jørgensen & Phillips, 1999). Identification of these power relations and awareness of the structures that maintain them can lead to change. According to Fairclough (1992), discursive practice can be seen as a hegemonic struggle and is part of social practice, including power relations. A discursive change occurs when discourse elements are articulated in new ways. A discursive practice might *reproduce* or *restructure* earlier discursive practices. Restructuring leads to change, whereas reproduction maintains the structures.

A reason for choosing Morgan and Sfard's (2016) theory for the analysis in article 2 was the emphasis on the endorsed narratives. The purpose of the study was to get a deeper understanding of how mathematical modelling was expressed in the teaching and learning of mathematics. Article 2 aimed to point out the endorsed narratives in the framework discourse and teacher discourse. Other theoretical approaches were considered to identify the teachers' understanding of mathematical modelling, e.g. Ernest's (1989) about teachers' knowledge, beliefs and attitudes. But these concepts are less tangible than the word use, visual mediators, routines and endorsed narratives with textual markers within discourse theory as presented by Morgan and Sfard (2016). I also considered a discursive approach which was based only on the textual bundles, but compared to Morgan and Sfard (2016), the focus on the understanding of mathematical modelling became indistinct. The teachers communicated differently, e.g., wordiness, modifiers in their language and preciseness. The analysis of textual bundles concerning their understanding of mathematical modelling was therefore not seen as a good choice. Another suggestion was Grounded Theory Approach, but since an interviewer's questions are not neutral, it might be challenging to use a Grounded Theory Approach analysing the constructed data from the interviews.

When analysing classroom discourse, the dialogues between the teacher and students as well as the interactions and roles in the classroom while working on mathematical modelling are relevant. How the interactions and dialogues reflect students' opportunities to make choices in the modelling process is the focus of Positioning Theory. This theory, which is developed as a lens in article 3 of this dissertation, is introduced in the following section.

3.3 Positioning Theory

Davies and Harré (1990) and Langenhove and Harré (1999) have drawn on Discourse Theory when developing Positioning Theory. According to Tait-McCutcheon and Loveridge (2016), "people's talk, text and actions are public, socially constructed, and normatively guided" (p. 329). Tait-McCutcheon and Loveridge (2016) are interested in how communications "position" people and the implications of particular constructions for people in their daily life. *Positions* refer to what one may meaningfully say and do. With every position goes a *storyline*. In this way, positioning may diminish the domain of what one does among the possibilities of what one can do (Harré & Slocum, 2003). A social episode is shaped by "things done by a person" and "things done to a person" (Davies & Harré, 1990). All social episodes are created and sustained by the people that participate in them, the positions they have, the storylines they develop and the social power of their words and actions on a particular occasion (Davies & Harré, 1990). Positioning theory and the associated constructs of *position*, *storyline* and *social acts* provide a useful framework and tools for examining the actions, interactions and participation opportunities of those involved in solving mathematical tasks (Tait-McCutcheon & Loveridge, 2016).

Herbel-Eisenmann et al. (2015) evaluated the use of Positioning Theory in mathematics education research. They found that the three long-standing precepts of Positioning Theory (Davies & Harré, 1990; Langenhove & Harré, 1999) – *position/positioning*, *speech*/*acts* and *storylines* – had no common use. The use of these three terms in this thesis is presented in article 3 (see Appendix C). The students and teacher have different roles in classrooms, but the teacher and student roles might change within different classroom situations. The positioning in the classroom affects how the storylines develop. In article 3, Positioning Theory is built on to form a lens to analyse students' opportunities to make their own decisions in the work of mathematical modelling and to explore the storylines about mathematical modelling that evolved through the teaching of mathematical modelling. Expressed in terms of discourse theory, the Positioning Theory helps to identify social constructions that affect power relations in the classroom. I will now first present other theoretical frameworks which I have considered for the analysis of the classroom observations, before the theories drawn on in this thesis are discussed in relation to each other in Section 3.4.

Valsiner’s (1997) zone theory draws on Vygotsky (1962) and expresses the social constructions as the ZPA/ZFM complex; which motions are allowed by the teacher, and which actions are promoted. The zone of proximal development is dependent on that complex. Blanton et al. (2005) draw on this zone theory and analysed utterances made by the teacher and the students in the classroom by identifying what the participating teacher promoted and allowed in the classroom, as within the Positioning Theory. Within Positioning Theory, the language is not only seen as a mediating artefact as within the socio-cultural learning theory which Valsiner (1997) draws on. Individuals’ understanding of mathematics is seen as created through discourse as the storylines develop from the speech/act and the positionings, which follows the research paradigm of this study.

Tropper et al. (2015) developed a framework focusing on teacher behaviour and instructional approaches in teaching mathematical. Within this framework, the perspective on mathematical modelling is given, and the modelling activities are expressed based on this perspective. However, this dissertation aimed to grasp which understanding of mathematical modelling is communicated in the discourse. The framework by Tropper et al. (2015) was therefore not found suitable. Also within the framework by Adler and Ronda (2015), the aim of the teaching is decided in advance. It includes studying mathematical discourse in instruction and dividing between exemplification, explanatory talk, learner participation and the object of learning. The learners’ participation is relatable to the focus on agency in the Positioning Theory framework.

The framework from Durandt et al. (2022) evaluates how teachers’ support affects the students learning of mathematical modelling by measuring students’ performance on tests. Durandt et al. (2022) defined mathematical modelling and chose parameters to test, but by using Positioning Theory the understanding of mathematical modelling is seen as revealed from the discourse rather than measuring the achievement of different teaching styles. Therefore, I rather chose the Positioning Theory for the analysis of the classroom observation. Now, I will present how the different theories included in this thesis are seen in relation to each other.

3.4 Curriculum, discourses, modelling perspectives and Positioning Theory

"If we are to understand discourses and their effects, we must also understand the context in which they arise" (Phillips & Hardy, 2002, p. 4). This echoes Fairclough (1992) regarding his three-dimensional model for Critical Discourse Analysis, which includes the textual dimension, an interpretation of discursive practices (including the production and consumption of texts) and an analysis of social practice. In this study, connecting discourse theory and curriculum assessment is useful for analysing mathematical modelling in mathematics education. The different parts of the curriculum within curriculum assessment are seen as different curriculum discourses. Analysing different discourses is part of the investigation of intertextuality (Fairclough, 1992), which allows us to see the interactions between the different discourses as discursive and social practices.

According to Porter (2006), if the content in the *intended*, *enacted* and *assessed* curriculum is similar, they are said to be aligned. From a critical discourse theorist's point of view, the different curriculum discourses cannot be considered similar, as the practices are different, but the same perspectives on mathematical modelling could be identified within the different curriculum discourses. Cary (2006) described discourse analysis in education as "peeling back the layers of discourse that frame our lives and the lives of others" (p. 19). Furthermore, she argued that "if we peel back the layers, we can gain a more adequate understanding and thus negotiate the effects of power" (p. 19). This relates to what is expressed by Jørgensen and Phillips (1999) about Critical Discourse Analysis. The aim is to identify social structures in discourses and to reveal power relations. In other words, we might be able to reflect upon the students' and teachers' experienced opportunities to make choices in the work of mathematical modelling by identifying the power relation in the classroom discourse, in the *enacted* curriculum, as well as how the other curriculum discourses affect the teaching of mathematical modelling.

In article 1 where the *instructional* and *assessed* curriculum were analysed, a cognitive modelling cycle was included as a tool. This specific theory has a different character than discourse theory. Within discourse theory, you seek to identify storylines or endorsed narratives developed in the social context, by analysing text or speech/act and identifying discursive and social structures. In article 1, however, the modelling cycle was chosen in advance, and the textbook tasks and exam tasks were compared with this to identify the endorsed narratives of mathematical modelling communicated from the tasks. The social structures connected to the "consumption of the text", as expressed by Fairclough (1992), were not included in article 1. But the textbooks and exam tasks are consumed in other discourses. Although the *instructional* and *assessed* curriculum are analysed by a cognitive theory of mathematical modelling, the intertextuality and discursive and social practices of these curriculum discourses are identified through the *perceived* and *enacted* curriculum.

Article 2 investigates how the words used by teachers when describing mathematical modelling communicated their understanding of mathematical modelling. The endorsed narratives about mathematical modelling were identified by discourse analysis. In article 3, the focus was on how the discourse positioned the students, the speech and the act and how this affected the storyline about mathematical modelling. The endorsed narratives expressed in article 2 are in line with the understanding of storylines in article 3, the constructed understanding of mathematical modelling. The storylines might change from episode to episode in a classroom, but the endorsed narratives are seen as established truths within a discourse.

To include the six modelling perspectives by Blum (2015) in this extended summary made the identification of differences in the storylines of mathematical modelling clearer, and thus perform a curriculum assessment regarding mathematical modelling. Structures within the modelling perspectives could be revealed by the aims, activities or processes of mathematical modelling. E.g., within the epistemological modelling perspective, the students ought to use mathematics as a tool to solve real-world problems finding the best mathematical means. The teacher should therefore position the students giving them agency to find out how to solve it using different mathematical approaches.

The lens of Critical Discourse Analysis allows the analysis of the different parts of the curriculum to be seen as different discourses. Seeing the discourses in relation to each other provides possibilities for reflection on why some meanings are taken for granted and others become marginalised in the teaching of mathematical modelling. Social events and their texts do not randomly occur. Certain factors shape the events and the texts (Maftoon & Sabbaghan, 2010). In the extended discussion of this thesis' overarching summary, the discursive practice and social practice dimensions of Fairclough's (1992) three-dimensional model for Critical Discourse Analysis are considered. The *reproduction* or *restructuring* of the discursive practices; change or maintenance of the structures could be identified. E.g., identifying discursive practices within the teaching of mathematical modelling and seeing it in relation to the activities included in a given perspective helps identify if and how discursive practices should be *reproduced* or *restructured* to include that perspective on mathematical modelling.

4 Empirical background

In this thesis, the focus is on mathematical modelling within mathematics education. Mathematical modelling within scientific disciplines (e.g. biology, medicine, physics or statistics) is included in the research field of mathematical modelling, but will not be included here. Mathematical modelling is also part of mathematical literacy across the curriculum, but in this study, the focus is on modelling within mathematics education in schools. The Norwegian *intended* curriculum is presented as a whole, from first to thirteenth grade, including the *core competence* of “mathematical modelling and application”. Studies from lower grade levels than upper secondary are also included in the empirical background if they are providing findings which are relevant to this study. I present studies from each of the curriculum discourses analysed in the articles of this thesis: The *instructional, perceived, enacted* and *assessed,* and relate the studies to Blum’s (2015) six perspectives on mathematical modelling.

4.1 Modelling in the *instructional* curriculum – types of modelling tasks

Results from studies concerning modelling textbook tasks will now be presented. First, I will present studies of textbooks from countries where mathematical modelling is more recently introduced in the *intended* curriculum, and then a study comparing the textbooks from two countries, before presenting studies conducted in Nordic countries where mathematical modelling has been an explicit part of the *intended* curriculum for decades, most alike the Norwegian context.

Urhan and Dost (2018) analysed a textbook made for the new *intended* curriculum in Turkey, which included mathematical modelling, based on model-eliciting principles developed by Lesh, Cramer, Doerr, Post, and Zawojewski (2003). These principles could be placed within the epistemological modelling perspective because modelling is seen as making use of learned mathematical knowledge to solve real-world problems (Blomhøj, 2009). They found that "only 22% of the activities in the course book possess the features of model-eliciting activities" (Urhan & Dost, 2018, p. 998). Within the epistemological modelling perspective, modelling is suggested to be integrated into one's entire mathematical learning and study process (Xu et al., 2022). However, this was not found in the Turkish textbook. In the study by Gatabi et al. (2012), Australian and Iranian mathematics textbooks were compared by content analysis of mathematical literacy, where the aim is to identify and understand the role that mathematics plays in the world. This points to the epistemological modelling perspective. It also involves people's ability "to make well-founded judgements" (Gatabi et al., 2012, p. 406), which points to the perspective of socio-critical modelling, as well as "to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen" (Gatabi et al., 2012, p. 406), which reflects the applied modelling perspective. This had been included in the Australian *intended* curriculum for years, but in the Iranian *intended* curriculum, mathematical modelling was a new topic. Gatabi et al. (2012) found that students had few opportunities to engage in mathematical modelling in the Iranian textbook. The Australian textbooks presented more problems in a diverse range of contexts.

Frejd (2013) analysed 15 Swedish upper secondary school textbooks to determine how the notion of mathematical modelling was treated. He found that mathematical modelling was not treated as a central notion, as it was in the Swedish *intended* curriculum. Intra-mathematical aspects, and not developing a holistic modelling ability, were the focus (Frejd, 2013). "[N]one of the textbooks analysed did really support the fulfilment of the Swedish *intended* curriculum concerning mathematical models and modelling" (Frejd, 2013, p. 91), where mathematical modelling is described as interpreting a realistic situation and designing a mathematical model as well as using and assessing a model's properties and limitations. These descriptions in the *intended* curriculum point to educational modelling, as it focuses on the holistic modelling ability. Jessen and Kjeldsen (2021) evaluated whether any relationship existed between mathematical modelling in scientific contexts and Danish upper secondary education. This scientific context points to the perspective of epistemological modelling – modelling as a natural path for people's cognition process. Results showed that the mathematics tasks from exams and textbooks did not require engagement in real-world contexts. The tasks could be solved simply by performing the given mathematical procedures. The students were not given opportunities to engage in the construction of models and there was no need to understand the situation through the perspectives of other disciplines (Jessen & Kjeldsen, 2021).

The different studies or textbook modelling tasks have taken different perspectives on mathematical modelling. If the textbook tasks are representative of classroom activities, then the findings confirm the gap discussed in Blum and Pollak (2018), in which the educational debate does not align with what happens in the classroom in terms of mathematical modelling. The tasks presented as modelling tasks in many countries' textbooks do not provide student engagement in the entire modelling process. Some of these studies were conducted shortly after mathematical modelling was implemented in the countries' curricula, such as in Iran and Turkey. In Australia, mathematical literacy has been part of the school for years, and the textbooks also included modelling tasks in a diverse range of contexts and more non-routine modelling tasks (Gatabi et al., 2012). Even if mathematical modelling has been included in the Swedish and Danish *intended* curriculum for years, the textbooks did not provide student engagement in the holistic modelling process (Frejd, 2013; Jessen & Kjeldsen, 2021).

4.2 The *perceived* curriculum – teachers' conceptions

If the *intended* curriculum does not comprehensively describe mathematical modelling, teachers have a significant impact on how it is implemented in mathematics education (Ärlebäck, 2010), and the teachers' understanding is therefore interesting to study.

Sahin et al. (2019) investigated Turkish teachers' criteria for evaluating whether a task was a mathematical modelling task. Four tasks were presented, and only one was a modelling task according to the researchers ("being suitable to real-life, being open-ended, complex or thought-provoking and being able to be solved according to the modelling process" (Sahin et al., 2019, p. 731)). This points to the perspective of educational modelling because of the focus on solving real-world problems throughout the modelling process. The criterion of being thought-provoking also underpins the aim of the educational perspective: realising one's own growth in competency. Many of the teachers identified a task as a modelling task solely because it was related to real life. Some of the teachers also emphasised that a modelling task should be open-ended and complex or thought-provoking. The researchers highlighted the importance of developing teachers' theoretical knowledge about teaching mathematical modelling and noted that this lack of knowledge affected their skills in the cognitive analysis of modelling tasks (Sahin et al., 2019). In a Swedish study, Frejd (2012) interviewed 18 upper secondary school teachers about their conceptions of mathematical modelling. The interview included different modelling tasks from the perspective of educational modelling for the teachers to evaluate their conceptions of mathematical modelling. Results showed that the teachers were only familiar with mathematical modelling in physics education but did not give priority to integrating mathematical modelling in their teaching of mathematics. They did not see all the mathematical modelling tasks to be relevant for learning mathematics and did not consider modelling competence as part of mathematical competence. The above findings actualise a barrier to the implementation of modelling presented by Burkhardt (2006) – limited professional development. The teachers were expected to be able to teach based on their pre-service education, even if this did not include mathematical modelling.

Teachers participating in teachers' modelling courses developed their understanding of mathematical modelling, as in an Israeli study by Shahbari and Tabach (2016). They found that the teachers saw the modelling process as linear before participating, and they did not pay attention to the real results, the validating process and the cyclical nature of the mathematical modelling process. Afterwards, their reports indicated that they were able to recognise all parts of the modelling process and distinguish the cyclical process. The teachers were aware of the changes in their descriptions as a result of their participation in the modelling course, where they performed all mathematical modelling processes when solving modelling tasks and discussed them. There are many choices to be made by teachers regarding the teaching of mathematical modelling, and the change of discursive practices is seen as challenging (Burkhardt, 2006). In Norway, studies concerning teachers' understanding of modelling have, to my knowledge, not been conducted. Therefore, in this dissertation, it was interesting to investigate teachers' understanding of modelling, where modelling has been a part of the *intended* curriculum for decades, to find if modelling was familiar to them and which perspective on mathematical modelling they were leaning towards.

4.3 Experiences from the *enacted* curriculum

I will now present studies presenting how mathematical modelling is taught in classrooms, and I also include studies from lower grades than upper secondary school. The reason for finding them relevant is because they contrast the first study and show possibilities in the teaching and learning of mathematical modelling.

Grigoraş et al. (2011) pointed out that in school, modelling involves “directing students to a specific mathematical topic, usually previously introduced or dealt with in the classroom” (p. 85). Therefore, they argue, the “process of mathematising is often eased by the fact that students are implicitly pointed to go in a particular direction. But this facility can have as effect a superficial treating of the problem, or even hinder developing modelling competencies” (p. 86). This does not reflect any of the six modelling perspectives and is in contrast to how mathematical modelling is taught in the studies by Brown and Stillman (2017) and Boaler (2001).

Brown and Stillman (2017) aimed to develop students’ conceptions of mathematics by working on modelling activities as a means to develop a sense of mathematics as a way of thinking about life. Brown and Stillman (2017) maintained that mathematical modelling should be seen as a part of mathematics rather than as an optional addition to it, as expressed in the *intended* curriculum. They suggested that this should “not be limited to a conception of mathematical modelling as a way of handling problems […] but rather as taking its rightful place within a broader conception of mathematics as an approach to life and a way of thinking” (p. 354). The intervention study was conducted in Grade 6 in Australia. The researchers’ view of modelling points to the perspective of epistemological modelling, in which modelling is seen as a natural path in the cognitive process. At the end of the project, most of the students saw modelling as a way of handling problems, whereas some saw it as a way of understanding the world. The students’ understanding of mathematics and mathematical modelling developed during this classroom interaction, and the researchers’ presentation of mathematical modelling and the use of a modelling cycle affected the students’ perceptions.

Boaler (2001) performed a longitudinal three-year study in England involving 300 students aged 13–16 years from two different schools who were learning mathematics in very different ways. One of the schools was traditional, where students were taught mathematics using textbooks that asked a series of short, closed questions. In the other school, they taught mathematics using a series of open-ended projects based on the philosophy that students should encounter situations in which they need to use and apply mathematical methods, including mathematical modelling. This study found that the students from the traditional school did not recognise school mathematics as relevant outside of school. However, students from the other school expressed that applying school mathematics came naturally and that they could connect it to situations outside of school. Boaler (2001) claimed, “[i]f students only ever reproduce standard methods that they have been shown, then most of them will only learn that particular practice of procedure repetition, which has limited use outside the mathematics classroom” (Boaler, 2001, p. 126). Although the tasks on the national examination were traditional, the students who were not working on such tasks in school still outperformed those from the traditional school (Boaler, 2001). She argued that this was attributable to the deeper learning of mathematical concepts when working on open-ended questions, and it also provided students with an opportunity to engage in important mathematical practices that had value beyond the mathematics classroom. The teaching aimed to allow students to see the application of mathematics in different situations. This points to the perspective of epistemological modelling, where the modelling activity is integrated into the entire mathematical learning and study process: “If the students encountered a need to know a method that they had not met before, the teachers taught it to them within the context of their projects” (Boaler, 2001, p. 122). This approach contrasts with the approaches in which the teacher focuses on one procedure at a time within a mathematical content that is not connected to other contents.

The studies by Brown and Stillman (2017) and Boaler (2001) show that students can experience school mathematics as relevant to everyday life. Moreover, owing to modelling, the students experienced a deeper understanding of mathematical concepts. To my knowledge mathematical modelling in the *enacted* curriculum is not studied in Norway, and neither Frejd (2014) nor Jessen and Kjeldsen (2021) included the *enacted* curriculum in their studies of mathematical modelling in Swedish and Danish upper secondary schools, respectively.

4.4 The *assessed* curriculum – assessment of mathematical modelling

As stated by Niss (1993), "[w]hat is not assessed in education becomes invisible or unimportant" (p. 27). Therefore, to show its importance, an assessment of mathematical modelling competence in mathematics education should be included in national exams. However, according to the PISA framework (OECD, 2013), assessing the entire modelling process of holistic tasks in written standardised tests is challenging. In the PISA test, "[i]t is often the case that significant parts of the mathematical modelling cycle have been undertaken by others […] and many PISA items involve only parts of the modelling cycle" (OECD, 2013, p. 26) owing to the challenges in assessment.

Frejd (2011) analysed Swedish upper-secondary mathematics exams concerning mathematical modelling. The study employed the educational modelling perspective as presented in the Swedish *intended* curriculum to develop modelling competence for solving real-world problems. Frejd (2011) found that the test items required the use of an already existing model to calculate a result and the assigning of variables to formulate a mathematical statement to calculate a result. The test items did not include an assessment of the student's ability to proceed with the first steps of the modelling cycle – to make simplifying assumptions to clarify what facts are most important. The national exams did not emphasise the critical assessment of the conditions, the interpretation of the result or the relation of the result to the real situation. This study showed that only fragments of the modelling process were assessed, and most often, the assessment included only a mathematical result instead of other parts of the modelling process. No holistic tasks were included in the tests. If Niss' (1993) statement about assessment is true – that what is not assessed becomes invisible or unimportant – parts of the modelling process could be seen as unimportant.

Test instruments to assess all parts of a modelling process through a written test have been developed. Hankeln et al. (2019) present a test instrument for assessing the sub-competencies of mathematical modelling in a German context. Here, one sub-competency is assessed in each task, and they argued that all parts of the modelling process could be evaluated through written tests without involving holistic modelling tasks. A similar instrument was developed in the USA by Leong (2012), who created scoring rubrics for each of the sub-processes in the modelling cycle: identifying variables, formulating a model, performing mathematical operations, interpreting the results, validating the conclusion and reporting the conclusion. This was a new field in American state standards at that time, and this instrument was considered to be a good beginning for finding appropriate methods of assessing the processes of mathematical modelling. However, a comprehensive assessment as described by Jensen (2007) must be conducted in other forms than a written test. According to Jensen (2007), at least three dimensions should be included in a valid assessment of modelling competence. The first is the *degree of coverage*, which evaluates a person's ability to systematise a real-world situation and evaluate both the modelling process and its results. The second is the *radius of action*, used to evaluate if a person can model all kinds of situations and contexts. The third is the *technical level*, which concerns how advanced the mathematical content is and how flexible it is used. Kartal et al. (2016) investigated whether a conventional standardised test used in the

USA could serve as a reliable "predictor" of students' potential for mathematical modelling competence. Their study sought to find the relation between students' scholastic aptitude test (SAT) scores in mathematics and two model-creation problems. They found that the students' SAT scores did not predict their performance on a modelling task. They concluded that "traditional assessments fail to identify students who can powerfully and effectively apply mathematics to real-world problems" (Kartal et al., 2016, p. 250). Further research is required on methods to assess students' modelling competence.

4.5 Research in the Norwegian context

Although mathematical modelling has been a part of the *intended* curriculum in Norway for decades, no extensive research projects about mathematical modelling have been conducted in Norwegian schools. In this thesis, perspectives on mathematical modelling are identified within the *ideological*, *intended*, *instructional*, *perceived, enacted,* and *assessed* curriculum discourses in Practical mathematics 2P, where mathematical modelling has been a part of the *intended* curriculum for decades. The research questions addressed in the three articles within this thesis are as follows:

- Which steps in the modelling cycle are needed to solve textbook modelling tasks and tasks from national examinations?
- What possible tensions in the approaches to mathematical modelling can be identified from a discourse analysis of relevant framework documents and interviews with four teachers?
- What are the storylines of the teacher–student dialogues in the teaching of mathematical modelling in which students take the initiative to positioning?
- How do teachers position the students to give them agency when working with mathematical modelling?

In the next chapter, I outline the methods included to answer these and the overarching research question.

5 Methodology

5.1 Research paradigm

A paradigm, a term derived from the history of science, describes a cluster of beliefs that influences what should be studied, how research should be conducted and how results should be interpreted by scientists in a given discipline (Bryman, 2016). It concerns ontology, epistemology, methodology and ethics (Denzin & Lincoln, 2017).

This study draws on critical realism. In terms of epistemology, knowledge is seen as a social product that is independent of those who produce it (Yucel, 2018). An aim within the paradigm of critical realism is to recognise the reality of the natural order and the events and discourses of the social world, and it holds that we will only be able to understand and subsequently change the social world if we identify the structures at work that generate those events and discourses (Bryman, 2016). Within critical realism, it is acceptable if the processes that are constitutive of the phenomenon of interest are not directly observable but can only be observed via their effects. In terms of this study, perspectives on mathematical modelling cannot be directly observed; however, through the analysis of the curriculum discourses, one can identify how it is expressed.

Within Critical Discourse Analysis, the aim is to reveal the role of discursive practice in the maintenance of the social world, including social relations, which include power relations (Jørgensen & Phillips, 1999). According to Laclau and Mouffe (2014/1985), a constructed reality theoretically exists, and the discourse reflects this reality. However, reality can also be seen as constructed through discourse (Bryman, 2016). At the same time, social structures affect power relations and limit individual agency (Jørgensen & Phillips, 1999).

In this dissertation, drawing on Stake (1995), social structures are identified through an analysis of the curriculum discourses to reveal power relations through case study research. The aim is to explore how the different curriculum discourses influence others and how this affects the way in which mathematical modelling is communicated.

Here, multiple perspectives or views, not just one best view, of the case study need to be represented; according to Stake (1995), a precise definition of cases or case studies is not possible when different disciplines are being studied. Nonetheless, a case could be seen as “an integrated system” that “has a boundary and working parts” and that is purposive (Stake, 1995, p. 2). He described a qualitative case study as “a study of the particularity and complexity of a single case, coming to understand its activity within important circumstances” (Stake, 1995, p. xi), and defined four characteristics: holistic, empirical, interpretive and emphatic. Its holistic quality means that researchers should see the phenomenon in relation to its context or, as Fairclough (1992) emphasised in discourse analysis, its intertextuality. Empirically, researchers base their studies on observations in the field. Interpretive here means that “researchers rest upon their intuition and see research basically as interaction” (Yazan, 2015, p. 139). Finally, emphatic means that “researchers reflect the vicarious experiences of the subjects in an emic perspective” (Yazan, 2015, p. 139). In this study, I, as a researcher, reflect upon the teachers’ experiences of mathematical modelling from their point of view – that is, from an emic perspective. I also adopt the etic perspective as an outsider in the classroom. The case is mathematical modelling within school mathematics 2P, and the different discourses of the case are expressed as different curriculum discourses.

5.2 Research design development

This study aimed to investigate the teaching of mathematical modelling. I started reading the *intended* curriculum searching for mathematical modelling on all grade levels and both in the recent and earlier versions to get a historical perspective. In Practical mathematics 2P in upper secondary school, mathematical modelling was expressed as a fundamental process, and I decided to place the study here. It can therefore be seen as a case study, as presented by Bryman (2016), of a school mathematics course in which mathematical modelling has, according to the *intended* curriculum, been expressed as a part of upper-secondary mathematics for 30 years.

A flexible research design allows researchers to make changes during the enquiry process, and according to Stake (1995), this is preferable. Stake's concept of flexibility in research design builds upon the notion of "progressive focusing", which leads to the assumption that the course of the study cannot be charted in advance (Yazan, 2015). Stake (1995) argued that "there is no particular moment when data collection begins since data collection can lead to alterations and reveal subjects of interest during the process" (p. 49). In an intrinsic case study, the case is dominant and of the highest importance. In an instrumental case study, the issue is dominant: the case is of secondary interest; it plays a supportive role and facilitates our understanding of something else. Nevertheless, the case is "looked at in-depth, its contexts scrutinized, and its ordinary activities detailed" (Stake, 2008, p. 123), as in an intrinsic case study, because this helps us to pursue the external interest. In this dissertation, the issue is the teaching of mathematical modelling, and this is of utmost interest. The issue is studied in the case of mathematics in Norwegian upper secondary schools. Therefore, it is an instrumental case study. Although the overarching aim of the study was formulated in advance, the research questions were formulated as the study was being conducted, following a flexible research design.

Because of the flexible research design, there has not been a strict process which could have been presented in advance. I will therefore briefly explain the process as it developed and present an overview at the end of this subchapter (see Figure 4). During the whole process, I have been reading research literature regarding mathematical modelling, theoretical perspectives and methods of analysis.

This study employs Stake's (1995) suggestion of using observation, interviews and document reviews in qualitative case study research. As pointed out by Grevholm (2017), textbooks have been extensively used in mathematics education in Nordic countries. Therefore, I found it relevant to include a document review of the textbook tasks (*instructional* curriculum) in this case study. As referred to in Chapter 4.4, what is not assessed in education becomes invisible and unimportant. Based on this, also exam tasks are relevant to analyse, and I included tasks from the national examinations in the analysis (*assessed* curriculum). This made a basis for the further development of the case study.

Teachers' perceptions of mathematical modelling affect how mathematical modelling is implemented in classrooms (Ärlebäck, 2010). The next step was to explore teachers' perceptions of mathematical modelling, the *perceived* curriculum. A questionnaire was developed for upper secondary school mathematics teachers based on so far known research literature, the results from the analysis of textbook tasks and exam tasks, and my own experience as a mathematics teacher. I sent the questionnaire to three mathematics teachers whom I knew, asking them to answer the questions as well as to provide feedback on the questions and suggestions for improving the formulations, and to indicate the duration of the process of answering the questionnaire. Based on the responses from the three teachers, I rephrased a question as two multiple choice questions but also included "others" as an option, which opened for typing their answers. I included one question concerning “theoretical mathematics” and one about “practical mathematics” (which are two separate courses of mathematics for the students to choose from in upper secondary school). I also specified some formulations based on the responses from these three teachers. Since the questionnaire was sent to all mathematics teachers at a school and not only for the specific course 2P, I included markers for which courses they had been teaching. If they answered 2P they were asked to answer the rest of the questionnaire based on the course 2P.

The data from the questionnaire was seen as a part of the enquiry process of the case study. The answers to one of the questions in the questionnaire were evaluated in depth: “What is required in a task to call it a “modelling task”? Give one concrete example”. First, the processes and the starting point of the presented tasks were identified. For each of the teachers’ answers to the questionnaire, the verbs/processes and the associated adverbial/object were identified. The answers were sorted according to whether they expressed the data to be presented in the task or if the students had to provide the data themselves (e.g. through experiments or measurements). The teachers’ answers were also sorted by whether they expressed specific mathematical content or mathematical processes. These different ways of working with the material gave me as a researcher insight into the *perceived* curriculum, the teachers’ understanding of mathematical modelling. I became also aware that identifying the depth of engagement in real-world situations was difficult based on the questionnaire. Therefore, I decided to rather base the analysis of the *perceived* curriculum on interviews.

In the design of the interview guide, I draw on the experiences from the questionnaire and earlier research. To gain experience with the teachers' view on engagement in real-world situations, I included three different tasks with different starting points in the interview guide, inspired by Frejd (2012). In the questionnaire, the teachers were asked what, in their opinion, was the most important thing in the teaching of practical mathematics, 2P. One of the teachers responded, “Important for what? For their grades or everyday life?”. This question was included in the interviews; if it is experienced as a conflict, to teach for the exam or to emphasise relevant mathematics for the students’ everyday lives. The aim was to be able to identify discursive and social practices of the teaching of mathematical modelling.

A semi-structured interview allows flexibility and emphasises the interviewee’s views and understanding of issues (Bryman, 2016); therefore, it is seen as relevant within a flexible research design, where the teachers’ understanding of mathematical modelling is of interest. The teachers were individually interviewed, as I was interested in how they discussed the teaching of mathematical modelling as individuals and not as a member of a group of teachers, which would rather be a focus group interview. The interviews were conducted in the teachers’ schools for practical reasons – to make it easier and less time-consuming for the teachers to participate; moreover, to be interviewed in familiar surroundings would be less stressful for the teachers.

As I worked with the analysis of the teacher interviews, I also wanted to include the *ideological* curriculum to unfold more aspects of the teaching of mathematical modelling. Therefore, article 2 includes both interviews and document reviews, following Stake’s (1995) suggestion for qualitative case study research and providing the study of intertextuality in the discourse analysis.

To engage in the *enacted* curriculum, I participated as an observer in the interviewed teachers’ classrooms when they were teaching mathematics 2P. Both the day they were interviewed, and some of the days the next weeks. During the observations, I wrote down what I noticed, and I also talked to the teachers and students whom I met, asking clarifying questions. After every visit to the schools, I wrote down my reflections and experiences. In these first observations, no audio recordings were made. However, to conduct a discourse analysis of the dialogues between the students and teachers, I decided to audio-record the conversations in observations for the analysis of article 3 in addition to using observation schemes.

In the design of the observation scheme, I added the columns "time" and "activity" to make track of the audio recording. During the earlier observations, I noticed that the teachers were referring to real-life situations, exams, textbooks, everyday life, logical structures of mathematics or the *intended* curriculum when they motivated the students. Therefore, in the observation scheme for article 3, I also added the column "refers to" to facilitate the understanding of the discursive and social structures. During and after the observations I also wrote down my reflections, and questions to ask the teacher or students.

To synthesise the findings from this case study and answer the overall research question, a meta-analysis was included in this extended summary drawing on Blum's (2015) six modelling perspectives, and the interactions between the different curriculum discourses were discussed as discursive and social practices in this extended summary.

Summarised, the research design is presented in Figure 4.

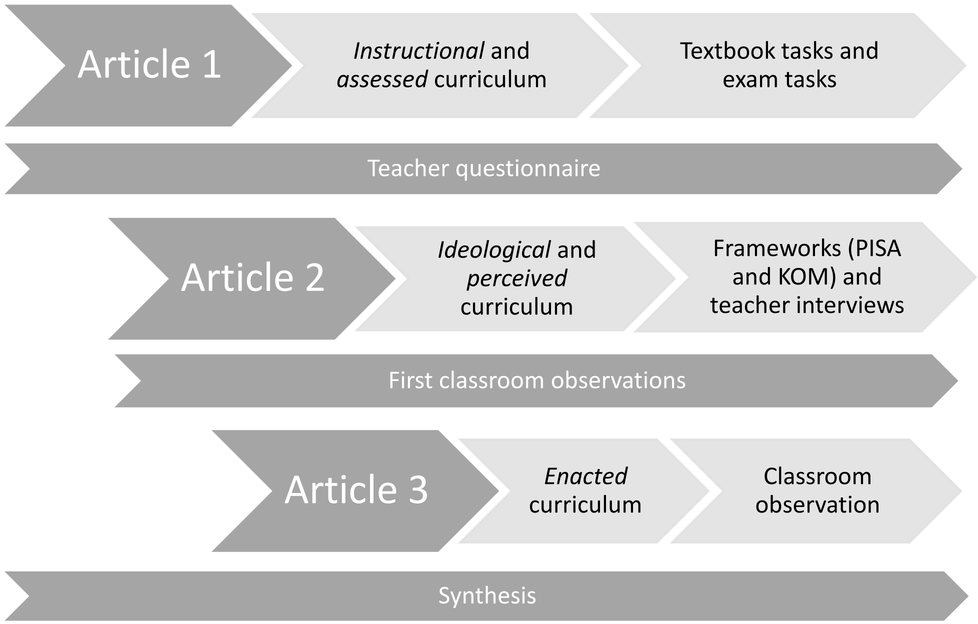

Figure 4: Overview of the research design

5.3 Research samples and participants

The students in the specific mathematics course 2P are about 17 years old, in their second year of upper secondary school and their 12th school year. This course is *practical mathematics* (named 2P), which is an alternative to *scientific mathematics* and *mathematics for social studies and economics*. In Norway, the students in upper secondary school choose one mathematics course to follow for a whole school year. The students have mathematics 5 hours a week during the first year of upper secondary school. Those who chose 2P have only 3 hours a week in the second year and no mathematics in the third year of upper secondary school. The alternative courses have 5 hours a week both in second and third grade. According to the teachers answering the questionnaire, it can be challenging to motivate the whole student group for doing mathematics both at school and at home. One of the interviewed teachers saw the student's (lack of) mathematical knowledge as a challenge, and he needed to focus on the basics. Another teacher described the student group as diverse, and it was challenging to follow up with both the students struggling to pass the course and also the students which needed more challenges. All the data were collected before 2021 when the *intended* curriculum LK06 was still current in 2P. The *main subject areas* were “Number and algebra in practice”, “Statistics”, “Modelling”, and “Functions in practice”. Each week the students in the observed classrooms got a plan for the week, where textbook tasks were listed. The students were encouraged to solve the rest of the tasks at home which they did not manage to do in the mathematics classes.

5.3.1 Textbooks and exams (article 1)

To analyse both the *instructional* and *assessed* curricula and see them in relation to each other, the tasks in the textbooks, and not the entire textbooks, were chosen to be analysed, as they could be analysed the same way as the exam tasks. In total, 514 $(108 + 149 + 257)$ tasks from textbooks by the three largest textbook publishers were included in the sample of textbook tasks. Each textbook aims to cover all the *intended* curriculum’s main subject areas. In two of the books, there was an own modelling chapter, and the sample of tasks includes all the tasks from this chapter. The last book has two chapters combining the main subject areas “Modelling” and “Functions in practice”, and I, therefore, included all the mathematics tasks from these two chapters. Most Norwegian upper secondary schools use one of these three textbooks. In informal conversations, teachers mentioned that one of the textbooks included step-by-step instructions in GeoGebra, which they preferred. Another textbook included some open-ended tasks, which the teachers found challenging for the students. They rather preferred tasks which the students could recognise from the examples given in the book. All three books include earlier given exam tasks, and most of the other tasks are formulated like these. The textbooks in Norway are not government approved but are published by independent publishers.

The sample of exam tasks included all exams available at the website of the Directorate for Education and Training when this first part of the study was conducted, the 10 exams from the years 2014–2018 (two exams per year) – 112 tasks in total.

5.3.2 Teachers participating in the survey

A request to participate in the study was sent to the principals of 15 schools. These 15 schools were chosen because of their location – within three hours of travelling distance for the researcher and in two different counties. The plan was to later request some of the teachers if they could be observed in their classroom over time, and it, therefore, had to be possible to travel back and forth to the school within a day. Of the 15 schools, 8 responded positively. The 8 schools are rural public schools, most of them situated in small communities where the population is quite homogenous in terms of cultural and social background. The questionnaire, as presented in Appendix A, was sent to all 49 mathematics teachers in these 8 schools. 39 of the teachers responded, and 35 of those had been teaching mathematics 2P. These 35 answers were further interpreted.

5.3.3 Frameworks, teachers interviewed (article 2) and first observations

Parts from two different frameworks were included in the analysis in article 2. Because the KOM framework by Niss and Jensen (2002) has influenced the Norwegian *intended* curriculum (Botten, 2016), and the PISA framework (OECD, 2003) also has had an impact (Breakspear, 2012), they were chosen for the purpose. Both frameworks include mathematical modelling as an important part of mathematics. The parts from the KOM framework concerning Modelling competence were included in the analysis and Mathematisation from the PISA 2003 framework.

My original plan for data collection for the *perceived* curriculum in article 2 was to contact the participants of the questionnaire and invite them to also participate in interviews. However, as a new school year was about to begin, some of the teachers were assigned to teach different grade levels than those from the previous year. Therefore, I rather contacted four schools and asked who was teaching mathematics 2P that year. The four schools were randomly selected from the fifteen schools after excluding four schools where I knew one or more of the teachers, to avoid possible ethical conflicts. The first teachers named in each of the four schools were contacted and asked to participate in interviews. I also asked them if I could participate in their classrooms as an observer. All four teachers responded positively. The sample of the four teachers who were interviewed was diverse in terms of gender, age and educational background. Their teaching experience in mathematics ranged from 4 to 20 years. The teachers were interviewed based on the interview guide (see Appendix A).

I also participated in the four teachers' classrooms as an observer in $5 + 4 + 3 + 1 = 13$ lessons and an additional lesson by a substitute teacher in one of the schools. There were practical reasons for the different numbers of observations. About $20 - 25$ students participated in each of the lessons which I observed. The duration of each lesson was from 0.75 to 1.5 hours, depending on the schools' timetable. These first observations were not included in the analysis of either of the articles.

5.3.4 Observations (article 3)

Two of the four teachers who participated in the interviews, chosen based on the interviews and first observations, were asked to participate in the next study by being audio-recorded and observed during their teaching of mathematical modelling in the next school year. Including two teachers allowed in-depth analysis of the observations. The two teachers expressed their teaching aims in different terms. One said that the aim was for the students to pass the exam. This teacher did not expect the students to understand all the mathematics behind the mathematical algorithms and procedures, but the aim was to enable them to perform the procedures adequately enough to solve the exam tasks. The other teacher aimed to ensure that the students experienced being knowledgeable about mathematics. Both teachers demonstrated different approaches in their teaching during the first observations. One seemed focused on the students to get the correct answer, and often gave it to them. The other was asking questions instead of answering them. The data analysed in article 3 were collected through classroom observations of these two teachers when they were planning to teach mathematical modelling. Data were collected over a period of about five weeks in each of the two classrooms, in a total of 13 ($8 + 5$) observations lasting 0.75-1.5 hours, dependent on the schools' timetable. It included audio recordings of the teacher and his/her dialogues with students.

During the observations, I used an observation scheme, and the teachers also gave me the tasks they gave the students. I sometimes took a picture of the blackboard in the classroom or of student work. This was not directly included in the analysis, but it was useful in the enquiry process of the case study.

5.4 Analysis

According to Stake (1995), "analysis essentially means taking our impressions, our observations, apart" (p. 71). The researchers' impressions are seen as the main source of data and making sense of them is the analysis.

5.4.1 Analysis of tasks through a modelling cycle (article 1)

In article 1, I analysed the textbook and exam tasks based on the modelling cycle of Blum and Leiß (2007). This cycle and the development of the analysis instrument are presented in article 1 (Appendix C). The analysis instrument was used to identify different steps of the modelling cycle needed to solve the tasks.

The modelling cycle by Blum and Leiß (2007), as outlined in article 1, could be placed under the perspective of applied modelling. However, it can be considered to have a cognitive perspective from the student's view while solving a task, as it includes cognitive steps (Borromeo Ferri, 2018). This points to the educational modelling perspective, focusing on the meta-perspective of the modelling process and developing modelling competence. This cycle was chosen because the whole cycle could represent the processes within some of the perspectives. If only parts of the steps were identified, this could point to other perspectives. For example, if most tasks only needed step 5, interpretation, and step 6, validation, this would have indicated a focus on critical assessment models made by others and pointed towards the perspective of socio-critical modelling, where critical assessment is presented as important. I also registered if the textbook tasks had a given correct answer. This revealed discursive practices and structure of the book.

The development of the analysis was inspired by earlier research, e.g., Frejd's (2011) eleven categories when analysing exam tasks concerning processes of a modelling cycle. The analysis helped "taking [my] observations apart", as noted by Stake (1995, p. 71). My impression was that not all parts of the modelling process were emphasised in the textbook tasks and the exam. Therefore, I wanted to analyse the tasks to find out more about them. Following critical realism, it exists a reality. By analysing elements of it, one can unfold parts of this reality. By analysing other aspects of the tasks, one would have unfolded other parts of the reality concerning textbook tasks and exam tasks. This analysis revealed which parts of the modelling cycle were emphasised, and the intertextuality of the two discourses.

The results from the task analysis in article 1 are discussed within the framework of curriculum assessment as expressed by Porter (2006): the *intended*, *enacted* and *assessed* curriculum and the two overarching perspectives on mathematical modelling – modelling for the sake of mathematics and mathematics for the sake of modelling. In article 1, I did refer to the textbook tasks as the *enacted* curriculum, but as the case study developed, this curriculum discourse was instead analysed through classroom observations; the curriculum discourse represented by textbook tasks is called *instructional* curriculum in this extended summary.

5.4.2 Analysis of interviews and frameworks (article 2)

The interviews were transcribed by the researcher shortly after they were conducted, as recommended by Porter (2006), and the teachers' gestures were described, when seen as relevant, and written in parentheses. The four interviews were analysed using the four categories of communication used by Morgan and Sfard (2016), as explained in article 2 of this thesis. They made a table containing guiding questions and relating textual indicators for each of the four categories: *word use*, *routines*, *visual mediators* and *endorsed narratives*. These four categories are seen as relevant categories of a mathematical discourse (Sfard, 2008).

The analysis tool used by Morgan and Sfard (2016) included guiding questions along with textual indicators to identify answers to these questions. They analysed the discourse of mathematics exam tasks by comparing exam tasks from different decades. Although their analysis tool was originally developed to evaluate changes in mathematics exam texts over time, they suggested that the instrument could be adapted to compare discourses in written or spoken forms within mathematics education (such as teacher discourse from teacher interviews and written frameworks concerning mathematical modelling within mathematics education). Phillips and Hardy (2002) argued that studying individual texts only gives clues into the nature of the discourse but that bodies of texts should also be evaluated (e.g. interrelations between text and change), along with the social context in which the texts are found and the discourses produced in these contexts. Within word use, Morgan and Sfard (2016) investigated specialised vocabulary by looking at lexical items using a coding tree and noting the frequency. This was not seen as relevant for this study because, in article 2 of this thesis, the focus was on oral language, not exam tasks. Moreover, the four teachers expressed themselves differently, and lexical bundles could not have been compared or counted the same way as they were for the exam tasks. Morgan and Sfard's (2016) study analysed the evolution of school mathematics as a change in discourse. However, in this study teacher interviews were analysed to explore the teaching of mathematical modelling in mathematics 2P and not the change over time. To compare the findings in the teacher interviews and the description of mathematical modelling in education that originated outside of the classroom was the interest of this study. The descriptions of modelling competence and mathematisation within the frameworks were also analysed using the adjusted analysis tool by Morgan and Sfard (2016) as described in article 2 (see Appendix C).

Certain adjustments to the questions guiding the analysis were made. First, the original analysis tool aimed to analyse mathematics exam tasks, in which logical complexity was of interest. This was not included in this study. Furthermore, a question asking about the specialised mathematical language used was changed to one asking about the specialised language used within mathematical modelling in education (e.g. mathematising). Two guiding questions were added. Within *word use* a question about specific mathematics terms related to mathematical modelling was added based on the responses to the questionnaire, where the example tasks mostly referred to specific mathematical content. The *routines* section already had a question on this topic, but the added question was specifically about modelling and mathematical content. The final added question was whether a modelling cycle was included within visual mediators. This is connected to the question about specialised language used within mathematical modelling in education, as scholars often describe the process as a cycle. Both teacher discourse and framework discourse were analysed using the guiding question and textual indicators presented in article 2.

The analysis was useful for identifying the endorsed narratives about mathematical modelling by taking my impression apart. Within this analysis, there was both the language, how the teachers expressed themselves about mathematical modelling, and the students' routines, processes and use of visual mediators described by the teachers were considered. The four textual indicators gave a width, and when also including the frameworks, this was opening more about the reality concerning mathematical modelling in mathematics education.

5.4.3 Analysis of classroom discourse through Positioning Theory

The analysis for article 3 began when taking notes on the observation schema in the classrooms. Immediately after each observation, I listened through the entire recording and completed the notes on the observation schema, filling out anything I had missed and highlighting interesting conversations. Next, I transcribed the recordings using black text for the teacher and green for the students. When transcribing group conversations, I used italics but not when the teacher and students spoke in a plenary. The students were named s1, s2, s3 … when several students were involved in the same conversation. In subsequent conversations, other students were given the same names. Sometimes, when the teacher was using the blackboard, I took pictures of it; at other times, I wrote down the text on the board on the observation schema. If the students were given worksheets, I got a copy and marked them with the school number and an observation number. In the transcriptions, explanations were included in parentheses if the teachers' or students' actions seemed to be an important part of the communication.

The focus of the observation was classroom discourse when teaching and learning mathematical modelling. The findings in articles 1 and 2 revealed a lack of emphasis on students' decision-making, assumptions and engagement in real-world situations. In the process of deciding on the analytic framework for article 3, different approaches were considered. An aim was to emphasise both student initiatives and how mathematical modelling was communicated in the classrooms. Therefore, the theoretical lens chosen for the analysis was narrowed down to Positioning Theory because speech was evaluated together with the participants' positioning and how this created a storyline – in this case, about mathematical modelling. An analysis tool was developed based on the positioning categories by Langenhove and Harré (1999) presented in article 3 of this thesis. The focus was on student agency; therefore, episodes in which a student said something other than (1) giving the one correct answer to a question asked by the teacher, (2) asking what to do or (3) asking if their answer was correct, were exposed and evaluated by identifying the different types of positioning during the dialogue in the episodes. The identification of the type of positioning is related to the storylines of the episodes concerning mathematical modelling.

Identifying the positionings in the classroom, both by students and teachers, revealed by their speech and act, helped me to identify discursive structures. It revealed actions they had to do or chose to do, which formed the storylines about mathematical modelling. Also, possibilities for students' agency were identified.

5.4.4 Meta-analysis of modelling perspectives

The six perspectives on mathematical modelling by Blum (2015) presented in Chapter 2.4 were included to present differences in how mathematical modelling is presented within the research field of mathematical modelling in mathematics education. It was also included in the meta-analysis presented in this extended summary to take apart the researcher's impressions regarding mathematical modelling in all parts of this case study. Including the perspectives provided more precise descriptions of mathematical modelling in the different discourses, and also helped discuss differences in the curriculum discourses included in this case study. The meta-analysis is not based on a systematic analysis of the data from each of the curriculum discourses regarding the six modelling perspectives. The results from each of the articles were synthesized with the researcher's experiences from the case study and analysed by identifying aims, tasks, and described processes to find traces of the different perspectives. Table 3 was used in this meta-analysis for each of the six curriculum discourses.

The […] curriculum	Aim	Activities	Modelling process
Applied modelling	Develop skills to model and understand authentic real-world scenarios	Authentic, messy real-life tasks that require the use of the modelling cycle.	Real-world and mathematics are seen as separate parts. A cyclic multistep process.
Educational modelling	Realising one's own competency growth. Develop modelling competency and understand mathematics.	Analyse and solve real-world problems, and meta-reflections. Both holistic and atomistic tasks.	Modelling process as sub-processes. All parts are important.
Socio-critical	Understand the role	Analyse and assess	Parts of a modelling

modelling	of mathematics, and develop mathematical modelling skills to make decisions in society.	given models from society. De-mathematise.	cycle are emphasised, including validation, and interpretation.
Epistemological modelling	Comprehending mathematics as a science. Forming a mathematical scheme.	Using mathematics as a tool to solve real-world problems, finding the best mathematical means.	Focus on the inner mathematical structures.
Pedagogical modelling	To make mathematics better marketable for students and facilitate motivation and creativity.	Open-ended tasks could be expressed in constructed fantasy-contexts.	Mathematisation from, and interpretation to the context, and working mathematically.
Conceptual modelling	To understand and learn mathematics.	Mathematically rich examples	Model of, model for, as in van den Heuvel-Panhuizen (2003).

Table 3: Analysis tool to identify modelling perspectives by aim, task or cycle. From the descriptions given in Chapter 2.4 and inspired by Abassian et al. (2020).

In the meta-analysis, each of the cells was assessed and marked by colour if it could be identified within that curriculum discourse. If all three indicators were coloured within a perspective, this was shaded in the first column for this perspective and toned if two of the indicators were marked. If only one of the three indicators was coloured within a perspective, then it was tinted for this curriculum discourse in the first column. The cells in the first column of Table 3 were then coloured from dark to light, depending on how many of the indicators were identified for each of the perspectives. The first columns for each of the tables for the different curriculum discourses were inserted as a row in Table 4, showing the result of the meta-analysis in Chapter 6.4. The indicator of the modelling cycles was identified by a description of the modelling process, and not by an included illustration. In curriculum discourses which did not include modelling activities, descriptions of such activities were analysed. One must recognise that if a cell is not coloured it only means that it was not identified within the curriculum discourse. That is not necessarily equivalent to this perspective not being present within the given discourse. E.g., in the textbooks, only the tasks were analysed, and not the introduction to the modelling chapter in the textbooks, which might include aims. But the tasks do both describe the process, which is emphasised through the analysis in article 1, and the type of tasks. That is, if a perspective is included in the textbooks, it will at least be identified through two of the three indicators.

A reason for including this meta-analysis was to be able to present perspectives on mathematical modelling identified in the different curriculum discourses of this case study explicitly, to form a basis for discussion of the interactions between the discourses, and to visualise the identification of perspectives.

5.4.5 The relations of the different analyses

The Positioning Theory applied in article 3 is developed from discourse theory. In this study, the storylines within Positioning Theory are seen as locally endorsed narratives of a specific conversation, as described in Chapter 3.4. The storylines were identified by the positioning and speech/act in the teacher-student dialogues. That is, the positioning was included in the identification of the storylines. When identifying the endorsed narratives, rather the textual indicators about word use, routines and visual mediators were used as an analysis tool.

The analysis of the textbook tasks and exam tasks by the modelling cycle differs from these other two analyses since it in advance defined what the processes of modelling are, and the analysis is to identify if these are present. This is the same for the meta-analysis, where aims, activities and processes are already expressed. But even if the modelling cycle was chosen in advance, this was used to identify how mathematical modelling was expressed by the tasks. Also in the meta-analysis, the descriptions were used for identifying how mathematical modelling was expressed, and not only if it was expressed in one or the other way.

Both the *ideological*, *instructional* and *assessed* curricula were analysed based on written text produced by authors. Here only the consumption of the text is included in the case study. The *perceived* and *enacted* curriculum were represented by transcriptions of oral language, and here also the production of the text is taken into account in the discourse analysis in the discussion of this extended summary. The text produced through oral language is expected to be of another genre than the curriculum discourses of written text. But the aim of the different analyses is the same, to unfold parts of the reality of the teaching and learning of mathematical modelling.

5.5 Trustworthiness

Trustworthiness is evaluated to assess the quality of qualitative research. Bryman (2016) referred to Guba and Lincoln (1994), who described the four criteria of trustworthiness: credibility, transferability, dependability and confirmability. Each of these will now be examined in the context of this study. Credibility parallels internal validity, and triangulation is recommended (Bryman, 2016). Stake (1995) presented four strategies for triangulation: data source triangulation, investigator triangulation, theory triangulation and methodological triangulation. This case study used data source triangulation, as all the data sources recommended by Stake (1995) were included: interviews, observations and documents. Article 1 used investigator triangulation, as six master's students analysed parts of the tasks that I also analysed, to confirm trustworthiness. This thesis also uses theoretical triangulation, as it draws on different theories in the three articles – discourse theory, Positioning Theory and curriculum assessment – and the different theoretical perspectives on mathematical modelling. The methods used in this thesis include content analysis and Discourse Analysis. To measure credibility, respondent validation is also recommended (Bryman, 2016). This was done during the interviews by asking for confirmation from the interviewees that I, as an interviewer, had correctly understood their responses as well as by requesting clarification from the teachers and students about episodes after observations. For example, I asked a teacher about his reasons for a decision made in a certain situation, and students about their earlier experiences using manipulatives.

Transferability parallels external validity (Bryman, 2016). The external validity or generalisability of case studies cannot be verified; nor is this an aim for case studies. In this study, the case of mathematics 2P was chosen because of the prevalence of mathematical modelling in the *intended* curriculum, where mathematical modelling is expressed as a fundamental process. The rationale was that if mathematical modelling was included in mathematics teaching, it should be in this specific course. The focus is the teaching of mathematical modelling. One can only say that some teachers communicate about mathematical modelling as presented in this study, but one cannot conclude that this is how all teachers communicate about mathematical modelling in general. Within transferability, researchers performing qualitative studies are encouraged to produce thick descriptions that provide others with a basis for making judgements about the possible transferability of the findings to other milieux (Bryman, 2016). Examples of the analysis showing parts of the data are therefore presented in each of the articles in this thesis.

Dependability parallels reliability (Bryman, 2016). Researchers should adopt an auditing approach to meet this criterion of trustworthiness. Stake (1995, p. 107) also pointed out the importance of validation: “All the way through our case study work, we wonder, “Do we have it right?” Not only “Are we generating a comprehensive and accurate description of the case?" but "Are we developing the interpretations we want?". This includes asking if it is correctly understood during the interviews and also after the observations are made.

Confirmability parallels objectivity. Within qualitative research, complete objectivity is impossible. Nonetheless, regarding confirmability, the researcher can be shown to have acted in good faith. As such, it should be apparent that he or she has not overtly allowed personal values or theoretical inclinations to sway the conduct of the research and the findings derived from it (Bryman, 2016). My background as a teacher piqued my interest in the teaching of mathematical modelling and has also influenced my values. As a researcher, it is important to be aware of one's own values, to keep in touch with the data and theory and to be aware of the possibility of seeing what one hopes to find instead of what is actually there. This research was conducted in good faith in an attempt to learn more about the teaching of mathematical modelling.

Being aware of what information is given to the participants is also important. The participants might act in a way they think is preferable. For example, when performing the first observations, I informed the teacher about my interest in mathematical modelling in the practical mathematics course. In the class I was observing, the students were given tasks in which they were supposed to use manipulatives as a learning resource. I asked some of the students if they were used to working with manipulatives, and they said they had never done it before. When I asked the teacher about it afterwards, she explained that she thought I was interested in observing practical mathematics and mathematical models, and therefore she chose this activity. Further, in my subsequent visits to different schools, I said I was interested in how they usually taught. I did sometimes get questions from the teachers about my opinions on different aspects of teaching mathematical modelling in informal conversations before or after observations. I was careful not to express any specific opinion and explained that I did not want to lead them in a specific direction.

Because qualitative research is based on the researcher's subjective interpretations, reliability and validity are important to assess. Within Critical Discourse Analysis, trustworthiness depends on the transparent articulation of the researcher's standpoint (Mullet, 2018). Bryman (2016) pointed out that the researcher assessing the curriculum should be familiar with student work and develop a clear framework for assessment. He noted how challenging it would be to analyse the *assessed* curriculum from reading a test item and how students will approach that item. Because of the students' different experiences in the classrooms, they might face different challenges with the same test item. This challenge also applies to textbook tasks. As a former upper secondary school teacher, I believe that I am familiar with student work and other curriculum discourses. As a lecturer in teacher education for a course including mathematical modelling and problem-solving, I am also familiar with the teaching of mathematical modelling. Therefore, I see myself as familiar with it, in line with Bryman (2016). Furthermore, for each of the three studies, I adjusted or developed a framework for analysis, as suggested by Bryman (2016).

According to Stake (1995), a qualitative case study researcher should "consciously and unconsciously test out the veracity of their eyes and robustness of their interpretations. It requires sensitivity and scepticism" (p. 50). This I have strived for in this study.

5.6 Ethical considerations

Ethical considerations have received increasing awareness in social research and should be addressed by all researchers (Bryman, 2016). These considerations can be broken down into four main areas (Dinener & Crandall, 1978, as cited in Bryman

(2016)): harm to participants, lack of informed consent, invasion of privacy and involvement of deception. In social research, the researcher and participants might develop a close relationship, and it is important to treat the participants with respect.

Harm to participants can include physical harm, harm to participants' development, their loss of self-esteem, stress and allowing them to perform reprehensible acts. In this study, the participants are teachers, and indirectly the textbook authors. Even if I reveal shortcomings concerning holistic modelling tasks in the textbooks, this does not mean I advise against the use of certain books. I have only studied the tasks presented in the modelling chapter and do not highlight all the positive aspects of the textbooks. I have no intention of harming the textbook authors. Because of my experience as a mathematics teacher, I could understand if the teachers were experiencing stress during the observations or interviews. One of the teachers apologised during the interview, admitting shortcomings in his education regarding mathematical modelling. He might have experienced a loss of self-esteem while I, as a researcher, was asking him about teaching mathematical modelling, which he might not have reflected upon earlier. Therefore, I tried to communicate that I was also a teacher who was trying to learn about the teaching of mathematical modelling. I saw their different backgrounds as strengths and was interested in how they were teaching in their classrooms. During the observations, the teachers expressed their nervousness, but after the first lesson, they said that during the lesson they had forgotten that they were being observed. Because I was interested in how they generally acted in the classroom, there was no need to induce subjects to perform any acts, much less reprehensible acts. I also communicated that my interest was not to compare teachers and determine the good or bad ways of teaching but rather to find out what mathematical modelling could "look like" in the classroom. Teachers' beliefs are the products of their experiences. They are not fixed and cannot be said to be right or wrong.

Interviews and observations may involve information about a third party not directly involved in the study. For example, teachers might mention colleagues, and students might talk about family members or friends. It was therefore important to ensure that the records remained confidential and that the participants remained anonymous in the published report.

To this end, the schools were represented by numbers 1–4, as were the four teachers at these schools who participated in interviews and observations. If they referred to a certain place that could indicate the location of the school, this was transcribed as [place]. The students were not distinguished unless they were participating in the same conversation. Then, they were given numbers S1, S2, S3 …, but the same numbers were used for other students in other episodes.

The second area of ethical consideration is informed consent. This includes, according to Cohen et al. (2017), four elements. First, the participants must be competent when deciding whether to participate. Second, participation must be voluntary. Third, the participants must fully understand the situation in which they are putting themselves by participating. Fourth, participants must have full information about the study in which they are participating. The teachers were asked to participate and were informed that if they changed their minds about participation, they could withdraw from the study without any consequences for them. They did not receive any favours for participating in the research project. The participants were informed about the researcher's role in the classroom while observing, how the recordings would be handled and that the transcriptions would be anonymised (see information letters in Appendix B). The participants signed a document to confirm their informed consent.

The third issue, trustworthiness and invasion of privacy, is linked to issues of anonymity and confidentiality in the research process. These, in turn, are related to the issue of harming participants (Bryman, 2016). In this study, personal information, such as health conditions, religious beliefs or income, was not relevant. However, during the periods of observation, which included informal talks before and afterwards, the researcher and the teachers sometimes shared personal thoughts and opinions. In such cases, the researcher needs to be aware and distinguish between information given in confidence and information relevant to the study.

The role of the Norwegian national research ethics committees is to promote high-quality, ethical research. The Norwegian National Centre and Archive for Research Data (NSD) facilitates the sharing and reuse of data about people and society and advises on data management and data protection in research (Norwegian centre for research data, 2021). If a researcher is processing the participants' personal data, the research project is subject to notification to NSD. Note that this study was approved by the NSD.

The final issue, deception, occurs when researchers represent their work as something other than what it is (Bryman, 2016). Bryman (2016) pointed out that it is rarely feasible or desirable to provide participants with a complete account of what the research is about. In this research, there was no reason to hide information from the participants. But no detailed information was given about the first findings of the analysis of textbook and exam tasks, as doing so could influence the teachers and encourage them to focus on these issues in their teaching, even if they normally would not.

6 Presentation of results

In this chapter, the results of each of the articles are presented as part of the *ideological, intended, instructional, perceived, enacted* and *assessed* curricula. Further, the result from the meta-analysis concerning the six modelling perspectives is presented.

6.1 Results article 1

The *instructional* curriculum was analysed through modelling textbook tasks, and the *assessed* curriculum was analysed through exam tasks.

Mathematical modelling in the *instructional* curriculum

In article 1, textbook tasks were placed within the *enacted* curriculum; however, in this extended summary, these are understood as the *instructional* curriculum.

Textbook tasks were analysed in terms of students' opportunities to perform the different steps of a modelling cycle while solving them. Results showed that in the three textbooks, few or no holistic tasks required the students to go through all the steps of the modelling cycle to solve them. Most tasks included working mathematically, and over half of the tasks also included interpreting the results in a real-world context. In about 20% of the modelling tasks in the textbooks, validation was asked for. Most of the tasks could be solved by following a given procedure, and a correct answer was provided for nearly all of the tasks in an own section of the textbooks. Mathematical modelling was communicated as performing regression analysis in GeoGebra from a given set of numbers and using the mathematical function to answer questions posed in everyday language by reading the graph. The students were expected to evaluate the range of the mathematical function in the given context.

Mathematical modelling in the *assessed* curriculum

In the *assessed* curriculum, the exam tasks comprised closed and narrow questions with only one correct answer, and these could be solved by applying certain algorithms. The percentage of exam tasks including mathematical work and interpretation was about the same as that of the textbook tasks, but only 3% of the tasks asked for the validation of a model. The exam tasks had one correct answer also for the mathematical models to be developed. Therefore, according to the *assessed* curriculum, it can be argued that mathematical modelling involves solving word problems by performing a given mathematical procedure and interpreting the mathematical answer in the given context.

Summary of findings in article 1

The textbook and exam tasks were similarly formulated, and the same steps of the modelling cycle were generally needed to solve the tasks. The steps included working mathematically (step 4), interpreting (step 5) and, in the textbook tasks, validating (step 6). Earlier exam tasks were included in the textbooks, which show the consumption of the *assessed* curriculum when producing the *instructional* curriculum.

Even if the *intended* curriculum presented the starting point of the modelling process as something that actually exists, the starting point of the tasks was to work mathematically. The real-world situations were already mathematised by the textbook authors, and a mathematical problem was presented in an everyday context.

6.2 Results article 2

The *ideological* curriculum was explored through KOM and PISA framework, and the *perceived* curriculum was explored through teacher interviews.

Mathematical modelling in the *ideological* curriculum

In the *ideological* curriculum represented by the PISA and KOM frameworks, students' involvement in the modelling process is emphasised. The first steps of the modelling cycle, which are seen as a crucial part of mathematical modelling in the PISA framework, include making assumptions and generalisations (OECD, 2003) and are presented as steps 1–3 in the modelling cycle in Figure 1. The connection to "the real world" is emphasised. According to the KOM framework, if students are not really involved in real-life situations, it is not considered mathematical modelling (Niss & Jensen, 2002). Modelling competency involves the ability to use mathematics to manage real-world problems by building mathematical models and critically assessing mathematical models presented by others (Niss & Jensen, 2002). This includes both *modelling for the sake of mathematics* and *mathematics for the sake of modelling* (Ferri, 2013).

Mathematical modelling in the *perceived* curriculum

According to a teacher, if students are given open-ended tasks, they would take too long to figure out what to do. The teacher would therefore guide the students in the direction she has in mind, and finds it more efficient to tell them what to do. This may indicate that the teacher's focus was on solving mathematical problems rather than on the earlier steps in the modelling cycle, as expressed in the *ideological* curriculum. This was also confirmed by another teacher, who pointed out that the starting point of the modelling process was within mathematics: to find a mathematical answer and to interpret it in the context given in the task. The teachers did not describe the students' opportunities to make choices when building models or de-mathematising models in society and did not refer to any modelling cycle.

Within the *perceived* curriculum, mathematical modelling was expressed as functions: to find functions using regression analysis in GeoGebra or to express given information as a linear, quadratic or exponential function. Modelling was also seen as figurate numbers expressed as functions. In such tasks, a correct answer was given; therefore, there is no room for students' own choices. The teachers expressed that the students were not given problems from daily life situations, but the mathematical tasks were formulated in an everyday context.

The teachers participating in the interviews had not been exposed to mathematical modelling in their mathematics education or their teacher education. They had only encountered the notion of mathematical modelling in mathematics education through the textbook and the *intended* curriculum.

For the teachers, the aim was for the students to be able to solve the tasks given on the exam as well as to master mathematics. They based their choices on how mathematical modelling was presented in the textbooks and exam tasks. Mathematical modelling was not by the teachers expressed as a fundamental process in school mathematics; instead, as a part of school mathematics connected to the content area functions.

Summary of findings in article 2

From the analysis in article 2, five tensions between the *ideological* and *perceived* curricula were identified:

1) Students' engagement in real-world situations, the first steps of the modelling cycle in Figure 1, is seen as important in the *ideological* curriculum. This is not emphasised by the teachers – the *perceived* curriculum. The teachers rather lead the students into the world of mathematics so that they do not waste time on the process of mathematisation.
2) Modelling is in the *perceived* curriculum seen as an extended part of the mathematical content area *function in practice* and not related to other

mathematical contents. In the *intended* curriculum, modelling is not connected to any specific content area of mathematics, but examples are given from diverse areas.

3) The teachers were not familiar with the theoretical knowledge of teaching and learning mathematical modelling. This involves different perspectives on mathematical modelling, aims, different types of tasks (holistic/atomistic) and modelling cycles as a tool for the meta-reflection of the modelling process. In the *ideological* curriculum, modelling competence is defined by a modelling cycle and is explained by theoretical perspectives.
4) The *ideological* curriculum presents mathematical modelling as complex cognitive processes (analyse, evaluate, create), and the *perceived* curriculum presents it as less complex cognitive processes (remember, apply).
5) In the *ideological* curriculum, one of the aims for the teaching and learning of mathematical modelling is to better master life. Mathematical modelling is seen as highly relevant also outside of school mathematics. However, within the *perceived* curriculum, mathematical modelling is not experienced as relevant to students' everyday life.

6.3 Results article 3

The *enacted* curriculum was investigated through observations in two classrooms. First, I will present the structure of the lessons observed, before presenting a summary of the findings.

Teaching of mathematical modelling in the *enacted* curriculum

The teachers often started by giving the students a task, which they jointly discussed in class. Often, the teacher asked closed questions and a student answered. Further, the students were most often given textbook tasks to solve. The most common questions asked by the students were "Is this correct?" or "What am I supposed to do here?”, which may indicate that validation and decision-making were part of the teacher’s role and not the students’ role.

Exams were often referred to as a reason they should work on a given type of task. Even when students asked why they should interpret their mathematical results to a given everyday context, this was explained by connecting these tasks to the types of tasks given on exams. The students were not introduced to a modelling cycle, and the teachers did not include meta-reflections about the modelling process. A clear aim for working with mathematical modelling was not presented to the students.

Summary of findings in article 3

The findings showed that the teachers provided students with opportunities for engagement in real-world situations by allowing them to critically validate given models. Then, the students used their everyday life experiences as part of their argumentation and discussed the data from which the model originated. They were given opportunities to deviate from the task by discussing related topics. The students were positioning the teacher by questioning the mathematical model when their everyday experiences did not fit the model. The data on which the model was based was questioned when the students were formulating the model by regression analysis in GeoGebra. They experienced that some mathematical models are only valid within a given domain, and because of variations in real life, the output might not be precise. The students were presented with situations where they had personal experiences, which enabled them to actively participate in the discussion. The teacher seemed to highlight the real-world aspect and validation of the model in these tasks.

For the tasks in which the students were asked to build a model, they were not expected to make assumptions or engage in real-world situations. The teachers led the students to solve these tasks using a given procedure – regression analysis in

GeoGebra. The students were either given the necessary data or instructed on how to collect the data. They were not expected to decide what data to collect.

In these tasks, it seemed as if the teacher emphasised the formal aspects of the tasks and how to find an answer. In one of the episodes, a student raised questions within a related real-world context presented by the teacher. Instead of connecting these questions to the mathematical domain, the teacher did not discuss the questions further. In an episode where the students should provide the data for the regression analysis themselves, a student questioned the model as they experienced the real-world situation to be less precise than the mathematical function. The teacher acknowledged this but further focused on solving the task as planned to provide an answer instead of letting the student explore an alternative approach.

The episodes in which the students took the initiative to deviate from the teacher’s plan show that the teacher’s positioning of the students is decisive for the students’ opportunities for further engagement. If the teacher emphasises formal aspects and how to find an answer, they might miss students’ initiative for engaging in the processes of mathematical modelling.

The storylines about mathematical modelling in these episodes are suggested as the process of solving given tasks by following specific procedures, such as GeoGebra regression analysis. This does not include the process of mathematising, where the students make assumptions and choices for their way of expressing a real-world situation in mathematical terms.

The positioning and opportunities for students’ agency were found to influence the storyline about mathematical modelling.

6.4 Synthesizing results

I will now present the results from the meta-analysis regarding the six modelling perspectives by Blum (2015). Identifying the perspectives in each of the curriculum discourses is an answer to the first part of the main research question of this extended summary. The other part, regarding the interactions between the different curriculum discourses, is included in the discussion in the next chapter.

Table 4, which is developed as described in Chapter 5.4.4, shows that all the perspectives are identified in the *ideological* (which here also includes Chapters 2 and 4) and *intended* curriculum, while only the socio-critical modelling perspective is found traces of in the *instructional* and the *enacted* curriculum.

	Blum's (2015) six modelling perspectives					
	Applied	Educational	Socio-critical	Epistemological	Pedagogical	Conceptual
Ideological						
Intended						
Instructional						
Perceived						
Enacted						
Assessed						

Table 4:Identification of modelling perspectives in the different curriculum discourses.

In the *instructional* curriculum, interpretation and validation were identified as important processes, which is important within the socio-critical perspective. Tasks in the textbooks included analysis of models in society, but the students were only asked to assess the range of the model, and not to de-mathematise and analyse how the models were constructed. The tasks did therefore not fulfil the description of a task within the socio-critical modelling perspective and the perspective is therefore only tinted in Table 4 for the *instructional* curriculum.

As we can see from Table 4, mathematical modelling is communicated as something else than within the six perspectives presented by Blum (2015) in the *instructional*, *perceived*, *enacted,* and *assessed* curriculum. In the next chapter, I will discuss discursive and social structures to reveal power relations between the discourses. Further, I will explore how these affect the teaching and learning of mathematical modelling.

7 General discussion and conclusions

Through Discourse Analysis, this study was conducted for investigating how mathematical modelling is expressed in mathematics education and to reveal discursive practices to identify reasons for the structures of the teaching of mathematical modelling. In the results presented in Chapter 6, there are found reasons to believe there is a gap between how mathematical modelling is expressed in the *ideological* and *intended* curriculum compared with how it is *perceived* and *enacted*. As presented in Chapter 3.4, within critical discourse theory, social events and their texts are not seen to happen randomly. Certain factors shape them and are shaped by them. Here, I discuss social and discursive practices of the different curriculum discourses; factors within and between each of the discourses that may shape their production. The discussion is based on the results of the three articles, the synthesized results, the literature presented in earlier chapters, and experiences from this case study as seen through the lens of discourse theory and interpreted by the researcher.

7.1 Discursive practices and social practices of the curriculum discourses

The holistic character (Stake, 1995) of this case study is taken into account by seeing how mathematical modelling is expressed in relation to its context. All the discourses are connected through discursive practices including intertextuality. And social structures affect the created understandings of mathematical modelling in the curriculum discourses. The production and consumption of the different discourses will be discussed, and those consumed in several of the other discourses will therefore be mentioned several times.

7.1.1 The *ideological* and the *intended* curriculum

Analysis of the production of the *ideological* and *intended* curriculum discourse is not included in this case study, but the consumption is recognised in other curriculum discourses. As shown in the presentation of the Norwegian *intended* curriculum (see Chapter 2.5), the different modelling perspectives from the *ideological* curriculum could be identified here, and as earlier mentioned, frameworks within the *ideological* curriculum (PISA and KOM) have influenced the *intended* curriculum (Botten, 2016; Breakspear, 2012). That is, the *ideological* curriculum is consumed in the production of the *intended* curriculum. There are not found any traces of consumption of the *ideological* curriculum in the other curriculum discourses as manifest intertextuality, that is, the influence of the *ideological* curriculum is not explicitly expressed.

Since "[m]athematical modelling is rather vaguely defined as a curriculum concept" (Jablonka & Gellert, 2010, p. 31), it can be challenging for teachers to grasp its meaning through the *intended* curriculum. The *intended* curriculum was only mentioned by the teachers twice in the interviews. It once came up when a teacher was talking about where they had learned the concept of mathematical modelling, and the answer was "the textbook and the curriculum", the *instructional* and the *intended* curriculum. It was again mentioned when another teacher was asked about what the students spent the most time on when working with mathematical modelling, and while she leafed through the textbook, she said, "According to the curriculum, or?". This was not further discussed. In the interviews, the teachers were asked if they acknowledged mathematical modelling to be a "fundamental process of [mathematics]". This was a phrase from the *intended* curriculum, but neither of the teachers recognised it; they did not see modelling as an overarching process in mathematics. The *intended* curriculum was only mentioned once in the *enacted* curriculum discourse when a student asked which types of regression were relevant for the exam. Then, the teacher answered that they only had to focus on the types of functions mentioned in the [*intended]* curriculum.

There was not to a great extent identified consumption of the *ideological* and the *intended* curriculum in the other curriculum discourses. This is also shown in the meta-analysis; the different perspectives on mathematical modelling could not be recognised in the other discourses.

7.1.2 The *instructional* curriculum

Discursive structures in the *instructional* curriculum were pointed out by a teacher answering the questionnaire: In the textbooks, mathematical modelling is presented as a chapter instead of being a part of all the different content areas. And modelling is seen as a part of the content area of functions. As mentioned in Chapter 4.3, Grigoraş et al. (2011) claimed that if mathematical modelling is taught within a given mathematical topic, the students are pointed towards a certain direction in the mathematising of real-world situations. The structure of the textbook, therefore, influences the students' experienced opportunities to make their own choices in the process of mathematising. That is, the *instructional* curriculum discourse maintains discursive practices in the *enacted* curriculum discourse. This was demonstrated in a teacher interview when the open-ended modelling task was presented: "[The students] had not managed to answer this. They would just … Tasks like this are never given. They are completely open, without connection to anything". Such tasks were not included in their textbook, and the teacher did not see them as relevant. As pointed out in Chapter 4.1, even if mathematical modelling is included in the *intended* curriculum, the focus on modelling activities in the textbooks might not be clear (Frejd, 2013; Jessen & Kjeldsen, 2021; Urhan & Dost, 2018). Also in the Danish textbooks it was found that the tasks could be solved by given mathematical procedures (Jessen & Kjeldsen, 2021), as found in this Norwegian case study. Open-ended modelling tasks allow students to experience the process of mathematising, thus enabling them to connect mathematics to everyday situations (Boaler, 2001). That is, discursive practices of the *instructional* curriculum should be restructured to include these processes. And also, the textbook should present problems in a diverse range of contexts, as pointed out by Gatabi et al. (2012).

The production of the *instructional* curriculum discourse was not included in this case study. This could have been given insight by interviewing textbook authors, which is discussed in subchapter 7.2. But intertextuality between the *assessed* curriculum and the *instructional* curriculum is recognised. Since earlier exam tasks are included in the textbooks, the *assessed* curriculum is consumed in the *instructional* curriculum. This may point to the influence of the *assessed* curriculum in the production of the *instructional* curriculum.

The *instructional* curriculum discourse is widely consumed in the *enacted* curriculum discourse, where the students are working through the textbooks during the school year following the structure of the textbook. On the other side, two of the 35 responding teachers to the questionnaire stated that they made their own modelling problems for the students, and a few others gave the students additional modelling problems to the ones presented in the textbooks.

7.1.3 The *perceived* curriculum

As already mentioned, the teachers pointed to the *intended* curriculum and *instructional* curriculum from where they had formed their understanding of mathematical modelling in mathematics education, and towards the *assessed* and the *instructional* curriculum when justifying their choices of the tasks to give the students – that is, manifest intertextuality of the *assessed* and the *instructional* curriculum in the *perceived* curriculum. The reason for not giving the students open-ended tasks was that such tasks do not appear on the exam or in the textbooks. This social construction can be identified in which the teacher is guided by the exam and textbook tasks when deciding how mathematical modelling is implemented in their teaching. Also in the Swedish study by Frejd (2012) the teachers did not recognise mathematical modelling as a part of mathematical competence when they were presented with open modelling problems. Morgan and Sfard (2016) noted that the national examinations influence the *perceived* and *enacted* curricula and claimed that a change in national assessment "is one of the most effective vehicles for bringing curriculum change to schools" (p. 92). However, when it comes to mathematical modelling, this can be challenging because of the holistic nature of the modelling process, which is difficult to assess using a written test (OECD, 2013). Holistic modelling was not included in the exam tasks; atomistic tasks that include the parts of the modelling process where assumptions and choices are made were also not included, according to the findings of article 1. In this light, it is problematic for teachers to use the exam tasks as guides for what to include in their teaching regarding mathematical modelling.

Within the *perceived* curriculum, teachers were unfamiliar with the *ideological* curriculum when it came to mathematical modelling. As presented in article 2, mathematical modelling was not included in the teachers' mathematics education. Most of the teachers who participated in the questionnaire conducted in this study had no experience with mathematical modelling from their own mathematics education. That is, the teachers may not have reflected on why mathematical modelling is included in the *intended* curriculum, its aim and how different types of tasks and modelling cycles are developed to accomplish these aims. As Burkhardt (2006) pointed out, it is a barrier to the implementation of mathematical modelling in mathematics education that the teachers are expected to be able to teach based on their pre-service education, even if this did not include mathematical modelling.

Even if the teachers were not familiar with the term mathematical modelling in an educational context, some of them were familiar with the processes from their education in natural sciences, yet they did not recognise these processes from how they experienced mathematical modelling in mathematics 2P.

Although the *ideological* curriculum suggests that being able to structure and mathematise a situation is part of modelling competence and considers the process of describing something from the real world in a mathematical language, this does not seem to be emphasised in the discourses in the other curriculum discourses. Each mathematical topic, and also mathematical modelling, is presented within a limited period, following the *instructional* curriculum. The teachers invited me to observe the teaching of mathematical modelling which was limited to a given period. They also expressed in the interviews that they did not teach this topic throughout the year. Mathematical modelling is included in textbooks as one (or two) of four chapters, the same as for the other *main content areas* in the *intended* curriculum. This might entail that modelling is not included as an overarching process. Within the *perceived* curriculum, mathematical modelling is rather expressed as an extension of the content area of function, and this connection could also be found in the textbooks, as pointed out in article 1. In the Turkish study by Sahin et al. (2019), it was found that many of the teachers identified a task as a modelling task solely because it was related to real life. This was also found in this case study, if a context and a function were given, it was seen as a modelling task. Sahin et al. (2019) point out the importance of developing teachers' theoretical competence regarding the teaching of mathematical modelling.

As expressed in the *perceived* curriculum, not all teachers expected the students to understand what was taught. The aim was for the students to be able to solve the tasks by performing the given procedures. The findings in article 2 showed that the teachers made the decisions in the modelling processes, leading the students to solve the tasks in a certain way. Within the *perceived* curriculum, students were expected to not “waste time” finding out how to mathematise the problem; the teachers would rather tell them how to approach it. This was also seen in examples of the *enacted* curriculum in article 3. The teacher led the students in a given direction in the modelling process by guiding them to use a specific method. Most tasks in the textbooks could be solved by memorising and applying certain procedures. These cognitive processes are less complex processes than analysing, evaluating and creating (Anderson et al., 2001), as referred to in article 2. The findings in article 1 showed that within the *instructional* and *assessed* curricula for the teaching of mathematical modelling, enabling students to make decisions themselves is not an aim, as the tasks can be solved by following strict procedures. As pointed out in article 2, one teacher explained that if a student asked for a reason for learning mathematics, his only answer was “for the tests and the exam”. One of the tensions identified in article 2 was that in the *ideological* curriculum, an aim for the implementation of mathematical modelling in mathematics education was to better equip students to master everyday life by teaching them mathematics in such a way that it is experienced as relevant in everyday situations. However, as shown in article 2, within the *perceived* curriculum, the teachers did not experience mathematical modelling as relevant for the students. Even if they understood the aim of Practical mathematics as learning relevant mathematics useful in everyday life situations, they did not experience mathematical modelling as relevant. Mathematical modelling in the *enacted* curriculum was understood as finding the correct answer by a given procedure as regression analysis in GeoGebra, based on the *instructional* and *assessed* curriculum.

In the Israeli study by Shahbari and Tabach (2016), teachers participated in a modelling course where they solved holistic modelling tasks and discussed their solutions. During the course, they developed their view on mathematical modelling and were aware of the changes. Providing courses for teachers to develop their theoretical knowledge is important, and equip them to make well-founded decisions regarding the teaching of mathematical modelling (Sahin et al., 2019). That is, the *perceived* curriculum should rather draw on the understanding of mathematical modelling from the *ideological* curriculum. The *perceived* curriculum has a significant impact on the *enacted* curriculum (Ärlebäck, 2010).

7.1.4 The *enacted* curriculum

As pointed out in article 3, the students often asked, "What are we supposed to do?" and "Is this correct?". The discursive practices in the *enacted* curriculum discourse seem to draw on the "traditional teaching of mathematics" (Boaler, 2001), where the students solve closed tasks by performing some given procedures to find the correct answer. The discourses when teaching and learning mathematical modelling *reproduce* this discursive practice of the "traditional teaching of mathematics", where the teacher helps the students to find the correct answer when solving tasks. The findings in article 3 showed that even if the students got involved in a given situation, the teacher led them in the direction he had planned for. The students were not given opportunities to make their own decisions regarding how to solve the tasks. The discursive practices were not *restructured* but *reproduced* by continuing the structures of the "traditional teaching of mathematics".

Brown and Stillman (2017) argued that mathematical modelling should be presented as a way of understanding the world through mathematics, instead of an optional part of mathematics. But as pointed out by a teacher within the *perceived* curriculum, although adopting an overarching modelling focus through the year would be preferred, he had to focus on the procedures owing to the exam. That is, a power relation between the *assessed* curriculum and the *enacted* curriculum is revealed.

The discursive structure of the exam hinders the teacher to include open-ended modelling tasks in the *enacted* curriculum. However, according to Boaler (2001), students who are working with open-ended tasks may outperform others in a traditional exam. She argued that when working with such open-ended modelling tasks the students gain deeper learning of mathematical concepts, and also experience mathematics as relevant outside of school.

Even if the students were given few opportunities to make their own choices when solving modelling tasks within the *enacted* curriculum, *restructured* discursive practices were identified in situations where the students were validating given models. Here, the teacher was not providing answers but asking students more questions, and encouraging the students to do so. As pointed out by Blum (2015), this is how mathematical modelling should be taught and learned: teacher interventions should allow students to continue their work without losing their independence and content-related interventions should not be adapted to prevent mistakes before they occur. Even if the students were not exposed to the whole modelling process including making assumptions and mathematising real-world problems within the *enacted* curriculum, they gained experiences regarding certain aspects of the modelling process. The findings in article 3 show that they used their own everyday experiences to discuss given mathematical models and questioned the development of the model if their experiences did not fit the mathematical model. They also experience that even if the real world is messy, it could be expressed in mathematical terms if assumptions are made. Students also posed questions from everyday life and questioned mathematical methods during the work of mathematical modelling.

7.1.5 The *assessed* curriculum

In article 1 it was found that the exam tasks did not assess the holistic modelling process, and only some of the steps of a modelling cycle were included. This was also found in Swedish exams, where the assessment items most often only included a mathematical result (Frejd, 2011). Modelling competence cannot be assessed only by a written test, because it is too comprehensive (Jensen, 2007). Therefore, the *assessed* curriculum should not be used as a guide to understanding the processes and aims for the teaching and learning of mathematical modelling, as was expressed in the *perceived* curriculum. One of the teachers mentioned the exam 15 times during the classroom observation, and each time, it was mentioned to motivate the students to concentrate on the tasks or on using a specific notation. The other observed teacher did not mention the exam, except when a student asked if something was relevant for the exam, and stated that they would spend time solving earlier exam tasks in the period before the exam. The influence of the *assessed* curriculum could be identified to different extents in the *enacted* curriculum discourse.

A student's score on a standardised test does not indicate their ability to solve mathematical modelling tasks (Kartal et al., 2016). If students' modelling competence is to be assessed, a change in the national examination is required. Some studies, such as those conducted in the USA (Leong, 2012) and Germany (Hankeln et al., 2019), have presented frameworks assessing each of the sub-processes of modelling in different tasks. These frameworks allow us to assess the parts of the modelling process in confined tasks even if the students are not involved in holistic modelling processes. This shows the possibility of assessing each of the sub-processes even in a written test. However, such tasks were not included in Norwegian examinations, as presented in article 1. The Norwegian national examination has the same form, a five-hour written test, as that before mathematical modelling was an explicit part of the *intended* curriculum. Although digital tools have been included in the exam in the last decades, the social practices of the exam allow the *reproduction* of the earlier discursive practices in the *enacted* curriculum discourse. These social practices influence the implementation of mathematical modelling in the *perceived* and *enacted* curriculum discourses.

7.2 Limitations, strengths and implications for practice and further research

Qualitative research can be subject to various criticisms, such as being too subjective, being difficult to replicate and lacking generalisability and transparency (Bryman, 2016). Here, I engage in some critical reflections on the limitations and implications of the research reported in this study.

The findings in this thesis are based on my interpretations as a researcher, leaning on the interpretative character of a case study. Owing to my many visits to the four schools, totalling about 30 days, I was included in the collegiums, and I participated during coffee breaks and lunches and had informal conversations also with other teachers at the schools. These opportunities helped me become better acquainted with the teachers, which reduced the chance of any misunderstandings in my interpretations.

This study was conducted on mathematics 2P in upper secondary schools in Norway, and the data were collected from textbooks, national examinations, interviews with four teachers and observation of classrooms. The data would not have been the same if data collection was done over another year or included other schools and teachers, and the sample of interviewees cannot be claimed to be representative. However, a case study provides deeper insights into the study topic. One strength of this study is that it includes six curriculum discourses, which provided a holistic character of the case study. It allows us to not only address the speech and act in the classroom but also the discursive and social practices concerning all the included curriculum discourses.

The production of the *intended, instructional* and *assessed* curriculum was not included in this case study. To involve this, I could rather have interviewed key participants in curriculum development, textbook authors and participants from the exam task developing group. This might have provided data for identifying other discursive practices, and the reasons and aims within these curriculum discourses would have been easier accessible. However, a strength in the current design of the study is that the curriculum discourses included are available for the teachers. While the intended curriculum, textbook and exam tasks are directly available for the teachers, the authors' reflections, reasons, and aims are not. The research design of the study presented in this thesis provides an analysis of how different aspects are brought into play in the classrooms, and how the different curriculum discourses interact.

In another study, it would have been interesting to further analyse how reasons for including mathematical modelling in school mathematics are changing in separated curriculum discourses. This could have been investigated through interviews of key participants in the production of the different curriculum discourses as suggested. Also, it could have been developed a research design focusing on the cognitive aspects of the different curriculum discourses by including more cognitive modelling theories and conducting student tests to also include the *learned* curriculum.

7.3 Retrospective reflections

In the foreword, I shared my experience as a substitute teacher where mathematical modelling, in the *intended* curriculum, was expressed as a fundamental process. I did not recognise the aim of mathematics 2P by leafing through the textbook and had no explicit experience with mathematical modelling from my education. Now, some years later, I understand some of the aims of *practical mathematics*: for the students to be able to use mathematics to better master real-life situations, make good decisions as democratic citizens and critically assess information presented by others. The emphatic characteristic of the case study provided reflections on the vicarious experiences of the teachers from an emic perspective. Because of my experiences as an upper-secondary mathematics teacher, this emic perspective was more accessible. At the same time, when I look back now, I see that my own experiences as an upper secondary teacher also imposed limitations on me as an inexperienced researcher. At the start of this case study, my impression was that mathematical modelling was not fully implemented in the teaching of mathematics. And because of my experience shared in the foreword of this extended summary, I thought the reasons were that the teachers were not aware of mathematical modelling and its place in mathematics education. Some of the questions in the questionnaire may bear the mark of this, as the aim is to reveal the teachers’ lack of theoretical knowledge about mathematical modelling. Fortunately, the responding teachers unfolded parts of the reality for me, showing a diversity in the understanding of mathematical modelling. And this did further awaken my interest in the discursive and social practices which form and maintain the endorsed narratives of the discourses.

In the early stages of this case study, I did not acknowledge how my role as a researcher affected the situations I interfered. In the production of the *perceived* curriculum, an interviewer and interviewees participated. It was my aim as a researcher to grasp the teachers' understanding of mathematical modelling. Within the research paradigm of this thesis, discourse is seen as a process through which meaning is created. That is, the meaning of mathematical modelling is created through texts and the practices of their production in the curriculum discourses. That is, the meaning of mathematical modelling was created through interviews where I as a researcher participated in the discourse as an interviewer. There were identified changes in the understanding of mathematical modelling through the interviews. When a teacher was presented with the three tasks at the end of the interview, she first rejected the third task as a modelling task. But through the interview she changed her mind, stating that maybe this task was more a modelling task than the other two. This change would probably not have happened if she had not participated in the interview. Through the work on this thesis, I have also developed as a researcher, and I might have made different choices in the process of research design development today. After learning more about Discourse Analysis, I believe it could have been interesting to perform Discourse Analysis as in the study by Morgan and Sfard (2016) also on the textbook and exam tasks, and not only the interviews and frameworks. This might have made it easier to see the findings in relation to each other, and I believe it would have provided a richer analysis of the tasks. In the analysis in article 1, I did identify steps of a modelling cycle, but if I had adjusted the analysis tool by Morgan and Sfard (2016) for the analysis of modelling tasks this would also have provided insight into other aspects. Epistemologically the research paradigm of critical realism draws on fallibilism (Yucel, 2018). The different approaches can reveal different layers of reality. In the analysis of the *assessed* and *instructional* curriculum, I implemented a cognitive modelling cycle, and in the analysis of the *perceived* and the *enacted* curriculum, I included social aspects. But in the discussion of this extended thesis also social aspects of the *assessed* and *instructional* curriculum are pointed out.

Over the course of conducting this study, I have developed a more nuanced understanding of mathematical modelling and acknowledge teachers' challenges and dilemmas. Even if they desire to facilitate their students to experience the relevance of mathematics, the discursive and social structures do not always allow them to do so.

7.4 Conclusions and closure

The main research question addressed in this thesis has been: Which perspectives on mathematical modelling can be identified in the different discourses of the mathematics curriculum, and which discursive and social practices can be identified within and between the discourses?

Mathematical modelling includes, according to the *ideological* curriculum, complex cognitive processes such as analysing, creating and evaluating as described within the six perspectives by Blum (2015). Students should be given opportunities to engage in analysing real-world situations and to mathematise by making their own choices when creating a mathematical model. Aspects within all the six modelling perspectives could be identified in the *intended* curriculum, even if the different perspectives are not written out explicitly. Both perspectives concerning to learn mathematical concepts, development of the use of mathematics in everyday life situations and recognising and critically assessing the use of mathematics in society. It was found that even if these aims for mathematical modelling are included in the *intended* curriculum, mathematical modelling was expressed differently in the other curriculum discourses. From the analysis of the textbook tasks within the *instructional* curriculum and the exam tasks within the *assessed* curriculum, it was found that most of the tasks were already mathematised, and they could be solved by following the given procedures. Some of the textbook tasks also involved validation of the answers, but most of the tasks had a correct given answer in an answer section of the textbooks. Within the *perceived* curriculum teachers wanted the students to experience mathematics as useful for everyday life, but they did not acknowledge mathematical modelling as relevant. Mathematical modelling was identified as finding the correct answer to mathematical tasks by a given procedure, for example, regression analysis in GeoGebra. The identification of the perspectives in the different curriculum discourses revealed a disappearance of the aims for including mathematical modelling in the teaching and learning of mathematics.

The discursive and social practices revealed that the *assessed* curriculum influenced both the *instructional*, *perceived* and *enacted* curriculum. Even if the discursive practices of the *assessed* curriculum of a 5-hour written examination are not suitable for assessing modelling competence (Jensen, 2007). The teachers, which did not have mathematical modelling as a part of their education, justified their choices in the *enacted* curriculum by the *assessed* and the *instructional* curriculum. The discursive practice of the *instructional* discourse was reproduced from the other main areas of mathematics. The modelling tasks were mainly closed tasks with one correct answer and modelling was placed in its own textbook chapter.

This case study has shown there is still a gap between the ideals of educational debate about mathematical modelling on the one hand, and everyday teaching practice on the other hand. As Fairclough (1992) pointed out, the first step of change is to identify the social structures that maintain earlier discursive practices. Based on the interpretations I as a researcher have made by a holistic view of this case study, I will permit myself to outline three suggestions for how to change the discursive practices of the teaching and learning of mathematical modelling to include more comprehensive modelling perspectives. First, the *ideological* curriculum should become more apparent in the *instructional* curriculum, that is, activities from diverse modelling perspectives should be included in textbooks and teaching material regarding mathematical modelling should be developed. Next, the *assessed* curriculum should let in light from everyday life both in the social and discursive practices. That is, the form of the exam as a 5-hour exam should be more like everyday situations, and also the tasks which are given. In addition, provision should be made for development within the *perceived* curriculum discourse. That is, the development of the teachers' theoretical dimension of the teacher knowledge of mathematical modelling should be provided for more comprehensive perspectives. This could equip them to develop their own modelling problems and based on earlier research make arguments for their choices in the teaching of mathematical modelling.

I would like to participate further to also bring this educational debate about mathematical modelling into schools and classrooms.

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9 Appendices

Appendix A: Questionnaire and interview guides

Spørjeundersøking / *Questionnaire*

	Norwegian	English
1	Kva matematikkurs tok du som elev i vgs?	*What is your school background in mathematics?*
2	Jobba du med matematisk modellering då du sjølv var elev? Korleis? Gje minst eitt eksempel	*Did you work with modelling as a school student? How? Give at least one example.*
3	Kva utdanning har du?	*What is your educational background?*
*4a	(Viss lærardanning) Kor mykje matematikk har du frå lærarutdanninga?	*(If teacher education) How many credits did you have in mathematics in your teacher education?*
*4b	(Viss ikkje lærarutdanning) Kva matematikkutdanning har du?	*(If not teacher education) What is your educational background in mathematics?*
5	Kva lærte du om matematisk modellering under utdanninga di?	*What did you learn about mathematical modelling during your years of education?*
6	I kor mange år har du undervist matematikk?	*How many years of experience do you have as a mathematics teacher?*
7	Kvifor vart du matematikklærar?	*Why did you become a mathematics teacher?*
8	Kva fag underviser du/har du undervist?	*Which mathematics subjects have you taught?*
9	Kva for lærebok/ressursar bruker de i 2P?	*Which textbook/learning resources do you use in 2P?*
10	For undervising av P-matematikk, kva er dei tre viktigaste punkta?	*For teaching practical mathematics, what are the three most important things?*
11	For undervising av meir teoretisk matematikk, kva er dei tre viktigaste punkta?	*For teaching theoretical mathematics, what are the three most important things?*
12	Kva opplever du som hovudutfordringane med å undervise P-matematikk?	*What are the main challenges when teaching practical mathematics?*
13	Korleis vil du definere kva matematisk modellering er?	*How will you define mathematical modelling?*
14	Forklar kva som må til for at ei oppgåve skal vere ei «modelleringsoppgåve», og kom med eit konkret eksempel.	*Explain what is needed for a task to be a 'modelling task' and give a concrete example.*
15	Bruker du modellering i matematikkundervisinga?	*Do you include mathematical modelling in your teaching?*
*16	(viss ja) På kva måte bruker du modellering i undervisinga?	*(If yes) How?*
17	Vurderer du modelleringskompetanse i sluttvurderinga (standpunktkarakteren)	*Is modelling competence included in your assessment of the students?*

	til elevane dine?	
18	Vil du seie at skriftleg eksamen vurderer modelleringskompetanse?	*Would you say modelling competence is assessed in the national written examinations?*
*19	(viss nei) Kvifor meiner du at modelleringskompetanse ikkje blir vurdert i skriftleg eksamen?	*(If no) Why?*
20	Kva kan vere utfordringar med å vektlegge modellering i undervisinga?	*Could there be challenges in emphasising on mathematical modelling in your teaching?*
21	Kor ofte snakkar du og dine kollegaer om modellering i matematikk	*How often do you and your colleagues discuss mathematical modelling?*
22	Opplever du at leiinga lyfter fram modellering som viktig	*Are your school leaders emphasising on mathematical modelling?*

*only given if…

Intervjuguide lærarar i 2P / Interview guide for teachers in 2P

A	Bakgrunn / *Educational background*	
1	Kva matematikkutdanning har du?	*Mathematics education?*
2	I kor mange år har du undervist matematikk? Og 2P?	*Years of experience as a mathematics teacher? In 2P?*
3	Kva lærte du om modellering i eiga utdanning?	*What did you learn about modelling during your own years of education?*
4	Kan du hugse å ha jobba med modellering som elev?	*Do you have modelling experience as a school student?*

B	Faget 2P / *2P*	
1	Kva er skilnaden på 2P og andre matematikkfag – slik du ser det?	*What are, as you see it, the differences between the 2P and other courses of mathematics?*
2	Kva er mest utfordrande med å undervise 2P?	*What is most challenging when teaching 2P?*
3	Kva trur du elevane ser på som mest utfordrande i faget 2P?	*What do you think the students see as most challenging in 2P?*
4	Kva vil praktisk matematikk seie?	*What is 'practical mathematics'?*
5	Kva sit elevane igjen med etter at dei har hatt 2P?	*What is the students' learning outcome 2P?*
6	Kva er poenget med 2P?	*What is the aim of 2P?*

C	Modellering i undervisinga / *Teaching mathematical modelling in 2P*	
1	Kva bok bruker de i faget 2P?	*What textbook do you use for 2P?*
2	Underviser du overordna kva modellering er? Kva legg du vekt på då? Frå boka, eller ikkje?	*Do you include a meta perspective when teaching mathematical modelling? What are you emphasising on?*
3	På kva måte jobbar de med modellering i undervisinga? Når? Berre når de har kapitlet?	*How do you include mathematical modelling in your teaching? When? Only when going through the 'modelling chapter' in the textbook?*
4	Kva bruker elevane mest tid på når dei jobbar med modellering?	*What are the students spending most of their time on when working with mathematical modelling?*
5	Kva er utfordrande for elevane når det gjeld modellering?	*What is challenging for the students concerning mathematical modelling?*
6	Vil du seie at matematisk modellering er ein fundamental prosess i faget 2P?	*Would you characterise mathematical modelling as a fundamental process of 2P?*
7	På kva måte vurderer du elevane sin matematiske kompetanse?	*How are you assessing the students' mathematical competence?*

D	Definisjonen av modellering / *The definition of mathematical modelling*	
1	Kva har fått innverknad på di oppfatning av kva modellering er?	*What has influenced your understanding of mathematical modelling?*
2	Kva meiner du er det viktigaste i ein modelleringsprosess?	*What do see as the most important in a modelling process?*
3	Vil du seie at modellering skil seg ut frå andre tema i matematikk? På kva måte?	*Would you say that modelling is different from other topics in mathematics? In what way?*
4	Kva skal til for at noko kan kallast modellering?	*What is required to identify a task as a modelling task?*
5	Er der nokon samanheng mellom problemløysingsoppgåver og modellering, slik du ser det?	*Is there any relation between problem-solving and modelling?*
6	Er modelleringskompetanse ein viktig del av det å kunne matematikk?	*Is modelling competence an important part of mathematical knowledge?*

E	Om 3 spesifikke oppgåver / *About three specific tasks*	
	Kven av desse oppgåvene er ei modelleringsoppgåve? Kommenter skilnadar. Kva for ei av desse oppgåvene legg mest tilrette for at elevane utviklar modelleringskompetanse?	*Which of the following tasks are modelling tasks?* *Comment on the differences between them. Which of the tasks are most preferable to develop students' modelling competence?*

Oppgåve 1 / Task 1

Ein modell for temperaturen i celsiusgradar x timar etter midnatt ein vinterdag er gitt ved

A model for the temperature in Celsius degrees x hours after midnight; a winter day is given:

$T(x) = -\frac{3}{8}x^2 + \frac{21}{2}x - \frac{135}{2}$,
når x er mellom 8 og 20.
where x ranges between 8 and 20.

a) Teikn grafen til T for hand / *Graph T by hand*
b) Kva tid på dagen var temperaturen $0°C$? / *At what time of the day was the temperature $0°C$?*
c) Kva tid på dagen var temperaturen høgast? / *At what time of the day was the highest temperature recorded?*
d) Kva var temperaturen då? / *What was the temperature then?*

Oppgåve 2 / Task 2

Per målte temperaturen kvar fjerde time gjennom eit døgn. Tabellen viser klokkeslett med tilhøyrande temperatur T.

Paul measured the temperature at four-hour intervals through one day and night. The table shows the time and the corresponding temperature T.

Klokkeslett / *Time*	14.00 2 PM	18.00 6 PM	22.00 10 PM	02.00 2 AM	06.00 6 AM	10.00 10 AM	14.00 2 PM
Temperatur T i $°C$	2.5	0.3	-1.4	-2.0	-2.6	-2.1	-0.2

a) Bruk regresjon og finn den andregradsfunksjonen som passar best til punkta i tabellen. Lat x vere tal på timar etter kl 14.00. / *Use regression analysis and find the best quadric function that fits the data in the table. Let x be the time in hours after 2 PM.*

b) Legg inn punkta i eit koordinatsystem og teikn grafen til uttrykket du fann i a). Korleis passar grafen med temperaturmålingane? / *Plot the points in a coordinate system and plot a graph of the function you found in a). How does the function fit the temperature measurements?*

c) Kva vil temperaturen i fylgje modellen vere 30 timar etter at Per starta med målingane. / *What will the temperature, according to the model, be 30 hours after Paul started to measure the temperature?*

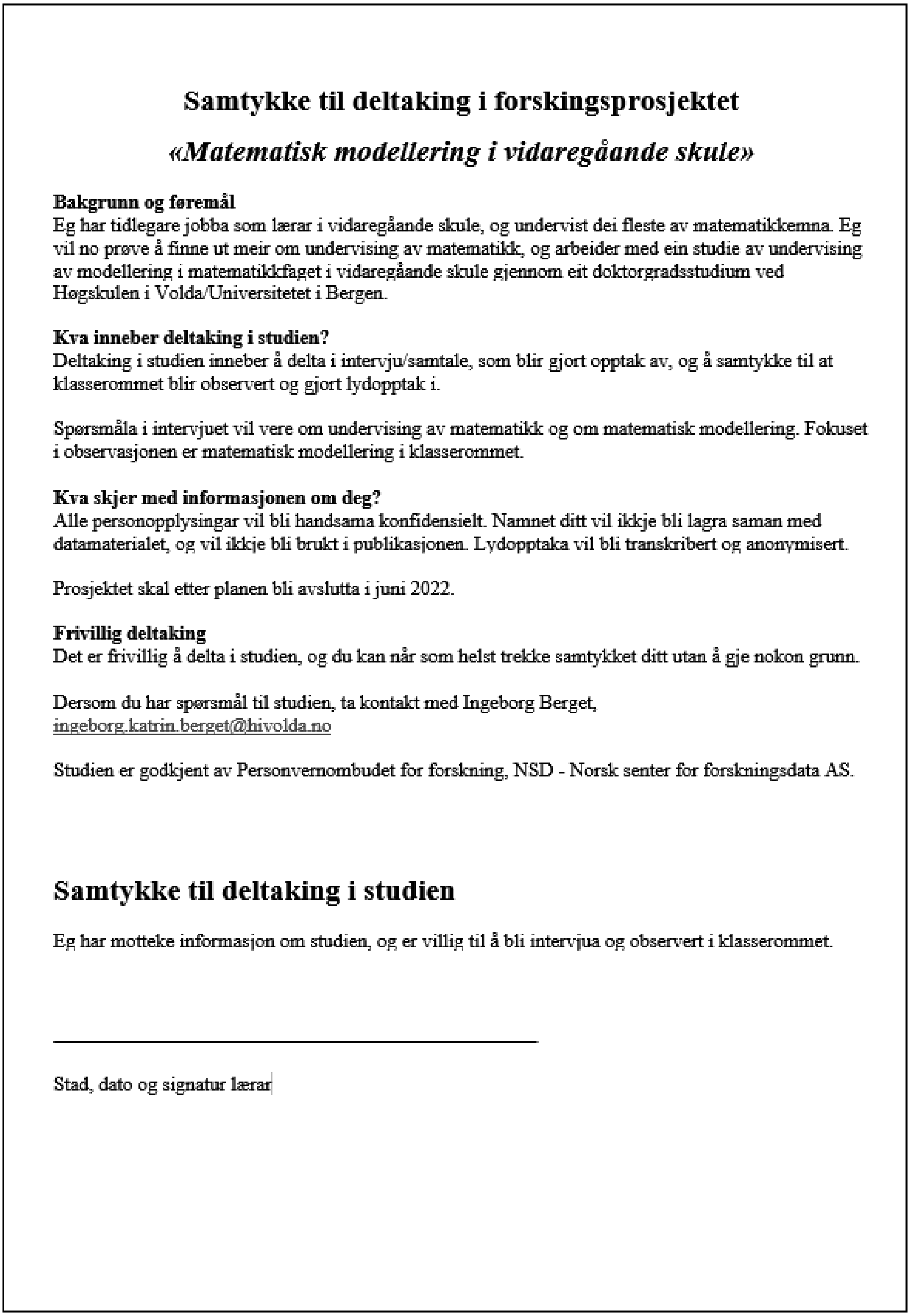
d) Kva vil temperaturen i fylgje modellen vere 48 timar etter at Per starta målingane?
Vurder kor realistisk modellen er. / *What will the temperature, according to the model, be 48 hours after Paul started to measure the temperature? Assess how realistic the model is.*

Oppgåve 3 / Task 3

Kva er den beste måten å kome seg til skulen på?
What is the best way of travelling to school?

Appendix B: Information letters with letters of consent

Teachers



Samtykke til deltaking i forskingsprosjektet

«Matematisk modellering i vidaregåande skule»

Bakgrunn og føremål

Eg har tidlegare jobba som lærar i vidaregåande skule, og undervist dei fleste av matematikkemna. Eg vil no prøve å finne ut meir om undervising av matematikk, og arbeider med ein studie av undervising av modellering i matematikkfaget i vidaregåande skule gjennom eit doktorgradsstudium ved Høgskulen i Volda/Universitetet i Bergen.

Kva inneber deltaking i studien?

Deltaking i studien inneber å delta i intervju/samtale, som blir gjort opptak av, og å samtykke til at klasserommet blir observert og gjort lydopptak i.

Spørsmåla i intervjuet vil vere om undervising av matematikk og om matematisk modellering. Fokuset i observasjonen er matematisk modellering i klasserommet.

Kva skjer med informasjonen om deg?

Alle personopplysingar vil bli handsama konfidensielt. Namnet ditt vil ikkje bli lagra saman med datamaterialet, og vil ikkje bli brukt i publikasjonen. Lydopptaka vil bli transkribert og anonymisert.

Prosjektet skal etter planen bli avslutta i juni 2022.

Frivillig deltaking

Det er frivillig å delta i studien, og du kan når som helst trekke samtykket ditt utan å gje nokon grunn.

Dersom du har spørsmål til studien, ta kontakt med Ingeborg Berget,
ingeborg.katrin.berget@hivolda.no

Studien er godkjent av Personvernombudet for forskning, NSD - Norsk senter for forskningsdata AS.

Samtykke til deltaking i studien

Eg har motteke informasjon om studien, og er villig til å bli intervjua og observert i klasserommet.

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Stad, dato og signatur lærar

Students

Samtykke til deltaking i forskingsprosjektet

«Matematisk modellering i vidaregåande skule»

Bakgrunn og føremål
Eg har tidlegare jobba som lærar i vidaregåande skule, og undervist dei fleste av matematikkemna. Då eg skulle undervise i faget 2P og modellering, fekk ikkje heilt tak på kva dette var. Dette vil eg no prøve å finne ut meir om, og arbeider med ein studie av undervising av modellering i matematikkfaget i vidaregåande skule, gjennom eit doktorgradsstudium ved Høgskulen i Volda/Universitetet i Bergen.

Kva inneber deltaking i studien?
Deltaking i studien inneber at klasserommet blir observert og det vil bli gjort lydopptak i nokre matematikktimar. Fokuset i observasjonen er matematisk modellering i klasserommet.

Kva skjer med informasjonen om deg?
Namnet ditt vil ikkje bli lagra saman med datamaterialet, og vil ikkje bli brukt i publikasjonen. Lydopptak vil bli transkribert og anonymisert.

Prosjektet skal etter planen bli avslutta i juni 2022.

Frivillig deltaking
Det er frivillig å delta i studien, og du kan når som helst trekke samtykket ditt utan å gje nokon grunn.

Dersom du har spørsmål til studien, ta kontakt med Ingeborg Berget, ingeborg.katrin.berget@hivolda.no

Studien er godkjent av Personvernombudet for forskning, NSD - Norsk senter for forskningsdata AS.

Samtykke til deltaking i studien

Eg har motteke informasjon om studien, og er villig til å bli observert i klasserommet

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Stad, dato og signatur elev

Appendix C: Articles 1–3

Article 1

Berget, I. K. L. (2022). Mathematical modelling in textbook tasks and national examination in Norwegian upper secondary school. *Nordic Studies in Mathematics Education*, *27*(1), 51–70. http://ncm.gu.se/media/nomad/enomad/nomad-subscribers/27_1_051070_berget.pdf

Article 2

Berget, I. K. L. (2023). Mathematical modelling in the discourses of the KOM and PISA frameworks and teacher interviews. *Research in Mathematics Education (RRME), 2023.* https://doi.org/10.1080/14794802.2023.2165536

Article 3

Berget, I. K. L. (2022). Identifying positioning and storylines about mathematical modelling in teacher–student dialogues in episodes from two upper secondary classrooms. *Teaching Mathematics and its Applications: An International Journal of the IMA*, 2022. https://doi.org/10.1093/teamat/hrac020

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