ABSTRACT

Title of Thesis: **MODELING THE DYNAMICS OF OPINION FORMATION AND PROPAGATION: AN APPLICATION TO MARKET ADOPTION OF TRANSPORTATION SERVICES**

**Aaron T.C.Y. Kozuki, M.S., 2007**

Thesis Directed By: **Professor Hani S. Mahmassani, Department of Civil and Environmental Engineering**

The objective of this research is to present a model that utilizes social and learning mechanisms to first explore the underlying dynamics of opinion formation and propagation, and then applies those mechanisms to an application of freight mode choice to investigate the effect that opinions have on choice set considerations, attribute perceptions, and the market adoption of a new rail freight service. Primary contributions of this research include the explicit modeling of social and learning mechanisms and their effects on opinion formation and propagation, the evolution of these opinions over time, and an exploration of the role that opinion dynamics have in choice processes. Research findings will offer insight to the process of evolving attitudes, perceptions, and opinions and the effects on individuals' judgment and decision making. It will also offer insight to the effects of attribute distortion on decision making.

# MODELING THE DYNAMICS OF OPINION FORMATION AND PROPAGATION: AN APPLICATION TO MARKET ADOPTION OF TRANSPORTATION SERVICES

By

Aaron T.C.Y. Kozuki

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Advisory Committee:
Dr. Hani S. Mahmassani, Chair
Dr. Elise J. Miller-Hooks
Dr. Cinzia Cirillo

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# Table of Contents

# List of Tables

## List of Figures

# Chapter 1: Introduction

*The Role of Opinions in the Choice Process*

Current travel demand models employ a random utility maximization (RUM) framework to represent choices made by tripmakers. Within this RUM framework, it is assumed that each individual is a rational, fully-informed decision maker that chooses an alternative that maximizes his utility. It is further assumed that the analyst knows what elements of the choice set (i.e., alternatives) are available to each decision maker. However, both economics and psychology studies have shown irrational decision making in the reversal of preferences, that decisions may not be based on correct information or based on limited information (e.g., systematic distortion of attributes, quality of information, see for example Tversky, 1981; Sammer et al., 2006), and that decisions are made without knowing the true probabilities of other alternatives, and thus the consequences of the decision are also unknown (Tversky and Kahneman, 1992). Other transportation studies have shown that ignoring these realisms leads to biased parameter estimates (see for example, Williams and Ortuzar, 1982; Basar and Bhat, 2003; Vythoulkas and Koutsopoulos, 2003; Swait, 2001; Cantillo and Ortuzar, 2005; Cantillo et al., 2006).

These limitations are exacerbated when trying to forecast future market shares when a new service is introduced. The current research practice is to elicit stated preference (SP) responses from current users to capture attitudes towards a new service, and then combine those responses with revealed choices, or revealed preferences (RP) to estimate a mixed RP/SP choice model. However, since the new service does not exist, there is no RP data (e.g., level of service attributes, performance measures) on the new service, and thus the entire forecast relies on the robustness of the RP data on existing services and the SP data collected from survey experiments. These SP exercises may not capture correctly the likelihood or willingness to shift to the new service due to a lack of real consequences and benefits. While much consideration and research has focused on constructing these SP experiments (e.g., in psychology experiments, payoffs and losses are commonly used as real benefits and consequences to decision making), it is difficult to capture behavior under real situations. Further, due to individuals' tendency to be over-optimistic about their future (Sjoberg and Biel, 1983), and the tendency to respond positively to an innovation that has potential to improve the status quo, the projected use of the new service tends to be over-inflated (Ben-Akiva and Morikawa, 1990).

Research has tried to incorporate concepts of limited attribute information, choice set generation, and intentions into comprehensive choice models that account for attitudes, perceptions, and preferences (see for example, Ben-Akiva et al., 1999; Walker and Ben-Akiva, 2002). These models present a general behavioral framework using latent constructs for perceptions, which are the individual's belief of an attribute value, attitudes, which reflect an individual's needs or ideal (goal) situation, and preferences, which represent the desirability of an alternative. However, because these parameters are endogenously estimated and not directly observable, it is difficult to explicitly capture the dynamics of choice behavior due to fuzzy choice sets, attribute value distortions, and intentions.

## 1.1 The Dynamics of Opinions and Choice

Much of the research to date on the random utility model development has focused on the estimation procedure and the accuracy of the parameter estimates. Driving this research is the argument that accurate parameter estimates will yield the correct sensitivities to changes in the attributes due to policy changes or the introduction of a new service. While unbiased parameters are essential to provide policy-makers with an accurate decision-making tool, little has been done to explore the dynamics of the process by which the market share adjusts to changes in the environment stemming from such policy decisions (e.g., decrease in travel time, increases in parking costs). It is important to evaluate the process of these market shifts since it is not realistic to expect the entire population to become aware of changes in the attributes instantaneously. Rather, individuals may learn about these changes through various social and interpersonal mechanisms over time, such as word-of-mouth, mass-media, direct experience, and belief learning. From a policy standpoint, policy-makers would like to know how long and at what rate it would take for the market shares to reach a desired goal, or the effect a period of policy change has on the demand (e.g., road closures, seasonal pricing) (Lerman and Manski, 1982). It is also important to consider whether the information diffusion is ultimately affecting an individual's choice, especially when considering market adoption of a new service.

Concerning the effect of information on choice, consumer and marketing science has made progress on exploring these dynamic processes through the study of opinion formation and propagation. Recent advances in information and communication technology (ICT) have greatly enhanced the visibility and accessibility of market products. Widespread use of e-mail and the internet facilitate the propagation of information to vast numbers of the population. Despite the massive amount of information available, there is concern that it may have little or no effect on altering an individual's opinion on the subject. Most people have opinions on a variety of topics that have formed either from internal reasoning and logic, or from external interactions with others with views on this topic. When introduced to new information or innovations in a certain product class, individuals associate the information with their current opinion on that class. If the opinion on the topic is negative, it may be difficult for the new information to change the individual's opinion to a positive one, and vice versa. Intuitively, having a positive opinion towards a product will most likely increase the likelihood of adopting it. This challenge to alter an individual's opinion – thoughts and ideas – has been a focal point for both commercial and private sectors in marketing to uncover the mechanisms behind opinion formation.

The formation of favorable and unfavorable opinions has been of great interest in problems of diffusion of innovation and technology adoption (Wu and Huberman, 2005). To draw a parallel to transportation, consider developments in intelligent transportation systems (ITS), traffic management centers, and real-time traffic navigation global positioning systems (GPS) units and the information made available to users about current conditions on transportation facilities. Information provided by these technological advances intends to direct system users to available facilities (in the case of congestion) or alternative services. Yet, as in the marketing field, it is difficult to determine whether information provided is actually affecting an individual's opinion, and ultimately his choice of an alternative.

Opinions differs from attitudes and perceptions in that an opinion reflects an evaluation of a current situation, whereas attitudes may reflect needs, desires, and values, and perceptions reflect convictions of a proposition. Attitudes and perceptions tend to form through belief learning and direct experience, while opinions, due to their lack of foundation since they are considered neither fact nor fiction, form through social mechanisms in addition to learning processes. Most importantly, one can elicit an opinion, whereas attitudes are generally latent and unobservable. The ability to observe opinions (i.e., elicit real responses) have made them the focus of marketing and consumer studies. As in the case of attitude formation, researchers are interested in the underlying mechanisms and dynamics of opinion formation, how these opinions evolve over time, and how these opinions propagate through a population.

Within the context of travel demand, however, there has been no known research that explicitly accounts for the role of opinions in a choice framework. Opinions research has shed some insight that a favorable opinion does not necessarily result in an observed choice; rather, a favorable opinion towards an alternative may encourage an individual to try the alternative for a trial period. Once the trial period is complete, the individual decides either to continue utilizing the alternative or go back to the previous alternative (Rogers, 2003; Garling, 1998). The implied two-phase process is also reflected in economic theory, e.g. Manski (1977), through the development of a two-stage choice framework that first selects the most important alternatives in a non-compensatory procedure (e.g., elimination by aspects, see Tversky, 1972), then evaluates the remaining set through compensatory rules. These findings suggest that opinions may have a role in screening relevant alternatives for consideration in the final choice set. If an individual has a significantly positive opinion (i.e., significantly different from neutral or some internal threshold) towards an alternative, the alternative is placed in the choice set for final consideration according to some decision rule. If the opinion is negative, or less than some internal threshold, the alternative does not enter the choice set. Such two-phase choice framework, which first articulates mechanisms for the choice set generation, then applies the choice mechanism, has been proposed in several transportation studies (see, for example, Ben-Akiva and Boccara, 1995; Swait and Ben-Akiva, 1987a; Swait and Ben-Akiva, 1987b), but these have not explicitly investigated the dynamics of opinions in choice set generation.

Investigating opinion dynamics in the context of a choice framework may also offer insight into how information about the attributes, and more importantly the changes in the attributes, spreads amongst a population. Since opinions are social and environmentally contextual (i.e., subject to social interaction, awareness of the environment), the mechanisms behind their formation may also correlate with information exchange. Exploring these information exchange mechanisms will offer insight into how individuals may react to changes in the level of service of a particular alternative, and more importantly, how over time the information about a new alternative or service may propagate through a population. This research seeks to construct mathematical representations of these social and learning interactions to approximate opinion formation and propagation and information exchange within a population. To extend this to a choice framework, this research intends to explore the relationships between opinions and information to choice set generation, perceptions of attribute information, and the market adoption of a new service.

## 1.2 Objective of Research

The objective of this research is to present a model that utilizes social and learning mechanisms to first explore the underlying dynamics of opinion formation and propagation, and then applies those mechanisms to an application of freight mode choice to investigate the effect that opinions have on choice set considerations, attribute perceptions, and the market adoption of a new rail freight service.

To do this, an extensive review of the literature spanning current discrete choice model developments, social interactions and learning processes, to mathematical opinion formation models will identify major processes or heuristics governing these interactions and give insight to modeling approaches and techniques.

Using this information, the research formulates a conceptual opinion framework that accounts for information interaction and opinion formation and revision. Incorporated in this conceptual framework are elements of the utility maximizing choice framework currently used in transportation. Within this framework are notions that govern the mechanisms behind choice set consideration and attribute information exchange. A constant theme in the framework is the idea that thresholds govern individuals' action; first, in considering whether to interact with the social or learning mechanism, and second, whether to give an alternative serious consideration in the choice set. Following the two-stage choice paradigm developed by Manski (1977), the conceptual framework contains two main components; the first considers receptivity to social and learning mechanisms, while the second component represents the final consideration stage in the choice process. Four classes of mechanisms are incorporated into the conceptual opinion framework: word-of-mouth, mass-media, belief learning, and direct experience. Additionally, the framework considers the interrelation between social classes, personality types, and issues of trust and credibility.

To implement the framework, the study proposes a simulation tool and associated experiments utilizing agent-based simulation of agents that interact with one another based on the mechanisms defined in the conceptual framework. The four classes of mechanisms quantify the means through which agents choose to interact or communicate with each other, while an opinion revision component serves to measure the change in opinion. A consideration mechanism is designed to place the alternative with a positive threshold greater than a random internal threshold into a final consideration set, taken to be the choice set considered when making a decision. The simulation program allows the testing of different scenarios, such as the effect of different social classes or personality types, as well as the physical characteristics of the population, such as the number of agents, the number holding a certain opinion, and the number of interactions.

As an application to current discrete choice models in transportation, this research synthesizes the opinion framework with a binary freight choice model calibrated for a mode choice study on a north-south corridor in Eastern Europe. Using the parameter coefficients estimated for level-of-service variables as an initial condition and the information about the experienced level-of-service variables observed from individuals, the opinion simulation framework intends to allow these agents to interact and evolve their opinions towards the two alternatives. The research considers an opinion scenario where there is no *a priori* assumption about the opinions of the individuals (i.e., chosen alternative evokes a positive opinion that is above the threshold, non-chosen alternatives evoke an indifferent or negative opinion), and a outlines a scenario where survey data on the evaluation of the current systems could be incorporated as the initial values for opinions toward the chosen alternative. The latter case allows for a correlation between opinions and the availability of information, in that there may be cases where individuals do not have a favorable opinion towards an alternative, but due to a lack of knowledge, perceives that alternative as the only feasible method of shipping freight goods. After a time period has expired, the base market shares obtained by the choice model will be compared to the market shares obtained through the opinion model.

As this research is also interested in the interaction between opinions towards an alternative and corresponding attribute perception; as such, the opinion model incorporates systematic distortions in attribute values that depend on the individual's opinion and memory capacity. An extremely positive opinion may tend to exaggerate the benefits of the service or alternative, whereas an extremely negative opinion may tend to exaggerate the shortcomings of the service. As per the psychology literature, individuals are not full-information decision makers, and so over time, depending on the opinion towards the alternative, attribute values may be exaggerated.

To explore the role of opinions in the market adoption of a new transportation service, the research utilizes a generalized random utility approach that will approximate individuals' sensitivities to the new service in addition to the current services. The proposed "new" service is a rail freight service that intends to improve level-of-service performance measures. To differentiate the new service, an error term is introduced and added to the systematic component of the utility equation for all alternatives. Using Monte Carlo simulation techniques, a new value is drawn from the error distribution each time an individual receives new information about that alternative, but remains at the previous value otherwise. This allows for the introduction of serial correlation and state dependence (i.e., the analyst can induce correlations in choice over time) through this process. Following the random utility framework, the alternative with the highest utility is chosen. This process model allows for the evaluation of the path to market share equilibrium and gives insight to the forecasting capability of the logit model.

## 1.3 Main Contributions

The primary contributions of this research are the explicit modeling of social and learning mechanisms and their effects on opinion formation and propagation, the evolution of these opinions over time, and an exploration of the role that opinion dynamics have in choice processes. This research offers insight to the processes of evolving attitudes, perceptions, and opinions and the effects on individuals' judgment and decision making. It will also offer insight to the effects of attribute distortion on decision making. The added value of this research will hopefully inspire policy makers to consider social and environmental contexts into account when modeling individual choice.

Concerning the word-of-mouth mechanism, results from the scenario testing could offer public agencies insight into different strategies of disseminating information about a new policy change or an innovation to a target group of individuals who are influential in forming a favorable opinion towards the innovation and in propagating their opinion to others. The mass-media mechanism scenarios could offer insight to strategies to better match the consumers' needs in order to evoke a response. With the internet becoming a more prevalent marketplace for idea and opinion exchange, the belief learning mechanism may show policy makers how important it is to be sensitive of the information published on the services of interest. Finally, the direct experience mechanism seeks to reveal users' sensitivity to the variability in level-of-service attributes and emphasize to policy makers the need to reduce uncertainties and variances in the experience of the service. Interaction of the different mechanisms may offer insight to developing strategies to shift users' choice and give insight to strategies to developing long-term choice.

A secondary contribution of this research is the development of the freight mode choice model for a freight corridor in Eastern Europe. To the author's knowledge, there have been few quantitative freight mode choice studies in the transportation literature, mainly due to the lack of data from logistics companies that regard such information as proprietary. Combined with data on the evaluation of the system, this research offers insight to shippers' sensitivities to current level-of-service attributes and projected future services through the mode choice model and the simulation experiment.

## 1.4 Plan of Discussion

This thesis is structured as follows. After this introductory chapter, which explains the motivation and objective for this research, as well as the expected contributions, chapter 2 presents an extensive literature review of relevant research on choice modeling and opinion formation and propagation. There are three components in the literature review. The first investigates developments in choice models that account for choice set generation, information exchange and attribute distortion, and market adoption in the context of RP-SP models. The second component explores qualitative research on opinion formation including literature on opinion leaders, social diffusion, and diffusion mechanisms, and examines recent developments in mathematical formulations of opinion formation and propagation models, as well as information flow in social groups, group dynamics, and opinion diffusion. To close chapter 2, the final section synthesizes research findings from the different fields of study and presents a general approach to the research problem.

Chapter 3 presents the methodology of this research and details a theoretical approach to modeling opinion formation and propagation. Preliminary information on the implications of social and learning mechanisms are defined, followed by the role these mechanisms have in the opinion formation and propagation process. A discussion on the formulation of the utility maximizing binary logit model is presented as the basis for the estimation of parameters of the attributes, which is then followed by the development of a conceptual framework that incorporates individuals' choices, the social and learning mechanisms, an opinion revision process, an attribute revision process, and a choice consideration set. The final section of chapter 3 details the implementation of the conceptual framework so that it may be made operational in a simulation experiment.

Chapter 4 describes the simulation framework designed to explore empirical equations, heuristics, and mechanisms for opinion formation and propagation. It first explains the simulation system features, and then discusses the general experimental factors considered. Scenarios to be implemented and tested in the simulation experiments are defined in this chapter. Also considered are the performance measures and properties governing the simulation. Chapter 5 then presents the simulation results from this exploration of the designed heuristics governing social and learning mechanisms.

Chapter 6 presents the freight mode choice problem for a freight corridor in Eastern Europe. Data used in the binary logit model is described, as well as data describing the evaluation of the transport system and data measuring the willingness to adopt a new service. Chapter 7 then presents the estimation results for the binary logit model and also presents the results from the simulation experiments and compares the market shares over time to the base market shares predicted by the binary logit model.

As a conclusion to the thesis, chapter 8 presents a summary of the contributions and discoveries. It also evaluates the use of the findings from the research, as well as assesses the limitations and future avenues of research.

# Chapter 2: Background Review and Synthesis

*Previous Research in Mode Choice Issues and Opinion Formation*

While there is little research on the explicit formulation of social and learning mechanisms, their effects on opinions, and the role that opinions play in a choice framework, there has been much investigation on the separate issues of clarifying the choice process, recognizing the evolution of choices, as well as research on social and learning mechanisms, and research on mathematical representations of opinion formation and propagation. The challenge for this research is to review the large literature encompassing fields of psychology, sociology, econometrics, marketing science, applied mathematics, applied physics, public policy, communications, epidemiology, and transportation and synthesize the information for the purpose of this study. This chapter is structured around two major sections: the first considers developments in choice models and the understanding of the choice process, while the second investigates developments in social and learning mechanisms and models of opinion formation and propagation. The final section synthesizes the wealth of information and outlines the general lessons learned from previous research that can be applied to this study.

## 2.1 Discrete Choice Models and Issues

Choice plays a role in transportation decision making in route choice, mode choice, activity scheduling, driving decisions, departure time, auto ownership, and household location, among the major research topics found in the literature. In general, given a set of transportation service alternatives (e.g., private car, bus, and light rail system for mode choice, different paths in route choice, and differing neighborhoods for household location), researchers are interested in the underlying factors contributing to the choice of the individual. Conventional methods to identify these factors and determine its impact on the aggregate system utilize mathematical models based on utility theory, which is founded on the principle that people maximize their utility value. These models attempt to circumvent the limitations noted in the expected utility theory literature (see for example, Tversky, 1969; Grether & Plott, 1979; Ellsberg, 1961; Lichtenstein & Slovic, 1971; Tversky et al., 1990; Tversky, 1967a; Tversky, 1967b) by randomizing utility and incorporating a random utility framework into the models. Current state-of-the-art choice models are of the general class of random utility models (RUMs) that attempt to capture personal tastes and preferences in decision-making at a disaggregate level. Recent advances in these models have made them more sophisticated and robust, enabling the examination of variations in personal tastes and preferences across the population (see for example, Ben-Akiva et al., 1999; Walker and Ben-Akiva, 2002).

One of the most widely used RUM in discrete choice is the logit, due largely to the fact that the formula for the choice probabilities takes a closed form and the parameter estimates are relatively straightforward. Luce (1959) derived the basic logit model in formulating constant utility. Given that Luce's choice axioms held, there was a "strict" utility model in which the probability of choosing an alternative *i* was the utility of that alternative over the sum of all the alternatives in the choice set. Marschak (1960) showed that Luce's axioms implied that the utility model was consistent with utility maximization. Research by Marley into the relation between the logit formulation and the distribution of the error term (the unobserved utility) proved that having an extreme value distribution leads to the logit formula (Train, 2003). However, it was the research of McFadden (1974) that proved the converse – that the logit model implied the error term is extreme value distributed.

To derive the logit model, assume that the error term of the utility function is IID extreme value, also known as the Gumbel distribution or the Type I extreme value distribution. Following the random utility approach argument that individuals are assumed to maximize utility, the objective function to maximize an individual's utility given *J* alternatives is written in equation 2.1.

$$Max\, U_{jn} = \frac{1}{\mu} \ln \sum_{j=2}^{J_n} e^{\mu V_{jn}}, \mu \qquad \textbf{(2.1)}$$

where μ is a scale parameter that for the logit model, is set to 1. The probability of an individual *n* choosing alternative *i* then becomes the formula written in equation 2.2.

$$P_n(i) = \Pr(\varepsilon_{jn} \le V_{in} - V_{jn} + \varepsilon_{in}, \forall j \in C_n, j \ne i) \qquad \textbf{(2.2)}$$

Equation 2.2 states that the choice probability depends on the probability that the error term of all the alternatives not including the alternative under consideration is less than or equal to the difference in the systematic components of the alternative *i* and the sum of all alternatives not including *i* plus the error term for alternative *i*. This probability specification is the cumulative distribution for the error terms of all alternatives not including *i*. Thus the choice probability is the integral of the error terms of all alternatives not including *i* evaluated at the right side of the less than or equal to sign in equation 2.2 over all values of the error term of *i* weighted by its density. Equation 2.3 displays this cumulative probability function.

$$P_n(1) = \int_{\varepsilon_{1n}=-\infty}^{\infty} \int_{\varepsilon_{2n}=-\infty}^{V_{1n}-V_{2n}+\varepsilon_{1n}} \cdots \int_{\varepsilon_{J_n n}=-\infty}^{V_{1n}-V_{J_n n}+\varepsilon_{1n}} f\left(\varepsilon_{1n}, \varepsilon_{2n}, \ldots, \varepsilon_{J_n n}\right) d\varepsilon_{J_n n} d\varepsilon_{J_n -1,n} \ldots d\varepsilon_{1n} \quad \textbf{(2.3)}$$

This is a *J*-dimensional integral that is fortunately tractable due to the assumption that the error terms are IID extreme value. Algebraic manipulation of equation 2.3 yields a simple closed form expression shown in equation 2.4.

$$P_n(i) = \frac{e^{Vin}}{\sum_{j \in C_n} e^{V_{jn}}}, \; 0 \le P_n(i) \le 1, \; \sum_{i \in C_n} P_n(i) = 1 \quad \textbf{(2.4)}$$

While an attractive, tractable method of determining market share probabilities, the logit model has limitations in that it cannot account for random taste variations, it assumes proportional substitution patterns, and it cannot account for unobserved correlations across panel data or repeated choice samples. Logit models can represent taste variations across a population that vary systematically with respect to the observed variables, but it cannot represent taste variations that vary with unobserved variables or vary purely randomly. Introducing a term in the parameter coefficient to account for unobservable factors creates a new random term in the utility specification that is no longer IID. The unobserved attributes (and their effects) enter each alternative, and as a result, the combined IID error term and new error term are correlated over alternatives. This violates the fundamental assumption that the error term in the utility specification follows the Gumbel distribution (Train, 2003).

Another limitation of the logit model is that it implies proportional substitution across alternatives. As shown in equation 2.4, the sum of the probabilities across alternatives is always equal to 1. When the attributes of an alternative improve, the utility of that alternative increases, and the probability of the alternative being chosen rises consequently. This implies that the probability of choosing another alternative in the choice set decreases. Researchers are concerned with how probabilities of other alternatives change with respect to an improvement in an attribute, or an introduction of a new alternative. However, the logit model assumes that there is a constant ratio preserved between alternatives. It also exhibits the IIA property in that for any two alternatives, the ratio of the logit probability is independent of the other alternatives in the choice set (Train, 2003).

The logit model is also limited in that it cannot account for the correlation amongst unobserved factors in repeated choice situations. If the unobserved factors affecting the individual's choice are independent over repeated choices, then the logit model is suitable to capture panel effects in the same fashion it would capture variations in cross-sectional data. Logit can account for state dependence, a factor that determines how an individual's past choices influence the current choices, and can account for lagged responses to changes in attributes (Train, 2003). However, assuming the errors are independent over repeated choices or over time is a strong assumption. If there are dynamics within the systematic or observable factors, it is likely there are also dynamics within the unobserved factors.

Much research has been focused on overcoming the limitations of logit to better understand and analyze individual behavior and improve the accuracy of the parameter estimates for better forecasts. More advanced RUMs have relaxed the logit assumptions. The nested logit model relaxes the assumption of proportional substitution and allows for correlations across alternatives through the introduction of a nesting structure. Probit and mixed-logit models have relaxed the limitation of random taste variations and unobserved correlations in repeated choice experiments by allowing flexible specification of the error terms (e.g., allowing for correlations across individuals as well as across alternatives) and allowing for random coefficients for the parameter estimates.

Despite the advances to account for judgment and decision making within an economic framework, RUMs have been largely criticized by both economists and psychologists as to whether they adequately account for behavioral realisms in decision making. RUMs mentioned in the previous paragraph do not account for latent factors such as attitudes and perceptions, availability constraints (i.e., which alternatives are available to an individual), different decision protocols, or stated preference information. This led to the development of a generalized random utility model that relaxes the restrictions of previous RUMs through extensions that account for flexible disturbances, latent variables (e.g., the explicit representation of the formation of attitudes and perceptions), latent classes (e.g., latent segmentation of taste variation, choice sets, and decision protocols), and combined revealed and stated preference data (Walker and Ben-Akiva, 2002).

Other criticisms have focused on the decision process of RUMs. Of interest to this research is the skepticism behind whether utility maximization is an approximation of individuals' objectives (e.g., are individuals always profit-driven or do they consider social welfare), and more importantly, whether the compensatory trade-offs imposed by utility theory on all alternatives in the choice set is an accurate depiction of individuals' decision process. Additionally, individuals are rarely viewed to be fully informed or aware of the full set of alternatives the analyst specifies in the choice model specification, nor are they fully cognizant of the correct values for attributes of the alternatives. These perceptions or beliefs may stem from experience, or interaction with information received from other individuals or a media conduit, and may lead to distortion of these values. Researchers have attempted to address these concerns through the investigation of choice set generation, quality of attribute information, and sensitivity to attribute changes. As this research is more concerned about the choice process and the evolution of choice rather than the estimation of choice parameters, the next three sections will focus on findings from studies on choice set formation, perception of attributes, and work on RP-SP models.

### 2.1.1 Choice Set Generation

Much of the research on choice set formation has stemmed from a seminal paper by Manski (1977) on the structure of random utility models. That work presents a framework of choice theory, by which the generation of alternatives within a choice set forms as an outcome of a two-step recursive process. In the first stage, exogenous forces pose a choice problem involving an individual and the associated choice set. Given that the choice set is well-defined, the individual chooses from a set of available alternatives. Until this work, most of the research on choice theory had focused on the second stage of the choice process. Manski formulates a probabilistic mechanism for the generation of the choice problem. Formally, given a choice set space $\Gamma$, and decision maker space $T$, choice set $C$ is a subset of the choice set space, and decision maker $t$, the choice problem is defined as a pair ($C$, $t$) drawn from the product space $\Gamma \times T$ according to some probability measure $M_{\Gamma T}$ that is defined over the product space. From this choice problem generating process, two probabilistic components, the decision maker generating process, $M_T$, and the choice set generating process, $M_\Gamma$, are derived. This yields the following equation:

$$M_{\Gamma T} = M_{\Gamma}(C \mid t) M_T(t) \tag{2.5}$$

In this formulation, the choice set generation is interpreted as an exogenous party or set of decision makers who control the availability of alternatives to each decision maker $t$. An example given in Manski (1977) is the college admissions process, where a set of students $T$ have an $A$ set of colleges to choose from. Admission administrators make the decision whether or not to admit a particular student, thereby making their college available to the student for final consideration.

The decision maker process is interpreted as a sampling strategy under the random utility framework (i.e., the analyst draws a sample from $T$ and observes their choices), or a set of decision rules from which the individual draws under the psychological model of random utility (Manski, 1977).

The interpretation of Manski's two-stage choice paradigm has been adapted to reflect the stochasticity of an individual's choice set (i.e., an analyst rarely knows the actual choice set from which an individual chooses an alternative), and to represent the limited information decision maker (i.e., an individual may not know about all the feasible alternatives, or may not seriously consider some known alternatives). The latter interpretation is demonstrated in Swait and Ben-Akiva (1987a) where the choice set generation process is constructed as a random constraint in the choice set space. Here, the choice set is a constrained subset of all possible alternatives that depend on socio-demographical, psychological, informational, societal, and cultural restrictions. However, it is improbable that the analyst can properly specify deterministic constraints that describe the restrictions on an individual's consideration set. Thus Swait and Ben-Akiva formalize the concept of probabilistic constraints that has a deterministic component consisting of a constraint index set, a vector of parameters, a vector of characteristics of the individual, and attributes of the alternatives, and a random variable that is additive to the deterministic portion. The alternative is considered available to the decision maker if all the constraints are satisfied. This is specified as a binary random variable that is 1 if the relevant constraints are satisfied, 0 otherwise (Swait and Ben-Akiva, 1987a). The interpretation of this approach is that if the choice set element or alternative is greater than some threshold level (denoted by the random parameter) it is placed into a consideration set. Since the analyst cannot observe the threshold, the inclusion or exclusion of an alternative becomes probabilistic.

Operationalizing this theoretical framework proved computationally difficult given the large choice set space as the number of alternatives increased. Much of the work attempting to model probabilistic choice set generation have implemented a constant consideration probability across individuals (Basar and Bhat, 2003). One of the limitations of the work in Swait and Ben-Akiva (1987b) is that although they allow for the threshold to vary across individuals, the construction of a parameterized logit captivity (PLC) model restricts the evaluation to a model where the consumers are captive to single alternative, or to a model where they choose from the full set of alternatives. Basar and Bhat (2003) formulate a parameterized choice set model that relaxes this restriction, and allows for individuals to choose from all possible choice set sizes. Once the choice set is formed by the probabilistic mechanism, the second stage of the model estimates attribute parameters using the multinomial logit formulation. This model allows for the evaluation of the effect of an attribute change on both the consideration set (i.e., the impact of placing an alternative into a choice set) and the choice (i.e., the impact on choosing an alternative, given the alternative is considered by the individual) (Basar and Bhat, 2003).

Basar and Bhat (2003) apply the probabilistic choice set multinomial logit model (PCMNL) to an airport choice problem. An assumption of this formulation is that all airports are feasible for each traveler, however, not all airports may be considered by each traveler. Given that $U_{qi}$ is the consideration utility for airport $i$ and individual $q$, the alternative is included in the choice set if the consideration utility exceeds a threshold. This threshold is assumed to be a random variable that is standard logistically distributed. As in the Swait and Ben-Akiva (1987a) formulation, the probability of an alternative $i$ appearing in the consideration is modeled as a binary process: 1 if the utility exceeds the threshold, 0 otherwise. Estimation results of an MNL model and the PCMNL model show substantial differences in parameter values for the two most important attributes, access time and flight frequency. Individuals are much more sensitive to access time at the choice stage in the PCMNL model as the MNL model assumes that all airports are available to all individuals. While the comparison of trade-off values between access time and flight frequency between both models reveal the dominance of access time at the choice stage, the PCMNL model also shows that flight frequency is the dominating factor for the consideration set. The added analytical power of the PCMNL has implications for marketing strategies to boost market share; for areas within close proximity of an airport, marketing campaigns would focus on improved access time (since given the proximity, an individual will most likely have that airport already in his choice set), while for those outside this area, the campaigns would focus on improved flight frequency to ensure that those individuals consider the airport (Basar and Bhat, 2003).

Where Basar and Bhat (2003) assume that all alternatives are feasible, though the individual may not know about them or consider them, this may not be an appropriate model specification for a choice problem where some of the alternatives are not feasible. The PCMNL formulation also does not explicitly account for latent processes that govern an alternative's availability. Research by Ben-Akiva and Boccara (1995) attempts to explicitly capture individual perception of the availability of an alternative. Using the basic model framework developed in Swait and Ben-Akiva (1987a), they incorporate choice set generation response indicators related to the latent variables and latent constraints to enable the explicit analysis of latent psychological factors that would yield information on individual behavior not normally inferred from observed behavior alone (Ben-Akiva and Boccara, 1995). Such indicators are derived from survey questions concerning the availability of the alternative. Realizing that these responses would not only have availability implications, but would also affect the desirability of the unchosen alternatives, they alter the constraint-based rule to include both the latent criteria $H_n$, and the latent utilities $U_n$ for a given alternative. The rationale for this correlation is that individuals may indicate that an alternative is unavailable, when in reality, it is an unchosen alternative (i.e., it is feasible or available) with a low utility.

Estimation results show that the probabilistic choice set (PCS) model has a better fit than the MNL model. The model improves prediction power over the MNL model in situations where there is substantial heterogeneity of choice sets actually considered, and it contains information on the loyalty or captivity of individuals to specific alternatives. Such additional information will allow marketing and advertising to design better strategies that focus on specific consumer segments. The implication of disregarding the availability of an alternative is reflected in the elasticities of the market shares; the elasticities may be over or understated in the absence of availability considerations (Ben-Akiva and Boccara, 1995).

Given the large implication that choice set generation has on marketing strategies, research in marketing science has taken interest in investigating the formation of choice sets with hopes of understanding how certain marketing campaigns may affect individuals' consideration of an alternative. Much of the marketing literature concurs with the two-stage choice paradigm and defines the full set of alternatives to be the awareness or feasible set, and the choice set to be the consideration set of alternatives. Gensch and Soofi (1995) use an information-theoretic algorithm for estimating the consideration set. They use information entropy as the uncertainty function over the choice sets and attribute ratings as a source of information for reducing uncertainty. The algorithm estimates the choice using the maximum entropy principle, which maximizes the uncertainty inherent in the distribution for predicting the outcomes. Maximum uncertainty occurs when all outcomes are equally likely. The objective of the research is to partition the awareness set into two subsets that have a high level of entropy within the set, but low entropy between sets. Alternatives with high probabilities are assumed to be in the consideration set, and low probability alternatives are in the non-consideration set. The authors argue that the consideration set includes the alternatives with only the higher non-negligible probabilities, so the set is much more homogenous than the non-consideration set. The key finding in this research is the predictive power of the logit model is concentrated in identifying the non-consideration set alternatives, or alternatives in the awareness set (Gensch and Soofi, 1995).

Horowitz and Louviere (1995) argue that in the absence of direct observations of the choice process, there are many situations in which standard operational indicators of consideration provide no additional information than that provided in the utility function, and are really just reflections of individual preferences. Modeling choice with a consideration stage, they say, may lead to a misspecified model that would produce inaccurate forecasts. To demonstrate this, they test the hypothesis that an individual's preferences towards all available alternatives can be described by a utility function that identifies both the consideration set and the choice against alternative models that model choice as a sequential, two-stage process. The study operationalizes two definitions of consideration: the first corresponds to measurement that evoked brands, while the second corresponds to measurement that aided in recollection of brands. Results suggest that the consideration stage may be a reflection of preferences, rather than the first stage in the two-stage choice process. Knowing the individual's consideration set would not offer additional information to that provided in the utility function. Given that an analyst knows the individual's utility function, the consideration set could be perfectly predicted given the knowledge of their size. However, Horowitz and Louviere recognize that utilities are latent, and that information about consideration sets could be used to improve efficiency in estimation and prediction given that the model is properly specified (Horowitz and Louviere, 1995).

There may be situations where individuals' consideration sets contain only one alternative. Lapersonne et al. (1995) argues that individuals construct their consideration set through a search process of alternatives in the awareness set and stop searching if a further search for feasible alternatives is perceived to be not cost-effective. Identifying the marketing factors and socio-demographic characteristics that lead to this situation has the potential to ensure a manufacturer has the ability to trigger further search processes from competitors (thus attracting new business) or to preclude active search of other brands by current customers (thus maintaining a loyal customer base). Using a survey of consumer purchases of automobiles, they find that 17% of the interviewees only considered the brand of their previous vehicle. Of this fraction, 48% actually ended up purchasing that previous brand. To investigate this phenomenon, Lapersonne et al. use a binary variable to indicate whether a consumer considered only his previous brand, and then analyze the variable using logistic regression.

Estimation results showed that consumer satisfaction was the dominant factor in purchasing the automobile. If an individual is very satisfied with his previous car (and dealer), the perceived benefit of collecting and analyzing data on other brands is trivial, and not worth the effort. On the other hand, dissatisfied individuals see a very high benefit to investigating different brands or alternatives and are willing to incur extra costs or efforts to attain the benefits. Socio-demographic factors that increase the propensity to consider only one brand include old age, low income, and low education. However, these explanatory variables are only proxies for latent effects hypothesized, such as a lower ability to process information, a lower propensity to explore unknown paths, a lower social ability, or a lower ability to control one's life. Considering product experience, they find that people who have "broader" previous experiences with the class of products are more likely to consider several brands. Concerning risk and involvement, it is interesting to note that buyers who considered only their previous brand do not feel they are taking a high risk of making a wrong decision (Lapersonne et al., 1995).

A number of managerial implications can be drawn from the Lapersonne et al. (1995) work. Improving satisfaction with the current product and current distributor is essential to maintaining a loyal customer base. Providing these loyal customers with positive and well-specified information on the brands sold by the distributor also helps to maintain customer loyalty. To attract new customers, distributors should focus on providing potential markets with information that convinces those customers that the potential utility of its products are high enough to warrant an active search or merit consideration of the product (Lapersonne et al., 1995). These implications raise interest on how direct marketing and segmentation and targeting strategies can affect an individual's consideration set. In light of the shift of marketing strategies to brand differentiation instead of the traditional brand placement in areas of highest demand, Allenby and Ginter (1995) investigate the effect of in-store displays and feature advertising on consideration sets. A critical issue for implementing the product differentiation strategy is to determine the extent to which advertising – marketing mechanisms – influence household consideration sets. They hypothesize that consideration sets reflect more than just preferences, even in simple product choices. Using information on household purchases of canned tuna brands, they specify a heteroscedastic RUM that contains explanatory variables such as price, the presence of an in-store display, and another dummy variable that is one when the product is featured in a major advertisement. The model allows for a flexible pattern of cross elasticities at the household level, allowing the analyst to describe patterns of price sensitivity among competing products. Findings show that, by allowing for the price sensitivity to depend on the in-store display and feature activity, the marketing variables serve to decrease household price sensitivity. The decrease is substantial for feature advertisements, which may suggest that households first identify their brand before going to the store and observing the array of prices. In-store displays and feature advertisements also influence inter-brand competition. These findings imply that effective management of these marketing mechanisms can offer an immediate, short-term reduction in price sensitivity, which is similar to long-term effects sought in product differentiation strategies (Allenby and Ginter, 1995).

Mitra (1995) investigates the effect advertising has on the stability of consideration sets over multiple purchase occasions. Using stability measures such as the number of brands considered at least once, the standard deviation of consideration set frequencies, and the average discordance in consideration set composition, she examines the stability of consideration sets when individuals are subject to advertising. She hypothesizes that advertising information affects consideration set stability by causing individuals to see larger differences among brands than would be without advertising. More simply, advertising has an effect on differentiating product perceptions. Findings show that differentiating advertising that contained diverse information on brand-attributes led to a greater dispersion of perceived brand utilities compared to no advertising conditions, and to a reminder advertising situation. Increasing gaps between brands lead to fewer brands in the consideration set. However, advertising by itself did not decrease the average consideration set size, suggesting that the effects of advertising on the composition of the consideration set is not captured by consideration set size (Mitra, 1995).

While most of the decision theory literature has utilized RUMs or information theory to model choice behavior, alternative formulations of choice have been proposed. Vythoulkas and Koutsopoulos (2003) incorporate concepts of fuzzy set theory and neural networks in a model of mode choice. This alternative approach to RUMs uses fuzzy sets and linguistic prompts to model individual perceptions of the attributes, and uses approximate reasoning and fuzzy control to model the decision process. In the fuzzy decision framework, individual perceptions are the inputs and modeled as fuzzy sets since they can capture the vagueness of perceptions associated with attributes of alternatives. Here, individuals are assumed to make decisions based on simple rules rather than maximizing a complicated utility function. Approximate reasoning reduces the number of rules, and the composition stage combines the fuzzy preferences from all rules that were triggered and calculates the overall fuzzy preference of an alternative. Finally, the preference is defuzzified, and a choice is revealed (Vythoulkas and Koutsopoulos, 2003).

To test the effectiveness of the fuzzy logic approach to choice modeling, they apply the framework to a mode choice problem. In the application, the main variables modeled were differences in travel time and costs by each alternative (rail and car), as well as an access-egress ratio for rail. They then assign linguistic labels very low, low, moderate, high, and very high to the explanatory variables. Corresponding output labels, or individual preferences, are represented as not preferred, probably not preferred, indifferent, probably preferred, and preferred, for each alternative. They used a deterministic rule-based procedure, as well as a probabilistic rule-based procedure to obtain the market shares, and compare those to a logit model for the same mode choice problem. Findings show that the fuzzy model compares favorably to the logit model, suggesting that there is potential for neuro-fuzzy models to model discrete choice problems in addition to modeling decisions made under time pressure (Vythoulkas and Koutsopoulos, 2003).

### 2.1.2 Attribute Value Perception

Utility is only one of several classes of decision rules studied in the decision theory literature. As mentioned previously, critics of the utility decision rule disagree with the compensatory nature of the decision maker (i.e., individuals are cognizant of the trade-offs implied by different attributes). Among the other prominent classes of decision rules in the literature include dominance, satisfaction, and lexicographic rules. Dominance and satisficing rules do not necessarily lead to choices, however, since it is rare to find an alternative that is dominant or satisfactory for all attributes. Lexicographic rules imply a choice by rank ordering the attributes by the level of importance and then eliminate the alternatives that do not yield the highest return for that attribute. One of the widely cited decision rules in the decision theory literature is the elimination-by-aspects by Tversky (1972), which is a combination of both lexicographic and satisficing rules. Here, the process begins with the most important attribute and eliminates the alternatives that do not meet a criterion. If there is more than one alternative left, the process continues with the second most important attribute, until a unique choice is revealed (Tversky, 1972).

The elimination-by-aspects model is better received by the decision theory community since it is a non-compensatory rule, thereby avoiding issues with individual perceptions of trade-offs between the attributes. Many studies have attempted to operationalize elimination-by-aspects, but this task has proven to be difficult. Within the realm of transportation and travel demand models, Williams and Ortuzar (1982) introduced the concept of a joint-compensatory, non-compensatory (hybrid) model, which is based on the importance of the attributes (i.e., the attribute rank order via the lexicographic decision rule) and a probabilistic choice component given the decision rule. The model formulation reflects the critical tolerance principle that relates to the psychological concept of just noticeable differences (Williams and Ortuzar, 1982). In this framework, alternatives are eliminated from further consideration in the decision process if the difference between attribute values exceeds an individually defined threshold or critical tolerance. The behavioral concept of the model is as follows. Individuals first rank the attributes in order of importance. Available or feasible alternatives are considered with reference to the most important attribute, and alternatives are eliminated if the threshold constraint for the attribute is violated. This process continues for the second most important attribute and so on, until either a single alternative remains, or the list of attributes used to assess the alternatives is exhausted, leaving more than one alternative remaining. In the case that the attribute rank list is exhausted, the decision between the remaining alternatives is made using a compensatory utility maximizing rule. Williams and Ortuzar argue that in the case where individuals fail to easily discriminate amongst alternatives using thresholds and a rank-ordered attribute list, they move to a more scrutinizing phase, where a compensatory rule is needed to distinguish the best alternative (Williams and Ortuzar, 1982).

Swait (2001) attempts to incorporate this concept of attribute thresholds or cutoffs within a non-compensatory choice model framework. Benefits (in terms of improved behavioral prediction) of incorporating these cutoffs are greater for decisions in which non-compensatory considerations are more prevalent, such as in the case of residential location or vehicle type choice. Swait formulates a model that incorporates a wide variety of decision strategies (e.g., compensatory, conjunctive, disjunctive) without imposing them on the model, instead inferring the strategies from the observed outcomes. This model is tested on an SP experiment considering vehicle rental from rental car companies. Results show that the specification of soft cutoffs (i.e., individuals may violate these constraints but the effect is a diminished return on the utility function) increase the statistical and explanatory power of utility models.

Cantillo and Ortuzar (2005) loosely interpret the model proposed by Williams and Ortuzar (1982) in formulating a semi-compensatory discrete choice model that explicitly accounts for attribute thresholds of perception. These thresholds represent acceptance levels of the attributes in the process of discrete choice, and could be random, systematically varying through the population, or a function of socio-economic features and choice conditions. The formulation they use allows for the estimation of the parameters of the threshold's probability distribution. Their model is able to consider correlations amongst thresholds, but findings from the estimation results show that the correlation effect is minimal. When applying the model to real data, they find that specifying the thresholds as a function of socio-economic characteristics and choice conditions provides a better fit than a single distribution over the entire sample. Estimation results also showed that in choice situations where thresholds do exist, the use of compensatory models such as the MNL or mixed logit can lead to errors in model estimation, and therefore provide erroneous subjective value of time (Cantillo and Ortuzar, 2005).

A similar study by Cantillo et al. (2006) incorporated thresholds for perception in attribute values. This work focused more on individuals' sensitivity to attribute changes, whereas in the previous study the focus was on the choice process. They argue that if thresholds are not accounted for, especially when policy changes to the attributes are small, it could lead to errors in prediction. Here, the thresholds are implemented as the minimum perceptible change in attributes. The model was applied to synthetic data as well as real data collected from a SP survey. Results show that if perception thresholds exist, the use of models that do not incorporate thresholds leads to errors in estimation and prediction, although the magnitude of the error depends on the weight of an attribute in the utility function. Data from the SP experiment provide support for the existence of thresholds in travel time, since a few seconds of added or loss time will probably not faze an individual (Cantillo et al., 2006).

Concepts of individuals' sensitivity to attribute change reveal a different critique of choice models. Choice models do not reflect the dynamic process by which travel demand adjusts to policy changes in the attributes. This is due to the fact that change in behavior is not instantaneous; rather it takes a period of time before the demand equilibrates. Another way to interpret this is that the population affected by these policy changes is not instantly made aware of the improved transportation level of service. In one of the seminal papers that frames the motivation for this research, Lerman and Manski (1982) formulate a model that examines the demand dynamics that arise when information about changes in the transportation environment spread through the population. Information flow is modeled by three distinct sources or mechanisms: direct observations from using the alternatives, word-of-mouth communication through casual interaction with informed individuals, and various forms of media communication including news outlets and advertising. Individuals constantly receive information from these three sources and consequently alter their perceptions and possibly alter their actual choices (Lerman and Manski, 1982).

To implement the model, they assume that there is a once-and-for-all change in the transportation system and that individuals use their most recent information obtained from any of the three sources when making their decision. The model considers two alternatives, $i$ and $j$, and contains a sequence of discrete time periods and at time $t = 0$, a steady state prevails. This means that the attribute values have remained stable and consistent long enough for decision makers to have perfect information. At time $t = 1$ a once-and-for-all change occurs in the attributes of one or both alternatives. This change could also be the introduction of a new alternative. Lerman and Manski specify three cases of aggregate effects that generate a change favoring one of the alternatives. In one case, $i$ and $j$ both deteriorate, but $j$ more so than $i$, while in the second case, $i$ and $j$ both improve, but $i$ more so than $j$. The third case is when $i$ improves and $j$ deteriorates. They evaluate the phases of the choice path (i.e., the path from the old steady state to the new one), monotonicity of the path, and the speed of adjustment (Lerman and Manski, 1982).

Qualitative results drawn from the investigation of the choice path show that the analyst can infer little about the location of the new steady state from observation of only one of two phases of the choice path (Lerman and Manski, 1982). Concerning monotonicity, the path to the new steady state need not be monotonic, even in the second phase of the choice path. For choice situations where there is deterioration in service, the speed of adjustment is higher than in situations in which there is improvement in service. They give the example that raising parking fees would be more effective (i.e., quicker, in terms of speed of response) in getting auto users to switch to bus than if the transit agency lowered bus fares. Overall results demonstrate the existence of two phases in the path over time for the aggregate choice frequencies. In the first phase, there may be choice reversals, but in the second phase, there are none. For situations where the change favors an alternative, the second phase is shown to be monotonic (Lerman and Manski, 1982).

Sammer et al. (2006) investigates how the quality of information about the attributes of the alternatives plays a role in decision making for a mode choice problem. One of the key contributions was a measure of knowledge. Here they developed four instruments to evaluate the level of knowledge of the individual being surveyed: a precise description of the trip, estimated door-to-door travel time, estimated range of or expected deviation from travel time, and estimated costs (Sammer et al., 2006). Each response was rated on a scale, and then normalized by taking the arithmetic average. To investigate the effectiveness of information technology, they apply the knowledge variable formulation to a mode choice case study in Austria, using the rating of the information and knowledge and the degree to which the information has penetrated the population. Results show that the inclusion of this knowledge and information status is statistically significant, supporting the argument that knowledge variables should be included in the model specification to increase its explanatory power (Sammer et al., 2006).

### 2.1.3 Revealed Preference – Stated Preference Models

The current state-of-the-art for estimating individuals' sensitivities towards the attributes of a new alternative is to estimate a model using both RP data and SP data. Respondents to the SP survey are presented with a hypothetical situation in which a future service is introduced and are asked how they would modify their current choices in response to this change. One of the first studies to estimate a joint model is by Ben-Akiva and Morikawa (1990). They developed a framework with two components, the RP model, which is the traditional RUM, and a stated intention model that compares the utility of the current chosen alternative to the stated intention alternative. To validate the application of this model, they administered two surveys: one before the addition of a new subway line, and the other a few months after the opening of the new line. The main findings show that combining SP and RP data increased the accuracy of the parameter estimates of the model, that if scaled correctly, the SP data would yield the same coefficients as the RP model, the SP data contained more random noise than did the RP data, and that the threshold value for switching was negative, implying that individuals overstated their intention to switch to the new alternative (Ben-Akiva and Morikawa, 1990).

One concern with RP-SP models is fitting the alternative specific constants (ASC) to properly forecast the market shares of new alternatives. Cherchi and Ortuzar (2006) investigate methods to properly specify the ASC for such RP-SP models. They develop four recommendations for specifying the ASC. If the RP and SP alternatives are the same, the ASC should be adjusted to match the base market shares. For cases where the SP data include new alternatives and if there is evidence that the data correctly predicts the market shares, then the ASC should be adjusted to match the SP market share for both existing and new alternatives. When market shares to match are unknown, the analyst should rely on estimation results and use the ASC specification for the model that provides the best statistical fit. If the SP design implies significant changes (e.g., alternatives sharing the same label represents new options) and it is difficult to distinguish those alternatives, then the best statistical model and analyst judgment is used (Cherchi and Ortuzar, 2006).

## 2.2 Research on Opinion Dynamics

While mathematical models of opinion formation and propagation are relatively new, the subject has been of long interest to social scientists, mainly sociologists, psychologists, and political scientists. Early studies on public opinion and propaganda explored human factors and interactions from a qualitative perspective (see, for example, Dodd, 1958-1959; Dodd and Winthrop, 1953). Economists, mathematicians, and physicists constructed mathematical representations of decisions to update opinion, group opinion consensus, and opinion diffusion using different assumptions. While quantitative research has provided interesting simulated results, they generally lack some real-world human factors, or mechanisms, through which opinions form and propagate (Wu and Huberman, 2005). Thus, it is necessary to review both qualitative and quantitative research to understand the contribution and limitations of each, and subsequently attempt to reconcile inconsistencies or deficiencies in improving the mathematical model. On the qualitative side, the review will focus on descriptions of the human mechanisms and human social factors through which opinions form and propagate. For the quantitative section, the review investigates innovations in modeling techniques. As there are similarities between opinion formation and propagation and traffic formation and propagation, especially in considering macro- and micro-level relationships, the review then focuses on models in traffic flow theory that potentially account for human mechanisms in opinion formation and propagation. This is followed by a synthesis of the findings that will help set the basis for the model and simulation experiment proposed for this research.

### 2.2.1 Factors Influencing Opinion Formation and Propagation

Understanding how an individual's opinion can form and then diffuse through a population is a central theme in investigating voter behavior and propaganda. Organizers of propaganda machines or political campaigns are eager to know how effectively their message or viewpoint (i.e., opinion) will spread amongst people. In a study to test the effects of a World War II propaganda machine, Dodd investigated rules for the distribution of propaganda. Measures of effectiveness included distance, extent (percentage of population), speed, and accuracy or consistency (Dodd, 1958-

1959). Using a combination of controlled experiments, empirical data, and expert judgment, Dodd found relationships between diffusion and spatial, temporal, population, activity, values, stimulation, and residual factors.

For the spatial factors, the further the individual is from the opinion source, the less likely the individual is to hear about the opinion. The percentage of opinion holders decreased at a rate dependent on an aggregate mode of travel (Dodd, 1958-1959). In general, the percentage of opinion holders is greatest when the source is in high-density areas such as urban zones. Temporally, opinion diffusion can be characterized by growth curves, with exponential and logistic curves found to fit best. Population factors exhibited that small towns seem to interact more and gossip, thus increasing the propagation of opinions. Younger people, children in particular, tended to collect the leaflets (propaganda source) with enthusiasm, which may indicate a propensity towards an open mind (Dodd, 1958-1959). Within the activity context, Dodd defines a "potency" index which denotes the "first-time hearers per tellers" in a given time period. The values and motivational factors examine how well the opinion matches individual values and preference rankings. For stimulation factors, the percentage of opinion holders increased as the strength of stimulation increased, but there is mention of diminished returns or an increase in insensitivity (Dodd, 1958-1959).

An earlier study by Dodd and Winthrop investigated some of the opinion interaction from a one-way interaction perspective. Along with specifying many of the same factors in the study above, they also investigate group network structure as an important aspect of opinion diffusion. They considered three types of group structures: an "iadic" structure where every member is interrelated with each other; a ramified structure in which members are not related to the friends of their own friends (i.e., all of one member's friends are strangers to each other); and a mixed ramified and partially iadic structure where members are partially interrelated with each other. Intrinsically, iadicity, or complete interrelatedness, means that everyone is acquainted with each other to a point where there are equal opportunities to share opinions (Dodd and Winthrop, 1953). Through experiments with small groups, they find that the number of converters generally follow a specified distribution.

What Dodd and Winthrop also implicitly mention is that the opinion source type has a role in the diffusion of opinions. In an experiment involving boys at summer camp, they chose as sources three boys identified as a leader, an isolate, and a "middler", respectively, as previously identified by a sociometric test administered prior to the experiment (Dodd and Winthrop, 1953). This follows the school of thought that leader-type personalities tend to have a larger impact on the diffusion of opinions than follower-type personalities. Research on opinion leaders focused on identifying socio-demographic factors, character traits, and social positions of these influential individuals (see for example, Weimann, 1991). Wright and Cantor build upon this issue by offering the concept of the opinion seeker and the opinion avoider. Their research complemented opinion leaders and explore the dynamics between leaders, followers, and isolationists, as well as the interrelationships between personality types, including leader-seekers, seeker-only characterizations (Wright and Cantor, 1967).

Concepts of opinion leaders and opinion seekers played a significant role in the formulation of the diffusion of innovation theory developed by Rogers in 1962. Drawing on sociological notions of imitation and innovation interaction among individuals, Rogers characterized five classes of adopters: the innovators, the early adopters, the early majority, the late majority, and the laggards. Innovators are the first 2.5% of a population to adopt an innovation, are generally very knowledgeable about the innovation subject, and have a higher propensity to take risks. Early adopters, consisting of the next 13.5% of the population to adopt an innovation, are social (opinion) leaders who, through their social network, learn from the innovators. The early majority, the next 34.5% of the population, are potential adopters who look to the early adopters for new ideas or opinions. The next 34.5% of the population to adopt is the late majority, and are generally conservatives, skeptics of innovations, or have a lower socio-economic status and are not as capable of taking risks. Laggards are the last 16% of the population to adopt an innovation. These people tend to be socially isolated, receive the information about the innovation from neighbors and friends, and are averse to losses incurred by risk taking (Wikipedia contributors, 2006).

Rogers (2003) defines the diffusion process as the communication of an innovation through various channels over time among members of a social system.

As such, there are four main elements to consider in the diffusion process: the innovation, the communication channels, the time, and the social system (Rogers, 2005). According to Rogers, the rate of adoption depends on five factors: relative advantage, compatibility, complexity, trialability, and observability. Relative advantage is the perceived or subjective advantage that the innovation has over the previous or status quo idea. Compatibility is individual's perception of the degree to which the innovation is consistent with an individual's existing values, attitudes, experiences, and needs. Complexity is an innovation's degree of difficulty to comprehend and utilize. Trialability is an indicator of the degree to which an innovation can be experimented with for a limited period. Observability is the degree of the innovation's visibility to others. It follows that innovations with greater relative advantage, compatibility, trialability, observability and less complexity will have a faster adoption rate (Rogers, 2005). An individual's path to adoption consists of a five-step decision process that progresses from the initial knowledge of the innovation, to the formation of an attitude or opinion towards the innovation, to an initial decision to adopt or reject the innovation, to the implementation of the innovation, and finally, to long-term use or a confirmation of the innovation (Rogers, 2005).

Rogers also states that marketing and mass media channels tend to foster knowledge of the innovation while interpersonal or word-of-mouth channels facilitate forming attitudes and opinions of the innovation and consequently influences the initial decision to adopt. Rogers's innovation decision process also implies a learning process from the initial adoption decision to the final decision of commitment to the innovation. The majority of the population (early majority, late majority, and laggards) will form attitudes and opinions based on subjective evaluations of peers and neighbors who have already adopted the innovation (Rogers, 2005).

Rogers's work on the diffusion of innovations highlighted the need to study the mechanisms of interactions between the different social categories. Subsequently, research has focused on clarifying the three mechanisms mentioned in Rogers's work: word-of mouth, mass marketing, and personal observation or learning (trialability). The following sections will consider studies that examine the dynamics of these mechanisms and the effect on opinion formation and propagation.

#### 2.2.1.1 Word-of-mouth Mechanisms

Communication of opinions has long been a research interest in public policy. A study by Glock (1953) highlights the important factors to consider in understanding the complexity of oral communication dynamics and its interdependence on the role of mass media. An increased media presence in a community facilitates an increased exposure of some individuals to mass media. These individuals have a higher propensity to communicate with others in their social group and use their high exposure to develop an opinion leadership. For communities where mass media is not accessible, opinion leadership forms around individuals who have traveled between communities and are exposed to varying opinions (Glock, 1953).

Feick and Price (1987) advanced the concept of opinion leaders by the introduction of the market maven, an individual who is an expert or is extremely knowledgeable and passionate about certain classes of market products. These individuals are influential because of their general marketplace expertise, and have a high propensity to seek information about new products or new information. Early knowledge of information and attention to details lead these people to early adoption. They distinguish between an opinion leader, whose expertise stems from involvement in a particular product class, an early adopter, whose expertise stems from personal purchases and product usage, and market mavens, whose expertise stems from personal interest in accumulating knowledge about the marketplace in general, and can therefore provide information across product classes (Feick and Price, 1987). This research introduced notions of different personality types among opinion leaders and early adopters and highlighted the need to determine the kind of information each type provided as well as the effectiveness of the information exchange.

Utilizing the concept of the market maven, Gladwell (2002) introduced the concept of the law of the few that identifies three personality types essential to social epidemics: connectors, mavens, and salesmen. Connectors are a class of opinion leaders who have a large social network that spans different classes, groups, and communities. What distinguishes these connectors from individuals who know many people is that these individuals actively seek to make new connections regardless of social class or social type and will seek to connect people in their social network. Mavens follow the definition of Feick and Price (1987). These individuals do not just possess a wealth of knowledge and expertise on a particular market class, they also actively try to help others get the best deals on a certain product. Further, while mavens generally hold their opinion given their expertise, they still actively seek feedback from other individuals to add to their wealth of information. Finally, salesmen are a class of opinion leaders that are highly persuasive. These individuals are not the stereotypical car salesman in that these salesmen have such charisma and presence that other individuals gravitate towards these people or attempt to mimic these people. Thus, a salesman's opinion is highly contagious since his personality draws people to mimic the same opinion (Gladwell, 2002).

In general, these opinion leaders actively seek to communicate by word-of-mouth with other individuals. Mavens generally have extensive knowledge of a particular market class, and individuals who seek this type of opinion leader take their opinion seriously. Thus, for a particular product, mavens have a high rate of effectiveness in altering an opinion seeker's opinion. Connectors tend to have a broader view of different market classes due to their high connectivity in a large social network. Their rate of effectiveness may not be as high as a maven's, but because they have a large social network at their disposal, they can gain information or opinions from several maven types, then spread their opinion amongst thousands of individuals. Salesmen are so charismatic and persuasive that their effectiveness may be the highest out of the three types, provided the opinion seeker develops a good rapport with the salesman (Gladwell, 2002).

### 2.2.1.2 Mass Media Mechanisms

Effects of mass media on opinion formation and propagation have been studied since newspapers and other medium to spread propaganda in the early 1900s. With the advent of the television in the 1950s, however, there was an exponential growth in individual exposure to marketing ads and campaigns. Subsequently, there was heightened interest in how these individuals would use this information and how it would effect their opinions.

Dimmick and Wang (2005) investigate how mass media medium diffuses through the population. Their research suggests that a logistic growth equation is the underlying mechanism behind media diffusion. A logistic curve is a good fit to the diffusion of media in the U.S. such as radio, television, cable, VCR, and personal computers. They propose that the $r$ and $K$ parameters of the logistic equation represent anticipated gratification utilities and economic conditions, respectively. Using curve-fitting methods, they showed that the acceptance of the medium followed a logistic curve rather than a linear curve, but only for cases when the medium has been slow to accept for a period of time after its introduction. Some event (e.g., the advent of the internet) created a spark in a sale of a particular medium (e.g., personal computers), which cause the penetration rate to follow the logistic curve. For innovations with a quick adoption onset, they suggest an exponential curve would provide a better fit (Dimmick and Wang, 2005). However, the conclusions were based on percent ownership of a particular medium, and not the actual penetration of a marketing campaign.

One role of mass media is to inform an individual of the general public opinion. This is prevalent in voting polls during election campaigns. Joslyn (1997) looks at the effect that public opinion has on individual opinion. Through survey instruments, findings support the notion that public opinion has a large influence on an individual's opinion, but the magnitude, direction, and significance depends on the predisposition of the individual. Of more interest is the context effects in representing public opinion can alter opinions through concepts of assimilation, resistance, or synergy. Assimilation is the scenario in which the individual opinion conforms to the external information source and takes on two variants: adaptation and acceptance. Resistance stems from the predisposition of the individual and occurs when the external information conflicts with internal beliefs or personal values. Individuals respond in this scenario with a reaction or backlash where the individual opinion becomes even more negative, or they ignore the information altogether. Finally, synergy occurs when external information complements or resonates internal values. Through a reinforcement effect, individuals' opinions deepen and move towards the held predisposition (Joslyn, 1997).

Following Rogers' proposal that mass media was a vehicle for information collection, Krishnan and Smith (1998) investigated the role information source and time has on attitudes and confidence. Two variables that may cause a decline in attitudes towards a specific brand are the source of information, specifically advertising or trial period, and the time of measurement. Findings from the study show that the confidence of an individual's attitude decreases over time for individuals exposed to advertising, that attitudes toward a brand decreases over time for individuals exposed to trial periods, that a decrease in attitude confidence leads to an increase in inconsistencies in behavior based on the attitude, and that reactivating attitudes will prevent declines in both attitudes and attitude confidence (Krishnan and Smith, 1998). These findings seem to imply that individuals become impervious to advertising over time.

Based on such notions, Leskovec et al. (2006) investigated the dynamics of viral marketing. Viral marketing utilizes existing social networks by encouraging consumers to share product information with their friends. Using an online retailer's incentivised viral marketing program, they were able to directly measure and model the effectiveness of recommendations. A simple stochastic model attempted to explain the propagation of recommendations and information cascade sizes. This research also investigated how user behavior varies within online communities that is defined by a recommendation network, established how the recommendation network grows over time, and determined how effective the network is from the perspective of the sender and receiver of the recommendations (Leskovec et al., 2006).

Findings show that viral marketing contradicts epidemic models in that the probability of being infected (by a recommendation) decreases with repeated interaction. The probability of purchasing a product increases with the first few recommendations received, but after a threshold, the probability drops to a constant and relatively low level, supporting the idea that individuals become impervious to recommendations of their friends and resist buying items they do not need or want. Once past a certain threshold of the number of recommendations, the success per recommendation declines. Characteristics of product reviews and the effectiveness of the recommendations were shown to vary by category and price, and it was shown the number of successful recommendations varies by product. Finally, model results show that smaller and more tightly knit groups tend to be more conducive to viral marketing (Leskovec et al., 2006).

#### 2.2.1.3 Learning and Experience Mechanisms

Evolving opinions due to experience or observation has long been of interest to psychologists, who model these learning effects through Bayesian updating rules. In transportation, learning mechanisms have generally followed Bayesian statistics. Chen and Mahmassani (2004) investigate models of travel time perception and learning mechanisms. In this study, perceived travel time and its variance were updated using a Bayesian updating mechanism, and an increase in variability of perceived travel time decreased the individual's confidence. Agent-based microsimulation was used to study the collective effects on day-to-day flows and how varying certain parameters affected the dynamics of travel time perception and learning. Simulation results showed that individuals' overall travel time perceptions strongly influenced the convergence of the traffic system. Holding constant all other factors, as the inter-update period increases, the number of days until convergence initially decreases, but then increases. However, the number of updates until convergence decreases. Traffic systems with individuals who update every travel time experience will converge slower than a system with selective individuals.

Finally, results show that a premature end to the learning process produces larger variances in perceived travel times (Chen and Mahmassani, 2004).

However, research has shown that individuals do not necessarily update probability estimates according to Bayes' rule due to conservatism or overconfidence (see for example,). Chen and Mahmassani (2006) address this by investigating reinforcement learning and belief learning and comparing results to Bayesian learning. Under reinforcement learning, previous payoffs "reinforce" the selection of alternatives once the alternative is selected (for complete discussion, see Erev et al., 1999). Belief learning assumes that individuals create and update beliefs about the choices of other individuals, and that choices are made based on these beliefs (for complete discussion, see Crawford, 1995). Simulated results show that for reinforcement learning mechanisms, convergence is difficult to achieve compared to Bayesian learning since there are no assumptions that individuals perceive less dispersion in travel times across the population as time progresses and more experiences gained. While reinforcement learning explains the integration of travel times experienced, its limitation is that it does not explain how uncertainty changes over time. Belief learning also does not explain how individuals update uncertainty, because it considers the experiences of all users, the system tends to converge faster compared to reinforcement learning, but still slower than Bayesian learning (Chen and Mahmassani, 2006).

Brenner (1997) investigated learning and experience mechanisms in modeling interactions between decision makers when exchanging information. The use of a variation-imitation-decision (VID) model draws from cognitive psychology concepts of dissonance, satisficing, and learning. It assumes individuals learn from their experiences, are motivated to change behavior if they are unsatisfied with the current situation, and are able to imitate successful strategies of others (Brenner, 1997). In the simulated experiments, the population of individuals search for satisfactory strategies without any initial information. Results show that it is important for an exchange of information to occur to find a satisfactory solution quickly. It supports previous empirical findings that the rate of learning increases, then converges with the number of individuals involved. Additionally, the findings suggest that groups evolve and stabilize once they determine a satisfactory strategy. This may demonstrate why two different innovations with no strong externalities may establish clientele within a population, or why in different social groups, there may be different solutions that evolve in each group (Brenner, 1997).

### 2.2.2 Mathematical Developments in Modeling Opinions

While laboratory experiments and empirical testing provided a good foundation for the issues surrounding opinion formation and propagation, it did not provide quantifiable evidence in recognizing patterns and interrelations between individuals. In furthering the understanding of the phenomenon of opinion formation and propagation, mathematical models were formulated in which an individual holds a specific opinion and decides whether to change opinion or keep the same one. Each field has used different techniques – economics with information cascades, psychology with Bayesian updating mechanisms for opinion revision, biology with evolutionary models, and applied physics with probabilistic or deterministic network structures, as well as kinetic theories and continuum models. Within the applied physics models of opinion formation and propagation, the research has utilized agent-based simulation to investigate the micro-level interactions between individuals. Many studies have used different forms of agent-based simulation including network graphs, cellular automata, opinion space, Brownian motion, and molecular dynamics.

Schweitzer (2006) provides a review of collective decisions in multi-agent systems. The review discusses how the classical approach of a rational agent that maximizes utility does not apply when there is incomplete information or ambiguous solutions or consequences of the outcome. Social elements such as imitation strategies, information contagion, and herding behavior need to be account for in considering internal beliefs or opinions. Suggested guidelines for a baseline model include having a decision between discrete alternatives, considering local neighborhoods modeled by a network or grid, calculating a utility that reflects maximizing consensus with neighborhood (i.e., adopt opinion of local majority), and considering scenarios where there is consensus versus coexistence of opinions. An advance model would consider social relationships, a continuous spectrum of opinions, and a more complex utility function (Schweitzer, 2006).

Several studies of opinion formation have utilized a network-graph representation for agent-based simulation models. Network graphs have nodes representing individuals and arcs connecting nodes denoting there is a relationship between the two individuals. Boudourides (2003) presents a review of social and political network theory with collective action and with voting choices. The work develops a model of public opinion formation that utilizes an adaptive culture model developed by Axelrod (1997) with rules of adaptation based on similarities of traits. Gil and Zanette (2006) developed a stochastic co-evolutionary process for opinion formation. A pair of connected nodes was chosen at random and is subject to three outcomes that govern these interactions. If the opinions of the two nodes were the same, nothing happened, but if they were different, either agent adopts the other agent's opinion with probability *p1*, with a complementary probability of both not changing opinions (1 – *p1*). At the same time, there is a probability *p2* that the link connecting the nodes is broken, reducing the connectivity of the network (and the number of connected pairs left to randomly sample). This continues until no further changes are possible (Gil and Zanette, 2006).

Bartolozzi et al. (2006) investigates stochastic opinion formation in scale-free networks in which the dynamics of opinion formation in a group of agents is modeled by a stochastic response of each agent to the opinion of its neighbors and a feedback loop of the average opinion of the population. Kuperman and Zanette (2002) examines stochastic resonance in a model of opinion formation on small-world networks that is subject to noise effects and external modulation which is interpreted as opinion formation by imitation under the effects of a fashion wave. Gaston and desJardine (2005) look at the performance aspect of multi-agent social systems.

Gonzalez et al. (2004) model opinion formation on a deterministic pseudo-fractal network that utilizes the Sznajd rule, which states that only a group of people sharing the same opinion can convince their neighbors. Analytical results for the Sznajd model are presented in Slavina and Lavicka (2003). Sousa and Sanchez (2006) work on the outward-inward information flux in an opinion formation model of different network topologies explores the differences in network topologies specified by the Sznajd model and the Galam majority rule model. Deffuant (2006) considers differing network topologies and uncertainty effects in modeling how extremist opinions propagate through a population. Tessone and Toral (2004) incorporate mechanisms of imitation, influences of fashion, and randomness into a model of opinion formation and found that in the absence of fashion, the model behaves as bi-stable system having random jumps between the two opinion states with a distribution of times following Kramer's law.

A number of studies utilized cellular automata (CA) to model opinion formation and propagation. CA consists of an infinite number of cells, each with a finite number of states, and each cell can be in one state for a given time step. Updating rules are a function of a cell's neighborhood states in the previous time step. Alves et al. (2002) used a CA model to examine the influence of electoral surveys on voting processes. In the model, each voter updates his vote intention using political strategies which depend on political ideology and a social impact parameter (Alves et al., 2002). Boccara and Fuks (2005) use a one-dimensional CA model to model the diffusion of an innovation using a number of deterministic rules and find that the number of individuals who keep adopting the innovation depends strongly on the connectivity between the individuals. Bagnoli et al. (2002) use a probabilistic CA model with two absorbing states to explore social pressure effects on opinion formation as well as the phase transitions between states. Tessone et al. (2004) investigates how minority opinions spread in neighborhoods in a model that accounts for local spatial effects.

Kacperski and Holyst (1999) used CA and a theory of social impact developed by Latane (1981) to formulate a model of opinion formation with a strong leader and external impact. In this system there are two states: one where there is a cluster around a leader sharing the same opinion, and the other is where every agent has the same opinion. Varying deterministic parameters like the strength of the leaders and strength of the external impact can change the cluster size or, once a certain threshold is reached, the system jumps to another phase (Kacperski and Holyst, 1999). The work also considers effects of social temperature, modeled as noise effects. Bordogna and Albano (2006) investigated phase transitions and improved on Kacperski and Holyst's model of opinion formation in social groups. Results show that there are first-order transitions between two states of opinions when there is a strong leader and an external mass media effect that have opposing opinions (Bordogna and Albano, 2006).

Another technique used to model opinion formation and propagation is the construction of opinion space. In opinion space, agents are distributed across space with finite boundaries according to their opinion value. Distances between agents represent the difference in opinion. Laguna et al. (2003) utilize opinion space in examining a model of social influence in which an agent's opinion is represented by a binary vector. Agents adjust their opinion values as a result of random interactions and exchange opinions whenever the difference in opinion is below a threshold. Evolution of opinions to a steady state relies on the threshold and a convergence parameter of the model (Laguna et al., 2003). Lorenz (2006) explores consensus building in opinions through a model utilizing opinion space. Simulation results showed that for consensus of opinions, one needs to bring more interrelated issues into discussion and initiate large group meetings, and for coexistence of opinions, bring more issues that are independent into discussion and prevent large group meetings (Lorenz, 2006). Consensus is analogous to outcomes of the decision of a board of directors, while coexistence is analogous to gossip. Lorenz (2006) further explores the phenomena of consensus in looking at a model of continuous opinion dynamics under bounded confidence.

A major critique of these mathematical models is that they often fail to incorporate many of the social network and class structure aspects of real-world environments. One of the seminal papers on the role social networks has on opinions is by Wu and Huberman. Using a general network structure, they show that the expected weighted fraction of the population that holds an opinion is constant over time (Wu and Huberman, 2005). The concept of weighted fraction interprets as the fraction of individuals holding a given opinion averaged over their social connectivity

(Wu and Huberman, 2005). Findings show that a relatively small number of individuals with high social ranks (the number of social connections is denoted by the degree of a node on a network) have a larger impact on opinion formation than those with low ranks. This finding helps to explain the fragility phenomenon, in which an opinion held by a large group of people can become nearly extinct in a short time (Wu and Huberman, 2005).

A similar study on management fads, pedagogies, and other soft technologies by Bendor et al. (2006) looked at a model for the diffusion of management fads and technologies that lack clear objective evidence about their merits. Here, agents do not adopt through a Bayesian mechanism, but instead the choice to adopt depends on personal evaluation and the social influence of their peers. Simulated results show that choice dynamics lead to outcomes that appear deterministic even though the process is stochastic. When objective evidence about a technology is weak, evolution of the process rapidly settles to a fraction of adopters not predetermined, and when objective evidence is strong, the proportion of adopters depends on the quality of evidence and the competence of the adopters (Bendor et al., 2006).

## 2.3 Summary and Synthesis

This chapter examined a number of previous research on discrete choice models and the implications on travel behavior processes. In learning about the behavioral mechanisms behind decision making and choice, a two-stage choice paradigm was discussed, where individuals form a consideration set which is a subset of the universal choice set. Research on choice dynamics in marketing were discussed in an effort to understand the effect that marketing campaigns or marketing behavior such as brand loyalty have on consumer decision making. Additionally, research on choice process considerations was evaluated, as were works that investigated the effects of attribute value distortion.

This chapter has also investigated a number of works investigating the concepts of opinion leaders and seekers, social classes, and different mechanisms that govern opinion formation and propagation. Characteristics of opinion leaders were discussed. Three main social mechanisms were found in the literature: communication, mass media, and personal experience. Additionally, social class and social group effects were explored. The literature also implied that there were issues of the quality of information as well as the reputation and credibility of the information. These are all elements to consider in a model of social opinion dynamics.

Previous works revealed that a number of different techniques were utilized in models of opinions. Information cascades, Bayesian learning, evolutionary models, and agent-based simulation models were prevalent in different fields of study. A number of agent-based models used network graphs, CA, and opinion space. Other techniques available but not yet widely used in the literature are Brownian motions and molecular dynamics. Each method has its strengths and weaknesses in modeling opinion dynamics, and each captures a different aspect of social dynamics.

However, as mentioned previously, individuals do not update opinions through Bayes' theorem, and thus Bayesian learning makes unrealistic assumptions. Information cascades, which utilizes Bayesian learning, predicts that given a set of initial opinions, one opinion will eventually dominate, which is counterintuitive to reality where opinions are fostered in groups (Wu and Huberman, 2005). Evolutionary models and epidemic models do not account for differences in social connectivity, social class, and personality type, and that not every individual has the same probability for infection or evolution. Thus, this research will utilize agent-based simulation to model opinion formation and propagation. Modeling the dynamics will incorporate heuristics that reflect social class, social type, social mechanism, and social networks.

To extend the modeling of opinion dynamics to a transportation choice context, this research will incorporate an opinion-choice dynamics framework that has as components the opinion formation and propagation model as well as a choice mechanism. The specification of the choice mechanism will draw on the research investigating consideration set phenomenon, as well as research that explores how the choice process evolves over time. Chapter 3 will detailed the development of the opinion formation and propagation model, while Chapter 6 will explain the extension of the opinion formation and propagation model to the opinion-choice dynamics framework.

# Chapter 3: Modeling Framework

*Theoretical Development, Framework, and Methodology*

From the literature, the dynamics of opinion formation and propagation stem from variations in individual characteristics (e.g., social class, social types), social credibility (e.g., trust, quality of information, confidence), as well as in social mechanisms (e.g., word-of-mouth, mass media, belief learning, and direct experience). These parameters, however, are loosely defined in scope and broadly interpreted by the literature. Thus, this chapter sets the foundation for a modeling framework for the formation and propagation of opinions. Preliminaries (i.e., definition and scope of parameters) are first specified, followed by the hypothesized social mechanism that most likely influences opinion revision and adoption for social class and social type parameters. Next, the research develops a conceptual framework consisting of social mechanism components and an opinion revision model. Finally, a theoretical exploration on the interaction model component and opinion revision model examines different modeling equations.

## 3.1 Preliminaries

A review of the literature on opinion formation and propagation gives an overview of elements that govern interactions. Each study, however, includes only a partial number of these elements in the model. This research intends to build on previous work to incorporate all identified social elements into the model of opinion formation and propagation. First, however, groundwork is needed to define these elements to fit in the context of this model.

### 3.1.1 Social Mechanisms

Consider a freshman in college who recently arrived in a town far from familiar surroundings and has until this point been oblivious to the community and campus's opinions. Suppose this individual does not own any mode of transportation (e.g., car, bike) and is faced with deciding how he will get from place to place during his tenure in this community. Further suppose that this individual is not impulsive and instead of purchasing something outright, seeks other opinions and information before deciding to purchase. What are the different ways this individual can exchange opinions? Perhaps he asks older students (e.g., residential assistant) their opinion about the best mode of transportation. Perhaps during orientation week, marketing teams are advertising their products as the best mode of transportation around campus. Or perhaps the individual carefully surveys the campus, the parking lots, the shuttle services, public transport, the accessibility to necessities like groceries and shopping malls before developing an opinion about the best mode of transportation. This research attempts to model these vehicles or mechanisms of interacting opinions.

Social mechanisms of communications, external impact, and belief and hedonic learning found in the literature are redefined as a word of mouth mechanism, mass media mechanism, and personal experience learning mechanisms. The word-of-mouth mechanism is a subset of communication mechanisms and investigates the dynamics between individuals who physically interact with each other and communicate through dialogue each personal opinion. A scenario explaining this phenomenon would be if an individual were walking down the street, bumped into a friend, and began to discuss a topic for which both have an opinion on, such as the political elections. An essential element of the word-of-mouth mechanism is that information is passed from an individual to a friend or to someone in an individual's social network. It is rare that an individual will communicate with a complete stranger. Thus, this mechanism implies a certain threshold of selectivity. An extension of the word-of-mouth mechanism is communication via the telephone, chat rooms, or e-mail. However, conversations over the phone do not explicitly detail body language and facial expressions (one could presumably guess the expressions by tone of voice, however, this is perceived), and e-mail has little intonation or expression. While chat rooms with video and audio capabilities do enable individuals to see and hear expressions, it is not clear they have the same effect as if meeting an individual in person. Certain characteristics such as imitation or persuasiveness may not have the same impact over video and audio feeds as they would in face-to-face conversations. For this research, only physical interpersonal interactions are considered to account for the full effects of social class, social types, and social credibility.

Mass media mechanisms are a specific class of external impact mechanisms, which targets certain groups in populations and subject the group members to information flow. Unlike other external impact effects like social temperature (see for example, Kacperski and Holyst, 1997), social pressure, and environmental characteristics that are constant with time, mass media mechanisms are time varying and population varying. Mass media exposure is periodic, and individuals have the ability to choose what media they give their attention. It also inflicts different social groups in different ways to target individuals in those groups. An example of this is by Geico Corporation, who has nearly a dozen different television commercials concepts advertising the same auto-insurance coverage in an attempt to reach out to all social demographics. For some segments of the population, certain commercials are regarded as inane and have either no bearing on an individual's opinion or a negative effect on the opinion. However, other segments of the population may regard those commercials as captivating and attention grabbing, influencing a favorable shift in opinion. In addition to television commercials, other traditional forms of mass media include radio, newspapers, and magazines. As individuals have become impervious to these traditional forms of media, companies have turned to online viral marketing. E-mail is also considered a form of mass media with the creation of large distribution lists for services. This research will focus on a general form of mass media with attention to the effectiveness, reliability, and credibility of each individual form.

In learning processes, individuals are assumed to work towards obtaining satisfaction through collective observation and direct experience. In collective observation, an individual observes trends, fashion fads, and other individuals' behavior in an attempt to form an opinion about a product or service. Here, the individual may also perceive a social temperature or a social pressure to conform to the majority of the entire population or just a subset of the population. With the advent of the internet, individuals have greater access to information on what other people are thinking about a particular topic, which may increase the rate of imitation. As demonstrated in Bendor (2006) and Wu and Huberman (2005), the presence of instant gratification information conduits tend to make collective observation impacts ephemeral and dynamic, especially if the opinion lacks a clear objective or merit. Intuitively, collective observation is most persuasive when the majority of the population has adopted a different opinion than a single individual's opinion. In direct experience, an individual forms an opinion through experimentation and trial and error. Often, an individual who is very unsatisfied with their current alternative (and thus have a low corresponding opinion towards that alternative) has a high propensity to try something else (Brenner, 1997). However, there may be situations in which an individual may choose to, out of curiosity, try a new product or alternative. The trial experience, good or bad, influences the opinion towards the product, and subsequent trials may reinforce or weaken the individual's opinion.

Collective observation, or belief learning, and direct experience have implications for other social mechanisms, as well as implications for risk personalities. Belief learning and direct experience may draw from word-of-mouth conversations or mass-media outlets. For example, two individuals or a group of individuals may engage in gossip, which is a form of word-of-mouth communication, but in which the information discussed is perceived, hypothetical, or exaggerated. Study statistics transmitted via mass media that are not sound research (e.g., propaganda) may lead to false pretenses. Likewise another individual's direct experience with a product may be explicitly discussed in word-of-mouth conversations (e.g., it took 45 minutes to travel by train this morning, and there was a delay of 3 minutes at the transfer station). Concerning risk characterizations, both the belief learning and direct experience components are mechanisms to gain additional information on the product or service, however risk-seeking individuals are more inclined to try something impulsively while risk-averse individuals tend to collect as much information as possible, perhaps from collective observation, before adopting an opinion. While both belief learning and direct experience will be incorporated into the model, these elements will be modeled as distinct, mutually exclusive mechanisms.

### 3.1.2 Social Parameters

What makes an individual highly regarded or influential to someone else? Often times, influence is correlated with an individual's social status. People tend to respect the opinion of those who are more wealthy and powerful and have experienced many different facets of society due to their resources. For example, when talk-show host Oprah Winfrey recommends or endorses a book during her book club segment, most people would take that recommendation seriously. One can postulate that this effectiveness is due to her celebrity status and fan base. However, most individuals who watch the show know that Oprah is one of the wealthiest women in the world and that she has the resources to be an authority on certain products. Is it simply because Oprah is wealthy that millions of Americans regard her opinion highly, or is it her glamorous personality and charm that sells people her opinion? This research considers both aspects as factors for opinion formation and propagation by investigating two components of social status in this research: social class and social type.

Social class refers to different groups of individuals generally related by an index of wealth. In this model, social class is not just an indication of how much assets and income an individual has, but also on the amount of resources available to that individual. Resources in this research generally focus on the number of friends in an individual's network. Though an individual may not have much in terms of assets and income, he may have in his social network friends from whom he can draw resources. Certainly one would consider the President of the United States in a higher social class than Oprah Winfrey or Bill Gates even though the latter two have a much higher income simply because the President has ample resources and power. This, however, does not mean that the more friends one has, the higher one is in social class. Additionally, the concept of prestige is highly correlated with social status and is considered in social class component of this model. Occupational prestige may have an effect on social class, more so for men than do for women (Richardson and Mahoney, 2004). The highest degree earned in education also factors into social class, as individuals with doctorates or medical degrees are well regarded in society. Ethnicity, gender, and religion also play a role in social class, albeit less significant than occupation and education.

Social type describes an individual's personality and determines whether he is characterized as an opinion leader or an opinion follower or seeker. Within the opinion leader category, there are connectors, mavens, and salesmen with the same traits defined in Gladwell (2002). There are two distinguishing traits of an opinion leader. First, though opinion leaders are present in each social class, these individuals tend to be outgoing and are insensitive to class restrictions. Opinion leaders easily connect with individuals from different classes. Second, opinion leaders are altruistic, and actively try to connect people, inform people, or persuade people. Just knowing many people does not make one a connector. To be a connector, an individual has in addition to a large social network the inclination to connect people within the network or to maintain contact with everyone in the network. A maven is more than just an individual with a plethora of knowledge on a subject; he is passionate about the subject, is an authority on the subject, and actively elicits information from others to add to his collection of knowledge. Salesmen are not individuals who manage to successfully sell their products; rather they are able to sell you their products because their personalities are so likeable and contagious, that others will emulate these opinion leaders. Connectors, mavens, and salesmen are characteristics or personalities of an opinion leader and are not necessarily mutually exclusive. For example, Oprah Winfrey would most likely be classified as a connector and a salesman since she has many people in her social network and actively seeks new connections and has a personality that is contagious and emulative.

There are also opinion followers to consider in this model of opinion formation and propagation. Two types exist in the literature: opinion seekers and isolationists. Opinion seekers actively look to opinion leaders to form their own opinions about an innovation, trend, or fashion fad. They lack the personality traits and the social network resources of an opinion leader to be on the forefront of innovative development. Additionally, these individuals will seek each other out to exchange information. A hypothesis for the seeker's motivation is that they try to collect as much information to form a strong opinion in an attempt to become a pseudo-opinion leader (by virtue of appearing to be a maven on the subject) (Rogers, 2005). These individuals are open to new ideas and concepts and willingly exchange information to form their opinions. Isolationists on the other hand, tend to be narrow-minded and skeptical of innovations, preferring the status quo to new ideas and concepts. These individuals do not freely exchange information with others; rather, the opinion seekers and leaders are the ones who persuade these individuals to change their opinion. Since they prefer the status quo, they have very strong opinions towards current conditions and are resistant to change and acceptance. It may also be that these individuals are almost impervious to others' opinions and form opinions only by their experiences.

While Oprah Winfrey's social status certainly contributes to the effectiveness of her opinion propagating through an audience, another component helps to maintain her recommendation success and longevity as the "queen" of talk shows. The number of people who take Oprah's opinion seriously is also a function of her credibility and credentials in the product or service. If Oprah recommends a book, and her fans find out that it was not very enjoyable, they may not regard Oprah's opinion as seriously the next time she recommends a product. Likewise, if Oprah does not have sound reasoning to support her book of the month, her recommendation effectiveness may decrease. These factors constitute a social credibility parameter that this research intends to investigate. Within social credibility are three factors: the quality of knowledge, trust, and confidence. Quality of knowledge is the level to which an argument is supported by evidence or logical reasoning. In gossip, the knowledge is based on hearsay and rumors; thus, the knowledge is of low quality and many people do not take this information seriously. While the information may be exciting initially, it has little bearing on forming long-term opinions (see for example, Lorenz, 2006; Bendor, 2006). High quality (or certain) knowledge may be more persuasive and believable, and may be more effective at forming and revising opinions. Likewise, when an information source is trustworthy, an individual is more likely to form or revise an opinion.

Confidence has two components, confidence in society and confidence in self, which have implications on learning mechanisms. The more confident one is of his opinion, the less likely he is to revise his opinion during future interactions. As confidence in society (a proxy would be the competence of or faith in society) increases, an individual is more likely to conform or adopt the opinion of society. If self-confidence is low and social confidence is low, one's opinion may oscillate over time. Alternatively, if both self-confidence and social confidence are high, one's opinion will depend on his social personality (e.g., opinion seeker conforms to societal opinion).

## 3.2 The Role of Mechanisms

Having set the definitions of the social mechanisms and social parameters, it is time to address how these mechanisms affect different individuals varying by social class, social type, factoring in social credibility as well. For this model, the research hypothesizes that the role of mechanisms vary primarily by social types and then by social class. There are issues of social credibility for each mechanism, since if the quality of information is not high, it will have the same effect on opinions regardless of social class or social type. Social class affects the role of mechanisms in that the higher one is in class, the more resources he is exposed to for gathering information and for opinion interaction. Social type has the greatest impact since personalities have implications on adoption of an innovation (Rogers, 2003). Figure 3-1 shows the hypothesized relationships between the individual personality type and the primary mechanism that will successful change his opinion.

In general, most personality types are influenced by word of mouth, with the exception of the maven. This is because the maven is what Rogers (2003) considers the innovative group, or the first 2.5% to adopt an innovation or opinion. As they are the first to change their opinion, there is little chance for mavens to change their opinion because someone else had communicated via word of mouth their opinion. Thus, mavens are more likely to be influenced by mass media, belief learning, or personal experience. Since mavens have such high interest in their product class or service, they are more willing to pay attention to mass media campaigns for a product class than the majority of the population. Likewise, they will actively search the news media for any information on innovations or modifications made in that product class. Alternatively, the mavens are themselves the innovators of a new product or service and are convinced through personal experience to change their opinion.

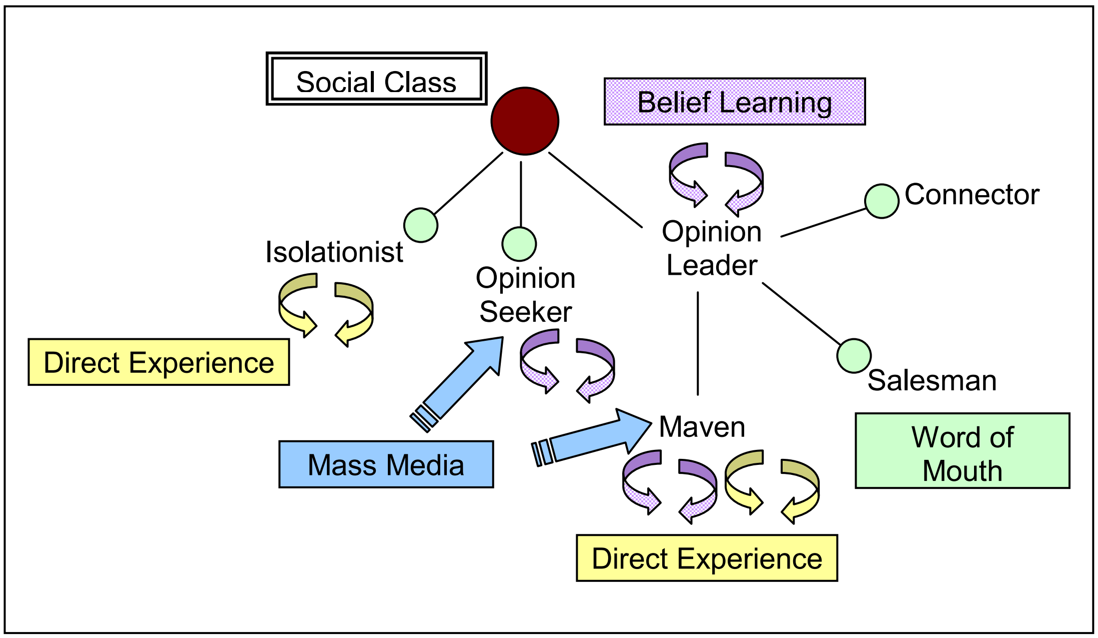


**Figure 3-1. Primary Mechanism Affecting Different Individual Type Opinion**

Connectors and salesmen are primarily influenced through word of mouth mechanisms since it is probable that within their social network, these opinion leaders know a maven. These individuals are what Rogers (2003) term the early adopters, or the next 13.5% of the population to adopt an opinion. Further, it is hypothesized that since these individuals are "trend setters", they are not under the influences of mass media, or may in fact be unreceptive to mass media since that mechanism, in addition to advertising innovations, may also reflect the opinion of the population or current trends in the population.

Opinion seekers are most likely to be influenced by word of mouth and mass media. Since they seek information about a product or service to form their opinion, they will look to opinion leaders and the mass media as their information sources. It is possible that since these individuals seek opinions, they rarely consider their own personal experiences or hedonic learning processes. Direct experience on the other hand, will primarily influence isolationists. Word of mouth and belief learning most likely play a role in opinion formation for isolationists, but only after much time has passed and a majority of the population has already adopted the opinion. Isolationists are synonymous with the term laggards (Rogers, 2003) and are the last 16% of the population to adopt an opinion. These individuals are stubborn and prefer the status quo, but once social pressure reaches a certain point, isolationists may observe the effect the opinion has on the population through internet blogs, recommendation web sites, and other media by which one does not physically interact nor does one receive the information. An example is through an internet web search, where unlike television or mass media outlets, the individual initiates the information search. Thus, this research distinguishes between mass media, which is an inward information flow (receiving or exposure), and belief learning, which is an outward information flow (pursuit or research).

## 3.3 Conceptual Framework

With knowledge of social mechanisms and social parameters governing opinion formation, and how these mechanisms play a role in the development and revision of opinions, this research develops a conceptual framework of opinion formation and propagation. Within this framework are three components, the interaction model, the opinion revision model, and the consideration model. This section explores mathematical functions and representations of the social mechanisms and opinion revision processes in the ideal scenario. Some inspiration for these representations is drawn from traffic flow theory, which also has concepts of leader-follower situations, acceptance of gaps, social pressure, and learning.

### 3.3.1 Lessons from Traffic Flow Theory

Macroscopic and microscopic theories of traffic flow provide a mathematical representation of the physical properties of the real-world traffic state, much like fluid dynamics, thermodynamics, and kinematics equations attempt to model hydraulics, heat transfer, and bodies in motion. What is particularly attractive about traffic flow theory is that it incorporates human behavior into mathematical equations, whereas the other theories do not explicitly. Further, traffic flow theory has a mathematically derived relationship between the microscopic and macroscopic levels. In modeling complex systems, Schweitzer (2006) asks how the properties of the elements and their interactions on a microscopic level related to the dynamics and properties of the whole system at the macroscopic level. Much of the research thus far has made the claim that the collective decisions are the result of aggregating individual decisions

(Schweitzer, 2006). Traffic flow theory has equations that represent behavioral interactions at the microscopic level, as well as a fundamental identity that represents the state of the aggregate system at the macroscopic level. As an exploration, this research attempts to redefine the state variables of macroscopic traffic flow to fit the context of opinion formation and propagation. This creates a basis for relating system-level states (i.e., general opinion trends in the population) to microscopic mechanisms that model the behavior of individuals as they form and transmit opinions. It also affords the use of microscopic mechanisms of gap acceptance and car following in modeling micro-level behavior.

Macroscopic traffic flow theory stems from the fundamental identity shown in equation 3.1

$$q = ku \tag{3.1}$$

where $q$ is the flow of vehicles per hour, $k$ is the density of vehicles per mile per lane, and $u$ is the velocity of the vehicle in miles per hour. In the context of aggregate-level opinion formation and propagation, $q$ would be the propagation of opinion, $k$ would be the density of opinion, and $u$ would be the speed at which the opinion travels. Density of opinion would be the number of individuals holding an opinion in a unit of opinion space. Speed of an opinion represents the propensity to travel from one individual to another. In this framework, $u$ is defined as a function of the opinion and the likelihood for an individual to adopt the opinion. One could interpret this parameter as openness to different opinions. If one assumes a logarithmic speed-density relationship of the density of opinions and the propensity to adopt, as given by the Greenberg (1959) model shown in equation 3.2, one can derive the microscopic car-following stimulus-response behavior developed by Gazis (1959) in equation 3.3.

$$u = \lambda k \ln \frac{k}{k_j} \quad \textbf{(3.2)}$$

where $u$ is the speed of opinions, $\lambda$ is a constant, $k$ is the density of opinion, and $k_j$ is the jam density which could reflect the fragility phenomenon shown in Wu and Huberman (2005) in which an opinion held by large group of individuals can become extinct in a short time.

$$\ddot{x}_B = \frac{\lambda_1}{[x_A(t) - x_B(t)]}[\dot{x}_A(t) - \dot{x}_B(t)] \quad \textbf{(3.3)}$$

where $\ddot{x}$ is the rate of change in opinion, $\lambda_1$ is a function of social status and social credibility, $\dot{x}$ is the speed of opinion, and $x$ is the position of the individual in opinion space. A logarithmic opinion density to adoption propensity curve is a reasonable assumption if one were to consider an individual with no prior information or opinion on the product and is captive towards his current product (i.e., there is a possibility that the individual is unsatisfied with the current product but has little or no other choice). Given this premise, an individual initially is open to a new opinion when there are few entities in his vicinity. As the frequency of interactions with entities having the same opinion increases, the individual will likely change opinions, and the propensity to adopt begins to decrease since the individual shares the same opinion.

Using this theoretical background, an aggregate model can investigate the wave propagation phenomenon of opinions in a population. One can visualize at the aggregate level how an opinion moves outward from initial information sources and see the retreat (or the fragility phenomenon) of opinions as a shockwave through the system. At the microscopic level, car following encapsulates concepts of leaders and followers, and the stimulus response equation is appropriate for the opinion revision model. The next three sections detail the components of the general conceptual framework for opinion formation and propagation model.

### 3.3.2 Interaction Model

This level of the framework investigates the interaction phenomenon that occurs when exchanging or conferring opinions. There are four mechanisms considered: word of mouth, mass media, belief learning, and direct experience. The interaction model governs the interface between two entities. This could be an individual communicating with another individual, or an individual exposed to mass media, or an individual with his belief of the opinion of the majority, or an individual with his personal experiences. For this model, it is assumed that individuals are subjected to one mechanism at any time (i.e., the effects of the mechanisms are mutually exclusive). This allows the analyst to capture the direct effect of mechanisms without having to specify correlations between interacting mechanisms. Group effects in the mechanisms (i.e., communication between more than two individuals for the word of mouth mechanism) could be incorporated with the addition of a group desirability parameter, but is beyond the scope of this research.

Word-of-mouth communication is modeled as two agents meeting and discussing their opinion on a particular product or topic. The model assumes that during the interaction, both individuals reveal their opinion (i.e., it is not implied). This mechanism only intends to capture physical, face-to-face interactions, as communication via e-mails, telephone conversations, and instant messaging applications do not explicitly convey an individual's social characteristics. Mass media is a periodic mechanism that targets certain groups of individuals at certain times with an opinion or information about an opinion. For a given time of exposure to mass media, there is a function of information flow. Here, the media element conveys an opinion, through some form of advertisement, to an individual. Direct experiences reflect trial and error and satisfaction with the status quo. This incorporates components of the satisfaction model used in Brenner (1997) where if individuals are unsatisfied with the status quo, they have a propensity to try a new product or service.

Common to all four components is an acceptance or connectivity parameter by which an individual judges whether to pay attention to or ignore incoming information on an opinion. Previous models have investigated systems where agents exchange information and opinions freely. However, humans are inherently selective in who they choose to interact with, what media they will pay attention to, what social group to emulate, and what experiences to recall. Thus, in this model, not every individual interacts with an entity that comes into contact. Each individual has a connectivity threshold that indicates how far from personal identity (e.g., social class, social type) he is willing to accept in order to engage in opinion exchange. If the difference between the individual and the entity is below the internal connectivity threshold, the individual will choose to engage in interaction with the intent of revising his opinion. If the difference is above the threshold, the individual will choose not to interact and stay with his current opinion. Additionally, as time progresses, the individual threshold will decrease as confidence in the opinion increases, decreasing the number of accepted interactions over time. This process intends to reflect the phenomenon that individuals tend to interact with similar individuals and that over time, individuals become less sensitive to differences in opinions (i.e., they become more confident in their own opinion).

### 3.3.3 Opinion Revision Model

Given that at least one individual (if there are two individuals interacting in the word of mouth mechanism) decides to revise an opinion, the individual choosing to revise his opinion will enter the opinion revision model which utilizes the non-linear stimulus-response mechanism for car following developed by Gazis (1959). To utilize opinion space, the model assumes that each individual in the system can be mapped to a position in space based on his opinion value. This model also postulates that the rate of change in opinion depends on how similar an individual is to the interacting entity. For individuals who interacted through word of mouth or through belief learning, the similarity function most likely depends on social class and social type. For those who interacted with mass media and direct experience, the similarity function would most likely depend on the difference in opinion (i.e., the marketing perspective or the opinion formed by direct observation). Knowing this, equation 3.3 is rewritten as:

$$\ddot{x}_B(t+T) = \frac{g(s_1, s_2, \ldots, s_n)}{[x_A(t) - x_B(t)]}[\dot{x}_A(t) - \dot{x}_B(t)] \quad \textbf{(3.4)}$$

where $g(s_1, s_2, \ldots, s_n)$ is the function of similarity that takes social parameters into account depending on the mechanism involved. To further evolve this into the context of opinion revision, suppose that acceleration, or the rate of change in speed, is synonymous with the rate of change in propensity to adopt, which for implementation purposes within this framework, is equivalent to the incremental change in opinion. Let the difference in opinion space also be a component of the general impedance function, λ, which either dampens or enhances the effect of the interaction. Thus the car-following equation is transformed into an opinion-following mechanism shown in equation 3.5.

$$\dot{O}p_B(t+T) = \lambda_{T,C,M,B,P}[Op_A(t) - Op_B(t)] \quad \textbf{(3.5)}$$

where $\dot{O}p_B(t+T)$ is the change in opinion for individual *B* for time $t+T$, $Op_A$ and $Op_B$ are the current opinion of individuals *A* and *B*, respectively, and $\lambda_{T,C,M,B,P}$ is an impedance function that is dependent on social type (T), social class (C), marketability (M), social behavior (B), and personality (P). To find the new opinion of individual *B*, utilize the velocity-equivalent transformation, shown in equation 3.6.

$$Op_B(t+T) = Op_B(t) + \dot{O}p_B(t+T) \quad \textbf{(3.6)}$$

where $Op_B(t+T)$ is the new opinion value and $Op_B(t)$ is the previous opinion value.

### 3.3.4 Consideration Model

In this framework, a post-interaction response occurs immediately following updates to the opinion speed and opinion position. Since opinion position reflects a continuous value for an opinion, this research considers values between positive, negative, or indifferent opinions to be intermediate values. Thus, an interaction may bring about a change in opinion value, but may not change it enough to form a confident opinion. Individuals can adopt a certain opinion once an internal threshold is exceeded. That is, once the individual is convinced of an opinion, he will adopt that opinion. In terms of choice, the adoption of an opinion implies that the individual considers the product or alternative seriously enough to warrant consideration in the choice set. Thus, this component of the opinion formation and propagation model is termed the consideration model.

The structure of the consideration model is much like a gap-acceptance model in microscopic traffic flow theory where individuals accept a gap that exceeds the critical threshold. In this situation, individuals choose to accept an opinion once the current opinion value exceeds an internal threshold. By using the gap-acceptance model, one can calculate a probability of accepting an opinion given the current opinion value and internal threshold value. Equation 3.7 shows the gap-acceptance model developed by Miller (1967) and interpreted in the context of opinion formation and propagation.

$$\begin{aligned}\Pr(Adopt) &= \Pr\left(t_i \geq \bar{t}_{cr} + \varepsilon_i\right) \\ &= \Pr\left(\varepsilon_i \geq t_i - \bar{t}_{cr}\right)\end{aligned} \tag{3.7}$$

where $t_i$ is the opinion value for individual $i$, $t_{cr}$ is the internal threshold value averaged over time, and $\varepsilon_i$ is an error term that has a distribution associated with it. The distribution assumption of the error term will yield different probability density functions; for this framework, the gap-acceptance equation is left in generic form.

As a synthesis of the framework, Figure 3-2 depicts the conceptual framework for opinion formation and propagation. An individual is initialized with an opinion value and "meets" another entity. Judging by intuition, the individual determines if the difference between social identities or media is greater than or less than their internal threshold. If the connectivity parameter is greater than the threshold, the individual moves on to the next entity. Should the connectivity parameter fall below the threshold value, the individual will choose to interact with the entity with the intent to revise his opinion. The rate of change in opinion will depend on a social parameter function as well as the distance between opinion positions and the opinion speeds. Once the new rate of change is determined, the opinion speed and opinion position are updated to provide a new opinion value. If this new opinion value is greater than an internal threshold, the individual will adopt a real, discrete opinion and will no longer seek a new opinion through word of mouth, mass media, or belief learning mechanisms. Only direct experiences, specifically dissatisfaction with the status quo, can convince an individual to seek new opinions. For an opinion value less than the internal threshold, the individual will continue to seek an opinion from other entities. Individuals keep a record of what entities they have encountered, and for those they interacted with, record their opinions. Over time, as the number of interactions increase, an individual tends to become more confident of his opinion, and thus his propensity to continue to enter to revision model decreases.

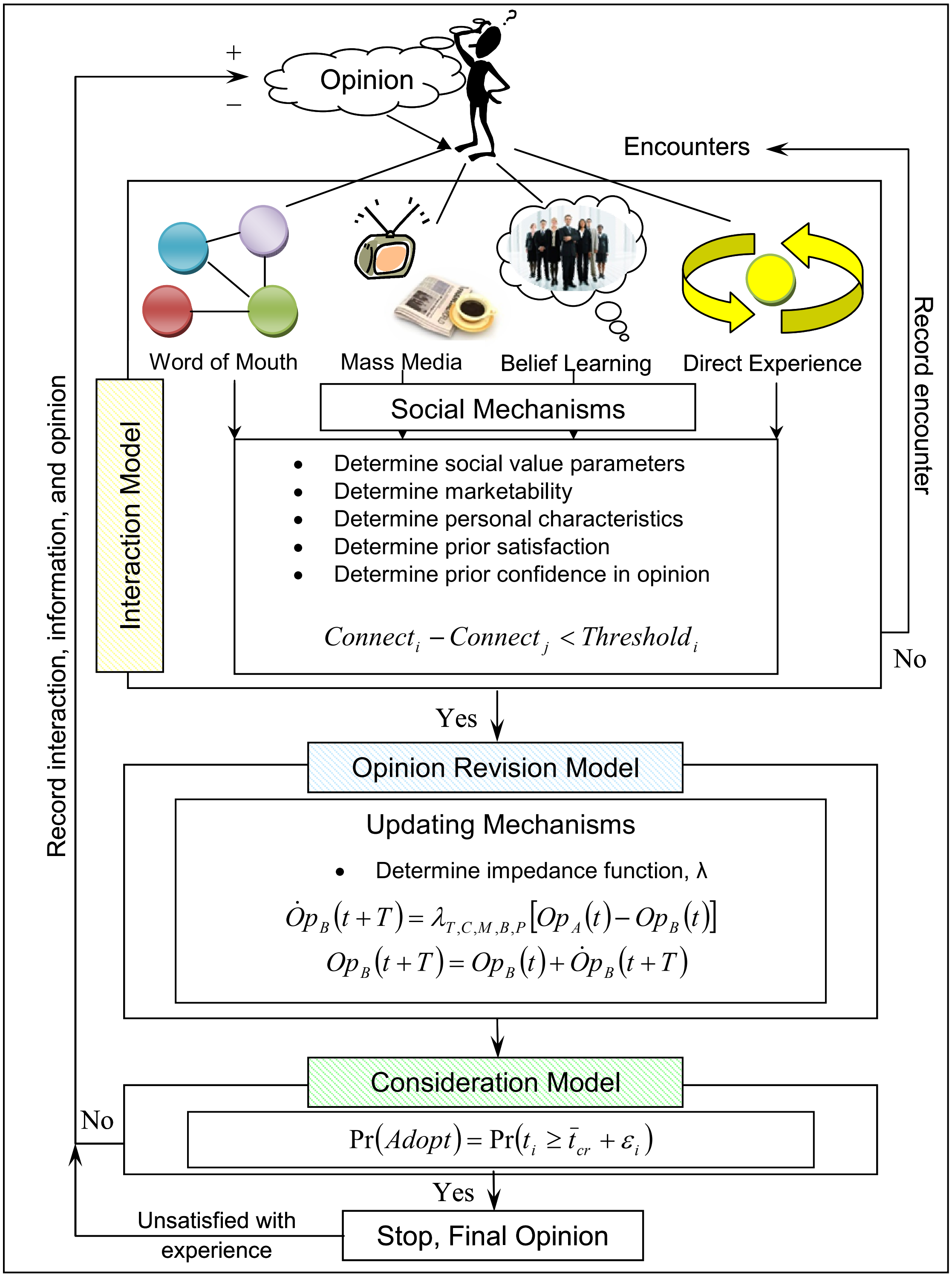


**Figure 3-2. Conceptual Framework of Opinion Formation and Propagation**

## 3.4 Model Implementation

To implement the conceptual framework of opinion formation and propagation, it is necessary to develop mathematical representations of the connectivity threshold, the impedance function, and the consideration threshold. Further, this section discusses specific factors that affect the connectivity of an individual to the interacting entity. For word-of-mouth, it may be social class and social type influences, but for mass-media, the connectivity criteria may be the product desirability and the exposure rate. The next four subsections discuss factors influencing the connectivity for each social mechanism, factors influencing the impedance function, as well as defining mathematical representations for interaction connectivity and opinion updating. This section concludes with a general specification of the consideration model.

### 3.4.1 Word-of-Mouth Mechanism

As mentioned previously, the interaction model represents an information process that resembles a screening process. An individual does not choose to communicate with everyone he meets; rather the individual is selective and bases his judgment on similarities in social class, social type, and perceived opinion. Literature has postulated that opinions tend to harbor in social groups with similar social status (see for example, Wu and Huberman, 2005, Bendor et al., 2006, Brenner, 1997). An opinion seeker does not always adopt an opinion of the individual with the highest social status; there are scenarios where he may take a relative's opinion more seriously than Oprah Winfrey's opinion. This model accounts for the similarities concept by incorporating the connectivity parameter and an interaction threshold parameter. The word-of-mouth connectivity measure of an individual $i$ is represented by equation 3.8.

$$Connect_i = \beta_1(SC_i) + \beta_2(ST_i) + \beta_3(OP_i) \qquad \textbf{(3.8)}$$

where $SC_i$ is the social class of individual $i$, $ST_i$ is the social type of individual $i$, and $OP_i$ is the opinion value of individual $i$. Equation 3.8 implies that the more similar two individuals are in connectivity, the higher chance of deciding to interact and revise opinions.

The interaction threshold incorporates a type-dependent component that follows the distribution of the diffusion of innovation theory for opinion seekers, and exaggerates the tolerance for opinion leaders. A class-dependent component models the tendency for individuals to look to those with the highest social class in pursuit of achieving the American dream. Part of Oprah's appeal is that she embodies the American dream and thus people are willing to tolerate disproportionate differences in social classes in hopes of improving their status. The lower the social class of an individual, the threshold parameter increases. Finally, a declining parameter represents an individual's confidence in their opinion. As confidence increases, the threshold to interact with other in pursuit of revising an opinion decreases. Equation 3.9 gives the equation for the interaction threshold.

$$TH_i = TT_i(C_{\max+1} - C_i)(1 - Conf_i) \qquad \textbf{(3.9)}$$

where $TH_i$ is the interaction threshold for individual $i$, $TT_i$ is the type-dependent tolerance, $C_{max+1}$ is the highest social class value plus one to represent admiration or emulation of those with elite status, $C_i$ is the social class of the individual, and $Conf_i$ is the confidence of the individual.

For this research, confidence is a measure of the sum of number of interactions over the number of encounters (collisions) multiplied by a memory-effect parameter. Formulating confidence in this manner in combination with the threshold parameter implies that the more interactions an individual has, the more likely he is to form a firm opinion. The memory-effect parameter accounts for some of the psychological phenomenon such as the availability heuristic (Tversky and Kahneman, 1974) where individuals tend to recall only recent events. In some instances, recollection is limited, in which case the record of interactions is shortened or reduced. Other times, the recollection may be exaggerated (e.g., an individual feels that he has interacted with hundreds of other individuals when in reality, the number of interactions was much fewer). The confidence parameter is shown in equation 3.10.

$$Conf_i = \left( \frac{\sum_{j=1}^{J} NumInteract}{J} \right) \times MET_i \qquad \textbf{(3.10)}$$

where *NumInteract* is the number of interactions, $J$ is the number of encounters (collisions), and $MET_i$ is the type-dependent memory effect of individual $i$.

Governing the decision of whether to engage in conversation or ignore the individual is the model equation. Currently, the formulation is deterministic;

however, it can be modified to reflect stochasticity and the probabilistic nature of behavior (e.g., preference reversals, intransitivity). Equation 3.11, where the difference in connectivity is below the interaction threshold, is the case where the individual will engage with intent to revise opinion. Equation 3.12, in which the difference in connectivity exceeds the interaction threshold, is the case where the individual will ignore the other individual and revert to the status quo (i.e., maintain the same opinion).

$$Connect_i - Connect_j < TH_i,\ i \neq j \tag{3.11}$$

$$Connect_i - Connect_j > TH_i,\ i \neq j \tag{3.12}$$

For the impedance function, $\lambda_{T,C,M,B,P}$, the word-of-mouth opinion updating simplifies the function to $\lambda_{T,C}$, as social class and social type are the main influences in how an individual updates his opinion. This is implemented as a combination of four trigger mechanisms with a binary variable: class-type similarity, opinion leader, opinion follower, and status quo.

#### 3.4.1.1 Class-Type Similarity

For this trigger mechanism to be invoked, the two interacting individuals must be either similar in social class, or similar in social type. This is expressed as a binary variable in equation 3.13.

$$\eta_i = \begin{cases} 1 & \text{if } |C_A - C_B| \leq 1 \text{ or if } T_A = T_B \\ 0 & \text{otherwise} \end{cases} \tag{3.13}$$

where $C_A$ and $C_B$ are the social class of individual $A$ and $B$ respectively, and $T_A$ and $T_B$ are the social type of individual $A$ and $B$. This mechanism implies that if two individuals are similar in social class or the same social type, they will simply average the difference in opinions between them. Both individuals have similar resources and influences, so they can convince each other half of each individual's opinion.

#### 3.4.1.2 Opinion Leader

If one of the individuals is an opinion leader (e.g., connector, maven, salesmen), the opinion-leader trigger mechanism is invoked. This is shown as a binary variable in equation 3.14.

$$\tau_i = \begin{cases} 1 & \text{if } T_A \text{ or } T_B \text{ is an opinion leader} \\ 0 & \text{otherwise} \end{cases} \qquad \textbf{(3.14)}$$

As opinion leaders seem to be altruistic, for this model, they are considered to be impervious to the social class of the interacting individual. In other words, the opinion exchanged between the opinion leader and the other individual does not depend on the other individual's social class. If the interacting individual is another opinion leader of a different type, however, the individual becomes aware of social class differences only. The leader trigger mechanism is interacted with a function denoting this revision process for individual $A$ in equation 3.15.

$$\tau_i \left( \frac{1}{\rho C_A} \right)^{\alpha Z}, \quad C_A \geq 1 \qquad \textbf{(3.15)}$$

where $C_A$ is the social class of the opinion leader $A$, $\rho$ is a user-specified constant, α is a user-specified parameter that is at least 1 if the interacting individual is an opinion follower, and less than 1 if the interacting individual is a different type of opinion leader, and $Z$ is another user-specified parameter that takes the value of greater than 1 if the interacting opinion leader is of a lower social class, 1 otherwise. For this model, the research considers $\rho = 2$, $\alpha = 2$ for opinion followers, 0.8 for connectors, 0.7 for mavens, and 0.6 for salesmen, and $Z = 2$ if interacting opinion leader has lower social class. With this specification, the best revision that can occur for the opinion leader is a simple averaging of the two opinions. The opinion revision depends on the opinion leader's social class, as the more resources he has in terms of social connections, the smaller the effect of a single opinion. Most of the time, the revision will be insignificant since opinion leaders have many social connections and a single individual has little effect on the opinion leader's opinion.

#### 3.4.1.3 Opinion Follower

To invoke this trigger mechanism, one of the two interacting individuals is an opinion follower (e.g., opinion seeker or isolationist). This is shown as a binary variable in equation 3.16.

$$\omega_i = \begin{cases} 1 & \text{if } T_A \text{ or } T_B \text{ is an opinion follower} \\ 0 & \text{otherwise} \end{cases} \qquad \textbf{(3.16)}$$

Opinion followers, particularly opinion seekers, tend to be sensitive to the social class and social type of the interacting individual. They also pay attention to similarities, in that if the difference in social class is minimal, they are more likely to take the opinion seriously. Seekers, however, sometimes weigh the interacting individual's opinion seriously if the latter is an opinion leader. Additionally, opinion followers are less inclined to revise an opinion if the interacting individual has a lower social class. Equation 3.17 displays the function that represents these behaviors.

$$\omega_i\left(\frac{\kappa}{\gamma\left|C_A - C_B\right|}\right)^{\beta} \tag{3.17}$$

where $\left|C_A - C_B\right|$ is the absolute value of the difference in social class between individual $A$ and $B$, $\kappa$ is a user-specified parameter that takes a value of greater than 1 if the interacting individual is an opinion leader and the individual is an opinion seeker, 1 otherwise, $\gamma$ is a user-specified constant, and $\beta$ is a user-specified parameter that takes a value of greater than 1 if the interacting individual has a lower social class, 1 otherwise. In this model, $\kappa$ takes a value of 2 if the interacting individual is an opinion leader, $\gamma$ is equal to 2, and $\beta$ is 2 if the interacting individual has a lower social class. Given this representation of a follower's opinion revision process, the best revision possible is a full adoption of the interacting individual's opinion. This occurs for an opinion seeker when the difference between social class is slightly more than one, which implies that the individuals are from distinct classes, and when the interacting individual is an opinion leader with a higher social class. For isolationists, the best revision possible is a simple averaging of opinions, which is the same as the class-type similarity mechanism.

#### 3.4.1.4 Status Quo

For certain scenarios, the research seeks to find the effect that some opinion leaders, once they have formed their opinion, are impervious to other individuals' opinions. Likewise, some isolationists may only prefer to revise their own opinion from the opinions of individuals like them. This status-quo mechanism allows for the possibility of interaction without taking anything away. The interacting individual may revise his opinion based on the opinion of the individual. Equation 3.18 shows this trigger mechanism as a binary variable.

$$\delta_i = \begin{cases} 1 & \text{if } T_A \text{ or } T_B \text{ is an opinion follower} \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(3.18)}$$

For this research and this mechanism, we consider mavens and salesmen to have firm opinions once they have formed. Mavens are the first 2.5% to adopt an opinion, and generally stay with that opinion since they are the experts on that product class. Salesmen, due to their charismatic personality, generally are hard to convince otherwise once they have formed an opinion. Once the trigger mechanism is invoked, the opinion revision equation reduces to zero.

From the four general trigger mechanisms, there are eight parameters to consider in the model. This yields 56 different combinations. Equation 3.19 presents the generalized opinion revision model with the mechanisms discussed in the last four sections. The equation takes the perspective of individual *A*.

$$O\dot{p}_A(t+T) = (1-\delta_A)\left[\frac{1}{2}\eta_A + \tau_A\left(\frac{1}{\rho C_A}\right)^{\alpha Z} + \omega_A\left(\frac{\kappa}{\gamma\left|C_A - C_B\right|}\right)^{\beta}\right](Op_B - Op_A) \quad \textbf{(3.19)}$$

### 3.4.2 Mass-Media Mechanism

With the internet, television, personal digital assistants (PDAs), and now even cell phones conveniently at their disposal, today's generation is constantly exposed to media advertising and marketing. Whereas earlier studies looked at the phenomenon of mass media through newspaper, radio, and television as a periodic source, perhaps today's media more closely represents a constant source through multi-media outlets. Though the source of marketing and advertising may be constant, individuals only pay attention to a small fraction of this information. This model of mass-media broadcasting is captured in this model by the assumption that media sources are constantly relaying information to individuals. Individuals may choose to "listen" to the information based on their need for a specific product and their exposure to the advertisement campaign. In this model, an individual's propensity to listen to mass-media is captured by how well the advertisement matches the individual's need and desirability for a product. The literature supports the notion that product differentiation and segmentation serve to disperse perceptions of brand utilities (Mitra, 1995) and offer more compelling information to switch products (Allenby and Ginter, 1995); thus, an individual's receptiveness to mass media depends also on his social class characteristics. On one hand, there is the mass media mechanism attempting to connect with individuals to advertise and sell a product (by convincing the individual to enter into the opinion revision model), while on the other hand, individuals are modeled as agents that give mass media attention only when there is a need for such a product, and even in such instances, will only want to pay attention (and thus revise his opinion) if there is a desire for that product.

An individual's mass-media connectivity (i.e., connectivity attributes from the perspective of the individual) is represented by equation 3.20.

$$Connect_m = Need_i + \frac{1}{\ln(1 + SC_i)} \qquad \textbf{(3.20)}$$

where $Need_i$ represents the individual's need for a specific product and $SC_i$ indicates the social class of individual $i$. Here, need for a product is represented stochastically; individuals are assumed to need some product 25% of the time. In epochs where an individual needs a product, it may be for a specific product, not just a product class. This is represented by adding a normally distributed product-class term with the product class designation as the mean. The second component of the individual mass-media connectivity represents a diminishing social class effect on the individual connectivity with media. Intuitively, given that everyone has the same needs for a specific product, those with more financial and social resources may be more likely to pay attention to mass-media campaigns since they are more capable of purchases.

From the media's perspective, connectivity depends on the product information advertised. Equation 3.21 represents mass media connectivity.

$$Media_p = Product + \varepsilon_{ip} \qquad \textbf{(3.21)}$$

where $Media_p$ is the media connectivity for product $p$, *Product* represents the product class advertised by the media, and $\varepsilon_{ip}$ characterizes randomness or white noise that may occur during the transmission from the media to the individual. The error could represent misconstrued or misinterpreted information on the behalf of the individual (i.e., the information might have been ambiguous). A normally distributed random variable with a mean of 0 and a variance of 0.1 is used for this model. In the experiments, *Product* is implemented as a product-class oriented variable for simplicity; however, it is possible to incorporate a number of specific products within a product class into the model.

Equation 3.21 intends to represent a one-message or one-product class marketing campaign in which the same media is transmitted to individuals repetitively. This is equivalent to the reminder ads Mitra (1995) discusses. As marketing has expanded to product differentiation (e.g., the different array of Geico insurance commercials discussed earlier in this chapter), this research also incorporates a media connectivity function that replicates this effort to appeal to different social classes. Equation 3.22 represents mass media connectivity with product differentiation.

$$Media_p = Product + \frac{1}{\ln(1 + MC_p)} + \varepsilon_{ip} \qquad \textbf{(3.22)}$$

The difference between equation 3.21 and equation 3.22 is the addition of a media class parameter, $MC_p$, that represents the different array of media classes a marketing campaign utilizes to advertise a product. The second term is constructed in the same manner as the individual media connectivity to allow diminishing sensitivities for higher social classes, but differs in that media classes are discrete (unlike social class which is continuous). This representation allows a marketing campaign the ability to systematically target specific social classes to focus the product advertising. For these experiments, if a marketing campaign decides to use product differentiation strategies, their product will be advertised using a random distribution of media classes.

The media interaction threshold incorporates concepts of product desirability, media exposure, and exposure proportionate to the sample size. Product desire stems from an individual's need for that product, given that the product is available and is being advertised. Individuals do not need something constantly; consequently, this research assumes individuals need one of the products 20% of the time. With need for a specific product being a continuous variable, and the product advertisements being discrete classes, desire is modeled as the confluence of "supply and demand" within a certain threshold. If an individual desires a product, the model assumes the individual will likely expose himself to media pertaining to that product. As the number of exposures increases, an individual is less likely to revise his opinion. Thus, the model assumes that individuals are cognizant of the number of times he is exposed to a product advertisement, given a desire for that product. These behaviors under the media connectivity threshold are captured in equation 3.23.

$$TH_{Media} = Desire_i \times \left(1 - \left(\xi + Expose_i\right)^{-1.1}\right) \times e^{-\frac{Expose_i}{\left(\phi \cdot NumUsers_p\right)}} \qquad \textbf{(3.23)}$$

where $TH_{Media}$ is the media connectivity threshold, $Desire_i$ is a binary variable that is 1 if the individual desires the product, 0 otherwise, $\xi$ is a user-specified constant that is greater than 1, $Expose_i$ is the number of times individual *i* has been exposed to the product prior to this encounter, $\phi$ is a user-specified constant that is greater than 1, and $NumUsers_p$ is the number of individuals within the target group of the product advertising. For these experiments, $\xi$ = 1.25, $\phi$ = 2, and the number of individuals within the target group is the number of agents. This threshold was constructed with the intention that initially, individuals would have a low threshold for product advertisements that they have never been exposed to. Once the advertisement becomes more familiar, individuals have the highest propensity to revise their opinion towards that product. However, if the number of exposure is too high, the individual begins to saturate on the product information and becomes less likely to revise an opinion. The user-specified constants in this equation allow the analyst to modify the threshold curve to match this process. Equation 3.24 shows the resulting media connectivity threshold criteria used to evaluate whether an individual will revise his opinion based on receiving information through product advertisements. If equation 3.24 is satisfied, the individual will enter the opinion revision model.

$$Media_p - Connect_m < TH_{Media} \tag{3.24}$$

Once in the opinion revision model, the individual will revise his opinion based on the impedance function. The framework assumes that the media advertises an opinion of 1 for that product to encourage consideration and choice. The impedance function could involve the concept of the stickiness factor discussed in Gladwell (2002) in which minute details of an advertisement may cause the message to resonate stronger with certain individuals. However, as it is difficult to quantify this in a manner that applies systematically to a population, for these experiments, the impedance function is assumed to be a constant, with a value of 0.5. This implies that there is a simple averaging of opinions once the connectivity threshold has been met. Equation 3.25 shows the opinion revision equation for mass media.

$$Op_{im}(t+T) = Op_i(t) + \lambda_m\left[Op_{Media}(t) - Op_i(t)\right] \tag{3.25}$$

where $Op_{im}(t+T)$ is the new opinion of individual $i$ due to media $m$ at time $t+T$, $Op_i(t)$ is the previous opinion towards that product, $\lambda_m$ = 0.5, and $Op_{Media}(t)$ is the advertisement opinion that usually takes a value of 1.

### 3.4.3 Belief Learning Mechanism

Under belief learning, individuals are improving their utility towards a product through the observation of the opinions of others. The essential problem to examine in these experiments is determining who the other individuals are. A general rule for these experiments is that the set of other individuals comprise a real, distinct subset of the population (whereas in reality, belief learning could occur under virtual conditions in which individuals observe other opinions through blogs or other information-based websites). This subset of the population could be determined randomly, or it could be class-based or friends-based. In belief learning, a representation of social temperature or social pressure needs to be accounted for to capture the behavior that, as an individual sees more people adopting an opinion, there is increasing pressure to change or mimic society. Thus, the belief learning connectivity threshold criteria is constructed in a manner that allows for a low threshold initially, and once a proportion of the population subset chooses a different alternative, the threshold grows relatively quickly until reaching some level of saturation in which the marginal increase in social pressure (i.e., the connectivity criteria threshold) due to the conversion of another person in the subset is negligible.

From the individual's perspective, connectivity is defined as how different the individual's opinion is from the opinions of the observed population subset. Thus the individual connectivity is simply the individual's current opinion towards the product or alternative. To model the connectivity of the perceive opinion of the subset, this framework takes the weighted average of opinions multiplied with a perception factor that could reflect information or product attribute distortion. To determine the subset, an individual is assumed to have a list of friends that have similar interests. This model uses social class as a proxy for individual with similar interests and resources. As an illustration, if the absolute difference between an individual A's social class and individual B's social class is less than some threshold, individual B is considered a friend of individual A. For these experiments, this threshold is a user-specified constant that takes a value of 0.5. Equation 3.26 shows the belief learning connectivity function.

$$Connect_{bl} = \frac{\sum_{1}^{S_n} Op_j}{S_n} \cdot \eta \qquad \textbf{(3.26)}$$

where $Op_j$ is the opinion of individual *j* in the subset not equal to individual *i*, $S_n$ is the number of individuals in the subset not including individual *i*, and $\eta$ is the perception or belief factor. In this model, the perception factor is defined as a uniform random variable with a range between 0.5 and 2.0.

To capture the effects of social temperature and social pressure, the belief learning threshold criteria was specified using a logit function. The logit function produces an S-shaped curve in which the "tail" ends change slowly, while near the mean the function rapidly increases. The axis of the function is defined as the difference of the number of individuals in the subset choosing a different alternative divided by the weighted number of other alternatives available and the number of individuals in the subset choosing the same alternative. This follows intuition as the more people choosing the same alternative as the individual of interest, the less likely that individual is to believe he should revise his opinion based on other individuals' opinions. This formulation also specifies a penalty for scenarios with many different alternatives or products in that an individual is sensitive to only intensity in choosing an alternative, not diversity. Thus, the implication is the more products of similar nature on the market, the less likely the individual is to revise the opinion even though the majority is not choosing the individual's alternative. The belief learning connectivity threshold is shown in equation 3.27.

$$TH_{bl} = \frac{1}{(1+\mu)\left[1+e^{-\left(\frac{N_D}{\theta \cdot N_a}-N_S\right)}\right]} \tag{3.27}$$

where $\mu$ is the absolute difference in the average social class of the subset and the individual's social class, $N_D$ is the number of individuals in the subset choosing a different alternative than individual $i$, $\theta$ is a weighing parameter that is based on how similar the alternatives are to each other, $N_a$ is the number of alternatives or products available, and $N_S$ is the number of individuals in the subset choosing the same alternative as individual $i$. For these experiments, since the products or alternatives are assumed to be adequately different, $\theta = 1$. Equation 3.28 presents the belief learning threshold criteria that, if satisfied, will induce opinion revision based on the belief learning mechanism.

$$Connect_{bl} - Op(t) < TH_{bl} \tag{3.28}$$

Once in the opinion revision model, the individual revises his opinion by adjusting it to the perceived opinion of others. While the impedance function could represent a weighing criteria that reflects influential individuals in both social class and social type, this model assumes simple averaging of the two opinion elements. This opinion revision process is shown in equation 3.29.

$$Op_i(t+T) = Op_i(t) + \lambda_{bl}\left[Op_{bl}(t) - Op_i(t)\right] \tag{3.29}$$

where $\lambda_{bl}$ = 0.5, $Op_{bl}$ is the perceived opinion of individuals in the subset, and $Op_i$ is individual *i*'s previous opinion value.

### 3.4.4 Direct Experience Mechanism

In many instances, individuals will use trial and error to decide whether they like or dislike a product. It is possible that driving mechanism behind this behavior is the individual's satisfaction with the current and past experiences with the product. The literature implies that if there is low satisfaction, an individual may be more likely to switch products or try a new alternative, while if the individual is highly satisfied with the current product or alternative he is more likely to continue choosing that product (see for example, Lapersonne et al., 1995; Brenner, 1997). Trial and error behavior may also be attributed to an individual's risk characteristics (see for example Chancelier et al., 2007) in that risk-seeking individuals are more inclined to try different products or alternatives than risk-averse individuals. The combination of these mechanisms imply that risk-seeking individuals are more likely to switch (or, in this case revise an opinion) alternatives for a given level of satisfaction than are risk-averse individuals.

To implement the direct experience mechanism, this research defines satisfaction as the deviation from an individual's expectation. If the difference between the expected value of an attribute (e.g., travel time, cost) and the experienced value is high, there is low satisfaction. If the difference between expectation and experience is low, there is high satisfaction. Reinforced satisfaction leads to product or brand loyalty, while reinforced dissatisfaction incurs a change in product choice. If an individual has tried a different alternative previously and that alternative appears better in hindsight, there is a prospective benefit in switching back to that alternative. Likewise, other new alternatives may appear better than the current alternative. To characterize an individual's expectation, this research looks at three different criteria: an individual's historical average with the alternative, the last observed experience, and the mean of last observed experience amongst users of that alternative. Equation 3.30 presents a generalized representation of the direct experience connectivity that illustrates the different criteria of expectation.

$$Connect_{d\exp} = Expect_{i,hlm}, \quad \textbf{(3.30)}$$

where,

$$Expect_{i,h} = \frac{\sum_1^t Observe_{in}}{N_{used,n}}, \quad \textbf{(3.30a)}$$

$$Expect_{i,l} = LastObs_{in}, \quad \textbf{(3.30b)}$$

$$Expect_{i,m} = \frac{\sum_1^t LastObs_n}{N_{j,n}} \quad \textbf{(3.30c)}$$

Here, $Observe_{in}$ is the observed experiences of alternative *n* by individual *i*, $N_{used,n}$ is the number of experiences with that alternative, $LastObs_{in}$ is the last observation of alternative *n* by individual *i*, and $N_{j,n}$ is the number of individuals not including *i* that have experienced alternative *n*. For the individual direct experience element, this model uses the individual's current experience. This implies that the opinion revision takes place post-experience, not during the actual experience as with the other mechanisms. To simply the model, this research assumes post-evaluation to be no different than live evaluation.

The direct experience threshold is specified as the interaction between an individual's risk characterization and his acceptable variance of the attributes. It is slightly different than the other three mechanism thresholds in that if the threshold criterion is met, it represents a positive experience and thus has a positive effect on the opinion. If the criterion is not met, and it is exceeded by a certain amount, it represents a negative experience, and thus has a negative effect on the opinion. Thus, for these experiments risk-averse individuals will have a higher threshold value as they will be less inclined to revise their opinion even if they have had an unsatisfactory experience. For a similar acceptable variance, risk-seekers will have a lower threshold value and are more likely to revise their opinion accordingly. These concepts are illustrated in equation 3.31.

$$TH_{de} = Risk_i \times \left(Expect_{i,h} + Var_i\right) \quad \textbf{(3.31)}$$

where $Risk_i$ is a uniform random variable that ranges from 0.0 to 0.74 for risk-seeking individuals, and from 0.75 to 1.5 for risk-averse individuals, $Expect_{i,h}$ is the expected value based on an individual's historical experiences, and $Var_i$ is a normally-distributed random variable. For these experiments, the mean is specified at 0, and the variance is 5.

Equation 3.32 presents the direct experience connectivity threshold, which if satisfied, means the individual will enter the opinion revision model and revise his opinion through the direct experience mechanism.

$$Connect_{d\exp} - Observe(t) < TH_{de} \qquad \textbf{(3.32)}$$

Once in the opinion revision model, the individual will revise his opinion based on the satisfaction with the experience. It is difficult to quantify an opinion from the direct experience, unlike the mass media mechanism in which the media broadcasts a specific opinion through an advertisement, or belief learning in which the individual perceives a collective opinion value. Thus, this research uses the proxy of satisfaction, or the deviation from expectation, to derive an opinion value by which an individual revises his opinion. To ensure that the opinion value is between -1 and 1, the satisfaction term is divided by the individual's acceptable variance, which includes the historical average of experiences. Equation 3.33 shows the opinion revision function for the direct experience mechanism.

$$Op_i(t+T) = Op_i(t) \pm \lambda_{de}\left[\frac{Connect_{d\exp} - Observe(t)}{Expect_{i,h} + Var_i} - Op_i(t)\right] \qquad \textbf{(3.33)}$$

where $\lambda_{de}$ is the impedance function that determines how influential the direct experience is on the individual's opinion. Note that the change in opinion depends on whether the experience was satisfactory (i.e., met the threshold criterion), or unsatisfactory (i.e., exceeded the threshold by a user-specified amount). For these experiments, the buffer amount is a uniform random variable that ranges from 0.05 to 0.2 that represents a buffer of 5% to 20% of the historical average. This threshold implementation implies that direct experience, depending on an individual's risk characteristic, has the potential to move the individual directly into the opinion revision model.

### 3.4.5 Consideration Mechanism

Following the opinion revision process, post interaction, an individual instinctively decides whether the opinion towards a product or alternative is positive enough to warrant placing it in his consideration set. For the opinion formation and propagation framework, this implies that if the opinion does exceed some opinion threshold, the individual will consider the opinion to be final and will not continue to revise it, unless he has an unsatisfactory experience (i.e., direct experience mechanism yields negative effect). To implement this, the model assumes each individual to have a latent threshold for each alternative, represented by a uniform random variable with a range between 0.5 and 1.0. If the opinion towards a specific alternative exceeds the threshold plus some random error, the individual places the alternative into his consideration set and no longer revises his opinion in subsequent interactions with the exception of direct experiences. Equation 3.34 shows the consideration mechanism criterion that if met, the individual's opinion remains constant.

$$Op_i > Opthres_{in} + \varepsilon_i \qquad \textbf{(3.34)}$$

where $Opthres_{in}$ is the opinion threshold for alternative *n* and individual *i*, and $\varepsilon_i$ is a normally-distributed variable that for these experiments, has a mean of 0 and variance of 0.07.

## 3.5 Synthesis of Model Development

Using qualitative and quantitative observations from the literature on opinion dynamics, this chapter formulated a conceptual framework for opinion formation and propagation, and then developed mathematical relationships to implement the different elements of the framework in a simulation program. Much of the early discussion focused on clearly defining concepts of opinion leaders and followers, social class, and the hypothesized influences that social personality and social class has on interaction and opinion revision. Drawing from concepts of leader-follower and gap-acceptance equations in traffic flow theory, Section 3.3 developed a general conceptual framework of opinion formation and propagation that has three components: an interaction model, an opinion revision model, and a consideration model. To make the framework operational in a simulation environment, the next section develops mathematical functions that attempt to mimic the effects of the social mechanisms in real-world scenarios, taking into account hypothesized sensitivities to interactions given personality and class traits.

While not the overall goal of this research, it is important to implement the opinion formation and propagation model to be able to draw insights on whether the mathematical relationships developed in this chapter produce “behaviors” similar to what is observed in the real world. It would be difficult to identify the effect of the social mechanisms had the opinion formation and propagation model been expanded to include choice dynamics. Thus Chapter 4 develops several classes of scenarios to be tested, and Chapter 5 presents the results of these simulation experiments.

# Chapter 4: Simulation Framework

*Description of Simulation Experiments*

Having conceptualized the theoretical relationships between individuals communicating via word of mouth and opinion revision, this section describes the simulation experiments used to test variations of parameters and different scenarios. System features of the simulation program are first described, followed by the identification of the principal factors tested. Finally, this chapter presents an explanation of the performance measures used to evaluate the experiments.

## 4.1 System Features

This research utilizes a hard-sphere particle molecular dynamics simulation program developed by Chen (working paper). Molecular dynamics offers good insight between the microscopic and macroscopic properties (e.g., molecules in a liquid) and utilizes concepts of acceleration, velocity, and position much like car-following theories. Under this program, individuals or agents are modeled as atoms or molecules and allowed to interact for a period under the laws of physics. Since the agents are hard spheres, the conservation of momentum includes only mass and velocity parameters. One of the advantages of the molecular dynamics program is the ability to change the mass of certain agents to reflect social mobility (e.g., larger masses indicate higher inertia and propensity to stay with the status quo). For simplicity though, this research assumes that all agents have the same mass, so that only the velocities vary. Through kinematics, one can solve for the magnitude and direction of the velocities of the agents.

In the initial stage, the program evenly distributes the agents according to the cell structure face-centered-cubit (fcc). As such, the number of agents needs to follow the equation below.

$$N_A = 2S^2 + 2S, \quad S \in \Re > 0 \tag{4.1}$$

where $N_A$ is the number of agents in the simulation, and $S$ is a user-specified number in the set of real numbers greater than zero. Thus, for $S$ = 5, $N_A$ = 60 agents. The system is bounded by a periodic boundary which allows the agents to move freely in the system (i.e., there are no collisions with the boundary). Bounded space has no implications for opinion; it only concerns the distances between agents.

Initialization of the program assigns random velocities, including magnitude and direction, to each agent. Each agent, given this velocity magnitude and direction calculates the distance and time to the nearest agent. The program searches for the closest pair of agents and updates collision time and the overall time to the time of that collision. If the interaction model is satisfied (i.e., connectivity is below the threshold), the program modifies the opinions according to the heuristics designed in the previous chapter. This process iterates until the number of collisions specified by the user occurs, at which the program terminates. More formally, the steps in the algorithm are outlined below, generally adopted from Haile (1992):

Initialization: Establish the thermodynamic state and assign initial positions and velocity to the hard spheres. Using these initial conditions, the analyst can then build a table of collision times.

Step 1: Specify a number of spheres $N$ and a packing fraction $\eta$.

Step 2: Calculate the area of the boundary space and adopt the length of one edge of the square as the unit of distance (i.e., set $L = 1$). Having established this unit distance, compute the sphere diameter $\sigma$.

Step 3: Assign initial positions $\mathbf{r}_i(0)$, $i = 1,\ldots,N$, at the sites on a face-centered-cubic lattice, or utilize positions taken from the end of an earlier simulation run.

Step 4: Assign initial velocities $\mathbf{v}_i(0)$, $i = 1,\ldots,N$ from a distribution (here the velocities are randomly assigned from a uniform distribution).

Step 5: Construct the table of collision times for each of the $\frac{1}{2}N(N-1)$ pairs of spheres using the following equation:

$$t_c = t_0 + \frac{(-\mathbf{v}_{12}\cdot\mathbf{r}_{12}) \pm \sqrt{(\mathbf{v}_{12}\cdot\mathbf{r}_{12})^2 - \mathbf{v}_{12}^2(\mathbf{r}_{12}^2 - \sigma^2)}}{\mathbf{v}_{12}^2} \qquad \textbf{(4.2)}$$

Step 6: Using the constructed collision time table, determine the duration $\Delta t$ until the next collision and identify the colliding spheres $i$ and $j$. This is equivalent to finding the nearest pair of spheres and calculating the time until collision.

Step 7: From the current positions, advance all spheres by $\Delta t$ to the nearest collision,

$$\mathbf{r}_k(t_0 + \Delta t) = \mathbf{r}_k(t_0) + \mathbf{v}_k(t_0)\Delta t \qquad k = 1,\ldots,N \qquad \textbf{(4.3)}$$

Step 8: Apply periodic boundary conditions to any sphere leaving the boundary space during $\Delta t$.

Step 9: Use the following equations to determine the postcollision velocities of $i$ and $j$.

For sphere 1, $v_1 = \mathbf{v}_1 - [(\mathbf{v}_1 - \mathbf{v}_2)\cdot \hat{\mathbf{r}}_{12}]\hat{\mathbf{r}}_{12}$ **(4.4)**

For sphere 2, $v_2 = \mathbf{v}_2 + [(\mathbf{v}_1 - \mathbf{v}_2)\cdot \hat{\mathbf{r}}_{12}]\hat{\mathbf{r}}_{12}$ **(4.5)**

Step 10: Calculate the new entries for the table of collision times by apply equation 4.2 to $i$, to $j$, and to any other sphere that would have collided with $i$ or $j$ had those sphere not collided.

Step 11: Repeat steps 6-10 until the desired number of collisions have occurred.

The next section explains the factors that were varied in the experiments.

## 4.2 Experimental Factors

For this model, the research considers three different levels of modeling. Models can be classified as the i) system properties, ii) basic, and iii) complex. The system properties models investigate the simulation system effects of varying initial opinion values and agent density, while the basic model investigates threshold variation, confidence variation, interaction requirements, and consensus versus coexistence.

### 4.2.1 System Properties

In the system properties model, the program assigns each agent an initial opinion value and the agents revise their opinion by a simple averaging of the opinions of the two agents in an interaction. Sixteen percent of the population, consistent with early adopters in a population, is initialized with a value of one. There are two factors evaluated in this model: the effect of initial opinion values and the density of agents.

Initial Opinion Values. Per the literature on opinion formation and propagation, there are two scenarios in initializing opinion values. One is for a fraction of agents to have a positive opinion value and the rest of the population to be indifferent at zero. The other is for a fraction of agents to have a positive opinion, with the rest of the population holding a negative opinion, denoted as –1. To evaluate the effect of different initial opinion values, this research tested a model initialized with [0,1] and [-1,1] opinion values. In both cases, 16% of the population held the positive opinion, and the number of collisions for both cases was set to 1,000.

Density of Agents. From the literature, it is hypothesized that the higher the density of agents in an area, the quicker the opinions will propagate. At lower densities, it will take longer for the agents to interact and for opinions to propagate. To investigate this hypothesis, this research considers different numbers of agents in the system. For this program, the higher the number of agents, the higher the density since all agents are initialized within the periodic boundary. A case where $N_A = 144$ and a case where $N_A = 12$ was tested and compared to the base case of $N_A = 40$. The number of collisions specified for each case depended on the time for opinion values to converge; for $N_A = 144$, the number of collisions was set to 3,000, for $N_A = 12$, there were 300 collisions, and for $N_A = 40$, there were 1,000 collisions.

### 4.2.2 Basic Models

For the basic models, the 16% of the population with a positive opinion were also initialized as opinion leaders. Each type of opinion leader was specified and represented in the model (~ 5% of the population, or two agents with $N_A = 40$). Only the indifference-positive model of opinions was considered in the basic model. Opinion leaders are assumed to maintain their opinion throughout the simulation (i.e., their opinion values were constant). Other agents updated their opinions through a simple averaging of the two opinions in an interaction. Within this model, the research investigated the variation of the threshold specification, memory effects on confidence, the interaction requirements, and consensus versus coexistence of opinion leaders.

Threshold Specification. In the model development, the interaction threshold has a constant component, the social class and social type parameters, and a dynamic component consisting of the confidence parameter that varies as the number of interactions increase. To test this specification of the interaction threshold, the research considers two cases, one where the threshold depends only on the constant component, and the other including the dynamic component. Results should show that with the dynamic component, as the number of interactions increase, agents' threshold decreases so that they become less willing to interact with other agents to revise their opinion. One would expect the number of interactions over the specified number of collisions would decrease under the dynamic component.

Memory Effects on Confidence. Within the confidence specification, there is a memory-type parameter representing an individual's memory of past interactions. For some individuals, the ability to recall all interactions is not possible; there is literature that supports the notion that individuals may recall a limited amount of recent interactions (see for example, Tversky and Kahneman, 1974, Lorenz, 2006). In some cases, individuals keep an accurate record of the interactions they have experienced; perhaps these are connectors or other opinion leaders. Other times, individuals may recall a recent number of interactions, but exaggerate those interactions, or, individuals may exaggerate interactions beyond their capacity to remember accurately. For this case, the memory-type parameter is varied to reflect pure interaction, where individuals remember all interactions, as well as exaggerated interactions, when history is dramatized or heightened. One would expect that both pure interaction and exaggerated interaction would make an individual more confident of their opinion, and thus reduce the number of interactions until their opinion value converges or stabilizes.

Interaction Requirements. In the opinion revision model, one of the requirements is that at least one of the two individuals in an encounter must meet the required connectivity constraint in the social mechanism level of the conceptual framework. This variation investigates the effect of both individuals must agree to interact on opinion values and their convergence. Results should show that imposing this restriction would further reduce the number of interactions in the system as well as reduce the convergence rate.

Consensus versus Coexistence of Opinion Leaders. This factor investigates the effect of having opinion leaders divided over different opinion values. For this case, the research tests two scenarios: one where half the opinion leaders (3) have an opinion value of 1, and the other half (3) have an opinion value of –1, and the other, where half the opinion leaders have an opinion value of 1, and the other half have an opinion value of 0. One would expect there to be little convergence in opinions; perhaps there may be some stratification effects, but that would depend on which agents interact with opinion leaders. There may also be huge variations in opinion values as one opinion seeker may interact with an opinion leader with a value of 1, and immediately thereafter interact with an opinion leader with a value of 0.

### 4.2.3 Complex Models

In the complex models, the research implements the interaction model and opinion revision model of the conceptual framework as discussed in the last chapter. Utilizing the word-of mouth mechanisms, mass media mechanism, belief learning mechanisms and direct experience mechanisms, these experiments intend to explore the effect of the opinion dynamics in the simulation environment. Additionally, these complex models allow for scenario design in which the analyst can compare and evaluate different policy implications. These scenarios are described in the following subsections. For models involving social type characteristics, it is important to note that mavens and salesmen do not update their opinion values (i.e., they remain constant), while connectors are allowed to revise their opinions.

#### 4.2.3.1 Word-of-Mouth Scenarios

In the word-of-mouth scenarios, 16% of the agents are initialized as opinion leaders with opinion values of 1. Opinion leaders who were mavens and salesmen did not revise their opinions, while connectors were able to revise if the connectivity threshold was satisfied. These scenarios employed the characteristics explored in basic models (e.g., dynamic threshold, memory-type characteristics, at least one agent agrees to interact). The main purpose of these scenarios is to determine the effect of accounting for influential opinion leaders in opinion formation and propagation. Additionally, this research is interested in evaluating the number of individuals who have placed the alternative into their consideration set. By accounting for these mechanisms, these experiments can offer insight as to whether certain policies or scenarios can influence additional individuals to consider the alternative.

Simple Averaging versus Incorporating Trigger Mechanisms. This scenario intends to determine whether incorporating the social type and social class trigger mechanisms and corresponding opinion revision functions reflect the intuition that influential individuals have a larger magnitude on an individual's opinion, and that these individuals gravitate towards the opinion of the leaders. The expectation is that the model that accounts for the trigger mechanisms will converge more quickly than the simple averaging model. In addition, incorporating the trigger mechanisms is expected to increase the number of individuals that consider the alternative when compared to the simple averaging model. For this case, the research considers the indifferent-positive initial opinion condition and the negative-positive initial opinion condition.

#### 4.2.3.2 Mass Media Scenarios

For mass media scenarios, agents are assumed to be equally receptive to marketing and advertising regardless of their social type. Thus, all agents are initialized with an indifferent opinion (i.e., an opinion of 0). In addition to exploring the effect of different mass media strategies (e.g., constant or reminder ads, product differentiation), these scenarios also intend to offer insight to competing products and their marketing strategies. This model assumes that in a competing product scenario, one product "advertises" an opinion of 1, while the other product "advertises" an opinion of -1.

Reminder Ads versus Product Differentiation. In this scenario, this research compares the simulated results from employing a constant reminder ad strategy and a differentiation strategy based on media class, which is highly correlated with social class and intends to replicate marketing segmentation and targeting strategies. The expectation is that differentiation strategies (i.e., advertisements that appeal to different social classes) will increase the number of agents considering that product when compared to the reminder ads.

Two Competing Products Using Reminder Ads. Here, agents are exposed to two competing products, one at an opinion value of 1, the other at opinion value of -1. Both employ the reminder ads strategy. The intent of this scenario is to evaluate the direct effects of competing products on opinion dynamics. One would intuitively expect that both marketing campaigns should get an equal number of agents to consider their products.

One Product Uses Segmentation versus Other Product Uses Reminder Ads. Building on the competing product scenario, this research is interested in exploring the effect of employing different strategies to influence more agents to consider one alternative or product. The product with an opinion of 1 utilizes a product differentiation strategy while the product with an opinion of -1 uses reminder ads. One would expect that the product using segmentation and targeting strategies would appeal to more agents, and would thus garner a higher number of agents considering the product.

Best Case Segmentation for Competing Products. Even though product differentiation and segmentation strategies attempt to appeal to different segments of the population, these strategies can only advertise through a discrete class. In other words, there are not enough resources to create a marketing campaign that closely matches an individual's social class and preferences. While not a realistic scenario, it is of interest to examine what the effect on an ideal segmentation strategy (i.e., one that can match individual preferences through matching his social class) is on opinion formation and propagation. Here, the two competing products advertise with full knowledge of an agent's social class and subsequently tailor the ad to the agent. The expectation is that agents will quickly respond to such campaigns, but then become saturated by the exposure and will no longer respond to the ads (i.e., consideration phase). While not a real occurrence in today's marketing strategies, internet websites like Amazon.com are utilizing individual preferences to customize an advertisement (e.g., product recommendations), so this scenario may not be too far-fetched even in the near future.

#### 4.2.3.3 Belief Learning Scenarios

By the design of the belief learning mechanism, belief learning does not have a pronounced effect unless there are an adequate number of individuals holding a different opinion, or if they perceive a distorted opinion amongst other individuals. Thus, for these experiments to reveal the effects of the belief learning mechanism, the research initializes opinion leaders with an opinion of 1 or -1. There are essentially three effects this research would like to explore: acquiring insight on how different opinion distributions affect the belief learning of agents, how different constructs of the observation group affects belief learning, and the role of opinion or attribute distortion.

One Opinion versus Two Opinions. This scenario is structured similarly to the consensus versus coexistence basic model in that for one case, the opinion leaders have an opinion of 1, with the rest of the agents initially having an indifferent opinion, while in the second case, half of the opinion leaders have an opinion of 1 while the other half has an opinion of -1. One would expect that given a higher concentration of leaders advocating a certain opinion (and thus a higher chance that the leader is in an agent's circle of friends) would yield a greater effect than having an equal number of leaders supporting two opinions.

Random Group Construct versus Similarity Group Construct. For this scenario, the intent is to explore the effect that different heuristics for constructing the observation group has on the opinion revision process. In the random case, agents choose at random a number of other agents to observe. This behavior is similar to observing celebrities or a group of individuals at an attraction (e.g., at a shopping mall). For the similarity case, agents search amongst the population to find individuals with similar social class and determine these individuals to be "friends." This is similar to online shopping website or utilizing social networking sites. Modeling these cases may also offer perspective on a combination of strategies, since there may be times when an individual's friend may be from a different social class, but intrinsically has some connection to that individual.

Opinion or Attribute Distortion. This scenario intends to offer insight to how the belief or perception of the group may distort an opinion of a product. To explore these effects, the model incorporates systematic biases in judgment by inflating or deflating an agent's opinion. The expectation is that the opinion distortion will exacerbate the opinion revision process, or cause an agent to prematurely consider the alternative (prematurely is defined as earlier product consideration than in the case where there is no distortion).

#### 4.2.3.4 Direct Experience Scenarios

For direct experience scenarios, agents' opinions are altered according to their experiences with the product. The model initializes the agents to have an indifferent opinion towards the alternative. Here, the research is interested in investigating the different ways in which individuals may define or establish their performance expectations. Three scenarios are generated to evaluate the effect on opinion revision: historical average expectation, last observation expectation, and the mean of the users. Implementation details for these three scenarios are explained in Chapter 3. The expected results are that expectation based over an increasing number of observations may temper a bad experience. In other words, one would expect that expectation based on the last observation would have the most volatile effect on opinion revision.

#### 4.2.3.5 Interactive Mechanisms Scenarios

As an objective of this research is to explore the interactive effect that word-of-mouth, mass-media, belief learning, and direct experience mechanisms have on opinion formation and propagation, these scenarios are designed to employ varying frequencies and combinations of these mechanisms in opinion revision. As a general rule, no agent can interact with more than one mechanism at a given epoch. This includes multiple agents interacting through word of mouth, or simultaneously being exposed to mass media while communicating via word of mouth to another agent. Through this rule, all mechanisms are considered independent entities; considering correlations between mechanisms is beyond the scope of this research.

As direct experience and word of mouth are considered to be the stronger mechanisms, this research models three scenarios: 1) each mechanism has an equal share or frequency of interacting with an agent (i.e., 25% of the interactions to each mechanism); 2) direct experience accounts for 60% of the interactions, belief learning accounts for 20% of the interactions, mass media and word of mouth each account for 10% of the interactions, and; 3) word of mouth accounts for 50% of the interactions, belief learning accounts for 20% of the interactions, mass media accounts for 20%, and direct experience accounts for 10% of the interactions. One would expect that the revision process of the dominant mechanism will prevail. Much of the opinion revision process will depend on the initial conditions; for all of these scenarios the research initializes agents to have indifferent opinions. Thus, the process most likely to occur in any scenario is that direct experiences or mass media will first affect an agent's opinion in the "formation phase", and then word of mouth and belief learning mechanisms will begin to have a measurable effect and continue to propagate opinions in the "propagation phase".

## 4.3 Performance Measures and Properties

Three principal performance criteria are evaluated for each scenario tested in the simulation experiments: 1) the time for opinion values to converge, 2) the value to which the opinions converge, and 3) the number of interactions until convergence.

1) Time for Opinion Values to Converge. It is important to investigate how long it takes the population to converge to an opinion, or if the opinion converges at all. Convergence is met when each agent's opinion value remains within a $\pm$ 0.001 range of opinion values. Although system time is in generic units, there are numerical values for comparison.

2) Value of Convergence. The value of convergence is the opinion value that is reached when the system reaches convergence, or when the program terminates.

3) Number of Interactions until Convergence. It is equally important to measure the number of interactions until convergence as an evaluation tool. As previously mentioned, an interaction occurs when the opinion revision model is invoked (i.e., the difference in connectivity is less than the interaction threshold). For different cases, this research interprets fewer interactions as an increase in confidence or a developing resistance to social temperature (i.e., the opinions of other individuals).

## 4.4 Summary of Simulation Design

This chapter has set the foundation for the implementation of the opinion formation and propagation conceptual framework using an agent-based simulation framework. After reviewing the literature on agent-based applications, a system featuring agents that behave according to the laws of kinematics (i.e., agents follow molecular dynamics) was utilized for this research. The basic algorithm derived from Haile (1992) was presented, along with a description of the simulation program features.

Several classes of scenarios were developed to test different aspects of the simulation program and the rules governing agents' interactions. The system properties class intends to look at how different initial states affect the opinion outcomes. Basic models investigate how rules governing an agent's behavior (e.g., varying connectivity criterion, different levels of confidence) may affect interactions and the opinion revision process. Finally, the complex models intend to focus on the four interaction mechanisms (word of mouth, mass media, belief learning, and direct experience) and examine their effects on opinions. A variety of scenarios were developed for each interaction mechanism to highlight some of the simulation program capabilities as well as incorporate real-world situations (e.g., media segmentation). Implementing these scenarios in Chapter 5 will offer insight to strategies or policies to follow in order to achieve a desired aggregate opinion value.

A brief discussion on the performance measures of the simulation program followed the scenario design to offer some evaluation basis for determining the state of the system. It is hypothesized that these measures will play a large role in evaluating and interpreting results from the system properties models as those models reflect system characteristics. More complex models (basic and complex scenarios) will probably not achieve system stability during the simulation period in that there are many mechanisms in place that prevent opinion convergence.

# Chapter 5: Simulation Experiments

*An Exploration of the Framework for Opinion Dynamics*

In this chapter, results from the simulation experiments, each corresponding to a different factor as discussed in the last chapter, are presented and discussed. First, the research investigates the base case and the effect of varying factors of the initial opinion values and the density of agents. Next, the basic model is considered, discussing the effect of varying factors of threshold specification, memory-effect on confidence, interaction requirements, and coexistence of opinion leaders. Finally, the last section considers the set of complex models, specifically the effects of word-of-mouth, mass-media, belief learning, and direct experience mechanisms. These complex models may also offer insight to the effect of different policies or strategies in the opinion revision process.

## 5.1 Base Case Results

Figures 5-1 through 5-4 display results from simulation experiments varying the initial opinion values. In the experiment where initial opinion values were set to 0 and 1, with 16% of the population harboring the opinion value 1, the opinion value converged to 0.1574, a value that one would expect given the *a priori* distribution of opinion values. However, when considering the case of initializing opinion values to –1 and 1, where 16% of the population harbored a positive opinion, the opinion value converged to -0.6852, which is not a reflexive result shown in Figure 5-3 (i.e., the opinion value did not converge to -0.84). This suggests that the convergence of opinion value depends on whom the positive individual first encountered. If these positive individuals encountered other positive individuals, or perhaps clustered together, the final opinion value would have been different. This is an interesting result because it suggests that the convergence of the base case is not pre-determined by the proportion of the population harboring a certain opinion.

Comparing the two experiments in terms of convergence time, the initial case of 0 and 1 values generally converged faster than did the case of –1 and 1 values. Specifically, a selected trajectory in the [0,-1] case reached convergence at a time of 6.438, while the same trajectory in the [-1,-1] case reached convergence at a time of 8.174. This confirms the hypothesis that the farther the population is divided in opinion values, the longer it takes the population to come to a consensus. An implication of this result is that to reach consensus in a short period, it is desirable to have individuals with diverse opinion values within the opinion range.

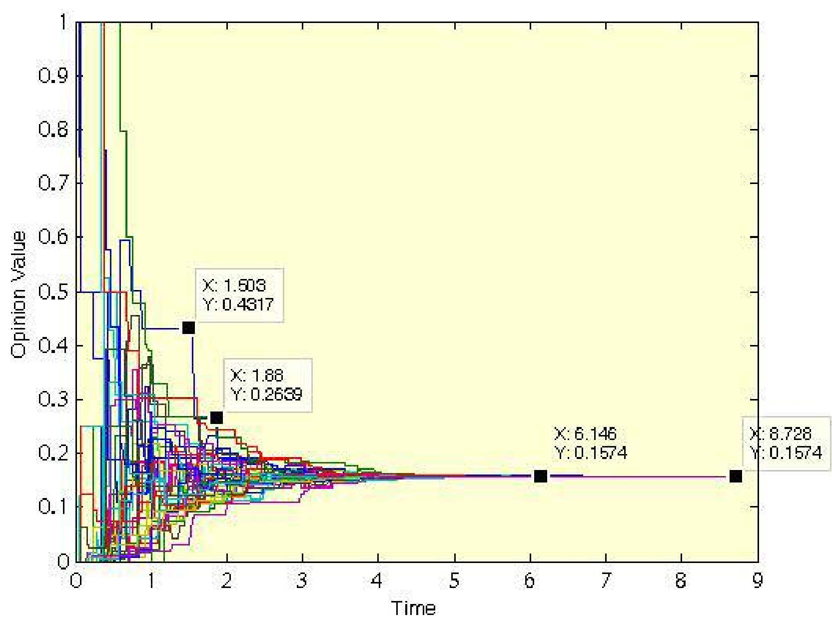


**Figure 5-1. Base Case, Initial Opinion Values [0, 1]**

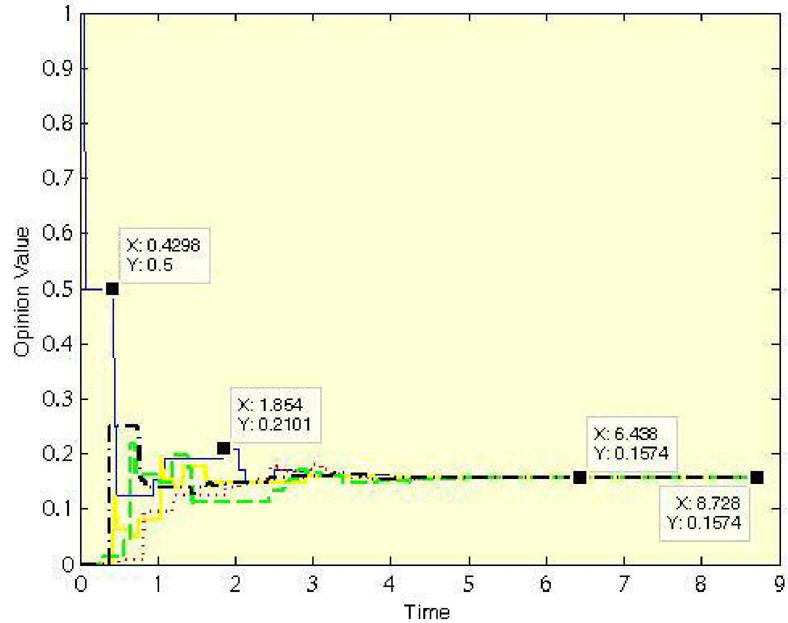


**Figure 5-2. Selected Trajectories of Initial Opinion Values [0, 1]**

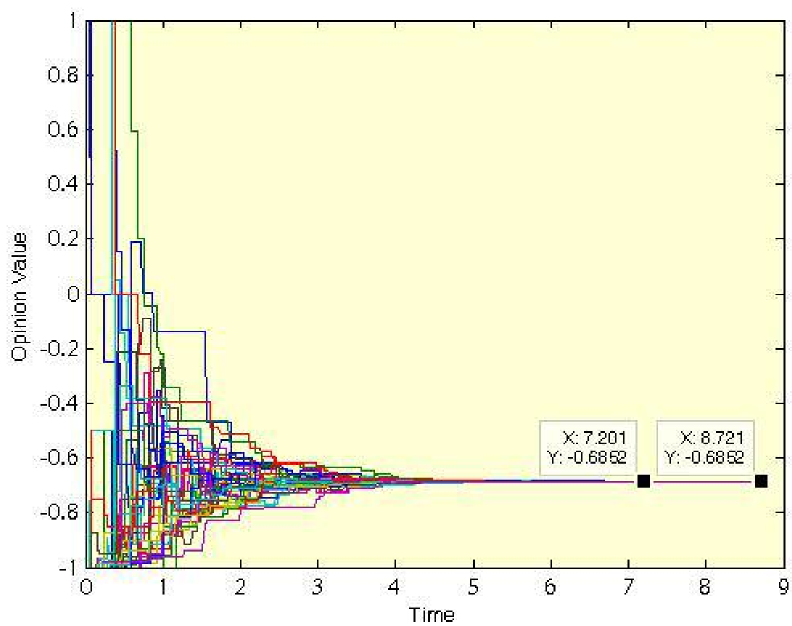


**Figure 5-3. Base Case, Initial Opinion Value [-1, 1]**

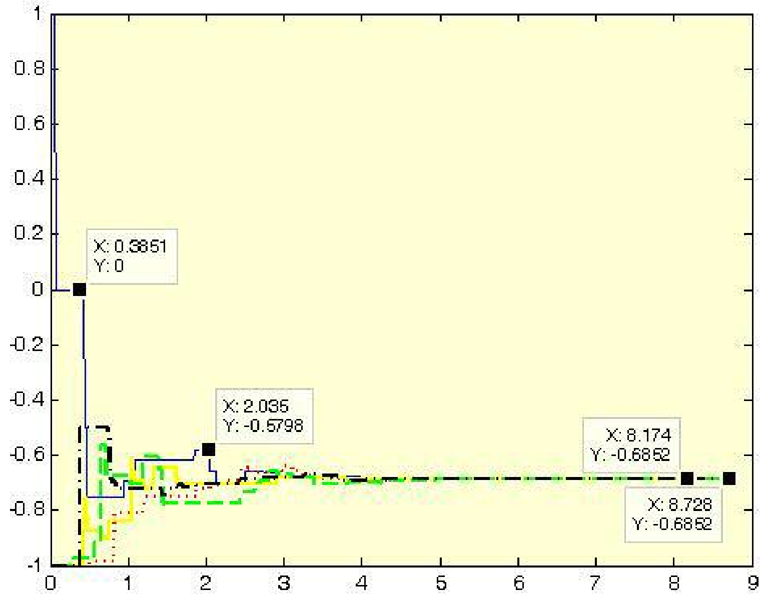


**Figure 5-4. Selected Trajectories of Initial Opinion Value [-1, 1]**

Figures 5-5 through 5-8 show the results for the experiments varying the density of agents (approximated by the number of agents within the system boundary). As expected, for an area with lower density ($N$ = 12), the time until convergence is longer than for an area with higher density ($N$ = 144). For the low-density case, a particular trajectory converges at a time of 13.84, but it is interesting to note that at time 6.601, the opinion value of that individual was 0.1159, a difference of only 0.0015. This supports previous literature that convergence of an individual's opinion takes a very long time, even under simple conditions. In the high-density case, the opinions converged at a time of 4.431. To ensure that there was ample time allowed for the agents to reach consensus, the number of collisions were varied (300 in the low-density, 4,000 in the high-density).

Another interesting result from these experiments is the different opinion values that the agents converged (reached consensus) upon. Both cases had 16% of the population with positive opinions, yet the value converged to in the low-density was 0.1174, while in the high-density case the converged value was much higher, at 0.2376. Such results imply that convergence values also depend on the number of individuals in the system.

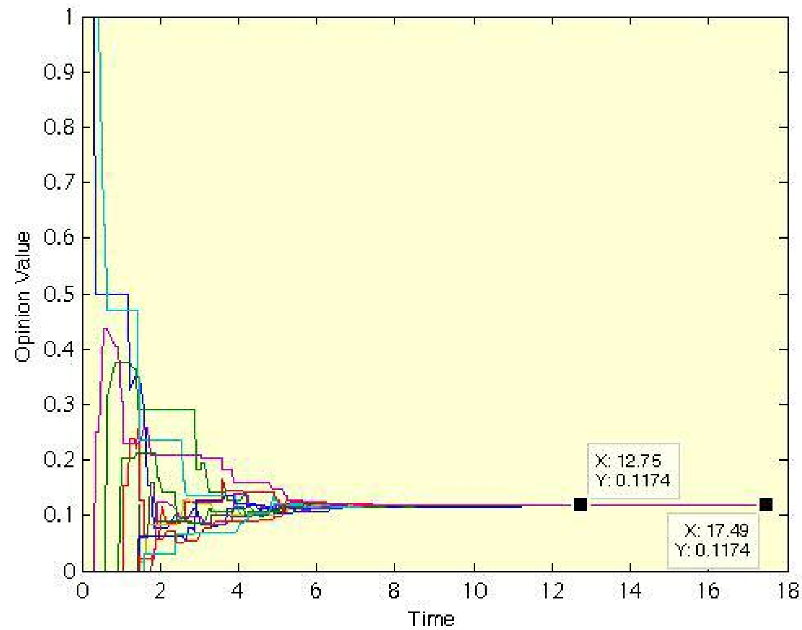


**Figure 5-5. Base Case, Density of Agents, $N = 12$**

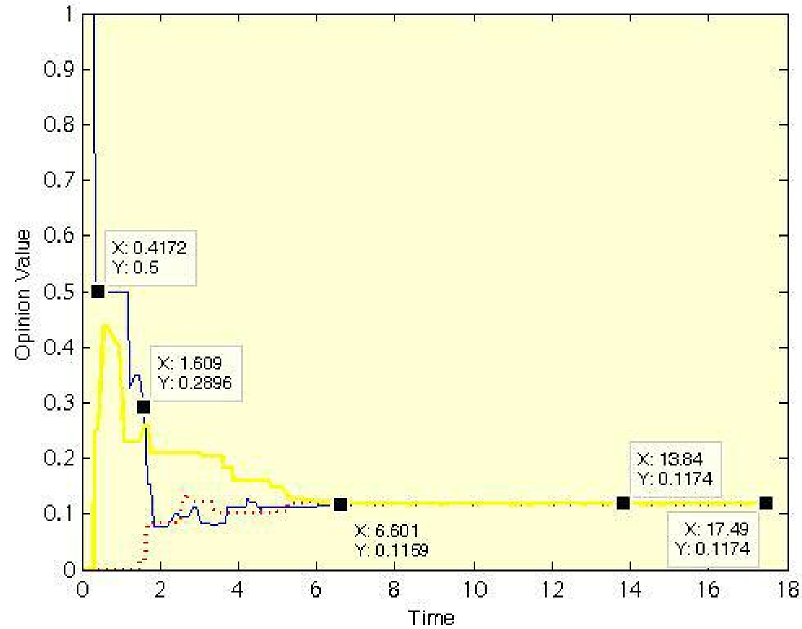


**Figure 5-6. Selected Trajectories for Density of Agents, *N* = 12**

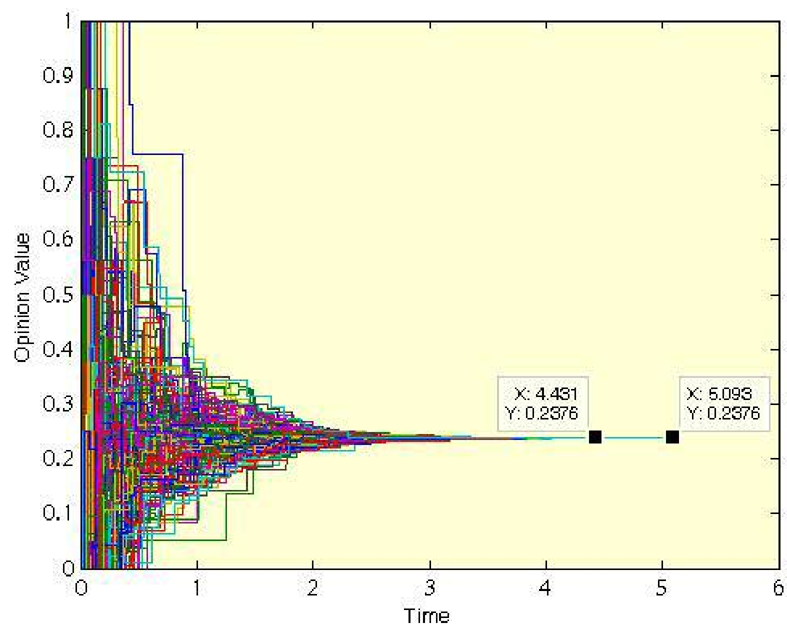


**Figure 5-7. Base Case, Density of Agents, *N* = 144**

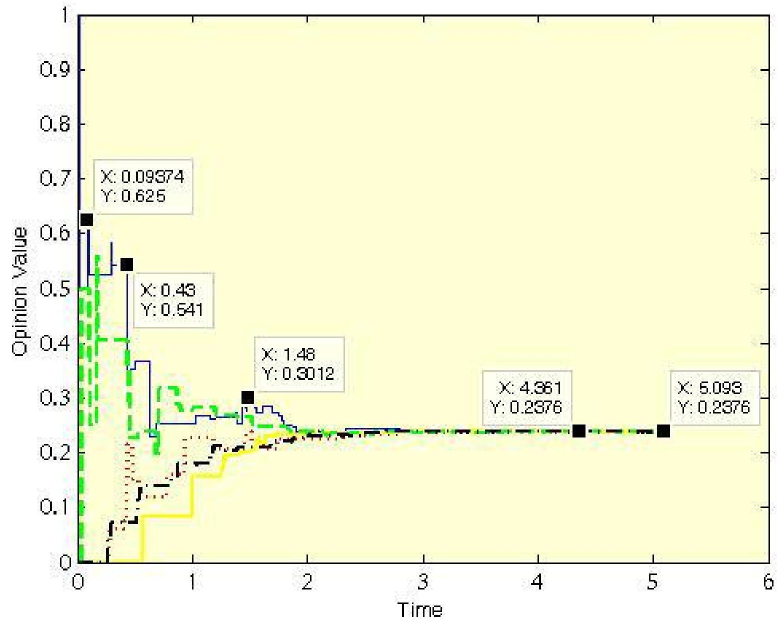


**Figure 5-8. Selected Trajectories, Density of Agents, *N* = 144**

## 5.2 Basic Model Results

In the basic model, all opinion leaders were initialized with a positive opinion (value of 1) that they maintained throughout the simulation. Opinion leaders consisted of 16% of the population; thus, the distribution of opinions remains the same as in the base case. The remaining agents were initialized with an indifferent opinion (value of 0) and were allowed to update their opinions by averaging the opinions of the two agents interacting.

Results from considering different threshold specifications are shown in figures 5-9 and 5-10. Figure 5-9 shows results from the case where the threshold remained constant (i.e., it depended only on class-threshold and type-threshold parameters, which remain constant throughout the simulation), while Figure 5-10 shows results from the case where the confidence parameter is included, making the threshold time-dependent. In both cases, the system does not reach convergence in 1,000 collisions, again supporting the idea that opinion convergence is a long process. Results show that under a constant threshold, agents trended towards a higher opinion value (0.963 for a particular trajectory) than under a dynamic threshold (0.9096 for the same trajectory). Results also show that under constant threshold, agents are closer to consensus (measured by the variance in the opinion values) than in the case of dynamic threshold. For the dynamic threshold case, the lower values and the greater dispersion of opinion values signify that it takes longer for an individual to encounter another individual that meets the interaction threshold requirement.

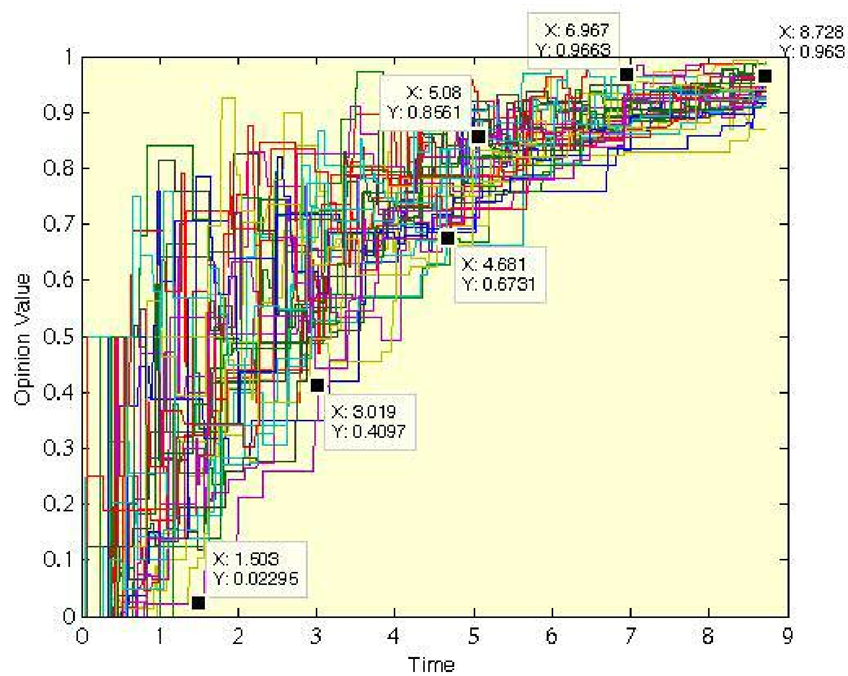


**Figure 5-9. Basic Model, Constant Threshold**

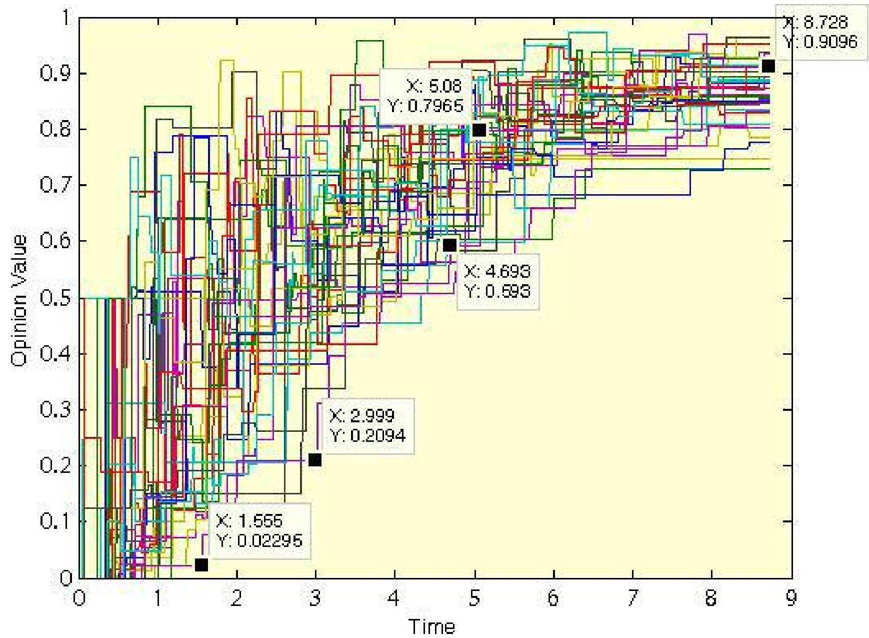


**Figure 5-10. Basic Model, Dynamic Threshold**

One of the components in the confidence equation is a memory-type parameter that is to reflect cognitive capacities. In the basic model, this research assumes that opinion seekers recall half of the number of interactions due to memory capacity or limitations, and assigns a value of 0.5 for the memory-type parameter. Connectors, on the other hand, remember all their interactions because they are individuals who make a record of these events and constantly refer back to them. These connectors were assigned a memory-type of 1, indicating what this research terms pure-memory effects.

Consider on the other hand, that individuals have large cognitive capacities and do remember the interactions, perhaps because there were not many interactions in the first place. Each agent thus is assigned a memory-type value of one. This is represented in the case where there are pure memory effects on individuals' confidence. Figures 5-11 and 5-12 display the simulation results from these experiments. Results show that there is a large dispersion of opinion values, and that the system does not converge. However, on an individual scale, opinions seem to stabilize for many individuals, suggesting that there is individual opinion convergence. There are still individuals that continue to seek opinions and revise their opinion values.

There may be instances where individuals may recall portions of the interaction list, but do so in an exaggerated manner. This may occur when recalling distant events (i.e., events that happened long ago), and may be made more compelling and convincing than the actual opinions exchanged. Thus, for this case of exaggeration, a value of 2 is assigned as the memory-type parameter. Figures 5-13 and 5-14 show the simulation results from these experiments. There is a large dispersion of opinion values, and the system does not converge, much like the pure memory case. Unlike the pure memory case, there is a much more pronounced effect on individual convergence of opinions. A good majority of the population reached a stabilized opinion value approximately halfway through the simulation. Figure 5-14 shows that three out of the four varying trajectories (the fifth is an opinion leader who remains constant at 1) have stabilized and perhaps reached an individual convergence. The dynamics of this memory effect on confidence, coupled with the threshold specification, offers interesting insights on how the model captures reality. More work is needed to find out whether confidence and the interaction threshold are correctly specified.

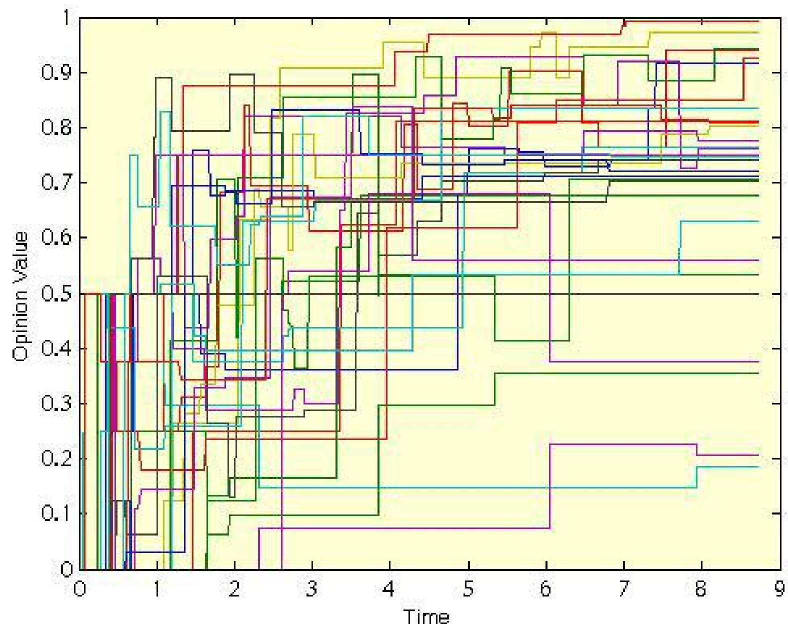


**Figure 5-11. Basic Model, Pure Memory Effect on Confidence**

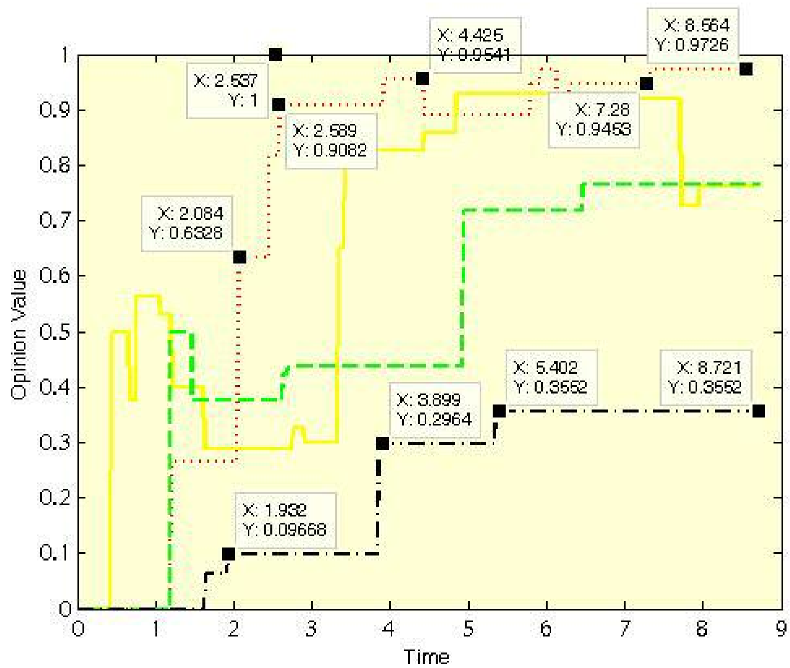


**Figure 5-12. Selected Trajectories of Pure Memory Effect on Confidence**

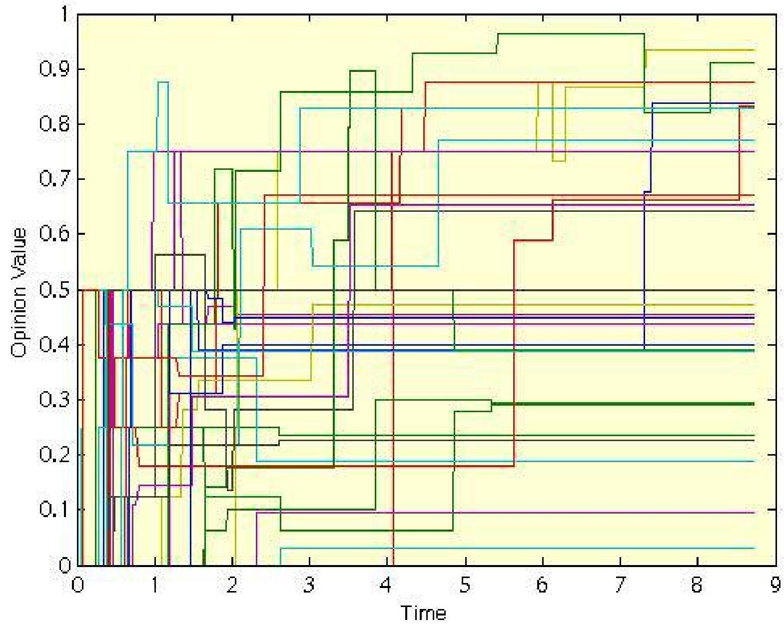


**Figure 5-13. Basic Model, Exaggerated Memory Effect on Confidence**

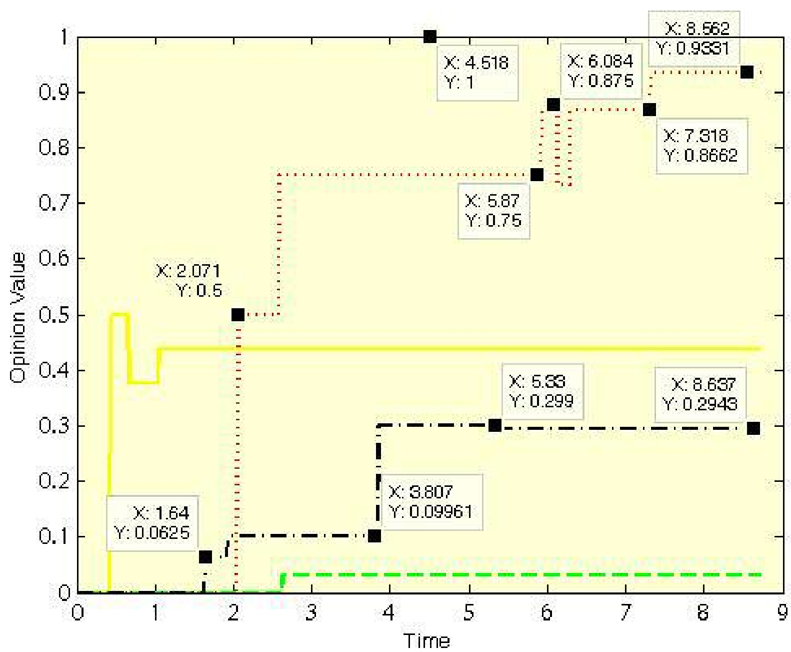


**Figure 5-14. Select Trajectories for Exaggerated Memory Effect on Confidence**

Results from considering scenarios where both agents need to meet the interaction threshold criteria are shown in Figure 5-15. In all other cases, at least one agent needed to "agree" to interact with the other and subsequently revise his opinion. These results can be compared to the results shown in Figure 5-10. Imposing this requirement of both agents meeting the criteria is much more restrictive, and has a pronounced effect on individual opinion convergence and overall convergence. There is greater dispersion of opinion values and no overall convergence on an opinion. However, it is interesting to note that several individuals may have reached an individual convergence on an opinion value, since the requirement is so restrictive that they will not meet another individual who satisfies the interaction criteria along with these individuals. Others seem not to be affected in terms of individual convergence, as they continue to seek and revise opinions, but perhaps not as dramatically as in earlier experiments.

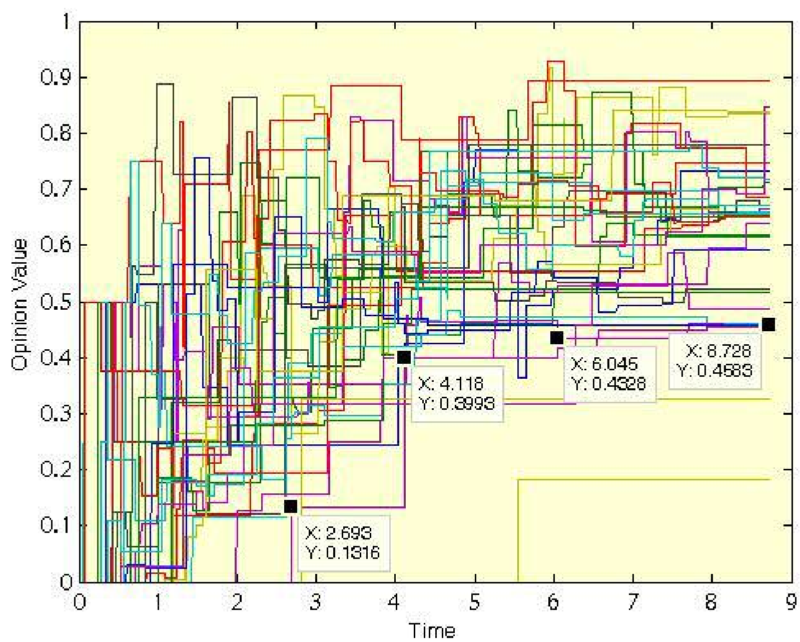


**Figure 5-15. Basic Model, Both Individuals Meet Interaction Threshold Criteria**

Also considered in the basic model is the effect of coexisting opinions of polarized opinion leaders. Here, half of the opinion leaders have a positive opinion, and half the opinion leaders have a negative opinion (case 1) or are indifferent (case 2) and remain at those opinions throughout the simulation. Figures 5-16 and 5-17 show results of an experiment considering a negative-positive coexistence of opinion leaders, while Figures 5-18 and 5-19 show results of an experiment considering an indifferent-positive coexistence of opinion leaders. In both cases, the system fails to converge. A number of individuals seem to build confidence, and consequentially reduce their inclination to interact with others. In the negative-positive case, the final opinion values seem to be evenly distributed about the mean (0). However, in the indifferent-positive case, more agents have a final opinion value below the mean of 0.5. This is an interesting result that may suggest that having opinion leaders with an indifferent opinion may dampen the positive opinions more than if the opinion leaders were polarized (i.e., negative opinion leaders versus positive opinion leaders).

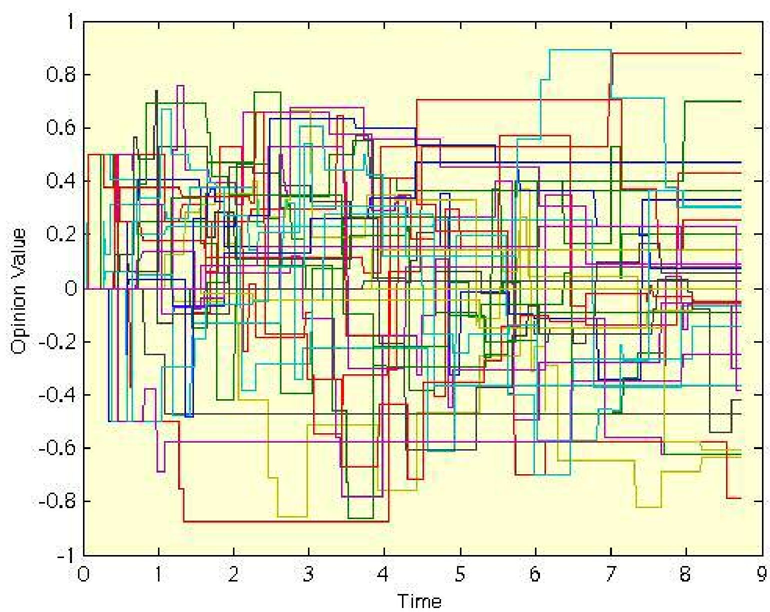


**Figure 5-16. Basic Model, Coexistence of Opinion Leaders at [-1, 1]**

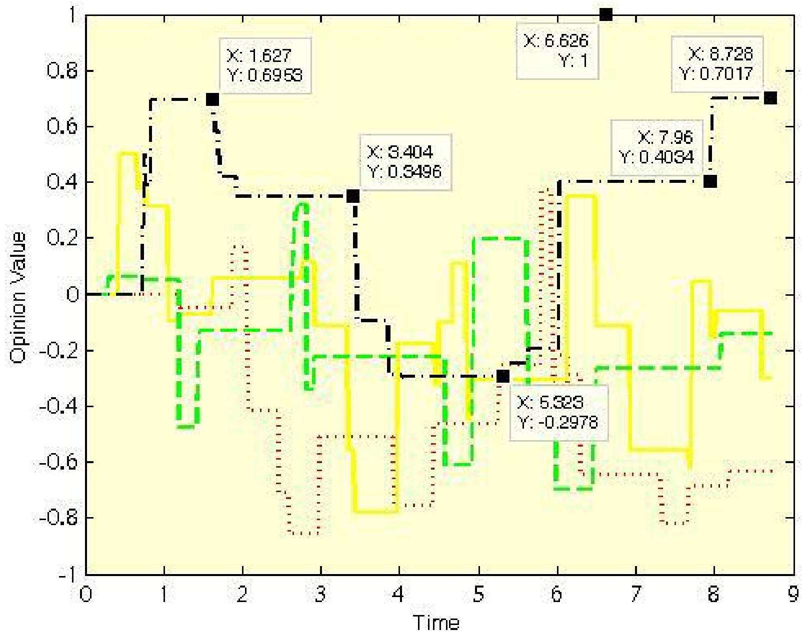


**Figure 5-17. Selected Trajectories of Coexistence of Opinion Leaders [-1, 1]**

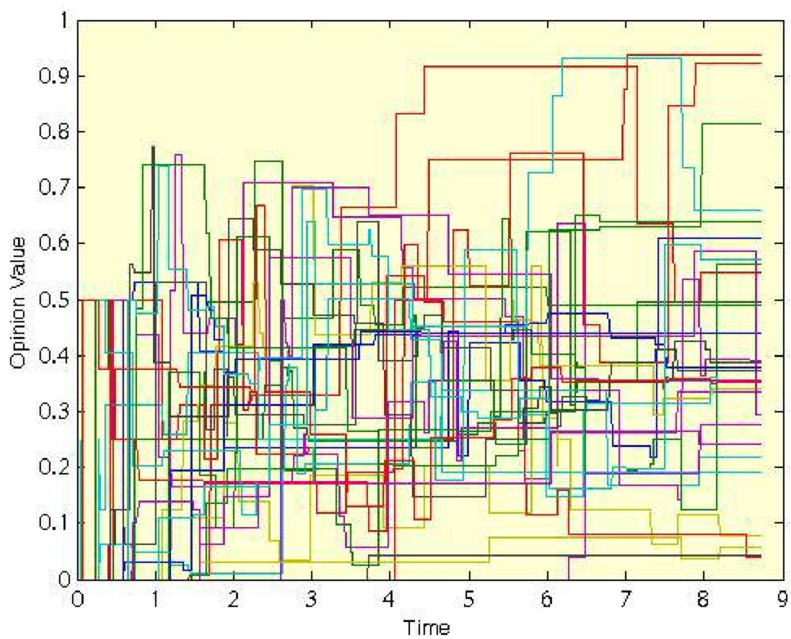


**Figure 5-18. Basic Model, Coexistence of Opinion Leaders at [0, 1]**

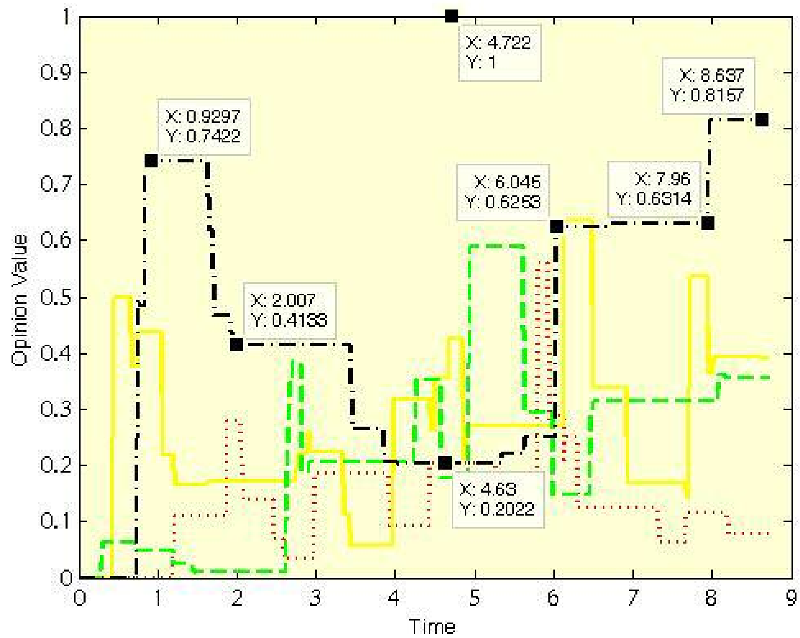


**Figure 5-19. Selected Trajectories of Coexistence of Opinion Leaders [0, 1]**

## 5.3 Complex Model Results

For these models, the research looks at two main elements: opinion values of the individual agents as they evolve over time, and the evolution of the number of agents considering the alternative. As mentioned previously, consideration means that an individual has adopted the opinion value and does not revise it unless a direct experience convinces the individual to think otherwise (i.e., the individual has a significant unsatisfactory experience). These models also shed insight on the effects of incorporating mathematical representations of word-of-mouth, mass-media, belief learning, and direct experience behavior.

### 5.3.1 Word of Mouth Model Results

Figure 5-20 through Figure 5-27 depict the simulation results employing the word-of-mouth mechanisms. One hypothesis concerning these mechanisms is that when compared to a base model (i.e., simple averaging, ignoring social class and social type effects), the social and personality trigger mechanisms will account for a "social gravitation" effect in which opinion followers are sensitive to influential individuals and will adopt more of the leaders' opinion than a simple averaging function. This research evaluates the gravitation effect by how effectively (i.e., time to convergence, value of convergence) the trigger mechanisms incorporate these social interaction characteristics.

The baseline case of simple averaging is illustrated in Figure 5-20, while Figure 5-21 shows the opinion trajectories for a model implementing the social and personality trigger mechanisms, with opinion leaders harboring an opinion value of 1, and all other agents have an opinion value of 0. While neither case reaches convergence (a recurring theme throughout these models), agents in the trigger mechanism model have a noticeable increase in the rate of increase of the opinion value, as it takes less simulation time for the aggregate average to reach an opinion of 0.5. Further, the attractiveness of the opinion leaders based on their social class and personality seem to draw agents' opinions increasingly towards a value of 1. The average opinion of agents in the trigger mechanism model is greater than the average opinion of the agents in the baseline model.

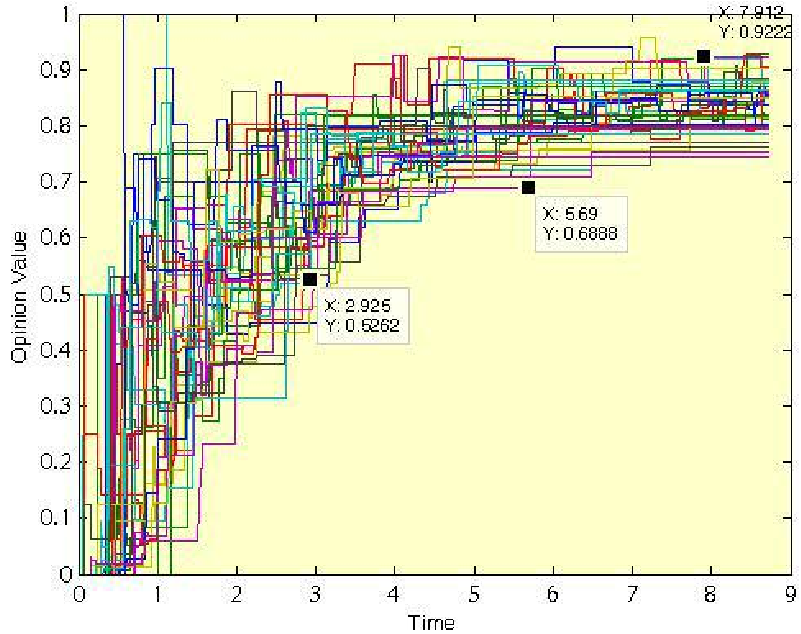


**Figure 5-20. Word of Mouth with Simple Averaging Opinion Revision**

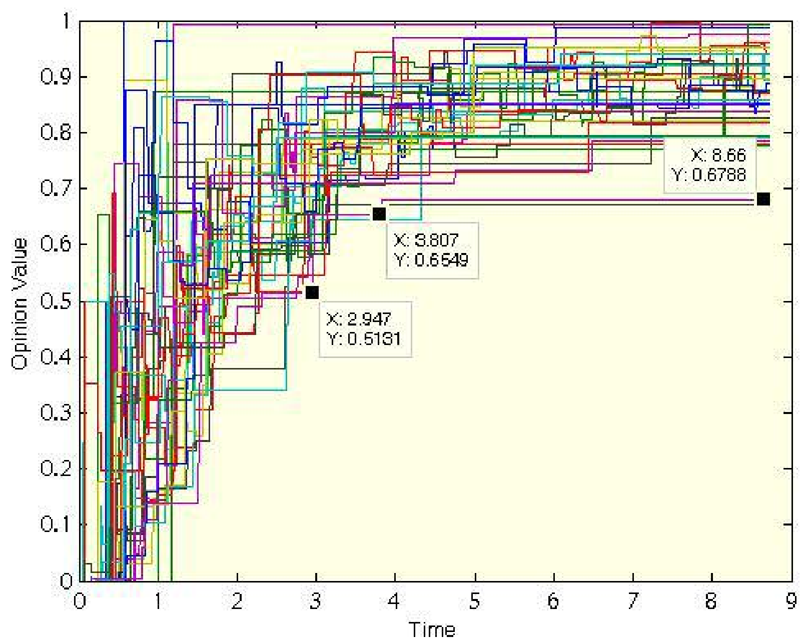


**Figure 5-21. Word of Mouth with Trigger Mechanisms Opinion Revision**

Another evaluation of the mechanism performance is its effectiveness in convincing individuals to consider the alternative. As mentioned earlier, an individual considers an alternative if his opinion on that alternative exceeds some internal threshold. In the opinion formation and propagation model, considering an alternative is equivalent to choosing an alternative in a choice framework, in that agents considering the alternative no longer revise their opinion unless a direct experience convinces them to do otherwise. The hypothesis then, for the word-of-mouth mechanism is that when compared to the baseline, the social and personality trigger mechanisms should increase the number of individuals considering the alternative, since the weighted attractiveness of the opinion leaders should draw more individuals towards an opinion of 1, and thus increasing the likelihood of exceeding the individual internal threshold.

Figure 5-22 looks at the consideration set for the simple averaging word-of-mouth model, while Figure 5-23 looks at the consideration set for the social and personality trigger mechanism model. At its peak, the simple averaging model has convinced 33 out of 40 agents to consider the alternative, and at the end of the simulation period, 29 agents considered the alternative. In the trigger mechanism model, a maximum of 36 agents considered the alternative, and 34 agents consider the alternative at the end of the simulation period. This suggests that incorporating the social and personality trigger mechanisms increases the number of agents considering the alternative when compared to the baseline model.

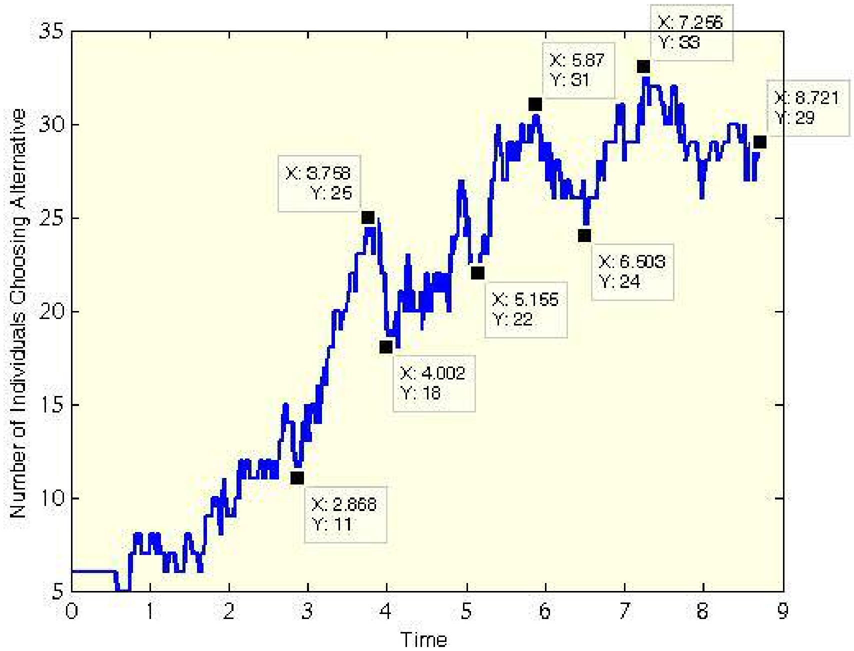


**Figure 5-22. Consideration Number for Word of Mouth Simple Averaging**

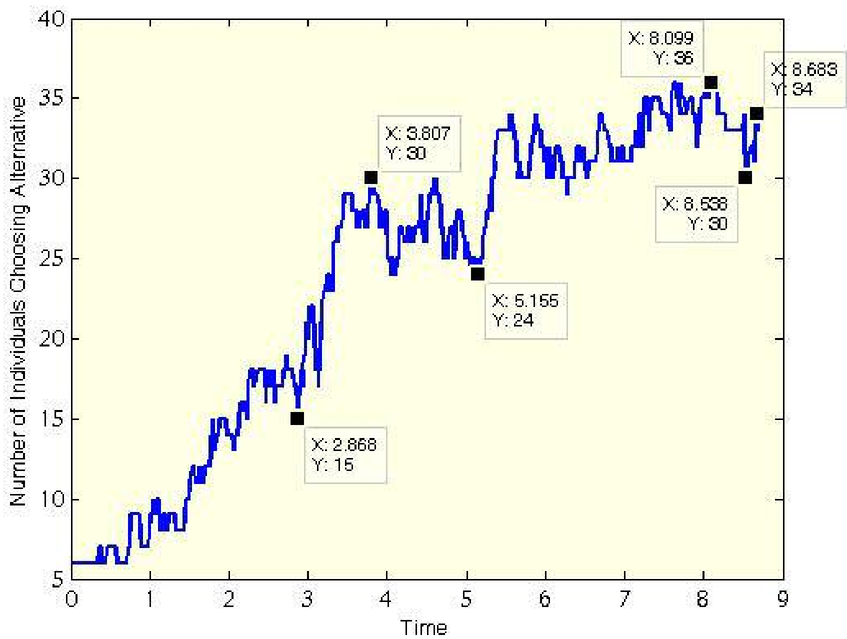


**Figure 5-23. Consideration Number for Word of Mouth with Trigger Mechanisms**

As an opinion of 0 represents indifference, another scenario to test was the effect of the mechanisms on a system with initial opinions at -1 and 1 (with opinion leaders having the opinion of 1). Figure 5-24 shows the simulation results from this scenario. The opinion trajectories appear very similar to the trigger mechanism model with initial opinions at 0 and 1, but due to the larger spectrum of opinion values, take slightly longer for the aggregate opinion value to reach 0.5. However, it achieves a very similar range of opinions as the trigger mechanism 0 and 1 scenario at the end of the simulation. This may suggest that the opinion separation between the opinion leaders and opinion followers only affect the convergence rate, and that if the simulation period is long enough, the opinions may converge to the same value regardless of the initial opinions of the followers.

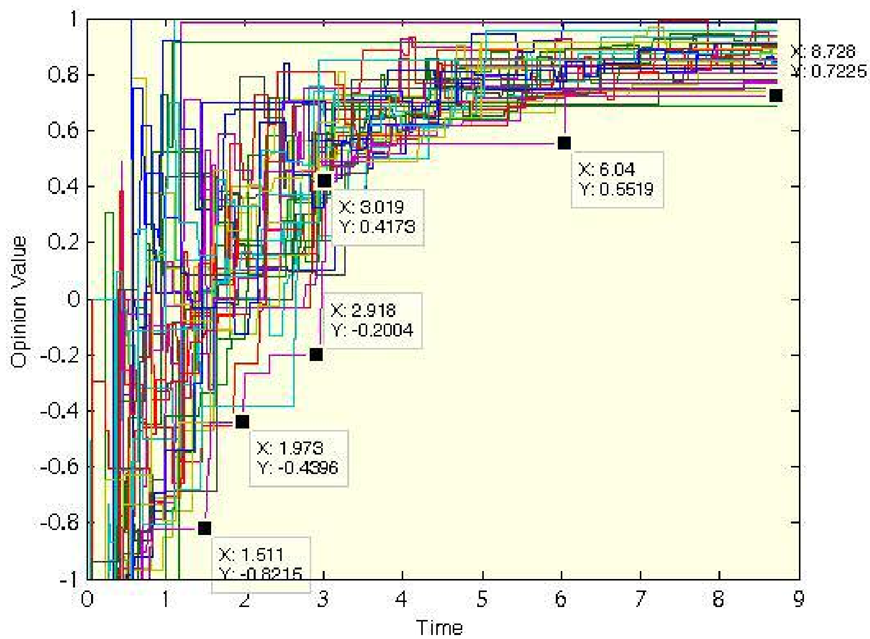


**Figure 5-24. Word of Mouth with Trigger Mechanisms Opinion Revision, Opinion Leaders at 1, Others at -1**

Another scenario of interest is where agents' opinions are initialized to some random value between -1 and 1. Opinion leaders are also included in this random opinion assignment. The simulation result is shown in Figure 5-25. It is interesting to note the linear trend in the opinion trajectories, and that the dispersion decreases as simulation time elapses.

Figure 5-26 and Figure 5-27 display the consideration sets for the opinions at -1 and 1 scenario and the random opinion scenario, respectively. In comparing the consideration set of the opinions at -1 and 1 scenario to the consideration set of the opinions at 0 and 1 scenario, the results show that it takes longer for the -1 and 1 case to convince a majority of agents to consider the alternative. However, at the end of the simulation period, the number of agents considering the alternative is 33, a number that is only a few less agents than in the opinions at 0 and 1 scenario.

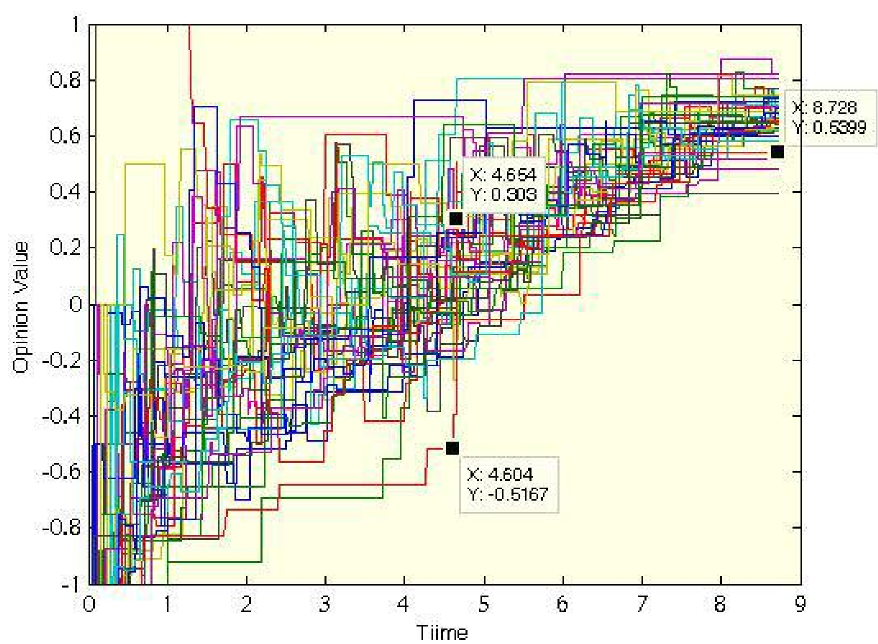


**Figure 5-25. Word of Mouth with Trigger Mechanisms Opinion Revision, Random Opinions**

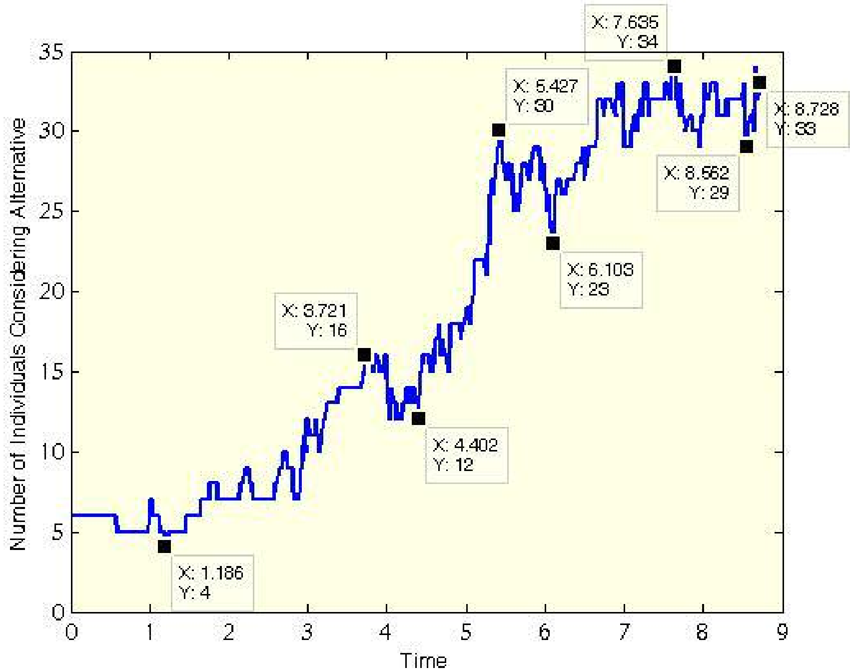


**Figure 5-26. Consideration Number for Word of Mouth with Mechanisms, Opinions at -1 and 1**

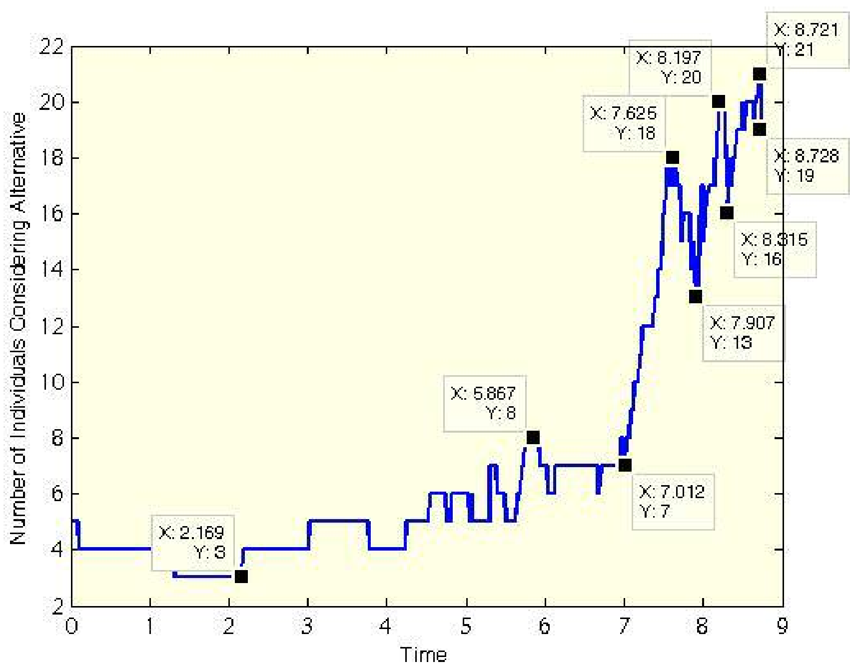


**Figure 5-27. Consideration Number for Word of Mouth with Mechanisms, Random Opinions**

For the random opinions scenario, the consideration set evolution shown in Figure 5-27 shows a sudden increase, as it is probable that many agents reach their internal opinion threshold at the same time. This sudden shift pattern may replicate the social phenomenon termed as the "tipping point" when a small change or a piece of new information may cause a large proportion of the population to consider something.

### 5.3.2 Mass Media Model Results

Figure 5-28 through Figure 5-35 depict simulation results for models incorporating mass media mechanisms. Here, agents are assumed to be captive to a product or alternative once the media has met the threshold criteria. Thus, it is important that the media mechanism reaches an agent early in the simulation period to capture the attention and eventually get the agent to consider the product. This specification allows for the evaluation of the exposure of agents to media, and the frequency that agents pay attention to media.

Figure 5-28 shows the results for a scenario with reminder ads for one product. The opinion trajectories of these agents give insight to the exposure rate of the agent to the media, and to how receptive that agent is to the media being broadcast. Individuals who have an exact need for the product advertised have a higher rate of increase in opinions. Notice also that the agents affected by the media mechanism (i.e., agents with a positive opinion) were convinced to revise their opinion early on. Yet, only 6 agents out of 40 consider the alternative, as shown in Figure 5-29.

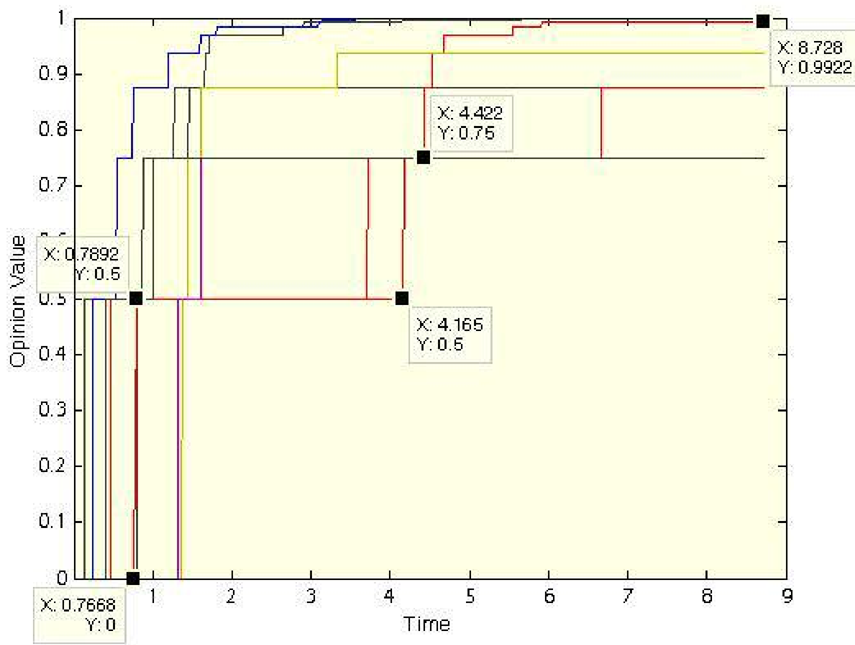


**Figure 5-28. Mass Media Mechanism, One Alternative, Reminder Ads**

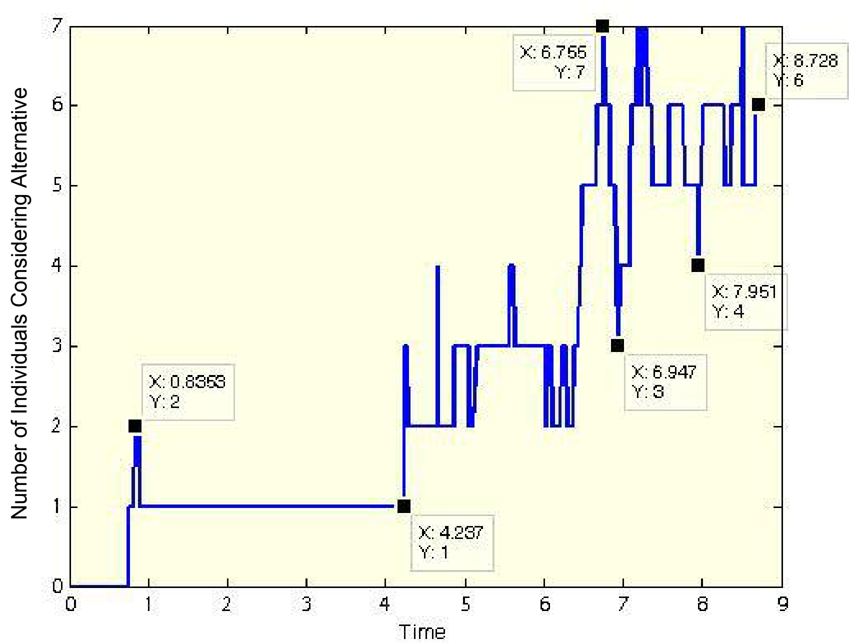


**Figure 5-29. Consideration Number for Mass Media, One Alternative, Reminder Ads**

Figure 5-30 and Figure 5-31 show the opinion trajectories resulting from the simulation runs for scenarios where there are competing alternatives. In Figure 5-30, there are two competing alternatives using reminder ads to capture a share of the agents. Agents are assumed to have equal need for both alternatives, and so it becomes a competition of which media can reach out and draw the attention of the agent first. Results of the competing reminder ads strategy show few agents actually paying attention to the media advertisement. Contrast this result with the results obtained in Figure 5-31, where one alternative utilizes a product differentiation or segmentation strategy in which the media is diversified to take into account different social classes. Those results show that there is a significant increase in the number of agents holding a favorable opinion (i.e., positive opinion) towards the alternative using the segmentation strategy, while the number of agents holding an opinion towards the alternative using reminder ads remains approximately the same. This may suggest that segmentation strategies may help an alternative to "win over" more individuals.

Consideration sets of the scenarios of competing alternatives with reminder ads and competing alternatives with one alternative using segmentation strategies are shown in Figure 5-32 and Figure 5-33, respectively. The consideration set is taken with respect to the alternative advocating an opinion value of 1. It is interesting to note that there are fewer agents considering the alternative than in the scenario with one alternative using reminder ads. This may be due to the competition for attention from the other alternative. Although it appears from Figure 5-31 that many agents have formed a positive opinion towards the positive opinion alternative, only 7 agents actually consider the alternative. This might suggest that the mass media mechanism with segmentation strategies is effective for capturing the attention of individuals, but may need help (either through more time, or through another mechanism) to persuade agents to consider the alternative.

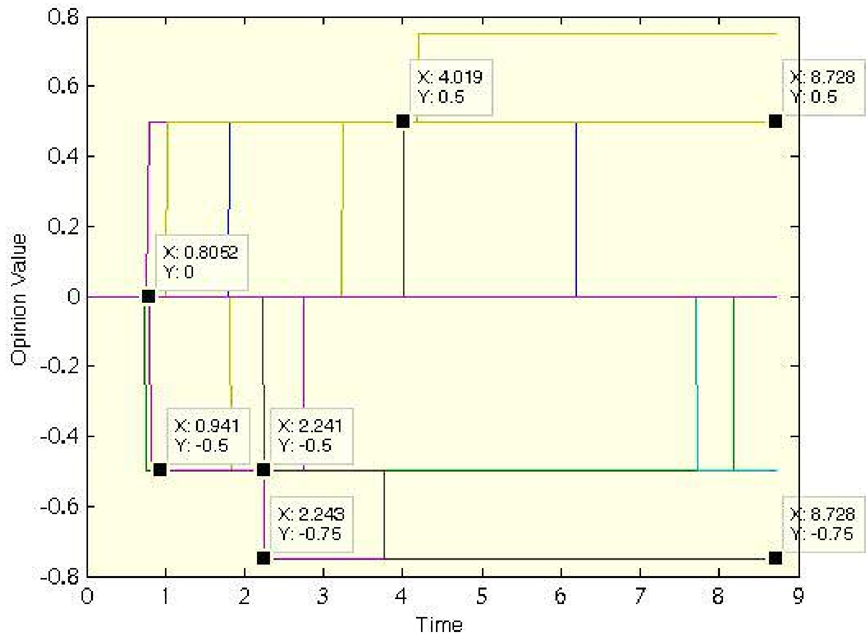


**Figure 5-30. Mass Media Mechanism with Competing Alternatives, Reminder Ads**

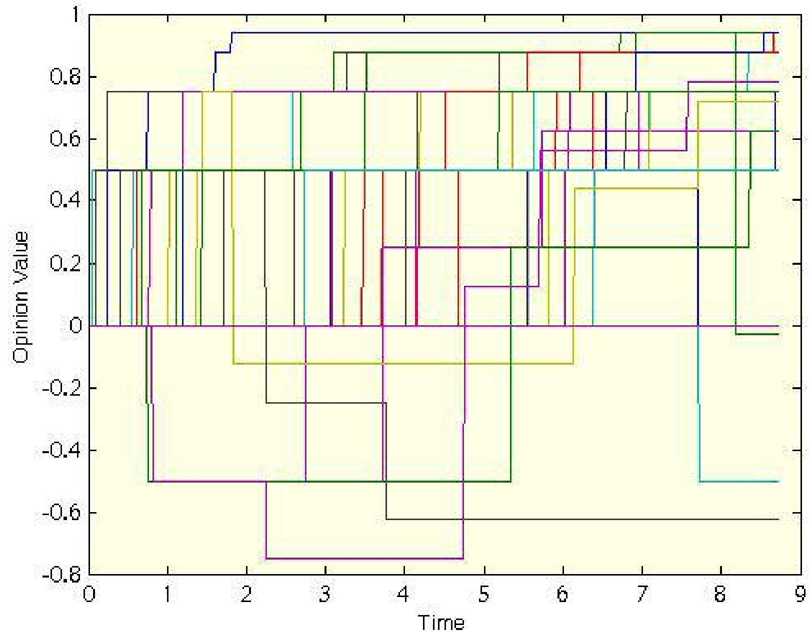


**Figure 5-31. Mass Media Mechanism, Competing Alternatives, One Uses Segmentation Strategy**

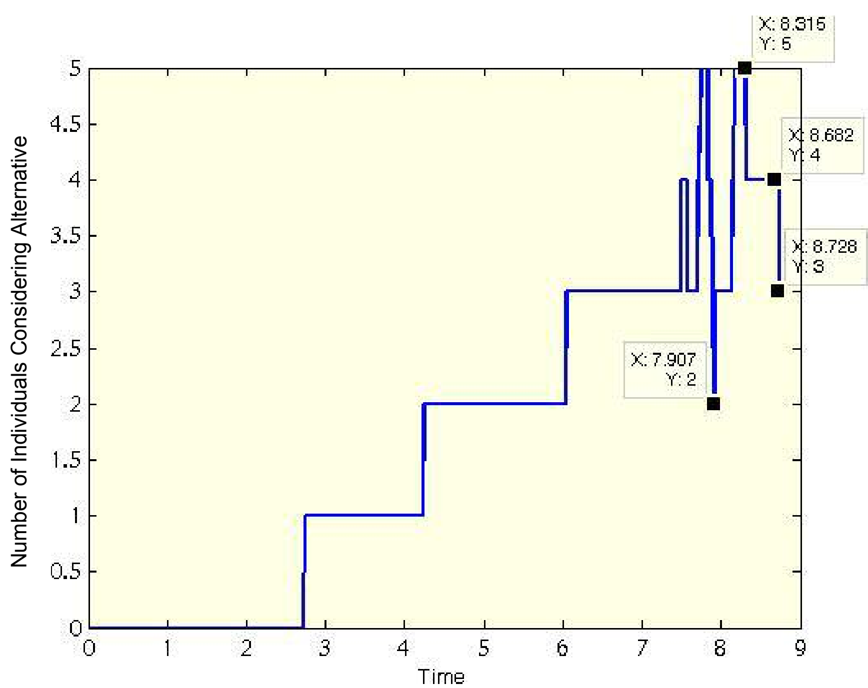


**Figure 5-32. Consideration Number for Mass Media, Competing Products, Reminder Ads**

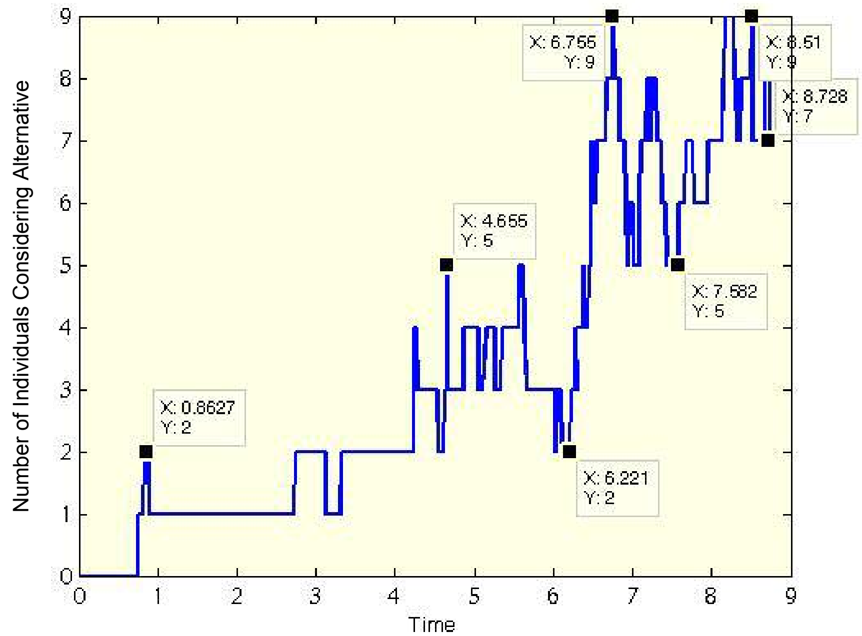


**Figure 5-33. Consideration Number for Mass Media, Competing Products, One Uses Product Segmentation**

To test the ideal mass media scenario where the segmentation strategies meet users' needs perfectly, the model assumes that the agents' needs are revealed as preferences to the media campaign, and that the media mechanism is capable of matching agent classes. The result of this scenario is shown in Figure 5-34 and Figure 5-35. As one would expect, the effectiveness of this strategy is significant, as agents are quickly captured by the alternative they need, and quickly revise their opinion. The consideration set evolution shown in Figure 5-35 shows the number of agents considering the positive opinion alternative increases quickly, and then tapers off as agents may have reached a point of saturation. The much higher proportion of agents considering the alternative suggests that best-case segmentation is the most effective media strategy in garnering agents' consideration.

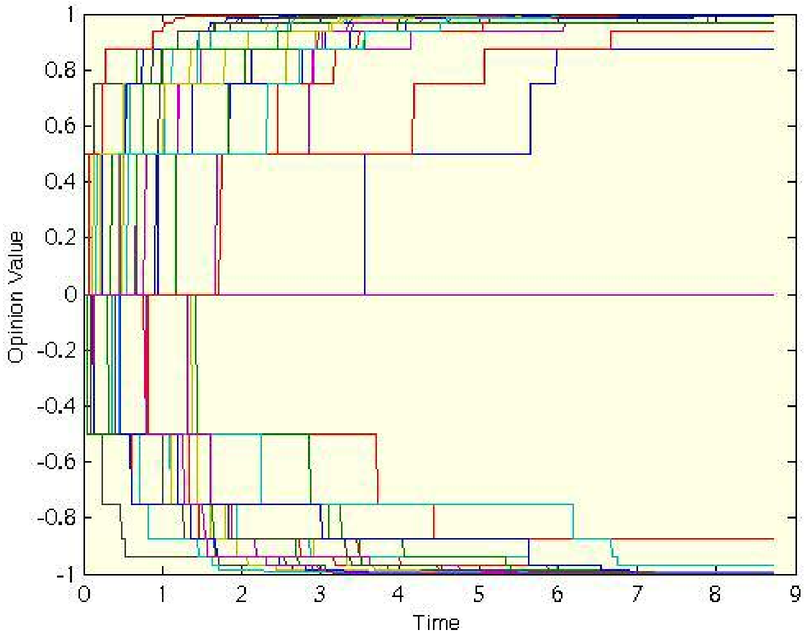


**Figure 5-34. Mass Media Mechanism, Competing Products, Best Case Segmentation**

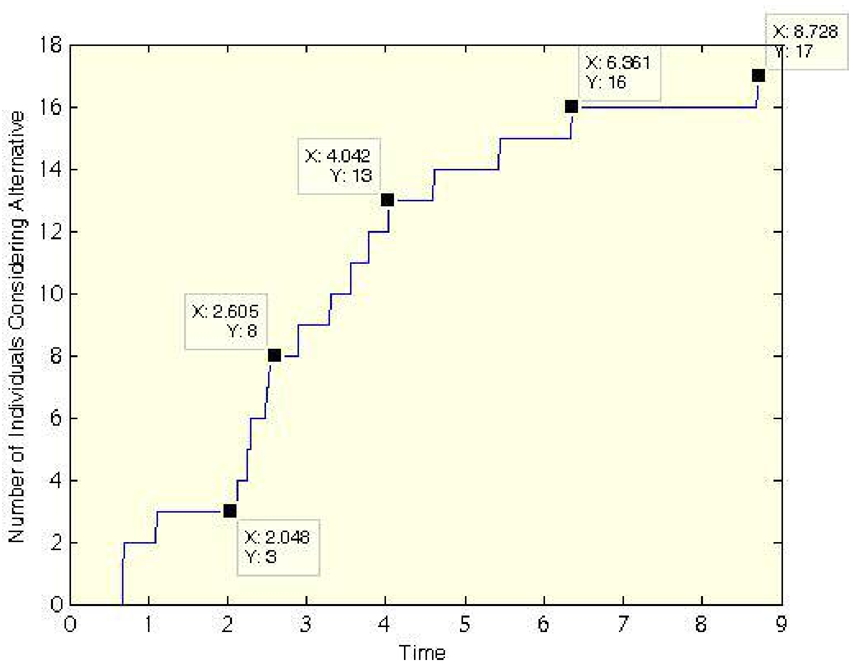


**Figure 5-35. Consideration Number for Mass Media, Competing Products, Best Case Segmentation**

### 5.3.3 Belief Learning Model Results

Figure 5-36 through Figure 5-42 display the simulation results obtained with the belief learning mechanism. As the mechanism entails observing "friends" and their opinions, all of these scenarios involve opinion leaders with some initial opinion value. Figure 5-36 shows the results from the scenario where agents construct their friends by social class similarity, with opinion leaders at 1. The opinion trajectories suggest that the belief mechanism effect is dynamic and ephemeral, a pattern that is supposed to mimic real-world social fads. It is interesting to note that no agent considers the alternative in this scenario, suggesting that the belief mechanism may be more effective given a population with a range of opinions. Figure 5-37 shows results from the scenario where opinion leaders have opinions of -1 and 1. Again, the opinion trajectories show that the process is dynamic, but does not leave a lasting impression, as no agent considers the alternative.

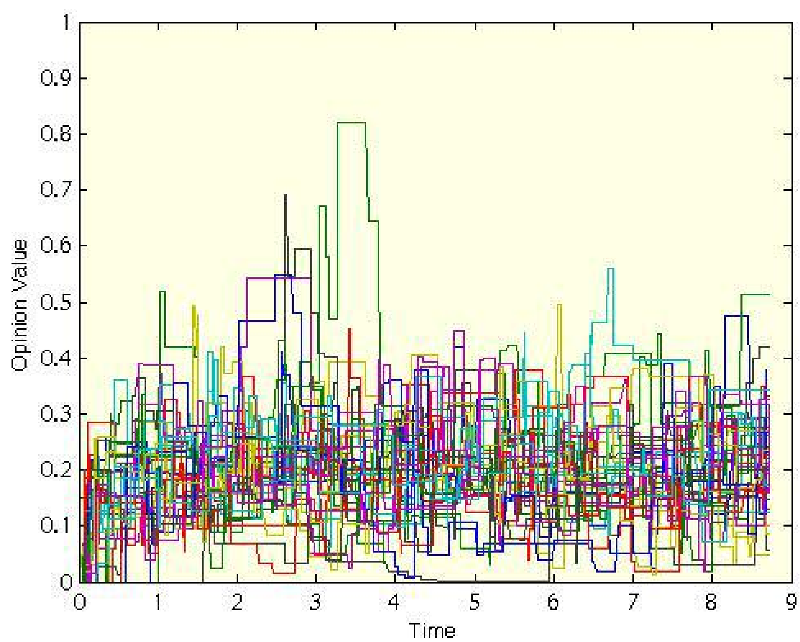


**Figure 5-36. Belief Learning Mechanism, Similar Social Class, Opinion Leaders at 1**

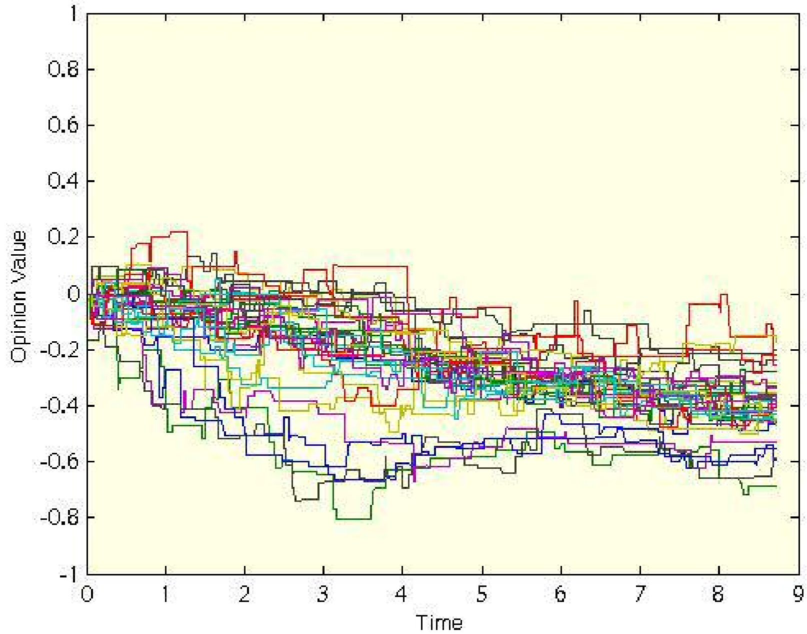


**Figure 5-37. Belief Learning Mechanism, Similar Social Class, Opinion Leaders at -1 and 1**

The previous two scenarios considered constructed social networks based on the similarity between agents' classes. Figure 5-38 and Figure 5-39 show results for scenarios where social networks are created at random, with opinion leaders at 1, and opinion leaders at -1 and 1, respectively. Since agents no longer need to meet a class similarity criterion, there are social class differences that play a role in the opinion revision. As shown in Figure 5-38, the opinion trajectories follow a logarithmic curve, in that opinions revise quickly at first, then taper to an opinion value of approximately 0.65 as the simulation time progressed. In the case where opinion leaders are at -1 and 1, as Figure 5-39 shows, the absolute impact is not as pronounced, most likely due to conflicting opinion values of the opinion leaders. Figure 5-40 shows the consideration set evolution for belief learning with random friends with opinion leaders at 1. It seems as though influential individuals play a more significant role in the belief learning mechanism with random friends as a construct for social networks than they do in the similar friends construct.

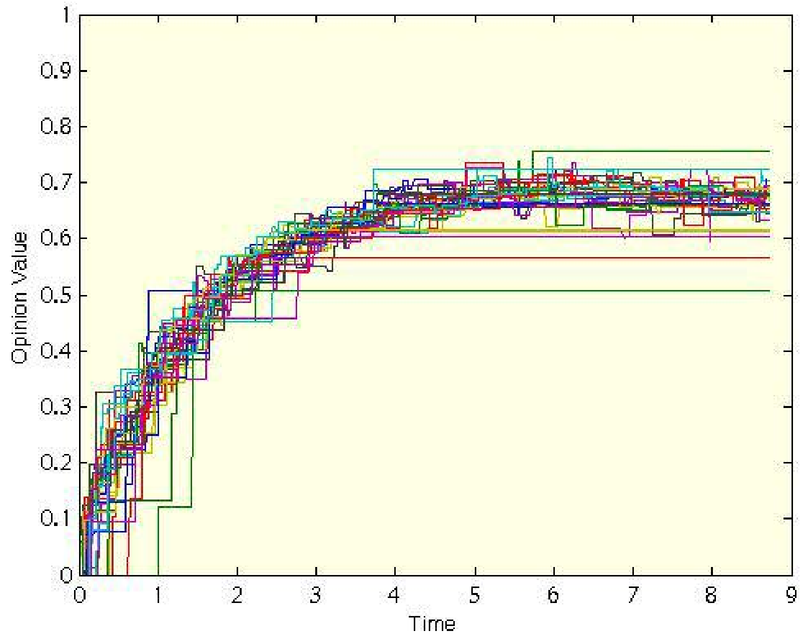


**Figure 5-38. Belief Learning Mechanism, Random Friends, Leaders at 1**

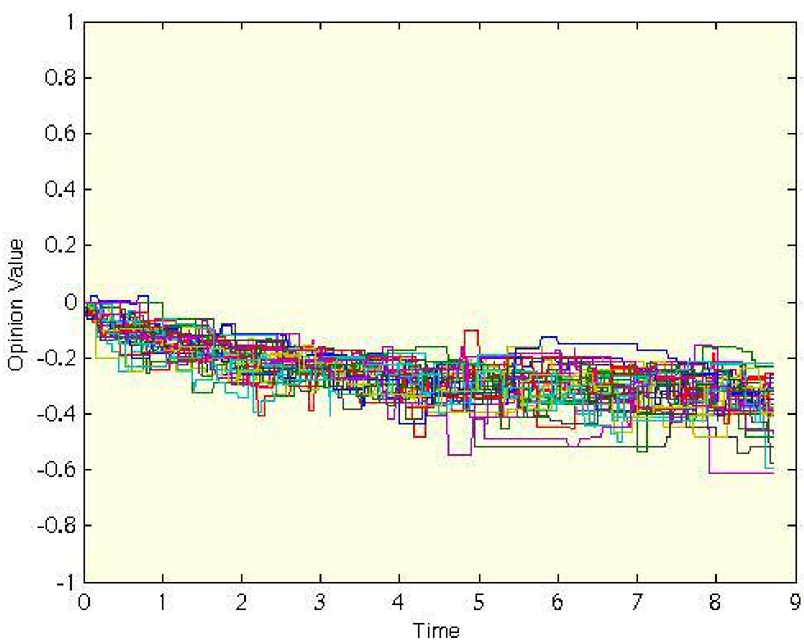


**Figure 5-39. Belief Learning Mechanism, Random Friends, Leaders at -1 and 1**

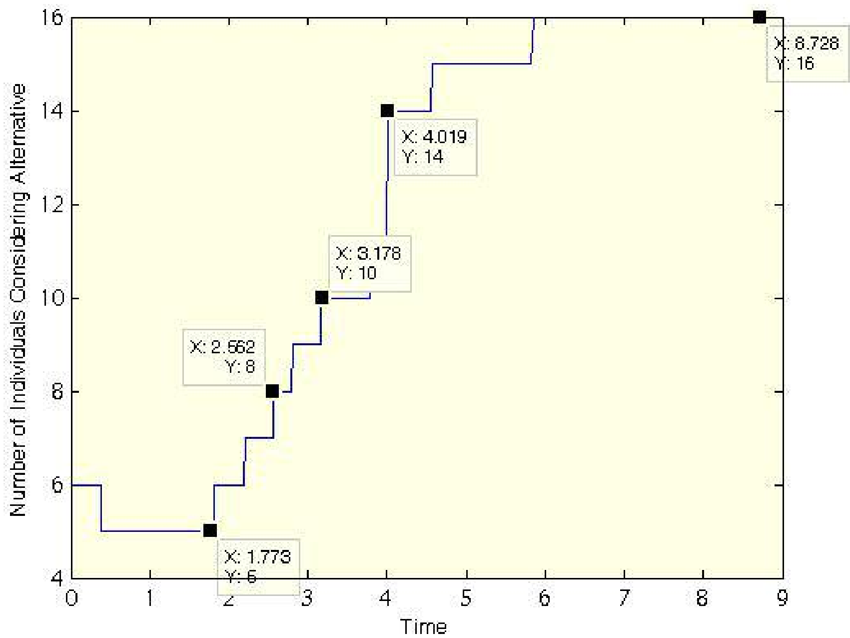


**Figure 5-40. Consideration Number for Belief Learning, Random Friends, Opinion Leaders at 1**

As a first step to investigating the effect of attribute distortion on opinion, Figure 5-41 and Figure 5-42 explore the scenario where agents have a perception factor on what they believe the opinions are of the agents that belong in their social network. This is implemented with a simple uniformly distributed random factor ranging between 0.5, which may represent agents who do not totally believe or trust the opinions of their network, and 1.5 which intends to replicate agents who exaggerate opinions. Opinion trajectories shown in Figure 5-41 suggest that this distortion effect heightens the opinion revision process (i.e., opinion values are higher), and disperses the range of opinions for higher values of simulation time. The dispersion at the end is most likely due to agents reaching their internal thresholds.

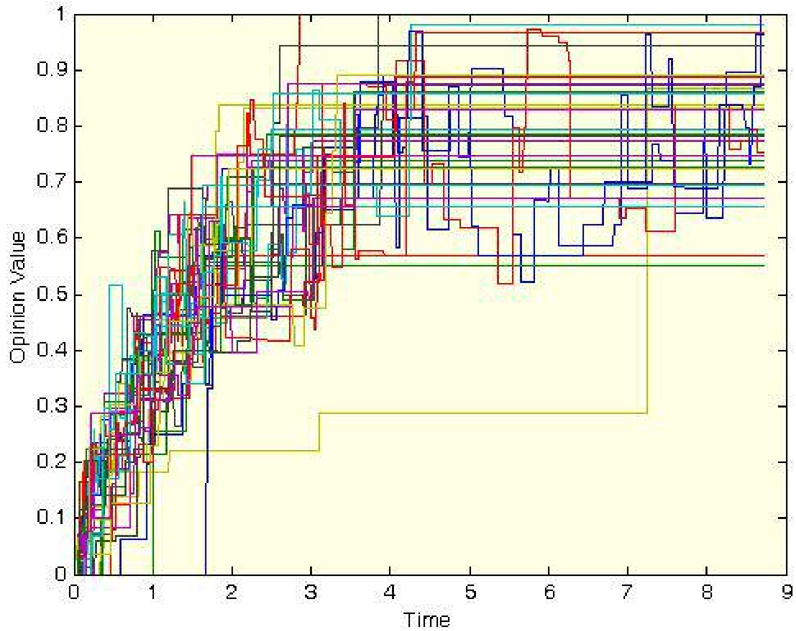


**Figure 5-41. Belief Learning Mechanism, Distorted Perception, Opinion Leaders at 1**

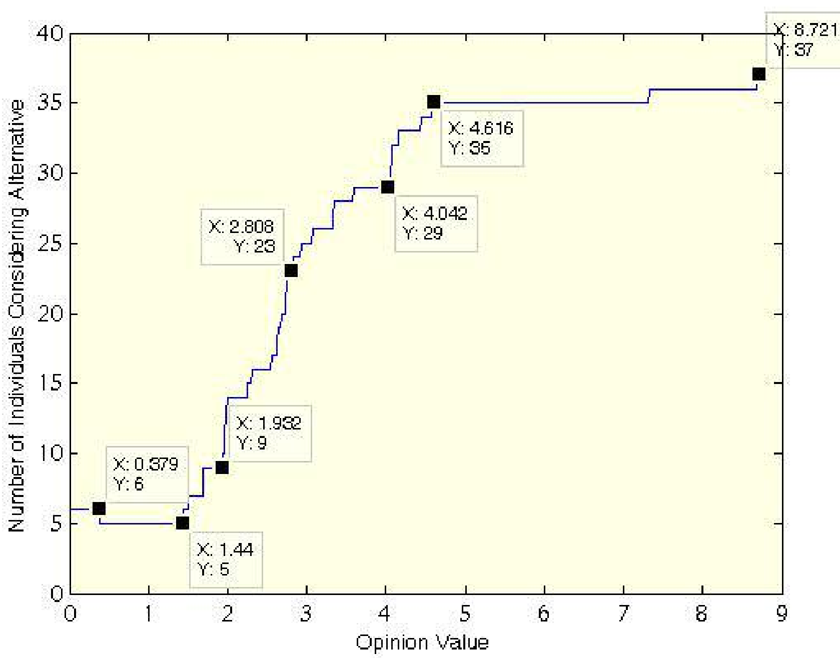


**Figure 5-42. Consideration Number for Belief Learning, Distorted Perception, Opinion Leaders at 1**

As shown in Figure 5-42, 37 of 40 agents are considering the alternative, which suggests that distortions with a positive net effect may actually help increase the number of individuals considering an alternative.

### 5.3.4 Direct Experience Model Results

For the direct experience mechanism, the research intent was to explore the different methods of constructing experiences on which agents compare their current observation to some expected value. Figure 5-43 through Figure 5-46 show the results of modeling expectation as a historical average, a sample average, and a last observation. In the historical average scenario, depicted in Figure 5-43, the dispersion of opinions is greater towards the end of the simulation period. This may be attributed to more observations concatenated to an agent's history. Thus, as time goes on, a large deviation from the history may have a larger impact.

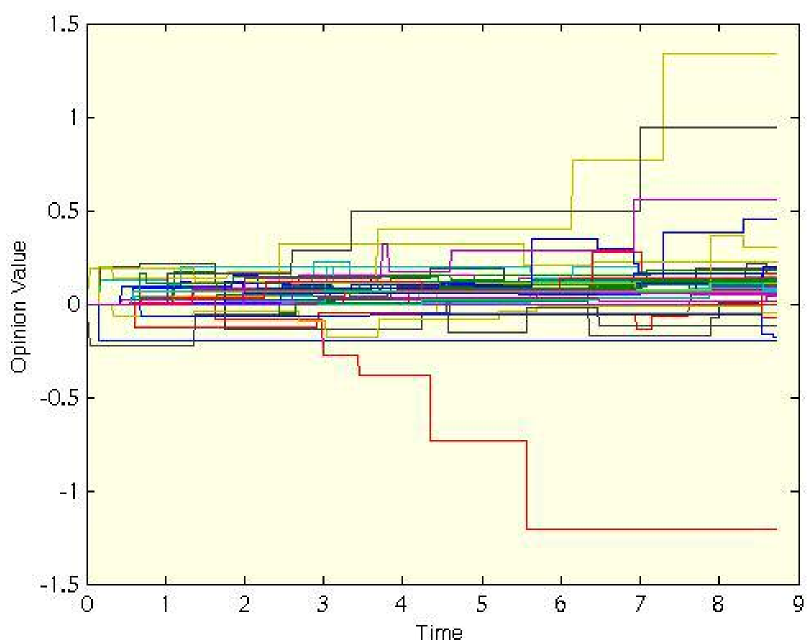


**Figure 5-43. Direct Experience Mechanism, Expect Historical Average**

When considering the sample average scenario, it is important to note that the model assumes that the agent is able to obtain the sample average information. This is not unreasonable, as websites and other information sources often publish average travel times across users. If agents should expect this value, the resulting effect is approximated by the model results in Figure 5-44. There is a much larger variance in opinion values than in the historical average since the sample average takes into account agents who may not have the same opinion patterns as the agent considering the direct experience. Thus, agents who have, for example a shorter travel time because they travel a short distance, may skew the information for the decision making agent.

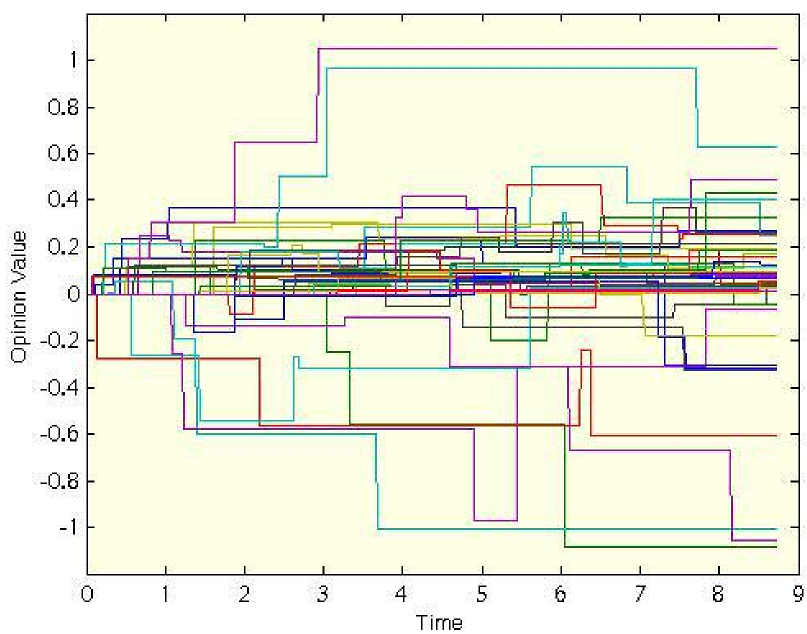


**Figure 5-44. Direct Experience Mechanism, Expect Sample Average**

Figure 5-45 illustrates the simulation results from modeling the last observation scenario. It was hypothesized that this would be the scenario with the highest impact in terms of opinion revision, but the results show that the impacts are relatively mild. This is inherently a characteristic of the experiences generated by the model. This scenario is relevant to real-world situations in that individuals often do not remember their historical observations with high accuracy, and rather consider the last observation. Given that the individual is entrenched in a pattern (e.g., traveling the same route every day), a deviation from the last observation may not have much of an impact on the opinion of the agent.

Figure 5-46 depicts the consideration set evolution for the scenario of the historical average. Very few agents consider the alternative, even though the variance is very high towards the end of the simulation period. This also supports the notion that to encourage consideration of an alternative, that alternative must consistently provide good experiences (i.e., low deviations to produce high satisfaction).

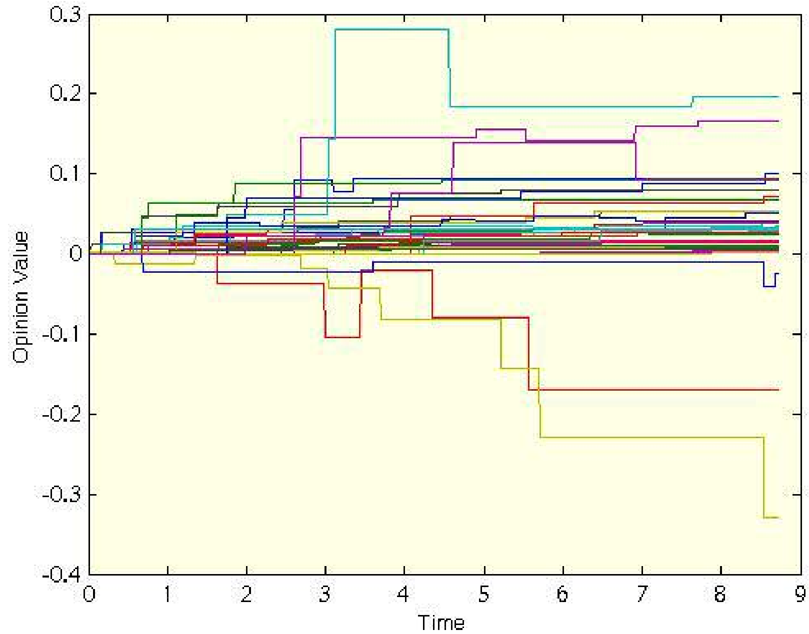


**Figure 5-45. Direct Experience Mechanism, Expect Last Observation**

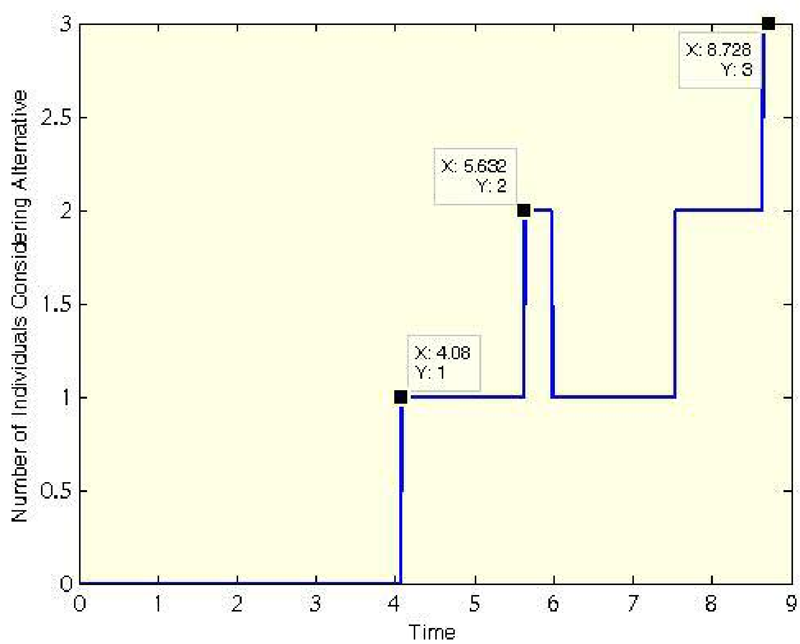


**Figure 5-46. Consideration Number for Direct Experience, Historical Average**

### 5.3.5 Interactive Mechanism Model Results

The interactive mechanism models shown in Figure 5-47 through Figure 5.52 allow for a combination of the mechanisms to interact with each other. These experiments also allow the analyst to explore how these mechanisms work together, which mechanisms may be most effective at encouraging individuals to consider an alternative, and which mechanisms serve as opinion formation mechanisms and which serve as opinion propagation mechanisms. To do this, the opinion of every agent is initialized to 0.

Figure 5-47 looks at the scenario where the agents have an equal probability of interacting with one of the four mechanisms: word-of-mouth, mass media, belief learning, or direct experience. In this scenario, the opinion trajectories are dispersed and follow a variety of updating patterns (e.g., some may be through word of mouth processes while others may be through mass media or direct experience). Figure 5-48 shows the consideration set evolution over the simulation period. At the end of the simulation, only 4 agents are considering the alternative, which suggests that the process of convincing an agent to consider an alternative takes much longer than the simulation period specified by these experiments.

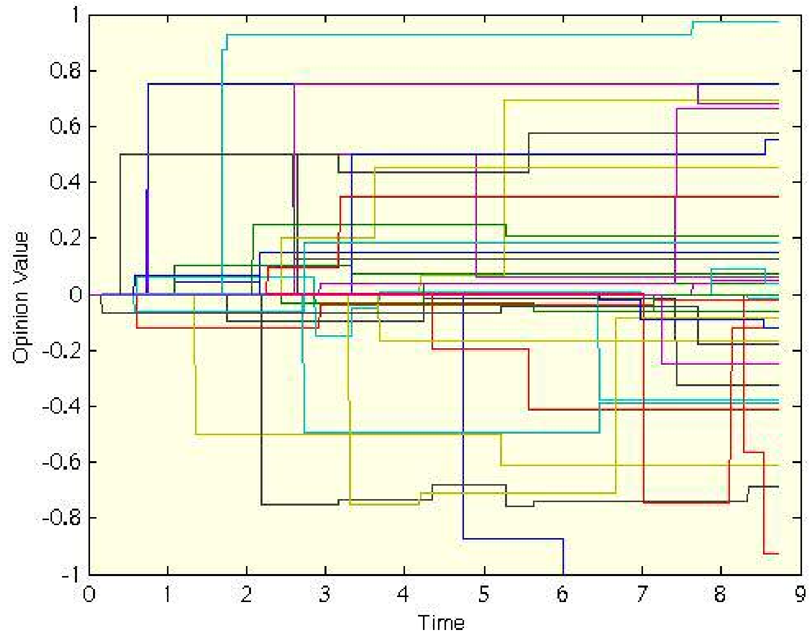


**Figure 5-47. Interactive Mechanisms, Equal Frequencies**

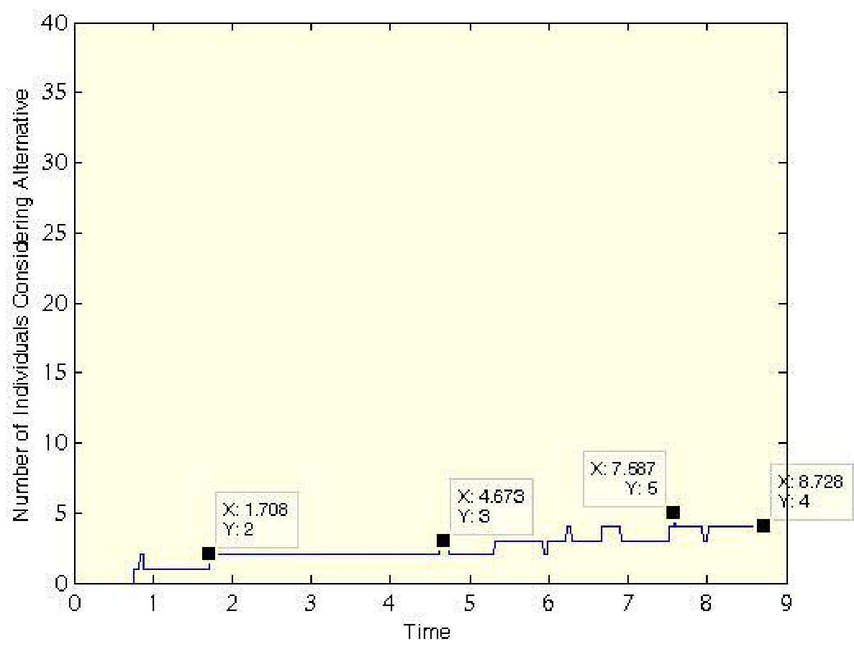


**Figure 5-48. Consideration Number for Interactive Mechanisms, Equal Frequencies**

A scenario where 60% of interactions are through direct experience, 20% of the interactions are through belief learning, and both mass media and word of mouth account for 10% of the interactions each is shown in Figure 5-49 and Figure 5-50. Here, direct experience receives a majority of the interactions as a test to determine whether it plays a role in the formation phase of the opinion process. It is surmised that since agents do not have an initial opinion, the only mechanisms that can help form opinions are the direct experience and mass media mechanisms. Figure 5-49 shows the resulting opinion trajectories for this scenario. It would appear that a majority of the agents have revised their opinion, making a strong case that the direct experience mechanism is one that forms opinions. However, judging from the consideration set evolution in Figure 5-50 it seems that it is not a very effective mechanism for convincing agents to consider the alternative.

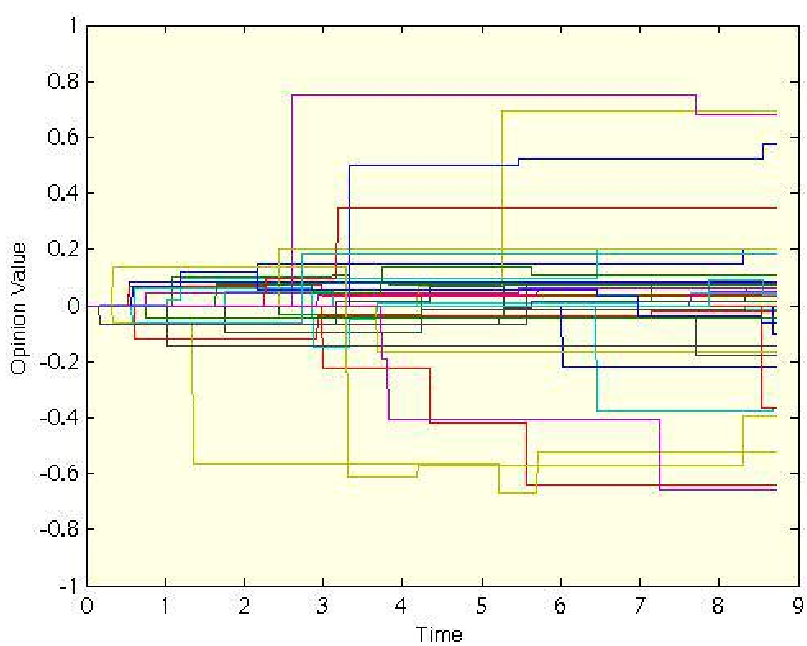


**Figure 5-49. Interactive Mechanisms, 60% Direct Experience, 20% Belief Learning, 10% Word of Mouth, 10% Mass Media**

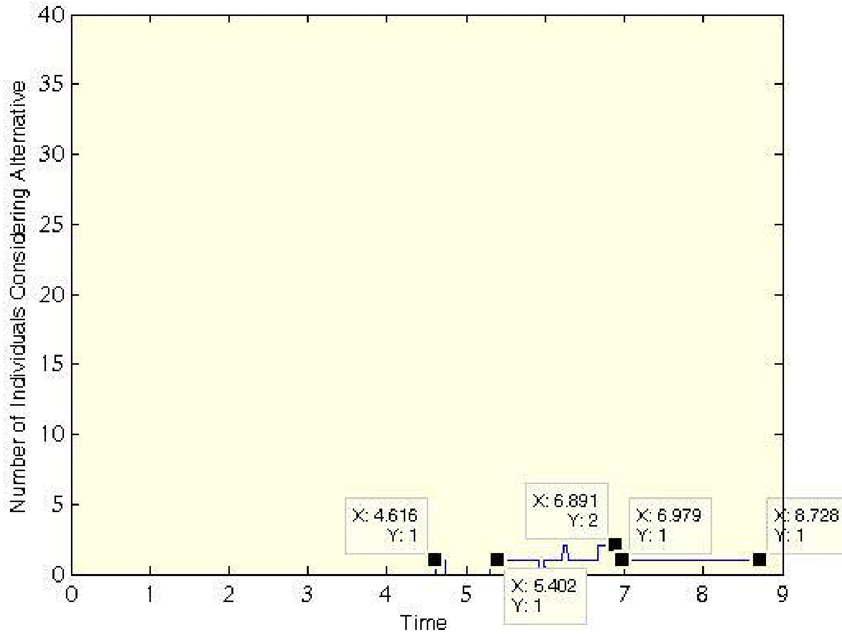


**Figure 5-50. Consideration Number for Interactive Mechanisms, 60% DE, 20% BL, 10% WoM, 10% MM**

To complete the hypothesis that direct experience and mass media are the two primary mechanisms for the formation phase in the opinion process, the research considered a scenario where word-of-mouth consisted of 50% of the interactions, mass media and belief learning each consisted of 20% of the interactions, and direct experience comprising the remaining 10% of the interactions. Results are shown in Figure 5-51. The opinion trajectories suggest that the opinion revisions stem largely from mass media, belief learning, or direct experience, and that word of mouth does not play a major role in the early phase of the simulation period. In looking at the consideration set evolution in Figure 5-52, it is interesting to note that there are more agents considering the alternative than in the previous scenario (2 versus 1). These results might suggest that direct experience and mass media mechanisms affect the formation phase of the opinion process, while word of mouth and belief learning serve as mechanisms to propagate the opinion.

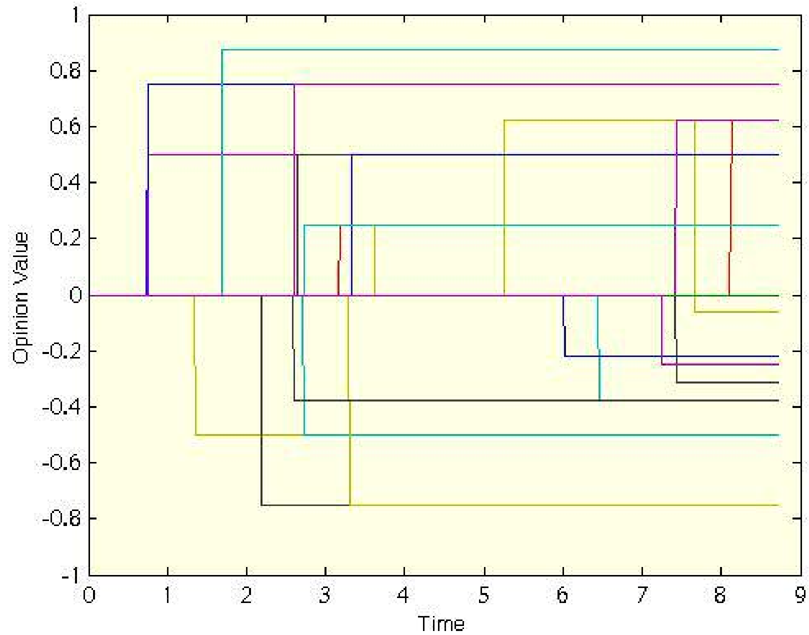


**Figure 5-51. Interactive Mechanisms, 50% Word of Mouth, 20% Mass Media, 20% Belief Learning, and 10% Direct Experience**

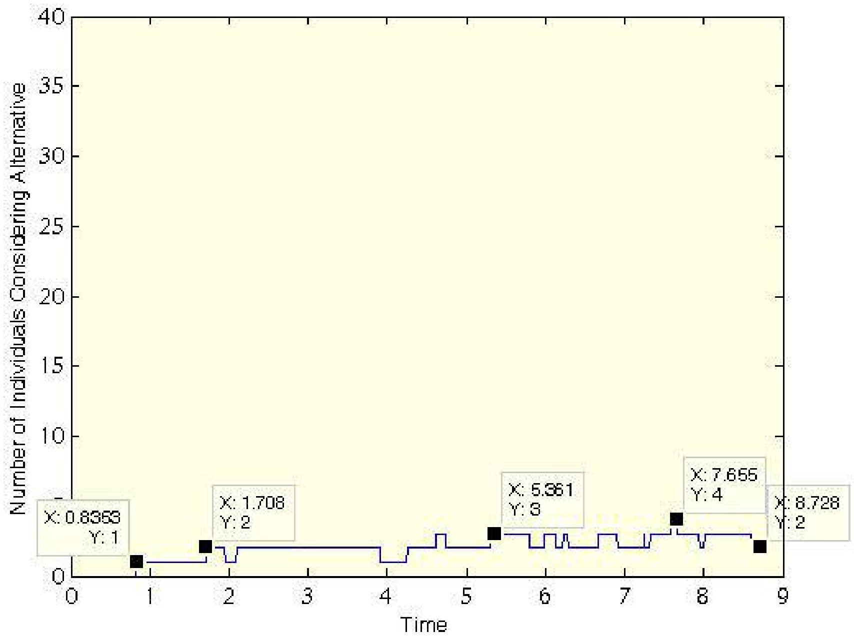


**Figure 5-52. Consideration Number for Interactive Mechanisms, 50% WoM, 20% MM, 20% BL, 10% DE**

## 5.4 Summary of Experimental Results

This chapter implemented the scenarios designed in Chapter 4 and made operational the opinion formation and propagation conceptual framework defined in Chapter 3. System properties scenarios revealed insight to how the simulation program environment governs the rate of interaction and showed that opinion value convergence varies depending on the initial distribution of opinions and the density of agents. Results from the basic model showed the effect of different connectivity criteria, confidence and memory, and coexistence of opinions. In general, imposing a more restrictive connectivity criterion resulted in a lowered rate of opinion revision. Increasing confidence or exaggerating past interactions tended to disperse opinions as some agents had very low connectivity thresholds due to the confidence or exaggeration and subsequently refused to exchange opinions in future interactions.

Coexisting opinion leaders served to disperse opinions as agents oscillated between the opposing views.

Overall results from the complex model results were that accounting for opinion leaders introduced a "gravitational pull" effect towards the leaders when compared with the baseline of a simple-averaging impedance function. This impact is measured by a combination of the difference in opinion trajectories and more distinctly, the difference in the number of agents considering the new service. Leaders have a high effectiveness through word of mouth mechanisms when harboring an opinion of 1. The only real-world scenario this case would apply to is when leaders have been previously informed through some mechanism, and at the start of the simulation, is convinced and an advocate of that opinion. This suggests that it would be to one's advantage to first build a coalition of opinion leaders and convince them of an opinion. Results from the mass media simulation experiments suggest that if media campaigns utilize product differentiation and segmentation strategies to diversify their message to reach different social class or classes in general, the advertisement may have a greater chance at reaching an individual and a greater impact on the opinion revision. Belief learning results suggests that a variety of opinions have a more significant effect than when only a few opinion leaders have an opinion of 1, while everyone else is indifferent. Finally, direct experience results suggest that as a general rule to convincing an individual to consider the alternative, one should keep the variance of the attributes low so as to meet an individual's expectation criterion.

These results will be utilized in Chapter 6 and Chapter 7 as a basis for comparing trends in the evolution of opinions and the number of individuals considering the new service. As these are controlled experiments (i.e., all data and parameters are user-specified), variations are pronounced and directly interpretable, and they offer direct insight to the measurable effects of the mathematical functions on the interaction dynamics. In the application of the opinion formation and propagation framework to a real choice problem, the data and the aggregate sensitivities come from exogenous sources, which could be a source of error. Thus, these experiments are needed as a reference to ensure that results obtained in the application models reflect the simulation results from this chapter.

# Chapter 6: Mode Choice Application

*An Application to Market Adoption of Freight Transport Services*

To apply the opinion formation and propagation model in the context of decision making in transportation, this research utilizes a freight mode choice problem developed for rail-based intermodal services in a trans-European corridor. This application will offer insight into the interrelationship between information and opinion dynamics and choice processes in situations where there is a new policy change or a new alternative. As the opinion formation and propagation framework does not make explicit assumptions about the connections between information, opinions, and choice, a first step in this development is to extend the opinion formation and propagation framework to that of an opinion-choice dynamics framework. This expanded framework will incorporate the entire process of market adoption of a new service. As the mode choice estimation results are an important part of the opinion-choice dynamics framework, the next section of this chapter discusses the choice problem, estimation methodology, and describes the variable statistics. Finally, the last section is devoted to detailing the assumptions of the opinion-choice dynamics models and outlining the scenarios tested.

## 6.1 Extension of Opinion Formation and Propagation Framework

As there is no direct connection between how information about a new alternative spreads through a population, how opinions form and propagate, and how individuals' choice sets and ultimately, choices are formed, it is necessary to extend the opinion formation and propagation framework beyond the discussion in chapter 3 to apply to a transportation choice context. This expanded framework, the opinion-choice dynamics framework, intends to bridge the gap between opinions and choice in situations where there is a new service introduced to the market. Several of the additional components of this framework are derived from Lerman and Manski (1982) in which they describe and model the effect of information diffusion on travel behavior.

### 6.1.1 Initial Conditions

One of the key assumptions in Lerman and Manski (1982) is that at $t = 0$, the market shares are at steady state. This means that those alternatives or policies have been present long enough for the market shares to equilibrate through a choice process and remain stable for some time. In this state, individuals are assumed to have perfect information on the existing alternatives. From a different standpoint, if an individual was sampled from the population, and was given multiple decision making scenarios, he would consistently choose the same alternative under repeated sampling. The opinion-choice dynamics framework incorporates this initial condition of steady-state market shares as this assumption allows the formulation of models that focus only on the new information stemming from the policy change or new alternative. Additionally, the steady-state market share assumption implies that the analyst can capture individual preferences through observations, and can introduce static sensitivities to attributes (i.e., coefficients of travel time, cost) for known alternatives to calculate the corresponding utilities. Since the focus of this research is on the choice evolution process, not on the estimation of choice sensitivities, another assumption is that attribute sensitivities (i.e., coefficients) are exogenously estimated using a discrete choice model (e.g., in this research, a binary logit model is utilized) and are used throughout the simulation as static coefficients.

### 6.1.2 Policy Change, Introduction of Alternative

Once the steady-state market shares are obtained for t = 0, a new policy or new alternative is introduced at t = 1. There are some important notes to observe for the introduction of new information. First, it is assumed that individuals have no *a priori* knowledge of this new policy or alternative, thus all individuals' opinions would be initialized to 0. While it is possible to model a scenario where there is prior knowledge of the new policy or alternative (e.g. initial public offers, insider information), this would require gathering observed information on opinions towards the current chosen alternative, which this research does not have, nor is it in the scope of this research. Rather, it is more interesting to focus on the true effect of opinion formation and propagation from a baseline of indifference or lack of knowledge. Second, it is assumed that the introduction of the policy or alternative is a once-and-for-all change, which means that once introduced, the change affects all individuals, and the change exists until the demand equilibrates. This research assumes that all individuals in these experiments are affected in some way by the introduction of a new service alternative, and that this new service intends to operate permanently.

### 6.1.3 Consideration Set Implications

In the opinion formation and propagation framework, each individual is assumed to have some internal threshold that when exceeded, results in the individual considering the alternative. To extend this to a choice context, the opinion threshold is equated to a consideration set formation mechanism similar to those defined by the literature (for example, see Manski, 1977; Basar and Bhat, 2003; Swait and Ben-Akiva, 1987a, Swait and Ben-Akiva, 1987b). Thus, if the opinion threshold is exceeded, the alternative is placed in the individual's consideration set for further contemplation and is a potential choice outcome.

### 6.1.4 Choice Mechanism

For the opinion-choice dynamics framework, this research utilizes a generalized random utility approach in which the alternative with the highest utility is chosen. This imposes no distributional assumptions about the error structure (e.g., Gumbel distributed for logit, normal for probit). The choice process is thus determined by the specification of the error term, as all other variables are features of the service performance and shipment attributes. Drawing from Lerman and Manski (1982), an individual utilizes the last new piece of information when making a choice. Thus, if new information on an alternative was received in the epoch, there is a possible change in preference towards that alternative. To calculate the resulting utility, the analyst needs to determine the error term. Within the random utility framework, it is specified that the alternative specific constant is the mean of the error term. This research uses Monte Carlo simulation to draw a number of times from a distribution to calculate the alternative specific constant. As this occurs only when new information on an alternative is transferred to an individual, this flexible specification allows for the introduction of serial correlation and panel effects.

As this research seeks to establish some insight into opinion dynamics and their effect on choice, it is assumed that opinions are captured in the error term, as the error term is the residual effect of individual preferences not explicitly specified in the utility equation. It is also assumed that positive opinions have a positive impact on choice, and likewise, negative opinions have a negative impact on choice. To incorporate this into the framework, if an individual has a positive opinion towards that alternative, the draw for the error term is positive, and if an individual has a negative opinion towards the alternative, the draw is negative. The direct effect of this specification is that positive opinions will generate a positive alternative specific constant, increasing the likelihood that the alternative will be chosen, and vice versa for the negative opinions.

Once the alternative specific constants are determined, the framework evaluates the alternatives if they are in the individual's consideration set, and then chooses the alternative with the highest utility given the observed or experienced attribute values. The new aggregate market shares are determined, and these shares are used as input to the initial condition for the next iteration of the framework.

### 6.1.5 Opinion-Choice Dynamics Framework

Figure 6-1 shows the opinion-choice dynamics framework. The system is initialized with steady-state market shares, and individuals having no *a priori* knowledge of the new service alternative. At $t$ = 1, the new service is introduced into the market, and affects all individuals. Individuals are allowed to interact with one another via the opinion formation and propagation model. If an individual's opinion threshold towards an alternative is exceeded, that alternative is placed in the consideration set. If new information on an alternative is received, a number of draws from a distribution given that individual's opinion towards the alternative will determine the new alternative-specific constant. Evaluating the alternatives in the consideration set, a choice is made by choosing the alternative with the highest utility. The choice outcomes are then aggregated over the population, and the new aggregate market shares are utilized as input for the next iteration.

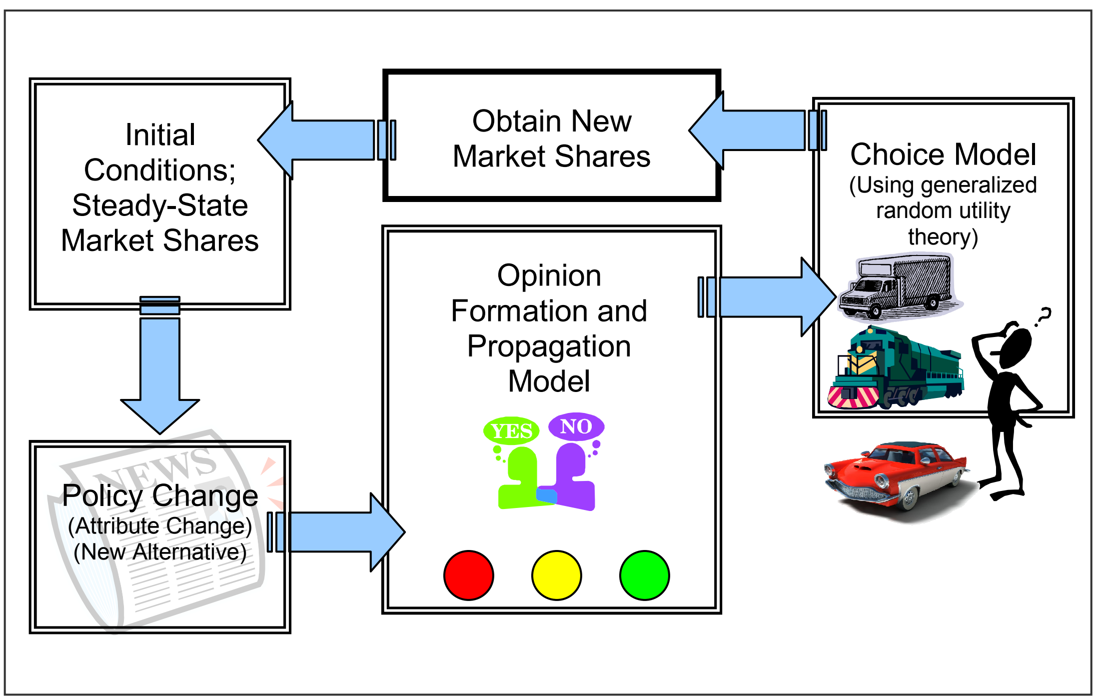


**Figure 6-1. The Opinion-Choice Dynamics Conceptual Framework**

## 6.2 Mode Choice Problem Description

As one of the initial assumptions of the opinion-choice dynamics framework is that individuals' sensitivities to level of service parameters are exogenously estimated, this section is devoted to describing the mode choice problem used to calibrate coefficient estimates for the utility specification. The research utilizes data from the REORIENT – Implementation of Change in the European Railway Area project performed for the European Commission (REORIENT Consortium, 2007). As one of the main goals of this related study was to investigate the potential customer base new rail-based intermodal services could attract, a disaggregate discrete choice model was developed to determine shippers' sensitivities to rail quality indicators (e.g., level of service indicators). This problem offered an interesting opportunity to implement the opinion-choice dynamics framework as it is interested in demand dynamics following the introduction of a new service. A more thorough description of the problem follows in the subsequent subsections, along with the model estimation methodology, and qualitative discussion on the descriptive statistics of the dataset.

### 6.2.1 Problem Description

REORIENT is a Coordination Action funded by the European Commission within the Sixth Framework Programme that addresses Strategic Objective 3.3.1 "Research to Support the European Transport Policy, Research Domain 3.1, Implementation of Change in the European Railway Area". The REORIENT project is examining the effects of the EU's legislation on rail interoperability, which is transforming the European rail freight industry from closed, monopolistic, nationally-oriented businesses insulated from market realities into market players where newcomers both from the rail and logistics industry can find new opportunities, and from non-interoperable nationally-fragmented railway subsystems into an internationally integrated pan-European system (REORIENT Consortium, 2007).

From a research perspective, these massive changes pose a host of challenges in monitoring and understanding how common legislation is transposed under diverse national political and economic conditions, industry changes, and social support and opposition to the changes. From a global perspective, these changes are taking place in the midst of a serious transformation of the transport industry as a whole, and where old solutions rapidly are becoming obsolete (REORIENT Consortium, 2007).

The project focuses on a trans-European transport corridor through eleven countries (called the REORIENT Corridor) stretching from Scandinavia in the north to Greece in the south, and works toward three main objectives (REORIENT Consortium, 2007):

1. Assessing and monitoring the progress toward the development of an integrated freight railway system in the countries located along the REORIENT Corridor, explaining the variation in the status of interoperability across these countries, assessing the degree of political and social support for improving interoperability in these countries, identifying barriers to seamless

rail freight transport through these countries, and recommending ways to overcome the barriers.

2. Identifying and assessing the market potential for new international rail freight transport services through these countries.
3. Evaluating the relevant internal and external effects that will result from implementing the new services, including the effects on rail companies and shippers, and the effects that bear on the whole society and the environment.

The mode choice work satisfies the second and third objectives of the REORIENT project, mainly to understand shippers' quality factors to assist in the identification of market potential for new international rail-based intermodal freight services, and to comprehend the subsequent market shift in demand following the introduction of the new services. Estimating shippers' sensitivities to the quality indicators offers insight to European policymakers of different policies to construct that will attract freight shipments to rail-based transport solution away from truck-based transport solution.

Thus, the focus of developing a mode choice model for policy scenario development involved land-based alternatives, a truck-only mode and a rail-based intermodal mode. While there was information on sea-based or other modes of transport, the simulation platform used to evaluate the policy scenarios focused on land-based strategies to shift freight demand from truck-based solutions to rail-based. It was assumed that among land-based alternatives, the discrete alternatives had error terms that were independently, identically distributed (IID). Consequently, a binary logit mode choice model was specified and estimated.

The REORIENT project considered developing a mode choice model for only one type of commodity which is characteristic for the market for intermodal transport solutions. General cargo is the dominant ETIS manifestation type of the shipper survey typical shipments which are representative for this market. However, because general cargo is a diverse commodity type, because there are small shares of other commodity groups within the segment and because of consistency with the network models we decided to estimate a mode choice model that takes into account the type of commodity by NST/R 11 classification.

### 6.2.2 Estimation Methodology

The shippers' mode choice problem is formulated as a discrete choice model, where a shipper *n*, chooses amongst discrete service alternatives *i*, to transport his shipment. The discrete choice framework assumes that the decision maker (e.g., the shipper) is choosing an alternative (e.g., a mode of transport) that maximizes his utility. By assuming the attributes are commensurate, the attractiveness of an alternative expressed by a vector of attributes values is reducible to a scalar. From this, one can define a single objective function describing the attraction of an alternative in terms of its attributes. The objective function is founded on the notion of trade-offs, or compensatory offsets, that a decision maker uses to compare different attributes, a characteristic of utility that distinguishes it from the other decision rules.

Choice models utilize a random utility maximization approach, in which observed behavioral inconsistencies in choice are taken to be a result of observational deficiencies on part of the analyst. In the random utility framework, individuals are assumed to choose the alternative with the highest utility. However, these utilities are latent (i.e., they are not known to the analyst with certainty) and are treated as random variables. In general, random utility is expressed as an additive function of an observable or systematic component and an unobservable component as shown in equation 6.5. Following this specification, the choice probability is rewritten as equation 6.6.

$$U_{in} = V(z_{in}, S_n) + \varepsilon(z_{in}, S_n) = V_{in} + \varepsilon_{in} \tag{6.5}$$

$$P(i \mid C_n) = \Pr\left[V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn}, all \;\; j \in C_n\right] \tag{6.6}$$

Equation 6.6 has two implications. First, the derivation of specific random utility models stem from the assumption of a joint probability distribution for the error term. Second, if the assumption of the error term distribution is IID, the choice probability of alternative $i$ is only a function of the differences in the systematic component of the utilities. As mentioned previously, a key assumption is that land-based alternatives (i.e., truck-only and rail-based modes) have errors that are IID. To derive the logit model, assume that the error term of the utility function is IID extreme value, also known as the Gumbel distribution or the Type I extreme value distribution. Following the random utility approach argument that individuals are assumed to maximize utility, the objective function to maximize an individual's utility given $J$ alternatives is written in equation 6.7.

$$Max\,U_{jn} = \frac{1}{\mu}\ln\sum_{j=2}^{J_n} e^{\mu V_{jn}}, \mu \quad \textbf{(6.7)}$$

where μ is a scale parameter that for the logit model, is set to 1. The probability of an individual *n* choosing alternative *i* then becomes the formula written in equation 6.8. In evaluating equation 6.8, one needs to take the cumulative distribution of all alternatives not equal to *i*. Given the IID assumption, this *J*-dimensional integral (*J* being the number of alternatives) reduces to the closed-formed expression shown in equation 6.9.

$$P_n(i) = \Pr\left(\varepsilon_{jn} \le V_{in} - V_{jn} + \varepsilon_{in}, \forall j \in C_n, j \ne i\right) \quad \textbf{(6.8)}$$

$$P_n(i) = \frac{e^{Vin}}{\sum_{j \in C_n} e^{V_{jn}}}, 0 \le P_n(i) \le 1, \sum_{i \in C_n} P_n(i) = 1 \quad \textbf{(6.9)}$$

The systematic component of the utility is decomposed into the following components:

$$\mathrm{V}_{\mathrm{n_i}} = \mathrm{V}_{\mathrm{n}} + \mathrm{V}_{\mathrm{n_i}} + \mathrm{V'}_{\mathrm{n_i}} \quad \textbf{(6.10)}$$

where $V_n$ is the portion of the utility associated with characteristics of the shipment, $V_{ni}$ is the portion of the utility associated with alternative *i*, $V'_{ni}$ is the portion of the utility which results from the interaction between alternative *i* and shipment *n*. As mentioned earlier, by assuming the errors are IID only the differences in the systematic component of the utility are relevant in the logit model. To specify the systematic component, it is common to use a functional relationship that is linear in the parameters, as shown in equation 6.11:

$$\mathrm{V}_{\mathrm{n_i}} = \sum_{\mathrm{l}} \alpha_{\mathrm{i,l}} \cdot \mathrm{s}_{\mathrm{n_i,l}} + \sum_{\mathrm{k}} \beta_{\mathrm{i,k}} \cdot \mathrm{x}_{\mathrm{n_i,k}} + \sum_{\mathrm{k,l}} \gamma_{\mathrm{i,k,l}} \cdot \mathrm{g}(\mathrm{x}_{\mathrm{n_i,k}}, \mathrm{s}_{\mathrm{n_i,l}}) \quad \textbf{(6.11)}$$

where *k* represent the dimension of transport quality, *i* denote the transport solution, *l* characterises the shipments (i.e., size variables). The interaction term *g* scales selected variables for transport quality $x_{n_i,k}$ with factors characterizing the commodity (e.g., the unit value of the commodity). This alleviates the drawback that the same parameter estimates are used for several commodity groups. Still, common parameters estimates are more representative for the commodities that are well represented in the data set. Weighing of data is an option to correct for this, but reduces the representativeness of the model for those commodity groups that are most important within the market for intermodal transport solutions.

Due to scarcity of data for some of the commodity groups, the model structure was formulated such that parameter estimates for some dimensions of transport quality were common for several modes and commodities. It was also decided to merge variables characterizing the shipment, variables for quality factors and interaction terms between shipment characteristics and transport quality into a single nest for the REORIENT network platform that considers a choice between two alternatives – truck-based and rail-based or intermodal.

Ideally the freight mode choice model for the REORIENT network model should take into account the causal decision process behind freight mode choices, where a mode is not an option if certain transport quality requirements are not satisfied. There may be instances where, if the quality requirements are not satisfied, then the mode is not chosen regardless of the price. As an example, for fresh perishable food, a minimum commercial speed and a minimum temperature are required. It is possible to take this into account by simply using an infinitely low utility for routes of transport solutions that doesn't satisfy the minimum requirements on the specific lanes. On the other hand, as long as the model uses aggregate commodity groups that consists of commodities with different minimum requirements, the result is either a biased model (if the minimum is set to the least demanding commodity within the group) or a model where the mode is unavailable to a part of the commodities despite the minimum requirement is satisfied (e.g., if the average minimum requirement for the commodity group is chosen). Consequently, for the REORIENT work it was decided not to make explicit representation of minimum requirements. It was assumed that each decision maker is fully informed about the alternatives (i.e., informed about the attribute values and the alternatives) and is a rational decision maker (i.e., preferences are transitive).

#### 6.2.2.1 Method for Imputing Missing Values

To implement a mode choice model within the REORIENT network model it was important to retain the logit structure, which makes it possible to predict mode shifts with respect to possible changes in the level of quality factors for several modes simultaneously at a disaggregate level. In order to estimate a disaggregate mode choice model based on the logit structure, it was necessary to establish data for cases of observed mode choice and data for corresponding variables for antecedent and independent variables in the utility function for all available transport solutions with sufficient transport quality.

From a total of 425 typical shipments in the shipper survey, a data set was prepared in which 27 typical shipments from Lithuania, 3 truck-only shipments to distant destinations across Ural Mountains in Russia, and 1 airborne transport observation were excluded. The transport solution used for the typical shipments were classified as truck-only, rail-based or other, where truck-only transport comprised shipments defined as single modal truck-shipments, rail-based transport was used for those shipments where rail transport was part of the transport chain and other transport for the remaining observations (i.e., shipments with no rail, but designed as ship as part of the transport chain).

The shipper survey data contains observations of mode choice and antecedent variables that describe the shipment, but only data for elements of the observed transport quality of the chosen transport solution. In some cases there are also missing data for elements of quality for the chosen transport solution. For the estimation of the logit model, however, data for the variables representing quality factors in the utility functions is needed for both the chosen and the un-chosen modes. To complete the shipper survey data, an instrument was developed to replace the missing data for quality factors for the un-chosen mode. Only quantitative service quality factors were considered, as there were no variables in the network model corresponding to shipper survey data for qualitative factors. The instrument was developed in terms of one prediction models for each type of transport quality dimension *k* and mode *i*:

$$\tilde{x}_{n_j,k} = f_{i,k}(\gamma_{i,k}) + \varepsilon_k \qquad \textbf{(6.12)}$$

where $\tilde{x}_{n_j,k}$ are data from the shipper survey for the levels of transport quality of typical shipment n with chosen mode *i* and $\boldsymbol{\gamma}_{i,k}$ is a vector of parameters for corresponding predictor variables. Candidates of predictor variables may represent characteristics of the shipments *n*. Distance is a characteristic that is correlated with several quality factors of the shipment lanes. GIS tools established in the REORIENT project were used to determine distances by road between Nuts zones of origin and destination of all shipments *n*. Also included were variables for the shares of truck-only, rail-based and other transport solutions of the flows between origins and destinations of the shipment lanes ($P(T)$, $P(R)$ and $P(O)$). Data for these variables were obtained from the ETIS Base. The typical shipments were classified by NST/R 11 commodity groups. Functional forms of the instruments that represented the extracted ETIS probabilities and distance by road between geographical zones were formulated as independent variables. One function per quality dimension and mode were required, and to ensure that there were non-negative values throughout the imputed data set, the log was taken of all values in the following functional forms of instruments for truck-only, rail-based and other transport solutions:

$$\begin{aligned}
\ln \tilde{x}_{n_T,k} &= f_{T,k}(\boldsymbol{\beta}_{T,k}) = \beta_{0,n_T,k} + \beta_{1n_T,k} \ln P(R) + \beta_{2,n_T,k} \ln P(O) + \beta_{3,n_T,k} \ln \text{dist} + \varepsilon_{n_T,k} \\
\ln \tilde{x}_{n_R,k} &= f_{R,k}(\boldsymbol{\beta}_{R,k}) = \beta_{0n_R,k} + \beta_{1,n_R,k} \ln P(T) + \beta_{2,n_R,k} \ln P(O) + \beta_{3,n_R,k} \ln \text{dist} + \varepsilon_{n_R,k} \\
\ln \tilde{x}_{n_O,k} &= f_{O,k}(\boldsymbol{\beta}_{O,k}) = \beta_{0n_O,k} + \beta_{1,n_O,k} \ln P(T) + \beta_{2,n_O,k} \ln P(R) + \beta_{3,n_O,k} \ln \text{dist} + \varepsilon_{n_O,k}
\end{aligned} \qquad \textbf{(6.13)}$$

where T denotes truck-only, R denotes rail-based and O denotes other transport solutions. It was assumed that regression of these instruments follows general ordinary least squares principles, mainly: (1) the specification is linear and the functional form is correct; (2) the error term has a mean of zero; (3) errors for different observations have the same variance; (4) the errors for different observations are not correlated; and (5) the probability parameters/ variables remain fixed in repeated sampling. In the literature the instrument variables $\hat{x}_{n_j,k} = f_j(\hat{\beta}_{\mathbf{j,k}}, P_T, P_R, P_O, \text{dist})$ are commonly referred to as 2SLS estimates which are statistically consistent**.** The instrument was estimated for the following dimensions of transport quality:

1) Travel Time (hr) (TT)
2) Cost (€) (TC)
3) Booking Time (hr) (BK)
4) Probability of Delay (% of shipments) (DL)
5) Probability of Damage (% of shipments) (DM)
6) Number of Tracking and Tracing Facilities (FC)
7) Transport price per Ton (€ /ton) (CPT)
8) Harbor Time (hr) (HT)
9) Border Time (hr) (BT)
10) Storage Time (hr) (ST)
11) Terminal Time (hr) (TM)

There were 117 observations with either an ETIS probability of 1.0 in one mode (probability of 0 for the other two modes) or ETIS probabilities of 0.5 and 0.5 in two modes (and 0 in the third). These observations were not included in the instrument regression, but when re-introducing those observations for the imputation, the imputed values were frequently negative and inconsistent with chosen alternative attributes, most likely due to the multicollinearity of the alternatives. For cases with ETIS probabilities of 1.0, we removed them from the data set, as this implies that the shipper would always choose the current alternative. For cases where including two probabilities lead to multicollinearity (i.e., the sum of the two probabilities is 1.0), the regression includes only one probability.

### 6.2.3 Survey Data Description

Following the removal of cases where the ETIS probabilities were 1.0 for the chosen mode, there were 318 cases remaining. Table 6-1 displays summary statistics for the level-of-service variables of interest, and Table 6-2 shows statistics for transformed level-of-service variables that were used in the model.

**Table 6-1. Descriptive Statistics for Level of Service Variables**

| | Transport Mode Alternatives | | | | | |
|---|---|---|---|---|---|---|
| | *Truck-Based* | | *Rail-Based* | | *Others* | |
| *PARAMETERS* | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. |
| *LEVEL OF SERVICE (LOS)* | | | | | | |
| **Travel Time (hours)** *(Total door-to-door transport time)* | 14.503 | 25.901 | 22.731 | 40.099 | 50.121 | 89.179 |
| **Price (Eur)** *(Total door-to-door transport price)* | 338.205 | 612.95 | 1481.48 | 3811.9 | 1550.63 | 3930.5 |
| **Booking Time (hours to departure)** *(Deadline for booking transport)* | 8.583 | 15.778 | 8.331 | 14.423 | 16.336 | 38.111 |
| **Tracking Devices** *(Tracking devices/services available)* | 1.217 | 2.772 | 1.079 | 1.993 | 1.244 | 5.817 |
| **% of Shipments Delayed** *(Delayed delivery - % of yearly shipments)* | 0.014 | 0.043 | 0.036 | 0.088 | 0.022 | 0.036 |
| **% of Shipments Damaged** *(Damage/loss of goods - % of yearly shipments)* | 0.0034 | 0.007 | 0.0044 | 0.011 | 0.0045 | 0.007 |
| **Harbor Time (hours)** *(Total time in harbors/ ferry harbors)* | 0.804 | 3.051 | 3.474 | 5.707 | 7.219 | 12.596 |
| **Border Time (hours)** *(Total time at border crossings/ custom posts)* | 1.420 | 4.220 | 1.151 | 1.904 | 12.187 | 104.551 |
| **Terminal Time (hours)** *(Total time at terminals - except at origin and destination)* | 4.944 | 9.829 | 9.629 | 23.555 | 27.339 | 61.995 |

It is important to note that Table 6-2 and Table 6-3 present summary statistics for both real survey responses and imputed values. Table 6-4 presents summary statistics on indicator variables, and Table 6.5 contains statistics on the shipment-specific variables. Both indicator and shipment-characteristic variables were real survey values.

**Table 6-2. Descriptive Statistics for Transformed Level-of-service Variables**

| PARAMETERS | Transport Mode Alternatives | | | | | |
|---|---|---|---|---|---|---|
| | *Truck-Based* | | *Rail-Based* | | *Others* | |
| | Mean | Std. Dev. | Mean | Std. Dev. | Mean | Std. Dev. |
| ***LOS INTERACTIONS*** *(with shipment-specific variables)* | | | | | | |
| **Speed (km/hr)** *(Total distance divided by travel time)* | 11.875 | 23.125 | 7.443 | 15.354 | 4.263 | 11.221 |
| **Price per ton/ Value per ton** *(Intends to capture commodity effects)* | 0.022 | 0.166 | 0.115 | 0.870 | 0.136 | 1.283 |
| **Low Travel Time x Value/ ton** *(For travel time < 70 hours)* | 175767.2 | 2110339 | 339503.2 | 4182498 | 155862.4 | 3294682 |
| **High Travel Time x Value/ ton** *(For travel time ≥ 70 hours)* | 178464.8 | 2839484 | 119429.7 | 1911343 | 532534.1 | 4150727 |
| **Value/ ton** *(For travel time < 70 hours)* | 20779.51 | 141623.9 | 28738.62 | 162127.7 | 86145.54 | 342781.1 |
| **Value/ ton** *(For travel time ≥ 70 hours)* | 42208.31 | 146327.8 | 12229.31 | 71669.54 | 14036.73 | 65490.77 |

**Table 6-3. Descriptive Statistics for Indicator Variables**

| PARAMETERS | Transport Mode Alternatives | | | | | |
|---|---|---|---|---|---|---|
| | *Truck-Based* | | *Rail-Based* | | *Others* | |
| | Freq. | Percent | Freq. | Percent | Freq. | Percent |
| **Chosen Mode** *(1 if mode was chosen, 0 otherwise)* | 202 | 63.52% | 78 | 24.53% | 38 | 11.95% |
| **Hazard** *(1 if shipment is hazardous, 0 otherwise)* | 21 | 36.84% | 36 | 63.16% | 0 | 0.00% |
| **Long Distance** *(1 if total distance is > 2000 km, 0 otherwise)* | 84 | 48.28% | 48 | 27.59% | 42 | 24.14% |
| **High Value** *(1 if value/ton > 20000, 0 otherwise)* | 78 | 78.79% | 9 | 9.09% | 12 | 12.12% |

**Table 6-4. Descriptive Statistics for Shipment Characteristics**

| PARAMETERS | *All Modes* | |
|---|---|---|
| | Mean | Std. Dev. |
| **Distance (km)** | 445.112 | 891.63 |
| **Frequency** | 0.208 | 0.331 |
| **Tons** | 72.346 | 933.2 |
| **Value per ton (Eur/ton)** | 8004.68 | 82857.8 |

Considering the statistics presented in Table 6-3, 63.52% of the shippers chose to ship using a truck-based transport service, 24.53% chose to ship using a rail-based service, and 11.95% chose to ship using either a ship-based or an air-based service. Approximately half of the shippers (48.28%) utilize truck for shipments travelling greater than 2,000 kilometers. Conforming to expectation, a large majority of shippers (78.79%) ship using a truck-based service for high valued goods (shipments having a value per ton of greater than €20,000. No hazardous shipments utilize an others-based service; this supports the hypothesis that truck and rail alternatives are correlated (by certain "land" characteristics) and are perceived differently than the others alternatives.

Examining the summary shown in Table 6-1 for the level-of-service variables, travel time is lowest for the truck mode and highest for the others mode. Likewise, the corresponding speed is highest for truck and lowest for others. Total door-to-door transport price is much lower for truck than is the price for rail-based transport services and others-based services; it is interesting to note that the average price of rail-based services is similar to the prices of others-based services. Price differentials are also reflected in the parameter price per ton / value per ton, since the weight and value are attributes of the shipment and are the same across all alternatives. Statistics for the booking time, border time, and terminal time have similar values for truck and rail, but significantly different values for the others alternative, supporting the argument that there is a perceived difference between shipping by land and by sea.

Of particular interest are the values in Table 6-2 for low travel time (below 70 hours) interacted with the value per ton and high travel time (70 hours or greater) interacted with value per ton. Since the shipper may be faced with a choice set consisting of both low and high travel times, one cannot directly infer that the value per ton is the same across all alternatives and that the effect is due purely to variations in travel time. Instead, the variable values are evaluated across the time threshold (i.e., compare the low travel time with the high travel time for one mode). Summary statistics are evaluated for value per ton within low and high travel time categories. For the truck alternative, it is interesting to note the high value per ton for the high travel times. It may be that these high-value shipments need to travel long distances and that shippers choose truck as it may be the quickest or best quality service. Despite the high value per ton, the average high travel time by value per ton is similar to the low travel time category, which means that most of these observations have travel times just above 70 hours.

It is also interesting to note that for rail, the low travel time by value per ton is much greater than the high travel time by value per ton (339,503 hr-€/ton vs. 119,430 hr-€/ton, respectively). When looking at the value per ton of the shipments segmented by low and high travel time, the average value per ton for travel times less than 70 hours is greater than the value per ton for truck. The rail shipments in the high travel time category are of low value. For the others alternative, the low travel time by value per ton is similar to truck, but high travel time by value per ton is much greater. However, from the value per ton for low travel times for others, it is apparent that very high value goods take other modes (e.g., air-based) that have shorter travel time.

### 6.2.4 Utility Specification

The utility specification for the binary logit model is shown in Table 6.5 and only considers differences between truck-based and rail-based alternatives (i.e., the others-based alternative is dropped). This model was specified in conjunction with the REORIENT network platform development (i.e., the specification utilized attributes that could be modeled in the network). The main considerations for mode choice in the network platform are travel times, price per ton, and value per ton. Since travel times were found to be non-linear, it was chosen to specify a piecewise linear travel time by value per ton parameter that would have a parameter for observations below a time threshold and another parameter for observations above that threshold. By inspection of the travel times in accumulating order, it was determined that a threshold of 70 hours was appropriate.

**Table 6-5. Utility Specification for Binary Freight Mode Choice**

| | $\beta_1$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_3$ | $\beta_4$ | $\beta_5$ |
|---|---|---|---|---|---|---|
| $U_{Truck}$ | 0 | $travel\ time \times (travel\ time < 70) \times \frac{value}{ton}$ | 0 | $travel\ time \times (travel\ time \geq 70) \times \frac{value}{ton}$ | $\frac{price}{value}$ | |
| $U_{Rail}$ | 1 | 0 | $travel\ time \times (travel\ time < 70) \times \frac{value}{ton}$ | $travel\ time \times (travel\ time \geq 70) \times \frac{value}{ton}$ | $\frac{price}{value}$ | $hazard$ |

## 6.3 Description of Model Scenarios

In implementing the opinion-choice dynamics framework, these experiments intends to address the original research objectives of investigating information propagation dynamics, opinion formation and propagation, and interactive opinion-choice dynamics with the inclusion of attribute distortion effects. To satisfy each of these objectives, this research develops three models: 1) an Information Propagation model; 2) an Opinion Formation-Choice model; and 3) an Interactive Opinion-Choice model. Each model builds upon the preceding model, growing in complexity and incorporating more of the opinion-choice dynamics framework components. In this section, each model is described in detail, with the objective each model intends to accomplish, an outline of the model assumptions, and the description of the scenarios to be tested. This research implements the information propagation model and the opinion formation-choice model; the interactive opinion-choice model is described, but the complex formulation and data requirements are beyond the scope of this research and may be considered a future research development.

### 6.3.1 Information Propagation Model

The first component of the objective of this research is to investigate how opinion formation and propagation affects individuals' consideration sets. Thus, the information propagation model looks at how information passes through a population and how agents utilize this information in constructing consideration sets. For this model, the analyst does not make any assumptions about inter-period choice. This model focuses on investigating how agents become aware of the introduction of a new alternative and whether they place the new alternative into their consideration set.

In keeping with the initial condition assumptions prescribed by the opinion-choice dynamics framework, it is assumed that agents' choice sets are stable and well-defined prior to the introduction of REORIENT services (i.e., the agents have perfect information on the truck-only and rail-based alternatives). Agents are also assumed to have no *a priori* information about the new REORIENT services and thus know nothing of the proposed level of service, history of the service or any other performance or evaluation criteria. As such, the agents have an initial opinion of 0. Additionally, as this model is interested in how agents receive new information and consider a new alternative, it does not track the opinions towards other alternatives as it assumes that those opinions are well-formed given that they are in the consideration set. As there is presently no information on the company name, size, and industry presence, this model formulation makes no assumptions on opinion leaders and instead randomly assign the leader designation to 16% of the observations. To determine shipper agents' social class, the model utilizes survey information on the frequency of shipments as a proxy of company size. It is reasonable to believe that larger companies or companies with more resources ship more often. Finally, it is assumed that the new REORIENT service is introduced to the system at $t = 1$.

Following the introduction of the REORIENT service, it is assumed that all agents are affected by this system change, and will interact with other agents with the instinctive intent to restore demand equilibrium. That is, agents are assumed to be open to hearing or exchanging information about the new service. As the opinions are initialized to 0, it is presumed that the initial phase of opinion formation will occur through mass media and direct experience. For mass media, it is assumed that the mechanism positively affects agents' opinions as there are no competing ads for the other alternatives. As there are no new experiences in this experiment, an agent's satisfaction is assumed to be based on the last observation experienced and the mean experience of the users interacted with. This implies that agents, in addition, to communicating their opinions, are also exchanging information. Thus satisfaction is derived by the travel time and price attributes in the utility function. An agent's expected value (for the direct experience mechanism) is drawn from word of mouth interactions and belief learning, where both opinion and information are now exchanged by word-of-mouth, and belief learning entails the perceived opinion and perceived level of service experiences. This experiment also implements a switching mechanism for low levels of satisfaction. In the opinion formation and propagation model, if satisfaction is low, an agent would negatively revise his opinion. This makes no implication on choosing a different alternative. Thus, for this model, if satisfaction is low (judging from current experience and the experiences of others), a simple threshold is implemented to characterize a switching behavior, in addition to revising opinion values. For simplicity, it is assumed agents can only switch to the new service.

Three scenarios are constructed within the information propagation model. The first is a baseline scenario, with no mass media and word-of-mouth mechanisms involved, and only belief learning as a proxy for the direct experience mechanism (since there are no new experiences). The second is the implementation of word-of-mouth, belief learning, and direct experience mechanisms to evaluate the effect of peer-to-peer interactions relative to the base line. Finally, the third scenario is the implementation of all four mechanisms with equal frequencies (i.e., at each epoch, there is equal likelihood of interacting with one of the four mechanisms) to determine the mass media effects relative to the second scenario.

### 6.3.2 Opinion Formation-Choice Model

The next two components of the research objective are to explore the effect of opinions on attribute perception and distortion, as well to explore the effect opinions have on the market adoption of a new service. Building upon the development of the information propagation model, for this model, inter-period choices are permitted using a generalized random utility choice framework described in the opinion-choice dynamics framework. Incorporating the choice mechanism allows for the evaluation of the effects opinion dynamics, as governed by the opinion formation and propagation model, affect choice outcomes. This model also considers a simple implementation that correlates opinions with attribute information distortion.

As this is an extension of the information propagation model, all assumptions from that model apply for the opinion formation-choice model. Coefficients from the mode choice model estimation are utilized to construct utility functions for each alternative and for each agent. Since REORIENT services are rail services, it is assumed that the parameter coefficients from the rail-based intermodal utility equation is applicable to the new service. The model assumes that agents have ten experiences over the course of the simulation, and that all agents are forced to choose an alternative at the same time. In generating experiences for agents to choose an alternative, truck and rail alternative attributes are generated from the descriptive statistics of those observations (mean and variance) using a normal distribution. For the new REORIENT service, the alternative attributes utilize the proposed level of service criteria outlined in the network platform simulation scenarios. Shipment specific characteristics (e.g., shipment tonnage, value of shipment) are generated from the descriptive statistics using a normal distribution.

An important distinction (and limitation) of this model is that it is concerned only about the effects of opinion dynamics on choice of the new alternative. The implication of this statement is that the problem scope is simplified to the generation of the error term for the new REORIENT service only, as the opinion effect is postulated to be contained in the error term. Thus, for this model, the alternative specific constant for the truck-only and rail-based alternatives remain constant throughout the simulation. To generate an alternative specific constant for the REORIENT service, at each choice, agents will instinctively look to see whether they have received new information on the REORIENT service through an interaction with a mechanism. If the agent has received new information during the inter-choice period and the REORIENT service is in the agent's consideration set, the model draws from a normal distribution a user-specified number of times (for these experiments, the number is 100) to construct the error term for the alternative. By taking the mean of the draws, the result is the alternative specific constant. It is important to emphasize that the distributional assumption for the error term draws is not restricted to one distribution as it is not necessary to preserve the IIA assumption as one does for the logit model.

To incorporate the opinion impacts, the model assumes that if the agent has a positive opinion towards the new service, the mean of the error distribution will be positive; likewise, if the agent has a negative opinion towards the new service, the mean of the error distribution is negative. Here, the model sets the mean of the error distribution directly to the opinion of the agent. Once the alternative specific constant is determined from the draws, the model calculates the utilities of each alternative and a choice is made on the utility with the highest alternative. In specifying this, one does not need to impose the logit expression for discrete choice, and one can introduce serial correlation and panel effects to account for the interaction dynamics commonly observed in real populations. To account for the effect opinions may have on attribute distortion, the model imposes an assumption that if an agent's opinion is highly positive or highly negative (10% greater than the agent threshold for consideration), the attribute values will be distorted by some factor (20% improvement or worsening of the attribute value).

For this model, three scenarios are constructed. The first investigates opinion dynamic effects on choices by incorporating the choice mechanism, but allowing agents to interact with the direct experience mechanism only. This scenario reflects many real-world situations in which agents in a competitive business environment may not be willing to share information. The second scenario implements the choice mechanism with all four mechanisms equally likely to occur in any interaction. A second consideration is to proportion the distribution of interactions to reflect a 25% chance of interacting through word-of-mouth, a 10% chance of interacting with mass media, a 25% chance of encountering belief learning, and a 40% chance of interacting with direct experiences. Finally, the third scenario models the effects of correlating opinions with attribute values in the attempt to explore how extreme opinions may distort attribute perceptions.

### 6.3.3 Interactive Opinion-Choice Model

As mentioned in the development of the opinion formation-choice model, the main limitation is that opinion dynamics evolve only for the new alternative, restricting the model to use estimated results for the alternative specific constants of the existing alternatives. The interactive opinion-choice model builds upon the previous two models and relaxes this assumption by allowing opinions towards all alternatives to evolve throughout the simulation. Although very flexible in terms of specification, this model is conceptually very complex and difficult to implement; hence, only a discussion on the theoretical implementation is given here. Making this model operational is a future research avenue.

As in the previous model, all assumptions from the information propagation model hold for the interactive opinion-choice model. Likewise, all assumptions of the opinion formation-choice model except the assumption that the alternative specific constants of the existing alternatives remain constant are maintained in this model. By allowing for the flexible specification of opinions for each alternative, and thus inherently allowing for a flexible specification of the error term, there must be some *a priori* knowledge of the opinion values of the existing alternatives. One could make a general assumption that the existing alternatives garner enough of a favorable opinion to be considered in the choice set, and that the alternative chosen has a higher opinion.

Allowing opinions to vary for each alternative introduces additional dimensions to the social and learning mechanisms employed in the opinion formation and propagation model. The flexibility of specifying opinions transfers to the specification of the mechanisms. For example, one scenario could be to have mass media of the existing rail-based services compete with the mass media of the new REORIENT service, while another scenario could compare a non-competing mass media strategy where each has a separate, positive-only campaign.

Agent interactions via word of mouth become very complex when introducing the exchange of opinions of more than one alternative. There are several possible scenarios for capturing these interactions. One way is that each agent advocates his chosen alternative, and the interacting agent adjusts the opinion for that alternative accordingly. Another way is to allow the agents to exchange opinions on all three alternatives. The latter method is easier to implement, and does not require commensurate adjustments to the opinions of the other alternatives as does the first method. When faced with a choice, the agent examines whether he has received new information on all alternatives in the consideration set. For alternatives which the agent has received new information through interaction, the model draws from a uniform distribution and the mean of those draws is taken to determine the alternative specific constant. Once all alternative specific constants are determined, the utility equations are evaluated using the latest observed experience (i.e., last observed travel time, price, shipment characteristics), and the alternative with the highest utility is then chosen.

## 6.4 Summary of Framework Extension and Scenarios

To apply the context of opinion formation and propagation to a choice framework, this chapter expanded the conceptual framework developed in Chapter 3 by incorporating additional components. This chapter attempts to build the connection between opinion dynamics and choice applications found in transportation studies. The first section discusses the implications of the additional components and presents a generalized opinion-choice dynamics framework that postulates the opinion formation and propagation model as a screening mechanism for the choice set. As one of the main assumptions of the generalized framework is that initial market shares and choice sensitivities are estimated exogenously, the second section discussed the mode choice problem and described the dataset used in this application. Special attention was given to transforming the original data, which contained only observed choices, into a dataset containing information on the non-chosen alternatives. This transformation was necessary to be able to use the utility maximizing decision rule commonly found in transportation choice problems.

The final section discussed varying levels of complexity for implementing the generalized opinion-choice dynamics framework. These levels are parallel to the different classes of model defined in Chapter 4 as a basis for comparison. The information-propagation model scenarios intend to examine how information flows through the system as governed by the mechanisms and interactions. Incorporating repeated choices, the opinion formation-choice model intends to examine whether the mechanisms exhibit the same patterns as do the experimental complex models in Chapter 5. Finally, the interactive opinion-choice model intends to capture the correlated effects between opinions and choice. Chapter 7 will implement the scenarios for the information-propagation and opinion formation-choice models only, as the level of complexity of the interactive opinion-choice model is beyond the scope of this research.

# Chapter 7: Estimation and Simulation Results

*Discrete Choice Model Results and Simulation Output*

In this chapter, results from the mode choice model estimation as well as the results from simulating the complex opinion-choice models outlined in the previous chapter are presented and discussed. Mode choice estimation results are presented first, as the parameter estimates are used in implementing the opinion-choice dynamics framework. Next, the simulation results of the two models, the information propagation model and the opinion formation-choice model, are presented and discussed. Finally, a synopsis of the findings and comments on their implications complete this chapter.

## 7.1 Mode Choice Estimation Results

With missing levels of transport quality as replaced by imputed values ($\tilde{x}_{ink}$), we used the Newton-Raphson algorithm for maximum likelihood in LIMDEP econometric software to simultaneously estimate the linear-in-parameters utility function for each mode. Some trial and error was required to find suitable level of aggregation to obtain significant estimates and to include variation in types of shipments (e.g., commodities) and transport quality. Table 7-1 shows the parameter estimates for the binary logit model obtained at the maximum of the log-likelihood function of the utility.

For the binary logit model considering only truck and rail alternatives, the model rejects 38 cases where the shippers chose the others alternative, resulting in

280 remaining observations. The resulting model produced a log-likelihood for constants only of -165.648 and a log-likelihood at convergence of -136.683. $\rho^2$, an indicator of how well the model fits, higher being better, was 0.2803. The alternative-specific constant for rail is negative, which implies that holding all other attributes constant (e.g., travel time, price), shippers prefer truck-based alternative over rail-based alternatives.

**Table 7-1. Estimation Results for the Binary Logit Model of Mode Choice**

| | Transport Mode Alternatives | | | |
|---|---|---|---|---|
| | *Truck-Based* | | *Rail-Based* | |
| **Variable** | Coeff. | Std. Err. | Coeff. | Std. Err. |
| ***Alternative Specific Constant*** | | | | |
| Rail | | | -0.5215 | 0.2105* |
| ***LOS Attributes*** | | | | |
| Low Travel Time (Travel Time < 70 hrs) x value/ton | -3.038E-06 | 1.337E-06* | -4.616E-06 | 1.289E-06** |
| High Travel Time (Travel Time ≥ 70 hrs) x value/ton | -2.151E-07 | 3.949E-07 | -2.151E-07 | 3.949E-07 |
| Price per ton/ Value per ton | -1.8411 | 1.0094 | -1.8411 | 1.0094 |
| ***Indicator Variables*** | | | | |
| Hazard | | | 1.6001 | 0.5702** |
| Log-likelihood for constants only | -165.648 | | | |
| Log-likelihood at convergence | -136.683 | | | |
| $\rho^2$ | 0.2803 | | | |
| Number of observations | 280 | | | |

Level of significance: All greater than 80%, * > 95%, ** > 99%
Coefficients that weren't significant at the 80% level were restricted to zero and omitted from the table.
Exception is high travel time x value/ton, network platform needs a value of travel time for all values of time.

From the parameter estimates shown, an increase in travel time up to 70 hours decreases the likelihood of shipping by a mode. The interaction with the value per ton attribute implies that the higher the value per ton of the shipment, the more sensitive the shipper is to travel time. It is noteworthy that below 70 hours of travel time, rail is more negatively affected by an increase in travel time than truck. Rail's increased sensitivity to increases in travel time may be attributed to high-value shipments that have a high priority, but are taking longer on average than truck. Above 70 hours, truck and rail alternatives are equally, negatively affected by increases in travel time; however, it is not significant at the 80% level. The magnitude of the parameter is more than 10 times smaller than the low travel time estimate for truck, and over 20 times smaller than the low travel time estimate for rail. It is reasonable to believe that, due to the combination of a significantly smaller parameter estimate and that it is not significant, shippers perceive long periods indifferently since it will take a long time no matter what service they choose.

As expected, price over value is negative and a generic coefficient (i.e., it is the same for both truck and rail alternatives), meaning the higher the price, the less likely the shipper will choose to ship by a mode. Since shippers pay upfront out-of-pocket costs for transport services regardless of mode, it is reasonable to assume that an increase in unit price will be perceived equally onerous across alternatives. We divide price per ton by value per ton (thus cancelling the tons) to again account for commodity effects. This formulation implies that as the value of the shipment increases, the shipper will be less sensitive to the price of shipping the good. It is synonymous with cost divided by income formulations seen in the literature regarding urban mode choice models.

Finally, an indicator for hazardous shipments in the utility for rail has a positive coefficient, which is interpreted as if a shipment is hazardous, it will increase the likelihood of a shipper choosing rail when compared to truck. This is reasonable since rail has fewer accidents (i.e., derailments) than trucks; shippers may perceive rail to be a safe mode for hazardous material transport.

## 7.2 Information Propagation Model Simulation Results

The information propagation model serves two main purposes; one is from the system perspective in that it allows the analyst to see how the agents are interacting using real data, and two stems from the data input in that the analyst can gain insight to the system performance of the current alternatives. As the computation complexity increases non-linearly as the number of agents in the system increases, it was decided for time efficiency to take a random subset of the original mode choice data and have that subset evolve within the model framework. As mentioned in Chapter 4, the simulation program has a limitation in that to guarantee that agents are evenly distributed across the interaction space, the number of agents needs to follow a face-centered cubic rule. It was decided that 84 out of 280, or 30% of the data was to be randomly sampled. This subset has approximately 70% of the agents choosing the truck alternative, and 30% of the agents choosing the rail-based alternative, which is not too different from the original market shares.

Figure 7-1 shows the results of investigating the model where only belief learning, considered a proxy for the direct experience mechanism since there are no new experiences generated for the simulation period, affects the opinion formation and propagation process. Judging from the opinion trajectories, it seems that the initial opinion revisions occur with the switching from the current alternatives to the new service (in that the information received or perceived on travel time and price may be similar to or significantly different than the best of the current alternatives). Belief learning appears to occur once some opinions have formed, an occurrence that was reflected in the simulation experiments in Chapter 5. While it may seem that belief learning mechanism is effective at persuading agents to revise their opinions away from the baseline value of 0, it is not very effective in intensifying the revision to a point of consideration. The consideration set evolution shown in Figure 7-2 illustrates this claim, with only 4 out of 84 agents considering the alternative at the end of the simulation period. This also however, could reflect that there is an inequality in level-of-service factors between truck and rail-based (including the new service) alternatives. It is noticeable that a majority of agents harbor a negative opinion towards the new service, which may stem from the fact that it just does not perform as well as their last (and only) observation, which for the large proportion of agents is a truck alternative experience.

In the second scenario, this research examines the effect of implementing peer-to-peer interaction mechanism, where information is relayed between agents instead of being relayed from a source. Thus the mass media mechanism is left out in this scenario, and direct experience, belief learning, and word-of-mouth mechanisms are employed with equal frequencies. Figure 7-3 depicts the opinion trajectories resulting from the simulation of this scenario. Here, if the agent switches from the current experiences, and is an opinion leader, it is assumed that there will be some positive opinion value at which they will communicate via word of mouth.

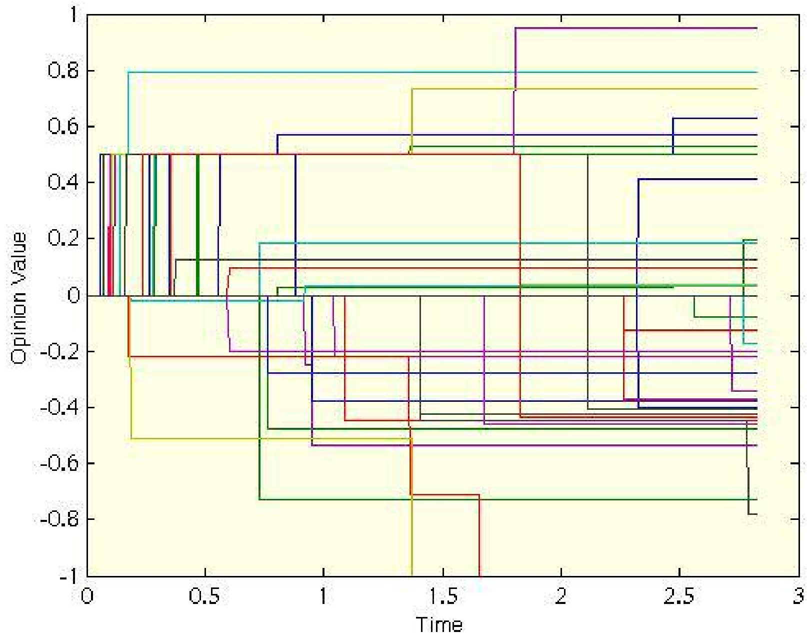


**Figure 7-1. Information Propagation Model, Direct Experience and Belief Learning**

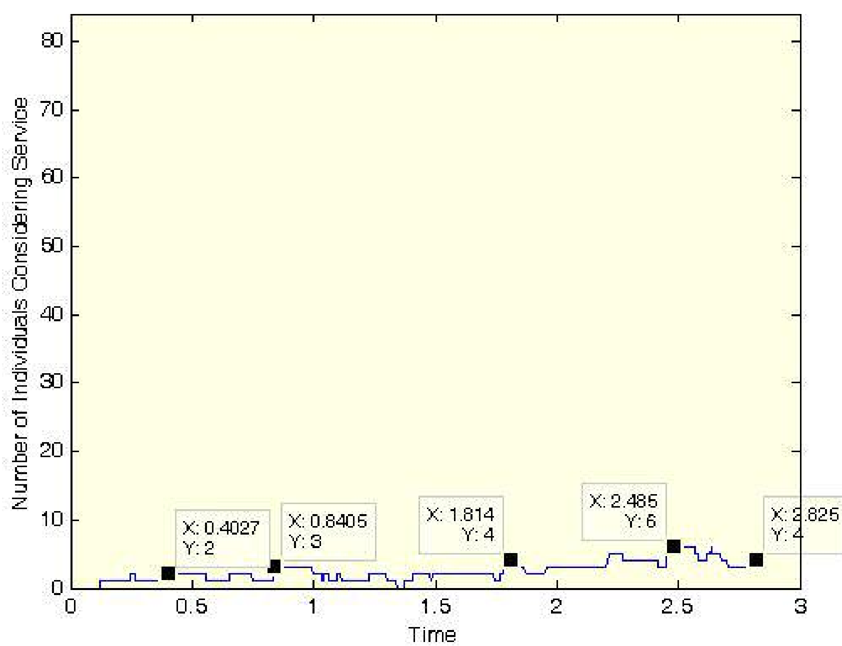


**Figure 7-2. Consideration Number for Information Propagation Model, Direct Experience, Belief Learning**

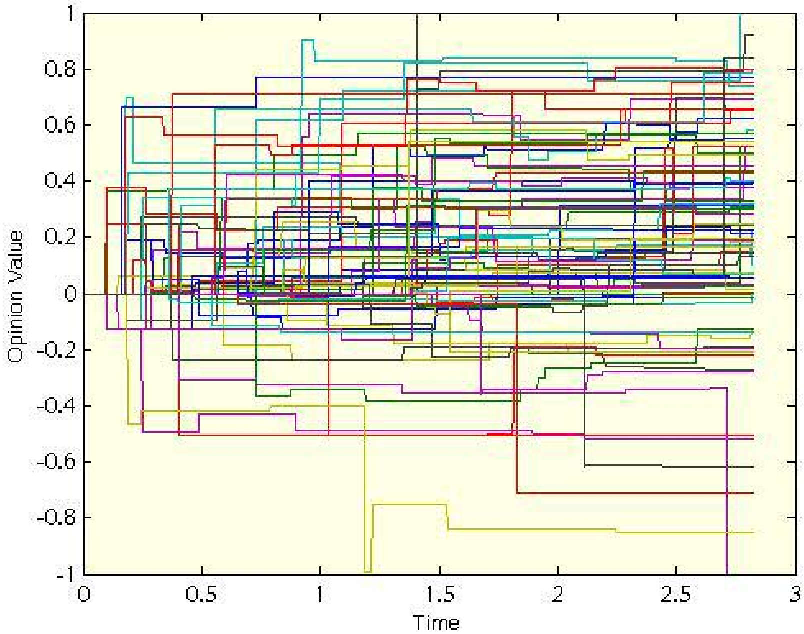


**Figure 7-3. Information Propagation Model, with Word of Mouth, Belief Learning, and Direct Experience**

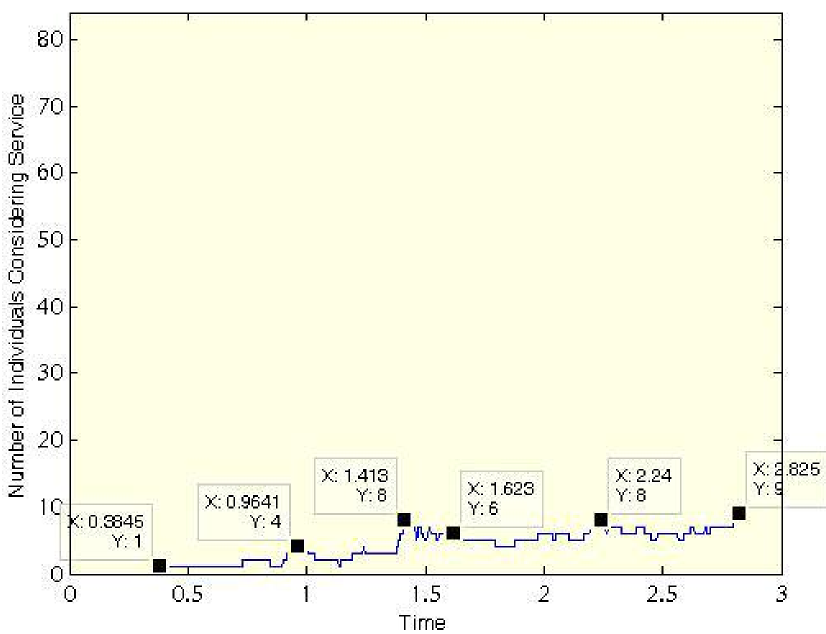


**Figure 7-4. Consideration Number for Information Propagation Model, WofM, BL, and DE**

The opinion trajectories in Figure 7-3 suggest that the word-of-mouth mechanism is responsible for creating a dynamic opinion revision process once an opinion has been formed. Most of the initial opinions seem to form through direct experience as the opinion value initially moves to a target value governed by the mechanism, but then does not change for some time. The interaction of these three mechanisms seems to elevate the opinion revision rate, and increase the overall opinion value towards the new service. Figure 7-4 shows the consideration set evolution for this scenario. More than double the number of agents now considers the alternative. Accounting for influential individuals (i.e., opinion leaders) is important to galvanizing the population towards consideration (or non-consideration) of the alternative.

Simulation results from the scenario where all four mechanisms are employed and have an equal share in the interaction frequency is shown in Figure 7-5 and Figure 7-6. Opinion trajectories shown in Figure 7-5 show less dynamic opinion revisions when compared to the previous scenario, but interestingly, fewer agents harbor a negative opinion. Since mass media is included as a vehicle of information exchange, the corresponding share of word-of-mouth interactions (and other mechanism interactions) drops. Perhaps the inclusion of the mass media mechanism is helping to persuade agents to form a favorable opinion towards the new service, but does not form a convinced or strong opinion. Figure 7-6 shows that there are slightly fewer agents considering the alternative than in the previous scenario.

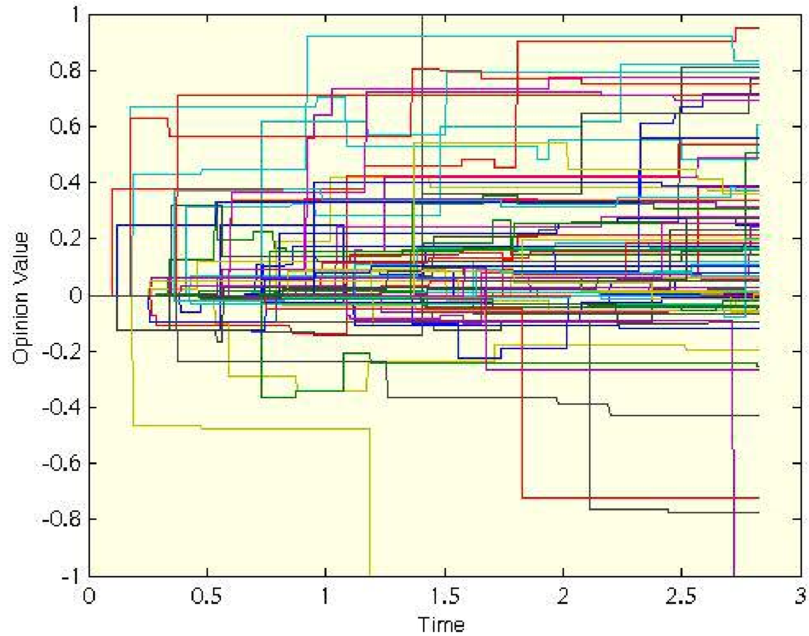


**Figure 7-5. Information Propagation Model, Equal Mechanisms**

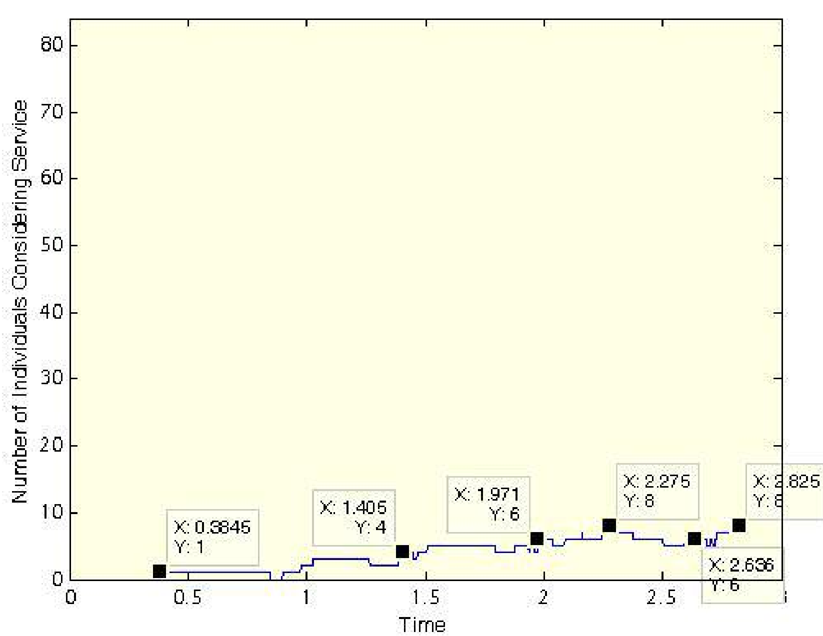


**Figure 7-6. Consideration Number for Information Propagation Model, Equal Mechanisms**

## 7.3 Opinion Formation-Choice Model Simulation Results

For scenarios utilizing the opinion formation-choice model, there are three criteria by which to examine how the mechanisms affect agents' information, opinion, and choice. The first is the opinion value trajectories for the new service. Since this model is only concerned with the opinion towards the new service, the revision process is driven by experience and satisfaction with the new service only, and interaction with other mechanisms that allow for an exchange of opinions. This is slightly different from the information propagation model in which agents' opinion revision was based on their last observation, which is from a current alternative and not the new service. Second, the number of individuals considering the alternative is evaluated as a precursor to investigating the choice dynamics. As the literature suggests (see for example, Ben-Akiva and Swait, 1987a, Ben-Akiva and Boccara, 1995), if an alternative is not in the consideration set, it does not appear in the choice set. Thus, the number of individuals considering the new service is an upper bound on the number of individuals that can choose to utilize the new service. Finally, the choice dynamics as seen through the market shares of agents taking each alternative is observed over time to see if market shares reach convergence or equilibrium, and to see how the choice sets evolve over time.

The first scenario modeled under the opinion formation-choice framework is one where agents revise opinions toward the new service only through direct experiences. Figure 7-7 shows the opinion trajectories resulting from this simulation. A general inference from the trajectories suggests that the design of the new service is within the agents' acceptable level-of-service parameter variance. This is due to the majority of agents having a favorable opinion towards the new service, which indicates that the performance over time has deviated little from the agents' expectation of that alternative. In evaluating the consideration set evolution shown in Figure 7-8, there is double the number of agents considering the service than in the information propagation model scenario where agents interacted exclusively through a direct experience proxy. This may suggest that real observations may be more influential than hearsay (i.e., information exchanged through word of mouth, mass media, or belief learning).

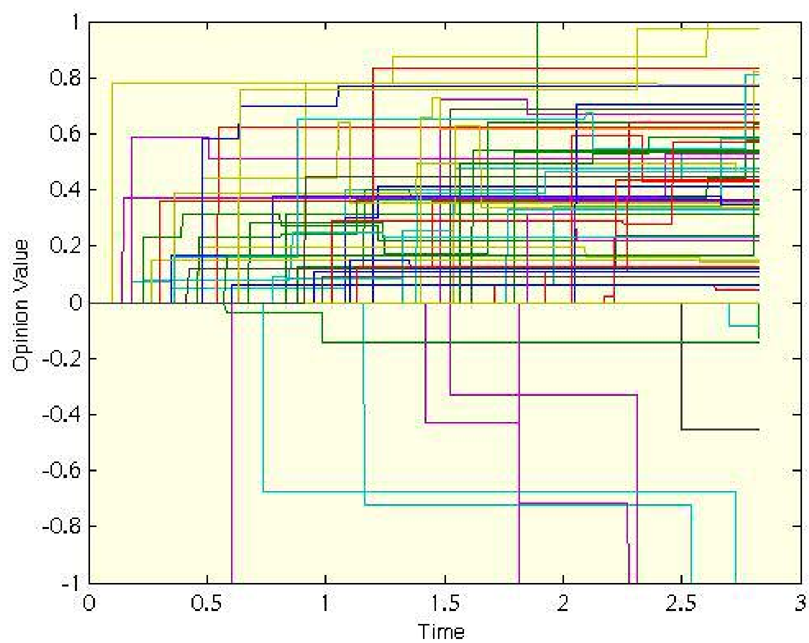


**Figure 7-7. Opinion Formation-Choice Model with Direct Experience Only**

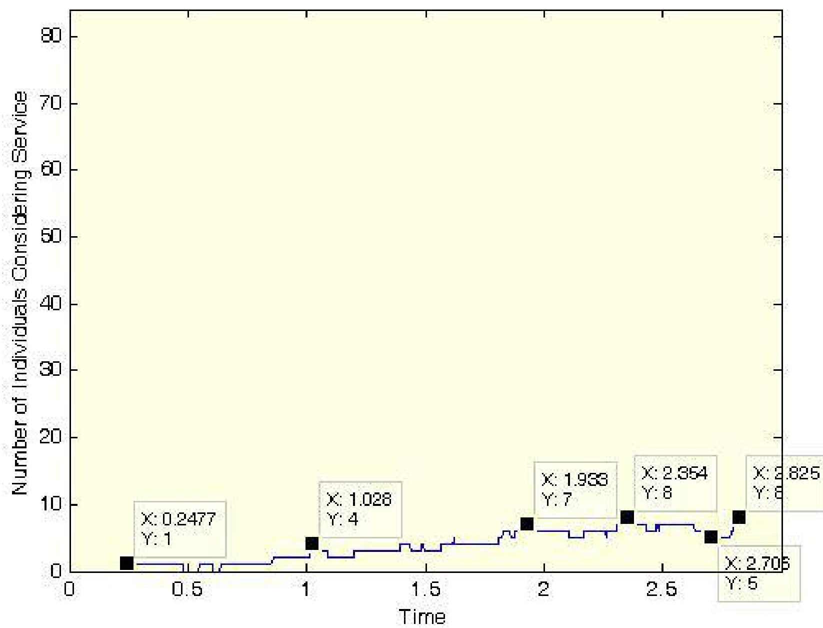


**Figure 7-8. Consideration Number for Opinion Formation-Choice, DE Only**

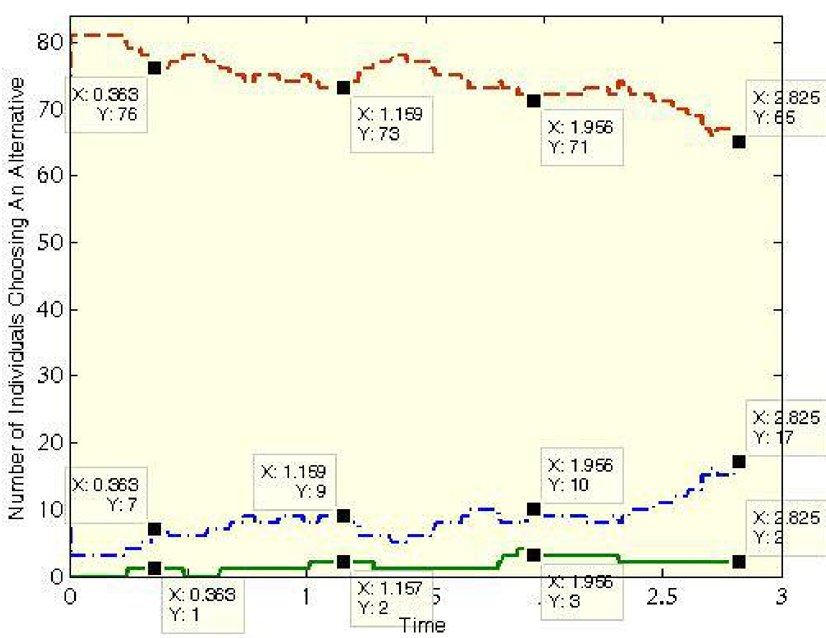


**Figure 7-9. Market Share Evolution for Opinion Formation-Choice Model, DE Only**

Market share dynamics are shown in Figure 7-9. It is interesting to note that even though at most 8 agents consider the new service, only 2 agents actually choose to utilize the new service. This may suggest that even though agents may consider the service, often times the new service is not as attractive (i.e., has a lower utility value) as an existing mode, particularly truck. Another noteworthy observation is that immediately after initializing shares to the data, the mode shares become a reflection of the generated experience. While the mode shares are significantly different at first, towards the end of the simulation period, it appears as though the simulated shares are approaching the initial shares.

It is important to note that by retaining the alternative specific constants for existing alternatives (i.e., truck-only and rail-based alternatives) in the utility equations used in the simulation, only improvements in travel time, costs, and opinion towards the new service will give the new service an edge over the existing alternatives. While this is practical (e.g., opinions of the truck-only and rail-based alternatives are inherently captured in the alternative specific constant), it is limiting in that the model cannot make any distinction on whether there is a compensatory or non-compensatory effect on the opinion values towards the existing alternatives (e.g., if an agent's opinion towards the new service goes up, what happens to the opinion towards the current chosen solution?). Variations in the opinions towards existing alternatives may increase the propensity to switch from existing services to the new service, and also in the propensity to choose the new service.

The next scenario considers agents' interacting with all four mechanisms with equal frequencies. Figure 7-10 shows the resulting opinion trajectories. Again, the trajectories suggest that the initial opinions are formed through direct experience or mass media, and then the opinion values propagated by the belief learning and word-of-mouth mechanisms. Again, fewer agents harbor a negative opinion, which may suggest that a power of persuasion exists through word of mouth, mass media, and belief learning. The persuasion or convincing done by the inclusion of peer-to-peer mechanisms seems to have increased the number of agents considering the new service, as shown by the resulting consideration set evolution plot in Figure 7-11. A maximum of 14 agents consider the service, and at the end of the simulation period, there are 12 agents considering the service. Of these 12 agents, 6 actually choose the new service, as shown by the market share dynamics plot in Figure 7-12.

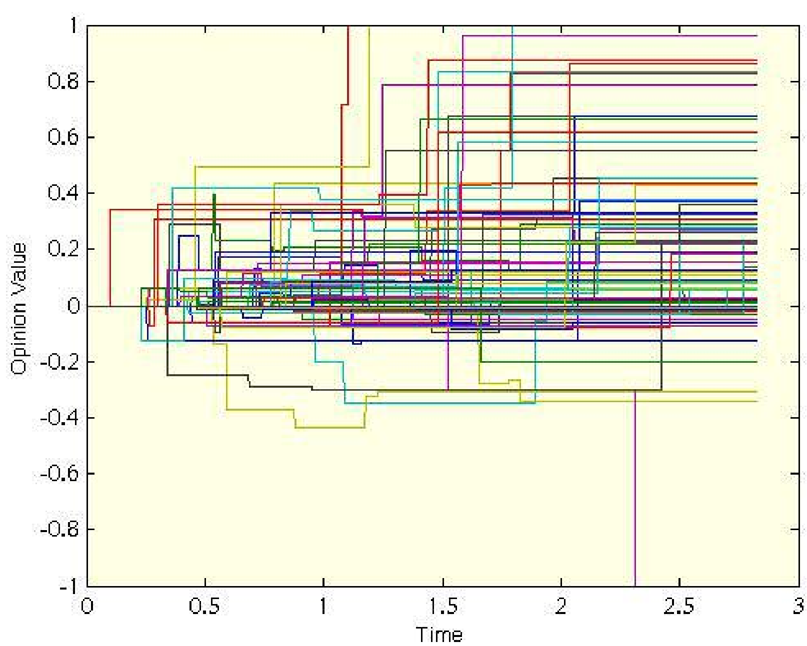


**Figure 7-10. Opinion Formation-Choice Model with Equal Mechanisms**

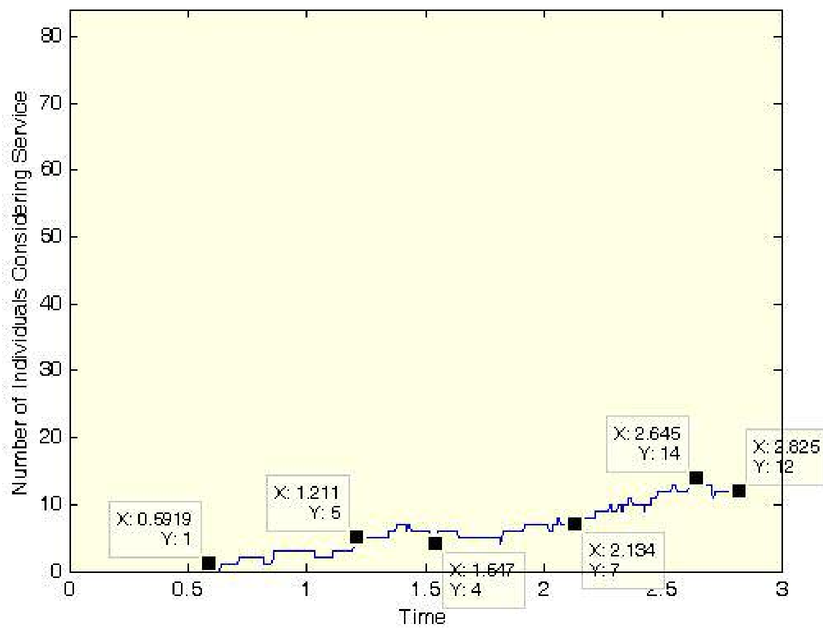


**Figure 7-11. Consideration Number for Opinion Formation-Choice Model with Equal Mechanisms**

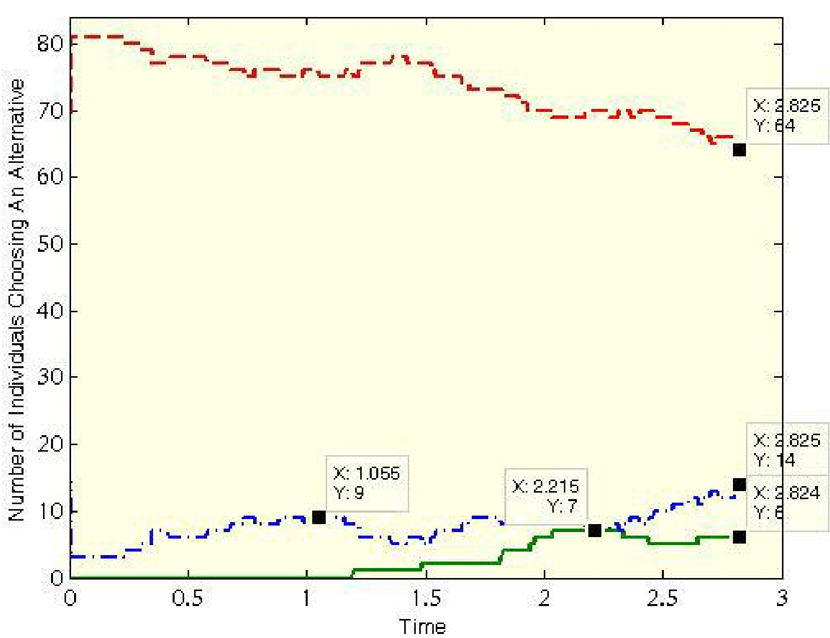


**Figure 7-12. Market Share Evolution for Opinion Formation-Choice Model, Equal Mechanisms**

The final scenario modeled using the opinion formation-choice framework is one where agents with a highly positive or highly negative opinion may distort or bias attribute values to conform to their opinions. Thus, an agent with a high positive opinion is expected to bias the attributes of the new service by improving the level-of-service parameters. It is also assumed that this bias in attributes will bias the agents' opinion as well. Figure 7-13 shows the opinion trajectories resulting from this simulation. Opinion values are very similar to the scenario where there is no attribute distortion; this is as expected since it is the attribute values that are distorted by 30%, while opinion values are only slightly biased more positive. The effect of the slight bias in opinion value on consideration, however, is not insignificant as shown in Figure 7-14. There are 15 agents considering the service, which are three more than in the scenario without attribute distortion.

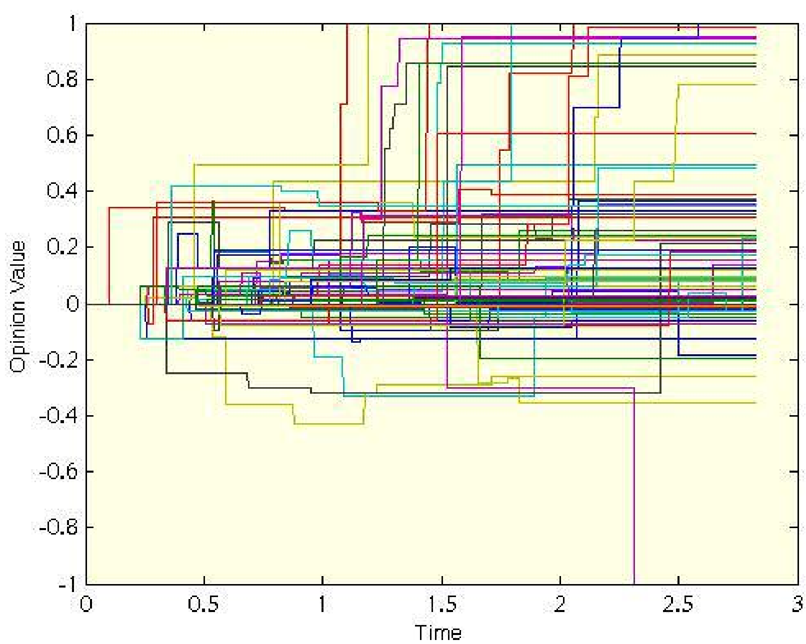


**Figure 7-13. Opinion Formation-Choice Model, Attribute Distortion, Improve Attributes by 30%**

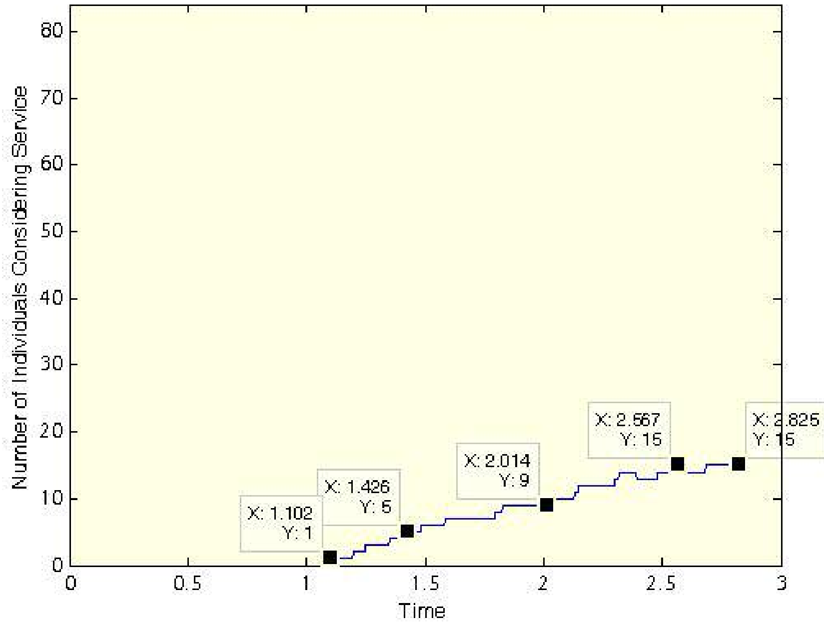


**Figure 7-14. Consideration Number for Opinion Formation-Choice Model with Attribute Distortion**

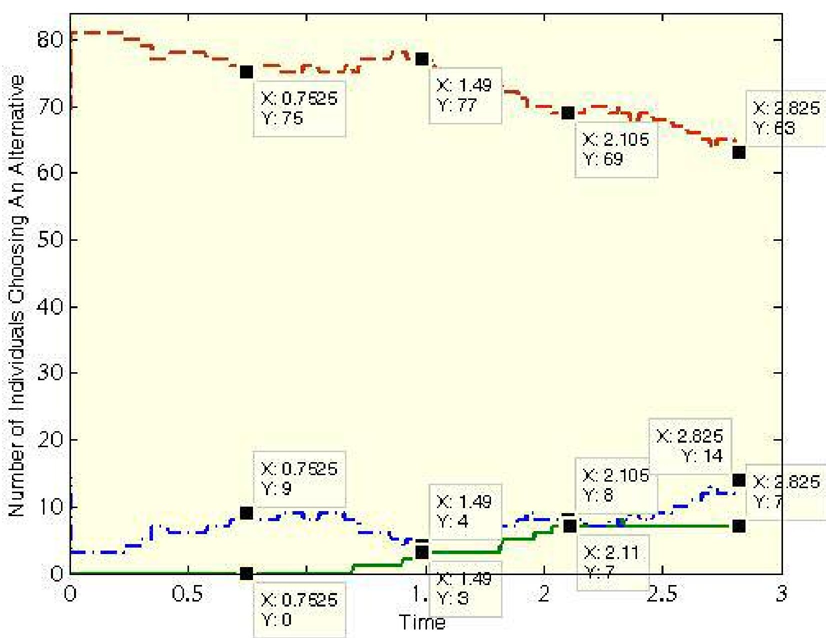


**Figure 7-15. Market Share Evolution of Opinion Formation-Choice Model with Attribute Distortion**

The market share evolution of the scenario with attribute distortion illustrated in Figure 7-15 shows that there is a marginal improvement in the share of agents choosing the new service. These results suggest that by incorporating attribute distortion into the simulation model, one may increase the likelihood of an agent choosing the new service through the perception of improving the attribute values. Through distortion of opinions given a high positive opinion, the agent is more likely to consider the alternative, while through distortion of the new service's attributes given a high positive opinion, the agent is more likely to choose the alternative.

## 7.4 Synopsis of Results

This chapter implemented different aspects of the opinion-choice dynamics framework outlined in Chapter 6. It also presents estimation results for the freight mode choice problem for the REORIENT corridor in Eastern Europe. Overall, the mode choice estimation results conformed to expectations. Travel time parameters were negative, and the travel time parameter for times under 70 hours was more negative for the rail-based alternative than truck. Travel times over 70 hours had a generic parameter estimate and was not significant, suggesting that at high travel times, the marginal disutility of another hour is imperceptible (i.e., the experience is unsatisfactory to the point where additional time does not make it worse). Price sensitivities were also negative. Commodity effects were incorporated to show that for shipments of higher value per ton, the more sensitive the shipment was to travel time variations, and the less sensitive it was to price variations.

The first model, the information propagation model, considered how agents relayed information about their last experience with their current alternative. Several scenarios explored different combinations of the interacting mechanisms by which these agents exchanged opinions and information. Results show that including peer-to-peer mechanisms (e.g., word of mouth, mass media, belief learning) in the interactions tends to have a greater impact on opinion revision than does direct experience. Results also suggest that direct experience is an opinion formation mechanism in that it is effective at convincing agents to revise their opinion away from 0. Depending on the exposure rate and connectivity, mass media may also be an opinion formation mechanism. Word of mouth and belief learning on the other hand, may tend to propagate opinions and intensify them towards the consideration threshold.

In the opinion formation-choice model, agents who considered the new service were assumed to be able to construct a choice set including the service. Utilizing random draws to construct an error distribution, an alternative specific constant was determined as the mean of the error and added to the utility of the new service. A new draw for the alternative specific constant occurred only if the individual received new information on the service in the inter-choice period. Results suggested that real observations may be more influential than secondhand information, and that the incorporation of all four mechanisms garner a higher number of agents considering the service than by the incorporation of direct experience alone. Of those that considered the new service, a fraction actually chose to utilize the new service. This suggests that even though an agent may have reached his opinion threshold and considers the new service, it may have been that the new service was outperformed by an existing service. It also reveals an underlying limitation in assuming alternative specific constants of the existing alternatives to be fixed. In each scenario, the market shares did not converge to a specific value which suggests that had the simulation time continued, more revisions might continue to alter the market shares until at some point, the shares reach equilibrium. The relation between the number of agents considering the new service and the market share dynamics shed insight on consideration sets and choice set stability. Intuitively, once the number of agents considering the new service stabilizes or reaches some saturation level, the choice sets will also stabilize and the market shares will reach equilibrium.

# Chapter 8: Conclusions

*Evaluation of Findings, Future Research Avenues*

This chapter presents concluding remarks on the research findings and discusses future direction for further research on the topics covered. General conclusions drawn from the various components of this research are summarized in the first section. Section 8.2 presents the author's perspective on the research contributions of the study of opinion formation and propagation and opinion-choice dynamics. Finally, the last section discusses some of the limitations of this research, as well as potential avenues for future research and development in this research area.

## 8.1 General Conclusions

This section first presents the state-of-art of utility models, decision making in transportation, and the role of opinions in choice. It then focuses on the major finds of this research.

Much of the research to date on the random utility model development has focused on the estimation procedure and the accuracy of the parameter estimates. Driving this research is the argument that accurate parameter estimates will yield the correct sensitivities to changes in the attributes due to policy changes or the introduction of a new service. While unbiased parameters are quintessential to provide policy-makers with an accurate decision-making tool, little has been done to explore the dynamics of the process by which the market share adjusts to the changes in the environment stemming from such policy decisions (e.g., decrease in travel time, increases in parking costs). It is important to evaluate the process of these market shifts since it is not realistic to expect the entire population to become aware of changes in the attributes instantaneously. Rather, individuals may learn about these changes through various social and interpersonal mechanisms over time, such as word-of-mouth, mass-media, direct experience, and belief learning. It is also important to consider whether the information diffusion is ultimately affecting an individual's choice, especially when considering market adoption of a new service, and to what extent attribute distortion plays a role in choice mechanisms.

Within the context of travel demand, to the author's knowledge, there has been no known research that explicitly accounts for the role of opinions in a choice framework. The literature on opinions has shed some insight that a favorable opinion does not necessarily result in an observed choice; rather, a favorable opinion towards an alternative may encourage an individual to try the alternative for a trial period. The connection in these findings form the basis for constructing opinions to have a role in screening relevant alternatives for consideration in the final choice set. If an individual has a significantly positive opinion (i.e., significantly different from neutral or some internal threshold) towards an alternative, the alternative is placed in the choice set for final consideration according to some decision rule. If the opinion is negative, or less than some internal threshold, the alternative does not enter the choice set.

Investigating opinion dynamics in the context of a choice framework also offers insight to how information about the attributes, and more importantly the changes in the attributes, spreads amongst a population. Since opinions are social and environmentally contextual the mechanisms behind their formation may correlate with information exchange. Exploring these mechanisms of information exchange offers insight to how individuals may react to changes in the level of service of a particular alternative, and more importantly, how over time the information about a new alternative or service may propagate through a population. This research intended to construct mathematical representations of these social and learning interactions to approximate opinion formation and propagation and information exchange within a population. As an extension to a choice framework, this research explored the relationships between opinions formation and information propagation to choice set generation, perceptions of attribute information, and the market adoption of a new service.

To accomplish the research objectives, this research compiled qualitative and quantitative pieces of studies that helped define elements of the opinion formation and propagation process seen in real-world situations. The goal was to develop mechanisms that would reflect real-world phenomenon such as social gravitation effects due to influential individuals, marketing and advertising behaviors, collective opinion effects due to social networks, and satisfaction and learning. Mathematical relationships were introduced in an attempt to model these situations. A general conceptual framework of the opinion formation and propagation process was then developed and implemented in an agent-based simulation program.

Overall results from these experimental results were that accounting for opinion leaders introduced an effective "gravitational pull" towards the leaders. Leaders have a high effectiveness through word of mouth mechanisms when harboring an opinion of 1, which may suggest that it would be to one's advantage to first build a coalition of opinion leaders and convince them of an opinion. Mass media experiments suggests that if media campaigns diversify their message among different social class or classes in general, the advertisement may have a greater chance at reaching an individual and a greater impact on the opinion revision. Belief learning results suggests that a variety of opinions have a more significant effect than when only a few opinion leaders have an opinion of 1, while everyone else is indifferent. Finally, direct experience results suggest that as a general rule to convincing an individual to consider the alternative, one should keep the variance of the attributes low.

To apply this opinion formation and propagation framework to a choice context, the next phase of this research focused on extending it to an opinion-choice dynamics framework. Here, the opinion formation and propagation model serves as a screening mechanism for the choice set. Opinions are correlated with choices in that the alternative specific constant of the alternatives are computed from an error distribution constructed only when the individual has received some new information about that alternative. This allows the analyst to incorporate serial correlation and panel effects without needing to specify the error term for specific interactions. As an application, a mode choice problem was described and the research utilized a real dataset on freight mode choice to estimate attribute sensitivities (i.e., parameter coefficients). Findings from the mode choice estimation results conformed to expectations. Travel time parameters were negative, and the travel time parameter for times under 70 hours was more negative for the rail-based alternative than truck. Travel times over 70 hours had a generic parameter estimate and was not significant, suggesting that at high travel times, the marginal disutility of another hour is imperceptible (i.e., the experience is unsatisfactory to the point where additional time does not make it worse). Price sensitivities were also negative. Commodity effects were incorporated to show that for shipments of higher value per ton, the more sensitive the shipment was to travel time variations, and the less sensitive it was to price variations. The estimated coefficients were then used in the utility equations for the model implementation of the opinion-choice dynamics framework.

In the implementation of the opinion-choice dynamics framework, this research investigated two models that incorporated different aspects of the framework. The first model, the information propagation model, considered how agents relayed information about their last experience with their current alternative. Several scenarios explored different combinations of the interacting mechanisms by which these agents exchanged opinions and information. Results show that including peer-to-peer mechanisms (e.g., word of mouth, mass media, belief learning) in the interactions tends to have a greater impact on opinion revision than does direct experience. Results also suggest that direct experience is an opinion formation mechanism in that it is effective at convincing agents to revise their opinion away from 0. Depending on the exposure rate and connectivity, mass media may also be an opinion formation mechanism. Word of mouth and belief learning on the other hand, may tend to propagate opinions and intensify them towards the consideration threshold.

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## 8.2 Research Contributions

The primary contributions of this research are the explicit modeling of social and learning mechanisms and their effects on opinion formation and propagation, the evolution of these opinions over time, and an exploration of the role that opinion dynamics have in choice processes. This research offers insight to the process of evolving attitudes, perceptions, and opinions and the effects on individuals' judgment and decision making. It also offers insight to the effects of attribute distortion on decision making. The added value of this research will hopefully compel policy makers to consider social and environmental contexts into account when modeling individual choice.

Concerning the word-of-mouth mechanism, results from the scenario testing could encourage public agencies into adopting strategies of spreading information about a new policy change or an innovation to a select target group of individuals who are influential in forming a favorable opinion towards the innovation and in propagating their opinion to others. Results from the mass-media mechanism scenarios offers insights to the effectiveness of segmentation versus reminder ads strategies to better match the consumers' needs in order to evoke a response. With the internet becoming a more prevalent marketplace for idea and opinion exchange, the belief learning mechanism results support the notion that it is important to be sensitive to the information published on the services of interest. Finally, the direct experience mechanism results reflect users' sensitivity to the variability in level-of-service attributes and could emphasize to policy makers the need to reduce uncertainties and variances in the experience of the service. Interaction of the different mechanisms offers insight to developing strategies to form opinions (i.e., employ mass media and direct experience mechanisms initially) and other strategies to propagation and intensify opinions (i.e., employ word-of-mouth and belief learning mechanisms).

A secondary contribution of this research is the development of the freight mode choice model for a freight corridor in Eastern Europe. Combined with data on the evaluation of the system, this research offers insight to shippers' sensitivities to current level-of-service attributes and projected future services through the mode choice model and the simulation experiment.

## 8.3 Limitations, Future Research Directions

While this research sheds many insights to opinion formation and propagation and opinion-choice dynamics, as well as freight mode choice estimation results, there are several limitations to the findings of this largely exploratory research. One of the major limitations is the absence of empirically validated results. There is limited effort in collecting data on actual opinions towards alternatives in transportation decision making applications, such as mode choice; as such, opinions are largely latent in transport choices. This research has assumed random distributions in assigning opinion values and attributes of social class and more importantly social types. Another limitation is that social networks and opinions constitute a rapidly growing field; since the beginning of this study, several new works have surfaced that have looked at additional aspects of opinion formation and propagation. In real-world situations, there may be other mechanisms to consider when forming or propagating opinions. Yet another limitation of this research is that the simulation program used to evolve opinion and choices did not incorporate group meetings or instances where more than one molecule collide. Further, the simulation results did not explicitly consider inter-collision evolution of opinions. Finally, this research has not implemented the full opinion-choice dynamics framework discussed in conceptual form. The latter would allow the flexibility of specifying opinions of all alternatives, and constructing error terms for the calculation of the alternative specific constant to account for inconsistent behaviors (i.e., individual would pick an alternative despite having a lower level of service due to harboring a positive opinion towards that alternative).

Many possibilities exist to further expand and develop the ideas articulated in this research. On the opinion formation and propagation side, one future avenue would be to investigate real opinion evolution over social networks. Another avenue would be to collect data to empirically validate the mathematical functions representing the interaction mechanisms. Concerning mode choice and opinions, one possible avenue would be to investigate opinions (through some data collection effort) and the direct correlations they have with choice. Yet another research avenue would be the implementation of the fully-flexible opinion-choice dynamics framework model, where there is an opinion on all alternatives and there are rules governing how agents exchange opinions on alternatives.

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