

ABSTRACT

Title of Document: STRATEGIC BEHAVIORS AND MARKET OUTCOMES: TWO ESSAYS

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This dissertation is comprised of two essays related, broadly, to themes of competitive dynamics and economic consequences. In Essay One, “Many Fields of Battle: How Cost Structure Affects Competition across Multiple Markets,” a conjectural variation model is developed to examine what role cost structure and product differentiation play in affecting the mutual forbearance outcome arising from multi-market contact. The analytical results show that the degree of collusion (as measured by the price level) enhanced through multimarket contact is greater when multimarket contact occurs between firms with similar production costs and undifferentiated products. This hypothesis is then tested using data from the U.S. airline industry. The empirical results provide support for the view suggesting that multimarket contact blunts the edge of competition between firms. Moreover, it is found that rival carriers with similar production costs are more likely to experience

such collusion facilitating effects from multimarket contact than those with dissimilar production costs. The second essay in this dissertation is entitled, “A Two-Location Inventory Model with Transshipments in a Competitive Environment.” In this study, an analytical model is developed to assess the impact of transshipments on inventory replenishment decisions and the implications for firm profitability in a competitive, uncertain market environment. To incorporate the competition between stocking locations, the analytical model developed in this paper uses a marketing variable, *customer’s switching rate*, to measure the probability of an individual consumer choosing an alternative source of supply in the event of stockout. In such an environment, firms not only cooperate through the practice of transshipments but also compete for business. A number of interesting conclusions are drawn from numerical optimization results. For instance, it is found that when firms differ in market demand, small firms benefit more from transshipments than do large firms. In addition, it is shown that there is an inverted u-shaped relationship between transshipment price and the profit improvements that large firms gain through transshipments, whereas such benefits are monotonically decreasing with transshipment price for small firms. These findings provide several managerial implications with regard to the role of transshipment price in creating benefits for participating firms.

STRATEGIC BEHAVIORS AND MARKET OUTCOMES:
TWO ESSAYS

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Chapter 1: Introduction

Multimarket contact competition describes a situation where the same firms compete with each other simultaneously in multiple markets. As the foundation of multimarket competition theory, the *mutual forbearance view* suggests an inverse relationship between multimarket contact and the intensity of interfirm rivalry. According to this view, as compared to a single market competition, multimarket contact endows firms with more opportunities to act in response to the strategic behaviors of rival firms. In other words, multimarket contact competition provides firms with greater opportunities to reward competitors if they “behave” by sustaining collusive outcomes, and to enact punishment if rival firms deviate from the collusive outcomes.

There has been an extensive body of research empirically investigating mutual forbearance and its ability to reduce competitive intensity. For example, Evans and Kessides (1994) estimate the effects of multimarket contact on pricing in the U.S. airline industry. They find that airfares are higher in those city-pair markets served by carriers with extensive inter-route contacts. This result provides support for the mutual forbearance hypothesis, suggesting that multimarket contact reduces the rivalry intensity between firms, thus leading to a high market price. In an analytical study of multimarket contact and tacit collusion, Bernheim and Whinston (1990) also find evidence that multimarket contact facilitates collusive behaviors. Moreover, they show that the market price sustained among multimarket competitors is even higher when the rival firms have dominant market positions in different markets, an effect known as *sphere of influence*. A simple illustration of sphere of influence is as

follows. When two firms (e.g., Firm 1 and 2) compete in Markets A and B, and the two firms have dominant positions in different markets (i.e., Firm 1 is the main player in Market A, and Firm 2 is the key player in Market B), each firm's incentive to compete aggressively in the other firm's focal market is restrained by the retaliatory threat of its rival in the market where the firm has a strong position.

The development of theories about multimarket contact competition has benefited from a growing body of empirical literature and from many well-established theoretical models analyzing firm collusive behaviors. However, none of previous studies has examined the moderating role that cost plays in the relationship between multimarket contact and competitive intensity. An important question is whether the mutual forbearance outcome will be achieved when rival firms incur substantially different production costs and have differentiated products. The rationales for viewing production cost as an important moderating factor are two-fold. First, the conjectural variation¹ one firm has with respect to another is presumed to be higher when the two rival firms incur similar production costs than when their production costs are dissimilar. Moreover, it is expected that the cross-price demand elasticities between products provided by firms having similar production costs will be greater than between firms with dissimilar production costs. This presumption is based on the rationale that products have a great degree of substitutability when they are produced by firms with similar production costs. Conjectural variation and cross-price elasticity are two main factors affecting the degree of tacit-collusion that firms sustain in the multimarket contact setting. As a result, the tacit cooperation opportunities

¹ Conjectural variation measures the extent of price movement that one firm expects or perceives the other to make in responding to its own price change.

enlarged by multimarket contact may be related to the relative costs of the competing firms.

The first essay in this dissertation theoretically and empirically examines the occurrence of multimarket contact between firms with different production costs and its impact on the market price sustained. In the analytical section of Essay 1, a conjectural variation model is developed to explore the pricing decisions made by firms competing simultaneously across multiple markets. In comparing tacit-colluding prices firms sustain in single market competition with those that occur through multimarket contact, the analytical results suggest that the degree of collusion (as measured by the price level) facilitated by multimarket contact is greater between firms with similar production costs. This proposition is then tested using airline data from the top 1,000 U.S. domestic origin and destination routes in 2002. The empirical findings suggest that multimarket contact reduces competitive intensity between carriers and leads to higher airfares. This result confirms the long-standing view of mutual forbearance. The findings also suggest that the degree to which multimarket contact impacts airfares depends on the relative costs of the carriers in a market. It is found that multimarket contact has a greater positive effect on price when rival carriers have similar production costs; when rival carriers have dissimilar production costs, multimarket contact has little impact on a carrier's *yields* (i.e., average one-way airfare divided by non-stop route distance).

The second strategic behavior addressed in this dissertation is the practice of transshipments between competing firms selling in markets with uncertain, asymmetrical demands. Transshipments refer to the practice of transferring goods from the location with excess stock to satisfy the demand at the location with insufficient stock. As a risk-pooling strategy, it has been widely applied in several industries, especially in those industries where the distribution lead time is long, the product selling season is short, the products are high-valued goods, and the local consumer market is unpredictable. Under these circumstances, transshipments are often observed to be made between stores that belong to the same chain. Take fashion or upper-end clothes store as an example. Suppose one Gap store in a local mall is stocked out of a particular size or style of an item, then another Gap store in a nearby mall might transship the product to the out-of-stock store. In this case, transshipments are implemented between firms that operate under the same corporate umbrella. Transshipments are initiated either voluntarily, or mandated by the company's headquarter. This type of transshipment has been well studied in previous literature.

An alternative setting for transshipments is examined by the second essay in this dissertation. In this setting, transshipments are implemented among firms that compete with one another. An example would be the transshipment of auto vehicles between independent car dealers. In this case, the two dealers may be located in fairly close proximity and distribute the same brand of automobile, but are independently owned. More importantly, the two car dealers not only cooperate through transshipments, but also compete with one another. If one dealer is stocked

out of a particular model, potential customers of this dealer might simply divert to a neighboring dealer and make purchase there. From this perspective, these two car dealers are head-to-head competitors. In this case, a critical decision facing each car dealer is whether to implement transshipments with other dealers.

In Essay 2, a two-location distribution model is developed to explore the following questions: (1) How do transshipments between rival firms affect their inventory decision-making and what are the performance outcomes? (2) When does a transshipment strategy benefit firms that are head-to-head competitors? (3) How are the benefits from transshipments shared among the firms? (4) Is there a transshipment price that will allow both competitors to increase their profits?

Through transshipments, firms can save inventory costs without impairing customer service levels, as measured by fill rates or stockouts. It has been well recognized that transshipments enable firms to share inventories and pool demand variability.

However, the question that remains whether the strategy of transshipments will provide benefits when firms are direct rivals. In this setting, firms' *ex ante* inventory replenishment decisions are interrelated through the implementation of transshipments. Specifically, one firm's stock level decision has an external negative impact on another firm's inventory decision. For example, when one firm carries a large inventory, the other firm tends to hold a small inventory because transshipments make it possible for the firm with small inventory to rely upon the large inventory held by the other firm in the event of stockout. As well, when one firm carries a

small inventory, the other firm tends to hold a large inventory because transshipments make it easier for the firm with large inventory to dispose of its extra stock when overstocks occur. In these situations, firms will, in most cases, benefit from a coordinated inventory policy. In a competitive setting, however, inventory coordination may not be as feasible. Thus, it is important to see if there exists an incentive mechanism (e.g., the use of an appropriate transshipment price) that can lead to positive outcomes for both firms. Building upon the analytical model, several numerical examples are used in Essay 2 to compare the performance outcomes under various competitive environments. The results suggest that first, transshipment price matters in a competitive environment; secondly, when the two firms are identical, there exists a transshipment price that is optimal for both firms; and finally, when the two firms are not identical, the smaller firm will prefer a lower transshipment price, and will achieve greater benefits from transshipments.

The remainder of this dissertation is organized as follows. Chapter II presents the study of multimarket contact for firms having different production costs and selling differentiated products. In Chapter III, the practice of transshipments between two rival firms is modeled and the results from numerical examples are provided. Chapter IV summarizes a number of key findings and managerial implications that are drawn from this dissertation.

Chapter 2: Essay One - Many Fields of Battle: How Cost Structure Affects Competition across Multiple Markets

1. Introduction

Multimarket contact refers to situations when the same firms simultaneously compete in multiple markets. This type of competition occurs when firms produce multiple product lines, diversify into several industries, or operate in different geographical markets. When firms compete in a multimarket context, potential and actual interactions across markets serve to affect the strategic behaviors of firms. Edwards (1955) is the first to make the point:

When two firms meet in multiple product or geographic markets, they may hesitate to contest a given market vigorously for fear of retaliatory attacks in other markets that erodes the prospective gain in that market.

Since then, this *mutual forbearance* view has become the fundamental theory of multipoint competition research and has found consistent support in the context of many industries, especially in the airline industry (e.g., Evans & Kessides 1994; Morrison et al 1996; Baum & Korn 1996 and 1999; Gimeno 1999). According to mutual forbearance theory, firms that meet simultaneously in multiple markets will compete less intensely with one another. Evans and Kessides (1994) are among the first authors to examine empirically the effect of multimarket contact on pricing in the U.S. airline industry. They find that airfares are higher on city-pair routes served by carriers with more overlapping routes in common.

As an extension of mutual forbearance theory, the *spheres of influence* view suggests that the inverse relationship between multimarket contact and rivalry intensity is greater when multimarket competitors have dominant positions in different markets.

The presence of asymmetric territorial interests endows firms with opportunities to retaliate in markets that are more important to their competitors. In this way, a firm behaves less aggressively in a rival firm's dominant market in exchange for the rival firm's similar subordination in its turf market. Gimeno (1999) offers empirical evidence for the spheres of influence argument. Using data from the U.S. airline industry, Gimeno (1999) finds that airlines restrain their competitive behaviors in their rival firms' important markets so as to reduce the competitive intensity of those rival firms in the airline's own dominant markets.

The cooperation facilitating effect of multimarket contact has also been extended to study the dynamic characteristics of competitive interactions among multimarket competitors. Morrison et al. (1996) estimate the effect of multimarket contact on the probability of an airline fare war. According to the mutual forbearance theory, multimarket contact facilitates carrier cooperation and thus reduces the occurrence of fare wars. On the other hand, multimarket contact exposes carriers to competition over more routes on which one carrier's price cuts could initiate retaliation from rival carriers on other routes, thereby leading to a greater likelihood of fare wars.

Analyzing the quarterly fare changes on the top 1,000 U.S. domestic routes in 1993, Morrison et al. (1996) find no empirical evidence for the mutual forbearance hypothesis. Instead, their results indicate that multimarket contact increases the likelihood of a fare war on a given route.

Recently, an inverted U-shape relationship between multimarket contact and competitive interactions among multimarket rivals has been proposed and received empirical support (Baum & Korn 1996 and 1999; Fuentelsaz and Gomez 2006). Baum and Korn (1996 and 1999) find that an increase in multimarket contact raises an airline's rate of market entry into and exit from other airlines' markets when the level of multimarket contact between rival carriers is low; multimarket contact, however, has a negative impact on an airline's rates of entry into and exit from other airlines' routes when multimarket contact between rival carriers grows beyond a threshold level. As pointed out by Fuentelsaz and Gomez (2006), the strategies of entry into new markets or exit from existent markets are purposefully utilized by firms to increase or decrease the extent of multimarket contact with their rivals. The findings of an inverted U-shape relationship between multimarket contact and entry rates provide support for the argument that when the level of multimarket contact between two rival firms is low, both firms intentionally use entry strategy to establish a foothold in the rival's markets so as to signal capabilities to retaliate against any aggressive attacks. Once the level of multimarket contact rises beyond a certain level, rival firms get more familiar with one another and are better able to recognize the interdependence of competing simultaneously across multiple markets. As such, multimarket contact serves to restrain aggressive actions and deter further entries of multimarket rivals (Fuentelsaz and Gomez 2006; Karnani and Wernerfelt 1985).

Most anecdotal evidence so far provides empirical support for a negative relationship between multimarket contact and the intensity of rivalry (e.g., Heggestad and

Rhoades, 1976; Feinberg 1985; Singal 1996; Jan and Rosenbaum 1996; Parker and Roller 1997; Fernandes and Marin 1998; Gimeno and Woo 1999). In these prior studies, several moderating factors have been incorporated into studying the negative effect of multimarket contact on rivalry intensity, as measured by the price level. Such factors include firm size (Baum and Korn 1999), market concentration (Jans and Rosenbaum 1996; Fernandes and Marin 1998), and spheres of influence (Gimeno 1999). However, there has been no attempt to investigate the impact of firm cost structure on the relationship between multimarket contact and the intensity of competition.

In this article, we first investigate the question as to whether multimarket contact reduces competitive intensity when it occurs between firms producing outputs at the same marginal cost, which is invariant throughout markets, and when markets are identical. Under these circumstances, Bernheim and Whinston (1990) suggest that multimarket contact is irrelevant and does not facilitate collusion. On the contrary, we show that when the conjectural variations firms have with respect to each other and the cross price elasticity between rival firms are positive, multimarket contact always restrains competitive behavior, thus facilitating tacit collusion. The non-zero values for conjectural variation and cross price elasticity make strategic interactions between multimarket rivals interdependent across markets: the optimal price firms choose in one market depends on prices realized in other markets. The question of whether aggressive pricing by a firm in one of its markets leads to loss or gain in other markets depends on two factors. First, the positive value for conjectural variation a

firm has with respect to its rival firms indicates the degree of retaliation that the firm perceives or expects its rival firms might take in any market. Second, the positive cross-price elasticities between the firm and its competing firms imply that their products are strategic substitutes, rather than complements. As such, the counterattacking prices initiated by rival firms lead to demand loss and reduce the firm's profitability in other markets. Under these two conditions, the punishing effects occurring simultaneously in more than one market are greater than the aggregate effects of those retaliations arising from any individual market and, as a result, multimarket contact serves to restrain competitive behaviors and fosters implicit colluding actions.

The second question to be addressed in this paper is whether multimarket contact between firms with similar production costs has a different competitive effect than multimarket contact between firms with dissimilar production costs. To analyze the collusion-facilitating effect of multimarket contact, we develop a conjectural variation model in which the tacit-colluding price in the single market setting is compared with the price in a multimarket contact setting. The analytical results reveal first, firms benefit more from a high tacit-colluding price in the multimarket contact setting, as compared to the single market setting; and second, the profit improvements resulting from tacit collusive pricing in the context of multimarket contact are greater when multimarket rivals have similar production costs than the case when multimarket rivals have dissimilar production costs.

Our empirical analysis in the context of the U.S domestic airline industry bears out the theoretical propositions. The results support the longstanding view that multimarket contact reduces interfirm rivalry intensity. Moreover, the collusion-enhancing effect of multimarket contact is more likely to be found between carriers with similar production costs. By contrast, there is no such effect when multimarket contact occurs between carriers having dissimilar production costs.

The remainder of this essay is organized as follows. Section 2 presents the analytical model and the results drawn from a series of numerical examples. In Section 3, we discuss hypotheses, empirical models, data and methodology. Section 4 summarizes the findings from our empirical analysis. The final section concludes and discusses implications for management and regulations.

2. Theoretical Model

In this section, we develop a conjectural variation model² to analyze firm collusive behavior in the setting of multimarket contact. To examine the potential effects of multimarket contact on collusive behavior, we compare the single market tacit colluding price with the tacit colluding price under multimarket contact. Although we focus on the case of two firms competing in two markets, the analysis and its conclusion can be extended to the case where n -firms meet with one another in m -markets.

² In this model, the conjectural variable is incorporated into the price equilibrium analysis.

2.1 Model Setup

Consider two firms, referred to as Firm 1 and Firm 2, competing in Market A or Market B, when they meet in a single market; and competing in Markets A and B when they meet in multimarkets. First, we assume that Firms 1 and 2 have identical production costs and that their products are highly substitutable. By comparing the tacit-colluding prices that firms sustain in single and multimarket settings, we investigate the question of whether multimarket contact facilitates collusive behavior when firms produce outputs at the same marginal cost, which is invariant across markets. Then, we consider that Firm 1 is a low-cost firm producing inferior goods, whereas Firm 2 is a high-cost firm providing superior goods. In this case, we assume that their products become less substitutable and the conjectural variations one firm has with respect to the other are lower as compared to those associated with the first scenario. The demand functions we use for Firms 1 and 2 in Market A or B are:

$$q_1 = a_1 - e_1 p_1 + d p_2 \quad (1.1)$$

$$q_2 = a_2 - e_2 p_2 + d p_1 \quad (1.2)$$

where p_1 and p_2 are the prices charged by Firms 1 and 2, respectively. These demand functions have been used by Singh and Vives (1984) to study the price and quantity competition in a differentiated duopoly setting and by Dixit (1979) to analyze the entry choice of new firms producing differentiated products and facing an established firm with demand (cost) advantage. To derive demand structures in a duopoly setting for firms producing differentiated products, we follow Dixit (1979) and Singh and Vives (1984) by assuming that there is an economy consisting of two sectors: a monopolistic sector in which two firms each produce a differentiated product and a

competitive *numeraire*³ sector. Since there is no income effect on the duopoly section, the demand for each firm can be determined by partial derivative equilibrium analysis of the utility function (i.e., $U(q_1, q_2)$), which represents the level of satisfaction that consumers derive from consuming q_i amount of goods of Firm i ($i = 1, 2$). The quadratic and strictly concave utility function, $U(q_1, q_2)$, is specified as the following when the products are produced by two firms with identical production costs.

$$U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - (\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2) / 2 \quad (1.3)$$

where α_i, β_i , and γ are all positive, indicating, respectively, that these two products are normal goods, satisfy the property of decreasing marginal utility, and substitute with one another. Building upon this utility formation, we can express the parameters

in demand functions (1.1) and (1.2) as the following: $a_i = \frac{\beta_j \alpha_i - \gamma \alpha_j}{\beta_i \beta_j - \gamma^2}$, $e_i = \frac{\beta_j}{\beta_i \beta_j - \gamma^2}$,

and $d = \frac{\gamma}{\beta_i \beta_j - \gamma^2}$ for $i, j = 1, 2$, and $i \neq j$.

In these demand functions, the positive sign on parameters e_1, e_2 suggests that market demand for the products of Firm 1 (2) decreases with the firm's own price p_1 (p_2); the positive parameter d indicates that demand for the products of Firm 1 or 2 increases with the price of the competitor, as their products are substitutes.

³In partial equilibrium analysis, the entire economy is considered as a two-good model. In this model, the expenditure on all commodities other than that under consideration is assumed as a single composite commodity. Such a hypothetical composite commodity is known as numerical commodity and the assumption on numerical commodity helps the exclusion of income effect, thereby simplifying the market equilibrium analysis.

For the second scenario where Firms 1 and 2 sell products of different qualities incurring different production costs, we incorporate service premium, s , and substitution degrading factor, h , into the utility function (1.3). Herein, the revised utility function that applies to the case when two firms produce outputs with different production costs can be written as:

$$U'(q_1, q_2) = \alpha_1 q_1 + (\alpha_2 + s)q_2 - [\beta_1 q_1^2 + 2(\gamma - h)q_1 q_2 + \beta_2 q_2^2]/2 \quad (1.4)$$

This utility function differs from (1.3) in two aspects. First, the positive parameter, s , represents the quality/service premium that is associated with products provided by the high-cost firm, or Firm 2. Secondly, the positive value for parameter h implies that as a result of such product differentiation, the degree of substitutability between a high quality product and a low quality product is less than that between two high or two low quality products.

Using utility Function (1.4), we get the inverse demand functions for Firms 1 and 2 under the condition that the two firms have different production costs and their products are different in service or product quality. The inverse demand functions are:

$$q_1 = a'_1 - e'_1 p_1 + d'_1 p_2 \quad (1.5)$$

$$q_2 = a'_2 - e'_2 p_2 + d'_1 p_1 \quad (1.6)$$

In these functions, $a'_1 = \frac{\beta_2 \alpha_1 - (\gamma - h)(\alpha_2 + s)}{\beta_1 \beta_2 - (\gamma - h)^2}$, $a'_2 = \frac{\beta_1 (\alpha_2 + s) - (\gamma - h)\alpha_1}{\beta_1 \beta_2 - (\gamma - h)^2}$,

$e'_i = \frac{\beta_j}{\beta_i \beta_j - (\gamma - h)^2}$, and $d' = \frac{(\gamma - h)}{\beta_i \beta_j - (\gamma - h)^2}$ for $i, j = 1, 2$, and $i \neq j$. Comparing

these parameters with those derived in the scenario where Firms 1 and 2 have identical production cost, we find first, $d' < d$, indicating that the cross-price effects on demand are smaller when two firms have distinct production costs; second, $e_i' < e_i$, implying that the own-price effects on demand are smaller when two firms have different production costs; and finally, the sign for the difference between a_i and a_i' is determined by service premium, s , and substitution degrading factor, h . For example, if $h=0$, and $s>0$, then the demand for Product 1 decreases by $\frac{\gamma s}{\beta_1\beta_2 - \gamma^2}$ as the production costs and the resulting product qualities of the two firms become dissimilar. On the other hand, the demand for Product 2, which is a high quality good, increases by $\frac{\beta_1 s}{\beta_1\beta_2 - \gamma^2}$. Table 1, below, presents the values for parameters a_i and a_i' , as derived from utility Functions (1.3) and (1.4), respectively.

Table II-1: The Values for Market Demand Parameters, a_i and a_i' , under Various Scenarios

	Low-cost Firm (Firm 1)	High-cost Firm (Firm 2)
Non-differentiated products	$a_1 = \frac{\beta_2\alpha_1 - \gamma\alpha_2}{\beta_1\beta_2 - \gamma^2}$	$a_2 = \frac{\beta_1\alpha_2 - \gamma\alpha_1}{\beta_1\beta_2 - \gamma^2}$
Differentiated products	$a_1' = \frac{\beta_2\alpha_1 - (\gamma - h)(\alpha_2 + s)}{\beta_1\beta_2 - (\gamma - h)^2}$	$a_2' = \frac{\beta_1(\alpha_2 + s) - (\gamma - h)\alpha_1}{\beta_1\beta_2 - (\gamma - h)^2}$

The parameters shown in the top row of Table 1 are for the case when the products provided by Firms 1 and 2 are of same quality. The bottom row in the table presents the parameters associated with differentiated products under the assumption that the low-cost firm provides low quality goods, while the high-cost firm provides high quality goods. Based on the results in Table 1, we find that the product differentiation

strategy implemented by the high-cost firm has both positive and negative impacts on its market demand. On the positive side, the product produced by the high-cost firm becomes more appealing to customers because of the enhanced quality; on the other side, such added quality or service premium makes the high-end products less substitutable with the low-end products. Jointly, these two effects might enlarge or shrink the market demand for the products provided by the high-cost firm, depending upon the increased value for service premium, s , relative to the degree of decreased substitutability, as measured by h .

We also find that although the low-end products are less attractive to consumers, the low-cost firm might, instead, face an enlarged market demand as a result of the reduction in product substitutability. Specifically, it can be shown that the low-cost firm has greater demands in the differentiated product market as compared to the non-differentiated product market if the following condition holds: $\frac{s(\gamma - h)}{h(\beta - \gamma + h)} < \frac{\alpha}{\beta + \gamma}$

when $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$; in addition, the high-cost firm has greater demands in the differentiated product market as compared to the non-differentiated product market if the following condition holds: $\frac{s}{h(\gamma - \beta - h)} < \frac{\alpha}{\beta(\beta + \gamma)}$ when

$\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$. Finally, it can be shown that when the following equality $\beta_2\alpha_1 - \beta_1\alpha_2 = \gamma(\alpha_2 - \alpha_1)$ holds, Firms 1 and 2 have identical demand parameters (i.e. $a_1 = a_2$) for the scenario where the two firms compete in the non-differentiated product market. For the scenario where these two firms compete in the

differentiated product market, they have identical demand parameters (i.e., $a_1' = a_2'$) when $\alpha_1(\beta_2 + \gamma - h) = (\alpha_2 + s)(\beta_1 + \gamma - h)$.

Now we assume that firms have constant marginal costs in Markets A and B. Firm 1's marginal cost is denoted by c_1 . The marginal cost for Firm 2 is c_2 . There are no fixed costs for Firms 1 and 2 in either Market A or B.

Given the above cost assumptions and inverse demand Functions (1.1) and (1.2), the profit for Firm 1 in Market A can be written as the following when Firms 1 and 2 are assumed to have identical production costs:

$$\pi_1^A = (a_1 - e_1^A p_1^A + d^A p_2^A)(p_1^A - c_1) \quad (1.7)$$

Under the scenario where Firms 1 and 2 have identical production costs, Firm 2's profit in Market A is:

$$\pi_2^A = (a_2 - e_2^A p_2^A + d^A p_1^A)(p_2^A - c_2) \quad (1.8)$$

For Firm i ($i=1, 2$) to achieve positive profit outcomes, it is required that the price charged by Firm i be greater than its marginal cost (i.e., $p_i^A - c_i > 0$) and the demand for Firm i 's output be positive (i.e., $a_i - e_i^A p_i^A + d^A p_j^A > 0$).

In the following section, we first derive the tacit-colluding prices for firms having identical production costs in both settings of single market and multimarket contact. In a similar way, we next derive the tacit-colluding prices for firms having different production costs in both single and multimarket contexts. As for the latter case, where

the two firms have different production costs, we use inverse demand functions specified in (1.5) and (1.6) to derive the profit functions for Firms 1 and 2, separately.

We make a further assumption that each firm perceives the price set by its rival as a function of its own price. Thus, each firm has an expectation on the direction and magnitude of the rival firm's price movement in responding to its own price change.

Two variables, denoted as v_1^A , and v_2^A , measure, respectively, the conjectural

variation for Firm 1 and Firm 2 in Market A. Specifically, we have $v_1^A = \frac{dp_2^A}{dp_1^A} \neq 0$,

and $v_2^A = \frac{dp_1^A}{dp_2^A} \neq 0$. Then the first-order condition for Firm 1 to maximize its profit in

Market A can be written as:

$$\frac{\partial \pi_1^A}{\partial p_1^A} = (-e_1^A + d_1^A v_1^A)(p_1^A - c_1) + (a_1 - e_1^A p_1^A + d_1^A p_2^A) \quad (1.9)$$

For Firm 2, the corresponding condition is:

$$\frac{\partial \pi_2^A}{\partial p_2^A} = (-e_2^A + d_2^A v_2^A)(p_2^A - c_2) + (a_2 - e_2^A p_2^A + d_2^A p_1^A) \quad (1.10)$$

Solving a system of equations (1.9) = 0, and (1.10) = 0, we get the single market tacit-colluding prices for Firms 1 and 2 as the following:

$$\begin{aligned} \bar{p}_1^A &= f_1^A(a_1, a_2, e_1^A, e_2^A, d_1^A, d_2^A, v_1^A, v_2^A, c_1, c_2) \\ &= \frac{e_2^A(2e_1^A c_1 + d_1^A c_2 + 2a_1) + d_1^A(a_2 - 2e_2^A c_1 v_1^A) - d_2^A v_2^A(e_1^A c_1 + a_1 + (c_2 - c_1 v_1^A)d_1^A)}{4e_1^A e_2^A - 2e_2^A d_1^A v_1^A - 2e_1^A d_2^A v_2^A - d_1^A d_2^A + d_1^A d_2^A v_1^A v_2^A} \end{aligned}$$

(1.11)

$$\begin{aligned}\bar{p}_2^A &= f_2^A(a_1, a_2, e_1^A, e_2^A, d_1^A, d_2^A, v_1^A, v_2^A, c_1, c_2) \\ &= \frac{e_1^A(2e_2^A c_2 + d_2^A c_1 + 2a_2) + d_2^A(a_1 - 2e_1^A c_2 v_2^A) - d_1^A v_1^A(e_2^A c_2 + a_2 + (c_1 - c_2 v_2^A)d_2^A)}{4e_1^A e_2^A - 2e_2^A d_1^A v_1^A - 2e_1^A d_2^A v_2^A - d_1^A d_2^A + d_1^A d_2^A v_1^A v_2^A}\end{aligned}$$

(1.12)

The second-order condition requires that the conjectural variations for Firms 1 and 2 satisfy the following inequalities: $d^A v_1^A < e_1^A$, and $d^A v_2^A < e_2^A$. The restrictions imposed on these parameters imply that the firm's own price change impacts its demand level more than does the rival firm's follow-up retaliating price movement. Similar results can be drawn for Market B.

Now, consider the case when Firms 1 and 2 compete in multiple markets; i.e., Markets A and B. Given the above assumptions on demand functions and marginal costs, the total profit that Firm 1 obtains from selling to both Markets A and B is written as:

$$\pi_1^{A\&B} = (a_1 - e_1^A p_1^A + d_1^A p_2^A)(p_1^A - c_1) + (b_1 - e_1^B p_1^B + d_1^B p_2^B)(p_1^B - c_1) \quad (1.13)$$

When Firms 1 and 2 compete in Markets A and B simultaneously, Firm 1's pricing behavior in Market A might initiate its rival's reaction not only in Market A but Market B as well. We use v_1^{AA} to denote the pricing responses that Firm 1 perceives Firm 2 would take in Market A as a reaction to its price change made in Market A, and v_1^{BA} to denote the perceived pricing response of Firm 2 in Market B following Firm 1's pricing action in Market A. Further, v_1^{AB} and v_1^{BB} represent the expected pricing reaction of Firm 2 in Market A, and B, respectively, as a response to Firm 1's

pricing action in Market B. Similarly, Firm 2 has its conjectural variations denoted by: v_2^{AA} , v_2^{BA} , v_2^{AB} , and v_2^{BB} .

Differentiating (1.13) with respect to p_1^A , p_1^B , we obtain the first-order-conditions for Firm 1 to maximize its joint profit in Markets A and B as:

$$\frac{\partial \pi_1^{A\&B}}{\partial p_1^A} = (-e_1^A + d^A v_1^{AA})(p_1^A - c_1) + (a_1 - e_1^A p_1^A + d^A p_2^A) + (d^B v_1^{BA})(p_1^B - c_1) \quad (1.14)$$

$$\frac{\partial \pi_1^{A\&B}}{\partial p_1^B} = (-e_1^B + d^B v_1^{BB})(p_1^B - c_1) + (b_1 - e_1^B p_1^B + d^B p_2^B) + (d^A v_1^{AB})(p_1^A - c_1) \quad (1.15)$$

Equation (1.14) shows us how the aggregate profit for Firm 1 in Markets A and B changes with the price that Firm 1 sets in Market A. By comparing expression (1.14) with (1.9), we can easily find that the profit effect of the price change by Firm 1 in Market A when Firm 1 competes with Firm 2 in both markets is different from the effect when these two firms meet merely in a single market; i.e., Market A.

In the single market setting, Firm 1's price change affects its profit in two ways. First, the market demand for Firm 1's output varies with its price, as determined by the firm's own price elasticity, and indirectly, the demand also shifts resulting from the reaction of the rival firm in responding to Firm 1's price movement, as determined by cross-price demand elasticity. Second, the net profit margin per output is affected by the unit price. In comparison, when two firms simultaneously meet in more than one market, the price change for Firm 1 in one of these markets (e.g., Market A) has an extra impact on its profit as a result of the potential price responses taken by its rival firm in other markets (e.g., Market B). Specifically, when Firm 1 cuts its price in

Market A, it gets more demand in Market A, while the demand for its output in Market B might be reduced, as Firm 2 might decrease its price in Market B to retaliate against Firm 1's aggressive pricing in Market A. This counterattack by Firm 2 in Market B is taken into consideration by Firm 1 when deciding its price in Market A. The part $d^B v_1^{BA}$ in Expression (1.14) measures the magnitude of the demand loss that Firm 1 is expected to suffer in Market B if it cut its price in Market A. The greater the perceived loss of demand in Market B, the less incentive for Firm 1 to price aggressively in Market A. According to the same rationale, Firm 1's aggressive pricing behavior in Market B is restrained by the potential demand deteriorating effect arising from Firm 2's counterattack in Market A. Therefore, the rivalry experienced by firms meeting in two markets simultaneously is less intense than when they compete in any of the two markets alone.

To find tacit-colluding prices in the setting of multimarket contact, we use the total profit expression for Firm 2 in Markets A and B and then differentiate this equation with respect to p_2^A , and p_2^B .

$$\pi_2^{A\&B} = (a_2 - e_2^A p_2^A + d^A p_1^A)(p_2^A - c_2) + (b_2 - e_2^B p_2^B + d^B p_1^B)(p_2^B - c_2) \quad (1.16)$$

Given a set of non-zero conjectural variations for Firm 2, we get the first-order-condition for Firm 2's total profit maximization problem as:

$$\frac{\partial \pi_2^{A\&B}}{\partial p_2^A} = (-e_2^A + d^A v_2^{AA})(p_2^A - c_2) + (a_2 - e_2^A p_2^A + d^A p_1^A) + d^B v_2^{BA}(p_2^B - c_2) \quad (1.17)$$

$$\frac{\partial \pi_2^{A\&B}}{\partial p_2^B} = (-e_2^B + d^B v_2^{BB})(p_2^B - c_2) + (b_2 - e_2^B p_2^B + d^B p_1^B) + d^A v_2^{AB}(p_2^A - c_2) \quad (1.18)$$

Solving a system of equations (1.14) = 0, (1.15) = 0, (1.17) = 0, and (1.18) = 0, we get the tacit-colluding prices $\hat{p}_1^A, \hat{p}_1^B, \hat{p}_2^A$ and \hat{p}_2^B for Firms 1 and 2 in a multimarket contact setting. The second-order condition for the profit maximization problem requires that the underlying parameters satisfy the following inequalities:

$$(i) v_1^{AA} d^A < e_1^A; (ii) v_1^{BB} d^B < e_1^B; (iii) v_2^{AA} d^A < e_2^A; (iv) v_2^{BB} d^B < e_2^B; (v)$$

$$(2d^A v_1^{AA} - 2e_1^A)(2d^B v_1^{BB} - 2e_1^B) > (d^A v_1^{AB} + d^B v_1^{BA})^2; \text{ and (vi)}$$

$$(2d^A v_2^{AA} - 2e_2^A)(2d^B v_2^{BB} - 2e_2^B) > (d^A v_2^{AB} + d^B v_2^{BA})^2.$$

Now, we can study the competition restraining effects of multimarket contact by comparing tacit colluding prices in a single market setting with those determined in a multimarket contact setting. Moreover, we can investigate whether multimarket contact has differential effects on collusive behavior when firms have dissimilar costs rather than identical costs. For this purpose, several numerical examples are drawn to show under what conditions the collusion enhancing effects of multimarket contact are different when firms produce differentiated goods at different levels of production costs.

□ **Proposition 1.** Multimarket contact facilitates tacit collusion and thus restrains aggressive pricing behavior when the cross-price demand effect in both Markets A and B are positive (i.e., $d^A, d^B > 0$) and firms have positive conjectural variations with respect to one another (i.e., $v_i^{AB}, v_i^{BA} > 0, i = 1, 2$).

To prove this proposition, we simply need to examine whether tacitly colluding in price makes both firms more profitable in a multimarket contact setting than in a single market setting. By comparing Equation (1.14) with Equation (1.9), we find that the presence of term $d^B v_1^{BA} (p_1^B - c_1)$ in Equation (1.14) suggests that the price change of Firm 1 in Market A has a different effect on its profitability when Firm 1 competes with Firm 2 in both Markets A and B as compared to when the two firms compete only in Market A. For a positive profit outcome, it is reasonable to assume that Firm 1's price in Market B, p_1^B , is greater than its marginal production cost, c_1 . Therefore, the positive value for $d^B v_1^{BA}$ implies that Firm 1 would get more profits through a tacit-colluding price in Market A when it meets Firm 2 in both markets as compared to when it meets Firm 2 in Market A alone. Similarly, the positive sign for $d^A v_1^{AB}$ suggests that the benefits arising from a tacit-colluding price in Market B are greater for Firm 1 when it meets Firm 2 in both markets than when it meets Firm 2 in Market B alone. The comparison of Equation (1.17) with (1.10) leads to the same results for Firm 2 as long as the following conditions hold: $d^B v_2^{BA} > 0$ and $d^A v_2^{AB} > 0$.

2.2 Numerical Examples

We start a series of numeral examples with a symmetric one, in which the demand structure, marginal production cost and conjectural variations for Firm 1 are identical to those for Firm 2. We also assume the cross-price demand effect, the own-price demand effect and the conjectural variations for each firm are constant across markets. As for the conjectural variation, we make a further assumption that when a firm competes in multiple markets, its conjectural variation in a given market (e.g., v_1^{AA}) would be the same as if it only competed in a single market (e.g., v_1^A). In fact, an empirical question remains as to whether the values for conjectural variation become smaller or larger when there is multimarket contact formed between rival firms.

To help make the example realistic, we use the calculated average expense/available seat from our U.S. airline dataset as the value for marginal production cost c_i , and the average market yield to derive values for the market size-related variables a_i and b_i .

The set of parameters are assumed to have values as follows:

$$c_1 = c_2 = 142 \text{ (dollars per passenger)}$$

$$a_1 = a_2 = 107, b_1 = b_2 = 107$$

$$e_1^A = e_2^A = 1, e_1^B = e_2^B = 1$$

$$d^A = 0.6, d^B = 0.6$$

$$v_1^{AA} = v_2^{AA} = 0.6, v_1^{BA} = v_2^{BA} = 0.6, v_1^{AB} = v_2^{AB} = 0.6, v_1^{BB} = v_2^{BB} = 0.6$$

Using these values, we get the tacit-colluding prices for Firm 1 in Market A, $\bar{p}_1^A = 190.27$ in a single market setting; $\hat{p}_1^A = 215.82$ under a multimarket contact setting. This result shows that the tacit-colluding price for Firm 1 in Market A is greater when Firm 1 competes with Firm 2 in two markets than the tacit-collusive price sustained in a single market setting. Further, we can calculate the non-tacit-colluding Nash equilibrium in both single and multimarket contact settings. By assuming that all conjectural variations are zero, i.e., $v_i^{jk} = 0$ ($i = 1, 2, j, k = A, B$), we follow the expression for \bar{p}_1^A to get the non-tacit-colluding price of Firm 1 in Market A in the single market setting, and the expression for \hat{p}_1^A to get the corresponding non-tacit-colluding price for a multimarket contact setting. It can be easily shown that Equation (1.14) has the same expression as Equation (1.9) when all conjectural variations are equal to zero. Under this particular case, the competitive price for Firm 1 in the single market setting has the same value as the price sustained in a multimarket contact setting. Using the assumed set of parameters, we get the competitive price $\tilde{p}_1^A = 177.86$, which is less than the tacit colluding price in both single and multimarket environments.

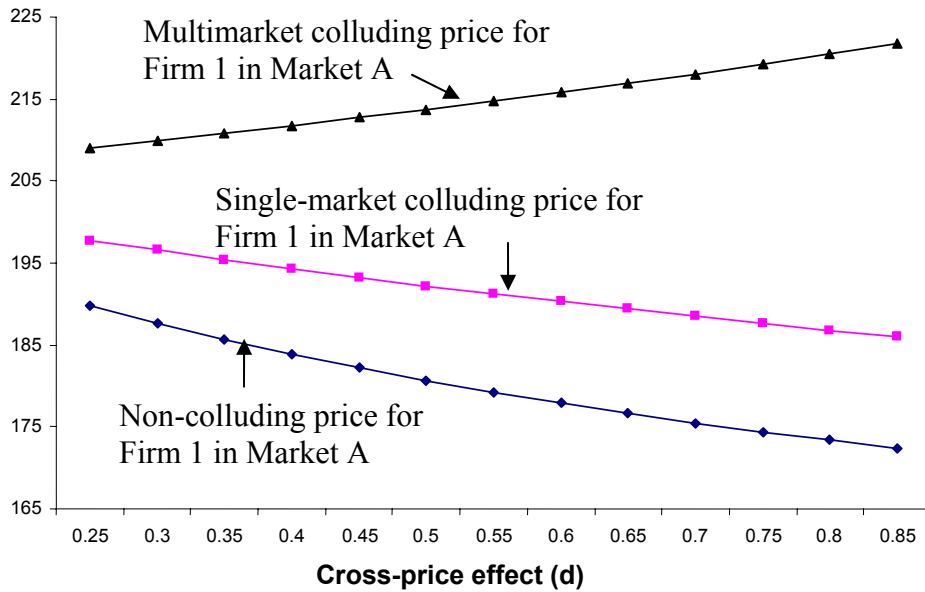
Next, we decrease the assumed values for the cross-price demand effect from 0.6 to 0.4, holding other parameters unchanged⁴. Using $d^A = 0.4, d^B = 0.4$, and other parameters assumed herein, we get the tacit-colluding price in the single market

⁴ From the utility function (see Equation 1.3), we derive the expression for parameters including own-price demand effect, e_i and cross-price demand effect denoted as d . It can be shown that the difference between e_i and d is a constant. Therefore, the value for e_i changes with d . As in the baseline example, the difference between e_i and d is invariantly fixed at 0.4.

setting, $\bar{p}_1^A = 194.29$; the tacit-colluding price in a multimarket contact setting, $\hat{p}_1^A = 211.72$; and the competitive price $\tilde{p}_1^A = 183.83$. Consistent with Proposition 1, this result implies that multimarket contact leads to a higher tacit-colluding price, or, lower rivalry intensity than does single market competition. Moreover, we observe that the tacit colluding price enhanced through multimarket contact declines as cross-price demand effects (i.e., d^A, d^B) decrease while holding other parameters invariant.

The above results suggest that the reduction in competition from multimarket contact, as evidenced by the higher equilibrium price, is increasing with the value for cross-price elasticity, *ceteris paribus*. As the products of Firm 1 and Firm 2 get more substitutable, firms obtain greater additional benefits from tacit collusion in the multiple market context compared to the single market context. Figure 1 graphically illustrates this point, showing that the difference in the tacit colluding price between the multimarket and single market settings enlarges with the parameter for the cross-price demand effect, or, d^j ($j = A, B$). Note that the feasible range for d^j ($j = A, B$) is within $[0.25, 0.85]$ when the set of conjectural variation parameters takes the value of 0.6. The restrictions imposed on d^j ($j = A, B$) serve to guarantee the strictly concave property of the profit function and a price that is, at least, as great as marginal cost.

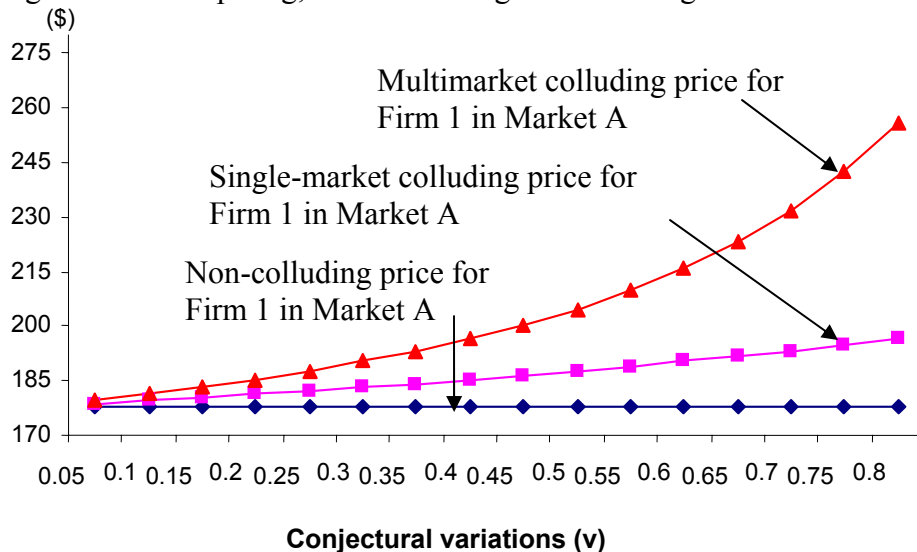
Figure II-1: Competing, Tacit-colluding Prices in Single and Multimarket Settings



It can also be shown that that the difference in tacit-colluding prices between the multimarket context and the single market setting varies with the levels of conjectural variation, *ceteris paribus*. When two rival firms form high conjectural variations with respect to each other, they perceive greater response threats. As a result, the benefits from tacit-collusion in the multimarket context increase. Figure 2 presents the tacit-colluding prices for Firm 1 in Market A for the multimarket contact and single market settings. It reveals that the tacit-colluding price facilitated through multimarket contact rises with conjectural variation, which ranges from 0.05 to 0.8 given a fixed cross-price parameter d^j of 0.6 ($j=A, B$). Under the assumed values for other key parameters, the non-tacit-colluding price for Firm 1 in Market A is constant at $\tilde{p}_1^A = 177.857$ and is always less than the tacit colluding prices associated with both multimarket and single market settings.

Collusion, when successful, will raise price above the competitive level under single market competition. The more inelastic the demand for the product at the competing price level, the higher the collusive price that is expected to hold in the market (Rosenbaum and Manns 1994). As shown in Figure 3, the tacit-colluding price in a single market context rises when consumer demands for each firm become more inelastic. In contrast, the tacit-colluding price in a multimarket contact setting decreases as price elasticity, e_i , falls from 1.35 to 0.8, *ceteris paribus*⁵. Figure 3 also reveals that at a given level of own-price elasticity, the tacit-colluding price for Firm 1 in a multimarket contact setting is higher than when Firm 1 competes with Firm 2 in a single market. Moreover, such an increase in price due to multimarket contact gets larger as the demands for Firm 1 (2)'s products become less elastic with Firm 1(2)'s own price.

Figure II-2: Competing, Tacit-colluding Prices in Single and Multimarket Settings



⁵ As indicated in the previous note, the value for the cross-price demand effect, d , varies with own-price demand effect, e_i . The difference between these two parameters is fixed at 0.4, as prescribed in the baseline model.

The finding that the collusion enhancing impact of multimarket contact is greater when a firm has a higher own-price demand effect can be explained as follows. In a single market setting, the nature of demand price-elasticity affects firms in making collusive or aggressive decisions. In a particular market environment, where demand is less sensitive to price, it is more likely for firms to collude. On the contrary, firms tend to compete more intensely in price when demand has a higher price elasticity. Under this situation, there will be greater potential for collusion enhancing effects from multimarket contact. Therefore, the increase in tacit-colluding prices as a result of multimarket contact will get larger as demands throughout markets become more sensitive to product price.

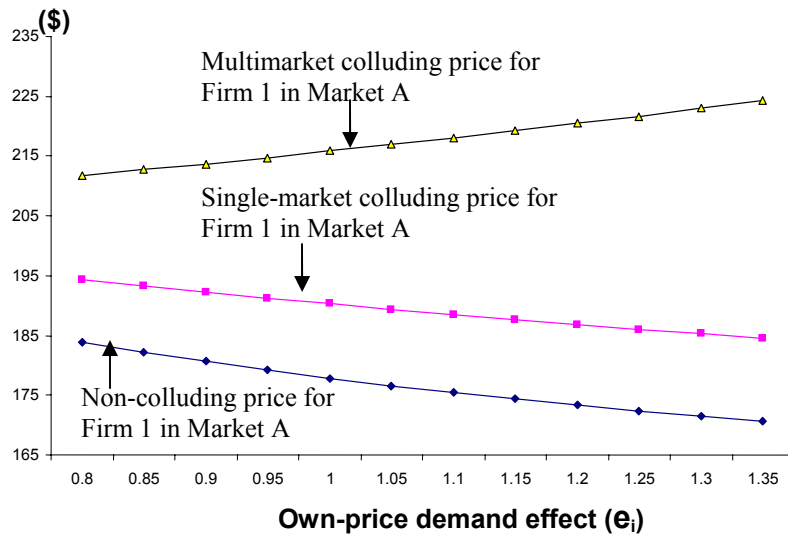


Figure II-3: Competing, Tacit-colluding Prices in Single and Multimarket Settings

So far, we have shown that when markets are identical, and rival firms have identical positive conjectural variations and positive cross-price effects on their demand functions, the tacit-colluding price for firms meeting simultaneously in two markets is always higher than what the price when the two firms meet in only one market. In the

following section, we explore whether multimarket contact has a greater impact on competitive behavior between two rival firms having similar production costs as compared to firms having dissimilar production costs.

□ **Proposition 2.** The tacit-collusion enhancing effects of multimarket contact are greater when it occurs between firms with similar production costs than when it occurs between firms with dissimilar productions costs.

In the preceding section, we showed that when two firms produce differentiated products incurring different levels of production costs, the degree of substitutability between a high quality and a low quality product is less than that between two high quality products or two low quality products. With the presence of a positive substitution degrading factor, h , the cross-price effect on demand (as denoted by d) will be smaller when firms have dissimilar production costs compared to when they have identical production costs. As products provided by the two rival firms become less substitutable, it is also reasonable to assume that the conjectural variation one firm has with respect to another declines in value. From the analytical results, we find that the collusion enhancing effects of multimarket contact are jointly determined by the value of the cross-price demand parameter, d , and the conjectural variation, v . The greater these parameters, the more effective multimarket contact is at facilitating collusive behavior. As such, the degree of implicit collusion enhanced through multimarket contact is greater when multimarket contact is formed between firms with similar production costs than between firms with dissimilar production costs.

To graphically illustrate Proposition 2, we develop numerical examples in which the differences in production costs between Firms 1 and 2 are gradually amplified from zero to \$56.80 (i.e., 40% of the marginal cost incurred by the high-cost firm). Table 2 lists a set of values for conjectural variation, v , that decrease in sync with the cross-price demand parameter, d , as the cost differences between the two firms become larger. For example, when Firms 1 and 2 have identical costs of \$142 per passenger, the values for d and v are assumed to be at the highest level of 0.8. As Firms 1 and 2 become dissimilar in their production costs, the values for d and v are linearly reduced from 0.8 to zero.

Table II-2: The Values for Key Parameters Associated with Firms 1 and 2

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
C_1	142	135.5	129	122.5	116	109.5	103	96.5	90
C_2	142	142	142	142	142	142	142	142	142
d^j	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
v_i^{jk}	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

Other parameters used in these examples are assumed to have the following values.

$$a_1 = a_2 = 185, b_1 = b_2 = 185$$

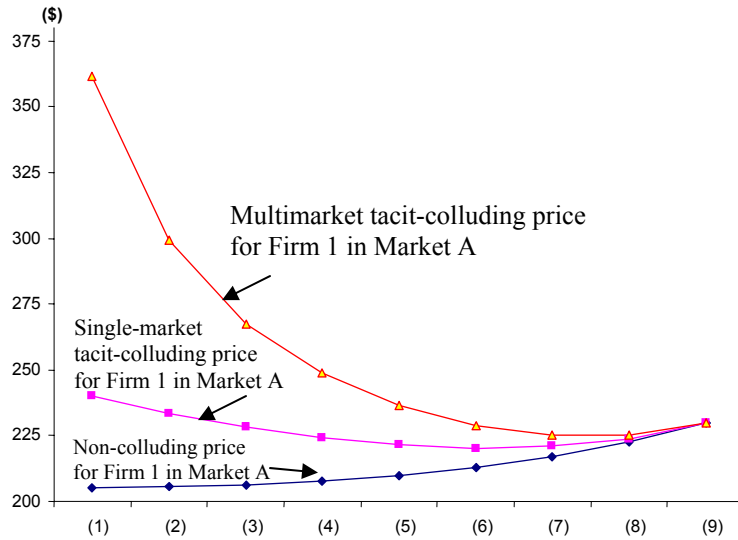
$$e_1^A = e_2^A = 1.3, e_1^B = e_2^B = 1.3^6$$

Using this specification, we calculate, for each pair of firm production cost levels, the competing prices for Firms 1 and 2 in Market A, and their corresponding tacit-colluding prices in both single market and multimarket contact settings. Results

⁶ In the baseline Example (1), the value for own-price demand effect e_i^j is 1.3, which ensures the satisfaction of the second-order condition for the profit maximization problem. In other examples from (2) to (9), the values for e_i^j decline with d^j , holding the difference between these two parameters fixed at 0.5.

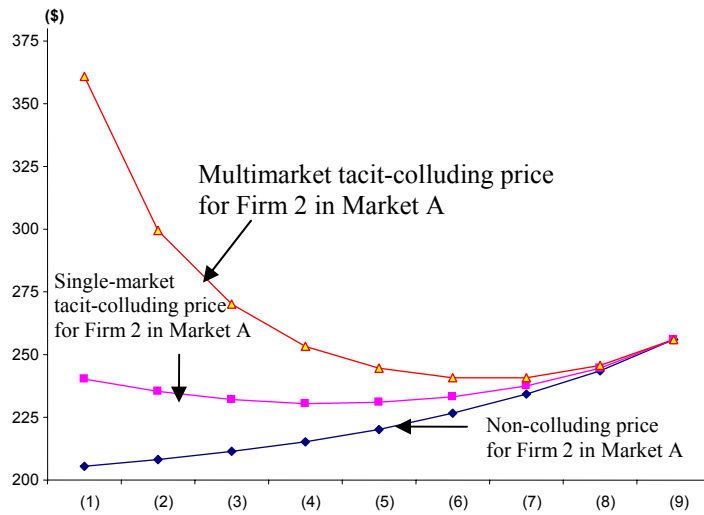
plotted in Figure 4 are for Firm 1 under various scenarios from (1) to (9). Figure 5 presents the results for Firm 2.

Figure II-4: Competing, Tacit-colluding Prices for Low-cost Firm in Single and Multimarket Settings



Note that the left-most example in Figure 4 represents Scenario (1), where Firms 1 and 2 have identical marginal production costs. With the reduction in Firm 1's production cost from Scenario (2) through (9), Firms 1 and 2 have widening differences in their production costs, holding Firm 2's production cost at a fixed level. Consistent with Proposition 2, we find from Figure 4 that the collusion facilitating effects of multimarket contact are greater when rival firms have identical production costs than when they have different levels of production cost. We also find that such impacts of multimarket contact on firm collusive behavior erode as the production costs of the two rival firms become more dissimilar.

Figure II-5: Competing, Tacit-colluding Prices for High-cost Firm in Single and Multimarket Settings



As shown in Figure 5, similar results hold for the high-cost firm, Firm 2. Comparing results in Figure 5 with those shown in Figure 4, we find that the high-cost firm experiences a similar pattern of collusion decreasing impacts from multimarket contact; that is, multimarket contact becomes less effective in facilitating collusive pricing behavior when the rival firms have greater dissimilarity in their production costs.

3. Empirical Analysis

The conjectural variation model and several numerical examples developed in Section 2 illustrate how multimarket contact serves to facilitate tacit-collusive pricing behaviors for firms under various scenarios. The analytical results suggest that the mutual forbearance effect arising from multimarket contact is moderated by market-related characteristics such as own-price and cross-price demand elasticities, and the level of a firm's production cost, relative to its rival's. In the following section, we

develop two hypothesis and use data from the U.S. airline industry to test them. Empirical evidence provides validation of the analytical results.

3.1 Hypotheses

The mutual forbearance view suggests that rival firms with a high degree of multimarket contact tend to collude rather than compete as a result of the mutual deterrence effect (see Proposition 1). According to this view, the rivalry intensity experienced by an airline on a given route is negatively related to the extent of multimarket contact the carrier has with its focal market rivals. In the airline industry, one widely used measure for rivalry intensity is *yields*, or airfares per mile flown (e.g., Evans and Kessides 1994, Gimeno 1999, and Gimeno et al. 1999). Generally speaking, the more intense the competition between carriers, the lower the yields that are expected on a given route, *ceteris paribus*. Therefore, Hypothesis 1 is formally stated as:

H1. The yields for a carrier on a given route are higher when the carrier has a greater extent of multimarket contact with its rival carriers.

Through multimarket contact, firms are endowed with more opportunities to deter their rivals from pricing aggressively. The mutuality of such forbearance actions, however, may not hold when firms differ substantially in their cost structures (see Proposition 2). In the context of the U.S. airline industry, Dresner and Windle (1996 and 1999) find empirical support for the point that a low-cost carriers, or LCCs, focus on price-sensitive passengers providing no-frills service, whereas high-cost “full

service carriers”, or FSCs, offer superior service to travelers who are not as sensitive to airfares. As the services offered by high-cost and low-cost carriers become more differentiated in quality, smaller cross-price effects on demand would be expected. Therefore, the following hypothesis is proposed:

H2. Multimarket contact between carriers with similar production costs has a greater positive effect on yields than that between carriers with different levels of production costs.

3.2 Empirical Models

The two hypotheses are tested using data from the U.S. domestic airline market. The reasons why we focus on the airline industry are two-fold. First, airport-pair routes can be used to specify market scope without causing ambiguity. Thus, we can follow existing empirical studies (e.g., Evans and Kessides 1994, Baum and Korn 1999, Gimeno 1999, and Gimeno and Woo 1999) to measure the extent of multimarket contact by counting the overlapping routes served by any two carriers. More importantly, carrier costs in the airline industry are not private information, and cost differences between carriers are relatively consistent across routes. Typically, a low-cost carrier has its cost advantage over a high-cost carrier on all the routes in which they compete. The main input factors, such as labor and fuel, are invariant on a per unit basis across routes. Some route-related costs (e.g., airport landing fees) do not vary across carriers for the same aircraft type. All these characteristics make this industry ideal to test the differential competitive effects of multimarket contact, depending upon cost differences between carriers.

To test Hypothesis 1, we follow the modeling approach by Evans and Kessides (1994) to estimate the reduced-form price model specified as Equation (1.19). The unit analysis in our study is the yield for an individual airline on an airport-pair route.

$$\ln(\text{Yield})_{ir} = \alpha_0 + \alpha_1 \ln(\text{Route HHI})_r + \alpha_2 \ln(\text{Route Market Share})_{ir} + \alpha_3 \ln(\text{Airport HHI})_r + \alpha_4 \ln(\text{Airport Market Share})_{ir} + \alpha_5 \ln(\text{Route Distance})_r + \alpha_6 \ln(\text{Market Size})_r + \alpha_7 (\text{Slot Controlled})_r + \alpha_8 \ln(\text{MMC})_{ir} + \alpha_9 (\text{Low-Cost Rival})_{ir} + \sum_{i=1, \dots, N-1} \alpha_{10i} (\text{Carrier})_i + \varepsilon_{ir} \quad (1.19)$$

The dependent variable $(\text{Yield})_{ir}$ is the average one-way airfare for airline, i , on route, r , divided by the route non-stop distance. To control for the impact of market concentration on airfare, we include the Herfindahl indices on both route and airport levels, denoted by *Route HHI*, and *Airport HHI*. The degree of market concentration for a given route is calculated as the sum of squared market shares for all carriers flying on the route. Similarly, Airport HHI calculates the summed squares of the market shares for all the airlines at a given airport. Then we use the maximum HHI at the two endpoint airports to measure the airport-based market concentration level for a particular route.

A number of studies have found that an airline's fare is positively related to its operation size at the route endpoint airports, well known as the hub premium effect (Borenstein 1989). We control for this market power effect by using *(Airport Market Share)*_{ir}, which is the maximum of the market shares for carrier, i , at its endpoint airports on route, r . We also take into account the market power effect for carriers

having dominant positions on a particular route by using *(Route Market share)_{ir}*, which measures the percentage of all passengers flying on route, *r*, that travel with airline, *i*. Moreover, Windle and Dresner (1995) find that the presence of low-cost carriers in an air traveling market results in significantly lower average fares for all carriers on the route. Hence, we include a dummy variable, *(Low-cost Rival)_{ir}*, to indicate whether the focal carrier, *i*, has a low-cost rival on route, *r*. Market concentration, market power, and the low-cost carrier's participation are factors all affecting the actual competitive level in the airline market.

The airline market is disciplined by potential competition as well. For instance, average airfares have been found to be higher on routes with slot-controlled endpoint airports. This finding supports the point that in an airline market, potential entrants are effectively deterred by slot control restrictions imposed on the airport.

Accordingly, we control for this potential deterrence effect by using the dummy variable, *(Slot Controlled)_r*, to indicate whether one or both endpoints on route, *r*, are slot-controlled.

The other two control variables included in the reduced-form price equation are *Route Distance*, and *Market Size*. *Route Distance* refers to non-stop distance, and *Market Size* measures the total number of passengers on a given route. It is widely known that airline operations are characterized by economies of distance and economies of density, and as a result, the average cost per passenger mile decreases with flight

distance and with traffic volume. In Equation (1.19), we expect the coefficients for $\ln(\text{Route Distance})_r$ and $\ln(\text{Market Size})_r$ to be negative.

The independent variable $(MMC)_{ir}$ measures the degree of multimarket contact for airline i on route r . As suggested in Hypothesis 1, the greater the $(MMC)_{ir}$, the lower the rivalry intensity between carrier i and its competitors, and thus the higher the airfare for carrier, i , or $(Yield)_{ir}$. To take into account carrier heterogeneity, we incorporate firm dummy variables, $(Carrier)_i$, in Equation (19). After controlling for these fixed carrier effects and market-related effects on airfares, we interpret the coefficient for $(MMC)_{ir}$ as the impact of multimarket contact on a carrier's yield.

The above empirical model is developed to estimate the overall effects of multimarket contact on pricing behaviors of carriers. Hypothesis 2 goes one step further by investigating the differential impacts of multimarket contact between carriers with similar cost levels, and between carriers with dissimilar cost levels. For a focal carrier on a given route, two additional multimarket contact variables are constructed. One measures the extent of the overlapping routes between the focal carrier and all of its rival carriers ranked in the same group according to operating expenses; the other measure captures the degree of multimarket contact between the focal carrier and all of its rival carriers belonging to the different group on the basis of operating costs. This approach requires that the sample carriers be grouped into low- and high-cost categories. The price equation to be estimated is specified as follows:

$$\begin{aligned} \ln(\text{Yield})_{ir} = & \alpha_0 + \alpha_1 \ln(\text{Route HHI})_r + \alpha_3 \ln(\text{Route Market Share})_{ir} + \alpha_3 \ln(\text{Airport} \\ & \text{HHI})_r + \alpha_4 \ln(\text{Airport Market Share})_{ir} + \alpha_5 \ln(\text{Route Distance})_r + \alpha_6 \ln(\text{Market Size})_r \\ & + \alpha_7 (\text{Slot Controlled})_r + \alpha_{hh} (\text{HHMMC})_{ir} + \alpha_{hl} (\text{HLMMC})_{ir} + \alpha_{ll} (\text{LLMMC})_{ir} + \alpha_8 \\ & (\text{Low-Cost Rival})_{ir} + \sum_{i=1, \dots, N-1} \alpha_{9i} (\text{Carrier})_i + \varepsilon_{ir} \end{aligned} \quad (1.20)$$

where $(\text{HHMMC})_{ir}$ measures the multimarket contact between high-cost carrier i and its high-cost rivals on route, r ; the variable (LLMMC) measures the multimarket contact between low-cost carrier i , and its low-cost rivals on route, r ; and $(\text{HLMMC})_{ir}$ is the multimarket contact measure for high- or low-cost carrier i with all of its rivals positioned in the opposite cost group. From the estimated coefficients for these variables, we can examine whether multimarket contact between carriers with similar cost levels impacts collusive behaviors differently from when multimarket contact occurs between carriers with dissimilar cost levels.

3.3 Data

The data used in this study are from Department of Transportation - DB1A (as provided by *Database Product Inc*), and from the *Bureau of Transportation Statistics* (BTS). DB1A contains the 10% Origin & Destination ticket survey data that can be used to determine a number of airline-route specific variables, such as yields for airport-pair markets, route distance, the average number of coupons per ticket, and the number of passengers on the route. BTS provides airline financial data and airport-related data, for example, the total number of passengers traveling into and out of an airport. The sample we collected includes the top 1,000 U.S. domestic routes in the year 2002. We use the complete dataset to calculate route specific characteristics, such as *Route HHI* and *Airport HHI*. Then, we exclude those carriers flying less than

1% of the total passengers on a route, and carriers flying fewer than 10 routes. This sampling approach follows the data filtering procedure used by Evans and Kessides (1994). The final sample includes 4,667 observations from 998 routes and 19 carriers. There are 89 endpoint airports. The 4 slot-controlled airports in the year 2002 are: Chicago O'Hare (ORD), New York City's John F. Kennedy and LaGuardia (JFK, LGA) and Washington D.C.'s Reagan National (DCA).

3.4 Measurement of Multimarket Contact – MMC

Multimarket contact has been measured in several different ways. The most widely used approach is to count the number of markets in which firms compete against one another. In the context of the airline industry, the number of overlapping routes served by airlines is used to measure the extent of multimarket contact between carriers (e.g., Evans and Kessides 1994; Gimeno and Woo 1996). Building on this measurement, we construct a carrier-route specific *MMC* index capturing the extent of multimarket contact for each carrier on the route. First, we count the number of contacts between any pair of carriers i and j across all routes ($r=1 \dots R$) as A_{ij} :

$$A_{ij} = \sum_{r=1}^R D_{ir} D_{jr}, \text{ where } D_{ir} \text{ is a dummy variable that equals 1 if airline } i \text{ flies on route } r, \text{ and 0 otherwise, and } D_{jr} \text{ equal 1 if airline } j \text{ flies on route } r, \text{ and 0 otherwise.}$$

Next the multimarket contact MC_{ij} between airlines i and j , A_{ij} , is scaled by the summation of the number of routes each carrier flies. The formula for MC_{ij} is:

$$MC_{ij} = \frac{A_{ij}}{\left(\sum_{r=1}^R D_{ir}\right) + \left(\sum_{r=1}^R D_{jr}\right)}.$$

Using this formula, the range of the value for MC_{ij} is within $[0, 0.5]$. Finally, the multimarket contact for carrier i on route r , MC_{ir} , is averaged across all of its competitors on route r . The expression for MC_{ir} is:

$$MC_{ir} = \frac{\sum_{j=1}^N D_{jr} * MC_{ij}}{\sum_{j=1}^N D_{jr}}, \text{ where } N \text{ is the total number of carriers in the dataset } (i \neq j).$$

Table 3, below, presents the description and summary statistics for all the variables we use in the estimation.

Table II-3: Descriptive Statistics for the Key Variables

Variable	Description	Mean and (Std. Deviation)
Yield	Average one-way airfare charged by airline i on route r . Non-stop route distance is used to obtain Yield as price measure. [Dollar/Mile]	0.1407 (9.031e-02)
Route HHI	Sum of squared market shares of all carriers flying on route r .	0.4352 (0.1870)
Route Market share	The percentage of passengers on route r that fly with airline i .	0.1944 (0.2421)
Airport HHI	Sum of squared market shares of all carriers at the airport. For carrier i on route r , the maximum HHI of the two endpoints is carrier i 's airport HHI on route r .	0.3523 (0.1470)
Airport Market share	The maximum market share for carrier i on the two endpoint airports of route r .	0.2108 (0.2082)
Distance	Non-stop distance on route r . [Miles]	1,296.20 (656.45)
Market size	Total number of passengers on route r .	21,529.16 (18120.38)
Slot controlled	Dummy variable (1-either one or both endpoint airports are slot controlled). In 2002, there were four slot controlled airports: ORD, JFK, LGA and DCA.	0.15 (0.35)
MMC	Multimarket contact index for carrier i on route r .	0.2948 (0.1038)
Expenses/ASM	Adjusted operating cost for carrier i on route r . [Dollar/seat-mile]	0.1108 (3.1283e-02)

3.5 Estimation of Airline Expenses/ASM

Airlines annually report to the DOT their total operating expenses. We use operating expenses per available seat-mile as an overall cost measure for each carrier.

Operating Expenses/ASM is an approximate assessment of the carrier's cost level. In the airline industry, operating characteristics such as *stage length* contribute to economies of distance. Stage length is the distance of a flight leg. On average, the longer a carrier's average stage length, the lower the average cost per mile incurred. To account for such economies of distance, we modify *Expenses/ASM* by using the elasticity of *Expenses/ASM* to stage length. The elasticity can be estimated by the following log-linear model: $\ln(\text{Exp} / \text{ASM})_i = \beta \ln(D)_i + \varepsilon_i$, in which $(D)_i$ is the average flight length for carrier i and ε_i is the error term.

The estimated elasticity $\hat{\beta}$ equals -0.365. Using the estimated value for $\hat{\beta}$, we adjust each carrier's overall expenses per available seat mile by the formula:

$$(\text{Exp} / \text{ASM})_i^* = (\text{Exp} / \text{ASM})_i * \left(\frac{D_i}{\bar{D}}\right)^{-0.365}$$

where \bar{D} is the average stage length for all carriers (i.e., $\bar{D} = \sum_{i=1}^{19} D_i / 19$). After the stage length adjustment, $(\text{Exp} / \text{ASM})_i^*$ reflects the overall unit cost for each carrier.

Taking the average $(\text{Exp} / \text{ASM})_i^*$ for all carriers as a cutoff value, we classify the sampled carriers into high-cost and low-cost groups. Table 4 presents the ranking results based on adjusted $(\text{Exp} / \text{ASM})_i^*$.⁷

⁷ The high-cost group represents, roughly, the pre-deregulation or "legacy" carriers while the low-cost group represents, roughly, the post-deregulation entrants into the U.S. interstate air transport market.

Table II-4: Low-Cost and High-Cost Carriers

Carrier	EXP/ASM (Dollar/Seat -Miles)	Adjusted (Exp/ASM) (Dollar/Seat -Miles)	Num of Rts Served	Average Flight Distance (Miles)	High-cost/ Low-cost
US Airways	0.1390	0.1253	480	684.20	H
United Airlines	0.1139	0.1194	595	1036.05	H
American Airlines	0.1114	0.1176	648	1054.30	H
Midway Airlines	0.1386	0.1106	12	490.13	H
Midwest Express	0.1154	0.1071	27	742.36	H
Continental Airlines	0.1015	0.1054	442	1008.22	H
Delta Airlines	0.1032	0.0979	743	788.66	H
Northwest Airlines	0.1062	0.0958	477	686.70	H
Alaska Airlines	0.0988	0.0891	53	686.50	H
American Trans	0.0769	0.0851	115	1200.33	L
America West Airlines	0.0809	0.0840	319	1009.32	L
Frontier Airlines	0.0832	0.0813	126	854.92	L
Vanguard Airlines	0.0807	0.0798	41	881.75	L
Spirit Airlines	0.0735	0.0753	35	970.77	L
Jet Blue Airlines	0.0643	0.0737	28	1323.50	L
Airtran	0.0847	0.0718	154	577.72	L
Southwest Airlines	0.0739	0.0667	399	686.30	L
National Airlines	0.0472	0.0554	26	1408.82	L
Sun County Airlines	0.0249	0.0235	12	775.80	L

4. Results

Tables 5 and 6, respectively, present the estimation results for Models 1 and 2, as shown by Equations (1.19) and (1.20). Three versions of each of the models are estimated. For Model 1, the classic OLS results show a positive and significant coefficient for Ln(MMC), supporting the tacit-collusion facilitating effect of multimarket contact. Consistent with Hypothesis 1, the airfare is found to be higher when a carrier has extensive market contact with its rivals, holding other variables constant. In addition, the results show that the presence of a low-cost rival has a significantly negative effect on yields, suggesting that airfares are lower when a carrier has low-cost rivals on a route, as compared to the situation where all of its competing carriers are high-cost, *ceteris paribus*.

From the classic OLS estimation results for Model 1, we also find that airport concentration, and airport market share, as expected, contribute to airfare premiums of various magnitudes. The airport concentration variable endows the airline with the most pricing power, followed by airport market share. The price elasticity associated with *Airport HHI* is 0.1919, which is 3.67 times as large as the elasticity related to *Airport Market Share*. On the other side, *Route HHI* is found to be negative, but insignificant. Moreover, we find that airfares are, *ceteris paribus*, higher on routes where either or both endpoint airports are slot controlled. Also implied is that the airfares decrease as route distance, or market size increases, holding other variables constant.

Table II-5: Estimation Results for Model One

	OLS estimates	Fixed-effects estimates	Random-effects estimates
Independent Variables	Coefficient (t-statistics)	Coefficient (t-statistics)	Coefficient (t-statistics)
Constant	3.3709 ^a (42.48)	4.0197 ^a (41.32)	3.5436 ^a (43.20)
Ln (Route HHI)	-0.0003 (-0.03)	0.0041 (0.41)	0.00346 (0.34)
Ln (Route Market Share)	0.00735 ^c (1.72)	-0.0084 ^b (-1.97)	-0.0056 (-1.32)
Ln (Airport HHI)	0.1919 ^a (20.68)	0.1661 ^a (18.76)	0.1681 ^a (18.99)
Ln (Airport Market Share)	0.05234 ^a (9.93)	0.1018 ^a (16.92)	0.09576 ^a (16.32)
Ln (Distance)	-0.61827 ^a (-80.29)	-0.6163 ^a (-84.44)	-0.61670 ^a (-84.38)
Ln (Market Size)	-0.06707 ^a (-11.12)	-0.0675 ^a (-11.79)	-0.06913 ^a (-12.07)
Slot Controlled	0.1427 ^a (13.57)	0.1134 ^a (11.24)	0.1151 ^a (11.39)
Ln (MMC)	0.08212 ^a (10.54)	0.1482 ^a (8.36)	0.1073 ^a (7.67)
Low-cost Rival	-0.0509 ^a (-5.63)	-0.0793 ^a (-8.90)	-0.08223 ^a (-9.31)
Number of observations	4667	4667	4667
R ²	0.7486	0.7816	0.7421
Significant at 0.01 level ^a , Significant at 0.05 level ^b , Significant at 0.1 level ^c ; The estimated coefficients for the carrier dummy variables are omitted in the column for the fixed-effects model.			

Table II-6: Estimation Results for Model Two

Independent Variable	Fixed-effects estimates	Random-effects estimates
	Coefficient (t-statistics)	Coefficient (t-statistics)
Constant	3.639 ^a (42.57)	3.3112 ^a (41.77)
Ln (Route HHI)	0.0071 (0.70)	0.00582 (0.58)
Ln (Route Market Share)	-0.00741 ^c (-1.73)	-0.0056 (-1.33)
Ln (Airport HHI)	0.1625 ^a (18.37)	0.1642 ^a (18.56)
Ln (Airport Market Share)	0.0998 ^a (16.59)	0.0957 ^a (16.41)
Ln (Distance)	-0.6217 ^a (-84.14)	-0.6211 ^a (-84.17)
Ln (Market Size)	-0.0725 ^a (-12.67)	-0.0727 ^a (-12.68)
Slot Controlled	0.10630 ^a (10.42)	0.1074 ^a (10.53)
Low-cost rival	-0.1025 ^a (-7.57)	-0.1031 ^a (-7.81)
Multimarket Contact between high-cost and high-cost carrier	0.5695 ^a (8.91)	0.5006 ^a (8.67)
Multimarket Contact between low-cost and low-cost carrier	0.4687 ^a (4.38)	0.5298 ^a (5.15)
Multimarket Contact between high-cost and low-cost carrier	-0.05173 (-0.78)	-0.0492 (-0.76)
Number of observations	4667	4667
R ²	0.7833	0.7476
Significant at 0.01 level ^a , Significant at 0.05 level ^b , Significant at 0.1 level ^c ; The estimated coefficients for the carrier dummy variables are omitted in the column for the fixed-effects model.		

Since Market Size is defined as the total number of passengers on a given route, this aggregate measure of demand is most likely to be independent of the error term in the airfare regression. Nevertheless, other market structure variables, such as *Route HHI* and *Airport Market Share*, are potentially endogenous and thus may be correlated with the error term, ε_{it} , in airfare regressions. To address this potential issue, we include carrier-specific dummy variables in our model. Columns 3 and 4 of Table 5 report the estimation results using fixed-effects and random-effects models.

Comparing the classic OLS results (Column 2 of Table 5) to the results from the fixed-effects model (Column 3), we find that the coefficients for $\text{Ln}(MMC)$ and for $\text{Ln}(\text{Airport Market Share})$ estimated by the fixed-effect model are twice as large as the respective coefficients estimated by the classic OLS regression. The fixed-effects estimation shows that the airline-specific effects account for 35.02% of the sample variation in the average airfare per mile. A comparison of the R-squared values indicates that the fixed-effects model provides a better goodness of fit than does the OLS model, which is not including carrier specific effects. It is further found that the coefficient for *Route Market Share* is, unexpectedly, negative in the estimation of the fixed-effects regression. The high correlation between *Route Market Share* and *Airport Market Share* (see Table 7) likely contributes to this result⁸.

Table II-7: Correlation among Market Structure Variables

	Airport HHI	Airport Market Share	Route HHI	Route Market Share
Airport HHI	1.0000			
Airport Market Share	0.2334	1.0000		
Route HHI	0.3878	0.2074	1.0000	
Route Market Share	0.0918	0.7786	0.2349	1.0000

In the estimation of the fixed-effects model, variance inflation factors (VIF) are computed to diagnose whether collinearity among some independent variables poses a serious concern for estimation reliability. We find that the values of VIF for all predictors are less than 10, indicating that multicollinearity is likely not a concern (Mason et al. 1991).

⁸ The alternative regression is also run for Model 1 after removing the variable, *Route Market Share*. The estimated coefficients for all other predicating variables are similar to the regression results in the original fixed-effects model.

The unit observation for the dependent variable in our estimation is average one-way airfare per mile for an individual airline on a given route. In this situation the errors for the same carrier are likely to be correlated across routes, or $\text{cov}(e_{ir}, e_{it}) \neq 0$.

Within-route errors are likely correlated between carriers as well, or $\text{cov}(e_{ir}, e_{jr}) \neq 0$.

If either of these cases occurs, the i.i.d. assumption for the error term will be violated, and the variance-covariance matrix estimated by the fixed-effects model will be biased, thereby making the further inferences invalid. Therefore, it is important to examine whether the results based on the fixed-effects model are robust to alternative estimation procedures, such as a GLS random-effects model.

The GLS random-effects model allows for stochastic regressors but relies upon the assumption of no correlation between predictors and the error term. Its estimators are asymptotically unbiased and more efficient. Column 5 of Table 5 presents the results for the random-effects model. A Hausman specification test is performed to examine whether the coefficients estimated by the fixed-effects model are statistically different from those estimated by the GLS random-effects model. The resulting chi-square statistic is 4.28 with 8 degree of freedom, which is insignificant at the 5 percent level. Thus, we cannot reject the null hypothesis that the coefficients estimated by the two procedures are the same.

Table 6 presents the results for Model 2, which examines whether the impact on firm collusive behavior is the same when multimarket contact occurs between firms with similar cost levels compared to when it takes place between firms with different cost

levels. Two variations of the model are estimated. From the fixed-effects estimation results, we find that the multimarket contact variables for carriers with similar costs have the expected signs and are statistically significant. Specifically, the positive coefficient for *Multimarket Contact between high-cost and high-cost carriers* indicates that the airfares for a high-cost carrier are higher on routes where it has more overlapping contacts with its high-cost rivals. Similarly, the positive coefficient for *Multimarket Contact between low-cost and low-cost carriers* suggests that airfares for a low-cost carrier are higher on routes where it has more overlapping markets with its low-cost counterparts. To see whether these two coefficients are statistically different, we run a restricted model where the two coefficients are constrained to be identical. The relevant F statistic comparing the unrestricted with the restricted regression is derived as $F_{1, 4639} = 0.573$, which is less than the critical value at the 5% level of significance. Therefore, we cannot reject the null hypothesis that the coefficient for multimarket contact between high-cost and high-cost carriers is the same as that for multimarket contact between low-cost and low-cost carriers. This result implies that multimarket contact between firms with similar cost levels has a positive effect on airfares, and it may not matter whether these rival firms are both high-cost, or both low-cost.

In comparison, the coefficient for *Multimarket contact between high-cost and low-cost carrier* is statistically insignificant. This finding supports our hypotheses that when multimarket contact is between carriers with dissimilar cost levels, there is no significant impact on airfares. In this case, firms make their pricing decisions

independent of multimarket contact; that is, multimarket contact does not facilitate tacit-colluding behavior. From Table 6, it is also found that the coefficient for *Low-Cost Rival* is negative and statistically significant. This finding is in line with the widely-held view that the presence of low-cost rivals on a given route intensifies price competition, thereby pulling down airfares on the route. The estimation results for other variables, such as market structure, route distance, and airport slot-controlled status, are similar to those found in Model 1.

5. Conclusions and Implications

This article theoretically and empirically investigates the differential impacts of multimarket contact on tacit-collusive behaviors for firms facing varying market characteristics, and for the rival pairs having similar/dissimilar production costs. The analytical results suggest that firms obtain more tacit collusion benefits when they compete simultaneously in multiple markets rather than in a single market. Therefore, multimarket contact facilitates tacit-colluding behavior and reduces the intensity of rivalry between multimarket competitors. A key contribution of our analytical study is to show that the collusion enhancing effects of multimarket contact hold true when markets are identical and firms produce outputs with identical marginal costs, which is constant throughout markets. Under this condition, Bernheim and Whinston (1990) suggest that multimarket contact is irrelevant and does not facilitate collusion. Their findings are based upon an analytical model studying infinitely repeated Bertrand price competition between firms in the multimarket contact setting. In comparison, the conjectural variation model we develop focuses on explaining how multimarket contact restrains the competitive intensity between multimarket rivals.

Our analytical model demonstrates that multimarket contact is more effective in facilitating tacit collusive pricing when it occurs between rival firms having similar production costs than when it occurs between rival firms with dissimilar production costs. It is the formation of higher conjectural variation and the presence of greater product substitutability that reinforces the collusion-facilitating effects of multimarket contact for firms having similar production costs. This finding may have implications for firms competing across multiple markets. For example, when two firms compete in a local market with a single product, one option for a firm to avoid fierce competition is to distinguish itself from the rival firm by introducing differentiated products. The more dissimilar the products, the less likelihood for the occurrence of a pricing war. However, under a multimarket scenario, tacit collusion and lower rivalry intensity may be more likely to sustain when the product lines firms develop are similar to one another. Consequently, the competitive implications of a product differentiation strategy may be dramatically different for single market and multimarket contact settings.

The empirical findings verify the propositions developed in our theoretical analysis. As expected, the estimation results support the longstanding view that multimarket contact reduces interfirm rivalry intensity. Using data from the U.S. airline market, we find that airfares are higher on routes where carriers have more overlapping contacts with rival carriers, *ceteris paribus*. Moreover, our estimations suggest that the positive impact of multimarket contact on airfares is present in the situation when

rival carriers have similar production costs; when rival carriers have dissimilar production costs, multimarket contact has little impact on a carrier's yield.

From this paper, it is found that low-cost carriers have positive reasons to engage in mutual forbearance when their rivals are also low-cost carriers. As a result, it may not be sufficient to just open airline markets to low-cost competition without any regulatory oversight. Since low-cost carriers appear to engage in tacit collusion, some regulatory oversight might still be needed. It is also important to realize that although multimarket contact enhances tacit collusive prices for both low-cost and high-cost carriers, it matters less as their products become more differentiated within and between markets.

Chapter 3: Essay Two - A Two-Location Inventory Model with Transshipments in a Competitive Environment

1. Introduction

Due to the random nature of demand, stock discrepancies (i.e., the difference between the product amount available and the product amount demanded) are commonly observed in single-location and multiple-location distribution systems. In a single-location environment, this gap can be filled by having an amount of safety stock; i.e., by operating with a higher inventory level to protect against demand uncertainty. Compared with the single-location system, the multiple-location network has more alternatives to deal with demand uncertainty. Among these methods, the implementation of lateral transshipments among outlets is an effective and efficient way for firms to reduce inventory carrying cost and to improve customer service levels at the same time.

Transshipments is a practice of transferring goods from one location with excess stock to satisfy demand at another location with insufficient stock (Dong and Rudi, 2004). In many industries with long lead-times, short selling seasons, great demand uncertainty, high inventory carrying costs, and/or high penalty stockout costs, transshipments have been widely used to reallocate inventory from an overstocked outlet to an out-of-stock outlet. This practice is logically equivalent to other types of risk-pooling strategies, such as inventory centralization, postponed differentiation, component commonality, and product substitution.

The literature on modeling transshipments in a multi-location inventory system dates back to Krishnan and Rao (1965). The distribution network they investigate has N warehouses, characterized by centralized control, independent demand, and identical inventory holding and shortage costs for all of the locations. Their analysis indicates that under these conditions, transshipments equalize optimal inventory and service levels among stocking locations. Krishnan and Rao (1965) present a fundamental framework to study the transshipment problem in several aspects. First, they identify a sequence of events with which transshipments are assumed to occur after the demand is realized, but before it is satisfied. Second, they set the transshipment size as the minimum of the excess stock at one location and the shortage at another location. When all locations are out of stock, or all have surplus stock, transshipments do not occur. The third contribution of their analysis is that they take into account the fact that there is certain transshipment costs incurred when units are delivered from the overstocked location to the under-stocked location. Within this framework, the practice of transshipments is modeled to enable the tradeoff between the total transshipment cost and the summation of inventory holding and shortage costs.

Since then, there has been a growing body of work published on the topic of transshipments. Tagaras (1989), for example, generalizes the analysis developed by Krishnan and Rao (1965) and investigates the effect of transshipments on the customer service level, measured by both non-stockout probability and fill rate. Tagaras (1989) builds a two-location inventory distribution system, having two locations replenished by a common supplier. Unlike the model developed by

Krishnan and Rao (1965), Tagaras (1989) relaxes the constraints on cost structure by allowing for different ordering and holding costs at different locations. Tagaras (1989) also extends the findings by Krishnan and Rao (1965) and argues that a *complete pooling* policy⁹ improves the service levels at both locations, even if the two stocking locations have different inventory ordering and holding costs. A complete pooling policy is consistent with the transshipment policy specified by Krishnan and Rao (1989). Moreover, Tagaras (1989) finds that under certain conditions a complete pooling policy is optimal in that it achieves the minimum total expected cost in a two-location distribution system, and it equalizes the service levels at both locations. However, such an equality of service levels for the cost minimization solution would hold only if the demand and cost structures are the same at the two locations.

The approach of minimizing the total expected cost serves as the building block for the traditional *newsvendor problem*. A newsvendor must determine the order size of the newspaper before observing the actual demand for today's paper. If the order size is more than the actual demand, the newsvendor suffers a loss because the current issue has little salvage value in the future; on the other hand, if the order is less than the actual demand, the newsvendor bears a direct loss from the stock insufficiency for the current period, and an indirect loss because some of those dissatisfied customers will switch to other newsvendors in the future. For a known distribution over demand, the probabilities of stocking-out and over-stocking will depend on the inventory level

⁹ According to complete pooling policy, the number of units transshipped from one firm to another is the minimum of the excess stock at one location and the shortage at the other location. In addition, no transshipments occur if both locations are stocked out or if neither of them is out of stock.

chosen by the newsvendor. Therefore, the optimal inventory level can be determined by minimizing the expected costs of stockouts and overstocks, given the cost structure as assumed.

Many transshipment studies have applied and extended the classic newsvendor problem to a multi-newsvendor setting or a multi-period context. Of these studies, Robinson (1990) examines the optimal ordering policy (i.e., the order-up-to point) in a multi-period and multi-location system, allowing transshipments among retail outlets. Robinson (1990) demonstrates that as a recourse action, transshipping products among retail outlets is an alternative to retailers making orders at the beginning of each period, and thus has an effect on the choice of the order-up-to level. By using a heuristic technique, Robinson (1990) verifies that if the base stock order-up-to point is nonnegative in the final period, it will be the optimal order-up-to level for all other periods, assuming that transshipments occur after demands are realized and before they are satisfied for each period. In Robinson's model, the size of transshipments is consistent with the complete pooling policy; i.e., the amount of goods transshipped between a pair of outlets is just enough to meet the shortage at the outlet with insufficient stock, but not more than what is available at the surplus location after demands are realized.

Most of these previous studies on transshipments follow the line that transshipments, as a mechanism to reallocate resources among locations at the same echelon level, benefit both the sending outlet and the receiving outlet. Through transshipments, the

sending outlet reduces its surplus inventory that otherwise would be less valuable, while the receiving outlet satisfies customers who might not be served otherwise. One limitation of these studies is that the two locations considered are independent, and isolated from each other. Because of this constraint, many interesting, dynamic, and strategic interactions between locations are ignored. As Herer and Reshit (1999) point out, in a two-location inventory system allowing the implementation of transshipments, each location serves as a secondary, random source of supply to the other. Therefore, the employment of transshipments between locations must have a nontrivial impact on their replenishment decisions. Herer and Reshit (1999) further demonstrate that in a two-location inventory system with nonnegligible fixed and joint replenishment costs, the traditional replenishment policy, i.e., the order-up-to-point (s, S) at each location, is no longer optimal. Instead, coordination in inventory replenishment activities is necessary to leverage the benefits from transshipments.

When inventory coordination and the implementation of transshipments are jointly considered, a central “parent” agent is assumed to help determine the order size and the transshipment quantity at each location (Rudi et al. 2001). In reality, however, transshipments are also common in a decentralized environment, in which the inventory replenishment and transshipment decisions are made locally, rather than globally. The work by Rudi et al. (2001) addresses the transshipment problem in such a local decision-making context, in which each location aims to maximize its own profit. Their analysis demonstrates that joint profits would not be maximized in the decentralized environment, without using a transshipment price. It is also found that

an intermediate transshipment price makes it possible for each location to choose an order size, which would lead to the maximal joint profits. According to Rudi et al. (2001), the choices of order quantities at each location in a localized decision making environment are interrelated because of the externality effect arising from the transshipment practice. Specifically, when one location orders a large inventory, it becomes easier for other locations to rely on this inventory in case of stock outs; when one location makes a small-size order, it is easier for other locations to dispose of their surplus stock.

Recently, the impact of transshipments on supplier performance has received growing attention. Dong and Rudi (2004), for instance, examine the effect of transshipments on manufacturer profits with two distinctive assumptions: (1) An exogenous wholesale price, which is the same regardless of whether transshipments occur or not; and (2) An endogenous wholesale price, which is set by the manufacturer as the best response to the optimal order quantities and the transshipment decisions made by retailers. Moreover, Dong and Rudi (2004) investigate the role that the number of retailers, and the demand correlation among them, might play in affecting the relationship between the implementation of transshipments and the profits for manufacturers and retailers. They find that in the case of an exogenous wholesale price, the transshipment practice provides retailers with greater gains as the number of retailers increases, and as demand correlation among them decreases. Both of these factors contribute to the risk-pooling effects. In the case of an endogenous wholesale price setting, however, transshipments are found to make retailers worse

off. Finally, their analytical results show that the benefit to the manufacturer from the implementation of transshipments is conditional upon the wholesale price in an endogenous price-setting situation. In other words, it is found that the manufacturer can achieve more profits by charging a higher wholesale price. Under such an exogenous wholesale price setting, the manufacturer is further found to benefit more from the transshipment practice when more retailers are participating and when their demands are less correlated.

In the prior studies, the practice of transshipments among retailers is either voluntary or motivated by an appropriate transshipment price. One notable example is the study by Dong and Rudi (2004), in which a transshipment price is set by a Stackelberg game between the supplier and retailers. In their study, Dong and Rudi (2004) investigate a unique distribution system, in which a common manufacturer sells to n retailers that are owned or operated by the same entity. The optimal inventory level choice for each retailer is made to maximize the total expected profits of these retailers, on the condition of having an either exogenous or endogenous wholesale price. In an earlier work by Rudi et al. (2001), the authors consider the scenario where each location makes the order quantity decision to maximize its own profit, namely a local decision setting. They find that there exists a transshipment price that enables firms to achieve the same profit outcome in a local decision setting as that in a joint-decision setting, under which the order quantities are determined to maximize the joint profits of firms. These studies, however, assume away the possibility that transshipments might be implemented among neighboring, competing locations. In

reality, many retail outlets are head-to-head rivals. For example, when a customer cannot get a particular automobile at one dealer site, he/she might switch to a nearby dealer to make the purchase there. Several questions remain in this situation: (1) Will transshipments occur when retailers are vying for the same pool of customers? (2) Will transshipments always lead to win-win outcomes in such a competitive environment? and (3) Will there be a transshipment price that enables a large firm to achieve the same benefits from the practice of transshipments as does a small firm?

In this essay, the transshipment problem is explored in a variety of competitive environments consisting of firms with symmetrical and asymmetrical market demands, and the comparisons are made in the performance effects of transshipments for firms operating in local and joint decision-making contexts. The model is built upon the work by Krishnan and Rao (1965), Rudi et al. (2001), and Dong and Rudi (2004). The remainder of this essay is organized as follows. The next section presents the basic modeling framework. Section 3 analyzes the case where inventory decisions are made by two firms independently. Section 4 examines the case where inventory choices are coordinated. Numerical examples are then used in Section 5 to illustrate these findings. Section 6 offers conclusions and discusses limitation and future research.

2. The Model

The analysis of the classic newsvendor problem provides a useful framework to study transshipments among competing firms. In the traditional newsvendor problem setting, the research focus is mainly on specifying an optimal order level for a single

newsvendor. The order quantity is determined to maximize the newsvendor's expected profit, which is composed of the revenue from selling goods, the salvage value of the unsold stock, the penalty cost for the unmet demand, and the inventory purchasing cost. It has been found that for any type of monotonic, continuous demand distribution, there always exists a unique order quantity at which the expected marginal cost of adding another unit of order stock equals the expected marginal benefit. The newsvendor's expected profit is therefore maximized at this inventory level.

The approach of modeling a single newsvendor problem has been extended to a two-location environment. In the context of two independent locations, the previous studies have presented a good illustration of how transshipments affect a firm's inventory management. In reality, transshipments sometimes might occur between two firms that are coincidentally competing with each other. In this setting, it is important to incorporate interfirm rivalry intensity into the analysis of the transshipment problem. Indeed, the nature of market competition can have a significant consequence on firm inventory decisions. Therefore, the conclusion drawn from previous models with respect to the implication of transshipments for performance outcomes would not hold in a competitive environment.

Consider a distribution system consisting of two distributors and a common manufacturer (or, more generally, a supplier) that produces and sells to these two firms

noted as i and j . At the beginning of a single order cycle, each firm decides a nonnegative inventory level $Q_{i(j)}$ before observing the actual demand $D_{i(j)}$.

To capture the extent to which one firm is a direct or close competitor of the other, the model proposes the use of a customer's switching rate (i.e., the probability of an individual consumer switching firms in the event of stockout). Formally, let λ_{ij} stand for the switching rate from Firm i to j . It represents the probability of an individual customer switching to Firm j , when his/her demand at Firm i 's site cannot be satisfied because of insufficient stock. At an aggregate level, λ_{ij} indicates the percentage of customers switching to Firm j when the primary supplier, Firm i , is stocked out. The more intense the competition between firms, the higher the switching rate. Further assume that the variable λ_{ij} is exogenously given and can take any value between 0 and 1.

Table 1 presents the notation for a series of cost parameters that are used in the model. These parameters include the unit retailing price of Firms i and j , the manufacturing cost, the wholesale price, the average inventory carrying cost, the salvage value for each unsold unit, the penalty cost for each unmet demand, the unit transshipment cost, and transshipment price. Table 1 also provides the notation for other relevant variables such as the level of transshipments, the amount of actual sales, unmet demand, and unsold stock.

Table III-1: Notations for Key Parameters and Variables used in the Model

Notation	<i>Definition</i>
p_i, p_j	The unit price for goods sold by Firms i and j, respectively.
m	The unit manufacturing cost incurred by the supplier.
w_i, w_j	The wholesale price paid by Firms i and j, respectively.
j	The average inventory carrying and holding costs.
s	The salvage value for each unsold stock.
p	The penalty cost for each unmet unit of demand.
τ	The cost for transshipping a unit of goods from Firm i to j, and vice versa.
c_{ij}, c_{ji}	The transshipment price paid by Firm j to i, and by Firm i to j.
X_{ij}, X_{ji}	The level of transshipments from Firm i to j, and from Firm j to i.
R_i, R_j	The actual sale of Firm i, and j.
U_i, U_j	The amount of unsold stock of Firm i, and j.
Z_i, Z_j	The amount of unmet demand of Firm i, and j.
$\lambda_{ij}, \lambda_{ji}$	The consumer's switching rate from Firm i to j, and from Firm j to i.

The model builds upon the sequence of events described as follows. At the beginning of the single replenishment period, shipments from the common supplier are ordered by Firms i and j to bring their stock levels to Q_i and Q_j . At some point in the period, all of the demands at both locations are realized and observed. Before this point in time, neither Firm i nor Firm j has complete knowledge of the actual demand for the current period. Then Firm i(j) compares the demand $D_{i(j)}$ with the stock level $Q_{i(j)}$. Four events may arise. In the first event, both firms have sufficient inventory to satisfy their demand, and thus all of the consumers are served instantly; in the second event, one firm, but not both, is out-of-stock, and the aggregate demand at the two locations exceeds the summed stock of Firms i and j; the third event specifies the situation similar to the previous one except that the joint inventory of the two locations is greater than the aggregate demands for these two firms; in the fourth

event, both firms are stocked out, and, as a result, the satisfied demand at each location is up to the amount of stock that Firms i, and j, hold.

The model further assumes that when one firm is out of stock while another firm has stock in redundancy, the firm having excess stock has two options to choose. Either it can transship its extra stock to the firm that is in short of supply, or it can sell to consumers switching from the out-of-stock firm. Figures 1 and 2 provide graphical illustrations of these two alternative scenarios. In Figure 1, transshipments are implemented between Firms i and j in the event of stock out and in this case, consumers will not switch but rather wait for the arrival of the transshipped goods. Figure 2 shows what happens when no transshipments are implemented. In this scenario, it is likely that consumers switch in the event of stock out.

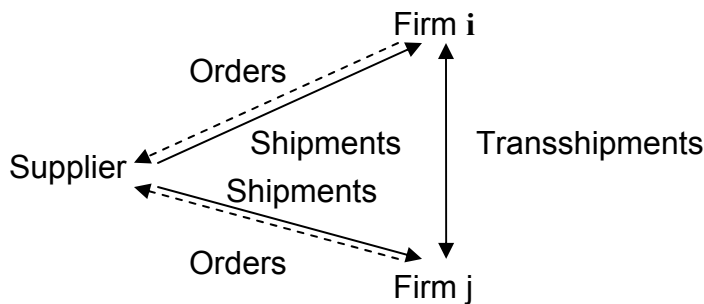


Figure III-1: An Illustration of the Scenario with Transshipments

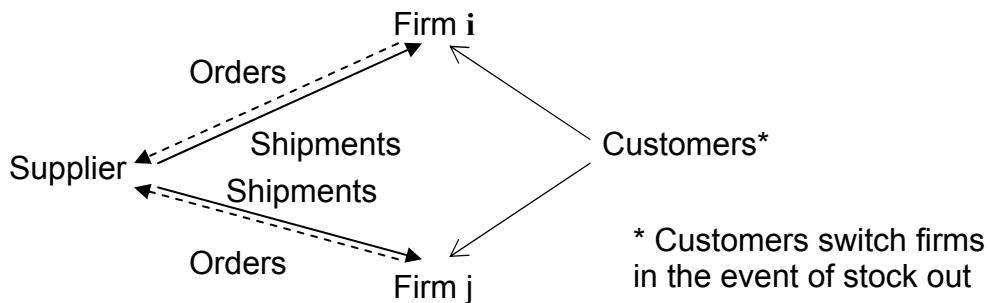


Figure III-2: An Illustration of the Scenario without Transshipments

Since the model considers only one replenishment period, it is reasonable to assume that firms adopt the *complete pooling* policy. Suppose Firm i is out of stock, while Firm j has extra stock. According to this policy, all of the transshipments requests that Firm i makes to Firm j are honored if inventory at j 's location is available. In other words, Firm j will not transship more than it has available; on the other side, Firm i will not accept more than it requests. Moreover, no transshipments will occur if both locations are out of stock or if both have extra stock.

Most previous studies have set the revenue per goods sold by the retailer as a constant r (e.g., Dong and Rudi 2004). In contrast, this model assumes that the unit price charged by Firms i and j may be different, denoted by p_i and p_j , respectively. The manufacturer produces to order with unit cost m and sells to Firm $i(j)$ at the wholesale price $w_{i(j)}$. In addition to purchasing costs, each unit of inventory incurs carrying costs. Let j be the average holding cost for each replenishment cycle, then the carrying costs associated with the inventory level Q can be written as jQ . Further assume that the salvage value for each unsold unit is s , and the penalty cost for each unmet unit of demand is p . For the goods transshipped from Firm i to j , Firm i is responsible for the delivery and thus bears a transshipment cost τ , for each unit transshipped. In return, Firm j pays the transshipment price to Firm i , c_{ij} per unit transshipped.

No short answer can be found for the questions of whether, when, and how much to transship from one Firm to the other. The *complete pooling* policy, nevertheless,

provides a straightforward solution to this problem. Following this rule, the transshipments from Firm i to j are determined by

$$X_{ij} = \min[[Q_i - D_i]^+, [D_j - Q_j]^+], \text{ where } [z]^+ = \max [z, 0].$$

In this expression, the notation $[Q_i - D_i]^+$ represents the extra supply at location i and $[D_j - Q_j]^+$ is for the excess demand at location j. According to the *complete pooling* rule, the transshipment size takes the lower value of the overstock at one location and the stock shortage at another. Similarly, the level of transshipments from Firm j to i can be written as: $X_{ji} = \min[[Q_j - D_j]^+, [D_i - Q_i]^+]$.

Adding the transshipments from Firm j, Firm i's sales are the followings:

$$R_i = \min(D_i, Q_i) + X_{ji}, \text{ and its unmet demand equals: } Z_i = (D_i - Q_i - X_{ji})^+.$$

Deducting the transshipments to Firm j, Firm i's unsold stock is the followings:

$$U_i = (Q_i - D_i - X_{ij})^+.$$

Correspondingly, the following expressions hold for Firm j:

$$R_j = \min(D_j, Q_j) + X_{ij}, \quad U_j = (Q_j - D_j - X_{ji})^+, \text{ and } Z_j = (D_j - Q_j - X_{ij})^+.$$

With transshipments, the expected profits for Firms i and j can be written as:

$$\pi_i(Q_i, Q_j, c_{ij}, c_{ji}) = E[p_i R_i + (c_{ji} - \tau)X_{ij} - c_{ij}X_{ji} + sU_i - pZ_i] - w_i Q_i - jQ_i \quad (2.1)$$

$$\pi_j(Q_i, Q_j, c_{ij}, c_{ji}) = E[p_j R_j + (c_{ij} - \tau)X_{ji} - c_{ji}X_{ij} + sU_j - pZ_j] - w_j Q_j - jQ_j \quad (2.2)$$

The profit for the supplier is a function of order quantities made by Firms i and j, shown by the expression below:

$$\pi^s = (w_i - m)Q_i + (w_j - m)Q_j \quad (2.3)$$

To see the role that transshipments play in reallocating resources and thus affecting *a priori* inventory decisions, it is necessary to draw a comparison between the cases with and without transshipments. Consider a setting when Firm i is overstocked by $(Q_i - D_i)$, whereas Firm j is out of stock by $(D_j - Q_j)$. Assume no transshipments are implemented from Firm i to j. Firm j would end with $(D_j - Q_j)$ unmet demand and of those dissatisfied customers, $\lambda_{ji}(D_j - Q_j)$ would switch to Firm i. The demand at Firm i's site would therefore increase to $D_i + \lambda_{ji}(D_j - Q_j)$ provided sufficient stock is available to serve consumers switching from Firm j. Alternatively, Firm i can transship part or all of its extra stock to Firm j.

Although transshipments, viewed as an inventory pooling practice, have been studied mostly from the retailer's perspective, it is important to see how the supplier fares as well. Indeed, Rudi et al. (2001) suggest that an important extension of the transshipment study in the future is to incorporate the manufacturer into the analysis. A follow-up question arises as to how to align the supplier's interest with the retailers' concerns or profitability. For example, if the supplier designs a constant wholesale price, its profit grows invariably with the order quantities from retailers. The more goods retailers order, the more profits the supplier gains. However, a large-size order is inevitably accompanied by high inventory-carrying costs for retailer, resulting in potential losses during periods of low demand. Therefore, a large-size order benefiting supplier may not be favored by retailer. Moreover, retailers may

prefer small order sizes when transshipments provide them with the opportunity to replenish their stock, incurring costs no greater than those related to lost sales or backorders. The above discussion ignores the key role of the transshipment price in realigning retailer inventory decisions with supplier profitability. In fact, the joint order quantities of retailers change nonlinearly with the transshipment price.

Therefore, it is in the supplier's interest to coordinate the retailer inventory decisions through the use of transshipment price. Moreover, if it is found that a small retailer benefits more through transshipments than its large rival counterpart, some additional incentive mechanisms might be necessary in order for the large firm to participate in the practice of transshipments with the small firm.

Previous studies of the transshipment problem have established a number of constraints on cost parameters. These assumptions are used to ensure that the results are nontrivial and feasible (Tagaras 1989, Robinson 1990, and Dong and Rudi 2004). Following their approach, the cost parameters in the model are restricted by the conditions below.

$$(1) \quad s < w_i + j, s < w_j + j$$

$$(2) \quad w_i + j < p_i + p, w_j + j < p_j + p$$

$$(3) \quad m < w_i, m < w_j$$

$$(4) \quad s + \tau + j < p_i + p, s + \tau + j < p_j + p$$

$$(5) \quad c_{ij} \in [s + \tau, p_i + p], c_{ji} \in [s + \tau, p_j + p]$$

Conditions (1) and (2) together exclude two extreme possibilities: Firms ordering an infinite amount, and firms not ordering at all. When the salvage value per good is

greater than the summation of the unit wholesale price and the inventory carrying cost, firms can always recover the cost of adding additional stock, no matter whether it is sold or not. On the other hand, when the cost of ordering additional stock is greater than the summation of the unit price and the penalty cost; i.e., the marginal value per unit sold, firms have no willingness to distribute the product.

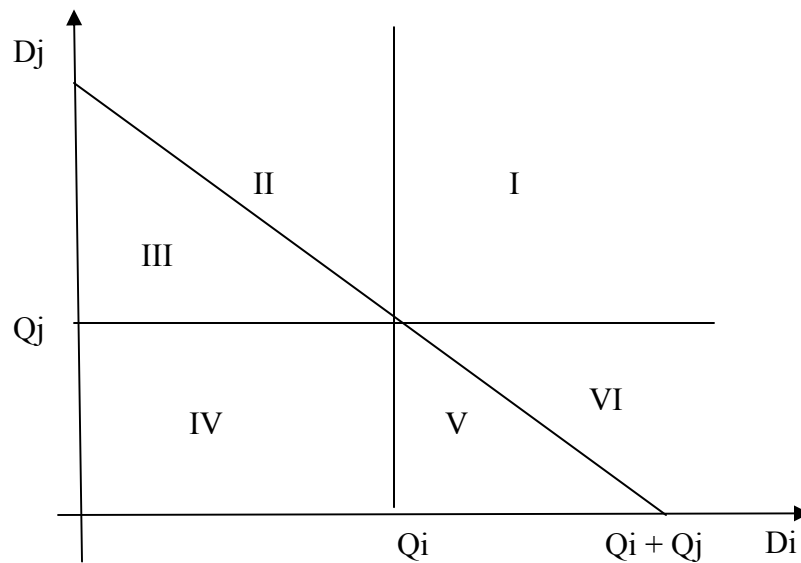
For the manufacturer or the common supplier to participate in the system, it is necessary that the unit manufacturing cost be less than the wholesale price, as shown in Condition (3). The inequality in Condition (4) suggests that transshipments, in general, are beneficial because the salvage value of the unit stock at one location, if not transshipped, is less than the revenue from selling it to another location, minus the cost incurred in transshipping and holding the good.

Condition (5) provides a feasible range for the transshipment price. The lower bound of this range is the total of the salvage value, and the transshipment cost, per unit good. It can be viewed as a reservation price for the firm sending the transshipped goods. From the sender's perspective, it will not transship the goods to another location unless it gets a payment exceeding such a reservation price. The upper bound of this range represents the marginal value of an additional sale, which is the summation of the unit market price and the penalty cost per lost sale. Similarly, it can be viewed as a reservation price for the firm accepting the transshipped goods. On the recipient's side, it will accept the transshipped goods only if the transshipment price it pays is less than such a reservation value. Therefore, only a transshipment price

restricted within this range can satisfy both the sender and the recipient (Note: c_{ij} represents the transshipment price paid by Firm i to j , and c_{ji} is the other way).

Although the following sections focus on studying expected profits for firms and their inventory decisions in a stochastic demand environment, it is necessary to describe the occurrence of transshipments in each of various scenarios, as presented in Figure 3.

Figure III-3: Graphic Illustration of Scenarios



In this graph, D_i , D_j represent the demand size at Firm i 's and Firm j 's locations, respectively. Q_i and Q_j are the order quantities that Firms i and j replenish at the beginning of the order cycle. In a single-period planning horizon, six scenarios can arise. Event I represents the scenario where Firms i and j are both out of stock. Event IV, on the contrary, is the scenario where Firms i and j are both overstocked. Intuitively, these two events involve no transshipments. Of particular interest are the other four events.

In Event II, Firm j has demand D_j that is more than its inventory stock Q_j , while Firm i has demand D_i that is less than its inventory stock Q_i . In other words, Firm i has surplus stock $(Q_i - D_i)$, and Firm j is out of stock by $(D_j - Q_j)$. Therefore, a transshipment is directed from Firm i to j. Similarly, in Event III, Firm j is stocked out, whereas Firm i is overstocked; thus Firm i transships its extra stock to Firm j. Although the directions of transshipments in Events II and III are the same, the quantities transshipped are different. A close look at the graph shows that in Event II, the total demand for Firms i and j, denoted as $(D_i + D_j)$, is greater than their aggregate stock $(Q_i + Q_j)$. This implies that the surplus stock at Firm i's location $(Q_i - D_i)$ is not sufficient to fill the stock shortage of Firm j, denoted by $(D_j - Q_j)$. Therefore, the maximum quantity transshipped is $(Q_i - D_i)$.

In Event III, however, the aggregate stock of Firms i and j is more than the joint demand at two locations. The system-wide inventory abundance implies that the short position at Firm j's location can be fulfilled with the transshipment from Firm i. In this case, the transshipment quantity is $(D_j - Q_j)$.

Figure 3 also shows that Event VI is a situation parallel to Event II. The transshipment directions are opposite in these two events. Nevertheless, the two events result in the same amount of unmet demand $(D_i + D_j - Q_i - Q_j)$. Similarly, Event V and Event III are counterparts in that these two scenarios end with the equivalent overall stock surplus $(Q_i + Q_j - D_i - D_j)$.

3. Transshipments in a Competitive Decision-Making Environment

In the competitive decision-making environment, each firm chooses its inventory level to maximize its own expected profit. The analysis is first conducted in the scenario where no transshipments are implemented between two competing firms, and then in the scenario where the complete pooling policy is employed to initiate transshipments between firms.

3.1 Optimal Inventory Decision without Transshipments

Suppose two firms are independent of each other, and the surplus stock at one firm's site is not allowed to transship to the coincidentally out-of-stock location. The expected profits for Firms i and j can be written as

$$\pi_i(Q_i, Q_j) = E[p_i R_i + sU_i - pZ_i] - w_i Q_i - jQ_i \quad (3.1.1)$$

$$\pi_j(Q_i, Q_j) = E[p_j R_j + sU_j - pZ_j] - w_j Q_j - jQ_j \quad (3.1.2)$$

For each firm, its expected profit includes the expected revenue from sales, revenue from the salvage value for unsold stock, the expected penalty cost for the unmet demand, and the inventory purchasing and holding costs. To simplify the analysis, some cost parameters, such as the unit salvage value, the unit penalty cost, and the average inventory carrying cost, are assumed to be the same for both firms; others are unique for each firm, e.g., the retail price and the wholesale price. Tables 2 and 3 display the amount of sales (R), unmet demand (U), and unsold stock (Z) for Firms i and j in each of the six Events from I to VI.

Table III-2: Key Values for Events I, II, and III

	I	II		III
	$D_j > Q_j$ $D_i > Q_i$	$D_j > Q_j$ $D_i + D_j > Q_i + Q_j$		$D_j > Q_j$ $D_i + D_j < Q_i + Q_j$
		$\lambda_{ji} < \frac{Q_i - D_i}{D_j - Q_j}$	$\lambda_{ji} > \frac{Q_i - D_i}{D_j - Q_j}$	
R_i	Q_i	$D_i + \lambda_{ji}(D_j - Q_j)$		$D_i + \lambda_{ji}(D_j - Q_j)$
R_j	Q_j	Q_j		Q_j
U_i	0	$Q_i - D_i - \lambda_{ji}(D_j - Q_j)$	0	$Q_i - D_i - \lambda_{ji}(D_j - Q_j)$
U_j	0	0		0
Z_i	$D_i - Q_i$	0	0	0
Z_j	$D_j - Q_j$	$D_j - Q_j$		$D_j - Q_j$

Table III-3: Key Values for Events IV, V, and VI

	IV	V	VI	
	$D_j < Q_j$ $D_i < Q_i$	$D_j < Q_j$ $D_i + D_j < Q_i + Q_j$	$D_j > Q_j$ $D_i + D_j < Q_i + Q_j$	
			$\lambda_{ij} < \frac{Q_j - D_j}{D_i - Q_i}$	$\lambda_{ij} > \frac{Q_j - D_j}{D_i - Q_i}$
R_i	D_i	Q_i	Q_i	
R_j	D_j	$D_j + \lambda_{ij}(D_i - Q_i)$	$D_j + \lambda_{ij}(D_i - Q_i)$	
U_i	$Q_i - D_i$	0	0	
U_j	$Q_j - D_j$	$Q_j - D_j - \lambda_{ij}(D_i - Q_i)$	$Q_j - D_j - \lambda_{ij}(D_i - Q_i)$	0
Z_i	0	$D_i - Q_i$	$D_i - Q_i$	
Z_j	0	0	0	0

In Tables 2 and 3, λ_{ji} and λ_{ij} denote the percent of unsatisfied consumers switching from Firm j to i, and from Firm i to j, respectively. The value of the switching rate represents the degree of competition between firms. When two firms are close rivals, the switching rate from one to the other is expected to be higher. Consumer switching behavior can be explained by several factors, such as product substitutability, consumer loyalty, price difference, geographical distance between firms, and so on. In the following analysis, the switching rate is assumed to be an exogenously given variable, which is unrelated to the retail price.

To see how the values for R, U, and Z are determined, take Event II as an example. In this case, Firm i has $(Q_i - D_i)$ stock left after satisfying its own consumers (D_i); Firm j, in contrast, cannot sell more than its own stock (Q_j) without getting transshipments from Firm i. Of those unserved consumers at j's location, $\lambda_{ji}(D_j - Q_j)$ switch to Firm i. Therefore, the units sold at Firm i's location equals $D_i + \lambda_{ji}(D_j - Q_j)$, while the sales by Firm j are Q_j . Further, for a relatively low switching rate λ_{ji} , Firm i still has unsold stock of $Q_i - D_i - \lambda_{ji}(D_j - Q_j)$ even after satisfying all the switching consumers from Firm j; when the switching rate is above the threshold value $\frac{Q_i - D_i}{D_j - Q_j}$, the extra stock left after Firm i sells products to its own market is not sufficient to satisfy all the switching consumers from Firm j. Nevertheless, those unsatisfied switching consumers will not incur penalty costs for Firm i, and thus the unmet demand at Firm i's location can be identified as zero.

The optimal inventory levels for Firms i and j are derived by solving the expected profit maximization problem in (3.1.1) and (3.1.2), with respect to Q_i and Q_j .

$$\begin{aligned} \frac{\partial \pi_i}{\partial Q_i} &= p_i \text{Prob}(I \cup V \cup VI) + s \left[\text{Prob}(II) \text{Prob}\left(\lambda_{ji} < \frac{Q_i - D_i}{D_j - Q_j}\right) + \text{Prob}(III) + \text{Prob}(IV) \right] \\ &\quad + p \text{Prob}(I \cup V \cup VI) - w_i - j \\ &= (p_i + p) [1 - \text{Prob}(D_i < Q_i)] + s \left[\text{Prob}(Q_i + Q_j - D_j < D_i < Q_i) \text{Prob}(\lambda_{ji}(D_j - Q_j) + D_i < Q_i) \right. \\ &\quad \left. + \text{Prob}(D_i < Q_i) - \text{Prob}(Q_i + Q_j - D_j < D_i < Q_i) \right] - w_i - j \end{aligned} \quad (3.1.3)$$

$$\begin{aligned} \frac{\partial \pi_j}{\partial Q_j} &= p_j \text{Prob}(I \cup II \cup III) + s \left[\text{Prob}(IV) + \text{Prob}(V) + \text{Prob}(VI) \text{Prob}\left(\lambda_{ij} < \frac{Q_j - D_j}{D_i - Q_i}\right) \right] \\ &\quad + p \text{Prob}(I \cup II \cup III) - w_j - j \end{aligned}$$

$$\begin{aligned}
&= (p_j + p)(1 - \text{Prob}(D_j < Q_j)) + s[\text{Prob}(D_j < Q_j) - \text{Prob}(Q_i + Q_j - D_i < D_j < Q_j)] \\
&\quad + \text{Prob}(Q_i + Q_j - D_i < D_j < Q_j)\text{Prob}(\lambda_{ij}(D_i - Q_i) + D_j < Q_j)] - w_j - j \quad (3.1.4)
\end{aligned}$$

Equation (3.1.3) shows the marginal expected profitability of increasing the inventory level for Firm i. The right hand side of (3.1.3) consists of three parts. First, the value $(p_i + p)$, or the summation of the retail price and the penalty cost, corresponds to the marginal benefit of adding an additional unit in stock when the demand happens to be greater than the stock level, denoted by $\text{Prob}(D_i > Q_i)$. Second, when the actual demand falls short of the inventory stock (i.e., in Events II, III, and IV), additional stock contributes to the revenue; increasing the unsold stock allows Firm i to obtain the product salvage value s . In Event II, only when the switching rate $\lambda_{ji} < \frac{Q_i - D_i}{D_j - Q_j}$, an additional unit in stock will be accompanied by an increase in the unsold stock, which in turn results in the fulfillment of the salvage value s . Finally, the value $w_i + j$ is the cost for acquiring and carrying an additional inventory unit. Similarly, Equation (3.1.4) gives the marginal expected profitability of increasing the inventory level for Firm j.

Table 4 provides a list of shorthand variables that can be used to simplify Equations (3.1.3) and (3.1.4). Suppose the demand for each firm, D_i and D_j , has continuous distribution, then the probability functions denoted as $\alpha_{i(j)}$, $\beta_{i(j)}$, $\gamma_{i(j)}$ and $y_{i(j)}$ are continuous, given fixed levels for Q_i and Q_j .

Table III-4: Variable Notations and Definitions

Notation	Variable	Description
$\alpha_i(Q_i)$	$Pr ob(D_i < Q_i)$	The probability for Firm i to be overstocked
$\gamma_i(Q_i, Q_j)$	$Pr ob(Q_i + Q_j - D_j < D_i < Q_i)$	The probability in the occurrence of Event II
$y_i(Q_i, Q_j, \lambda_{ji})$	$Pr ob(D_i < Q_i - \lambda_{ji}(D_j - Q_j))$	The probability for Firm i to have leftover stock after selling to switching consumers from Firm j
$\beta_i(Q_i, Q_j)$	$Pr ob(Q_i < D_i < Q_i + Q_j - D_j)$	The probability in the occurrence of Event V
$\alpha_j(Q_j)$	$Pr ob(D_j < Q_j)$	The probability for Firm j to be overstocked
$\gamma_j(Q_i, Q_j)$	$Pr ob(Q_i + Q_j - D_i < D_j < Q_j)$	The Probability in the occurrence of Event VI
$y_j(Q_i, Q_j, \lambda_{ij})$	$Pr ob(D_j < Q_j - \lambda_{ij}(D_i - Q_i))$	The probability for Firm j to have leftover stock after selling to switching consumers from Firm i
$\beta_j(Q_i, Q_j)$	$Pr ob(Q_j < D_j < Q_i + Q_j - D_i)$	The probability in the occurrence of Event III

Using the notations in Table 4, Equations (3.1.3) and (3.1.4) can be rewritten as:

$$\frac{\partial \pi_i}{\partial Q_i} = (p_i + p)(1 - \alpha_i(Q_i)) + s[\alpha_i(Q_i) - \gamma_i(Q_i, Q_j) + \gamma_i(Q_i, Q_j)y_i(Q_i, Q_j, \lambda_{ji})] - w_i - j \quad (3.1.5)$$

$$\frac{\partial \pi_j}{\partial Q_j} = (p_j + p)(1 - \alpha_j(Q_j)) + s[\alpha_j(Q_j) - \gamma_j(Q_i, Q_j) + \gamma_j(Q_i, Q_j)y_j(Q_i, Q_j, \lambda_{ij})] - w_j - j \quad (3.1.6)$$

Next, the optimal inventory level for Firm i, Q_i^* , can be derived by solving the

equation $\frac{\partial \pi_i}{\partial Q_i^*} = 0$, assuming Firm j's inventory level Q_j as given. Similarly, the

optimal inventory level for Firm j, Q_j^* , can be derived by solving the equation

$\frac{\partial \pi_j}{\partial Q_j^*} = 0$, given Firm i's inventory choice Q_i . The conditions characterizing optimal

inventory levels for Firms i and j are, therefore, as follows.

$$\alpha_i(Q_i) - \gamma_i(Q_i, Q_j)y_i(Q_i, Q_j, \lambda_{ji})\left(\frac{s}{p_i + p - s}\right) + \gamma_i(Q_i, Q_j)\left(\frac{s}{p_i + p - s}\right) = \frac{p_i + p - w_i - j}{p_i + p - s} \quad (3.1.7)$$

$$\alpha_j(Q_j) - \gamma_j(Q_i, Q_j) y_j(Q_i, Q_j, \lambda_{ij}) \left(\frac{s}{p_j + p - s} \right) + \gamma_j(Q_i, Q_j) \left(\frac{s}{p_j + p - s} \right) = \frac{p_j + p - w - j}{p_j + p - s} \quad (3.1.8)$$

Rearrange Equation (3.1.7). The condition characterizing the optimal inventory for

Firm i can be written as

$$\alpha_i(Q_i) = \frac{p_i + p - w_i - j}{p_i + p - s} - \gamma_i(Q_i, Q_j) \left(\frac{s}{p_i + p - s} \right) + \gamma_i(Q_i, Q_j) y_i(Q_i, Q_j, \lambda_{ji}) \left(\frac{s}{p_i + p - s} \right) \quad (3.1.7')$$

Given a continuous distribution over the demand, the value for Q_i increases

with $\alpha_i(Q_i)$, which represents the probability of the demand D_i being less than the

inventory level Q_i . Therefore, as the value for the right-hand side of Equation (3.1.7')

increases, the optimal inventory level goes up. From the above discussion,

Propositions 3.1.1 and 3.1.2 are stated as follows.

Proposition 3.1.1:

Without transshipments, Firm i's optimal inventory level is increasing with the salvage value, holding the summation of the retail price and the stockout cost constant, for a continuous demand distribution.

Proposition 3.1.2:

Without transshipments, Firm i's optimal inventory level is decreasing with the wholesale price and the inventory carrying cost, holding other cost parameters constant, for a continuous demand distribution.

Proposition 3.1.3:

Without transshipments, there exists a static Nash equilibrium (Q_i^*, Q_j^*) for two competing Firms i, and j, in the inventory-decision game; Firm i's optimal inventory level decreases with j's optimal inventory, for $p_i + p > s$.

Proof:

According to Fudenberg and Tirole (1991), a unique Nash equilibrium exists if the reaction function is monotonic, and its absolute slope value is less than 1. Therefore, to prove Proposition 3.1.3, it is sufficient to show that the reaction function $\partial Q_i / \partial Q_j$ satisfies the monotonic and less-than-one slope value requirements. Transform Equation (3.1.7) to the function $F(Q_i, Q_j, \lambda_{ji})$, as shown in (3.1.9). By taking the implicit differentiation of Equation (3.1.9) with respect to Q_i , and Q_j , the expression for

$$\partial Q_i / \partial Q_j \text{ can be derived as } \frac{\partial Q_i}{\partial Q_j} = - \frac{F'_{Q_j}}{F'_{Q_i}}.$$

The following equation is transformed from (3.1.7):

$$F(Q_i, Q_j, \lambda_{ji}) = \alpha_i(Q_i) - \gamma_i(Q_i, Q_j) y_i(Q_i, Q_j, \lambda_{ji}) \left(\frac{s}{p_i + p - s} \right) + \gamma_i(Q_i, Q_j) \left(\frac{s}{p_i + p - s} \right) - \frac{p_i + p - w - j}{p_i + p - s} \quad (3.1.9)$$

For the continuous and differentiable functions $\alpha_i(Q_i)$, $\gamma_i(Q_i, Q_j)$, and $y_i(Q_i, Q_j, \lambda_{ji})$, use the following symbols to represent the relevant marginal probabilities, in which f_t is the probability density function for the variable t .

Let $a_i = f_{D_i}(Q_i)$

$$m_{ij}^1 = \text{Pr ob}(D_i > Q_i) f_{D_i+D_j|D_i>D_j}(Q_i + Q_j)$$

$$m_{ij}^2 = \text{Pr ob}(D_i + D_j < Q_i + Q_j) f_{D_i+D_j|D_i+D_j<Q_i+Q_j}(Q_i)$$

$$n_{ij}^1 = \text{Pr ob}(D_i < Q_i) f_{D_i+D_j|D_i<Q_i}(Q_i + Q_j)$$

$$n_{ij}^2 = \text{Pr ob}(D_i + D_j > Q_i + Q_j) f_{D_i+D_j|D_i+D_j>Q_i+Q_j}(Q_i)$$

$$q_{ij} = f_{D_i+\lambda_{ji}D_j}(Q_i + \lambda_{ji}Q_j)$$

Therefore, the partial derivatives of $F(Q_i, Q_j, \lambda_{ji})$ with respect to Q_i , and Q_j , are:

$$F'_{Q_j} = \left(\frac{s}{p_i + p - s}\right)(n_{ij}^1 y_i - q_{ij} \lambda_{ji} \gamma_i) - \left(\frac{s}{p_i + p - s}\right)n_{ij}^1 \quad (3.1.10)$$

$$F'_{Q_i} = a_i - \left(\frac{s}{p_i + p - s}\right)[y_i(-n_{ij}^1 + n_{ij}^2) + \gamma_i q_{ij}] + \left(\frac{s}{p_i + p - s}\right)(-n_{ij}^1 + n_{ij}^2) \quad (3.1.11)$$

Hence, the reaction function $\partial Q_i / \partial Q_j$ is:

$$\frac{\partial Q_i}{\partial Q_j} = -\frac{s(y_i - 1)n_{ij}^1 - s\lambda_{ji}\gamma_i q_{ij}}{(p_i + p - s)a_i + s(y_i - 1)(n_{ij}^1 - n_{ij}^2) - s\gamma_i q_{ij}} \quad (3.1.12)$$

It can be shown that the absolute value for the right-hand-side of Equation (3.1.12) is less than 1.

3.2 Optimal Inventory Decision with Transshipments

This section first examines the inventory level each firm chooses in order to maximize its own expected profit after allowing for the implementation of transshipments. The optimal inventory level is then compared with the results drawn from Section 3.1 (i.e., the inventory choice in the setting of no transshipments). When the practice of transshipments is incorporated into the analysis, the expected profits for Firms i and j have the following expressions.

$$\pi_i(Q_i, Q_j, c_{ij}, c_{ji}) = E[p_i R_i + (c_{ji} - \tau)X_{ij} - c_{ij}X_{ji} + sU_i - pZ_i] - w_i Q_i - jQ_i \quad (3.2.1)$$

$$\pi_j(Q_i, Q_j, c_{ij}, c_{ji}) = E[p_j R_j + (c_{ij} - \tau)X_{ji} - c_{ji}X_{ij} + sU_j - pZ_j] - w_j Q_j - jQ_j \quad (3.2.2)$$

Consider Firm i 's expected profit. The revenue side has three parts: the expected sales to meet its demand, the expected revenue from transshipping its extra stock to Firm j , and the expected salvage value for unsold items; the expected costs include: the transportation cost involved to make transshipments to Firm j , the payment to Firm j for receiving its transshipped items, the penalty cost for lost sales, and the inventory purchasing and carrying costs. Table 5 shows the amount of transshipments (X), sales (R), unmet demand (U), and unsold stock (Z) for Firms i and j in each of the six Events from I to VI.

Table III-5: Events and Associated Key Values

	I	II	III	IV	V	VI
	$D_j > Q_j$ $D_i > Q_i$	$D_j > Q_j$ $D_i + D_j > Q_i + Q_j$	$D_j > Q_j$ $D_i + D_j < Q_i + Q_j$	$D_j < Q_j$ $D_i < Q_i$	$D_j < Q_j$ $D_i + D_j < Q_i + Q_j$	$D_j < Q_j$ $D_i + D_j > Q_i + Q_j$
X_{ji}	0	0	0	0	$D_i - Q_i$	$Q_j - D_j$
X_{ij}	0	$Q_i - D_i$	$D_j - Q_j$	0	0	0
R_i	Q_i	D_i	D_i	D_i	D_i	$Q_i + Q_j - D_j$
R_j	Q_j	$Q_i + Q_j - D_i$	D_j	D_j	D_j	D_j
U_i	0	0	$Q_i + Q_j - D_i - D_j$	$Q_i - D_i$	0	0
U_j	0	0	0	$Q_j - D_j$	$Q_i + Q_j - D_i - D_j$	0
Z_i	$D_i - Q_i$	0	0	0	0	$D_i + D_j - Q_i - Q_j$
Z_j	$D_j - Q_j$	$D_i + D_j - Q_i - Q_j$	0	0	0	0

Event II, again, is used as an example to show how these values are determined. In this example, Firm i has surplus stock $(Q_i - D_i)$ available to transship to Firm j, whose inventory stock (Q_j) is not sufficient to satisfy all its demand (D_j) . Moreover, the amount of the surplus stock at Firm i's location is less than Firm j's shortage.

According to the complete pooling policy, the size of the transshipment from Firm i to j, X_{ij} , is $(Q_i - D_i)$. Therefore, sales by Firm i, R_i , are D_i and sales by Firm j, R_j , are equal to $(Q_j + (Q_i - D_i))$. With the transshipment, X_{ij} , Firm i consumes all of its extra inventory and thus the unsold stock, U_i , is zero; Firm j, however, still has unmet demand of $(D_j + D_i - Q_i - Q_j)$, denoted by Z_j .

Take the derivative of the expected profit in (3.2.1) and (3.2.2), with respect to Q_i , and Q_j , respectively. The results are shown in (3.2.3) and (3.2.6).

$$\begin{aligned}
\frac{\partial \pi_i}{\partial Q_i} &= p_i \text{Prob}(I \cup VI) + (c_{ji} - \tau) \text{Prob}(II) + c_{ij} \text{Prob}(V) + s \text{Prob}(III \cup IV) \\
&\quad + p \text{Prob}(I \cup VI) - w_i - j \\
&= (p_i + p)[1 - \text{Prob}(D_i < Q_i) - \text{Prob}(Q_i < D_i < Q_i + Q_j - D_j)] + (c_{ji} - \tau) \text{Prob}(Q_i + Q_j \\
&\quad - D_j < D_i < Q_i) + c_{ij} \text{Prob}(Q_i < D_i < Q_i + Q_j - D_j) + s[\text{Prob}(D_i < Q_i) - \text{Prob}(Q_i + Q_j \\
&\quad - D_j < D_i < Q_i)] - w_i - j \tag{3.2.3}
\end{aligned}$$

Equation (3.2.3) presents the marginal expected profitability of increasing the inventory level for Firm i. The right hand side of (3.2.3) has five parts. First, the value of $(p_i + p)$ represents the marginal benefit of increasing the inventory level when transshipments from Firm j are impossible or not sufficient to fill the oversized demand; i.e., either Event I or Event IV occurs. Second, in Event II, Firm i transships

all of its surplus stock to Firm j. As such, each additional inventory unit increases Firm i's revenue by the transshipment price minus the transportation cost (as denoted by $c_{ji} - \tau$). Third, an additional unit in stock can save Firm i the transshipment price, c_{ij} , which otherwise would be paid to Firm j when Event V occurs. Fourth, when either Events III or IV occurs, raising Firm i's inventory level only leads to an increase in unsold items, and thus contributes to the bottom line with the salvage value, s . Lastly, the marginal cost of increasing inventory stock equals the summation of the purchasing and carrying cost (as denoted by $w_i + j$).

By using notations α , β , γ and y in Table 4, Equation (3.2.3) is simplified as:

$$\frac{\partial \pi_i}{\partial Q_i} = (p_i + p)(1 - \alpha_i(Q_i) - \beta_i(Q_i, Q_j)) + (c_{ji} - \tau)\gamma_i(Q_i, Q_j) + c_{ij}\beta_i(Q_i, Q_j) + s(\alpha_i(Q_i) - \gamma_i(Q_i, Q_j)) - w_i - j \quad (3.2.4)$$

Next, the optimal inventory level for Firm i, Q_i^* , is determined by solving the

equation $\frac{\partial \pi_i}{\partial Q_i} = 0$, given Firm j's inventory choice of Q_j . At the optimal inventory

level Q_i^* , the following condition holds.

$$\alpha_i(Q_i) + \beta_i(Q_i, Q_j) \left(\frac{p_i + p - c_{ij}}{p_i + p - s} \right) - \gamma_i(Q_i, Q_j) \left(\frac{c_{ji} - s - \tau}{p_i + p - s} \right) = \frac{p_i + p - w_i - j}{p_i + p - s} \quad (3.2.5)$$

Similarly, for Firm j, the marginal expected profitability of increasing the inventory

level is:

$$\frac{\partial \pi_j}{\partial Q_j} = p_j \Pr ob(I \cup II) + (c_{ij} - \tau) \Pr ob(VI) + c_{ji} \Pr ob(III) + s \Pr ob(IV \cup V) + p \Pr ob(I \cup II) - w_j - j$$

$$\begin{aligned}
&= (p_j + p)[1 - \text{Pr ob}(D_j < Q_j) - \text{Pr ob}(Q_j < D_j < Q_i + Q_j - D_i)] + (c_{ij} - \tau) \text{Pr ob}(Q_i + Q_j \\
&- D_i < D_j < Q_j) + c_{ji} \text{Pr ob}(Q_j < D_j < Q_i + Q_j - D_i) + s[\text{Pr ob}(D_j < Q_j) - \text{Pr ob}(Q_i + Q_j \\
&- D_i < D_j < Q_j)] - w_j - j \\
&= (p_j + p)(1 - \alpha_j(Q_j) - \beta_j(Q_i, Q_j)) + (c_{ij} - \tau)\gamma_j(Q_i, Q_j) + c_{ji}\beta_j(Q_i, Q_j) \\
&+ s(\alpha_j(Q_j) - \gamma_j(Q_i, Q_j)) - w_j - j \tag{3.2.6}
\end{aligned}$$

And the condition for $\frac{\partial \pi_j}{\partial Q_j} = 0$, assuming Firm i's inventory level Q_i as given, is:

$$\alpha_j(Q_j) + \beta_j(Q_i, Q_j) \left(\frac{p_j + p - c_{ji}}{p_j + p - s} \right) - \gamma_j(Q_i, Q_j) \left(\frac{c_{ij} - s - \tau}{p_j + p - s} \right) = \frac{p_j + p - w_j - j}{p_j + p - s} \tag{3.2.7}$$

Rearrange Equations (3.2.5), and (3.2.7). The conditions characterizing the optimal inventory choices for Firms i, and j, are shown below.

$$\alpha_i(Q_i) = \frac{p_i + p - w_i - j}{p_i + p - s} - \beta_i(Q_i, Q_j) \left(\frac{p_i + p - c_{ij}}{p_i + p - s} \right) + \gamma_i(Q_i, Q_j) \left(\frac{c_{ji} - s - \tau}{p_i + p - s} \right) \tag{3.2.5'}$$

$$\alpha_j(Q_j) = \frac{p_j + p - w_j - j}{p_j + p - s} - \beta_j(Q_i, Q_j) \left(\frac{p_j + p - c_{ji}}{p_j + p - s} \right) + \gamma_j(Q_i, Q_j) \left(\frac{c_{ij} - s - \tau}{p_j + p - s} \right) \tag{3.2.7'}$$

For a continuous demand distribution, the optimal inventory level Q_i^* increases with the value for the right-hand side of Equation (3.2.5'). Accordingly, Propositions 3.2.1 and 3.2.2 always hold.

Proposition 3.2.1:

With transshipments, Firm i's optimal inventory level is increasing with the salvage value, holding the summation of the retail price and the stockout cost constant, for a continuous demand distribution; with transshipments, Firm i's optimal inventory level is

decreasing with the wholesale price and the inventory carrying cost, holding other cost parameters constant, for a continuous demand distribution.

Proposition 3.2.2:

With transshipments, Firm i's optimal inventory level is increasing with the transshipment price, when the transshipment price $c_{ij} < p_i + p$ and $c_{ji} > s + \tau$.

It is a commonly accepted assumption that the summation of the retail price and the penalty cost is greater than the salvage value. According to Proposition 3.2.2, Firm i chooses to have more inventory when the transshipment price it pays to the sender becomes greater; on the other side, Firm i chooses to have more inventory when the transshipment price paid by the recipient becomes greater. By comparing Equation (3.2.5') with (3.1.7'), it is found that the key determinants for the optimal inventory level without transshipments include the salvage value and the switching rate; when transshipments are implemented, the optimal inventory choice is determined mainly by other factors, such as transshipment price and transshipment cost.

Proposition 3.2.3:

With transshipments, there exists a unique Nash equilibrium (Q_i^*, Q_j^*) for two competing Firms i, and j, in the inventory-decision game; Firm i's optimal inventory level decreases with j's optimal inventory, when the transshipment price $c_{ij} < p_i + p$ and $c_{ji} > s + \tau$.

The proof for Proposition 3.2.3 is similar to that for Proposition 3.1.3. By checking the sign and the slope value for $\partial Q_i / \partial Q_j$, as indicated in (3.2.11), the existence of the equilibrium can be proved.

Proof:

$$F(Q_i, Q_j) = \alpha_i(Q_i) + \beta_i(Q_i, Q_j) \left(\frac{p_i + p - c_{ij}}{p_i + p - s} \right) - \gamma_i(Q_i, Q_j) \left(\frac{c_{ji} - s - \tau}{p_i + p - s} \right) - \frac{p_i + p - w_i - j}{p_i + p - s}$$

(3.2.8)

$$\frac{\partial Q_i}{\partial Q_j} = - \frac{F'_{Q_j}}{F'_{Q_i}}$$

Let $a_i = f_{D_i}(Q_i)$

$$m_{ij}^1 = \text{Prob}(D_i > Q_i) f_{D_i + D_j | D_i > D_j}(Q_i + Q_j)$$

$$m_{ij}^2 = \text{Prob}(D_i + D_j < Q_i + Q_j) f_{D_i | D_i + D_j < Q_i + Q_j}(Q_i)$$

$$n_{ij}^1 = \text{Prob}(D_i < Q_i) f_{D_i + D_j | D_i < Q_i}(Q_i + Q_j)$$

$$n_{ij}^2 = \text{Prob}(D_i + D_j > Q_i + Q_j) f_{D_i | D_i + D_j > Q_i + Q_j}(Q_i)$$

$$F'_{Q_j} = \left(\frac{p_i + p - c_{ij}}{p_i + p - s} \right) m_{ij}^1 + \left(\frac{c_{ji} - s - \tau}{p_i + p - s} \right) n_{ij}^1 \quad (3.2.9)$$

$$F'_{Q_i} = a_i + \left(\frac{p_i + p - c_{ij}}{p_i + p - s} \right) (m_{ij}^1 - m_{ij}^2) + \left(\frac{c_{ji} - s - \tau}{p_i + p - s} \right) (n_{ij}^1 - n_{ij}^2) \quad (3.2.10)$$

$$\frac{\partial Q_i}{\partial Q_j} = - \frac{(p_i + p - c_{ij}) m_{ij}^1 + (c_{ji} - s - \tau) n_{ij}^1}{(p_i + p - s) a_i + (p_i + p - c_{ij}) (m_{ij}^1 - m_{ij}^2) + (c_{ji} - s - \tau) (n_{ij}^1 - n_{ij}^2)} \quad (3.2.11)$$

It can be shown that the absolute value for the right-hand-side of Equation (3.2.11) is less than 1.

4. Transshipments in a Cooperative Decision-Making Environment

In a cooperative decision-making environment, the optimal inventory level at each firm's location is determined by maximizing the joint profits of the two firms. This scenario has been studied by Robinson (1990), and Rudi et al. (2001). The analysis developed in this section differs from previous research in two aspects. First, the model takes into account the fact that consumers switch firms in case of stockout when transshipments are not implemented between firms. Second, the model investigates and compares the inventory decisions and profit outcomes of firms that are implementing transshipments in two different cooperative mechanisms.

In the first setting, the inventory level at each location is optimized to maximize the aggregate expected profits of the two firms. For each firm, its expected profit is based on the demand forecast of the local market. This case has been studied by Rudi et al. (2001). The second cooperative scenario modeled in this section differs from previous research (e.g., Rudi et al. 2001) in that the aggregate profits of the two firms are determined by the total demand forecasts in the markets of the two firms. In this case, the two firms determine their joint order quantities based on the aggregate market demand forecasts and then allocate the total inventory according to the respective local market demands. In both cases, the practice of transshipments can be viewed as an intra-firm reallocation of inventories and it allows firms to reduce inventory investments without lowering customer service level. The analysis in this section provides a framework to examine whether the practice of order coordination enhances the profit benefits that firms achieve from transshipments.

The rest of this section is organized as follows. Section 4.1 investigates the scenario where no transshipments are implemented between two cooperative firms. Consistent to the approach used by Rudi et al. (2001), the inventory levels are optimized to maximize the joint profits of the two firms. As discussed in the preceding section, the “without transshipments” cooperative scenario is distinct from the one studied by Rudi et al. (2001) in that it takes the likelihood of consumers’ switching between firms into consideration when stockout occurs at one but not both locations. Section 4.2 investigates two cooperative scenarios when transshipments are implemented between firms. In Section 4.2.1, the optimal inventory level at each location is determined by maximizing the summation of the expected profits of the two firms. The expected profit for each firm is based on the anticipated demand in the local market. In comparison, Section 4.2.2 makes a further assumption that the two firms coordinate their inventory decisions so that aggregate profits of the two firms are maximized basing on their joint demand forecasts. In this section, firms first make their joint order quantity decision and then allocate the optimal inventory. Since such a practice of order coordination enables firms to pool the demands at the local market, it is expected to find that firms will achieve greater profits in the scenario modeled by Section 4.2.2, as compared to in the previously studied scenario, in which firms simply share their inventories through the employment of transshipments.

4.1 Optimal Inventory Decision without Transshipments

In Expression (4.1.1), $\pi^j(Q_i, Q_j)$ is the joint expected profit for Firms i and j, in the case when no transshipments are implemented. Without transshipments, some of the consumers at Firm j’s location switch to Firm i, for Events II, and III, in which Firm j is stocked out

while i is overstocked. The size of the switching consumer group is determined by the value of the switching rate, and the number of unserved consumers. Tables 6 and 7 list the number of units sold by each firm, the total number of unsold stock units, and unmet demand for Firms i and j , that are associated with each of the six events.

$$\pi^j(Q_i, Q_j) = E[p_i R_i + p_j R_j + sU_i + sU_j - pZ_i - pZ_j] - w_i Q_i - jQ_i - w_j Q_j - jQ_j \quad (4.1.1)$$

Table III-6: Key Values for Events I, II, and III

	I	II		III
	$D_j > Q_j, D_i > Q_i$	$D_j > Q_j$ $D_i + D_j > Q_i + Q_j$		$D_j > Q_j$ $D_i + D_j < Q_i + Q_j$
		$\lambda_{ji} < \frac{Q_i - D_i}{D_j - Q_j}$	$\lambda_{ji} > \frac{Q_i - D_i}{D_j - Q_j}$	
R_i	Q_i	$D_i + \lambda_{ji}(D_j - Q_j)$		$D_i + \lambda_{ji}(D_j - Q_j)$
R_j	Q_j	Q_j		Q_j
$U_i + U_j$	0	$Q_i - D_i - \lambda_{ji}(D_j - Q_j)$	0	$Q_i - D_i - \lambda_{ji}(D_j - Q_j)$
$Z_i + Z_j$	$D_i + D_j - Q_i - Q_j$	$D_j - Q_j$		$D_j - Q_j$

Table III-7: Key Values for Events IV, V, and VI

	IV	V	VI	
	$D_j < Q_j, D_i < Q_i$	$D_j < Q_j$ $D_i + D_j < Q_i + Q_j$	$D_j < Q_j$ $D_i + D_j > Q_i + Q_j$	
			$\lambda_{ij} < \frac{Q_j - D_j}{D_i - Q_i}$	$\lambda_{ij} > \frac{Q_j - D_j}{D_i - Q_i}$
R_i	D_i	Q_i	Q_i	
R_j	D_j	$D_j + \lambda_{ij}(D_i - Q_i)$	$D_j + \lambda_{ij}(D_i - Q_i)$	
$U_i + U_j$	$Q_i + Q_j - D_i - D_j$	$Q_j - D_j - \lambda_{ij}(D_i - Q_i)$	$Q_j - D_j - \lambda_{ij}(D_i - Q_i)$	0
$Z_i + Z_j$	0	$D_i - Q_i$	$D_i - Q_i$	

Then the optimal inventories for Firm i and Firm j are derived from solving the joint profit maximization problem with respect to the stock levels Q_i , and Q_j .

$$\begin{aligned}
\frac{\partial \pi^J}{\partial Q_i} &= (p_i + p) \text{Prob}(I \cup V \cup VI) - \lambda_{ij} p_j \text{Prob}(V \cup VI) + s[\text{Prob}(II) \text{Prob}(\lambda_{ji} < \frac{Q_i - D_i}{D_j - Q_j}) + \text{Prob}(III) \\
&\quad + \text{Prob}(IV) + \lambda_{ij} \text{Prob}(V) + \lambda_{ij} \text{Prob}(VI) \text{Prob}(\lambda_{ij} < \frac{Q_j - D_j}{D_i - Q_i})] - w_i - j \\
&= (p_i + p)(1 - \text{Prob}(D_i < Q_i)) - p_j \lambda_{ij} [\text{Prob}(Q_i < D_i < Q_i + Q_j - D_j) + \text{Prob}(Q_i + Q_j \\
&\quad - D_i < D_j < Q_j)] + s[\text{Prob}(Q_i + Q_j - D_j < D_i < Q_i) \text{Prob}(\lambda_{ji} < \frac{Q_i - D_i}{D_j - Q_j}) + \text{Prob}(D_i < Q_i) \\
&\quad - \text{Prob}(Q_i + Q_j - D_j < D_i < Q_i) + \lambda_{ij} \text{Prob}(Q_i < D_i < Q_i + Q_j - D_j) + \lambda_{ij} \text{Prob}(Q_i + Q_j \\
&\quad - D_i < D_j < Q_j) \text{Prob}(\lambda_{ij} < \frac{Q_j - D_j}{D_i - Q_i})] - w_i - j \tag{4.1.2}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi^J}{\partial Q_j} &= -p_i \lambda_{ji} \text{Prob}(II \cup III) + (p_j + p) \text{Prob}(I \cup II \cup III) + s[\lambda_{ji} \text{Prob}(II) \text{Prob}(\lambda_{ji} < \frac{Q_i - D_i}{D_j - Q_j}) \\
&\quad + \lambda_{ji} \text{Prob}(III) + \text{Prob}(IV) + \text{Prob}(V) + \text{Prob}(VI) \text{Prob}(\lambda_{ij} < \frac{Q_j - D_j}{D_i - Q_i})] - w_j - j \\
&= (p_j + p)(1 - \text{Prob}(D_j < Q_j)) - p_i \lambda_{ji} [\text{Prob}(Q_i + Q_j - D_j < D_i < Q_i) + \text{Prob}(Q_j < D_j \\
&\quad < Q_i + Q_j - D_i)] + s[\text{Prob}(Q_i + Q_j - D_i < D_j < Q_j) \text{Prob}(\lambda_{ij} < \frac{Q_j - D_j}{D_i - Q_i}) + \text{Prob}(D_j < Q_j) \\
&\quad - \text{Prob}(Q_i + Q_j - D_i < D_j < Q_j) + \lambda_{ji} \text{Prob}(Q_j < D_j < Q_i + Q_j - D_i) + \lambda_{ji} \text{Prob}(Q_i + Q_j \\
&\quad - D_j < D_i < Q_i) \text{Prob}(\lambda_{ji} < \frac{Q_i - D_i}{D_j - Q_j})] - w_j - j \tag{4.1.3}
\end{aligned}$$

Using the notation given in Table 4, the above equations can be simplified as the following:

$$\begin{aligned}
\frac{\partial \pi^J}{\partial Q_i} &= (p_i + p - w_i - j) + (-p_i - p + s)\alpha_i(Q_i, Q_j) + (-p_j + s)\lambda_{ij}\beta_i(Q_i, Q_j) + (y_i - 1)s\gamma_i(Q_i, Q_j) \\
&\quad + (-p_j + sy_j)\lambda_{ij}\gamma_j(Q_i, Q_j) \tag{4.1.4}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi^J}{\partial Q_j} &= (p_j + p - w_j - j) + (-p_j - p + s)\alpha_i(Q_i, Q_j) + (-p_i + s)\lambda_{ji}\beta_j(Q_i, Q_j) + (y_j - 1)s\gamma_j(Q_i, Q_j) \\
&\quad + (-p_i + sy_i)\lambda_{ji}\gamma_i(Q_i, Q_j) \tag{4.1.5}
\end{aligned}$$

Now let $\frac{\partial \pi^J}{\partial Q_i} = 0$ and $\frac{\partial \pi^J}{\partial Q_j} = 0$. Using the notation in Table 4, the solution for the

optimal inventory levels can be simplified as follows.

$$\begin{aligned} & \alpha_i(Q_i) + \beta_i(Q_i, Q_j) \left(\frac{p_j \lambda_{ij} - s \lambda_{ij}}{p_i + p - s} \right) + \gamma_i(Q_i, Q_j) \left(\frac{s - y_i s}{p_i + p - s} \right) + \gamma_j(Q_i, Q_j) \left(\frac{p_j \lambda_{ij} - s y_j \lambda_{ij}}{p_i + p - s} \right) \\ &= \frac{p_i + p - w_i - j}{p_i + p - s} \end{aligned} \quad (4.1.6)$$

$$\begin{aligned} & \alpha_j(Q_j) + \beta_j(Q_i, Q_j) \left(\frac{p_i \lambda_{ji} - s \lambda_{ji}}{p_j + p - s} \right) + \gamma_j(Q_i, Q_j) \left(\frac{s - y_j s}{p_j + p - s} \right) + \gamma_i(Q_i, Q_j) \left(\frac{p_i \lambda_{ji} - s y_i \lambda_{ji}}{p_j + p - s} \right) \\ &= \frac{p_j + p - w_j - j}{p_j + p - s} \end{aligned} \quad (4.1.7)$$

Rearrange (4.1.6) and (4.1.7), and the amount for the optimal stock Q_i , and Q_j can be obtained by solving Equations (4.1.6') and (4.1.7').

$$\alpha_i(Q_i) = \frac{p_i + p - w_i - j}{p_i + p - s} - \beta_i(Q_i, Q_j) \left(\frac{p_j \lambda_{ij} - s \lambda_{ij}}{p_i + p - s} \right) - \gamma_i(Q_i, Q_j) \left(\frac{s - y_i s}{p_i + p - s} \right) - \gamma_j(Q_i, Q_j) \left(\frac{p_j \lambda_{ij} - s y_j \lambda_{ij}}{p_i + p - s} \right) \quad (4.1.6')$$

$$\alpha_j(Q_j) = \frac{p_j + p - w_j - j}{p_j + p - s} - \beta_j(Q_i, Q_j) \left(\frac{p_i \lambda_{ji} - s \lambda_{ji}}{p_j + p - s} \right) - \gamma_j(Q_i, Q_j) \left(\frac{s - y_j s}{p_j + p - s} \right) - \gamma_i(Q_i, Q_j) \left(\frac{p_i \lambda_{ji} - s y_i \lambda_{ji}}{p_j + p - s} \right) \quad (4.1.7')$$

Proposition 4.1.1:

Without transshipments, Firm i's optimal inventory level is decreasing with the switching rate, λ_{ij} , when the retail price charged by its rival firm is greater than the salvage value.

Proposition 4.1.2:

Without transshipments, Firm j 's optimal inventory level is decreasing with the switching rate, λ_{ji} , when the retailer price charged by its rival firm is greater than the salvage value.

4.2 Optimal Inventory Decision with Transshipments

4.2.1 Joint Decision-Making without Order Coordination

This section examines the case where the inventory level decisions are made by two firms individually to maximize their joint expected profit. The expected profit for each firm is determined by the demand distribution in the local market. The joint expected profit, $\pi^{JT}(Q_i, Q_j)$, is expressed by (4.2.1), in which $X_{ji} + X_{ij}$ represents the total number of units transshipped between Firms i and j , and τ is the unit transshipment cost. In this model, the order stock levels Q_i , and Q_j , are optimized to maximize the joint expected profits for Firms i and j . Therefore, the transshipment payment made between firms is cancelled out in (4.2.1).

$$\pi^{JT}(Q_i, Q_j) = E[p_i R_i + p_j R_j - \tau(X_{ji} + X_{ij}) + s(U_i + U_j) - p(Z_i + Z_j)] - w_i Q_i - j Q_i - w_j Q_j - j Q_j \quad (4.2.1)$$

Table III-8: Events and Associated Key Values

	I	II	III	IV	V	VI
	$D_j > Q_j$ $D_i > Q_i$	$D_j > Q_j$ $D_i + D_j > Q_i + Q_j$	$D_j > Q_j$ $D_i + D_j < Q_i + Q_j$	$D_j < Q_j$ $D_i < Q_i$	$D_j < Q_j$ $D_i + D_j < Q_i + Q_j$	$D_j < Q_j$ $D_i + D_j > Q_i + Q_j$
$X_{ji} + X_{ij}$	0	$Q_i - D_i$	$D_j - Q_j$	0	$D_i - Q_i$	$Q_j - D_j$
$R_i + R_j$	$Q_i + Q_j$	$Q_i + Q_j$	$D_i + D_j$	$D_i + D_j$	$D_i + D_j$	$Q_i + Q_j$
$U_i + U_j$	0	0	$Q_i + Q_j - D_i - D_j$	$Q_i + Q_j - D_i - D_j$	$Q_i + Q_j - D_i - D_j$	0
$Z_i + Z_j$	$D_i + D_j$ $-Q_i - Q_j$	$D_i + D_j - Q_i - Q_j$	0	0	0	$D_i + D_j - Q_i - Q_j$

Table 8 shows that for each of the six events, the total number of units sold by Firms i, and j, the total number of units transshipped between Firms i and j, the aggregate unsold stocks and unmet demands of the two firms. Their optimal stock levels are derived by simultaneously solving Equations (4.2.2) and (4.2.3).

$$\begin{aligned}
\frac{\partial \pi^{JI}}{\partial Q_i} &= (p_i + p)Prob(I \cup VI) + (p_j - \tau + p)Prob(II) + (\tau + s)Prob(V) + sProb(III \cup IV) - w_i - j \\
&= (p_i + p)[1 - Prob(D_i < Q_i) - Prob(Q_i < D_i < Q_i + Q_j - D_j)] + (p_j - \tau + p)Prob(Q_i + Q_j \\
&\quad - D_j < D_i < Q_i) + (\tau + s)Prob(Q_i < D_i < Q_i + Q_j - D_j) + s[Prob(D_i < Q_i) \\
&\quad - Prob(Q_i + Q_j - D_j < D_i < Q_i)] - w_i - j \tag{4.2.2}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi^{JI}}{\partial Q_j} &= (p_j + p)[1 - Prob(D_j < Q_j) - Prob(Q_j < D_j < Q_i + Q_j - D_i)] + (p_i + p - \tau)Prob(Q_i \\
&\quad + Q_j - D_i < D_j < Q_j) + (\tau + s)Prob(Q_j < D_j < Q_i + Q_j - D_i) + s[Prob(D_j < Q_j) \\
&\quad - Prob(Q_i + Q_j - D_i < D_j < Q_j)] - w_j - j \tag{4.2.3}
\end{aligned}$$

By using the notation in Table 4, the above optimization problem is simplified as

(4.2.4) and (4.2.5). To get (4.2.4) and (4.2.5), let $\frac{\partial \pi^{JI}}{\partial Q_i} = 0$, and $\frac{\partial \pi^{JI}}{\partial Q_j} = 0$.

$$\alpha_i(Q_i) + \beta_i(Q_i, Q_j) \left(\frac{p_i + p - \tau - s}{p_i + p - s} \right) - \gamma_i(Q_i, Q_j) \left(\frac{p_j + p - \tau - s}{p_i + p - s} \right) = \frac{p_i + p - w_i - j}{p_i + p - s} \tag{4.2.4}$$

$$\alpha_j(Q_j) + \beta_j(Q_i, Q_j) \left(\frac{p_j + p - \tau - s}{p_j + p - s} \right) - \gamma_j(Q_i, Q_j) \left(\frac{p_i + p - \tau - s}{p_j + p - s} \right) = \frac{p_j + p - w_j - j}{p_j + p - s} \tag{4.2.5}$$

Therefore, the optimal stock levels when Firms i and j implement transshipments in a cooperative setting are determined by (4.2.4') and (4.2.5').

$$\alpha_i(Q_i) = \frac{p_i + p - w_i - j}{p_i + p - s} - \beta_i(Q_i, Q_j) \left(\frac{p_i + p - \tau - s}{p_i + p - s} \right) + \gamma_i(Q_i, Q_j) \left(\frac{p_j + p - \tau - s}{p_i + p - s} \right) \tag{4.2.4'}$$

$$\alpha_j(Q_j) = \frac{p_j + p - w_j - j}{p_j + p - s} - \beta_j(Q_i, Q_j) \left(\frac{p_j + p - \tau - s}{p_j + p - s} \right) + \gamma_j(Q_i, Q_j) \left(\frac{p_i + p - \tau - s}{p_j + p - s} \right) \quad (4.2.5')$$

Propositions 4.2.1:

With transshipments, Firm i's optimal inventory level is decreasing with transshipment cost, holding other cost parameters constant, for a continuous demand distribution.

Proposition 4.2.2:

With transshipments, Firm i's optimal inventory level is decreasing with the product price in its own market relative to the product price in Firm j's market, for a continuous demand distribution.

Finally, the solution for the optimal (Q_i, Q_j) is unique, proved as follows.

$$F(Q_i, Q_j) = (p_i + p - s)\alpha_i(Q_i) + (p_i + p - \tau - s)\beta_i(Q_i, Q_j) - (p_j + p - \tau - s)\gamma_i(Q_i, Q_j) - (p_i + p - w_i - j) \quad (4.2.6)$$

Let $a_i = f_{D_i}(Q_i)$

$$m_{ij}^1 = Prob(D_i > Q_i) f_{D_i + D_j | D_i > D_j}(Q_i + Q_j)$$

$$m_{ij}^2 = Prob(D_i + D_j < Q_i + Q_j) f_{D_i | D_i + D_j < Q_i + Q_j}(Q_i)$$

$$n_{ij}^1 = Prob(D_i < Q_i) f_{D_i + D_j | D_i < Q_i}(Q_i + Q_j)$$

$$n_{ij}^2 = Prob(D_i + D_j > Q_i + Q_j) f_{D_i | D_i + D_j > Q_i + Q_j}(Q_i)$$

$$F'_{Q_j} = \left(\frac{p_i + p - \tau - s}{p_i + p - s} \right) m_{ij}^1 + \left(\frac{p_j + p - \tau - s}{p_i + p - s} \right) n_{ij}^1 \quad (4.2.7)$$

$$F'_{Q_i} = a_i + \left(\frac{p_i + p - \tau - s}{p_i + p - s} \right) (m_{ij}^1 - m_{ij}^2) + \left(\frac{p_j + p - \tau - s}{p_i + p - s} \right) (n_{ij}^1 - n_{ij}^2) \quad (4.2.8)$$

$$\frac{\partial Q_i}{\partial Q_j} = - \frac{F'_{Q_j}}{F'_{Q_i}} = - \frac{(p_i + p - \tau - s)m_{ij}^1 + (p_j + p - \tau - s)n_{ij}^1}{(p_i + p - s)a_i + (p_i + p - \tau - s)(m_{ij}^1 - m_{ij}^2) + (p_j + p - \tau - s)(n_{ij}^1 - n_{ij}^2)} \quad (4.2.9)$$

4.2.2 Joint Decision-Making with Order Coordination

This section examines an alternative cooperative decision making scenario, in which the joint inventory for the two firms is optimized to maximize their total expected profits that are determined by the aggregate market demands. Then the two firms allocate the joint inventory. The order quantity assigned to an individual firm is commensurate with its market demand. To simplify the analysis, we make a further assumption that Firm i and j have identical retailing price (i.e., $p_i = p_j = r$). The joint inventory level of Firm i and j is denoted as Q (i.e., $Q = Q_i + Q_j$), and their aggregate demand is represented by D (i.e., $D = D_i + D_j$). In Expression (4.2.10), the joint profits for Firms i and j are composed of five parts: The revenue from selling R units of goods, the salvage value associated with U units of unsold stock, the costs of transshipping $X_{ij} + X_{ji}$ amount of goods, the penalty cost of having Z units of unmet demand (i.e., $Z = Z_i + Z_j$), and the costs of purchasing and carrying Q (i.e., $Q = Q_i + Q_j$) units of inventory.

$$\pi_{(Q)}^{JIII} = E[rR + sU - \tau(X_{ij} + X_{ji}) - pZ] - (w + j)Q \quad (4.2.10)$$

Table 9 presents the values for R, U, Z, and $X_{ij} + X_{ji}$ that are associated with each of the six events. Of these variables, the number of goods transshipped from one firm to another is determined by the quantities of stock assigned to Firms i and j, and the actual demand realized in the markets of the two firms. The model further assumes that the allocation is based on the market demand one firm has relative to another.

Let $Q_i = \frac{Q}{n}$, and $Q_j = (\frac{n-1}{n})Q$. The value for parameter n can be derived by solving

the equation: $\frac{1}{n-1} = \frac{D_i}{D_j}$. According to this allocation rule, the stock levels allocated

to Firms i and j is proportionate to the ratio of market demands for these two firms;

$$\text{i.e., } \frac{Q_i}{Q_j} = \frac{D_i}{D_j}.$$

To illustrate this rule, consider a special example where the two firms have identical demand distribution. In this case, the value for n equals to 2, and Firms i and j each has an inventory level of $Q/2$.

Table III-9: Events and Associated Key Values

	I	II	III	IV	V	VI
	$D_j > Q_j$ $D_i > Q_i$	$D_j > Q_j$ $D_i + D_j > Q_i + Q_j$	$D_j > Q_j$ $D_i + D_j < Q_i + Q_j$	$D_j < Q_j$ $D_i < Q_i$	$D_j < Q_j$ $D_i + D_j < Q_i + Q_j$	$D_j < Q_j$ $D_i + D_j > Q_i + Q_j$
R	Q	Q	D	D	D	Q
U	0	0	Q - D	Q - D	Q - D	0
Z	D - Q	D - Q	0	0	0	D - Q
$X_{ij} + X_{ji}$	0	$\frac{Q}{n} - D_i$	$D_j - (\frac{n-1}{n})Q$	0	$D_i - \frac{Q}{n}$	$(\frac{n-1}{n})Q - D_j$

Given the values provided in Table 9, the optimal aggregate stock level is derived by solving the equation (4.2.11) = 0.

$$\begin{aligned} \frac{\partial \pi^{III}}{\partial Q} = & (r+p)Prob(I \cup II \cup VI) + sProb(III \cup IV \cup V) - \frac{\tau}{n}Prob(II) - \left(\frac{n-1}{n}\right)\tau Prob(VI) \\ & + \left(\frac{n-1}{n}\right)\tau Prob(III) + \frac{\tau}{n}Prob(V) - w - j \end{aligned} \quad (4.2.11)$$

Table III-10: Notations for Key Parameters

$\beta_i(Q) = Prob\left(\frac{Q}{n} < D_i < Q - D_j\right)$	$\beta_j(Q) = Prob\left(\frac{n-1}{n}Q < D_j < Q - D_i\right)$
$\gamma_i(Q) = Prob\left(Q - D_j < D_i < \frac{Q}{n}\right)$	$\gamma_j(Q) = Prob\left(Q - D_i < D_j < \frac{n-1}{n}Q\right)$
$\eta(Q) = Prob(D < Q)$	

Using the notation in Table 10, the Expression (4.2.11) can be simplified as (4.2.12).

$$\frac{\partial \pi^{III}}{\partial Q} = (r+p)(1-\eta(Q)) + s\eta(Q) - \frac{\tau}{n}\gamma_i(Q) - \left(\frac{n-1}{n}\right)\tau\gamma_j(Q) + \left(\frac{n-1}{n}\right)\tau\beta_j(Q) + \frac{\tau}{n}\beta_i(Q) - w - j \quad (4.2.12)$$

Therefore, the optimal joint inventory level for Firms i and j is determined by solving the following equation.

$$\eta(Q) = \frac{r+p-w-j}{r+p-s} - \frac{\tau}{n(r+p-s)}\gamma_i(Q) - \frac{(n-1)\tau}{n(r+p-s)}\gamma_j(Q) + \frac{(n-1)\tau}{n(r+p-s)}\beta_j(Q) + \frac{\tau}{n(r+p-s)}\beta_i(Q) \quad (4.2.13)$$

In the previous section, the optimal inventory level for Firm i and j are jointly determined by solving Equations (4.2.4') = 0 and (4.2.5') = 0. In comparison, the optimal total inventory in the Scenario II is determined by solving Equation (4.2.13) = 0. It would be interesting to examine whether the inventory level decisions that Firms i and j make in these two scenarios are identical, and whether these two scenarios leads to an equivalent profit outcome. If not, then a follow-up question is: Of these two cooperative settings, which scenario is more efficient and provides greater benefits to the participating firms?

Now consider a special case, in which Firms i and j are assumed to face identical demand distribution, and the values for other parameters are symmetrical. In this case, Firms i and j evenly share the optimal total inventory. In other words, the following conditions hold: $n = 2$, $\gamma_i(Q) = \gamma_j(Q)$, and $\beta_i(Q) = \beta_j(Q)$. Hence,

Expression (4.2.13) can be rewritten as:

$$\eta(Q) = \frac{r + p - w - j}{r + p - s} + \frac{\tau}{r + p - s} \beta_i(Q) - \frac{\tau}{r + p - s} \gamma_i(Q) \quad (4.2.14)$$

The optimal joint inventory Q^* is determined by solving (4.2.14) = 0, and for each firm, its optimal inventory equals to $Q^*/2$. To illustrate these analytical results, several numerical examples are developed in the following section.

5. Numerical Examples

In this section, a series of numerical examples are used to illustrate the analytical results. In Example 1, Firm i and Firm j are assumed to face an identical uniform demand distribution within [0, 200]. In the second example, Firms i and j have asymmetrical demands; the demand for Firm i is uniformly distributed between [0, 200], and the demand for Firm j is uniformly distributed between [0, 300]. In Example 3, the difference in the mean demand for these two hypothetical firms becomes larger; Firm j has its demand distributed within [0, 400] and the demand for Firm i remains within [0, 200]. In all of these examples, the values for relevant cost parameters are invariant for Firms i and j (see Table 11).

Table III-11: Assumed Values for Relevant Cost Parameters

Cost Parameters	Notation	Assumed Value
Wholesale price	w_i, w_j	20
Retail price	p_i, p_j	40
Salvage value	s	10
Penalty cost	p	8
Inventory carrying cost	j	4
Transshipment cost	τ	2
Transshipment price	c_{ij}, c_{ji}	[0, 48]
Switching rate	$\lambda_{ij}, \lambda_{ji}$	[0, 1]

5.1 Results for Example 1

In this section, the analysis is drawn from Example 1, in which Firms i and j are assumed to have identical, uniformly distributed demands within [0, 200].

5.1.1 Results for the Scenario with Transshipments

Figure 4 shows the relationship between the optimal inventory level and the value of the switching rate when no transshipments are implemented between the two firms. In the competitive setting, each firm's optimal stock is invariant with the switching rate, as shown by the flat line in Figure 4. In our examples, the switching rate values are assumed to be symmetrical for Firms i and j. For each firm, therefore, the potential loss of consumers to its competitor is counteracted by the same number of consumers diverting from the rival firm, as the two firms are assumed to have identical demand distribution. In such a symmetrical scenario, the competitive intensity between rival firms has no impact on optimal stock levels, individually and jointly.

By contrast, the optimal stock level decreases with the switching rate in the cooperative setting, as shown in Figure 4. Without transshipments, consumers divert away from one firm to another, thereby providing two cooperating firms with opportunities to share their stocks. As the value for the switching rate increases, firms are better able to pool their inventories and thus more likely to hold fewer stock units.

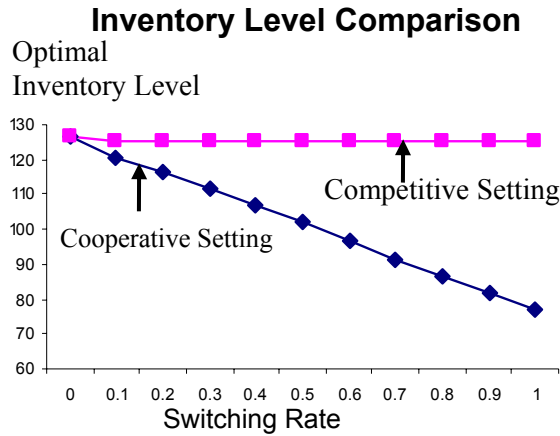


Figure III-4: Inventory Level Comparison

In this example, the optimal inventory level for Firm i is calculated by using the formulas (3.1.7') and (4.1.6'), given the assumed demand distributions for Firms i and j . To obtain an average profit outcome for each firm under various scenarios, we first derive the mean demand for each of these six events and then compute the probability associated with an occurrence of each individual event, given the stock levels determined. For example, when the switching rate between two rival firms is 0.5, the mean demands for Firms i and j under competitive settings are: (162.58, 162.58) for Event I; (103.25, 178.08) for Event II; (43.88, 154.94) for Event III; (62.58, 62.58) for Event IV; (154.94, 43.88) for Event V; and (178.08, 103.25) for Event VI.

The graphs in Figure 5 show that when no transshipments are implemented, firms are better off as the switching rate gets higher in both competitive and cooperative settings. Moreover, the improved performance outcomes as a result of cooperation between the two firms become enlarged when these two firms have a higher switching rate. The degree to which coordinated inventory decisions lead to a better performance outcome than noncooperative inventory decisions is determined by the opportunities for firms to share their inventories and reduce the demand-related risks. The more opportunities for firms to pool their inventories and serve customers, the greater the performance outcome enhanced through cooperative replenishment decisions.

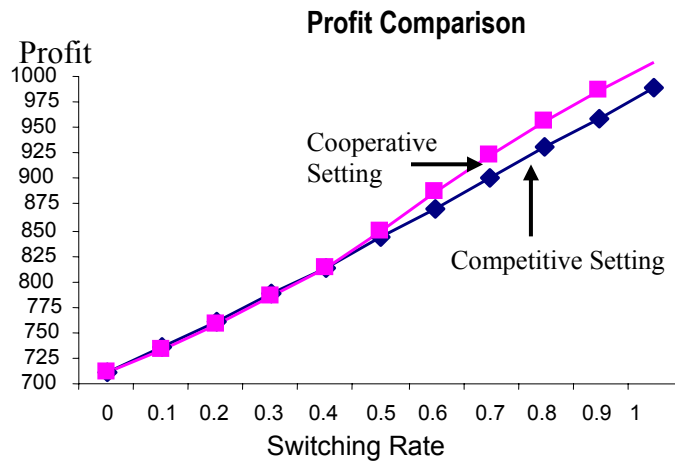


Figure III-5: Profit Comparison

When the two firms cooperate in their replenishment decisions to maximize joint profits, it would be more profitable for them to hold fewer stock units as their consumers become more likely to divert from one firm to another. At a high level for the switching rate, the two firms are better able to reduce their inventory investments without losing a great number of consumers to stockouts, and as a result, the two firms benefit more from inventory coordination than from decentralized inventory

decisions. On the contrary, these two firms do not benefit much from the use of coordinated inventory mechanism when the switching rates between them are relatively low. The graph in Figure 5 shows that for switching rates greater than 0.5, firms achieve more profits under cooperative setting than competitive setting; however, there is little difference in the profit outcome between coordinated inventory policy and localized inventory decision when the level of switching rate is below 0.5.

5.1.2 Results for the Scenario without Transshipments

The extent to which transshipments affect inventory replenishment decisions and performance outcomes is determined by the level of transshipment price implemented between rival firms. Under the competitive setting, Equation (3.2.5') in Section 3 sets up the condition characterizing the inventory choice for Firm i. In this formula, $\alpha_i(Q_i)$ on the left-hand-side of Formula (3.2.5') represents the likelihood that the market demand for Firm i is less than the chosen inventory level. The right-hand-side in this formula sets the value for the threshold probability of having extra stock, which can be used to derive the optimal inventory level.

$$\alpha_i(Q_i) = \frac{p_i + p - w_i - j}{p_i + p - s} - \beta_i(Q_i, Q_j) \left(\frac{p_i + p - c_{ij}}{p_i + p - s} \right) + \gamma_i(Q_i, Q_j) \left(\frac{c_{ji} - s - \tau}{p_i + p - s} \right) \quad (3.2.5')$$

By solving this equation along with (3.2.7'), the optimal stock levels are derived for Firms i and j under the scenario where transshipments are implemented between the two rival firms. The Formula (3.2.5') clearly reveals that the optimal inventory level for Firm i increases with the transshipment price c_{ij} (c_{ji}), holding other cost

parameters constant. Transshipment price drives up the inventory level in two ways.

First, as transshipment price gets higher, the net revenue that Firm i earns per unit transshipped to Firm j rises, and as a result, Firm i tends to hold more inventories.

The part $\gamma_i(Q_i, Q_j) \left(\frac{c_{ji} - s - \tau}{p_i + p - s} \right)$ of the right-hand-side in (3.2.5') is the expected revenue Firm i can obtain from transshipping goods to Firm j, and it increases with the transshipment price c_{ji} for a given occurrence of transshipment. On the other hand, as transshipment price becomes higher, Firm i can reduce its transshipment payment by holding more stock and thus avoiding transshipment requests from Firm j. In Equation (3.2.5'), the part $\beta_i(Q_i, Q_j) \left(\frac{c_{ij} - p_i + p}{p_i + p - s} \right)$ is the expected cost that Firm i incurs when getting the transshipments from Firm j, and it increases with the transshipment price, c_{ij} , for a given occurrence of transshipment. Consequently, the optimal stock level for each firm increases with the transshipment price, as illustrated below in Figure 6.

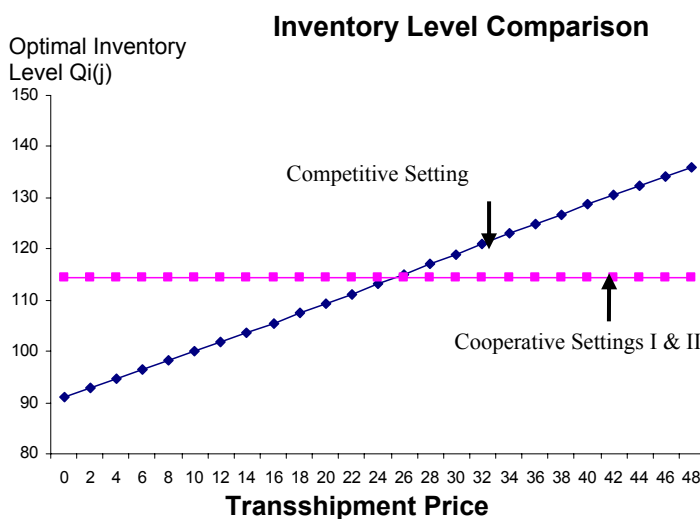


Figure III-6: Inventory Level Comparison

Figure 6 also shows that under the two cooperative settings, a firm's optimal inventory level does not change with the level of transshipment price. When firms coordinate their replenishment decisions to maximize joint profits, transshipment payments from one firm to another can be viewed as the same as an internal monetary transfer. Under this circumstance, transshipment price does not affect the optimal inventory levels that are determined by maximizing the joint profits of the two firms.

It is not surprising to find that the inventory level choices firms make are the same in the two cooperative settings. In this example, the two firms are assumed to have identical uniform demand distribution. As a result, their aggregate demand has a symmetrical triangular distribution, denoted as $[0, 400, 200]$. The optimal stock levels in the cooperative setting I are determined by solving a pair of Equations $(4.2.4') = 0$, and $(4.2.5') = 0$. In comparison, the optimal joint inventory in the cooperative setting II is determined by solving Equation $(4.2.14) = 0$. Since the two firms are assumed to have identical demand in this example, it is not surprising to find that these two cooperative scenarios lead to the equivalent inventory level choice. In other words, the optimal order quantities that the two firms choose based on their aggregate market demand forecast are the same as those determined by the accumulation of the expected profits from each of their market demands. It would be interesting to study whether these two cooperative mechanisms still give rise to an equivalent inventory and profit outcomes when the two firms are assumed to have asymmetrical demand distributions.

Figure 7 presents the profit outcomes for Firm $i(j)$ when per unit transshipment prices charged between these two rival firms range from 0 to \$48 under two cooperative and non-cooperative inventory decisions. As shown in this graph, Firm $i(j)$'s profit has an inverted U-shape relationship with respect to transshipment price. Although the optimal inventory level chosen by each firm increases steadily with transshipment price, the profit outcome for Firm $i(j)$ rises with transshipment price when it is less than \$28 per transshipped unit; the profit outcome for Firm $i(j)$ declines with transshipment price when it is greater than \$28 per transshipped unit. An intuitive explanation for this nonlinear effect of transshipment price on profits is as follows. At a high level of transshipment price, firms tend to hold a great amount of stock. Thus, the likelihood for the occurrence of transshipments decreases as both firms keep more inventory to prevent the chance of stockouts. Under these circumstances, the expected transshipment revenue declines, despite an increased level for the transshipment price. The expected revenue from transshipments is not sufficient to compensate for the expected increase in inventory-related costs, which is associated with a greater level of transshipment price. As a result, a high transshipment price has a negative impact on firm performance. On the contrary, at a low transshipment price, firms tend to hold fewer stocks in order to take advantage of opportunities for inventory sharing. Under this circumstance, transshipments are more likely to be employed between firms. Thus, the expected revenue through implementing transshipments rises with transshipment price. Although the inventory level increases with transshipment price, the increased revenues from transshipments are great

enough to cover the incremental inventory-related costs. As such, a low transshipment price has a positive impact on firm performance.

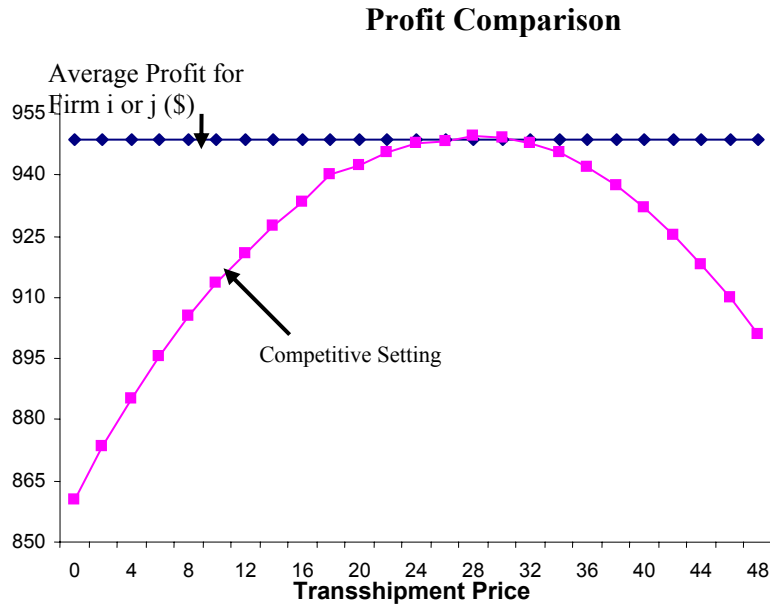


Figure III-7: Profit Comparison

The next section examines how the implementation of transshipments affects a firm’s inventory replenishment decision and the resulting performance outcome when firms meet in various competitive settings.

5.1.3 The Impacts of Transshipments on Firm Inventory Level Choice and the Profit Outcome in Various Competitive Settings

Figure 8 compares the inventory levels with and without transshipments for two firms competing in the setting where the switching rate from one to another is 0.5 and the unit transshipment price between them varies between 0 and \$48. The results indicate that within a certain range of low transshipment prices (i.e., from 0 to \$36), the inventory levels chosen by the two rival firms are greater when no transshipments are employed between them than when transshipments are implemented. The stock level

held by each of the two rival firms rises as transshipment price increases. It is shown that for any transshipment price beyond \$36, the optimal inventory each firm holds with transshipments is greater than without the implementation of transshipments.

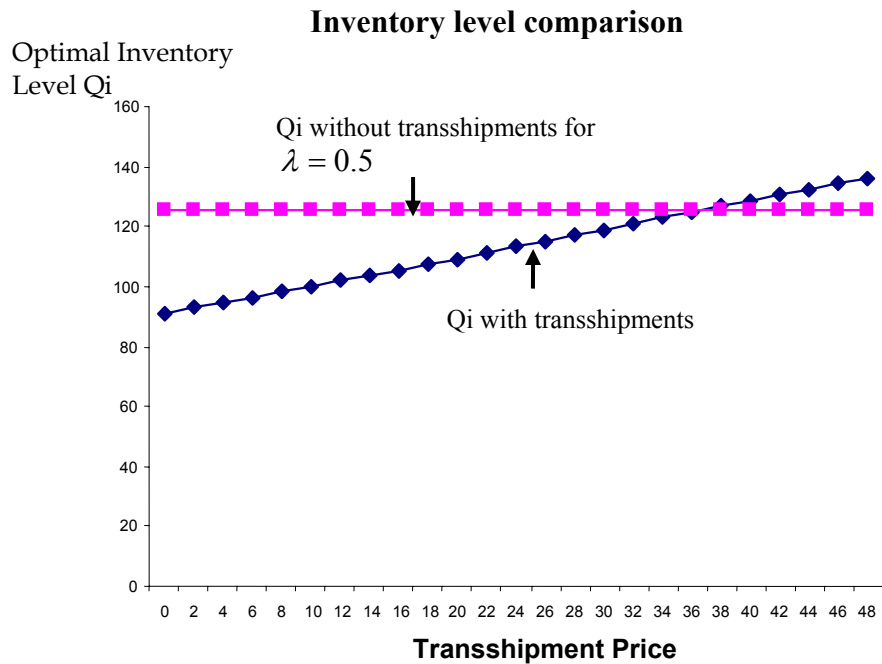


Figure III-8: Inventory Level Comparison

Previous studies have found that transshipments, when implemented among stocking locations operating on the same echelon, improve their service levels and performance outcomes through risk sharing and safety stock reduction. Nevertheless, the practice of transshipments has not been investigated in a competitive setting, in which rival firms might face various market demands and set different services levels. Several interesting questions remain as to: (1) Whether the practice of transshipments still provides benefits when firms compete with one another; (2) what factors have potential to affect the improved performance outcomes arising from transshipments in the competitive setting; (3) whether transshipments have differential impacts on profit

outcomes depending on firm service levels and demands; and (4) what transshipment price firms should set to achieve the greatest benefits under various competitive environments. Figure 9, below, graphically illustrates the performance impact of transshipments implemented between firms that are intense competitors, moderate rivals, and non competitors.

From this graph, we can draw several interesting conclusions. First, whether the implementation of transshipments improves the performance outcome depends on the intensity of competition between rival firms. When two firms are perfect rivals (i.e., the switching rate =1), transshipments will not improve the profitability of firms. Instead, firms will earn more profits if they choose not to employ transshipments and make their inventory decisions accordingly. This is because transshipments add costs to firms. When switching rate between two firms is high, it is less costly to just let consumers divert to the alternative firm in the event of stock out. In comparison, transshipments will improve the profitability of firms when the two firms are moderately competitive or non-rivals. The degree of performance improvements is affected by the level of transshipment price. Specifically, it is found that the transshipment price has a non-linear impact on the profit improvement for a given rivalry intensity. For example, when the two rival firms have switching rates of 0.5, the profit benefit that each firm gains from the practice of transshipments increases with transshipment price, when the level of transshipment price is below \$28. In contrast, the profit benefits decrease with transshipment price when transshipment prices are greater than \$28.

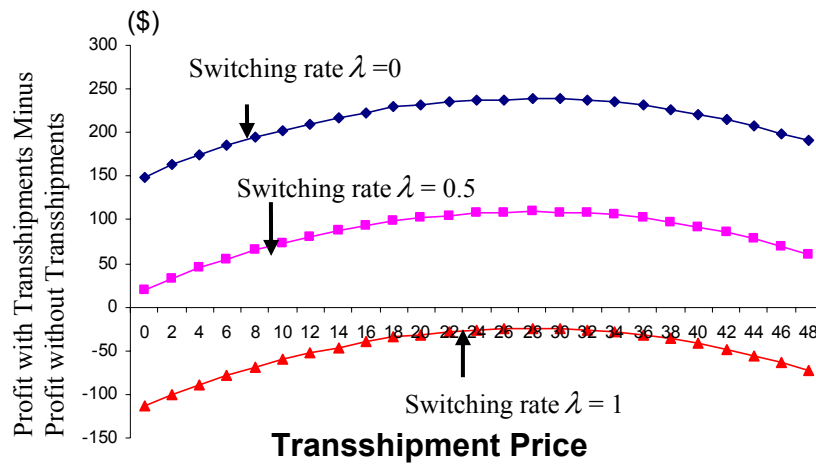


Figure III-9: The Performance Impacts of Transshipments

The forgoing discussions are based on Numerical Example 1, in which the market demands for the two rival firms are identical. In Example 2, we consider a situation where the two rival firms differ in their mean demands, while other parameters remain unchanged.

5.2 Results for Example 2

In Example 2, the mean demands are assumed to be different for the two rival firms. The purpose of analyzing an asymmetric demand scenario is to show whether firm size affects the performance outcome of transshipments; i.e., how does the practice of transshipments affect the profit for a large firm relative to a small firm in various competitive environments.

In this example, we assume that Firms i and j differ only in the mean demand and that the switching rates between these two rivals are symmetric. Specifically, the demand for the small firm, or Firm i , is assumed to have a uniform distribution within $[0,$

200]. In comparison, the large firm, or Firm j , has its demand distributed within $[0, 300]$. To assess the impact of transshipments on firm performance (as measured by profits), given the unit transshipment price of \$24, we first calculate the optimal stock levels for the “without transshipment” scenario by solving a set of equations (3.1.7) and (3.1.8). For the “with transshipments” scenario, we solve Equations (3.2.5') and (3.2.7') to obtain the optimal stock levels for Firms i and j , respectively. Then we compute the expected profits associated with each of these scenarios. For example, at the unit transshipment price of \$24, the optimal stock levels determined for Firms i and j are (124.746, 188.239). Given this pair of inventory levels, the probabilities for Firm i falling into each of the six events are: 0.1864 for Event I; 0.1238 for Event II; 0.0987 for Event III; 0.3217 for Event IV; 0.2001 for Event V; and 0.0692 for Event VI.

At the inventory level of (124.746, 188.239), the mean demands associated with each of these six events are as follows. In Event I, the mean demands for Firms i and j are 154.428, and 238.661, respectively. In Event II, they are 73.4733 for Firm i , and 263.5161 for Firm j . In Event III, they are 31.8832 for Firm i , and 209.2052 for Firm j . In Event IV, they are 54.428 for Firm i , and 88.661 for Firm j . In Event V, they are 146.769 for Firm i , and 65.875 for Firm j . Finally, in Event VI, they are 173.3045 for Firm i , and 150.6265 for Firm j . Given the mean demands presented above, we then calculate event revenues for Firms i and j based on the following variables: units transshipped (i.e., X_{ij} , X_{ji}), sales (i.e., R_i , R_j), unmet demands (i.e., Z_i , Z_j), and unsold stocks (i.e., U_i , U_j) (See Table 5). For Firm i , the revenue associated with each of

these six events is derived as follows: 3989.664 for Event I; 3717.351 for Event II; 2427.654 for Event III; 2721.4 for Event IV; 4960.848 for Event V; and 4479.344 for Event VI. Multiplying event revenues by the derived probability associated with each of these six events, we get an amount of \$3621.96, the expected revenue that Firm i achieves at the unit transshipment price of \$24. After the deduction of inventory acquisition and carrying costs, the average profit for Firm i equals \$1009.426. In a similar way, we use this procedure to calculate the expected profit for Firms i and j, respectively, under various competitive scenarios (as indicated by different switching rates ranging between 0 and 1). Table 12, below, presents some of these results.

5.2.1 The Impacts of Transshipments on Firm Inventory Level Choice and the Profit Outcomes in Various Competitive Settings

Table III-12: The Impacts of Transshipments on Performance Outcomes

Profit Outcome	Switching Rate = 0		Switching Rate = 0.5		Switching Rate = 1	
	Firm i (small)	Firm j (large)	Firm i (small)	Firm j (large)	Firm i (small)	Firm j (large)
With transshipments*	1009.416	1145.332	1009.416	1145.332	1009.416	1145.332
Without transshipments	710.886	1070.604	911.786	1203.634	1144.006	1347.524
Change in absolute term	298.53	74.728	97.63	-58.302	-134.59	-202.192
% Change	41.99%	6.98%	10.708%	-4.844%	-11.765%	-15.005%

* The unit transshipment price is assumed to be \$24.

Several interesting results can be drawn from this table. First, the implementation of transshipments benefits both the large firm and small firm, when the rivalry intensity between these two rivals is relatively low. However, transshipments make neither firm better off when their competition intensifies to a higher level, as indicated by the switching rate approaching to 1. This result reveals that the profit benefits arising

from the implementation of transshipments are decreasing as the two rival firms become more competing. More interestingly, we find that the small firm always benefits more from transshipments than does the large firm at various levels of switching rates ranging between 0 and 1. This is true both in actual monetary terms and in percentage terms. For example, the results show that the performance improvements the small firm (i.e., Firm i) achieves through transshipments are 3.995 (6.016) times in absolute (percentage) terms as the benefits for the large firm, when switching rate between these two firms is 0. As the intensity of rivalry between these two firms increases, the small firm achieves more profit benefits through transshipments than does the large firm. When the switching rate is 0.5, the practice of transshipments endows the small firm (i.e., Firm i) with an improved performance outcome, while making the large firm (i.e., Firm j) fare worse. These findings suggest that the positive impacts from transshipments depend on the relative demand levels of the firms, and on the degree of competition between the two rivals.

In Table 12, we present the differential impacts of transshipments on performance outcomes for Firms i and j at a given unit transshipment price of \$24. The table shows how firm profitability is affected by transshipments given different levels of transshipment price. The results from the tables are illustrated in Figures 11 and 12. They illustrate the performance impacts of transshipments for large and small firms, respectively, given different levels of transshipment price, from 0 to \$48. Before discussing the impact of the practice of transshipments on firm performance, it is necessary to investigate whether the stock levels that firms choose in the “with

transshipments” scenario differ from those chosen in the “without transshipment” scenario, and how transshipment price affects inventory decision-making for firms with different market demands.

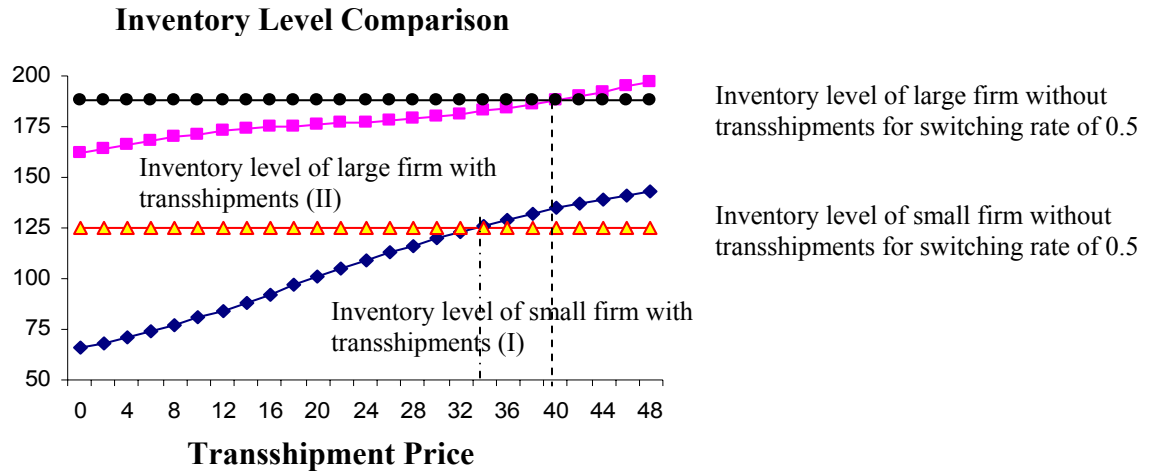


Figure III-10: The Impact of Transshipment on Firm’s Inventory Level

The graphs in Figure 10 reveal the positive impact of transshipment price on firm inventory decisions. As transshipment price rises, the optimal stock levels chosen by both the small firm and the large firm increase. Moreover, we find there is a broader range of transshipment price, under which the implementation of transshipment facilitates the reduction of inventory investment for the firm with greater market demand. As shown in Figure 10, the institution of transshipments decreases the inventories that the large firm holds when transshipment price is less than \$41 (given a switching rate of 0.5). In comparison, the implementation of transshipments enables the small firm to hold less stock only when transshipment price is less than \$32. Finally, simply comparing Curves I with II in Figure 10, we find that the growth rate of stock level with respect to transshipment price is greater for the small firm (i.e., Firm i) than for the large firm (i.e., Firm j). This result suggests that the optimal

inventory level that a large firm holds is less sensitive to transshipment price than for a small firm. In other words, a large firm is less likely than the small firm to take into account transshipment price when making inventory decisions.

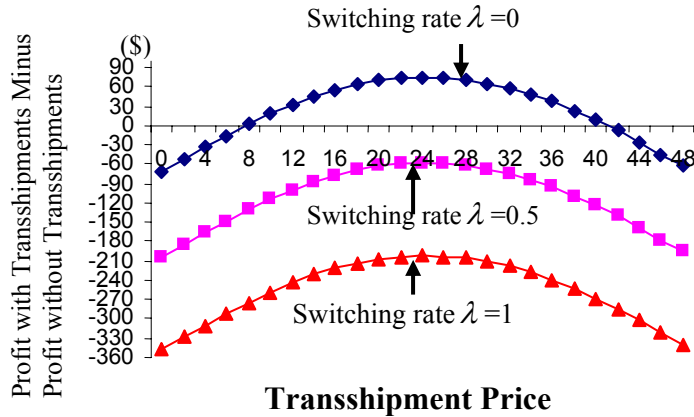


Figure III-11: The Performance Impacts of Transshipments for Large Firm

Figure 11 compares the profit outcomes that a large firm (i.e., Firm j) incurs with and without the implementation of transshipments for three scenarios: The large and small firms are close competitors, moderate rivals, or non-competitors. The impacts of transshipments on the profits of the small firm (i.e., Firm i) are provided in Figure 12. From Figure 11, two factors are found to determine whether a large firm benefits from the implementation of transshipments: the level of transshipment price and the competitive intensity between the two rivals. When two firms compete intensely (as indicated by the switching rate of 1), transshipments will not benefit the large firm in terms of the profit outcome. The large firm achieves higher profits without transshipments. As well, transshipment price also affects profits. As shown in Figure 11, there is an inverted U-shape relationship between transshipment price and the profits of the large firm for the various intensity levels.

In comparison, the performance improvement for the small firm, or Firm i, continuously declines with transshipment price, as shown in Figure 12. We also find that the small firm, unlike the large firm, benefits from transshipments even under moderate competition (as indicated by the switching rate of 0.5).

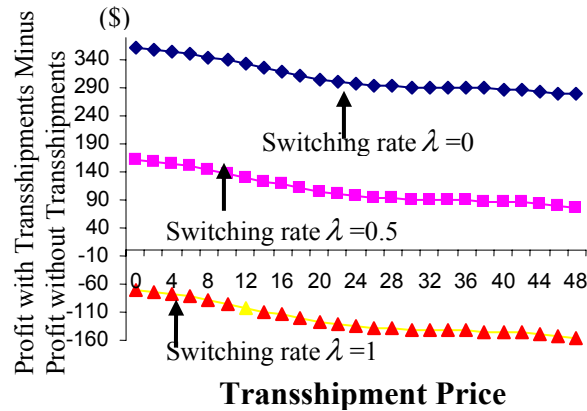
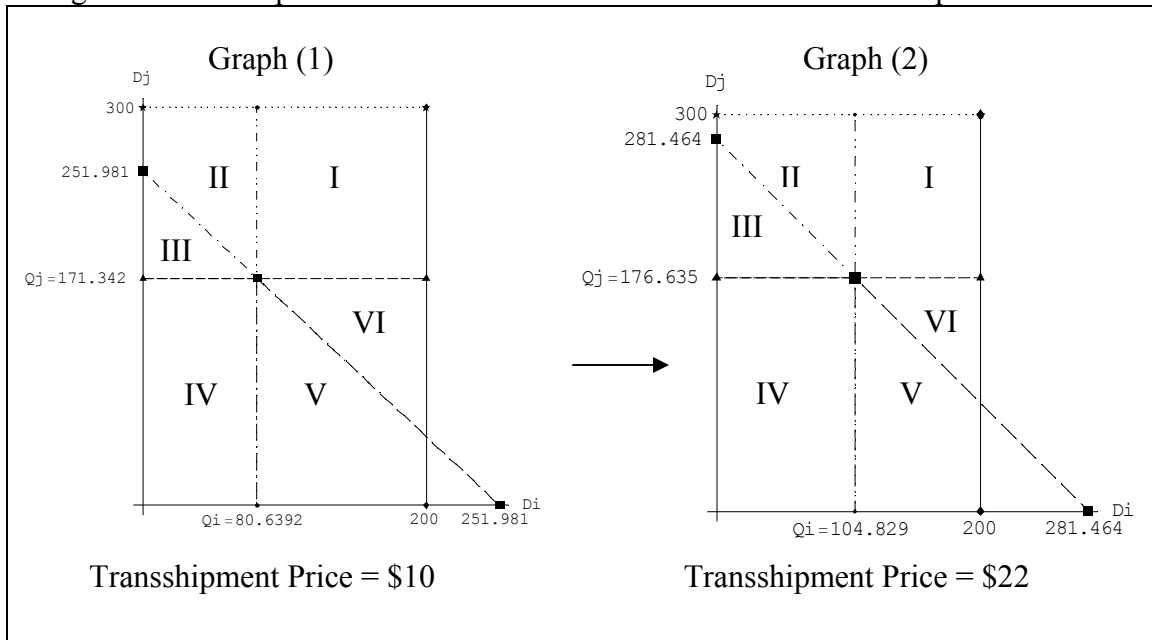


Figure III-12: The Performance Impacts of Transshipments for Small Firm

The main point from the forgoing analysis is that the performance improvements (as measured by the increase in profits) resulting from transshipments diminish with competitive intensity between rivals. This is true for both the large and the small firm. The reason for this relationship is that under high levels of competitive intensity, switching consumers “substitute” for transshipments, thereby reducing the benefits of a transshipment policy. A more difficult question is what causes transshipment price to differentially impact the large and small firms. To understand why this might occur, we start by comparing the probabilities associated with each of the six events (see Figure 13). We have noted from Figure 10 that the optimal stock levels for both the small and large firm (a.k.a. Firms i and j, respectively) increase steadily with transshipment price. The increasing rate of optimal inventory with respect to transshipment price, however, is greater for the small firm than for the large

firm. For example, Firm i would hold an additional 24 stock units if transshipment price rises from \$10 to \$22. By contrast, Firm j only adds 5 stock units. The fact that a small firm increases its inventory to a greater extent than does a large firm in response to an increase in transshipment price leads to asymmetric occurrences of transshipments between the firms. Figure 13 plots the optimal stock levels for Firms i and j (as noted by Q_i and Q_j), and the six scenarios (as noted by I, II, III, IV, V, and VI) that are associated with two levels of transshipment price, \$10 and \$22.

Figure III-13: Graphic Illustration of Scenarios for Different Transshipment Prices



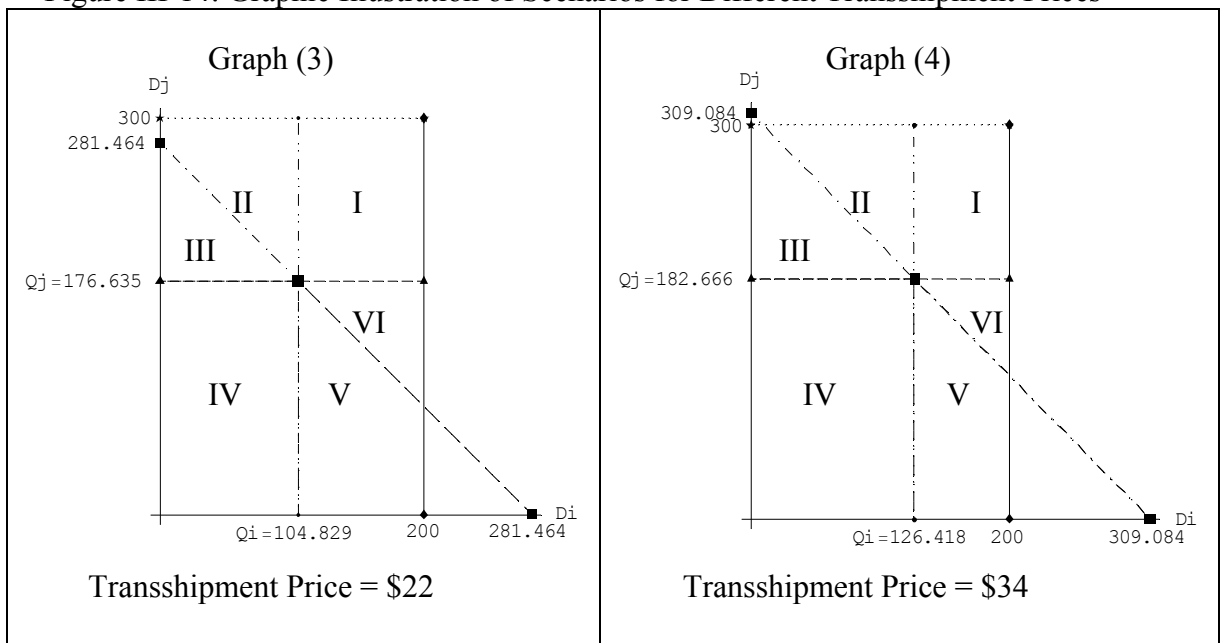
From Figure 13, we can see that as Firms i and j increase their inventories in response to a rising transshipment price, it becomes less likely for the two firms to both stock out, as indicated by the shrinking area for Event I from Graph (1) to (2). On the contrary, there is an increased probability of having the two firms both overstocked when transshipment price rises from \$10 to \$22, as shown by the expanded region for Event IV. As for the amount of transshipments between these two firms, it is intuitive to find that a large firm tends to transship more to a small firm than the

reverse. Such asymmetric occurrences are simply because the large firm tends to hold greater inventories, on average, than does the small firm at any given transshipment price. However, the likelihood of transshipments from the small firm to the large firm increases with transshipment price, while the occurrence of transshipments from the large firm to the small firm becomes less likely.

In the two graphs in Figure 13, Event II represents a scenario when Firm i transships all of its redundant stock to Firm j to satisfy Firm j's stockout demand. The goods transshipped from Firm i to j, nevertheless, are not sufficient to fully cover all of the under-stocked demand at Firm j's location. In comparison, Event III represents the situation when the transshipments from Firm i to j are great enough to fully satisfy the extra demand occurring in Firm j's market. Under this circumstance, Firm i still has some units left over after transshipping to Firm j. In a similar way, Events V and VI identify the two scenarios for transshipping goods from Firm j to i. Comparing each of these four regions (i.e., II, III, IV, and V) between Graphs (1) and (2), we can draw two conclusions. First, the likelihood that Firm i transships to Firm j is greater at a higher transshipment price. The goods that Firm i transships to Firm j are either all of its extra stock (as represented by Event II), or part of the additional inventories it has (as indicated by Event III). Second, the probability of transshipping goods from Firm j to i is lower with an increase in transshipment price. In other words, a high transshipment price suppresses the probability of transshipping goods from the large firm to the small firm, while making it more likely for the occurrence of transshipments from the small firm to the large firm.

We can find that the changing patterns in the probabilities associated with the series of Events from I to VI hold under various transshipment prices. The only exception occurs with Event II, which refers to the scenario when the transshipments from Firm i to j are not sufficient to completely meet Firm j 's excess demands. The graphs in Figure 14 illustrate a reduction in the probability of Event II when transshipment price rises from \$22 to \$34.

Figure III-14: Graphic Illustration of Scenarios for Different Transshipment Prices

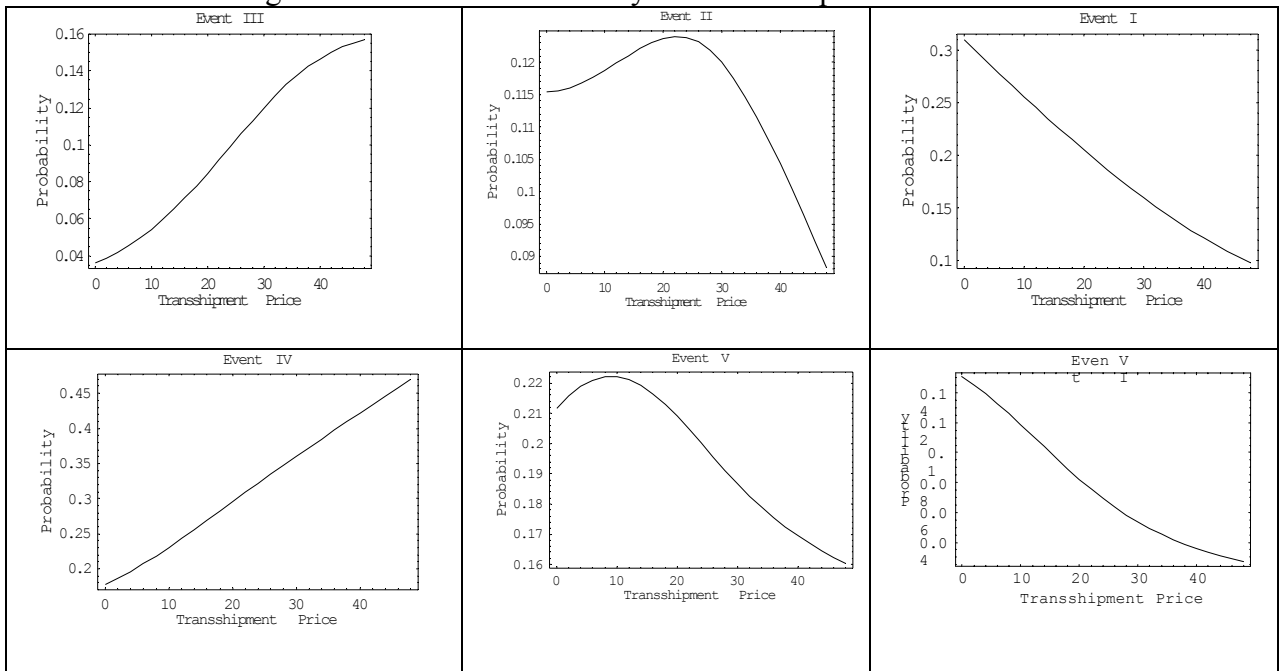


A closer look at the two graphs in Figure 14 helps explain why the probability associated with Event II decreases as transshipment price increases from \$22 to \$34. At a transshipment price of \$34, the joint inventories the two firms hold include 309.084 units, greater than the maximum demand (equal to 300) that Firm j has in its market. As a result, the goods transhipped from Firm i to j are sufficient to completely satisfy the excess demand for Firm j . In other words, it is the expanded

area associated with Event III that suppresses the occurrence of Event II, as shown by the smaller region II in Graph (4) compared to that in Graph (3).

Figure 15 provides an in-depth illustration of the relationship between transshipment price and the resulting probabilities related to each of the six events. These graphs extend the forgoing discussion based on three different transshipment prices to the whole range of transshipment price from 0 to \$48, and provide consistent arguments with regard to the relationship between transshipment price and the likelihood of each of the six events. As shown in the graph (top-right), the probabilities for the occurrence of Event I steadily decrease with transshipment price, suggesting it becomes less likely that the two firms will both stock out as transshipment price increases. On the other side, the probability for the two firms to be both overstocked increases with transshipment price, as shown in the graph (Event IV).

Figure III-15: Event Probability and Transshipment Price



As discussed in the preceding section, the likelihood for the occurrence of Event II increases with transshipment price to a threshold level. As transshipment price gets higher than the threshold value, the probability that Firm i and j fall into Event II declines with transshipment price. The probability of Event III, however, shows a continuous increase with transshipment price. By contrast, the occurrence of Events V and VI shows an opposite trend with respect to transshipment price. Overall, transshipments from a large firm to a small firm become less likely as transshipment price increases.

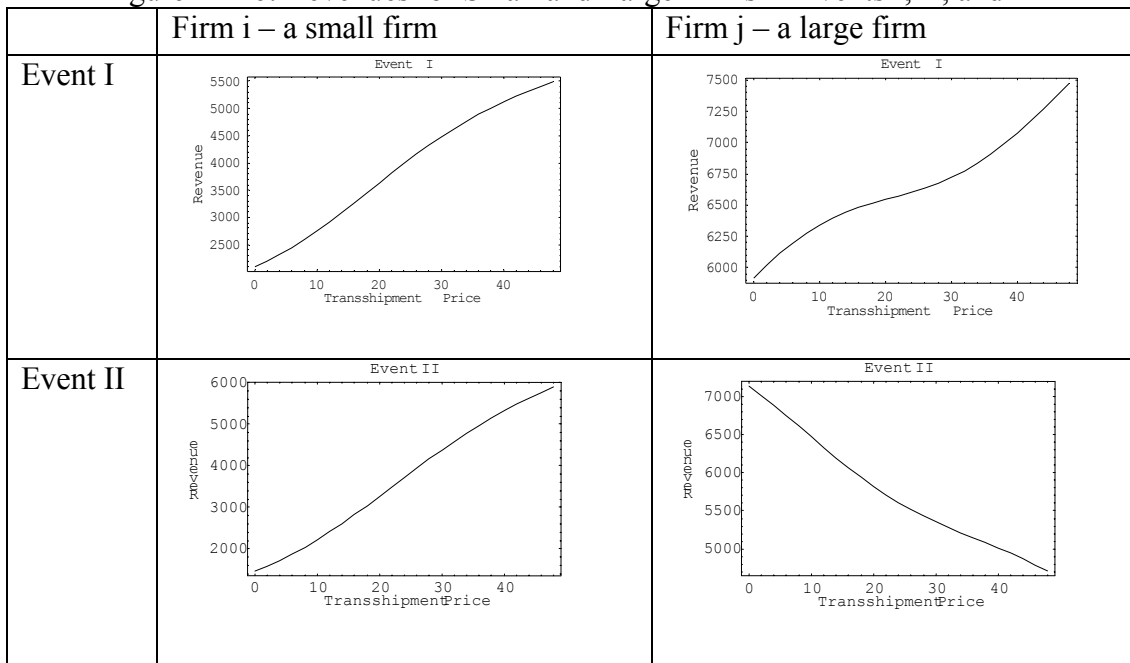
The analysis, so far, has shown how transshipment price affects the likelihood for the occurrence of each of the six events that Firms i and j experience in a single-period replenishment cycle. To study the impact of transshipments on firm profits, we also need to assess the revenues that firms achieve under various scenarios. Table 13, below, provides the formula to calculate revenues associated with Events I to VI. The notations in Table 13 are the same as those used in Section 2.

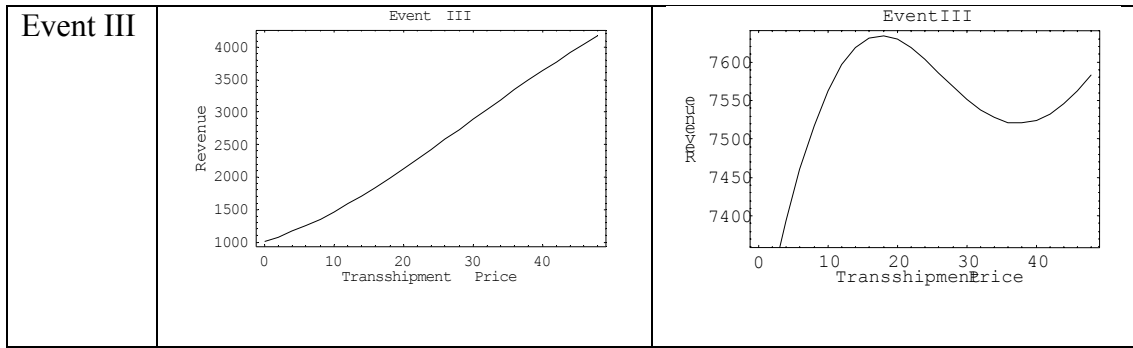
Table III-13: Event Revenues for Large and Small Firms

	Firm i	Firm j
Event I	$p_i \times Q_i - p \times (D_i - Q_i)$	$p_j \times Q_j - p \times (D_j - Q_j)$
Event II	$p_i \times D_i + (c_{ji} - \tau) \times (Q_i - D_i)$	$p_j \times (Q_i + Q_j - D_i) - c_{ji} \times (Q_i - D_i) - p \times (D_i + D_j - Q_i - Q_j)$
Event III	$p_i \times D_i + (c_{ji} - \tau) \times (D_j - Q_j) + s \times (Q_i + Q_j - D_i - D_j)$	$p_j \times D_j - c_{ji} \times (D_j - Q_j)$
Event IV	$p_i \times D_i + s \times (Q_i - D_i)$	$p_j \times D_j + s \times (Q_j - D_j)$
Event V	$p_i \times D_i - c_{ij} \times (D_i - Q_i)$	$p_j \times D_j + (c_{ij} - \tau) \times (D_i - Q_i) + s \times (Q_i + Q_j - D_i - D_j)$
Event VI	$p_i \times (Q_i + Q_j - D_j) - c_{ij} \times (Q_j - D_j) - p \times (D_i + D_j - Q_i - Q_j)$	$p_j \times D_j + (c_{ij} - \tau) \times (Q_j - D_j)$

From Table 13, we can see that there are transshipments that occur in Events II, III, V, and VI. In these events, transshipment price plays a double-role in affecting event revenue. Above all, transshipment price determines the unit revenue received by the firm that transships goods, and the payment per transshipment from the firm that receives a shipment. Secondly, transshipment price has an indirect impact on revenues since the optimal inventory levels that firms adopt are related to transshipment price. The combination of these effects makes it hard to draw straightforward conclusion regarding the effect of transshipment price on revenues. Moreover, the problem becomes more complicated in the context of two firms with different mean demands. Figures 16 and 17 present the calculated average revenues associated with events from I and VI for Firms i and j.

Figure III-16: Revenues for Small and Large Firms in Events I, II, and III





From Figure 16, we observe that transshipment price affects revenues differently for the small firm (i.e. Firm i) than for the large firm (i.e., Firm j). For example, Firm i's revenue in Event II increases with transshipment price, whereas Firm j's revenue decreases. As shown in Table 13, Firm i's revenue in Event II consists of two parts: The revenue from selling to its own customers, and the revenue it receives from transshipping all overstocked goods to Firm j. As transshipment price rises, Firm i tends to hold a greater inventory and thus has more extra stock to transship. As a result, transshipment revenue for Firm i is positively related to transshipment price. In comparison, the event revenue for Firm j under this scenario is composed of three segments: 1) The revenue from selling to its own market; 2) the transshipment payment it makes to Firm i; and 3) the penalty cost it incurs from not being able to satisfy all of the market demand, even after receiving transshipments from Firm i. Transshipment price, in this example, has both positive and negative impacts on the revenue for Firm j. On the negative side, the transshipment payments that Firm j makes to Firm i increase with the unit transshipment price and with the greater transshipment quantities associated with a higher transshipment price. On the positive side, Firm j tends to hold greater inventories as transshipment price rises and thus gains more revenues from selling to its own market and reducing the penalty cost

it would otherwise incur. The negative effect of transshipment price on revenue, nevertheless, dominates. Therefore, overall revenue for Firm j in Event II decreases with transshipment price.

In the case of Event III, the revenue for Firm i steadily increases with transshipment price, whereas the revenue for Firm j is affected by transshipment price in a more complicated way. From Figure 10, we note that the marginal increase in optimal inventory for Firm j is decreasing with transshipment price at a lower range of transshipment price. In contrast, it is increasing with transshipment price when transshipment price rises beyond a threshold level. This non-uniform pattern can help explain why the growth rate of Firm j's revenue in Event II is first decreasing with transshipment price and then increasing when transshipment price rises above a critical level. The revenue for Firm j in Event III can be written as:

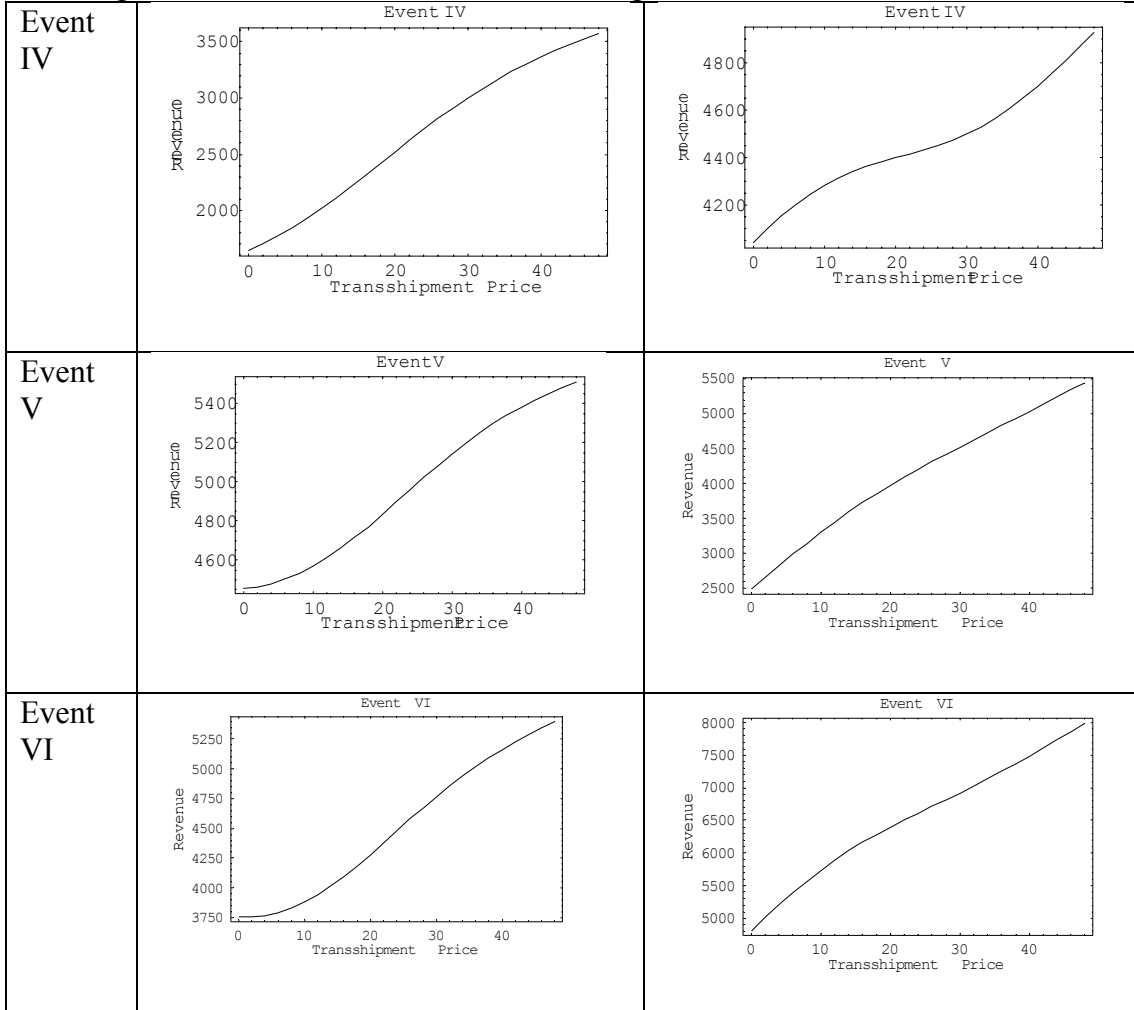
$p_j \times D_j - c_{ji} \times (D_j - Q_j)$, as shown in Table 13. In the numerical example, the market price p_j is given at \$40, and transshipment price is denoted by c_{ji} . To examine the question as to how the event revenue changes with transshipment price, we first use $f'_{(Q_j)}$ to represent the growth rate of optimal inventory for Firm j with respect to transshipment price. Drawing upon the forgoing analysis, we have the following inequalities: 1) $f'_{(Q_j)} > 0$ for transshipment price within $[0, 48]$; 2) $f''_{(Q_j)} < 0$ when transshipment price is less than a critical level; and 3) $f''_{(Q_j)} > 0$ when the transshipment price is greater than the critical level. Taking the first derivative of the revenue expression with respect to transshipment price, we get the growth rate of

revenue as: $-D_j + Q_j + c_{ji} \times f'_{(Q_j)}$. In Event III, it is known that Firm j's inventory level Q_j is always less than its market demand D_j . Therefore, the growth rate of revenue is positive when the following inequality holds: $c_{ji} \times f'_{(Q_j)} + Q_j > D_j$; on the other side, the growth rate of revenue is negative when $c_{ji} \times f'_{(Q_j)} + Q_j < D_j$. These inequality expressions can be used to explain the findings that first: Firm j's revenue in Event 3 increases with transshipment price when the transshipment price is less than \$18; then decreases with transshipment price when the transshipment price is within (\$18, \$40); and it increases with transshipment price when the transshipment price is greater than \$40. In Section 2, we have discussed how to set a feasible range for the transshipment price to make sure that the transshipment price within this range is acceptable to both sender and recipient. Given the cost parameters assumed in this example, the feasible range for the transshipment price is [\$12, \$48].

So far, we have analyzed the scenarios when the direction of transshipments is from Firm i to j, as represented by Events II and III. It may be expected that the revenues associated with Events V and VI for Firm i(j) have a similar pattern as those for Firm j(i) in Events II and III, given that Events V and VI differ from Events II and III only in the direction of transshipments. However, transshipments between Firms i and j are not symmetric. It is reasonable to assume that more goods may be expected to be transshipped from the large firm to the small firm than the other way, simply because the large firm, on average, holds greater inventory than the small firm. Actually, the answer to this question is not that simple. What makes the question complicated is that the small firm increases its inventory more than does the large firm as

transshipment price rises. Figure 17 (below) graphically presents the relationship between transshipment price and revenues for Firms i and j for Events IV, V, and VI.

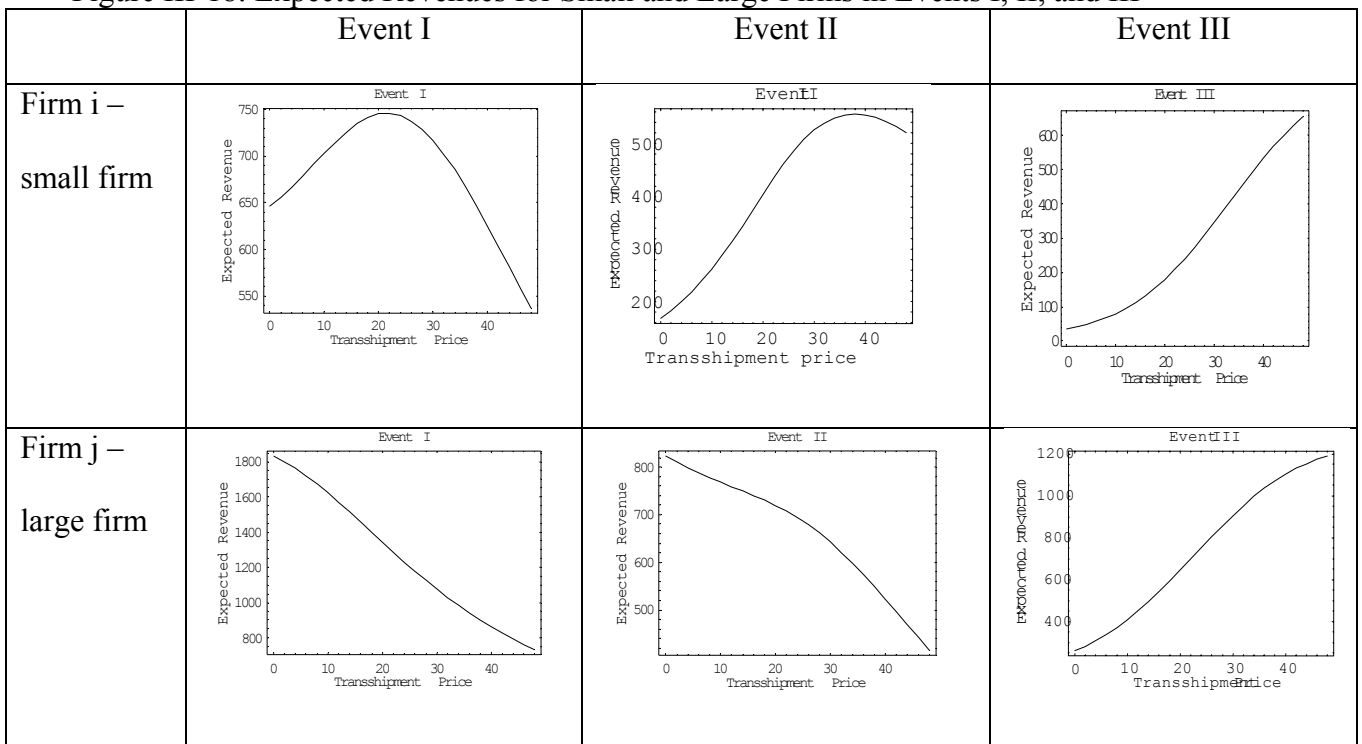
Figure III-17: Revenues for Small and Large Firms in Events IV, V, and VI



In comparison to Events II and III, Figure 17 suggests that when the large firm transships its extra stock to the small firm, the revenue for both the small and large firm increases steadily with transshipment price. This result is distinct from those related to Events II and III.

The revenues reported in Figures 16 and 17 are deterministic values in that these results have not taken into account different probabilities associated with the occurrence of various events from I to VI. In Figure 15, we presented the probabilities for Events from I to VI at transshipment prices ranging from 0 to \$48. Multiplying the event probability by its respective revenue, we get the expected event revenues for Firms i and j. These results are summarized in Figures 18 and 19.

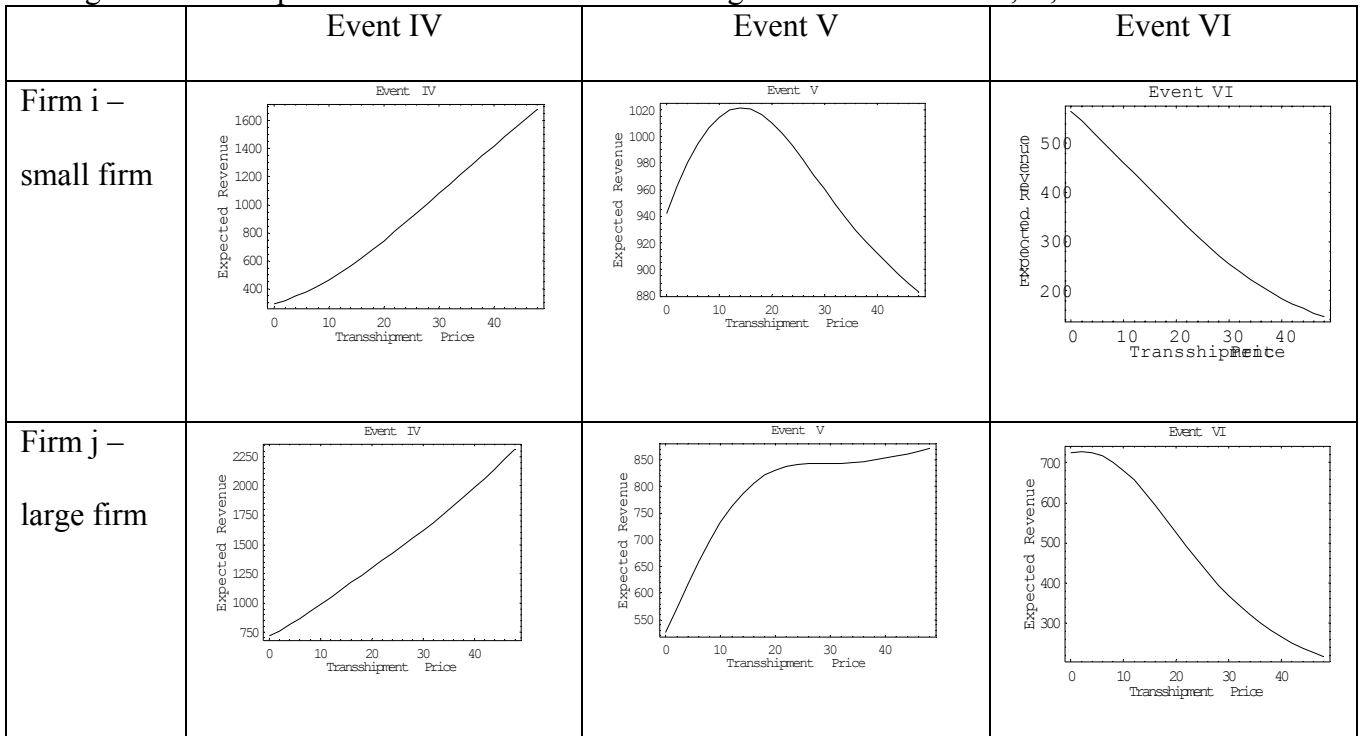
Figure III-18: Expected Revenues for Small and Large Firms in Events I, II, and III



The graphs in Figure 18 support the notion that transshipment price has differential impacts on expected event revenues for the small firm and the large firm. As transshipment price increases, the probability that Firms i and j both stock out is steadily reduced, as represented by Event I. Although the revenues related to Event I increase with transshipment price for the two firms, the expected revenue for the small firm (i.e., Firm i) has an inverted-U shaped relationship with transshipment

price. In comparison, the expected revenue for the large firm (i.e., Firm j) under this scenario is monotonically declining with transshipment price. Such differences are also present in the relationship between transshipment price and expected revenues for Event II. In the case of Event III, the expected revenues for both small and large firms monotonically increase with transshipment price. Similarly, the consistent relationship between transshipment price and expected revenues are found in Events IV and VI, as revealed in Figure 19. In these two events, the expected revenues for Firms i and j decrease with transshipment price.

Figure III-19: Expected Revenues for Small and Large Firms in Events IV, V, and VI

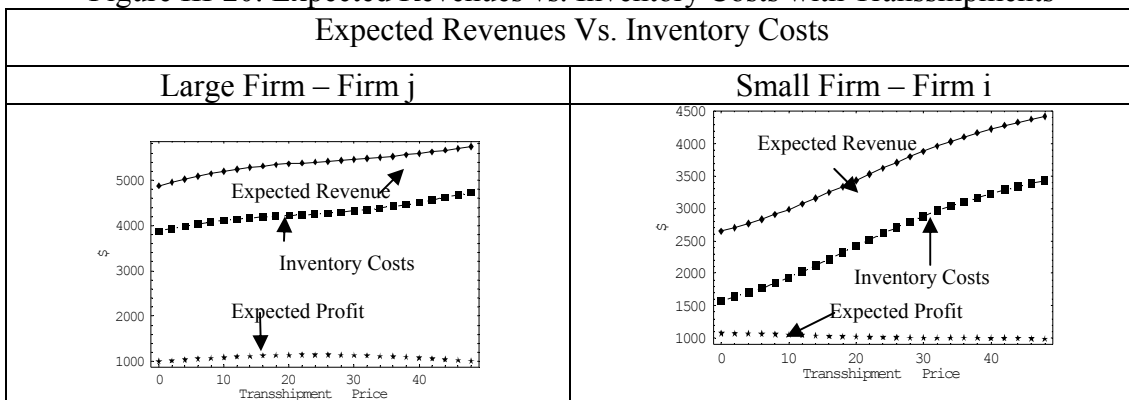


Event V refers to a scenario where the large firm (i.e., Firm j) transships some of its extra stock to Firm i, a small firm. With the occurrence of transshipments, the market demands of Firm i are completely satisfied. It is interesting to find that the expected revenue that Firm j gets from the implementation of transshipments

increases with transshipment price, but at a decreasing rate. On the other hand, the expected revenue that Firm i gets after accepting the transshipments from Firm j is increasing with transshipment price, and then decreasing as transshipment price rises from 0 to \$48. Comparing the results in Event V with those in Event II, we argue that transshipment price impacts the expected revenue for the small firm differently from it does for the large firm within and across events.

Overall, the expected revenues for Firms i and j are the summation of event revenues, weighted by the probability related to each of the six events. The expected profits for Firms i and j are calculated by subtracting inventory costs from the expected revenue. Inventory expenses include wholesale purchasing costs and inventory carrying costs. Figure 20 presents the expected revenue, inventory costs, and expected profits for Firms i and j at various transshipment prices. The steeper curves in the graph for Firm i suggests that the expected revenue and inventory costs are more elastic for the small firm with respect to transshipment price compared to those for the large firm, or Firm j. To investigate how the expected profits for Firms i and j are affected by the level of transshipment price, the values for the expected profits are redrawn in Figure 21.

Figure III-20: Expected Revenues vs. Inventory Costs with Transshipments



The two curves in Figure 21 represent the expected profits that Firms i and j achieve in the scenario when transshipments are implemented, and the transshipment price varies from 0 to \$48.

Figure III-21: Expected Profits with Transshipments

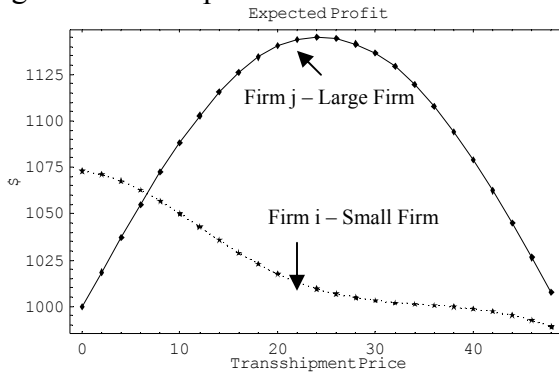


Figure 21 clearly suggests that transshipment price has differential effects on the expected profits for the small and large firms. Specifically, there is an inverted-U shaped relationship between transshipment price and the expected profits of the large firm, or Firm j. The expected profits of the small firm, or Firm i, are negatively related to transshipment price. Moreover, Figure 21 reveals that the small firm has greater expected profits under a transshipment policy than does the large firm when transshipment price is relatively low, or less than \$8. On the contrary, the expected profits under a transshipment policy are greater for the large firm than for the small firm when transshipment price rises above \$8. To examine whether firms benefit from the implementation of transshipments, Figure 22 compares the expected profits for firms when transshipment are implemented to the expected profits without transshipments.

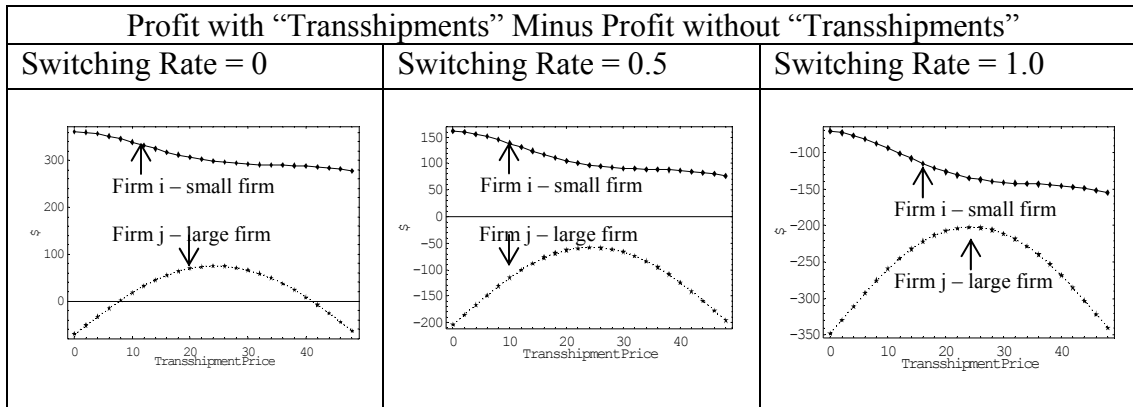


Figure III-22: The Performance Impacts of Transshipments in Various Competitive Settings

Interestingly, we find that the small firm consistently benefits more from the implementation of transshipments than does the large firm no matter the competitive intensity (as measured by switching rate). Moreover, the result that the small firm benefits more than large firm from transshipments always holds true independently of the transshipment price level. These findings complement those from Figure 21. It seems that the large firm gains greater profits from the practice of transshipments than does the small firm. However, the relative benefits from the practice of transshipments are greater for the small firm. For example, the graph illustrates that only the small firm benefits from transshipments when the switching rate between firms is 0.5. When the switching rate rises to 1.0, neither firm is found to benefit from transshipments.

Based on the forgoing discussion, we conclude that there are several factors affecting the performance benefits that firms achieve through the strategy of transshipments. These factors include: the rivalry intensity between firms, the transshipment price, and the market demand that one firm has relative to the other.

5.2.2 The Impacts of Transshipments on Joint Inventory Level and Aggregate Profit Outcomes in Various Competitive Settings

In the previous section, we have discussed the impacts of transshipments on the inventory levels and the profit outcomes for Firms i , and j . In this section, we focus on how transshipments impact the joint inventory of the two firms and the differences in aggregate profits that firms achieve from the implementation of transshipments under various competitive environments.

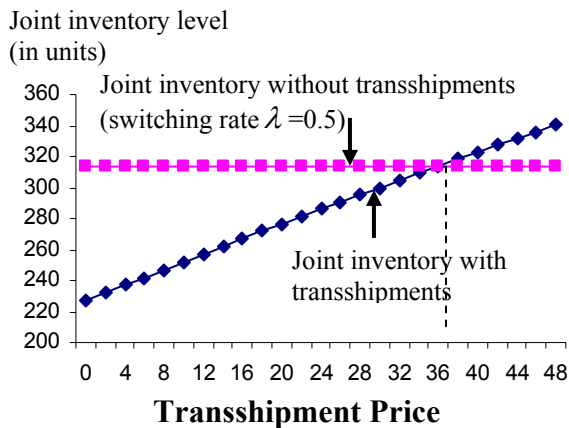


Figure III-23: The Impacts of Transshipments on Joint Inventories of Firms

In Figure 23, the flat line represents the joint inventory that Firms i and j hold when the two firms compete in a market with a switching rate of 0.5, and when no transshipments take place between the two firms. In comparison, the upward sloping line presents the inventories that Firms i and j jointly carry when the two firms transship their stocks at various transshipment prices ranging from 0 to \$48. Two conclusions are drawn from this graph. First, transshipment price has a positive effect on the joint inventory held by the two firms; in other words, the amount of inventory that Firms i and j jointly hold is greater at a higher transshipment price. Moreover, the inventory that the two firms jointly hold when no transshipments take place is

different from the inventory levels under a transshipment policy. Specifically, firms hold greater inventories when there are no transshipments, as compared to in the scenario with transshipments when the unit transshipment price is less than \$36. With the unit transshipment price rising above \$36, the joint inventories that firms hold with transshipments are greater than without transshipments.

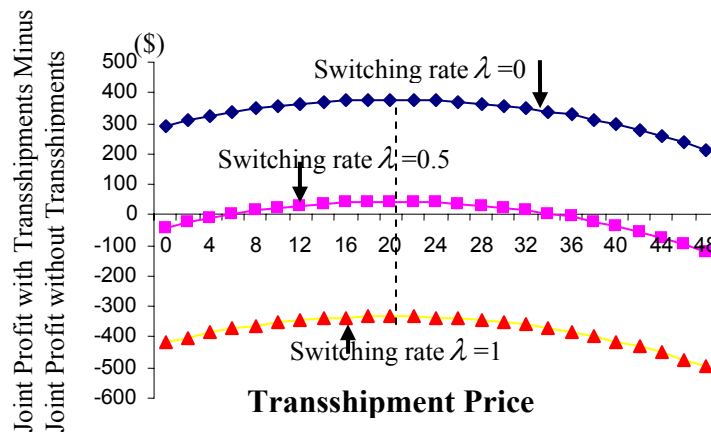


Figure III-24: The Impacts of Transshipments on Joint Profits of Firms

The graphs in Figure 24 shows the differences in the joint profits that Firms i and j achieve through the practice of transshipments under various competitive settings. The results suggest first, the joint performance benefits from transshipments are greatest when the two firms experience no competition, as indicated by the switching rate of 0. As the switching rate between the two firms rises from 0 to 1.0, the joint benefits that the firms achieve through the practice of transshipments are declining and then becoming negative. In particular, it is found that the implementation of transshipments no longer improves the joint profits of the two firms, relative to the scenario without transshipments, when there is a switching rate between 0.5 and 1. These findings are in line with the suggestion that transshipments are more profitable

when the participating firms compete less intensely with one another. Furthermore, it is shown that in any of the three settings illustrated in Figure 24, joint benefits from transshipments are greatest when the unit transshipment price is \$20.

The above result raises a follow-up question: What impacts does the practice of transshipments have on small and large firm, respectively, when the two firms, viewed together, achieve maximal performance benefits through transshipments.

Figure 25 presents the performance impacts of transshipments for Firms i and j when transshipments are implemented at the price level of \$20.

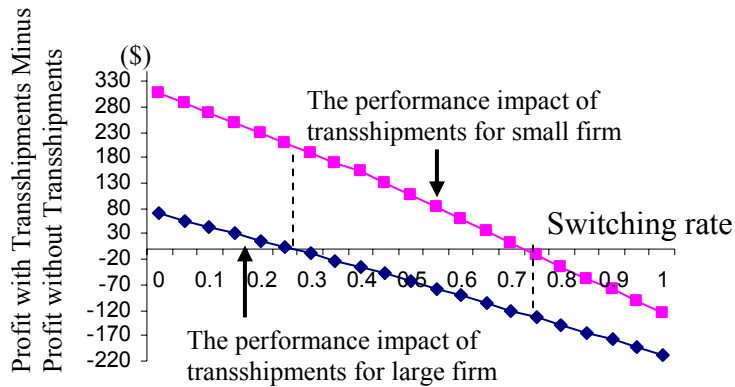


Figure III-25: The Profit Impact of the Transshipment Strategy that Maximizes Joint Benefits

The main findings from Figure 25 are three-fold. First, at the transshipment price of \$20, both small and large firms benefit from the implementation of transshipments when the switching rate between them is less than 0.25. Secondly, transshipments benefit the small firm only, when the switching rate between the two firms is greater than 0.25, but less than 0.75. Finally, neither firm benefits from the implementation of transshipments when the switching rate is greater than 0.75. These results suggest that it is necessary to provide the large firm, or Firm j, with an extra incentive to

implement transshipments when the switching rate between the two firms is within $[0.25, 0.5]$. As shown in Figure 24, the implementation of transshipments improves the system-wide profits when the switching rate is below 0.5. However, what Figure 25 reveals is that the large firm, or Firm j , does not benefit from the practice of transshipments when the switching rate is between $[0.25, 0.5]$. Under this circumstance, it is, therefore, important to reduce the asymmetric benefits (costs) between small and large firms. One approach is to use asymmetric transshipment prices between the small and large firms. An alternative solution is to employ side payments from the small firm to the large firm. The question of how to design an appropriate, effective mechanism remains for future research.

5.2.3 The Impacts of Order Coordination on Firm Inventory Level Choice in the Joint Decision-Making Environment with Transshipments

In Section 4.2, we have presented the analysis focusing on how the optimal inventory level decisions of firms are made when two firms operate in a joint decision-making environment, and when transshipments are implemented between them. Following Rudi et al. (2001), we first investigate the scenario, in which the inventory level decisions of the two firms are determined to maximize their aggregate profits. In this case, there is no effort to coordinate the ordering decisions made by firms, and the optimal order quantities for Firms i and j are derived by solving Equations (4.2.4') and (4.2.5'), jointly. In comparison, the second scenario we developed assumes that the two firms make their joint inventory level decision based on the expected aggregate demands, and then allocate such an optimal inventory in proportion to their

relative market demands. In this setting, the optimal total inventory for the two firms is determined by solving Equation (4.2.13). The ratio of the inventory allocated to each location (i.e., Q_i/Q_j) equals to the relative forecasted market demands of the two firms (i.e., D_i/D_j). As compared to the previous setting, this scenario involves order coordination between firms.

The results based on Numerical Example 1 in Section 5.1.2 suggest that these two joint decision-making mechanisms are equivalent when firms have identical demand distribution and other cost parameters are symmetric. Nevertheless, it remains unknown whether order coordination makes difference in the inventory decision and profit outcomes when firms have different market demands. This section compares the optimal inventory levels and the profit outcomes that are determined by these two decision-making rules.

In this numerical example, the market demands for Firms i and j are assumed to have uniform distribution denoted as follows: $D_i \sim U[0, 200]$, $D_j \sim U[0, 300]$. Table 14 provides the expressions for those key probability parameters included in (4.2.4') and (4.2.5').

Table III-14: Expressions for Key Parameters

$\alpha_i(Q_i) = Q_i / 200$	$\alpha_j(Q_j) = Q_j / 300$
$\beta_i(Q_i, Q_j) = \frac{[Q_j(200 - Q_i) - 0.5(200 - Q_i)^2]}{60000}$	$\beta_j(Q_i, Q_j) = \frac{0.5Q_i^2}{60000}$
$\gamma_i(Q_i, Q_j) = \frac{[0.5Q_i^2 + Q_i(300 - Q_i - Q_j)]}{60000}$	$\gamma_j(Q_i, Q_j) = \frac{0.5(200 - Q_i)^2}{60000}$

In Table 14, the probabilities associated with various events are expressed as functions of order quantities for Firms i and j. Using these notations and the assumed values for relevant cost parameters, the optimal inventory levels for Scenario I (see Section 4.2.1), Q_i^* , Q_j^* are calculated by solving Equations (4.2.4') = 0, and (4.2.5') = 0. The results are: $Q_i^* = 116.129$ and $Q_j^* = 174.194$. Thus, their total inventory equals to 290.323.

Next, we use Equation (4.2.13) and Table 10 to calculate the joint optimal inventory level for Scenario II (see Section 4.2.2). In this scenario, Firms i and j are assumed to make their joint order quantity decision based on their aggregate demand forecasts. Then the two firms allocate the optimal inventory. In this example, the ratio of the inventory allocated to the two firms Q_i/Q_j equals to 2/3. Thus, the value for n is 5/2. Table 14 provides the expressions for the probability parameters included in Equation (4.2.13). Given the assumption that Firms i and j have uniform demand distributions, their aggregate demand has a trapezoidal distribution. Table 15 shows the expressions for the key probability parameters used in Equation (4.2.13).

Table III-15: Expressions for Key Parameters

$\beta_i(Q) = \frac{[0.6Q(200 - 0.4Q) - 0.5(200 - 0.4Q)^2]}{60000}$	$\beta_j(Q) = \frac{0.5(0.4Q)^2}{60000}$
$\gamma_i(Q) = \frac{[0.5(0.4Q)^2 + 0.4Q(300 - Q)]}{60000}$	$\gamma_j(Q) = \frac{0.5(200 - 0.4Q)^2}{60000}$
$\eta(Q) = \frac{200^2}{2 \times 60000} + \frac{Q}{300} - \frac{200}{300}$	

Using the notations in Table 14, the optimal joint inventory Q^* is derived by solving Equations (4.2.13) = 0. The order quantities allocated to Firms i and j are:

$Q_i^* = 0.4 \times Q$, and $Q_j^* = 0.6 \times Q$. The results are as follows: $Q^* = 291.991$, $Q_i^* = 116.796$, and $Q_j^* = 175.195$. It is shown that firms hold similar amount of stock in Scenarios I and II. This finding suggests that the two cooperative mechanisms lead firms to have equivalent inventory decisions and profit outcomes.

5.3 Results for Example 3

In Section 5.2, we analyzed the case where the demand differences between a large firm and a small firm are moderate. We find that transshipments benefit small firms more than large firms in reducing inventory investments and in improving performance outcomes. These differences are present under various competitive settings and become greater as firms compete more vigorously. In this section, we further investigate to what extent transshipments reward small firms disproportionately to large firms when the scale of demand differences becomes further enlarged. In Example 3, the demand for the large firm (Firm j) is assumed to be uniformly distributed within $[0, 400]$, and the demand for the small firm (Firm i) is within $[0, 200]$.

The numerical results based on Example 3 are consistent with those from Example 2. In Section 5.3.1, we first present the inventory and profit outcomes in the scenario when no transshipments are implemented between Firms i and j. Next, Section 5.3.2 provides the inventory level and profit outcomes in the scenario when transshipments are implemented between the two firms. Finally, the impacts of transshipments on inventory and profit of firms, overall and separately, are investigated in Section 5.3.3.

5.3.1 Results for the Scenario without Transshipments

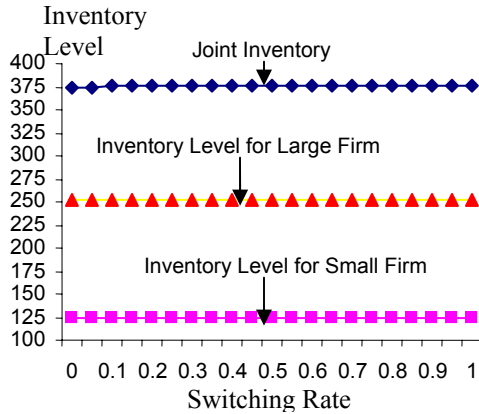


Figure III-26: Inventory Levels Under Various Competitive Settings

Consistent with the previous findings, the optimal inventory levels that Firms i and j choose under various competitive settings remain nearly the same. Given the assumption of symmetrical switching rates, it is expected that firms make their inventory decisions regardless of the level of switching rate. This argument is verified by the flat lines in Figure 26 suggesting the constant inventory levels that are chosen by firms for various switching rates within $[0, 1]$. Although firms maintain constant inventory levels, the profit outcomes for both small and large firms increase with the switching rate. As shown in Figure 27, Firms i and j both have greater profits when the switching rate is higher.

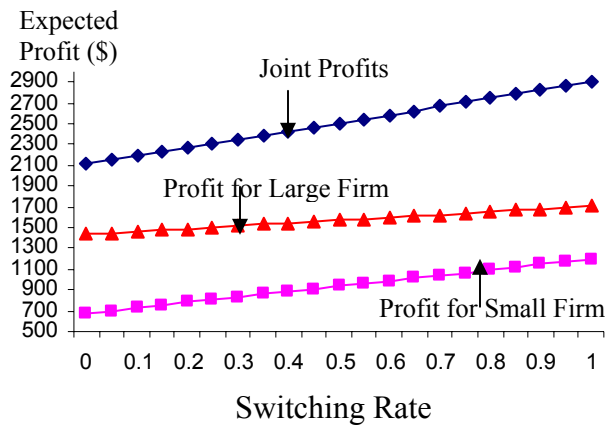


Figure III-27: Profit Outcomes under Various Competitive Settings

A potential explanation for this result is that firms have more opportunities to share their inventories when consumers are more likely to switch from one firm to another. As a consequence, the expected revenues for both firms are greater when the switching rate between the two firms is higher. Since the inventory-related costs are constant for different switching rates, the increase in the expected revenue contributes to the performance improvements, as indicated by the higher profit outcome.

5.3.2 Results for the Scenario with Transshipments

In the scenario, when transshipments are implemented between Firms i and j , the results are also consistent with those in the previous example. As shown in Figure 28, both small and large firms increase their inventory levels with transshipment price. However, the inventory level chosen by the small firm is more elastic with respect to transshipment price than it is for the large firm. This finding further suggests that transshipment price has differential impacts on the inventory replenishment decisions for firms facing differing market demands.

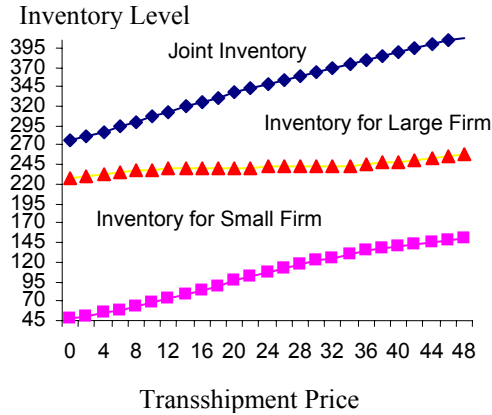


Figure III-28: Inventory Levels at Various Transshipment Prices

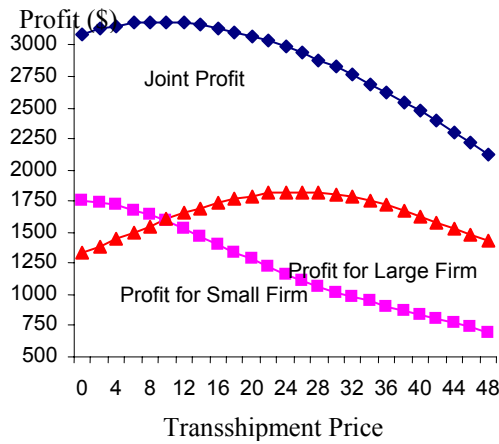


Figure III-29: Profit Outcomes at Various Transshipment Prices

In Example 2, we have found the presence of an inverted-U shaped relationship between transshipment price and the profit outcome of the large firm. Moreover, it is found that there is a negative relationship between transshipment price and the profit outcome for the small firm. These findings also hold true in Example 3, where the difference in the mean demand between large and small firm increases to 100 units.

5.3.3 The Impacts of Transshipments on Firm Inventory Level Choice and the Profit Outcomes in Various Competitive Settings

The graphs in Figures 30 and 31 compare the optimal inventory levels with and without transshipments for Firm i and j, respectively, assuming the switching rate of 0.5.

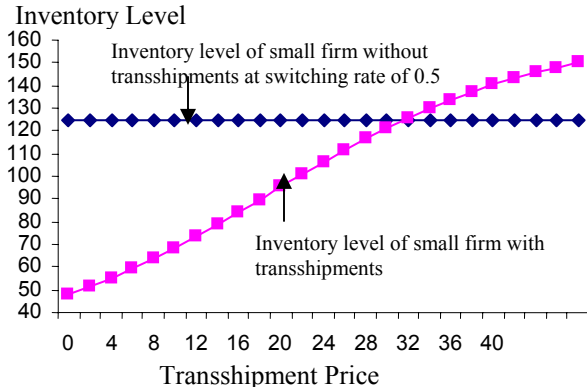


Figure III-30: Inventory Levels for Small Firm with and without Transshipments

The upward-sloping curve in Figure 30 suggests a positive impact of transshipment price on Firm i's inventory level. As transshipment price rises, the optimal stock levels chosen by the small firm increase. Similar results are found for Firm j, as shown in Figure 31.

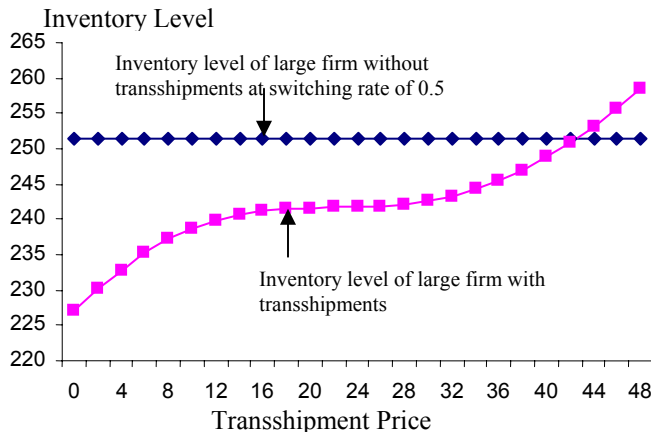


Figure III-31: Inventory Levels for Large Firm with and without Transshipments

Comparing Figure 30 with Figure 31, we find that although the inventory levels for both small and large firms increase with transshipment price, the growth rate for the small firm is increasing with transshipment price, and the growth rate for the large firm is decreasing with transshipment price. Such differences in the growth rate of inventory levels were also found in the previous example, in which the mean demand for the large firm is greater than that for the small firm by 50 units.

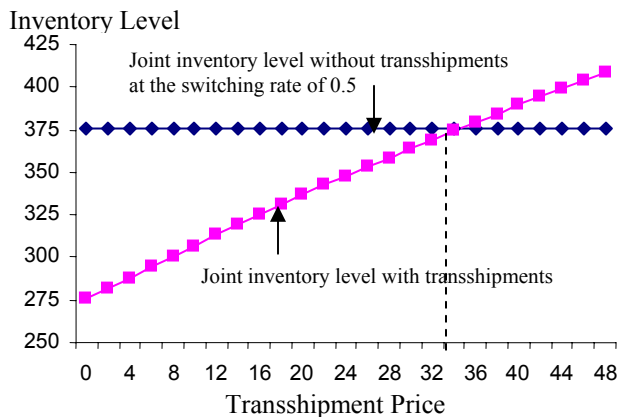


Figure III-32: Joint Inventories with and without Transshipments

In Figure 32, the flat line represents the joint inventory that Firms *i* and *j* hold when the two firms compete in the market with a consumer's switching rate of 0.5, and when no transshipments are implemented between the two firms. In comparison, the upward sloping line displays the inventories that Firms *i* and *j* jointly hold when the two firms implement transshipments under various transshipment prices from 0 to \$48. It is found that the joint inventories that Firms *i* and *j* hold with transshipments are greater than those held by the two firms without transshipments, when the unit transshipment price is above \$34. Note that in Example 2, it was found that the threshold transshipment price was \$36. In these two examples, we have held everything else constant. Therefore, the larger gap in the mean demand between the

two firms might lead to a lower threshold transshipment price in Example 3, compared to that in Example 2.

The performance impacts of transshipments for Example 3 are presented in the following table. Table 16 indicates that the implementation of transshipments benefits the small firm more than the large firm at various competitive settings. This finding is consistent with the results presented in Table 12 for Example 2.

Table III-16: The Impacts of Transshipments on Performance Outcomes

Profit Outcome	Switching Rate = 0		Switching Rate = 0.5		Switching Rate = 1	
	Firm i (small)	Firm j (large)	Firm I (small)	Firm j (large)	Firm i (small)	Firm j (large)
With transshipments*	1529.543	1650.422	1529.543	1650.422	1529.543	1650.422
Without transshipments	673.794	1433.404	933.926	1569.586	1197.774	1708.898
Change in absolute term	855.749	217.018	595.617	80.836	331.769	-58.476
% Change	127.005%	15.14%	63.776%	5.15%	27.699%	-3.421%

* The unit transshipment price is assumed to be \$12.

6. Conclusions and Future Research

The analytical model developed in this essay investigates the practice of transshipments between two competing stocking locations. Many previous studies in supply chain management have modeled the performance impacts of uncertainties, such as demand and lead time variability, and their implications for inventory management. The role of competition in affecting the performance outcomes of transshipments has not received much attention. This essay examines inventory replenishment decisions and the application of the transshipments strategy in various competitive environments. The analysis can be used to predict under what conditions

transshipments are likely to be employed between rival firms (e.g., auto dealerships). The results suggest that there exist opportunities for rival firms to collaborate through transshipments. In other words, the practice of transshipments is able to improve firm performance (as measured by profits), even when the participating firms are direct rivals. Moreover, the numerical results are used to illustrate how firms with asymmetric demands benefit differently from transshipments under various competitive settings.

It is found that transshipments provide more performance improvements for the small firm than for the large firm. Such imbalanced benefits become more substantial as the competition intensity between the two firms increases. These results suggest that it is important to design an effective, appropriate incentive mechanisms (e.g., monetary transfers, asymmetric transshipment prices) to initiate transshipments between rival firms with varying demands.

A few interesting questions remain for further research. One extension is to investigate what the performance impacts of transshipments are for firms operating under the competitive environment, in which the prices of their products vary with the rivalry intensity between firms. In this study, the retail prices of product are held constant under various competitive settings. It would be more interesting to characterize the market of greater competition with both a lower level of retail price and a higher value of switching rate. The use of endogenous price and switching rate would provide us with an opportunity to view the pricing decision of firms conjointly

with their inventory decision. The second extension of this essay is to relax the assumption that the demands at different locations are independent. The study of transshipments between rivals firms with correlated demands would help us understand to what extent the factor of competition moderates the benefits from the risk-pooling strategy enabled through transshipments.

Chapter 4: Conclusions

This dissertation explores two types of strategic behaviors and market outcomes: (1) How strategic interactions across markets affect multimarket competitors in their pricing behaviors and collusive outcomes, and (2) how transshipments in a competitive market affect rival firms in their inventory decisions and profit outcomes.

In Essay 1, a conjectural variation model is developed to examine how a firm makes product pricing decisions when taking into account the strategic contacts the firm has with its rivals across multiple markets. An insight that has not received much attention, but revealed from our formal analysis of competitive behavior in a multimarket contact setting, is that similarity in production costs plays an important moderating role in the inverse relationship between multimarket contact and rivalry intensity. That is, multimarket contact is more effective in facilitating tacit collusive pricing when it occurs between rival firms having similar production costs than when it occurs between rival firms with dissimilar production costs.

Such differential impacts of multimarket contact on collusive behavior arise from the consideration that rival firms of similar production costs have greater conjectural

variation with respect to one another, and more importantly, they have less degree of vertical product differentiation. It is the formation of higher conjectural variation and the presence of greater product substitutability that reinforce the collusion facilitating effects of multimarket contact for firms having similar production costs.

This finding gives firms competing in single market and multimarket contexts different implications with respect to product development strategy. For example, when two firms compete in a local market with a single product, one option for a firm to avoid fierce competition is to distinguish itself from the rival firm by introducing differentiated products. The more dissimilar the products are, the less likelihood for the occurrence of a pricing war. However, it would be a different story if firms competed simultaneously in multiple markets. Under this scenario, tacit collusion and lower rivalry intensity are more likely to sustain when the product lines firms develop are similar with one another. Consequently, the competitive implications of product differentiation strategy are dramatically different in single market and multimarket contact settings.

Competitive interactions across multiple markets and their economic consequences have received growing interest in the marketing and strategic management literatures. In a review paper by Jayachandran et al. (1999), the authors discuss, in detail, the implications of multimarket competition for marketing strategies, in particular, product line rivalry and market entry decision. The notion of mutual forbearance and multimarket competition also applies to conglomerate firms with diversified

businesses (Hughes and Oughton 1993). Along with these studies, our work extends and enhances the understanding of the questions: What nature of multimarket contact facilitates tacit collusion and how these collusion enhancing effects differ in various market circumstances.

Moreover, the finding that the tacit-colluding opportunities endowed with multimarket contact are more likely to hold between carriers with similar production costs has policy implications. Traditionally, it has been well recognized that high cost carriers tend more than low cost carriers to engage in tacit collusive pricing. In the multimarket contact setting, however, low cost carriers also have positive reasons to engage in mutual forbearance when their rivals are also low cost carriers. As a result, it may not be sufficient to just open airline markets to low-cost competition without any regulatory oversight. Since low-cost carriers appear to engage in tacit collusion, some regulatory oversight might still be needed.

Finally, it is also important to realize that although multimarket contact enhances tacit collusive prices for both low cost and high cost carriers, it matters less as their products become more differentiated within and between markets.

There are several research extensions from the multimarket contact essay. Since our empirical analysis is based on the U.S. airline market in the year 2002, one concern of using this dataset is that the U.S. airline market had not yet fully recovered from the 9/11 shock. The airfare impact of multimarket contact consequently might be

overestimated or underestimated. In future research, it would be worthwhile to attempt to estimate the airfare impact of multimarket contact during other time periods.

Another limitation of the current study is that it investigates the competitive effects of multimarket contact only in a static setting. An interesting question remains how multimarket contact affects airlines in making route entry and exit decisions. In other words, it would be of our particular interest to examine whether airlines select routes that enable them to avoid or seek contact with “rival” carriers. An investigation of the competitive effect of multimarket contact in a dynamic setting will also provide an insight into the question: Under what circumstances does multimarket contact contribute to stable or unstable outcomes after new entries.

The second essay in this dissertation focuses on the implementation of lateral transshipments among competing firms. The analytical model developed in this paper investigates the practice of transshipments between two competing stocking locations. It contributes to the existing literature on transshipments in several ways.

First, many previous studies have modeled the performance impacts of environmental uncertainties, such as demand and lead time variability, and their implications for inventory management. The role of competition between firms in affecting the transshipment strategy and profitability outcomes has not received much attention. This paper examines inventory replenishment decisions and the application of the

transshipment strategy in various competitive environments. The results suggest that transshipments may not be cost effective if the firms are operating in an environment that allows consumers to easily switch between firms. In such an environment, firms compete more intensely with one another and consumers have lower loyalty towards firms, both of which result in a high consumer's switching rate.

Second, the analysis incorporates the role that transshipment price plays in reallocating the benefits from transshipments between firms. It is found that the use of an appropriate level of transshipment price is an effective tool for firms to optimize their inventory level decision and maximize the performance improvement from transshipments. In particular, there exists a unique transshipment price that is optimal for both firms when the two firms are identical in market demand and inventory-related cost parameters. However, it is shown that when the two firms are not identical, the smaller firm will prefer a lower transshipment price, and will achieve greater benefits from transshipments.

Third, the consideration of asymmetric firm characteristics into the study of transshipments adds to the previous literature and enriches the managerial implications. The finding that transshipments are likely to provide asymmetric benefits when firms have asymmetric market demands suggests that transshipments actually enable the small firm to take a "free" ride on the great amount of inventory that the large firm holds. Moreover, the opportunity for such a free ride makes it difficult for the two firms having asymmetric demands to reach a common

transshipment price. Under these circumstances, it is necessary to design an effective incentive mechanism (e.g., side payments, flexible transshipment price) to make transshipments pay off for both firms.

In the transshipment essay, both consumer switching rate and product price are considered as exogenous variables. The use of fixed, constant values for these variables makes the analytical solutions tractable and explicable. As an immediate future research, the practice of transshipments can be viewed in a multi-stage sequential game modeling framework. For example, firms make stock level decision in the first stage; the pricing decisions are made in the second stage by firms competing in the same market; and finally, firms decide whether to implement transshipments and what policy to follow in terms of the transshipment volume. In such a three-stage game theoretical model, both the price and inventory level decisions can be thought of as endogenous variables.

On one side, the price of one firm relative to the other's might affect the probability of a consumer's switching firms. On the other side, the inventory level one firm holds relative to the other's might affect the direction and the magnitude of transshipments. By incorporating the consumer's utility function into the classic newsvendor model (see Dana and Petruzzi 2001 as an example), the price decision of firms can be jointly analyzed with their inventory and transshipment decisions from the strategic perspective.

The analysis developed in the transshipment essay relies on a major assumption on the consumer switching behavior. According to this assumption, no consumers switch firms when transshipments are implemented in the event of stockout. However, it would be useful to extend the current model into other more complicated and dynamic situations. For example, consumers might switch before or after the occurrence of transshipments. Since the performance impacts of transshipments are subject to the specification in the sequence of events, it would be important and necessary to examine transshipments in other hypothetical settings. In this aspect, there are great potentials for the simulation work to be developed in quantifying and validating the analytical results.

Furthermore, it would be interesting to explore the implementation of transshipments in an empirical setting. Several interesting hypotheses can be developed and tested by integrating various perspectives in the fields such as operations management, consumer behavior, marketing, and industrial organization economics.

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