ABSTRACT

Title of dissertation: A SUSY SO(10) GUT MODEL WITH LOPSIDED STRUCTURE

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The standard model (SM) of elementary particles has been established for more than 30 years and tested by a large number of experiments. However, because of the naturalness problem of the electroweak symmetry breaking scale and a large number of unexplained parameters in SM, physicists have been looking for a more fundamental theory. Supersymmetry (SUSY) and grand unification are two appealing concepts that have been mostly implemented to build candidates for beyond SM theories. SUSY helps to stabilize the scale of electroweak symmetry breaking, and grand unification embeds the SM gauge groups into larger and more fundamental gauge groups.

Neutrino oscillations, signaling massive neutrinos, are the first direct evidence of beyond SM physics. A tiny neutrino mass can be elegantly explained by the seesaw mechanism. The neutrino masses from this mechanism are of Majorana type and therefore break the $B-L$ (baryon number minus lepton number) symmetry. A favorable framework of studying neutrino masses and oscillations is the SO(10) grand unification theory (GUT) which naturally accommodates a $B-L$ breaking.

The same $B-L$ breaking can also facilitate baryogenesis via a leptogenesis scenario. This provides an interesting correlation between these two pieces of phenomenology. This thesis presents a realistic SUSY SO(10) GUT model with lopsided structure, which generates the correct masses and mixing of neutrinos and produces the right amount of baryon asymmetry.

One of the most characteristic features of this model is the lopsided mass matrices structure. We examine observables in B decays that are sensitive to this structure, and find a specific pattern of predictions that can be used to test this type of models.

A SUSY SO(10) GUT MODEL WITH LOPSIDED STRUCTURE

by

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Chapter 1

INTRODUCTION

1.1 The Standard Model

There have been discovered, so far, four fundamental interactions in nature. They include, in the order of increasing strengths, gravity, weak interaction, electromagnetic interaction, and strong interaction.

The gravity is the earliest studied interaction, yet still the most mysterious one. It caused the apple to drop on Newton's head, and governs the structure of galaxies. It keeps the moon moving around the earth, and the earth around the sun. Its strength is proportional to masses, and therefore the gravitational interaction between microscopic particles is significantly weaker than the other three forces.

Electromagnetic interaction is another interaction that we are familiar with. Most of the physical phenomena in our ordinary life are related to this interaction. It binds electrons to the nuclei to form atoms, and it binds atoms into molecules. All the chemical and biological processes are conducted by this interaction. Our feelings of cold and hot, tastes of sweet and sour, and emotions of happiness and sadness are all electromagnetic signals in our nervous systems. At microscopic levels, this interaction should be quantized, leading to the quantum electrodynamics (QED), a $U(1)_{EM}$ gauge theory, in particle physicists' jargon.

While the formation of an atom is due to the electromagnetic interaction, the formation of the much tinier object — the nucleus — can only be possible by a much stronger force, the strong interaction. It has taken a long time and a series of nontrivial discoveries to realize that the strong interaction among hadrons is only the residual effect of a more fundamental interaction, the quantum chromodynamics (QCD), which is a $SU(3)_C$ gauge theory of more fundamental particles — quarks. The most remarkable properties of QCD are the asymptotic freedom [1] and quark confinement — no free quark found in experiments.

The weak interaction is responsible for the processes like decays, fissions, and fusions of atomic nuclei. Without this interaction, the sun would not be powered and we would face the most serious energy crisis. This interaction is rather unique. First, it was found to break the parity maximally [2] — only left-handed fields participating in the weak interaction. Second, the interactions of QED and QCD are through the exchange of massless gauge bosons: photons and gluons in the case of QED and QCD, respectively. The weak interaction, however, was first proposed as a four-fermion pointlike interaction by E. Fermi [3]. Although it was found later that this pointlike interaction is only a low-energy effective theory and the exchange of vector bosons $W^{\pm}, Z^0$ is indeed involved in the weak interaction, these vector bosons are massive (their heavy masses, in fact, explain why this interaction is so weak). The existence of fundamental massive vector particles in the quantum field theory breaks unitarity and leads to non-renormalizability of the theory, which indicate the ultraviolet inconsistency.

This problem of weak interaction is tightly related to that it is not a gauge theory, where gauge bosons are guaranteed to be massless by gauge symmetries. This suggests that one prospering way of unitarizing the weak interaction and making it renormalizable is to restore the gauge symmetry at high energy. It was realized by P. W. Higgs [4] that although there are Goldstone bosons arising from the spontaneous breaking of continuous global symmetry [5], these Goldstone boson degrees of freedom, in the case of spontaneous breaking of gauge symmetry, are "eaten" by gauge particles and become their longitudinal components. Thus, the spontaneous breaking of gauge symmetry makes gauge particles massive. At high energy, the gauge symmetry is restored and the theory behaves well ultravioletly. This is the Higgs mechanism and has been applied to the case of the weak interaction. The gauge symmetry $SU(2)_L \times U(1)_Y$, where the subscript L denotes the left-handed and the subscript Y denotes the hyper-charge, is assumed to be broken to $U(1)_{EM}$ by the vacuum expectation value (vev) of the Higgs fields. Three generators of the $SU(2)_L \times U(1)_Y$ gauge symmetries are broken, and three corresponding gauge bosons $W^{\pm}, Z^0$ become massive, while the other gauge boson corresponding to the unbroken gauge symmetry $U(1)_{EM}$, i.e. the photon, remains massless. The Dirac masses of fermions which couple left-handed and right-handed components are forbidden before the symmetry breaking since they are not invariant under the gauge group $SU(2)_L$. The spontaneous breaking of the gauge symmetry $SU(2)_L \times U(1)_Y$ also generates fermion masses. In the end, all the fermion masses and the $W^{\pm}, Z^0$ boson masses are proportional to the Higgs vev. This application of Higgs mechanism to the case of weak interaction leads to a unified picture of electromagnetic and weak interaction: they belong to the unbroken and the broken parts of the same gauge groups $SU(2)_L \times U(1)_Y$. This is the main point of the standard model (SM), in which three of the fundamental interactions are described in the framework of gauge theory of three gauge groups $SU(3)_C \times SU(2)_L \times U(1)_Y$. The $SU(3)_C$ describes the strong interaction. The $SU(2)_L \times U(1)_Y$, i. e. the electroweak sector, is the origin of the electromagnetic and weak interactions.

SM is so far the most successful model of elementary particles. Tremendous amount of experimental data have been found to support it. However, although it is renormalizable and hence does not manifestly require a ultraviolet completion, SM still suffers the naturalness problem and has a large number of parameters, and beg for a more fundamental understanding at high energy.

1.2 Problems of the Standard Model

1.2.1 Hierarchy Problem of Standard Model

There are two fundamental scales in the SM. One is represented by the mass of proton, or equivalently Λ_{QCD} — the scale at which the running coupling of QCD [1] becomes non-pertubative, quarks become confined, and chiral symmetry is broken. This scale is generated through dimensional transmutation from a theory without any scale at the first place — QCD is formally a conformal theory with quark masses neglected. This scale, although signalling tremendous difficulties in understanding the strong interaction at low energy, is perfectly a natural one by itself.

The other scale, which is of totally different type, is represented by the mass of fundamental fermions, including both quarks and leptons. This scale is associated with the electroweak symmetry breaking (EWSB). In SM, the EWSB is induced by the Higgs mechanism and the scale of it depends on the Higgs potential. The problem is that the Higgs field is scalar whose quadratic coupling is not protected by any symmetry in SM and therefore its mass is quadratically divergent. In quantum field theory, the quadratic divergence, like the logarithmic divergence, can be renormalized by expressing the Greens functions in terms of physical observables and absorbing the divergence into bare couplings which could be infinite. However, the existence of a physical cut-off, which could be the grand unification scale or the Planck scale, makes this quadratical divergence unnatural. This is because one can only choose bare couplings once to absorb the infinities. A very large physical cut-off, however, requires the theory to be adjusted accordingly after it is already renormalized. This adjustment is to make two large scales cancel with each other with a much smaller difference left. For example, given the grand unification scale of the order of 10^{16} GeV, the fine-tuning required to make a electroweak scale of the order of 100 GeV is one part of 10^{28}!

To solve this problem, there are generally two approaches. One is of the technicolor type which identifies the Higgs particle as a condensate of fermions. In this way, the electroweak scale is generated in the similar way as the Λ_{QCD} is generated. However, although this approach is conceptually appealing, no technicolor model has ever been constructed to produce right fermion masses and mixing.

The other way of solving this fine-tuning problem is to impose some kind of symmetry forbidding the scalar mass. The supersymmetry (SUSY) theory is of this type. The bosonic and fermionic sector are connected by SUSY and therefore the scalar mass is protected from quadratical radiative correction since the corresponding fermion mass is protected from it by chiral symmetry.

Not only does it solve the fine-tuning problem of SM, SUSY, by itself, is conceptually attractive as it offers way to get around [7] the Coleman-Mandula "no-go" theorem [8] which claims that there is no way to extend the Poincaré group to include space-time transformation that connects particle states with different spins. As this is realized by the SUSY algebra, it would be a surprise if nature does not use it.

Besides solving the fine-tuning problem and being conceptually favorable, SUSY is an attractive candidate for beyond SM physics in many other aspects: it provides a dark matter candidate, as the lightest SUSY partner of SM particle (LSP); the local SUSY requires the inclusion of gravity for consistency; the introduction of SUSY partner particles (sparticle) modifies the running of gauge couplings and makes three of them meet at a single point (the grand unification scale), which persuade physicists to believe in both SUSY and grand unification theory (GUT).

1.2.2 Problems with the Gauge Groups of Standard Model

As the breaking of electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ introduces the fine-tuning problem of SM, the SM gauge groups have many unjustified aspects:

1. The hyper-charge Y is quantized.

   The quantum number of an Abelian group, in contrast to that of a non-Abelian group, is not required to be quantized. This can be easily seen by looking at the angular momentum and the momentum as examples. The rotations along different directions do not commute with each other and therefore the angular momentum is quantized. The space-time translations in different directions commute with each other and therefore momentum can take continuous values. However, in the case of hyper-charge, the $U(1)_Y$ is an Abelian group, yet hyper-charge is quantized. This fact indicates that the hyper-charge should have a non-Abelian origin at a more fundamental level.

2. The electron and the proton have exactly the opposite electric charge.

   If the electric charges of a electron and a proton are not exactly opposite to each other, atoms would not be neutral, which would have disastrous consequence. However, the electric charges of particles in SM are due to the specific assignment of hyper charges, which requires a more fundamental explanation.

3. SM is only accidentally anomaly free.

   The anomaly in SM cancels due to the way of the assignment of quantum numbers. It is not due to the nature of the gauge groups of SM. The fact of the accidental anomaly cancellation implies that the SM gauge groups might be embedded in some larger group which is automatically anomaly free because of the group structure. SO(10) group is such an example because it is a real group which is automatically anomaly free.

4. Left and right are different in SM.

   The gauge group $SU(2)_L$ only acts on the left-handed fields. Its right-handed counterpart $SU(2)_R$ is not present in SM. Does nature really prefer left to right or is it only true at low energy? Although it is acceptable in terms of phenomenology, parity breaking is not conceptually satisfying.

5. There are three separate gauge groups instead of one.

   The gauge groups of SM appear to be rather complicated. Is it possible to have a simpler theory at more fundamental level which has only one gauge group and all the fermions are unified into one multiplet of this unification group? As mentioned in the last section, the three gauge couplings do meet at one point at grand unification scale with SUSY. This fact encourages physicists to believe that these three gauge groups are indeed unified at that scale.

6. There are 19 parameters in SM, whose values are fixed by experimental data.

   There are three uncorrelated gauge couplings for SM gauge groups. There are six quark masses and three charged lepton masses. There are four parameters in Cabibbo-Kobayashi-Maskawa (CKM) matrix including three mixing angles and one CP violation phase. In addition, there are two parameters in the Higgs potential: Higgs vev v and the quartic coupling λ. The remaining one parameter is the strong CP violation parameter θ. These 19 parameters could be correlated in a more fundamental theory. In SO(10) GUT, not only are three SM gauge couplings unified into a single one, but also the masses and mixing angles in the quark sector and the lepton sector are correlated, further reducing the number of parameters.

All these problems point in one direction: SM should be embedded into some GUT. There are many versions of GUT, such as $SU(2)_L \times SU(2)_R \times SU(4)_C$ [9],

$SU(5)$ [10], and SO(10) [11]. All of them involve the unification of gauge groups and the unification of quarks and leptons . Taking the SO(10) as example, all the 15 fermions of SM plus the right-handed neutrino are unified into a single 16 dimensional spinor representation; left and right are on the same footing; the anomaly automatically cancels since SO(10) is real; the hyper-charge is the linear combination of I_{R3} and $B - L$ which are both generators of SO(10) and therefore it is quantized and the assignment is not arbitrary.

1.3 Various Places to Probe the Beyond Standard Model Physics

The progress of the theoretical physics can never be driven solely by theory itself. The only way to tell these concepts are relevant is to test them in experiments. There are many ways to probe the beyond SM physics. For instance,

1. One may observe those heavier particles in a more fundamental theory directly in colliders such as the Large Hadron Collider (LHC) which is planed to start to run in 2007.

2. One may see the effects of the beyond SM physics in cosmology, which provides a unique probe of very high energy physics at the early Universe.

3. One may probe the beyond SM physics through processes sensitive to loops, like the penguin-dominated channels in B decays.

4. One may see the signature of beyond SM physics through processes revealing new small violation of accidental symmetries, such as B, L, and $B-L$, of SM.

1.3.1 Neutrino Physics

The neutrino oscillations belong to the fourth type of probes discussed above. They provide the first direct evidence of beyond SM physics in that it violates the accidental symmetry $L_{e,\mu,\tau}$ of SM.

Moreover, the neutrino oscillations imply that neutrinos are massive. The average mass of neutrinos has been constrained from cosmology to be of order 0.05eV or smaller [12], a scale much lower than the masses of other fermions. This tiny neutrino mass can be most naturally explained by the seesaw mechanism [13], indicating the Majorana nature of neutrinos [14]. This brings the violation of another accidental symmetry, $B - L$, of SM at a very high scale that is close to the grand unification scale. The $B - L$, as an accidental symmetry of SM, is anomaly free and therefore can be gauged. (A more precise statement is that $B - L$ is anomaly free as a global symmetry in SM and it is anomaly free as a gauged symmetry with right-handed neutrino added to SM.) In fact, it is one of the gauge symmetries of SO(10) and $SU(2)_L \times SU(2)_R \times SU(4)_C$. When the SO(10) and $SU(2)_L \times SU(2)_R \times SU(4)_C$ are broken to SM, this gauge symmetry is broken spontaneously. As a result, these GUTs provide a natural framework of $B - L$ breaking which is a necessary condition for the seesaw mechanism to work.

Even if one assumes that neutrinos are actually Dirac particles, whose masses are the coupling between left-handed and right-handed Weyl spinors, the fact that there are no right-handed neutrino in SM signifies the involvement of beyond SM physics in the explanation of neutrino masses. moreover, the SO(10) and $SU(2)_L \times$

$SU(2)_R \times SU(4)_C$ GUTs, being left-right symmetric, require the existence of the right-handed neutrino. For these reasons, studying neutrino oscillation in the GUT framework is particularly interesting regardless the Majorana or Dirac nature of neutrinos.

While it is well motivated to study neutrino physics in the GUT framework, it is rather nontrivial to do this in practice. The reason is that the lepton sector and quark sector are correlated in GUT. Like in SM, the fermion masses in GUT are generated from Yukawa interactions. The Yukawa interactions in GUT are among the GUT multipletes, which involve both the quark and the lepton fields. In such a way, the masses of quarks and leptons are related. For example, the SO(10) operator $16_f 16_f 10_H$, where 16_f's are fermion fields and 10_H is a Higgs field, contributes to Dirac masses of all the quarks (including both the up-type and the down-type) and the leptons (including both the charged ones and the neutrinos). Therefore, the masses and mixing of quarks and leptons are dependent on the same set of parameters. There are totally 18 fermion masses and mixing angles that have been measured. They include six quark masses, three charged lepton masses, four CKM elements, three lepton mixing angles (there is so far only an upper limit on the reactor mixing angle θ_{13}), and two neutrino mass-squared difference ΔM_{12}^2 and ΔM_{13}^2. A significant question is whether one can construct a realistic GUT model explaining the masses and mixing in both the quark sector and the lepton sector in terms of less than 18 parameters. If this goal can be successfully achieved, it means some nontrivial GUT relations among the quark and lepton masses and mixing angles are discovered. Given those conceptually attractive properties of

GUT discussed in the previous section, it is an important task to investigate if the unification of quarks and leptons can be realized in practice by constructing realistic GUT models.

1.3.2 Baryogenesis via Leptogenesis

The current observation of the Universe depends on its early history when it was very hot and thus was described by a more fundamental theory at high energy. Because of this, cosmology provides a unique probe of beyond SM physics. One of the puzzles in cosmology is where the baryon asymmetry observed in the Big Bang nucleosynthesis (BBN) and the cosmic microwave background (CMB) comes from.

It was realized long time ago by Sakharov [15] that, in order to produce the baryon asymmetry, three conditions have to be satisfied. They include: (1) baryon number violation; (2) both C and CP violation; (3) out of thermal equilibrium condition.

There are many scenarios of baryogenesis satisfying these three conditions. Initially, it was thought that this can be realized by the baryon number violating decays of heavy particles in GUT [16, 17]. But later, this scenario was ruled out after it was realized that the baryon asymmetry produced in such way would be completely washed out by the in-equilibrium sphaleron process [18, 19].

With the GUT scenario of baryogenesis abandoned, the next candidate is to generate the baryon asymmetry in the sphaleron process itself. Since the out of equilibrium condition has to be satisfied, this baryogenesis, referred as electroweak baryogenesis, can only happen in the bubble wall of the electroweak phase transition. The requirements of strong first order phase transition and enough CP violation rule out this scenario in SM and only leave small possibility in SUSY.

Since the electroweak baryogenesis is difficult, the scenario of baryogenesis via leptogenesis [20] becomes a very attractive candidate. In this scenario, the right-handed neutrinos, whose Majorana masses violate the $B-L$ number, decay and produce lepton number asymmetry. This lepton number asymmetry is converted to baryon number asymmetry by the sphaleron process in such a way that $B-L$ number is conserved. The baryogenesis via leptogenesis scenario requires the $B-L$ breaking at very high energy, which is also a necessary condition for explaining the small neutrino masses in terms of seesaw mechanism. This makes both neutrino physics and baryogenesis via leptogenesis tightly related to the GUT since it provides a natural scheme of $B-L$ breaking.

Due to the connection among the baryogenesis via leptogenesis, neutrino physics, and GUT, it is important to investigate if such a GUT model can be constructed to explain fermion masses and mixing, including the neutrino sector, and at the same time produce right amount of baryon asymmetry.

1.3.3 Low Energy Precision Test of SUSY GUT Model

As mentioned earlier, SUSY helps to realize the gauge coupling unification which encourages physicists to trust both SUSY and GUT. Therefore, when an SO(10) GUT model is constructed, one would like to supersymmetrize it and make it a SUSY GUT model.

While one certainly wishes to see those sparticles directly in the collider, one may also detect their effects indirectly in the quantum loops. These heavy sparticles in loops could induce new flavor violations and CP violations.

In SM, the flavor violation is only in the quark sector and all the CP violations can be explained by a single CP violation phase in the CKM matrix [21]. Going beyond the SM, one expects new flavor and CP violations in the new physics above the electroweak scale M_{EW}. Constraints of lepton flavor violations such as $\mu \to e\gamma$, $\tau \to \mu\gamma$, and $\tau \to e\gamma$ have been established in experiments, with the constraint on the first one ($\mu \to e\gamma$) being the most restrictive. Constraints of various flavor violations and flavor-changing/conserving CP violations in the quark sector have also been established from experiments. Although the hadronic uncertainties make the precision test in quark sector more complicated, there exist some golden channels, such as $B_d \to \phi K_s$ and $B_d \to \eta' K_s$ that attract lots of interest. At quark level, these processes involve $b \to s s \bar{s}$, where the SM contribution starts at the loop level (penguin diagrams) [22]. These penguin dominated processes have attracted a lot of experimental and theoretical efforts in hope to see the signature of beyond SM physics.

With a SUSY GUT model that fits all the fermion masses and mixing and produces right amount of baryon number asymmetry, it would be interesting to investigate its impact on the low-energy precision tests.

1.4 What is the Thesis about

In this thesis, we discuss an SO(10) GUT model with lopsided structure and various phenomenological aspects of it, including the fermion masses and mixing, baryogenesis via leptogenesis, and precision tests in B physics.

In constructing an SO(10) GUT model for fermion masses and mixing, the first problem that one faces is how to produce large neutrino mixing and small quark mixing simultaneously. The lopsided structure of SO(10) solves this problem in an elegant way. However, the realistic SO(10) GUT models based on the lopsided structure in the literature suffer the fine-tuning problem. We construct a new model with lopsided structure avoiding the fine-tuning problem. The new model explains 18 measured fermion masses and mixing angles in terms of 13 parameters. Moreover, being rather different from the previous model with lopsided structure in the literature, our prediction of reactor neutrino mixing θ_{13} is sizable and within the reach of the next generation of reactor neutrino experiments.

In the baryogenesis via leptogenesis scenario, the right-handed neutrino decays and produces the lepton asymmetry which is converted to the baryon asymmetry by the sphaleron process. The lepton asymmetry, however, gets washed out by inverse decays and scattering processes. The washout effect can not be too strong in order for enough lepton asymmetry to be left. We investigate in detail how much lepton number asymmetry can be produced and how much of them gets washed out and find that right baryon asymmetry can be produced in our model.

The lopsided structure which characterizes our model implies that there is a large right-handed down-type quark mixing associated with the large neutrino mixing in the 2-3 sector. This could induce large $b \to s$ transition in the SUSY context. We study observables $S_{\phi K_S}$ and $S_{\eta' K_S}$ which are indirect CP violations in the $B_d \to \phi K_S$ and $B_d \to \eta' K_S$ decays and find that predictions from our model with lopsided structure show a specific pattern. This pattern can be used to differentiate the models with lopsided structure from others.

The thesis is organized as follows. In chapter 2, we discuss the basics of the SUSY and GUT, especially the SO(10) GUT. Chapter 3 is devoted to a detailed discussion of the construction of SO(10) GUT model with lopsided structure. In chapter 4, we present the study of the baryogenesis via leptogenesis in our model. In chapter 5, the investigation of the signature of this SO(10) GUT model in penguin dominated processes in B decays is presented. Chapter 6 presents the conclusion.

Chapter 2

BASICS OF SUPERSYMMETRY AND GRAND UNIFICATION THEORY

2.1 SUSY Basics

SUSY is a symmetry of space-time. It generalizes the Poincaré group to the super-Poincaré group by introducing the SUSY generators Q_α and Q^*_β that satisfy the anticommutation rules

$$\begin{aligned} \{Q_\alpha, Q_\beta\} &= \{Q^*_{\dot\alpha}, Q^*_{\dot\beta}\} = 0, \\ \{Q_\alpha, Q^*_{\dot\beta}\} &= \bar\sigma^\mu_{\alpha\dot\beta}\frac{\partial}{\partial x_\mu}. \end{aligned} \tag{2.1}$$

These generators act like the annihilation operator and creation operator of a one-dimensional fermionic harmonic oscillator. They change the spin of a state by 1/2, thus establishing a connection between bosonic and fermionic degrees of freedom.

The basic rules of writing down the Lagrangian of SUSY invariant action and soft SUSY breaking terms are presented in this section. This part is written without referring to a specific gauge group. The minimal supersymmetric standard model (MSSM) at low energy and the SUSY GUT at high energy follow straightforwardly by identifying the gauge group as those of SM and GUT, respectively.

2.1.1 SUSY Invariance and Soft Breaking

There are two kinds of supermultiplets that are used to construct the SUSY invariant action. One is the chiral supermultiplet and the other is the vector supermultiplet. The SUSY transformation rules tell that the space-time integration of the supermultiplet component with the highest dimension is SUSY invariant and therefore can be used to construct the action. The reason is simply that the SUSY transformation of highest dimensional components has to involve space-time derivative ∂_μ acting on a lower dimensional component, which becomes the surface term after space-time integration and hence vanishes.

The chiral supermultiplet (Wess-Zumino supermultiplet) has the component fields $\phi(x)$, $\psi(x)$, and $F(x)$, which describe a complex scalar, a Weyl spinor, and an auxiliary complex scalar, respectively. These components form the chiral superfield

$$\Phi(x,\theta) = \phi + \theta\psi(x) + \frac{1}{2}\theta\theta F(x), \tag{2.2}$$

where the Grassmann variable θ is introduced. The dimension of this chiral superfield is assigned to be 1 and the dimension of θ is $-\frac{1}{2}$. The dimension of each component is easily computed to be $[\phi] = 1$, $[\psi] = \frac{3}{2}$, and $[F] = 2$. The scalar field $F(x)$ is the auxiliary field needed for the SUSY algebra to close off-shell, i.e. without referring to the equation of motion. In the end, the equation of motion for F always relates it back to the scalar field ϕ.

The product of chiral superfields with opposite chirality $\Phi^*\Phi$ is a real superfield ($(\Phi^*\Phi)^* = \Phi^*\Phi$). The free lagrangian of chiral superfield is obtained by taking the highest dimensional component (the D term) of this real superfield

$$\mathcal{L}_0^{WZ} = \int d^2\bar{\theta} d^2\theta \Phi^*\Phi = -\partial_\mu\phi^*\partial^\mu\phi - i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^*F. \tag{2.3}$$

On the other hand, the products of chiral superfields with the same chirality, such as the terms in the superpotential

$$W = \frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{6}y_{ijk}\Phi_i\Phi_j\Phi_k, \tag{2.4}$$

are still chiral superfields. The lagrangian of SUSY invariant action that describes the Yukawa interactions among chiral superfields is obtained by taking the highest dimensional component (the F term) of the above chiral superfield

$$\mathcal{L}_{Yukawa} = \int d^2\theta W + \text{H.C.}. \tag{2.5}$$

The higher-order terms involving more than three chiral superfields are non-renormalizable operators and are omitted here.

The quadratic term in the superpotential gives

$$\int d^2\theta \Phi_i\Phi_j = \phi_i F_j + \phi_j F_i - \psi_i\psi_j, \tag{2.6}$$

while the cubic term gives

$$\int d^2\theta \Phi_i\Phi_j\Phi_k = \phi_i\phi_j F_k + \phi_i F_j\phi_k + F_i\phi_j\phi_k - \phi_i\psi_j\psi_k - \phi_j\psi_i\psi_k - \phi_k\psi_i\psi_j. \tag{2.7}$$

The kinetic term and the superpotential together yield the equation of motion for the auxiliary field F

$$F_i = -\left(\frac{\partial W}{\partial\phi_i}\right)^*. \tag{2.8}$$

Thus the scalar potential is

$$\begin{aligned} V_F(\phi,\phi^*) &= -\left(F_i^*F_i + F_i^*\left(\frac{\partial W}{\partial\phi_i}\right) + \left(\frac{\partial W}{\partial\phi_i}\right)^* F_i\right) = F_i^*F_i \\ &= m_{ik}^* m^{kj}\phi^{*i}\phi_j + \frac{1}{2}m^{in}y_{jkn}^*\phi_i\phi^{*j}\phi^{*k} + \frac{1}{2}m_{in}^* y^{jkn}\phi^{*i}\phi_j\phi_k \\ &\quad + \frac{1}{4}y^{ijn}y_{kln}^*\phi_i\phi_j\phi^{*k}\phi^{*l}. \end{aligned} \tag{2.9}$$

The vector supermultiplet has the component fields $A^\mu(x)$, $\lambda(x)$, $D(x)$, which describe a 4-vector, a 2 component Weyl spinor, or equivalently, a 4-component Majorana spinor, and an auxiliary scalar, respectively. For simplicity, the Abelian gauge field is taken as an example here, while the extension to the non-Abelian case is straightforward.

In the non-SUSY case, a spin-one massless particle is described by a Lorentz 4-vector, which has non-physical degrees of freedom — the gauge degrees of freedom. Similarly, in the SUSY case, the vector supermultiplet is embedded into a real superfield ($V^* = V$)

$$\begin{aligned} V(x,\theta,\bar\theta) &= A(x) - i(\theta\psi(x) + \bar\theta\psi^\dagger(x)) - i\theta\theta C(x) - i\bar\theta\bar\theta C^*(x) + i\bar\theta\sigma^\mu\theta A_\mu(x) \\ &\quad + \theta\theta\bar\theta\lambda^\dagger(x) + \bar\theta\bar\theta\theta\lambda(x) + \bar\theta\bar\theta\theta\theta D(x), \end{aligned} \tag{2.10}$$

which involves the gauge degrees of freedom $A(x)$, $\psi(x)$, and $C(x)$. These gauge degrees of freedom can be eliminated by choosing the Wess-Zumino gauge ($A(x) = 0$, $\psi(x) = 0$, $C(x) = 0$). As a result, the SUSY transformation on the vector superfield is accompanied by a gauge transformation

$$V(x,\theta,\bar\theta) \to V(x,\theta,\bar\theta) + i(\Lambda(x,\theta) - \Lambda^*(x,\bar\theta)), \tag{2.11}$$

where the $\Lambda(x,\theta)$ is a chiral superfield.

Just like the real superfield can be constructed in terms of chiral superfields, the chiral superfield can also be constructed in terms of vector superfields

$$\mathcal{W}(x,\theta) = -i\lambda(x) + \left[D(x) - \frac{i}{2}\sigma^{\mu\nu}F_{\mu\nu}(x)\right]\theta + \theta\theta\bar{\sigma}^{\mu}\partial_{\mu}\lambda^{\dagger A}(x), \tag{2.12}$$

which transforms under the SUSY transformation as

$$\mathcal{W}(x^{\mu},\theta) \to \mathcal{W}(x^{\mu} + \xi^{\dagger}\sigma^{\mu}\theta, \theta + \xi), \tag{2.13}$$

and is invariant under gauge transformation. This is the generalized gauge field strength, from which the kinetic term of the gauge field can be constructed

$$\mathcal{L}_{gauge} = \frac{1}{4}\left(\int d^2\theta \mathcal{W}\mathcal{W} + \text{H.C.}\right) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda^{\dagger}\sigma^{\mu}\partial_{\mu}\lambda + \frac{1}{2}D^2. \tag{2.14}$$

The interactions between the chiral and vector supermultiplets are obtained by making the kinetic term of the chiral supermultiplet gauge invariant. Under the local gauge transformation

$$\Phi(x,\theta) \to e^{i\xi\Lambda(x,\theta)}\Phi(x,\theta), \tag{2.15}$$

where $\Lambda(x,\theta)$ is a chiral superfield, the kinetic term $\Phi^*\Phi$ is not invariant

$$\Phi^*\Phi \to e^{i\xi(\Lambda-\Lambda^*)}\Phi^*\Phi. \tag{2.16}$$

The gauge invariant kinetic term can be obtained as

$$\begin{aligned} \mathcal{L}_{gauge-int.} &= \int d^2\bar{\theta}d^2\theta\Phi^* e^{gV}\Phi \\ &= -(\mathcal{D}_{\mu}\phi)^*(\mathcal{D}_{\mu}\phi) - i\psi^{\dagger}\bar{\sigma}^{\mu}\mathcal{D}_{\mu}\psi + F^*F \\ &\quad -\sqrt{2}g\left[\phi^*\psi\lambda + \lambda^{\dagger}\psi^{\dagger}\phi\right] + g\phi^*\phi D \end{aligned} \tag{2.17}$$

where $\mathcal{D}_\mu = \partial_\mu + igA_\mu$ is the covariant derivative. The terms in the last line are new in SUSY. The last term, together with the $\frac{1}{2}D^2$ in $\mathcal{L}_{gauge}$, gives the equation of motion of D

$$D = -g\phi^*\phi. \tag{2.18}$$

Thus D term contribution to the scalar potential is

$$V_D(\phi, \phi^*) = -\left(\frac{1}{2}D^2 + g\phi^*\phi D\right) = \frac{1}{2}D^2 = \frac{1}{2}g^2(\phi^*\phi)^2 \tag{2.19}$$

To summarize, the lagrangian of a SUSY invariant action is

$$\begin{aligned} \mathcal{L} &= \frac{1}{4}\left(\int d^2\theta \mathcal{W}\mathcal{W} + \text{H.C.}\right) + \int d^2\bar{\theta} d^2\theta \Phi^\dagger e^{gV}\Phi \\ &+ \int d^2\theta \left[\left(\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{6}y_{ijk}\Phi_i\Phi_j\Phi_k\right) + \text{H.C.}\right], \end{aligned} \tag{2.20}$$

and the scalar potential from this lagrangian is

$$V(\phi, \phi^*) = F^*F + \frac{1}{2}D^2. \tag{2.21}$$

The masses for bosons and fermions in a supermultiplet have to be exactly equal. Such a mass relation is not observed in reality and therefore SUSY has to be broken. In order not to destroy the ultraviolet properties of SUSY that solve the hierarchy problem, the SUSY has to be softly broken. There are many scenarios of SUSY soft breaking and all of them should lead to the soft terms in the low-energy effective Lagrangian

$$\begin{aligned} \mathcal{L}_{soft} &= -\frac{1}{2}(M_\lambda\lambda\lambda + \text{H.C.}) - (m^2)^i_j\phi^{j*}\phi_i \\ &- \left(\frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{H.C.}\right), \end{aligned} \tag{2.22}$$

where the first and second terms in the first line are gaugino masses and scalar masses in the chiral supermultiplet, respectively. These terms create a difference among the masses of different components of a supermultiplet and break SUSY manifestly. The terms in the second line are couplings among scalar components of chiral supermultiplets, which are allowed by gauge invariance if the corresponding terms in the superpotential are allowed.

2.1.2 RG Running in SUSY

In this section, the one-loop renormalization group (RG) equations for the SUSY invariant terms and soft terms are presented. The complete two-loop RG equations can be found in Ref. [23]

The RG equation of the gauge coupling is

$$\frac{d}{dt}g = \frac{1}{16\pi^2}\beta_g, \tag{2.23}$$

with

$$\beta_g = g^3[S(R) - 3C(G)] \tag{2.24}$$

where the Dynkin index $S(R)$ for representation R is

$$\mathrm{Tr}_R(t^A t^B) \equiv S(R)\delta^{AB}, \tag{2.25}$$

and $C(G)$ is the Casimir invariant $C(R)$ of the adjoint representation. The Casimir invariant $C(R)$ for the representation R is

$$(t^A t^A)_i^j \equiv C(R)\delta_i^j. \tag{2.26}$$

Specialized to the adjoint representation, one has $C(G)\delta^{AB} = f^{ACD} f^{BCD}$ with f^{ABC} the structure constant of the group.

The RG equation of the gaugino mass is

$$\frac{d}{dt}M = \frac{1}{16\pi^2}\beta_M \tag{2.27}$$

with

$$\beta_M = g^2[2S(R) - 6C(G)]M. \tag{2.28}$$

The superpotential is not renormalized due to the holomorphicity. The running of Yukawa coupling is due to the wave-function renormalization of the chiral superfields, which gives rise to the anomalous dimensions

$$\frac{d}{dt}Y^{ijk} = \frac{1}{16\pi^2}Y^{ijp}\gamma_p^k + (k \leftrightarrow i) + (k \leftrightarrow j), \tag{2.29}$$

where the anomalous dimension

$$\gamma_p^k = \frac{1}{2}Y_{ipq}Y^{ipq} - 2\delta_i^j g^2 C(i). \tag{2.30}$$

The RG equation of the soft trilinear term a^{ijk} is

$$\frac{d}{dt}a^{ijk} = \frac{1}{16\pi^2}\beta_a{}^{ijk}, \tag{2.31}$$

where

$$\begin{aligned} \beta_a{}^{ijk} &= \frac{1}{2}a^{ijl}Y_{lmn}Y^{mnk} + Y^{ijl}Y_{lmn}a^{mnk} \\ &\quad -2(a^{ijk} - 2MY^{ijk})g^2C(k) + (k \leftrightarrow i) + (k \leftrightarrow j). \end{aligned} \tag{2.32}$$

The RG equation of the soft scalar mass is

$$\frac{d}{dt}(m^2)_i^j = \frac{1}{16\pi^2}\beta_{m^2}{}_i^j, \tag{2.33}$$

where

$$\begin{aligned}\beta_{m^2{}_i^j} &= \frac{1}{2}Y_{ipq}Y^{pqn}(m^2)_n^j + \frac{1}{2}Y^{ipq}Y_{pqn}(m^2)_i^n + 2Y_{ipq}Y^{jpr}(m^2)_r^q \\ &+a_{ipq}a^{jpq} - 8\delta_i^j MM^\dagger g^2 C(i) + 2g^2 t^{Aj}_{\ i}\mathrm{Tr}[t^A m^2]. \qquad (2.34)\end{aligned}$$

2.2 GUT Basics

As briefly discussed in Chapter 1, the problems of SM gauge groups imply that SM should be embedded into GUT. The first attempt in this direction was made by Pati and Salam [9] in their $SU(2)_L \times SU(2)_R \times SU(4)_C$ model (G(224) model), where quarks and leptons are unified — a lepton is the fourth color of a quark. The discovery of neutrino oscillation strongly supports this type of extension of SM since the right-handed neutrino, a singlet of SM, has to be present in G(224) model to form a complete representation, together with the right-handed charge leptons and the right-handed quarks. This model, although has both $SU(2)_L$ and $SU(2)_R$ gauge groups, does not have to be left-right symmetric since the gauge couplings g_L and g_R could be different. It was proposed in Refs. [24, 25, 26] to make this model left-right symmetric by requiring the couplings of $SU(2)_L$ and $SU(2)_R$ to be equal, $g_L = g_R$. In this way, there are only two gauge couplings in the left-right symmetric G(224) model.

Another model was proposed by Georgi and Glashow [10] in the same year. This model unifies three gauge groups of the SM into a single rank-4 group SU(5) and therefore there is only one gauge coupling. However, the left-handed and right-handed fields are intrinsically different in this type of grand unification models and hence parity cannot be a good symmetry at GUT scale. Another disadvantage of the SU(5) model is that, unlike the G(224) model, the right-handed neutrino, as a singlet of SU(5), does not occur naturally.

It was first noted by Georgi, Fritzsch, and Minkowski [11] that the SO(10) can serve as the grand unification group. The groups of both the G(224) model and the SU(5) model are subgroups of SO(10), and therefore, the SO(10) model exhibits nice features of both models. Moreover, the SO(10) goes beyond them in the following aspects:

1. The SO(10) model is automatically parity-conserving in the gauge interaction sector, whereas G(224) model can only have this as an assumption and SU(5) model does not conserve parity.

2. In the SO(10) model, all 16 fermions, including the right-handed neutrino, are embedded into a single 16-dimensional spinor representation, whereas in G(224) model, the fermions are divided into left and right sectors, and in SU(5) model, 16 fermions belong to 1, $\overline{5}$, and 10 representations.

3. The SO(10) group, being a real group, is automatically anomaly free.

4. Because of the single gauge group, the SO(10) model imposes more constraints on the quark and lepton masses and mixing, and therefore is more predictive than the G(224) and the SU(5) models.

In the following, we first briefly discuss the G(224) and SU(5) models, and then focus on the SO(10) model in more details.

2.2.1 Left-right Symmetric Models G(224) and G(221)

The $G(224) = SU(2)_L \times SU(2)_R \times SU(4)_C$ model is the first that assumes the existence of right-handed neutrinos. Two key ideas of this model purely based on aesthetic reasons are:

1. the quark-lepton unification based on extending the $SU(3)_C$ of SM to $SU(4)_C$ which has the lepton as the fourth color of the quark;

2. the left-right symmetry based on extending the $SU(2)_L$ in SM to $SU(2)_L \times SU(2)_R$. (The exact left-right symmetry relies on the equality of two gauge couplings $g_L = g_R$).

To realize the above two ideas, the right-handed neutrino has to be postulated since it fits together with the three right-handed up-type quarks to form a complete multiplet of $SU(4)_C$ and it fits together with the right-handed charged lepton to form a doublet of $SU(2)_R$.

By including the right-handed neutrino, 15 fermions in each family (we take the first family as example here) are extended to 16 fermions which form two complete representations of G(224):

$$F_{L,R} = \begin{pmatrix} u_1 & u_2 & u_3 & \nu_e \\ d_1 & d_2 & d_3 & e \end{pmatrix}_{L,R} \tag{2.35}$$

where the subscripts $1 \sim 3$ denote three colors. F_L and F_R transform as (2,1,4) and (1,2,4), respectively, under G(224).

The G(224) model provides a natural explanation for the hyper-charge $Y = I_{3R} + (B - L)/2$, which is assigned arbitrarily in SM. The hyper-charge is naturally quantized since I_{3R} and $B-L$ are generators of non-Abelian groups $SU(2)_R$ and $SU(4)_C$, respectively. The electric charge $Q_{EM} = I_{3L}+Y = I_{3L}+I_{3R}+(B-L)/2$ is completely fixed and those of the proton and electron are guaranteed to be opposite in this assignment.

Alternatively, one could extend the SM to becoming left-right symmetric without quark-lepton unification. The left-right symmetric model $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ (G(221)) [24, 25, 26] was proposed in this spirit. The G(221) model is the minimal model that restores left-right symmetry. Probing the physics associated with the right-handed sector has drawn a lot of effort [27] since the model was proposed.

In G(221) model, fermions are:

$$\begin{aligned} l_L &\equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L : (2,1,-1); \qquad & l_R \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_R : (1,2,-1); \\ q_L^{1,2,3} &\equiv \begin{pmatrix} u \\ d \end{pmatrix}_L^{1,2,3} : (2,1,\frac{1}{3}); \qquad & q_R^{1,2,3} \equiv \begin{pmatrix} u \\ d \end{pmatrix}_R^{1,2,3} : (1,2,\frac{1}{3}); \end{aligned} \tag{2.36}$$

where the representation labels refer to $(SU(2)_L, SU(2)_R, U(1)_{B-L})$. It is easy to check that the assignment of $U(1)$ charge as the $B-L$ number, -1 for leptons and $1/3$ for quarks, gives exactly the correct hyper charge in SM. Here the right-handed neutrino is required not only because it is a component of $SU(2)_R$ doublet, but also because the $B-L$ gauge symmetry of G(221) must be anomaly free.

The Higgs sector, in the simplest form, includes the bi-doublet $\phi = (2, 2^*, 0)$ and the triplets $\Delta_R = (1, 3, 2)$ and $\Delta_l = (3, 1, 2)$. The bi-doublet transforms as

$$\phi \to U_L \phi U_R^\dagger, \tag{2.37}$$

and the triplets transform as

$$\begin{aligned} \Delta_R &\to U_R \Delta_R U_R^\dagger, \\ \Delta_L &\to U_L \Delta_L U_L^\dagger. \end{aligned} \tag{2.38}$$

The explicit forms of ϕ and Δ_R are

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \tag{2.39}$$

and

$$\Delta_R = \begin{pmatrix} \Delta_R^+/\sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 & \Delta_R^+/\sqrt{2} \end{pmatrix}, \tag{2.40}$$

with the superscript denoting the electric charge $Q_{EM} = I_{3L} + I_{3R} + (B - L)/2$.

Since the electric charge is not spontaneously broken, the vev of the bi-doublet has to be

$$\langle \phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}, \tag{2.41}$$

and the vev of the triplet is

$$\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ V & 0 \end{pmatrix}. \tag{2.42}$$

The $\langle \Delta_R \rangle$ breaks the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to $SU(2)_L \times U(1)_Y$, which is further broken to $U(1)_{EM}$ by the vev of ϕ.

The left-right symmetric model G(221) is the minimal model that introduces the right-handed neutrino and realizes the seesaw mechanism naturally. This can be seen straightforwardly from the Yukawa couplings

$$\mathcal{L} = g\bar{l}_R\phi l_L + g'\bar{l}_R\tilde{\phi}l_L + f\bar{q}_R\phi q_L + f'\bar{q}_R\tilde{\phi}q_L + h\tilde{l}^c_R\Delta_R l_R + \text{H.C.} \tag{2.43}$$

where

$$\tilde{\phi} \equiv \tau_2\phi^*\tau_2 = \begin{pmatrix} \phi_2^{0*} & -\phi_2^+ \\ -\phi_2^- & \phi_1^{0*} \end{pmatrix} \tag{2.44}$$

and

$$\tilde{l}^c_R \equiv \overline{l^c_R}i\tau_2 = \begin{pmatrix} \overline{e^c_R} & -\overline{\nu^c_R} \end{pmatrix} \tag{2.45}$$

The vev of Δ_R, which breaks both $SU(2)_R$ and $U(1)_{B-L}$, induces a Majorana mass term for the right-handed neutrino:

$$\mathcal{L}_{Majorana} = hV(\overline{\nu^c_R}\nu_R + \text{H.C.}). \tag{2.46}$$

On the other hand, the neutrino Dirac mass is induced from the vev of ϕ

$$\mathcal{L}_{Dirac} = (gv_1 + g'v_2^*)\bar{\nu}_R\nu_L + \text{H.C.}, \tag{2.47}$$

which is the scale of the Dirac masses of other fermions.

In terms of Majorana fields $\nu = \nu_L + \nu^c_L$ and $N = \nu_R + \nu^c_R$, the Dirac and Majorana masses can be put together in the following form:

$$\mathcal{L} = \frac{1}{2}\begin{pmatrix} \overline{\nu} & \overline{N} \end{pmatrix} \begin{pmatrix} 0 & (gv_1 + g'v_2^*) \\ (gv_1 + g'v_2^*) & hV \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix} + \text{H.C..} \tag{2.48}$$

Assuming the $SU(2)_R$ and $U(1)_{B-L}$ are broken at very high energy scale $V \gg v_1, v_2$, the two mass eigenvalues are

$$\begin{aligned} m_1 &= -(gv_1 + g'v_2^*)^2/(hV); \\ m_2 &= hV. \end{aligned} \tag{2.49}$$

As a result, one of the two Majorana neutrinos is very heavy and the other is highly suppressed by the ratio $(gv_1 + g'v_2^*)/(hV)$. This suppression provides a natural explanation why the neutrino mass is so light compared with charged leptons and quarks. This is called the seesaw mechanism. We omit the discussion of the type II seesaw mechanism, in which the vev $\langle \Delta_L \rangle$ is suppressed by $\langle \phi \rangle / \langle \Delta_R \rangle$ according to the Higgs potential.

2.2.2 SU(5) Model

In contrast to the left-right symmetric models, the SU(5) model goes in the direction of gauge group unification. It is the minimal semisimple group that unifies the three gauge groups of SM.

The $8+3+1 = 12$ generators in SM are embedded into the $(N^2-1)|_{N=5} = 24$ generators λ_i ($i = 1 \sim 24$) of $SU(5)$, which are traceless 5×5 hermitian matrices. In a commonly used convention, the first 8 generators λ_i ($i = 1 \sim 8$) are identified as the generators of $SU(3)_C$ (Gell-Mann matrices), the $\lambda_{21,22,23}$ are identified as generators of $SU(2)_L$ (Pauli matrices), the λ_{24} is identified as the SM hyper-charge, which takes the following form:

$$\lambda_{24} = \frac{2}{\sqrt{15}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix}. \tag{2.50}$$

The remaining 12 generators are

$$\lambda_9 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \lambda_{10} = \begin{pmatrix} 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \lambda_{12} = \begin{pmatrix} 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{13} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \lambda_{14} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{15} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad \lambda_{16} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{17} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \lambda_{18} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \\ 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{19} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \qquad \lambda_{20} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \end{pmatrix} \qquad (2.51)$$

The 15 fermions in SM are in representations $\bar{5}$ and 10 of SU(5):

$$\psi_{\bar{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix}_L ; \psi_{10} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}_L \tag{2.52}$$

It is straightforward to see why 15 fermions are embedded in such a way by looking at the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ contents of $\bar{5}$ and 10. Since 5 is identified as $(3, 0) + (0, 2)$ under $SU(3)_C \times SU(2)_L$, we know that

$$\bar{5} = (\bar{3}, 0) + (0, \bar{2}) \tag{2.53}$$

where

$$\bar{3} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \tag{2.54}$$

(the reason that it is the d^c instead of u^c is obvious by looking at the SM hypercharge) and

$$\bar{2} = i\tau_2 \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} e_L \\ -\nu_L \end{pmatrix}. \tag{2.55}$$

Similarly the 10 of $SU(5)$ can be obtained as antisymmetric tensor of two 5's. It is useful to use the notation $5 = \psi^\alpha$, $\bar{5} = \psi_\alpha$ with $\alpha = 1 \sim 5$. In the explicit form of the generators, it is easy to see that the first three indices $\alpha = 1, 2, 3$ are the color indices under $SU(3)_C$, while the last two indices $\alpha = 4, 5$ are the indices under $SU(2)_L$. Thus we have $10 = \psi^{\{\alpha\beta\}}$, where the $\{\alpha\beta\}$ means the α and β are anti-symmetrized. From different combinations of α and β, it is easy to assign the quarks and leptons in the specific form shown in Eq. (2.52).

In the minimal scenario, the SU(5) is broken to $SU(3)_C \times SU(2)_L \times U(1)_Y$ by the vev of Higgs field Σ_{24} of the adjoint representation, which should be a singlet of $SU(3)_C \times SU(2)_L$ and carry no $U(1)_Y$ hyper-charge. The only choice is

$$\langle \Sigma_{24} \rangle = \begin{pmatrix} 2/3 & 0 & 0 & 0 & 0 \\ 0 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} V. \tag{2.56}$$

The breaking of the SM group is induced by the vev of fundamental Higgs field H_5

$$\langle H_5 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}. \tag{2.57}$$

In the above minimal Higgs scenario, the fermion masses are generated from the Yukawa couplings with H_5

$$\mathcal{L} = f^{ij} (\psi_{10}^{\alpha\beta})_i (\psi_{\bar{5}\alpha})_j H_{5\beta}^* + h^{ij} \epsilon_{\alpha\beta\gamma\delta\rho} (\psi_{10}^{\alpha\beta})_i (\psi_{10}^{\gamma\delta})_j H_5^\rho, \tag{2.58}$$

where the $i, j = 1, 2, 3$ are family indices and the $\alpha, \beta, \gamma, \delta, \rho$ are $SU(5)$ indices. While the second Yukawa coupling h^{ij} only gives the masses of up quarks, the first Yukawa coupling f^{ij} gives masses to both the down-type quarks and the charged leptons. In fact, f^{ij} yields equal masses for down-type quarks and charged leptons at the GUT scale

$$
\begin{aligned}
m_e &= m_d, \\
m_\mu &= m_s, \\
m_\tau &= m_b.
\end{aligned}
\tag{2.59}
$$

Including the running effect, the last relation, referred as the $b-\tau$ unification, is well satisfied. However, the first two equations are badly violated by the experimental data. Therefore, the fermion masses require an extended Higgs sector, which is also demanded by the proton decay experiments, since the minimal SU(5) model predicts a decay rate larger than the experiment bound [28].

The right-handed neutrino, a singlet of SU(5), needs not to be present in the SU(5) model. One must extend SU(5) to $SU(5) \times U(1)_X$ in order to have the right-handed neutrino as a necessary element of the model. The anomaly free conditions for $U(1)_X^3$ and $SU(5)^2 U(1)_X$ triangle diagrams require the introduction of singlet of SU(5) with the $U(1)_X$ charge assigned to be +5. A linear combination of $U(1)_X$ and a diagonal generator of $SU(5)$ gives the $B-L$. The breaking of $U(1)_{B-L}$ induces a Majorana mass term for the right-handed neutrino. While the $U(1)_{B-L}$ global symmetry is not violated anomalously in SM, the $U(1)_{B-L}$ gauge symmetry is anomaly free only with the right-handed neutrino present. Therefore, the introduction of right-handed neutrino is intrinsically related to the $U(1)_{B-L}$ gauge symmetry. In $SU(5) \times U(1)_X$ model, $U(1)_{B-L}$ is introduced as combination of $U(1)_X$ and a diagonal $SU(5)$ generators. In contrast, in the left-right symmetric models discussed in the last section and the SO(10) model in the next section, the $U(1)_{B-L}$ gauge symmetry is naturally introduced in the first place.

2.2.3 SO(10) GUT

The SO(10) group is a rank 5 group, while the rank of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ is 4. The extra diagonal generator of SO(10) is the $B-L$, whose breaking is tightly related to neutrino masses. The $B-L$ breaking must be realized when the SO(10) is broken to SM. Thus, the SO(10) model provides natural framework of studying neutrino masses. On the other hand, since the SO(10) group contains the G(224) group and the SU(5) group as the maximal subgroup, the SO(10) model provides much stronger constraints on the relations of masses and mixing of quarks and leptons. This makes it highly nontrivial to build realistic SO(10) models. We will postpone the discussion of building realistic SO(10) models to the next Chapter, and focus on the basic aspects of SO(10) in this section.

We follow the Refs. [29, 30] for this part of discussion.

2.2.3.1 Clifford Algebra

Starting with a set of operators χ_i and $\chi_i^\dagger$ with $i = 1 \sim N$ that satisfy the anticommutation relations:

$$\begin{aligned} \{\chi_i, \chi_j^\dagger\} &= \delta_{ij}, \\ \{\chi_i, \chi_j\} &= 0, \end{aligned} \tag{2.60}$$

it can be shown that the operators T^i_j defined as

$$T^i_j = \chi^\dagger_i \chi_j \tag{2.61}$$

satisfy the algebra of $U(N)$ group

$$[T^i_j, T^k_l] = \delta^k_j T^i_l - \delta^i_l T^k_j. \tag{2.62}$$

On the other hand, the operator Γ_μ with $\mu = 1 \sim 2N$ defined as

$$\begin{aligned} \Gamma_{2j-1} &= -i(\chi_j - \chi^\dagger_j), \\ \Gamma_{2j} &= (\chi_j + \chi^\dagger_j), \qquad j = 1 \sim N, \end{aligned} \tag{2.63}$$

satisfy the anticommutation rule

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu} \tag{2.64}$$

and thus form a Clifford algebra. The generators of the SO(2N) group can be constructed in terms of Γ's

$$\Sigma_{\mu\nu} = \frac{1}{2i}[\Gamma_\mu, \Gamma_\nu]. \tag{2.65}$$

The $\Sigma_{\mu\nu}$ can also be written in terms of χ_i and $\chi^\dagger_i$

$$\begin{aligned} \Sigma_{2j-1,2k-1} &= \frac{1}{2i}[\chi_j, \chi^\dagger_k] - \frac{1}{2i}[\chi_k, \chi^\dagger_j] + i(\chi_j\chi_k + \chi^\dagger_j\chi^\dagger_k), \\ \Sigma_{2j,2k-1} &= \frac{1}{2}[\chi_j, \chi^\dagger_k] + \frac{1}{2}[\chi_k, \chi^\dagger_j] - (\chi_j\chi_k - \chi^\dagger_j\chi^\dagger_k), \\ \Sigma_{2j,2k} &= \frac{1}{2i}[\chi_j, \chi^\dagger_k] - \frac{1}{2i}[\chi_k, \chi^\dagger_j] - i(\chi_j\chi_k + \chi^\dagger_j\chi^\dagger_k). \end{aligned} \tag{2.66}$$

2.2.3.2 Spinor Representation

While the tensor representations of SO(2N) group are simple to construct, the spinor representations are more complicated. The easiest way to obtain the spinor representations is to explicitly construct it for the Γ's in the Clifford algebra [30]. This can be done by iterative construction. For N=1, the Γ matrices can be chosen as

$$\Gamma_1^{(N=1)} = \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$\Gamma_2^{(N=1)} = \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{2.67}$$

To go from N to N+1, the $\Gamma^{(N+1)}$'s are constructed as

$$\Gamma_i^{N+1} = \begin{pmatrix} \Gamma_i^{(N)} & 0 \\ 0 & -\Gamma_i^{(N)} \end{pmatrix} \quad \text{for} \quad i = 1 \sim 2N,$$
$$\Gamma_{2N+1}^{N+1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\Gamma_{2N+2}^{N+1} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{2.68}$$

The generators of SO(2N) defined in terms of Γ's as in Eq. (2.65) are obtained correspondingly as

$$\Sigma_{ij}^{(N+1)} = \begin{pmatrix} \Sigma_{ij}^{(N)} & 0 \\ 0 & \Sigma_{ij}^{(N)} \end{pmatrix}, \quad i,j = 1-2N$$

$$\Sigma_{i,2N+1}^{(N+1)} = i \begin{pmatrix} 0 & \Gamma_i^{(N)} \\ -\Gamma_i^{(N)} & 0 \end{pmatrix}, \quad i = 1-2N$$

$$\Sigma_{i,2N+2}^{(N+1)} = \begin{pmatrix} 0 & \Gamma_i^{(N)} \\ \Gamma_i^{(N)} & 0 \end{pmatrix}, \quad i = 1-2N$$

$$\Sigma_{2N+1,2N+2}^{(N+1)} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.69}$$

By constructing the $2^N \times 2^N$ matrices for generators of $SO(2N)$ group, the spinor representation of dimension 2^N is explicitly constructed. Since the Γ_{FIVE} matrix constructed as

$$\Gamma_{\mathrm{FIVE}} = (-i)^N(\Gamma_1\Gamma_2...\Gamma_{2N}) \tag{2.70}$$

anticommutes with all the Γ's, the 2^N dimensional spinor representation of SO(2N) group is reducible. To recover irreducible representations, one can apply the operators $(1+\Gamma_{\mathrm{FIVE}})$ and $(1-\Gamma_{\mathrm{FIVE}})$ to project out the "right-handed" and"left-handed" spinor representations, respectively. In the following, we denote the s-dimensional right-handed spinor by s^+, and s-dimensional left-handed spinor by s^-.

In order to obtain the invariant by coupling two spinors, it is necessary to introduce the charge-conjugation operator. Under the group transformation, the spinors transform as

$$\begin{aligned}\delta\psi &= i\epsilon_{\mu\nu}\Sigma_{\mu\nu}\psi, \\ \delta\psi^\dagger &= -i\epsilon_{\mu\nu}\psi^\dagger\Sigma_{\mu\nu}, \\ \delta\psi^T &= i\epsilon_{\mu\nu}\psi^T\Sigma^T_{\mu\nu}. \end{aligned} \tag{2.71}$$

Thus, the ψ^T does not transform as the conjugate spinor of SO(2N). The charge-conjugation operator B of SO(2N) group is defined to satisfy

$$B^{-1}\Sigma^T_{\mu\nu}B = -\Sigma_{\mu\nu}, \tag{2.72}$$

thus

$$\delta(\psi^T B) = -i\epsilon_{\mu\nu}(\psi^T B)\Sigma_{\mu\nu}. \tag{2.73}$$

With B, the invariant of SO(2N) can be made in terms of ψB and ψ, such as $\psi BC\Gamma_\mu\psi\phi^\mu$, $\psi BC\Gamma_{\mu\nu\tau}\psi\phi^{\mu\nu\tau}$, where ϕ^μ and $\phi^{\mu\nu\tau}$ are tensor representations of SO(2N) and C is the charge-conjugation operator of Lorentz spinors. The simplest example $\psi BC\psi$, although invariant under SO(2N), is identically zero.

It is often helpful to construct the space of 2^N-dimensional spinor representation as the direct product of N 2-dimensional spinors, $|\epsilon_1\epsilon_2...\epsilon_N\rangle$, where $\epsilon_i = \pm$. The Γ matrices are then

$$\begin{aligned}\Gamma_{2k-1} &= 1\times 1\times 1...\times\tau_1\times\tau_3\times\tau_3...\times\tau_3 \\ \Gamma_{2k} &= 1\times 1\times 1...\times\tau_2\times\tau_3\times\tau_3...\times\tau_3 \end{aligned} \tag{2.74}$$

where 1 denotes the unit matrix and it appears $k-1$ times and τ_3 appears $N-k$ times. The Γ_{FIVE} matrix takes the form

$$\Gamma_{\mathrm{FIVE}} = \tau_3\times\tau_3...\times\tau_3 \tag{2.75}$$

with τ_3 appearing N times. Obviously, the Γ_{FIVE} acting on the spinor $|\epsilon_1\epsilon_2...\epsilon_N\rangle$ gives

$$\Gamma_{\mathrm{FIVE}}|\epsilon_1\epsilon_2...\epsilon_N\rangle = \left(\prod_{i=1}^{N}\epsilon_i\right)|\epsilon_1\epsilon_2...\epsilon_N\rangle. \tag{2.76}$$

Thus $\prod_{i=1}^{N}\epsilon_i = +1$ for the right-handed spinor and $\prod_{i=1}^{N}\epsilon_i = -1$ for the left-handed spinor. Taking the $N = 5$ case as an example, the $|+++++\rangle$ is a right-handed spinor and $|-----\rangle$ is a left-handed spinor. Flipping even number of signs of ϵ of a spinor leads to one with the same chirality, while flipping odd number of signs leads to a spinor with opposite chirality.

In this explicit notation, the charge-conjugation operator takes the form

$$B = i\tau_2 \times i\tau_2 \times ... \times i\tau_2, \tag{2.77}$$

with N $i\tau_2$ matrices. Obviously, the charge-conjugation operator acting on the spinor $|\epsilon_1\epsilon_2...\epsilon_N\rangle$ flips all the signs of ϵ's. For SO(2N) with even N, the left-handed and right-handed spinors are self-conjugate. For odd N, like SO(10), the left-handed and right-handed spinors are conjugate of each other.

2.2.3.3 How SU(N) is Embedded into SO(2N)

It is quite instructive to study how the SU(N) is embedded into SO(2N), since the SM groups are all unitary.

The U(N) group involves transformations on N-dimensional complex vectors a and b with the inner product $\Sigma_{i=1}^{N} b_i^* a_i$ invariant. Each N-dimensional complex vector can be decomposed as two N-dimensional real vectors, $a_j = x_j + iy_j$, $b_j = x'_j + iy'_j$. The transformation of U(N) group leaves both real and imaginary part of $\Sigma_{i=1}^{N} b_i^* a_i$

invariant. Therefore one has both

$$\Sigma_{j=1}^{N}(x_j' x_j + y_j' y_j), \tag{2.78}$$

and

$$\Sigma_{j=1}^{N}(x_j' y_j - y_j' x_j), \tag{2.79}$$

invariants. On the other hand, the two N-dimensional real vectors can be combined together to obtain one 2N-dimensional real vector

$$v = \begin{pmatrix} x \\ \\ y \end{pmatrix}. \tag{2.80}$$

The SO(2N) group is defined to have the invariant inner product

$$\Sigma_{j=1}^{N}(x_j' x_j + y_j' y_j). \tag{2.81}$$

Comparing Eq. (2.78), Eq. (2.79) and Eq. (2.81), it is obvious that the U(N) group is more restricted than the SO(2N) group. Thus it is sensible to consider embedding of the U(N) group in the SO(2N) group. Moreover, the 2N-dimensional vector representation of SO(2N) can be decomposed into N and $\bar{N}$ of SU(N) which correspond to $x + iy$ and $x - iy$, respectively.

Γ_μ forms a vector representation of SO(2N) and the linear combinations χ and $\chi^\dagger$ are N and $\overline{N}$ of SU(N), respectively. The $N(2N-1)$ generators of SO(2N) are antisymmetrized combinations $((N+\overline{N}) \times (N+\overline{N}))_A$. Taking the N=5 case as example, the antisymmetrized combinations $(5 \times 5)_A$ and $(\bar{5} \times \bar{5})_A$ are 10 and $\overline{10}$ under SU(5), respectively; the antisymmetrized combination $(5 \times \bar{5})_A$ decomposes to 1 (singlet) and 24 (adjoint representation) under SU(5). In terms of χ_i and $\chi_i^\dagger$, the 10 and $\overline{10}$ are $\chi_i\chi_j$ and $\chi_i^\dagger\chi_j^\dagger$ $(i \neq j)$, respectively; 1 is $\Sigma_{i=1}^5\chi_i^\dagger\chi_i$ and 24 is $(\chi_i^\dagger\chi_j - \text{trace})$. The singlet is the generator of $U(1)_X$ in the reduction $SO(10) \rightarrow SU(5) \times U(1)_X$.

This decomposition of SO(2N) generators to SU(N) multiplets helps to understand the decomposition of SO(2N) spinor to multiplets of SU(N). The generators of $U(N)$ have one raising and one lowering operators. Besides these, the other generators of SO(2N) have two lowering or two raising operators. This means that the components of a SU(N) multiplet have the same number of + and −. For example, the $|++--+\rangle$ and $|++-+-\rangle$ are components of the same SU(5) multiplet. The SO(2N) generators that do not belong to SU(N) change the number of " + " and " − " by 2. Denoting an SU(N) multiplet by the number of " − ", k, as $[k]$, we get the following decomposition of right-handed and left-handed spinors

$$
\begin{aligned}
s^+ &\rightarrow [0] + [2] + [4] + ... \quad , \\
s^- &\rightarrow [1] + [3] + [5] + ... \quad .
\end{aligned}
\tag{2.82}
$$

For example, the 16-dimensional left-handed spinor of SO(10) is the sum of 1, 10, and $\bar{5}$ in SU(5), which corresponds to [5], [3], and [1], respectively.

2.2.3.4 Identify Fermions as Spinors

The 16 fermions, including the 15 of SM and the right-handed neutrino, form the 16-dimensional spinor representation of SO(10). In the following, we discuss how these 16 fermions are identified as components of the 16-dimensional spinor of SO(10).

Since we know the quantum numbers of each fermion under the gauge groups of

SM, we shall study how the components of the spinor carry these quantum numbers of SM. The SO(10) can be decomposed into the product of two orthogonal groups $SO(6) \times SO(4)$. We choose the convention of letting the SO(6) acting on the first 6 vector indices and SO(4) acting on the last 4 vector indices. The $SU(3)_C$ is embedded into SO(6) and the $SU(2)_L$ into $SO(4)$.

For SO(6), the 2^2-dimensional right-handed spinor 4^+ consists of $|+++\rangle$, $|+--\rangle$, $|-+-\rangle$, and $|--+\rangle$. Under $SU(3)_C$, it decomposes as

$$4^+ \rightarrow [0] + [2] = 1 + \bar{3} \tag{2.83}$$

where 1 and $\bar{3}$ are the singlet and the conjugate of the fundamental representation of $SU(3)_C$, respectively. Obviously, the singlet 1 is $|+++\rangle$, and the $\bar{3}$ consists of $|+--\rangle$, $|-+-\rangle$, and $|--+\rangle$. The left-handed spinor 4^- consists of $|---\rangle$, $|++-\rangle$, $|+-+\rangle$, and $|-++\rangle$. Under $SU(3)_C$, it decomposes as

$$4^- \rightarrow [3] + [1] = 1 + 3 \tag{2.84}$$

where 1 and 3 are the singlet and the fundamental representations of $SU(3)_C$, respectively. Obviously, the singlet 1 is $|---\rangle$, and the fundamental representation 3 consists of $|++-\rangle$, $|+-+\rangle$, and $|-++\rangle$. As expected, the charge-conjugation operator B which flips all the $\pm$ signs connects 4^+ and 4^-.

On the other hand, the decomposition of SO(4) spinor into $SU(2)_L$ representations is more subtle than the case of SO(6). It is well know that there are two ways of embedding $SU(2)$ into SO(4). One is the "magnetic minus electric"

$$\tau_k \Leftrightarrow \epsilon_{ijk}\sigma_{ij} - \sigma_{k4}, \quad i, j, k = 1, 2, 3, \tag{2.85}$$

while the other is "magnetic"

$$\tau_k \Leftrightarrow \epsilon_{ijk}\sigma_{ij} \quad i,j,k = 1,2,3, \tag{2.86}$$

It turns out [30] that the right way is "magnetic minus electric", so that the right-handed 2^+ and left-handed 2^- decompose as

$$\begin{aligned} 2^+ &\rightarrow [0] + [2] = 1 + 1 \\ 2^- &\rightarrow [1] = 2 \end{aligned} \tag{2.87}$$

where 1 and 2 are singlet and doublet of $SU(2)_L$, respectively. Each of the 2^+ and 2^- is conjugate to itself. Obviously, the two singlets of $SU(2)_L$ in 2^+ are $|++\rangle$ and $|--\rangle$, and the doublet in 2^- consists of $|+-\rangle$ and $|-+\rangle$.

The left-handed spinor 16^- of SO(10), expressed in terms of left-handed and right-handed spinors of $SO(6)$ and $SO(4)$, is

$$16^- = 4^+2^- + 4^-2^+. \tag{2.88}$$

Given 4^+, 4^-, 2^+, and 2^- expressed in the explicit notation $|\epsilon_1\epsilon_2\epsilon_3\rangle$ and $|\epsilon_4\epsilon_5\rangle$, one can easily identify the 16 left-handed fermions as the components of 16-dimensional spinor 16^- as

$$\begin{aligned} &d^1 = |+--+-\rangle; \quad d^2 = |-+-+-\rangle; \quad d^3 = |--++-\rangle; \\ &u^1 = |+---+\rangle; \quad u^2 = |-+--+\rangle; \quad u^3 = |--+-+\rangle; \\ &\bar{d}^1 = |++-++\rangle; \quad \bar{d}^2 = |+-+++\rangle; \quad \bar{d}^3 = |-++++\rangle; \\ &\bar{u}^1 = |++---\rangle; \quad \bar{u}^2 = |+-+--\rangle; \quad \bar{u}^3 = |-++--\rangle; \\ &e^- = |++++-\rangle; \quad \nu = |+++-+\rangle; \\ &e^+ = |---++\rangle; \quad \nu^c = |-----\rangle. \end{aligned} \tag{2.89}$$

The various $U(1)$ charges can be read directly from the explicit expression of fermions. The $U(1)_X$ in the $SU(5) \times U(1)_X$ decomposition of SO(10) is

$$\begin{aligned} X &= \sigma_{12} + \sigma_{34} + \sigma_{56} + \sigma_{78} + \sigma_{910} \\ &= \tau_3 \times 1 \times 1 \times 1 \times 1 + 1 \times \tau_3 \times 1 \times 1 \times 1 + ... + 1 \times 1 \times 1 \times 1 \times \tau_3, \end{aligned} \tag{2.90}$$

which gives $\Sigma_{i=1}^{5}\epsilon_i$ when acting on spinor $|\epsilon_1\epsilon_2\epsilon_3\epsilon_4\epsilon_5\rangle$. The commonly used normalization is $X = \frac{1}{5}\Sigma_{i=1}^{5}\epsilon_i$. The $U(1)_{B-L}$ is embedded into the $SU(3)_C \times U(1)_{B-L}$ decomposition of SO(6), therefore it appears quite similar to $U(1)_X$, with the only difference being that the indices are restricted to be $i = 1 \sim 3$ instead of $i = 1 \sim 5$. Therefore we have

$$B - L = \frac{1}{3}(\epsilon_1 + \epsilon_2 + \epsilon_3). \tag{2.91}$$

Similarly, the hypercharge is

$$\frac{Y}{2} = -\frac{1}{4}(\epsilon_4 + \epsilon_5) + \frac{1}{6}(\epsilon_1 + \epsilon_2 + \epsilon_3), \tag{2.92}$$

and the electric charge is

$$Q = -\frac{1}{2}\epsilon_4 + \frac{1}{6}(\epsilon_1 + \epsilon_2 + \epsilon_3). \tag{2.93}$$

Obviously, the $B - L$, $Y/2$, and X are linearly dependent

$$B - L = X + \frac{4}{5}\left(\frac{Y}{2}\right). \tag{2.94}$$

As in the SM, the fermion masses arise from Yukawa couplings of fermions to Higgs bosons. The vev of Higgs fields that are responsible for the spontaneous gauge symmetry breaking can also induce fermion masses. The most general Yukawa couplings that are responsible for fermion masses take the form $\psi_i BC\Gamma_\mu\psi_j\phi^\mu$,

$\psi_i BC\Gamma_\mu\Gamma_\nu\Gamma_\tau\psi_j\phi^{\mu\nu\tau}$, and $\psi_i BC\Gamma_\mu\Gamma_\nu\Gamma_\tau\Gamma_\delta\Gamma_\rho\psi_j\phi^{\mu\nu\tau\delta\rho}$, where i and j are family indices and μ, ν, τ, δ, and ρ are SO(10) vector indices. The ϕ^μ, $\phi^{\mu\nu\tau}$, and $\phi^{\mu\nu\tau\delta\rho}$ could be fundamental Higgs fields or composite Higgs fields. In the case that ϕ^μ, $\phi^{\mu\nu\tau}$, and $\phi^{\mu\nu\tau\delta\rho}$ are fundamental, the operators are renormalizable, while in the case that they are composite (these vector indices are carried by more than one Higgs field), the operators are non-renormalizable ones.

The reason that only odd number of Γ matrices are allowed is because the total number of raising and lowering operators ($\chi_i^\dagger$ and χ_i) must be even, and the charge-conjugation operator already has odd number of them. Due to the anti-commutation rule between fermion fields, it is easy to see that the family indices i and j are symmetric in operators $\psi_i BC\Gamma_\mu\psi_j\phi^\mu$ and $\psi_i BC\Gamma_\mu\Gamma_\nu\Gamma_\tau\Gamma_\delta\Gamma_\rho\psi_j\phi^{\mu\nu\tau\delta\rho}$, and anti-symmetric in $\psi_i BC\Gamma_\mu\Gamma_\nu\Gamma_\tau\psi_j\phi^{\mu\nu\tau}$. This symmetry property is manifest in the case that ϕ^μ, $\phi^{\mu\nu\tau}$, and $\phi^{\mu\nu\tau\delta\rho}$ are fundamental Higgs fields. On the other hand, in the case of non-renormalizable couplings such as $16_i 16_j 16_H 16_H$, the symmetry property is not manifest, since two 16_Hs can be coupled to form any one of these three tensors.

Chapter 3

SO(10) GUT MODEL BUILDING

As shown in Chapter 1, many problems and puzzles of SM can be elegantly solved and explained in the framework of GUT, especially the SO(10) GUT. But so far, everything is still at the conceptual level. If GUT is indeed a correct idea, it should be tested by experiments.

The fermion masses and mixing, although showing a interesting pattern, are merely inputs in SM. One of the important features of GUT is that the quarks and the leptons are tightly correlated. Hence one of the best places of testing the idea of GUT is to check if realistic GUT models can be constructed to explain masses and mixing of both quarks and leptons.

Among all the fermions, the neutrino is rather unique. In SM, while all the other fermions have both left-handed and right-handed components, the neutrino has only left-handed component; while all the other fermions are massive, the neutrino is massless. The discovery of neutrino oscillations has opened up a fascinating window for physics beyond the SM. Experimental data on the neutrino mass differences and mixing help to constrain various theoretical models of new physics.

Assuming three light flavors, the lepton mixing is described by the Pontecorvo-

Maki-Nakagawa-Sakata (PMNS) matrix [31]

$$
\begin{aligned}
U_{PMNS} &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \\
&\times diag(e^{i\phi_1}, e^{i\phi_2}, 1)
\end{aligned}
\tag{3.1}
$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The unitary matrix U_{PMNS} is characterized by three mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ plus three CP-violating phases if assuming neutrinos are Majorana fermions. The atmospheric and accelerator neutrino experiments [32, 33, 34, 36] have determined θ_{23}, which is often referred as atmospheric angle $\theta_{23} \equiv \theta_{\rm atm}$, and mass splitting $\Delta m^2_{\rm atm}$, at 3σ,

$$
\begin{aligned}
\sin^2 2\theta_{\rm atm} &> 0.87, \\
1.4 \times 10^{-3}{\rm eV}^2 < \quad \Delta m^2_{\rm atm} &< 3.3 \times 10^{-3}{\rm eV}^2.
\end{aligned}
\tag{3.2}
$$

The θ_{12}, usually referred as solar angle $\theta_{12} \equiv \theta_{\rm sol}$, has been measured by the solar and reactor neutrino experiments, with an even better precision [35, 36, 37].

$$
\begin{aligned}
0.70 < \quad \sin^2 2\theta_{\rm sol} &< 0.94, \\
7.1 \times 10^{-5}{\rm eV}^2 < \quad \Delta m^2_{\rm sol} &< 8.9 \times 10^{-5}{\rm eV}^2.
\end{aligned}
\tag{3.3}
$$

These results have already facilitated elimination of a large class of neutrino mass matrix models in the literature. The CHOOZ reactor experiment has discovered that $\sin^2 2\theta_{13}$, if non-zero, should be smaller than 0.1 [38]. The next generation of neutrino experiments under proposal aims to push the limit to $\sin^2 2\theta_{13} \sim 0.01$ [39, 40], which undoubtedly will reveal a great deal about the mechanism of neutrino mass generation.

If small neutrino masses are assumed to arise from the seesaw mechanism [13], the first thing one recognizes from the present data is that the seesaw scale (the scale where the $B-L$ symmetry is broken) must be substantially high. This strongly suggests that the seesaw scale may be connected with one of the leading ideas for new physics beyond the standard model, i.e., supersymmetric GUT according to which all forces and matter unify at a truly short distance scale corresponding to an energy of order 10^{16} GeV. Moreover, as discussed in Chapter 1, the most important ingredient of the seesaw mechanism is the $B-L$ breaking which can be naturally realized in the framework of SO(10) GUT. Therefore, neutrino oscillation is a suitable testing ground of GUT physics.

This Chapter is devoted to the SO(10) GUT model building which aims to explain the masses and mixing of both quarks and leptons, including neutrinos. It starts with a general discussion to motivate the introduction of lopsided structure. Following that, it presents the details of an SO(10) GUT with lopsided structure.

3.1 Motivation for Lopsided Structure

The SO(10) gauge symmetry is spontaneously broken by the Higgs vevs to $SU(3)_C \times SU(2)_L \times U(1)_Y$, then further to $SU(3)_C \times U(1)_{EM}$. The second step of the breaking is induced by the vev of a vector representation 10. However, the scheme of the first step breaking is not unique.

Since the SO(10) is a rank 5 group (SM is rank 4) with the extra diagonal generator $B-L$, the breaking of SO(10) to SM requires the breaking of $B-L$

symmetry. In the framework of SO(10) GUT, there are two choices of Higgs fields to accomplish this — an antisymmetric 5-index tensor $\overline{126}$ or a spinor $\overline{16}$. Whichever Higgs field is used, it generates the heavy seesaw scale — the scale of Majorana mass of right-handed neutrinos. These two alternatives of Higgs field are distinguished in numerous aspects. Using $\overline{126}$, its vev breaks $B-L$ by two units and thus leaves the matter parity unbroken, however the unified gauge coupling blows up and becomes non-pertubative above the GUT scale. The $\overline{16}$ vev breaks the $B-L$ by one unit and therefore breaks the matter parity. A number of models utilizing these two classes of Higgs have been constructed [41, 42, 43, 44, 45, 46, 47, 49, 50, 51]. While most of these models are successful in fitting to the experimental data of masses and mixing angles of leptons and quarks, they predict quite different values for the poorly-known neutrino mixing angle θ_{13}. Majority of models [41, 42, 43] with high-dimensional Higgses tend to yield θ_{13} close to the current experimental upper bound, and majority of those with low-dimensional Higgses [44, 45, 46, 47, 49, 50, 51] predict a small θ_{13}. Thus it appears that θ_{13} might be an excellent observable to differentiate between the two classes of SO(10) models.

We will concentrate on the model using a pair of spinors 16 and $\overline{16}$. In this way of breaking SO(10) to SM, the vev of 16 is in the direction of right-handed neutrino. Apparently, this vev breaks both I_{3R} and $B-L$ and provides the necessary ingredient of the seesaw mechanism.

The breaking of SO(10) to SM requires the presence of an adjoint representation 45 and/or a symmetric second order tensor representation 54, in addition to spinors 16 and $\overline{16}$. It has been shown [52] that a minimal scheme of breaking the

SO(10) to SM involves a single adjoint representation 45 plus some 16 and $\overline{16}$ pairs.

In the framework of SO(10), the colored partners of the Higgs $SU(2)_L$ weak-doublet need to be super heavy whereas the weak-doublet itself is light. This doublet-triplet spitting can be most naturally realized by the Dimopoulos-Wilczek mechanism [53]. The idea of Dimpopoulos-Wilczek mechanism is based on the observation that the color-triplet has $B - L = \pm 2/3$ and weak-doublet has $B - L = 0$. Therefore, in the coupling $10_H 45_H 10_H$, the vev of adjoint 45 in the direction of $B - L$ will make the color-triplet components heavy, and leave the weak-doublet component light. Hence, the vev of adjoint Higgs 45 is constrained to be in the direction of $B - L$.

Given the set of Higgs fields involving 10_H, 16_H, $\overline{16}_H$, and 45_H, various operators associated with fermion masses can be constructed. These operators involve two fermion spinors 16_i, with $i = 1, 2, 3$ the family index. In general, each of these operators gives masses to all the quarks and the leptons. For instance, the simplest operator, which is the only renormalizable operator in this set, is $16_i 16_j 10_H$. (Notice that, for simplicity, the charge-conjugation operators B and C are omitted from now on.) This single operator generates masses to both up-type and down-type quarks and both charged leptons and neutrinos (of Dirac type) in the coherent pattern (taking $i = j = 3$ as example)

$$\begin{aligned} m_t &= m_{\nu_\tau}^{Dirac}, \\ m_b &= m_\tau. \end{aligned} \tag{3.4}$$

The other operators have higher dimension. They are not renormalizable and should be considered as effective operators of a more fundamental theory. As an example, two fermion spinors and two Higgs spinors can be coupled together $16_i 16_j 16_{1H} 16_{2H}$. The 16_{1H} and 16_{2H} could be the same Higgs field or the different ones. Depending on how these spinors are coupled, this operator can produce masses to quarks and leptons with variant patterns.

The first step of building the SO(10) GUT model is to have a clear picture of the pattern of fermion masses and mixing. In fact, the masses of down-type quarks and charged leptons are observed to have the following approximate relations at the GUT scale [54]

$$m_b \simeq m_\tau, \quad (3.5)$$

$$m_s \simeq \frac{1}{3} m_\mu, \quad (3.6)$$

$$m_d \simeq 3 m_e. \quad (3.7)$$

The first relation is the well known $b-\tau$ unification, which can be naturally realized in SO(10) GUT. The operator $16_3 16_3 10_H$ leads to it as discussed above. With this operator alone, the mass matrices of fermions are

$$
\begin{aligned}
U &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_U, \\
D &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_D, \\
N &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_U, \\
L &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_D,
\end{aligned}
\tag{3.8}
$$

where U, D, N, and L denote the Dirac mass matrices of the up-type quark, the down-type quark, the neutrino, and the charged lepton, respectively.

The second mass relation $m_s \simeq \frac{1}{3} m_\mu$ also appears natural from the SO(10) group structure. The vev of the adjoint representation 45 Higgs is in the direction of $B-L$ direction. Notice that the $B-L$ is -1 for leptons and $1/3$ for quarks. Therefore, the operator involving the 45_H tends to generate the masses of leptons three times heavier than the masses of quarks. Thus it seems that one can get the relation $m_s \simeq \frac{1}{3} m_\mu$ by putting 2-2 elements to mass matrices of D and L. However, the operator involving 45_H is $(16_i 16_j)_{120} 10_H 45_H$, where two fermion spinors are coupled to a 3-index tensor 120. This operator generates antisymmetric mass matrices. It can only give off-diagonal elements to D and L. Taking family indices i and j of the operator $(16_i 16_j)_{120} 10_H 45_H$ to be 2 and 3, respectively, one has

$$\begin{aligned} U &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, \\ D &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_D, \\ N &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, \\ L &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_D. \end{aligned} \tag{3.9}$$

However, this form of mass matrices is not exactly what we want. In fact, by diagonalizing the mass matrices, one has $m_s \simeq \frac{1}{9} m_\mu$ instead of $m_s \simeq \frac{1}{3} m_\mu$. Thus, although the SO(10) group structure can potentially lead to the lepton mass three times larger than the quark mass, it requires additional elaboration. This puzzle is left here and will be revisited when discussing how to generate different mixing in the quark and the lepton sector.

The third relation $m_d \simeq 3 m_e$ seems to be in contradiction to the SO(10) group structure. However, this relation can be realized based on the second relation $m_s \simeq \frac{1}{3} m_\mu$. For simplicity, let us concentrate on the 1-2 sector of D and L. Assuming the 2-2 elements can be made to satisfy the relation $m_s \simeq \frac{1}{3} m_\mu$, then one can get the relation $m_d \simeq 3 m_e$ by leaving 1-1 element empty and putting 1-2 and 2-1 elements nonzero

$$\begin{aligned} D^{(12)} &\propto \begin{pmatrix} 0 & a \\ b & 1 \end{pmatrix}, \\ L^{(12)} &\propto \begin{pmatrix} 0 & a' \\ b' & 3 \end{pmatrix}. \end{aligned} \tag{3.10}$$

After diagonalizing D and L, one has the mass of the electron and the down-type quark as $m_e = a'b'/3$ and $m_d = ab$, respectively. Evidently, as far as the product ab is equal to the product $a'b'$, the mass relation $m_d \simeq 3m_e$ follows naturally.

In summary, both the down-type quarks and the charged leptons show a hierarchical mass pattern, and there exist empirical relations between their masses for each family. Two of these mass relations can be realized naturally. On the other hand, there is no obvious numerical relation between masses of up-type quarks and neutrinos. This is expected if one assumes the neutrino mass is of Majorana type.

The CKM matrix for the quark mixing and the PMNS matrix for the lepton mixing are rather different, especially in the 2-3 sector. The 2-3 mixing in the CKM matrix is described by

$$V_{cb} \simeq 0.04, \tag{3.11}$$

which is very small. However, the 2-3 mixing in the PMNS matrix, measured from the atmospheric and accelerator neutrino experiments, is very large

$$\sin^2 2\theta_{\text{atm}} > 0.87 \tag{3.12}$$

at 3σ, with the central value to be maximum $\sin^2 2\theta_{\text{atm}} = 1$.

This difference is not expected. First, the hierarchical pattern of mass eigenvalues tends to be associated with small mixing angles, which is indeed the case in the quark sector. Second, it is very hard to reconcile this big difference between the quark and the lepton sector in the unification scheme.

To see how this puzzle can be naturally explained in the framework of SO(10) GUT, it is helpful to see what the CKM and PMNS mixing matrices mean first. The Dirac mass matrices U, D, and L are diagonalized by two unitary transformations U and V, acting on left-handed and right-handed fields, respectively

$$\begin{aligned} \hat{U} &= V_U^\dagger U U_U, \\ \hat{D} &= V_D^\dagger D U_D, \\ \hat{L} &= V_L^\dagger L U_L, \end{aligned} \tag{3.13}$$

and the Majorana mass matrix of the left-handed neutrino is diagonalized by one unitary transformation acting on the left-handed field

$$\hat{m}_\nu = U_\nu^T m_\nu U_\nu. \tag{3.14}$$

The mixing matrices describe the mismatch between mass eigenstates of upper and lower components of the weak-doublet, which are left-handed fields

$$\begin{aligned} U_{CKM} &= U_D^\dagger U_U, \\ U_{PMNS} &= U_L^\dagger U_\nu. \end{aligned} \tag{3.15}$$

On the other hand, the right-handed transformations $V_{U,D,L}$ are not directly observable. (They are observable indirectly in the quantum loops in the SUSY context as will discussed in the Chapter 5.)

Since the quark mixing and the lepton mixing involve left-handed fields only, there emerges one natural way of explaining the small quark mixing and the large lepton mixing in the quark-lepton unified scheme. The idea is that, although the quarks and the leptons are unified in the framework of GUT, the direct connection might exist only between the quark field and the lepton field with opposite chirality. In this way, it is the mixing of right-handed quark fields that is associated with the mixing of left-handed leptons, and vice versa. As a result, the large mixing of left-handed leptons in the PMNS matrix indicates the large mixing of right-handed quarks, which is not directly observable, on the other hand, the small mixing of left-handed quarks in the CKM matrix indicates the small mixing of right-handed leptons, which is again not directly observable.

Let us check if the connection between the quarks and the leptons of opposite chirality can be realized in GUT. Obviously, this is not true in the left-right symmetric model G(224), where the quark and the lepton fields of the same chirality are unified. However, this is indeed the case in the left-right asymmetric model of SU(5), where it is the charge-conjugate of the right-handed down-type quarks that sit together with the left-handed lepton doublet in the $\bar{5}$ and it is the charge-conjugate of right-handed charged lepton that sit together with the left-handed quark-doublet in the 10. Considering, for example, the minimal toy model of SU(5), which has the following Yukawa couplings

$$\mathcal{L}_{Yukawa} = \lambda_{33}(\bar{5}_3 10_3)\bar{5}_H + \lambda_{23}(\bar{5}_2 10_3)\bar{5}_H + \lambda_{32}(\bar{5}_3 10_2)\bar{5}_H \tag{3.16}$$

with $\lambda_{32} \ll \lambda_{23} \sim \lambda_{33}$, the 2-3 sector of mass matrices of down-type quarks and charged leptons follow

$$D^{(23)} = \begin{pmatrix} 0 & \sigma \\ \epsilon & 1 \end{pmatrix} M_D,$$

$$L^{(23)} = \begin{pmatrix} 0 & \epsilon \\ \sigma & 1 \end{pmatrix} M_D, \tag{3.17}$$

with $M_D = \lambda_{33}\langle\bar{5}_H\rangle$, $\epsilon M_D = \lambda_{32}\langle\bar{5}_H\rangle$, and $\sigma M_D = \lambda_{23}\langle\bar{5}_H\rangle$, which leads to $\epsilon \ll \sigma \sim 1$. The mass matrices of down-type quarks and charged leptons are related, but up to a left-right transposition $D = L^T$ due to the group structure. In such a way, the left-handed quark mixing and right-handed lepton mixing in the 2-3 sector are both of the size of ϵ, which is small, and the left-handed lepton mixing and right-handed quark mixing in the 2-3 sector are both of the size of σ, which is large.

Although the SO(10) GUT is a left-right symmetric theory, it can be broken in the left-right asymmetric way, and has the potential of realizing the structures of SU(5). In fact, the SU(5) operator $(\bar{5}_i 10_j)\bar{5}_H$ can be embedded into the SO(10) operator $(16_i 16_H)_{10}(16_j 16'_H)_{10}$, where 16_H pick up the vev in the right-handed neutrino direction (1 of SU(5))and the other 16-dimensional spinor $16'_H$ pick up the vev in the component of $\bar{5}$ of SU(5). Thus, in the SO(10) model, with the following operators

$$\mathcal{L} = \lambda_{33} 16_3 16_3 10_H + \lambda_{23}(16_2 16_H)_{10}(16_3 16'_H)_{10} + \lambda'_{23}(16_2 16_3)_{120} 10_H 45_H \tag{3.18}$$

the mass matrices take the form

$$
\begin{aligned}
U &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, \\
D &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma+\epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_D, \\
N &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, \\
L &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \sigma+\epsilon & 1 \end{pmatrix} M_D,
\end{aligned} \tag{3.19}
$$

with $\epsilon \ll \sigma \sim 1$. The introduction of a large σ brings two benefits. First, it realizes the lopsided structure of SU(5). Second, the mass relation between the down-type quark and the charged lepton in the second family is changed. Instead of

$$
\begin{aligned}
m_s &\simeq \left(\frac{\epsilon}{3}\right)^2 M_D, \\
m_\mu &\simeq \epsilon^2 M_D,
\end{aligned} \tag{3.20}
$$

which leads to the problematic mass relation

$$
\frac{m_s}{m_\mu} = \frac{1}{9}, \tag{3.21}
$$

as we mentioned earlier, the introduction of large σ changes the masses to be approximately

$$\begin{aligned} m_s &\propto \frac{\epsilon}{3}\left(\sigma + \frac{\epsilon}{3}\right) \simeq \frac{\epsilon}{3}\sigma, \\ m_\mu &\propto \epsilon(\sigma + \epsilon) \simeq \epsilon\sigma, \end{aligned} \tag{3.22}$$

which leads to the correct empirical mass relation

$$\frac{m_s}{m_\mu} = \frac{1}{3}. \tag{3.23}$$

The above analysis of masses and mixing of quarks and leptons shows that the lopsided structure, which is intrinsically embedded in SU(5) but also realizable in SO(10) model, can not only naturally explain the large 2-3 mixing of leptons and small 2-3 mixing of quarks, but also help to give the correct empirical relation between masses of down-type quarks and charged leptons in the second family. In the following section, we discuss the realistic models based on this idea.

3.2 Realistic SO(10) GUT Models with Lopsided Structure

The first realistic SO(10) model with lopsided structure is constructed in a series of papers by Albright, Barr, and Babu [45, 46, 47, 48]. At the first stage [45], the model was established to explain the large atmospherical mixing in the lepton sector versus small 2-3 mixing V_{cb} in the quark sector. Soon after, the model was extended by Albright and Barr to include the first family [46]. It was found that the model naturally predicts the 1-2 neutrino mixing angle θ_{12} (solar angle) to be either small, or very near to the maximum value, corresponding to the so called "bimaximum mixing" scenario. After the solar neutrino experiments in which the θ_{12}

is found to be around 30^o, Albright and Barr have attempted to modify their model to accommodate this large solar angle. But in doing so, there must be considerable fine-tuning. The issue is that, in order to produce the large solar angle, different free parameters must be made to be exactly equal to each other.

This can be seen from the explicit presentation of their original model. Via couplings with a set of Higgs multiplets $\mathbf{10}_H$, $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ and $\mathbf{45}_H$ and the constraints from the flavor $U(1) \times Z_2 \times Z_2$ symmetry, the Dirac fermion mass matrices have the following forms in the original model of Albright and Barr,

$$
\begin{aligned}
U &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} M_U, \\
N &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} M_U, \\
D &= \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D, \\
L &= \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix} M_D,
\end{aligned}
\tag{3.24}
$$

where U, D, L, and N, as in the last section, denote up-type quark, down-type quark, charged lepton, and neutrino Dirac mass matrices, respectively. The various entries in the mass matrices come from different SO(10) Yukawa operators, e.g., η from $\mathbf{16}_1\mathbf{16}_1\mathbf{10}_H$; ϵ from $\mathbf{16}_2\mathbf{16}_3\mathbf{10}_H\mathbf{45}_H$, δ, δ' from $\mathbf{16}_1\mathbf{16}_{2,3}\mathbf{16}_H\mathbf{16'}_H$; and σ from $\mathbf{16}_2\mathbf{16}_H\mathbf{16}_3\mathbf{16'}_H$. It is worth to mention that all the relations between different entries, such as minus signs and coefficients of 1/3, are naturally due to the group structure. The model so far is completely natural. The parameter σ is of order one, signaling the lopsidedness between the second and third families in D and L. This feature leads to a large left-handed neutrino mixing in the PMNS matrix and a small left-handed quark mixing in the CKM matrix. The parameter ϵ is one order-of-magnitude smaller than σ and generates the hierarchy between the second and third families. In extending to the first family, δ and δ' were introduced into the D and L.

The large solar mixing angle is constructed from the left-handed neutrino Majorana mass matrix, which in turn depends on a very specific structure in right-handed Majorana mass matrix M_R

$$M_R = \begin{pmatrix} c^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda_R, \tag{3.25}$$

with all the entries in M_R generated from the operator $16_i 16_j \overline{16}_H \overline{16}_H$. The fine-tuning problem is obvious — the entries in the Majorana mass matrix M_R and the Dirac mass matrices U, D, L, and N are from different operators, and therefore they are not connected by the group structure. The appearance of ϵ in both Majorana mass matrix M_R and Dirac mass matrices U, D, L, N surely signifies the fine-tuning problem.

By varying the four parameters in the M_R [49], the predication of θ_{13} from this model was found to lie in the range of $10^{-5} \leq \sin^2\theta_{13} \leq 10^{-2}$. A narrower range of $0.002 \leq \sin^2\theta_{13} \leq 0.003$ is obtained when constraints are imposed on the parameter space. Therefore, the prediction of θ_{13} from the original lopsided model supports the empirical rule that model using large dimensional Higgses predict large θ_{13} and these using small dimensional Higgses predict small θ_{13} [55]. If $\bar{\nu}_e$ disappearance at short baseline experiments is observed in the next generation of short baseline reactor experiments [40], the original lopsided model would be ruled out.

Given that the SO(10) GUT model with lopsided structure is one of the most successful GUT theories incorporating all the known experimental facts, two obvious questions arise immediately. First, is there a more natural way to realize the large solar-neutrino mixing angle without fine-tuning? And second, if such an alternative model exists, is θ_{13} consistently small?

Since the fine-tuning problem is associated with the specific way of generating the solar angle, we look for other ways of producing it. The lepton mixing matrix describes the mismatch between mass eigenstates of left-handed neutrino and charged lepton as shown in Eq. (3.15). The large solar mixing can either be generated from $U_L^\dagger$ or U_ν or a combination of both. If there is a non-vanishing 1-2 rotation from U_ν, it can either be generated from the Dirac mass matrix or from the Majorana mass matrix of the right-handed neutrinos or a combination of both. In the following, we focus on the possibilities in which one of the matrices generates a large solar-neutrino mixing angle, keeping in mind though that a general situation might involve a mixture of the extreme cases.

In the original lopsided model, the large solar-neutrino mixing is induced mainly by the right-handed neutrino mass matrix. Thus, an alternative possibility is to produce the large solar-neutrino mixing from the charged lepton matrix. In fact, in Ref. [56], a model was proposed in which both large solar and atmospheric neutrino mixings are generated from the lopsided charged-lepton mass matrix. The value of $\sin^2 2\theta_{13}$ is again found to be small, 0.01 or less.

Here we study yet a third possibility of generating a large 1-2 rotation in the lepton mixing from the neutrino Dirac mass matrix N. The easiest way to achieve this might be to use a lopsided structure in the 1-2 entries of N. However, this is impossible in group theory of SO(10). A large rotation, however, can be generated through 1-3 and 2-3 entries without affecting, for example, the quark mass hierarchy between the first and second generations. Thus the modified mass matrices are

$$\begin{aligned}
U &= \begin{pmatrix} \eta & 0 & \kappa+\rho/3 \\ 0 & 0 & \omega \\ \kappa-\rho/3 & \omega & 1 \end{pmatrix} M_U, \\
D &= \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma+\epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix} M_D, \\
N &= \begin{pmatrix} \eta & 0 & \kappa-\rho \\ 0 & 0 & \omega \\ \kappa+\rho & \omega & 1 \end{pmatrix} M_U, \\
L &= \begin{pmatrix} \eta & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma+\epsilon & 1 \end{pmatrix} M_D, \\
M_R &= \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R, \qquad (3.26)
\end{aligned}$$

The symmetric entries ω and κ in U and N can be generated from the dimension-5 operator $\mathbf{16}_i\mathbf{16}_j[\overline{\mathbf{16}}_H\overline{\mathbf{16}}'_H]_{10}$, and the antisymmetric ρ entries in U and N are from dimension-6 operator $\mathbf{16}_i\mathbf{16}_j[\overline{\mathbf{16}}_H\overline{\mathbf{16}}'_H]_{10}\mathbf{45}_H$, where the subscript 10 indicate that the spinor Higgses are coupled to 10 of SO(10). Because of the modification, the ϵ entries in D and L now must be generated from dimension-6 operator $\mathbf{16}_i\mathbf{16}_j[\mathbf{16}_H\mathbf{16}'_H]_{10}\mathbf{45}_H$. We assume as in the past that $\mathbf{45}_H$ Higgs develops a vacuum expectation value (VEV) in the $B-L$ direction. $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ are the Higgs spinors which break the SO(10) to SU(5) by taking the VEV in the singlet direction of SU(5). The second pair of $\mathbf{16}'_H$ and $\overline{\mathbf{16}}'_H$ develop VEV in $\overline{\mathbf{5}}$ and $\mathbf{5}$ of SU(5), respectively, and therefore the operators involving $\overline{\mathbf{16}}'_H$ and $\mathbf{16}'_H$ contribute to up and down sectors of weak doublets, respectively.

Usually a rotation is connected with the mass spectrum. However, in our case the 1-2 rotation angle from U will be combined with the 1-2 rotation from D to obtain the Cabibbo angle θ^c, and a constraint from the up-type quark spectrum must be avoided. Thus, the first two families in the U and N cannot be coupled to each other directly, but can be coupled indirectly through the third family. The 1-2 rotations in U and N generated from this way are proportional to the ratios $\gamma \equiv (\kappa - \rho/3)/\omega$ and $\gamma' \equiv (\kappa + \rho)/\omega$, respectively.

Taking the approximation $\eta = 0$, the dependence of various mass ratios and CKM elements on parameters can be seen roughly from the following approximate expressions (the superscript 0 indicates the relevant quantities are at GUT scale)

$$\begin{aligned}
m_b^0/m_\tau^0 &\simeq 1-\frac{2}{3}\frac{\sigma}{\sigma^2+1}\epsilon\ ,\\
m_u^0/m_t^0 &\simeq 0\ ,\\
m_c^0/m_t^0 &\simeq (1+\gamma^2)\omega^2\ ,\\
m_\mu^0/m_\tau^0 &\simeq \epsilon\frac{\sigma}{\sigma^2+1}\ ,\\
m_s^0/m_b^0 &\simeq \frac{1}{3}\epsilon\frac{\sigma}{\sigma^2+1}\ ,\\
m_e^0/m_\mu^0 &\simeq \frac{1}{9}t_L t_R\ ,\\
m_d^0/m_s^0 &\simeq t_L t_R,\\
V_{cb}^0 &\simeq -\sqrt{1+\gamma^2}\omega-\frac{1}{\sqrt{1+\gamma^2}}\frac{\epsilon}{3(1+\sigma^2)}\ ,\\
V_{us}^0 &\simeq \frac{1}{\sqrt{1+\gamma^2}}(-\gamma+t_L e^{i\theta})\ ,\\
V_{ub}^0 &\simeq \frac{1}{\sqrt{1+\gamma^2}}\frac{\epsilon}{3(\sigma^2+1)}(\gamma-t_L e^{i\theta}+\sqrt{1+\sigma^2}t_R)\ ,
\end{aligned}\tag{3.27}$$

where t_L, t_R and θ are defined as $t_L e^{i\theta} \equiv 3(\delta-\sigma\delta' e^{i\phi})/(\sigma\epsilon)$ and $t_R \equiv 3\delta\sqrt{\sigma^2+1}/(\sigma\epsilon)$. The expressions for mass ratios in down-type quark and charged lepton sectors are the same as those in the original lopsided model. The expressions for m_c^0/m_t^0 and elements in CKM matrix are new. These approximations allow us to design strategies to fit various parameters to experimental data.

First, we use the up-type quark and lepton spectra and the parameters in the CKM matrix to determine 10 parameters σ, ϵ, δ, δ', ϕ, ω, γ, η, M_U and M_D. Our best fit yields σ and ϵ approximately the same as those in the original lopsided model, and thus the successful prediction for the mass ratios m_μ^0/m_τ^0 and m_s^0/m_b^0 are kept. The two CKM elements $|V_{us}^0|$ and $|V_{ub}^0|$, together with the CP violation phase δ_{CP} and the constraint on the product $t_L t_R$ from mass ratio m_e^0/m_μ^0, can fix the t_L, t_R, γ and θ. Then ω and η can be fixed from m_c^0/m_t^0 and m_u^0, respectively. The down-type quark mass spectrum comes out as predictions.

To see the dependence of the lepton mixing PMNS matrix on various parameters, we construct the Majorana mass matrix of the left-handed neutrino from the seesaw mechanism[13], $m_\nu = -N M_R^{-1} N$,

$$m_\nu = -\begin{pmatrix} \eta^2/a + (\kappa+\rho)^2 & (\kappa+\rho)\omega & \eta(\kappa-\rho)/a + (\kappa+\rho) \\ (\kappa+\rho)\omega & \omega^2 & \omega \\ \eta(\kappa-\rho)/a + (\kappa+\rho) & \omega & 1 + (\kappa-\rho)^2/a + \omega^2/b \end{pmatrix} M_U^2/\Lambda_R \quad (3.28)$$

which depends on the four unknown parameters, γ', Λ_R, a and b. With parameter a taking a reasonably large value, say, order of 0.001 or larger, the η dependent terms can be neglected. Then one readily sees that the m_ν matrix can be diagonalized by a 1-2 rotation of angle θ_{12}^ν with tan $\theta_{12}^\nu = \gamma'$, and followed by a 2-3 rotation by angle θ_{23}^ν, with

$$\tan 2\theta_{23}^\nu = \frac{2\sqrt{1+\gamma'^2}\omega}{1 + (\kappa-\rho)^2/a + \omega^2/b - (1+\gamma'^2)\omega^2} \ . \quad (3.29)$$

The neutrino Majorana masses of the second and the third families are

$$\begin{aligned} m_{\nu 2} &= -\left[(1+\gamma'^2)\omega^2 + \sqrt{1+\gamma'^2}\omega\,(\mathrm{cot}2\theta_{23}^\nu - \mathrm{csc}2\theta_{23}^\nu)\right] M_U^2/\Lambda_R \ , \\ m_{\nu 3} &= -\left[(1+\gamma'^2)\omega^2 + \sqrt{1+\gamma'^2}\omega\,(\mathrm{cot}2\theta_{23}^\nu + \mathrm{csc}2\theta_{23}^\nu)\right] M_U^2/\Lambda_R \ , \end{aligned} \quad (3.30)$$

with $m_{\nu 1} = 0$ as the result of the approximation $\eta = 0$. Therefore, the present model constrains the neutrino mass spectrum as hierarchial, which means that the parameters in the light-neutrino mass matrix, the mass eigenvalues and mixings, do not run significantly from GUT to low-energy scales. The mass difference $\Delta m^2_{\nu 12}$ can be used to fix the right-handed neutrino mass scale Λ_R.

Taking into account rotations from matrices m_ν and L, we arrive at the elements in the PMNS matrix,

$$
\begin{aligned}
U_{e2} &= \left(\frac{\gamma'}{\sqrt{1+\gamma'^2}} - \frac{t_R}{3}\frac{1}{\sqrt{1+\gamma'^2}}\frac{1}{\sqrt{1+\sigma^2}}\right)\cos\theta^\nu_{23} - \frac{t_R}{3}\frac{\sigma}{\sqrt{1+\sigma^2}}\sin\theta^\nu_{23}\ , \\
U_{\mu 3} &= -\frac{\sigma}{\sqrt{1+\sigma^2}}\cos\theta^\nu_{23} + \frac{1}{\sqrt{1+\gamma'^2}}\left(\gamma'\frac{t_R}{3} + \frac{1}{\sqrt{1+\sigma^2}}\right)\sin\theta^\nu_{23}\ , \\
U_{e3} &= \frac{t_R}{3}\frac{\sigma}{\sqrt{1+\sigma^2}}\cos\theta^\nu_{23} + \left(\frac{\gamma'}{\sqrt{1+\gamma'^2}} - \frac{t_R}{3}\frac{1}{\sqrt{1+\sigma^2}\sqrt{1+\gamma'^2}}\right)\sin\theta^\nu_{23}
\end{aligned}
\tag{3.31}
$$

The data on the solar-neutrino mixing U_{e2}, together with the ratio of mass differences, $\Delta m^2_{\nu 12}/\Delta m^2_{\nu 23} = m^2_{\nu 2}/(m^2_{\nu 3} - m^2_{\nu 2})$, can fix γ' and θ^ν_{23}, where the latter depends on a combination of a and b. Having fixed γ' and parameters in M_R, the atmospheric-neutrino mixing $U_{\mu 3}$ and U_{e3} are obtained as predictions.

We summarize our input and detailed fits as follows. For CKM matrix elements, we take $|V_{us}| = 0.224$, $|V_{ub}| = 0.0037$, $|V_{cb}| = 0.042$, and $\delta_{CP} = 60^\circ$ as inputs at electroweak scale. With a running factor of 0.8853 for $|V_{ub}|$, and $|V_{cb}|$ taken into account, we have $|V^0_{ub}| = 0.0033$ and $|V^0_{cb}| = 0.037$ at GUT scale. For charged lepton masses and up-type quark masses, we take the values at GUT scale corresponding to $\tan\beta = 10$ from Ref. [54]. For neutrino oscillation data, we take the solar neutrino angle to be $\theta_{\rm sol} = 32.5^\circ$ and mass square differences as $\Delta m^2_{\nu 12} = 7.9 \times 10^{-5}\text{eV}^2$ and

$\Delta m^2_{\nu 23} = 2.4 \times 10^{-3}\text{eV}^2$. The result for the 12 fitted parameters is

$$\begin{aligned}
\sigma &= 1.83, \\
\epsilon &= 0.1446, \\
\delta &= 0.01, \\
\delta' &= 0.014, \\
\phi &= 27.9^\circ, \\
\eta &= 1.02 \times 10^{-5}, \\
\omega &= -0.0466, \\
\rho &= 0.0092, \\
\kappa &= 0.0191, \\
M_U &= 82.2 \text{ GeV}, \\
M_D &= 583.5 \text{ MeV}, \\
\Lambda_R &= 1.85 \times 10^{13} \text{ GeV}
\end{aligned} \tag{3.32}$$

There is a combined constraint on a and b, and thus the right-handed Majorana mass spectrum is not well determined. As examples, if $a = b$, $a = -2.039 \times 10^{-3}$; and if $a = 1$, $b = -1.951 \times 10^{-3}$.

We show the result for the down-type quark masses and right-handed Majorana neutrino masses (taking a and b as those values in Eq. (4.21) which are fitted to produce the baryon asymmetry) as follows,

$$
\begin{aligned}
m_d^0 &= 1.08 \text{ MeV}, \\
m_s^0 &= 25.97 \text{ MeV}, \\
m_b^0 &= 1.242 \text{ GeV}, \\
M_1 &= 2.27 \times 10^{10} \text{GeV}, \\
M_2 &= 3.61 \times 10^{10} \text{GeV}, \\
M_3 &= 1.85 \times 10^{13} \text{GeV} \ .
\end{aligned}
\tag{3.33}
$$

The predictions for the mixing angles in the PMNS matrix are,

$$
\begin{aligned}
\sin^2\theta_{\text{atm}} &= 0.49, \\
\sin^2 2\theta_{13} &= 0.074 \ .
\end{aligned}
\tag{3.34}
$$

The result for θ_{atm} is particularly interesting: Although the lopsided mass matrix model is built to generate a large atmospheric-neutrino mixing angle, the charged lepton mass matrix alone produces a 2-3 rotation of 63° instead of 45° because of the constraint from the lepton mass spectrum. With an additional rotation $\theta_{23}^{\nu} \simeq 21°$ fixed mainly from the ratio of mass differences $\Delta m^2_{\nu 12}/\Delta m^2_{\nu 23}$, the nearly maximal atmospheric mixing 44.6° comes out as a prediction. If one releases the best-fit value of $\Delta m^2_{\nu 12}$ and $\Delta m^2_{\nu 23}$ and only imposes the 3σ constraint as $7.1 \times 10^{-5}\text{eV}^2 \leq \Delta m^2_{\nu 12} \leq 8.9 \times 10^{-5}\text{eV}^2$ and $1.4 \times 10^{-3}\text{eV}^2 \leq \Delta m^2_{\nu 23} \leq 3.3 \times 10^{-3}\text{eV}^2$, one would obtain, as shown in Fig. 3.1, $0.44 \leq \sin^2\theta_{\text{atm}} \leq 0.52$ which is well within the 1σ limit, and $0.055 \leq \sin^2 2\theta_{13} \leq 0.110$ which, as a whole region, lies in the scope of next generation of reactor experiments.

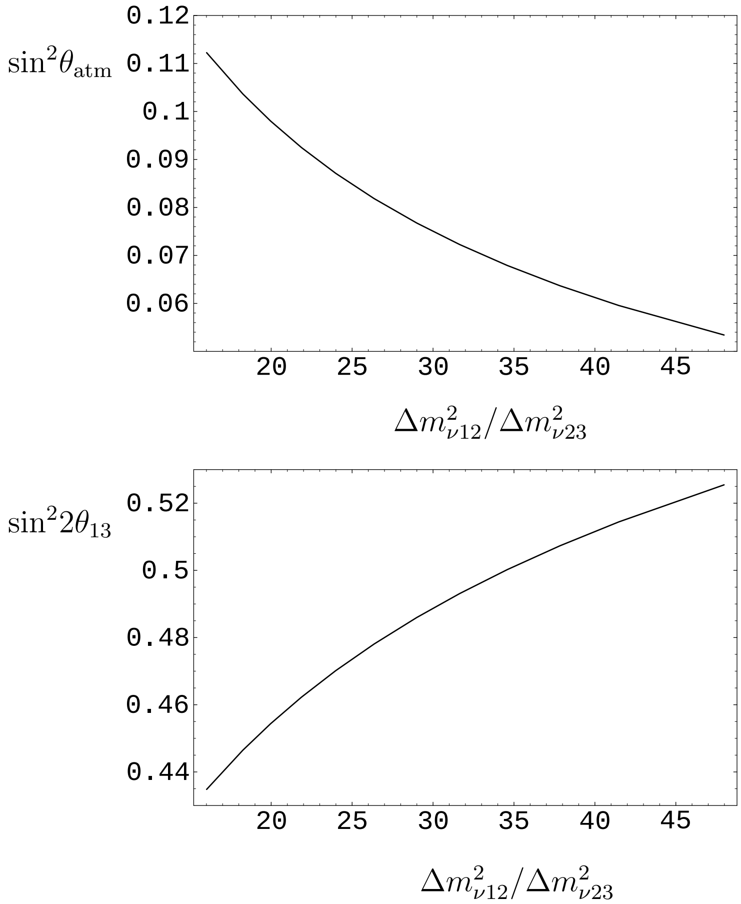

Figure 3.1: The predictions of $\sin^2\theta_{\rm atm}$ and $\sin^2 2\theta_{13}$ against the mass square difference ratio $\Delta m^2_{\nu 23}/\Delta m^2_{\nu 12}$. The region of $\Delta m^2_{\nu 23}/\Delta m^2_{\nu 12}$ is obtained from the values of $\Delta m^2_{\nu 23}$ and $\Delta m^2_{\nu 12}$ within their 3σ limits.

In summary, an SUSY SO(10) GUT model with lopsided structure is constructed to overcome the fine-tuning problem associated with the original lopsided model of Albright, Babu and Barr. It contains 13 parameters. After fitting them to experimental data, it yields a number of predictions. Whenever the experimental data are available, they work well. Most interestingly, the model predicts a $\sin^2 2\theta_{13}$ around 0.074, which is significantly larger than that from any of previous lopsided models. It can surely be tested through the next generation of reactor neutrino experiments.

Chapter 4

BARYOGENESIS VIA LEPTOGENESIS IN LOPSIDED MODEL

4.1 Introduction: Various Scenarios of Baryogenesis

One of the most fundamental questions in modern cosmology is where the baryon number asymmetry in today's Universe comes from. Baryon asymmetry of the Universe is customarily defined as the ratio of the baryon density to the photon density after the recombination, and has been measured to very good precision from the WMAP experiment [58]:

$$\eta_B = \frac{n_B}{n_\gamma} = (6.1 \pm 0.2) \times 10^{-10}. \tag{4.1}$$

The big-bang nucleosynthesis is completely consistent with this determination. Interestingly, if the baryon density in the Universe is larger or smaller than this number by two orders of magnitude, galaxies would not form and human beings would not exist. Instead of assuming this asymmetry as the initial condition, modern physicists believe it is generated dynamically in the early epoch of the Universe.

It has been realized long time ago by Sakharov [15] that there are three necessary conditions of generating a baryon asymmetry. The first condition is that there must be a fundamental process that violates baryon number. The second condition is that both C and CP must be simultaneously violated. This is because the processes producing particles and antiparticles are connected by both C and CP, with the only difference between C and CP being related to polarizations. Thus with either C or CP conserved in a process, the total numbers of particles and antiparticles produced in the process, regardless of the polarization, are guaranteed to be the same. The third condition is that the baryon number violating process must be out of equilibrium. This is because the number densities of particles in equilibrium are completely fixed by their masses. Since the particle and antiparticle have the same mass, they are of the same abundance in equilibrium.

There are many scenarios of baryogenesis in which three Sakharov conditions are satisfied. The GUT baryogenesis scenario is the earliest one. The advent of GUT realized the first condition — baryon number violation [16, 17] (which was thought not to be satisfied in SM). Like in SM, the second condition, C and CP violations, can be naturally satisfied in GUT. The third condition, out of equilibrium condition, can be satisfied if the baryon asymmetry is generated in the delayed decay of the heavy particles that are out of equilibrium [59].

Soon after this GUT baryogenesis scenario was proposed, it was realized that the baryon asymmetry generated in this scenario would be entirely washed out by the sphaleron process. As noted by 't Hooft [18], the baryon number is not conserved in SM due to the quantum corrections. There are many vacua corresponding to different topological charges. The tunnelling between different vacua causes the violation of the baryon number. However, this tunnelling, being suppressed by the factor of $e^{-16\pi/g^2}$, was thought to be very weak until Kuzimin, Rubakov, and Shaposhnikov [19] noted that the tunnelling is not weak at a temperature close to or higher than the electroweak scale. The solution corresponding to this transition is called “sphaleron”, which means “ready to fall”.

The presence of the sphaleron process makes the GUT scenario not a viable one any more. Nevertheless, the sphaleron process itself makes another scenario possible, that is, electroweak baryogenesis. The idea is that the baryon asymmetry can be generated in the sphaleron process itself. Since the sphaleron process is in equilibrium above the electroweak scale, the only place that it can generate the asymmetry is in the first-order elctroweak phase transition. However, in order for the electroweak phase transition to be of a strong first-order, the Higgs mass need be significantly smaller than the critical value, which is 73.3 ± 6.4 GeV from lattice simulation [60]. This is in odd with the empirical lower limit 114 GeV of the Higgs mass obtained at LEP [61]. SUSY relaxes this tension by introducing extra scalar particles [62]. In this case, a strong first-order electroweak transition requires $m_\phi < 100 \sim 115$ GeV, which has a small overlap with the empirical lower bound 91 GeV in MSSM [61]. Another problem of electroweak baryogenesis in SM is that the CP violation in SM is too small. This problem is also solved in the presence of SUSY which introduces extra CP violation phases. To summarize, the electroweak baryogenesis is ruled out in SM but still possible in the presence of SUSY.

Given the problems of the GUT baryogenesis and the difficulty of electroweak baryogenesis, baryogenesis via leptogenesis [20] emerges as a rather attractive scenario and has been extensively studied [63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78]. The idea of baryogenesis via leptogenesis is based on the observation that although baryon number and lepton number are both violated in the sphaleron process, the $B - L$ number is conserved since it is anomaly free. Thus, if the asymmetry of $B-L$ can be generated at high energy, the subsequent sphaleron process will not erase this asymmetry. What the in-equilibrium sphaleron process does is just to redistribute the $B-L$ asymmetry into baryon asymmetry and lepton asymmetry with a fixed proportion [74, 75]

$$\Delta B_f = \frac{8N_G + 4N_H}{22N_G + 13N_H}\Delta(B-L), \tag{4.2}$$

where the $\Delta(B-L)$ is the initial $B-L$ number produced at high energy and also the final $B-L$ number after the sphaleron process, the ΔB_f is the final baryon number, N_H is number of Higgs doublets, and N_G is the number of families. In SM, one has $N_G = 3$ and $N_H = 1$, and therefore

$$\Delta B_f = \frac{28}{79}\Delta(B-L). \tag{4.3}$$

Obviously, the final baryon number is zero if $\Delta(B-L) = 0$ initially. This is the case in GUT baryogenesis scenario, where the baryon and lepton asymmetries are generated in such a way that the $B-L$ asymmetry is zero. On the other hand, the baryon asymmetry would be generated if the $B-L$ asymmetry can be produced at high energy. Therefore, the $B-L$ violation is the necessary condition for this to happen. The SO(10) GUT naturally realizes this because $B-L$, as a gauge symmetry of SO(10), has to be broken when SO(10) is broken to SM. Moreover, in order for the $B-L$ violation process to be out of equilibrium, it has to happen at a very high energy scale. This is consistent with the small neutrino mass in the seesaw mechanism. Thus, the baryogenesis via leptogenesis and the small neutrino masses are correlated and both of them can be studied in the SO(10) GUT framework.

In the following, we study the baryogenesis via leptogenesis in the SO(10) models with the lopsided structure, which have been shown to produce neutrino masses and mixing among other things.

4.2 Baryogenesis via Leptogenesis in SO(10) Models with Lopsided Structure

In the baryogenesis via leptogenesis scenario, the lepton number asymmetry, which is also the $B - L$ asymmetry, is produced through the out-of-equilibrium decay of heavy right-handed neutrinos. This lepton number asymmetry is converted to the baryon number asymmetry by the in-equilibrium sphaleron process, in which the $B - L$ asymmetry remains conserved.

In this scenario, several considerations have to be made. First, what is the number of right-handed neutrinos decaying out of thermal equilibrium? The answer to this question in principle depends on the thermal history of the right-handed neutrinos. In our model, it turns out that this dependence is rather weak because of the strong washout. Second, what is the lepton density generated from a right-handed neutrino decay? This, of course, is related to the CP asymmetry of the decay which in turn depends on the Yukawa interactions. Third, a part of the generated lepton number asymmetry gets washed out by inverse-decay processes and scattering. This effect can be rather important, particularly in the so-called strong washout region. Finally, one must calculate the percentage of lepton number density converted into the baryon number density through the electroweak sphaleron process. The answers to some of the questions are less model-dependent and are standard in the literature [76]. Here we focus on the parts depending on a particular model.

4.2.1 Leptogenesis in the New SO(10) Lopsided Model

The density of leptons from right-handed neutrino decays is

$$n_L = \frac{3\zeta(3) g_N T^3}{4\pi^2} \sum_{i=1}^{3} \kappa_i \epsilon_i \ , \tag{4.4}$$

where the first factor is the thermal density of a relativistic fermion with $g_N = 2$ and the sum is over the number of right-handed neutrinos. The ϵ_i is the CP asymmetry in the decay of the i-th right-handed neutrino; κ_i is the corresponding efficiency factor, taking into account the fraction of out-of-equilibrium decays and the washout effect. Both factors depend on the effective mass defined as

$$\widetilde{m}_i = \frac{(M'_{\nu_D} M'^{\dagger}_{\nu_D})_{ii}}{M_i} \ . \tag{4.5}$$

where M'_{ν_D} denotes the neutrino Dirac mass matrix in the basis in which the right-handed neutrino matrix is real and diagonal and M_i is the mass of the right-handed neutrino.

The lepton number is converted into the baryon number through the $B - L$ conserving electroweak sphaleron effect as shown in Eq. (4.2, 4.3). The photon density can be calculated from the entropy density $s = \frac{2}{45} g_* \pi^2 T^3$, where g_* is the effective number of degrees of freedom, through the relation

$$s = \frac{4}{3} \frac{\pi^2}{30} \left(2 + \frac{21}{11} \right) \frac{\pi^2}{2\zeta(3)} n_\gamma \tag{4.6}$$

where the second factor takes into account the neutrino contribution. Ignoring the lightest right-handed neutrino contribution, g_* is 106.75 in SM.

The final ratio of baryon to photon number density through leptogenesis is

$$\eta_B = \frac{n_B}{n_\gamma} = -\frac{602}{53009}\sum_i \kappa_i \epsilon_i = -0.0114 \sum_i \kappa_i \epsilon_i \ . \tag{4.7}$$

The right-handed neutrinos are assumed to be CP eigenstates in the absence of the Yukawa type of weak interactions. Through the interactions, they can decay into both left-handed leptons (neutrino and charged leptons) plus Higgs bosons and right-handed antileptons plus Higgs bosons. In the leading order, the decay rate is

$$\Gamma_i = \frac{1}{8\pi}(Y'Y'^\dagger)_{ii} M_i \ , \tag{4.8}$$

in SM. Here again, Y' is the Yukawa matrix in the basis where the right-handed neutrinos are in the mass eigenstates.

At next-to-leading order, the decay rates into leptons and antileptons are different due to the complex phases in the Yukawa couplings. The decay asymmetry is defined as

$$\epsilon_i = \frac{\Gamma(N_i \to l_j H) - \Gamma(N_i \to \bar{l}_j H^\dagger)}{\Gamma(N_i \to l_j H) + \Gamma(N_i \to \bar{l}_j H^\dagger)} \ . \tag{4.9}$$

In one-loop approximation, one finds,

$$\epsilon_i = \frac{1}{8\pi}\sum_{j\neq i} F\left(\frac{M_j^2}{M_i^2}\right)\frac{\mathrm{Im}[(Y'Y'^\dagger)_{ij}^2]}{(Y'Y'^\dagger)_{ii}} \ , \tag{4.10}$$

where the decay function [77] is given by

$$F(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\frac{1+x}{x}\right] \ . \tag{4.11}$$

In the limit of large x, this become $-3/2\sqrt{x}$. The first term in F is singular when two right-handed neutrinos become degenerate in mass, in which case, one must resum the self-energy corrections, which leads to the so-called resonant leptogenesis [67, 68].

As mentioned in the last Chapter, the complex parameters a and b are not completely fixed by fermion masses and mixing. Here, their values are fitted to produce right amount of baryon asymmetry

$$a = 0.00129 e^{-1.808i} , \qquad b = 0.00198 e^{-3.210i} , \tag{4.12}$$

which leads to the following masses of three right-handed neutrinos

$$M_1 = 2.27 \times 10^{10} \text{ GeV}, \quad M_2 = 3.61 \times 10^{10} \text{GeV}, \quad M_3 = 1.85 \times 10^{13} \text{ GeV} . \tag{4.13}$$

We see that M_1 and M_2 are of the same order, but there is no need to fine-tune their values to obtain the resonant enhancement. The Yukawa matrix in the basis in which the right-handed neutrino mass matrix is diagonal and real is

$$Y'_{ij} = \begin{pmatrix} 4.8 \times 10^{-6} e^{-0.9i} & 0 & 0.0046 e^{-0.9i} \\ 0 & 0 & -0.022 e^{-1.6i} \\ 0.013 & -0.022 & 0.472 \; . \end{pmatrix} \tag{4.14}$$

Plugging in the Yukawa matrix and mass ratios, we find the following CP asymmetries,

$$\begin{aligned} \epsilon_1 &= -0.92 \times 10^{-5} , \\ \epsilon_2 &= -0.24 \times 10^{-5} . \end{aligned} \tag{4.15}$$

Here we have shown the CP asymmetry from the second right-handed neutrino as well because its mass is close to the first one, and hence is potentially important for leptogenesis. The result for ϵ_1 exceeds slightly the bound derived by Davidson and Ibarra [69] because the masses are not so hierarchical.

In our model, M_1 is close to M_2, and M_3 is much heavier. Thus, it is a good approximation to neglect the CP asymmetries and lepton number generated from the heaviest right-handed neutrinos (those with mass M_3). However, since $\delta_2 \equiv (M_2 - M_1)/M_1 = 0.53$ is less than 1, one has to consider the full decay and washout effects from the two light right-handed neutrinos.

The efficiency factor can be calculated by solving the Boltzmann equation for the right-handed neutrinos and lepton densities. The result depends on the effective mass $\widetilde{m}_i$. In the present case, we find,

$$\widetilde{m}_1 = 29.1 \text{ meV} , \qquad \widetilde{m}_2 = 406 \text{ meV} , \tag{4.16}$$

The effective masses determine the so-called decay parameters $K_i = \widetilde{m}_i/m^*$ where $m^* = 16\pi^{5/2}\sqrt{g^*}v^2/(3\sqrt{5}M_{pl}) = 1.08 \times 10^{-3}$eV. In our case

$$K_1 = 27.0 , \qquad K_2 = 376.2 . \tag{4.17}$$

Since $K_i \gg 1$, we are in the so-called strong washout region. In this region, the effective factor has little dependence on the thermal history of the right-handed neutrinos. One can assume for instance that they are not present in the beginning but are produced entirely by the inverse scattering processes.

Since the M_1 and M_2 are close to each other, there exists mutual washout due to the two lightest right-handed neutrinos. This situation has been discussed recently in Ref [70], where analytical formulas have been derived from numerical solutions of the Boltzmann equations,

$$\kappa_1 = \frac{2}{z_B(K_1 + K_2^{(1-\delta)^3}) \cdot (K_1 + K_2^{(1-\delta)})} \ , \tag{4.18}$$

$$\kappa_2 = \frac{\left[1 + 2\ln\left|\frac{1+\delta}{1-\delta}\right|\right]^2}{z_B(K_2 + K_1^{(1-\delta)^3}) \cdot (K_2 + K_1^{(1-\delta)})} e^{-\frac{8\pi}{3}K_1\left(\frac{\delta}{1+\delta}\right)^{2.1}} \ , \tag{4.19}$$

where $z_B = M_1/T_B$ is the inverse temperature at which the washout effects are minimized and κ_2 is valid when $\delta \equiv (M_2 - M_1)/M_1 < 1$ [71]. Plugging in the parameters, we find,

$$\kappa_1 = 6.8 \times 10^{-3} \ , \qquad \kappa_2 = 1.3 \times 10^{-4} \ . \tag{4.20}$$

Thus, because $K_2 \gg K_1$, one has $\kappa_1 \gg \kappa_2$. Therefore, the number of out-of-equilibrium decays from the second lightest right-handed neutrino is more than an order of magnitude smaller than that from the lightest one.

Putting everything together, the baryon asymmetry in our model is

$$\eta_B = -0.96 \times 10^{-2} \sum_i \kappa_i \epsilon_i = 6.0 \times 10^{-10} \ , \tag{4.21}$$

which agrees with the present observation.

Having shown that our model can have enough CP violation at high energy for leptogenesis, we are interested to see the size of the CP violation at low energy. The low energy CP violation is encoded into one Dirac CP phase δ_{CP} and two Majorana phases ϕ_1 and ϕ_2 in the PMNS matrix. It has been shown by Branco, Morozumi, Nobre and Rebelo in [78] that there is no model-independent relation between the CP violations at high and low energies. The predictions of the low energy CP phases from our model are all small. The scatter plots of these CP phases versus

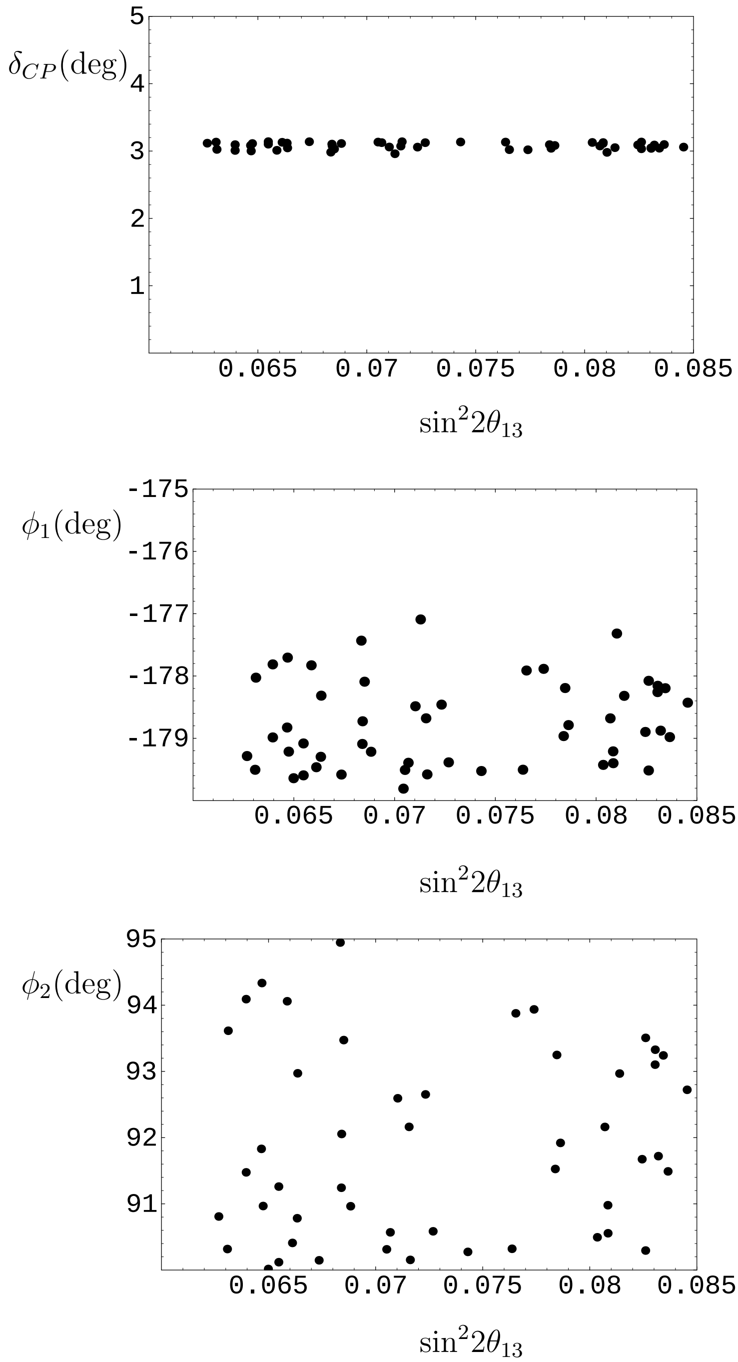

Figure 4.1: The predictions of δ_{CP}, ϕ_1, and ϕ_2 against $\sin^2 2\theta_{13}$. The plots are chosen according to the requirement of producing enough leptogenesis and satisfying the 3σ range of neutrino oscillation data as described in the context.

the $\sin^2\theta_{13}$ are shown in Fig. 4.1. Those points are chosen to satisfy the 3σ range of neutrino oscillation data: $0.7 \leq \sin^2 2\theta_{12} \leq 0.92$; $\sin^2 2\theta_{23} \geq 0.87$; $\sin^2\theta_{13} \leq 0.051$; $7.1 \times 10^{-5}\text{eV}^2 \leq \Delta m^2_{\nu 12} \leq 8.9 \times 10^{-5}\text{eV}^2$; $1.4 \times 10^{-3}\text{eV}^2 \leq \Delta m^2_{\nu 23} \leq 3.3 \times 10^{-3}\text{eV}^2$. As shown in these scatter plots, the δ_{CP} is constrained to be around 3 degree, and the ϕ_1 and ϕ_2 are within 3 degree and 5 degree deviation from -180 and 90 degree, respectively, indicating very small CP violation at low energy. The prediction of $\sin^2 2\theta_{13}$ is shown to be within the range $0.06 \leq \sin^2 2\theta_{13} \leq 0.085$.

4.2.2 Leptogenesis in Albright and Barr's SO(10) Lopsided Model

The main structure of the lopsided of Albright and Barr has been presented in Eq. (3.24,3.25) in Chapter 3. A set of parameters which reproduce the quark and charged-lepton spectra and mixings are,

$$\begin{aligned}
\epsilon &= 0.147, \\
\eta &= 6 \times 10^{-6}, \\
\delta &= 0.00946, \\
\delta' &= 0.00827, \\
\sigma &= 1.83, \\
\phi &= 2\pi/3, \\
m_U &= 113 \text{ GeV}, \\
m_D &= 1 \text{ GeV}.
\end{aligned} \tag{4.22}$$

Given the above, additional parameters, a, b, c and Λ_R, can be easily found to fit the neutrino mass differences and mixing. However, the model generates a very large $\widetilde{m}_i$, which in turn produces a very large decay width for the lightest right-handed neutrino. As a consequence, the efficiency factor κ is too small. To enhance the lepton number production, the masses of the two lightest right-handed neutrinos are forced to a near degeneracy, yielding a large resonant decay asymmetry.

In a recent publication, a very extensive search in the parameter space was conducted to find a viable leptogenesis in the model [50]. One of the solution has the following parameters,

$$\begin{aligned}
\eta &= 1.1 \times 10^{-5}, \\
\delta_N &= -1.0 \times 10^{-5}, \\
\delta'_N &= -1.5 \times 10^{-5}, \\
\Lambda_R &= 2.85 \times 10^{14} \text{ GeV}, \\
a &= c = 0.5828i, \\
b &= 1.7670i.
\end{aligned} \tag{4.23}$$

These parameters lead to the following right-handed neutrino masses,

$$M_1 \sim M_2 = 5.40 \times 10^8 \text{ GeV} , \qquad M_3 = 2.91 \times 10^{14} \text{ GeV} . \tag{4.24}$$

The η_B we calculate from these parameters, however, is 2.5×10^{-6}, roughly a factor of 2 smaller than that quoted in Ref. [50]. The difference comes from the CP asymmetry of the decay. When the masses of the two right-handed neutrinos are close, one cannot use the one-loop result in Eq. (4.10) directly. One has to resum the self-energy correction [67] to arrive at

$$\epsilon_1 \approx \frac{\mathrm{Im}[(Y'Y'^{\dagger})^2_{12}]}{8\pi(Y'Y'^{\dagger})_{11}} \frac{r_N}{r_N^2 + [(Y'Y'^{\dagger})_{11}/8\pi]^2} , \tag{4.25}$$

$$\epsilon_2 \approx \frac{\mathrm{Im}[(Y'Y'^{\dagger})^2_{21}]}{8\pi(Y'Y'^{\dagger})_{22}} \frac{r_N}{r_N^2 + [(Y'Y'^{\dagger})_{22}/8\pi]^2} .$$

where $r_N = (M_1^2 - M_2^2)/(M_1 M_2)$ is the degeneracy parameter.

It is worth pointing out that although the CP asymmetry tends to be enhanced in the case of two lightest right-handed neutrinos being quasi-degenerate, the washout effect is also enlarged in this case. Fortunately, in the present model, $\widetilde{m}_2 \sim \widetilde{m}_1$, so the effect is not particularly large. The modified numerical results are listed in Table 4.1.

From the above discussions, it is obvious that the key parameter which controls the main features of the leptogenesis is the effective mass $\widetilde{m}_1$: For a small $\widetilde{m}_1$, the efficiency factor is large, and one only needs a moderate value of the decay asymmetry ϵ to accomplish leptogenesis. For a large values of $\widetilde{m}_1$, the out-of-equilibrium decays are rare, and a successful leptogenesis requires a large decay asymmetry, which is possible when the masses of right-handed neutrinos become degenerate. A list of parameters relevant to leptogenesis of various realistic SO(10) models are presented in Table 4.1 [72], which illustrate this trend.

	BPW	GMN	JLM	DMM	AB
$M_1(GeV)$	10^{10}	10^{13}	3.77×10^{10}	10^{13}	5.4×10^8
$-\epsilon$	$2.0 \times 10^{-6} \sin 2\phi$	1.94×10^{-6}	1.0×10^{-5}	$10^{-4} \sin 2\phi$	9.4×10^{-4}
$\widetilde{m}_1(eV)$	0.003	0.006	0.026	0.1-0.4	5.4
κ	6×10^{-2}	1.2×10^{-2}	6.3×10^{-3}	10^{-3}	1.4×10^{-5}
η_B	$12 \times 10^{-10} \sin 2\phi$	4.97×10^{-10}	6.2×10^{-10}	$10^{-9} \sin 2\phi$	2.6×10^{-10}
$\sin^2 2\theta_{13}$	≤ 0.1	0.12	0.06-0.085	$0.014 - 0.048$	0.008

Table 4.1: Predicted mass M_1 of the lightest right-handed neutrino, CP asymmetry ϵ, effective mass $\widetilde{m}_1$, efficiency factor κ and baryon asymmetry η_B, and θ_{13} in various SO(10) models. The order is arranged according to the size of $\widetilde{m}_1$. The BPW, GMN, JLM, DMM, and AB denote the models in Ref. [44], [42], [57], [43], and [50], respectively.

Chapter 5

TESTING LOPSIDED STRUCTURE IN B PHYSICS

5.1 Introduction

It has been shown that an SO(10) GUT model with lopsided structure can be constructed to naturally explain the large atmospheric neutrino mixing and small quark mixing in the 2-3 sector. This model can fit to all the fermion masses and mixing, and give an amount of baryon asymmetry consistent with the experiment data. Nevertheless, there are many other realistic SO(10) GUT models that are also successful in terms of fitting fermion masses. Given this situation, we are motivated to investigate the following question: what is the most characteristic feature of the models with lopsided structure and where to test it?

From the lopsided structure itself, it is clear that the most characteristic feature is the large right-handed 2-3 mixing of down-type quarks associated with the large atmospheric neutrino mixing angle. The question is where and how to see its signature. Clearly this right-handed mixing has nothing to do with the quark CKM matrix. However, in the presence of SUSY, this large right-handed mixing has the potential of generating sizable off-diagonal elements in the soft mass matrices of squarks which, in turn, can be manifested in the flavor-changing neutral current interaction of down-type quarks, for example, the $b \to s$ transition.

The penguin dominated $b \to s$ transition ($b \to ss\bar{s}$) has long been regarded as a golden channel for probing new physics. Moreover, if there are phases associated with the new physics contributing to this transition, there could be new CP violations in B decays.

To facilitate our discussion of CP violations in B decays, we follow the commonly used notation of defining A_f, $\overline{A}_f$, $A_{\overline{f}}$, and $\overline{A}_{\overline{f}}$ as decay amplitudes of $P \to f$, $\overline{P} \to f$, $P \to \overline{f}$, and $\overline{P} \to \overline{f}$, respectively, where P and f denote decaying meson and final multi-particle states, respectively, and $\overline{P}$ and $\overline{f}$ denote their CP conjugate states. The CP violations in decays depend on $|\overline{A}_{\overline{f}}/A_f|$. For charged meson, CP violations are only manifested in decays. For neutral meson P^0 and $\overline{P}^0$ which mix with each other, the CP violations also show up in mixing. The mass eigenstates P_L and P_H of neutral meson system, where subscripts L and H denote light and heavy, respectively, are linear combinations of strong interaction eigenstates P^0 and $\overline{P}^0$

$$|P_{L,H}\rangle = p_{L,H}|P^0\rangle \pm q_{L,H}|\overline{P}^0\rangle, \tag{5.1}$$

with

$$|p_{L,H}|^2 + |q_{L,H}|^2 = 1. \tag{5.2}$$

The Hamiltonian of this system is

$$\mathcal{H} = M - \frac{i}{2}\Gamma \tag{5.3}$$

in the basis of P^0 and $\overline{P}^0$. The conservation of either CP or CPT leads to

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}}. \tag{5.4}$$

If either CP or T is conserved, M_{12} and Γ_{12} are relatively real and we have

$$|q/p| = 1. \tag{5.5}$$

The three types of CP violations in meson decays include [22]:

1. CP violations in decays which depend on

$$|\overline{A}_{\overline{f}}/A_f| \neq 1. \tag{5.6}$$

This is the only source of CP asymmetries in charged meson decays

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma(P^- \to f^-) - \Gamma(P^+ \to f^+)}{\Gamma(P^- \to f^-) + \Gamma(P^+ \to f^+)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2 - 1}{|\overline{A}_{f^-}/A_{f^+}|^2 + 1}. \tag{5.7}$$

2. CP violations in mixing which depend on

$$|q/p| \neq 1. \tag{5.8}$$

This is the only source of CP violations in the time-dependent asymmetry of charged-current semileptonic neutral meson decays

$$\mathcal{A} \equiv \frac{d\Gamma/dt[\overline{P}^0_{phy}(t) \to l^+X] - d\Gamma/dt[P^0_{phy}(t) \to l^-X]}{d\Gamma/dt[\overline{P}^0_{phy}(t) \to l^+X] + d\Gamma/dt[P^0_{phy}(t) \to l^-X]} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \tag{5.9}$$

3. CP violations in interference between a decay without mixing $P^0 \to f$ and a decay with mixing $P^0 \to \overline{P}^0 \to f$ which depend on

$$\mathcal{I}m(\lambda_f) \neq 0, \tag{5.10}$$

where

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f}. \tag{5.11}$$

This can be observed in the time-dependent asymmetry of neutral meson decays into final CP eigenstates f_{CP}

$$\mathcal{A}_{f_{CP}(t)} \equiv \frac{d\Gamma/dt[\overline{P}^0_{phy}(t) \to f_{CP}] - d\Gamma/dt[P^0_{phy}(t) \to f_{CP}]}{d\Gamma/dt[\overline{P}^0_{phy}(t) \to f_{CP}] + d\Gamma/dt[P^0_{phy}(t) \to f_{CP}]}, \tag{5.12}$$

which has a simple form in the case of B mesons [79]

$$\mathcal{A}_f(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \tag{5.13}$$

with

$$S_f \equiv \frac{2\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}, \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}. \tag{5.14}$$

The neutral B meson decays $B_d \to \phi K_S$ and $B_d \to \eta' K_S$ involve $b \to s s \bar{s}$ at the quark level and therefore is penguin dominated. Within SM, the indirect CP asymmetry parameter $S_{\phi K_S}$ and $S_{\eta' K_S}$ are essentially the same as that of $B \to J/\psi K_S$: $S^{SM}_{\phi K_S} \simeq S^{SM}_{\eta' K_S} \simeq S_{J/\psi K_S} = \mathrm{Sin}2\beta = 0.685 \pm 0.032$. However, the experimental values of $S_{\phi K_S}$ and $S_{\eta' K_S}$ from BaBar and Belle [80] show large deviations from the SM prediction:

$$\begin{aligned} S^{\text{exp.}}_{\phi K_S} &= 0.50 \pm 0.25^{+0.07}_{-0.04} \quad \text{(BaBar)}, \\ &= 0.06 \pm 0.33 \pm 0.09 \quad \text{(Belle)}, \\ S^{\text{exp.}}_{\eta' K_S} &= 0.27 \pm 0.14 \pm 0.03 \quad \text{(BaBar)}, \\ &= 0.06 \pm 0.18 \pm 0.04 \quad \text{(Belle)}, \end{aligned} \tag{5.15}$$

with the average of $S^{\text{exp.}}_{\phi K_S} = 0.34 \pm 0.20$ and $S^{\text{exp.}}_{\eta' K_S} = 0.41 \pm 0.11$, which display 1.7σ and 2.5σ deviations from the SM predictions, respectively. This significant discrepancy between SM prediction and experiment data has generated tremendous amount of effort in searching for beyond SM physics [81, 82].

It has been pointed out in Ref. [82] that there could exist the correlation between the large atmospheric mixing and the large $b \to s$ transition, based on the connection between left-handed charged leptons and right-handed down-type quarks in the framework of SO(10) GUT. However, it should be noted that this correlation depends exclusively on the lopsided structure, which is the only way of realizing the possible connection between left-handed charged leptons and right-handed down-type quarks. Within other realistic SO(10) models [41, 42, 43, 44] without lopsided structure, a set of parameters of the same order are combined constructively and destructively to give large atmospheric angle and small quark 2-3 mixing angle, respectively. In these SO(10) models [41, 42, 43, 44], there is typically no large right-handed down-type quark mixing associated with the large atmospheric mixing.

We propose to test this correlation by investigating CP-conserving and CP-violating observables in B decays [57]. Our study [83] shows that the $S_{\phi K_S}$ and $S_{\eta' K_S}$ could indeed have large deviations from their SM values in the SUSY context. Moreover, we find a particular pattern of correlation between $S_{\phi K_S}$ and $S_{\eta' K_S}$, which makes this class of models distinguishable from other types. We expect the similar result from Albright and Barr model [45, 46, 50], because these two models are nearly the same in the down-type quark sector.

This Chapter is organized as follows. The Sec. 5.2 is devoted to the calculation of flavor changing parameters from the our SUSY SO(10) GUT model. In Sec. 5.3, we present the predictions of $S_{\phi K_S}$ and $S_{\eta' K_S}$ with the constraints from $b \to s\gamma$ as well as the recent measurement of ΔM_{B_S}, and discuss how the particular pattern of predictions can be used to differentiate models with lopsided structure from others by future experiments.

5.2 SUSY Flavor Violation Parameters from the SO(10) GUT Model

If SUSY is assumed, the low energy phenomenology is governed by the MSSM, which introduces more particles and parameters than those in SM.

In SM, there is only one Higgs field. The anomaly cancellation does not involve Higgs fields because they are scalars. However, in SUSY theories, the Higgs superfields involve fermionic degrees of freedom (Higgsino). In order for the anomaly cancellation, two Higgs superfields H_u and H_d with opposite hyper-charge need be introduced. This introduces two more parameters. One of them is

$$\tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}, \tag{5.16}$$

which arises in the electroweak breaking as the ratio of vevs of two Higgs fields H_u and H_d. Thus

$$\langle H_u \rangle = v\sin\beta, \quad \langle H_d \rangle = v\cos\beta, \tag{5.17}$$

with $v = 175$ GeV as fixed from the $W^{\pm}$ and Z^0 masses. The reasonable range of $\tan\beta$ is $1 \leq \tan\beta \leq 60$ [84].

The other parameter is the coupling between these two Higgs superfields in the superpotential

$$\mu H_u H_d, \tag{5.18}$$

which is referred as the μ term. Although the coupling μ can be a very large scale, the radiative electroweak breaking condition requires it to be of the same order as the soft SUSY breaking scale.

While there are only two extra parameters needed in the unbroken MSSM, the soft-broken MSSM introduces more parameters

$$
\begin{aligned}
-\mathcal{L}_{soft} \;=\;& (m^2_{\tilde{l}})_{ij}\tilde{l}^\dagger_{Li}\tilde{l}_{Lj} + (m^2_{\tilde{e}})_{ij}\tilde{e}^\dagger_{Ri}\tilde{e}_{Rj} + (m^2_{\tilde{q}})_{ij}\tilde{q}^\dagger_{Li}\tilde{q}_{Lj} + (m^2_{\tilde{u}})_{ij}\tilde{u}^\dagger_{Ri}\tilde{u}_{Rj} \\
& + (m^2_{\tilde{d}})_{ij}\tilde{d}^\dagger_{Ri}\tilde{d}_{Rj} + (m^2_{H_u})H^\dagger_u H_u + (m^2_{H_d})H^\dagger_d H_d \\
& + \left(A^{ij}_u H_u \tilde{q}_{Li}\tilde{u}^\dagger_{Rj} + A^{ij}_d H_d \tilde{q}_{Li}\tilde{d}^\dagger_{Rj} + A^{ij}_e H_d \tilde{l}_{Li}\tilde{e}^\dagger_{Rj} + B_h H_u H_d + \text{H.C.}\right) \\
& + \left(\frac{1}{2}M_1\tilde{B}_L\tilde{B}_L + \frac{1}{2}M_2\tilde{W}^a_L\tilde{W}^a_L + \frac{1}{2}M_3\tilde{g}^a_L\tilde{g}^a_L + \text{H.C.}\right). \qquad (5.19)
\end{aligned}
$$

Here the first few quadratic terms are soft masses of doublet-slepton $\tilde{l}_L$, the right-handed charged slepton $\tilde{e}_R$, the doublet-squark $\tilde{q}_L$, the right-handed up-type squark $\tilde{u}_R$, the right-handed down-type squark $\tilde{d}_R$, and the Higgs bosons. The terms in the third lines are soft trilinear A terms and bilinear B term. The terms in the last line are gaugino mass terms, where $\tilde{B}$, $\tilde{W}$, and $\tilde{g}$ are B-ino, W-inos, and gluinos, respectively.

The soft masses and A terms are 3×3 matrices in the family space, with off-diagonal terms causing flavor violations. The parameters in soft terms could be complex and therefore induce new CP violations. In fact, the soft terms in its most general form introduce too many extra parameters, leading to too large flavor violations and CP violations. This is the infamous flavor problem and CP problem of SUSY. To solve this problem, the universality condition has to be imposed at the SUSY breaking scale M_*. This universality condition, which can be naturally realized in some SUSY breaking scenario, requires the soft mass matrices to be proportional to the unit matrix, and the soft trilinear matrices to be proportional to the corresponding Yukawa coupling (alignment condition) with universal coefficients m_0^2 and A_0, respectively. Under this, the number of parameters of MSSM is considerably reduced. Moreover, the three gaugino masses M_1, M_2, and M_3 can be traced back to a single gaugino mass $M_{1/2}$ at the GUT scale. Thus with the universality condition imposed, there are only 5 extra parameters in MSSM

$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \mu. \tag{5.20}$$

Although the number of parameters is dramatically reduced with the universality condition imposed on the SUSY breaking scale M_*, the RG running to the electroweak scale M_{EW} still generates flavor violation terms.

The SUSY flavor violation and flavor-violating CP violation in the quark sector are induced by the off-diagonal elements of squark mass-squared matrices m_{AB}^2 with $A, B = L, R$ indicating the chirality. Taking the down-type squark as an example, the mass-squared matrix $(m^d)_{ij}^2$ is

$$-\mathcal{L} = \begin{pmatrix} \tilde{d}_{Li}^\dagger, & \tilde{d}_{Ri}^\dagger \end{pmatrix} \begin{pmatrix} (m_{LL}^d)_{ij}^2 & (m_{RL}^d)_{ij}^2 \\ (m_{LR}^d)_{ij}^2 & (m_{RR}^d)_{ij}^2 \end{pmatrix} \begin{pmatrix} \tilde{d}_{Lj} \\ \tilde{d}_{Rj} \end{pmatrix}, \tag{5.21}$$

where $i, j = 1, 2, 3$ are family indices. The $(m_{LL}^d)^2$ and $(m_{RR}^d)^2$ are Hermitian matrix. They are given by

$$\begin{aligned} (m_{LL}^d)_{ij}^2 &= (m_{\tilde{q}}^2)_{ij} + m_{d_i}^2\delta_{ij} + m_Z^2\delta_{ij}\cos2\beta\left(-\frac{1}{2} + \sin^2\theta_W\right), \\ (m_{RR}^d)_{ij}^2 &= (m_{\tilde{d}}^2)_{ij} + m_{d_i}^2\delta_{ij} - m_Z^2\delta_{ij}\cos2\beta\sin^2\theta_W, \end{aligned} \tag{5.22}$$

where the $(m_{\tilde{q}}^2)_{ij}$ and $(m_{\tilde{d}}^2)_{ij}$ are soft masses, and the remaining terms originate from F-term and D-term and arise after the electroweak breaking. The $(m_{LR}^d)^2$ is given by

$$(m^d_{LR})^2_{ij} = A^{ij}_d v\cos\beta - m_{d_i}\mu\delta_{ij}\tan\beta \tag{5.23}$$

where the first term originates from soft trilinear coupling, and the second term is from the F-term in the scalar potential. The $(m^d_{RL})^2$ is the Hermitian conjugate of $(m^d_{LR})^2$.

In the mass insertion approximation (MIA) approach, the relevant parameters are δ_{AB}'s, which are the m^2_{AB} divided by the average squark mass-squared. We restrict ourselves to studying the gluino contribution, which is believed to be the dominant one due to the enhancement by the large gauge coupling α_S [85]. In the gluino-induced contribution, the relevant parameters for the $b \to s$ transition are the $(\delta^d_{LL,RR,LR,RL})_{23}$ of down-type quarks. We are going to show how these parameters are calculated in the SO(10) GUT model in this section.

Among the five SUSY parameters: $(m_0, m_{1/2}, A_0, \tan\beta, \mu)$, we assume $A_0 = 0$ at M_* (see [88] for justification). A nonzero A_0 does not bring any significant change to the results since the alignment condition is assumed. The magnitude of μ is fixed from the radiative electroweak breaking, whereas its phase ϕ_μ is constrained from the electric dipole moment (EDM) bounds, which, if cancellation exists, restrict ϕ_μ to be within $\pm\pi/10$ deviation from 0 or π [86, 87]. We take the ϕ_μ to be in the range of $(-\pi/10, \pi/10)$ for concreteness. The $\tan\beta$ is fixed to be 10 when our SO(10) model is constructed to fit fermion masses [57]. The two soft masses m_0 and $m_{1/2}$ are set to be within 1 TeV.

The off-diagonal elements of squark mass-squared matrices are generated from the RG running between the SUSY breaking scale M_* and the electroweak scale $M_{\rm EW}$. The GUT symmetry breaking scale, $M_{\rm GUT} = 2 \times 10^{16}$GeV, divides this running into two parts: above -$M_{\rm GUT}$ and below -$M_{\rm GUT}$ runnings.

In the following discussion, we will stick to the super-KM basis for the squark fields, in which the neutral current quark-gaugino-squark vertices are diagonal.

Below the $M_{\rm GUT}$, there are two Yukawa couplings in the quark sector: $\bar{d} Y_d Q H_d$ and $\bar{u} Y_u Q H_u$. The running of $(m^d_{RR})^2$ is proportional to $Y_d Y_d^\dagger$ which is diagonal in the super-KM basis of right-handed down-type squarks. Therefore no off-diagonal element of $(m^d_{RR})^2$ should be generated from the below-$M_{\rm GUT}$ running. Nevertheless, the running of $(m^d_{LL})^2$ involves both $Y_d^\dagger Y_d$ and $Y_u^\dagger Y_u$. While the former is diagonal in the super-KM basis of left-handed down-type squarks, the latter is not and could generate the off-diagonal elements of $(m^d_{LL})^2$. We have

$$(\delta^d_{LL})^{\rm below-GUT}_{ij} = -\frac{3}{8\pi^2}(Y_u^\dagger Y_u)_{ij} \ln(\frac{M_{GUT}}{M_{EW}}) \tag{5.24}$$

where again the Y_u is in the basis of $SU(2)_L$ doublet that Y_d is diagonal.

Above the $M_{\rm GUT}$, all the 16 fermions, including the right-handed neutrino, are in the 16 spinor representation of SO(10). The soft mass-squared m^2_{16} is renormalized by the single renormalizable operator $f_{33} 16_3 16_3 10_H$ in the model. As discussed in Ref. [89], the initial universal soft mass-squared $(m^2_{16})|_{M_*} = {\rm diag}(m_0^2, m_0^2, m_0^2)$ is not kept at $M_{\rm GUT}$: $(m^2_{16})|_{M_{GUT}} = {\rm diag}(m_0^2, m_0^2, m_0^2 - \Delta m^2)$. The change of 3-3 element Δm^2 is due to the renormalization by the operator $f_{33} 16_3 16_3 10_H$:

$$\Delta m^2 = \frac{60 m_0^2}{16\pi^2} f_{33}^2 \ln(\frac{M_*}{M_{GUT}}). \tag{5.25}$$

The parameter f_{33} is not completely fixed in the model and we choose it to be 1/2, which is in the reasonable range. This non-universal, diagonal, soft mass-squared matrix is in the GUT basis. After being rotated to the super-KM basis, off-diagonal elements of $(m^d_{RR,LL})^2$ are generated:

$$(m^d_{LL,RR})^2|_{\text{super-KM}} = U^\dagger_{L,R}(m^2_{16})|_{M_{GUT}}U_{L,R} \tag{5.26}$$

where $U_{L,R}$ are the unitary transformations that diagonalize the down-type quark mass matrix $M_d^{\text{digonal}} = U^\dagger_R M_d U_L$. The $(\delta^d_{LL,RR})^{\text{above-GUT}}$ is obtained as $(m^d_{LL,RR})^2|_{\text{super-KM}}$ divided by the average of its diagonal elements. Finally, the δ^d_{LL} is the sum of $(\delta^d_{LL})^{\text{below-GUT}}$ and $(\delta^d_{LL})^{\text{above-GUT}}$, while the $\delta^d_{RR} = (\delta^d_{RR})^{\text{above-GUT}}$ is only from above-GUT running.

That the correlation between atmospheric neutrino mixing and $b \to s$ transition depends exclusively on the lopsided structure can be explicitly seen here: In the lopsided flavor structure, the 2-3 element of M_d is large, and induces a large 2-3 rotation θ^R_{23} in U_R. This large rotation in turn produces a large $(\delta^d_{RR})^{above-GUT}_{23}$ as shown in Eq. (5.26). Finally, the large off-diagonal squark masses can generate a large $b \to s$ transition.

Although we set $A_0 = 0$ at M_*, it could be generated through radiative corrections. For $(m^d)^2_{RL,LR}$, the running below M_{GUT}, being proportional to Y_d, only induces diagonal elements in the super-KM basis. However, running from M_* to M_{GUT} does generate off-diagonal elements of $A_{LR,RL}$. In the GUT basis

$$A^d_{RL}|_{\text{GUT}} = c \begin{pmatrix} \frac{63}{2}\eta & 45\delta & 45\delta' \\ 45\delta & 0 & 45\sigma + 61\epsilon/3 \\ 45\delta' & -61\epsilon/3 & \frac{63}{2} \end{pmatrix} M_D, \tag{5.27}$$

where $c = \frac{1}{8\pi^2} g_{10}^2 M_{1/2} \ln(\frac{M_*}{M_{GUT}})$ and η, δ, δ', ϵ, and M_D are parameters fixed in the model [57]. The pre-coefficients 63/2, 45, and 61 are sums of Casimirs of SO(10) representations involved in the operators $16_i 16_j 10_H$, $16_i 16_j [\overline{16}_H \overline{16}'_H]_{10}$, and $16_i 16_j [\overline{16}_H \overline{16}'_H]_{10} 45_H$, respectively. Again, one simply applies $U_{R,L}$ on both sides of A^d_{RL} to go to the super-KM basis

$$A^d_{RL}|_{\text{super-KM}} = U^\dagger_R A^d_{RL}|_{\text{GUT}} U_L, \tag{5.28}$$

which, together with the diagonal μ term contribution, gives the full $(m^d_{RL})^2$: $(m^d_{RL})^2 = A^d_{RL}|_{\text{super-KM}} - \mu \tan\beta \ \text{diag}(m_d, m_s, m_b)$. Finally, $\delta^d_{RL,LR}$ is obtained as $\delta^d_{RL,LR} = (m^d_{RL,LR})^2/m_0^2$.

To see the characteristic feature of the lopsided structure, it is instructive to look at the size of all the δ's. Taking $m_0 = 300$ GeV, $m_{1/2} = 500$ GeV, and $\phi_\mu = \pi/10$ as an example, we have $(\delta^d_{RR})_{23} = 0.28 e^{-0.05i}$, $(\delta^d_{LL})_{23} = 0.0028 e^{-0.07i}$, $(\delta^d_{LR})_{23} = 0.00003 e^{-0.05i}$, $(\delta^d_{RL})_{23} = -0.0009 e^{-0.05i}$. Obviously, the $(\delta^d_{RR})_{23}$ is of several orders of magnitude larger than all the other δ's. Moreover, given this large $(\delta^d_{RR})_{23}$, a large effective $(\delta^d_{RL})^{\text{eff}}_{23}$ is generated from the double mass insertion: $(\delta^d_{RL})^{\text{eff}}_{23} = (\delta^d_{RL})_{23} + (\delta^d_{RR})_{23} \mu \tan\beta m_b / m_0^2$, which is $0.064 e^{-0.38i}$ for the same set of parameters. The effective $(\delta^d_{LR})^{\text{eff}}_{23}$ still remains small in the case of double mass insertion due to the smallness of $(\delta^d_{LL})_{23}$. As a result, the lopsided model predicts large $(\delta^d_{RR})_{23}$ and $(\delta^d_{RL})^{\text{eff}}_{23}$, and small $(\delta^d_{LL})_{23}$ and $(\delta^d_{LR})^{\text{eff}}_{23}$.

5.3 $S_{\phi K_S}$ and $S_{\eta' K_S}$ Predictions

The most general effective Hamiltonian $H_{eff}^{\Delta B=1}$ for the non-leptonic $\Delta B = 1$ processes is

$$\begin{aligned} H_{\text{eff}}^{\Delta B=1} &= \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) \\ & +\{Q_i \to \tilde{Q}_i, C_i \to \tilde{C}_i\}, \end{aligned} \tag{5.29}$$

where $\lambda_p = V_{pb}^{CKM} V_{ps}^{CKM*}$ and $C_i \equiv C_i(m_b)$ is the Wilson coefficient at the energy scale of b quark mass. The basis operators in $H_{eff}^{\Delta B=1}$ are

$$\begin{aligned} Q_1^p &= (\bar{p}_L^\alpha \gamma^\mu b_L^\beta)(\bar{s}_L^\beta \gamma^\mu p_L^\alpha), & Q_2^p &= (\bar{p}_L^\alpha \gamma^\mu b_L^\alpha)(\bar{s}_L^\beta \gamma^\mu p_L^\beta), \\ Q_3 &= (\bar{s}_L^\alpha \gamma^\mu b_L^\alpha) \sum_q (\bar{q}_L^\beta \gamma^\mu q_L^\beta), & Q_4 &= (\bar{s}_L^\alpha \gamma^\mu b_L^\beta) \sum_q (\bar{q}_L^\beta \gamma^\mu q_L^\alpha), \\ Q_5 &= (\bar{s}_L^\alpha \gamma^\mu b_L^\alpha) \sum_q (\bar{q}_R^\beta \gamma^\mu q_R^\beta), & Q_6 &= (\bar{s}_L^\alpha \gamma^\mu b_L^\beta) \sum_q (\bar{q}_R^\beta \gamma^\mu q_R^\alpha), \\ Q_7 &= (\bar{s}_L^\alpha \gamma^\mu b_L^\alpha) \sum_q \frac{3}{2} e_q (\bar{q}_R^\beta \gamma^\mu q_R^\beta), & Q_8 &= (\bar{s}_L^\alpha \gamma^\mu b_L^\beta) \sum_q \frac{3}{2} e_q (\bar{q}_R^\beta \gamma^\mu q_R^\alpha), \\ Q_9 &= (\bar{s}_L^\alpha \gamma^\mu b_L^\alpha) \sum_q \frac{3}{2} e_q (\bar{q}_L^\beta \gamma^\mu q_L^\beta), & Q_{10} &= (\bar{s}_L^\alpha \gamma^\mu b_L^\beta) \sum_q \frac{3}{2} e_q (\bar{q}_L^\beta \gamma^\mu q_L^\alpha)), \\ Q_{7\gamma} &= \frac{e}{8\pi^2} m_b \bar{s}_L^\alpha \sigma^{\mu\nu} b_R^\alpha F_{\mu\nu}, & Q_{8g} &= \frac{g_s}{8\pi^2} m_b \bar{s}_L^\alpha \sigma^{\mu\nu} t_{\alpha\beta}^A b_R^\beta G_{\mu\nu}^A, \end{aligned} \tag{5.30}$$

where $\sigma^{\mu\nu} = \frac{1}{2} i[\gamma^\mu, \gamma^\nu]$ and $\sum_q \equiv \sum_{q=u,d,s,c,b}$. The α and β are color indices, and $t_{\alpha\beta}^A$ are the SU(3) Gell-Mann matrices. Moreover, the $\tilde{Q}_i \equiv \tilde{Q}_i(m_b)$ are obtained from Q_i by the exchange of chirality $L \leftrightarrow R$. In SM, due to that charged current interaction only involves left-handed fields, the Wilson coefficients $\tilde{C}_i$ vanishes, while in MSSM, both C_i and $\tilde{C}_i$ exist.

The effective Hamiltonian $H_{eff}^{\Delta B=2}$ for the $\Delta B = 2$ processes is

$$H_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu) + \sum_{i=1}^{3} \tilde{C}_i(\mu) \tilde{Q}_i(\mu). \tag{5.31}$$

(Notice that the Wilson coefficients in Eqs. (5.29) and (5.31), although both denoted by C_i and $\tilde{C}_i$, are in fact different ones.) The basis operators in $H_{eff}^{\Delta B=2}$ are

$$\begin{aligned} Q_1 &= \bar{s}_L^\alpha \gamma_\mu b_L^\alpha \bar{s}_L^\beta \gamma^\mu b_L^\beta, \\ Q_2 &= \bar{s}_R^\alpha b_L^\alpha \bar{s}_R^\beta b_L^\beta, \\ Q_3 &= \bar{s}_R^\alpha b_L^\beta \bar{s}_R^\beta b_L^\alpha, \\ Q_4 &= \bar{s}_R^\alpha b_L^\alpha \bar{s}_L^\beta b_R^\beta, \\ Q_5 &= \bar{s}_R^\alpha b_L^\beta \bar{s}_L^\beta b_R^\alpha, \end{aligned} \tag{5.32}$$

and the operators $\tilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by the exchange of chirality $L \leftrightarrow R$.

All the relevant contributions of the high energy physics above W mass, including the SUSY particle contribution, enter the Wilson coefficients at $\mu = m_W$: $C(m_W)$ and $\tilde{C}(m_W)$. The matrix elements of local operators are, however, obtained at the energy scale of the bottom quark mass m_b. Therefore, one needs to obtain the Wilson coefficients at low energy by solving the RG equations of QCD and QED in SM:

$$C_i(m_b) = \sum_j \hat{U}(m_b, m_W) C_j(m_W) \ , \tag{5.33}$$

where the evolution matrix $\hat{U}(m_b, m_W)$ for $\Delta B = 1$ and $\Delta B = 2$ Wilson coefficients can be found in Ref. [90] and Ref. [91], respectively.

The SM and SUSY contributions to Wilson coefficients can be found in Refs. [92, 93]. It is worth noting that the SUSY contribution depends on the squark mass $m_{\tilde{q}}$ and gluino mass $m_{\tilde{g}}$, which are larger than the universal soft scalar mass m_0 and gaugino mass $m_{1/2}$ due to the RG running. We use the matrix elements of local operators evaluated in QCD factorization (QCDF), developed in Ref. [94], which makes the strong phase calculable, yet introduces undetermined parameters $\rho_{H,A}$ and phases $\phi_{H,A}$.

To make prediction of $S_{\phi K_S}$, we first impose constraints on the parameter space by requiring the prediction of branching ratio and CP asymmetry of $b \to s\gamma$ and ΔM_{B_S} to be within the experimental bounds.

The gluino contribution to the branching ratio $b \to s\gamma$ is [92]

$$
\begin{aligned}
BR(b \to s\gamma)_{\tilde{g}} &= \frac{\alpha_s^2 \alpha}{81\pi^2 m_{\tilde{q}}^4} \left\{ |m_b M_3(x)(\delta^d_{LL})_{23} \right. \\
&\quad \left. + m_{\tilde{g}} M_1(x)(\delta^d_{LR})_{23}|^2 + L \leftrightarrow R \right\} ,
\end{aligned} \tag{5.34}
$$

where the loop functions $M_1(x)$ and $M_3(x)$ with $x = m_{\tilde{q}}^2/m_{\tilde{g}}^2$ can be found in Ref. [92]. As discussed in Ref. [92], the experimental bound and the SM uncertainty together require that the gluino contribution $BR(b \to s\gamma)_{\tilde{g}} < 4 \times 10^{-4}$. The bound on the CP asymmetry $A^{CP}_{b\to s\gamma}$ plays no significant role in constraining the parameter space. Therefore we neglect its discussion here, although we have included it in the calculation carried out in the same way as in Ref. [93].

The D0 and CDF Collaborations [95] have reported new results for ΔM_{B_S}:

$$
\begin{aligned}
17 \text{ ps}^{-1} < \Delta M_{B_S} < 21 \text{ ps}^{-1} &\qquad \text{(D0)}, \\
\Delta M_{B_S} = 17.33^{+0.42}_{-0.21} \pm 0.07 \text{ ps}^{-1} &\qquad \text{(CDF)},
\end{aligned} \tag{5.35}
$$

while the best fit value in SM is $\Delta M_{B_S} = 17.5$ ps^{-1}. Much effort on the theory side [96, 97] has been generated by this recent experimental result. The result imposes the constraint $|R_M| \equiv |M_{12}^{SUSY}/M_{12}^{SM}| \leq 4/17$, where $M_{12} = \langle B_s^0 | H_{\rm eff}^{\Delta B=2} | \overline{B}_s^0 \rangle$. One should notice that this bound remains valid if one considers the uncertainty in the SM value and assumes $\Delta M_{B_S} = 21\ ps^{-1}$ [97].

The decay amplitude of $B_d \to \phi K_s$ is given by [93]

$$A_{B_d\to\phi K_s} = -i\frac{G_F}{\sqrt{2}} m_{B_d}^2 F_+^{B_d\to K_s} f_\phi \times \sum_{i=1\sim 10,7\gamma,8g} H_i(\phi)(C_i + \tilde{C}_i) \tag{5.36}$$

where $f_\phi = 0.233$GeV, and $F_+^{B_d\to K_s} = 0.35$ is the transition form factor evaluated at transferred momentum of order of m_ϕ. The $H_i(\phi)$'s are dependent on QCDF parameters $\rho_{H,A}$ and $\phi_{H,A}$ in the following way [93]

$$\begin{aligned}
H_1(\phi) &\simeq -0.0002 - 0.0002i, \\
H_2(\phi) &\simeq 0.011 + 0.009i, \\
H_3(\phi) &\simeq -1.23 + 0.089i - 0.005X_A - 0.0006X_A^2 - 0.013X_H, \\
H_4(\phi) &\simeq -1.17 + 0.13i - 0.014X_H, \\
H_5(\phi) &\simeq -1.03 + 0.053i + 0.086X_A - 0.008X_A^2, \\
H_6(\phi) &\simeq -0.29 - 0.022i + 0.028X_A - 0.024X_A^2 + 0.014X_H, \\
H_7(\phi) &\simeq 0.52 - 0.026i - 0.006X_A + 0.004X_A^2, \\
H_8(\phi) &\simeq 0.18 + 0.037i - 0.019X_A + 0.012X_A^2 - 0.007X_H, \\
H_9(\phi) &\simeq 0.62 - 0.037i + 0.003X_A + 0.0003X_A^2 + 0.007X_H, \\
H_{10}(\phi) &\simeq 0.62 - 0.037i + 0.007X_H, \\
H_{7\gamma}(\phi) &\simeq -0.0004, \\
H_{8g}(\phi) &\simeq 0.047.
\end{aligned} \tag{5.37}$$

The decay amplitude of $B_d \to \eta' K_s$ is given by [93]

$$\begin{aligned} A_{B_d\to\eta' K_s} &= -i\frac{G_F}{\sqrt{2}} m_{B_d}^2 F_+^{B_d\to K_s} f_{\eta'}^s \\ &\times \sum_{i=1\sim 10,7\gamma,8g} H_i(\eta')(C_i - \tilde{C}_i) \end{aligned} \quad (5.38)$$

where the $H_i(\eta')$'s are dependent on QCDF parameters $\rho_{H,A}$ and $\phi_{H,A}$ in the following way [93]

$$\begin{aligned} H_1(\eta') &\simeq 0.44 + 0.0005i, \\ H_2(\eta') &\simeq 0.076 - 0.064i + 0.006X_H, \\ H_3(\eta') &\simeq 2.23 - 0.15i + 0.009X_A + 0.0008X_A^2 + 0.014X_H, \\ H_4(\eta') &\simeq 1.76 - 0.29i + 0.026X_H, \\ H_5(\eta') &\simeq -1.52 + 0.004X_A + 0.008X_A^2, \\ H_6(\eta') &\simeq 0.54 - 0.029i + 0.006X_A + 0.027X_A^2 + 0.026X_H, \\ H_7(\eta') &\simeq 0.078 + 0.001X_A - 0.004X_A^2, \\ H_8(\eta') &\simeq -0.58 + 0.02i + 0.004X_A - 0.014X_A^2 - 0.004X_H, \\ H_9(\eta') &\simeq -0.44 + 0.054i + 0.005X_A - 0.0004X_A^2 - 0.007X_H, \\ H_{10}(\eta') &\simeq -0.80 + 0.02i - 0.004X_H, \\ H_{7\gamma}(\eta') &\simeq 0.0007, \\ H_{8g}(\eta') &\simeq -0.089. \end{aligned} \quad (5.39)$$

The SUSY contribution modifies the CP asymmetries as

$$\begin{aligned} S_{\phi K_S} &= \frac{\sin 2\beta + 2R_\phi \cos\delta_\phi \sin(\theta_\phi + 2\beta) + R_\phi^2 \sin(2\theta_\phi + 2\beta)}{1 + 2R_\phi \cos\delta_\phi \cos\theta_\phi + R_\phi^2}, \\ S_{\eta' K_S} &= \frac{\sin 2\beta + 2R_{\eta'} \cos\delta_{\eta'} \sin(\theta_{\eta'} + 2\beta) + R_{\eta'}^2 \sin(2\theta_{\eta'} + 2\beta)}{1 + 2R_{\eta'} \cos\delta_{\eta'} \cos\theta_{\eta'} + R_{\eta'}^2}. \end{aligned} \quad (5.40)$$

The $R_{\phi,\eta'}$, $\theta_{\phi,\eta'}$, and $\delta_{\phi,\eta'}$ are defined in the ratio

$$\begin{aligned} A^{SUSY}_{B_d\to\phi K_S}/A^{SM}_{B_d\to\phi K_S} &\equiv R_\phi e^{i\theta_\phi} e^{i\delta_\phi} \\ A^{SUSY}_{B_d\to\eta' K_S}/A^{SM}_{B_d\to\eta' K_S} &\equiv R_{\eta'} e^{i\theta_{\eta'}} e^{i\delta_{\eta'}} \end{aligned} \tag{5.41}$$

where $R_{\phi,\eta'}$ is the absolute value of the ratio, $\theta_{\phi,\eta'}$ is the SUSY CP violating weak phase which depends on the phases in δs, and $\delta_{\phi,\eta'} = \delta^{SM}_{\phi,\eta'} - \delta^{SUSY}_{\phi,\eta'}$ is the CP conserving strong phase depending on $\phi_{H,A}$.

Contrary to the $B \to \phi K$ transition, the initial and final states in $B \to \eta' K$ transition have opposite parity, therefore $\langle \eta' K|Q_i|B\rangle = -\langle \eta' K|\tilde{Q}_i|B\rangle$, and C_i and $\tilde{C}_i$ appear as $(C_i - \tilde{C}_i)$ in the amplitude $A_{B_d\to\eta' K_s}$ instead of $(C_i + \tilde{C}_i)$ in $A_{B_d\to\phi K_s}$ as shown in Eq. (5.36, 5.38). Since $C_{7\gamma,8g}$ depends on $(\delta^d_{LR})_{23}$, whereas $\tilde{C}_{7\gamma,8g}$ depends on $(\delta^d_{RL})_{23}$, this difference makes the correlation between $S_{\phi K_S}$ and $S_{\eta' K_S}$ in the case with large $(\delta^d_{LR})_{23}$ different from the case with large $(\delta^d_{RL})_{23}$. In fact, as shown in the general analysis in Ref. [93], the deviations of $S_{\phi K_S}$ and $S_{\eta' K_S}$ from the SM values are in the same direction if $(\delta^d_{LR})_{23}$ is large and in the opposite direction if $(\delta^d_{RL})_{23}$ is large. This turns out to be important in the following discussion of the correlation of $S_{\phi K_S}$ and $S_{\eta' K_S}$ predictions from the model with lopsided structure.

Besides the SUSY parameters $(m_0, m_{1/2}, A_0, \tan\beta, \phi_\mu)$, the undetermined parameters $\rho_{H,A}$ are constrained by $BR(B_d \to \phi K_S)$ to be within $\rho_{H,A} \leq 2$ [93], and the strong phase $\phi_{H,A}$ is not constrained.

By scanning over the allowed ranges of undetermined parameters $(m_0, m_{1/2}, \phi_\mu, \rho_{H,A}, \phi_{H,A})$ and imposing the bound of ΔM_{B_S}, as well as $BR(b \to s\gamma)$ and $A^{CP}_{b\to s\gamma}$, we find the allowed $(m_0, m_{1/2})$ shown in Fig. 5.1: There is a large parameter space

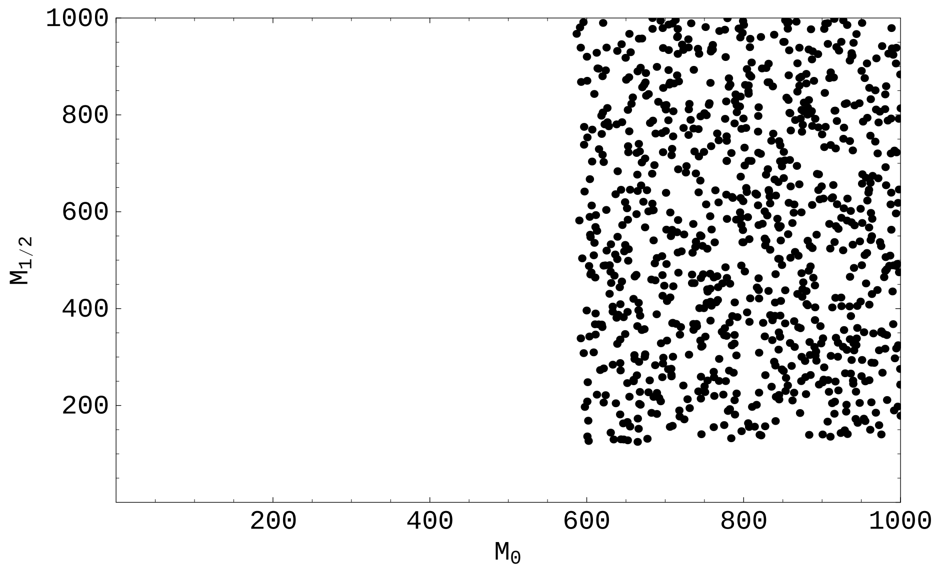

Figure 5.1: Scatter plot of the parameter space of m_0 and $m_{1/2}$ with the constraints from $BR(b \to s\gamma)$, $A^{CP}_{b\to s\gamma}$ and ΔM_{B_S}.

of $m_0, m_{1/2}$ satisfying the bound.

The corresponding predictions of $S_{\phi K_S}$ and $S_{\eta' K_S}$ are shown in Fig. 5.2, from which we see that the large $(\delta^d_{RR})_{23}$ does push the $S_{\phi K_S}$ and $S_{\eta' K_S}$ off their SM value 0.685. For the purpose of comparison, we set by hand the $(\delta^d_{RR})_{23}$ to be of the size of $(\delta^d_{LL})_{23}$, which would be the case without lopsided structure, and present the corresponding prediction of $S_{\phi K_S}$ and $S_{\eta' K_S}$ in Fig. 5.3, which, together with Fig. 5.2, shows clearly that the large deviation of $S_{\phi K_S}$ and $S_{\eta' K_S}$ from their SM values are exclusively due to the large $(\delta^d_{RR})_{23}$ of the lopsided models.

While Fig. 5.2 shows that the lopsided structure may explain the anomalies of both $S_{\phi K_S}$ and $S_{\eta' K_S}$, the correlation between these two quantities, shown in Fig. 5.4, indicates an interesting pattern: the large $(\delta^d_{RL})_{23}$ push $S_{\phi K_S}$ and $S_{\eta' K_S}$ in opposite directions. For points where $S_{\phi K_S}$ falls below the SM value, the $S_{\eta' K_S}$ becomes

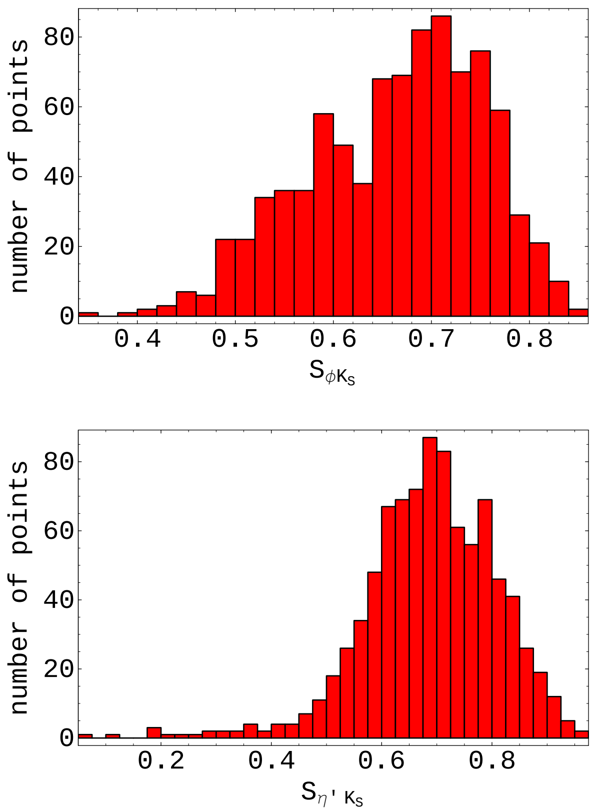

Figure 5.2: Predictions of $S_{\phi K_S}$ and $S_{\eta' K_S}$ corresponding to the points in Fig. 5.1.

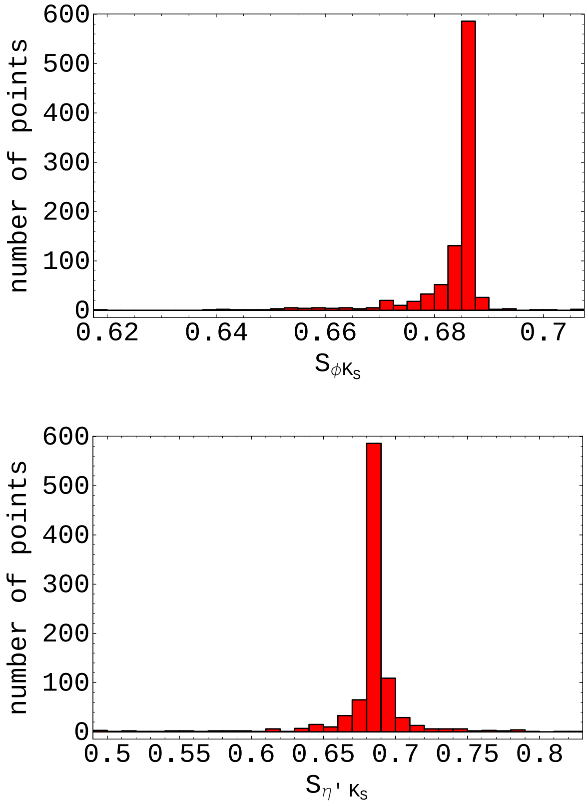

Figure 5.3: Predictions of $S_{\phi K_S}$ and $S_{\eta' K_S}$ in the case that there is no large $(\delta^d_{RR})_{23}$ and its induced $(\delta^d_{RL})_{23}$.

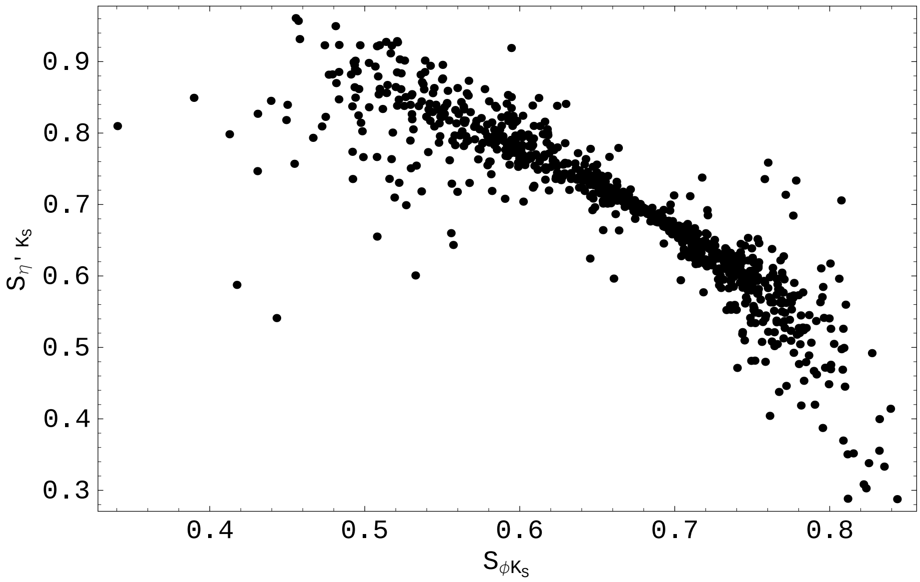

Figure 5.4: Scatter plot of predictions of $S_{\phi K_S}$ and $S_{\eta' K_S}$ corresponding to the points in Fig. 5.1.

larger than SM value, and vice versa. As discussed above, this specific pattern is intrinsic to the large $(\delta^d_{RR})_{23}$, which induces large $(\delta^d_{RL})_{23}$ yet leaves $(\delta^d_{LR})_{23}$ small. Therefore, it is tightly associated with the lopsided structure. This specific pattern of correlation between $S_{\phi K_S}$ and $S_{\eta' K_S}$ means that the lopsided flavor structure cannot be responsible for both anomalies simultaneously. If future experiments confirm that $S_{\phi K_S}$ and $S_{\eta' K_S}$ are indeed both significantly smaller than the SM values, the lopsided SO(10) model is ruled out, unless one assumes that SUSY parameters are such that large $(\delta^d_{RR})_{23}$ from the lopsided structure makes no significant contribution to the $b \to s$ transition and the $S_{\phi K_S}$ and $S_{\eta' K_S}$ anomalies are from other beyond SM physics sources. On the other hand, if future experiments show that the deviations of $S_{\phi K_S}$ and $S_{\eta' K_S}$ from their SM values are in opposite directions, it would be a strong evidence for the lopsided flavor structure.

In summary, we point out that a possible correlation between large atmospheric neutrino mixing and the $b \to s$ transition, first discussed in Ref. [82], in fact, depends exclusively on the lopsided SO(10) structure. We study the prediction of $S_{\phi K_S}$ and $S_{\eta' K_S}$ from a realistic SO(10) model with lopsided flavor structure with the constraints from ΔM_{B_S}, and $b \to s\gamma$ applied. We find that both quantities can show significant deviations from their SM values due to the lopsided structure, but with a specific type of correlation. We discuss that the specific correlation of the two quantities can be used to test the flavor structure in future experiments.

Chapter 6

CONCLUSIONS

As SUSY and GUT are two of the most promising ideas on beyond SM physics, the details of how these ideas are realized depend on specifics. It is hard to test these ideas without building realistic models and comparing their predictions with experiments. The work presented in this thesis follows this spirit.

We present a SUSY SO(10) GUT model with lopsided mass structure. This model naturally produces large neutrino mixing and small quark mixing. It does not have the fine-tuning problem present in the previous model of the same type in the literature. This model fits all the quark and lepton masses, mixing angles, as well as the CP violation phase in the CKM matrix. The reactor neutrino mixing is predicted to be large $0.055 \leq \sin^2 2\theta_{13} \leq 0.110$, and is within the reach of the next generation of reactor experiments.

We further investigate the baryogenesis via leptogenesis in this model and find right order of magnitude of baryon asymmetry can be produced. Compared with the previous model on this, which requires another fine-tuning to generate the resonant enhancement, this model has another advantage.

The most characteristic property of the models with lopsided structure is that there is a large mixing of right-handed down-type quarks associated with the large atmospheric neutrino mixing. We propose to probe this mixing in the $b \to s$ transition. A specific pattern of predictions of the indirect CP violation parameters $S_{\phi K_S}$ and $S_{\eta' K_S}$ in the $B_d \to \phi K_S$ and $B_d \to \eta' K_S$ decays is found to be associated with the lopsided structure. This pattern can be used to confirm or rule out this class of model with future precision measurements of these observables.

There are also other aspects of the model that need be studied in the future. For instance, it is interesting to study the proton decay. It would place the model on a more solid basis if one can construct the Higgs potential that generates the needed Higgs vevs, and if one can find extra global symmetries that lead to those operators in the model. These topics are beyond the scope of this thesis.

There is a long journey in front of us to pin down the beyond SM physics. In the past 30 years, the progress of the theoretical particle physics has been mainly driven by itself. The discovery of neutrino oscillations was certainly a breakthrough. Hopefully, we will start to know a lot more about the TeV physics with the LHC producing enough data. We believe a joint effort from both theorists and experimentalists will eventually reveal a new layer of the exciting truth of nature.

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