TECHNICAL RESEARCH REPORT

Lifetime Maximizing Adaptive Traffic Distribution and Power Control in Wireless Sensor Networks

by Fangting Sun, Mark Shayman

TR 2006-14

Lifetime Maximizing Adaptive Traffic Distribution and Power Control in Wireless Sensor Networks

Fangting Sun and Mark Shayman

Department of Electrical and Computer Engineering

University of Maryland, College Park, MD 20742

Email: {ftsun, shayman}@eng.umd.edu

Abstract

In this paper we study how to maximize the lifetime of randomly deployed wireless sensor networks by applying adaptive traffic distribution and power control. We model this problem as a linear program by abstracting the network into multiple layers. First we focus on the scenario where transmission energy consumption plays the dominant role in overall energy consumption. After ignoring the receiving energy consumption, we observe that: in order to maximally extend the lifetime, each node should split its traffic into two portions, where one portion is sent directly to the sink, and the other one to its neighbor in the next inner layer. Next we consider the effect of incorporating the processing energy consumption. In this case, we have similar observation: for each packet to be sent, the sender should either transmit it using the transmission range with the highest energy efficiency per bit per meter, or transmit it directly to the sink. Besides studying the upper bound of maximum achievable lifetime extension, we discuss some practical issues, such as how to handle the signal interference caused by adaptive power control. Finally, we propose a fully distributed algorithm to adaptively split traffic and adjust transmission power for randomly deployed wireless sensor networks. We also provide extensive simulation results which demonstrate that the network lifetime can be dramatically extended by applying the proposed approach in various scenarios.

I. Introduction

Wireless sensor networks have drawn extensive attention in recent years due to their potential applications, such as environmental monitoring, industrial sensing and diagnostics, battlefield surveillance, target tracking, search and rescue, and disaster relief [1], [2]. However, owing to their compact form and extremely low cost, sensor nodes are usually severely energy constrained. Furthermore, in a large-scale wireless sensor network with thousands of nodes, replacing batteries will be either extremely difficult or even impossible, especially in harsh environments. This leads to one of the most critical performance measures for wireless sensor networks: *network lifetime*.

In the literature, various types of techniques have been proposed to maximize the lifetime of wireless sensor networks under energy constraints. One approach is to design energy-aware routing protocols, such as the schemes proposed in [3]–[5]. Another approach is to apply energy-aware sleep scheduling, described in [6]–[8], which have addressed how and when to put the sensor nodes into sleep mode to reduce energy consumption. As demonstrated in [9]–[11], the network lifetime can also be extended by performing in-network data aggregation to exploit the redundancy among the collected data. By applying energy-efficient clustering and hierarchical routing, that is, dividing the network into multiple clusters and routing in a hierarchical structure, the network lifetime can also be extended, as shown in [12]–[14]. Another approach is to move the sink around in order to collect data from sensors to balance the energy consumption in different sensor nodes is also a possible option [15]. Another straightforward way to extend the network lifetime is to allocate more resources into specific areas to relieve the bottleneck effect, as illustrated in [16]–[18].

In this paper we investigate how to extend the lifetime of a randomly deployed wireless sensor network by applying adaptive traffic distribution and transmission range (power) adjustment. This is motivated by the following observation: in a sensor network where nodes need to send data to the sink and all nodes use the same transmission power, nodes around the sink will experience much higher power consumption rate than faraway nodes because of the extra relaying burden. As a consequence, these nodes will run out of energy very quickly, resulting in the death of the network, though there is still considerable unused energy left in those faraway nodes. If nodes can adaptively adjust their transmission range, nodes far away from the sink can at least send data directly to the sink to reduce relaying burden of down stream nodes, and consequently increase the network lifetime. Such an approach has also

been considered in [19]–[22] from the energy balancing aspect. The connection between our work and their work as well as the differences will be further discussed in Section VI.

As in [3], we define the network lifetime as the time elapsing between network deployment and the moment when the first node dies. After abstracting the network using a layered model, we can model the lifetime maximizing adaptive traffic distribution and power control problem as a linear program. In order to help better understand the problem and meanwhile shed light on the solution to more complicated scenarios, we first study the scenario by ignoring the processing energy consumption (e.g., circuit-level energy consumed during transmission and receiving). In this case, both numerical results and theoretical analysis have confirmed the following important finding: in order to maximally extend the network lifetime, for each packet to be sent, the node should transmit it either directly to the sink, or to the immediate next inner hop. The significance of such a finding lies in fact that it can lead to very simple and efficient distributed algorithms for splitting the traffic and adjusting the transmission power adaptively.

We then study the effect of incorporating the processing energy consumption into our problem. In this case a similar finding is obtained: for each packet to be sent, the sender should transmit it either directly to the sink, or to the certain inner layers with the highest energy efficiency per bit per meter. Moreover, the results indicate that incorporating processing energy consumption will not decrease the effectiveness of the proposed adaptive traffic distribution and power control approach. Furthermore, incorporating the processing energy consumption can even make the maximally extensible network lifetime increase. The results also show that the variation of the processing energy consumption will not significantly affect the extensible lifetime. In other words, the proposed approach can work in various scenarios under various sensor node settings.

Although adaptive transmission power adjustment can lead to significant lifetime extension, in practice, we may not be able to reach the maximally achievable extension that it has promised. One reason is that such an adaptive transmission power adjustment scheme can introduce extra signal interference, especially when long transmission ranges are used. To combat this issue, instead of focusing on designing complicated scheduling and medium access control schemes, in this work we propose a very simple yet effective approach: limiting the number of nodes that are allowed to adjust their transmission range. Specifically, only a certain number of nodes nearest to the sink are allowed to perform adaptive transmission range adjustment, and all other nodes will keep their transmission power fixed. Although this may reduce the maximally achievable lifetime extension, the simulation results demonstrate that the lifetime extension is still significant.

Decentralization is one key feature of wireless sensor networks. In order to make the proposed approach practically applicable, we need to implement it in a fully distributed way. Towards this goal, we propose Energy-Aware Data Propagation Algorithm (EADPA), a fully distributed algorithm, to perform online adaptive traffic distribution and transmission range adjustment. Our extensive results demonstrate that EADPA is very efficient and significantly extending the lifetime of randomly deployed wireless sensor networks in various scenarios and network settings.

The rest of this paper is organized as follows. Section II presents the network model and the problem formulation. Section III describes the numerical solutions to the problem as well as the theoretical analysis. The distributed algorithm is described in Section IV. Section V presents the simulation results. Some related works are discussed in Section VI. Finally Section VII concludes the paper.

II. System Model and Problem Formulation

We consider randomly deployed wireless sensor networks consisting of a set of homogeneous wireless sensors. Each sensor needs to submit the collected information to the sink which is roughly located at the center of the area. We assume that all sensors have the same amount of initial energy, denoted by E. This is usually true in randomly deployed wireless sensor networks. However, we do not put energy constraint on the sink, which also makes sense in practice. Given the network to be deployed, some Quality of Service (QoS) requirements, and specific types of sensors, we also pose a minimum and maximum transmission range limitation for each sensor, denoted by r_{min} and r_{max}. The value of r_{max} is usually decided by hardware constraint, while the value of r_{min} can be determined by both hardware limitation and QoS requirements, such as network connectivity.

In this paper, similar as in [17], [23], we model the transmission energy consumption at each node as follows:

$$P_t(r) = \beta \cdot r^\alpha \quad \text{per bit.} \tag{1}$$

Here α is the path loss exponent, r is the targeted transmission range, and β is a scalar indicating the energy needed to successfully convey an information bit to a unit distance.

Besides transmission energy consumption, circuit-level energy consumption, such as energy consumed during encoding, decoding, modulation, and demodulation, also play an important role in many scenarios. In this paper we refer to this as the processing energy. We first consider processing energy consumed during transmission. In general this contributes a constant additive term to the overall energy consumption, which can be molded as follows:

$$P_c = \gamma_1 \text{ per bit.} \tag{2}$$

The value of γ_1 is determined by the underlying communication technologies, such as the encoding and modulation schemes used. Similarly, the processing energy consumed per bit during receiving stage is modeled as follows:

$$P_r = \gamma_2 \text{ per bit.} \tag{3}$$

The value of γ_2 is also determined by the underlying technologies, such as the decoding and demodulation schemes used.

Next we model the lifetime maximization problem. If the exact distances between all pairs of nodes are known, it is possible to model the problem precisely. However, in randomly deployed wireless sensor networks, such information is usually impossible to obtain. To make the problem tractable, in this paper we first focus on a sensor network deployed inside a circular area with the sink located in the center. We divide the network into multiple layers: a node belongs to the lth layer if and only if its distance to the sink lies in the range $((l-1) \cdot r_{min}, l \cdot r_{min}]$, and the layer 0 is the sink. Here r_{min} also specifies the width of each layer. We also assume that the sensors in each layer are equally-spaced deployed. These assumptions will be relaxed later when we conduct performance evaluation.

Let R denote the radius of the network and let L denote the total number of layers in the network, that is, $L = \lceil \frac{R}{r_{min}} \rceil$. For any integers l, k with $0 \le k < l \le L$, let $x_{l,k}$ denote the average number of bits that a node in the lth layer needs to request nodes in the kth layer to forward per unit time. Let g denote the average number of bits each node will generate per unit time. We can readily check that the ratio between the number of nodes in the kth layer and the number of nodes in the lth layer $(k > l)$ is $\frac{2k-1}{2l-1}$. Thus the average number of bits that a node in the lth layer will receive from nodes in the kth layer $(k > l)$ should be $\frac{2k-1}{2l-1}x_{k,l}$. Let T_{life} denote the network lifetime and $P = \frac{E}{T_{life}}$ be the average energy consumption rate. Here maximizing T_{life} is equivalent to minimizing P. Then we can model the lifetime maximization problem as the following MIN-MAX linear program:

$$\min_{\{x_{l,k}, \ 1 \le l \le L, \ 0 \le k \le L\}} P \quad s.t. \tag{4}$$

$$\sum_{k=l+1}^{L} \frac{2k-1}{2l-1}x_{k,l} + g = \sum_{k=0}^{l-1} x_{l,k}, \quad 1 \le l \le L \tag{5}$$

$$\sum_{k=0}^{l-1} x_{l,k}P_t^{l,k} + \sum_{k=l+1}^{L} \frac{2k-1}{2l-1}x_{k,l}P_r \le P, \quad 1 \le l \le L \tag{6}$$

$$x_{l,k} \ge 0, \quad 0 \le k < l, \quad 1 \le l \le L \tag{7}$$

$$x_{l,k} = 0, \quad (l-k)r_{min} > r_{max}, 0 \le l, k \le L \tag{8}$$

$$x_{l,k} = 0, \quad 0 \le l \le k \le L \tag{9}$$

where $P_t^{l,k} = P_c + P_t((l-k)r_{min})$. Eqn. (5) models the traffic conservation, i.e., for each node the amount of transmitted traffic should be equal to the traffic received plus the traffic generated. Eqn. (6) poses the energy constraint. Eqn. (7) guarantees the feasibility of the solutions. Eqn. (8) limits each node's maximum transmission range. Eqn. (9) prevents nodes sending traffic further away from the sink.

III. NUMERICAL RESULTS AND THEORETICAL ANALYSIS

A. Results without Considering Processing Energy

To help better understand the problem and shed light on the solutions to more complicated scenarios, we first study the lifetime maximization problem (4)-(9) by setting $\gamma_1 = \gamma_2 = 0$, i.e., ignoring processing energy consumption. This
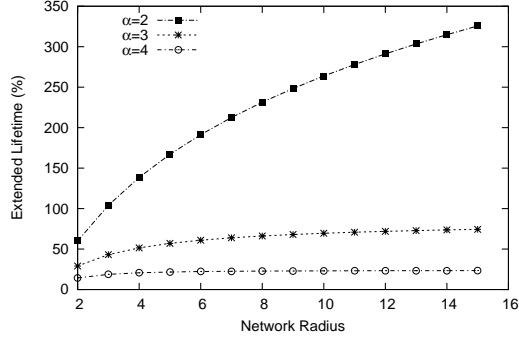
Fig. 1.   Possible Lifetime extension comparison for different path loss exponent and network radius
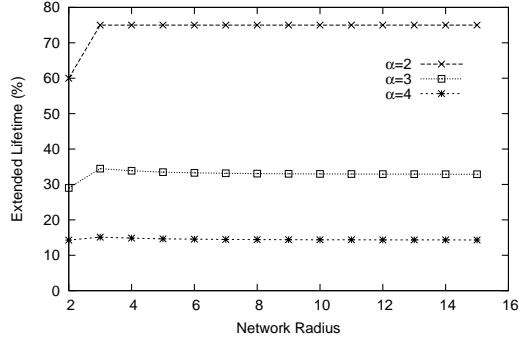

Fig. 2.   Lifetime extension under the constraint $r_{max} = 2r_{min}$

applies to the situations where the transmitting power plays a dominant role, such as in long range communication. Now for each node the energy consumed to transmit a bit r meters away is:

$$P_t(r) = \beta \cdot r^\alpha = \beta \cdot r_{min}^\alpha \cdot \left(\frac{r}{r_{min}} \right)^\alpha \qquad (10)$$

Without loss of generality we normalize $r_{min} = 1$.

Before presenting the theoretical results, we first examine the numerical solutions, and compare the lifetime extension under different settings. The baseline approach is as follows: each layer is only allowed to transmit to its next immediate inner layer, that is, $r_{max} = r_{min}$. In the following comparison, *extended lifetime (or lifetime extension)* denotes the ratio between the extended lifetime by other approach over the lifetime obtained by the baseline approach. In other words, if the extended lifetime is x%, the whole lifetime is (1+x%) times the lifetime of the baseline approach.

We first study the maximum possible lifetime extension that can be achieved by applying adaptive traffic distribution and power control. Fig. 1 illustrates the numerical results for different network radii and path loss exponents by setting $r_{max} \geq R$ and assuming interference-free medium access scheduling. From these results we can see that the network lifetime can be extended about 75% when the path loss exponent is 3, and around 25% when the path loss exponent is 4. When the path loss exponent is 2, the lifetime extension can be up to 325% for the network size 15 (here network size denotes the number of layers in the network). From these results we can observe that the larger the network size, the more the network lifetime can be extended, especially when the path loss exponent is low. When the path loss exponent becomes high, the benefit of increasing transmission range will be reduced due to the fact that longer transmission range results in lower energy efficiency per bit per meter.

As we mentioned before, in practice nodes usually have maximum transmission range constraint. So we next study the numerical solutions by imposing maximum transmission range constraint. Fig. 2 illustrates the results for one case: $r_{max} = 2r_{min}$. First, from these results we can see that even after imposing such a restrictive constraint, significant lifetime extension can still be achieved: 75%, 33% and 14% for path loss exponent being 2, 3, and 4 respectively. Furthermore, given a fixed path loss exponent, the extended network lifetime percentage remains almost the same for different network sizes.
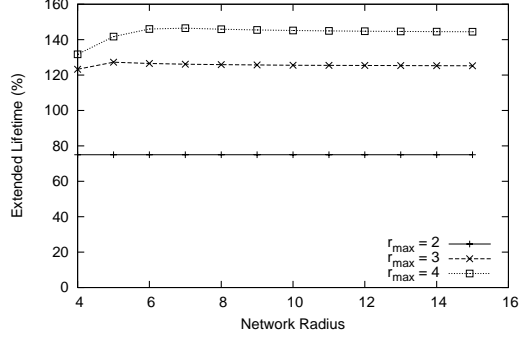
Fig. 3. Lifetime extension under different r_{max} constraint, $\alpha = 2$
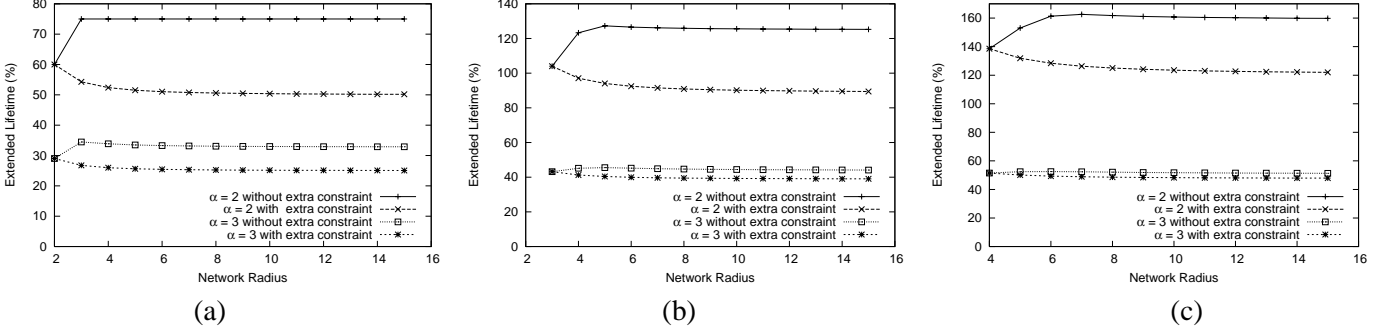

Fig. 4.   Comparison between constraining transmission power adjustment to innermost several layers versus imposing a maximum transmission range constraint. (a) $r_{max} = 2$ (b) $r_{max} = 3$ (c) $r_{max} = 4$

To better understand this phenomena, we slightly modify the original problem by putting an extra constraint: only the inner k layers are allowed to adjust their transmission range, while all the other layers keep their transmission range fixed at r_{min}. Then we can theoretical prove the following statements: when $\alpha = 2$, setting $k = 3$ can result in 75% lifetime extension; when $\alpha = 3$, setting $k = 4$ can result in 32.8% lifetime extension; when $\alpha = 4$, setting $k = 3$ can result in 128.% lifetime extension. Now we use $\alpha = 2$ as an example to explain how this can be achieved. We assume there are m nodes in the first layer. Then there are $(2i - 1)m$ nodes in the ith layer. The bound 75% can be achieved by applying the following strategy: the nodes in the 1st layer transmit $\frac{4}{7}L^2mg$ traffic to the sink; the nodes in the 2nd layer transmit $\frac{3}{7}L^2mg$ traffic to the sink and transmit none to the 1st layer; the nodes in the 3rd layer transmit $(\frac{4}{7}L^2 - 1)mg$ traffic to the 1st layer and $(\frac{3}{7}L^2 - 3)mg$ traffic to the 2nd layer. This suggests that when the maximum transmission range is constrained to 2, the network lifetime is mainly determined by how the inner four layers distribute traffic.

Now we study the more general cases by varying r_{max}. Fig. 3 illustrates the numerical results under different r_{max} values, where $\alpha = 2$. We can see that with the increase of r_{max}, the network lifetime extension also increases. This is easy to understand: with the increase of r_{max}, each node has more choices to send traffic to, and the optimization problem becomes less constrained. Similar to the results in Fig. 2, we can also observe that the extended lifetime remains almost the same for different network size, 125% for $r_{max} = 3$ and 160% for $r_{max} = 4$. The reason is as before: in this case the network lifetime is mainly determined by how those innermost layers behave.

To give a better idea of how an important role the innermost layers act as, we come back to the following simple strategy by imposing an extra constraint: only those innermost layers that can directly reach the sink are allowed to adaptively adjust their transmission range, while all the other layers fix the transmission range at r_{min}. Fig. 4 illustrates the numerical results as well as the comparison to the results without imposing this extra constraint, that is, all nodes can adjust their transmission range up to r_{max}. From these results we can see that although there is performance loss comparing to the case where all nodes are allowed to adjust transmission range, the lifetime extension is still significant: 50%, 91% and 126% when only innermost 2, 3, and 4 layers are allowed to adjust their transmission range, respectively. As will be mentioned many times later, the attraction of this extra constraint
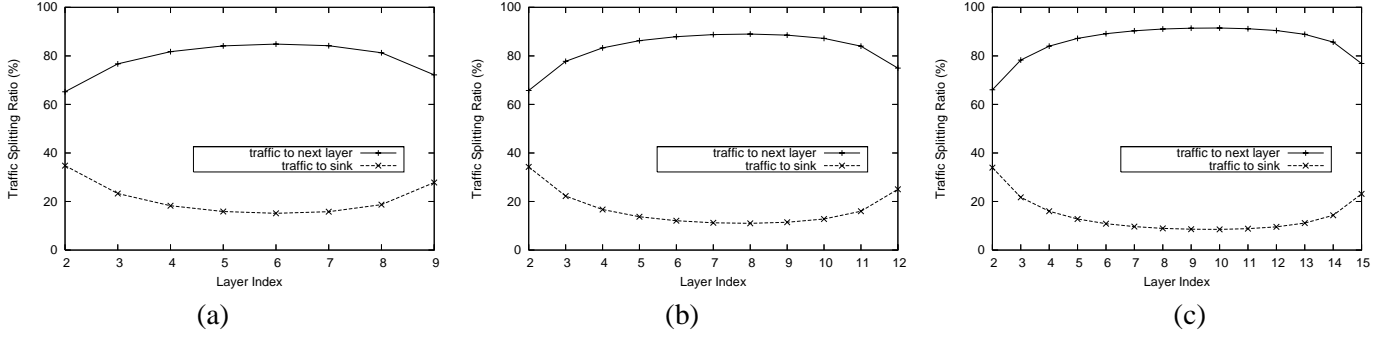
Fig. 5. Traffic splitting ratio for each layer, where the whole traffic is split between the next inner layer and the sink. Path loss exponent $\alpha = 2$. (a) network radius = 9 (b) network radius = 12 (c) network radius = 15

lies in that it can greatly simplify the implementation: if only some innermost layers are allowed to adjust their transmission power, the scheduling and medium access control protocol can be greatly simplified and the extra signal interference caused by power adjustment can be significantly reduced.

If we take a further look at the numerical solutions to the problem (4)-(9), we can see for each node its whole traffic will be split into two portions, one is directly sent to the sink and one is sent to the next inner layer , as illustrated in Fig. 5. This can be translated into the following statement: when there is no maximum transmission range constraint (e.g., $r_{max} \geq R$), the optimization problem (4)-(9) should have at least one optimal solution with the following simple form:

$$\{x_{i,i-1} \geq 0, \ x_{i,0} \geq 0, \ x_{i,j} = 0, \quad 1 \leq i \leq L, \ 1 \leq j \leq i - 2\} \tag{11}$$

The attraction of this form lies in that it can direct us to design efficient and fully distributed algorithm to perform online traffic distribution and power adjustment. Now the question is whether this can hold in general. To answer this question, we have formally prove the following theorem:

Theorem 1: **When $\alpha \geq 1$, $\gamma_1 = \gamma_2 = 0$, and each node can reach the sink by adjusting its transmission power, there always exists an optimal solution to the optimization problem (4)-(9) with the simple form (11).**

Proof: We assume there exist one optimal solution with arbitrary form for LP (4)-(9). We then redistribute the traffic on the links other than $\{x_{i,i-1}, x_{i,0}, 1 \leq i \leq L\}$ repeatedly until no such link exists. In this way, the original optimal solution is transformed into a simple form solution. Finally, we prove that this transformed simple form solution does not have shorter lifetime than original optimal solution. For detail proof, please refer to [24] ■

Lemma 1: **When $\gamma_1 = \gamma_2 = 0$, by applying the optimal solution for the optimization problem (4)-(9), the nodes in all layers will use energy at the same rate.**

Proof: Suppose by applying the optimal solution, the nodes in different layers use the energy at different rate. Considering the innermost layer j from those layers with the highest energy consumption rate, there are two cases:
I. If $j \geq 2$, let the energy consumption rate for nodes in layers j and $j - 1$ be p_1 and p_2 respectively. Let the nodes in layer j send $0 < \Delta x < \frac{(p_1 - p_2)(2j-3)}{(j-1)^\alpha (2j-1)}$ more traffic to layer $j - 1$ and the nodes in layer $j - 1$ send $\frac{2j-1}{2j-3}\Delta x$ more traffic to the sink. In this way the nodes in the layer j will reduce their energy consumption rate, and the nodes in the layer $j - 1$ will increase their energy consumption rate but smaller than p_1. Notice that since all nodes have the same initial energy and same traffic generation rate, this adjustment is always applicable. By applying this adjustment iteratively, the MIN-MAX power will be reduced, thus the original optimal solution is not optimal. Lemma holds;
II. If $j = 1$, we denote layer i as the layer where all layers between layer 1 and layer i (including layer 1 and layer i) have same energy consumption rate and layer $i + 1$ has smaller energy consumption rate. Let the energy consumption rate for nodes in layers i and $i + 1$ be p_1 and p_2 respectively. Let the nodes in layer $i + 1$ send $0 < \Delta x < \frac{(p_1 - p_2)}{(i+1)^\alpha - 1}$ more traffic to the sink then the nodes in layer i have less relay traffic. In this way the nodes in the layer i will reduce their energy consumption rate, and the nodes in the layer $i + 1$ will increase their energy consumption rate but smaller than p_1. Similar to case I, the Lemma holds. ■

Corollary 1: **When $\alpha \geq 1$, $\gamma_1 = \gamma_2 = 0$, the simple form (11) solution is the unique optimal solution to the optimization problem (4)-(9)**

Proof: According to the proof for Theorem 1, any other solution form can be transformed into simple form without decreasing the network lifetime. During the transformation, there always exist some layers whose energy consumption rate is lowered. It indicates that any other solution form except simple form can not be optimal solution according to Lemma 1. ∎

Now we take a further look at the results in Fig. 5. We can see that although the splitting ratio is different, all three subfigures have a similar shape: the nodes in the middle layers will send traffic to sink with a lower ratio, while the nodes in the layers either near the sink or near the boundary will send traffic to the sink with a higher ratio. The reason is that nodes in faraway layers near the boundary have less traffic, so they can afford to send a higher percentage of traffic directly to the sink, and nodes in innermost layers can afford to send a higher percentage of traffic directly to the sink becuase their distance to the sink is small.

B. Results Considering Processing Energy

Now we study the effect of processing energy consumption on the lifetime maximization problem (4)-(9). When no processing energy consumption is considered, due to the nonlinear (e.g., quadratic for $\alpha = 2$) increase of transmission power consumption with respect to the transmission range, shorter transmission range is usually preferred. In other words, as long as the network connectivity can be maintained and certain QoS requirements can be satisfied, the smaller the value of r_{min}, the higher the energy efficiency per bit per meter, and consequently the higher the maximum achievable lifetime extension. However, when processing energy consumption is also considered, shorter transmission range may not always be preferred to longer transmission range. Further, there should exist an optimal transmission range such that the energy consumption per bit per meter can be minimized, which is referred to as the characteristic distance [25]. In our model, it is easy to check that the characteristic distance d_{char} is:

$$d_{char} = \left(\frac{\gamma_1 + \gamma_2}{\beta(\alpha - 1)} \right)^{\frac{1}{\alpha}}. \tag{12}$$

Later we can see that d_{char} plays a critical role in the solutions to the optimization problem (4)-(9).

In order to obtain the optimal solution to (4)-(9), similar as in Section III-A, our first step is to apply numerical analysis. To make the results solid, our analysis is based on the typical energy consumption parameters as well as their deviations [26], [27]. To have a better understanding how processing energy consumption affects the results, we impose different constraints on the original problem (4)-(9). Specifically, four sets of constraints are imposed separately, as described in Table I. To compare the lifetime obtained under different constraints, we regard the lifetime obtained by imposing constraint C1 as the baseline.

TABLE I

EXTRA CONSTRAINTS IMPOSED ON THE ORIGINAL PROBLEM (4)-(9)

C1:	always transmit using r_{min}.
C2:	always transmit using d_{char}.
C3:	either transmit using d_{char}, or directly to the sink.
C4:	no extra constraint.

We set $\gamma_1 = 45$nJ/bit, $\gamma_2 = 135$nJ/bit, and $\beta = 10$pJ/bit/m^2 for $\alpha = 2$ [26]. Thus, $d_{char} \simeq 134m$. We then fix r_{min} to be the characteristic distance calculated based on the above parameters, and decrease the value of β to get different characteristic distance. Such decrease happens when the receiving technologies advances, or when some special decoding techniques are applied. We fix $R = 20r_{min}$. Fig. 6 illustrates the results obtained by imposing different constraints.

First, Fig. 6 shows that dramatic lifetime extension can be achieved by C3 and C4. For example, when $d_{char} = r_{min}$, more than 700% extension is achieved, while when $d_{char} = 5r_{min}$, the lifetime extension can be up to 5000%. The results indicate that instead of decreasing the effectiveness of adaptive traffic distribution and power control approach, the introduction of processing energy consumption can even further increase the maximum achievable network lifetime extension. Second, we can see that the lifetime extension also increases with the increase of characteristic distance. For example, even for C2, when $d_{char} = 5r_{min}$, the lifetime extension can reach 1200%. This suggests that transmitting using characteristic distance is much more energy efficient than transmitting using
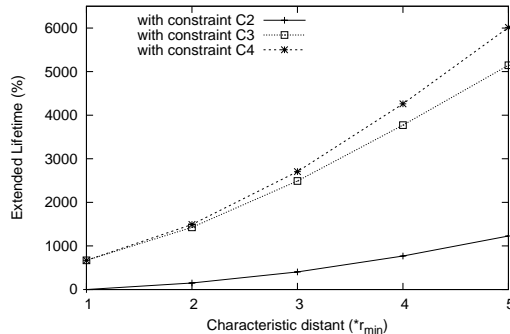
Fig. 6. Lifetime comparison among different scenarios. Both r_{min} and the network radius are fixed. $\alpha = 2$
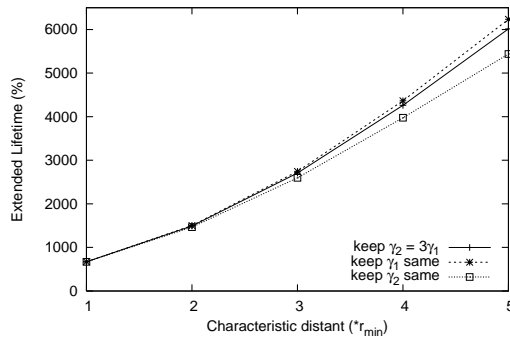

Fig. 7. Lifetime extension comparison by varying the ratio of γ_1/γ_2, $\alpha = 2$.

r_{min}. In other words, shorter transmission range is not always preferred with the existence of processing energy consumption. The reason is that when processing energy consumption play a more important role in overall energy consumption (i.e., $\frac{\gamma_1+\gamma_2}{\beta}$ increases), transmitting using a short range becomes less energy efficient per bit per meter.

After examining the numerical solutions, we observe that when no extra constraint is imposed (corresponding to C4), in most situations each node either directly transmit the traffic to the sink, or to some inner layers around d_{char} away. This also explains why the lifetime gap between C3 and C4 is small, where the lifetime obtained by imposing C4 is only slightly longer than the lifetime obtained by imposing C3. Due to its simplicity and the concern of distributed implementation, the constraint C3 is usually preferred.

So far when we change d_{char}, we fixed the values of γ_1 and γ_2. However, with the advance of technology, both γ_1 and γ_2 may change. For example, applying sophisticated decoding techniques may lead to the increase of γ_2, while applying sophisticated encoding techniques may increase γ_1, and either can lead to the decrease of β. Now the question is whether the change of γ_1 and γ_2 will result in the dramatic change of the solutions. To study this issue, we consider the following three scenarios when increasing d_{char}: 1) simultaneously increase γ_1 and γ_2 by keeping $\gamma_1/\gamma_2 = 1/3$; 2) increase γ_2 only by fixing γ_1; 3) increase γ_1 only by fixing γ_2. Fig. 7 illustrates the numerical results obtained under 3 different scenarios by applying constraint C4. From Fig. 7 we can see that although the maximum achievable lifetime extension is slightly different among the three scenarios, the three curves look very similar. This suggests that varying γ_1 and γ_2 will not affect the effectiveness of the adaptive traffic distribution and power control approach.

In the above analysis we focus on studying the effect of different transmitting and receiving technologies (i.e., different γ_1, γ_2 and β settings). Now we fix the values of γ_1, γ_2, and β, and study the effect of r_{min}. As we mentioned before, besides physical limitation, r_{min} is also determined by certain QoS requirements, such as network connectivity. For example, in a dense network we can use a small r_{min}, while in a sparse network we need a large r_{min} to maintain necessary connectivity. Fig. 8 illustrates the results for various r_{min}. As before, we set $\alpha = 2$, $\gamma_1 = 45$nJ/bit, $\gamma_2 = 135$nJ/bit, $\beta = 10$pJ/bit/m^2. Two network sizes are studied: $10d_{char}$ and $5d_{char}$. Since we have observed that imposing constraint C1 is not energy efficient, here we use the lifetime obtained by imposing constraint C2 as baseline. The results illustrated in Fig. 8 is the extended lifetime obtained by imposing no extra
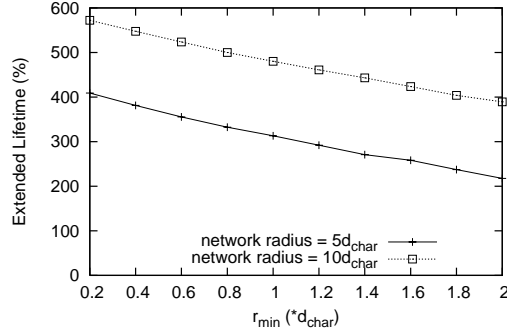
Fig. 8. Lifetime comparison for different network size $5 * d_{char}$ and $10 * d_{char}$ by fixing d_{char} and changing r_{min}, $\alpha = 2$.
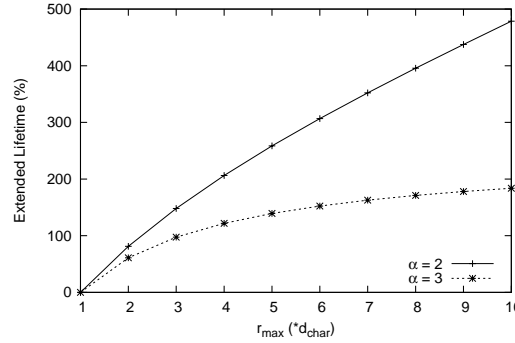

Fig. 9. Lifetime extension for different r_{max} under extra constraint

constraints (C4).

First, Fig. 8 shows that higher lifetime extension can be obtained with the increase of network size. This is similar as the results illustrated in Fig. 1. Second, with the decrease of r_{min}, the extensible lifetime increases. This is because smaller r_{min} allows nodes to adjust their transmission power in a finer way. On the other hand, though the extensible lifetime decreases when r_{min} increases, there is still considerable lifetime extension available. For example, when the network size is $10d_{char}$ and r_{min} is $2d_{char}$, the extended lifetime is about 400%. This also suggests that the approach of adaptive traffic distribution and power control effectively extend the network lifetime under different network size and node density.

When we further examine the numerical solutions to optimization problem (4)-(9), we find that when $r_{min} \geq 1.2d_{char}$, the optimal solution also exhibit a simple form. This has been generalized by the following theorem:

Theorem 2: **When all nodes can reach the sink by adjusting their transmission power, as long as $\frac{r_{min}}{d_{char}} \geq \max\{(\frac{(\alpha-1)(l^\alpha - r^\alpha)}{[r(l-r+1)]^\alpha + (r-1)^\alpha - l^\alpha - [(l-r+1)\cdot(r-1)]^\alpha})^{\frac{1}{\alpha}}, 3 \leq l \leq L, 2 \leq r \leq l-1\}$, there always exists an optimal solution to the problem (4)-(9) with the simple form (11).**

Proof: Similar to the proof of Theorem 1, we assume there exist one arbitrary optimal solution then we transform it to a simple form solution according to some procedure. To prove the transformed solution has no less lifetime than original one, we need r_{min} is large enough which is described in the above statement. For detail, please refer to Appendix. ∎

The Theorem 2 shows that as long as r_{min} is wider enough, the simple form solution is always the optimal solution for LP (4)-(9). At the same time, according to Fig. 6, though the simple form solution is not optimal when r_{min} is small, it can still approximate the optimal solution very well.

Corollary 2: **When all nodes can reach the sink by adjusting their transmission power, and $\alpha = 2$, as long as $\frac{r_{min}}{d_{char}} \geq \frac{\sqrt{5}}{2}$, there always exists an optimal solution to the problem (4)-(9) with the simple form (11).**

Proof: It is easy to prove that when $\alpha = 2$, the max value in the formulation in Theorem 2 is achieved at $l = 3, r = 2$. ∎

As we mentioned before, allowing nodes to adaptively adjust transmission power and transmit using a long range may cause significant signal interference. In Section III-A we combat this issue by only allowing a small number

of sensors that are nearest to the sink to adjust their transmission range, while all other nodes fix their transmission range to be r_{min}. In this subsection we also adopt a similar approach with the difference being that now outside nodes will fix their transmission range to be d_{char}. Fig. 9 illustrates the results by limited the number of nodes that are allowed to adjust transmission range. The baseline is the lifetime obtained by imposing constraint C2. The results are obtained by fixing the network size to be $10d_{char}$, setting $r_{min} = d_{char}$, changing r_{max}, using constraint C3, and imposing the extra constraint that only nodes who can directly reach the sink are allowed to adjust transmission range.

From the results presented in Fig. 9 we can see that even when only several innermost layers are allowed to adjust their transmission power, the lifetime extension can still be significant. When α is large, the performance loss compared to the case without the extra constraint is small. For example, when $\alpha = 3$, which is a typical path loss exponent value, by only allowing innermost 3 layers to adjust transmission power, we can have 100% lifetime extension. The conclusion is similar as those obtained from Fig. 3 and Fig. 4. The difference is that the network lifetime extension has been increased by incorporating the processing energy consumption. As discussed before, the significance of such constraint lies in that it can greatly simplify the distributed algorithm implementation and reduce the extra signal interference.

IV. DISTRIBUTED ALGORITHM

In Section III we found that imposing constraint C3 (either transmitting using d_{char} or to the sink directly) can significantly simplify the implementation without much performance loss. Further, in order to combat the negative effect of long transmission range, we impose an extra constraint: only those nodes that can directly reach the sink are allowed to adjust their transmission range. As long as r_{max} is small, the extra signal interference caused by adaptive power adjustment should be limited. However, all these numerical results are obtained through centralized computation which is not appropriate to wireless sensor networks.

In this section we propose *Energy Aware Data Propagation Algorithm* (EADPA), a fully distributed algorithm, to adaptively split traffic and adjust transmission power for each node. For a given node, if it is allowed to adjust its transmission range, it needs to determine how to split the traffic to be sent between the sink and the next relay respectively. It is not efficient to let some sensors relay a lot since it will make the network die fast. Thus when a node has higher residual energy than its relays, it should send the traffic directly to the sink.

The basic idea of EADPA is as follows: suppose each node has selected a set of nodes (possibly one)) as its relays where the relays are around d_{char} away from it. Each node keeps record of the residual energy of its relays. When a node has a packet to send, it first compares its residual energy to the residual energy of its relays. If its residual energy is more than the residual energy of all relays and it can directly reach the sink, then it sends the packet directly to the sink; otherwise, it sends the packet to one of the relays that has the maximum residual energy.

The algorithmic description of EADPA is illustrated in Algorithm 1. In Algorithm 1 we assume that for each node the set of its relays $\mathcal{P}$ has been given. This can be obtained in the following way: for each node, if the sink is within d_{char} distance, then set the sink as its only relay; otherwise, pick k nodes within its $c \cdot d_{char}$ distance, who have the shortest distances to the sink, as its relays. Here $k \geq 1$ and $c \geq 1$ are system parameters that can be determined empirically. Another issue is how a node maintains the residual energy of its relays. This can be done by letting all nodes broadcast their residual energy periodically.

Algorithm 1 describes the procedure for one single data transaction. Most steps are executed by the sender except steps 14-17 which are executed by the receiver.

V. SIMULATION RESULTS

This section evaluates the performance of proposed distributed algorithm EADPA, which is the distributed implementation of proposed adaptive traffic distribution and power control approach, in randomly deployed wireless sensor networks. We first focus on randomly deployed circular sensor network with radius R. The sink is located at the center of the area. We set $r_{min} = 75m$, and set the node density to be $20/\pi r_{min}^2$. Initially, each sensor has 2000 Joule energy. In each unit time (round) each node will generate a 25-Byte message to be sent to the sink. The network radius varies from 600m to 1000m. Next we evaluate the performance of the proposed EADPA algorithm. The baseline network lifetime is obtained by applying constraint C1. In the simulations, only the nodes that can reach the sink directly execute EADPA. We then set r_{max} to different value to test different scenarios.

Algorithm 1 Energy Aware Push Algorithm

Input: E denotes a node's initial energy, $\mathcal{P}$ denotes the set of its relays, r denotes its distance to the sink;

1: $E_{residual} = E$; $d = \min\{r, d_{char}\}$;
2: **while** ($E_{residual} > 0$) **do**
3:     Let T denote the total traffic need to be sent this time;
4:     **if** ($T \cdot (P_c + P_t(d)) > E_{residual}$) **then**
5:       **break;**
6:     **end if**
7:     Find the relay p with the maximum residual energy from $\mathcal{P}$, and use $E^p_{residual}$ to denote p's residual energy;
8:     **if** ($E^p_{residual} < E_{residual}$ AND $r \leq r_{max}$) **then**
9:       Directly send the packet to the sink;
10:       $E_{residual} = E_{residual} - T \cdot (P_c + P_t(r))$;
11:     **else**
12:       send the packet to p;
13:       $E_{residual} = E_{residual} - T \cdot (P_c + P_t(d_{char}))$;
14:       $E^p_{residual} = E^p_{residual} - T \cdot P_r$;
15:       **if** ($E^p_{residual} < 0$) **then**
16:         **break;**
17:       **end if**
18:     **end if**
19: **end while**

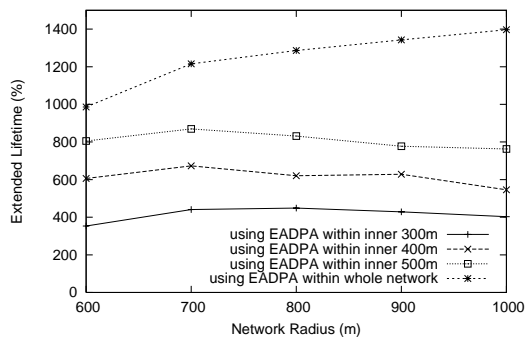

Fig. 10. Lifetime extension for EADPA in circular network

In the first setting, for each sensor, we set $\gamma_1 = 45$nJ/bit, $\gamma_2 = 135$nJ/bit, $\beta = 10$pJ/bit/m^2 for $\alpha = 2$, therefore $d_{char} \simeq 134m$. Fig. 10 illustrates the simulation results, where the four curves correspond to the lifetime extension obtained under four different r_{max} setting: 300m, 400m, 500m, and R. It is worth pointing out that in our simulation, interference has not been considered separately.

First, from Fig. 10 we can see that after applying adaptive traffic distribution and power control, the network lifetime can be significantly extended. For example, when $r_{max} = R = 700m$, more than 1000% lifetime extension has been achieved, which also agrees with our numerical results. These results also confirm that even only a small portion of nodes are allowed to adjust transmission range, the lifetime extension can still be significant. For example, when R = 1000m and $r_{max} = 300m$, only 9% of the nodes are allowed to adjust their transmission power, while 400% lifetime extension can be achieved. Third, we can see that the lifetime extension is mainly determined by r_{max}, and almost keep unchanged with the variation of network radius R, which also agrees with our numerical analysis.

Next we decrease the value of γ_2 from 135nJ/bit to 45nJ/bit to see the effect. All the other simulation parameters are kept the same. The simulation results are illustrated in Fig. 11. By comparing the results in Fig. 10 and Fig. 11 we can see that the results look very similar. The difference lies in that when we decrease γ_2 from 135nJ/bit to
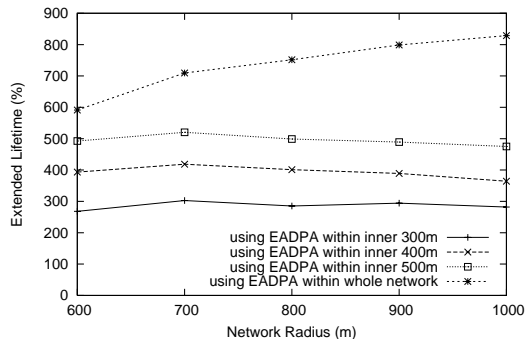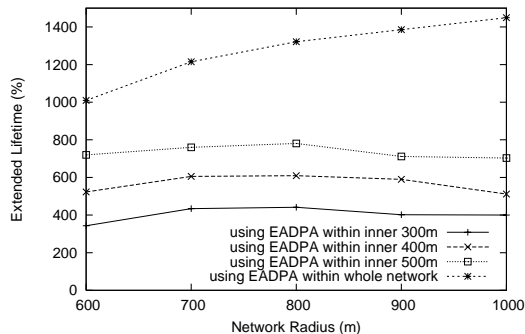
Fig. 11. Lifetime extension for EADPA in circular network

Fig. 12. Lifetime extension for EADPA in square network

45nJ/bit, the extended lifetime also decreases. For example, when $r_{max} = 300$m and R = 1000m, the extended lifetime ratio decrease from 400% to 300% after we decreasing γ_2 from 135nJ/bit to 45nJ/bit. The reason is that when processing energy consumption is decreased, the characteristic distance also decreases. When r_{min} is fixed, comparing to a longer characteristic distance, less energy has been saved when d_{char} is decreased. This also indicates that the proposed adaptive traffic distribution and power control approach can work even better under higher processing energy consumption situation.

Until now we have mainly focused on circular area. Next we change the network area from circular to square. In this set of simulations, nodes are randomly deployed in a square with the sink lies in the center. The length of each edge is $2R$. All the other parameters are the same as in the first set of simulations. The simulation results are illustrated in Fig. 12. Comparing the results in Fig. 12 with the results in Fig. 10 we can see that the results are almost identical except some minor difference. One difference is that when $r_{max} = R$, the squared case results are slightly higher in lifetime extension due to that more nodes are in the squared network. Another difference is that when $r_{max} < R$, the squared case results are slightly lower in lifetime extension due to the less portion of nodes are allowed to adjust their transmission range in the squared case. These results suggest that the proposed scheme EADPA can also work well in networks with non-circular shape.

VI. RELATED WORK

In [19], [20], Rolim et. al. have studied energy-balanced data propagation in sensor networks. In their work, they try to derive schemes which can make all nodes in the network die at the same speed. They have focused on a special case: for each node, when it has a packet to send, it only has two options: either send to the next hop, or directly send to the sink. In [21], the authors have derived an algorithm and prove that it can compute the traffic split ratio optimally so that all nodes will die at the same speed.

However, there exist several major differences to distinguish our work with those in [19]–[21]. First, they have not considered processing energy consumption, which limits their contribution. Second, instead of maximizing the network lifetime, their goal is to let nodes die at the same speed, which may lead to a suboptimal solution. Third, they have only considered the situation that the path loss exponent is 2. Fourth, they have not considered

any practical issues related to their strategies, such as how to handle interference. Fifth, they do not provide any distributed algorithms, and all their solutions need to be calculated in a centralized way.

In [22] also, the authors study energy-balanced data propagation by taking into consideration the processing energy consumption. But again, their goal is to let nodes consume energy at same speed instead of maximizing network lifetime.

It is also worth pointing out that our proposed approach can work together with existing techniques to further extend the network lifetime, such as in-network data aggregation, sleep scheduling, and so on.

VII. CONCLUSION

In this paper we have demonstrated that adaptive traffic distribution and power control can significantly extend the lifetime of wireless sensor networks. We have also demonstrated that by incorporating the processing energy consumption, the lifetime can be further extended comparing to only considering the transmission energy consumption. When investigating the optimal solutions to the lifetime maximizing adaptive traffic distribution and power control problem, one important finding is that nodes should either transmit in the most energy efficient way, or directly transmit to the sink. This has been verified by both numerical results as well as theoretical analysis. We have also shown that the network lifetime can still be dramatically extended even if only a small portion of innermost nodes are allowed to adjust their transmission power. This has great practical implication since it can significantly simplify the medium access control and scheduling protocol design. Finally, we have proposed a fully distributed algorithm to perform adaptive traffic distribution and transmission power adjustment for randomly deployed wireless sensor networks. Extensive simulation have also been conducted, and the results have demonstrated that the network lifetime can be dramatically extended by applying the proposed approach in various scenarios.

APPENDIX

Theorem 2: When all nodes can reach the sink by adjusting their transmission power, as long as $\frac{r_{min}}{d_{char}} \geq$ $\max \{(\frac{(\alpha-1)(l^\alpha - r^\alpha)}{[r(l-r+1)]^\alpha + (r-1)^\alpha - l^\alpha - [(l-r+1)\cdot(r-1)]^\alpha})^{\frac{1}{\alpha}}, 3 \leq l \leq L, 2 \leq r \leq l-1 \}$, there always exists an optimal solution to the problem (4)-(9) with the simple form (11).

Proof: We let $r_{min} = c \cdot d_{char}$, where c is constant. Then $P_t^{l,k} = \gamma_1 + \beta((l-k)cd_{char})^\alpha = \gamma_1 + \frac{\gamma_1 + \gamma_2}{\alpha - 1} \cdot (c(l-k))^\alpha$. We will show that any optimal solution can be transformed into a solution in simple form without losing optimality.

Let $\{x_{i,j}\}$ be an optimal solution. If this optimal solution is not in the simple form, then we can transform $\{x_{i,j}\}$ to $\{\tilde{x}_{i,j}\}$ such that $\{\tilde{x}_{i,j}\}$ is in the simple form (11). The whole procedure is as follows:

We iteratively apply the following procedure: find the first link $x_{l,l-r}$ with the following properties:

- $\{x_{i,i-1} \geq 0, x_{i,0} \geq 0, x_{i,j} = 0, 0 < i < l, 1 \leq j \leq i-2\}$, that is, for all $i < l$, except $x_{i,0}$ and $x_{i,i-1}$, no other links can have non-zero value.
- For all $0 < j < l-r$, $x_{l,j} = 0$.

Next we show how to redistribute $x_{l,l-r}$ to the other links without increasing the MIN-MAX power. For layer $l-r$, its initial power is:

$$P_{l-r} = x_{l-r,0}P_t^{l-r,0} + x_{l-r,l-r-1}P_t^{l-r,l-r-1} + P_r \sum_{i=l-r+1}^{L} \frac{2i-1}{2l-2r-1}x_{i,l-r}$$

$$\geq (P_t^{l-r,l-r-1} + P_r)\left(\frac{x_{l-r+1,l-r}(2l-2r+1)}{2l-2r-1} + \frac{x_{l,l-r}(2l-1)}{2l-2r-1} + \sum_{i=l+1}^{L} \frac{2i-1}{2l-2r-1}x_{i,l-r} \right) \quad (13)$$

where $\frac{x_{l-r+1,l-r}(2l-2r+1)}{2l-2r-1} + \frac{x_{l,l-r}(2l-1)}{2l-2r-1} + \sum_{i=l+1}^{L} \frac{2i-1}{2l-2r-1}x_{i,l-r} + g_{l-r} = x_{l-r,0} + x_{l-r,l-r-1}$ is the traffic that layer $l-r$ needs to transmit.

We split traffic $x_{l,l-r}$ into two parts $\Delta x_{l,l-r+1}$ and $\Delta x_{l,0}$ which will be sent to the layer $l-r+1$ and the sink respectively. To conserve traffic and to keep the layer l power consumption unchanged, we need to have

$$\begin{cases} \Delta x_{l,0} + \Delta x_{l,l-r+1} = x_{l,l-r} \\ P_t^{l,0}\Delta x_{l,0} + P_t^{l,l-r+1}\Delta x_{l,l-r+1} = P_t^{l,l-r}x_{l,l-r} \end{cases}$$

$$\Rightarrow \begin{cases} \Delta x_{l,l-r+1} = \frac{l^\alpha - r^\alpha}{l^\alpha - (r-1)^\alpha} x_{l,l-r} \\ \Delta x_{l,0} = \frac{r^\alpha - (r-1)^\alpha}{l^\alpha - (r-1)^\alpha} x_{l,l-r} \end{cases} \tag{14}$$

After this traffic rerouting, the power consumed by layer l does not change. However, the incoming traffic of layer $l - r + 1$ has been increased. Therefore we need to adjust layer $l - r + 1$'s traffic too. We intend to keep the power consumption of layer $l - r + 1$ the same, so we try to increase $x_{l-r+1,l-r}$ by $\Delta x_{l-r+1,l-r}$ and decrease $x_{l-r+1,0}$ by $\Delta x_{l-r+1,0}$. Traffic conservation and power consumption invariance imply that

$$\begin{cases} \Delta x_{l-r+1,l-r} - \Delta x_{l-r+1,0} = \frac{2l-1}{2l-2r+1} \Delta x_{l,l-r+1} \\ P_t^{l-r+1,l-r} \Delta x_{l-r+1,l-r} + P_r \frac{2l-1}{2l-2r+1} \Delta x_{l,l-r+1} = P_t^{l-r+1,0} \Delta x_{l-r+1,0} \end{cases}$$

$$\Rightarrow \begin{cases} \Delta x_{l-r+1,l-r} = \frac{\alpha - 1 + (c(l-r+1))^\alpha}{(c(l-r+1))^\alpha - c^\alpha} \frac{2l-1}{2l-2r+1} \Delta x_{l,l-r+1} \\ \Delta x_{l-r+1,0} = \frac{\alpha - 1 + c^\alpha}{(c(l-r+1))^\alpha - c^\alpha} \frac{2l-1}{2l-2r+1} \Delta x_{l,l-r+1} \end{cases} \tag{15}$$

Then there are two possible scenarios:

- Scenario 1: $\Delta x_{l-r+1,0} \leq x_{l-r+1,0}$
- Scenario 2: $\Delta x_{l-r+1,0} > x_{l-r+1,0}$

For scenario 1, $\{x_{i,j}\}$ is updated as follows:

$$\begin{aligned} x_{l,l-r}^1 &= 0 \\ x_{l,l-r+1}^1 &= x_{l,l-r+1} + \Delta x_{l,l-r+1} \\ x_{l,0}^1 &= x_{l,0} + \Delta x_{l,0} \\ x_{l-r+1,l-r}^1 &= x_{l-r+1,l-r} + \Delta x_{l-r+1,l-r} \\ x_{l-r+1,0}^1 &= x_{l-r+1,0} - \Delta x_{l-r+1,0} \\ x_{i,j}^1 &= x_{i,j}, \text{for other } i,j \text{ and } i > l - r \\ x_{i,j}^1 &= \frac{\sum_{k=i+1}^{L} \frac{2k-1}{2i-1} x_{k,i}^1 + g_i}{\sum_{k=i+1}^{L} \frac{2k-1}{2i-1} x_{k,i} + g_i} x_{i,j}, i \leq l - r \end{aligned}$$

After updating, the traffic for layers over $l - r + 1$ keeps same except layer l, so their power consumption do not change. The power consumption of layers l and $l - r + 1$ do not change, and the incoming traffic of layer $l - r$ is changed by $\frac{2l-2r+1}{2l-2r-1} \Delta x_{l-r+1,l-r} - \frac{2l-1}{2l-2r-1} x_{l,l-r} = \frac{2l-1}{2l-2r-1} (\frac{\alpha - 1 + (c(l-r+1))^\alpha}{(c(l-r+1))^\alpha - c^\alpha} \cdot \frac{l^\alpha - r^\alpha}{l^\alpha - (r-1)^\alpha} - 1) x_{l,l-r}$. If $\frac{\alpha - 1 + (c(l-r+1))^\alpha}{(c(l-r+1))^\alpha - c^\alpha} \cdot \frac{l^\alpha - r^\alpha}{l^\alpha - (r-1)^\alpha} \leq 1$, the incoming traffic of layer $l - r$ will not increase. It is readily to check that

$$\frac{\alpha - 1 + (c(l-r+1))^\alpha}{(c(l-r+1))^\alpha - c^\alpha} \cdot \frac{l^\alpha - r^\alpha}{l^\alpha - (r-1)^\alpha} \leq 1 \Leftrightarrow c \geq \left(\frac{(\alpha - 1)(l^\alpha - r^\alpha)}{[r(l-r+1)]^\alpha + (r-1)^\alpha - l^\alpha - [(l-r+1) \cdot (r-1)]^\alpha} \right)^{\frac{1}{\alpha}}$$

Thus the incoming traffic of layer $l - r$ and the layers insider layer $l - r$ will not increase, thus the MIN-MAX power will not increase.

Now let us consider scenario 2: $\Delta x_{l-r+1,0} > x_{l-r+1,0}$. In this scenario, we cannot decrease $x_{l-r+1,0}$ by the whole amount $\Delta x_{l-r+1,0}$. Consequently, $\{x_{i,j}\}$ is updated as follows:

$$\begin{aligned} x_{l,l-r}^1 &= 0 \\ x_{l,l-r+1}^1 &= x_{l,l-r+1} + \Delta x_{l,l-r+1} \\ x_{l,0}^1 &= x_{l,0} + \Delta x_{l,0} \\ x_{l-r+1,l-r}^1 &= x_{l-r+1,l-r} + x_{l-r+1,0} + \frac{2l-1}{2l-2r+1} \Delta x_{l,l-r+1} \\ x_{l-r+1,0}^1 &= 0 \\ x_{i,j}^1 &= x_{i,j}, \text{ for other } i,j \text{ and } i > l - r \\ x_{i,j}^1 &= \frac{\sum_{k=i+1}^{L} \frac{2k-1}{2i-1} x_{k,i}^1 + g_i}{\sum_{k=i+1}^{L} \frac{2k-1}{2i-1} x_{k,i} + g_i} x_{i,j}, \ i \leq l - r \end{aligned}$$

After updating, the power consumption of layer l keep same. The power consumption of layer $l - r + 1$ is

$$
\begin{aligned}
P_{l-r+1}^1 &= P_t^{l-r+1,l-r}(x_{l-r+1,l-r} + x_{l-r+1,0} + \frac{2l-1}{2l-2r+1}\Delta x_{l,l-r+1}) \\
&+ P_r(\sum_{k=l-r+1}^{L} \frac{2k-1}{2l-2r+1}x_{k,l-r+1} + \frac{2l-1}{2l-2r+1}\Delta x_{l,l-r+1}) \\
&\leq (P_t^{l-r+1,l-r} + P_r)(x_{l-r+1,l-r} + x_{l-r+1,0} + \frac{2l-1}{2l-2r+1}\Delta x_{l,l-r+1})
\end{aligned}
$$

Next we will show $P_{l-r+1}^1 \leq P_{l-r}$. Since $\Delta x_{l-r+1,0} > x_{l-r+1,0}$, we have

$$
x_{l-r+1,0} + \frac{2l-1}{2l-2r+1}\Delta x_{l,l-r+1} < \Delta x_{l-r+1,0} + \frac{2l-1}{2l-2r+1}\Delta x_{l,l-r+1} = \Delta x_{l-r+1,l-r} \leq \frac{2l-1}{2l-2r+1}x_{l,l-r}, \quad (16)
$$

where the final inequality has been approved in scenario 1. We then have

$$
P_{l-r+1}^1 < (P_t^{l-r+1,l-r} + P_r)(x_{l-r+1,l-r} + \frac{2l-1}{2l-2r+1}x_{l,l-r}) \leq P_{l-r} \quad (17)
$$

So the power consumption of layer $l - r + 1$, P_{l-r+1}^1, is smaller than the original MIN-MAX power.

The incoming traffic of layer $l - r$ is changed by $\frac{2l-2r+1}{2l-2r-1}x_{l-r+1,0} + \frac{2l-1}{2l-2r-1}\Delta x_{l,l-r+1} - \frac{2l-1}{2l-2r-1}x_{l,l-r} < 0$. Thus, the incoming traffic of layer $l - r$ is decreased, so the power consumption of layer $l - r$ will not increase. The power consumption of all other layers do not increase either. Therefore, in scenario 2, after updating, the MIN-MAX power does not increase either.

Thus, after this procedure, $\{x_{i,j}\}$ is updated to $\{x_{i,j}^1\}$ by redistributing traffic to delete the traffic on $(l, l - r)$, and the MIN-MAX power does not increase. We keep executing this procedure until the solution is in the standard form. Since each application of this procedure does not increase the MIN-MAX power, the theorem is proved. ∎

Corollary 2: When all nodes can reach the sink by adjusting their transmission power, and $\alpha = 2$, as long as $\frac{r_{min}}{d_{char}} \geq \frac{\sqrt{5}}{2}$, there always exists an optimal solution to the problem (4)-(9) with the simple form (11).

Proof: When $\alpha = 2$,

$$
\left(\frac{(\alpha-1)(l^\alpha - r^\alpha)}{[r(l-r+1)]^\alpha + (r-1)^\alpha - l^\alpha - [(l-r+1)\cdot(r-1)]^\alpha}\right)^{\frac{1}{\alpha}} = \left(\frac{l+r}{2(l-r+1)(r-1)}\right)^{\frac{1}{\alpha}}
$$

Regard r as constant, let $f(l) = \frac{l+r}{2(l-r+1)(r-1)}$, then we have

$$
f(l) \geq f(l+1) \Leftrightarrow \frac{l+r}{2(l+1-r)(r-1)} \geq \frac{l+1+r}{2(l+1+1-r)(r-1)} \Leftrightarrow 2r \geq 1 \quad (18)
$$

So when r keeps same, $f(l)$ is nonincreasing function.

Regard l as constant, let $g(r) = \frac{l+r}{2(l-r+1)(r-1)}$, and let $l - 3 \geq w \geq 1$, then we have

$$
g(2) \geq g(2+w) \Leftrightarrow \frac{l+2}{2(l-1)} \geq \frac{l+2+w}{2(l-w-1)(w+1)} \Leftrightarrow l^2 \geq lw + l + 2w + 3 \Leftrightarrow w \leq \frac{l^2 - l - 3}{l+2} \quad (19)
$$

At the same time, $l - 3 \leq \frac{l^2-l-3}{l+2} \Leftrightarrow -3 \leq 0$. It guarantees that (19) holds when $w \leq l - 3$. So the max of $g(r)$ is achieved at $r = 2$

Thus when $\alpha = 2$, $\max\{(\frac{(\alpha-1)(l^\alpha - r^\alpha)}{[r(l-r+1)]^\alpha + (r-1)^\alpha - l^\alpha - [(l-r+1)\cdot(r-1)]^\alpha})^{\frac{1}{\alpha}}, 3 \leq l \leq L, 2 \leq r \leq l - 1\}$ is achieved at $l = 3, r = 2$ which makes it as $\frac{\sqrt{5}}{2}$. According to Theorem 2, the Corollary holds. ∎

REFERENCES

[1] C. Perkins, *Ad Hoc Networking*, Addison-Wesley, 2000.

[2] C. K. Toh, *Ad Hoc Mobile Wireless Networks: Protocols and Systems*, Prentice Hall PTR, 2001.

[3] J.-H. Chang and L. Tassiulas, "Energy conserving routing in wireless ad-hoc networks," in *IEEE INFOCOM*, 2000.

[4] P.-H. Hsiao, A. Hwang, H. T. Kung, and D. Vlah, "Load-balancing routing for wireless access networks," in *IEEE INFOCOM*, 2001, pp. 986–995.

[5] A. Boukerche, X. Cheng, and J. Linus, "Energy-aware data-centric routing in microsensor networks," in *ACM MSWiM*, 2003, pp. 42–49.

[6] V. Rajendran, K. Obraczka, and J. Garcia-Luna-Aceves, "Energy-efficient, collision-free medium access control for wireless sensor networks," in *ACM Sensys*, 2003.

[7] W. Ye, J. Heidemann, and D. Estrin, "Medium access control with coordinated adaptive sleeping for wireless sensor networks," *IEEE/ACM Transactions on Networking*, vol. 12, no. 3, pp. 493–506, 2004.

[8] Q. Cao, T. Abdelzaher, T. He, and J. Stankovic, "Towards optimal sleep scheduling in sensor networks for rare-event detection," in *International Conference on Information Processing in Sensor Networks (IPSN)*, April 2005.

[9] C. Intanagonwiwat, R. Govindan, and D. Estrin, "Directed diffusion: A scalable and robust communication paradigm for sensor networks," in *MobiCom*, August 2000.

[10] B. Krishnamachari, D. Estrin, and S. Wicker, "Modelling data-centric routing in wireless sensor networks," in *InfoCom*, 2002.

[11] V. P. Mhatre, C. Rosenberg, D. Kofman, R. Mazumdar, and N. Shroff, "A minimum cost heterogenous sensor network with a lifetime constraint," *IEEE Transactions on Mobile Computing*, vol. 4, no. 1, January/February 2005.

[12] W. R. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for wireless microsensor networks," in *IEEE Hawaii International Conference on System Sciences*, 2000.

[13] S. Bandyopadhyay and E. J. Coyle, "An energy efficient hierarchical clustering algorithm for wireless sensor networks," in *IEEE INFOCOM*, 2003, pp. 1713–1723.

[14] O. Younis and S. Fahmy, "Distributed clustering in ad-hoc sensor networks: A hybrid, energy-efficient approach," in *IEEE INFOCOM*, 2004.

[15] J. Luo and J.-P. Hubaux, "Joint mobility and routing for lifetime elongation in wireless sensor networks," in *IEEE INFOCOM*, 2005.

[16] J. Lee, B. Krishnamachari, and C.C. J. Kuo, "Impact of heterogeneous deployment on lifetime sensing coverage in sensor networks," in *IEEE SECON*, October 2004.

[17] Y. T. Hou, Y. Shi, H. D. Sherali, and S. F. Midkiff, "Prolonging sensor network lifetime with energy provisioning and relay node placement," in *IEEE SECON*, September 2005.

[18] Y.Xin, T. Guven, and M.A. Shayman, "Relay deployment and power control for lifetime elongation in sensor networks," in *IEEE ICC*, June 2006.

[19] C. Efthymiou, S. Nikoletseas, and J. D.P. Rolim, "Energy balanced data propagation in wireless sensor networks," in *4th Workshop on Algorithms for Wireless, Mobile, Ad Hoc and Sensor Networks*, 2004.

[20] P. Leone, S. Nikoletseas, and J. D. P. Rolim, "An adaptive blind algorithm for energy balanced data propagation in wireless sensor networks," in *Proceedings of the First International Conference (DCOSS)*, 2005, pp. 35–48.

[21] O. Powell, P. Leone, and J. Rolim, "Energy optimal data propagation in wireless sensor networks," Tech. Rep., arXiv.org automated e-print archives, Report CS-0508052, 2005.

[22] J. Wang and I. Howitt, "Optimal traffic distribution in minimum energy wireless sensor networks," in *Globecom*, 2005.

[23] S. Vasudevan, C. Zhang, D. Goeckel, and D. Towsley, "Optimal power allocation in wireless networks with transmitter-receiver power tradeoffs," in *Proceedings of IEEE INFOCOM*, April 2006.

[24] F. Sun and M. Shayman, "Lifetime maximizing adaptive power control in wireless sensor networks," Tech. Rep., University of Maryland, The Institute for Systems Research, 2006.

[25] M. Bhardwaj and A. P. Chandrakasan, "Bounding the lifetime of sensor networks via optimal role assignments," in *IEEE Infocom*, March 2002.

[26] W. Heinzelman, *Application-Specific Protocol Architectures for Wireless Networks*, Ph.D. thesis, Massachusetts Institute of Technology, 2000.

[27] M. Bhardwaj and A. P. Chandrakasan, "Bounding the lifetime of sensor networks via optimal role assignments," in *IEEE Infocom*, March 2002.